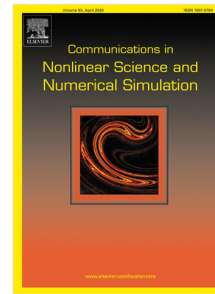


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Optimal Time-Fixed Impulsive non-Keplerian Orbit to Orbit Transfer Algorithm based on Primer Vector Theory

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Abstract

The work applies a systematic method to compute an optimal transfer trajectory between two non-Keplerian halo orbits in the Circular Restricted Three-body formulation of the Earth-Moon dynamics. The paper exploits the knowledge of the natural non-linear dynamics and the primer vector theory applied to the Circular Restricted Three Body Problem to design optimal multi-impulsive Halo-to-Halo transfers with fixed and limited Time of flight, from every point of the departure orbit to every point of the arrival orbit with an optimal number of impulses. The used method is constituted by three systematic steps with the goal of facilitating the mission design process and the on-board autonomous guidance implementation of future missions in highly non-linear dynamics.

Keywords: CR3BP, Halo Transfer, Optimisation, Astrodynamics

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1. Introduction

Future Moon exploration has become one of the primary goals among the private sector and space agencies around the world, as testified by the planned launch of the mission Artemis I [1], the mission Capstone [2] and the Global Space Exploration Road-map [3] in addition to specific national programs [4, 5, 6].

The extensive and sustainable return to our natural satellite requires in many cases dealing with the high complexity of the Earth-Moon non-linear dynamics, which introduces many difficulties in the design of new efficient missions. In the literature many non-linear models are used to describe the dynamics of space vehicles moving in such gravitational environment, the most common one being the Circular Restricted Three Body problem (CR3BP) [7, 8, 9], which provides significant results for the preliminary trajectory design and for the design of Guidance Navigation and Control (GNC) algorithms to perform tasks such as proximity operations, Earth-Moon transfers or formation flight. Other models commonly used in this context are the Elliptic Restricted Three Body Problem [10, 11] and the full-Ephemeris model [12], for instance.

Under the hypothesis of CR3BP in the Earth-Moon co-rotating reference frame the existence of five equilibrium points called Lagrangian points has been mathematically proved [13]. Three of them are unstable (L1, L2, L3) and aligned with the primaries, the other two, L4 and L5 are instead stable and located on the third vertex of an equilateral triangle with the other two vertexes centred on the Earth and the Moon. Under the same hypothesis, the existence of periodic orbits is also demonstrated; in addition, the CR3BP is a time-independent dynamic representation in the co-rotating frame, which makes it the most common model for preliminary interplanetary mission design [14, 15].

For the above reasons, this work is based on the CR3BP model, in particular, it concentrates on the Lagrangian point L2, since its energy property makes it of particular interest because it can open the way to future Mars travels [16], and its stability characteristics allow the existence of important non-Keplerian orbits to be used in the colonisation of the Moon[17].

The main focus of the paper is on a specific family of non-Keplerian periodic orbits around the point L2, the south Halo orbits; in particular, the Near Rec-

tilinear Halo Orbits. The choice is motivated by the literature [7], since one such orbit is the proposed location of the lunar gateway station. The paper proposes to apply a method to design an optimal multi-impulsive strategy to minimise fuel consumption when transferring between two Halo orbits.

Transfer between orbits is crucial to ensure the success of future complex missions and for fuel/time efficiency. The capability of easily planning the transfer between orbits in a highly non-linear environment will increase the efficiency of the mission design process, providing a useful tool for planning phasing manoeuvres, station keeping, formation flying etc... Moreover, an efficient way of transferring in a highly non-linear environment will contribute in enhancing the autonomy of orbiting space vehicles since it could be potentially adapted for on-board implementation.

The topic of orbit-to-orbit transfer has widely been studied in the past. A common approach to designing a transfer between orbits is the one based on the Lambert theorem [18], since it is one of the most widely used in the two-body problem model [19, 20, 21], it can be also used in the three body dynamics [22] but in the nonlinear dynamics based on three (or more) attracting bodies it can represent a non-reliable solution since either provides no-solution or unfeasible solutions in many cases [23]. An alternative that takes advantage of the peculiarities of the CR3BP model is based on the numerical computation of stable and unstable manifolds intersections [24, 25, 26]. This approach is very fuel efficient and very suitable in the case of low-thrust spacecraft architectures, but it requires a large amount of time of flight. Due to the weak instability of the NRHO this implies a very slow diverging/converging dynamics in their proximity. In addition, the intersection between stable/unstable manifolds is usually very difficult to find, therefore it is often searched through Montecarlo analysis [27].

Other approaches use classical optimal control theory [28] or artificial intelligence nonlinear optimizers [29], these can be efficient, however in space, the largest parts of the manoeuvres are open-loop, the shape trajectory is rarely predictable and the computation might be very energy consuming due to non-linearities, perturbations and other model approximations. These statements are also confirmed by [30] and [31] where the problem of fuel-optimal Earth-Moon and orbit-to-orbit transfer with low thrust engines is computed only in the planar problem and with many constraints because of the high sensitivity of the problem.

An interesting methodology for orbit-to-orbit transfer is described in [32], where a multi-impulsive optimal algorithm is presented, based on the knowl-

edge of the Floquet modes and the natural stable dynamics of the system. The proposed solution is efficient, but in the original formulation requires large transfer times. A modification to the above can be found in [33, 34]. The authors formulate a bi-impulsive transfer strategy in the ER3BP exploiting the natural intersection between Halo and Lissajous orbits and then running a *primer vector-based* optimisation to select the best departure and arrival point. This proposed solution provides a method to compute orbit-to-orbit transfers in highly non-linear, keeping the method clear, straightforward and applicable to different missions within the CR3BP framework. In [35] another indirect optimization method is proposed to optimize transfers between orbits, which mainly differs from the proposed one for the use of continuous thrust, the way the initial guess is computed and the fact that the final time is also optimised, resulting in an overall more computationally demanding procedure.

More recently, the work described in [36] is based on the primer vector theory and the knowledge of the halo orbital family to compute the initial guess for the transfer propagation. The advantages of the proposed approach lay in the fact that the time of flight is limited to a few days, the shape of the trajectory can be somehow established thanks to the knowledge of the complex dynamics of the system and the results are promising in terms of ΔV and time of flight.

The primer vector theory is not a new concept for space trajectory optimisation, in fact, it was formulated in [37] and used by [38] and [39] among others, to find the optimal trajectory to move a satellite between Earth orbits, or as in [36], to optimise the trajectory for a mission from Earth to the Jovian satellite Europa. However, in this work, as rarely seen in the current literature, to the best of the authors' knowledge, the formulation of the primer vector theory for the CR3BP is given, which differs from the standard formulation given in [40].

The goal of the present paper is not to modify or extend the theory, but rather to apply it as a new, simple method to compute multi-impulsive optimal transfers in a highly non-linear environment such as the CR3BP, which introduces many numerical convergences problems [41]. The paper is organised as follows: the non-linear dynamics of the restricted three-body problem is presented first, with the details of the characteristics of a halo orbital family. Next some details of the primer vector theory are reviewed and the numerical algorithms are described. Finally, the proposed algorithm is applied to a case study, with a critical analysis of the results and conclusions.

2. Working Framework

A detailed working framework is described in this section, in order to provide the reader with the background needed to explain the proposed algorithm. In particular the equations of motion, some hints on the primer vector theory and the software used to simulate the results

2.1. Circular Restricted Three Body - CR3BP

Moving a spacecraft from one point in the solar system to another requires solving the non-linear problem of its motion under the influence of many celestial bodies. In general, the N-body problem consists of predicting the motion of N bodies under mutual gravitational influence. When just 2 bodies are taken into account with only one main gravitational body, we can refer to the Two-Body Problem (2BP).

In the case of the Earth-Moon framework, instead, the two-body model is not sufficient since it introduces important discrepancies with reality. Therefore, it is necessary to move to a more accurate model to recover the reliability of the results and to this scope, the CR3BP is used in this work. In this framework, two main gravitational bodies are considered, M_1 and M_2 , and a spacecraft M_3 which has a negligible mass and it is moving under the influence of the other two masses. Some other simplification hypotheses can be made to remove the dependence on the time and treat the system as autonomous.

In particular:

- The mass of the spacecraft is negligible compared with the M_1 and M_2 . In other words $M_3 \ll M_1, M_2$.
- M_1 and M_2 are revolving in a circular motion around the common centre of mass with a constant angular velocity.

2.2. Rotating reference frame

A very useful reference frame to describe the motion of bodies under their mutual attraction is the *synodic reference frame*. In this paper all the results are reported and plotted in this reference frame, where the

$$\mathcal{S} : \{O; x_s, y_s, z_s\} \quad (1)$$

It is a non-inertial rotating frame, centred in the primaries centre of mass O and with axis:

- x_s : along the intersection between the two bodies, pointing towards the smaller body
- z_s : perpendicular to the plane where the two bodies move and positive

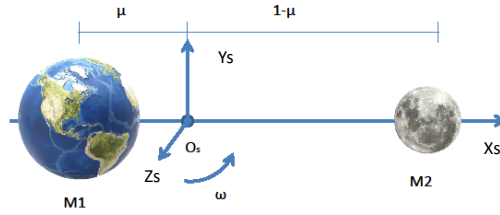


Figure 1: Synodic rotating frame

2.3. Equations of motion

In the standard formulation, the equations of motion (EOMs) are derived using the augmented or effective potential $\bar{U}$

$$\bar{U}(x, y, z) = -\frac{1}{2}(x^2 + y^2) + U(x, y, z) \quad (2)$$

In the synodic frame we have:

$$U(x, y, z) = -\frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} - \frac{1}{2}\mu_1\mu_2 \quad (3)$$

where $\mu_1 = Gm_1$, $\mu_2 = Gm_2$ and U is the gravitational potential experienced by the third object, the spacecraft, as a result of the gravitational pull of M_1 and M_2 . With r_1 and r_2 representing the distances between the third body and the two primaries, in rotating coordinates as:

$$r_1 = \sqrt{(x + \mu_2)^2 + y^2 + z^2}$$

$$r_2 = \sqrt{(x - \mu_1)^2 + y^2 + z^2}$$

After substituting the gravitational potential (3) in equation (2), the effective potential can be expressed by:

$$\bar{U}(x, y, z) = -\frac{1}{2}(\mu_1 r_1^2 + \mu_2 r_2^2) - \frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} \quad (4)$$

In order to obtain the correct EOMs, we use the Euler-Lagrange approach, which yields:

$$\begin{aligned} \ddot{x} &= 2\dot{y} - \frac{\partial \bar{U}}{\partial x} \\ \ddot{y} &= -2\dot{x} - \frac{\partial \bar{U}}{\partial y} \\ \ddot{z} &= -\frac{\partial \bar{U}}{\partial z} \end{aligned} \quad (5)$$

Being these equations Hamiltonian and independent of time, they admit an energy integral of motion:

$$E(x, y, z, \dot{x}, \dot{y}, \dot{z}) = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \bar{U}(x, y, z)$$

the above physically represents the value of energy associated with the object's motion. Typically, the scientific community uses $-2E$, which is the Jacobi constant, defined as:

$$C = -2E = -(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - 2\bar{U}(x, y, z)$$

The Jacobi integral is known as the only integral of motion of the CR3BP. It means that there is no analytic solution to this problem and due to this, the use of numerical methods is required. In order to obtain numerically stiff results, some form of normalisation is used to avoid ill-conditioned situations during the numerical solution of the equation of motions. More details on the normalisation technique can be found in [7].

2.3.1. Lagrangian Points and Halo orbital Family

In the CR3BP context, as a result of the attraction to both the two primaries, five equilibrium points called Lagrange points exist. Here, the forces acting on the third body are perfectly balanced such that they let the spacecraft maintain a stable position relative to the two primaries. They are denoted by $L_1 - L_5$ and divided into collinear and equilateral equilibrium points, as shown in Figure 2.

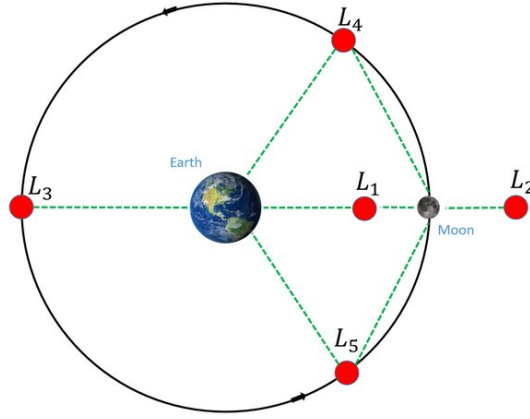


Figure 2: Equilibrium points of the CR3BP [42]

By using the effective potential, the following three equations can be solved to determine the location of the five points:

$$\begin{cases} \partial \bar{U} / \partial x = 0 \\ \partial \bar{U} / \partial y = 0 \\ \partial \bar{U} / \partial z = 0 \end{cases} \quad (6)$$

The stability characteristics of these points guarantee the existence of periodic orbits in their proximity, in particular, the linearized dynamics around Lagrangian points L_1 , L_2 and L_3 show that there are two stable modes, associated to purely imaginary eigenvectors, which imply the existence of periodic motions around them, the first periodic families which could be found are the planar Lyapunov and the vertical orbital family.

Moving along the space of planar Lyapunov periodic solutions, the bifurcation with the Halo orbits is found and the Halo orbital family can be computed starting from there.

The great advantage of Halo orbits is that they allow a satellite or space station placed there to be able to communicate with both the Earth and the Moon and to open a passage towards Mars.

Figure 3 shows the family of southern halo orbits around the Lagrangian point L_2 . The closer such orbits get to the Moon, the more their period decreases. In particular, one can distinguish the NRHO, a subfamily of halo orbits characterised by the Moon's eclipse-avoidance properties and weak instability that makes them suitable for exploitation as parking orbits for space

missions.

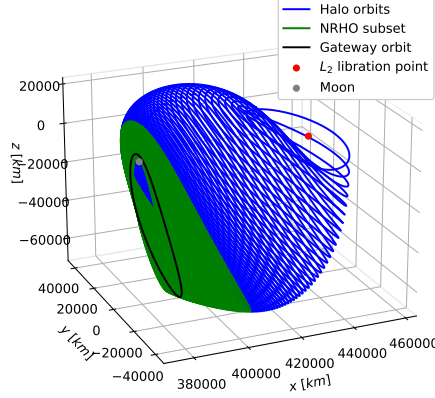


Figure 3: L_2 Halo and NRHOs orbit family [43]

3. Primer Vector Theory

In this section, an indirect method for trajectory optimisation based on primer vector theory [44, 45] is described and extended for the CR3BP framework [36], which allows for the identification of locally optimal point-to-point transfers between different Halo orbits. Optimality here refers to the minimum velocity change (Δv) to be provided by the spacecraft propulsion system to perform the transfer that ultimately is linked to the propellant mass to be stored on-board computed using the well-known rocket equation, for instance. [46]. The dynamics equation described in Eq. (5) can be generalised by including the effect of such thrust system and compacted as follows:

$$\dot{\vec{X}} = \begin{bmatrix} \dot{\vec{r}} \\ \dot{\vec{v}} \end{bmatrix} = \begin{bmatrix} \vec{v} \\ \vec{g}(\vec{r}) + \vec{c}(\vec{v}) + \vec{u} \end{bmatrix} \quad (7)$$

where: $\vec{X}$ is the state vector encoding the position $\vec{r} = [x \ y \ z]^T$ and velocity $\vec{v} = [\dot{x} \ \dot{y} \ \dot{z}]^T$ of the spacecraft; $\vec{g}(\vec{r})$ is a vector made by the quantities on the right-hand side of Eq. (5) describing the effect of the gravity field on the spacecraft motion; the vector $\vec{c}(\vec{v})$ describes the quantities

on the right-hand side of Eq. (5), which are related to the use of a non-inertial reference frame; the vector $\vec{u} = \Gamma \hat{u}$ is the control vector, representing the acceleration provided by the thrust system. This is composed of the acceleration magnitude Γ and by the thrust direction $\hat{u}$.

The overall Δv to be minimised can then be written as the integral of the acceleration Γ :

$$J = \int_{t_0}^{t_f} \Gamma(t) dt \quad (8)$$

on which t_0 and t_f are the start and final time of the transfer, respectively. The optimisation problem is then completed with user-defined initial conditions:

$$\vec{X}(t_0) = \vec{X}_0 \quad (9)$$

encoding the position and velocity of the spacecraft on the first Halo orbit, as well as terminal constraints in the form of:

$$\Psi(X(t_f), t_f) = \vec{X}(t_f) - \vec{X}_T = 0 \quad (10)$$

where X_T encodes the target position and velocity on the arrival Halo orbit. One can then define the Hamiltonian associated to the problem:

$$H = \Gamma + \vec{\lambda}_r \cdot \vec{v} + \vec{\lambda}_v \cdot [\vec{g}(\vec{r}) + \vec{c}(\vec{v}) + \Gamma \hat{u}] \quad (11)$$

where $\vec{\lambda}_r$ and $\vec{\lambda}_v$ are Lagrange multipliers. The adjoint equations are then:

$$\begin{aligned} \dot{\vec{\lambda}}_r &= -\frac{\partial H}{\partial \vec{r}} = -G(\vec{r})\vec{\lambda}_v \\ \dot{\vec{\lambda}}_v &= -\frac{\partial H}{\partial \vec{v}} = -\vec{\lambda}_r - C(\vec{v})\vec{\lambda}_v \end{aligned} \quad (12)$$

on which $G(\vec{r}) = \partial \vec{g}(\vec{r}) / \partial \vec{r}$ is the 3x3 symmetric gravity gradient matrix and $C(\vec{v}) = \partial \vec{c}(\vec{v}) / \partial \vec{v}$. The terminal constraints on the adjoint variables are then:

$$\begin{aligned} \vec{\lambda}_r(t_f) &= \left[\frac{\partial \Psi}{\partial \vec{r}} \right]_{t_f} \\ \vec{\lambda}_v(t_f) &= \left[\frac{\partial \Psi}{\partial \vec{v}} \right]_{t_f} \end{aligned} \quad (13)$$

Eqs. (7) to (13) constitute a two-point boundary-value problem. The control $\vec{u} = \Gamma \hat{u}$ is chosen according to the Pontryagin minimum principle [47], stating that the optimal control is the one that minimises the Hamiltonian in Eq. (11) with respect to the control variables at any time. The first observation from Eq. (11) is that the direction of the control $\hat{u}$ should be chosen to be anti-parallel to the opposite adjoint to the velocity $\vec{\lambda}_v$. Therefore, by defining the vector $\vec{p}(t) = -\vec{\lambda}_v$, we have:

$$\hat{u} = \frac{\vec{p}(t)}{p(t)} \quad (14)$$

where $\vec{p}(t)$ is called primer vector [44, 45]. By using Eq. (12) and (14), we can rewrite Eq. (11) as:

$$H = -(p-1)\Gamma + (\dot{\vec{p}} - C(\vec{v})\vec{p}) \cdot \vec{v} - \vec{p} \cdot \vec{g}(\vec{r}) - \vec{p} \cdot \vec{c}(\vec{v}) \quad (15)$$

Minimising H with respect to the choice of Γ thus depends upon the sign of $p-1$. Therefore, the minimum H occurs when $\Gamma = \Gamma_{max}$ if $p > 1$ (Γ_{max} being the maximum acceleration provided by the thrust system), and $\Gamma = 0$ (i.e., no thrust) if $p < 1$. The case $p = 1$ leaves the Γ undefined, and corresponds to the ignition/switching off of the thrust system [44].

3.1. Primer vector theory for impulsive transfers

If the spacecraft is equipped with a high-thrust propulsive system, then the thrust durations are small compared with the time between thrusts. Therefore, we can approximate the velocity changes during such small intervals as impulsive $\Delta\vec{v}$ manoeuvres, characterised by a propellant burning time $t_b \rightarrow 0$ and $\Gamma_{max} \rightarrow \infty$. Thus, the primer vector determines the time and directions of impulses when $p = 1$ and $p < 1$ when the thrust system is off. At any impulse i along the trajectory, one has that the primer vector is a unit vector pointing towards the optimal thrust direction:

$$\vec{p}_i = \frac{\Delta\vec{v}_i}{\Delta v_i} \quad (16)$$

Necessary conditions for optimality for impulsive transfers, firstly derived in [44], then become:

- 1 The primer vector and its first derivative are always continuous during the transfer.

- 2 The magnitude of the primer vector is $p < 1$ with $p = 1$ at the time of impulses.
- 3 At the time of impulses, the primer vector is a unit vector in the direction of the optimal thrust.
- 4 From previous conditions, the primer vector first derivative $\dot{p} = \dot{\vec{p}} \cdot \vec{p} = 0$ at the moment of mid-course impulses (this condition applies to the initial and final ones in the case of time-free problems).

One then notices that from Eq. (12), using $\vec{p}(t) = -\vec{\lambda}_v$, the primer vector dynamics can be written as:

$$\ddot{\vec{p}} = G\vec{p} + C\dot{\vec{p}} \quad (17)$$

or alternatively:

$$\frac{d}{dt} \begin{bmatrix} \vec{p} \\ \dot{\vec{p}} \end{bmatrix} = \begin{bmatrix} O_3 & I_3 \\ G & C \end{bmatrix} \begin{bmatrix} \vec{p} \\ \dot{\vec{p}} \end{bmatrix} \quad (18)$$

where O_3 and I_3 are the 3x3 zero and identity matrices, respectively and G and C are the jacobian of g and c , where the dependencies are removed to simplify the notation. The solution of Eq. (18) is:

$$\begin{bmatrix} \vec{p} \\ \dot{\vec{p}} \end{bmatrix} = \Phi(t, t_0) \begin{bmatrix} \vec{p} \\ \dot{\vec{p}} \end{bmatrix}_{t_0} \quad (19)$$

where $\Phi(t, t_0)$ is the state transition matrix (STM) of the system. It should be noted that differentiating Eq. (7) for zero control (i.e., between consecutive impulses), the same matrix as from Eq. (18) is obtained:

$$\delta \dot{X} = \begin{bmatrix} \delta \dot{\vec{r}} \\ \delta \dot{\vec{v}} \end{bmatrix} = \begin{bmatrix} O_3 & I_3 \\ G & C \end{bmatrix} \begin{bmatrix} \delta \vec{r} \\ \delta \vec{v} \end{bmatrix} \quad (20)$$

Therefore, the primer vector propagates with the same STM $\Phi(t, t_0)$ of the state disturbances.

The primer vector history along an impulsive trajectory as from Eq.(19) indicates the way of lowering the cost in the rewritten form of Eq. (8):

$$J = \sum_{i=1}^N \Delta v_i \quad (21)$$

where N is the number of impulses. The minimum number of impulses needed to perform the transfer is $N=2$, and the maximum number of impulses is automatically selected by the algorithm. A spacecraft that has state $[\vec{r} \ \vec{v}_{1-}]^T$ on the departing orbit at a given time changes its velocity from $\vec{v}_{1-}$, before the impulsive manoeuvre, to $\vec{v}_{1+}$, after the impulsive manoeuvre, by means of an impulse $\Delta\vec{v}_1 = \vec{v}_{1+} - \vec{v}_{1-}$. The spacecraft state $[\vec{r}_1 \ \vec{v}_{1+}]^T$ is then propagated along the transfer trajectory for a time $TOF = t_f - t_0$ until the final position $\vec{r}_N$ is reached on the next orbit. The state is now $[\vec{r}_N \ \vec{v}_{N-}]^T$. To match the velocity with that of the target orbit $\vec{v}_{N+}$, a second $\Delta\vec{v}_N = \vec{v}_{N+} - \vec{v}_{N-}$ is performed. Velocities and are usually solutions of the orbital two-point boundary value problem, whose solutions follow well-known strategies for simplified dynamical models [18, 21], or numerical multiple-shooting techniques for more complex dynamical frameworks [36]. The process used in this paper to generate two-impulse trajectories between different Halo orbits is described in section 4.1.2.

The primer vector at the initial and final impulse can then be evaluated:

$$\begin{aligned}\vec{p}(t_0) &= \vec{p}_0 = \frac{\Delta\vec{v}_1}{\Delta v_1} \\ \vec{p}(t_f) &= \vec{p}_f = \frac{\Delta\vec{v}_N}{\Delta v_N}\end{aligned}\tag{22}$$

To obtain the primer vector history with time, we need $\dot{\vec{p}}(t_0) = \dot{\vec{p}}_0$. From Eq. (19) the primer vector propagates with the STM of the state disturbances, that is:

$$\Phi(t_f, t_0) = \begin{bmatrix} M_{f0} & N_{f0} \\ S_{f0} & T_{f0} \end{bmatrix}\tag{23}$$

where for each sub-matrix M, N, S, T the abbreviated notation is used that $M(t_f, t_0) = M_{f0}$ and so on. By using Eqs. (19), (23) one has that:

$$\vec{p}_f = M_{f0}\vec{p}_0 + N_{f0}\dot{\vec{p}}_0\tag{24}$$

and thus:

$$\dot{\vec{p}}_0 = N_{f0}^{-1}[\vec{p}_f - M_{f0}\vec{p}_0]\tag{25}$$

where the inverse matrix N_{f0}^{-1} exists always except for isolated values of the time of flight (TOF) [38]. The vectors $\vec{p}_0$ and $\dot{\vec{p}}_0$ are then known, and

thus $[\vec{p}(t) \quad \dot{\vec{p}}(t)]^T$ can be evaluated at any time t within $t_0 \leq t \leq t_f$ using Eq. (19).

3.2. Insertion of extra impulses to lower the cost

From necessary conditions of optimality for impulsive transfers (see section 3.1), if the primer vector magnitude exceeds 1 along the transfer then the transfer is not optimal. Following the derivation in [48, 38], criteria for adding either initial or terminal coast arcs or mid-course manoeuvres can be derived that may lower the cost as from Eq. (21). For the purpose of the present paper, only the latter is relevant, as the transfers are assumed to be TOF-fixed. Such criterion states that if $p(t_i) > 1$ for a given $t_0 \leq t_i \leq t_f$, then a manoeuvre placed at $t = t_i$ lowers the cost [38]. This can be seen intuitively by differentiating the cost of a two-impulse transfer ($N=2$):

$$J = \Delta v_1 + \Delta v_2 \quad (26)$$

by adding a i^{th} manoeuvre $\Delta \vec{v}_i$ that modifies the cost by [48]:

$$dJ = \Delta v_i \left(1 - \vec{p}_i \cdot \frac{\Delta \vec{v}_i}{\Delta v_i}\right) \quad (27)$$

In other words, an additional impulse is to be placed that maximises the reduction of the cost variation of Eq. (27). This is done by a gradient descent method explained in later sections 4.1.3 and 4.1.4, on which the position $\vec{r}_i$ and time t_i of the impulse are found such that the cost with the additional impulse is minimized.

3.3. Multiple Shooting Algorithm

Another tool which is necessary to be able to compute the optimal trajectory is an efficient multiple shooting algorithm to numerically solve the Two Value Boundary Problem (TVBP) which is used both in the computation of the bi-impulsive initial guess and in the computation of the connecting arcs between the impulses during the optimisation process and in the optimal trajectory.

The solution to the TVBP is often not immediate this means, that no analytical solution exists when the dynamic is strongly non-linear and complex, such as the one of the CR3BP. For those cases, the solving procedure consists of an iterative process, at the end of which the algorithm either converges to a desired level of precision (tolerance) or it does not.

Let's assume to have a set of L points in our CR3BP framework that we want to connect in order to complete a certain task. These are called *nodes*, or equivalently *patch points*. Each patch point j is represented by the state $\vec{x}_j = [\vec{r}_j \ \vec{v}_j]$ and by the time t_j . By integrating from the initial condition $\mathbf{x}_j$ in the temporal arc $[t_l, t_{l+1}]$, a new state is obtained at the end, which is generally different from the state of the successive patch point $\vec{x}_{l+1}$. Is now introduced a notation to solve this conflict: we denote with $\vec{x}_i^+$ the fixed state of point i , while with $\vec{x}_i^-$ the one obtained from the integration of the state before, both at the unique time t_i .

Designing a trajectory means properly positioning the patch points, depending on the state that is aimed to be achieved. In the most general case, one could define a vector of design variables as:

$$\vec{z} = \begin{bmatrix} \vec{x}_0^+ \\ \vdots \\ \vec{x}_{L-1}^+ \\ t_0 \\ \vdots \\ t_{N-1} \end{bmatrix} \in \mathbb{R}^{7 \times L} \quad (28)$$

and a vector of constraints to verify expressed as in Eq. (29), which aims to ensure the continuity in position and velocity for the full trajectory:

$$\vec{g}(\vec{z}) = \begin{bmatrix} \vec{x}_1^- - \vec{x}_1^+ \\ \vdots \\ \vec{x}_{N-1}^- - \vec{x}_{L-1}^+ \end{bmatrix} \in \mathbb{R}^{6 \times (L-1)} \quad (29)$$

Each element g_i is also called *residual*, since represents the error committed at each iteration. The resolution procedure consists of solving the system of equations:

$$\vec{g}(\vec{z}) = \vec{0} \quad (30)$$

via a Newton-like root-finding method. In the presented work the used multiple shooting method is the Levenberg-Marquardt algorithm [49] for its convergence speed and the dimensions of its region of convergence.

4. Methods

This section describes the algorithm to compute the optimal three impulsive transfers for a Halo-to-Halo case. It is based on SEMpy toolbox, an open-source Python 3 library, developed at ISAE Supaero, Toulouse, France. ISAE Supaero's SACLab (Space Advanced Concept Laboratory) team is responsible for developing and maintaining the software package. It provides user-friendly tools for advanced astrodynamics studies and mission analysis in non-Keplerian environments [50].

The algorithm is composed of three steps: the computation of the bi-impulsive initial guess, the initial guess of the location of the intermediate impulse and finally the optimization step to select the best location in position and time, of the newly inserted impulse. A schematic of the algorithm is reported in Figure 4.

4.1. Algorithm

The step-by-step algorithm is analysed in the following.

step0: Initial, Final State and Time of Fight Definition

input: x_0 , x_f , TOF:

step1: Two Impulses Trajectory computation (initial transfer for the optimization)

While: $p > 1$ or $\dot{p} \notin \mathbb{C}$ or $\dot{p}(t_i) \neq 0$

step2: i-Impulse Insertion and Guess Computation

step3: Iterating position and time of intermediate manoeuvres

output: $r_i, t_i \forall i \in [0, N]$

4.1.1. Step 0 - Initial, Final State and Time of Fight Definition

The objective of the proposed algorithm is to provide a technique to compute the optimal transfer from any point of the starting orbit to any point of the arrival orbit.

A specific point is identified by a number in degrees (Θ) that is the equivalent of the mean anomaly for Keplerian orbits. Eq. (31) gives the definition of Θ and Figure 5 shows different values of this quantity.

$$\Theta = \frac{\tau}{T} 360^\circ \quad (31)$$

where τ is the time passed from the last perilune passage of the orbit and T is the orbital period, $M = 0^\circ$ corresponds to the perilune, $M = 180^\circ$

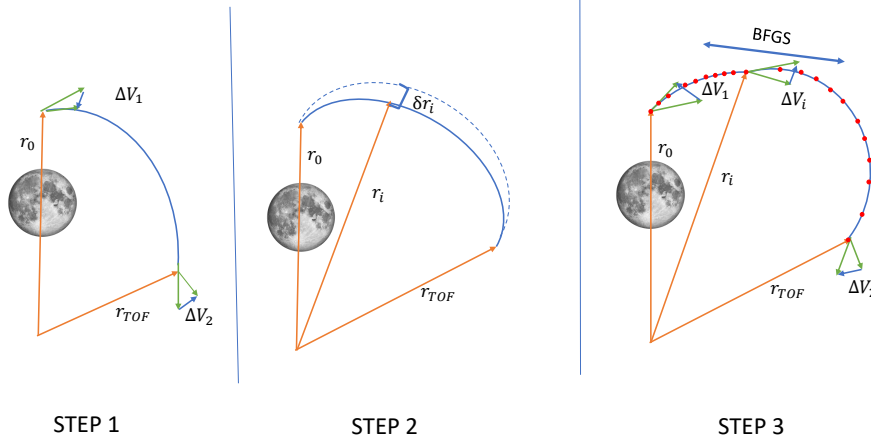


Figure 4: Orbit to Orbit algorithm

corresponds at the apolune.

In this paper, the Time of Flight (TOF) is selected through the following

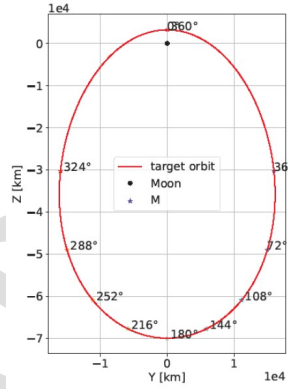


Figure 5: Example of different Θ angles

empirical formula [43, 23]:

$$TOF = \frac{T_{dep} + T_{arr}}{2} (\Theta_{arr} - \Theta_{dep}) \quad (32)$$

where Θ_{dep} and Θ_{arr} are the departure and arrival points on the two Halo orbits and T_{dep} and T_{arr} are the periods of the departure and arrival orbit.

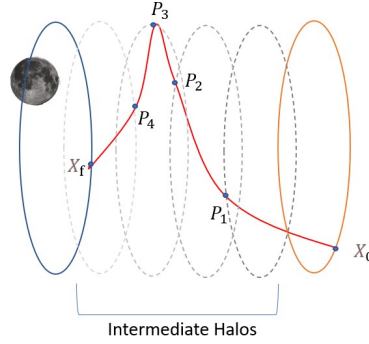


Figure 6: Schematics of initial patch points allocation to compute the bi-impulsive transfer.

In previous studies of the same authors [43, 23] the impact of TOF on ΔV was addressed. It was shown that Eq. (32) results in a good compromise between propellant cost and transfer duration for practical applications when the departure and arrival orbits have a similar shape, i.e. they belong to the same orbital family. Nevertheless, the TOF can also be selected freely if needed by the specific application.

4.1.2. Step 1 - Two Impulses Trajectory Computation

The computation of the bi-impulsive transfer is based on the knowledge of other intermediate orbits, which belongs to the same orbital family of the arrival and departure orbit. The high non-linearity of the system makes it very difficult to build an initial transfer that solves the two-point boundary value problem based only on the knowledge of the arrival and departure conditions and of the time of flight.

This difficulty was overcome by selecting $M-2$ subsequent orbits are characterised by a constant increment in the corresponding periods - at least 3 - in the between the arrival and the departure orbits and selecting other $M-2$ patch points on the intermediate orbits equally spaced in Θ . Then a multiple shooting procedure is run to ensure the continuity of the trajectory at each point [50]. A two impulses trajectory is used as starting point for the optimization since two is the minimum number of impulses necessary to solve the TVBP [7]. A conceptual scheme of the initial sequence of patch points is given in Figure 6. The more the departure and arrival orbits are distant in the family the more M increases - empirically between 3 and 10 - to obtain

a smoother and natural transfer, which is critical for the convergence of the algorithm due to its high sensitivity to small perturbations. The choice of the parameter M significantly influences the shape of the guess and therefore the goodness of the final result.

4.1.3. Step 2 - i -Impulse Insertion and Guess Computation

As discussed in section 3.2, one needs to find the values of $[\vec{r}_i \ t_i]$ that maximise the cost reduction from Eq. 27. A guess for those values is to be provided to initialise the local gradient descent method (see section 4.1.4). From Eq. 27, one identifies the initial guess for t_i as the time at which the magnitude $\vec{p}_i$ has the maximum value, as shown in Figure 10b, for the case study described later on. Regarding the initial guess on $\vec{r}_i$, the following analytical procedure is employed [38].

For each value of the primer vector, there is a corresponding position r_n in the nominal N-1 impulses trajectory. Consequently, the value of the primer vector at time t_i is associated with a position r_{in} in the nominal trajectory. The initial guess of the extra impulse position is obtained by perturbing the nominal position r_{in} by a certain δr_i .

$$\vec{r}_i = \vec{r}_{in} + \delta \vec{r}_i \quad (33)$$

The choice over the perturbation δr_i (and thus over $\vec{r}_i$) should be such that the product $\vec{p}_i \cdot \Delta \vec{v}_i / \Delta v_i$ has a maximum. It is thus necessary that:

$$\Delta \vec{v}_i = \varepsilon \vec{p}_i \quad (34)$$

where $\varepsilon > 0$ is a scalar. It can be shown that [48]:

$$\Delta \vec{v}_i = \vec{A} \delta \vec{r}_i \quad (35)$$

where the matrix A is defined by:

$$A \equiv - (N_{i+1,i}^T M_{i+1,i}^{-T} + T_{i,i-1} B_{i,i-1}^{-1}) \quad (36)$$

and is composed by sub-matrices of the STM from Eq. (23) defined between the previous impulse time t_{i-1} to the i^{th} one t_i ($STM_{i,i-1}$) and between t_i to the final one ($STM_{i,i-1}$).

Combining Eq. (34) and (35) and assuming that the matrix A is invertible, yields:

$$\delta \vec{r}_i = \varepsilon A^{-1} \frac{\vec{p}_i}{\|\vec{p}_i\|} \quad (37)$$

By introducing the parameter $\beta = \delta r_i / r_{in}$ (a user-defined positive small number), we can rewrite ε as:

$$\varepsilon = \frac{\beta r_i}{\|A^{-1}\vec{p}_i\|} \quad (38)$$

The position r_m of the mid-course impulse is then given by applying the perturbation $\delta\vec{r}_m$ as:

$$\delta\vec{r}_i = \beta r_i \frac{A^{-1}\vec{p}_i}{\|A^{-1}\vec{p}_i\|} \quad (39)$$

To determine the size of such perturbation, the parameter β has to be chosen such that the resulting dJ is negative (i.e., the cost decreases by adding the Δv_i). Typical values for β are in the 0.0005-0.05 range. Note that from the Eq (39) the inverse of the matrix A is required to compute the perturbation $\delta\vec{r}_m$. In turn A , recalling Eq. (36), also depends on the inverse of two matrices: the STM sub-matrix N computed once from the $i-1^{th}$ state to the i^{th} state ($N_{i,i-1}$) and the other from the i^{th} state to the next one ($N_{i+1,i}$). So a check on the invertibility of such matrices must be made.

4.1.4. Step 3 - Iterating position and time of intermediate manoeuvres

Once that initial guess of position and time of the i^{th} impulse has been computed, the next step is to find the proper $k = [\vec{r}_i \ t_i \ \dots]$, $\forall i \in [1, N-1]$ that minimizes the cost of the N -impulse trajectory as from Eq. (21). It can be shown that the variations of the cost function with respect to the optimization variables is given by [36]:

$$\begin{aligned} \frac{dJ}{d\vec{r}_i} &= \dot{p}_{i+}^\Gamma - \dot{p}_{i-}^\Gamma \\ \frac{dJ}{dt_i} &= \dot{p}_{i-}^\Gamma v_{i-} - \dot{p}_{i+}^\Gamma v_{i+} \\ \nabla J &= \left[\frac{dJ}{d\vec{r}_i} \ \frac{dJ}{dt_i} \ \dots \right] = \left[\frac{dJ}{dx_i} \ \frac{dJ}{dy_i} \ \frac{dJ}{dz_i} \ \frac{dJ}{dt_i} \ \dots \right] \forall i \in [1, N-1] \end{aligned} \quad (40)$$

where $\vec{r}_i = [x_i \ y_i \ z_i]$ and $\vec{p}_{i-}$ and $\vec{p}_{i+}$ correspond to the primer vector before and after the application of the i^{th} impulse $\Delta\vec{v}_i$, respectively. The availability of the gradient for the cost function naturally drives the choice towards a gradient-based method to optimize the cost with respect to $\vec{k} =$

$[\vec{r}_i \ t_i] \forall i \in [1, N - 1]$. In this work, a Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm is employed [51]. This is a quasi-Newton line-search method that at any iteration it selects the search direction $\mathbf{s}$ and step length α such that $k_{it+1} = k_{it} + \alpha \mathbf{s}$ minimises the cost along that direction. At every step of the minimisation process, the full trajectory is recomputed and a multiple shooting algorithm, with a number K of patch point, is used to compute every arch which composes the trajectory, from the impulse i to the on $i + 1$. This number K is a critical parameter for the convergence of the algorithm, numerical details are provided later in the paper.

4.2. Exit conditions

The algorithm terminates when the optimality conditions on the primer vector and its derivative are satisfied:

- $p(t) < 1, \forall t \in [0, TOF]$
- $\dot{p}(t) \in \mathbb{C}, \forall t \in [0, TOF]$
- $\dot{p}(t_i) = 0, \forall i \in [1, N - 1]$

According with the literature [44] [45] [36], if this terminal conditions are all verified the found trajectory is a locally optimal trajectory.

5. Study Case and Results

The proposed algorithm is applied to a specific study case in this section. Note that all the simulations were run on PANDO, a supercomputer available at ISAE SUPAERO [52]

5.1. Study case

The presented method is applied to a specific study case, which was selected as a realistic application related to the Artemis program [53]. The arrival orbit is chosen as the orbit of the Gateway, a 9:2 resonant L2 Halo orbit with the characteristics given in Table 1, the departure orbit belongs to the same family of the previous one but it has a period of 1.2 times larger (see Table 2). The latter was selected due to the relative phase dynamics that can be exploited, for example, to perform phasing manoeuvres. The two orbits are shown in Figure 7.

Table 1: Gateway Orbit Characteristics-Arrival Orbit

Name	Symbol	Value
Vertical extension	A_z	69958.505 km
Periselene radius	r_p	3225.211 km
Aposelene radius	r_a	71170.507 km
Period	T_{NRHO}	6.562 days
Jacobi constant	C_{jac}	3.059

Table 2: Departure Orbit Characteristics

Name	Symbol	Value
Vertical extension	A_z	74611.556 km
Periselene radius	r_p	7412.961 km
Aposelene radius	r_a	77697.041vkm
Period	T_{NRHO}	7.874 days
Jacobi constant	C_{jac}	3.040

In this scenario, the maximum available ΔV in $100m/s$ ([43]) for the entire transfer and the maximum time of flight is limited to the largest orbital period, 7.874 days. The latest constraint is not a limitation in fact, if the space mission allows more time to transfer two solutions can be implemented: the presented technique can be applied with a larger TOF allowing the trajectory to perform an extra tour around the Moon or the spacecraft can wait on the departure orbit until a feasible configuration is reached.

5.2. Results

The main results are described in the following. Figures 8 and 12 indicate the ΔV consumption for the two impulses trajectories and the gain obtained in N-impulses cases as a function of starting and arrival points. There are some cases in which the algorithm does not find the solution for the bi-impulsive case to satisfy the two-point boundary value problem and those cases are indicated with "inf", and the other cases are coloured in relation to their value. The values that are marked in green are the lowest, then the colourmap moves towards the red colour, and all the transfers that are

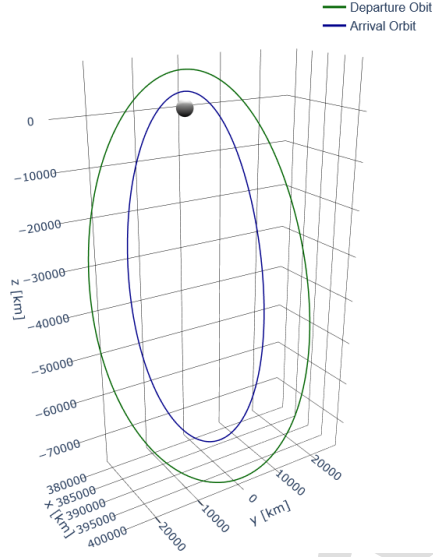


Figure 7: arrival and Departure Orbits

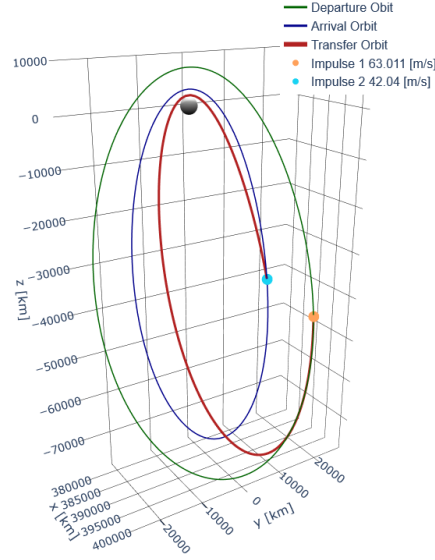
marked in red are unfeasible or almost unfeasible. The points of 5 intermediate orbits were used to generate the initial bi-impulsive transfer. Figure 9 also reports the TOF for every computed transfer in days. Figure 8 shows two green stripes, which correspond to an interval of phase differences that stays between 70° and 310° . In these intervals the trajectory has a shape similar to the one reported in Figure 10a. From the same figure is clear that when the mean anomalies of arrival and departure have about the same value, the cost is very high, as proven by the red stripe in the middle of the table. In this area, the bi-impulsive trajectories have the shape reported in Figure 11a, this behaviour can be mitigated if the limit on the TOF is removed and the trajectory is left free to do another turn around the Moon (see Figure 11b) [43, 23]. The results could be improved if the limit on TOF is removed, see Figure 11b.

By analysing again the results for the N impulse case in Figure 12 in terms of saved ΔV . It can be noticed that where the bi-impulsive trajectory is not already optimal nor "inf", the gain goes from 0.5 m/s up to 1300 m/s. The performance changed smoothly, thanks also to the regular shape of the Halo family. However, there are still some isolated points where the optimization is not able to recover any ΔV , the poor performance is related to the bad shape of the bi-impulsive transfer.

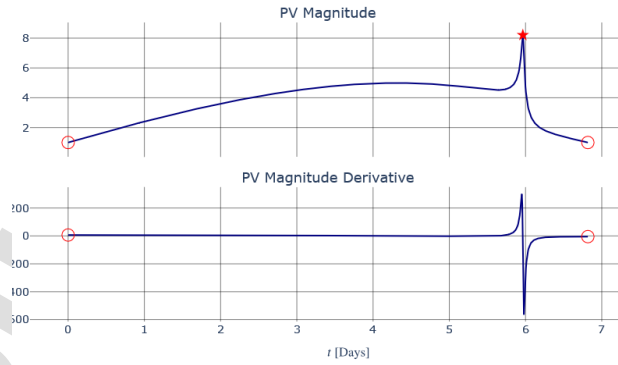
$\theta_{arr} [^\circ]$	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300	310	320	330	340	350	
0	inf	725	421	307	246	207	179	159	142	129	117	108	99.5	93.6	90.4	90.5	94.4	102	113	127	143	161	182	205	230	257	286	317	351	388	429	474	527	inf	inf	inf	
10	inf	1938	1026	540	374	289	237	201	175	155	139	126	115	105	96.6	89.3	82.9	77.1	72.1	67.8	64.1	61.2	59.1	57.8	57.5	58.1	59.6	62	65.2	69.3	74.5	81.3	90.3	103	121	174	
20	inf	inf	1391	1113	572	389	297	241	203	176	155	138	124	113	103	95	87.7	81.4	75.8	71	66.8	63.2	60.3	58.1	56.6	55.7	55.6	56.2	57.4	59.3	62.1	65.8	71	78.6	91.3	120	
30	inf	200	238	302	1164	591	399	302	244	205	176	154	137	124	112	102	93.9	86.7	80.4	75	70.3	66.3	62.9	60.2	58.1	56.6	55.7	55.4	55.6	56.5	58	60.3	63.7	68.9	77.9	98.7	
40	inf	147	162	184	212	1194	602	405	305	245	205	176	154	137	123	111	101	92.9	85.8	79.7	74.4	69.9	66	62.9	60.3	58.3	56.8	55.9	55.5	55.6	56.3	57.6	59.8	63.5	70.2	86.2	
50	inf	117	123	135	150	168	1212	609	407	306	245	204	175	153	135	121	110	100	91.9	84.9	79	73.9	69.5	65.9	62.8	60.4	58.5	57	56.1	55.6	55.6	56.2	57.6	60.1	65.1	77.7	
60	inf	97.8	100	107	117	129	143	1221	611	408	306	244	203	174	151	134	120	109	99	91	84.1	78.3	73.4	69.2	65.7	62.8	60.5	58.6	57.2	56.2	55.7	55.7	56.3	57.9	61.5	71.6	
70	inf	84.1	84.4	89.1	96.3	106	114	846	1222	610	408	304	243	202	172	150	133	119	107	98	90.1	83.4	77.8	73.1	69	65.7	62.8	60.5	58.7	57.2	56.2	55.7	55.6	56.5	59	66.9	
80	inf	73.9	73.1	76.5	81.8	88.9	98	inf	123	1218	607	404	302	240	200	170	148	131	117	105	97	89.3	82.9	77.4	72.8	68.9	65.6	62.8	60.5	58.7	57.2	56.1	55.6	55.7	57.3	63.4	
90	inf	66.1	64.8	67.3	71.6	77.4	85	94.9	108	124	1210	603	400	299	238	197	168	147	130	116	105	96.3	88.7	82.4	77.1	72.6	68.8	65.5	62.8	60.5	58.6	57.1	56	55.5	56.2	60.7	
100	inf	60	58.4	60.4	64.1	68.1	75.7	84.3	95.5	110	228	1199	597	396	296	235	195	166	145	128	115	104	95.6	88.3	82.1	76.9	72.5	68.7	65.5	62.7	60.4	58.4	56.9	55.8	59		
110	inf	55.2	53.6	55.3	58.5	62.9	68.8	76.4	86.4	796	inf	757	1186	590	392	292	233	193	165	144	127	114	104	95.2	88	81.9	76.8	72.4	68.7	65.4	62.7	60.3	58.3	56.8	56.1	58.1	
120	inf	51.6	49.9	51.5	54.4	58.3	63.6	70.4	79.3	799	778	760	208	1173	584	387	289	230	191	163	143	127	114	103	94.9	87.9	81.9	76.8	72.5	68.7	65.5	62.6	60.2	58.3	57	58	
130	inf	48.9	47.3	48.7	51.4	55	59.6	65.7	inf	803	inf	764	750	203	1160	578	383	286	228	190	162	142	126	113	103	94.9	87.9	82	77	72.6	68.9	65.6	62.7	60.3	58.6	58.9	
140	inf	47.3	45.5	46.8	49.2	52.5	56.7	62.1	inf	inf	785	768	753	742	203	1149	572	380	284	227	189	162	142	126	113	103	95.1	88.2	82.3	77.3	72.9	69.1	65.9	63.1	60.9	60.6	
150	inf	46.5	44.6	45.7	47.8	50.7	54.4	59.3	837	811	85.7	101	758	746	736	216	1140	568	378	283	226	188	162	142	126	114	104	95.6	88.7	82.8	77.8	73.4	69.7	66.4	63.9	63.2	
160	280	46.6	44.3	45.1	46.9	49.5	52.8	inf	841	70.4	80.6	94.6	114	750	741	738	726	1133	566	377	282	226	189	162	142	127	114	104	96.2	89.3	83.5	78.5	74.2	70.5	67.6	66.6	
170	238	47.4	44.5	45	46.5	48.7	51.6	55.4	60.4	67.1	76.1	88.5	106	inf	inf	inf	inf	731	725	1129	564	376	283	226	189	163	143	127	115	105	97.1	90.3	84.4	79.5	75.3	72	70.8
180	205	48.8	45.3	45.3	46.4	48.2	50.7	54.6	58.4	64.3	72.2	83	97.9	118	inf	742	736	inf	1238	1129	555	377	284	228	190	164	144	129	116	107	98.8	91.4	85.6	80.8	77.1	75.6	
190	172	50.7	46.4	45.9	46.6	48	50.1	52.9	56.7	61.8	68.7	78.1	90.9	109	133	167	211	236	inf	727	1132	567	379	286	229	192	165	145	130	118	108	99.7	92.8	87.3	83	81.2	
200	159	52.9	47.7	46.7	47	48	49.7	52.1	55.3	59.7	65.6	73.6	84.4	98.3	120	149	inf	inf	737	731	inf	1139	571	382	288	232	194	167	147	132	119	109	101	94.7	89.6	87.5	
210	147	55.5	49.4	47.8	47.6	48.3	49.5	51.4	54.1	57.8	62.8	69.5	78.5	90.7	108	132	inf	213	745	740	737	478	1148	576	386	291	234	196	169	149	133	121	111	103	97.3	94.5	
220	inf	58.4	51.2	49.1	48.4	48.6	49.4	50.9	53.1	56.1	60.3	65.8	73.1	82.9	96.2	115	142	181	237	749	745	742	inf	1159	582	390	294	237	199	171	151	135	123	113	106	103	
230	210	61.6	53.3	50.5	49.4	49.2	49.5	50.5	52.2	54.6	58	62.5	68.3	76	86.1	99.7	119	148	193	759	inf	752	749	inf	1172	589	395	298	240	201	173	153	137	125	116	112	
240	240	65.1	55.6	52.2	50.5	49.8	49.8	50.3	51.5	53.4	56	59.6	64.1	69.9	77.4	87.1	100	119	148	198	768	764	761	759	756	1185	596	399	301	242	203	176	155	140	128	122	
250	251	68.9	58.2	54	51.9	50.7	50.2	50.3	51.1	52.4	54.4	57.1	60.5	64.8	70.2	76.9	85.6	97.2	114	140	190	287	776	773	770	inf	1198	602	403	304	245	205	177	157	142	134	
260	262	73.2	61.1	56.1	53.4	51.7	50.8	50.5	50.8	51.7	53.1	55.1	57.6	60.7	64.4	69	74.7	81.9	91.3	105	125	165	277	426	548	786	783	1209	607	406	306	246	207	179	160	149	
270	276	78.1	64.3	58.5	55.2	53	51.7	51	50.9	51.3	52.2	53.5	55.2	57.4	60	63	66.7	71.2	76.9	84.1	94	106	184	209	442	inf	708	803	1218	611	409	308	248	208	181	168	
280	284	83.6	68.1	61.3	57.3	54.7	52.9	51.8	51.3	51.3	51.7	52.5	53.5	54.9	56.6	58.6	60.9	63.8	67.3	71.7	77.2	84.5	94.9	111	836	851	822	830	913	1223	613	409	308	248	210	187	
290	295	90.2	72.5	64.7	60	56.7	54.5	53	52.2	51.7	51.9	52.4	53.1	54	55.2	56.7	58.5	60.7	63.4	66.9	87.8	77.2	821	96.8	116	160	864	248	861	1222	613	408	307	249	215		
300	302	98.1	77.9	68.8	63.2	59.4	56.7	54.8	53.5	52.6	52	51.8	51.7	51.9	52.2	52.8	53.5	54.6	55.9	57.7	59.9	62.9	66.7	71.8	78.6	88.2	101	124	inf	784	905	1215	607	405	306	252	
310	318	108	84.6	74	67.4	62.9	59.6	57.1	55.3	53.9	52.9	52.1	51.5	51.1	50.9	50.9	51.1	51.6	52.3	53.4	54.9	57	59.7	63.2	68	74.4	83.1	95.8	116	158	964	966	1200	598	399	307	
320	327	121	93.3	80.8	73	67.5	63.4	60.3	57.8	55.8	54.2	52.8	51.7	50.8	50.1	49.6	49.3	49.3	49.5	50.1	51.1	52.5	54.6	57.3	60.9	65.7	72.1	80.7	92.9	111	145	inf	1053	1172	583	394	
330	330	139	106	90.4	80.8	74	68.9	64.8	61.4	58.7	56.3	54.3	52.6	51.1	49.8	48.8	48	47.5	47.3	47.5	48	49.1	50.7	53	56	59.9	65	71.6	80.4	92.7	111	142	216	1185	1126	563	
340	343	167	126	106	93.7	84.8	77.9	72.4	67.8	64	60.7	57.7	55.2	52.9	50.9	49.2	47.7	46.6	45.9	45.6	45.9	46.7	48.1	50.1	52.9	56.4	60.8	66.3	73.4	82.6	95.3	114	144	206	1408	inf	
350	845	233	173	143	124	110	99.3	90.8	83.8	77.8	72.7	68.1	64.1	60.5	57.2	54.3	51.8	49.6	48.1	47.1	47	47.6	49.1	51.3	54.1	57.7	62	67.2	73.5	81.3	91.2	104	123	153	210	441	

Figure 8: Initial transfer ΔV [m/s] consumption with respect to the arrival and departure point

25

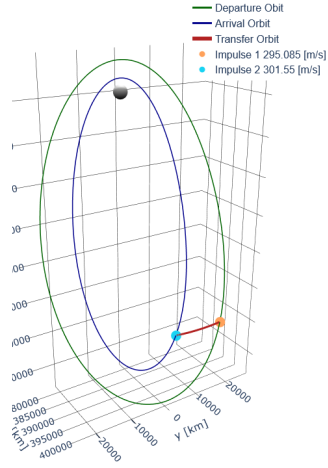


(a) Initial transfer - $\Theta_{dep} = 70^\circ$, $\Theta_{arr} = 50^\circ$ - $\Delta V = 105 \text{ m/s}$, TOF = 6.80 days

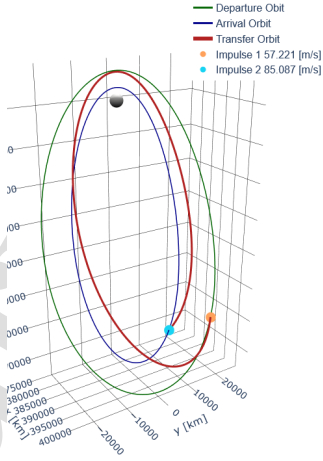


(b) Primer vector associated with the Initial transfer trajectory and its derivative. The red star indicated the maximum of the primer vector

Figure 10: Example of Initial bi-impulsive Trajectory and its associated primer vector.



(a) Initial transfer - $\Theta_{dep} = 100^\circ$, $\Theta_{arr} = 120^\circ$ - $\Delta V = 596.64 m/s$, TOF = 0.40 days



(b) Initial bi-impulsive Trajectory where multiple revolutions are allowed - $\Theta_{dep} = 100^\circ$, $\Theta_{arr} = 120^\circ$ - $\Delta V = 142.31 m/s$, TOF = 7.62 days

Figure 11: Example of Two Impulses Trajectory with different TOF

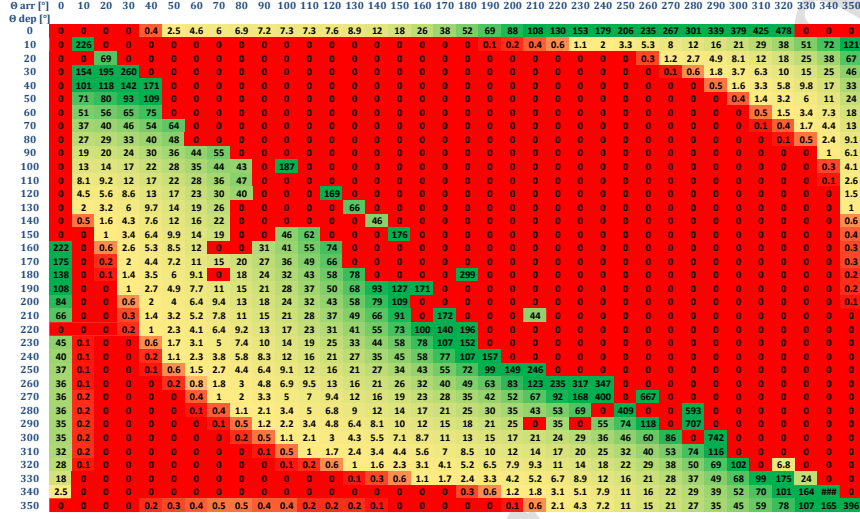


Figure 12: N-impulse optimal ΔV consumption with respect to the arrival and departure points

To help the convergence of the algorithm each transfer arc, from X_{i-1} to X_i and from X_i to X_{i+1} , is computed through a multiple shooting method with a number of patch points which gets increasingly high if the convergence is not reached, more precisely 2, 6 or 10. This solution was applied to avoid a major complexity of the algorithm since more patch points enhance the accuracy of the algorithm and help in its convergence, but they require a larger computational complexity. In the final optimal trajectory, the number of impulses for the optimal trajectory goes from 2 impulses up to 5.

Figure 15 shows an optimised trajectory associated with the primer-vector of Figure 14, which is obtained through a sequence of 4 impulses, which initial guess is located every time at the maximum of the $\vec{p}$ and it is optimized through a BFGS algorithm. The shape of the optimised trajectory starting from Figure 11b is reported in Figure 12.

The algorithm is capable to find the optimal solutions in all the tested cases, even if, in some rare cases the computer inaccuracy leads to a partial accomplishment of the optimality conditions, in detail, it might happen that the derivative of the primer vector ($\dot{\vec{p}}$) is not perfectly continuous at the impulse

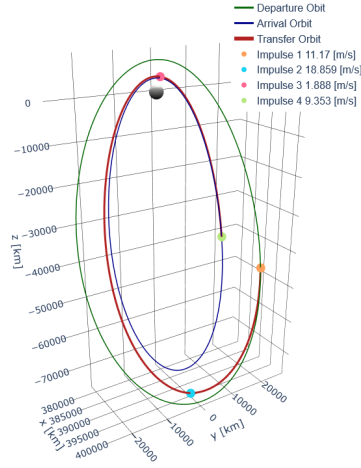


Figure 13: An example of optimal trajectory - $\Theta_{dep} = 70^\circ$, $\Theta_{arr} = 50^\circ$ $\Delta V = 41.30 m/s$, TOF = 6.80 days, 4 impulses

time.

The same algorithm was also proven to work with orbits which are further away in the family, for example, to transfer from a Halo orbit with a period which is double of the arrival orbit one to the same arrival orbit, it is possible to gain up to 1800 m/s. The only difference with respect to the previous application case is the number of intermediate orbits to generate the initial bi-impulsive transfer, in this case, was selected to be 10, instead of 5, which implies the generation of a smoother initial transfer due to the larger distance of the orbits in the family. In Figure 16 a transfer trajectory related to this last example is reported for coherence.

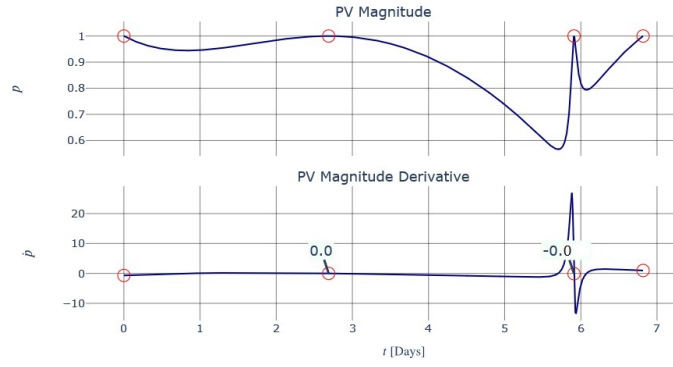


Figure 14: Example of primer vector and its derivative after the optimisation process - $\Theta_{dep} = 70^\circ$, $\Theta_{arr} = 70^\circ$

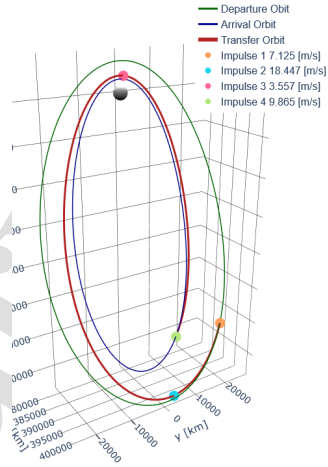
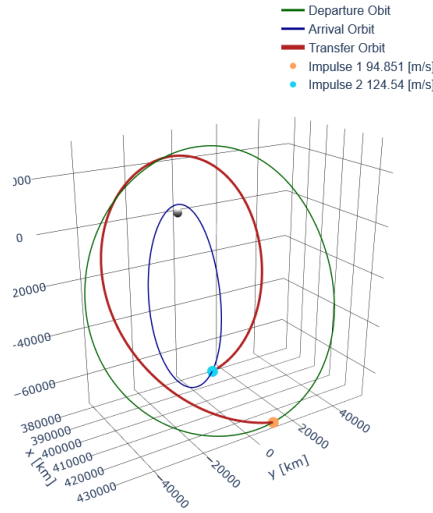
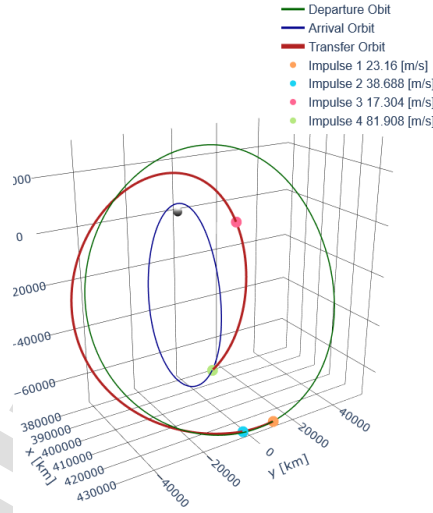


Figure 15: An example of optimal trajectory - $\Theta_{dep} = 100^\circ$, $\Theta_{arr} = 120^\circ$ $\Delta V = 38.99 \text{ m/s}$, TOF = 7.62 days, 4 impulses



(a) Initial bi-impulsive transfer - $\Theta_{dep} = 170^\circ$, $\Theta_{arr} = 180^\circ$ - $\Delta V = 272.85 m/s$, TOF = 8.75 days



(b) Optimal trajectory - $\Theta_{dep} = 170^\circ$, $\Theta_{arr} = 180^\circ$
- $\Delta V = 161.06 m/s$, TOF = 8.75 days

Figure 16: Example of trajectories in the case of distant departure orbit

6. Conclusions

The paper presents a method for the automated computation of optimal transfers of any pair of points of a Halo-to-Halo scenario in the circular restricted three-body problem formulation.

The main result is a systematic, simple effective algorithm to compute transfers in this highly non-linear framework, with the goal of overcoming standard approaches, where the transfer is designed ad hoc for each mission and no decisional power is given to the satellite. The ultimate goal of this study, is in fact, to pave the way to satellites that will be able to plan and execute autonomous transfer and orbit reconfiguration using onboard simple and reliable software. To overcome the computationally extensive calculation for optimal transfers with time constraints, the shape of a two-impulse initial transfer was selected. Secondly, the systematic optimisation method formalised herein for the CR3BP provides a wide range of optimal and sub-optimal solutions to target any arrival point, this can be considered a promising result, compared with the current state of the art.

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Highlights

Optimal Time-Fixed Impulsive non-Keplerian Orbit to Orbit Transfer Algorithm based on Primer Vector Theory

Giordana Bucchioni, Gabriele Geminiani, Francesco Lombardi, Andrea Bellome, João Pedro Fernandes Leitão, Stèphanie Lizy-Destrez, Mario Innocenti

- Research highlights 1: The paper proposes an optimal and effective way to transfer between non-keplerian orbit which belong to the same orbital family. The algorithm relies on the knowledge on the natural dynamics of the system and on well consolidated technique as the primer vector theory, which was clearly stated in this paper to a formulation suitable for the circular restricted three body problem dynamics.
- Research highlights 2: The paper describes a simple and effective technique to design bi-impulsive initial guesses for the optimisation process, which are based on the knowledge of the dynamics of the Halo orbital family.
- Research highlights 3: The paper proposes an effective, feasible and procedure, based on previous research results, to reduce the ΔV consumption to transfer between non Keplerian orbits adding a number N of intermediate impulses which is automatically selected by the algorithm.
- Research highlights 4: In the paper is shown an application of the technique to a halo-halo transfer, which is a plausible scenario of future space missions, which constitutes an alternative method to other indirect optimization methods.

Credit author statement

- Giordana Bucchioni: Conceptualization, Methodology, Formal analysis, Investigation, writing original draft and editing, supervision, Project administration.
- Gabriele Geminiani: methodology, Software, Validation, Formal analysis, editing.
- Francesco Lombardi: methodology, Software, Validation, Formal analysis.
- Andrea Bellome: methodology, Formal analysis, editing.
- Joao Fernandez Leita: methodology, Software, Validation, Formal analysis,
- St phanie Lizy-Destrez: Supervision, funding acquisition
- Mario innocenti ; Supervision and editing

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:



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