



Large strain zero Poisson's ratio spring cell metamaterial with critical defect analysis and variable stiffness distributions

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ARTICLE INFO

Keywords:

Mechanical metamaterial
large strain zero Poisson's ratio
Spring structure
3D printing
Tuneable stiffness

ABSTRACT

This paper presents novel metamaterial skins formed by 3D Spring Cells exhibiting zero Poisson's ratio in two directions with no stress concentration on joints. Precise CAD models are generated to perform numerical and experiment analysis. High-fidelity Finite element models are developed to assess the homogenisation study of zero Poisson's ratio. Moreover, the analytical method is also used to present for normalised Young's modulus. Parametric study for the effect of parameter on normalised Young's modulus and Poisson's ratio is demonstrated based on analytical method. Pure shear analysis is demonstrated to show the off-axis loading behaviour. Structural defect analysis is investigated with regarding to its deformation mechanism under tensile strain. Furthermore, variable stiffness distribution of Spring Cell metamaterials is demonstrated while maintaining large strain zero Poisson's ratio. The frequency and buckling analysis of metamaterials formed by spring cells are investigated and compared with equivalent shell. Moreover, their mechanical behaviours including buckling and frequency are investigated. Experiment analysis is performed to validate the force-displacement, Poisson's ratio and deformation mechanisms discussed in numerical simulations. Finally, construction material analysis is taking to investigate the relations between metamaterial Poisson's ratio and Young's modulus and variety types of construction material.

1. Introduction

The mechanical metamaterial is an artificial material with superior mechanical behaviours. It frequently consists of periodically arranged building blocks whose properties and functions differ from and exceed those of their constituent materials [1]. Manufacturing technologies, such as additive manufacturing (AM), also known as 3D printing, have enabled the fabrication of cellular materials with complex structures across multiple length scales, with feature sizes in nanometer scale for a wide range of materials [2].

Mechanical metamaterial can be classified as ultra-property, negative and extreme varieties with a variety of exotic properties [3]. Poisson's ratio metamaterials can be classified as Negative Poisson's ratio (NPR), Positive Poisson's ratio (PPR), Zero Poisson's ratio (ZPR), or tailored Poisson's ratio. Zero Poisson's ratio (ZPR) metamaterials exhibit no transverse displacement under in-plane longitudinal loadings and no double curvature under out of plane bending due to the lack of coupling between stresses and strains in two orthogonal directions [4]. Therefore, ZPR metamaterials exhibit a single curvature under bending

(monoclastic) in contrast to the Positive Poisson's ratio with anticlastic behaviour and synclastic curvature of negative Poisson's ratio [5]. This results in enhanced integration characteristics for complex geometries and boundary conditions. Consequently, they are ideally suited for a wide range of complicated topologies, such as morphing wing and soft robotics. In tissue engineering applications, scaffolding with a ZPR behaviour may be more suitable for simulating native tissue behaviour, accommodating and transmitting force to the host tissue site during wound healing (or tissue regrowth) [6]. In the aviation industry, ZPR star shape structure have been designed to overcome the Poisson's ratio variation, which can lead to a substantial increase in the 'effective' in-plane axial stiffness in the morphing direction, resulting in a proportional increase in the force required for morphing actuation [7].

Some semi re-entrant design which are combination of PPR and NPR cells also exhibits ZPR in one direction, i.e. Del Broccolo et al. [8] presented a kirigami inspired cellular structure combined hexagonal cell with positive Poisson's ratio and re-entrant cell has negative Poisson's ratio to neutralise the Poisson's ratio. A multi-stable mechanical metamaterial with double-U array of the parallel fine crack mechanism

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<https://doi.org/10.1016/j.compstruct.2023.117102>

Received 24 November 2022; Received in revised form 17 March 2023; Accepted 28 April 2023

Available online 5 May 2023

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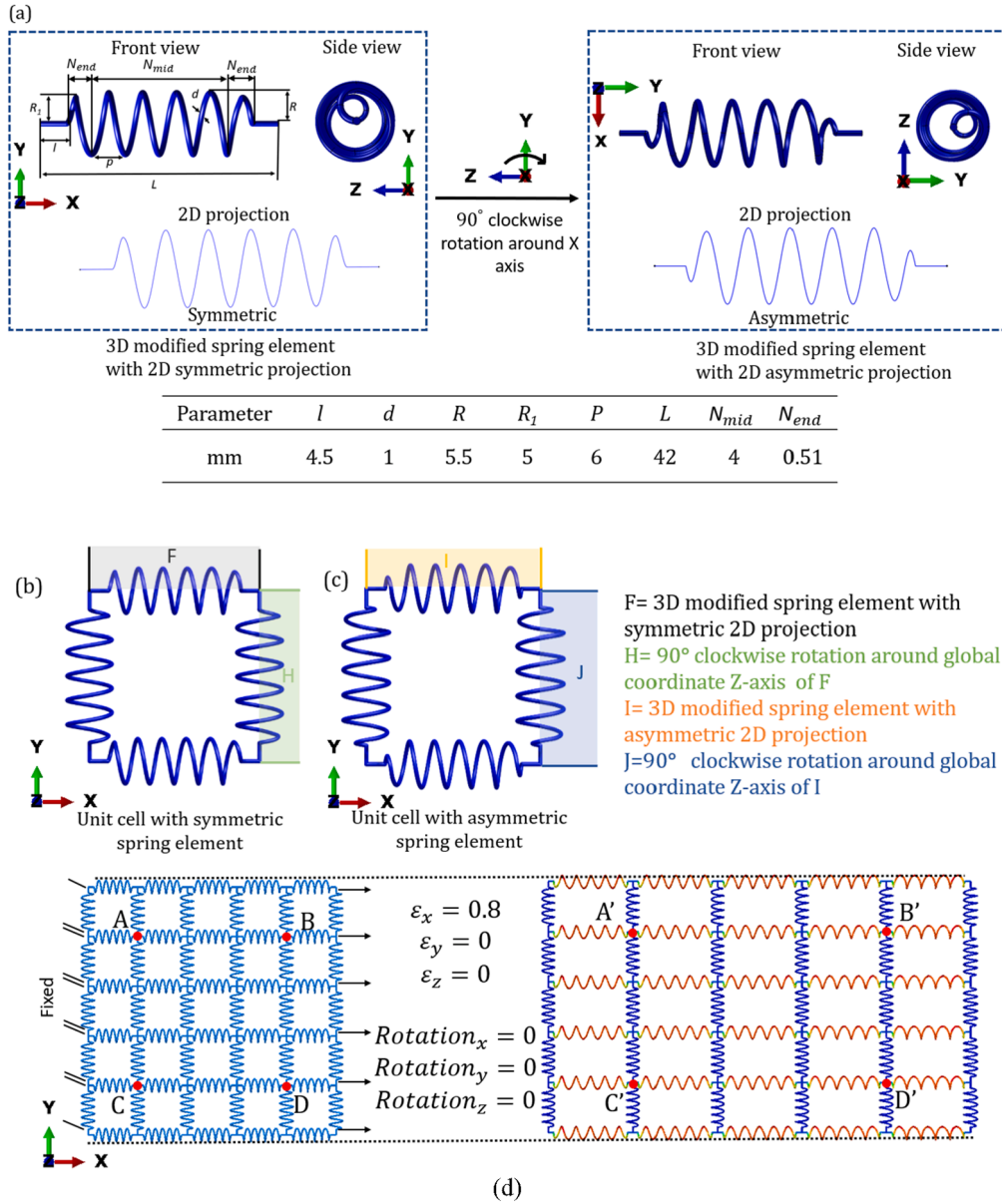


Fig. 1. Large strain ZPR spring cell metamaterial (a) 3D modified Spring element and geometry parameters, (b) ZPR unit cell with 3D modified symmetric spring elements (c) ZPR unit cell with 3D modified asymmetric spring elements, (d) ZPR demonstration in two directions, tensile boundary conditions and measuring points for Poisson's ratio.

created by Yang et al. showed ZPR behaviour in one direction with around 40% tensile strain in the experimental and numerical analysis [9]. Furthermore, Feng et al. [10] presented a bio-inspired 2D Gardenia-Shaped Honeycomb (GSH). They developed theoretical models for calculating the in-plane Young's modulus and Poisson's ratio based on Euler-Bernoulli beam theory. Their results demonstrated that Poisson's ratio behaviour can be changed by editing the angle between cell element and connectors.

Some cell designs with an identical shape between longitudinal and transverse directions exhibit ZPR behaviour in two directions. Mizzi et al. [11] demonstrated mechanical metamaterials formed by n-pointed symmetric star-shaped pores exhibit zero to negative Poisson's ratios up to a tensile strains exceeding 0.15. However, some asymmetric cells can also achieve ZPR behaviour in two directions by specially designed connection arrangements. Naghavi Zadeh et al. [12] introduced a 2D metamaterial formed by Fish-Cell with Zero Poisson's ratio behaviour in two orthogonal directions. The detailed discussion on the effect of the joints was presented by numerical analysis showed the strains

concentration limits the tensile strain of the metamaterial. Jha et al. [13] presented a Genetic Algorithm (GA) shape optimisation to improve the ZPR characteristics of the Fish-Cell metamaterial and a detailed joint design study was proposed to maintain ZPR behaviour over a maximum strain range. Their results shows when tensile strain in -y direction within 0.01, the optimised metamaterial has 88.1% lower absolute value for Poisson's ratio compared to baseline metamaterial and a 25% lower maximum Poisson's ratio was observed at 46% higher maximum strain range. Moreover, 3D cylindrical Fish-Cell zero Poisson's ratio metamaterials in two different arrangements was presented by Qin et al. [14]. Their work showed the elastic tensile range is smaller than 2D Fish-Cell metamaterial, however, the deformation mechanism and location of stress concentration are identical with the 2D Fish-Cell metamaterial. Also, the results showed the cellular structure and the assembly direction of the unit cells affect the magnitude and the mode shape for the buckling and frequency results.

Helical structure is an efficient mechanism to increase the elastic tensile range and reduce the stress concentration of the metamaterial. It

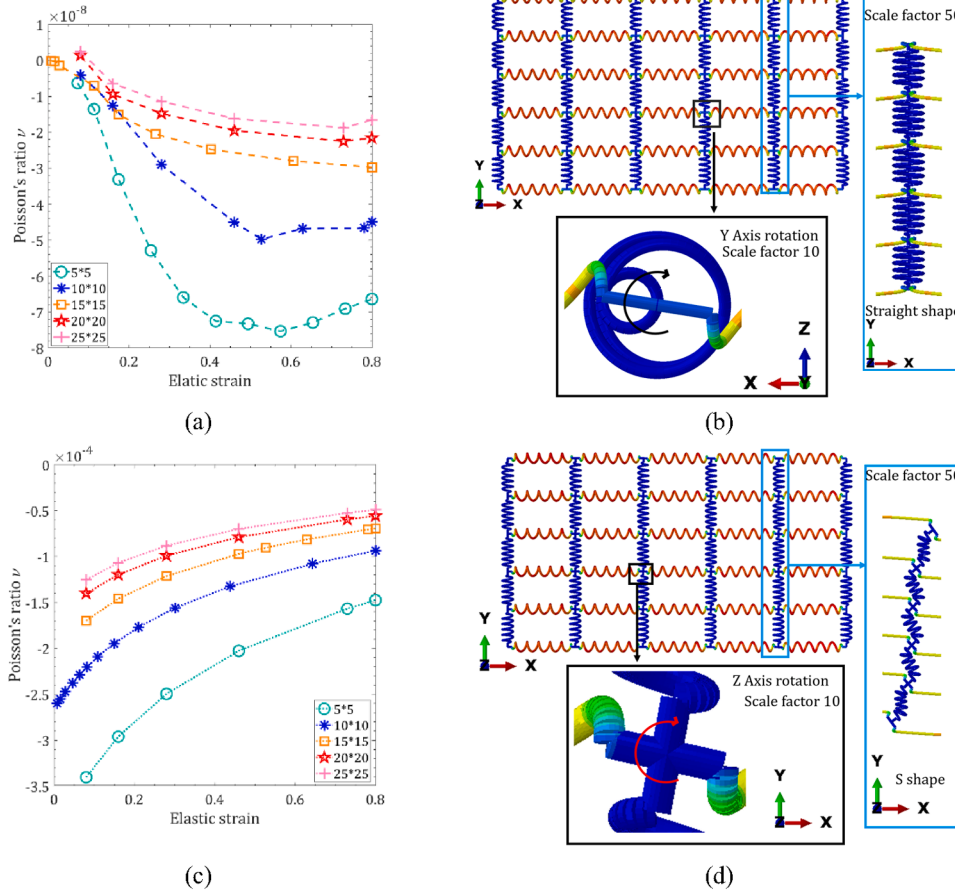


Fig. 2. Large strain homogenisation studies of Poisson's ratio and deformation mechanisms of two spring tessellations (a) ZPR trends for tessellations with symmetric spring elements (b) Deformation mechanism of tessellations with symmetric spring elements, (c) ZPR trends for tessellations with asymmetric spring elements, (d) Deformation mechanism of tessellations with asymmetric spring elements.

is one of the fundamental mechanisms with superior strain energy storage and release capability with large strain deformation performance used widely in different research fields such as sound absorption and medical use [15,16]. Moreover, Cimolai et al. [17] demonstrated a GA shape optimisation for helical structure for metamaterial applications to achieve about 39% and 33% increment in tensile and compression elastic strain capability with lower buckling load and less frequency than the conventional linear spring. Although many structures can achieve zero Poisson's ratio, ZPR behaviour in two directions under large strain is barely stated due to most of the cellular ZPR metamaterials suffering from stress concentration.

In this paper, metamaterial skins formed by spring cells with no stress concentration on joints and exhibit ZPR behaviour under large strain in two directions is introduced for the first time. The homogenisation study for Poisson's ratio behaviour is revealed by the numerical method. Also, the homogenisation study on Young's modulus is taken by numerical and analytical analysis. Parametric study for influence on normalised Young's modulus and Poisson's ratio is conducted based on analytical method. Pure shear analysis is demonstrated with normalised shear modulus and shear deformation to show off-axis deformation behaviour. The structural defect model is presented with the deformation mechanism stated to demonstrate the capability to maintain ZPR behaviour under extreme damage. Variable stiffness for zero Poisson's ratio spring cell metamaterial is presented with different Young's modulus assigned to each spring element in gradient and random stiffness distributions. Moreover, linear buckling and vibration behaviours of the spring cell metamaterial are studied and compared with equivalent shell. Furthermore, experimental, numerical and analytical analysis

is used to demonstrate Poisson's ratio behaviour and deformation mechanism. Finally, construction material analysis is investigated by numerical method with experimental CAD samples to show the mechanical behaviour of the metamaterial.

2. Problem definition

The nano or micro scale architecture of metamaterials plays a decisive role in increasing the strain within the construction material elastic deformation range. The geometry of the unit cell in this paper is inspired by the deformation of optimised Fish-Cell and the semi re-entrant structure [18,19]. Both structures combine the hexagonal and re-entrant structures, to have a similar mechanism for cancelling the Poisson's ratio. The large tensile deformation of optimised Fish-Cell 2D skins show the Fish-Cell deforms into a rectangular shape with an entire rotation of inclined ligaments connected to each unit cell [13]. The rotation and stress concentration on the joints of these structures increase the risk for structural failure and limit the strain range. Thus, it is important to design a metamaterial with reduced stress concentration on joints as well as enhanced flexural mechanics to increase the deformation capability under actuation loads. The helical structure with superior axial deformation capability is used to distribute the stress evenly on the coils and obtain large strains through transforming tensile displacement to torsion.

2.1. Spring cells architecture

3D spring architecture for minimising the stress concentration and

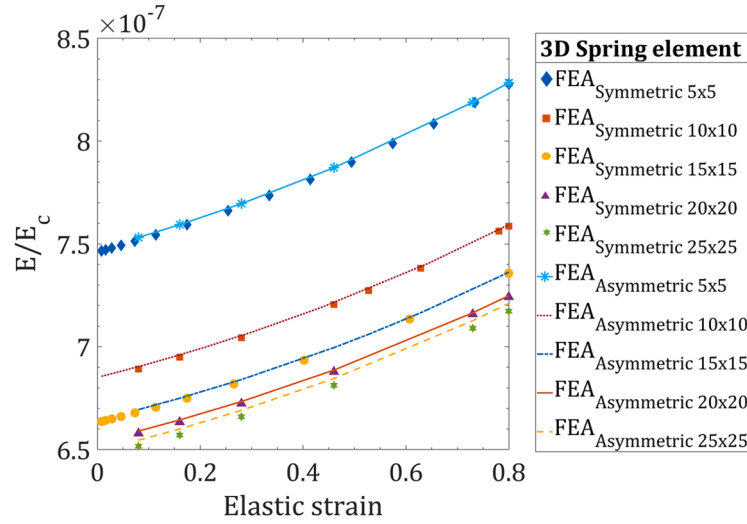


Fig. 3. Large strain homogenisation studies of Young's modulus for 3D Symmetric and 3D asymmetric tessellations in two directions.

maximise the tensile strain is shown in Fig. 1 (a). The spring element can be defined by nine parameters: the total length L , length of straight beam l , pitch of the spring P , diameter for cross-section d , radius of main coils R , radius of coils at both ends R_l , numbers of the main coils N_{mid} and numbers of the coils at both ends N_{end} . The projection of the 3D spring element shown in Fig. 1(a) is symmetric in 2D. 3D spring element with 2D symmetric projection is rotated 90 degrees along X axis in the clockwise direction to have a 2D asymmetric projection to investigate the influence of assembly direction on mechanical behaviour. The spring cells formed by 3D spring elements with symmetric and asymmetric projections are presented in Fig. 1 (b) and (c) respectively. Furthermore, the unit cell comprises four of 3D modified spring elements to demonstrate zero Poisson's ratio with assembly direction highlighted in Fig. 1 (c). It is noticed that the radius of the main and end coils i.e. R and R_l are limited by the distance between the coils at the corners of the unit cell to avoid contact. Thus, the radius R_l is 0.5 mm smaller than the main coil radius R .

3. Zero Poisson's ratio skin metamaterial, concept validation

In 2D continuum mechanics, there are two Poisson's ratios, ν_{XY} and ν_{YX} , with respect to longitudinal and transverse loading directions, as shown in Fig. 1. The Poisson's ratio is the ratio between the transverse displacement to the tensile displacement i.e. $\nu_{XY} = -\frac{\epsilon_Y}{\epsilon_X}$ and $\nu_{YX} = -\frac{\epsilon_X}{\epsilon_Y}$, where the ϵ_X is defined as the average strain for the distance between AB and CD. Moreover, ϵ_Y is defined as the average strain for the distance between AC and BD. The location of the measuring points has a significant effect on Poisson's ratio calculation as shown in Appendix A. Tessellation numbers for numerical and experimental models are presented in 'm*n' format, where 'm' presents the unit cell numbers in the longitudinal direction, and 'n' shows unit cell numbers in the transverse direction. For example, tessellation 5*3 corresponds to five unit cells in the X direction and three unit cells in the Y direction. Therefore, when tessellation numbers m is identical to n, a consistent geometric ratio in square unit cells will be achieved with identical Poisson's ratio value in longitudinal and transverse directions, thus $\nu_{xy} = \nu_{yx} = \nu$.

ABAQUS V6.18 is used for simulations of Poisson's ratio analysis with Nylon PA2200 as construction material with Young's modulus $E_c = 1130$ MPa, yield stress $\sigma_{yield} = 14$ MPa and Poisson's ratio $\nu_c = 0.4$. The details plastic behaviour for this material is explained in previous author research [14] and listed in Appendix B. The initial simulations are nonlinear analysis with three nodes quadratic beam elements B32 with circular cross-section shape. A very fine mesh with a maximum global seed size 0.5 is used to ensure the accuracy of the simulation.

Boundary conditions for tensile analysis are shown in Fig. 1 (d) where one end of model is fully fixed and the other end has tensile strain 0.8. Measuring Points A, B, C and D located at the centre of metamaterial are used to measure the displacement in longitudinal and transverse directions to obtain minimised boundary condition effects.

4. Homogenisation study for zero Poisson's ratios

Tessellation numbers, load and boundary conditions effects have significant influence on the structural behaviour of skin metamaterial. Thus, it is crucial to perform homogenisation study to introduce tessellation numbers demonstrating a behaviour convergence independent from these effects. Thus, reaching a tessellation with the right number of unit cells is crucial to demonstrate a behaviour convergence independent of these effects. Therefore, different tessellation numbers 5*5, 10*10, 15*15, 20*20, 25*25 are considered to analyse the tessellations with 3D Spring symmetric model and asymmetric spring elements.

Fig. 2 shows large strain homogenisation studies of Poisson's ratio and deformation mechanisms of two spring tessellations. Fig. 2 (a) demonstrates decreasing trends of negative Poisson's ratio by increasing the elastic tensile strain for tessellations with symmetric spring elements. Moreover, the negative Poisson's ratio is increasing with tessellation numbers with the same tensile strain and converge at tessellation 20*20.

The deformation mechanism for tessellations with 3D symmetric spring in Fig. 2(b) shows the clockwise rotation of the joints on XZ plane as highlighted in the zoom area. The magnitude of joint rotation proportionally increases with tensile strain. However, this rotation does not affect the deformation on XY plane as highlighted in Fig. 2(b) with a straight shape deformation scaled 50 times larger.

Fig. 2 (c) presents increasing trends of negative Poisson's ratio by increasing the elastic tensile strain for tessellations with asymmetric spring elements. Furthermore, the negative Poisson's ratio is increasing with tessellation numbers increase at the same tensile strain and converge at tessellation 20*20.

It can be seen in Fig. 2(d), that the joints highlighted in the black box with 10 times deformation scale applied to show the rotation at joints in a clockwise direction in XY planar and increase with tensile displacement increase. Fig. 2(d) shows the blue box with 50 times the deformation scale for the vertical coils. The deformation shows the vertical coils have inclined S shape behaviour, leading to the change in Poisson's ratio behaviour. Moreover, this joint rotation mechanism is similar to chiral structure, which can cause expansion [20–23]. However, springs

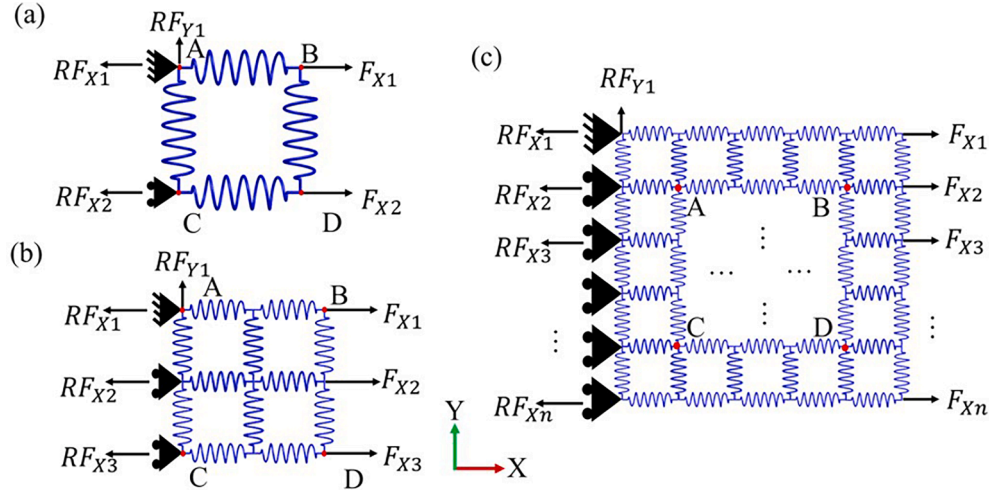


Fig. 4. Analytical models and boundary conditions for Poisson's ratio study with different tessellations (a) single unit cell, (b) tessellation 2×2 , (c) general large tessellation $n \times n$.

are placed next to the joints to minimise the expansion.

Both tessellations with symmetric spring and asymmetric elements show zero Poisson's ratio behaviours. The Poisson's ratio for tessellation 20×20 with symmetric spring element is 2.6×10^3 times smaller than tessellation with asymmetric spring elements. Consequently, the spring element assembly orientation influences the deformation mechanism, resulting in distinct Poisson's ratio behaviours. The design for spring cell metamaterial significantly minimised the stress concentration at joints compared to semi re-entrant and Fish-Cell metamaterials [8,12]. The maximum elastic strain for spring cell metamaterials is almost four times larger than optimised Fish-Cell metamaterial [13] and star-shaped metamaterial [11]. Unlike Fish-Cell metamaterial, the mechanical properties in two directions are identical. However, the limitation for this design is the thickness of the metamaterial might change under large in-plane deformation.

4.1. Homogenisation study for Young's modulus

Normalised Young's modulus with respect to original constructing material Young's modulus is illustrated in Fig. 3 for different tessellation numbers with 3D symmetric and asymmetric springs. This shows that the equivalent Young's modulus of the spring models is dramatically smaller than the original constructing material and the difference between 3D symmetric and asymmetric spring elements is very small. The trends of normalised Young's modulus are increasing by increasing the elastic tensile strain. This is because the diameter of the coils decreases with tensile strain increases where the spring stiffness for coils is changing. Moreover, the normalised Young's modulus is decreasing with tessellations increase at the same tensile strain and converge at tessellation 20×20 . These very low stiffness and zero Poisson's ratio characteristics can apply to soft robots, medical stents, and morphing wings.

To sum up, the tessellations with symmetric spring elements have smaller Young's modulus and Poisson's ratio than those with asymmetric spring elements. However, tessellations with symmetric spring elements are more difficult for manufacturing because the spring coils at the corner are very close as shown in Fig. 1(b). Thus, the following sections are focused only on tessellations with asymmetric spring elements. In other words, the simplified terms of spring element refer to the asymmetric spring element.

5. Analytical analysis for mechanical properties under large tensile strain

5.1. Analytical model and discussion for zero Poisson's ratio

Fig. 4 (a) shows a single isolated cell with measuring points ABCD, where the boundary conditions are considered with a fixed degree of freedom at point A, enabling displacement in the Y direction at point C. At the other end, the displacement in the X direction at points B and D are identical. From equilibrium equation $\sum F_Y = 0$ at point A, the reaction force in Y direction will be zero i.e. $RF_{Y1} = 0$. Similarly, the reaction force in Y direction for larger tessellations demonstrated in Fig. 4 (b) and (c) will be obtain zero. Therefore, there will be no displacement in Y direction with respect to Castigliano theory. Thus, the strain in Y direction will be zero i.e. $\epsilon_Y = 0$ and the Poisson's ratio for the metamaterial will be obtain zero i.e. $\nu_{YX} = -\frac{\epsilon_Y}{\epsilon_X} = 0$. As demonstrated in Section 3, $\nu_{XY} = \nu_{YX}$, therefore, $\nu_{XY} = 0$. Conclusively, the tessellation exhibit ZPR behaviour analytically within the elastic range independent of spring element parameters and construction material.

Naghavi Zadeh et al [18] demonstrated the reliability of predicting the Poisson's ratio on tessellation by estimating the Poisson's ratio on a single unit cell. They also reported there is no lateral displacement in the Fish Cell Zero Poisson's ratio skin metamaterial under loading in the X direction.

5.2. Analytical model for Young's modulus

The analytical formulas developed for Young's modulus under large tensile strain is very beneficial for parametric study as it reduce significantly computation times.

Equivalent Young's modulus in the X direction i.e. E_X of Spring metamaterials can be found based on the Hooke's law for isotropic material as:

$$E_X = \frac{\sigma_X}{\epsilon_X} = \frac{F_{\text{metamaterial-X}}/A}{\delta_X/L_{\text{metamaterial}}} \quad (1)$$

Where A is the equivalent cross-section area of the metamaterial tessellations i.e. $A = 2RL_{\text{metamaterial}}$ and δ_X stands for the displacement of the metamaterial in X direction. Moreover, $L_{\text{metamaterial}} = mL$, where tessellation numbers m and length of the spring element L are indicated in Fig. 1. Also the total reaction force ($F_{\text{metamaterial-X}}$) for the tessellation can be calculated as:

$$F_{\text{metamaterial-X}} = k_{\text{metamaterial-X}} \cdot \delta_X \quad (2)$$

Table 1

Normalised equivalent Young's modulus compared with FEA simulations for different tessellations.

Analytical results					
Tessellation	5*5	10*10	15*15	20*20	25*25
E/E_c	7.64×10^{-7}	7.00×10^{-7}	6.79×10^{-7}	6.69×10^{-7}	6.62×10^{-7}
Error rate compared to FEA results demonstrated in Fig. 3					
Strain 0.2	0.174%	0.175%	0.179%	0.180%	0.331%
Strain 0.4	2.243%	2.241%	2.238%	2.237%	2.107%
Strain 0.6	5.167%	5.142%	5.058%	5.161%	5.573%
Strain 0.8	8.415%	8.411%	8.410%	8.410%	8.883%

The stiffness of the metamaterial ($k_{\text{metamaterial-X}}$) can be calculated depending on the tensile force direction based on the series and parallel connection law of the spring:

$$k_{\text{metamaterial-X}} = (m+1) \cdot \left(n \frac{1}{k_{\text{spring}}} \right)^{-1} \quad (3)$$

To calculate the total stiffness of the spring (k_{spring}), the 3D spring element shows in Fig. 1 (a) is formed with four constant diameter coils at the middle and one variable diameter coil at both ends. Thus from spring series equation

$$k_{\text{spring}} = \frac{1}{\frac{1}{k_{\text{mid}}} + 2 \frac{1}{k_{\text{end}}}} \quad (4)$$

By applying early study of Wahl [24] for curved beam theory, the stiffness constant for the middle linear spring (k_{mid}) as shown:

$$k_{\text{mid}} = \frac{Gd^4}{8(2R)^3 N_{\text{mid}}} = \frac{E_c d^4}{128R^3 N_{\text{mid}} (1 + \nu_c)} \quad (5)$$

Where N_{mid} shows the active coil numbers with value $N_{\text{mid}} = 4$, and G presents the shear modulus of construction material as $G = \frac{E_c}{2(1+\nu_c)}$.

Moreover, Gu et al. [25] presented an analytical method for spring with variable pitch and constant wire diameter based on Wahl theory.

They separate the spring into small twist beams with constant length but the variable distance between the centre of the wire cross-section to the spring axis.

Depending on their calculation, the following relations for spring stiffness of variable diameter at end of the coil (k_{end}):

$$k_{\text{end}} = \frac{G\pi d^4}{4} \cdot \frac{1}{\int_0^{2\pi N} (R_1 \alpha)^3 d\alpha + \int_0^{2\pi N} ((R - R_1) \alpha)^3 d\alpha} = \frac{E_c d^4}{32\pi^3 N_{\text{end}}^4 (R_1^3 + (R - R_1)^3) (1 + \nu_c)} \quad (6)$$

Where N_{end} shows half of the active coil number at each end with a value $N_{\text{end}} = 0.51$. By combining equations (4), (5) and (6), the spring stiffness for 3D edited spring will be formed as:

$$k_{\text{spring}} = \frac{E_c d^4}{64(1 + \nu_c) (2R^3 N_{\text{mid}} + \pi^3 N_{\text{end}}^4 (R_1^3 + (R - R_1)^3))} \quad (7)$$

Finally Young's modulus in the X direction of the tessellation can be found by combining equations (1), (2), (3) and (7) as:

$$E_X = \frac{(m+1)E_c d^4}{128nR(1 + \nu_c) (2R^3 N_{\text{mid}} + \pi^3 N_{\text{end}}^4 (R_1^3 + (R - R_1)^3))} \quad (8)$$

Similarly, as demonstrated in Section 3 due to identical geometry distribution and loading, Young's modulus in Y direction will be identical to E_X i.e. $E_Y = E_X$.

Table 1 presents the normalised Young's modulus obtained by the analytical method, and the error rate between the average value from the FEA analysis shown in Fig. 3. The results for the analytical method with different tessellation numbers show that the normalised Young's modulus decreases with tessellation increases with a homogenised value at tessellation 20*20.

Moreover, the error rate between analytical and numerical methods ranges from 0.174% at strain 0.2 for tessellation 5*5 to 0.883% at strain 0.8 with tessellation 25*25. The results are more accurate in the smaller tensile range because the analytical method does not consider the change of the coil diameter under large tensile deformations. Moreover, the overall results show that less tessellations are more accurate than large tessellations. This is because analytical results do not consider the stiffness of the straight connector beams. This error rate increases with the number of straight connector beams increase. Thus, this method is only applied for small deformation with negligible diameter change up to approximate strain 0.6, and the effect of the stiffness on the straight connector beams should be negligible.

5.3. Parametric study for Young's modulus

Equation (8) implies that nine variables can influence the value of

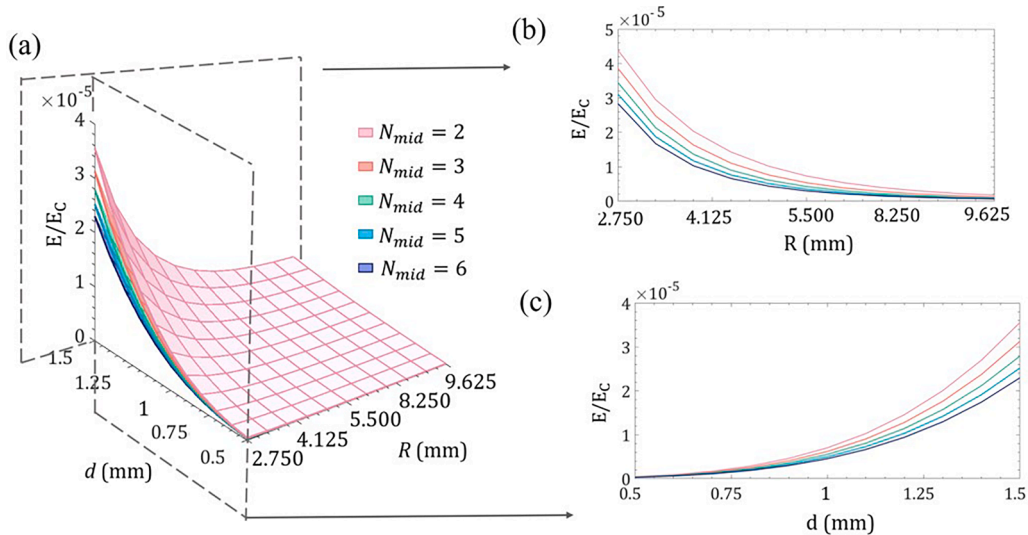


Fig. 5. Parametric study for the effect on normalised Young's modulus with details (a) effect of parameters on normalised Young's modulus, (b) radius of main coils VS normalised Young's modulus with 1.5d, (c) diameter of cross-section VS normalised Young's modulus with 0.5R.

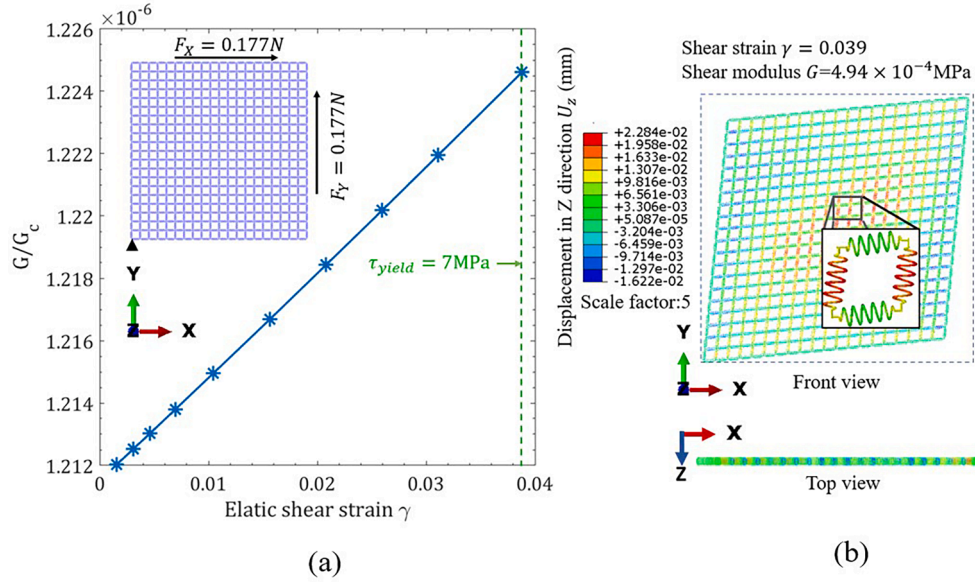


Fig. 6. Normalised shear behaviour and deformation for spring metamaterial with tessellation 20*20, (a) shear boundary conditions and normalised shear modulus as a function of elastic shear strain. (b) deformation under shear loading with scale factor five.

Young's modulus, and five of these variables are geometrical parameters for spring elements: R , R_1 , N_{mid} , N_{end} and d , however, the number and the radius for coils at both ends (N_{end} , R_1) are constrained by the unit cell shape. Fig. 5 indicates the parameter effect on the normalised Young's modulus for tessellation 20*20 including R , N_{mid} and d with a variation range $\pm 50\%$ to assure the validity of the analytical model.

Fig. 5(a) shows the stiffness of the tessellations decreases with N_{mid} increases with R and d fixed, i.e. the normalised Young's modulus with a value 3.6×10^{-5} at $N_{mid} = 2$ is 1.57 times larger than the value $N_{mid} = 6$ when cross-section thickness of the spring element $d = 1.5$, main coil radius $R = 2.750$.

Moreover, Fig. 5 (b) demonstrates the stiffness of the tessellation is decreasing with the radius of the main coil R increases when N_{mid} and d fixed. For instance, normalised Young's modulus 1.4×10^{-6} with $R = 9.625$ is 25.6 times larger than normalised Young's modulus with $R = 2.750$ at $d = 1.5$, $N_{mid} = 2$.

Furthermore, Fig. 5 (c) indicates that the normalised Young's modulus has increasing trend with the cross-section of spring element d increases when N_{mid} and R fixed, i.e. normalised Young's modulus 2.9×10^{-7} with $d = 0.5$ is 124.1 times smaller than the normalised Young's modulus with $d = 1.5$ at $R = 0.5$ and $N_{mid} = 2$.

All of the parameters can affect Young's modulus value. The cross-section diameter d has the largest impact on Young's modulus property. However, the influence on the coil number at the middle N_{mid} at a large coil radius or small cross section area is insignificant comparing the impact for other parameters.

6. Pure shear analysis

Shear analysis is considered with pure shear loading with boundary conditions on the tessellation demonstrated in Fig. 6 (a). Maximum shear force $F_s = F_x = F_y = 0.177N$ has been evenly distributed to the tessellation edges by applying the concentrated force on corresponding nodes of each edge. Displacement of one node at the corner of the tessellation is constrained to avoid rigid body motion. The shear stress is obtained by the shear force divided the area of the edge: $\tau = \tau_{YX} = \tau_{XY} = \frac{F_s}{2mL}$, where $m = n$. Moreover, due to the shear theory, yield point of the shear stress can be calculated as $\tau_{yield} = \frac{\sigma_{yield}}{2} = 7$ MPa. The shear strain can be found by $\gamma = \frac{\delta Y}{mL} + \frac{\delta X}{nL}$, where δY and δX record the displacement of the edges in Y and X directions. Thus, the shear modulus

can be calculated by $G = \frac{\tau}{\gamma}$, and the shear modulus is normalised by the material shear modulus $G_c = \frac{E_c}{2(1+\nu)}$.

Fig. 6 (a) demonstrates that the normalised shear modulus is linearly increasing with shear strain increases with a maximum value 1.25×10^{-6} at maximum elastic strain 0.039. The normalised shear modulus for spring cell metamaterial is approximately ten times larger than Fish-Cell metamaterial[18]. Fig. 6 (b) presents the deformation shape for unit cell and tessellation at maximum shear strain 0.039 exaggerated with scale factor five. The detailed view of the spring cell in tessellation illustrates that the main deformation under pure shear loading is the springs deflect into inclined shape, while the joints have no obverse deformation. The rotation of the joints under shear behaviour leads to the compressive and tensile behaviour on springs. Moreover, out of plane deformation for tessellation is observed with maximum negligible displacement 2.28×10^{-2} mm.

The normalised shear modulus is extremely smaller than the constrained material, showing that the metamaterial is easy to deform under shear loading. For applications that require stiffer shear resistance, the construction material can be replaced with a stiffer material, such as composite and metallic materials. Furthermore, the spring cell metamaterial can be applied as a sandwich morphing skin core with two stiffer layers covered to increase the shear strength. However, the systematic study of shear behaviour of this metamaterial can be an independent paper, which is out of the scope of this paper.

7. Large strain zero Poisson's ratio behaviour with structural defects

The mechanical properties of metamaterials are dependent to defect shapes, quantity, distributions, density, edge locations, relative defect orientations with respects to loading and tessellation number. As a novelty of the research, this design maintains zero Poisson's ratio under large elastic strains even with the 10% defect distributions. However, the detailed defect analysis of Poisson's ratio is beyond the scope of this paper and will be studied in next paper.

Fig. 7 (a) demonstrates an extreme distribution with 10% defects for tessellation 20*20. This is because 10% quantity, edge location and large density of defects create an excessive condition. Three types of the defect are categorised as regular, semi regular and irregular shapes. The difference between semi regular and regular shapes is existence of spring

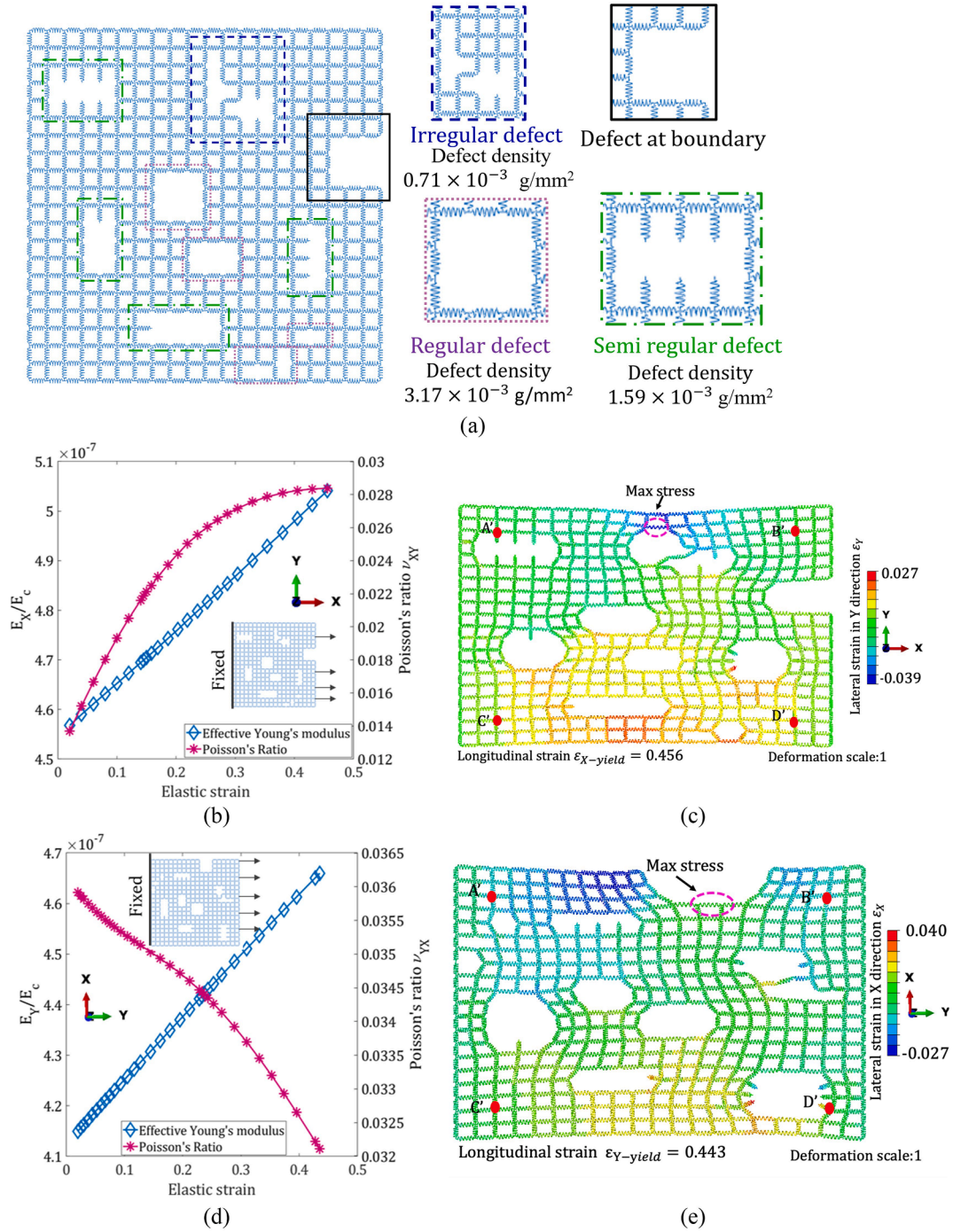


Fig. 7. Extreme distribution with 10% defects for tessellation 20*2 and its impact on large strain ZPR metamaterial (a) different defect types with details (b) Poisson's ratio and Young's modulus in X direction, (c) deformation mechanism in X direction loading (d) Poisson's ratio and Young's modulus in Y direction, (e) deformation mechanism in Y direction loading.

elements with free boundary conditions and loading, which do not contribute to structural stiffness. Irregular type is combined by different shapes other than a single rectangular shape, which makes the deformation mechanism more complex. Defect density is the quantity of element absence in a single local domain. For example, the highlighted regular shape shows the defect density of $3.17 \times 10^{-3} \text{ g/mm}^2$, which corresponds to absence of 15 spring elements in nine unit cell area. Similarly, highlighted semi-regular and irregular shapes are having defect density of $1.59 \times 10^{-3} \text{ g/mm}^2$ and $0.71 \times 10^{-3} \text{ g/mm}^2$ respectively. Locations of the defect may be classified as inner places and at boundaries. The regular, semi-regular and irregular shapes are located

at the inner places. Moreover, highlighted defect at the boundary increases the complexity of the damage condition relative to loading direction.

An important impact of the defect distributions is the change of initial identical stiffness E_x and E_y as shown in Fig. 1(d) to anisotropic situation shown in Fig. 7 (b) and (d). This is because the identical defect has different deformation mechanism with respects to relative loading orientation, i.e. perpendicular and parallel loading directions. Fig. 7 (b) shows 43% reduction of elastic strain due to defects in X-direction loading, in compared to intact model shown in Fig. 2 (c). Unlike the negative Poisson's ratio trend of the intact model, the defect model

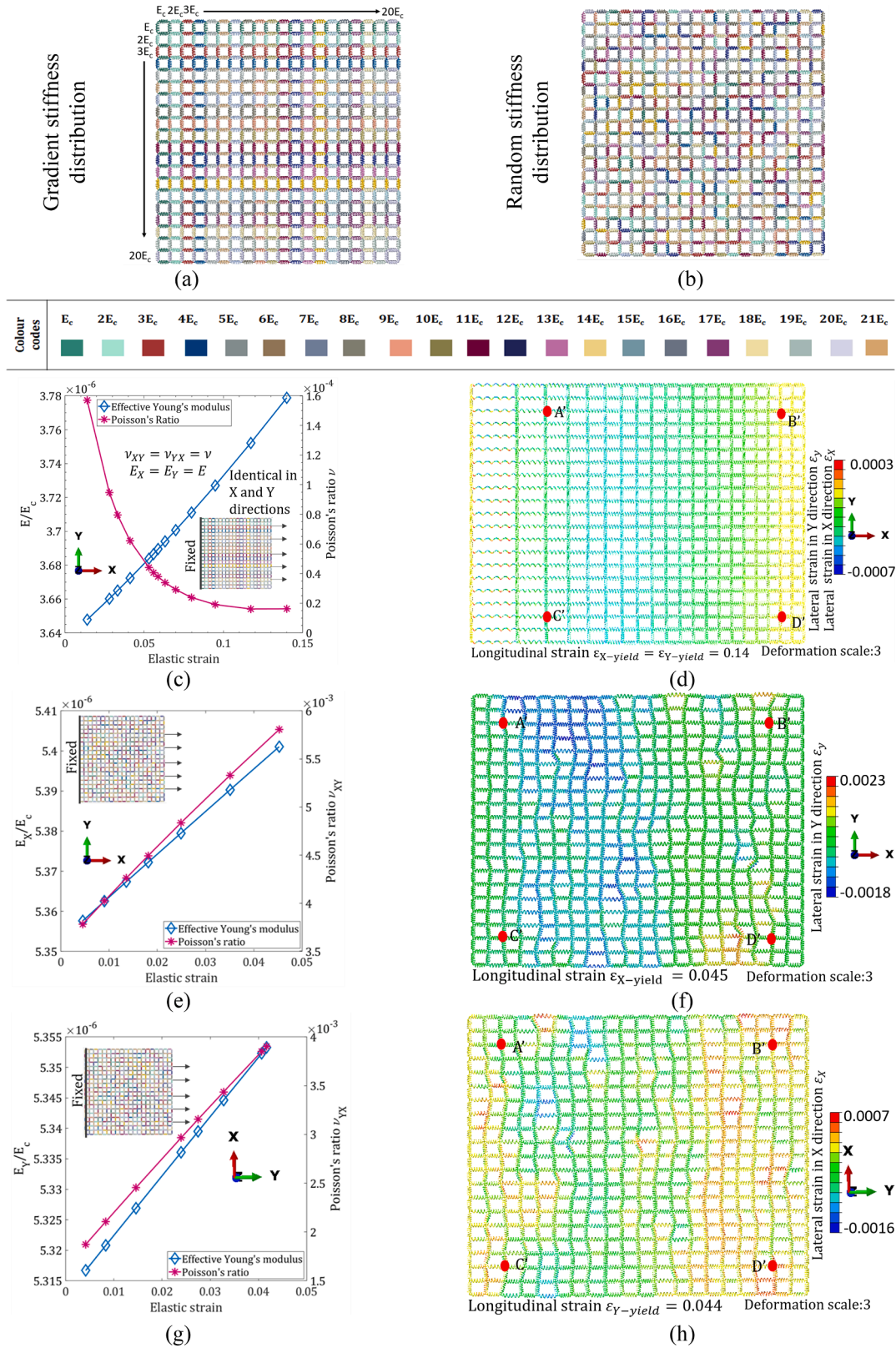
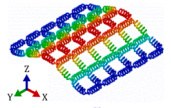
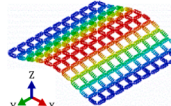
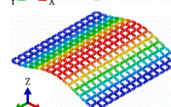
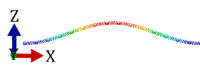
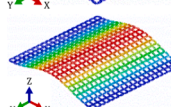
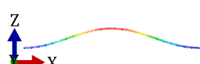
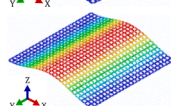
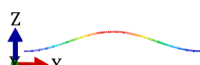
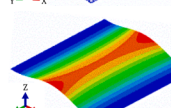


Fig. 8. Young's modulus and Poisson's ratio for tessellations with variable stiffness distributions (a) gradient stiffness distribution, (b) random stiffness distribution (c) Poisson's ratio and Young's modulus for gradient stiffness distribution in two directions, (d) tensile deformation for gradient stiffness distribution in two directions, (e) Poisson's ratio and Young's modulus for random stiffness distribution in X direction, (f) tensile deformation for random stiffness distribution in X direction, (g) Poisson's ratio and Young's modulus for random stiffness distribution in Y direction, (h) tensile deformation for random stiffness distribution in Y direction.

Table 2

Critical buckling load and mode shapes for different tessellations and equivalent shell model.

Tessellation	Critical Buckling load (N)	Normalised critical load to weight	Buckling mode shape	Symmetry	Side view
5*5	5.32×10^{-2}	6.07		Yes	
10*10	2.47×10^{-2}	7.86×10^{-1}		Yes	
15*15	1.60×10^{-2}	5.09×10^{-1}		Yes	
20*20	1.18×10^{-2}	9.83×10^{-2}		Yes	
25*25	9.36×10^{-3}	4.97×10^{-2}		Yes	
Equivalent shell model	2.31×10^{-5}	7.57×10^{-3}		Yes	

shows increasing positive trend to a converged value of 0.028, which is 35.5 times larger than the intact model Poisson's ratio. Although the normalised Young's modulus has a similar nonlinear increasing trend, the stiffness reduce 7% compared to intact model. Fig. 7(c) shows the position of maximum yield stress with a value of 14 MPa at highlighted irregular defect. The major deformation mechanism is in-plane shear as the loading is parallel to the defects orientations, for example, the irregular defect has deformed into '8' shape and the edge defect has stretched due to shear.

Fig. 7(d) illustrates 45% reduction of elastic strain due to defects in Y-direction loading, compared to intact model in Fig. 2(c). Unlike the negative Poisson's ratio trend of the intact model, the defect model shows decreasing positive trend to a value of 0.032, which is 45.8 times larger than the intact model Poisson's ratio. Although the normalised Young's modulus has a similar nonlinear increasing trend, the stiffness reduce 6% compared to intact model. Fig. 7(d) presents the position of maximum yield stress with a value of 14 MPa at highlighted edge defect. The major deformation mechanism is opening mode as the loading is perpendicular to the defect orientations, for example, the edge defect has opened significantly due to tensile stress normal to defect.

Furthermore, for both tension in X and Y directions, the regular defects and semi-regular defects deform to oval shapes, where the width of defects reduces, and the length increases parallel to the loading direction.

8. Zero Poisson's ratio behaviour with variable stiffness distributions

ZPR metamaterial with variable stiffness can be used in designing and tailoring effective elastic modulus through continuous carbon fibres using 3D printing [26]. To demonstrate the zero Poisson's ratio behaviour with variable stiffness distributions, 21 Young's modulus values with identical yield points are allocated in various locations for the spring elements on tessellation 20*20 to form 'gradient stiffness distribution' and 'random stiffness distribution' shown in Fig. 8(a) and (b), respectively. Although the elastic strain is also affected by the yield point of each material in realistic cases, study of plasticity in different

materials is beyond the scope of this paper.

The stiffness of a single spring can be modified by the geometry of the spring element, cross-section thickness and changing material. Considering recent progress in additive manufacturing technology, it is highly feasible to fabricate hybrid structures by Selective Laser Melting (SLM) 3D printing technology with various material power blended at different weight ratios [27] in a small scale as well as multi material welding [28] with different metallic materials in large scale.

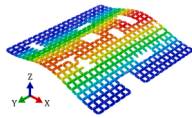
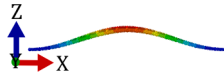
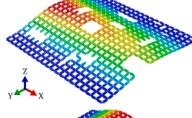
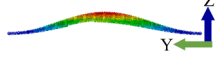
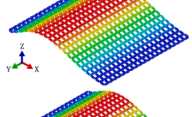
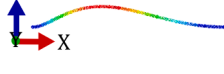
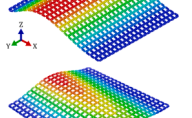
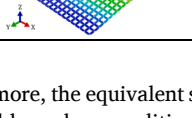
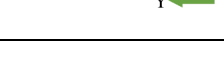
While Fig. 8(a) shows gradient distributions with identical stiffness, i.e. $E_X = E_Y$ and Poisson's ratio $\nu_{XY} = \nu_{YX}$, the random stiffness distribution result in anisotropic behaviour as shown in Fig. 8(b). The random stiffness distribution of Young's Modulus in the range of [1–21] *Es is generated by MATLAB function 'randi' for uniformly distributed pseudorandom integers in two directions. Furthermore, Python script was written by MATLAB to automatically assign massive numbers of the materials to the appropriate spring elements in ABAQUS.

Fig. 8(c) shows 5.71 times reduction of elastic strain due to stiffness increase in X and Y direction loading, in comparison to the original model shown in Fig. 2(c). Unlike the negative Poisson's ratio trend of the original model, the gradient stiffness distribution model shows a reducing positive trend to a converged value of 1.62×10^{-5} , which is 10.57 times smaller than the original model Poisson's ratio. Although the normalised Young's modulus has a similar nonlinear increasing trend, the stiffness increased 5.7 times compared to original model. Fig. 8(d) illustrates the deformation mechanism with uneven displacements for unit cells in loading direction, where the unit cells with smaller stiffness have larger displacements. Moreover, the lateral distances between points 'AC' slightly increases because they are in softer zone, while the lateral distance between points 'BD' reduces because they are in stiffer zone. Therefore, the average Poisson's ratio stays positive between measuring points ABCD.

Fig. 8(e) shows 17.8 times reduction of elastic strain due to random increase of stiffness in X direction loading, compared to original model in Fig. 2(c). Unlike the negative Poisson's ratio trend of the original model, the random stiffness distribution model shows increasing positive trend with a maximum value of 5.8×10^{-3} , which is 36.9 times larger than the original model Poisson's ratio. Although the normalised

Table 3

Critical buckling load and mode shapes for 10% critical defect distribution and variable stiffness distributions of tessellation 20*20.

	Critical Buckling load (N)	Normalised critical load to weight	Buckling mode shape	Symmetry	Side view
10% critical defect distribution	$F_{cr-loadXdirection} = 9.75 \times 10^{-3}$	9.05×10^{-2}		No	
	$F_{cr-loadYdirection} = 9.56 \times 10^{-3}$	8.88×10^{-2}		No	
Gradient stiffness distribution	$F_{cr-loadXdirection} = F_{cr-loadYdirection} = 7.85 \times 10^{-2}$	6.54×10^{-1}		Yes	
Random stiffness distribution	$F_{cr-loadXdirection} = 1.16 \times 10^{-1}$	9.7×10^{-1}		No	
	$F_{cr-loadYdirection} = 1.14 \times 10^{-1}$	9.5×10^{-1}		No	

Young's modulus has a similar nonlinear increasing trend, the stiffness increased 1.84 times compared to original model. Fig. 8(f) with three times graphical scale factor shows the deformation mechanism with distorted spring elements due to random stiffness distribution in X direction loading. Furthermore, the random deformation for unit cells leads to shrinking of the distance between AC, and expansion for BD, but Poisson's ratio has an overall positive value by averaging the displacement between measuring points ABCD.

Fig. 8 (g) presents 18.2 times reduction of elastic strain due to random increase of stiffness in Y direction loading compared to original model in Fig. 2 (c). Unlike the negative Poisson's ratio trend of the original model, the random stiffness distribution model shows increasing positive trend with a maximum value of 3.90×10^{-3} , which is 25.01 times larger than the original model Poisson's ratio. Although the normalised Young's modulus has a similar nonlinear increasing trend, the stiffness increased 1.78 times compared to original model. Fig. 8(h) with three times graphical scale factor shows the deformation mechanism with distorted spring elements due to random stiffness distribution in Y direction loading. Moreover, the lateral displacement between 'AC' and 'BD' are different, however, the tessellation shows an overall positive Poisson's ratio by averaging measuring points ABCD. These simulations demonstrate that even with significant variable stiffness distributions, the tessellations maintain ZPR behaviour under smaller elastic strain range.

9. Eigenvalue buckling analysis

Eigenvalue buckling analysis is essential for the limitation study of the applications [13]. Some Poisson's ratio metamaterials such as self-recovering and multi-stable achieve the desired deformation by designing the buckling shape [29–32]. Thus, buckling analysis of metamaterial is investigated in this section. The effect of tessellation numbers on the buckling characteristics of these structures is also investigated. Numerical analysis for eigenvalue buckling behaviour of metamaterials uses beam elements to minimise computational complexity with tessellation numbers 5*5, 10*10, 15*15, 20*20 and 25*25. One end of the metamaterial is entirely constrained and the opposite end of the material is subjected to 1 N compression force. Moreover, the reliability of buckling analysis using FEA analysis with beam element was approved in the authors previous work [14].

Furthermore, the equivalent shell for tessellation 25*25 is analysed with identical boundary conditions, mass and dimensions, but with reduced equivalent shell thickness of 0.181 mm.

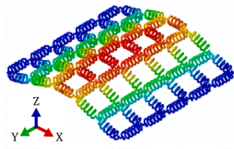
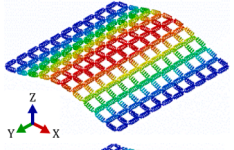
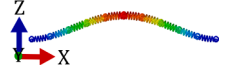
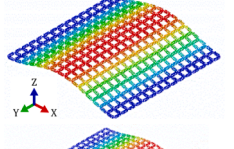
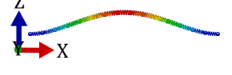
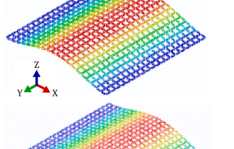
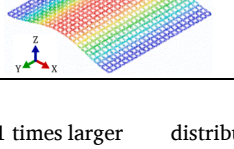
Table 2 shows the first buckling loads and mode shapes for metamaterial with different tessellations and the equivalent shell model. The buckling loads show a nonlinear decrease as tessellation numbers increase, for example, the buckling load for tessellation 5*5 with a value 5.32×10^{-2} N is 5.7 times larger than tessellations 25*25 with the value of 9.36×10^{-3} N. This is comparable to the weight ratio of 21.6 for these two structures. For better inception, the buckling loads are normalised to the weight of each tessellation model. The trend also has a nonlinear decreasing behaviour, for instance, tessellation number 25*25 with the normalized ratio of 4.97×10^{-2} is 122 times smaller than the tessellation number 5*5 with a normalised ratio of 6.07. Arch shapes are formed for all tessellations buckling modes demonstrating out of plane bending at the middle. Moreover, the critical buckling mode for metamaterial skins with all tessellations are symmetric from X and Y axis. Similar to the buckling modes of 2D Fish cell metamaterial skin [13], the free edges of the spring cell tessellations exhibit monoclastic behaviour due to ZPR characteristics.

The magnitude of buckling loads for the equivalent shell is 2.31×10^{-5} N, which is about 405 times smaller than the metamaterial skin with tessellation 25*25. Moreover, the normalised critical load to weight for the equivalent shell has a value of 7.57×10^{-3} , which is 6.6 times smaller than tessellations 25*25. The buckling mode shapes for equivalent shell in Table 2 illustrates out of plane bending at the middle of the shell and the hump form into a hourglass shape. Unlike ZPR tessellation has single curvature, shell planer has a double curvature due to the anticlastic behaviour.

Table 3 shows the buckling loads and a detailed description of mode shapes for the structural defect and variable stiffness models with tessellations 20*20. The gradient distributions with identical stiffness, i.e. $F_{cr-loadYdirection} = F_{cr-loadXdirection}$, the random stiffness distribution and critical defect distribution result in anisotropic behaviour $F_{cr-loadYdirection} \neq F_{cr-loadXdirection}$. The critical buckling load for the structural defect model with compression in X direction $F_{cr-loadXdirection} = 9.75 \times 10^{-3}$ N, which is 1.01 times larger than in the Y direction with a value $F_{cr-loadYdirection} = 9.56 \times 10^{-3}$ N. Thus, the critical buckling load for the structural defect model with compression in X and Y directions are 1.21 and 1.23 times less than the original model, respectively. Similarly,

Table 4

The first frequency mode and magnitude for different tessellations.

Tessellations	1st Frequencies (Hz)	First mode shapes	Symmetric	Cross-section
5*5	3.27		Yes	
10*10	0.80		Yes	
15*15	0.35		Yes	
20*20	0.19		Yes	
25*25	0.13		Yes	

the normalised load to weight in the X direction is also 1.01 times larger than in the Y direction and 0.92 times of the original model. The critical buckling load for the variable stiffness model with gradient stiffness distribution has critical buckling load in X and Y directions with value 7.85×10^{-2} N is 6.65 times larger than the original model with tessellation 20*20. Also, the normalised load to weight ratio is 6.65 times larger than the original model due to higher Young's modulus. Moreover, the critical buckling load for the variable stiffness model with random stiffness distribution in the X direction with value $F_{cr-loadXdirection} = 11.6 \times 10^{-2}$ which is 1.03 times bigger than in Y direction with a value $F_{cr-loadYdirection} = 11.4 \times 10^{-2}$. The critical buckling loads for the random stiffness distribution model in the X and Y direction are 9.83 and 9.86 times larger than the original model. The ratio for the normalised load to weight in the X direction is 1.03 times larger than in Y direction and 9.87 times larger than original model.

Moreover, the critical buckling modes for all models except the variable stiffness model with gradient stiffness distribution are asymmetric from the X and Y axis. All models have hump shape formed; however, most of them are irregular. For instance, the humps have gradient height for the structural defect model and random variable

distribution model with compression in the Y direction. Furthermore, the humps switched closer to the softer side of the tessellation, such as in the gradient variable stiffness model. Moreover, all models exhibit monoclastic behaviour due to their ZPR characteristics.

10. Modal analysis

Structural vibration techniques are essential due to their domination in application quality, personal health, product lifetime and operating accuracy [33]. Thus, modal analysis is presenting in this section to reveal the fundamental vibration characteristics of metamaterial skin with spring cell. The tessellation numbers for the frequency analysis are set to be identical with those in buckling analysis. Eigenvalue frequency analysis is performed by FEA simulations with beam element and fixed-fixed boundary conditions. The free vibration behaviour of the tessellation 25*25 is compared with equivalent shell cylinders with identical dimensions and mass with modified density 167.8 kg/m^3 .

Table 4 shows the first frequencies and detailed description of mode shapes for different tessellations of metamaterial skin. The trend of the frequencies shows a nonlinear decrease as tessellation numbers increase,

Table 5

Comparison of first five frequencies and mode shapes between tessellation 25*25 and equivalent shell.

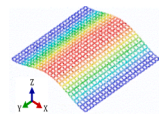
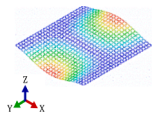
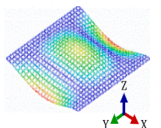
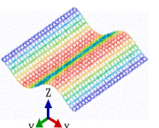
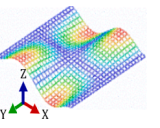
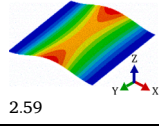
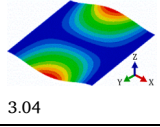
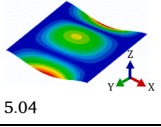
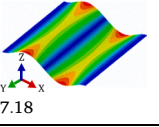
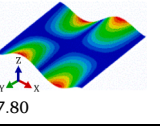
	1st Frequency (Hz)	2nd frequency (Hz)	3rd frequency (Hz)	4th frequency (Hz)	5th frequency (Hz)
Tessellation 25*25					
Equivalent shell	0.13	0.16	0.26	0.34	0.39
					
	2.59	3.04	5.04	7.18	7.80

Table 6

First five frequencies and mode shapes for 10% critical defect distribution and different variable stiffness distributions.

	Structural defect distribution		Variable stiffness distributions	
	10% critical defect		Gradient stiffness distribution	Random stiffness distribution
Mode shape 1				
1st Frequency (Hz)	$f_x = 0.19$	$f_y = 0.19$	$f_x = f_y = 0.52$	$f_x = 0.62$
Mode shape 2				
2nd Frequency (Hz)	$f_x = 0.25$	$f_y = 0.24$	$f_x = f_y = 0.69$	$f_x = 0.80$
Mode shape 3				
3rd Frequency (Hz)	$f_x = 0.39$	$f_y = 0.39$	$f_x = f_y = 1.17$	$f_x = 1.30$
Mode shape 4				
4th Frequency (Hz)	$f_x = 0.52$	$f_y = 0.53$	$f_x = f_y = 1.47$	$f_x = 1.75$
Mode shape 5				
5th Frequency (Hz)	$f_x = 0.6$	$f_y = 0.59$	$f_x = f_y = 1.71$	$f_x = 1.98$

for instance, the frequency for tessellations 5*5 with a value 3.27 Hz is 25.1 times larger than tessellations 25*25 with a value of 0.13 Hz. The vibration mode shapes for all tessellations have a bump formed at the middle of the metamaterial skin. For tessellation number 5*5, the highest deformation is located at the centre of the hump. However, for the rest of the tessellations, the highest deformation area is throughout the middle of the hump. Moreover, the frequency modes for all tessellations are symmetric through the X and Y axis.

Table 5 compares the first five frequencies and mode shapes between tessellation 25*25 and equivalent shell to study the effect of spring cell architecture on free vibration. Frequencies for tessellations are approximately 20 times smaller than the equivalent shell model. However, the increment of the frequencies for tessellation is almost the same as equivalent shell. For example, the fifth frequency is three and 3.1 times larger than the first frequency for the tessellation 25*25 and equivalent shell, respectively. Also, the increment trend for frequency of tessellation and equivalent shell models are very similar. The magnitude increased 1.23 and 1.17 times for tessellation and equivalent shell models for first to second frequencies, respectively. From second to third frequencies, the frequency increases 1.63 times for the tessellation and 1.65 times for the equivalent shell. Moreover, from fourth to third, the frequency increases 1.31 and 1.42 times for tessellation and equivalent shell, respectively. Finally, the fifth frequency is 1.15 and 0.99 times larger than the fourth frequency for the tessellation and equivalent shell model.

All mode shapes for metamaterial and equivalent shell are identical, excluding the first and fourth model shapes. The first mode shape for the equivalent shell shows synclastic behaviour at the free edges of the bump. However, the first mode shape for tessellation 25*25 exhibits monoclastic behaviour when the bump forms. The second mode shape shows a hump and depression on both sides for the metamaterial and equivalent shell models. The third mode shapes for both modes have depressions on both sides and a hump at the middle of the planer. The general mode shapes for both models at the fourth frequency are formed into a wave shape. However, the free edges for the metamaterial are flat

and smooth, while the free edges for the equivalent shell model have small depressions. The fifth mode shapes for both models are identical, they formed into a wave shape and have depressions on the free edges.

Table 6 presents the modal analysis with the first five frequencies and mode shapes for structural defect and variable stiffness models. The boundary condition for frequency analysis considers fixed-fixed with edges perpendicular to X axis (f_x) and Y (f_y) axis for structural defect model and gradient variable stiffness model. The first frequencies for structural defect model are identical with original tessellation 20*20, while variable stiffness model with gradient and random stiffness distribution are approximately 2.7 and 3.3 times larger than original model with tessellation 20*20. The increments of the frequencies for structural defect model with different fixed edges is very similar. For example, the fifth frequencies are 3.16 and 3.11 times larger than the first frequencies for the structural defect model with fixed-fixed edges perpendicular to the X-axis and Y-axis. Moreover, the increment of the frequencies for the variable stiffness model with random stiffness distribution with different fixed edges is different. For example, the fifth frequency is 3.19 times larger than the first frequency for fixed-fixed edges perpendicular to X axis, for fixed-fixed edges perpendicular to Y axis, the fifth frequency is 3.11 times larger than the first frequency.

The increment trend for frequency of structural defect model with fixed edges perpendicular to X and Y axis are nonlinear. The magnitude increased 1.26 and 1.32 times from first to second frequencies with fixed edges perpendicular to the X axis and Y axis fixed, respectively. From second to third frequencies, the frequency increases 3.72 and 3.88 times for the structural defect model with fixed edges perpendicular to the X and Y axes. Moreover, from fourth to third, the frequency magnitude increased 1.33 and 1.36 times for the structural defect model with fixed edges perpendicular with X axis and perpendicular to Y axis, respectively. Finally, the fifth frequency is 1.15 and 1.11 times larger than the fourth frequencies for the structural defect model with fixed edges perpendicular to X and Y axis.

The increment trends for frequency of variable stiffness models with gradient and random distributions are nonlinear and dissimilar with

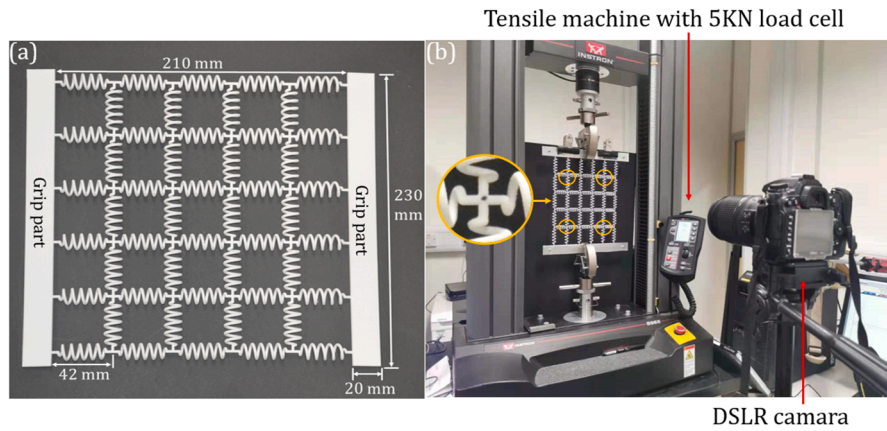


Fig. 9. Experimental set up with measuring points (a) 3D printed 5*5 tessellation with highlighted dimensions (b) tensile experiment apparatus and data acquisition system.

structural defect model. The magnitude from first to second frequencies increased 1.33 times for the gradient stiffness distribution, while the random stiffness distribution increased 1.29 and 1.27 times with fixed edges perpendicular to X and Y axis. For second to third frequencies, gradient stiffness distribution increased 1.70 times and random stiffness distribution increased 1.63 times for both boundary conditions. The difference of the magnitude for fourth and third is 1.26 times for frequencies gradient stiffness distribution while for random stiffness distribution is 1.15 and 1.29 times with fixed edges perpendicular to X and Y axis. Moreover, from fourth to fifth, gradient stiffness distribution increased 1.16 times which is identical to the random stiffness distribution with fixed edges perpendicular to the Y axis, while the frequency increased 1.13 times with fixed edges perpendicular to the X axis.

The overall mode shapes for the structural defect and variable stiffness models are very similar to the mode shapes with constant stiffness shown in Table 6. However, the hump length and shape are affected by the defects and stiffness of the spring elements. For example, the width of the hump is not constant in the first frequency mode shape for the structural defect model and the random stiffness distribution model with fixed edges perpendicular to X axis. Moreover, for the first frequency mode shape, the location of the hump is shifting closer to cells with lower Young's modulus. For the second frequency mode shape, the height of the hump and depression on the free edges are different for the structural defect model with fixed edges perpendicular to the X axis and gradient stiffness model. In fourth frequency mode shapes, the humps and depressions have wave shape for all models except the gradient stiffness distribution model. The area and the displacement for all depressions and humps are different for all models in fifth frequency mode shapes.

11. Tensile experimental analysis

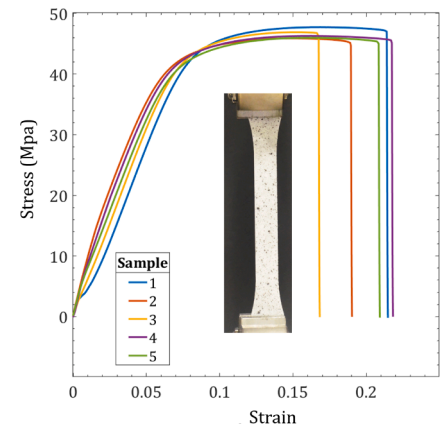
11.1. Additive manufacturing method

The complexity of curvature for the small-scale spring metamaterial with homogenised tessellation and minimised boundary condition effects challenges the manufacturing. Selective Laser Sintering (SLS) technology is applied for 3D printing of the metamaterial with accomplished surface finish and extremely low tolerance. The main limitation of additive manufacturing is the minimum achievable thickness of the cross-section radius for the cell and the size of printing area. Moreover, Molten Polymer Deposition (MPD) may not be suitable for the 3D Spring metamaterial due to the massive numbers of support needed to generate, causing a coarse finish and uneven thickness. Considering these limitations and the tensile test area, the 3D Spring Symmetric model cannot manufacture due to the gap between the coils at the cell corners being too small. However, a 3D Spring Asymmetric model with tessellation

Table 7

Material elastic properties and plasticity for the 3D printed dog bone samples.

Elastic material property	
Young's modulus	756.175
E (MPa)	
Poisson's ratio	0.4
Plastic material property	
Strain	Stress (MPa)
0	20.6587
2.2099×10^{-5}	20.76372
0.00265	29.27555
0.00868	37.78737
0.12931	45.64755



5*5 was manufactured with a spring cross-section 2 mm as illustrated in Fig. 9.

It is feasible to fabricate helical structures in micro/nano scale. A micro coil array with coil diameter of 100 μm and the wire diameter 25 μm made from UV curable resin were fabricated by high-resolution projection micro-stereolithography system (P μ SL)[34]. Moreover, both linear and nonlinear helical structures were successfully manufactured on micro scale using a solvent-cast direct-write (SC-DW) fabrication method using PLA[35]. However, considering the structural test procedure and equipment, the 3D printing sample for spring cell metamaterial maintains a normal scale.

11.2. Material characteristics

The material properties of the constructing material PA12 needs to be examined to validate the numerical models. For this purpose, five 3D printed test specimens under ASTM D638-14 Type 2 standard were tested by tensile experiment to assess the mechanical properties. The average material properties for PA12 from the tensile test are listed in Table 7.

11.3. Tensile structural tests

Tensile experiments evaluate the Poisson's ratio and the stiffness of 3D printed models. Grip parts on the sample, as shown in Fig. 9 (a) are designed on both sides of the model for installation on aluminium plates and a tensile machine to simulate the boundary conditions shown in

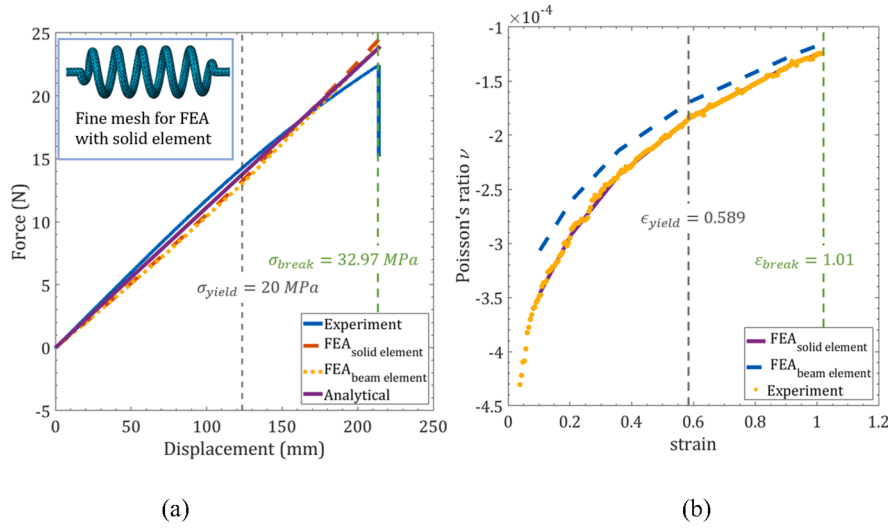


Fig. 10. Comparison of experimental, numerical and analytical analysis (a) force-displacement behaviour, (b) Poisson's ratio behaviour as a function of strain.

Fig. 1(d). The specially designed aluminium plates are installed on the model to ensure the tensile force is evenly distributed on the grip parts. The models are installed on a tensile machine as shown in Fig. 9 (b). Digital Single Lens Reflex (DSLR) camera Nikon D7000 is used for capturing the mechanical behaviour of the structure every 30 s to measure the transverse displacements for the calculation of Poisson's ratio. It should be mentioned that the cross-section of the model is too thin to recognise by Digital Image Correlation (DIC) technique. The DSLR camera lens is placed horizontally to the centre of the model to ensure the ratio of the length and pixels are correct. The tension is applied at a 2 mm/min displacement rate to simulate a static tensile test scenario. Poisson's ratio is measured based on images taken by a DSLR camera, while a tensile machine fulfils Force-Displacement. The displacement in the transverse direction is calculated and collected based on pixel in the software 'Plot Digitiser', where the ratio between the length and pixel is calculated.

11.4. Experimental results validation and discussion

A Poisson's ratio validation study between experiment and

numerical models is presented in this section to determine the reliability of numerical models. 3D continuum quadratic solid elements C3D10 with material property list in Table 7 are used for numerical analysis in ABAQUS. Mesh convergence studies are performed, and a fine mesh with a global size of 0.5 mm is tested and used for discretisation. To track the position of measuring points, A, B, C and D are painted on the model using a marker pen with reverse colour highlighted by orange circles in Fig. 9 (b). The position of Points A, B, C and D in experimental results is determined by 'Plot Digitizer' software, which converts the pixel of the experimental images into mathematical coordinates.

Fig. 10 (a) present tensile force-displacement relations for 3D printed 3D Spring Asymmetric model with tessellation 5×5 . The results show that the experimental model reached a yield point of 123.7 mm with a tensile strain of 58.9% and experienced structural failure at 213 mm with a strain of 1.01. The experimental results are validated with FEA and analytical results. The FEA with solid elements shows excellent agreement with experimental results with a maximum error rate of around 6.5% at a displacement 213 mm. Furthermore, FEA results with beam element have a maximum error rate of approximately 6.0% at displacement 213 mm compared to the experiment model. The

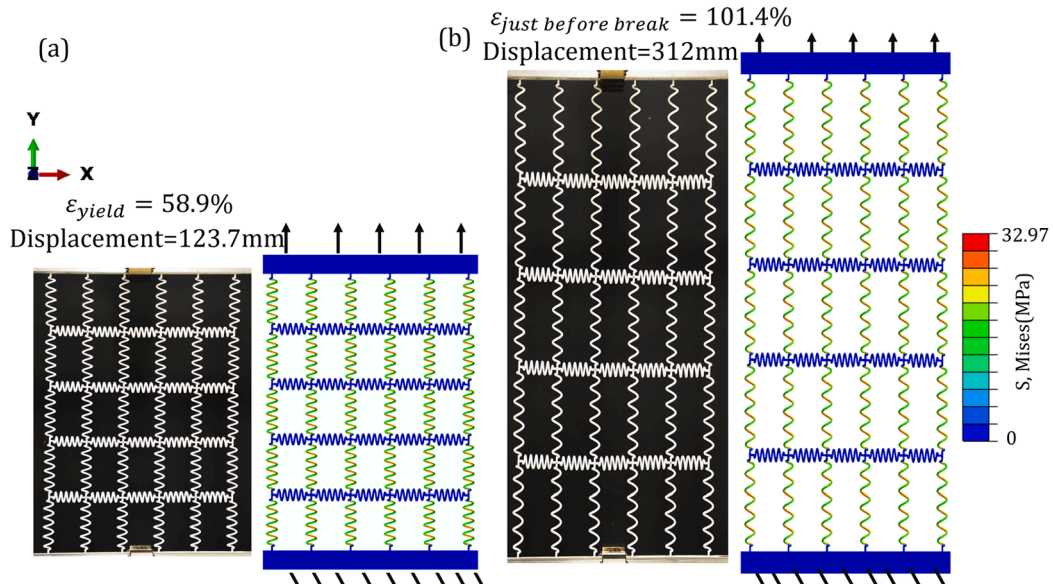


Fig. 11. Validation of deformation mechanism in experiment and simulation (a) tensile displacement at yield (b) tensile displacement at failure.

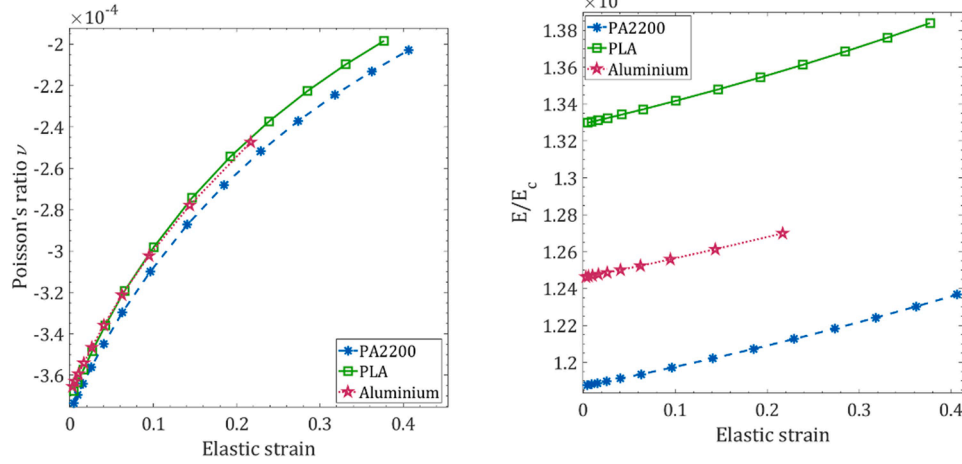


Fig. 12. Different construction material behaviours of experiment tessellation as function of strain (a) Poisson's ratio analysis, (b) Normalised Young's modulus behaviour.

maximum error rate between FEA with solid and beam elements is around 2% at a displacement 213 mm. Moreover, the error rate between the analytical and experimental results increases with displacement growth, thus, the maximum error rate 5.8% at a displacement 213 mm.

Fig. 10 (b) shows Poisson's ratio as a function of tensile strain for the 3D Spring Asymmetric model. The experimental and FEA results with solid elements exhibit an excellent correlation for Poisson's ratio values. The maximum error rate between FEA with solid element and experiment is 2.7% at tensile strain 0.8. The Poisson's ratio result for simulation with beam element shows a maximum 11.4% error with FEA with solid element and experiment results at strain 0.1. However, the error is reduced with the tensile increases, and the Poisson's ratio trends between FEA with beam element, solid element and experiment are very similar. Chen and Naghavi Zadeh [12,36] also reported that the results from FEA with beam and solid elements are different. However, the simulation by FEA with beam elements with less complexity shows the reliability for predicting the results.

11.5. Deformation mechanism validation

The deformation mechanism of metamaterial skin formed by 3D asymmetric spring cell made from PA12 is compared between experimental and numerical models to reveal the reliability of numerical simulations in a detailed study of the structural deformations. As previously mentioned, the experimental and numerical models use identical data for the characteristics of tensile displacement and solid elements. The FEA simulation in Fig. 11 (a) and (b) with the solid element has the exact deformation mechanism as the FEA with the beam element listed in Section 2.4. Both simulation and experimental results show shear behaviour on the XY plane, however, the shear behaviour is more observable in the experimental sample. Furthermore, the joints which formed by straight beams have a chiral shape that causes the joints rotate under the tensile deformation. It must emphasise that the centre of the joints where the measuring points are located slightly moves after the spring has tensile deformation. Furthermore, the diameter of the vertical 3D spring elements reduces with the tensile displacement increase. This deformation is stiffening the metamaterial with tensile displacement increases. Furthermore, the FEA analysis at yield strain in Fig. 11 (a) and strain just before break in Fig. 11 (b) shows the maximum stress distributed at the centre of the inner surface of the coil. Moreover, the design has no stress concentration at joints and is evenly distributed through the coils.

Fig. 11 (b) presents similar fracture locations with maximum stress and identical deformation mechanism in experiment and simulation. Furthermore, the fracture mechanism study is constrictive material

dependent. Thus, further research might be performed with metallic, composites and elastomer materials. This is beyond the scope of this paper as the focus is only on metamaterial architecture.

12. Construction material effect

Constructive material is one of the decisive factors for metamaterial properties such as Poisson's ratio and elastic strain range. FEA with beam element has been performed on the tessellation model with specifications identical to the experimental model listed in Fig. 9 (a). Most of all, PA2200, where the material properties listed in section 2, Polylactic Acid PLA and Aluminium with the material property listed in work from Guglielmo et al. [17] are selected as construction materials.

Fig. 12(a) demonstrates the similarity of Poisson's ratio for all investigated construction materials. The maximum Poisson's ratio for tessellation with aluminium is -2.4×10^{-4} at elastic strain 0.22, which is 4.16% and 2.8% difference from PA2200 and PLA at identical elastic ranges. Moreover, the maximum Poisson's ratio for PA2200 and PLA is -2.0×10^{-4} and -1.9×10^{-4} respectively. The yield stress limits the maximum elastic strain for the construction material property, thus, aluminium has a maximum value of 0.22 while PLA and PA2200 have 0.38 and 0.41, respectively.

Fig. 12 (b) shows plots of normalised Young's modulus vs elastic strain for the three construction materials obtained. The maximum normalised Young's modulus for tessellation with aluminium is 1.27×10^{-5} at elastic strain 0.22, which is 4.4% and 6.9% difference from PA2200 and PLA at identical elastic ranges. Moreover, the maximum normalised Young's modulus for PA2200 and PLA are 1.24×10^{-5} and 1.38×10^{-5} respectively. In particular, the Young's modulus of each case has been normalized to the Young's modulus of the construction materials to confirm the similarities between different materials. Furthermore, the similar Poisson's ratio behaviour of PA2200, PLA and Aluminium as construction material for identical tessellation model is evidenced. Although tessellation models present identical geometric characteristics for varying material properties, the Young's modulus values at yielding are generally different.

Moreover, the elastic strain range can be improved by applying different materials to metamaterial or spring elements. Thus, an optimisation study for construction material is beneficial for further study. However, it is beyond the scope of this paper.

13. Conclusion

This paper introduced novel metamaterial skins formed by 3D spring cells exhibiting zero Poisson's ratio under large elastic tensile strain with

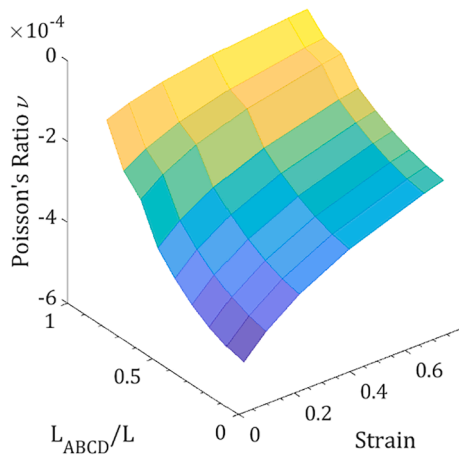


Fig. A1. Effect of different locations of ABCD measuring points on Poisson's ratio as a function of strain.

Table B1

Material properties of construction material Nylon PA2200 [14].

Material Elasticity	
Young's modulus (E)	1130 MPa
Poisson's ratio (ν)	0.4
Material Plasticity	
Yield Stress (MPa)	Plastic Strain
14.0	0.0000
34.1	0.0025
42.7	0.0500
46.7	0.0750
49.0	0.1000
50.5	0.1250
51.3	0.1500
51.6	0.1750

no stress concentration on joints in two directions. Comparing with other designs such as semi re-entrant and Fish-Cell shapes, this novel metamaterial is able to achieve identical mechanical behaviour in two directions and stiffness is variable with tensile strain changes. Compelling numbers of FEA simulations with fine mesh were performed on high-performance computing clusters for different tessellations for all models to investigate Poisson's ratio and Young's modulus. Finite element simulations with beam elements illustrate the homogeneity and ideal value of Poisson's ratio and Young's modulus for constant size ratio with different tessellation numbers, while FEA models with solid elements present the reliability for deformation mechanisms in detail. Moreover, an analytical method was used to estimate Young's modulus,

reaction force for the metamaterial. Parametric study for effect on normalised Young's modulus and Poisson's ratio was also based on analytical method. The pure shear behaviour obtained by FEA analysis demonstrate the low shear modulus, but the shear stiffness can be increased by changing the construction material and use as core for sandwich structure. The structural analysis obtained by FEA analysis shows that the metamaterial can maintain ZPR behaviour with 10% defects under elastic tensile strain. Variable stiffness study on the metamaterial was investigated with gradient and random stiffness distributions. Furthermore, buckling and modal behaviours of different tessellations with asymmetric 3D spring elements were studied and compared with equivalent shell, illustrating a lower range of buckling load than equivalent skin due to the lower stiffness of the metamaterials. The unique cellular shape and tessellation homogeneity lead to similar buckling and vibration mode shapes with equivalent shells. Moreover, the buckling and modal behaviour for the structural defects model and variable stiffness models were investigated to show the effect on modes, buckling load and frequency. Tensile experiments were performed to validate Poisson's ratio and deformation mechanism between the experiment and numerical models. Finally, a construction material analysis showed the Poisson's ratio independent from the construction material and the effect on normalised Young's modulus.

This low stiffness spring cell metamaterial capable maintain ZPR behaviour under large tensile strain in two directions is beneficial for large strain morphing application such as morphing wing to gain more morphing range without structural failure.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

CRediT authorship contribution statement

Qing Qin: Conceptualization, Software, Validation, Formal analysis, Writing – original draft, Writing – review & editing. **Iman Dayyani:** Conceptualization, Writing – review & editing, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix A

The boundary condition limits the deformation of some unit cells, affecting the Poisson's ratio. Thus, nonlinear analysis of Poisson's ratio is studied with constant boundary conditions but changing the location of the measuring point. Measuring points are putted in different locations to assess the sensitivity of the Poisson's ratio. It must be emphasised that the measure points ABC and D formed into a square shape, and the centre coincides with the centre of the metamaterial. The location and the length of the measuring points A,B,C and D can be fined by L_{ABCD}/L where L_{ABCD} is the length between AB, AC, BC and CD. Thus, the value for L_{ABCD}/L is from 0.1 to 0.9, where 0.1 locate close to the centre of the metamaterial and 0.9 locate near the boundary. The results in Figure A indicate that the Poisson's ratio increases with the elastic tensile strain increase when the location of measuring points remains the same. Furthermore, Poisson's ratio behaviour is more stable when measuring points are closer to the boundary. When the tensile strain is the same, it can be observed that the magnitude of Poisson's ratio increases with measuring points closer to the centre of the metamaterial.

See Fig. A1.

Appendix B

See Table B1.

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Large strain zero Poisson's ratio spring cell metamaterial with critical defect analysis and variable stiffness distributions

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2023-08-15

Elsevier

<https://doi.org/10.1016/j.compstruct.2023.117102>

Qin Q, Dayyani I. (2023) Large strain zero Poisson's ratio spring cell metamaterial with critical defect analysis and variable stiffness distributions. Composite Structures, Volume 318, August 2023, Article number 117102