

CRANFIELD UNIVERSITY

C. HATHAWAY

GIBSON'S CRITERIA: LECTURES AND EXERCISES FOR TEST
PILOTS AND FLIGHT TEST ENGINEERS

SCHOOL OF ENGINEERING

MSc THESIS
Academic Year: 2013 - 2014

Supervisor: D. LEE
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the degree of Master of Science

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ABSTRACT

Empire Test Pilots' School (ETPS) trains experienced military pilots, engineers and civilian engineers as Test Pilots and Flight Test Engineers. Over a year, ETPS broadens the experience of each student through exposure to a wide variety of aircraft in addition to increasing their depth of understanding of how aerodynamics affect the handling qualities and ultimately the mission effectiveness of aircraft.

The advent of computerized flight control systems has reduced the role of the aerodynamicist to a necessary one but no longer sufficient to achieve excellent handling qualities. Complex software programming performed by a control engineer is now required to realize this goal. Consequently, second order handling qualities criteria such as the longitudinal short period natural frequency and damping ratio are no longer strictly applicable to the significantly higher order modern flight control systems. Gibson's Criteria was developed to provide control engineers the ability to design in excellent handling qualities from the outset, rather than waiting for pilots to identify failures during testing.

While ETPS students are introduced to Gibson's Criteria as part of the graduate course, the emphasis is on exposure and not an in-depth understanding. As the expense and time to develop modern high order aircraft grows exponentially, the importance of getting the handling qualities correct from the start only becomes more acute. It would be highly beneficial for graduates of ETPS to have a comprehensive understanding of how Gibson's Criteria is applied early in software development to ensure excellent handling qualities are designed in early and effectively. This thesis is a survey of current material on Gibson's Criteria and existing aircraft data / handling qualities problems formatted as a series of lectures and Matlab based problems designed to be given as part of the ETPS graduate course to achieve that level of knowledge.

Keywords:

High Order Flight Control Systems, Short Period Pitch Response, Natural Frequency and Damping, Matlab, Designing in Excellent Handling Qualities, Modern Handling Qualities Criteria, Tornado, F-16, Hawk, A-7A

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LIST OF ABBREVIATIONS

CU	Cranfield University
ETPS	Empire Test Pilots' School
FW	Fixed Wing
FTE	Flight Test Engineer
GT	Gibson's Thesis
HOFCS	High Order Flight Control System
TP	Test Pilot
PIO	Pilot Induced Oscillations, occasionally also referred to as Aircraft Pilot Coupling in the USA

1 INTRODUCTION

1.1 Thesis Objectives

1. To review the literature available concerning Gibson's Criteria.
2. To develop a classroom lecture suitable for presenting the background, theory and application of Gibson's Criteria to an ETPS class.
3. To develop 3 new case studies involving the introduction and application of the Gibson criteria to ETPS class A FW TP and FTE.

1.2 Background

The author of this thesis attended ETPS as an exchange student from the US Navy. Cranfield University offers a Master's of Science degree to individuals attending the full year program at ETPS who complete additional requirements. The author enrolled in this program, completed additional assignments during his year at ETPS and attended several in resident courses at Cranfield in the four months subsequent to graduation from ETPS. This thesis was written after the author returned to the United States and was serving as an F/A-18 Air Vehicle & Propulsion project officer and test pilot at NAS Patuxent River, Maryland.

Just as Cranfield University strives to be on the leading edge of technology and science, ETPS endeavours to be the global leader in the education and training of professional Test Pilots and Flight Test Engineers. This requires constant update to the syllabus in response to industry developments and best practices. While Gibson's Criteria are introduced as handling quality criteria for 4th/5th generation fighter and attack aircraft during the graduate level ETPS course, there is insufficient focus on the topic to develop an appreciation of the critical nature of these criteria in the development of modern, computer controlled, high order flight control systems. The development of a classroom lecture to present the background, theory and application of Gibson's Criteria suitable for use in the ETPS graduate level flight test professional course would help bring the ETPS syllabus to the leading edge of current aviation knowledge.

Additionally, A number of ETPS 'Flight Dynamics' and Cranfield 'Flight Dynamics' and 'Flying Qualities and Flight Control' assignments from recent years are available and cover the initial steps required to allow the development of more advanced case studies where the Gibson Criteria can be introduced. The development of existing assignments into a small set of case studies introducing Gibson Criteria to graduate level students at ETPS is a natural and beneficial thesis topic for a recent ETPS graduate to undertake as part of the MSc program at Cranfield University.

1.3 Report Structure

This thesis is structured around a series of classroom lectures presented as slides with accompanying lecture script, with three supplementary exercises designed to challenge and expose flight test professionals to a control engineering problem by integrating a Gibson command path pre-filter into a simplified aircraft model at a given flight condition by using Matlab.

Accomplishing the thesis objectives within the standard Cranfield University thesis format has presented a minor challenge. Following this introduction are two chapters, the first providing an overview of the breadth and depth of the literature reviewed as part of the development of the lecture series and exercises, and the second briefly describing the methodology used.

Chapter four serves as the analysis and discussion portion of this thesis and contains the focus of effort and objectives: the lectures and exercises. There are two lectures and three assignments. The first lecture is split into two parts, each approximately 55 minutes long to accommodate the traditional hour long classroom lecture period with a short break before proceeding onto part two. This first lecture takes the students through the background and history of handling qualities and control engineering as well critical considerations such as the control loop variable, pilot variability and controls system hardware before defining Gibson's criteria.

The second lecture could be used as a standalone presentation for students with a solid grasp of the aforementioned topics, or to be given on a subsequent

day to the first lecture to less experienced students. It is approximately 40 minutes long and contains a review of critical attitude parameters followed by a method to integrate a Gibson pre filter into the command path of a simplified aircraft model using Matlab and ends with a detailed worked solution of Exercise B, the A-7A.

These two lectures are followed by the three exercises: A, the F-16; B, the A-7A; C, the Hawk. These exercises are all formatted the same. The assignment to be given to the students is contained in the main body of the thesis, while the full solution set and accompanying Matlab files are in an appendix.

2 LITERATURE SEARCH & REVIEW

2.1 Breadth & depth of current knowledge

In 1999, John Gibson, a British aerodynamicist, submitted work describing a lifetime of work dedicated to “Development of a methodology for excellence in handling qualities design for fly by wire aircraft” to the Delft University of Technology as a doctoral thesis. This nearly 250 page document summarized his method and solution to several PIO incidents he encountered during development of the Tornado attack aircraft. These incidents caused him to look more specifically at the effect of high order dynamics of hydraulically powered and computer controlled aircraft on handling qualities. He found the contemporarily accepted standards that were based on conventional second order aircraft to be wanting and developed a set of criteria to be used during the design phase to ensure the absence of PIO and the presence of excellent pitch handling qualities. ‘Gibson’s Criteria’ were designed to be understood intuitively by pilots and control system engineers alike.

While avoiding PIO is certainly critical to the safe and effective operation, good handling qualities do not end with merely the elimination of PIO. “It performs an essential part in task performance. The combat pilot must devote all possible attention to the management and operation of a variety of complex weapons and defensive systems against an increasingly capable opposition. The ability actually to fly the aircraft with minimum attention to safety and structural limits releases considerable mental capacity to the combat task.” [GT, para 10.1, pg 200]. Making the aircraft and pilot more effective at the mission task is the final and complete reason for improving handling qualities.

In 1903, the Wright brothers conducted the first powered, heavier than air, controlled flight. While the Wright Flyer was actually statically unstable in the longitudinal axis, the brothers’ “greatest and fundamental achievement was their invention of three-axis aerodynamic control” [Culick, F.E.C., *Wright Brothers: The First Aeronautical Engineers and Test Pilots*, reference 6] achieved through wing warping and control tabs.

This was all the more remarkable due to the minimal understanding of the equations of motion at work in their 6 degree of freedom problem called an airplane.

From that starting point, there have been vast improvements in the performance of aircraft. There are two notable technological developments that are pertinent to this discussion: First, the advent of the jet engine, requiring the inclusion of hydraulic powered flight controls to overcome the aerodynamic forces as well as requiring artificial feel devices for the pilot; and Second, the arrival of computers, allowing for less aerodynamically stable aircraft that require the speed and processing power of flight control computers to be flyable and presenting the possibility of carefree handling of flight control and engine inputs.

A high order computer controlled flight control system (HOFCS) can be called fly by wire or, as Gibson referred to them, super augmented. It is “any flight control system that provides behaviour almost independent of the aerodynamic qualities through the use of” software translating pilot “demanded responses such as normal acceleration, angle of attack and flight path angle” into preprogramed control surface movements to produce the desired response. [GT, para 4.1, pg 37]

“The principal driver for the development of fly by wire technology has been the improvement of combat aircraft performance by reduced or negative angle of attack stability....[Aircraft with this instability] cannot be controlled by a human pilot, it is necessary to employ continuously operative stability and command augmentation using multiple signal paths for reliability and safety.” [GT, para 4.1, pg 37]

HOFCS also “...enable the ready implementation of functions such as maneuver limiting to protect the airframe from overstressing or from stall, regardless of weight or centre of gravity variations.....allow[ing] the pilot to concentrate on tactics and weapon operation without distraction by concern for the safety of the aircraft itself.” [GT, para 4.2, pg 38]

“Since fly by wire systems can be made to provide any desired response dynamics to a reasonably close degree, it is a simple task to design the control laws to [Gibson’s pitch handling qualities] criteria, providing good handling from the start rather than after [an expensive and lengthy] trial and error process of flight development. The basic tools for this task are a direct feed forward path from inceptor to control actuation to provide instantaneous accelerations and a command path pre-filter as appropriate to tune the response dynamics. The usual aim is to provide an aircraft open loop response that resembles closely, or as closely as is chosen, the idealized K/s integrator-like dynamics with which the pilot can easily interact with low workload.” [GT, initial summary, no page given]

2.2 Relate to objectives

The first objective of this thesis was to review the literature available concerning Gibson’s Criteria generally and specifically how it is currently taught at Test Pilot Schools. The main sources literature for the theory and application of the criteria come from Gibson’s 1999 doctoral thesis for Delft University and his earlier 1995 technical paper that was the first complete elucidation of his ideas. Furthermore, the author of this thesis used material presented in Cranfield and Empire Test Pilot School lectures for review as to the current methods and details of presenting the theory and then conducting Matlab based exercises for test pilot and flight test engineer students. Gibson’s Criteria is not currently taught at the other globally recognized test pilot school the author has access to, US Navy TPS at Patuxent River, Maryland. Additional references and sources are used throughout the thesis to round out the history of handling qualities and are noted in the classroom lectures and the references at the end of this thesis.

3 WORK CARRIED OUT & RESULTS

3.1 Methodology

3.1.1 Lectures

The lectures presented in the next chapter serve to distil and focus the material presented in Gibson's 1999 doctoral thesis as well as his earlier 1995 paper outlining the historical development of handling qualities. Additionally, other referenced works were used to round out the material used in the development of the lecture material. The end goal of the creation of this thesis was to take the high level material Gibson presents and concentrate it in such a manner to be presented to students at ETPS who most likely do not have advanced degrees in aerodynamics of aircraft designed but do have quite extensive experience and in depth knowledge in flying and testing high performance modern aircraft. Part of Gibson's work points out that pilots and engineers needed to understand each other and his criteria as both parties were necessary to achieve success in designing in excellent handling qualities. While Gibson's focus was directed to other aerodynamicists and control engineers, this author's work is targeted to the other side of the aircraft development coin: test pilots and flight test engineers.

3.1.2 Exercises

The exercises were built upon the format of existing assignments used in the Flight Dynamics Principles lecture conducted by the thesis advisor, Mr. D. Lee. These were modified by the thesis author in content, but not significantly in form, to include the application of a command path pre-filter in accordance with Gibson's Criteria. Simplified aircraft models at a given flight condition were as given in the Flight Dynamics Principles assignments or as found in Mr Cook's second edition to Flight Dynamics Principles book, currently taught at Cranfield University.

3.2 Results

The final product of these lectures and exercises proved to be a careful balance. The material that Gibson developed was a lifetime of knowledge and worth years or decades of study to understand the full nuances of his arguments. The attempt of this thesis is to distil that into a simplified, applied product. While the theory behind HOFCS handling qualities can and does inform the assessment of the flying qualities of an aircraft, there is insufficient capacity in the ETPS graduate course syllabus to spend weeks or even days on this particular topic. This author feels enormous gratitude that he had the opportunity to spend additional months after the ETPS course at Cranfield to dive into the material and years to develop and compile the correct level of depth and breadth of material on this topic, suitable to fit into the ETPS curriculum.

The objectives of this thesis were:

1. To review the literature available concerning Gibson's Criteria.
2. To develop a classroom lecture suitable for presenting the background, theory and application of Gibson's Criteria to an ETPS class.
3. To develop 3 new case studies involving the introduction and application of the Gibson criteria to ETPS class A FW TP and FTE.

A summary of the literature review performed during the undertaking of this thesis is in the previous chapter (2) and serves to satisfy the first objective.

The background and theory of Gibson's Criteria in objective two is satisfied via the two part lecture in sections 4.1 and 4.2. The application of those criteria to an aircraft model is covered in the second lecture, given in section 4.3.

The 3 case studies are presented in sections 4.4: The F-16, 4.5: the A-7A, and 4.6: The Hawk. Full solution and accompanying Matlab files are in the appendices. These fulfil the third objective of the thesis.

4 ANALYSIS & DISCUSSION

4.1 GIBSON'S CRITERIA: A PRIMER FOR TEST PILOTS AND FLIGHT TEST ENGINEERS, PART ONE

This lecture is part one of two and contains 45 slides and accompanying lecture designed to last approximately 55 minutes. It covers the first three topics:

- Control Engineering & Handling Qualities Background
- Historical Handling Qualities Criteria Review
- The Control Loop Variable

The lecture notes are shown first, followed by the accompanying slides.

4.1.1 Part One Lecture

(direct quotes from John Gibson's 1999 Thesis (source 8) are noted in the following commentary as [GT, para X.X, pg] indicating Gibson's Thesis, paragraph and page number)

Slide 1

Good afternoon. My name is LT Jack Hathaway. (*Biographical statement*)

Slide 2 (2 minutes for this slide/ :02 overall)

John Gibson was a British aerodynamicist closely associated with Delft University of Technology. He worked on the design and development of stability and control systems for aircraft from the dawn of the jet age through the advent of modern fly by wire, high order aircraft including the BAE Lightning the Tornado, European Airplane project, and Eurofighter. During the Tornado's development, several PIO incidents caused him to look more specifically at the effect of high order dynamics of hydraulically powered and computer controlled aircraft on handling qualities. He found the contemporarily accepted standards that were based on conventional second order aircraft to be wanting and developed a set of criteria to be used during the design phase to ensure the absence of PIO and the presence of excellent pitch handling qualities. 'Gibson's Criteria' were designed to be understood intuitively by pilots and control system engineers alike.

This lecture is split into two parts. This lecture is the first component and is a two hour overview, with a break in the middle, designed to expose the flight test engineer and test pilot students into the world of control engineering and handling qualities criteria, and how improving technology and aircraft performance has affected the longitudinal flying qualities of modern combat fighters ultimately driving John Gibson to develop his criteria to design in excellent short period pitch handling qualities and eliminate pilot induced oscillations. The second half is a shorter lecture covering the basic application of the Gibson's criteria to some simplified aircraft using MATLAB software.

Slide 3 (2 min/ :04 overall)

A High Order computer controlled Flight Control system can be called fly by wire or, as Gibson referred to, Superaugmented. It is "any flight control system that provides behavior almost independent of the aerodynamic qualities through the use of" software translating pilot "demanded responses such as normal acceleration, angle of attack and flight path angle" into preprogrammed control surface movements to produce the desired response. [GT, para 4.1, pg 37]

"The principal driver for the development of fly by wire technology has been the improvement of combat aircraft performance by reduced or negative angle of attack stability...[Aircraft with this instability] cannot be controlled by a human pilot, it is necessary to employ continuously operative stability and command augmentation using multiple signal paths for reliability and safety." [GT, para 4.1, pg 37]

HOFCS also “...enable the ready implementation of functions such as maneuver limiting to protect the airframe from overstressing or from stall, regardless of weight or centre of gravity variations.....allow[ing] the pilot to concentrate on tactics and weapon operation without distraction by concern for the safety of the aircraft itself.” [GT, para 4.2, pg 38]

“Since fly by wire systems can be made to provide any desired response dynamics to a reasonably close degree, it is a simple task to design the control laws to [Gibson’s pitch handling qualities] criteria, providing good handling from the start rather than after [an expensive and lengthy] trial and error process of flight development. The basic tools for this task are a direct feed forward path from inceptor to control actuation to provide instantaneous accelerations and a command path pre-filter as appropriate to tune the response dynamics. The usual aim is to provide an aircraft open loop response that resembles closely, or as closely as is chosen, the idealized K/s integrator-like dynamics with which the pilot can easily interact with low workload.” [GT, initial summary, no page given]

Slide 4 (1.5 min/ :05.5 overall)

“Good handling is not just a means for giving enjoyment to the pilot. It performs an essential part in task performance. The combat pilot must devote all possible attention to the management and operation of a variety of complex weapons and defensive systems against an increasingly capable opposition. The ability actually to fly the aircraft with minimum attention to safety and structural limits releases considerable mental capacity to the combat task.”

“ Governments, the owners of combat aircraft, enhance the efficiency of their forces when the less skilled and experienced pilots can fly their aircraft as aggressively as their more skilled colleagues.”

“Up to the 1940s, aircraft could be tuned to give desirable handling, though sometimes only by expending considerable effort. [Combat aircraft could require] hundreds of hours of flight testing spent developing the control system balance, tab and spring gearing and control circuit[s]. In the post-1940s era, the handling qualities were dominated by the increasingly poor aerodynamic and inertial properties of aircraft. These could be ameliorated at best by stability augmentation that was sometimes mechanically quite complex, but they could seldom be made perfect even assuming that such a state could be adequately defined. In the rapidly growing sphere of fly by wire technology, in both military and civil applications, the means for good handling resides largely (but certainly not only) in software code. It costs little more in software overheads to write "good" code rather than "bad" or merely careless code, assuming again that the desirable end result is known. The overall effort required to write the software is very large and should not be wasted to any degree on inferior handling code.” [GT, para 10.1, pg 200]

Slide 5 (1 min/ :06.5 overall)

The discussion of handling qualities and how various criteria can be used to evaluate aircraft fully are both topics that can only be fully explored to their full depth through many years of academic study, industrial application and perhaps regulatory experience.

The scope of this lecture is limited to a few hours of discussion and is designed to be part of a course to qualify flight test professionals. By nature then, we cannot cover every nuance and detail of the topics. Specifically, we are limited to fixed wing combat fighters and attack aircraft as larger bombers, tactical and strategic transports require different assumptions and considerations to address their flying qualities as well as how pilots apply different techniques and concepts to operation and mission execution.

Additionally, as we will discuss a bit further, this lecture and follow on problems are limited to the pitch axis in general and the short period pitch response specifically. Since most of the flying qualities encountered in modern fighters are driven by the SPPR, this is the dominant area of concern for both control engineers and pilots. Other simplifications, such as setting airspeed to a fixed value or negating the effect of elevator lift effects pop up. In general, however, the top level nature of this lecture should provide some understanding to the nature, problems and solutions of the effect of high order flight controls on a wide variety of aircraft and this will be of good value to any flight test professional, regardless of your background.

Slide 6 (0.5 min / :07 overall)

Our topics of presentation today include a background on control engineering and handling qualities, followed by a historical review of handling qualities criteria. We'll then discuss options for the control loop variable, how pilot variability can make the control engineers' problem more difficult, and the effect of control system hardware. Finally, we will go over Gibson's pitch handling qualities criteria, starting with the Drop Back criteria to design in excellent short period pitch handling qualities and then his criteria to eliminate low order PIO, Phase Rate, PIO frequency and gain limits. We'll finish the lecture with a few slides on non-linear PIO.

Slide 7 (2 min / :09 overall)

In 1903, the Wright brothers conducted the first powered, heavier than air, controlled flight. While the Wright Flyer was actually statically unstable in the longitudinal axis, the brothers' "greatest and fundamental achievement was their invention of three-axis aerodynamic control"[1] achieved through wing warping and control tabs. This was all the more remarkable due to the minimal understanding of the equations of motion at work in their 6 degree of freedom problem called an airplane.

From that starting point, there have been vast improvements in the performance of aircraft. There are two notable technological developments that are pertinent to this discussion: First, the advent of the jet engine, requiring the inclusion of hydraulic powered flight controls to overcome the aerodynamic forces as well as requiring artificial feel devices for the pilot; and Second, the arrival of computers, allowing for less aerodynamically stable aircraft that require the speed and processing power of flight control computers to be flyable and presenting the possibility of carefree handling of flight control and engine inputs.

“The achievement of good handling qualities has always depended on a variety of mechanical design aspects as well as the basic aerodynamics and inertial properties of aircraft. At first the search was for appropriate pilot’s inceptors, settling quite early to the universally adopted stick and pedals in various guises. Tuning of the handling by means of aerodynamic tabs and simple spring devices followed. Jet propulsion brought radical changes to aircraft design necessitating the use of hydraulic power actuation, artificial feel devices and stability augmentation. The roles of the mechanical controls engineer assumed a equal importance to that of the aerodynamicist.

“It was possible to manipulate stick characteristics to a very large extent by mechanical means under difficult and diverse flight conditions of speed mach, center of gravity, under severe design constraints of long control circuits, large accelerations, structural deformations, temperature changes and so on. The advent of fly by wire systems has replaced most of the mechanical complexity associated with good handling with computer control law complexity, requiring new skills once again.” [GT, para 6.4, pg 86]

[1] Culick, 45th SETP, Sep '01. wrightflyer.org

Slide 8 (2.5 min / :11.5 overall)

In concert with the vast improvement in performance and mission effectiveness, the complexity of modern aircraft design increased dramatically as well. This complexity is now hidden inside millions of lines of computer code instead of a physically intuitive system of cables, pulleys, cranks and aerodynamic tabs.

While designers and engineers can create an aircraft with sufficient stability and control, or in the modern fly-by-wire sense, designed with neutral or slight instability but sufficient computer power to create apparent stability; we must not lose the critical understanding that the underlying purpose of all these technological improvements is to make the aircraft better at performing its task.

“Handling Qualities relate directly to the ease with which a task can be performed. They describe the manner in which the aircraft responds both to the demands of the pilot and the influence of turbulence, gusts, weapon release, airbrake, flap and undercarriage operation, reheat selection, engine failure and other disturbances.” [GT, para 1.1, pg 1]

When evaluating how easily the task can be performed, the aircraft cannot be divorced from the pilot. There is no such thing as an aircraft that has perfect handling qualities before a pilot steps into it. The combination and interaction of the pilot and aircraft must be tested together. While modern technology allows excellent modeling of how the aircraft will respond to various disturbances and stimulus, there is no perfect model of a pilot. “Pilots are not machines, and they tend to display some variability of their own and relative to other pilots’ opinions. All that can be said is that there is a high statistical probability that some qualities will be found satisfactory to most pilots. At the other end of the scale, a [handling] quality which could lead to loss of control might not reveal itself until after thousands of flights over many years. Both extremes need to be understood.” [GT, para 1.3, pg 3]

“A workload that is too high will lead to a collapse in performance at any skill level...If the FCS provides excellent handling qualities and protection from overstressing or loss of control, the pilot is relieved from the task of maintaining continual observation of the appropriate instruments and limits, can therefore absorb more effectively the details of the task displays, and can thus devote the maximum attention to the task in hand for greater efficiency. This is the absolute justification for optimization of handling qualities.” [GT, para 5.2.2, pg 59]

Figure 1-2 from Gibson’s 1995 essay

Slide 9 (0.5 min / :12 overall)

“To describe the motions of an aircraft completely requires the solution of six simultaneous non-linear force and moment equations of such complexity that it is only feasible by computer. The reduction of this problem to manageable proportions for the study of generic handling qualities, divorced from direct representation of a particular aircraft and capable of simple physical interpretation, requires very considerable simplification.”[GT, para 3.2, page 21]

Slide 10 (2 min / :14 overall)

A 1911 [2] study showed that these six equations are separable into two independent groups identified by the axis in which they operate. Motion in the longitudinal axis can be described by a pair of second order equations, one governing short term oscillations and one governing long terms ones, commonly referred to as the SPPO or SPPR, short period pitch response and the Phugoid. Motions in the lateral axis and directional axis were inherently coupled and could only be described together, the single order roll mode, single order spiral mode and the second order Dutch Roll mode. The roll mode can be defined by its time constant and maximum roll rate and is a description of the aircraft’s ability to roll about its longitudinal axis. The spiral mode is a balance between the lateral and directional stability in addition to the yaw damping and roll damping. It assesses if the aircraft will return to wings level or diverge into a spiral if left open loop with an angle of bank. The dutch roll is a nuisance mode excited by the presence of sideslip and is an exchange of sideslip and angle of bank.

With the advent of computer controlled flight controls, the modern control engineer can eliminate many undesirable motions that were inherent to the nature of second order aircraft throughout the last century. The introduction of feedback loops closed by a computer instead of a human pilot can eliminate the speed and pitch angle oscillations of the phugoid, the sideslip that sets off the Dutch Roll, and spiral stability can be created however the designer likes. The roll mode continues to be driven primarily by actual aerodynamic capabilities of the aircraft, but the designer can account for changes in airspeed, angle of attack to ensure that lateral stick commands the same roll rate regardless of the flight condition, so the pilot does not have to concern himself with changes in performance in different parts of the envelope.

[2] Bryan, G. H., 1911, *Stability in Aviation*, MacMillan, London, referenced in GT, para 2.2.1, pg 9

Slide 11 (1 min / :15 overall)

This leaves us with the SPPO. Traditionally referred to as oscillations, some texts now refer to this mode as a short period pitch response, as modern flight control computers can largely eliminate the oscillation, and discussion is more focused on how to shape the pitch response of the system. The rest of this discussion will focus primarily on this mode.

The SPPR in a basic sense, can be under damped, critically damped or over damped. This feature has a decisive impact on the handling qualities the pilot feels. An under-damped system would feel very sensitive and respond quickly, but would be difficult to control precisely. An over-damped system would be sluggish, but once it reached its target, would be quite precise. Conventional second order systems often tried to balance the desire for quick response with the need for precise handling by making the aircraft aerodynamically critically damped.

Modern flight control computers give the promise of the best of both worlds, and in this case, they can provide the quickness and maneuverability of an under-damped system, while simultaneously providing the precision and deadbeat nature of an over-damped one.

Slide 12(4 min / :19 overall)

This figure is a depiction of an idealized stick input and aircraft response. The top graph is a step input of aft longitudinal stick to some arbitrary, but constant displacement at time zero that is taken out at some later time.

The middle graph shows pitch rate of the aircraft that instantly reaches a steady state value with no overshoots or oscillations as the stick input is made at time zero, then subsequently goes instantly to zero when the stick input is removed.

The bottom graph is aircraft pitch attitude, represented as θ , which goes up linearly, until the stick input is removed, and then remains at the final value with no overshoots or oscillations.

There are many ways in which this is an idealized scenario. First, there is no pilot that can instantly move a control stick from zero displacement to a new commanded position in zero time. While some modern aircraft have the ability to digitally control this via flight test aid computer software, we are designing for excellent handling qualities with the pilot in the loop performing real tasks, not a flight test aid computer.

Second, the pitch rate response is exactly linear, with no lag, time delay, or second order like response at all, both at the input and the removal of the input. While we can get close to this type of response, it is not fully achievable.

This perfect pitch rate leads to the perfectly linear pitch attitude over time. Displacement is the integral of rate. In this case, pitch attitude is the integral of pitch rate. The perfectly rectangular rate integrates to a perfectly linear attitude.

This relationship is the idealized pilot-aircraft interaction. No delays, lags or second order (or higher) response artifacts: Predictable and repeatable output for a given input. This integrator response type is referred to in state space mathematics as K/s , where $1/s$ is an integrator function on the frequency plane and K is a gain value governing the input to output.

Additionally, this graph highlights an important consideration: How direct is the link between the pilot and the pitch attitude that he is using to close the loop

around and control the aircraft? It appears from this idealized presentation that the link is direct, a control stick is moved, rate is instantaneous and steady state, and the attitude is a perfectly integrated function of the rate.

Clearly, in the real world this is not the case. In conventional, second order aircraft, the pilot's stick, yoke or wheel would command tension on a wire which would then pull on the distant flight control surface it was attached to. This surface would deflect in the commanded direction, resulting in a change of aerodynamic force, which would then act upon the center of gravity causing an acceleration, resulting in a rate and finally a noticeable aircraft response that the pilot would then have to assess for its magnitude and direction before applying any corrections. This process is more automated in modern fly by wire aircraft, as the pilot input is sensed electronically via displacement or force, and the computer commands often hydraulically powered surfaces to move in such a manner to generate the motion commanded by the pilot.

In both examples, though, some sort of transfer function is occurring. In the second order aircraft, the primary transfer function for the longitudinal axis, specifically the short period pitch response, that would be evaluated would be the control input over the elevator control surface movement. In computer controlled high order aircraft, the transfer function will be more specific, as the pilot would be demanding pitch rate, normal acceleration or 'g', or angle of attack, and the flight control computers would assume the role of determining how to move the control surfaces to achieve that. These transfer functions are critical to control engineers, as the modification of them through software is how excellent handling qualities can be designed in. We will cover that process in more depth later, but an understanding that modification of the transfer function via software in a flight control computer is the method by which a control system engineer can take what a pilot is commanding and program the aircraft to respond in the desired manner.

Figure 5-2 from GT, 1999

Slide 13 (1 min / :20 overall)

A simplification that is important to make is the removal of airspeed changes from the longitudinal model of an aircraft. By assuming that speed will remain the same, we can reduce a three degree of freedom problem to a two degree problem, making the math much simpler. Clearly, when a longitudinal control input is made, the speed of the aircraft will be affected, but the magnitude of the change will be small in the short term. Since we aren't really interested in long term dynamics, like the phugoid, only the short term pitch response characteristics, this assumption is reasonable.

We can further convince ourselves of the validity of this assumption by looking at the gain component of a Bode diagram, such as the figure shown. It depicts a comparison of a two and three degree of freedom longitudinal only generic aircraft model, the extra degree of freedom being horizontal speed. On the vertical axis is the gain applied to pitch attitude commands for a given control stick input. The horizontal axis is control stick inputs in hertz.

Figure 3-2, Gibson's 1999 Thesis, pg 34

Slide 14 (1 min / :21 overall)

Since we will encounter a number of Bode Diagrams, here is a quick review of them: The X axis is pilot input frequency, typically in radians, but occasionally in hertz or another unit. The Y axis shows two variables, the top graph is gain in decibels, which is of course, a logarithmic scale. The bottom graph shows phase angle and is typically in degrees. Negative values are lags and positive values are leads.

On the left side of this particular bode diagram at very slow input rates, on the order of historical phugoid modes, there is a large difference between the models highlighted in red. Since the phugoid is a long term mode exchange of speed and altitude, we expect speed changes to be critical and thus a significant difference.

Figure 3-2, Gibson's 1999 Thesis, pg 34

Slide 15 (1 min / :21 overall)

However, if we consider input frequencies "about midway between the phugoid and short period modes," approximately 0.1 hz and 10 hz, the two and three degree of freedom transfer functions graphed are not all that dissimilar. This should convince us that we can proceed with the assumption that speed can be held constant in any analysis of short term pitch dynamics. [GT, para 3.2.2, page 24]

Figure 3-2, Gibson's 1999 Thesis, pg 34

Slide 16 (for 16 through 21, 1 min / :22 overall)

Typical pilot inputs are between approximately 0.1 rad and 10 rad. This is equivalent to roughly 0.5 to 2 hertz. This can be difficult to visualize since most pilot inputs don't make a full revolution and input frequency is based off how long it would have taken the input to make a full revolution. [CLICK PLAY] In this example, it takes the red circle 2 seconds to complete a full revolution.

Slide 17(for 16 through 21, 1 min / :22 overall)

A full revolution is 360 degrees, but we will talk in terms of hertz, cycles per second, or radians per second since those are the dominant units when assessing pilot inputs in the frequency domain, such as on the Bode plot we just looked at or the Nichols plot we will see later. [CLICK PLAY] One revolution is one cycle. Since it takes 2 seconds for the red circle to complete a cycle, that's 0.5 hertz. Converting to radians, it's about 3 radians per second.

Slide 18(for 16 through 21, 1 min / :22 overall)

Converting the basic idea of a full revolution or cycle into a linear stick input is shown here. [CLICK PLAY] Regardless of whether the motion is circular or linear, a full cycle is complete and the frequency of input is half a hertz or about 3 radians per second.

Slide 19(for 16 through 21, 1 min / :22 overall)

The assessment of frequency is independent of the magnitude of the input, as long as the input is the same magnitude both fore and aft of neutral, or symmetric. [CLICK PLAY] Even though the magnitude of the input on the right

is about half as large as the input on the left, the frequency of both is still the same half a hertz or 3 radians per second.

Slide 20(for 16 through 21, 1 min / :22 overall)

Most pilot inputs aren't fully symmetric and therefore you couldn't time a full cycle of input to get the frequency. It is possible to just look at part of the cycle to determine the frequency however. [CLICK PLAY] In this example, the left input is a full cycle and the right only the forward then neutral. Quick visual inspection though tells us that they are moving at the same frequency. Although I am showing an input going forward and then back to neutral, the same logic applies to an input that only goes forward, such as a step input that is used ubiquitously in control engineering and flight test.

Slide 21(for 16 through 21, 1 min / :22 overall)

When looking at Bode diagrams or other frequency domain plots, it can be useful to understand the physical limits of pilot inputs. On the left is an input of 2 hertz, or about 12 radians per second. On the right is the same input we've been looking at of half a hertz or about 3 radians per second. [CLICK PLAY MULTIPLE TIMES] I will click play a few times here. Imagine attempting to put an input in faster than the one on the left, or slower than the one on the right. While it's perhaps possible to go faster or slower, it's pretty difficult to move your hand or arm that fast. Rounding off the numbers, you can usually pay attention to between 0.1 hertz or 0.1 radians on the slow side, and 2 hertz or 10 radians on the fast side. That will typically include the long and short period longitudinal motions and vast majority of pilot inputs on any frequency plot you encounter.

Slide 22 (1 min / :23 overall)

In 1976, during the development of the Panavia Tornado, a project on which John Gibson worked, a landing accident occurred due to a PIO event. Gibson "identified a specific area in the dynamic attitude behavior as the root cause...There were no suitable criteria or requirements available at that time to explain the problem...[The Tornado's flight control laws] contain additional feed forward and feedback filters and gains that rendered meaningless the conventional pitch handling requires in the official specifications of the time." [GT, 7.1, page 93].

In the 1980's, the handling criteria for the FCS augmented BAE Jaguar "was that its handling criteria should be as acceptable as that of the standard Jaguar." [GT 7.1, page 93, implying Daley, 1984] Initial designs were not as successful as desired, and drove an FCS redesign based on simulator evaluations rather than a formalized method using industry or regulator accepted handling quality criteria. "The formal specifications for the short period ...were based primarily on its frequency and damping. These are associated with the flight path response through control of the angle of attack, and lack detailed definition of the attitude response which dominations the pilot's actual perception of aircraft control." [GT 7.1, page 94]

Picture used with permission: Sepecat Jaguar T2; Royal Air Force / ETPS; RAF Fairford (FFD / EGVA), UK - England; 21st July 1989; Photographer: Shaun Connor; <http://www.abpic.co.uk/contact.php?q=1372366>

Slide 23 (1 min / :24 overall)

This figure shows an early pitch handling quality criteria called a thumbprint. It is a plot of natural frequency vs the damping ratio characteristics of the SPPO. In general, these are formed by plotting a point at the intersection of the natural frequency and damping ratio of a tested set of flight controls. This plotted point is notated with the pilot comments and a rating of some sort. With a wide enough set of tested parameters for a given flight condition and task, lines can be drawn to encapsulate all of the similar pilot ratings. In this example, areas of unacceptable, poor, acceptable and satisfactory handling qualities are defined. A new developmental aircraft program might then take this graph and attempt to design in satisfactory handling qualities aerodynamically or through a FCS by making sure the natural frequency and damping ratio would plot in the satisfactory region. However, this thumbprint style criteria is based on a second order considerations. A higher order aircraft does not have explicit conventional short period parameters of natural frequency & damping ratio. While it is possible to calculate, with many assumptions and simplifications, 'apparent' short period parameters to compare to this historical criteria, there are unknown and possibly dangerous regions of flight condition where the higher order of the FCS combined with pilot gain and inputs attempting a task will likely drive poor handling qualities or a PIO, such as the incident Gibson encountered in the Tornado landing accident.

Figure from Cranfield lecture, FCS 14

Slide 24 (2 min / :26 overall)

As the complexity of higher order flight controls and their effect on handling qualities became more apparent in the mid to late 20th century, efforts were undertaken to revise and update the regulatory handling qualities criteria. A study[4] in the mid sixties on precision flight path response was performed that "proposed that the pilot's ability to predict a flight path response was related to the ratio of the initial pitching acceleration to the steady state normal g following a step control input. This ratio, the Control Anticipation Parameter or CAP, connects the short period natural frequency, flight condition, wing loading, lift slope and maneuver margin." [GT, para 7.3.1, pg 91] With the advent of flight control systems, aircraft could now be pitch rate demand, flight path demand, not just the AOA demand that second order aircraft are.

CAP was an effort to take this into account and defined ranges of CAP and damping ratio that would produce level 1, 2, 3 handling qualities for various tasks. While it became a component of the specification for handling qualities, as this graph shows, various tests of in flight and simulator based flight control systems produced areas of satisfactory handling qualities outside the recommended areas, and some poor results within the specified level 1 areas.

[4] Bihrlé, W., 1966, "A Handling Qualities Theory for Precise Flight Path Control", AFFDL-TR-65-198, referenced throughout GT

Slide 25 (1 min / :27 overall)

By the late 1960's and 70's, efforts were being made to address the dearth of effective handling qualities criteria that could address the significant control issues being seen in the modern fighters being developed, including the Saab Gripen, LM F16 and Boeing F18.

One of the routes of investigation was identified as a Low Order Equivalent System by Hodkinson in 1976. The concept behind this was to match “complex high order frequency response dynamics with classical low order dynamics and a time delay to derive equivalent short period frequency and damping mode parameters.” ...“normal acceleration at the instantaneous centre of rotation of the actual aircraft are matched as closely as possible with the simple two degree of freedom approximations with an added time delay, in the frequency range of 1 to 10 radians per second. The resulting equivalent short period frequency and damping are then compared with the standard requirements to assess the general handling, though these parameters strictly define only the flight path qualities.” [GT, para 7.2.1, pg 97]

Figure from Cranfield Flying Qualities & Flight Controls Course Lecture, FCS 16.

Slide 26 (3 min / :30 overall)

These figures show the output of a low order equivalent matching algorithm in a time response on the left and a Bode plot on the right. The original, high order system is shown, as well as two attempts to match with a low order equivalent, one where the aircraft parameter $t_{\theta 2}$ is fixed and one where it is free to vary. While a varying $t_{\theta 2}$ generally produced a better fit, there is also a loss of connection to the physical reality of the aircraft as $t_{\theta 2}$ doesn't vary in flight in conventional aircraft.

Low Order Equivalent Systems provided a simple way to stick with known criterion systems. Existing criteria were such that many different combinations of damping and natural frequency would produce handling qualities that most pilots could handle. Traditional second order aircraft had to make design compromises to take into account that the human pilot had physical and mental limitations in terms of what he could control. Flight control computers solved that problem by being able to provide apparent stability to the pilot, who could then focus on the mission tasks that a computer couldn't do. As aircraft became easier to fly, the minimum accepted standard of handling qualities rose. The Wright brothers accepted a standard of handling qualities that included neutral longitudinal stability. Constant oscillation was acceptable, as long as it didn't diverge with a pilot in the loop. By the second world war, oscillations were unacceptable as the pilot was required to track targets. With the complexity of modern aircraft, controlling the aircraft has to be almost an afterthought as pilots are required to operate complex and demanding weapons and sensors to accomplish the mission.

With the expected standard of handling qualities rising, many control engineers including Gibson “found LOES unattractive. It is not itself a handling qualities criterion, but merely provides a model that can then be assessed against the older specifications for conventional aircraft. There is a wide variety of possible conventional response shapes that satisfy these older specifications but do not actually guarantee good or optimum handling.” [GT, para 7.2.1, pg 97] Especially since the best fits require $t_{\theta 2}$ to vary, these simpler descriptions are more divorced from the physical reality of the aircraft thus increasing the risk of not identifying unwanted and possibly dangerous dynamics hidden in the higher order response.

With precise control of the handling qualities available via software in a flight control computer rather than conventional modification of aerodynamic control

surfaces, it should not only be possible to design in excellent handling qualities, but prevent unwanted dynamics such as PIO from interfering.

Figure from Cranfield Flying Qualities & Flight Controls Course Lecture, FCS 16.

Slide 27 (1 min / :31 overall)

Understanding the complex nature of high order aircraft requires looking deeper than the conventional aerodynamic properties of damping and natural frequency. In the early 1970's, Neal and Smith published a new criteria for closed loop pitch tracking tasks in fighter aircraft. Historically, handling qualities investigation was limited to the time domain. Researchers applied an input to an aircraft, either in flight test or through the limited simulation capabilities available at the time, and looked at a graph of properties over a period of time. These properties might be pitch attitude, pitch rate, normal acceleration or some other value pertinent to the investigation of longitudinal handling qualities. This was effectively showing the aircraft's aerodynamic properties open loop, that is without the pilot involved. Neal and Smith recognized that the most critical tasks were closed loop with the pilot very involved in achieving success. In order to account for the role of the pilot in closed loop tasks, the investigation of handling qualities also had to include the phase and gain the pilot brought to the table. Neal and Smith used frequency domain tools to quantify the bandwidth, droop and closed loop resonance required for good handling qualities with the pilot in the loop.

Slide 28 (1 min / :32 overall)

For their criteria, Neal and Smith identified 3 characteristics in the frequency domain, minimum bandwidth frequency, droop and closed loop resonance. They defined these characteristics with a real world, physical description as well as mathematically.

The figure shown is a Bode Plot with gain in dB and phase in degrees across a spectrum of frequencies from 0.1 rad/sec to 100 rad/sec. The plotted red line is a generic representation of the longitudinal response characteristics of an aircraft over the given frequency spectrum, with pictorial descriptions of Neal & Smith's 3 characteristics.

Bandwidth is "a measure of how quickly the pilot can manoeuvre the nose of the aircraft on to the target pitch attitude." Measured as the frequency at which 90 deg phase lag occurred, it could not be less than 3.5 rad/sec to predict good handling qualities.

Low frequency droop is "a measure of how slowly the nose of the aircraft settles on to the target pitch attitude." It is measured from the 0 dB line to the lowest point of the gain plot before the rise to the peak, and shouldn't be lower than -3 dB.

The closed loop resonance is "a measure of the pitch damping." It is measured from the 0 dB line to the highest point of the gain plot peak, and should be the minimum achievable.

Like all Bode plots, this figure shows open loop aircraft response. Neil & Smith assessed these limits as the maximum acceptable limit such that with the addition of a pilot in the loop, the combined gain and phase values of the aircraft pilot closed loop system would not exceed desirable handling qualities limits.

Slide 29 (1 min / :33 overall)

This is a plot of closed loop resonance against the pilot phase compensation required at 3.5 rad/sec, Neal & Smith's critical bandwidth value, overlaid with the familiar level 1/2/3 flying qualities determined by Cooper-Harper pilot ratings. Since the Bode plot we just looked at showed the open loop aircraft phase response and the minimum bandwidth was -90, or 90 degrees of lag, the pilot phase required to compensate for that would be +90 or 90 degrees of lead. This plot ends on the right just less than 90, as that is the Neal & Smith bandwidth limit.

As their criteria indicate, the closer to zero phase compensation required by the pilot, the better the pilot ratings are. As closed loop resonance increases, the pilot rating decreases, hence the N&S limit to minimize this resonance value.

Slide 30 (1 min / :34 overall) (slides 30-33: 4 min / :37 overall)

The Bandwidth criteria was developed in the early 1980's, based on the work of Neal & Smith. Neil and Smith looked at a fixed bandwidth of 3.5 rad/sec. Mitchell and Hodgkinson recognized that this limited the generality of its application and looked to create a criteria that was more suitable to a wide variety of applications. This criteria relies heavily on the low order equivalent system model to draw the gain and phase response shapes as well as determine the phase delay, which is a measure of the accumulated phase lags due to the high order nature of the system.

"Augmented aircraft with high bandwidth generally exhibit good pilot-in-the-loop tracking characteristics. Stability margins are large enough to accommodate variations in pilot gain-phase in all workload situations. High pilot workload situations in low bandwidth aircraft can lead to erosion of phase margin in particular to the point of instability. A PIO is usually the result."

Figure & quote from Cranfield Flying Qualities & Flight Controls Course Lecture, MODULE 3/17 "THE BANDWIDTH CRITERION".

Slide 31 (1 min / :35 overall)

These figures show a generic aircraft reduced order longitudinal model for typical gain and phase responses on a Bode Plot where we can break out the Bandwidth criteria for phase & gain bandwidth as well as the high order phase delay lag.

The phase bandwidth is evaluated where the Bode plot phase response crosses the 135 degree line. Bode plots are open loop response shapes, so this leaves 45 degrees of phase margin before crossing the 180 degree lag where PIO likelihood greatly increases. In this case, drawing a line down from where the response shape crosses 135 degrees phase lag yields a phase bandwidth frequency of about 3.8 rad/sec.

The gain bandwidth criteria is evaluated in multiple steps. First, draw a line up from where the phase response crosses 180 degrees, notated on this figure by the letter A.

Follow that vertical line to where it crosses the gain response shape and note that gain level, shown in letter B on this figure. Then follow the gain response

shape to where the gain value equals the gain level at 180 deg phase angle plus 6 dB, and read off the frequency, in this case, about 1.4 rad/sec.

The overall bandwidth of the aircraft is determined by whichever is lower, the phase bandwidth frequency or the gain bandwidth frequency. In this case, the gain bandwidth frequency is lower, so the aircraft bandwidth is considered 1.4 rad/sec

Figure from Cranfield Flying Qualities & Flight Controls Course Lecture, MODULE 3/17 "THE BANDWIDTH CRITERION".

Slide 32 (1 min / :36 overall)

This figure shows a comparison of the aircraft bandwidth and a parameter called an equivalent phase delay, τ_{ph} . Plotting those two values will show what the bandwidth criterion predict the handling qualities will be.

Phase delay is usually calculated by the low order equivalent matching algorithm that we discussed earlier. If that data is not available, an equivalent phase delay can be calculated by comparing the slope of the response on a bode phase plot between the 180 degree point and twice that frequency. Regardless of the actual calculation, though, this value is effectively a way of saying how much delay or lag is caused by the higher order nature of a super augmented system, whether that comes from time delays due to digital processing requirements, actuator lag or some other source.

In general, the bandwidth criterion is saying that the larger the effective delay or lag, the worse the flying qualities will be. Additionally, the lower the bandwidth number, the more likely the pilot will encounter PIO. As we discussed earlier, most pilots can drive their input frequency up to about 10 rad/sec. And since they physically can, at some point they most likely will, which is why bandwidth frequency's in the low single digits result in level 2 or worse flying qualities.

Figure from Cranfield Flying Qualities & Flight Controls Course Lecture, MODULE 3/17 "THE BANDWIDTH CRITERION".

Slide 33 (1 min / :37 overall)

The bandwidth criterion is designed to minimize the probability of PIO due to insufficient bandwidth margins in the aircraft. However, there was a belief that attitude characteristics hadn't been sufficiently taken into account by looking at bandwidth and delay alone. An additional component was added to take into account pitch attitude response characteristics in the context of PIO. In this case, a bode gain plot of the pitch rate transfer function is shown at the top, of the poorly damped A7A Corsair II, at 35,000 ft and 0.6 M. You can visually see the ratio of the resonant peak at the short period frequency and determine the 'overshoot; from the transfer function value between the longitudinal mode resonant peaks. Here, the overshoot is about 18 Db minus 3, or about 15 dB of gain overshoot in the pitch rate transfer function.

On the bottom left, I've placed the same figure from the previous slide for comparison. The bottom right graph has the same information on it as the bottom left, pitch attitude bandwidth vs the phase delay. The level 1,2 & 3 flying qualities lines have been replaced by boxes representing where the likelihood of PIO is increased. The level 1 flying qualities on the bottom left figure correspond roughly with the No PIO category on the bottom right figure.

Above a 0.2 sec phase delay, both figures point to poor flying qualities. In the level 2 area, though, the bottom right figure puts in some caveats based upon the pitch rate overshoot just discussed, as well as a flight path bandwidth value that we haven't looked at specifically, but would be calculated in the same manner as the pitch attitude bandwidth.

Figure from Cranfield Flying Qualities & Flight Controls Course Lecture, MODULE 3/17 "THE BANDWIDTH CRITERION".

Slide 34 (1 min / :38 overall)

Handling Criteria development has two important aspects. The criteria can be used to look backwards: I have an airplane, now I can evaluate it against the criteria, or it can be to design in good handling qualities from the beginning. C* was developed as a method to improve handling qualities from the beginning. Historically, handling qualities criteria could be used on any sort of flight control system, whether wires and pulleys, a stability augmentation system or a full fly by wire design. C* is different in that it is based upon an idea that Tobie, Elliot and Malcom "postulated that pilots responded not only to acceleration cues but also to pitch rate and pitch acceleration cues." It was the first FCS design and handling qualities criteria that used a control approach that explicitly blended closed loop control around several variables. This created a "synthetic variable" that was dominated by pitch rate and pitch acceleration below a critical crossover speed, but dominated by normal acceleration above that speed.

Quote from Cranfield Flying Qualities & Flight Controls Course lecture, MODULE 3/9 "THE C* CRITERION".

Slide 35 (1 min / :39 overall)

This idea of a blended FCS system is a concept that continues to prove its worth. Most modern fighter attack aircraft have some sort of blended system that takes into account multiple variables. This has been refined over the decades as more understanding of what the pilot is actually using to control the aircraft in various sections of the flight envelope.. The ideal scenario is one where the variable around which most pilots close the loop in a particular area of the envelope is the dominant variable in the control system at that point.

Sectioning off the areas of the flight envelope in which the FCS transitions from one dominant variable to the next can be difficult and identifying the transition area is still part experimentation through simulation and flight test.

In developing C star, it was concluded that below a certain speed, termed a crossover velocity, pitch rate was dominant in closed loop pilot control technique. Above that speed, normal acceleration was the dominant control variable. This speed was determined by the authors to 237kts.

Slide a modified version of slide 3 from Cranfield Flying Qualities & Flight Controls Course lecture, MODULE 3/9 "THE C* CRITERION".

Slide 36 (0.5 min / :39.5 overall)

"Acceptable time response shapes for the synthetic C* variable were identified from flight test experiment. The criterion defines step response boundaries for

normalised C^* response. The boundaries correlate approximately with Flight Phase Categories”

Quote and figures from Cranfield Flying Qualities & Flight Controls Course lecture, MODULE 3/9 “THE C^* CRITERION”.

Slide 37 (0.5 min / :40 overall)

Having discussed historical handling qualities criteria, we’ve seen a trend towards the selection of a control loop variable other than conventional second order angle of attack. The ability to change this variable is a key benefit of high order systems, so we will spend some time looking at the options.

I showed this figure at the beginning of the presentation to discuss how handling qualities were a function of the aircraft, its environment and how the pilot perceived and reacted to all of the stimulus.

Figure 1-2 from Gibson’s 1995 essay

Slide 38 (0 min / :40 overall)

We’ll look at just a couple components of this dynamic now, specifically, the center row.

Figure 1-2 from Gibson’s 1995 essay, modified by thesis author

Slide 39 (3 min / :43 overall)

In this simplification, we have a task that a pilot is performing. He does this by displacing an inceptor in the cockpit. In a traditional second order aircraft, that displacement is transmitted directly to the flight control surfaces causing them to deflect and a change in the resultant aerodynamic forces and yielding an overall aircraft response, usually described as a change in attitude, rate or acceleration in the pitch roll or yaw axis.

Historically, pilots can close the loop around angle of attack for a large number of tasks quite well, using AoA as the control loop variable produced very controllable aircraft with impressive performance all the way through the Second World War.

However, there are a number of deficiencies that are present as well. While pilots continually prove that they are capable of using AoA as the control loop variable, the workload can be very high. The AoA required for steady flight changes with speed, such that every speed has a unique stick & trim position for steady flight. As an aircraft changes speed during a mission, this requires the pilot to either hold a constantly changing force on the stick to maintain a desired attitude during accelerations such as a dive bomb or gun attack, or constantly trim the forces away. During the high workload of a dive attack, this dramatically reduces the capacity of the pilot to focus on target tracking.

Additionally, while pilots could, in general, maintain control of AoA demand aircraft, there is a physiological limit to the number and size of deviations a pilot can perceive and how quickly the brain can understand and react to it. While the Wright brothers could fly an aircraft that was statically unstable but dynamically neutral, this would be a difficult airframe to perform any other mission than simply maintaining control. To give the pilot the necessary capacity to perform other tasks, from simple navigation to complex target identification and tracking, aircraft designers had to maintain a minimum amount

of stability in conventional aircraft. Since stability and manoeuvrability are traded off in conventional aircraft development, requiring a minimum amount of stability put a ceiling on the manoeuvrability.

Finally, a traditional aircraft control system provided no protection against overstress and departure from controlled flight. Limits were imposed upon the pilot and were flight condition dependent and could be quite complex. This forced the pilot to be well versed in the structural and departure related limitations of the aircraft and constantly monitor the instruments as the limits were approached. Since combat demands the maximum performance of the aircraft, the pilot was forced to divert attention to maintaining structural integrity and within departure limits just when his focus should be primarily devoted to target tracking or avoiding hostile fire.

Figure 1-2 from Gibson's 1995 essay, modified by thesis author

Slide 40 (1 min / :44 overall)

With the advent of flight control systems run on computers, it became possible to improve the performance of the aircraft/pilot team by offloading tasks from the pilot to the computer that were calculation heavy, repetitive and required constant monitoring. This dramatically reduced the pilot's required workload to control the aircraft and allowed him to focus on mission and task accomplishment. The ideal flight control system performs 3 tasks:

it provides more maneuverability by allowing for reduced or even negative static stability because the computer can monitor for, detect and correct far smaller deviations at a significantly faster speed than the human can.

It provides envelope wide similarity. A pilot no longer has to take into account how high and how fast (or the opposite) before knowing how large of an inceptor input has to be made to create the desired movement.

It provides carefree handling in terms of structural integrity and stall/departure resistance. Similarly, control logic is used in engines to provide carefree handling in throttle movements, but that is outside the scope of this discussion.

While these are general goals, each type of demand system has its own set of benefits and negatives that must be carefully balanced by control engineers when designing and implementing a high order flight control system.

Figure 1-2 from Gibson's 1995 essay, modified by thesis author

Slide 41 (1 min / :45 overall)

While the control loop variable in conventional aircraft is angle of attack, it is also possible to close the loop around angle of attack with a computer controlled FCS. This allows the static stability to be reduced or made negative to improve manoeuvrability as the computer is now maintaining the closed loop stability of the aircraft instead of the pilot. Additionally, it is possible to provide structural and stall/departure carefree handling by limiting the AOA available to the pilot at each flight condition.

This method is very intuitive for pilots exactly because there is no handling difference from the conventional aircraft they and most pilots are still trained on. Airspeed and flight path can still be held very precisely for most areas of the envelope.

However, the phugoid is still present. This exchange of airspeed and altitude with a constant angle of attack at a low frequency is more often considered to be an anachronism that can and should be designed out of modern HOFCS aircraft since it requires the pilot to intervene and thus increases his workload.

Additionally, angle of attack demand systems still required the pilot to trim out forces or maintain force of the stick to deviate from the trim angle of attack. This increases pilot workload by not maintaining envelope wide control similarity

This slide paraphrase from GT, para 4.2.3, page 40

Slide 42 (3 min / :48 overall)

As with all HOFCS, pure pitch rate demand systems allows the static stability to be reduced or made negative to improve manoeuvrability. Additionally, it is possible to provide structural and stall/departure carefree handling by limiting the pitch rate available to the pilot at each flight condition.

Physiologically, A pilot responds to a sensed movement such as 'G' acceleration or a pitch rate. In AoA demand systems, the change in acceleration or rate that a pilot senses is a consequence of the change in AoA. He can't actually perceive the AoA change directly. A Pitch Rate Demand system has a much more direct link. The pilot is simultaneously perceiving the pitch rate while adjusting his demand for it by moving the inceptor. This immediate connection can make the aircraft simpler to control.

Unlike AOA demand systems, pitch rate system provide envelope wide control similarity. Each inceptor position corresponds to a demand for a specific pitch rate. As the pilot selects that position, the FCS must use current aircraft altitude and airspeed in combination with a look up table to find the correct control surface deflection to command to achieve the pilot's requested pitch rate. Since the neutral inceptor position demands zero pitch rate, a pitch rate demand system eliminates the phugoid since the FCS will always zero out the pitch rate and stop the phugoid quicker than the pilot can detect it. Additionally, the neutral inceptor always demands zero pitch rate and therefore constant attitude, there is no need to trim out any forces like an AoA demand system and the pilot's workload is reduced.

While the removal of the phugoid and trim requirement by having a neutral stick demand zero rate can be a benefit, it can also be a detriment. By removing speed stability, a cue to the pilot of reduction in airspeed has been removed. If insufficient thrust is present, either because the pilot has reduced the throttle to initiate a deceleration to a particular speed or some other reason, the pilot has to carefully monitor airspeed. If it isn't monitored, then the FCS will maintain the current altitude while the aircraft decelerates. The FCS will increase the AoA to balance out the reduction in airspeed until the max lift AOA is reached and then the aircraft will stall.

This is a problem not present in conventional aircraft control systems, and it usually greatly reduced by an auto-throttle function, but accidents still happen because pilots think an auto throttle system is engaged when it isn't and the aerodynamic cue of increasing trim force has been removed by the FCS.

If an angle of bank, however small, is present in an aircraft with a Pitch Rate Demand FCS without any aft inceptor input to demand a pitch rate, a zero pitch rate spiral can develop. General aerodynamics tells us that when an aircraft is performing a level turn, it is the horizontal component of the lift vector causing the turn. This horizontal component has a non-zero Z axis component, such that a level turn always includes a pitch rate.

With any angle of bank and no commanded pitch rate through aft stick command, the lift is reduced and is insufficient to maintain level flight path, so the flight path falls, creating a rate of descent. In a pitch rate demand system, with neutral stick, the FCS maintains zero pitch rate by reducing the angle of attack. This exasperates the rate of descent, increasing the downward flight path angle.

These problems are unique to FCS demand systems and can be overcome by various means, including the use of autopilot and autothrottle functions. It is important to realize that changing the control loop variable is not a panacea to all flight control problems. Each method simultaneously solves some problems while creating unintended ones.

This slide paraphrase from GT, para 4.2.1, page 39

Slide 43 (2 min / :50 overall)

'G' demand systems, or Nz, Normal acceleration, are to a large extent similar to pitch rate demand systems in their benefits and negatives. 'G' demand allows the static stability to be reduced or made negative to improve manoeuvrability and it is possible to provide structural and stall/departure carefree handling by limiting the 'G' available to the pilot at each flight condition. "Normal Acceleration demand can be expected to show quite conventional control response characteristics, since it commands the parameter from which flight path follows directly" [GT, para 4.3.2, pg 42] thus making it quite intuitive for the pilot to fly.

Since this demand system is commanding 1 'G' at the neutral inceptor position and some increment above and below that for each inceptor position fore or aft of neutral, it also provides envelope wide similarity. This will damp out any phugoid tendency as the flight control computers null out any deviation from 1g at any indication of phugoid development. Just like a PRD system, this also removes any requirement to trim throughout the envelope, as neutral stick is always 1g, regardless of airspeed. The negative corollary of this is the loss of speed stability indications: the first indication of stall won't be an increase in stick trim force, but rather the actual stall as the FCC's fight to keep 1g up until the stall cliff and it is no longer aerodynamically possible.

As the aircraft decelerates towards the max lift/stall AoA, a 'G' demand system can make control for precise tasks, such as target tracking, difficult. In this scenario, max G available may only be 1.5 or 2 before the stall AoA is surpassed. Since neutral stick is always 1.0g and available g could be as low as 1.5, it may be difficult for the pilot to command the exact g desired.

This is not typically a problem in pitch rate or AoA demand systems as there is still a reasonable envelope of pitch rates and AoA's achievable before the lift limit is met.

This slide paraphrase from GT, para 4.2.2, page 40

Slide 44 (2 min / :52 overall)

"None of the basic manoeuvre demand systems on its own has completely favourable qualities. Most FBW aircraft employ some combination of demand parameters, attempting to combine the elements which are optimum for specific flight regimes and requirements." [GT, para 4.5, pg 45]

“This can be accomplished by blending functions which transfer control from one to the other as the stick input and manoeuvre amplitude increase.

Transition between the regimes was completely unnoticed by the pilots.” [GT, para 4.5.3, pg 47]

We have “discussed the variety of ways in which aircraft with manoeuvre demand flight control systems may differ in their response from conventional types. These differences have led to unexpectedly poor handling in some aircraft. They are significant in both the long term and short term response regimes equivalent to the conventional phugoid and short period response. An appreciation of the differences is essential when attempting to design good handling qualities in FBW systems, so that those features of conventional handling that are desired by pilots may be retained, emulated or enhanced.” [GT, para 4.7, pg 50]

It is critical to base the selection of the control loop variable on its effect on handling qualities. “If the FCS provides excellent handling qualities and protection from overstressing or loss of control, the pilot is relieved from the task of maintain continual observation of the appropriate instruments and limits, can therefore absorb more effectively the details of the task displays, and can thus devote the maximum attention to the task at hand for greater efficiency. This is the absolute justification for optimization of handling qualities.” [GT, para 5.2.2, pg 59]

Slide 45 (1 min / :55 overall) *BREAK SLIDE*

Over the last hour, we covered background on control engineering and handling qualities, followed by a historical review of handling qualities criteria. We discussed options for the control loop variable. This is a good spot to answer any questions up to this point and then take a break before going on to the second half of the lecture on pilot variability and physiology and the effect of control system hardware before finishing with Gibson’s pitch handling qualities criteria.

4.1.2 Part One Slides

Gibson's Criteria

A primer for Test Pilots and Flight Test Engineers on John Gibson's Handling Qualities Criteria for High Order FCS

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LT Jack Hathaway, USN

Empire Test Pilots' School, No70 Fixed Wing Course
Cranfield University, School of Engineering, Aerospace

Gibson's HQ Criteria

Introduction & Background

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Gibson's Criteria is a method to design in excellent handling qualities for modern fly by wire high order aircraft



Benefits of a High Order Flight Control Systems (HOFCS)

- Reduction in aerodynamic stability can improve performance, but requires computer controlled stability and command augmentation
- Excellent Handling Qualities for any mission task
- Envelope wide similarity
- Care free handling
 - Departure resistance
 - Structural limits

3

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- “Good handling is not just a means for giving enjoyment to the pilot. It performs an essential part in task performance. The combat pilot must devote all possible attention to the management and operation of a variety of complex weapons and defensive systems against an increasingly capable opposition. The ability actually to fly the aircraft with minimum attention to safety and structural limits releases considerable mental capacity to the combat task.”

-John Gibson

Development of a design methodology for handling qualities excellence in fly by wire aircraft
Doctoral Thesis for Delft Technical University, Netherlands
February 1999

4

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Scope of the Lecture/Criteria

- Pitch axis & short period
- Fighter/attack sized and designed aircraft

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Topics of Presentation

- Control Engineering & Handling Qualities Background
- Historical Handling Qualities Criteria Review
- The Control Loop Variable
- Pilot Variability
- Control System Hardware
- Dropback Criteria for excellent SPPR Handling Qualities
- Eliminating linear PIO through Phase Rate, PIO frequency and gain limits
- Non-Linear PIO
- Summary

6

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Control Engineering and
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Stability & Control



7

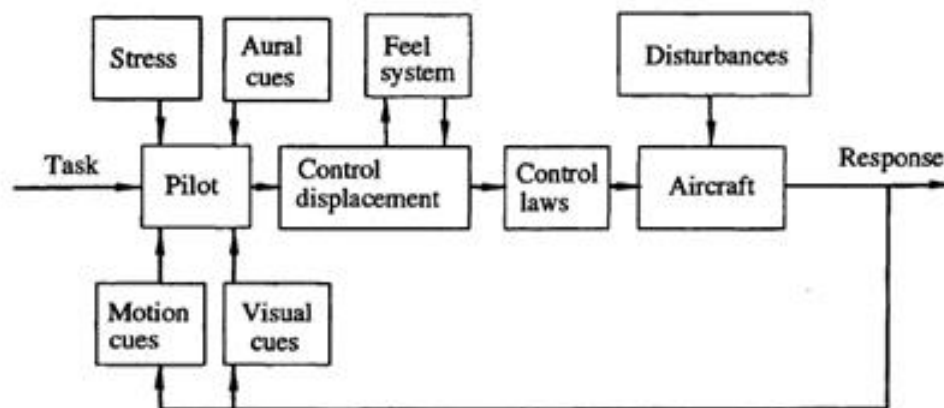
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Control Engineering and
Handling Qualities Background

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Handling Qualities = Pilot + Aircraft

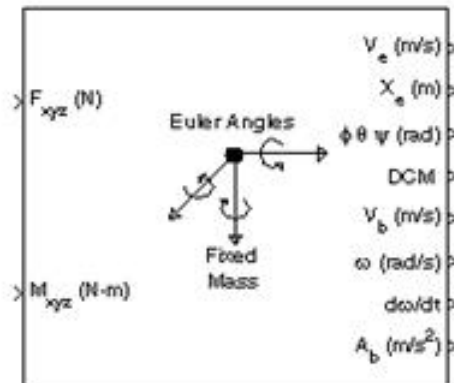


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Gibson's HQ Criteria

Control Engineering and
Handling Qualities Background



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Gibson's HQ Criteria

Control Engineering and
Handling Qualities Background

Longitudinal Modes:

- Short Period Pitch Oscillations (SPPO)
- Phugoid

Lateral-Directional Modes:

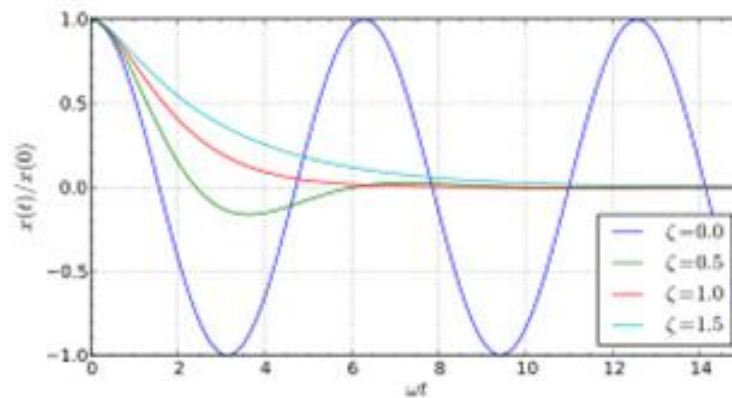
- Roll Mode
- Spiral Mode
- Dutch Roll

10

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Control Engineering and
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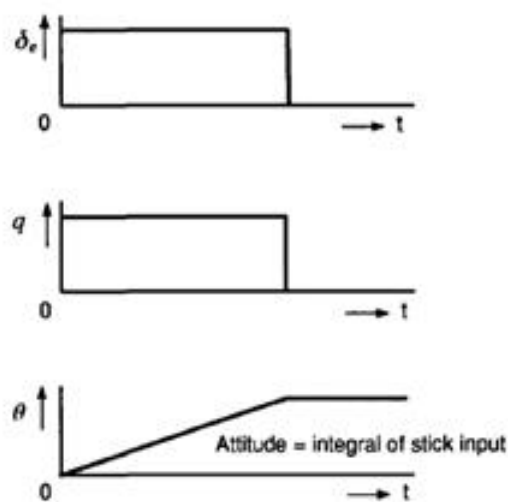
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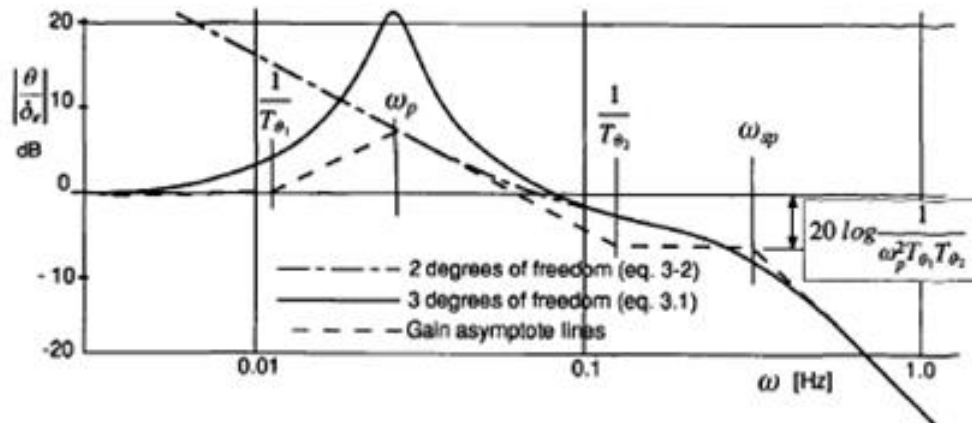
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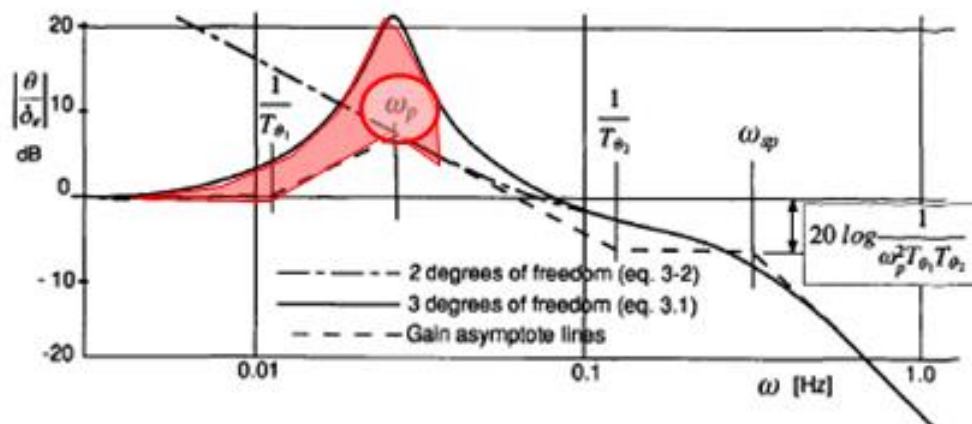


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Control Engineering and
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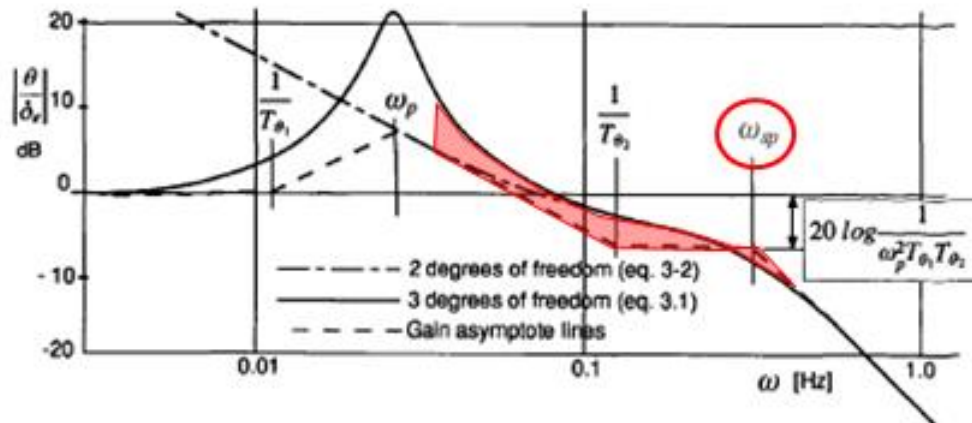
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Control Engineering and
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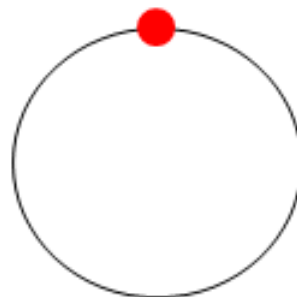
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Determining the frequency of individual pilot inputs



Time: 0 1 2

16

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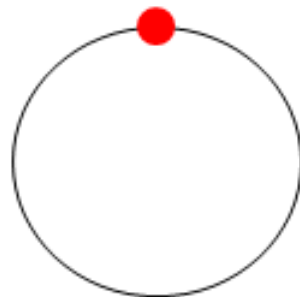
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Determining the frequency of individual pilot inputs



Time: 0 1 2

1 rotation in 2 seconds

1 rotation = 1 cycle, so

$\frac{1 \text{ cycle}}{2 \text{ seconds}} = 0.5 \text{ hertz}$

or

1 rotation = 2π radians, so

$\frac{2\pi \text{ radians}}{2 \text{ seconds}} = 3.14 \text{ rad/sec}$



17

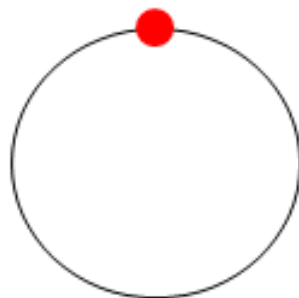
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Control Engineering and
Handling Qualities Background

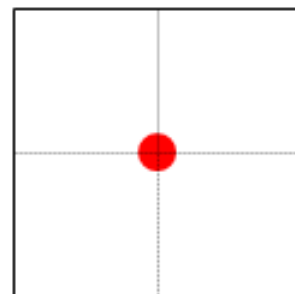
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Determining the frequency of individual pilot inputs



Time: 0 1 2



0 1 2



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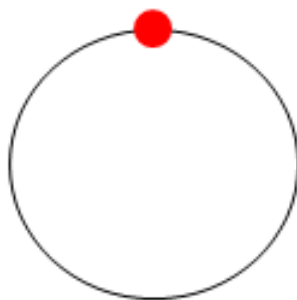
Gibson's HQ Criteria

Control Engineering and
Handling Qualities Background

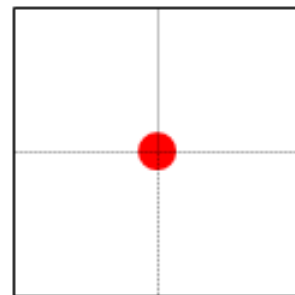
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Determining the frequency of individual pilot inputs



Time: 0 1 2



0 1 2



19

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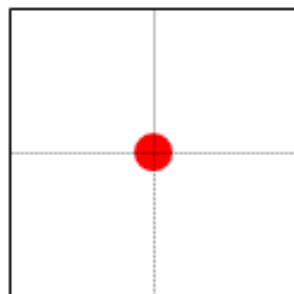
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Control Engineering and
Handling Qualities Background

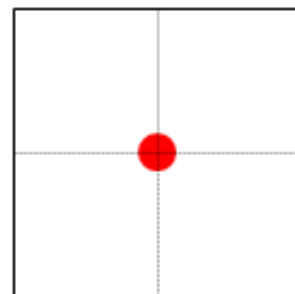
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Determining the frequency of individual pilot inputs



0 1 2



0 1 2



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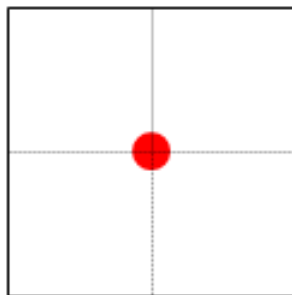
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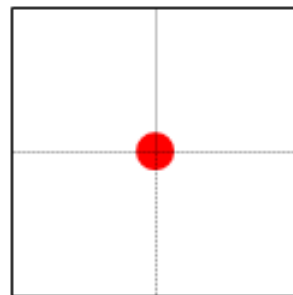
Determining the frequency of individual pilot inputs

2 hertz or 12 rad/sec



0 1 2

0.5 hertz or 3 rad/sec



0 1 2



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Gibson's HQ Criteria

Historical Handling Qualities
Criteria Review

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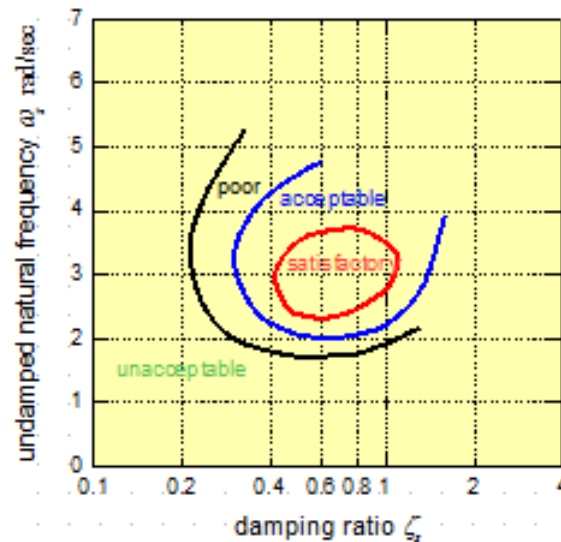


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Gibson's HQ Criteria

Historical HQ Criteria Review
Thumbprint



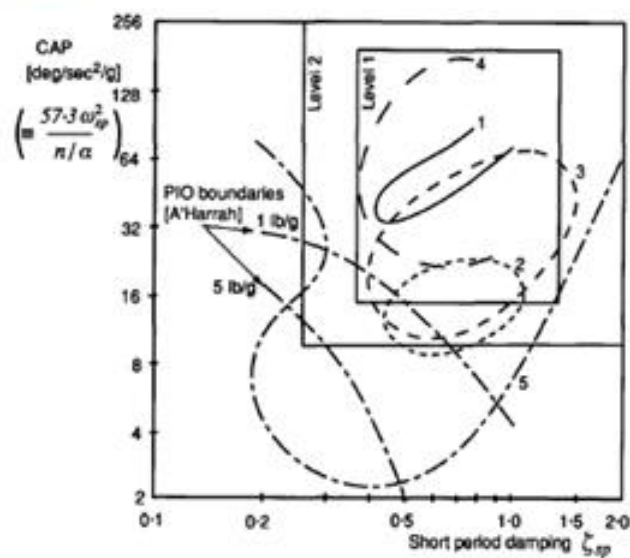
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Gibson's HQ Criteria

Historical HQ Criteria Review
Control Anticipation Parameter



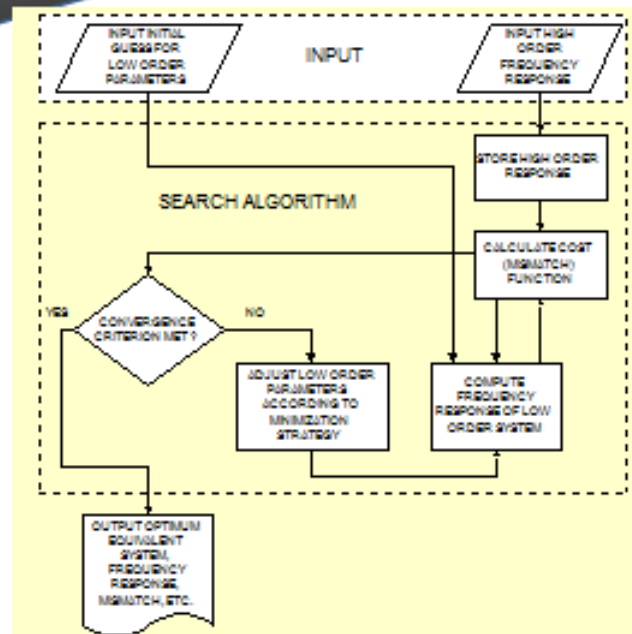
From:
1 Newell [1954]
2 Harper [1955]
3 Chalk [1958]
4 Eney [1969]
5 A'Harrish [1964]

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Historical HQ Criteria Review
Low Order Equivalent System

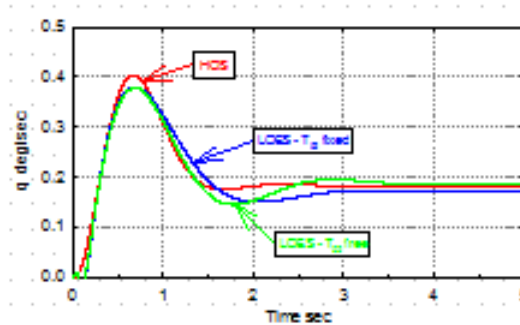
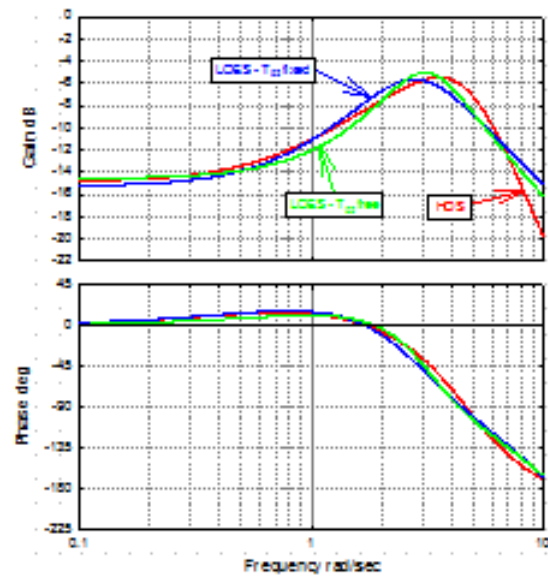


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Historical HQ Criteria Review
Low Order Equivalent System



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Gibson's HQ Criteria

*Historical HQ Criteria Review
Neal & Smith Criteria*

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Neal, T.P. and Smith, R.E.

A Flying Qualities Criterion for the Design of Fighter Flight Control Systems.

AIAA Journal of Aircraft, Vol. 8, No. 10, October 1971.

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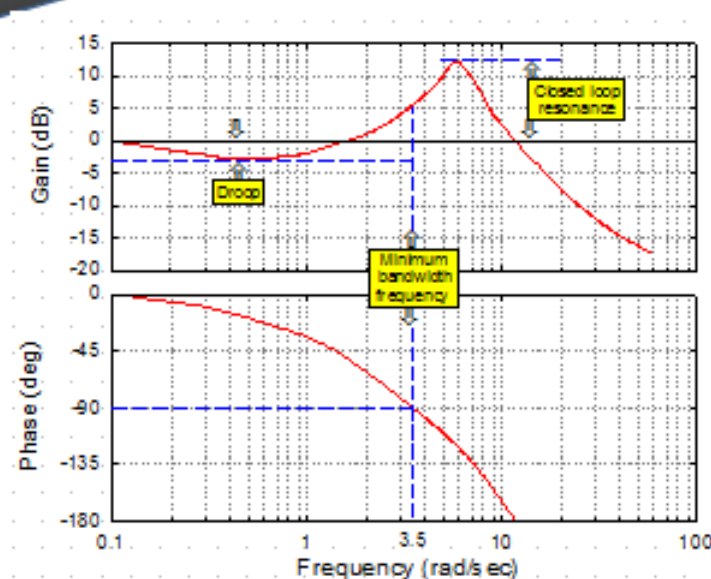
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*Historical HQ Criteria Review
Neal & Smith Criteria*

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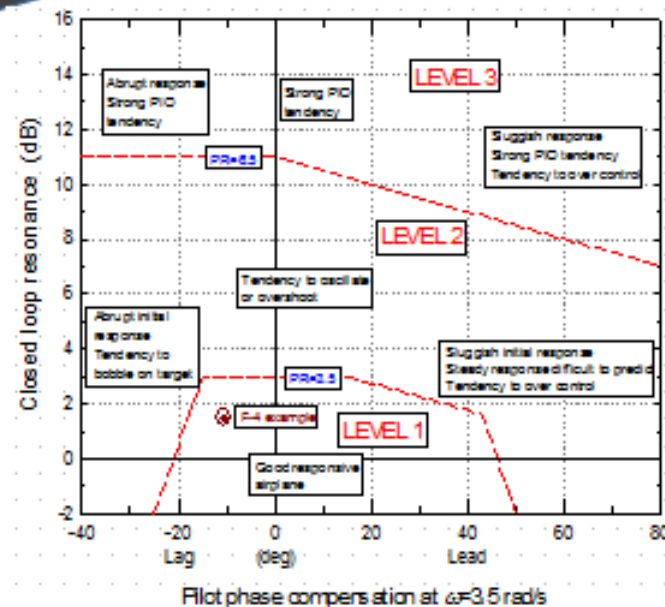


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Historical HQ Criteria Review
Neal & Smith Criteria



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Gibson's HQ Criteria

Historical HQ Criteria Review
Bandwidth Criteria

Hoh, R.H., Mitchell, D.G. and Hodgkinson, J.

Bandwidth - A Criterion for Highly Augmented Airplanes.

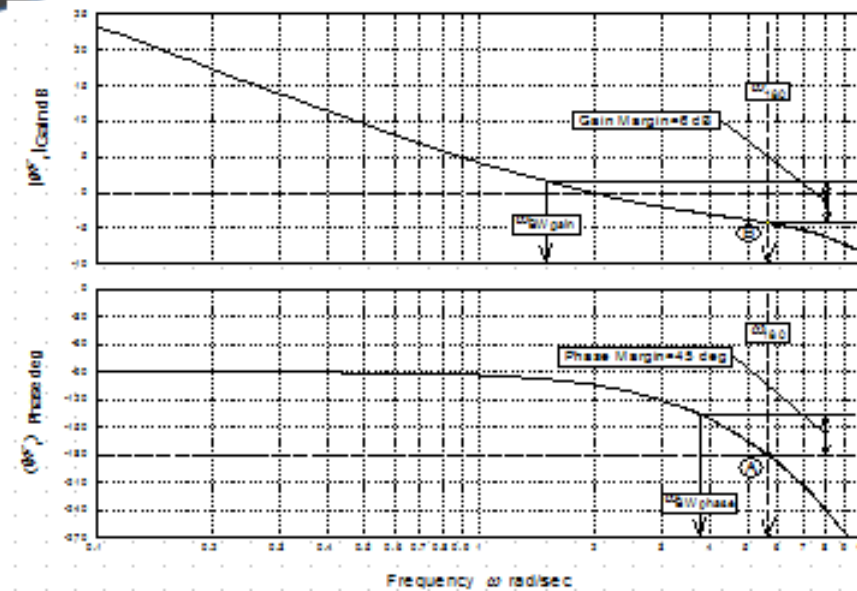
AGARD Conference Proceedings No.333, Criteria for Handling Qualities of Military Aircraft. Flight Mechanics Panel Symposium, Fort Worth, USA. April 1982.

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Gibson's HQ Criteria

Historical HQ Criteria Review
Bandwidth Criteria

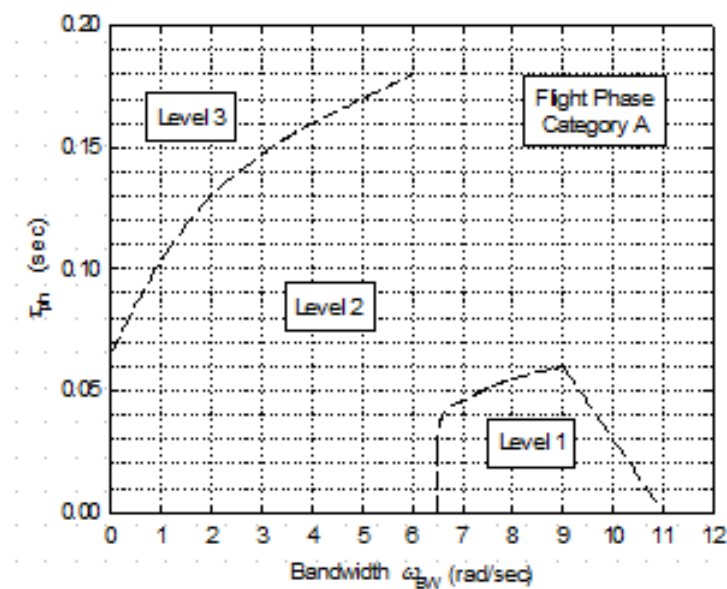


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Bandwidth Criteria



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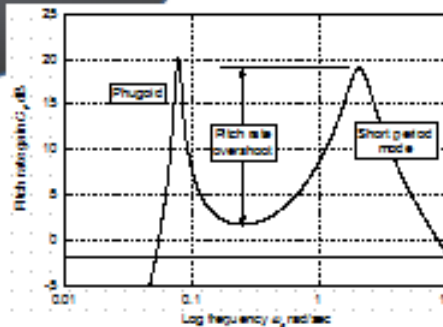
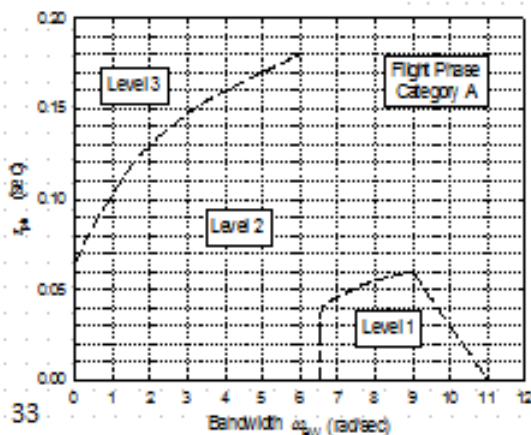
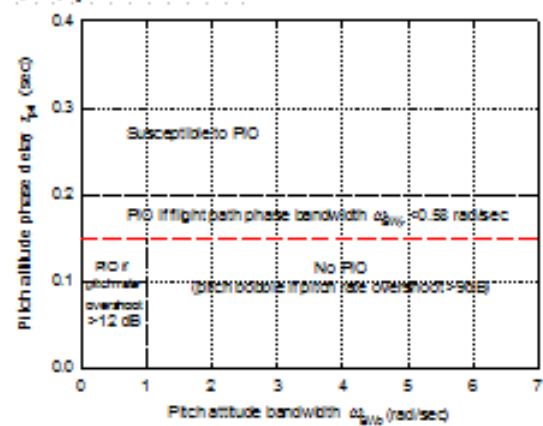
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Historical HQ Criteria Review
C* Development

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ETPS

Tobie, H.N., Elliott, E.M. and Malcom, L.G. (Boeing)
1966:

A New Longitudinal Handling Qualities Criterion.

Proc NAECON, Dayton, Ohio.

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Historical HQ Criteria Review
C* Development

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$$C^*(t) = n_z(t) + K_1 q(t)$$

- Crossover velocity V_{co}
 - At speeds below crossover velocity pitch cues dominate.
 - At speeds above crossover velocity normal acceleration cues dominate.
 - At the crossover velocity pitch cues and acceleration cues are equally weighted.

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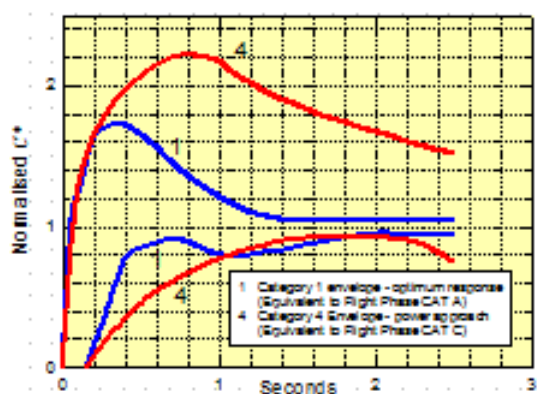
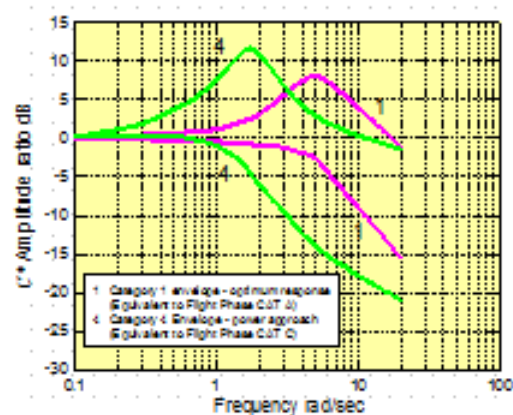
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Historical HQ Criteria Review
C* Development

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C* Time & FREQUENCY RESPONSE BOUNDARIES

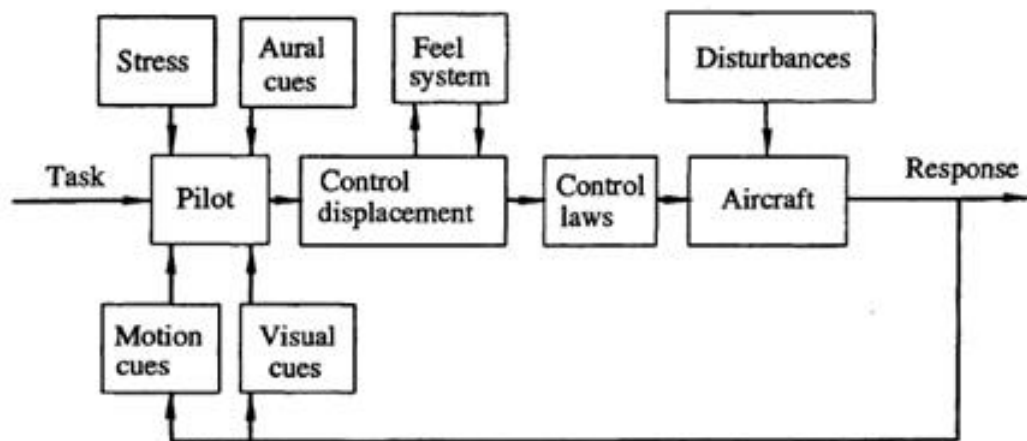


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Gibson's HQ Criteria

Control Loop Variable: What does the pilot demand from his Inceptor?

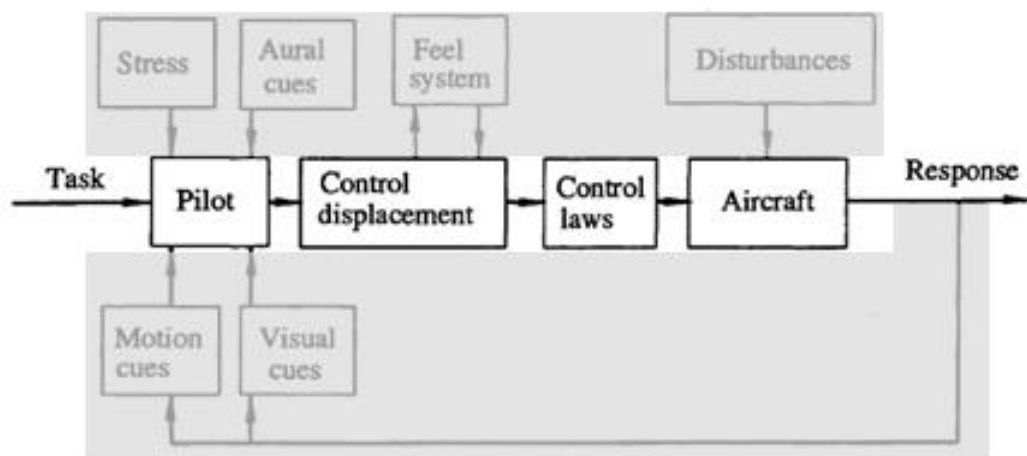


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Gibson's HQ Criteria

Control Loop Variable: What does the pilot demand from his Inceptor?



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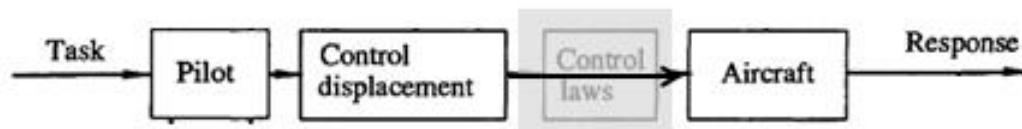
Gibson's HQ Criteria

Control Loop Variable: What does the pilot demand from his Inceptor?

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Conventional Second Order Aircraft Control



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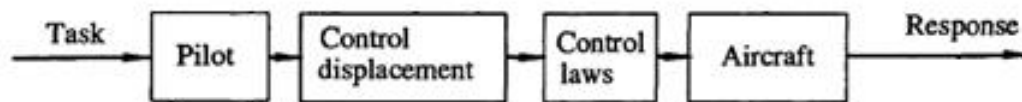
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Control Loop Variable: What does the pilot demand from his Inceptor?

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High Order Aircraft Control Systems



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Gibson's HQ Criteria

Control Loop Variable: What does the pilot demand from his Inceptor?

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Angle of Attack (via HOFCS)

Benefits

- Common to traditional aircraft response.
- Airspeed and Flight Path can be held precisely

Negatives

- Phugoid still present
- Trimming still required through speed envelope

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Gibson's HQ Criteria

Control Loop Variable: What does the pilot demand from his Inceptor?

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Pitch Rate Demand

Benefits

- Can be simpler for the pilot to control
- Phugoid and trimming requirement removed

Negatives

- Angle of Attack can diverge easily
- Zero PRD spiral

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Gibson's HQ Criteria

Control Loop Variable: What does the pilot demand from his Inceptor?

Normal Acceleration, 'G', Demand

Benefits

- Can be simpler for the pilot to control
- Phugoid and trimming requirement removed

Negatives

- Angle of Attack can diverge easily
- Can be difficult to control precisely near the aerodynamic stall

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Gibson's HQ Criteria

Control Loop Variable: What does the pilot demand from his Inceptor?

High Order Aircraft Control Systems



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Gibson's Criteria

A primer for Test Pilots and Flight Test Engineers on John Gibson's Handling Qualities Criteria for High Order FCS

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Part 1 - Complete Questions & Break

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4.2 GIBSON'S CRITERIA: A PRIMER FOR TEST PILOTS AND FLIGHT TEST ENGINEERS, PART TWO

This lecture is part two of two and contains 43 slides and an accompanying lecture designed to last approximately 55 minutes. It covers the remaining 5 topics.

- Pilot Variability
- Control System Hardware
- Dropback Criteria for excellent SPPR Handling Qualities
- Eliminating linear PIO through Phase Rate, PIO frequency and gain limits
- Non-Linear PIO

The slides and lecture are numbered sequentially after part one. Since part one ended on slide 45, part two starts on slide 46.

4.2.1 Part Two Lecture

Slide 46 (2 min / :02 post break / :57 overall)

The pilot is currently an inextricable part of the system. While some aircraft may claim to be unmanned at the moment, they are all actually remotely manned or preprogrammed. That is because the human brain still vastly outpaces the ability of computers to perform many critical tasks, be adaptable and make moral/ethical decisions in the heat of combat. While we avidly embrace computers to provide apparent stability to aerodynamically unstable aircraft or reduce the pilot's workload by minding structural and stall/departure limits for us, the time when we authorize an unmanned air vehicle to identify targets in combat and autonomously decide to employ weapons has not yet arrived.

Within current understanding, it is not possible to model the pilot accurately. Despite the knowledge modern science has of physiology and brain chemistry, and the ability of control engineers to develop complex simulations on powerful computers, creating an accurate representative model of the pilot to develop the perfect flight control system is not possible

However, "development of fly by wire technology has changed the situation to one where the aircraft dynamics can be shaped to a considerable extent to suit the pilot. For this purpose the earliest and simplest pilot models suffice, leading to the ultimate simplicity in desirable dynamics. This is the K/s - like response applicable to a wide range of controlled variables primarily including attitude and flight path." [GT, para 5.4, pg 67]

Figure 1-2, from Gibson's 1995 essay

Slide 47 (0 min / :02 post break / :57 overall)

To understand how the flight control system influence the aircraft dynamics, let's take a look at what does a pilot do in a conventional aircraft.

Figure 1-2, from Gibson's 1995 essay, modified by thesis author

Slide 48 (1 min / :03 post break / :58 overall)

This figure "illustrates the basic longitudinal response parameters controlled by a pilot. [Gibson argued that] all pilots use visual attitude references for their primary control cues. Their opinion is sometimes based entirely on the attitude characteristics and in the limit is always influenced by them even where flight path control is the primary task...

"In a complex modern aircraft there is a mass of information about the on-board systems, navigation and flight conditions, and so on, which are presented to the pilot by an array of instruments and displays. In a military combat aircraft there is even more information to be absorbed from weapon and attack displays. Under high workload conditions the ability to absorb this information becomes less and less. This is evidenced by a tendency to fixate on the most important instrument and for the scan pattern and rate to become disordered. [6]

"Experienced pilots are able to maintain a more normal scan, an ability acquired from hundreds of hours of training and which distinguishes them from the inexperienced. A workload that is too high will lead to a collapse in performance at any skill level, and indeed attempts have been made to measure workload from observation of pilots' visual scanning patterns. " [GT, para 5.2.2, page 58]

Figure 1-1, GT, para 1-3, page 3

Slide 49 (5 min / :08 post break / 1:03 overall)

"Flying by the seat of the pants" is a well-worn phrase but it is almost meaningless. If this were the only source of information available to the pilot, flight would be impossible. The dominant sensors possessed by the human are the visual, tactile and vestibular systems, by which motions and attitudes relative to the environment can be detected..... An understanding of the sensing abilities of the pilot helps to focus attention on the particular features of aircraft response characteristics which are of primary importance to him. The characteristics of these sensors are the natural result of evolution to operate in an essentially 1 g environment for the purposes of keeping the human balanced upright, the visual environment stable and keeping precise track of targets. The human can physically generate only quite low linear accelerations and velocities, but high angular accelerations and visual target rates are possible through rapid head and eye rotation.

In the unnatural flight environment the capabilities of these sensing systems result in an acute consciousness of attitude displacements, rates and angular accelerations, but only a rather generalized and sometimes misleading sense of linear velocity and acceleration.

"The eyes are by far the most important sensors the pilot has. It would be possible to maintain control with these alone and a view of the outside world, as can be demonstrated in any fixed base flight simulator. The fundamental method of flying any aircraft is to adjust its attitude against a visible background, which would be the earth's horizon if it can be seen... or an attitude reference instrument. Without attitude cues, control can last only for a very brief period. The only additional item of information usually necessary for routine flight is airspeed, although even this can be dispensed with in some simple low performance aircraft. The skilled pilot can infer a level attitude from vertical velocity and turn rate information alone, though only within limited departures from straight and level flight.

"The vestibular system contains the semi-circular canals which detect angular accelerations, and the otoliths which detect specific force. Located in the inner ear, their signals are processed by the central nervous system, which interprets them as angular and linear velocities over a substantial range of frequencies, and also the otoliths provide orientation to the gravitational vertical. Neither sensor can maintain a perception of steady velocity for more than a few seconds, and when the velocity ceases it is then perceived as motion in the opposite direction. Both the canals and otoliths are subject to thresholds in detection of accelerations and a latency time before the sensation becomes apparent...

"Manoeuvring flight requires an increase in normal acceleration which can be sensed by the otolith system and by the tactile sensors which measure the pressure on the parts of the anatomy supporting the body [the seat of the pants feel].

Combat pilots learn to estimate the level of normal acceleration quite closely, though sometimes at the cost of cues such as pain in the lower arms at 9 g.....Control of a coordinated level turn is achieved not by direct application of the required g but by whether the aircraft tends to climb or descend after the

desired bank angle is achieved, requiring respectively a decrease or increase in applied g.

“In striking contrast, the performance of the semi-circular canals is an order better. Many passengers will be familiar with the sensation of rolling and turning induced by even the modest maneuvers of an airliner, followed by an apparent ability to “see” the actual motion visually even when nothing is visible outside and the g sensation is negligible. Known as “the oculogyral illusion”, this is the result of the angular acceleration and rate sensing compensation to the visual system, although it is impossible to determine the actual bank angle.

“Although the flight path is altered by normal acceleration, this is produced by angle of attack changes not directly perceptible to the pilot. Aspects of the normal acceleration transient response have to be respected to achieve satisfactory short term control of flight path changes, but they give little clue to longer term changes. From the pilot's point of view, these are generally effected by attitude rates and displacements. Most handling qualities are to a great extent perceived through the qualities of the attitude response of an aircraft. These are clearly very appropriate for the tasks which require precision pointing of the airframe.

“In flight, an abnormal condition for the human, the senses reinforce each other so long as well coordinated visual flight is possible. Regardless of the attitude, if the normal acceleration remains vertical with respect to the airframe then all feels well. Lateral specific force gives a feeling of falling over sideways, extremely uncomfortable to the human. When the outside world or its representation is not visible, the short-term nature of the vestibular sensors ensures that the pilot will lose control after a very brief period. Flight in instrument conditions with only limited visual cues is made more difficult if the handling is unpredictable and badly coordinated, causing potential conflict of the senses. A sudden loss of visual cues during the course of maneuvers can lead to complete disorientation and loss of control.” [GT. Para 5.2.1-2, pg 57-59]

Slide 50 (1 min / :09 post break / 1:04 overall)

Understanding the basic physiology of a pilot helps us understand how his sensors help him control the aircraft, but a mathematical construct of a pilot would be useful so that we can combine that with the frequency, damping and bandwidth and other mathematical descriptions of the aircraft to find out what the result of the whole system will be.....

“The describing function, Y_p , of the standard pilot model” that Gibson accepted was developed by researchers in 1960 and is shown on this slide. [7]

“The pilot gain K_p is expressed with the same unit dimensions as the aircraft frequency response ...The reaction time delay τ represents the finite time which elapses between the pilot's decision to act and the physical act itself. It is always present when tracking a random-seeming target, typical values being around 0.2 to 0.3 seconds. Its effect is to add phase lag to the pilot's output proportional to the frequency but it does not alter the amplitude. “

“The lead time constant T_L represents the pilot's ability to predict and advance a control action, required when the aircraft response is slow or unresponsive so that it has to be speeded up by anticipatory inputs...The amount of lead which

can be generated easily is quite small, about a one second time constant being the largest without serious degradation of pilot opinion.

"The lag time constant $T_{little\ L}$ represents the pilot's ability to apply smooth inputs to overcome an oscillatory aircraft response.

The latter may be the result either of low damping or of an excessively "jumpy" response which makes it difficult to achieve a steady closed loop tracking solution, both requiring some attenuation of the activity...

"The gain, lag and lead comprise the pilot equalization or compensation efforts. These were not assumed *a priori*, but were found to be the consequence of pilots' efforts to minimise the... tracking error." Pilots were compensating for aircraft characteristics to produce a closed loop aircraft/pilot response that was K/s like. [GT, para 5.2.4, page 60-61]

Equation 5.1 and comments from para 5.2.4, pg 60 of GT

[7] McRuer, D. T., Ashkenas, I. L. and Guerre, C. L., 1960, "A Systems Analysis View of Longitudinal Flying Qualities", W ADD TR-60-43

Slide 51 (0.5 min / :09.5 post break / 1:04.5 overall)

"It has long been recognized that the pilot gain required to perform precision tracking tasks is a dominant influence on the pilot's opinion of the handling qualities." The figure shown [7] is a result from the standard pilot model just described. "Typically there is an optimum aircraft gain at which the pilot can readily achieve a desired" handling quality rating or as close a possible performance to the idealized K/s. "The pilot can accommodate moderate variations in aircraft gain from this level by personal gain adjustments...with an acceptable penalty in workload. Larger gain variations exact increasing workload and worsening pilot opinion, leading ultimately even to PIO problems for aircraft gains that are either too large or too small." [GT, 5.2.8, page 63]

Figure 5-7 from GT, page 71, original data from [7] McRuer, D. T., Ashkenas, I. L. and Guerre, C. L., 1960, "A Systems Analysis View of Longitudinal Flying Qualities", W ADD TR-60-43

Slide 52 (1.5 min / :11 post break / 1:06 overall)

Finally in this section, it is important to recognize how difficult it can be to communicate between pilots and control engineers. This poor communication can lead to difficulty in development of excellent handling qualities as both parties are critical.

"Pilots and control engineers don't always speak the same language, which can lead to considerable misunderstanding or even mistrust between them." [GT, para 11.0, pg 223] For example, "to an aerodynamicist, "trim" may mean the balancing of an aircraft state by appropriate choice of control surface settings to achieve a zero pitching, rolling or yawing moment overall. To a pilot, it is simply the condition of the control system in which flight can be maintained with the hands off the controls, hence in which a stable condition results." [GT, para 6.2.5, pg 81]

The figure on the bottom is a contrived example where the lack of shared experience can make this communication very difficult. It shows a plot of a pilot doing a pitch doublet at time zero and the resultant response.

The short period response is very small and squashed on the left, but the subsequent phugoid motions are readily apparent. It appears as if the phugoid is a vastly more significant factor than the short period. While the relative importance of these two modes is usually well understood by pilots and

engineers alike, the way they experience them is not. Engineers see and experience these modes, and by extrapolation all aircraft motions and issues through plots and equations. The pilot's experience is more direct, being in the cockpit at the time.

Having a much different experience of these aircraft motions can have an effect on how each party makes conclusions on the importance and acceptability of aircraft motions. While the comparison of short period and phugoid importance is a bit contrived, towards the end of this presentation I will show you some figures of a major pilot induced oscillations during the landing phase of a Tornado that ends up damaging the aircraft. Good communication and understanding despite the vastly different experience of that PIO is critical to working towards a solution.

We have just been discussing the reduction of a pilot into a set of mathematical equations. While some test pilots out there will understand when an engineer refers to them as all gain, no bandwidth, most pilots will struggle to understand what that means and why it is important to the control engineer. As part of his development of criteria and design methodology, Gibson laid out 5 rules that we covered in the opening slides of this presentation. The last two were that the methodology:

- 1) "should enable the control design engineer to synthesize directly the handling characteristics desired for specific tasks as perceived visually or physically by pilots.
- 2) "Their meaning should be equally clear to both pilots and control design engineers." [GT, para 7.1, page 95]

Slide 53 (1 min / :11 post break / 1:06 overall)

In the first half of the 20th century, "given sufficient stability through proper aerodynamic configuration design, handling qualities were influenced primarily by the detailed design of control surface aerodynamic balance and trim devices. Their development was pursued intensively during World War 2 to cope with the rapidly increasing power, size and performance of military combat aircraft.

It became increasingly difficult to achieve consistent and satisfactory results, and these devices represented a large overhead in the flight testing of many designs because of the repeated adjustments that were often necessary, even during production flight testing. In the post-World War 2 period, further increases in aircraft performance led to the rapid development of hydraulically boosted and fully powered controls ... The response of the actuators introduced a new factor in the equations of motion and had some unexpected effects on the feel qualities. It became necessary to replace the effect of the control surface aerodynamic hinge moments with artificially generated "feel" forces, such as stick force/deflection gradients proportional to dynamic pressure as in q-feel. The role of the aerodynamicist in providing good handling qualities began to be supplemented by, and was eventually almost subordinated to, the new flight control engineer." [GT, 6.1, page 73]

Slide 54 (3 min / :14 post break / 1:09 overall)

"With the ability to pick off pilot stick inputs of either force or displacement by electrical means, modern control engineers may be tempted by the apparent

simplicity of a rigid or "force" stick. The absence of stick motion acting effectively as a filter between pilot and command allows higher precision in certain tracking tasks. Similarly, the use of force pick-offs on a stick with conventional displacements offers a more lag-free interface between the pilot and the task. Such conceptions do not accurately represent the constraints of real flight.

In several cases, abrupt, inadvertent, or turbulence-induced stick inputs degraded the handling so severely that a change to displacement sensors was necessary, as in the McDonnell Douglas F-18 [Walker 1982].

"The General Dynamics F-16 has been the only production aircraft equipped with a rigid side stick. Although most pilots adapted to its use, it proved difficult to optimise as an interface.

For some flight tasks the absence of stick motion was a hindrance to good control. Also, because there was no physical indication of the limits, pilots often applied twice as much as the maximum signal force, which was very tiring in sustained manoeuvres. Small aft and lateral deflections were introduced, approximately 4.5 mm in pitch and ± 3 mm in roll [Garland 1978]. This was just enough to give a sense of movement and to enable the maximum travel limits to be felt.

"Wrist action" sticks, that is with a very short pivot length and significant angular motion, suffer from cross-talk between the pitch and roll inputs unless the pivot axis is skewed to be in line with the fore-arm. This makes them generally unsuitable for central mounting, as the large skew needed can lead to a roll input being applied when only pitch is intended. This was a concern for the Shuttle Orbiter which has such sticks skewed by 18 degrees, but its limited manoeuvre requirements have not shown a problem in the flight environment [Gilbert, 1983]. Wrist inceptors can work effectively in the side location, where experiments have shown that there is a certain minimum displacement and maximum stiffness for optimum function [Black, 1979].

"Arm action sticks, pivoted further from the hand, can also work well at the side. In the Lockheed YF-22, travels of 13 mm aft and 6 mm forwards were used successfully [Domheim, 1992]. Arm action is generally essential in the centre position, with a substantial length between the pivot and grip to reduce the angular motions. It is also less tiring in sustained manoeuvres where large forces have to be applied. The wrist muscle system is not well suited to this [Myers *et al*, 1987]. Another example is the original small centre stick in the SAAB Gripen, which caused muscular fatigue as well as oversensitivity [Knotts *et al*, 1990]. A travel of 6 mm from neutral to the aft stop typifies a reasonable choice for a combat aircraft, with about half this for lateral travel, though "local tradition" usually determines the choice. In the Grumman X-29, this led to an initial design with an aft travel of 20 centimetres, which was found to give sluggish handling. With the stick travel halved and the feel force stiffness doubled to maintain unchanged stick force per g, a far more satisfactory pitch control resulted [Anon, 1991]." [GT, para 6.3.2, pg 84]

Slide 55 (1 min / :15 post break / 1:10 overall)

In the middle of the 20th century, combat aircraft performance was rapidly increasing. While the mechanical engineers and aerodynamicists of the time

period did excellent work producing controllable and effective high performance jet aircraft, the solutions were increasingly complex and relied heavily on expensive developmental and production flight test to tune. The advent of computers fast, small and light enough to be used on tactical aircraft allowed a change in design philosophy. No longer did the pilot need to adjust to various artifacts of the aircraft's control and handling characteristics.

By exchanging the complexity of mechanical designs into millions of lines of software code, the aircraft could be simultaneously designed for the best possible aerodynamic performance regardless of human limitations, and tuned during the design phase to present excellent handling qualities to the pilot throughout the envelope.

Additionally, the pilot's workload would be reduced because the computer would now be primarily responsible for structural load limits, departure/spin resistance in much of the envelope, as well as the continual requirement of keeping the nose pointed where the pilot initially told it to be. This is the ultimate justification for the time, effort and cost of introducing a computer controlled flight control system; That the aircraft and pilot system are more mission effective with it than without.

Slide 56 (2 min / :17post break / 1:12 overall)

Of course, with the introduction of these systems came new challenges to surmount. Conventional aircraft were second order and pilots over the decades have come to understand and become adept at employing those systems and predicting how they will respond in a large part of the envelope. Flight control computers, hydraulically or electrically driven actuators and indirect stick to control surface connections dramatically increased the mathematical order of the transfer functions that describe the connection from the pilot applying an input to the inceptor and the aircraft moving as desired.

This figure shows a comparison of a conventional second order aircraft response to a step input in pitch with the response of a super augmented or high order flight control system. The two graphs on the right are in the time domain, with pitch rate the vertical axis on the top graph, and angle of attack the vertical axis on the bottom. The pitch rate graph depicts a massive pitch rate overshoot on the conventional aircraft and a very minor overshoot on the super augmented version. If a pilot was tracking a target, he would make inputs very fast. In doing that, the pitch rate overshoot and the time to settle to steady state would be of vital importance in the assessment of the handling qualities associated with the target tracking task. Most pilots would prefer the super-augmented version as that response has a smaller overshoot and reaches steady state much quicker.

On the bottom right graph, the conventional aircraft angle of attack increases significantly faster. Combined with the excessive pitch rate overshoot in the top graph, we can infer that this rapid AoA increase onset results in a bobbling or over sensitivity.

However the slow rise of the AoA in the super augmented version may, in fact, be appear to be sluggish from the pilot perspective. It's hard to know from these graphs alone which control system will be more preferred by the pilot.

On the left side, is a the gain portion of a Bode Diagram comparing the same conventional aircraft vs a super-augmented aircraft in attitude at the top and flight path angle on the bottom. It is critical to look at the control system and aircraft response in both the time and frequency domains as handling qualities issue can arise in both domains. At the natural frequency, ω_n , of the aircraft, there is significantly more lag/lead dynamics present in the super augmented version. This is, of course, a result of the higher mathematical order of the super-augmented control system.

These dynamics may affect the handling qualities in standard mission tasks, and may also cause significant issues of PIO that aren't readily apparent.

Figure 4-2 from GT, page 52

Slide 57 (3 min / :20post break / 1:15 overall)

Gibson believed a new criteria and design methodology for these super augmented flight control systems was necessary. While it was generally agreed that the short period pitch response characteristics were the fundamental determinant of good handling qualities, formal specifications still relied heavily on the accepted regions of SPPO frequency and damping, or the even more generally unhelpful specification that handling qualities of super augmented aircraft should be as good or better than their mechanically controlled brethren. [paraphrased from GT, para 7.1, pg 94]

This deficient formal guidance in the specifications was based on lack of widely accepted criteria, so regulators relied on what they knew from the past. This was clearly inadequate for super augmented systems. With these systems, response characteristics and handling qualities were “dominated by the flight control system and the aerodynamic parameters tend to have greatly reduced influence, the simple modal parameters equivalent to the conventional frequency and damping are usually absent. There may be several poles and zeros clustered quite close to frequencies that are typical of the short period mode of conventional aircraft, and it is often impossible to decide which one should be taken to quantify the handling. Even if there is only one response mode with a suitable “short period” frequency, it may not be in angle of attack, the parameter conventionally operated on by pilots' control inputs.

It will be the parameter controlled by the maneuver demand system, for example pitch rate.” It became “of the utmost important to be able to quantify desirable characteristics in ways other than by the traditional expressions of the short period mode parameters. “[GT, para 7.1, pg 93]

Gibson used existing data by searching the “available literature in papers, journals and reports for results of handling qualities flight research. Only in-flight data was considered to be suitable, and this was mostly provided by variable stability aircraft studies such as those of Calspan as early as the 1950s.” [para 7.1, page 94]

Following his experience with PIO in the Tornado's development, Gibson pursued this goal with the following “Criteria and Design Methodology:

“a) They would have to be expressed in both frequency and time domains, as neither on its own give a sufficiently complete design definition.

b) They should in particular define limits on the shape of the responses in these domains without reference to numerical mode parameters or mathematical formula, so that they could be applied to any controlled response

however high its order, and independent of the type of model available, i.e. parametric or non-parametric.

c) They should not depend on the assumption of any explicit pilot model parameters, as it is very difficult to apply these reliably.

d) They should enable the control design engineer to synthesize directly the handling characteristics desired for specific tasks as perceived visually or physically by pilots.

e) Their meaning should be equally clear to both pilots and control design engineers.”

Gibson determined “the significant attitude parameters were ... the attitude dropback, pitch rate overshoot ratio ...and sensitivity....explicitly connected to the classical short period by the parameter $T\text{-}\theta\text{-}2$.” [GT, para 7.1, pg 94-95]

Slide 58 (1 min / :25 post break / 1:20 overall)

Approximations for $T\text{-}\theta\text{-}2$ were derived in the mid to late 1960's by Bihrlé and then Chalk during development of the Control Anticipation Parameter we discussed earlier. They assume zero tail lift effect.

The classical short period, from an aerodynamic derivative vantage point, is driven by $M\alpha$, AOA stability and Mq , pitch damping. These derivatives are an aerodynamic description of the wing and tail plane geometry and the current flight condition. $T\text{-}\theta\text{-}2$ is also driven by these underlying fundamentals and therefore retains the connection to the short period that we are interested in for handling qualities, but is independent of mathematical order of the control system.

As we will see over the next few slides, $t\text{-}\theta\text{-}2$ turns out to be more relevant and appropriate to the high order super augmented systems than the conventional parameters of frequency and damping.

Equations 7.1 & 7.2 from GT, para 7.3.1, pg 99

Slide 59 (Slides 59&60, 1 min / :26 post break / 1:21 overall)

Here are four time domain plots, flight path, pitch attitude, rate and acceleration using three individual generic longitudinal aircraft models with speed removed, label a, b, and c. Each aircraft model has the same natural frequency and damping ratio, but the $T\text{-}\theta\text{-}2$ values are such that there is one smaller than, equal to and greater than a value called τ_y or flight path delay. In each plot, a step input has been applied t time zero and then removed at time t about two thirds down the plot. In order to make the flight path responses identical to help compare, the three models have different trim airspeeds.

Figure 7-4 from GT, page 121

Slide 60 (Slides 59&60, 1 min / :26 post break / 1:21 overall)

The first plot is flight path, γ . What you may notice first is that the responses of all three aircraft are identical. This might appear to make sense at first, since their second order flying quality parameters, ω_n and λ are identical, and conventional theory would predict the same handling qualities ratings. However, all three aircraft, in fact, produce noticeably different handling qualities ratings for the same pitch task. Gibson argued that pilots' are using

pitch attitude not flight path for the vast majority of high gain tasks. Implicit in his argument, as we discussed earlier, is that the human is physiologically tuned to be particularly sensitive to changes in angles due to peripheral vision, inner ear functions and the way our brains process this information. Thus over the next slides, we will see that the pitch attitude behaving different, even though the end result of flight path is the same, will have significant impacts on pilot ratings during test and evaluation.

Additionally, the value of flight path delay, or τ_y , is shown here. "It notionally represents the path angle response by an initial delay followed by a constant slope." [GT, para 7.1, page 100]

Figure 7-4 from GT, page 121

Slide 61 (2 min / :28 post break / 1:23 overall)

This plot shows pitch attitude in the time domain. The three different aircraft response shapes are visible and labeled as a, b, and c. Let's focus on response A first.

The pilot applies an aft stick step input at time zero. There is an initial transient as the aerodynamic forces shift and rebalance until a steady state pitch rate is established in the middle third of the plot.

The pilot removes the step input at the point indicated and the pitch attitude reduces, or drops back down as the initial aerodynamic equilibrium is restored. This is a conventional angle of attack demand system response type and most pilots will be very familiar with it having been trained on a conventional aircraft. From an engineering perspective, it makes sense as well. The initial conditions of the aircraft had some arbitrary trim angle of attack. When the pilot applied the step input, the AoA demand increased and resulted in the initial AoA plus an increase resulting in the pitch rate.

When that pilot demand is removed, the aircraft must return from its AoA trim plus extra AoA demanded, to its AoA trim. The pitch angle dropping back down to its trim position seems a natural consequence of the AoA with a pitch rate being higher than at trim.

In response shape B, there is just a transient after the control demand is removed, but zero actual drop back. In response shape C, there is actually a negative dropback, or a pitch overshoot.

The value of the dropback or overshoot can be measured in degrees (or radians), by comparing the pitch attitude at the moment of pilot input removal to the steady state pitch rate after the transient smooths out.

Additionally, if you extend the steady state pitch rate slope back down, you can measure the dropback where that line crosses the y axis. Extending the line further to cross the x axis, you have identified the ratio of dropback to steady state pitch rate, or DB over Q_{ss} . By convention, dropback is positive and overshoots are negative, thus making this ratio positive to the left of the origin and negative to the right.

Figure 7-4 from GT, page 121

Paraphrasing from GT, para 7.1, page 100

Slide 62 (2 min / :308 post break / 1:25 overall)

Gibson's last two critical parameters are the pitch rate overshoot ratio and pitch sensitivity which he initially treated separately. On the left are the pitch rate and

acceleration plots of the three models we've been looking at showing the pitch rate overshoot ratio. On the right is a plot of pitch rate per pound of stick force the pilot has to apply, a measure of pitch sensitivity.

In general, a pilot is looking for a predictable pitch response with the appropriate sensitivity. Any bobbling tendency around the intended attitude can be seen as a predictability issue. This assumes that the short period response itself is not causing the bobbling oscillation through very poor damping or instability.

If that is not the case, the bobbling is caused because the pilot has difficulty knowing when to stop the input to achieve a desired attitude; he can't predict where the attitude will end up, thus requiring constant corrective inputs or a bobbling.

There could be a balance between predictability, short period damping and stability, but short period characteristics that poor can be identified in open loop tests and should be well understood and hopefully removed before attempting pilot in the loop testing.

The sensitivity can be understood as a ratio of input over output: How much pilot command produces how much pitch acceleration. This was difficult to tune in conventional aircraft and required many hours of flight test and re-test and was often a trade off between great performance in one area of the envelope for average or less performance in another. In a super augmented system, this control engineer can vary the sensitivity based on flight condition to achieve envelope wide similarity. However, the ideal sensitivity was still difficult to design in from the start because conventional criteria had such a wide band of acceptability.

This importance of this sensitivity ratio was soon overridden "by the empirical observation that, at least for well damped highly augmented fly by wire response types, there is a reasonably good correspondence between [it] and the gain of the initial peak of the pitch rate time response." [GT, para 7.6.2, 115-116] This can be seen by on the right plot. The response with the lowest sensitivity ration, 0.74 also has the lowest pitch rate peak to steady state ratio.

The pitch rate time response can be characterized by a ratio of max pitch rate in the transient to the steady state pitch rate. This observation dramatically simplified Gibson's handling quality criteria by subsuming pitch sensitivity into the pitch rate overshoot ratio. Any aircraft with excessive pitch sensitivity could be identified by the drop back criteria rather than requiring a separate, additional criterion.

Figure 7-4 from GT, page 121

Figure 7-30 from GT, page 137

Slide 63 (2 min / :32 post break / 1:27 overall)

This figure is a generic time domain plot of a longitudinal step input. It "adds [Gibson's] new time response metrics of attitude dropback ratio and flight path time delay with their relationship to the classical short period parameters." [GT, para 7.3.1, pg 100]

As we have discussed, Gibson theorized that since a pilot senses attitude most precisely, he is most sensitive to these cues and thus they drive the opinion ratings given in handling qualities tests. The size of the dropback is directly related to the predictability of where the nose will end up when the pilot removes his demand input. $T_{\theta 2}$, which is a fundamental description of the

physical properties of the aircraft's aerodynamics, is clearly a critical player in how large the dropback will be.

In conventional aircraft, $t_{\theta 2}$ cannot vary, it stays at whatever the aerodynamicist designed from the outset. The metrics of natural frequency and damping are key descriptions of the short period pitch response characteristics and thus can quantify how the transients at the beginning and end of the pilot input will behave.

Gibson, however, believed that they weren't actually the critical variable. Constraining them to certain values as handling qualities criteria have done for most of the 20th century limited the size of the flight path delay. This, in turn, in second order systems, constrained the dropback and produced the good handling qualities in many conventional aircraft.

Gibson's breakthrough lied in his recognition that not only did the aircraft design team want to control dropback directly instead of indirectly, but that was now possible with computer controlled systems. The aerodynamicist could produce a wing and aircraft shaped to perform aerodynamically ideally without regard to its effect on the short period or $t_{\theta 2}$, as those values could be contrived by the control engineer during the flight control software design phase.

This goes a long way towards Gibson's stated methodology. This is time domain, not frequency yet, but it is response shape dependent, so therefore independent of FCS order. The physical effect of flight path delay, drop back, q steady state, peak pitch rate can be simultaneously seen on a graph by an engineer but be intuitively understood by a pilot and has a direct physical connection to the aircraft response.

Figure 7-5 from GT, page 122

Slide 64 (1 min / :33 post break / 1:28 overall)

After gathering the critical parameters from a time domain plot of the pitch response, the actual application of the dropback criteria is performed on a figure Gibson developed and is shown here.

"Negative attitude dropback (i.e. [when pitch rate continues after control input removed] was associated with sluggish, unpredictable response both in flight path control and in tracking. Attitude dropback ratio from 0 to about 0.25 seconds was excellent for fine tracking ... Increasing attitude dropback [values beyond 0.25] led to abrupt response and 'bobbling', from 'slight tendency' to 'continuous oscillations' in tracking tasks...

"The pitch rate overshoot seemed to qualify the dropback behavior [possibly because it implied an overly pitch sensitive control system]. A ratio greater than 3.0 making a dropback ratio as small as 0.25 seconds unacceptable." [GT, para 7.3.1, pg 100]

Slide 65 (1 min / :34 post break / 1:29 overall)

Bringing up the attitude and pitch rate response plots again, we can see how they would workout on this figure. Response A has a positive dropback and a large pitch rate overshoot, putting it somewhere in quadrant 1. Response B has zero dropback and a medium pitch rate overshoot, putting it on the upper half of the y axis. Response C has negative dropback or overshoot and no overshoot, putting it on the left x axis.

“This chapter has presented the author’s core discoveries of a new metric for pitch handling qualities. Derived from conventional aircraft dynamics, they explain the confusion contradictions in pilot opinion in many past variable stability aircraft research experiments. They offer a coherent range of parameters that constrain the satisfactory area of handling on the natural frequency and damping ratio plane, for long the primary handling parameters for conventional aircraft, to an area that varies with the aircraft parameter $T\text{-}\theta\text{-}2$. The parameters are the attitude dropback ration, ...pitch rate overshoot ratio with associated sensitivity gain The ‘shape’ of both time and frequency domain responses are found to provide important aids to the recognition and quantification of satisfactory handling dynamics, including the desirable crossover model characteristics....Requiring no mathematics for their application and providing a direct visual representation of the response characteristics, they are ideally suited to fostering the all-important dialogue between control law design engineers and pilots, the lack of which has caused many mistakes in the past.” [GT, para 7.7, pg 117]

Figure 7-4 from GT, page 121

Figure from Cranfield Flying Qualities & Flight Controls Course Lecture, FCS12

Slide 66 (1 min / :35 post break / 1:30 overall)

While excellent short period pitch handling characteristics greatly improve the mission effectiveness of the pilot and his aircraft, achieving that goal is not sufficient to remove all undesirable motions and artifacts from the control law.

Pilot Induced Oscillations, sometimes referred to as Aircraft-Pilot Coupling, is “caused by an unstable or neutrally damped closed loop coupling between the aircraft dynamics and the pilot.” [GT, para 8.1, pg 138]

“A PIO can range from small undamped oscillations to a grossly unstable divergence, and can occur even where the normal aircraft response is stable and heavily damped...Most PIOs begin only when the pilot attempts tight closed loop control, and will stop instantly if the pilot abandons the task. Unfortunately, in many cases the pilot feels impelled not to abandon the task because of the circumstances...” [GT, 8.1.1, pg 139] such as of proximately to the ground and fear of safely landing or because succeeding at the task is the immediate and necessary goal of the pilot as when attempting to refuel airborne, maintaining precise close formation or achieving a tracking solution on a target.

The example trace shown here depicts “a roll PIO in the landing flare of an early Tornado prototype version...caused by the high stick to control surface gain.” [GT, para 9.2, pg 178] This roll PIO is good explanatory example of a fully developed PIO. “PIO is usually initiated following what has become widely known as a trigger event.

This can take many forms, all creating the need for tightened pilot control. The increased pilot gain employed just before a landing touchdown is a classic trigger.” [GT, 9.6, pg 192] In this example, a highly experienced test pilot enters a stop to stop roll PIO 3-4 seconds prior to touchdown.

Figure 9-2 from GT. Page 194

Slide 67 (2 min / :37 post break / 1:32 overall)

Now showing just the stick input and bank angle of the jet. 3-4 seconds prior to touchdown, the trigger event occurs as the pilot suddenly becomes aware of a small angle of bank to the left and applies right stick to correct it. The stick input

is quickly reversed as the roll attitude cross zero or wings level. The PIO becomes effectively locked in by the time of the second stick reversal and only ceases following touchdown. In the final seconds before landing, pilots often raise their gain, sometimes dramatically, in response to the increase stimulus provided by the ground rush and the need to make final corrections prior to touchdown.

However, it is unknown why the landing on this particular sortie triggered a PIO and not during the previous sorties.

Gibson believed that triggers could not be accurately predicted because “it must be assumed that at some time a pilot may use enough gain to enter a PIO in response to some stimulus not yet thought of.” [GT, 10.3.1, pg 201] Additionally, “reliance on flight test procedures simulating mission tasks to uncover PIO may well give false comfort to the testing authorities. Apparent success may mean only that the critical trigger has been overlooked or not identified.”

“It is essential to grasp that pilots have absolutely no training to cope with severe PIO, and they have never experienced anything remotely similar. It is extremely dangerous to assume that they would not, or even should not, employ very large control inputs when plunged into such a situation, or to expect them to be able to limit their inputs to some arbitrary extent. Because such events are traumatic and bizarre and because pilots invariably believe that there has been a system failure, they will apply whatever control seems to be necessary at the time, even though the best input is none at all.” [GT, 9.2, 197]

Slide 68 (1 min / :38 post break / 1:33 overall)

Gibson also postulated that “the pilot would act in a highly synchronized way with the visual perception of the oscillation. Usually this is linked directly to the attitude response but can also be linked to the motion of a symbol in a pilot's display“, [GT, para 9.6, pg 190]

Following the trigger event, the pilot becomes a pure gain machine and stops applying lead or lag when closing the loop around the aircraft response, operating 180 degrees out from the rate in response to attitude reversals. Pilots have spent careers adapting to the controls and response of aircraft and when presented with severe PIO, it is often impossible to identify whether a catastrophic failure has occurred or the scenario is indeed PIO. In the example shown, the pilot moves the stick stop to stop in response to the aircraft reversing roll rate at the largest amplitude attitudes, applying maximum gain to stop the roll. This, of course, does not work as the aircraft roll rate is 180 out from his commands.

Not directly related to the PIO shown, but an interesting artifact, “it may be noted that the form of the pilot action differs between left stick (positive) and right stick (negative) inputs. There is a more positive control of the former, in which the pilot can apply the full pressure from the arm muscles via the palm of the hand. In right stick inputs, the force can only be applied through the pull of the fingers, and the input peaks are more rounded. [GT, 9.6, 191]

Slide 69 (2 min / :40 post break / 1:45 overall)

While we just looked at an example of a roll PIO, the predominant number of PIO scenarios occur in the pitch axis and are “generally found only in combat aircraft, where the aircraft gain may be high because the stick forces are low in

order to reduce pilot effort in manoeuvring at very high normal accelerations, in tasks genuinely requiring precise pointing of the airframe towards a non-stationary target.”

There plenty of examples of severe PIO in the first decade of aviation, including the Wright Flyer. Researchers (Jex and Culick [1985]) “estimated that its highly unstable airframe, with a time to double amplitude of 0.6 seconds, was controllable if the pilot closed a tight attitude loop, with a neutrally damped attitude oscillation [or intentional PIO]...

The Flyer was unusual in that, at the pilot gain value for stable closed loop control, either increased or reduced pilot gain would have resulted in loss of control. This [may be] the only example ...of a PIO condition that could be said to be necessary for flight.” [GT, 8.1, 138]

As understanding of aerodynamic stability increased, aircraft became more controllable and easier to fly. “Up to the 1939-1945 war, aircraft had moderate wing loading and moderate or high wing lift slope. Consequently they tended to small or moderate values of $T\text{-}\theta\text{-}2$...With moderate shelf width and negligible dropback, hence little tendency towards the “PIO syndrome”, and with high damping, they possessed good K/s-like attitude response behaviour. PIO was almost unknown in them. Later swept wing types with increased wing loading, lower lift slopes due to high wing sweep, and higher short period frequencies particularly at supersonic speeds, were more prone to” the problem of linear high order PIO. [GT, 8.2.1, 144].

Slide 70 (2 min / :427 post break / 1:37 overall)

During the development of the Tornado in the 1980's, Gibson analyzed the problem of these severe PIOs. He separated them into cases of linear PIO and non-linear PIO. While non-linear cases, such as actuator rate limiting, are significant, we will treat the linear cases first. Linear PIO's were further divided into low order and high order cases.

Low order PIOs “are typical of conventional aircraft and essentially result in poorly damped coupling with the pilot. They can be subdivided further:

- The “PIO syndrome”, a result of poorly shaped aircraft open loop dynamics that lead to problems of closed loop droop and resonance. These force the pilot to adopt considerable compensation dynamics to avoid oscillatory coupling when tight attitude tracking is attempted.
- The “bobble PIO”, a result of an excessively abrupt angle of attack response that creates a sharp attitude disturbance every time the pilot initiates or ends a pitch control input. It seems to be more a problem of precognitive pursuit tracking than of compensatory closed loop tracking, but is often described as PIO by pilots.
- The “overdriving PIO”, a result of low bandwidth aircraft open loop dynamics. The pilot applies excessive control to speed up the initial response, but the response overshoots because it is so slow that the end result cannot be predicted, setting up a continuous cycle of slow over control when tight tracking is attempted.” [GT, 8.8, 162]

High order PIOs “are typical of fly by wire aircraft and often resulted in unstable coupling with loss of control. With response dynamics dominated by the flight control system, designers inadvertently introduced non-classical handling

qualities often typified by excessive phase lags. [Gibson] found that the offending high order dynamics are characterized thus:

- a) PIO always occurs at or slightly higher than the frequency where the open loop phase lag reaches 180° , the "PIO frequency".
 - b) The open loop phase lag reaches 180° at low frequency, typically well under 1.0 Hz or about 6 rad/sec
 - c) The phase lag increases rapidly with frequency in this region.
 - d) The open loop response gain at the PIO frequency is large, allowing large amplitude oscillations to be forced within the limits of the control stick travel."
- [GT, 8.8, 163]

Gibson believed and showed through development of the Tornado, the FBW Jaguar, and the EAP and the Eurofighter 2000 unstable canard deltas, that the low order PIO scenarios could be eliminated through the use of his dropback criteria for pitch handling. The high order PIOs needed additional criteria including limits on phase rate, PIO frequency and gain limits

Slide 71 (1 min / :43 post break / 1:38 overall)

This figure is a bode plot of a generic aircraft's pitch response characteristics. Gibson believed that there was "no point ...in trying to develop a theory of transitional pilot behavior between normal control and the full PIO." Pilots, in a sense, operated in their typically manner of applying appropriate lead/lag and gain compensation to achieve desired results for a specific task up until they reached the cliff of a PIO trigger event. At that point, they transitioned completely to a synchronous control method operating entirely as high gain, 180° degrees out from the control variable, typically a rate or attitude reversal. He dealt "with PIO as a specific problem of aircraft dynamics complete in itself, separately from considerations of general handling qualities..." and "decided to concentrate only on the aircraft dynamics in the 180° degree phase lag region." [GT, 8.4, 148]

Slide 72 (slides 72-78, 1 min / :44 post break / 1:39 overall)

"In aircraft prior to the use of fly by wire control systems, the basic response dynamics were essentially low order...despite the addition of actuator dynamics and a certain amount of stability augmentation. " [GT, 8.1.1, 139]

They didn't cross the 180° deg phase angle line. This limited these aircraft to the low order PIO scenarios that could be addressed through the drop back handling criteria discussed earlier.

Slide 73 (slides 72-78, 1 min / :44 post break / 1:39 overall)

With the advent of higher order flight control systems, the phase lag easily exceeds 180° degree line. The question now becomes at what frequency does this happen, what is the phase rate, and how much has the gain attenuated.

Slide 74 (slides 72-78, 1 min / :44 post break / 1:39 overall)

Identifying the PIO frequency is done by seeing where the phase lag first crosses the 180° degree line

Slide 75 (slides 72-78, 1 min / :44 post break / 1:39 overall)

The Phase Rate can be calculated by comparing the slope between the phase lag at the PIO frequency and the phase lag at twice the PIO frequency

Slide 76 (slides 72-78, 1 min / :44 post break / 1:39 overall)

The gain level at the PIO frequency can be read off the top graph

Slide 77 (slides 72-78, 1 min / :44 post break / 1:39 overall)

As with his dropback criteria, Gibson's PIO criteria require picking values off a response graph, in this case a frequency response shown on a Bode plot, independent of mathematical order and applying those values to a specific criteria figure. This allows any order aircraft to be analyzed and compared as well as easily understood on a figure by pilots and control engineers alike.

Actual values for the average phase rate and PIO frequency are shown here. As typical for Gibson, he believed very strongly that control engineers should never strive for the minimum acceptable standard. They should always strive for the perfect solution, so he defined a Level 1* area to target for optimum design.

Slide 78 (slides 72-78, 1 min / :44 post break / 1:39 overall)

A Nichols plot is another method of viewing an aircraft's pitch response characteristics against Gibson's PIO criteria. While he developed the special diagram that we just saw to measure the average phase rate against the PIO frequency, a standard Nichols plot was used to evaluate the levels of handling qualities for the gain attenuation at the PIO frequency. Average phase rate can also be seen on these plots.

Slide 79 (1 min / :45 post break / 1:40 overall)

A "1976 Tornado accident that introduced [Gibson] to the problem is illustrated in [in this slide]. The pilot had the impression of a small turbulence disturbance followed by an uncontrollable pitch-up despite the fully forward stick input. The record showed a PIO of gross proportions in which the aircraft was responding to the controls, though out of phase with the stick." [GT, 8.4, 146]

"The record shows that shortly before touchdown the pilot initiated an oscillatory stick input... This is actually a commonplace activity known as "stick pumping", widely seen in landing records of many aircraft... This phenomenon is frequently seen and is a predictable activity identified by [another researcher](Bihrlé [1966] while working at Republic Aircraft in the 1950s).

"He noted that in the few seconds just before touchdown, pilots would often engage in a rapid pitch control oscillation in phase with pitch acceleration... The frequency was typically about 6 to 10 radians per second, well above the short period, where the pitch acceleration had little or no phase lag relative to stick inputs. Due to the high frequency there was little attitude and even less flight path disturbance. The pitch acceleration amplitude was consistently around ± 6.5 deg/sec², which [was] found to coincide with available data on the threshold of vestibular detection in humans. He concluded that pilots acted this

way to generate confidence in pitch control as the speed reduced towards the stall when very precise flight path control was needed for a smooth and safe landing. The activity was also quite subconscious, all pilots being unaware of it.” [GT, 8.4, 149]

“The most likely trigger for the accident and the subsequent incident seemed to be that the pilots suddenly became aware of the attitude oscillation.” [GT, 8.4, 150]

Slide 80 (1 min / :46 post break / 1:41 overall)

While the solution for the Tornado PIO event was developed in house, Gibson continued to look at the problem and strove to identify a general solution for the problem. A release of flight test data for high order systems done by Calspan became available a short time after the PIO event in the Tornado development and Gibson used it to reveal “a consistent pattern of differences between cases free of PIO and those highly prone to it.” [GT, para 8.5, pg 151]

This slide shows a Nichols plot on the left figure and time domain responses to a step input on the right. They all show 3 cases: The solid line depicts a low order response, while the two dashed examples show increasing phase lag that Calspan achieved by a lag pre-filter. Cooper-Harper and PIO ratings for all three are in the bottom center of the slide. The ratings drop from an HQR 2/ PIO 1 in the case of the low order (no phase lag) to an HQR 10 / PIO 4 for the largest phase lag case.

Slide 81 (slide 81&82, 1 min / :47 post break / 1:42 overall)

While the Nichols plot makes it difficult to identify what the exact PIO frequency is, the other two PIO criteria of Gibson’s, phase rate and gain amplitude are clearly depicted. The low phase lag case has a gain attenuation of about 12, and a steep phase rate slope, and thus a small change in phase lag per unit increase in pilot input frequency.

Slide 82 (slide 81&82, 1 min / :47 post break / 1:42 overall)

The large phase lag case has a gain increase! of about 2, and a much shallower phase rate slope, and thus a much higher phase rate. These agree with Gibson’s PIO criteria to be strong indicators of potential PIO. Indeed, the pilot ratings agree closely with Gibson’s predictions.

Slide 83 (1 min / :48 post break / 1:43 overall)

A hallmark of any successful criteria is that it should be able to explain incidents that were not used in the development of the criteria. While Gibson’s work in developing this PIO criteria was rooted in the Tornado landing PIO, that particular data had not been used in the development of the criteria. His work was based entirely on the Calspan high order experiments.

This slide is a Nichols plot of the 4 relevant versions of the Tornado control laws. The 1st version in the dotted line was the control law in place during the accident shown earlier. The dashed line represents the second control law version, in which a second PIO incident happens, though the pilot recognized the transition to full PIO, froze the controls and no accident occurred. The third

version, a solid line, was the final version. The un-augmented, effectively low order response, is shown in the longer dashed line.

Looking at the phase rate slope and gain values from the 1st to the third version, there is a clear trend from low gain attenuation and shallow phase rate slope (thus high phase rate numbers) to larger attenuation and steeper slopes, thus significantly less PIO prone. Gibson's PIO criteria is successful in predicting the PIO susceptibility in this initial version and its removal by the third.

Slide 84 (1 min / :49 post break / 1:44 overall)

Having discussed linear high order PIO, it is important to also spend some time on the non-linear sort.

Gibson "considered applying [the] linear phase rate [and other PIO criteria] to non-linear responses... [but] eventually realized that the idea was actually inappropriate. [Those criteria are] a valid measure as a symptom of adverse dynamics in normal linear control." However, most non-linear PIO is a result of saturating the rate controls of the system, either software rate limitations in the flight control computers or hardware rate limitations such as the hydraulic pumps or actuators physically moving the control surfaces. "Severely rate saturated dynamics do not affect the handling in normal circumstances and are not involved in control tasks. These dynamics are themselves the root cause of severe [non-linear] PIO." [GT, 9.2, 180]

Slide 85 (2 min / :51 post break / 1:46 overall)

In this figure, the sinusoidal line is a generalized pilot's inceptor command. It is presented as sinusoidal to represent a center stick with large displacement. If a small displacement center or side stick were the inceptor, the behavior would likely be more switch or relay like, but the conclusions would be the same. The rate limited output line represents the actual control surface movement, limited in rate or output due to physical or software limitations.

In this generic example of a rate saturation non-linear PIO, the pilot commands a rate that exceeds the acceptable limit, and thus the surface moves slower than the demand. When the pilot reverses input at line 1, probably in response to a rate or attitude reversal similar to previous discussions, "The [flight control surface] actuator cannot reverse direction instantly.

"The servo device has to first decelerate the actuator and then accelerate it by a physical movement of the flow control valve from one end of its stroke to the other. " This results in some non zero amount of time, shown from line 1 to line 3, between when the pilot is commanding opposite rate to when the actuator deflects in that direction.

"Since the valve displacement controls the actuator rate, its own velocity controls the acceleration of the actuator. Unless this is high enough, the output lag in a saturated oscillation is significantly increased and the amplitude ratio is also increased." This lag increase is apparent in this figure by noting that the distance from peak inceptor input to peak surface output increase, the distance from lines 3 to 4 is larger than the distance from 1 to 2. The amplitude ratio

increase can be seen as well, the ratio of A to C is larger than the ratio of A to B.

“As an example, a seemingly rapid servo valve movement from centre to its maximum in one fortieth of a second represents an acceleration of 4000 deg/sec² in an actuator with a maximum rate of 100 deg/sec., but introduces an additional 5 *degrees of control surface* position error in a maximum rate reversal through a total rate change of 200 deg/sec. [GT, para 9.2, 179]

This is a telling example of how control engineers cannot avoid PIO potential all by themselves. Actuator demands must be accounted for.

Slide 86 (1 min / :52 post break / 1:47 overall)

“Good handling is not just a means for giving enjoyment to the pilot. It performs an essential part in task performance. The combat pilot must devote all possible attention to the management and operation of a variety of complex weapons and defensive systems against an increasingly capable opposition. The ability actually to fly the aircraft with minimum attention to safety and structural limits releases considerable mental capacity to the combat task.”

“Up to the 1940s, aircraft could be tuned to give desirable handling, though sometimes only by expending considerable effort. [Combat aircraft could require] hundreds of hours of flight testing spent developing the control system balance, tab and spring gearing and control circuit[s]. In the post-1940s era, the handling qualities were dominated by the increasingly poor aerodynamic and inertial properties of aircraft. These could be ameliorated at best by stability augmentation that was sometimes mechanically quite complex, but they could seldom be made perfect even assuming that such a state could be adequately defined. In the rapidly growing sphere of fly by wire technology, in both military and civil applications, the means for good handling resides largely (but certainly not only) in software code. It costs little more in software overheads to write “good” code rather than “bad” or merely careless code, assuming again that the desirable end result is known. The overall effort required to write the software is very large and should not be wasted to any degree on inferior handling code.” [GT, para 10.1, pg 200]

Slide 87 (1 min / :53 post break / 1:48 overall)

Our topics of presentation today have been a background on control engineering and handling qualities, followed by a historical review of handling qualities criteria. We discussed options for the control loop variable, how pilot variability can make the control engineers’ problem more difficult, and the effect of control system hardware. Finally, we introduced Gibson’s pitch handling qualities criteria, starting with the Drop Back criteria to design in excellent short period pitch handling qualities and then his criteria to eliminate low order PIO, Phase Rate, PIO frequency and gain limits and finished with a quick overview of non-linear PIO.

This lecture exposed the flight test engineer and test pilot students into the world of control engineering and handling qualities criteria, and how improving technology and aircraft performance such as high order fly by wire flight control systems with powered actuators has affected the longitudinal flying qualities of

modern combat fighters, ultimately driving John Gibson to develop his criteria to design in excellent short period pitch handling qualities and eliminate pilot induced oscillations in a way that both pilots and engineers can understand.

Slide 88

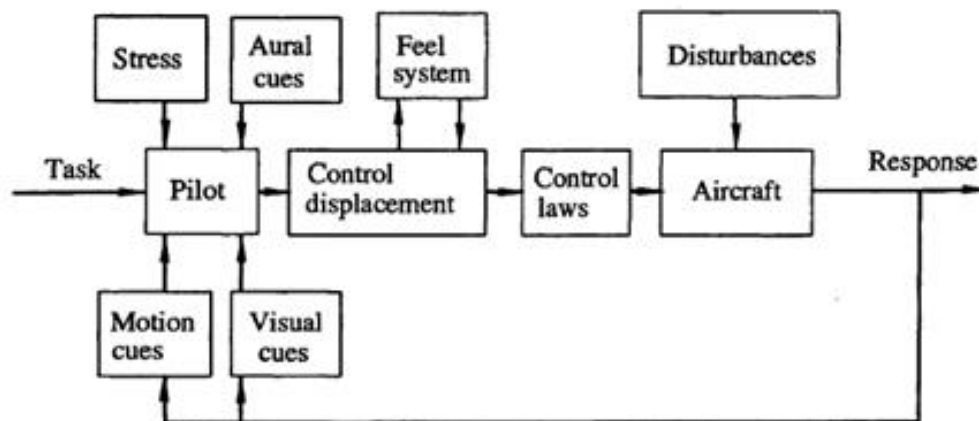
Questions

4.2.2 Part Two Slides

Gibson's HQ Criteria

Modeling Pilot Physiology, Variability and Individual behavior

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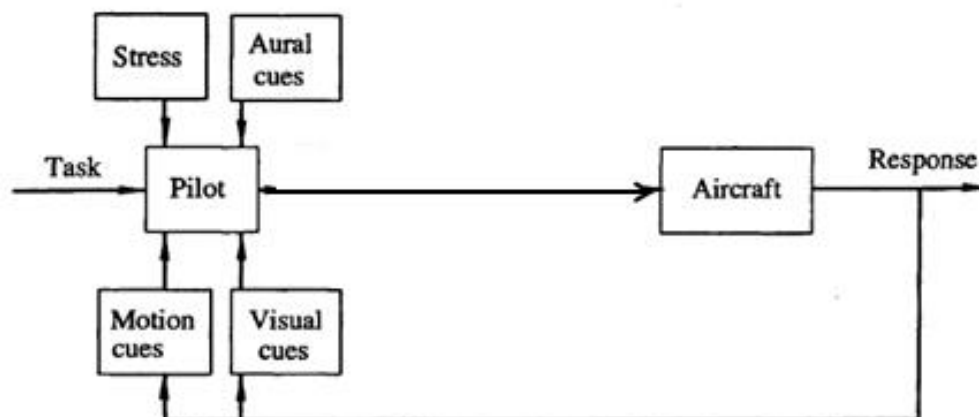
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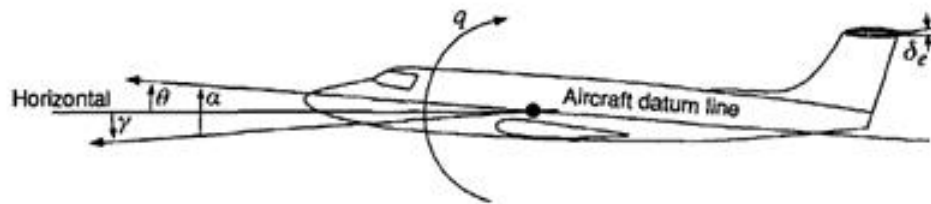


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Gibson's HQ Criteria

Modeling Pilot Physiology, Variability and Individual behavior



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Gibson's HQ Criteria

Modeling Pilot Physiology, Variability and Individual behavior

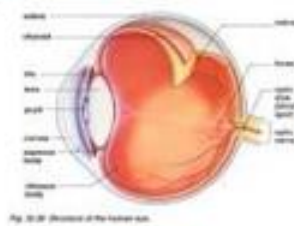


Fig. 10.20 Structure of the human ear.



- These produce an acute consciousness of attitude displacements, rates and angular accelerations, but a poor sense of linear velocity and acceleration.
- Most handling qualities are to a great extent perceived through the qualities of the attitude response of an aircraft.

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Gibson's HQ Criteria

Modeling Pilot Physiology, Variability and Individual behavior

Mathematical description of Pilot's role in a closed loop system

$$Y_p = \frac{K_p e^{-\tau s} (T_L s + 1)}{(T_I s + 1)(T_N s + 1)}$$

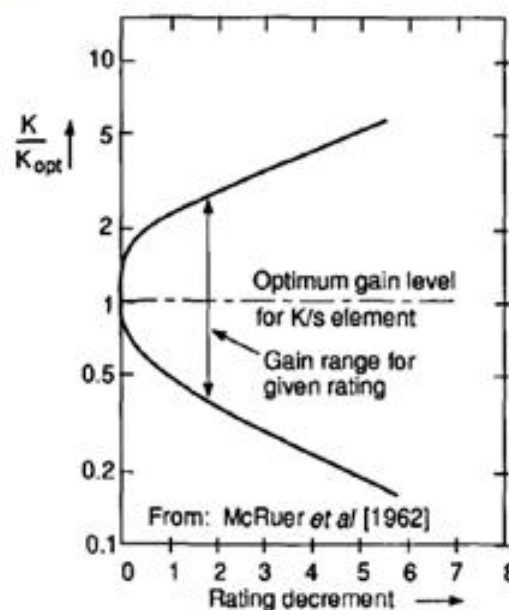
- The gain, lag and lead comprise the pilot equalization or compensation efforts. These were found to be the consequence of pilots' efforts to minimize the tracking error.
- Pilots were compensating for aircraft characteristics to produce a closed loop aircraft/pilot response that was K/s like.

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Gibson's HQ Criteria

Modeling Pilot Physiology, Variability and Individual behavior



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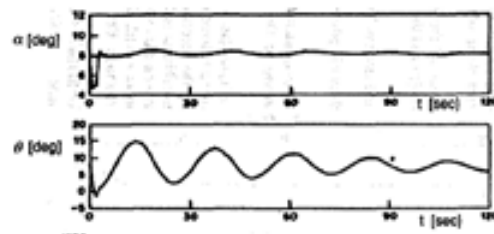
Modeling Pilot Physiology, Variability and Individual behavior

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Pilot – Engineer communication

- The control design engineer can synthesize directly the handling characteristics desired for specific tasks as perceived visually or physically by pilots.
- Their meaning should be equally clear to both pilots and control design engineers."



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Control System Hardware effect on Handling Qualities

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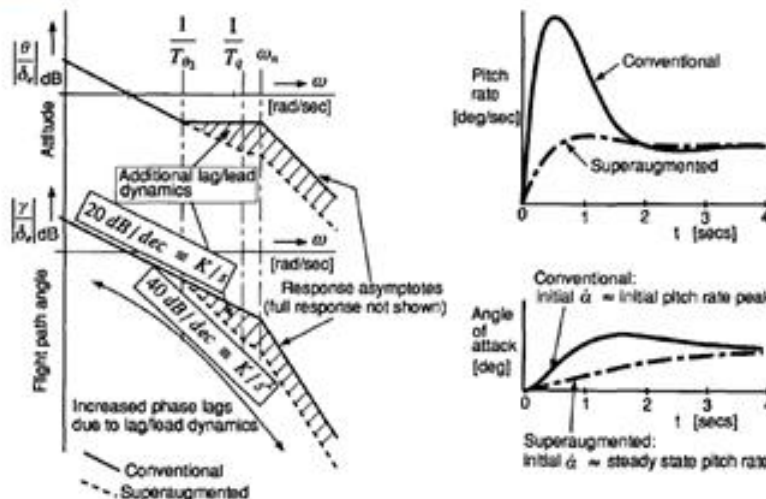


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Gibson's HQ Criteria

Drop back Criteria: Designing in Excellent Short Period Pitch Handling Qualities



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Gibson's HQ Criteria

Drop back Criteria: Designing in Excellent Short Period Pitch Handling Qualities

Criteria and Design Methodology

- Expressed in frequency and time domain
- Define limits on the shape of the response not numbers/formula, so independent of the order of the system
- No dependence on explicit pilot models
- Synthesize directly the handling qualities desired
- Equally understandable by pilots and control engineers.

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Gibson's HQ Criteria

Drop back Criteria: Designing in Excellent Short Period Pitch Handling Qualities

$$\frac{1}{2} \rho V^2 S C_{L\alpha} \approx 1/T_{\theta 2}$$

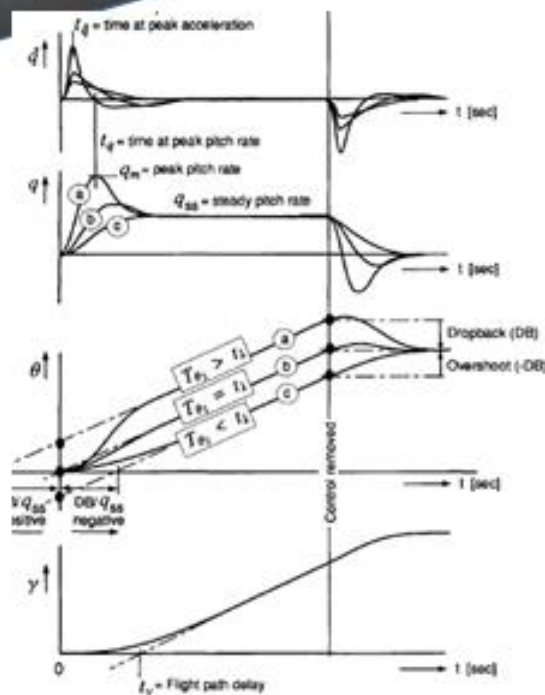
$$T_{\theta 2} \approx 2 / (\rho V^2 S C_{L\alpha})$$

Where: ρ = air density
 V = true air speed
 S = wing area
 $C_{L\alpha}$ = wing lift coefficient

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Gibson's HQ Criteria

Drop back Criteria: Designing in Excellent Short Period Pitch Handling Qualities



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Gibson's HQ Criteria

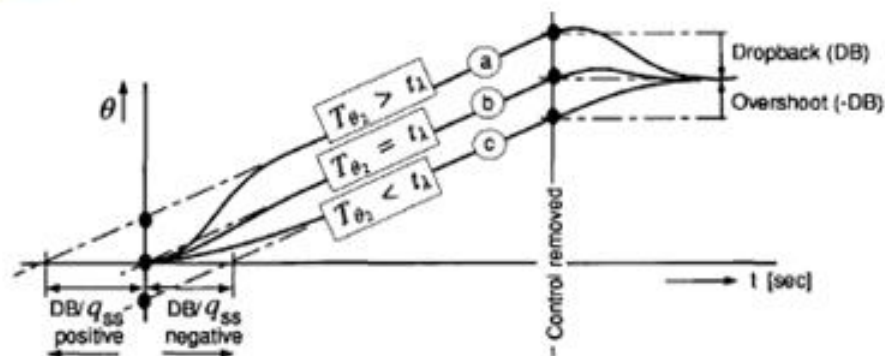
Drop back Criteria: Designing in Excellent Short Period Pitch Handling Qualities



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Gibson's HQ Criteria

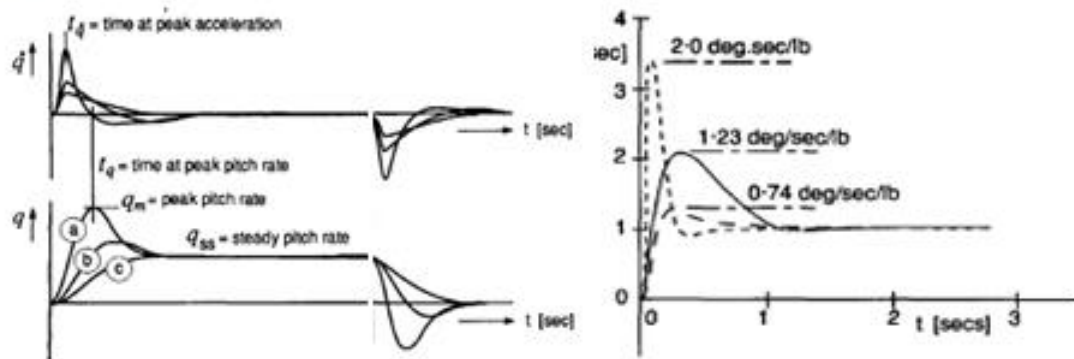
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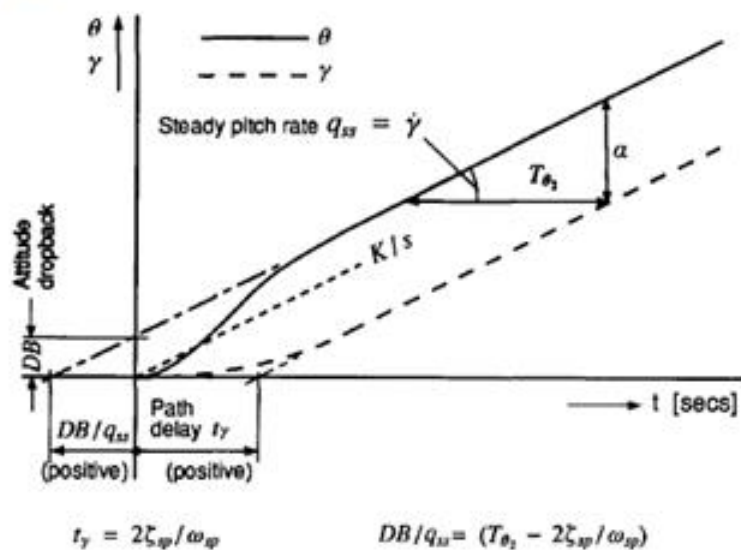
Drop back Criteria: Designing in Excellent Short Period Pitch Handling Qualities



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Gibson's HQ Criteria

Drop back Criteria: Designing in Excellent Short Period Pitch Handling Qualities



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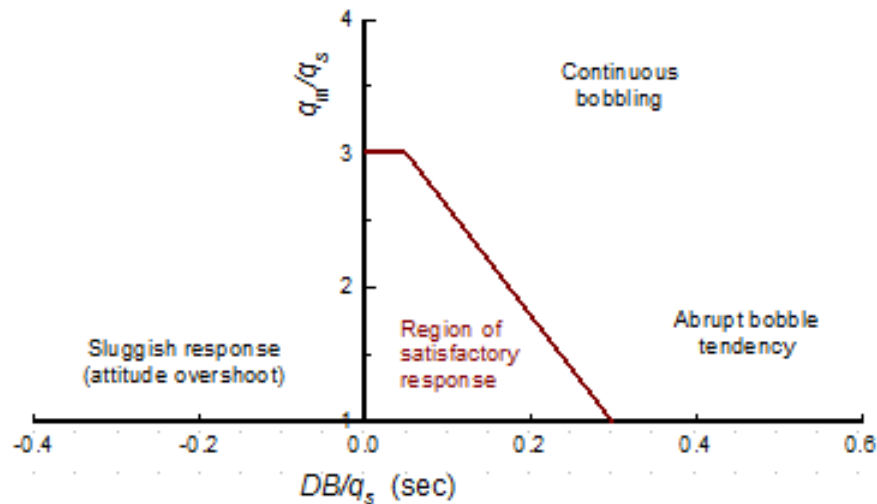
Slide 65 (2 min / :30 post break / 1:25 overall)

Gibson's HQ Criteria

Drop back Criteria: Designing in Excellent Short Period Pitch Handling Qualities

Time Response Analysis results:

Dropback Criteria



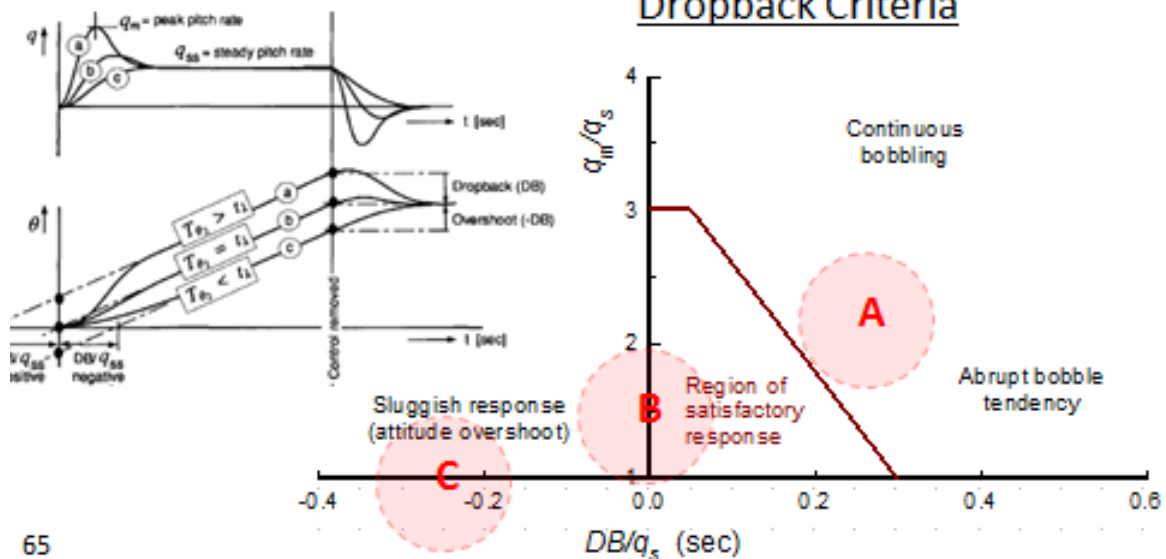
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Gibson's HQ Criteria

Drop back Criteria: Designing in Excellent Short Period Pitch Handling Qualities

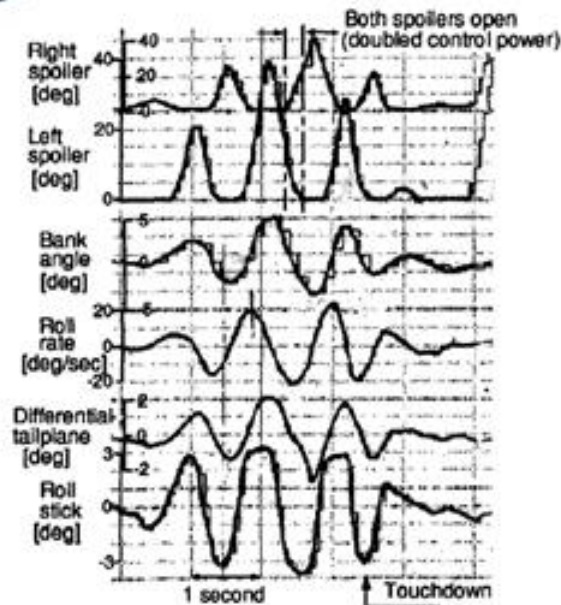
Dropback Criteria



65

Gibson's HQ Criteria

*Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits*

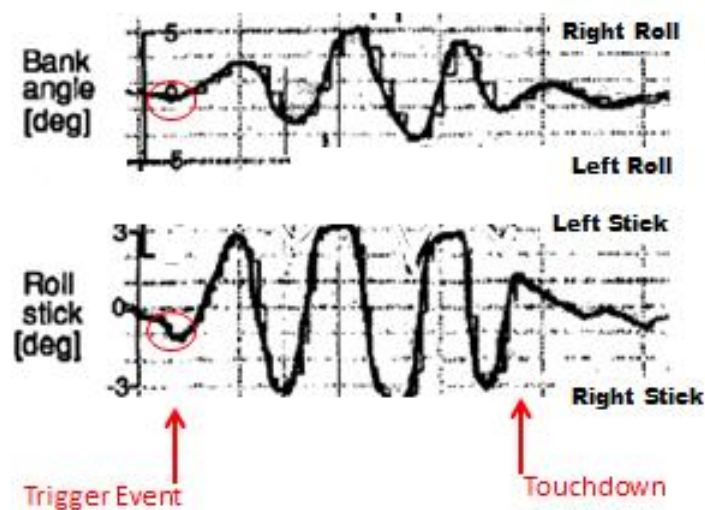


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Gibson's HQ Criteria

*Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits*



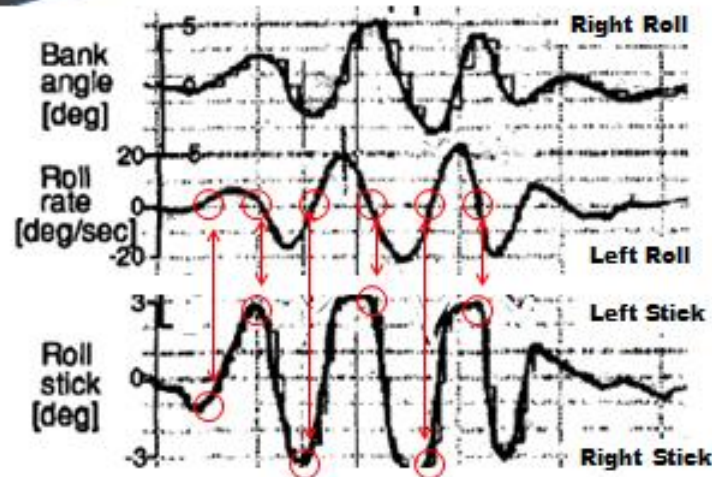
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Gibson's HQ Criteria

*Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits*

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Stick reversals at roll
rate reversal

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Gibson's HQ Criteria

*Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits*

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Gibson's HQ Criteria

*Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits*

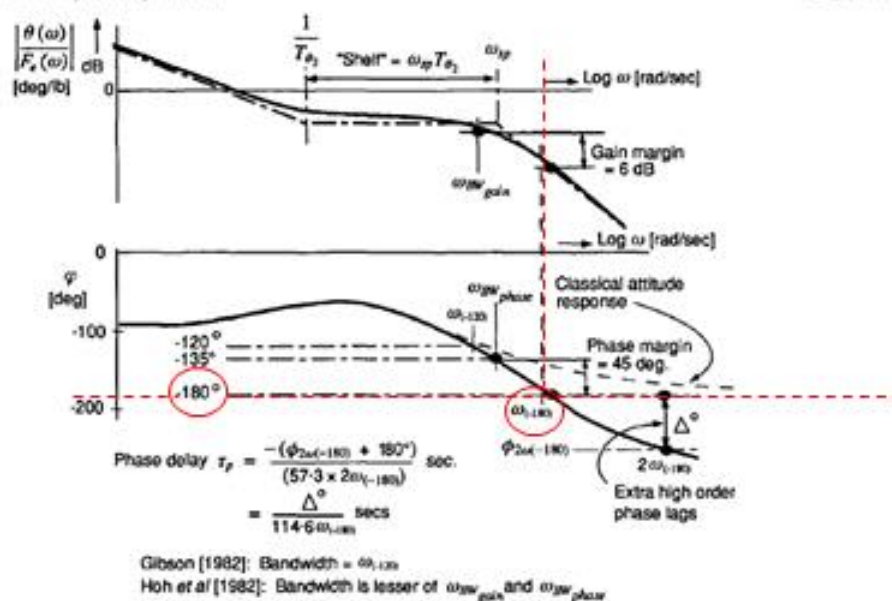
- Low Order PIO
 - PIO Syndrome: Poorly shaped open loop aircraft dynamics
 - Bobble PIO: excessively abrupt AoA response typically called PIO by pilots
 - Overdriving PIO: low bandwidth open loop aircraft dynamics
- High Order PIO
 - Occurs at or slightly higher frequency where open loop phase lag reaches 180°
 - open loop phase lag reaches 180° at low frequency, <1.0 Hz or 6.3 rad/sec
 - Rapid increase in phase lag at this frequency
 - Open loop response gain is large

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Gibson's HQ Criteria

*Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits*

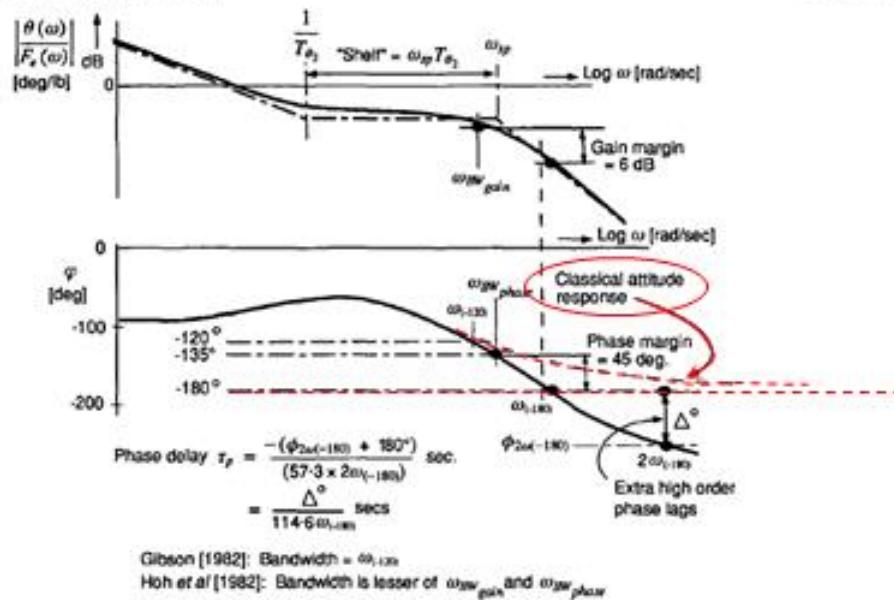


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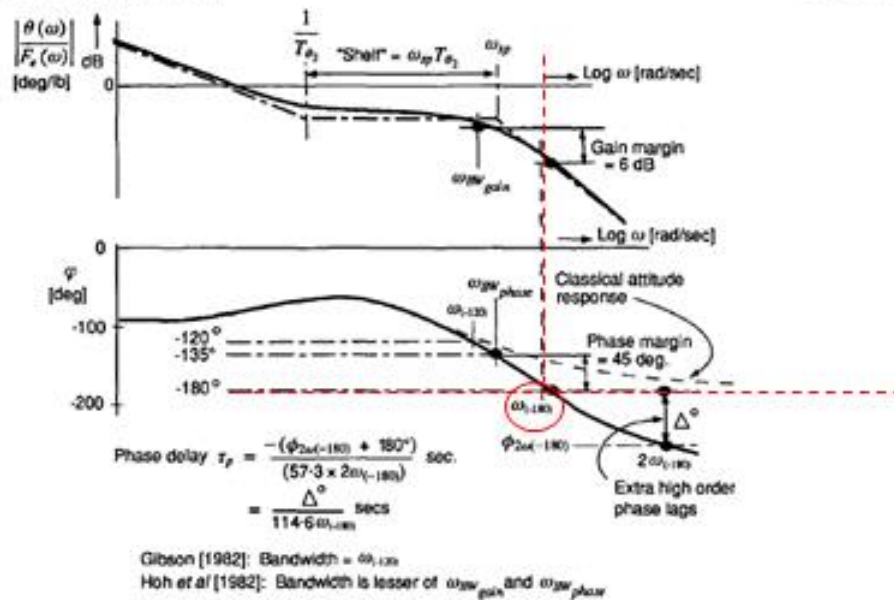
Gibson's HQ Criteria

Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits



Gibson's HQ Criteria

Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits

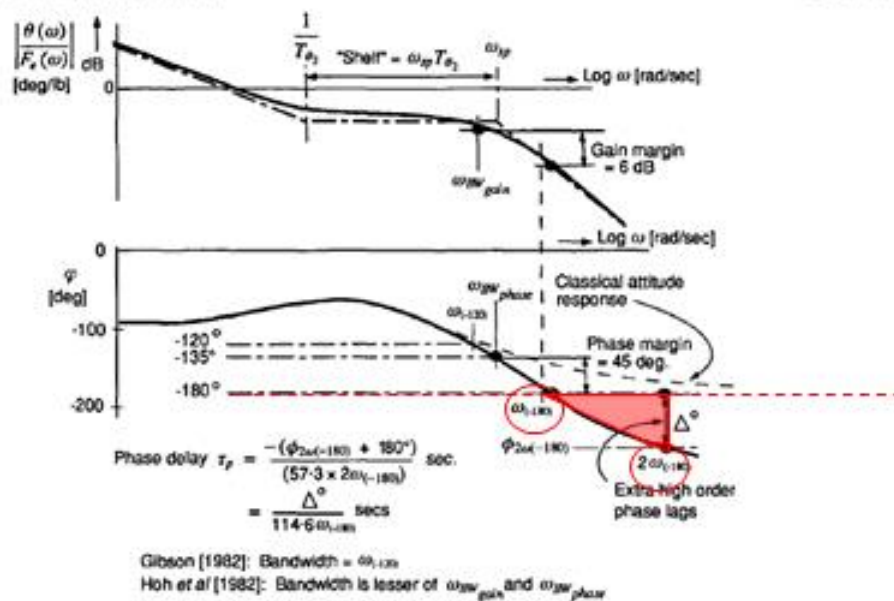


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Gibson's HQ Criteria

Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits

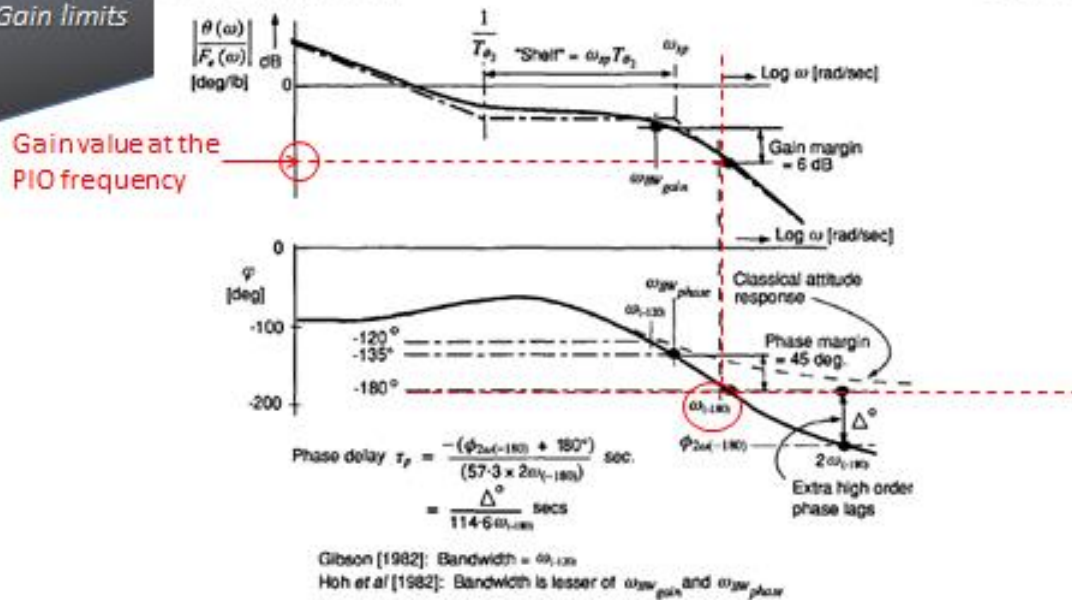


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Gibson's HQ Criteria

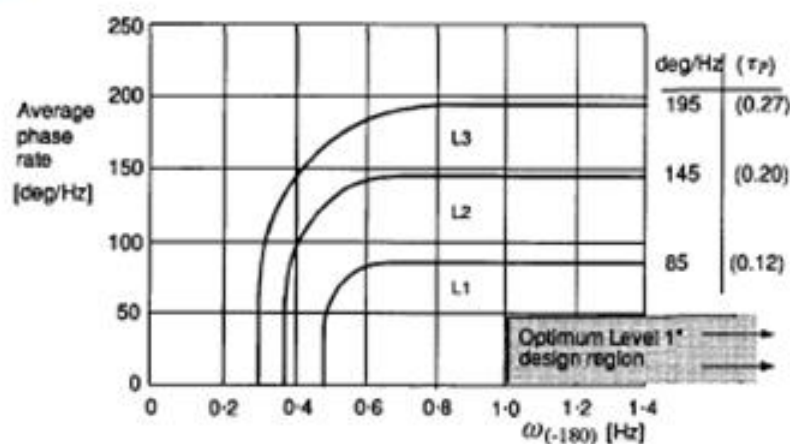
Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits



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Gibson's HQ Criteria

Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits

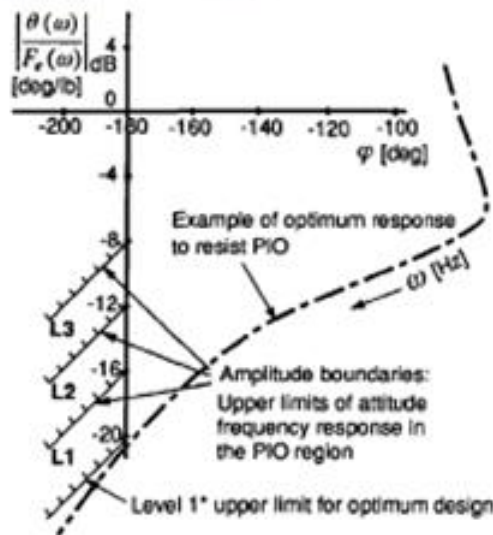


(c) Phase rate and frequency criterion

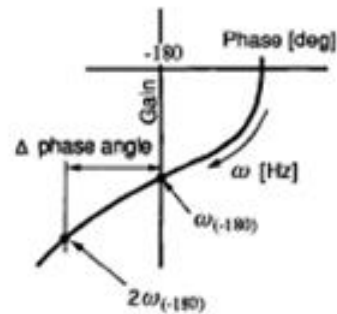
77

Gibson's HQ Criteria

Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits



(a) PIO gain limit criterion



$$\text{Average phase rate} = (-\Delta \text{ phase angle} + \omega_{(-180)}) \text{ deg/Hz}$$

$$\text{Phase delay } \tau_p = \lceil \text{Average phase rate} + 720 \rceil$$

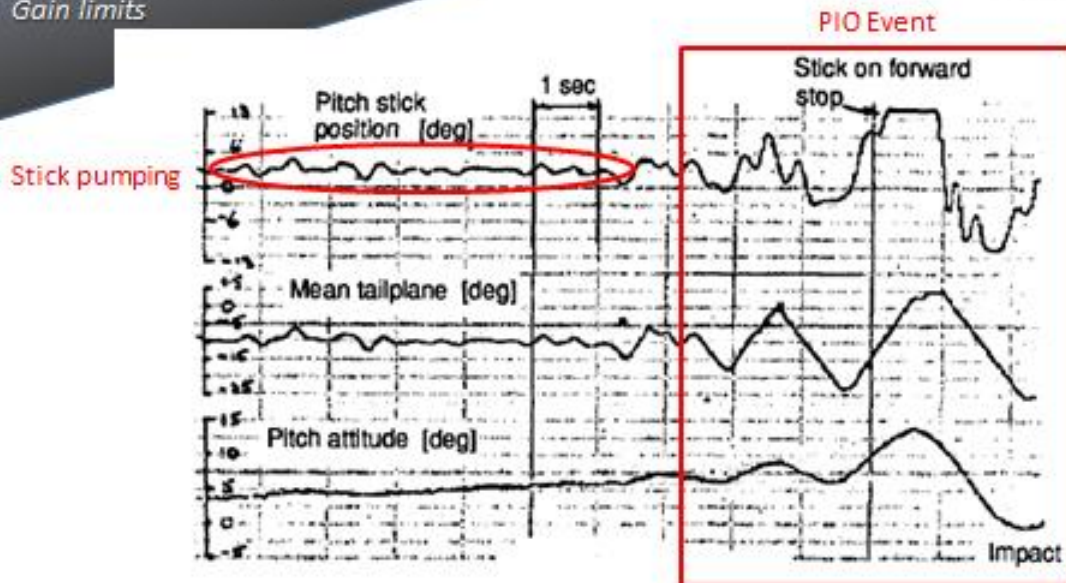
(b) Definition of average phase rate

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Eliminating linear high order PIO by
Phase Rate, PIO Frequency and
Gain limits

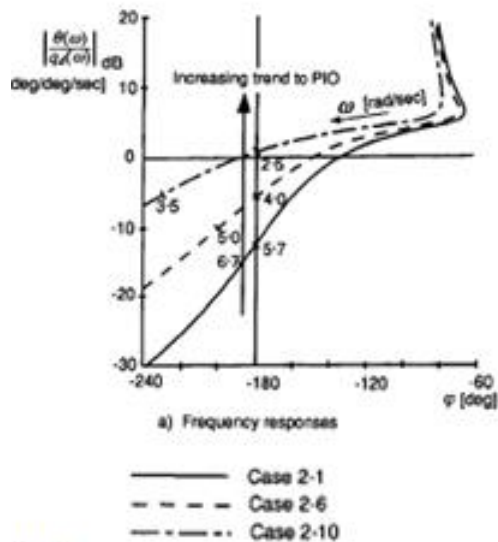


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Gibson's HQ Criteria

Eliminating linear high order PIO by Phase Rate, PIO Frequency and Gain limits

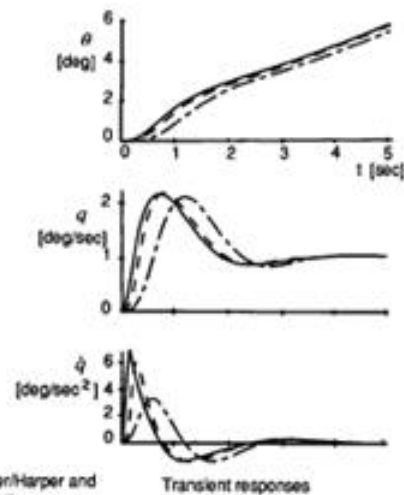


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Cooper/Harper and
PIO ratings:

2-10 PR=10 PIOR=4
Unavoidable PIO
(High lag pre-filter)

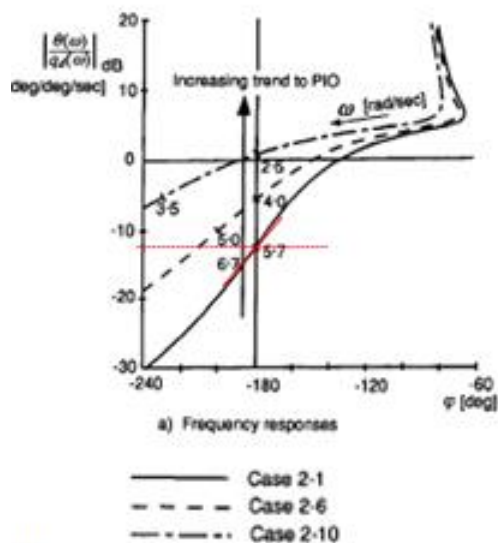
2-6 PR=5 PIOR=2½
(Low lag pre-filter)

2-1 PR=2 PIOR=1
Excellent, no PIO
(No pre-filter)

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Gibson's HQ Criteria

Eliminating linear high order PIO by Phase Rate, PIO Frequency and Gain limits

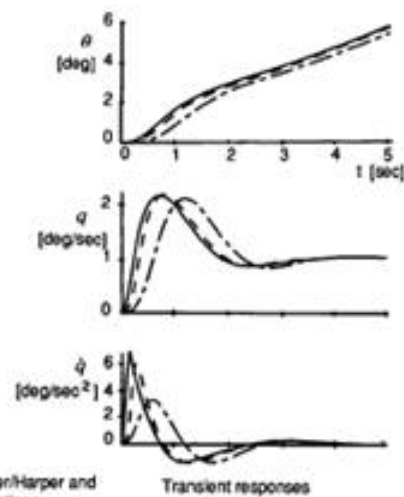


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Cooper/Harper and
PIO ratings:

2-10 PR=10 PIOR=4
Unavoidable PIO
(High lag pre-filter)

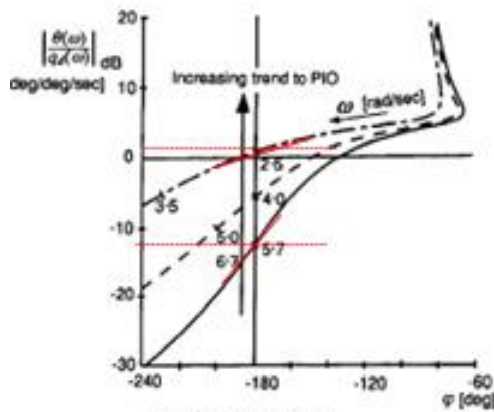
2-6 PR=5 PIOR=2½
(Low lag pre-filter)

2-1 PR=2 PIOR=1
Excellent, no PIO
(No pre-filter)

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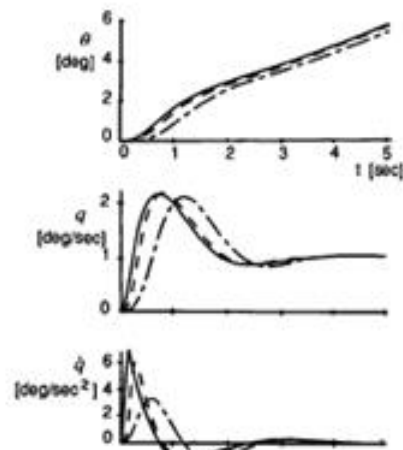
Gibson's HQ Criteria

Eliminating linear high order PIO by Phase Rate, PIO Frequency and Gain limits



a) Frequency responses

— Case 2-1
- - - Case 2-6
- · - · Case 2-10



Transient responses

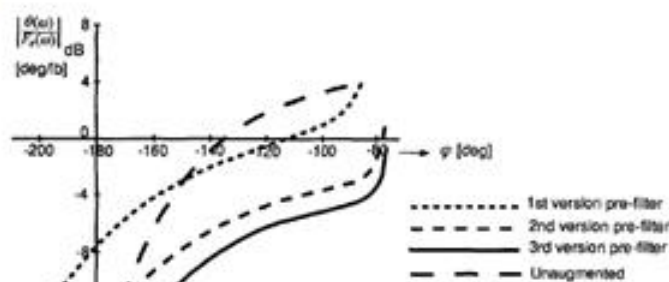
Cooper/Harper and PIO ratings:

- 2-10 PR=10 PIOR=4 Unavoidable PIO (High lag pre-filter)
- 2-6 PR=5 PIOR=2½ (Low lag pre-filter)
- 2-1 PR=2 PIOR=1 Excellent, no PIO (No pre-filter)

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Gibson's HQ Criteria

Eliminating linear high order PIO by Phase Rate, PIO Frequency and Gain limits



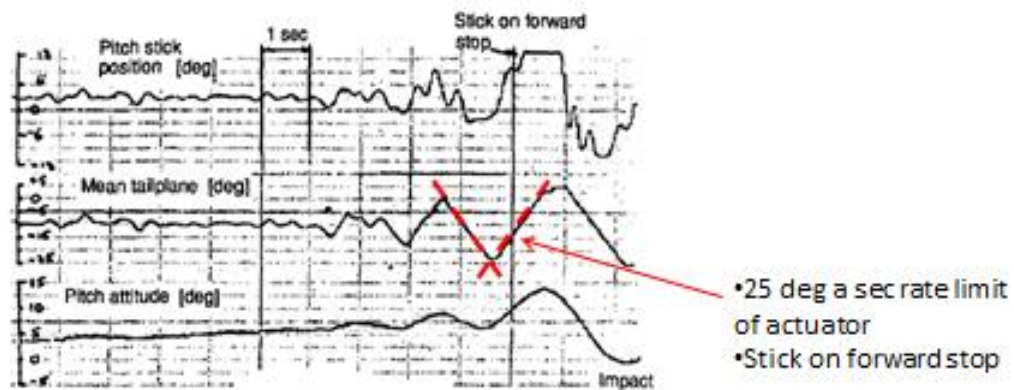
1st version pre-filter
2nd version pre-filter
3rd version pre-filter
Unaugmented

Version	Bandwidth ω_{10} rad/sec	PIO frequency ω_{PIO} rad/sec	Time to transient acceleration peak $t_{1/2}$ sec	Pre-filter characteristics
1st	0.245	0.43	0.35	$(1 + 0.157s)(1 + 0.47s)$
2nd	0.245	0.455	0.3	$(1 + 0.133s)(1 + 0.454s)(1 + 0.0625s)$
3rd	0.25	0.535	0.22	$(1 + 0)$
Unaug	0.145	0.44	0.12	$(1 + 0)$

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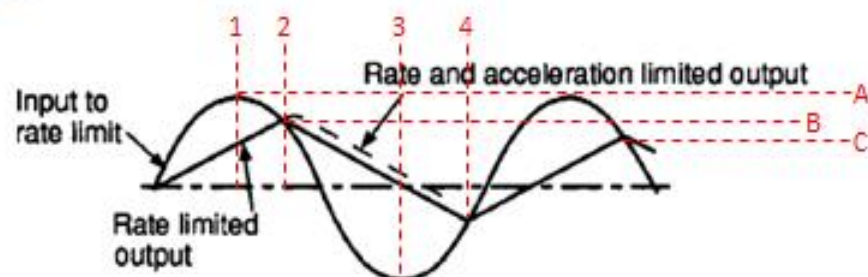
Understanding non-linear PIO



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Gibson's HQ Criteria

Understanding non-linear PIO

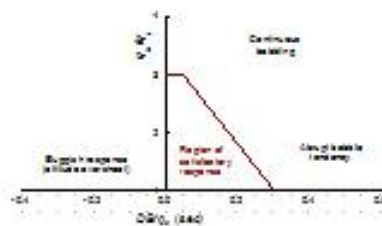
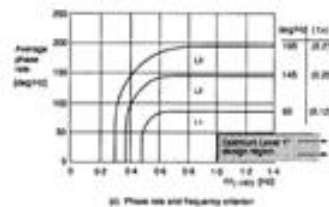
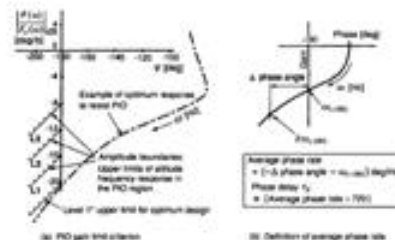


(a) Non-linear gain and phase effects due to rate and acceleration limiting.

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Gibson Criteria

A primer for Test Pilots and Flight Test Engineers on John Gibson's Handling Qualities Criteria for High Order FCS



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Gibson's HQ Criteria

Introduction & Background

Topics of Presentation

- Control Engineering & Handling Qualities Background
- Historical Handling Qualities Criteria Review
- The Control Loop Variable
- Pilot Variability
- Control System Hardware
- Dropback Criteria for excellent SPPR Handling Qualities
- Eliminating linear PIO through Phase Rate, PIO frequency and gain limits
- Non-Linear PIO

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Gibson's Criteria

A primer for Test Pilots and Flight Test Engineers on John Gibson's Handling Qualities Criteria for High Order FCS

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Part 2 - Complete Questions

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4.3 APPLYING GIBSON'S CRITERIA: A WORKED EXAMPLE OF THE A-7A

This is the second lecture of the series and is designed as a general example of how to approach implementing a Gibson pre-filter to improve an aircraft's performance according to Gibson's Criteria. It is 38 slides and approximately 40 minutes long. Additionally, exercise B, the A-7A, is worked through. The topics covered are:

- Review of Critical Attitude Parameters
- Gibson Pre-Filter, Control System & Transfer Functions
- Worked Exercise of the A-7A

4.3.1 Applying Gibson's Criteria Lecture

Slide 1

Good afternoon. My name is LT Jack Hathaway. (*Biographical statement*)

Slide 2 (1 minutes for this slide/ :01 overall)

John Gibson was a British aerodynamicist closely associated with Delft University of Technology. He worked on the design and development of stability and control systems for aircraft from the dawn of the jet age through the advent of modern fly by wire, high order aircraft including the BAE Lightning, the Tornado, European Airplane project, and Eurofighter. During the Tornado's development, several PIO incidents caused him to look more specifically at the effect of high order dynamics of hydraulically powered and computer controlled aircraft on handling qualities. He found the contemporarily accepted standards that were based on conventional second order aircraft to be wanting and developed a set of criteria to be used during the design phase to ensure the absence of PIO and the presence of excellent pitch handling qualities. 'Gibson's Criteria' were designed to be understood intuitively by pilots and control system engineers alike.

This lecture is designed to follow the two parts of the previous lecture: Gibson's Criteria: A primer for test pilots and flight test engineers. We will be working a simple example at the end of this lecture.

Slide 3 (2 min/ :03 overall)

A High Order computer controlled Flight Control system can be called fly by wire or, as Gibson referred to it, superaugmented. It is "any flight control system that provides behaviour almost independent of the aerodynamic qualities through the use of" software translating pilot "demanded responses such as normal acceleration, angle of attack and flight path angle" into pre-programmed control surface movements to produce the desired response. [GT, para 4.1, pg 37]

"The principal driver for the development of fly by wire technology has been the improvement of combat aircraft performance by reduced or negative angle of attack stability[Aircraft with this instability] cannot be controlled by a human pilot, it is necessary to employ continuously operative stability and command augmentation using multiple signal paths for reliability and safety." [GT, para 4.1, pg 37]

HOFCS also "...enable the ready implementation of functions such as manoeuvre limiting to protect the airframe from overstressing or from stall, regardless of weight or centre of gravity variations.....allow[ing] the pilot to concentrate on tactics and weapon operation without distraction by concern for the safety of the aircraft itself." [GT, para 4.2, pg 38]

"Since fly by wire systems can be made to provide any desired response dynamics to a reasonably close degree, it is a simple task to design the control laws to [Gibson's pitch handling qualities] criteria, providing good handling from the start rather than after [an expensive and lengthy] trial and error process of flight development.

The basic tools for this task are a direct feed forward path from inceptor to control actuation to provide instantaneous accelerations and a command path pre-filter as appropriate to tune the response dynamics. The usual aim is to provide an aircraft open loop response that resembles closely, or as closely as is chosen, the idealized K/s integrator-like dynamics with which the pilot can easily interact with low workload.” [GT, initial summary, no page given]

Slide 4 (1.5 min/ :04.5 overall)

“Good handling is not just a means for giving enjoyment to the pilot. It performs an essential part in task performance. The combat pilot must devote all possible attention to the management and operation of a variety of complex weapons and defensive systems against an increasingly capable opposition. The ability actually to fly the aircraft with minimum attention to safety and structural limits releases considerable mental capacity to the combat task.”

“ Governments, the owners of combat aircraft, enhance the efficiency of their forces when the less skilled and experienced pilots can fly their aircraft as aggressively as their more skilled colleagues.”

“Up to the 1940s, aircraft could be tuned to give desirable handling, though sometimes only by expending considerable effort. [Combat aircraft could require] hundreds of hours of flight testing spent developing the control system balance, tab and spring gearing and control circuit[s]. In the post-1940s era, the handling qualities were dominated by the increasingly poor aerodynamic and inertial properties of aircraft. These could be ameliorated at best by stability augmentation that was sometimes mechanically quite complex, but they could seldom be made perfect even assuming that such a state could be adequately defined. In the rapidly growing sphere of fly by wire technology, in both military and civil applications, the means for good handling resides largely (but certainly not only) in software code. It costs little more in software overheads to write "good" code rather than "bad" or merely careless code, assuming again that the desirable end result is known. The overall effort required to write the software is very large and should not be wasted to any degree on inferior handling code.” [GT, para 10.1, pg 200]

Slide 5 (1.5 min/ :06 overall)

The discussion of handling qualities and how various criteria can be used to evaluate aircraft fully are both topics that can only be fully explored to their full depth through many years of academic study, industrial application and perhaps regulatory experience.

The previous lecture you received was a few hours of discussion on control engineering in general and Gibson's Criteria more specifically as part of an extensive course for flight test professionals. We did not cover every nuance and detail of the topics. Specifically, Gibson's criteria is limited to fixed wing combat fighters and attack aircraft as larger bombers, or tactical and strategic transports require different assumptions and considerations to address their flying qualities as well as how pilots apply different techniques and concepts to operation and mission execution.

This lecture assumes that the heavy legwork of developing the complete framework and definition of the control structure has been completed. The transfer functions have been linearized and are valid for small perturbations from trim. We are only applying the criteria to the pitch axis and of that, only really the short period. When we apply the criteria, we will reduce the order of the transfer functions by fixing speed, a simplification necessary to avoid contaminating the pitch response with speed effects and an assumption valid in that we understand that speed will vary a miniscule amount if at all for the time frames and maneuvers we are looking at.

Slide 6 (1 min / :07 overall)

Our topics of presentation today include a review of critical attitude parameters, the placement and effect of Gibson based Command Path pre-filter on control systems and transfer functions, and finally a worked example on evaluating the A-7A Corsair II according to Gibson's criteria and then applying the pre-filter to assess its effect.

Slide 7 (3 min / :10 overall)

Gibson believed a new criteria and design methodology for super augmented flight control systems was necessary. While it was generally agreed that the short period pitch response characteristics were the fundamental determinant of good handling qualities, formal specifications still relied heavily on the accepted regions of short period natural frequency and the damping ratio, or the even more generally unhelpful specification that handling qualities of super augmented aircraft should be as good or better than their mechanically controlled brethren. [paraphrased from GT, para 7.1, pg 94]

This deficient formal guidance in the specifications was based on lack of widely accepted criteria, so regulators relied on what they knew from the past. This was clearly inadequate for super augmented systems. With these systems, response characteristics and handling qualities were "dominated by the flight control system and the aerodynamic parameters tend to have greatly reduced influence, the simple modal parameters equivalent to the conventional frequency and damping are usually absent. There may be several poles and zeros clustered quite close to frequencies that are typical of the short period mode of conventional aircraft, and it is often impossible to decide which one should be taken to quantify the handling. Even if there is only one response mode with a suitable "short period" frequency, it may not be in angle of attack, the parameter conventionally operated on by pilots' control inputs. It will be the parameter controlled by the maneuver demand system, for example pitch rate." It became "of the utmost important to be able to quantify desirable characteristics in ways other than by the traditional expressions of the short period mode parameters. "[GT, para 7.1, pg 93]

Gibson used existing data by searching the "available literature in papers, journals and reports for results of handling qualities flight research. Only in-flight data was considered to be suitable, and this was mostly provided by variable stability aircraft studies such as those of Calspan as early as the 1950s." [GT, para 7.1, page 94] Gibson pursued his goal with the following "Criteria and Design Methodology:

“a) They would have to be expressed in both frequency and time domains, as neither on its own give a sufficiently complete design definition.

b) They should in particular define limits on the shape of the responses in these domains without reference to numerical mode parameters or mathematical formula, so that they could be applied to any controlled response however high its order, and independent of the type of model available, i.e. parametric or non-parametric.

c) They should not depend on the assumption of any explicit pilot model parameters, as it is very difficult to apply these reliably.

d) They should enable the control design engineer to synthesize directly the handling characteristics desired for specific tasks as perceived visually or physically by pilots.

e) Their meaning should be equally clear to both pilots and control design engineers.”

Gibson determined “the significant attitude parameters were ... the attitude dropback, pitch rate overshoot ratio ...and sensitivity....explicitly connected to the classical short period by the parameter $T_{\theta 2}$.” [GT, para 7.1, pg 94-95]

Slide 8 (Slides 8-10, 1 min / :11 overall)

As a brief review, these figures are time domain plots of aircraft response to a longitudinal step input at time zero that is removed about two thirds of the way along the time axis. There are 3 different aircraft responses on these figures, each with a different $T_{\theta 2}$ value, labeled a, b and c. The top plot is pitch rate and the bottom plot is pitch attitude.

Slide 9 (Slides 8-10, 1 min / :11 overall)

Dropback, or its opposite negative dropback or overshoot, is a measure of how much the pitch attitude changes from the moment the pilot input is removed to its steady state value.

Slide 10 (Slides 8-10, 1 min / :11 overall)

The pitch attitude overshoot is evaluated as a ratio between the peak pitch rate, q_{max} , and the steady state pitch rate, q_{ss} . Appropriate selection of this value inherently solves any handling issues associated with the pitch sensitivity.

Slide 11 (0.5 min / :11.5 overall)

This figure is a time domain plot of pitch attitude, the solid line, and flight path, the dashed line following a step input at time zero. The step input removal is not shown on this plot.

After much research and analysis, Gibson came to the conclusion that, in general, the best handling qualities occurred when the pitch attitude response closely mirrored a “K/S-like attitude response. [Thus,] a step control input should result in a ramp-like profile commencing at time zero, apart from the inevitable initial transient dynamics.” [GT, para 10.4.1, pg 206]

Slide 12 (1.5 min / :13 overall)

"In the special case of optimized precision attitude control with a K/s -like response, little attitude dropback and moderate pitch rate overshoot ...the flight path time delay closely matches the attitude response numerator coefficient $T_{\theta 2}$." [GT, para 10.4, pg 206]

In the case shown, we can compare $T_{\theta 2}$ to the path delay [Click Play for 'Move Path Delay'] and see that $T_{\theta 2}$ is larger by a noticeable amount [Click Play for ' θ_2 - Path delay '], equivalent to the dotted line here. Moving that line down to the dropback measured on the x axis [Click Play for 'Compare to DB'], we can see the difference is largely a measure of the dropback inherent in this control system.

In conventional aircraft, $t_{\theta 2}$ cannot vary. It stays at whatever the aerodynamicist designed from the outset. The metrics of natural frequency and the damping ratio are key descriptions of the short period pitch response characteristics and thus can quantify how the transients at the beginning and end of the pilot input will behave. Gibson, however, believed that they weren't actually the critical variable. Constraining them to certain values as handling qualities criteria have done for most of the 20th century limited the size of the flight path delay. This, in turn, in second order systems, constrained the dropback and produced the good handling qualities in many conventional aircraft.

Gibson's breakthrough lied in his recognition that not only did the aircraft design team want to control dropback directly instead of indirectly, but that was now possible with computer controlled systems. The aerodynamicist could produce a wing and aircraft shaped to perform aerodynamically ideally without regard to its effect on the short period or $t_{\theta 2}$, as those values could be contrived by the control engineer during the flight control software design phase.

Figure 7-5 from GT, page 122

Slide 13 (1 min / :14 overall)

This is a block diagram of a simple control system that can be used to show where the command path pre filter that Gibson advocated will be placed. The system starts with an input from a pilot inceptor on the left, an inceptor gain is applied, then the Gibson pre filter is applied outside the closed loop. Inside the closed loop are some proportional gain blocks and an integrator that are summed prior to being applied to the aircraft dynamics which results in the output, in this example, pitch rate. Pitch rate is then fed back into the system. Gibson believed that there must be a direct a path as possible from the input to the output to maintain the physical connection from the pilot to the motions. His pre filter lies along that command path, not in the feedback loop.

Slide 14 (1 min / :15 overall)

As we have covered before, the mathematical method by which the control engineer accomplishes the insertion of the Gibson command path pre filter is via the transfer function. On the left side of this slide is a generic pitch attitude over inceptor input transfer function. The conventional short period parameters of natural frequency and the damping ratio are in the denominator and $T_{\theta 2}$ is in the numerator. This is a second order transfer function and we are using it to keep the math simple and accessible.

In general, as you increase the order of the transfer function, the denominator becomes significantly more complex as the additional modes are included and it may become impossible to determine what the correct short period description is. However, the numerator will always contain a term for the basic aerodynamic value of $T_{\theta 2}$. Using a second order example is a useful simplification for these exercises as we don't lose any of the underlying physics while leaving out unnecessary complexity.

On the right side of this slide is the generic form of Gibson's command Path pre-filter function. The actual aerodynamic $t_{\theta 2}$ goes in the denominator, while the new $T_{\theta 2}$ we have selected to minimize dropback and the pitch overshoot ratio goes in the numerator.

Slide 15 (Slides 15-17, 1 min / :16 overall)

Since the pre filter will be in series with the transfer function as we saw in the block diagram a few slides ago, we multiply the two functions together.

Slide 16 (Slides 15-17, 1 min / :16 overall)

The aerodynamic $t_{\theta 2}$ terms will cancel out as designed and the control engineer designed $t_{\theta 2}$ value will appear in the numerator of the new transfer function.

Slide 17 (Slides 15-17, 1 min / :16 overall)

By putting in this pre filter, the only difference in the pitch attitude transfer function will be that the aerodynamic $t_{\theta 2}$ will be replaced with the new specifically chosen value.

"The resulting attitude transfer function remains classical in form, but behaves in response to pilot commands as though the wing loading and/or lift slope have been altered." [GT, para 10.4.1, pg 206]

Slide 18 (1 min / :17 overall)

This may seem like a great idea, but nothing happens without some sort of penalty. While the pitch attitude transfer function has not increased in order or had adverse effect, not all the transfer functions have the same format

On this slide, the original normal acceleration transfer function is shown on the left. It does not have $T_{\theta 2}$ in the numerator, but instead velocity. There is no way to apply the Gibson pre filter to the pitch attitude transfer function alone. It is applied to the system, and therefore affects every transfer function, and in several cases such as this one or the flight path transfer function, it increases the order of the transfer function. This should not be done carelessly and must be considered and investigated.

As we have discussed before, it is critical to understand what variable the pilot is closing the loop around in what part of the envelope. If a pilot is using flight path or normal acceleration and the control engineer has applied a Gibson pre filter and let attitude be the dominant control variable, there may be a dangerous high order effect hidden in the complexity. This concern is why the entire control law must be mapped out and careful consideration applied to which part of the envelope is governed by what mix of control variables.

In our simple examples, this is not a factor, but the possible unintended consequences should be investigated in a real world system.

Slide 19 (Slides 19 & 20, 1 min / :18 overall)

Having discussed the theory, it is now time for a practical example. Given a set of linearized equations of motion matrices for a specific flight condition of an A7 Corsair II aircraft, design a command path pre filter in accordance with Gibson's Criteria. Use of software like Matlab to perform the calculations will be required.

Slide 20 (Slides 19 & 20, 1 min / :18 overall)

Before we can design and apply a pre-filter, the provided equations of motion must be evaluated for their stability characteristics, and feedback introduced to stabilize the system if required. Gibson's criteria requires speed to be fixed at the initial value, so a reduced order model to eliminate speed variability and the phugoid will be used. An actuator lag will be added to notionally simulate the higher order nature of the systems Gibson created this criteria for. We will assess the system against both the dropback time domain criteria and the frequency response criteria of PIO frequency, phase rate and PIO gain limits. Finally, we will design and apply a pre-filter and then re-assess according to the criteria.

Slide 21 (Slides 21 & 22, 1 min / :19 overall)

Full 6 degree of freedom non-linear models of modern aircraft are extremely complex and unwieldy. In this example problem and the subsequent assignments, you will be provided the equations of motion that have been linearized around a specific flight condition in a simple format. In this case, the state space equations have been provided in full, and simply need to be typed into matlab. You'll notice the C matrix has 6 rows versus the 4 rows for the A and B matrices. It has already been modified to output to additional values, angle of attack and flight path. The C matrix is very useful for this type of activity and you should be aware of this option going forward.

Slide 22 (Slides 21 & 22, 1 min / :19 overall)

Here is a screen shot of what I have typed into Matlab for the A, B, C, and D matrices compared to the given values in the upper right. There are only a few small differences. Notably, the C matrix has a factor in multiple places called F2M or feet to meters. In addition to being able to calculate other output values, you can use the C matrix to convert to the correct units, in this case feet to meters, but often this is a place to convert from radians to degrees or vice versa.

Slide 23 (1 min / :20 overall)

The last part of Task 1 is to identify the stability characteristics. The Damp function in Matlab will take the collection of aerodynamic derivatives in the state matrix and pull out the eigenvalue roots and compute the damping ratio and natural frequency. In this case, all the roots are negative, telling us that the system is stable.

The short period natural frequency is 1.63 rad/sec with a damping ratio of 0.276. The phugoid natural frequency is 0.14 rad/sec with a damping ratio of 0.119.

So we have a stable system with relatively low frequencies and damping ratios. If there was a positive root, feedback would be required to stabilize, or we could simply add feedback to attempt to change the stability characteristics closer to idealized values of perhaps 0.7 damping ratio and a higher natural frequency. For this example, we have omitted that step to focus on applying Gibson's Criteria, but some of the follow on exercises include a requirement to introduce feedback.

Slide 24 (1 min / :21 overall)

One of the critical assumptions when applying Gibson's Criteria is that airspeed is not allowed to vary following the step input. This assumption allows the control engineer to eliminate the contribution of speed changes to pitch handling and look solely at the aircraft and pilot closed loop system. Additionally, by eliminating speed and theta from the control law, the phugoid mode is eliminated.

This is not complex with the matrix format, the U and theta lines can be crossed out and the reduced order format written and put into matlab in its smaller format. In this case, the output matrix, C also includes a third row to output angle of attack.

Slide 25 (1 min / :22 overall)

Having the state functions still requires conversion to a transfer function so we can operate with the system by looking at time and frequency domain plots and applying example inputs such as a step function. Matlab uses a function called ss2tf and the inputs are the A, B, C, and D matrices. The ss2tf function converts the matrices into two matrices that describe the coefficients of the numerator and denominators of the transfer functions. Each row of the numerator matrix corresponds to the rows of the output matrix, in this case, vertical velocity, pitch rate and angle of attack. These can be separated into individual transfer functions by the tf function, which divides the assigned row of the numerator by the denominator.

You can also see the zpk function wrapped around the tf function. The zpk stands for zero pole gain, and will attempt to factor the numerator and denominators of the transfer function rather than simply output them as coefficients for the various orders of s.

This process works for any variable in the output matrix. To get pitch attitude and flight path, which aren't, some calculation is required.

Slide 26 (1 min / :23 overall)

As always, displacement is the integral of velocity, so pitch attitude is the integral of pitch rate. In the frequency domain in which transfer functions operate, a transfer function can be integrated by multiplying it by 1 over s. This is shown at the top of this slide.

Flight path is the remainder when subtracting angle of attack from pitch attitude.

So the flight path transfer function is the result of the pitch attitude transfer function minus the angle of attack transfer function, as shown in the bottom half of this slide.

The denominators do not match, so the subtraction cannot be performed yet. Multiplying the second term by s over s makes the denominators the same. This raises the s order by one in the second term numerator.

This can be accounted for in the matrix by moving each of the numerator coefficients one spot to the left in the matrix. The first coefficient term is zero and is omitted to make the matrix dimensions agree for the subsequent subtraction.

The denominator is equal to theta denominator, which is equal to the pitch rate denominator multiplied by $1/s$. This can be accomplished in a similar manner, by moving each of the pitch rate denominator coefficients one spot to the left in the matrix.

Several more times in the exercise, it is required to create transfer functions from the state space format. This is accomplished in the same manner as just discussed.

Slide 27 (1 min / :24 overall)

Gibson's Criteria was designed to handle any system, regardless of its mathematical order. The addition of multiple feedback loops, computer processing, multiple actuators, etc will create a system with very high mathematical order. While we want to keep the system simple, including an actuator lag can give us a notionally higher order system in order to analyze the system according to Gibson's Criteria. The actuator lag is similar in format to the basic integrator we used earlier to transform pitch rate to pitch attitude, albeit with a larger time constant. This makes sense, as they both add an overall lag to the system. The actuator lag can be incorporated into the state space matrices as shown on the right. The `ss2tf` function can then be used again to convert to transfer functions.

Slide 28 (2 min / :26 overall)

As you may remember from the lectures, one of Gibson's design goals was to create a set of criteria that was easily understandable and assessable from plots and didn't require heavy mathematics. Conventional criteria required the eigenvalue solutions and then some further maths to calculate the damping ratio and natural frequency. For Gibson's time domain criteria, all we need is a plot of a step function and a good ruler to read off values.

I've used MatLab to create some labeled plots to make it easy to see on a power point slide. The critical Gibson parameters in the time domain are the max pitch rate during the transient, steady state pitch rate, dropback and the flight path delay. The pitch rate values are gleaned from the pitch rate transfer function on the left. For this example, the pitch rate hasn't perfectly converged to a steady state solution yet, the damping is quite low, but I selected an average value.

On the right, are the pitch attitude and flight path transfer functions. The dropback value we are looking for is actually dropback divided by steady state

pitch rate. That is determined by continuing the slope of the steady state attitude back down to the x axis.

I sample the attitude transfer functions at 5 and 8 seconds here and had Matlab draw the dashed intercept line back down. There is nothing magical about sampling at 5 and 8 seconds, just after the initial transient has settled. The dropback divided by steady state pitch rate is read as the x intercept, and in this case is about 1.2, as by convention, dropback is positive to the left of the origin. Flight Path delay is identified in the same manner, by drawing an intercept line back down from the flight path transfer function. The value is not important now, but we will use it later in the command path pre-filter.

Slide 29 (1 min / :27 overall)

The frequency domain parameters are read off a bode plot as discussed in the main lecture. The three critical values here are the PIO frequency, defined by the point at which the phase lag first crosses 180 degrees, the gain magnitude at the PIO frequency, and the phase lag at twice the PIO frequency. Those can all be read off the chart here. We are actually not looking for the phase lag values but the slope between them, called Phase Rate, but Mat Lab will easily calculate them for you once they are read off the plot.

Slide 30 (31 min / :30 overall)

Here are the Gibson Criteria charts that you have seen in the lecture. Top right is the time domain, dropback criteria. The pitch overshoot ratio was calculated to be 2.8 and the dropback over steady pitch rate as 1.2. The criteria predict poor pitch handling qualities characterized by continuous and abrupt bobbling. That seems intuitive based on the low damping and long oscillatory pitch rate trace on the last slide.

On the bottom left, is the PIO frequency gain limit on a frequency domain Nichols Chart. The negative 8.1 dB is on the bottom border of level 3 qualities, indicating a dangerous amount of gain available to the pilot at precisely the frequency PIO will occur at. This is a serious safety of flight issue and must be addressed.

On the bottom right, the average phase rate was calculated to be 27 with a PIO frequency of 0.6 hz. This lies within the level 1 criteria. This may seem counter intuitive since the previous two limits were failed. You may be surprised that the PIO frequency may be as low as 0.5 hertz, on the very slow side of average pilot inputs and still be considered level 1 flying qualities. However, Gibson designed the criteria to work in concert. A PIO frequency that slow, and therefore likely to be achieved in normal flying could be acceptable, as long as the phase rate slope was quite shallow, so the pilot could effectively see the PIO coming, not fall off a cliff, and the gain was attenuated drastically by a large negative gain magnitude preventing the sort of stop to stop, large magnitude PIO's that break airplanes and kill pilots, the gain attenuation would ensure that even if the pilot's inceptor went stop to stop, the aircraft would hardly respond and no attitude reversals would be readily apparent for the pilot to couple with.

In this case, while the phase rate and PIO frequency are acceptable, the gain attenuation is not, and it is all or nothing, so the system fails both the time and frequency domain criteria.

Slide 31 (2 min/ :32 overall)

Task 4 requires us to design and apply a command path pre-filter in accordance with Gibson's method to create better pitch handling qualities. With a system this poor, it is unlikely that this alone will produce a perfect solution, as this type of pre filter usually gets added at the end to fine tune handling qualities to be excellent after the major deficiencies have been addressed, but the process is the same.

Gibson theorized that by using a prefilter to cancel out the aerodynamic $t_{\theta 2}$ value and replace it with a new $t_{\theta 2}$ value equivalent to the flight path delay, the dropback could be reduced close to zero and the pitch overshoot ratio reduced. By making the new $t_{\theta 2}$ match the flight path delay, the pitch attitude response transfer function will be much closer to the desired K/s like performance of a predictable integration of pitch rate.

As you can see around the origin of this plot, $t_{\theta 2}$ is equal to the dropback/q plus the flight path delay. Due to a less than perfect linear fit on the time domain plot that we used earlier to calculate both dropback and flight path delay, that value for $t_{\theta 2}$ is off by about a few hundredths of a second from the value in the pitch attitude/tail plane demand transfer function. For clarity in cancelling terms, the $t_{\theta 2}$ value from the transfer function is used in this exercise.

Slide 32 (1 min/ :33 overall)

The pitch attitude transfer function is shown on the left side of this slide, with the existing $t_{\theta 2}$ term we are trying to cancel circled in red. You could simply read the term off the transfer function in the command window and type it into a script, or you can get Matlab to pull it for you.

These values are in the coefficients of the numerator matrix output by the ss2tf function. On the right of this slide, I have the code I used to create the existing and new $t_{\theta 2}$ values. If you are quick to notice, I've used row 2 of the numerator matrix, which is the pitch rate line, since pitch attitude isn't in the output matrix or the numerator matrix, we created it through math. Of course, attitude is the integral of rate, so the transfer functions only differ by a 1 over s term and therefore the numerators are the same.

Slide 33 (1 min/ :34 overall)

The command path pre-filter is designed and will be multiplied against all the transfer functions. In the case of the pitch attitude transfer function, this will cancel out the existing $t_{\theta 2}$ term and apply the new one, improving the pitch handling qualities. There is no explicit $t_{\theta 2}$ term in the flight path transfer function and so multiplying the pre-filter in will only increase the complexity of that transfer function. It is critical that this is carefully thought about. Gibson theorized that pilots most often use pitch attitude when accomplishing tasks, but occasionally close the loop around other variables such as flight path when flaring to land. Control engineers must be careful to ensure that the pre filter is present in areas of the envelope where pilots will be using attitude to accomplish tasks and fade it out when other variables are the primary control method.

Slide 34 (1 min/ :35 overall)

With the command path pre-filter inserted into the control law by multiplying it through all the transfer functions, we can now see it's effect on the same plots we used before to assess against Gibson's Criteria. On the left is the pitch rate response, with the blue line being post filter response and the green line the previous response. The filter has caused the response to quicken and reduced the overshoot ratio, but hasn't affected the settling time significantly. On the right are the pitch attitude and flight path traces. The pitch rate is higher, so the attitude and flight path are steeper, but more critically, the pitch attitude steady state case lies nearly atop the k/s integrator line. This predicts much better handling qualities according to the criteria.

Slide 35 (1 min/ :36 overall)

In the frequency domain, the pre filter has actually made the situation must worse on all three counts. The PIO frequency, where the response crosses 180 degrees is actually much lower, the phase rate slope has steepened indicating a more cliff like transition to PIO and the gain magnitude is actually now positive! Thus the control law is adding to the pilot's demand and making the PIO scenario worse than previously. As we discussed before, however, this exercise is a part task trainer in applying a command path pre-filter. In a control system this poor, many steps would be taken to increase effective damping as well as improve the frequency response before the command path filter was used to fine tune the short period pitch handling characteristics.

Slide 36 (1 min/ :37 overall)

The response with the pre filter applied is in red, with the previous results in blue. There is a drastic improvement in drop back performance, and thus short period pitch handling characteristics, with the result now lying near the origin in the region of satisfactory response. As just discussed, the frequency response criteria have still failed and are in fact worse than before.

Slide 37

That brings us to the end of the worked example of applying a Gibson command path pre-filter and assessing the system against Gibson's Criteria. There are additional problem sets to work through, including an F-16 and a Hawk exercise.

Slide 38

Questions

4.3.2 Applying Gibson's Criteria Slides

Applying Gibson's Criteria

*Designing in excellent handling qualities
with a command path pre-filter
based on Gibson's Criteria*

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LT Jack Hathaway, USN

Empire Test Pilots' School, No70 Fixed Wing Course
Cranfield University, School of Engineering, Aerospace

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Applying Gibson's Criteria

Introduction & Background

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Design in excellent handling qualities and
eliminate the possibility of PIO in modern fly
by wire high order aircraft by including a
well designed command path pre-filter to
achieve K/S attitude response



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Benefits of a High Order Flight Control Systems (HOFCS)

- Reduction in aerodynamic stability can improve performance, but requires computer controlled stability and command augmentation
- Excellent Handling Qualities for any mission task
- Envelope wide similarity
- Care free handling
 - Departure resistance
 - Structural limits

3

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"Good handling is not just a means for giving enjoyment to the pilot. It performs an essential part in task performance. The combat pilot must devote all possible attention to the management and operation of a variety of complex weapons and defensive systems against an increasingly capable opposition. The ability actually to fly the aircraft with minimum attention to safety and structural limits releases considerable mental capacity to the combat task."

-John Gibson

Development of a design methodology for handling qualities excellence in fly by wire aircraft
Doctoral Thesis for Delft Technical University, Netherlands
February 1999

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Scope & Assumptions

- Fighter/attack sized and designed aircraft
- Complete definition of control law structure
- Linear small perturbation transfer functions
- Pitch axis & short period
- 2 degree of freedom (fixed speed approximation)

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Topics of Presentation

- Review of Critical Attitude Parameters
- Gibson Pre-Filter, Control System & Transfer Functions
- Worked Exercise of the A-7A Corsair II

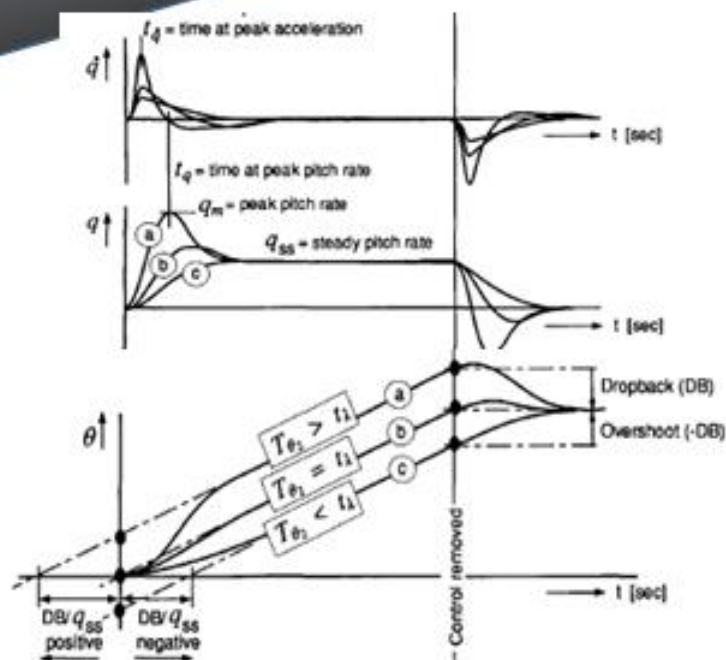
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Criteria and Design Methodology

- Expressed in frequency and time domain
- Define limits on the shape of the response not numbers/formula, so independent of the order of the system
- No dependence on explicit pilot models
- Synthesize directly the handling qualities desired
- Equally understandable by pilots and control engineers.

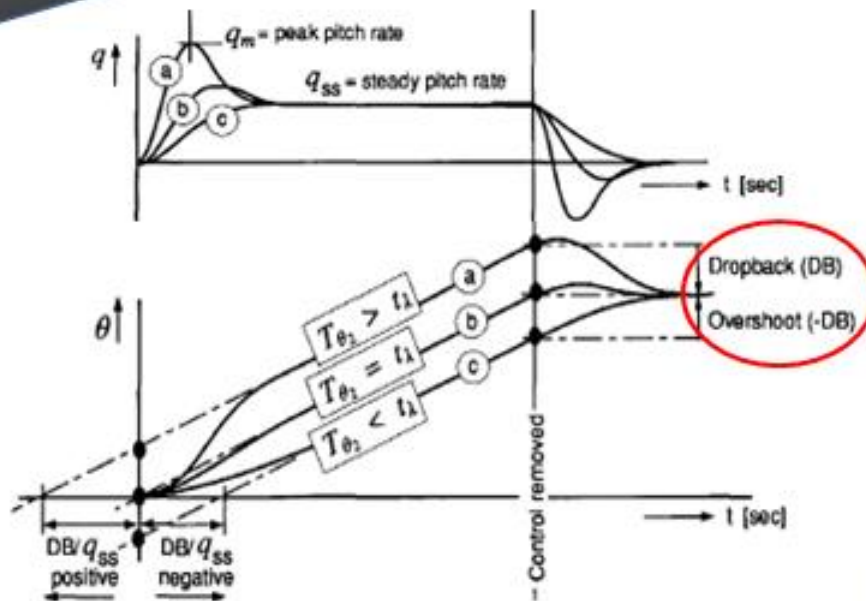
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Applying Gibson's Criteria

Review of critical attitude parameters

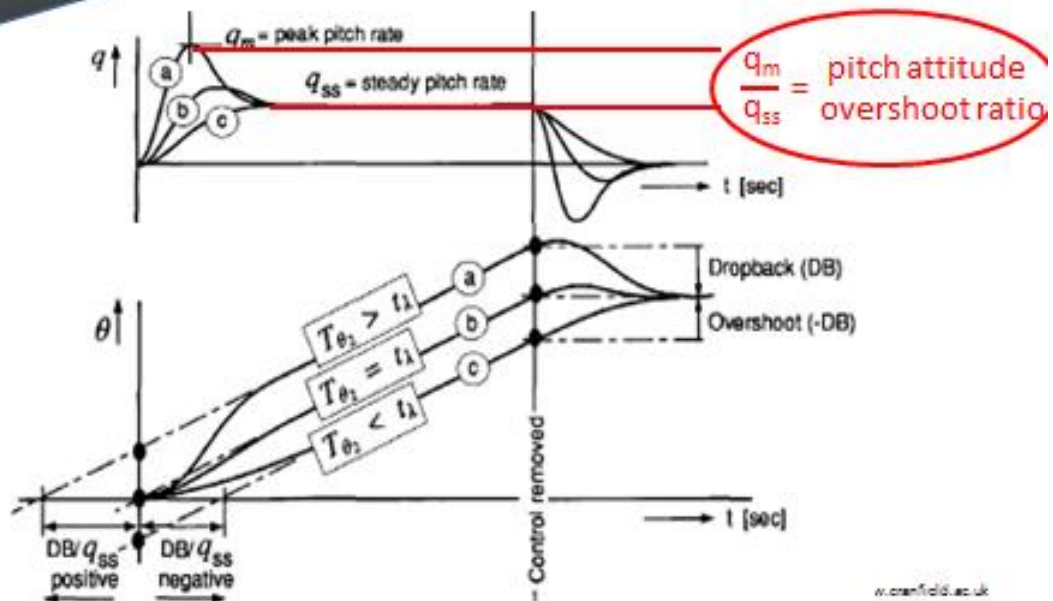


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Review of critical attitude parameters

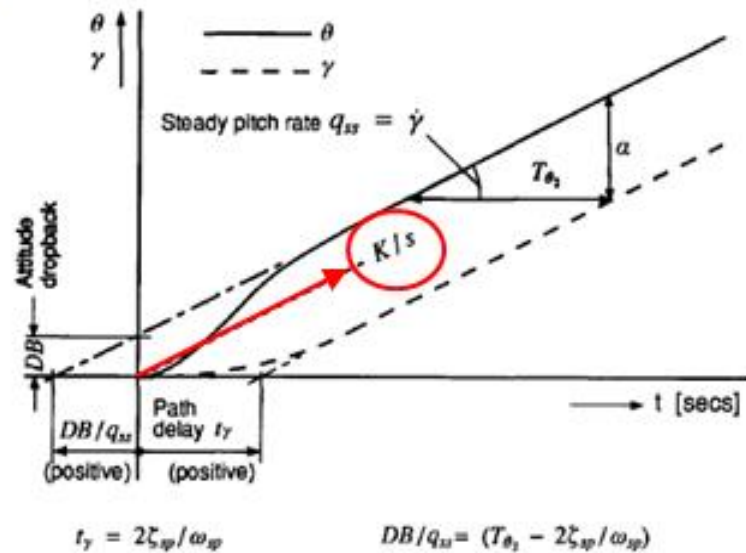


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Applying Gibson's Criteria

Review of critical attitude parameters

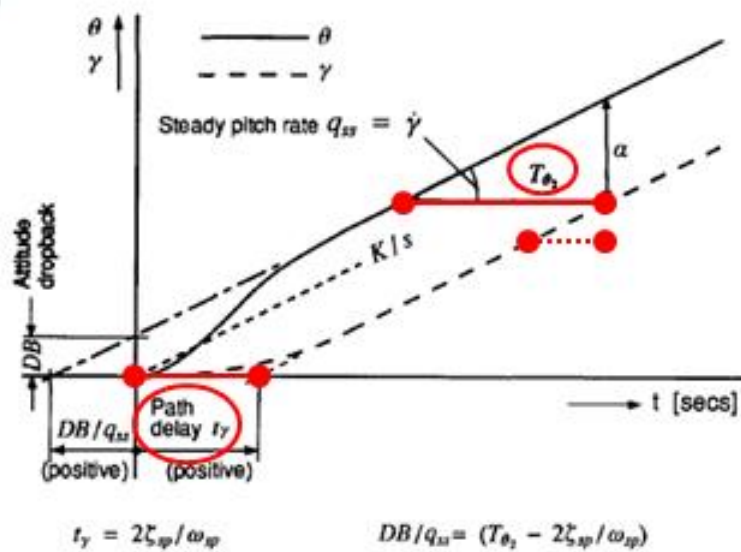


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Applying Gibson's Criteria

Review of critical attitude parameters



Move Path delay



T_{e2} - Path delay

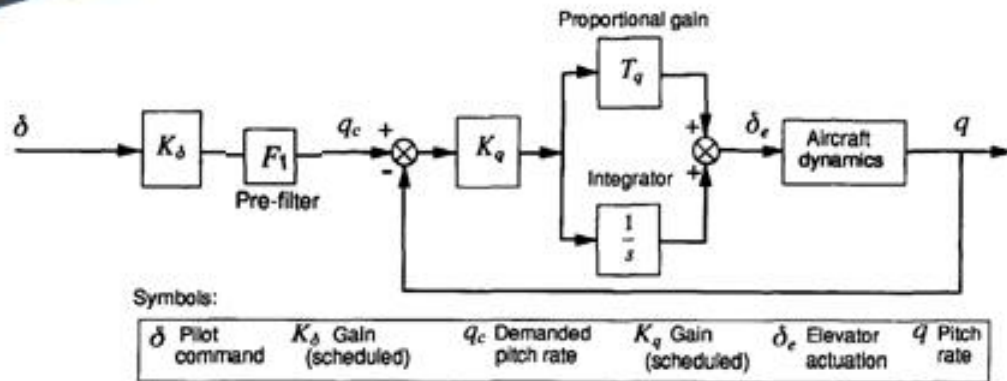


Compare to DB

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Applying Gibson's Criteria

Control System Block Diagram



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Applying Gibson's Criteria

Effect on Transfer Functions

Pitch attitude / Inceptor input
Transfer Function

$$\frac{\theta(s)}{\delta_e(s)} = \frac{K_q(1 + T_{\theta 2}s)}{s[1 + (2\zeta/\omega_{\theta})s + (s^2/\omega_{\theta}^2)]}$$

Gibson Command Path
Pre-Filter Function

$$F_1(s) = \frac{(1 + T_{\theta 2}new s)}{(1 + T_{\theta 2}s)}$$

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Applying Gibson's Criteria

Effect on Transfer Functions

Pitch attitude / Inceptor input
Transfer Function

$$\frac{\theta(s)}{\delta_e(s)} = \frac{K_q(1 + T_{\theta 2}s)}{s[1 + (2\zeta/\omega_p)s + (s^2/\omega_{sp}^2)]}$$

Gibson Command Path
Pre-Filter Function

$$F_1(s) = \frac{(1 + T_{\theta 2 new}s)}{(1 + T_{\theta 2}s)}$$

$$\frac{\theta(s)}{\delta_e(s)} F_1(s) = \frac{(1 + T_{\theta 2 new}s)}{(1 + T_{\theta 2}s)} \frac{K_q(1 + T_{\theta 2}s)}{s[1 + (2\zeta/\omega_p)s + (s^2/\omega_{sp}^2)]}$$

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Applying Gibson's Criteria

Effect on Transfer Functions

Pitch attitude / Inceptor input
Transfer Function

$$\frac{\theta(s)}{\delta_e(s)} = \frac{K_q(1 + T_{\theta 2}s)}{s[1 + (2\zeta/\omega_p)s + (s^2/\omega_{sp}^2)]}$$

Gibson Command Path
Pre-Filter Function

$$F_1(s) = \frac{(1 + T_{\theta 2 new}s)}{(1 + T_{\theta 2}s)}$$

$$\begin{aligned} \frac{\theta(s)}{\delta_e(s)} F_1(s) &= \frac{(1 + T_{\theta 2 new}s)}{\cancel{(1 + T_{\theta 2}s)}} \frac{K_q \cancel{(1 + T_{\theta 2}s)}}{s[1 + (2\zeta/\omega_p)s + (s^2/\omega_{sp}^2)]} \\ &= \frac{K_q(1 + T_{\theta 2 new}s)}{s[1 + (2\zeta/\omega_p)s + (s^2/\omega_{sp}^2)]} \end{aligned}$$

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Applying Gibson's Criteria

Effect on Transfer Functions

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Pitch attitude / Inceptor input
Transfer Function
Without pre filter

$$\frac{K_q (1 + T_{\theta 2} s)}{s [1 + (2\zeta / \omega_{sp}) s + (s^2 / \omega_{sp}^2)]}$$

Pitch attitude / Inceptor input
Transfer Function
With Gibson Pre Filter

$$\frac{K_q (1 + T_{\theta 2 new} s)}{s [1 + (2\zeta / \omega_{sp}) s + (s^2 / \omega_{sp}^2)]}$$

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Applying Gibson's Criteria

Effect on Transfer Functions

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Normal Acceleration/ Inceptor input
Transfer Function
Without pre filter

$$\frac{K_q V}{g (1 + \frac{2\zeta_{sp}}{\omega_{sp}} s + \frac{s^2}{\omega_{sp}^2})}$$

Normal Acceleration/ Inceptor input
Transfer Function
With Gibson Pre Filter

$$\frac{K_q V}{g (1 + \frac{2\zeta_{sp}}{\omega_{sp}} s + \frac{s^2}{\omega_{sp}^2})} \cdot \frac{(1 + T_{\theta 2 new} (s))}{(1 + T_{\theta 2} (s))}$$

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Applying Gibson's Criteria

A7 Corsair II Practice Problem

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- Given a set of linearized equations of motion matrices for a specific flight condition of an A7 Corsair II aircraft, design a command path pre filter in accordance with Gibson's Criteria



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Applying Gibson's Criteria

A7 Corsair II Practice Problem

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- Identify initial stability characteristics at the given flight condition
- Use a reduced order model and introduce a lag to represent the control surface actuators
- Assess the system against Gibson's Criteria
- Apply a command path pre-filter and re-assess.

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Applying Gibson's Criteria

A7 Corsair II Practice Problem
Task 1: Input State Space Equations
And identify stability characteristics

State space format:

$$\dot{x}/dt = Ax + Bu, \text{ and}$$

$$Y = Cx + Du, \text{ where:}$$

x is the state vector, in this case, a 4 row/1 column matrix of $[u, w, q, \theta]$

u is the input vector, in this case consisting solely of η

A is the state matrix, a collection of the aerodynamic derivatives

B is the input matrix, a collection of the control derivatives

C is the output matrix, and is used to manipulate the outputs

D is the direct transmission matrix, and is almost always a null matrix

A-7A

Given Values:

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} 0.00501 & 0.00464 & -72.90000 & -31.34000 \\ -0.08570 & -0.54500 & 309.00000 & -7.40000 \\ 0.00185 & -0.00767 & -0.39500 & 0.00132 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} 5.63000 \\ -23.80000 \\ -4.51576 \\ 0 \end{pmatrix} \eta$$

$$\begin{pmatrix} u \\ w \\ q \\ \theta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0.00316 & 0 & 0 \\ 0 & -0.00316 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \eta$$

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Applying Gibson's Criteria

A7 Corsair II Practice Problem
Task 1: Input State Space Equations
And identify stability characteristics

Input state space equations into Matlab

```
A = [0.00501 0.00464 -72.9 -31.34  
      -0.08570 -0.54500 309.000 -7.400  
      0.00185 -0.00767 -0.39500 0.00132  
      0 0 1 0];
```

```
B = [5.63  
      -23.8  
      -4.51576  
      0];
```

```
C = [F2M 0 0 0  
      0 F2M 0 0  
      0 0 1 0  
      0 0 1  
      0 0.00316*F2M 0 0  
      0 -0.00316*F2M 0 1]; % last two lines are augmenting the output to include alpha and gamma
```

```
% F2M is being used to convert all outputs that have units of feet (u, w) into units of meters  
% F2D is being used to convert all outputs that have units of radians (q, theta) into units of degrees
```

```
D = zeros(6,1);
```

A-7A

Given Values:

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} 0.00501 & 0.00464 & -72.90000 & -31.34000 \\ -0.08570 & -0.54500 & 309.00000 & -7.40000 \\ 0.00185 & -0.00767 & -0.39500 & 0.00132 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} 5.63000 \\ -23.80000 \\ -4.51576 \\ 0 \end{pmatrix} \eta$$

$$\begin{pmatrix} u \\ w \\ q \\ \theta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0.00316 & 0 & 0 \\ 0 & -0.00316 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \eta$$

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Applying Gibson's Criteria

A7 Corsair II Practice Problem
Task 1: Input State Space Equations
And identify stability characteristics

Identify Stability Characteristics

	Eigenvalue	Damping	Freq. (rad/s)
Damp (A) yields	-4.51e-001 + 1.57e+000i	2.76e-001	1.63e+000
	-4.51e-001 - 1.57e+000i	2.76e-001	1.63e+000
	-1.66e-002 + 1.39e-001i	1.19e-001	1.40e-001
	-1.66e-002 - 1.39e-001i	1.19e-001	1.40e-001

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Applying Gibson's Criteria

A7 Corsair II Practice Problem
Task 2: Use a reduced order model
And introduce control
Actuator lag

Transfer Functions

```
Integration_TF = tf(1,[1 0]); % define 1/s integration transfer function;
disp('Pitch Attitude/Tailplane Reduced Order Transfer Function:')
theta_eta_reduced_order = Integration_TF*q_eta_reduced_order
```

```
% gamma(s)/eta(s) = theta(s)/eta(s) - alpha(s)/eta(s),
%
% or in generic terms:
% gamma(s)/eta(s) = A(s+B)/[s(s^2+Xs+Y)] - C(s+D)/(s^2+Xs+Y)

gamma_numerator_second_term = [num_reduced_order(3,2), num_reduced_order(3,3), 0];
gamma_numerator = num_reduced_order(2,:) - gamma_numerator_second_term;

gamma_denominator=[den_reduced_order(1), den_reduced_order(2), den_reduced_order(3)], 0];
```

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Applying Gibson's Criteria

A7 Corsair II Practice Problem
Task 2: Use a reduced order model
And introduce control
Actuator lag

Introduce an actuator with a lag defined by a time constant of 20 seconds

$$\frac{\eta(s)}{\delta(s)} = \frac{20}{s+20} \frac{\text{deg}}{\text{deg}}$$

```
A_ro_actuator = [A(2,2) A(2,3) B(2,1)
                  A(3,2) A(3,3) B(3,1)
                  0      0      -20]
```

```
B_ro_actuator = [0
                  0
                  20]
```

```
C_ro_actuator = [1 0 0
                  0 1 0
                  0 0 1
                  0.00316*F2M 0 0];
```

```
D_ro_actuator = zeros(4,1);
```

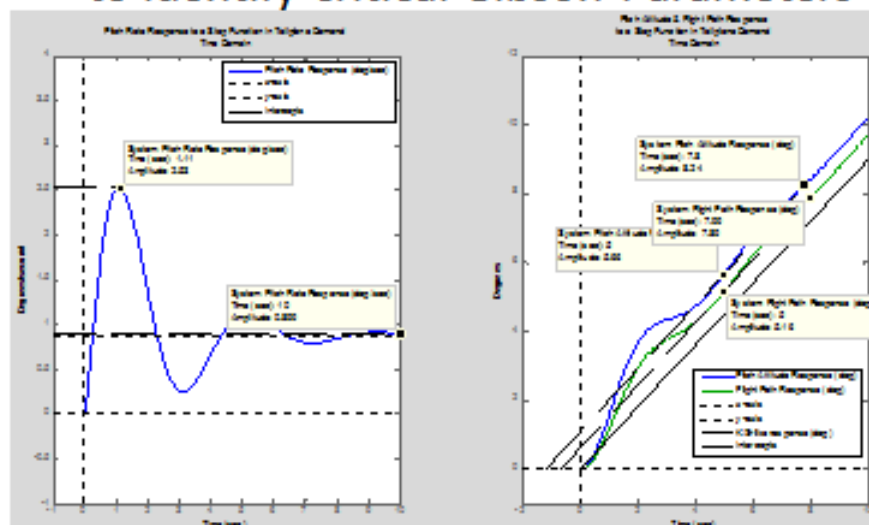
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Applying Gibson's Criteria

A7 Corsair II Practice Problem
Task 3: Assess system against
Gibson's Criteria

Use a step function graph
to identify critical Gibson Parameters



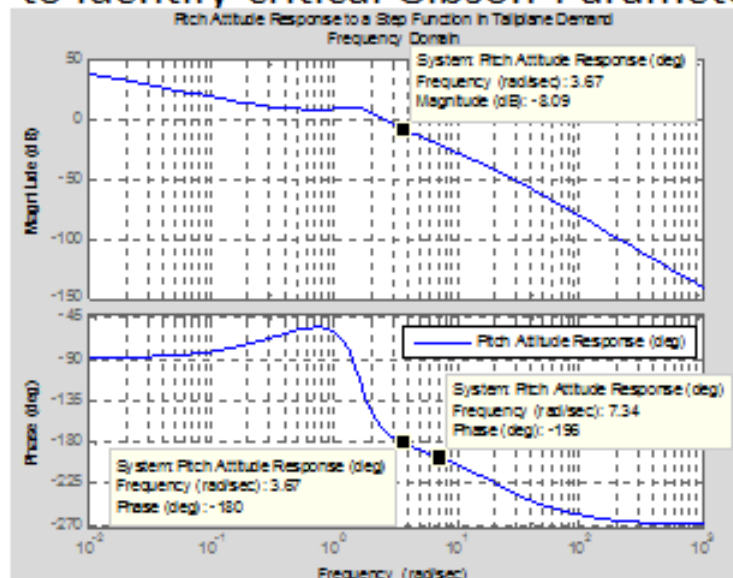
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Applying Gibson's Criteria

A7 Corsair II Practice Problem
Task 3: Assess system against
Gibson's Criteria

Use a step function graph
to identify critical Gibson Parameters



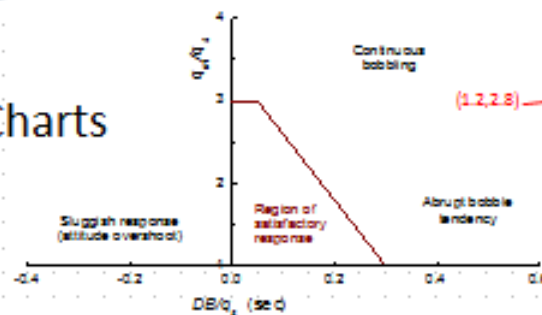
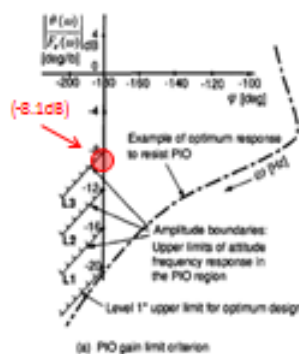
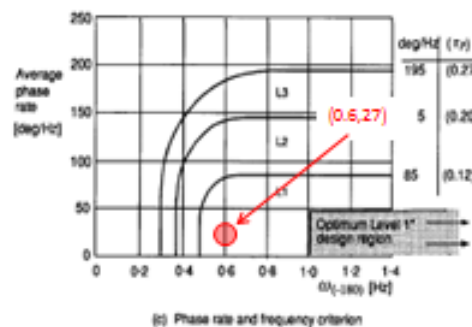
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Applying Gibson's Criteria

A7 Corsair II Practice Problem
Task 3: Assess system against
Gibson's Criteria

Gibson Criteria Charts



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Applying Gibson's Criteria

A7 Corsair II Practice Problem

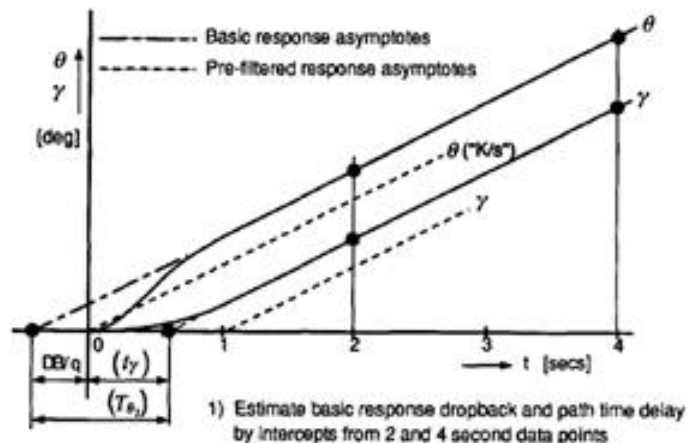
Task 4: Design/apply a command path

pre-filter and reassess IAW

Gibson Criteria

Design Gibson pre-filter

$$\text{Gibson Command Path Pre-Filter Function} = F_1(s) = \frac{(1 + T_{\theta 2 \text{ new}} s)}{(1 + T_{\theta 2} s)}$$



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Applying Gibson's Criteria

A7 Corsair II Practice Problem

Task 4: Design/apply a command path

pre-filter and reassess IAW

Gibson Criteria

Design Gibson pre-filter

$$\text{Gibson Command Path Pre-Filter Function} = F_1(s) = \frac{(1 + T_{\theta 2 \text{ new}} s)}{(1 + T_{\theta 2} s)}$$

Pitch Attitude/Tail plane demand
transfer function

$$-90.3152 (s+0.5046)$$

$$s (s+20) (s^2 + 0.94s + 2.585)$$

t_theta_2 = 1/(num_ro_actuator(2,4)/num_ro_actuator(2,3))
new_t_theta_2 = Flight_Path_Delay % new value of T_theta

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Applying Gibson's Criteria

A7 Corsair II Practice Problem

Task 4: Design/apply a command path

pre-filter and reassess IAW

Gibson Criteria



Design Gibson pre-filter

Pitch Attitude/Tail plane demand transfer function

$$-90.3152 (s+0.5046)$$

Command Path Pre-Filter

New T_{θ_2} term ($s+1.492$)

cancel

Existing $T_{\theta 2}$ term $(s+0.5046)$

doesn't cancel →

$$s(s+20)(s^2 + 0.94s + 2.585)$$

Flight Path/Tail plane demand
transfer function

$$0.45847 (s-138.7) (s+0.7167)$$

$$s(s+20)(s^2 + 0.94s + 2.585)$$

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Applying Gibson's Criteria

A7 Corsair II Practice Problem

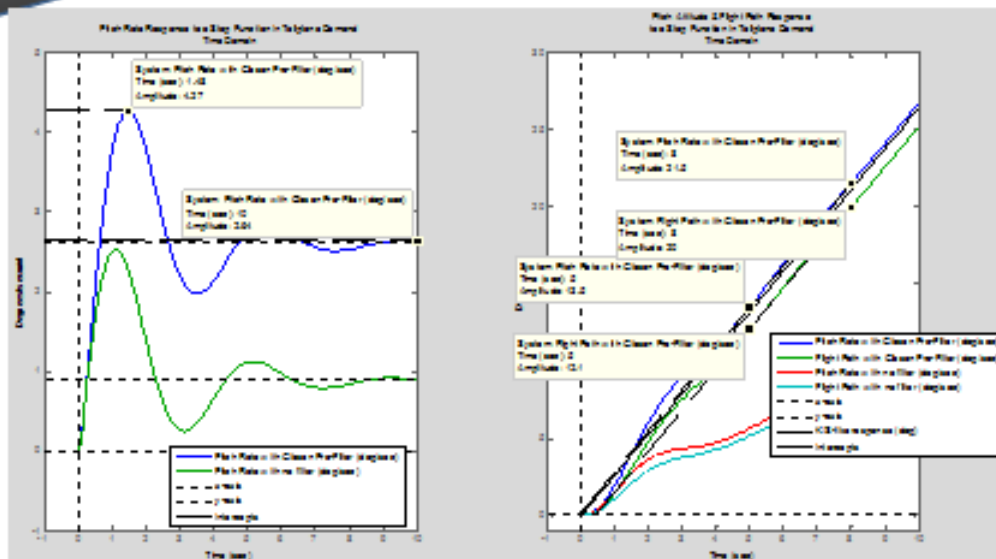
Task 4: Design/apply a command path

pre-filter and reassess IAW

Gibson Criteria



Critical Gibson Parameters



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Applying Gibson's Criteria

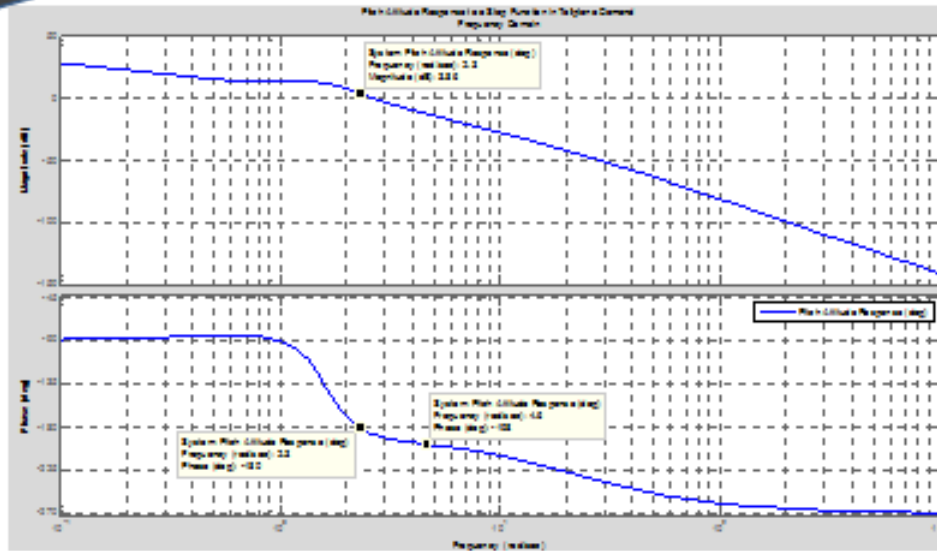
A7 Corsair II Practice Problem

Task 4: Design/apply a command path

pre-filter and reassess IAW

Gibson Criteria

Critical Gibson Parameters



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Applying Gibson's Criteria

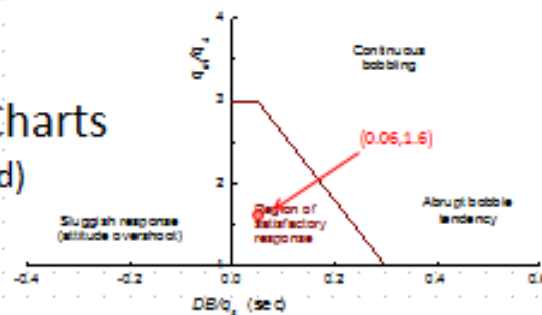
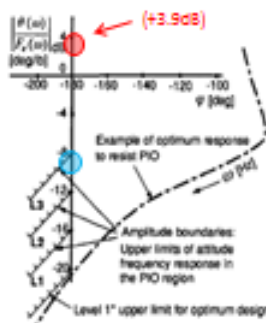
A7 Corsair II Practice Problem

Task 4: Design/apply a command path

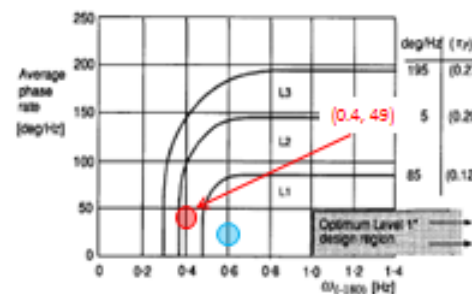
pre-filter and reassess IAW

Gibson Criteria

Gibson Criteria Charts (Pre-Filter applied)



(a) PIO gain limit criterion



(b) Phase rate and frequency criterion

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Applying Gibson's Criteria

*Designing in excellent handling qualities
with a command path pre-filter
based on Gibson's Criteria*

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Additional problems to work through:

Exercise A: F-16

Exercise B: A-7A Corsair

Exercise C: Hawk

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Applying Gibson's Criteria

*Designing in excellent handling qualities
with a command path pre-filter
based on Gibson's Criteria*

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Questions

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4.4 EXERCISE A: THE F-16



School of Engineering

Using Gibson's Criteria to design in excellent longitudinal pitch handling qualities

Exercise A – F16



Assignment notes:

1. Reference [1] provides a six-degrees-of-freedom non-linear F-16 aircraft model to explore various features of aircraft stability and control. A linear state space model at a defined flight condition is given by [1] and used in this assignment for a simplified opportunity to investigate stability and control issues associated with a fourth generation fighter aircraft.
2. This assignment is split into several tasks based upon information presented in [4] and culminates in the application of a command path pre-filter in accordance with Gibson's Criteria to design in excellent longitudinal handling qualities.
 - A. Identify initial stability characteristics at the given flight condition
 - B. Introduce feedback to stabilize the system
 - C. Use a reduced order model and introduce a lag to represent the control surface actuators
 - D. Assess the system against Gibson's Criteria
 - E. Apply a command path pre-filter and re-assess.
3. The tasks to be completed for this assignment are shown in appendix A. Assemble a summary of your responses to tasks 1 to 5 in a concise report of not more than 20 pages including essential figures. Limited material and calculations (including, for example, MATLAB outputs) may be appended to the report but must be properly ordered and referenced in the report where appropriate.
4. Initial data to use in this assignment are provided in appendices B, C and D. In your submission the notation, theoretical background and other relevant information that you present should be consistent with [2] wherever possible. The use of system analysis and design tools provided by MATLAB is essential for this assignment.

References:

1. Stevens, B.L. and Lewis, F.L. *Aircraft Control and Simulation*. John Wiley & Sons, New Jersey, 2003.
2. Cook, M.V. *Flight Dynamics Principles, 2nd Edition*. Butterworth-Heinemann, Oxford, 2007.
3. Cook, M.V. *The Gibson Criteria*. Flying Qualities and Flight Control lecture notes, Cranfield University, 2003.
4. Hathaway, J. *Gibson's Criteria: A primer for Test Pilots and Flight Test Engineers on John Gibson's Handling Qualities Criteria for High Order FCS*, Cranfield University MSc Thesis, January 2014.

Appendices:

A: Assignment tasks.

B: F-16 aircraft model – flight condition and relevant derivative data.

C: Assignment requirements for predicted level 1 handling qualities.

D: Gibson Dropback criterion.

Appendix A: Assignment Tasks

In all of the tasks that follow, whenever you are asked to list or present a transfer function, do so in factorised form with no redundant terms and annotated with the correct units

Task 1 (10 marks):

a. Use the data available to you in appendix B to assemble and then solve the longitudinal equations of motion of the F-16 aircraft model to obtain the response transfer functions. List transfer functions for:

$$\frac{u(s)}{\eta(s)}, \frac{\alpha(s)}{\eta(s)}, \frac{\theta(s)}{\eta(s)}, \frac{q(s)}{\eta(s)}, \frac{\gamma(s)}{\eta(s)}$$

with corresponding units of:

$$\frac{\text{m}}{\text{sec} \cdot \text{deg}}, \frac{\text{deg}}{\text{deg}}, \frac{\text{deg}}{\text{deg}}, \frac{\text{deg}}{\text{sec} \cdot \text{deg}}, \frac{\text{deg}}{\text{deg}}$$

(5 marks)

b. Summarise the significance of the predicted stability modes that are evident from your solution. Explain why it is essential that some augmentation is introduced to change the predicted stability modes.

(5 marks)

Task 2 (15 marks)

a. Investigate and report on the possibilities for introducing angle of attack feedback or/and pitch rate feedback to modify the predicted stability modes. Select a combination of feedback gains for angle of attack feedback (k_α) and pitch rate feedback (k_q) to satisfy the dynamic stability criteria shown in appendix C – target your selection to meet a short period natural frequency of $2.25 \text{ rad sec}^{-1}$ and a short period damping ratio of 0.7. Explain the justification for your gain selections.

(10 marks)

b. Present the augmented transfer functions for $\frac{\alpha(s)}{\eta(s)}$ and $\frac{q(s)}{\eta(s)}$ and summarise the significance of the predicted stability modes that are now evident from your solution.

(5 marks)

Task 3 (25 marks):

A major assumption within Gibson's Criteria is that the system being assessed has fixed speed. Since we are looking at the short period pitch response over a short time period, any disturbance or pilot input will not have any significant effect on speed, so we can safely eliminate it from the system by looking at a reduced order version of the current model. This also eliminates the phugoid from the system.

a. Construct and present the reduced order equations of motion that describe the short term dynamics of the un-augmented F-16 aircraft model in the form: (5 marks)

$$\begin{pmatrix} \dot{\alpha} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} z_{\alpha} & z_q \\ m_{\alpha} & m_q \end{pmatrix} \begin{pmatrix} \alpha \\ q \end{pmatrix} + \begin{pmatrix} z_{\eta} \\ m_{\eta} \end{pmatrix} \eta$$

b. Use your values of k_q and k_{α} from task 2a with the reduced order equations of motion to develop reduced order augmented transfer functions for $\frac{\alpha(s)}{\eta(s)}$ and $\frac{q(s)}{\eta(s)}$ develop the corresponding reduced order transfer function for the

pitch attitude perturbation $\frac{\theta(s)}{\eta(s)}$ and the flight path angle perturbation $\frac{\gamma(s)}{\eta(s)}$. (10 marks)

Additionally, Gibson's Criteria was designed to handle any system, regardless of its mathematical order. The addition of multiple feedback loops, computer processing, multiple actuators, etc will create a system with very high mathematical order. We can achieve this higher order notionally by adding a lag due to the tail plane actuator.

c. For the purposes of this assignment, the F-16 aircraft model tail plane actuator is modelled by the following transfer function:

$$\frac{\eta(s)}{\delta(s)} = \frac{20}{s + 20} \frac{\text{deg}}{\text{deg}}$$

In this, δ represents the tail plane angle demand via the actuator in degrees and, as before, η is the tail plane angle deflection in degrees. Modify the reduced order state equation from task 3(a) to include the actuator dynamics and present the amended reduced order state equation. Use your values of k_q and k_a from task 2a with the modified reduced order equations of motion to

$$\frac{\alpha(s)}{\delta(s)}, \frac{\theta(s)}{\delta(s)}, \frac{q(s)}{\delta(s)}, \frac{y(s)}{\delta(s)}$$

develop reduced order augmented transfer functions for

(10 marks)

Task 4 (25 marks):

(15 marks)

a. Use your results from task 3c and apply the 'Gibson Dropback criterion' to determine the predicted handling qualities for a pitch tracking task for the actuator included, augmented aircraft model. Summarise the results of your analysis and your conclusion about the predicted handling qualities.

(10 marks)

b. Continuing to use your results from task 3c, specifically the pitch attitude transfer function, apply the frequency domain portion of Gibson's Criteria: Phase Rate, PIO frequency and PIO gain limit criterion. Compare your results with the values shown in appendix D and comment on any 'specification compliance' issues.

Task 5 (25 marks):

(15 marks)

a. Design a Command Path pre-Filter in accordance with Gibson's guidance as discussed in [4]. Present transfer functions for the pre-filter, pitch attitude and flight path and comment on any significant features.

(10 marks)

b. Re-assess against the time domain and frequency domain Gibson Criteria and comment on the effectiveness of your command path pre-filter

Appendix B: F-16 aircraft model - flight condition and relevant derivative data

The notation used throughout this appendix is consistent with that used in [2].

The following data for the F-16 aircraft model is taken from the specified operating condition defined on p 197 of [1] with a brief summary of the relevant data reproduced in table B-1. However, the **derivative** data from [1] presented in this assignment has been changed from US imperial to SI units.

Altitude	h	0	ft
Mach number	M	0.45	
Velocity	V_0	297	KTAS
CG position	'Nominal' at $0.35\bar{c}$		

Table B-1: F-16 aircraft model operating condition data

The longitudinal state equation is presented in the following form:

$$\begin{pmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} x_u & x_\alpha & x_\theta & x_q \\ z_u & z_\alpha & z_\theta & z_q \\ 0 & 0 & 0 & 1 \\ m_u & m_\alpha & m_\theta & m_q \end{pmatrix} \begin{pmatrix} u \\ \alpha \\ \theta \\ q \end{pmatrix} + \begin{pmatrix} x_\eta \\ z_\eta \\ 0 \\ m_\eta \end{pmatrix} \eta$$

Equation B-1

The notation used in equation (B.1) is consistent with [2] and the symbol η is used to represent the tail plane angle deflection from trim (positive for trailing edge down) measured in **degrees**.

The units of the derivatives are such that speeds and changes in speeds are in **m s⁻¹**; changes in angular displacements are in **rad** and changes in angular rates in **rad s⁻¹**.

The derivative values for the given flight condition are shown in table B-2. Derivatives that are not shown may be assumed to be negligible and taken as equal to zero.

Flight path is equal to pitch attitude minus angle of attack.

Range of available feedback gains:

Gains selected for α -feedback and q -feedback should be as low as possible and in particular:

$$|k_q| \leq 0.5 \frac{\text{rad}}{\text{rad sec}^{-1}} \text{ or } \frac{\text{deg}}{\text{deg sec}^{-1}}$$

$$|k_\alpha| \leq 0.5 \frac{\text{rad}}{\text{rad}} \text{ or } \frac{\text{deg}}{\text{deg}}$$

Concise dimensional derivative	Value for given flight condition	Units for concise dimensional derivative
X_u	-0.019311	sec^{-1}
X_α	2.6870	m sec^{-2}
X_θ	-9.8054	m sec^{-2}
X_q	-0.17526	m sec^{-1}
Z_u	-0.00083297	sec^{-1}
Z_α	-1.0189	m sec^{-2}
Z_q	0.90506	m sec^{-1}
m_α	0.82225	sec^{-2}
m_q	-1.0774	sec^{-1}
x_η	0.05294	$(\text{m sec}^{-2})/\text{deg}$
z_η	-0.0021499	$(\text{m sec}^{-2})/\text{deg}$
m_η	-0.17555	$(\text{rad sec}^{-2})/\text{deg}$

Table B-2: F-16 aircraft model derivative data

Appendix C: Assignment requirements for level 1 handling qualities

In this assignment, the F-16 aircraft is classified as class IV (high manoeuvrability) and is operating in flight phase category A (a non-terminal flight phase that requires rapid manoeuvring, precision tracking or precision flight path control). There are many criteria for predicted level 1 handling qualities that might be considered. However, for the purposes of this assignment, assume that the following requirements are to be met, as far as possible:

Dynamic stability

Short period natural frequency:

$$2.0 \leq \omega_{n_s} \leq 5.0 (\text{rad sec}^{-1})$$

Short period damping ratio:

$$0.35 \leq \zeta_s \leq 1.0$$

Phugoid damping ratio

$$\zeta_p \geq 0.04$$

Ratio of short period to phugoid natural frequencies:

$$\frac{\omega_s}{\omega_p} \geq 10$$

Gibson Criteria: (See Appendix D)

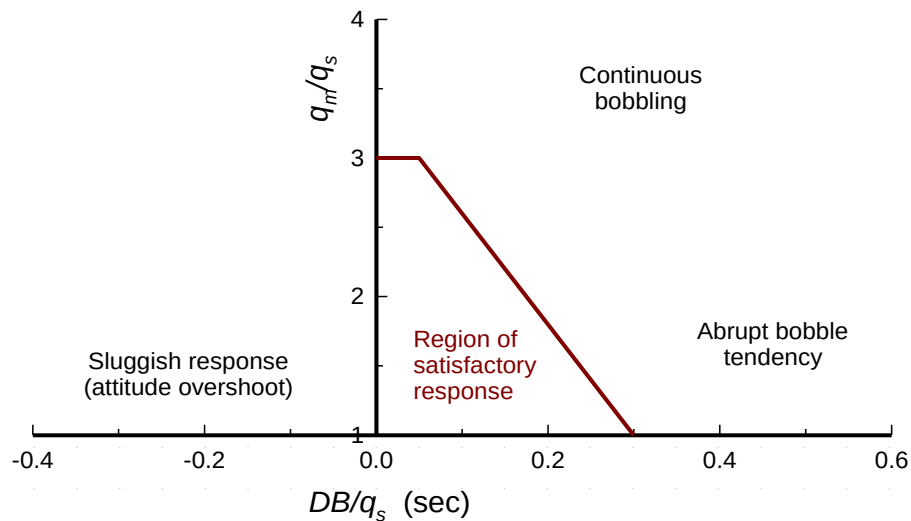
Within the region of satisfactory response on
the Dropback Criterion

Level 1* Phase Rate and PIO Frequency
criteria

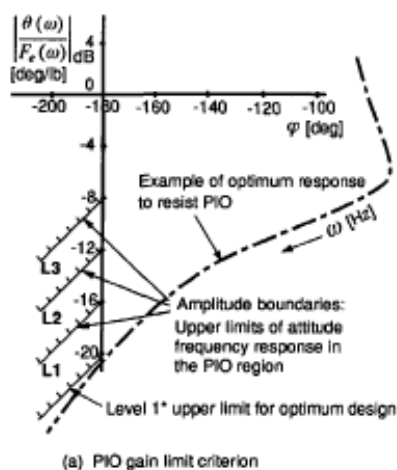
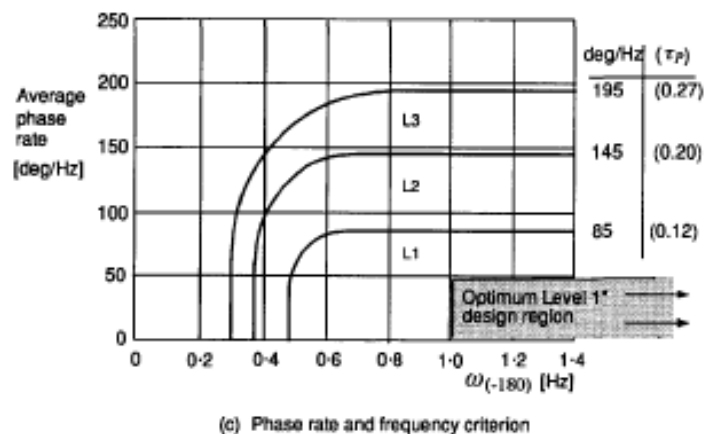
Level 1 PIO Gain criterion

Appendix D: Gibson Dropback Criterion

The background to the Gibson criteria – consisting of the Dropback criterion and the Frequency Domain criteria of PIO Frequency, Phase Rate and PIO Gain – is summarised in [4] and a copy of this will be made available to you in support of this assignment. The Following criteria chart are provided for convenience.



Time Domain Criteria: Dropback



Frequency Domain Criteria: PIO Frequency, Phase Rate and PIO Gain

4.5 EXERCISE B: THE A-7A



School of Engineering

Using Gibson's Criteria to design in excellent longitudinal pitch handling qualities

Exercise B – A-7A Corsair II

(Presented in the Applying Gibson's Criteria lecture)



Assignment notes:

1. Reference [1], cited in [2] provides a linear state space model at a defined flight condition for the Ling-Temco-Cought A-7A Corsair II which is used in this assignment for a simplified opportunity to investigate stability and control issues.
2. This assignment is split into several tasks based upon information presented in [4] and culminates in the application of a command path pre-filter in accordance with Gibson's Criteria to design in excellent longitudinal handling qualities.
 - F. Identify initial stability characteristics at the given flight condition
 - G. Use a reduced order model and introduce a lag to represent the control surface actuators
 - H. Assess the system against Gibson's Criteria
 - I. Apply a command path pre-filter and re-assess.
3. The tasks to be completed for this assignment are shown in appendix A. Assemble a summary of your responses to tasks 1 to 4 in a concise report of not more than 20 pages including essential figures. Limited material and calculations (including, for example, MATLAB outputs) may be appended to the report but must be properly ordered and referenced in the report where appropriate.
4. Initial data to use in this assignment are provided in appendices B, C and D. In your submission the notation, theoretical background and other relevant information that you present should be consistent with [2] wherever possible. The use of system analysis and design tools provided by MATLAB is essential for this assignment.

References:

1. Teper, G.L. . *Aircraft Stability and Control Data*. Systems Technology, INC., STi Technical Report 176-1. NASA Contractor Report, National Aeronautics and Space Administration, Washington, D.C. 20546
2. Cook, M.V. *Flight Dynamics Principles, 2nd Edition*. Butterworth-Heinemann, Oxford, 2007.
3. Cook, M.V. *The Gibson Criteria*. Flying Qualities and Flight Control lecture notes, Cranfield University, 2003.
4. Hathaway, J. *Gibson's Criteria: A primer for Test Pilots and Flight Test Engineers on John Gibson's Handling Qualities Criteria for High Order FCS*, Cranfield University MSc Thesis, January 2014.

Appendices:

A: Assignment tasks.

B: A-7A Corsair II aircraft model – flight condition and relevant data.

C: Assignment requirements for predicted level 1 handling qualities.

D: Gibson Dropback criterion.

Appendix A: Assignment Tasks

In all of the tasks that follow, whenever you are asked to list or present a transfer function, do so in factorised form with no redundant terms and annotated with the correct units

Task 1 (10 marks):

a. Use the data available to you in appendix B to assemble and then solve the longitudinal equations of motion of the A-7A aircraft model to obtain the response transfer functions. List transfer functions for:

$$\frac{u(s)}{\eta(s)}, \frac{w(s)}{\eta(s)}, \frac{q(s)}{\eta(s)}, \frac{\theta(s)}{\eta(s)}, \frac{\alpha(s)}{\eta(s)}, \frac{\gamma(s)}{\eta(s)}$$

with corresponding units of:

$$\frac{\text{m}}{\text{sec} \cdot \text{deg}}, \quad \frac{\text{m}}{\text{sec} \cdot \text{deg}}, \quad \frac{\text{deg}}{\text{sec} \cdot \text{deg}}, \quad \frac{\text{deg}}{\text{deg}}, \quad \frac{\text{deg}}{\text{deg}}, \quad \frac{\text{deg}}{\text{deg}}$$

(6 marks)

b. Summarise the significance of the predicted stability modes that are evident from your solution.

(4 marks)

Task 2 (30 marks):

A major assumption within Gibson's Criteria is that the system being assessed has fixed speed. Since we are looking at the short period pitch response over a short time period, any disturbance or pilot input will not have any significant effect on speed, so we can safely eliminate it from the system by looking at a reduced order version of the current model. This also eliminates the phugoid from the system.

a. Construct and present the reduced order equations of motion of pitch rate and angle of attack that describe the short term dynamics of the A-7A aircraft model.

(10 marks)

b. Use the reduced order equations of motion to develop reduced order transfer functions for $\frac{\alpha(s)}{\eta(s)}$ and $\frac{q(s)}{\eta(s)}$ and develop the corresponding reduced order

transfer function for the pitch attitude perturbation $\frac{\theta(s)}{\eta(s)}$ and the flight path angle

perturbation $\frac{\gamma(s)}{\eta(s)}$. (10 marks)

Additionally, Gibson's Criteria was designed to handle any system, regardless of its mathematical order. The addition of multiple feedback loops, computer processing, multiple actuators, etc will create a system with very high mathematical order. We can achieve this higher order notionally by adding a lag due to the tail plane actuator.

c. For the purposes of this assignment, the A-7A aircraft model tail plane actuator is modelled by the following transfer function:

$$\frac{\eta(s)}{\delta(s)} = \frac{20}{s + 20} \frac{\text{deg}}{\text{deg}}$$

In this, δ represents the tail plane angle demand via the actuator in degrees and, as before, η is the tail plane angle deflection in degrees. Modify the reduced order state equation from task 2(a) to include the actuator dynamics and present the amended reduced order state equation. Use the amended reduced order equations of motion to develop reduced order transfer functions for

$$\frac{\alpha(s)}{\delta(s)}, \frac{\theta(s)}{\delta(s)}, \frac{q(s)}{\delta(s)}, \frac{\gamma(s)}{\delta(s)}.$$

(10 marks)

Task 3 (30 marks):

(15 marks)

a. Use your results from task 2c and apply the 'Gibson Dropback criterion' to determine the predicted handling qualities for a pitch tracking task for the actuator included aircraft model. Summarise the results of your analysis and present your conclusions about the predicted handling qualities.

b. Continuing to use your results from task 2c, specifically the pitch attitude transfer function, apply the frequency domain portion of Gibson's Criteria: Phase Rate, PIO frequency and PIO gain limit criterion. Compare your results with the values shown in appendix D and comment on any 'specification compliance' issues.

Task 4 (30 marks):

(15 marks)

- a. Design a Command Path pre-Filter in accordance with Gibson's guidance as discussed in [4]. Present transfer functions for the pre-filter, pitch attitude and flight path and comment on any significant features.
(15 marks)
- b. Re-assess against the time domain and frequency domain Gibson Criteria and comment on the effectiveness of your command path pre-filter

Appendix B: A-7A aircraft model - flight condition and relevant derivative data

The notation used throughout this appendix is consistent with that used in [2].

The following data for the F-16 aircraft model is taken from the specified operating condition defined on p 139 of [2] with a brief summary of the relevant data reproduced in table B-1. Distances are in feet, angles are in degrees

Altitude	h	15,000	ft
Mach number	M	0.3	
Speed of Sound at 15k = 626.4Speed of			

A-7A aircraft model operating condition data

The longitudinal state equation is presented in the following form:

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} 0.00501 & 0.00464 & -72.90000 & -31.34000 \\ -0.08570 & -0.54500 & 309.00000 & -7.40000 \\ 0.00185 & -0.00767 & -0.39500 & 0.00132 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} 5.63000 \\ -23.80000 \\ -4.51576 \\ 0 \end{pmatrix} \eta$$

And

$$\begin{pmatrix} u \\ w \\ q \\ \theta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0.00316 & 0 & 0 \\ 0 & -0.00316 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \eta$$

Longitudinal State Equation

The notation used in equation (B.1) is consistent with [2] and the symbol η is used to represent the tail plane angle deflection from trim (positive for trailing edge down) measured in degrees.

Appendix C: Assignment requirements for level 1 handling qualities

In this assignment, the A-7A aircraft is classified as class IV (high manoeuvrability) and is operating in flight phase category A (a non-terminal flight phase that requires rapid manoeuvring, precision tracking or precision flight path control). There are many criteria for predicted level 1 handling qualities that might be considered. However, for the purposes of this assignment, assume that the following requirements are to be met, as far as possible:

Gibson Criteria: (See Appendix D)

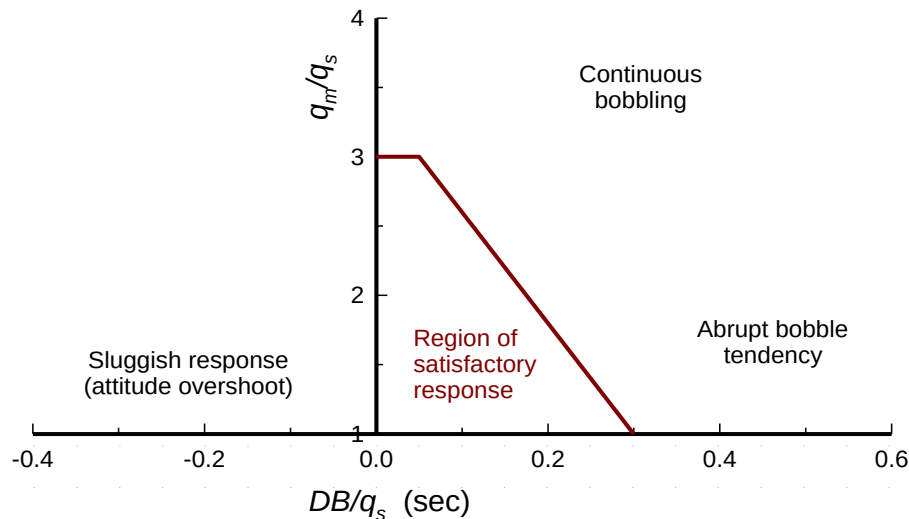
Within the region of satisfactory response on
the Dropback Criterion

Level 1* Phase Rate and PIO Frequency
criteria

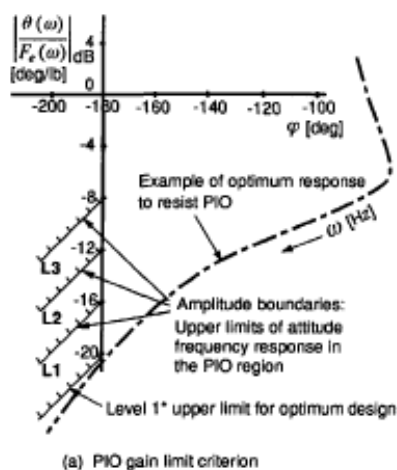
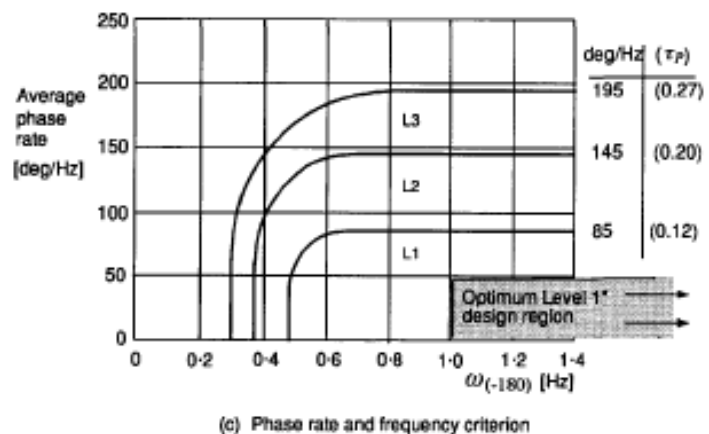
Level 1 PIO Gain criterion

Appendix D: Gibson Dropback Criterion

The background to the Gibson criteria – consisting of the Dropback criterion and the Frequency Domain criteria of PIO Frequency, Phase Rate and PIO Gain – is summarised in [4] and a copy of this will be made available to you in support of this assignment. The Following criteria chart are provided for convenience.



Time Domain Criteria: Dropback



Frequency Domain Criteria: PIO Frequency, Phase Rate and PIO Gain

4.6 EXERCISE C: THE HAWK



School of Engineering

Using Gibson's Criteria to design in excellent longitudinal pitch handling qualities

Exercise C – Hawk T1 Mk 1



Assignment notes:

1. A three degree of freedom (3 DOF) non-linear longitudinal model of a Hawk T Mk1 was created by Dr G Mullen between January and March 2009 [1]. A particular feature of the model is an output of a linear state space model at a defined operating point for use in further 'off-line' analysis. This assignment provides an opportunity to use such a linear model, understand some of the stability and control problems predicted by the model and propose possible ways of overcoming them.
2. This assignment is split into several tasks based upon information presented in [4] and culminates in the application of a command path pre-filter in accordance with Gibson's Criteria to design in excellent longitudinal handling qualities.
 - A. Identify initial stability characteristics at the given flight condition
 - B. Introduce feedback to stabilize the system
 - C. Use a reduced order model and introduce a lag to represent the control surface actuators
 - D. Assess the system against Gibson's Criteria
 - E. Apply a command path pre-filter and re-assess.
3. The tasks to be completed for this assignment are shown in appendix A. Assemble a summary of your responses to tasks 1 to 5 in a concise report of not more than 20 pages including essential figures. Limited material and calculations (including, for example, MATLAB outputs) may be appended to the report but must be properly ordered and referenced in the report where appropriate.
4. Initial data to use in this assignment are provided in appendices B, C and D. In your submission the notation, theoretical background and other relevant information that you present should be consistent with [2] wherever possible. The use of system analysis and design tools provided by MATLAB is essential for this assignment.

References:

1. Mullen G. *Hawk T Mk 1 Longitudinal Simulink Model – Description and Rough User guide*. ETPS internal note, 20 April 2009.
2. Cook, M.V. *Flight Dynamics Principles, 2nd Edition*. Butterworth-Heinemann, Oxford, 2007.
3. Cook, M.V. *The Gibson Criteria*. Flying Qualities and Flight Control lecture notes, Cranfield University, 2003.
4. Hathaway, J. *Gibson's Criteria: A primer for Test Pilots and Flight Test Engineers on John Gibson's Handling Qualities Criteria for High Order FCS*, Cranfield University MSc Thesis, January 2014.

Appendices:

A: Assignment tasks.

B: Hawk aircraft model – flight condition and relevant derivative data.

C: Assignment requirements for predicted level 1 handling qualities.

D: Gibson Dropback criterion.

Appendix A: Assignment Tasks

In all of the tasks that follow, whenever you are asked to list or present a transfer function, do so in factorised form with no redundant terms and annotated with the correct units

Task 1 (10 marks):

a. Use the data available to you in appendix B to assemble and then solve the longitudinal equations of motion of the Hawk aircraft model to obtain the response transfer functions. List transfer functions for:

$$\frac{u(s)}{\eta(s)}, \frac{w(s)}{\eta(s)}, \frac{q(s)}{\eta(s)}, \frac{\theta(s)}{\eta(s)}, \frac{\alpha(s)}{\eta(s)}, \frac{\gamma(s)}{\eta(s)}$$

with corresponding units of:

$$\frac{\text{m}}{\text{sec} \cdot \text{deg}}, \frac{\text{m}}{\text{sec} \cdot \text{deg}}, \frac{\text{deg}}{\text{sec} \cdot \text{deg}}, \frac{\text{deg}}{\text{deg}}, \frac{\text{deg}}{\text{deg}}, \frac{\text{deg}}{\text{deg}}$$

(6 marks)

b. Summarise the significance of the predicted stability modes that are evident from your solution. Explain how augmentation might change the stability modes.

(4 marks)

Task 2 (15 marks)

a. Investigate and report on the possibilities for introducing pitch rate feedback to modify the predicted stability modes. Select a feedback gain for pitch rate feedback (k_q) to satisfy the dynamic stability criteria shown in appendix C. Explain the justification for your gain selections.

(10 marks)

b. Present the augmented transfer function for pitch rate and summarise the significance of the predicted stability modes that are now evident from your solution.

(5 marks)

Task 3 (25 marks):

A major assumption within Gibson's Criteria is that the system being assessed has fixed speed. Since we are looking at the short period pitch response over a short time period, any disturbance or pilot input will not have any significant effect on speed, so we can safely eliminate it from the system by looking at a reduced order version of the current model. This also eliminates the phugoid from the system.

a. Construct and present the reduced order equations of motion that describe the short term dynamics of the un-augmented Hawk aircraft model in the form: (5 marks)

$$\begin{pmatrix} \dot{w} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} z_w & z_q \\ m_w & m_q \end{pmatrix} \begin{pmatrix} w \\ q \end{pmatrix} + \begin{pmatrix} z_\eta \\ m_\eta \end{pmatrix} \eta$$

b. Use your values of k_q from task 2a with the reduced order equations of motion to develop reduced order augmented transfer function for $\frac{q(s)}{\eta(s)}$, develop the corresponding reduced order transfer functions for $\frac{\theta(s)}{\eta(s)}$, $\frac{\alpha(s)}{\eta(s)}$, $\frac{\gamma(s)}{\eta(s)}$. (10 marks)

Additionally, Gibson's Criteria was designed to handle any system, regardless of its mathematical order. The addition of multiple feedback loops, computer processing, multiple actuators, etc will create a system with very high mathematical order. We can achieve this higher order notionally by adding a lag due to the tail plane actuator.

c. For the purposes of this assignment, the Hawk aircraft model tail plane actuator is modelled by the following transfer function:

$$\frac{\eta(s)}{\delta(s)} = \frac{20}{s + 20} \frac{\text{deg}}{\text{deg}}$$

In this, δ represents the tail plane angle demand via the actuator in degrees and, as before, η is the tail plane angle deflection in degrees. Modify the reduced order state equation from task 3(a) to include the actuator dynamics and present the amended reduced order state equation. Use your value of k_q from task 2a with the amended reduced order equations of motion to develop reduced order augmented transfer

functions for $\frac{\alpha(s)}{\delta(s)}$, $\frac{\theta(s)}{\delta(s)}$, $\frac{q(s)}{\delta(s)}$, $\frac{\gamma(s)}{\delta(s)}$.

Task 4 (25 marks):

(15 marks)

a. Use your results from task 3c and apply the 'Gibson Dropback criterion' to determine the predicted handling qualities for a pitch tracking task for the actuator included, augmented aircraft model. Summarise the results of your analysis and your conclusion about the predicted handling qualities.

(10 marks)

b. Continuing to use your results from task 3c, specifically the pitch attitude transfer function, apply the frequency domain portion of Gibson's Criteria: Phase Rate, PIO frequency and PIO gain limit criterion. Compare your results with the values shown in appendix D and comment on any 'specification compliance' issues.

Task 5 (25 marks):

(15 marks)

a. Design a Command Path pre-Filter in accordance with Gibson's guidance as discussed in [4]. Present transfer functions for the pre-filter, pitch attitude and flight path and comment on any significant features.

(10 marks)

b. Re-assess against the time domain and frequency domain Gibson Criteria and comment on the effectiveness of your command path pre-filter

Appendix B: Hawk T Mk1 aircraft flight condition and relevant derivative data

The notation used throughout this appendix is consistent with that used in [2].

The following data for the Hawk T Mk 1 aircraft at a specified operating condition was used with [1] to create a linear state space model as shown below. Some of the flight condition and aircraft data are provided in the table below

Altitude	h	20000	ft
Mach number	M	0.7	
Velocity	V_0	221	m sec ⁻¹
Aircraft mass	m	5000	kg
Pitch Inertia	I_{yy}	20026	kg m ²
Wing area	S	16.689	m ²
Standard mean chord	$\bar{c}$	1.777	m
CG position	'forward' (CG = 24 see [1])		

Hawk T Mk 1 data

The longitudinal state equation is presented in the following form:

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} x_u & x_w & x_q & x_\theta \\ z_u & z_w & z_q & z_\theta \\ m_u & m_w & m_q & m_\theta \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} x_\eta \\ z_\eta \\ m_\eta \\ 0 \end{pmatrix} \eta$$

This equation is consistent with [2] and the symbol η is used to represent the tail plane angle deflection from trim (positive for trailing edge down) measured in **degrees**.

The units of the derivatives are such that speeds and changes in speeds are in **m s⁻¹**; changes in angular displacements are in **rad** and changes in angular rates in **rad s⁻¹**.

The derivative values for the given flight condition are shown in table B-2. Derivatives that are not shown may be assumed to be negligible and taken as equal to zero.

Important Definitions:

$$\alpha \approx \frac{w}{V_0}$$

$$\gamma \approx \theta - \alpha$$

$$a_z \approx \dot{w} - qV_0$$

$$n_z \approx \frac{a_z}{g}$$

$$n_\alpha \approx \frac{n_z}{\alpha}$$

Concise dimensional derivative	Value for given flight condition	Units for dimensional derivative
x_u	-0.0115	sec ⁻¹
x_w	0.0574	sec ⁻¹
x_q	-8.7001	m sec ⁻¹
x_θ	-9.8022	m sec ⁻²

z_u	-0.0505	sec^{-1}
z_w	-1.3595	sec^{-1}
z_q	218.4078	m sec^{-1}
z_θ	-0.3905	m sec^{-2}
m_u	0.0090	$(\text{m sec})^{-1}$
m_w	-0.1008	$(\text{m sec})^{-1}$
m_q	-1.2046	sec^{-1}
x_η	0.0395	$(\text{m sec}^{-2})/\text{deg}$
z_η	-0.9921	$(\text{m sec}^{-2})/\text{deg}$
m_η	-1.0701	$(\text{rad sec}^{-2})/\text{deg}$

Hawk T Mk 1 derivative data

Appendix C: Assignment requirements for level 1 handling qualities

In this assignment, the Hawk Mk T1 aircraft is classified as class IV (high manoeuvrability) and is operating in flight phase category A (a non-terminal flight phase that requires rapid manoeuvring, precision tracking or precision flight path control). There are many criteria for predicted level 1 handling qualities that might be considered. However, for the purposes of this assignment, assume that the following requirements are to be met, as far as possible:

Dynamic stability

Short period natural frequency:

$$2.0 \leq \omega_{n_s} \leq 5.0 (\text{rad sec}^{-1})$$

Short period damping ratio:

$$0.35 \leq \zeta_s \leq 1.0$$

Phugoid damping ratio

$$\zeta_p \geq 0.04$$

Ratio of short period to phugoid natural frequencies:

$$\frac{\omega_s}{\omega_p} \geq 10$$

Gibson Criteria: (See Appendix D)

Within the region of satisfactory response on
the Dropback Criterion

Level 1* Phase Rate and PIO Frequency
criteria

Level 1 PIO Gain criterion

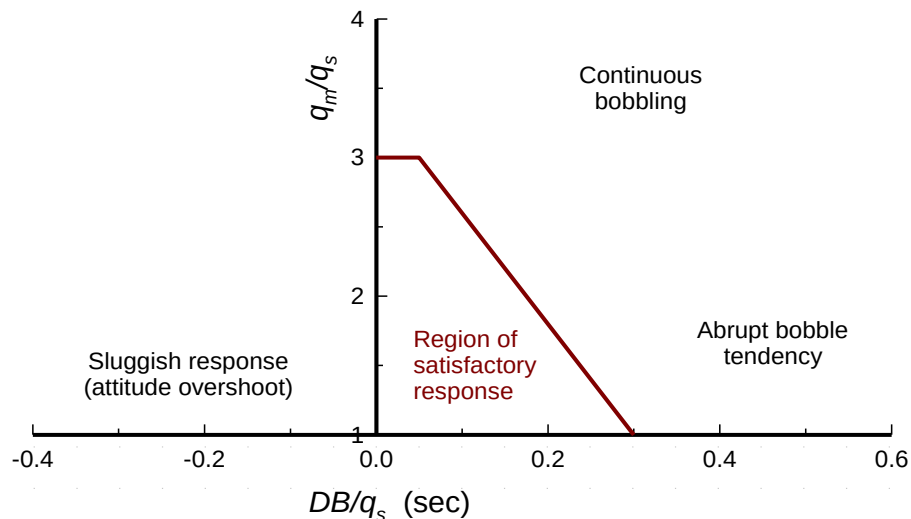
Range of available feedback gains:

Gains selected for q -feedback should be as low as possible and in particular:

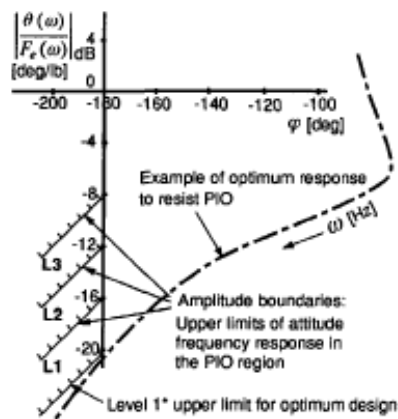
$$|k_q| \leq 0.1 \frac{\text{rad}}{\text{rad sec}^{-1}} \text{ or } \frac{\text{deg}}{\text{deg sec}^{-1}}$$

Appendix D: Gibson Dropback Criterion

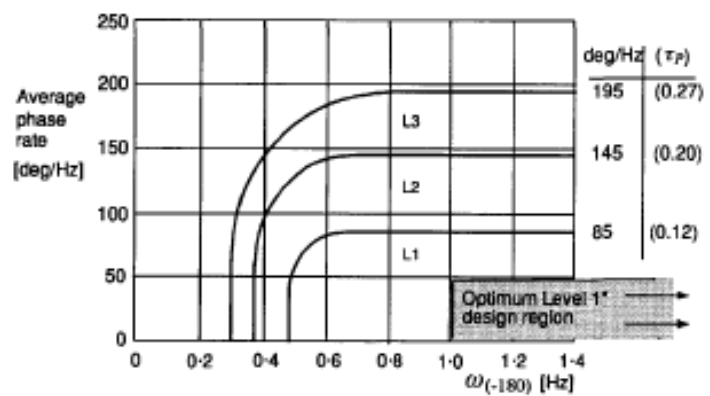
The background to the Gibson criteria – consisting of the Dropback criterion and the Frequency Domain criteria of PIO Frequency, Phase Rate and PIO Gain – is summarised in [4] and a copy of this will be made available to you in support of this assignment. The Following criteria chart are provided for convenience.



Time Domain Criteria: Dropback



(a) PIO gain limit criterion



(c) Phase rate and frequency criterion

Frequency Domain Criteria: PIO Frequency, Phase Rate and PIO Gain

5 Conclusion

5.1 Summary of the work

Literature Review

Methodology & Results

Lecture 1: Gibson's Criteria: A Primer for Test Pilots and Flight Test Engineers

Part 1 is 45 slides and approximately 55 minutes long

Part 2 is 43 slides and approximately 55 minutes long

Lecture 2: Applying Gibson's Criteria: a worked example of the A-7A

38 slides and approximately 40 minutes long

Exercise A: The F16

Exercise B: The A-7A

Exercise C: The Hawk

Appendices with full solutions and accompanying Matlab files

5.2 Statement of conclusions

It is the conclusion of the author that these lectures and accompanying assignments will serve as a concise and effective teaching tool at Test Pilot Schools around the world. The material provides a solid background and history, sufficient depth and breadth of knowledge such that future test pilots and flight test engineers will be more effective at their jobs in the early design phases of high order flight control systems as well as understanding the critical impact of the control law in affecting handling qualities in later test efforts.

The material in the lectures and assignments successfully achieves the three objectives of this thesis:

1. To review the literature available concerning Gibson's Criteria.

2. To develop a classroom lecture suitable for presenting the background, theory and application of Gibson's Criteria to an ETPS class.
3. To develop 3 new case studies involving the introduction and application of the Gibson criteria to ETPS class A FW TP and FTE.

5.3 Further suggested work

It is recommended to present the material during the graduate course of study at ETPS and potentially other leading Test Pilots' Schools and gather feedback to improve the lecture series and assignments

It is recommended to develop the Hawk assignment further to apply Gibson's Criteria to improve handling qualities on a baseline Hawk in the 6 DOF simulator at ETPS and potentially even the AstraHawk variable stability aircraft.

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Appendix A Exercise A: F-16, Solution & Matlab Files

A.1 Exercise A: F-16 Full Solution



School of Engineering

Using Gibson's Criteria to design in excellent longitudinal pitch handling qualities

Exercise A – F16

SOLUTION



Task 1 (Part A)

a. Use the data available to you in appendix B to assemble and then solve the longitudinal equations of motion of the F-16 aircraft model to obtain the response transfer functions. List transfer functions for:

$$\frac{u(s)}{\eta(s)}, \frac{\alpha(s)}{\eta(s)}, \frac{\theta(s)}{\eta(s)}, \frac{q(s)}{\eta(s)}, \frac{\gamma(s)}{\eta(s)}$$

with corresponding units of:

$$\frac{\text{m}}{\text{sec} \cdot \text{deg}}, \frac{\text{deg}}{\text{deg}}, \frac{\text{deg}}{\text{deg}}, \frac{\text{deg}}{\text{deg}}, \frac{\text{deg}}{\text{sec} \cdot \text{deg}}$$

The response transfer functions are:

Speed/Tailplane Transfer Function: $\text{m}/(\text{sec} \cdot \text{deg})$

$$\frac{0.05294 (s+1.415) (s^2 + 1.154s + 23.65)}{(s+1.912) (s-0.09755) (s^2 + 0.3014s + 0.03601)}$$

Alpha/Tailplane Transfer Function: deg/deg

$$\frac{-0.12318 (s+75) (s^2 + 0.01964s + 0.008892)}{(s+1.912) (s-0.09755) (s^2 + 0.3014s + 0.03601)}$$

Pitch Attitude/Tailplane Transfer Function: deg/deg

$$\frac{-10.0583 (s+1.027) (s+0.02174)}{(s+1.912) (s-0.09755) (s^2 + 0.3014s + 0.03601)}$$

Pitch Rate/Tailplane Transfer Function: $\text{deg}/(\text{sec} \cdot \text{deg})$

$$\frac{-10.0583 s (s+1.027) (s+0.02174)}{(s+1.912) (s-0.09755) (s^2 + 0.3014s + 0.03601)}$$

Flight Path/Tailplane Transfer Function: deg/deg

$$\frac{0.12318 (s-13.08) (s+6.426) (s+0.01375)}{(s+1.912) (s-0.09755) (s^2 + 0.3014s + 0.03601)}$$

Task 1 (Part B)

b. Summarise the significance of the predicted stability modes that are evident from your solution. Explain why it is essential that some augmentation is introduced to change the predicted stability modes.

The predicted stability modes are derived by setting the denominator of the transfer functions equal to zero and solving for the roots:

$$(s + 1.912)(s - 0.09755)(s^2 + 0.3014s + 0.03601) = 0$$

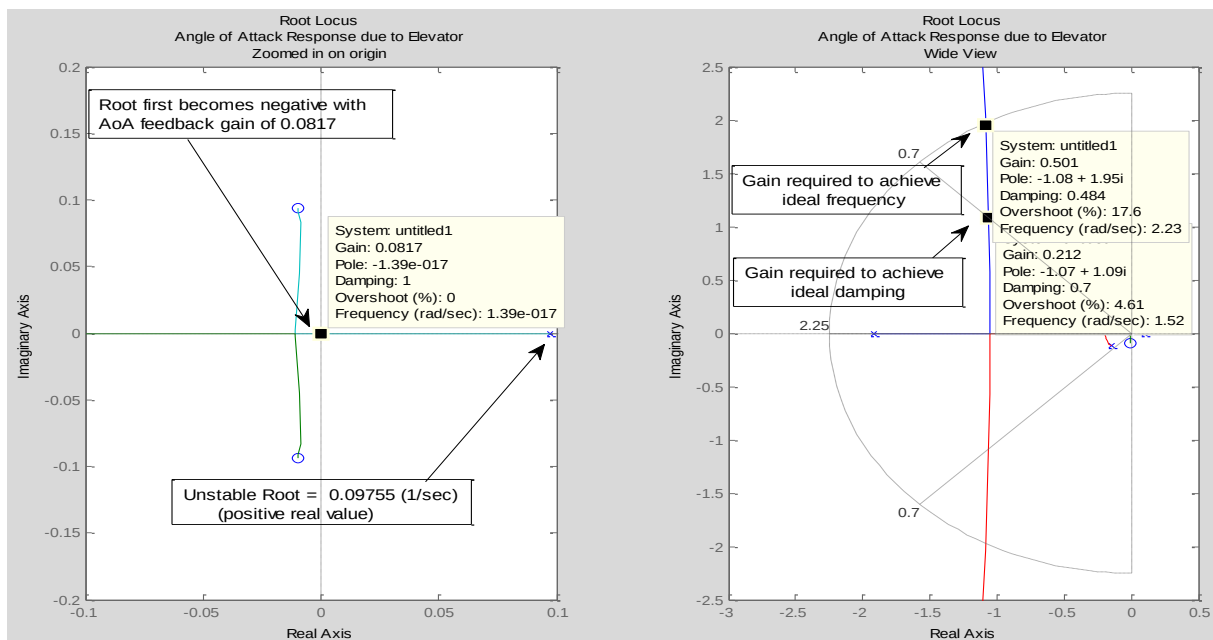
$$s = -0.1912 \text{ or } 0.09755 \text{ or } -0.151 \pm 0.115i$$

A positive root indicates instability in the system. In this case, the 0.09755 root is positive and real, indicating it is an exponential mode that is unstable. While unstable systems are not inherently impossible to control, and the relatively small magnitude here indicates that control may be possible, it is certainly not desirable. The pilot's focus must be on the mission tasks required, not on simply maintaining control of the aircraft. Stability augmentation via a computer controlled feedback loop is required to make this control system acceptable for the fighter role.

Task 2 (Part A)

a. Investigate and report on the possibilities for introducing angle of attack feedback or/and pitch rate feedback to modify the predicted stability modes. Select a combination of feedback gains for angle of attack feedback (k_a) and pitch rate feedback (k_q) to satisfy the dynamic stability criteria shown in appendix C – target your selection to meet a short period natural frequency of $2.25 \text{ rad sec}^{-1}$ and a short period damping ratio of 0.7. Explain the justification for your gain selections.

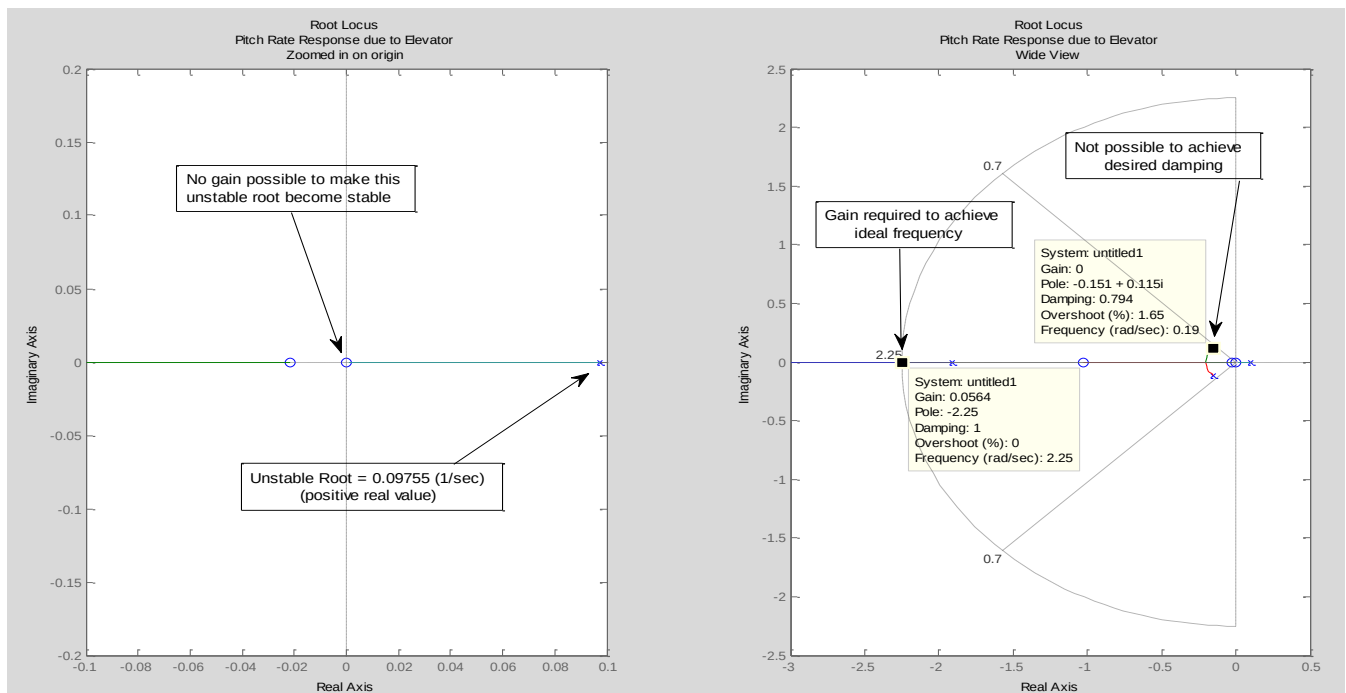
Angle of attack feedback only was investigated first via a root locus diagram:



Looking at the left diagram first, a zoomed in view of the origin, the unstable root is visible, as well as the minimum AoA feedback gain required to achieve stability, 0.0817, that is to move the root from the right or unstable side of the diagram to the left or stable side.

Looking to the right diagram, a wider view, it is possible to identify the gain required for 0.7 damping (0.212) or the gain to achieve the intended natural frequency of 2.25 rad/sec (0.501). It is also clear that there exists no gain such that the system passes through or near simultaneously achieving the desired natural frequency and damping.

Next, pitch rate feedback only was investigated first via a root locus diagram:



Looking at the left diagram first, a zoomed in view of the origin, the unstable root is visible. No amount of gain will move that unstable root to the left/stable side of the diagram. Pitch rate feedback alone cannot stabilize the system.

Looking to the right diagram, a wider view, it is possible to identify the gain required to achieve the intended natural frequency of 2.25 rad/sec (0.564), but that value exceeds the specified limit of 0.5 for this problem. The system never crosses the 0.7 damping ratio line so it is not possible to achieve that with any gain value.

Having identified that neither feedback type is sufficient to achieve desired stability characteristics alone, variations combinations of the two feedbacks were investigated. This achieved by picking values from zero to 0.5 at 0.1 interval for each feedback type and trying each possible combination. A loop was created via matlab to do this quickly and output the values to the command window. Some engineering judgement is required to pick the root that most closely resembles the short period pitch response and use that. A table was created by D.A. Lee and is used here to summarize the matlab results.

By inspection of the table and matlab results, it is clear that a gain magnitude of 0.4 for angle of attack feedback and a gain magnitude of 0.1 for pitch rate feedback achieve the desired natural frequency and damping values simultaneously.

All in $\frac{\text{deg}}{\text{deg}}$ or $\frac{\text{deg}}{\text{deg sec}^{-1}}$ as appropriate	$ k_\alpha = 0.0$	$ k_\alpha = 0.1$	$ k_\alpha = 0.2$	$ k_\alpha = 0.3$	$ k_\alpha = 0.4$	$ k_\alpha = 0.5$
$ k_q = 0$	Unstable due to positive root	$\omega_n = 1.13 \text{ rad sec}^{-1}$ $\zeta = 0.931$	$\omega_n = 1.49 \text{ rad sec}^{-1}$ $\zeta = 0.715$	$\omega_n = 1.77 \text{ rad sec}^{-1}$ $\zeta = 0.606$	$\omega_n = 2.01 \text{ rad sec}^{-1}$ $\zeta = 0.535$	$\omega_n = 2.23 \text{ rad sec}^{-1}$ $\zeta = 0.485$
$ k_q = 0.1$	Unstable due to positive root	$\omega_n = 1.22 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 1.8 \text{ rad sec}^{-1}$ $\zeta = 0.868$	$\omega_n = 2.04 \text{ rad sec}^{-1}$ $\zeta = 0.771$	$\omega_n = 2.25 \text{ rad sec}^{-1}$ $\zeta = 0.7$	$\omega_n = 2.45 \text{ rad sec}^{-1}$ $\zeta = 0.647$
$ k_q = 0.2$	Unstable due to positive root	$\omega_n = 1.11 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 2.06 \text{ rad sec}^{-1}$ $\zeta = 0.999$	$\omega_n = 2.28 \text{ rad sec}^{-1}$ $\zeta = 0.909$	$\omega_n = 2.47 \text{ rad sec}^{-1}$ $\zeta = 0.84$	$\omega_n = 2.66 \text{ rad sec}^{-1}$ $\zeta = 0.787$
$ k_q = 0.3$	Unstable due to positive root	$\omega_n = 1.08 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 1.43 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 1.94 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 2.67 \text{ rad sec}^{-1}$ $\zeta = 0.965$	$\omega_n = 2.84 \text{ rad sec}^{-1}$ $\zeta = 0.911$
$ k_q = 0.4$	Unstable due to positive root	$\omega_n = 1.07 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 1.31 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 1.59 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 1.94 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 2.42 \text{ rad sec}^{-1}$ $\zeta = 1$
$ k_q = 0.5$	Unstable due to positive root	$\omega_n = 1.06 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 1.25 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 1.45 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 1.68 \text{ rad sec}^{-1}$ $\zeta = 1$	$\omega_n = 1.93 \text{ rad sec}^{-1}$ $\zeta = 1$

Task 2 (Part B)

b. Present the augmented transfer functions for $\frac{\alpha(s)}{\eta(s)}$ and $\frac{q(s)}{\eta(s)}$ and summarise the significance of the predicted stability modes that are now evident from your solution.

The augmented response transfer functions are:

Alpha/Tail plane Transfer Function: *deg/deg*

$$\frac{-0.12318 (s+75) (s^2 + 0.01964s + 0.008892)}{(s^2 + 0.01737s + 0.005139) (s^2 + 3.153s + 5.087)}$$

Pitch Rate/Tail plane Transfer Function: *deg/(sec*deg)*

$$\frac{-10.0583 s (s+1.027) (s+0.02174)}{(s^2 + 0.01737s + 0.005139) (s^2 + 3.153s + 5.087)}$$

The augmented stability modes are derived by setting the denominator of the transfer functions equal to zero and solving for the roots:

$$(s^2 + 0.01737s + 0.0005139)(s^2 + 3.153s + 5.087) = 0$$

$$s = -0.158 \pm 0.161i \text{ or } -0.00869 \pm 0.0712i \text{ (units of inverse seconds)}$$

These yield short period pitch response characteristics of:

$$\omega_n = 2.26 \text{ rad/sec}$$

Matches desired criteria

$$\zeta = 0.699 \text{ unitless}$$

Matches desired criteria

And phugoid characteristics of:

$$\omega_n = 0.0717 \text{ rad/sec}$$

$$\omega_{n-sh} / \omega_{n-ph} = 2.26/0.0717 = 31.5 \geq 10$$

Matches desired criteria

$$\zeta = 0.121 \text{ unitless}$$

Matches desired criteria

The introduction of both AoA and pitch rate feedback has removed the instability, matched the short period characteristics to the ideal values and ensured the phugoid characteristics are well within the desired specifications. This solution meets the criteria and should produce good pitch handling qualities according to the conventional criteria we just assessed it against.

Task 3 (Part A)

a. Construct and present the reduced order equations of motion that describe the short term dynamics of the un-augmented F-16 aircraft model in the form:

$$\begin{pmatrix} \dot{\alpha} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} z_{\alpha} & z_q \\ m_{\alpha} & m_q \end{pmatrix} \begin{pmatrix} \alpha \\ q \end{pmatrix} + \begin{pmatrix} z_{\eta} \\ m_{\eta} \end{pmatrix} \eta$$

The reduced order equations of motion are:

$$\begin{pmatrix} \dot{\alpha} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} -1.0189 & 0.9051 \\ 0.8223 & -1.0774 \end{pmatrix} \begin{pmatrix} \alpha \\ q \end{pmatrix} + \begin{pmatrix} -0.0021 \\ -0.1756 \end{pmatrix} \eta$$

Task 3 (Part B)

b. Use your values of k_q and k_{α} from task 2a with the reduced order equations of motion to develop reduced order augmented transfer functions for $\frac{\alpha(s)}{\eta(s)}$ and $\frac{q(s)}{\eta(s)}$ develop the corresponding reduced order transfer function for the pitch attitude perturbation $\frac{\theta(s)}{\eta(s)}$ and the flight path angle perturbation $\frac{\gamma(s)}{\eta(s)}$.

The augmented, reduced order transfer functions are:

Alpha/Tail plane Augmented, Reduced Order Transfer Function: *deg/deg*

$$\frac{-0.12318(s+74.98)}{(s^2 + 3.151s + 5.083)}$$

$$(s^2 + 3.151s + 5.083)$$

Pitch Rate/Tail plane Augmented, Reduced Order Transfer Function: *deg/(sec*deg)*

$$\frac{-10.0583(s+1.029)}{(s^2 + 3.151s + 5.083)}$$

$$(s^2 + 3.151s + 5.083)$$

Pitch Attitude/Tail plane Augmented, Reduced Order Transfer Function:
deg/deg

$$\frac{-10.0583 (s+1.029)}{s (s^2 + 3.151s + 5.083)}$$

Flight Path/Tail plane Augmented, Reduced Order Transfer Function:
deg/deg

$$\frac{0.12318 (s-13.09) (s+6.417)}{s (s^2 + 3.151s + 5.083)}$$

Task 3 (Part C)

c. Modify the reduced order state equation from task 3(a) to include the actuator dynamics and present the amended reduced order state equation. Use your values of k_q and k_α from task 1c with the modified reduced order equations of motion to develop reduced order augmented transfer functions for:

$$\frac{\alpha(s)}{\delta(s)}, \frac{\theta(s)}{\delta(s)}, \frac{q(s)}{\delta(s)}, \frac{\gamma(s)}{\delta(s)}$$

The original reduced order state equation can be combined with the given actuator model transfer function into the following combined state equation:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} -1.0189 & 0.9051 & -0.0021 \\ 0.8223 & -1.0774 & -0.1756 \\ 0 & 0 & 20 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \eta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} \delta$$

The augmented, reduced order, actuator included transfer functions are calculated in a similar manner as the previous question and are:

Alpha/Inceptor Augmented, Reduced Order, Actuator Included Transfer Function: *deg/deg*

$$\frac{-2.4636 (s+74.98)}{(s+19.05) (s^2 + 3.047s + 5.337)}$$

Pitch Rate/Inceptor Augmented, Reduced Order, Actuator Included Transfer Function: *deg/(sec*deg)*

$$\frac{-201.1655 (s+1.029)}{(s+19.05) (s^2 + 3.047s + 5.337)}$$

Pitch Attitude/Inceptor Augmented, Reduced Order, Actuator Included Transfer Function: *deg/deg*

$$\frac{-201.1655 (s+1.029)}{s (s+19.05) (s^2 + 3.047s + 5.337)}$$

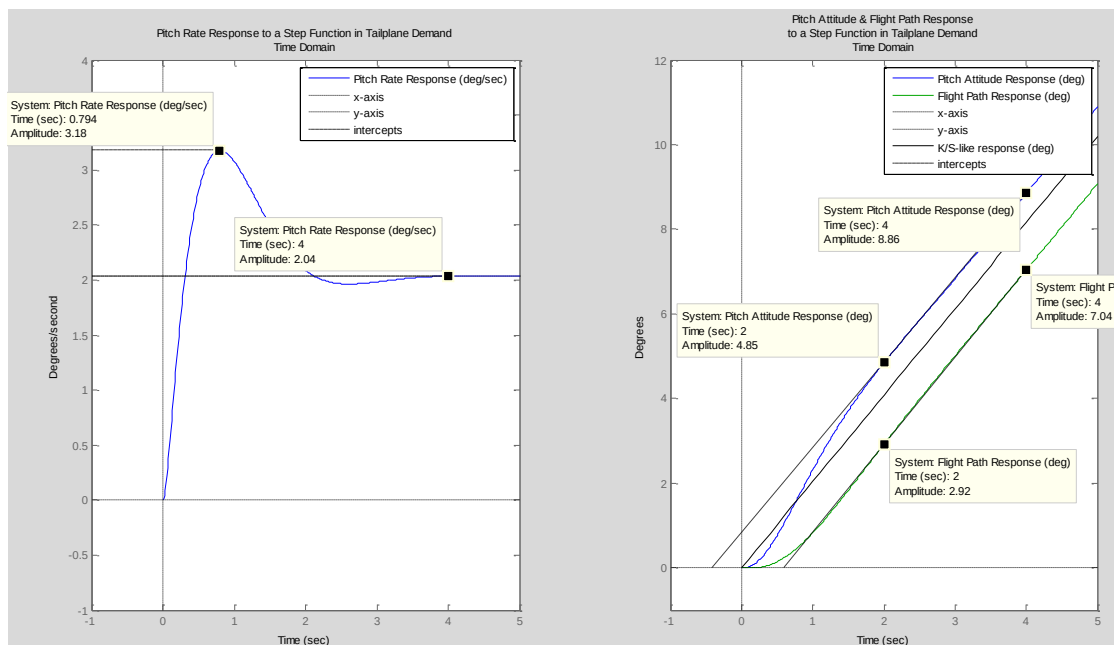
Flight Path/Inceptor Augmented, Reduced Order, Actuator Included Transfer Function: *deg/deg*

$$\frac{2.4636 (s-13.09) (s+6.417)}{s (s+19.05) (s^2 + 3.047s + 5.337)}$$

Task 4 (Part A)

a. Use your results from task 3c and apply the 'Gibson Dropback criterion' shown in Appendix D to determine the predicted handling qualities for a pitch tracking task for the actuator included, augmented aircraft model. Summarise the results of your analysis and your conclusion about the predicted handling qualities.

Gibson's dropback criteria was applied by plotting the transfer functions of pitch rate, pitch attitude and flight path in response to the tailplane demanded by the inceptor on a time domain plot. The plot is shown:



Critical parameters of pitch rate maximum, steady state pitch rate, the dropback/steady state pitch rate and the flight path delay were pulled off the plots:

$$q_{\max} = 3.18 \text{ deg/sec}$$

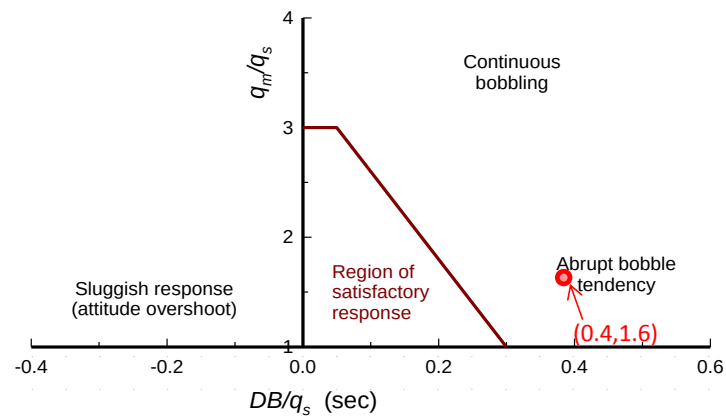
$$q_{ss} = 2.04 \text{ deg/sec}$$

$$\text{pitch rate overshoot ratio: } q_{\max} / q_{ss} = 1.6 \text{ unitless}$$

$$\text{Drop Back/ } q_{ss} = 0.4 \text{ sec}$$

$$\text{Flight path delay} = 0.5874 \text{ sec}$$

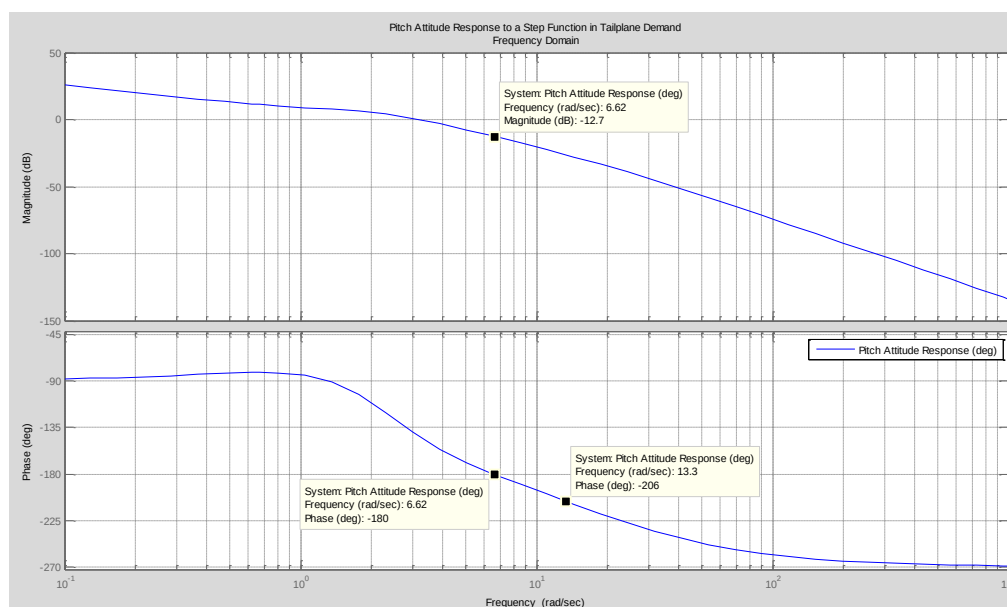
On a Gibson Dropback chart, this results in a prediction of poor handling qualities characterized by an abrupt bobble tendency.



Task 4 (Part B)

b. Continuing to use your results from task 3c, specifically the pitch attitude transfer function, apply the frequency domain portion of Gibson's Criteria: Phase Rate, PIO frequency and PIO gain limit criterion. Compare your results with the values shown in appendix D and comment on any 'specification compliance' issues.

Gibson's frequency domain criteria were applied by plotting the pitch attitude transfer function on a Bode Plot. The plot is shown:



Critical parameters of PIO frequency, the phase lag at twice the PIO frequency, the gain magnitude at the PIO frequency were pulled off the plot

PIO Frequency = 6.62 rad/sec

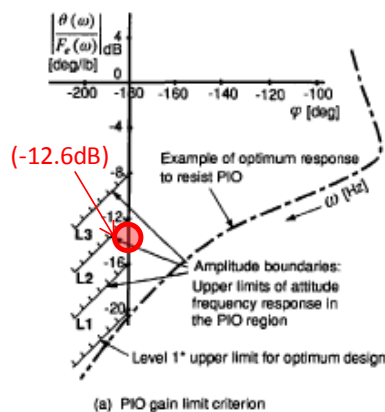
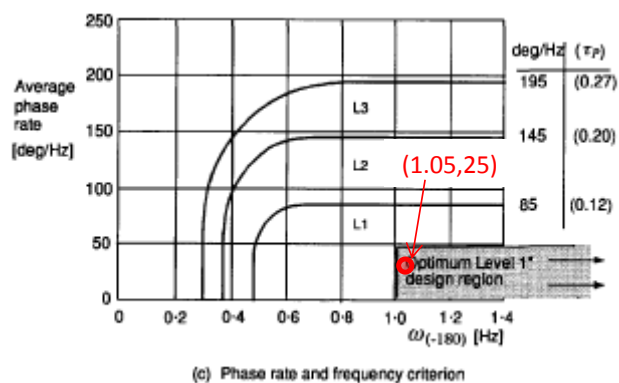
converted to hz = 1.05 Hz

Phase Lag at twice the PIO frequency = 206 degrees

Phase Rate = 25 deg/hz

PIO Frequency Gain Magnitude = -12.6 dB

On a Gibson frequency criteria chart, the PIO frequency and phase rate indicate level 1* handling qualities. The gain magnitude comes out as only Level 2 handling qualities, indicating that there is still substantial gain available to the pilot at the PIO frequency. This does not meet specification and should be corrected.



Task 5 (Part A)

b. Design a Command Path pre-Filter in accordance with Gibson's guidance as discussed in [4]. Present transfer functions for the pre-filter, pitch attitude and flight path and comment on any significant features.

A command path pre filter was designed to cancel out the existing $T_{\theta 2}$ value and replace it with the measured flight path delay value to reduce the drop back to zero to achieve excellent pitch handling qualities.

Existing $T_{\theta 2}$ value = 0.9718 seconds

New $T_{\theta 2}$ value = flight path delay = 0.5874 seconds

Command Path Pre-Filter Transfer Function: (sec/sec)

$$\frac{(s+1.702)}{(s+1.029)}$$

Pitch Attitude/Tail plane demand transfer function: (deg/deg)

$$\frac{-201.1655 (s+1.029)}{s (s+19.05) (s^2 + 3.047s + 5.337)}$$

Flight Path/Tail plane demand transfer function: (deg/deg)

$$\frac{2.4636 (s-13.09) (s+6.417)}{s (s+19.05) (s^2 + 3.047s + 5.337)}$$

The design of the pre-filter ensures that the $t_{\theta 2}$ term in the numerator of the pitch attitude will cancel out completely and be replaced with the new designed value. This does not happen in the flight path transfer function. While Gibson theorized that many of the pilot's tasks involved him closing the loop tightly around pitch attitude response and thus the handling qualities would be greatly associated with drop back characteristics, not all tasks are performed that way. For example, flaring to land is a flight path activity, not a pitch attitude one. The control engineer must be careful to use this type of pre filter when appropriate and fade it out when not, so that there are no hidden higher order effects as the control law's dominant variable is not the one the pilot is primarily using to accomplish the task.

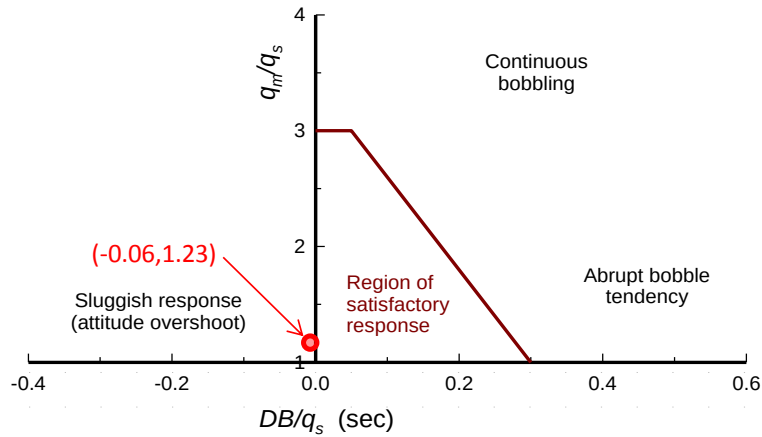
Task 5 (Part B)

b. Re-assess against the time domain and frequency domain Gibson Criteria and comment on the effectiveness of your command path pre-filter

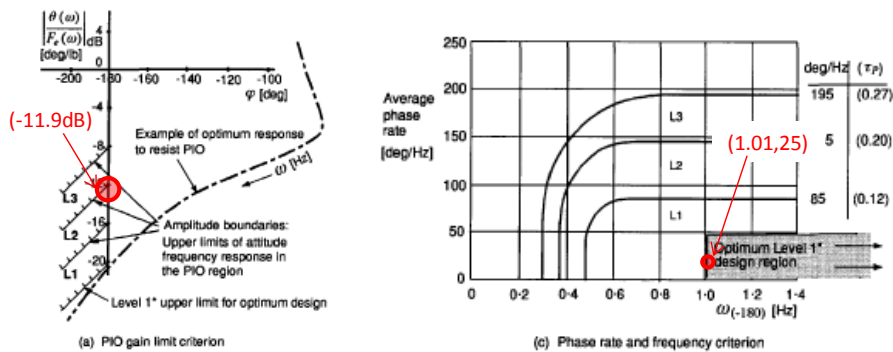
The command path pre filter was inserted into the control law and the same procedure as Task 4a and 4b was performed, resulting in the following key data pulled from the time and frequency domain plots:

$q_{\max}$	= 4.16 deg/sec
q_{ss}	= 3.37 deg/sec
pitch rate overshoot ratio: $q_{\max} / q_{ss}$	= 1.23 unitless
Drop Back/ q_{ss}	= -0.006 sec
Flight path delay	= 0.8389 sec
PIO Frequency	= 6.35 rad/sec
converted to hz	= 1.01 Hz
Phase Lag at twice the PIO frequency	= 205 degrees
Phase Rate	= 25 deg/hz
PIO Frequency Gain Magnitude	= -11.9 dB

On a Gibson Dropback chart, this results in a nearly perfect solution in the bottom left of the region of satisfactory response. Both the DB/ q_{ss} and pitch rate overshoot have been dramatically improved. However, the DB/ q_{ss} is actually just negative, which indicates a trend towards sluggish response or attitude overshoot. Prior to the final control law, the new $T_{\theta 2}$ value should be tweaked so the DB/ q_{ss} is zero or a very small positive number



On a Gibson frequency criteria chart, there is little change to the frequency response. The phase rate and frequency still indicate excellently, but the gain amplitude at the PIO frequency should be reduced by at least 4 db.



Overall, the command path pre-filter did an excellent job improving the predicted short period pitch handling qualities, but had a negligible effect on the frequency response.

A.2 Exercise A: F-16 Matlab Files

A.2.1 Task 1 M File

```
% Applying Gibson's Handling Qualities. Exercise A, F16
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2013

% Task 1a and 1b
% Solve the full order state equation and produce transfer
functions

% Start with all extra matlab windows closed from a fresh screen
with all stored values cleared
close all
clear all
clc

% Conversion factors and initial conditions
D2R = pi()/180; % Degress to Radians conversion
R2D = 180/pi(); % Radians to Degrees conversion
KTS2MS = 0.51444; % Knots to Meters per second conversion
V0 = 297*KTS2MS; % Freestream velocity in kts true

% Define state space matrices
%
% The longitudinal equations of motion corresponding to the given
flight
% condition were given as concise dimensional deriviatives. These
% derivatives have been divided through by the mass, so the state
space
% format is:
%  $dx/dt = Ax + Bu$ , and
%  $Y = Cx + Du$ , where:
%  $x$  is the state vector, in this case, a 4 row/1 column matrix
of  $[u, w, q, \theta]$ 
%  $u$  is the input vector, in this case consisting solely of  $\eta$ 
%  $A$  is the state matrix, a collection of the aerodynamic
derivatives
%  $B$  is the input matrix, a colelction of the control
derivatives
%  $C$  is the output matrix, and is used to manipulate the
outputs
%  $D$  is the direct transmission matrix, and is almost always a
null matrix

% Careful selection of variable names to keep track of what is what
can be
```

```

% difficult in writing code. I used A, A_reduced_order, etc to
keep track
% of which version of the A matrix is being operated on, rather
than
% simply saving over A everytime it changes. This also helps
identifying
% variables properly when outputting to the workspace.

A = [-0.019311 2.6870 -9.8054 -0.17526
     -0.00083297 -1.0189 0 0.90506
     0 0 0 1
     0 0.82225 0 -1.0774];

B = [0.05294
     -0.0021499
     0
     -0.17555];

C = [1 0 0 0;
     0 R2D 0 0;
     0 0 R2D 0;
     0 0 0 R2D];
% R2D is being used to convert all outputs that have units of
radians (alpha, q, theta) into units of degrees

D = zeros(4,1);

%Calculate full order Transfer Functions from equations of motion

[num,den] =ss2tf(A,B,C,D,1); % go from equations of motion to
transfer functions

%the code below removes the extra small number and extra zero that
matlab produces
[row,col] = size(num);
num_new = zeros(row,col);
for i =1:row
    for j=1:col
        num_new(i,j) = num(i,j);
        if abs(num_new(i,j)) < 1e-10
            num_new(i,j) = fix(num_new(i,j));
        end
    end
end

end
num = num_new;
%the code above removes the extra small number and extra zero that
matlab produces

disp('Task 1a:')
disp('Speed/Tailplane Transfer Function:')
u_eta = zpk(tf(num(1,:),den)) % isolate individual transfer
functions and output
disp('Alpha/Tailplane Transfer Function:')
alpha_eta = zpk(tf(num(2,:),den))
disp('Pitch Attitude/Tailplane Transfer Function:')

```

```

theta_eta      = zpk(tf(num(3,:),den))
disp('Pitch Rate/Tailplane Transfer Function:')
q_eta  = zpk(tf(num(4,:),den))

% Since flight path is pitch attitude minus angle of attack and we
have
% both of those transfer functions, we can subtract one from the
other to
% get a flight path transfer function:

num_gamma=num(3,:)-num(2,:);
disp('Flight Path/Tailplane Transfer Function:')
gamma_eta  = zpk(tf(num_gamma,den))

damp(A)  % outputs the short and long period damping ratio and
natural frequency

% This is the master file. Remaining tasks are run from here.

run task_2
run task_3
run task_4a
run task_4b
run task_5

```

A.2.2 Task 2 M File

```
% Applying Gibson's Handling Qualities. Exercise A, F16
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2013

% Task 2a and 2b
% Add feedback loops and reassess transfer functions and stability

% Task 2a
% Examine alpha and / or pitch rate feedback for stability
augmentation

% Produce root locus diagrams of AoA/eta Transfer function
% to identify possible gain values

figure(1)

subplot(1,2,1)
rlocus(-alpha_eta) %negative sign required to account for sign
convention
sgrid(0.7, 2.25)
axis([-0.1 0.1 -0.2 0.2])
title({'Root Locus';'Angle of Attack Response due to
Elevator';'Zoomed in on origin'})

subplot(1,2,2)
rlocus(-alpha_eta) %negative sign required to account for sign
convention
axis([-3 0.5 -2.5 2.5])
title({'Root Locus';'Angle of Attack Response due to
Elevator';'Wide View'})
sgrid(0.7, 2.25)

% Produce root locus diagrams of q/eta Transfer function
% to identify possible gain values

figure(2)

subplot(1,2,1)
rlocus(-q_eta) %negative sign required to account for sign
convention
sgrid(0.7, 2.25)
title({'Root Locus';'Pitch Rate Response due to Elevator';'Zoomed
in on origin'})
axis([-0.1 0.1 -0.2 0.2])

subplot(1,2,2)
rlocus(-q_eta) %negative sign required to account for sign
convention
axis([-3 0.5 -2.5 2.5])
title({'Root Locus';'Pitch Rate Response due to Elevator';'Wide
View'})
```

```

sgrid(0.7, 2.25)

% Neither feedback alone produces a solution near to 0.7 damping
and
% 2.25 deg/sec natural frequency. The following loop runs
iterates
% through combinations of pitch rate and angle of attack damping
from 0
% to 0.5 (the limit in appendix B) and outputs the result to the
% workspace. A table can then be created to identify the best
pair.

disp('Task 2a:')

for K_alpha = [0,0.1,0.2,0.3,0.4,0.5]
    for K_q = [0,0.1,0.2,0.3,0.4,0.5]
        K_alpha
        K_q
        K=[0,-K_alpha,0,-K_q]; %feedback is negative
        A_augmented=A-B*180/pi*K;
        damp(A_augmented)
    end
end

% Following inspection of the above output, the gain magnitudes
required to
% achieve the desired natural frequency and damping ratio are
alpha: 0.4
% and q: 0.1
% Those values are put in as the gain values and new transfer
functions
% created below in task 2b

K_alpha = -0.4;
K_q = -0.1;
K=[0,K_alpha,0,K_q];
A_augmented=A-B*180/pi*K;

[num_augmented,den_augmented] =ss2tf(A_augmented,B,C,D,1); % go
from equations of motion to transfer functions

%the code below removes the extra small number and extra zero that
matlab produces
[row,col] = size(num_augmented);
num_new = zeros(row,col);
for i =1:row
    for j=1:col
        num_new(i,j) = num_augmented(i,j);
        if abs(num_new(i,j)) < 1e-10
            num_new(i,j) = fix(num_new(i,j));
        end
    end
end

end
num_augmented = num_new;

```

```
%the code above removes the extra small number and extra zero that  
matlab produces
```

```
disp('Task 2b:')  
disp('Alpha/Tailplane Augmented Transfer Function:')  
alpha_eta_augmented = zpk(tf(num_augmented(2,:),den_augmented))  
disp('Pitch Rate/Tailplane Augmented Transfer Function:')  
q_eta_augmented      = zpk(tf(num_augmented(4,:),den_augmented))
```

```
damp(A_augmented) % outputs the short and long period damping  
ratio and natural frequency
```

A.2.3 Task 3 M File

```
% Applying Gibson's Handling Qualities. Exercise A, F16
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2013

% Task 3a, 3b and 3c
% Reduce the order, develop the transfer functions,
% introduce an actuator lag

% The reduced order equation is the same format as previous, but
with U and
% theta removed. In this case, rows 1 & 3 and columns 1 & 3
removed.

disp('Task 3a:')
A_reduced_order = [A(2,2) A(2,4)
                  A(4,2) A(4,4)]

B_reduced_order = [B(2,1)
                  B(4,1)]

C_reduced_order = [R2D 0
                  0 R2D];

D_reduced_order = zeros(2,1);

% Now with the reduced order state space equations defined, apply
the gains
% determined in task 2a to get an augmented, reduced order model

K_alpha = -0.4;
K_q      = -0.1;
K_reduced_order=[K_alpha,K_q];
A_ro_aug=A_reduced_order-B_reduced_order*180/pi*K_reduced_order;
%A_ro_aug stands for A matrix, reduced order, then augmented

[num_ro_aug,den_ro_aug]
=ss2tf(A_ro_aug,B_reduced_order,C_reduced_order,D_reduced_order,1);

%the code below removes the extra small number and extra zero that
matlab produces
[row,col] = size(num_ro_aug);
num_new = zeros(row,col);
for i =1:row
    for j=1:col
        num_new(i,j) = num_ro_aug(i,j);
        if abs(num_new(i,j)) < 1e-10;
            num_new(i,j) = fix(num_new(i,j));
        end
    end
end
end
```

```

num_ro_aug = num_new;
%the code above removes the extra small number and extra zero that
matlab produces

disp('Task 3b:')
disp('Alpha/Tailplane Augmented, Reduced Order Transfer Function:')
alpha_eta_ro_aug = zpk(tf(num_ro_aug(1,:),den_ro_aug))
disp('Pitch Rate/Tailplane Augmented, Reduced Order Transfer
Function:')
q_eta_ro_aug      = zpk(tf(num_ro_aug(2,:),den_ro_aug))

% Pitch attitude is the integration of Pitch Rate. Since 1/s is an
% integration term, we simply need to multiply the augmented,
reduced order
% pitch rate transfer function by 1/s to develop the augmented,
reduced
% order pitch attitude transfer function

Integration_TF = tf(1,[1 0]); % define 1/s integration transfer
function:
disp('Pitch Attitude/Tailplane Augmented, Reduced Order Transfer
Function:')
theta_eta_ro_aug = Integration_TF*q_eta_ro_aug

% Flight path is the remainder when subtracting angle of attack
from
% pitch attitude.
%  $\gamma(s)/\eta(s) = \theta(s)/\eta(s) - \alpha(s)/\eta(s)$ ,
% or in generic terms:
%  $\gamma(s)/\eta(s) = A(s+B)/[s(s^2+Xs+Y)] - C(s+D)/(s^2+Xs+Y)$ 
% multiply the second term on the right by s/s to make the
% denominators the same. This raises the s order by one in
the
% second term numerator. This can be easily accomplished by
moving each of
% the numerator coefficients one spot to the left in the
matrix.
% The first coefficient term is zero and is omitted to make
the
% matrix dimensions agree for the subsequent subtraction.

gamma_numerator_second_term = [num_ro_aug(1,2), num_ro_aug(1,3),
0];

% now subtract that from the theta numerator to get the full gamma
numerator
% keep in mind the theta TF numerator equals the q TF
numerator.
gamma_numerator = num_ro_aug(2,:) - gamma_numerator_second_term;

% The denominator is equal to theta denominator, which is equal
to the
% pitch rate denominator multiplied by 1/s. This can be
easily
% accomplished by moving each of the pitch rate denominator
coefficients one spot to the left in the matrix.

```

```

gamma_denominator=[den_ro_aug(1), den_ro_aug(2), den_ro_aug(3), 0];

% Compile the gamma/eta transfer function
disp('Flight Path/Tailplane Augmented, Reduced Order Transfer
Function:')
gamma_eta_ro_aug = zpk(tf(gamma_numerator,gamma_denominator))

% Task 3C
% Introduce an actuator lag with a time constant of 20 seconds into
the
% reduced order model, then apply the gains from Task 2 to augment
and
% stabilize the system, then output the new transfer functions.

A_ro_actuator = [A(2,2) A(2,4) B(2,1)
                 A(4,2) A(4,4) B(4,1)
                 0      0      -20]

B_ro_actuator = [0
                 0
                 20]

C_ro_actuator = [R2D 0 0
                 0 R2D 0
                 0 0 1];

D_ro_actuator = zeros(3,1);

[num_ro_act,den_ro_actuator] =
ss2tf(A_ro_actuator,B_ro_actuator,C_ro_actuator,D_ro_actuator,1);
% go from equations of motion to transfer functions

K_alpha = -0.4;
K_q      = -0.1;
K_ro_actuator=[K_alpha,K_q, 0];
A_ro_actuator=A_ro_actuator-B_ro_actuator*180/pi*K_ro_actuator;
%A_ro_actuator stands for A matrix, reduced order, then augmented

[num_ro_actuator,den_ro_actuator]
=ss2tf(A_ro_actuator,B_ro_actuator,C_ro_actuator,D_ro_actuator,1);

%the code below removes the extra small number and extra zero that
matlab produces

[row,col] = size(num_ro_actuator);
num_new = zeros(row,col);
for i =1:row
    for j=1:col
        num_new(i,j) = num_ro_actuator(i,j);
        if abs(num_new(i,j)) < 1e-10;
            num_new(i,j) = fix(num_new(i,j));
        end
    end
end
end

```

```

end
num_ro_actuator = num_new;
%the code above removes the extra small number and extra zero that
matlab produces

disp('Task 3c:')
disp('Alpha/Inceptor Augmented, Reduced Order, Actuator Included
Transfer Function:')
alpha_eta_ro_actuator =
zpk(tf(num_ro_actuator(1,:),den_ro_actuator))
disp('Pitch Rate/Inceptor Augmented, Reduced Order, Actuator
Included Transfer Function:')
q_eta_ro_actuator =
zpk(tf(num_ro_actuator(2,:),den_ro_actuator))

Integration_TF = tf(1,[1 0]); % define 1/s integration transfer
function:
disp('Pitch Attitude/Inceptor Augmented, Reduced Order, Actuator
Included Transfer Function:')
theta_eta_ro_actuator = Integration_TF*q_eta_ro_actuator

gamma_numerator_second_term = [num_ro_actuator(1,2),
num_ro_actuator(1,3), num_ro_actuator(1,4),0];

gamma_numerator = num_ro_actuator(2,:) -
gamma_numerator_second_term;

gamma_denominator=[den_ro_actuator(1), den_ro_actuator(2),
den_ro_actuator(3), den_ro_actuator(4), 0];

disp('Flight Path/Inceptor Augmented, Reduced Order, Actuator
Included Transfer Function:')
gamma_eta_ro_actuator = zpk(tf(gamma_numerator,gamma_denominator))

```

A.2.4 Task 4A M File

```
% Applying Gibson's Handling Qualities. Exercise A, F16
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2013

% Task 4a
% Assess the system against Gibson's Dropback Criteria

% Run this file one time, then look at the figure 3 to identify Max
pitch rate,
% steady state pitch rate, and evaluate the pitch attitude and
flight path
% values at 2 and 4 seconds.

figure(3)
t=5;

subplot(1,2,1)
grid off
step(-q_eta_ro_actuator,t);
hold all
axis([-1 t -1 4])
title({'Pitch Rate Response to a Step Function in Tailplane
Demand';'Time Domain'})
ylabel('Degrees/second')
line([0,0],[-1,4],'color','k','linestyle',':')
line([-1,t],[0,0],'color','k','linestyle',':')

% Run this file one time, then look at the plot identify Max pitch
rate,
% steady state pitch rate, and evaluate the pitch attitude and
flight path
% values at 2 and 4 seconds.
% Input those values below here to draw the intercept lines

% Identify Pitch Rate Max amplitude and time:
q_max_amplitude = 3.18;
q_max_time = 0.794;
% Identify Pitch Rate steady state amplitude and time:
q_ss_amplitude = 2.04;
q_ss_time = 4.0;
% Evaluate pitch attitude at 2 seconds and 4 seconds:
theta_2_seconds = 4.85;
theta_4_seconds = 8.86;
% Evaluate Flight Path at 2 seconds and 4 seconds:
gamma_2_seconds = 2.91;
gamma_4_seconds = 7.03;

% Pitch Rate Max intercept line
```

```

line([-1,q_max_time],[q_max_amplitude,
q_max_amplitude],'color','k','linestyle','--')
legend('Pitch Rate Response (deg/sec)','x-axis','y-
axis','intercepts');

% Steady State Pitch Rate intercept line
line([-1,q_ss_time],[q_ss_amplitude,q_ss_amplitude],'color','k','linestyle
','--')

subplot(1,2,2)
grid off
step(-theta_eta_ro_actuator,t);
hold all
step(-gamma_eta_ro_actuator,t) ; ylabel('Degrees')
hold all
axis([-1 t -1 12])
title({'Pitch Attitude & Flight Path Response';'to a Step Function
in Tailplane Demand';'Time Domain'})
line([0,0],[-1,12],'color','k','linestyle',':')
line([-1,t],[0,0],'color','k','linestyle',':')

% Attitude integrator function (k/s performance)
t_int=[0:0.1:t];
integrator=t_int*q_ss_amplitude;
plot(t_int,integrator,'color','k','linestyle','-')
% Steady state pitch attitude slope line (used to identify
dropback)
theta_slope = (theta_4_seconds-theta_2_seconds)/2;
theta_y_intercept = theta_2_seconds - 2*theta_slope;
theta_x_intercept = -theta_y_intercept/theta_slope;
line([theta_x_intercept,4],[0,
theta_4_seconds],'color','k','linestyle','--')
% Steady state flight path slope line (used to identify flight
path delay)
gamma_slope = (gamma_4_seconds-gamma_2_seconds)/2;
gamma_y_intercept = gamma_2_seconds - 2*gamma_slope;
gamma_x_intercept = -gamma_y_intercept/gamma_slope;
line([gamma_x_intercept,4],[0,
gamma_4_seconds],'color','k','linestyle','--')
legend('Pitch Attitude Response (deg)','Flight Path Response
(deg)','x-axis','y-axis','K/S-like response (deg)','intercepts');

%Output critical attitude parameters to workspace
disp('Task 4a:')
Pitch_overshoot_ratio = q_max_amplitude/q_ss_amplitude
Dropback_over_qss = -theta_x_intercept % negative sign to account
for sign convection of dropback
Flight_Path_Delay = gamma_x_intercept

```

A.2.5 Task 4B M File

```
% Applying Gibson's Handling Qualities. Exercise A, F16
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2013

% Task 4b
% Assess the system against Gibson's Frequency Domain Criteria

% Run this file one time, then look at the figure 4 to identify the
pio
% frequency, the gain at the pio frequency, and twice the pio
frequency
%

figure(4)

bode(-theta_eta_ro_actuator)
grid on
title({'Pitch Attitude Response to a Step Function in Tailplane
Demand'; 'Frequency Domain'})
legend('Pitch Attitude Response (deg)')

% Run this file one time, then look at the figure 4 to identify the
pio
% frequency, the gain at the pio frequency, and twice the pio
frequency
% Input those values below here to output the Gibson frequency
criteria to
% the command window

% Identify the PIO Frequency, where the phase lag crosses 180
degrees
PIO_Frequency = 6.62; %units of radians
% Double the PIO frequency, and evaluate the phase lag there
Double_PIO_Frequency_Lag = 206; %units of degrees
% Read the gain magnitude off the top plot at the pio frequency
PIO_Gain = -12.6; %units of dB

%Output critical attitude parameters to workspace
disp('Task 4b:')
PIO_Frequency_hz = PIO_Frequency/(2*pi) %convert to hertz
Phase_Rate = (Double_PIO_Frequency_Lag - 180)/(2*PIO_Frequency_hz-
PIO_Frequency_hz)
PIO_Gain
```

A.2.6 Task 5 M File

```
% Applying Gibson's Handling Qualities. Exercise A, F16
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2013

% Task 5a and 5b
% Develop and apply a command path pre-filter IAW Gibson's guidance
and
% then evaluate its effectiveness in improving handling qualities
prediction

% Run task_1 first to get all the tasks to run through
sequentially. Then
% identify critical values on figures 5 & 6 to type back into here
and then
% run again from task_1 to get the final plots

% Task 5a
% Design Command Path Pre-Filter by replacing the existing T Theta
2 value
% with the value of the system's flight path delay

% t_theta_2 is equal to the dropback/qss plus the flight path delay
% (This relationship is apparent by looking at slide 64 of [4])
% Due to a less than perfect linear fit in task 4a, that value is
off by
% about 3 hundredths of a second from the value in the
% pitch attitude/tail plane demand transfer function. For clarity
in
% cancelling terms, the t_theta_2 value from the transfer
function is
% used below.

% The t_theta_2 value in the pitch attitude transfer function can
be
% identified by inspection, or pulling it directly from the
numerator of
% the transfer function, as I do below. Remember the theta TF has
the same
% numerator as the pitch rate TF
disp('Task 5a:')

t_theta_2 = 1/(num_ro_actuator(2,4)/num_ro_actuator(2,3))
new_t_theta_2 = Flight_Path_Delay % new value of T_theta_2 equal to
flight path delay should reduce attitude dropback to zero, but not
negative

a=1/new_t_theta_2;
b=1/t_theta_2;

num5 = [1 a];
den5 = [1 b];
```

```

disp('Command Path Pre-Filter')
command_path_pre_filter = zpk(tf(num5,den5))
disp('Pitch Attitude/Tail plane demand transfer function')
theta_eta_ro_actuator
disp('Flight Path/Tail plane demand transfer function')
gamma_eta_ro_actuator

% Task 5b
% Evaluate the effectiveness of the pre-filter

% Apply the command path pre-filter
theta_eta_gibson = command_path_pre_filter*theta_eta_ro_actuator;
q_eta_gibson = command_path_pre_filter*q_eta_ro_actuator;
gamma_eta_gibson = command_path_pre_filter*gamma_eta_ro_actuator;

figure(5)

subplot(1,2,1)
grid off
step(-q_eta_gibson,t);
hold all
step(-q_eta_ro_actuator,t);
hold all
axis([-1 t -1 5])
title({'Pitch Rate Response to a Step Function in Tailplane Demand';'Time Domain'})
ylabel('Degrees/second')
line([0,0],[-1,5],'color','k','linestyle',':')
line([-1,t],[0,0],'color','k','linestyle',':')

% Run this file one time, then look at the figure 5 identify Max
pitch rate,
% steady state pitch rate, and evaluate the pitch attitude and
flight path
% values at 2 and 4 seconds.
% Input those values below here to draw the intercept lines

% Identify Pitch Rate Max amplitude and time:
q_max_amplitude = 4.16;
q_max_time = 1.02;
% Identify Pitch Rate steady state amplitude and time:
q_ss_amplitude = 3.37;
q_ss_time = 4.0;
% Evaluate pitch attitude at 2 seconds and 4 seconds:
theta_2_seconds = 6.64;
theta_4_seconds = 13.3;
% Evaluate Flight Path at 2 seconds and 4 seconds:
gamma_2_seconds = 3.82;
gamma_4_seconds = 10.4;

% Pitch Rate Max intercept line
line([-1,q_max_time],[q_max_amplitude,
q_max_amplitude],'color','k','linestyle','--')

```

```

legend('Pitch Rate with Gibson Pre-Filter (deg/sec)', 'Pitch Rate
with no filter (deg/sec)', 'x-axis', 'y-axis', 'intercepts');

% Steady State Pitch Rate intercept line
line([-
1, q_ss_time], [q_ss_amplitude, q_ss_amplitude], 'color', 'k', 'linestyle
', '--')

subplot(1,2,2)
grid off
step(-theta_eta_gibson, t);
hold all
step(-gamma_eta_gibson, t) ; ylabel('Degrees')
hold all
step(-theta_eta_ro_actuator, t);
hold all
step(-gamma_eta_ro_actuator, t) ; ylabel('Degrees')
hold all
axis([-1 t -1 20])
title({'Pitch Attitude & Flight Path Response'; 'to a Step Function
in Tailplane Demand'; 'Time Domain'})
line([0,0], [-1,20], 'color', 'k', 'linestyle', ':')
line([-1,t], [0,0], 'color', 'k', 'linestyle', ':')

% Attitude integrator function (k/s performance)
t_int=[0:0.1:t];
integrator=t_int*q_ss_amplitude;
plot(t_int, integrator, 'color', 'k', 'linestyle', '-')
% Steady state pitch attitude slope line (used to identify
dropback)
theta_slope = (theta_4_seconds - theta_2_seconds)/2;
theta_y_intercept = theta_2_seconds - 2*theta_slope;
theta_x_intercept = -theta_y_intercept/theta_slope;
line([theta_x_intercept, 4], [0,
theta_4_seconds], 'color', 'k', 'linestyle', '--')
% Steady state flight path slope line (used to identify flight
path delay)
gamma_slope = (gamma_4_seconds - gamma_2_seconds)/2;
gamma_y_intercept = gamma_2_seconds - 2*gamma_slope;
gamma_x_intercept = -gamma_y_intercept/gamma_slope;
line([gamma_x_intercept, 4], [0,
gamma_4_seconds], 'color', 'k', 'linestyle', '--')
legend('Pitch Rate with Gibson Pre-Filter (deg/sec)', 'Flight Path
with Gibson Pre-Filter (deg/sec)', 'Pitch Rate with no filter
(deg/sec)', 'Flight Path with no filter (deg/sec)', 'x-axis', 'y-
axis', 'K/S-like response (deg)', 'intercepts');

% Output critical attitude parameters to workspace
disp('Task 5b:')
Pitch_overshoot_ratio = q_max_amplitude/q_ss_amplitude
Dropback_over_qss = -theta_x_intercept % negative sign to account
for sign convention of dropback
Flight_Path_Delay = gamma_x_intercept

% Task 5b continued
% Assess the system against Gibson's Frequency Domain Criteria

```

```

% Run this file one time, then look at the figure 6 to identify the
pio
% frequency, the gain at the pio frequency, and twice the pio
frequency

figure(6)

bode(-theta_eta_gibson)
grid on
title({'Pitch Attitude Response to a Step Function in Tailplane
Demand'; 'Frequency Domain'})
legend('Pitch Attitude Response (deg)')

% Run this file one time, then look at the figure 4 to identify the
pio
% frequency, the gain at the pio frequency, and twice the pio
frequency
% Input those values below here to output the Gibson frequency
criteria to
% the command window

% Identify the PIO Frequency, where the phase lag crosses 180
degrees
PIO_Frequency = 6.35; %units of radians
% Double the PIO frequency, and evaluate the phase lag there
Double_PIO_Frequency_Lag = 205; %units of degrees
% Read the gain magnitude off the top plot at the pio frequency
PIO_Gain = -11.9; %units of dB

%Output critical attitude parameters to workspace
PIO_Frequency_hz = PIO_Frequency/(2*pi) %convert to hertz
Phase_Rate = (Double_PIO_Frequency_Lag - 180)/(2*PIO_Frequency_hz-
PIO_Frequency_hz)
PIO_Gain

```


Appendix B A7, Full Solution

B.1 Full Solution



School of Engineering

Using Gibson's Criteria to design in excellent longitudinal pitch handling qualities

Exercise B – A-7A Corsair II

(Presented in the Applying Gibson's Criteria lecture)

SOLUTION



Task 1 (Part A)

a. Use the data available to you in appendix B to assemble and then solve the longitudinal equations of motion of the A-7A aircraft model to obtain the response transfer functions. List transfer functions for:

$$\frac{u(s)}{\eta(s)}, \frac{w(s)}{\eta(s)}, \frac{q(s)}{\eta(s)}, \frac{\theta(s)}{\eta(s)}, \frac{\alpha(s)}{\eta(s)}, \frac{\gamma(s)}{\eta(s)}$$

with corresponding units of:

$$\frac{\text{m}}{\text{sec} \cdot \text{deg}}, \frac{\text{m}}{\text{sec} \cdot \text{deg}}, \frac{\text{deg}}{\text{deg}}, \frac{\text{deg}}{\text{deg}}, \frac{\text{deg}}{\text{sec} \cdot \text{deg}}, \frac{\text{deg}}{\text{deg}}$$

Speed/Tailplane Transfer Function: $m/(\text{sec} \cdot \text{deg})$

$$1.716 (s+58.44) (s+0.5866) (s+0.3691)$$

$$(s^2 + 0.03329s + 0.01972) (s^2 + 0.9017s + 2.665)$$

Vertical Speed/Tailplane Transfer Function: $m/(\text{sec} \cdot \text{deg})$

$$-7.2542 (s+59.05) (s^2 - 0.00877s + 0.009786)$$

$$(s^2 + 0.03329s + 0.01972) (s^2 + 0.9017s + 2.665)$$

Pitch Rate/Tailplane Transfer Function: $\text{deg}/(\text{sec} \cdot \text{deg})$

$$-4.5158 s (s+0.5055) (s-0.008233)$$

$$(s^2 + 0.03329s + 0.01972) (s^2 + 0.9017s + 2.665)$$

Pitch Attitude/Tailplane Transfer Function: *deg/deg*

$$-4.5158 (s+0.5055) (s-0.008233)$$

$$(s^2 + 0.03329s + 0.01972) (s^2 + 0.9017s + 2.665)$$

Alpha/Tailplane Transfer Function: *deg/deg*

$$-0.022923 (s+59.05) (s^2 - 0.00877s + 0.009786)$$

$$(s^2 + 0.03329s + 0.01972) (s^2 + 0.9017s + 2.665)$$

Gamma/Tailplane Transfer Function: *deg/deg*

$$0.022923 (s-138.7) (s+0.7239) (s-0.01392)$$

$$(s^2 + 0.03329s + 0.01972) (s^2 + 0.9017s + 2.665)$$

Task 1 (Part B)

b. Summarise the significance of the predicted stability modes that are evident from your solution.

The predicted stability modes are derived by setting the denominator of the transfer functions equal to zero and solving for the roots:

$$(s^2 + 0.03329s + 0.01972) (s^2 + 0.9017s + 2.665) = 0$$

$$s = -0.451 \pm 1.57i \text{ or } -0.0166 \pm 0.139i$$

Eigenvalue	Damping	Freq. (rad/s)
-4.51e-001 + 1.57e+000i	2.76e-001	1.63e+000
-4.51e-001 - 1.57e+000i	2.76e-001	1.63e+000
-1.66e-002 + 1.39e-001i	1.19e-001	1.40e-001
-1.66e-002 - 1.39e-001i	1.19e-001	1.40e-001

These stability modes represent the conventional short period and phugoid and are all stable. The short period natural frequency is 1.63 rad/sec with a damping ratio of 0.276. The phugoid natural frequency is 0.14 rad/sec with a damping ratio of 0.119. The system is stable with relatively low frequencies and damping ratios. If there was a positive root, feedback would be required to stabilize, or we could simply add feedback to attempt to change the stability characteristics closer to idealized values of perhaps 0.7 damping ratio and a higher natural frequency. For this exercise, we have omitted that step to focus on applying Gibson's Criteria and this simplification will be apparent in the frequency domain criteria later on.

Task 2 (Part A)

a. Construct and present the reduced order equations of motion of pitch rate and angle of attack that describe the short term dynamics of the A-7A aircraft model.

The reduced order equations of motion are:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -0.54500 & 309.00000 \\ -0.00767 & -0.39500 \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} -23.80000 \\ -4.51576 \end{bmatrix} \eta$$

Task 2 (Part B)

b. Use the reduced order equations of motion to develop reduced order transfer functions for $\frac{\alpha(s)}{\eta(s)}$ and $\frac{q(s)}{\eta(s)}$ and develop the corresponding reduced order transfer function for the pitch attitude perturbation $\frac{\theta(s)}{\eta(s)}$ and the flight path angle perturbation $\frac{\gamma(s)}{\eta(s)}$.

The reduced order transfer functions are:

Pitch Rate/Tailplane Reduced Order Transfer Function: $\text{deg}/(\text{sec} \cdot \text{deg})$

$$\frac{-4.5158 (s+0.5046)}{(s^2 + 0.94s + 2.585)}$$

Alpha/Tailplane Reduced Order Transfer Function: deg/deg

$$\frac{-0.022923 (s+59.02)}{(s^2 + 0.94s + 2.585)}$$

Pitch Attitude/Tailplane Reduced Order Transfer Function: *deg/deg*

$$\frac{-4.5158 (s+0.5046)}{s (s^2 + 0.94s + 2.585)}$$

Flight Path/Tailplane Reduced Order Transfer Function: *deg/deg*

$$\frac{0.022923 (s-138.7) (s+0.7167)}{s (s^2 + 0.94s + 2.585)}$$

Task 2 (Part C)

c. Modify the reduced order state equation from task 2(a) to include the actuator dynamics and present the amended reduced order state equation. Use the amended reduced order equations of motion to develop reduced order transfer functions for:

$$\frac{\alpha(s)}{\delta(s)}, \frac{\theta(s)}{\delta(s)}, \frac{q(s)}{\delta(s)}, \frac{\gamma(s)}{\delta(s)}$$

The original reduced order state equation can be combined with the given actuator model transfer function into the following combined state equation:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} -0.54500 & 309.00000 & -23.80000 \\ -0.00767 & -0.39500 & -4.51576 \\ 0 & 0 & 20 \end{bmatrix} \begin{bmatrix} w \\ q \\ \eta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} \delta$$

The reduced order, actuator included transfer functions are calculated in a similar manner as the previous question and are:

Pitch Rate/Inceptor Reduced Order, Actuator Included Transfer Function:
*deg/(sec*deg)*

$$\frac{-90.3152 (s+0.5046)}{(s+20) (s^2 + 0.94s + 2.585)}$$

Alpha/Inceptor Reduced Order, Actuator Included Transfer Function: *deg/deg*

$$-0.45847 (s+59.02)$$

$$(s+20) (s^2 + 0.94s + 2.585)$$

**Pitch Attitude/Inceptor Reduced Order, Actuator Included Transfer Function:
*deg/deg***

$$-90.3152 (s+0.5046)$$

$$s (s+20) (s^2 + 0.94s + 2.585)$$

**Flight Path/Inceptor Reduced Order, Actuator Included Transfer Function:
*deg/deg***

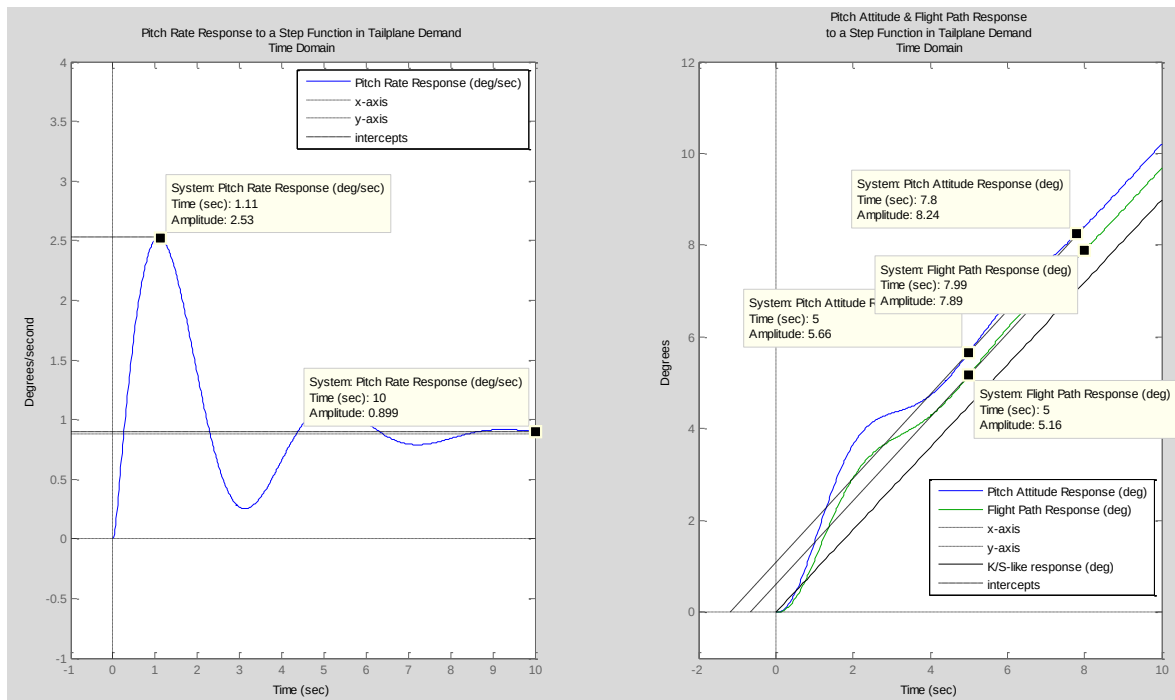
$$0.45847 (s-138.7) (s+0.7167)$$

$$s (s+20) (s^2 + 0.94s + 2.585)$$

Task 3 (Part A)

a. Use your results from task 2c and apply the 'Gibson Dropback criterion' to determine the predicted handling qualities for a pitch tracking task for the actuator included aircraft model. Summarise the results of your analysis and present your conclusions about the predicted handling qualities.

Gibson's dropback criteria was applied by plotting the transfer functions of pitch rate, pitch attitude and flight path in response to the tailplane demanded by the inceptor on a time domain plot. The plot is shown:



Critical parameters of pitch rate maximum, steady state pitch rate, the dropback/steady state pitch rate and the flight path delay were pulled off the plots:

$$q_{\max} = 2.53 \text{ deg/sec}$$

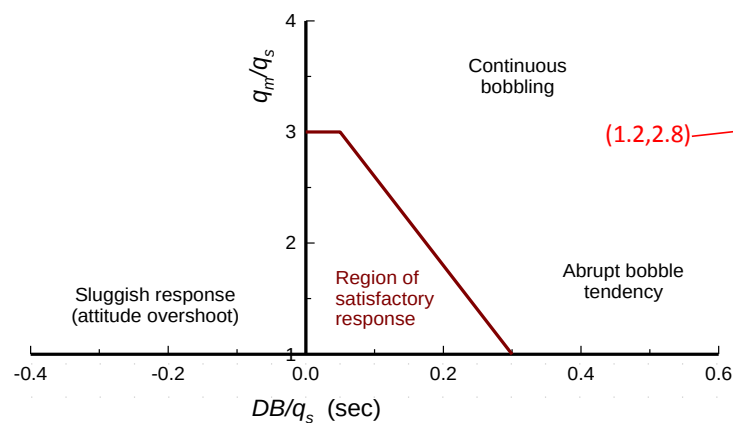
$$q_{ss} = 0.899 \text{ deg/sec}$$

$$\text{pitch rate overshoot ratio: } q_{\max} / q_{ss} = 2.8 \text{ unitless}$$

$$\text{Drop Back/ } q_{ss} = 1.2 \text{ sec}$$

$$\text{Flight path delay} = -0.6703 \text{ sec}$$

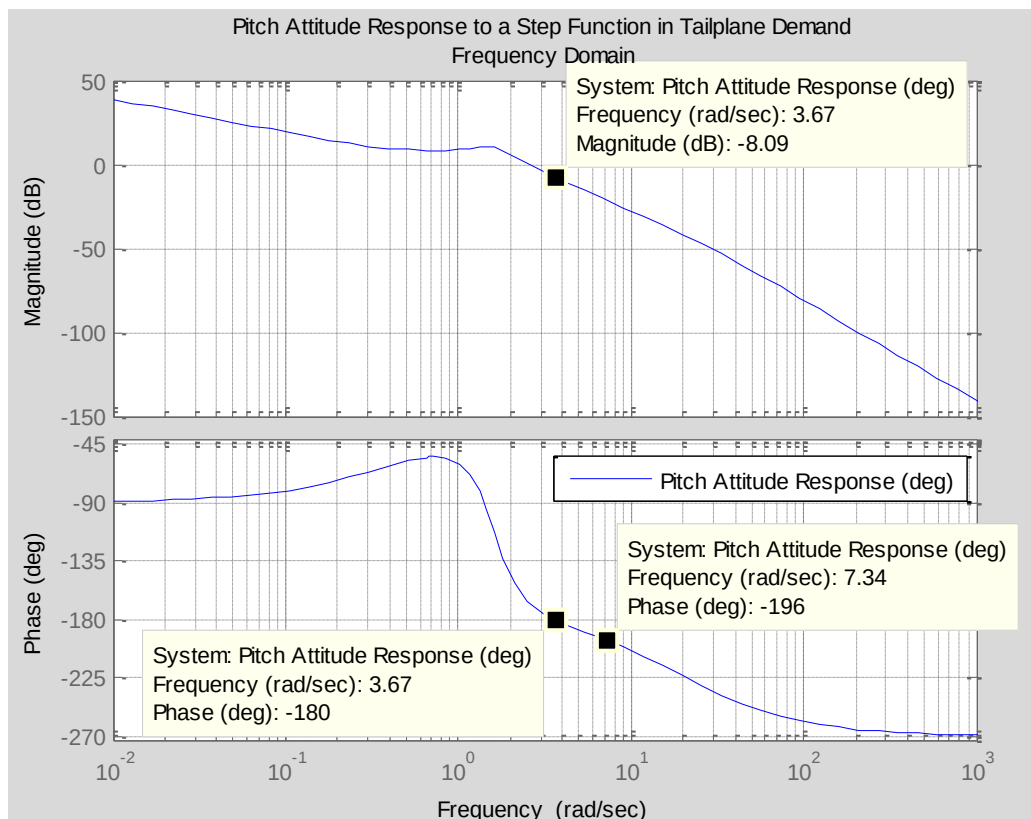
On a Gibson Dropback chart, this results in a prediction of poor handling qualities characterized by continuous and abrupt bobbling.



Task 3 (Part B)

b. Continuing to use your results from task 2c, specifically the pitch attitude transfer function, apply the frequency domain portion of Gibson's Criteria: Phase Rate, PIO frequency and PIO gain limit criterion. Compare your results with the values shown in appendix D and comment on any 'specification compliance' issues.

Gibson's frequency domain criteria were applied by plotting the pitch attitude transfer function on a Bode Plot. The plot is shown:



Critical parameters of PIO frequency, the phase lag at twice the PIO frequency, the gain magnitude at the PIO frequency were pulled off the plot

PIO Frequency = 3.7 rad/sec

converted to hz = 0.6 Hz

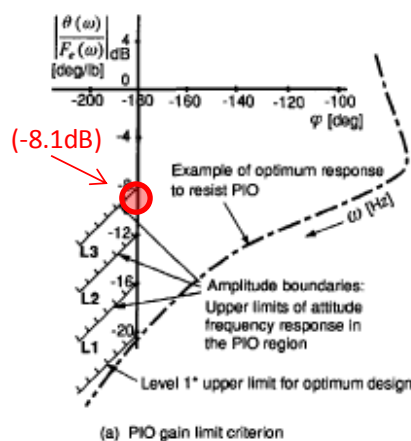
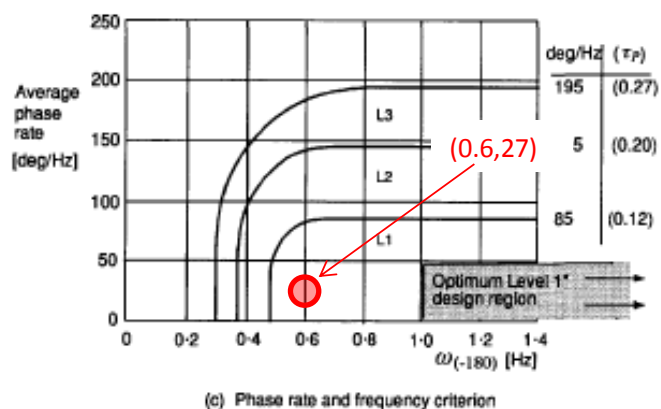
Phase Lag at twice the PIO frequency = 196 degrees

Phase Rate = 27 deg/hz

PIO Frequency Gain Magnitude = -8.1 dB

The -8.1 dB PIO frequency gain limit is on the bottom border of level 3 qualities indicating a dangerous amount of gain available to the pilot at precisely the frequency PIO will occur at. This is a serious safety off flight issue and must be addressed.

The average phase rate was calculated to be 27 with a PIO frequency of 0.6 hz. This lies within the level 1 criterion. This may seem counter intuitive since the previous two limits were failed. The PIO frequency may be as low as 0.5 hertz, on the very slow side of average pilot inputs and still be considered level 1 flying qualities. Gibson designed the criteria to work in concert. A PIO frequency that slow, and therefore likely to be achieved in normal flying could be acceptable, as long as the phase rate slope was quite shallow, so the pilot could effectively see the PIO coming, not fall off a cliff, and the gain was attenuated drastically by a large negative gain magnitude preventing the sort of stop to stop large magnitude PIO's that break airplanes and kill pilots, the gain attenuation would ensure that even if the pilot's inceptor went stop to stop, the aircraft would hardly respond and no attitude reversals would be readily apparent for the pilot to couple with.



Task 4 (Part A)

c. Design a Command Path pre-Filter in accordance with Gibson's guidance as discussed in [4]. Present transfer functions for the pre-filter, pitch attitude and flight path and comment on any significant features.

A command path pre filter was designed to cancel out the existing $T_{\theta 2}$ value and replace it with the measured flight path delay value to reduce the drop back to zero to achieve excellent pitch handling qualities.

Existing $T_{\theta 2}$ value = 1.9819 seconds

New $T_{\theta 2}$ value = |flight path delay| = 0.6703 seconds

The effective flight path delay in this system is negative due to the poor stability characteristics. This wouldn't be the case if we had improved the system stability with feedback. Again, this is a simplistic example to focus on applying Gibson's Criteria and designing/applying a pre-filter. In this case, the magnitude of the flight path delay should be used as the new $T_{\theta 2}$ value.

Command Path Pre-Filter Transfer Function: (sec/sec)

$$\frac{(s+1.492)}{(s+0.5046)}$$

Pitch Attitude/Tail plane demand transfer function: (deg/deg)

$$\frac{-90.3152 (s+0.5046)}{s (s+20) (s^2 + 0.94s + 2.585)}$$

Flight Path/Tail plane demand transfer function: (deg/deg)

$$\frac{0.45847 (s-138.7) (s+0.7167)}{s (s+20) (s^2 + 0.94s + 2.585)}$$

The design of the pre-filter ensures that the $T_{\theta 2}$ term in the numerator of the pitch attitude will cancel out completely and be replaced with the new designed value. This does not happen in the flight path transfer function. While Gibson theorized that many of the pilot's tasks involved him closing the loop tightly

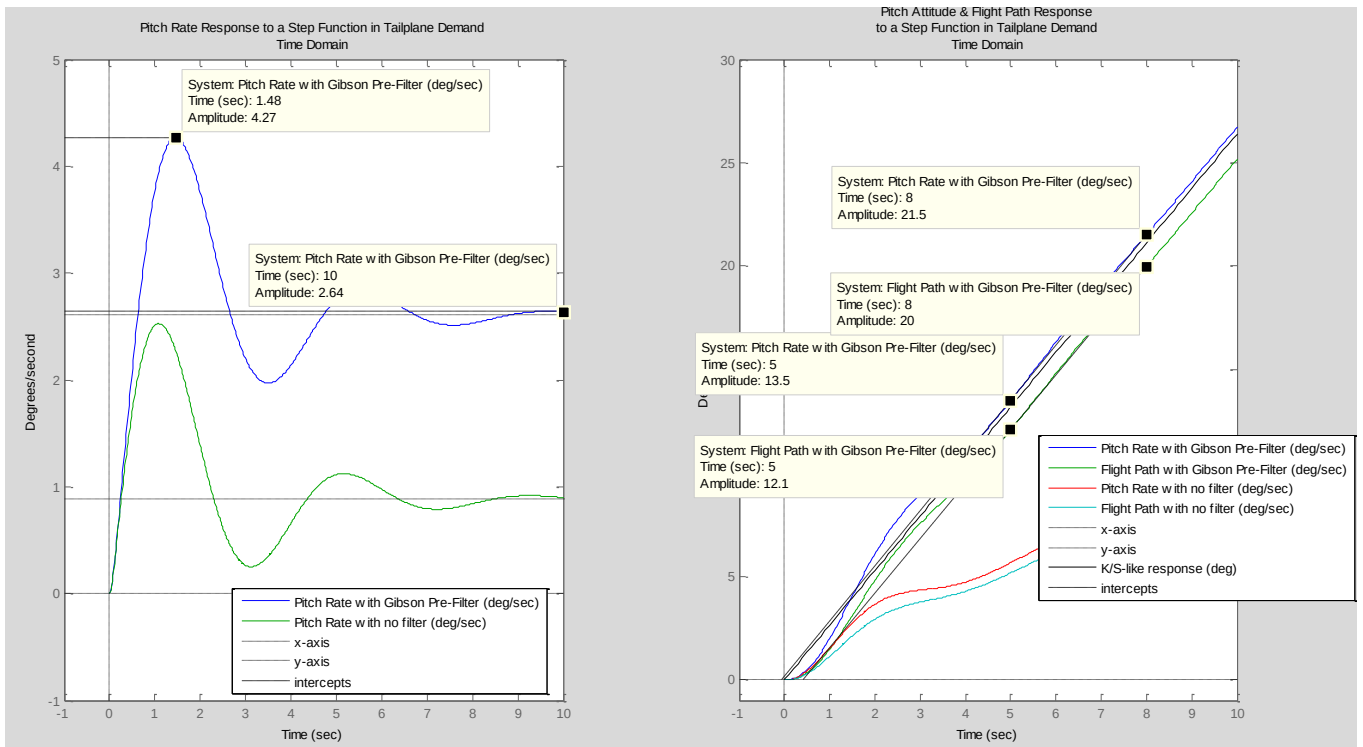
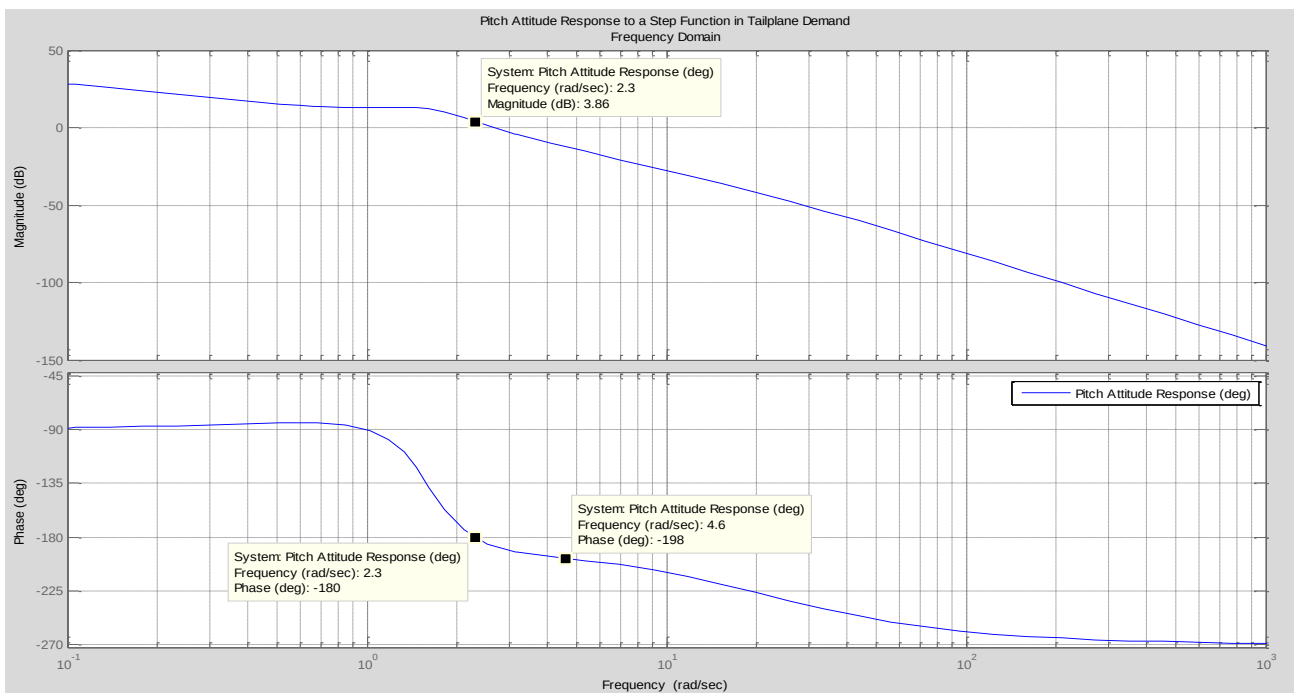
around pitch attitude response and thus the handling qualities would be greatly associated with drop back characteristics, not all tasks are performed that way. For example, flaring to land is a flight path activity, not a pitch attitude one. The control engineer must be careful to use this type of pre filter when appropriate and fade it out when not, so that there are no hidden higher order effects as the control law's dominant variable is not the one the pilot is primarily using to accomplish the task.

Task 4 (Part B)

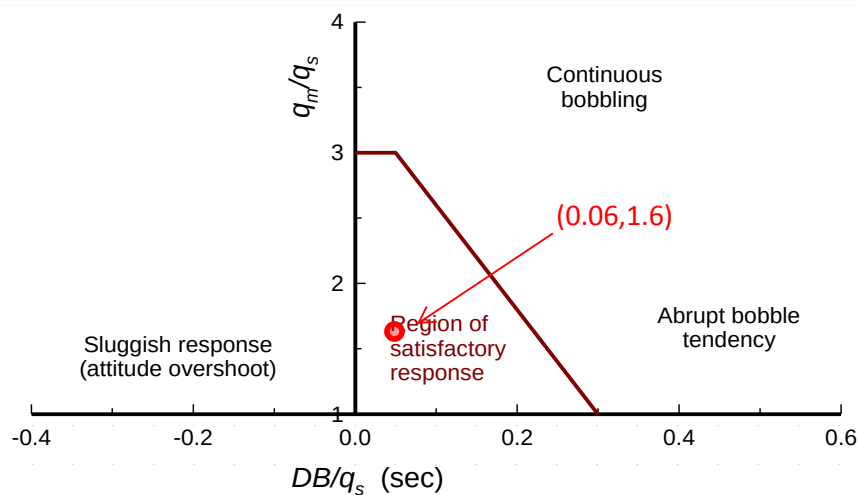
b. Re-assess against the time domain and frequency domain Gibson Criteria and comment on the effectiveness of your command path pre-filter

The command path pre filter was inserted into the control law and the same procedure as Task 3a and 3b was performed, resulting in the following key data pulled from the time and frequency domain plots:

$q_{\max}$	= 4.27 deg/sec
q_{ss}	= 2.64 deg/sec
pitch rate overshoot ratio: $q_{\max} / q_{ss}$	= 1.6 unitless
Drop Back/ q_{ss}	= 0.06 sec
Flight path delay	= 0.4051 sec
PIO Frequency	= 2.3 rad/sec
converted to hz	= 0.4 Hz
Phase Lag at twice the PIO frequency	= 198 degrees
Phase Rate	= 49 deg/hz
PIO Frequency Gain Magnitude	= +3.9 dB

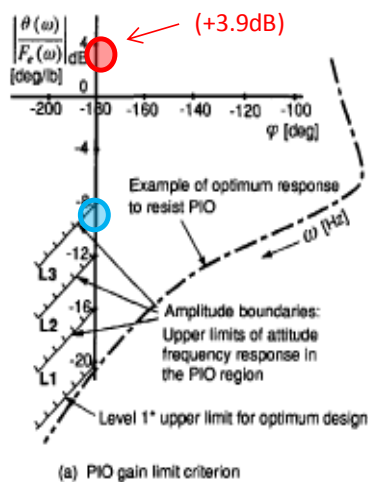
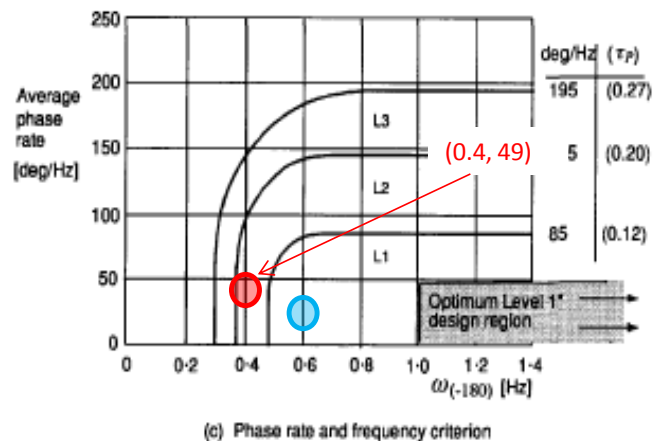


On a Gibson Dropback chart, this results in a solution in the region of satisfactory response. Both the DB/q_s and pitch rate overshoot have been dramatically improved. The solution with a pre-filter is shown in red, and the previous unfiltered solution in blue, well off to the right.



In the frequency domain, the pre filter has actually made the situation must worse on all three counts. The Gibson pre-filter solution is shown in red and the prior response in blue.

The PIO frequency, where the response crosses 180 degrees is actually much lower, the phase rate slope has steepened indicating a more cliff-like transition to PIO and the gain magnitude is actually now positive. Thus the control law is adding to the pilot's demands and making the PIO scenario worse than before. As we discussed before, however, this exercise is a part task trainer in applying a command path pre-filter. In a control system this poor, many steps would be taken to increase effective damping as well as improve the frequency response before the command path filter was used to fine tune the short period pitch handling characteristics.



Overall, the command path pre-filter did an excellent job improving the predicted short period pitch handling qualities, but had a worsening effect on the frequency response.

B.1.1 Task 1 M File

```
% Applying Gibson's Handling Qualities. Exercise B, A-7A Corsair II
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2013

% Task 1a and 1b
% Solve the full order state equation and produce transfer
functions

% Start with all extra matlab windows closed from a fresh screen
with all stored values cleared
close all
clear all
clc

% Conversion factors and initial conditions
D2R = pi()/180; % Degress to Radians conversion
R2D = 180/pi(); % Radians to Degrees conversion
F2M = 0.3048; % Feet to Meters conversion

V0_mach = 0.3; % given initial speed as 0.3 M
% Assuming a standard day, the speed of sounds at the initial
condition of
% 15,000 is 626.4 kts
V0=V0_mach*626.4; % convert to kts
KTS2MS = 0.51444; % Knots to Meters per second conversion
V0 = V0*KTS2MS; % Freestream velocity in kts true

% Define state space matrices
%
% The longitudinal equations of motion corresponding to the given
flight
% condition were given as concise dimensional deriviatives. These
% derivatives have been divided through by the mass, so the state
space
% format is:
%  $dx/dt = Ax + Bu$ , and
%  $Y = Cx + Du$ , where:
%  $x$  is the state vector, in this case, a 4 row/1 column matrix
of  $[u, w, q, \theta]$ 
%  $u$  is the input vector, in this case consisting solely of  $\eta$ 
%  $A$  is the state matrix, a collection of the aerodynamic
derivatives
%  $B$  is the input matrix, a colelction of the control
derivatives
%  $C$  is the output matrix, and is used to manipulate the
outputs
%  $D$  is the direct transmission matrix, and is almost always a
null matrix

% Careful selection of variable names to keep track of what is what
can be
```

```

% difficult in writing code. I used A, A_reduced_order, etc to
keep track
% of which version of the A matrix is being operated on, rather
than
% simply saving over A everytime it changes. This also helps
identifying
% variables properly when outputting to the workspace.

A = [0.00501 0.00464 -72.9 -31.34
     -0.08570 -0.54500 309.000 -7.400
     0.00185 -0.00767 -0.39500 0.00132
     0 0 1 0];

B = [5.63
     -23.8
     -4.51576
     0];

C = [F2M 0 0 0
     0 F2M 0 0
     0 0 1 0
     0 0 0 1
     0 0.00316*F2M 0 0
     0 -0.00316*F2M 0 1]; % last two lines are augmenting the
output to include alpha and gamma

% F2M is being used to convert all outputs that have units of feets
(u, w) into units of meters
% R2D is being used to convert all outputs that have units of
radians (q, theta) into units of degrees

D = zeros(6,1);

%Calculate full order Transfer Functions from equations of motion

[num,den] =ss2tf(A,B,C,D,1); % go from equations of motion to
transfer functions

%the code below removes the extra small number and extra zero that
matlab produces
[row,col] = size(num);
num_new = zeros(row,col);
for i =1:row
    for j=1:col
        num_new(i,j) = num(i,j);
        if abs(num_new(i,j)) < 1e-10
            num_new(i,j) = fix(num_new(i,j));
        end
    end
end

end
num = num_new;
%the code above removes the extra small number and extra zero that
matlab produces

disp('Task 1a:')

```

```

disp('Speed/Tailplane Transfer Function:')
u_eta = zpk(tf(num(1,:),den)) % isolate individual transfer
functions and output
disp('Vertical Speed/Tailplane Transfer Function:')
w_eta = zpk(tf(num(2,:),den))
disp('Pitch Rate/Tailplane Transfer Function:')
q_eta = zpk(tf(num(3,:),den))
disp('Pitch Attitude/Tailplane Transfer Function:')
theta_eta = zpk(tf(num(4,:),den))
disp('Alpha/Tailplane Transfer Function:')
alpha_eta = zpk(tf(num(5,:),den))
disp('Gamma/Tailplane Transfer Function:')
gamma_eta = zpk(tf(num(6,:),den))

damp(A) % outputs the short and long period damping ratio and
natural frequency

% This is the master file. Remaining tasks are run from here.

run task_2
run task_3a
run task_3b
run task_4

```

B.1.2 Task 2 M File

```
% Applying Gibson's Handling Qualities. Exercise B, A-7A Corsair II
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2013

% Task 2a, 2b and 2c
% Reduce the order, develop the transfer functions,
% introduce an actuator lag

% The reduced order equation is the same format as previous, but
with U and
% theta removed. In this case, rows 1 & 4 and columns 1 & 1
removed.

disp('Task 2a:')
A_reduced_order = [A(2,2) A(2,3)
                  A(3,2) A(3,3)]

B_reduced_order = [B(2,1)
                  B(3,1)]

C_reduced_order = [1 0
                  0 1
                  0.00316*F2M 0];

D_reduced_order = zeros(3,1);

[num_reduced_order,den_reduced_order]
=ss2tf(A_reduced_order,B_reduced_order,C_reduced_order,D_reduced_or
der,1);

%the code below removes the extra small number and extra zero that
matlab produces
[row,col] = size(num_reduced_order);
num_new = zeros(row,col);
for i =1:row
    for j=1:col
        num_new(i,j) = num_reduced_order(i,j);
        if abs(num_new(i,j)) < 1e-10;
            num_new(i,j) = fix(num_new(i,j));
        end
    end
end
num_reduced_order = num_new;
%the code above removes the extra small number and extra zero that
matlab produces

disp('Task 2b:')

disp('Pitch Rate/Tailplane Reduced Order Transfer Function:')
```

```

q_eta_reduced_order =
zpk(tf(num_reduced_order(2,:),den_reduced_order))
disp('Alpha/Tailplane Reduced Order Transfer Function:')
alpha_eta_reduced_order =
zpk(tf(num_reduced_order(3,:),den_reduced_order))

% Pitch attitude is the integration of Pitch Rate. Since 1/s is an
% integration term, we simply need to multiply the augmented,
reduced order
% pitch rate transfer function by 1/s to develop the augmented,
reduced
% order pitch attitude transfer function

Integration_TF = tf(1,[1 0]); % define 1/s integration transfer
function:
disp('Pitch Attitude/Tailplane Reduced Order Transfer Function:')
theta_eta_reduced_order = Integration_TF*q_eta_reduced_order

% Flight path is the remainder when subtracting angle of attack
from
% pitch attitude.
%  $\gamma(s)/\eta(s) = \theta(s)/\eta(s) - \alpha(s)/\eta(s)$ ,
% or in generic terms:
%  $\gamma(s)/\eta(s) = A(s+B)/[s(s^2+Xs+Y)] - C(s+D)/(s^2+Xs+Y)$ 
% multiply the second term on the right by s/s to make the
% denominators the same. This raises the s order by one in
the
% second term numerator. This can be easily accomplished by
moving each of
% the numerator coefficients one spot to the left in the
matrix.
% The first coefficient term is zero and is omitted to make
the
% matrix dimensions agree for the subsequent subtraction.

gamma_numerator_second_term = [num_reduced_order(3,2),
num_reduced_order(3,3), 0];

% now subtract that from the theta numerator to get the full gamma
numerator
% keep in mind the theta TF numerator equals the q TF
numerator.
gamma_numerator = num_reduced_order(2,:) -
gamma_numerator_second_term;

% The denominator is equal to theta denominator, which is equal
to the
% pitch rate denominator multiplied by 1/s. This can be
easily
% accomplished by moving each of the pitch rate denominator
% coefficients one spot to the left in the matrix.
gamma_denominator=[den_reduced_order(1), den_reduced_order(2),
den_reduced_order(3), 0];

% Compile the gamma/eta transfer function
disp('Flight Path/Tailplane Reduced Order Transfer Function:')

```

```

gamma_eta_reduced_order =
zpk(tf(gamma_numerator, gamma_denominator))

% Task 2C
% Introduce an actuator lag with a time constant of 20 seconds into
the
% reduced order model, then output the new transfer functions.

A_ro_actuator = [A(2,2) A(2,3) B(2,1)
                  A(3,2) A(3,3) B(3,1)
                  0      0      -20]

B_ro_actuator = [0
                 0
                 20]

C_ro_actuator = [1 0 0
                 0 1 0
                 0 0 1
                 0.00316*F2M 0 0];

D_ro_actuator = zeros(4,1);

[num_ro_actuator, den_ro_actuator] =
ss2tf(A_ro_actuator, B_ro_actuator, C_ro_actuator, D_ro_actuator, 1);
% go from equations of motion to transfer functions

%the code below removes the extra small number and extra zero that
matlab produces
[row,col] = size(num_ro_actuator);
num_new = zeros(row,col);
for i = 1:row
    for j = 1:col
        num_new(i,j) = num_ro_actuator(i,j);
        if abs(num_new(i,j)) < 1e-10;
            num_new(i,j) = fix(num_new(i,j));
        end
    end
end
num_ro_actuator = num_new;
%the code above removes the extra small number and extra zero that
matlab produces

disp('Task 2c:')

disp('Pitch Rate/Inceptor Reduced Order, Actuator Included Transfer
Function:')
q_eta_ro_actuator =
zpk(tf(num_ro_actuator(2,:), den_ro_actuator))
disp('Alpha/Inceptor Reduced Order, Actuator Included Transfer
Function:')

```

```

alpha_eta_ro_actuator =
zpk(tf(num_ro_actuator(4,:),den_ro_actuator))

Integration_TF = tf(1,[1 0]); % define 1/s integration transfer
function:
disp('Pitch Attitude/Inceptor Reduced Order, Actuator Included
Transfer Function:')
theta_eta_ro_actuator = Integration_TF*q_eta_ro_actuator

gamma_numerator_second_term = [num_ro_actuator(4,2),
num_ro_actuator(4,3), num_ro_actuator(4,4),0];

gamma_numerator = num_ro_actuator(2,:) -
gamma_numerator_second_term;

gamma_denominator=[den_ro_actuator(1), den_ro_actuator(2),
den_ro_actuator(3), den_ro_actuator(4), 0];

disp('Flight Path/Inceptor Reduced Order, Actuator Included
Transfer Function:')
gamma_eta_ro_actuator = zpk(tf(gamma_numerator,gamma_denominator))

```

B.1.3 Task 3A M File

```
% Applying Gibson's Handling Qualities. Exercise B, A-7A Corsair II
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2013

% Task 3a
% Assess the system against Gibson's Dropback Criteria

% Run this file one time, then look at the figure 3 to identify Max
pitch rate,
% steady state pitch rate, and evaluate the pitch attitude and
flight path
% values at 2 and 4 seconds.

figure(1)
t=10;

subplot(1,2,1)
grid off
step(-q_eta_ro_actuator,t);
hold all
axis([-1 t -1 4])
title({'Pitch Rate Response to a Step Function in Tailplane
Demand';'Time Domain'})
ylabel('Degrees/second')
line([0,0],[-1,4],'color','k','linestyle',':')
line([-1,t],[0,0],'color','k','linestyle',':')

% Run this file one time, then look at the plot identify Max pitch
rate,
% steady state pitch rate, and evaluate the pitch attitude and
flight path
% values at 2 and 4 seconds.
% Input those values below here to draw the intercept lines

% Identify Pitch Rate Max amplitude and time:
q_max_amplitude = 2.53;
q_max_time = 1.11;
% Identify Pitch Rate steady state amplitude and time:
q_ss_amplitude = 0.899;
q_ss_time = 10.0;
% Evaluate pitch attitude at 5 seconds and 8 seconds:
theta_5_seconds = 5.66;
theta_8_seconds = 8.41;
% Evaluate Flight Path at 5 seconds and 8 seconds:
gamma_5_seconds = 5.16;
gamma_8_seconds = 7.89;

% Pitch Rate Max intercept line
```

```

line([-1,q_max_time],[q_max_amplitude,
q_max_amplitude],'color','k','linestyle','--')
legend('Pitch Rate Response (deg/sec)','x-axis','y-
axis','intercepts');

% Steady State Pitch Rate intercept line
line([-
1,q_ss_time],[q_ss_amplitude,q_ss_amplitude],'color','k','linestyle
','--')

subplot(1,2,2)
grid off
step(-theta_eta_ro_actuator,t);
hold all
step(-gamma_eta_ro_actuator,t) ; ylabel('Degrees')
hold all
axis([-2 t -1 12])
title({'Pitch Attitude & Flight Path Response';'to a Step Function
in Tailplane Demand';'Time Domain'})
line([0,0],[-1,12],'color','k','linestyle',':')
line([-2,t],[0,0],'color','k','linestyle',':')

% Attitude integrator function (k/s performance)
t_int=[0:0.1:t];
integrator=t_int*q_ss_amplitude;
plot(t_int,integrator,'color','k','linestyle','-')
% Steady state pitch attitude slope line (used to identify
dropback)
theta_slope = (theta_8_seconds-theta_5_seconds)/(8-5);
theta_y_intercept = theta_5_seconds - 5*theta_slope;
theta_x_intercept = -theta_y_intercept/theta_slope;
line([theta_x_intercept,8],[0,
theta_8_seconds],'color','k','linestyle','--')
% Steady state flight path slope line (used to identify flight
path delay)
gamma_slope = (gamma_8_seconds-gamma_5_seconds)/(8-5);
gamma_y_intercept = gamma_5_seconds - 5*gamma_slope;
gamma_x_intercept = -gamma_y_intercept/gamma_slope;
line([gamma_x_intercept,8],[0,
gamma_8_seconds],'color','k','linestyle','--')
legend('Pitch Attitude Response (deg)','Flight Path Response
(deg)','x-axis','y-axis','K/S-like response (deg)','intercepts');

%Output critical attitude parameters to workspace
disp('Task 3a:')
Pitch_overshoot_ratio = q_max_amplitude/q_ss_amplitude
Dropback_over_qss = -theta_x_intercept % negative sign to account
for sign convection of dropback
Flight_Path_Delay = gamma_x_intercept

```

B.1.4 Task 3B M File

```
% Applying Gibson's Handling Qualities. Exercise B, A-7A Corsair II
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
%   Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
%   given by D.A. Lee at ETPS in August 2013

% Task 3b
% Assess the system against Gibson's Frequency Domain Criteria

% Run this file one time, then look at the figure 4 to identify the
pio
% frequency, the gain at the pio frequency, and twice the pio
frequency
%

figure(2)

bode(-theta_eta_ro_actuator)
grid on
title({'Pitch Attitude Response to a Step Function in Tailplane
Demand'; 'Frequency Domain'})
legend('Pitch Attitude Response (deg)')

% Run this file one time, then look at the figure 4 to identify the
pio
%   frequency, the gain at the pio frequency, and twice the pio
frequency
% Input those values below here to output the Gibson frequency
criteria to
%   the command window

% Identify the PIO Frequency, where the phase lag crosses 180
degrees
PIO_Frequency          = 3.67; %units of radians
% Double the PIO frequency, and evaluate the phase lag there
Double_PIO_Frequency_Lag = 196; %units of degrees
% Read the gain magnitude off the top plot at the pio frequency
PIO_Gain = -8.1; %units of dB

%Output critical attitude parameters to workspace
disp('Task 3b:')
PIO_Frequency_hz = PIO_Frequency/(2*pi) %convert to hertz
Phase_Rate = (Double_PIO_Frequency_Lag - 180)/(2*PIO_Frequency_hz-
PIO_Frequency_hz)
PIO_Gain
```

B.1.5 Task 4 M File

```
% Applying Gibson's Handling Qualities. Exercise B, A-7A
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2013

% Task 4a & 4b
% Develop and apply a command path pre-filter IAW Gibson's guidance
and
% then evaluate its effectiveness in improving handling qualities
prediction

% Run task_1 first to get all the tasks to run through
sequentially. Then
% identify critical values on figures 3 & 4 to type back into here
and then
% run again from task_1 to get the final plots

% Task 5a
% Design Command Path Pre-Filter by replacing the existing T Theta
2 value
% with the value of the system's flight path delay

% t_theta_2 is equal to the dropback/qss plus the flight path delay
% (This% relationship is apparent by looking at slide 64 of [4])
% Due to a less than perfect linear fit in task 4a, that value is
off by
% about a few hundredths of a second from the value in the
% pitch attitude/tail plane demand transfer function. For clarity
in
% cancelling terms, the t theta 2 value from the transfer
function is
% used below.

% The t theta 2 value in the pitch attitude transfer function can
be
% identified by inspection, or pulling it directly from the
numerator of
% the transfer function, as I do below. Remember the theta TF has
the same
% numerator as the pitch rate TF
disp('Task 4a:')

t_theta_2 = 1/(num_ro_actuator(2,4)/num_ro_actuator(2,3))
new_t_theta_2 = -Flight_Path_Delay % new value of T_theta_2 equal
to flight path delay should reduce attitude dropback to zero, but
not negative

a=1/new_t_theta_2;
b=1/t_theta_2;
```

```

num5 = [1 a];
den5 = [1 b];
disp('Command Path Pre-Filter')
command_path_pre_filter = zpke(tf(num5,den5))
disp('Pitch Attitude/Tail plane demand transfer function')
theta_eta_ro_actuator
disp('Flight Path/Tail plane demand transfer function')
gamma_eta_ro_actuator

% Task 4b
% Evaluate the effectiveness of the pre-filter

% Apply the command path pre-filter
theta_eta_gibson = command_path_pre_filter*theta_eta_ro_actuator;
q_eta_gibson = command_path_pre_filter*q_eta_ro_actuator;
gamma_eta_gibson = command_path_pre_filter*gamma_eta_ro_actuator;

figure(3)

subplot(1,2,1)
grid off
step(-q_eta_gibson,t);
hold all
step(-q_eta_ro_actuator,t);
hold all
axis([-1 t -1 5])
title({'Pitch Rate Response to a Step Function in Tailplane Demand'; 'Time Domain'})
ylabel('Degrees/second')
line([0,0],[-1,5], 'color','k','linestyle',':')
line([-1,t],[0,0], 'color','k','linestyle',':')

% Run this file one time, then look at the figure 3 identify Max
pitch rate,
% steady state pitch rate, and evaluate the pitch attitude and
flight path
% values at 2 and 4 seconds.
% Input those values below here to draw the intercept lines

% Identify Pitch Rate Max amplitude and time:
q_max_amplitude = 4.27;
q_max_time = 1.48;
% Identify Pitch Rate steady state amplitude and time:
q_ss_amplitude = 2.64;
q_ss_time = 10.0;
% Evaluate pitch attitude at 2 seconds and 4 seconds:
theta_5_seconds = 13.5;
theta_8_seconds = 21.5;
% Evaluate Flight Path at 2 seconds and 4 seconds:
gamma_5_seconds = 12.1;
gamma_8_seconds = 20.0;

% Pitch Rate Max intercept line

```

```

line([-1,q_max_time],[q_max_amplitude,
q_max_amplitude],'color','k','linestyle','--')
legend('Pitch Rate with Gibson Pre-Filter (deg/sec)','Pitch Rate
with no filter (deg/sec)','x-axis','y-axis','intercepts');

% Steady State Pitch Rate intercept line
line([-1,q_ss_time],[q_ss_amplitude,q_ss_amplitude],'color','k','linestyle
','--')

subplot(1,2,2)
grid off
step(-theta_eta_gibson,t);
hold all
step(-gamma_eta_gibson,t) ; ylabel('Degrees')
hold all
step(-theta_eta_ro_actuator,t);
hold all
step(-gamma_eta_ro_actuator,t) ; ylabel('Degrees')
hold all
axis([-1 t -1 30])
title({'Pitch Attitude & Flight Path Response';'to a Step Function
in Tailplane Demand';'Time Domain'})
line([0,0],[-1,30],'color','k','linestyle',':')
line([-1,t],[0,0],'color','k','linestyle',':')

% Attitude integrator function (k/s performance)
t_int=[0:0.1:t];
integrator=t_int*q_ss_amplitude;
plot(t_int,integrator,'color','k','linestyle','-')
% Steady state pitch attitude slope line (used to identify
dropback)
theta_slope = (theta_8_seconds-theta_5_seconds)/3;
theta_y_intercept = theta_5_seconds - 5*theta_slope;
theta_x_intercept = -theta_y_intercept/theta_slope;
line([theta_x_intercept,8],[0,
theta_8_seconds],'color','k','linestyle','--')
% Steady state flight path slope line (used to identify flight
path delay)
gamma_slope = (gamma_8_seconds-gamma_5_seconds)/3;
gamma_y_intercept = gamma_5_seconds - 5*gamma_slope;
gamma_x_intercept = -gamma_y_intercept/gamma_slope;
line([gamma_x_intercept,8],[0,
gamma_8_seconds],'color','k','linestyle','--')
legend('Pitch Rate with Gibson Pre-Filter (deg/sec)','Flight Path
with Gibson Pre-Filter (deg/sec)','Pitch Rate with no filter
(deg/sec)','Flight Path with no filter (deg/sec)','x-axis','y-
axis','K/S-like response (deg)','intercepts');

%Output critical attitude parameters to workspace
disp('Task 4b:')
Pitch_overshoot_ratio = q_max_amplitude/q_ss_amplitude
Dropback_over_qss = -theta_x_intercept % negative sign to account
for sign convection of dropback
Flight_Path_Delay = gamma_x_intercept

```

```

% Task 4b continued
% Assess the system against Gibson's Frequency Domain Criteria

% Run this file one time, then look at the figure 4 to identify the
pio
% frequency, the gain at the pio frequency, and twice the pio
frequency

figure(4)

bode(-theta_eta_gibson)
grid on
title({'Pitch Attitude Response to a Step Function in Tailplane
Demand';'Frequency Domain'})
legend('Pitch Attitude Response (deg)')

% Run this file one time, then look at the figure 4 to identify the
pio
% frequency, the gain at the pio frequency, and twice the pio
frequency
% Input those values below here to output the Gibson frequency
criteria to
% the command window

% Identify the PIO Frequency, where the phase lag crosses 180
degrees
PIO_Frequency = 2.3; %units of radians
% Double the PIO frequency, and evaluate the phase lag there
Double_PIO_Frequency_Lag = 198; %units of degrees
% Read the gain magnitude off the top plot at the pio frequency
PIO_Gain = 3.9; %units of dB

%Output critical attitude parameters to workspace
PIO_Frequency_hz = PIO_Frequency/(2*pi) %convert to hertz
Phase_Rate = (Double_PIO_Frequency_Lag - 180)/(2*PIO_Frequency_hz-
PIO_Frequency_hz)
PIO_Gain

```

Appendix C Exercise C, Hawk, Full Solution

C.1 Full Solution



School of Engineering

**Using Gibson's Criteria to design in excellent
longitudinal pitch handling qualities**

Exercise C – Hawk T1 Mk 1

SOLUTION



Task 1 (Part A)

a. Use the data available to you in appendix B to assemble and then solve the longitudinal equations of motion of the Hawk aircraft model to obtain the response transfer functions. List transfer functions for:

$$\frac{u(s)}{\eta(s)}, \frac{w(s)}{\eta(s)}, \frac{q(s)}{\eta(s)}, \frac{\theta(s)}{\eta(s)}, \frac{\alpha(s)}{\eta(s)}, \frac{\gamma(s)}{\eta(s)}$$

with corresponding units of:

$$\frac{\text{m}}{\text{sec} \cdot \text{deg}}, \frac{\text{m}}{\text{sec} \cdot \text{deg}}, \frac{\text{deg}}{\text{sec} \cdot \text{deg}}, \frac{\text{deg}}{\text{deg}}, \frac{\text{deg}}{\text{deg}}, \frac{\text{deg}}{\text{deg}}$$

The response transfer functions are:

Speed/Tailplane Transfer Function: $m/(\text{sec} \cdot \text{deg})$

$$\frac{0.0395 (s+235.8) (s^2 + 1.038s + 1.428)}{(s^2 + 0.0145s + 0.007149) (s^2 + 2.561s + 23.72)}$$

Vertical Velocity/Tailplane Transfer Function: $m/(\text{sec} \cdot \text{deg})$

$$\frac{-0.9921 (s+236.8) (s^2 + 0.01172s + 0.002608)}{(s^2 + 0.0145s + 0.007149) (s^2 + 2.561s + 23.72)}$$

Pitch Rate/Tailplane Transfer Function: $\text{deg}/(\text{sec} \cdot \text{deg})$

$$\frac{-61.3122 s (s+1.264) (s+0.01369)}{(s^2 + 0.0145s + 0.007149) (s^2 + 2.561s + 23.72)}$$

Pitch Attitude/Tailplane Transfer Function: deg/deg

$$\frac{-61.3122 (s+1.264) (s+0.01369)}{(s^2 + 0.0145s + 0.007149) (s^2 + 2.561s + 23.72)}$$

Angle of Attack/Tailplane Transfer Function: deg/deg

$$\frac{-0.0044891 (s+236.8) (s^2 + 0.01172s + 0.002608)}{(s^2 + 0.0145s + 0.007149) (s^2 + 2.561s + 23.72)}$$

Flight Path/Tailplane Transfer Function: *deg/deg*

$$\frac{-1.0656 (s-0.07727) (s^2 + 0.3623s + 0.03366)}{(s^2 + 0.0145s + 0.007149) (s^2 + 2.561s + 23.72)}$$

Task 1 (Part B)

b. Summarise the significance of the predicted stability modes that are evident from your solution. Explain how augmentation might change the stability modes.

The predicted stability modes are derived by setting the denominator of the transfer functions equal to zero and solving for the roots:

$$(s^2 + 0.0145s + 0.007149) (s^2 + 2.561s + 23.72) = 0$$

$$s = -1.28 \pm 4.7i \text{ or } -0.00725 \pm 0.0842i$$

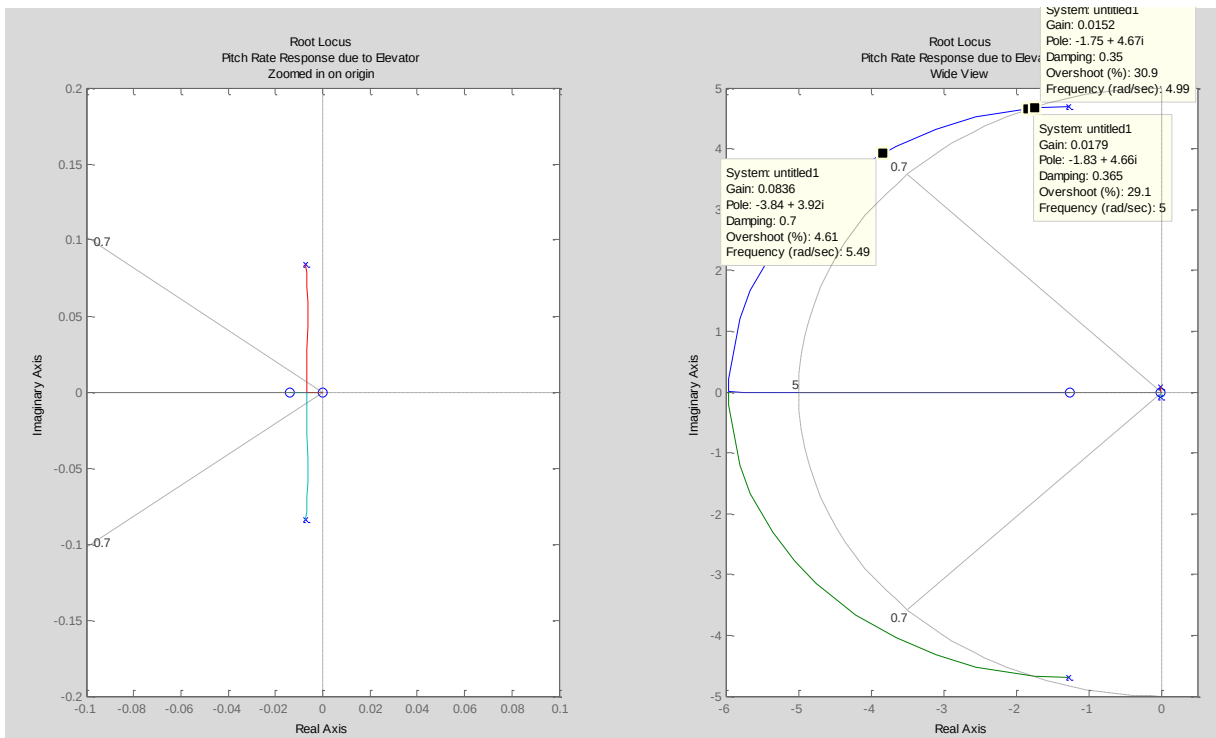
These roots correspond to a short period natural frequency of 4.87 rad/sec and a damping ratio of 0.26; and for the phugoid 0.0846 rad/sec and 0.0857.

Since all the roots are negative, the system is stable and does not require feedback for that purpose. However, feedback should be used to increase the damping and natural frequencies of the short period to ideal values.

Task 2 (Part A)

a. Investigate and report on the possibilities for introducing pitch rate feedback to modify the predicted stability modes. Select a feedback gain for pitch rate feedback (k_q) to satisfy the dynamic stability criteria shown in appendix C. Explain the justification for your gain selections.

Pitch rate feedback was investigated via a root locus diagram:



The left plot shows a zoomed in view around the origin with the roots associated with the phugoid. We intend to eliminate these roots through a reduced order model, so there is no need to investigate them further

The right plot shows the short period roots. Grid lines for critical damping (0,7 ratio) and a natural frequency of 5 have been drawn. Appendix B gives limits of 0.35-1.0 for the damping and 2.0 to 5.0 for the natural frequency. The Hawk's natural frequency starts at 4.87 and only increases as gain is applied. The hawk's damping ratio starts at 0.26 (outside the desired values) and increases to 1 as natural frequency increases to 6 rad/sec.

Gain can be used to increase both the damping and natural frequency:

Option 1) Increase damping ratio to the minimum required level.

$$K_q \text{ Gain} = 0.0152 \text{ deg*sec/deg} \quad \zeta = 0.35 \quad \omega_n = 4.99 \text{ rad/sec}$$

Option 2) Increase damping ratio until natural frequency is at it's maximum desirable value:

$$K_q \text{ Gain} = 0.0179 \text{ deg*sec/deg}$$

$$\zeta = 0.37 \quad \omega_n = 5.00 \text{ rad/sec}$$

Option 3) Increase damping ratio to ideal level of critical damping regardless of effect on natural frequency:

$$K_q \text{ Gain} = 0.0836 \text{ deg*sec/deg}$$

$$\zeta = 0.7 \quad \omega_n = 5.49 \text{ rad/sec}$$

To keep the values within the desired range, we will go with option 2 for now, but

some consideration should be given to re-addressing this value if simulator or aircraft testing indicates that the damping is not yet high enough. It may be that better damping is worth the cost of a natural frequency slightly above the upper limit.

Task 2 (Part B)

b. Present the augmented transfer function for pitch rate and summarise the significance of the predicted stability modes that are now evident from your solution.

The augmented response transfer function is:

Pitch Rate/Tail plane Transfer Function: $\text{deg}/(\text{sec*deg})$

$$\frac{-61.3122 (s+1.264) (s+0.01369)}{(s^2 + 0.0142s + 0.006754) (s^2 + 3.659s + 25.11)}$$

Eigenvalue	Damping	Freq. (rad/s)
-1.83e+000 + 4.66e+000i	3.65e-001	5.01e+000
-1.83e+000 - 4.66e+000i	3.65e-001	5.01e+000
-7.10e-003 + 8.19e-002i	8.64e-002	8.22e-002
-7.10e-003 - 8.19e-002i	8.64e-002	8.22e-002

The augmented stability modes are derived by setting the denominator of the transfer functions equal to zero and solving for the roots:

$$(s^2 + 0.0142s + 0.006754) (s^2 + 3.659s + 25.11) = 0$$

$$s = -1.83 \pm 4.66i \text{ or } -0.00710 \pm 0.0819i$$

These yield short period pitch response characteristics of:

$$\omega_n = 5.0 \text{ rad/sec}$$

Matches desired criteria

$$\zeta = 0.37 \text{ unitless}$$

Matches desired criteria

And phugoid characteristics of:

$$\omega_n = 0.0822 \text{ rad/sec}$$

$$\omega_{n-sh} / \omega_{n-ph} = 5.0/0.0822 = 61 \geq 10 \text{ Matches desired criteria}$$

$$\zeta = 0.09 \text{ unitless}$$

Matches desired criteria

The introduction of pitch rate feedback has moved the short period characteristics to the within the desired ranges and ensured the phugoid characteristics are well within the desired specifications. This solution meets the criteria and should produce good pitch handling qualities according to the conventional criteria we just assessed it against.

Task 3 (Part A)

a. Construct and present the reduced order equations of motion that describe the short term dynamics of the un-augmented Hawk aircraft model in the form:
(5 marks)

$$\begin{pmatrix} \dot{w} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} z_w & z_q \\ m_w & m_q \end{pmatrix} \begin{pmatrix} w \\ q \end{pmatrix} + \begin{pmatrix} z_\eta \\ m_\eta \end{pmatrix} \eta$$

The reduced order equations of motion are:

$$\begin{pmatrix} \dot{w} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} -1.3595 & 218.4078 \\ -0.1008 & -1.2046 \end{pmatrix} \begin{pmatrix} w \\ q \end{pmatrix} + \begin{pmatrix} -0.9921 \\ -1.0701 \end{pmatrix} \eta$$

Task 3 (Part B)

b. Use your values of k_q from task 2a with the reduced order equations of motion to develop reduced order augmented transfer function for $\frac{q(s)}{\eta(s)}$, $\frac{\theta(s)}{\eta(s)}$, $\frac{\alpha(s)}{\eta(s)}$, $\frac{\gamma(s)}{\eta(s)}$.
develop the corresponding reduced order transfer functions for $\frac{\theta(s)}{\eta(s)}$, $\frac{\alpha(s)}{\eta(s)}$, $\frac{\gamma(s)}{\eta(s)}$.

The augmented, reduced order transfer functions are:

Pitch Rate/Tail plane Augmented, Reduced Order Transfer Function:
*deg/(sec*deg)*

$$-61.3122 (s+1.266)$$

$$(s^2 + 3.662s + 25.04)$$

Alpha/Tail plane Augmented, Reduced Order Transfer Function: *deg/deg*

$$-0.0044891 (s+236.8)$$

$$(s^2 + 3.662s + 25.04)$$

Pitch Attitude/Tail plane Augmented, Reduced Order Transfer Function:
deg/deg

$$-61.3122 (s+1.266)$$

$$s (s^2 + 3.662s + 25.04)$$

Flight Path/Tail plane Augmented, Reduced Order Transfer Function:
deg/deg

$$0.0044891 (s-1.342e004) (s+1.288)$$

$$s (s^2 + 3.662s + 25.04)$$

Task 3 (Part C)

c. Modify the reduced order state equation from task 3(a) to include the actuator dynamics and present the amended reduced order state equation. Use your value of k_q from task 2a with the amended reduced order equations of motion to develop reduced order augmented transfer functions for

$$\frac{\alpha(s)}{\delta(s)}, \frac{\theta(s)}{\delta(s)}, \frac{q(s)}{\delta(s)}, \frac{\gamma(s)}{\delta(s)}$$

The original reduced order state equation can be combined with the given actuator model transfer function into the following combined state equation:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} -1.3595 & 218.4078 & -0.9921 \\ -0.1008 & -1.2046 & -1.0701 \\ 0 & 0 & -20 \end{bmatrix} \begin{bmatrix} w \\ q \\ \eta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} \delta$$

The augmented, reduced order, actuator included transfer functions are calculated in a similar manner as the previous question and are:

Pitch Rate/Inceptor Augmented, Reduced Order, Actuator Included Transfer Function: *deg/(sec*deg)*

$$\frac{-21.402 (s+1.266)}{(s+18.83) (s^2 + 3.732s + 26.6)}$$

Alpha/Inceptor Augmented, Reduced Order, Actuator Included Transfer Function: *deg/deg*

$$\frac{-0.089783 (s+236.8)}{(s+18.83) (s^2 + 3.732s + 26.6)}$$

Pitch Attitude/Inceptor Augmented, Reduced Order, Actuator Included Transfer Function: *deg/deg*

$$\frac{-21.402 (s+1.266)}{s (s+18.83) (s^2 + 3.732s + 26.6)}$$

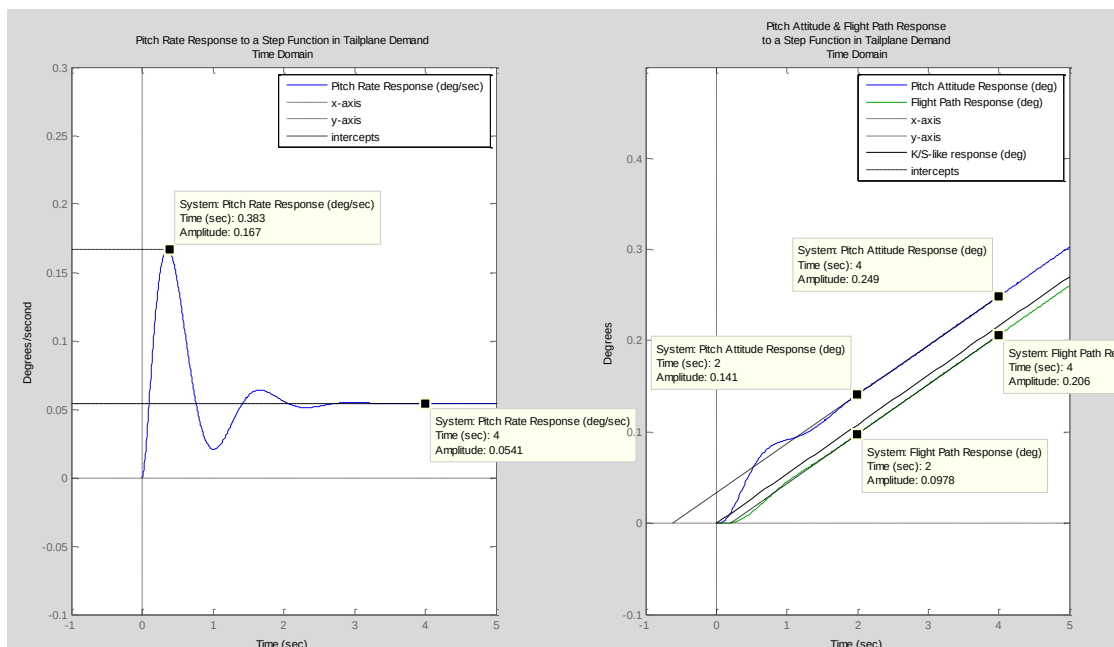
Flight Path/Inceptor Augmented, Reduced Order, Actuator Included Transfer Function: *deg/deg*

$$\frac{0.089783 (s-18.19) (s+16.59)}{s (s+18.83) (s^2 + 3.732s + 26.6)}$$

Task 4 (Part A)

a. Use your results from task 3c and apply the 'Gibson Dropback criterion' shown in Appendix D to determine the predicted handling qualities for a pitch tracking task for the actuator included, augmented aircraft model. Summarise the results of your analysis and your conclusion about the predicted handling qualities.

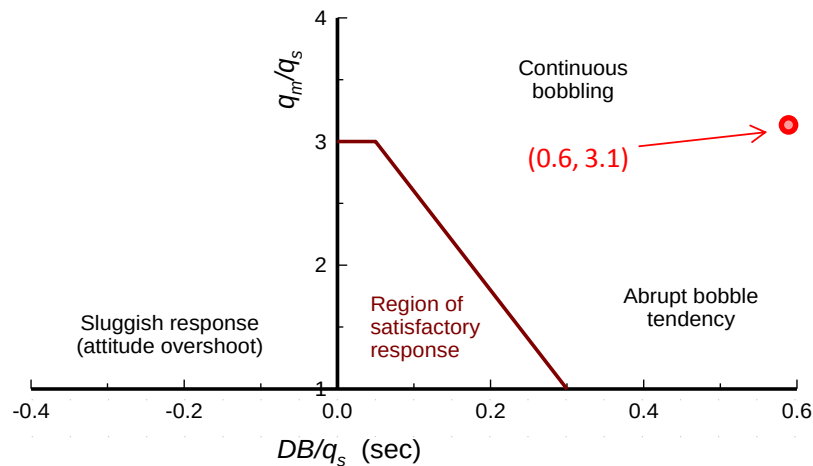
Gibson's dropback criteria was applied by plotting the transfer functions of pitch rate, pitch attitude and flight path in response to the tailplane demanded by the inceptor on a time domain plot. The plot is shown:



Critical parameters of pitch rate maximum, steady state pitch rate, the dropback/steady state pitch rate and the flight path delay were pulled off the plots:

$$\begin{aligned}
 q_{\max} &= 0.167 \text{ deg/sec} \\
 q_{ss} &= 0.0541 \text{ deg/sec} \\
 \text{pitch rate overshoot ratio: } q_{\max} / q_{ss} &= 3.1 \text{ unitless} \\
 \text{Drop Back/ } q_{ss} &= 0.6 \text{ sec} \\
 \text{Flight path delay} &= 0.1993 \text{ sec}
 \end{aligned}$$

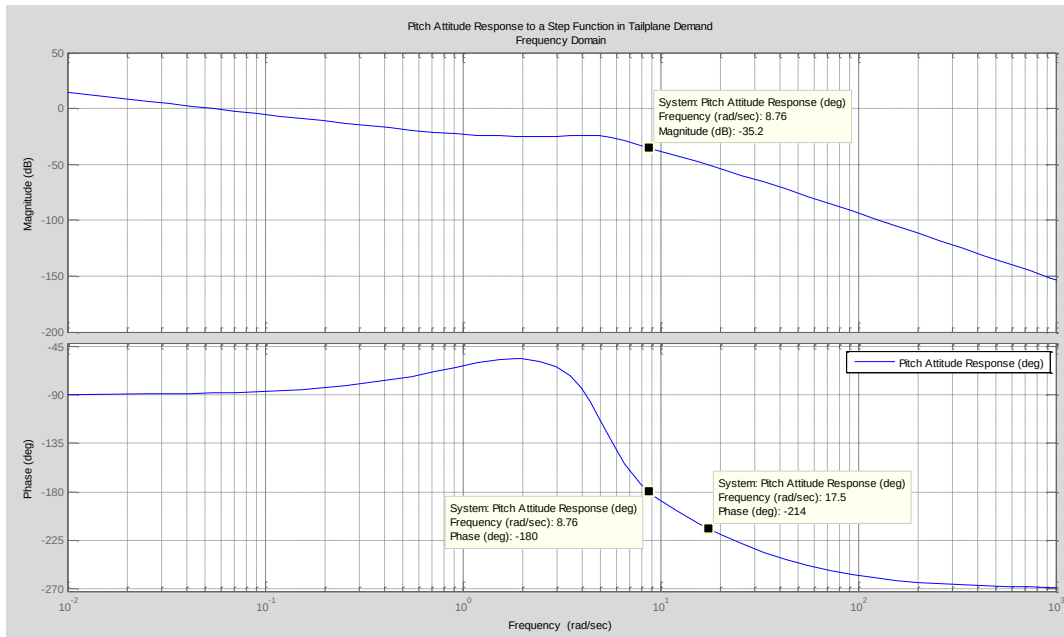
On a Gibson Dropback chart, this results in a prediction of poor handling qualities characterized by an abrupt/continuous bobble tendency.



Task 4 (Part B)

b. Continuing to use your results from task 3c, specifically the pitch attitude transfer function, apply the frequency domain portion of Gibson's Criteria: Phase Rate, PIO frequency and PIO gain limit criterion. Compare your results with the values shown in appendix D and comment on any 'specification compliance' issues.

Gibson's frequency domain criteria were applied by plotting the pitch attitude transfer function on a Bode Plot. The plot is shown:



Critical parameters of PIO frequency, the phase lag at twice the PIO frequency, the gain magnitude at the PIO frequency were pulled off the plot

PIO Frequency = 8.76 rad/sec

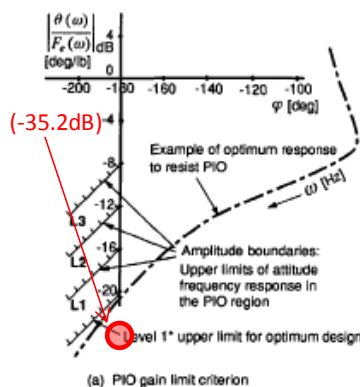
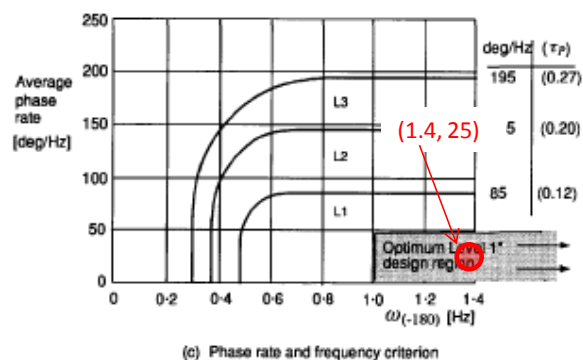
converted to hz = 1.39 Hz

Phase Lag at twice the PIO frequency = 214 degrees

Phase Rate = 24 deg/hz

PIO Frequency Gain Magnitude = -35.2 dB

On a Gibson frequency criteria chart, the PIO frequency and phase rate indicate level 1* handling qualities. The gain magnitude is also well within the level 1 criteria. The system appears to be well protected from linear PIO.



Task 5 (Part A)

d. Design a Command Path pre-Filter in accordance with Gibson's guidance as discussed in [4]. Present transfer functions for the pre-filter, pitch attitude and flight path and comment on any significant features.

A command path pre filter was designed to cancel out the existing $T_{\theta 2}$ value and replace it with the measured flight path delay value to reduce the drop back to zero to achieve excellent pitch handling qualities.

Existing $T_{\theta 2}$ value = 0.7899 seconds

New $T_{\theta 2}$ value = flight path delay = 0.1993 seconds

Command Path Pre-Filter Transfer Function: (sec/sec)

$$\frac{(s+5.019)}{(s+1.266)}$$

Pitch Attitude/Tail plane demand transfer function: (deg/deg)

$$\frac{-21.402 (s+1.266)}{s (s+18.83) (s^2 + 3.732s + 26.6)}$$

Flight Path/Tail plane demand transfer function: (deg/deg)

$$\frac{0.089783 (s-18.19) (s+16.59)}{s (s+18.83) (s^2 + 3.732s + 26.6)}$$

The design of the pre-filter ensures that the $t_{\theta 2}$ term in the numerator of the pitch attitude will cancel out completely and be replaced with the new designed value. This does not happen in the flight path transfer function. While Gibson theorized that many of the pilot's tasks involved him closing the loop tightly around pitch attitude response and thus the handling qualities would be greatly associated with drop back characteristics, not all tasks are performed that way. For example, flaring to land is a flight path activity, not a pitch attitude one. The control engineer must be careful to use this type of pre filter when appropriate and fade it out when not, so that there are no hidden higher order effects as the control law's dominant variable is not the one the pilot is primarily using to accomplish the task.

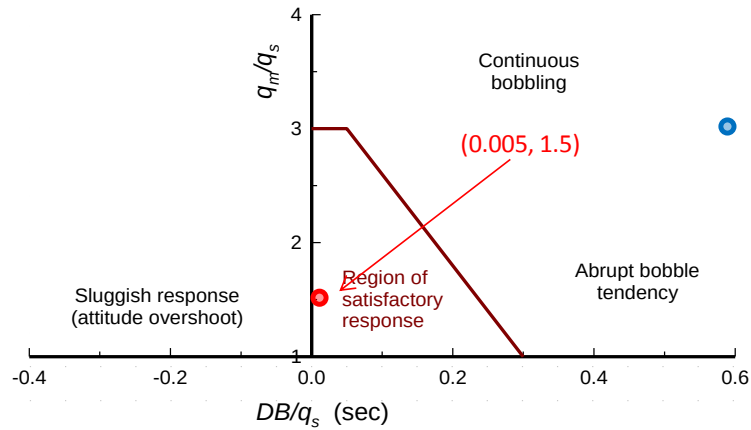
Task 5 (Part B)

b. Re-assess against the time domain and frequency domain Gibson Criteria and comment on the effectiveness of your command path pre-filter

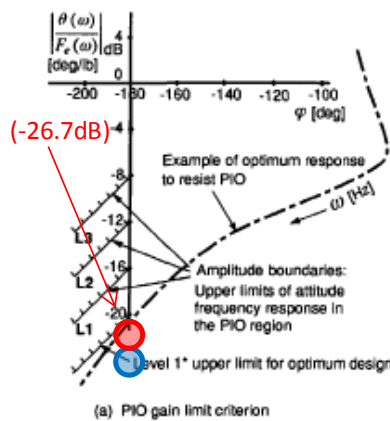
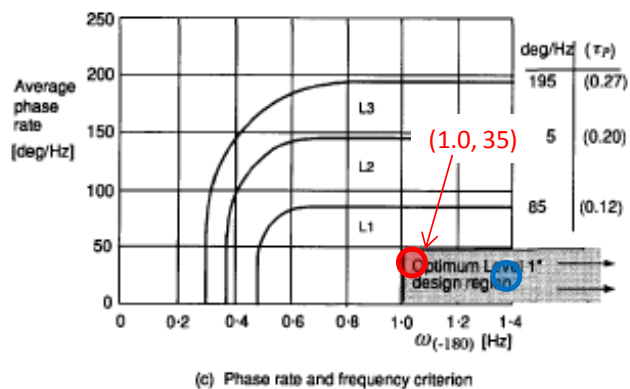
The command path pre filter was inserted into the control law and the same procedure as Task 4a and 4b was performed, resulting in the following key data pulled from the time and frequency domain plots: [see next page for plots]

$q_{\max}$	= 0.317deg/sec
q_{ss}	= 0.214 deg/sec
pitch rate overshoot ratio: $q_{\max} / q_{ss}$	= 1.5 unitless
Drop Back/ q_{ss}	= 0.0047 sec
Flight path delay	= 0.6890 sec
PIO Frequency	= 6.55 rad/sec
converted to hz	= 1.04 Hz
Phase Lag at twice the PIO frequency	= 217 degrees
Phase Rate	= 35 deg/hz
PIO Frequency Gain Magnitude	= -26.7 dB

On a Gibson Dropback chart, this results in a nearly perfect solution in the bottom left of the region of satisfactory response. Both the DB/ q_{ss} and pitch rate overshoot have been dramatically improved. This predicts a drastic improvement in pitch handling qualities.



On a Gibson frequency criteria chart, there was a little reduction in performance to the lower ends of the level 1 and 1* areas. However, both the gain attenuation and phase rate and PIO frequency criteria are met.



Overall, the command path pre-filter improved the short period pitch handling qualities, and predicts excellent HQR's when the system is tested in the simulator or aircraft.

C.1.1 Task 1 M File

```
% Applying Gibson's Handling Qualities. Exercise C, Hawk T1 Mk1
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2011

% Task 1a and 1b
% Solve the full order state equation and produce transfer
functions

% Start with all extra matlab windows closed from a fresh screen
with all stored values cleared
close all
clear all
clc

% Conversion factors and initial conditions
D2R = pi()/180;      % Degrees to Radians conversion
R2D = 180/pi();      % Radians to Degrees conversion
V0 = 221;             % Freestream velocity in meters per second

% Define state space matrices
%
% The longitudinal equations of motion corresponding to the given
flight
% condition were given as concise dimensional derivatives. These
% derivatives have been divided through by the mass, so the state
space
% format is:
%  $\frac{dx}{dt} = Ax + Bu$ , and
%  $Y = Cx + Du$ , where:
%  $x$  is the state vector, in this case, a 4 row/1 column matrix
of  $[u, w, q, \theta]$ 
%  $u$  is the input vector, in this case consisting solely of  $\eta$ 
%  $A$  is the state matrix, a collection of the aerodynamic
derivatives
%  $B$  is the input matrix, a collection of the control
derivatives
%  $C$  is the output matrix, and is used to manipulate the
outputs
%  $D$  is the direct transmission matrix, and is almost always a
null matrix

% Careful selection of variable names to keep track of what is what
can be
% difficult in writing code. I used  $A$ ,  $A_{\text{reduced\_order}}$ , etc to
keep track
% of which version of the  $A$  matrix is being operated on, rather
than
% simply saving over  $A$  everytime it changes. This also helps
identifying
% variables properly when outputting to the workspace.
```

```

% u
% w
% q
% theta

A = [-0.0115 0.0574 -8.7001 -9.8022
     -0.0505 -1.3595 218.4078 -0.3905
     0.0090 -0.1008 -1.2046 0
     0 0 1 0];

B = [0.0395
     -0.9921
     -1.0701
     0];

C = [1 0 0 0
     0 1 0 0
     0 0 R2D 0
     0 0 0 R2D
     0 1/V0 0 0
     0 -1/V0 0 1];
% R2D is being used to convert all outputs that have units of
radians (alpha, q, theta) into units of degrees
% Row 5 adds alpha to output matrix by dividing w by V0: alpha =
w/V0
% Row 6 adds gamma to output matrix by subtracting alpha from
theta: gamma = theta - alpga = theta - w/V0

D = zeros(6,1);

%Calculate full order Transfer Functions from equations of motion

[num,den] =ss2tf(A,B,C,D,1); % go from equations of motion to
transfer functions

%the code below removes the extra small number and extra zero that
matlab produces
[row,col] = size(num);
num_new = zeros(row,col);
for i =1:row
    for j=1:col
        num_new(i,j) = num(i,j);
        if abs(num_new(i,j)) < 1e-10
            num_new(i,j) = fix(num_new(i,j));
        end
    end
end

end
num = num_new;
%the code above removes the extra small number and extra zero that
matlab produces

disp('Task 1a:')
disp('Speed/Tailplane Transfer Function:')

```

```

u_eta      = zpk(tf(num(1,:),den)) % isolate individual transfer
functions and output
disp('Vertical Velocity/Tailplane Transfer Function:')
w_eta      = zpk(tf(num(2,:),den))
disp('Pitch Rate/Tailplane Transfer Function:')
q_eta      = zpk(tf(num(3,:),den))
disp('Pitch Attitude/Tailplane Transfer Function:')
theta_eta  = zpk(tf(num(4,:),den))
disp('Angle of Attack/Tailplane Transfer Function:')
alpha_eta  = zpk(tf(num(5,:),den))
disp('Flight Path/Tailplane Transfer Function:')
gamma_eta  = zpk(tf(num(6,:),den))

damp(A) % outputs the short and long period damping ratio and
natural frequency

% This is the master file. Remaining tasks are run from here.

run task_2
run task_3
run task_4a
run task_4b
run task_5

```

C.1.2 Task 2 M File

```
% Applying Gibson's Handling Qualities. Exercise C, Hawk T1 Mk1
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2011

% Task 2a and 2b
% Add feedback loops and reassess transfer functions and stability

% Task 2a
% Examine pitch rate feedback for stability augmentation

% Produce root locus diagrams of q/eta Transfer function
% to identify possible gain values

figure(1)

subplot(1,2,1)
rlocus(-q_eta) %negative sign required to account for sign
convention
sgrid(0.7, 2.25)
title({'Root Locus';'Pitch Rate Response due to Elevator';'Zoomed
in on origin'})
axis([-0.1 0.1 -0.2 0.2])

subplot(1,2,2)
rlocus(-q_eta) %negative sign required to account for sign
convention
axis([-6 0.5 -5 5])
title({'Root Locus';'Pitch Rate Response due to Elevator';'Wide
View'})
sgrid(0.7, 5)

disp('Task 2a:')

% Following inspection of the above output, the gain options are:
% 0.0152 damping 0.35 freq 4.99
% 0.0179 damping 0.37 freq 5.00
% 0.0836 damping 0.7 freq 5.49

K_q = -0.0179 % gain values are typically negative
K=[0,0,K_q,0];
A_augmented=A-B*180/pi*K;

[num_augmented,den_augmented] =ss2tf(A_augmented,B,C,D,1); % go
from equations of motion to transfer functions

%the code below removes the extra small number and extra zero that
matlab produces
[row,col] = size(num_augmented);
num_new = zeros(row,col);
for i =1:row
    for j=1:col
```

```

        num_new(i,j) = num_augmented(i,j);
        if abs(num_new(i,j)) < 1e-10
            num_new(i,j) = fix(num_new(i,j));
        end
    end

end

num_augmented = num_new;
%the code above removes the extra small number and extra zero that
matlab produces

disp('Task 2b:')
disp('Pitch Rate/Tailplane Augmented Transfer Function:')
q_eta_augmented = zpk(tf(num_augmented(4,:),den_augmented))

damp(A_augmented) % outputs the short and long period damping
ratio and natural frequency

```

C.1.3 Task 3 MFile

```
% Applying Gibson's Handling Qualities. Exercise C, Hawk T1 Mk1
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2011

% Task 3a, 3b and 3c
% Reduce the order, develop the transfer functions,
% introduce an actuator lag

% The reduced order equation is the same format as previous, but
with U and
% theta removed. In this case, rows 1 & 4 and columns 1 & 4
removed.

disp('Task 3a:')
A_reduced_order = [A(2,2) A(2,3)
                  A(3,2) A(3,3)]

B_reduced_order = [B(2,1)
                  B(3,1)]

C_reduced_order = [1    0
                  0    R2D
                  1/V0 0];
% Row 3 adds alpha to output matrix by dividing w by V0:  alpha =
w/V0

D_reduced_order = zeros(3,1);

% Now with the reduced order state space equations defined, apply
the gains
% determined in task 2a to get an augmented, reduced order model

K_reduced_order=[0,K_q];
A_ro_aug=A_reduced_order-B_reduced_order*180/pi*K_reduced_order;
% A_ro_aug stands for A matrix, reduced order, then augmented

[num_ro_aug,den_ro_aug]
=ss2tf(A_ro_aug,B_reduced_order,C_reduced_order,D_reduced_order,1);

% the code below removes the extra small number and extra zero that
matlab produces
[row,col] = size(num_ro_aug);
num_new = zeros(row,col);
for i =1:row
    for j=1:col
        num_new(i,j) = num_ro_aug(i,j);
        if abs(num_new(i,j)) < 1e-10;
            num_new(i,j) = fix(num_new(i,j));
        end
    end
end
end
```

```

end
num_ro_aug = num_new;
%the code above removes the extra small number and extra zero that
matlab produces

disp('Task 3b:')
disp('Pitch Rate/Tailplane Augmented, Reduced Order Transfer
Function:')
q_eta_ro_aug = zpk(tf(num_ro_aug(2,:),den_ro_aug))
disp('Alpha/Tailplane Augmented, Reduced Order Transfer Function:')
alpha_eta_ro_aug = zpk(tf(num_ro_aug(3,:),den_ro_aug))

% Pitch attitude is the integration of Pitch Rate. Since 1/s is an
% integration term, we simply need to multiply the augmented,
reduced order
% pitch rate transfer function by 1/s to develop the augmented,
reduced
% order pitch attitude transfer function

Integration_TF = tf(1,[1 0]); % define 1/s integration transfer
function:
disp('Pitch Attitude/Tailplane Augmented, Reduced Order Transfer
Function:')
theta_eta_ro_aug = Integration_TF*q_eta_ro_aug

% Flight path is the remainder when subtracting angle of attack
from
% pitch attitude.
%  $\gamma(s)/\eta(s) = \theta(s)/\eta(s) - \alpha(s)/\eta(s)$ ,
% or in generic terms:
%  $\gamma(s)/\eta(s) = A(s+B)/[s(s^2+Xs+Y)] - C(s+D)/(s^2+Xs+Y)$ 
% multiply the second term on the right by s/s to make the
% denominators the same. This raises the s order by one in
the
% second term numerator. This can be easily accomplished by
moving each of
% the numerator coefficients one spot to the left in the
matrix.
% The first coefficient term is zero and is omitted to make
the
% matrix dimensions agree for the subsequent subtraction.

gamma_numerator_second_term = [num_ro_aug(3,2), num_ro_aug(3,3),
0];

% now subtract that from the theta numerator to get the full gamma
numerator
% keep in mind the theta TF numerator equals the q TF
numerator.
gamma_numerator = num_ro_aug(2,:) - gamma_numerator_second_term;

% The denominator is equal to theta denominator, which is equal
to the
% pitch rate denominator multiplied by 1/s. This can be
easily
% accomplished by moving each of the pitch rate denominator

```

```

%      coeffeicients one spot to the left in the matrix.
gamma_denominator=[den_ro_aug(1), den_ro_aug(2), den_ro_aug(3), 0];

%      Compile the gamma/eta transfer function
disp('Flight Path/Tailplane Augmented, Reduced Order Transfer
Function:')
gamma_eta_ro_aug = zpk(tf(gamma_numerator,gamma_denominator))


% Task 3C
% Introduce an actuator lag with a time constant of 20 seconds into
the
% reduced order model, then apply the gains from Task 2 to augment
and
% stabilize the system, then output the new transfer functions.

disp('Task 3c:')
A_ro_actuator = [A(2,2) A(2,3) B(2,1)
                  A(3,2) A(3,3) B(3,1)
                  0      0      -20]

B_ro_actuator = [0
                  0
                  20]

C_ro_actuator = [1  0  0
                  0  1  0
                  0  0  1
                  1/V0 0  0];

% Row 4 adds alpha to output matrix by diving w by V0:  alpha =
w/V0

D_ro_actuator = zeros(4,1);

%[num_ro_act,den_ro_actuator] =
ss2tf(A_ro_actuator,B_ro_actuator,C_ro_actuator,D_ro_actuator,1);
% go from equations of motion to transfer functions

K_ro_actuator=[0,K_q, 0];
A_ro_actuator=A_ro_actuator-B_ro_actuator*180/pi*K_ro_actuator;
%A_ro_actuator stands for A matrix, reduced order, then augmented

[num_ro_actuator,den_ro_actuator]
=ss2tf(A_ro_actuator,B_ro_actuator,C_ro_actuator,D_ro_actuator,1);

%the code below removes the extra small number and extra zero that
matlab produces

[row,col] = size(num_ro_actuator);
num_new = zeros(row,col);
for i =1:row
    for j=1:col
        num_new(i,j) = num_ro_actuator(i,j);
        if abs(num_new(i,j)) < 1e-10;
            num_new(i,j) = fix(num_new(i,j));
        end
    end
end

```

```

        end
    end
end
num_ro_actuator = num_new;
%the code above removes the extra small number and extra zero that
matlab produces

disp('Pitch Rate/Inceptor Augmented, Reduced Order, Actuator
Included Transfer Function:')
q_eta_ro_actuator =
zpk(tf(num_ro_actuator(2,:),den_ro_actuator))
disp('Alpha/Inceptor Augmented, Reduced Order, Actuator Included
Transfer Function:')
alpha_eta_ro_actuator =
zpk(tf(num_ro_actuator(4,:),den_ro_actuator))

Integration_TF = tf(1,[1 0]); % define 1/s integration transfer
function:
disp('Pitch Attitude/Inceptor Augmented, Reduced Order, Actuator
Included Transfer Function:')
theta_eta_ro_actuator = Integration_TF*q_eta_ro_actuator

gamma_numerator_second_term = [num_ro_actuator(4,2),
num_ro_actuator(4,3), num_ro_actuator(4,4),0];

gamma_numerator = num_ro_actuator(2,:) -
gamma_numerator_second_term;

gamma_denominator=[den_ro_actuator(1), den_ro_actuator(2),
den_ro_actuator(3), den_ro_actuator(4), 0];

disp('Flight Path/Inceptor Augmented, Reduced Order, Actuator
Included Transfer Function:')
gamma_eta_ro_actuator = zpk(tf(gamma_numerator,gamma_denominator))

```

C.1.4 Task 4A M File

```
% Applying Gibson's Handling Qualities. Exercise C, Hawk T1 Mk1
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2011

% Task 4a
% Assess the system against Gibson's Dropback Criteria

% Run this file one time, then look at the figure 2 to identify Max
pitch rate,
% steady state pitch rate, and evaluate the pitch attitude and
flight path
% values at 2 and 4 seconds.

figure(2)
t=5;

subplot(1,2,1)
grid off
step(-q_eta_ro_actuator,t);
hold all
axis([-1 t -0.1 0.3])
title({'Pitch Rate Response to a Step Function in Tailplane
Demand';'Time Domain'})
ylabel('Degrees/second')
line([0,0],[-0.1 0.3],'color','k','linestyle',':')
line([-1,t],[0,0],'color','k','linestyle',':')

% Run this file one time, then look at the plot identify Max pitch
rate,
% steady state pitch rate, and evaluate the pitch attitude and
flight path
% values at 2 and 4 seconds.
% Input those values below here to draw the intercept lines

% Identify Pitch Rate Max amplitude and time:
q_max_amplitude = 0.167;
q_max_time = 0.382;
% Identify Pitch Rate steady state amplitude and time:
q_ss_amplitude = 0.0541;
q_ss_time = 4.0;
% Evaluate pitch attitude at 2 seconds and 4 seconds:
theta_2_seconds = 0.141;
theta_4_seconds = 0.249;
% Evaluate Flight Path at 2 seconds and 4 seconds:
gamma_2_seconds = 0.0976;
gamma_4_seconds = 0.206;

% Pitch Rate Max intercept line
```

```

line([-1,q_max_time],[q_max_amplitude,
q_max_amplitude],'color','k','linestyle','--')
legend('Pitch Rate Response (deg/sec)','x-axis','y-
axis','intercepts');

% Steady State Pitch Rate intercept line
line([-1,q_ss_time],[q_ss_amplitude,q_ss_amplitude],'color','k','linestyle
','--')

subplot(1,2,2)
grid off
step(-theta_eta_ro_actuator,t);
hold all
step(-gamma_eta_ro_actuator,t) ; ylabel('Degrees')
hold all
axis([-1 t -0.1 0.5])
title({'Pitch Attitude & Flight Path Response';'to a Step Function
in Tailplane Demand';'Time Domain'})
line([0,0],[-0.1 0.5],'color','k','linestyle',':')
line([-1,t],[0,0],'color','k','linestyle',':')

% Attitude integrator function (k/s performance)
t_int=[0:0.1:t];
integrator=t_int*q_ss_amplitude;
plot(t_int,integrator,'color','k','linestyle','-')
% Steady state pitch attitude slope line (used to identify
dropback)
theta_slope = (theta_4_seconds-theta_2_seconds)/2;
theta_y_intercept = theta_2_seconds - 2*theta_slope;
theta_x_intercept = -theta_y_intercept/theta_slope;
line([theta_x_intercept,4],[0,
theta_4_seconds],'color','k','linestyle','--')
% Steady state flight path slope line (used to identify flight
path delay)
gamma_slope = (gamma_4_seconds-gamma_2_seconds)/2;
gamma_y_intercept = gamma_2_seconds - 2*gamma_slope;
gamma_x_intercept = -gamma_y_intercept/gamma_slope;
line([gamma_x_intercept,4],[0,
gamma_4_seconds],'color','k','linestyle','--')
legend('Pitch Attitude Response (deg)','Flight Path Response
(deg)','x-axis','y-axis','K/S-like response (deg)','intercepts');

%Output critical attitude parameters to workspace
disp('Task 4a:')
Pitch_overshoot_ratio = q_max_amplitude/q_ss_amplitude
Dropback_over_qss = -theta_x_intercept % negative sign to account
for sign convection of dropback
Flight_Path_Delay = gamma_x_intercept

```

C.1.5 Task 4B M File

```
% Applying Gibson's Handling Qualities. Exercise C, Hawk T1 Mk1
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
% Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
% given by D.A. Lee at ETPS in August 2011

% Task 4b
% Assess the system against Gibson's Frequency Domain Criteria

% Run this file one time, then look at the figure 4 to identify the
pio
% frequency, the gain at the pio frequency, and twice the pio
frequency
%

figure(3)

bode(-theta_eta_ro_actuator)
grid on
title({'Pitch Attitude Response to a Step Function in Tailplane
Demand'; 'Frequency Domain'})
legend('Pitch Attitude Response (deg)')

% Run this file one time, then look at the figure 4 to identify the
pio
% frequency, the gain at the pio frequency, and twice the pio
frequency
% Input those values below here to output the Gibson frequency
criteria to
% the command window

% Identify the PIO Frequency, where the phase lag crosses 180
degrees
PIO_Frequency = 8.76; %units of radians
% Double the PIO frequency, and evaluate the phase lag there
Double_PIO_Frequency_Lag = 214; %units of degrees
% Read the gain magnitude off the top plot at the pio frequency
PIO_Gain = -35.2; %units of dB

%Output critical attitude parameters to workspace
disp('Task 4b:')
PIO_Frequency_hz = PIO_Frequency/(2*pi) %convert to hertz
Phase_Rate = (Double_PIO_Frequency_Lag - 180)/(2*PIO_Frequency_hz-
PIO_Frequency_hz)
PIO_Gain
```

C.1.6 Task 5 M File

```
% Applying Gibson's Handling Qualities. Exercise C, Hawk T1 Mk1
%
% written by LT Jack "Radio" Hathaway, USN; Cranfield University
MSc
%   Candidate, Jan 14. Based on a Flight Dynamics Principles
assignment
%   given by D.A. Lee at ETPS in August 2011

% Task 5a and 5b
% Develop and apply a command path pre-filter IAW Gibson's guidance
and
% then evaluate its effectiveness in improving handling qualities
prediction

% Run task_1 first to get all the tasks to run through
sequentially. Then
% identify critical values on figures 4 & 5 to type back into here
and then
% run again from task_1 to get the final plots

% Task 5a
% Design Command Path Pre-Filter by replacing the existing T Theta
2 value
% with the value of the system's flight path delay

% t_theta_2 is equal to the dropback/qss plus the flight path delay
%   (This relationship is apparent by looking at slide 64 of [4])
%   Due to a less than perfect linear fit in task 4a, that value is
off by
%   about 3 hundredths of a second from the value in the
%   pitch attitude/tail plane demand transfer function. For clarity
in
%   cancelling terms, the t theta 2 value from the transfer
function is
%   used below.

% The t theta 2 value in the pitch attitude transfer function can
be
%   identified by inspection, or pulling it directly from the
numerator of
%   the transfer function, as I do below. Remember the theta TF has
the same
%   numerator as the pitch rate TF
disp('Task 5a:')

t_theta_2 = 1/(num_ro_actuator(2,4)/num_ro_actuator(2,3))
new_t_theta_2 = Flight_Path_Delay % new value of T_theta_2 equal to
flight path delay should reduce attitude dropback to zero, but not
negative

a=1/new_t_theta_2;
b=1/t_theta_2;

num5 = [1 a];
den5 = [1 b];
```

```

disp('Command Path Pre-Filter')
command_path_pre_filter = zpk(tf(num5,den5))
disp('Pitch Attitude/Tail plane demand transfer function')
theta_eta_ro_actuator
disp('Flight Path/Tail plane demand transfer function')
gamma_eta_ro_actuator

% Task 5b
% Evaluate the effectiveness of the pre-filter

% Apply the command path pre-filter
theta_eta_gibson = command_path_pre_filter*theta_eta_ro_actuator;
q_eta_gibson = command_path_pre_filter*q_eta_ro_actuator;
gamma_eta_gibson = command_path_pre_filter*gamma_eta_ro_actuator;

figure(4)

subplot(1,2,1)
grid off
step(-q_eta_gibson,t);
hold all
step(-q_eta_ro_actuator,t);
hold all
axis([-1 t -0.1 0.4])
title({'Pitch Rate Response to a Step Function in Tailplane Demand';'Time Domain'})
ylabel('Degrees/second')
line([0,0],[-0.1 0.4],'color','k','linestyle',':')
line([-1,t],[0,0],'color','k','linestyle',':')

% Run this file one time, then look at the figure 5 identify Max
pitch rate,
% steady state pitch rate, and evaluate the pitch attitude and
flight path
% values at 2 and 4 seconds.
% Input those values below here to draw the intercept lines

% Identify Pitch Rate Max amplitude and time:
q_max_amplitude = 0.317;
q_max_time = 0.504;
% Identify Pitch Rate steady state amplitude and time:
q_ss_amplitude = 0.214;
q_ss_time = 4.0;
% Evaluate pitch attitude at 2 seconds and 4 seconds:
theta_2_seconds = 0.43;
theta_4_seconds = 0.859;
% Evaluate Flight Path at 2 seconds and 4 seconds:
gamma_2_seconds = 0.274;
gamma_4_seconds = 0.692;

% Pitch Rate Max intercept line
line([-1,q_max_time],[q_max_amplitude,
q_max_amplitude],'color','k','linestyle','--')
legend('Pitch Rate with Gibson Pre-Filter (deg/sec)','Pitch Rate
with no filter (deg/sec)','x-axis','y-axis','intercepts');

```

```

% Steady State Pitch Rate intercept line
line([-
1,q_ss_time],[q_ss_amplitude,q_ss_amplitude],'color','k','linestyle',
'--')

subplot(1,2,2)
grid off
step(-theta_eta_gibson,t);
hold all
step(-gamma_eta_gibson,t) ; ylabel('Degrees')
hold all
step(-theta_eta_ro_actuator,t);
hold all
step(-gamma_eta_ro_actuator,t) ; ylabel('Degrees')
hold all
axis([-1 t -0.1 1])
title({'Pitch Attitude & Flight Path Response';'to a Step Function
in Tailplane Demand';'Time Domain'})
line([0,0],[-0.1 1],'color','k','linestyle',':')
line([-1,t],[0,0],'color','k','linestyle',':')

% Attitude integrator function (k/s performance)
t_int=[0:0.1:t];
integrator=t_int*q_ss_amplitude;
plot(t_int,integrator,'color','k','linestyle','-')
% Steady state pitch attitude slope line (used to identify
dropback)
theta_slope = (theta_4_seconds-theta_2_seconds)/2;
theta_y_intercept = theta_2_seconds - 2*theta_slope;
theta_x_intercept = -theta_y_intercept/theta_slope;
line([theta_x_intercept,4],[0,
theta_4_seconds],'color','k','linestyle','--')
% Steady state flight path slope line (used to identify flight
path delay)
gamma_slope = (gamma_4_seconds-gamma_2_seconds)/2;
gamma_y_intercept = gamma_2_seconds - 2*gamma_slope;
gamma_x_intercept = -gamma_y_intercept/gamma_slope;
line([gamma_x_intercept,4],[0,
gamma_4_seconds],'color','k','linestyle','--')
legend('Pitch Rate with Gibson Pre-Filter (deg/sec)','Flight Path
with Gibson Pre-Filter (deg/sec)','Pitch Rate with no filter
(deg/sec)','Flight Path with no filter (deg/sec)','x-axis','y-
axis','K/S-like response (deg)','intercepts');

%Output critical attitude parameters to workspace
disp('Task 5b:')
Pitch_overshoot_ratio = q_max_amplitude/q_ss_amplitude
Dropback_over_qss = -theta_x_intercept % negative sign to account
for sign convetion of dropback
Flight_Path_Delay = gamma_x_intercept

% Task 5b continued
% Assess the system against Gibson's Frequency Domain Criteria

```

```

% Run this file one time, then look at the figure 6 to identify the
pio
% frequency, the gain at the pio frequency, and twice the pio
frequency

figure(5)

bode(-theta_eta_gibson)
grid on
title({'Pitch Attitude Response to a Step Function in Tailplane
Demand'; 'Frequency Domain'})
legend('Pitch Attitude Response (deg)')

% Run this file one time, then look at the figure 4 to identify the
pio
% frequency, the gain at the pio frequency, and twice the pio
frequency
% Input those values below here to output the Gibson frequency
criteria to
% the command window

% Identify the PIO Frequency, where the phase lag crosses 180
degrees
PIO_Frequency = 6.55; %units of radians
% Double the PIO frequency, and evaluate the phase lag there
Double_PIO_Frequency_Lag = 217; %units of degrees
% Read the gain magnitude off the top plot at the pio frequency
PIO_Gain = -26.7; %units of dB

%Output critical attitude parameters to workspace
PIO_Frequency_hz = PIO_Frequency/(2*pi) %convert to hertz
Phase_Rate = (Double_PIO_Frequency_Lag - 180)/(2*PIO_Frequency_hz-
PIO_Frequency_hz)
PIO_Gain

```