

Dual-Wavelength Laser Speckle Technique for Absolute Angle Sensing in Two Dimensions

by

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Abstract

This thesis presents the theory, development, and construction of a novel absolute angle sensor employing dual-wavelength laser speckle for use in measuring two dimensional surface tilt angles. The targeted application is robotic manufacturing, specifically robot drilling where perpendicularity to the workpiece is particularly important and an accuracy of up to 0.1° is desired. The proposed technique uses two collinear beams of differing wavelengths, illuminating the surface of interest. The generated speckle patterns are shifted with respect to each other, with the position and orientation of the shift being directly related to the two dimensional tilt of the surface. This relationship is derived mathematically, and a number of experimental results are presented to confirm its validity.

Two full implementations of the sensor were tested and characterised. The first used a single laser outputting two longitudinal modes with a wavelength separation of 0.5nm. Initial results in one dimension presented measurements over the wide angle range of $\pm 45.00^\circ$, with an average absolute bias error of around 0.51° . These errors were smaller at smaller magnitudes, averaging around 0.22° between $\pm 11.2^\circ$. Further experiments in two dimensions focused on the magnitude range $\pm 7.50^\circ$, finding average absolute bias errors of 0.15° in θ_x and 0.08° in θ_y . The estimated standard deviations in these measurements were 0.10° in θ_x and 0.18° in θ_y .

The second sensor used two separate lasers with a wavelength separation of 21.5nm, increasing the sensitivity by a factor of 40. This sensor showed greater wavelength instability due to the two independent lasers and an increased mode instability. As such, the overall errors in experimental results were greater than expected, and the data were filtered to demonstrate the achievable performance if this laser instability were resolved. Results for surface tilts up to $\pm 0.25^\circ$ in two dimensions showed average absolute bias errors of 0.007° in θ_x and 0.009° in θ_y after filtering, and standard deviations of 0.003° in θ_x and 0.017° in θ_y . These results demonstrate that the dual-wavelength speckle technique has the potential to provide high accuracy absolute angle measurements in two dimensions.

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List of Acronyms

1D	one dimensional
2D	two dimensional
3D	three dimensional
APC	angled physical contact
BF	bilateral normal filtering
BRDF	bidirectional reflectance distribution function
CAD	computer aided (or assisted) design
CCD	charge-coupled device
CMOS	complementary metal-oxide semiconductor
CNC	computer numerical control
CW	continuous-wave
DOF	degrees of freedom
DWS	dual-wavelength speckle
ESPI	electronic speckle pattern interferometry
FDM	fused deposition modelling
FFT	fast Fourier transform
FRT	fractional Fourier transform

List of Acronyms

FST	Fresnel transform
FT	Fourier transform
FWHM	full width half maximum
InGaAs	indium gallium arsenide
IR	infrared
LCT	linear canonical transform
MCT	mercury cadmium telluride
MEMS	microelectromechanical systems
MWA	mean weighted by angle
MWAAT	mean weighted by areas of adjacent triangles
MWE	mean weighted equally
MWELR	mean weighted by edge length reciprocals
MWRELR	mean weighted by square root of edge length reciprocals
MWSELR	mean weighted by sine and edge length reciprocals
OFRT	optical fractional Fourier transform
OFT	optical Fourier transform
OSA	optical spectrum analyser
PCA	principal component analysis
PDF	probability density function
PIV	particle image velocimetry
PLA	polylactic acid
PPR	product process and resource model
RGB	red green blue
RMS	root mean square

SCARA	selective compliance assembly (or articulated) robot arm
SLM	selective laser melting
TOF	time of flight
UV	ultraviolet

List of Symbols

$\mathbf{A} = (A_x, A_y, 0)$	speckle shift at detector plane
$A_0 = (A_{x0}, A_{y0})$	speckle shift at zero tilt (calibration constant)
α	phasor amplitude
$\boldsymbol{\alpha}$	resultant phasor amplitude
C	responsivity (calibration) constant
χ	bias error/residual
$\mathbf{D} = (D_x, D_y, D_z)$	position vector of detector
$\hat{\mathbf{d}} = (d_x, d_y, d_z)$	unit vector to detector
d_a	lens aperture diameter
$\Delta\lambda$	wavelength separation
E	electric field
f	phasor
F	random phasor sum, or resultant phasor
$I(\mathbf{P})$	intensity at $\mathbf{P}$
k	wavenumber
$K(\mathbf{P}, \mathbf{D})$	transmission function
$L_D(\mathbf{P}, \mathbf{D})$	vector from $\mathbf{P}$ to detector

List of Symbols

L_D	detector (working) distance, magnitude of $\mathbf{L}_D(\mathbf{P}, \mathbf{D})$
$\mathbf{L}_S(\mathbf{P})$	vector from $\mathbf{P}$ to source
L_S	source distance, magnitude of $\mathbf{L}_S(\mathbf{P})$
λ	wavelength
M	speckle pattern magnification due to wavelength difference
m_x	surface gradient in x
m_y	surface gradient in y
$\mathbf{O} = (0, 0, 0)$	laser spot position, origin of geometry coordinates
$\mathbf{P} = (x, y, z)$	position vector of surface point
$\mathbf{P} = (x, y, (m_x x + m_y y))$	position vector of surface point with surface gradient
$\mathcal{P}$	degree of polarisation
p	probability density function
ϕ	surface tilt direction
ψ	surface tilt magnitude
φ	(phasor) phase
φ	resultant phasor phase
r	resolution
R	correlation contribution
RR	cross correlation
ρ	correlation of intensities
$\mathbf{S} = (S_x, S_y, S_z)$	position vector of source
$\hat{\mathbf{s}} = (s_x, s_y, s_z)$	unit vector to source
s	camera pixel size
σ	standard deviation

θ_{max}	practical sensor range
θ_R	theoretical sensor range
θ_x	surface tilt around x axis
θ_y	surface tilt around y axis
$U(\boldsymbol{P}, \boldsymbol{D})$	complex amplitude at detector
w_{long}	longitudinal speckle size
w_{spot}	spot diameter
w_{tran}	transverse speckle size

Glossary of Terms

General

accuracy	How close a measurement is to the true value, generally defined during experimental work with respect to a known reference or an instrument of known accuracy
precision	The number of digits or decimal places a value can be measured to
repeatability	How close repeated measurements from an instrument are to each other
resolution	The minimum perceptible change in a measured variable for a particular instrument
responsivity	The degree to which a particular variable is affected by a given change, or more specifically the factor relating one variable to another (see also ‘responsivity constant’)
sensitivity	Whether a particular variable is affected by a given change, eg. cross-component sensitivity or insensitivity to tilt/translation
uncertainty	The range of values around an experimental measurement in which the true value is expected to lie

Robotics

Abbe error	The magnification of angular errors at increasing distance
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end effector	The tool mounted on the end of a robot's wrist
exteroceptive sensors	External, or exteroceptive, sensors of a robot are those used to measure the robot's environment, and are also generally externally mounted. This can include contact, vision, sound, range, and force sensors
proprioceptive sensors	Internal, or proprioceptive, sensors of a robot are built-in sensors that monitor the position and orientation of the robot's joints, linkages, and end effector

Speckle

speckle correlation	Direct comparison of speckle patterns, using correlation
speckle interferometry	Interference between two beams from an optically rough surface, or one beam with a reference, produces fringes to be measured; measures displacements smaller than speckle size
speckle photography	Uses lenses to image a subjective speckle pattern; often places image in a specific plane of the lens, such as the Fourier plane; measures displacements greater than speckle size

Angle Sensing

bias error	The difference between the measured angle and the applied stage angle, also called the residual
Cartesian angle frame	Uses Cartesian coordinate system to describe angle measurements; angles are given by components of rotation around axes, where θ_x is the rotation around the x axis, θ_y is the rotation around the y axis
centred	The sensor component (source or detector) is directly opposite the spot, along the z axis
detector	The camera and/or its position within the sensor geometry

mode power stability	The changeability in the power (height) of a laser mode
planar	The source and detector are in-plane, in particular both in the xz plane
polar angle frame	Uses polar coordinate system to describe angle measurements; angles are given in terms of magnitude, ψ , and direction, ϕ
responsivity constant	The constant C , which relates the speckle shift to surface angle for a given sensor geometry
sensor	The full sensor setup, including the camera, lens tube, and laser spot on the surface
sensor constants	Also referred to as the calibration constants; the three values A_{x0} , A_{y0} , and C
sensor components	The source (practically the lens tube used to focus the source) and the detector (camera)
sensor characteristics	The characteristics of a completed sensor encompass its range, sensitivity, and resolution
sensor implementation	The combination of sensor arrangement and geometry, laser(s) and wavelengths used, specifications of the camera, and signal processing method; all resulting in a complete sensor
sensor arrangement	The general layout of the sensor components in relation to each other; e.g. detector above spot, or sensor above spot, etc.
sensor geometry	The specific distances between the sensor components and the surface of interest
sensor range	The maximum measurable angle by the sensor, derived from the minimum distance in the correlation image between the central and measurement peaks where each are still distinct

sensor resolution	The minimum angle resolvable by a complete sensor, derived from the accuracy of the 3-point Gaussian fit used on the correlation peak during signal processing
source	The lens tube and/or its position within the sensor geometry
speckle offset	The speckle shift of the detector at a surface angle of zero (i.e. surface parallel to detector plane), also written as the constants (A_{x0}, A_{y0})
speckle shift	The distance at the detector plane between the two different speckle patterns generated by DWS, also written as (A_x, A_y)
surface gradient	The gradient of the tilted surface, given by $m = \tan(\theta)$
surface tilt/angle	The angle of tilt of the surface of interest, described using either a Cartesian or polar reference frame
wavelength separation	The difference between the two wavelengths of light used in DWS
wavelength stability	The changeability in the wavelength (position) of a laser mode
working distance	The distance between the camera's detector plane and the surface of interest

Chapter 1

Introduction

1.1 Background and Context

Industrial robotic arms are increasingly widely used in areas of manufacturing such as drilling, riveting, welding, and milling. Robots are particularly well-suited to highly repetitive tasks such as hole drilling and riveting for large parts or on construction lines. As such, they are designed to have a very high repeatability, but their absolute accuracy is often lower. In other words, when returning to a previous position they can achieve a very high relative accuracy, matching the previous position very closely. However, when moving to an arbitrary position, the accuracy as compared to the expected position is much lower.

The ABB IRB4400 robotic arm for example has a stock position accuracy of around $\pm 3.00\text{mm}$, as measured by Olsson et al. [1]. An optional high accuracy calibration when purchasing raises this to $\pm 0.50\text{mm}$. The repeatability of the robot, however, is more than ten times higher, at $\pm 0.05\text{mm}$.

Orientation accuracy is particularly important in robotic drilling, which has applications in the automotive and aerospace industries. In aerospace, 80% of fatigue cracks originate from riveting holes [2], while a 2.0° misalignment of drilled holes can shorten the fatigue life of bolts by 47% [3]. An often quoted aim for position and orientation accuracy in aerospace is 0.10mm and 0.5° [4]. However, these values are not currently achievable with off-the-shelf commercial robots [5].

The source of the errors in robot positioning accuracy are varied, originating di-

rectly from the robot, from the workpiece, or as the machining process is carried out [6]. For example, the robot itself may have been inaccurately calibrated, or have deformations in the joints caused by wear. Then, the workpiece could have manufacturing defects or have been deformed in a previous processes. Finally, the force feedback during processes such as drilling or milling can lead to deformations in the robot joints or in the workpiece.

As a result of the relatively low accuracy, robots are often aided by additional, external sensors. These monitor the robot's environment, providing feedback that can be used to correct position and orientation. This external compensation can raise the positioning accuracy to 0.10mm and the orientation accuracy to 0.2° [6]. Using dedicated tilt sensors, the orientation can be improved even further, reaching 0.15° [7] or 0.10° [8].

Current approaches to tilt sensing in robotics mainly use two techniques: an arrangement of distance meters used to measure the tilt of the surface directly [7], or surface normal estimation from three dimensional (3D) point cloud data [9]. Though both techniques are capable of achieving the required orientation accuracy for drilling in aerospace, they also both have their drawbacks. The first technique involves combining three or four individual distance sensors, resulting in a relatively bulky instrument. The second approach, using 3D point clouds, requires the use of surface normal estimation algorithms, which can take up to 2.6s to determine surface normal to an accuracy of 0.5° [9].

The motivation behind this work, therefore, was to develop an online, low-cost, absolute tilt measurement system using a single sensor head and suitable for applications such as robot drilling. Given the focus on robotic drilling in the literature, and the accuracies achieved by other techniques, the target accuracy of the sensor will be 0.1° , with a minimum 2D range of $\pm 5.0^\circ$.

1.2 Project Novelty and Aims

The novel technique underpinning the sensor is dual-wavelength speckle (DWS), wherein a surface is illuminated with laser light of two wavelengths, and the resulting speckle patterns have a relative shift that relates to the tilt of the surface. The technique can be used to measure angles in two dimensions, acquiring all the

information needed for a measurement in a single camera frame and thus giving an absolute tilt angle.

While the application targeted for the sensor is robotic manufacturing, the technique has novelty as a speckle-based tilt sensor. Existing techniques for tilt sensing using laser speckle rely on relative or referenced measurements, recording the speckle pattern before and after the surface deformation [10, 11], or using two cameras [12, 13]. The ability of the proposed sensor to measure absolute angles provides a significant improvement over these techniques

The basis of the technique is the displacement in position of a speckle pattern when the illuminated surface undergoes a translation or tilt. Methods that use before and after images make use of this effect. However, the displacement is also affected by the wavelength of the illumination. For two wavelengths illuminating a surface simultaneously, two similar but offset speckle patterns will be generated, as shown in Figure 1.1. As the surface tilts, the patterns move independently by an amount proportional to their wavelength, and so their relative displacement (or shift) also changes. It is this shift that can be measured, using a single image, and used to directly calculate the surface tilt.

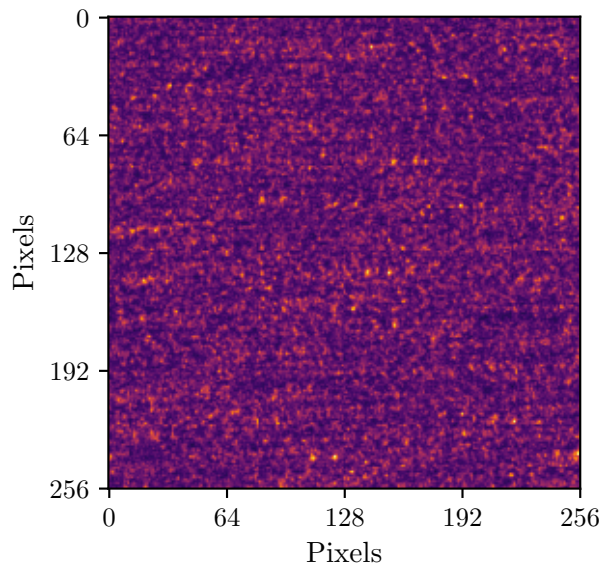


Figure 1.1: Example dual speckle pattern; the image contains two similar but displaced speckle patterns, one from each source wavelength.

The theory behind the deformation of a speckle pattern generated by a single wavelength from a tilted surface was presented in Charrett and Tatam [14], deriving a mathematical relation between the two factors. The theory is extended here to describe the relative speckle shift resulting from a dual-wavelength illumination. The additional variables in the theoretical equations come from the geometry of the sensor setup, specifically the positions of the source and detector in relation to the illuminated spot on the surface. By changing the sensor geometry, the sensitivity, range, and resolution of the sensor can all be adjusted, thus demonstrating the flexibility of the technique for angle measurements.

The main objectives of the project were:

- i Develop and experimentally test the mathematical theory relating the dual-wavelength speckle pattern shift to surface tilt angles.
- ii Develop the image processing method for extracting the speckle shift between the two patterns present in recorded frames.
- iii Investigate the types of laser instability and how these affect angle measurements.
- iv Construct and characterise a tilt sensor using a single laser operating with two longitudinal modes, looking at one and two dimensional surface tilts.
- v Construct and characterise a two dimensional tilt sensor using a wider wavelength separation for a higher sensitivity.

1.3 Thesis Outline

The remainder of the thesis is structured as follows:

Chapter 2 focuses on the intended application of robotic drilling and the need for external sensors to improve accuracy. The chapter reviews the current approaches to tilt sensing within robotic drilling and similar applications such as riveting and milling.

Chapter 3 introduces laser speckle, including how it is formed and the effects surface properties have on speckle pattern generation. Current approaches to both position and tilt sensing using speckle-based techniques are reviewed,

with the chapter ending with a summary of tilt sensing techniques employing speckle.

Chapter 4 describes the concept and mathematical theory of DWS for surface tilt sensing. This includes a formal derivation of the equations relating speckle shift to surface tilt. The chapter also looks in detail at how the sensor setup affects its range, sensitivity, and other characteristics.

Chapter 5 covers in detail the methodology behind constructing a DWS sensor. This includes the image processing, techniques for investigating laser stability, the calibration method developed for the sensor, and the stages used to test the sensor performance.

Chapter 6 provides a proof-of-concept demonstration of the theory, showing that the mathematics derived in Chapter 4 apply in practice, and investigating the effect of factors such as surface roughness and spot size.

Chapter 7 presents a sensor using a single laser in a dual-mode operation, resulting in a relatively small sensitivity but with the advantage of only one laser module. Results are presented for one dimensional (1D) angle measurements over the range $\pm 45.00^\circ$, and for two dimensional (2D) measurements with magnitudes up to $\pm 7.50^\circ$.

Chapter 8 presents the second DWS sensor, which uses two lasers to provide a much higher wavelength separation and thus a significantly higher sensitivity. Results for 2D angle measurements are presented for the range $\pm 0.25^\circ$, which reflects the higher sensitivity, and the characteristics and accuracy of the sensor are compared with the implementation in Chapter 7.

Chapter 9 summarises the results and novelty of the technique and its implementations. This chapter also discusses possible improvements and extensions to the implementation of the technique, including a proposal for online determination of the working distance.

Appendix A collects the calculation of the various partial derivatives used in the theory formulation in Chapter 4.

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Chapter 2

Industrial Robotics and its Applications Requiring Tilt Sensing

This chapter will provide a broad overview of industrial robotics, before focusing more directly on tilt sensing using external sensors as it applies to applications such as robotic drilling, riveting, and milling.

Firstly Section 2.1 introduces the types of stationary robots used in industry, using the common classifications of articulated, SCARA (selective compliance assembly (or articulated) robot arm), Cartesian, cylindrical, spherical, and delta. Then, an overview is given of the difference between internal and external sensors in robotics, and why external sensors are often required to improve a robot's accuracy.

Building on this background, Section 2.2 gives a more detailed look at tilt sensing in robotics, focusing on robotic drilling. The section describes current techniques, such as using an arrangement of distance meters, and proposed techniques, including the sensors based on MEMS (microelectromechanical systems) technology.

2.1 Overview of Robotics in Industry

A robot is defined as “a machine controlled by a computer that is used to perform jobs automatically” [1]. This definition is fairly broad, covering a huge variety of robots from the ‘turtle’ robots used to teach simple programming to children, to the robotic arms used in car factories, to NASA’s space probes and Mars rovers.

Industrial robots are stationary robots used in manufacturing. Although factories often use mobile robots to transport materials between areas, these are not included under the category of industrial robots. Industrial robots have a manipulator arm that is used to manoeuvre a tool or gripper, known as the end effector, into position to perform a task. The tasks can range from pick-and-place, where the robot is used to pick up objects and place them in a particular position and/or orientation, to manufacturing processes such as welding, milling, or drilling [2, 3].

2.1.1 Types of Industrial Robots

Industrial robots are generally split into six types based on their arm configuration [2, 4]: articulated, selective compliance assembly (or articulated) robot arm (SCARA), Cartesian (or gantry), cylindrical, spherical (or polar), and delta (or parallel) robots.

Articulated robots, such as the one shown in Figure 2.1a, are the most commonly used type. They consist of a number of rigid parts connected by rotary actuators. Typically, the actuators at the base and at the end effector spin, like a shoulder or wrist joint, while those in between function like elbow joints. Most industrial robotic arms have four or six actuators, but they can have anywhere between two and ten [3]. Each joint adds a further degree of movement for the robot, but also increases the complexity of its kinematics. Kinematics describe the layout of the rigid parts and moving joints, and governs how each part moves in order to get the end effector from point to point. More complex kinematics combined with the additional mass results in a slower robot, so articulated robots are not ideal for applications relying on speed [6]. However, their versatility makes them suitable for a wide range of manufacturing applications, including welding, metal cutting, pick-and-place tasks, material handling, spray painting, and more.

The second type of robot is the selective compliance assembly (or articulated) robot arm, shown in Figure 2.1b. The name refers to its movement, as the robot is

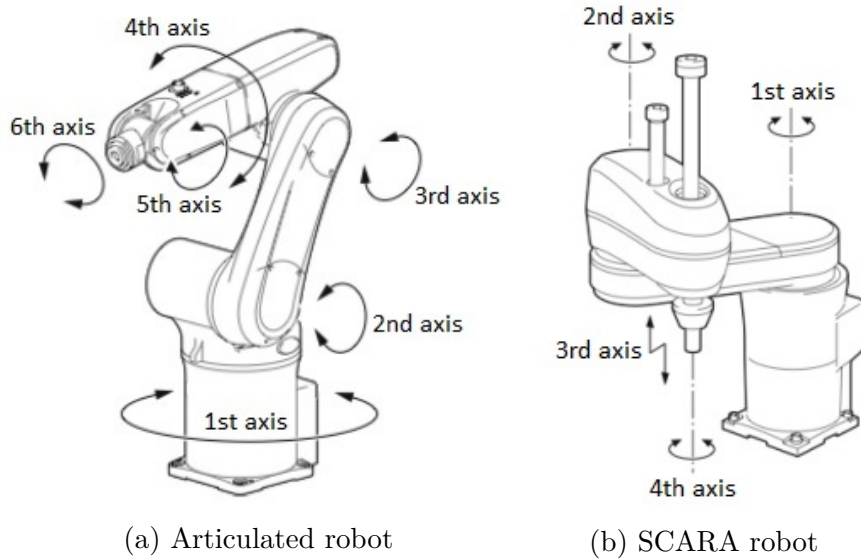


Figure 2.1: (a) Schematic of an articulated robot, showing the axes of rotation starting from the base. This image demonstrates the increased degrees of freedom introduced by successive actuators. (b) Schematic of a SCARA robot, with the axes labelled from the base to the end effector. The arm can move freely in the xy plane, but is rigid in the z axis. Images taken from [5].

compliant (i.e. flexible) in the x and y axes but rigid in the z axis [7]. SCARA robots have a central base, from which extends an arm at right angles. The arm has two rotary joints on vertical axes, so the arm moves laterally in one plane. In addition, the end effector can move vertically and rotate. This configuration gives SCARA robots very high speed and accuracy, with a relatively small footprint. They are also well-suited to working in enclosed or confined spaces, as the jointed arm enables the robot to fold up into a small space when needed. SCARA robots are most commonly used for pick-and-place, dispensing, and assembly tasks.

Cartesian, cylindrical, and spherical robots are all named for the shape of their work envelope [2, 4]. All three are shown in Figure 2.2. Cartesian robots operate on a gantry-like structure, moving laterally in up to three dimensions. Their kinematics are relatively simple, and due to the rigid gantry are particularly useful for carrying heavy loads. Laser cutters and 3D printers use Cartesian robots to manipulate the tool head.

Cylindrical robots are also fairly simple; they have a central column with an arm at right angles containing a linear actuator. The arm can swivel and move vertically

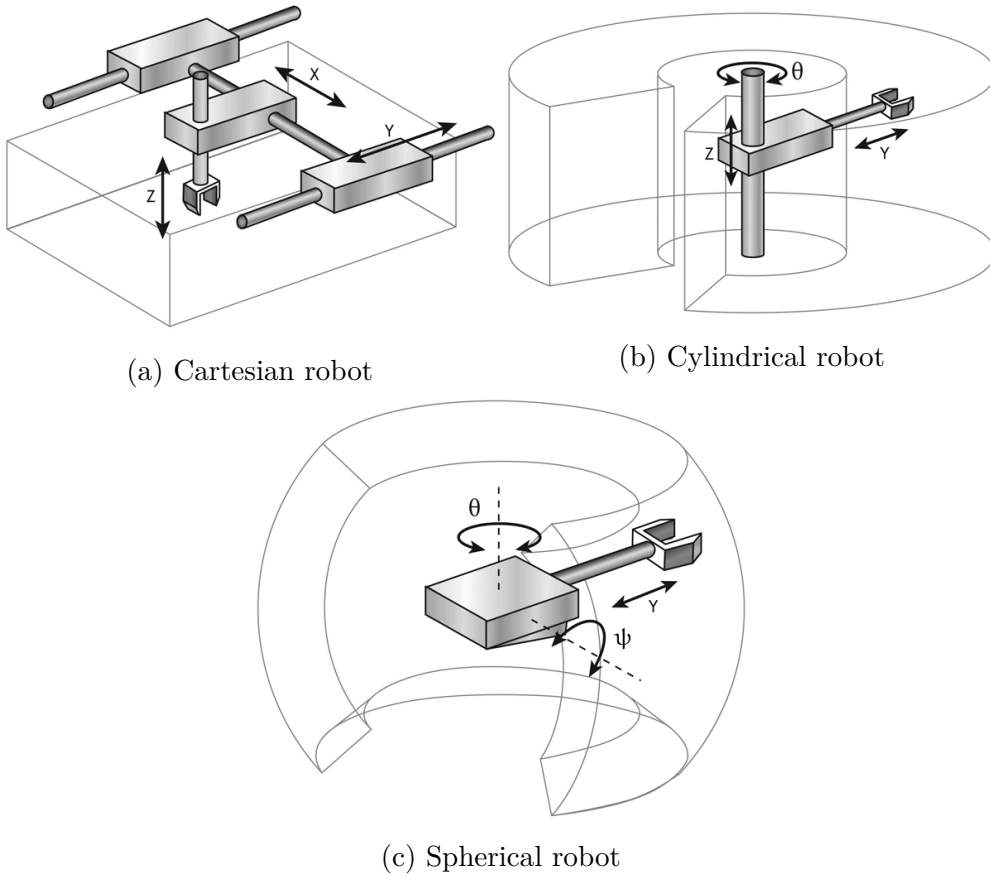
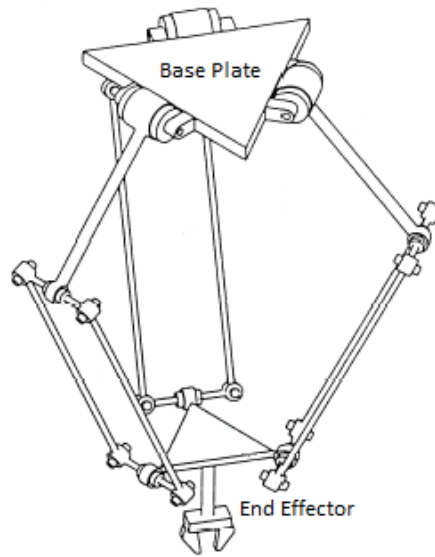


Figure 2.2: (a) Schematic of a Cartesian or gantry robot. Linear actuators combine to give a range of motion along the three axes X, Y, and Z. (b) Schematic of a cylindrical robot. The degrees of freedom of the robot are described by the angle of the rotary actuator at θ , and the linear actuators at Y and Z. (c) Schematic of a spherical robot. Similarly to the cylindrical robot, the degrees of freedom are given by the rotary axes at θ and ψ , and the linear actuator at Y. Images taken from [4].



(a) Delta robot

Figure 2.3: Schematic of a delta robot. The base plate is positioned above the rest of the robot, connected to the end effector via three arms. Image taken from [8].

along the base, and the linear joint allows it to extend along the radius of the work envelope. These are fast and compact, and are often used for pick-and-place tasks. They are also able to handle heavy payloads.

Spherical, or polar, robots have a rotational joint at the base, with two further rotary joints and a linear joint along the arm. These combine to give a spherical range of motion for the end effector. Their complexity requires a more sophisticated control system than a Cartesian or cylindrical robot, but their flexibility and large work volume mean they can be used for a wide variety of tasks, such as welding, injection moulding, die casting, and material handling [3, 4].

The final type of robot, the delta robot, is also called a parallel or parallel-link robot [2]. They use parallel joint linkages to manoeuvre a single end effector very precisely. The most common delta robots use three arms, as in Figure 2.3, each with a pair of linking rods to minimise twisting motions and ensure the end effector remains parallel to the base plate [9]. This configuration gives the robot a dome shaped work envelope. Many delta robots also have a rotation motor on the end effector, providing an additional degree of freedom. A major advantage of the delta robot is that the heavy motors used to manipulate the arms are positioned in the base, meaning that the arms themselves can be very light. This gives delta robots their

high speed and manoeuvrability. Compare this to an articulated robot, where each successive joint adds the weight of an extra motor, reducing its overall speed. A disadvantage of the delta robot, however, is that the payload mass must be limited in order to maintain its high speed. Thus, delta robots are most widely used in fields requiring a high degree of precision, such as food, pharmaceuticals, and electronics [3].

2.1.2 Internal and External Sensors in Robotics

Robots use an array of sensors to monitor and aid their movement. One of the main advantages of robotics as a manufacturing solution is their ability to react to their environment, such as manoeuvring around changing obstacles and avoiding humans [10], or adapt to a changing workpiece, for example by following a part on a moving conveyor belt. In order to facilitate this flexibility, sensors are necessary to track a robot's position, velocity, its proximity to other objects, the status of its end effector tool, and more. Position and orientation tracking is particularly useful in applications where the robot's end effector makes contact with the work surface. Tasks requiring physical contact, such as drilling and milling, are more prone to errors in movement and alignment than free space applications where the robot should not normally be disturbed [11].

Sensors are split into two categories. Internal, or proprioceptive, sensors are those built in to the robot, and are used to monitor its state and aid in movement control [12, 13]. They track the robot's joints and end effector, measuring one or more of their position, velocity, acceleration, and torque.

Most robots are also aided by external, or exteroceptive, sensors. In this context, external refers to sensors that monitor the environment, where internal sensors monitor the robot itself. Exteroceptive sensors facilitate tasks such as workpiece recognition and obstacle avoidance, for both the end effector and the robot itself. External sensors are more varied, including force, visual, tactile, range, contact, or sound sensors [13]. For some specific examples, contact and proximity sensors can be used to detect and avoid collisions, and force and tactile sensors are used in robots with grasping end effectors for picking objects up and moving them around.

Another important benefit of external sensors are in improving the accuracy of a robot's position and orientation. Although robots have a very high repeatability,

their absolute accuracy is often significantly lower. For example, a KUKA KR210 robotic arm has a quoted repeatability of $\pm 0.06\text{mm}$, but experimentally demonstrates an accuracy of around $\pm 3.00\text{mm}$ [14]. Another robot arm, the ABB IRB4400, offers a high accuracy calibration, increasing its accuracy from around $\pm 3.00\text{mm}$ to $\pm 0.50\text{mm}$. Its repeatability remains an order of magnitude higher, at $\pm 0.05\text{mm}$ [15].

Part inspection for quality control is an important process in the aerospace and automotive industries. The required position accuracy for the scanners used in these processes is generally around $0.05\text{-}0.10\text{mm}$, while the orientation accuracy is around $0.1^\circ\text{-}0.5^\circ$. Currently, commercial robots are unable to meet this level of accuracy [16].

The errors in robotic machining are typically classified using the product process and resource model (PPR) [17, 18]. Product related errors arise from the part being machined or manufactured, including deformation during processing and uncertainties in previous manufacturing stages. Process errors refer to errors introduced by the machining process, such as the tolerances and uncertainties of the process, or joint deformation of the robot due to process forces. Finally, resource errors are those intrinsic to the robot, such as calibration errors, joint deformations due to wear, and robot translations or interruptions from external factors. The three types of errors are linked, as the three elements of manufacturing are linked: “a product is manipulated by a resource following a process” [17].

Generally speaking, the types of errors that fall under those described by PPR cannot be accounted for using the robot’s internal sensors. Resource errors due to wear and inaccurate calibration in particular are inherently linked to the internal sensors. External sensors can be used to account for some errors that fall under the product and resource categories. For example, feature referencing on a flat surface can improve a robot’s position accuracy (a product error) to $\pm 15\mu\text{m}$, and give an angular accuracy of $\pm 0.48^\circ$ [18]. In terms of resource errors (those linked to the robot), compensation through external sensors can result in positional accuracies of $\pm 0.1\text{mm}$ and orientation accuracies of $\pm 0.2^\circ$ [18].

Vision Sensors

Vision sensors are a particularly common category of external sensor, essentially encompassing any sensor that incorporates a camera [19]. Most of the tilt sensors

described in the next section fall under vision sensors. Some vision sensors are active, including time of flight (TOF) sensors that emit a laser pulse and measure the time it takes to return, or triangulation-based laser distance meters (discussed in Section 2.2.1). Others are passive, most commonly using cameras to simply image the local environment. Stereo (binocular) and trinocular cameras are particularly useful, as they can be used to determine depth information. These can be used alongside additional processing methods such as object and feature tracking algorithms, or 3D point cloud generation (see Section 2.2.2 below). As vision sensors can be used for both movement and position control, and in aiding the work of the end effector, they have a particularly wide range of applications and uses.

For robotic arms, vision sensors can be categorised in two ways [20]. Eye-in-hand sensors are affixed to the robot arm itself, often on the wrist of the robot, near the end effector. Eye-to-hand sensors are positioned alongside the robot, such that it can view the whole robot and its work space at once. Each type has a different use case: an eye-in-hand sensor has a very precise but restricted view of the workpiece and tool, whereas an eye-to-hand camera can better monitor complete motions of the entire robot but with a generally lower accuracy.

Often these two types of sensors are used in tandem, in which case synchronising the two becomes an important factor. One way of achieving this is through using a global coordinate system, and using known reference points to calibrate the multiple sensors [20]. Another approach uses servoing, a technique that iteratively adjusts the robot's position until it reaches the desired point. In terms of eye-in-hand and eye-to-hand sensors, the servoing algorithm incorporates feedback from all sensors to improve the robot position [21].

2.2 Tilt Sensing for Robotics

The focus of this work is on applications that require tilt sensing, most often for ensuring perpendicularity with the work surface. In terms of robotics, this generally means articulated robot arms, for tasks including coating application [22], instrument alignment for non-destructive testing [23], and drilling or riveting [24, 25].

Robotic drilling is a particularly common application, and is especially utilised in the aerospace industry. A misorientation of riveting holes along an aeroplane wing

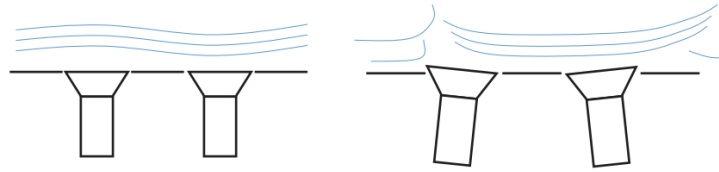


Figure 2.4: Effect of non-perpendicular rivets on air flow, from Santos et al. [28].

can have major impacts on strength and safety; a study by Boeing found that a misalignment of 2.0° between the bolt axis and the applied load will shorten the fatigue life of the bolt by 47%, and a misalignment of 5.0° will shorten it by 95% [26]. Furthermore, 80% of fatigue cracks in aeroplane bodies originate from riveting holes [27].

Aerodynamics are also affected by non-perpendicularity. Figure 2.4 demonstrates the difference in airflow for perpendicularly aligned rivets and those with angular errors [28]. Considering the thousands of rivets on an aircraft fuselage, the combined effect of many misalignments can lead to a noticeable increase in drag and fuel consumption. As such, the commonly quoted standard for accuracy in aerospace is 0.1mm for position, and 0.5° for orientation [28].

Some of the measurable faults resulting from non-perpendicularity after drilling are position misalignment, elliptical holes, and burring [29, 30]. Elliptical holes are an obvious consequence of the drill not being perpendicular to the surface. A slightly less obvious consequence are position errors; if the position on the top surface is correct but the orientation is not, the position of the hole on the bottom surface will be incorrect. These are effectively Abbe errors, which are used to describe the magnification of angular errors with increasing distance. The final example given, burring, is the formation of an uneven ridge around the edge of drilled holes. Poor orientation during drilling can lead to larger burrs, which can in turn affect factors such as assembly or part fatigue.

2.2.1 Plane Measurements with Distance Meters

The most common approach for orientation monitoring in robotic drilling is an arrangement of distance meters. Laser distance meters are a relatively inexpensive option for distance monitoring, and by combining multiple sensors they can be used to measure the angle of a surface from a robotic arm. Figure 2.5 shows a photo of

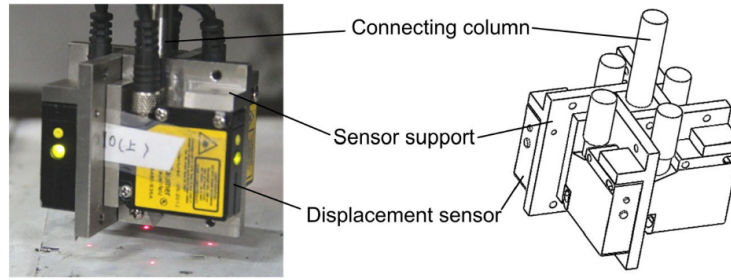


Figure 2.5: Tilt sensor using an arrangement of four distance meters (labelled displacement sensors), used by Gao et al. [11].

a setup using four sensors in a rhombus layout [11].

This technique assumes the surface is flat, using the distance measurements acquired from the sensors to construct a plane through each point, and thus determining the direction of the surface normal relative to the end effector. Even for a curved surface, if the sensors are close together this assumption gives a reasonable approximation of the surface normal. One area of research within this technique is testing specific arrangements of (generally three or four) distance meters. The rhombus arrangement shown in Figure 2.5 had an accuracy of $\pm 0.15^\circ$ when measuring the surface normal of a convex workpiece, and $\pm 0.30^\circ$ for a concave workpiece [11]. The same group later developed an iterative method that could achieve an accuracy of less than 0.3° , taking two iterations for planar workpieces and three for curves surfaces [31].

Tian et al. [24] used a similar rectangular arrangement, focusing on the auto-normalization algorithm developed for correcting the robot's orientation. A series of test movements were made with the robot, and the orientation was measured for the initial position and then again for the corrected position after applying the auto-normalization algorithm. The average deviation before correction was 5.22° , and the average after was 0.32° with a maximum of 0.47° .

Yu et al. [32] used another four sensor setup, this time measuring the surface height as well as the surface normal. Their results showed an accuracy in angle measurements of less than 0.5° , and in the surface height an accuracy of $\pm 0.04\text{mm}$.

Yet another four sensor setup, this time in a square arrangement, was used by Chen et al. [25], with a focus on testing two calibration methods. Over 20 drilling points, the mean angle error was 0.54° before calibration, 0.22° with the first calibration

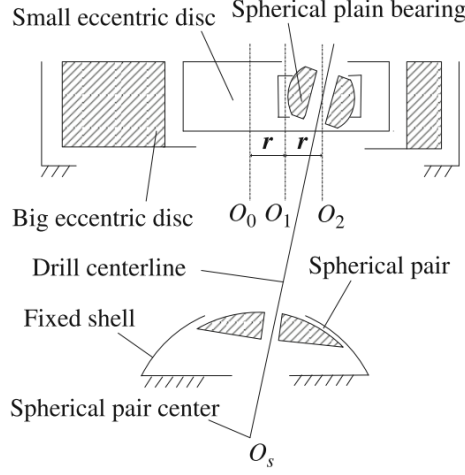


Figure 2.6: Diagram of a drill re-orientation system using two eccentric discs and a spherical bearing to adjust the upper position of the drill bit while keeping the tip on the surface. O_0 , O_1 , and O_2 give the centre of rotation of the big eccentric disc, small eccentric disc, and spherical bearing, while O_s is the centre of rotation of the drill bit. Image taken from Yuan et al. [33] but initially covered in [34].

method, and 0.10° with the second calibration method.

Another aspect of this problem is correcting the angle of the drill bit. One approach is the use of two double-eccentric discs, along with a spherical bearing, in the arrangement shown in Figure 2.6 [33]. The two discs can be rotated to reposition the upper section of the drill bit, while the spherical bearing ensures the point remains in contact with the surface. Yuan et al. [34] initially described the technique, showing it could correct angles of up to 5.0° in magnitude with an accuracy of $\pm 0.1^\circ$, and using an arrangement of distance meters for the angle measurements. Shi et al. [30] further demonstrated the effect of the improved perpendicularity for drilling, the results of which were increased hole circularity and decreased burr height, drilling heat, and tool wear.

Tilt measurement based on distance meters was combined into a six dimensional (6D) pose measurement system for robotic drilling by Frommknecht et al. [14]. Three laser distance meters with a range of 100mm and resolution of $20\mu\text{m}$ were used to measure the initial tilt of the surface, then four non-optical distance sensors with a smaller range (3mm) but higher resolution ($10\mu\text{m}$) were used to measure the tilt once the robot was in place. The system also used a 2D vision camera to monitor position. The average position error using the system was 0.334mm, with

a minimum error of 0.055mm and a maximum of 0.724mm. For orientation, the average angular error was 0.29° , with a minimum of 0.04° and maximum of 0.63° .

A similar multi-sensor system was used to track the 6D position of the end effector of a cable-suspended delta robot by Korayem et al. [35]. Unlike most delta robots with rigid linkages, this cable-suspended version cannot rely on the positioning information relayed by the robot's encoders. The proposed tracking method uses three laser distance meters mounted on the underside of the end effector to measure its pitch, yaw, and out-of-plane position. Roll, pitch, and yaw are the standard descriptors for three dimensional rotations; in this context the pitch and yaw are the tilt of the end effector. The top of the end effector has three markers, and a camera is mounted above this on the robot base. Image processing software is used to track the markers on the end effector, giving its 2D in-plane position and its roll. The system was tested by tracking the robot through an inclined circle, and with a number of point-to-point movements. In the inclined circle test, the errors of the proposed system were 10%, compared to 30% for the robot encoders. For the point-to-point tests, the errors were 2% for the proposed method, and 10% for the encoders.

Curved Surface Measurements

A slightly more complex version of this technique can be used to measure the surface normal of curved surfaces. The basis of this technique is the same as described above, using distances taken from point measurements. However, instead of fitting a plane to the measured positions, these techniques fit one or more curves through multiple points.

Chen et al. [29], the same group that developed the double-eccentric disc adjustment method, used four distance meters to measure the surface normal while the drill bit was in contact with the surface. This gave five known points on the surface, including the drill point, in a cross shape. Each set of three points along the line of the cross are used to construct a circle, and calculate the tangent vector through the drill bit. With two tangent vectors from the two sets of points, their cross product gives the surface normal. The paper did not directly calculate the improvement in orientation using this method, but the drilled holes were shown to have less burring after applying a correction based on the measurement.

Another version of the technique developed by Lee and Shiou [36] uses a camera

to capture five laser spots projected on the surface of interest, then applies image processing to determine the 3D position of the points. As in the above technique, the spots form a cross shape, or two intersecting curves in 3D space. The particular curve fitting used in this technique is a quadratic Bezier curve, and the output is the same as above, with tangents to the two curves being calculated and the cross product applied to find the surface normal. The accuracy of this method was found to be 1.8° , though with a wide range of 40.0° . The height between the surface and the sensor was also measured, and this aspect of the sensor had an accuracy of $30\mu\text{m}$ and range of 2.4mm .

A slightly more complex version of this technique involves using 2D laser scanners. Zhang et al. [37] mounted two of these scanners perpendicularly around the drill bit, together projecting a cross shape onto the surface (Figure 2.7a). Each scanner makes 21 distance measurements along the projected line. The demonstration of the technique in the paper used an ellipsoid surface, and so the measured points are used to construct the tangent to the ellipsoid along that line, as shown in Figure 2.7b. Then, the surface normal is calculated as the line orthogonal to the two tangents. The measurement error of the technique was found to be less than 0.2° , taking less than 0.5s to make the necessary calculation of surface normal. The technique also measured the distance to the surface, finding a measurement error of less than $\pm 0.02\text{mm}$.

This final measurement technique provides more accurate measurements of surface normal for curved surfaces, but its disadvantage is that 2D laser scanners are larger than single point distance meters and cannot be mounted directly on the robot arm (as demonstrated in Figure 2.7a). In each of the methods using distance meters, the sensors are directly mounted on the end effector or on the wrist of the robot, functioning as eye-in-hand sensors. However, the combination of multiple sensors does result in a somewhat bulky sensor, especially as the sensors are required to surround the end effector in order to take a representative measurement of the surface.

The accuracies of all the techniques covered in this section are summarised in Table 2.1. The methods are split by whether they use a plane approximation to measure the surface normal, or a curved surface approximation. The highest accuracies were achieved by Yuan et al. [34] and the second calibration method proposed by

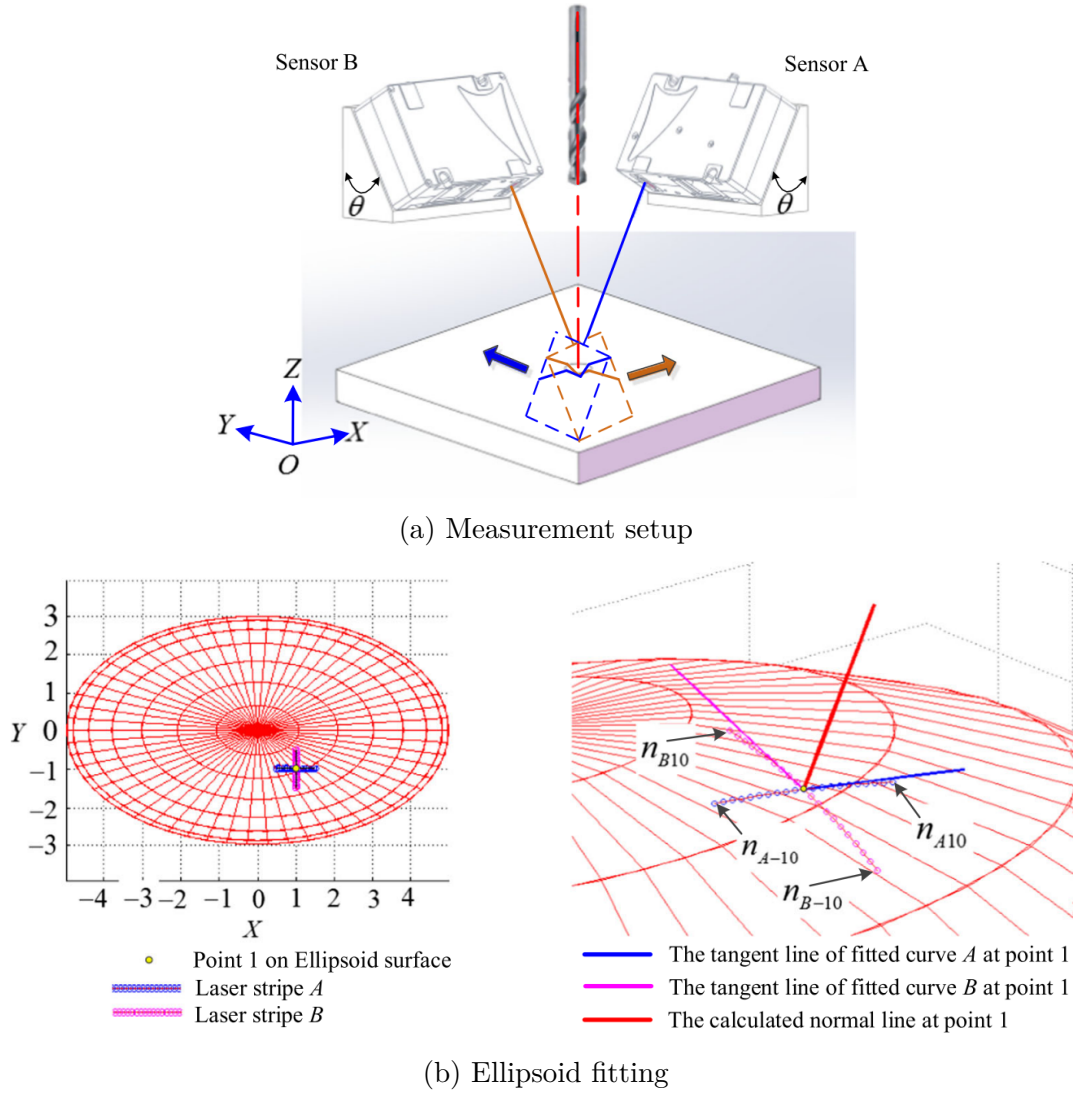


Figure 2.7: Tilt sensor employing two 2D laser scanners used by Zhang et al. [37], mounted alongside rather than on the drilling robot as shown in (a). The points acquired from the scanners (n_A from the first scanner, n_B from the second) are used to fit to an ellipsoid, finding the surface normal of the shape at the point where the scanners cross, as shown in (b).

Table 2.1: Summary of tilt sensing techniques using distance meters, including those using a plane approximation and a curved surface approximation.

Method	Accuracy (deg)
Planar surface	
Gao et al. [11]	0.15 convex, 0.30 concave
Gao et al. [31]	0.30
Tian et al. [24]	0.32
Yu et al. [32]	0.50
Chen et al. [25]	0.22 method 1, 0.10 method 2
Yuan et al. [34]	0.10
Frommknecht et al. [14]	0.29
Korayem et al. [35]	2% errors
Curved surface	
Lee and Shiou [36]	1.80
Zhang et al. [37]	0.20

Chen et al. [25], both reaching 0.1° . The main differences between the techniques were the specific arrangement of the distance meters, with the shape affecting the mathematics of the plane or curve calculation, and the proximity to the drill bit affecting the validity of the approximation.

2.2.2 3D Point Cloud Modelling

A more intensive technique for calculating surface normals is full surface mapping. There are two aspects to this approach: firstly scanning the surface to create a 3D point cloud of data, and then secondly interpolating the data to find the surface location and normal at any point. This method is often used to compare any deviations in a manufactured component from its computer aided (or assisted) design (CAD) model. As an example, turbine blades can be manufactured using an additive manufacturing technique called selective laser melting (SLM). The resulting surface is quite rough, which can lead to airflow problems inside the blade. A point cloud measurement of the surface can be used along with the CAD model to plan a trajectory for a robotic milling arm, correcting the blade shape by milling away the extra material [38].

Chen et al. [40] provides an overview of optical methods used to measure the 3D shape of an object. One method is laser scanning, using 1D, 2D, or 3D laser scanners such as the ones described above that measure the distance to a surface at one or more points. An example of a 2D laser scanner mapping a surface is shown in Figure 2.8. The use of structured light, sometimes referred to as active triangulation, can also be used. A pattern, normally fringes, is projected onto the surface and then images of the deformed pattern are used to derive the surface structure. Another method is the use of stereo vision techniques, using two offset cameras and image

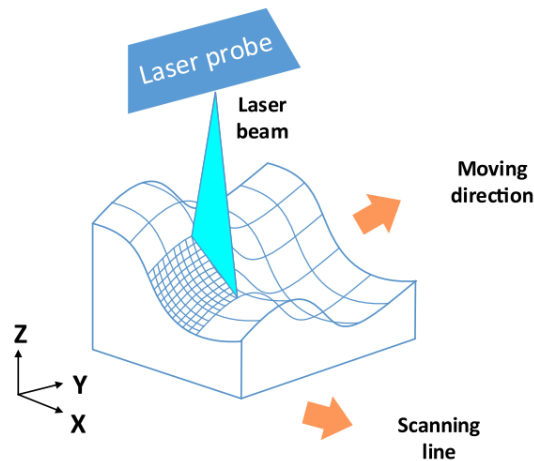


Figure 2.8: Diagrammatic demonstration of a 2D laser scanner being used to create a 3D point cloud by moving across a surface, from Song et al. [39].

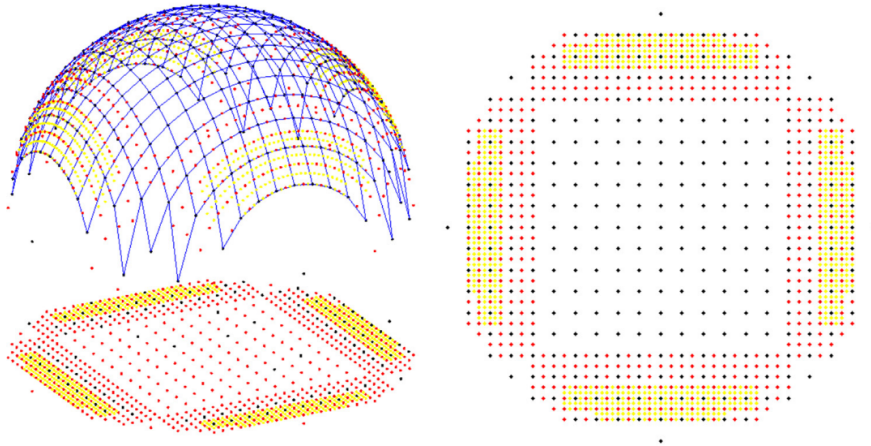


Figure 2.9: Demonstration of the iterating point cloud density algorithm used by Song et al. [39] to calculate surface normals from a spherical shape. The dark blue points show the sparsest cloud used in the initial fitting, and the red and then yellow demonstrate the increasing density used in subsequent normal calculations.

processing to reconstruct the surface. The robotic milling application mentioned above uses a stereo vision system [38].

A fringe projection based method was used by Rao et al. [41], implemented as an eye-in-hand sensor mounted on a drilling robot. A 3D point cloud was acquired at each drilling position, and then compared to the measured orientation of the drilled holes. The average difference in angle between the point cloud estimations and the measured holes was 0.20° . A previous method from the same group used dual-frequency phase-shifting patterns, again measuring surface normals for robotic drilling [42]. The average angle errors for this technique were 0.51° for convex work surfaces, and 0.52° for concave surfaces.

Much of the research in this area focuses on the processing of the point cloud data to reconstruct the full surface. This is a complex field of research, with many variables such as point density, neighbourhood size [43], processing time, noise levels [44], and processing algorithm. Song et al. [39] looked at six similar algorithms for surface normal determination, using a dynamic sampling method. The process was iterative, starting with a relatively sparse point cloud of the model, and increasing the density until the target accuracy was achieved. A demonstration of this is shown in Figure 2.9, with the dark blue points showing the initial point cloud, then the red showing the points added in the next step, and the yellow an additional step. The

models investigated were chosen with aeronautical drilling in mind, using a sphere to approximate a cockpit, a cylinder for the fuselage, and an ellipsoid for the wing.

The six algorithms tested by Song et al. [39] were: mean weighted equally (MWE), mean weighted by angle (MWA), mean weighted by sine and edge length reciprocals (MWSELR), mean weighted by areas of adjacent triangles (MWAAT), mean weighted by edge length reciprocals (MWELR), and mean weighted by square root of edge length reciprocals (MWRELR). They were each used to estimate the surface normals of the three models, iterating until the average angular errors across the model were below a certain threshold. The thresholds tested were 5.0° , 2.0° , and 0.5° ; the results for the first three algorithms (MWE, MWA, MWSELR) for the 0.5° threshold are given in Table 2.2. The results for MWAAT, MWELR, and MWRELR are very similar to the results for MWA and MWSELR. The first algorithm, MWE, is the clear outlier, taking 17s to reach the error threshold for the sphere and 264s for the ellipsoid. The other five algorithms all took less than 0.3s for the sphere and ellipsoid, and less than 2.8s for the cylinder. These five also only required up to one iteration of increasing the point cloud density; the MWE algorithm required up to five iterations. These results give an idea of the achievable accuracy of 3D point cloud modelling, and some of the factors involved in choosing a normal determination algorithm.

The field of surface normal estimation from point cloud data has a much broader application than just robotics. Lu et al. [45] tested five deep learning algorithms on a wide variety of point cloud models, including “CAD-like” models such as the one shown in Figure 2.10a, and smooth models such as the Stanford bunny (a standard test model) given in Figure 2.10b. The five algorithms used here were: principal component analysis (PCA), bilateral normal filtering (BF), Hough method, NestiNet, plus their proposed method. The errors in the calculated normals (averaged across the model) for each algorithm are given in Table 2.2 for both the CAD part and the bunny. Overall, the normal estimation is more accurate for the smoother surface of the bunny. For the CAD part, the sharp changes in surface normal at the hard edges present a problem for neighbourhood estimation, which is one of the main areas of optimization for surface normal determination algorithms. This selection of results gives an idea of the difference in performance on different types of model, as well as the variation between algorithms.

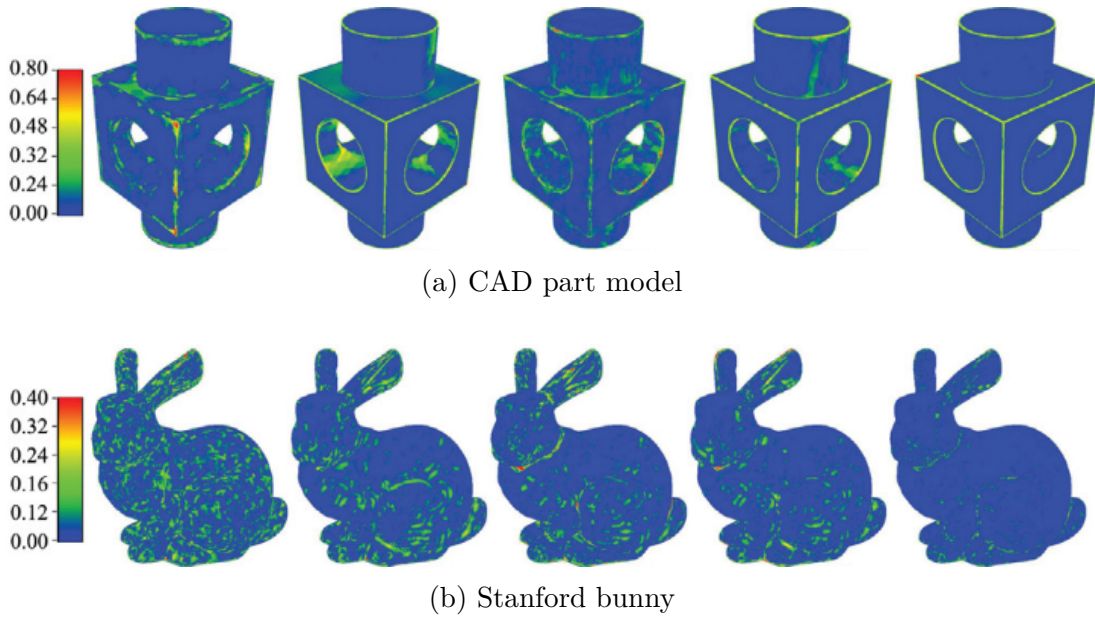


Figure 2.10: The results of five surface normal determination algorithms for a CAD-like part and the Stanford bunny, from Lu et al. [45]. The colourmap shows the error in the surface normal determination in degrees at each point on the surface. The five algorithms are: PCA, BF, Hough method, Nesti-Net, and the method proposed by the paper.

Table 2.2 summarises the data presented here regarding surface normal determination using 3D point cloud measurements. Much of the data focuses on the testing of different algorithms, used with constructed point cloud models to evaluate their accuracy. The results presented by Song et al. [39] focus on models that approximate surfaces in aerospace that require drilling and riveting operations, such as an ellipsoid for an aeroplane wing. The results from Rao et al. [42, 41] are particularly notable, as they presented a full system of 3D point cloud acquisition, surface normal determination, and experimental results from robotic drilling.

Table 2.2: Summary of tilt sensing techniques employing 3D point clouds. The first two results are from experimental tests of robotic drilling, the others are simulation testing of various surface normal estimation algorithms using point cloud models.

Method	Accuracy (deg)
Rao et al. [42]	0.51 convex, 0.52 concave
Rao et al. [41]	0.20
Algorithm testing	
MWE, Song et al. [39]	0.5 in 17.1s sphere, 0.5 in 2.4s cylinder, 0.5 in 263.6s ellipsoid
MWA, Song et al. [39]	0.5 in 0.2s sphere, 0.5 in 2.6s cylinder, 0.5 in 0.2s ellipsoid
MWSELR, Song et al. [39]	0.5 in 0.3s sphere, 0.5 in 2.6s cylinder, 0.5 in 0.2s ellipsoid
PCA, Lu et al. [45]	5.56 CAD part, 2.92 bunny
BF, Lu et al. [45]	1.26 CAD part, 0.34 bunny
Hough, Lu et al. [45]	2.46 CAD part, 0.52 bunny
Nesti-Net, Lu et al. [45]	0.86 CAD part, 0.23 bunny
Proposed, Lu et al. [45]	0.63 CAD part, 0.11 bunny

2.2.3 MEMS Inclinometers

Microelectromechanical systems (MEMS) are another common method for measuring surface tilt. Broadly, MEMS are microscopic devices comprising of electrical and mechanical components. The components are between 1-100 μm , and a complete device is generally around 20 μm -1mm. MEMS inclinometers use an accelerometer to measure tilt angle with respect to gravity [46].

A single MEMS inclinometer measures the angle magnitude from the reference point of the gravity vector, with no direction information. Łuczak [47] described a number of mathematical approaches that can be used to reframe this measurement as a value of roll. In terms of an accelerometer, roll and pitch are the perpendicular angles from the gravity vector, and yaw is the rotation around the vector. The three equations proposed by Łuczak [47] give a value of the roll, regardless of the amount of pitch, with varying levels of accuracy. The first method had a maximum error of 2.00° , the second an error of 0.27° , and the third an error of 0.18° . Another approach from Łuczak [48] used a triaxial MEMS accelerometer to measure pitch and roll, again with a focus on the mathematical underpinnings of the system. This technique could measure pitch and roll around the full angular range with an accuracy of 0.20° .

Qian et al. [49] proposed a system that uses three orthogonal MEMS accelerometers to measure the tilt of a single axis with greater accuracy than an individual MEMS sensor. The proposed application of the sensor is borehole drilling for oil and gas exploration. The axis of the sensor, therefore, would be aligned with the drill direction, and so the drill's angle with respect to gravity can be obtained. A model of the instrument is shown in Figure 2.11, where the axis of measurement is given by PQ . A calibration method was also applied to account for any non-orthogonality in the setup. Experimental results gave an error of 0.10° over a measurement range of 0° - 120° .

An extension of this technique from the same group used three fluxgate magnetometers [51] alongside the inclinometers, used to measure the angle of the sensor with respect to the Earth's magnetic field [50]. The measurement principle is the same as for the MEMS sensors, using three orthogonal fluxgate sensors to measure the angle of the instrument axis. This system effectively gives a measurement of the combined pitch and roll (from the accelerometers), and the yaw (from the magnetometers). Figure 2.12 demonstrates these two angles, defining the first as tilt

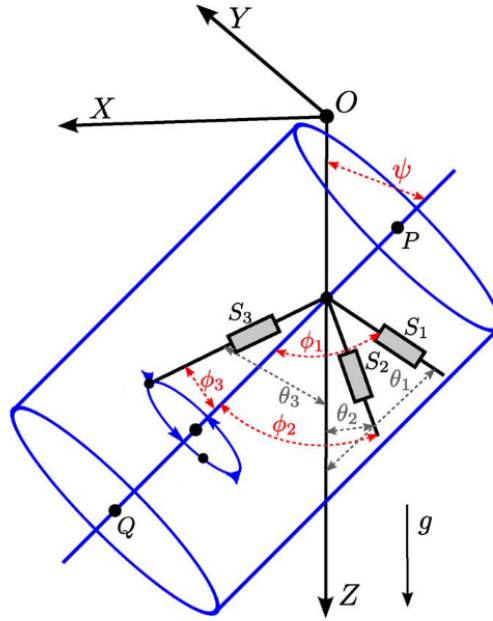


Figure 2.11: Diagram of a tilt sensor using three orthogonally arranged MEMS sensors, labelled S_1 , S_2 , and S_3 , from Qian et al. [49]. The MEMS sensors are mounted at angles θ and ϕ from PQ , the centre line of the complete sensor. The measured angle, ψ , is that made between PQ and the gravitational vector, given by g .

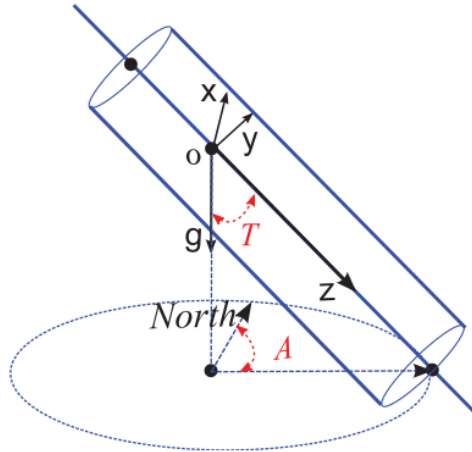


Figure 2.12: Diagram showing the definition of tilt angle, T , as given by the inclination from the gravitational vector g , and the azimuth angle, A , describing the orientation angle from North. The sensor proposed by Yang et al. [50] uses MEMS accelerometers to measure the tilt angle, and magnetometers to measure the azimuth.

(T) and the second as azimuth angle (A). Two different calibration methods were tested for the sensor, each giving similar results for determination of the tilt (the accelerometer-based sensors). However for the azimuth angle, measured using the magnetometers, method 2 performed slightly better. Method 1 had an average error in the tilt of 0.08° , with a maximum of 0.11° , and an average error of 0.47° in the azimuth, with a maximum of 0.52° . For method 2, the average tilt error was 0.08° with a maximum of 0.09° , and for the azimuth the average error was 0.36° with a maximum of 0.39° .

One of the disadvantages of MEMS sensors is a tendency to drift. Łuczak et al. [52] investigated the stability of a commercial MEMS accelerometer over 48 hours, finding that the drift in acceleration measurements corresponded to an angular change of around 0.43° . Furthermore, a study of the same accelerometer over a 4.5 year period found that ageing effects can result in an angular error of around 0.86° [52, 53].

Another consideration for the use of MEMS sensors in applications like robotic drilling is that they measure angle in relation to a global coordinate system, as defined by gravity. For drilling along, say, an aeroplane wing, they provide no information about the angle orientation as it relates to the wing surface, unless the surface angle is also measured. The exception to this is the application of oil drilling given by Yang et al. [50]. The referencing system in this case is well-suited to the application, as in most cases the drill should be aligned with the direction of gravity.

The various methods incorporating MEMS sensors are summarised in Table 2.3. Łuczak [47] demonstrated that the measurements obtained from MEMS devices can depend heavily on the mathematics used to derive a useful tilt value, with accuracies ranging from 2.00° to 0.18° . The highest accuracy method was presented by Yang et al. [50], reaching 0.08° for measurements of combined pitch and roll, while also measuring yaw using a fluxgate-based sensor to an accuracy of 0.36° .

Table 2.3: Summary of tilt sensing techniques using MEMS inclinometers. This includes the tilt direction measurement from Yang et al. [50] that uses fluxgate magnetometers.

Method	Accuracy (deg)
Luczak [47]	2.00 method 1, 0.27 method 2, 0.18 method 3
Luczak [48]	0.20
Qian et al. [49]	0.10
Accelerometer (pitch/roll), Yang et al. [50]	0.08 method 1, 0.08 method 2
Magnetometer (yaw), Yang et al. [50]	0.47 method 1, 0.36 method 2

2.2.4 Autocollimators

Another well-established method for measuring surface angle is the autocollimator. These non-contact optical sensors measure two dimensional angles to a very high precision, up to fractions of arc seconds, but with an accordingly low range. For example, TriOptics' universal electronic autocollimator has a number of options for range and accuracy, with the highest accuracy model reaching 0.2arcsec with a 610x490arcsec range, while the largest range model has a range of 6170x4940arcsec and an accuracy of 2.5arcsec [54]. Their higher precision models can reach 0.05arcsec, while the wide range model has a 3°x3° (or 10800x10800arcsec) field of view.

Despite being a widely used angle measurement technique, autocollimators are not commonly applied in robotics applications. Referencing some of the above techniques using distance meters [24, 34], the angle offset needing to be corrected in robotic drilling could be up to 5.0°, beyond the range of the commercial autocollimators quoted here. The high precision of autocollimators is also unnecessary in robotic drilling; the commonly referenced threshold for hole drilling accuracy in aerospace is 0.5° [28]. Another aspect likely factoring into the unpopularity of autocollimators in manufacturing is that they require a smooth, highly reflective surface, often a specific reflective target. The use of targets would be impractical for a repetitive application such as drilling, and the surface alone is likely not smooth enough

to provide an adequate reflection.

Autocollimators are employed in manufacturing for quality control, however. A popular application is measuring the straightness of precision stages and guideways [55, 56]. A component with a reflector is moved along the component to be tested and the autocollimator is used to measure any angular deviation. Ideally, a perfectly straight component should show no angular deviation, but small deviations will be picked up by the precise autocollimator measurements. For example, Saito et al. [56] used an autocollimator-based system to reduce deviations in a linear stage to less than 0.5% over a 500mm range. The angular resolution of the sensor was under 0.01arcsec. A slightly different application was described by Li et al. [57], using an autocollimator coupled with a laser distance meter to monitor manufacturing and assembly. The combined system has a position accuracy of 1.1mm and angular accuracy of 0.025°.

2.2.5 Summary of Tilt Sensing in Robotics

The two most widely used techniques for angle sensing in robotics are tilt sensing using distance meters, and 3D point cloud modelling. The first is a relatively low-tech solution that can monitor tilt to an accuracy of up to 0.1° [34] using a simple arrangement of laser distance meters. One of the challenges with this technique is surface curvature, with convexity and concavity affecting measurement accuracy [11], and new methods being developed specifically for curved surfaces [36, 37]. Calibration is another factor, with Chen et al. [25] demonstrating the improvement in accuracy from 0.54° to 0.22° with one calibration method, and 0.10° for another. The accuracies of all the techniques using distance meters are summarised in Table 2.1.

For 3D point cloud measurements, the main research focus is in improving surface normal determination algorithms used to calculate the surface normal at any point on the model, interpolating between the measured cloud. Rao et al. [41] demonstrated a complete measurement method using structured light and an eye-in-hand sensor on a robotic drill to capture point clouds during drilling. The calculated surface normals were compared with the orientation of the drilled holes, finding an accuracy of 0.20°. This section also looked at research into the accuracy of various surface normal estimation algorithms. Song et al. [39] demonstrated a number of

algorithms that could meet an accuracy of 0.5° in 2.6s or less for spherical, cylindrical, and ellipsoidal models. The models were specifically chosen with the aerospace industry in mind. These results, along with a selection from Lu et al. [45] for broader applications of surface normal estimation, are given in Table 2.2.

For further background, the widely used MEMS sensors and autocollimators were also discussed, although they are not common in robotic applications as yet. MEMS-based techniques can reach accuracies of 0.08° [50], but a fundamental aspect of the technique is that angles are referenced with the gravitational frame, rather than the frame of the workpiece. Autocollimators can reach precisions significantly higher than any of the other methods, but are limited by their comparatively low measurement range. For robotic drilling, the literature covered here suggests that angle misalignments of up to 5.0° should be able to be measured and corrected [24, 34], which is outside the widest measurement range of autocollimators (3.0° [54]).

Overall, there remain significant disadvantages to the most commonly used tilt sensing techniques in robotics, despite them meeting the 0.5° accuracy requirement of aerospace. Distance meters are bulky devices, and multiple are needed for a complete sensor. In addition, the curvature of the surface can have significant effects on the measurement accuracy, as the surface normal is calculated from only a small number of discrete measurement points (3 to 4, or up to 21 depending on the technique). 3D modelling techniques use many more measurement points to determine surface normal, but have a corresponding increase in calculation time and computing power. An ideal tilt sensing technique would be able to measure directly at the point of interest, giving a real time measurement with minimal computation time, while still meeting the 0.5° accuracy standard stipulated in aerospace.

To conclude this chapter, the four different categories of tilt sensing techniques and their advantages and disadvantages are summarised in Table 2.4.

Table 2.4: Summary of types of tilt sensing techniques for robotic drilling and their advantages and disadvantages.

Technique	Advantages	Disadvantages
Arrangement of distance meters	Up to 0.1° accuracy [34]; simple, off-the-shelf sensor components; on-line measurements; can be improved with calibration methods and processing	Bulky sensor head; measurement accuracy depends on suitability of plane estimation; more complex curved surface estimations require more measurement points
Surface normal estimation from 3D point cloud	Up to 0.2° accuracy [41]; can determine surface normal at any point; can be used as an on-line sensor [41]	Accuracy depends heavily on surface normal estimation algorithm; algorithms can take up to 2.6s second to calculate normal measurements [39]; point cloud scanning requires more complex vision sensors (eg. structured light, scanning displacement sensor)
MEMS sensors	Up to 0.08° accuracy [50]; inexpensive and very small sensor components; wide measurement range (0° - 120° [49])	Measures angle with respect to gravitational vector only; accuracy depends on mathematics used to extract tilt [47]; MEMS devices have a tendency to drift [52]
Autocollimators	Very high accuracy, up to 0.05arcsec [54]; can be bought off-the-shelf	Small measurement range, maximum of $3^\circ \times 3^\circ$ [54]; require highly reflective surfaces or targets; expensive

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Chapter 3

Laser Speckle and its Applications in Detecting Surface Deformations

This chapter will review the basics of laser speckle, including how it forms and what it looks like, as well as the various factors that affect how a speckle pattern appears. Then, some uses of speckle in sensing will be explored briefly, ending with a more detailed look at tilt sensing using laser speckle.

Section 3.1 covers the introduction to speckle, including an in-depth look at the statistics of how speckle patterns are produced. The section moves on to the differences between objective and subjective speckle, and how speckle patterns are affected by illumination wavelength. The effect of surface properties on speckle is also explored, including how the changing the angle of a surface changes the speckle pattern produced.

Following this, Section 3.2 investigates using speckle to measure surface deformations. The focus of the section is in-plane translations and out-of-plane tilts, both of which have similar effects on speckle patterns and can be measured using various techniques. A number of methods for separating the measurements of tilt and translation are also discussed.

3.1 Introduction to Speckle

Speckle is an interference pattern produced by coherent light, such as that from a laser, incident on an optically rough surface [1, 2]. An “optically rough surface” refers to the roughness on the scale of the wavelength of the illuminating light. This includes a great many surfaces such as paper, many plastics, concrete, ground glass, unpolished metal, and many painted surfaces. The roughness of the surface causes the light to scatter randomly, interfering to produce the irregular patterns of speckle.

A speckle pattern is a 3D field emanating from the surface. The speckles themselves are considered as long, cigar-shaped Gaussian intensity distributions [1, 3], i.e. the result of constructive interference. The low intensity areas where there are no speckles result from destructive interference. The length of the cigar shapes is along the direction from the illuminated spot on the surface.

In 2D, such as when viewing or photographing speckles, the pattern becomes an irregular collection of different sized circular Gaussian distributions. This is effectively a 2D slice through the three dimensional speckle field, slicing through the cigar shapes at different points along their length. An example of laser speckle is shown in the image in Figure 3.1.

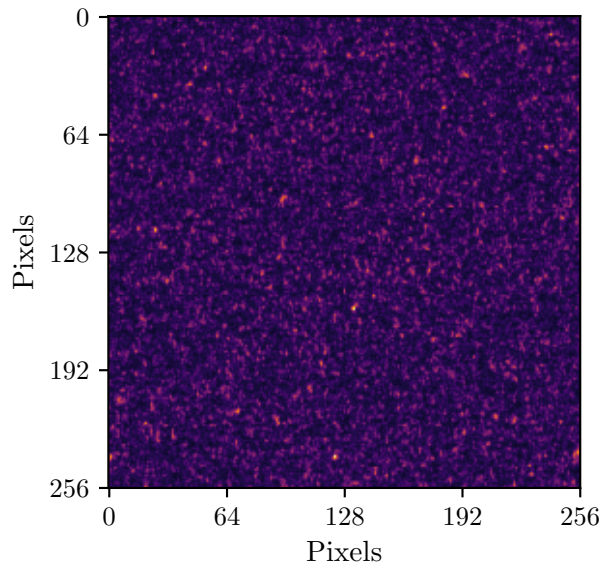


Figure 3.1: Example speckle pattern, taken using a camera with no lens (objective pattern).

3.1.1 Statistics of Speckle

Speckle can be considered statistically as the result of the additive effect of the many individual complex components, which together make up the light reflected from a rough surface [4]. Each component has some random phase and amplitude, and can be described as a phasor with the length representing the amplitude and its direction the phase. The n th phasor, f_n , is described by the following equation, where the amplitude is given by α_n and the phase by φ_n .

$$f_n = \alpha_n \exp(i\varphi_n) \quad (3.1.1)$$

A speckle pattern formed at a surface is the result of the sum of these random phasor components at each point on the surface. This concept is also called a random walk. Figure 3.2, taken from Goodman [4], shows two examples of random walks, where the individual phasors are depicted with the thinner arrows, and the thick arrow shows the overall sum. Figure 3.2a demonstrates a scenario in which the sum, i.e. the interference, is largely constructive, resulting in a large magnitude (intensity) vector overall. Figure 3.2b shows destructive interference, where the total vector sum has a much smaller magnitude than the first figure.

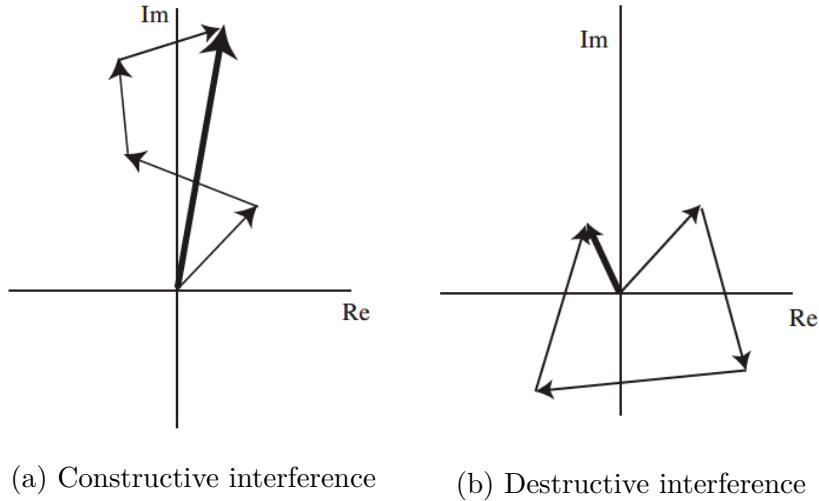


Figure 3.2: Diagram of two example random walks, i.e. sums of complex phasors. The component phasors are shown by the thinner arrows, with the thicker arrows giving the resultant phasor sum. The sum describes the interference of the phasors, with (a) being mostly constructive, and (b) destructive.

Mathematically, this random phasor sum is given by:

$$F = \frac{1}{\sqrt{N}} \sum_{n=1}^N \alpha_n \exp(i\varphi_n) = \alpha \exp(i\varphi) \quad (3.1.2)$$

where α is the amplitude of the resultant sum, i.e. the magnitude of the phasor, and φ its phase. Additionally, three statistical assumptions are made when considering this sum:

- For any n , α_n and φ_n are statistically independent of each other.
- For any $n \neq m$, α_n and φ_n are statistically independent of α_m and φ_m .
- Over all N , the phases φ_n are evenly distributed between $\pm\pi$.

These assumptions are considered to hold true when N is very large, and as $N \rightarrow \infty$ the sum in equation (3.1.2) tends to an asymptotic Gaussian. Goodman [4] (Chapter 2: Random Phasor Sums) considers the probability density functions (PDFs) of both α and φ under this condition. Their joint probability density function, $p_{\alpha,\varphi}$, is given by:

$$p_{\alpha,\varphi}(\alpha, \varphi) = \frac{\alpha}{2\pi\sigma^2} \exp\left(-\frac{\alpha^2}{2\sigma^2}\right) \quad (3.1.3)$$

where σ is the standard deviation of the distribution. This applies when $\alpha \geq 0$ and $-\pi \leq \varphi < \pi$, with $p_{\alpha,\varphi} = 0$ otherwise.

Now considering the individual variables, the PDF for α is:

$$p_{\alpha}(\alpha) = \int_{-\pi}^{\pi} p_{\alpha,\varphi}(\alpha, \varphi) d\varphi = \frac{\alpha}{\sigma^2} \exp\left(-\frac{\alpha^2}{2\sigma^2}\right) \quad (3.1.4)$$

This is equivalent to the Rayleigh density function, whose shape is shown in Figure 3.3. The figure is taken from Goodman [4] with variable names changed for consistency.

For φ , the PDF is equal to:

$$p_{\varphi}(\varphi) = \int_0^{\infty} p_{\alpha,\varphi}(\alpha, \varphi) d\alpha = \frac{1}{2\pi} \quad (3.1.5)$$

This equation demonstrates that the third assumption stated above does hold true for a sufficiently large N . In addition, the first assumption is proven by the factori-

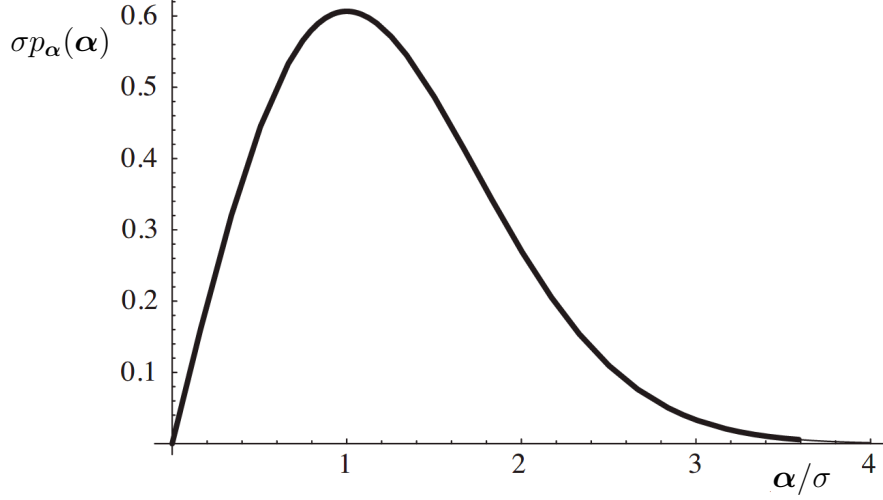


Figure 3.3: Probability density function of α , p_α , equivalent to the Rayleigh distribution function. Image taken from Goodman [4], with labels changed for consistency.

sation of the joint probability distribution $p_{\alpha,\varphi}$ into p_α and p_φ .

When a camera or detector captures a speckle pattern, it records only the time averaged intensity, with no information about the phase and amplitude of the incoming wave. Intensity is equal to the square of the magnitude of the electric field, E , and so taking the Cartesian components:

$$I = |E_x|^2 + |E_y|^2 + |E_z|^2 \quad (3.1.6)$$

For waves travelling along the z axis, the E_z component can be ignored. Then, in terms of the phasor sums described above, the intensity can be considered as:

$$I = \begin{cases} |F_x|^2 + |F_y|^2, & \text{circularly polarised, partially polarised, unpolarised} \\ |F|^2, & \text{linearly polarised} \end{cases} \quad (3.1.7)$$

Note that in the linearly polarised case, F is along the direction of polarisation.

Now, the PDF of I can be calculated from p_α (equation (3.1.4)) using a standard relation between PDFs [4], giving:

$$p_I(I) = \frac{1}{2\sigma^2} \exp\left(-\frac{I}{2\sigma^2}\right) \quad (3.1.8)$$

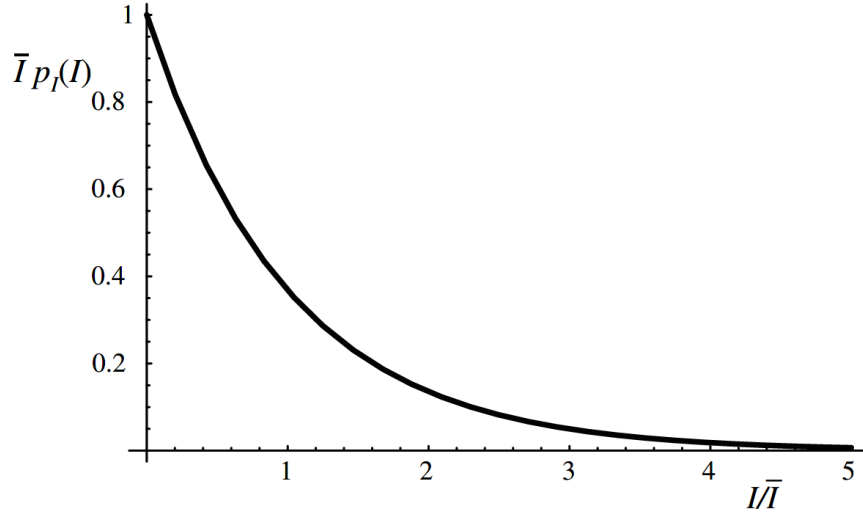


Figure 3.4: Probability density function of I , p_I , showing a negative exponential distribution. Image taken from Goodman [4].

As the mean intensity, $\bar{I}$, is equal to $2\sigma^2$, this can be simplified to:

$$p_I(I) = \frac{1}{\bar{I}} \exp\left(-\frac{I}{\bar{I}}\right) \quad (3.1.9)$$

This negative exponential relation is shown in Figure 3.4, from Goodman [4].

A speckle pattern with the above distribution function is considered to be “fully developed speckle”. For partially developed speckle, the phasors are not uniformly distributed between $\pm\pi$, violating the third assumption made above. In this case, there are fluctuations in the intensity which can be characterised by their standard deviation, denoted by σ_I .

Additionally, speckle contrast is frequently used as a measure of the degree of these fluctuations as compared to the mean intensity, and is given by $\sigma_I/\bar{I}$. Returning to fully developed speckle, the speckle contrast of such a pattern is 1, meaning that the intensity fluctuations are of the same order as the overall intensity.

Sums of Speckle Patterns

Goodman [4] (Chapter 3: First-Order Statistical Properties of Optical Speckle) considers the sum of two independent speckle patterns, where “independent” can be considered the same as “uncorrelated” for fully developed speckle patterns. Speckle

patterns experience gradual decorrelation (i.e. a change in the construction of the speckles) when the illuminated surface shifts, when the wavelength or angle of incidence changes, and when the polarisation of the incident light changes. As such, independent speckle patterns are generated from light incident on non-overlapping parts of a surface, or can arise from significant changes in the incident wavelength, angle of illumination, or polarisation.

Two speckle patterns, of total intensities I_1 and I_2 , recorded at a detector will give rise to the summed intensity I_s :

$$I_s = I_1 + I_2 \quad (3.1.10)$$

If both patterns are fully developed, they will each have a probability density function as follows.

$$p_1(I_1) = \frac{1}{\bar{I}_1} \exp\left(-\frac{I_1}{\bar{I}_1}\right) \quad (3.1.11)$$

$$p_2(I_2) = \frac{1}{\bar{I}_2} \exp\left(-\frac{I_2}{\bar{I}_2}\right) \quad (3.1.12)$$

Through manipulation of the characteristic functions of I_1 , I_2 , and I_T , the PDF of I_T can be found as:

$$p_s(I_s) = \begin{cases} \frac{1}{\bar{I}_1 - \bar{I}_2} \left(\exp\left(-\frac{I_s}{\bar{I}_1}\right) - \exp\left(-\frac{I_s}{\bar{I}_2}\right) \right), & \bar{I}_1 > \bar{I}_2 \\ \frac{I_s}{\bar{I}^2} \exp\left(-\frac{I_s}{\bar{I}}\right), & \bar{I}_1 = \bar{I}_2 = \bar{I} \end{cases} \quad (3.1.13)$$

This PDF is shown in Figure 3.5, for four different values of the ratio $r = \bar{I}_2/\bar{I}_1$. At $r = 0$, the curve is simply the PDF of a single speckle pattern (as the second pattern has an intensity of zero), whereas at $r = 1$, the intensities of the two patterns are equal and the second case of equation (3.1.13) applies.

Additionally, this summing of speckle pattern results in a decrease of the speckle contrast of the sum, $\sigma_s/\bar{I}_s$. This is given mathematically by:

$$\frac{\sigma_s}{\bar{I}_s} = \frac{\sqrt{\bar{I}_1^2 + \bar{I}_2^2}}{\bar{I}_1 + \bar{I}_2} \quad (3.1.14)$$

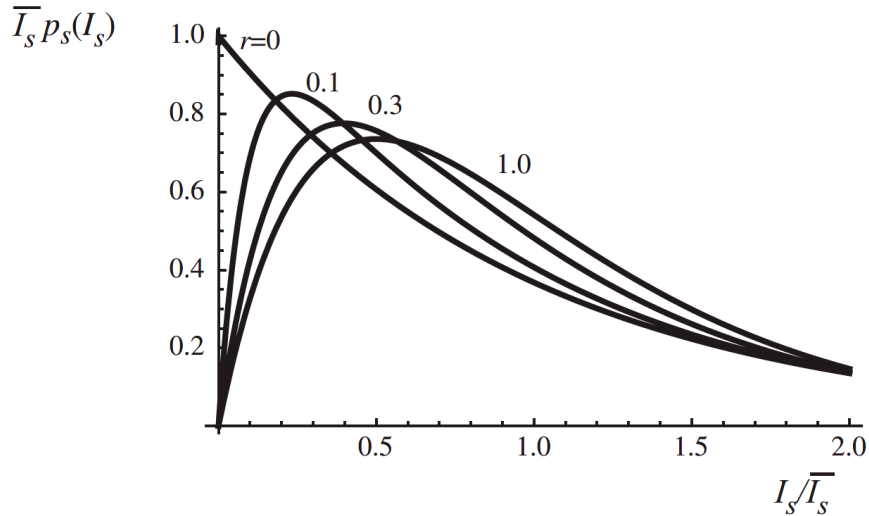


Figure 3.5: Probability density functions of I_s , $p_s(I_s)$, for different values of $r = \bar{I}_2/\bar{I}_1$. Image taken from Goodman [4].

The expression is minimised at $\bar{I}_1 = \bar{I}_2$, where the speckle contrast is $1/\sqrt{2}$. This can be useful for suppressing speckle, in applications where laser speckle is a hindrance to measurements.

When considering sums of correlated, i.e. non-independent, speckle patterns, the approach taken by Goodman [4] is to use a transformation of the speckle fields to convert the two correlated patterns into two uncorrelated ones with an equal summed intensity. Goodman [4] (Chapter 3: First-Order Statistical Properties of Optical Speckle) proves that such a transformation matrix exists, and as such the sum of two correlated speckle patterns can then be treated the same mathematically as the sum of two independent patterns. Though the total summed intensity, I_s , is preserved via the transformation, the individual intensities, I_1 and I_2 , will likely change. However, the distribution of the intensities, i.e. $p_1(I_1)$ and $p_2(I_2)$, will retain the same shape, only being scaled by the change in I_1 or I_2 .

One particular example of correlated speckle patterns are those from orthogonally polarised light. If the detection of a speckle pattern is polarised, the pattern observed when the polariser is oriented in the x direction will be different to the pattern observed for the y direction. As with speckle patterns generated for different wavelengths or incidence angles, the difference between the patterns can be quantified by the correlation between the two. For instance, linearly polarised light incident

on a metallic surface results in more highly correlated speckle patterns between the two polarisation states than those produced by paper [4].

This case can be considered by adapting the general case of a sum of two speckle patterns to the sum of the x and y components of the pattern. These components, I_x and I_y , each represent a fully developed speckle pattern. Thus in this case, the total intensity is given by:

$$I_0 = I_x + I_y \quad (3.1.15)$$

As an aside, this is equal to the first Stokes parameter, a common quantification of the polarisation of a wave. Further discussion on the Stokes parameters is detailed in Goodman [4] (Chapter 4: Higher-Order Statistical Properties of Speckle). The precise statistics of I_0 depend on the correlation between I_x and I_y , a quantity given by ρ_{xy} . This can then be used to find the degree of polarisation, $\mathcal{P}$.

A partially polarised wave can be considered as a sum of two components, where one component is linearly polarised and the other completely unpolarised [5]. These two components are not necessarily those along the x and y axes. From this, the degree of polarisation can be defined generally as the ratio given by the intensity of the polarised component over the total wave intensity. In terms of I_x , I_y , and ρ_{xy} , $\mathcal{P}$ is given by:

$$\mathcal{P} = \frac{\sqrt{(\bar{I}_x - \bar{I}_y)^2 + 4\bar{I}_x\bar{I}_y\rho_{xy}}}{\bar{I}_x + \bar{I}_y} \quad (3.1.16)$$

When the correlation between components, ρ_{xy} , is equal to one, the value of $\mathcal{P}$ is also one. However, if $\rho_{xy} = 0$, $\mathcal{P}$ is only zero when the intensities of the two components are equal. More generally, the complete relationship between $\mathcal{P}$, ρ_{xy} , and the difference of the intensities I_x and I_y is shown in the graph in Figure 3.6. This last factor is quantified as the ratio $\bar{I}_x/\bar{I}_0$ of the average intensity in x divided by the total average intensity $\bar{I}_0$.

Finally, the probability density function of I_0 , $p_0(I_0)$, is equal to:

$$p_0(I_0) = \frac{1}{\mathcal{P}\bar{I}_0} \left(\exp\left(-\frac{2}{1+\mathcal{P}}\frac{I_0}{\bar{I}_0}\right) - \exp\left(-\frac{2}{1-\mathcal{P}}\frac{I_0}{\bar{I}_0}\right) \right) \quad (3.1.17)$$

Given the dependence of $p_0(I_0)$ on $\mathcal{P}$ and the ratio $I_0/\bar{I}_0$, this PDF is rendered in 3D, shown in Figure 3.7.

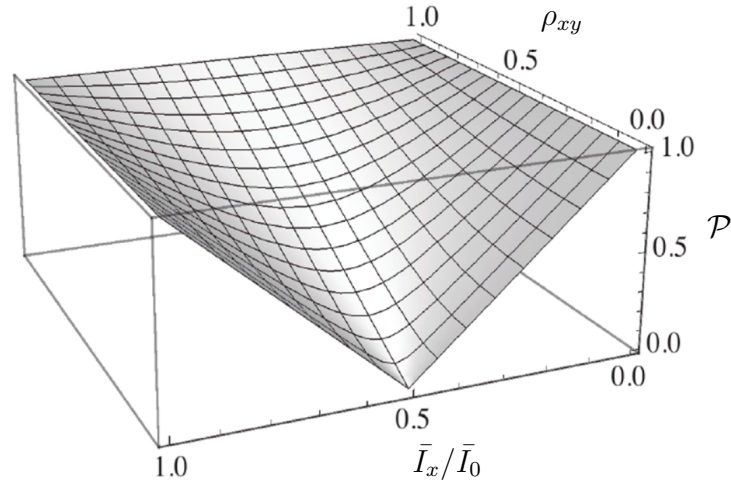


Figure 3.6: Relationship between polarisation $\mathcal{P}$, correlation ρ_{xy} , and the intensity ratio $\bar{I}_x/\bar{I}_0$ for the x and y components of a partially polarised speckle pattern. Image taken from Goodman [4], with labels changed for consistency.

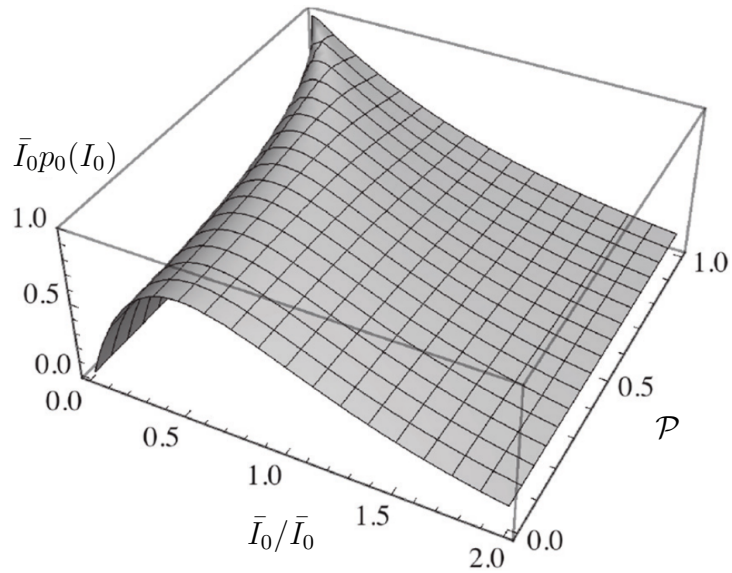


Figure 3.7: Probability density function of I_0 , $p_0(I_0)$, as it relates to polarisation $\mathcal{P}$, and the intensity ratio $I_0/\bar{I}_0$. Image taken from Goodman [4], with labels changed for consistency.

3.1.2 Subjective vs. Objective Speckle

When the light is scattered from the rough surface directly onto another surface, whether a wall or an optical sensor with no lens, an objective speckle pattern is formed. The key factor is the absence of any imaging lenses. The speckle pattern in Figure 3.1 is an objective speckle pattern. Additionally, the pattern recorded at the detector plane has contributions from the scattered light from the whole of the initial surface. In this case, the characteristics of the speckle pattern are a result of the system's geometry and the wavelength of the light.

More specifically, the size of the speckles are related to the wavelength of light, λ , the spot size, w_{spot} , and the separation between the surface producing the speckles and the plane they are imaged in. This separation is referred to as the working distance, L_D . Using the description above of cigar-shaped speckles in 3D space, each speckle has a longitudinal size w_{long} , given by the length of the cigars, and a transverse size w_{tran} , given by their width. These dimensions are shown in Figure 3.8, from Shirley and Hallerman [3].

Shirley and Hallerman [3] relates the “average speckle size”, both longitudinal and transverse, to the spot size via the following equations. Note that the relationship between the speckles and the spot is an inverse proportionality; as the spot size

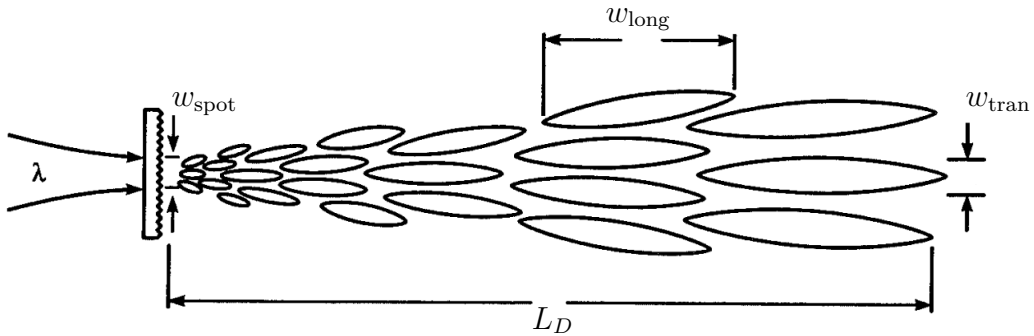


Figure 3.8: Diagram of a speckle field showing the longitudinal, w_{long} , and transverse, w_{tran} , speckle size from a spot of size w_{spot} at a working distance L_D from the surface. Image taken from Shirley and Hallerman [3], with labels changed for consistency.

increases, the speckle size decreases, and vice versa.

$$w_{\text{long}} = \frac{4\lambda L_D^2}{w_{\text{spot}}^2} \quad (3.1.18)$$

$$w_{\text{tran}} = \frac{\lambda L_D}{w_{\text{spot}}} \quad (3.1.19)$$

There are a number of different ways to consider speckle size in the literature, but these essentially just apply a scaling factor to the above equations, preserving the inverse relationship between spot size and speckle size, and the dependence on wavelength. Dainty et al. [2] describes the transverse speckle size as the average distance between adjacent regions of minimum and maximum brightness, resulting in the following relation:

$$w_{\text{tran}} = \frac{1.2\lambda L_D}{w_{\text{spot}}} \quad (3.1.20)$$

It is also important to note that these equations assume the spot is circular. An elliptical spot also produces elliptical speckles. In terms of the transverse speckle size, the major axis of the elliptical spot will be perpendicular to the major axis of the speckles, due to the inverse relationship between the two.

In terms of imaging, an objective speckle pattern will appear the same no matter how it is imaged, as long as the detector has no lens(es). A speckle pattern imaged using a lens system is a subjective speckle pattern. In this case, the transverse speckle size depends on the diameter of the lens aperture, d_a , and (as per Dainty et al. [2]) has a slightly different scaling factor:

$$w_{\text{tran}} = \frac{0.6\lambda L_D}{d_a} \quad (3.1.21)$$

For a subjective speckle pattern, each point of the image is illuminated by a finite region of the surface. As a result, the speckle pattern will look different if imaged at different positions. This is a gradual change; if the detector is moved continuously, the recorded pattern will show a corresponding translation. Eventually, the detector is imaging a completely different part of the 3D speckle field, at which point the pattern is considered decorrelated from the original. This decorrelation happens gradually, as individual speckles leave or enter the image plane.

Goodman [4] (Chapter 5: Spatial Structure of Speckle) defines speckle size more

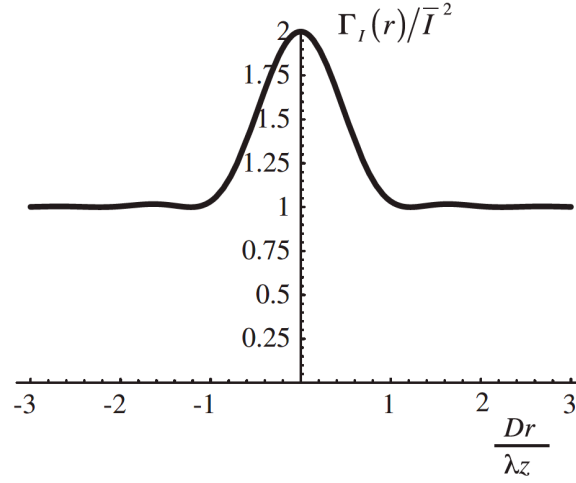


Figure 3.9: Cross-section of the autocorrelation in x and y of a speckle pattern from a circular spot, from Goodman [4] with labels changed for consistency.

rigorously, by considering the autocorrelation function of the intensity distribution of a given speckle pattern. A correlation function describes the degree of similarity between two sets of variables, and an autocorrelation compares the same set of values with itself. Goodman [4] shows that the normalised autocorrelation function of a speckle pattern from a circular spot can be expected to have the distribution shown in Figure 3.9.

Taking the full width half maximum (FWHM) of this peak, the transverse speckle size can be regarded as:

$$w_{\text{tran}} = \frac{1.0\lambda L_D}{w_{\text{spot}}} = \frac{1.0\lambda L_D}{d_a} \quad (3.1.22)$$

Note that the equation is the same for objective speckle, using the spot size w_{spot} , or subjective speckle, for an aperture of d_a .

Similarly, the longitudinal speckle size can be defined by considering the distance over which a speckle pattern remains correlated in the direction perpendicular to the surface. In this case, the cross correlation between the intensity distribution at two closely spaced distances from the surface are considered. The resulting function is shown in Figure 3.10, with the FWHM of this peak given as:

$$w_{\text{long}} = \frac{7.1\lambda L_D^2}{w_{\text{spot}}^2} = \frac{7.1\lambda L_D^2}{d_a^2} \quad (3.1.23)$$

Again, the longitudinal speckle size is the same for both objective and subjective

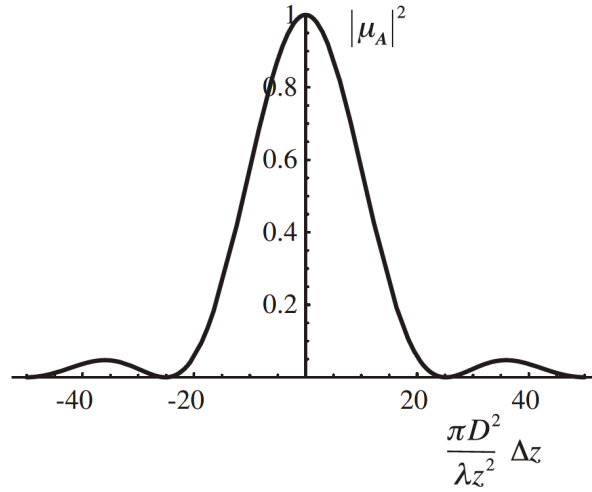


Figure 3.10: Normalised covariance of intensities in z for a speckle pattern from a circular spot, from Goodman [4] with labels changed for consistency.

speckle, with the only change being the definition of the spot size, w_{spot} , or the aperture diameter, d_a .

Fresnel and Fraunhofer Regimes

There are two regimes to consider when investigating diffraction patterns (including speckle): Fresnel, or near-field, and Fraunhofer, or far-field. Mathematically, these two approaches each apply a different approximation when evaluating the diffraction integral for a particular diffraction pattern. In both cases this results in a significant simplification of the mathematics involved.

The cut-off between the two regions is described by the following relation [6], where the Fresnel approximation is considered valid when the inequality holds.

$$z^3 > \frac{25 \left(d + \sqrt{x^2 + y^2} \right)^4}{\lambda} \quad (3.1.24)$$

Here d describes the spot or aperture size, λ the wavelength of light, and (x, y, z) simply the Cartesian coordinates of the point of interest.

A key feature of speckle in the far-field is that the longitudinal size of each speckle can be considered infinite under this regime [3]. In other words, the equations (3.1.18) and (3.1.23) hold only in the Fresnel regime, i.e. in the near-field. Under

the Fraunhofer regime, the intensity of the speckle pattern in the radial direction does not change, aside from the expected reduction with $1/z^2$.

Shirley and Hallerman [3] give the transition between near- and far-field at the following distance from the spot (described by its diameter w_{spot}) or aperture (diameter d_a):

$$z = \frac{w_{\text{spot}}^2}{\lambda} = \frac{d_a^2}{\lambda} \quad (3.1.25)$$

This is an approximate distance, as it depends on the specific locations of the speckles in the 3D field. Broadly speaking, however, the value of w_{long} outside of this range is so large that it can be considered functionally infinite.

The implication of this for measurements of speckle patterns is that, in the Fresnel region, there will be a change in the speckle pattern when recording at different working distances. Many techniques employ a fixed working distance regardless, but this is an important factor to consider.

3.1.3 Effects of Surface Properties on Speckle

Although the size of the speckles in a pattern are determined by the working distance, wavelength, and either the spot or aperture diameter, a number of other factors can affect the speckles' positions. For both objective and subjective speckle, changes in the wavelength of the light source, its angle of incidence, the position or tilt of the surface, or the surface strain all produce a translation in the speckle pattern as recorded by a static sensor [1, 4]. As with the movement of the detector for subjective speckle, these changes also result in a decorrelation of the speckle pattern, which introduces an inherent limitation on the range of techniques using speckle pattern displacement measurements.

Yamaguchi [7] derived a comprehensive series of mathematical relations describing the effect of translation, rotation, and strain on speckle displacement. The paper then demonstrated the movement of the speckle pattern for the three surface deformations. A lens imaging system was used to capture the subjective speckle patterns before and after deformation, and the movement of the speckles are shown in Figure 3.11. Figure 3.11a shows an in-plane rotation of 0.167° , 3.11b shows an in-plane strain of 2×10^{-5} in x and 1×10^{-5} in y , and finally 3.11c shows a two-component translation of $6.63\mu\text{m}$ in y and $63.3\mu\text{m}$ towards the detector in z .

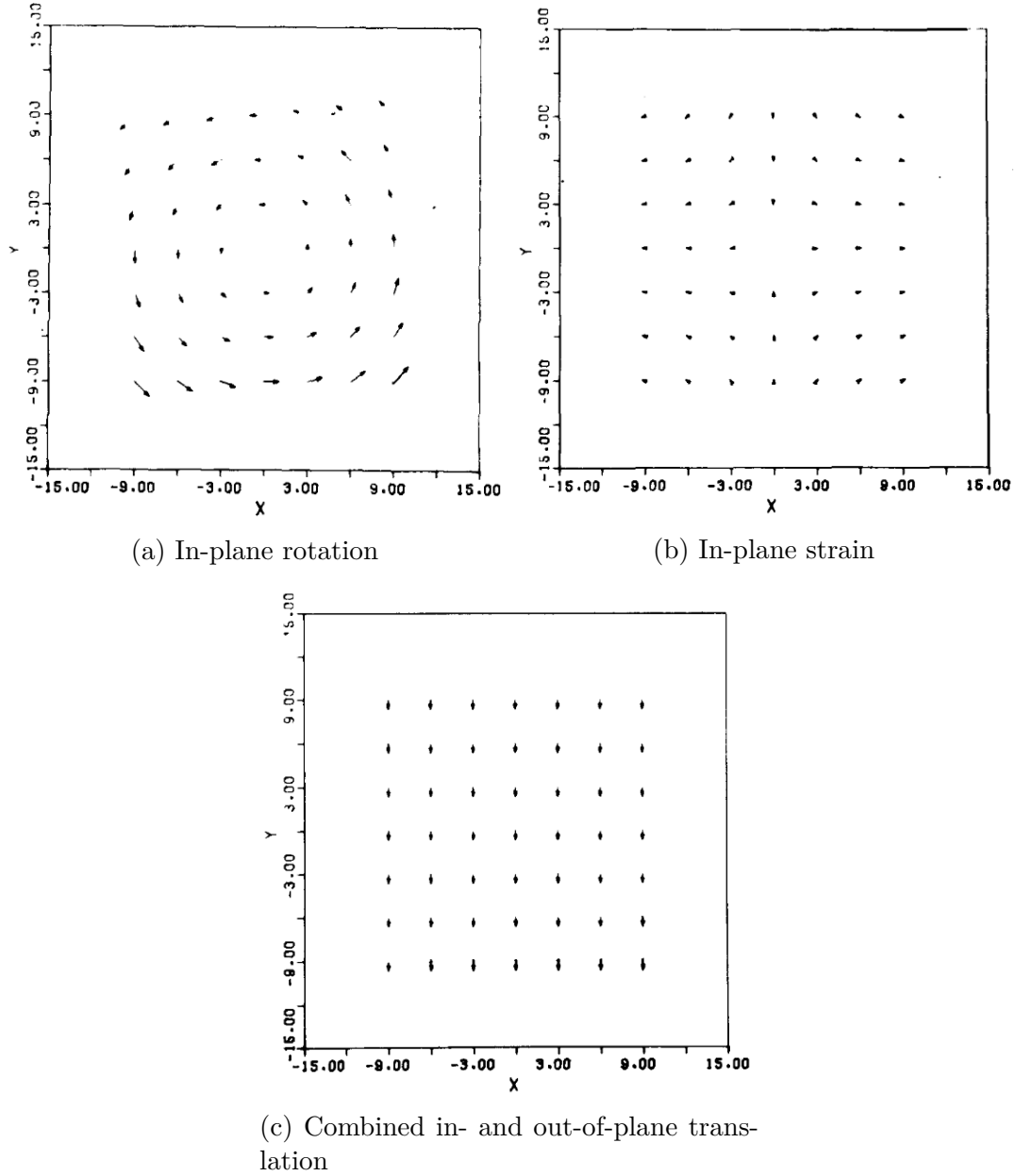


Figure 3.11: Speckle displacement measured by Yamaguchi [7] for (a) an in-plane rotation of 0.167° , (b) an in-plane strain of 2×10^{-5} in x and 1×10^{-5} in y , and (c) a combined translation of $6.63\mu\text{m}$ in y and $63.3\mu\text{m}$ in z .

Using these simple relations between surface translation and the displacement in speckle patterns, a variety of measurement systems can be constructed. For example, velocity can be measured using continuous, timestamped translation measurements [8]. In this case, new images are correlated with a reference image, monitoring the translation of the surface at the 500fps frame rate of the camera. For out-of-plane translations, laser speckle can be used in depth sensing [9], where a reference image of a flat surface parallel to the detector is compared against real-time images to measure distance from the detector for various features in the image, thus building up a complex depth profile.

Another application of these relationships is in vibration sensing, using the fast-changing angle of a surface as it vibrates [10]. This results in a linearly vibrating speckle pattern in the detection plane. Wu and Haruyama [11] expanded on this application by using a speckle-based sensor to detect audio. The sensor monitors the movement of an object caused by sound vibrations, reconstructing their frequency. The paper presented results showing the real-time reconstruction of a piece of music using the technique.

Another surface property that can be investigated using speckle is surface roughness. However, unlike the previous surface changes mentioned, surface roughness affects the correlation and contrast of speckles [12]. The degree of surface roughness changes the decorrelation between speckle patterns from different wavelengths. The two patterns are fully decorrelated when the following relation is satisfied [1], where σ is the standard deviation of the surface roughness.

$$\sigma \geq \frac{\lambda^2}{2\Delta\lambda} \quad (3.1.26)$$

Applying this to measurements, a surface is illuminated with light of wavelength λ and the speckle pattern imaged. Then, the surface is illuminated again with a wavelength $\lambda + \Delta\lambda$, with the speckle pattern imaged and compared to the first. By increasing $\Delta\lambda$ gradually and comparing the speckle patterns to that from the original wavelength, the point at which the patterns are decorrelated will satisfy the equality in equation (3.1.26), thus giving a measurement of surface roughness as defined by σ .

Considering speckle contrast, the contrast of a subjective speckle pattern can be directly related to surface roughness [4]. For a completely flat surface, such as a

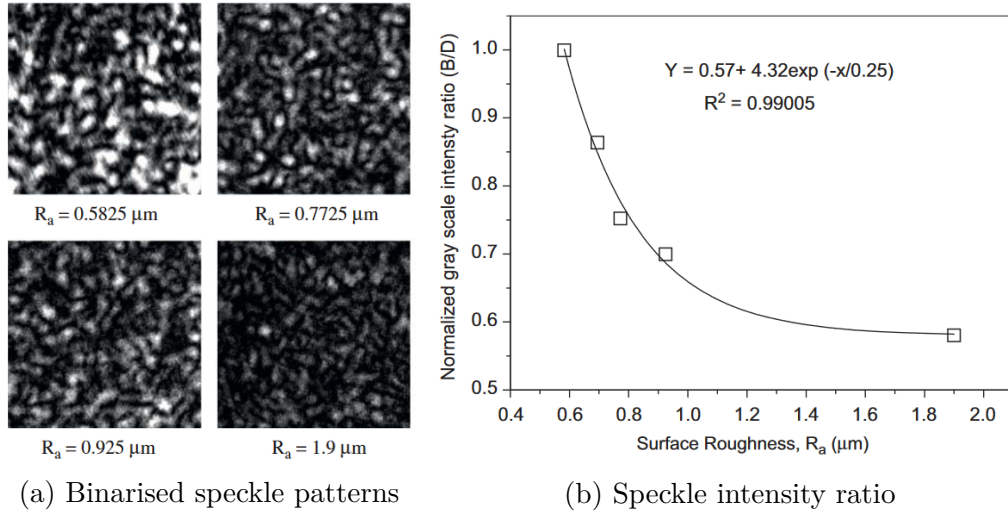


Figure 3.12: Demonstration of the relationship between surface roughness, R_a , and speckle contrast, from Kayahan et al. [13]. This is shown directly through the binarised speckle patterns in (a), and with the processed data plotting the ratio of bright to dark pixels (B/D) in the binary pattern against surface roughness in (b).

mirror, no speckle pattern is formed and so the speckle contrast is 0. Then, broadly speaking, as the surface roughness increases so does the speckle contrast, levelling off at 1 when the pattern is fully developed. The particular surface roughness for which a pattern reaches a contrast of 1 depends on the size of the aperture and the wavelength of the light.

Figure 3.12a shows speckle patterns from surfaces of varying roughness [13]. The measure of surface roughness used here is R_a , which is the average height deviation from the mean line. The technique described in the paper converts these images to binary only, and uses the ratio of bright to dark pixels to demonstrate the relationship between speckle contrast and surface roughness. This relationship is exponential, as shown in Figure 3.12b.

A very different but very popular measurement technique using speckle is blood flow monitoring [14, 15]. In this technique, laser speckle contrast is used to measure the speed of the flow. A recorded speckle pattern from a blood vessel is influenced by the movement of blood cells; the cells act as scattering particles, and the pattern fluctuates as they move. By capturing images with an exposure time longer than the time scale of these fluctuations, the speckle pattern images show a blurring effect, or a decrease in the contrast. With a constant exposure time, the degree of speckle

blur is proportional to the speed of blood flow.

For all of these measurement techniques, one of the main advantages of laser speckle is that it results in a non-contact sensor. This is particularly important in applications such as strain or surface roughness testing. For example, laser speckle can be used to measure the strain in concrete railway sleepers, replacing the “tedious” installation of individual foil strain gauges or the “costly and time-consuming” maintenance of fibre optic sensors embedded in the concrete [16]. For surface roughness monitoring, the most common technique uses a stylus moving over the surface while maintaining contact with it. This can cause surface damage, is often time-consuming and difficult to perform, especially in-situ, and its resolution is limited by the needle diameter [17]. Non-contact sensing improves on, and in some cases eliminates, these various drawbacks.

3.2 Detecting Surface Deformation Using Speckle

This more in-depth review focuses on using speckle to measure in-plane translations and out-of-plane tilts. Both of these surface deformations result in a shift in the speckle pattern that is seen at the detector as a linear displacement [18, 19]. One inherent difficulty for these techniques is that the speckle pattern displacement is additive for a combined translation and tilt. In some cases, the surface movement is restricted to one or another and so this limitation is irrelevant. For other cases, a number of the methods described below aim to separate each component. However, a common downside of these techniques is the increase in complexity and number of sensor components, such as a second detector. As with all sensors, the requirements of the application must be balanced with the limitations of the technique.

The differences in speckle techniques lie in how the displacement, or speckle shift, is measured. The techniques described here will be broadly separated into three categories using the distinctions drawn by Smíd et al. [20]: photography, interferometry, and correlation. In speckle photography, a double exposure image is taken before and after a surface deformation. An optical Fourier transform performed on this image reveals fringes, which are used to determine the displacement of the speckle pattern between deformations [21]. This technique measures displacements larger than the speckle size. Speckle interferometry, on the other hand, requires that displacements be smaller than the speckle size [20]. The interference between two

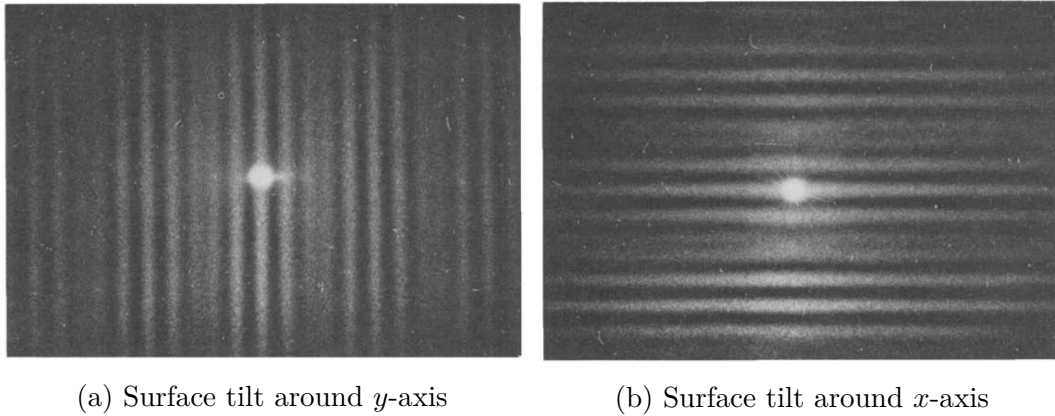


Figure 3.13: Images of doubly-exposed photographic plates used by Tiziani [21] to measure surface tilt changes, showing fringes resulting from a $60''$ tilt around the (a) y -axis, and (b) x -axis.

wavefronts, either both from the surface of interest or one plus a reference beam, is used to produce fringes. Similarly to speckle photography, the separation between the fringes is used to determine the speckle shift and thus the surface deformation. Finally, the speckle shift can be determined through directly correlating the two patterns. A correlation image with a peak is produced, and the location of the peak gives the displacement between the patterns [18].

As the focus of this project is angle measurement, this section will conclude with a summary of all speckle techniques that relative to tilt or rotation sensing.

3.2.1 Speckle Photography

The earliest speckle measurement techniques, developed in the 1970s and 80s, used glass photographic plates to record speckle patterns [21]. Often techniques would require the plate be doubly-exposed, recording two speckle patterns on one plate. The plate would then be illuminated to produce fringes, where the fringe spacing showed the separation between the two speckle patterns. With the invention of digital cameras, many of these techniques could be revisited using modern cameras and digital image correlation. As such, the optical systems used in these earlier papers would be equally effective using more modern imaging and processing techniques.

One method of independently measuring tilt is to perform an optical Fourier transform (OFT) on the light reflected from the surface of interest [1]. Images are taken in the Fourier plane, at the focal distance of the lens, before and after the tilt.

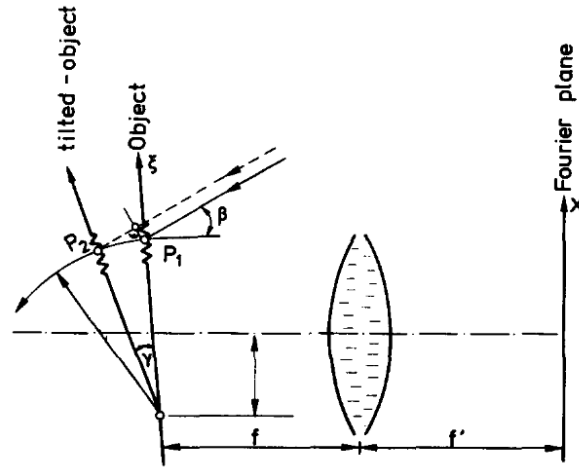


Figure 3.14: Experimental setup used by Tiziani [21] to measure surface tilts in the Fourier plane, making the technique insensitive to object translation. The focal length of the lens is f , and the distance to the Fourier plane f' . The object is tilted by γ between positions P_1 and P_2 . β gives the angle of incidence of the source.

Taking the sum or difference of the images results in an interference pattern whose fringe spacing is inversely proportional to the tilt angle.

Tiziani [21] used this technique with doubly exposed photographic plates to measure the fringe patterns, and despite the “not very sophisticated” recording method found an agreement with expected results of better than 4%. Photographs of the fringes are shown in Figure 3.13; in each case the applied surface tilt was 60arcsec, but the direction of tilt was perpendicular. The experimental results also demonstrated the insensitivity to translation; as shown in the experimental setup in Figure 3.14, tilting the surface also changed its distance from the lens. This change in target distance had no effect on the results obtained.

Gregory [22] later demonstrated that the insensitivity of this technique to translation only held true for collimated light. However the paper extended the concept, demonstrating that for any source there are two planes behind the lens in a speckle imaging system, one insensitive to translation and the other to tilt.

The basis of Gregory’s theory was to formulate an optically rough surface as a collection of mirror facets. In order to find a plane insensitive to translation, called the tilt plane, the incident light is considered to have an apparent source behind the “mirror” of the surface. When the lens is focused on this source, all rays are

brought to the same point behind the lens regardless of their scattering position on the surface. In this plane, any speckle displacement from translation will not cause a corresponding translation in the tilt plane.

The transverse plane, where images are insensitive to tilt, is simply the imaging plane of the lens. In this case, tilt speckles are considered to originate from the surface, as they are unaffected by the distance to the source. Thus, as all rays from the surface are focused to the same point in the image plane, they cannot be detected in this transverse plane. A diagram of these planes can be seen in Figure 3.15. The arrows show the direction of movement and the corresponding speckle shift both objectively before the lens, and subjectively in each plane. It is important to note that this theory holds for small translations and tilts only. For larger surface transforms, as the illumination point on the surface changes, so will the generated speckle pattern.

The results demonstrated by Tiziani [21] are a special case of the theory developed by Gregory [22]. For collimated light, the source position is considered to be infinity, and so the tilt plane corresponds to the Fourier plane of the lens.

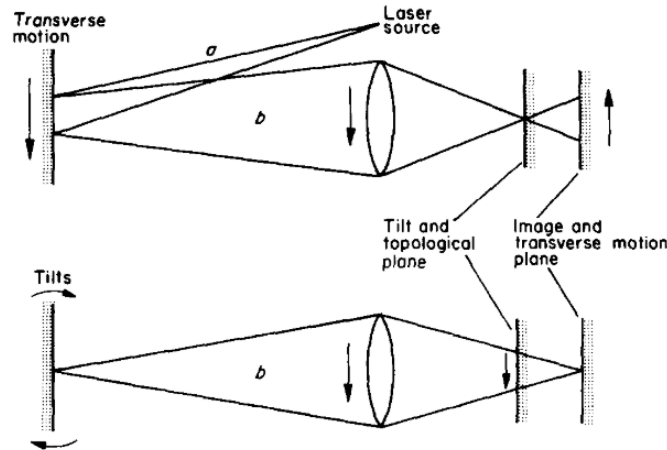


Figure 3.15: Diagram from Gregory [22] showing the tilt and topological plane, where measurements are insensitive to translation, and the image and transverse motion plane, where measurements are insensitive to tilt. The distance to the source is a , and the distance from the surface to the lens is b . The arrows show the direction of motion and resulting direction of speckle shift.

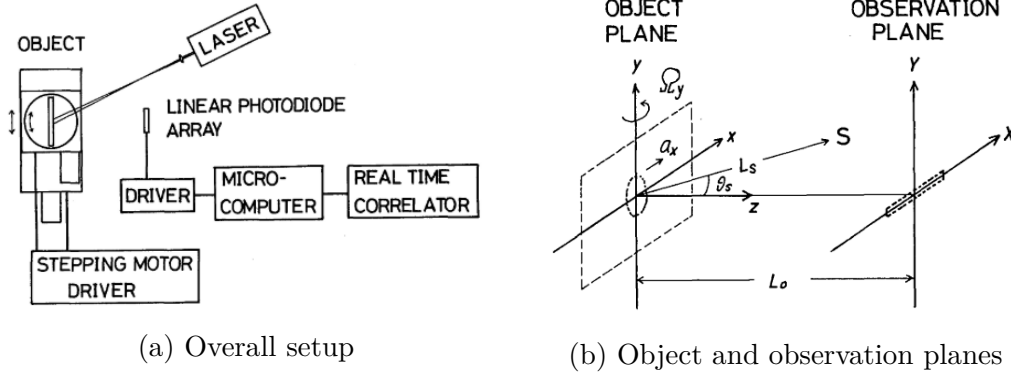


Figure 3.16: Experimental setup used by Yamaguchi [18] to measure translation and tilt of a surface using speckle pattern images and electronic speckle correlation. The labels in (b) describe the distance, L_S , and angle, θ_S , to the source, S ; the distance to the observation plane, L_O ; and the translation, a_x , and tilt, Ω_y , of the surface (object).

3.2.2 Speckle Correlation

Yamaguchi [18] first demonstrated how electronic speckle correlation could be used to measure a translation or tilt of a surface by monitoring the speckle pattern directly. Correlation is a mathematical function that compares the similarity of two series, and is also called the sliding dot product or sliding inner-product. When comparing two images, a template from one image is compared pixel-by-pixel with the second, measuring the similarity at each point. The result is a correlation image with a peak at the point where the images are most similar. A mathematical description of correlation is given later in Chapter 5.

The technique from Yamaguchi [18] images an objective speckle pattern generated by the surface of interest using a linear photodiode array, shown in Figure 3.16. Images are recorded before and after a movement, and a digital 1D cross-correlation is applied to find the relative displacement between the two. The technique was used to measure in-plane translations covering a range of $\pm 500\mu\text{m}$, and the experimental results were found to be within 1% of the predicted values. For angle measurements, the surface was tilted through $\pm 0.3^\circ$, and measurements were within 0.5% of the theoretical values expected. Note that this technique cannot be used to measure surface transformations that combine translation and tilt, as the resulting speckle displacements are additive and cannot be separated. The use of a linear detector gives another limitation, as movement could only be measured in one dimension.

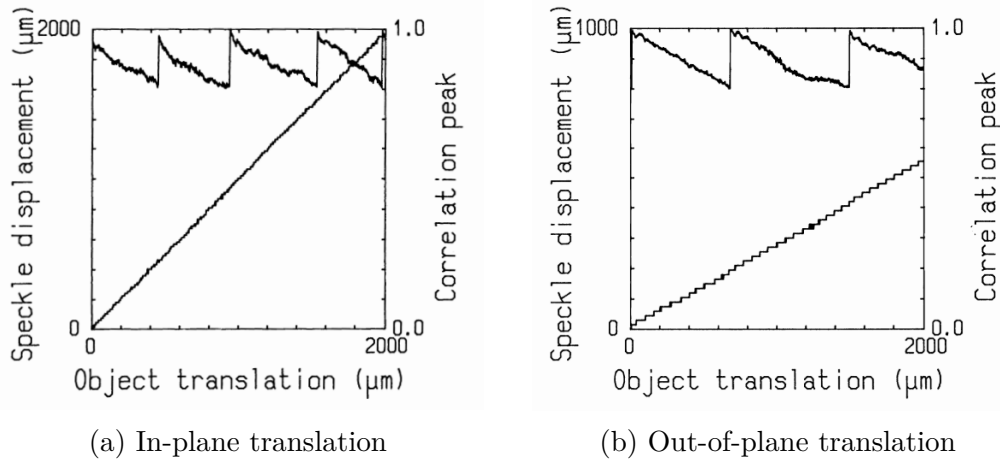
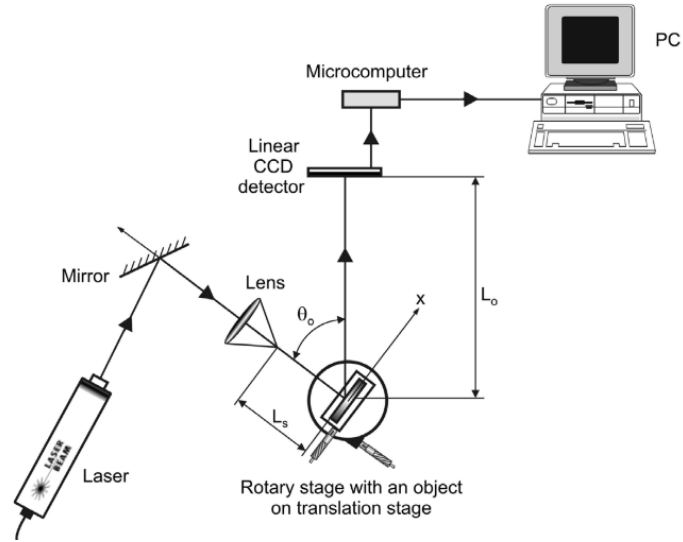


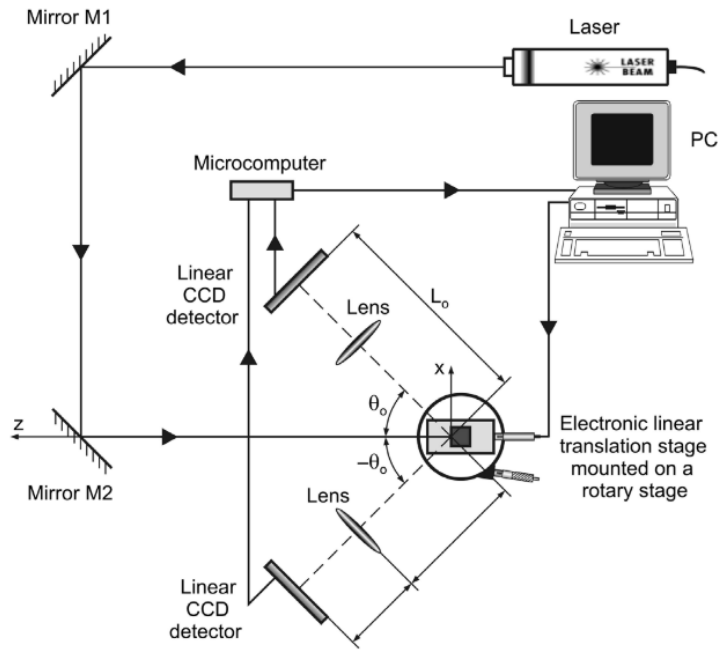
Figure 3.17: Translation measurements from Yamaguchi and Fujita [23] using relative speckle correlation. The right-hand axis shows the height of the correlation peak; when this dropped below 0.8, the reference image was replaced. (a) Measured in-plane translation of an aluminium plate, moved in steps of $1\mu\text{m}$ across 0-2000 μm . (b) Measured out-of-plane translation of a paper sheet, moved in steps of $15\mu\text{m}$ across 0-2000 μm .

As with most speckle techniques, the movement range is limited by the speckle pattern decorrelating from the reference image. A second paper from Yamaguchi and Fujita [23] investigated this decorrelation for in-plane and out-of-plane translations. Figure 3.17 shows experimental results for the continuous speckle displacement as a stepped translation is applied to an object. The right-hand axis shows the height of the correlation peak, normalised against an autocorrelation of the original image. As the object is translated, the new image is compared to an initial reference until the height of the correlation peak falls to 0.8. At that point, the process switches to a new reference image and continues the measurements. For in-plane translations, the surface was moved around $500\mu\text{m}$ before the reference image was replaced. The speckle displacement due to out-of-plane translations is smaller, so the surface was able to move around $700\mu\text{m}$ in this direction before a new reference image was required.

Building on Yamaguchi's work, Horváth et al. [24] also looked at using speckle correlation to measure in- and out-of-plane translations. For in-plane translations the concept was unchanged; a reference image was acquired and correlated with an image after translation to find the relative speckle shift, used to calculate the translation magnitude. They also used a linear charge-coupled device (CCD) detector,



(a) In-plane translation



(b) Out-of-plane translation

Figure 3.18: Experimental setup from Horváth et al. [24], used to measure (a) in-plane, and (b) out-of-plane translations of an aluminium block by speckle correlation. In each case, L_s describes the distance to the source, L_o the distance to the observation plane, and θ_o the angle between the two.

Table 3.1: Summary of results from Horváth et al. [24], where relative speckle correlation was used to measure in-plane and out-of-plane translations of an aluminium block.

Translation (μm)	Mean (μm)	Standard deviation (μm)	Percentage error (%)
<hr/> In-plane <hr/>			
2.00	2.08	0.04	4.0
10.00	10.00	0.10	0.0
30.00	32.48	0.04	8.3
200.00	202.80	0.10	1.4
<hr/> Out-of-plane <hr/>			
5.00	4.78	0.02	4.4
50.00	49.10	0.30	1.8
100.00	100.00	0.40	0.0
600.00	599.40	0.1	0.1

shown in Figure 3.18a, measuring in one dimension. A summary of their results can be seen in the top half of Table 3.1, along with calculated percentage errors for comparison to other techniques.

For out-of-plane-measurements Horváth et al. [24] used two linear detectors, positioned in the same plane symmetrically around the light source. The speckle shifts were measured by each detector in the same way, but in this case the difference of the two speckle shifts were used to calculate the translation. The results from this measurement series are in the lower half of Table 3.1, again with calculated percentage errors.

The same group extended the out-of-plane measurement technique for use in detecting an object's slope (Smíd et al. [20]). The same setup was used, shown in diagrammatic form in Figure 3.19. For this technique, the object is translated along the f'_x -axis in increments of $40\mu\text{m}$, with images taken at each point on both detectors. Each successive image is correlated with the previous, determining the speckle shift

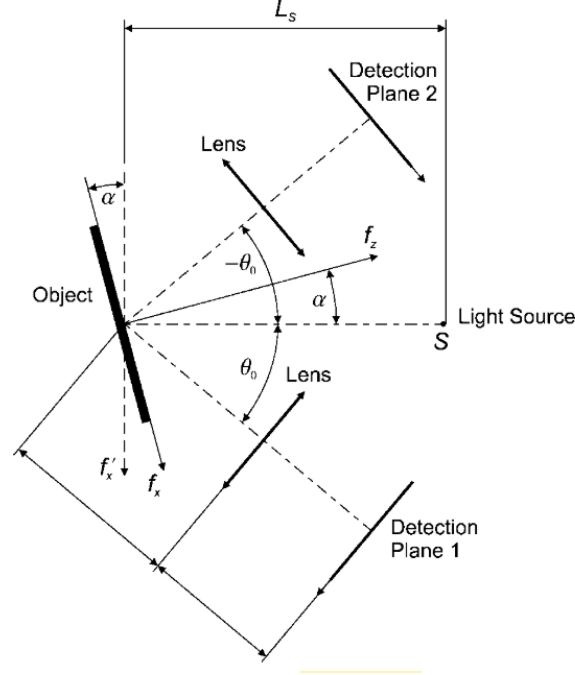


Figure 3.19: Diagram of the experimental setup used by Smíd et al. [20], equivalent to that used in Horváth et al. [24] and shown in Figure 3.18b. This setup was used to measure an object's slope by imaging its translation using relative speckle correlation. L_S gives the distance to the source, and θ_O the angle between it and the observation plane(s); the surface tilt is α , resulting in translations of the speckle shift in x (f_x) and in z (f_z).

between movements. This information, along with the known translation length $40\mu\text{m}$, is used to calculate the slope of the surface between each two imaged points. A set of measurements for repeated translations describes the shape of the object along the line imaged. For a flat surface, this dataset can be averaged to give the overall value of the surface slope.

To demonstrate the technique, Smíd et al. [20] investigated the tilt of a flat surface positioned at various known angles. A theoretical analysis on the choice of sensor geometry indicated that the range of the detector would be 8° - 30° , with the smallest resolvable difference 2° . Accordingly, the sensor was used to measure a series of angles between 8° and 30° , separated by 2° . Sixty translation steps of $40\mu\text{m}$ were used to image the surface, and from these a mean tilt value and its standard deviation were calculated. For angles between 12° and 30° , the percentage error on the mean measurement was less than 3%, with standard deviations of less than 4.4° . These errors were somewhat higher at the lower bound of the measurement range, with

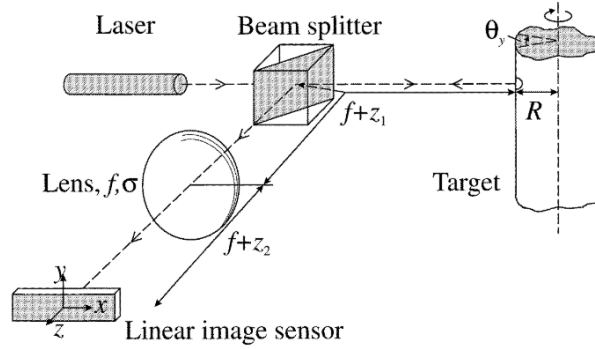
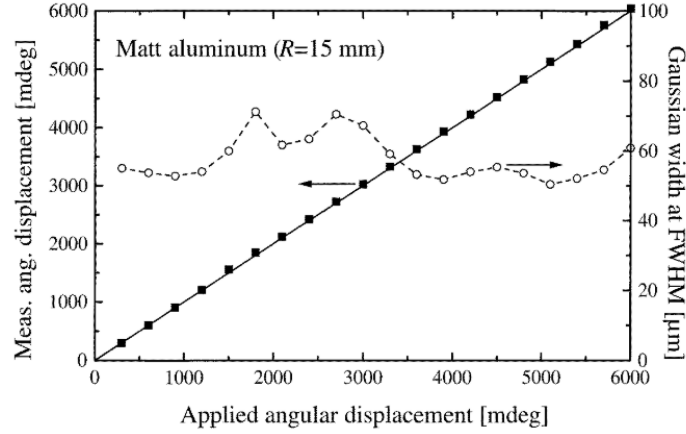


Figure 3.20: Experimental setup used by Rose et al. [25] to measure the rotation (θ_y) of an aluminium shaft with radius of curvature R . A lens of focal length f and aperture σ is positioned such that the image sensor is in the Fourier plane (i.e. $z_2 = 0$) and so the technique is insensitive to the target distance z_1 .

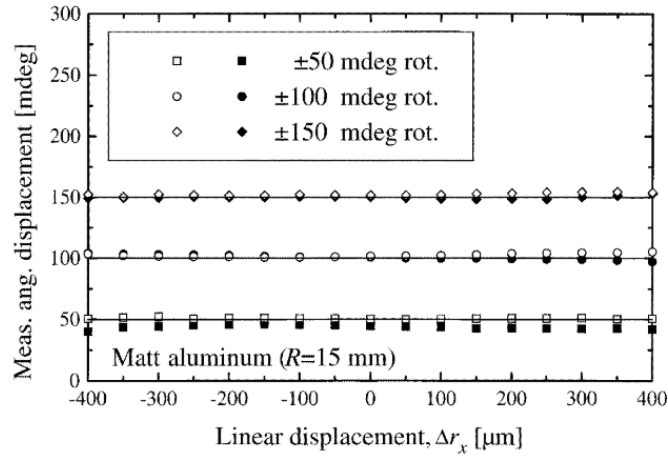
a 13% error on the mean at 10° , and an exceptionally high standard deviation of 19.2° for a surface tilt of 8° . The paper concluded that the results were satisfactory within the range 10° - 30° .

The authors also described the possibility of extending the technique further with a third detector. As implemented, the translation is determined to be along the constructed f'_x -axis, and the magnitude of the translation must be known. With a third detector, the components of the translation in f_x and f_z could be calculated from measurements, and then used to calculate the surface slope without prior knowledge of the translation step.

Speckle correlation can also be used to monitor the rotation of a cylinder or shaft [23, 25]. The technique used by Rose et al. [25] took advantage of the effect described above in Section 3.2.1 whereby placing the detector in the Fourier plane renders it insensitive to surface translation (see Figure 3.20). This additionally means that the object under measurement need not be a regular cylinder. Experimental results for an aluminium shaft are shown in Figure 3.21. Figure 3.21a shows the angular displacement measurements for a series of rotations in steps of 0.3° across the range 0.0° - 6.0° , while the right-hand axis gives the FWHM of the correlation peak. For these measurements, each successive image was used as the reference for the next. The maximum measurable angle step was found to be 0.5° ; a rotation larger than this would result in complete decorrelation of the speckle pattern. Figure 3.21b demonstrates the system's insensitivity to target distance, i.e. translation. A refer-



(a) Rotation measurement only

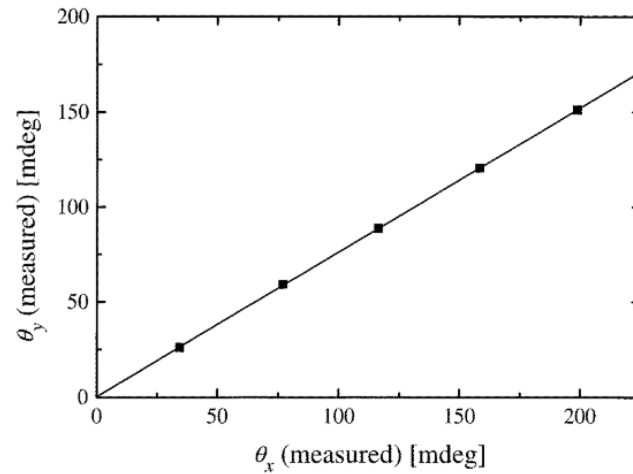


(b) Rotation measurement with object translation

Figure 3.21: Experimental results from Rose et al. [25], showing the rotation of an aluminium shaft measured with speckle correlation. (a) Measured angular displacement for applied rotations in steps of 0.3° across the range 0.0° - 6.0° . (b) Measured angular displacement for three applied rotations with the shaft at linear displacements between $\pm 400\mu\text{m}$.

θ_{appl}	θ_x	θ_y	θ_{meas}
50.0	36.1	27.7	45.5
100.0	76.9	58.0	96.3
150.0	120.8	90.8	151.1
200.0	160.9	121.4	201.6
250.0	200.8	152.3	252.1

(a) Applied and measured rotation



(b) Components of rotation in x and y

Figure 3.22: Experimental results from Rose et al. [25] for the rotation of an aluminium shaft with a 2D sensor inclined at 52.0° . The measured rotation has x and y components in the image sensor plane. (a) Table of results, showing measured components and total rotation. (b) Plot of measured rotation in x vs. that in y .

ence pattern was obtained, then a rotation and linear displacement along the x -axis applied. The measured angular displacement was unaffected across multiple linear displacements between $\pm 400\mu\text{m}$.

Rose et al. [25] also demonstrated the ability to extend the technique into two dimensions. For these measurements, the linear image sensor was replaced by a 2D sensor inclined at 52.0° . Using this setup, the speckle displacement due to rotation of the shaft had components in both the x and y axes of the sensor. The rotation can still be calculated by taking the hypotenuse of the two angular components, as shown in the table in Figure 3.22a. Additionally, by plotting the x and y components against each other, the inclination of the sensor can be recovered from the graph's slope. The measured inclination from Figure 3.22b was 52.4° , in good agreement with the applied 52.0° .

Another approach to tilt sensing was proposed by Fricke-Begemann [26] (also described in [27]). This technique uses a subjective setup with a circular aperture, as shown in Figure 3.23. When a tilt is applied to the surface, the speckle pattern shifts at the aperture, resulting in a different section of the pattern passing through the aperture. A demonstration of this is given in Figure 3.24. Figure 3.24a shows the before and after images of the speckle pattern, while 3.24b demonstrates the overlap in the pattern of the two images. A cross power spectrum, which is the Fourier transform of the cross-correlation, is calculated from the before and after images. This operation results in an image such as the one in Figure 3.25, where the elliptical shape (comparable to the diagram in Figure 3.24b) is proportional to the tilt of the surface. The width of the ellipsoid's minor axis is inversely proportional to the magnitude of the tilt, while its orientation gives the tilt direction. A limitation

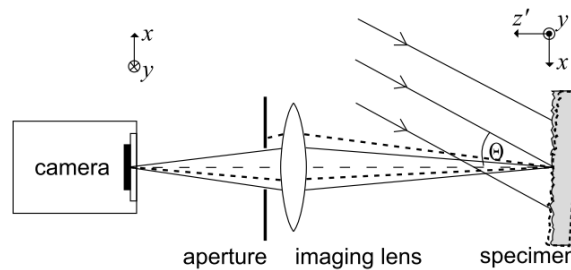


Figure 3.23: Experimental setup used by Fricke-Begemann [26] to measure surface tilts using a subjective speckle setup. Θ gives the angle of incidence of the source.

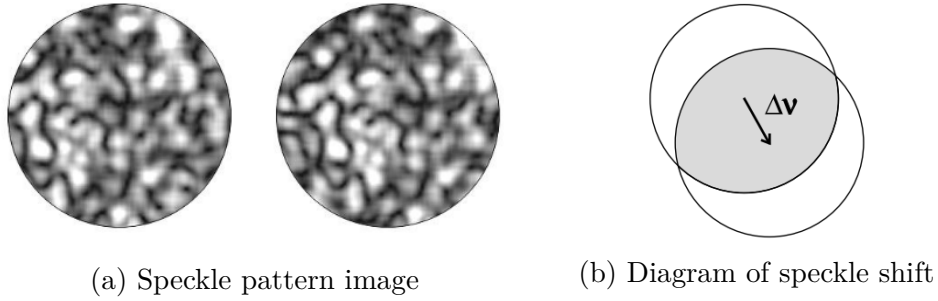


Figure 3.24: Demonstration of the speckle shift after surface tilt as seen through the aperture in the technique used by Fricke-Begemann [26]. (a) Two speckle patterns before and after tilt, directly showing the displacement of the pattern. (b) Diagram illustrating the pattern displacement, $\Delta \mathbf{v}$, and the shape of the overlap between the two images.

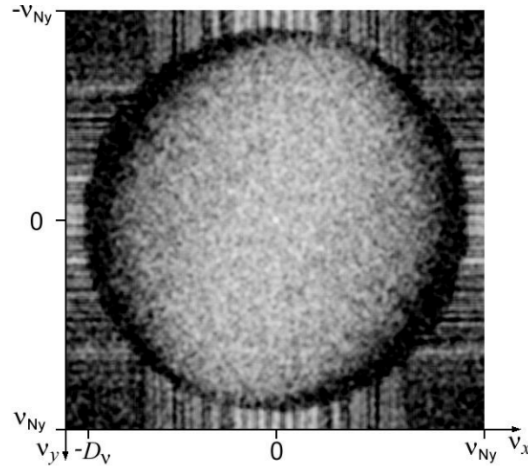


Figure 3.25: Cross power spectrum from a before and after image of a surface tilt of 0.11° , from Fricke-Begemann [26]. The central area shows an ellipsoid whose minor axis gives the magnitude of tilt (via its length) and the direction (via orientation).

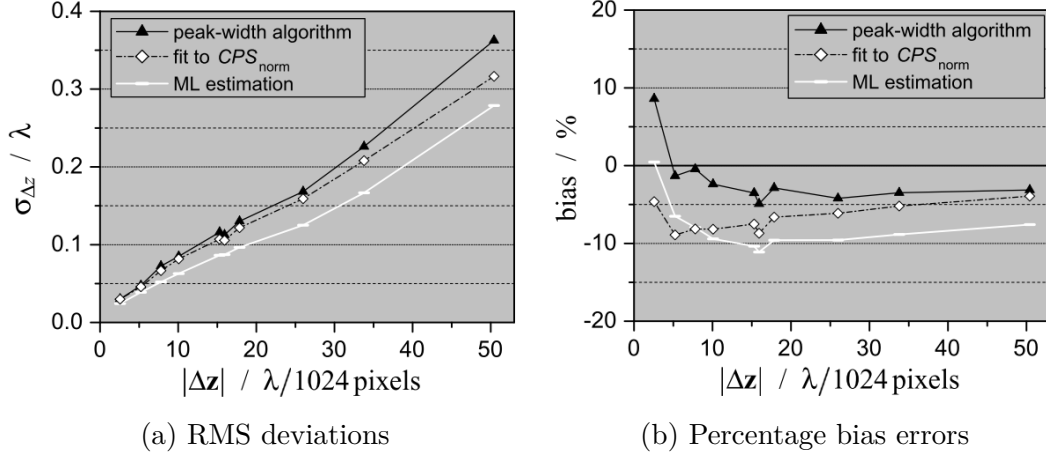


Figure 3.26: Experimental errors from tilt measurements between 0.00° and 4.22° using cross power spectra, from Fricke-Begemann [26]. A uniform tilt was applied, given in terms of the resulting height difference Δz , and multiple measurements were made at different subimage pairs of the full before and after images. Three different algorithms were tested for fitting to the cross power spectra. The RMS deviations in measurements from different subimages are shown in (a), and the percentage bias errors in the measurements are given in (b).

of this measurement is that the sign of the tilt (i.e. positive or negative orientation) cannot be determined directly from the cross power spectrum. The solution to this was to take a second reference image, at a slightly different surface tilt than the first [27]. By comparing subsequent images with both references, the tilt direction can be determined.

An additional element of this technique is the splitting of each image into smaller subimages. For a uniform tilt, the results from these subimages can be averaged to give a root mean square (RMS) deviation on the measurement. Fricke-Begemann [26] presented results for increasing tilts up to around 4.22° , given in terms of the height difference over each subimage, Δz , as caused by the surface tilt. These data are shown in Figure 3.26, for three different algorithms that were tested for extracting the tilt information from the cross power spectrum. For reference, the subimages were each 64×64 pixels, with a pixel width of $6.7 \mu\text{m}$, and the wavelength used during measurements was 632.8 nm . The RMS deviation on measurements (Figure 3.26a) was found to increase with increasing angle, reaching no more than 0.03° . However, there was a consistent bias error on results (Figure 3.26b) of around -5% , equivalent to -0.21° at 4.22° .

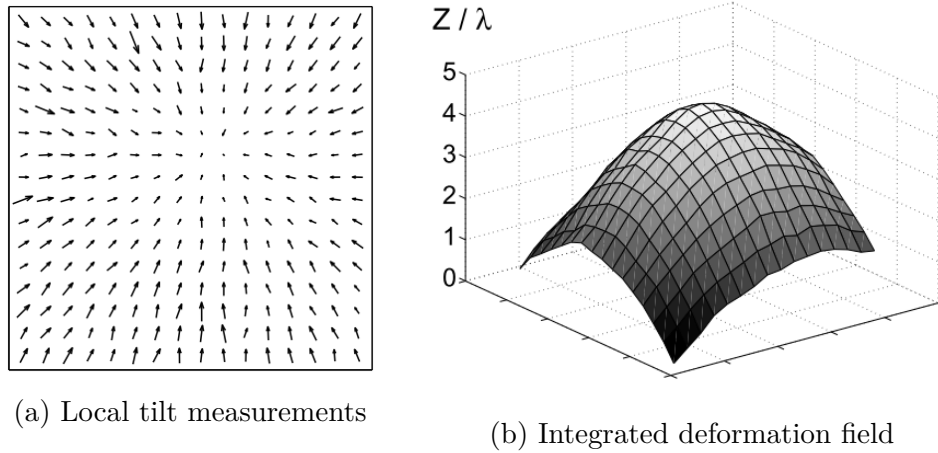


Figure 3.27: Measurement of an out-of-plane deformation field from Fricke-Begemann [26]. (a) Local tilt measurements at each point of the surface. (b) Integrated deformation field across the surface.

An alternative application of the segmentation into subimages is to extend the method to the measurement of an out-of-plane deformation field over a curved surface. The local surface tilt is measured for each subimage, resulting in data such as those shown in Figure 3.27a. The tilt measurements can then be integrated across the surface, producing a full deformation field as shown in Figure 3.27b.

Fricke-Begemann [26] also proposed measuring in-plane surface translations using the phase of the cross power spectrum. An example of this is given in Figure 3.28, which shows the phase of the cross power spectrum for a surface translation of $2.68\mu\text{m}$ in x and $4.02\mu\text{m}$ in y . This measurement technique was used to measure the translation of a high precision stage over the range $0\text{--}4\mu\text{m}$, with an average error over measurements of 3nm .

The use of OFTs, which are a specialised form of linear canonical transforms (LCTs), can also be applied to detecting surface transforms with speckle. Kelly et al. [28] used optical fractional Fourier transforms (OFRTs) to measure both surface tilt and translation. The technique places the detector in a plane corresponding to a fractional Fourier transform (FRT) of known order (i.e. domain). Then, an image of the surface is taken before and after the deformation. A cross correlation is performed on the two images, and the position of the resulting correlation peak directly corresponds to the change in position. This technique has the advantage of being able to determine the direction of movement as well as the magnitude, something a purely

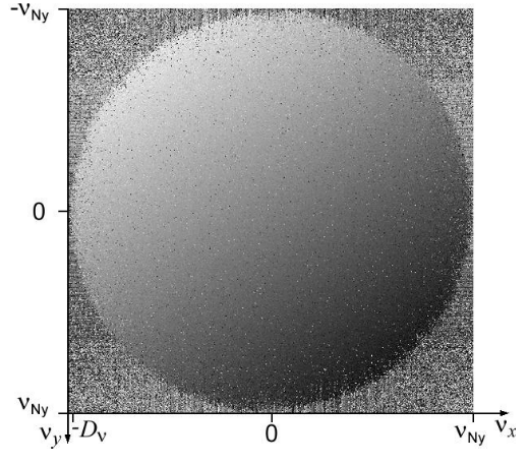


Figure 3.28: Phase of the cross power spectrum for a before and after image of a surface translation of $2.68\mu\text{m}$ in x and $4.02\mu\text{m}$ in y , from Fricke-Begemann [26].

OFT-based system cannot do. In addition, by varying the fractional order of the OFRT, the range and sensitivity of the detector can be customised depending on the requirements of the application. For example, an OFRT of order 1.2 can detect a maximum angle of 0.154° (0.0027rad), whereas for order 1.6 the maximum angle is 0.498° (0.0087rad). This tripling of range, however, is offset by a corresponding de-

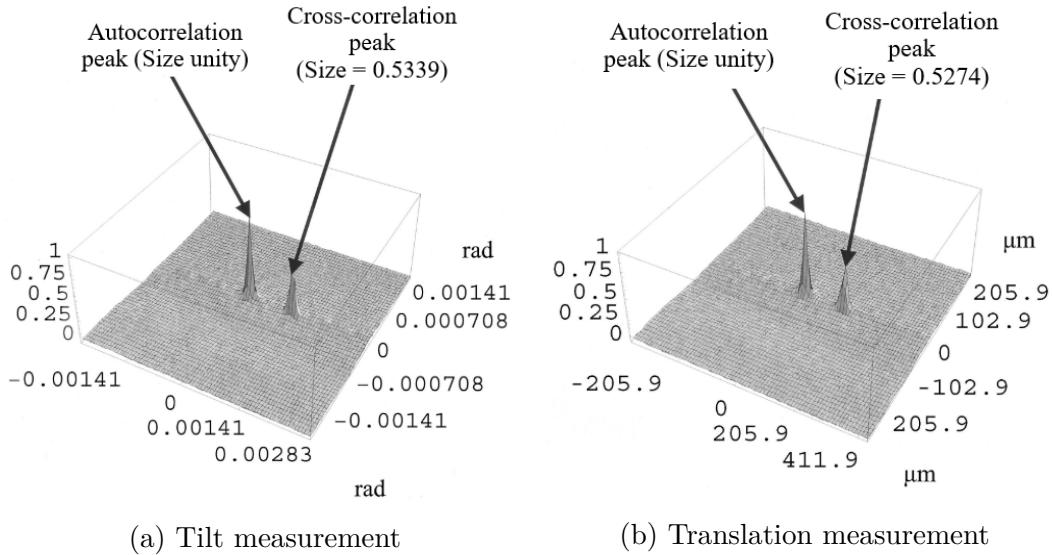


Figure 3.29: Individual measurements of tilt and translation using an OFRT system of order 1.6, from Kelly et al. [28]. (a) Applied clockwise rotation of 0.0011rad , measured at 0.0011rad . (b) Applied positive translation of $150\mu\text{m}$, measured at $145\mu\text{m}$.

crease in sensitivity of one third. Figure 3.29 shows results from a 1.6 order system, for (a) a tilt of 0.0011rad measured at 0.0011rad, and (b) a translation of 150 μm measured at 145 μm .

Simultaneous Measurement of Translation and Tilt

The technique demonstrated by Kelly et al. [28] can also be used to measure combined surface translation and tilt. This involves two cameras, each arranged with a different order of OFRT, as shown in Figure 3.30. In this case, the equations used to relate the position of the correlation peak to the surface tilt/translation are treated as simultaneous equations, with the peak positions from each camera image used to solve them. This system was used to measure a combined tilt of 0.0011rad and translation of 150 μm , giving a result of 0.0011rad and 147 μm , comparable to the individual measurements. For a larger transformation of 0.0022rad and 300 μm , the errors were slightly larger, with the system measuring 0.0021rad and 317 μm . These second measurements correspond to a percentage error of around 5%.

The same group went on to implement a similar system using a more general LCT [29], of which fractional Fourier transforms are just one example. The generalised system can also independently measure combined surface transforms, but has the advantage of requiring only one camera and optics system. However, as with the previous technique, measurements in two different domains are required in order to solve the formulation of simultaneous equations for both translation and tilt. In this

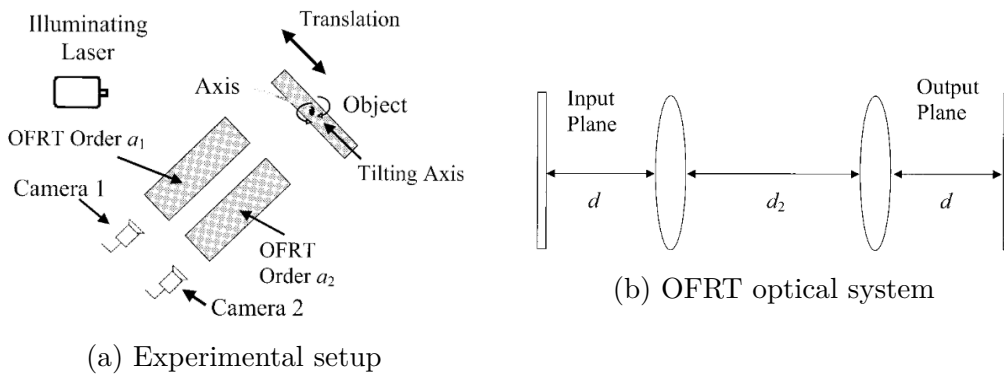


Figure 3.30: Experimental setup used by Kelly et al. [28] employing two OFRT systems of differing order to measure simultaneous tilt and translation movements. One system had OFRT order $a_1 = 1.4$ and the other $a_2 = 1.6$. The order is varied by changing the distance d_2 .

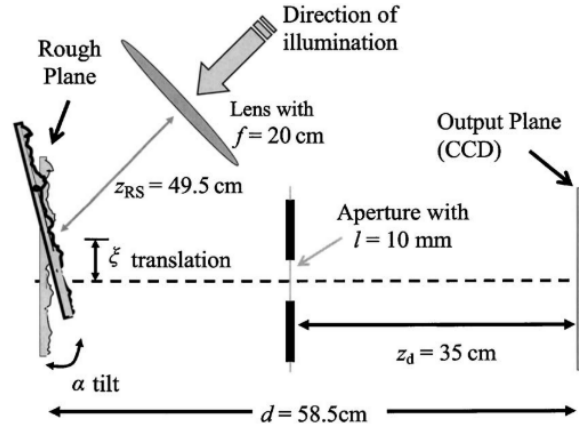


Figure 3.31: Experimental setup used by Kelly et al. [29] to measure simultaneous tilt and translation movements using different LCT domains. The domain is changed by removing the lens and thus changing the curvature of the illumination.

case, the domain of the LCT is varied by changing the curvature of the illumination. This is controlled by adding or removing a lens, shown at the top of the experimental setup in Figure 3.31. With the lens present the incident beam is diverging, and when the lens is removed the light reverts to a plane wave. This arrangement means that the two images needed cannot be captured simultaneously, so the surface must be stationary. The system was tested with a combined surface transformation of 0.54rad and $50.0\mu\text{m}$, which was measured at 0.54rad and $46.2\mu\text{m}$. The larger error in the translation measurement (7.6%) was predicted by the particular LCT parameters used; it would be possible to balance the errors by adjusting these parameters. Adjustment of the LCT can also be used to change the range and sensitivity of the system, as with changing the order of the OFRT in the previous technique.

Another example of a LCT is the Fresnel transform. Hwang and Han [30] proposed using Fresnel transforms (FSTs) to measure tilt and translation without requiring lenses, as shown in their optical setup in Figure 3.32. Each path describes a different FST. Compared to the setup in Figure 3.31 used by Kelly et al. [29], this arrangement uses two cameras to measure in both FST domains simultaneously, thus eliminating the time delay in adjusting the lens system. Each camera captures a double exposure from before and after the transformation, and the resulting images are Fourier transformed to give a fringe pattern. The period of the fringe pattern can be used to calculate both the tilt and translation with direction. The results presented by Hwang and Han [30] are from simulated measurements only,

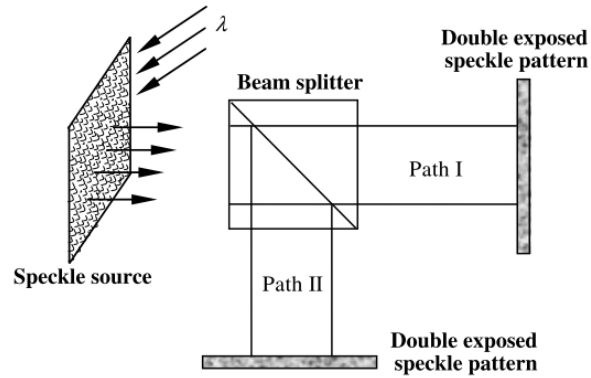
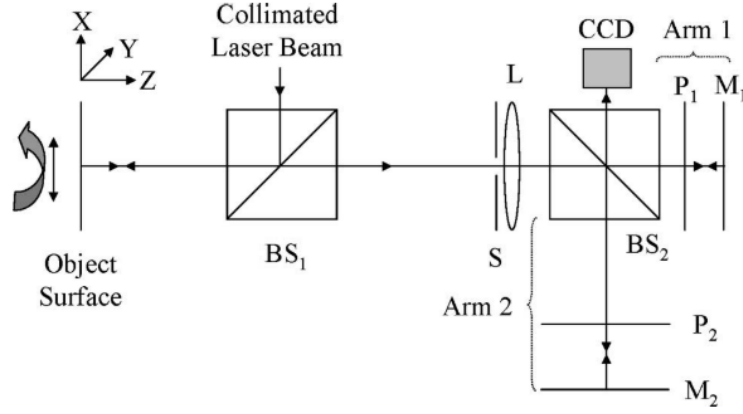


Figure 3.32: Theoretical setup from Hwang and Han [30] for simultaneous translation and tilt measurements using two different FST domains (paths I and II).

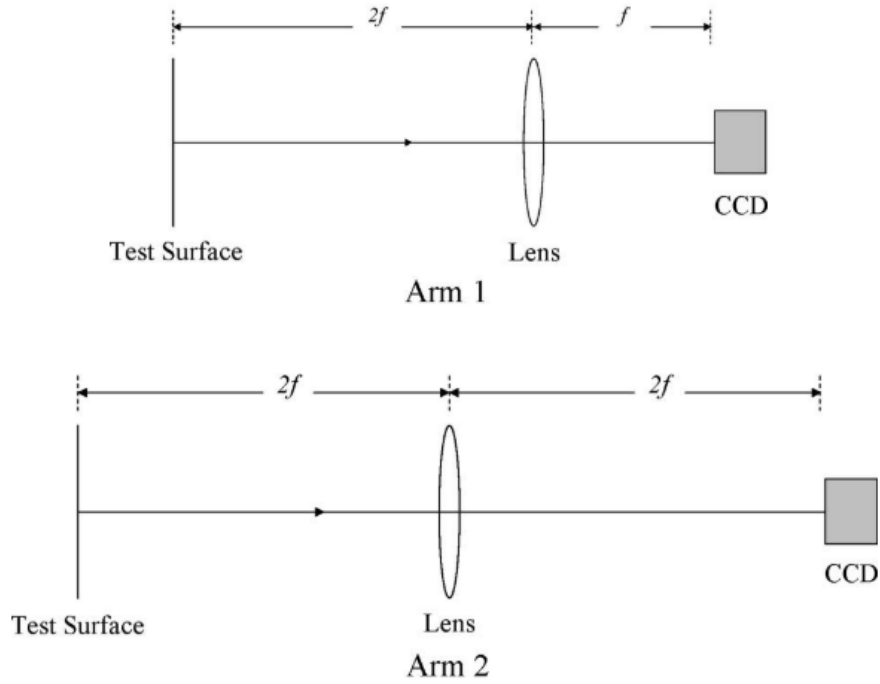
constructing an image with random phase information at each pixel to represent the reflection from an optically rough surface. A combined 2D translation and 2D tilt were applied to the simulated image, and the measured deformations had errors of less than 2% across all values.

Bhaduri et al. [19] aimed to measure combined surface transforms using a “Michelson-style” arrangement, with one arm measuring tilt and the other translation. The first uses an OFT system and so can measure tilt while being insensitive to translation. The second arm is effectively an imaging system with unit magnification, and measures only the surface translation. The two beams are superimposed on the same CCD detector, with each beam polarised differently to avoid interference. This method applies a cross-correlation to a before and after image to determine the transformation of the surface. The resulting correlation image has two peaks, one for each arm of the setup. In this way, the translation and tilt measurements remain fully separated.

The paper presented experimental measurements for tilts and translations around and along the x axis, which were determined from the location of each peak along the x axis of the correlation image. These results demonstrate an issue with the technique, which is that a certain degree of knowledge of the surface transformation is required to distinguish the two peaks. Two methods were suggested by the authors to circumvent this, the first being to introduce a known translation along the y axis, moving the translation peak off the x axis in the correlation image. The second method is to use individual reference images for tilt and translation. This entails



(a) Experimental setup



(b) Measurement arms

Figure 3.33: Experimental setup (a), and expanded measurement paths (b) from Bhaduri et al. [19] for taking individual measurements of tilt (arm 1) and translation (arm 2) using a single system. Components: BS₁,BS₂, beam splitters; L, imaging lens; S, aperture stop; M₁,M₂, mirrors; P₁,P₂, polarisers; CCD, charge-coupled device (camera).

closing each arm of the system and recording a reference for the other. Then, measurement images (taken using both arms) can be correlated with both reference images, each producing a correlation image with a single peak and thus no ambiguity. Using the first method, measurements taken using the setup were found to deviate from expected values by less than 2% for translation and less than 1% for tilt.

3.2.3 Speckle Interferometry

Speckle interferometry involves combining two beams of light from an optically rough surface to create an interference pattern. One example of this is electronic speckle pattern interferometry (ESPI), which has been used to measure tilts [31] and translations [32], vibrations [33], and stress and strain [34]. Another example is speckle shear interferometry, or shearography. This technique can be used to measure surface tilts [35, 36], as well as strain [37] and vibration [38].

ESPI has been most widely used for translation measurements, including in- and out-of-plane displacements [32]. Figure 3.34 shows typical ESPI setups for both measurement regimes, taken from Yang et al. [32]. For out-of-plane displacements

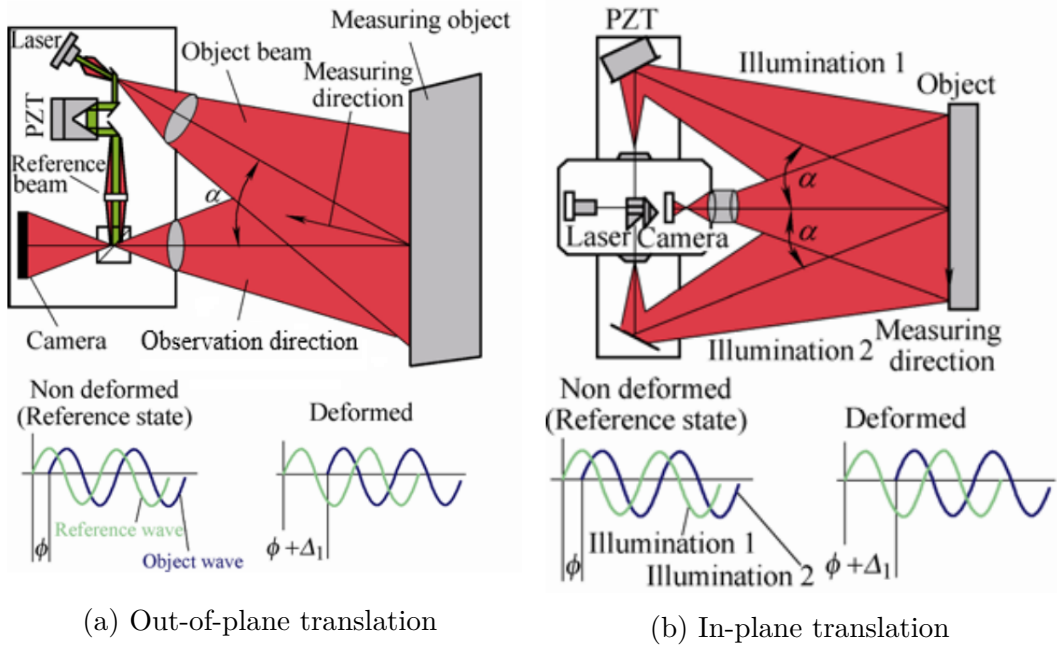


Figure 3.34: Typical ESPI setups for measuring (a) out-of-plane, and (b) in-plane surface translations, taken from Yang et al. [32]. Components: PZT, piezoelectric transducer.

(Figure 3.34a) a single laser beam is used, which is split into a reference beam and an object beam and then recombined at the detector, giving a speckle pattern. If the object undergoes an out-of-plane displacement, the phase, φ , of the object beam changes and so, therefore, does the amplitude of the combined beams. An image is taken before and after the movement, and the subtraction of the two speckle patterns results in a fringe pattern related to the surface displacement.

As for in-plane measurements (Figure 3.34b), the object is illuminated with two symmetrical beams, both of which are similarly split and recombined. An in-plane translation causes the phase of one object beam to decrease, and the other to increase. The relative phase changes, once again resulting in a fringe pattern when comparing a before and after image of the displacement.

By combining in- and out-of-plane ESPI measurements, it is possible to measure full 3D surface displacements. Broetto and Schajer [39], for instance, used a compact interferometer design incorporating diffraction gratings to split and recombine beams from a single laser. The technique was found to measure displacements in the x and y axes to an accuracy between 0.01-0.05 μm , and in z to an accuracy between 0.03-0.09 μm .

Before publishing the correlation technique described in the previous section, Fricke-Begemann and Burke [31] worked on a 3D deformation field measurement that incorporated tilt measurements using ESPI. The technique uses a speckle interferometer with out-of-plane sensitivity and an off-axis reference beam to measure surface tilt using a before and after image. The phase change in the amplitude field is used to calculate the surface tilt, with various phase-reconstruction algorithms investigated in the paper. Overall, the accuracy of the method was found to be better than 0.053mdeg “under good conditions”, which included the preferred phase-reconstruction algorithm and optimising the speckle correlation. To obtain the full 3D deformation of the surface, a speckle correlation was used to measure the in-plane displacement, using reconstructions of the object intensity fields. This in-plane measurement was found to be accurate to within 0.6 μm .

Looking at shearography, Mohanty et al. [35] used a rotational shearing technique to measure the strain on an eccentrically loaded diaphragm. They noted that the same technique could be used to measure the change in tilt of an object by Fourier filtering a double exposure from before and after the tilt. The technique measures at the

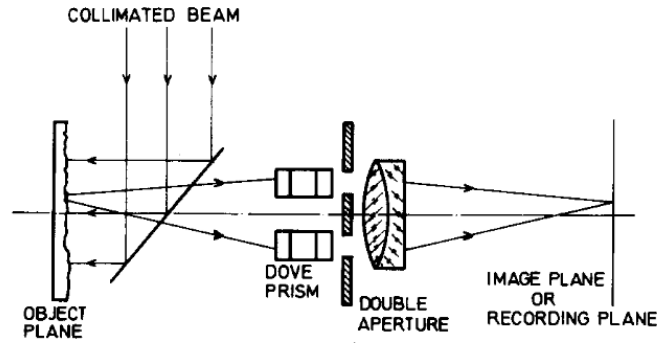


Figure 3.35: Experimental setup used by Mohanty et al. [35] to measure surface tilt using shearography; the Dove prisms induce the shear on each beam.

image plane, rather than the methods by Tiziani [21] and Gregory [22] described above, where the measurement is made in a defocused plane. The experimental setup can be seen in Figure 3.35; it uses two Dove prisms and a double aperture to generate the two imaging beams. The Dove prisms can be translated to change the magnitude of the rotational shear and thus the sensitivity of the sensor.

An adjustment to this system used additional apertures to simultaneously apply an inversion and folding shear [40]. In this setup, adjustment of the relative orientation of the folding and tilt axes changes the sensitivity of the sensor. The system was tested for a surface tilt of 0.050mrad (2.9mdeg), determining the tilt by means of the inversion shear, and two different sensitivities of folding shear. The measured angles were 0.049mrad for the inversion shear, and 0.049mrad and 0.048mrad for the two folding shears. These correspond to percentage errors of 2.0% and 4.0%. An additional advantage of the multi-aperture technique is the ability to measure out-of-plane displacement alongside tilt [41]. The displacement is measured using speckle interferometry with a ground or plain glass aperture, while the prisms apply the shearing effect to measure tilt.

An alternative implementation from this group for tilt measurements only used two right angle prisms to apply the shears [42]. The experimental setup is shown in Figure 3.36; prisms A and B are mounted on rotation/translation stages, which allows the applied shear to be varied while using the same setup. A 0.020mrad tilt was measured at 0.021mrad using an inversion shear (5.0% error), and a tilt of 0.040mrad was measured at 0.039mrad with a folding shear (2.5% error).

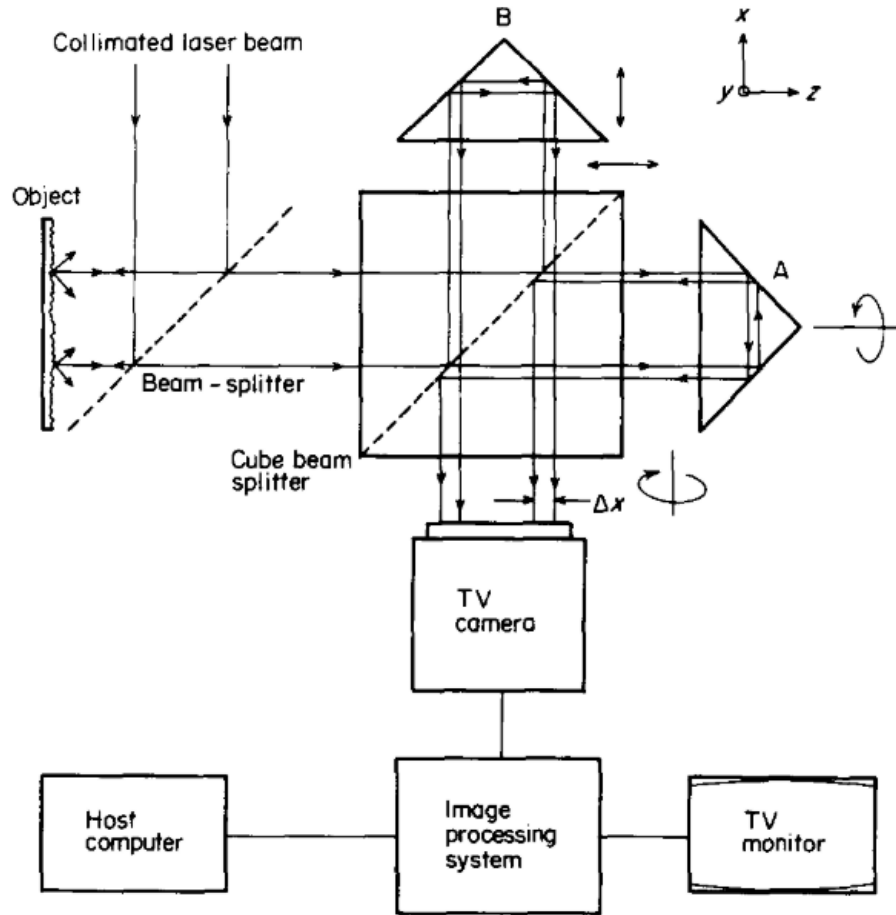


Figure 3.36: Experimental setup used by Sirohi et al. [42] to measure surface tilt using shearography. The two right angle prisms (A and B) can be translated and rotated to apply various shears.

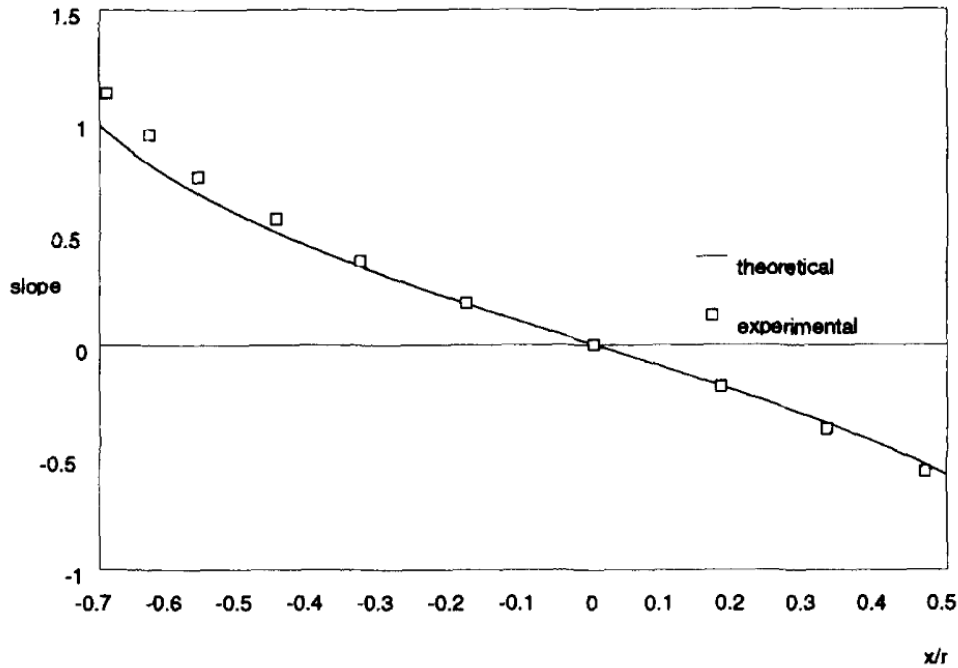


Figure 3.37: Experimental vs. theoretical slope of a cross section of a cylinder, measured by Huang et al. [43] using shearography.

Shearography can also be used to measure the changing slope across an object such as a cylinder. Huang et al. [43] used a two wavelength approach, taking an image with one wavelength and then another, while applying a 4mm lateral shear. A correlation of the two images produces fringes whose features relate to the object's slope. The technique was used to measure a cross section of the slope of a cone, cylinder, and sphere. The measurement also depended on the working distance of the collinear source and detector, and so for a measurement distance of 1000mm, the maximum percentage errors were 6.24%, 4.80%, and 6.20% for the cone, cylinder, and sphere respectively. Doubling the working distance to 2000mm halved these errors, resulting in maximum deviations of 3.12%, 2.40%, and 3.10% for the three shapes.

A similar approach from Groves et al. [44] used a change in the angle of incidence of the source to measure the shape of an object. An image was taken before and after the source displacement, each with a lateral shear of 10mm applied to the light from the object. The experimental setup is shown in Figure 3.38. The resulting fringes from subtracting the two images describe the two dimensional shape of the

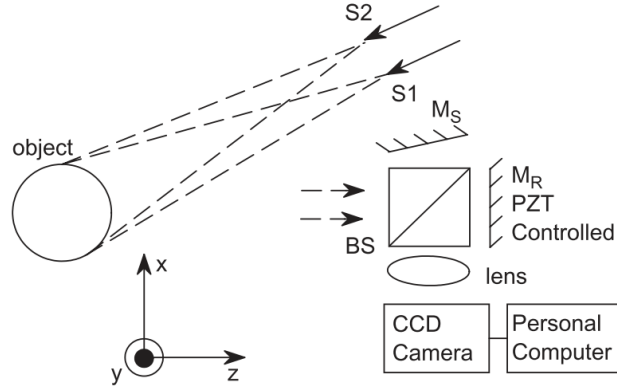


Figure 3.38: Experimental setup used by Groves et al. [44] to measure object shape using shearography. Components: S1,S2, source positions; BS, beam splitter; M_S , shearing mirror; M_R , reference mirror; PZT, piezoelectric transducer.

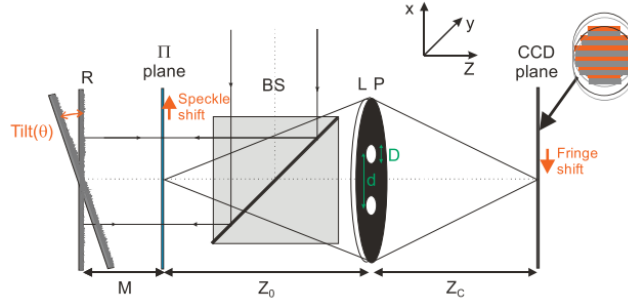


Figure 3.39: Experimental setup used by Molina Prado et al. [36] to measure surface tilt with a double-aperture pupil mask. Components and dimensions: R, surface; L, lens; P, double aperture pupil mask; D, pupil diameter; d, aperture separation; θ , surface tilting angle; M, distance from surface to plane P; Z_0 , object distance; Z_C , image distance.

object (as compared to the single dimension measurement from the above technique). A 900mm pipe with diameter 90mm was measured using the source displacement technique, finding a maximum deviation in measurements of 2mm, or 10%.

Another interferometric method for tilt measurements was developed by Molina Prado et al. [38]. The technique was first developed for use on vibrating surfaces, and used to map the regions where the slope of the vibrations had the same pitch. They moved on to using a similar system to measure static surface tilts [36]. The technique uses a double-aperture pupil lens system to effectively convert the speckle pattern generated from the surface into a fringe pattern. These are recorded using a CCD camera, and converted into intensity profiles. Their experimental setup can

be seen in Figure 3.39. As the surface tilt changes, the intensity profile of the fringe pattern shifts linearly in the camera plane. By tracking the displacement of the peaks in the intensity profile, the change in angle of the surface can be calculated. As the technique looks at the difference in position of the intensity peak, it is only able to measure relative changes in tilt from a known reference angle. The experimental data presented looked at angles in the range $\pm 1.20\text{mrad}$, using a rotation stage with an accuracy of 0.20mrad . The reported margin of error on angles measured using the technique was 0.02mrad , or 1.1mdeg .

3.2.4 Summary of Tilt Sensing Techniques using Speckle

All of the techniques outlined in this section measure relative motion, requiring either two images or a known reference to compare. Initial speckle techniques used photographic plates, often employing a double exposure to capture both speckle patterns. Tiziani [21] measured surface tilt of 60arcsec to an accuracy of 4% using speckle photography.

For correlation techniques, Smíd et al. [20] had a unique approach, applying known translations to the object in order to map its shape. The presented results measured the tilt of a flat surface, but the technique could also be used to map an irregular object. The addition of a third detector would remove the requirement to know the applied translation, but would introduce additional components to an already relatively complex system. The method demonstrated by Rose et al. [25] could be used in two dimensions, measuring both the rotation of a shaft and its tilt. The paper also demonstrated its insensitivity to target distance, showing that it could be used for irregular objects as well as smooth cylinders. Finally, the technique proposed by Fricke-Begemann [26] using cross power spectra could be used for measuring very small angles, with an error in the RMS deviation of less than 0.03° , despite a significantly higher 5% bias error.

A number of speckle correlation techniques have been adapted to measure both tilt and translation simultaneously, though most require either two cameras or, in the case of Kelly et al. [29], a change in the experimental layout to capture multiple images. The method used by Bhaduri et al. [19] was particularly notable, using a single camera and achieving the smallest percentage error on measurements of 1%.

The speckle interferometry technique from Fricke-Begemann and Burke [31] was

used for measuring small angles, with a stated accuracy of around 0.053mdeg. The shearography technique proposed by Mohanty et al. [40] was also notable for the variable sensitivity afforded by changing the applied shear, plus the ability to measure out-of-plane translation with the same system.

Each of the speckle-based tilt sensing techniques are summarised in Table 3.2. The angle range, if given, is stated for each method, along with the percentage error or resolution/accuracy in some cases.

Table 3.2: Summary of speckle-based tilt sensing techniques, separated into photography, correlation, and interferometry.

Technique	Angle range	Percentage error	Notes
Photography			
OFT, Tiziani [21]	X	4%	Relative
Correlation			
1D imaging, Yamaguchi [18]	$\pm 0.3\text{deg}$	1%	Relative, one dimension
Two cameras, maps surface using translation, Smíd et al. [20]	10-30deg	2deg resolution	Absolute, but requires relative translation, two cameras
Detector in Fourier plane, Rose et al. [25]	0-6deg	0.3mdeg resolution	Relative
Cross power spectrum, Fricke-Begemann [26]	0-4.22deg	5%	Relative, also measures translation using phase
OFRT, Kelly et al. [28]	$\pm 0.498\text{deg}$	5%	Relative, simultaneous tilt and in-plane translation, two cameras

Generalised LCT, Kelly et al. [29]	X	7.6%	Relative, simultaneous tilt and in-plane translation, two before and two after images
FST, Hwang and Han [30]	X	2%	Relative, simultaneous tilt and in-plane translation, two cameras, modelled data only
OFT, Bhaduri et al. [19]	X	1%	Relative, simultaneous tilt and in-plane translation
<hr/> Interferometry <hr/>			
ESPI, Fricke-Begemann and Burke [31]	X	0.053mdeg accuracy	Relative, also measures translation using correlation
Shearography, inversion and folding shears, Mohanty et al. [40]	2.86mdeg (single measurement)	4%	Relative, variable sensitivity, simultaneous out-of-plane translation
Shearography, inversion and folding shears, Sirohi et al. [42]	1.15&2.30mdeg (single measurements)	5.0&2.5%	Relative, measured with various shears in same setup
Shearography, lateral shear, measures slope, Huang et al. [43]	X	3.12% max.	Relative, changes source wavelength, can measure cross sections of objects with varying slope

Shearography, lateral shear, X measures shape, Groves et al. [44]	10% max.	Relative, uses changing source position, full shape measurement (with varying slope)
Double aperture pupil, $\pm 68.8\text{mdeg}$ Molina Prado et al. [36]	1.7%	Relative

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Chapter 4

Concept and Formulation of Dual-Wavelength Speckle

This chapter will outline the principles of the technique of dual-wavelength speckle for angle sensing, and derive the theory supporting the technique. Concisely, the technique uses a dual-wavelength laser source to illuminate a surface, producing two speckle patterns. The separation between these patterns (the speckle shift) will be shown to be mathematically related to the angle of the surface.

Section 4.1 gives a conceptual overview of how speckle patterns are affected by surface angle, and how the separate patterns produced by two distinct wavelengths allow an absolute measurement of angle.

Following this, Section 4.2 gives a detailed derivation of the theoretical equations relating surface angle to speckle shift. Note that an experimental demonstration of the theory can be found in Chapter 6.

The final section builds on the general theory introduced in Sections 4.1 and 4.2 by considering the specifics of the sensor arrangement and geometry. These terms are used here to describe the positioning of the sensor elements in relation to each other, in both general (arrangement) and more specific (geometry) terms. Section 4.3 also outlines the advantages and disadvantages of different camera and laser types, describing how these will affect the characteristics of the sensor such as range, resolution, cost, and complexity.

4.1 Description of the Technique

An objective speckle pattern generated by a surface is dependent on the angle of the surface, and the wavelength of the light source used. The sensor concept uses these two factors, illuminating the surface with two wavelengths simultaneously to generate two distinct but correlated speckle patterns. Each of these patterns will then be affected slightly differently by a change in the tilt of the surface. Specifically, the separation between the two speckle patterns will change for different surface tilts. This forms the basis of the dual-wavelength speckle (DWS) technique.

A general diagram showing the setup and concept is given in Figure 4.1. S is the position of the dual-wavelength source, specifically the focus from which the beam appears to originate. The detector is positioned at D . Both are above a flat surface that can tilt in two dimensions. The origin of the coordinate system is placed at the laser spot, where the illumination is incident on the surface. The surface tilt is described by rotations around the x and y axes, given by the angles θ_x and θ_y respectively.

The similarity and displacement between the two speckle patterns can be seen in

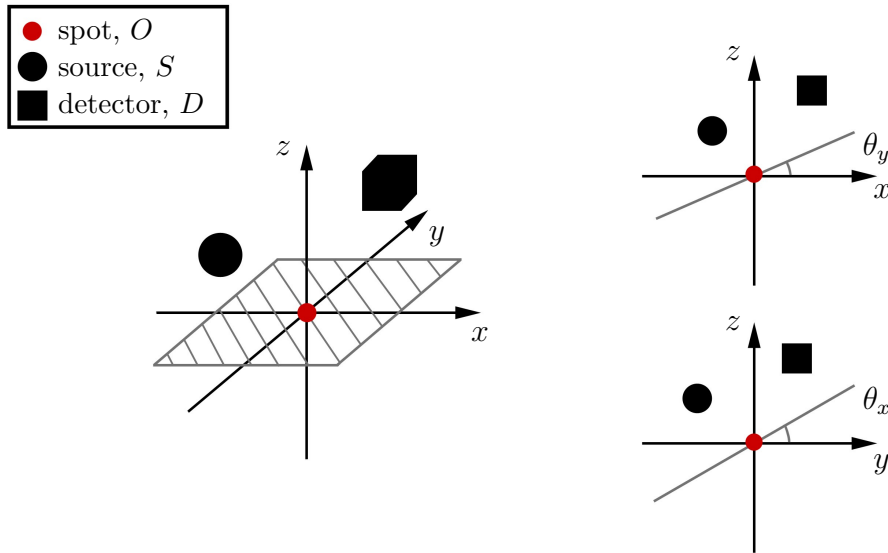


Figure 4.1: Diagram demonstrating the basic setup of DWS. A dual-wavelength source and detector are positioned above a surface, which can be tilted by θ_x and θ_y around the x and y axes respectively.

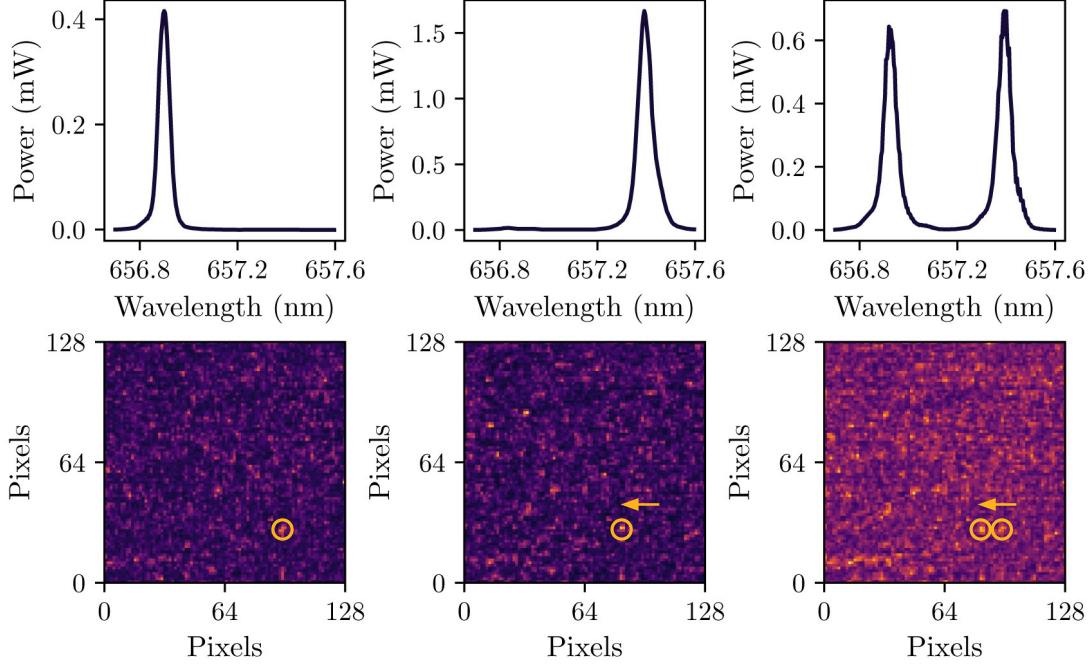


Figure 4.2: Speckle patterns generated by different wavelengths of light. Top: Laser spectra recorded using an OSA. Bottom: Objective speckle patterns recorded with a CMOS camera. The surface was first illuminated by light of wavelength 656.9nm, then 657.4nm, which caused the speckle pattern to shift slightly to the left. Finally, the surface was illuminated with both wavelengths simultaneously, generating the two offset speckle patterns at the same time.

Figure 4.2. The top row of the figure shows the laser spectra as recorded by an optical spectrum analyser (OSA), while the speckle patterns in the bottom row were captured with a complementary metal-oxide semiconductor (CMOS) camera. The first two columns show the speckle patterns generated by the individual illumination with the corresponding wavelength, 656.9nm for λ_1 and 657.4nm for λ_2 . In each case, the distribution of speckles is the same, although some are brighter or dimmer across the images. However, there is a horizontal shift between each pattern, shown by the highlighted speckle. The third image shows the speckle pattern recorded when the surface is illuminated with both wavelengths simultaneously. The two patterns are overlapping, and again the speckle from the previous images is highlighted in both.

The displacement between the two patterns can be found with a 2D autocorrelation. In an autocorrelation, a single image is compared with itself, used as both the template and the base. This produces a correlation image with three peaks; one

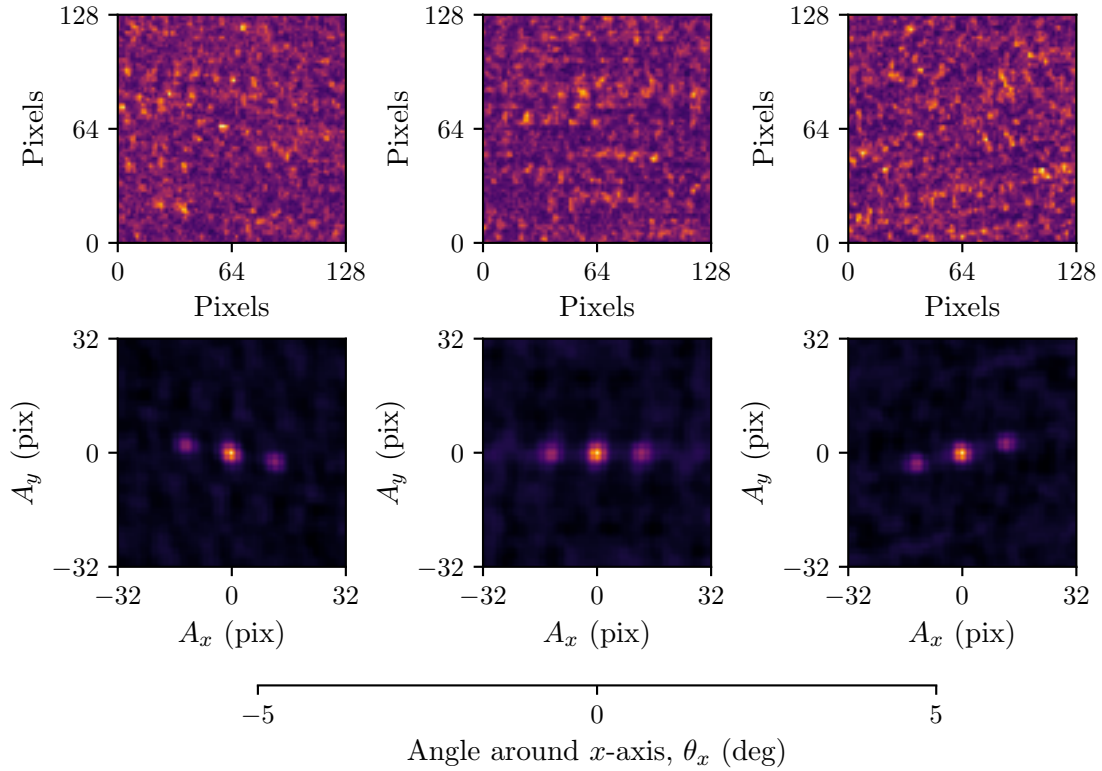


Figure 4.3: Speckle patterns and corresponding correlation images for a surface at various tilts around the x -axis and illuminated with two wavelengths simultaneously. As the surface moves, the magnitude and direction of the shift between the two speckle patterns changes accordingly.

central peak at zero shift representing the correlation between the full image and itself, plus two side peaks for the correlation of pattern 1 with pattern 2, and pattern 2 with pattern 1. The side peaks are symmetric about the centre of the image, and the distance between them and the central peak is equal to the separation of the two speckle patterns. This signal processing is explained in more detail in the following chapter in Section 5.1.

Figure 4.3 shows some example speckle patterns and their corresponding autocorrelations for three different tilts around the x axis. In each case, the relative separation of the two speckle patterns, referred to as the speckle shift, has a different magnitude and direction. This is shown in the correlation images by the change in position of the side peaks. The speckle shift can therefore be calculated by finding the position of one of the side peaks. It is important that the same side peak is used for this, as the opposite peak describes the negative shift and thus the inverse angle.

As demonstrated by Figure 4.3, a tilt around the x axis produces a change in the y component of the speckle shift A_y . The reverse is also true; a surface tilt around the y axis results in a change in speckle shift in the x direction, A_x . In this way, the surface tilt can be calculated in two dimensions.

4.2 Formalising the Technique

4.2.1 Definition of Variables

The precise relationship between source wavelengths, speckle shift, and surface tilt can be formalised mathematically. The derivation follows the methods described previously in Yamaguchi [1], Horváth et al. [2], and Charrett and Tatam [3]. Yamaguchi [1] first derived the theory, using it to investigate the change in speckle structure and displacement for object translations, rotations, strains, and combinations of these deformations. Horváth et al. [2] extended this theory, relating it to the small deformation tensors and applying it to more complex optical systems. Finally, Charrett and Tatam [3] looked more specifically at the deformation of a tilted surface as it relates to a single wavelength. Here, this theory is extended to apply it to two wavelengths.

The first step in any version of the derivation is to define the components of the system and their relative positions. Figure 4.4 shows a generalised surface illuminated by two collinear wavelengths of light, λ_1 and λ_2 , incident at an arbitrary point $\mathbf{P} = (x, y, z)$. Note that later in the derivation, an integration is performed over all points, $\mathbf{P}$, covered by the illumination spot. The surface is assumed to be parallel to the xy plane, as is the detector plane.

The source origin, the focal point of a diverging beam, is described by the position vector $\mathbf{S} = (S_x, S_y, S_z)$. The vector from the point $\mathbf{P}$ to the source is given by $\mathbf{L}_S(\mathbf{P})$. The scattered light from the surface is recorded by a detector at $\mathbf{D} = (D_x, D_y, D_z)$. The vector $\mathbf{L}_D(\mathbf{P}, \mathbf{D})$ defines the path from $\mathbf{P}$ to a speckle formed by λ_1 at $\mathbf{D}$. The equivalent speckle formed by λ_2 has a relative shift at the detector, given by $\mathbf{A} = (A_x, A_y, 0)$. The vector describing this path is $\mathbf{L}_D(\mathbf{P}, \mathbf{D} + \mathbf{A})$. As the speckle patterns are measured in the xy plane, the shift in z , A_z , is assumed to be zero. The full expressions for $\mathbf{L}_S(\mathbf{P})$, $\mathbf{L}_D(\mathbf{P}, \mathbf{D})$, and $\mathbf{L}_D(\mathbf{P}, \mathbf{D} + \mathbf{A})$ are given

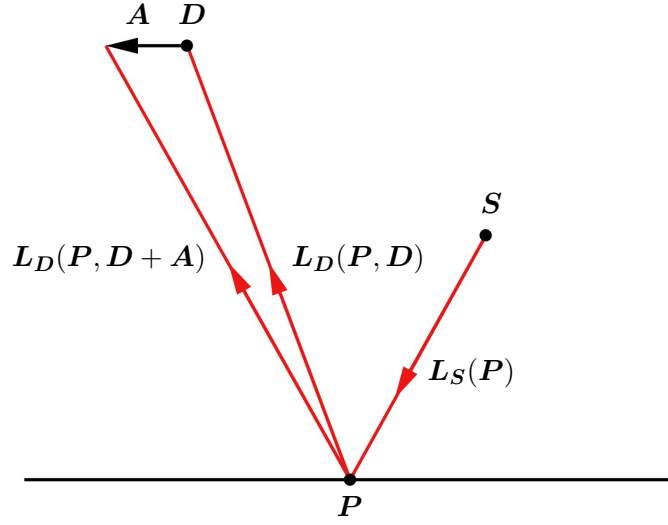


Figure 4.4: A surface is illuminated by a laser source, $\mathbf{S} = (S_x, S_y, S_z)$, comprised of two collinear wavelengths. The light follows the vector $\mathbf{L}_S(\mathbf{P})$ to an arbitrary point $\mathbf{P} = (x, y, z)$ on the surface. The two wavelengths are scattered from $\mathbf{P}$, forming two speckle patterns. Considering a corresponding speckle from both patterns, each follows a different path to the detector $\mathbf{D} = (D_x, D_y, D_z)$, $\mathbf{L}_D(\mathbf{P}, \mathbf{D})$ and $\mathbf{L}_D(\mathbf{P}, \mathbf{D} + \mathbf{A})$. $\mathbf{A} = (A_x, A_y, 0)$ describes the shift in the two speckle patterns at the detector.

below:

$$\mathbf{L}_S(\mathbf{P}) = \mathbf{S} - \mathbf{P} = (S_x - x, S_y - y, S_z - z) \quad (4.2.1)$$

$$\mathbf{L}_D(\mathbf{P}, \mathbf{D}) = \mathbf{D} - \mathbf{P} = (D_x - x, D_y - y, D_z - z) \quad (4.2.2)$$

$$\mathbf{L}_D(\mathbf{P}, \mathbf{D} + \mathbf{A}) = (\mathbf{D} + \mathbf{A}) - \mathbf{P} = ((D_x + A_x) - x, (D_y + A_y) - y, D_z - z) \quad (4.2.3)$$

It is also useful to define the unit vectors from $\mathbf{P}$ to the source, and $\mathbf{P}$ to the detector. These are given below, substituting the variables $|\mathbf{L}_S(\mathbf{P})| = L_S$ and $|\mathbf{L}_D(\mathbf{P}, \mathbf{D})| = L_D$.

$$\hat{\mathbf{s}} = \frac{\mathbf{S} - \mathbf{P}}{|\mathbf{S} - \mathbf{P}|} = \left(\frac{S_x - x}{L_S}, \frac{S_y - y}{L_S}, \frac{S_z - z}{L_S} \right) = (s_x, s_y, s_z) \quad (4.2.4)$$

$$\hat{\mathbf{d}} = \frac{\mathbf{D} - \mathbf{P}}{|\mathbf{D} - \mathbf{P}|} = \left(\frac{D_x - x}{L_D}, \frac{D_y - y}{L_D}, \frac{D_z - z}{L_D} \right) = (d_x, d_y, d_z) \quad (4.2.5)$$

For reference, the full values of L_S and L_D are given below.

$$L_S = \sqrt{(S_x - x)^2 + (S_y - y)^2 + (S_z - z)^2} \quad (4.2.6)$$

$$L_D = \sqrt{(D_x - x)^2 + (D_y - y)^2 + (D_z - z)^2} \quad (4.2.7)$$

For the purpose of this derivation, the tilt of the surface is described in terms of surface gradient. A tilt of θ_y around the y axis is described instead as the gradient in x , m_x . Similarly, a tilt θ_x around the x axis is described by the gradient m_y .

$$\tan \theta_y = m_x = \frac{\Delta z}{\Delta x} \quad (4.2.8)$$

$$\tan \theta_x = m_y = \frac{\Delta z}{\Delta y} \quad (4.2.9)$$

The surface gradients can be incorporated into the unit vectors $\hat{\mathbf{s}}$ and $\hat{\mathbf{d}}$ using the expression $z = m_x x + m_y y$. These forms of the variables will be used to describe the complete relationship between wavelengths, speckle shift, and surface gradient.

$$\hat{\mathbf{s}} = \left(\frac{S_x - x}{L_S}, \frac{S_y - y}{L_S}, \frac{S_z - (m_x x + m_y y)}{L_S} \right) \quad (4.2.10)$$

$$\hat{\mathbf{d}} = \left(\frac{D_x - x}{L_D}, \frac{D_y - y}{L_D}, \frac{D_z - (m_x x + m_y y)}{L_D} \right) \quad (4.2.11)$$

L_S and L_D are also rewritten as follows:

$$L_S = \sqrt{(S_x - x)^2 + (S_y - y)^2 + \left(S_z - (m_x x + m_y y) \right)^2} \quad (4.2.12)$$

$$L_D = \sqrt{(D_x - x)^2 + (D_y - y)^2 + \left(D_z - (m_x x + m_y y) \right)^2} \quad (4.2.13)$$

Finally, the derivation requires partial derivatives of L_D , L_S , d_y , and d_x with respect to x and y . The calculation of these expressions is described in full in Appendix A, and the results are summarised in Table 4.1, to be used throughout the next section.

Table 4.1: Summary of partial derivatives of L_S , L_D , d_x , and d_y in both x and y , for both flat and tilted surfaces.

Variable	Partial derivative in x , $\frac{\partial}{\partial x}$	Partial derivative in y , $\frac{\partial}{\partial y}$
Flat surface		
L_S	$\frac{\partial L_S}{\partial x} = -s_x$	$\frac{\partial L_S}{\partial y} = -s_y$
L_D	$\frac{\partial L_D}{\partial x} = -d_x$	$\frac{\partial L_D}{\partial y} = -d_y$
d_x	$\frac{\partial d_x}{\partial x} = \frac{1}{L_D}(d_x^2 - 1)$	$\frac{\partial d_x}{\partial y} = \frac{1}{L_D}(d_x d_y)$
d_y	$\frac{\partial d_y}{\partial x} = \frac{1}{L_D}(d_x d_y)$	$\frac{\partial d_y}{\partial y} = \frac{1}{L_D}(d_y^2 - 1)$
Tilted surface		
L_S	$\frac{\partial L_S}{\partial x} = -m_x s_z - s_x$	$\frac{\partial L_S}{\partial y} = -m_y s_z - s_y$
L_D	$\frac{\partial L_D}{\partial x} = -m_x d_z - d_x$	$\frac{\partial L_D}{\partial y} = -m_y d_z - d_y$
d_x	$\frac{\partial d_x}{\partial x} = \frac{1}{L_D}(m_x d_x d_z + d_x^2 - 1)$	$\frac{\partial d_x}{\partial y} = \frac{1}{L_D}(m_y d_x d_z + d_x d_y)$
d_y	$\frac{\partial d_y}{\partial x} = \frac{1}{L_D}(m_x d_y d_z + d_x d_y)$	$\frac{\partial d_y}{\partial y} = \frac{1}{L_D}(m_y d_y d_z + d_y^2 - 1)$

4.2.2 Speckle Shift Between Two Wavelengths

The aim is to derive expressions for the shift, $\mathbf{A} = (A_x, A_y, 0)$, between the two speckle patterns recorded at the detector. Yamaguchi [1] investigated the shift of a speckle pattern generated using a single wavelength of light between object deformation, deriving a full description of the speckle shift resulting from any combination of translation, rotation, and strain of the surface.

Following Yamaguchi's method, the derivation uses the complex amplitudes of the two light sources, U_1 and U_2 , to calculate the cross correlation of their intensities I_1 and I_2 .

$$\langle I_1 I_2 \rangle = \langle |U_1|^2 \rangle \langle |U_2|^2 \rangle + |\langle U_1 U_2^* \rangle|^2 \quad (4.2.14)$$

The second term of this expression is equal to the cross correlation of the change in the intensities. These values are each given by $\Delta I_1 = I_1(\mathbf{P}) - \langle I_1(\mathbf{P}) \rangle$ and $\Delta I_2 = I_2(\mathbf{P}) - \langle I_2(\mathbf{P}) \rangle$, and their cross correlation, taken from Yamaguchi [1], is:

$$\langle \Delta I_1 \Delta I_2 \rangle = |\langle U_1 U_2^* \rangle|^2 \quad (4.2.15)$$

The complex amplitude at the detector $\mathbf{D} = (D_x, D_y, D_z)$ is the integral of the light scattered from all illuminated points $\mathbf{P} = (x, y, z)$. The following equation uses $n \in [1, 2]$ to represent both wavelengths λ_1 and λ_2 .

$$U_n(\mathbf{P}, \mathbf{D}) = \iint K_n(\mathbf{P}, \mathbf{D}) \sqrt{I_n(\mathbf{P})} \exp(i\varphi) \exp(ik_n L_S(\mathbf{P})) d^2 \mathbf{P} \quad (4.2.16)$$

The intensity at the detector from $\mathbf{P}$ is given by $I_n(\mathbf{P})$. φ and k are the phase and wavenumber of the illuminating light, where $k = 2\pi/\lambda$. $L_S(\mathbf{P})$ is the magnitude of the vector $\mathbf{L}_S(\mathbf{P})$, i.e. $L_S(\mathbf{P}) = |\mathbf{L}_S(\mathbf{P})|$.

The variable $K(\mathbf{P}, \mathbf{D})$ is the transmission function from $\mathbf{P}$ to the detector. For objective speckle:

$$K_n(\mathbf{P}, \mathbf{D}) = \exp(ik_n L_D(\mathbf{P}, \mathbf{D})) \quad (4.2.17)$$

Unlike the derivation by Yamaguchi [1], there is no additional factor in the expressions (4.2.16) and (4.2.17) accounting for additional deformations of the surface, such as translation or strain. This would be represented as a speckle shift at the surface, but here both patterns are imaged simultaneously and so there would be no such change.

U_1 and U_2 for each wavelength λ_1 and λ_2 are given below, using the two distances $L_D(\mathbf{P}, \mathbf{D})$ and $L_D(\mathbf{P}, \mathbf{D} + \mathbf{A})$ which describe the paths of the corresponding speckle in each pattern. The expressions also substitute the wavenumbers k_1 and k_2 , where $k_1 = 2\pi/\lambda_1$ and $k_2 = 2\pi/\lambda_2$. The speckle shift $\mathbf{A} = (A_x, A_y, 0)$ is explicit in equation (4.2.19). Note also that the second form of equation (4.2.19) has the additional factor $k_1/k_1 = 1$, which will allow for a simplification of the equation later.

$$U_1(\mathbf{P}, \mathbf{D}) = \iint \exp\left(ik_1 L_D(\mathbf{P}, \mathbf{D})\right) \sqrt{I_1(\mathbf{P})} \exp(i\varphi) \exp\left(ik_1 L_S(\mathbf{P})\right) d^2 \mathbf{P} \quad (4.2.18)$$

$$\begin{aligned} U_2(\mathbf{P}, \mathbf{D} + \mathbf{A}) &= \iint \exp\left(ik_2 L_D(\mathbf{P}, \mathbf{D} + \mathbf{A})\right) \sqrt{I_2(\mathbf{P})} \exp(i\varphi) \\ &\quad \times \exp\left(ik_2 L_S(\mathbf{P})\right) d^2 \mathbf{P} \\ &= \iint \exp\left(ik_1 \frac{k_2}{k_1} L_D(\mathbf{P}, \mathbf{D} + \mathbf{A})\right) \sqrt{I_2(\mathbf{P})} \exp(i\varphi) \\ &\quad \times \exp\left(ik_1 \frac{k_2}{k_1} L_S(\mathbf{P})\right) d^2 \mathbf{P} \end{aligned} \quad (4.2.19)$$

Similarly to $L_S(\mathbf{P})$, $L_D(\mathbf{P}, \mathbf{D})$ is the magnitude of the vector from $\mathbf{P}$ to the detector, given by $L_D(\mathbf{P}, \mathbf{D}) = |\mathbf{L}_D(\mathbf{P}, \mathbf{D})|$. Unlike $L_S(\mathbf{P})$, however, $L_D(\mathbf{P}, \mathbf{D})$ is a function of $\mathbf{D}$, and so is affected by the speckle shift $\mathbf{A}$ of the second wavelength. Hence the two different terms $L_D(\mathbf{P}, \mathbf{D})$ and $L_D(\mathbf{P}, \mathbf{D} + \mathbf{A})$ for U_1 and U_2 .

In order to calculate the cross correlation, the complex conjugate of $U_2(\mathbf{P}, \mathbf{D} + \mathbf{A})$ is required. The full expression for this is:

$$\begin{aligned} U_2^*(\mathbf{P}, \mathbf{D} + \mathbf{A}) &= \iint -\exp\left(ik_1 \frac{k_2}{k_1} L_D(\mathbf{P}, \mathbf{D} + \mathbf{A})\right) \times \sqrt{I_2(\mathbf{P})} \times -\exp(i\varphi) \\ &\quad \times -\exp\left(ik_1 \frac{k_2}{k_1} L_S(\mathbf{P})\right) d^2 \mathbf{P} \end{aligned} \quad (4.2.20)$$

Combining this with equation (4.2.18), the value of $\langle U_1 U_2^* \rangle$ can be calculated and simplified. It is assumed that the intensities of the two beams are equal, giving the

substitution $I_1(\mathbf{P}) = I_2(\mathbf{P}) = I(\mathbf{P})$.

$$\begin{aligned}
\langle U_1 U_2^* \rangle &= \iint \exp(i k_1 L_D(\mathbf{P}, \mathbf{D})) \times \sqrt{I(\mathbf{P})} \times \exp(i\varphi) \times \exp(i k_1 L_S(\mathbf{P})) \\
&\quad \times -\exp\left(i k_1 \frac{k_2}{k_1} L_D(\mathbf{P}, \mathbf{D} + \mathbf{A})\right) \times \sqrt{I(\mathbf{P})} \\
&\quad \times -\exp(i\varphi) \times -\exp\left(i k_1 \frac{k_2}{k_1} L_S(\mathbf{P})\right) d^2 \mathbf{P} \quad (4.2.21) \\
&= \iint I(\mathbf{P}) \exp\left(i k_1 \left(L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P}) \right. \right. \\
&\quad \left. \left. - \frac{k_2}{k_1} L_D(\mathbf{P}, \mathbf{D} + \mathbf{A}) - \frac{k_2}{k_1} L_S(\mathbf{P}) \right) \right) d^2 \mathbf{P}
\end{aligned}$$

The next stage is to express $L_D(\mathbf{P}, \mathbf{D} + \mathbf{A})$ in terms of $L_D(\mathbf{P}, \mathbf{D})$. There are a number of approximations that can be used to achieve this. A Fresnel, or parabolic, approximation rearranges the expression to the form $a\sqrt{b+1}$, which can then be estimated using a Taylor expansion. This is the mathematical implementation of the Fresnel approximation, used for evaluating diffraction integrals in the near-field (see Chapter 3, Section 3.1.2). The derivation will be continued using this method, but the resulting equations only hold for a detector positioned opposite the laser spot at $\mathbf{D} = (0, 0, D_z)$.

Alternatively, the Taylor expansion can be applied directly, giving a closer approximation for off-axis detector positions as per Charrett and Tatam [3]. This was found to be equivalent to the Fraunhofer approximation for far-field diffraction, and as such is applicable to the general detector position $\mathbf{D} = (D_x, D_y, D_z)$, but reduces to the same expression for on-axis positions. The version of the derivation using the Taylor expansion is completed in the next subsection, following the version using the Fresnel approximation.

Fresnel Approximation

The Fresnel approximation, also called the parabolic approximation, makes use of the form of the definition of L_D , given in equation (4.2.7). The first step is to make the two substitutions $(D_z - z)^2 = L_0^2$ and $(D_x - x)^2 + (D_y - y)^2 = \rho^2$, leading to

the following rearrangement:

$$L_D = \sqrt{\rho^2 + L_0^2} = L_0 \sqrt{\frac{\rho^2}{L_0^2} + 1} \quad (4.2.22)$$

This is now in the form $a\sqrt{b+1}$, of which the first two terms of the Taylor expansion are $a(1 + b/2)$. Applying this to L_D :

$$L_D(\mathbf{P}, \mathbf{D}) \approx L_0 + \frac{L_0}{2} \frac{\rho^2}{L_0^2} = L_0 + \frac{(D_x - x)^2 + (D_y - y)^2}{2L_0} \quad (4.2.23)$$

This is the Fresnel approximation of $L_D(\mathbf{P}, \mathbf{D})$. Noting that $\rho^2 = L_D^2 - L_0^2$, the expression can be further rearranged:

$$\begin{aligned} L_D(\mathbf{P}, \mathbf{D}) &\approx L_0 + \frac{L_D^2 - L_0^2}{2L_0} \\ &= L_0 + \frac{(D_x - x)^2 + (D_y - y)^2 + (D_z - z)^2 - L_0^2}{2L_0} \\ &= L_0 + \frac{|\mathbf{D} - \mathbf{P}|^2 - L_0^2}{2L_0} \\ &= L_0 + \frac{\mathbf{D}^2 + \mathbf{P}^2 - 2\mathbf{D}\mathbf{P} - L_0^2}{2L_0} \end{aligned} \quad (4.2.24)$$

The Fresnel approximation of $L_D(\mathbf{P}, \mathbf{D} + \mathbf{A})$ can be obtained similarly.

$$L_D(\mathbf{P}, \mathbf{D} + \mathbf{A}) \approx L_0 + \frac{(D_x + A_x - x)^2 + (D_y + A_y - y)^2}{2L_0} \quad (4.2.25)$$

This expression uses the same value L_0 because in $\mathbf{A} = (A_x, A_y, 0)$, $A_z = 0$. Using the generalised A_z would give a value equal to $(D_z + A_z - z)^2$. As L_0 is the same, $L_D(\mathbf{P}, \mathbf{D} + \mathbf{A})$ can also be rearranged:

$$\begin{aligned} L_D(\mathbf{P}, \mathbf{D} + \mathbf{A}) &\approx L_0 + \frac{|\mathbf{D} + \mathbf{A} - \mathbf{P}|^2 - L_0^2}{2L_0} \\ &= L_0 + \frac{\mathbf{D}^2 + \mathbf{A}^2 + \mathbf{P}^2 + 2\mathbf{D}\mathbf{A} - 2\mathbf{D}\mathbf{P} - 2\mathbf{A}\mathbf{P} - L_0^2}{2L_0} \end{aligned} \quad (4.2.26)$$

Now equations (4.2.24) and (4.2.26) can be used to find the following relation:

$$\begin{aligned}
L_D(\mathbf{P}, \mathbf{D}) - L_D(\mathbf{P}, \mathbf{D} + \mathbf{A}) &= L_0 + \frac{\mathbf{D}^2 + \mathbf{P}^2 - 2\mathbf{D}\mathbf{P} - L_0^2}{2L_0} \\
&\quad - L_0 - \frac{\mathbf{D}^2 + \mathbf{A}^2 + \mathbf{P}^2 + 2\mathbf{D}\mathbf{A} - 2\mathbf{D}\mathbf{P} - 2\mathbf{A}\mathbf{P} - L_0^2}{2L_0} \quad (4.2.27) \\
&= \frac{-\mathbf{A}^2 - 2\mathbf{D}\mathbf{A} + 2\mathbf{A}\mathbf{P}}{2L_0} \\
&= -\frac{\mathbf{A}^2}{2L_0} - \frac{\mathbf{D}\mathbf{A}}{L_0} + \frac{\mathbf{A}\mathbf{P}}{L_0}
\end{aligned}$$

Rearranging:

$$L_D(\mathbf{P}, \mathbf{D} + \mathbf{A}) = L_D(\mathbf{P}, \mathbf{D}) + \frac{\mathbf{A}^2}{2L_0} + \frac{\mathbf{D}\mathbf{A}}{L_0} - \frac{\mathbf{A}\mathbf{P}}{L_0} \quad (4.2.28)$$

This expression can now be substituted into equation (4.2.21):

$$\begin{aligned}
\langle U_1 U_2^* \rangle &= \iint I(\mathbf{P}) \exp \left(ik_1 \left(L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P}) \right. \right. \\
&\quad \left. \left. - \frac{k_2}{k_1} L_D(\mathbf{P}, \mathbf{D}) + \frac{k_2}{k_1} \left(\frac{\mathbf{A}^2}{2L_0} + \frac{\mathbf{D}\mathbf{A}}{L_0} - \frac{\mathbf{A}\mathbf{P}}{L_0} \right) - \frac{k_2}{k_1} L_S(\mathbf{P}) \right) \right) d^2 \mathbf{P} \quad (4.2.29) \\
&= \iint I(\mathbf{P}) \exp \left(ik_1 \left(\left(1 - \frac{k_2}{k_1} \right) (L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P})) \right. \right. \\
&\quad \left. \left. - \frac{k_2}{k_1} \left(\frac{\mathbf{A}^2}{2L_0} + \frac{\mathbf{D}\mathbf{A}}{L_0} - \frac{\mathbf{A}\mathbf{P}}{L_0} \right) \right) \right) d^2 \mathbf{P}
\end{aligned}$$

Next, taking the expression $(1 - k_2/k_1)(L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P}))$, a Taylor expansion is applied around $\mathbf{P} = 0$.

$$\begin{aligned}
\left(1 - \frac{k_2}{k_1} \right) (L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P})) &\approx \left(1 - \frac{k_2}{k_1} \right) (L_D(0, \mathbf{D}) + L_S(0)) \\
&\quad + \left(1 - \frac{k_2}{k_1} \right) \nabla [L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P})]_0 \cdot \mathbf{P} \quad (4.2.30)
\end{aligned}$$

This is substituted into equation (4.2.29), with the constants taken outside of the integral.

$$\begin{aligned} \langle U_1 U_2^* \rangle = & \exp \left(i k_1 \left(\left(1 - \frac{k_2}{k_1} \right) (L_D(0, \mathbf{D}) + L_S(0)) - \frac{k_2}{k_1} \left(\frac{\mathbf{A}^2}{2L_0} + \frac{\mathbf{D}\mathbf{A}}{L_0} \right) \right) \right) \\ & \times \iint I(\mathbf{P}) \exp \left(i k_1 \left(\left(1 - \frac{k_2}{k_1} \right) \nabla [L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P})]_0 + \frac{k_2}{k_1} \left(\frac{\mathbf{A}}{L_0} \right) \cdot \mathbf{P} \right) \right) d^2 \mathbf{P} \end{aligned} \quad (4.2.31)$$

Now, it is clear that the integral of the cross correlation $\langle U_1 U_2^* \rangle$ is maximised under the following condition:

$$\left(1 - \frac{k_2}{k_1} \right) \nabla [L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P})]_0 + \frac{k_2}{k_1} \left(\frac{\mathbf{A}}{L_0} \right) = 0 \quad (4.2.32)$$

Therefore the speckle shift $\mathbf{A}$ can be calculated using equation (4.2.32), as so:

$$\mathbf{A} = -L_0 \left(\frac{k_2}{k_1} - 1 \right) \nabla [L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P})]_0 \quad (4.2.33)$$

The wavenumbers in the factor $(k_2/k_1 - 1)$ can be converted back into wavelengths, giving $(\lambda_1/\lambda_2 - 1) = \Delta\lambda/\lambda_1$, where $\Delta\lambda = \lambda_2 - \lambda_1$.

$$\mathbf{A} = -L_0 \frac{\Delta\lambda}{\lambda_1} \nabla [L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P})]_0 \quad (4.2.34)$$

At this stage, the divergence operator (∇) can be applied and the equation separated into the partial derivatives in x and y . This gives the individual components of the speckle shift, A_x and A_y .

$$A_x = -L_0 \frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial L_D}{\partial x} + \frac{\partial L_S}{\partial x} \right) \quad (4.2.35)$$

$$A_y = -L_0 \frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial L_D}{\partial y} + \frac{\partial L_S}{\partial y} \right) \quad (4.2.36)$$

Now, using the partial derivatives from Table 4.1, these equations can be expressed using variables describing the geometry of the system. Below is the relationship for

a flat surface, with no tilt.

$$A_x = L_0 \frac{\Delta\lambda}{\lambda_1} (d_x + s_x) \quad (4.2.37)$$

$$A_y = L_0 \frac{\Delta\lambda}{\lambda_1} (d_y + s_y) \quad (4.2.38)$$

The value L_0 is equal to the distance to the detector in the z direction, and the other variables are the y components of the unit vectors $\hat{\mathbf{d}} = (d_x, d_y, d_z)$ and $\hat{\mathbf{s}} = (s_x, s_y, s_z)$.

In order to apply this to tilt sensing, the substitution $z = m_x x + m_y y$ is made to introduce the surface gradients m_x and m_y . Now, using the second set of partial derivatives from Table 4.1, a pair of equations which fully relate the speckle shift, surface tilt, source wavelengths, and sensor geometry are obtained.

$$\begin{aligned} A_x &= L_0 \frac{\Delta\lambda}{\lambda_1} (d_x + m_x d_z + s_x + m_x s_z) \\ &= L_0 \frac{\Delta\lambda}{\lambda_1} (d_x + s_x) + L_0 \frac{\Delta\lambda}{\lambda_1} (d_z + s_z) m_x \end{aligned} \quad (4.2.39)$$

$$\begin{aligned} A_y &= L_0 \frac{\Delta\lambda}{\lambda_1} (d_y + m_y d_z + s_y + m_y s_z) \\ &= L_0 \frac{\Delta\lambda}{\lambda_1} (d_y + s_y) + L_0 \frac{\Delta\lambda}{\lambda_1} (d_z + s_z) m_y \end{aligned} \quad (4.2.40)$$

Comparing equations (4.2.37) and (4.2.38) with (4.2.39) and (4.2.40), it is clear that the second set is equal to the first in the case of zero surface tilt. Equations (4.2.39) and (4.2.40) have a constant term (equal to equations (4.2.37) and (4.2.38)) defined by the sensor geometry, and a common term multiplying the gradient. Also of note is the lack of co-dependence between the gradients for each component of the speckle shift. The shift in x , A_x , depends only on m_x , and A_y is dependent only on m_y .

Taylor Expansion

This method uses a Taylor expansion to find an expression for $L_D(\mathbf{P}, \mathbf{D} + \mathbf{A})$ in terms of $L_D(\mathbf{P}, \mathbf{D})$, giving a more general form for off-axis detector positions [3]. As mentioned above, this was found to equate to the implementation of the Fraunhofer approximation, though this approach is not used explicitly.

Firstly, as the speckle shift $\mathbf{A}$ at the detector is equivalent to a shift of $-\mathbf{A}$ at the surface, it follows that $L_D(\mathbf{P}, \mathbf{D} + \mathbf{A}) = L_D(\mathbf{P} - \mathbf{A}, \mathbf{D})$. Now, the Taylor expansion can be applied to $L_D(\mathbf{P} - \mathbf{A}, \mathbf{D})$ around $\mathbf{P} = (x, y, z)$.

$$L_D(\mathbf{P} - \mathbf{A}, \mathbf{D}) \approx L_D(\mathbf{P}, \mathbf{D}) + \left(\frac{\partial L_D}{\partial x} A_x + \frac{\partial L_D}{\partial y} A_y + \frac{\partial L_D}{\partial z} A_z \right) \quad (4.2.41)$$

Substituting the values of the partial derivatives given in Table 4.1, this expression can be rewritten using the unit vector $\hat{\mathbf{d}} = (d_x, d_y, d_z)$.

$$\begin{aligned} L_D(\mathbf{P} - \mathbf{A}, \mathbf{D}) &\approx L_D(\mathbf{P}, \mathbf{D}) + \left((-d_x)A_x + (-d_y)A_y + (0)A_z \right) \\ &= L_D(\mathbf{P}, \mathbf{D}) + \hat{\mathbf{d}}(\mathbf{P}, \mathbf{D}) \cdot \mathbf{A} \end{aligned} \quad (4.2.42)$$

This can now be substituted into equation (4.2.21), giving the following expression for $\langle U_1 U_2^* \rangle$:

$$\begin{aligned} \langle U_1 U_2^* \rangle &= \iint I(\mathbf{P}) \exp \left(ik_1 \left(L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P}) \right. \right. \\ &\quad \left. \left. - \frac{k_2}{k_1} L_D(\mathbf{P}, \mathbf{D}) + \frac{k_2}{k_1} \hat{\mathbf{d}}(\mathbf{P}, \mathbf{D}) \cdot \mathbf{A} - \frac{k_2}{k_1} L_S(\mathbf{P}) \right) \right) d^2 \mathbf{P} \\ &= \iint I(\mathbf{P}) \exp \left(ik_1 \left(\left(1 - \frac{k_2}{k_1} \right) (L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P})) \right. \right. \\ &\quad \left. \left. - \frac{k_2}{k_1} \hat{\mathbf{d}}(\mathbf{P}, \mathbf{D}) \cdot \mathbf{A} \right) \right) d^2 \mathbf{P} \end{aligned} \quad (4.2.43)$$

As with the derivation method above, the next step is to apply a Taylor expansion around $\mathbf{P} = 0$. The expansion of $(1 - k_2/k_1)(L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P}))$ was given previously

in equation (4.2.31). The expansion of $-(k_2/k_1)\hat{\mathbf{d}}(\mathbf{P}, \mathbf{D}) \cdot \mathbf{A}$ is outlined below.

$$\begin{aligned}
 -\frac{k_2}{k_1}\hat{\mathbf{d}}(\mathbf{P}, \mathbf{D}) \cdot \mathbf{A} &= \frac{k_2}{k_1}(-A_x d_x(\mathbf{P}, \mathbf{D}) - A_y d_y(\mathbf{P}, \mathbf{D})) \\
 &\approx \frac{k_2}{k_1} \left(-A_x (d_x(0, \mathbf{D}) - \nabla[d_x(\mathbf{P}, \mathbf{D})]_0 \cdot \mathbf{P}) \right. \\
 &\quad \left. - A_y (d_y(0, \mathbf{D}) - \nabla[d_y(\mathbf{P}, \mathbf{D})]_0 \cdot \mathbf{P}) \right) \quad (4.2.44) \\
 &= -\frac{k_2}{k_1}\hat{\mathbf{d}}(0, \mathbf{D}) \cdot \mathbf{A} - \frac{k_2}{k_1}A_x \nabla[d_x(\mathbf{P}, \mathbf{D})]_0 \cdot \mathbf{P} \\
 &\quad - \frac{k_2}{k_1}A_y \nabla[d_y(\mathbf{P}, \mathbf{D})]_0 \cdot \mathbf{P}
 \end{aligned}$$

These are now substituted into equation (4.2.43) and the constants taken outside of the integral.

$$\begin{aligned}
 \langle U_1 U_2^* \rangle &= \exp \left(ik_1 \left(\left(1 - \frac{k_2}{k_1} \right) (L_D(0, \mathbf{D}) + L_S(0)) - \frac{k_2}{k_1} \hat{\mathbf{d}}(0, \mathbf{D}) \cdot \mathbf{A} \right) \right) \\
 &\times \iint I(\mathbf{P}) \exp \left(ik_1 \left(\left(1 - \frac{k_2}{k_1} \right) \nabla[L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P})]_0 \right. \right. \\
 &\quad \left. \left. - \frac{k_2}{k_1} A_x \nabla[d_x(\mathbf{P}, \mathbf{D})]_0 - \frac{k_2}{k_1} A_y \nabla[d_y(\mathbf{P}, \mathbf{D})]_0 \right) \cdot \mathbf{P} \right) d^2 \mathbf{P} \quad (4.2.45)
 \end{aligned}$$

Now, the the integral of the cross correlation $\langle U_1 U_2^* \rangle$ is maximised when:

$$\left(1 - \frac{k_2}{k_1} \right) \nabla[L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P})]_0 - \frac{k_2}{k_1} A_x \nabla[d_x(\mathbf{P}, \mathbf{D})]_0 - \frac{k_2}{k_1} A_y \nabla[d_y(\mathbf{P}, \mathbf{D})]_0 = 0 \quad (4.2.46)$$

Splitting this into x and y components gives a set of two simultaneous equations describing A_x and A_y .

$$\left(1 - \frac{k_2}{k_1} \right) \frac{\partial L_D}{\partial x} + \left(1 - \frac{k_2}{k_1} \right) \frac{\partial L_S}{\partial x} - A_x \left(\frac{k_2}{k_1} \right) \frac{\partial d_x}{\partial x} - A_y \left(\frac{k_2}{k_1} \right) \frac{\partial d_y}{\partial x} = 0 \quad (4.2.47)$$

$$\left(1 - \frac{k_2}{k_1} \right) \frac{\partial L_D}{\partial y} + \left(1 - \frac{k_2}{k_1} \right) \frac{\partial L_S}{\partial y} - A_x \left(\frac{k_2}{k_1} \right) \frac{\partial d_x}{\partial y} - A_y \left(\frac{k_2}{k_1} \right) \frac{\partial d_y}{\partial y} = 0 \quad (4.2.48)$$

Dividing through by (k_2/k_1) and substituting $(k_2/k_1 - 1) = (\lambda_1/\lambda_2 - 1) = \Delta\lambda/\lambda_1$ gives

the expressions in terms of wavelength.

$$\frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial L_D}{\partial x} + \frac{\partial L_S}{\partial x} \right) - A_x \frac{\partial d_x}{\partial x} - A_y \frac{\partial d_y}{\partial x} = 0 \quad (4.2.49)$$

$$\frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial L_D}{\partial y} + \frac{\partial L_S}{\partial y} \right) - A_x \frac{\partial d_x}{\partial y} - A_y \frac{\partial d_y}{\partial y} = 0 \quad (4.2.50)$$

In order to solve the simultaneous equations, first rearrange equation (4.2.49).

$$A_y = \frac{\frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial L_D}{\partial x} + \frac{\partial L_S}{\partial x} \right) - A_x \frac{\partial d_x}{\partial x}}{\frac{\partial d_y}{\partial x}} \quad (4.2.51)$$

Then substitute into equation (4.2.50) and rearrange to give the full expression for A_x .

$$\begin{aligned} \frac{\partial d_y}{\partial y} \left(\frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial L_D}{\partial x} + \frac{\partial L_S}{\partial x} \right) - A_y \frac{\partial d_x}{\partial x} \right) &= \frac{\partial d_y}{\partial x} \left(\frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial L_D}{\partial y} + \frac{\partial L_S}{\partial y} \right) - A_x \frac{\partial d_x}{\partial y} \right) \\ A_x \left(\frac{\partial d_x}{\partial x} \frac{\partial d_y}{\partial y} - \frac{\partial d_y}{\partial x} \frac{\partial d_x}{\partial y} \right) &= \frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial d_y}{\partial y} \left(\frac{\partial L_D}{\partial x} + \frac{\partial L_S}{\partial x} \right) - \frac{\partial d_y}{\partial x} \left(\frac{\partial L_D}{\partial y} + \frac{\partial L_S}{\partial y} \right) \right) \\ A_x &= \frac{\frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial d_y}{\partial y} \left(\frac{\partial L_D}{\partial x} + \frac{\partial L_S}{\partial x} \right) - \frac{\partial d_y}{\partial x} \left(\frac{\partial L_D}{\partial y} + \frac{\partial L_S}{\partial y} \right) \right)}{\frac{\partial d_x}{\partial x} \frac{\partial d_y}{\partial y} - \frac{\partial d_y}{\partial x} \frac{\partial d_x}{\partial y}} \end{aligned} \quad (4.2.52)$$

Finally, substitute this back into (4.2.51) and cancel terms to give the complete expression for A_y .

$$A_y = \frac{\frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial d_x}{\partial x} \left(\frac{\partial L_D}{\partial y} + \frac{\partial L_S}{\partial y} \right) - \frac{\partial d_x}{\partial y} \left(\frac{\partial L_D}{\partial x} + \frac{\partial L_S}{\partial x} \right) \right)}{\frac{\partial d_x}{\partial x} \frac{\partial d_y}{\partial y} - \frac{\partial d_y}{\partial x} \frac{\partial d_x}{\partial y}} \quad (4.2.53)$$

As with the final result using the Fresnel approximation, the first set of partial derivatives from Table 4.1 can be used to find the relationship between speckle shift, source wavelengths, and sensor geometry for a flat surface with no tilt. First

the denominator of equations (4.2.52) and (4.2.53) is calculated:

$$\begin{aligned}
\frac{\partial d_x}{\partial x} \frac{\partial d_y}{\partial y} - \frac{\partial d_y}{\partial x} \frac{\partial d_x}{\partial y} &= \frac{1}{L_D} (d_x^2 - 1) \cdot \frac{1}{L_D} (d_y^2 - 1) - \frac{1}{L_D} (d_x d_y) \cdot \frac{1}{L_D} (d_x d_y) \\
&= \frac{1}{L_D^2} \left((d_x^2 d_y^2 - d_x^2 - d_y^2 - 1) - d_x^2 d_y^2 \right) \\
&= \frac{1 - d_x^2 - d_y^2}{L_D^2} \\
&= \frac{d_z^2}{L_D^2}
\end{aligned} \tag{4.2.54}$$

Note the final substitution of $1 - d_x^2 - d_y^2 = d_z^2$; as $\hat{\mathbf{d}} = (d_x, d_y, d_z)$ is a unit vector, $d_x^2 + d_y^2 + d_z^2 = 1$.

Now, the full expressions can be calculated, resulting in two symmetrical expressions for A_x and A_y .

$$\begin{aligned}
A_x &= \frac{L_D^2}{d_z^2} \frac{\Delta \lambda}{\lambda_1} \left(\frac{1}{L_D} (d_y^2 - 1) (-d_x - s_x) - \frac{1}{L_D} (d_x d_y) (-d_y - s_y) \right) \\
&= \frac{\Delta \lambda}{\lambda_1} \frac{L_D}{d_z^2} \left((-d_x d_y^2 - d_y^2 s_x + d_x + s_x) - (-d_x d_y^2 - d_x d_y s_y) \right) \\
&= \frac{\Delta \lambda}{\lambda_1} \frac{L_D}{d_z^2} (d_x + s_x + d_x d_y s_y - d_y^2 s_x)
\end{aligned} \tag{4.2.55}$$

$$\begin{aligned}
A_x &= \frac{L_D^2}{d_z^2} \frac{\Delta \lambda}{\lambda_1} \left(\frac{1}{L_D} (d_x^2 - 1) (-d_y - s_y) - \frac{1}{L_D} (d_x d_y) (-d_x - s_x) \right) \\
&= \frac{\Delta \lambda}{\lambda_1} \frac{L_D}{d_z^2} \left((-d_x^2 d_y - d_x^2 s_y + d_y + s_y) - (-d_x^2 d_y - d_x d_y s_x) \right) \\
&= \frac{\Delta \lambda}{\lambda_1} \frac{L_D}{d_z^2} (d_y + s_y + d_x d_y s_x - d_x^2 s_y)
\end{aligned} \tag{4.2.56}$$

Compared to the equations resulting from the Fresnel approximation ((4.2.39) and (4.2.40)), these expressions contain notably more geometry terms. Each of A_x and A_y are dependent on the detector's position in x , y , and z , and the source's position in x and y .

For a tilted surface, the second set of partial derivatives from Table 4.1 are used to

find the denominator of (4.2.52) and (4.2.53).

$$\begin{aligned}
 \frac{\partial d_x}{\partial x} \frac{\partial d_y}{\partial y} - \frac{\partial d_y}{\partial x} \frac{\partial d_x}{\partial y} &= \frac{1}{L_D} (m_x d_x d_z + d_x^2 - 1) \cdot \frac{1}{L_D} (m_y d_y d_z + d_y^2 - 1) \\
 &\quad - \frac{1}{L_D} (m_x d_y d_z + d_x d_y) \cdot \frac{1}{L_D} (m_y d_x d_z + d_x d_y) \\
 &= \frac{1}{L_D^2} \left((m_x m_y d_x d_y d_z^2 + m_x d_x d_y^2 d_z + m_x d_x d_z \right. \\
 &\quad \left. + m_y d_x^2 d_y d_z + d_x^2 d_y^2 - d_x^2 - m_y d_y d_z - d_y^2 + 1) \right. \\
 &\quad \left. - (m_x m_y d_x d_y d_z^2 + m_x d_x d_y^2 d_z \right. \\
 &\quad \left. + m_y d_x^2 d_y d_z + d_x^2 d_y^2) \right) \\
 &= \frac{1 - d_x^2 - d_y^2 - m_x d_x d_z - m_y d_y d_z}{L_D^2} \\
 &= \frac{d_z^2 - m_x d_x d_z - m_y d_y d_z}{L_D^2}
 \end{aligned} \tag{4.2.57}$$

The full expressions for a tilted surface contain many more terms than the previous

equations. The full expansion and simplification is given for A_x below:

$$\begin{aligned}
A_x &= \frac{L_D^2}{d_z^2 - m_x d_x d_z - m_y d_y d_z} \frac{\Delta \lambda}{\lambda_1} \\
&\quad \times \left(\frac{1}{L_D} (m_y d_y d_z + d_y^2 - 1) (-m_x d_z - d_x - m_x s_z - s_x) \right. \\
&\quad \left. - \frac{1}{L_D} (m_x d_y d_z + d_x d_y) (-m_y d_z - d_y - m_y s_z - s_y) \right) \\
&= \frac{\Delta \lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z - m_y d_y d_z} \\
&\quad \times \left((-m_x m_y d_y d_z^2 - m_y d_x d_y d_z - m_y m_x d_y d_z s_z - m_y d_y d_z s_x \right. \quad (4.2.58) \\
&\quad - m_x d_y^2 d_z - d_x d_y^2 - m_x d_y^2 s_z - d_y^2 s_x + m_x d_z + d_x + m_x s_z + s_x) \\
&\quad - (-m_x m_y d_y d_z^2 - m_x d_y^2 d_z - m_x m_y d_y d_z s_z - m_x d_y d_z s_y \\
&\quad \left. - m_y d_x d_y d_z - d_x d_y^2 - m_y d_x d_y d_z s_z - d_x d_y s_y) \right) \\
&= \frac{\Delta \lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z - m_y d_y d_z} \left((d_x + s_x + d_x d_y s_y - d_y^2 s_x) \right. \\
&\quad \left. + m_x (d_z + s_z + d_y d_z s_y - d_y^2 s_z) + m_y (d_x d_y s_z - d_y d_z s_x) \right)
\end{aligned}$$

The expression for A_y is obtained similarly:

$$\begin{aligned}
A_y &= \frac{L_D^2}{d_z^2 - m_x d_x d_z - m_y d_y d_z} \frac{\Delta \lambda}{\lambda_1} \\
&\quad \times \left(\frac{1}{L_D} (m_x d_x d_z + d_x^2 - 1) (-m_y d_z - d_y - m_y s_z - s_y) \right. \\
&\quad \left. - \frac{1}{L_D} (m_y d_y d_z + d_y^2 - 1) (-m_x d_z - d_x - m_x s_z - s_x) \right) \quad (4.2.59) \\
&= \frac{\Delta \lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z - m_y d_y d_z} \left((d_y + s_y + d_x d_y s_x - d_x^2 s_y) \right. \\
&\quad \left. + m_x (d_x d_y s_z - d_x d_z s_y) + m_y (d_z + s_z + d_x d_z s_x - d_x^2 s_z) \right)
\end{aligned}$$

These equations show greater dependence on the sensor geometry, and a cross-dependence between the two components of the surface gradient. This means that the 3D positions of the source and detector are required to use these expressions to measure surface tilt, and the two equations must be solved simultaneously.

This is the full version of the theory that will predominantly be used throughout

the project. The following section will outline some simplifications of the equations, including some that remove the cross-dependence of the tilt components.

4.3 Realisation of the Technique

4.3.1 Sensor Arrangement

This section will discuss the choice of general sensor arrangement. This refers to where the sensor elements are placed in relation to the coordinate system, e.g. having the source or detector position restricted to the z axis, rather than being positioned anywhere in the xyz space. Choosing an arrangement where one element is fixed in this way will allow a simplification of the mathematical equations derived above, making them easier to follow and implement.

The arrangements will use the terms centred and planar to describe the positions of the source and detector. Centred refers to the sensor component (either the source or detector) being centred directly opposite the spot, somewhere along the z axis. Planar describes the two components being in the same plane, in this case the xz plane.

Non-Centred, Non-Planar (General Arrangement)

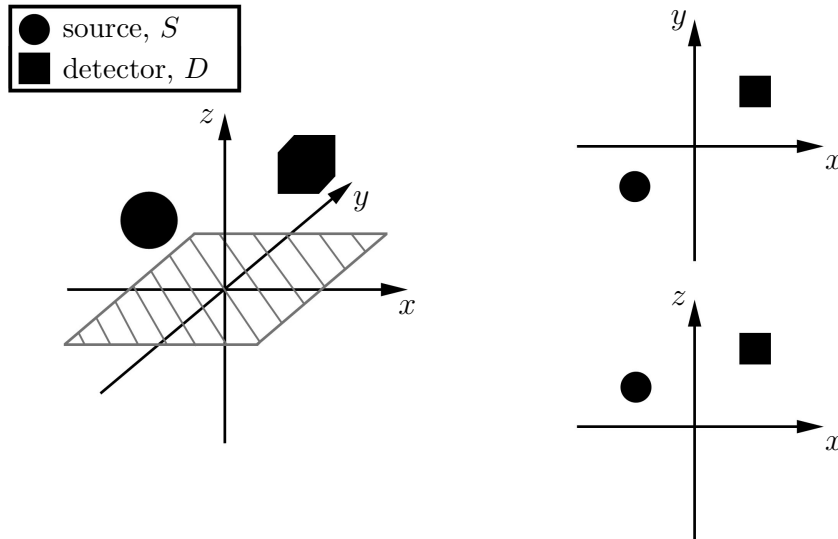


Figure 4.5: Diagram of a general sensor arrangement with no restrictions on source or detector position. This is described as a non-centred, non-planar arrangement.

The most general arrangement, shown diagrammatically in Figure 4.5, is described by the final equations derived above. These are restated here for completeness.

$$A_x = \frac{\Delta\lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z - m_y d_y d_z} \left((d_x + s_x + d_x d_y s_y - d_y^2 s_x) \right. \\ \left. + m_x (d_z + s_z + d_y d_z s_y - d_y^2 s_z) + m_y (d_x d_y s_z - d_y d_z s_x) \right) \quad (4.3.1)$$

$$A_y = \frac{\Delta\lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z - m_y d_y d_z} \left((d_y + s_y + d_x d_y s_x - d_x^2 s_y) \right. \\ \left. + m_x (d_x d_y s_z - d_x d_z s_y) + m_y (d_z + s_z + d_x d_z s_x - d_x^2 s_z) \right) \quad (4.3.2)$$

In this case, the source and detector can each have any position $\mathbf{S} = (S_x, S_y, S_z)$ and $\mathbf{D} = (D_x, D_y, D_z)$. The only restriction is that both must have a positive z position, i.e. be placed above the surface.

Non-Centred, Planar

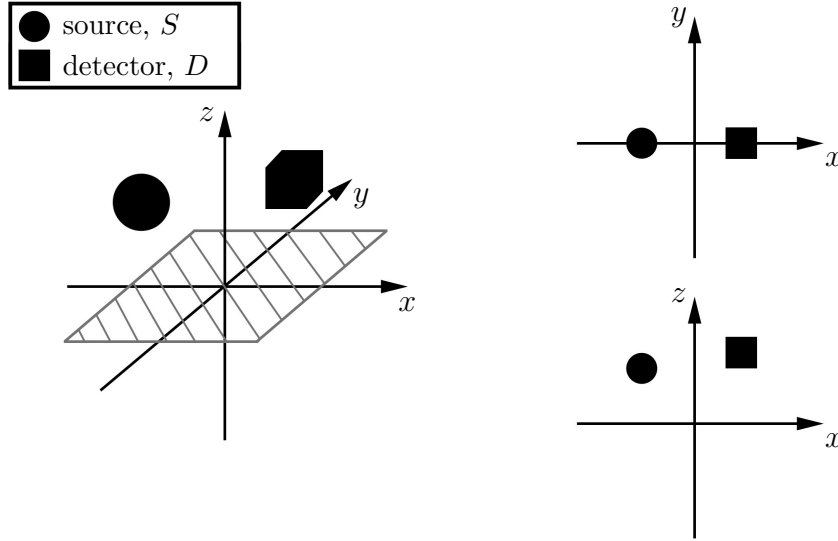


Figure 4.6: Diagram of a non-centred, planar sensor arrangement, where the source and detector are restricted to the xz plane.

The first and simplest restriction to place on the arrangement is to fix both source and detector to the xz plane, shown by Figure 4.6. This planar arrangement results in the y components of $\hat{\mathbf{d}} = (d_x, d_y, d_z)$ and $\hat{\mathbf{s}} = (s_x, s_y, s_z)$ becoming zero, removing

many of the terms in the above equations.

$$A_x = \frac{\Delta\lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z} (d_x + s_x + m_x(d_z + s_z)) \quad (4.3.3)$$

$$A_y = \frac{\Delta\lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z} (m_y(d_z + s_z - d_x^2 s_z + d_x d_z s_x)) \quad (4.3.4)$$

Although these equations have significantly fewer terms than (4.3.1) and (4.3.2), there is still a cross-dependence between the x and y components of the surface gradient. In other words, to find the surface gradient, these two equations must be solved simultaneously using both speckle shifts A_x and A_y .

Source Centred, Detector Non-Planar

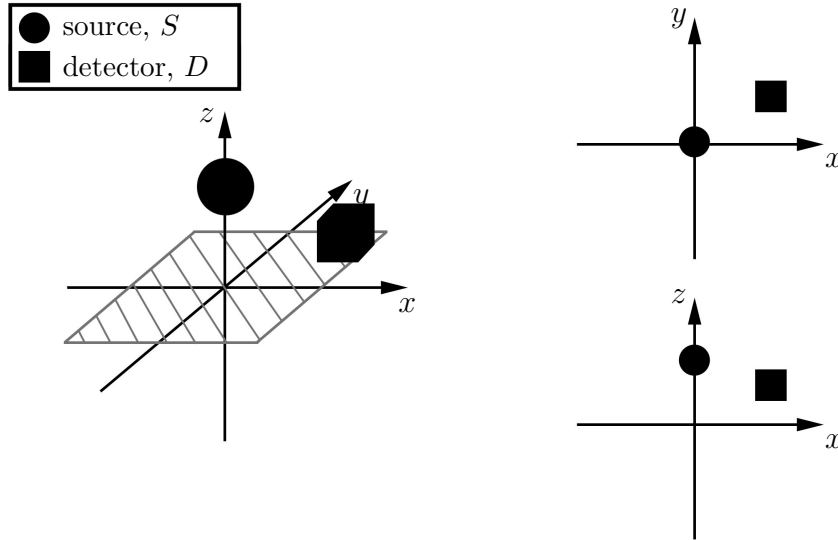


Figure 4.7: Diagram of a sensor arrangement with the source centred opposite the laser spot, and the detector anywhere in the xyz space.

Another approach would be to centre the source directly opposite the laser spot, at $\mathbf{S} = (0, 0, S_z)$. The unit vector components s_x and s_y are both now equal to zero, and the only remaining component becomes $s_z = 1$. This arrangement allows the detector to be positioned anywhere above the surface, so the components of

$\hat{\mathbf{d}} = (d_x, d_y, d_z)$ are all unchanged.

$$A_x = \frac{\Delta\lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z - m_y d_y d_z} (d_x + m_x(d_z - d_y^2 + 1) + m_y(d_x d_y)) \quad (4.3.5)$$

$$A_y = \frac{\Delta\lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z - m_y d_y d_z} (d_y + m_x(d_x d_y) + m_y(d_z - d_x^2 + 1)) \quad (4.3.6)$$

A practical advantage of positioning the source opposite the spot is that it is relatively easy to mount and align. In terms of the ease of use of the equations, however, this arrangement has no advantages over the previous one. Equations (4.3.5) and (4.3.6) have more terms than (4.3.3) and (4.3.4), and the cross-dependence of the surface gradients is still present.

Source Centred, Detector Planar

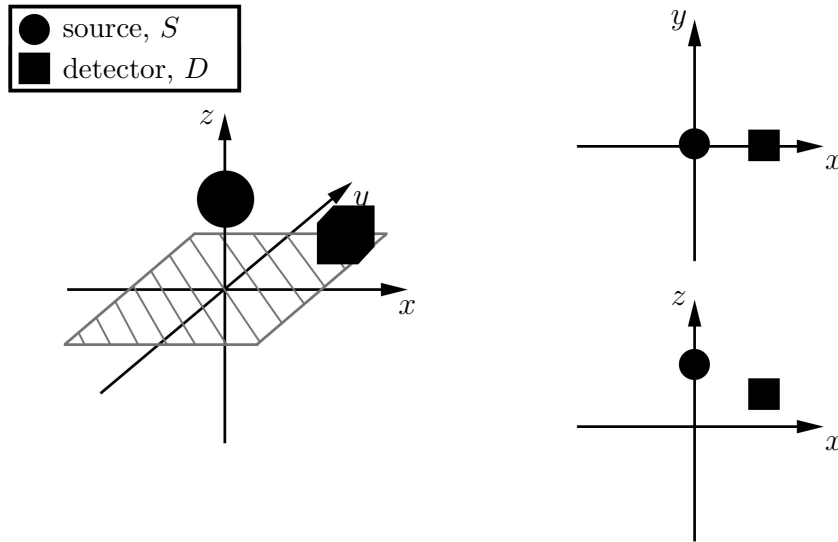


Figure 4.8: Diagram of a sensor arrangement with the source centred opposite the laser spot, and the detector restricted to the xz plane.

A further simplification can be achieved by keeping the source centred and fixing the detector in the xz plane, so $d_y = 0$.

$$A_x = \frac{\Delta\lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z} (d_x + m_x(d_z + 1)) \quad (4.3.7)$$

$$A_y = \frac{\Delta\lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z} (m_y(d_z - d_x^2 + 1)) \quad (4.3.8)$$

Mathematically, these equations are an improvement on the previous ones, as A_x is no longer sensitive to m_y . This makes the equations easier to solve, as they are no longer coupled as simultaneous equations. The cross-component sensitivity remains for A_y , though, and so the surface gradient must always be calculated in x first.

Detector Centred, Source Non-Planar

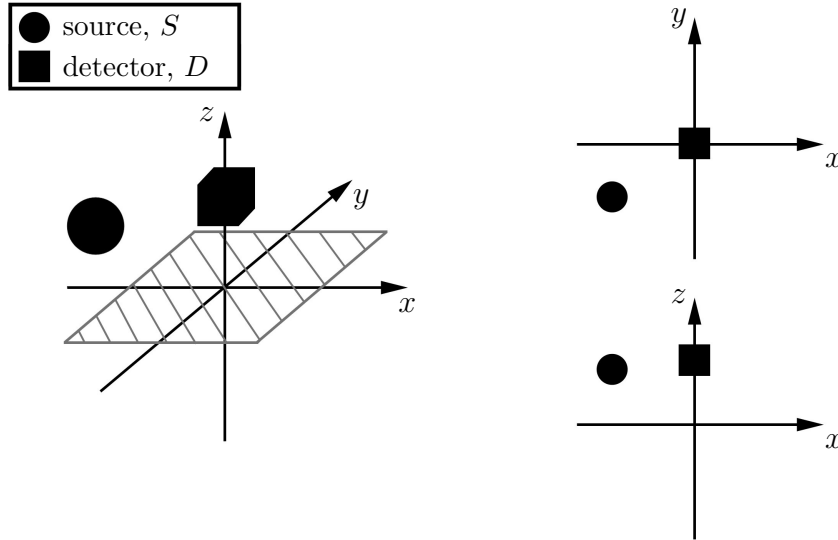


Figure 4.9: Diagram of a sensor arrangement with the detector centred above the laser spot, and the source anywhere in the xyz space.

Alternatively, the detector can be placed opposite the spot at $\mathbf{D} = (0, 0, D_z)$, and the source given any position $\mathbf{S} = (S_x, S_y, S_z)$. In this case, the x and y components

of $\hat{\mathbf{d}} = (d_x, d_y, d_z)$ are zero, and $d_z = 1$.

$$A_x = \frac{\Delta\lambda}{\lambda_1} L_D(s_x + m_x(1 + s_z)) \quad (4.3.9)$$

$$A_y = \frac{\Delta\lambda}{\lambda_1} L_D(s_y + m_y(1 + s_z)) \quad (4.3.10)$$

This arrangement has a particularly important advantage over the previous ones; the cross-component sensitivity of the surface gradients has been removed completely, and the equations have been simplified greatly. Another advantage of this arrangement is a practical one. As the detector is directly opposite the laser spot, there is no risk of shadowing on the captured image from the edges of any filter or lens on the front of the camera.

Detector Centred, Source Planar

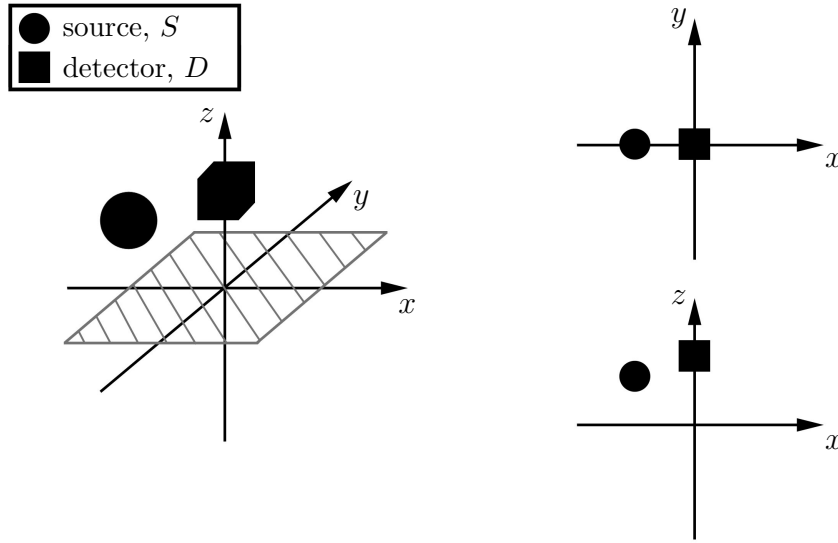


Figure 4.10: Diagram of a sensor arrangement with the detector centred above the laser spot, and the source restricted to the xz plane.

Another slight simplification can be achieved by fixing the source in the xz plane. s_y becomes zero, and the equations are now:

$$A_x = \frac{\Delta\lambda}{\lambda_1} L_D (s_x + m_x(1 + s_z)) \quad (4.3.11)$$

$$A_y = \frac{\Delta\lambda}{\lambda_1} L_D (m_y(1 + s_z)) \quad (4.3.12)$$

Source and Detector Centred (Collinear)

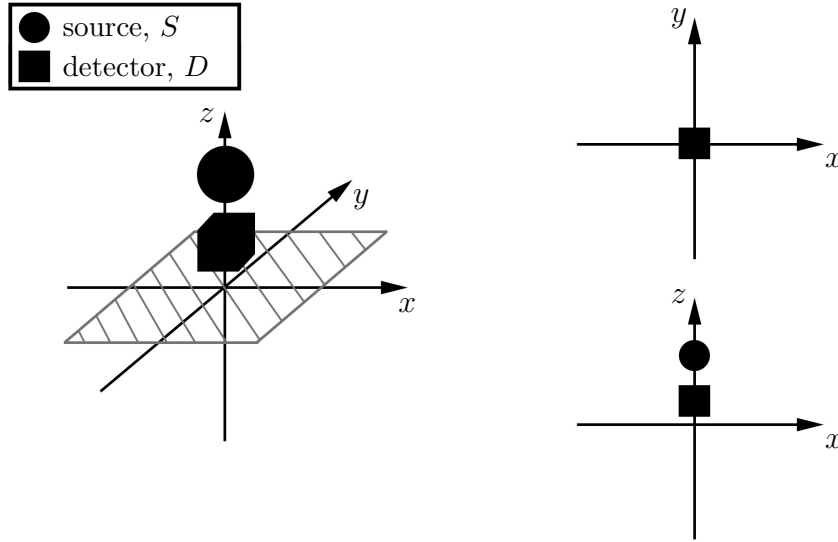


Figure 4.11: Diagram showing a collinear sensor arrangement, where both the source and detector are centred opposite the laser spot.

Finally, the source and detector could be arranged collinearly, both opposite the laser spot. This removes all x and y components of the two unit vectors, and results in a direct proportionality between speckle shift and surface gradient.

$$A_x = 2 \frac{\Delta\lambda}{\lambda_1} L_D m_x \quad (4.3.13)$$

$$A_y = 2 \frac{\Delta\lambda}{\lambda_1} L_D m_y \quad (4.3.14)$$

In practical terms, this arrangement is much harder to implement than any of the previous ones, as it requires more components and a more complex mount to achieve the collinearity. Another complication for this arrangement is that the speckle shift at zero tilt will also be zero, and so referring to the correlation image, the two side peaks will be merged with the central peak in an autocorrelation. This would make small tilts difficult or potentially impossible to distinguish using single frame capture and autocorrelation. This could be circumvented by separating the images of each of the two speckle patterns, for instance using multiple cameras or multiple laser colours with a colour camera, which would also increase the overall complexity of the sensor.

Summary of Sensor Arrangements

The seven arrangements outlined above are summarised in Table 4.2, along with their advantages and disadvantages. The final arrangement, with the source and detector collinear, results in the simplest formulation of the equations, with speckle shift directly proportional to surface gradient for each component x and y and no cross-sensitivity. However, this arrangement is more complex practically, with more components and care required to correctly mount the sensor elements.

The arrangement with the detector centred and a planar source was chosen for all versions of the tilt sensor presented in this thesis. The speckle shift equations are relatively very simple, and there is no cross-component sensitivity between the surface gradients. This choice also balances the practical considerations of building the sensor, with the elements much easier to mount than for a collinear arrangement.

Another interesting aspect of this arrangement is that the distance from the source to the spot (L_S) has no effect on the speckle patterns, as long as its angle of incidence remains the same (and the spot size remains constant). In other words, only the angle made by the vector to the source from the z axis is needed for the speckle theory equations. There are also practical advantages to the spot being directly opposite the detector, such as the removed possibility of shadowing from the front of the camera casing or an attached lens tube.

Although the chosen arrangement places the source in the xz plane, to account for any possible misalignment the equations for a non-planar source were used ((4.3.9) and (4.3.10)), retaining the variable s_y . This will be used in the following section to

define a sensor constant describing the speckle shift at a surface tilt of zero.

Converting the surface gradients to angles ($m_x = \tan \theta_y$ and $m_y = \tan \theta_x$) gives the final equations for A_x and A_y that will be used throughout this project.

$$A_x = \frac{\Delta\lambda}{\lambda_1} L_D (s_x + \tan \theta_y (1 + s_z)) \quad (4.3.15)$$

$$A_y = \frac{\Delta\lambda}{\lambda_1} L_D (s_y + \tan \theta_x (1 + s_z)) \quad (4.3.16)$$

Table 4.2: Summary of possible sensor arrangements and their advantages and disadvantages.

Arrangement	Advantages	Disadvantages
Non-centred, non-planar (generalised arrangement)	No restrictions on positioning	Complex equations, cross-component sensitivity
Non-centred, planar	Significantly simplified equations from general arrangement	Cross-component sensitivity
Source centred, detector non-planar	Simplified equations from general arrangement, source relatively easy to mount and align	Cross-component sensitivity
Source centred, detector planar	Significantly simplified equations from general arrangement, no cross-component sensitivity in A_x , source relatively easy to mount and align	Cross-component sensitivity in A_y
Detector centred, source non-planar	Significantly simplified equations from previous arrangements, no cross-component sensitivity	Source slightly more difficult to mount and align
Detector centred, source planar	Significantly simplified equations from previous arrangements, no cross-component sensitivity	Source slightly more difficult to mount and align
Source and detector centred (collinear)	Simplest form of equations, speckle shift and surface gradient are directly proportional	Significantly more difficult to mount and align

4.3.2 Definition of Sensor Constants

Starting from equations (4.3.15) and (4.3.16) and rearranging, terms describing three sensor constants can be extracted. The formulation of these constants is dependent on the chosen sensor arrangement of a centred detector. The precise value of the constants is given by the specific geometry of a sensor, meaning the exact locations of the components within the sensor arrangement.

$$A_x = \frac{\Delta\lambda}{\lambda_1} L_D (s_z + 1) \tan \theta_y + \frac{\Delta\lambda}{\lambda_1} L_D s_x \quad (4.3.17)$$

$$A_y = \frac{\Delta\lambda}{\lambda_1} L_D (s_z + 1) \tan \theta_x + \frac{\Delta\lambda}{\lambda_1} L_D s_y \quad (4.3.18)$$

The factor multiplying the $\tan \theta$ in each equation can be thought of as the responsivity, defined as C . Note that this is the same in both equations, i.e. for both angles. This factor is dependent on the working distance of the sensor, L_D , and the z component of the unit vector to the source, s_z .

$$C = \frac{\Delta\lambda}{\lambda_1} L_D (s_z + 1) \quad (4.3.19)$$

C is defined in units of length, but using the small angle approximation for $\tan \theta = \theta$ can be given in mm per radian. Thus it is explicitly linked to the degree of change in measured speckle shift per the normalised change in surface tilt.

The last term in each of equations (4.3.17) and (4.3.18) is equal to the speckle shift in that direction at zero surface tilt, i.e. when the surface is parallel to the detector plane. These are defined as A_{x0} for the x direction, and A_{y0} for y . $A_0 = (A_{x0}, A_{y0})$ is also defined.

$$A_{x0} = \frac{\Delta\lambda}{\lambda_1} L_D s_x \quad (4.3.20)$$

$$A_{y0} = \frac{\Delta\lambda}{\lambda_1} L_D s_y \quad (4.3.21)$$

These two terms are also given in units of length, specifically in mm. Their precise values depend heavily on the working distance of the detector L_D , plus the unit vector components in the matching direction. As the sensor will offset the source in the x direction, the speckle shift at zero surface tilt is in the x direction only and

A_{y0} should always be equal to zero.

This offsetting of the source restricts the measurable speckle shift in x to the range between the offset A_{x0} and the centre of the cross correlation image. As a speckle shift in x corresponds to an angle change in θ_y (and vice versa), the range of the sensor can be defined by the minimum measurable angle in θ_y , described by θ_R :

$$\tan \theta_R = \frac{A_{x0}}{C} = \frac{s_x}{s_z + 1} \quad (4.3.22)$$

This equation is a slight simplification, however. The true range of the sensor is limited by the point at which the correlation side peak is no longer distinguishable from the central peak. Therefore the range also depends on the width of these peaks, which is proportional to the average speckle width. As such, a more accurate value for this minimum angle would be θ_R minus half the width of the correlation peak, which will be called $\theta_{y,\min}$.

The minimum and maximum angles in the other three directions can also be defined. These use the quarter image rule, a convention that takes one quarter of the image width as the largest measurable shift in a correlation image [4]. This, along with the full correlation process, will be explained in more detail in Chapter 5.

A constructed example of a typical correlation image is shown in Figure 4.12. Focusing on the right-hand side peak, the angular range of the sensor corresponds to the measurable range of this peak. The dashed lines define the angle limits $\theta_{y,\min}$, $\theta_{y,\max}$, $\theta_{x,\min}$, and $\theta_{x,\max}$, boxing in the side peak. $\theta_{y,\min}$ is equal to θ_R (shown by the dotted line cutting through the central peak) plus 5 pixels, as the correlation peaks in this image have a full width half maximum (FWHM) of 10 pixels.

The practical sensor range will be defined as the smallest of the four values $\theta_{y,\min}$, $\theta_{y,\max}$, $\theta_{x,\min}$, and $\theta_{x,\max}$, denoted by $\theta_{\max}$. In reality, the smallest value was found to always be $\theta_{y,\min}$, and so this value effectively defines the sensor range.

4.3.3 Effect of Sensor Geometry on Range and Responsivity

As stated above, the sensor geometry describes the exact positions of the sensor elements and their distances from each other within the chosen arrangement. Altering the sensor geometry, for example increasing the separation between source and detector, can have a large effect on the range and responsivity of the sensor.

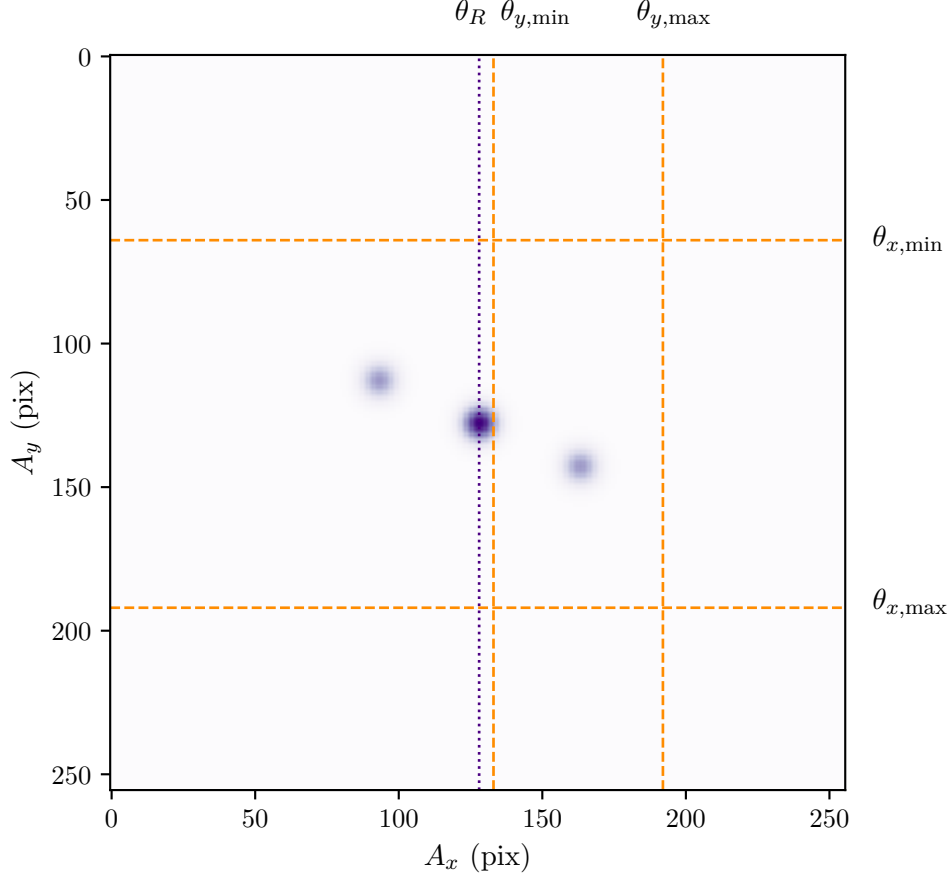


Figure 4.12: Constructed correlation image showing the various angle limits of the sensor. θ_R is the theoretical limit defined in equation (4.3.22). $\theta_{y,\min}$ is limited by the overlap with the correlation peak. The other three limits come from the quarter image rule for distinguishing correlation peaks.

The responsivity constant, C , depends predominantly on the two wavelengths of the source, plus the working distance L_D . Using the purely theory-derived range limit of θ_R , it is clear that the sensor range depends only on the source position. Thus changing the relative positions of the detector and source will change the range and responsivity, but this must be balanced with the practicalities of designing a compact and rigid sensor head.

A sequence of successive geometry adjustments and the corresponding effects on range and responsivity are outlined in Figure 4.13. These cover the three possible changes possible within the sensor arrangement, while considering the practical results on a sensor head design.

A large separation between the source and detector (S_x) is the obvious first approach,

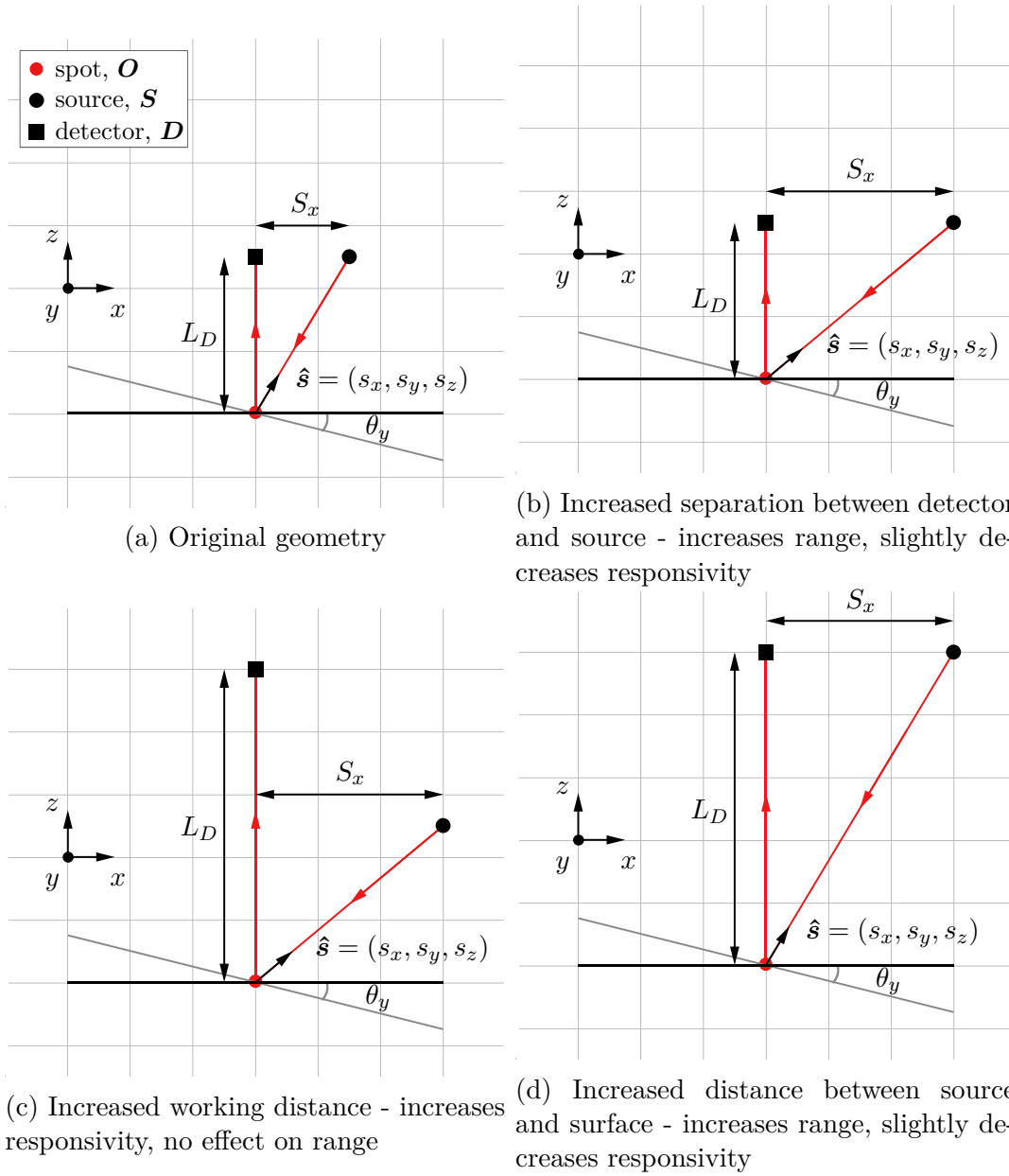


Figure 4.13: Schematic diagrams of (a) a general sensor geometry, and (b-d) three possible adjustments. The effect on range and responsiveness as compared to the previous geometry is summarised for each.

as increasing this distance increases both the range and responsivity (Figure 4.13b). Practically, however, this results in a wide and bulky sensor head that is intrusive and difficult to mount. This wide spacing is also likely to result in a relative decrease in the rigidity of the sensor head as compared to a smaller separation. Depending on how the sensor is mounted, this could lead to the position of the source and/or detector changing as the whole construction flexes, thus causing changes to all three of the sensor constants.

The next obvious solution from the equations is to increase the working distance L_D , shown in Figure 4.13c. By moving the detector further from the surface while keeping the source in the same position, the responsivity will increase while the range stays the same. Again this results in an impractical sensor design, however, as the source and detector are no longer in the same plane and so have to be mounted at an angle to each other. As the important factor for the source position is the angle it makes with the detector (see Section 4.3.1 above), this issue could be overcome by keeping the source in the same plane as the sensor but increasing their separation, keeping this angle the same. This leads to the same problems described above for the geometry in Figure 4.13b, caused by the wide spacing of the two components.

A more practical approach would be to bring the source back in line with the detector, as in Figure 4.13d. While this does increase the responsivity, it also decreases the maximum range. Combining these variables, the best approach when designing the sensor head is to maximise the separation of the source and detector within the bounds of a practical sensor head size, and then balancing the combined working distance to optimise both the range and responsivity.

Effect of Camera Sensor and Pixel Size on Range and Resolution

The other consideration for range and responsivity is the choice of detector. The previous section considers the sensor range as defined by θ_R , relating to the distance the side peak in the correlation image can move towards the central peak before it becomes undetectable (i.e. the smallest measurable speckle shift). This assumes that the detector's imaging sensor is large enough that no similar such restriction exists as the side peak moves in the opposite direction, towards the edge of the image. For the camera used in this project, this assumption is true and will be demonstrated in later chapters for the constructed sensors. However, it is theoretically possible for a smaller camera sensor or for a very high value of A_{x0} that there would be a

restrictive range limit for $\theta_{y,\max}$ as well as in $\theta_{y,\min}$, so this factor is important to consider.

The detector choice will also affect the resolution of the sensor, i.e. the smallest resolvable surface tilt. The highest achievable resolution will be limited by the physical size of the camera pixels and the method used to locate the height of the correlation peak. For instance, a method that locates the peak to the nearest pixel will result in a resolution defined precisely by the pixel size. Though, as will be described in more detail in the following chapter (Chapter 5), there are many peak fitting algorithms capable of finding sub-pixel peaks. This gives the resolution for the speckle shift, i.e in distance units.

Equations (4.3.17) and (4.3.18) are of the form $y = ax + b$, with C being the gradient a and the relevant part of $A_0 = (A_{x0}, A_{y0})$ being the intercept b . Therefore the resolution in the speckle shift can be converted into an angular resolution using the gradient of this line, C . So for a pixel size of s and a peak finding method with a resolution r_{fit} in pixels, the resolution of the sensor, r_{sens} , is:

$$r_{\text{sens}} = \frac{sr_{\text{fit}}}{C} \quad (4.3.23)$$

This can be used to give the expected sensor resolution in radians, using the $\tan \theta = \theta$ approximation for small angles, and will be used as a more intuitive benchmark to compare the performance of different sensors. This equation also makes it clear that increasing the responsivity results in a higher resolution, as would be expected.

4.3.4 Source Wavelengths

The choice of source wavelengths also has a large impact on the responsivity of the sensor. An advantage of adjusting these, however, is that they have no effect on the theoretical sensor range of θ_R , or the related range limit $\theta_{y,\min}$. The equations describing C and θ_R are restated below for convenience.

$$C = \frac{\Delta\lambda}{\lambda_1} L_D (s_z + 1) \quad (4.3.24)$$

$$\tan \theta_R = \frac{A_{x0}}{C} = \frac{s_x}{s_z + 1} \quad (4.3.25)$$

An increase in $\Delta\lambda$ and/or decrease in λ_1 will increase the responsivity, C , while keeping the ranges θ_R and $\theta_{y,\min}$ the same. The opposite range limit $\theta_{y,\max}$, which is effectively limited by the edge of the camera sensor, will be affected by a change in source wavelength. As $\Delta\lambda$ increases or λ_1 decreases, A_{x0} increases, reducing the available range in this direction (see Figure 4.12). In this case, however, A_{x0} can be decreased by moving the source and detector closer together, decreasing s_x . These factors can be balanced to ensure the range remains large enough in $\theta_{y,\max}$, and this will be demonstrated particularly in the sensor tested in Chapter 8.

The sensor range and responsivity are not the only consideration when deciding on the wavelengths. The cost, size, and mode of operation for the laser or lasers also need to be taken into account. The cost and size of the camera used are also important factors, as these are affected by the wavelength range of the camera sensor. As one of the potential aims for the project is that the sensor be low-cost, the prices of both the laser and camera were particularly important. Cameras operating in the visible region tend to be cheapest, and also often better value in terms of their sensor specifications such as pixel size and number of pixels.

Conversely, infrared (IR) lasers around 1000-2000nm are generally the preferred choice for low-budget applications. These wavelengths are commonly used in telecommunications, and as such there are a broader array of commercially available lasers and components for this wavelength range. However, for wavelengths greater than around 900nm, silicon-based cameras are no longer suitable and more expensive options such as indium gallium arsenide (InGaAs) or mercury cadmium telluride (MCT) are required [5, 6].

Balancing these factors, visible wavelengths were chosen for the sensor, specifically in the red region. This decision keeps the camera cost low, and red laser sources are cheaper than comparable options in the green or blue ranges. This also allowed lab development using existing laser systems and cameras. Choosing visible over IR also gives a lower λ_1 , which means a higher responsivity for the same value of $\Delta\lambda$.

Generating Two Wavelengths

The key feature of the DWS technique is the simultaneous illumination with two wavelengths. There are multiple methods that could be used to generate this illumination. One option is to use two single mode sources, coupling their outputs

together to create one dual-wavelength source. A second possibility is to drive one laser such that there are two longitudinal modes present in the output spectrum. Another method would be to modulate a laser between two drive currents that each produce a single mode at a different wavelength. In this case, the modulation frequency and the camera's exposure time must be considered in tandem to ensure each exposure contains multiple modulation periods. This will not only guarantee that each image captures both speckle patterns, but an exposure covering tens or hundreds of modulation periods will make the image more robust to vibration by averaging out any changes unrelated to the surface angle.

There are advantages and disadvantages to each of these approaches. As the goal is an inexpensive sensor, a single laser with a dual mode output would be the most cost-effective method. A single laser also has an advantage over two when considering wavelength stability. As the wavelength difference is smaller than the absolute wavelength, it can be seen mathematically from equation (4.3.24) that a change in $\Delta\lambda$ would have a greater effect on the responsivity than the same change in λ_1 . For two lasers, any instability in one will be independent of the other, which results in a larger uncertainty $\Delta\lambda$. When generating the two wavelengths with a single laser, any instability should in theory cause the two modes to drift together, leading to a lower uncertainty in $\Delta\lambda$. Laser modulation with a single laser would also offer this advantage. This method would be slightly more involved, though, requiring extra components in order to provide the modulating drive current.

On the other hand, using a single laser limits the potential wavelength separation to very small numbers - typically less than 1nm. Two lasers give much more control over the values of the wavelengths and their separation, and therefore more control over the characteristics of the sensor. For instance, using a red and green laser would give a very large $\Delta\lambda$, resulting in a high responsivity for the sensor. This particular approach would also provide an alternative solution to the directional ambiguity problem. This is caused by the two side peaks produced using autocorrelation, but the different coloured sources could be used with a colour camera, separating the red and green components from a single exposure. Correlating the two components with each other results in a correlation image with a single peak. This removes both the directional uncertainty and the range restriction caused by the central peak.

Ultimately, multiple implementations of the sensor were tested throughout the

project, demonstrating the flexibility of the sensor characteristics discussed above. Each used Fabry-Pérot laser diodes with continuous-wave (CW) operation. The first full implementation (Chapter 7) used a single laser, with the applied current such that the output spectrum had two distinct modes. The second implementation (Chapter 8) used two individual lasers, each operating with a single mode and combined using a fibre coupler. The particular lasers and the methods use to characterise them will be described in more detail in Chapter 5, which describes the methodology, and the aforementioned experimental chapters.

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Chapter 5

Constructing and Characterising a Dual-Wavelength Speckle Angle Sensor

This chapter will outline the experimental methods used to construct and characterise a dual-wavelength tilt sensor. These are used in the following chapter, which presents a practical demonstration of the theory, and in Chapters 7 and 8, which each demonstrate a complete sensor.

Section 5.1 details the image processing, following these steps: image capture, cross correlation, peak finding, and applying the speckle theory equations. An additional step is required for implementations with a large $\Delta\lambda$, which is also described in this section. Lastly, the expected error in the peak finding is estimated and equations for the uncertainty in angle measurements derived.

Following this, Section 5.2 describes the equipment and methods used to investigate laser stability, both qualitatively and quantitatively. As with the image processing, equations are derived describing the uncertainty in angle measurements relating to the laser instability.

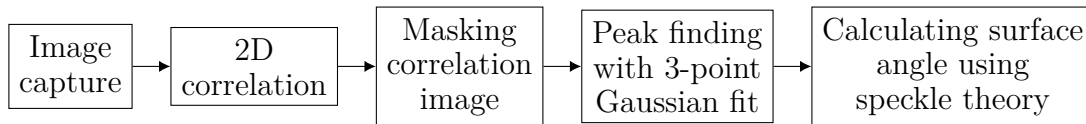
Finally, the calibration method for a complete sensor is described in Section 5.3. Three sensor constants (A_{x0} , A_{y0} , and C) were derived in the previous chapter, which can be calibrated practically. This section also outlines the specifications of the stages used to calibrate and then test the sensor.

5.1 Image Capture and Processing

The two basic components of the sensor are the dual-wavelength light source, and a camera to record speckle patterns. The camera transfers the images to a computer via USB 3.0, and these are then processed using a programme written in Python. The outcome of the image processing is the speckle shift, i.e. the separation between the two speckle patterns in the image. This can then be used in the theory equations derived in the previous chapter to calculate the surface tilt.

5.1.1 Image Processing Method

The basic elements of the image processing are as follows:



For each implementation of the sensor, the two speckle patterns generated by dual-wavelength speckle (DWS) are captured in a single frame. Following this, a 2D correlation is used to extract the distance between the two speckle patterns. There were two different methods used to correlate the patterns, chosen based on the wavelength separation of the source. Each method produces an image with a correlation peak whose location relative to the centre of the image is equal to the separation between the two speckle patterns, i.e. the speckle shift. Once the correlation image is obtained, it is masked in order to isolate the relevant correlation peak. The peak finding algorithm is a 3-point Gaussian fit, which starts from the highest pixel and uses the adjacent pixels in each direction to find the sub-pixel location of the peak. This fit is applied in x and y , over five points total, to get the 2D position of the Gaussian peak. From this, the speckle shift (A_x , A_y) is used along with the theory equations outlined in Chapter 4 to calculate the tilt angle of the surface.

Image Capture

All sensor versions used the same Ximea MQ013MG-ON camera [1]. This is a highly compact monochrome 1.3MP (megapixel) camera, with a total size of 26.4x26.4x26.5mm and weighing 27g. The largest image size is 1280x1024 pixels, at which the maximum frame rate is 210fps. Both of these values are adjustable, and so a higher

Table 5.1: Specifications of the Ximea MQ013MG-ON camera used in all implementations of the sensor.

Model number	MQ013MG-ON
Sensor type	CMOS
Image type	Monochrome
Lens mount	C-mount or CS-mount
Max. image size	1280x1024 pixels
Pixel size	4.8x4.8 μ m
Max. frame rate	210fps
Dimensions (C-mount)	26.4x26.4x26.5mm
Dimensions (CS-mount)	26.4x26.4x21.5mm
Weight	27g

frame rate is possible for smaller images. The full specifications of the camera are given in Table 5.1.

The detection is unpolarised, though the lasers are linearly polarised (see next section). This means that the speckle pattern from each wavelength is a combination of the patterns produced by each polarisation state of the reflected light, as described in Section 3.1.1. However, as the patterns are produced from a metallic surface, they will be highly correlated [2] and so the only difference between polarised and unpolarised detection would be the signal strength at the camera. The only optical element used with the detector is a laser line filter of centre 655nm and bandwidth 50nm is used to reduce background light. This was found experimentally to improve the correlation peak height by reducing the background correlation noise.

The required image size is highly dependent on the wavelength separation of the sensor. The size of the image must be at minimum twice the largest expected speckle shift; any smaller and the correlation peak would fall outside of the range of the image. However, a second consideration is the height of the correlation peak. As the peak moves further towards the edge of the image, its height falls. As such, a widely adopted limit of detection for the peak is one quarter of the image size [3]. For a sensor with small responsivity and range, this is possible with a fraction of

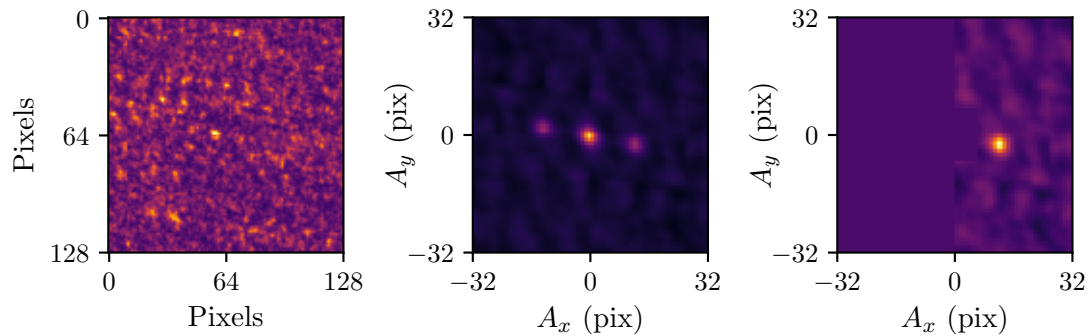


Figure 5.1: Sequence of steps in the image processing method for a sensor with closely spaced wavelengths, $\Delta\lambda = 0.5\text{nm}$. First, the image captured by the camera containing two offset speckle patterns, followed by the result of the 2D autocorrelation. Finally, the correlation image with the central peak and the half containing the extraneous side peak masked. The surface tilt for this image was $\theta_x = 5.0^\circ$, $\theta_y = 0.0^\circ$. Note: These images are cropped, the full images used in the correlation are 256x256 pixels.

the total camera area. For a larger responsivity, however, the required image size increases, decreasing the possible frame rate and increasing the processing time.

Correlation and Masking for Small Wavelength Separation

For a sensor with a small wavelength separation $\Delta\lambda$, a 2D autocorrelation is applied directly to the double speckle pattern recorded by the camera. In terms of the results presented here, this method is used throughout Chapter 6 and for the sensor in Chapter 7, where the wavelength separation is around 0.5nm. The correlation image produced by this method contains one normalised central peak where the image has fully correlated with itself, and two lower side peaks. This can be seen in the Figure 5.1. The wavelength separation in these images is 0.5nm, with source wavelengths of $\lambda_1 = 656.9\text{nm}$ and $\lambda_2 = 659.4\text{nm}$. One side peak is generated by the correlation of speckle pattern 1 with pattern 2, and the second is generated by the correlation of pattern 2 with pattern 1. The position of either of the side peaks relative to the central peak is equal to the speckle shift between the two patterns. Only one of the side peaks is used to calculate the surface angle, so one half of the correlation image is masked. The central peak is also masked, to ensure the highest value pixel found by the peak fitting belongs to the side peak. This is also shown

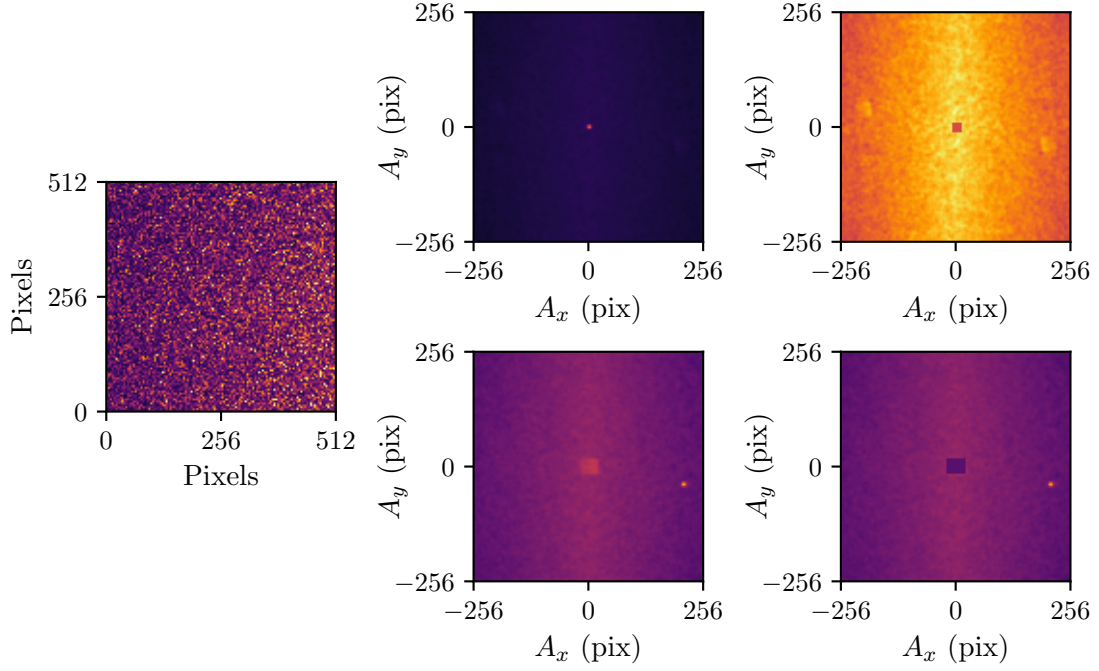


Figure 5.2: Sequence of steps in the image processing method for a sensor with widely spaced wavelengths, $\Delta\lambda = 21.1\text{nm}$. The first image is that taken by the camera, containing the two offset speckle patterns. Then, the first row shows the image processing using an autocorrelation only, resulting in very indistinct side peaks. The second row shows the processing with an extra magnification step. A magnified speckle image is correlated with the original, producing a correlation image with only one side peak. The final image in this row shows the indistinct central peak masked. The surface tilt for this image was $\theta_x = -3.5^\circ$, $\theta_y = 2.0^\circ$. Note: These images are cropped, the full images used in the correlation are 1280×1024 pixels.

in the final image of Figure 5.1.

Correlation and Masking for Large Wavelength Separation

If the sensor implementation uses a larger wavelength separation, the correlation process requires an extra step. This was introduced in the sensor presented in Chapter 8, where the wavelength separation is around 21.5nm . In this case, the two speckle patterns generated by the source wavelengths have a different magnification, as well as a different position [4]. This is the case at smaller values of $\Delta\lambda$ also, but the magnification is very small and can be neglected. The magnification factor is

given by the ratio of the two source wavelengths:

$$M = \frac{\lambda_1 + \Delta\lambda}{\lambda_1} = \frac{\lambda_2}{\lambda_1} \quad (5.1.1)$$

The magnification step becomes necessary when there is no longer enough overlap between corresponding speckles in the two patterns. Therefore the cut-off point for the introduction of this step depends on the speckle size as well as the wavelength separation. For further context, however, a version of the sensor was tested during the project with a wavelength difference of around 2.5nm, which did not require the magnification step. Thus the crossover point in the wavelength separation can be considered to be greater than 2.5nm.

For Figure 5.1, the two source wavelengths are 656.9nm and 657.4nm, and so the magnification factor between them is 1.0008. In Figure 5.2, the source wavelengths are 638.1nm and 659.2nm, giving a wavelength separation of 21.1nm and a magnification factor of 1.0329. The first row of Figure 5.2 shows the recorded speckle pattern processed using the simple autocorrelation method outlined above, though the third image has only the the middle peak masked, leaving both halves of the image visible. The magnification causes a variable shift across the image and thus blurred, indistinct side peaks.

The second row of Figure 5.2 shows the result of the image processing using a magnification step. The image taken by the camera, containing both speckle patterns, is magnified using the factor given above. This magnified speckle image is then cropped to the original size and correlated with the non-magnified image. Now the correlation image in Figure 5.2 shows one very distinct side peak. The single side peak is a direct result of the magnification; now only pattern 1 can correlate with the magnified pattern 2', and pattern 2 no longer correlates with the magnified 1'. This also results in a blurring of the central peak, similar to the blurred side peaks produced by an autocorrelation.

Although now lower than the single side peak, the central area is still masked to make sure it is not found during the peak finding step. This can be done very easily, as the size of the blurred central peak is equal to the difference in size between the original image and the magnified one. For the example images in Figure 5.2, the mask in the centre is 17x21 pixels, as the size of the magnified image was 1237x1045, compared to the original's 1280x1024. As there is no second side peak, there is no

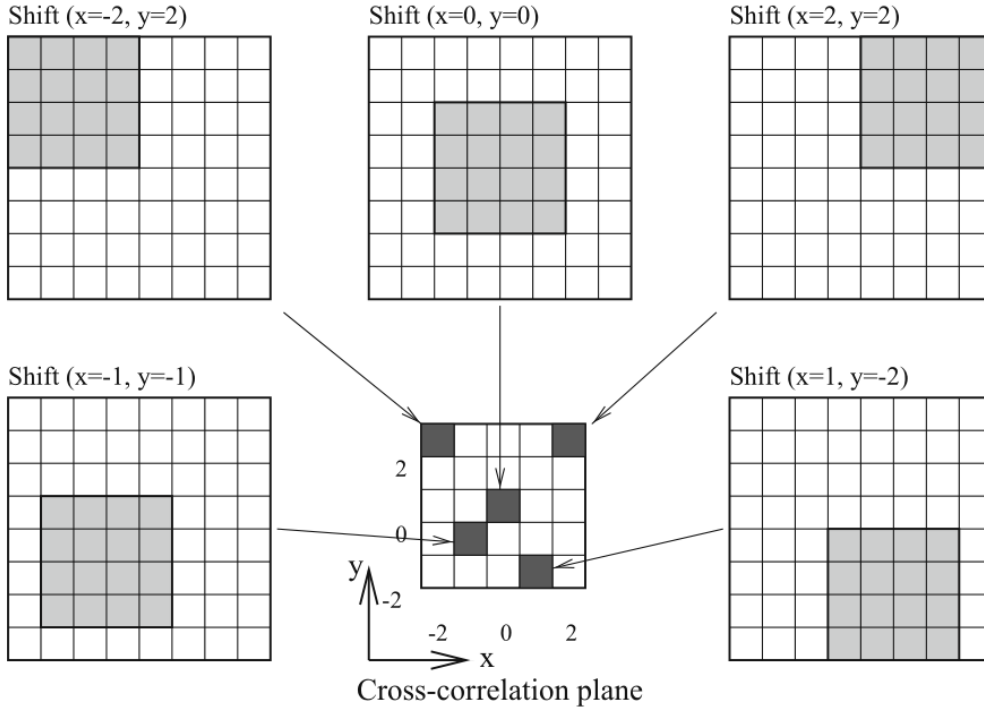


Figure 5.3: Pictorial demonstration of the cross correlation of an 8x8 pixel image using a 4x4 pixel template, taken from Raffel et al. [5]. The cross correlation image is formed pixel by pixel as the template is moved over the image.

need to mask half of the image.

Correlation and Peak Finding

Taking a closer look at the correlation step, Figure 5.3 shows pictorially how a correlation image is generated. A section of the original image, called the template, is moved over the original image, measuring the correlation at each point by summing the points under the template. Each position then corresponds to a pixel in the correlation image.

In equation form, each point (x, y) of the correlation image is equal to the sum over all points of the template at point (x, y) of the original image.

$$RR(x, y) = \sum_{i=-M}^M \sum_{j=-N}^N I(i, j) I'(i + x, j + y) \quad (5.1.2)$$

The template has size $M \times N$, with an intensity of $I(i, j)$ at each point. The intensity

of the corresponding point in the original image is given by $I'(i + x, j + y)$.

The particular correlation used throughout this project was a normalised cross correlation, meaning that the height of the central peak in each correlation image is always one. The equation for this normalised cross correlation is as follows, using the average values of $I(i, j)$ and $I'(i + x, j + y)$, denoted by $\bar{I}(i, j)$ and $\bar{I}'(i + x, j + y)$.

$$RR_{\text{norm}}(x, y) = \frac{\sum_{i,j} (I(i, j) - \bar{I}(i, j)) (I'(i + x, j + y) - \bar{I}'(i + x, j + y))}{\sqrt{\sum_{i,j} (I(i, j) - \bar{I}(i, j))^2 \sum_{i,j} (I'(i + x, j + y) - \bar{I}'(i + x, j + y))^2}} \quad (5.1.3)$$

Figure 5.3 uses a smaller template than the full image, but the image processing for the angle sensor uses the same image as the template, and so both are the same size. This introduces a problem; as the template moves further from the centre of the image, fewer and fewer pixels are contributing to the correlation, as the template further overhangs the underlying image.

One method of dealing with this issue is to pad the edges of one image with null data [5]. However, this approach introduces high frequency noise caused by the discontinuity at the edge of the image, as well as increasing the time and processing required for each correlation.

Another method, and the one used here, is to wrap the image, effectively correlating the overhang with itself. This is called a circular correlation [5]. The result of this method is random correlation noise at the points where the image wraps. This creates a smoother transition effect than using padding, resulting in more noise at the edge of the image where the percentage of pixels contributing to the true correlation is lower.

This circular correlation method also has an effect on the height and position of the correlation peak. Moving out from the centre of the image, the percentage of pixels contributing to the true correlation drops from 100% to almost zero. As a result, the correlation peak is skewed towards the centre, with a bigger effect the further from the middle of the image. This is demonstrated in Figure 5.4, taken from Raffel et al. [5].

The skewing of the peak resulting from the circular correlation can be accounted for during the peak finding. The basic peak finding method first locates the highest pixel

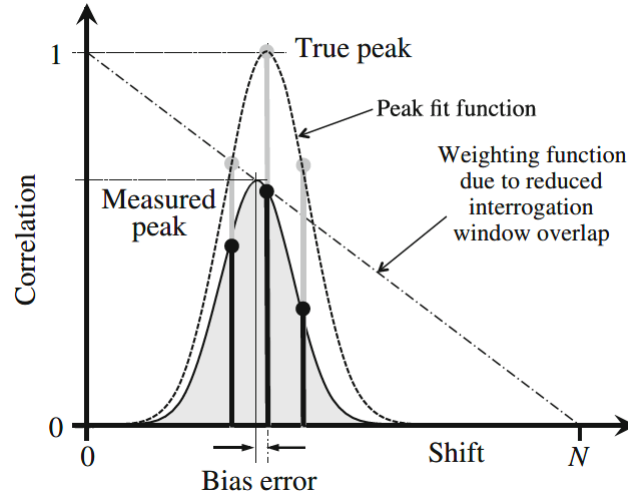


Figure 5.4: Demonstration of the bias error introduced by the circular correlation method for an image of size N , taken from Raffel et al. [5]. The weighting function skewing the peak has a greater effect the closer to the edge of the image the peak is positioned.

in the image, then uses this plus the adjacent pixels in each direction to calculate the true location of the peak. The equations used are considered peak estimators [5], as mathematically they are a simplification of the full Gaussian equations. However, the advantage of these estimators is that they can be solved directly, using the three points around the peak in each direction.

The equations use the height, R , at each pixel to find the peak centre (x_0, y_0) . The position of the highest pixel is described by (i, j) , with the adjacent pixels at $(i - 1, j)$, $(i + 1, j)$ and $(i, j - 1)$, $(i, j + 1)$.

$$x_0 = i + \frac{\ln R_{(i-1,j)} - R_{(i+1,j)}}{2 \ln R_{(i-1,j)} - 4 \ln R_{(i,j)} + 2 \ln R_{(i+1,j)}} \quad (5.1.4)$$

$$y_0 = j + \frac{\ln R_{(i,j-1)} - R_{(i,j+1)}}{2 \ln R_{(i,j-1)} - 4 \ln R_{(i,j)} + 2 \ln R_{(i,j+1)}} \quad (5.1.5)$$

As shown in Figure 5.4, the skewing of the peak by the circular correlation method is described by a weighting function. This function can be applied in reverse to the values calculated using the Gaussian estimators, restoring the true peak position.

5.1.2 Error Analysis for Image Capture and Processing

The detector used throughout this project is a complementary metal-oxide semiconductor (CMOS) camera, which can be susceptible to various types of camera noise during image capture. These include dark current, which refers to the accumulated of current in a pixel even when there is no light source, fixed pattern noise such as broken/dead pixels, shot noise, which refers to the resolution dictated by the discrete nature of photons, and electronic noise during readout. Shot noise is generally only considered at low light intensities, where the signal-to-noise ratio makes it a significant factor, which did not apply here. Experimentally, dark current and dead pixels were found to have a very minimal effect on captured images, with dead pixels in particular being averaged out by the correlation step.

In general, noise originating from image capture will manifest as background noise in the correlation image, which then will potentially affect the locating of the correlation peak. As such, the important consideration is the signal-to-noise ratio of the correlation peak and its effect on the accuracy of the peak finding method.

The correlation and peak finding techniques described above are taken from particle image velocimetry (PIV). PIV is a well-established field of research dedicated to measuring the velocity field of fluids [5]. Small tracer particles are added to the fluid in question, where small is used to mean that the particles are of a size that they follow the flow dynamics of the fluid. Successive images are taken with a known time interval and then correlated to determine the position shift, and thus velocity, of the particles. Typically, multiple windows of the image are correlated, which gives the velocity of particular regions of the flow, and so a complete velocity field can be constructed.

Within PIV, there are many different factors that can affect measurements and their errors, including particle size and density, interrogation window size, noise level, turbulent fluctuations, velocity gradients, and out-of-plane particle motion [5]. The last three are inherent to particle velocimetry, and so do not translate to sources of uncertainty for speckle pattern correlation. However, speckle size and density, image size, and correlation noise are important factors to consider when applying PIV techniques to speckle correlation.

Raffel et al. [5] (Chapter 6: PIV Uncertainty and Measurement Accuracy) looks extensively at the potential sources of uncertainty in PIV, including the quantitative

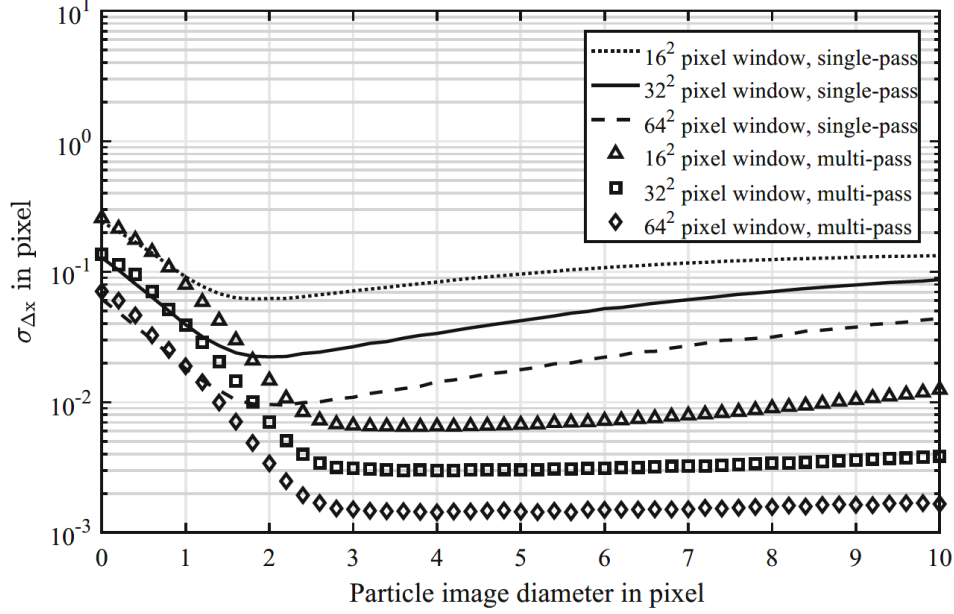


Figure 5.5: Measurement uncertainty in the Gaussian 3-point fit as applied to particle image velocimetry. The values are simulated results for changing particle diameter, determined for different window sizes and two different evaluation approaches. Figure taken from Raffel et al. [5].

effects of changes in various factors. Figure 5.5 shows the measurement uncertainty (random error) in the x direction, $\sigma_{\Delta x}$, as the particle diameter increases for the 3-point Gaussian fit described in the section above. The values were obtained from simulated PIV data and as such are roughly an order of magnitude smaller than real world experimental uncertainties, but demonstrate the shape of the error curve for changing particle diameter and the relative increase/decrease for changing window size. Results for two different evaluation approaches are shown, single- and multi-pass, but only the single-pass approach (i.e. a single correlation for each window) is applicable to speckle correlation.

The first conclusion that can be drawn from the figure is that a larger image size (pixel window) results in a smaller value of uncertainty. This is due to the increased number of particles in an image, which both increases the valid contributions to the correlation and thus the correlation peak height (i.e. signal strength), and decreases the effect of background noise.

Secondly, these results show that there is a minimum, or optimum, particle diameter for PIV. This is most clear in the single-pass results, where the minimum for each

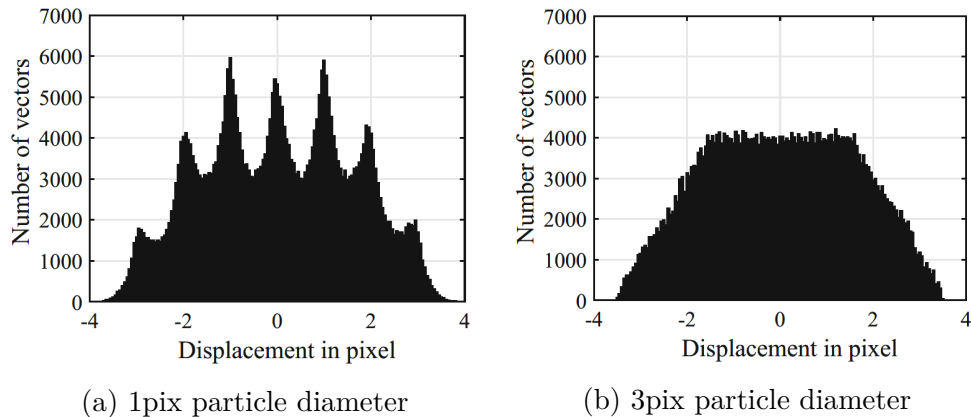


Figure 5.6: Histograms of the measured particle displacements for simulated PIV images, demonstrating the peak locking effect present for particle sizes for less than 3 pixels. Figures taken from Raffel et al. [5].

graph is at around 2 pixels. For particle sizes smaller than this, the particle is not well sampled by the discrete pixels of the sensor, and so the uncertainty on the 3-point fit rises sharply. As the particle diameter increases beyond 2 pixels, the variation of intensity around the maximum point decreases, and the 3-point sample is insufficient to make a good estimate of the peak position. This could be overcome by using a peak fitting method with more points, but at the cost of greater computing time.

As some examples of the numbers shown in Figure 5.5, the minimum uncertainty value for the 32x32 pixel window (single-pass) is 0.022pix. Looking at the larger 64x64 pixel window, the uncertainty ranges from around 0.01pix at a 2pix particle diameter, to roughly 0.02pix at a 6pix diameter, and then 0.03pix at 8pix.

Particle diameters smaller than around 3 pixels can also introduce a systematic error called “peak locking” or “pixel locking”, a periodic bias error in the measured particle displacement. This effect biases the displacement measurements towards integer values, with a greater effect the smaller the particle size. The two histograms in Figure 5.6 show the measured displacement for all particles (i.e. their vectors) for a simulated image with a single-pass correlation, with the vectors collected into 0.05pix width bins. Figure 5.6a uses a particle size of 1pix and shows an obvious bias towards integer pixel displacements, while Figure 5.6b uses a particle size of 3pix and shows no periodic features in the measured displacement. From this, it is clear that peak locking can be avoided by using a particle diameter, or speckle size,

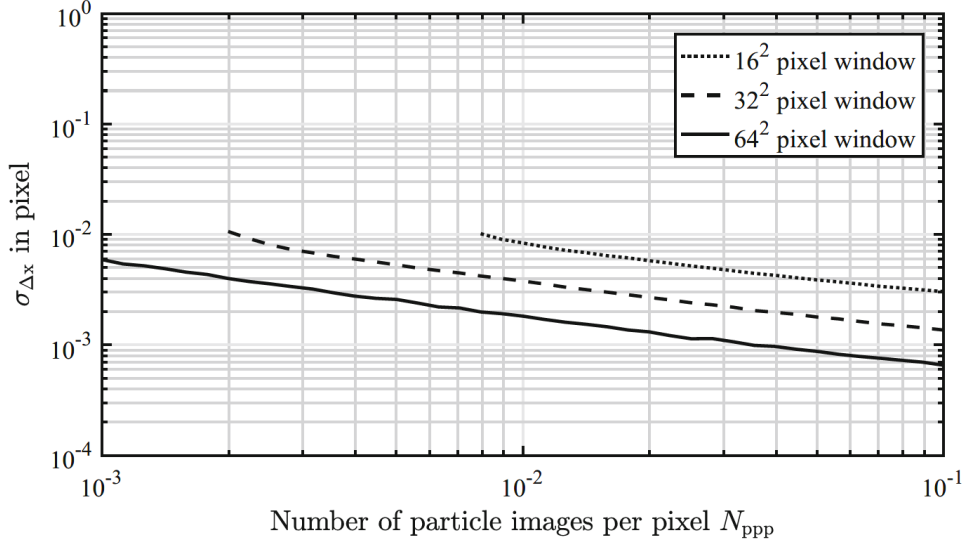


Figure 5.7: Measurement uncertainty in the Gaussian 3-point fit for simulated particle image velocimetry images. The results show the uncertainty for changing particle density, measured by the number of particles per pixel, for three different window sizes. Figure taken from Raffel et al. [5].

of greater than 3 pixels.

The number of particle pairs, i.e. particles that are present in both images, is another important factor in minimising random errors in the peak finding. This is affected by the particle density, image size, and particle displacement. A higher particle density and image size increases the number of particles in a single image, thus increasing the probability of capturing more particle pairs across two images. At the same time, the magnitude of the particle displacement affects this chance of capturing a complete pair, as a greater displacement increases the likelihood of a particle moving outside of the image window. A higher number of particle pairs increases the signal strength (height) of the correlation peak, thus reducing the uncertainty of the peak fitting applied to it.

Figure 5.7 shows the uncertainties derived from simulated data using different particle densities, quantified as the number of particles per pixel N_{ppp} , for different window (image) sizes. The graphs show the decrease in uncertainty as both image size and particle density increases. At an N_{ppp} of $10^{-2} = 0.01$, the value of the uncertainty is 0.008pix for the smallest window size of 16x16pix, then 0.004pix for a window of 32x32pix, and finally 0.002pix for a window size of 64x64pix.

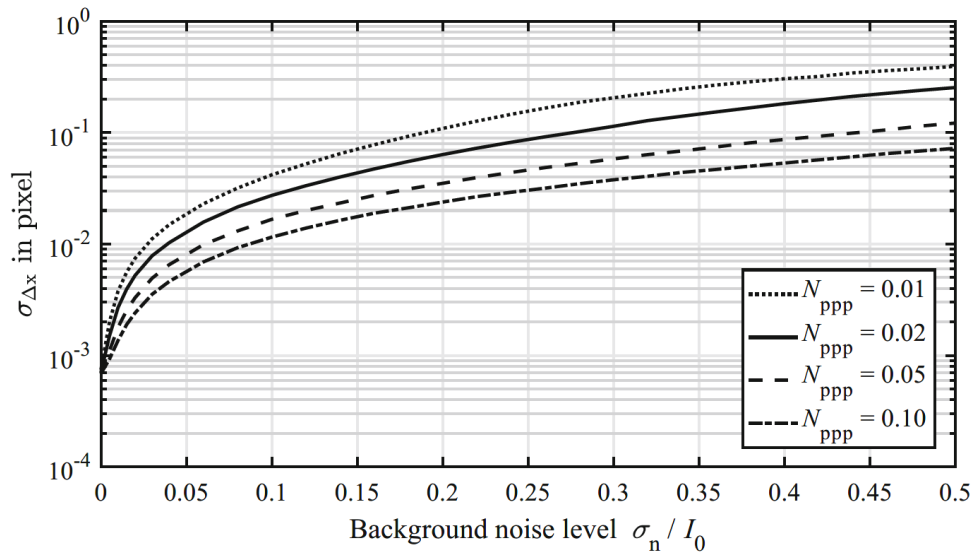


Figure 5.8: Measurement uncertainty in the Gaussian 3-point fit for simulated particle image velocimetry images. The figure shows the uncertainty for varying levels of background noise in the original images, defined as the standard deviation in the noise over the maximum intensity of the image, σ_n/I_0 . Results are shown for four different particle densities (number of particles per pixel, N_{ppp}). Figure taken from Raffel et al. [5].

Applying this to speckle correlation, the particle densities used in PIV (and considered by Raffel et al. [5]) tend to be smaller than the speckle density of a typical speckle pattern. In addition, the images sizes that will be used throughout this project are larger than those considered by Raffel et al. [5], at either 256x256pix or 1280x1024pix. Therefore, assuming the trend of decreasing uncertainty for increasing image size and increasing particle density continues past the data presented by Raffel et al. [5], the uncertainty due to speckle density and image size can be assumed to be lower than the values quoted here for PIV.

The final consideration from Raffel et al. [5] on measurement uncertainty that can be applied to speckle is background noise. An increase in background noise, simulated by adding normal-distributed noise at an increasing percentage of the maximum intensity of the noise-free image, increases the uncertainty in the peak finding and also decreases the height of the correlation peak. This change in the uncertainty, $\sigma_{\Delta x}$, is shown in Figure 5.8 with increasing background noise level, defined as the standard deviation of the noise, σ_n , over the maximum intensity of the noise-free image, I_0 . The specific uncertainty is also affected by the particle density, and so for

the median noise level of $\sigma_n/I_0 = 0.25$, the uncertainty is around 0.03 for $N_{ppp} = 0.10$, 0.05 for $N_{ppp} = 0.05$, 0.09 for $N_{ppp} = 0.02$, and 0.15 for $N_{ppp} = 0.01$.

Considering all these factors, the uncertainty in measurements of speckle shift can be minimised primarily by using a large image size, and also by ensuring the speckle size is close to 3 pixels. This second stipulation also meets the criterion for Nyquist sampling, whereby a periodic function should be sampled at more than twice its frequency in order to be fully reconstructed. Additionally, the effect of background noise is minimised by the use of a laser line filter to remove most ambient light.

Based on previous speckle work with similar setups [6, 7], the uncertainty value of the fit was assumed to be between 0.05 and 0.10 pixels. This inherent uncertainty limits the accuracy, repeatability, and resolution of the speckle shift measurement, and thus angle measurement, by the random errors introduced into measurements. More specifically, the assumed value of uncertainty will be used to define the resolution of the sensor, as described previously at the end of Chapter 4, as well as being used in the following section to estimate the uncertainty in angle measurements.

Error Propagation for Image Processing

The uncertainty on the 3-point peak fitting method described above is expected to be between one tenth and one twentieth of a pixel [5]. Taking the most conservative value, this gives an uncertainty in the speckle shifts A_x and A_y of 0.1 times the camera's pixel size. The Ximea camera used for the sensor has a pixel size of $4.8 \times 4.8 \mu\text{m}$, and so the uncertainty in speckle shift is expected to be $0.48 \mu\text{m}$ in each direction.

This can be converted to an expected error in surface tilt using conventional error propagation. As a starting point, the equations derived in Chapter 4 are restated below. These use the measured speckle shift, source wavelengths, and sensor geometry to calculate the tilt of the imaged surface.

$$\tan \theta_y = \frac{\lambda_1}{\Delta \lambda} \frac{A_x}{L_D (s_z + 1)} - \frac{s_x}{s_z + 1} \quad (5.1.6)$$

$$\tan \theta_x = \frac{\lambda_1}{\Delta \lambda} \frac{A_y}{L_D (s_z + 1)} - \frac{s_y}{s_z + 1} \quad (5.1.7)$$

It is easier to consider the errors in terms of the surface gradients m_x and m_y .

The gradients are linearly related to the speckle shifts A_x and A_y , whereas the tan operation results in a non-linear relationship between the angles and the shifts. As the angle θ_y is around the y axis, its corresponding gradient is m_x . The same applies for θ_x , and so for $\tan \theta_y = m_x$ and $\tan \theta_x = m_y$, Equations (5.1.6) and (5.1.7) become:

$$m_x = \frac{\lambda_1 A_x}{\Delta \lambda L_D (s_z + 1)} - \frac{s_y}{s_z + 1} \quad (5.1.8)$$

$$m_y = \frac{\lambda_1 A_y}{\Delta \lambda L_D (s_z + 1)} - \frac{s_x}{s_z + 1} \quad (5.1.9)$$

There is no cross-dependency in the speckle shifts between m_x and m_y , so the error propagation in x and y is independent of the other. The expected error in m_x , σ_{m_x} , relates to the error in A_x , σ_{A_x} , as so:

$$\sigma_{m_x}^2 = \left| \frac{\partial m_x}{\partial A_x} \right|^2 \sigma_{A_x}^2 \quad (5.1.10)$$

A similar equation applies for the error in m_y , σ_{m_y} :

$$\sigma_{m_y}^2 = \left| \frac{\partial m_y}{\partial A_y} \right|^2 \sigma_{A_y}^2 \quad (5.1.11)$$

Now, the partial derivatives are calculated:

$$\frac{\partial m_x}{\partial A_x} = \frac{\lambda_1}{\Delta \lambda L_D (s_z + 1)} \quad (5.1.12)$$

$$\frac{\partial m_y}{\partial A_y} = \frac{\lambda_1}{\Delta \lambda L_D (s_z + 1)} \quad (5.1.13)$$

The final equations for the expected uncertainties due to the signal processing are as follows:

$$\sigma_{m_x}^2 = \left| \frac{\lambda_1}{\Delta \lambda L_D (s_z + 1)} \right|^2 \sigma_{A_x}^2 \quad (5.1.14)$$

$$\sigma_{m_y}^2 = \left| \frac{\lambda_1}{\Delta \lambda L_D (s_z + 1)} \right|^2 \sigma_{A_y}^2 \quad (5.1.15)$$

As the uncertainty is the same in A_x and A_y , i.e. $\sigma_{A_x} = \sigma_{A_y}$, these two equations

give the same value. There is no dependence on speckle shift or therefore angle, and so the uncertainty in m_x and m_y is a constant value at all points. This uncertainty in the gradient can now be converted into an uncertainty in angle measurement using the relation $\sigma_\theta = \arctan \sigma_m$.

The uncertainty is also dependent on the geometry of the sensor, specifically the values $L_D(\mathbf{P}, \mathbf{D})$ and s_z . As such, the expected errors cannot be calculated without knowing the geometry of the sensor. Consider a sensor with the geometry $S = (100, 0, 100)\text{mm}$, $D = (0, 0, 100)\text{mm}$ and wavelengths $\lambda_1 = 658.0\text{nm}$, $\lambda_2 = 660.0\text{nm}$. This sensor has $L_D = 100\text{mm}$ and $s_z = 0.71$, as shown in Figure 5.9. The expected errors from the image processing for this sensor are 0.053° in both θ_x and θ_y .

5.2 Source Characterisation

5.2.1 Effect of Laser Instability on Angle Measurements

Characterising the laser or lasers used in each implementation is important for two reasons. Firstly, the precise wavelengths of the source are required for the speckle

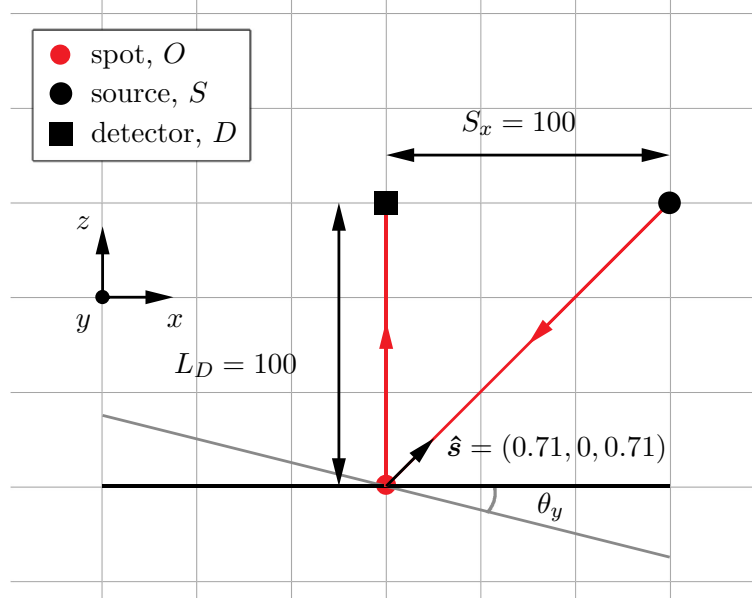


Figure 5.9: Diagram of an example sensor geometry. The distances S_x (between source and detector) and L_D (between detector and spot) are given in mm. $\hat{s}$ is the unit vector from the spot to the source. An example surface tilt around the y axis of θ_y is shown in grey.

theory equations used in the signal processing, specifically the values λ_1 and $\Delta\lambda$. Secondly, investigating the stability of the lasers can be useful for estimating and understanding any errors encountered during angle measurements.

In order to generate the two distinct wavelengths required for DWS, either one or two lasers are needed: one laser with an output of two distinct longitudinal modes, or two lasers each with a single mode. All sensor implementations used Fabry-Pérot diode lasers. This type of laser can have complex spectra, either single, few, or many longitudinal mode(s), depending on drive current and diode temperature, so understanding the behaviour of these lasers is particularly important. Some example spectra from one of the laser diodes used in the sensors are shown in Figure 5.10, measured using an optical spectrum analyser (OSA). The selection includes a single mode and a dual mode output.

Fluctuations in the applied current or diode temperature can arise from factors such as electronic noise or changes in ambient temperature, which can then lead

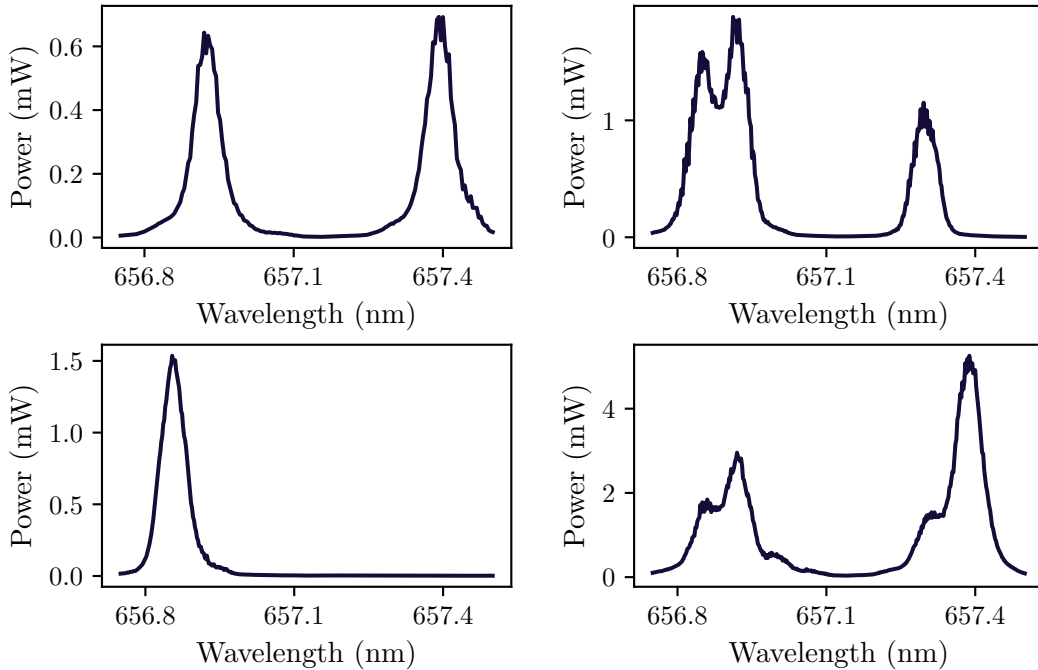


Figure 5.10: Example laser spectra from one of the laser diodes used in the project, each at four different input currents. They were measured using an OSA at a sampling resolution of 0.002nm. The laser is a FTEC2-658-30 FiberTec II diode, with a quoted wavelength of 658nm and maximum power of 30mW.

to fluctuations in the laser wavelength and/or intensity. In terms of DWS, changes in the intensity of one mode in relation to the other result in a change in the correlation peak height, thus affecting the peak fitting method as described in the previous section. As long as the intensity change does not result in the correlation peak dropping close to zero, the effect this will have on measurements is a slight increase in the peak finding uncertainty.

Any drift in the two wavelengths λ_1 and λ_2 , however, will result in changes to the separation between the two generated speckle patterns for a given surface angle, i.e. the measured speckle shift. This results in a difference in the calculated surface angle, despite the surface remaining fixed. Particularly important is the difference between the two modes, $\Delta\lambda$. $\Delta\lambda$ is smaller than λ_1 , so the same absolute change to both will result in a much larger relative change to $\Delta\lambda$. Thus instability in $\Delta\lambda$ has a larger effect on the ratio $\lambda_1/\Delta\lambda$ and therefore angle measurements as compared to instability in λ_1 .

Mode Power Stability and Wavelength Stability

Two different measures of laser stability were used to describe the changes observed in the lasers over time. The first was the mode power stability, defined as the change over time in the peak power of each longitudinal mode present at a particular laser tuning. Effectively, this measures which modes are dominant in the spectrum and how this changes over time. The second measure was wavelength stability, defined as the change, or drift, in the peak wavelength of a particular mode.

These two types of instability have different effects on angle measurements. A drift in the wavelength stability of a particular mode will cause a corresponding change in the speckle shift and therefore the angle measurement, introducing a random error into measurements. Instability in the mode power, however, can cause larger changes in angle measurements obtained by the sensor. If the expected mode (i.e. the wavelength used in the equations) drops in power and another becomes dominant, this will cause a discrete jump in the angle measurement away from the true value. This results in a systematic error in measurements for the duration of the mode switch.

These two types of stability are illustrated in Figure 5.11. These graphs show theoretical measurements for the example sensor defined above and shown in Figure

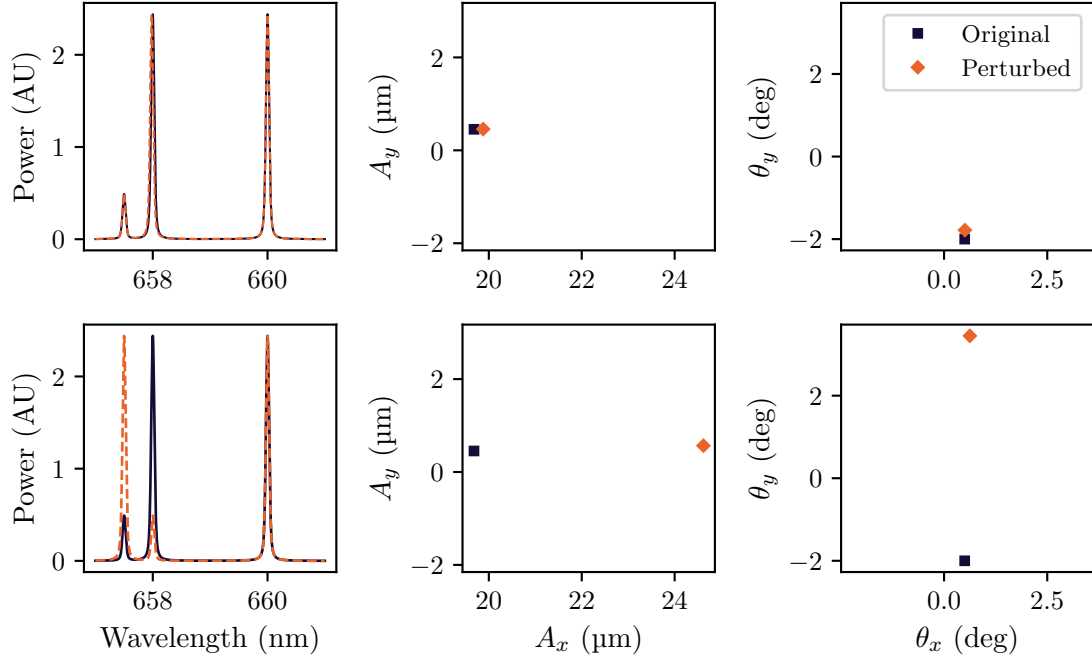


Figure 5.11: A demonstration of the difference between wavelength stability (top) and mode power stability (bottom). Each row shows the wavelengths of a theoretical sensor, followed by the resulting speckle shift and the calculated surface angle. The first row shows a small change in the wavelength λ_1 of 0.02nm, while the second shows a change in the mode power of the two modes in laser 1. The second instability results in a much bigger change in the measured tilt angle.

5.9. The source wavelengths of this sensor are $\lambda_1 = 658.0\text{nm}$ and $\lambda_2 = 660.0\text{nm}$. The first graphs of Figure 5.11 show the sensor wavelengths at 658.0nm and 660.0nm (solid lines), plus a parasitic mode in laser 1 with one fifth of the power at 657.5nm. The corresponding speckle shifts (A_x , A_y) and surface tilts (θ_x , θ_y) for the expected sensor parameters are calculated for an “applied” tilt angle of $\theta_x = +0.5^\circ$ and $\theta_y = -2.0^\circ$. These are shown in Figure 5.11 as the ‘Original’ points.

In the first row of the figure, a small wavelength perturbation of -0.02nm is applied to λ_1 , shown by the dotted line. Now calculating the speckle shift from the applied angles of $\theta_x = +0.5^\circ$ and $\theta_y = -2.0^\circ$ using the new value of λ_1 gives a slightly different result due to the changes in the constants A_{x0} , A_{y0} , and C . In a practical sensor, this value would simply be the measured speckle shift. Then, calculating the angle using the original values of A_{x0} , A_{y0} , and C (assuming λ_1 is as expected), gives a small error in the measured surface tilt. The new values after the wavelength

perturbation are given by the diamond markers in the figure. In θ_x this change is equal to $+0.005^\circ$ (a percentage error of 1%), and in θ_y the angle change is $+0.218^\circ$ ($\sim 10\%$ error).

The second row of Figure 5.11 shows the effect of a change in the dominant mode, i.e. a large perturbation in λ_1 . The dotted line in the spectra shows that the powers of each mode in laser 1 have reversed, with the mode at 657.5nm becoming dominant. The resulting change in the speckle shift and measured angle, calculated as for the previous wavelength perturbation, are much larger than for a small wavelength instability. The change in θ_x is now $+0.125^\circ$ (a 25% error), and in θ_y the angle change is $+5.449^\circ$ ($\sim 250\%$ error).

The mode power stability was used as the most important factor when conducting initial tuning scans. The aim at this stage was to identify possible operating conditions with either one or two dominant peak(s). The current and temperature were varied while monitoring the output spectrum by eye using an OSA, ensuring the output remained consistent over a few minutes for each input condition.

Once suitable values were found, long-term stability measurements were taken over several hours, using a PC to record OSA spectra and perform an automated peak fitting (described in the following section). For these measurements, the wavelength stability became the more important factor. Provided the mode power distribution remains the same, the drift in the centre wavelength of the dominant mode over the measurement period can be quantified as a standard deviation in the wavelength. This value is useful for calculating a precise estimate of the uncertainty in tilt measurements that can be expected from a given laser instability.

5.2.2 Error Propagation for Wavelength Stability

Once the standard deviation on wavelength stability has been determined, the effect on angle measurements can be calculated using the same error propagation methods used in the previous section. The starting point is again the initial equations relating

surface tilt to speckle shift.

$$\tan \theta_y = \frac{\lambda_1}{\Delta\lambda} \frac{A_x}{L_D (s_z + 1)} - \frac{s_x}{s_z + 1} \quad (5.2.1)$$

$$\tan \theta_x = \frac{\lambda_1}{\Delta\lambda} \frac{A_y}{L_D (s_z + 1)} - \frac{s_y}{s_z + 1} \quad (5.2.2)$$

Experimentally, only λ_1 and λ_2 are measured, so $\Delta\lambda$ should be rewritten as $(\lambda_2 - \lambda_1)$. Substituting this and both $\tan \theta_y = m_x$ and $\tan \theta_x = m_y$, these equations become:

$$m_x = \frac{\lambda_1 A_x}{(\lambda_2 - \lambda_1) L_D (s_z + 1)} - \frac{s_y}{s_z + 1} \quad (5.2.3)$$

$$m_y = \frac{\lambda_1 A_y}{(\lambda_2 - \lambda_1) L_D (s_z + 1)} - \frac{s_x}{s_z + 1} \quad (5.2.4)$$

Error propagation for the laser stability uses the partial derivatives of m_x and m_y with respect to λ_1 and λ_2 , along with the measured standard deviation in each variable.

$$\sigma_{m_x}^2 = \left| \frac{\partial m_x}{\partial \lambda_1} \right|^2 \sigma_{\lambda_1}^2 + \left| \frac{\partial m_x}{\partial \lambda_2} \right|^2 \sigma_{\lambda_2}^2 \quad (5.2.5)$$

$$\sigma_{m_y}^2 = \left| \frac{\partial m_y}{\partial \lambda_1} \right|^2 \sigma_{\lambda_1}^2 + \left| \frac{\partial m_y}{\partial \lambda_2} \right|^2 \sigma_{\lambda_2}^2 \quad (5.2.6)$$

Most of the variables in Equations (5.2.3) and (5.2.4) are still present in the partial derivatives of m_x and m_y . This means that the exact uncertainties are dependent on the geometry of the system and the wavelengths of the source. They are also dependant on the measured speckle shifts A_x and A_y , so the expected uncertainty is different at different surface gradients. Each of the partial derivatives with respect

to λ_1 and λ_2 are given below.

$$\frac{\partial m_x}{\partial \lambda_1} = \frac{\lambda_2 A_x}{(\lambda_2 - \lambda_1)^2 L_D (s_z + 1)} = \frac{\lambda_2 A_x}{\Delta \lambda^2 L_D (s_z + 1)} \quad (5.2.7)$$

$$\frac{\partial m_x}{\partial \lambda_2} = \frac{-\lambda_1 A_x}{(\lambda_2 - \lambda_1)^2 L_D (s_z + 1)} = \frac{-\lambda_1 A_x}{\Delta \lambda^2 L_D (s_z + 1)} \quad (5.2.8)$$

$$\frac{\partial m_y}{\partial \lambda_1} = \frac{\lambda_2 A_y}{(\lambda_2 - \lambda_1)^2 L_D (s_z + 1)} = \frac{\lambda_2 A_y}{\Delta \lambda^2 L_D (s_z + 1)} \quad (5.2.9)$$

$$\frac{\partial m_y}{\partial \lambda_2} = \frac{-\lambda_1 A_y}{(\lambda_2 - \lambda_1)^2 L_D (s_z + 1)} = \frac{-\lambda_1 A_y}{\Delta \lambda^2 L_D (s_z + 1)} \quad (5.2.10)$$

Combining the partial derivatives with equations (5.2.5) and (5.2.6), the full expressions for the estimated uncertainties are as follows:

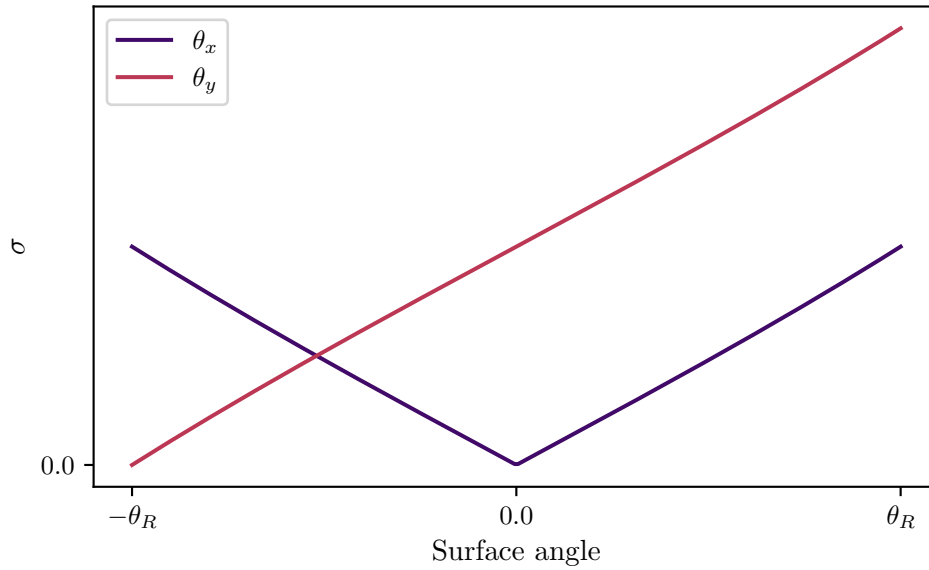
$$\sigma_{m_x}^2 = \left| \frac{\lambda_2 A_x}{\Delta \lambda^2 L_D (s_z + 1)} \right|^2 \sigma_{\lambda_1}^2 + \left| \frac{-\lambda_1 A_x}{\Delta \lambda^2 L_D (s_z + 1)} \right|^2 \sigma_{\lambda_2}^2 \quad (5.2.11)$$

$$\sigma_{m_y}^2 = \left| \frac{\lambda_2 A_y}{\Delta \lambda^2 L_D (s_z + 1)} \right|^2 \sigma_{\lambda_1}^2 + \left| \frac{-\lambda_1 A_y}{\Delta \lambda^2 L_D (s_z + 1)} \right|^2 \sigma_{\lambda_2}^2 \quad (5.2.12)$$

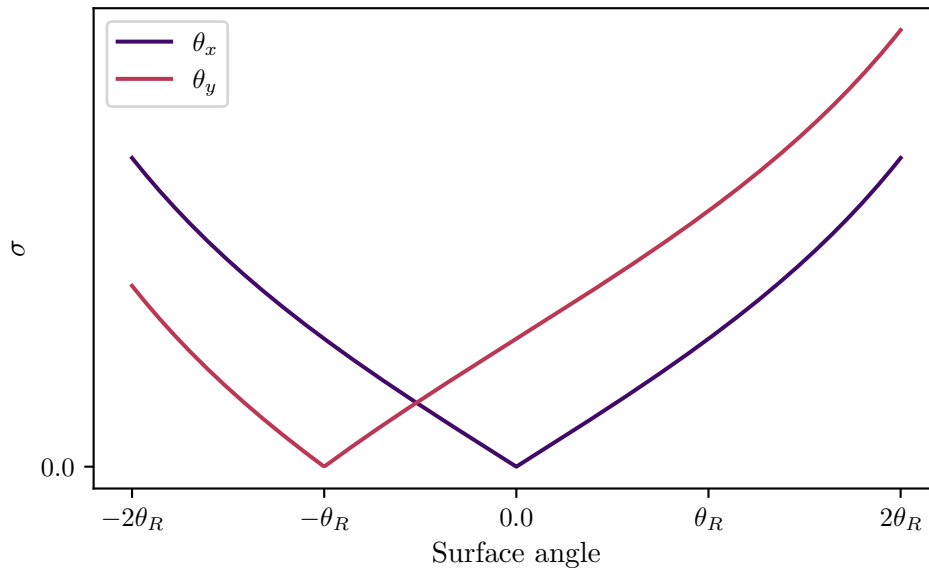
From these equations, it is clear that the errors in m_x depend on the value of A_x , and those in m_y depend on A_y , with no cross-dependence. At this stage, the relationships $\sigma_{\theta_y} = \arctan \sigma_{m_x}$ and $\sigma_{\theta_x} = \arctan \sigma_{m_y}$ can be applied, converting the uncertainties back to angles rather than gradients.

All versions of the sensor presented in this thesis are configured such that the source is offset in the x direction from the detector. In terms of the constants A_{x0} and A_{y0} defined in Chapter 4, this results in a non-zero value for A_{x0} , and an A_{y0} equal to zero. In other words, a speckle shift of zero in the y direction ($A_y = 0 = A_{y0}$) corresponds to a surface tilt around x of zero, or $\theta_x = 0$. However for the other dimension, zero speckle shift in x ($A_x = 0$) is achieved at $\theta_y = \theta_R$, the maximum range of the sensor.

This relationship can be seen clearly in the projected uncertainties from the laser stability in Figure 5.12. Figure 5.12a shows plots of the relative values of σ for a sensor with range $\pm \theta_R$. The errors are minimised at zero speckle shift, increasing as the magnitude of shift increases. In angular terms, this occurs at $\theta_x = 0^\circ$ and $\theta_y = \theta_R$. Therefore during measurements, the uncertainties in θ_x can be expected



(a) Laser error curves up to $\pm\theta_R$



(b) Laser error curves up to $\pm 2\theta_R$

Figure 5.12: Relative error curves for the standard deviation, σ , of θ_x and θ_y due to laser instability. The curve for θ_x is centred at zero tilt, while the curve for θ_y is centred at $-\theta_R$. The range of the first graph covers the defined maximum range of the sensor, $\pm\theta_R$, showing an apparently linear relationship between surface angle and σ . The second graph shows the relationship up to twice θ_R , and thus the non-linearity introduced by $\tan \theta$.

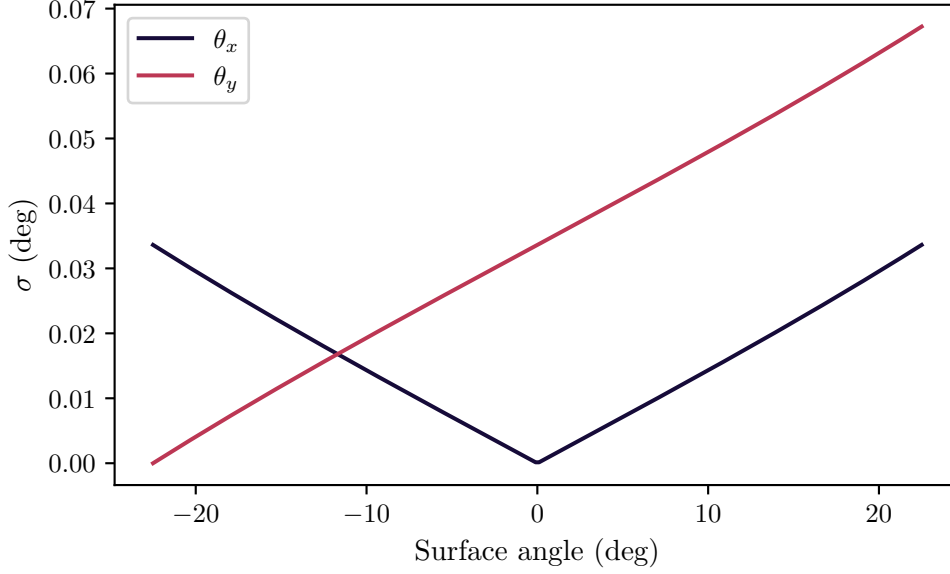


Figure 5.13: Expected error curves for the theoretical sensor described in Figure 5.9. The range, θ_R , of this sensor is 22.5° , and the assumed standard deviation on each wavelength is 0.02nm . The maximum uncertainties are 0.034° in θ_x and 0.067° in θ_y .

to increase as the magnitude of the surface tilt increases. For θ_y , however, the uncertainties increase continuously across the range of surface gradients from negative to positive values of θ_y .

Another consequence of this is that, in general, the expected uncertainties are larger in θ_y than in θ_x . This is also demonstrated by equations (5.2.11) and (5.2.12). As described in Section 5.1, A_x will always be greater than the size of the mask used on the central correlation peak. A_y , on the other hand, can have negative and positive values close to zero. So in many cases, A_x will be larger than A_y , leading to $\sigma_{m_x} > \sigma_{m_y}$, or in terms of angles $\sigma_{\theta_y} > \sigma_{\theta_x}$.

Despite the non-linearity introduced when converting from gradient (m) to angle (θ), the range of Figure 5.12a appears to show a linear relationship between surface angle and σ . This suggests that across the working range of the sensor, a linear relationship can be assumed. Figure 5.12b extends the range to twice the maximum value θ_R . The non-linearity is more clear in this figure, seen at the edges of the graphs. This figure also demonstrates clearly the minimum errors expected in θ_y at θ_R .

Using the theoretical sensor described above, the uncertainties σ_x and σ_y can be estimated for the example wavelength shift of +0.02nm used in Figure 5.11. This value is taken as the standard deviation in both of the source wavelengths. The resulting error curves are given in Figure 5.13. This sensor has a maximum range of $\theta_R = 22.5^\circ$. Note that this discounts the masking of the central peak during image processing. The errors approach zero at $\theta_x = 0$ and $\theta_y = 22.5^\circ$, and the maximum values of σ_x and σ_y are 0.034° and 0.067° respectively.

Directional Component of Wavelength Errors

Another way of looking at these errors is in the $A_x A_y$ plane of the speckle shifts. Figure 5.14 shows a circle of speckle shifts for the theoretical sensor. As the source is offset from the detector in x , the speckle shifts are also offset from zero in A_x . The shifts were generated using the example sensor geometry of $S = (100, 0, 100)\text{mm}$, $D = (0, 0, 100)\text{mm}$, and a series of surface tilts with magnitude $\psi = 3.0^\circ$ and direction increments of $\phi = 20.0^\circ$ around a circle. The angles can be converted between Cartesian and polar reference frames using the standard relations between the two coordinate systems:

$$\psi = \sqrt{\theta_x^2 + \theta_y^2} \quad (5.2.13)$$

$$\tan \phi = \frac{\theta_y}{\theta_x} \quad (5.2.14)$$

For each of the “applied” surface tilts, the speckle shift was calculated for the theoretical sensor using the expected wavelengths of $\lambda_1 = 658.0\text{nm}$ and $\lambda_2 = 660.0\text{nm}$, plus for changes to λ_1 of $\pm(0.01, 0.02, 0.03)\text{nm}$. As such, Figure 5.14 demonstrates the direction of the errors caused by laser instabilities. As the speckle shift is measured from zero, any changes in the measurements are along the line towards zero speckle shift. This is seen clearly in the lower part of Figure 5.14, which includes a series of lines drawn from the speckle shift at $\lambda_1 = 658.0\text{nm}$ to $(0, 0)$.

All implementations of the sensor tested in this project will offset the source in x , so the general conclusion from this error distribution is that errors in θ_y (relating to shifts in A_x) are likely to be higher than errors in θ_x . This broadly follows from the error analysis of wavelength instability above, but this framing shows clearly the reason for the discrepancy. It is also clear that the difference will be more

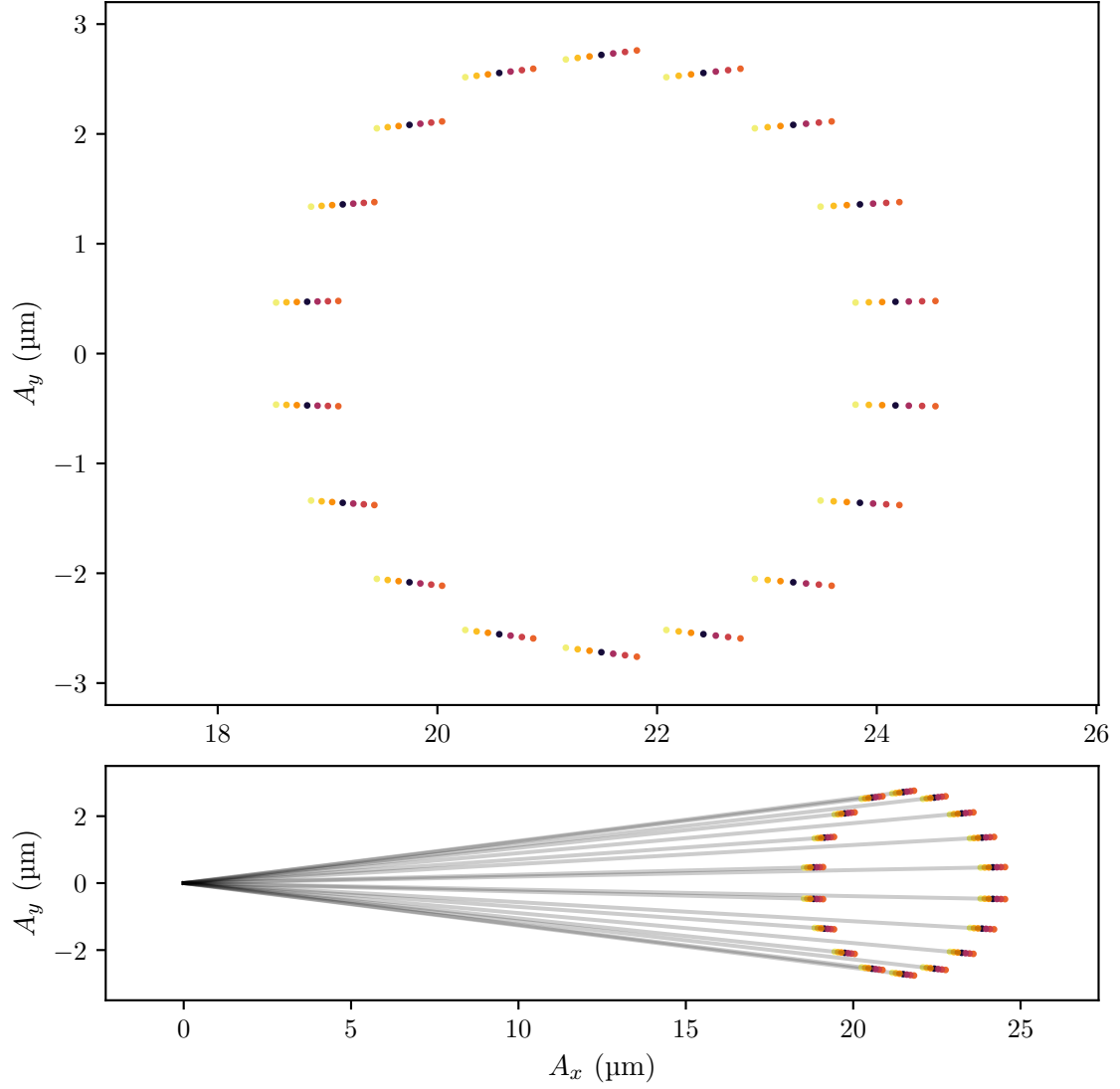


Figure 5.14: Demonstration of the directional nature of errors due to wavelength instability. The graphs show speckle shifts for a series of surface tilts of magnitude 3.0° and varying direction around a circle. The shifts were calculated using the theoretical sensor outlined in this chapter. At each point, a series of small wavelength changes were applied to λ_1 , $\pm(0.01, 0.02, 0.03)\text{nm}$, resulting in changes along the line from zero speckle shift. The lower graph adds these lines from the shift at λ_1 to $(0, 0)$.

Table 5.2: Summary of laser specifications for the three FiberTec II diodes used throughout the project.

Model number	Centre wave-length (nm)	Maximum power (mW)	Observed wavelength range (nm)
FTEC2-658-30	658	30	656-658
FTEC2-658-60	658	60	658-660
FTEC2-638-20	638	20	637-639

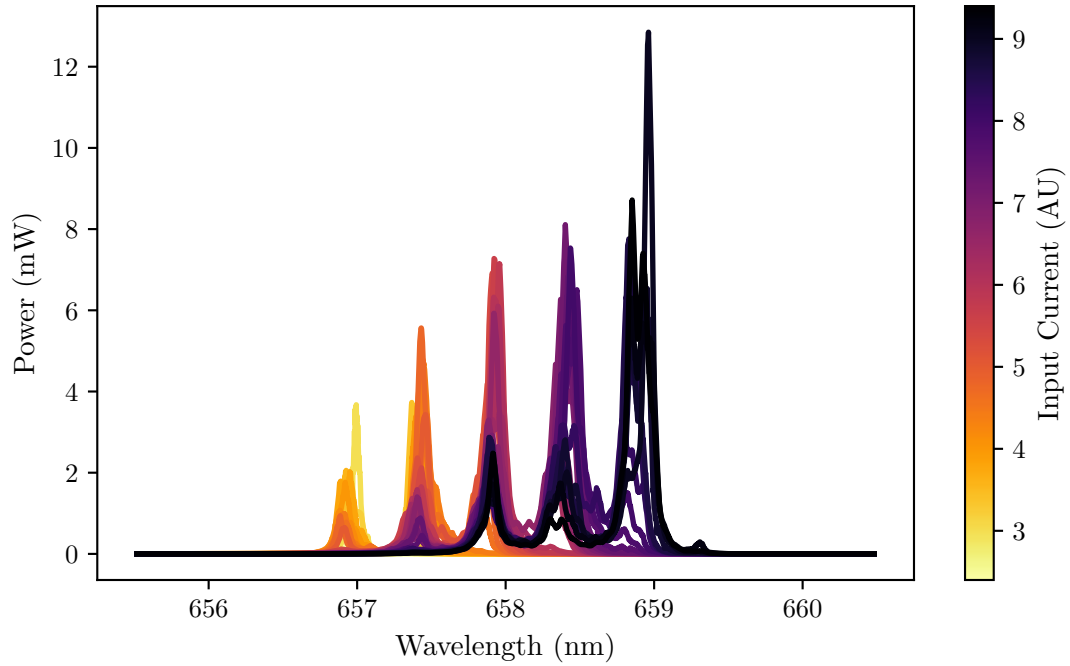
pronounced further from zero speckle shift, or for sensors with a larger separation between the detector and the source.

5.2.3 Investigating Laser Stability

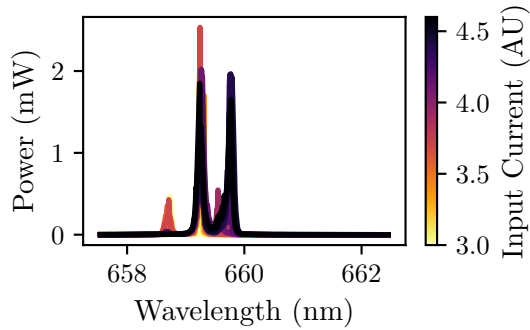
A total of three different lasers were used in the various sensor implementations throughout this work. All three are FiberTec II Fabry-Pérot laser diodes, manufactured by Blue Sky Research [8]. They each have a variable input current, allowing for manual control of the precise wavelength output, plus temperature stabilisation to 1°C. They all fall within the red region of the visible spectrum, with two having a quoted wavelength of 658nm, and the other quoted at 638nm. The lasers are all linearly polarised, with the axis aligned within 4° of the connector key.

Investigations into the output and stability of each laser diode were carried out using a Yokogawa AQ6370C optical spectrum analyser [9], which has a maximum resolution of 0.02nm and maximum sampling interval of 0.002nm, though both these values are variable. The real-time monitoring was used initially to get an understanding of which longitudinal modes were present at different input currents, and to determine the exact wavelengths of the laser diodes. Table 5.2 gives the model number and quoted specification of each laser, along with the observed wavelength range across the full range of input currents. Although the first two lasers have the same advertised wavelength, they were found to have slightly different wavelength ranges in testing, likely due to small discrepancies in the size of the laser cavity.

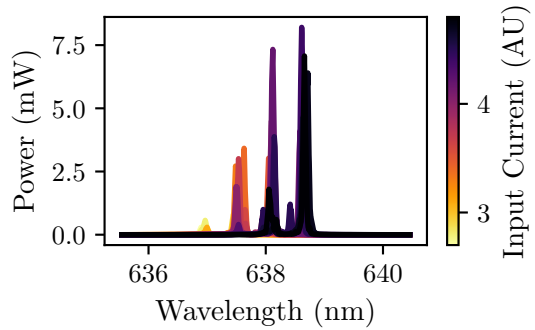
In general, as the input current to each laser was increased, the output wavelength also increased. However, the output was restricted to distinct longitudinal modes.



(a) FTEC2-658-30



(b) FTEC2-658-60



(c) FTEC2-638-20

Figure 5.15: Overlaid spectra at different operating currents for the three FiberTec II laser diodes used throughout the project. The successive spectra show the distinct longitudinal modes in each laser.

As the current increased, lower wavelength modes would fall in power, and higher wavelength modes would rise. This can be seen in Figure 5.15, and most clearly in Figure 5.15a; the increasing current gradually shifts the spectrum into longitudinal modes with higher wavelengths. The three graphs show the spectra at varying applied currents for each of the three lasers used throughout the project. The input current is given in arbitrary units, as the control mechanism uses a potentiometer to select a power, which is then converted into a current using an unknown factor. Thus the values used (shown by the colourbar) are equal to the chosen input power, rather than a direct measure of the applied current. The spectra were recorded by the OSA at the maximum resolution and sampling interval of 0.02nm and 0.002nm respectively.

For the sensor, each laser was to be operated in continuous-wave (CW) mode at a fixed current and temperature. Once a promising set of operating conditions had been found using the real-time monitoring, the OSA was used to record spectra at regular intervals over a number of hours. These spectra could then be processed to find the variability in wavelength of a particular mode, i.e. its wavelength stability. The data produced by the OSA are a series of spectra, with the measured power at each wavelength sampled. A peak fitting was applied to each spectrum to find the centre wavelengths of any modes present to a sub-datapoint precision. The fitting had two stages: a coarse peak finding method, followed by a least-squares curve fitting on each mode. The first pass finds local maxima, so is limited by the resolution of the data. The results from this stage are used to separate the spectrum into sections around each mode. The curve fit is applied to each section, with the peaks found in the first pass used as the initial guesses.

The peak fitting steps are outlined in Figure 5.16. The analysis is carried out using Python, with the coarse peak finding method taken from the module *detecta* [10]. This method finds local maxima in a dataset given parameters such as minimum peak height/separation and thresholds on neighbouring peaks. The parameters for the example figure were a minimum peak height of 0.4 and a minimum peak separation of 30 points, with no thresholds for neighbouring peaks. The first spectrum of Figure 5.16 shows the results of this fit, giving the highest point of each peak.

The next step is to split the spectrum into sections. Ideally, the spectrum would contain two distinct peaks (modes), separated by the desired wavelength spacing,

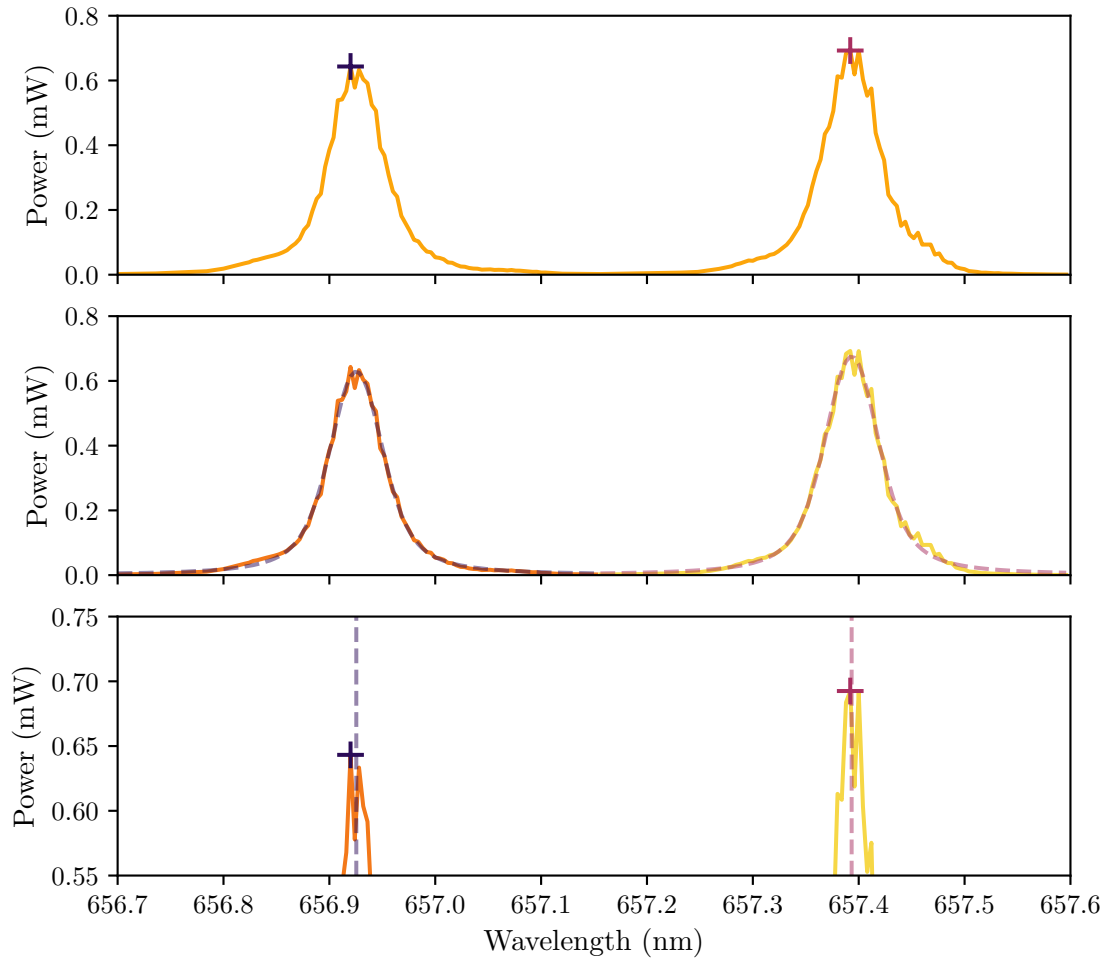


Figure 5.16: Steps of the peak fitting method used to determine the centre wavelengths of laser spectra. First a coarse peak finding method is used to locate the highest point in each peak. This is then used to apply an algorithmic curve fit to find the sub-datapoint value of the peak centre.

Table 5.3: Final fitting residual (sum of the square of the differences) for each curve profile applied to the three lasers, as shown in Figure 5.17.

Laser	Gaussian	Lorentzian	Voigt
FTEC2-658-30	7.26	26.41	5.01
FTEC2-658-60	0.37	1.92	0.52
FTEC2-638-20	1.97	10.87	4.73

and so the spectrum would be split in two, with the divide being halfway between the two peaks. This is true in Figure 5.16, where the second and third spectra are plotted in different colours according to the split point. The more precise curve fitting can now be applied to each section separately. This fitting method is taken from the Python module *lmfit* [11], and uses the Levenberg-Marquardt algorithm to perform a least squares fit to the data. The function returns a final sub-datapoint value for the height (power) and centre position (wavelength) of the peak.

The *lmfit* module can be used to fit to a variety of curve profiles. The Gaussian, Lorentzian and Voigt profiles were each fitted to an example spectra with one mode present, in order to find the closest match for each laser. The results of the fits are shown in Figure 5.17, and the residuals for each are given in Table 5.3. These residuals are the sum of the square of the differences of the spectrum and its fit.

For all three lasers, the Gaussian and Voigt profiles gave a similar result to each other, while the Lorentzian fit had a noticeably higher residual. For the FTEC2-658-30 laser diode, the Voigt fit gave a slightly better result, at 5.01 versus the Gaussian's 7.26 residual. For the other two lasers, the FTEC2-658-60 and FTEC2-638-20, the Gaussian profile was better, with residuals of 0.37 and 1.97 respectively. As mentioned above, the full results are in Table 5.3.

The results of this profile testing will be used to customise the final curve fit to each laser, ensuring the most accurate results. The complete spectra fitting method will be used to determine the wavelength stability of the laser(s) used in each sensor before angle measurements. Each laser will be monitored using the OSA over a period of multiple hours, and the spectra fitting used to find the standard deviation of the wavelength of the laser over this time. As described above, this can then be used to estimate the angular errors in the full sensor.

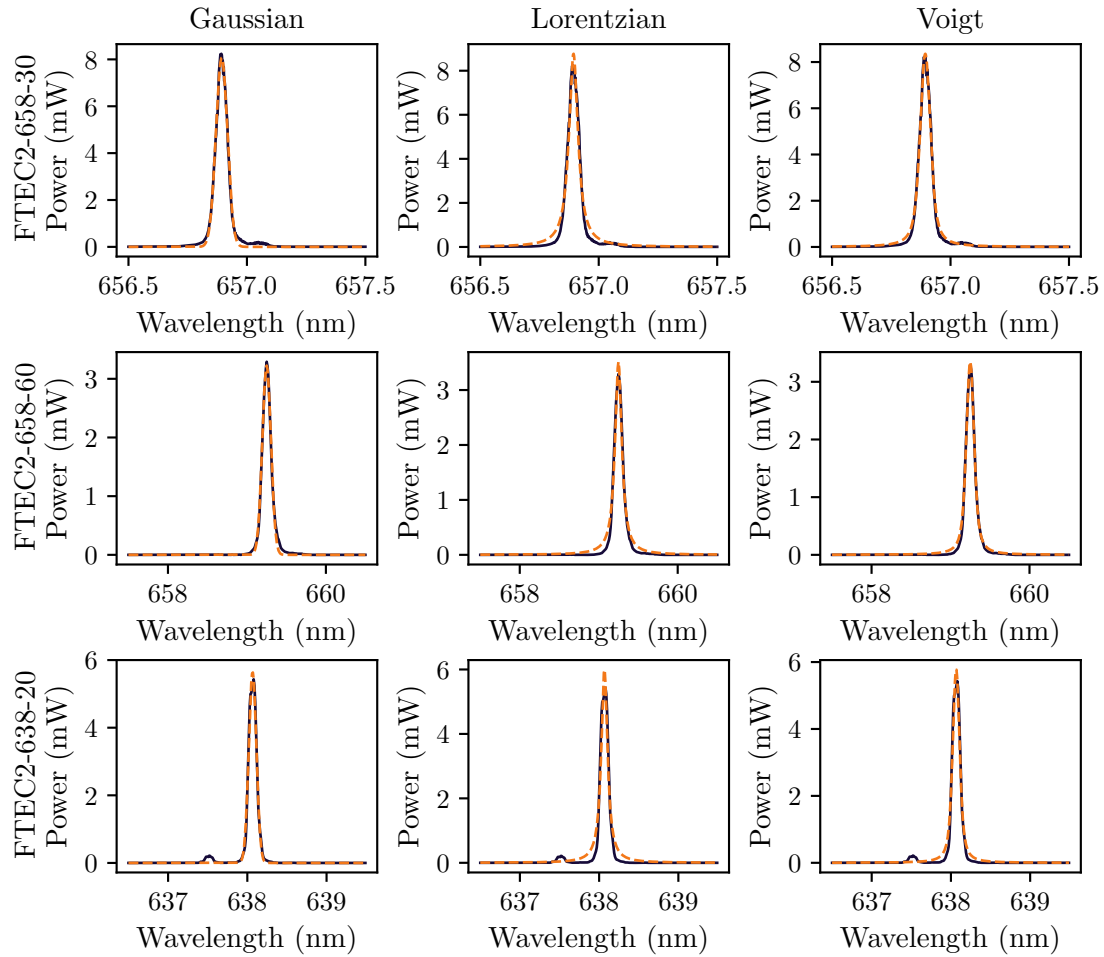


Figure 5.17: Curve fitting for each of the three lasers using a single mode tuning. A Gaussian, Lorentzian, and Voigt profile were tested with each spectrum, using a Levenberg-Marquardt least squares fit. For the first laser, the Voigt profile was found to give the best fit, but for the other two lasers the Gaussian profile gave the lowest fitting value (see Table 5.3).

5.2.4 Testing the Peak Finding Method with Modelled Data

The accuracy of the centre finding of the peak fitting method was tested using modelled data. The aim of this test was to determine the variability of the peak finding method, finding its standard deviation over a number of randomly generated spectra. This value can then be compared to the standard deviations calculated for real data. Ideally, the standard deviation on the fitting will be much lower than the standard deviation on laser measurements, thus confirming that the measured stability is an accurate representation of the laser's wavelength stability.

The curve used in the modelled data was the Voigt profile, and the models varied in complexity from a single mode to one with multiple extraneous peaks. The spacing between the datapoints in the model spectra was 0.002nm, as this is the finest sampling resolution used by the OSA. As a result, the centre point of the peak was defined to a precision of 0.1pm to ensure it fell between datapoints and thus test the sub-datapoint accuracy of the peak fitting method.

Five different levels of complexity were defined for the models, as detailed in Table 5.4 and Figure 5.18. In each case the centre position of the main peak remained fixed at 656.9457nm, but additional features were added at each level. The first model had only the main peak with no noise. The second added multiplicative noise across the whole spectrum. The multiplicative noise was generated using a normal distribution around 1.0 with a standard deviation of 0.05. This value seemed comparable by eye to the amount of noise present in a recorded spectrum.

From the third level upwards, extra side peaks were added, with a height below 30% of the main peak, alongside the multiplicative noise. The height of the main peak was randomised for all models using a normal distribution of centre 10 and standard deviation 3, and any side peaks were given a random height between 0% and 30% of the main peak height. The position of side peaks was determined by applying a discrete jump of a multiple of 0.1nm, plus a randomised offset generated from a normal distribution of standard deviation 0.02.

The offset value for side peaks was chosen to attempt to emulate the difference in apparent peaks in the second and fourth spectra in Figure 5.10, as this test was to estimate the effect of noise peaks, rather than the adjacent modes around 0.5nm away. During the fitting method (as described above), the two modes are separated and the fit applied individually, so each mode is assumed to have no biasing effect

Table 5.4: Descriptions of the different levels of complexity and noise features given to the model spectra generated to test the peak fitting method.

Model	Noise features
0	Input value of model
1	No noise
2	Multiplicative noise
3	One low fixed extra peak, multiplicative noise
4	One low random extra peak, right-hand side, multiplicative noise
5	One low random extra peak, any side, multiplicative noise
6	Up to three low random extra peaks, any side, multiplicative noise

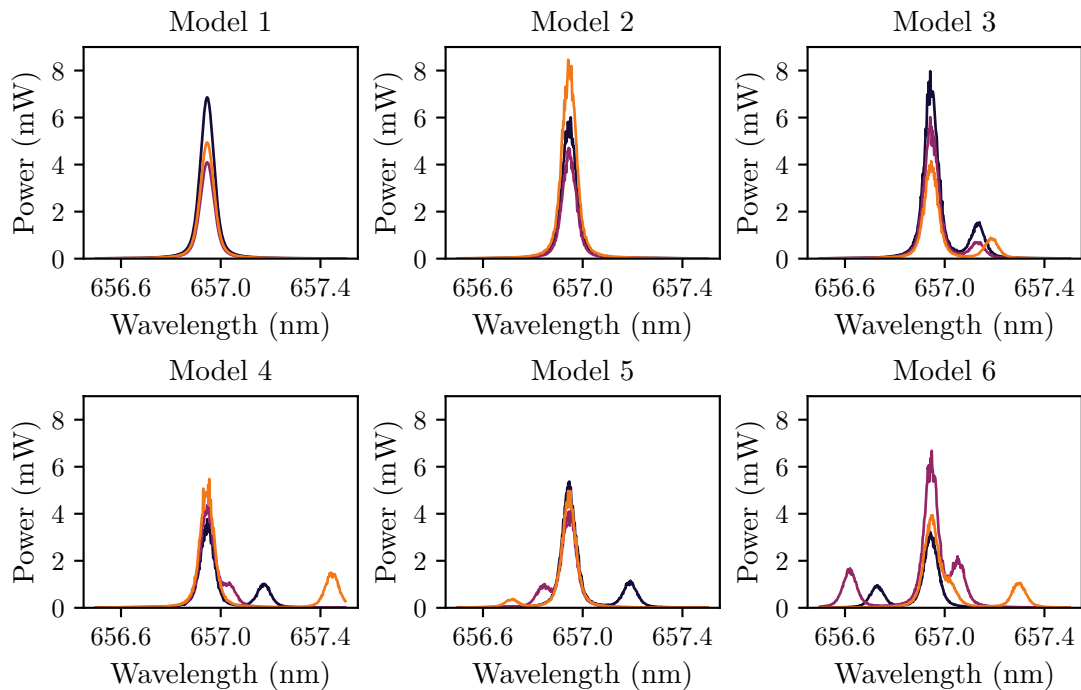


Figure 5.18: Example modelled spectra for each level of complexity described in Table 5.4. Each graph shows three modelled spectra at the same level of complexity.

Table 5.5: Summary of results from using the peak finding method on modelled peaks with a known centre and randomised noise. These values are from the centres calculated across a set of 5000 randomised spectra for each level of complexity. Note that all values are quoted to the same level of precision, but the mean wavelengths are given in nm, whereas the bias errors and standard deviations are in pm.

Model	Mean (nm)	Bias error (pm)	Standard deviation (pm)
0	656.945700	n/a	n/a
1	656.945700	0.000	0.000
2	656.945696	+0.004	0.267
3	656.945791	-0.091	0.304
4	656.945938	-0.238	0.664
5	656.945704	-0.004	0.690
6	656.945692	+0.008	0.746

on the other. Additionally, the closer noise peaks are more likely to affect the peak finding than a distinct mode further away, and so this test would overestimate any bias errors as compared to evaluating the effect of an adjacent mode.

Each of the five models was used to generate 5000 spectra with randomised noise features, and the peak fitting method was used to find the centre of the main peak. For each set of 5000, the mean centre, standard deviation, and bias error on the mean were calculated. These results are summarised in Table 5.5.

For model 1, with no noise, the peak finding method found the correct centre for every spectrum, no matter the height of the peak. At level 2, there is a small bias error of +0.004pm, and a standard deviation of 0.267pm. For the third level, the side peak was fixed within a small range. These data showed a clear biasing effect towards the main peak, with a mean centre -0.091pm from the true value. However, the standard deviation across the 5000 peaks is similar to the previous level at 0.304pm.

At the fourth level, this effect was magnified as the position of the extra peak could vary over a larger range, although always on the right-hand side of the main peak. This resulted in a much higher bias error of -0.238pm, as well as a higher standard

deviation of 0.664pm. For models 5 and 6, the extra peak(s) could be placed on either side of the main peak, in a large range of possible positions. The standard deviations at these levels are similar to that at level 4: 0.690pm for 5 and 0.746pm for 6. However, due to the equal likelihood of the peak(s) being placed on the left or right of the main peak, the average bias error was close to zero, at -0.004pm for 5 and +0.008pm for 6.

When investigating the wavelength stability of the lasers in later chapters, these data can be used to estimate any contributions of the fitting method to the measured uncertainty in the laser wavelength. For example, if a particular mode in a spectrum has an additional intermittent noise peak on one side, the data from model 3 can be compared with the real measurements to get a better understanding of the true uncertainty. Alternatively, taking the highest errors across all models, the upper limit on errors in the fitting method are a ± 0.238 pm bias error, and a 0.746pm standard deviation.

5.3 Sensor Calibration

5.3.1 Calibration Constants

From the mathematical theory supporting the technique (see Chapter 4, Section 4.3.2), there are three constants defined by the geometry of the sensor. These are two offsets A_{x0} , A_{y0} , plus a responsivity constant C . A_{x0} and A_{y0} are the speckle shifts A_x and A_y when the surface has no tilt, so is parallel to the camera plane. The responsivity constant describes how much the speckle shift changes when the surface angle changes. The equations defining the three constants are:

$$A_{x0} = \frac{\Delta\lambda}{\lambda_1} L_D s_x \quad (5.3.1)$$

$$A_{y0} = \frac{\Delta\lambda}{\lambda_1} L_D s_y \quad (5.3.2)$$

$$C = \frac{\Delta\lambda}{\lambda_1} L_D (1 + s_z) \quad (5.3.3)$$

These constants are defined by the geometry of the sensor build, plus the source wavelengths. The particular variables needed are: the two source wavelengths, the working distance (from the camera to the surface), and the distance between the

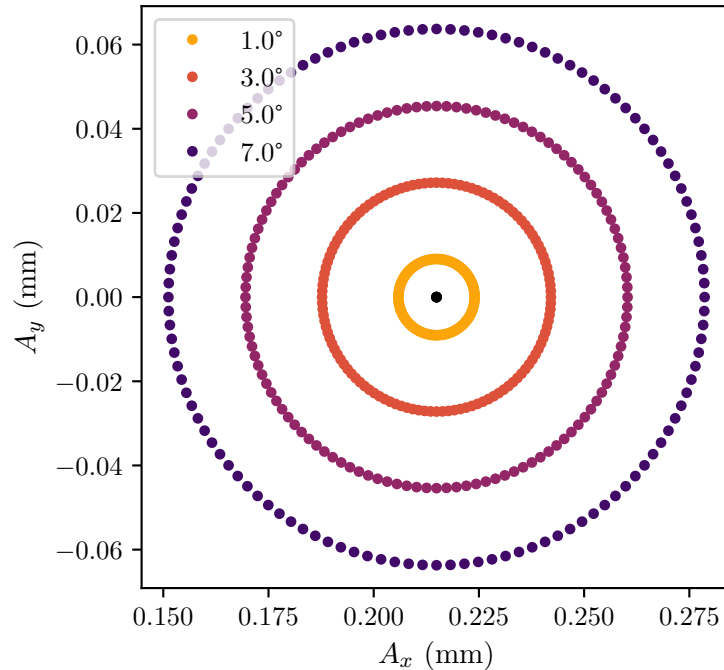


Figure 5.19: Generated calibration data using the theoretical sensor described by Figure 5.9, simulating speckle shift measurements of circles of fixed tilt magnitude and incrementing direction of 3.0° . The angle magnitudes used are: 1.0° , 3.0° , 5.0° , and 7.0° .

source and detector. These can all be measured to an accuracy of $\pm 1\text{mm}$ during setup, but for a high responsivity sensor even errors on this scale can have a large effect on angle measurements.

Another factor during setup is making sure the axes of the camera and the stage are aligned. When testing the sensor, the x and y axes of the stage and detector should be perfectly aligned along the z axis. In other words, the detector plane and the stage at zero tilt should be parallel, with their centres aligned along the z axis and the x and y axes pointing in the same direction. This is much more difficult to align by eye, and so errors are more likely in this part of the set up.

To mitigate these errors in assembly, the sensor can be calibrated. The fully assembled sensor is used to measure a series of surface tilts with a fixed, known magnitude, ψ , and varying direction, ϕ . The sensor measures the speckle shift at each point, drawing out a circle in the $A_x A_y$ plane. An example of this using generated data from the theoretical sensor from Figure 5.9 is shown in Figure 5.19. The centre of each circle, shown in black, is equal to the speckle shifts at zero tilt, or the constants

A_{x0} and A_{y0} . The radius of the circle, R_ψ , is related to the responsivity, C , by the following equation:

$$C = \frac{\tan \psi}{R_\psi} \quad (5.3.4)$$

The many points around the circle should give a reliable measure of A_{x0} and A_{y0} , but C is fully dependent on the accuracy of the applied tilt magnitude ψ . To get around this, tilts of multiple magnitude are applied, producing circles of varying radii. Figure 5.19 shows speckle shifts for angles of magnitude 1.0° , 3.0° , 5.0° , and 7.0° , and in each case the direction was incremented through one full circle in steps of 3.0° . Note that as this data is generated from a perfect set of angles, there are none of the variations that would be expected in real data.

Circles are fitted to the data using the function *leastsq* from Python's *scipy* module, which applies the Levenberg-Marquardt algorithm. The initial guesses for the fit are the values calculated using equations (5.3.1)-(5.3.3) above and the intended sensor geometry. In the initial sensor implementation in Chapter 7 these circles were fitted individually, with the final values of C (and A_{x0} and A_{y0}) found by averaging the results from each fit. This process was streamlined for the second sensor implementation (Chapter 8), where all circles were fitted at once, giving one overall value for the calibration constants.

5.3.2 Calibration Technique

The first technique for applying the calibration uses a series of 3D printed profiles, with a known tilt magnitude, as seen in Figure 5.20. These each use a 40x40mm aluminium plate as the surface. The mounts are affixed to a stage that can rotate the mounts around the z axis of the detector, continuously varying the tilt direction. The calibration data taken for the complete sensors in Chapters 7-8 used either 3 or 5 full rotations of the mounts, with increments of either 1.0° or 2.0° .

The angled profiles were printed in polylactic acid (PLA) plastic using an Ultimaker 2+ 3D printer [12]. This is a fused deposition modelling (FDM) printer, which builds objects with successive layers of molten plastic. The layer height for these mounts was 0.1mm, which means that the uncertainty in the height difference for the two edges of the tilted metal plate is ± 0.14 mm. This translates to a potential error in the tilt magnitude of 0.2° . For the larger magnitude mounts, this uncertainty is a small percentage of the expected angle, but for the smaller magnitude mounts this

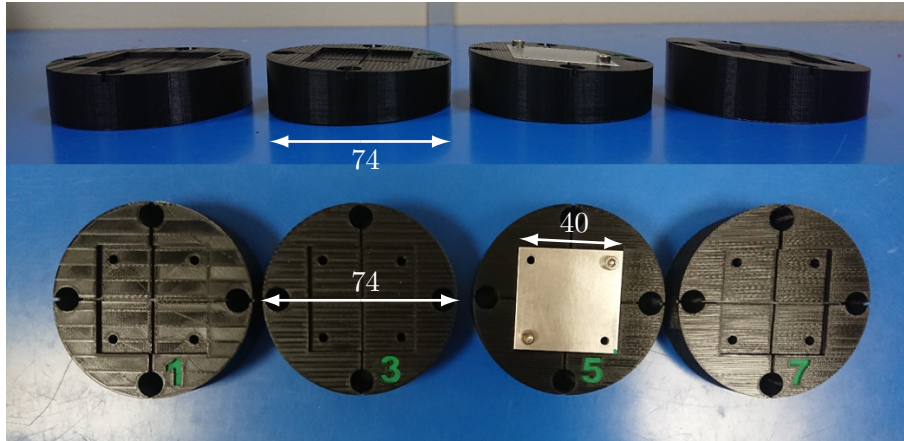


Figure 5.20: 3D printed tilt profiles at known angles used to calibrate the completed sensor, viewed from the side (top) and from above (bottom). A 40x40mm aluminium plate is attached to each to provide the working surface, which can be seen on the third mount. The width of the plate and diameter of the mounts are given in mm. Mount angle magnitudes: 1.0°, 3.0°, 5.0°, and 7.0°.

can have a large effect on the measured data. The full set of 3D printed mounts covers 0°-10° in 1° steps, giving a wide angle range for averaging.

The calibration setup using the 3D printed profiles is shown in Figure 5.21. The assembled sensor head is mounted directly opposite the centre of the metal plate mounted on the calibration profile. In this case, the profile is attached to an Aerotech AGR-50 rotation stage, positioned such that it can rotate around the z axis of the detector. As the stage rotates, the tilt of the surface is continuously moved through different directions, while the magnitude defined by the profile stays constant.

A second version of this calibration method was also used. The theory and processing were exactly the same, with the only difference being the method of generating the angles. One of the stages used throughout testing was a Hybrid Hexapod 6 degrees of freedom (DOF) stage, pictured in Figure 5.22. This stage has a tripod component, able to manoeuvre the surface attached to the stage through 2D tilts, without the need for the 3D printed profiles.

There are advantages and disadvantages of each approach. The maximum tilt magnitude able to be applied using the Hexapod is around 7°, but the 3D printed mounts have magnitudes up to 10° plus the ability to extend this further. However, a major advantage of using the Hexapod stage for calibration and testing is that the sensor does not need to be moved between the two series of measurements. When using the

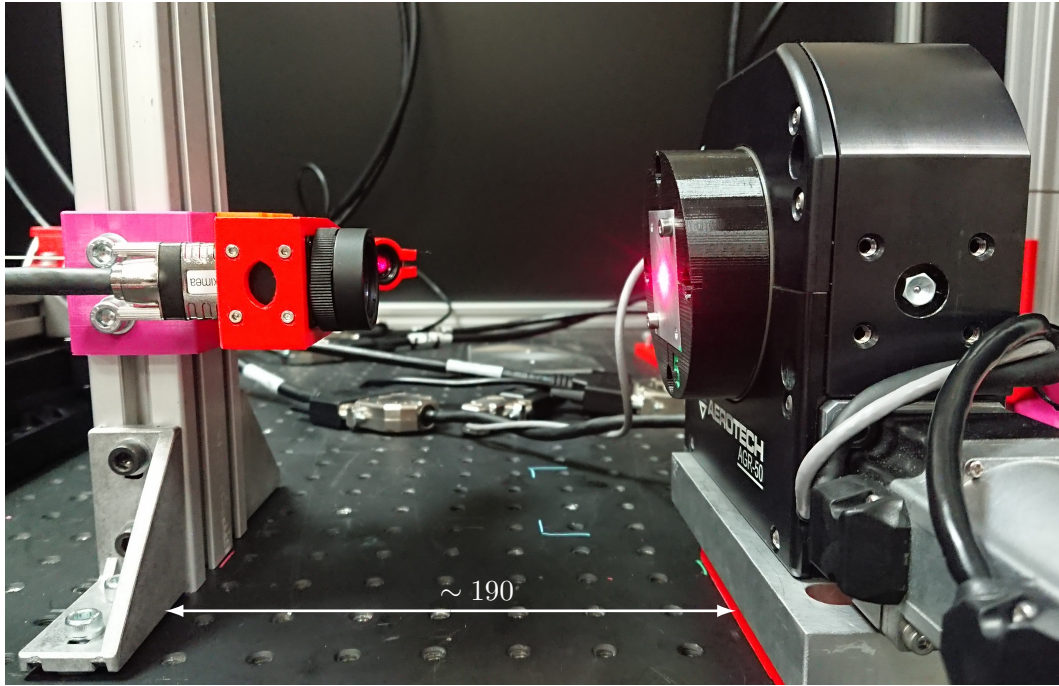


Figure 5.21: Calibration setup, consisting of the complete sensor head on the left, and on the right the Aerotech rotation stage with the calibration profile mounted facing the sensor. The rough distance between the sensor mounting and stage base is given in mm.

3D printed mounts with the Hexapod, the sensor needs to be moved further away to maintain the working distance. Another advantage is that using the Hexapod is easier; the printed mounts need to be changed over manually, whereas the Hexapod can run through multiple tilt magnitudes with no intervention.

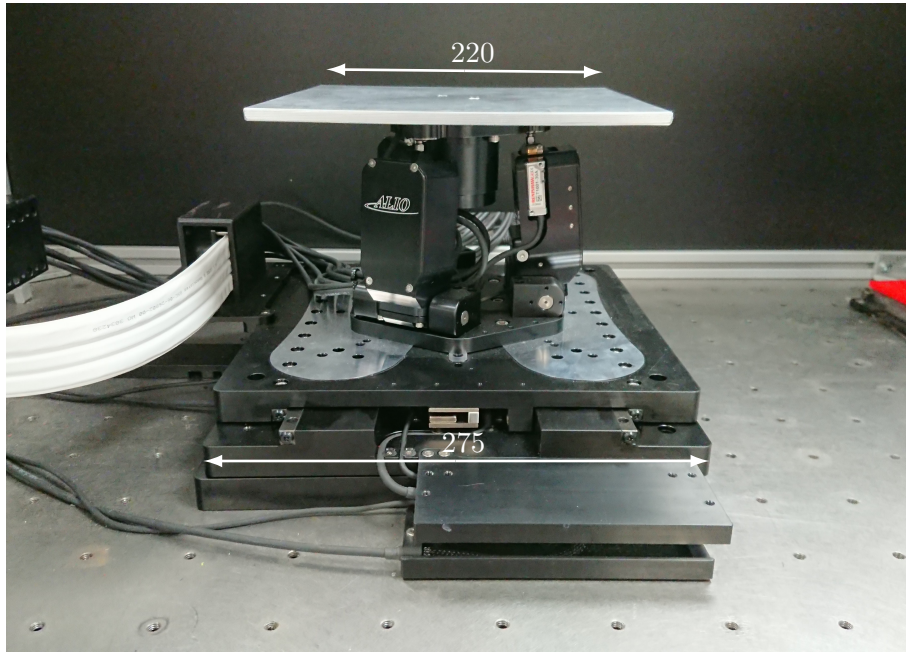


Figure 5.22: Photo of Hybrid Hexapod stage with aluminium plate mounted on top. The stage base consists of x and y linear stages (left/right and forward/backward in the photo), with a tripod stage mounted on the top linear stage. The tripod stage controls pitch and roll, or angles around the x and y axes. Finally, the top of the tripod contains a rotation stage, controlling the yaw and allowing the plate to be spun through multiple 360° rotations. The widths of the plate and linear stage are given in mm.

5.3.3 Stage Specifications

Aerotech Rotation Stage

There were two stages used for both calibrating and testing the sensors. The first stage is an Aerotech AGR-50 rotation stage, full model number AGR-50-UNLIMITED-9DU-NC-BM-R-3-NSL [13]. This stage was shown in the calibration setup in Figure 5.21. The stage has one axis of movement, able to rotate its central ring (inner diameter 50mm, seen in Figure 5.23) through a continuous 360°. The range, accuracy, and repeatability of this stage are given in Table 5.6.

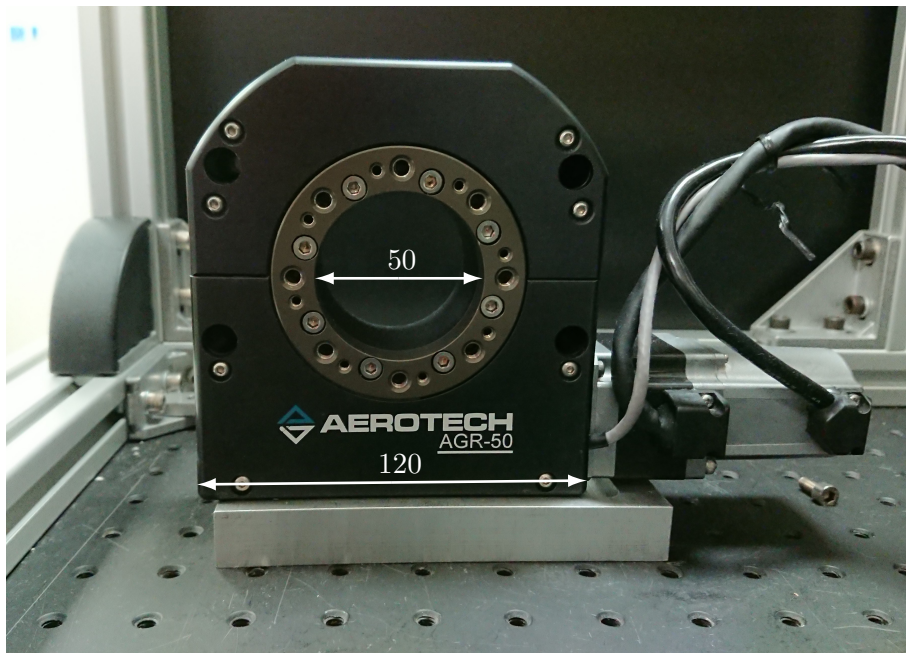


Figure 5.23: Photo of the Aerotech AGR-50 rotation stage. The central ring can be rotated through multiple 360° circles, and has multiple mounting points. The stage can also be used flat, or upright as in the photo. The width of the stage and inner diameter of the central ring are given in mm.

Table 5.6: Specifications of the Aerotech AGR-50 rotation stage [13].

Model number	AGR-50-UNLIMITED-9DU-NC-BM-R-3-NSL
Range	360°+
Accuracy	180arcsec
Unidirectional repeatability	10arcsec
Bidirectional repeatability	45arcsec
Tilt error motion	10arcsec
Axial error motion	5 μ m
Radial error motion	10 μ m

Hybrid Hexapod Stage

The second, more complex, stage is the Alio Hybrid Hexapod stage (model AI-HYBRID-HEX-60XY-15Z-56R-ACS) [14], introduced above in Figure 5.22. This is a 6 DOF stage, able to move the surface mounted on the top through x , y , and z motions, 2D tilts (pitch and roll), and continuous 360° rotations (yaw) about the z axis. The top of the Hexapod has multipurpose mounting points, so the aluminium plate shown in Figure 5.22 can be replaced with other surfaces, including the 3D printed calibration mounts.

The stage specifications of the Hybrid Hexapod are given in Table 5.7, including range, resolution, and accuracy in the various degrees of movement.

Table 5.7: Specifications of the Alio Hybrid Hexapod stage. Each section describes the linear stages (x and y movement), tripod (pitch, roll, and z movement), and rotation (yaw) stage [14].

Model number	AI-HYBRID-HEX-60XY-15Z-56R-ACS
Linear stages	
x/y range	$\pm 150\text{mm}$
Resolution	5nm
Linear accuracy	4 μm
Angular accuracy	15arcsec
Bidirectional linear repeatability	30nm
Straightness	3 μm
Flatness	3 μm
Orthogonality	20arcsec
Tripod stage	
Pitch/roll range	7.0°
z range	15mm
Linear resolution	5 μm
Angular resolution	0.02arcsec
Rotation stage	
Range	360°+
Resolution	0.039arcsec
Axial runout	13 μm
Radial runout	13 μm

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Chapter 6

Practical Demonstration of Dual-Wavelength Speckle

This chapter demonstrates the first practical implementation of dual-wavelength speckle. The theory derived in Chapter 4 is tested practically, and a number of factors affecting the sensor efficacy are investigated.

First, Section 6.1 presents data from a number of sensors with varying sensor geometry, specifically increasing the values S_x , L_D , and S_z . Measured speckle shifts are used to calculate the values of the sensor constants A_{x0} , A_{y0} , and C for the physical implementation, comparing these to the predicted values from speckle theory. Also compared are the sensor ranges and resolutions, giving an idea of the performance of each sensor.

Section 6.2 looks at the effect of changing spot size (which is directly related to speckle size) on the height of the correlation peak. The derivation of mathematical theory relating the two is presented, along with experimental results.

Lastly, the effect of surface roughness on correlation peak height is explored briefly in Section 6.3, looking at three of the different surfaces used throughout the project. This section also looks at the effect of correlation image size on the peak height and the noise in the correlation image.

6.1 Effect of Geometry on Sensor Constants

In order to further demonstrate the theory outlined in Chapter 4, this section will look at using various sensor geometries to measure speckle shifts for known 1D surface tilts in θ_x and θ_y . The focus in this section will be on investigating the three sensor constants and estimating the sensor range, rather than using the speckle theory to find measured surface angles.

As described in Section 4.3.3, and in particular Figure 4.13, changes to the geometry of the sensor will affect both its range and responsivity. This will be demonstrated practically using the same steps given in Figure 4.13, outlined again here for reference. First, increasing the separation between the source and detector (S_x) will increase the sensor range while slightly decreasing its responsivity. Then, the responsivity can be increased by increasing the working distance L_D . Finally, increasing S_z brings the source more in line with the detector, giving a more sensible geometry for assembling a compact sensor head. This change causes a large decrease in the range while slightly increasing the responsivity.

This section will demonstrate practically that these changes do have the expected effect on the sensor constants A_{x0} , A_{y0} , and C , given below:

$$A_{x0} = \frac{\Delta\lambda}{\lambda_1} L_D S_x \quad (6.1.1)$$

$$A_{y0} = \frac{\Delta\lambda}{\lambda_1} L_D S_y \quad (6.1.2)$$

$$C = \frac{\Delta\lambda}{\lambda_1} L_D (S_z + 1) \quad (6.1.3)$$

Five successive steps will be used to investigate each of the three geometry changes: increasing S_x , increasing L_D , and increasing S_z . This will show the change and rate of change of each of the constants as compared to the particular geometry parameter. Each of the other geometry values will remain fixed. The theory equations are stated once again below, in terms of the three constants A_{x0} , A_{y0} , and C .

$$A_x = C \tan \theta_y + A_{x0} \quad (6.1.4)$$

$$A_y = C \tan \theta_x + A_{y0} \quad (6.1.5)$$

These equations are in the form of a straight line $y = ax + b$, with C equal to the gradient a , and A_{x0} or A_{y0} equal to the constant b . As such, by measuring a series of speckle shifts for varying tilts in θ_x or θ_y only, the resulting data can be used to fit a straight line, thus finding A_{x0} , A_y and C directly from the data. Additionally, the $\tan \theta = \theta$ approximation can be used for angles up to 10° , where $\tan 10^\circ = \tan(0.175\text{rad}) = 0.176$. This means the sensor constants can be calculated without converting between θ and the gradient $\tan \theta$.

Also considered is the range of each of the sensors. The sensor range is defined as θ_R , which due to the sensor arrangement is limited by the measurable speckle shift in the correlation image between $(0, 0)$ and $(A_{x0}, 0)$. In equation form, this is given by:

$$\tan \theta_R = \frac{A_{x0}}{C} = \frac{s_x}{s_z + 1} \quad (6.1.6)$$

The actual predicted range of a given sensor is slightly smaller than this value, however, as it also takes into account the masking of the central correlation peak (see Chapter 5). The true predicted value will be denoted by $\theta_{\max}$, in theory equal to the minimum of the absolute values of $\theta_{x,\min}$, $\theta_{x,\max}$, $\theta_{y,\min}$, and $\theta_{y,\max}$, but in practice equal to $|\theta_{y,\min}|$.

Each sensor geometry tested used the FTEC2-658-30 red laser diode, operating such that it had an output of two longitudinal modes. The wavelengths of these modes were 656.9nm and 657.4nm. More detailed information regarding this laser and the optical setup are given in the following chapter, as the focus here is on the relationship between the sensor geometry and the derived speckle theory. The spot size for this work was around 3.5mm.

The detector was the Ximea MQ013MG-ON camera, running with an exposure time of 1ms. The camera frame rate was 100fps, however measurements were not taken continuously; individual images were only recorded once the given surface tilt had been applied. Processing of the images was carried out parallel to but out-of-sync with the image acquisition, making use of threading in Python. This approach reduced or eliminated the need for time spent post-processing data, while ensuring data acquisition was as fast as possible.

The source (lens tube) and detector were both affixed to 3D printed mounts that could be bolted to the optical table, giving a fairly large, but discrete, control of their

positions. The mounts were printed using the same Ultimaker 2+ 3D printer [1] used to make the calibration mounts shown in the previous chapter (Chapter 5). The mounts are printed in polylactic acid (PLA) plastic, a thermoplastic polymer with high strength and low thermal expansion, and one of the most common materials used for 3D printing [2].

For the camera mount the positioning steps were 12.5mm in x , and 25.0mm in z . For the lens tube mount, the steps of the mounts were 5.0mm in x , and 25.0mm in z . The heights of the mounts ensured that the centre of the lens tube and camera were aligned in y .

Additionally, the source mount had two separate pieces, such that the lens tube could be attached at different angles. This was necessary to align the spot opposite the detector when moving each sensor element around. The chosen geometries, therefore, were constructed around the possible angles of the lens tube, which were evenly spaced between 33.75° and 56.25° in steps of 5.625° (as per the 3D design, the printed tolerances may not be as precise). The ideal values of S_x and S_z were calculated for each source angle, and these were targeted during set up. Due to the discrete options for fixing the mounts to the optical bench, the ideal values were not always achievable, and so the stated geometries are those measured after set up.

For each constructed sensor, the Aerotech AGR-50 stage [3] was used to apply one dimensional tilts of up to $\pm 45^\circ$ in both θ_x and θ_y to an aluminium plate. The complete setup for the first geometry, given by $\mathbf{S} = (66.3, 0.0, 100.0)\text{mm}$, $\mathbf{D} = (0.0, 0.0, 99.0)\text{mm}$, is shown in Figure 6.1. Figure 6.1a shows the stage oriented such that the angles are applied in θ_x , and Figure 6.1b shows the stage oriented to produce surface angles in θ_y . In both cases the axis of rotation of the stage is parallel with the midpoint of the camera sensor, aligning it with the sensor axes.

Finally, although it won't be tested experimentally, the expected resolution, r_{sens} , for each of the sensor geometries will be stated. This value comes from the equation given below, relying on the responsivity and the resolution of the peak finding used in the image processing. This equation also makes use of the $\tan \theta = \theta$ approximation.

$$r_{\text{sens}} = \frac{pr_{\text{fit}}}{C} \quad (6.1.7)$$

The pixel size of the camera, p , is equal to $4.8 \times 4.8 \mu\text{m}$, and the resolution of the peak

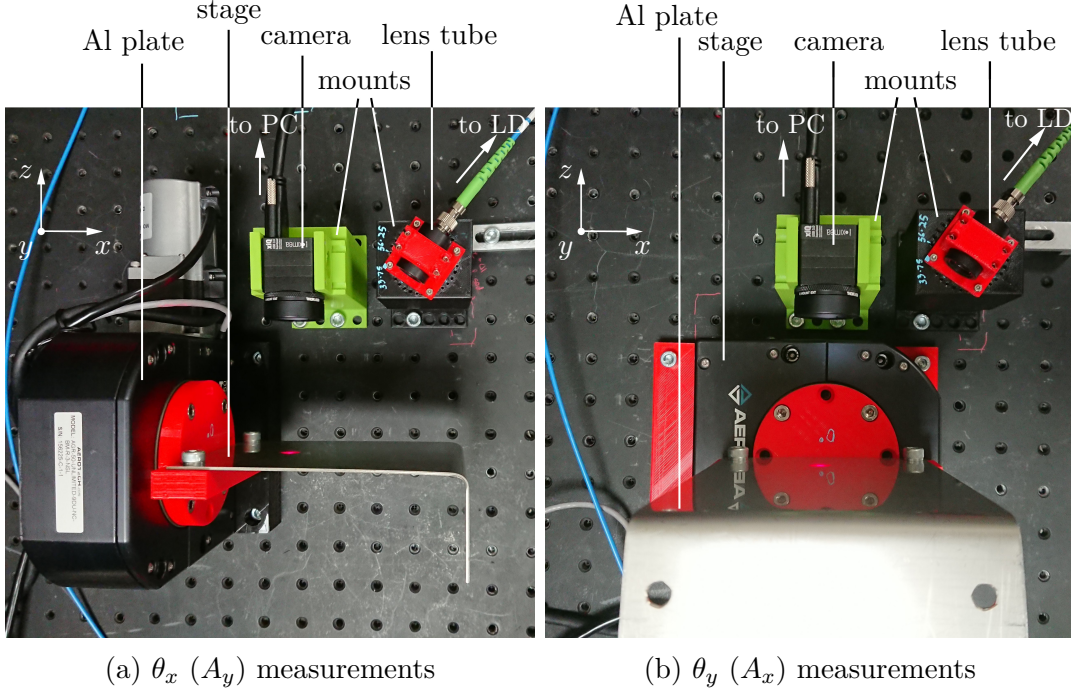


Figure 6.1: Labelled photos of the sensor in the initial geometry $\mathbf{S} = (66.3, 0.0, 100.0)\text{mm}$, $\mathbf{D} = (0.0, 0.0, 99.0)\text{mm}$, with the Aerotech stage used to apply tilt angles in θ_x , (a), and θ_y , (b), to an attached aluminium (Al) plate. [LD: laser diode]

fitting, r_{fit} is taken as 0.1pix. As discussed in Chapter 5, the expected uncertainty in the fit is 0.05-0.10 pixels [4]; the 0.1pix value is the more conservative limit.

6.1.1 Increasing S_x

The first geometry change is increasing S_x , the separation between the source and detector. This change increases A_{x0} and thus the range of the sensor θ_R , but also slightly decreases the responsivity C . Figure 6.2 shows the initial geometry used for these measurements, plus a dashed arrow indicating how S_x changes. The general sensor geometry for this section is $\mathbf{S} = (S_x, 0.0, 100.0)\text{mm}$, $\mathbf{D} = (0.0, 0.0, 99.0)\text{mm}$, with each of the variables other than S_x remaining fixed.

The initial sensor (also shown in the photos in Figure 6.1) has a geometry of $\mathbf{S} = (66.3, 0.0, 100.0)\text{mm}$, $\mathbf{D} = (0.0, 0.0, 99.0)\text{mm}$, measured after setup using callipers and a ruler. This gives the theoretical sensor constants $A_{x0} = 0.042\text{mm}$, $A_{y0} = 0.000\text{mm}$, and $C = 0.138\text{mm/rad}$. The minimum measurable angle in y , $\theta_{y,\text{min}}$, is limited by the masking in the image processing of the central correlation peak. For a

mask on the central peak of 5 pixels during processing, the theoretical sensor range is therefore $\theta_{\max} = 5.3^\circ$ (where $\theta_{y,\min} = -5.3^\circ$).

Figure 6.3 shows results for the initial geometry, plotting the applied angle, in radians along the bottom and degrees on the top, versus the measured speckle shifts in x and y . As a reminder, a speckle shift in y relates to an angle change in x , and vice versa. The sharp cut-off in measurable angles at $\theta_{y,\min}$ results in the flat line in the A_x data as the centre of the side peak has moved within the masked area of the correlation image. This cut off gives an estimate of the measured $\theta_{y,\min}$, though it is limited by the resolution of the data, which is 0.5° . Using the first point immediately above the flat line, the measured value of $\theta_{y,\min}$ for this sensor geometry is -3.5° , slightly less than the expected value of -5.3° .

The same cut-off point is used to apply a straight line fit to the A_x data above $\theta_{y,\min}$, along with the full dataset in A_y . Both of these are also shown in Figure 6.3. The fit is from the Python module *lmfit*, using a Levenberg-Marquardt minimisation algorithm and returning the slope and intercept of the fitted line. As there are data for both A_x and A_y , there are two measurements of the slope, C . For A_x

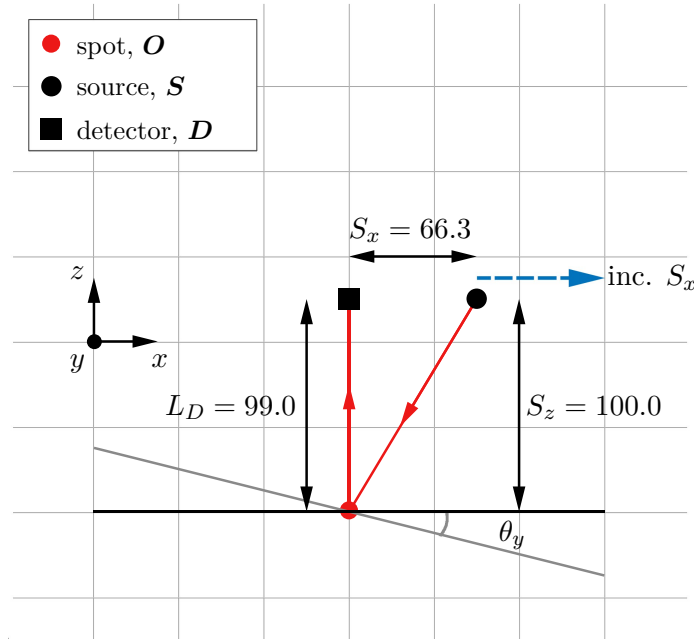


Figure 6.2: Schematic showing the initial sensor geometry and indicating the source movement for increasing (inc.) S_x . Measurements are given in mm. Increasing S_x increases the range (as A_{x0} increases), but slightly decreases the responsivity.

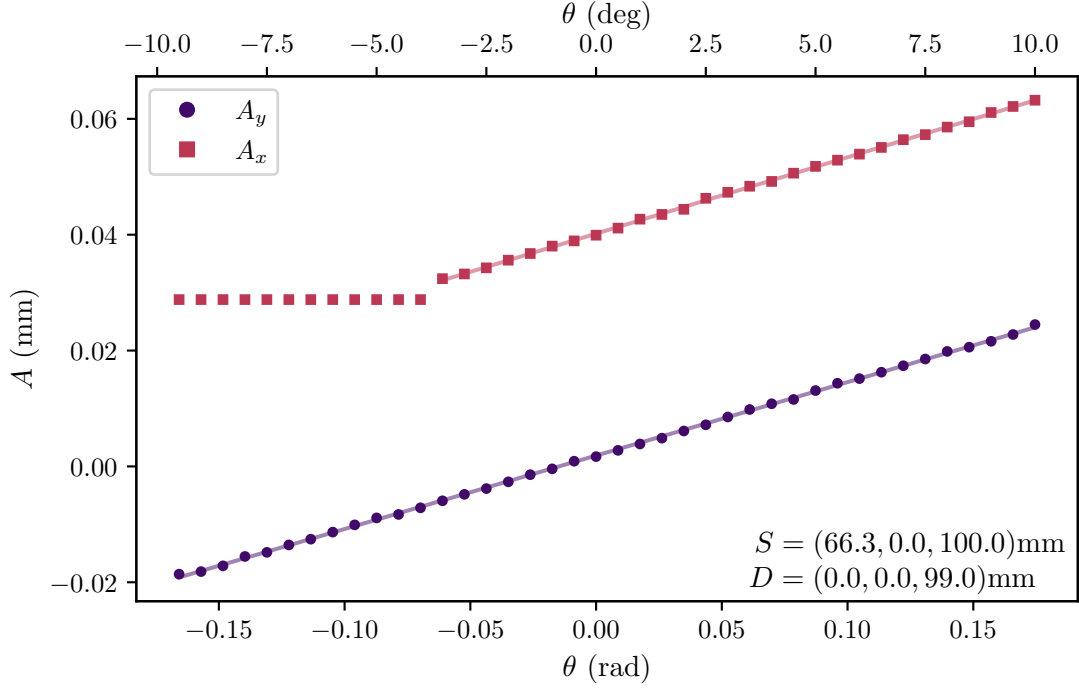


Figure 6.3: Measured speckle shifts for the geometry $\mathbf{S} = (66.3, 0.0, 100.0)\text{mm}$, $\mathbf{D} = (0.0, 0.0, 99.0)\text{mm}$. The applied surface tilt is shown in radians (lower axis) and degrees (upper axis). A straight line fit is plotted for the A_y data and the upper section of the A_x data.

the slope is 0.133mm/rad , and for A_y the slope is 0.127mm/rad , compared to the theoretical value of C at 0.138mm/rad . The measured value of A_{x0} is 0.040mm , with a theoretical value of 0.042mm , and the measured value of A_{y0} is 0.002mm , versus the theoretical 0.000mm .

This process was applied for each successive geometry change, using the straight line fit to find the sensor constants and the cut off to estimate the range. For increasing S_x , the values were: 66.3mm , 81.0mm , 100.9mm , 121.4mm , 151.2mm . The speckle shifts and straight line fit are shown in Figure 6.4a for the alternating values $S_x = 66.3, 100.9, 151.2\text{mm}$. The change in A_{x0} is seen clearly in the A_x data, with the intercept increasing as S_x increases. The change in C is smaller, and more clearly seen in the A_y data. As the intercept is close to zero for each of these datasets, the decreasing gradient, C , as S_x increases causes the speckle shifts at 10° (0.17 in radians) to decrease also.

In order to get a clearer picture of how the sensor constants change as the geometry

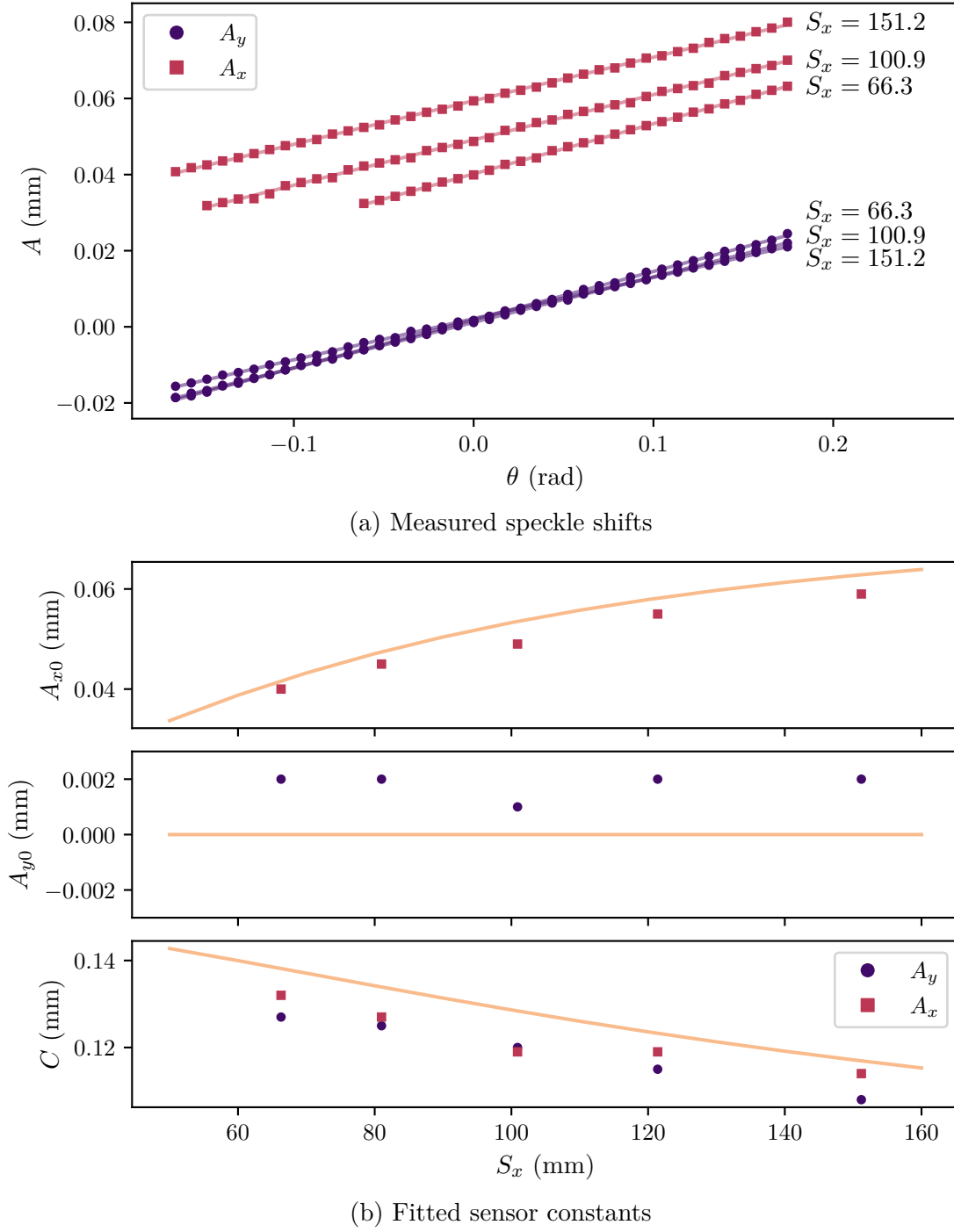


Figure 6.4: (a) Measured speckle shifts for changing surface tilts between $\pm 10^\circ$ (shown in radians) for three values of S_x (given in mm), with a straight line fit applied. (b) The intercepts and slope of the fits give the sensor constants A_{x0} , A_{y0} , and C , which are plotted against S_x for all five experimental values, along with a solid line showing the theoretical relationship.

changes, the three constants are plotted against S_x in Figure 6.4b. Each part of the figure has a line showing the theoretical change as S_x continuously increases. It is clear here that the changes to A_{x0} and C are non-linear, and the measured values from the practical sensors follow roughly the same trend. As the sensor arrangement stays the same, such that the source is only offset in x , A_{y0} remains zero for all geometries, with a very small deviation for the measured values. Note again that there are two measurements of C , one from each of the θ_x and θ_y measurement sets, but only one measurement each of A_{x0} and A_{y0} .

The discrepancies in A_{x0} and C are likely due to the spot being slightly off-centre with regards to the detector. The mounts used to arrange the sensor elements were designed to keep the centre of the lens tube (source) and camera (detector) at the same height, minimising any errors in their y positions. In x , however, the discrete fixing positions of the mounts limited the precision of the physical geometries. For the geometry at $S_x = 100.9\text{mm}$, the angle of the lens tube was 45° and the value of S_z was 100.0mm , which implies that the position of the spot was off by 0.9mm . There are similar errors in most of the other setups, ranging from -1.1mm at $S_x = 81.0$ to $+1.5\text{mm}$ at $S_x = 151.2\text{mm}$.

More precise control of the source and detector positions would be needed to test the non-linearity of the relationships between S_x and A_{x0} , and S_x and C . However, the data presented here show that even with the $\sim 1\text{mm}$ errors during setup, the sensor constants follow the trends of A_{x0} increasing and C decreasing as S_x increases.

To summarise these results, the theoretical (‘Theor.’) and measured (‘Meas.’) values of A_{x0} , A_{y0} , C , and $\theta_{\max}$ are all given in Table 6.1 for each of the values of S_x . The measured ranges follow the expected trend of increasing as S_x increases. However, there is a consistent $\sim -2.0^\circ$ difference in the theoretical and measured values. This is not surprising, as the theoretical estimates assume that the side peak remains measurable until it is obscured by the mask on the central peak; in reality there is likely an area of at least one or two pixels where the peak can no longer be measured reliably or accurately.

Also noted in Table 6.1 are the expected values of the sensor resolution, r_{sens} . These values were not measured experimentally, but illustrate the change in resolution caused by the geometry change. There is a small decrease as S_x increases, but broadly the resolution is very similar across each geometry.

Table 6.1: Theoretical and measured values of the three sensor constants A_{x0} , A_{y0} , and C , plus the sensor range, $\theta_{\max}$, for five steps of increasing S_x .

Sensor geometry		Speckle shifts at zero		Responsivity		Range	Resolution
S_x , mm		A_{x0} , mm	A_{y0} , mm	C , mm/rad		$\theta_{\max}$, deg	r_{sens} , deg
				A_x data	A_y data		
66.3	Theor.	0.042	0.000	0.138		5.3	0.20
	Meas.	0.040	0.002	0.132	0.127	3.5	
81.0	Theor.	0.047	0.000	0.134		7.9	0.21
	Meas.	0.045	0.002	0.127	0.125	6.0	
100.9	Theor.	0.054	0.000	0.128		10.9	0.21
	Meas.	0.049	0.001	0.119	0.120	8.5	
121.4	Theor.	0.058	0.000	0.123		13.4	0.22
	Meas.	0.055	0.002	0.119	0.115	11.5	
151.2	Theor.	0.063	0.000	0.117		16.2	0.24
	Meas.	0.059	0.002	0.114	0.108	14.0	

6.1.2 Increasing L_D

The next geometry change investigated was increasing D_z , normally referred to as the working distance L_D . As this experiment was following the outline of Figure 4.13, the changes to L_D were applied after the changes to S_x , so the first geometry used here is the same as the final geometry in the previous experiment series. Figure 6.5 shows this initial geometry, again with a dashed arrow indicating the planned changes to L_D .

The expected effect of increasing L_D is an increase in the responsivity C and a corresponding increase in A_{x0} , leading to no effect on the range θ_R . However, there is a slight effect on the range as defined by $\theta_{y,\min}$. The changes in C and A_{x0} change the range of non-measurable speckle shifts covered by the masking of the central peak during image processing. As a result, there is a small decrease in $\theta_{y,\min}$ (and thus increase in $\theta_{\max}$) with increasing L_D .

The data collection and analysis in this section was the same as for increasing S_x . Aside from the changes in L_D , the sensor geometry remained fixed at $\mathbf{S} = (151.2, 0.0, 100.0)\text{mm}$, $\mathbf{D} = (0.0, 0.0, L_D)\text{mm}$. The tested values of L_D were: 99.0mm,

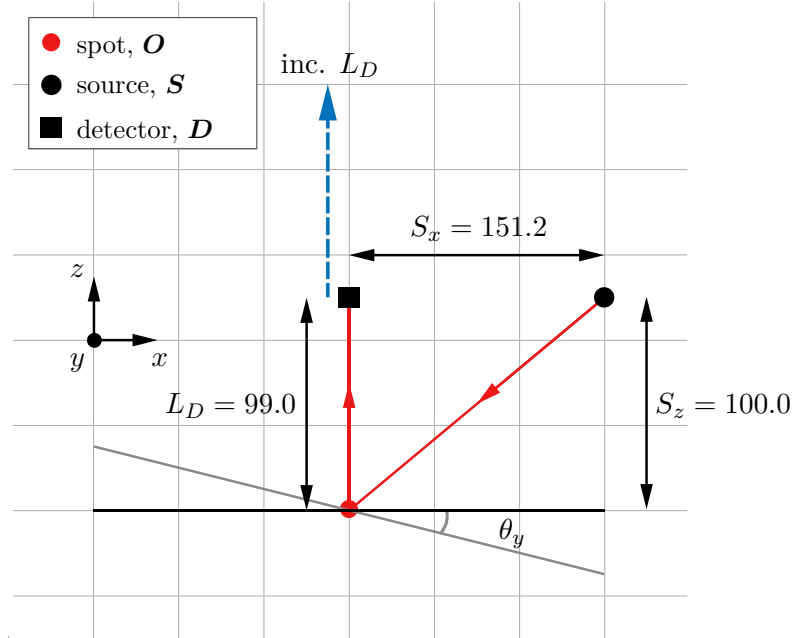
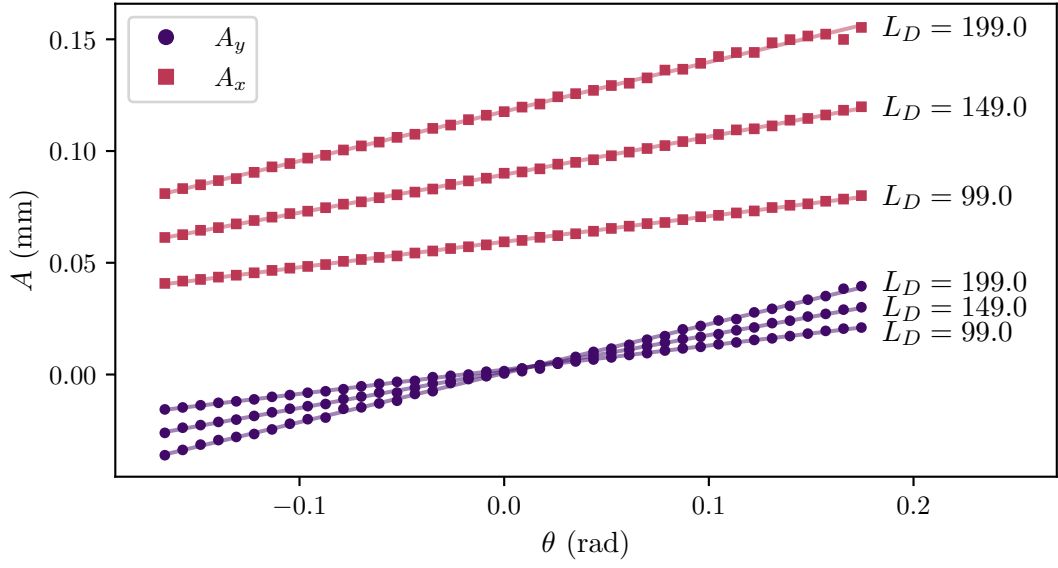
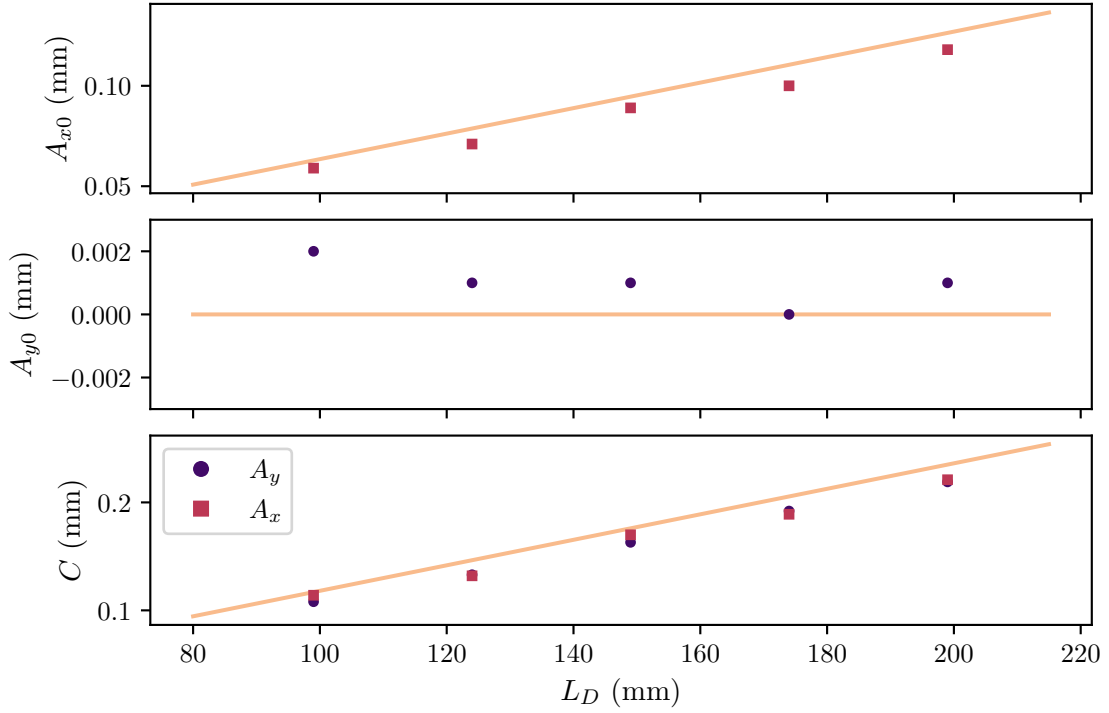


Figure 6.5: Schematic showing the initial sensor geometry and indicating the detector movement for increasing (inc.) L_D . Measurements are given in mm. Increasing L_D increases the responsivity with no effect on θ_R but a small increase in $\theta_{y,\min}$.



(a) Measured speckle shifts



(b) Fitted sensor constants

Figure 6.6: (a) Measured speckle shifts for changing surface tilts between $\pm 10^\circ$ (shown in radians) for three values of L_D (given in mm), with a straight line fit applied. (b) The intercepts and slope of the fits give the sensor constants A_{x0} , A_{y0} , and C , which are plotted against L_D for all five experimental values, along with a solid line showing the theoretical relationship.

124.0mm, 149.0mm, 174.0mm, and 199.0mm. The increases in A_{x0} and C are now seen clearly in the measured speckle shifts, shown for $L_D = 99.0\text{mm}$, 149.0mm, 199.0mm in Figure 6.6a.

The relationships between the three sensor constants and the change in L_D are shown in Figure 6.6b. The change in A_{x0} and C is linear, while A_{y0} is once again fixed at zero. As the source remained fixed during these experiments, the small offset in the spot position was also constant at +1.5mm.

The sensor ranges are now all outside of the $\pm 10^\circ$ range of the graphs in Figure 6.6a. However, the theoretical and measured values are given in Table 6.2, along with those for A_{x0} , A_{y0} , and C , and the expected sensor resolution r_{sens} . This geometry change led to a small increase in the sensor range as L_D increased. A comparison between the measured and theoretical range values shows the same $\sim -2.0^\circ$ difference as with the previous dataset, supporting the assumption that this is caused by a slight overestimation in the measurable range of the side peak in the correlation image.

Finally, the resolution shows a more noticeable change with increasing L_D than for the change in S_x . Between $L_D = 99.0$ and $L_D = 199.0$ the resolution doubles, from 0.24° to 0.12° .

Table 6.2: Theoretical and measured values of the three sensor constants A_{x0} , A_{y0} , and C , plus the sensor range, $\theta_{\max}$, for five steps of increasing L_D .

Sensor geometry		Speckle shifts at zero		Responsivity		Range	Resolution
L_D , mm		A_{x0} , mm	A_{y0} , mm	C , mm/rad		$\theta_{\max}$, deg	r_{sens} , deg
				A_x data	A_y data		
99.0	Theor.	0.063	0.000	0.117		16.2	0.24
	Meas.	0.059	0.002	0.114	0.108	14.0	
124.0	Theor.	0.079	0.000	0.146		18.8	0.19
	Meas.	0.071	0.001	0.132	0.133	16.0	
149.0	Theor.	0.095	0.000	0.176		20.5	0.16
	Meas.	0.089	0.001	0.170	0.163	19.0	
174.0	Theor.	0.110	0.000	0.205		21.7	0.13
	Meas.	0.100	0.000	0.189	0.192	19.5	
199.0	Theor.	0.126	0.000	0.235		22.5	0.12
	Meas.	0.118	0.001	0.221	0.219	21.0	

6.1.3 Increasing S_z

The final geometry adjustment tested was increasing S_z . This change decreases A_{x0} , also decreasing the sensor range (in terms of both θ_R and $\theta_{\max}$). It also slightly increases the responsivity C . As before, the initial geometry for this experiment, shown in Figure 6.7, is the same as the final geometry in the previous experiment. The dashed arrow indicates the increasing S_z .

Once again, the data collection and analysis is the same as for the increasing S_x experiment. The generalised sensor geometry was $\mathbf{S} = (151.2, 0.0, S_z)\text{mm}$, $\mathbf{D} = (0.0, 0.0, 199.0)\text{mm}$, with the values of S_z being: 100.0mm, 125.0mm, 150.0mm, 184.0mm, 218.0mm.

Figure 6.8a shows the decrease in A_{x0} with increasing S_z , having plotted the measured speckle shifts for the geometries $S_z = 100.0, 150.0, 218.0\text{mm}$. The small increase in C can be seen most clearly in the A_y data where, similarly to the data for changing S_x , the fixed intercept and increasing gradient causes an increase in the speckle shifts measured at 10° as S_z increases.

The changes in A_{x0} and C are non-linear, as shown in Figure 6.8b. As another

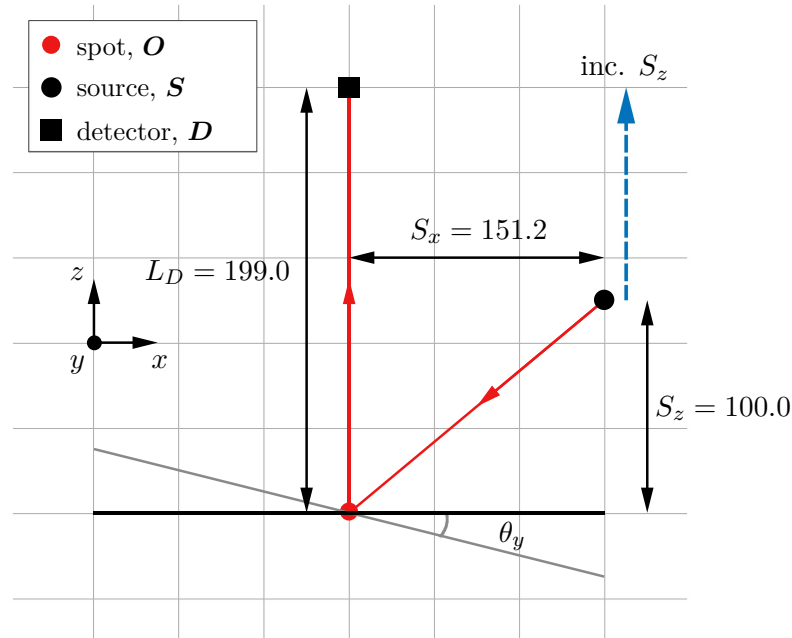


Figure 6.7: Schematic showing the initial sensor geometry and indicating the source movement for increasing (inc.) S_z . Measurements are given in mm. Increasing S_z decreases A_{x0} and thus the range, and slightly increases the responsivity C .

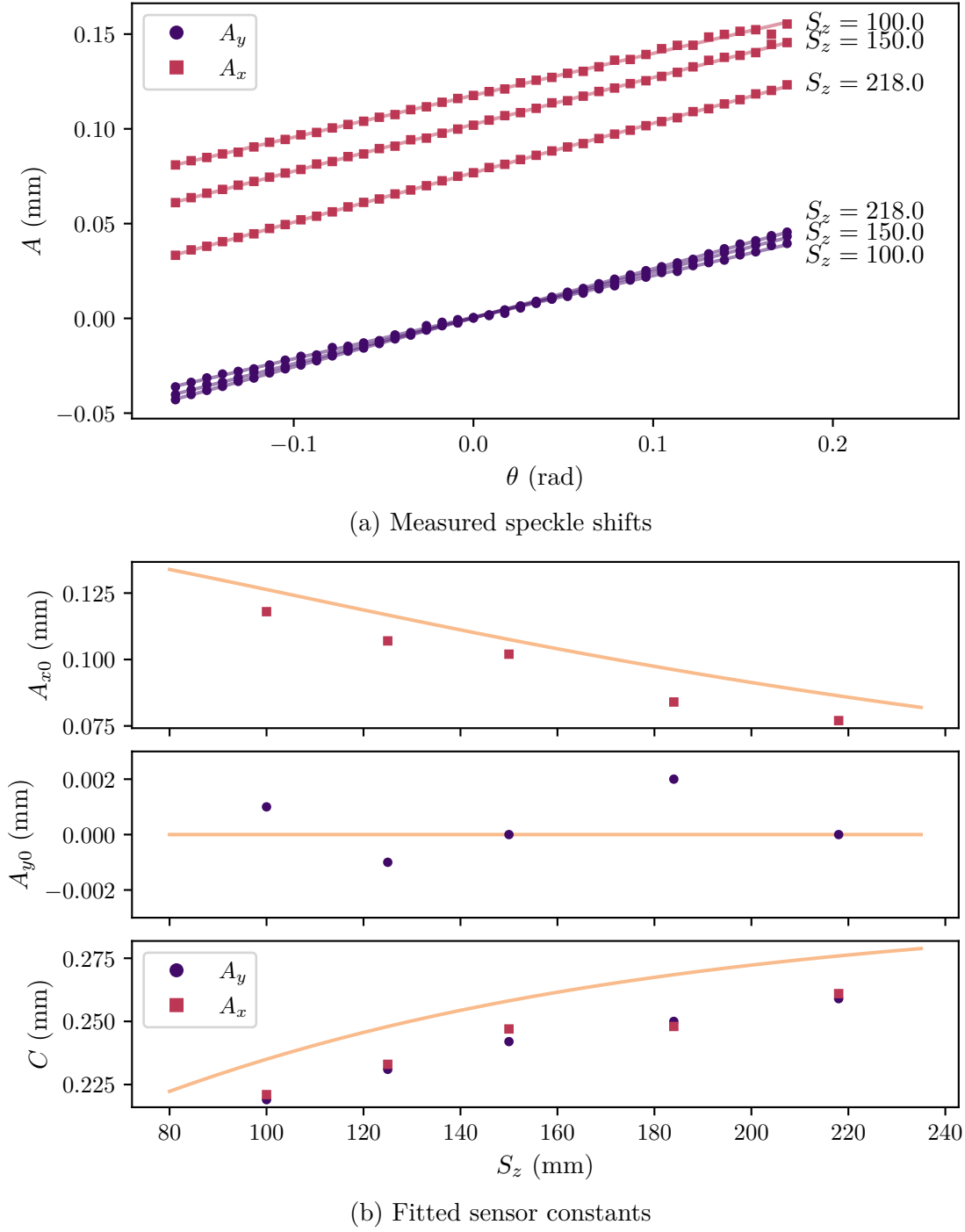


Figure 6.8: (a) Measured speckle shifts for changing surface tilts between $\pm 10^\circ$ (shown in radians) for three values of S_z (given in mm), with a straight line fit applied. (b) The intercepts and slope of the fits give the sensor constants A_{x0} , A_{y0} , and C , which are plotted against S_z for all five experimental values, along with a solid line showing the theoretical relationship.

similarity with the geometries used for testing S_x , there were small discrepancies in the spot position in relation to the detector. These offsets range from -1.1mm for $S_z = 125.0$ mm, to +1.5 at $S_z = 100.0$, and then to a particularly high +5.5mm at $S_z = 218.0$. However, the overall directions of the trends in A_{x0} and C with increasing S_z are as expected.

As with the previous two experiments, the data for the theoretical and measured values of the three sensor constants, the range $\theta_{\max}$, and the expected resolution r_{sens} are summarised in Table 6.3. The increase in S_z results in a decrease in the sensor range, demonstrated by the measured values in the table. As with the previous two sets of experiments, the $\sim -2.0^\circ$ difference in theoretical and measured values of $\theta_{\max}$ is still present. The resolution change is very small for increasing S_z , remaining roughly constant at 0.11° .

Table 6.3: Theoretical and measured values of the three sensor constants A_{x0} , A_{y0} , and C , plus the sensor range, $\theta_{\max}$, for five steps of increasing S_z .

Sensor geometry		Speckle shifts at zero		Responsivity		Range	Resolution
S_z , mm		A_{x0} , mm	A_{y0} , mm	C , mm/rad		$\theta_{\max}$, deg	r_{sens} , deg
				A_x data	A_y data		
100.0	Theor.	0.126	0.000	0.235		22.5	0.12
	Meas.	0.118	0.001	0.221	0.219	21.0	
125.0	Theor.	0.117	0.000	0.248		19.5	0.11
	Meas.	0.107	-0.001	0.233	0.231	18.0	
150.0	Theor.	0.108	0.000	0.258		17.0	0.11
	Meas.	0.102	0.000	0.247	0.242	16.0	
184.0	Theor.	0.096	0.000	0.268		14.1	0.10
	Meas.	0.084	0.002	0.248	0.250	11.5	
218.0	Theor.	0.086	0.000	0.276		11.8	0.10
	Meas.	0.077	0.000	0.261	0.259	9.5	

6.2 Effect of Spot Size on Image Correlation

One factor that became apparent during testing of the sensor implementation in Chapter 8 was a relationship between spot size and the height of the side peaks in the correlation image. The general trend was an increasing side peak height with smaller spot size. A short experiment was undertaken to investigate this, and a theoretical model developed in order to attempt to explain the relationship.

6.2.1 Theoretical Effect of Changing Spot Size

Individual speckles can be modelled as an elongated cigar-shaped Gaussian intensity distribution, along the direction of its path from the surface [5]. For objective speckle, the average speckle size is directly related to the spot size, w_{spot} . The longitudinal speckle size can be thought of as the length of the cigar shape, and the transverse size is its cross section. These two values are given by the following equations from Shirley and Hallerman [5], which also use the working distance, L_D , between the surface and the detector plane, and the source wavelength, λ .

$$w_{\text{long}} = \frac{4\lambda L_D^2}{w_{\text{spot}}^2} \quad (6.2.1)$$

$$w_{\text{tran}} = \frac{\lambda L_D}{w_{\text{spot}}} \quad (6.2.2)$$

A smaller spot size results in larger speckles (i.e. larger values of both w_{long} and w_{tran}), and vice versa. The hypothesised reason for the increase in cross correlation peak height for a smaller spot is that there is less decorrelation between the patterns from the two wavelengths as the speckle size is larger. Decorrelation is the change in the construction of a speckle pattern; if two patterns are completely decorrelated, there is no similarity between the distribution of the speckles in each.

To expand further, consider a single speckle appearing in each pattern. The shift caused by the difference in wavelengths has a z component, A_z , as well as an x and y shift (A_x and A_y). In the extreme case where the A_z shift is greater than the speckle's longitudinal size, there will be no correlation between the two speckles. For larger speckles, this becomes less likely to occur, as a greater shift (or here greater wavelength difference) is needed to achieve decorrelation. Conversely for smaller speckles, decorrelation is more likely, and so over the many speckles in an image the

maximum correlation between patterns is lower.

Note that this assumes that the angle sensor operates in the Fresnel regime, and so the longitudinal speckle size has a finite length, given by equation (6.2.1) above. Under the Fraunhofer regime, the longitudinal speckle size is considered infinite.

This relationship can be considered mathematically. An individual speckle will be recorded at the detector plan as a cross-section of the 3D cigar shape at some point along its length w_z , measured from the centre of the speckle. Thus the intensity at the detector of a single speckle, modelled by a Gaussian distribution, is given by:

$$I_1 = \exp \left(-\frac{\left(z - (L_D - w_z) \right)^2}{2w_{\text{long}}^2} \right) \quad (6.2.3)$$

The same speckle in the second pattern will be magnified by the following value:

$$M = \frac{\lambda + \Delta\lambda}{\lambda} \quad (6.2.4)$$

This magnification effectively shifts the speckle along the z -axis, and scales its length. As an aside, the magnification also causes a change in the x and y positions of each speckle, which is accounted for during the image processing described in Section 5.1. Here only the height change, related to the z position, is considered. Now, the intensity of the corresponding speckle from λ_2 is given by:

$$I_2 = \exp \left(-\frac{\left(z - M(L_D - w_z) \right)^2}{2(Mw_{\text{long}})^2} \right) \quad (6.2.5)$$

For this single speckle only, its contribution to the cross correlation peak height is equal to:

$$R_{12} = I_1 I_2 \quad (6.2.6)$$

Using I_1 and I_2 with a measurement plane of $z = L_D$, an expression for R_{12} can be found. The terms are rearranged to collect those that are factors of w_z^2 and w_z ,

plus those that are constant.

$$\begin{aligned}
R_{12} &= \exp \left(-\frac{(L_D - (L_D - w_z))^2}{2w_{\text{long}}^2} \right) \exp \left(-\frac{(L_D - M(L_D - w_z))^2}{2(Mw_{\text{long}})^2} \right) \\
&= \exp \left(-\left(\frac{M^2 w_z^2}{2M^2 w_{\text{long}}^2} + \frac{(L_D - M(L_D - w_z))^2}{2M^2 w_{\text{long}}^2} \right) \right) \\
&= \exp \left(-\left(\frac{M^2 w_z^2 + L_D^2 + M^2(L_D^2 + w_z^2 - 2L_D w_z) - 2ML_D(L_D - w_z)}{2M^2 w_{\text{long}}^2} \right) \right) \\
&= \exp \left(-\left(\frac{2M^2 w_z^2}{2M^2 w_{\text{long}}^2} + \frac{(2ML_D - 2M^2 L_D)w_z}{2M^2 w_{\text{long}}^2} + \frac{L_D^2 + M^2 L_D^2 - 2ML_D^2}{2M^2 w_{\text{long}}^2} \right) \right)
\end{aligned} \tag{6.2.7}$$

Over the whole image, i.e. for many speckles, the cross correlation of the two patterns is obtained by integrating with respect to w_z . The speckles are considered to have evenly distributed offsets from the detector plane, w_z , so the integration limits are $\pm\infty$.

$$RR_{12} = \int_{-\infty}^{+\infty} R(w_z) w_z \tag{6.2.8}$$

Due to the form of equation (6.2.7), and the integration limits of $\pm\infty$, the following definite integral expression can be used to perform the integration:

$$\int_{-\infty}^{+\infty} \exp \left(-\left(ax^2 + bx + c \right) \right) dx = \sqrt{\frac{\pi}{a}} \exp \left(\frac{b^2}{4a} - c \right) \tag{6.2.9}$$

x is equal to w_z , and the expressions for a , b , and c are given below. The first two are slightly simplified from equation (6.2.7).

$$a = \frac{1}{w_{\text{long}}^2} \tag{6.2.10}$$

$$b = \frac{L_D - ML_D}{Mw_{\text{long}}^2} \tag{6.2.11}$$

$$c = \frac{L_D^2 + M^2 L_D^2 - 2ML_D^2}{2M^2 w_{\text{long}}^2} \tag{6.2.12}$$

Now the result of the integration is equal to:

$$\begin{aligned}
 RR_{12} &= \sqrt{\pi w_{\text{long}}^2} \exp \left(\frac{(L_D - ML_D)^2 w_{\text{long}}^2}{M^2 w_{\text{long}}^4} - \frac{L_D^2 + M^2 - 2ML_D^2}{2M^2 w_{\text{long}}^2} \right) \\
 &= \sqrt{\pi} w_{\text{long}} \exp \left(\frac{(L_D - ML_D)^2 - 2(L_D^2 + M^2 - 2ML_D^2)}{4M^2 w_{\text{long}}^2} \right) \\
 &= \sqrt{\pi} w_{\text{long}} \exp \left(\frac{-L_D^2(M^2 - 2M + 1)}{4M^2 w_{\text{long}}^2} \right) \\
 &= \sqrt{\pi} w_{\text{long}} \exp \left(\frac{-L_D^2(M - 1)^2}{4M^2 w_{\text{long}}^2} \right)
 \end{aligned} \tag{6.2.13}$$

As the image processing uses a normalised cross correlation, this peak height needs to be normalised. This is achieved by dividing by the sum of the autocorrelations of the two individual speckle patterns, RR_{11} and RR_{22} .

$$RR_{\text{norm}} = \frac{RR_{12}}{RR_{11} + RR_{22}} \tag{6.2.14}$$

Following the same steps as above, the autocorrelations are both equal to:

$$RR_{11} = RR_{22} = \sqrt{\pi} w_{\text{long}} \tag{6.2.15}$$

Now RR_{norm} becomes:

$$\begin{aligned}
 RR_{\text{norm}} &= \frac{\sqrt{\pi} w_{\text{long}}}{2\sqrt{\pi} w_{\text{long}}} \exp \left(\frac{-L_D^2(M - 1)^2}{4M^2 w_{\text{long}}^2} \right) \\
 &= \frac{1}{2} \exp \left(\frac{-L_D^2(M - 1)^2}{4M^2 w_{\text{long}}^2} \right)
 \end{aligned} \tag{6.2.16}$$

The term $L_D(M - 1)$ is equal to the speckle shift in z , A_z , simplifying the equation further:

$$RR_{\text{norm}} = \frac{1}{2} \exp \left(-\frac{A_z^2}{(2Mw_{\text{long}})^2} \right) \tag{6.2.17}$$

This equation gives the normalised height of the cross correlation side peak, as related to the longitudinal speckle size w_{long} . The equation demonstrates the proposed hypothesis mathematically, namely that for fixed source wavelengths and working distance, an increase in longitudinal speckle size increases the maximum possible correlation peak height.

Using equations (6.2.1) and (6.2.2), the relationship between correlation peak height and spot size or transverse speckle size can also be obtained.

$$RR_{\text{norm}} = \frac{1}{2} \exp \left(-\frac{A_z^2 w_{\text{spot}}^4}{(8M\lambda L_D)^2} \right) \quad (6.2.18)$$

$$RR_{\text{norm}} = \frac{1}{2} \exp \left(-\frac{A_z^2 \lambda^2 L_D^2}{(8Mw_{\text{tran}}^2)^2} \right) \quad (6.2.19)$$

6.2.2 Experimental Results for Changing Spot Size

Experimental data was also gathered to investigate the link between spot size and cross correlation peak height. The setup placed the detector at a working distance of 180mm opposite a flat aluminium surface. The source used a microbench system and a series of lenses with focal lengths between 30mm and 80mm to vary the spot size on the surface. The laser light emitted from the fibre was collimated, then passed through the variable lens at a fixed distance from the surface, forming a different sized spot depending on the focal length of the lens. The angle of incidence of the source remained constant at roughly 35° , thus keeping the sensor geometry constant. The source wavelengths were the same as used in Section 6.1 above, at $\lambda_1 = 656.9\text{nm}$ and $\lambda_2 = 657.4\text{nm}$.

The standard image processing method (outlined in Chapter 5) returns the height of the correlation peak for each measurement. As for the speckle size, a 1D Gaussian fit using the Python module *lmfit* was applied to cross sections of the central correlation peak to find the root mean square (RMS) widths, i.e. standard deviations, of the peak in x and y . Assuming the peaks were circular, the average of the RMS widths were taken as the transverse speckle size. This gave a more precise value to compare the peak height to, rather than using an estimation of the spot size from the optical setup.

The experimental results are shown in Figure 6.9, along with a line plot of equation (6.2.19). The source wavelengths were 656.9nm and 657.4nm, giving a magnification between speckle patterns of 1.0008. The data roughly follows the exponential curve, although it is limited at the lower end as the speckle size approaches one pixel. Once the speckle size is below one pixel, the width of the correlation peak can no longer

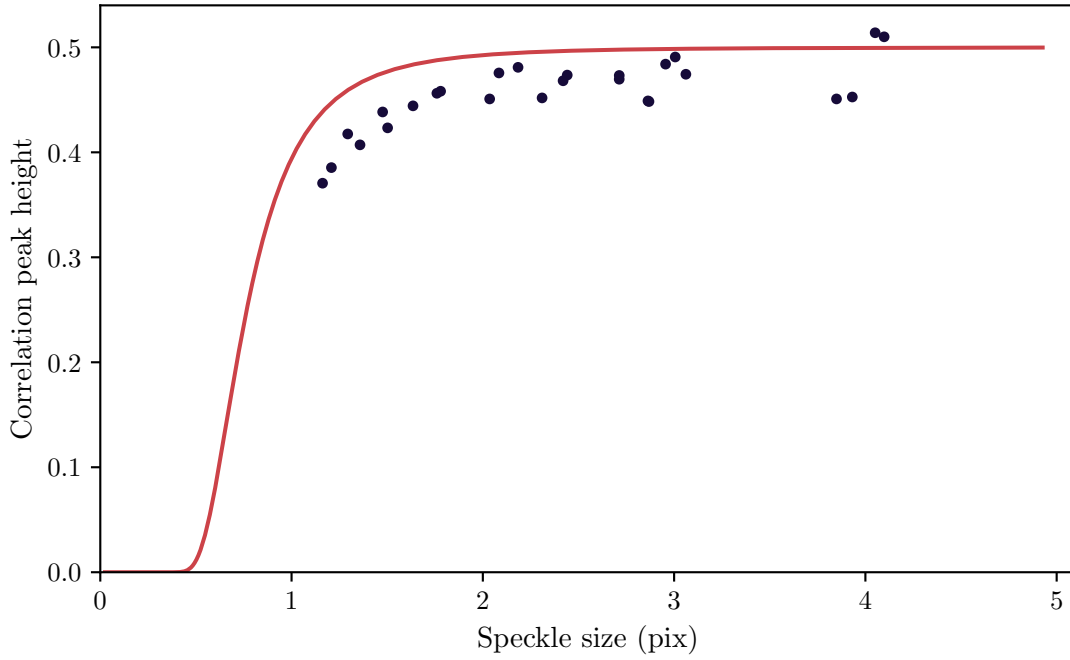


Figure 6.9: Transverse speckle size in pixels, measured as the average RMS width of the correlation peak, versus the correlation peak height. The points show measured data, while the line plots equation (6.2.19) for the sensor variables $L_D = 180\text{mm}$, $\lambda_1 = 656.9$, and $\Delta\lambda = 0.5$.

be measured.

There are other factors affecting the correlation peak height also, namely surface roughness and the size of the image correlation. Both of these factors will be explored further in the following section, but in terms of the data and theory presented here, each would have the effect of an additional scaling factor. This would reduce the maximum peak height approached by the exponential, accounting for the offset between data and theory seen in Figure 6.9.

The practical conclusion that can be drawn from this data is that there does appear to be an inverse relationship between spot size and the maximum height of the cross correlation peak. This can be used to increase a low peak height (by decreasing the spot size), which was one of the difficulties encountered during testing of the sensor with the $\Delta\lambda = 21.5\text{nm}$ wavelength separation and on certain surfaces. The relationship can be explained by the increased decorrelation for speckles of a smaller longitudinal size, where a smaller speckle size relates directly to a larger spot size.

6.3 Effect of Surface Roughness on Correlation

It was found during experiments that changing the surface also affected the height of the correlation peak. It is well known that surface roughness affects speckle contrast and decorrelation [6, 7]. The roughness of the various surfaces used throughout the project was unknown and could not be measured, but it was clear by eye that there was some variation between them. Three surfaces in particular were tested qualitatively for this short investigation, using the same sensor setup to compare the correlation peak heights and speckle contrast.

The first two surfaces tested, labelled Plate 1 and Plate 2 in Figure 6.10, are aluminium plates that can be attached to the Aerotech and Hexapod stages respectively. The third is a freestanding metal plate, also believed to be aluminium. By eye Plate 1 appears very smooth, Plate 2 is cast aluminium and has a more brushed metal texture, and Plate 3 is quite rough and scratched.

The experimental setup is shown in Figure 6.11, with Plate 3 in position. The geometry of the sensor was $\mathbf{S} = (151.2, 0.0, 218.0)\text{mm}$, $\mathbf{D} = (0.0, 0.0, 199.0)\text{mm}$, with wavelengths of $\lambda_1 = 656.9\text{nm}$ and $\lambda_2 = 657.4\text{nm}$.

Dual-wavelength speckle (DWS) measurements of each surface showed a noticeable

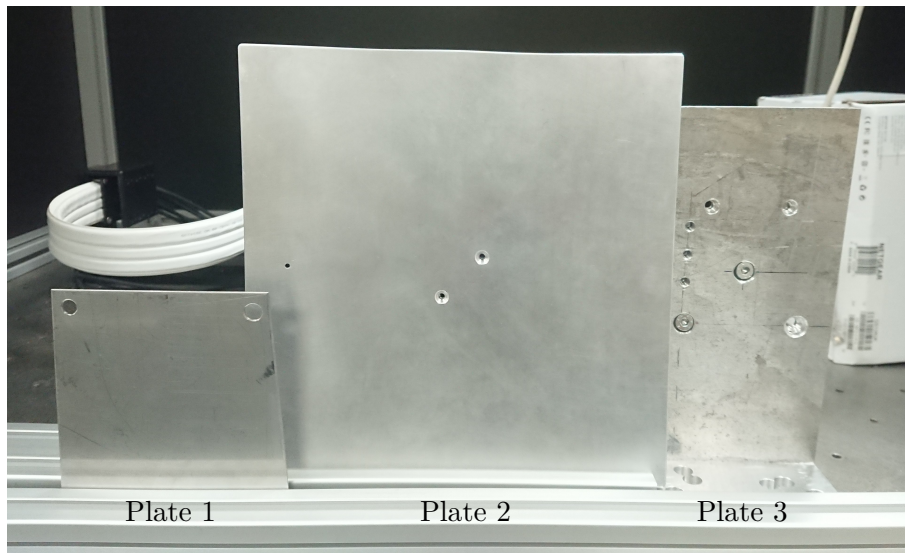


Figure 6.10: Photo of the three aluminium plates used to test surface roughness, arranged with the smoothest surface on the left (Plate 1), and the roughest on the right (Plate 3).

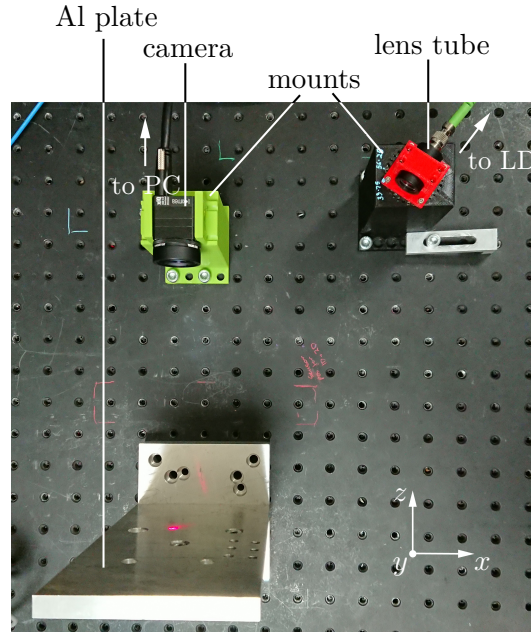


Figure 6.11: Labelled photo of the sensor used to measure plates with varying surface roughness. The sensor geometry is $\mathbf{S} = (151.2, 0.0, 218.0)\text{mm}$, $\mathbf{D} = (0.0, 0.0, 199.0)\text{mm}$, and the plate in the image is Plate 3. [LD: laser diode]

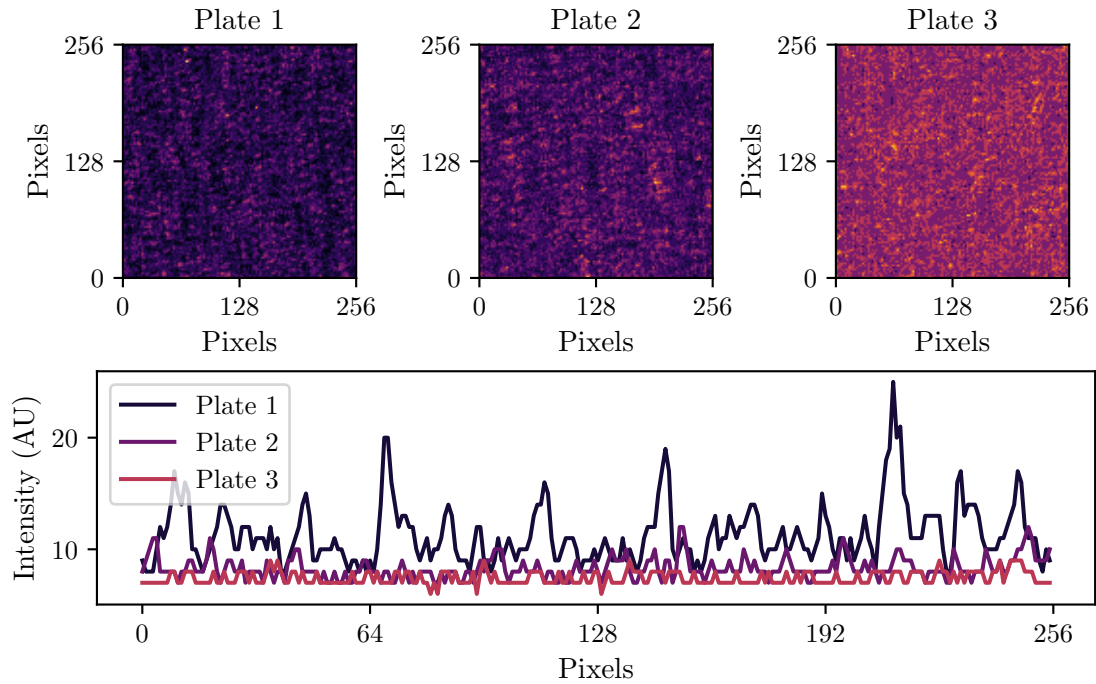


Figure 6.12: Double speckle patterns from the three different plates shown in Figure 6.10, each with a different surface roughness, plus a cross-section of each image.

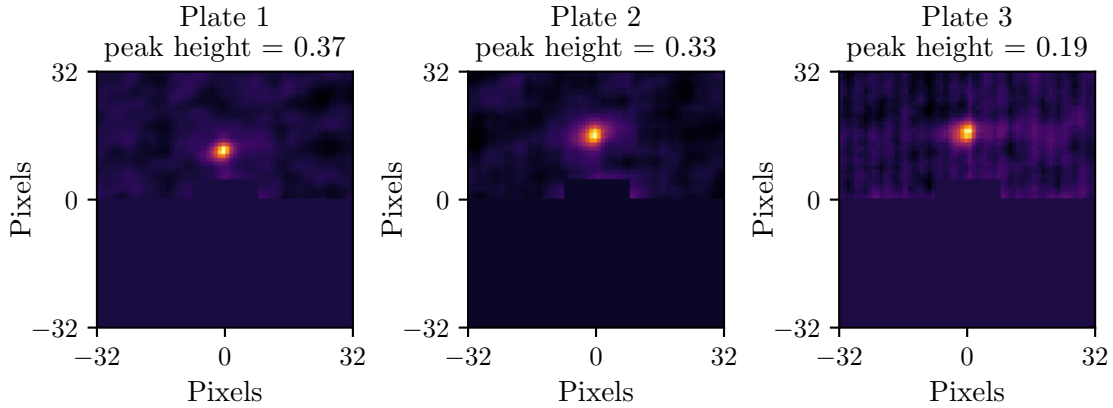


Figure 6.13: Correlation images for the three plates from Figure 6.10, with the central peak and the lower half masked. The height of each correlation peak is given in the plot titles, showing a decrease with increasing surface roughness.

difference in speckle contrast. The speckle patterns recorded by the sensor are shown in Figure 6.12, with a cross-section of each plotted in the lower half of the figure. There is a clear decrease in the standard deviation of each cross section, from 2.65 for Plate 1, to 1.01 for Plate 2, and then 0.60 for Plate 3. This supports the theory that the three plates have increasing surface roughness.

The decorrelation caused by surface roughness can be seen in the correlation images in Figure 6.13. The correlation peak heights are given above each image, showing a downwards trend with increasing surface roughness. This shows that surface roughness must be considered when constructing a DWS sensor; for instance the third surface could not be used with the $\Delta\lambda = 21.5\text{nm}$ sensor presented in Chapter 8. The large wavelength separation also introduces a degree of decorrelation, and combined with the high surface roughness of Plate 3 the resulting correlation peak was too low to measure.

A somewhat related consideration is the bidirectional reflectance distribution function (BRDF) of these surfaces. The BRDF describes the scattering angles of light from a surface, more specifically how much of the light is scattered in each direction. As shown in Figure 6.14, this can range from a Lambertian surface, where the reflected light is perfectly diffuse, to a mirrored or specular surface, where light is reflected in one direction only.

Although the BRDF of the surfaces used throughout this project could not be measured, it was observed experimentally that the intensity recorded at the detector

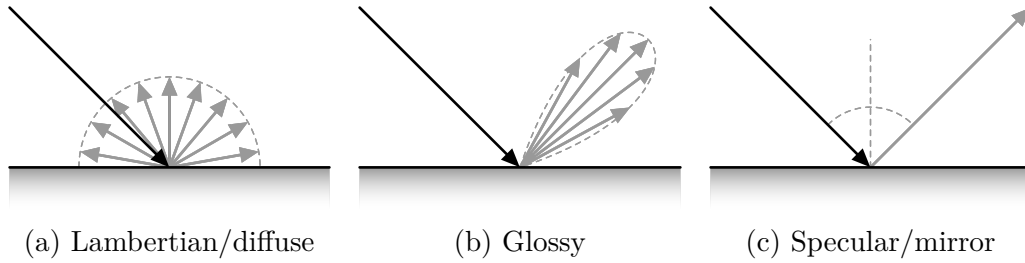


Figure 6.14: Diagrams showing the expected bidirectional reflectance distribution function of a Lambertian, also called diffuse, surface (a), a glossy surface (b), and a mirrored or specular surface (c). Images taken from [8].

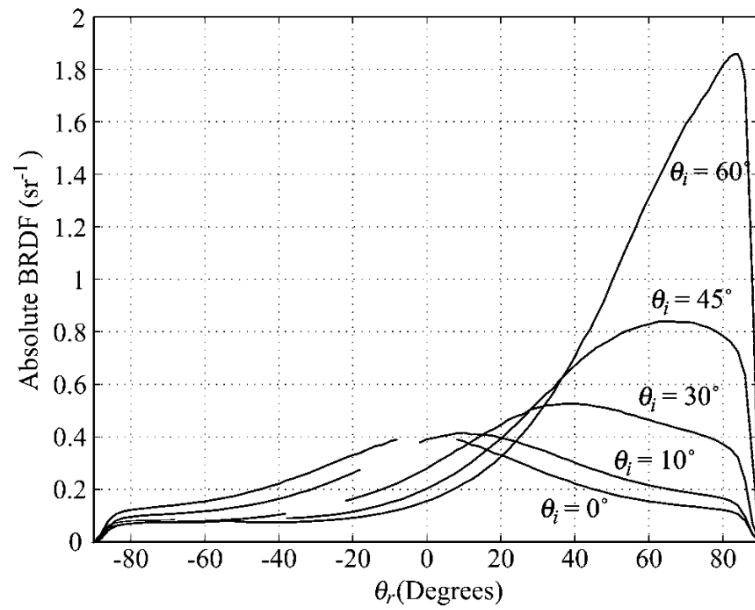


Figure 6.15: BRDFs of a rough aluminium surface, as measured from different angles of reflectance with differing angles of incidence, from Li [9].

had some cone of scattering around the specular reflection, similar to the diagram in Figure 6.14b. This was also observed by Li [9] through direct measurements of a rough aluminium surface using a gonireflectometer. Their experimental results for varying incidence angles θ_i are shown in Figure 6.15, with the BRDF measured across varying angles of reflectance θ_r .

Applying this to the angle sensor developed in this project, the intensity of recorded speckle patterns can be expected to change depending on the angle of incidence of the source and the tilt of the surface. However, the technique does not require or use an intensity measurement to determine surface angle, only measuring the displacement of the pattern as a whole. The signal processing also includes a normalisation step, neutralising any effect of changing intensity on the correlation peak finding. Thus the only potential effect on measurements would be either a loss of signal or an oversaturation of the detector. In either of these cases, the exposure time of the camera could be adjusted dynamically to account for the variation in intensity.

An avenue for further characterisation of this technique would be a more comprehensive review of surface roughness and BRDF as it applies to DWS. For the former, contour measurements using a Talysurf [10] for example could be combined with the speckle measurements to more quantitatively investigate the relationship shown in Figure 6.13. In particular, an investigation of the limiting value of surface roughness for a given DWS sensor would be a valuable experiment. As for the consideration of BRDFs, moving to a dynamic exposure time would make the sensor more robust against intensity fluctuations caused by non-Lambertian surfaces.

6.3.1 Correlation Image Size

Another factor affecting correlation peak height is the size of the correlation image. A smaller image has fewer speckles contributing to the correlation, and so statistically any decorrelated speckles, such as those at the edge of the image, will have a greater effect on the overall correlation. This can be seen in Figure 6.16, which shows the correlation peak heights for the three surfaces in Figure 6.10, as measured for correlation images of sizes 64x64, 128x128, 256x256, 512x512, and 1024x1024 pixels. The figure shows the trend in decreasing peak height for increased surface roughness, but also the trend for decreasing peak height with decreasing correlation image size.

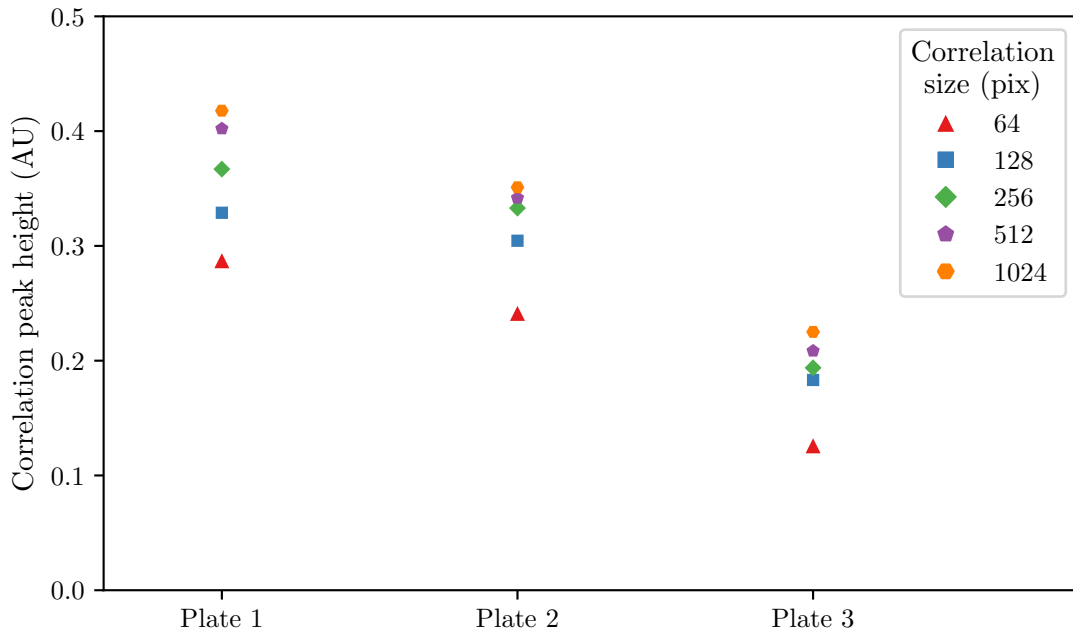


Figure 6.16: Correlation peak height for the three different surfaces in Figure 6.10, for correlation image sizes of 64x64, 128x128, 256x256, 512x512, and 1024x1024 pixels. The plot demonstrates both the trend of decreasing peak height with increasing surface roughness, and decreasing peak height with decreasing correlation image size.

In general, a larger correlation image is preferred, both due to the increase in peak height and a decrease in correlation noise. The noise in a correlation image is formed of non-corresponding speckles in the two patterns correlating slightly with each other. Similarly to the effect on peak height, a greater number of speckles in the image decreases the proportion of these random correlations.

This effect can be seen in Figure 6.17, which shows a slice through each size of correlation image, at a point away from the correlation peak. The smallest correlation image, at 64 pixels, has the highest standard deviation, at 0.038. Meanwhile the standard deviation of the largest correlation image, 1024 pixels, is about thirteen times lower at 0.003. The means and standard deviations of each noise slice are given in Table 6.4.

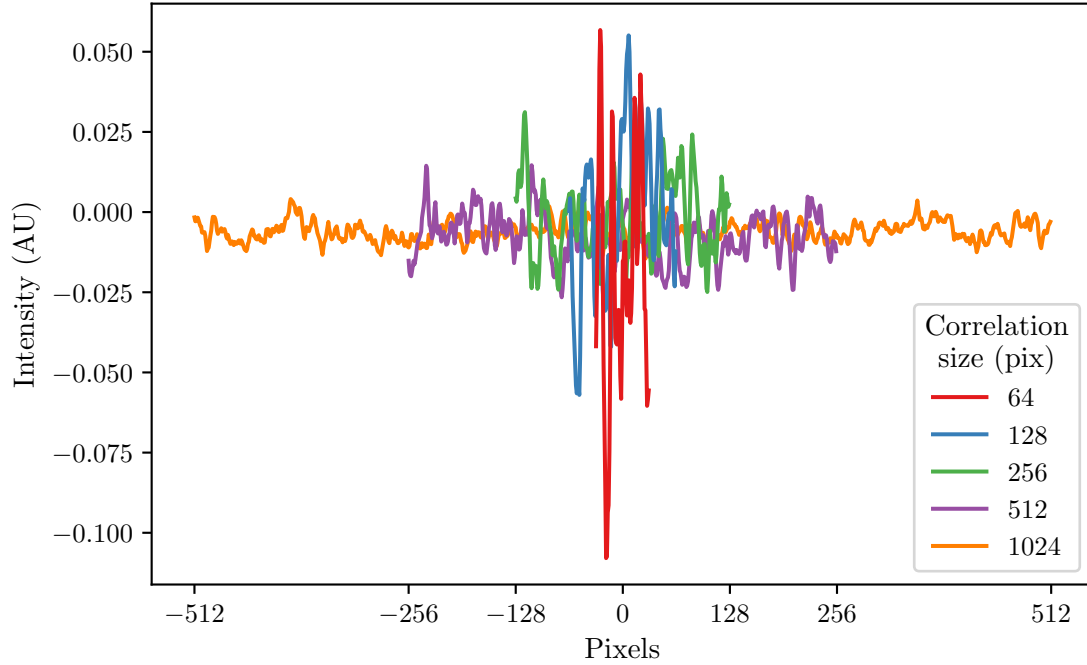


Figure 6.17: Cross sections of the noise (i.e. away from the correlation peak) in correlation images from Plate 1 of sizes 64x64, 128x128, 256x256, 512x512, and 1024x1024 pixels.

Table 6.4: Means and standard deviations of the noise cross sections in Figure 6.17.

Correlation size (pix)	Mean noise (AU)	Standard deviation (AU)
64	-0.020	0.038
128	-0.001	0.024
256	-0.001	0.012
512	-0.007	0.007
1024	-0.006	0.003

6.4 Summary of Factors Affecting DWS Sensor Operation

The results presented in this chapter demonstrate the range of factors that should be taken into account when constructing a DWS sensor. The first section demonstrated how the sensor geometry affects the sensor characteristics, including the range, responsivity, and resolution. A greater separation between the source and detector, S_x , gives a larger sensor range, but a slightly reduced responsivity and resolution. Increasing only the working distance L_D results in a higher responsivity and resolution, and a slightly increased range. Finally, an increased distance between the source and the surface, S_z , leads to a decreased range, with a slight increase in resolution and responsivity. These factors need to be balanced when constructing a sensor, and depending on the sensor requirements can be adjusted to prioritise a larger range, or a higher resolution.

This section also demonstrated the validity of the speckle theory derived in Chapter 4. The three sensor constants A_{x0} , A_{y0} , and C (the last of which is the responsivity, discussed above), were calculated using the theory, and determined from experimental measurements of speckle shift. The measured values were in good agreement with the theoretical ones, following the same expected trends with the changing geometry factors.

The other two sections in this chapter investigate some of the factors that affect the height of the correlation side peak during image processing. It was found during experiments that changing the spot size, which is directly related to speckle size, leads to a change in the correlation peak height. A decrease in the spot size increases the peak height, and a mathematical theory relating the two has been presented. The practical outcome of this relationship is that a smaller spot size can be used to maximise the height of the correlation peak, particularly in cases where other factors may be reducing the peak height.

One of these other factors is surface roughness. A higher surface roughness results in a lower correlation peak height, as the surface roughness increases the decorrelation between speckle patterns. Qualitative data was presented demonstrating the difference in surface roughness between various surfaces used throughout sensor experiments, including one metal plate whose surface roughness was too high to

produce a significant correlation peak for a sensor with a larger wavelength separation. The wavelength separation of the source is another factor that affects the correlation peak height; a larger wavelength separation also leads to decorrelation between the two generated speckle patterns.

The final factor investigated here was correlation image size. A smaller correlation image size leads to a smaller peak, as there are fewer speckles contributing to the correlation. The smaller image also results in higher noise, and so the signal-to-noise ratio can be greatly improved by using a larger correlation image.

These various factors should be considered together when constructing and operating a DWS sensor, in order to optimise the correlation peak height and thus the image processing. For example, a smaller spot size could be used to increase the peak height when using a sensor with a large wavelength separation on a particularly rough surface.

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Chapter 7

Single Laser Angle Sensor with Small $\Delta\lambda$

This chapter will outline the first full implementation of a dual-wavelength speckle (DWS) angle sensor. A description and overview of the sensor is given in Section 7.1, including a short discussion of the expected advantages of this version of the sensor.

Section 7.2 describes the construction of the sensor, beginning with the chosen geometry and expected sensor characteristics such as range and resolution, then moving on to look at the laser characterisation and expected uncertainty. Finally this section describes the results of the calibration of the complete sensor. The experimental methods used in this section are explained in detail in Chapter 5.

Experimental results are presented for the sensor in Sections 7.3 and 7.4. The first covers angle measurements in one dimension, testing the accuracy of the detector over the very large angle range of $\pm 45^\circ$. Section 7.4 looks at measuring two dimensional surface tilts, focusing on angles with magnitudes up to 7.5° .

Lastly, Section 7.5 summarises the results and performance of the sensor across all experimental measurements.

7.1 Sensor Description

The first implementation of the sensor used the fewest components and most basic processing, designed to demonstrate the technique and its simplicity. The wavelength spacing of this version was $\Delta\lambda = 0.46\text{nm}$. The two modes were generated with a single laser, operating with a dual-mode output. In theory, the use of a single laser should minimise errors, as there is a dependence between the wavelength stability of each mode. In an ideal case, the wavelength separation $\Delta\lambda$ should remain the same, while only the value of λ_1 changes. As instability in $\Delta\lambda$ has more of an effect mathematically on the calculation of the surface angle, this keeps the overall measurement uncertainty as low as possible.

The specific laser used was a Fabry-Pérot laser diode from Blue Sky Research [1], model number FTEC2-658-30. This was the first laser described in Section 5.2. It was used in continuous-wave (CW) operation, after finding a suitable operating condition for the required dual-mode output. The use of a single laser minimises the cost and complexity of the overall sensor.

Figure 7.1 shows the optical setup for the sensor, with the laser connected to one

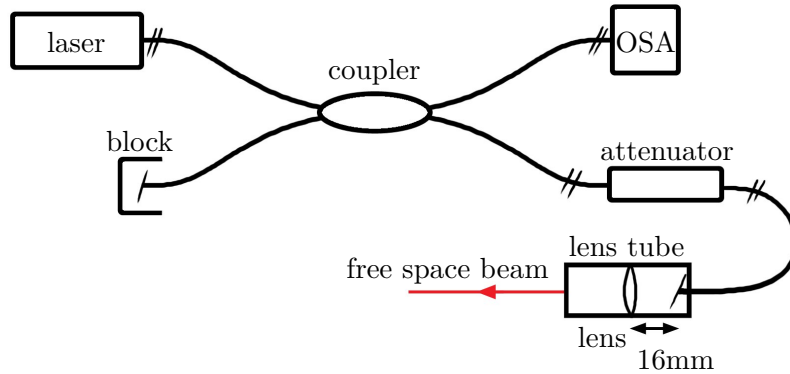


Figure 7.1: Diagram of the laser setup, showing the fibre-coupled Fabry-Pérot diode, connecting to the fibre coupler used to split the light between the OSA for monitoring and the lens tube in the sensor head. The lens tube contains a 15mm focal length lens placed 16mm from the fibre output. The fibre connectors (shown by the dashes) are all APC.

input arm of a 50:50 wideband fibre coupler (Thorlabs part number TW630R5A2). The coupler is centred at 630nm with a bandwidth of 50nm. Of the two output arms, one is connected to the Yokogawa optical spectrum analyser (OSA) [2] used to monitor the laser output, and the other leads to the sensor head. A variable beam blocking attenuator from OZ Optics is used on the sensor arm to regulate the output power to below 4mW. The full part number of the attenuator used is BB-500-11-650-4/125-P-50-3A3A-3-0.5. Each of the fibre connections use angled physical contact (APC) connectors, with polarisation maintaining fibre throughout. Finally, the light beam exiting the fibre is narrowed by a 15mm focal length lens positioned at 16mm from the fibre tip, producing a 6.8mm spot size.

The sensor camera was the Ximea MQ013MG-ON [3] described in Chapter 5, along with the laser line filter at $655 \pm 50\text{nm}$. The camera's exposure time was 1ms, running at 100fps. However, as with the data collection in the previous chapter, images were acquired individually once the stage was in position, with the image processing carried out in parallel to data capture. As the wavelength separation for this sensor was small, the basic signal processing outlined in Section 5.1 was used, without the additional magnification step.

Once the sensor wavelengths have been determined, the geometry can be chosen. The sensor arrangement positions the detector directly opposite the spot, with the source at the same z distance and a fixed x distance. The x distance to the source needs to balance the range and responsivity with the physical size of the sensor head. A larger value increases responsivity and range, but results in a less practical sensor. Because the small wavelength separation gives a small responsivity and range, the sensor head for this version was relatively large, maximising the sensor's performance through the geometry.

7.2 Sensor Construction

7.2.1 Geometry and Expected Performance

The geometry for this implementation set both the working distance (L_D) and the separation between the source and detector (S_x) at 103.5mm. This is shown in the diagram in Figure 7.2. As discussed in Section 4.3.1, the important factor when positioning the source is the angle of incidence; changing the distance L_S from the

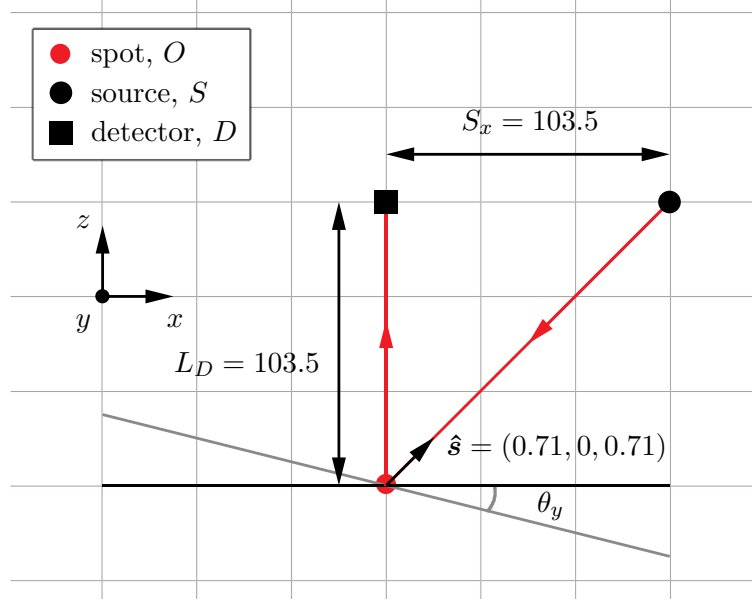


Figure 7.2: Diagram of the sensor geometry. The distances S_x (between source and detector) and L_D (between detector and spot) are given in mm. $\hat{s}$ is the unit vector from the spot to the source. An example surface tilt around the y axis of θ_y is shown in grey.

spot to the source while keeping the angle of incidence fixed has no effect on the sensor characteristics. This geometry results in a 45° angle for the source, which maintains a reasonable size for the sensor head and working distance. The particular value was dictated by the laboratory setup, specifically by the spacing on the optical bench.

This first implementation of the sensor used a fully 3D printed mount for the sensor head, shown in Figure 7.3. The printer was an Ultimaker 2+ [4], using polylactic acid (PLA) thermoplastic. This allowed the design to use the very precise value of $S_x = 103.5$, and achieve the 45° angle for the source. The expected accuracy on these values was given by the size of the printer nozzle, 0.2mm, and the layer height during printing, also 0.2mm.

The resolution for this geometry was calculated using equation (4.3.23) described in Section 4.3.3, using the sensor responsivity C plus the detector pixel size ($p = 4.8\mu\text{m}$ in each direction) and the resolution of the image processing ($r_{\text{fit}} = 0.1\text{pix}$ [5]). The sensor resolution, r_{sens} , was found to be 0.222° , falling short of the target of 0.1° but suitable for a proof-of-concept demonstration.

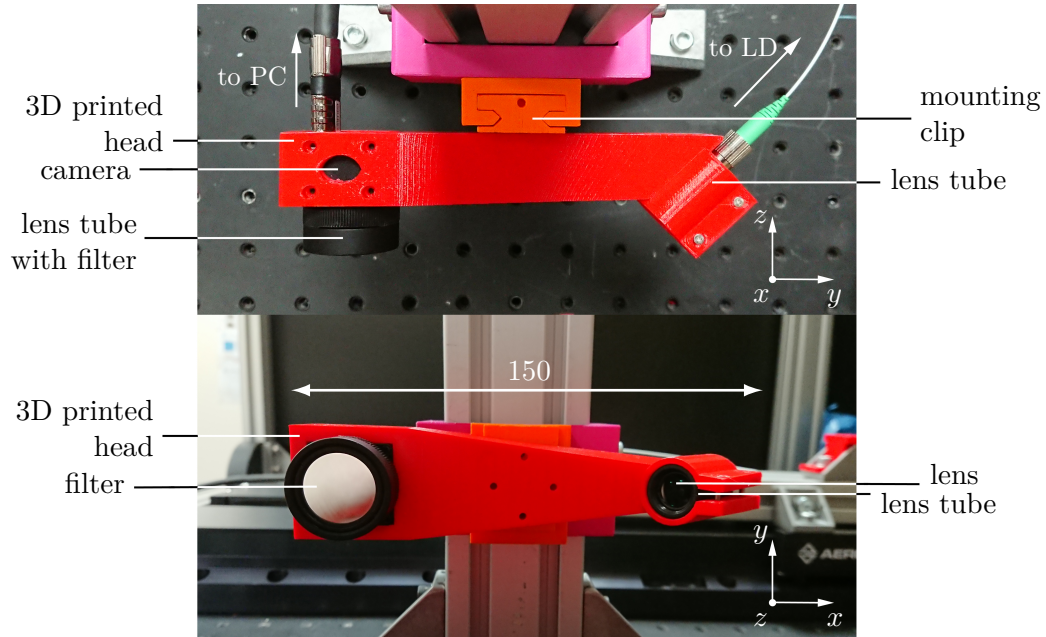


Figure 7.3: Photos of the 3D printed sensor head, from above (top) and from the front (bottom). The various components are labelled, and the total width of the head is given in mm.

Table 7.1: Maximum measurable surface tilts for positive and negative changes in both θ_x and θ_y . The absolute value of the angle $\theta_{y,\min}$ is taken as the sensor range, as this is the smallest value.

Sensor ranges	Expected value (deg)
θ_R	22.50
$\theta_{y,\min}$	-12.42
$\theta_{y,\max}$	+90.00
$\theta_{x,\min}$	-90.00
$\theta_{x,\max}$	+90.00
$\theta_{\max}$	12.42

Table 7.2: Summary of the expected performance of this sensor implementation, namely the theoretical range, resolution, and responsivity.

Sensor characteristic	Expected value
Range, $\theta_{\max}$	12.42°
Resolution, r_{sens}	0.222°
Responsivity, C	0.124mm/rad

As for the sensor range, θ_R was calculated at 22.50°. Experimental tests found that the size of the mask required to block out the central correlation peak was 5 pixels. This resulted in a final theoretical range of $\theta_{\max} = |\theta_{y,\min}| = 12.42^\circ$. The angle ranges in the other directions are given in Table 7.1.

Using the equations derived in Chapter 4, the expected uncertainty due to the signal processing can also be calculated. The values of both $\sigma_{A_x}^2$ and $\sigma_{A_y}^2$ come from the same 0.1 pixel uncertainty on the peak finding method as was used to calculate the sensor resolution. As a result, the expected uncertainty due to the signal processing (calculated from the equations in Section 5.1.2) is the same as the sensor resolution to three decimal places, with a value of $\sigma = 0.222^\circ$ in both θ_x and θ_y .

7.2.2 Laser Characterisation and Expected Uncertainty

This sensor used a single laser, the FTEC2-658-30 diode, operating such that there were two longitudinal modes in the output spectra. As described in Chapter 5, the operating condition for the laser was determined using the Yokogawa OSA [2], and the same OSA was used to monitor the stability of the laser over time.

Figure 7.4 is a typical spectrum for the laser, showing the two modes at around 656.9nm and 657.4nm. The OSA was set to a resolution of 0.02nm, with a sampling interval of 0.004nm. The fitted peak wavelengths of this spectrum are 656.93nm and 657.39nm.

The spectrum was monitored over a total period of around 6.5 hours, sampling the spectra roughly every two minutes. This was then processed using the method outlined in Chapter 5, giving the wavelength stability of each mode over time. This data is given in Figure 7.5, with the top two graphs showing the wavelengths of the

two modes, λ_1 and λ_2 , and the bottom graph showing the difference at each point in time, giving $\Delta\lambda$.

The standard deviation across the two modes over this time period was 0.002nm, or 2pm, and the standard deviation on the difference was 0.003nm, or 3pm. One of the theoretical advantages of using a single laser to generate both wavelengths was that any wavelength drift would be mitigated as the two wavelengths would drift in tandem. However the data suggests this is not necessarily true, as the standard deviation in $\Delta\lambda$ is the same as would be expected if the two wavelengths were independent. In this case, the standard deviation in $\Delta\lambda$ could be calculated by combining the individual standard deviations using the formula $\sigma_{\Delta\lambda} = \sqrt{\sigma_{\lambda_1}^2 + \sigma_{\lambda_2}^2}$. This gives the same result of 3pm. Either way, the deviation is much smaller than the values of λ_1 , λ_2 , and even $\Delta\lambda$.

Additionally, the standard deviations from the measured wavelengths are at least an order of magnitude higher than the bias errors and standard deviations found during the testing of the peak fitting method described in Section 5.2.4. This confirms that the errors in the stability analysis are significantly lower than the errors inherent to the peak fitting method, and so the standard deviations quoted above can be assumed to be an accurate representation of the wavelength uncertainty of the laser.

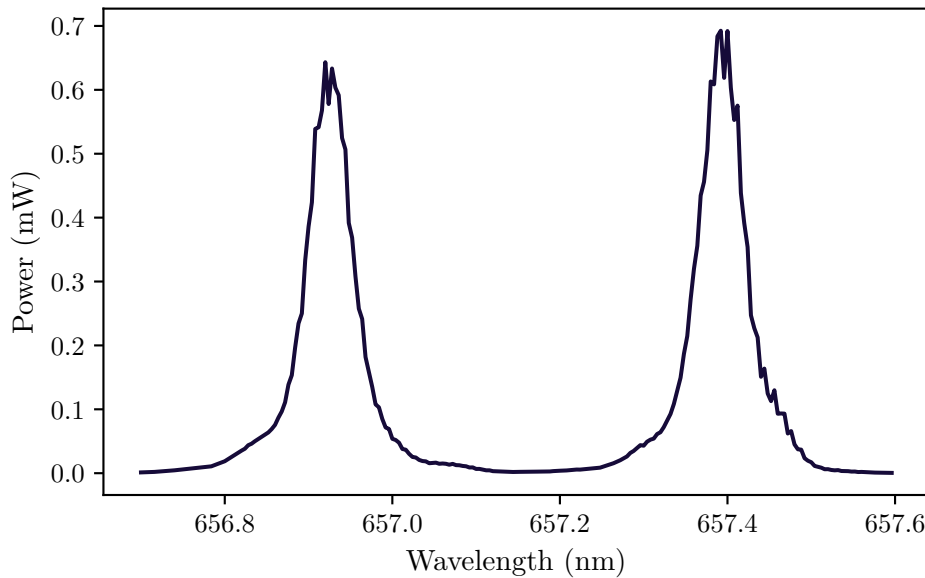


Figure 7.4: Typical spectrum for the FTEC2-658-30 laser diode operating with a dual mode output.

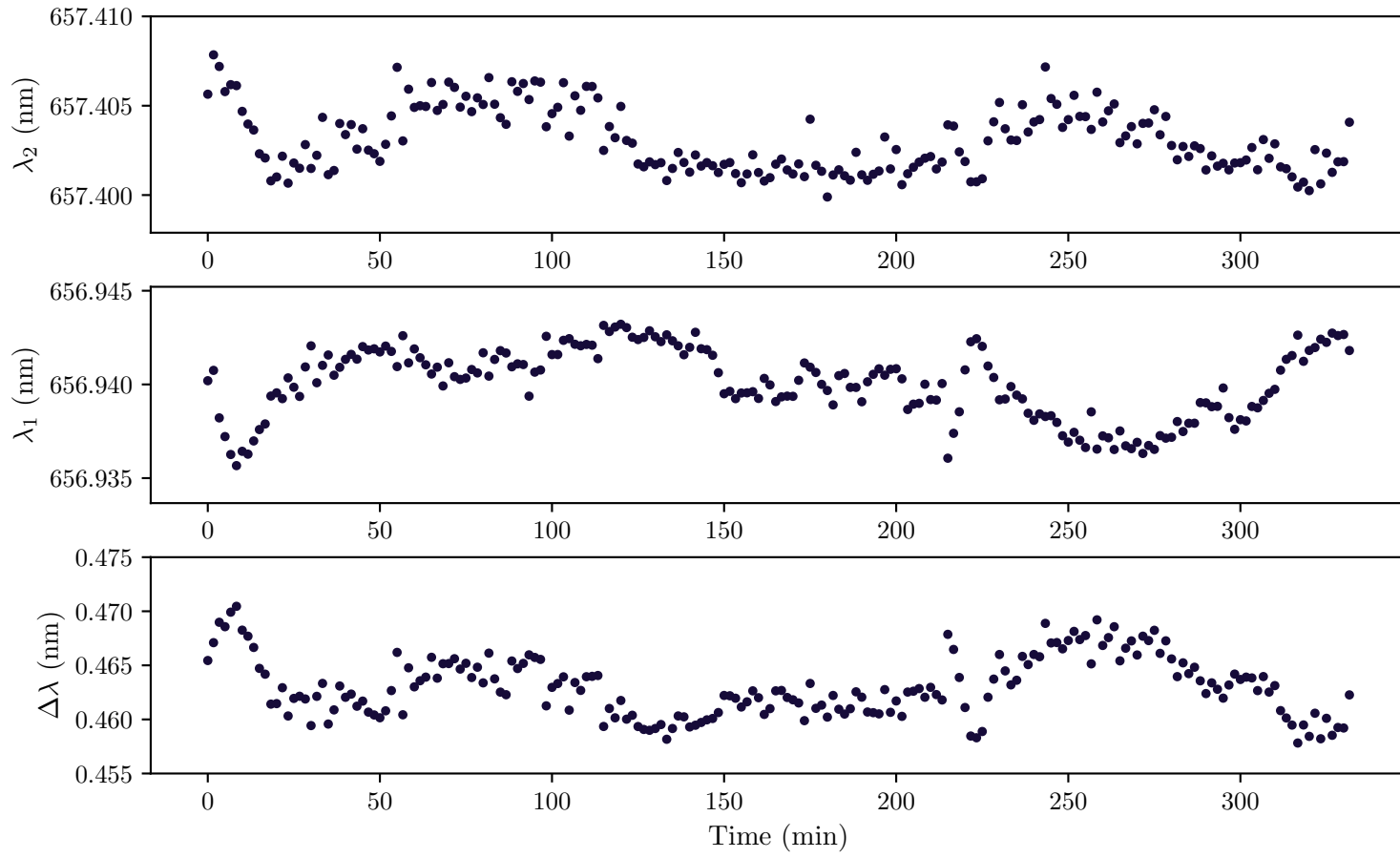


Figure 7.5: Wavelength stability over time of the two modes generated by the FTEC2-658-30 laser diode. The first two graphs show the two modes, λ_1 and λ_2 , and the final graph gives the difference between them at each point in time, $\Delta\lambda$.

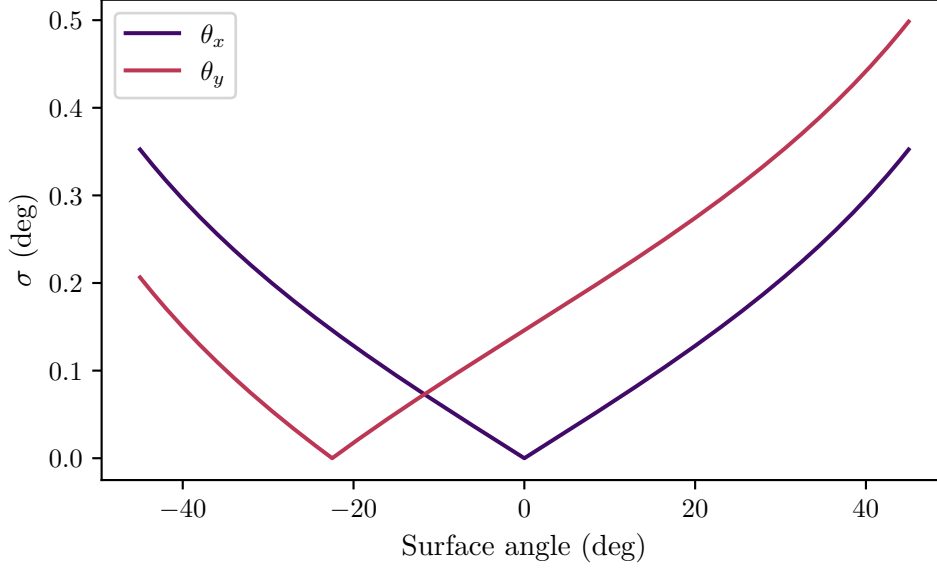
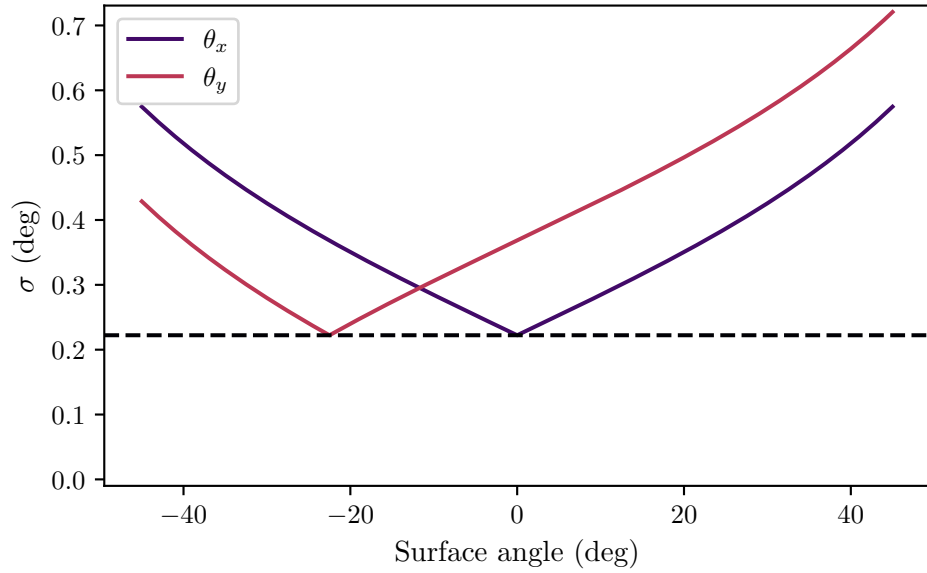


Figure 7.6: Expected uncertainty, σ , in angle measurements for θ_x and θ_y across the range $\pm 45^\circ$ for the laser wavelength instability only. The difference between θ_x and θ_y is caused by the sensor geometry offsetting the source in the x direction.

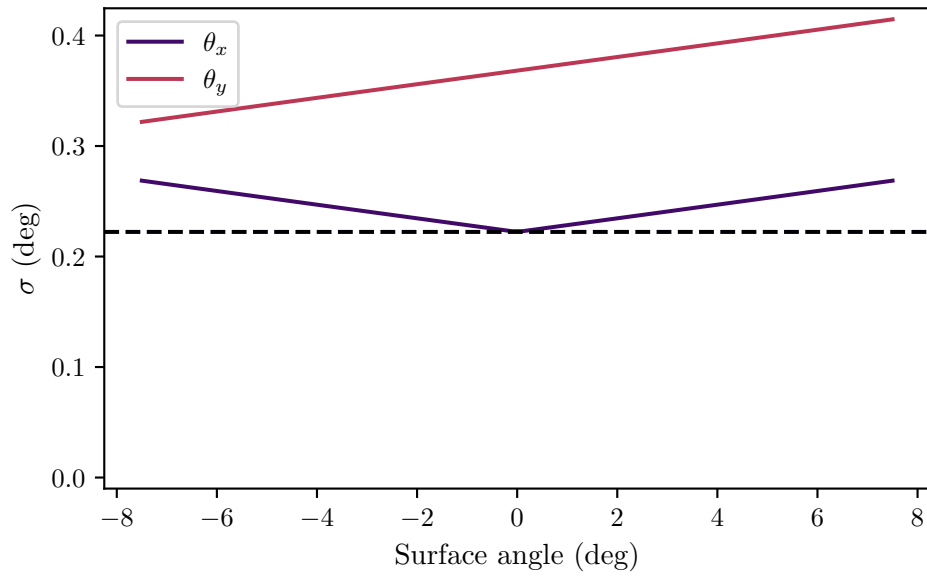
Using the calculated standard deviations from the wavelength stability data, the error propagation equations (see Section 5.2.2) can be used to estimate the expected errors in angle measurements. Figure 7.6 shows these errors over the angle range $\pm 45^\circ$. The errors reach zero at $\theta_R = 22.5$ for θ_y and 0° for θ_x . They are maximised at larger angles, reaching 0.325° at $\pm 45^\circ$ for θ_x and 0.498° at $+45^\circ$ for θ_x .

The expected errors from the signal processing also need to be taken into account. This uncertainty is consistent across the measurement range at 0.222° , so effectively adds an extra constant value to the graph of Figure 7.6.

Figure 7.7 shows the combined estimated uncertainties, covering both the wide range of $\pm 45^\circ$ and the smaller range of $\pm 7.5^\circ$, with the constant uncertainty from the image processing shown by the dashed line. These two angle ranges are those tested in the next sections; Section 7.3 looks at 1D angle measurements up to $\pm 45^\circ$ for θ_x , and then Section 7.4 investigates 2D angles up to a 7.5° magnitude. Including the uncertainties from the image processing, the expected minimum error for the 1D measurements is 0.222° , and the maximum is 0.575° . For the 2D measurements, the minimum is 0.222° in θ_x and 0.322° in θ_y , and the maximums are 0.269° in θ_x and 0.415° in θ_y .



(a) $\pm 45^\circ$ range



(b) $\pm 7.5^\circ$ range

Figure 7.7: Expected uncertainty, σ , in angle measurements for θ_x and θ_y across the ranges $\pm 45^\circ$ (a), and $\pm 7.5^\circ$ (b). The graphs show the constant uncertainty from the signal processing (given by the dashed line) combined with the uncertainty curve from the wavelength instability. The difference between θ_x and θ_y is caused by the sensor geometry offsetting the source in the x direction.

7.2.3 Calibration

The final step in constructing a sensor is the calibration. For this sensor, the calibration was carried out using the 3D printed profiles attached to the Aerotech rotation stage [6]. A photo of this setup, and more detail about the calibration process and stage, is found in Section 5.3 (Figure 5.21).

Angles of magnitude 1.0° , 2.0° , 3.0° , 4.0° , 5.0° , 6.0° , 7.0° , and 8.0° were used in this calibration, although for clarity only the odd numbered magnitudes are shown in Figure 7.8. Each calibration profile was rotated through five full circles at 1.0° increments, changing the direction of the applied angle, ϕ . The resulting circles in the speckle shift plane were each fitted to individually, giving a total of eight results for the three calibration constants A_{x0} , A_{y0} , and C . The fits plus their centres are shown in black in Figure 7.8. The averaged values are given in Table 7.3 in mm, along with the expected values using the geometry of the sensor.

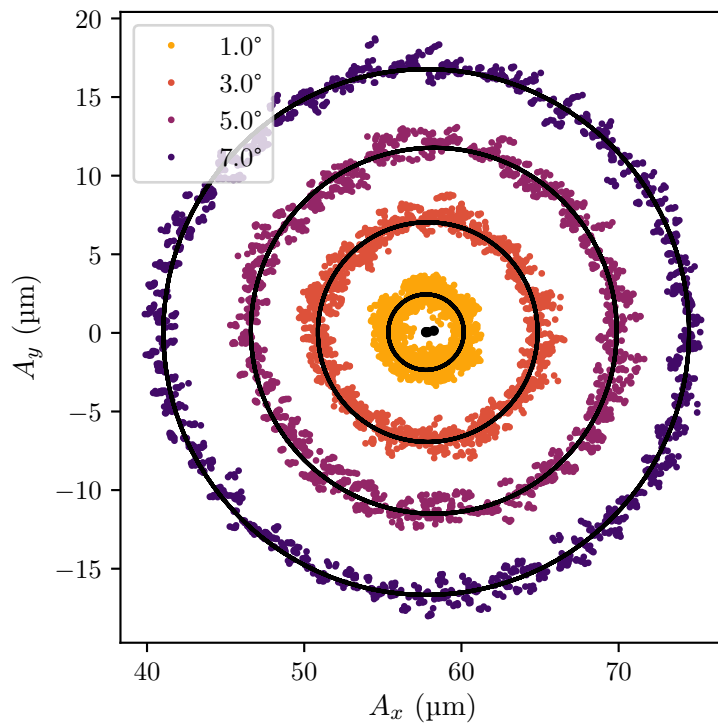


Figure 7.8: Calibration data, showing measured speckle shifts for applied angles of constant magnitude but varying direction. The angles were applied in direction increments of 1.0° , with five full rotations. The fitted circles and their centres are shown in black. For clarity, the magnitudes shown in the figure are 1.0° , 3.0° , 5.0° , and 7.0° , but data was also collected for 2.0° , 4.0° , 6.0° , and 8.0° .

Table 7.3: Expected and measured values of the three calibration constants: the offset in the speckle shift at zero tilt, $A_0 = (A_{x0}, A_{y0})$, and the responsivity constant, C .

Calibration constant	Expected value (mm)	Measured value (mm)
C	0.124	0.135
A_{x0}	0.051	0.058
A_{y0}	0.000	0.000

The differences between the expected and calibrated constants are very small; in pixel terms the difference in C is 2.3 pixels (11.0 μm), and in A_{x0} the difference is 1.5 pixels (7.2 μm). The discrepancies will be caused by small differences between the intended and actual geometries, which is why the calibration is employed after the sensor has been assembled.

7.3 1D Angle Measurements

The first angle measurements taken with the completed sensor were in one dimension only. The sensor was also not calibrated, as the calibration method was developed after these initial measurements. Therefore the expected calibration constants, calculated from the geometry of the sensor and the wavelengths of the source, were used to obtain the surface angle from the recorded speckle shift.

To generate angles in one dimension, the Aerotech rotation stage [6] was used to apply angles in x between $\pm 45^\circ$ to an aluminium plate. Referring back to Chapter 6, Section 6.3, this was Plate 1, believed to be the least rough of the three. An image of this experimental setup is shown in Figure 7.9. The angles were applied around the x axis, aiming to test the theoretical range limit of $\pm 90.0^\circ$ in that direction (see Table 7.1). Each angle was applied and measured successively, from -45° to $+45^\circ$ in steps of 0.2° , pausing for 0.5s between stage moves to ensure the stage had reached the expected position and had stopped moving.

The theoretical ranges presented in Table 7.1 only take into account the size of the correlation image. There are other practical factors likely to restrict these values. One is that the spot, and therefore the speckles themselves, becomes more elliptical

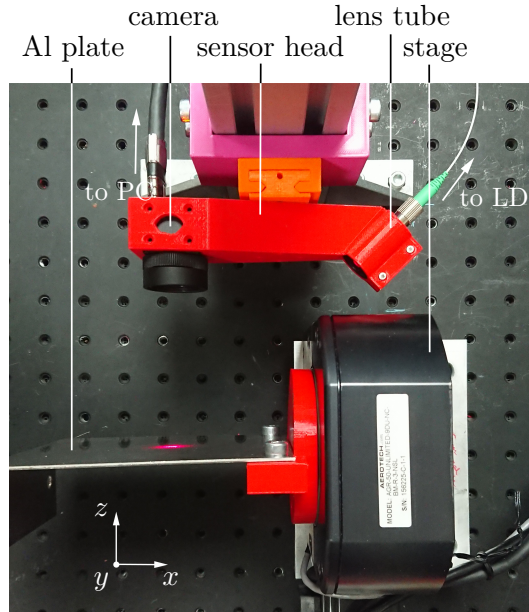


Figure 7.9: Photo of the setup used to take 1D angle measurements. The sensor head is mounted opposite an aluminium plate attached to the Aerotech stage, with the detector positioned directly opposite the spot. The stage is oriented such that angles are applied around the x axis. [LD: laser diode]

as the surface tilt increases. The Gaussian peak fitting used in the image processing assumes the spot is circular, and so an elliptical peak could induce irregularities and errors. Additionally, an elliptical peak covers a larger part of the surface, which could lead to problems if the surface is not perfectly flat. The angle could change slightly across the spot, which would have the ultimate effect of blurring the correlation peak during image processing. Another change caused by the increasing surface angle is a decrease in scattered light reaching the detector. Ultimately, as the angle approaches and surpasses 90° , the detector will be in line and then behind the surface, making angle detection impossible.

The results of the experiment are given in Figure 7.10. The focus of this experiment was investigating any bias errors, particularly at the extreme angles. The residuals, χ_x , are shown in the lower graph of the figure. The residuals do seem to show a trend of positive errors at negative applied angles, and negative errors for positive angles. The average residual error for negative angles is $+0.461^\circ$, while the average for positive angles is -0.549° . This could be due to the elliptical peak shape described above, interfering with the image processing and causing larger errors as the angle

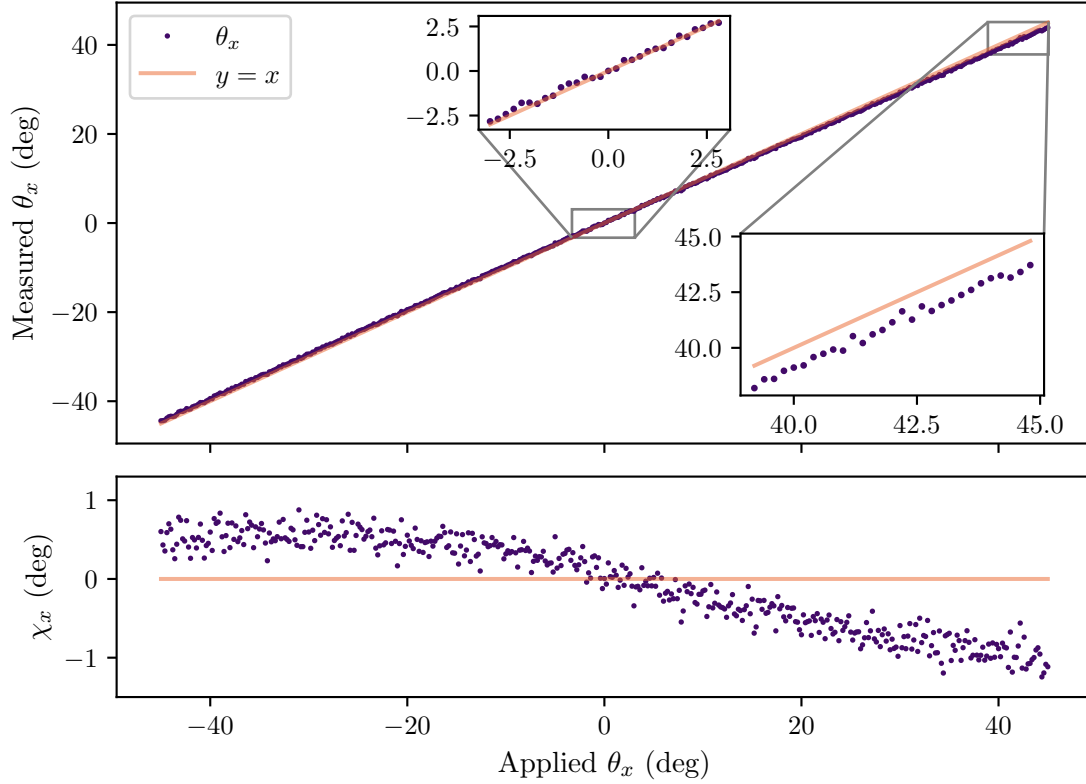


Figure 7.10: 1D angle measurements for θ_x over the range $\pm 45^\circ$. The first graph shows the measured angles, as compared to the expected values. The second graph gives the residual errors, χ_x , for each point.

increases.

The residual errors could also be due to errors in the sensor geometry. As the sensor was uncalibrated, the sensor constants may not fully match the actual sensor geometry. A change in the responsivity constant C , which is heavily affected by the working distance L_D , would affect the overall slope of the line in the residuals graph in Figure 7.10. To investigate this, Figure 7.11 shows the same speckle shift data processed using assumed working distances of $103.5 - 1 = 102.5\text{mm}$ and $103.5 - 2 = 101.5\text{mm}$. This resulted in values of C equal to 0.123mm/rad and 0.121mm/rad , compared to the expected value of 0.124mm/rad . As shown in the figure, this small change in the assumed working distance gives a significant change in the residuals across the dataset. For the second graph, with $L_D = 101.5\text{mm}$, the mean residuals are now $+0.107^\circ$ for negative angles, and -0.197° for positive angles. This shows the effect of a small, 2mm offset in the sensor geometry, emphasising the importance

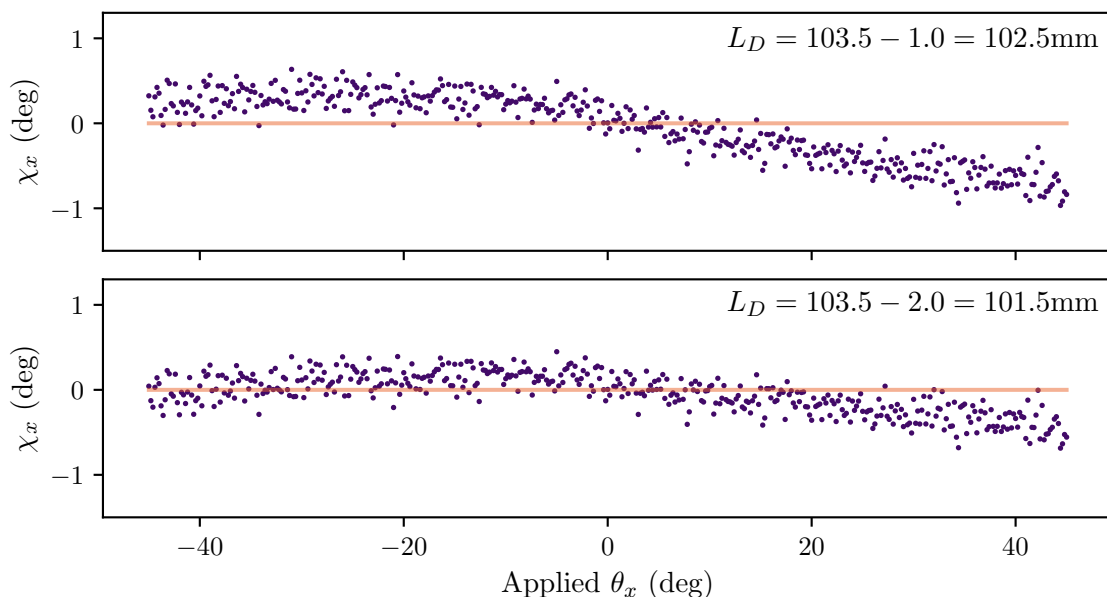


Figure 7.11: Residual errors for reprocessed speckle shift data using adjusted working distances $103.5 - 1 = 102.5\text{mm}$ and $103.5 - 2 = 101.5\text{mm}$. The change in assumed working distance changes the responsivity C and thus the calculated surface angle.

of the calibration procedure.

As the experiment only collected one run of data, a conventional standard deviation on repeat measurements could not be used. Thus to compare the collected data to the expected errors calculated above, a rolling standard deviation was applied to residuals in Figure 7.10, i.e. those calculated using the intended working distance of $L_D = 103.5\text{mm}$. The standard deviations, denoted by σ_{χ_x} , are shown in Figure 7.12. This approach calculated the standard deviation for a rolling sample of 10 data points on the residuals, with the full dataset having 451 points.

The resulting graph looks to show a slight trend towards higher values at larger magnitude tilts. This difference is much less extreme than that predicted by Figure 7.7a, however. The average value of σ_{χ_x} between $\pm 22.0^\circ$ is 0.113° , while for the outer half of the measurement series ($-45.0 - -22.0$ and $+22.0 - +45.0$) the average value is 0.131° . Both values are significantly smaller than the minimum expected value of 0.222° . This is potentially due to the small time period over which the measurements were taken; the single set of angle measurements was recorded in around five minutes, leaving little room for large variations in the laser output.

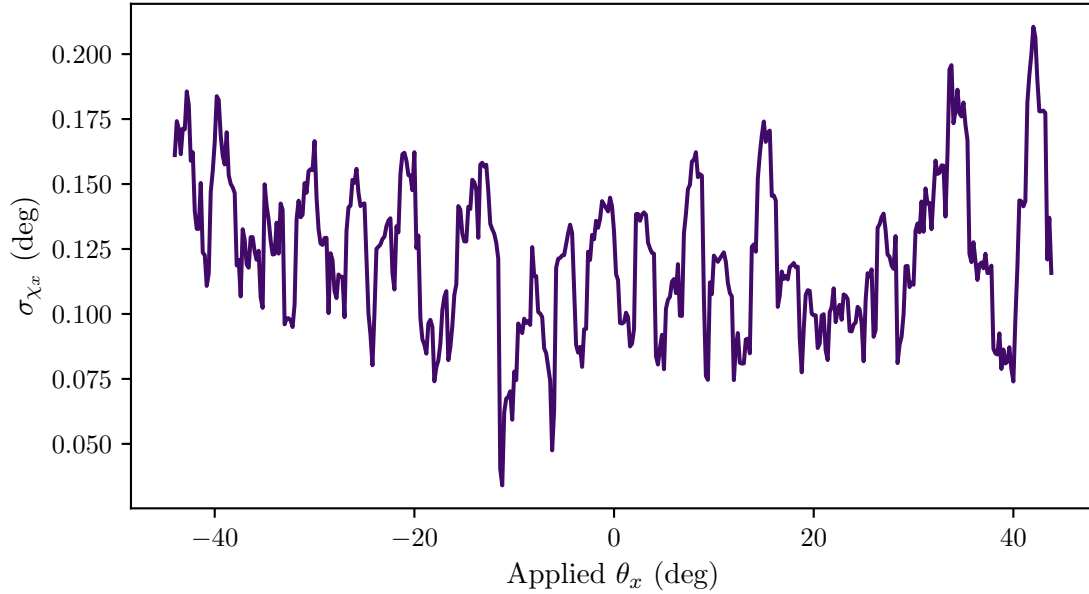


Figure 7.12: Results of a rolling standard deviation on the residuals in Figure 7.10 (for the intended working distance $L_D = 103.5\text{mm}$) denoted by σ_{χ_x} , as applied to rolling samples of 10 points on the bias errors, χ_x .

Overall, these initial measurements show that the sensor can perform reasonably well in the large angle range of $\pm 45^\circ$, though measurement errors increase as the magnitude of tilt increases. The presented results also demonstrate the effect a small change (1-2mm) in the working distance can have on the measured surface angle, indicating the utility of the calibration process for a complete sensor. Further experiments looking at the performance of the sensor focus on smaller surface angles, while also expanding the investigation into two dimensions.

7.4 2D Angle Measurements

For 2D angle measurements, the sensor was mounted above the Alio Hybrid Hexapod stage [7], as shown in Figure 7.13. The full specifications of this stage are given in Section 5.3.3, Table 5.7, but the angular resolution of the tripod is 0.02arcsec. The plate used with this stage is Plate 2 described in Section 6.3.

To reiterate, the sensor is exactly the same as was used for the 1D measurements above, with the same wavelengths and geometry. The 3D printed sensor head was also the same, though the photo of the experimental setup in Figure 7.13 is at a

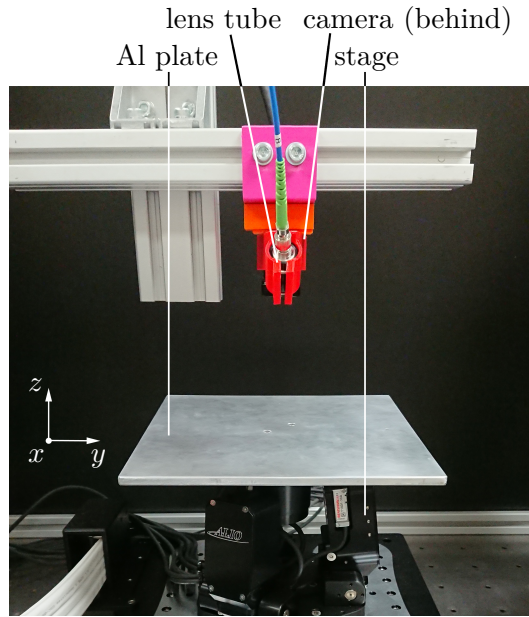


Figure 7.13: Photo of the setup used to take 2D angle measurements. The sensor head is mounted above the Hexapod stage, with the detector positioned above its centre. The tripod element of the stage can move the plate through 2D surface tilts in x and y . [LD: laser diode]

different angle and so this is not clear. The photo is taken along the x axis, where the sensor schematic in Figure 7.2 (and photo in Figure 7.9) is viewed along the y axis.

The surface angles were applied using the tripod portion of the Hexapod stage, which moves the attached aluminium plate through two dimensional tilts. The maximum tilt magnitude able to be applied using this stage is around 7.5° , which is within the expected 12.42° range of the sensor. The expected errors over the $\pm 7.5^\circ$ range are 0.222° - 0.269° in θ_x and 0.322° - 0.415° in θ_y (see Figure 7.7b).

The angles chosen to test the stage were a series of tilts with constant direction and varying magnitude. In each case, the stage started at zero tilt and increased the angle magnitude ψ in increments of 0.5° . Eight directions were tested, spread evenly around the circle. This gave four measurement sets with θ_x or θ_y tilts only, and four sets with compound angles. At each applied tilt, the angle was measured 50 times to investigate the repeatability.

Figure 7.14 shows the results of the 2D angle measurements, averaged over the 50 repeat measurements. The first tilt series measured was at the direction $\phi = 0$,

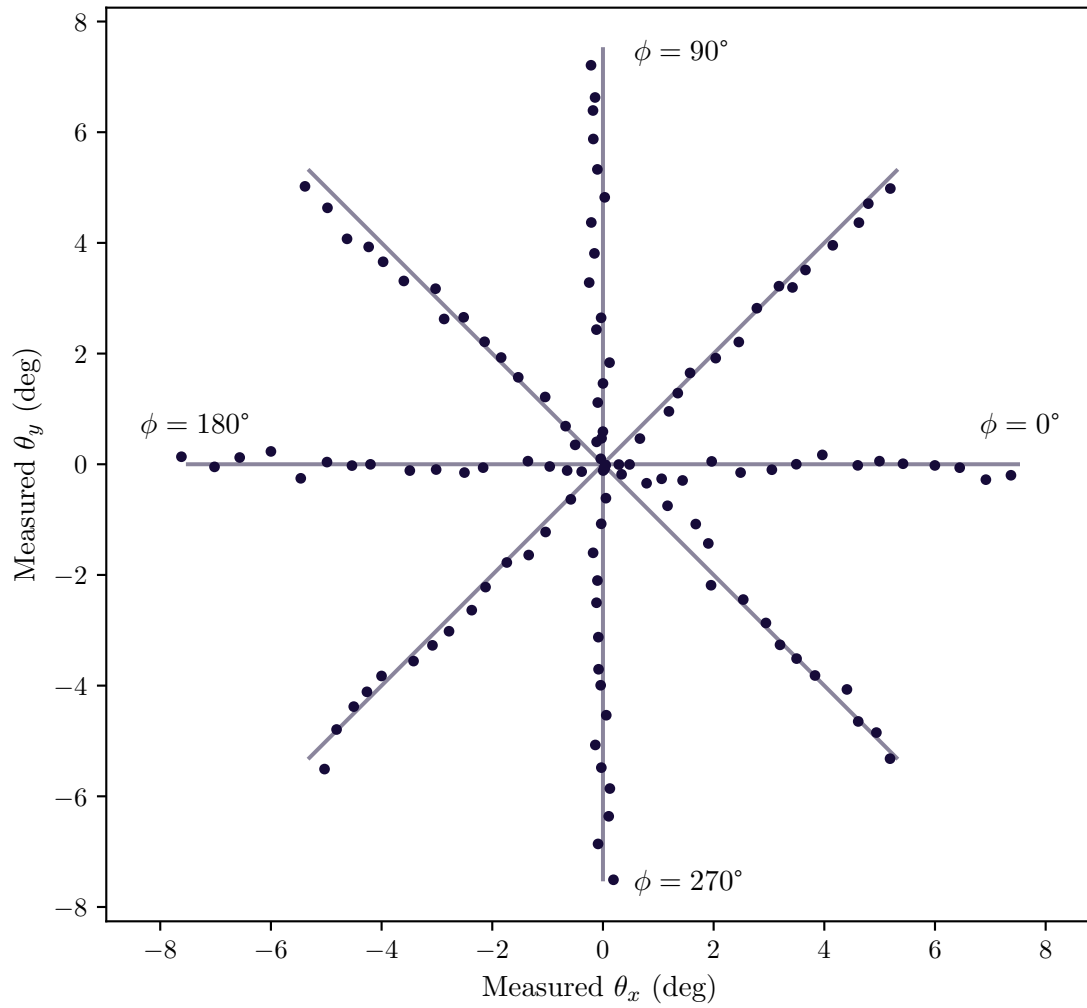


Figure 7.14: 2D angle measurements, showing means of 50 repeats at each angle. Each measurement set started at zero tilt and increased the angle magnitude by 0.5° in a constant direction up to a maximum of 7.5° . The tilt directions started at zero degrees along the positive x axis, incrementing anticlockwise in steps of 45° .

with $\theta_y = 0^\circ$ and positive θ_x tilts. The direction was then increased anticlockwise around the axes. As each measurement set started at zero tilt, there are eight different measurements clustered at zero, with the rest of the measurements moving outwards. Overall, the angle measurements follow the general expected shape, with some random variation from the expected values.

Figure 7.15 gives a closer look at the data for the direction $\phi = 315^\circ$. The two lower graphs in the figure show the individual bias errors χ_x and χ_y (i.e. residuals on the mean angle measurement), along with the standard deviation at each angle on the 50 repeats. The standard deviations are very small; the average across this measurement series is 0.017° for σ_x and 0.032° for σ_y . The bias errors are noticeably larger in magnitude, averaging 0.087° in θ_x and 0.165° in θ_y across these measurements.

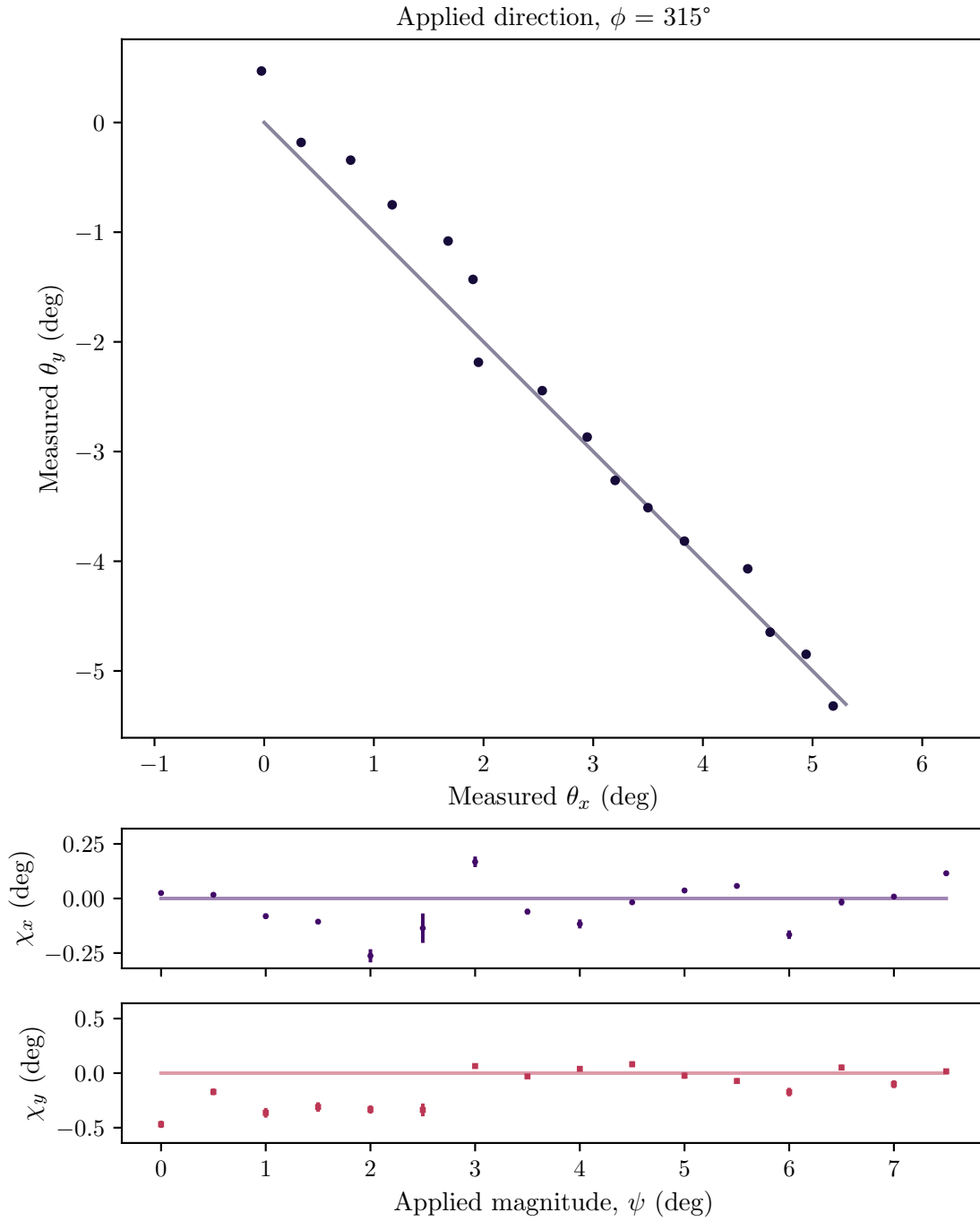


Figure 7.15: 2D angle measurements, showing means of 50 repeats, for the tilt direction $\phi = 315^\circ$. The first graph shows the mean measured angles, and the lower two graphs show the bias errors and standard deviations on these means in θ_x and θ_y .

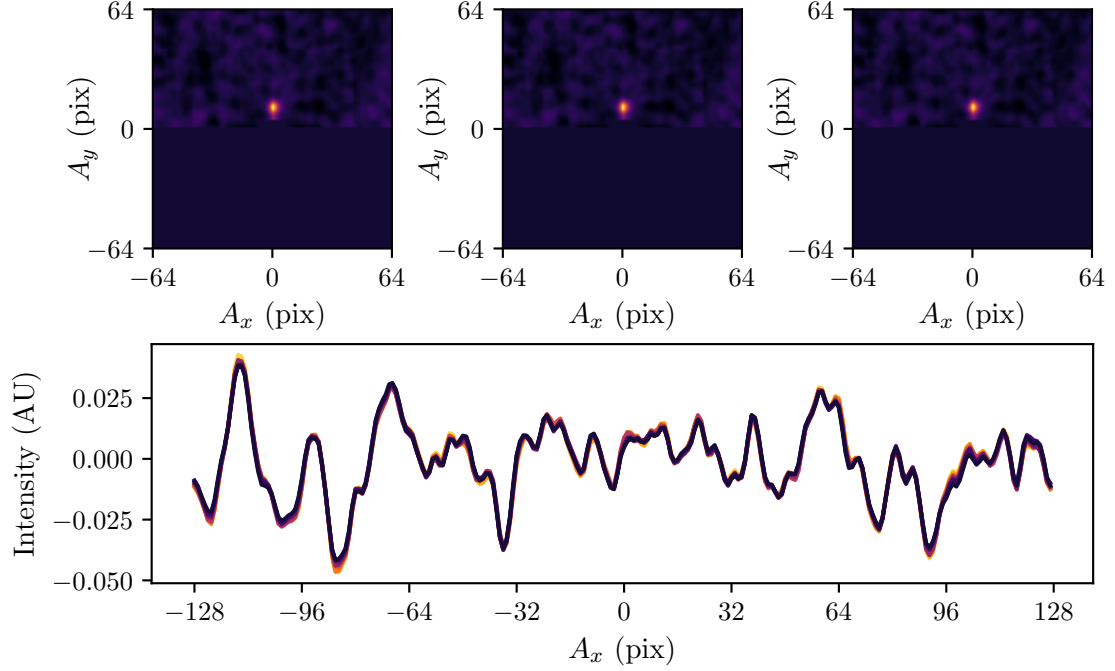


Figure 7.16: Top: Three example masked correlation images (out of the 50 repeats) from the initial measurement position, $\phi = 0^\circ$, $\psi = 0^\circ$ ($\theta_x = 0$, $\theta_y = 0$). Bottom: Cross sections at $A_y = 32\text{pix}$ of nine different correlation images from the same position, including the three images above. Note: The images are cropped to $128 \times 128\text{pix}$, but the cross section shows the full 256pix width.

Looking first at the standard deviations, there are two possible reasons the measured values are much lower than expected. The first is that the 50 repeat measurements are each at the same spot on the surface, and so the generated speckle patterns should be identical. This means that any error on the peak finding for the correlation images will be the same for all repeats, because the correlation image, and its noise distribution, is the same. This is shown in Figure 7.16, which shows three correlation images taken randomly from the 50 repeats of the first measurement at $\phi = 0^\circ$, $\psi = 0^\circ$ ($\theta_x = 0$, $\theta_y = 0$). The lower part of the figure shows cross sections of nine correlation images, including the three above, at $A_y = 32$ pixels. The cross-section in particular shows that the background correlation noise of each image, which is the main factor affecting the uncertainty in the peak fitting, is almost exactly the same.

As an aside, if the DWS technique were applied to measure the changing tilt in a fixed configuration (such as the tilt from the back surface of a mirror or moving

part), the bias errors introduced by the correlation noise would also remain fixed and so could be calibrated out.

In order to test the uncertainty of the correlation peak fitting, the speckle patterns would have to be randomised, which would mean moving the spot on the surface. This is difficult to accomplish with the limited tilting range of the Hexapod and the sensor mounted at a fixed working distance, but could be accomplished in future work with a dedicated setup.

The second probable reason for the lower measured standard deviation is that the expected uncertainties come from a long-term measurement of the wavelength stability - almost six hours. The angle measurements were taken over a much shorter time period, taking around 15 minutes for each arm of Figure 7.14 and 2 hours in total. Looking at Figure 7.5, the wavelength changes occur gradually, and the variation over a 15 minute time period can be quite small.

During data collection, the laser spectra were recorded alongside the detector images for monitoring purposes. Using these, the change in wavelength over the 50 repeat measurements at each angle can be determined. Figures 7.17 and 7.18 show the measured angles in x and y along with the laser wavelengths λ_1 and $\Delta\lambda$ for the two tilt magnitudes 2.5° and 3.0° along $\phi = 315.0^\circ$. Figure 7.15 shows that $\psi = 2.5^\circ$ has the highest standard deviation in θ_x (σ_x) for this measurement set at 0.068° , compared to 0.024° for $\psi = 3.0^\circ$. The values of σ_y are 0.059° for $\psi = 2.5$ and 0.024° for $\psi = 3.0$.

Looking at Figure 7.17 first, the standard deviations in λ_1 and $\Delta\lambda$ across this smaller time period are 0.72pm and 0.77pm . As a reminder, the values obtained from the long-term stability measurements were 2.0pm and 3.0pm respectively. These values can be used to calculate the expected uncertainties at this particular angle for the laser instability only. For the original values of 2.0pm and 3.0pm , the expected uncertainties were $\sigma_x = 0.011^\circ$ and $\sigma_y = 0.135^\circ$. For the new values, covering only these measurements, the expected uncertainties are $\sigma_x = 0.003^\circ$ and $\sigma_y = 0.034^\circ$, a full order of magnitude smaller and more in line with the measured results of 0.068° in x and 0.059° in y .

The values for $\psi = 3.0$ (Figure 7.18) show a similar result. Here, the standard deviations in λ_1 and $\Delta\lambda$ over the 50 measurements are 0.26pm and 0.28pm , giving estimated uncertainties of $\sigma_x = 0.001^\circ$ and $\sigma_y = 0.013^\circ$. For the original (larger)

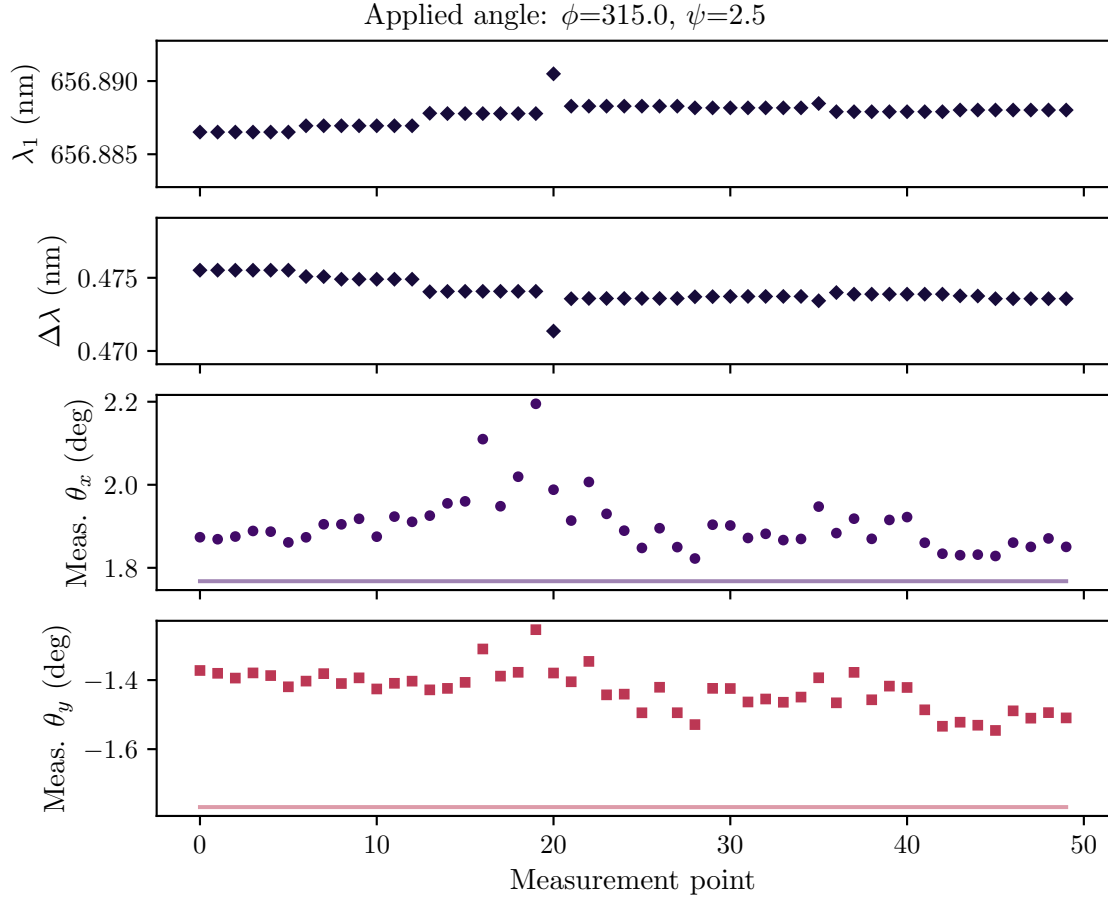


Figure 7.17: 50 repeat measurements at the applied angle $\phi = 315^\circ$, $\psi = 2.5^\circ$. The two upper graphs show λ_1 and $\Delta\lambda$ at the corresponding datapoint, while the lower graphs show the measured (Meas.) values of θ_x and θ_y , as compared to the expected value (solid line).

values these estimates were $\sigma_x = 0.013^\circ$ and $\sigma_y = 0.135^\circ$, and the measured values were 0.024° in both x and y .

This all suggests that for the full measurement range, the standard deviation in the bias errors may be more comparable to the expected error values. Though the error at each angle is roughly constant across repeats, across surface tilts there will be more long-term laser drift, and a change in the speckle patterns at each angle. Looking initially at the series of angles shown in Figure 7.15, the standard deviations in the bias errors χ_x and χ_y are $\sigma_{\chi_x} = 0.106^\circ$ and $\sigma_{\chi_y} = 0.173^\circ$. These can be compared to the expected uncertainty ranges using the long-term laser measurements, which are 0.222° - 0.269° in θ_x and 0.322° - 0.415° in θ_y .

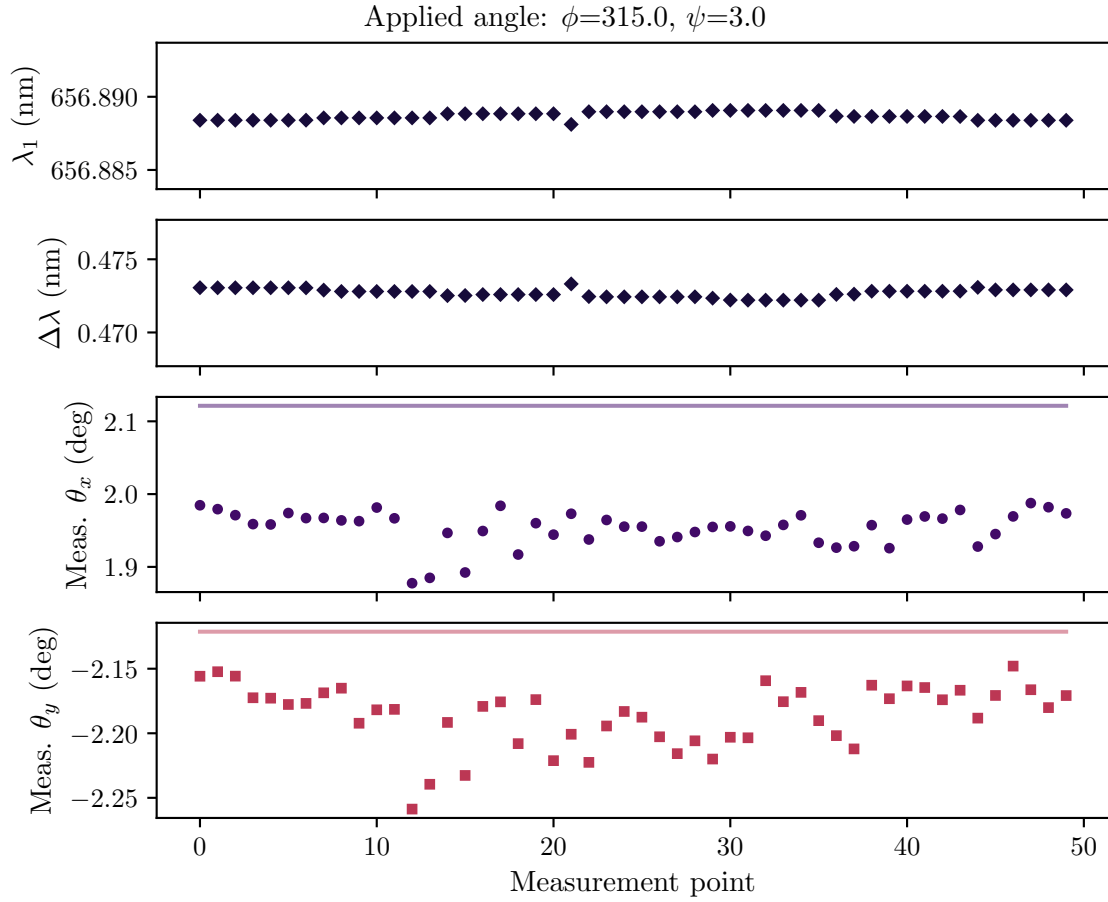


Figure 7.18: 50 repeat measurements at the applied angle $\phi = 315^\circ$, $\psi = 3.0^\circ$. The two upper graphs show λ_1 and $\Delta\lambda$ at the corresponding datapoint, while the lower graphs show the measured (Meas.) values of θ_x and θ_y , as compared to the expected value (solid line).

The values of σ_{χ_y} and σ_{χ_x} are very reasonable compared to these expected values, with the variation in θ_y being larger than that in θ_x as expected. Despite this, the measured standard deviations are still smaller than the lower limits of the expected uncertainties. Given that the long-term laser stability has been taken into account, this suggests that the errors in the image processing are being overestimated. The uncertainty in the correlation peak finding was estimated at 0.05-0.10 pixels [5], with the upper limit being used for the error analysis above in Section 7.2.2. Using the lower limit of 0.05 pixels, combined with the long-term wavelength stability, the expected range of uncertainties would now be 0.111° - 0.158° in θ_x and 0.211° - 0.304° in θ_y . This is now much more in line with the measured values, and demonstrates that the sensor is operating close to, if not better than, best estimates.

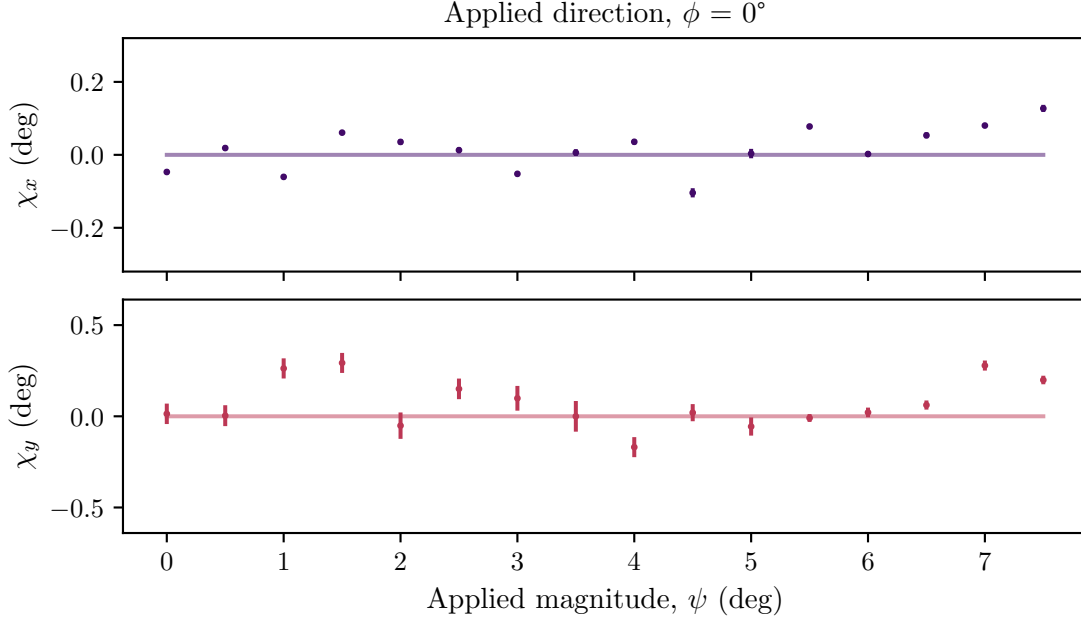


Figure 7.19: Results of the 2D angle measurements for the tilt direction $\phi = 0^\circ$. The two graphs show the bias errors and standard deviations on the 50 repeat measurements in θ_x and θ_y .

Looking again at the broader dataset, Figures 7.19, 7.20, and 7.21 show the bias errors and standard deviations for the directions 0° , 90° , and 225° . As in Figure 7.15, the standard deviations on the repeat measurements are much smaller than the values of χ_x and χ_y . For example, the direction $\phi = 0^\circ$ (Figure 7.19) has an average standard deviation of 0.008° in θ_x and 0.049° in θ_y , while the average magnitude of the bias errors, χ , in each direction are 0.049° and 0.105° .

Applying the same analysis described above, the standard deviations on the bias errors for these three directions were obtained, along with the same values for each of the other tilt directions. These are given in Table 7.4, plus the expected ranges for the two different values of the peak fitting uncertainty from the image processing for comparison. Each series follows the same trend, with the errors being higher in θ_y than in θ_x (as expected), and with the values being closer to those calculated from the 0.05 pixels error value than the original 0.10 pixels estimate.

The values presented in Table 7.4 are also approaching the target value of 0.1° resolution, performing better than the more conservative estimates described in Section 7.2.2. In addition, using the revised value of 0.05 pixels for the uncertainty during image processing leads to an improved theoretical sensor resolution of 0.111° .

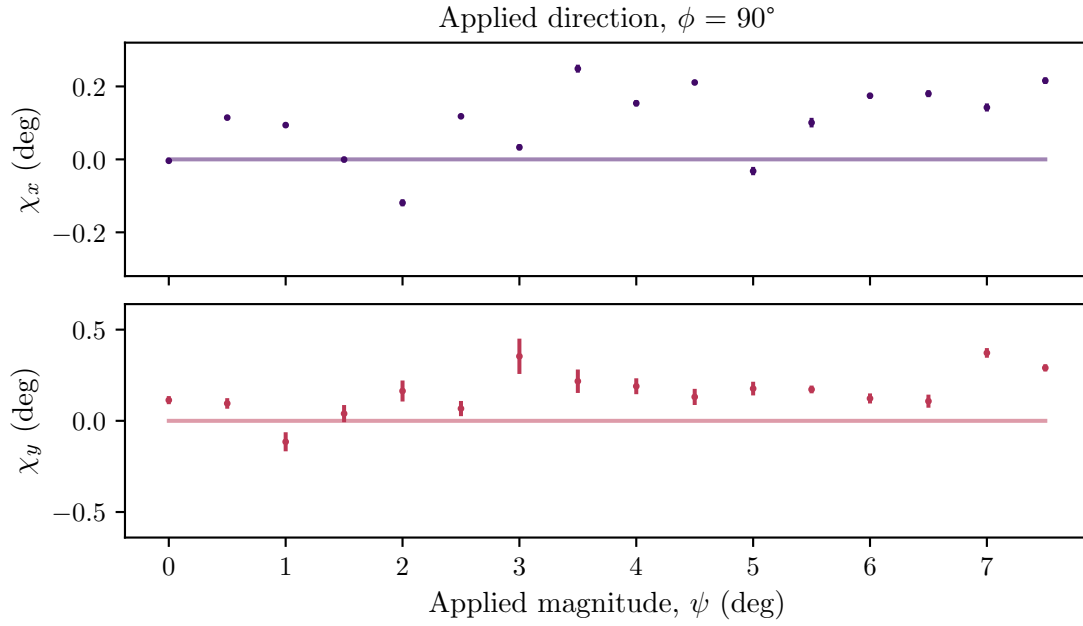


Figure 7.20: Results of the 2D angle measurements for the tilt direction $\phi = 90^\circ$. The two graphs show the bias errors and standard deviations on the 50 repeat measurements in θ_x and θ_y .

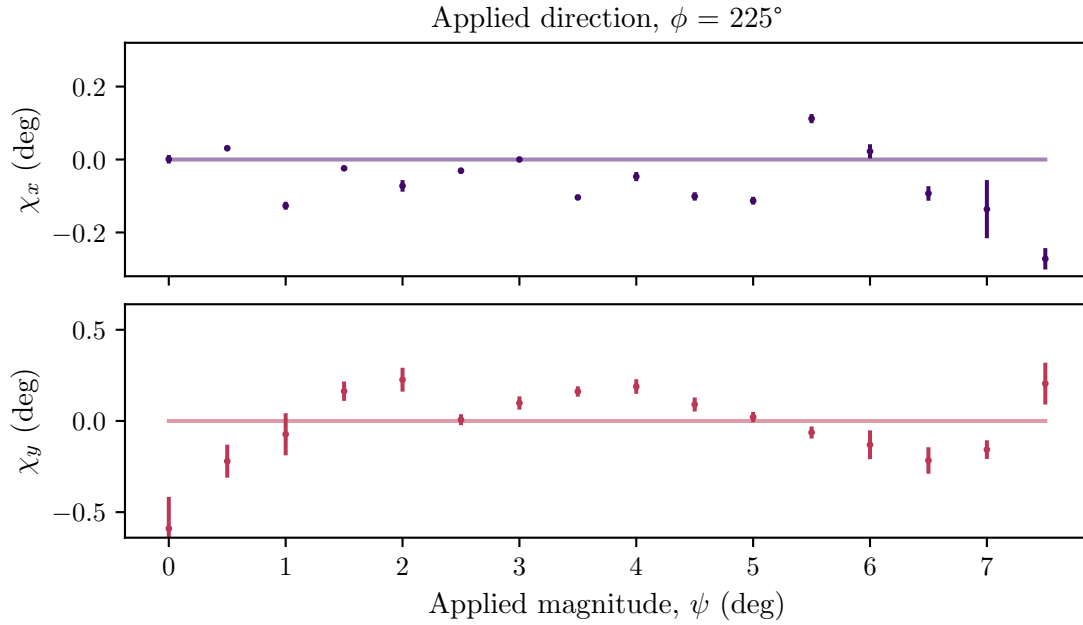


Figure 7.21: Results of the 2D angle measurements for the tilt direction $\phi = 225^\circ$. The two graphs show the bias errors and standard deviations on the the 50 repeat measurements in θ_x and θ_y .

Table 7.4: Top: Expected angle uncertainty ranges for the upper and lower limit of the expected error in the peak finding used during image processing. Bottom: The complete list of standard deviations on the bias errors for each of the eight measured tilt directions, plus the standard deviation across all measurements.

Expected image processing error (pix)	Expected angle uncertainty range (deg)	
	σ_x	σ_y
0.10	0.222 – 0.269	0.322 – 0.415
0.05	0.111 – 0.158	0.211 – 0.304
Angle measurement direction, ϕ (deg)	Standard deviation in bias errors (deg)	
	σ_{χ_x}	σ_{χ_y}
0	0.058	0.129
45	0.087	0.121
90	0.099	0.116
135	0.070	0.208
180	0.085	0.120
225	0.086	0.208
270	0.101	0.098
315	0.106	0.173
All	0.100	0.181

Overall, the angles measured by the sensor are in good agreement with the angles applied by the stage. The variation in measurements is as expected from the error analysis, though this is masked by the choice to take repeat measurements consecutively. The consecutive repeats produce constant speckle patterns, removing the ability to test the correlation peak fitting with randomised data, and demonstrate only the short term stability of the laser wavelengths. Over the longer time range of the different angle measurements, the variation in bias errors is much closer to the variation expected from the long-term wavelength instability of the laser. The data also suggest that, in this case at least, the smaller uncertainty of 0.05 pixels in the image processing step was more in line with measurements than the more

conservative original estimate of 0.10 pixels.

7.5 Results Summary and Conclusions

This sensor uses a single laser diode to generate two wavelength modes with a 0.46nm spacing. This resulted in a relatively low responsivity value of 0.124mm/rad, and an estimated resolution of 0.111°. This is close to the target resolution of 0.1°, and demonstrates a low cost approach to the technique using a single laser.

Two experiments were carried out with this sensor. The first was to investigate the performance over a large angle range ($\pm 45^\circ$) in θ_x only, and was undertaken before the calibration process was finalised. The second experiment focused on a smaller range of angles (up to 7.5°), but investigated the performance for 2D surface tilts, measuring both θ_x and θ_y .

The first conclusion that can be drawn from the 1D data is that there is an increase in the residual (bias) error at higher magnitude angles. The average absolute value of χ_x over the full measurement range was 0.510° , and this dropped to 0.215° for the smaller range $\pm 11.2^\circ$. This trend is possibly due to errors in the sensor geometry, as the data was performed without calibrating the sensor, but could also be due to the increased ellipticity of the spot and speckles. The image processing assumes the speckles, and therefore correlation peak, are circular, and so it is likely that the elliptical speckles are causing the increasing errors. Further research is necessary to establish this.

For this first experiment, no repeat measurements were taken, and so the uncertainty could not be determined directly. However an analysis using a rolling standard deviation suggests that the uncertainty increases at higher magnitude angles, as expected from the error propagation. For tilts between $\pm 22.0^\circ$, the average value of σ_{χ_x} was 0.113° . For the tilts of magnitudes larger than 22.0° , the average value was 0.131° .

It is also difficult to fully assess the measurement uncertainty from the 2D results, despite the 50 repeats at each angle. The repeat measurements were taken consecutively, over a much shorter time period than the variations found in the laser wavelength stability. An investigation into the angle uncertainty due to the laser instability would require that measurements be better distributed in time. Addi-

tionally, each repeat measurement was at the same point on the surface, leading to no variation in the speckle patterns. Ideally the pattern should be randomised, using a different section of the surface to change the speckle pattern, and thus better test the uncertainty in the peak fitting during image processing.

The calculated uncertainties from the experiment in two dimensions were therefore much lower than the expected values, reflecting the short time period in which repeat measurements were collected. The average standard deviations across the full measurement series were 0.012° in θ_x and 0.045° in θ_y , compared to the expected range of $0.222^\circ - 0.269^\circ$ in σ_x , and $0.322^\circ - 0.415^\circ$ in σ_y .

As the measurements of different surface angles were distributed over a larger time period, the standard deviation in the bias errors across all angles was calculated. The standard deviations in θ_x and θ_y , σ_{χ_x} and σ_{χ_y} , were 0.100° and 0.181° respectively. These values were determined to be more representative of the uncertainty in angle measurements.

Using these uncertainty measurements in σ_{χ_x} and σ_{χ_y} , the final conclusion that can be drawn from the second experiment is that the 0.10pix value used for the uncertainty in image processing is too conservative. The smaller value of 0.05 pixels was found to match more closely with the measured uncertainties, leading to the improved sensor resolution estimate of 0.111° .

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Chapter 8

Dual Laser Angle Sensor with Large $\Delta\lambda$ and Additional Image Processing

This chapter presents a second implementation of a dual-wavelength speckle (DWS) angle sensor. This version has a greater responsivity and improved resolution, though required some modifications to the initial signal processing. An overview of this sensor version is given in Section 8.1, including a brief comparison with the previous version.

Then, the characterisation of the lasers and construction of the complete sensor are described in Section 8.2. The difference in signal processing method is also described briefly in this section, though this is covered in more detail in Chapter 5, Section 5.1.

Experimental results are given in Section 8.3. This implementation of the sensor was used to measure a regular grid of combined θ_x and θ_y angles in 2D. The range of angle magnitudes measured was much smaller than in the previous chapter, with a focus on investigating the resolution of the sensor.

Finally, the results are summarised and compared with those from the previous implementation in Section 8.4. The advantages and disadvantages of each version are also summarised.

8.1 Sensor Description

The main aim of this implementation of the sensor was to greatly increase the resolution, namely by increasing the wavelength separation $\Delta\lambda$. As such, two separate lasers were used for this sensor, but still in the red region of the spectrum but giving a wavelength separation of 21.2nm. Each laser was running in a single-mode configuration, and their outputs combined via a fibre coupler.

The lasers used in this sensor were the second and third described in Section 5.2; both are FiberTecII Fabry-Pérot diodes from Blue Sky Research [1]. One laser was centred at 658nm (model FTEC2-658-60), referred to throughout this chapter as ‘laser 1’, the other was centred at 638nm (model FTEC2-638-20), referred to as ‘laser 2’.

The optical setup is given in Figure 8.1. The arrangement is similar to that used for the previous sensor, but with the additional laser on the second coupler input. The coupler is the same 50:50 wideband coupler at $630 \pm 50\text{nm}$ (Thorlabs part number TW630R5A2). The first output arm is used by the Yokogawa optical spectrum analyser (OSA) [2] to monitor the lasers’ spectra, while the second leads to the

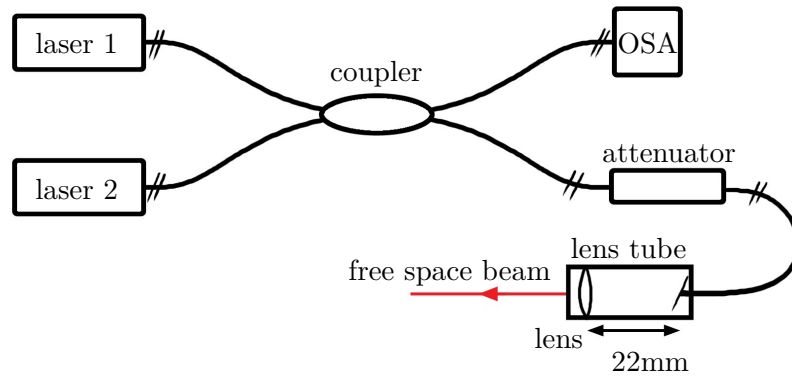


Figure 8.1: Diagram of the laser setup, showing the fibre-coupled Fabry-Pérot diode and the fibre coupler used to split the light between the OSA for monitoring and the lens tube in the sensor head. The lens tube contains a 15mm focal length lens placed 22mm from the fibre output. The fibre connectors (shown by the dashes) are all APC.

sensor head via the variable beam blocking attenuator. Although the operating bandwidth for the attenuator was not specified by the manufacturer (OZ Optics, part number BB-500-11-650-4/125-P-50-3A3A-3-0.5), it was found experimentally that it was suitable for both lasers used here. The attenuator was used to ensure the output light from the sensor fell below 4mW. Each of the connections are via angled physical contact (APC), with polarisation maintaining fibres. The configuration of the lens tube for this sensor used a 15mm focal length lens at 22mm from the fibre tip, bringing the beam to a 1.8mm spot size on the surface.

The smaller spot size (as compared to the previous sensor) was found to be necessary due to the increase in decorrelation caused by the wider wavelength separation. As discussed in Chapter 6, Section 6.2, a decrease in spot size results in an increase in correlation peak height, offsetting the decrease in peak height due to the wavelength separation.

The sensor camera was once again the Ximea MQ013MG-ON [3]. As with previous experiments, the camera was running with an exposure time of 1ms at 100fps, but with images captured individually when the stage was in position. Image processing was carried out in parallel to data acquisition, using Python's threading module. This approach was particularly useful for this series of experiments, as the data acquisition was especially time consuming. Additionally, the $> 20\text{nm}$ separation of the two wavelengths necessitated the use of the improved signal processing, incorporating the magnification step described in Section 5.1.

The large increase in $\Delta\lambda$ resulted in a correspondingly large increase in the theoretical range and responsivity of the sensor. So much so that these values had to be decreased by adjusting the geometry. For the same geometry used in Chapter 7 (sensor at $\mathbf{S} = (103.5, 0.0, 103.5)\text{mm}$, detector at $\mathbf{D} = (0.0, 0.0, 103.5)\text{mm}$), these source wavelengths would result in A_{x0} being equal to 504 pixels. This is beyond the assumed one quarter image limit [4], and close to the edge of the 1280 pixel width of the camera sensor.

To balance this increase, the separation between the detector and source was roughly halved from $S_x = 103.5\text{mm}$ to $S_x = 49.0\text{mm}$. Even with this adjustment, the expected responsivity was still 40 times that of the first, at $C = 4.925\text{mm/rad}$ versus the first sensor's $C = 0.124\text{mm/rad}$. In addition, the smaller separation allows for a smaller, more practical sensor head.

The other major difference with this sensor compared to the previous implementation is the inclusion of the extra magnification step in the image processing. The two speckle patterns produced by the source wavelength at the detector have a different relative magnification, as well as their positional speckle shift. The previous sensor version did not need to account for this, but this new version does. As a result, the image processing requires an additional step wherein the acquired frame, containing the two speckle patterns, is magnified and then correlated with the original, instead of performing a simple autocorrelation. This difference in processing is explained in full in Chapter 5, Section 5.1.

In terms of the correlation image, this results in an image with a blurred central correlation peak and a single, sharper side peak. This requires a larger mask for the central peak, reducing the range limit $\theta_{\max} = |\theta_{y,\min}|$ further as compared to θ_R . A detailed description of the various range limits can be found in Chapter 4, Section 4.3.2.

8.2 Sensor Construction

8.2.1 Geometry and Expected Performance

This sensor uses an improved metal sandwich head to mount the camera and lens tube, shown in Figure 8.2. The fully 3D printed sensor head was found to warp slightly due to the heat emitted by the camera, and so the new mount uses two metal plates to sandwich the camera and a 3D printed rotating mount for the lens tube together. Once again, the 3D printed elements were made from polylactic acid (PLA), using an Ultimaker 2+ printer [5].

The metal plates both help to disperse heat from the camera, and are more resistant to warping than the 3D printed plastic. This version of the sensor head is also more flexible in terms of the sensor geometry; as shown in Figure 8.2 the metal plates have mounting holes along the full length, allowing the camera and lens tube to be repositioned without reprinting any components.

As discussed above, the large increase in $\Delta\lambda$ resulted in a corresponding increase in C and A_{x0} , which was mitigated by altering the geometry. Thus the separation between source and detector (S_x) was set to the smallest allowed by the sandwich mount, 49mm. The working distance, L_D , was also reduced to 80mm; both of these

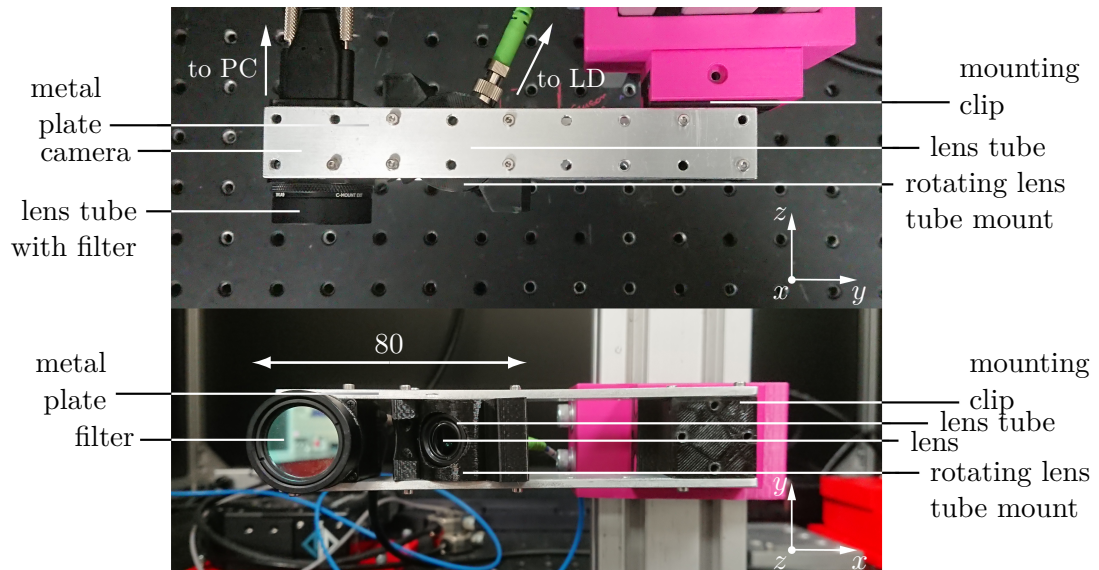


Figure 8.2: Photos of the metal sandwich sensor head, from above (top) and from the front (bottom). The various components are labelled, and the width of the part containing the camera and lens tube is given in mm.

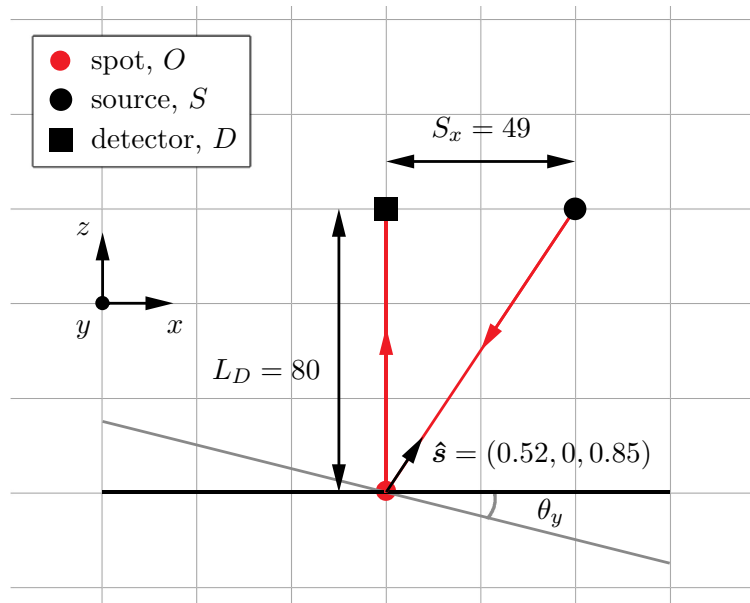


Figure 8.3: Diagram of the sensor geometry. The distances S_x (between source and detector) and L_D (between detector and spot) are given in mm. $\hat{s}$ is the unit vector from the spot to the source. An example surface tilt around the y axis of θ_y is shown in grey.

Table 8.1: Maximum measurable surface tilts for positive and negative changes in both θ_x and θ_y . The absolute value of the angle $\theta_{y,\min}$ is taken as $\theta_{\max}$, as this is the smallest value.

Maximum ranges	Expected value (deg)
θ_R	15.74
$\theta_{y,\min}$	-15.23
$\theta_{y,\max}$	+90.00
$\theta_{x,\min}$	-90.00
$\theta_{x,\max}$	+90.00
$\theta_{\max}$	15.23

values can be seen in the diagram in Figure 8.3.

Due to the geometry adjustments, the expected range of this sensor was similar to the previous version, at $\theta_{\max} = 15.23^\circ$. This value was also calculated using the more conservative size of 10 pixels for the mask on the central correlation peak. As this predicted range falls outside of the range of the Hexapod stage used to test this sensor, the mask could cover the whole of the blurred central peak without interfering with the sensor range. As with the previous sensor implementation, the maximum range in each of the other directions (summarised in Table 8.1) is 90° . However, these values only take into account the measurable range of the speckle shift given the correlation size, ignoring other factors such as distortion of the spot or speckle decorrelation.

The theoretical resolution of this sensor, calculated using equation (4.3.23) in Section 4.3.3 is significantly higher than the previous implementation, with a value of $r_{\text{sens}} = 0.003^\circ$. The sensor resolution and the measurement uncertainty introduced by the signal processing both relate closely to the 2D peak finding for the correlation peak, which has an estimated uncertainty of 0.05 pixels [6]. This is the lower end of the expected uncertainty range, as the results from the previous chapter found that using 0.10pix overestimated the uncertainty. The 0.05pix value is used in both calculations (sensor resolution and measurement uncertainty), and so the two values are very similar. The expected uncertainty in angle measurements from the signal

Table 8.2: Summary of the expected performance of this sensor implementation, namely the theoretical range, resolution, and responsivity.

Sensor characteristic	Expected value
Range, $\theta_{\max}$	15.2°
Resolution, r_{sens}	0.003°
Responsivity, C	4.925mm/rad

processing (using the equations in Section 5.1.2) is $\sigma = 2.806\text{mdeg}$ for both θ_x and θ_y .

The final important characteristic of the sensor is its responsivity, given by the calibration constant C . This implementation of the sensor has a much higher responsivity than the previous, which had a value of 0.124mm/rad. The expected value of the responsivity is 4.925mm/rad, calculated using the assumed geometry of the sensor. The responsivity, range, and resolution of the sensor are summarised in Table 8.2.

8.2.2 Laser Characterisation

The two lasers used in this version of the sensor were investigated in tandem using the Yokogawa OSA [2]. This meant that the conditions would be as similar as possible to the full sensor setup, and so any possible reflections/interference between the lasers would be seen in the characterisation measurements. The disadvantage of this approach is the time taken by the OSA to scan across the wavelength range. The OSA has to scan the full >20nm range, and so the time scanning the region between the two lasers is essentially wasted. To compensate for this, the settings used during this data collection prioritised speed over resolution.

The resolution of the OSA was set to 0.05nm, with a wavelength span of 24nm. Each scan took around 0.4s to complete, though most of this time was spent scanning the empty region between 638nm and 658nm. Spectra were collected every 0.5s, taking the most recent values for each wavelength. 10,000 spectra total were recorded, taking around 2.5hrs.

The first graph of Figure 8.4 shows all 10,000 spectra recorded during the stability

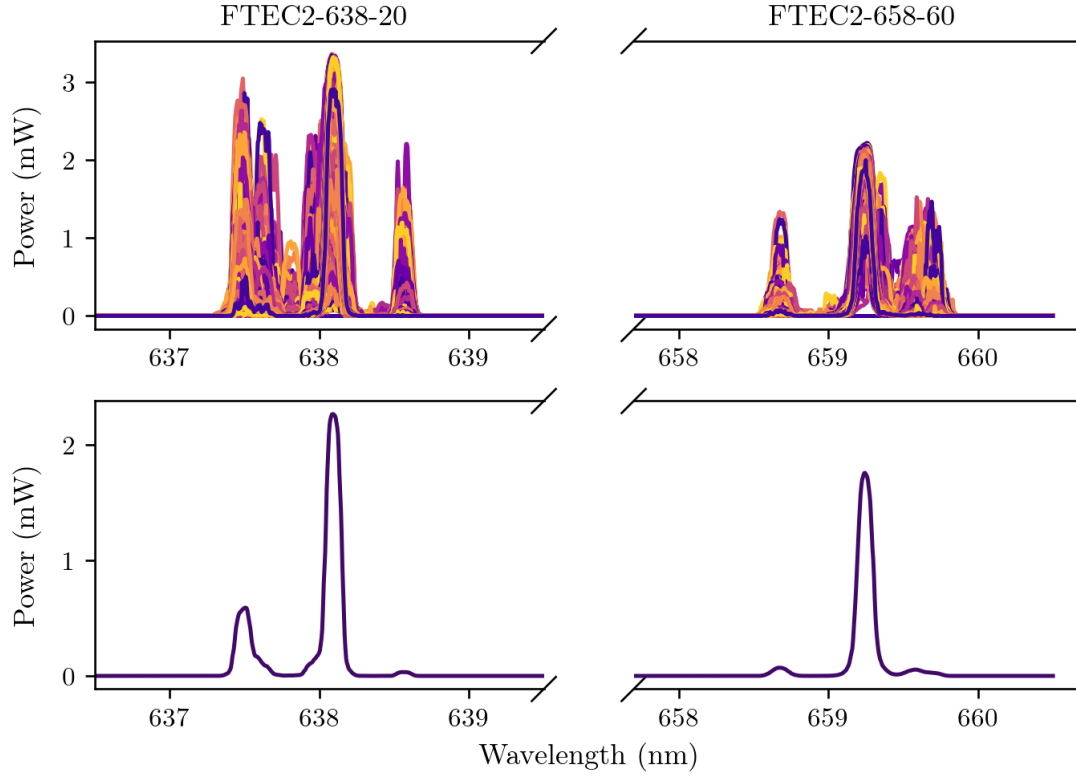


Figure 8.4: Spectra from the FTEC2-638-20 (laser 1, left) and FTEC2-658-60 (laser 2, right) laser diodes, recorded every 0.5s over roughly 2.5hrs. The first graph shows all 10,000 spectra overlaid, and the second shows the average of these spectra. The expected wavelength modes are at 638.1nm and 659.2nm.

experiment. The overlapping peaks give an idea of how many laser modes there are and their power (mode power stability), as well as their wavelength stability. The central mode in each laser is at the desired wavelength, but each laser has a parasitic mode on each side of the main peak. The second graph of Figure 8.4 shows the average of the 10,000 spectra. This view of the data shows that the parasitic modes are appearing much less frequently than the main modes.

These spectra were processed using the peak fitting method described in Chapter 5, and the results are presented in Figure 8.5. The colour of the points shows the power of the individual modes, and so it can easily be seen that the power of the central mode in each laser is consistently higher than either of the parasitic modes. These data were additionally sorted by mode, separating them into bins with the following bounds: [637.0, 637.8, 638.3, 648.0, 659.0, 659.4, 660.2]. The mean wavelength and power of each bin (mode) are given in Table 8.3, where each mode has been labelled.

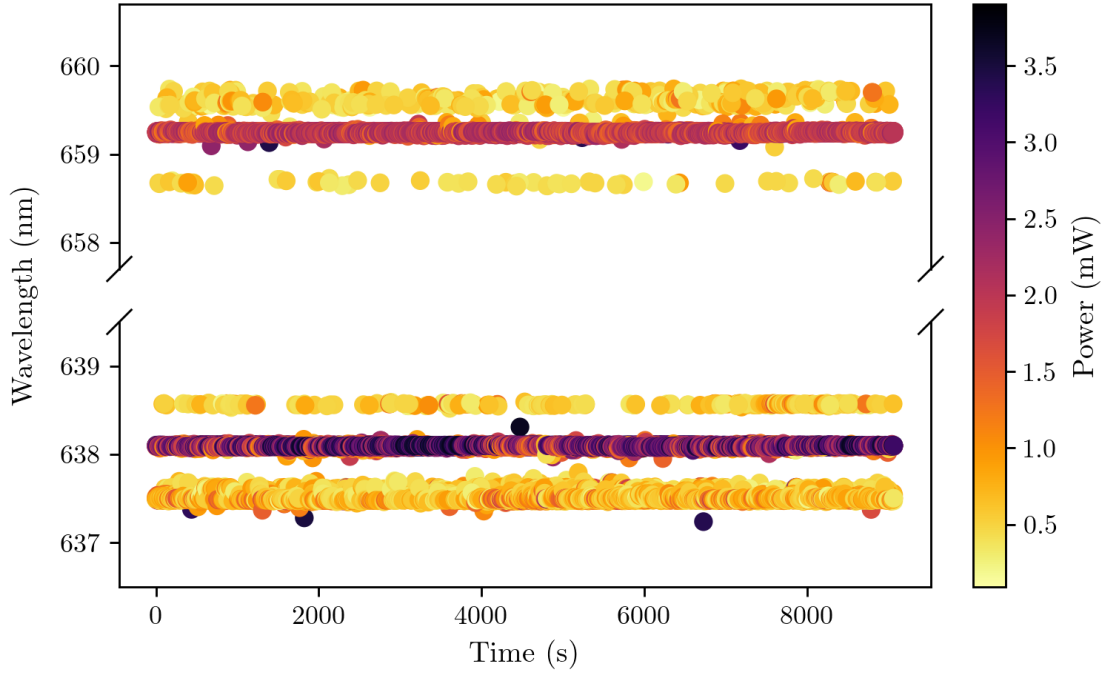


Figure 8.5: Laser stability data for the FTEC2-638-20 (laser 1, bottom) and FTEC2-658-60 (laser 2, top) laser diodes, recorded every 0.5s over 2.5hrs. Each laser has three main modes, though not all are consistently present. The central modes are at 638.1nm and 659.2nm, and on average have a much higher power than any of the parasitic modes (see Table 8.3).

Table 8.3: The average wavelength (nm) and power (mW) of the six total modes present in the two laser diodes FTEC2-638-20 (laser 1) and FTEC2-658-60 (laser 2) across roughly 2.5hrs. The two central, expected modes (638.1nm and 659.2nm) have the highest powers by a significant margin.

Mode	Wavelength (nm)	Power (mW)
1a	637.51	0.601
1b	638.09	2.630
1c	638.56	0.018
2a	658.68	0.004
2b	659.25	2.012
2c	659.62	0.024

Mode Power Stability

The data in Table 8.3 confirms that the central mode in each laser (each labelled 1b and 2b) is dominant for the majority of the time these measurements were taken over. Though, as shown in Figure 8.5, there are datapoints in time where another mode is equal to or higher than the central mode. This mode power instability can be quantified by finding the number of spectra, from the total 10,000, that have a parasitic mode with a higher power than the central mode. For the 638nm laser, there were 463 spectra where mode 1a was dominant, and 26 with mode 1c dominant. The 658nm was much more stable in this regard, having only 4 spectra with mode 2a dominant, and 8 with mode 2c. As there was no overlap between the parasitic modes appearing in each laser, these values can simply be totalled, giving the results that 5.01% of the 10,000 spectra had a dominant parasitic mode.

Mode power stability is an important factor as a change in the dominant mode while the sensor is operating will cause a discrete jump in the measured angle. For a detailed description of this effect, refer back to Section 5.2, and see particularly Figure 5.11. The precise angle change depends on exactly which modes are dominant, as well as the surface tilt being measured. Speckle theory can be used to estimate the change in angle measurement for various combinations of the six modes. As surface tilt is also a factor in these calculations, angles across the experimental range of $\pm 0.25^\circ$ (see Section 8.3 below) were investigated.

The factor with the largest effect on measured angle is $\Delta\lambda$, so the combinations of modes can be grouped based on their rough $\Delta\lambda$. The parasitic modes described in Table 8.3 are around 0.5nm from the central mode, and so one laser switching to a parasitic mode would cause a corresponding change in $\Delta\lambda$ of around 0.5nm (either positive or negative). These combinations of modes are 1a/2b, 1c/2b, 1b/2a, and 1b/2c. For 2D surface tilts ranging between $\pm 0.25^\circ$ in x and y , the expected differences from the true angle, $\Delta\theta_x$ and $\Delta\theta_y$, were calculated for these mode combinations. The range of errors in angle measurements are plotted in Figure 8.6, with the expected result at zero in yellow, and the results for these four combinations in light orange.

For $\Delta\theta_x$, the expected range of errors is symmetric about zero, with significant overlap between each pair of modes. For the four combinations with a $\Delta\lambda$ change close to $\pm 0.5\text{nm}$, the error ranges are roughly between $\pm 0.006^\circ$. These ranges are

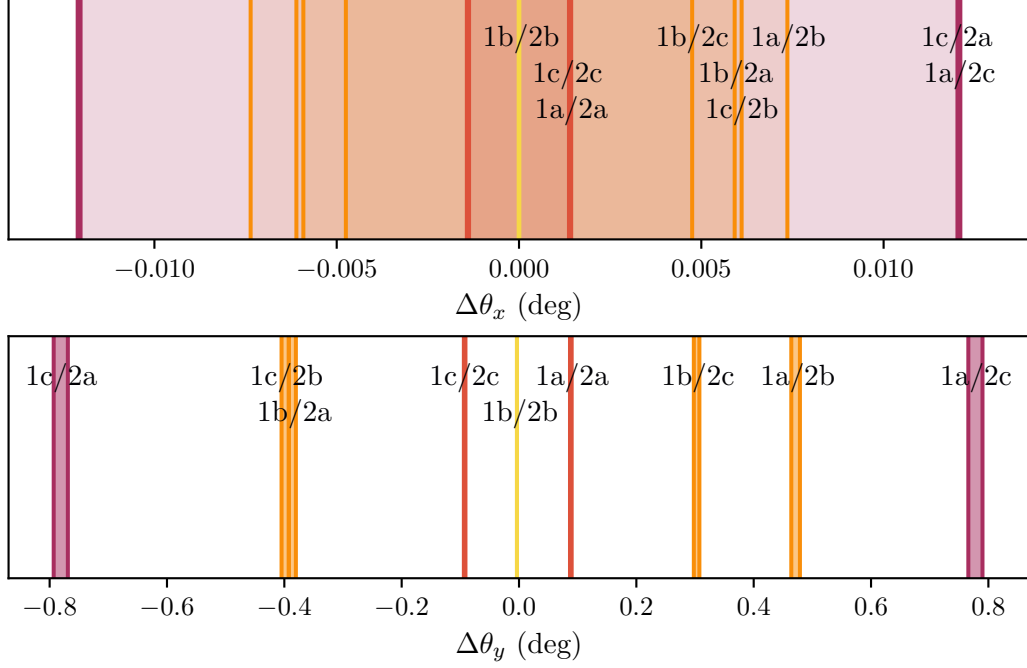


Figure 8.6: Expected changes in θ_x and θ_y , given by $\Delta\theta_x$ and $\Delta\theta_y$, for each combination of possible laser modes 1a, 1b, 1c, 2a, 2b, and 2c. For $\Delta\theta_x$, the error ranges are symmetric about zero, with the labels on the positive limit. For $\Delta\theta_y$, the ranges are distinct from each other, each covering a similar width to the corresponding band in $\Delta\theta_x$ but with a higher absolute value.

somewhat clearer for $\Delta\theta_y$ as the bands are distinct, though their width is similar to the ranges for θ_x . The ranges for θ_y are: -0.404° – -0.392° for the 1c/2b band, -0.392° – -0.380° for 1b/2a, 0.298° – 0.307° for 1b/2c, and 0.464° – 0.479° for 1a/2b. These values of $\Delta\theta_y$ are significant compared to the expected resolution of the sensor (0.003°), and so should be easy to spot in experimental data.

The second set of mode combinations are for a change of $\sim +0.5\text{nm}$ (or $\sim -0.5\text{nm}$) in both lasers. This will result in a similar value of $\Delta\lambda$ as expected, but even a small change will affect the measured angle, especially for this high-responsivity sensor. The predicted changes in θ_x and θ_y are shown in dark orange in Figure 8.6, the two regions closest to zero degrees. For $\Delta\theta_x$ the band falls between $\pm 0.0014^\circ$ for both mode pairs, and in $\Delta\theta_y$ the range is -0.094° – -0.091° for 1c/2c, and 0.087° – 0.090° for 1a/2a. Though these differences are much smaller than those outlined above, and more likely to fall within the random spread of the data, the laser stability measurements show that this scenario is highly unlikely; none of the 10,000 gathered

spectra exhibited this behaviour.

The final two combinations of modes are $1a/2c$ and $1c/2a$, leading to a change in $\Delta\lambda$ of $\sim +1.0\text{nm}$ and $\sim -1.0\text{nm}$ respectively. These have the largest change in $\Delta\lambda$ and as such have the highest expected deviation from the true angle, plus the largest spread, shown in dark red in Figure 8.6. $\Delta\theta_x$ is between $\sim \pm 0.0121^\circ$ and in $\Delta\theta_y$ the ranges are -0.793° – -0.769° for $1c/2a$ and 0.766° – 0.790° for $1a/2c$. These combinations of modes are both easy to spot (especially in θ_y) and highly unlikely to appear. As with the previous group of combinations, neither $1a/2c$ nor $1c/2a$ were present in the spectra collected during the laser stability testing.

Wavelength Stability

Having determined the mode power stability, the wavelength stabilities of the central modes need to be calculated. This is simply the standard deviation in the wavelength of the relevant modes. This value is used in the error propagation calculations, derived in Section 5.2.2, to determine the expected uncertainty in the finished sensor. The standard deviation of the two central modes across this dataset was 0.010nm for the 638nm laser, and 0.011nm for the 658nm laser. These result in expected errors of up to 0.011° in θ_x and 0.023° in θ_y for the full $\pm 15.2^\circ$ range of the sensor.

The final expected error curves using both the wavelength stability and peak finding uncertainty are shown in Figure 8.7a. The shape of the curves is determined by the uncertainty of the laser, while the uncertainty in signal processing adds a constant value of $\sigma = 2.806\text{mdeg}$ (shown by the dashed line). As discussed in Section 5.2, the curve in θ_x has a minimum at an applied angle of zero degrees, while the minimum for θ_y is offset at θ_R due to the geometry of the sensor. The minimum value for θ_x is $\sigma_x = 2.822\text{mdeg}$, and for θ_y is $\sigma_y = 3.207\text{mdeg}$. The maximum values, bounded by the maximum range of the sensor, are $\sigma_x = 13.956\text{mdeg}$ and $\sigma_y = 25.474\text{mdeg}$.

Figure 8.7b shows the expected error curves over the experimental range of $\pm 0.25^\circ$. This range was chosen to test the high resolution of this sensor, with angles spaced 0.05° apart. As the range is so small, the error curves are almost flat. The range of expected uncertainties in θ_x is $2.806 - 2.984\text{mdeg}$, and for θ_y is $14.179 - 14.536\text{mdeg}$.

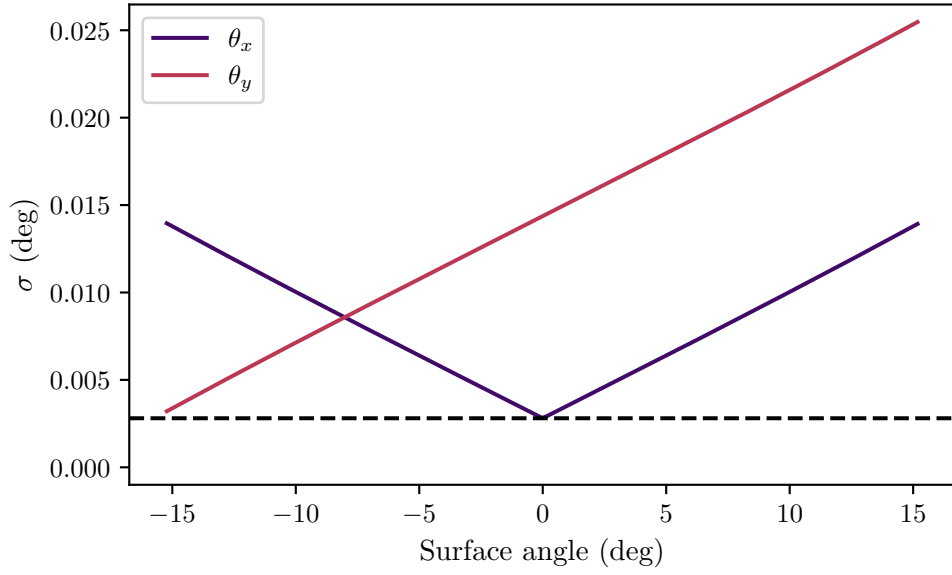
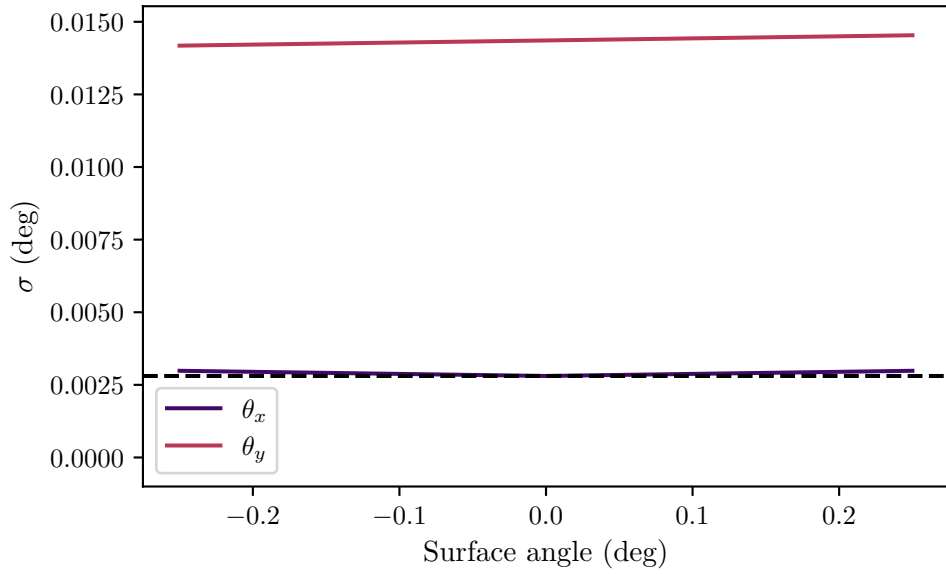
(a) $\pm 15.23^\circ$ range(b) $\pm 0.25^\circ$ range

Figure 8.7: Expected uncertainty, σ , in angle measurements for θ_x and θ_y across the theoretical range of the sensor $\theta_{\max} = \pm 15.23^\circ$ (a), and the experimental range $\pm 0.25^\circ$ (b). The graphs show the constant uncertainty from the signal processing (given by the dashed line) combined with the uncertainty curve from the wavelength instability. The difference between θ_x and θ_y is caused by the sensor geometry offsetting the source in the x direction.

8.2.3 Calibration

Initially, the calibration for this sensor was carried out using the printed calibration mounts, described in Chapter 5, with the Hexapod's rotation stage [7]. However, as this implementation has such a high responsivity, the human error involved in changing the working distance between the calibration and test setups became an important factor. The working distance of the sensor affects all calibration constants, but most strongly affects the responsivity. The intended working distance for the sensor was 80mm, giving an expected responsivity of 4.925mm/rad. This is measured using a ruler during setup, and a difference of 1mm from human error would result in a change of around ± 0.070 mm/rad in C , and a difference of around 0.15° in the calculated angle measurement.

Instead, the calibration was carried out using the tripod of the Hexapod stage,

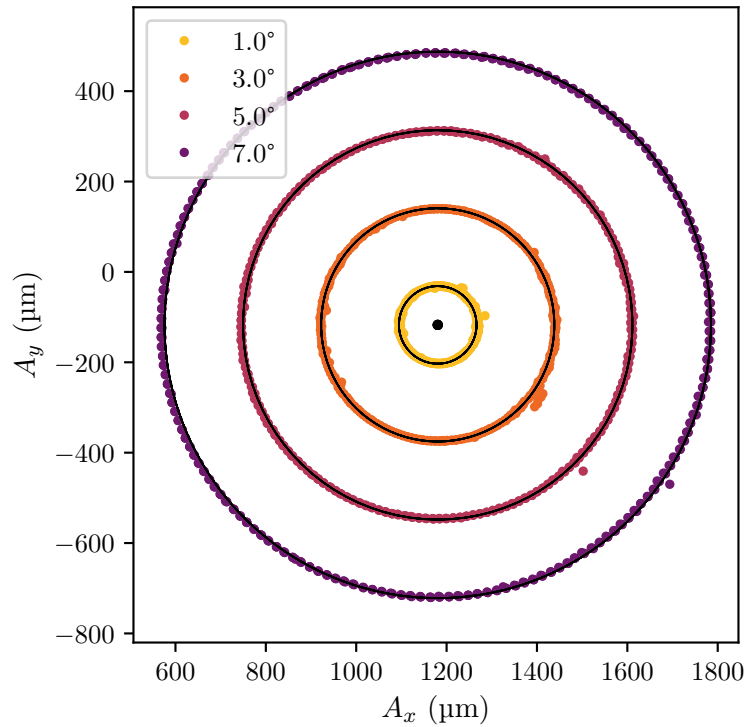


Figure 8.8: Calibration data, showing measured speckle shifts for applied angles of constant magnitude but varying direction. The angles were applied in direction increments of 2.0° , with three full rotations. The fitted circles and centre point are shown in black. For clarity, the magnitudes shown in the figure are 1.0° , 3.0° , 5.0° , and 7.0° , but data was also collected for 2.0° , 4.0° , and 6.0° .

Table 8.4: Expected and measured values of the three calibration constants: the offset in the speckle shift at zero tilt, $A_0 = (A_{x0}, A_{y0})$, and the responsivity constant, C .

Calibration constant	Expected value (mm)	Measured value (mm)
C	4.925	4.924
A_{x0}	1.388	1.180
A_{y0}	0.000	-0.117

though the only change is the method used to generate the calibration angles. The metal plate attached to the Hexapod is moved through a series of angles with a fixed magnitude and changing direction, and this is repeated for a number of different magnitudes. The direction increments were 2.0° , and a total of three full circles were applied at each magnitude. The magnitudes used were 1.0° , 2.0° , 3.0° , 4.0° , 5.0° , 6.0° , and 7.0° .

The calibration circles for the odd magnitude angles are given in Figure 8.8. The points show the measured speckle shifts in μm , and the solid black lines are the fitted circles. All circles are used in the fitting process, resulting in one final result (see Section 5.3 for more detail). The black point is the centre of the fit, i.e. (A_{x0}, A_{y0}) . The values of A_{x0} , A_{y0} are given in Table 8.4, along with the responsivity C , all in mm.

The differences in the expected and measured values of A_{x0} and A_{y0} are most likely due to a misalignment between the centre of the detector and the centre of rotation of the stage. As the difference in values is in the order of 0.1mm this would be very difficult to correct during setup, but is very easy to account for in the calibration, as demonstrated.

8.3 2D Angle Measurements

In order to test the sensor, it was mounted above the Hybrid Hexapod stage [7] (specifications given in Chapter 5), as shown in Figure 8.9. As per Chapter 6, the surface was Plate 2 from the measurements in Section 6.3. The tripod component of the Hexapod was used to tilt the aluminium surface, generating a ‘grid’ of tilt

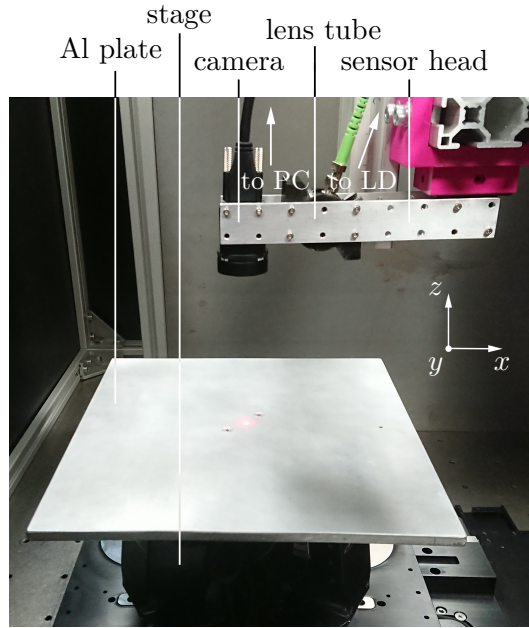


Figure 8.9: Photo of the setup used to take 2D angle measurements. The sensor head is mounted above the Hexapod stage, with the detector positioned above its centre. The tripod element of the stage can move the plate though 2D surface tilts in x and y . [LD: laser diode]

angles with components in both x and y . As this implementation should have a much higher resolution than the previous, the range of angles chosen was $\pm 0.25^\circ$, with a step size of 0.05° . The stage was paused for 0.5 seconds after each movement to ensure it had reached the applied angle and stopped moving. Once at each surface tilt, an image was taken using the camera, and the most recent spectrum was recorded from the Yokogawa OSA.

During data collection, the stage was moved through the full grid of angles for each repeat measurement, rather than taking repeat measurements with the stage stationary. This approach was a direct response to the 2D results from the previous chapter, where the consecutive repeats over a very short time period did not adequately demonstrate the repeatability of the sensor. A downside of this approach is the possibility of introducing stage repositioning errors, but this is also perhaps more accurate to a real-world application. Although the stage repeatability is not quoted in the specifications, the angular resolution of the tripod is 0.02arcsec , and its linear resolution is $5\mu\text{m}$, much smaller than the angles generated for the experiment.

The data presented here consist of 100 repeat measurements. These were collected

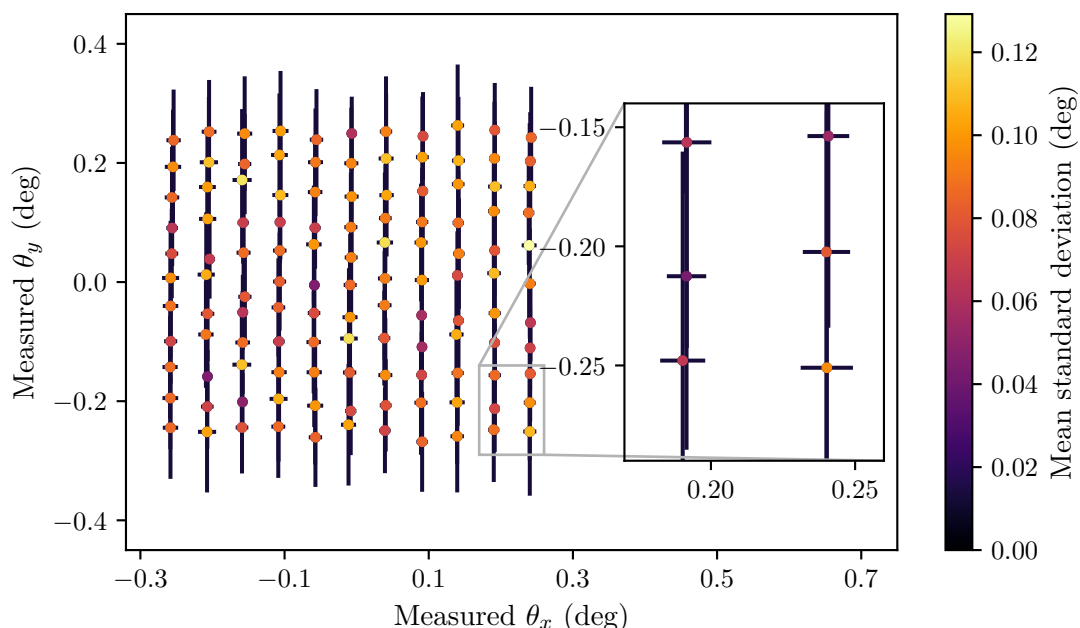


Figure 8.10: Angle measurements for a grid of surface tilts over the range $\pm 0.25^\circ$ in θ_x and θ_y , over 100 repeat measurements. The mean measured tilt in θ_x and θ_y is plotted, along with the standard deviations along each. The colour of the data points shows the mean of the two components of the standard deviation.

in two blocks of 50 across two different days, as one set of 50 measurements was found to take around 5.5 hours to collect. Initial processing of the data showed very high standard deviations across all measurement points - between two and ten times higher than the predicted errors of around 0.003° in θ_x and 0.014° in θ_y . This can be seen in Figure 8.10, which plots the mean angles in x and y over the 100 repeats, plus their standard deviations. The error bars showing standard deviation in θ_y (σ_y) are overlapping with those from other points. The highest value of σ_y is 0.129° , and the average across the grid is 0.088° . σ_x is much lower, however, with a maximum value of 0.013° and average of 0.009° .

The colour of the points in Figure 8.10 gives the standard deviation for each angle averaged in x and y , i.e. $\sigma = \sqrt{\sigma_x^2 + \sigma_y^2}$. This helps to show at a glance the spread and range of values, though is heavily dominated by the values in σ_y . The maximum and mean values of σ are the same as those for σ_y to the three decimal places quoted.

Despite this high standard deviation, the mean measured angle at each point was very close to the applied value. Figure 8.11 shows the bias errors in vector form,

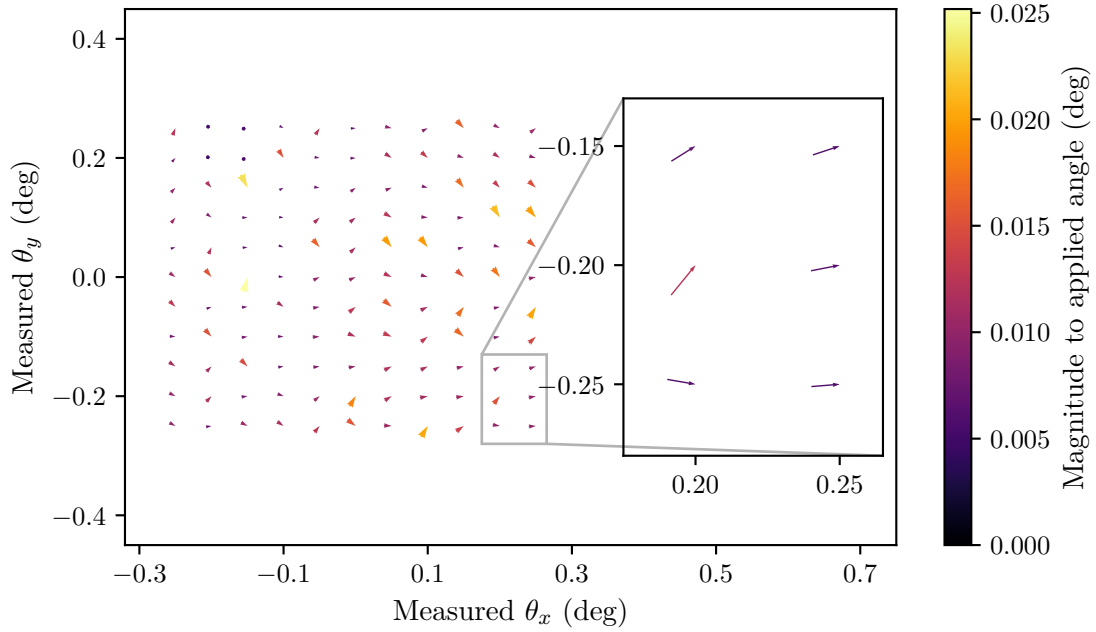


Figure 8.11: Angle measurements for a grid of surface tilts over the range $\pm 0.25^\circ$ in θ_x and θ_y , over 100 repeat measurements. The plot shows the vector from the mean measured angle to the applied angle at each tilt, with the magnitude of the vector given by its colour.

with the arrow pointing from the mean measured angle to the applied angle. Again, the colour of each point shows the mean bias error across x and y , or in this case the magnitude of the vector. Using the symbols χ_x and χ_y for the bias errors, the magnitude is equal to $\chi = \sqrt{\chi_x^2 + \chi_y^2}$. The largest magnitude error is around 0.025° , compared to the 0.129° maximum in the mean standard deviation, and the mean of these magnitudes is 0.011° .

Taking a closer look at the measurements before averaging demonstrates why the standard deviation is so high. Figure 8.12 shows the distribution of measurements across all repeats for the angle $(\theta_x, \theta_y) = (+0.25, -0.20)$. Most measurements are clustered around the expected (applied) angle, but many are scattered in the y direction by up to $\pm 0.4^\circ$. The mode power stability analysis above suggests that the extremes of this scatter are being caused by a parasitic mode becoming dominant in one of the lasers.

This trend was investigated for the full dataset by plotting a histogram of the bias errors across all individual measurements over all applied tilts. The result is given

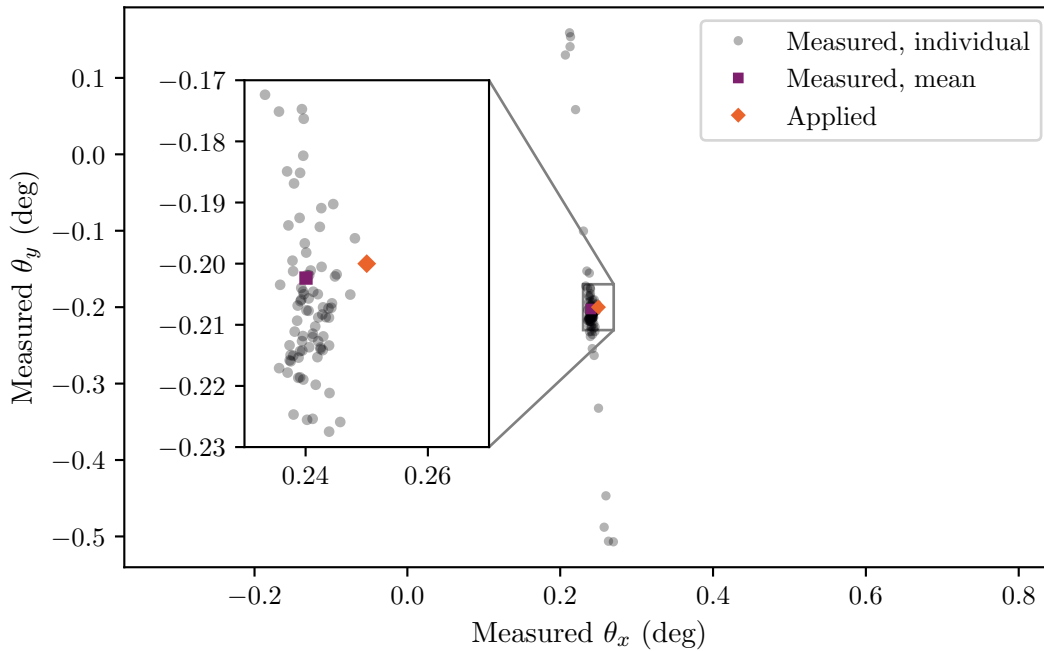


Figure 8.12: Measured angles for all 100 repeat measurements at a surface tilt of $\theta_x = +0.25$, $\theta_y = -0.20$. Also indicated are the mean of these results, $\theta_x = +0.240$, $\theta_y = -0.202$, and the applied angle.

in Figure 8.13. Most measurements are clustered close to zero in a roughly normal distribution, as would be expected. However there are two extra peaks of data on either side of the main peak, agreeing with the suggestion above that there is at least one parasitic mode causing discrete changes in the angle measurements.

This hypothesis is confirmed by looking at the spectra and cross-correlation images of specific points. Figure 8.14 shows images for three consecutive runs at a surface tilt of $\theta_x = +0.25$, $\theta_y = -0.20$, where the central run had a very large discrepancy between the applied and measured angles. The expected wavelengths of the two lasers were $\lambda_1 = 638.1\text{nm}$ and $\lambda_2 = 659.2\text{nm}$. The spectra from runs 38 and 40 each show a distinct peak at those values, with side peaks around a third or less than the height of the main peak. When run 39 was recorded, however, the first laser's highest peak was at around 637.5nm .

This is reflected in the correlation images, displayed on the right hand side of Figure 8.14. Runs 38 and 40 have a single correlation peak at $(882.08, 491.95)\text{pix}$ and $(882.07, 491.96)\text{pix}$ respectively, indicated by the black marker. These correspond

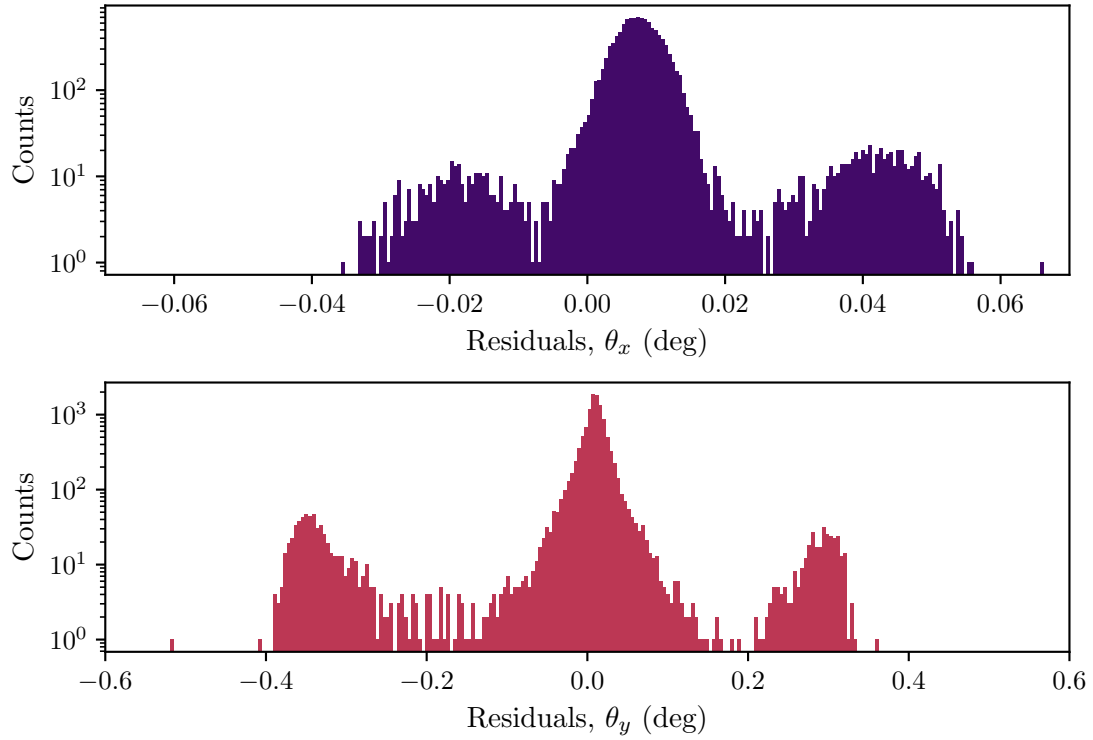


Figure 8.13: Histogram of residuals for individual measurements at all angles. The y axis uses a logarithmic scale to emphasise the two extra peaks of data on either side of the main peak around zero.

to angles of $(\theta_x = 0.243^\circ, \theta_y = -0.208^\circ)$ and $(\theta_x = 0.243^\circ, \theta_y = -0.209^\circ)$, close to the applied values of $(\theta_x = 0.25^\circ, \theta_y = -0.20^\circ)$. The correlation image for run 39 shows a strong correlation peak at a noticeably different A_x position, with a fainter peak at the same position as the other two images. The strong peak, located by the peak finding during processing, is at $(888.66, 491.44)\text{pix}$. This results in a measured angle of $(\theta_x = 0.212^\circ, \theta_y = 0.159^\circ)$, more than -0.3° away in θ_y from the applied value.

The experimental data included a measurement of the laser spectrum at each angle measurement, to be used for monitoring and error investigation on the data. This was used to filter out points where an unwanted mode was dominant in either of the lasers, with the aim of demonstrating the sensor performance for a more ideal laser stability. The spectra were analysed to find the power of all modes, assuming that each laser had a central peak at the desired wavelength, plus a side peak to the left and right. Not all peaks were present in every spectra, so the results were sorted

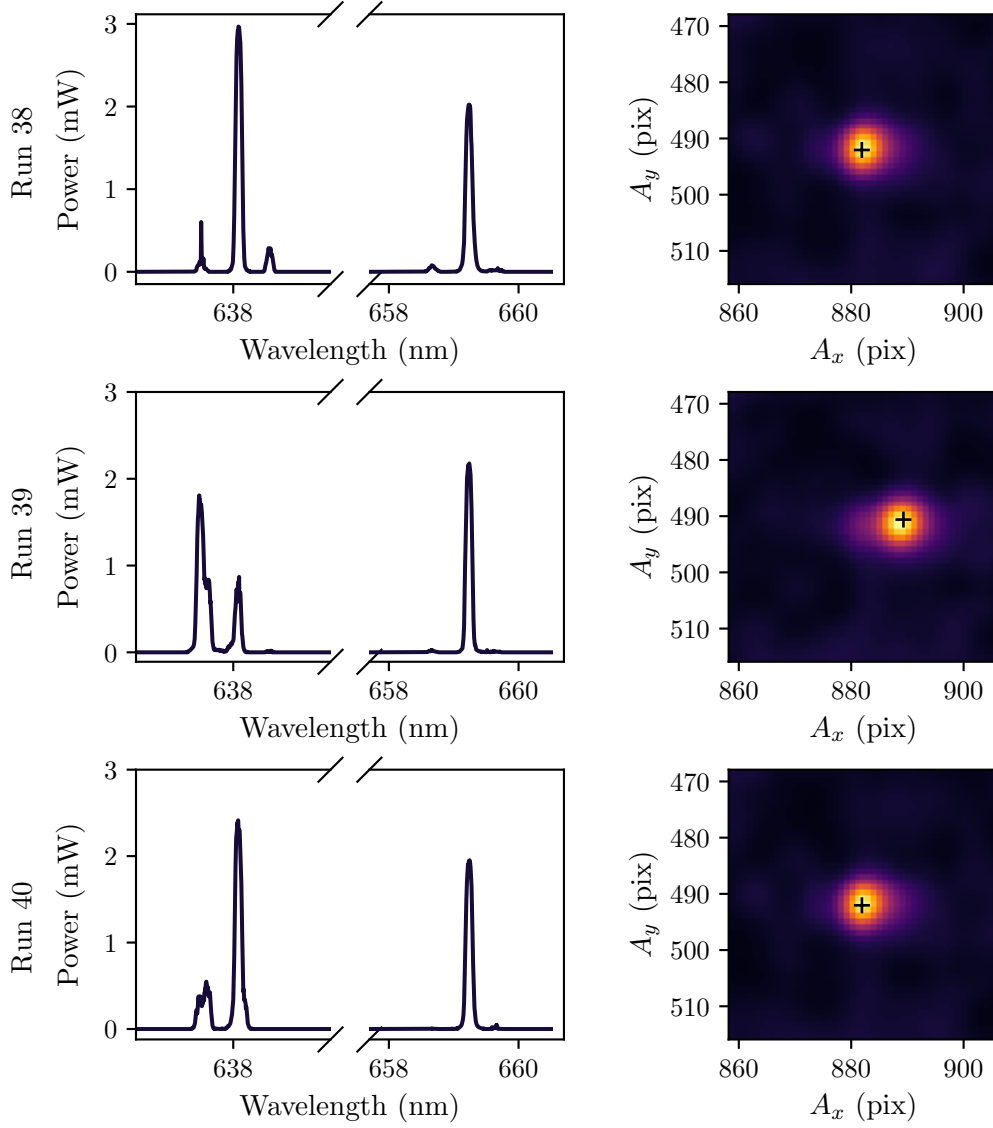


Figure 8.14: Spectra and corresponding correlation images for repeat measurements (runs) 38, 39, and 40. For run 39, the dominant modes in the laser output were not as expected, causing a large error in the angle measurement ($\theta_x = 0.212^\circ$, $\theta_y = 0.159^\circ$ versus an applied $\theta_x = +0.25$, $\theta_y = -0.20$).

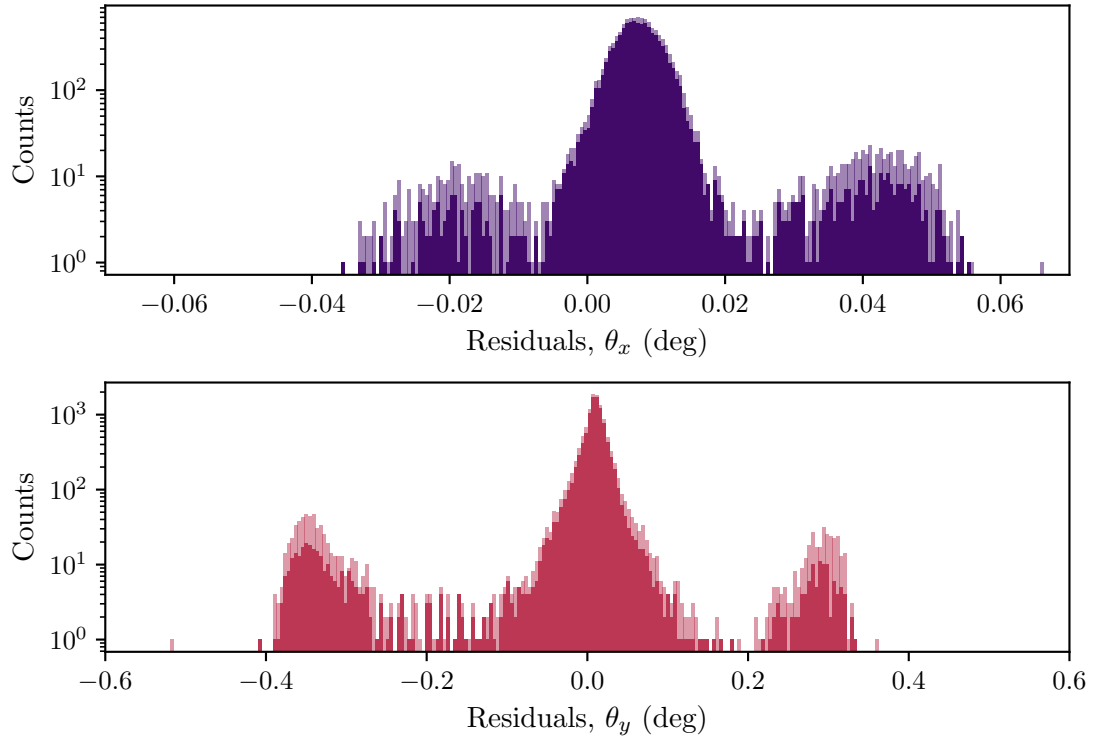


Figure 8.15: Histogram of residuals for individual measurements at all angles, both before (translucent bars) and after (opaque bars) filtering. The filter removed individual measurements where the powers of the expected laser modes were less than twice any extra side peaks. Note that the y axis uses a logarithmic scale.

such that modes that did not appear were given a power of 0. Finally, a filter was applied to discard measurements where any of the side peaks had a power greater than half that of their central peak.

Figure 8.15 shows a histogram of the residuals across all measurements before (translucent bars) and after (opaque bars) the laser filtering was applied. Although some of the points in the side peaks have been filtered out, a significant portion remains. The figure also shows that a small number of points that fall in the central peak have also been removed by the filter. Overall, the filter removed 17.3% of points from the dataset.

This filtering method does decrease the overall standard deviations and bias errors after averaging across runs. Figure 8.16 (comparable to Figure 8.10) shows the new means and standard deviations. The largest value of σ_y is now 0.098° , compared to 0.129° unfiltered. For σ_x , the maximum value after filtering is 0.012° , compared

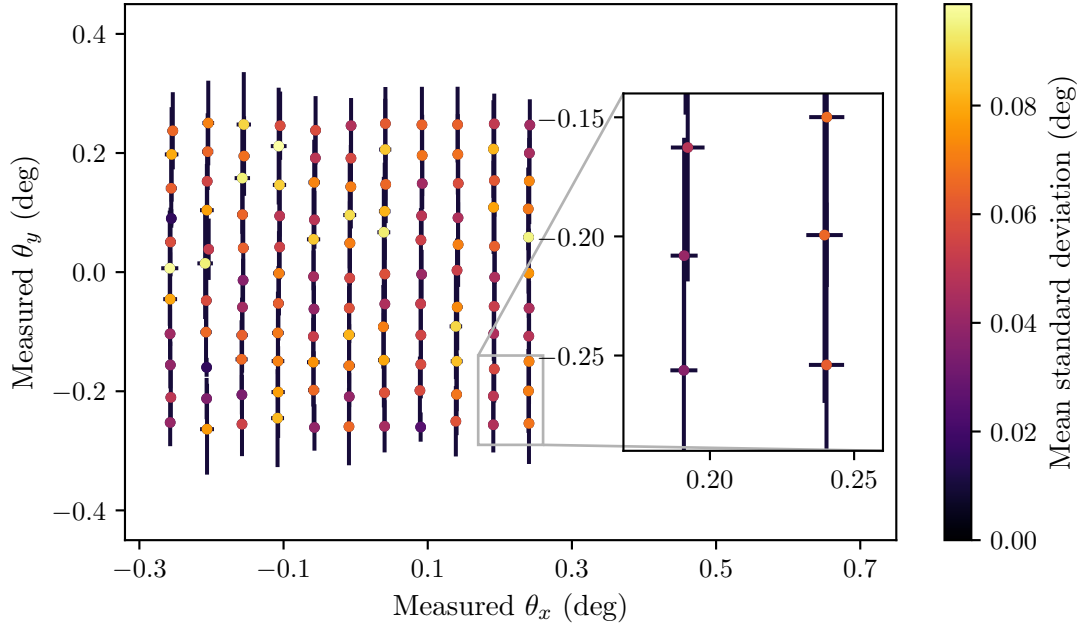


Figure 8.16: Angle measurements for a grid of surface tilts over the range $\pm 0.25^\circ$ in θ_x and θ_y . Individual measurements where the powers of the expected laser modes were less than twice any extra side peaks have been filtered out. The mean measured tilt in θ_x and θ_y is plotted, along with the standard deviations along each. The colour of the data points shows the mean of the two components of the standard deviation.

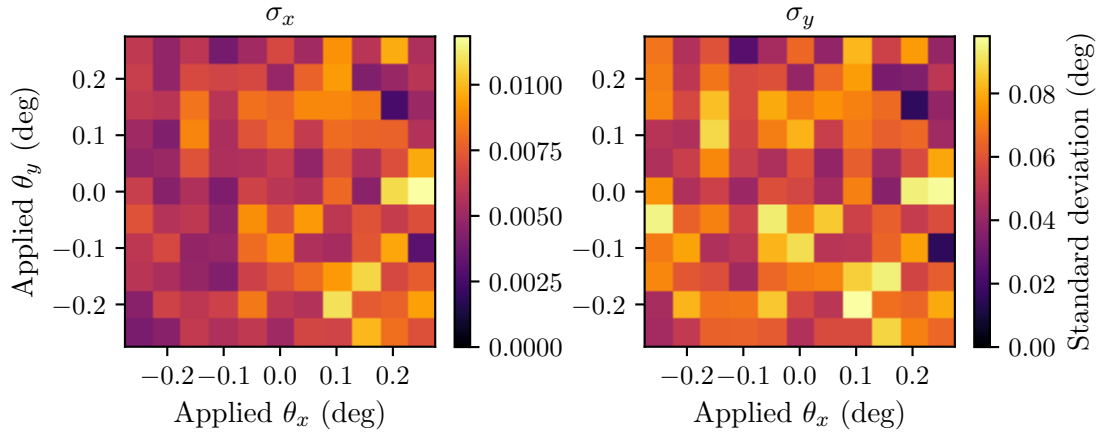


Figure 8.17: Heatmaps showing the individual standard deviation in θ_x and θ_y across the $\pm 0.25^\circ$ grid of angle measurements, after filtering using the recorded laser spectra at each measurement.

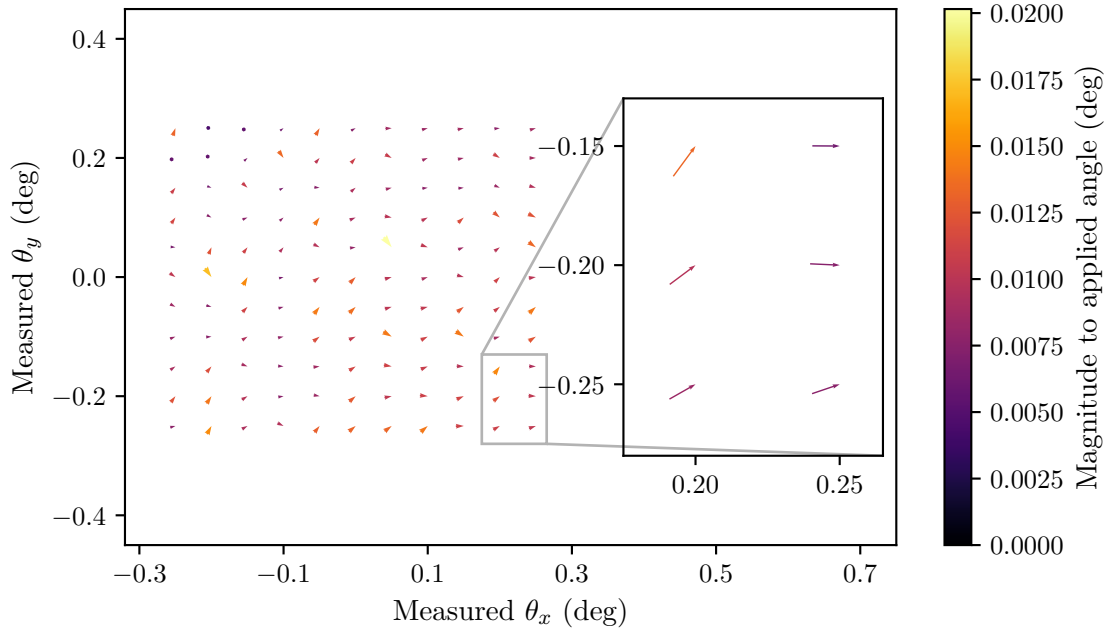


Figure 8.18: Angle measurements for a grid of surface tilts over the range $\pm 0.25^\circ$ in θ_x and θ_y . Individual measurements where the powers of the expected laser modes were less than twice any extra side peaks have been filtered out. The plot shows the vector from the mean measured angle to the applied angle at each tilt, with the magnitude of the vector given by its colour.

to 0.013° before. This is also reflected in the mean standard deviation σ , which now has a maximum value of 0.099° , and an average value of 0.062° . An alternative view of the same data is shown in Figure 8.17. This shows the distribution of the standard deviations in x and y separately using a heatmap. Most of the values of σ_x are clustered around 0.007° , and in σ_y most values are around an order of magnitude higher at 0.060° .

Figure 8.18 shows the same format vector plot as Figure 8.11 after the laser filtering has been applied. Overall, the bias errors are fairly similar between the filtered and unfiltered datasets. The largest magnitude error before filtering was 0.025° , and after filtering this value is equal to 0.020° . As with the standard deviations, the errors in x and y have been separated out in the heatmaps in Figure 8.19. Also similarly to the standard deviations, the values of χ_x are about an order of magnitude less than those in χ_y . Another interesting feature highlighted by these heatmaps is the clear positive bias error in θ_x . There also seems to be a positive bias in θ_y , with a higher number of positive errors and only a handful of significant negative errors. This

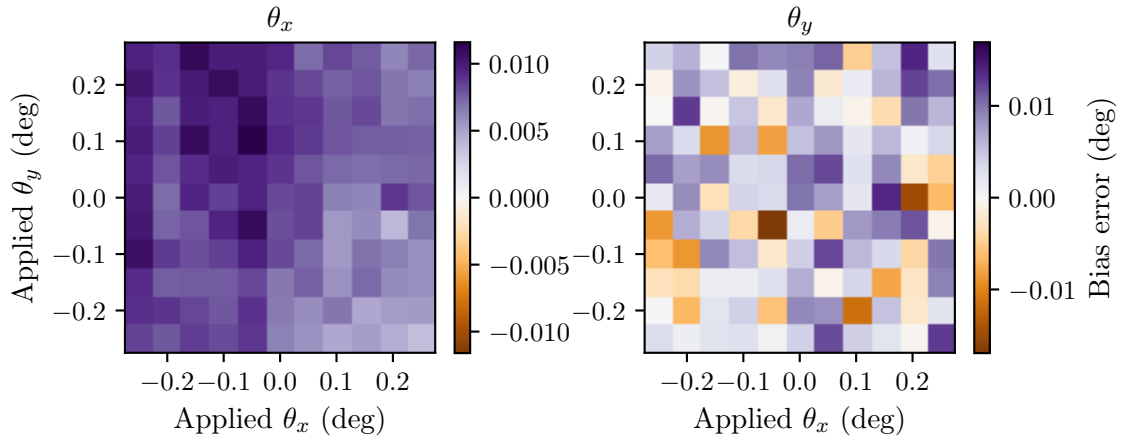


Figure 8.19: Heatmaps showing the individual bias errors in θ_x and θ_y across the $\pm 0.25^\circ$ grid of angle measurements, after filtering using the recorded laser spectra at each measurement.

systematic trend suggests a misalignment between the surface and detector; either the origins or the zero tilt angle are not aligned. However, as Figure 8.15 shows that there are still measurements with large angle errors, another cause could be these anomalous points influencing the data.

As with Figure 8.14, it is useful to look at the spectrum and cross-correlation image for one of these anomalous points. Figure 8.20 shows run 12 at the same surface angle as Figure 8.14, along with the two adjacent runs. Run 12 was measured at $\theta_y = +0.05$, while the applied angle was $\theta_y = -0.20$. The spectra for all three runs indicate that the lasers were behaving as expected, hence why none were removed when filtering using the laser spectra. However, the correlation image for run 12 shows two correlation peaks of similar height and very close to each other. The black marker indicates the result of the peak finding, at (886.86, 491.57)pix, which corresponds to an angle of $(\theta_x = 0.220^\circ, \theta_y = 0.059^\circ)$. For comparison, runs 11 and 13 gave a calculated tilt of $(\theta_x = 0.242^\circ, \theta_y = -0.214^\circ)$ and $(\theta_x = 0.240^\circ, \theta_y = -0.206^\circ)$.

As the spectra are collected as a sweep across the wavelengths, and only the most recent spectrum is recorded for an image, it is not guaranteed that the spectrum was the same at the moment the image was recorded. The correlation suggests that the image was taken as there was an extra mode in one of the lasers, of around equal height to the main peak. Therefore the spectrum is not representative, and so the

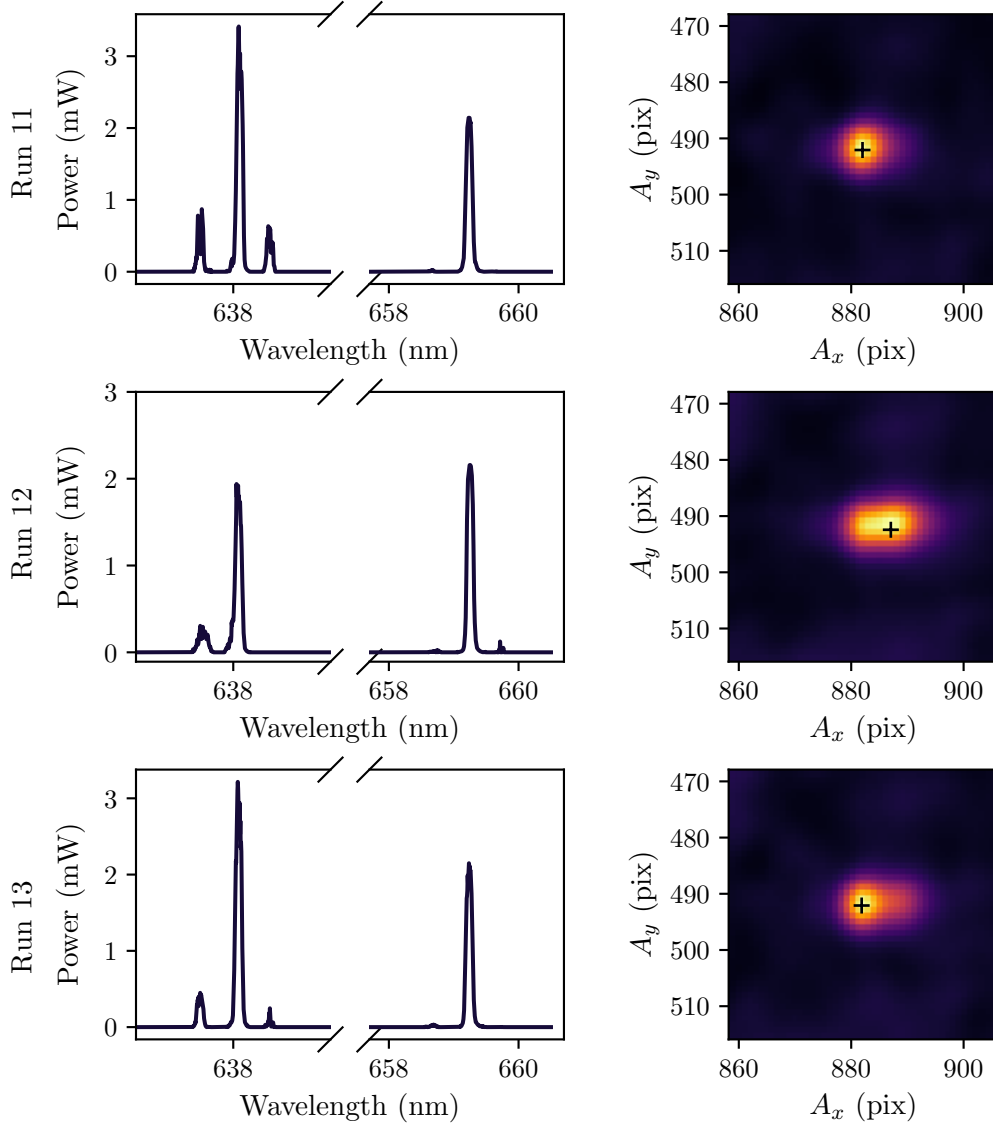


Figure 8.20: Spectra and corresponding correlation images for repeat measurements (runs) 11, 12, and 13. The calculated tilt for run 12 was $\theta_x = 0.220^\circ$, $\theta_y = 0.059^\circ$, compared to the applied angle of $\theta_x = +0.25$, $\theta_y = -0.20$. Although the spectrum for run 12 is as expected, the correlation image is not, showing two peaks instead of one.

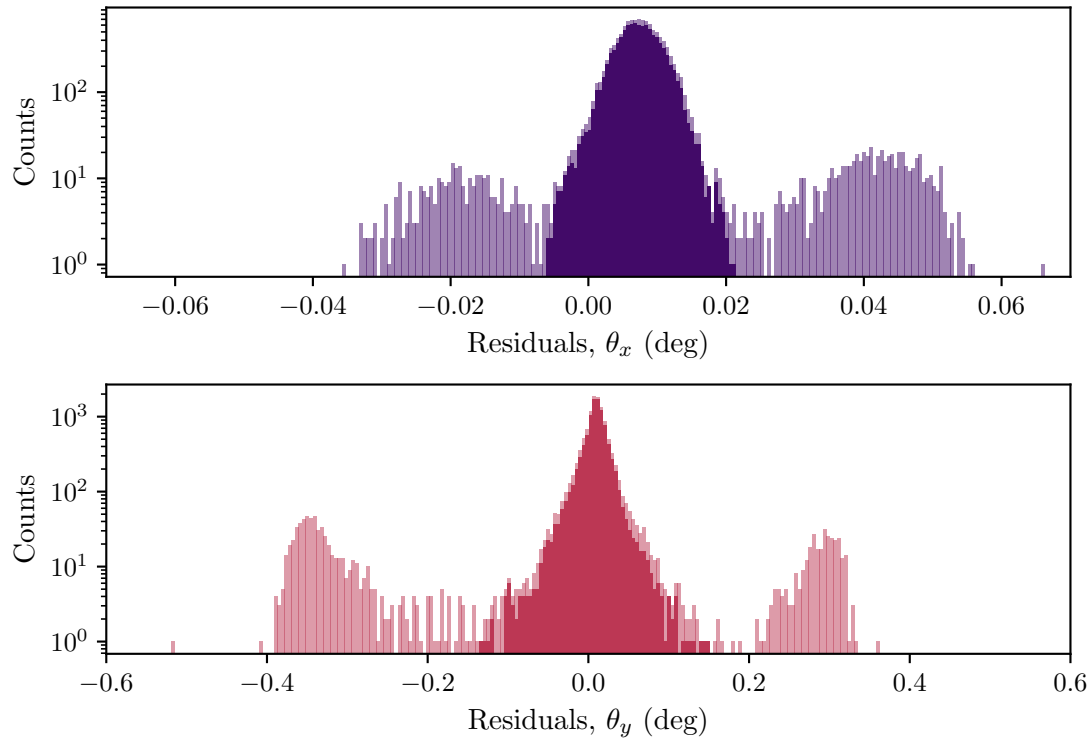


Figure 8.21: Histogram of residuals for individual measurements at all angles, both before (translucent bars) and after (opaque bars) filtering. Two filters were applied: the laser-based filtering and a cut-off filter removing points outside of the central distribution peak. Note that the y axis uses a logarithmic scale.

measurement cannot be filtered out using the method described above.

Due to the unreliability of using the recorded spectra to filter the data, the next approach was to use the peaks of the histogram in Figure 8.13 to apply a cut-off to the measurements before averaging across runs. Points with a bias error between -0.007° and 0.021° in θ_x , and -0.135° and 0.150° in θ_y were kept, and any points outside those ranges were removed. These values were chosen manually, looking at the edge of the central peak in the histograms in Figure 8.13. Note that these ranges reinforce the suggestion of positive bias errors in the measurements, especially in θ_x . Figure 8.21 shows the histogram after both the cut-off filter and the laser filter were applied (opaque bars), as compared to the original unfiltered values (translucent bars). The total percentage of points removed after applying both filters was 20.5%.

As expected, applying the cut-off filter hugely reduced the standard deviations on the measurements. The full dataset is presented in Figures 8.22 and 8.23, and a

Table 8.5: Maximum and mean standard deviations in θ_x and θ_y across all surface tilts, with the various filters applied. The application of each filter shows a successive reduction in the values of σ_x and σ_y .

Filter	Max σ_x (deg)	Mean σ_x (deg)	Max σ_y (deg)	Mean σ_y (deg)
<i>Expected range</i>	0.0028 – 0.0030		0.0142 – 0.0145	
None	0.013	0.009	0.129	0.088
Laser	0.012	0.007	0.098	0.061
Laser & Cut-off	0.004	0.003	0.025	0.017

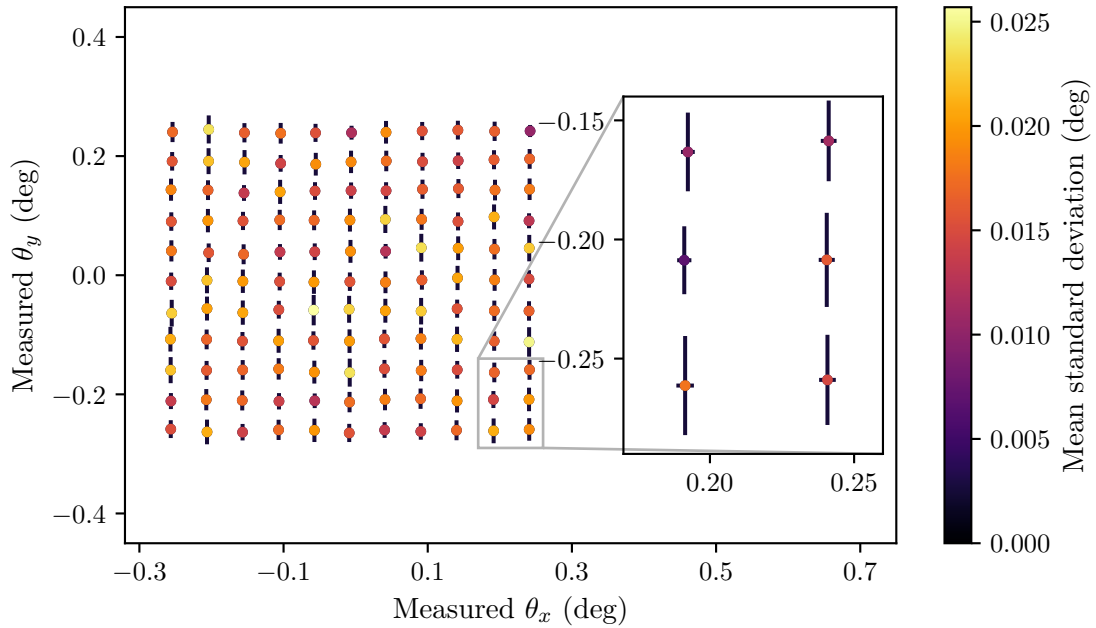


Figure 8.22: Angle measurements for a grid of surface tilts over the range $\pm 0.25^\circ$ in θ_x and θ_y . Both the laser-based filter and the cut-off filter based on the histogram of bias errors have been applied. The mean measured tilt in θ_x and θ_y is plotted, along with the standard deviations along each. The colour of the data points shows the mean of the two components of the standard deviation.

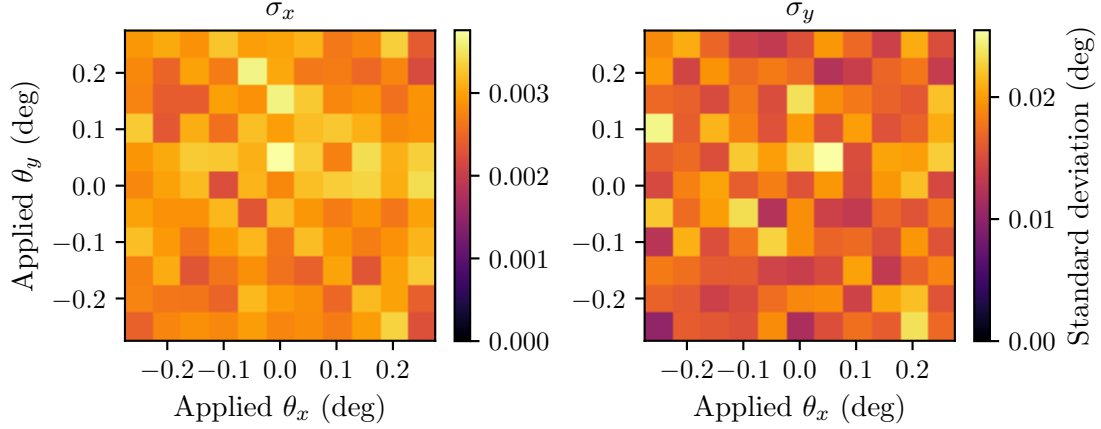


Figure 8.23: Heatmaps showing the individual standard deviation in θ_x and θ_y across the $\pm 0.25^\circ$ grid of angle measurements, after both laser and cut-off filtering.

Table 8.6: Maximum and mean absolute bias errors in θ_x and θ_y across all surface tilts, with the various filters applied. As the averages of θ_x and θ_y were taken over nearly 100 repeat measurements, even before filtering most measured tilts were less than 0.01° away from the applied angle.

Filter	Max χ_x (deg)	Mean χ_x (deg)	Max χ_y (deg)	Mean χ_y (deg)
None	0.012	0.008	0.025	0.007
Laser	0.012	0.008	0.017	0.006
Laser & Cut-off	0.010	0.007	0.015	0.009

summary of the changes to σ_x and σ_y are given in Table 8.5. The error bars in Figure 8.22 are now distinct, and the highest average standard deviation (shown by the colourbar) has dropped from 0.099° with just the laser-based filtering to 0.026° . The average value of σ also dropped, from 0.062° to 0.018° with the addition of the cut-off filter. Looking at Figure 8.23, the distribution of the standard deviations is much more uniform. The mean value of σ_x (from Table 8.5) is 0.003° , very close to the expected value of around 0.0029° . For σ_y , the mean standard deviation falls just outside the expected range of 0.0142° - 0.0145° , at 0.017° .

The bias errors on the average measurements have also improved slightly, as shown in Table 8.6. Comparing Figure 8.24 to Figure 8.18, the highest magnitude of the bias errors has dropped from 0.020° to 0.017° by applying the cut-off filter. More

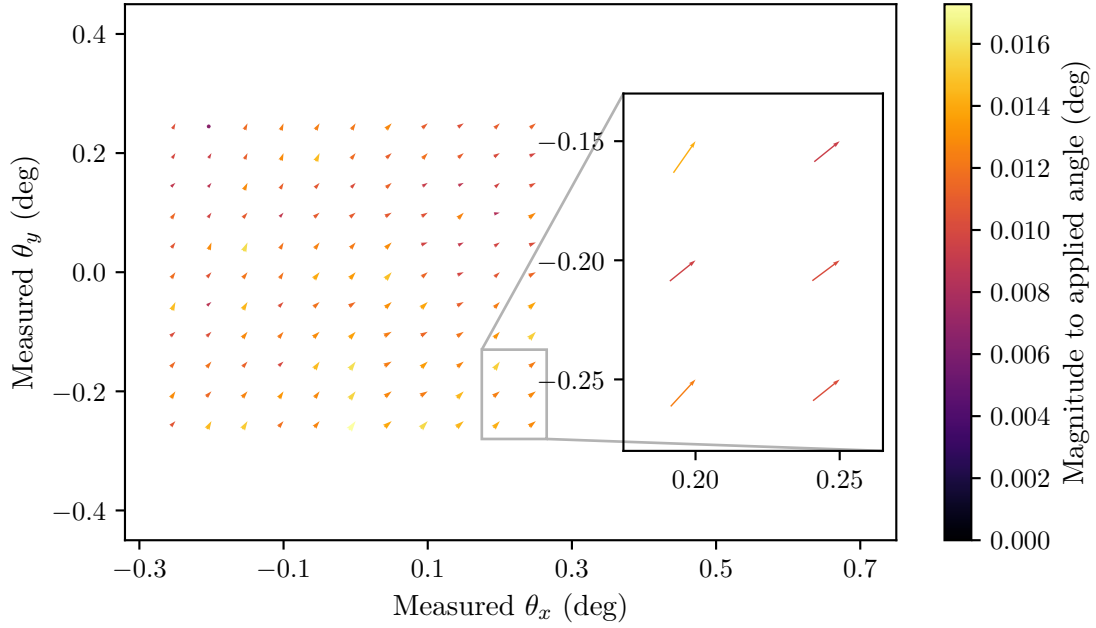


Figure 8.24: Angle measurements for a grid of surface tilts over the range $\pm 0.25^\circ$ in θ_x and θ_y . Both the laser-based filter and the cut-off filter based on the histogram of bias errors have been applied. The plot shows the vector from the mean measured angle to the applied angle at each tilt, with the magnitude of the vector given by its colour.

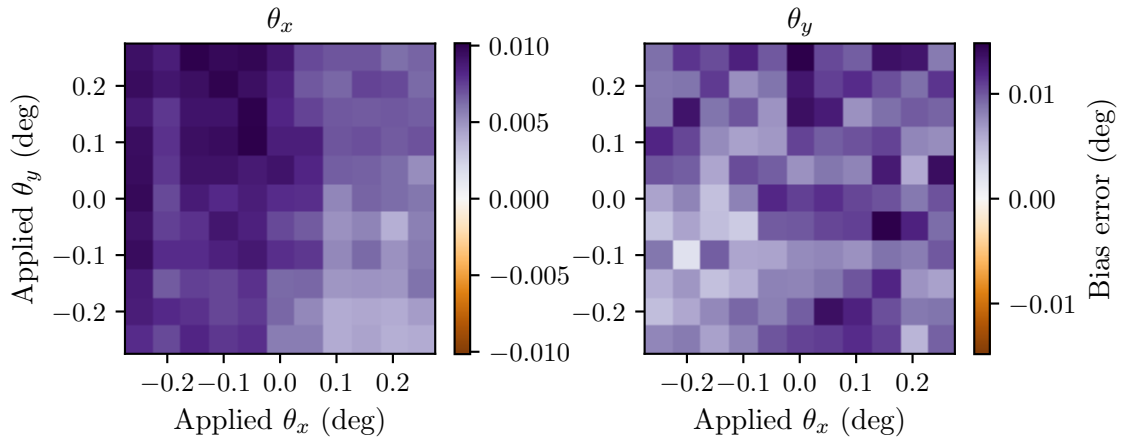


Figure 8.25: Heatmaps showing the individual bias errors in θ_x and θ_y across the $\pm 0.25^\circ$ grid of angle measurements, after both laser and cut-off filtering.

interestingly, Figure 8.25 now shows a strong positive bias error in both θ_x and θ_y . As the anomalous measurements have now been filtered out of the dataset, this suggests that there is indeed a misalignment between the surface and the detector. The most likely cause is that the detector and the surface at zero tilt are non-parallel. This cannot be accounted for in the calibration, as the nature of the technique is that it is an absolute angle measurement. The difference is very small, but still within the expected resolution of the sensor; the average bias is around 0.007° in χ_x , and 0.010° in χ_y . For a completed sensor with improved laser stability, one option to overcome this would be to “zero” the sensor at the desired surface tilt. This would both eliminate the bias error and allow for more flexibility in measurements.

Finally, an alternative, more conventional, view of the fully filtered data is presented in Figures 8.26-8.27. These graphs show a linear cross-section through the grid, at constant θ_y and varying θ_x , or vice versa. The top graph for each shows the mean and standard deviation for both θ_x and θ_y at each angle, and the bottom two graphs show the corresponding bias errors. The solid line in each graph describes the applied angles, or the expected bias of zero. These graphs make it clear that the positive bias errors in both θ_x and θ_y are a consistent trend. They also show the difference in the magnitudes of the standard deviation in x and y , with those in x being an order of magnitude smaller and very difficult to distinguish on this scale.

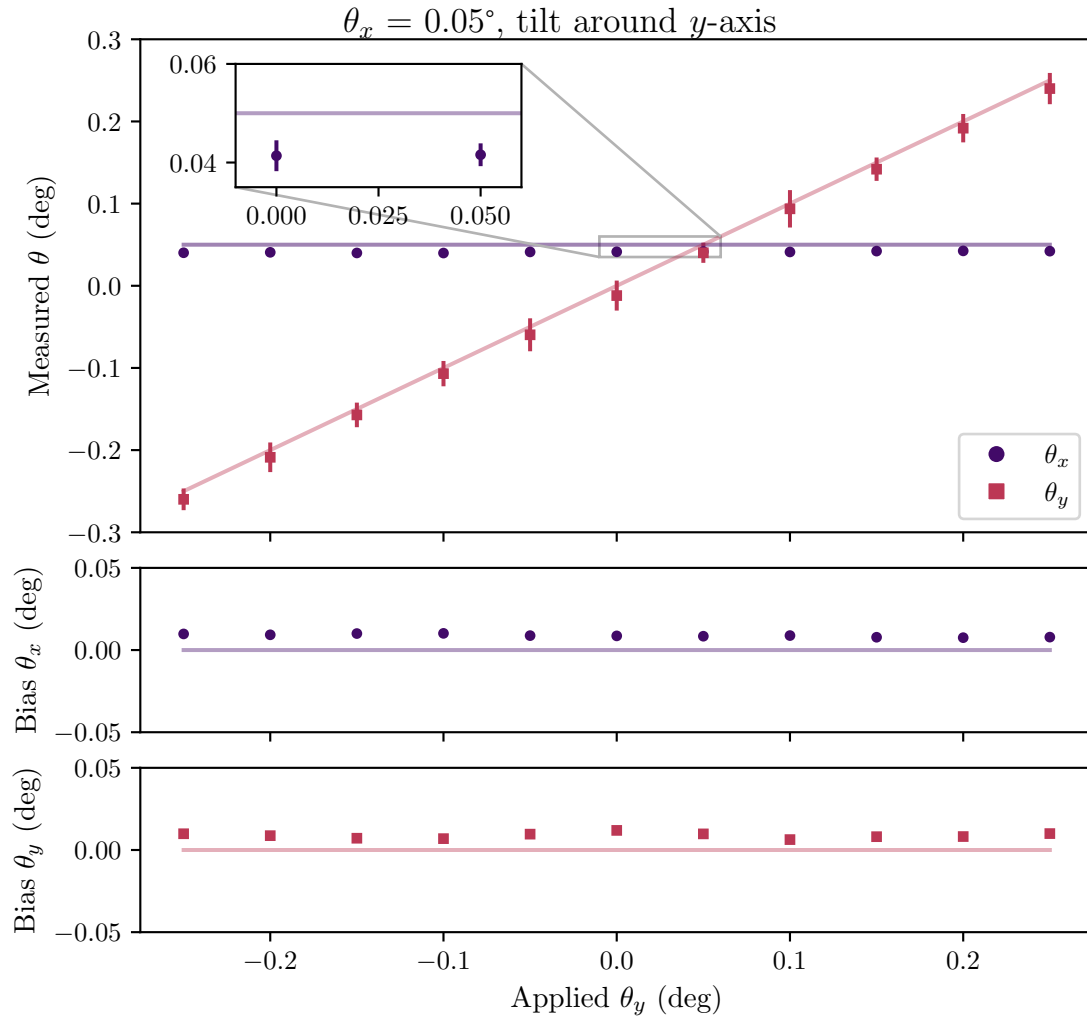


Figure 8.26: Cross-section of results from the $\pm 0.25^\circ$ grid of surface tilt measurements. The applied tilts have a fixed $\theta_x = 0.05^\circ$ and varying θ_y between $\pm 0.25^\circ$. The first graph shows the measured angle in θ_x and θ_y and their standard deviation component. In each case, the solid line gives the expected (applied) angle. The lower two graphs show the bias errors in θ_x and θ_y , with the solid line at the expected value of zero.

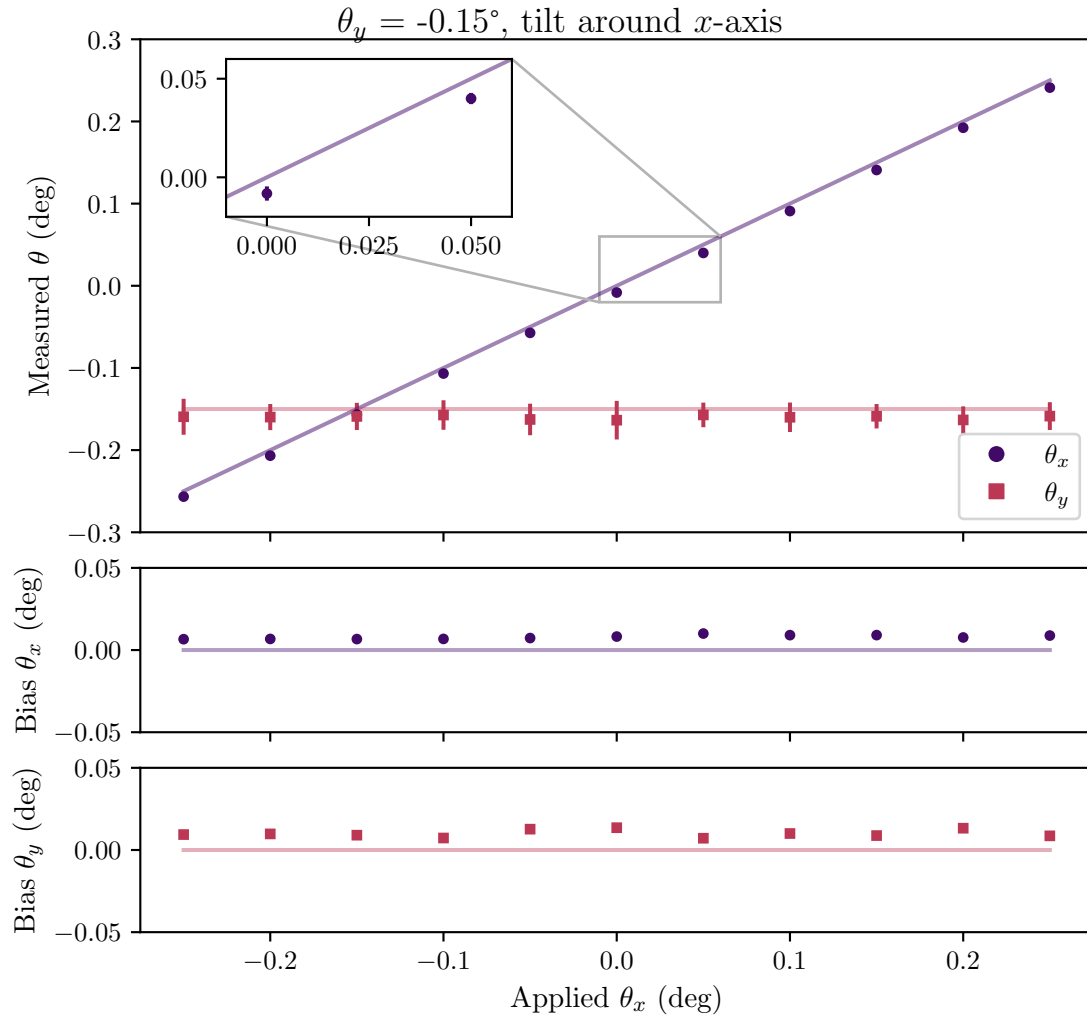


Figure 8.27: Cross-section of results from the $\pm 0.25^\circ$ grid of surface tilt measurements. The applied tilts have a fixed $\theta_y = -0.15^\circ$ and varying θ_x between $\pm 0.25^\circ$. The first graph shows the measured angle in θ_x and θ_y and their standard deviation component. In each case, the solid line gives the expected (applied) angle. The lower two graphs show the bias errors in θ_x and θ_y , with the solid line at the expected value of zero.

8.4 Results Summary and Conclusions

The sensor presented in this chapter used two laser diodes to achieve a large wavelength separation of 21.2nm. This gave a responsivity of 4.925mm/rad and an expected resolution of 0.003° . This surpasses the target resolution of 0.1° , while still maintaining an expected sensor range of 15.2° .

The experimental measurements undertaken in this chapter investigated a 2D grid of angles covering the range 0.25° in both θ_x and θ_y . Angles were incremented in steps of 0.05° in each direction. 100 repeat measurements were taken at each angle, with the full set of angles applied each repeat, rather than taking consecutive repeats at each position. This was building off the experimental work in the previous chapter, where taking consecutive repeats removed the influence of the long-term laser stability from the measured results, narrowing the possible analysis of the data.

The initial conclusion drawn from the results was that the laser stability, particularly the mode power stability, was an issue for this sensor iteration. An error analysis showed that measurement errors of up to $\pm 0.50^\circ$ in θ_y could be expected if a parasitic mode became dominant in one of the lasers. This was borne out by the data; a histogram plot of residuals of individual measurements showed a central peak with two side peaks clustered around $\pm 0.30^\circ$ from the central peak in θ_y and $\pm 0.03^\circ$ from the centre in θ_x .

The choice was made to filter the results, in order to demonstrate the sensor performance if the laser stability were improved. First, using the OSA data collected alongside measurements, points were filtered based on whether or not the expected modes were dominant in the measured spectrum. This removed some outlying results, but not all, as there were discrepancies between the timing of recorded speckle patterns and spectra. A second filter was applied, using the residuals histogram to exclude points that fell outside of the main peak. This, as expected, further improved both the measured standard deviation and bias error on filtered results.

Having isolated results where the sensor was operating as expected, the angle measurements were very favourable when compared with the expected performance. The predicted uncertainty in measurements across the $\pm 0.25^\circ$ range was $0.0028^\circ - 0.0030^\circ$ in θ_x and $0.0142^\circ - 0.0145^\circ$ in θ_y . The standard deviation was calculated at each angle in θ_x and θ_y , giving a maximum of 0.004° in σ_x and 0.025° in σ_y . The average

values across the grid were 0.003° for σ_x and 0.017° for σ_y .

The uncertainty value used for the peak fitting during image processing was 0.05 pixels, following the conclusion in the previous chapter that this was more representative of the measured results. Given the close agreement between predicted uncertainties and the measured standard deviations here, this conclusion seems to hold true for this sensor also.

In terms of the bias errors, there was a consistent positive bias in both θ_x and θ_y , likely caused by a non-parallelity of the surface and the detector plane at zero tilt. The aggregated bias errors looked at their absolute value, finding a maximum of 0.010° in χ_x and 0.015° in χ_y . The average absolute bias errors across the grid were 0.007° for χ_x and 0.009° for χ_y . These results suggest an option for further work from this project - in developing a calibration method for correcting or accounting for the non-parallel alignment of surface and detector.

8.4.1 Comparison of Sensor Implementations

This chapter and the previous chapter both presented a detailed construction and evaluation of a DWS sensor. The geometry, wavelength separation, and expected characteristics of each are summarised in Table 8.7. Both sensors performed in line with the expected uncertainties on angle measurements, though the second sensor

Table 8.7: Comparison of the geometry, wavelength separation, and characteristics of the two full sensor implementations tested (in this chapter and the previous).

Sensor variable	Value	
Wavelength, λ_1	656.9nm	638.1nm
Wavelength separation, $\Delta\lambda$	0.46nm	21.2nm
Source position, $\mathbf{S}$	(103.5, 0.0, 103.5)mm	(49.0, 0.0, 80.0)mm
Detector position, $\mathbf{D}$	(0.0, 0.0, 103.5)mm	(0.0, 0.0, 80.0)mm
Range, $\theta_{y,\min}$	12.42°	15.23°
Resolution, r_{sens}	0.222°	0.003°
Responsivity, C	0.124mm/rad	4.925mm/rad

had a bigger issue with laser instability than the first.

The main difference between the two sensors is the much greater wavelength separation for the second sensor, leading to a much higher responsivity and resolution. Despite this, both sensors have a similar maximum range, at 12.42° and 15.23° .

The first sensor, with the smaller wavelength spacing, was shown to be able to measure 2D surface tilts in increments of 0.5° across the range $\pm 7.5^\circ$ in both directions. The estimated standard deviations in these measurements was 0.100° in θ_x and 0.181° in θ_y , in line with expectations and close to the target resolution of 0.1° . The second sensor was used to measure angles across the much smaller range of 0.25° , in increments of 0.05° . Taking results where the laser was behaving as expected, the standard deviations on these measurements were 0.003° in θ_x and 0.017° , again in line with expectations. This sensor surpassed the 0.1° target resolution, but the problems with laser instability necessitate further development before it could be used to this precision for online measurements.

In terms of the sensor construction, the first sensor uses a single laser to generate the two wavelength modes, while the second uses two separate diodes. As a result,

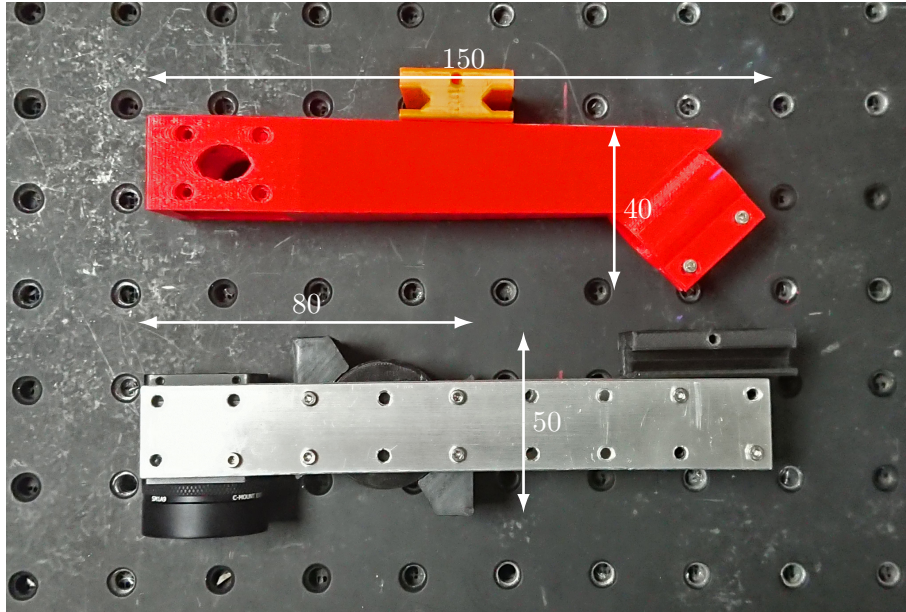


Figure 8.28: Comparison photo of the fully 3D printed sensor head used for the sensor in Chapter 7 (top), and the metal sandwich mount with modular 3D printed pieces used in for the sensor in this chapter (bottom). Dimensions are given in mm.

the first sensor is less expensive, and the laser component can be smaller with the use of only one diode.

However, due to the sensor geometry, the sensor head is smaller for the larger wavelength separation sensor. Figure 8.28 shows a direct comparison of the two sensor heads each used in Chapters 7 and 8. The full length of the 3D printed sensor head for the $\Delta\lambda = 0.5\text{nm}$ sensor is around 150mm, with the separation between source and detector, S_x , at 103.5mm. This length would be similar if using the metal sandwich mount for the same sensor geometry.

The sensor head for the $\Delta\lambda = 21.2\text{nm}$ sensor presented in this chapter used around half the length of the modular metal sandwich mount, totalling 80mm for the sensor components only (with $S_x = 49.0$). The more compact head is an advantage for this second sensor, especially for the targeted application of robotics where the sensor will have to be mounted alongside the end effector tool.

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Chapter 9

Conclusions and Further Work

9.1 Thesis Conclusions

This thesis has demonstrated the novel technique of dual-wavelength speckle (DWS), which can be used to measure the absolute tilt of a surface in two dimensions.

The basis of the technique is simultaneous illumination of an optically rough surface with two wavelengths of light from a collinear beam. Each wavelength produces a similar speckle pattern, with a positional offset between them, referred to as the speckle shift. A single frame image of the patterns is taken, and an autocorrelation applied to extract the speckle shift. This is directly related to the tilt of the surface, which can then be calculated.

In terms of existing angle sensing techniques employing speckle, the major advantage of this approach is its absolute measurement of surface angle. Previous techniques require before and after images [1], or two-camera setups [2] to make angle measurements. DWS obtains the surface angle from a single image. The technique as presented in this thesis is also low-cost, and capable of taking real-time measurements.

The thesis presented for the first time the full derivation of the mathematical theory supporting dual-wavelength speckle. Following this, the theory was tested with physical implementations of the sensor, demonstrating that the theoretical values of the sensor constants and their changes with different sensor geometries align well with those calculated from experimental results. Two complete implementations of

the sensor were constructed and characterised in full, with each having different sensitivities and tested over different ranges.

The intended application for the sensor was robotics, specifically applications such as robotic drilling and riveting that require perpendicularity with the work surface. The aerospace industry in particular has high tolerances for position and orientation, stipulating a 0.1mm accuracy for position and 0.5° for orientation [3]. As such, the target accuracy of the sensor was 0.1° .

Furthermore, the proposed DWS sensor would eliminate some of the disadvantages of widely used techniques in robotics. DWS measures directly at the point of interest, in contrast to approaches using distance meters that approximate the surface using measurements around a point. The second major technique, 3D modelling, uses a point cloud of the surface to calculate the normal direction at any point. This approach requires significant computing time and a 3D point scan of the surface. DWS can make real time measurements, making use of the fast correlation and peak finding methods used extensively in particle image velocimetry (PIV).

9.2 Fulfilment of Project Objectives

The introduction (Chapter 1) set out five main project objectives.

- i Develop and experimentally test the mathematical theory relating the dual-wavelength speckle pattern shift to surface tilt angles.

The full derivation of the theory supporting dual-wavelength speckle was presented in Chapter 4, Section 4.2. The final equations related the speckle shifts in x and y to the surface tilts θ_y and θ_x . The chapter further defined three sensor constants for the chosen sensor arrangement, which placed the detector directly opposite the surface spot and the source offset in x . These constants relate to the sensor geometry (specifically the working distance of the detector and the unit vector to the source) and a practical experiment in Chapter 6, Section 6.1, demonstrated that the constants determined from the speckle theory were in good agreement with those calculated from experimental data.

- ii Develop the image processing method for extracting the speckle shift between the two patterns present in recorded frames.

Chapter 5, Section 5.1 described in detail the image processing. The procedure is slightly different depending on the value of the wavelength separation $\Delta\lambda$, but broadly involves applying a correlation to the single image (containing both speckle patterns) captured by the camera. The correlation image contains a side peak whose position, once isolated, gives the speckle shift between the two patterns. The accuracy of the peak fitting method was discussed in Chapter 5, and the error propagation equations for surface angle derived. The experimental results from Chapter 7 suggested that the lower limit on the peak fitting, 0.05 pixels, was the most appropriate value given the measured standard deviations on results.

- iii Investigate the types of laser instability and how these affect angle measurements.

There were two types of laser instability encountered in the project: mode power stability, which refers to the power stability (height) of the one or more modes present in the laser spectra, and wavelength stability, referring to the wavelength (position) of a particular mode. Chapter 5, Section 5.2 detailed the effect of each of these on angle measurements, and described the methods used to monitor the laser output using an optical spectrum analyser. The three lasers used throughout the project, each being Fabry-Pérot laser diodes, were introduced in this chapter, but their stability was investigated further in the experimental chapters Chapter 7 and Chapter 8. The laser stability was found to be the main limitation on the sensor accuracy, as demonstrated especially in Chapter 8.

- iv Construct and characterise a tilt sensor using a single laser operating with two longitudinal modes, looking at one and two dimensional surface tilts.

Chapter 7 described the construction of a DWS sensor using a single laser, with a wavelength spacing of around 0.5nm. The sensor was used to measure one dimensional surface tilts over a large range of $\pm 45^\circ$, finding an increasing error with increasing tilt magnitude, theorised to be caused by the increasing ellipticity of the spot. Further measurements of two dimensional tilts focused on angle magnitudes up to 7.5° , and found an estimated standard deviation in results of 0.100° in θ_x and 0.181° in θ_y , which was in line with expected values.

- v Construct and characterise a two dimensional tilt sensor using a wider wavelength separation for a higher responsivity.

A second DWS sensor was presented in Chapter 8. This sensor used two lasers and had a wavelength separation of around 21nm, leading to a 40x increase in the responsivity compared to the version in Chapter 7. The sensor was tested using a 2D grid of angles across the range $\pm 0.25^\circ$ in both θ_x and θ_y . The laser stability, particularly the mode power stability, was found to be a problem for this sensor, and so results were filtered to demonstrate the potential performance of the sensor given a more stable source. The final, filtered, results showed a standard deviation in measurements of 0.003° in θ_x and 0.017° in θ_y , with average absolute bias errors of 0.007° and 0.009° in each direction.

In addition to the five objectives above, a calibration method was developed to determine the sensor constants from speckle shift measurements using a completed sensor. The method was described in full in Chapter 5, Section 5.3. It was shown in Chapter 7 that a mis-alignment of 1-2mm can have a large effect on the angles measured by the sensor, so the calibration method can refine the sensor constants given such misalignments, improving the sensor accuracy.

9.3 Further Work

9.3.1 Improving Sensor Performance

Laser Stability

As mentioned above, the laser stability was found to be the main limitation on sensor accuracy. The lasers, and especially their wavelengths, used in this project were chosen with the low-cost aim for the sensor in mind. The visible range is more cost-effective in terms of the camera, and the particular laser diodes used were relatively low-cost, with limited built-in stabilisation. One option for further work would be to invest in lasers with built-in stabilisation, or dedicated stabilisation components such as isolators, to improve the wavelength stability while staying in the visible region of the spectrum.

Alternatively, the laser spectra could be measured in real time using a low-cost spectrometer, feeding this information back into the determination of surface angle. This could have been achieved with the Yokogawa optical spectrum analyser (OSA) used throughout the project, but the decision was made to only use it for monitoring and, in Chapter 8, filtering data in post-processing. A real-time wave-

length measurement would increase the processing needed for the sensor, especially for online angle measurements, so this consideration would have to be balanced with the benefit of an ongoing wavelength measurement. This could be achieved by only updating the wavelength values occasionally, or by limiting the measurement rate of the angle sensor.

Another option would be to explore atomic absorption referencing, which uses one or more gas absorption lines as a reference to lock the laser wavelength. The wavelength of the absorption lines are an inherent, well-documented property of the gas [4], and sealed gas cells can be purchased as off-the-shelf optical components. However, application of the technique is limited by the availability of gas lines in the desired wavelength region, as well as the practicalities and safety considerations of the gas itself. If this approach were to be applied to DWS, an initial review would be required into the available options for gas lines, laser components, and the associated camera requirements.

All of these options would help to reduce the uncertainty in the value of the wavelengths used for DWS. A decrease in this uncertainty leads to more accurate angle measurements at a higher precision.

Infrared Wavelengths

Given the limitations of the options outlined above, another approach would be to switch to the near-infrared (IR) region. This would provide many options in terms of atomic absorption referencing, but there are also many broader options and commercially available components for improving laser stability in this region. This is due to the widespread use of IR in telecommunications, particularly around the wavelengths 1000-2000nm.

The main justification for using visible wavelengths throughout this project was the relatively better options and performance of cameras in this region over IR, but ongoing improvements in camera technology have improved the quality and prices of IR cameras, especially in the near-infrared. It is likely that the trade-offs associated with IR wavelengths that were established at the start of this project would no longer apply if it were starting now.

In terms of the theory of dual-wavelength speckle, increasing the absolute wavelength (λ_1) of a sensor while keeping the wavelength separation the same would decrease its

responsivity and resolution. For instance, a λ_1 of 1550nm (a wavelength commonly used in communications) is more than double that for the sensors presented here, resulting in a halving of the responsivity. However, as the important factor is the ratio $\Delta\lambda/\lambda_1$, this could be offset fully (or partially) by doubling (increasing) the value $\Delta\lambda$.

Some examples of this are shown in Table 9.1. Starting from the dual laser sensor presented in Chapter 8, increasing λ_1 to 1550nm with no other changes to the sensor results in a responsivity of 2.027mm/rad and resolution of 0.007°, compared to 4.925mm/rad and 0.003°. Then, increasing the wavelength separation, $\Delta\lambda$, to 50.0nm for example offsets this change, giving a responsivity of 4.781mm and a resolution of 0.003°.

Table 9.1: Expected sensor characteristics for example DWS sensors using IR wavelengths.

Sensor variable	Value		
Wavelength, λ_1	638.1nm	1550.0nm	1550.0nm
Wavelength separation, $\Delta\lambda$	21.2nm	21.2nm	50.0nm
Source position, $\mathbf{S}$	(49.0, 0.0, 80.0)mm	(49.0, 0.0, 80.0)mm	(49.0, 0.0, 80.0)mm
Detector position, $\mathbf{D}$	(0.0, 0.0, 80.0)mm	(0.0, 0.0, 80.0)mm	(0.0, 0.0, 80.0)mm
Range, $\theta_{y,\min}$	15.23°	15.74°	15.74°
Resolution, r_{sens}	0.003°	0.007°	0.003°
Responsivity, C	4.925mm/rad	2.027mm/rad	4.781mm/rad

Correlation Peak Finding

Although the 3 point Gaussian fit used throughout experiments is the widely accepted choice in PIV, it is possible that a fitting method employing more datapoints would be a more optimal solution for DWS. As with the improvements to the laser stability suggested above, a decrease in the uncertainty of the peak fitting will lead to more accurate and precise angle measurements. For example, the Levenberg-Marquardt algorithm used throughout the project in other areas could be applied to conduct a fit on the full 2D Gaussian. This approach lies at the opposite end of the scale in terms of complexity and processing as compared to the 3 point estimators, and in theory (due to the increase of points used) should have a lower uncertainty in results. An initial review of different approaches to 2D peak finding would be required to pursue this avenue of further work.

There are also iterative algorithms used in PIV and widely employed in astronomy that can improve the accuracy of peak fitting methods, specifically the 3 point Gaussian fit in the case of PIV [5]. Figure 9.1, also given earlier in the thesis, shows the measurement uncertainty in the 3 point fit for changing particle diam-

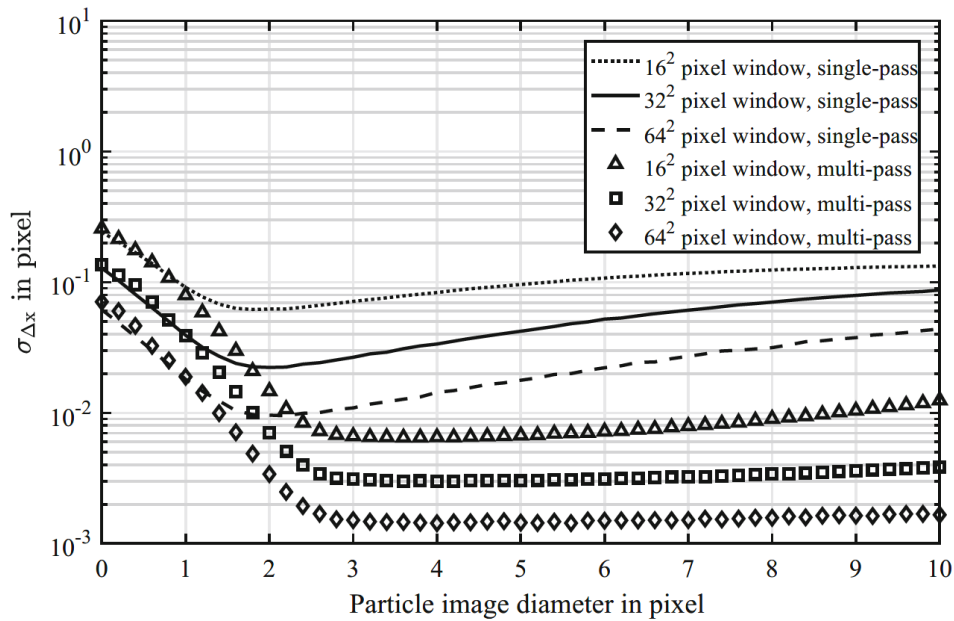


Figure 9.1: Measurement uncertainty in the Gaussian 3-point fit as applied to particle image velocimetry. The values are simulated results for changing particle diameter, determined for different window sizes and two different evaluation approaches. Figure taken from Raffel et al. [5].

eter. Results are given for a single-pass approach, most similar to the methods used throughout this project, and a multi-pass approach. The multi-pass method uses “iterative image deformation and Gaussian window weighting”, and shows a significantly lower uncertainty for the same window size as compared to the single-pass. For the 32x32pix window size, the minimum uncertainty is 0.022pix for the single-pass approach, and 0.003pix for the multi-pass. The multi-pass approach has the additional advantage of a much larger range of optimal particle diameters than single-pass, covering around 3-6 pixels compared to the 2 pixel optimum of single-pass.

Considering algorithms from other areas of physics, CLEAN is an iterative deconvolution algorithm frequently used in radio astronomy that removes noise and clutter from images of the sky [6]. The algorithm treats the image as a number of point sources, identifying and removing particular points using the point spread function. More broadly, the Lucy-Richardson (or Richardson-Lucy) deconvolution algorithm is widely used for reconstructing images blurred by a known point spread function [7, 8]. Algorithms such as these could be used to remove noise from the correlation image before applying the peak fitting, thus improving its accuracy.

Another factor that was theorised to affect the sensor performance was the elliptical spot at larger surface angles. This was discussed briefly in Raffel et al. [5], suggesting the use of a two dimensional Gaussian estimator [9]. This could be used to find the peak width in x and y separately, and the ellipticity of the peak, as well as the peak position in x and y and its height. With six unknowns to solve for, the estimator requires a minimum of nine points around the peak, increasing the computation time over the more widely used three point fit [5].

Alongside testing an elliptical peak fitting, further work should investigate the correlation peak fitting as applied to randomised speckle patterns. The results in Chapter 7 demonstrated that repeat measurements at the same point on the surface result in correlation images with identical noise distributions. This means that any bias errors in the peak fitting are consistent across repeats. A more thorough investigation of the peak fitting would take DWS measurements from different surface points, perhaps using a linear stage to move the surface, thus randomising the speckle pattern while keeping the surface angle constant.

Surface Roughness

Following on from the more thorough investigation of randomised speckle patterns, the technique would benefit from an investigation on the effect of surface roughness on sensor performance. As described in Chapter 6 (Section 6.3), surface roughness has an effect on the correlation of the two speckle patterns used in DWS. This relationship leads to a decrease of the correlation peak height as surface roughness increases. It follows that there will be a limiting value of surface roughness for a particular sensor, above which the correlation peak will be too low to reliably measure.

Given the complex relationships between the various sensor characteristics that can affect the correlation peak height, this limiting surface roughness will depend on other sensor variables. Additionally, changes to these other sensor variables, in particular spot size (and thus speckle size), can compensate to some degree for the drop in signal caused by a large surface roughness. These interdependent factors should be considered together during an investigation of surface roughness.

A good starting point for this study would be using a Talysurf [10] to make contour measurements of the surfaces used throughout this project, as well as any other surfaces that could be used as to test the sensor. Trials of different surfaces could also be incorporated in the investigation of the peak fitting method outlined above. Ideally, however, quantitative measurements using known samples of varying surface roughness would allow the effect of surface roughness on DWS to be fully characterised.

9.3.2 Expanding the Technique

RGB Wavelengths

One proposed implementation of the sensor, that was not able to be investigated within the time frame of this project, would use a colour camera and lasers in the red, green, and/or blue ranges. The colour camera would allow a readout of each red green blue (RGB) component of the image individually, still using a single frame of data. For, say, red and green source wavelengths, the red and green components of the image (each containing one speckle pattern) would be correlated, producing a correlation image with only one peak and so removing the need for masking. This

change would also increase the range of the sensor, as the implementations presented in this thesis have a range limit imposed by the central correlation peak and (for the small $\Delta\lambda$ sensor) potential interference from the mirrored side peak.

This approach would naturally have a much greater wavelength separation than the sensor presented here, leading to a large increase in the theoretical responsivity, but also an increase in the decorrelation between speckle patterns. Further work on this sensor implementation would require an initial feasibility study, incorporating the discussions in Chapter 6 on the factors affecting decorrelation and the correlation peak height, and some degree of modifications to the image processing technique.

Real Time Operation

One of the overall aims for the sensor was the ability to perform online angle measurements. This was tested successfully during the project, though none of the data was showcased in the thesis. However, a more extensive test of the real-time capabilities of the sensor would be beneficial, particularly in testing the computation speed of the image processing. One factor that was not tested was the difference in processing speed between the image processing with and without the additional magnification step.

Following on from the suggested adjustments to the peak finding above, the speed and accuracy of any changes to the peak finding method should be balanced with the required or desired measurement speed of the angle sensor. This should be compared with the existing sensor configuration as a benchmark.

Another possible investigation in this vein would be to test the sensor with moving surfaces, experimenting with small exposure times and perhaps the image processing algorithms described above to minimise blur. This could broaden the range of applications of the sensor, and/or make it more resilient against vibration or position misalignments. However, the working distance for these experiments would need to remain fixed, as this is a limiting feature of the technique.

Working Distance Determination

As mentioned above, inherent to dual-wavelength speckle is the requirement of a fixed working distance. Split dioptre lenses could be used to construct a sensor able to make online measurements of the working distance, adjusting the angle calcula-



Figure 9.2: Split dioptré lens, also called a split field filter [11].

tions accordingly. A split dioptré lens, also called a split field filter, is essentially a half lens, as shown in Figure 9.2. They are often used in photography and cinematography for shots that require subjects in different parts of the frame and at different distances from the camera to both be in focus. In other words, it allows each side of the image to be captured using a different focal length.

Applying this to DWS, a split dioptré lens would introduce a virtual image plane for half of the image, changing its working distance. Split dioptré lenses also introduce a blurred area at the centre of the image, at the edge of the half lens, so subsections of each half of the image would have to be chosen to avoid this region. Autocorrelations of these two subsections would give two measurements of the speckle shift, each at a different working distance but with a known difference between the two working distances. This is shown in the diagram in Figure 9.3. Put simply, this gives two measurements (two different speckle shifts) and two unknowns (the working distance and the surface tilt) which can be determined mathematically.

This approach could also be applied to the proposed RGB-based sensor. Using the full range of red, green, and blue lasers, correlations of red/green and green/blue patterns would also give two different measurements of speckle shift. Assuming the other geometry variables are known, such as the separation between source and detector and the angle of each, the working distance can be determined alongside the surface tilt.

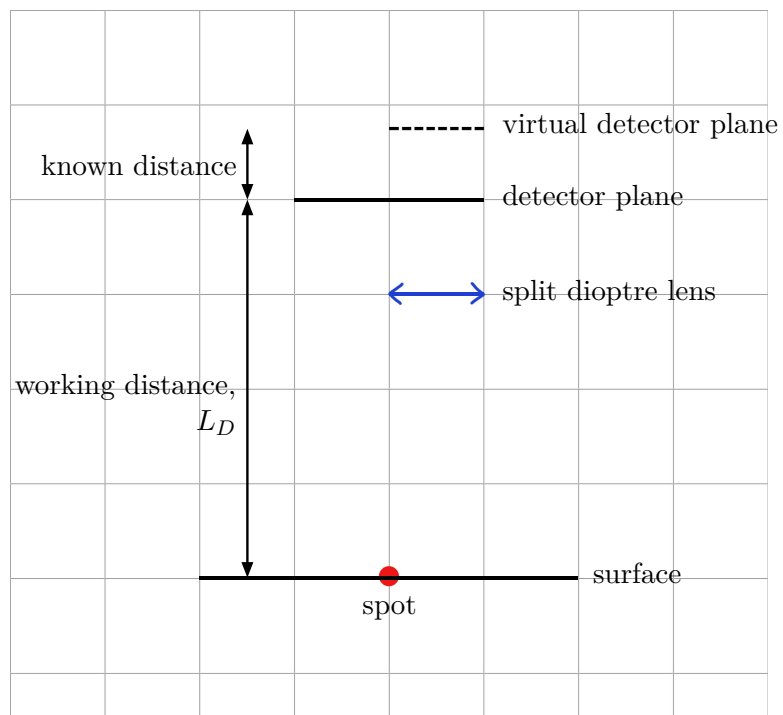


Figure 9.3: Diagram showing the application of a split dioptr lens for determining the working distance of a DWS sensor.

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List of Publications

Conferences

- S. J. Gibson, T. O. H. Charrett, and R. P. Tatam, “Absolute angle measurement using dual-wavelength laser speckle for robotic manufacturing,” in *Proceedings of SPIE*, vol. 11056, no. 110560K, 2019, pp. 122 – 120
 - Oral presentation given at SPIE Optical Metrology, Munich, Germany, 2019
- S. J. Gibson, T. O. H. Charrett, and R. P. Tatam, “Absolute angle measurement using correlation of dual wavelength laser speckle”
 - Oral presentation given at SPIE Photonex Europe, Coventry, UK, 2019
- S. J. Gibson, T. O. H. Charrett, and R. P. Tatam, “Two-dimensional absolute angle sensing using dual-wavelength laser speckle”
 - Submitted paper for oral presentation at I2MTC 2020, Dubrovnik, Croatia, 2020
 - Accepted after peer review, but withdrawn due to COVID-19

Appendix A

Calculation of Partial Derivatives

This appendix outlines the calculation of the various partial derivatives required for the speckle theory formulation in Chapter 4.

For a planar surface with zero tilt, the partial derivatives in x and y of $L_D(\mathbf{P}, \mathbf{D})$ and $L_S(\mathbf{P})$ can be calculated directly using their definitions. As an example, $\frac{\partial L_D}{\partial x}$ is calculated below:

$$\begin{aligned}\frac{\partial L_D}{\partial x} &= \frac{\partial}{\partial x} \sqrt{(D_x - x)^2 + (D_y - y)^2 + (D_z - z)^2} \\ &= -\frac{D_x - x}{\sqrt{(D_x - x)^2 + (D_y - y)^2 + (D_z - z)^2}} \\ &= -\frac{D_x - x}{L_D} \\ &= -d_x\end{aligned}\tag{A.1}$$

The result is the x component of the unit vector $\hat{\mathbf{d}} = (d_x, d_y, d_z)$ from $\mathbf{P}$ to the detector. The calculation and result for the other partial derivatives are similar.

$$\frac{\partial L_D}{\partial y} = -d_y\tag{A.2}$$

$$\frac{\partial L_S}{\partial x} = -s_x\tag{A.3}$$

$$\frac{\partial L_S}{\partial y} = -s_y\tag{A.4}$$

Appendix A. Calculation of Partial Derivatives

For a tilted surface, an alternative form of $L_D(\mathbf{P}, \mathbf{D})$ and $L_S(\mathbf{P})$ is used that incorporates the surface gradients m_x and m_y . In this case, the surface point is $\mathbf{P} = (x, y, (m_x x + m_y y))$, and $L_S(\mathbf{P})$ and $L_D(\mathbf{P}, \mathbf{D})$ are:

$$L_D = \sqrt{(D_x - x)^2 + (D_y - y)^2 + (D_z - (m_x x + m_y y))^2} \quad (\text{A.5})$$

$$L_S = \sqrt{(S_x - x)^2 + (S_y - y)^2 + (S_z - (m_x x + m_y y))^2} \quad (\text{A.6})$$

Therefore the z components of each of the unit vectors $\hat{\mathbf{s}}$ and $\hat{\mathbf{d}}$ are:

$$d_z = \frac{D_z - (m_x x + m_y y)}{L_D} \quad (\text{A.7})$$

$$s_z = \frac{S_z - (m_x x + m_y y)}{L_S} \quad (\text{A.8})$$

Now, the partial derivatives of $L_D(\mathbf{P}, \mathbf{D})$ and $L_S(\mathbf{P})$ have an extra term relating to the tilt of the surface. An example calculation for $\frac{\partial L_D}{\partial x}$ is outlined below:

$$\begin{aligned} \frac{\partial L_D}{\partial x} &= \frac{\partial}{\partial x} \sqrt{(D_x - x)^2 + (D_y - y)^2 + (D_z - (m_x x + m_y y))^2} \\ &= -\frac{m_x (D_z - (m_x x + m_y y))}{L_D} - \frac{(D_x - x)}{L_D} \\ &= -m_x d_z - d_x \end{aligned} \quad (\text{A.9})$$

The remaining partial derivatives are given below:

$$\frac{\partial L_D}{\partial y} = -m_y d_z - d_y \quad (\text{A.10})$$

$$\frac{\partial L_S}{\partial x} = -m_x s_z - s_x \quad (\text{A.11})$$

$$\frac{\partial L_S}{\partial y} = -m_y s_z - s_y \quad (\text{A.12})$$

The derivation using the Taylor expansion method requires additional expressions for the partial derivatives of the components of the unit vector $\hat{\mathbf{d}} = (d_x, d_y, d_z)$ with respect to both x and y . To compute these, the quotient rule for differentiating a

function $h(x) = \frac{f(x)}{g(x)}$ is used:

$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2} \quad (\text{A.13})$$

This is applied as follows:

$$\begin{aligned} \frac{\partial d_x}{\partial x} &= \frac{\partial}{\partial x} \frac{D_x - x}{L_D} \\ &= \frac{\frac{\partial(D_x - x)}{\partial x} L_D - \frac{\partial L_D}{\partial x} (D_x - x)}{L_D^2} \\ &= \frac{(-1)L_D - (-d_x)(D_x - x)}{L_D^2} \\ &= \frac{1}{L_D} (d_x^2 - 1) \end{aligned} \quad (\text{A.14})$$

For $\frac{\partial d_y}{\partial x}$, the outcome has a slightly different form, as $\frac{\partial(D_y - y)}{\partial x} = 0$:

$$\begin{aligned} \frac{\partial d_y}{\partial x} &= \frac{\partial}{\partial x} \frac{D_y - y}{L_D} \\ &= \frac{\frac{\partial(D_y - y)}{\partial x} L_D - \frac{\partial L_D}{\partial x} (D_y - y)}{L_D^2} \\ &= \frac{(-d_x)(D_y - y)}{L_D^2} \\ &= \frac{1}{L_D} (d_x d_y) \end{aligned} \quad (\text{A.15})$$

The remaining partial derivatives are listed below:

$$\frac{\partial d_z}{\partial x} = \frac{1}{L_D} (d_x d_z) \quad (\text{A.16})$$

$$\frac{\partial d_x}{\partial y} = \frac{1}{L_D} (d_x d_y) \quad (\text{A.17})$$

$$\frac{\partial d_y}{\partial y} = \frac{1}{L_D} (d_y^2 - 1) \quad (\text{A.18})$$

$$\frac{\partial d_z}{\partial y} = \frac{1}{L_D} (d_y d_z) \quad (\text{A.19})$$

The same method is used to calculate the partial derivatives for tilted surfaces under the Taylor expansion approximation:

$$\begin{aligned} \frac{\partial d_x}{\partial x} &= \frac{\partial}{\partial x} \frac{D_x - x}{L_D} \\ &= \frac{\frac{\partial(D_x - x)}{\partial x} L_D - \frac{\partial L_D}{\partial x} (D_x - x)}{L_D^2} \\ &= \frac{(-1)L_D - (-m_x s_z - d_x)(D_x - x)}{L_D^2} \\ &= \frac{1}{L_D} (m_x d_x d_z + d_x^2 - 1) \end{aligned} \quad (\text{A.20})$$

Again, $\frac{\partial d_y}{\partial x}$ has a slightly different form:

$$\begin{aligned} \frac{\partial d_y}{\partial x} &= \frac{\partial}{\partial x} \frac{D_y - y}{L_D} \\ &= \frac{\frac{\partial(D_y - y)}{\partial x} L_D - \frac{\partial L_D}{\partial x} (D_y - y)}{L_D^2} \\ &= \frac{(-m_y s_z - d_x)(D_y - y)}{L_D^2} \\ &= \frac{1}{L_D} (m_y d_y d_z + d_x d_y) \end{aligned} \quad (\text{A.21})$$

Finally, the remaining equations follow the same pattern:

$$\frac{\partial d_z}{\partial x} = \frac{1}{L_D}(m_x d_z^2 + d_x d_z) \quad (\text{A.22})$$

$$\frac{\partial d_x}{\partial y} = \frac{1}{L_D}(m_y d_x d_z + d_x d_y) \quad (\text{A.23})$$

$$\frac{\partial d_y}{\partial y} = \frac{1}{L_D}(m_y d_y d_z + d_y^2 - 1) \quad (\text{A.24})$$

$$\frac{\partial d_z}{\partial y} = \frac{1}{L_D}(m_y d_z^2 + d_y d_z) \quad (\text{A.25})$$