

Convexification in Energy Optimization of a Hybrid Electric Propulsion System for Aerial Vehicles

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Abstract. This paper concerns the energy management of a hybrid electric propulsion system for aerial vehicles, using convex optimization. The main contribution of this paper is the proposal of a new convexification, which simplifies the formation of the convexified problem, and the proof of equality between the original problem and the convexified problem. The primary energy management is formulated from first principles and using experimental data. The convexity of the original problem is clarified via investigating the approximation to the experimental data. Then, change of variables and equality relaxation are implemented to convexify the concave constraints. The introduced variable—battery internal energy, is proposed to convexify the battery model. The relaxation of a non-affine equality yields to new convex inequality constraints. Numerical examples and forward simulations were carried out to validate the convexified problem. The first test case verifies that the convex relaxation does not sacrifice the optimality of the solution nor does the variable change lose the original bounds. Also, the optimal control from convex optimization is demonstrated to be robust to a disturbance in power demand. Comparison with the benchmark optimization—dynamic programming, shows that convex optimization achieves a minimal objective value with less fluctuation of the optimal control value. Most significant is that the convexification reduces the optimization computation time to a level compatible with implementation in practical application.

Keywords: Hybrid Electric Vehicles; Aerial Vehicles; Energy Management; Convex Optimization; Convexification

1 Introduction

Over the last decades, the rapid growth in fossil fuel consumption, and resulting polluting emissions has generated a need for energy conservation. NASA N+3 concepts aim to reduce the fuel burn of aerial vehicles by 70% by 2025 [1]. To meet this challenge, the aeronautic industries show great interest in sustainable energy, which leads to electric propulsion systems [2–4]. However, the energy density of electrical energy storage sources, such as batteries, are much lower than that of fossil fuel. Currently, aerial vehicles with electric propulsion systems cannot achieve the same flight endurance as their conventional (i.e. combustion engine) powered counterparts.

The hybrid propulsion system is a new-generation concept for aerospace studies and industry. The hybrid rocket motor is created to take advantages of both solid and liquid propulsion systems [5]. The thrust throttling and multiple starts can be achieved for solid rocket propulsions and the performance of liquid rocket propulsions is stabilized [6–8]. With respect to

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aeronautical world, the Hybrid Electric Propulsion System (HEPS) represents the Electric Motor (EM) being combined with the Internal Combustion Engine (ICE) to provide propulsive power. The result is decreased fuel burn compared with ICE-powered aircraft, and an increased flying range compared with electric aircraft. For these reasons, there is a significant interest in research into this technology and its applications for aerial vehicles.

Studies carried out at the US Air Force Institute of Technology (AFIT), by Harmon [9] and Hiserote [10], concluded that the clutch-start parallel configuration is the most practically realizable. The research team from Queensland University of Technology (QUT) also demonstrated that aircraft climbing performance can be improved by a large margin (56%) [11] and fuel usage decreased by 6% [12], with only a 5% weight penalty compared to the non-hybrid system. The University of Cambridge developed a parallel hybrid powertrain for the ultra-light aircraft, SOUL [13]. The simulation results demonstrated that the fuel saving could be up to 50% if the powerplant hybridization was 60%. Panagiotou et al. studied the conceptual design of a hybrid solar UAV and increase 50% endurance during summertime [14]. Riboldi applied an optimal approach to design the small hybrid electric aircraft and investigated the sensitivity of the optimization results [15]. All the aforementioned researches show that HEPS can provide better fuel economy and lower emissions without substantially compromising vehicle performance, but those advantages come at the cost of increased complexity of energy management.

Energy optimization is one of the most popular topics regarding hybrid energy management [16]. Following the preliminary design of hybrid electric aircraft, Riboldi moved farther and investigated the off-line optimization of power management for aircraft HEPS [16]. One of popular optimization approach applied in hybrid aircraft is Dynamic Programming (DP) since it can guarantee the global optimality of the solution [17]. However, the downside of DP is the large computational cost due to the nonconvex characteristics of the energy management problem. Optimization results from DP are usually used as a benchmark for other strategies [18]. For practical applications, many simplification approaches have been studied since they can transfer the original problem to a new problem with lower level of complexity. Currently for energy optimization, the most popular simplified-based optimizations includes linear programming [19] and quadratic programming [20]. But linearization and quadrification suffer from over-simplification since the energy optimization problem is normally neither linear nor quadratic. Therefore, convexifying methods for HEPS energy optimization with little loss of accuracy is worth studying.

Convex optimization to solve the energy management of hybrid ground vehicles was firstly proposed by Murgovski, et al., [21]. The model of energy storage (capacitor or battery) was convexified by introducing the variable: pack energy. Furthermore, a constraint relaxation and proof of its being lossless was also provided. The lifetime of the battery was considered in another study by Murgovski, et al., [22]. One significant finding of this paper is that the optimal result of battery size is noticeably different, when the battery lifetime is considered in the sizing problem. The above-mentioned work indicates that the global minimum of the original problem can be obtained by solving the relaxed problem without loss of equality [23]. Another advantage of convexification is the significant reduction of optimization time, while being guarantee of the sub-optimal performance [24].

With the capability of real-time computing, the convex optimization is popularized to solve trajectory planning and guidance problems for space vehicles [25–32]. Yet the authors did not find in the open literature articles applying convexification technology and validating the lossless convexity specifically to the hybrid aerospace. This paper proposes a new convexification method, a technique that can not only retain the feature of lossless relaxation, but also lead to a more straightforward and clearer form of the convexified problem. Following the system described in Section 2, the paper starts by first covering the primary energy management formulation, in which the minimization of the total fuel used is selected as the objective function. The approximation method is employed to investigate the convexity of cost and constraints. In Section 4, two techniques—change of

variables and equality relaxation are implemented with the aim of convexification. Afterwards, the discretization is implemented on the new, convexified problem. Finally, the analysis of approximation and numerical simulations are presented in Section 5.

2 System Description

The HEPS presented in this section are part of the ongoing work performed as part of the AIRSTART project. AIRSTART is a collaborative research project to support long-endurance unmanned aircraft equipped with advanced propulsion systems. Cranfield is working on a hybrid powertrain, converting a Rotron engine [33] and Emrax motor/generator [34] into a hybrid combustion-electric system, in which the battery and its management system is customized by EMRAX and AXTER [35]. The aerial platform for testing the hybrid powertrain is an experimental medium-altitude long-endurance unmanned vehicles—the Aegis.

There are various hybrid powertrain configurations currently in use. The most commonly used configurations are series, parallel and series-parallel architecture. Among them, the series architecture enables the engine to operate at its ideal operating condition. However, its system efficiency is relatively low since large energy losses exist in the energy conversion [20]. Also, the series configuration cannot achieve a combination of combustion and electrical power. The series-parallel is the most functional structure of the three architectures. Nevertheless, its complicated clutch/gear mechanism raises the issue of reliability on real aircraft applications. Compared with the series-parallel configuration, the parallel configuration is lighter and less complex, whilst keeping the flexibility of hybridization. According to Hiserote’s work [10], the parallel configuration is best suited for long-endurance unmanned aircraft.

A parallel HEPS was designed to act as the propulsion system for the aircraft and its high-level component diagram is shown in Fig. 1. The black solid line without arrows represents the mechanical connection between different components, while the power supply is represented by the red solid line with arrows. The control and feedback signal are represented by black dotted lines with arrows. The supervisory controller is the high-level controller responsible for the energy optimization of HEPS. It receives power demands from flight controller (or throttle stick command directly from the pilot) and then optimizes the engine power request. To apply the optimal results on the propulsive units, the supervisory controller also takes charge of converting the optimal control to the engine throttle command and motor/generator torque/speed command. The output shaft of the engine is connected through a reduction gear and sprag clutch to drive the motor/generator input shaft.

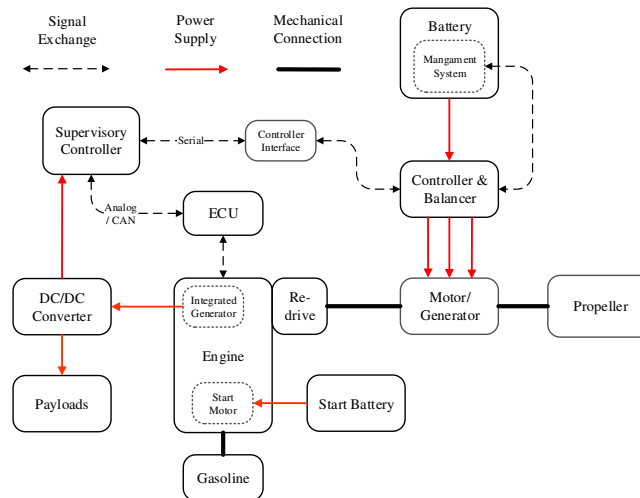


Fig. 1. Architecture of the parallel hybrid propulsion system for Aegis.

This setup enables the system to combine the power from engine and motor/generator, when the power demand exceeds that which can be provided by only one of the propulsive units. Otherwise, when the power demand is lower than the engine maximum power, the motor/generator can draw power from the engine to charge the battery. Moreover, this architecture also permits the motor to drive the propeller alone, if electric-only flight mode is required. The hybrid architecture has the property that it is fault-tolerant. In the case of a failure of one of the propulsion units, it can de-couple the combustion drivetrain or shut down the electric power supply, so that one of the working units will continue driving the propeller.

The convexification approach covered in the previous section of this paper was applied for the energy optimization of the hybrid powered aircraft system. In this architecture, the power requirement P_{req} is the sum of the power required to drive the propeller and the power demanded by the auxiliary devices of the aircraft (such as payloads, power electronics and supervisory controller). On the other hand, $P_{EM,loss}$ is the power loss of the single motor/generator. With the configured mechanical links, the relationship between the transmission ratio and gear ratio is $r = G$. The key parameters of our HEPS are listed in Table 1, with their values and the corresponding units. Some of the parameters are explicitly used in the problem formulation, so their symbols are also given in the table. The rotation speed following a power value via @ indicates that this is the power reached at the stated speed.

Table 1 Parameters of Hybrid-Electric Propulsion System

Parameter	Symbol	Value
Maximum engine speed (rpm)	-	7500
Maximum engine power (kW)	-	30@6500rpm
Maximum motor speed (rpm)	-	5000
Maximum motor power (kW)	-	30@5000rpm
Capacity (Ah)	Q_{max}	10
Internal resistance (Ω)	R_{int}	0.6708
Nominal voltage (V)	-	296
DC/DC output voltage (V)	-	300
Propeller diameter (inch)	-	28.0
Gear ratio	G	1.79
Fuel Energy Density (MJ/kg)	-	46.5

3 Original Problem Formulation

In order to reduce the environmental impact of aerial vehicles in general, the most straightforward approach is to minimize the fuel consumption of the engine by optimizing the power distribution. In the following section, the formulation of the original problem, including constraints and objective, is presented.

3.1 Powertrain Modelling

The HEPS combines the energy from the fuel and electric storage to power the vehicle, regardless of the hybrid powertrain configuration. Considering the instantaneous power flow in the powertrain, the power output of fuel combustion P_{fuel} in addition to electric storage, $P_{electric}$, always equals to the sum of system power requirement, P_{req} , and power loss, P_{loss} (see Eq. (1)).

$$P_{fuel} + P_{electric} = P_{req} + P_{loss}. \quad (1)$$

P_{fuel} is sometimes replaced by the engine power P_{ICE} , since the fuel map of the engine is normally represented as the function of power, P_{ICE} . If the minor mechanical power loss is omitted, the power loss, P_{loss} , needs contain only the electric power loss, then Eq. (1) can be re-written as:

$$P_{ICE} + P_{electric} = P_{req} + P_{electric,loss}. \quad (2)$$

If the battery is selected as the electrical energy source, the battery discharging/charging power will be the electric power displayed in Eq. (2), while the electric power loss will be the power loss from the electric motor:

$$P_{ICE} + P_{batt} = P_{req} + P_{EM,loss}. \quad (3)$$

Put differently, the power demand P_{req} , in addition to the electric motor power losses $P_{EM,loss}$, appears as the sum of the contributions of engine power, P_{ICE} , and battery power, P_{batt} .

The role of HEPS energy management of is to split the power demand appropriately between the combustion and electrical energy sources. Thus, the control variable of the powertrain is typically formulated relative to the engine output power, i.e. $u = P_{ICE}$.

When selecting the battery for energy storage, the state variable of the hybrid propulsion system is the battery's State-of-Charge (SoC), and is denoted by x . According to the definition of SoC [36], the system state transition equation can be written as:

$$\dot{x} = -\frac{I}{Q_{max}}, \quad (4)$$

where I is the current flowing supplied by the battery and Q_{max} is the battery maximum capacity.

In hybrid vehicle analysis, the battery model is commonly described by an ideal open-circuit voltage source in series with an internal resistance [37]. It uses the open circuit battery voltage V_b and internal resistance R_{int} to obtain the battery output power P_{batt} :

$$P_{batt} = I * V_b(x) - I^2 R_{int}, \quad (5)$$

in which the resistance R_{int} is assumed to be constant.

In addition to system dynamics, the optimization must consider the physical limitations of each component, which could be expressed by the bounds of variables:

$$P_{ICE,min}(\omega_{ICE}) \leq P_{ICE} \leq P_{ICE,max}(\omega_{ICE}), \quad (6)$$

$$x_{min} \leq x \leq x_{max}, \quad (7)$$

$$I_{min} \leq I \leq I_{max}, \quad (8)$$

where the engine rotational speed is ω_{ICE} , and the engine power upper and lower boundaries change with ω_{ICE} .

The engine rotational speed is determined by the reference speed due to the mechanical link constraint:

$$\omega_{ICE} = r * \omega_{req}, \quad (9)$$

where r is the transmission ratio of the drivetrain.

3.2 Objective

In this study, the objective of the energy optimization is to minimize the fuel consumption of the engine during the overall flight mission. The cost function can be expressed by:

$$J = \int_{t_0}^{t_f} \dot{m}_f(P_{ICE}) dt, \quad (10)$$

where $\dot{m}_f$ denotes the fuel consumption rate of the engine. The time $t \in [t_0, t_f]$, where t_f is the final time and the initial time t_0 generally starts at zero.

In sum, the Problem is to minimize the objective, J , in Eq. (11a)

$$J = \int_{t_0}^{t_f} \dot{m}_f(P_{ICE}) dt, \quad (11a)$$

subject to:

$$P_{ICE} + P_{batt} = P_{req} + P_{EM,loss}. \quad (11b)$$

$$\dot{x} = -\frac{I}{Q_{max}}, \quad (11c)$$

$$P_{batt} = I * V_b(x) - I^2 R_{int}, \quad (11d)$$

$$P_{ICE,min}(\omega_{ICE}) \leq P_{ICE} \leq P_{ICE,max}(\omega_{ICE}), \quad (11e)$$

$$x_{min} \leq x \leq x_{max}, \quad (11f)$$

$$I_{min} \leq I \leq I_{max}, \quad (11g)$$

$$\omega_{ICE} = r * \omega_{req}, \quad (11h)$$

3.3 Approximation of Experimental Data

Approximation is normally employed to reveal the inherent correlation between different variables using algebraic expressions. By analysing experiment data obtained from suppliers [33–35], the convexity of cost and constraints can be investigated thoroughly.

In this case, the fuel rate (the integrand of the objective function) is not a convex function of the control variable, but it is piecewise linear (affine) dependent on the engine torque at a given speed (see Fig. 2). In other words, the fuel rate at each speed can be addressed by:

$$\dot{m}_f = k(\omega_{ICE}) \frac{P_{ICE,0}}{\omega_{ICE}} + d(\omega_{ICE}), \quad (12)$$

where k and d are parameters that are estimated by piecewise linear approximation and will vary depending on engine speed, while $P_{ICE,0}$ refers to engine power at sea-level conditions. Generally, the output performances of one piston engine are strongly affected by the altitude, thus experiment data should be re-collected at different flying height [38]. For Rotron engine in this case, an altitude-compensating control has been adopted so that the power loss due to the increased altitude would not be large.

The results of piecewise linear approximation are plotted in Fig. 2 as the solid curve. The first turning point (gradient change point) indicates the best efficiency for a given speed, if the fuel rate curve is piecewise convex. This condition is not fulfilled at 6500 rpm and 7000 rpm, so their best efficiency points move to the second turning point due to the presence of nonconvex segments. The best efficiency points at different speeds are marked by mauve triangular symbols in Fig. 2. To avoid substantial distortion and shift of the best efficiency region, the best efficiency points need to be retained at their original values when implementing the approximation.

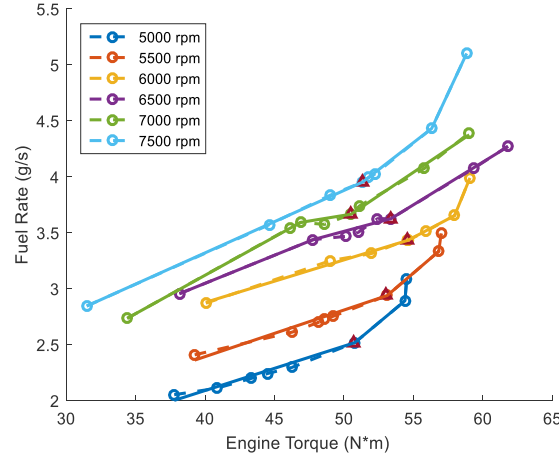


Fig. 2. Piecewise linear approximation of the fuel rate at different speeds.

In general, the plot of battery voltage as a function of SoC can be divided into three segments, with two turning points: the end of the exponential zone and the end of the nominal zone. The experimental data provided by the manufacturer [35] and turning points are shown in Fig. 3. It is clear that the function V_b is nonlinear and also nonconvex. Fortunately, to extend the lifetime of the battery, the SoC is typically limited between 20% and 80%, i.e. the nominal zone. The voltage in the nominal zone is convex and can be accurately fitted with a quadratic function of the form: $V_b = a_v x^2 + b_v x + c_v$. The convex approximation result is also marked in Fig. 3, using the dashed curve with red circles. When the coefficients (a_v, b_v, c_v) are (24.95, 9.319, 291.0), respectively, the coefficient of determination is 0.9993. Note that the voltage is broadly fitted with a constant or affine function, to further simplify the model. However, the constant or affine function is neither applied here due to its lower accuracy. On the other hand, the quadratic function performs well but leads to concave inequality constraints. This issue will be illustrated in Section 4.1.

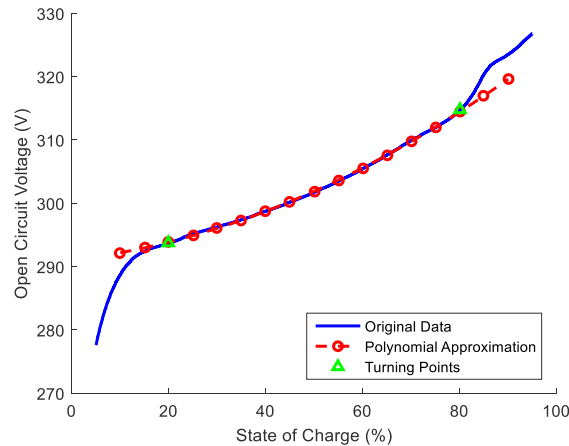


Fig. 3. Approximation of open circuit voltage along nominal zone.

The power loss of the electric motor is calculated using the efficiency map supplied by the manufacturer [34]. As shown in previous work [39], the power loss of the electric motor is actually dependent on output speed and torque. In this paper, the correlation between power loss and torque/speed was analysed first. It was found that the distance correlation between power loss and torque is only 0.0541. This means that the power loss is only slightly influenced by torque. This is demonstrated in Fig. 4 which presents the relationship between power loss and speed under different output torques.

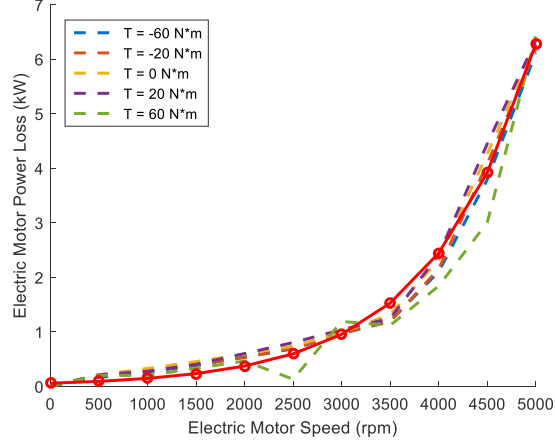


Fig. 4. Approximation of the power losses of the electric motor.

As a result, the power loss can be approximated in a formula which involves only the motor speed ω_{EM} , using the convex function $P_{EM,loss} = a_m * \exp(b_m \omega_{EM})$. The fitting results are also plotted in Fig. 4, with the red solid line marked with circles. The fitted coefficients are $a_m = 0.0563$, $b_m = 9.4248 * 10^{-4}$, when the speed unit is revolutions per minute (rpm). The coefficient of multiple determination is 0.9919. It is worth noting that the observed data used in the fitting is the mathematical expectation of original power loss data at different torques.

4 Convexification

The convexity of the original Problem represented by Eq. (11) is examined in Section 3.3 by evaluating the experimental data. The objective function is found to be convex with the piecewise linear integrand. The variable bounds (11e-11g) and mechanical constraint (11h) are affine, but equality constraints (11b-11d) do not satisfy the requirement of convex programming [40]. In the following section, a convexification including change of variables and equality relaxation, is carried out to convert the Problem represented by Eq. (11) into a convex Problem represented by Eq. (23).

4.1 Change of Variables

Internal power of battery P_b which does not include power loss is proposed in this paper, then the equality constraints (Eqs. (11b) and (11d)) become:

$$P_{ICE} + P_b = P_{req} + P_{EM,loss} + P_{b,loss}, \quad (13)$$

$$P_b = I * V_b(x). \quad (14)$$

The power loss of the battery, $P_{b,loss}$ defined here is mainly due to the power dissipated in the resistance, which is dependent on the internal battery power and battery SoC (see Fig. 5). Similar to $P_{EM,loss}$, the distance correlation between power loss and SoC is only 0.0810. In other words, the power loss is correlated mainly with battery power, rather than SoC. $P_{b,loss}$ can be fitted

with a simple convex quadratic function: $P_{b,loss} = a_l P_b^2$. The fitted coefficient is $a_l = 3.24 * 10^{-6}$, with its coefficient of multiple determination being 0.9998. The observed data used in the fitting is the mathematical expectation of the original power loss data at different SoC. Note that by introducing the battery internal power P_b , the power loss is transformed to a symmetrical graph across the y-axis, which effectively simplifies the formulation of battery model and the proof of *Lemma 1* as follows in Section 4.2.

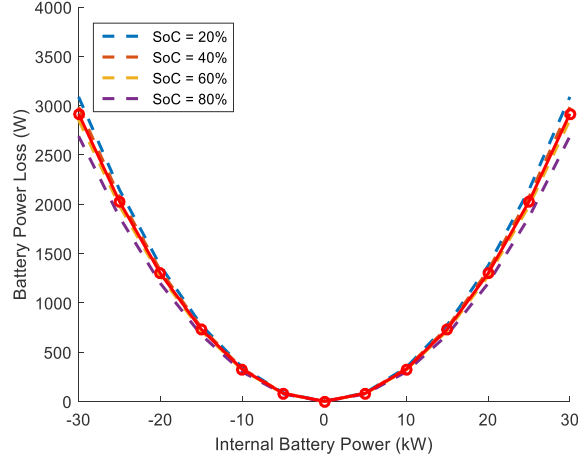


Fig. 5. Approximation of the battery power losses.

Now, the inequality (11g) becomes:

$$I_{min} \leq \frac{P_b}{V_b(x)} \leq I_{max}, \quad (15)$$

which is affine in the battery internal power P_b , but does not guarantee the convexity of the battery voltage $V_b(x)$.

Consider approximating $V_b(x)$ with two affine functions: $V_{lb}(x)$ and $V_{ub}(x)$, as illustrated in Fig. 6, with black dotted and solid lines, respectively. The $V_{lb}(x)$ is the Taylor expansion of $V_b(x)$ at $x = 0.5$, while the line defined by $V_{ub}(x)$ goes through two points at 20% and 80% SoC.

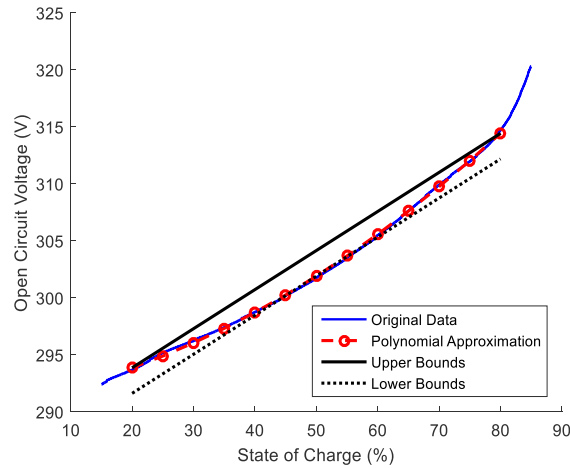


Fig. 6. Upper and lower bounds of the open circuit voltage.

Because of the convexity of $V_b(x)$, $V_{lb}(x)$ and $V_{ub}(x)$ suggest the lower and upper bounds of the voltage:

$$I_{min}V_b(x) \leq I_{min}V_{ub}(x), I_{max}V_b(x) \geq I_{max}V_{lb}(x) \quad (16)$$

Thus the following constraints need to be satisfied:

$$I_{min}V_{ub}(x) \leq P_b \leq I_{max}V_{lb}(x). \quad (17)$$

By solving the problem within the bounds of these constraints (see Eq. (17)), one never obtains solutions that violate the constraints imposed by the physical problem. The bounds on errors (e_{lb} , e_{ub}) introduced by the second approximation were computed and are displayed in Fig. 7. The figure clearly shows that the percentages of errors of upper and lower bounds are both smaller than 1%. Put differently, for all practical purposes, the inequalities Eq. (15) are not compromised when replaced by Eq. (17).

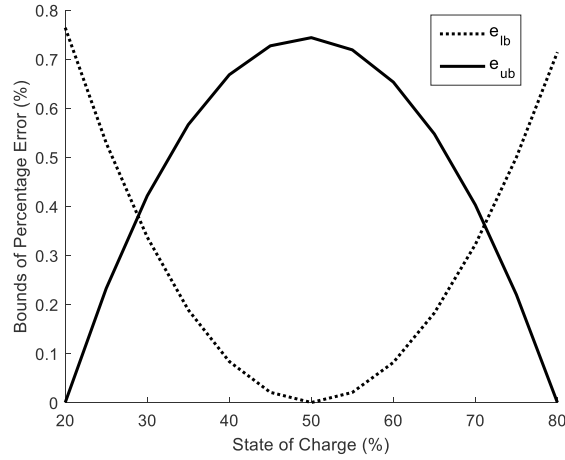


Fig. 7. Error bounds of the second approximation.

From Eq. (11c) and Eq. (23), the state equality function can be deduced:

$$\dot{x} = -\frac{P_b}{V_b(x)Q_{max}}, \quad (18)$$

which still does not meet the requirement of convex programming. Therefore, a new variable called battery internal energy E is introduced to replace the original state variable (SoC). The new state transition equality is formulated as:

$$\dot{E} := Q_{max}V_b(x)\dot{x} = -P_b, \quad (19)$$

while x still denotes the battery SoC to maintain consistency. Simple integration leads to:

$$E(x) = Q_{max} \int V_b(x)dx = Q_{max} \left(\frac{1}{3}a_v x^3 + \frac{1}{2}b_v x^2 + c_v x + d_v \right), \quad (20)$$

where d_v is set to zero without loss of generality. Meanwhile, the inequality (11f) is transformed into:

$$E_{min} \leq E \leq E_{max}. \quad (21)$$

Since $E(x)$ monotonically increases with x in the domain of definition, E_{min} , E_{max} are calculated using $E(x_{min})$ and $E(x_{max})$, respectively.

4.2 Constraint Relaxation

Similarly, by the substitution of variables, the equality (11b) is converted to the convex equality Eq. (13) to prepare for the formulation of the new convex problem. Subsequently, the relaxation of Eq. (13) yields to the inequality constraint:

$$P_{ICE} + P_b \geq P_{req} + P_{EM,loss} + P_{b,loss}. \quad (22)$$

As a consequence, the non-affine equality is successfully converted to a convex inequality. This relaxation does not prejudice the optimality of the solution. The detailed reasoning supporting this statement is given in [36]. Assuming that the convex solver finds the optimal solution while respecting $P_{ICE} + P_b > P_{req} + P_{EM,loss} + P_{b,loss}$, it would mean that some of the energy supplied by the fuel and battery was wasted, thus a better solution can be found via Eq. (22), while maintaining the equality. Proof of this proposition will be given in *Lemma 1*.

The constraint relaxation leads to a convex fuel optimization problem. The convex formulation of the original Problem (11) is re-constructed as Problem (23):

$$J = \int_0^{t_f} \left(k(\omega_{ICE}) \frac{P_{ICE}}{\omega_{ICE}} + d(\omega_{ICE}) \right) dt, \quad k > 0 \quad (23a)$$

subject to:

$$P_{ICE} + P_b \geq P_{req} + P_{EM,loss}(\omega_{EM}) + P_{b,loss}, \quad (23b)$$

$$\dot{E} = -P_b, \quad (23c)$$

$$P_{ICE,min}(\omega_{ICE}) \leq P_{ICE} \leq P_{ICE,max}(\omega_{ICE}), \quad (23d)$$

$$E_{min} \leq E \leq E_{max}, \quad (23e)$$

$$I_{min}V_{ub}(x) \leq P_b \leq I_{max}V_{lb}(x), \quad (23f)$$

$$\omega_{ICE} = G * \omega_{EM} = r * \omega_{req}, \quad (23g)$$

in which the motor speed ω_{EM} is related to the engine speed by the gear ratio G .

Lemma 1: The optimal solution of the original Problem (11) can be accomplished by solving the relaxed convex Problem (23). In other words, a sufficient condition for reaching the minimum of Problem (11) is that the constraint (23b) holds with equality at the optimum of Problem (23).

Proof: Assume that an optimal solution of Problem (23) is found that satisfies:

$$P_{ICE}^* + P_b^* > P_{req} + P_{EM,loss} + P_{b,loss}^*. \quad (24)$$

It is possible to construct a suboptimal solution with the same battery internal power and power loss trajectory, i.e. $\tilde{P}_b = P_b^*$, $\tilde{P}_{b,loss} = P_{b,loss}^*$. In the meantime, this feasible solution maintains the condition:

$$\tilde{P}_{ICE} + \tilde{P}_b = P_{req} + P_{EM,loss} + \tilde{P}_{b,loss}. \quad (25)$$

It can then be inferred that:

$$P_{ICE}^* > \tilde{P}_{ICE}. \quad (26)$$

Meanwhile, the cost, J , increases monotonically with P_{ICE} . Consequently, the following inequality can be deduced:

$$J^* > \tilde{J}, \quad (27)$$

which contradicts the assumption that P_{ICE}^* is the optimal solution of Problem (23). That means Eq. (23b) holds with the condition of equality at the optimum. Therefore, the relaxation does not lead to any loss of the original optimality.

4.3 Discretization

The discretization is implemented subsequently on the convexified Problem (23), since the numerical optimization algorithm requires the discrete version of the control problem. The discretization procedure converts the infinite dimensional optimization problem to a finite-dimensional one, by discretizing the system at specific time nodes. The constraints are also converted to a discrete formation and imposed at the discretization points.

For a given time interval $[t_0, t_f]$, and time increment array, Δt_k , the discretization nodes can be expressed as:

$$t_k = t_{k-1} + \Delta t_k, \quad k = 1, \dots, N, \quad (28)$$

in which t_0 is normally set to zero and N denotes the number of discretization points. Then, a priori known power requirement and speed reference are converted to $P_{req}(t_k)$ and $\omega_{req}(t_k)$. As illustrated in Section 3.3, the power loss of the EM is related only to the motor/generator speed, so it can be transformed to a pre-known variable $P_{EM,loss}(t_k)$.

The most important step in the discretization of the problem is to obtain the discretized control input u . Then, all other variables (like system states) can be represented by the control variable $u(t_k)$.

With the convex relaxation of equality (11b), the original control variable has to be extended to:

$$u(t_k) = \begin{bmatrix} \sigma_1(t_k) \\ \sigma_2(t_k) \end{bmatrix} = \begin{bmatrix} P_{ICE}(t_k) \\ P_b(t_k) \end{bmatrix}, \quad k = 1, \dots, N, \quad (29)$$

where $\sigma_1(t)$ and $\sigma_2(t)$ are introduced to represent the continuous formation of the control input u .

The control variable is broadly constructed on the basis of some prescribed continuous basis functions, $\phi_j, j = 1, \dots, M$:

$$\begin{bmatrix} \sigma_1(t) \\ \sigma_2(t) \end{bmatrix} = \sum_{j=1}^M (\eta_j \phi_j), \quad j = 1, \dots, M. \quad (30)$$

Thus, the control input $u(t_k)$ at the discretization points can be addressed by:

$$\begin{bmatrix} \sigma_1(t_k) \\ \sigma_2(t_k) \end{bmatrix} = I_k H, \quad k = 1, \dots, N, \quad (31)$$

where I_k is the matrix function of time index determined by the selected basis functions, and $H = [\eta_1, \dots, \eta_M]$. After acquiring the discrete control (engine and battery power), the cost and constraints can be represented by $u(t_k)$.

If the piecewise constant basis functions are applied:

$$\phi_j(t) = \begin{cases} 1, & t \in [t_{j-1}, t_j] \\ 0, & \text{otherwise} \end{cases}, \quad j = 1, \dots, M, \quad (32)$$

which indicates that:

$$\eta_j = \begin{bmatrix} \sigma_1(t_{j-1}) \\ \sigma_2(t_{j-1}) \end{bmatrix}, j = 1, \dots, N, \quad (33)$$

and $M = N$.

5 Results and Analysis

This section will verify the piecewise linear approximation of the fuel rate by comparison between the original efficiency map and fitted data. A second fitting result using the spline approximation is also presented for comparison.

Furthermore, to validate the proposed convexification and convex programming, two different flight test scenarios are considered. Also, two battery operating strategies are included: charge-depleting strategy and charge-sustaining strategy. The charge-sustaining strategy aims at maintaining the SoC of the battery at a certain level with on-board recharging. The charge-depleting strategy differs in which the battery SoC is allowed to decrease continuously until its minimum value is reached [41].

The first test case is a complete mission that includes take-off, climbing, cruising and landing phases, in which the battery charge-depleting and charge-sustaining strategies are both implemented. In the second, the charge-sustaining based strategy is employed for the cruising flight phase and the performance is compared with the results obtained using DP. Meanwhile, the optimized results for the two scenarios are employed and validated on a forward simulation platform developed in the previous study [39]. To integrate the optimal control to the forward simulation platform, a power-consuming model has been designed [39]. The power-consuming was estimated using simulation results of flight control and could convert the command of flight control to the request of power demand.

5.1 Approximation of Fuel Rate

A piecewise linear approximation of the engine fuel rate was introduced to convexify the cost function. The piecewise approximation was done by first specifying the best efficiency point at the current speed. The best efficiency point was kept at its original value when the fuel rate was approximated by a piecewise linear function, via the least square method. The efficiency map of the approximated data is shown in Fig. 8, with the original efficiency [33] also shown. The dotted curve represents the original fuel rate data, with fitted data denoted by the dashed curve. It can be concluded that the piecewise linear fitting reflects the feature of the original data, especially around the area of best efficiency. The reason is that minimizing the fitting error for this portion of the data takes priority.

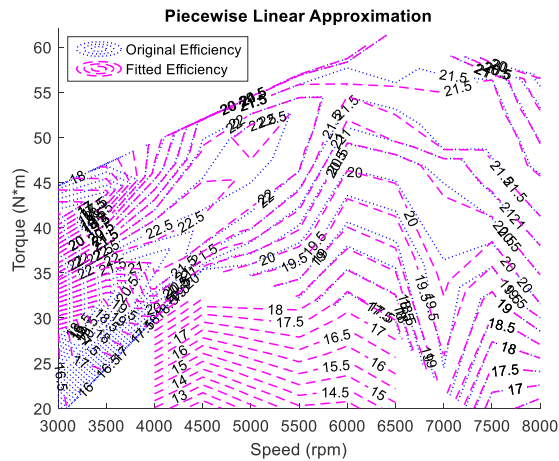


Fig. 8. Piecewise linear approximation of the engine map.

Another efficiency map from spline approximation is presented in Fig. 9 as a comparison of piecewise linear approximation. The comparison demonstrates that both approaches can avoid a large shift of the best efficiency region, whereas the piecewise linear fitting performs better in keeping its original shape. On the other hand, the two methods both underachieve in fitting data around the region between 3000-4000 rpm and 20-35 N*m. This may be due to the deficiency of the original experimental data. However, it can be observed that our piecewise linear approximation tries to follow the gradient of the original data, while there occurs obviously overfitting for the spline approximation. The two techniques express different understandings when predicting the missing original data between 3000-7000 rpm and 20-35 N*m. It is not straightforward to judge which technique is better, but the piecewise linear approximation seems more reliable than the spline approximation that overreacts to minor fluctuations.

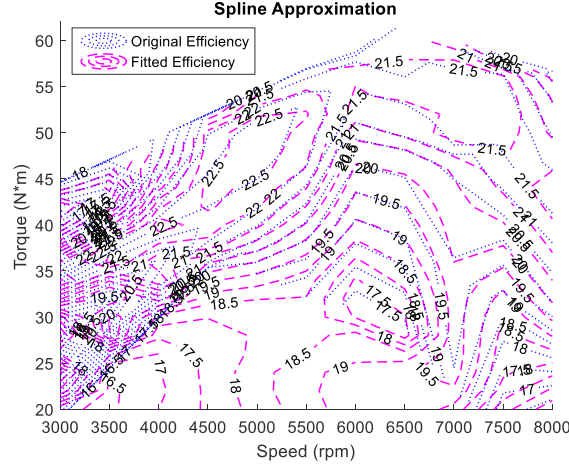


Fig. 9. Spline approximation of the engine map.

5.2 Test Case 1

The test case used in this section simulates a complete 12-minute flight mission that includes take-off, climbing, cruising and landing phases. The aircraft take-off run is the 1st minute, it then climbs to the cruising altitude in the time interval of the 1st - 5th minute. The aircraft commences the cruising phase at 5th minute. Cruises at constant speed and level flight until the 9th minute and finally in the last minute there is the landing phase, after which the mission ends. For a SoC-regulating strategy, the initial value of SoC is set to 60%. The maximum and minimum value of SoC are 80% and 20%, respectively, considered the nominal operating range of the battery. When the SoC depletes to 30%, the energy supervisory controller maintains the SoC around this value.

5.2.1 Lossless Optimal Results

The power requirement of this flight mission is shown in Fig. 10(a) as the dark blue dotted line, in addition the figure also shows the optimal engine power (ICE power), the battery internal power and the power consumed. The power consumed is the sum of useful power and power losses, namely $P_{ICE} + P_b - P_{EM,loss} - P_{b,loss}$. As shown in Fig. 10(a), the power consumption is equal to the power requirement, i.e. $P_{ICE} + P_b - P_{EM,loss} - P_{b,loss} = P_{req}$ being satisfied at each and every time node. That means Eq. (23b) holds under the equality condition at the optimal trajectory. In other words, the relaxation presented in Section 4.2 does not affect the results of the optimization.

Fig. 10(b) displays the optimal state variable E with a red dotted line. Note that this virtual variable E has no precise physical definition, and thus cannot reflect the actual remaining energy of the battery. The original state SoC is computed from internal energy E and plotted alongside the internal energy. In this case, the SoC value during the mission satisfies the original variable

bounds and meets the requirement of the SoC-regulating strategy. The optimal SoC demonstrates that the convex optimization can realize both depleting and sustaining strategies. It also demonstrates that the change of state variable in convexification retains the original variable bounds.

In short, the convexification procedures proposed in this paper do not lead to any loss of original optimality and the optimization of convexified problem gives the same solution as the one of original problem.

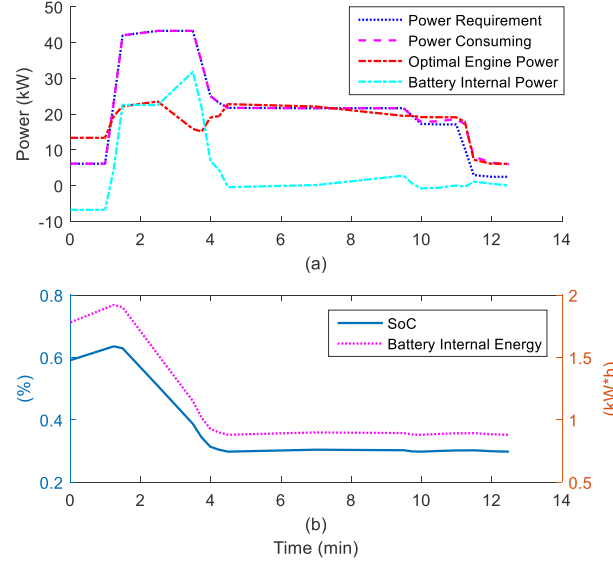


Fig. 10. Optimization results of convex programming in test case 1.

5.2.2 Forward Simulation

The optimal control is implemented in the forward simulation model. Since in real flight, external factors such as wind will cause fluctuation of power demands, to increase the fidelity of the model, a disturbance is added to the original power requirement, as shown in Fig. 11. Fig. 11(a) includes power request with presence of a disturbance and simulation results of the engine and motor/generator power, while Fig. 11(b) displays the SoC of the battery.

It can be seen from Fig. 11, that the simulated engine power is not exactly the same as for the optimal (reference) case, see Fig. (10)a. When the power demands show large variations, such as during the 1st and the 3rd minutes, the actual simulated power may overshoot its reference value for a few seconds. This phenomenon only occurs during transient processes and generally lasts for a short time. Actually, the amplitude and time length of the overshoot are both dependent on the performance of the engine control unit, so they can be improved if necessary. For most of the time during the mission, the engine power fluctuates with the power disturbance but maintains convergence and good performance. It implies that the optimal control is robust to the disturbances.

The motor/generator power is monitored in this simulation instead of the battery internal power, since the latter cannot be observed in real applications. This approach is reasonable since they both can reflect the consumption of electric power and the difference between their values comes from power losses. This fact is confirmed by comparing the motor/generator power in Fig. 11 and the battery internal power in Fig. 10.

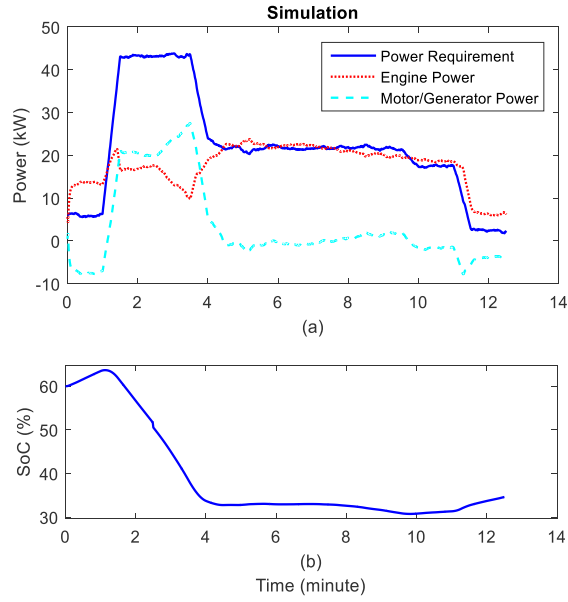


Fig. 11. Simulation results of convex programming in test case 1.

Fig. 11(b) displays the simulation results for SoC. The SoC increases in the first one minute, due to battery charging during the initial ground-roll run. Later, it drops by a large margin with the aircraft's demand for power for lift-off and climb. Once the SoC reaches the 30% level (set as the sustaining value in this test case), the controller maintains it at that level. The SoC value does not decrease any further and remains fairly constant, until the end of the mission. It starts to increase again around the 11th minute, since the power demands in this phase decrease so there is the extra power to charge the battery. With the fluctuation of control variable (engine power), the value of the state variable, SoC, also diverges from the optimal, but still meets the requirement of both depleting and sustaining strategies.

Hence it is concluded that the optimal control has the potential to be applied in real applications with a degree of robustness.

5.3 Test Case 2

The second test case compares the results from convex programming with ones from another global optimization—DP. The initial value of SoC is set to 50% and the value is supposed to be sustained in the 13-minute flight.

Different from the first test case, the flight begins when the aircraft has reached the cruising altitude and gets ready to start the mission. Thus, the trip excludes the take-off, climb and landing phases, as shown in Fig. 12. Assuming that the aircraft has approached the target at t_0 and then hovers over the target, the value of power requested for most of the time is slightly higher than the power needed for cruising [39]. Similar to test case 1, the disturbance to power demand is added to the profile when the forward simulation is applied. The two power profiles, respectively used in optimization and simulation, are provided and compared in Fig. 12.

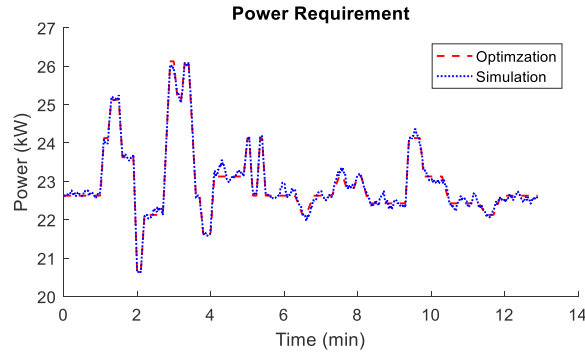


Fig. 12. Power requirement of test case 2 [39].

5.3.1 Comparison of Optimization Results

This section firstly compares optimization results from DP and convex optimization, with the variation of the number of discretization points. The corresponding values of final SoC, average SoC and optimization time are also presented, see Table 2. As illustrated in Table 2, both DP and convex optimization can meet the constraint of 50% final SoC, regardless of the number of discretization nodes. To further investigate the accuracy performance of two optimization methods, average SoC values are calculated and compared. It is apparent that the convex optimization could reach better accuracy than DP, in terms of SoC sustaining. On interesting fact is that DP is susceptible to the range settings of SoC values. For instance, if the up and limit of SoC is set quite closely to the final SoC value, DP easily converges to an infeasible region. Thus, it can be concluded that the convex optimization is more accurate and robust regarding SoC sustaining. The conclusion is expectable and mainly thanks to the convexity of optimization search space, especially battery state equality constraints.

Table 2 Comparison of optimization results between convex optimization and dynamic programming

	Dynamic Programming		Convex Programming		
Number of Discretization Points	774	387	387	55	16
Final SoC (%)	50.0001	50.0006	50.0000	50.0000	50.0000
Average SoC (%)	49.9723	49.7243	50.0000	50.0000	49.9814
Optimization Time (sec)	14.0185	7.9268	2.1707	0.6708	0.3263

With the reduction of the number of discretization points, the computation cost of DP is relieved while its accuracy is also weakened. Actually, it is generally accepted that global optimization such as DP normally has low efficiency and its (worst-case) complexity exponentially grows with the intenseness of discretization mesh. In contrast, the convex optimization is robust to the density of discretization grid. Even with only 16 number of discretization nodes, the convex optimization could achieve great final SoC accuracy with a little of sacrifice of average SoC accuracy. Its high accuracy and robustness are both resulted from the lossless convexity of original HEPS energy problem.

Another important advantage of convex programming is the reduction in the computational cost of global optimization, compared to DP. Note that all optimizations were conducted on processor Intel Core i7-3770K @ 3.40GHz with 8GB memory. Considering results with 387 discretization nodes, it took around 8 sec for the DP to converge, while the convex programming took approximately 2 sec, about three times faster. With the decrease of number of discretization nodes, the convergency time of

convex programming continues to drop, but its accuracy keeps. Confirming that lossless convexification simplifies the original problem and substantially reduces the complexity cost.

5.3.2 Comparison of Simulation Results

Two optimal results are applied and verified in the forward simulation. One with 387 discretization points for DP is selected since its accuracy is satisfied. Considering the simplification capability of convex optimization, it is expected that its reduction of optimization time is at least up to ten times. In other words, the time has to be reduced from 7.9 sec to lower than 0.7 sec. Thus, one convex optimization using 55 discretization nodes is employed to derive another optimal control.

Table 3 lists the overview performance of two compared simulations, including number of discretization points, final SoC, and fuel/energy consumptions, etc. With the impact of additional power disturbance, the simulation values of final SoC differs from the optimization ones (basically equalling to 50.00%). By contrast to the optimization, DP shows better final SoC accuracy in the simulation. This implies that the constraint-handling performance of DP is more robust to modelling uncertainties, in comparison with convex optimization. This robustness may come from the higher number of discretization points.

Table 3 Comparison of simulation results between convex optimization and dynamic programming

	Dynamic Programming	Convex Programming
Number of Discretization Points	387	55
Final SoC (%)	48.80	47.37
Fuel Consumption (kg)	1.8328	1.8235
Corrected Final SoC (%)	50.00	50.00
Corrected Fuel Consumption (kg)	1.8359	1.8304
Energy Consumption (MJ)	85.13	84.85

Another conclusion of final SoC value for convex programming lower than one for DP is that the convex programming implies greater consumption of electric energy. The extra electric energy stored in the battery with DP is the cause of the additional fuel consumed. Therefore, the remaining fuel does not represent real fuel consuming. To make fuel consumption of each optimization comparable, a correction method is introduced to adjust final SoC values to the same value. Here, both final SoC values are re-set to 50.00% and corrected (real) fuel consumptions are acquired. The corrected numbers are also displayed in Table 3 and it shows that the convex optimization has lower fuel usage compared with DP. Apart from the amount of fuel used, another criterion—energy consumption also indicates that the convex optimization demands less energy to complete the flight mission.

Fig. 13(a) details the simulation results of battery SoC, engine power and fuel consumption from both convex optimization and dynamic programming. The curves of SoC verify that those two optimizations can both achieve the charge-sustaining, but the DP obtains a more precise regulation of SoC. It can be demonstrated that both methods are practical for achieving optimal control in cruising applications. Yet the optimal control (engine power) derived from convex optimization achieve better SoC-sustaining, see Fig. 13(a); while the optimal control (engine power) derived from DP exhibits more intense variations, see Fig. 13(b).

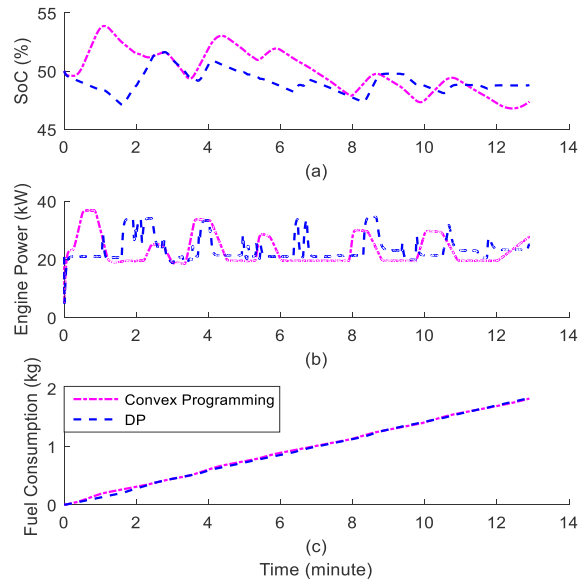


Fig. 13. Comparison of simulation results between convex optimization and DP in test case 2

6 Conclusion

This paper proposed a lossless convexification for energy optimization of a hybrid electric propulsion system, for aerial vehicles. The experimental data were firstly analysed and fitted to clarify the convexity of the original problem. It is concluded that the battery model and power request equality do not satisfy the requirement of convex programming. Hence, two techniques—change of variables and equality relaxation were implemented. A variable named battery internal voltage is proposed in this paper to convexify the state transition and power request equality. The relaxation of a non-affine equality yielded a new inequality constraint. A Lemma was derived to illustrate that this convexification does not lead to any loss of original optimality.

The developed HEPS system was briefly described and a parallel hybrid configuration was nominated for later optimization and simulation. To validate the proposed convexification, two different flight test scenarios were considered. The first test case verifies that the convex relaxation does not sacrifice the optimality of the solution and variable change does not lose the original bounds. With forward simulation, the optimal control from convex optimization was demonstrated to be robust to the disturbance in power requirement. By comparing with DP in test case 2, the convex optimization performs gentler optimal control and better optimal cost results. It even demands slightly less total energy to accomplish the flight mission. However, the most significant advantage of convex optimization is that it can converge to the optimal solution with a much lower time cost than the DP method. All those benefits make the convex optimization attractive for real applications.

Conflict of interest statement

The authors declared that they have no conflicts of interest to this work.

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