

CRANFIELD UNIVERSITY

CAMERON FRASER LESLIE

A NOVEL CRYOGENIC STORAGE SOLUTION FOR
ATMOSPHERE-BREATHING ELECTRIC PROPULSION
SYSTEMS

SCHOOL OF AEROSPACE, TRANSPORT AND
MANUFACTURING

Astronautics and Space Engineering

Submitted for MSc
Academic Year: 2020 - 2021

Supervisor: Dr Stephen Hobbs
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This report is submitted in partial fulfilment of the requirements for the
degree of Astronautics and Space Engineering.

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Abstract

Recent interest in missions to Very Low Earth Orbit (VLEO) has led to the emergence of Atmosphere Breathing Electric Propulsion (ABEP). In theory this technology could enable missions to VLEO that forego the need to launch with propellant. This innovative concept could allow the full potential of low altitude regimes to be exploited as propellant consumption no longer limits lifetime. The most promising designs use a passive intake system that requires no moving parts to operate. Natural phenomena caused by the flow conditions at orbital velocity are exploited to provide an electric thruster with an adequate supply of gas. However a previously identified shortfall in its capabilities is that no propellant is stored during operation. This leaves the system at the mercy of the unpredictable conditions witnessed in Earth's atmosphere.

This thesis proposes a solution to this problem. If molecules ingested by a passive intake can be cooled to cryogenic temperatures, they can be temporarily captured so long as they are kept cold. This phenomenon has the potential to be exploited as an in-orbit propellant storage system.

To investigate this concept, firstly a baseline mathematical model for an ABEP system was constructed using current design methodologies. Secondly, this system was adapted to incorporate the novel storage concept. The resulting model was then compared to the baseline along common performance parameters in order to assess its feasibility.

The results showed that during the collection of propellant the requirements on flow conditions for adequate performance became more stringent. Nevertheless a feasible operating region was found and a baseline system chosen. This design point allowed for a propellant collection rate of between 0.43 mg/s and 0.74 mg/s. At this rate of collection, calculations speculated a potential for >1000 m/s of Δv over an 8 year mission could be gained. If realised, the potential mission applications are vast and could not only complement the performance of ABEP systems but open up novel mission concepts such as in-orbit refuelling.

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List of Abbreviations

ABEP	Atmosphere-Breathing Electric Propulsion
ABIE	Air-Breathing Ion Engine
ECR	Electron Cyclotron Resonance
ESA	European Space Agency
GOCE	Gravity Field and Steady-State Ocean Circulation Explorer
JB2008	Jacchia-Bowman 2008 Model
LAN	Longitude of Ascending Node
MABHET	Mars Atmosphere Breathing Hall Effect Thruster
MHD	Magnetohydrodynamic Thruster
NRLMSISE-00	Naval Research Laboratory Mass Spectrometer, Incoherent Scatter Radar Extended Model - 2000
PROFAC	Propulsive Fluid Accumulator
RAM-EP	Rarefied Atmosphere Breathing Electric Propulsion
SSO	Sun Synchronous Orbit
TRL	Technology Readiness Level
VLEO	Very Low Earth Orbit

1 | Introduction

The requirement to carry propellant into orbit is the limiting factor during most space missions. Whether maintaining an orbit or travelling to another, an adequate supply of propellant must be provided to perform manoeuvres and counteract perturbations. In all missions thus far propellant is launched with the spacecraft, limiting the available mass that could otherwise be occupied by payload. For recent missions to Very Low Earth Orbit (VLEO) this fundamental principle remains the same. Missions such as the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) have provided valuable information about Earth's magnetic field and the behaviour of our atmosphere [1]. However the lifetime of this mission was limited by the requirement to continuously compensate for atmospheric drag. Further interest in missions to VLEO has led to the development of Atmosphere-Breathing Electric Propulsion systems or ABEP. In theory, an ABEP system could supply an electric thruster using rarefied atmospheric gases as propellant. By removing the requirement to carry propellant, missions to VLEO are no longer limited by propellant mass, providing valuable mass for Earth Observation payloads [2]. Most current system concepts propose the use of a "passive" intake that ingests rarefied gases, compresses them and then supplies them to a thruster [3]. Significant advances have been made in recent years with recent ground tests showing promise [4].

During operation, passive systems do not store the ingested gases. For this reason they are at the mercy of the local atmosphere and are required to keep within favourable flow conditions. Should a storage solution be found, ABEP systems could be improved as propellant could be kept for performing manoeuvres or dealing with fluctuations in the local atmosphere [3]. This idea has yet to be fully realised due to the challenges associated with increased mass and the complexity of actively compressing propellant [5].

Collecting and storing atmosphere as a source of propellant is not a recent concept. Proposals as early as the 1950's suggested using nuclear powered harvesters to collect and liquefy air for use on other missions [6]. Although this is a very different implementation when compared to the solar electric systems of today, many of the underlying benefits and mission applications remain the same.

A potential means of capturing molecules ingested by a passive intake system is through condensation. In order for a gas to condense on a surface, it must be sufficiently cold. In vacuum science, this phenomena is used generate ultra-low pressure environments by trapping molecules on surfaces that are cooled to cryogenic temperatures. This device is known as a cryopump and is a well established tool used in space simulation, particle accelerators and fusion experiments [7]. Gases stored on the surface are held in place as a solid condensate. In theory they will remain in this state as long as adequate cooling is supplied. This is the basic principle for the storage concept proposed in this study.

The following thesis aims to investigate the feasibility of this concept. Several challenges must be overcome to suggest a valid design. Firstly, the performance of passive intakes used in ABEP systems must be high enough to allow for a usable supply of gas to a cryopump while also supplying a thruster. Secondly, cooling requirements for the cryopump must fall within reasonable power limits. Both these criteria were investigated thoroughly. After a feasible design space was identified, the performance of the system was modelled to assess its potential as a complement to current systems and to enable new mission opportunities.

1.1 Project Objectives

The project objectives changed significantly throughout its duration. Initially the focus was placed on assessing the current state of Atmosphere Breathing Propulsion systems through reviewing current literature. Upon investigation, it was found that the current developments in ABEP systems have resulted in an ideal platform for the introduction of a propellant collection system. Given the promise that was seen in the cryopump concept, the focus was steered towards this as a primary objective. The following aim and objectives are the latest version reflecting the focus on a cryopump storage system.

- **Project Aim:** Demonstrate the feasibility of a cryopump based storage system for use with an ABEP platform.
- **Project Objectives:**
 1. Perform a literature review spanning three focus areas: Air Breathing Electric Propulsion, Cryopumps and Space Cryocoolers.
 2. Construct a baseline system model reflecting the current work on Air Breathing Electric Propulsion

3. Devise a storage concept by adding a cryopump to an ABEP system and identify key design constraints.
4. Build upon the baseline model by adding the storage system.
5. Test the performance of the new model and assess its feasibility by identifying a valid design space.

2 | Literature Review

The scope of the project involved a detailed survey of literature over a broad range of fields. The following section provides a summary of findings that provided the basis for later analysis.

Firstly a detailed investigation of Atmosphere-Breathing Electric Propulsion and its encompassing topics was conducted to better understand the current state of the technology, working principles and its shortfalls. In relation to propellant collection, early concepts are also explored. Secondly literature on Cryopumps were reviewed with specific interest in finding crossovers with expected conditions in an ABEP system. As an extension of the Cryopump investigation, further research was done on space cryogenics to gauge the feasibility of implementing later concepts.

2.1 Air-Breathing Electric Propulsion

Atmosphere-Breathing Electric Propulsion is a proposed concept to utilise residual atmosphere as propellant for electric propulsion systems. In theory, this removes the requirement to launch with propellant on-board, allowing for lower altitude orbits to be utilised without limitations governed by propellant consumption. Significant interest in enabling long term missions in VLEO has driven development towards a common design architecture:

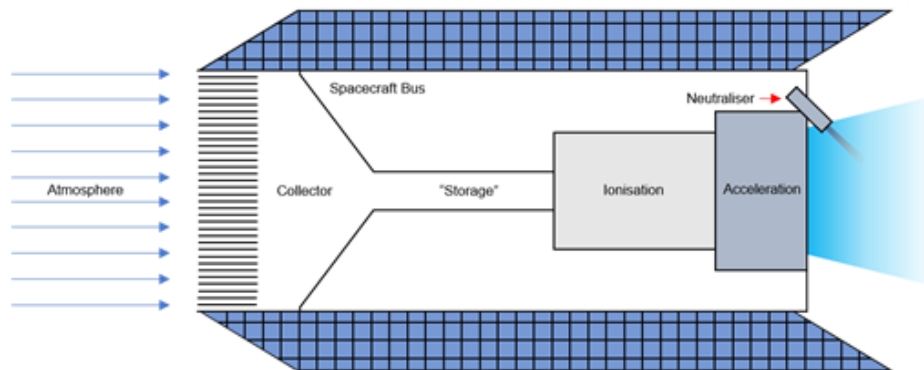


Figure 2.1: Example Atmosphere-Breathing Electric Propulsion Layout

Collection System

- *Intake*: An intake area that allows molecules to be ingested while minimising back-flow from the collection chamber.
- *Collection Chamber*: An open chamber or cavity that allows molecules to be "thermalised" before entering the thruster intake.

Thruster System

- *Ionisation*: When collecting neutral atmosphere, molecules must be ionised such that the resulting plasma can be accelerated.
- *Acceleration*: Thrust is produced by accelerating the charged particles.
- *Neutralisation*: To prevent the plume of charged particles returning to the body of the spacecraft, the exhausted propellant must be neutral. In the case of electrostatic propulsion this must be deliberately induced using a neutraliser. In other forms this is done as a part of the acceleration process.

Figure 2.1 illustrates a typical layout. Although this common architecture is shared among many concepts, specific configurations vary. The following sections discuss the literature on ABEP Intake theory, the implications for utilising atmosphere in electric thrusters, and current system designs.

2.1.1 Electric Propulsion for Air Breathing Applications

Extensive literature exists on the operation of electric thruster systems utilising atmosphere, or its individual components, as propellant. Although a detailed look at their design principles is beyond the scope of this study, a general discussion on their limitations driving intake design are important for later system models. Special consideration must be given to the operating conditions of a potential Atmosphere-Breathing thruster. In particular, the "propellant" (atmosphere) composition and density have direct implications on system choice.

To limit the scope of the survey, electric propulsion systems that have not yet been investigated for atmosphere-breathing applications were omitted. Of the systems that have been tested with atmospheric gases, the following sections outlined key findings of those that have been tested alongside a passive intake. All systems that have been tested with an intake system have been electrostatic thrusters.

Electrostatic thrusters utilise Coulomb forces to accelerate charged particles [8, 9]. Usually this is achieved through ionising a neutral propellant through electron bombardment, producing positively charged ions that can be easily accelerated using electrostatic fields. All electrostatic thrusters operate on the aforementioned thruster stages: ionisation, acceleration and neutralisation. Thrusters of this type can be further categorised based on variations of these stages:

- *Gridded Ion Thrusters (GITs)* inject propellant into the ionisation chamber before extracting ions from the plasma using a set of grids across which there is a voltage differential. Plasma can be generated in several ways, most often either through a thermionic cathode, or induction using microwave or radio frequency generators (Radiofrequency Ion Thrusters (RIT)). GITs typically have the higher efficiency out of electrostatic systems with Specific impulses of 2000-10,000 s.
- *Hall Effect Thrusters (HETs)* generate plasma in an annular chamber in which electrons are constrained by a magnetic field. Neutral gas is injected into the chamber and ionised through collision with the electrons and accelerated by an electric field perpendicular to the magnetic field, resulting in thrust. Typically these systems are simpler and have higher thrust to power ratios, at the cost of specific impulse.

The performance of a given thruster depends on these processes and as such the suitability of different systems for use with atmospheric gases has been explored in a number of studies. Existing RITs were investigated by Cifali et al. for use in an ABEP system through experimental testing of the RIT-10 with O₂ and N₂ [10, 11]. In general, an increase in power was required for the same thrust during air-breathing operation due to the higher ionisation energy of both gases. This higher energy requirement also forced the use of argon to ignite the the plasma discharge process. Upon ignition the run was successful, generating thrust of up to around 6 mN for a 450 W supply power.

Early works by Tagawa et al proposed the use of a gridded ion engine with an Electron Cyclotron Resonance (ECR) plasma source [12]. It was identified that the pressure of the gases available to an air breathing thruster would require special consideration when picking an ion source. The ECR source was chosen as it maintained a reasonable level of utilisation efficiency at low pressures of 0.005 to 0.5 Pa. This was then set as the target pressure for the intake. Other studies of a similar nature come to a consistent pressure requirement of 1×10^{-3} Pa [3, 13]

2.1.2 Passive Intakes

Passive intakes take advantage of the inherent flow conditions seen in a typical orbit to collect and compress the atmosphere using only geometric design [3]. The flow is usually assumed to be free molecular at altitudes above 120 km, where the mean free path is larger than the intake diameter and collisions are uncommon. These conditions require special consideration from an intake design perspective as the flow must be treated as a rarefied gas.

An important characteristic of these interactions is momentum accommodation; the extent to which a particle transfers its kinetic energy to a surface upon collision. Interactions where molecules transfer all or retain only a negligible portion of their kinetic energy are said to be diffuse. Under these conditions, molecules interacting with a surface are emitted at a velocity dependant on the temperature of the surface in a random direction. This condition is a common assumption in design studies.

To achieve the necessary compression and mass flow requirements for the ABIE, Nishiyama et al suggested the "collimator" configuration [12]; a series of small tubular ducts followed by a larger chamber. Given the flow is largely normal to the direction of the intake the likelihood of a collision with the tube walls is reduced. Particles that make it through the ducts interact with walls of the intake chamber and are diffusely emitted. Flow in this state is called "thermalised" flow as the velocity is entirely a function of the wall temperature. Since the flow is no longer travelling along the direction of the ducts there is a lower likelihood of transfer back through the intake. This produces a net compression.



Figure 2.2: Diagram of Collimator Principle [5]

Later work by on the ABIE by Tagawa et al [14] using DSMC simulations identified an important trade-off between compression ratio and intake efficiency: Increasing the length of the collimator tubes decreases their transmission probability, decreasing backflow and

increasing the density ratio. However since the flow is not perfectly collimated, wall interactions during inflow reduce the capture probability, hence reducing available mass flow. Extending the transmission probability principle to the thruster, theoretical work by Barral et al [5] drew further attention to this principle, noting that the overall performance of a passive intake depends on the probability of intake transfer, backflow, and thruster chamber transmission. Within this work Barral also identified a minimum chamber density of 10^{18} m^{-3} in line with current electric thruster requirements.

Extensive work determining optimum performance criteria for intake systems was performed by Binder et al. The study focused on methods for analytically determining the transmission probabilities within intakes using analytical expressions [15]. In the case of thermalised flow, Clausings method allows very simple calculation of transmission probabilities through a finite tube [16]. In the case of collimated flow, Hughes' method was used to determine the transmission through a finite tube with a non-zero entrance velocity [17]. These analytical methods were validated with DSMC studies and found to have good agreement.

2.1.3 Active Intakes & Compression

As outlined previously, a common trade-off in passive intake design is the balance between compression ratio and intake efficiency. One solution to overcome the limitations in this area is a form of active intakes, whereby components, usually pumps of some kind, actively compress the captured gas, allowing capture efficiency to be optimised while maintaining high thruster inlet pressures. Furthermore, the compression limits given passive methods can be exceeded so increased to levels akin to the working density of current thrusters.

Active compression methods often appear in preliminary discussions; however, few attempts at implementation were found. Analytical estimates for the required pumping speeds were performed by Barral et al [5]. This investigation suggested a passive intake for pre-compression followed by a second active compressor. Assuming a pre-compression ratio of 100 at 200 km altitude, a second stage inlet condition of $4 \times 10^{-3} \text{ Pa}$ was taken as reference. The results found a the required pumping speed to be $31 \text{ m}^3/\text{s}$ per 1 m^2 of pump area. Due to considerations of high pumping requirement and probable associated increase in mass and complexity, passive intakes were exclusively chosen for later studies.

A complete intake design was suggested by Li et al [18] with a vacuum air-intake Device. several pumps were placed in series, providing a total of four compression stages. The first stage consisted of a passive intake similar to those described previously. This was followed

by a large multi-stage turbo pump, a smaller turbomolecular pump, and finally a miniature scroll pump. A diagram of the system can be seen in Figure 2.3.

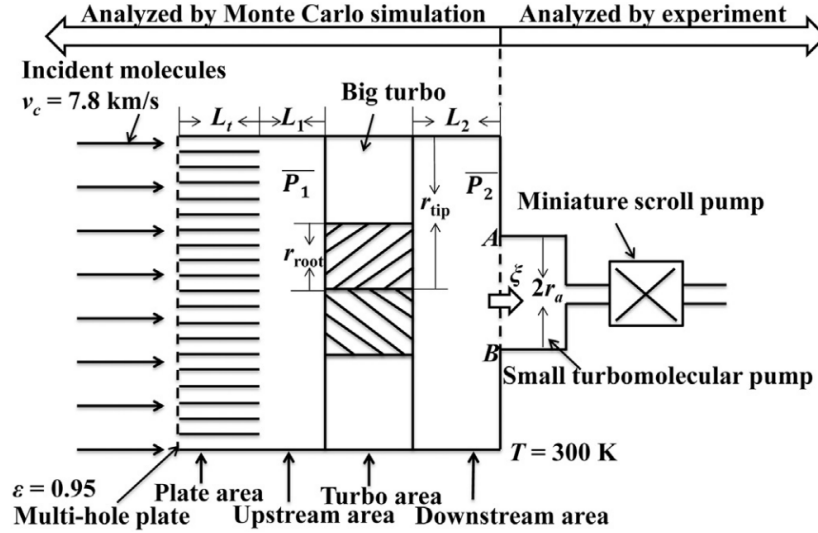


Figure 2.3: Schematic Diagram of vacuum air-intake device [18]

The first two stages were investigated using DSMC with the final two pumps tested experimentally. Results showed increases in efficiency over passive studies in the range of 41-58% depending on rotation speed and compression to $8.5 \cdot 10^4$ Pa. This performance was estimated to have reasonable power requirement in the range of 39-203 W. Although promising values, the complexity and mass of the system are the primary concerns illustrated in other papers. The experimentally tested pumps had a reasonable mass of 6.54 kg however operated at 60,000 rpm. Suggestions are made to improve reliability through friction-less magnetic bearings, however this was not investigated in detail.

2.1.4 Full System Concepts & Mission Design

The ground work for the design principles discussed previously were established by several important studies and feasibility concepts. These studies proved instrumental in the maturation of the technology and led to the mission proposals that exist today. A consistent finding is that performance requirement and hence entire system designs are highly mission specific. This combined with the low maturity of ABEP has led to wide variations in mission design and hence system design. The following section will outline the general scope of these proposed concepts, primarily identifying the key characteristics from a systems perspective.

Air Breathing Ion Engine (ABIE)

A preliminary design study followed by subsequent tests of the "Air Breathing Ion Engine" were performed by the University of Kobe starting in 2003 [12]. A general altitude range of 150 - 200 km was chosen for investigation of a propulsion platform consisting of an annular "collimator" intake surrounding the satellite bus with an Electron Cyclotron Resonance ion source, followed by conventional electrostatic acceleration using a set of grids.

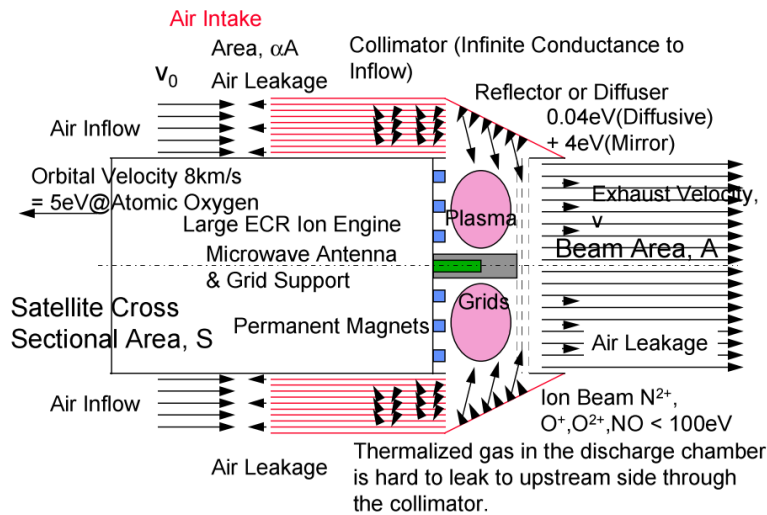


Figure 2.4: Air Breathing Ion Engine (ABIE) Concept, JAXA. [12]

To reduce losses in performance an Electron Cyclotron Resonance plasma generator to ionise the collected atmosphere given the lower requirements in feed pressure when compared to alternatives. The plasma is generated via a microwave antenna resonating with a local magnetic field. A set of Grids then accelerate the Ions, providing thrust. Experimental tests performed in 2013 successfully simulated a system operating below 200 km using an ECR plasma generator as an atomic oxygen plasma source.

ESA: RAM-EP

Initial system level requirements and a feasibility study was conducted by D. Di Cara et al with ESA in 2007 [3]. A focus was placed on passive intakes that would allow operation with existing electrostatic thrusters. Initial designs for an intake along with an extensive test campaign to assess the feasibility of operating Hall Effect Thrusters and Radio Frequency Ion Thrusters using atmospheric molecular propellants were carried out.

When investigating the applicable range for Atmosphere-Breathing Electric Propulsion systems, power was found to impose a lower limit of 180 km. Furthermore, above 250 km, conventional electric propulsion was found to be more competitive as less than 100 kg would be required for an 8 year mission. A system design study also assessed the possibility of a duty cycle for on/off thruster times, finding a ratio 5:6 ratio of thruster on time to be beneficial to account for eclipses. An example earth observation mission in a SSO was then proposed for further investigation.

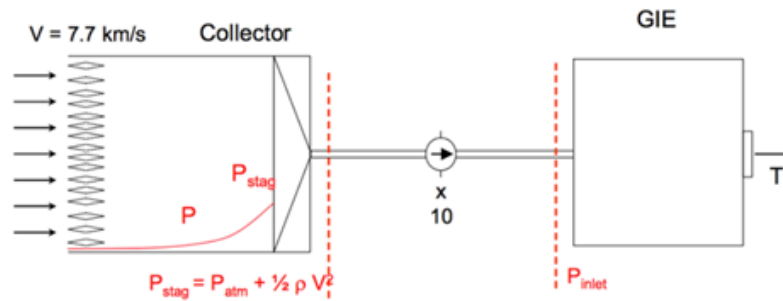


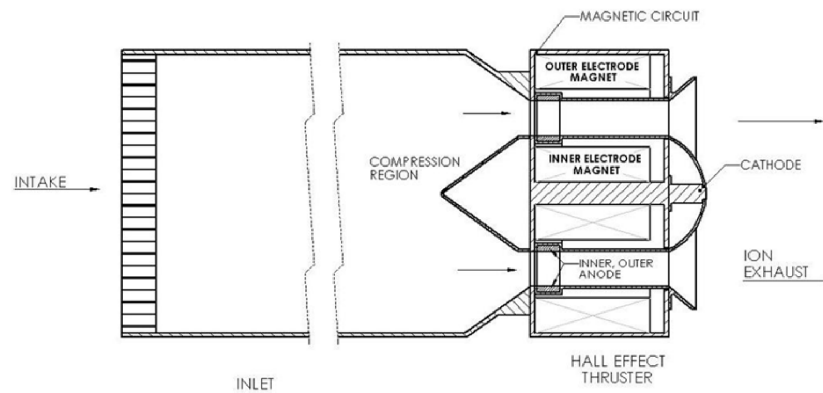
Figure 2.5: Preliminary RAM-EP Concept, ESA. [3]

The proposed collector design differed from the more common collimator, using a series of baffles to prevent back-flow. Simulations to evaluate the effect of collector length on pressure delivery performance were performed, finding an optimum length of 1m for a 0.6 m² intake to be adequate for providing a constant pressure of $>10^{-3}$ Pa to the thruster without significant fluctuation. Based on performance models provided by manufacturers, an assessment of potential candidate thrusters was conducted using drag calculations for a given altitude and an available power of 1 kW. A RIT10 gridded ion thruster was chosen as the primary candidate.

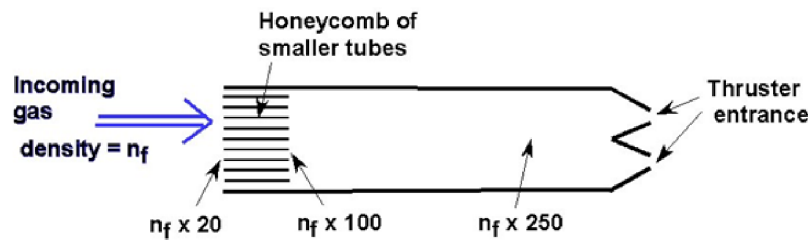
The RAM-EP system was applied to a reference mission intended for technology demonstration and to further the concept TRL. The mission profile attempted to account for fluctuations in atmospheric density due to the 11 year solar cycle by altering the operational altitude in accordance with changes. During the first three years, during a solar minimum, a Sun Synchronous Orbit at 200 km was chosen. During the rise in solar activity, another three years would be spent climbing to 250 km. Another year would be spent decreasing altitude at a constant rate of decay until before de-orbiting. A full breakdown of proposed sub-system specifications were theorised, most notably an operating power of 3297 W BOL and 2900 W EOL.

Mars Atmosphere Breathing Hall Effect Thruster (MABHET)

Another notable mission concept is the MABHET; pioneered by BUSEK Inc and NASA for a low altitude Mars observation mission. A target altitude of 120-180 km was chosen as the lowest possible given power requirements to overcome drag. The most notable changes given the martian atmosphere is the reduction in available solar power, and the majority CO_2 atmosphere. A system baseline of 1 kW was found to provide a good balance between solar panel drag estimates and available power for a 0.15 m^2 frontal area assuming a drag coefficient between 2.2 and 4.



(a) System Schematic



(b) Intake Principle

Figure 2.6: MABHET Concept Schematic and Intake Design. [19]

Notable comparisons can be drawn to the early study performed by Di Cara et al on the effects of long intake ducts after a collimator. This principle was also used by the MAHBET concept to increase compression to a suggested 250 as seen in Figure 2.6b

SITAEL: RAM-HET

Further to the feasibility analysis conducted by ESA, an extensive design and test campaign was conducted by Andreussi et al, eventually leading to SITAEL's RAM-EP system. The concept consists of a "split-ring" duct system that maintains a constant aspect ratio while maximising geometric packing [5]. Once gathered the flow is thermalised before transferring to the first stage of the RAM-HET. The system was successfully fired using a mixture of 1.27 $\text{N}_2\text{-O}_2$ for a simulated altitude of 200 km.

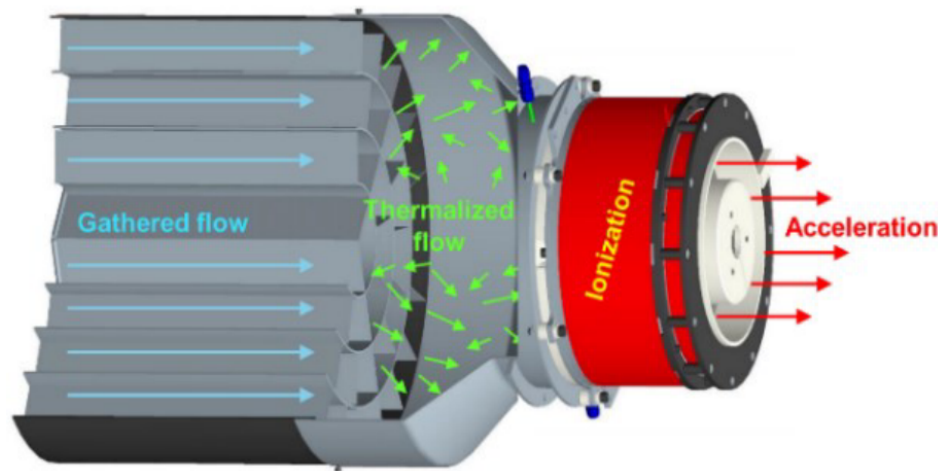


Figure 2.7: RAM-EP Concept with RAM-HET, SITAEL. [4]

A systems optimisation study was performed based on specific design altitudes of 210 km, 195 km, 180 km. Spacecraft parameters were optimised for Increased thrust, decreased drag, and minimum power. As a general trend, lower altitudes required increased power to compensate for drag however also resulted in a lower maximum Thrust/Drag ratio. Due to the dependence on solar panel size for power, the overall mass of the spacecraft decreased with increasing altitude during optimisation for particular design altitudes. For this reason a nominal target altitude of 210 km was chosen. Of the optimum solutions found thruster input power varied from 480 W to 1570 W while the full system power ranged from 770 W to 1970 W. The proposed mission profile further featured a descent to 180 km to test controlled re-entry, followed by an optional raising burn, returning to 210 km. This manoeuvre is intended to test the ability to increase the orbit when planning for a lack of thrust during an eclipse. This demonstrator mission is planned for a launch between 2024 and 2026.

2.2 Propellant Collection

Prior to the concepts that defined ABEP in the 21st century, several notable concepts from the 1950s and 1960s attempted to propose propellant collection methods that implemented storage solutions. Little comparisons can be drawn to the methods that are explored in this study as concepts of this era often suggest nuclear systems with power in the MW range. However it is worth noting the target altitudes and general concept architecture.

Air Scooping Vehicle

Berner and Comac proposed a refueling station concept called an "Air Scooping Vehicle" [20]. This would provide liquified atmosphere to other vehicles as a method of reducing launch costs.

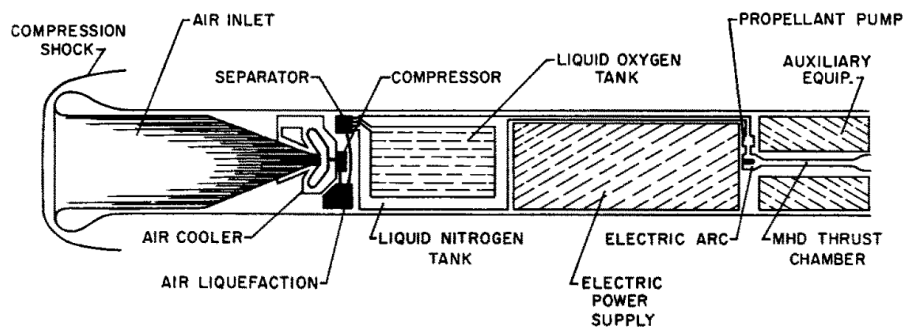


Figure 2.8: Air Scooping Vehicle [20]

Two systems were proposed. One powered by a nuclear reactor and the other solar panels. In both systems the air was "scooped" by an intake system at the front of the spacecraft before being liquefied. The oxygen and nitrogen are then separated into two tanks. The Nitrogen is used to propel the spacecraft using a magnetohydrodynamic (MHD) thruster. Oxygen is then kept for supplying an oxidiser for other missions. The nuclear system was found to have a minimum altitude of 300,000 ft (91 km). This is significantly below the considered region for modern ABEP studies. However the solar powered system was found to have a minimum altitude of 550,000 ft (168 km). This is very similar to findings in recent studies. To liquify the nitrogen at the inlet with commercial liquification units at the time, the inlet pressure at the end of the air scoop was required to be 10 cm Hg (Around 13 kPa). This is far above today's estimates for the capabilities of passive intakes.

2.2.1 Propulsive Fluid Accumulator

The Propulsive Fluid Accumulator or PROFAC was a similar concept proposed by Demetriades in the late 1950s [21, 6, 22].

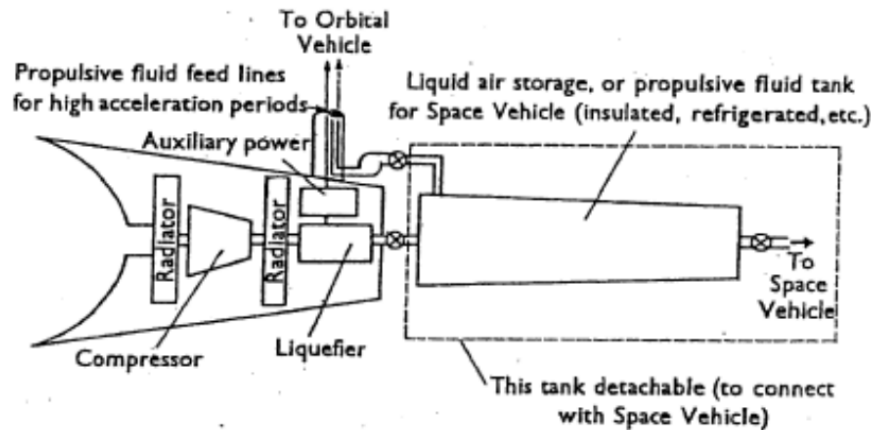


Figure 2.9: Propulsive Fluid Accumulator Concept [6]

The initial proposal is similar to that of the Air-Scooping Vehicle. A large intake collects air and delivers it to an air liquifyer. However prior to liquifying the air it is put through a compressor. This system is intended for use with several other vehicles. An "orbital vehicle" consisting of a large 10m^2 inlet provides thrust using a large MHD thruster. This orbital vehicle also uses atmosphere however similar to modern studies it is not collected, only used as ingested. PROFAC is connected to this larger vehicle, relying on it for thrust. Other vehicles then connect to PROFAC to replenish their propellant while in orbit. This collective system required 6 MW of power supplied by a nuclear reactor. In a later iteration of the design, other methods of air collection are suggested [22]. Most notably a cryopump. Once collected air is passed through a series of heat exchangers until it is frozen at 30 K. A cryocooling cycle is also proposed using helium gas in a modified brayton cycle.

2.3 Cryopumps

A cryopump is typically defined as a vacuum pump that captures molecules through the cooling of surfaces below 120 K [7]. As a form of capture pump, cryopumps retain the gas they are pumping. This means they temporarily store the gas as a means to remove them from a vacuum chamber as opposed to generating a continuous flow. This is done through two mechanisms [23]:

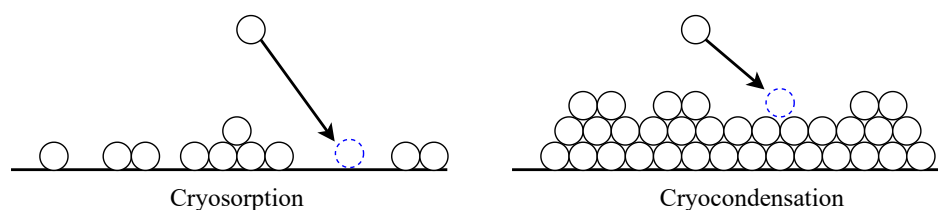


Figure 2.10: Cryopump Mechanisms. Adapted from [24]

- *Cryosorption* is the process in which a gas is pumped onto a surface or sorbent material. Gases impinging on surface lose energy in the form of heat. If the surface is cold enough, so much energy is lost that the weak intermolecular forces are enough to hold the molecule to the surface. This process is also called physisorption with "cryo" denoting this process as low temperatures [7].
- *Cryocondensation* occurs after sufficient surface coverage results in the molecules sticking to the molecules already on the surface. In this scenario the gas essentially undergoes a phase change directly to solid form [24].

In the case of cryocondensation, a gas can be pumped if the pressure above the surface is above the saturation pressure of the gas in question. Saturation pressure reduces with temperature. Hence to achieve the capture of a gas the cryosurface must be cooled such that the saturation pressure drops below the pressure in the environment. In the case of a vacuum chamber, the chamber pressure will continue to drop until the saturation pressure for said temperature is reached. For this reason the ultimate final pressure is dictated by the ability to cool the surface to low enough temperatures [7].

Smaller molecules such as hydrogen and helium tend to have a higher saturation pressure for a given temperature and so are more difficult to pump. For comparison the required temperature to condense hydrogen is 4 K, whereas only 100 K is needed for water. In the case of atmospheric components, 20 K is the commonly quoted requirement [7, 25, 24].

In general cryosorption is used to pump the smaller molecules. Interactions between the molecule and the surface allow for higher saturation pressures. However after a single monolayer is formed cryocondensation occurs and the effect is lost. For this reason, sorbent materials with large surface areas such as activated charcoal are often used to pump helium and hydrogen. This method is used alongside cryocondensation in a typical layout seen in Figure 2.12.

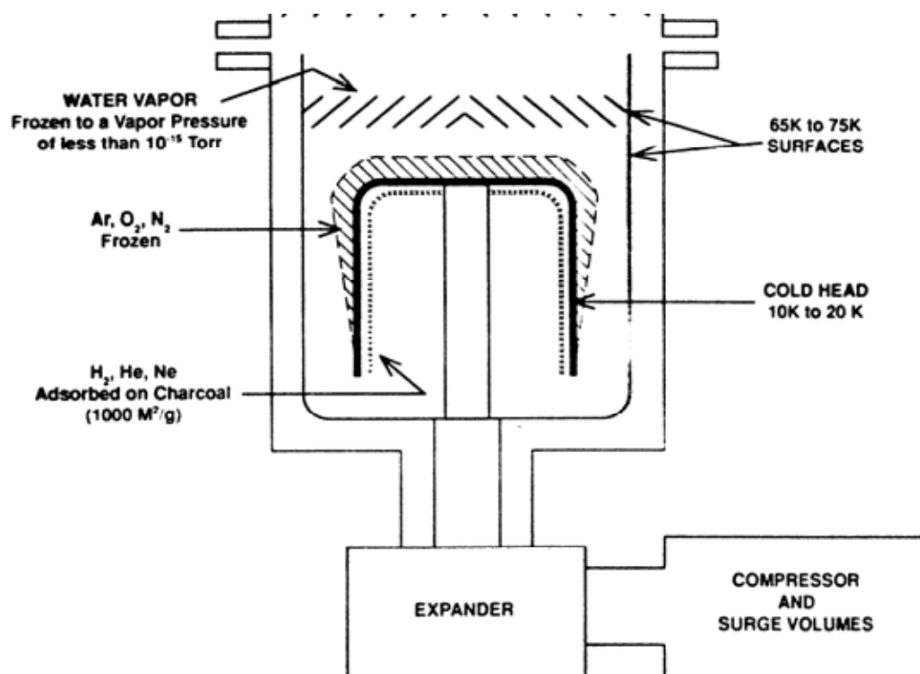


Figure 2.11: Typical cryopump configuration[7].

With specific reference to pumping components of the atmosphere, O₂ and N₂ are usually pumped using cryocondensation at 20 K. In the typical layout shown, a series of baffles can be observed at the entrance to the pump. In most cases in vacuum conditions, radiation is the largest contributor to heat load on the cryosurface. Furthermore, even if enough cooling power is supplied to the head, radiation will melt the upper layers of the collected ice and reduce the effectiveness of the pump. For this reason cooled baffles are used as a shield. These are usually cooled to between 60 - 100 K depending on the configuration.

2.4 Cryocoolers in Space

Cryocoolers occupy a crucial role in modern space missions. By far the most common application is the cooling of instrumentation used for astrophysics [26]. Many variants of cooling systems exist for different applications depending on temperature requirements.

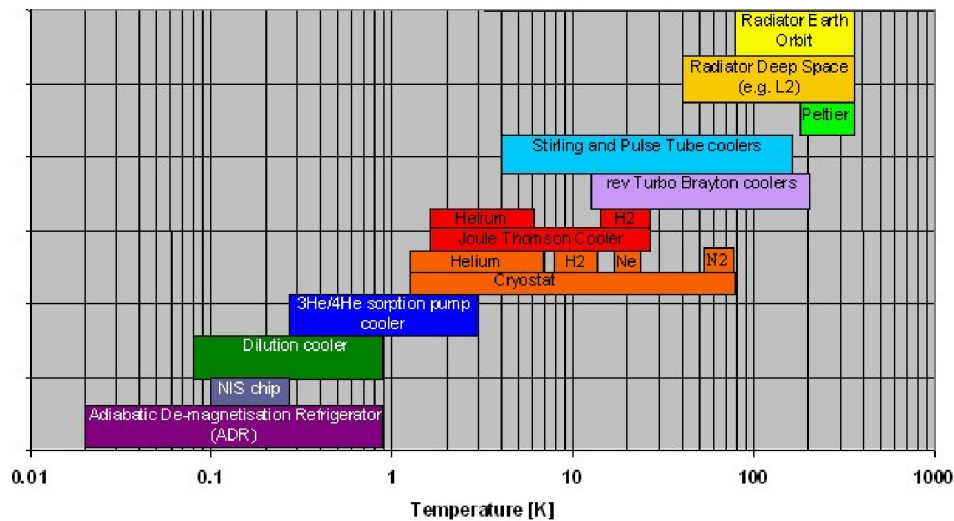


Figure 2.12: Space Cooler Operating Ranges [26]

Of the available temperature ranges for various technologies, the 10-100 K region is of most interest when applied to the cryocondensation of atmospheric gases. These systems can be roughly broken down into two categories [27]:

- *Stored Cryogen* systems use sublimation to adsorb the head load at the "cold end". Solid or liquid cryogens can be used depending on application. Solid cryogens are better suited for the microgravity environment however are limited to 10 K. Alternatively liquid systems allow <5 K. The main disadvantage is in lifetime, as the cryogen is vented as it is used. The requirement to carry cryogens limits their operational use several years or in some cases several months.
- *Mechanical* systems were primarily developed to overcome the lifetime constraints in stored cryogen coolers. Typically a compressor is used to generate a pressure difference in a conventional refrigeration cycle where heat is removed at one end as the gas expands and dissipated at the other when the gas is compressed.

For the application of continuous cooling for propellant storage an obvious benefit can be

seen in mechanical coolers. These systems are now a mature technology and have significant heritage and proven reliability. Multi stage systems are capable of providing temperatures as low as 1.7 K for over long periods of time however special consideration must be given to the vibrations produced by the compressor.

Two stage systems are particularly interesting from the perspective of a cryopump system. It is likely that multiple temperature levels will be required for the baffle and the cryopump head. Systems that operates with a first stage temperature of <100 K and a second stage temperature of 20 K are ideal for this application.

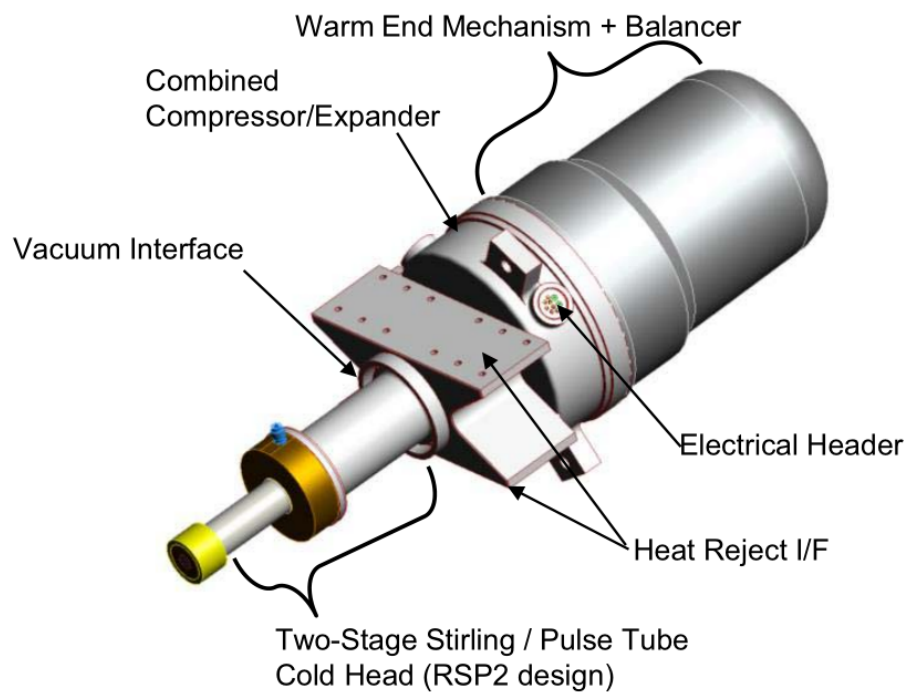


Figure 2.13: RSP2 Cryocooler [28]

The RSP2 cryocooler developed by Ratheon consists of a Two Stage Stirling Pulse Tube hybrid cycle [28]. Both stages are capable of independent temperature control. Several variants exist with varying power, mass and load performance. The most notable variant is the Compact Inline design, allowing a first stage heat load of 4 W and a second stage load of 0.25 W at 20 K with an input power of 200 W. Coolers of this class fit very well with the requirements for a cryopump system and as such were used as a baseline for feasibility later in the study.

3 | Atmospheric Model & Flow Conditions

The performance of any atmosphere breathing system depends on the local environment experienced throughout a given orbit. Earth's atmosphere is far from constant. Noticeable changes in composition, temperature and wind velocity are experienced over the course of an orbit and even larger changes throughout a mission due to solar activity. It is therefore important to choose and implement a suitable atmospheric model that provides a good measure of expected ambient conditions.

The changes in Earth's atmosphere stem primarily from the solar cycle, the day-night cycle of an orbit, and the geomagnetic properties of the Earth itself. This produces complex variations in important inlet parameters over the course of a single orbit and throughout a typical mission that must be considered in the design phase. It is common practice to evaluate the performance of a system through averaging the local atmosphere over a single orbit at a given altitude, providing an adequate baseline prior to more complex performance simulations [29, 30, 15] using orbit propagators. As a first step in evaluating system performance this method will be adopted by averaging the conditions over a specified circular orbit using simple analytical expressions. The resulting average can then be compared for different altitudes and the performance estimated. The following section will outline the method for determining the said average conditions.

3.1 Atmospheric Model

Earth's atmosphere has a distinct structure constituting regions of specific behaviour and composition. VLEO can be generally be defined as the region below 450 km, where the effects of atmospheric drag significantly reduce the life of a satellite [2]. VLEO is therefore entirely confined within the thermosphere located between the mesopause at 90 km and the exosphere, where the mean free path is approximately equal to the vertical scale height (400-600 km depending on geomagnetic and solar activity) [31].

In the majority of space applications, and in the case of ABEP, knowledge of the neutral

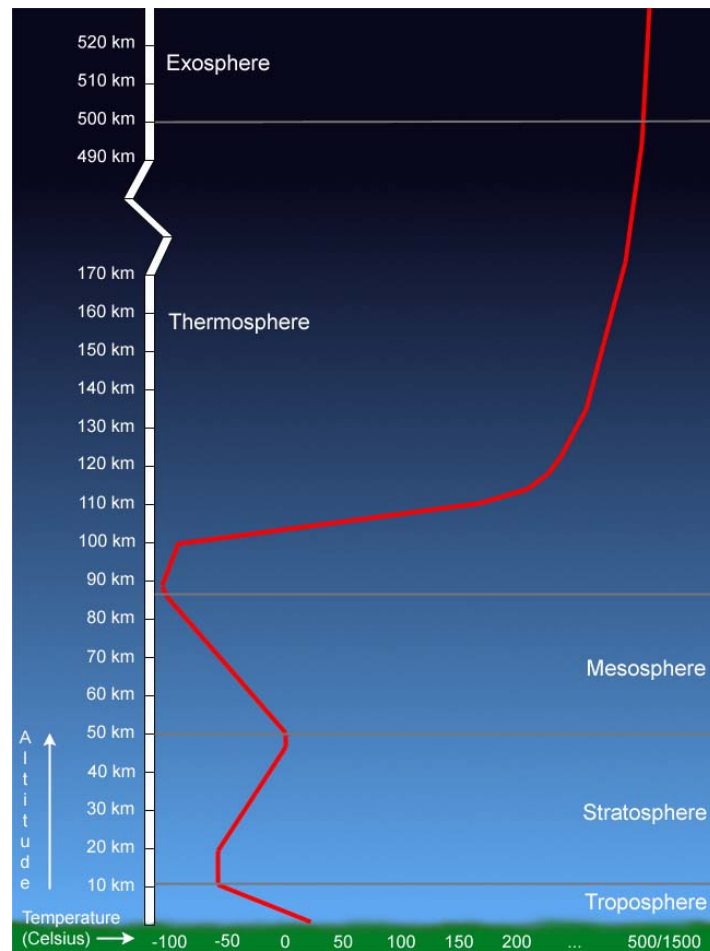


Figure 3.1: Regions of Earth's Atmosphere and corresponding temperature [31]

species composition, density and temperature is required. Specific standards exist to assist in selection of up to date models for the relevant application [31]. As of 2021 the accepted standard empirical models for neutral densities are JB2008 (Jacchia-Bowman 2008 Model) and NRLMSISE-00 (Naval Research Laboratory Mass Spectrometer, Incoherent Scatter Radar Extended Model - 2000). JB2008 describes the neutral temperature and total density within the thermosphere. It is currently the most up to date and accurate representation of the total density out of available models, however it does not include a representation of composition. NRLMSISE-00 also models the neutral temperature, however represents atmospheric density by its individual species. Despite the greater accuracy of JB2008 in total density, a description of composition is needed to determine the major species that are available to the system. Furthermore, both models have been compared in previous literature focused on ABEP and were found to have good agreement for total density [30, 29]. For this reason, along with the availability of native MATLAB toolboxes, the NRLMSISE-00 model was chosen.

3.1.1 NRLMSISE-00 Description and Use

The MSIS series of models have significant heritage in determining atmospheric composition for space applications [31]. Each iteration utilises data from various satellites and ground based radar systems to build an empirical model from ground to the exosphere. The latest version uses solar and geomagnetic activities as inputs to predict species number densities and neutral temperature.

Solar activity is primarily dictated by the 11 year solar cycle, a consistent pattern of approximately sinusoidal fluctuation reaching a maximum every 11 years. Incident solar flux is measured through ground observation and reported at specific wavelengths by the "solar indices". The F10.7 index, used by NRLMSISE-00, represents radio flux at 10.7 cm (2800 MHz) which has been shown to be a reliable indicator for general solar activity. Geomagnetic disturbances caused by complex interactions between the Earth's magnetic field and solar wind cause further fluctuations in the atmosphere. The intensity of such disturbances is reported through the K-index, representing the magnitude of geomagnetic storms on a logarithmic scale from 0-9 with 1 being calm behaviour and 5 or above indicating a geomagnetic storm. Collection of data throughout each day across a range of 13 mid latitude weather stations determines the "planetary" index K_p . NRLMSISE-00 Utilises the daily average A_p represented as an equivalent linear scale from 0-400.

Although modern atmospheric models show good agreement with atmospheric composition and temperature values for given solar and geomagnetic activities, the prediction of said activities is far less accurate. It is therefore common to remove time dependence from system design by assigning arbitrary levels of solar and geomagnetic activity. A summary table of recommended values for 81 day averaged low, moderate and high activity levels are shown in Table 3.1.

Table 3.1: Recommended long term solar and geomagnetic activity indices used by NRLMSISE-00

Index	Low	Moderate	High
$F_{10.7}$	65	140	250
$F_{10.7_{avg}}$	65	140	250
A_p	0	15	45

The model requires either a specific date and time or manual selection of solar indices along with position in altitude, latitude and longitude. The inputs are used to empirically

determine the total atmospheric density, individual species number densities and the absolute temperature at the corresponding position and activity. A breakdown of inputs and outputs to the model are shown in Figure 3.2. MATLAB's Aerospace Toolbox was used to implement the model for later analysis.

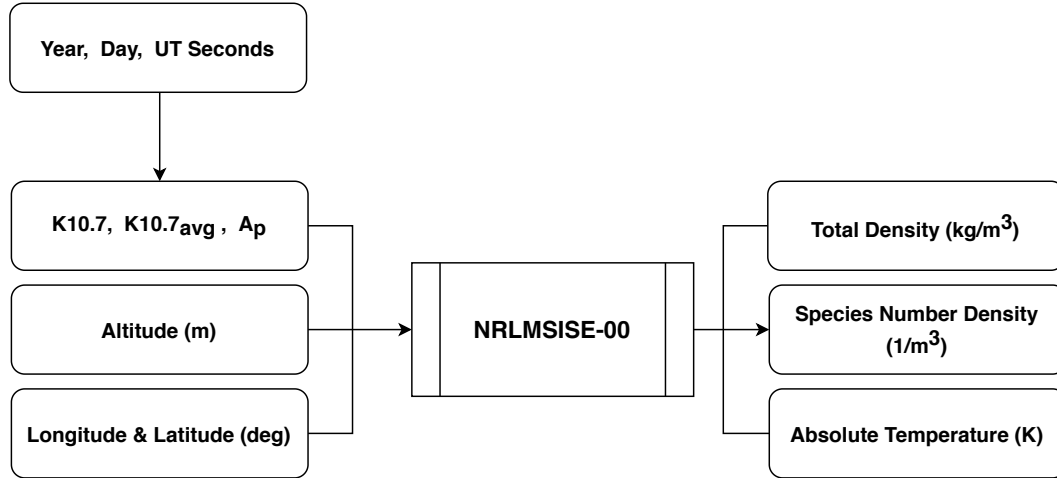


Figure 3.2: Input/Output diagram for NRLMSISE-00 Atmospheric Model

3.2 Orbit Definitions

Spatially averaging the variations in atmospheric conditions over an orbit for a given level of activity is often used as a first approximation of conditions at a design altitude [29, 30].

The majority of studies relating to ABEP assume a Sun Synchronous Orbit (SSO) to exploit access to continuous power. Given that the potential for continuous collection even during eclipse periods is a topic of interest in this project, several orbit profiles were chosen for reference performances. An equatorial circular orbit with an inclination of $i = 0$ and sun synchronous orbit with a Longitude of Ascending Node (LAN) of $\Omega = 0$, the inclination of which changes with altitude due to the J2 Perturbation. Note that the LAN will change with the time of year however since longitudinal variations are low, a constant LAN was set for later analysis.

For both orbit regimes, semi-major-axis a is discretised between altitudes of 120 km and 450 km. For each specified a the true anomaly θ is discretised between 0 and 2π at an equal number of intervals to a . Positions in Keplerian elements are then converted to Altitude h , Latitude Λ and Longitude φ . Although the ability to investigate an equatorial orbit was

Table 3.2: Considered Orbit Regimes in Keplerian Elements

Orbit	$\theta(\text{rad})$	$a(\text{km})$	$i(\text{rad})$	$e(\text{rad})$	$\Omega(\text{rad})$	$\omega(\text{rad})$
SSO	$0-2\pi$	6498-6828	$i(a)$	0	0	0
Equatorial	$0-2\pi$	6498-6828	0	0	0	0

Note: inclination in SSO is a function of semi major axis (See Appendix C)

implemented in the model, only one case was investigated due to time constraints. The sun synchronous case was chosen for all later analysis to improve comparisons to other studies.

3.3 Flow Characteristics & Composition

To investigate the behaviour of critical parameters relating to the performance of atmosphere-breathing systems, orbit averaged values for composition and temperature were determined using NRLMSISE-00 across a range of altitudes and solar activity. Several further important parameters defining the intake flow characteristics were calculated from the results. Firstly the mean free path of the molecules, to determine if free molecular flow conditions could be assumed in the region of interest, and the Speed Ratio of incident molecules along the spacecraft's trajectory.

3.3.1 Composition

Figure 3.3 shows the changes in species composition with altitude at moderate solar conditions. A total of eight species are present with molecular nitrogen (N_2) and atomic oxygen (O) constituting the majority at all altitudes of interest.

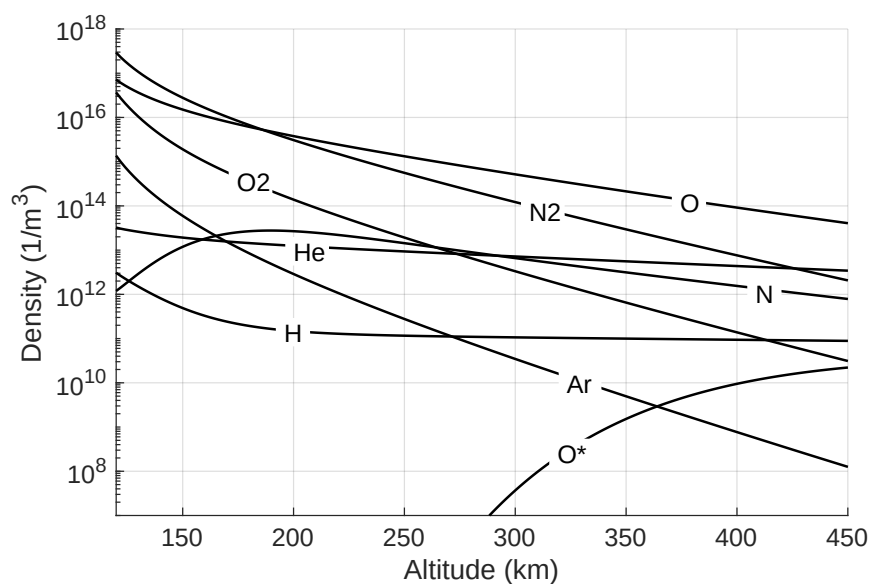


Figure 3.3: Changes in composition of the atmosphere with altitude during moderate solar activity

Figure 3.4 represents the proportion of each constituent species to the total number density. It can be seen that N_2 and O account for 90% of the composition at all altitudes however the ratio between them changes significantly.

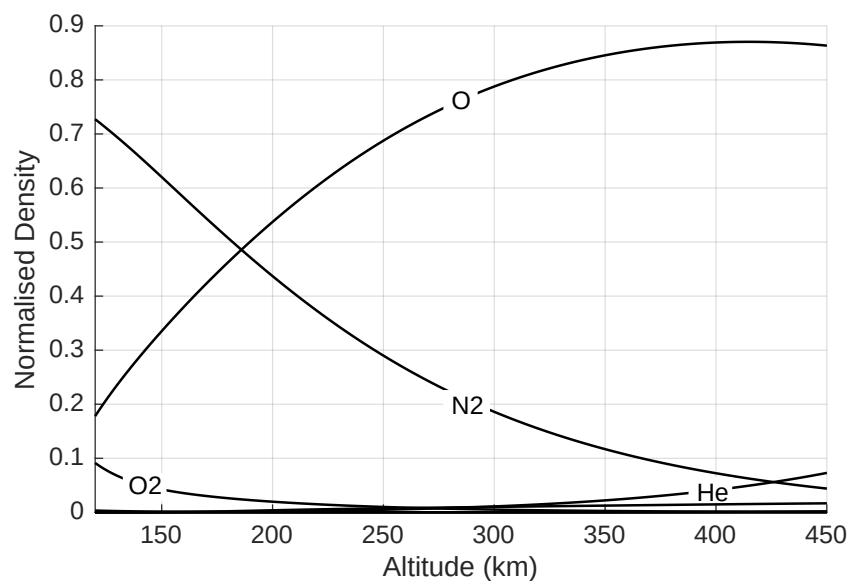


Figure 3.4: Normalised Changes in composition of the atmosphere with altitude during moderate solar activity

The relationship between N_2 and O is near inversely proportional. At lower altitudes a greater

presence of N₂ is seen. The number of oxygen atoms increases until an equal ratio is seen at 186 km. Above this altitude Atomic oxygen remains the majority constituent.

3.3.2 Effect of Solar Activity

Significant variation in the local flow conditions occur due to changes in solar activity throughout a typical mission. The relationship between flow conditions and altitude for varying solar activity was investigated in this section.

The local mass density determines the mass available to the intake of an atmosphere-breathing system as well as the drag force that must be counteracted. As such it must be carefully considered in the analysis so as to properly understand performance limitations. Figure 3.5 shows the relationship between mass density and altitude for select solar activities.

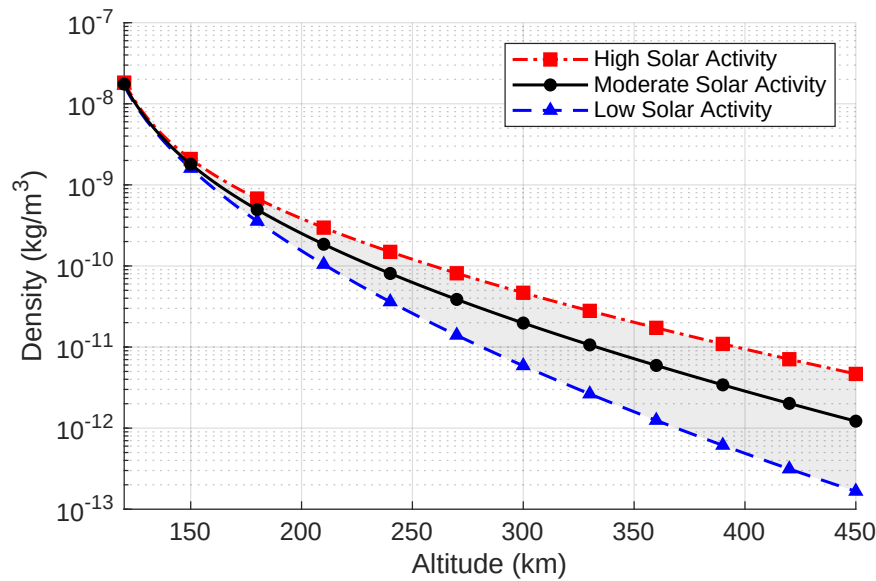


Figure 3.5: Averaged total density of the atmosphere with altitude during various levels of solar activity (See Table 3.1)

For all solar activities, density decreases exponentially with increasing altitude. At 120 km a maximum of around 1.75×10^{-8} kg/m³ occurs with little sensitivity to solar activity. As altitude increases the variations due to solar activity increases significantly. At 450 km a moderate solar activity results in an estimated 1.22×10^{-12} kg/m³ while High and Low activity see more than an order of magnitude change between them at 4.66×10^{-12} and 1.66×10^{-13} respectively. In general, higher solar activity results in higher local mass density and hence increasing available mass while also increasing drag.

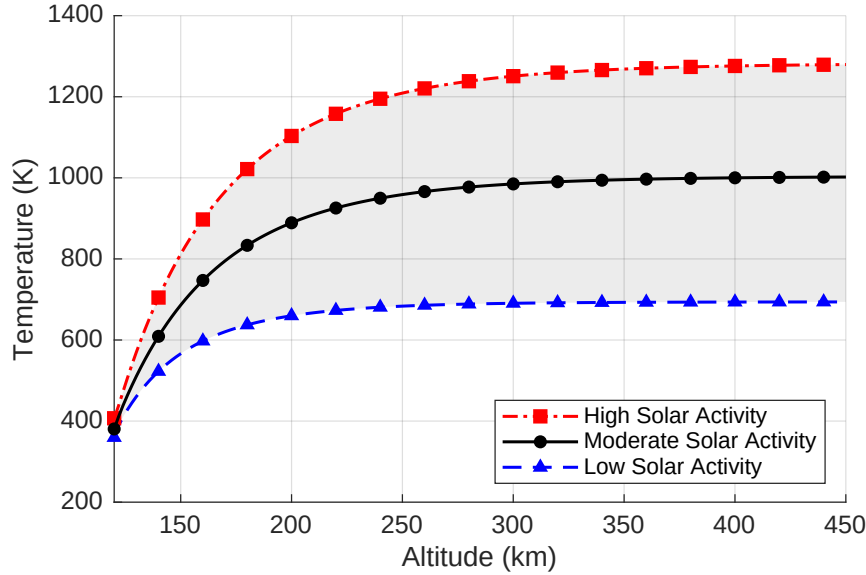


Figure 3.6: Absolute ambient temperature vs. altitude during various levels of solar activity (See Table 3.1)

Figure 3.6 shows solar activities affect on temperature over the same altitude range. As one might expect higher solar activities result in higher temperatures, indicating one contributing factor to the respective increase in density. With increasing altitude temperature grows rapidly from 360-410 K at 120 km until reaching a plateau approaching the exosphere. This plateau varies significantly with solar activity between 694 K for low activity and nearly doubling to 1280 K at during high activity. The absolute temperature plays an important role in determining the speed ratio and hence the transmission probability at the intake.

To justify the use of the free molecular flow assumption during intake analysis, it is important to assess the Mean Free Path of incoming molecules and hence the Knudsen number. Assuming a characteristic length of 1 m, $K_n = L_m$. The orbit averaged Mean Free Path was calculated using Equation 3.3.2 and plotted in Figure 3.7.

$$\lambda = \frac{1}{\sqrt{2}N\pi d^2}$$

Where N is the total number density and d is the average diameter of a given "particle". The assumption is usually made to assess molecules and atoms in rarified gases as a "hard sphere" of a set diameter to predict collision likelihood geometrically [32]. From hard sphere models typically available in textbooks this was taken as 355 pm (picometer). This is the

average between N_2 and O_2 kinetic diameters [33]. Note that this is a slight overestimation for collision likelihood as no single oxygen atom hard sphere references could be found.

Mean Free Path is inversely proportional to number density such that fewer "particles" per unit area results in a greater path length between collisions. This relationship is apparent from Figure 3.7 where L_m increases exponentially with altitude along with the sensitivity to solar activity. Inversely to density higher solar activity results in lower mean free paths for a given altitude. Above 133 km the ambient flow conditions can be assumed to be free molecular for all solar activities.

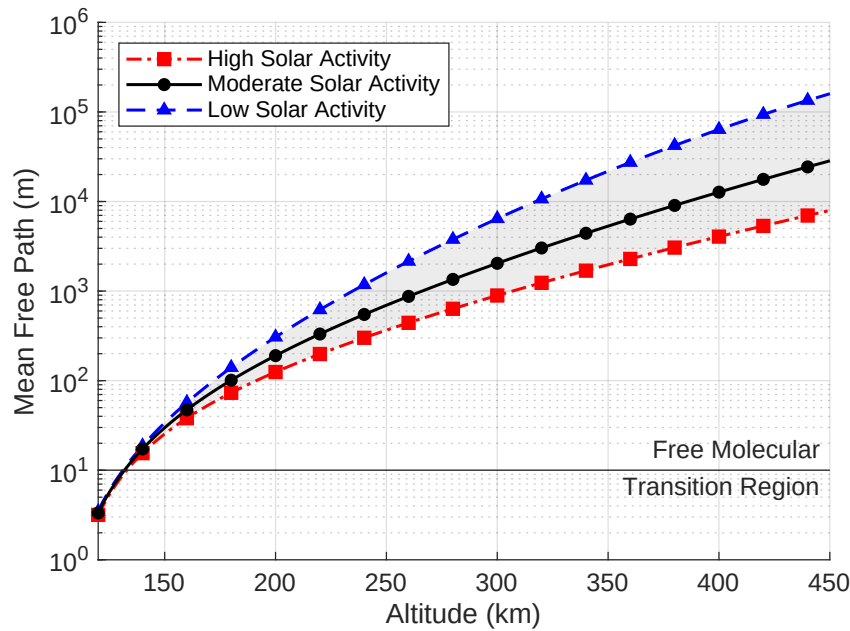


Figure 3.7: Mean Free Path vs. Altitude during various levels of solar activity

Another important flow condition is the Speed Ratio S_∞ ; the ratio between the normal velocity of the flow and the most probable thermal velocity (See Equation 4.3). S_∞ is a function of both Temperature T_∞ (increasing with altitude) and Orbital velocity V_∞ (Decreases with altitude), noting that with a circular orbit the flow velocity was assumed constant for each altitude. Figure 3.8 shows this relationship over the considered region. In general S_∞ decreases with altitude with a steep gradient observed below 200 km. Higher solar activity results in a higher absolute temperature, increasing the most probable velocity term in Equation 4.3

hence resulting in a net reduction in S_∞ . The opposite is seen for reduced activity.

$$S_\infty = \frac{V_\infty}{\sqrt{2kT_\infty \bar{M}_s^{-1}}} \quad (3.1)$$

Where $\bar{M}_s$ is the mean molecular mass of all species. The Speed Ratio is directly linked to the transmission probabilities across the intake ducts. Higher values of S_∞ result in better collimated flows and hence are favourable to overall intake performance. In general it can therefore be stated that higher solar activity will reduce the transmission probabilities.

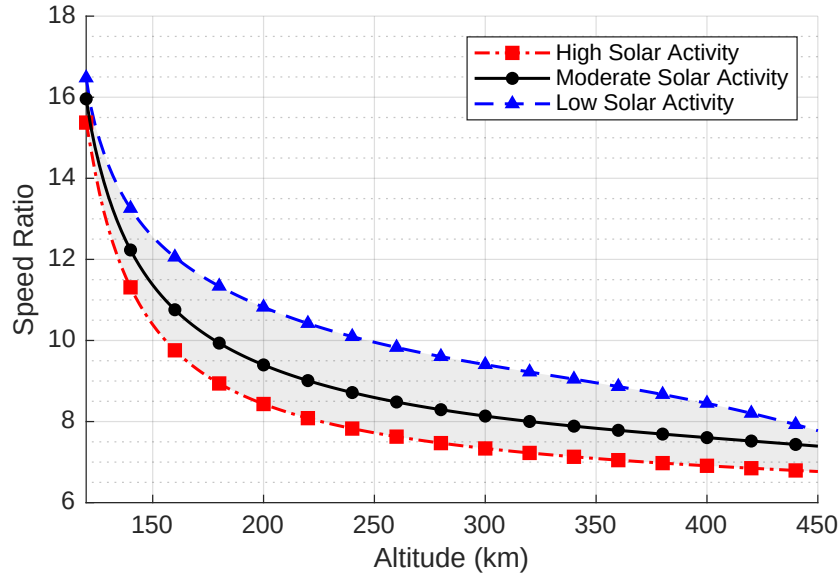


Figure 3.8: Speed ratio vs. Altitude for varying solar activities

In all plots there is a consistent pattern of behaviour whereby solar activity has a larger impact on flow conditions as altitude increases. As a means of understanding the sensitivity of critical variables to solar effects, the percentage mean absolute deviation due to solar activity of Mass Density, Absolute Temperature, Speed ratio and Mean Free Path were plotted with altitude. This represents the average maximum deviation from the moderate condition for high and low solar conditions at a particular altitude. The deviations were normalised as a percentage in reference to the moderate condition resulting in the following relation:

$$MAD(\%) = 50 \left\{ \frac{|X_{mod} - X_{high}| + |X_{mod} - X_{low}|}{X_{mod}} \right\} \quad (3.2)$$

Where $MAD(\%)$ is the mean absolute deviation in percent and X_{low} , X_{mod} and X_{high} are any one variable at a single altitude for low, moderate and high solar activities respectively. This measure of sensitivity was plotted in Figure 3.9.

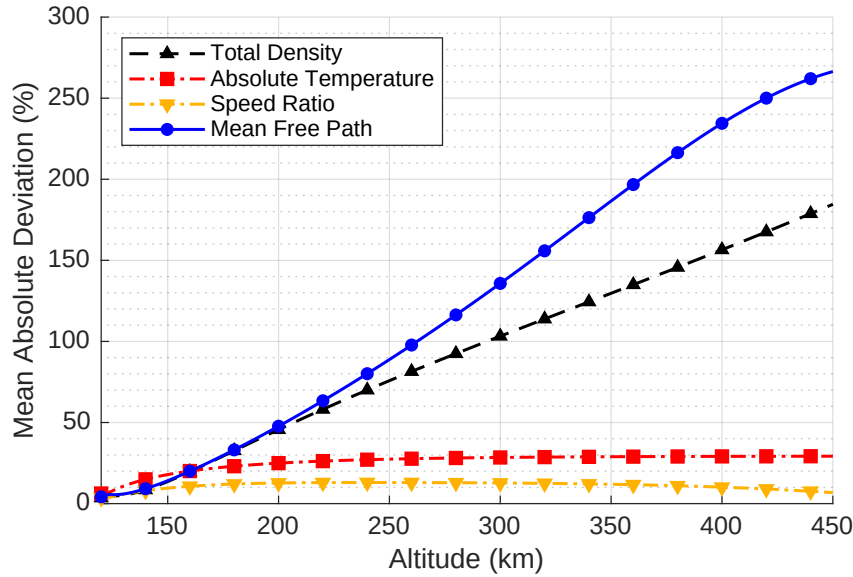


Figure 3.9: Mean Absolute Deviation of critical parameters due to varying solar activity vs. altitude

In correspondence with previous figures, changes in solar activity have significantly more impact at higher altitudes. Below 200 km, all evaluated parameters have an average deviation of less than 50% from the moderate condition. This remains true for Absolute Temperature and Speed Ratio never passing above 30% across all altitudes. The largest impacts can be seen in the mass density and mean free path which steadily increase in MAD with maximums of 185% and 266% respectively at 450 km. This provides valuable indications as to the degree of variations in flow parameters experienced over the 11 year solar cycle.

4 | Baseline System Model

The working principle for the majority of proposed ABEP concepts utilises a passive capture method; taking advantage of the atmospheric flow conditions and the behaviour of rarefied gases to provide a mass flow at the necessary pressure to an electric thruster. Further to the discussion in Chapter 2 on general working principles and common design features, this section outlines the derivation of a baseline system model for a typical ABEP system.

4.1 Model Definition

A common system architecture has emerged for ABEP providing a good baseline for comparison between studies [15, 34, 35, 30]. Figure 5.4 outlines this typical architecture:

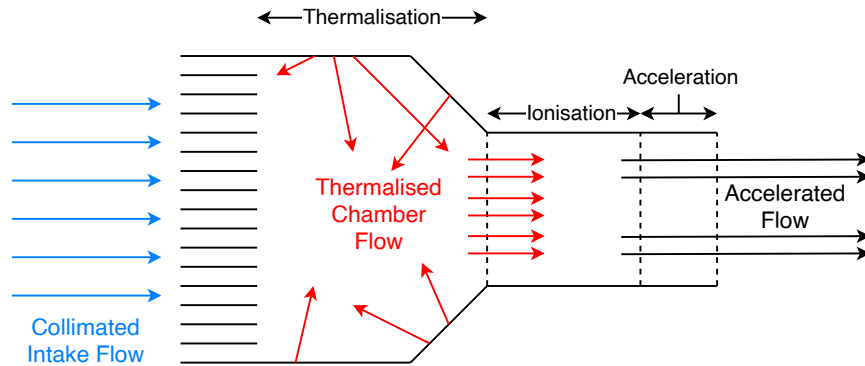


Figure 4.1: Common system architecture for ABEP utilised as the baseline model

In this baseline configuration, an intake is modelled as a steady state sum of fluxes, followed by a simplified thruster model. The overall drag of the spacecraft is assumed to be entirely a result of the intake area, with the solar panels aligned parallel to the incoming flow, with a frontal area assumed to be negligible in comparison. During the definition of the following model, several key simplifying assumptions were applied to the overall model:

- All flow is free molecular, negating the need to consider collisions between molecules in the analysis.
- Surface interaction is entirely diffuse in nature.
- The system is always in equilibrium: No transient flow effects are considered.
- Density distribution in the intake chamber is uniform everywhere.

4.1.1 Intake Flow Model

Consider an intake system consisting of a collection of collimator ducts totalling an intake area A_i . Incident flow transfers through the ducts before entering the collector region where wall interactions thermalise the flow. From the chamber, some molecules return back through the intake ducts, while the remainder transfer through the thruster. This series of flows can be approximated a net zero sum of particle flow rates.

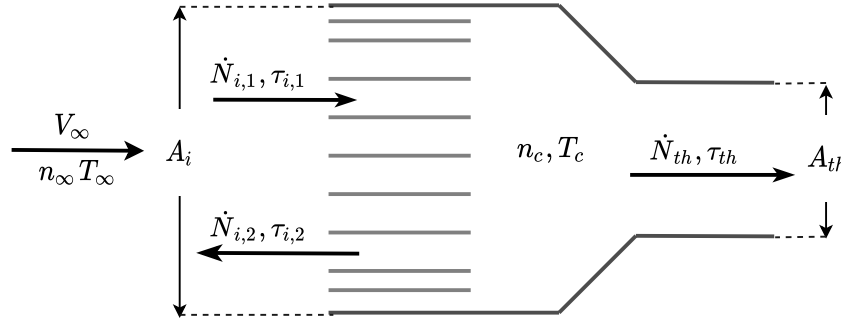


Figure 4.2: Baseline Flow Model. Adapted from [30]

Each intake duct is treated as a cylindrical tube. Due to the flow conditions in orbit, the incident molecules appear to the intake at orbital velocity V_∞ . The transmission probability $\tau_{i,1}$ for the incoming flow through each duct is therefore a function of the Aspect ratio $L/\sqrt{A}$ and the Speed Ratio S_∞ . This can be determined analytically using the method described by Hughes et al [17]. The program written to solve these methods can be seen in Appendix C. Assuming that $S_\infty \gg 1$ the flow can be treated as hyper-sonic, resulting in the ingested particle flow rate $\dot{N}_{i,1}$ [30, 32]:

$$\dot{N}_{i,1} = \phi_{i,1} A_i \tau_{i,1} = n_\infty V_\infty A_i \tau_{i,1} \quad (4.1)$$

Where $\phi_{i,1}$ is the particle flux and n_∞ is the atmospheric number density. Assuming all

ingested flow interacts with the chamber wall at temperature T_c , the resulting back-flow flux $\phi_{i,2}$ will be thermal in nature. The transmission probability for particles escaping the intake chamber $\tau_{i,2}$ with a Speed Ratio of unity can be approximated by the analytical expressions derived by Clausing as a function of Aspect Ratio [16] (See Appendix C). The resulting flow rate escaping through the intake ducts $\dot{N}_{i,2}$ will therefore be:

$$\dot{N}_{i,2} = \phi_{i,2} A_i \tau_{i,2} = \frac{1}{4} n_c \bar{u}_c A_i \tau_{i,2} \quad (4.2)$$

Where n_c is the number density in the chamber and $\bar{u}_c$ is the mean velocity of particles in the chamber according to the following equation:

$$\bar{u}_c = \sqrt{\frac{8kT_c}{\pi M_s}} \quad (4.3)$$

Where k is the Boltzmann constant, T_c is the intake chamber temperature and M_s is the molecular mass. The flow leaving through the thruster intake $\dot{N}_{th}$ is also assumed to be thermalised, therefore:

$$\dot{N}_{th} = \phi_{th} A_{th} \tau_{th} = \frac{1}{4} n_c \bar{u}_c A_{th} \tau_{th} \quad (4.4)$$

Where A_{th} is the thruster inlet area, ϕ_{th} is the thruster inlet flux and τ_{th} is the thruster transmission probability. By conservation of mass, the intake continuity equation can be derived:

$$\dot{N}_{i,1} - \dot{N}_{i,2} - \dot{N}_{th} = 0 \quad (4.5)$$

Substituting flow rates for terms in Equations 4.1, 4.2 and 4.4:

$$n_\infty V_\infty A_i \tau_{i,1} = \frac{1}{4} n_c \bar{u}_c (A_i \tau_{i,2} + A_{th} \tau_{th}) \quad (4.6)$$

For multiple species the intake, the mass flow through a given section $\dot{m}$ is:

$$\dot{m} = \sum_{s=1}^{n_s} \dot{N}_s M_s \quad (4.7)$$

Where n_s is the number of species, $\dot{N}_s$ is the species particle flow rate and M_s is the species particle mass in kg. Alternatively:

$$\dot{m} = \dot{N} \bar{M}_p = \dot{N} \frac{\sum_{s=1}^{n_s} (N_s M_s)}{\sum_{i=1}^{n_s} N_s} \quad (4.8)$$

Where $\bar{M}_p$ is the mean particle mass of the mixture, $\dot{N}$ is the total particle flow rate through the region of interest and N_s is the number of particles for a given species. As a simplifying assumption, the mean mass of the mixture was considered such that $M_s = \bar{M}_p$.

4.1.2 Intake Performance Parameters

The aforementioned density ratio and collection efficiency are commonly used figures of merit for the intake system. The trade-off between these two parameters is focused on meeting thruster requirements by increasing density ratio and minimising power by increasing collection efficiency. Determining these values allows for a comparison to other systems independent of system size.

The density ratio of the system is simply the ratio between the free stream number density n_∞ and the chamber density n_c [30]. Rearranging Equation 5.14:

$$v = \frac{n_c}{n_\infty} = \frac{V_\infty}{\frac{1}{4} \bar{v}_c \tau_{i,2} + \frac{A_{th}}{A_i} \tau_{th}} \tau_{i,1} \quad (4.9)$$

Here the reduced thruster transmission probability τ_{th}^* is introduced, relating the thruster flow to a function of the ratio between the intake and the thruster inlet area:

$$\tau_{th}^* = \frac{A_{th}}{A_i} \tau_{th} \quad (4.10)$$

By substitution with Equation 3.1 the density ratio can be simplified to a function of the temperature ratio, speed ratio and transmission probabilities:

$$v = 2\sqrt{\pi} S_\infty \sqrt{\frac{T_\infty}{T_c}} \frac{\tau_{i,1}}{\tau_{i,2} + \tau_{th}^*} \quad (4.11)$$

The collection efficiency η_c is defined as the amount of particles "used" in relation to the total available to the intake area. This in effect is the ratio between the thruster particle flux $\dot{N}_{th}$ and the available particle flux at the inlet. Using Equation 4.4 this can be reduced to a function of transmission probabilities only:

$$\eta_c = \frac{\dot{N}_{th}}{n_\infty V_\infty A_i} = \frac{\tau_{i,1}}{\tau_{i,2} + \tau_{th}^*} \tau_{th}^* \quad (4.12)$$

It is apparent that the thruster transmission term affects these performance parameters inversely for a fixed environmental condition. Increasing the thruster transmission reduces the density ratio and hence the pressure supplying the thruster. Conversely, the efficiency, and hence the mass flow through the thruster, is increased with increasing thruster transmission.

4.1.3 Thruster Characteristics

The characteristics of the thruster were defined using simple analytical expressions for electrostatic systems [9]. For a given particle flow rate through the thruster, the thruster mass flow rate $\dot{m}_{th}$ can be determined from Equation 4.8:

$$\dot{m}_{th} = \dot{N}_{th} \bar{M}_p \quad (4.13)$$

Where the thrust T produced by acceleration of said mass flow to exhaust velocity v_{ex} is given by:

$$T = \dot{m}_{th} v_{ex} \quad (4.14)$$

The kinetic thrust power required to accelerate a given propellant mass flow; jet power P_{jet} , is given by Equation 4.15:

$$P_{jet} = \frac{1}{2} \dot{m}_{th} v_{ex}^2 = \frac{T^2}{2\dot{m}_{th}} \quad (4.15)$$

The total efficiency of the thruster system η_{th} is the ratio of the required input power to the jet power:

$$\eta_{th} = \frac{P_{jet}}{P_{in}} \quad (4.16)$$

The thruster transmission probability is a measure of the amount of propellant that enters the ionisation chamber and transfers through the accelerating region. Since the ions in the chamber are affected by the electrostatic field, the probability of a neutral transmission and an ion transmission differs. It is useful to assume the following [36]:

- The flow entering the thruster that fails to be ionised is reflected back into the chamber.
- The flow that is successfully ionised is accelerated and exits the system.

The proportion of neutrals that are successfully ionised in the discharge chamber is known as the propellant utilisation efficiency η_m . The transmission probability can therefore be simply approximated as $\tau_{th} = \eta_m$. Relating this to reduced thruster transmission:

$$\tau_{th}^* = \frac{A_{th}}{A_i} \eta_m \quad (4.17)$$

The determination of propellant utilisation efficiency varies depending on the plasma source employed. It is generally accepted that a value of above 0.8 is desirable [9]. As discussed in Section 2, a lack of neutral density in the discharge chamber for likely candidate thrusters limits the ability for ionisation to occur to a satisfactory degree. Since modelling the utilisation efficiency itself across a range of potential ionisation schemes is beyond the scope of this study, an assumed minimum of 0.8 was chosen based on similar work on Air-Breathing thrusters by Garrigues et al [37].

4.1.4 Drag & Thrust Balance

The sum of forces on the system is a function of the drag caused by the body of the spacecraft, and the thrust produced by the propulsion system. As a first approximation for the forces on system, the following assumptions can be made:

- The non useful area of the intake is negligible, therefore the area available for ingestion A_i is equal to the total frontal area A_f
- Since orbital velocity is far greater than the velocity of particles leaving the intake, the momentum of these particles is assumed to be negligible.

These assumptions represent an idealised and simplified case that allows for a simple representation of the forces on the system in terms of the overall drag on the frontal area, and the thrust produced as a result of exhaust velocity. The drag on the system is a function of

orbital velocity V_∞ , local mass density ρ and drag coefficient C_d . The drag coefficient is often determined through panel method based analytical expressions, or through DSMC analysis for a given geometry. To limit the complexity of drag calculations, a common approach is to utilise experimentally determined values for drag coefficient and model the drag force through Equation 4.18. For the baseline case, a value of $C_d = 2.5$ was assumed [4].

$$F_d = \frac{1}{2} \rho V_\infty^2 A_f C_d \quad (4.18)$$

This results in a momentum equation summarising the forces on the system can be found using Equations 4.12, 4.14 and 4.18:

$$\sum F = T - F_d = \bar{M}_p n_\infty V_\infty \eta_c u_e - \frac{1}{2} \bar{M}_p n_\infty V_\infty^2 A_i C_d \quad (4.19)$$

4.2 Model Performance

Having defined a baseline model for an Atmosphere-Breathing system, the atmospheric model in Chapter 3 can be used to test the performance for given ambient conditions. This section will analyse performance of the intake and relate it to the requirements for the thruster so as to gauge the system limitations. The following criteria were used to assess limitations and define a valid design space:

- **Thrust:** The thrust produced by the system must be at least equal to the drag on the spacecraft.
- **System Power:** Based on previous missions to VLEO [1] and current proposed concepts outlined in Section 2, an upper limit to thruster input power of 1500 W was chosen.
- **Chamber Density** For feasible thruster operation, a neutral inlet density of 10^{18} m^{-3} was assumed [5].

In an ideal case, an exhaustive study could be done to optimise the system to a particular altitude or mission profile across a range of varying altitudes. This however is a complex multi-objective optimisation problem and as such is beyond the capability of this investigation given time constraints. As a method for determining the behaviour of the system in relation to geometric characteristics and in relation to flow conditions, the following strategy was adopted:

1. Assess the feasibility of the system for a series of potential altitudes, varying geometric parameters that effect performance: The intake duct aspect ratio $L/\sqrt{A}$ and thruster inlet to intake area ratio A_{th}/A_i .
2. Choose a feasible design point taking into account limitations in power and number density.
3. Test said design point across a range of altitudes to assess "off design point" limitations.

This method is intended to illustrate some typical trade-off steps in an iterative design process. In later sections, the same method will be mirrored with that of the proposed storage system, allowing a simple performance comparison using this section as a reference. Although not exhaustive in determining the feasible bounds of altitude bounds, the intended outcome is to establish general system behaviour for later comparison.

4.2.1 Analysis Software Design

The mathematical model described in the previous sections of Chapter 4 was implemented in MATLAB to perform all necessary calculations. Two programs were written for analysis of the baseline from the aforementioned strategy. The scripts for both programs can be found in Appendix C.

Constraint Analysis Method

The first analysis program consisted of three sections relating to part one of the strategy:

1. *Flow Conditions*: The flow conditions for a given solar activity and altitude were calculated using the method in Chapter 3. These calculations were used as constant inputs for the following section.
2. *Performance Calculations*: Using the baseline system model, the performance parameters of Density ratio and collection efficiency were calculated by varying the thruster-intake area ratio and the intake duct aspect ratio.
3. *Constraint Analysis*: Given the known performance parameters, the chosen feasibility limits of chamber density and thruster input power were chosen as "constraints". These were then plotted within the collection efficiency results to show the feasible design space.

The result of this method is an illustration of the feasible bounds for the thruster inlet area

and intake ducts. From this point, the results can be analysed to produce a design point with respect to these geometric parameters.

Off-Design Point Analysis

The design point chosen using constraint analysis was then frozen and implemented in a further program to assess the behaviour of the system when not at a known feasible point. As with the previous program, the off-design point analysis can be broken down into three sections:

1. *Flow Conditions*: The altitude related flow conditions were calculated using the same method as the previous section. However now a large range of altitudes were considered.
2. *Performance Calculations*: As in the constraint analysis section the performance of the system was calculated, however this time the geometry was fixed. Changes in performance could then be solely attributed to changes in flow condition.
3. *Off-design point analysis*: Having calculated the operating conditions of the thruster across a range of altitudes, a feasible operating altitude region for the chosen design geometry can be evaluated from results.

Results gathered from this program can be used to illustrate the influence that flow conditions have on the feasibility of a particular design point. Most notably: at what altitudes limitations in power and chamber density exceeded.

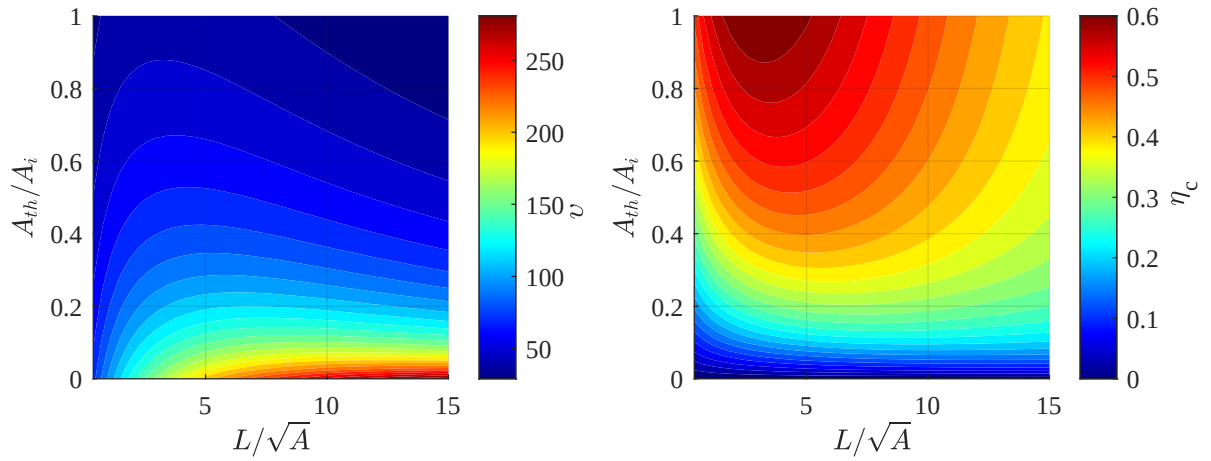
4.2.2 Constraint Analysis Results

Three set altitudes were chosen for the constraint analysis study with reference to the altitude ranges presented as feasible in the literature [36, 29]. For a chosen range of A_{th}/A_i and $L/\sqrt{A}$ the variables are discretised over an equally spaced grid. The "Grid Size" denotes the number calculations across the chosen ranges. All solar conditions were considered at moderate levels. Table 4.1 outlines the main inputs used for the program:

Table 4.1: Constraint Analysis program Inputs for the baseline case

Input	Value
Altitudes (km)	170,180,190
A_i (m ²)	1
T_c (K)	300
Power Limit (W)	1500
Minimum n_c (m ⁻³)	10^{18}
η_{th}	0.6
η_m	0.8
A_{th}/A_i	0 to 1
$L/\sqrt{A}$	0 to 15
Grid Size	30

Firstly, to validate the results and hence the model, the performance parameters for the intake were compared to previous works. Most notably the work by Parodi et al given the similarity in method [30]. Figure 4.4 shows the resulting contour plots for density ratio (left) and collection efficiency (right).

**Figure 4.3:** Density Ratio (Left) & Collection Efficiency (Right) at at 180 km with moderate solar conditions.

The contours produced are consistent with results produced by Parodi: An upper limit to density ratio can be seen around 250 occurring at $L/\sqrt{A} = 15$ and $A_{th}/A_i = 0$. Commenting firstly on the density ratio, increasing A_{th}/A_i for a given duct geometry causes a decrease in

number density within the intake chamber. This is to be expected as the larger the thruster area, the more molecules leave the intake chamber. Furthermore, increasing the duct aspect ratio has the inverse effect, resulting in an increase in density ratio. This can be explained by looking at the collimator principle, in particular the transmission probabilities. Increasing the aspect ratio of a duct reduces the transmission of the back flow. It also reduces the transmission of molecules entering the chamber, however the reduction is greater for the backflow relative to the inflow due to the high speed ratio. The result is a maximum density ratio where flow to the thruster is minimised and the aspect ratio is maximised.

When considering the collection efficiency, a near opposite effect is observed. As noted before, decreasing thruster area increases the density in the chamber. However, this also has the effect of reducing the total mass into the thruster. Since the flow is not ingested by the thruster it is eventually lost through the intake. This reduces the efficiency as fewer molecules available for thrust are used. The same can be said for the increases in $L/\sqrt{A}$: Reducing the transmission probability through the ducts increases the number of molecules that never make it to the intake chamber. For this reason these molecules only contribute to drag and are not used. This inverse relationship between Density Ratio and Collection Efficiency is an important phenomena when considering the performance of an Atmosphere-Breathing system.

Consider now limitations on power and density. Mapping these limits onto performance plots allows the parameter of interest to be optimised within a feasible design space. The choice of parameter to optimise is dependent on the mission goals. In the case of this study collection efficiency was chosen as this is the most relevant figure of merit for comparing other results to other studies.

Figure 4.4 shows the resulting design space taking limits into account. The chamber density limits correspond to a failure of the intake to compress the incoming flow to a level that is suitable for thruster operation. The Input power limit corresponds to the point where the required power to produce thrust equivalent to drag is exceeded.

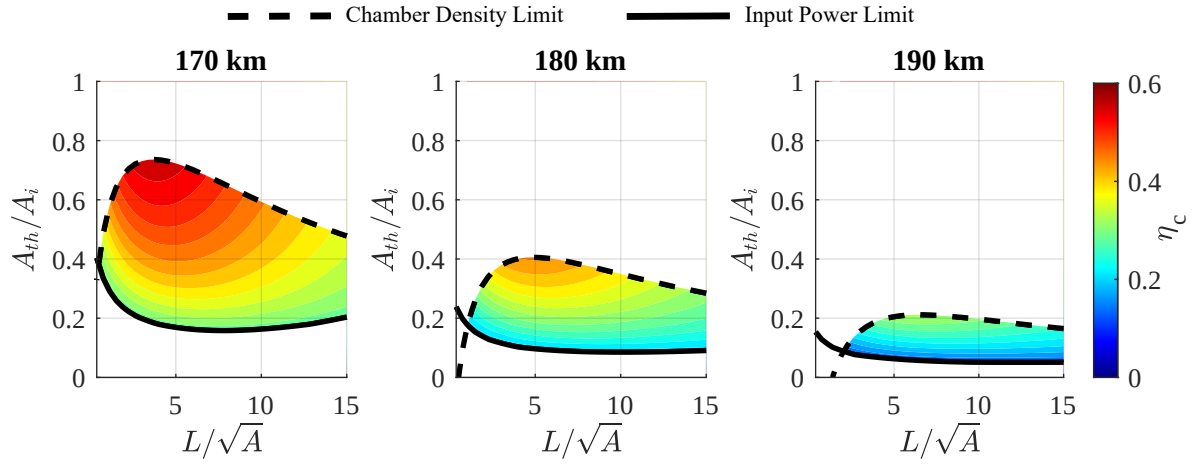


Figure 4.4: Collection efficiency constraint plot at moderate solar conditions for altitudes of 170 km, 180 km and 190 km

The following characteristics when assessing the performance results were observed:

- *Collection efficiency:* The maximum thruster area ratio for each altitude corresponds to the maximum collector efficiency, and hence minimum required power. Between the altitudes considered, this results in a A_{th} range of 0.21 to 0.72. The $L/\sqrt{A}$ range for maximum collection efficiency is between 4 and 7.
- *Density Ratio:* The optimum density ratio occurs where efficiency is minimised. This corresponds to a thruster area range of 0.05 to 0.10 and a $L/\sqrt{A}$ range of 7.5 and 12.4

To optimise for both parameters, it is challenging to choose a design point that overlaps all considered altitudes. For instance, it is clear that the feasible design space is less strict at lower altitudes whereas tighter requirements can be seen at higher altitudes. To maximise the available envelope of feasible altitudes it would be sensible to choose a parameters that fit within constraints at all altitudes. However there is obviously a more favourable region at 170 km than at 190 km. Table 4.2 outlines the final design point chosen as well as the corresponding optimums described previously:

Table 4.2: Constraint Analysis Results and Choice of Design Point

Altitude (km)	170	180	190
Best v	129	159	194
Best η_u	0.56	0.45	0.33
Best v	129	159	194
A_{th}/A_i (Best η_c)	0.72	0.38	0.21
A_{th}/A_i (Best v)	0.10	0.10	0.05
$L/\sqrt{A}$ (Best η_c)	3.8	4.9	7.1
$L/\sqrt{A}$ (Best v)	7.5	9.0	12.4
Chosen Design Point			
A_{th}/A_i		0.25	
$L/\sqrt{A}$		7	
Design Point Results			
v	108	104	n/a
η_c	0.38	0.37	n/a

4.2.3 Off Design Point Performance

It is reasonable to assume that the geometry of the intake will not change while in orbit for a fully passive intake system. For this reason, the choice of design point via the previous method does not consider a true mission scenario where a design will be forced to cope with significant changes in conditions as described in Chapter 3. To imitate this scenario the performance of the system was tested at the chosen design point over a range of altitudes corresponding to those assessed in Section 3.

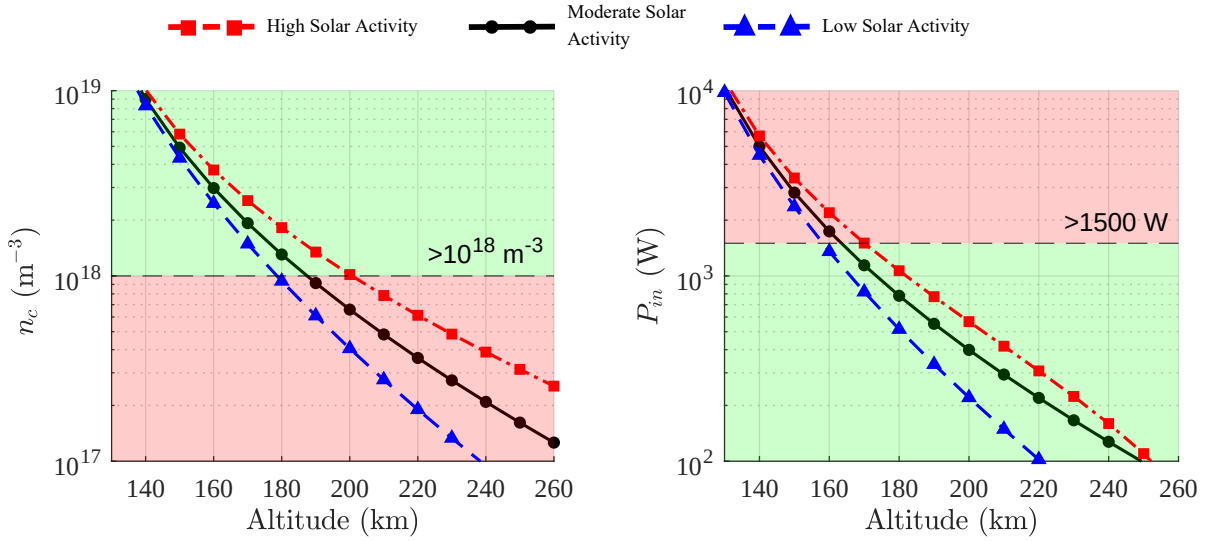


Figure 4.5: Chamber Density (left) and Required Input Power (Right) vs. Altitude for Low, Medium and Moderate Solar Conditions. Feasible regions are denoted in green

Figure 4.5 shows the chamber density and required input power compared to altitude. Looking at the results for input power it is clear that decreasing altitude results in increased power. The upper limit of 1500W is reached at a minimum altitude 164 km for moderate solar conditions below which the available mass flow fails to provide the required thrust for drag compensation at maximum power. In the case of chamber density, the upper limit to altitude was found to be 189 km. Above this point the density in the intake chamber is no longer high enough to meet thruster requirements. This results in an operational envelope of around 25 km.

The effects of solar activity further change the feasible ranges. low solar conditions are favourable from the perspective of power as less thrust is required to overcome the drag. The opposite can be seen for chamber density where high solar activity allows for more ingested mass and a larger speed ratio, resulting in better density ratios for a given altitude. These effects result in a "shift" in the feasible envelope throughout the solar cycle. For this reason, mission designs in the literature often incorporate changes in altitude to match this shift [3, 38, 36].

The minimum required power and chamber density give a good indication of operational envelope. However, if the system is designed to perfectly match the estimates for drag at a set altitude, any fluctuations exceeding the maximum allowable drag will result in a loss in altitude. For this reason the sum of forces on the spacecraft body during maximum thrust was plotted against altitude in Figure 4.6.

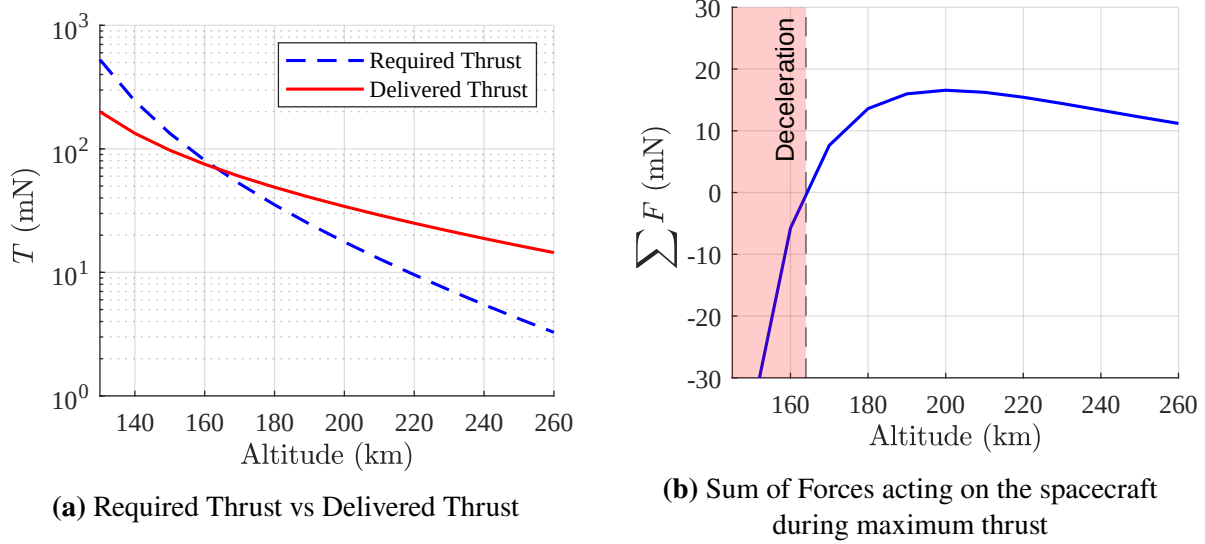


Figure 4.6: Required thrust, delivered thrust and sum of forces acting on the spacecraft at moderate solar conditions for the chosen design point in Table 4.2

The implications of dropping below the design altitude are clear when comparing the sum of forces acting on the spacecraft. As noted previously, the lower limit during solar conditions is 164 km. At this lower limit, the net forces on the spacecraft are zero. Below this point a sharp increase in net deceleration is seen. Above 164 km the available excess thrust increases to a maximum of 16.6 mN at 200 km. It is worth noting that this optimum occurs above the maximum altitude threshold of 189 km. Assuming the excess thrust can be used to counteract unforeseen fluctuations or for manoeuvres to increase altitude with respect to the solar cycle, remaining close to the chamber density limit is favourable.

5 | Storage Concept Model

Having outlined a baseline model for an Atmosphere-Breathing Propulsion System, the proposed storage concepts can be suggested based on similar underlying principles. This section will describe the proposed system designs and define a general model for feasibility analysis.

5.1 Storage System Concept

Consider the baseline model from Chapter 4. The thruster inlet placed within the intake chamber results in a flux of particles leaving the system to be used for thrust. If another inlet, this time to a theoretical storage vessel, were to also be placed within the intake chamber, a gas would escape through it. This in itself could provide a potential storage solution, however there is no compression of the gas and therefore the pressure in the vessel is limited by pressure in the intake chamber, which in itself is on the order of 10^{-3} Pa. Active compression methods discussed in the literature review such as turbo-molecular pumps could provide good pumping speeds, however require high speed rotating parts.

The proposed method of storage is a cryopump system, whereby cryocondensation of Molecular Nitrogen and Atomic Oxygen acts as a compression method while also retaining propellant in a highly dense form. Two similar concepts are explored:

Open System

In the *Open System*, the propellant enters through a series of baffles at the intake before "sticking" to the cryopump head. Upon release through heating of the cryosurface, the gas is released back into the intake chamber to be used by the thruster through the same mechanisms as the baseline design.

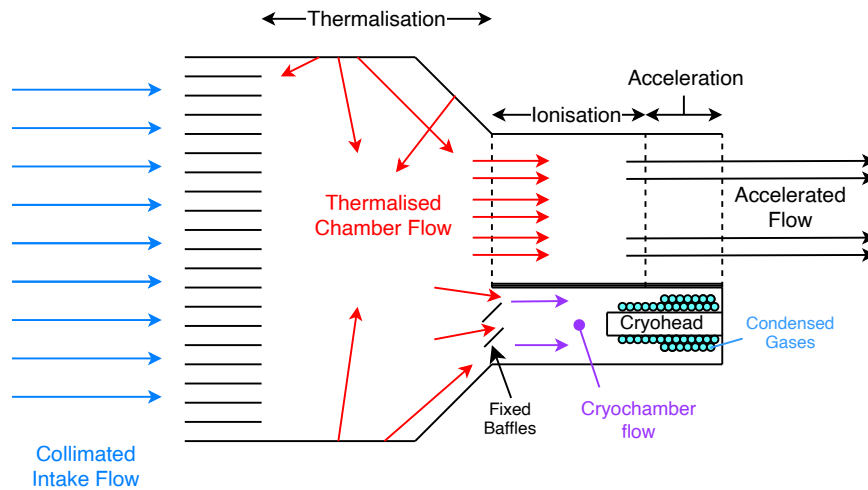


Figure 5.1: System architecture for open storage concept

This method has several distinct benefits; most notably the lack of moving parts. This method requires only sufficient cooling of the baffles and cryosurface and as such no mechanisms are involved in the storage process, discounting the cryocooler system. Since no moving parts are used however, the system is "Open" and a portion of the released molecules are lost through the intake ducts, reducing the utilisation of the collected propellant.

Closed System

In the *Closed System*, propellant is collected in exactly the same manner as the open system. However, when using propellant, the baffle system is *Closed* off from the intake through the same method used by some cryopumps during regeneration [39]. This is typically done through a single closing door at the entrance such as a mushroom valve. Once closed, the ideal system would release the collected propellant, creating a pressurised vessel. This can then be fed to the thruster using a traditional feed system used in all EP thrusters.

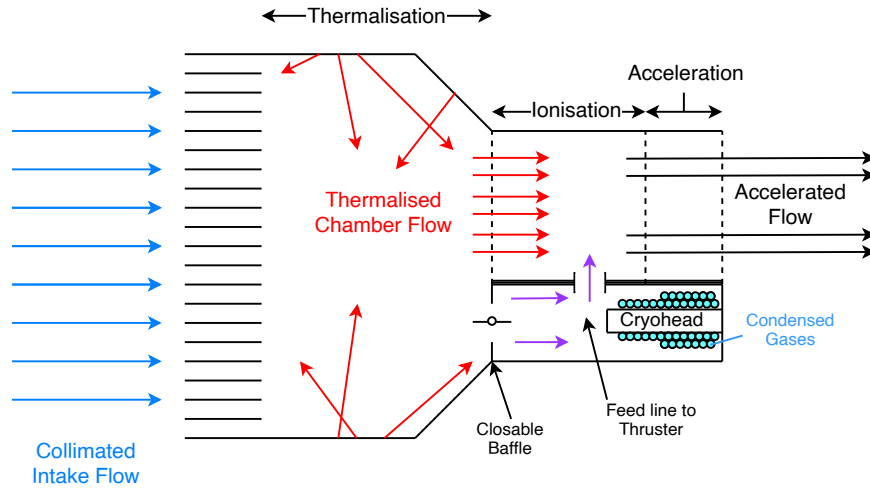


Figure 5.2: System architecture for closable storage concept

Unlike the Open system, theoretically all the propellant stored can be fed to a thruster and as such the system theoretically allows a higher overall efficiency. Furthermore, if the chamber can maintain high pressures, the cryocooler can be turned off and the system becomes a traditional EP system, allowing thrust generation independent of any ABEP technology. This comes at the likely cost of weight and complexity associated with a sturdy pressure vessel and large valve system.

Pumping Speed

The Pumping Speed is the rate at which a cryopump accumulates material on its surface [24]. Equation 5.1 defines ideal Pumping Speed S_{id} to a cryopump surface:

$$S_{id} = \frac{1}{4} \left(1 - \frac{P_s}{P_{vap}} \right) A_c \bar{u}_c \quad (5.1)$$

Where $\bar{u}_c$ is the mean velocity of the particles in the intake chamber, P_s is the pressure of the incident gas in the cryochamber and P_{vap} is the Vapour Pressure of the pumped material. The true pumping speed is a function of both the pump inlet geometry and the interactions between the gas and the cryosurface.

The probability of molecules sticking to the surface is known as the sticking probability α . Given a sufficiently cold surface, this probability can be forced to approach unity in most cases [7]. In most cases, the cryopump inlet will consist of a series of cooled baffles, shielding

the cryosurface and reducing heat load. The transmission probability of the baffles τ_b further reduces the true pumping speed. The total resulting decrease in the pump speed is accounted for by the capture coefficient c in Equation 5.2.

$$\frac{1}{c} = \frac{1}{\alpha} + \frac{1}{\tau_b} - 1 \quad (5.2)$$

Assuming a sticking probability of unity, a relation for the resulting number flux through the cryopump inlet $\dot{N}_s$ can be derived:

$$\dot{N}_s = \frac{1}{4} n_c \sqrt{\frac{8kT_c}{\pi M_s}} A_c \tau_b \left(1 - \frac{P_s}{P_{vap}} \right) = \frac{P_{vap} - P_s}{\sqrt{2\pi m k T_s}} A_c \tau_b \quad (5.3)$$

Note that the Chamber pressure P_c is not the pressure close to the surface of the pump P_s , therefore assuming an ideal gas, Equation 5.3 becomes:

$$\dot{N}_s = \frac{1}{4} n_c \sqrt{\frac{8kT_c}{\pi M_s}} A_c \tau_b \left(1 - \frac{P_c \sqrt{\frac{T_s}{T_c}}}{P_{vap}} \right) = \frac{P_{vap} - P_c \sqrt{\frac{T_s}{T_c}}}{\sqrt{2\pi M_s k T_s}} A_c \tau_b \quad (5.4)$$

Given sufficiently low temperatures, $P_{vap} \gg P_s$ can be assumed [7]. In this scenario, the pump can condense all material that enters through the baffle. This ideal case was assumed for the storage system during the collection phase, allowing the simplification of equation 5.3 such that:

$$\dot{N}_{s,c} = \frac{1}{4} n_c \sqrt{\frac{8kT_c}{\pi M_s}} A_c \tau_b \quad (5.5)$$

Where $\dot{N}_{s,c}$ denotes the particle flux during the collecting phase. This assumption does not hold when using the propellant as control of the flux through evaporation must carefully account for the changes in vapour pressure due to the cryosurface temperature. When vaporising the collected propellant in the case of the *Open System*, equation 5.4 can be

rearranged [40] for the feeding flux $\dot{N}_{s,f}$:

$$\dot{N}_{s,f} = \frac{P_{vap} - P_c \sqrt{\frac{T_s}{T_c}}}{\sqrt{2\pi M_s k T_s}} A_c \tau_b \quad (5.6)$$

The vapour pressure of the species condensing or evaporating from the cryosurface is a function of temperature, hence control of the cryosurface allows for the net flux to be chosen. Of the available species to condense, Nitrogen and Oxygen are the most prominent and also have the lowest vapour pressures for a given temperature. Other favourable condensates such as Argon are scarce while other species such as Helium require temperatures on the order of 1-5 K [25]. Given N_2 and O occupy >90% of the composition throughout VLEO as seen in Figure 3.4, these were considered as the target species. The vapour pressures for both gases were determined using the Clausius–Clapeyron Equation outlined in Appendix A [23]. The results can be seen in Figure 5.3.

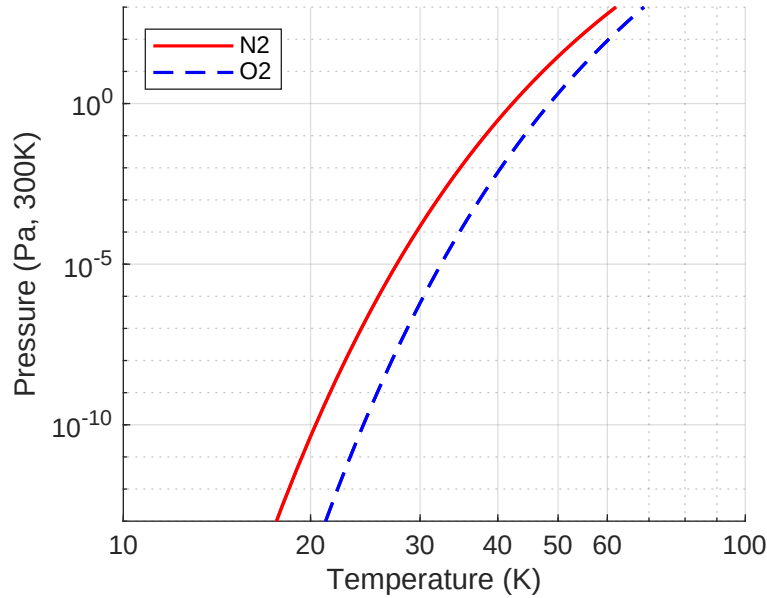


Figure 5.3: Vapour Pressures of N_2 and O_2 as a function of temperature

Scarce resources on the behaviour of atomic oxygen exist in the literature. As such, no information could be found on Atomic Oxygen's vaporisation pressure and so the Vapour pressure of O_2 was taken. For all temperatures, the vapour pressure of O_2 is below that of N_2 .

It was assumed that sufficiently low temperatures to result in condensation of N₂ will also result in the same for O₂. A commonly quoted temperature for guaranteed N₂ condensation is 20 K and as such this was chosen as a target T_s for all collection phases.

While vaporising propellant, the vapour pressure of N₂ and O₂ differs for a given temperature: O₂ has a smaller vapor pressure for the same temperature. From Equation 5.6, it can be seen that fewer oxygen atoms will be released from the condensate layers than nitrogen for the same head temperature. Determination of the total combined flux requires knowledge of the pressure in the chamber which is simultaneously a result of said flux. This inter-dependency makes analytical calculation of multiple species challenging. To simplify calculations, given the relatively close relations of N₂ and O₂, the vapour pressure was assumed to follow that of N₂ at all times.

Geometry & Baffle Transmission

In the case of a vacuum system, convective heat transfer, or transfer of energy carried by individual molecules, tends to be far smaller than radiative transfer from surfaces in the system [23]. For this reason the cryosurface is often isolated from the remainder of the system via a series of baffles or an obstructing wall that is cooled to an intermediary temperature. The heat transfer between each stage is largely a function of geometric view factors. As such a simplified geometry was established.

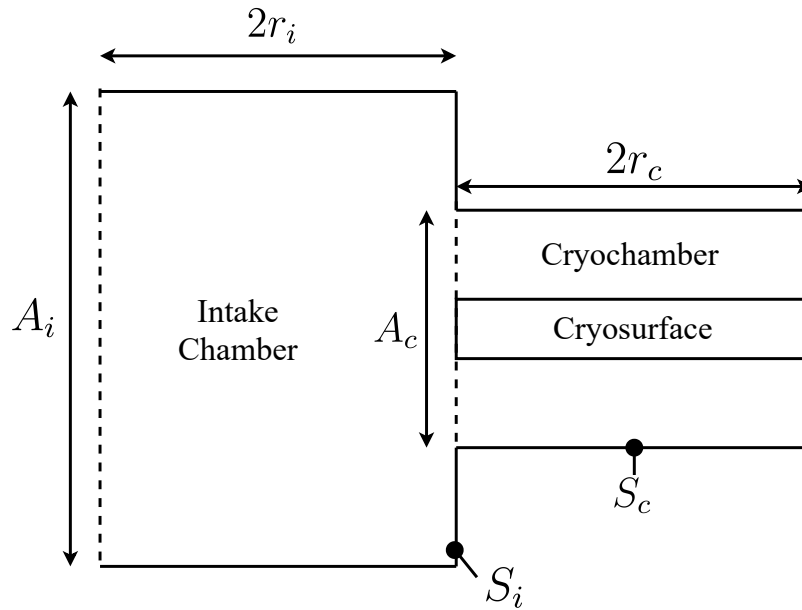


Figure 5.4: Cryopump Concept Geometry

Three stages were considered, each with defined temperatures and geometric parameters relative to the intake area:

- **Intake:** The intake consists of a cylindrical chamber of intake area A_i and radius r_i . The length of the intake chamber was assumed to be $2r_i$, with an interior surface area S_i .
- **Cryochamber:** The cryochamber was considered as a cylinder of radius r_c and inlet area A_c . This was assumed to be concentric with the intake to allow for analytically determined view factors. The total length of the cryochamber was assumed to be $2r_c$, with a surface area of S_c .
- **Cryosurface:** The final stage was the cryosurface itself, being considered as a concentric cylinder entirely enclosed within the cryochamber. To simplify the analysis, the pump surface was set to the same as the baffle area, hence the intake area and cryosurface are both A_c .

For later calculations requiring specific dimensions, an intake area of $A_i = 1 \text{ m}^2$ was assumed. Furthermore to simplify pressure calculations in the chamber, the length between the end of the ducts and the thruster inlet was assumed to be 1m, allowing an intake chamber volume of $V_i = 1 \text{ m}^3$.

A full breakdown of the geometry and calculation of view factors can be found in Appendix B. The exact geometry of the baffle system can be complex and as such usually require numerical methods to determine transmission probabilities. For preliminary calculations, 0.4 is often adopted for transmission of condensable gases such as N₂ and O₂ [7]. As such for all future calculations $\tau_b \approx 0.4$ was used.

Material Selection & Emissivity

The heat transfer between two surfaces as a result of radiation is in part a function of the emissivity of the material: the ratio between the heat radiated by the material and that radiated by a perfect emitter [41]. As such, the feasibility of the system depends on material requirements as well as heat loading. The following criteria were used to choose such a material:

- The material must have a low emissivity with tested performance at cryogenic temperatures.

- The material must be vacuum compatible ideally with space and cryogenics heritage.
- The material should have demonstrated resistance to atomic oxygen with in orbit testing.

From the literature investigation, several compatible materials were found with readily available information on their behaviour in vacuum conditions and at low temperatures. With the exception of Silver and osmium, most metals are generally resistant to atomic Oxygen at the macroscopic scale [42]. Of the materials regularly used in cryogenics, Aluminium and Copper were found to have emissivities below 0.1% at cryogenic temperatures [43]. Both materials are compatible for the application, although a full trade-off between the two was outside the scope of the project. For the initial concept, an emissivity of below 0.01 was deemed achievable for the low temperature baffles below 100 K while the intake ε_i was assumed to be 0.03.

The emissivity of the cryosurface itself is dependent initially on the material, however upon sufficient build up of monolayers, the emissivity is that of the condensate. Assuming the "ice" to consist of N_2 and O_2 , no specific studies could be found determining the emissivity of the condensate. For this reason, a conservative worst case estimate of 0.9 was assumed. Table 5.1 summarises the assumptions.

Table 5.1: System Emissivities

Parameter	Intake	Baffles	Condensate
Temperature (K)	300	<100	20
Emissivity	0.03	0.01	0.9

Radiative Heat Transfer

Given a reference geometry and estimates for emissivity, the heat load through each stage can be determined. Considering the radiative heat load on the baffles $Q_{R,b}$ for an intake chamber temperature of T_c and the load on the cryosurface $Q_{R,s}$ from a baffle temperature of T_b for a surface temperature T_s , the simple relations for two opaque surfaces in an enclosure were used [41]:

$$\dot{Q}_{R,b} = \frac{\sigma(T_c^4 - T_b^4)}{\frac{1 - \varepsilon_i}{S_i \varepsilon_i} + \frac{1}{S_i F_{i-b}} + \frac{1 - \varepsilon_b}{A_c \varepsilon_b}} \quad (5.7)$$

$$\dot{Q}_{R,s} = \frac{\sigma(T_b^4 - T_s^4)}{\frac{1 - \varepsilon_b}{S_c \varepsilon_b} + \frac{1}{S_b F_{b-s}} + \frac{1 - \varepsilon_s}{A_c \varepsilon_s}} \quad (5.8)$$

Where ε_i , ε_b and ε_s are the respective emissivities for the intake, baffle and cryosurface materials. The view factors between each stage, F_{i-b} and F_{b-c} , were determined using the aforementioned geometry using the methods found in Appendix B.

Conductive Heat Transfer from Gas Collisions

In addition to the radiation, heat is transferred to the baffle and cryosurfaces through gas-surface collisions. Assuming free molecular flow throughout all components and fully diffuse wall interactions, the conductive heat load on the baffle $\dot{Q}_{C,b}$ can be estimated as [44]:

$$\dot{Q}_{C,b} = \frac{1}{4} n_c \bar{u}_c A_c 2kT_c \quad (5.9)$$

For the cryosurface transfer $\dot{Q}_{C,s}$, the molecules that have transferred through the baffles were assumed to have interacted with the baffle itself or the cryochamber walls at the same temperature. The flux incident on the surface was assumed to be the number of molecules transferring through the baffle. Therefore the following relation can be derived:

$$\dot{Q}_{C,s} = \frac{1}{4} n_c \bar{u}_c 2A_c kT_b \quad (5.10)$$

5.2 Revised Flow Model

Considering again the baseline system, the cryopump storage model can be incorporated into a full system flow model. With reference to Figure 4.2, the additional flux into the pump system were considered according to Figure 5.5.

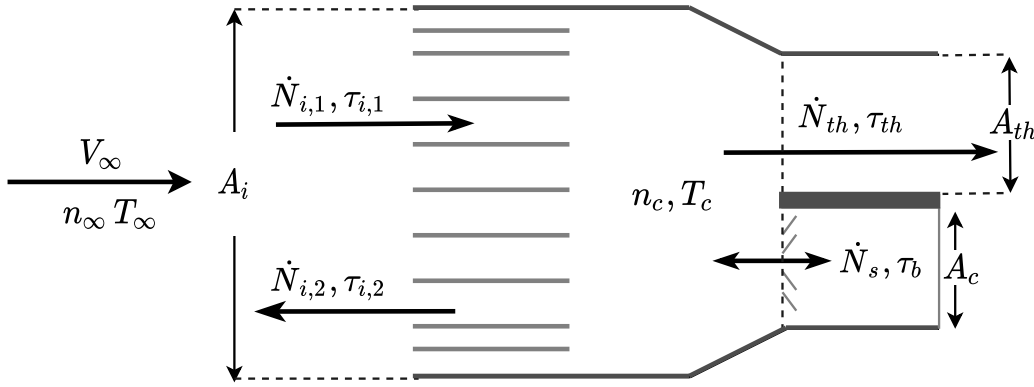


Figure 5.5: Cryopump Flow Model

Using the same method as outlined in Section 4.1.1 the flow through the intake, thruster and now the cryopump, can be summarised using the following continuity relations by adding the cryosurface particle flow rates to equation 4.5:

$$\dot{N}_{i,1} - \dot{N}_{i,2} - \dot{N}_{th} - \dot{N}_{s,c} = 0 \quad (5.11)$$

$$\dot{N}_{i,1} - \dot{N}_{i,2} - \dot{N}_{th} + \dot{N}_{s,f} = 0 \quad (5.12)$$

Where Equation 5.11 is the collecting phase and 5.12 is the open system flux leaving the cryochamber during a feed phase. Firstly dealing with the collecting case: Substituting the storage number flow rate with Equation 5.5 the continuity equation becomes:

$$n_\infty V_\infty A_i \tau_{i,1} = \frac{1}{4} n_c \bar{u}_c (A_i \tau_{i,2} + A_{th} \tau_{th} + A_c \tau_b) \quad (5.13)$$

Now deriving the continuity expression during propellant vaporisation using Equation 5.6:

$$n_\infty V_\infty A_i \tau_{i,1} = \frac{1}{4} n_c \bar{u}_c (A_i \tau_{i,2} + A_{th} \tau_{th}) - \frac{P_{vap} - P_c \sqrt{\frac{T_s}{T_c}}}{\sqrt{2\pi M_s k T_s}} A_c \tau_b \quad (5.14)$$

It is important to note that given the dependence on known chamber pressure, the calculation of net flux from cryochamber must take into account chamber geometry. In reality, the pressure distribution is non-uniform as described by [3]. Accurate determination of the pressure in the chamber requires numerical simulation to account for effects caused by complex geometry such as the intake ducts. For this preliminary investigation, the aforementioned simplification

of intake chamber volume $V_i = 1 \text{ m}^3$ allows for simple dependency on the chamber density using the ideal gas law:

$$P_c = n_c k T_c \quad (5.15)$$

The mass flow rates throughout the system can be determined using Equation 4.7 applied to any of the derived particle flow rates.

5.2.1 Revised Performance Parameters

As in Section 4.1.2, certain performance parameters can be defined to provide a figure of merit for future systems. As before, the density ratio is simply the ratio between the ambient flow number density and the chamber density. However the previously defined collection efficiency no longer represents the efficiency of the entire intake system. Furthermore, the terms for during ideal collection and for vaporisation differ as the pressure terms must be considered in the latter case. Two density ratios were therefore be defined for each case with further definitions to non-dimensionless performance:

Perfect Collector Density Ratio

In the case of a collector that has a sufficiently low temperature to ensure all molecules entering the baffles are captured, the following definition of density ratio applies:

$$v = \frac{n_c}{n_\infty} = \frac{V_\infty}{\frac{1}{4} \bar{v}_c \tau_{i,2} + \frac{A_{th}}{A_i} \tau_{th} + \frac{A_c}{A_i} \tau_b} \tau_{i,1} \quad (5.16)$$

As with the thruster in section 4.1.2 it is useful to denote the reduced transmission through the baffles as a function of the area ratio between the intake and cryopump intake.

$$\tau_b^* = \frac{A_c}{A_i} \tau_b \quad (5.17)$$

Again substituting for the speed ratio equation and rearranging:

$$v = 2\sqrt{\pi} S_\infty \sqrt{\frac{T_\infty}{T_c}} \frac{\tau_{i,1}}{\tau_{i,2} + \tau_{th}^* + \tau_b^*} \quad (5.18)$$

Open Feed Density Ratio

In the previously described Open System, the density ratio increases as propellant is fed into the intake chamber. This also can be solved analytically this time rearranging equation 5.12 for the ratio between ambient and chamber conditions:

$$v = \frac{n_c}{n_\infty} = \frac{V_\infty A_i \tau_{i,1} + \frac{P_{vap} n_\infty^{-1}}{\sqrt{2\pi k M_p T_s}}}{\frac{1}{4} \bar{u}_c (A_i \tau_{i,2} + A_{th} \tau_{th} + \frac{\sqrt{\frac{T_s}{T_c}} k T_c A_c \tau_b}{\sqrt{2\pi k M_p T_s}})} \quad (5.19)$$

Further simplifying using expressions Speed Ratio, Average chamber velocity, The ideal gas law and the reduced baffle transmission:

$$v = \frac{2\sqrt{\pi} S_\infty \tau_{i,1} \sqrt{T_\infty T_s} + P_{vap} n_\infty^{-1} \tau_b^*}{k \sqrt{T_c T_s} (\tau_{i,2} + \tau_{th}^*) + k \sqrt{\frac{T_s}{T_c}} T_c \tau_{th}^*} \quad (5.20)$$

Non-dimensionalised Collection Parameters and Efficiency

The previously defined collection efficiency η_c represented the proportion of the flow that was utilised by the thruster, hence measuring the efficiency of the intake. Given the flow to the thruster as well as the flow into the cryopump can be considered useful, the efficiency of the system is not defined entirely by the thruster. For this reason, the definition of η_c can be clarified as simply the ratio between inlet flow and thruster flow R_{th} , having the same derivation as Equation 4.12, this time accounting for the additional storage term:

$$R_{th} = \frac{\dot{N}_{th}}{n_\infty V_\infty A_i} = \frac{\tau_{i,1}}{\tau_{i,2} + \tau_{th}^* + \tau_b^*} \tau_{th}^* \quad (5.21)$$

Repeating the same principle, this time denoting the ratio between inlet flow and the flow stored by the cryopump:

$$R_s = \frac{\dot{N}_s}{n_\infty V_\infty A_i} = \frac{\tau_{i,1}}{\tau_{i,2} + \tau_{th}^* + \tau_c^*} \tau_b^* \quad (5.22)$$

Finally, the sum of both flows into the thruster and cryopump in relation to available inlet flow was denoted as Utilisation Efficiency η_u indicating the amount of propellant lost due to backflow.

$$\eta_u = \frac{\dot{N}_{th} + \dot{N}_s}{n_\infty V_\infty A_i} = R_{th} + R_s \quad (5.23)$$

Efficiency During Propellant Feeding

In the case of the closed feed system, it was assumed that all stored propellant would reach the thruster and as such the feed efficiency is always theoretically 100%. The remainder of the system reverts back to the baseline only with the additional mass flow from the cryochamber.

This cannot be assumed for the open case as flow is lost through the intake ducts when released back into the intake chamber. The efficiency of the feeding phase could reasonably be defined as the ratio between the collected molecules leaving the cryosurface and the portion of the thruster flux that initially started at the cryosurface. However given the cryosurface flux is dependent on the chamber density, determination of this ratio is challenging to derive analytically. As such the efficiency of the open feed phase was left for future work with DSMC methods.

5.3 System Performance

Following a similar method to Section 4.2 the newly proposed storage system can be analysed. The previously defined criteria for Thrust, Thruster Input Power and Chamber Density were continued. An additional two criteria were defined to incorporate the limitations of heat load on the system. Both criteria were taken from a reference cryocooler system: The 2-stage Inline RSP2 [28].

- **Baffle Load:** The baffle load limit was taken as the First Stage heat lift at 200 W input power. This allows for up to $\dot{Q}_b = 4$ W of total heat load on the baffle at $T_b = 70$ K.
- **Cryosurface Load:** The corresponding Second Stage Heat lift at 200 W is 0.2 W. This allows a maximum cryopump heat load of $\dot{Q}_s = 0.2$ W at 20 K.

Using the same strategy as Chapter 4, the performance of the new system can be compared to the baseline.

5.3.1 Collection Phase

Before assessing the performance of the storage system itself, the impact of its implementation was compared to the baseline by looking at the density ratio and "collection efficiency". For the sake of comparison the baseline definition for collection efficiency will be used. The results at 180 km for moderate solar conditions can be seen in Figure 5.6.

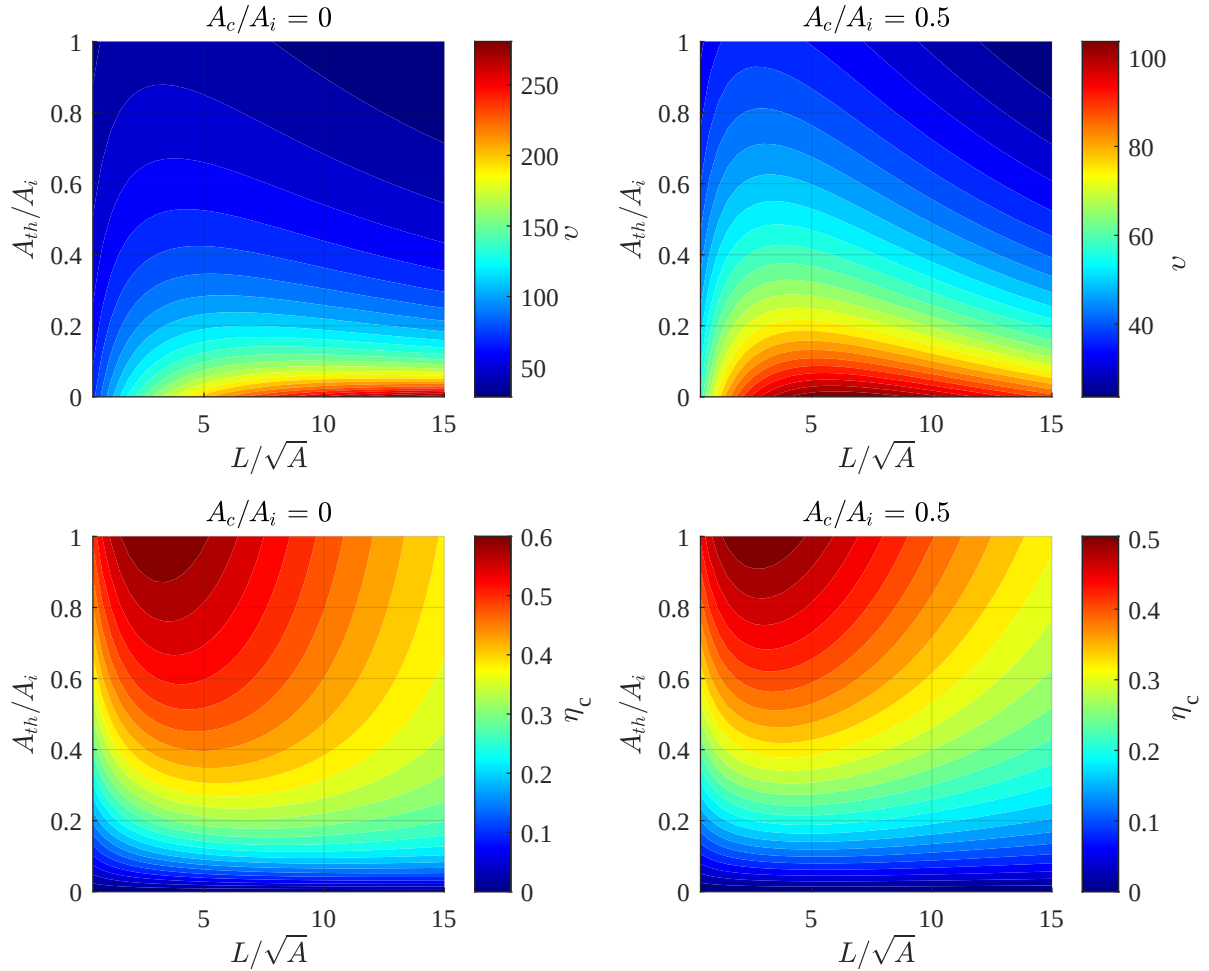


Figure 5.6: Density Ratio (top left and right) and collection efficiency (bottom left and right) with changing thruster area ratio A_{th}/A_i and intake aspect ratio $L/\sqrt{A}$ at cryopump inlet area ratios of $A_c/A_i = 0$ and $A_c/A_i = 0.5$.

The inverse relationship between collection efficiency and density ratio is still seen. However a loss in both density ratio and collection efficiency is observed when the intake area for the cryopump is increased. This is easily explained. More molecules are leaving the intake chamber than in the baseline for a given thruster inlet area resulting in a lower chamber

density. The loss in "collection efficiency" is a result of this same principle. The baseline definition of collection efficiency is the ratio between the number of molecules entering the thruster to those available to the intake area. Introducing the cryopump reduces the flux to the thruster hence reducing this ratio. This illustrates the need to define Utilisation Efficiency η_u to account for the stored mass in the efficiency of the system.

It is worth noting that there is a change in location of maximum density ratio. For $A_c/A_i = 0$ the optimum density ratio occurs at $A_{th}/A_i = 0$ and $L/\sqrt{A} = 15$. When introducing the cryopump with $A_c/A_i = 0.5$, the maximum density ratio occurs at $L/\sqrt{A} \approx 6$. The thruster inlet area remains the same. This can be explained by the net flux leaving the system via the cryopump reducing the density ratio in the chamber. It is indicated that further consideration is needed when optimising the intake duct area ratio for density ratio. Specifically, introducing a storage solution reduces the optimum aspect ratio. However to minimise the changes from the baseline for comparison purposes an aspect ratio of $L/\sqrt{A} = 7$ was retained. The following table outlines the input variables for the constraint analysis in the storage system case:

Table 5.2: Constraint analysis program inputs for the storage system case

Input	Value
Altitudes (km)	170,180,190
A_i (m ²)	1
T_c (K)	300
Power Limit (W)	1500
Minimum n_c (m ⁻³)	10^{18}
Max $\dot{Q}_b$ (W)	4
Max $\dot{Q}_s$ (W)	0.2
η_{th}	0.6
η_m	0.8
$L/\sqrt{A}$	7
A_{th}/A_i	0 to 1
A_c/A_i	0 to 1
Grid Size	30

Figure 5.7 illustrates the newly defined heat load constraints alongside the chamber density and required input power. The constraint lines are plotted within the design space with corresponding values. The example used was taken at 180 km for moderate solar conditions.

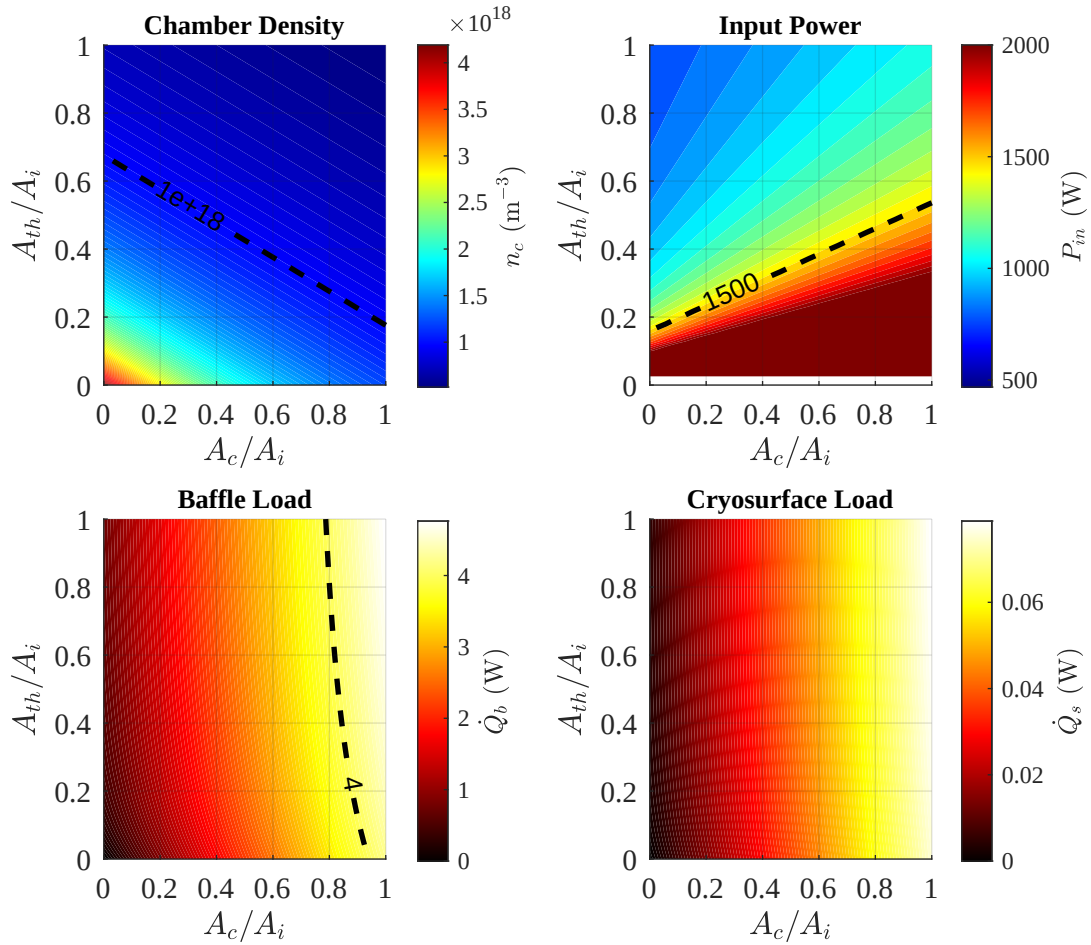


Figure 5.7: Contour plots of Intake Chamber Density, Required input power, Baffle Load and Cryosurface Load using the Cryopump Collection Model. Results correspond to an altitude of 180 km with moderate solar conditions.

Considering Intake Chamber Density, increases in both thruster inlet area and cryopump inlet area result in a decrease in density due to the increase in molecules leaving the system. In the case of required input power, increasing thruster area increases the mass flow rate to the thruster, hence decreasing power. Increasing the cryochamber inlet area has the opposite effect, as this effectively removes molecules from the system that could otherwise be used for thrust.

The heat loads are largely a function of cryopump inlet area. Generally, larger cryopump inlet areas result in larger heat loads. A slight affect can be seen from changes in thruster inlet area due to the heat transferred through collisions of the molecules with the baffles and

cryosurfaces: At low thruster inlet areas more molecules are impinging on both surfaces, resulting in a larger heat load. However it is likely that the heat loads are largely a result of radiation as this effect is minor. It is worth noting that in all design spaces considered, the load on the cryosurface was not a limiting factor as even at a maximum intake area of $A_c = A_i$ the load is not exceeded. This was observed for all future constraint analysis results.

Collection Rate

To investigate the performance of the storage system the rate of collection is critical as maximising the collection rate decreases the time required to collect propellant that can be used. The collection rate can be maximised by increasing the number flux at the baffle. Given the currently considered system, the following results were found given the same altitudes as in Section 4.2 and applying the limitations shown in the previous section. Firstly consider the non-dimensional parameters, R_s and η_u . Since they are independent of true geometry this provides a good figure of merit for future investigations.

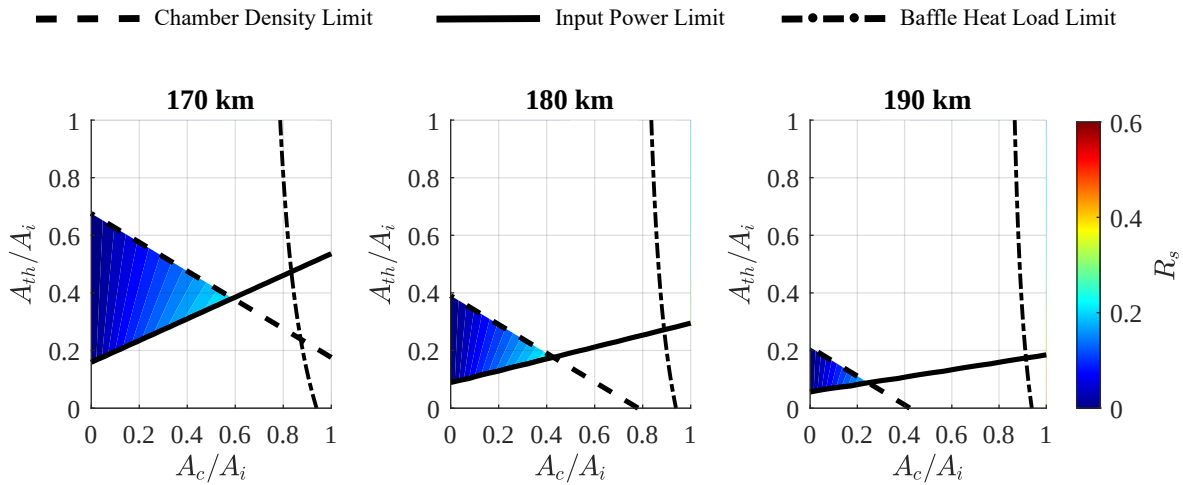


Figure 5.8: Ratio of inlet flow to pump flow R_s for 170 km, 180 km, and 190 km.

Figure 5.8 shows the changing design space with altitude when considering the ratio between flow into the cryopump and the flow incident on the intake area. The maximum flow to the storage system occurs at maximum cryopump intake area for all cases. This also represents the condition for maximum collection rate. For the conditions shown, the Power and Chamber Density constraints are the limiting factor in maximising the collection rate as the heat load does not restrict the design space in any of the analysis performed.

Now considering the constraints on the Utilisation Efficiency, the total "used" propellant can be investigated. The results are shown in figure 5.9.

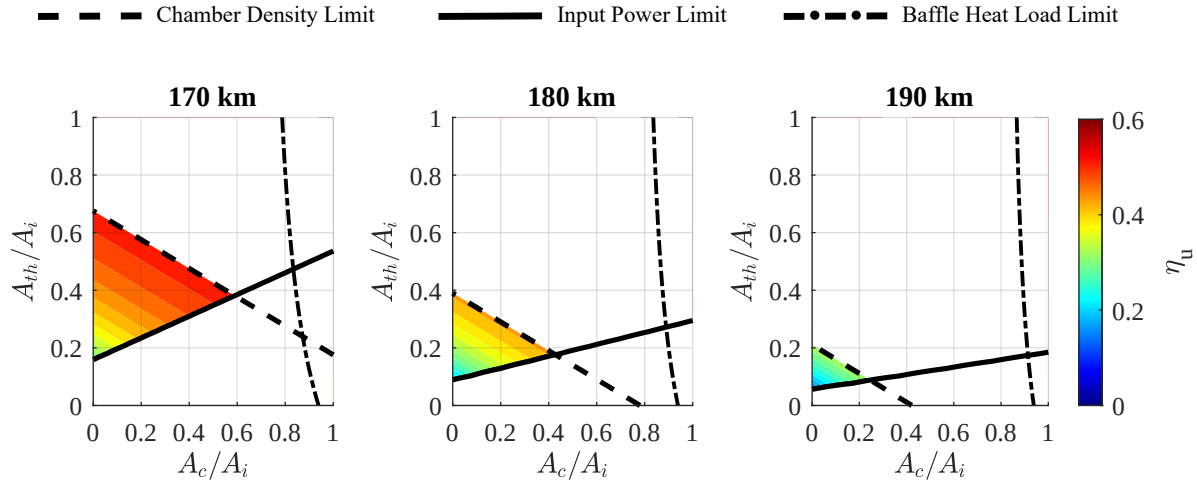


Figure 5.9: Utilisation Efficiency η_u for 170 km, 180 km, and 190 km.

The Utilisation efficiency is a measure of the total mass entering the thruster and the storage system. As such the maximum occurs where the thruster inlet area and cryopump inlet area are maximum. Since increasing these parameters results in a lower chamber density, the maximum possible utilisation efficiency drops significantly with altitude. This is due to the lower overall density available even at maximum density ratio.

Applying the constraints to the design case where $A_i = 1 \text{ m}^2$ the maximum collection rate can be determined. The results shown in Figure 5.10 are identical to the behaviour shown in by R_s : The maximum storage mass flow rate occurs where A_c is maximum for all cases.

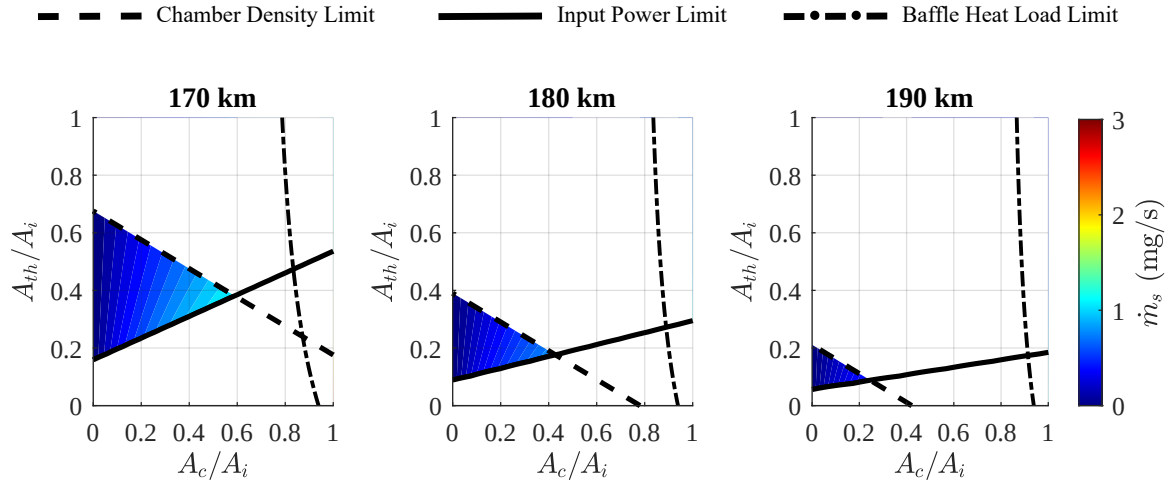


Figure 5.10: Storage mass flow rate for 170 km, 180 km, and 190 km.

The issue of choosing a design point that ideally occupies all design spaces is the same as the baseline case. However for the storage case less overlap is observed between altitudes. This is likely due to the reduced density ratio and thruster flow caused by the introduction of a storage system. This effectively reduces the envelope of operation when compared to the baseline.

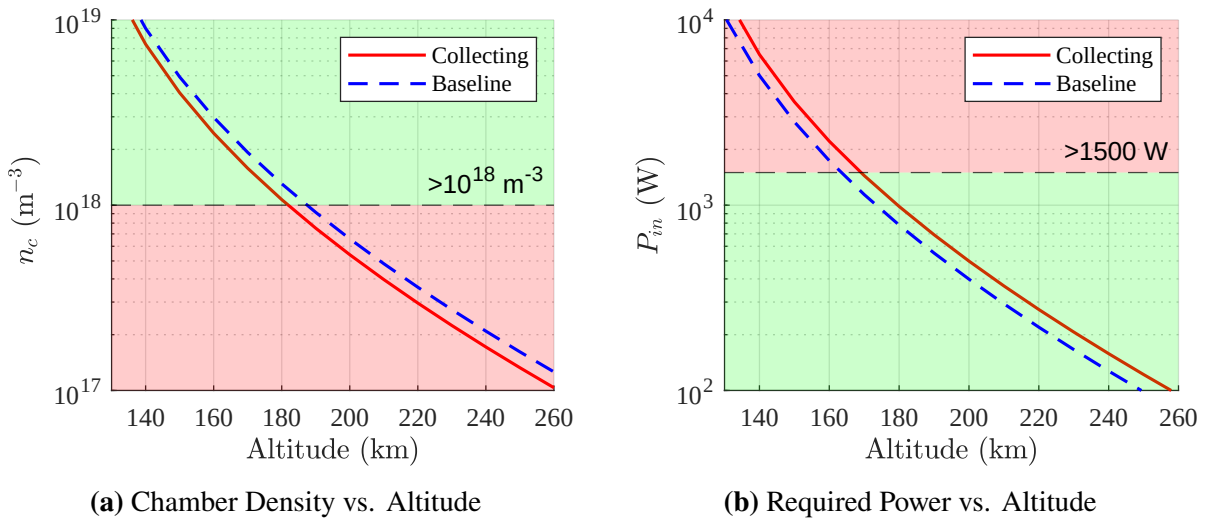
When selecting the design point for the cryopump inlet area, maximising collection efficiency seems a reasonable goal. From the constraint analysis it can be seen that as altitude decreases the opportunity for higher mass flow rates increases. For instance the maximum evaluated storage system mass flow of 1.17 mg/s occurs at 170 km. this point however will result in a small operational envelope as it occurs at the point where both the number density and power limits are reached. A conservative estimate of $A_c/A_i = 0.2$ was made as this point was present in all design spaces. For comparison purposes $A_{th} = 0.25$ was retained from the baseline despite its location outside the constraints shown for 190 km altitude. The constraint analysis results can be seen in Table 5.3.

Table 5.3: Constraint Analysis Results and Choice of Design Point for the storage case

Best Values Found			
Altitude (km)	170	180	190
Best R_s	0.23	0.23	0.16
Best η_u	0.52	0.44	0.32
Best $\dot{m}_s$ (mg/s)	1.17	0.81	0.47
Chosen Design Point			
A_{th}/A_i	0.25		
A_c/A_i	7		
Design Point Results			
R_s	0.12	0.12	n/a
η_u	0.43	0.42	n/a
$\dot{m}_s$ (mg/s)	0.63	0.43	n/a

5.3.2 Off Design Point Performance

To compare the performance of the new concept to the baseline the process for determining the off-design point performance was repeated.

**Figure 5.11:** Off design point performance with limits due to power and thruster density

A noticeable reduction in envelope can be seen as expected from constraint analysis. This is likely due to the reduction number density and thruster flow. Since there is a reduction in density ratio, the upper limit of the altitude envelope has dropped to 182 km compared to 189 km. Due to an increase in required thruster input power the minimum altitude has increased to 169 km compared to 164 km for the baseline. This results in an altitude envelope of 13 km. A reduction in 12 km from the baseline which is significant. Despite this reduction, a feasible operating region for the collection phase has been established.

Further investigating this operating region, the mass flow rate entering the storage system was plotted against altitude in Figure 5.12a. The resulting net force on the system was plotted with altitude and compared to the baseline as seen in Figure 5.12b. This allows an indication of the penalty in useful thrust for the storage system.

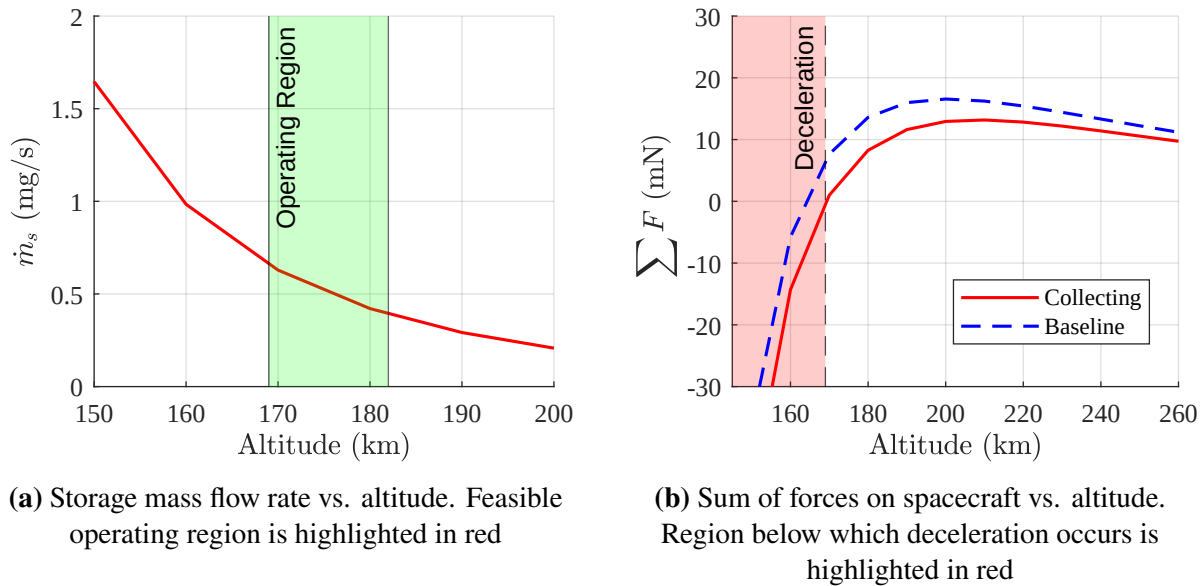


Figure 5.12: Performance of the system with altitude

As expected from constraint analysis a loss in mass flow to the storage system can be seen if altitude is increased. Within the operating region the $\dot{m}_s$ has a range of 0.74 mg/s to 0.43 mg/s. Within this region a similar behaviour in maximum thrust is seen between the baseline and the storage system concept however there is a net loss in available thrust for all altitudes considered. Between the maximum and minimum altitudes a reduction in available thrust of ≈ 5 mN was observed.

5.3.3 Feeding Phase

During the feed phases of a mission the propellant that has been collected is being fed either directly to the thruster (closed system) or indirectly through the intake chamber (open system). To determine the effectiveness of this phase the thrust performance was evaluated for both concepts.

Open System

In the case of the open system varying the temperature of the head will affect the net flux leaving the cryochamber. Therefore by solving equation 5.20 for T_s the relationship between head temperature, thrust and the mass flow rate leaving the storage system can be shown for a maximum power. Results for these parameters can be seen in Figure 5.13.

Figure 5.13a Shows the relationship between head temperature and the sum of forces for maximum available thrust. As the cryohead temperature increases the net thrust is increased. The "Zero Storage Flux" denotes the point where there is a net equilibrium between the flux of condensing molecules and those evaporating from the surface. As seen in Figure 5.13c below this point there is a net gain during the collection phase. Figure 5.13b shows the effective limit to pumping speed is reached when the head is below 24 K. Below this point the model for the perfect collector applies. Above the zero flux line propellant is fed to the intake chamber, increasing the thrust. Figure 5.13b shows the comparison between mass flow rate through both the thruster and through the cryopump inlet for varying cryohead temperatures. Since the thruster is still receiving flow from the intake thrust is still produced when the cryohead temperature is low enough to collect the gases. Once T_s is high enough there is a net increase of molecules added to the chamber, increasing the thruster mass flow rate. At a cryohead temperature of 29.5 K the majority of the thruster flux is supplied by the storage system.

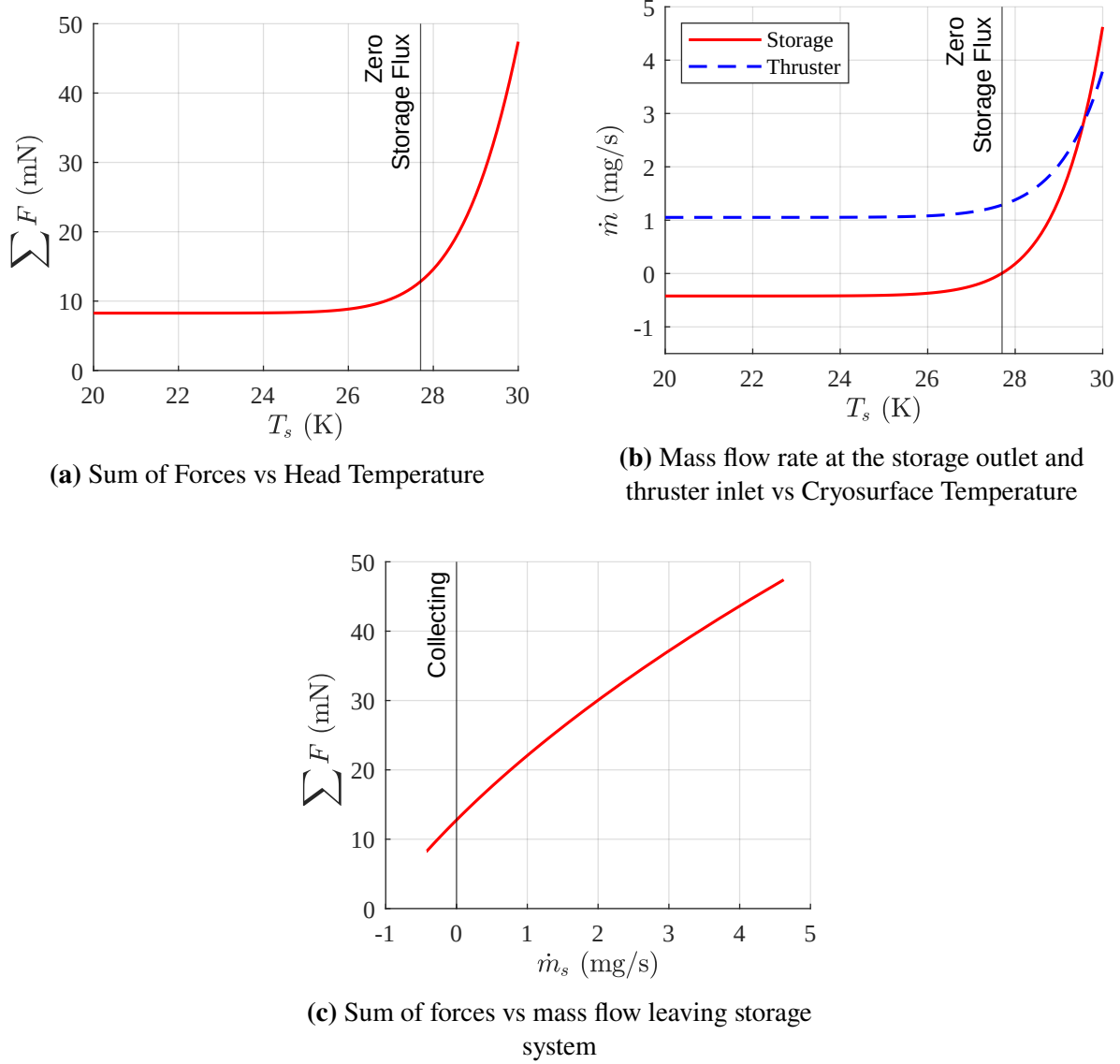


Figure 5.13: Chosen Performance Parameters for Feed System. Altitude = 180 km, $R_{th} = 0.25$, $R_s = 0.2$

In effect this constitutes control of the desired thrust through heating and cooling the cryohead. However alternatively, this could be used to constitute control of chamber density and minimum required power. For instance, looking at Figure 5.14a, the required P_{in} can be reduced by increasing mass flow to the thruster. Furthermore the number density can be controlled using the same principle as seen in figure 5.14b. Increasing the cryohead temperature increases the number density in the chamber.

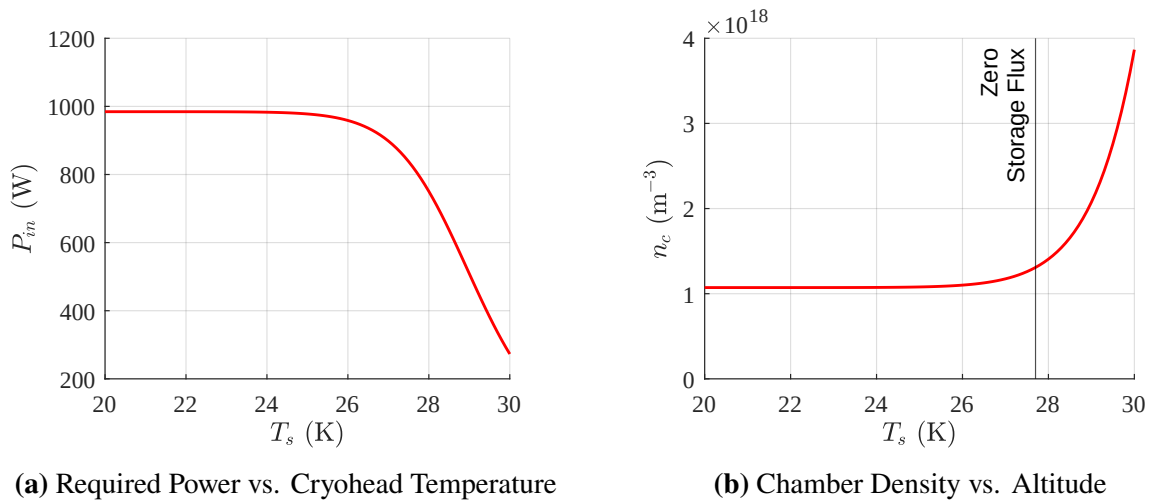


Figure 5.14: Required system power and chamber density according to chosen pump surface temperature

These two phenomena constitute two of the primary benefits beyond additional thrust. By control of the required power and number density, the upper and lower limits of altitude for a given mission can be expanded temporarily provided stored propellant is available. Alternatively, fluctuations due to solar activity, wind and location can be compensated.

Closed System

In the closed system the collected propellant is fed to the thruster through the same method as traditional feed systems. Since in a closed system the cryochamber can be thought of as a pressurised vessel, the mass flow to the thruster from the storage can be controlled using valves. Since no molecules are lost through the intake ducts, the system operates exactly like the baseline only with an additional mass flow added to the thruster. This is assumed to have no effect on the intake.

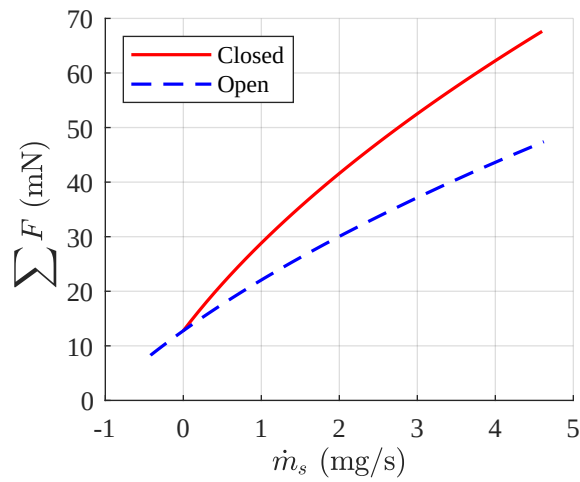


Figure 5.15: Collection Performance vs Altitude

Figure 5.15 Compares the resulting useful thrust in the case of the open and closed systems. It is apparent that since fewer molecules are lost, better thrust performance can be seen for the same rate of propellant burn.

6 | Further Discussion

In the previous two sections the method for each system was shown. This was followed by performance estimates and results for each system, each with specific discussions. This chapter will contain a more general discussion given what was learned from results in previous sections. Furthermore the method will be critiqued to assess limitations and hence the quality of the study.

Firstly it is useful to reassess the goals for the investigation. Early in the project the goals changed often as an understanding of ABEP systems matured. Once the storage system became the focus of the investigation many potential areas of analysis were uncovered which proved a daunting task for a short term project. To balance the depth of investigation with the scope of the study from an overall system perspective, the primary objectives became designing a model for the concept and proving a feasible design space existed. Before suggesting whether this was a success, it is important to take a critical look at the methods used in the previous sections.

6.1 Method Critique

In this section the method and choice of parameters used for analysis will be critiqued in a discussion about their limitations. Firstly the baseline will be compared to other similar studies. This will allow an indication of the scope of analysis. After this the newly developed model for a storage system will be discussed. Given this concept is novel, the quality of the method will be assessed by looking at the level of validation.

6.1.1 Environment Model

The atmospheric model choice and implementation has a direct impact on the confidence in results. Firstly addressing the choice of model: NRLMSISE-00 has been used extensively by the aerospace industry and by several papers on ABEP systems. Although seemingly a well validated choice, limitations in its accuracy exist. For instance the calculations implemented

81-day averaged values at set solar activity levels. The advantage of this method is that it is independent of time and can give a reasonable estimate of potential values however true fluctuations and mission specific conditions that change with time are omitted. This is a significant drawback in assessing the changes that the system will need to cope with over a short timescale. Nevertheless given this methods proven track record in mission design for previous space systems, it was deemed a suitable level of accuracy for the purpose of simply proving that the ABEP and corresponding storage concepts are at least feasible.

The implementation of this model is another consideration. The technique used averaged the conditions over a given circular orbit. This is essentially the result of a trade-off between the detail of results with respect to true fluctuations and the ability to assess conditions at differing altitudes. Spacially averaging the environmental conditions fails to take the fluctuations over an orbit into account. However given the time allocated, a detailed investigation of the fluctuations was not possible and the choice was made to focus on the system model and its implementation. For this reason future work assessing the performance over a single orbit will likely be needed.

Further to the averaging method, a detailed comparison between various orbits could allow for a better understanding of the performance of the system under different mission scenarios. The current study only considers a simple analytical approximation for a SSO

6.1.2 Baseline Model

The baseline model that was implemented is relatively well established in the literature. However there exists some assumptions in the current implementation. Considering some of the major simplifications in the model:

- *Free Molecular Flow*: The flows throughout all sections of the intake are assumed to be free molecular. This negates several factors that may be important to design decisions: Firstly collisions inside the chamber are not considered in any way. This allows the model to be purely analytical however in a true case the density in the chamber is higher than that in the free stream. As such in at low altitudes the increase in the density could cause the flow to enter the transition region below $Kn < 10$. Furthermore the flow in the chamber is likely non-uniform. This can be seen in the studies done by Di Cara et al where the extra compression caused by the length of the chamber is deliberately induced to reduce pressure fluctuations at the thruster inlet. Simply put this implies a density change between the ducts and the thruster inlet which is not considered by the

analytical model. The solution to this is to use DSMC solvers to numerically model the flow into the intake. For this reason, this is the most important of the future work.

- *Geometry and Collisions*: Even in free molecular flow where the majority of the flow behaviour is governed by wall interactions, the geometry itself is not considered by the current model. For instance the transmission probabilities and particle flux rates are all simplified to analytical expressions. Like in the previous point, Verification of these parameters using DSMC methods are required to take into account three dimensional geometry, thruster inlet position and inlet shape.
- *Thruster Model*: The method for evaluating thruster performance is very simplistic. Simple analytical formulas that generalise the phenomena in electrostatic thrusters are used as an initial estimate. Given the importance of thruster performance and thruster type on the feasibility of an ABEP intake this generalisation likely overlooks limitations to the overall system. For instance by simply assuming values for Mass Utilisation Efficiency η_m and for overall thruster efficiency η_{th} the effect of flow conditions at the inlet are not linked to the performance of the thruster. In the current study it is assumed that these values hold granted the an intake chamber density of $n_c = 10^{18}$ is achieved. An alternative to this would be to relate the mass utilisation efficiency to the mean free path in the thruster as done by Garrigues et al and Andreussi et al [37, 38, 36]. This would allow discharge power and thruster transmission to be modelled more accurately however the method would depend on a choice of plasma source.
- *Drag Model and Spacecraft Geometry*: A significant simplification can be seen in the model for drag and the geometry of the spacecraft. In particular, higher power levels will require larger solar panels and hence further increase drag. The current model assumes that the form factor of the solar panels allows a negligible impact on the drag. This is done in several preliminary studies [29, 30] however more recent studies incorporate this into full system designs through numerical optimisation methods [36]. Furthermore looking at the intake itself, the flow impinging on the sections in-between and around the intake ducts are not considered. Work by Boldini et al [34] refer to this as the packing efficiency: the ratio of total frontal area to open intake area governed by duct geometry. Incorporating these factors into system model would yield better results for considered drag.

The calculation of drag itself is also a simplification. By simply assuming a Drag coefficient of 2.5 this fails to take into account specific geometry. A common method for analytically calculating the drag coefficient the Schaaf and Chambre model which

relates the drag coefficient to specific angles of attack and the accommodation coefficients of the spacecraft bus materials [45]. An alternative is to again use a DSMC solver.

A common theme in the shortfalls of the model is a failure to take into account specifics of the mission, geometry or sub-system design of the spacecraft. This is a classic problem in systems engineering as a trade-off must be done between simplifying assumptions and detail for a given allocated investigation time. Simply put the more detail in the model, the more decisions about sub-system design are required. It is clear that ABEP systems are highly mission specific in their design and as such removing many of the simplifications would require detailed choices in geometry, thruster type, and mission design.

6.1.3 Storage System Model

The storage system concepts suggested are in many ways novel in application. However all sub-systems that have been suggested not novel individually and so the assumptions in the method are based on established engineering. This allows for a method critique by relating the assumptions made in the baseline to the new concepts and their components. Firstly it is worth noting that all critiques of the baseline also apply to the storage system as both utilise the same principles for the passive intake and thruster designs. Further to this, the rest of the system can be discussed:

- *Real System Configuration:* The proposal of the Open and Closed systems are highly conceptual and as such very few design specifics are presented. Since the concepts are first iterations, very little consideration was given to important factors such as location on the spacecraft, implications to mass and volume or lifetime. Further design work for assessing feasibility will require these considerations. This method however is typical of early design studies.
- *Pump Speed and Flux Model:* The method outlined in Chapter 5 to model the flux to and from the storage system was simplified on the same assumptions as the baseline model. Additionally certain instances of the design were assumed from the literature as opposed to calculation. One such instance is the baffle transmission probability. This value is used as an initial estimate in cryopump design [7] however in reality it is geometry specific. Careful consideration must be given to heat transfer when designing the baffles. As such it was deemed too detailed for the present study as too many assumptions would need to be made about the configuration of the system.

Using simple analytical expressions for the flux into the cryopump is used often in

design studies [7, 24, 25]. To simplify the model further, calculations assumed a mean molecular mass of the mixture instead of individual species. This approximation is reasonable when considering a perfect collector as the flow is largely independent of species mass as seen in equation 5.18. However when considering evaporation from the cryosurface this assumption is incorrect as different species have different vapour pressures. This choice along side the assumption that the vapour pressure is simply that of N_2 likely overestimates the rate of evaporation for atomic oxygen. To correct for this however numerical methods may be needed as since the evaporation rate is a function of cryochamber pressure (and hence intake chamber pressure).

- *Geometry and Heat Transfer:* To assess the feasibility of the system in relation to heat load on the baffles and cryosurface, a simplified geometry was adopted. The geometry was used to solve simple estimated for view factors. The geometry presented however is likely impossible if the thruster is assumed to be concentric with the intake as is done in most ABEP concepts. This fails to properly capture the true likely view factor of the baffles. However, to find this true view factor a specific geometry and a numerical solver would likely be needed.

Separate to radiative heat transfer, conduction through the components onboard is not considered. This will likely have a bearing on the heat load for each stage of the cryo-cooler and effect the performance. However as with calculation of accurate radiation heat transfer, the geometry and materials must be considered for any calculations. It is worth noting that a preliminary material selection was made with aluminium or copper being the prime candidates, however given time constraints this was not extended to conduction between various sub-systems. The heat transfer between the thruster and the cryo-cooler for instance is likely an important factor. It is assumed that the cryopump chamber is the same temperature as the baffles, if the system is housed close to the thruster this assumption could greatly underestimate loading on the 70 K first stage of the cooler.

6.1.4 Constraint and Off Design Point Methods

Properly understanding the performance taking into account all design parameters is very challenging without complex optimisation tools. Even then, a target must be set and so evaluating performance limitations in all scenarios is not feasible. For this reason, many potential optimums and feasible design spaces are overlooked.

The method employed to determine the limitations for fixed environmental conditions was constraint analysis, where the limiting parameters are mapped onto a target variable that the designer wishes to optimise for. This method has several drawbacks. Firstly, only conditions are constant, meaning the optimisation is limited to the number of conditions investigated. Secondly, the parameter of interest is mission specific. For instance if the mission demands high power, choosing a parameter that maximises thruster flow is a good option. Alternatively if power is in abundance and collection rate is of greater interest, this parameter could be chosen. This requires a choice that limits the optimisation to a single factor. A more sophisticated optimisation technique such as that used by Andreussi et al [36] or that proposed by Parodi et al [30] could provide a clearer picture of performance.

To help combat the previous methods inability to assess the performance over varying flow conditions, an off-design point study was used to test the limits of a given system within the same constraints. The disadvantages of this method are near opposite. Although it allows for an evaluation of performance over a range of flow conditions, a design point must be chosen. This removes any ability to clearly see the effect of change in geometry on the system.

When used in tandem these methods complement each other quite well however since both require "freezing" of certain design parameters the scope of the results are limited to how many freezing points are made. Simply repeating these calculations would result in a more complete picture however this is inefficient. The studies mentioned previously that employ optimisation algorithms solve this issue by theorising a mission application first. This provides requirements and targets for the algorithm.

6.2 Further Discussion and Feasibility

Taking into account the results and limitations discussed in the previous section, this section will theorise the potential of a cryopump based storage system.

6.2.1 Reduction in Performance During Collection

From the results in Chapter 5 an obvious loss in performance with respect to the baseline can be seen. As described before, taking part of the collected mass from the intake chamber that would otherwise supply the thruster has a similar effect as lowering intake performance. This has several important implications:

1. Increasing collection rate reduces the upper and lower limits to altitude, increasing

requirements for orbit control.

2. Alternatively, increasing collection rate required better performance from the intake or less strict limitations to chamber density and power in order to maintain the altitude envelope.

These two points illustrate an important trade-off when considering margins for error in the orbit. Additionally a second noticeable trade-off can be seen. Consider the design of a collection phase in terms of mass accumulation rate:

1. Reducing collection rate provides a more favourable flight envelope however this decreases collection rate. Hence more time must be spent in this envelope.
2. Increasing collection rate reduces the flight envelope. However the rate of accumulation is greater. Hence the time spent in non-ideal conditions is less.

Applying this trade-off to a potential duty cycle: The collection is done at a particular altitude over a short period of time during favourable solar conditions. For another portion of the mission the propellant is used to allow for mission objectives such as particular observation points that are not feasible with purely passive systems. Another important point is the impact that ABEP passive intakes have on the performance. Given the correlation seen in results between the performance of the intake, future advancements in this area will only improve the concept's potential.

6.2.2 Storage Capacity

Estimating the storage capacity of a cryopump proved a challenging investigation. The requirements for cryopumps used in their usual high vacuum application are drastically different to the prospect of using them as space propellant storage. For instance for an effective spacecraft storage system, one can assume that total system mass and maximum physical capacity are drivers in design. The requirements for industrial vacuum pumps are quite different. Mass only becomes an issue from the perspective of mounting one to a vacuum chamber and as such they are often heavy. Furthermore, vacuum pumps must maintain a sealed vacuum between atmosphere on ground and a high vacuum environment, further adding material that is wasted in space applications.

Nevertheless, some rough estimates were made with reference to existing systems once flow calculations had led to a choice in cryopump inlet area. Considering the design point of

$A_c/A_i = 0.2$, with an intake area of unity, the intake radius for the pump can be assumed to be around 0.25 m. A reference cryopump with a matching radius was identified from this value. The UH20P Ulvac system allows for a storage capacity of 5.8×10^8 Pa·l (Note that vacuum pump capacity is often measured in Pascal Litres). Using ideal gas law and presuming that the condensate will be at 20 K, the geometry described previously can be used to estimate the volume and hence the capacity in number of molecules. Note that there is an assumption that both pumps will have a similar volume. This results in an estimated number of captured molecules of 2.1×10^{27} . Assuming an average molecular mass of 3.6×10^{-26} kg this results in roughly 76 kg of capacity.

This is a substantial estimate however it was omitted from the main analysis as it is difficult to verify given the lack of available information. Given the likely unpredictable characteristics of condensate growth on the cryohead, capacity would likely need to be determined experimentally. Defining capacity with future research is an important factor as this will define the duty cycle between which propellant is used or collected.

To briefly assess the implications of storage capacity, consider a spacecraft with with a collection rate of 0.5 mg/s (between the design point values in Table 5.3). Assuming an even duty cycle of collection and evaporation, roughly half the mission will be spent collecting. In a typical 8 year mission that is 4 years of collection. At 0.53 mg/s that amounts to around 67 kg of propellant collected over the mission. Considering the same spacecraft has a mass of 800 kg and a thruster with a specific impulse of $I_{sp} = 2000$ s (A moderate value for today's ion thrusters [9]). Tsiolkovskis Equation can be used to estimate the excess Δv that would be available throughout the mission.

$$\Delta v = I_{sp} g \ln \frac{M_0}{M_f} \quad (6.1)$$

Where M_0 is the initial mass after collection, M_f is the dry mass of 800 kg and g is 9.81 m/s^2 . This results in a 1578 m/s of extra Δv . The usage of the propellant depends on the storage capacity, as if less propellant can be stored there will be a maximum in Δv per collection phase. It is also worth noting that a duty cycle may not be equal between collection and evaporation. Propellant required for minor corrections could be collected quickly in a few months before burning it slowly when needed for several years.

Feed Performance

It is apparent from results that in the case of an open system the rate at which propellant can be fed to a thruster is a function of head temperature. The ability to precisely control the cryohead is an important factor in feasibility. For instance if the goal is to counteract fluctuations for a more precise orbit, changing the mass flow to the thruster depends on the precision and speed of the temperature change. Alternatively a series of cryoheads could be set up to act as binary sections that either collect or feed. Balancing the number of active and inactive surfaces could provide control with less dependency on precise temperature control. This however would likely be more complex and the implication of temperature differentials on the rest of the pump would need to be carefully considered.

In the case of a closed system, a more complex system is required. Firstly the closing baffles or valve into the chamber would need to be reliable enough for operation in space. If considering a large mushroom valve this would constitute a single moving part. More complex systems exist in space such as solar panel drives and movable antennas. However the development and testing would likely be extensive when compared to the open system that requires no moving parts and can operate on existing cryocoolers. That being said it is worth noting that in the event the closed system fails while in an open position, this would simply become an open system.

6.3 Mission Applications

On a final note, the work produced will be used to give insight into possible mission scenarios in which a storage system such as this would be useful. Three scenarios can be proposed:

- *Improving Earth Observation with ABEP*: The benefits of using ABEP for earth observation have been well established in previous works. Lower Altitude Missions reduce launch costs, improve observations of Earth's surface, and can provide valuable insight into the climate and atmosphere. The current ABEP designs are a good solution to this problem. However if implementing a storage system, their shortfalls can be reduced.

For instance unforeseen fluctuations in flow conditions are very likely to occur given the unpredictable nature of the atmosphere. To cope with this, a small window of collection at the start of a mission could provide an alternative to simply increasing power.

From an orbit perspective, having a supply of propellant allows for a larger range of altitudes for a period of the mission. If a storage system was implemented on an ABEP system, regions of the atmosphere that were previously inaccessible due to limitations in power and chamber density could be accessed for short periods of time to perform observations before returning to the collecting region.

- *Refuelling Missions:* Although this idea is still far from realisable, collection of propellant for use on other missions is not a new concept. The work outlined briefly in the literature review proposed using large scale nuclear powered refuelling stations. This concept is unlikely to be approved given the dangers of nuclear reactor operating in a region where any failure results in re-entry. However solar panels that produce multiple kW of power are common today, and with ABEP systems developing quickly the opportunity to revisit propellant harvesting has re-emerged.

7 | Conclusions & Future Work

In summary, a baseline model for an Atmosphere Breathing Electric Propulsion system was constructed and tested. A viable storage solution was then proposed and successfully tested. The performance of both systems was compared and the feasibility of the storage solution assessed.

The baseline systems model adapted from previous works showed good agreement in performance to comparable studies. The effects leading to the common trade-off between density ratio and collection efficiency were clearly observed. A valid design point was found using constraint analysis. This resulted in an estimated density ratio between 104 and 107, with a collection efficiency between 0.37 and 0.38. Off-design point analysis led to a feasible operating region of 164 km to 189 km at moderate solar conditions. This is within estimates of previous studies adopting a similar method. This provided a good confidence in the baseline model for later comparison with the proposed storage system.

Two cryogenic storage concepts were investigated using the same method. Building upon the baseline system, a mathematical model was derived to describe the operation of the storage concepts. To assess feasibility a design space taking into account chamber density, thruster power, baffle heat load and cryosurface heat load was established. Several important observations were made:

- When analysing the limits on the system, the thermal loading on the cooled components was found not to be a primary limiting factor. This has several implications. Firstly the power required to supply the cryocooler is lower than that of the reference system. Provided the system can be flown within other constraints this shows good promise from the perspective of cooling requirements. It is implied that less than 200 W of power is needed to achieve the collection performance shown in later results. Furthermore if 200 W is afforded to the system, a larger collection rate may be possible.
- Constraint analysis further revealed that the primary limiting factors were the thruster power and the chamber pressure. Collection reduced the supply of gas to the thruster

resulting in a tighter design space. This must be carefully considered in the design of a storage system as high collection rates can lead to insufficient thrust to overcome drag.

- Given a design point of A_c/A_i of 0.2 a storage collection ratio of 0.12 and a system utilisation efficiency of between 0.42 and 0.43 calculated. At this design point a feasible region for collection was found between 169 km and 182 km. When compared to the baseline this results in a net loss of 13 km in operational envelope to operate the collector. The collection rate achieved in this region was between 0.43 mg/s and 0.74 mg/s.
- Finally the performance of feeding collected to the thruster was analysed for both the Open and Closed systems. A clear increase in performance was observed. By evaporating the propellant, an increase in chamber density led to a clear increase in thrust. When comparing the open and closed system, a clear benefit was seen in the performance of the closed system as less propellant was lost during the feeding process. Furthermore the usage of propellant successfully reduced requirements on thruster intake power and number density. This suggests it is feasible to use propellant storage to cope with unforeseen changes in flow condition or allowing access to otherwise infeasible orbits.

Reflecting on the project objectives, the model developed for in-orbit cryogenic storage gives a good indication that the system is feasible. However since the concept is in its infancy, adequate validation has not been given to confirm this. To further the investigation the following future areas of work could lead to a greater confidence in the current system and provide insight into its potential.

- A common next step in similar studies involves using DSMC methods to better assess the flow conditions in the system. A study of this nature could use the current design points or implement a similar method to assess new ones. A far greater confidence in results could be had if results from a well established numerical solver were in good agreement.
- General improvements in the system model itself would yield better understanding of the concepts limitations. For instance, the impact of local fluctuations in flow conditions throughout a single orbit have not been explored. Nor has the effects of wind and angle of attack. Furthermore, simple changes to the current method such as considering multiple species would increase accuracy of transmission probability.
- The design of a cryocooler cycle is not considered in the current analysis. The system

could be optimised for the cryogenic storage case and used to further assess the required power levels for cooling.

- Implementing a global optimisation method would improve the efficiency of assessing performance over the current constraint method.
- A detailed mission design would provide more specific requirements. This could provide a basis for decisions on system elements such as power, mass and feed control. Additionally certain concepts could be explored such as thrust-collection duty cycles and potential new orbit regimes.

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A | Saturation Pressure Model

Clausius-Clapeyron Equation: A and B are experimentally derived constants and T is the temperature of the surface [23].

$$\text{Log}(P_{sat}) = A - \frac{B}{T}$$

Table A.1: Clausius-Clapeyron Equation Constants

	O ₂	N ₂
A	9.082	8.362
B	480.871	387.428

B | View Factors

The following expressions outline the view factors used for each component in the storage system.

Intake Chamber to Baffle View Factor

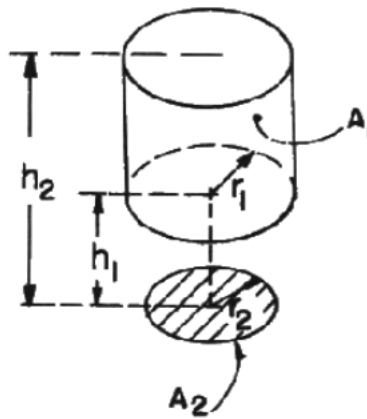


Figure B.1: View Factor from Intake Chamber to Baffle

$$F_{1-2} = \frac{1}{4R(H_2 - H_1)} \left[(X_1 - X_2) - (X_1^2 - 4R^2)^{1/2} + (X_2^2 - 4R^2)^{1/2} \right]$$

$$R = r_1/r_2 \quad H_1 = h_1/r_2$$

$$H_2 = h_2/r_2 \quad X = H^2 + R^2 + 1$$

Baffle and Cryochamber to Cryosurface View Factor

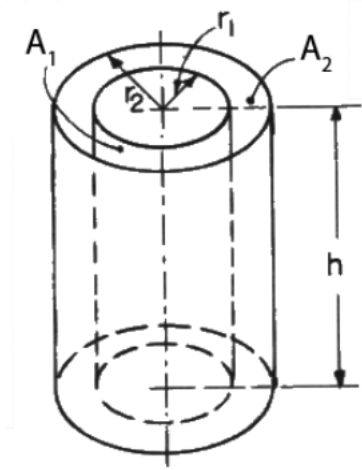


Figure B.2: View Factor from Cryochamber to Cryosurface

$$F_{1-2} = \frac{1}{\pi R_2} \left[\frac{1}{2} (R_2^2 - R_1^2 - 1) \cos^{-1} \frac{R_1}{R_2} + \pi R_1 - \frac{\pi}{2} AB - 2 R_1 \tan^{-1} (R_2^2 - R_1^2)^{1/2} + \left\{ (1 + A^2)(1 + B^2) \right\}^{1/2} \tan^{-1} \left\{ \frac{(1 + A^2)B}{(1 + B^2)A} \right\}^{1/2} \right]$$

$$R_1 = r_1/h \quad R_2 = r_w/h$$

$$A = R_2 + R_1 \quad B = R_2 - R_1$$

C | Code & Software Based Analysis

This section includes the code for all MATLAB based analysis conducted throughout this project. The major written programs are included alongside functions written for the analysis. Note that native MATLAB functions are not shown.

C.1 Space Environment Model

The following is the software written for...

```

1 % -----Space Envionrment in VLEO Model----- %
2 %
3 % Author:    Cameron Leslie
4 % Version:   V3 (02/05/21)
5 %
6 %
7 % Description: Utilises NRLMSISE-00 Atmpsheric model to determine
8 % atmpsheric parameters (Density, Temperature and molecular composition)
9 %
10 % V3 Analyses atmospheric composition, neutral density and temperature
11 % by averaging the determined parameters over a specified orbit.
12 %
13 % ----- %
14 clear
15 clc
16
17 restoredefaultpath
18 addpath(genpath('E:\1. IRP\2. Systems Study\ToolBoxM'));
19
20 %%
21
22 % ----- Constants ----- %
23 rE          = 6378;                % Earth radius
24 muE          = 3.986E+5;            % Gravitational parameter

```

```

25 J2          = 1.083E-3;          % J2 perturbation
26 tE          = 24*365.256*(60^2);  % Earth orbit period (Year)
27
28 % ----- Changing Parameter Scope ----- %
29 h           = (0:2:500)';        % Altitudes
30 theta       = ((2*pi)/(length(h)))...
31             :((2*pi)/length(h))...
32             :2*pi)';              % True Anomaly
33
34 % ----- Orbit Parameters ----- %
35 r           = h+rE;              % Orbit radial distance
36 e           = 0;
37 Om          = pi/2;
38 om          = 0;
39 dOmegaYear  = 0;                  % Omega change per year
40 t           = 2*pi*sqrt((r.^3)...
41             /muE);                % Orbital period
42 inc         = acosd(dOmegaYear...
43             ./((-1.*((3.*pi.*J2.*rE^2)...
44             ./ (r.^2))).*(tE./t))); % Inclination for SSO
45 vOrbit      = sqrt(muE./(h+rE));  % Orbital velocity
46
47 % Override Inclination
48 %inc = 0;
49
50 % ----- Environment Model Settings ----- %
51 year = 2022;
52 dayOfYear = 4;
53 UTseconds = 0;
54
55 f107a = [65 140 250]';
56 ap    = [0 2 3 4 5 6 7 9 12 15 18 22 27 32 39 48 56 67 80 94 111 312 ...
57         154 179 207 236 300 400]';
58 Kp    = [ 0 0+(1/3) 1-(1/3) 1 1+(1/3) 2-(1/3) 2 2+(1/3) 3-(1/3) 3 ...
59         3+(1/3) 4-(1/3) 4 4+(1/3) ...
60         5-(1/3) 5 5+(1/3) 6-(1/3) 6 6+(1/3) 7-(1/3) 7 ...
61         7+(1/3) 8-(1/3) 8 8+(1/3) 9-(1/3) 9]';
62 apHML = [0 15 45]';
63 KpHML = [interp1(ap,Kp,apHML(1))
64         interp1(ap,Kp,apHML(2))
65         interp1(ap,Kp,apHML(3))]' ;
66
67
68
69
70
71
72
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```

```

65
66 % ----- Section ----- %
67 for i= 1:length(h)
68
69     % // Keplerian -> Cartesian //
70     for ii = 1:length(theta)
71         if length(inc) == length(h)
72             kep = [r(i) e deg2rad(inc(ii)) Om om theta(ii)]';
73         else
74             kep = [r(i) e deg2rad(inc) Om om theta(ii)]';
75         end
76         car(ii,:) = kep2car(kep,muE);
77     end
78
79     % // Cartesian -> Spherical //
80     [long,lat,rad] = cart2sph(car(:,1),car(:,2),car(:,3));
81     long = rad2deg(long);
82     lat  = rad2deg(lat);
83
84     % ----- Atmosphere Model ----- %
85     for iii = 1:3
86         [T,rho] = atmosnrlmsise00(h(i)*1000, lat, long, year, ...
87             dayOfYear, UTseconds, f107a(iii), f107a(iii), KpHML(iii));
88
89     % // Log Average Data over Orbit for each solar condition //
90
91     % TEMPERATURE:
92     T_avg(i,iii) = mean(T(:,2));
93
94     % AVERAGE DENSITY:
95     rhoAvg(i,iii) = mean(rho(:,6));
96
97     % COMPOSITIONS:
98     avgSp_He(i,iii) = mean(rho(:,1));
99     avgSp_O(i,iii) = mean(rho(:,2));
100    avgSp_N2(i,iii) = mean(rho(:,3));
101    avgSp_O2(i,iii) = mean(rho(:,4));
102    avgSp_Ar(i,iii) = mean(rho(:,5));
103    avgSp_H(i,iii) = mean(rho(:,7));
104    avgSp_N(i,iii) = mean(rho(:,8));
105    avgSp_AO(i,iii) = mean(rho(:,9));
106 end

```

```

107
108
109 % ----- Parametric Analysis ----- %
110
111 % // Altitude vs Composition //
112 comps = [avgSp_He(:,2)...
113           avgSp_O(:,2)...
114           avgSp_N2(:,2)...
115           avgSp_O2(:,2)...
116           avgSp_Ar(:,2)...
117           avgSp_H(:,2)...
118           avgSp_N(:,2)...
119           avgSp_AO(:,2)];
120 compSum      = sum(comps,2);
121 normComp      = comps./compSum;
122
123 % ===== GAS MIXTURE ===== :
124 k      = 1.38064852E-23;
125
126 % Species mass in kg:
127 mO      = 2.65676264E-26;
128 mN2     = 4.6517E-26;
129 mHe     = 6.6464764063E-27;
130 mO2     = 5.31352528E-26;
131 mAr     = 6.6335209E-26;
132 mH      = 1.6737236E-27;
133 mN      = 2.32586705e-26;
134 mAO     = 2.65676264E-26;
135
136 % Total Mass and number density:
137 Mtot = avgSp_O.*mO...
138       + avgSp_N2.*mN2...
139       + avgSp_He.*mHe...
140       + avgSp_O2.*mO2...
141       + avgSp_Ar.*mAr...
142       + avgSp_H.*mH...
143       + avgSp_N.*mN...
144       + avgSp_AO.*mAO;
145
146 N = avgSp_N2...
147     +avgSp_O...
148     +avgSp_He...
149     +avgSp_O2...

```

```

150     +avgSp_Ar...
151     +avgSp_H...
152     +avgSp_N...
153     +avgSp_AO;
154
155 mp = Mtot./N;                                % Mean Molecular Mass of Mixture
156
157
158 % ===== Mean Free Path ===== %
159 d = 4.1e-10;
160 cL = 1;
161 mfp = 1./(sqrt(2).*N.*pi.*(d.^2));
162 Kn = mfp./cL;
163
164 % ===== Speed Ratio ===== %
165 sR = (vOrbit*1000)./sqrt(2*(k.*T_avg)./mp));
166 %sR = mean(sR);
167
168 % ===== Mean Absolute Percent Deviation ===== %
169
170 % Total Density:
171 rhoPD_low = abs(rhoAvg(:,2)-rhoAvg(:,1))./rhoAvg(:,2);
172 rhoPD_high = abs(rhoAvg(:,2)-rhoAvg(:,3))./rhoAvg(:,2);
173 rhoPMAD = (rhoPD_high+rhoPD_low).*50;
174
175 % Total Temperature:
176 T_PD_low = abs(T_avg(:,2)-T_avg(:,1))./T_avg(:,2);
177 T_PD_high = abs(T_avg(:,2)-T_avg(:,3))./T_avg(:,2);
178 T_PMad = (T_PD_high+T_PD_low).*50;
179
180 % Speed Ratio:
181 sR_PD_low = abs(sR(:,2)-sR(:,1))./sR(:,2);
182 sR_PD_high = abs(sR(:,2)-sR(:,3))./sR(:,2);
183 sR_PMad = (sR_PD_high+sR_PD_low).*50;
184
185 % Mean Free Path:
186 mfp_PD_low = abs(mfp(:,2)-mfp(:,1))./mfp(:,2);
187 mfp_PD_high = abs(mfp(:,2)-mfp(:,3))./mfp(:,2);
188 mfp_PMad = (mfp_PD_high+mfp_PD_low).*50;

```

C.2 Systems Model: Constraint Analysis program

```

1
2 % ----- Constraint Analysis ----- %
3 %
4 % Author:   Cameron Leslie
5 %
6 % ----- %
7 clear
8 clc
9
10 restoredefaultpath
11 addpath(genpath('E:\1. IRP\2. Systems Study\'))
12
13 % ----- Design Altitude ----- %
14 h = 180;
15 %%
16 % ----- Constants ----- %
17 rE      = 6378;           % Earth radius
18 muE      = 3.986E+5;      % Gravitational parameter
19 J2        = 1.083E-3;     % J2 perturbation
20 tE        = 24*365.256*(60^2); % Earth orbit period (Year)
21
22 % ----- Orbit Parameters ----- %
23 stepsize = 300;
24 theta     = ((2*pi)/(stepsize))...
25             :((2*pi)/stepsize)...
26             :2*pi)';      % True Anomaly
27 r          = h+rE;        % Orbit radial distance
28 e          = 0;
29 Om         = 0;
30 om         = 0;
31 dOmegaYear = 2*pi;        % Omega change per year
32 t          = 2*pi*sqrt((r.^3)...
33             /muE);        % Orbital period
34 inc        = acosd(dOmegaYear...
35             ./((-1.*((3.*pi.*J2.*rE^2)...
36             ./ (r.^2))).*(tE./t))); % Inclination for SSO
37
38 vOrbit     = sqrt(muE./(h+rE)); % Orbital velocity
39
40 % Override Inclination
41 %i = 90;
42 %inc = zeros(length(h),1)+ i;
43

```

```

44 % ----- Orbit Coordinates ----- %
45
46 % // Keplerian -> Cartesian //
47 for ii = 1:stepsize
48     kep = [r e deg2rad(inc) Om om theta(ii)]';
49     car(ii,:) = kep2car(kep,muE);
50 end
51
52 % // Cartesian -> Spherical //
53 [long,lat,rad] = cart2sph(car(:,1),car(:,2),car(:,3));
54
55 longs = rad2deg(long);
56 lats  = rad2deg(lat);
57
58
59 % ===== %
60 % ===== Environment Model ===== %
61 % ===== %
62
63 % Initialise Arrays:
64
65 % TEMPERATURE:
66 %T_Avg      = zeros(length(h),1);
67
68 % AVERAGE DENSITY:
69 %rhoAvg     = zeros(length(h),1);
70
71 % COMPOSITIONS:
72 % avgSp_He   = zeros(length(h),1);
73 % avgSp_O    = zeros(length(h),1);
74 % avgSp_N2   = zeros(length(h),1);
75 % avgSp_O2   = zeros(length(h),1);
76 % avgSp_Ar   = zeros(length(h),1);
77 % avgSp_H    = zeros(length(h),1);
78 % avgSp_N    = zeros(length(h),1);
79 % avgSp_AO   = zeros(length(h),1);
80
81 %%
82 % ===== ATMOSPHERE ===== %
83
84 % Time Settings - (Irrelevant when specifying solar activity indexes)
85 year = 2022;
86 dayOfYear = 4;

```

```

87 UTseconds = 0;
88
89 % Solar Indexes
90 activity = 2;
91 %%
92 if activity == 1
93     f107a = 65;
94     Kp = 0;
95 elseif activity == 2
96     f107a = 140;
97     Kp = 3;
98 elseif activity == 3
99     f107a = 250;
100    Kp = 4.89;
101 end
102
103 for ii = 1:stepsize
104
105     % ----- Atmosphere Model ----- %
106     [T,rho] = atmosnrlmsise00(h*1000, lats(ii), longs(ii), year, ...
107         dayOfYear, UTseconds, f107a, f107a, Kp);
108
109     % // Log Average Data over Orbit for each solar condition //
110
111     % TEMPERATURE:
112     T_Avg = mean(T(:,2));
113
114     % AVERAGE DENSITY:
115     rhoAvg = mean(rho(:,6));
116
117     % COMPOSITIONS:
118     avgSp_He = mean(rho(:,1));
119     avgSp_O = mean(rho(:,2));
120     avgSp_N2 = mean(rho(:,3));
121     avgSp_O2 = mean(rho(:,4));
122     avgSp_Ar = mean(rho(:,5));
123     avgSp_H = mean(rho(:,7));
124     avgSp_N = mean(rho(:,8));
125     avgSp_AO = mean(rho(:,9));
126 end
127
128

```

```

129 % ===== GAS MIXTURE ===== :
130 k    = 1.38064852E-23;
131
132 % Species mass in kg:
133 mO   = 2.65676264E-26;
134 mN2  = 4.6517E-26;
135 mHe  = 6.6464764063E-27;
136 mO2  = 5.31352528E-26;
137 mAr  = 6.6335209E-26;
138 mH   = 1.6737236E-27;
139 mN    = 2.32586705e-26;
140 mAO  = 2.65676264E-26;
141
142 % Total Mass and number density:
143 Mtot = avgSp_N2.*mN2...
144       + avgSp_O.*mO;
145
146 N = avgSp_N2...
147     +avgSp_O;
148
149 mp = Mtot./N;                % Mean Molecular Mass of Mixture
150
151 mixture_ratio = avgSp_N2/avgSp_O;
152
153 % ===== Mean Free Path ===== %
154 d    = 4.1e-10;
155 cL   = 1;
156 mfp   = 1./(sqrt(2).*N.*pi.*(d.^2));
157 Kn    = mfp./cL;
158
159 % ===== Speed Ratio ===== %
160 sR    = (vOrbit*1000)./sqrt(2*((k*T_Avg)./mp));
161
162
163 %% |Systems Model|
164 % ===== %
165 % ===== Systems Model ===== %
166 % ===== %
167
168 % ===== Parameter Scopes ===== %
169 % - Constants - %
170 k = 1.38064852e-23;
171

```

```

172 % - Chosen Constants - %
173
174 % Temps:
175 Tc = 300;
176 Tb = 70;
177 TsC = 20;
178 TsE = 28;
179
180 % Emissivities:
181 eC = 0.02;
182 eB = 0.01;
183 eS = 0.9;
184
185 % Transmissions:
186 etaM = 0.8;
187 c = 0.4;
188
189 % Spacecraft:
190 Af = 1;
191 Cd = 2.5;
192
193 % Discretisation:
194 gridsize = 30;
195
196 % === Preset Parameters === %
197 AR = 7;
198 Rth = 0.25;
199 Rc = 0.2;
200 % ===== %
201
202 % === Parameters to Vary === %
203 p1 = Rc;
204 p2 = Rth;
205 % ===== %
206
207 % - Duct Aspect Ratio - %
208 AR_max = 15;
209 AR_range = (AR_max/gridsize:AR_max/gridsize:AR_max)';
210
211 if p1 == AR
212     AR = AR_range;
213     p1 = AR;
214     p1_label = '$$L/{\sqrt{A}}$$';

```

```

215 elseif p2 == AR
216     AR = AR_range;
217     p2 = AR;
218     p2_label = '$$L/{\sqrt{A}}$$';
219 end
220
221 % - Thruster Area Ratio - %
222 Rth_range = (0:((1+1e-12)/(gridsize-1)):(1+1e-12))';
223
224 if p1 == Rth
225     Rth = Rth_range;
226     p1 = Rth;
227     p1_label = '$$A_{th}/A_{i}$$';
228 elseif p2 == Rth
229     Rth = Rth_range;
230     p2 = Rth;
231     p2_label = '$$A_{th}/A_{i}$$';
232 end
233
234 % - Collector Area Ratio - %
235 Rc_range = (0:1/(gridsize-1):1)';
236
237 if p1 == Rc
238     Rc = Rc_range;
239     p1 = Rc;
240     p1_label = '$$A_{c}/A_{i}$$';
241 elseif p2 == Rc
242     Rc = Rc_range;
243     p2 = Rc;
244     p2_label = '$$$A_{c}/A_{i}$$';
245 end
246
247
248 %% ===== Transmission Probabilities ===== %
249
250 D = 2./(sqrt(pi).*AR);
251
252 for i = 1:length(D)
253     tau1_i(i,1) = tubeflow(sR,D(i));
254     tau1_o(i,1) = tubeflow(0,D(i));
255     fprintf('\n Calc: %.0f | D = %.2f ',i,D(i))
256 end
257

```

```

258 %% |Performance Calculations|
259
260 tau2_s = etaM.*Rth;
261 tau3_s = c.*Rc;
262
263
264 % ===== Collector Performance ===== %
265
266 for i = 1:length(AR)
267     for ii = 1:length(Rth)
268         for iii = 1:length(Rc)
269
270             if p1 == AR
271                 n = i;
272             elseif p1 == Rth
273                 n = ii;
274             elseif p1 == Rc
275                 n = iii;
276             end
277
278             if p2 == AR
279                 m = i;
280             elseif p2 == Rth
281                 m = ii;
282             elseif p2 == Rc
283                 m = iii;
284             end
285
286             % ===== COLLECTION ===== %
287
288             % ----- Density Ratio ----- %
289             dRc(m,n) = (2.*sqrt(pi).*sR.*tau1_i(i).*k.*...
290             sqrt(T_Avg.*TsC)+(p_sat(TsC,'N2').*tau3_s(iii).*N.^(-1))).*...
291             (k.*sqrt(Tc.*TsC).*(tau1_o(i) ...
292             + tau2_s(ii))+k.*sqrt(TsC/Tc).*Tc.*tau3_s(iii)).^(-1);
293
294             ncC(m,n) = dRc(m,n).*N;
295
296             % ----- Efficiencies ----- %
297             etaC(m,n) = ...
298                 (tau1_i(i)./(tau1_o(i)+tau2_s(ii)+tau3_s(iii))).*tau3_s(iii);
299             etaTh(m,n) = ...
300                 (tau1_i(i)./(tau1_o(i)+tau2_s(ii)+tau3_s(iii))).*tau2_s(ii);

```

```

299
300     etaUc(m,n) = ...
301         ((ncC(m,n).*k.*Tc.*sqrt(TsC./Tc)-p_sat(TsC,'N2'))./...
302         (sqrt(2.*pi.*mp.*k.*TsC))).*tau3_s(iii)...
303         + ...
304         0.25.*ncC(m,n).*sqrt((8.*k.*Tc)./(pi.*mp)).*tau2_s(ii))...
305         ./ (N.*vOrbit.*1000.*Af);
306
307     etaThC(m,n) = ...
308         ((0.25.*ncC(m,n).*sqrt((8.*k.*Tc)./(pi.*mp)).*...
309         Rth(ii).*etaM)./(N.*vOrbit.*1000.*Af));
310
311     % ===== EVAPORATION ===== %
312
313     % ----- Density Ratio ----- %
314     dRe(m,n) = ...
315         (2.*sqrt(pi).*sR.*taul_i(i).*k.*sqrt(T_Avg.*TsE)+...
316         ((p_sat(TsE,'N2')).*tau3_s(iii).*N.^(-1)))).*...
317         (k.*sqrt(Tc.*TsE).*(taul_o(i) + ...
318         tau2_s(ii))+k.*sqrt(TsE/Tc)).*...
319         Tc.*tau3_s(iii)).^(-1);
320
321     ncE(m,n) = dRe(m,n).*N;
322
323     % ----- Efficiencies ----- %
324     etaE(m,n) = ...
325         ((ncE(m,n).*k.*Tc.*sqrt(TsE./Tc)-p_sat(TsE,'N2'))./...
326         (sqrt(2.*pi.*mp.*k.*TsE))).*tau3_s(iii))./(N.*vOrbit.*1000.*Af);
327
328     etaUe(m,n) = ...
329         ((0.25.*ncE(m,n).*sqrt((8.*k.*Tc)./(pi.*mp)).*Rth(ii).*...
330         etaM)...
331         ./(((p_sat(TsE,'N2')-(ncE(m,n).*k.*Tc.*sqrt(TsE./Tc)))/...
332         (sqrt(2.*pi.*mp.*k.*TsE))).*tau3_s(iii))...
333         + (N.*vOrbit.*1000.*Af)));
334
335     etaThE(m,n) = ...
336         ((0.25.*ncE(m,n).*sqrt((8.*k.*Tc)./(pi.*mp)).*...
337         Rth(ii).*etaM)./(N.*vOrbit.*1000.*Af));
338
339     % ===== Radiative Heat Transfer ===== %

```

```

334         [qRadB(m,n),qRadC(m,n)] = ...
            radHeat(Af,Rc(iii),Tc,Tb,TsC,eC,eB,eS);
335
336     end
337 end
338 end
339
340 % ----- Chamber Conditions ----- %
341 pc      = ncC.*k.*Tc;
342 dP      = ((p_sat(TsC,'O2')+p_sat(TsC,'N2'))/2)-pc;
343
344 % ----- Mass Flows ----- %
345 mDotTh  = mp.*((0.25.*ncC.*sqrt((8.*k.*Tc)/(pi.*mp)).*Rth.*etaM));
346 mDotC   = mp.*(((ncC.*k.*Tc.*sqrt(TsC./Tc)-p_sat(TsC,'N2'))./...
347             (sqrt(2.*pi.*mp.*k.*TsC))).*tau3_s');
348
349 % ===== Heat Transfer ===== %
350
351 % Molecular Heat Transfer:
352 Qb      = ((0.25.*ncC.*sqrt((8.*k.*Tc)/(pi.*mp)).*2.*k.*Tc.*Rc));
353 Qc      = ((0.25.*ncC.*sqrt((8.*k.*TsC)/(pi.*mp)).*2.*k.*TsC.*Rc.*c));
354
355 % Total loads on Baffle and Cryosurface:
356 Q_baffle = Qb+qRadB;
357 Q_cryosurface = Qc+qRadC;
358
359 % ===== Simple Drag Model ===== %
360 Fd = 0.5.*rhoAvg.*Af.*(vOrbit*1000).^2.*Cd;
361 uE = Fd./mDotTh;
362 isp = uE./9.81;
363 Pjet = 0.5.*mDotTh.*uE.^2;
364 Pin  = Pjet./0.6;

```

C.3 Systems Model: Off-Design Point program

```

1
2 % ----- Off-Design Point Study ----- %
3 %
4 % Author:   Cameron Leslie
5 %
6 % ----- %

```

```

7 clear
8 clc
9
10 restoredefaultpath
11 addpath(genpath('E:\1. IRP\2. Systems Study\'))
12
13 % ----- Constants ----- %
14 rE      = 6378;                % Earth radius
15 muE      = 3.986E+5;           % Gravitational parameter
16 J2       = 1.083E-3;           % J2 perturbation
17 tE       = 24*365.256*(60^2);   % Earth orbit period (Year)
18
19 % ----- Changing Parameter Scope ----- %
20 h        = (120:10:260)';      % Altitudes
21 theta    = ((2*pi)/(length(h)))...
22           :((2*pi)/length(h))...
23           :2*pi)';             % True Anomaly
24
25 % ----- Orbit Parameters ----- %
26 r        = h+rE;               % Orbit radial distance
27 e        = 0;
28 Om       = 0;
29 om       = 0;
30 dOmegaYear = 2*pi;             % Omega change per year
31 t        = 2*pi*sqrt((r.^3)...
32           /muE);               % Orbital period
33 inc      = acosd(dOmegaYear...
34           ./((-1.*((3.*pi.*J2.*rE^2)...
35           ./ (r.^2))).*(tE./t))); % Inclination for SSO
36
37 vOrbit   = sqrt(muE./(h+rE));   % Orbital velocity
38
39 % Override Inclination
40 %i = 0;
41 %inc = zeros(length(h),1)+ i;
42
43 % ----- Orbit Coordinates ----- %
44 for i= 1:length(h)
45
46     % // Keplerian -> Cartesian //
47     for ii = 1:length(h)
48         kep = [r(i) e deg2rad(inc(i)) Om om theta(ii)]';
49         car(ii,:) = kep2car(kep,muE);

```

```

50     end
51
52     % // Cartesian -> Spherical //
53     [long,lat,rad] = cart2sph(car(:,1),car(:,2),car(:,3));
54
55     longs(:,i) = rad2deg(long);
56     lats(:,i)  = rad2deg(lat);
57
58 end
59
60
61 % ===== %
62 % ===== Environment Model ===== %
63 % ===== %
64
65 % Initialise Arrays:
66
67 % TEMPERATURE:
68 T_Avg      = zeros(length(h),1);
69
70 % AVERAGE DENSITY:
71 rhoAvg     = zeros(length(h),1);
72
73 % COMPOSITIONS:
74 avgSp_He   = zeros(length(h),1);
75 avgSp_O    = zeros(length(h),1);
76 avgSp_N2   = zeros(length(h),1);
77 avgSp_O2   = zeros(length(h),1);
78 avgSp_Ar   = zeros(length(h),1);
79 avgSp_H    = zeros(length(h),1);
80 avgSp_N    = zeros(length(h),1);
81 avgSp_AO   = zeros(length(h),1);
82
83
84 % ===== ATMOSPHERE ===== %
85
86 % Time Settings - (Irrelevant when specifying solar activity indexes)
87 year = 2022;
88 dayOfYear = 4;
89 UTseconds = 0;
90
91 % Solar Indexes
92 activity = 2;

```

```

93
94 if activity == 1
95     f107a = 65;
96     Kp = 0;
97 elseif activity == 2
98     f107a = 140;
99     Kp = 3;
100 elseif activity == 3
101     f107a = 250;
102     Kp = 4.89;
103 end
104
105 for i = 1:length(h)
106     for ii = 1:length(h)
107
108         % ----- Atmosphere Model ----- %
109         [T,rho] = atmosnrlmsise00(h(i)*1000, lats(ii,i), longs(ii,i), ...
110             year, dayOfYear, UTseconds, f107a, f107a, Kp);
111
112         % // Log Average Data over Orbit for each solar condition //
113
114         % TEMPERATURE:
115         T_Avg(i,1) = mean(T(:,2));
116
117         % AVERAGE DENSITY:
118         rhoAvg(i,1) = mean(rho(:,6));
119
120         % COMPOSITIONS:
121         avgSp_He(i,1) = mean(rho(:,1));
122         avgSp_O(i,1) = mean(rho(:,2));
123         avgSp_N2(i,1) = mean(rho(:,3));
124         avgSp_O2(i,1) = mean(rho(:,4));
125         avgSp_Ar(i,1) = mean(rho(:,5));
126         avgSp_H(i,1) = mean(rho(:,7));
127         avgSp_N(i,1) = mean(rho(:,8));
128         avgSp_AO(i,1) = mean(rho(:,9));
129
130     end
131 end
132
133 % ===== GAS MIXTURE ===== :
134 k = 1.38064852E-23;

```

```

135 % Species mass in kg:
136 mO  = 2.65676264E-26;
137 mN2 = 4.6517E-26;
138 mHe = 6.6464764063E-27;
139 mO2 = 5.31352528E-26;
140 mAr = 6.6335209E-26;
141 mH  = 1.6737236E-27;
142 mN  = 2.32586705e-26;
143 mAO = 2.65676264E-26;
144
145 % Total Mass and number density:
146 Mtot = avgSp_N2.*mN2...
147       + avgSp_O.*mO;
148
149 N = avgSp_N2...
150     +avgSp_O;
151
152 mp = Mtot./N;                % Mean Molecular Mass of Mixture
153
154
155 % ===== Mean Free Path ===== %
156 d  = 3.55e-10;
157 cL = 1;
158 mfp      = 1./(sqrt(2).*N.*pi.*(d.^2));
159 Kn = mfp./cL;
160
161 % ===== Speed Ratio ===== %
162 sR = (vOrbit*1000)./sqrt(2*((k*T_Avg)./mp));
163
164
165 %%
166 % ===== %
167 % ===== Systems Model ===== %
168 % ===== %
169
170
171 % ===== Transmission Probabilities ===== %
172 AR = 7;
173 D = 2/(sqrt(pi)*AR);
174
175 for i = 1:length(h)
176     tau1_i(i,1) = tubeflow(sR(i),D);
177     tau1_o = tubeflow(0,D);

```

```

178     fprintf('\n Calc: %.0f | Sr = %.2f ',i,sR(i))
179 end
180
181 %% ===== Performance ===== %
182 clear dR nc pc mfp_c Kn_c etaC mDot Fd uE isp Pjet tau2 tau3 M_stored...
183     etaf mDotC mDotf
184
185 % ----- Constants ----- %
186 kb = 1.38064852e-23;
187
188 % ----- System Parameters ----- %
189
190 % INTAKE / THRUSTER
191 Tc = 300;
192 etaM = 0.8;
193 A2_Ai = 0.25;
194
195 % STORAGE
196 A3_Ai = 0.2;
197 c = 0.4;
198 Ts = 20;
199 Tb = 70;
200
201 % SPACECRAFT
202 Af = 1;
203 Cd = 2.5;
204 Ms = 1000;
205
206 % ----- Choose operational mode ----- %
207 % ===== %
208 % Mode 1: No Storage System
209 % Mode 2: Isolated Intake Storage
210 % Mode 3: Collection
211 % Mode 4: Feed
212 % ===== %
213
214 Mode = 3;
215
216 % ===== No Storage ===== %
217 if Mode == 1
218     tau2_s = etaM*A2_Ai;
219     % ----- Density Ratio ----- %
220     dR = 2*sqrt(pi).*sqrt(T_Avg/Tc).*sR.*(taul_i./(taul_o+tau2_s));

```

```

221
222     % ----- Chamber Conditions ----- %
223     nc      = dR.*N;
224     pc      = nc.*kb.*Tc;
225     mfp_c   = 1./(sqrt(2).*nc.*pi.*(d.^2));
226     Kn_c    = mfp_c./0.3;
227
228     % ----- Collection Efficiency ----- %
229     etaC = (taul_i./(taul_o+tau2_s)).*tau2_s;
230
231     % ----- Mass Flow @ Thruster ----- %
232     mDot = rhoAvg.*vOrbit.*1000.*etaC;
233     mf_thruster = mp.*0.25.*nc.*sqrt((8.*k.*Tc)./(pi.*mp)).*A2_Ai.*etaM;
234
235     % ===== Simple Drag Model ===== %
236     Fd = 0.5.*rhoAvg.*Af.*(vOrbit*1000).^2.*Cd;
237     Pin = 1500;
238     Pjet = 0.6*Pin;
239     uE = sqrt(Pjet./(2.*mDot));
240     isp = uE./9.81;
241     T = sqrt(2.*Pjet.*mDot);
242     T_D = T./Fd;
243     T_D2 = T-Fd;
244
245     uE_required = Fd./mDot;
246     isp_required = uE_required./9.81;
247     Pjet_required = 0.5.*mDot.*uE_required.^2;
248     Pin_required = Pjet_required/0.6;
249 end
250
251 % ===== Isolated Intake Storage ===== %
252 if Mode == 2
253     tau3 = c*A3_Ai;
254     % ----- Density Ratio ----- %
255     dR = 2*sqrt(pi).*sR.*(taul_i./(taul_o+tau3));
256
257     % ----- Chamber Conditions ----- %
258     nc      = dR.*N;
259     pc      = nc.*kb.*Tc;
260     mfp_c   = 1./(sqrt(2).*nc.*pi.*(d.^2));
261     Kn_c    = mfp_c./0.3;
262
263     % ----- Collection Efficiency ----- %

```

```

264     etaC = (tau1_i./(tau1_o+tau3)).*tau3;
265
266     % ----- Mass Flow @ Collector ----- %
267     mDot = rhoAvg.*vOrbit.*1000.*etaC;
268
269     % ----- Collected Mass after time 't'----- %
270     M_stored = mDot.*t;
271
272     % ----- Drag ----- %
273     Fd = 0.5.*rhoAvg*Af.*(vOrbit*1000).^2*Cd;
274 end
275
276 % ===== Collector in loop ===== %
277 if Mode == 3
278     % ----- Density Ratio ----- %
279     tau2_s = etaM*A2_Ai;
280     tau3_s = c.*A3_Ai;
281     dR = ...
282         2*sqrt(pi).*sR.*sqrt(T_Avg/Tc).*(tau1_i./(tau1_o+tau2_s+tau3_s));
283
284     % ----- Chamber Conditions ----- %
285     nc      = dR.*N;
286     pc      = nc.*kb.*Tc;
287     mfp_c   = 1./(sqrt(2).*nc.*pi.*(d.^2));
288     Kn_c    = mfp_c./0.3;
289
290     % ----- Collection Efficiency ----- %
291     etaf = (tau1_i./(tau1_o+tau2_s+tau3_s)).*tau2_s;
292     etaC = (tau1_i./(tau1_o+tau2_s+tau3_s)).*tau3_s;
293
294     etaU = etaf+etaC;
295
296     % ----- Mass Flows ----- %
297     mDotf = mp.*N.*vOrbit.*1000.*etaf;
298     mDotC = mp.*N.*vOrbit.*1000.*etaC;
299
300     % --- Collection per orbit --- %
301     M_orbit = mDotC.*t;
302     Dv_orbit = 3000*9.81.*log((Ms+M_orbit)./Ms);
303
304     % --- Collection in 8 year life --- %
305     Dv_life = (Dv_orbit/t)*60^2*24*365*8;

```

```

306     % ===== Simple Drag Model ===== %
307     Fd = 0.5.*rhoAvg*Af.*(vOrbit*1000).^2*Cd;
308     Pin = 1500;
309     Pjet = 0.6*Pin;
310     uE = sqrt(Pjet./(2.*mDotf));
311     isp = uE./9.81;
312     T = sqrt(2.*Pjet.*mDotf);
313     T_D = T./Fd;
314     T_D2 = T-Fd;
315
316     uE_required = Fd./mDotf;
317     isp_required = uE_required./9.81;
318     Pjet_required = 0.5.*mDotf.*uE_required.^2;
319     Pin_required = Pjet_required/0.6;
320 end
321
322 % ===== Evaporation in loop ===== %
323 if Mode == 4
324     % ----- Density Ratio ----- %
325     tau2_s = etaM*A2_Ai;
326     tau3_s = c.*A3_Ai;
327     dR = (2.*sqrt(pi).*sR.*tau1_i.*k.*sqrt(T_Avg.*Ts)+...
328     ((p_sat(Ts, 'N2').*tau3_s.*N.^(-1)))).*...
329     (k.*sqrt(Tc.*Ts).*(tau1_o + ...
330     tau2_s)+k.*sqrt(Ts/Tc).*Tc.*tau3_s).^(-1);
331
332     % ----- Chamber Conditions ----- %
333     nc = dR.*N;
334     pc = nc.*kb.*Tc;
335     mfp_c = 1./(sqrt(2).*nc.*pi.*(d.^2));
336     Kn_c = mfp_c./0.3;
337
338     % ----- Net Mass flow to Condenser ----- %
339     mf_condensed = mp*((nc.*k.*Tc.*sqrt(Ts./Tc)-p_sat(Ts, 'N2'))./...
340     (sqrt(2.*pi.*mp.*k.*Ts))).*tau3_s);
341
342     % ----- Net Mass flow to Thruster ----- %
343     mf_thruster = mp.*0.25.*nc.*sqrt((8.*k.*Tc)./(pi.*mp)).*A2_Ai.*etaM;
344
345     % ===== Simple Drag Model ===== %
346     Af = 1;
347     Cd = 2.5;

```

```

348     Fd = (0.5.*rhoAvg*Af.*(vOrbit*1000).^2*Cd);
349     uE  = Fd./mf_thruster;
350     isp = uE./9.81;
351     Pjet = 0.5.*mf_thruster.*uE.^2;
352     Pin  = Pjet/0.6;
353 end

```

C.4 Functions

C.5 Intake Transmission Probabilities

```

1
2 % ----- Intake Transmission Function ----- %
3 %
4 % Author:   Cameron Leslie
5 %
6 %
7 % Description: Uses expressions derived by Hughes et al.
8 % Determines transmission probabilities given speed
9 % ratio S and aspect ratio L/D. (In function form).
10 %
11 % ...
12
13 ----- ...
14 %
15
12 function [W_out] = tubeflow(S,D)
13
14 % ===== Clausing ===== %
15 % Determines Clausing's Factor a(D):
16
17 A      = 3+(7^(1/2)).*D;
18 A1     = (9+(A.^2).*D.^2).^(1/2);
19 A2     = (7+A.^2).^(1/2);
20
21 a_D    = (A1.*A2.*(3.*A1-9-7^(1/2)).*D.^2.*(A2-7^(1/2))))./...
22         (A1.*A2.*(3-7^(1/2)).*D).*(A1+A2.*D)+A.*(7.*A1.*D-9.*A2));
23
24
25 % ===== Auxilliary Functions ===== %
26 % Auxilliary analytical expressions:
27

```

```

28 psi_D = (2/(D^2))*((1+D^2)^(1/2)-1);
29 zeta_D = (2/(3*D^2))*((1+D^2)^(3/2)-D^3-1);
30
31
32 % ===== Zero Entrance Velocity ===== %
33 % Clausings solution to the case of S=0:
34
35 W_D = 1-a_D*psi_D-(1-2*a_D)*zeta_D;
36
37 if S == 0
38     W_out = W_D;
39 end
40
41
42 if S ≠ 0
43     % ===== Non-Zero Entrance Velocity ===== %
44     syms phi
45
46     split = atan(D);
47
48     % ----- T < 1 ----- %
49     R0l = @(phi) (1+((4*(1-2*a_D))./(3*pi.*(tan(phi)./D))).*...
50     (1-(1-(tan(phi)./D).^2).^(3/2)).*...
51     -(2.*(1-a_D))./pi).*(tan(phi)./D).*(1-(tan(phi)./D).^2).^...
52     (1/2)+asin((tan(phi)./D))).*...
53     (sin(phi).*cos(phi).*(1+S.^2.*(cos(phi)).^2).*exp(-S.^2)+...
54     sqrt(pi).*S.*cos(phi).*exp(-S.^2.*sin(phi).^2).*(3/2)+S.^2.*...
55     cos(phi).^2).*...
56     (1+erf(S).*cos(phi))));
57
58     R0l_solved = vpaintegral(R0l,phi,[0 split], 'RelTol', 1e-32, ...
59     'AbsTol', 1e-32);
60
61     R0l_solved = double(R0l_solved);
62
63     % ----- T > 1 ----- %
64     Fm = @(phi) sin(phi).*cos(phi).*(1+S.^2.*(cos(phi)).^2).*exp(-S.^2)+...
65     sqrt(pi).*S.*cos(phi).*exp(-S.^2.*sin(phi).^2).*(3/2)+S.^2.*cos(phi).^2).*...
66     (1+erf(S*cos(phi))));
67
68     R0m = @(phi) (a_D+(4.*(1-2.*a_D))./(3.*pi.*(tan(phi)./D))).*...
69     (sin(phi).*cos(phi).*(1+S.^2.*(cos(phi)).^2).*exp(-S.^2)+...
70     sqrt(pi).*S.*cos(phi).*exp(-S.^2.*sin(phi).^2).*(3/2)+S.^2.*...

```

```

70         2.*cos(phi).^2).*...
71         (1+erf(S).*cos(phi))));
72
73
74     R0m_solved = vpaintegral(R0m,phi,[split pi/2], 'RelTol', 1e-32, ...
75         'AbsTol', 1e-32);
76     R0m_solved = double(R0m_solved);
77
78     % ----- Orifice Probe Condition ----- %
79     R_S_inf = 2*vpaintegral(Fm,phi,[0 pi/2], 'RelTol', 1e-32, ...
80         'AbsTol', 1e-32);
81     R_S_inf = double(R_S_inf);
82
83     % ----- Pressure Ratio ----- %
84     R = (2/W_D)*(R0m_solved+R0l_solved);
85
86     % ----- "Reduced" Pressure Ratio ----- %
87     PHI = R./R_S_inf;
88
89     % ----- Probability ----- %
90     W_SD = PHI.*W_D;
91
92     W_out = W_SD;
93
94 end
95
96
97 end

```

C.6 Saturation Pressures

```

1
2 % --- Saturated Vapour Pressure of Considered Species --- %
3 %
4 % Author:    Cameron Leslie
5 %
6 % ----- %
7 function [P_sat] = p_sat(T,Sp)
8

```

```

9  if strcmpi('N2',Sp) == 1
10     % --- N2 (1) --- %
11     A = 8.362;
12     B = 387.428;
13 elseif strcmpi('O2',Sp) == 1
14     % --- O2 (2) --- %
15     A = 9.082;
16     B = 480.871;
17 else
18     error('Error: Species undefined')
19 end
20
21
22 % --- Clausius-Clapeyron equation --- %
23 P_sat = 133.333.*(T.^0.5.*10.^(A(1,1)-B(1,1)./T));
24
25 end

```

C.7 Radiative Heat Transfer

```

1
2 % ----- %
3 % View Factor Estimates;
4
5 % Assumes concentric cylinders in series as an estimate for view ...
   factors in
6 % the system
7
8
9
10 % INPUTS:
11 % A_i    - Intake Area
12 % Rc     - Ratio between intake and collector inlet (baffle area)
13 % Rs     - Ratio between radius of baffle intake and radius
14
15 % ----- %
16 function [Qb,Qs] = radHeat(A_i,Rc,T_c,T_b,T_s,e_c,e_b,e_s)
17
18 if Rc == 0
19     Qb = 0;
20     Qs = 0;

```

```

21     return
22 end
23
24 % ===== GEOMETRY ===== %
25
26 % Main intake chamber Dimensions:
27 r_chamber = sqrt(A_i/pi);
28 S_i = 6*pi*r_chamber^2;
29
30 % Baffle Dimensions:
31 A_c = Rc.*A_i;
32 r_baffle = sqrt(A_c./pi);
33
34 % ----- %
35 % View Factor for Chamber-Baffle Interface:
36 R = r_chamber./r_baffle;
37 H2 = (2.*r_chamber)./r_baffle;
38 X1 = R.^2 +1;
39 X2 = H2.^2 + R.^2 +1;
40
41 Fc_b = ...
         (1./(4.*R.*H2)).*( (X1-X2)-sqrt(X1.^2-4.*R.^2)+sqrt(X2.^2-4.*R.^2) );
42 % ----- %
43
44 % Cryopump Chamber Dimensions:
45 S_c = 6.*pi.*r_baffle.^2;
46
47 % Cryosurface Dimensions:
48 % -- Assumes the Cryosurface area = Baffle inlet area
49 r_s = sqrt(A_c./(6*pi));
50
51 % ----- %
52 % View factor of Concetric cylinder cryosurface:
53 R1 = r_s./(2.*r_baffle);
54 R2 = 1/2;
55 A = R1+R2;
56 B = R2-R1;
57
58 Fb_s = (A_c./S_c).*( (1./(pi.*R1)).*(0.5.*(R2.^2 - R1.^2 - ...
         1).*acos(R1./R2) + pi.*R1 -(pi./2).*A.*B...
59         -2.*R1.*atan(sqrt(R2.^2-R1.^2)) + ...
         sqrt((1+A.^2).*(1+B.^2)).*atan(sqrt(((1+A.^2).*B)./((1+B.^2).*A)))));
60 % ----- %

```

```
61
62
63 % ===== Heat Transfer ===== %
64 sig = 5.670374419e-8;
65
66 Qb = (sig.*(T_c.^4 - ...
        T_b.^4))./(((1-e_c)./(S_i.*e_c))+(1./(S_i.*Fc_b))+(1-e_b)./(A_c.*e_b)));
67
68 Qs = (sig.*(T_b.^4 - ...
        T_s.^4))./(((1-e_b)./(S_c.*e_b))+(1./(S_c.*Fc_s))+(1-e_s)./(A_c.*e_s)));
69
70 end
```