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Ph.D. THESIS

JAMES D. LANG

The Interaction of an Oscillating Airfoil
and/or Flap with a Separating Flow

Supervisor: Dr. P.A.T. Christopher

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SUMMARY

At present little is known of the dynamics of a separating and reattaching flowfield. This unsteady flow topic relates to a wide range of aerodynamic problems such as dynamic stall and control-surface-buzz.

An experiment was performed with the purpose of investigating the dynamics of a separated region and the effect of the region on the motion of a flap-type control surface. A NACA 0012 airfoil section was fitted with a movable spoiler and flap. Measurements were made of the unsteady, non-linear hinge-moment due to spoiler oscillation. Aerodynamic hinge-moment was also investigated with the spoiler fixed and the flap oscillated harmonically. Flap response to spoiler-induced forcing was also observed in addition to observations of coupled spoiler and flap system behavior. The coupled system exhibited limit-cycle oscillations.

Flap response to spoiler-induced loading can be predicted well by use of a linear equation for flap motion which is forced by the effect of spoiler motion. The equation of motion includes attached flow values of aerodynamic damping and stiffness.

A theory is developed which models separation bubble dynamics. It includes a quasi-steady model for the external shear-layer, and flow of mass within the bubble at a mean reversed-flow velocity. Pressure perturbations along the bubble are modeled also.

The theory is used with the method of Beecham and Titchener in order to predict limit-cycle behavior of the coupled system. Good agreement with experimental results is achieved.

The instant of instability and of growth of a laminar separation bubble at the leading-edge of an oscillating airfoil is also investigated theoretically. The beginning of dynamic stall is seen to be related to the dynamics of a separation bubble and the inviscid flowfield. The theory is applied also to a study of the phenomena of control-surface-buzz, where separation is induced by shock waves.

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NOTATION

a	a constant, e.g., the slope of a linear velocity profile or a non-dimensional pitch-axis location
a_i	coefficient of a polynomial
a_∞	free-stream speed of sound
A	flap system moment of inertia or separation bubble area or a gain factor
AR	aspect ratio of separation bubble, $AR \equiv \frac{\text{bubble span}}{\text{bubble length}}$
b	airfoil semichord, $b \equiv \frac{c}{2}$
b_f	flap span
b_{sp}	spoiler span
B	flap system mechanical damping
c	airfoil chord or non-dimensional flap hinge location
c_f	flap chord
C	flap system structural stiffness
$C(\nu_a)$	Theodorsen function
C_H	non-dimensional hinge-moment coefficient, $C_H \equiv \frac{H}{q_\infty b_f c_f^2}$
C_p	pressure coefficient, $C_p \equiv \frac{p - p_\infty}{q_\infty}$
$C_{H\dot{\beta}}, C_{H\ddot{\beta}}, C_{p\dot{\alpha}}, C_{p\ddot{\alpha}}, C_{p\dot{\beta}}, C_{p\ddot{\beta}}, \text{ etc.}$	non-dimensional derivatives, e.g., $C_{p\ddot{\alpha}} \equiv \frac{\partial C_p}{\partial \alpha''}$, $C_{H\dot{\beta}} \equiv \frac{\partial C_H}{\partial \beta'}$ (Note: $C_{H\dot{\beta}} = -2 k_\beta = 2 \left(\frac{c}{c_f}\right)^2 h_\beta$, $C_{H\ddot{\beta}} = -2 k_\beta' = 2 \left(\frac{c}{c_f}\right)^3 h_\beta'$ where the k 's are AGARD notation and the h 's are conventional English notation based on airfoil chord)

CL_{α}	lift-curve slope
ΔC_H	incremental hinge-moment coefficient
$\Delta C_{H_n}, C_{P_n}$	coefficients of Fourier series
C_{PT_i}	pressure coefficient due to circulation effects
d_i	bubble length versus spoiler height curve slope, $d_i \equiv \frac{\partial x_i}{\partial h}$, $i = b, r$
d_R	slope of curve, $d_R = d_r \left(\frac{c}{c_f}\right)$
h_s, h	actual and non-dimensional spoiler height, $h \equiv \frac{h_s}{c_f}$
$h_{os}, \Delta h_s, h_o, \Delta h$	actual and non-dimensional spoiler parameters, e.g., $h = h_o + \Delta h \cos \omega t = \frac{h_{os}}{c_f} + \frac{\Delta h_s}{c_f} \cos \omega t$
h_{eff}	effective spoiler height
H	hinge-moment or shear-layer shape factor, $H \equiv \frac{\delta^*}{\theta}$
$H_{\beta}, H_{\dot{\beta}}$	dimensional derivatives, e.g., $H_{\dot{\beta}} = \frac{\partial H}{\partial \dot{\beta}}$
ΔH	incremental hinge-moment
$H_i^{(2)}$	Hankel function of the second kind of order i
I	integral related to circulation effects
k	a constant, e.g., a coupling parameter
k_i	a constant
l	total separation bubble length or laminar shear layer length
m	turbulent mixing and reattachment region length

M	Mach number or ratio of unsteady and steady flow circulation effects
N_i	describing function coefficients, $i = 1, 2$
P, P_0	static and total pressure
P_∞	free-stream static pressure
q_n, q_s	normal and tangential components of velocity at bubble boundary
q_∞	free-stream dynamic pressure, $q_\infty = \frac{1}{2} \rho U_\infty^2$
Q	parameter used to evaluate circulation
Re_x	Reynolds number based on distance x (velocity is either U_s or U_∞)
t	time
u	x-component of velocity
u_B	velocity at bubble dividing streamline at station b
U	x-component of velocity at the edge of a shear layer (also U_e)
U_c	apparent convective velocity
U_0	value of U due to mean airfoil incidence
U_∞	free-stream speed
v_e	entrainment velocity
W	rate of energy dissipation within bubble
x	non-dimensional chordwise distance based on airfoil chord or (in section 4) based on semichord with origin at midchord, i.e., $x = \frac{X}{c}$ or $x = \frac{X}{b} - 1$
x_0	location in wake of vortex element
x_f	flap hinge-line location
x_{sh}	shock-wave location
x_{sp}	spoiler location
X	chordwise distance from airfoil leading-edge or (in part of section 3) from spoiler

y_d, y_e, y_z	distance normal to airfoil surface of dividing streamline, edge of shear-layer, and "zero" streamline
α	airfoil angle-of-attack or a phase angle
α_o	airfoil mean incidence angle
$\Delta \alpha$	amplitude of airfoil oscillation
$\Delta \alpha_s$	stall overshoot angle, $\Delta \alpha_s = \alpha_{\text{dynamic stall}} - \alpha_{\text{static stall}}$
β	flap deflection angle
β_n	coefficients of Fourier series
$\Delta \beta$	amplitude of flap oscillation
$\Delta \beta_{LC}$	amplitude of limit-cycle oscillation
$\Delta \beta_s$	stall overshoot angle
γ	density of vorticity or gas constant
Γ	circulation
δ	shear-layer thickness
δ^*	shear-layer displacement thickness
δC_p	difference between pressure coefficients in separated and attached flow
ϵ	error parameter
ζ	damping ratio
θ	shear-layer momentum thickness
θ_n	phase angle in Fourier series for pressure
κ	a constant, $\kappa \equiv \frac{\rho b_f c_f^4}{2A}$
λ	stability parameter, $\lambda = \frac{\Delta \dot{\beta}}{\Delta \beta}$
μ	viscosity
ν	frequency parameter based on flap chord, $\nu \equiv \frac{\omega c_f}{U_\infty}$

ν_a	frequency parameter based on airfoil semi-chord, $\nu_a \equiv \frac{\omega c}{2 U_\infty} = \frac{\omega b}{U_\infty}$
ξ	non-dimensional chordwise distance, $\xi = \frac{X}{c}$
ρ	free-stream density
σ	pressure recovery coefficient or Crabtree's parameter, $\sigma = \frac{C_p - C_{ps}}{1 - C_{ps}}$ or $\sigma = \frac{C_{pr} - C_{ps}}{1 - C_{ps}}$
σ_{CR}	critical value of Crabtree's parameter
$\tau, \tau_a, \tau_o, \tau_\epsilon$	non-dimensional time, i.e., $\tau = \frac{U_\infty t}{c_f}$, $\tau_a = \frac{U_\infty t}{b}$, $\tau_o = \frac{l_o}{2 U_c }$, $\tau_\epsilon = \frac{l - X}{ U_c }$
ϕ	velocity potential or phase angle
ϕ_n	phase angle in Fourier series for flap angle
ψ	phase angle for circulation effects
ψ_n	phase angle in Fourier series for hinge-moment
ω	angular frequency of oscillation
$(\dot{}) =$	$\frac{d()}{dt}$
$(') =$	$\frac{d()}{d\tau}$ or $\frac{d()}{d\tau_a}$
$\langle p \rangle, \langle u \rangle$	RMS values of pressure and velocity fluctuations

superscripts

$\tilde{i}$	perturbation quantity
$\hat{i}$	describing functions for transient motion
i'	fluctuating quantity due to turbulence
$\bar{i}$	time average value in turbulent flow

subscripts

a	associated with airfoil dimensions
---	------------------------------------

b	at location where transition and reattachment begins
l	on lower surface
m	at location of maximum velocity
o	mean value
r	at location of reattachment
s	at location of separation
u	on upper surface

1. INTRODUCTION

At present little is known of the dynamics of a separating and reattaching flowfield. This unsteady flow topic relates to a wide range of aerodynamic problems such as control-surface-buzz; dynamic stalling of lifting surfaces or bodies; "wing rock", "departure" and autobuffeting of aircraft; "stall flutter" of helicopter rotor blades, engine compressor blades, and possibly V/STOL aircraft wings and lifting body configurations.^{1,2,3}

The above examples of interaction between an oscillating surface (or body) and a separating flow represent significant problems. Unfortunately, there exists a void in knowledge which precludes the development of valid models for the related flowfield. The above examples usually involve low rates of change of properties within the flowfield, thus dynamic viscous effects play an important role. Flutter in one degree of freedom may exist in which there is a mutual interaction between the motion of a body and the separating and reattaching flow. Shock waves may be involved as the mechanisms for inducing flow separation. The topic of flutter in one degree of freedom is discussed by Lambourne in reference 1. Jones, in reference 4, concludes that in this type of flow the flow field is deterministic and the appropriate analytical model is autonomous and nonlinear.

One of the problems associated with these inviscid-viscous flow interactions is that there must be a matching of conditions at the boundaries of the rotational and irrotational flow regions. An iterative process is usually involved. At extremes of this matching problem are cases where either the inviscid or viscous region dominates the other. For example, a thin boundary layer on an airfoil may be neglected completely in many cases of steady and unsteady flow.

In this paper two cases will be investigated which are seen to lie near the above extremes. The first case involves a fence-type spoiler which can be thought of as an overwhelming perturbation and is used to 'control' a sizable separated region. The growth of the separated region is studied. It is found that viscous effects are of primary importance. Near the other extreme is the interaction between a small laminar leading-edge separation bubble and the outer inviscid flow field on an airfoil. Results obtained during these analyses are extended in order to study the complex problem of control-surface-buzz, which lies somewhere between the two extremes of interaction types.

The first part of this paper then deals primarily with a controlled separated region. The purpose is to gain further insight into the dynamics of an unsteady separating flow by significantly reducing the influence of the inviscid flow field. Section 2 elaborates on an experiment which was designed to do this. It involves an airfoil with flap and a spoiler which can

be mechanically driven in harmonic motion in order to control an unsteady separating flow. The experiment is also designed so that the flap, which lies downstream of the spoiler, may be fixed or be allowed to respond freely to the effects of spoiler forcing, or be coupled to the spoiler.

An excellent survey of work related to separated flows is found in Chang's book (Ref. 3). Other important contributions in the study of steady or quasi-steady separated flows are found, for example: (i) in the analytical work of Crimi and Reeves (Ref. 6) which deals with a separated laminar shear-layer which transitions to a turbulent shear-layer with mixing and reattachment onto a surface, (ii) in the experimental studies of Barnes (Ref 7) and Newman (Ref 8) which deal with a separated region aft of a fence-type spoiler, (iii) in the experiment by DeBroderode and Bradshaw (Ref 9) which deals with a separated region aft of a rearward-facing step and discusses three-dimensional effects and the dependence of bubble length on step height and initial boundary-layer thickness and shape, (iv) in the analysis by Bradshaw and Wong (Ref 10) which deals with strong and overwhelming perturbations to a shear-layer (caused by steps or spoilers, for example) and the subsequent downstream reattachment and relaxation of the shear-layer, (v) in the analytical work of Norbury and Crabtree (Ref 11) which deals with the mixing and reattachment region of a separation bubble, and (vi) in the studies by McGregor (Ref 12), Gaster (Ref 13), Young and Horton (Ref 14), Horton (Ref 15) and more recently Woodward (Ref 16) which are concerned with the growth and bursting of laminar leading-edge separation bubbles.

Trilling (Ref 17), East and Wilkinson (Ref 18) and Theisen (Ref 19) have offered models of a separated region in which unsteady mass flow into the region is considered in order to determine growth characteristics in unsteady flow.

In section 3 a mathematical model of a separated region is derived which is used in comparison with the experimental results of section 2. Further analysis is offered in section 4 which extends the analytical model of the experiment to the study of dynamic stall. In particular the stability of a laminar leading-edge separation bubble on an airfoil is studied and when instability is achieved, for example, after an airfoil has pitched upward past the static stall incidence angle, a prediction is made of the instant at which the bubble begins to grow. Subsequent growth and loading due to bubble growth is not analyzed. Also included in section 4 is an extension of the analysis to include the study of control-surface-buzz.

2. EXPERIMENTS ON AN AIRFOIL WITH OSCILLATING SPOILER AND FLAP

An experimental program was designed in order to study a separating and reattaching flowfield in incompressible flow. The program included a series of tests which involved an airfoil model with a movable fence-type spoiler.

In the first series of testing the spoiler, which is mounted on the model upper surface, is oscillated and thus used to "control" flow separation and reattachment. Pressures along the airfoil centerline are recorded as well as unsteady hinge-moment which acts on a flap-type control surface (mounted aft of the spoiler). A second series of tests involves the spoiler at fixed heights while the flap is oscillated mechanically. The flap is allowed to respond freely to spoiler induced loading in a third series of tests, and a final series of tests is performed with the spoiler and flap mechanically coupled.

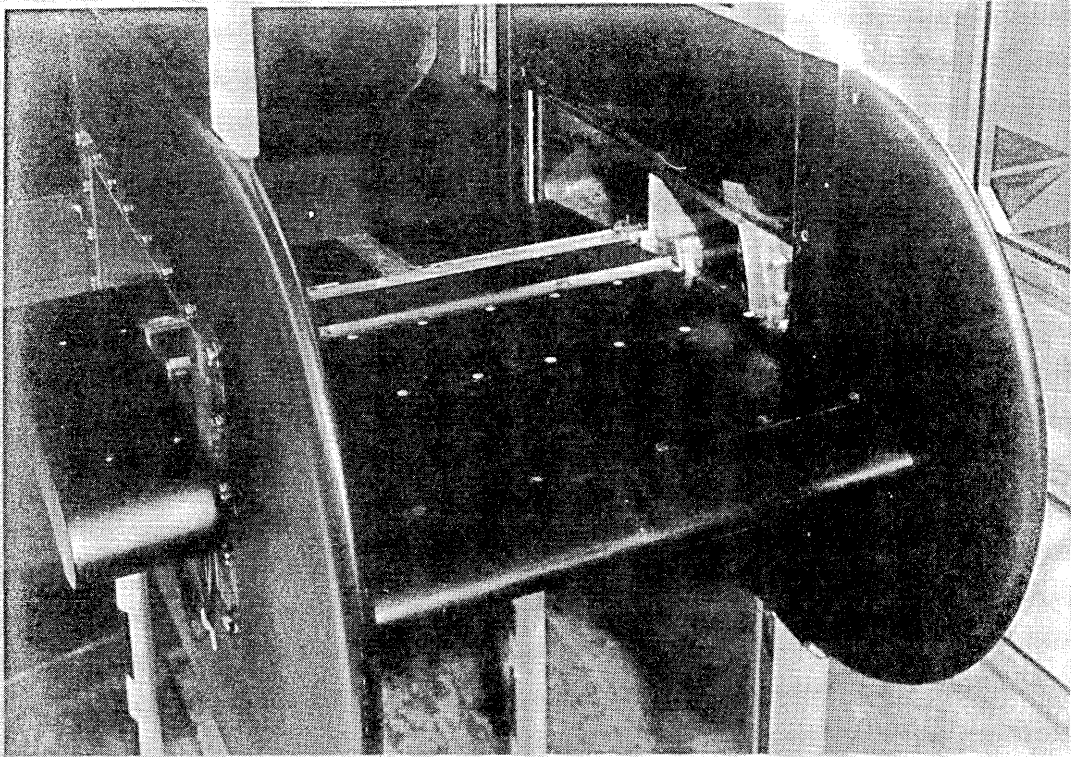
Spoiler chordwise position and maximum height were chosen from results of an earlier experiment so that at maximum height a region of near constant pressure "dead air" extended to the trailing edge of the model. The program was designed to isolate and concentrate on the least understood elements of the separating flow field. These elements relate to the dynamics of a separated region.

The spoiler induces a region of separated flow which exists on the airfoil aft portion. Since regions of separated flow are known to break down into "stall cells" if model aspect ratio is large, a model of fairly low aspect ratio is used so that a single "stall cell" exists and the flow field is symmetrical about the model centerline.

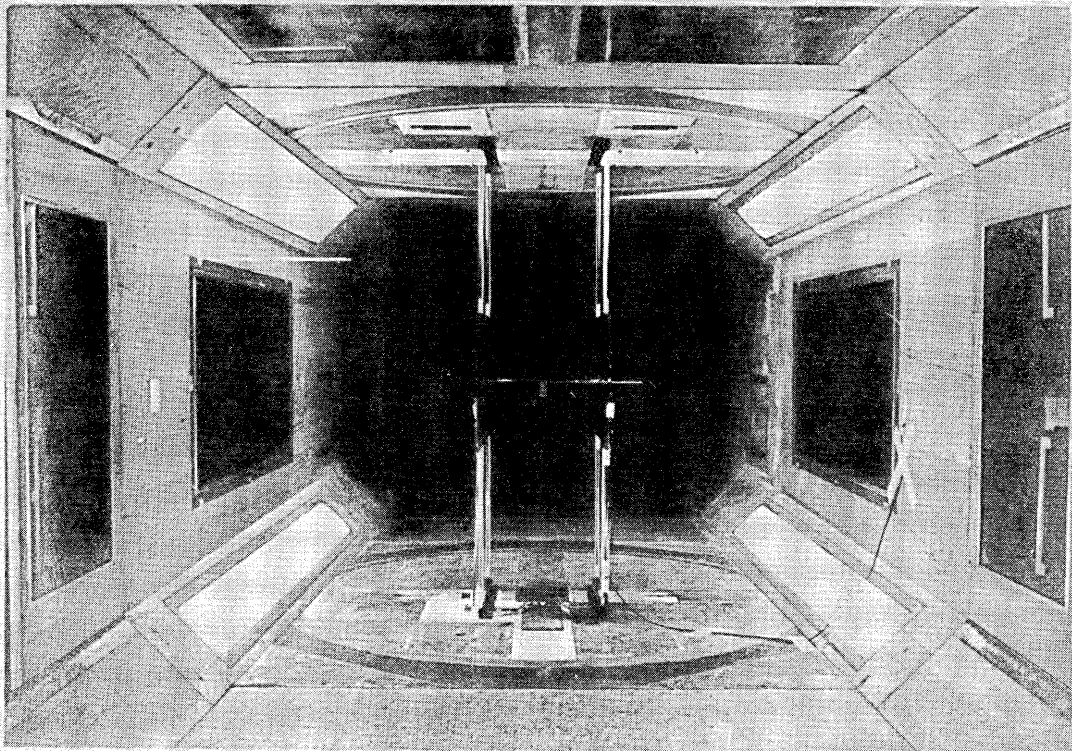
Unsteady pressures and hinge-moments were expected to depend on the following quantities: airfoil angle-of-attack, flap angle, spoiler height (mean height and oscillation amplitude), frequency of oscillation, air density and viscosity, free-stream speed, airfoil chord, flap chord, spoiler chordwise position, spoiler span, boundary-layer displacement thickness, free-stream turbulence, and (for flap-free testing) flap system inertia, mechanical damping and structural stiffness. In addition, pressures depend on chordwise location. Dimensional analysis then shows that non-dimensional pressure and hinge-moment coefficients depend on appropriate non-dimensional parameters. The experimental program served to investigate the effects of these parameters over a limited range of parameter variations. Reference 20 contains a detailed description of the experiment and results. Some of that information is included here.

2.1. Apparatus and Methods of Measurement

A modified NACA 0012 airfoil model of 24" chord and 18" span was constructed, assembled and mounted in the 8' x 6' sub-



CLOSE-UP OF MODEL ASSEMBLY
(OBLIQUE VIEW FROM UPSTREAM)



REAR VIEW OF MODEL IN 8' x 6' WORKING SECTION
(LOOKING UPSTREAM)

FIGURE 2.1. GENERAL VIEWS OF MODEL INSTALLED
IN WIND TUNNEL.

sonic wind tunnel of the Department of Aerodynamics, Cranfield Institute of Technology (C.I.T.). (See Figure 2.1.)

The airfoil model included a $\frac{1}{2}$ chord plain flap (with hinge-line located at $\frac{3}{4}$ chord) and a fence-type spoiler located 14" from the airfoil leading-edge (L.E.), i.e., at 58.3% chord. The spoiler was free to move in a direction normal to the airfoil chord. The airfoil is nominally a NACA 0012 section up to 75% chord. The flap has flat upper and lower surfaces and a blunt (.045" thick) trailing edge (T.E.).

Figure 2.2 shows a top view of the airfoil model, including flap, with top surface panels removed. Not shown is the fact that the airfoil "center" and rear spars and the flap shaft extend beyond the model span. The airfoil spars extend through perspex end-plate windows on both sides of the model and are attached to brackets on external stiffeners. The stiffeners allow adjustment of airfoil incidence and can be adjusted over a range of incidence angles $-5^\circ < \alpha < +22^\circ$.

The flap shaft also extends in span through the end plate windows and attaches to adjustable support brackets via Bendix flexure pivots. The attachment can be adjusted to alter the gap at the flap hinge-line. The gap was adjusted to .008" by use of shims giving a gap to chord ratio of .00033. A gap of .026" existed between the flap and end-plates.

A special balance bracket and rod was attached to the flap shaft on the port side and was used to mass balance and vary the moment of inertia of the flap. Also a "locking bar" with strain gages could be used to secure the flap at desired flap angles and to measure hinge-moment.

An alternate assembly could be used on the flap shaft for flap-forcing tests. The flexure pivot and bracket, and balance bracket were removed and a connecting arm with strain gages, bearing and bracket were connected directly to the flap shaft. The connecting arm was then attached to a drive rod (to be described).

The starboard extension of the flap shaft was connected, in addition to a flexure pivot and bracket, to a low-torque, precision wire-wound potentiometer which was used for flap angle measurement. A 12:1 gear ratio was provided between potentiometer and flap. An adjustable mechanical limit was provided to limit flap travel to amplitudes of 20° or less.

The spoiler was driven through a lever and shaft assembly located on the model port side. A rack and pinion arrangement was used within the airfoil in order to move the .082" thick nylon spoiler normal to the airfoil chord line. Brackets could be adjusted for best gear alignment and to eliminate backlash.

Two spoilers were constructed, one a "full-span" version of 18" span and one "part-span" of 15" span. The "full-span"

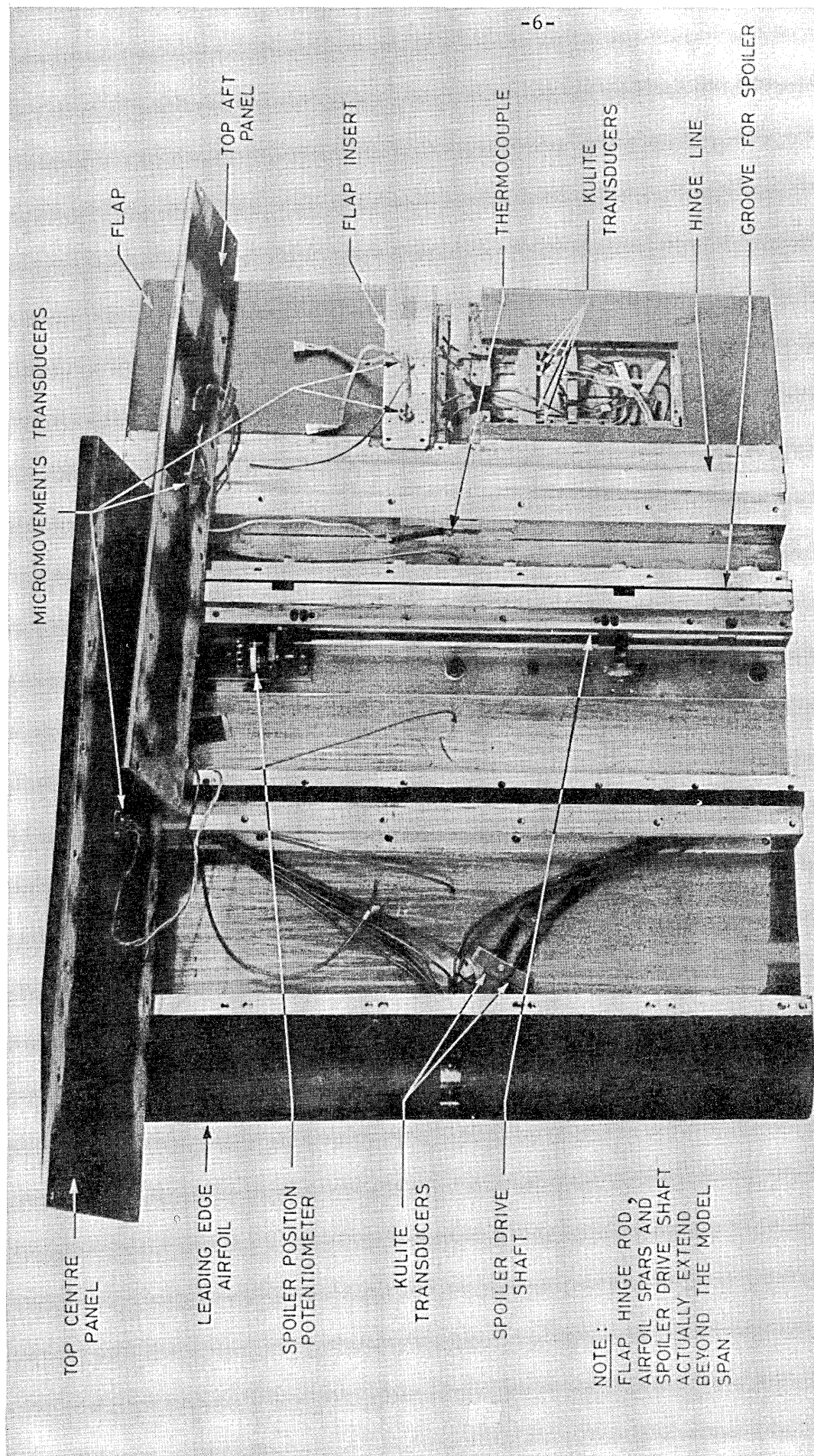


FIGURE 2.2. TOP VIEW OF AIRFOIL INTERIOR

spoiler did not completely fill the distance between end-plates. A .041" gap existed on both sides.

A drive rod, which was used for spoiler and flap forcing, was constructed of light alloy and connected under the tunnel floor to a variable eccentric cam. The cam could be altered to vary drive rod stroke from zero to about four inches. The variable cam/flywheel assembly was driven through an infinitely variable hydraulic transmission connected to an electric motor. Rotary speed of the flywheel assembly was held constant by a self-contained hydraulic servo mechanism and could be adjusted over a frequency range of zero to about seven revolutions per second. The drive rod delivered a nearly perfect sinusoidal linear motion at the model. It was also used to force the flap through sinusoidal variations of flap angle up to $\pm 10^\circ$. The apparatus allowed for adjustment of either spoiler or flap mean position, amplitude of oscillation and frequency.

A simple linkage mechanism was also provided in order to couple the flap to the spoiler. A slotted lever was attached to the spoiler drive shaft and coupled to the mass balance rod of the flap assembly. A coupling pin and bushing could be moved in order to vary the "gearing" between flap and spoiler motion. Also mean spoiler height could be altered.

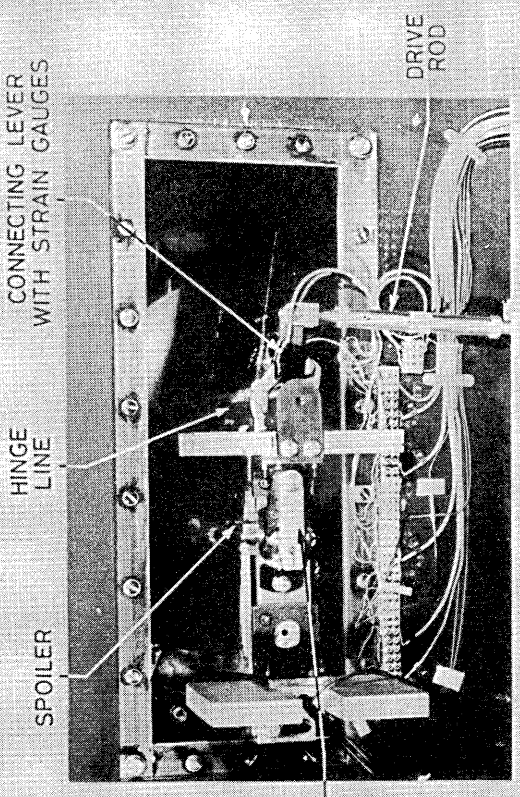
Refer to Figure 2.3 for views showing different configurations.

End plates were constructed of wood. They were bolted to the perspex end windows and four steel struts which were used to secure the apparatus to the tunnel ceiling and floor. Streamlined fairings were attached to the struts. Cowlings were fabricated of sheet metal and used to cover apparatus which was exposed at the end plates. The cowlings were shaped in order to reduce wake disturbances.

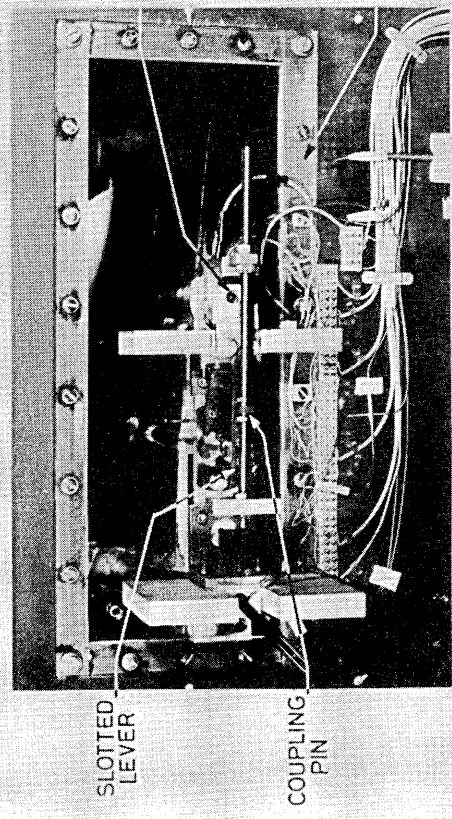
The model, end plates and cowlings were painted a dull black.

The airfoil was constructed with static pressure taps initially at 8 chordwise positions on the upper and lower surface along the model centerline, i.e., at .625, 1.25, 2.5, 5, 15, 25, 45 and 65% chord. Five static pressure taps at 76.25, 85 and 95% chord on the upper surface and at 82.5 and 90% on the lower surface were also provided on the flap for initial testing. Plastic tubing was led out of the model through the spars and flap shaft, down the struts through the tunnel floor to a multiple-tube manometer.

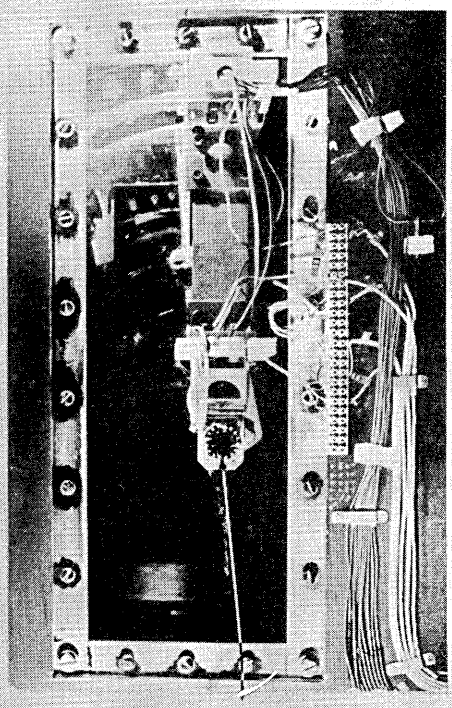
A pitot-static tube was positioned approximately 2' above and to the left of the airfoil with pitot-tube opening at the model leading-edge. Leads were connected to the manometer. Most of the static pressure taps were connected to the manometer throughout the series of experiments, however, some were later connected to pressure transducers as explained shortly.



A. SPOILER DRIVEN, FLAP FIXED (PORT SIDE)



B. FLAP DRIVEN, SPOILER FIXED (PORT SIDE)



C. COUPLED FLAP & SPOILER (PORT SIDE)

(STARBOARD SIDE)

FIGURE 2.3. SIDE VIEWS OF MODEL SHOWING DIFFERENT CONFIGURATIONS

Initially two Micromovements Ltd. type 8-612/s3 differential pressure transducers were purchased by the Department of Aerodynamics and installed on the flap insert upper surface at the 80 and 87.5% chord positions. They were bonded with Aroldite adhesive to the lower surface of the insert beneath small holes. The reference pressure side of each transducer was connected to a common manifold that was kept at 'tunnel static' pressure via a connection to the pitot-static tube. Two additional Micromovements transducers were later purchased and mounted close to the surface at 45 and 65% chord on the airfoil upper surface. Also, six Kulite model CQH-125-5D pressure transducers were purchased by the Frank J. Seiler Research Laboratory (FJSRL) and connected by plastic tubing to taps at 1.25 and 2.5% chord on the airfoil upper surface, to 76.25 and 95% chord on the flap upper surface, and to 82.5 and 90% chord on the flap lower surface. Protective brass housings were constructed for the Kulite transducers. Refer to Figure 2.4 for details of transducer mounting in the flap.

All transducers were connected on the reference pressure side to the common manifold. The wiring associated with each transducer was led through the model spars or flap shaft to a connector strip on the end plates. Screened four-conductor cables were used as leads to amplifiers and power supplies located at the control bench.

Spoiler position was sensed by a precision wire-wound potentiometer that was located at the spoiler drive shaft in the model. Flap position was sensed by the precision potentiometer mentioned previously. All potentiometer electrical connections were led to a connector strip on the starboard end-plate and from there to the control bench by four-conductor screened cables. Electrical connector strips and leads may be seen in Figure 2.3.

Hinge-moments were measured by one of three strain-gage devices. Each device was instrumented with an additive bridge arrangement of four "polyester" strain gages. The strain gages were bonded with adhesive to sand blasted surfaces. For flap-forcing the measurement of hinge-moment is taken from the strain-gages on a thick, steel connecting lever. For flap fixed testing a locking bar was used to secure the flap.

A thermocouple (which can be seen in Figure 2.2) was secured to the rear airfoil spar and electrical connections were led to a thermocouple multimeter located on the control bench.

A single-element hot wire anemometer was used to measure tunnel free-stream turbulence levels. It was positioned on a probe that extended 6" in front of the airfoil leading-edge. Measurements were taken both at the airfoil center-line and upstream of the starboard end plate. Later the probe was moved to position the hot-wire 7" aft of the trailing-edge along the

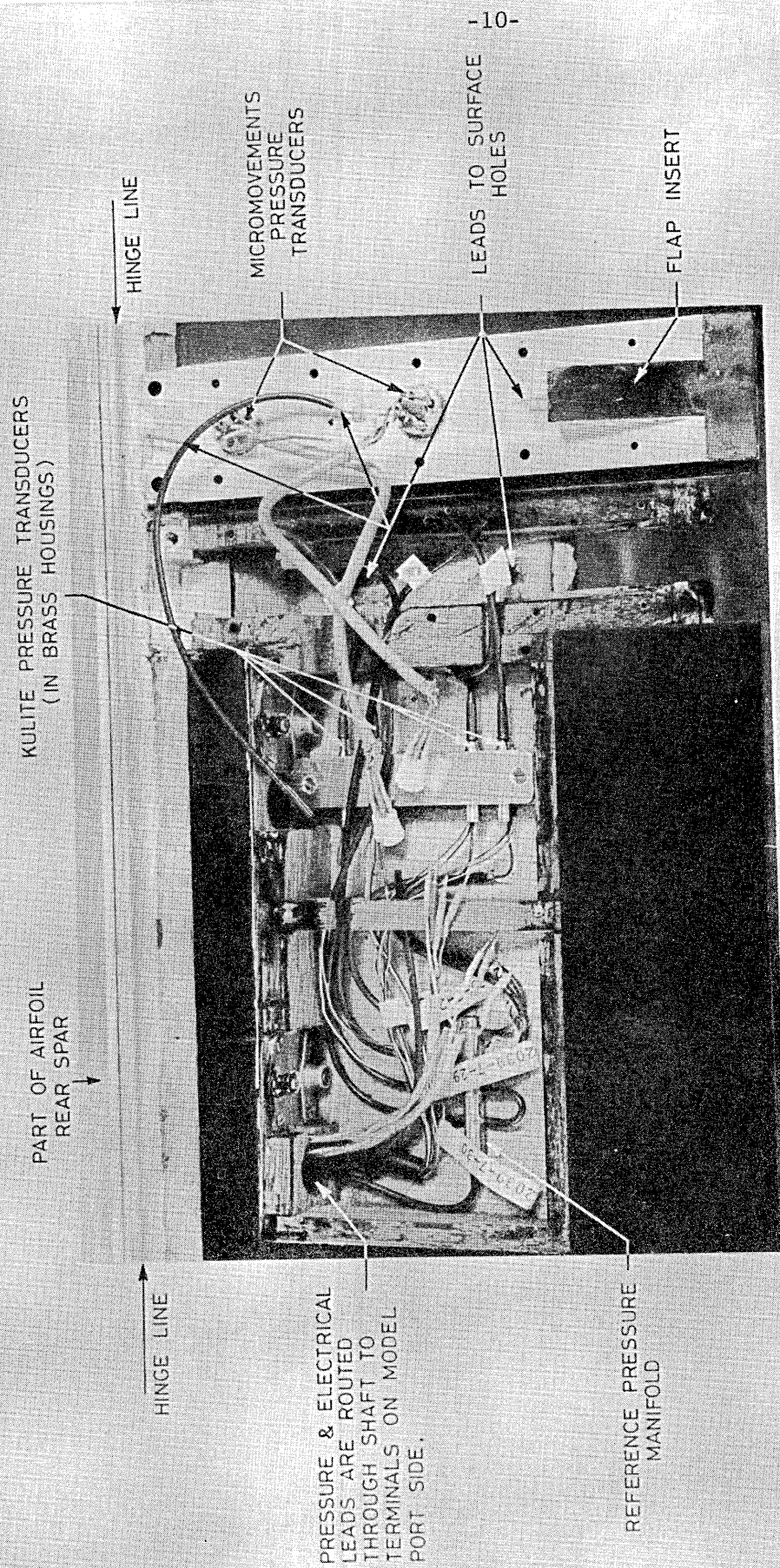


FIGURE 2.4. FLAP INTERIOR LAYOUT.

center-line and 1" above the airfoil. The hot-wire electrical lead was brought out to the control bench to associated DISA equipment.

A nine tube boundary-layer rake was also used during a test series. The tubes measured total pressures with 3/4" of the airfoil surface. Plastic tubing connected the boundary-layer rake to the multiple-tube manometer.

Most data was available in the form of electrical signals from the various sensors. These signals were brought to the control bench where most of the data acquisition apparatus was located. (See Figure 2.5.)

Four differential D.C. amplifiers with individual bridge power supplies were constructed by the Instrumentation Unit at C.I.T. and made available for the experiment. They were used to supply bridge power and to amplify the output of the Micro-movements pressure transducers. Burr Brown amplifiers with a stabilizing feedback network comprised the core of the amplifier units. Eight Ectron Differential D.C. Amplifiers and a bridge excitation supply purchased by the Frank J. Seiler Research Labs were available for the experiment. The single excitation supply was used to power the six Kulite pressure transducers and a strain gage. The Kulite transducer outputs, strain gage output and the thermocouple output were fed into the eight Ectron amplifiers.

The hot-wire element was connected directly to a constant temperature anemometer (CTA). The CTA output was fed to a DC-voltmeter and RMS unit. For some tests the CTA output was also fed into an Ectron amplifier in place of the thermocouple signal. All amplifiers had one side of the output set at earth potential. The other output leads were fed into a junction strip.

The spoiler and flap position potentiometers were supplied with a stabilized D.C. voltage by separate supplies. The center tap of each potentiometer was set at earth potential and the wiper arm outputs were fed into the junction strip. Either spoiler or flap position signals could be fed into a digital counter in order to monitor frequency of oscillation. A speed control was provided at the control bench in order to vary either spoiler or flap frequency of oscillation.

The 14 channels of data available at the junction strip were fed into an output monitor panel. Also available at the monitor panel were the 2 stabilized voltage signals. These sixteen channels of data were fed into a digital voltmeter and scanning unit. The unit also contained a paper tape punch. The output monitor panel allowed the operator to select one of 16 output voltages to be displayed on a digital voltmeter. Also direct pressure transducer outputs were available at the panel. In addition, any four of the 14 amplifier and position signals could be displayed on a four channel oscilloscope. A Polaroid camera with scope mount was also used on occasion.

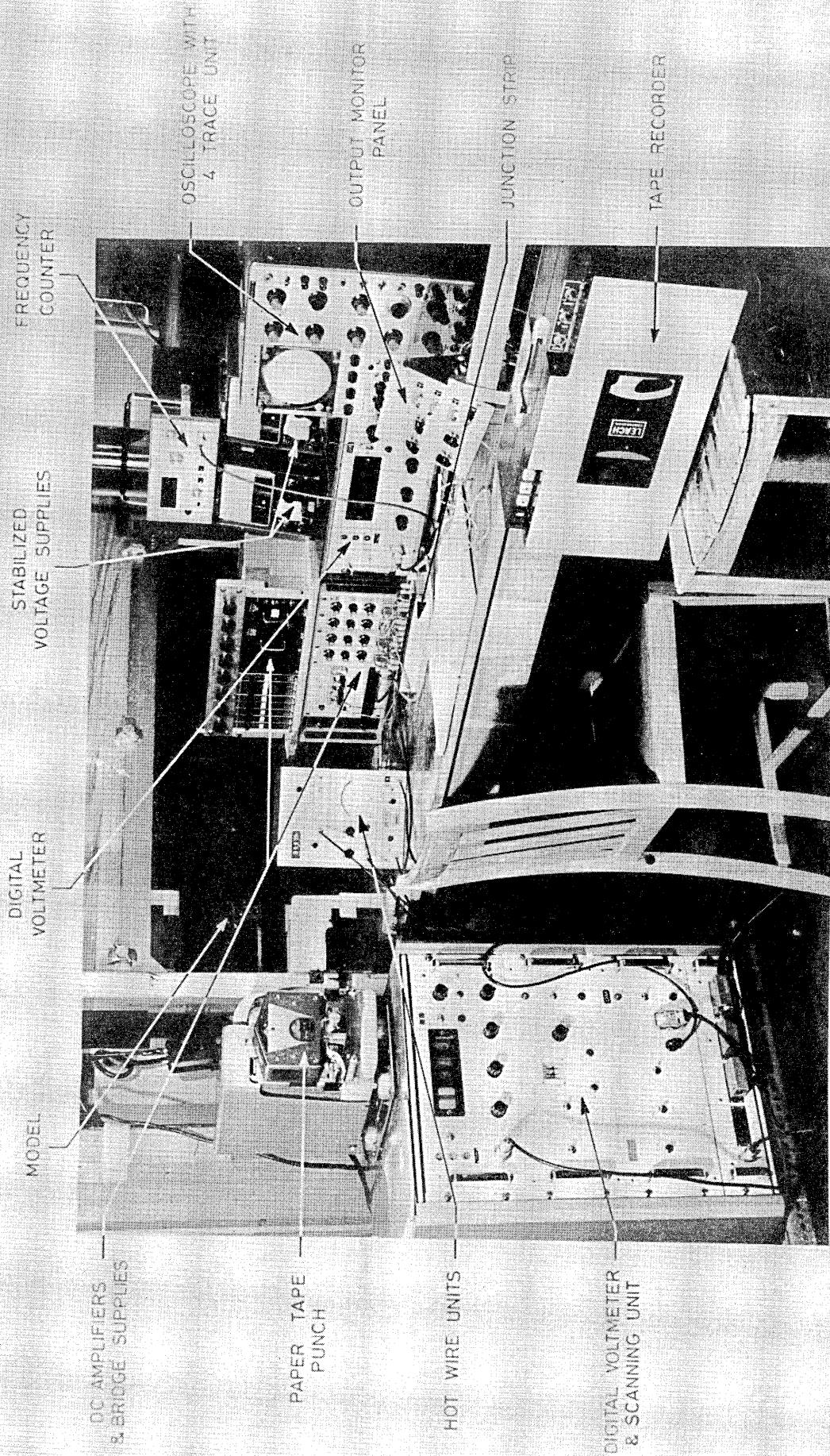


FIGURE 2.5. DATA ACQUISITION APPARATUS.

A Leach tape recorder type MTR-3200A was loaned for the duration of the experiment by the U.S. Air Force Flight Dynamics Lab (AFFDL). It was used to record 14 channels of data for subsequent off-line reduction. A voice channel was also utilized during each recording session.

A schematic diagram of the on-line data acquisition apparatus is shown in Figure 2.6.

A series of flow visualization tests involved the use of a titanium dioxide and oil mixture which was thinned with kerosene. For these tests, a hand-held camera was used to record the flow patterns.

Cine-film recordings were made to record the response of small tufts attached to the airfoil upper surface. The camera was loaned by the Photographic Unit at C.I.T. A film speed of 64 frames per second was used.

Details of calibration procedures for all test equipment, results, error analyses and discussion may be found in reference 20.

The calibration runs also involved determination of flap system inertias, mechanical damping and structural stiffness. Various configurations were tested and are described below.

Flap system inertia could be varied by use of lead weights fastened to the flap balance bracket and rod. Three inertia configurations were tested with the flap-free or fixed. Two represented a mass-balanced flap system. The inertia configurations are described below:

A_1 : balance bracket, rod and three .547 lbf lead weights

A_2 : balance bracket, rod and one lead weight (shown in Fig. 2.3)

A_3 : balance bracket only, no mass balance

An additional configuration, A_4 , was involved in flap-forced testing and includes the flap assembly and bearing used for flap-forced tests.

Mechanical damping for the flap-free and forced, B, was also found. The locking bars were found to introduce negligible damping. Existing mechanical damping was primarily due to lead flexing and found to be small. It was nearly constant for all configurations.

Stiffness was provided, in the flap-free case, solely by the flexure pivots. Two types of flexure pivots were available, one with $\pm 30^\circ$ freedom of rotation and a stiffer type with $\pm 15^\circ$ freedom. With the flap fixed by use of locking bars,

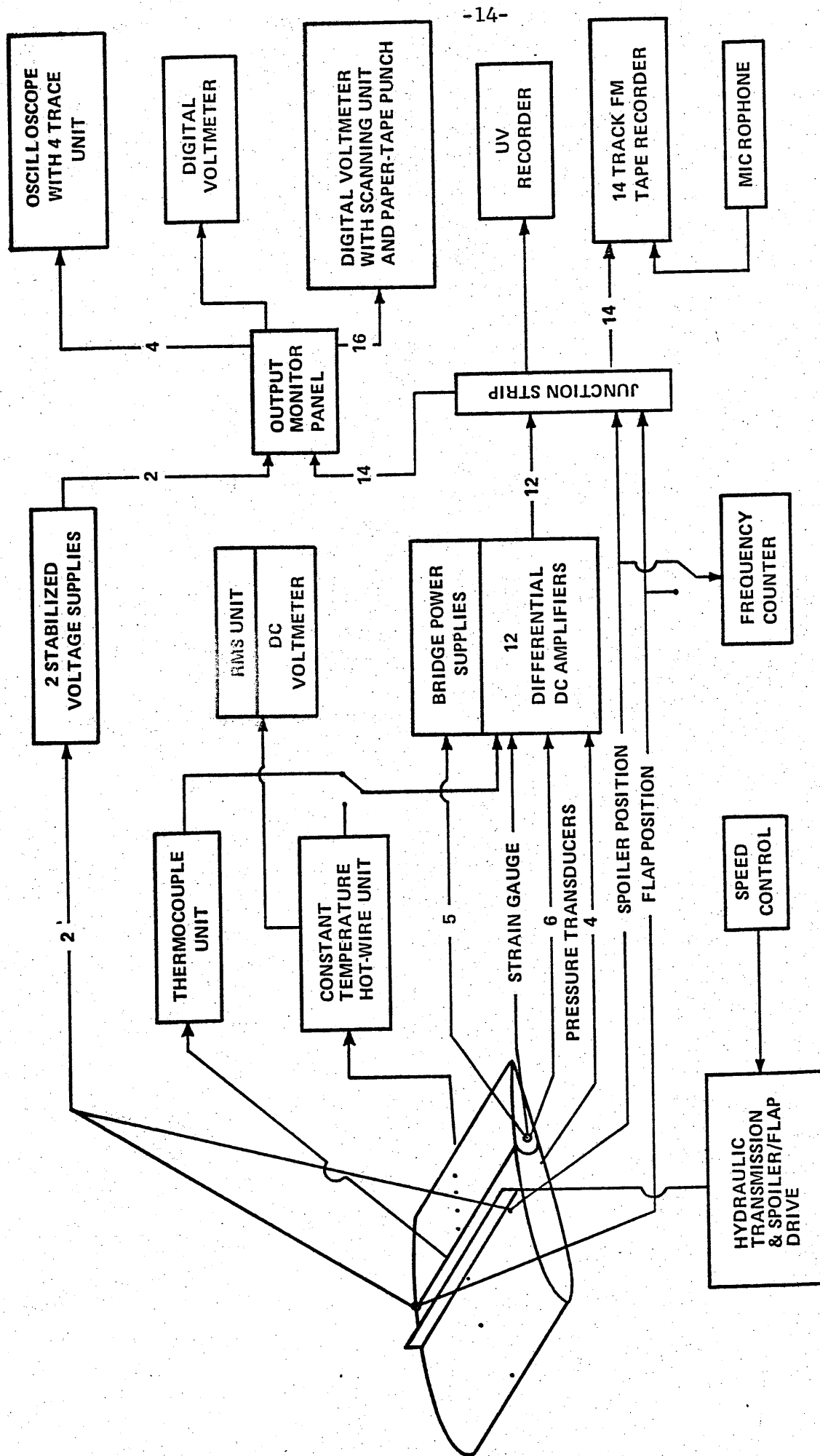


FIGURE 2.6. ON-LINE DATA ACQUISITION SCHEMATIC

their stiffness was included. The stiffness configurations are:

C_1 : two $\pm 30^\circ$ flexure pivots

C_2 : one $\pm 15^\circ$ (stiff) and one $\pm 30^\circ$ flexure pivot

C_3 : SG_1 locking bar and two $\pm 30^\circ$ flexure pivots

C_4 : SG_2 locking bar and two $\pm 30^\circ$ flexure pivots

C_5 : one $\pm 30^\circ$ pivot (used for flap-forced tests)

SG_2 locking bar is thicker and stiffer than the SG_1 locking bar.

Data was collected, reduced and corrected according to the following schemes.

Dynamic pressure was corrected for solid and wake blockage. Pressure transducer data was also corrected since reference pressure was not free-stream static. Hinge-moment data was corrected for locking bar dynamic effects. Corrected values of dynamic pressure were used to non-dimensionalize corrected pressures and hinge-moments.

$$C_p \equiv \frac{P - P_\infty}{q_\infty}$$

$$C_H \equiv \frac{H}{q_\infty b_f c_f^2}$$

where

b_f is flap span, 1.5 ft
 c_f is flap chord, 0.5 ft

A reduced frequency, based on flap chord length, is defined:

$$\nu \equiv \frac{2\pi f c_f}{U_\infty} = \frac{\omega c_f}{U_\infty}$$

where

f is frequency of spoiler or flap oscillation

No dynamic corrections were applied to pressure transducer signals.

Most unsteady flow data was recorded and reduced off-line by use of the data reduction facility of the C.I.T. Department of Flight. An example of recorded data from several channels is shown in Figure 2.7. This is a rather poor copy of recorded

FLAP - FIXED

$$h = .052 + .052 \cos U \tau$$

$$U \cong .006 \quad Re_c = 1.92 \times 10^6$$

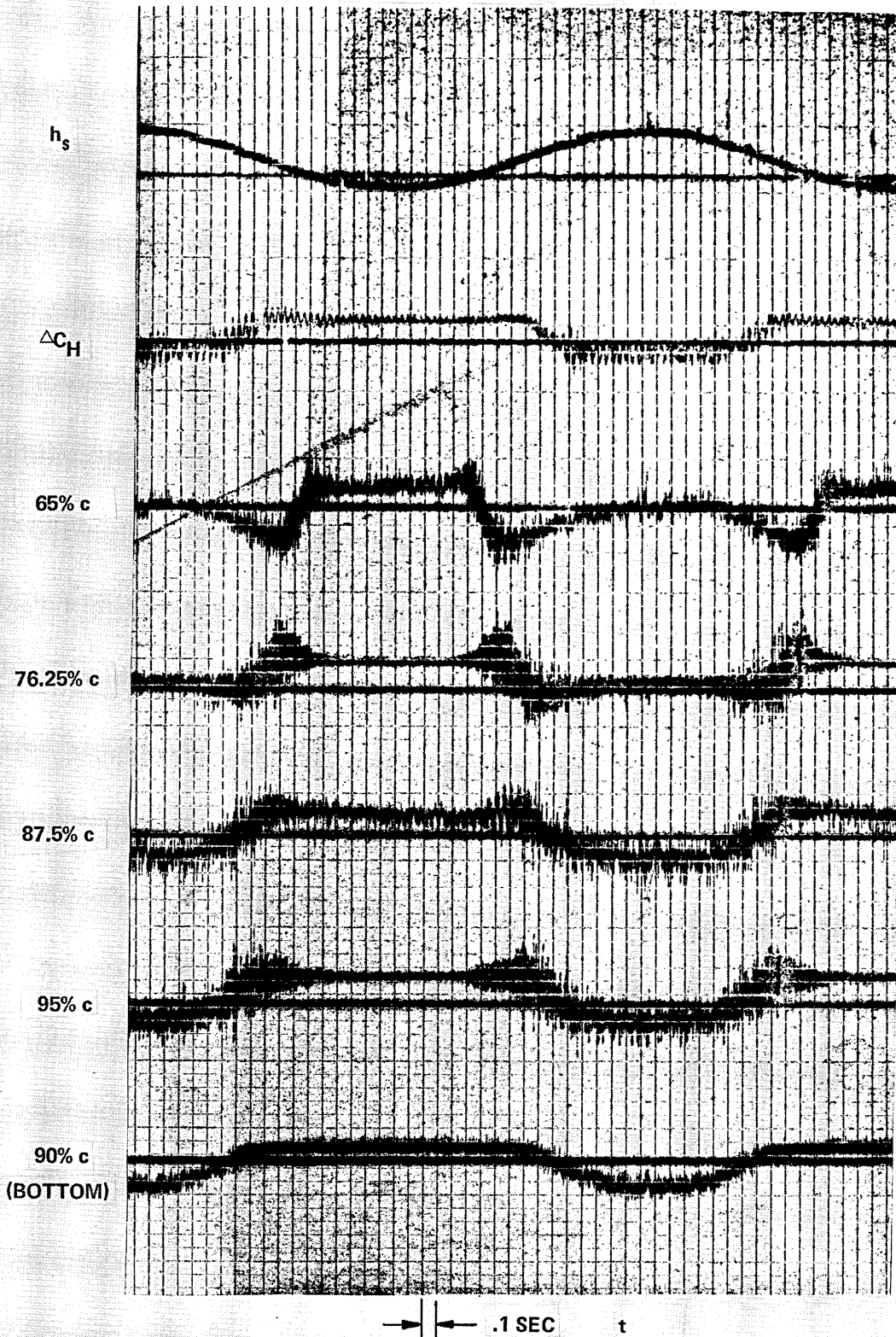


FIGURE 2.7. ACTUAL DATA TIME HISTORIES

data which was collected during a flap-fixed test with spoiler-induced separation.

The data reduction apparatus was utilized to solve for components of the Fourier series which represented each unsteady signal. A schematic diagram of the off-line data apparatus is shown in Figure 2.8. An analog computer was wired to provide a modified sawtooth waveform which was triggered by the sinusoidal forcing signal (i.e., either spoiler or flap position signal). It contained higher harmonic content as well as the fundamental forcing frequency. The sawtooth and forcing signals were filtered and digitized and evaluated by a Fourier transform program. Thus, a Fourier series representation of the sawtooth waveform was generated and stored within the computer. The magnitude and phase of higher harmonics was adjusted for filter effects. Other channels in turn were fed into the computer along with the sawtooth waveform. A cross-spectral density analysis was performed which, in effect, isolated the coherent portions of each time history. Thus, a Fourier series could be formed for each of the data channels. In practice the Fourier series was truncated after eight terms since all higher frequency components were of very low magnitude. Recording times of one minute were used for all data collection in order to provide a large sample. This sample was divided into separate "buffers" for analysis. Results from all buffers were averaged. Details of the reduction procedure are found in reference 20.

For spoiler-forcing tests spoiler height was taken as:

$$h \equiv \frac{h_s}{c_f} = h_o + \Delta h \cos \omega t = h_o + \Delta h \cos \nu \tau$$

where

$$\tau \equiv \frac{U_\infty t}{c_f}$$

With the flap free, flap angle is

$$\beta = \beta_o + \sum_{n=1}^q \beta_n \cos (n \nu \tau + \theta_n)$$

where $q \leq 8$, and β_o and β_n are primarily functions of h_o , Δh and ν .

For the flap fixed or free, non-dimensional pressures are:

$$C_p = C_{p_o} + \sum_{n=1}^q C_{p_n} \cos (n \nu \tau + \phi_n)$$

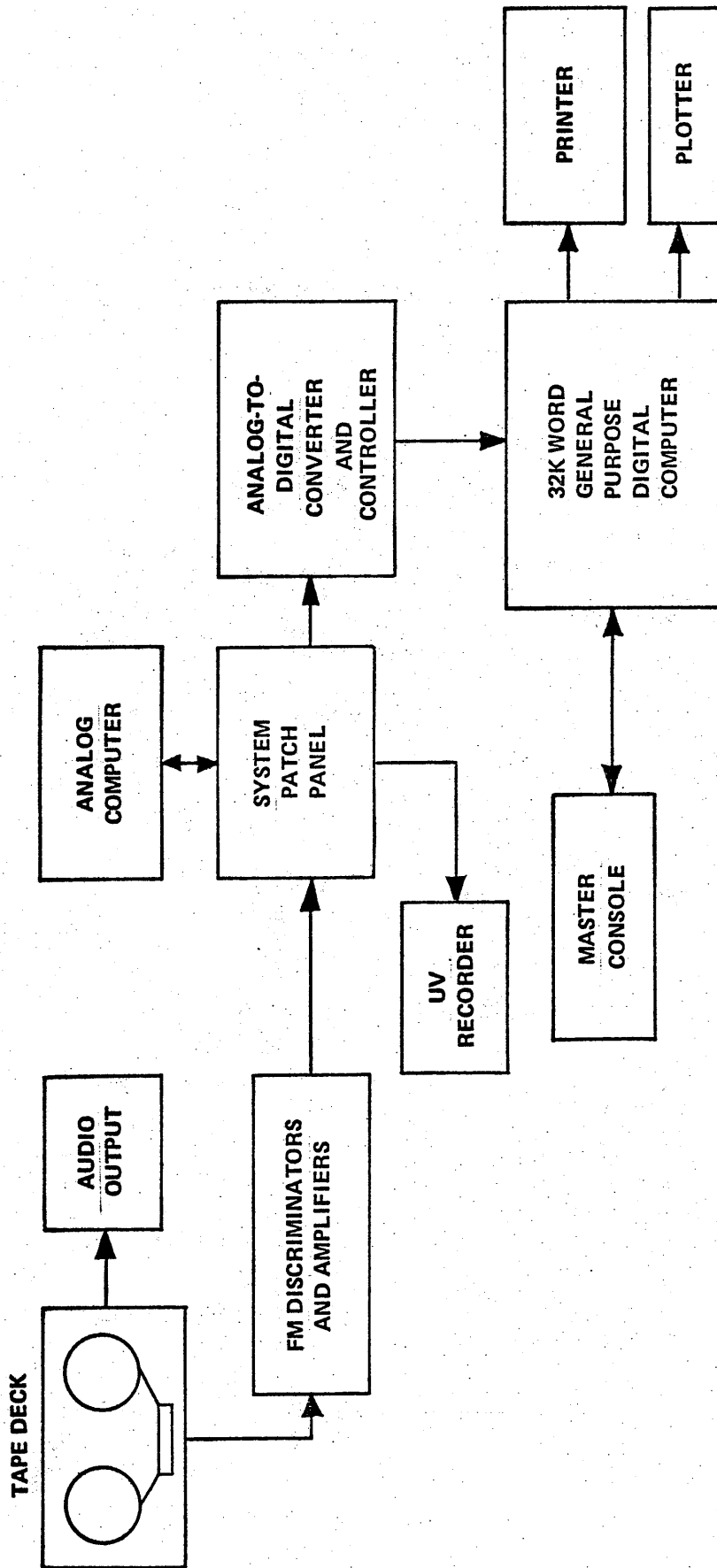


FIGURE 2.8. OFF-LINE DATA REDUCTION SCHEMATIC

where C_{p_0} and C_{p_n} are primarily functions of x , h_0 , Δh and ν .

Pressure fluctuations were also measured in steady-flow. R.M.S. voltages from appropriate pressure transducers were measured either on-line or off-line from a recording. In all cases the voltages were corrected for the wind-off noise level. Results will be displayed as R.M.S. pressure coefficient fluctuations, i.e., $\langle p \rangle / q_\infty$.

Cine-film recordings were observed in order to gather qualitative and quantitative results. Small tufts of thread were found to yield good response for the frequencies of testing and were attached primarily near and aft the spoiler. The cine-film recordings were analyzed frame-by-frame. The growth and decay of the "separation bubble" produced by the spoiler could be observed. The position of reattachment along the centerline was measured during these runs.

For spoiler forcing with the flap fixed by locking bars, hinge moment coefficient was found and represented by:

$$\Delta C_H = \Delta C_{H_0} + \sum_{n=1}^q \Delta C_{H_n} \cos (n \nu \tau + \psi_n)$$

where ΔC_{H_0} and ΔC_{H_n} are primarily functions of h_0 , Δh and ν .

For flap forcing tests, flap angle was taken as:

$$\beta = \Delta \beta \cos \omega t = \Delta \beta \cos \nu \tau$$

Hinge-moment response was found and it was noted that higher harmonic distortion could be neglected.

2.2. Details of Tests

The primary purpose of the experimental program was to investigate the dependence of pressure and hinge-moment coefficients on the following non-dimensional parameters:

Reynolds number

$$Re_c = \frac{\rho U_\infty c}{\mu}$$

Frequency parameter

$$\nu = \frac{\omega c_f}{U_\infty}$$

Mean spoiler height

$$h_0 \equiv \frac{h_{0s}}{c_f}$$

Spoiler oscillation amplitude	$\Delta h \equiv \frac{\Delta h_s}{c_f}$
Flap angle	β
Chordwise position (for pressure coeff.)	$x = \frac{X}{c}$

Other pertinent parameters were briefly investigated:

Spoiler chordwise position	$x_{sp} = \frac{X_{sp}}{c}$
Ratio of mean spoiler height to boundary-layer displacement thickness	$\frac{h_{os}}{\delta^*}$
Free-stream turbulence level	$\frac{\langle u \rangle}{U_\infty}$
Airfoil angle-of-attack	α
Flap inertia	A
Flap structural stiffness	C
Spoiler span	b_{sp}

The form of the cyclic pressure and hinge-moment variation can be written in terms of Fourier components as:

$$C_p = C_{p_0} \{x, h_o, \Delta h, \nu, \beta, Re_c\} + \sum_{n=1}^{\infty} C_{p_n} \{x, h_o, \Delta h, \nu, \beta, Re_c\} \cos [n \omega t + \theta_n \{x, h_o, \Delta h, \nu, \beta, Re_c\}]$$

and as:

$$C_H = C_{H_0} \{h_o, \Delta h, \nu, \beta, Re_c\} + \sum_{n=1}^{\infty} C_{H_n} \{h_o, \Delta h, \nu, \beta, Re_c\} \cos [n \omega t + \psi_n \{h_o, \Delta h, \nu, \beta, Re_c\}]$$

Flap and Spoiler-Fixed Testing

A series of tests was made with variations in model angle-of-attack, flap angle and spoiler height in order to collect steady-flow pressure and hinge-moment data. All tests were

performed at a nominal free-stream speed of 150 ft/sec ($Re_c \cong 1.92 \times 10^6$). Combinations of the following nominal parameter settings were used with the full-span spoiler.

α ($^\circ$)	β ($^\circ$)	h_s (in)
+12	+12	0
+8	+8	.125
+4	+4	.250
0	0	.375
-4	-4	.500
	-8	.625
	-12	

A series of steady flow tests was also performed using a titanium dioxide and oil mixture for flow visualization. Free-stream speed was $U_\infty = 100$ fps ($Re_c \cong 1.28 \times 10^6$). Airfoil incidence, flap angle and spoiler height were varied during this series.

For various fixed spoiler heights testing was accomplished in order to measure RMS pressure fluctuations at four chordwise locations aft of the spoiler.

Steady-flow tests were performed with the boundary-layer rake and the following parameter variations.

α ($^\circ$)	U_∞ (ft/sec)	$Re_c \times 10^{-6}$	h_s (in)	B.L. Rake Position	
				spanwise	chordwise, x
0	150	1.92	0	ϕ	.582
4	100	1.28	.125	$\downarrow$	.997
8	75	.96	.250	$\downarrow$	.900 (facing aft)
	50	.64	.375	On STBD endplate	.583
			.500	$\downarrow$	$\downarrow$
			.625	$\downarrow$	$\downarrow$

A trip-wire of .013" diameter was placed at 5% chord on upper and lower surfaces during some of the tests.

Free-stream turbulence levels were measured in a series of tests involving free-stream speeds of 50, 75, 100 and 150 ft/sec.

The effect of sealing gaps at the end-plates for the spoiler and flap was investigated by collecting steady-flow pressure data and by use of surface-flow visualization.

Flap-Fixed, Spoiler-Forced Testing

A series of tests was recorded with the thin locking bar (SG_1) in place and one of two flap inertia configurations (A_1 or A_2). Angle-of-attack and flap-angle were zero. A series was also recorded using the thick locking bar (SG_2) and a lower inertia configuration (A_3). The following actual parameter settings were used for the spoiler in this series

Case	h_{O_s} (in)	h_o	Δh_s (in)	Δh
A	.160	.027	.159	.027
B	.320	.053	.157	.026
C	.458	.076	.159	.027
D	.235	.039	.235	.039
E	.372	.062	.237	.040
F	.310	.052	.311	.052
G	.311	.052	.312	.052

where Case G involved the part-span spoiler.

The following additional nominal parameter settings were used.

U_∞ (ft/sec)	150		100		75		
$Re_c \times 10^{-6}$	1.92		1.28		.96		
ν	.015	.030	.045	.060	.075	.090	.105
	.120	.135	.150	.175	.200	.240	.280

For each free-stream speed a frequency limit existed due to limitations of the drive system. A max frequency of 6.7 cps relates to

U_∞ (ft/sec)	ν_{max}
150	.140
100	.210
75	.280

During an early series of testing, before installation of most pressure transducers, measurements of mean pressure were taken from the manometer.

Cine-film recordings were made of tuft response both with the flap fixed and free. Incidence was set at zero. The following parameter variations were used.

U_{∞} (ft/sec)	h_{0s} (in)	Δh_s (in)	Spoiler
150	.24	.24	Full-Span
125	.31	.31	Part-Span
100			
50			

The effect of trip wires located near the leading-edge at 5% chord was investigated with free-stream speeds of 75, 100 and 150 ft/sec.

With the flap-fixed and full-span spoiler motion of the form: $h = .052 + .052 \cos \nu \tau$ hinge-moment coefficients were measured for incidence of 4° and 8° ($\beta = 0$) and for fixed flap angles of -4° and $+4^\circ$ ($\alpha = 0$). This involved a free-stream speed of 150 ft/sec.

Also a short test series involved spoiler motion of the form: $h = .017 + .017 \cos \nu \tau$.

Spoiler-Fixed, Flap-Forced Testing

The flap was forced with the spoiler fixed and airfoil angle-of-attack set at zero. Oscillation amplitudes of 5° and 10° were used. Mean flap angle was zero. Parameter settings were as follows.

U_{∞} (ft/sec)	$Re_c \times 10^{-6}$	ν	h_s (in)	h
150	1.92	.02	0	0
100	1.28	.04	.125	.021
75	.96	.06	.250	.042
		.08	.375	.063
		.10	.500	.083
		.12	.625	.104
		.15	.750	.125
		.18		
		.21		
		.24		

Fixed, fence-type spoilers were also attached at two other chordwise positions and two spoiler heights were involved in a series of testing with flap-forcing at a free-stream speed of 150 ft/sec. The parameter combinations were

h_s (in)	Chordwise Position, x
.750	.30
.500	.30
.500	.45

The effect of trip wires at 5% chord was also observed at free-stream speeds of 75 and 150 ft/sec.

Spoiler-Forced, Flap-Free Testing

The flap was free to respond with airfoil angle-of-attack set at zero. Flap system configuration primarily involved stiffness and inertia parameters C_1 and A_2 . The actual spoiler parameters are as shown below. The case letter refers to a nearest corresponding flap-fixed case.

Case	h_{o_s} (in)	h_o	Δh_s (in)	Δh
A	.157	.026	.157	.026
B	.324	.054	.165	.028
C	.468	.078	.169	.028
D	.240	.040	.238	.040
D'	.250	.042	.160	.027
E	.397	.066	.247	.041
F	.316	.053	.316	.053
F'	.311	.052	.312	.052
G	.311	.052	.312	.052

Inertia configuration A_1 was used for F' and G. Case G also involved the part-span spoiler. The tests were accomplished at $U_\infty = 150$ ft/sec ($Re_c = 1.92 \times 10^5$) with nominal reduced frequencies of: $\nu = .020, .040, .050, .060, .070, .080, .090, .100, .110$ and $.130$.

Cine-film recordings of tuft response were also accomplished.

Several runs were made at high airfoil incidence (up to 19°) in order to study spoiler and flap oscillation effects and to observe flap-free buffeting. The effect of airfoil stalling was also investigated.

Coupled Spoiler and Flap Testing

Tests were conducted with the flap coupled to the spoiler in order to study possible self-excited oscillations and limit-cycle behavior. This series of testing involved a range of system inertias and coupling parameter variations (h_{o1} and k) where:

$$h \equiv \frac{h_s}{c_f} = h_{o1} + k \beta$$

and

$$0 \leq h_{o1} < .097$$

$$.30/\text{rad} < k < .40/\text{rad}$$

Also two structural stiffness values were involved. It was found that dry friction existed in the coupled system because of contact between the spoiler and spoiler groove. Furthermore, the dry friction was a function of the drag force on the spoiler, which depends on spoiler height and free-stream speed. Because of the amount of dry friction, self-excited oscillations were not possible; therefore, an initial condition was applied to the system via a simple attachment to the flap balance rod in order to study limit-cycle behavior.

2.3. Test Results

Flap and Spoiler-Fixed Testing

Cavalcanti (Ref. 21) conducted an experimental program on a similar NACA 0012 airfoil and spoiler model. His work serves as a background to the present program and offers a comprehensive analysis of steady-flow pressure and hinge-moment effects.

Most of the steady-flow analysis gathered in the present experiment is not included here since it serves primarily to confirm Cavalcanti's results. See reference 20 for a summary of his results.

Steady-flow hinge-moment coefficient as a function of flap angle and spoiler height is plotted in Figure 2.9. Hinge-moment coefficient is once again defined as:

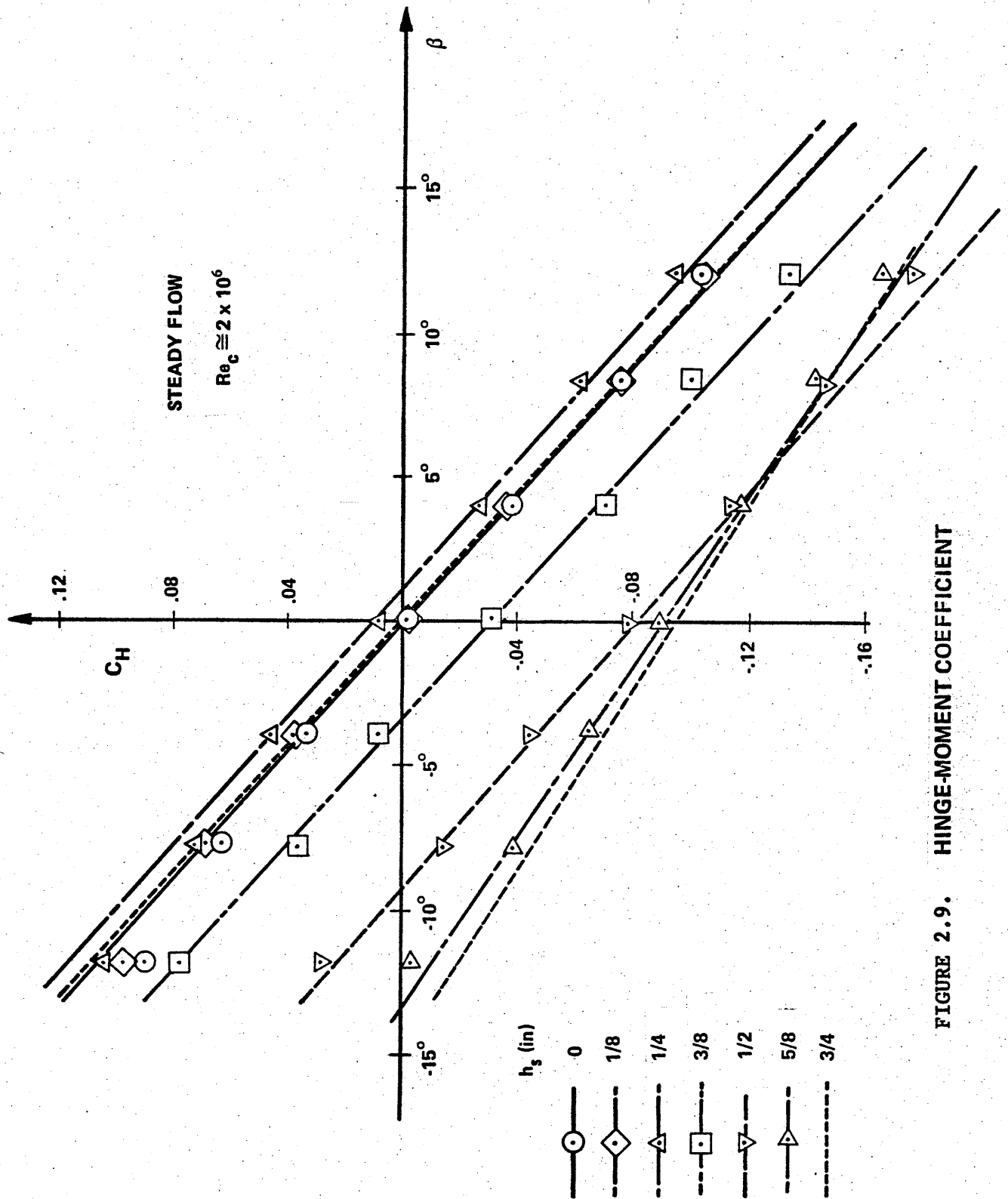


FIGURE 2.9. HINGE-MOMENT COEFFICIENT

$$C_H \equiv \frac{H}{q_\infty b_f c_f^2}$$

where positive is T.E. down.

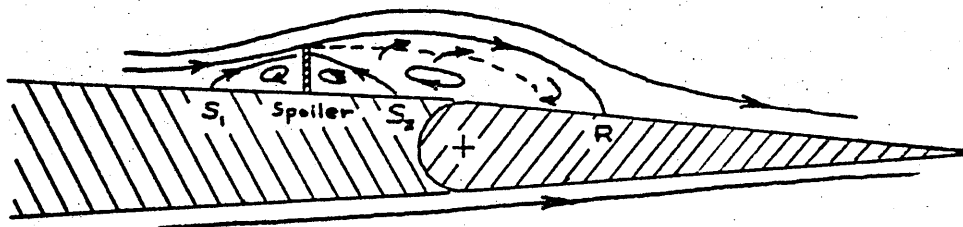
It can be seen that $C_{H\beta}$ is nearly independent of spoiler height for spoiler heights less than about 5/8". This relates to the geometry and pressure distribution of the shear layer aft of the spoiler and is associated with turbulent mixing and recompression which occurs on the flap surface. The straight-line segments shown in Figure 2.9 are plotted from the flap-forcing data for $C_{H\beta}$ extrapolated to zero frequency. The extrapolated value of $C_{H\beta}$ for low spoiler heights is $C_{H\beta} \cong -.520/\text{rad}$ as compared to the two dimensional thin airfoil theory result $C_{H\beta} = -.944/\text{rad}$. The two-dimensional "real flow" prediction (from the Royal Aeronautical Society Data Sheets) is $C_{H\beta} \cong -.561/\text{rad}$.

In reference 22 it is explained that when reattachment or recompression of the separated shear-layer occurs on the airfoil (or flap) surface "... the surface is controlling the mainstream," as it does for attached flow. "For reattachment well aft in the wake, the wing [or flap] can hardly control the flow ..."

Control of the flow is evident in the data of reference 21. For a fixed spoiler height of 1/4", flap deflection can cause large velocity variations at the spoiler position. For a downward flap deflection, velocity at the spoiler increases and separated region length decreases. For larger spoiler deflections the flap has less effect.

Steady-flow pressures and hinge-moment coefficient are plotted as functions of spoiler height in Figure 2.10. The data points are connected by curves which are reconstructed from data obtained during spoiler oscillation tests. The curves represent average data from cases A-F extrapolated to zero frequency. No evidence of aerodynamic hysteresis was found in all the testing, possibly because of the large turbulent intensities associated with the flow field.

The flow-field associated with the spoiler is extremely complex and not well understood. However, a sketch of the flow pattern along the airfoil centerline can be postulated as: 5,7-10



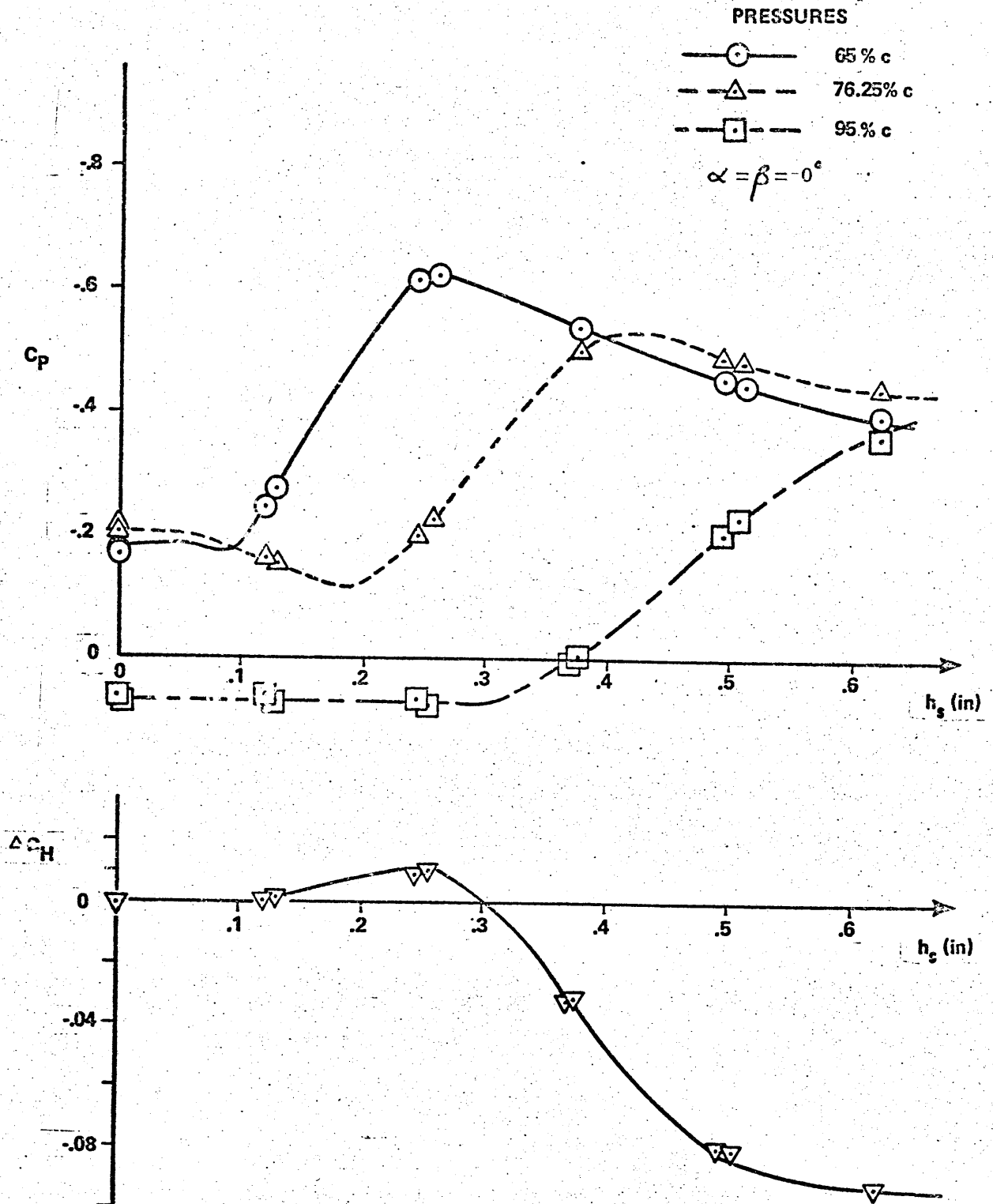
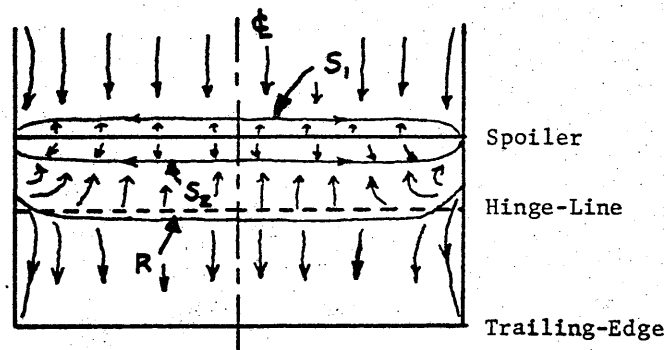


FIG. 2.10. STEADY-FLOW PRESSURES AND HINGE-MOMENT

As spoiler height increases, the length of the separation "bubble" increases. For $h_s < .3$ " there is a negligible pressure effect at the trailing-edge.

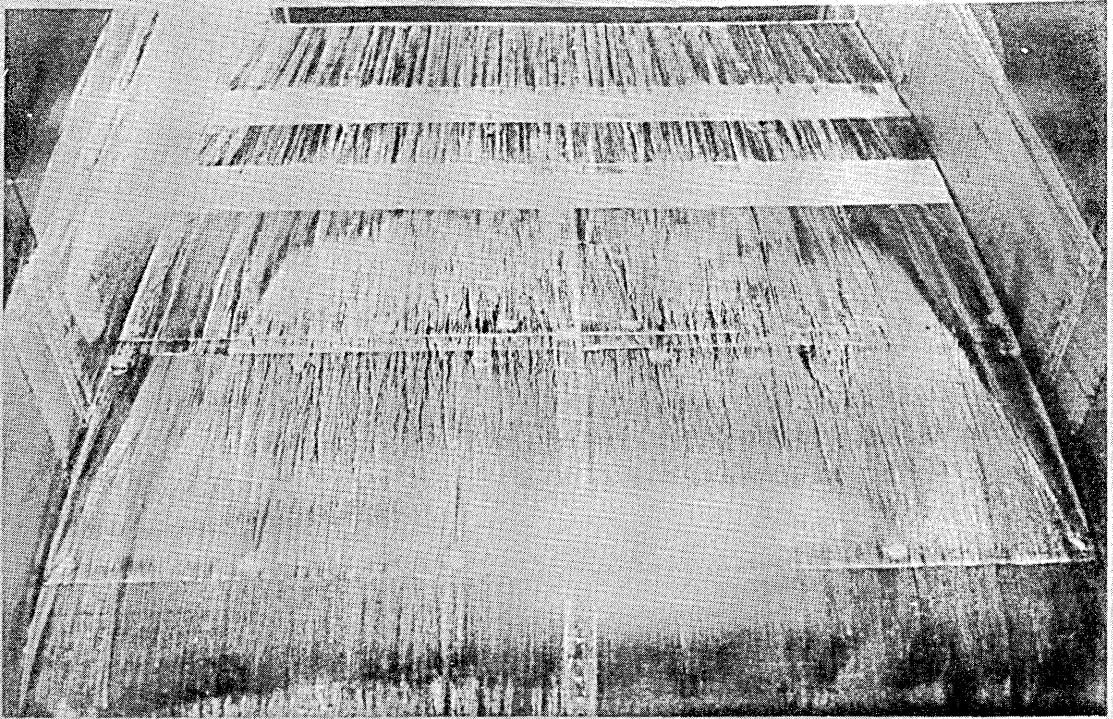
Refer to Figures 2.11 and 2.12 for some results of flow-visualization testing. With the spoiler retracted, natural transition occurs over most of the span at about 50% chord (for $Re_c \cong 1.28 \times 10^5$). The end-plates provoke early transition and a spread of the associated turbulent boundary-layer. Also shown is the transition effect of a small piece of plasticene located near the L.E. (later removed). With the spoiler extended $1/2$ " the effect of counter-rotating vortices is evident near the end-plates. Figure 2.12 shows that the vortex effect increases as spoiler height increases. When the gaps along the spoiler and flap are sealed at the end-plates, the vortex centers were observed to move slightly closer to the end-plates. Pressures along the centerline in the separated region are only slightly affected. In all cases a centerline symmetry is apparent which was desired in the experiment.

The surface flow associated with the vortices is sketched below for a spoiler height of $1/4$ " (refer to Figure 2.12).

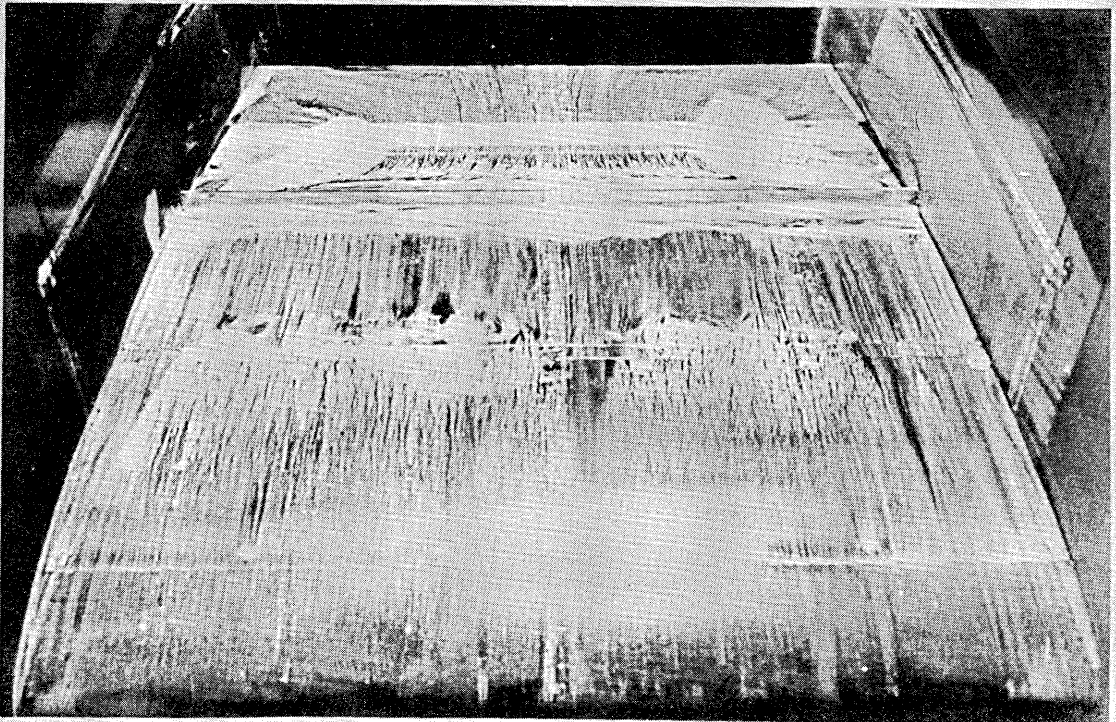


Two lines of separation and a reattachment line near the hinge-line are evident for $h_s = 1/4$ ". All lines show little curvature across the span. For larger spoiler heights the vortices grow in size and the separation and reattachment lines move farther from the spoiler and have more curvature. For spoiler heights of .44 and .63" reversed flow occurs along the flap centerline. For $h_s = .44$ " a line of "confluence" probably exists just aft of the trailing-edge in the wake. The separation lines in front and aft of the spoiler show evidence of pooling of the fluid used for testing. The irregularity in these lines is probably associated with irregular application of the fluid and not primarily due to spoiler effects.

The flow visualization testing, to include the cine-film records, provided measurements of reattachment point location. These are plotted in Figure 2.13 where $\Delta x_{sp} \equiv (x - x_{sp}) \frac{c}{c_f}$. Cine-film results extrapolated to zero frequency lie in the shaded regions shown.



SPOILER RETRACTED
($h_s = 0''$)



SPOILER EXTENDED 0.5"
($h_s = 0.5''$)

FIGURE 2.11. SURFACE FLOW PATTERNS AS SEEN FROM UPSTREAM.

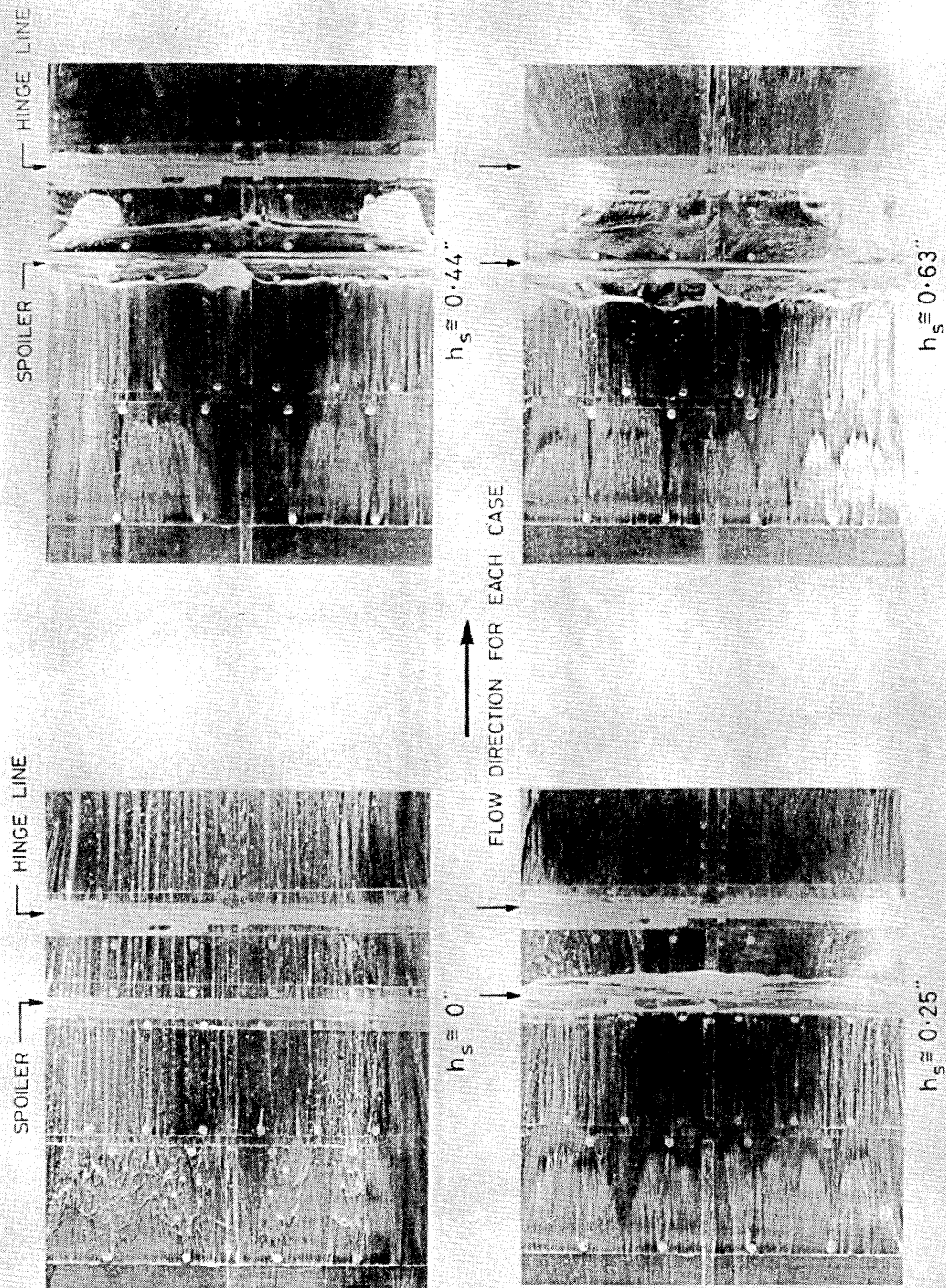


FIGURE 2.12. SURFACE FLOW PATTERNS FOR DIFFERENT SPOILER HEIGHTS.

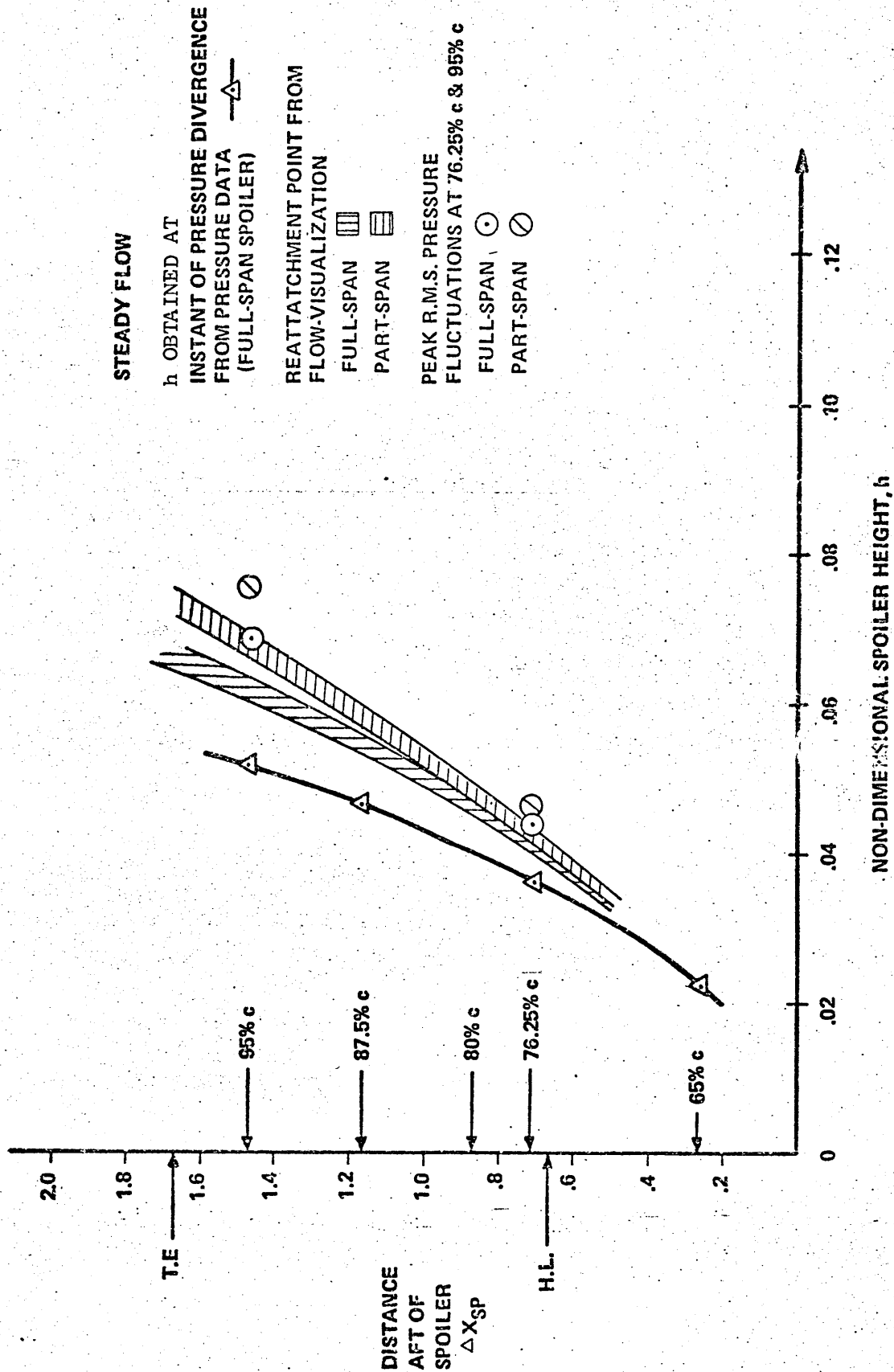
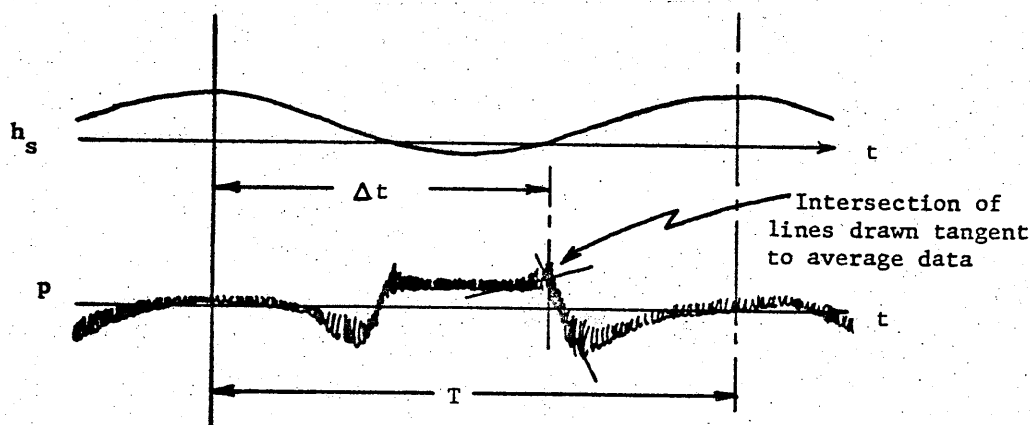


FIGURE 2.13. SPOILER HEIGHT EFFECT ON SEPARATION BUBBLE LENGTH

RMS pressure fluctuations at 76.25 and 95% chord are shown in Figure 2.14. The data is plotted versus $\frac{h - .02}{\Delta x_{sp}}$ and shows that peak levels occur at near constant values of $\frac{h - .02}{\Delta x_{sp}}$. Data taken at 65 and 87.5% is not plotted due to wide scatter, but generally follows the trends shown. These results follow those found by Heller, et. al.²³, Lawford & Beauchamp,²⁴ Fricke and Stevenson²⁵ and Mabey²⁶ in the study of separated regions. Also spectral density analysis of the fluctuations shows that as the bubble grows, the high frequency content of the fluctuations decreases. For zero spoiler height, values of $\frac{\langle p \rangle}{q_\infty}$ are close to the expected value for a turbulent boundary-layer, i.e., $\frac{\langle p \rangle}{q_\infty} \cong 6 \times 10^{-3}$.

Also shown in Figure 2.13 are data points taken from recorded data of the instant of pressure divergence at a transducer location. This instant was found from the raw data as follows:



Data was found for all frequencies and extrapolated back to zero frequency (see Figure 2.17). The corresponding spoiler height was found from:

$$h = h_0 + \Delta h \cos 2\pi \frac{\Delta t}{T}$$

and plotted for all cases on Figure 2.13.

In relation to the reattachment point, the point of pressure divergence lies downstream at about 130% of bubble length and the peak value of RMS pressure fluctuations occurs at about 85% of bubble length. For spoiler height greater than about .2" (i.e., $h > .033$) the separation bubble grows with spoiler height at a constant rate. For the full-span spoiler $d_R \cong 38$ and for

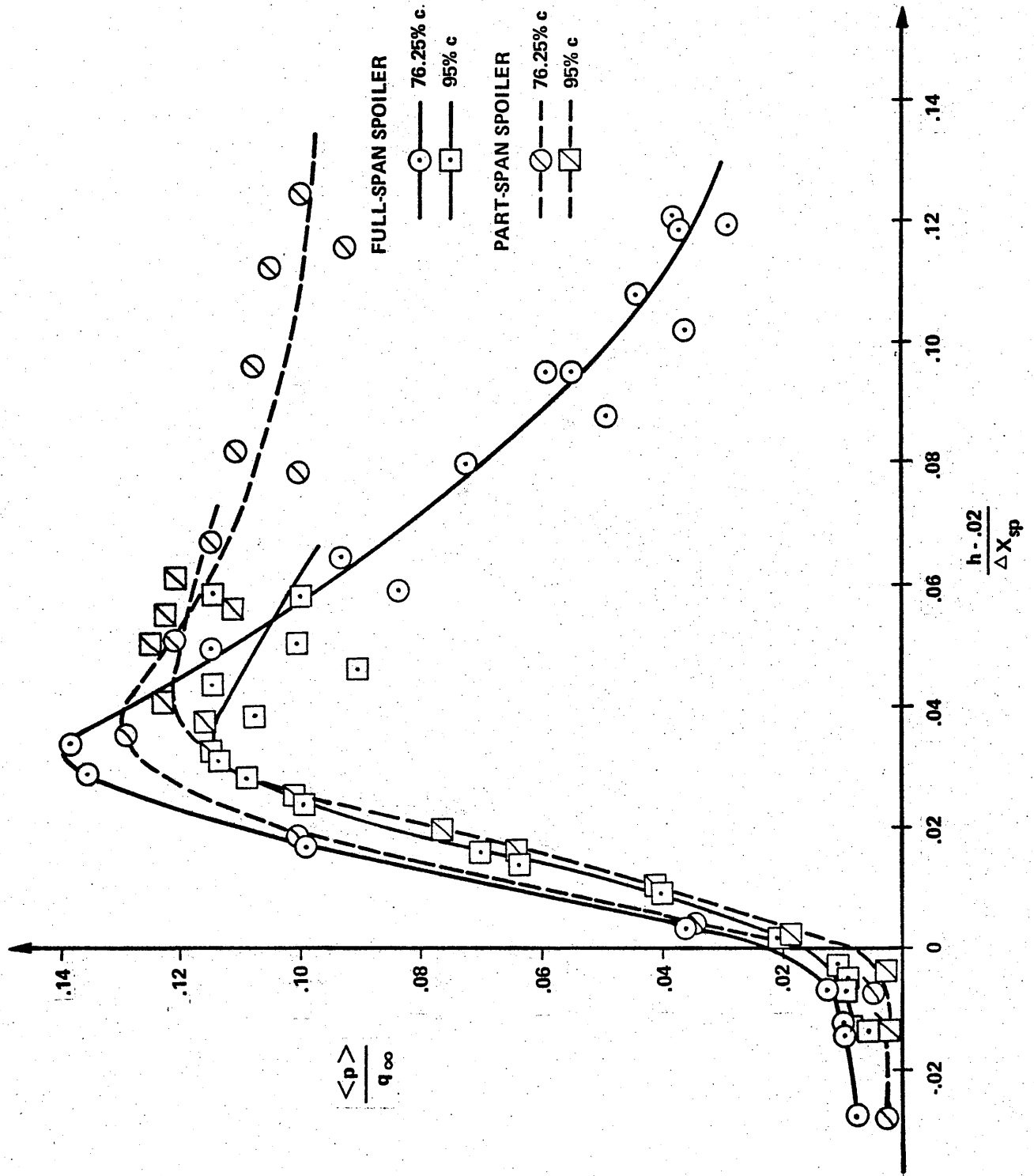


FIGURE 2.14. SHEAR-LAYER "NOISE" INTENSITY

the part-span spoiler $d_R \cong 31$ where $d_R \equiv \frac{\delta \Delta x_{sp}}{\delta h}$. For low spoiler heights (i.e., $h < .03$) the relation for both spoilers is approximately $d_R \cong 10$.

Boundary-layer analysis yields data which is tabulated in reference 20. Some of the displacement thickness data is summarized in Figure 2.15. For zero incidence and no spoiler deflection with no trip wire, boundary-layer thickness and displacement thickness at the spoiler location are found to be $\delta \cong .170''$ and $\delta^* \cong .026''$.

Free-stream turbulence intensities are shown in the table below as a function of mean free-stream speed and Reynolds number.

U_∞ (ft/sec)	$Re_c \times 10^{-6}$	$\frac{\langle u \rangle}{U_{avg}} \cdot 100$	$\frac{\langle u \rangle}{U_{max}} \cdot 100$
50	.64	.165	.223
75	.96	.199	.276
100	1.28	.250	.410
150	1.92	.410	.775

The maximum values are associated with "bursts" of turbulence that occurred at irregular intervals. The average value at 150 ft/sec corresponds to a turbulence factor of 1.33 which is considered high. In reference 27, Evans shows that high levels of free-stream turbulence can modify and even eliminate the laminar separation bubble on an airfoil. In the present experiment, so-called Reynolds number effects could contain effects of free-stream turbulence, but it will be seen that these are relatively minor.

Pressures on the airfoil wetted surface depend on spoiler height. Pressure coefficient as a function of spoiler height is plotted in Figure 2.16 for four chordwise locations; i.e., 1.25 and 45% chord on the top surface and at 82.5 and 90% chord on the bottom surface. In the figure a pressure coefficient, δC_p , is shown where:

$$\delta C_p \equiv C_p - (C_p)_{h=0}$$

Steady-flow theories do not account for the shear-layer reattachment or recompression characteristics but could be useful for predicting loading on the wetted surface.²⁸⁻³⁰ In Woods' theory, found in reference 28, pressure coefficients on the wetted surface are said to be proportional to:

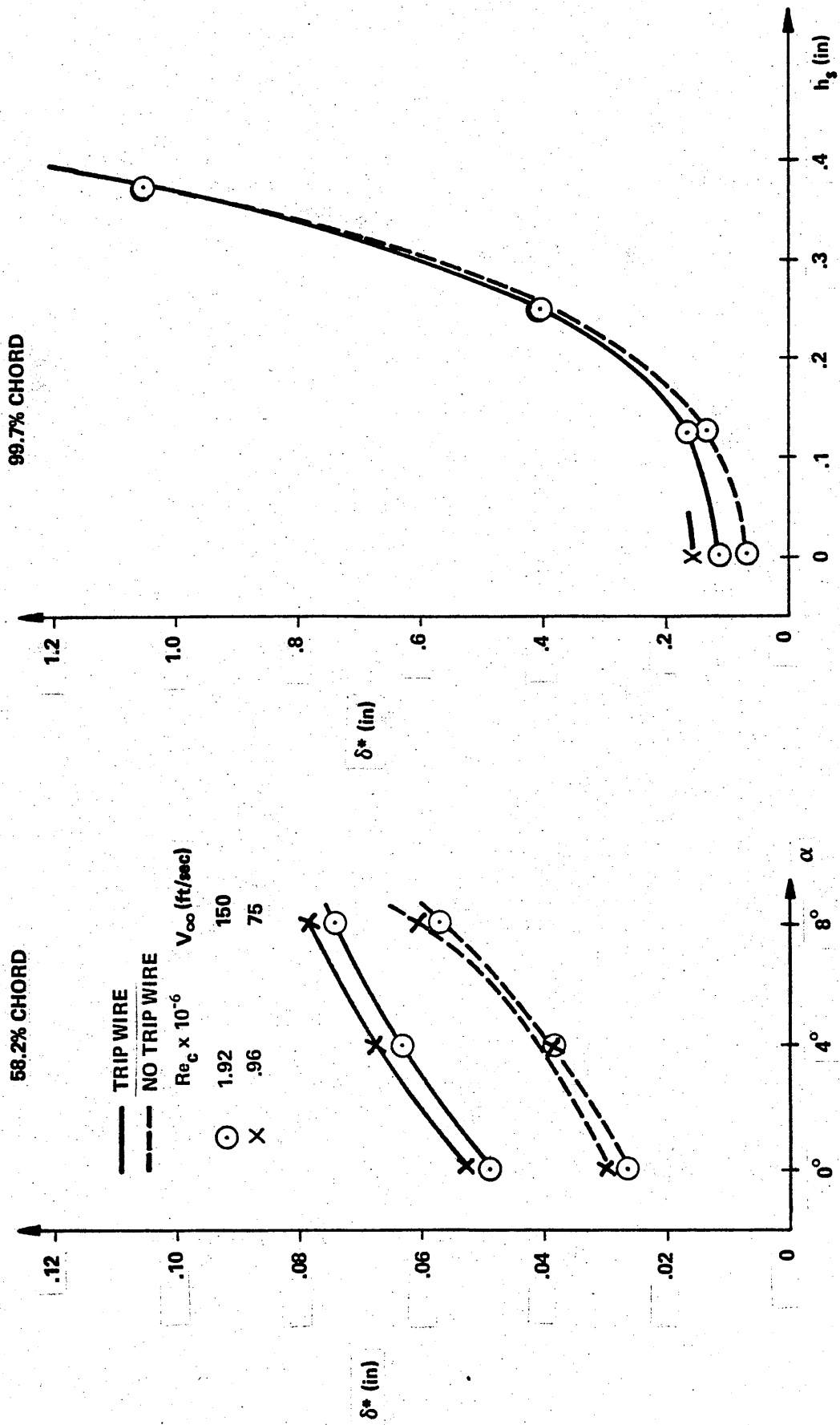


FIGURE 2.15. BOUNDARY - LAYER DISPLACEMENT THICKNESS

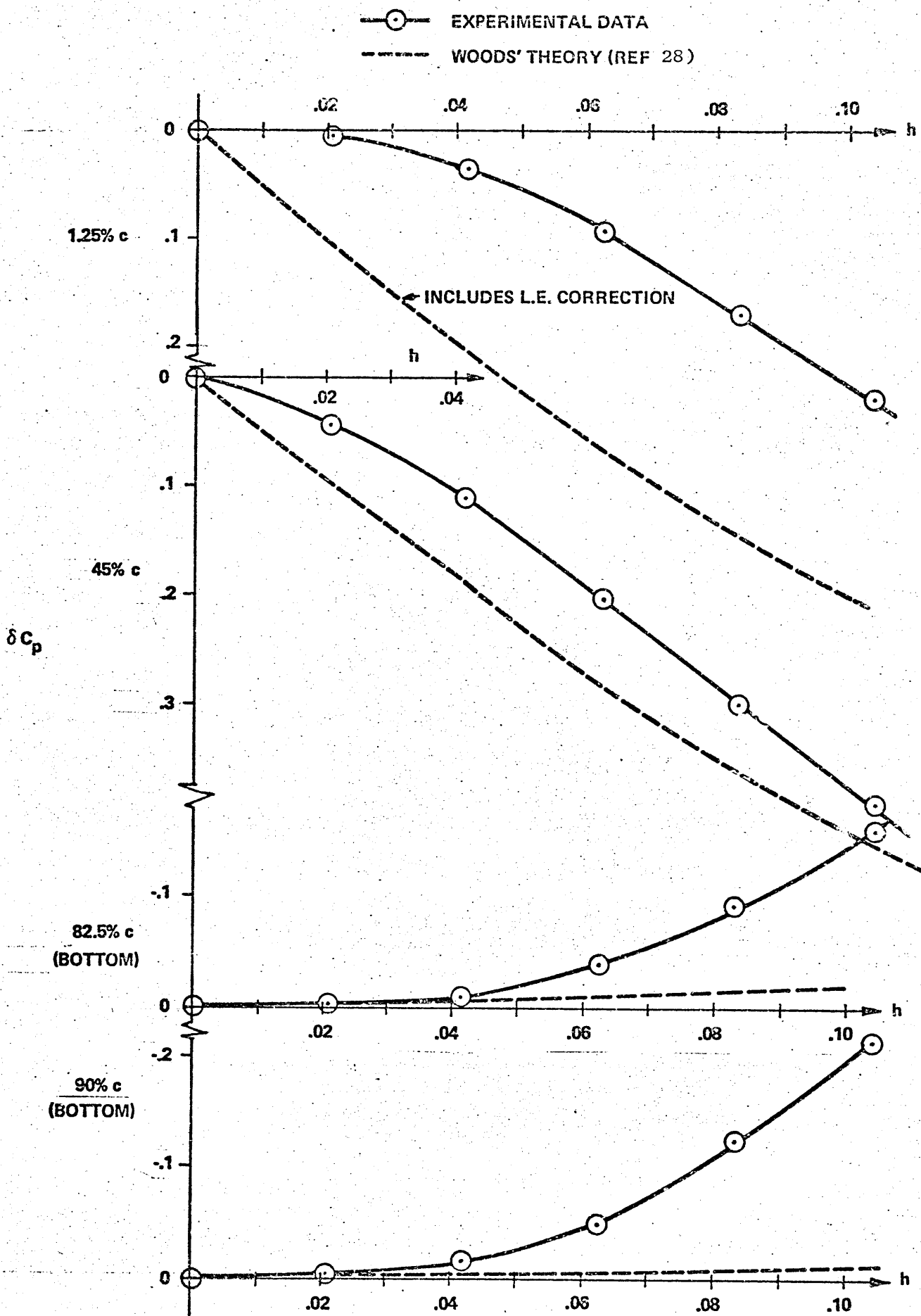


FIG 2.16. STEADY-FLOW PRESSURES ON WETTED SURFACE

$$\frac{h_s}{\sqrt{\delta}} \quad \text{for} \quad 0 \leq h_s \leq 2\delta$$

and

$$\sqrt{h_s - \delta} \quad \text{for} \quad 2\delta < h_s$$

Woods' predictions are plotted in Figure 2.16 along with the experimental results. Near the leading-edge Lighthill's correction factor has been applied to the theory.³¹ There are major discrepancies at the L.E. and on the flap under surface. Woods' theory does not account for a divergence in pressure at the T.E., hence the large discrepancy on the under surface.

At present there is no theory which is valid for spoiler heights which involve reattachment or recompression of the shear layer on the airfoil surface. Theoretical predictions of $C_{H\beta}$ in the presence of this type of spoiler effect are also not available at present.

Flap-Fixed, Spoiler-Forced Results

With the flap fixed the hinge-moment coefficient due to unsteady spoiler effects was evaluated. For spoiler motion of the form:

$$h = h_0 + \Delta h \cos \nu \tau$$

the incremental hinge-moment coefficient is expressed in Fourier series form as:

$$\Delta C_H = \Delta C_{H_0} + \sum_{n=1}^q \Delta C_{H_n} \cos (n \nu \tau + \psi_n)$$

Values of $\frac{\Delta C_{H_0}}{\Delta h}$, $\frac{\Delta C_{H_n}}{\Delta h}$ and ψ_n for n up to 4 are shown as a function of reduced frequency for cases A-G in Figures 2.17A-G. The relatively large scatter on the data points is inherent in the method of data reduction (explained in reference 20.); however, averaged curves could be drawn through the data points as shown. Note that all phase angles extrapolate to either 0° or 180° as frequency approaches zero.

Extrapolation to zero frequency was done for all cases involving the full-span spoiler and an average "steady-flow" curve was constructed (see Figure 2.10). From the averaged curve a prediction was made of the Fourier components for all

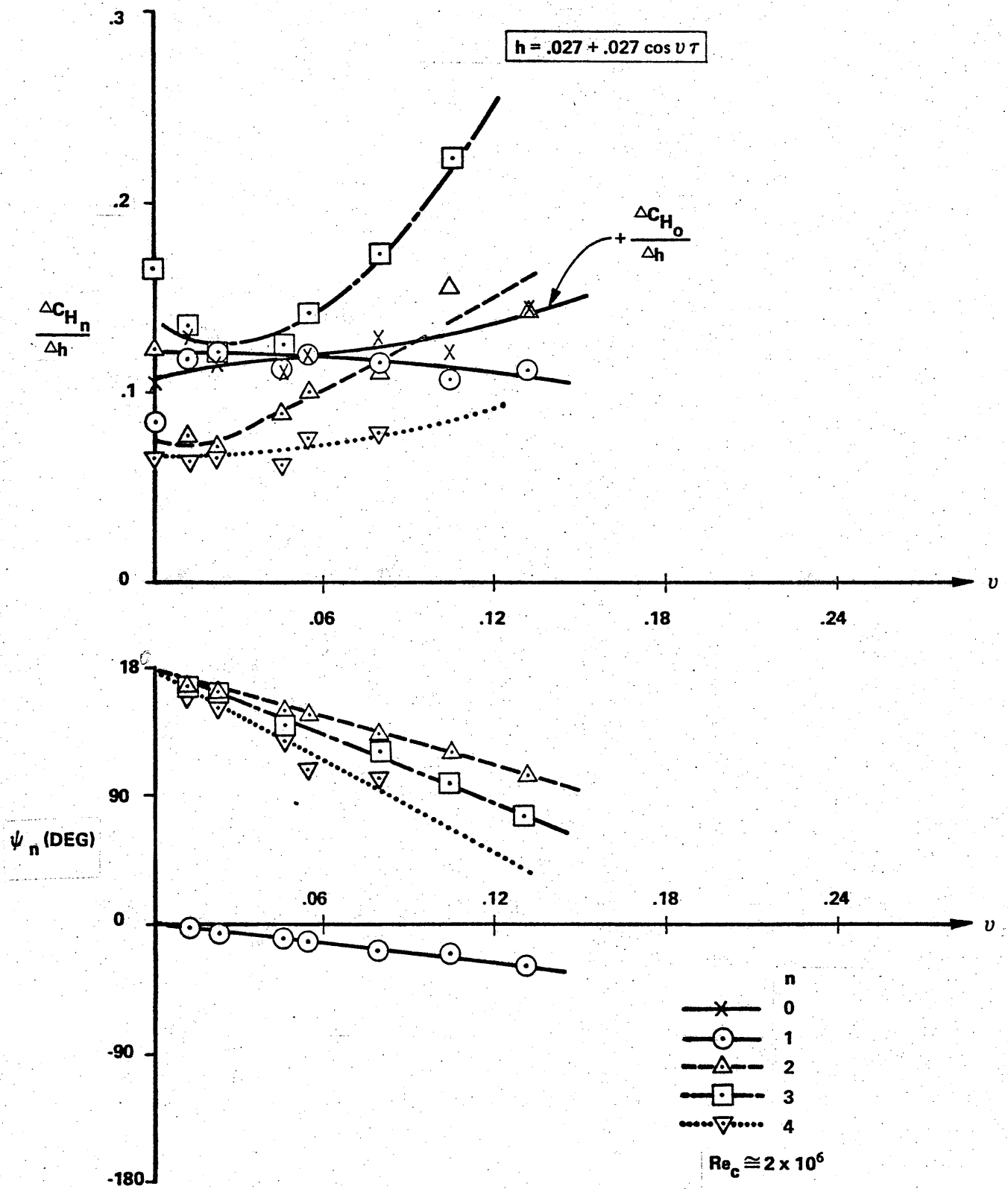


FIGURE 2.17A. COMPONENTS OF HINGE-MOMENT COEFFICIENT

$$h = .053 + .026 \cos v \tau$$

-40-

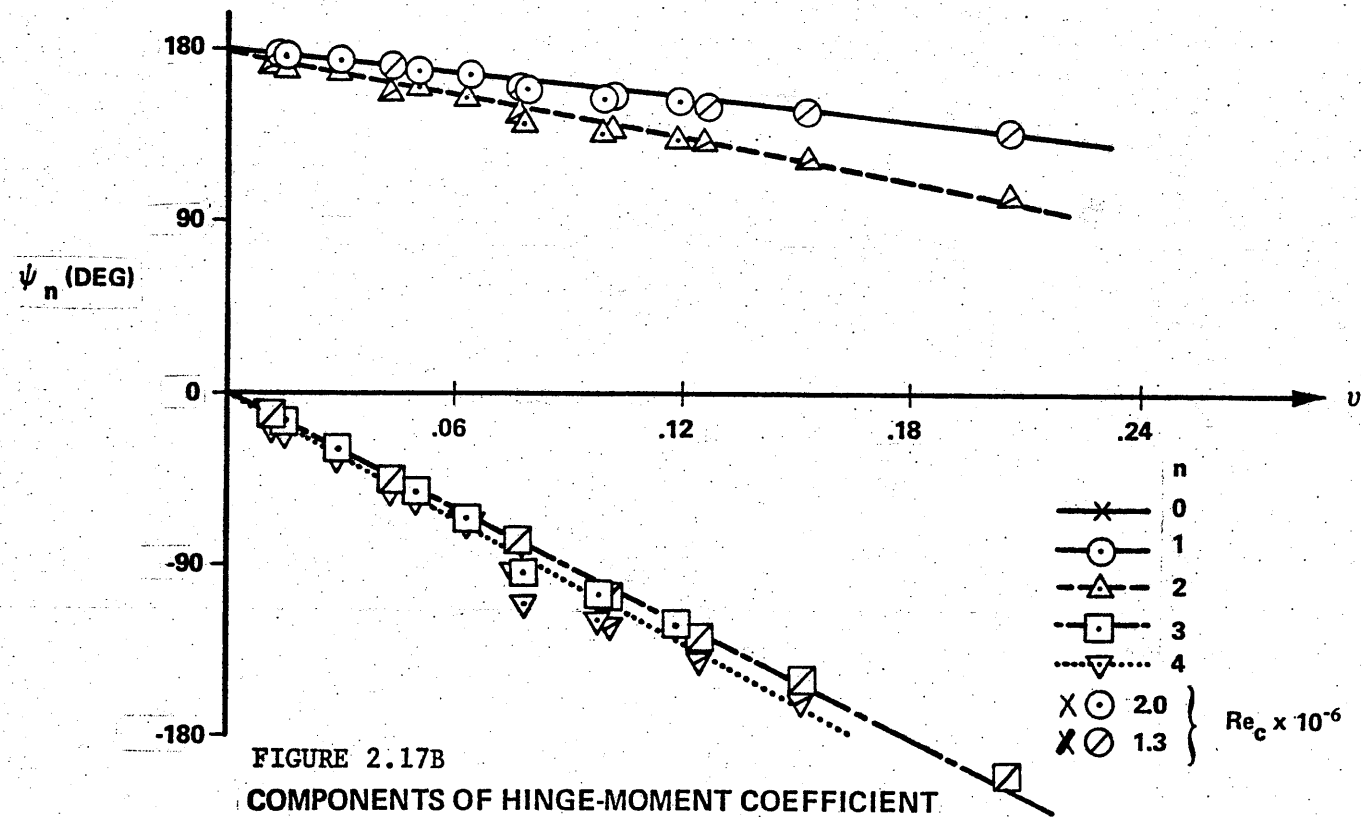
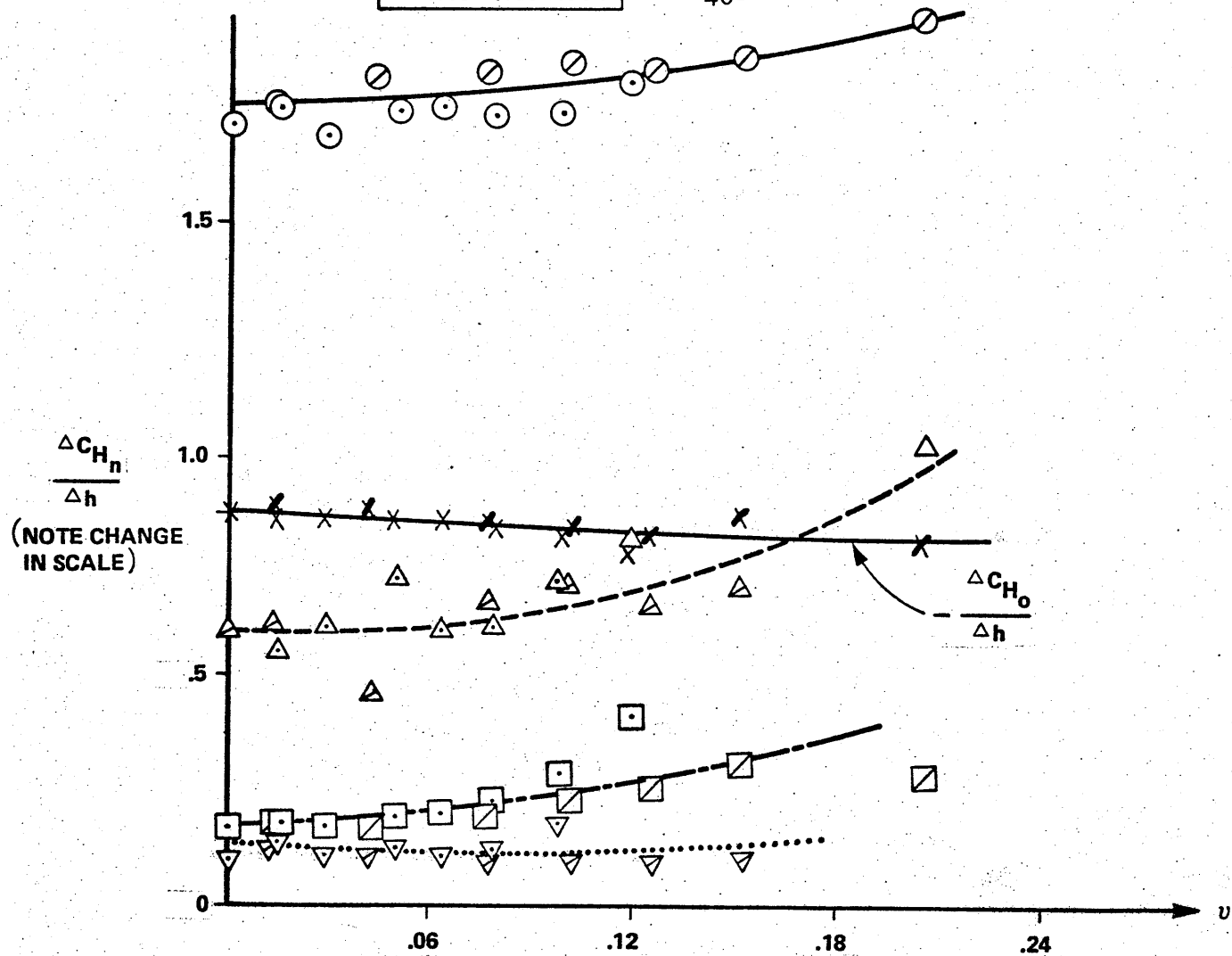


FIGURE 2.17B
COMPONENTS OF HINGE-MOMENT COEFFICIENT

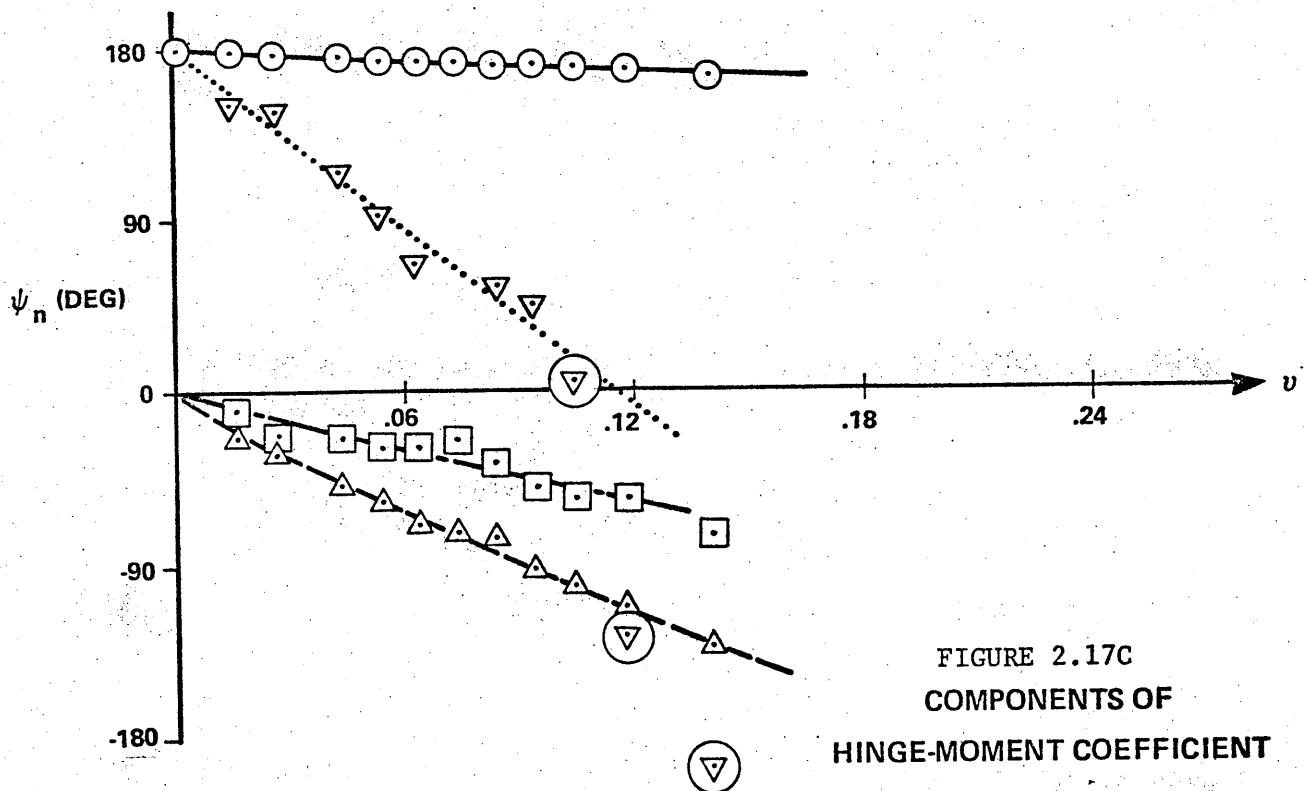
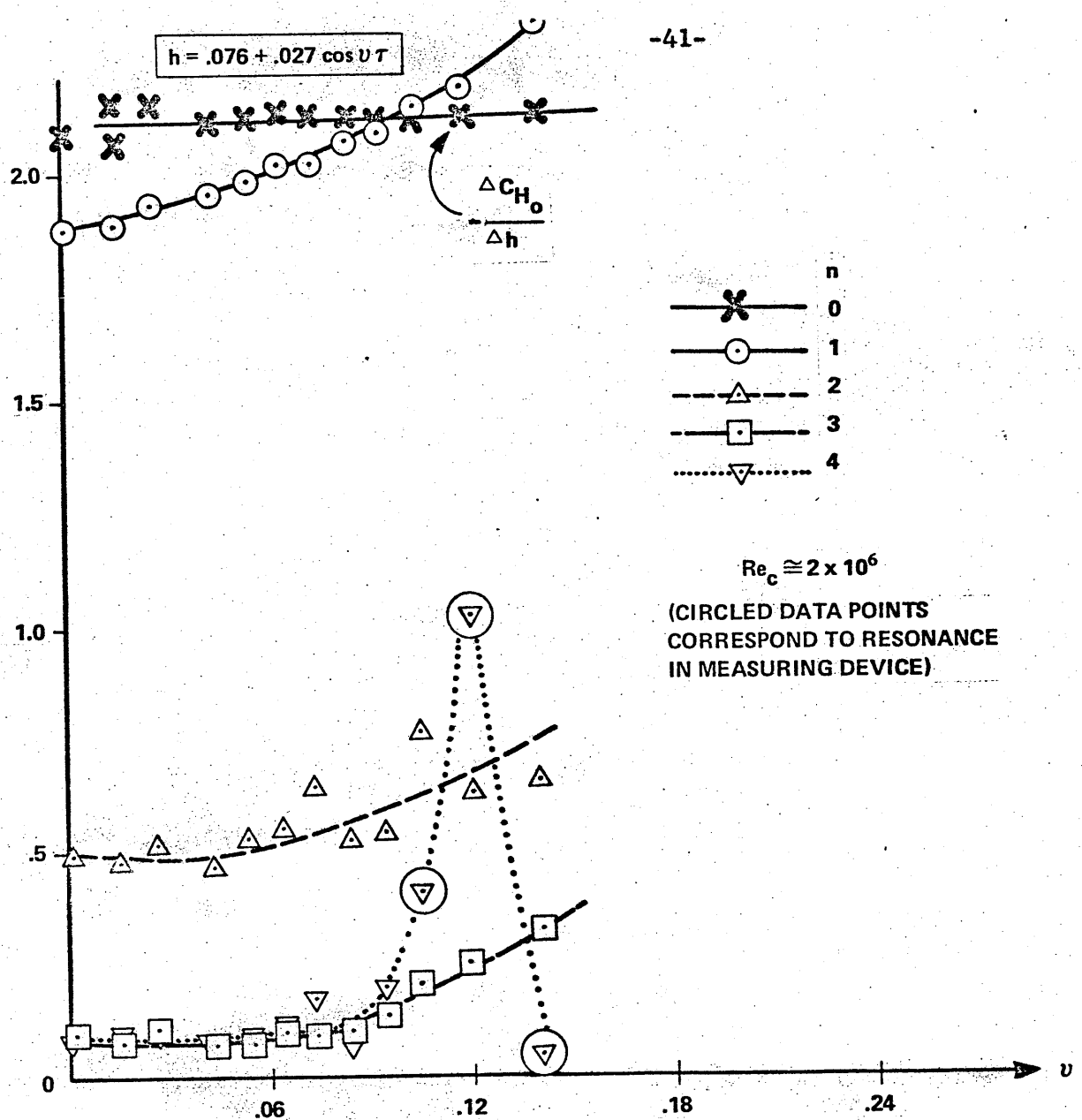


FIGURE 2.17C
COMPONENTS OF
HINGE-MOMENT COEFFICIENT

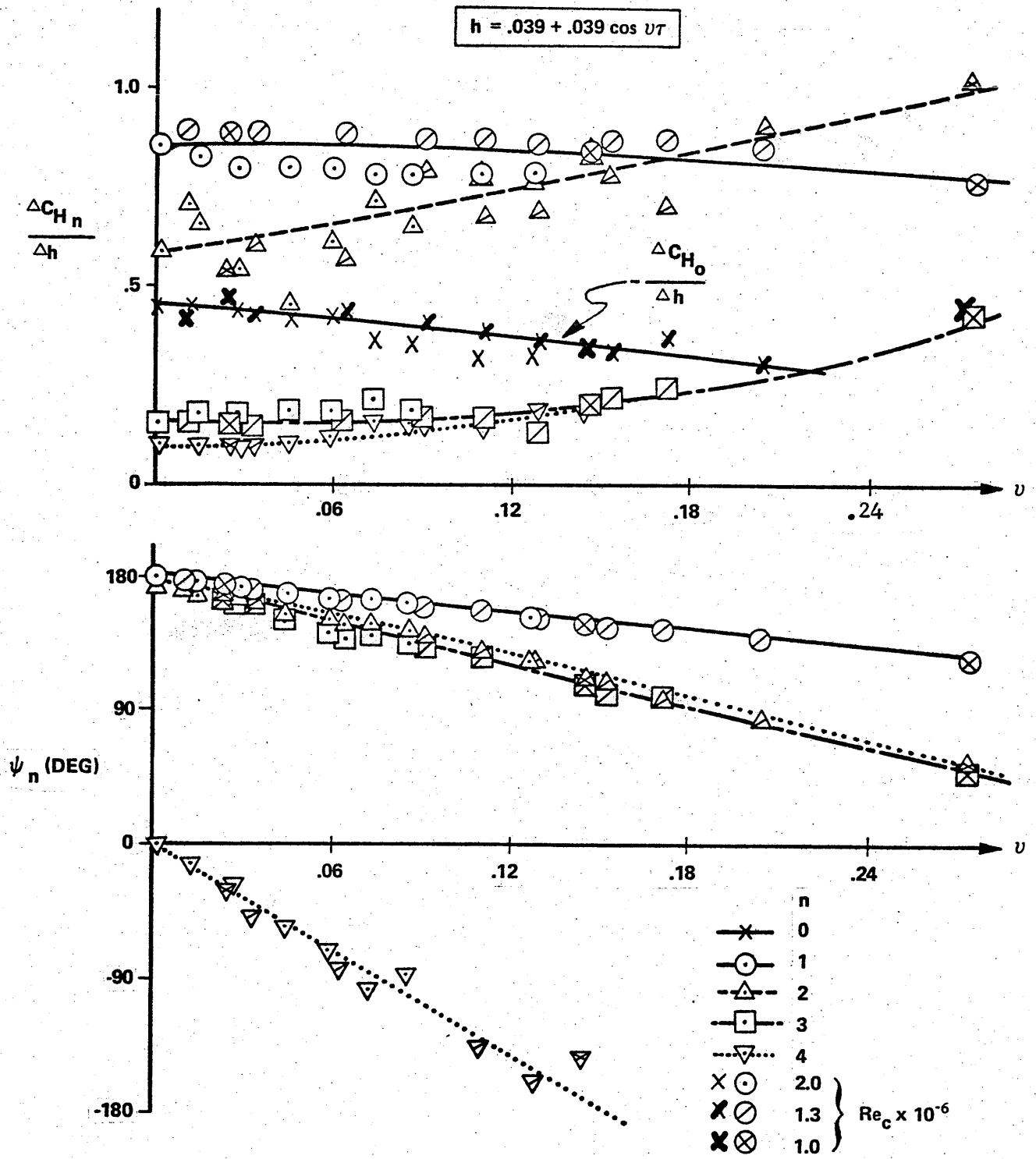


FIGURE 2.17D. COMPONENTS OF HINGE-MOMENT COEFFICIENT

$h = .062 + .040 \cos U T$

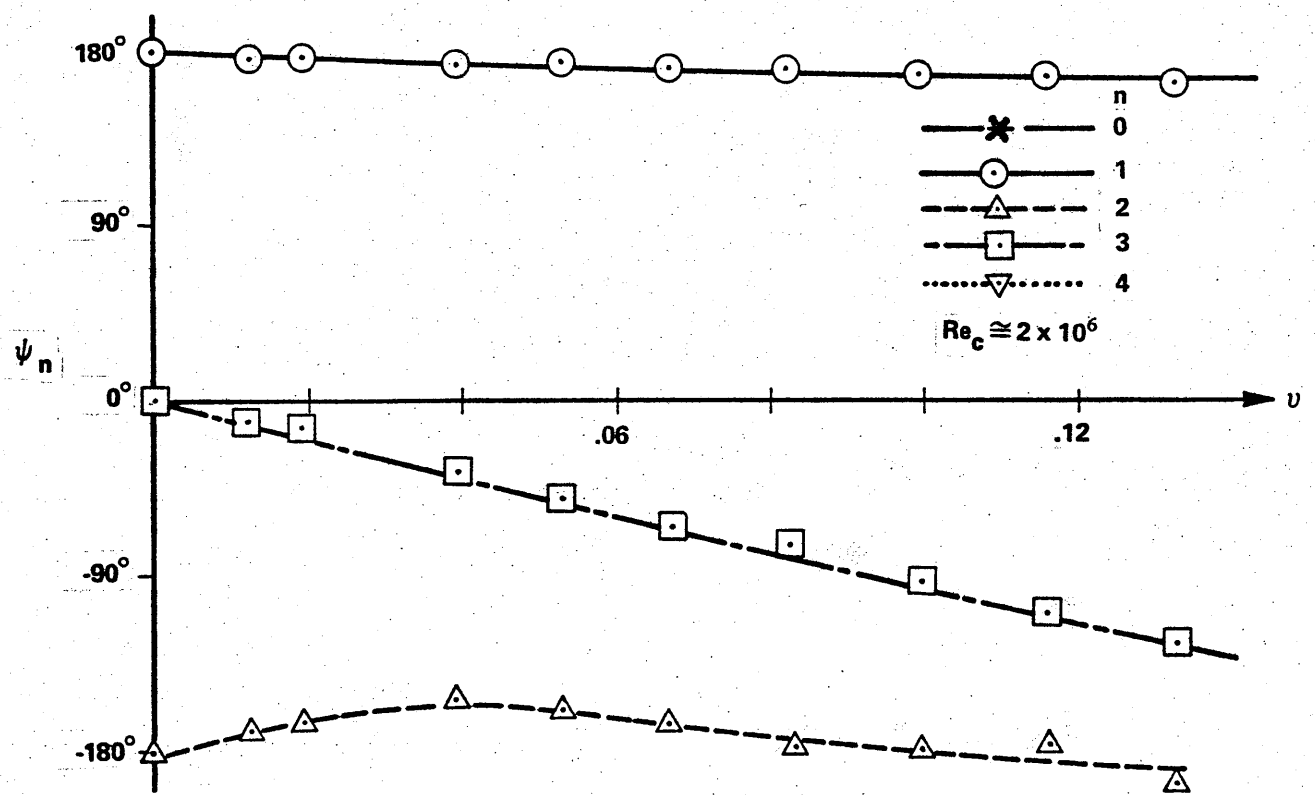
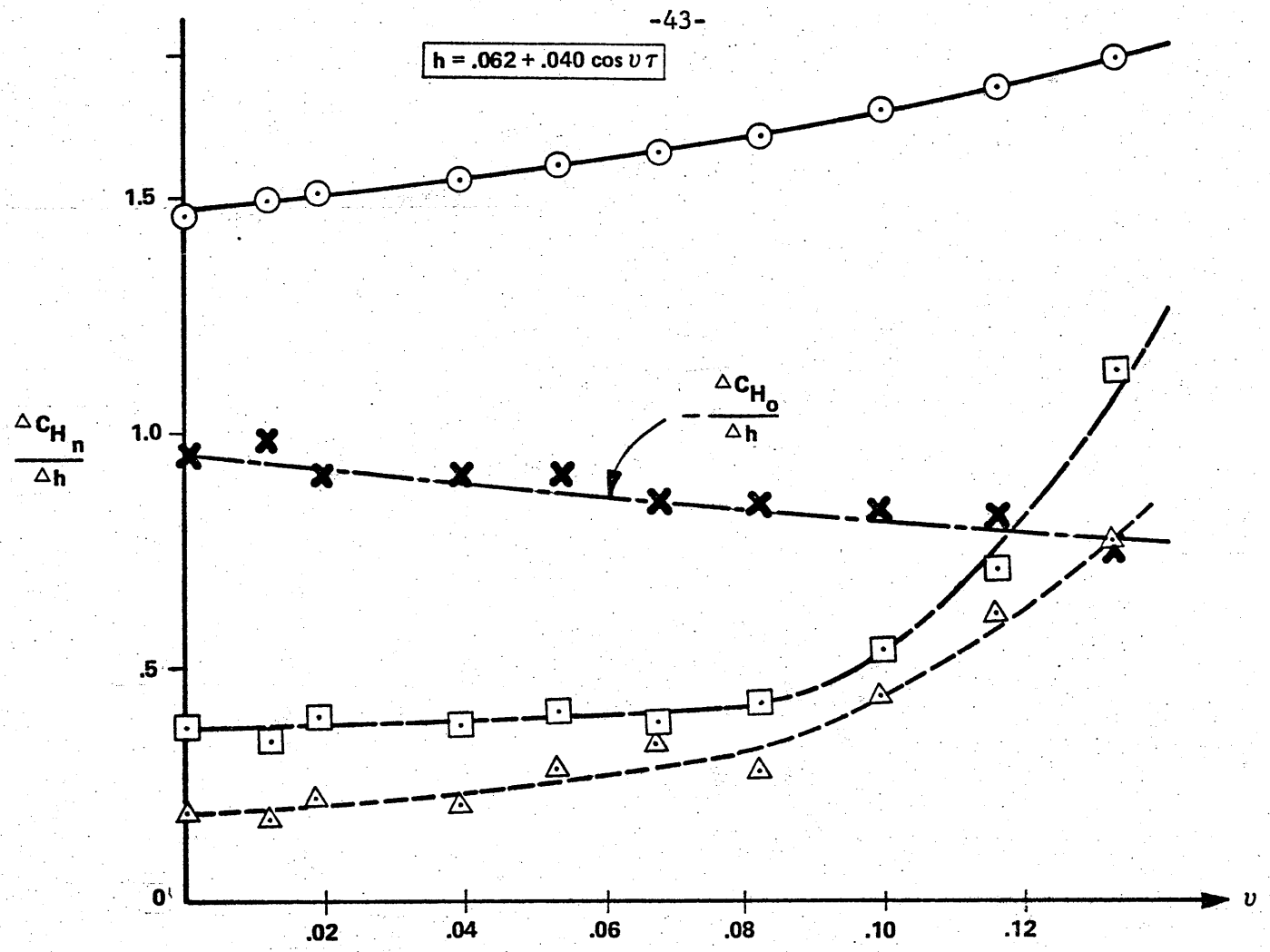


FIG. 2.17E. COMPONENTS OF HINGE-MOMENT COEFFICIENT

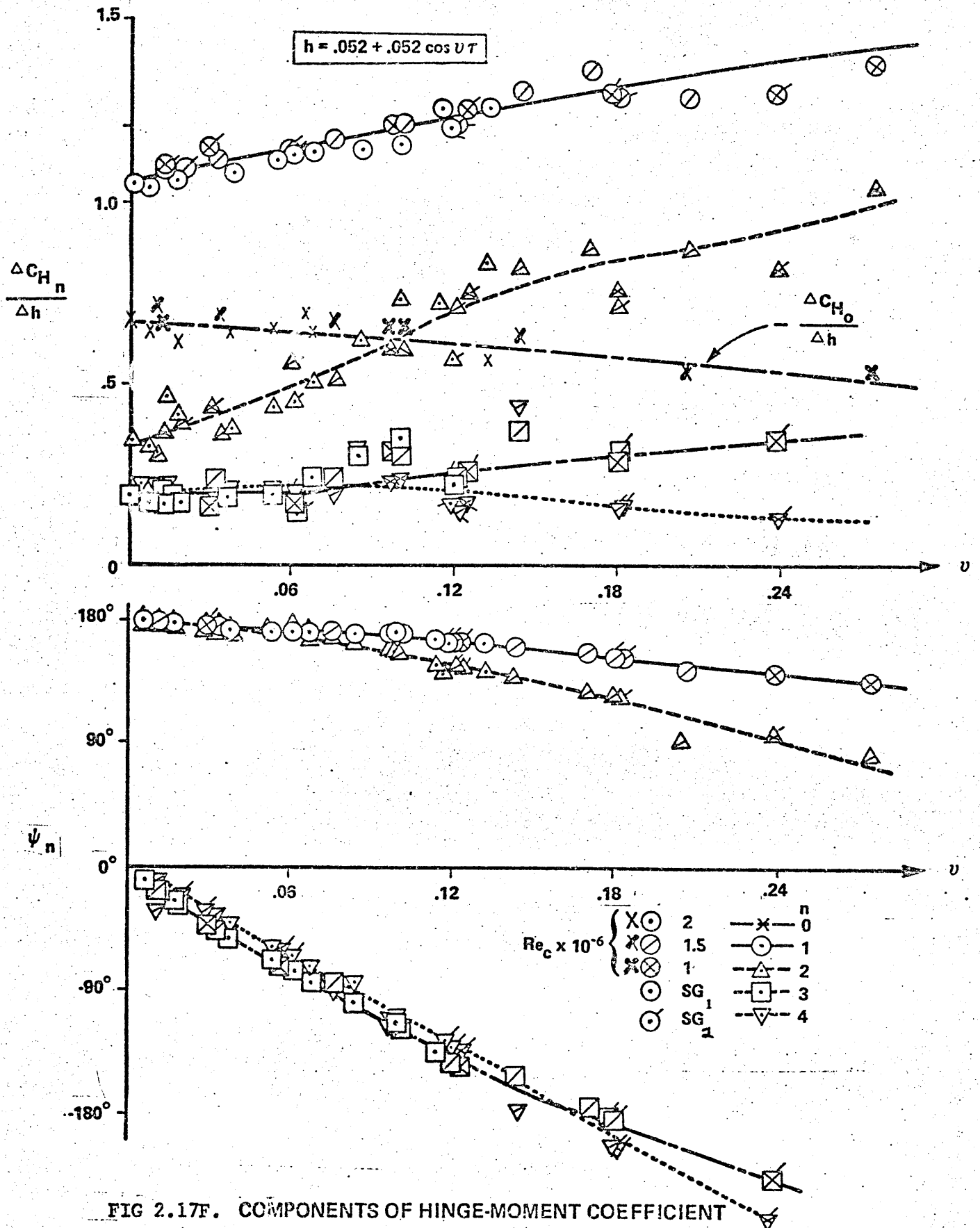


FIG 2.17F. COMPONENTS OF HINGE-MOMENT COEFFICIENT

PART-SPAN SPOILER

$$h = .052 + .052 \cos \nu \tau$$

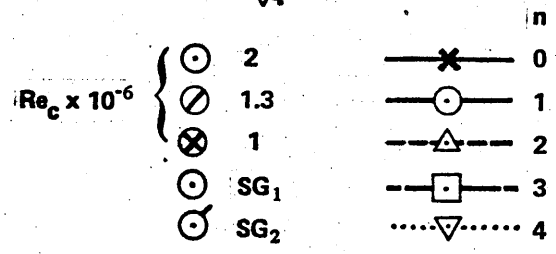
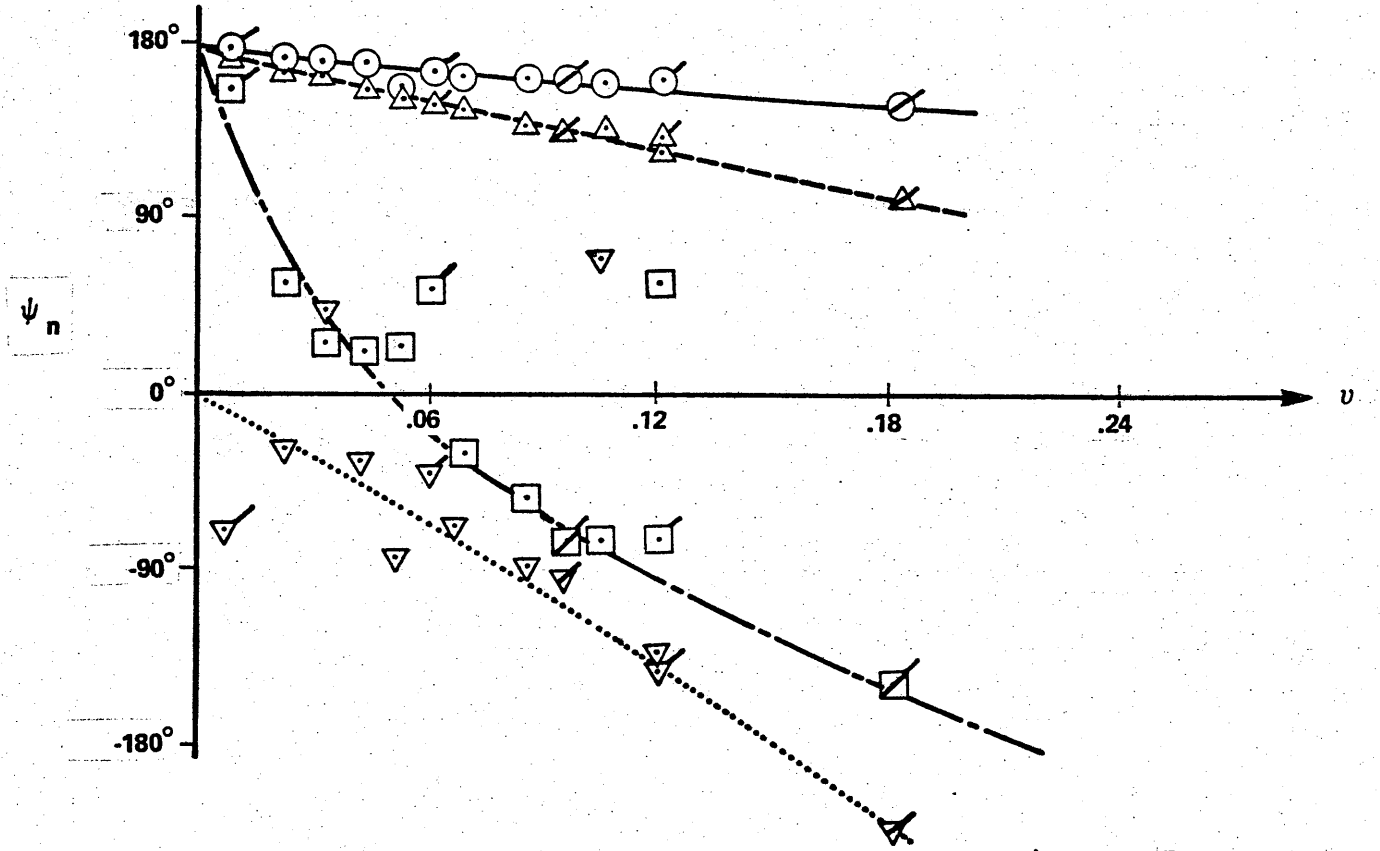
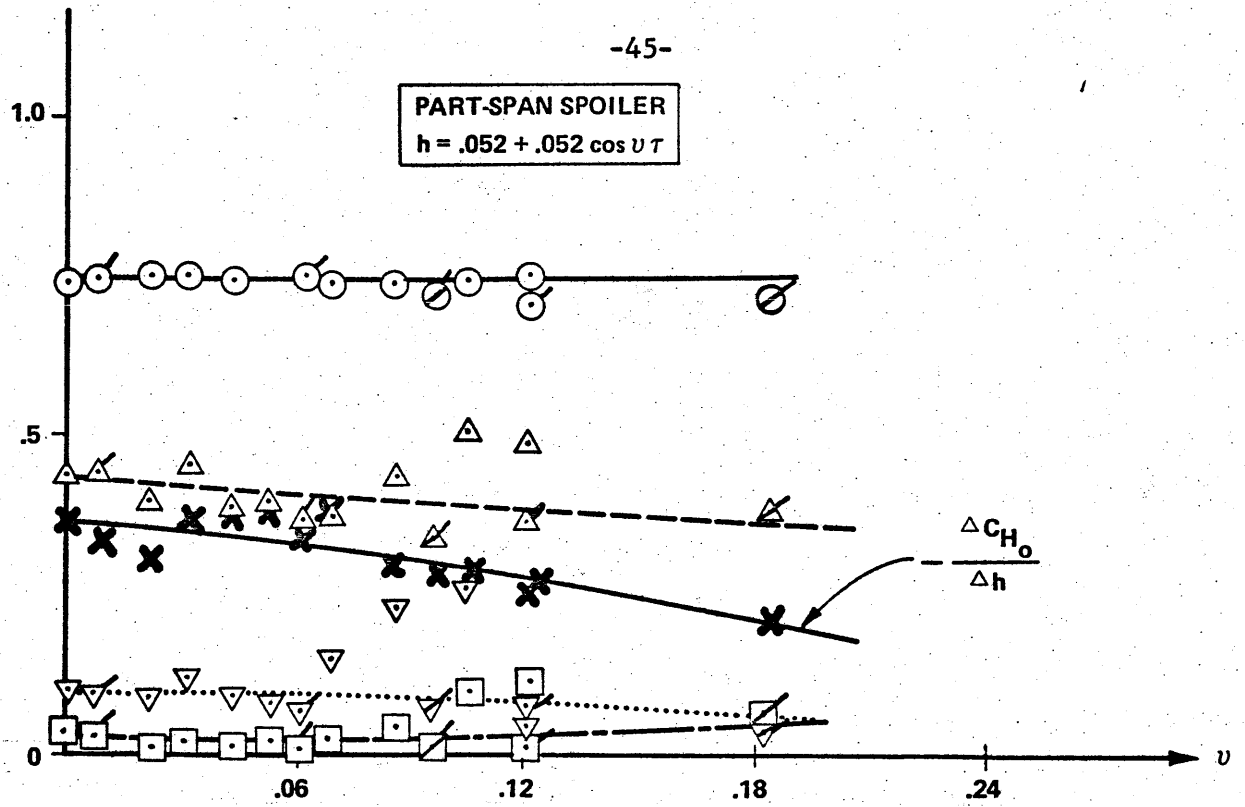


FIG. 2.17G. COMPONENTS OF HINGE-MOMENT COEFFICIENT

cases and the results were plotted as zero frequency "data." The zero frequency predictions are good except for lowest values of h_0 and Δh , i.e., Figure 2.17A. The zero frequency averaged curve also allowed prediction of a "describing function" for quasi-steady flow defined as $\frac{\Delta C_{H_1}}{\Delta h}$ and shown as a function of h_0 and Δh in Figure 2.18.

A lag effect is apparent for all components of the series. The phase lag, for all components, increases with frequency in a near linear manner. There is no fixed relation between the lag effect of the various components which would relate simply to a time lag of a basic waveform.

Shown in Figure 2.17C are data points of the 4th harmonic which were not corrected (or removed) for locking bar resonance effects.

The zeroth harmonic component generally shows a "rectification" effect with frequency. There is a small change towards a flap-down hinge moment as frequency increases for all but the largest value of h_0 .

Reynolds number effects are apparent, especially in the fundamental component, in Figures 2.17B and D. A possible explanation for the increase in magnitude of the fundamental component with a decrease in Reynolds number is that both the thinner boundary-layer and decreased free-stream turbulence (for the lower Reynolds number) allow the spoiler to be more effective in separating the flow.

For case F (as in Figure 2.17F) the effect of a L.E. trip wire, which thickens the boundary-layer, is to reduce the magnitude of the fundamental component. Also for case F, if the flap is fixed at $+4^\circ$ or -4° there is a negligible effect on the fundamental component except at very low frequencies where there is a minor effect, i.e., for upward flap deflection a small increase in magnitude occurs and for downward deflection a decrease is apparent. This minor effect is possibly due in part to the upstream flap influence on the boundary-layer and to a change in the boundary conditions downstream on the separating shear layer.

The effect of an increase in airfoil incidence to 4° and 8° is to slightly increase the magnitude of the fundamental. Airfoil incidence affects boundary-layer thickness and the external, inviscid flow-field. It appears that the inviscid effect dominates in this case. In reference 32 it is argued that the dominance of one of these two effects depends primarily on spoiler chordwise position. In the present case it appears that the spoiler is at a location where the two effects nearly cancel.

Data for the trip wire, flap angle and incidence effects mentioned above also shows the zeroth harmonic rectification

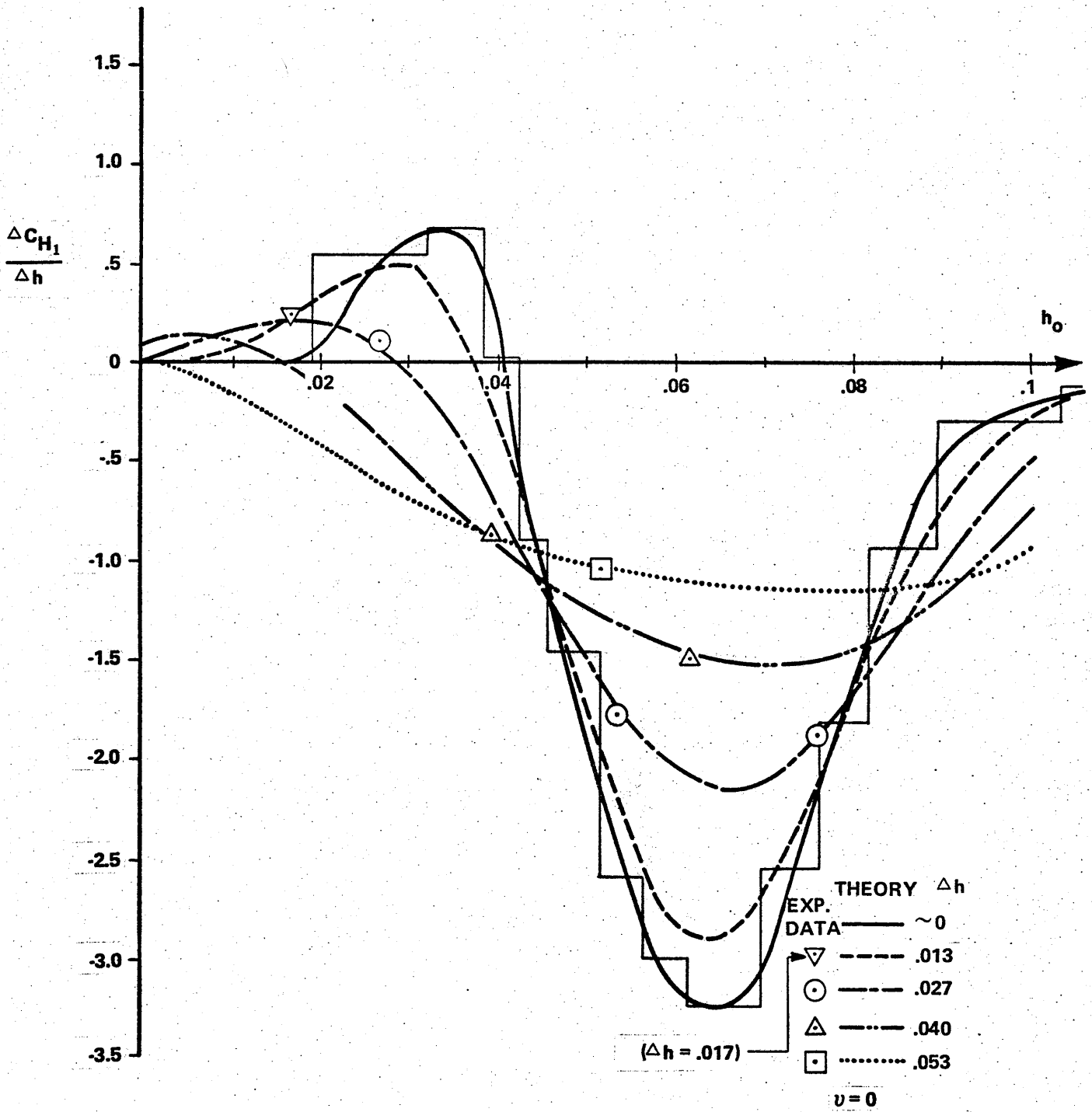


FIG. 2.18. DESCRIBING FUNCTION FOR QUASI - STEADY FLOW

effect mentioned previously. For all three effects there is no apparent change in phase angle data. These effects are not shown on Figure 2.17F.

Data for the part-span spoiler, shown in Figure 2.17G, shows wide scatter of phase angles for the third and fourth harmonic components. This is at present unexplained. Other data plots fairly well as in the full-span spoiler case. Note, however, that magnitudes and frequency dependence of the components differs for the two spoilers.

Not shown is data for the spoiler oscillation case: $h = .017 + .017 \cos \nu \tau$. Trends mentioned previously were again noted. At very high frequencies all component amplitudes decreased markedly except the zeroth harmonic which showed a flap-down trend.

Actual recordings of raw pressure data were evaluated to find the instant of unsteady pressure divergence on the spoiler upstroke. This data is plotted in Figure 2.19 and labeled "reattachment point position on spoiler upstroke." Actually, the point measured is at about 130% of reattachment point position (measured from the spoiler). The lag effect, however, was verified by measurements made from cine-film recordings.

Figure 2.20 is an example of a sequence of cine-film frames over a cycle of spoiler motion. The quality of recording shown in the figure is much poorer than actually achieved, also the figure shows a negative image. Response of the tufts to flow reversal is apparent. Movement of the reattachment line along the airfoil center-line was observed with respect to the small tufts shown on the flap insert (at left edge of each frame). Spoiler height was measured indirectly via measurements of its shadow.

In the unsteady cases the separation bubble did not contract in size on the spoiler downstroke as in quasi-steady flow. Movement of the reattachment point toward the spoiler on the downstroke lagged the quasi-steady position. At high enough frequencies, for some cases, reattached flow appeared simultaneously along the airfoil aft of the spoiler, and at very high frequencies the bubble appeared to be "washed off" the airfoil as it dissipated.

The lag effect, shown in Figure 2.19, is the same as measured from cine-film and can be converted to an apparent convective lag since:

$$\left(\frac{\nu \tau}{2\pi}\right)_r = \left(\frac{\nu \tau}{2\pi}\right)_{r_0} + \frac{\partial \left(\frac{\nu \tau}{2\pi}\right)_r}{\partial \nu} \nu$$

where

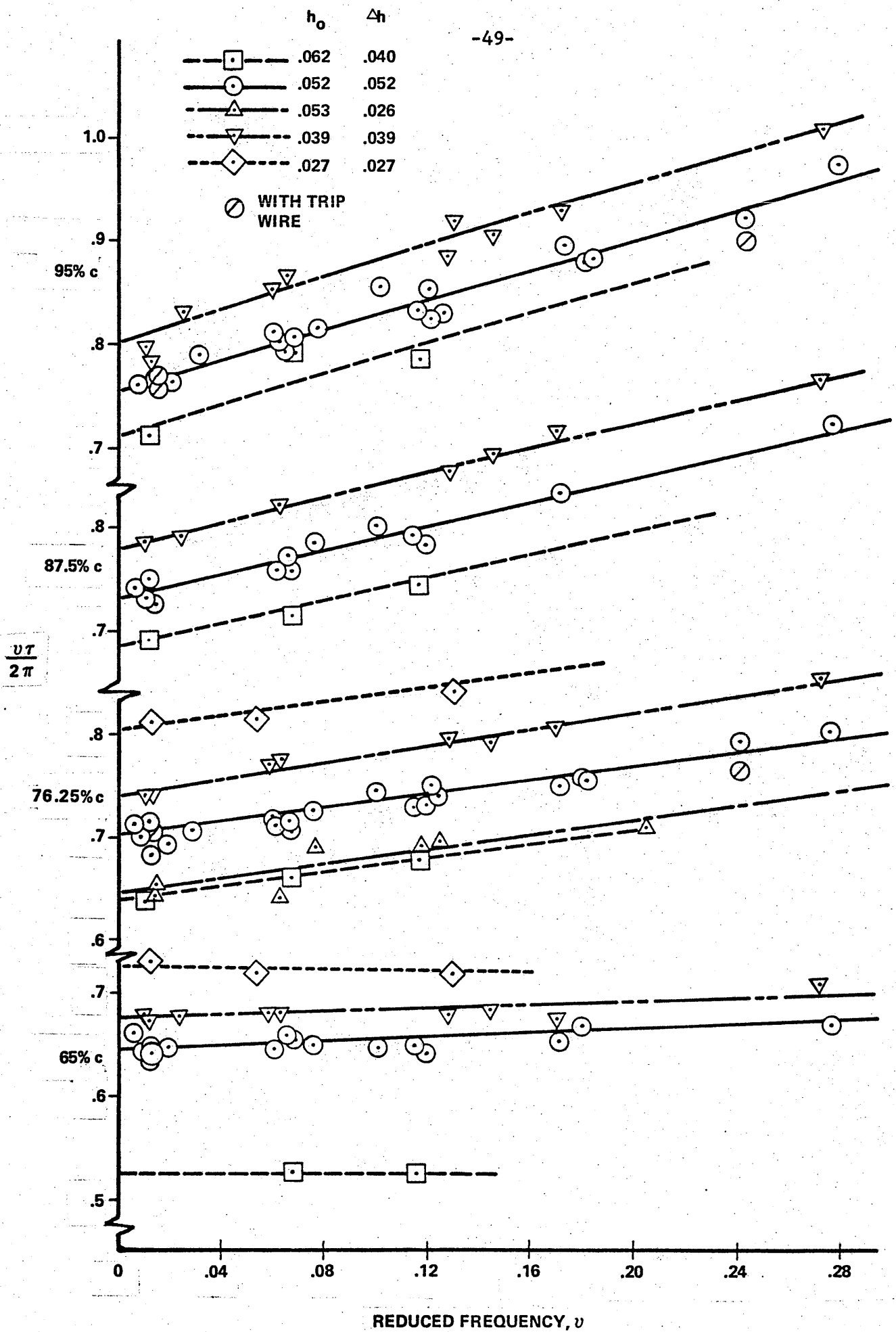
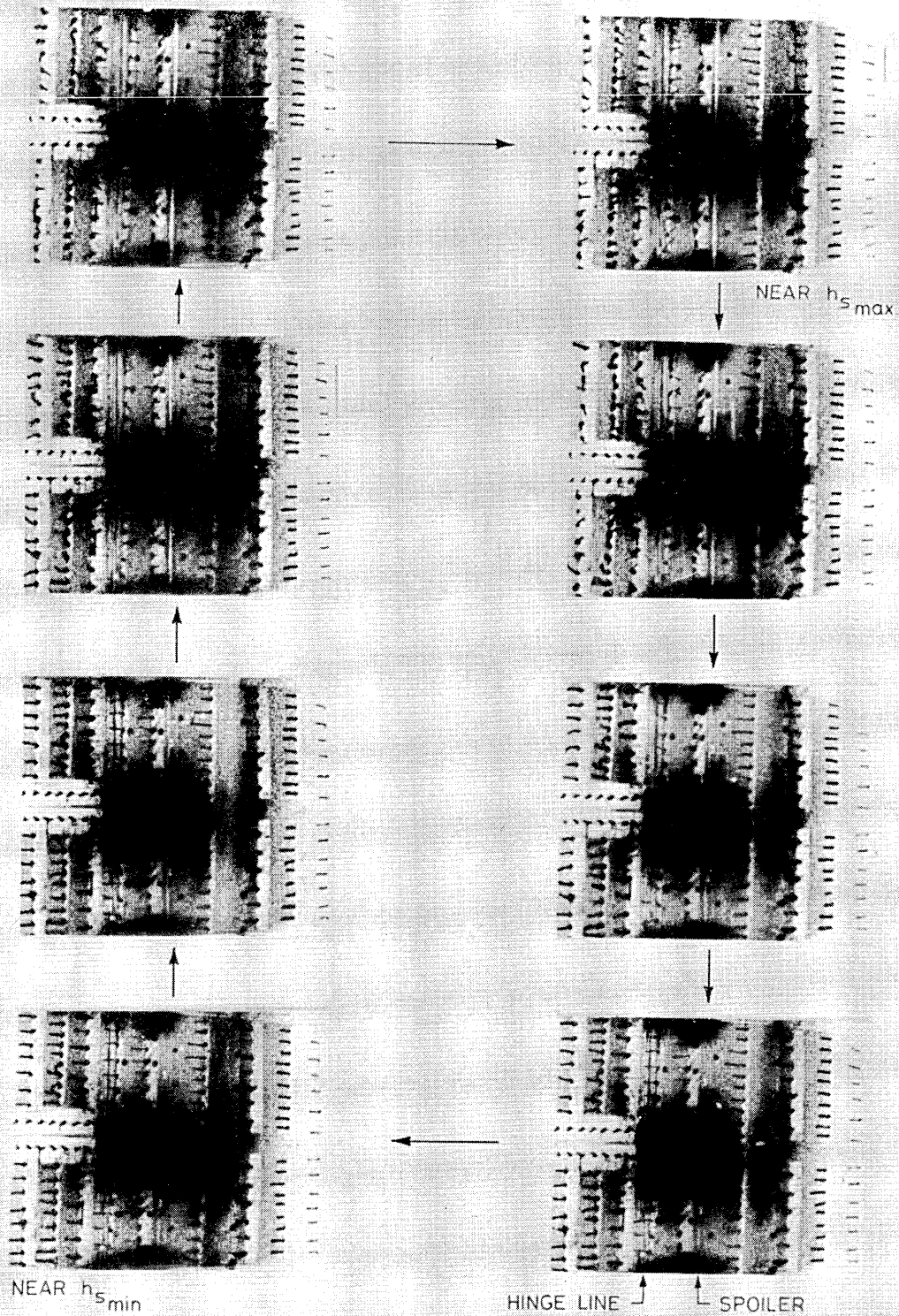


FIGURE 2.19. REATTACHMENT POINT POSITION ON SPOILER UPSTROKE



$$h_s \approx 0.31 + 0.31 \cos \psi$$

FLAP FIXED; $\psi \approx 0.03$

FIGURE 2.20. TOP VIEW OF AIRFOIL SHOWING TUFT RESPONSE.

r_o relates to reattachment point location in quasi-steady flow

r is for unsteady flow

Now

$$\frac{\partial \left(\frac{\nu \tau}{2\pi} \right) r}{\partial \nu} = \frac{\Delta \tau}{2\pi} \cong \begin{cases} .125 \text{ for } 65\%c \\ .365 \text{ for } 76.25\%c \\ .575 \text{ for } 87.5\%c \\ .735 \text{ for } 95\%c \end{cases}$$

A convective lag would take the form:

$$\frac{(x - x_{sp})c}{U_c} = \Delta t = \frac{c_f}{U_\infty} \Delta \tau$$

or

$$\frac{U_c}{U_\infty} = \frac{(x - x_{sp})c}{\Delta \tau c_f} \cong \begin{cases} .339 \text{ for } 65\%c \\ .312 \text{ for } 76.25\%c \\ .323 \text{ for } 87.5\%c \\ .318 \text{ for } 95\%c \end{cases}$$

where U_c is an apparent convective velocity. The average value is $\frac{U_c}{U_\infty} \cong .323$. This velocity ratio could possibly be related to the rate of mass flow into the growing separated region.

Unsteady pressures in the region aft of the spoiler appear to reflect a localized effect due to influence of the separation bubble. Near the trailing-edge, on the flap under surface, negligible pressure effects are apparent except for the portion of each cycle involving T.E. pressure divergence.

Over the wetted upper surface ahead of the spoiler there are increasing phase lags and magnitude reductions in the components of pressure coefficient as frequency increases. Pressures near the leading-edge have a phase lag that increases with frequency at about three times the rate of lag increase for 45% chord pressure. In general, leading edge pressures suffer the greatest magnitude reduction as frequency increases. At present there is no theory, including that of reference 33, which can predict these unsteady pressure effects.

Spoiler-Fixed, Flap-Forced Results

Results of testing with the spoiler fixed at various heights are plotted as in-phase and in-quadrature components in Figures 2.21A and B.

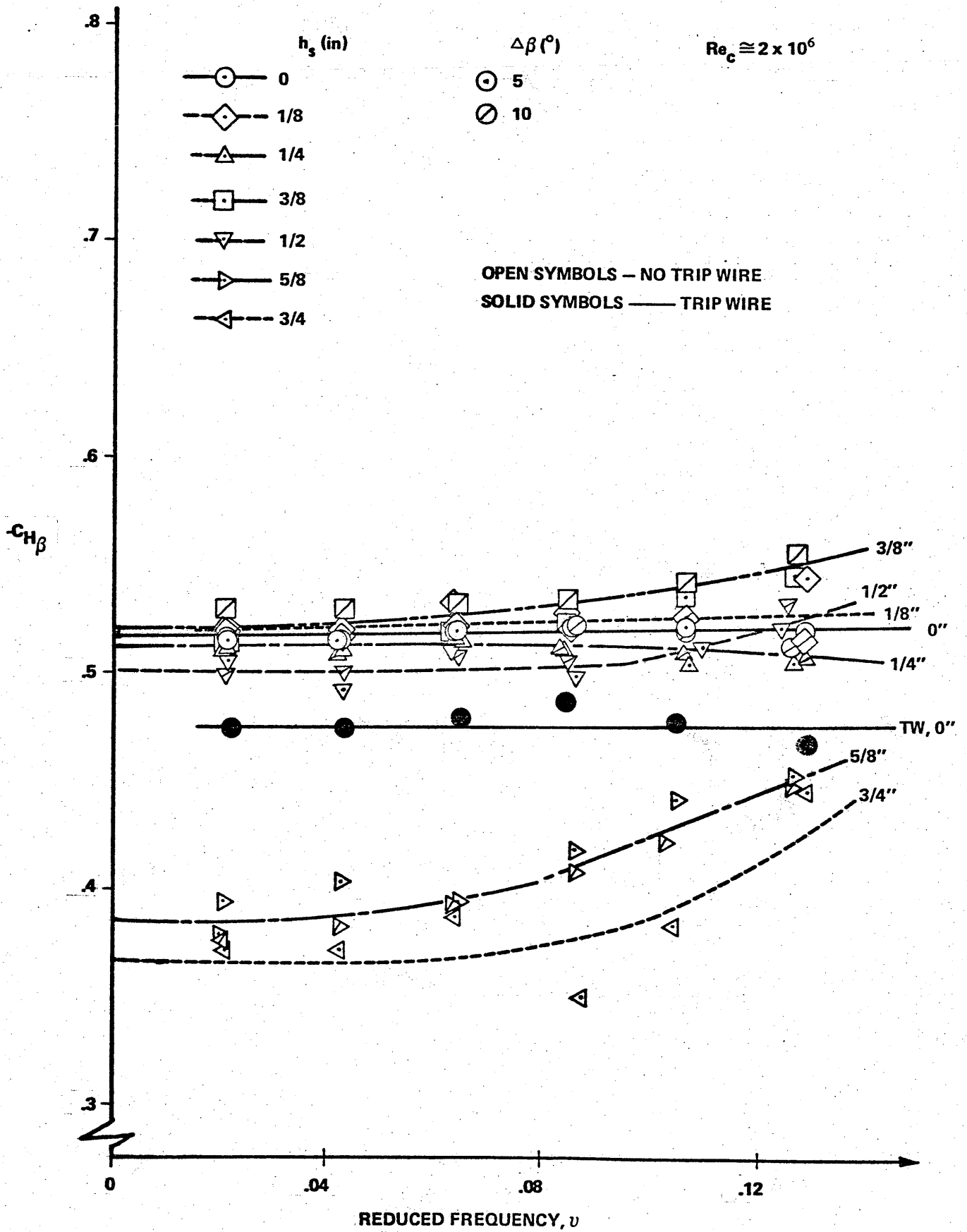


FIG. 2.21A. IN-PHASE COMPONENT OF HINGE-MOMENT COEFFICIENT

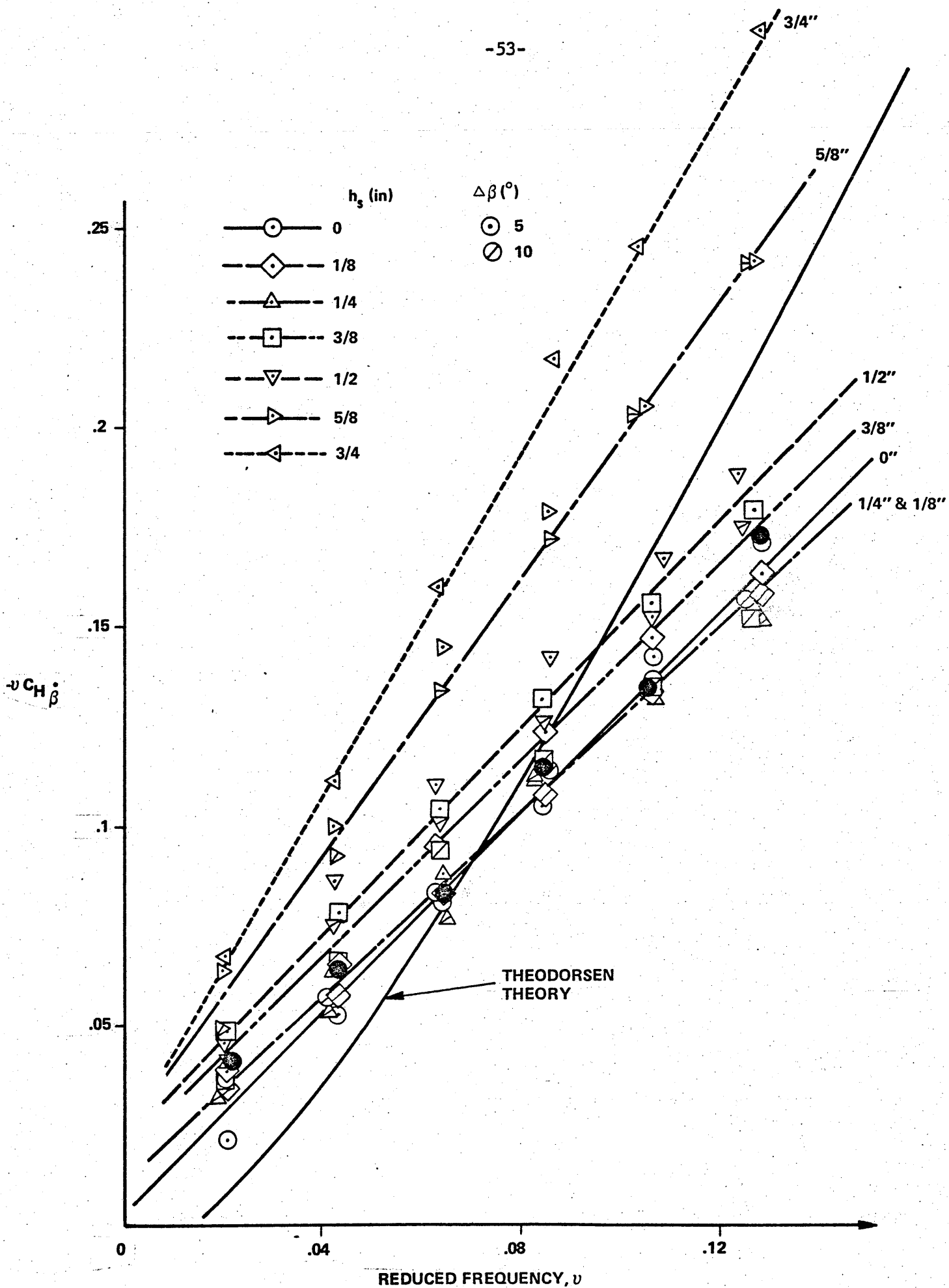


FIGURE 2.21B. IN-QUADRATURE COMPONENT OF HINGE-MOMENT COEFFICIENT

Values of $C_{H\beta}$ from Figure 2.21A are relatively insensitive to spoiler height for $h_s < .5"$. This is also true for values of $\nu C_{H\beta}^\circ$ shown in Figure 2.21B. This may be explained as in reference 22, where for recompression or reattachment on the flap surface, the flap is able to "control" the external flow field as it does in attached flow. The relatively low values of $|C_{H\beta}|$ attained with the trip wires in place may be due in part to the fact that both upper and lower surface boundary-layers were thickened whereas the spoiler primarily affects the upper surface boundary layer.

Theodorsen's theory predicts a value of $C_{H\beta} = -.943/\text{rad}$ at zero frequency and $C_{H\beta} = -.822/\text{rad}$ at $\nu = .2$. At zero frequency the value of $C_{H\beta}$ may be taken as $C_{H\beta} \cong -.516/\text{rad} \pm .8\%$ for $0 \leq h_s < .5"$. This differs by 8% from the two-dimensional "real flow" prediction of $C_{H\beta} = -.561/\text{rad}$, possibly due to end-plate and gap three-dimensional effects. The discrepancy between theory and actual $\nu C_{H\beta}^\circ$ values is probably related to viscous and three-dimensional effects. None of the theories found in references 34-36 are applicable in this flow situation.

Not shown on Figures 2.21A and B are data found at lower Reynolds numbers and higher frequencies and data for the other fixed spoiler configurations.

Lower Reynolds number is associated with a thinner boundary-layer for this experiment*, and as in the case of incremental hinge-moment coefficient, results in slightly higher values of $|C_{H\beta}|$. In all cases the higher frequency data showed that $C_{H\beta}$ was nearly independent of frequency. (The upward trend shown for 3/8" and 1/2" spoiler heights as frequency increased is not as pronounced when these other data points are included.) Results for large spoiler deflections show an increase in $|C_{H\beta}|$ as frequency increases, with $C_{H\beta} \rightarrow -.52$ as $\nu \rightarrow .26$. There are no significant Reynolds number effects on the behavior of $\nu C_{H\beta}^\circ$. The data for frequencies up to $\nu = .26$ lies on or near the extrapolated lines shown in Figure 2.21B.

Chordwise position as well as spoiler height has a profound effect on hinge-moment due to flap deflection. For the 3/4" spoiler at 30% chord results were similar to those for the 5/8" spoiler at 58.3%c. A separation bubble extended into the wake with recompression occurring over most of the flap. Also for the 1/2" spoiler at 45% chord, results followed the trends above. But for the 1/2" spoiler at 30% chord, where reattachment of the shear-layer occurred ahead of the flap, $C_{H\beta}$ reached a value of $-.25$ near zero frequency and showed increased values of $|\nu C_{H\beta}^\circ|$. As frequency increased, $C_{H\beta}$ and $\nu C_{H\beta}^\circ$ values

*For low Reynolds numbers the boundary-layer was laminar at the spoiler position when spoiler height was zero.

approached the aft spoiler values. It appears that the primary effect of the 1/2" spoiler at 30% chord was to greatly thicken the boundary-layer over the flap and so reduce its effectiveness primarily at low frequencies.

Although there were no large effects on $C_{H\beta}$ and $\nu C_{H\beta}$ for $h_s < 1/2"$, the magnitude and phase angles of pressure response varied significantly with changes in spoiler height. No unexpected changes occurred with an increase in frequency.

As spoiler height was increased from zero to about 1/4", for a fixed frequency, there were no large changes in the first harmonic component of pressure response over the airfoil, except directly downstream of the spoiler. The magnitude of first harmonic component at 76.25% chord decreased, in this spoiler range, by an order of magnitude. The near-singular behavior at the flap-hinge line associated with attached flow gave way to a greatly reduced flap-motion effect. As spoiler height was further increased to about 1/2" most components (even on the wetted surface) went through a transition to generally lower magnitudes of pressure response. Only over the flap upper surface was there an increase in magnitudes, i.e., at 87.5% and 95% c. The magnitude of response increased near the trailing-edge by a factor of three. As spoiler height was further increased to 3/4", the component magnitudes leveled off at low values. The magnitudes of response near the T.E. also decreased but to a level nearly twice the zero spoiler height value. These results are comparable to the steady flow flap effects found in reference 21. Large changes in phase angle occurred at some locations for spoiler heights above 1/4". Near the airfoil leading-edge phase angles changed from about a 195° lag to a 185° lead. At the 90% chord position, on the flap under surface, a 24° phase lead increased to a 82° lead (for a spoiler height increase from 1/4" to 1/2"), then decreased to a 49° lead for 3/4" spoiler height.

Spoiler-Forced, Flap-Free Results

Flap response to spoiler motion was represented in Fourier series form as:

$$\beta = \beta_0 + \sum_{n=1}^q \beta_n \cos (n \nu \tau + \phi_n)$$

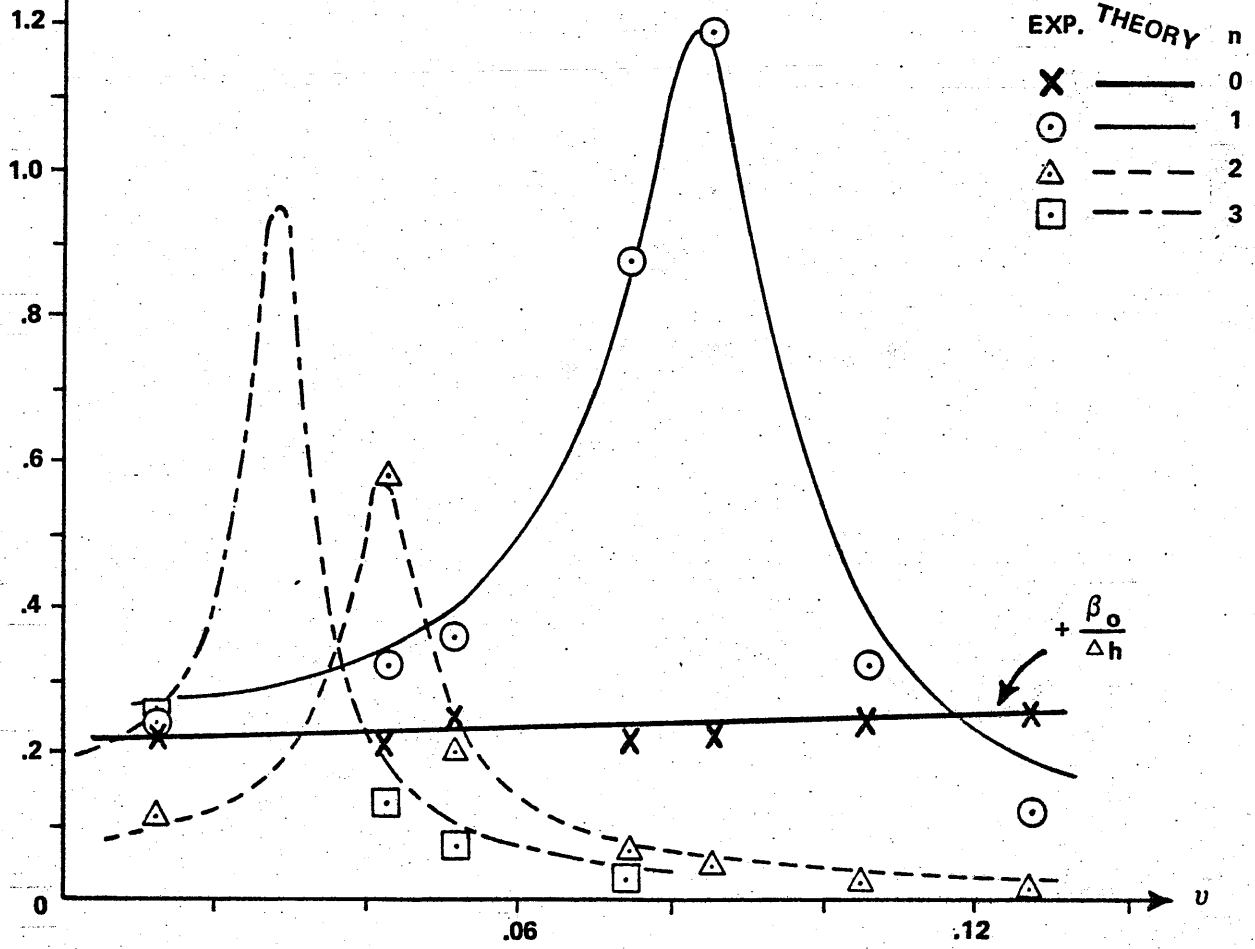
where

$$h = h_0 + \Delta h \cos \nu \tau$$

Data points corresponding to cases A-G are plotted in Figures 2.22A-G. The curves are predictions which are explained in section 2.4. The flap system acted as a very good filter with a natural frequency that depended primarily on mechanical

$$h = .026 + .026 \cos v\tau$$

$\frac{\beta_n}{\Delta h}$



ϕ_n (DEG)

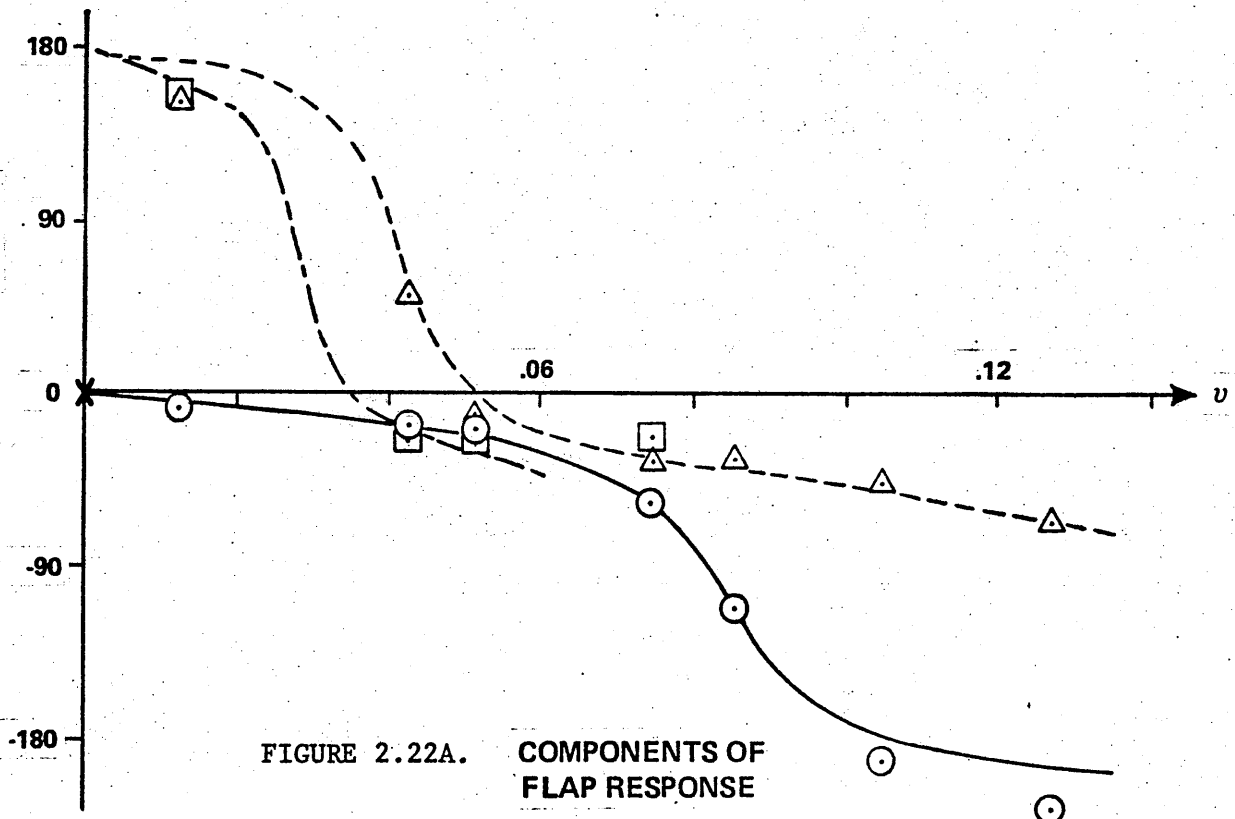


FIGURE 2.22A. COMPONENTS OF FLAP RESPONSE

$$h = .054 + .028 \cos v \tau$$

EXP.	THEORY	n
×	—	0
○	—	1
△	- - -	2
□	- - -	3

NOTE: CIRCLED DATA IS SUSPECT

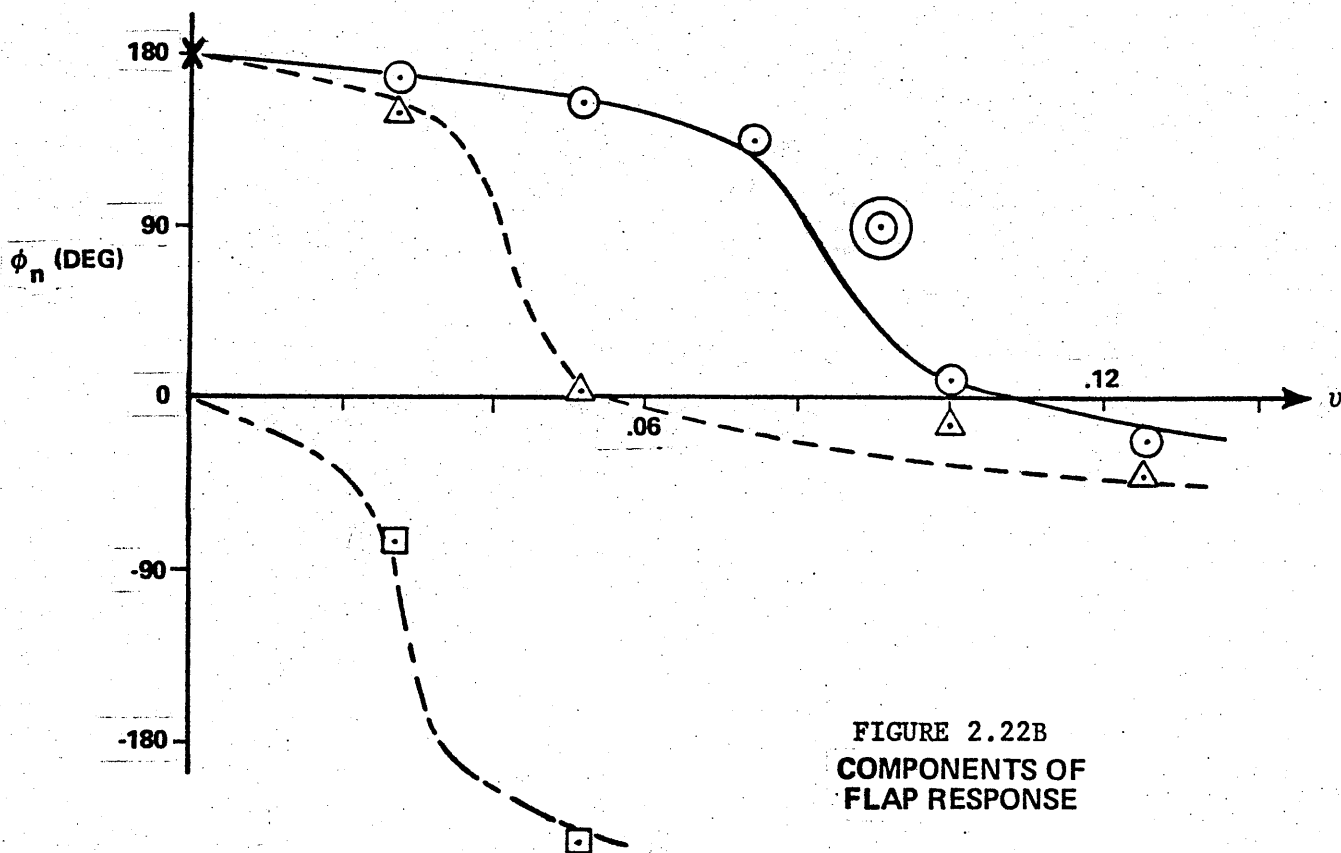
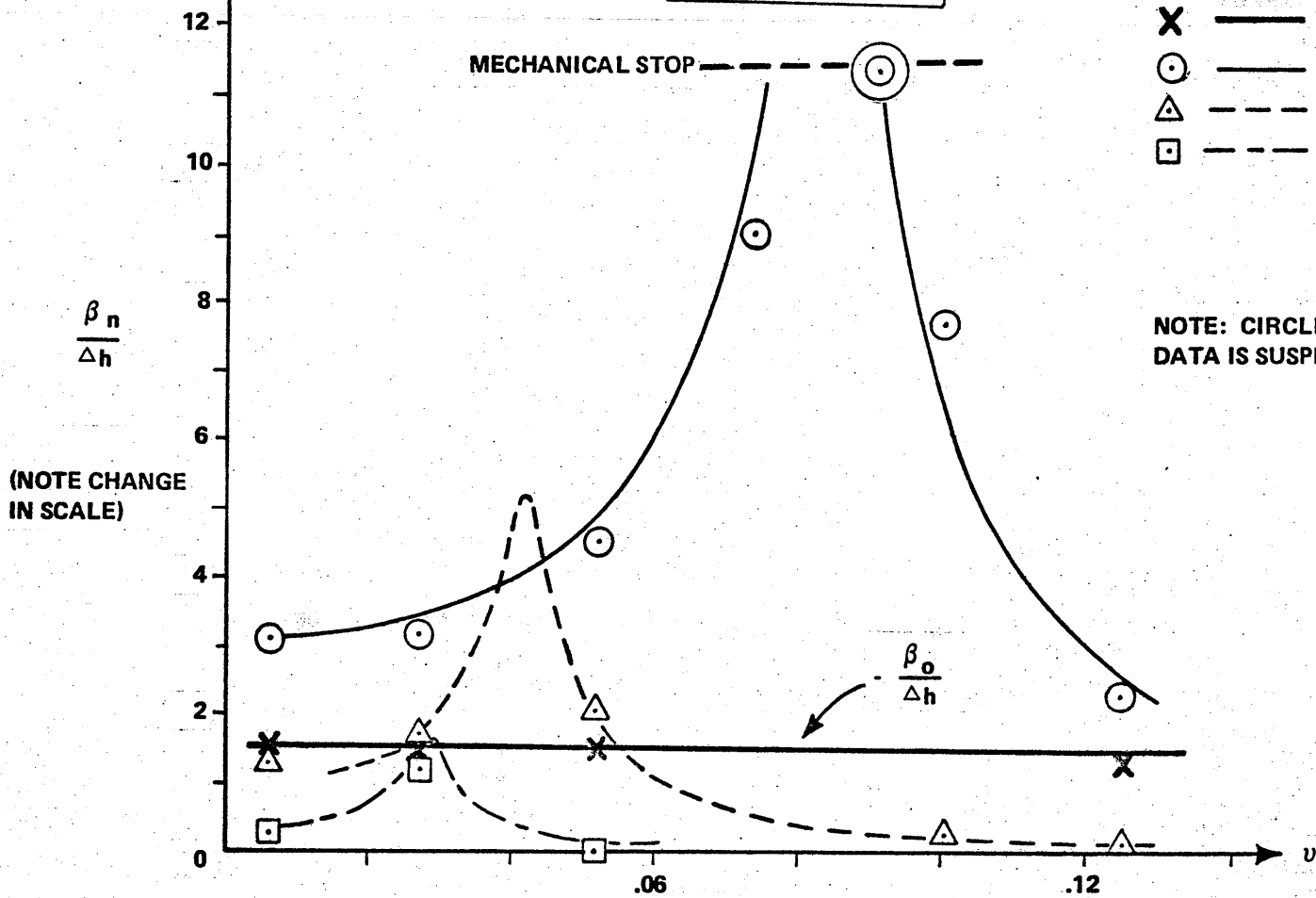


FIGURE 2.22B
COMPONENTS OF
FLAP RESPONSE

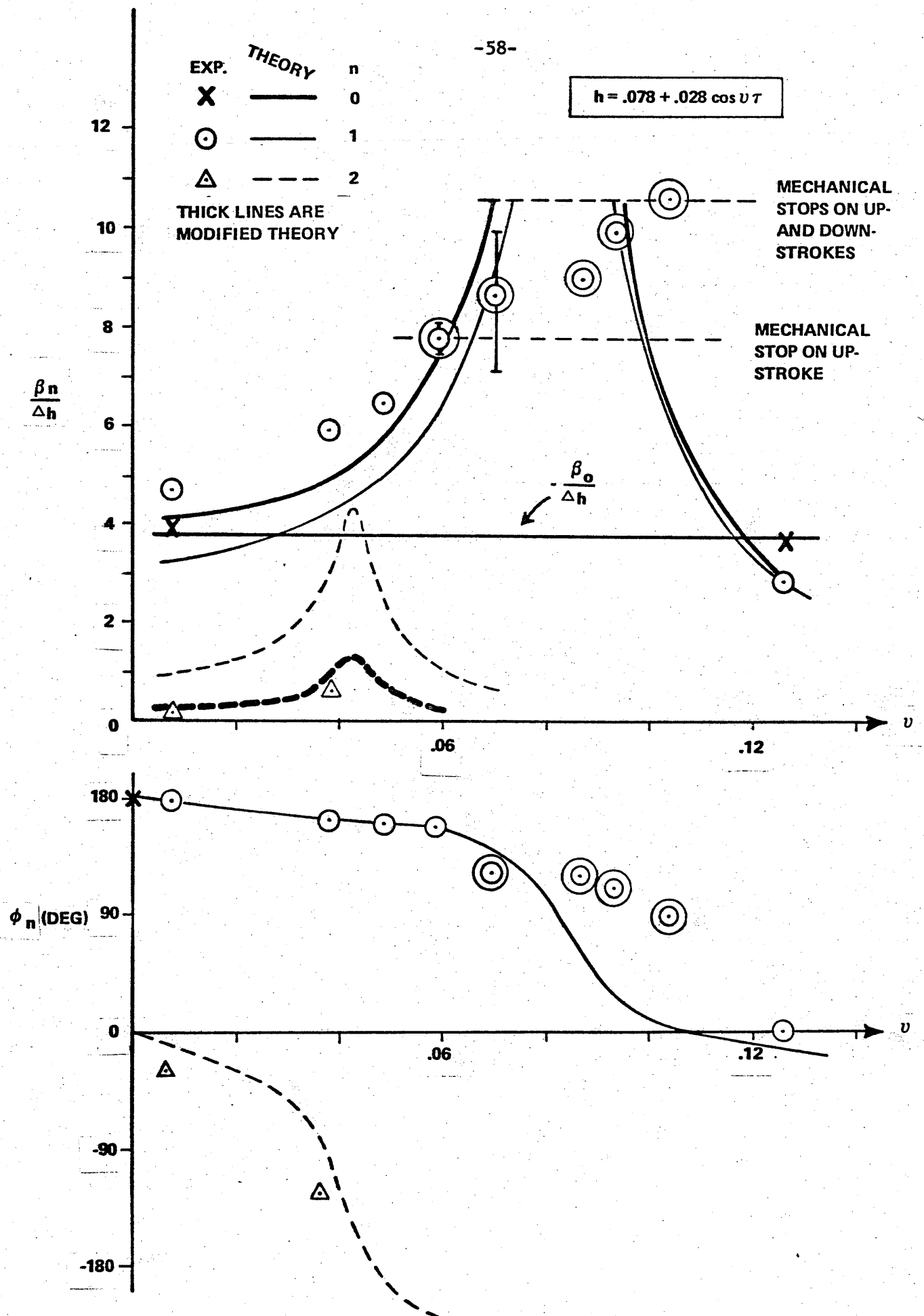


FIGURE 2.22C. COMPONENTS OF FLAP RESPONSE

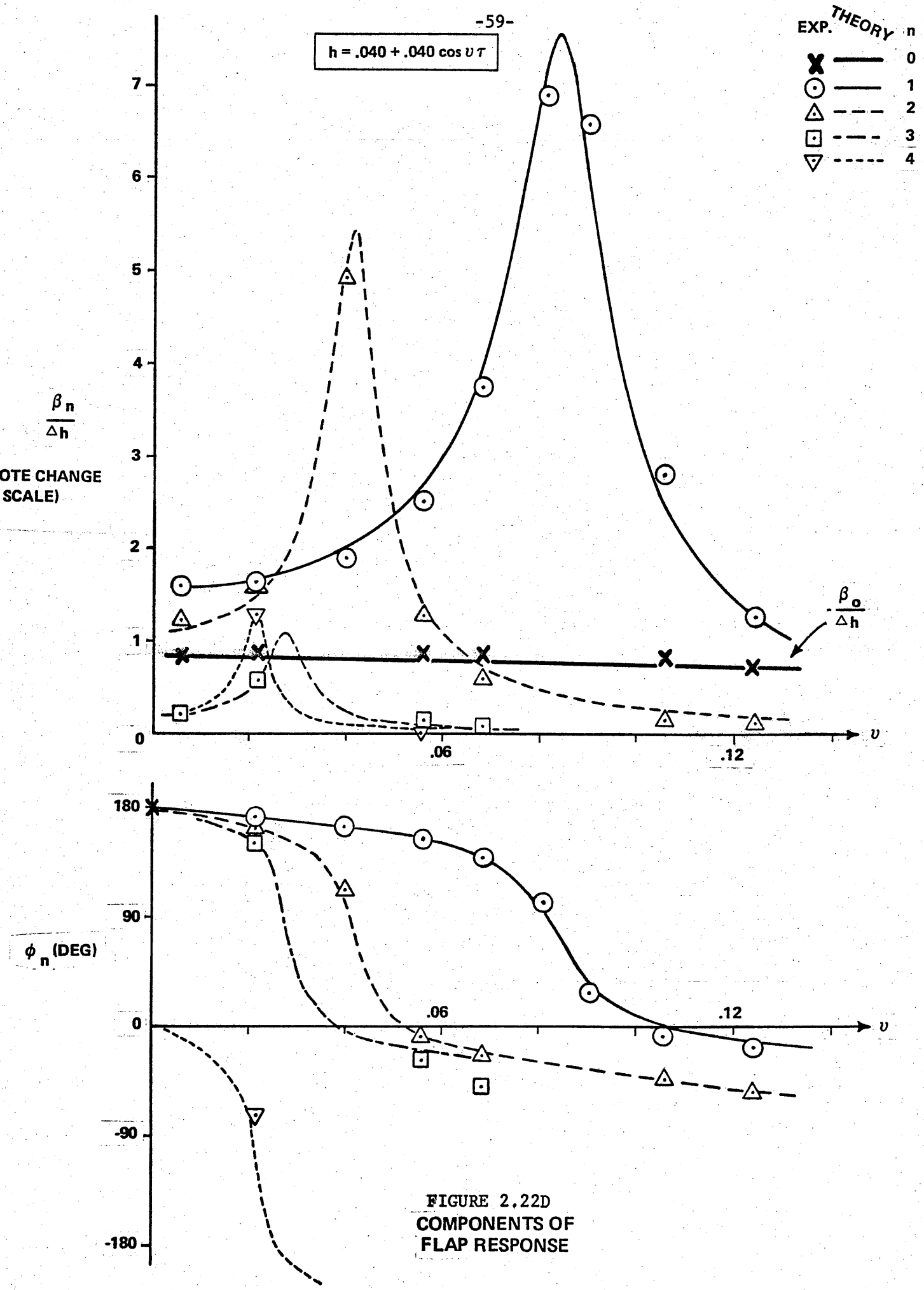


FIGURE 2.22D
COMPONENTS OF
FLAP RESPONSE

$$h = .042 + .027 \cos v \tau$$

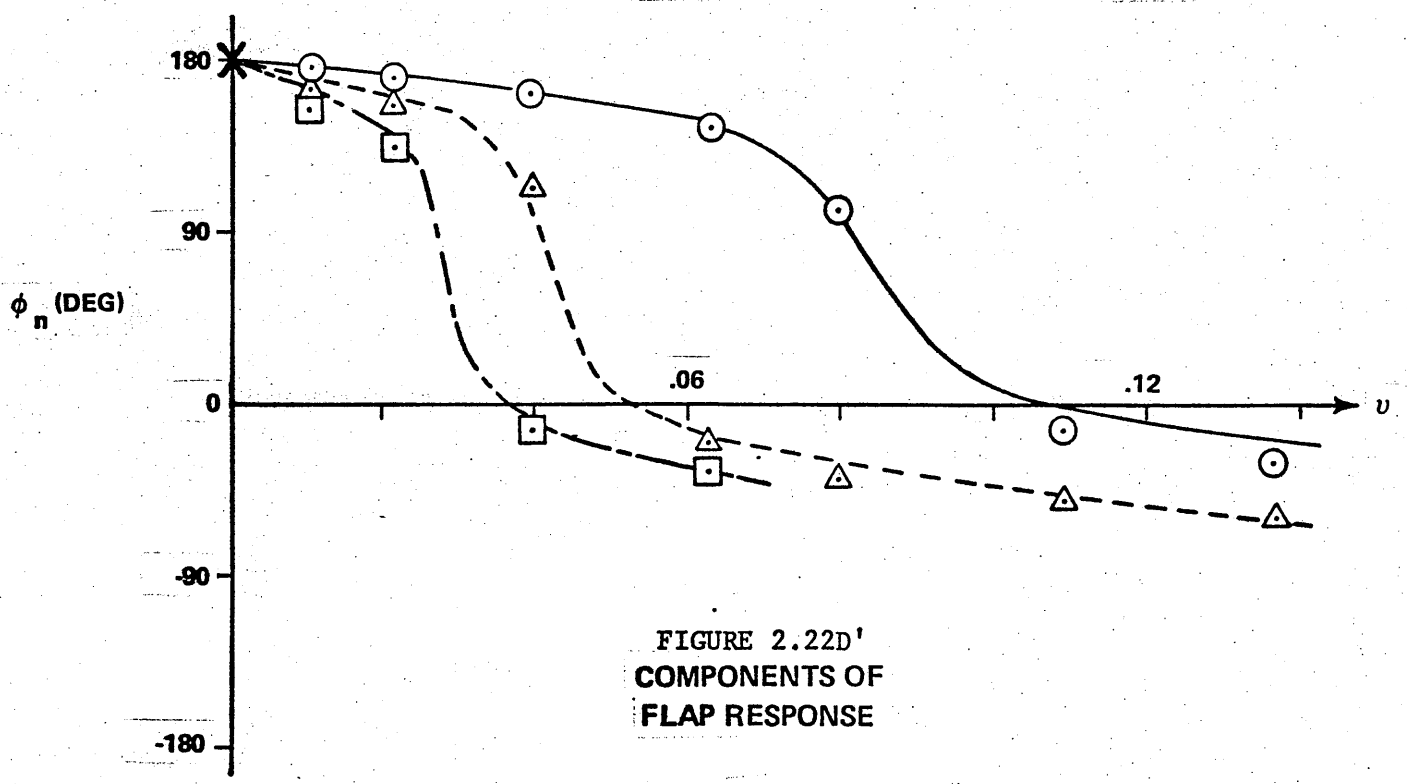
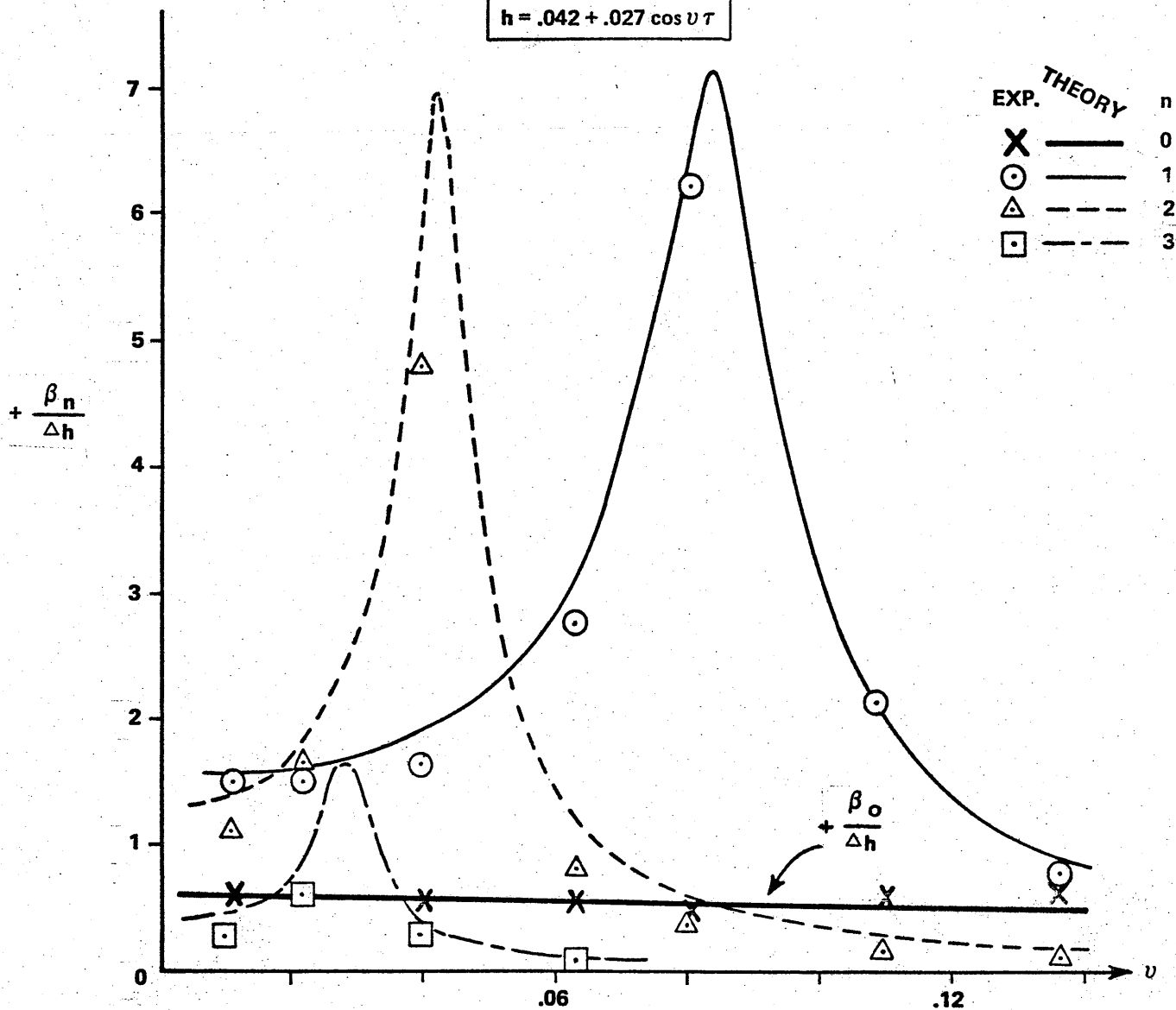


FIGURE 2.22D'
COMPONENTS OF
FLAP RESPONSE

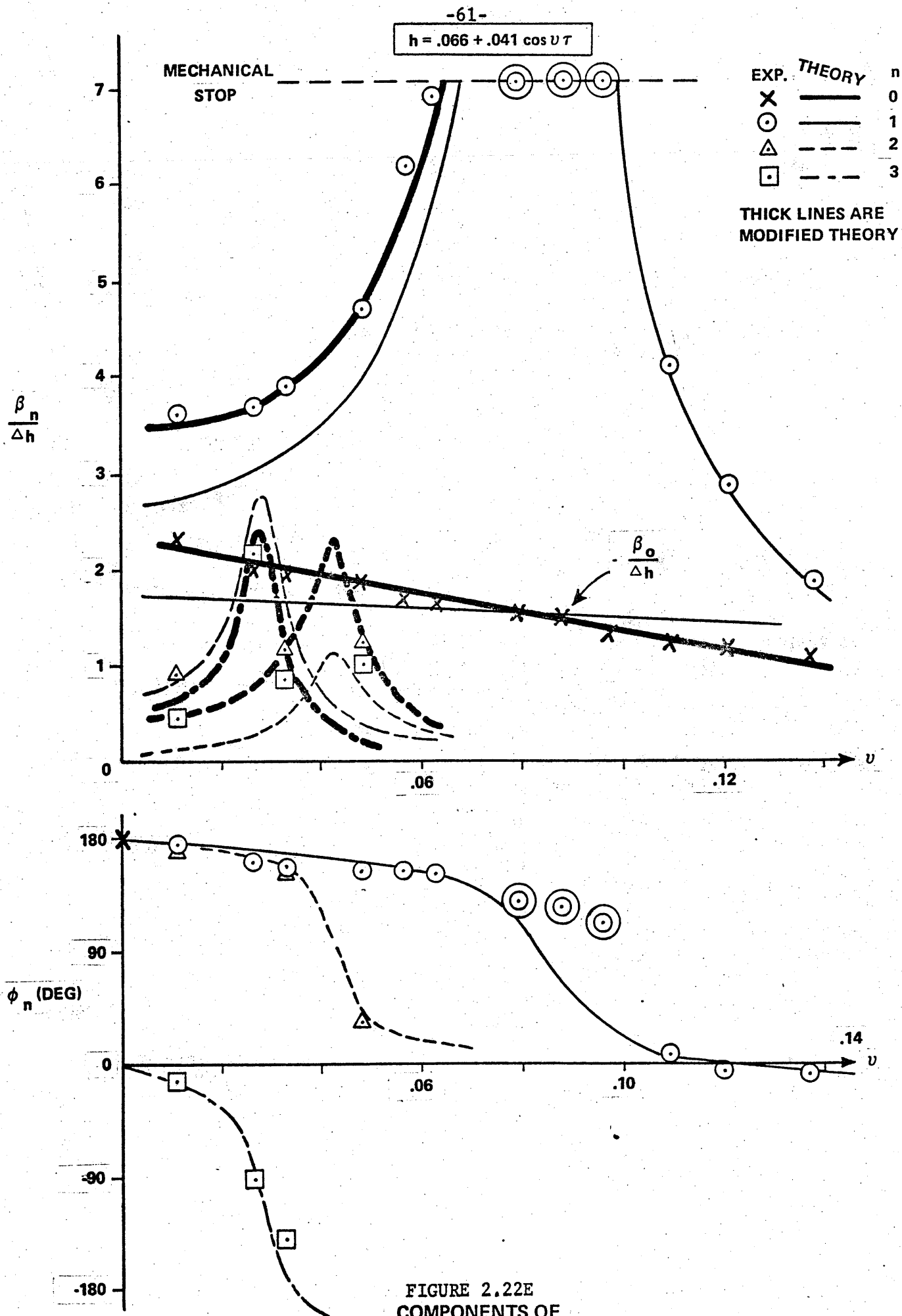


FIGURE 2.22E
COMPONENTS OF
FLAP RESPONSE

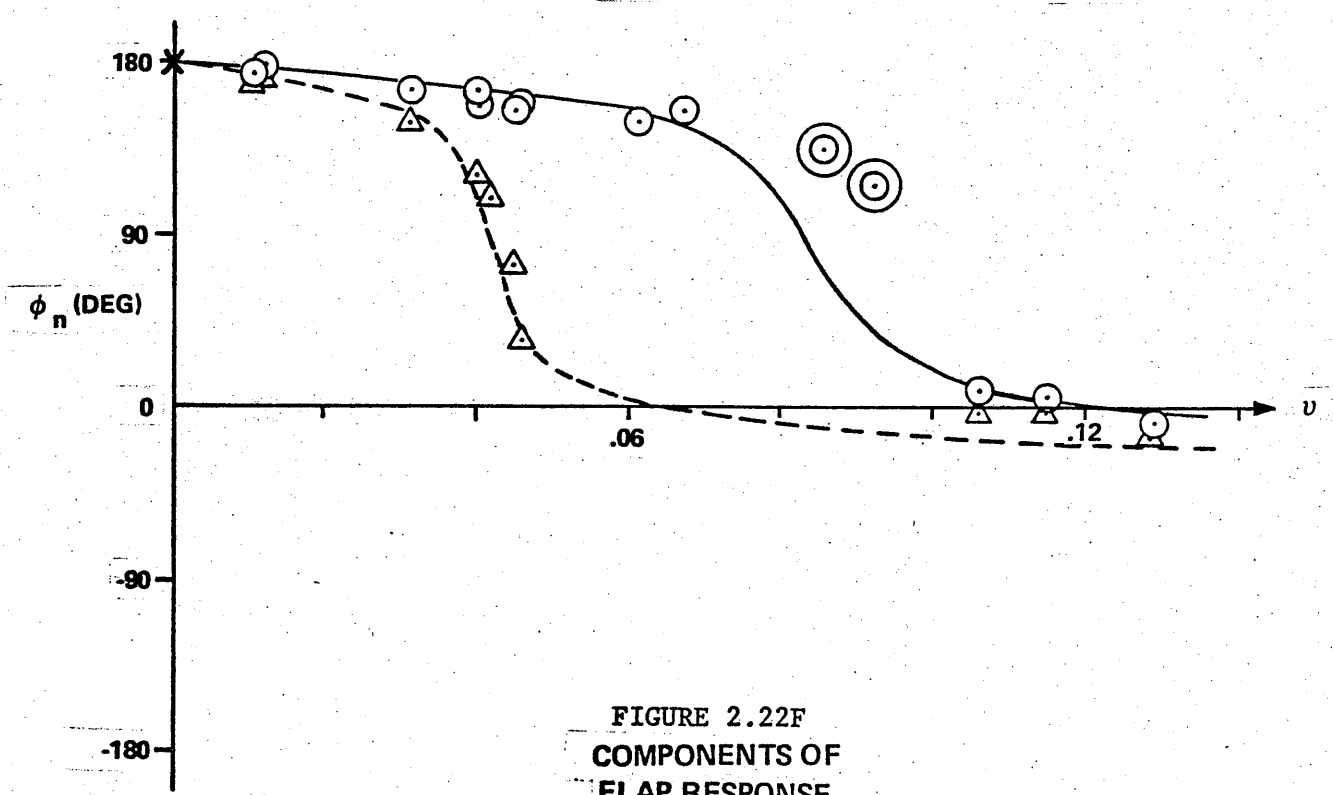
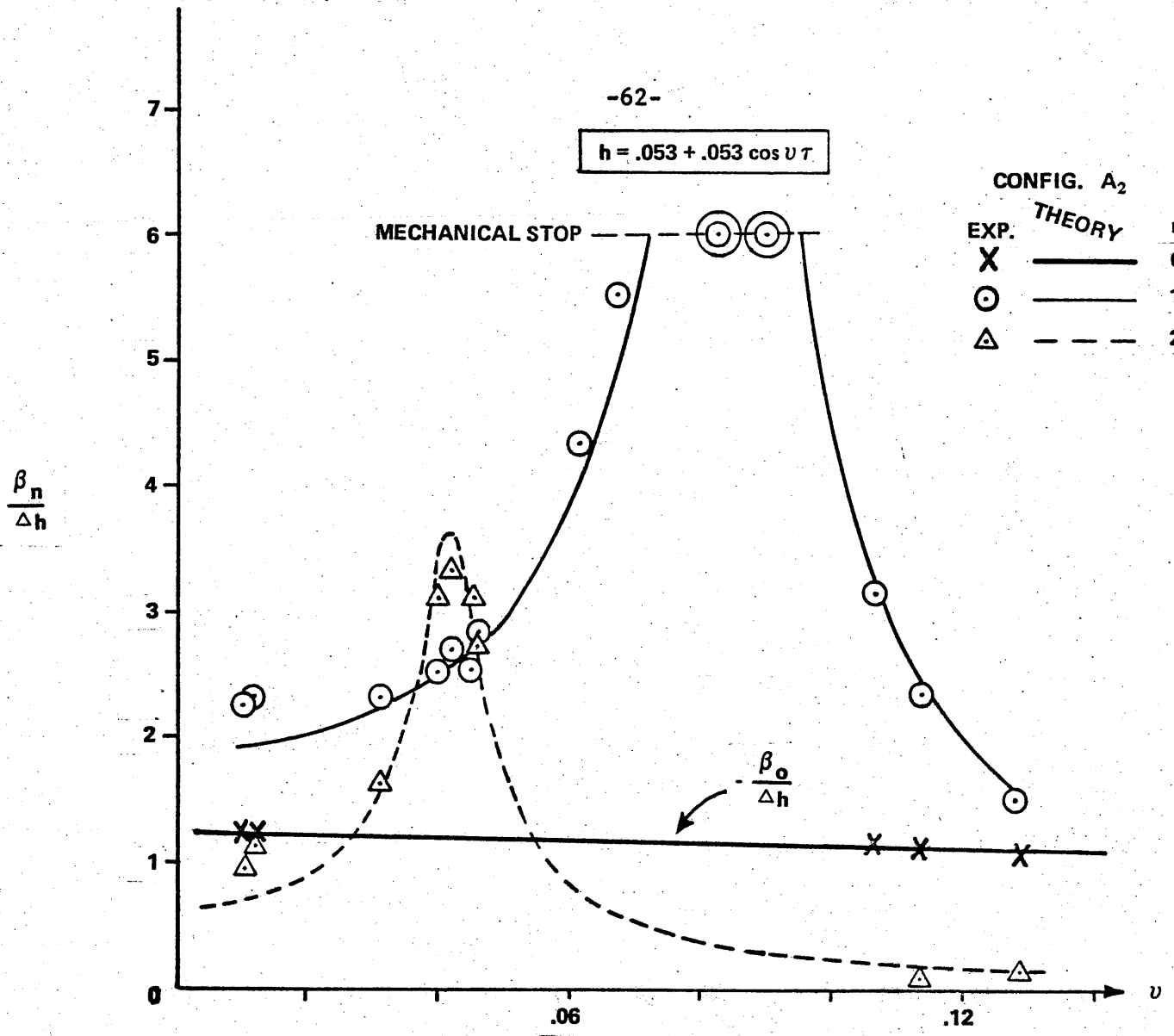


FIGURE 2.22F
COMPONENTS OF
FLAP RESPONSE

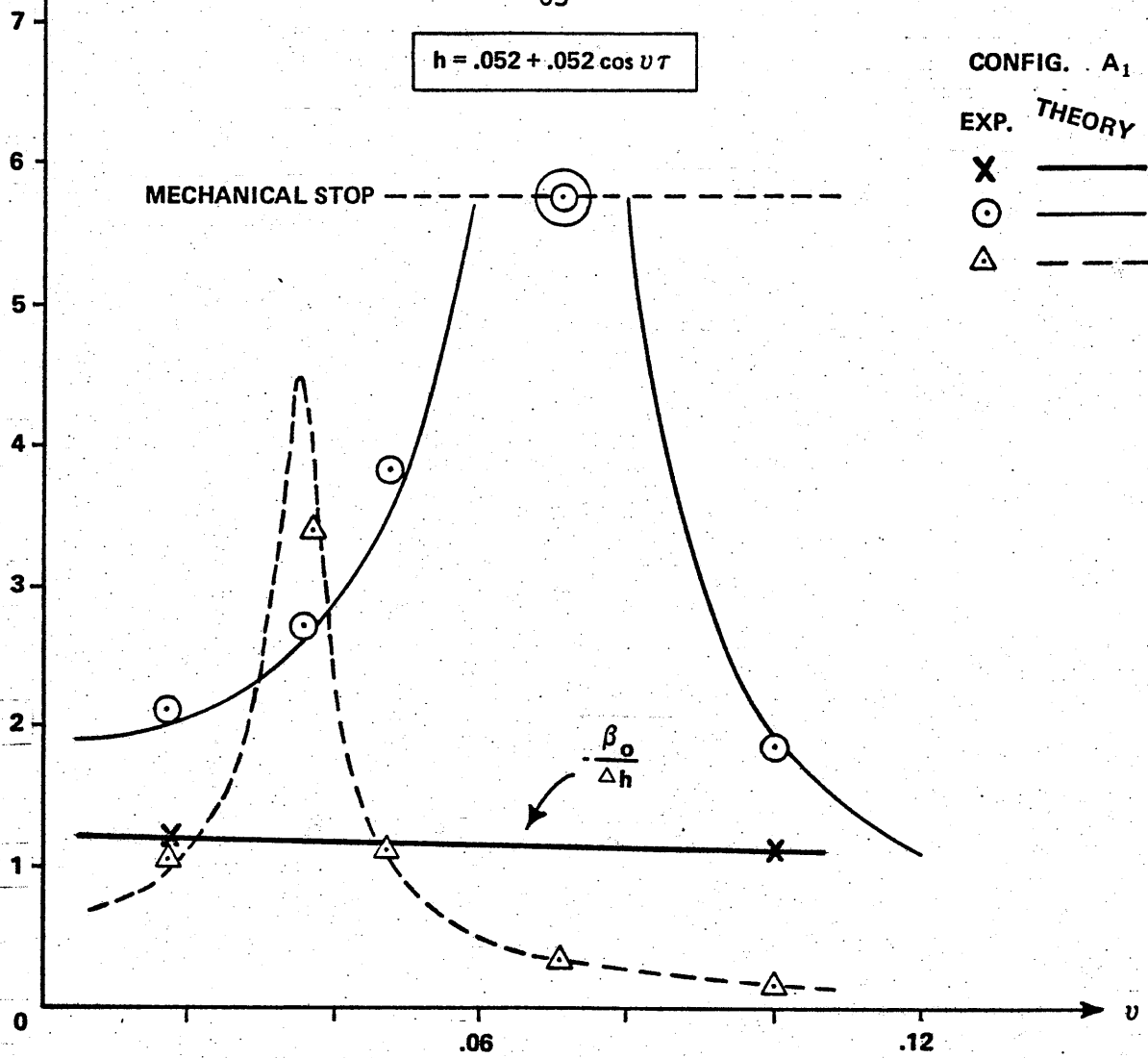
$$h = .052 + .052 \cos v \tau$$

CONFIG. A₁

EXP.	THEORY	n
X	—	0
○	—	1
△	- - -	2

MECHANICAL STOP

$$\frac{\beta_n}{\Delta h}$$



$$\phi_n \text{ (DEG)}$$

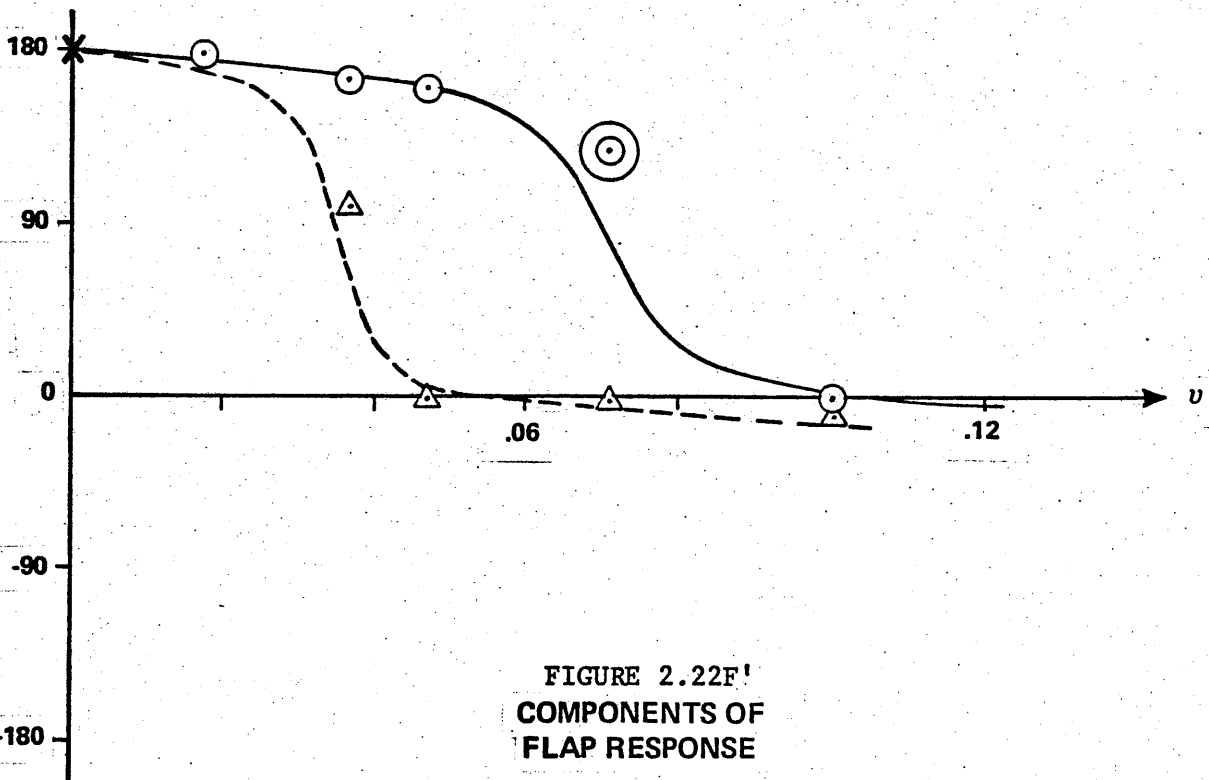


FIGURE 2.22F'
COMPONENTS OF
FLAP RESPONSE

PART-SPAN SPOILER

$$h = .052 + .052 \cos v \tau$$

CONFIG. A₁

EXP.	THEORY	n
×	—	0
⊙	—	1
△	- - -	2

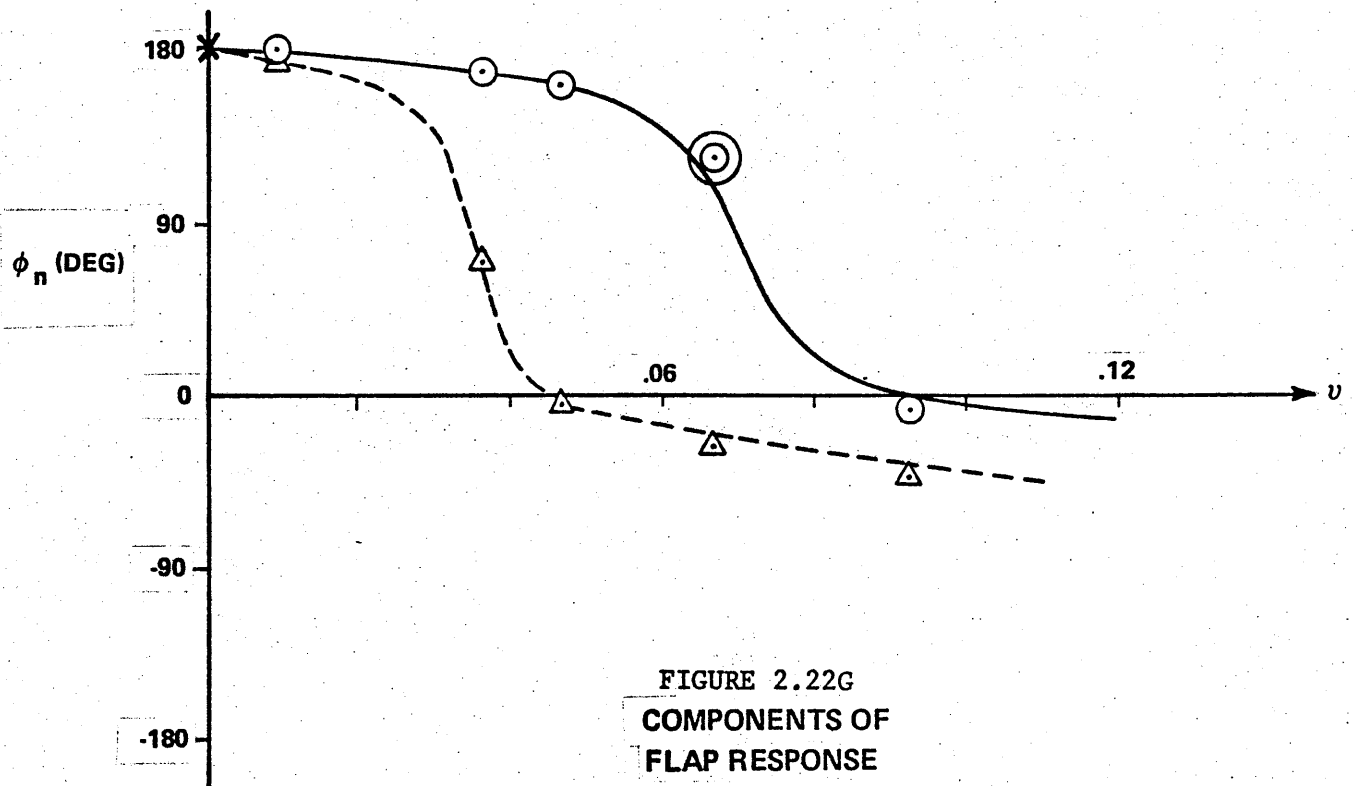
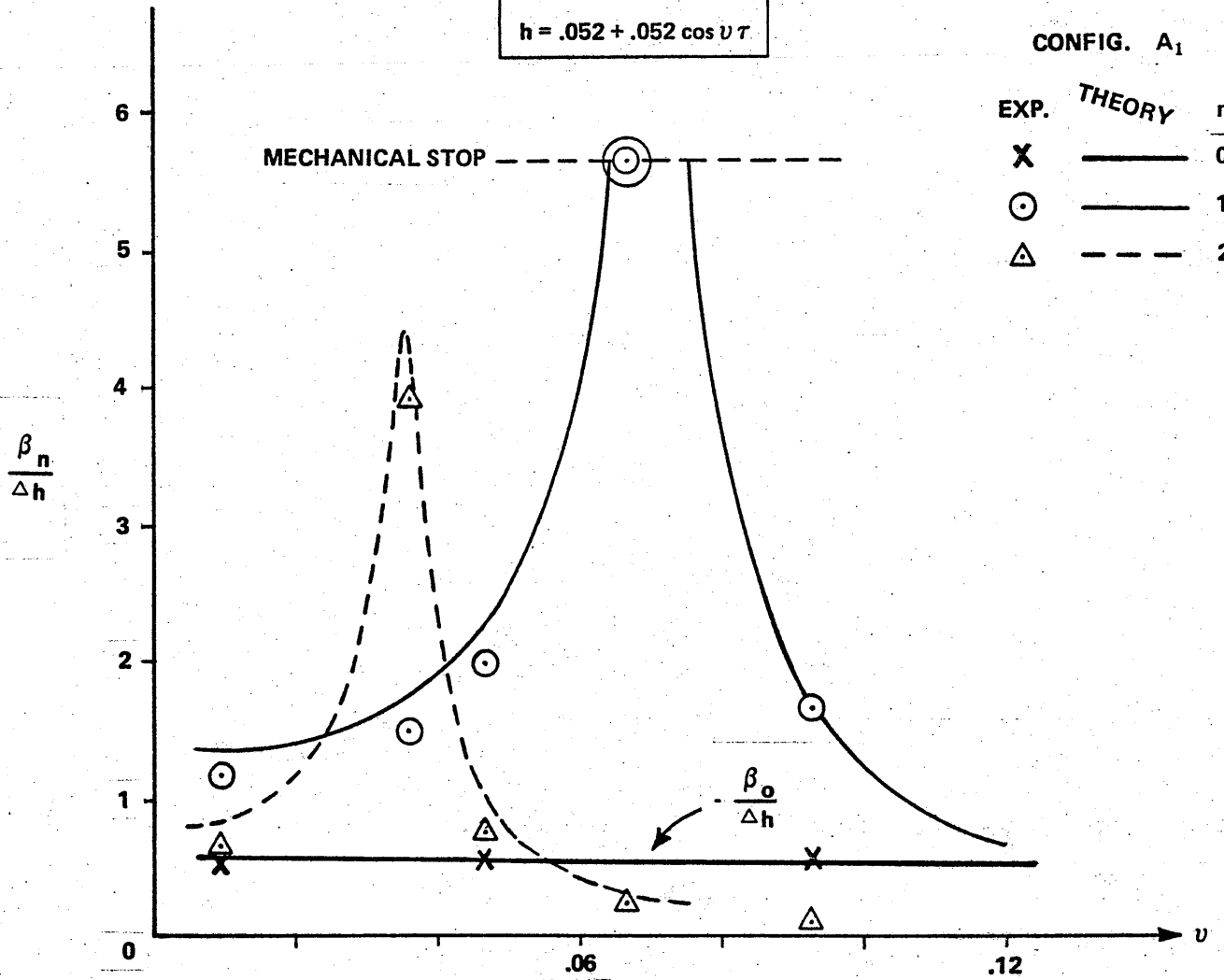


FIGURE 2.22G
COMPONENTS OF
FLAP RESPONSE

and attached flow aerodynamic stiffness so that harmonics greater than order three were greatly suppressed in most cases. In some cases (primarily those involving large values of h_0 and/or Δh) the mechanical stop(s) prevented large amplitude oscillations near resonance. Data which was affected by the stops is shown circled and is suspect because of the effect of the stops.

Since the forcing function, ΔC_H , was a non-linear function of spoiler height and contained higher harmonic content, ultraharmonic resonance was expected and observed. Figures 2.23A-C show cyclic histories of flap response for various frequencies of spoiler motion. These figures serve to illustrate flap response as a function of frequency. Figure 2.23A shows an example of an ultraharmonic response. Here the flap is responding primarily to the second harmonic, i.e., $\nu \cong \frac{1}{2} \nu_t$ where ν_t is flap natural frequency. Also the response of tufts on the model can be observed. Several tufts are located on the perspex end windows as well. Figure 2.23B shows the typical large amplitude response at resonance to the fundamental component. Figure 2.23C shows high frequency flap response. No evidence of sub-harmonic resonance was observed nor expected.

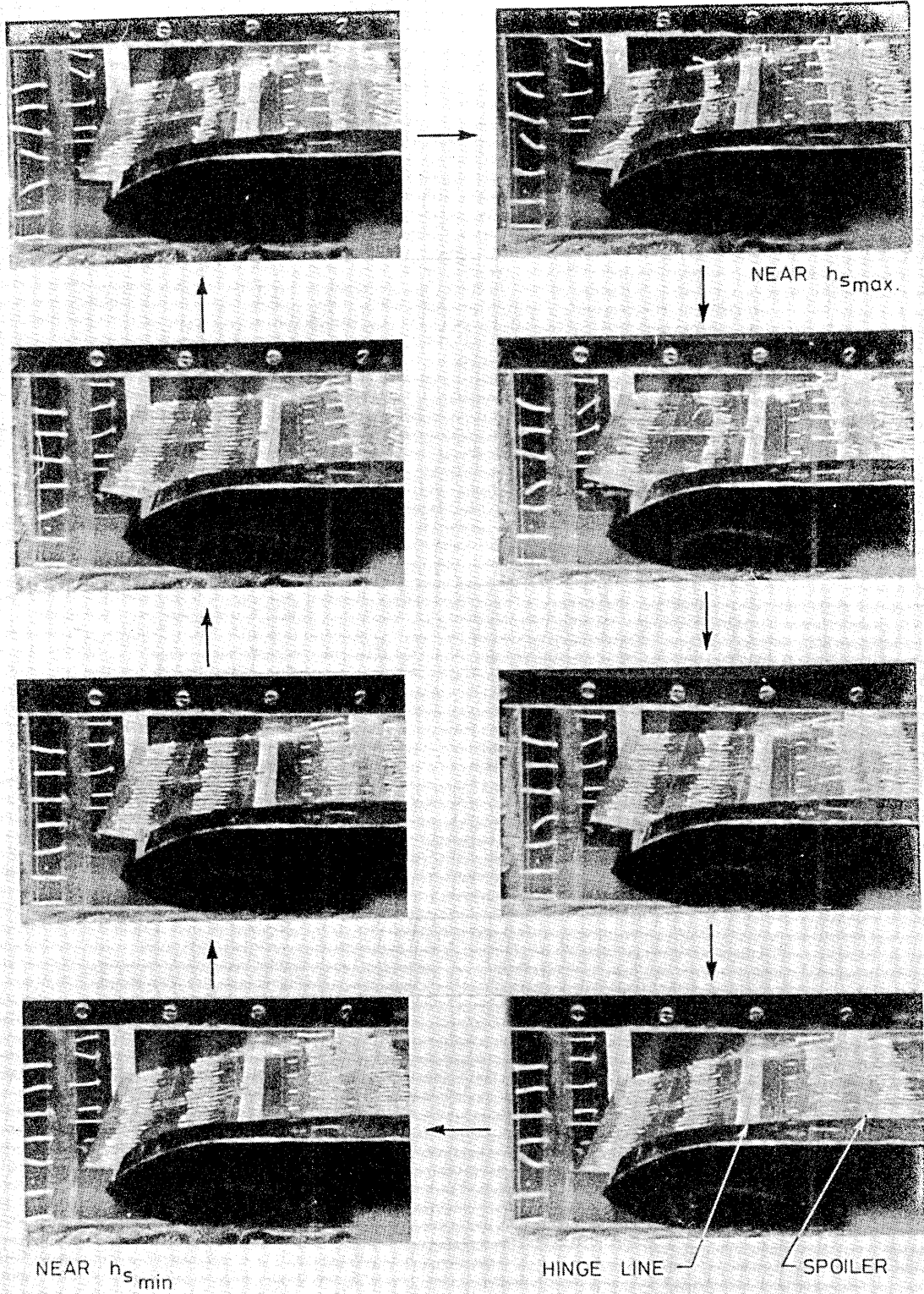
Figure 2.24 contains photographs of oscilloscope traces which show unsteady pressure effects during flap motion. The unsteady pressures appear to represent a linear superposition of flap and spoiler motion effects. That this is not actually the case follows from the discussions of flap-fixed and flap-forced results. The combination of spoiler and flap-motion-induced pressure effects is highly non-linear.

The series of testing which involved the airfoil at high incidence did not yield significant results. An attempt was made to achieve self-excited oscillations which would involve flap-motion coupling to airfoil leading-edge separation and reattachment. Flap-forcing was also tried in order to force the leading-edge laminar separation bubble to burst (see reference 37). The fact that neither of these attempts was successful is probably due to a basic model design deficiency. Three-dimensional effects were significant near the airfoil leading-edge, due primarily to end-plate design. Also an unsteady blockage effect, due to flap motion, had a large adverse effect on the L.E. bubble. The bubble actually increased in length on flap downstroke.

Flap buffeting was observed at incidence and spoiler motion effects were found to be small due to the thickened boundary-layer. Bursts of free-stream turbulence caused hysteresis phenomena in airfoil stall and unstall that masked other motion-induced effects.

Coupled Spoiler and Flap Results

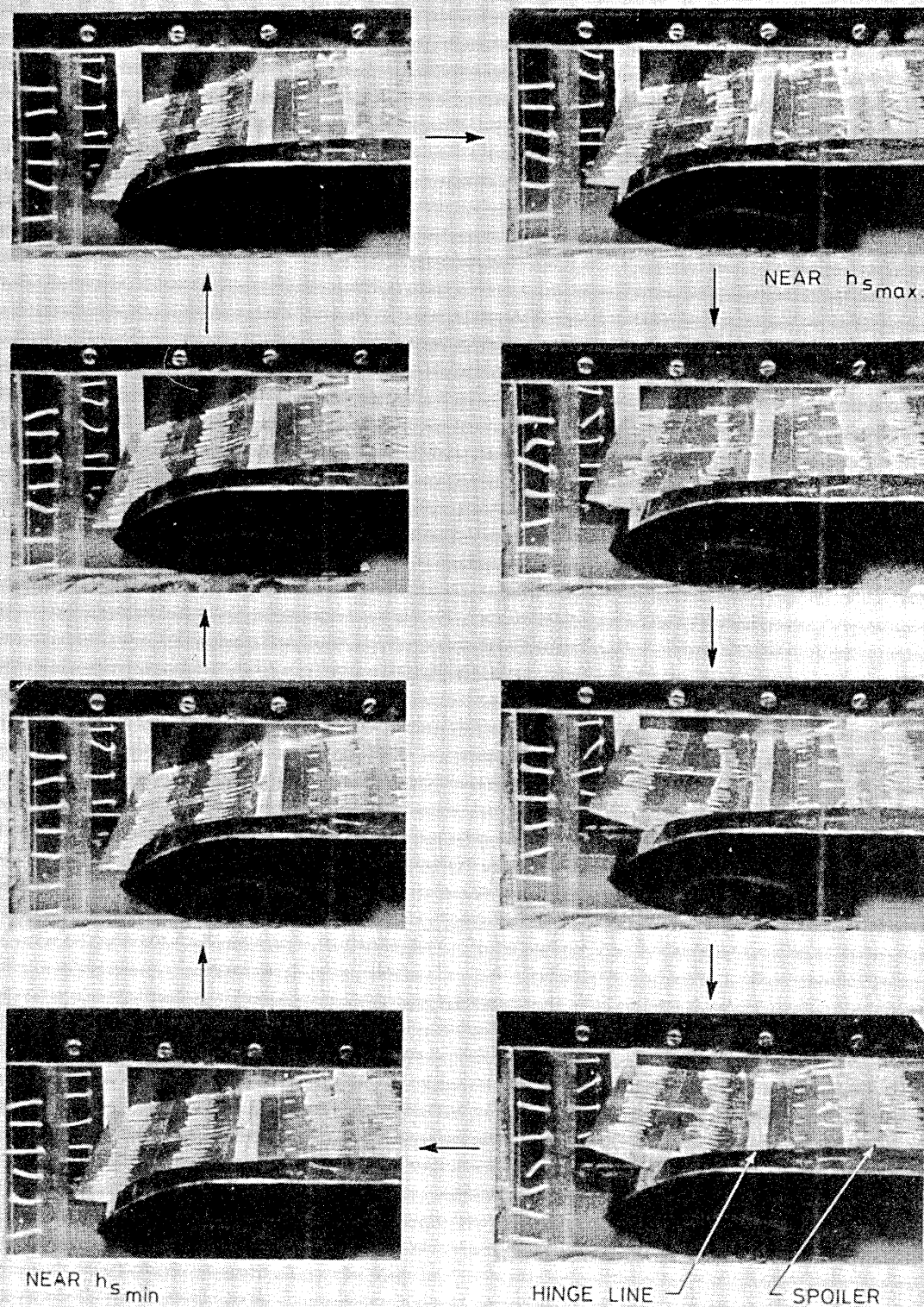
Most of the results from coupled system testing are presented in section 3 and compared to analytical predictions.



$$h_s \cong 0.24 + 0.24 \cos \nu T$$

$$\nu \cong 1/2 \nu_t$$

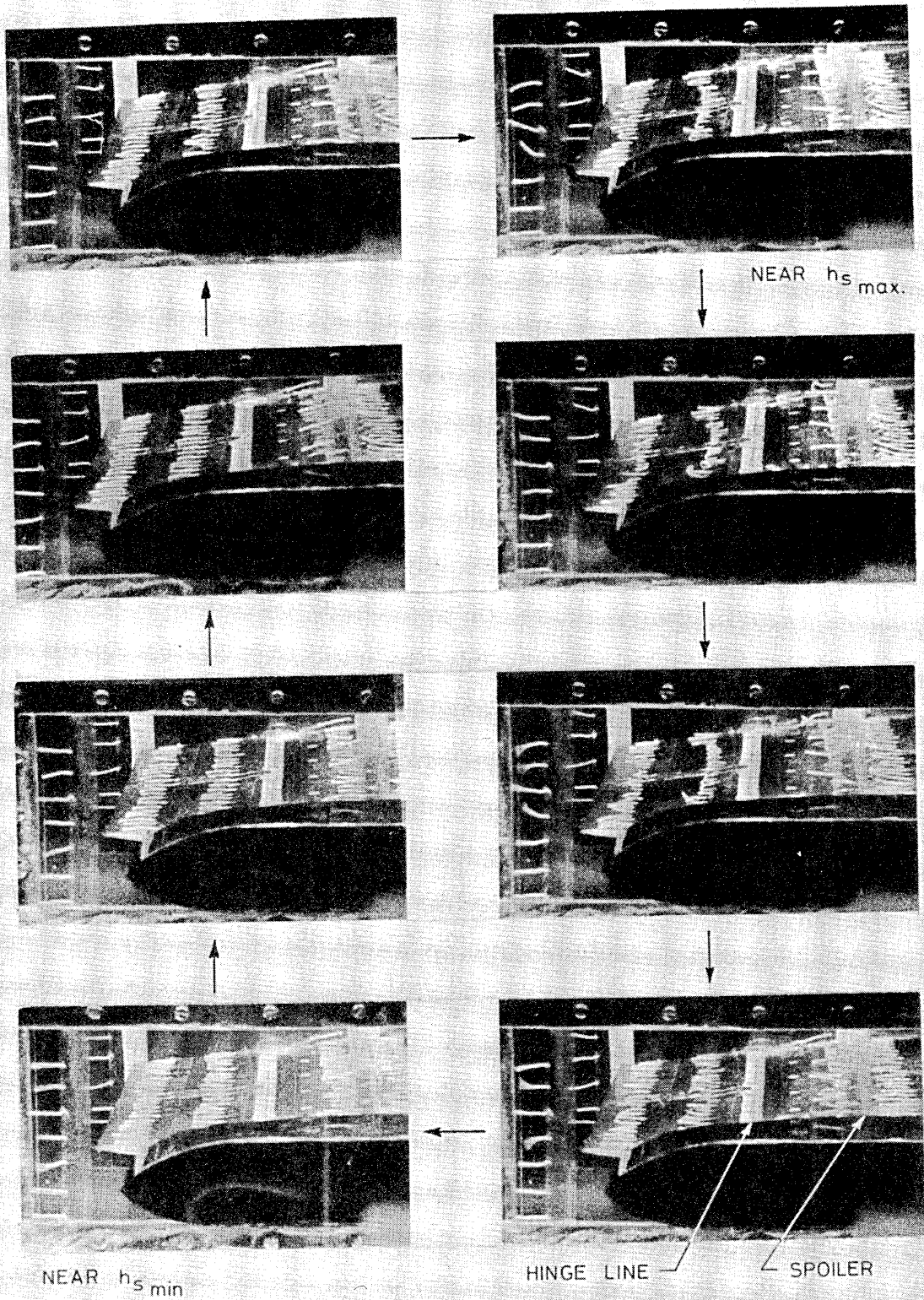
FIGURE 2.23A. ULTRA-HARMONIC FLAP RESONANCE.



$$h_s \approx 0.24 + 0.24 \cos \nu \tau$$

$$\nu \approx \nu_t$$

FIGURE 2.23B. FLAP RESONANCE.

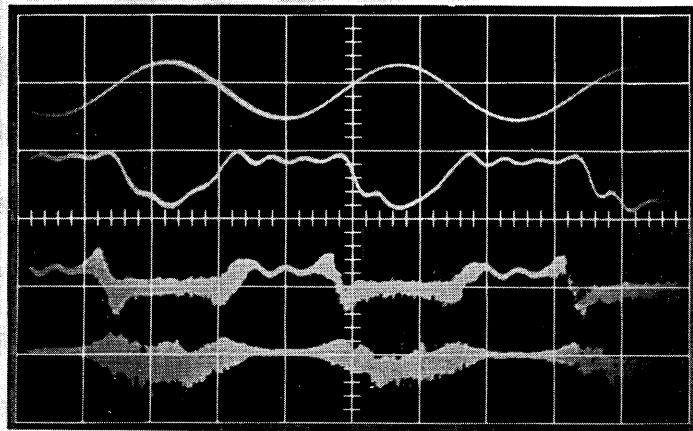


$$h_s \cong 0.24 + 0.24 \cos \nu T$$

$$\nu \cong \frac{3}{2} \nu_t$$

FIGURE 2.23C. HIGH FREQUENCY FLAP RESPONSE.

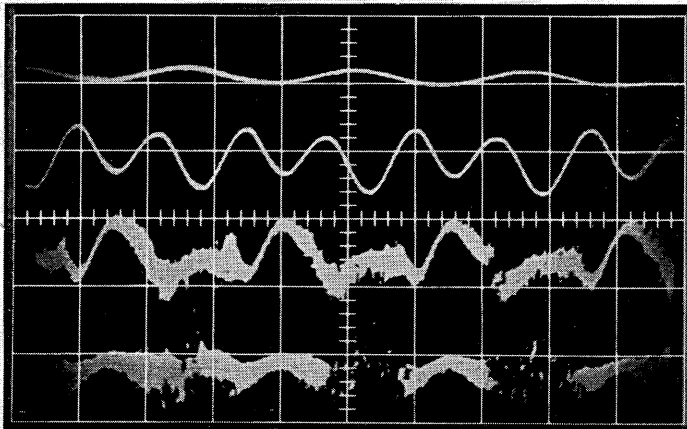
h_s $\begin{matrix} + \\ - \end{matrix}$
 β $\begin{matrix} + \\ - \end{matrix}$
 C_p $\begin{matrix} + \\ - \end{matrix}$
 76.25%
 95%



CASE F

$$h = .053 + .053 \cos \nu \tau \quad \nu \cong .012$$

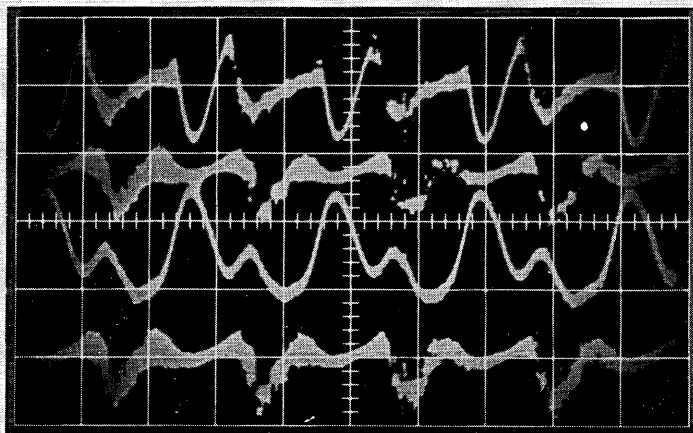
h_s
 β
 C_p
 76.25%
 95%



CASE D'

$$h = .042 + .027 \cos \nu \tau \quad \nu \cong .040$$

C_p
 76.25%
 (Top)
 95%
 (Top)
 82.5%
 (Bottom)
 90%
 (Bottom)



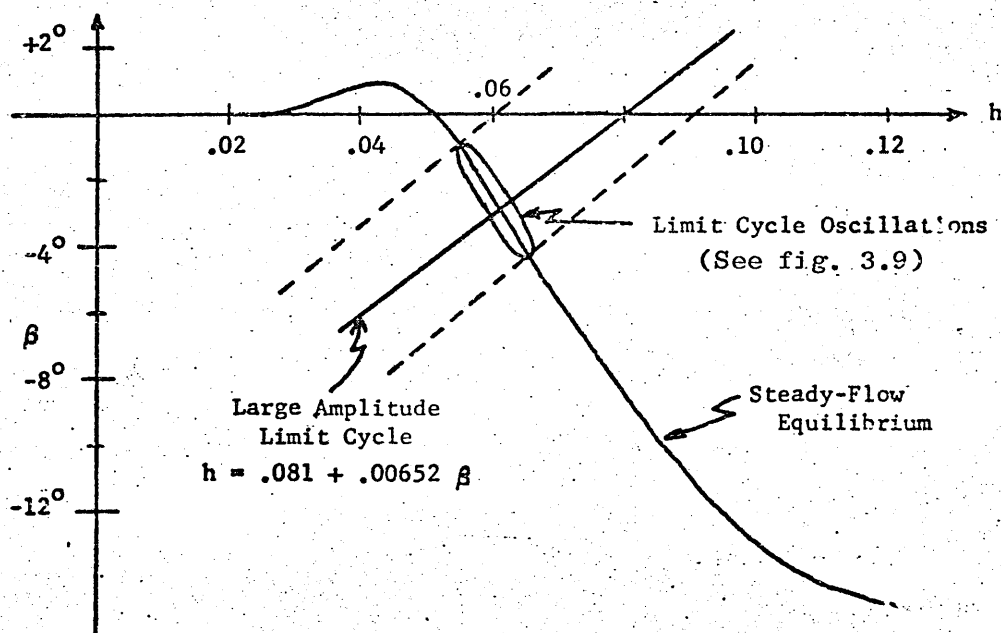
CASE F

$$h = .053 + .053 \cos \nu \tau \quad \nu \cong .046$$

Figure 2.24. Examples of Flap and Pressure Response

Self-excitation of the system was not observed due to the presence of dry friction, but limit-cycle motion could be provoked over a range of coupling parameters, system inertias and free-stream speeds.

The following sketch shows the steady-flow equilibrium relation between h and β ; that is, $C_H = 0$ along the curve shown. Also shown are straight line segments which represent test conditions at the extremes of the limit-cycle region. The region of steady-flow solutions which relate to limit-cycles is circled. Coupled motion showed no significant trace of higher harmonic distortion.



2.4. Discussion of Results

As in reference 10, the ratio of spoiler height and boundary-layer thickness may be used to classify the strength of the spoiler perturbation. That is:

- (1) Weak perturbation: $h_s/\delta \ll 1$
- (2) Strong perturbation: $h_s/\delta = O(1)$
- (3) Overwhelming perturbation: $h_s/\delta \gg 1$

It can be seen that the range of spoiler deflections relates primarily to the strong perturbation case which is recognized as "... the hardest of all to understand."¹⁰

A further classification and description of the spoiler effect can be accomplished with reference to the pressure effects (see Figures 2.10, 2.16 and 2.27A-G). The range of spoiler heights in each category is approximate.

- (1) Short bubble: $0 < h_s < .25''$
 $(0 < h_s/\delta < 1.47 \text{ and } 0 < h_s/\delta^* < 9.6)$
 Very low pressures and varying pressure gradients are attained in the separated region. "Overshoot" of attached flow pressure occurs near reattachment.
 Bubble growth law is roughly $\frac{(x_r - x_{sp})c}{h_s} \equiv d_R \cong 10.$
 Negligible pressure effect at T.E. Minor pressure effect on airfoil "wetted surface."

- (2) Long bubble: $h_s > .25''$
 $(h_s/\delta > 1.47 \text{ and } h_s/\delta^* > 9.6)$
 Relatively constant, low pressure and near zero pressure gradient over most of the bubble. Pressure recovery of near constant gradient to reattachment or to the T.E. No significant overshoot effect.
 Bubble growth law is roughly $\frac{(x_r - x_{sp})c}{h_s - .12} = d_R \cong 38.$
 Pressure effect at T.E. Stronger pressure effect over wetted surface.

The long bubble category can be broken down into 3 components as follows:

- (2) Long bubble: $h_s > .25''$
 - i. Reattachment point lies on flap surface:
 $.25'' < h_s < .31''$
 - ii. Recompression occurs on flap surface:
 $.31'' < h_s < .6''$
 - iii. Near constant pressure aft of spoiler. Recompression (to free-stream static pressure) occurs in the wake: $h_s > .6''$

In reference 8, Newman investigated a spoiler on an airfoil that involved reattachment onto the airfoil surface. He stated that this spoiler effect resembled effects which have been observed behind shock waves. Significant transverse pressure gradients were also found to exist in the separated region.

Barnes in reference 7 investigated the effect of normal fences on a flat plate. A significant finding was that spoiler tip geometry is not critical. In the present experiment the tip is flat.

DeBroderode and Bradshaw in reference 9 discuss the flow behind a rearward facing step. Pertinent conclusions from references 5 and 7-10 can now be summarized in order to explain observed steady-flow behavior.

Upstream of the spoiler pressure coefficients are functions of $\frac{h_s}{\delta^*}$ and $\frac{(x_{s1} - x_{sp})c}{h_s} \equiv d_{s1}$ where x_{s1} is the location of the upstream separation point. As $\frac{h_s}{\delta^*}$ increases, d_{s1} decreases. The process of reattachment of the separated shear-layer downstream of the spoiler is governed largely by the rate of entrainment of fluid from the separated region. The rate of entrainment is largely independent of shear-layer thickness at the spoiler tip.^{5,7} Spoiler base pressure (C_{ps}) (i.e., pressure on the downstream face) decreases very rapidly as $\frac{h_s}{\delta^*}$ increases, for small values of $\frac{h_s}{\delta^*}$, and finally remains fairly constant for large $\frac{h_s}{\delta^*}$. The fluid in the separated region beneath the lower edge of the separated shear layer will be accelerated from near rest due to entrainment into the shear layer, hence the pressure in this region will decrease relative to the surroundings. The resultant transverse pressure gradient causes the shear-layer to curve towards the surface and, possibly, to reattach. Near the reattachment point an amount of fluid is returned to the separated region to balance the amount entrained.

An increase in entrainment will cause an increase in velocities inside the bubble with a resulting decrease in C_{ps} . The larger transverse pressure gradient will cause an increase in shear-layer curvature, and so a decrease in bubble length. An increase occurs in the angle of incidence of the dividing streamline, so an increase occurs in fluid returned to the bubble. Hence, lower base pressures are associated with smaller non-dimensional reattachment distances, d_R .

Three-dimensional flow effects may also be significant. (Refer to Figures 2.11 and 2.12.) The corner flow can have an influence on C_{ps} and reattachment distance, d_R , even though the crossflow is too small to measure directly. In separated flows a spanwise supply of fluid to the recirculating region can greatly affect its size. De Broderode and Bradshaw⁹ suggest that the large three-dimensional effects found on airfoils are due mainly to the presence of a curved separation line and to crossflow via spanwise lift grading. Barnes⁷ suggests that the degree of three-dimensionality in this type of flow is related to the ratio:

$$\frac{\text{wall boundary-layer thickness}}{\text{model span}}$$

measured at the spoiler position. For this experiment the ratio is:

$$\frac{\delta_w}{b} \cong .038$$

In reference 9 it is stated that end effects are virtually negligible for aspect ratios greater than 2, where

$AR \equiv \frac{\text{bubble span}}{\text{bubble length}} > 1.8$ for the present experiment with reattachment on the flap surface.

In order to explain the effect of three-dimensionality we refer to reference 9. "If the flow near the center-line were independent of details of end-plate conditions we would expect, on a two-dimensional argument with respect to base bleed, that C_{p_s} and d_R would rise together if fluid were injected into the bubble from the corner region." The effect of open gaps along the spoiler and flap/end-plate intersections did result in this behavior. On the basis of this suggestion a decrease in AR allows a greater fluid flow through the gaps into the bubble and should cause an increase in d_R and C_{p_s} . This was observed.

A further observation concerning the reattaching shear-layer is that the short and long bubbles described herein appear to relate to the short and long bubbles which have been defined in discussions of airfoil stalling. It is possible that the short bubble exists for low spoiler heights (i.e., for a strong perturbation) but "bursts" and becomes a long bubble when the shear-layer cannot overcome a large adverse pressure gradient. In the present case "bursting" relates to a modification of streamline curvature, base pressure and the bubble growth law.

A pressure recovery coefficient for the bubble can be defined as:

$$\sigma \equiv \frac{p - p_s}{\frac{1}{2} \rho U_s^2} = \frac{C_p - C_{p_s}}{1 - C_{p_s}}$$

where the subscript s refers to conditions near the point of separation at the spoiler tip, but in the present case cannot be measured directly.

In reference 38, Kao shows that reduced coordinates based on the pressure recovery coefficient defined above and a non-dimensional distance based on bubble length, i.e.,

$$d = \frac{x - x_{sp}}{x_r - x_{sp}}, \text{ may be used to represent pressure distributions}$$

on airfoil long bubbles. For the present experiment this representation appears valid for long bubbles.

The short bubble defined in this experiment does not fit the above representation primarily because of the occurrence of

large suction peaks and overshoot of pressure at reattachment. However, if σ is evaluated at reattachment, a value of $\sigma \cong .30$ is found. The significance of this factor is discussed in reference 20 and later in sections 3 and 4.

Results of pressure measurements also show the effect of the secondary separation, observed within the separated region. Surface pressures measured near and under the secondary separation region are expected to be higher than those measured in the "primary" separation bubble. The pressure data for 65% c in Figure 2.10 best illustrates the effect of the secondary separation. For spoiler heights greater than about .3" pressure increases due to the effect. The 76.25% c data appears to be affected for $h_s > .45"$.

The effect of pressure overshoot on hinge-moment coefficient is shown in Figure 2.10 as a positive moment effect for $.125" < h_s < .31"$.

In Figures 2.25A-F and 2.26A-F incremental hinge-moment and pressure data are reconstructed for zero frequency and $\nu = .12$. Reconstructed pressure data is also found in Figures 2.27A-G. Reconstructed pressure coefficients represent the difference between actual pressures and zero spoiler height pressures, i.e.:

$$\delta C_p \equiv C_p - (C_p)_{h=0}$$

Components up to the 4th harmonic, in most cases, were used to reconstruct the actual data. In some cases of pressure data a fairly abrupt divergence (see Figure 2.7) was "rounded off"; therefore, Figures 2.25, 2.26 and 2.27 can be interpreted only in a limited sense.

Figures 2.25A-F are cyclic (or "time") histories plotted so as to show two periods of reconstructed data on the same scale. It is apparent that unsteady curves show a lag and distorted waveform. The lag in separated flow effects is greater near the trailing-edge. Closer to the hinge-line, increased suction associated with the separation bubble is obvious.

In the unsteady case, once the bubble affects the pressure at a point, there follows a rapid decrease in pressure. Then follows a monotonic, near constant rate increase to attached-flow pressure. As expected, unsteady incremental hinge-moment illustrates the weighted integral effect of unsteady pressures.

At present there is no theory, including that of reference 33, which can predict these unsteady loading effects. A possible description can be offered, however, to explain the observed behavior of the unsteady separated region. On the spoiler upstroke an amount of fluid from the shear-layer must

$$h = .027 + .027 \cos v\tau$$

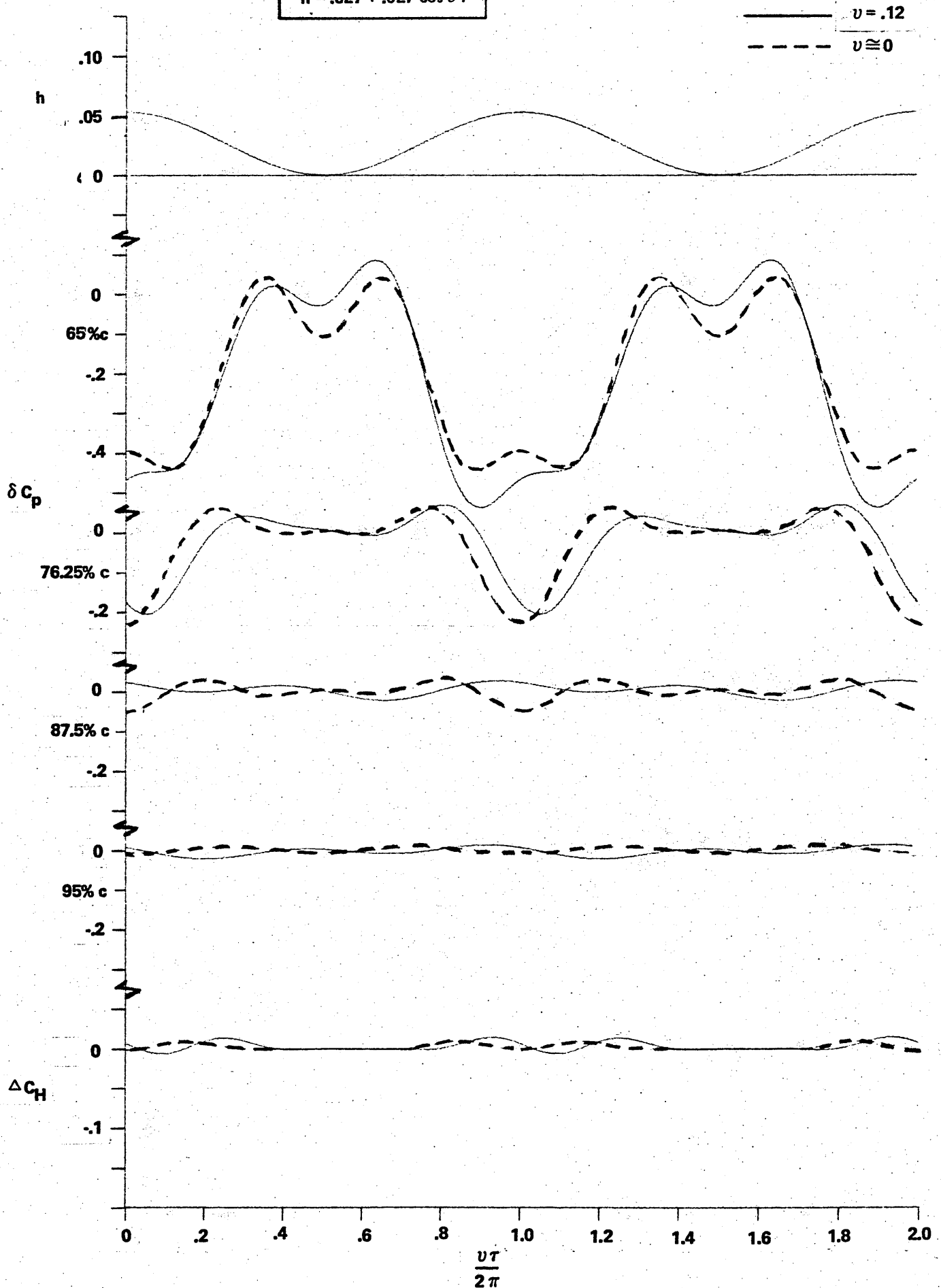


FIGURE 2.25A. RECONSTRUCTED TIME HISTORIES OF SPOILER HEIGHT, PRESSURES AND HINGE-MOMENT

$$h = .053 + .026 \cos v \tau$$

— $v = .12$

- - - $v \cong 0$

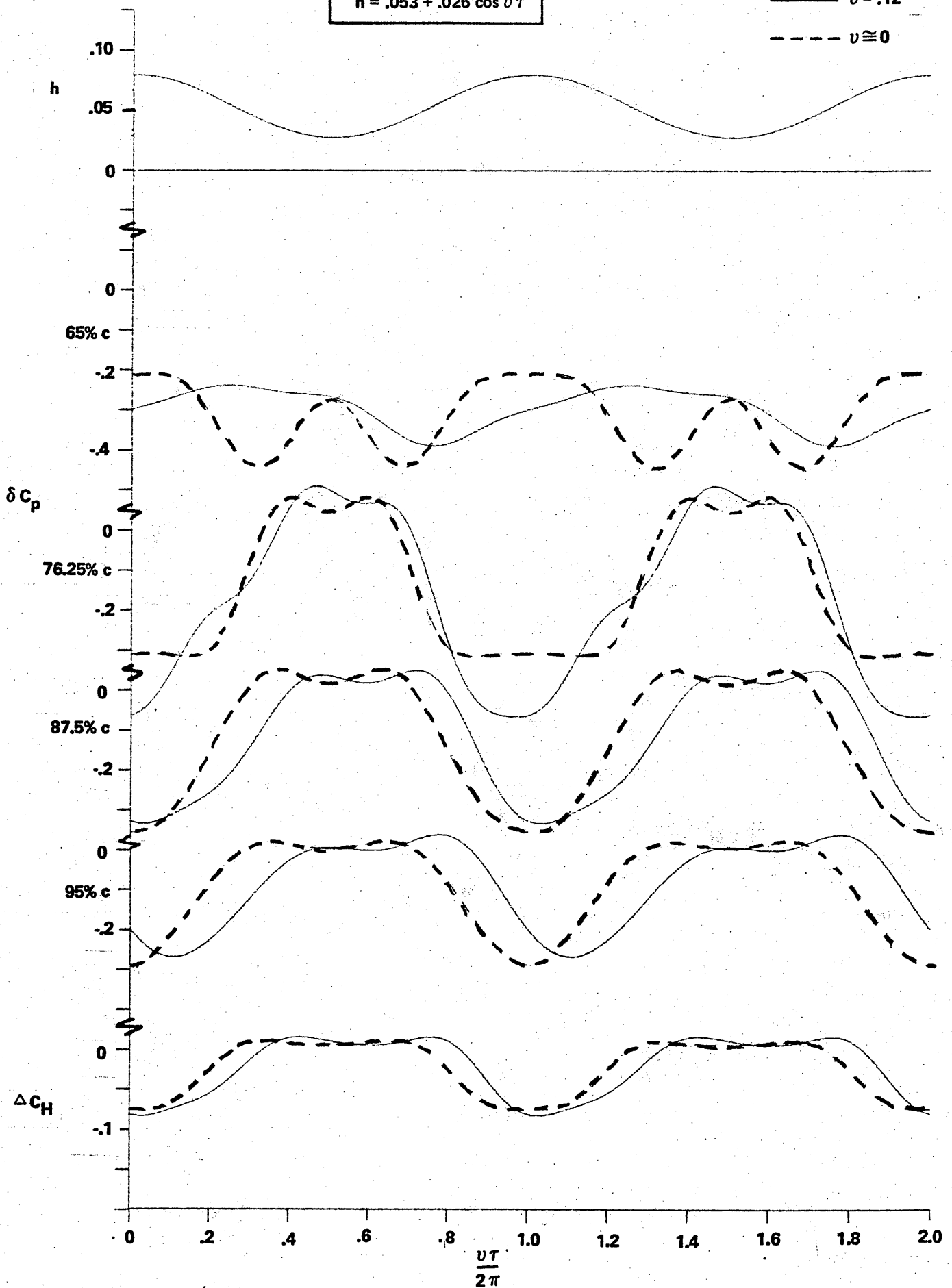


FIGURE 2.25B. RECONSTRUCTED TIME HISTORIES OF SPOILER HEIGHT, PRESSURES AND HINGE-MOMENT

$$h = .076 + .027 \cos v\tau$$

— $v = .12$

- - - $v \cong 0$

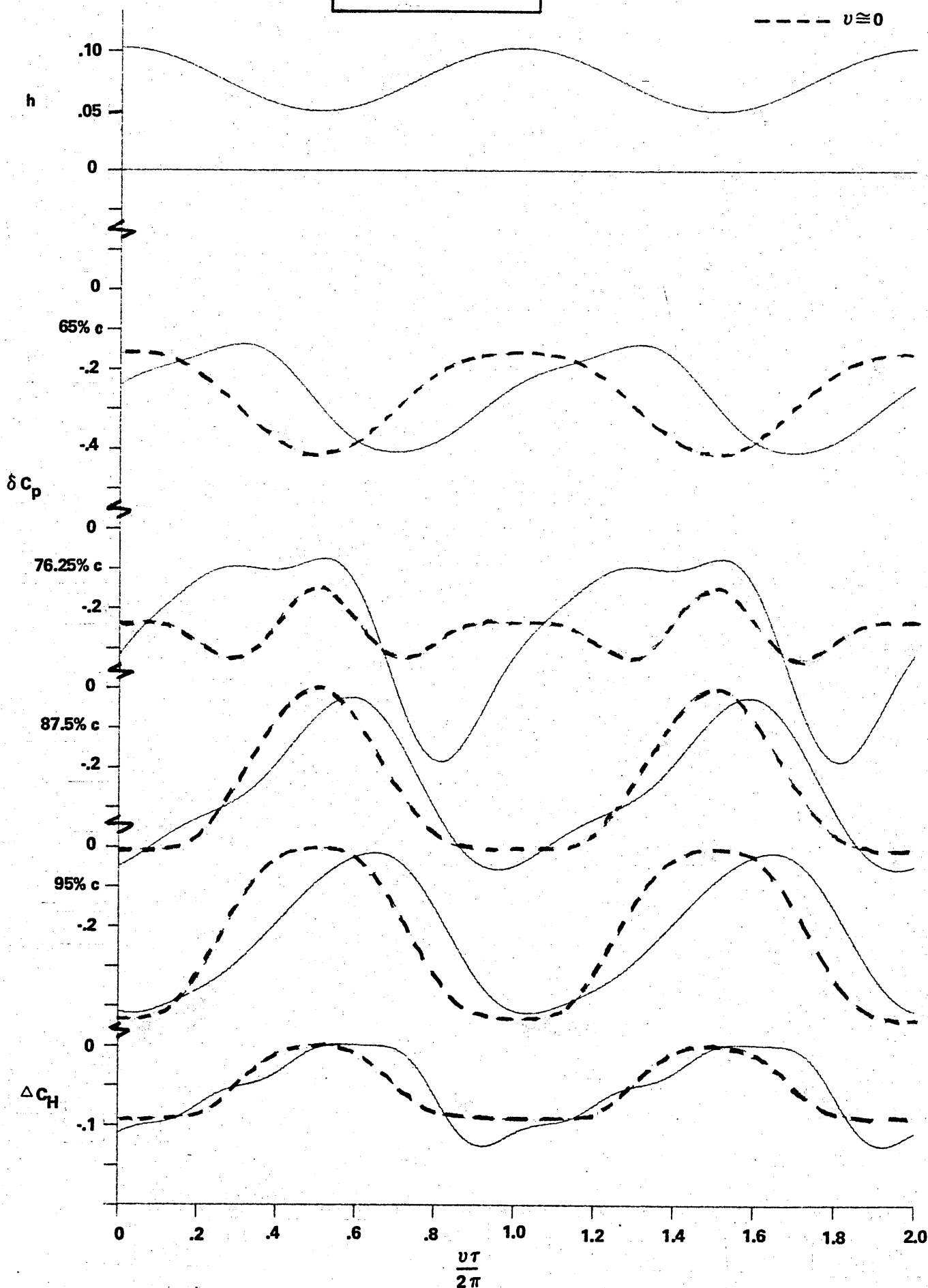


FIGURE 2.25C. RECONSTRUCTED TIME HISTORIES OF SPOILER HEIGHT, PRESSURES AND HINGE-MOMENT

$$h = .039 + .039 \cos v\tau$$

— $v = .12$

- - - $v \cong 0$

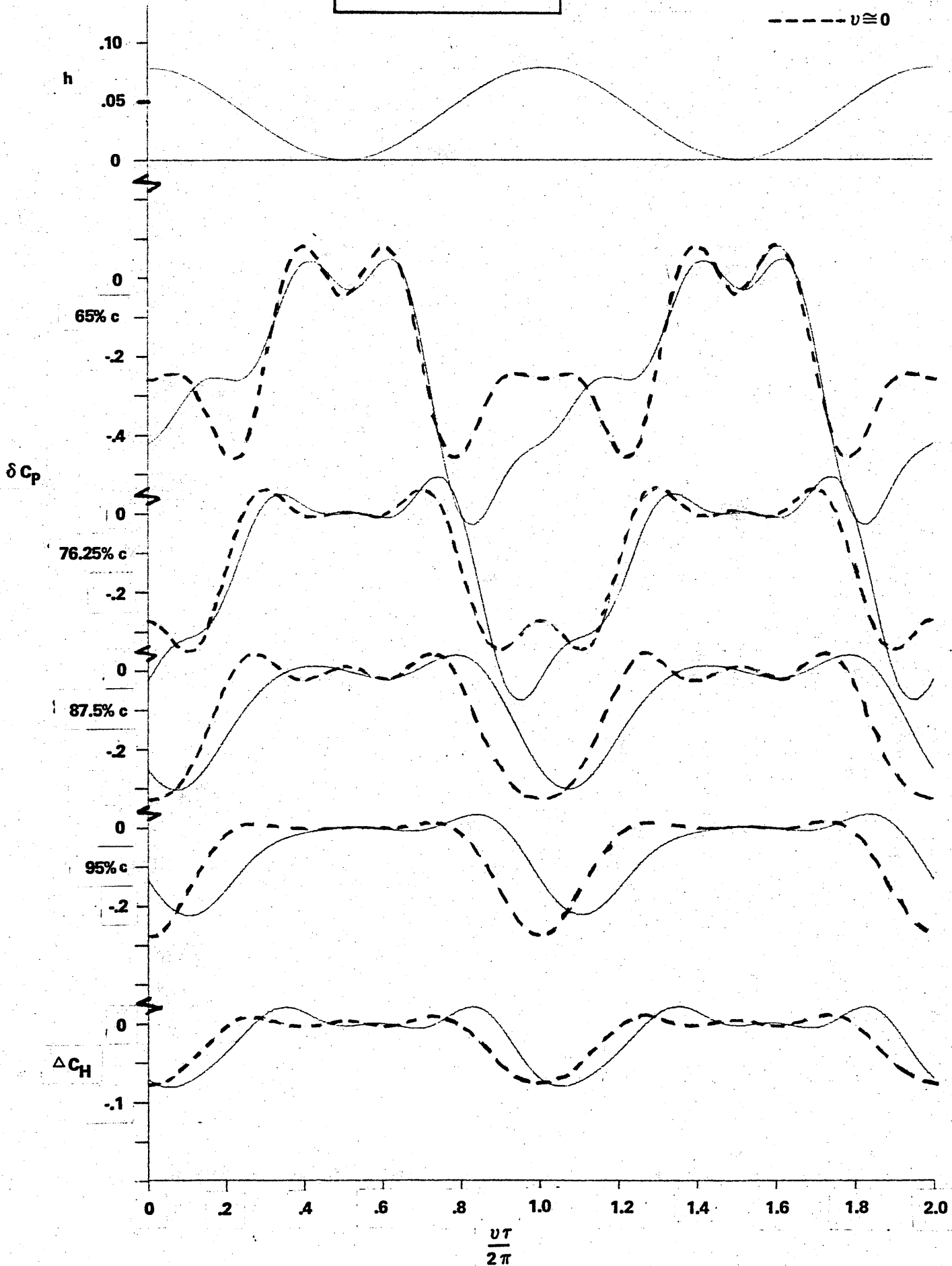


FIGURE 2.25D. RECONSTRUCTED TIME HISTORIES OF SPOILER HEIGHT, PRESSURES AND HINGE-MOMENT

$$h = .062 + .040 \cos v \tau$$

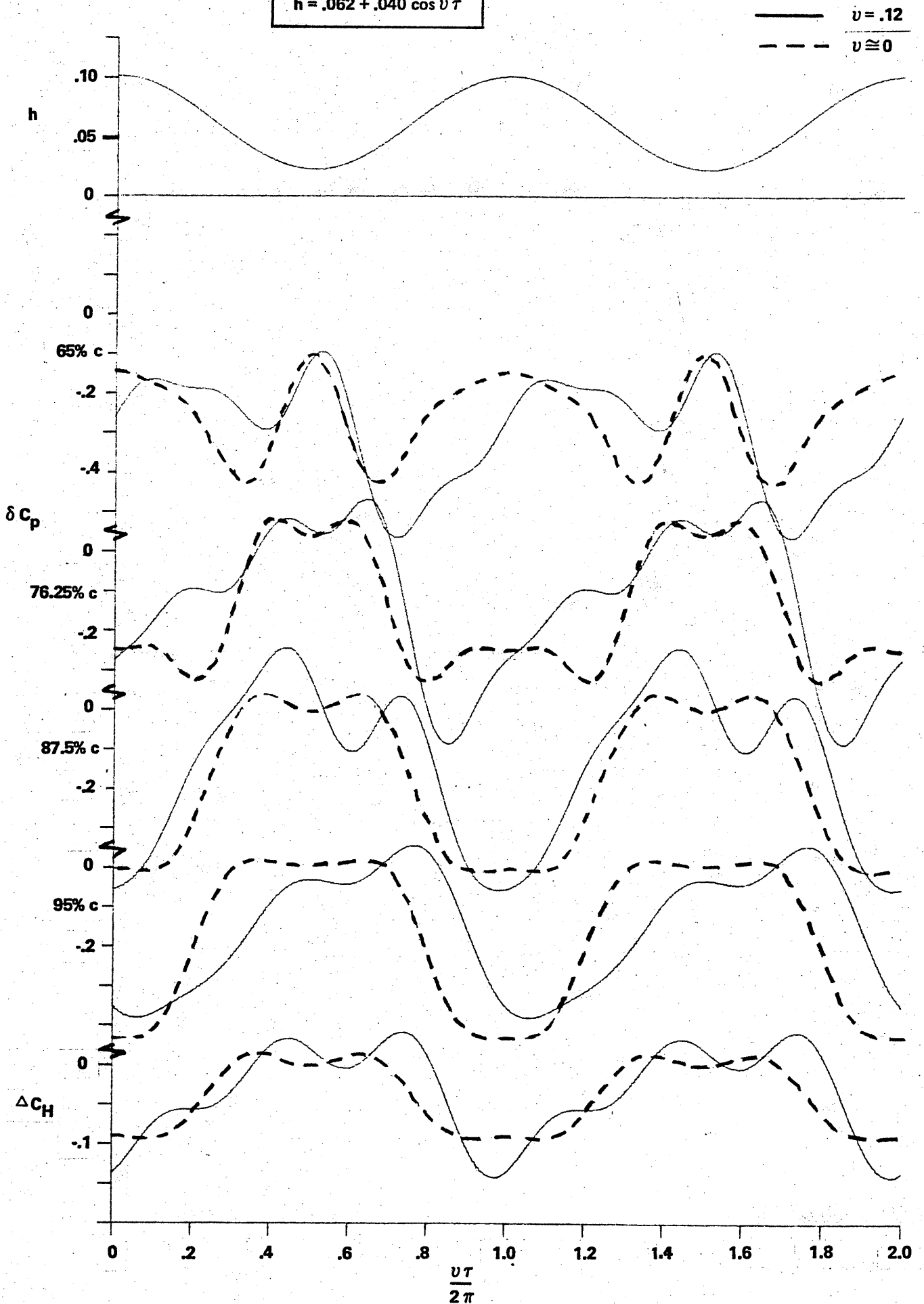


FIGURE 2.25E. RECONSTRUCTED TIME HISTORIES OF SPOILER HEIGHT, PRESSURES AND HINGE-MOMENT

$$h = .052 + .052 \cos v \tau$$

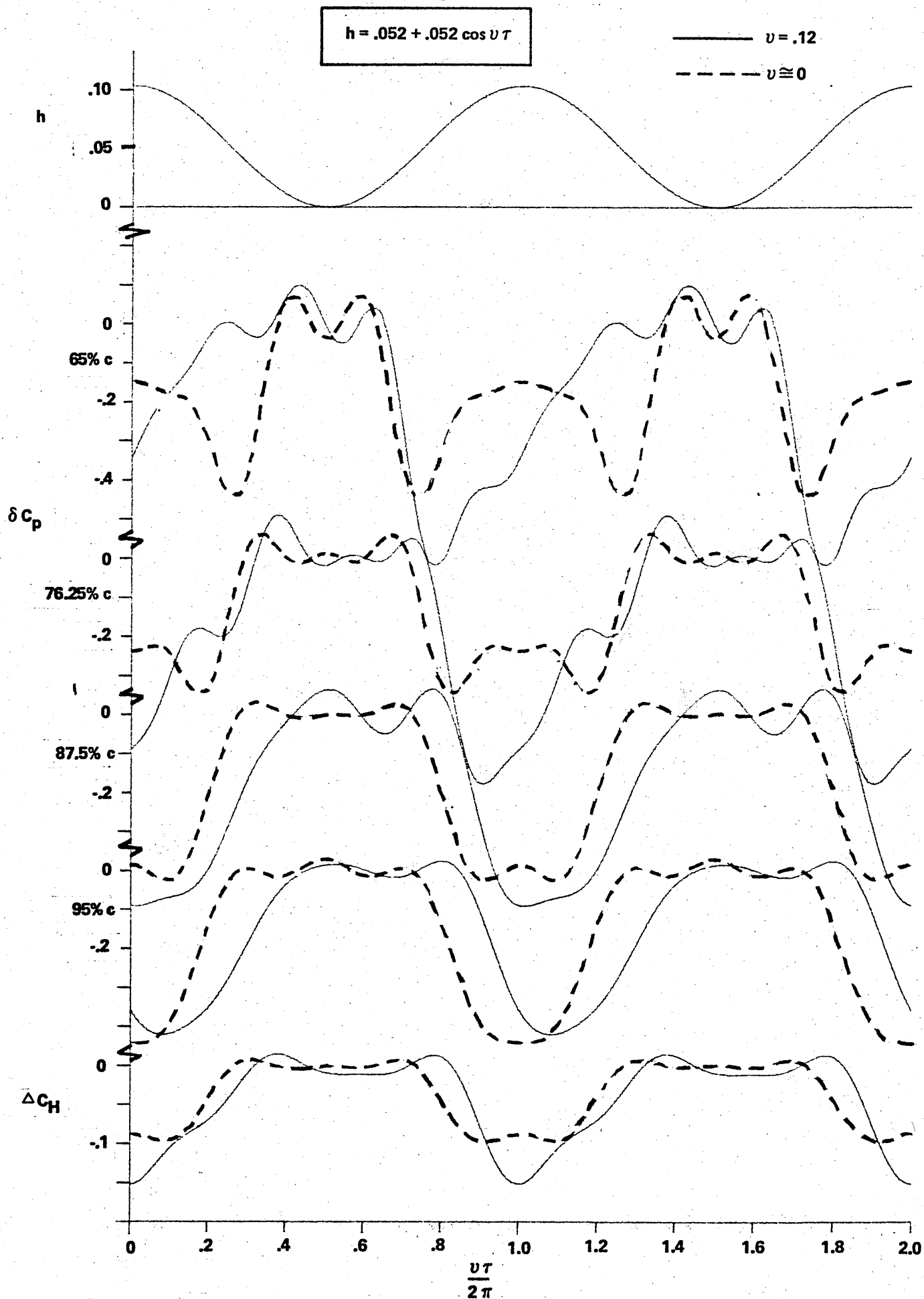


FIGURE 2.25F. RECONSTRUCTED TIME HISTORIES OF SPOILER HEIGHT, PRESSURES AND HINGE-MOMENT

be returned to the separated region so that it can grow in size. The returned fluid will contain vorticity of predominately clockwise sense. An increased positive pressure gradient must exist over the downstream portion of the bubble in order to direct the greater mass flux which is required in the unsteady case. The increased pressure gradient relates to greater "dividing streamline" curvature, lower base pressures and a shorter bubble length. Thus, on the spoiler upstroke, the bubble becomes shorter and taller with lower pressures that feed an increasing supply of fluid to the interior. The vorticity supplied to the bubble results in a greater magnitude of circulation, which is compatible with the induced pressure and velocity fields. Of course, bubble boundary conditions must match the unsteady external flow field, but the explanation above allows for such a matching. A more detailed model of these fluid dynamic interactions will be found in section 3.

Figures 2.26A-F illustrate the dynamic effects associated with spoiler motion. Also shown are reproduced, average curves for quasi-steady flow. The reproduced pressure and hinge-moment data is plotted versus spoiler height over a cycle of motion. Actual cyclic histories would contain evidence of turbulent fluctuations and would show cycle-to-cycle variations due to random fluctuations.

On the spoiler upstroke evidence of low pressure "overshoot" is apparent near the spoiler, even for cases involving small values of h_o and Δh . These traces bear a striking resemblance to those associated with dynamic stalling of an airfoil.

Figures 2.27A-G show the reconstructed data for pressures at points on the airfoil surface. Data "points" are joined by curves which are estimated with reference to steady-flow chordwise distributions and to time histories of pressure data. Only two points are available upstream of the spoiler so the distribution shown there is expected to be less accurate than that shown downstream, where six data points--four on the upper and two on the lower surface--are used.

The chordwise pressure distributions clearly illustrate the spatial as well as temporal distribution of pressure. Also apparent is the change in pressures and length associated with an unsteady bubble.

The part-span spoiler curves shown in Figure 2.27G are based on less reliable data than that for the full-span spoiler. On the spoiler upstroke, in the unsteady case, higher than quasi-steady values of pressure are initially found under the separation bubble. This could be caused by an inflow of fluid from the relatively high pressure region at the spoiler ends. When the bubble becomes more fully developed, mass flux into the bubble may occur primarily at the reattachment line. It must be emphasized that this explanation is speculative at present.

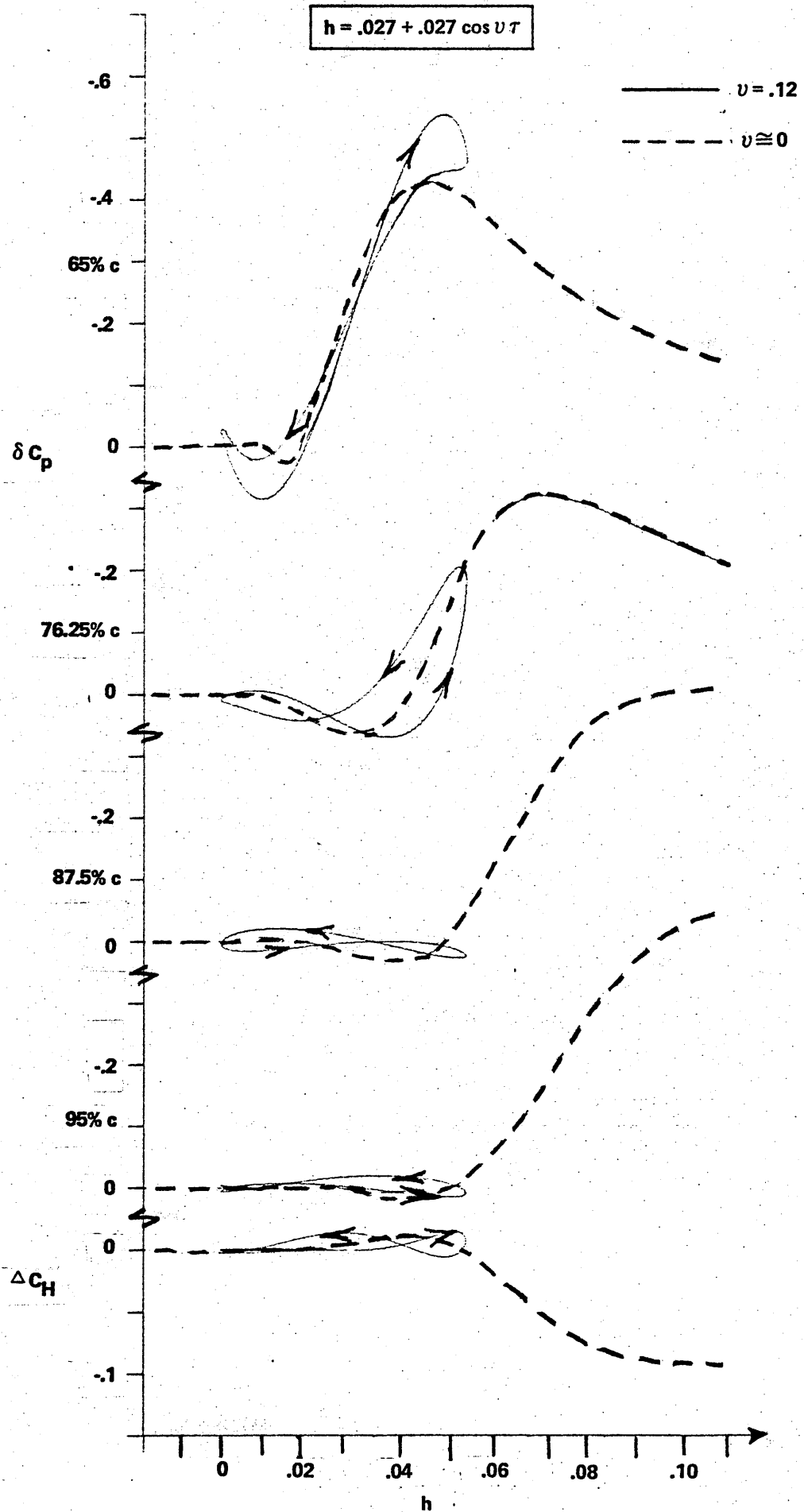


FIGURE 2.26A. RECONSTRUCTED DYNAMIC VARIATIONS OF PRESSURES AND HINGE-MOMENT

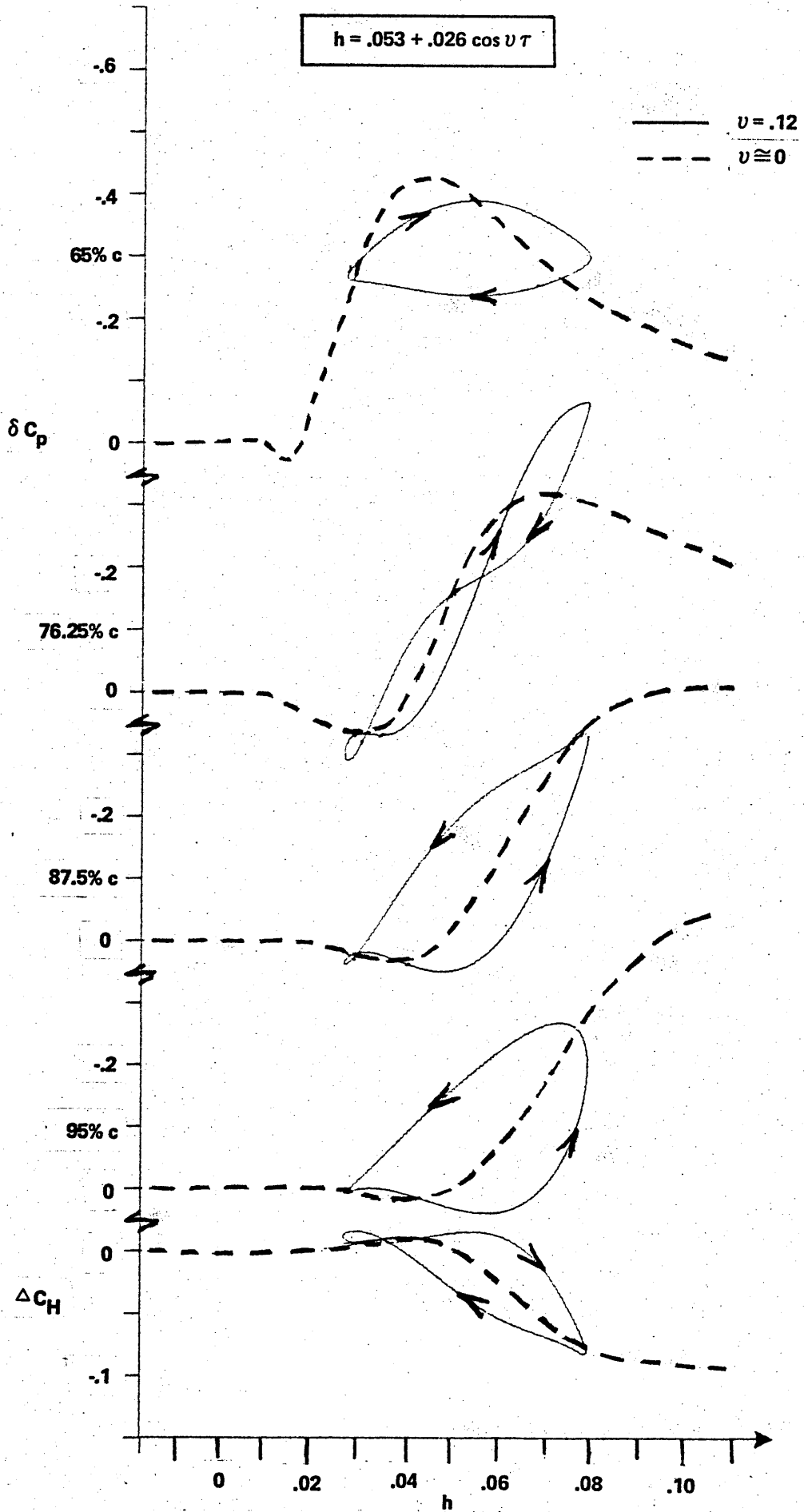


FIGURE 2.26B. RECONSTRUCTED DYNAMIC VARIATIONS OF PRESSURES AND HINGE-MOMENT

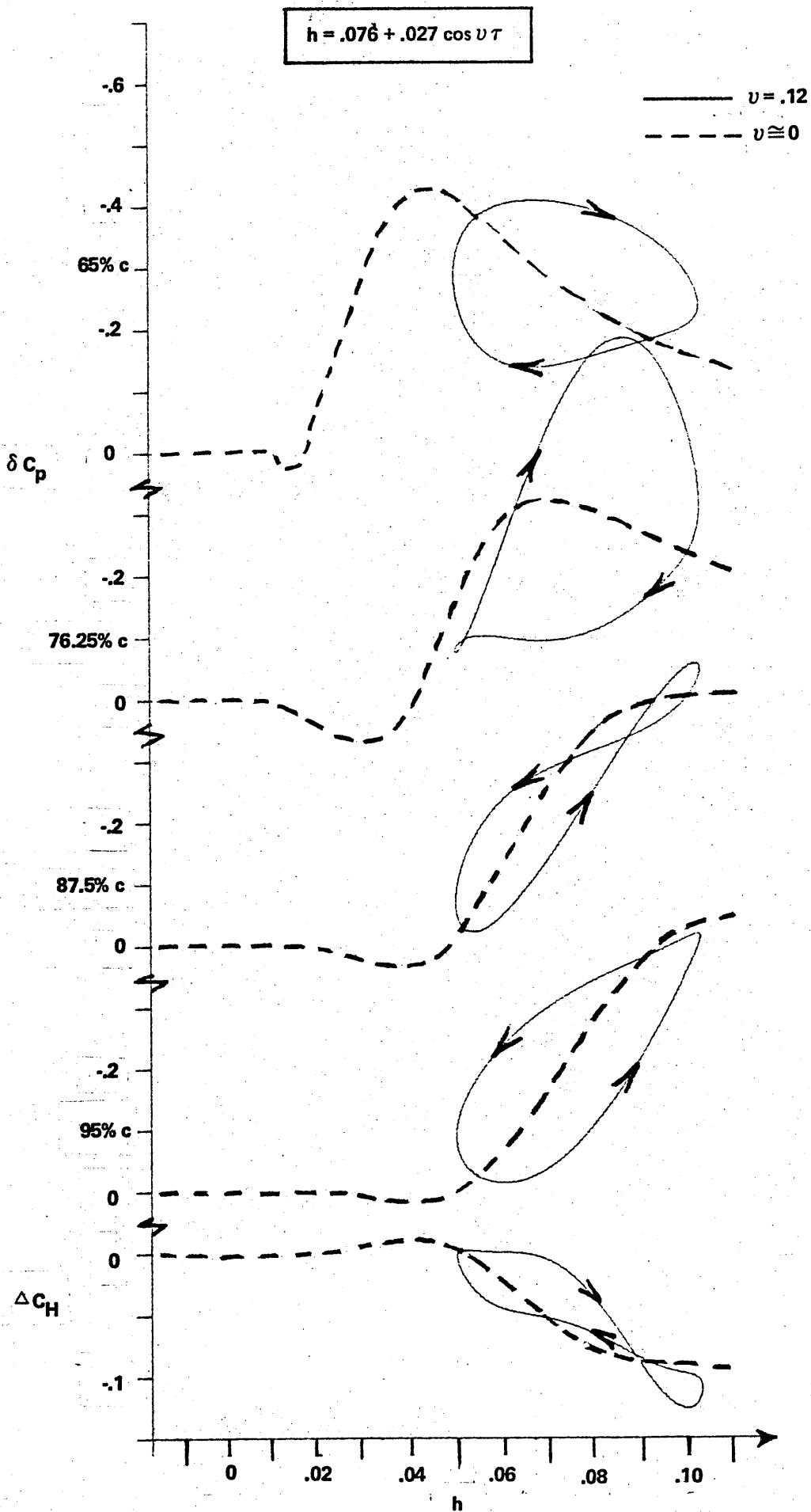


FIGURE 2.26C. RECONSTRUCTED DYNAMIC VARIATIONS OF PRESSURES AND HINGE-MOMENT

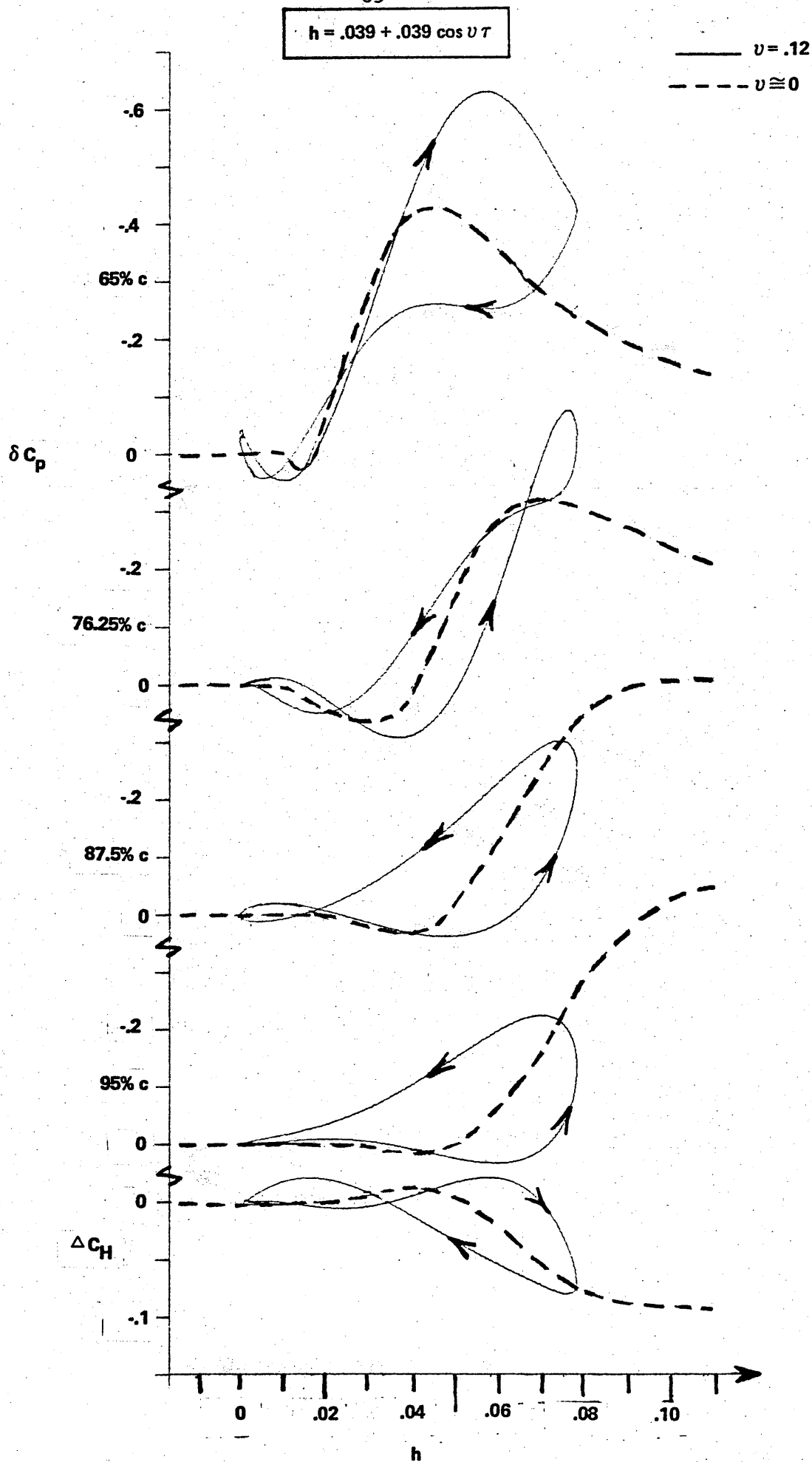


FIGURE 2.26D. RECONSTRUCTED DYNAMIC VARIATIONS OF PRESSURES AND HINGE-MOMENT

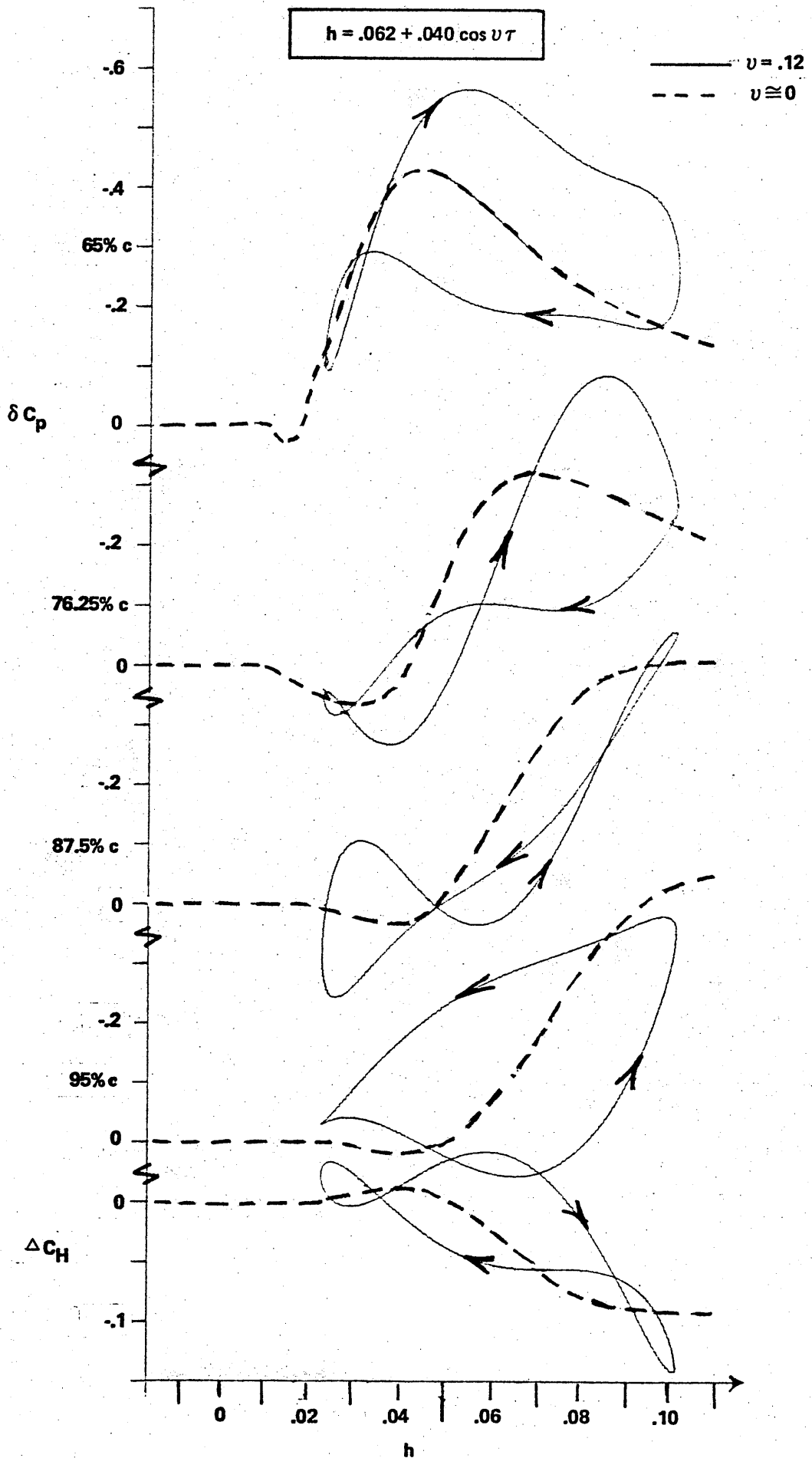


FIGURE 2.26E. RECONSTRUCTED DYNAMIC VARIATIONS OF PRESSURES AND HINGE-MOMENT

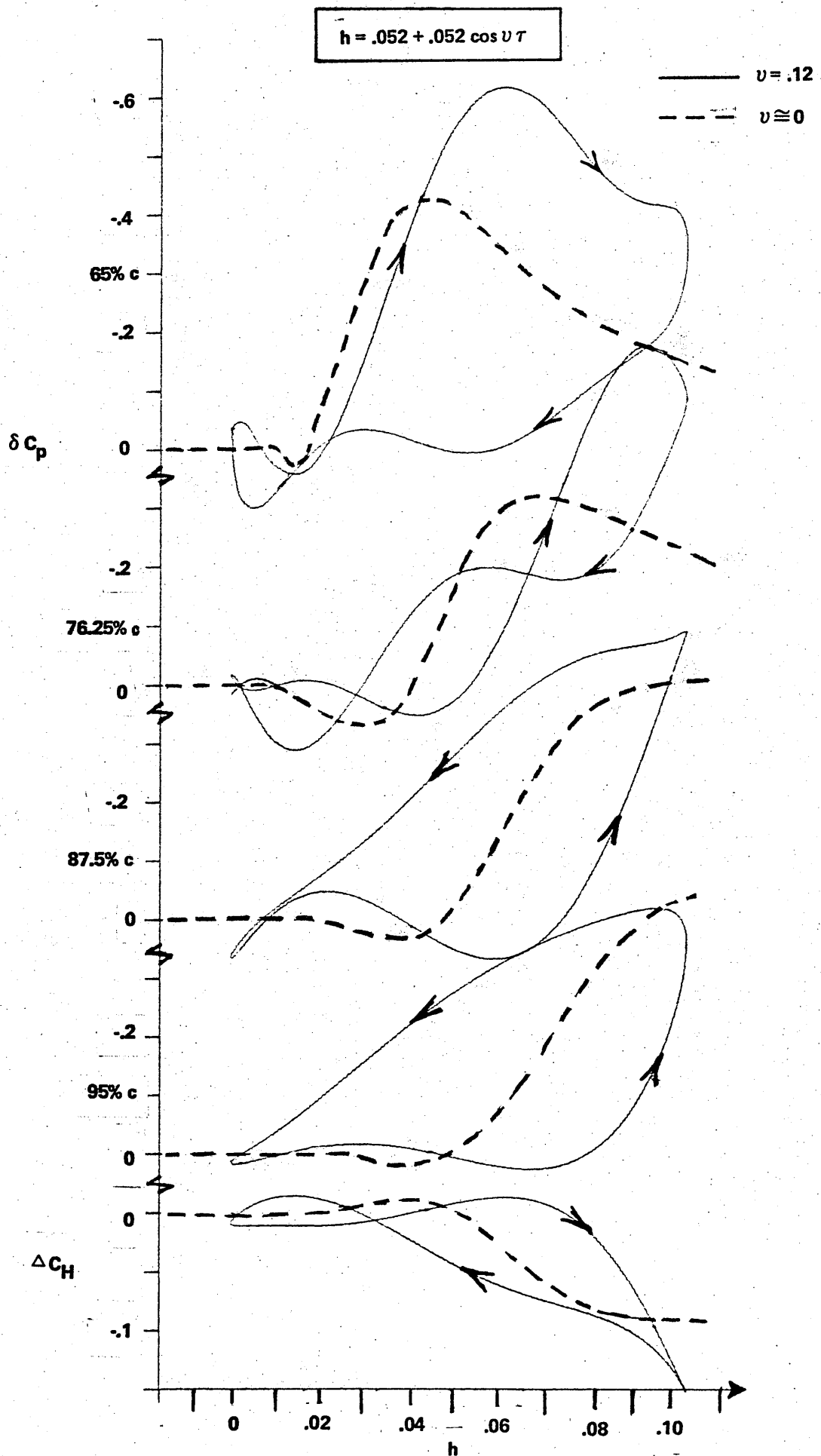


FIGURE 2.26F. RECONSTRUCTED DYNAMIC VARIATIONS OF PRESSURES AND HINGE-MOMENT

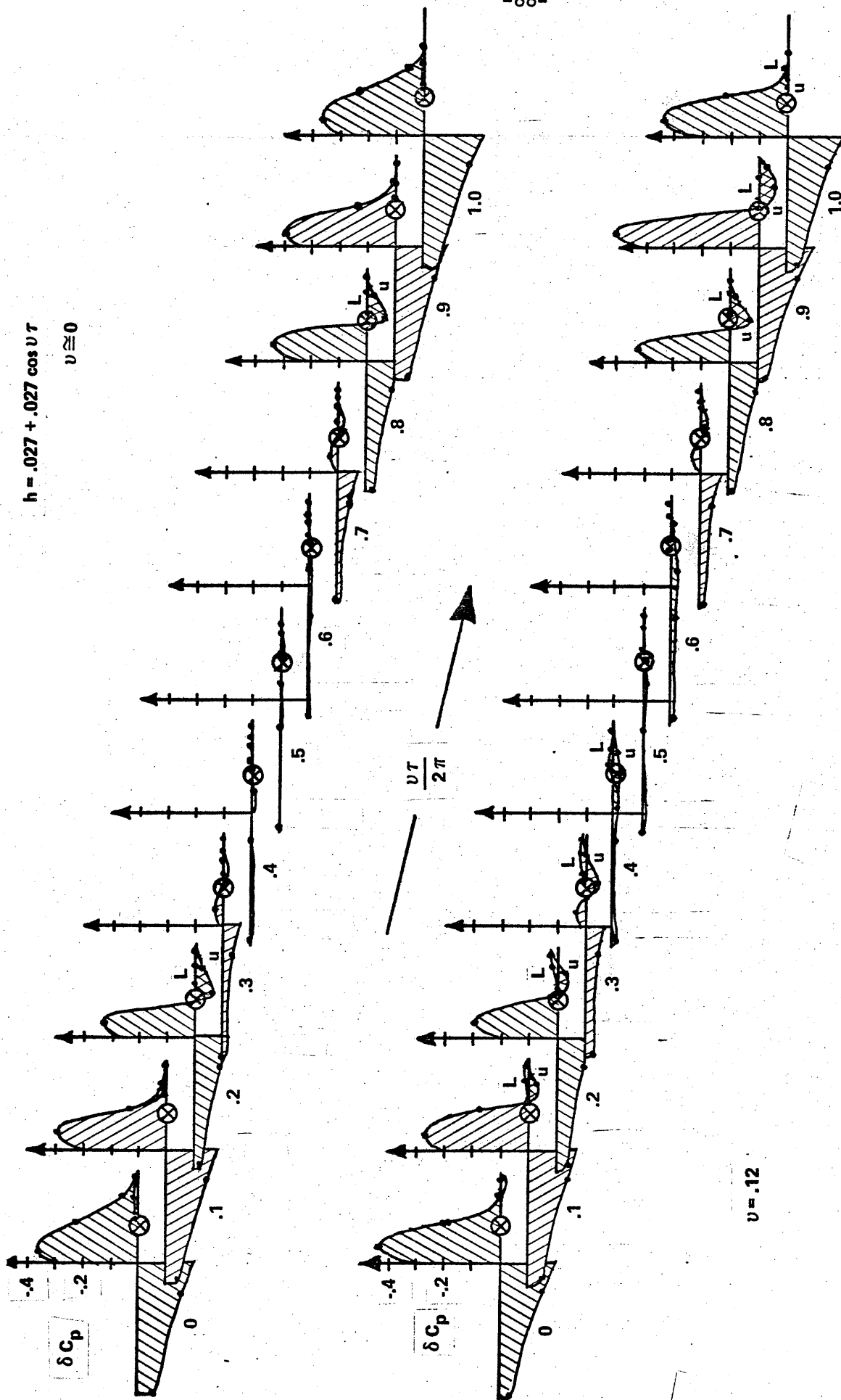


FIGURE 2.27A. INSTANTANEOUS PRESSURE DISTRIBUTION

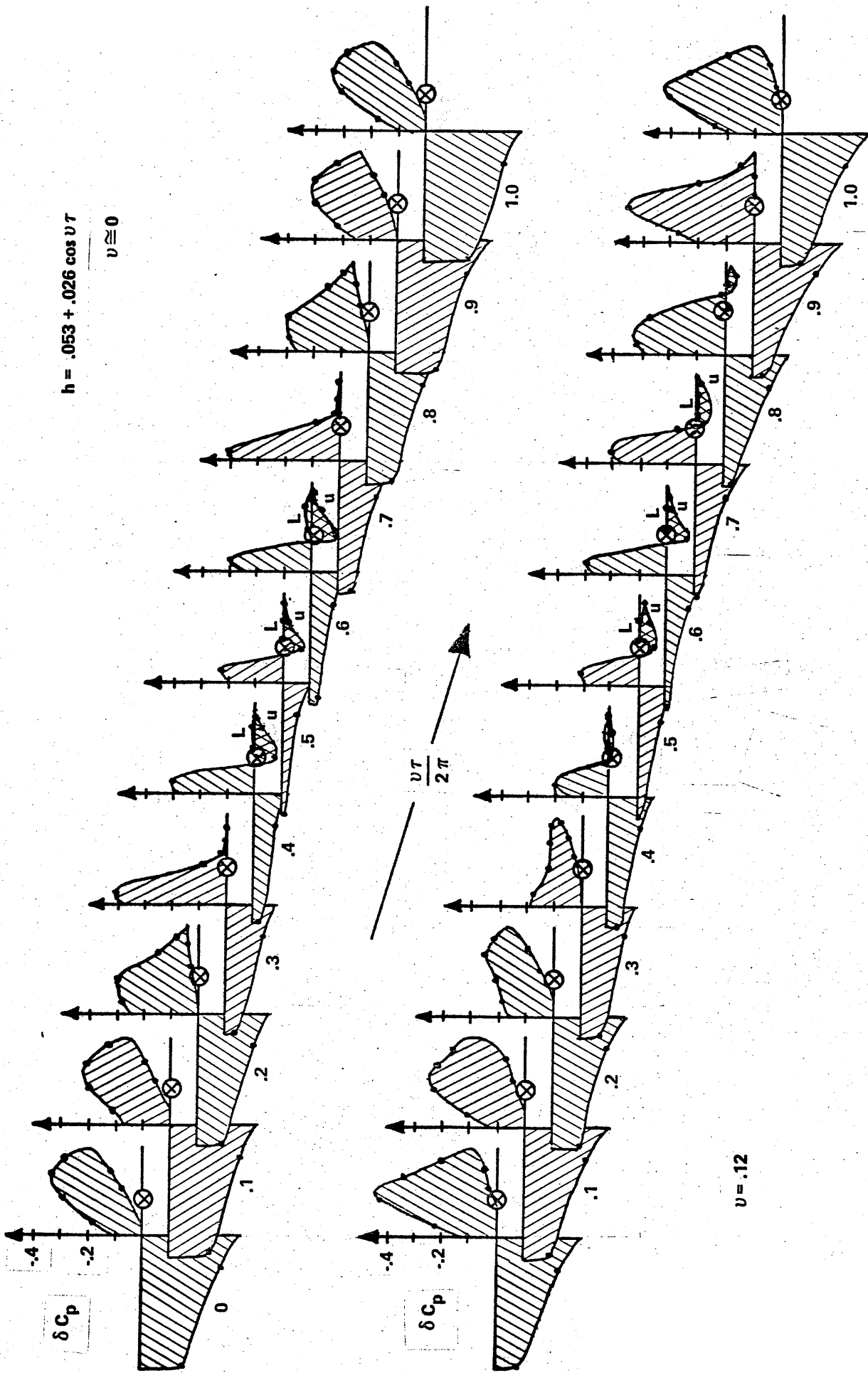


FIGURE 2.27B. INSTANTANEOUS PRESSURE DISTRIBUTION

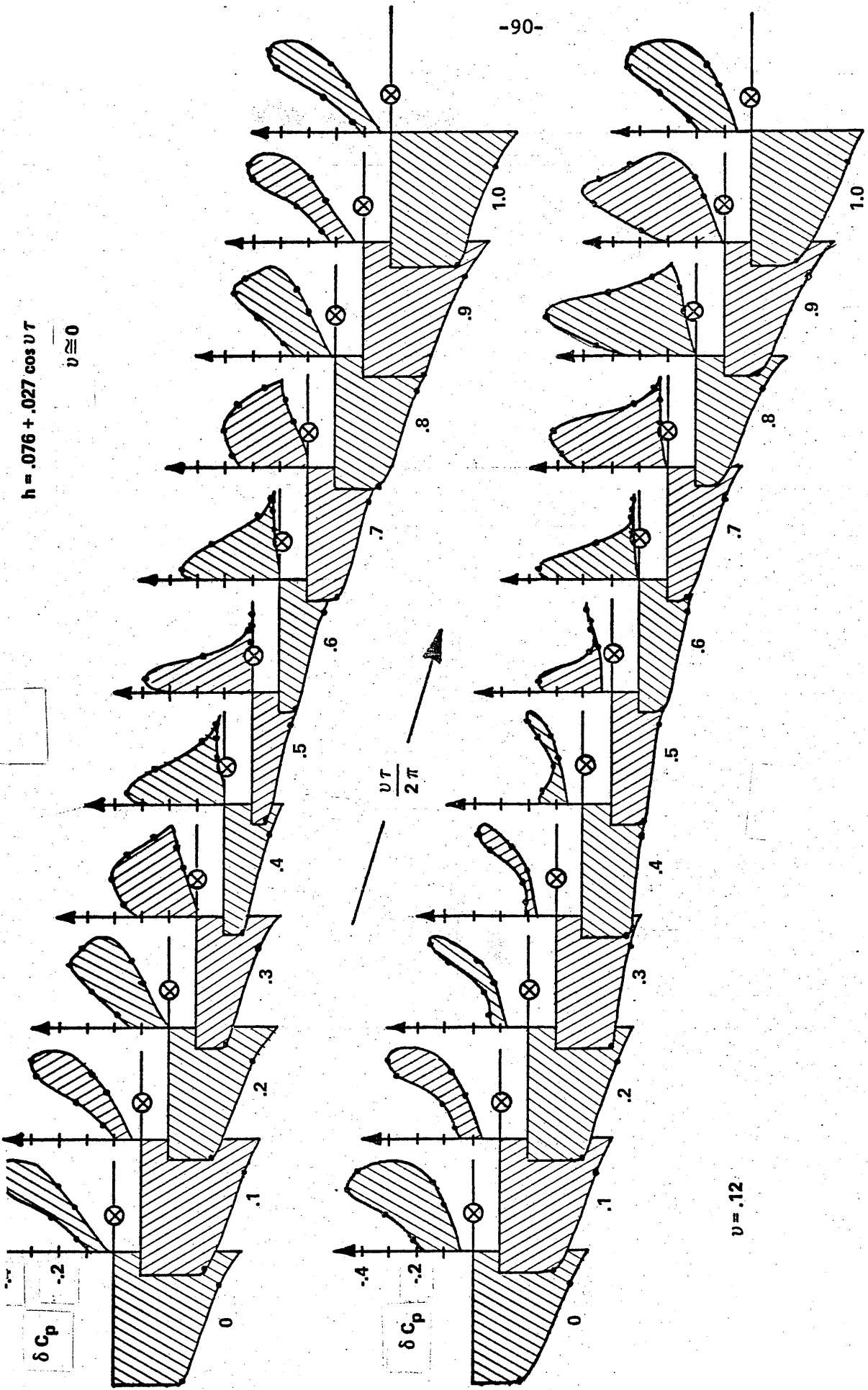


FIGURE 2.27C. INSTANTANEOUS PRESSURE DISTRIBUTION

$$h = .039 + .039 \cos v\tau$$

$$v \approx 0$$

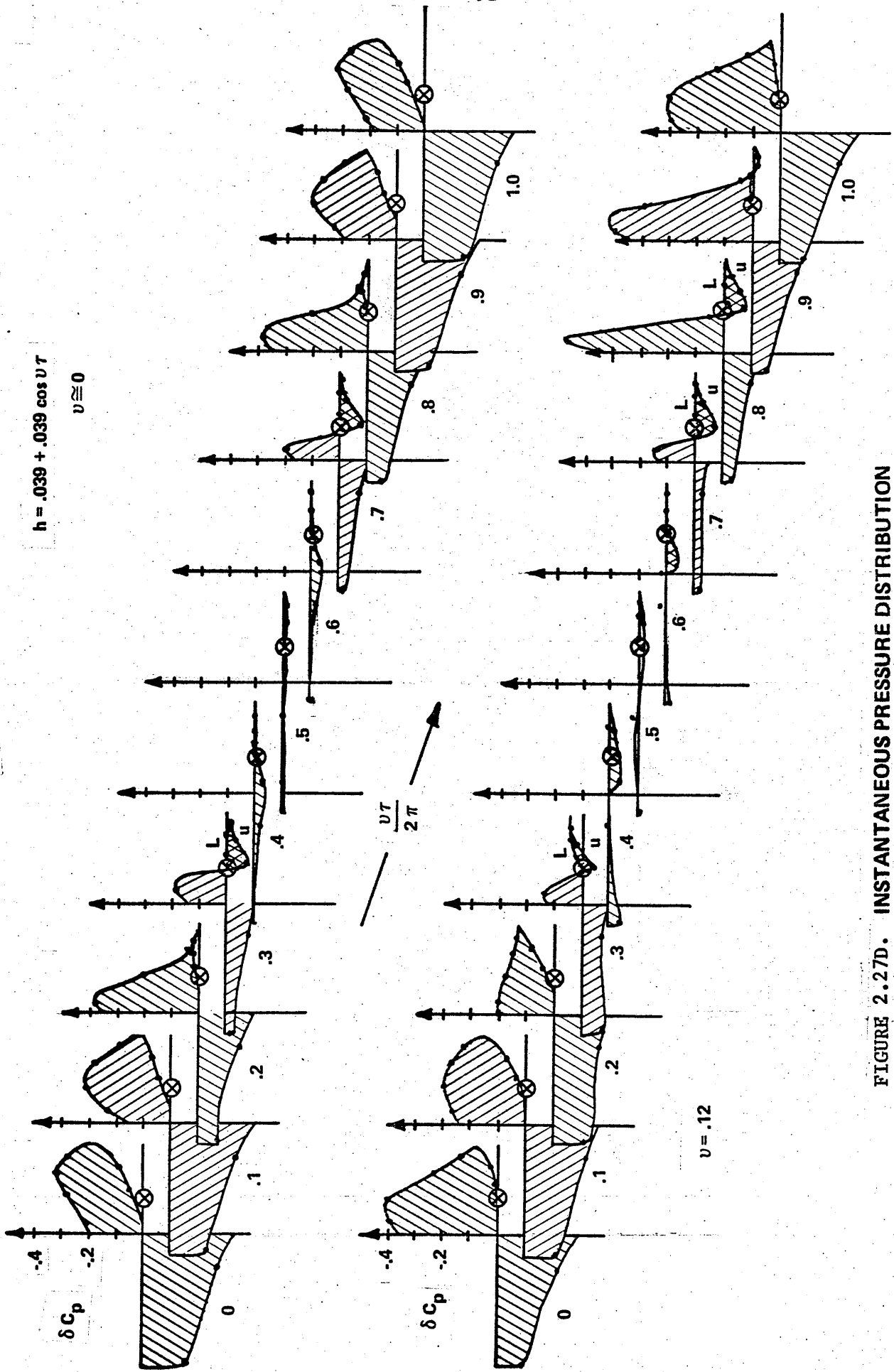


FIGURE 2.27D. INSTANTANEOUS PRESSURE DISTRIBUTION

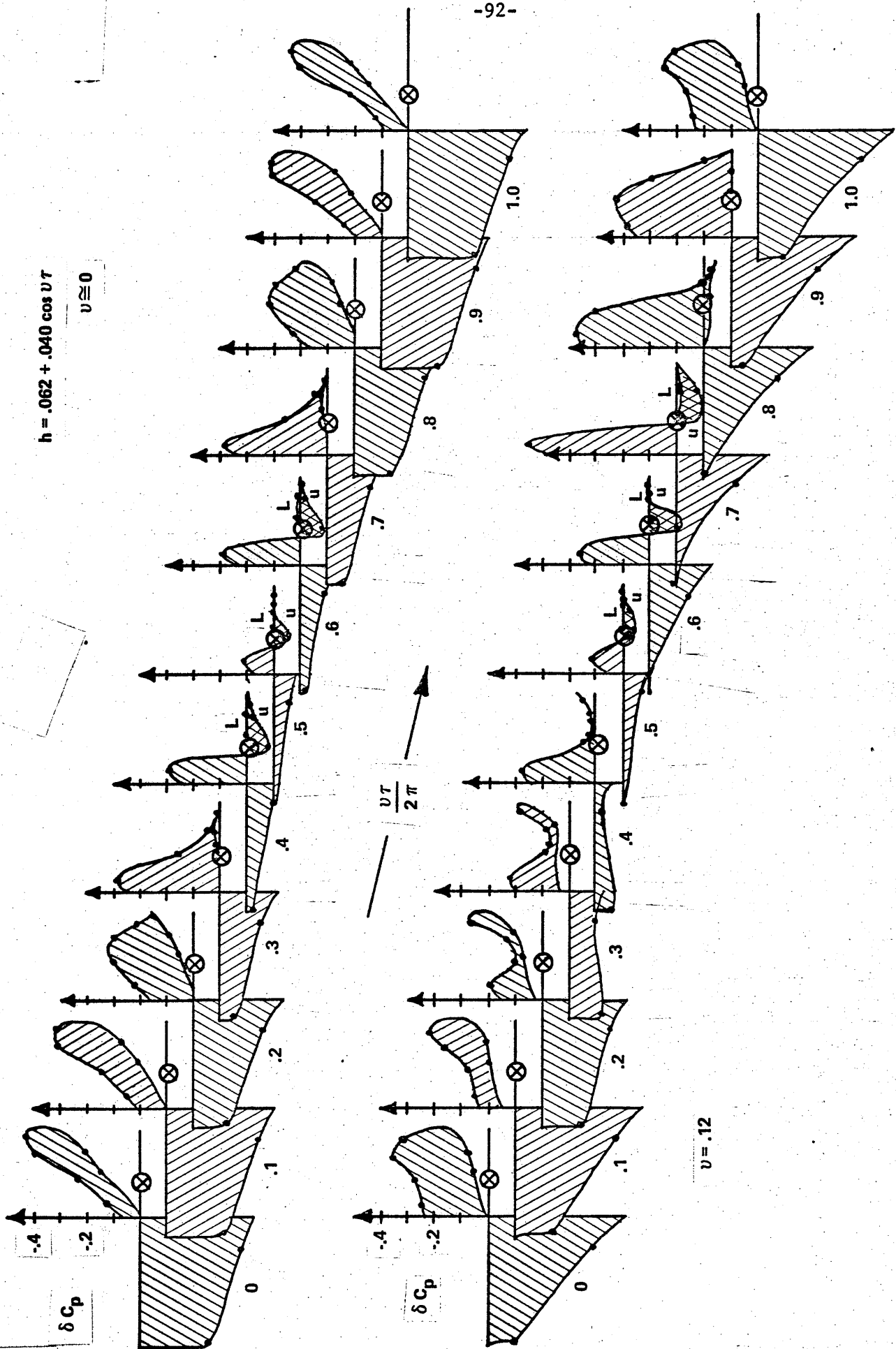


FIGURE 2.27E

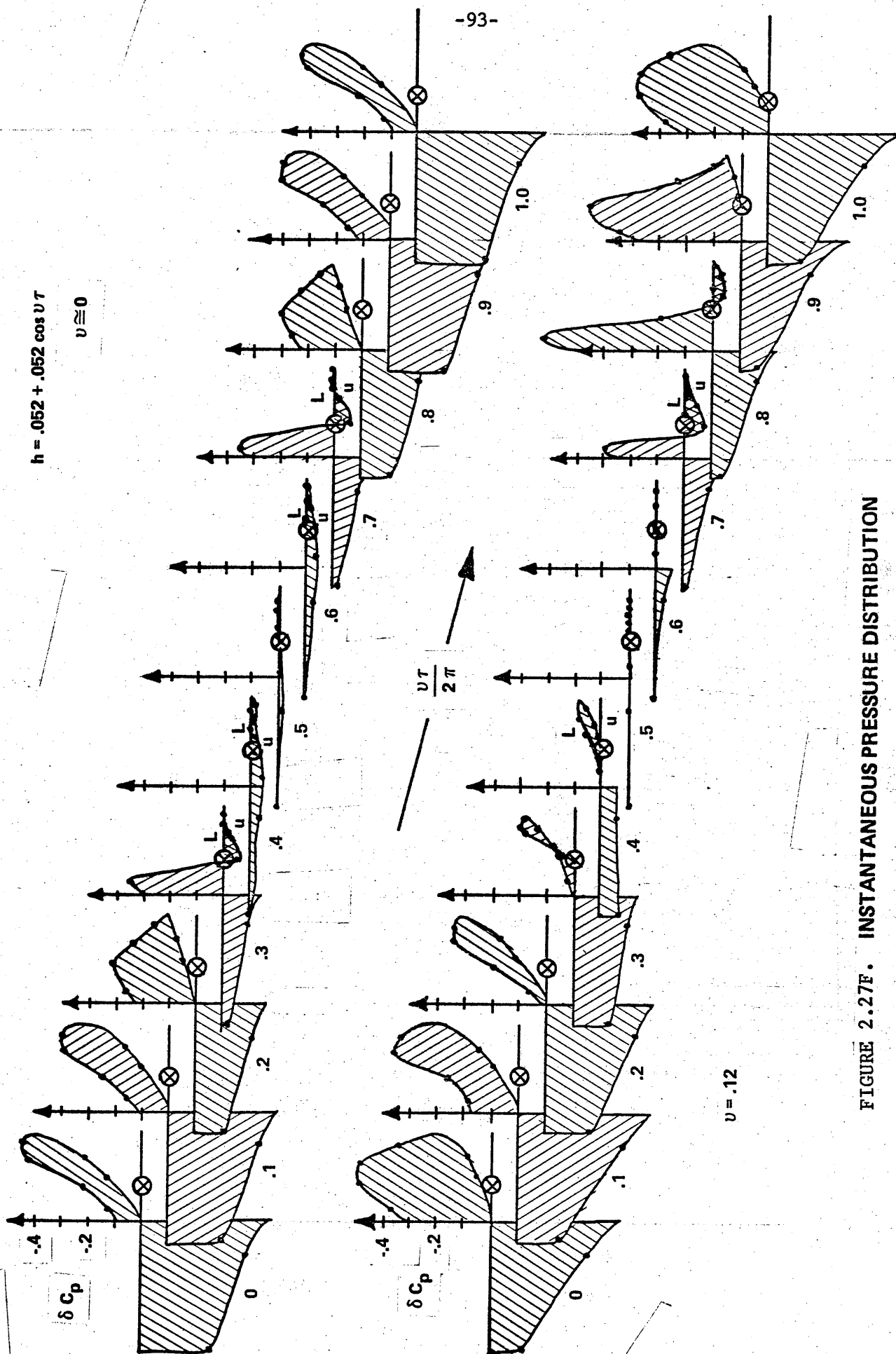


FIGURE 2.27f. INSTANTANEOUS PRESSURE DISTRIBUTION

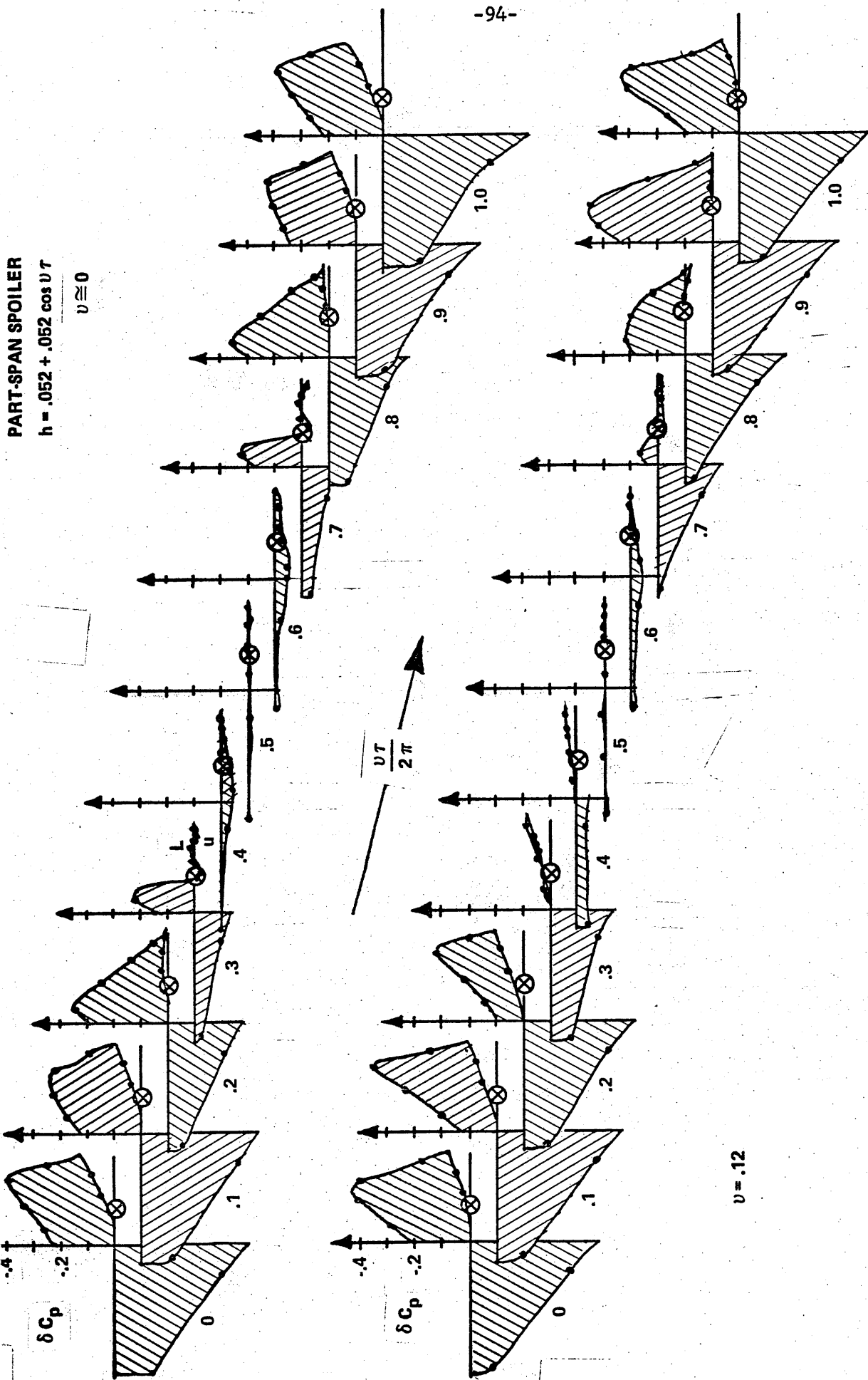


FIGURE 2.27G. INSTANTANEOUS PRESSURE DISTRIBUTION

The unsteady effects shown in Figures 2.25A-F, 2.26A-F and 2.27A-G for $\nu = .12$ show trends that appear to be nearly linearly related to frequency, therefore, only a typical frequency in the range of interest is represented. For $\nu = .06$, for example, one could "average" data between $\nu = 0$ and $\nu = .12$ in order to find the probable real behavior.

Figures 2.22A-G contain curves which are predictions of flap response to spoiler forcing. These are based solely on an assumed equation of motion of the form:

$$A\ddot{\beta} + B\dot{\beta} + C\beta = H$$

where it is assumed:

$$H = H_{\dot{\beta}} \dot{\beta} + H_{\beta} \beta + \Delta H$$

(since $\frac{H_{\ddot{\beta}}}{A} \ll 1$) with $H_{\dot{\beta}}$ and H_{β} based on spoiler-fixed results and with:

$$\Delta H \equiv \Delta C_H q_{\infty} b_f c_f^2$$

and

$$H_{\dot{\beta}} \equiv C_{H\dot{\beta}} q_{\infty} b_f c_f^2 \left(\frac{c_f}{U_{\infty}}\right)$$

$$H_{\beta} \equiv C_{H\beta} q_{\infty} b_f c_f^2$$

The equation of motion for the forced system is:

$$\ddot{\beta} + \frac{(B - H_{\dot{\beta}})}{A} \dot{\beta} + \frac{(C - H_{\beta})}{A} \beta = \frac{\Delta H}{A}$$

or

$$\ddot{\beta} + 2 \zeta_t \omega_t \dot{\beta} + \omega_t^2 \beta = \frac{\Delta H(t)}{A}$$

For a given frequency of forcing, the flap will respond as a linear second order system; however, the aerodynamic terms $H_{\dot{\beta}}$ and H_{β} are functions of the frequency of motion and mean spoiler height.

Therefore, flap steady-state response components can be found as follows:

$$\beta_n \equiv \frac{\frac{\kappa}{\nu_t^2} \Delta C_{Hn}}{\{[1 - (\frac{n\nu}{\nu_t})^2]^2 + [2 \zeta_t (\frac{n\nu}{\nu_t})]^2\}^{\frac{1}{2}}}$$

and

$$\phi_n = \psi_n - \tan^{-1} \left[\frac{2 \zeta_t (\frac{n\nu}{\nu_t})}{1 - (\frac{n\nu}{\nu_t})^2} \right]$$

where

$$\kappa \equiv \frac{\rho_\infty b_f c_f^4}{2A}$$

$$\nu_t = (\nu_s^2 + \nu_o^2)^{\frac{1}{2}}$$

$$\nu_s = \left(\frac{c_f}{U_\infty}\right) \sqrt{\frac{C}{A}}$$

$$\nu_o = \sqrt{-\kappa C_{H\beta}}$$

$$\zeta_t = \zeta_s \left(\frac{\nu_s}{\nu_t}\right) + \zeta_o \left(\frac{\nu_o}{\nu_t}\right)$$

$$\zeta_s = \frac{B}{2 \sqrt{AC}}$$

$$\zeta_o = -\frac{1}{2} C_{H\beta} \sqrt{\frac{\kappa}{-C_{H\beta}}}$$

For predictions based on the above formulation it is assumed that:

$$C_{H\beta} = -.518/\text{rad}$$

and

$$C_{H\beta} = -1.29 - \frac{.001}{n\nu} \quad \text{for } n > 0$$

These are characteristics found for $h = 0$ and $0 < \nu < .14$. Parameter values are found to be:

$$\nu_o \cong \begin{cases} .0667 & \text{for } A_1 \\ .0805 & \text{for } A_2 \end{cases}$$

$$\nu_s \cong \begin{cases} .0208 & \text{for } A_1 \\ .0249 & \text{for } A_2 \end{cases}$$

$$\nu_t \cong \begin{cases} .0699 & \text{for } A_1 \\ .0843 & \text{for } A_2 \end{cases}$$

$$.0830 < \zeta_o < .1160$$

$$\zeta_s = \begin{cases} .0247 & \text{for } A_1 \\ .0301 & \text{for } A_2 \end{cases}$$

$$.0865 < \zeta_t < .1200$$

Since parameter values h_o and Δh for cases A-G in the flap-fixed tests differ somewhat from those in the flap-free tests it was necessary to correct the measured values of ΔC_{H_n} and ψ_n for h_o and Δh variations. Also, it was desired to express them as functions of frequency.

A Taylor series expansion for $\Delta C_{H_n} = \Delta C_{H_n} \{h_o, \Delta h, \nu\}$ and for $\psi_n = \psi_n \{\nu\}$ was accomplished as:

$$\Delta C_{H_n} = (\Delta C_{H_n})_{\substack{h_{o1} \\ \Delta h_1 \\ \nu=0}} + \frac{\partial \Delta C_{H_n}}{\partial h_o} (h_o - h_{o1}) + \frac{\partial \Delta C_{H_n}}{\partial \Delta h} (\Delta h - \Delta h_1) + \frac{\partial \Delta C_{H_n}}{\partial \nu} \nu + \{ \text{Higher Order Terms} \}$$

and

$$\psi_n = (\psi_n)_{\substack{h_{o1} \\ \Delta h_1 \\ \nu=0}} + \frac{\partial \psi_n}{\partial \nu} \nu + \{ \text{H.O.T.} \}$$

The subscript 1 refers to flap-fixed results. First terms in each series were evaluated at zero frequency in the appropriate flap-fixed case. $(\psi_n)_{\nu=0}$ was either 0° or 180° . The partial derivatives with respect to frequency were approximated as constants by straight line fits to the flap-fixed data. ΔC_{H_n} variation with h_o and Δh was evaluated analytically by the technique of reference 20. Values of the constants in these

expansions are tabulated for all components in all cases in reference 20.

Predictions based on the foregoing theory are shown for all cases (including the part-span spoiler) in Figures 2.22A-G. Reasonable agreement with data is apparent except in Figures 2.22C and E, which involve extreme mean spoiler heights. In all other cases including those shown in Figures 2.22F and F', which involve a change in flap system inertia, component magnitudes and phase angles are predicted fairly well even near resonance.

The theory could be modified to account for large values of h_0 by two means. First the system "natural" aerodynamic damping and stiffness terms could be altered and values for large values of spoiler height used. Secondly, the values of $(\Delta C_{H_n})_{\nu=0}$, $\frac{\partial \Delta C_{H_n}}{\partial h_0}$ and $\frac{\partial \Delta C_{H_n}}{\partial \Delta h}$ could be altered and based on steady-flow data for mean flap-deflections of $\beta < 0$. This second method was attempted. The steady-flow ΔC_H versus h curve for a flap angle of -8° was constructed and used as an input to the procedure of reference 20. The procedure yielded new values of ΔC_{H_n} which reflected the difference in shape of this new curve. New values of $\frac{\partial \Delta C_{H_n}}{\partial h_0}$ and $\frac{\partial \Delta C_{H_n}}{\partial \Delta h}$ were not used in the modified theory, however. Values of $\partial \Delta C_{H_n} / \partial \nu$ were assumed. The modified theory predictions show better agreement with data.

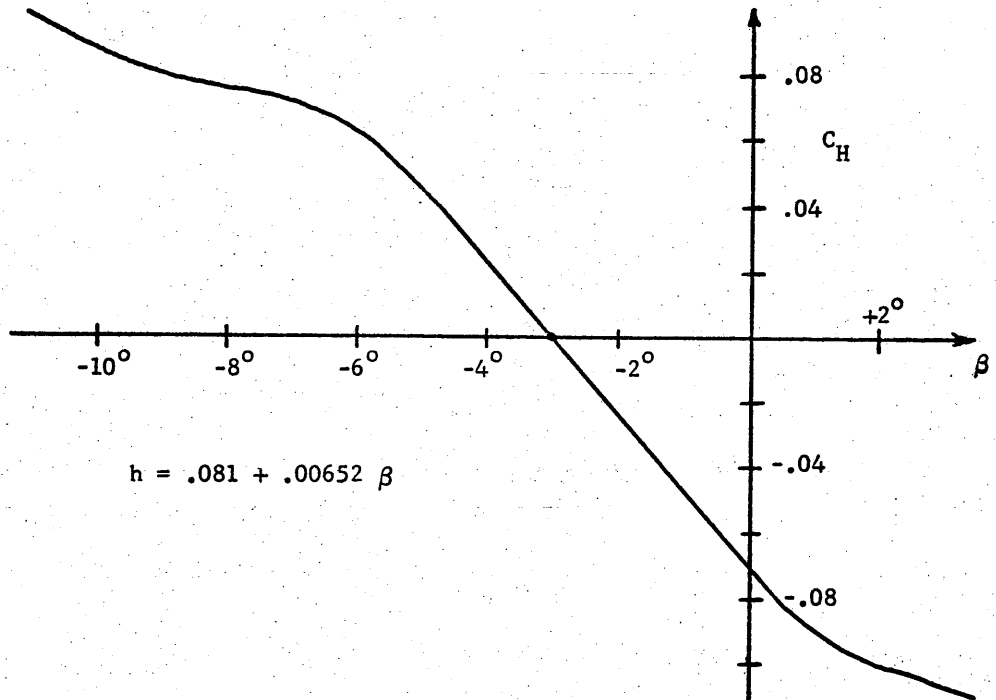
The generally good agreement between the above theory and experiment is useful to show that the flap-free system has a low-pass filtering property. Higher harmonic content of the forcing signal is greatly attenuated when the forcing frequency is near the flap system "natural" frequency, i.e., for $\omega \cong \sqrt{\frac{C - H_\beta}{A}}$.

Consider the coupled system with coupling parameters such that:

$$h = .081 + .00652 \beta$$

where β is measured in degrees. This relation is shown in a previous sketch.

A plot of C_H versus β can be constructed as shown in the following sketch.



Over the region $-6 \leq \beta \leq 1^\circ$ the relation is nearly linear. The "steady-state" solution is found as $\beta_0 \cong -3^\circ$ and $h = .0612$ ($h_s = .367''$). For this case the limit cycle amplitude was found to be $\Delta\beta_{LC} \cong 4.1^\circ$, which relates to $\Delta h = .026$ ($\Delta h_s = .156''$). By use of Figure 2.18, a describing function for spoiler effects is found:

$$\frac{\Delta C_{H1}}{\Delta h} \cong -2.12$$

or, in the coupled case:

$$\frac{\Delta C_{H1}}{\Delta h} \frac{dh}{d\beta} = \frac{\partial \Delta C_{H1}}{\partial \beta} \equiv \Delta C_{H\beta} = -.791/\text{rad}$$

When this is added to the value of $C_{H\beta}$ found at $h = .0625$, i.e., $C_{H\beta} = -.520/\text{rad}$ we find an estimate for total system aerodynamic stiffness:

$$C_{H\beta_{\text{tot}}} = C_{H\beta} + \Delta C_{H\beta} = -1.312/\text{rad}$$

This is in very close agreement with the slope of the linear portion of the sketch. The resultant prediction of reduced frequency (based on total system inertia and including mechanical stiffness) is $\nu = .124$ which agrees well with the measured

value of $\nu = .122$. A consideration of dry-friction (which was measured in a series of tests) and spoiler-fixed aerodynamic damping, results in a predicted lag angle for the ΔC_{H_1} component of 187.52° which appears reasonable. (See Figures 2.17C and E.) Section 3 contains a more detailed account of coupled system behavior.

Phillips and Adams in reference 39 offer the results of a similar experiment. They used a NACA 65-009 airfoil section of 6" chord with a 1/4 chord plain flap. The flap was coupled to two spoilers, one on each surface at 66% chord, in a symmetrical arrangement. The coupling relation was:

$$h \equiv \frac{h_s}{c_f} = .0668 + .00616 \beta$$

where β is measured in degrees.

Limit-cycle behavior was observed over an airfoil incidence range of 1° to 6° . Incidence had little effect on reduced frequencies of oscillation or limit-cycle amplitude. (Refer to the prior discussion of incidence effects and see reference 40.) Limit-cycle behavior was observed for $.17 < \nu < .24$ with a maximum amplitude of $\Delta \beta_{LC} \cong 4^\circ$.

When the data of reference 39 is further analyzed it is found that for a typical limit-cycle case where $\nu = .20$, an incremental hinge-moment coefficient can be found, i.e., $\Delta C_{H\beta} \cong -1.28/\text{rad}$.

If one assumes that data obtained for the case mentioned previously in the present experiment can be extrapolated by allowing for the effect of two spoilers and applying corrections from reference 40 for the differences in airfoil thickness and spoiler chordwise location, then a predicted value of $\Delta C_{H\beta} = -1.20/\text{rad}$ is obtained. This large spoiler effect is responsible for the higher reduced frequencies associated with the Phillips and Adams experiment. With little spoiler effect (i.e., for $h_{o_1} = 0$) their flap system had a high natural frequency of $\nu \cong .136$.

Results obtained for steady-flow testing can be summarized as follows:

- (1) Hinge-moment coefficient is approximately linearly related to flap angle for fixed spoiler heights.
- (2) Hinge-moment coefficient exhibits a pronounced non-linear relation to spoiler height for fixed flap angles. The relation is slightly affected by flap angle. A "reversal" of spoiler effect occurs at low heights and is associated with the "overshoot" of pressure aft of reattachment.

- (3) The separation bubble lengthens as spoiler height increases, but can be termed a "short bubble" for low spoiler heights and a "long bubble" for moderate heights. Bubble growth law differs for the two types.
- (4) Pressures in regions directly influenced by the separation bubble exhibit a non-linear relation to spoiler height. On the "wetted surface" pressures are more linearly related to spoiler height.
- (5) Pressure variations with flap angle show that, for a short bubble pressure within the bubble and upstream of the spoiler as well as bubble length are affected, for a long bubble upstream influence and changes to bubble pressures are slight.
- (6) RMS pressure fluctuations reach a peak within the bubble at about 85% of bubble length.

Results obtained for unsteady-flow testing can be summarized:

- (1) With the flap fixed and spoiler forced to oscillate:
 - i. Unsteady pressures and hinge-moment show cyclic waveform distortion that does not relate to a simple time lag.
 - ii. On the spoiler upstroke, bubble length lags its steady flow position and low pressures are achieved in the bubble.
 - iii. On the spoiler downstroke, pressures increase nearly linearly (with respect to time) to attached flow values and the bubble length lags its steady flow position.
 - iv. The above characteristics change in a near linear fashion as frequency changes. The lag effect of reattachment position can be related to an apparent convective velocity where $\frac{U_c}{U_\infty} \approx .323$.
 - v. The effect of a trip wire, which increases boundary-layer thickness, is to reduce spoiler effectiveness.
 - vi. Reynolds number changes act to alter boundary-layer thickness and so affect spoiler effectiveness. Free-stream turbulence effects are contained within these Reynolds number effects.
 - vii. Airfoil angle-of-attack affects boundary-layer thickness and velocity at the spoiler. For this configuration the two effects nearly cancel.

- viii. Part-span spoiler effects are qualitatively similar to those for the full-span spoiler.
- ix. A "rectification" effect is apparent but small. Mean spoiler effectiveness diminishes as frequency increases.

(2) With the spoiler fixed and the flap forced to oscillate:

- i. Hinge-moment coefficient is nearly linearly related to flap angle for $|\beta| \leq 10^\circ$.
- ii. In-phase and in-quadrature components of hinge-moment coefficient are weak functions of spoiler height for low-to-moderate heights.
- iii. A thicker boundary-layer reduces flap effectiveness.
- iv. Pressures vary in a near linear manner as flap angle changes, but are functions of spoiler height.
- v. Large spoiler heights reduce flap effectiveness, especially at low frequencies.
- vi. Spoiler chordwise position may alter flap effectiveness.

(3) With the spoiler forced to oscillate and the flap free to respond:

- i. The flap system responds approximately as a linear second-order system with stiffness and damping that contain attached-flow aerodynamic contributions.
- ii. The flap system responds primarily to the Fourier components of the unsteady hinge-moment due to spoiler motion.
- iii. Ultra-harmonic resonance is observed.
- iv. The Fourier components of flap response can be predicted with good accuracy by simple superposition of response to the forcing function components. The forcing function components are found with the flap fixed.

(4) With the spoiler and flap coupled and free to oscillate:

- i. Limit-cycle oscillations were observed over a range of coupling parameters, system inertias and free-stream speeds.

- ii. Results are qualitatively similar to those of reference 39.
- iii. Reduced frequencies of oscillation can be predicted fairly well from steady-flow, uncoupled test results.

Results of these tests confirm that a separating and reattaching flow field is extremely complex. At very low frequencies unsteady effects are significant, but over the range of frequencies considered the modifications to this complex, non-linear phenomenon follow a near linear trend.

It is relevant to remark that the kind of non-linear effects (for example, hinge-moment due to spoiler motion and limit-cycle behavior for the coupled system) are not peculiar to this specific test configuration. Analogous examples have been mentioned which involve shock-induced separation or dynamic stalling of a lifting surface. Phillips and Adams in reference 39 offer suggestions which imply that the coupled spoiler and flap results can be thought of as a subsonic analogy to the phenomenon of control-surface buzz. Although the analogy is not direct, spoiler-induced loading on a flap is known to resemble shock-induced loading.⁸ Spoiler-induced loading also appears similar to that observed in airfoil dynamic stalling.

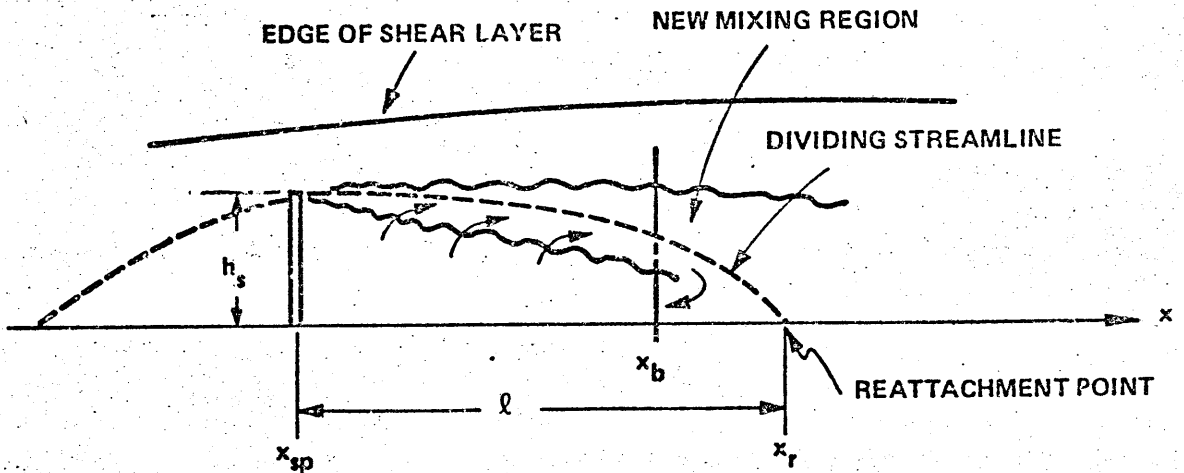
In section 3 a mathematical model is developed which can be used to predict unsteady loading induced by the spoiler and to predict conditions necessary for limit-cycle oscillation of the coupled system. An extension of the model is attempted in section 4 in order to study the above mentioned "analogous" flow situations.

3. THEORETICAL PREDICTION OF SPOILER EFFECTS

In section 2.4 it was shown that two categories of separation bubbles could be defined. The ratio of spoiler height to boundary-layer thickness (h_s/δ) relates to the strength of the spoiler perturbation and to a resultant bubble type. In the following development only the "long bubble" type, which probably results from an overwhelming perturbation, will be considered. It will be shown that unsteady effects can serve to modify the character of the long bubble in such a way that it may resemble a short bubble.

3.1. A Simple Steady-Flow Bubble Model

Consider the sketch below, which serves to define several parameters.



Based on experimental observations and other works (Ref 5) the length of the bubble is represented as a linear function of spoiler height as follows: (See fig. 2.13)

$$l = (x_r - x_{sp}) = d_r (h_s - h_{sc}) \frac{c}{c_f}$$

or in non-dimensional form as:

$$\begin{aligned} \frac{l}{c_f} &\equiv \Delta x_{sp} = (x_r - x_{sp}) \frac{c}{c_f} = d_r \left(\frac{h_s}{c_f} - \frac{h_{sc}}{c_f} \right) \frac{c}{c_f} \\ &= d_r (h - h_c) \frac{c}{c_f} \end{aligned}$$

or

$$(x_r - x_{sp}) = d_r (h - h_c)$$

and

$$x_r = d_r (h - h_c) + x_{sp} = d_r h + x_c$$

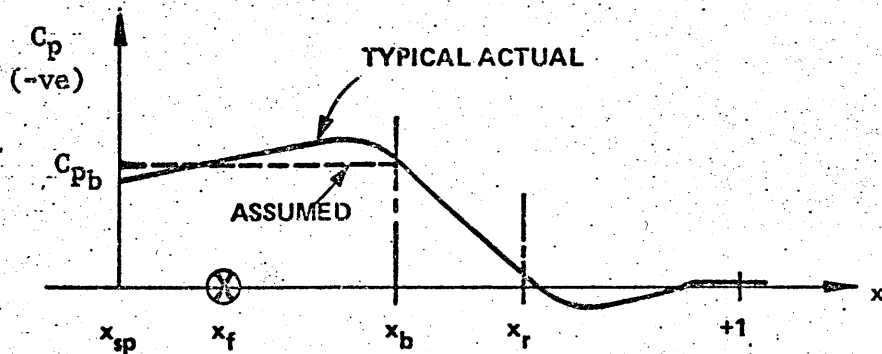
where

$$x_c = x_{sp} - d_r h_c$$

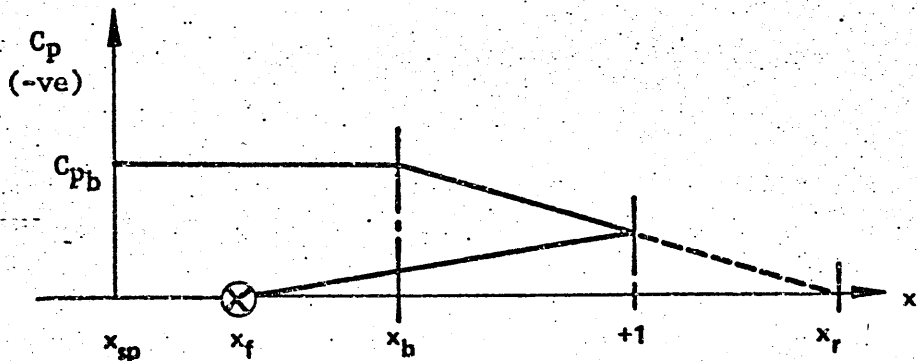
also we have:

$$x_b = d_b h + x_c$$

The chordwise pressure distribution along the surface is approximated by a "roof-top" distribution as shown below, with $C_{pr} = 0$. (Note: C_p in the following is actual C_p minus C_p due to airfoil thickness.)



For long bubbles with recompression on the flap surface and pressure divergence at the trailing edge, a linear pressure distribution which recovers to zero at the flap hinge-line is assumed on the lower surface. This simplification is justified by measurements made on the flap lower surface.

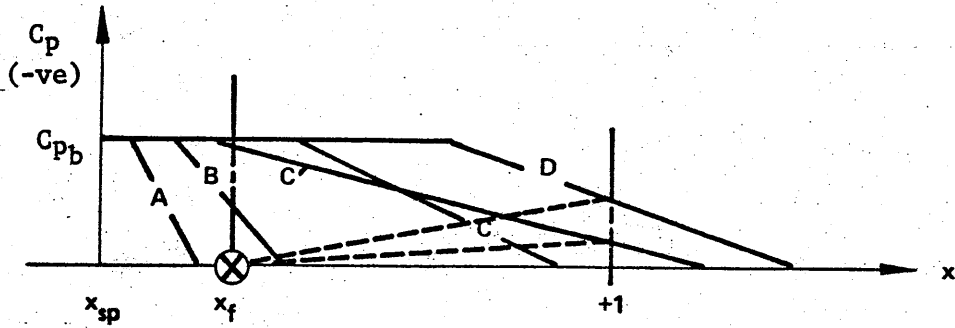


The incremental hinge-moment due to spoiler deflection may be found by integrating pressures over the flap.

$$\Delta C_H \equiv \frac{\Delta H}{q b_f c_f^2} = - \left(\frac{c}{c_f} \right)^2 \int_{x_f}^1 (C_{p\ell} - C_{pu}) (x - x_f) dx$$

where pressure coefficients are approximated as discussed above.

We have five cases to consider:



Case A

$$x_r \leq x_f ; \quad \text{i.e.,} \quad h \leq \frac{x_f - x_c}{d_r}$$

$$\therefore C_{p\ell} = C_{pu} = 0 \quad \text{and} \quad \Delta C_H = 0$$

Case B

$$x_b \leq x_f \quad \text{and} \quad x_f \leq x_r \leq 1$$

$$\text{i.e.,} \quad h \leq \frac{x_f - x_c}{d_b} \quad \text{and} \quad \frac{x_f - x_c}{d_r} \leq h \leq \frac{1 - x_c}{d_r}$$

$$\therefore C_{p\ell} = 0 ; \quad C_{pu} = C_{pb} \frac{(x_r - x)}{(x_r - x_b)} \quad \text{for } x_f \leq x \leq x_r$$

and

$$\Delta C_H = C_{pb} \frac{\left(\frac{c}{c_f} \right)^2}{(d_r - d_b)} \left[\frac{1}{6} d_r^3 h^2 + \frac{1}{2} d_r^2 (x_c - x_f) h \right. \\ \left. + \frac{1}{2} d_r (x_c - x_f)^2 + \frac{1}{6} \frac{(x_c - x_f)^3}{h} \right]$$

Case C

$$x_f \leq x_b \quad \text{and} \quad x_r \leq 1 ;$$

$$\text{i.e.,} \quad \frac{x_f - x_c}{d_b} \leq h \quad \text{and} \quad h \leq \frac{1 - x_c}{d_r}$$

$$\therefore C_{p\ell} = 0 ;$$

$$C_{pu} = \begin{cases} C_{pb} & x_f \leq x \leq x_b \\ C_{pb} \frac{(x_r - x)}{(x_r - x_b)} & x_b \leq x \leq x_r \end{cases}$$

and

$$\begin{aligned} \Delta C_H &= C_{pb} \left(\frac{c}{c_f} \right)^2 \left[\frac{1}{6} (d_b^2 + d_b d_r + d_r^2) h^2 \right. \\ &\quad \left. + \frac{1}{2} (d_b + d_r) (x_c - x_f) h + \frac{1}{2} (x_c - x_f)^2 \right] \end{aligned}$$

Case C' (in lieu of Case C)

$$x_b \leq x_f \quad \text{and} \quad 1 \leq x_r$$

$$\text{i.e.,} \quad h \leq \frac{x_f - x_c}{d_b} \quad \text{and} \quad \frac{1 - x_c}{d_r} \leq h$$

$$\therefore C_{p\ell} = C_{pb} \frac{(x_r - 1)(x - x_f)}{(x_r - x_b)(1 - x_f)} \quad x_f \leq x \leq 1$$

$$C_{pu} = C_{pb} \frac{(x_r - x)}{(x_r - x_b)} \quad x_f \leq x \leq 1$$

and

$$\Delta C_H = C_{pb} \left(\frac{c}{c_f} \right)^2 \frac{(1 - x_f)^2}{6 (d_r - d_b)} \left[d_r + \frac{(x_c - x_f)}{h} \right]$$

Case D

$$x_f \leq x_b \leq 1 \quad \text{and} \quad 1 \leq x_r$$

$$\text{i.e., } \frac{x_f - x_c}{d_b} \leq h \leq \frac{1 - x_c}{d_b} \quad \text{and} \quad \frac{1 - x_c}{d_r} \leq h$$

$$\therefore C_{p_l} = C_{p_b} \frac{(x_r - 1)(x - x_f)}{(x_r - x_b)(1 - x_f)} \quad x_f \leq x \leq 1$$

$$C_{p_u} = \begin{cases} C_{p_b} & x_f \leq x \leq x_b \\ C_{p_b} \frac{(x_r - x)}{(x_r - x_b)} & x_b \leq x \leq 1 \end{cases}$$

and

$$\begin{aligned} \Delta C_H = C_{p_b} \frac{(\frac{c}{c_f})^2}{(d_r - d_b)} & \left\{ -\frac{1}{6} d_b^3 h^2 - \frac{1}{2} d_b^2 (x_c - x_f) h \right. \\ & + \left[\frac{1}{6} d_r (1 - x_f)^2 - \frac{1}{2} d_b (x_c - x_f)^2 \right] \\ & \left. + \left[-\frac{1}{6} (x_c - x_f)^3 + \frac{1}{6} (x_c - x_f)(1 - x_f)^2 \right] \frac{1}{h} \right\} \end{aligned}$$

Experimental data found in Figure 2.13 yields the following values:

$$d_r = 8.538; \quad d_b = 4.838; \quad x_c = .4167$$

Cases A, B, C' and D are considered with $C_{p_b} = -.4$ (this corresponds to a pressure recovery coefficient value of $\sigma = .286$) and results are plotted in Figure 3.1. Further approximate results with parameter values of $d_r = 5.50$; $d_b = 3.00$ and $x_c = x_{sp}$ are also shown.

The experimental data is not predicted accurately; however, the non-linear nature of the functional relation $\Delta C_H = f\{h\}$ is firmly established. The above simple model also demonstrates that the hinge-moment functional relation may also be written as $\Delta C_H = C_{p_b} f\{h\}$. Furthermore, we find that for all results

$\left| \frac{\partial \Delta C_H}{\partial h} \right|_{\max}$ occurs just prior to trailing-edge pressure divergence. For the curve based on experimental data the value of $\frac{\partial \Delta C_H}{\partial h}$ at T.E. pressure divergence (an average since slope is discontinuous there) is -3.548 at $h \approx .068$. This compares

fairly well with the experimental value of $\frac{\partial \Delta C_H}{\partial h} = -3.3$ at $h \approx .066$.

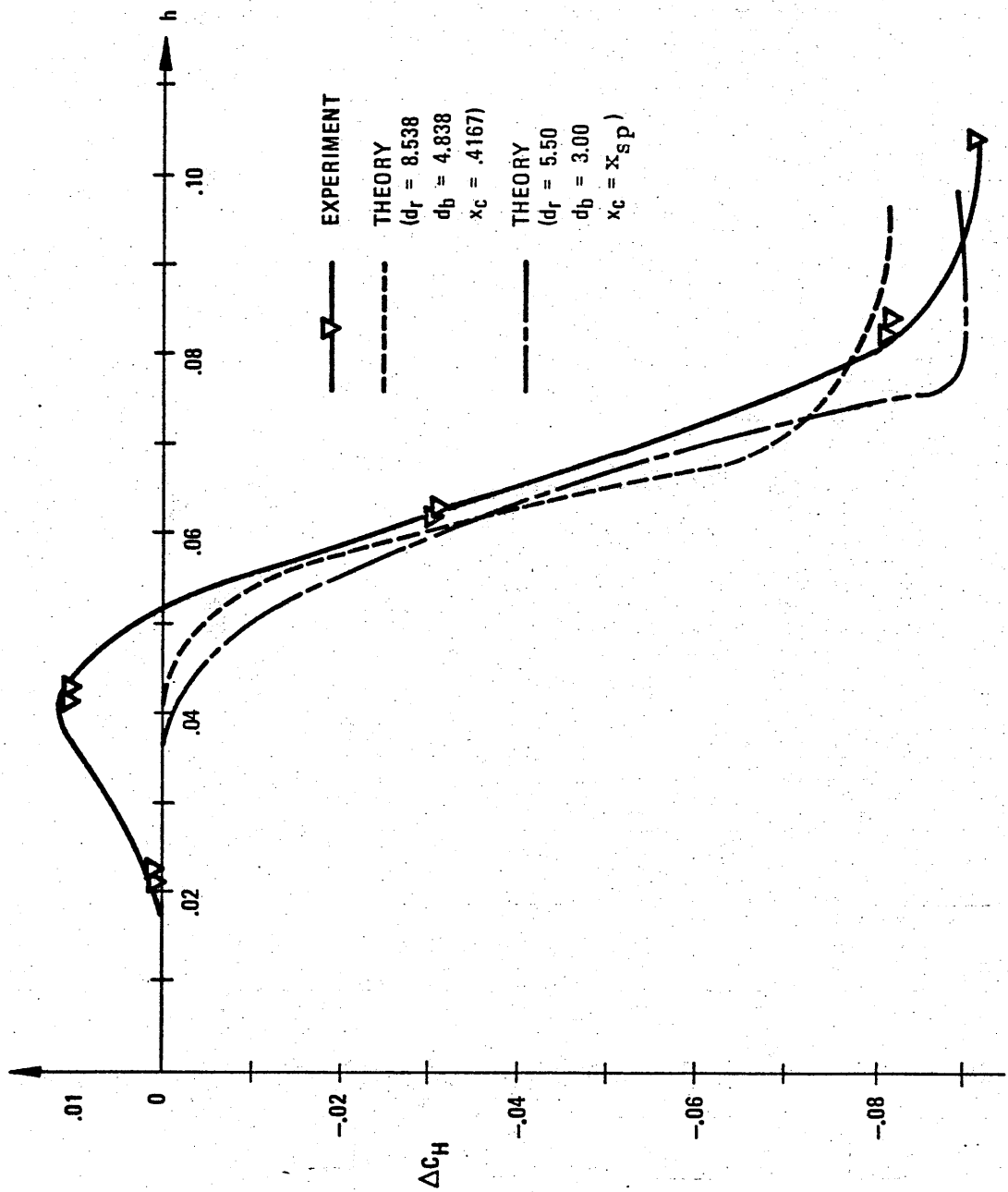


FIG. 3.1 ACTUAL AND APPROXIMATE HINGE-MOMENT COEFFICIENTS

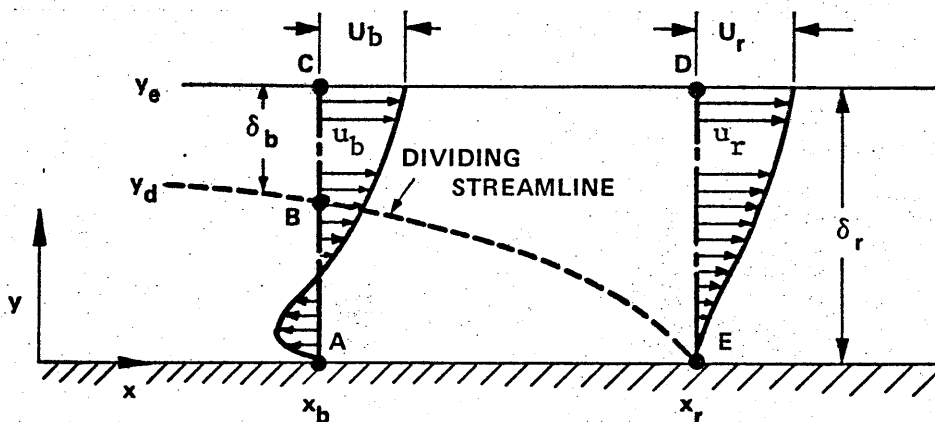
For quasi-steady spoiler motion of the form $h = h_0 + \Delta h \cos \phi$ we find that the above formulation does not properly represent the functional dependence of ΔC_H on h_0 and Δh . Accordingly, a polynomial "curve fit" to the experimental data was attempted in order to duplicate the steady-flow functional relation $\Delta C_H = f \{h_0, \Delta h\}$ over a range of h_0 and Δh values of interest. A fifth order polynomial in h was needed. This polynomial will be used in the unsteady-flow theory as the steady-flow functional relation.

$$\Delta C_H = a_5 h^5 + a_4 h^4 + a_3 h^3 + a_2 h^2 + a_1 h + a_0$$

The justification for this step lies in the fact that prediction of accurate unsteady effects is based in large part on a good steady-flow model.

Let us now consider the separation bubble in more detail. First we will investigate the pressure rise over the aft portion of the bubble.

A detailed view of the aft portion of the bubble, to include mean velocity profiles, is shown below.



Norbury and Crabtree (Ref 11) have represented the turbulent mixing, and reattachment region on a separation bubble as above. They derived an expression for the pressure recovery coefficient by assuming that the principal mixing process and corresponding pressure rise occur in the region BCDE as shown on the sketch. The pressures across ABC and DE are assumed uniform and equal to p_b and p_r . The portion of the bubble to the left of station x_b is assumed to be a region of constant pressure "dead air" with pressure p_b . Further assumptions are that: CD is a streamline parallel to AE, no fluid enters or leaves AB on the average, and that curvature of the surface AE may be neglected.

p_b and p_r are related to U_b and U_r by Bernoulli's equation:

$$p_b + \frac{1}{2} \rho U_b^2 = p_r + \frac{1}{2} \rho U_r^2$$

By use of the steady flow momentum equation applied to the control volume ABCDE, the following expression is derived for the pressure difference ($p_r - p_b$).

$$(p_r - p_b) = \frac{\rho}{\delta_r} \left[\int_{y_d}^{y_e} u_b^2 dy - \int_0^{\delta_r} u_r^2 dy \right]$$

Viscous and turbulent stresses are neglected in this development as they were in reference 11, thus an error is introduced with an effect that cannot be determined. However, it appears reasonable to postulate that pressure effects dominate the process of momentum transfer in the x-direction in the shear layer. Assuming a linear velocity profile across BC of the form:

$$\frac{u_b}{U_b} = a + (1 - a) \frac{y - y_d}{\delta_b}$$

At the dividing streamline ($y = y_d$) and

$$u_b = a U_b \equiv u_B$$

so we have:

$$(p_r - p_b) = \frac{\rho}{\delta_r} \left[\frac{1}{3} u_B^2 \delta_b \left(1 + \frac{1}{a} + \frac{1}{a^2} \right) + U_r^2 (\theta_r + \delta_r^* - \delta_r) \right] \quad (3.1)$$

where θ_r and δ_r^* are the momentum and displacement thicknesses of the shear layer at reattachment.

The steady flow continuity equation is:

$$\int_{y_d}^{y_e} u_b dy = \int_0^{\delta_r} u_r dy$$

or

$$\frac{1}{2} u_B \delta_b \left(1 + \frac{1}{a} \right) = U_r (\delta_r - \delta_r^*) \quad (3.2)$$

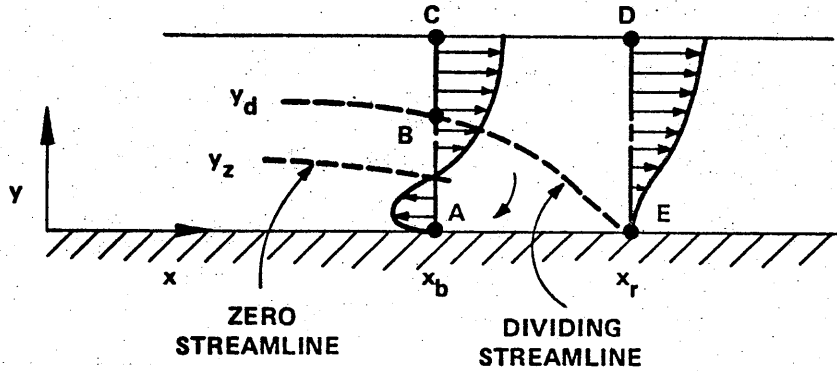
Norbury and Crabtree proceeded to develop an expression for the pressure recovery coefficient, σ , where:

$$\sigma \equiv \frac{p_r - p_b}{\frac{1}{2}\rho U_b^2} = \frac{C_{p_r} - C_{p_b}}{1 - C_{p_b}}$$

This parameter will be discussed in detail in section 4. For now we assume that σ remains constant as spoiler height varies in steady flow. Actually the bubble parameters d_r , d_b and σ are expected to be functions of Re_{δ}^* as discussed in reference 20, but for an overwhelming spoiler perturbation we expect the Reynolds Number dependence to be minimal.

The pressure rise in the reattachment region has also been modeled by many authors with the assumption of isentropic recompression along the dividing streamline (Ref 5). This approach will not be attempted here.

Now consider the steady flow continuity equation applied to the interior of the separation bubble, i.e., the so-called "dead air" region. (See also Ref 18). We have $\dot{m}_{in} = \dot{m}_{out}$. At the dividing streamline on the average $\dot{m}_{in} = \dot{m}_{out} \cong 0$, but now consider the region near reattachment.



In the region ABE we have $\dot{m}_{in} = \dot{m}_{out}$ and the mass flux per unit span is:

$$\dot{m}_{out} = -\rho \int_0^{y_z} u \, dy$$

and

$$\dot{m}_{in} = \rho \int_{y_z}^{y_d} u \, dy = \dot{m}_{out}$$

Fluid to the left of region ABE is entrained from under the zero streamline into the new mixing layer such that:

$$\dot{m}_{out} = \rho c \int_{x_{sp}}^{x=x_b} v_e dx = \rho \int_{y_z}^{y_d} u dy$$

where v_e is the entrainment velocity of fluid into the mixing region. Vasilu (in Ref 5) assumes $\frac{v_e}{U_e} = \text{const.}$ where U_e is the velocity at the edge of the shear layer along the bubble.

Since C_p is assumed constant over much of the bubble we have $\frac{U_e}{U_\infty} = \text{const}$ and

$$\dot{m}_{out} = \rho c v_e \int_{x_{sp}}^{x_b} dx \cong \rho v_e \ell \quad \begin{array}{l} \text{since } v_e \rightarrow 0 \\ \text{as } x \rightarrow x_r \end{array}$$

Fluid recirculated into the dead air region from the region near reattachment can be represented as:

$$\dot{m}_{in} = \rho \int_{y_z}^{y_d} u dy = \rho u_B \delta_o$$

where u_B is the velocity along the dividing streamline and δ_o is a resultant length. (For a linear velocity profile from y_z to y_d , $\delta_o = \frac{y_d - y_z}{2}$.)

And for the separated region we have:

$$\rho u_B \delta_o = \rho v_e \ell$$

and

$$\delta_o = \frac{v_e}{u_B} \ell$$

i.e., bubble length parameters scale to ℓ in steady flow.

Also within the bubble at any station x we find that mass flux recirculated into the region bounded by the spoiler, station x and the zero streamline is:

$$\begin{aligned}\dot{m}_{in}(x) &= \dot{m}_{out}(x) = \rho c \int_{x_{sp}}^x v_e dx \\ &= \rho c v_e (x - x_{sp}) \\ &= \rho c v_e \ell \frac{(x - x_{sp})}{\ell} = \dot{m}_{in}(x_r) \frac{(x - x_{sp})}{\ell}\end{aligned}$$

These and previous equations describe approximate steady-flow bubble behavior. In the following pages, an attempt is made to account for the effect of harmonic oscillations of the spoiler about a mean position which involves a long bubble type of separation.

3.2. Unsteady Flow Modifications

3.2.1. Unsteady Bubble Growth

Spoiler motion of the form $h = h_0 + \Delta h \cos \omega t$ will be considered. Use of a reduced frequency parameter ν results in:

$$h = h_0 + \Delta h \cos \nu \tau$$

Consider now the "quasi-steady" assumption that separation bubble growth is in phase with spoiler height. The bubble growth relation would be:

$$\rho \frac{d}{dt} (A) = \dot{m}_{in}(t) - \dot{m}_{out}(t)$$

where we approximate:

$$A \cong \frac{1}{2} h_s \ell$$

and

$$\rho \frac{d}{dt} (A) \cong \frac{\rho}{2} (\dot{h}_s \ell + h_s \dot{\ell})$$

but in the quasi-steady assumption:

$$\ell = d_r \frac{c}{c_f} (h_s - h_{sc}) \cong d_r \frac{c}{c_f} h_s$$

and

$$\dot{l} = d_r \frac{c}{c_f} \dot{h}_s$$

$$\therefore \rho \frac{d}{dt} (A) \cong \rho d_r \frac{c}{c_f} h_s \dot{h}_s = \rho d_r c c_f h \dot{h}$$

Define this as the quasi steady mass flux $\dot{m}_\epsilon$ so that

$$\rho \frac{d}{dt} (A) = \dot{m}_\epsilon (t) = \rho d_r c c_f h \dot{h} \quad (3.3)$$

In reference 18, East and Wilkinson postulate that a similar mass flux may result when a new filament line is "selected" at the spoiler tip. Piston theory is used in their hypersonic flow problem in order to account for the unsteady effect.

In the quasi-steady model we also assume that $\dot{m}_{outQS} = \dot{m}_{outSS}$ where the "steady-state" mass flux is as before:

$$\dot{m}_{outSS} \cong \rho v_e l$$

And that $\dot{m}_{inQS} = \dot{m}_{inSS} + \dot{m}_\epsilon$ where $\dot{m}_{inSS} = \rho u_B \delta_o$ and where modifications to u_B and δ_o account for the quantity $\dot{m}_\epsilon$ in quasi-steady flow. That is, in the assumed model $\dot{m}_\epsilon$ is a mass flux that is forced into or out of the bubble in the reattachment region. Later it will be shown how this may be achieved.

As done for the steady flow quantity $\dot{m}_{in}(x)$ we assume in the quasi-steady case

$$\dot{m}_\epsilon (x) = \dot{m}_\epsilon (x_r) \frac{(x - x_{sp}) c}{l}$$

The above quasi-steady result involves a bubble that grows in phase with spoiler motion. Since this is at odds with experimental data we are led to the conclusion that a quasi-steady model is not appropriate. In reference 17, for example, it is argued that since a characteristic time for the outer shear layer is δ/U_∞ as compared to $l/|U_c|$ for the separated flow (where U_c is the mean reversed flow velocity), an appropriate assumption would be that the outer layer reacts instantaneously to spoiler motion; i.e., the return mass flow is $\dot{m}_\epsilon$ from the quasi-steady assumption.

Now consider an elemental area, δA , within the bubble where we make the unsteady-flow assumption (as in reference 17) that the backflow mass fluctuations, $\dot{m}_\epsilon$, travel upstream at a mean

flow velocity, U_c . That is, $\dot{m}_\epsilon(X) = \dot{m}_\epsilon(X, t - \tau_\epsilon) = \dot{m}_\epsilon(X, t - \frac{\ell - X}{|U_c|})$ where now X is a distance measured aft from the spoiler. Then

$$\begin{aligned} \rho \frac{d}{dt} [\delta A(X, t)] &= \dot{m}_{in_\epsilon} - \dot{m}_{out_\epsilon} \\ &= \dot{m}_\epsilon(X + dX, t - \frac{\ell - (X + dX)}{|U_c|}) \\ &\quad - \dot{m}_\epsilon(X, t - \frac{\ell - X}{|U_c|}) \end{aligned}$$

For long wavelength (i.e., $\frac{\omega \ell}{|U_c|} \ll 2\pi$) we find from a Taylor series expansion that

$$\begin{aligned} \rho \frac{d}{dt} (\delta A) &\cong \frac{\partial}{\partial X} \dot{m}_\epsilon(X, t - \tau_\epsilon) dX \\ &\cong \frac{\partial}{\partial X} \dot{m}_\epsilon(\ell, t - \tau_\epsilon) \frac{X}{\ell} dX \\ &\cong \dot{m}_\epsilon(\ell, t - \tau_\epsilon) \frac{dX}{\ell} \end{aligned}$$

Now we integrate over the bubble area

$$\rho \int_{\text{bubble}} \frac{d}{dt} (\delta A) = \rho \frac{d}{dt} (A) = \frac{1}{\ell} \int_0^\ell \dot{m}_\epsilon(\ell, t - \frac{\ell - X}{|U_c|}) dX$$

where $\dot{m}_\epsilon(\ell, t - \tau_\epsilon) = \rho d_r cc_f h(t - \tau_\epsilon) \dot{h}(t - \tau_\epsilon)$. For $h = h_0 + \Delta h \cos \omega t$; $\dot{h} = -\omega \Delta h \sin \omega t$ we find:

$$\begin{aligned} \frac{d}{dt} (A) &= - \frac{|U_c| d_r cc_f}{\ell} [2h_0 \Delta h \sin \omega(t - \frac{\ell}{2|U_c|}) \\ &\quad \sin \omega \frac{\ell}{2|U_c|} + \frac{\Delta h^2}{2} \sin 2\omega(t - \frac{\ell}{2|U_c|}) \sin \omega \frac{\ell}{|U_c|}] \end{aligned}$$

Consider now small perturbations so that $h = h_0 + \tilde{h}$; $\ell = \ell_0 + \tilde{\ell}$ where $\tilde{h} = \Delta h \cos \omega t$ and $|\tilde{h}| \ll h_0$ & $|\tilde{\ell}| \ll \ell_0$.

We have:

$$\frac{d}{dt} (A) \cong \frac{2 |U_c| d_r c c_f}{l_o \omega} h_o \sin \left(\omega \frac{l_o}{2|U_c|} \right) \\ [- \omega \Delta h \sin \omega \left(t - \frac{l_o}{2|U_c|} \right)]$$

but $d_r c h_o = l_o$

$$\therefore \frac{d}{dt} (A) \cong \frac{2|U_c| c_f}{\omega} \sin \left(\frac{\omega l_o}{2|U_c|} \right) \tilde{h} \left(t - \frac{l_o}{2|U_c|} \right)$$

But since the area is $A \cong \frac{1}{2} h_s l = \frac{1}{2} h l c_f$ then $\frac{d}{dt} (A) \cong \frac{1}{2} [\tilde{h} l_o + \tilde{l} h_o] c_f$ and we find the unsteady flow result:

$$\tilde{l} \cong \frac{l_o}{h_o} \left[\frac{2}{\omega \tau_o} \sin \omega \tau_o \tilde{h} (t - \tau_o) - \tilde{h} \right]$$

where $\tau_o \equiv \frac{l_o}{2|U_c|}$. For $\tilde{h} = \Delta h \cos \omega t$ we find:

$$\tilde{l} \cong \frac{l_o}{h_o} [(k \cos \omega \tau_o - 1) \tilde{h} + (k \omega \sin \omega \tau_o) \tilde{h}]$$

where

$$k = \frac{4|U_c|}{l_o \omega} \sin \omega \frac{l_o}{2|U_c|} = \frac{2}{\omega \tau_o} \sin \omega \tau_o$$

Integrating to find $\tilde{l}(t)$ we have:

$$\tilde{l}(t) - \tilde{l}(o) = \frac{l_o}{h_o} [(k \cos \omega \tau_o - 1) \Delta h (\cos \omega t - 1) \\ + (k \sin \omega \tau_o) \Delta h \sin \omega t]$$

Assume that over a cycle the average length perturbation is zero so that

$$\tilde{l}(o) = \frac{l_o}{h_o} (k \cos \omega \tau_o - 1) \Delta h$$

and

$$\begin{aligned}\tilde{l}(t) = \frac{\ell_o}{h_o} [(k \cos \omega \tau_o - 1) \Delta h \cos \omega t \\ + (k \sin \omega \tau_o) \Delta h \sin \omega t]\end{aligned}$$

This can be written as:

$$\tilde{l}(t) = \frac{\ell_o}{h_o} A \Delta h \cos (\omega t - \alpha) = d_r c A \Delta h \cos (\omega t - \alpha)$$

where

$$A = (k^2 - 2 k \cos \omega \tau_o + 1)^{\frac{1}{2}}$$

and

$$A \geq 0$$

also

$$\alpha = \tan^{-1} \left[\frac{\sin \omega \tau_o}{\cos \omega \tau_o - \frac{1}{k}} \right]$$

$$\alpha \geq 0$$

Now:

$$\omega \tau_o = \omega \frac{\ell_o}{2|U_c|} = \frac{\omega c_f}{U_\infty} \left| \frac{U_\infty}{U_c} \right| \frac{d_r c}{2 c_f} h_o = 2 \nu h_o \frac{\frac{d_r}{U_c}}{\left| \frac{U_c}{U_\infty} \right|}$$

Note that for quasi-steady bubble growth:

$$\tilde{l}(t) = \frac{\ell_o}{h_o} \tilde{h}(t) = d_r c \Delta h \cos \omega t$$

so that the unsteady result above involves an effective spoiler height perturbation that is amplitude and phase dependent on a time lag; τ_o , where

$$\tau_o = \frac{\ell_o}{2|U_c|}$$

That is,

$$\tilde{h}_{\text{eff}} = A \Delta h \cos (\omega t - \alpha)$$

This result can be achieved by alternative assumptions about the mechanisms of unsteady bubble growth. Inherent in the present result is a model for unsteady entrainment of fluid from the bubble into the outer shear layer. For small perturbations the present model is expected to be correct to first order.

Let us investigate this linear time lag model in detail. First consider small bubbles at low frequency with

$$\omega \tau_0 \ll 1 \quad (\text{where } |U_c| \sim 0 \left(\frac{U_\infty}{3} \right))$$

We find

$$k = \frac{2}{\omega \tau_0} \sin \omega \tau_0 \cong 2$$

and

$$A \cong 1$$

$$\alpha \cong \tan^{-1} (2 \omega \tau_0) = \tan^{-1} \left(\omega \frac{l_0}{|U_c|} \right)$$

Therefore, the length perturbation is:

$$\tilde{l}(t) \cong d_r c \Delta h \cos \omega \left(t - \frac{l_0}{|U_c|} \right)$$

That is, the length lags its quasi-steady location by a time lag which is the "bubble characteristic time."

Now consider a larger bubble and higher frequency where the relation is more complex. For example, consider the following values:

$$\nu = .12; \quad l_0 = 6''; \quad \left| \frac{U_c}{U_\infty} \right| = \frac{1}{3}$$

then $\omega \tau_0 = .18 \text{ rad}$; $k \cong 1.99$; $A \cong 1.02$ and $\alpha \cong 20.3^\circ = .354 \text{ rad}$

$$\therefore \tilde{l}(t) = d_r c \Delta h (1.02) \cos (\omega t - 20.3^\circ)$$

The linear approximation yields:

$$\tilde{x}(t) \cong d_r c \Delta h \cos(\omega t - 19.8^\circ)$$

The unsteady bubble growth relation also shows that the region of the bubble near reattachment grows in phase with spoiler motion.

3.2.2. The Pressure Rise Near Reattachment

The mass flux term, $\dot{m}_\epsilon$, which is included in the unsteady flow model can be represented as

$$\dot{m}_\epsilon = -\rho u_B \tilde{\delta}_b$$

that is, a perturbation near x_b at the "dividing streamline" results in the additional mass flux. If we assume that conditions at reattachment are unaffected then from the continuity equation (3.2) we find:

$$\begin{aligned} \frac{1}{2} u_B \delta_b \left(1 + \frac{1}{a}\right) &= \frac{1}{2} (u_B + \tilde{u}_B) (\delta_b + \tilde{\delta}_b) \left(1 + \frac{1}{a}\right) \\ &= U_r (\delta_r - \delta_r^*) \end{aligned}$$

and

$$\tilde{u}_B \delta_b \cong -u_B \tilde{\delta}_b$$

That is, a perturbation of δ_b results in a proportional change to u_B .

The momentum equation (3.1) yields an appropriate perturbation to p_b .

$$\begin{aligned} -\tilde{p}_b \delta_r &\cong \rho \left[\frac{2}{3} u \tilde{u}_B \delta_b \left(1 + \frac{1}{a} + \frac{1}{a^2}\right) \right. \\ &\quad \left. + \frac{1}{3} u_B^2 \tilde{\delta}_b \left(1 + \frac{1}{a} + \frac{1}{a^2}\right) \right] + \dot{m}_\epsilon u_B \\ &= \rho \left[-\frac{1}{3} u_B^2 \tilde{\delta}_b \left(1 + \frac{1}{a} + \frac{1}{a^2}\right) \right] - \rho u_B^2 \tilde{\delta}_b \end{aligned}$$

or

$$\tilde{p}_b = \rho u_B^2 \frac{\tilde{\delta}_b}{\delta_r} \left[\frac{1}{3} \left(4 + \frac{1}{a} + \frac{1}{a^2}\right) \right]$$

and in coefficient form:

$$\tilde{C}_{pb} \equiv \frac{\tilde{P}_b}{\frac{1}{2} \rho U_\infty^2} = - \left[\frac{1}{3} \left(4 + \frac{1}{a} + \frac{1}{a^2} \right) \right] \frac{\dot{m}_\epsilon u_B}{\frac{1}{2} \rho U_\infty^2 \delta_r}$$

Now $\delta_r = 0$ (h_s) and continuing the assumption that conditions at reattachment are unaffected by small perturbations to h we have:

$$\delta_r = \delta_{r0} \cong k_1 d_r c h_0$$

From the quasi-steady equation for mass flux (3.3) we have:

$$\dot{m}_\epsilon \cong \rho d_r c c_f h_0 \dot{\tilde{h}}$$

Therefore, the pressure perturbation can be written as:

$$\tilde{C}_{pb} = - \left[\frac{1}{3} \left(4 + \frac{1}{a} + \frac{1}{a^2} \right) \right] \frac{2 c_f u_B}{k_1 U_\infty^2} \dot{\tilde{h}}$$

or if we assume a constant relation

$$k_2 = \frac{u_B}{|U_c|}$$

then, since $\dot{\tilde{h}} = -\omega \Delta h \sin \omega t$,

$$\begin{aligned} \tilde{C}_{pb} &= + \left[\frac{1}{3} \left(4 + \frac{1}{a} + \frac{1}{a^2} \right) \right] 2 \frac{k_2}{k_1} \left(\frac{\omega c_f}{U_\infty} \right) \left| \frac{U_c}{U_\infty} \right| \Delta h \sin \omega t \\ &= k_3 \nu \left| \frac{U_c}{U_\infty} \right| \Delta h \sin \omega t \end{aligned}$$

where

$$k_3 = \left[\frac{2}{3} \left(4 + \frac{1}{a} + \frac{1}{a^2} \right) \right] \frac{d_r c h_0 u_B}{\delta_{r0} |U_c|}$$

Thus, we see that the pressure perturbation depends on spoiler rate and a shear-layer parameter, k_3 . This corresponds to a perturbation to the pressure recovery coefficient, σ , and indicates that the normally constant value of σ for a reattaching shear-layer may be modified in unsteady flow.

In order to gain insight into the mechanisms involved in these unsteady flow perturbations, let us consider and extend a treatment of vorticity within the separation bubble.

McGregor (Ref 12) investigated the conditions necessary for equilibrium of the separation bubble in steady-flow. Constancy of circulation and the maintenance of kinetic energy are required.

Circulation around the bubble is

$$\Gamma = \iint_{\text{bubble}} \gamma \, dx \, dy = \oint_C q_s \, ds$$

where γ is the vorticity $(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})$ and C is a curve, described in the counterclockwise sense, which encloses the bubble. It coincides with the dividing streamline, spoiler aft face and airfoil surface, in steady flow. q_s is the tangential velocity component around the dividing streamline.

If we consider turbulent flow so that u , v and γ consist of mean values $\bar{u}$, $\bar{v}$, and $\bar{\gamma}$ on which are superimposed random fluctuating components u' , v' and γ' due to turbulence and whose time average values are zero then we find (Ref 12):

$$\frac{d\Gamma}{dt} - \oint \bar{\gamma} \, \bar{q}_n \, ds = - \nu \oint \frac{\partial \bar{\gamma}}{\partial n} \, ds + \oint \overline{\gamma' q_n'} \, ds \quad (3.4)$$

In steady flow the terms on the L.H.S. are zero since $\bar{q}_n = 0$ on the bubble boundary. Then we have:

$$\nu \oint \frac{\partial \bar{\gamma}}{\partial n} \, ds = \oint \overline{\gamma' q_n'} \, ds$$

On the surface of the model $q_n' = 0$ and we may write

$$\nu \int_{\text{sepn}}^{\text{reatt}} \frac{\partial \bar{\gamma}}{\partial y} \, dx_{\text{surf}} + \left[\int_{\text{reatt}}^{\text{sepn}} \left(\nu \frac{\partial \bar{\gamma}}{\partial n} - \overline{\gamma' q_n'} \right) ds \right] = 0$$

The first term is related to the pressure distribution and the first part of the second term may be neglected so that:

$$-\frac{1}{\rho} (p_{\text{sepn}} - p_{\text{reatt}}) \cong \int_{\text{sepn}}^{\text{reatt}} \overline{\gamma' q_n'} \, ds$$

This gives the relation between the fluctuating vorticity and normal velocity at the bubble boundary and the pressure

rise.¹² The rate of transfer of vorticity is very high in the reattachment region since the pressure rise occurs primarily there.

Consider now an unsteady flow assumption that involves the LHS of equation (3.4). Assume that the sum of the terms is zero, as in steady flow, but in unsteady flow we have:

$$\frac{d\Gamma}{dt} = \oint \bar{\gamma} \bar{q}_n ds$$

where $\oint \bar{\gamma} \bar{q}_n ds = \bar{\gamma}_{avg} \frac{d}{dt} (A_{bubble})$ with $\bar{\gamma}_{avg}$ an average value of vorticity along the dividing streamline.

Previous experiments have shown that for low spoiler heights (roughly for $h_s \leq .015 c$) airfoil lift is relatively unaffected by spoiler projection.^{8,29,40} This is thought to be associated with shear layer reattachment on the airfoil surface for low spoiler heights.

It appears reasonable to postulate, therefore, that for low spoiler heights (which do not involve T.E. pressure divergence) total airfoil circulation remains constant so that

$$\Gamma_{airfoil} = \Gamma_{wetted \ surface} + \Gamma_{bubble} = \Gamma_o$$

Now Wood's theory (Ref 28) predicts that lift generated by the wetted surface varies in direct proportion to spoiler height, so that

$$\Gamma_{wetted \ surface} \cong \Gamma_o + k_4 h$$

where $k_4 > 0$. This results in:

$$\Gamma_{bubble} \cong - k_4 h$$

and

$$\frac{d\Gamma_{bubble}}{dt} \cong - k_4 \dot{h}$$

Also we have

$$\frac{d\Gamma_{bubble}}{dt} = \oint \bar{\gamma} \bar{q}_n ds \cong \bar{\gamma}_{avg} \frac{d}{dt} (A_{bubble})$$

If we neglect phase lags then both $\bar{q}_n$ and A are proportional to $\dot{h}$ and $\bar{\gamma}(s) < 0$, which is to be expected. Thus, on the spoiler upstroke ($\dot{h} > 0$), we find that vorticity of clockwise sense is added to the bubble interior, and conversely on the downstroke, vorticity of clockwise sense is removed from the bubble interior.

If we consider now the maintenance of kinetic energy of the bubble, as in Ref 12, we may look at the steady flow equilibrium relation:

$$W = + \mu \oint \bar{\gamma} \bar{q}_s ds + \oint \bar{q}_n' p' ds - \oint \bar{q}_s (-\rho \bar{q}_s' \bar{q}_n') ds \quad (3.5)$$

where W is the rate of dissipation of energy due to viscosity and

$$W \cong \mu \iint \bar{\gamma}^2 dx dy$$

The terms on the RHS of equation (3.5) represent, in turn, the rate at which work is done: on the bubble by the viscous stresses, on the turbulent motion by the pressure fluctuations, and on the bubble by the turbulent shearing stress tangential to the bubble boundary. All but the last term may be neglected as an approximation.

$$\therefore W \cong \mu \iint \bar{\gamma}^2 dx dy \cong - \oint \bar{q}_s (-\rho \bar{q}_s' \bar{q}_n') ds$$

as the energy equilibrium relation for the bubble.¹²

Note that the bubble contains both positive and negative vorticity. The negative vorticity largely diffuses into the bubble in steady flow and the positive vorticity is created at the surface. The rate of dissipation, W , is least for any given circulation, Γ_{bubble} , when the vorticity is all of the same sign.

In unsteady flow where we have found that negative vorticity (of clockwise sense) is either transferred into or out of the bubble we expect the following:

$$\frac{dW}{W} < \frac{d\Gamma}{\Gamma} \quad \text{for } \dot{h} > 0$$

and

$$\frac{dW}{W} > \frac{d\Gamma}{\Gamma} \quad \text{for } \dot{h} < 0$$

That is, the energy dissipation increases less rapidly than circulation on the upstroke and more rapidly than circulation on the downstroke. Thus, on the upstroke an increased pressure rise across the bubble, with corresponding increase in energy dissipation, can be maintained. The increased magnitude of circulation on the upstroke is associated with this pressure rise.

Other qualitative judgements can be made. On the upstroke, an increased pressure rise and increased magnitude of circulation are expected to result in a more highly curved reattachment "streamline." On the other hand, our quasi-steady mass flux model results in near quasi-steady growth for the region of the bubble near to reattachment, thus we expect that the height of this region increases faster than its length, on the upstroke. This deformation to bubble shape is also compatible with increased inviscid external flow velocities and decreased pressures near the point of maximum bubble thickness, that is, probably near x_b .

One factor has not been considered yet. That is, on the spoiler upstroke when vorticity is transferred to the bubble and circulation magnitude increases, there is likely to be an additional integral effect on circulation. This follows if energy is being added to the bubble faster than it is dissipated. A more comprehensive unsteady flow model would probably yield this result.

The previous discussion of bubble vorticity is not offered as a rigorous study of actual conditions. The quasi-steady and unsteady flow assumptions are not well supported in general; however, the qualitative results appear to explain much of the observed behavior. Further testing is absolutely necessary in order to justify and/or refine these assumptions.

3.3. Unsteady Flow Model

Two significant unsteady flow effects will be included in the steady flow equation for ΔC_H . A polynomial approximation is used for the steady flow relation and modified to account for unsteady pressure perturbations, which are taken to act over the dead air region. An effective spoiler height is used, which lags actual spoiler motion.

Thus, the steady-flow relation:

$$\Delta C_H = a_5 h^5 + a_4 h^4 + a_3 h^3 + a_2 h^2 + a_1 h + a_0$$

is modified to the unsteady flow approximation for $h = h_0 + \Delta h \cos \omega t$:

$$\begin{aligned} \Delta C_H(t) \cong \left[1 + \frac{\tilde{C}_{p_b}}{C_{p_{b_0}}} \right] [a_5 h_{eff}^5 + a_4 h_{eff}^4 + a_3 h_{eff}^3 \\ + a_2 h_{eff}^2 + a_1 h_{eff} + a_0] \end{aligned} \quad (3.6)$$

where $C_{p_{b_0}}$ is the steady flow value of C_{p_b}

$$\tilde{C}_{p_b} = k_3 \nu \left| \frac{U_c}{U_\infty} \right| \Delta h \sin \omega t$$

and

$$h_{eff} = h_0 + A \Delta h \cos (\omega t - \alpha)$$

The following parameter values are chosen based on experimental observations and other results.⁵

$$C_{p_{b_0}} \cong - .4$$

which yields

$$\frac{u_B}{U_b} \cong .535 = a$$

Also, we have

$$\frac{U_c}{U_\infty} \cong \frac{1}{3}$$

so that

$$\frac{u_B}{|U_c|} \cong 2.0$$

But,

$$k_3 = \left[\frac{2}{3} \left(4 + \frac{1}{a} + \frac{1}{a^2} \right) \right] \frac{d_r^{ch_o}}{\delta_{r_o}} \frac{u_B}{|U_c|}$$

$$\therefore k_3 \cong 12.0 \frac{d_r^{ch_o}}{\delta_{r_o}}$$

Now, based on boundary layer measurements, assume $3 h_o c_f \cong \delta_{r_o}$ so that

$$k_3 \cong 4.0 \frac{d_r^c}{c_f} \cong 16.0 d_r = 88$$

where $d_r = 5.5$ and $x_r = d_r h + x_{sp}$. A best steady-flow curve fit yields:

$$\begin{aligned} a_5 &= -4 \times 10^5; a_4 = 1.3 \times 10^5; a_3 = -1.48 \times 10^4; \\ a_2 &= 689; a_1 = -12.38375; a_0 = .06190375 \end{aligned}$$

Figure 3.2 shows a comparison of the unsteady prediction and experiment (reconstructed curves) for the following three cases:

<u>Case</u>	<u>h_o</u>	<u>Δh</u>
B	.053	.026
C	.076	.027
E	.062	.040

$\nu = .12$ for all curves.

Also shown on Figure 3.2 are unsteady predictions which include an empirical modification to allow for the integral circulation effect mentioned previously.

The unsteady "dead-air" pressure perturbations, $\tilde{C}_{pb}$, are integrated from the start of the spoiler upstroke, multiplied by a factor of $\frac{1}{2}$ (to approximately allow for dissipation of energy due to viscous and turbulent stresses) and superimposed on the quasi-steady value of $\tilde{C}_{pb}$; that is,

$$(\tilde{C}_{pb})_{mod} = k_3 \nu \left| \frac{U}{U_\infty} \right| \Delta h \left[\sin \omega t - \frac{1}{2} (1 + \cos \omega t) \right]$$

This empirical modification improves the otherwise fair agreement which is demonstrated in Figure 3.2; however, it is

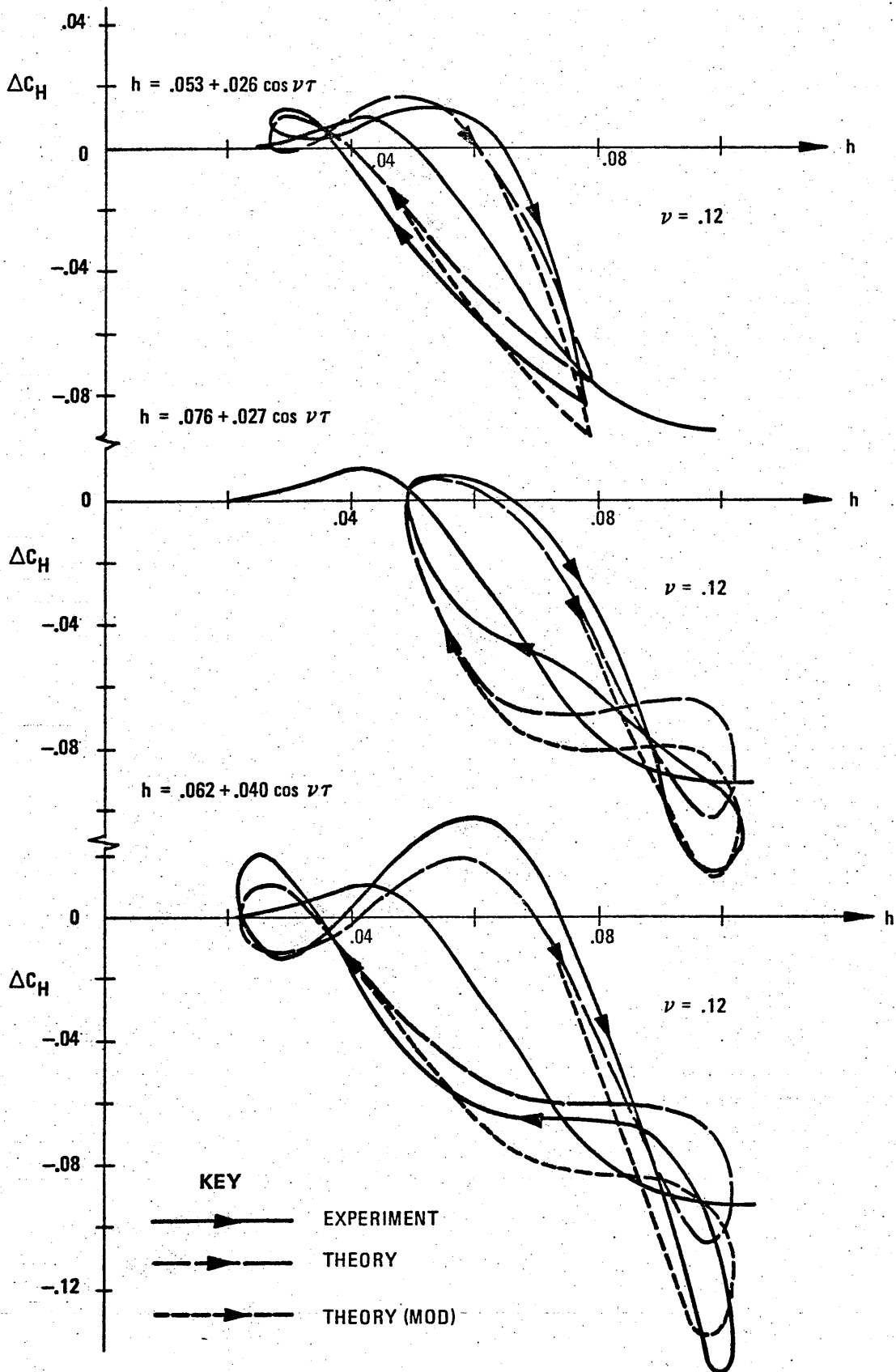


FIG. 3.2 DYNAMIC HINGE-MOMENT VARIATIONS, EXPERIMENT AND THEORY

not used in further analysis since it is a rather speculative addition to the unsteady theory.

The present theory cannot be compared with other flap-fixed, spoiler-forced results for two reasons:

1. The steady-flow curve fit is only valid for $.03 \leq h \leq .10$.
2. For large bubbles which involve recompression on the flap surface and "reattachment" at a point (line in 3-D flow) of confluence in the wake, the mass flux relations are no longer valid for the bubble.

However, it will be seen that the theory can be used in order to predict limit-cycle behavior of the coupled flap/spoiler system.

3.4. The Describing Functions

For spoiler motion of the form $h = h_0 + \Delta h \cos \nu \tau$, with h_0 , Δh and ν constant, the resultant incremental hinge-moment coefficient may be represented in Fourier series form as:

$$\Delta C_H = \Delta C_{H_0} \{h_0, \Delta h, \nu\} + \sum_{n=1}^{\infty} \Delta C_{H_n} \{h_0, \Delta h, \nu\} \cos [n\omega t + \psi_n \{h_0, \Delta h, \nu\}]$$

Results obtained from the unsteady relation (equation 3.6) are as follows:

$$\Delta C_{H_0} = \frac{1}{2\pi} \int_0^{2\pi} \Delta C_H \{h_0 + \Delta h \cos \omega t, -\omega \Delta h \sin \omega t\} d(\omega t)$$

Thus:

$$\begin{aligned} \Delta C_{H_0} = & [(a_5 h_0^5 + a_4 h_0^4 + a_3 h_0^3 + a_2 h_0^2 + a_1 h_0 + a_0) \\ & + \frac{1}{2} (10a_5 h_0^3 + 6a_4 h_0^2 + 3a_3 h_0 + a_2) \Delta h^2 A^2 \\ & + \frac{3}{8} (5a_5 h_0 + a_4) \Delta h^4 A^4] + \frac{k_3 \nu |U_c/U_\infty|}{2 C_{pb_0}} \Delta h \sin \alpha \\ & [(5a_5 h_0^4 + 4a_4 h_0^3 + 3a_3 h_0^2 + 2a_2 h_0 + a_1) \Delta h A \\ & + \frac{3}{4} (10a_5 h_0^2 + 4a_4 h_0 + a_3) \Delta h^3 A^3 + \frac{5}{8} (a_5) \Delta h^5 A^5] \end{aligned} \quad (3.7)$$

Now define the describing function $\frac{\Delta C_H}{\Delta h}$,

$$\frac{\Delta C_H}{\Delta h} \equiv N_1 + i N_2$$

where

$$\begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \frac{1}{\pi \Delta h} \int_0^{2\pi} \Delta C_H \{h_0 + \Delta h \cos \omega t, -\omega \Delta h \sin \omega t\} \begin{bmatrix} \cos \omega t \\ \sin \omega t \end{bmatrix} d(\omega t)$$

We find:

$$\begin{aligned} N_1 = & [(5a_5 h_0^4 + 4a_4 h_0^3 + 3a_3 h_0^2 + 2a_2 h_0 + a_1) \\ & + \frac{3}{4} (10a_5 h_0^2 + 4a_4 h_0 + a_3) \Delta h^2 A^2 + \frac{5}{8} (a_5) \Delta h^4 A^4] \\ & A \cos \alpha + \frac{k_3 \nu |U_c / U_\infty|}{2 C_{pb_0}} \sin 2\alpha \left[\frac{1}{2} (10a_5 h_0^3 + 6a_4 h_0^2 \right. \\ & \left. + 3a_3 h_0 + a_2) \Delta h^2 A^2 + \frac{1}{2} (5a_5 h_0 + a_4) \Delta h^4 A^4 \right] \end{aligned} \quad (3.8)$$

and

$$\begin{aligned} N_2 = & [(5a_5 h_0^4 + 4a_4 h_0^3 + 3a_3 h_0^2 + 2a_2 h_0 + a_1) \\ & + \frac{3}{4} (10a_5 h_0^2 + 4a_4 h_0 + a_3) \Delta h^2 A^2 + \frac{5}{8} (a_5) \Delta h^4 A^4] \\ & (-A \sin \alpha) + \frac{k_3 \nu |U_c / U_\infty|}{2 C_{pb_0}} \cos 2\alpha \left[\frac{1}{2} (10a_5 h_0^3 \right. \\ & \left. + 6a_4 h_0^2 + 3a_3 h_0 + a_2) \Delta h^2 A^2 + \frac{1}{2} (5a_5 h_0 + a_4) \Delta h^4 A^4 \right] \\ & - \frac{k_3 \nu |U_c / U_\infty|}{C_{pb_0}} [(a_5 h_0^5 + a_4 h_0^4 + a_3 h_0^3 + a_2 h_0^2 + a_1 h_0) \end{aligned}$$

$$+ a_0) + \frac{1}{2} (10a_5 h_0^3 + 6a_4 h_0^2 + 3a_3 h_0 + a_2) \Delta h^2 A^2 \\ + \frac{3}{8} (5a_5 h_0 + a_4) \Delta h^4 A^4] \quad (3.9)$$

Also we find:

$$\left| \frac{\Delta C_{H_1}}{\Delta h} \right| = (N_1^2 + N_2^2)^{\frac{1}{2}}$$

and

$$\psi_1 = 180^\circ + \tan^{-1} \frac{N_2}{N_1}$$

Thus, the first two terms of the Fourier series representation are:

$$\Delta C_H \cong \Delta C_{H_0} + \Delta C_{H_1} \cos (\omega t + \psi_1)$$

or

$$\Delta C_H \cong \Delta C_{H_0} + N_1 \Delta h \cos \omega t + \frac{N_2}{\omega} (-\omega \Delta h \sin \omega t)$$

Plots of ΔC_{H_0} , $\frac{\Delta C_{H_1}}{\Delta h}$ and ψ_1 , N_1 and N_2 are shown in Figures 3.3 to 3.5 and may be compared to the experimental steady flow results, which are also shown.

Unsteady predictions for ΔC_{H_0} are generally confirmed by experimental results. (See data in section 2 and that found in reference 20.) Frequency effect is clearly duplicated; that is, there is a reduction in magnitude of ΔC_{H_0} as frequency increases. Existing discrepancies, which are minor, can be attributed primarily to the approximate fifth order polynomial curve fit for steady flow.

Predictions of the describing function, $\frac{\Delta C_{H_1}}{\Delta h}$, are also dependent on the goodness of fit of the polynomial. For high values of h_0 there is obvious discrepancy and the complicated dependency of $\frac{\Delta C_{H_1}}{\Delta h}$ on Δh is not reproduced. Furthermore, frequency effect on magnitude is not accurately predicted for high h_0 , and phase shift ψ_1 is not predicted well over some of the h_0 range. Several reasons exist for these discrepancies:

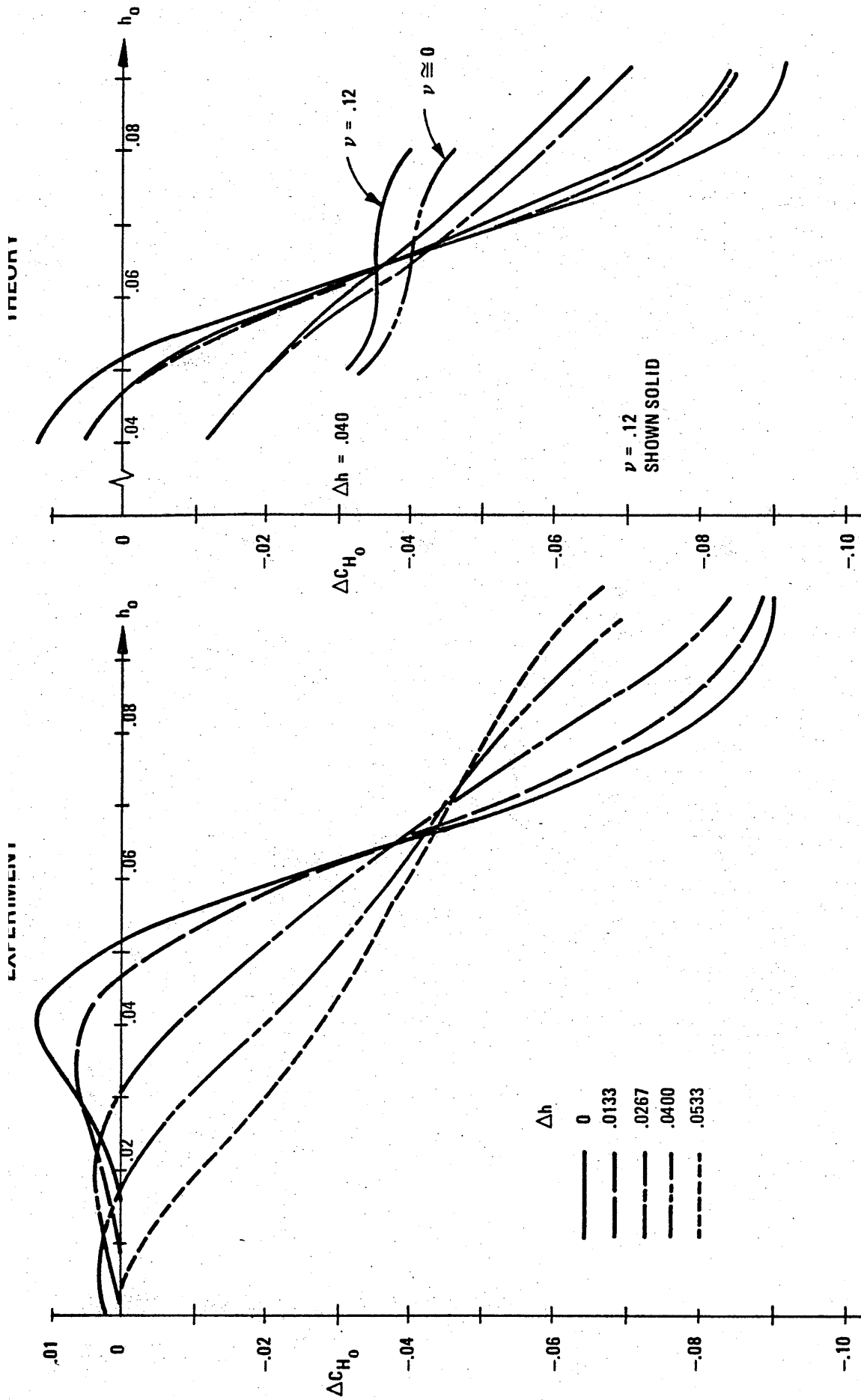


FIG. 3.3 VARIATION OF Δc_{H_0} WITH h_0 , Δh AND ν

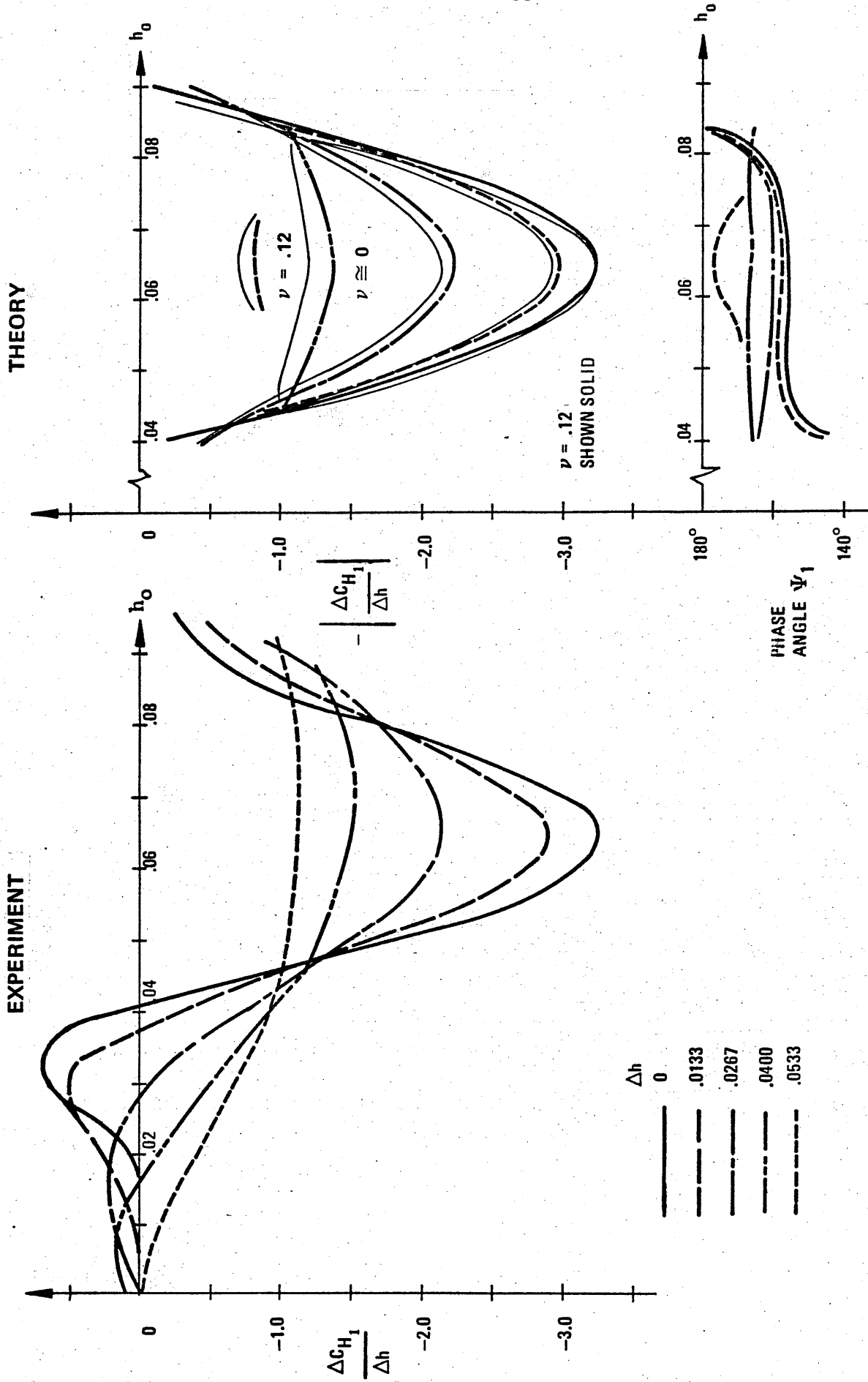


FIG. 3.4 VARIATION OF DESCRIBING FUNCTION, $\frac{\Delta c_{H1}}{\Delta h}$

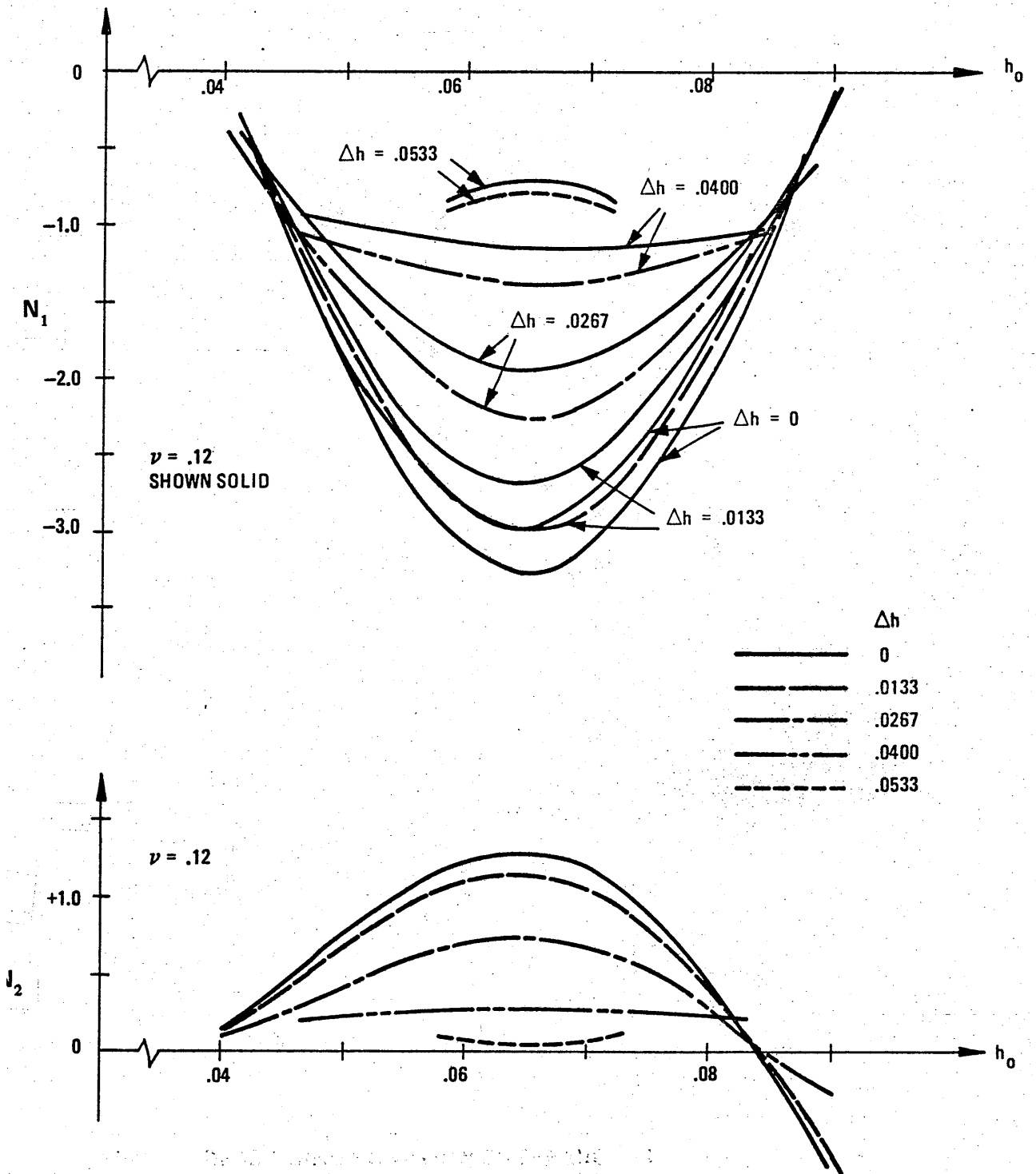


FIG. 3.5 DESCRIBING FUNCTION COEFFICIENTS

1. The experimental results reflect the integral effect on circulation, which would tend to increase $\frac{\Delta C_{H_1}}{\Delta h}$ as frequency and h_0 increase.
2. For spoiler motion which involves very low spoiler heights during part of the cycle, the short bubble type is formed at the onset of the upstroke. Pressure recovery to high values aft of reattachment can influence the hinge-moment greatly. This, plus the low pressures in the bubble near reattachment (which relate to large negative incremental moments) combine to increase $\left| \frac{\Delta C_{H_1}}{\Delta h} \right|$ as frequency increases.
3. The theory does not allow for large bubbles where reattachment, hence mixing and additional mass flux, occurs in the wake.
4. In the limit at very high frequencies, the bubble is washed off the airfoil into the wake. It does not dissipate as in our model. The effect of bubble "wash off" is that a large region of high suction is able to affect hinge-moment greatly.

Although there appear to be large discrepancies in the unsteady describing function in general, it appears that the functional relation is in fair, if not good, agreement in the region near $h_0 = .065$; that is, for small perturbations with reattachment near the T.E. This region is of prime interest as will be shown when coupled system behavior is considered.

Before leaving this topic, it is necessary to consider the evaluation of describing functions which can be used when spoiler motion is of the form:

$$h = h_0(t) + \Delta h(t) \cos \omega(t) t$$

That is, for transient motion, where h_0 , Δh and ω are assumed to vary slowly and can be averaged over a cycle of motion. We have: $\dot{h} \cong \Delta \dot{h} \cos \omega t - \omega \Delta h \sin \omega t$ and now proceed to form an expression for $\Delta C_H(t)$ which includes the first "damped harmonic."⁴¹ The following expressions are evaluated:

$$\Delta \hat{C}_{H_0} = \frac{1}{2\pi} \int_0^{2\pi} \Delta C_H \{ h_0 + \Delta h \cos \omega t, \dot{\Delta h} \cos \omega t - \omega \Delta h \sin \omega t \} d(\omega t)$$

$$\begin{bmatrix} \hat{N}_1 \\ \hat{N}_2 \end{bmatrix} = \frac{1}{\pi \Delta h} \int_0^{2\pi} \Delta C_H \{h_o + \Delta h \cos \omega t, \dot{\Delta h} \cos \omega t - \omega \Delta h \sin \omega t\} \begin{bmatrix} \cos \omega t \\ \sin \omega t \end{bmatrix} d(\omega t)$$

And we find:

$$\begin{aligned} \Delta \hat{C}_{H_0} &= \Delta C_{H_0} - \frac{k_3 c_f |U_c|}{C_{pb_0} U_\infty^2} \Delta h \left[\frac{1}{2} (5a_5 h_o^4 + 4a_4 h_o^3 \right. \\ &\quad \left. + 3a_3 h_o^2 + 2a_2 h_o) \Delta h A + \frac{3}{8} (10a_5 h_o^2 + 4a_4 h_o + a_3) \right. \\ &\quad \left. \Delta h^3 A^3 + \frac{5}{16} (a_5) \Delta h^5 A^5 \right] \cos \alpha \end{aligned} \quad (3.10)$$

where ΔC_{H_0} is given in equation (3.7).

$$\begin{aligned} \hat{N}_1 &= N_1 - \frac{k_3 c_f |U_c|}{C_{pb_0} U_\infty^2} \frac{\Delta h}{\Delta h} \{ [(a_5 h_o^5 + a_4 h_o^4 + a_3 h_o^3 \\ &\quad + a_2 h_o^2 + a_1 h_o + a_0) + \frac{1}{2} (10a_5 h_o^3 + 6a_4 h_o^2 + 3a_3 h_o \\ &\quad + a_2) \Delta h^2 A^2 + \frac{3}{8} (5a_5 h_o + a_4) \Delta h^4 A^4] + [\frac{1}{4} (10a_5 h_o^3 \\ &\quad + 6a_4 h_o^2 + 3a_3 h_o + a_2) \Delta h^2 A^2 + \frac{1}{4} (5a_5 h_o + a_4) \\ &\quad \Delta h^4 A^4] \cos 2\alpha \} \end{aligned} \quad (3.11)$$

where N_1 is given in equation (3.8), and

$$\begin{aligned} \hat{N}_2 &= N_2 + \frac{k_3 c_f |U_c|}{C_{pb_0} U_\infty^2} \frac{\Delta h}{\omega \Delta h} \left[\frac{1}{4} (10a_5 h_o^3 + 6a_4 h_o^2 + 3a_3 h_o \right. \\ &\quad \left. + a_2) \Delta h^2 A^2 + \frac{1}{4} (5a_5 h_o + a_4) \Delta h^4 A^4 \right] \sin 2\alpha \end{aligned} \quad (3.12)$$

where N_2 is given in equation (3.9).

The above relations will be used in order to investigate system stability and transient behavior for the coupled flap/spoiler system. The first terms in the Fourier series representation for ΔC_H would be:⁴¹

$$\begin{aligned}\Delta C_H &\cong \Delta \hat{C}_{H_0} + \hat{N}_1 \Delta h \cos \omega t + \frac{\hat{N}_2}{\omega} (\dot{\Delta h} \cos \omega t - \omega \Delta h \sin \omega t) \\ &\cong \Delta \hat{C}_{H_0} + [\hat{N}_1 + \frac{\hat{N}_2}{\omega} \frac{\dot{\Delta h}}{\Delta h}] \Delta h \cos \omega t + \frac{\hat{N}_2}{\omega} (-\omega \Delta h \sin \omega t)\end{aligned}$$

3.5. Prediction of Coupled System Behavior

When the spoiler and flap are mechanically coupled we expect, in some cases, that self-excited oscillations may occur which lead to stable limit cycle motion.

Consider now the linear coupling relation:

$$h = h_{o_1} + k\beta \quad (3.13)$$

which involves the coupling parameters h_{o_1} and k . We seek to investigate the dependence of system stability on these parameters.

The system equation of motion is of the form:

$$A\ddot{\beta} + B\dot{\beta} + C\beta = H$$

where

$$H = H_{\dot{\beta}}\dot{\beta} + H_{\beta}\beta + \Delta H$$

This form of the equation has been justified by experimental results. (See section 2.4 and reference 20.) So we have:

$$\ddot{\beta} + \frac{(B - H_{\dot{\beta}})}{A} \dot{\beta} + \frac{(C - H_{\beta})}{A} \beta = \frac{\Delta H}{A} \quad (3.14)$$

In the flap-free case ΔH , the forcing function, was due solely to spoiler motion, but now it is a function of flap motion through the coupling relation (3.13). We have:

$$\frac{\Delta H}{A} = \frac{\Delta C_H q_{\infty} b_f c_f^2}{A} \quad (3.15)$$

where $\Delta C_H = f\{\beta_0, \beta_1, \omega\}$ for spoiler motion of the form $\beta \cong \beta_0 + \beta_1 \cos \omega t$.

This form of motion is assumed since the LHS of equation (3.14) relates to a system with low-pass filter characteristics. That is, higher harmonic content of $\beta(t)$ may be neglected. Therefore, it is appropriate to represent $\Delta C_H(t)$, in order to determine system stability, as:

$$\Delta C_H \cong \Delta \hat{C}_{H_0} + \left[\hat{N}_1 + \frac{\hat{N}_2}{\omega} \frac{\dot{\beta}_1}{\beta_1} \right] k \beta_1 \cos \omega t + \frac{\hat{N}_2}{\omega} (-\omega k \beta_1 \sin \omega t)$$

since

$$\Delta h = k \beta_1$$

But now $\Delta \hat{C}_{H_0}$, $\hat{N}_1$ and $\hat{N}_2$ are functions of the assumed slowly varying parameters β_0 , β_1 and ω .

In order to study sustained (limit-cycle) oscillations we represent ΔC_H as:

$$\Delta C_H \cong \Delta C_{H_0} + N_1 k \beta_1 \cos \omega t + \frac{N_2}{\omega} (-\omega k \beta_1 \sin \omega t)$$

A direct solution of equation (3.14) is not attempted, instead (as in references 42 and 43) we consider an approximate method of solving non-linear differential equations.

$$\text{First define } \lambda \equiv \frac{\dot{\beta}_1}{\beta_1} = \frac{\dot{\Delta h}}{\Delta h}.$$

Then from equations (3.14) and (3.15) we find the following approximate relations for β_0 , λ and ω^2 :⁴²

$$\beta_0 \cong \frac{\Delta \hat{C}_{H_0} q_\infty b_f c_f^2}{(C - H_\beta)} = \frac{\Delta \hat{C}_{H_0}}{\left(\frac{C}{q_\infty b_f c_f^2} - C_{H\beta} \right)} \quad (3.16)$$

$$\lambda \cong - \frac{(B - H_\beta) - \frac{\hat{N}_2}{\omega} k q_\infty b_f c_f^2}{2A}$$

$$= \frac{1}{2A} \left[\left(\frac{c_f}{U_\infty} C_{H\beta} + k \frac{\hat{N}_2}{\omega} \right) q_\infty b_f c_f^2 - B \right] \quad (3.17)$$

$$\begin{aligned}\omega^2 &\cong -\lambda^2 + \left[\frac{(C - H_\beta) - k \hat{N}_1 q_\infty b_f c_f^2}{A} \right] \\ &= -\lambda^2 + \left[\frac{C - (C_{H\beta} + k \hat{N}_1) q_\infty b_f c_f^2}{A} \right] \quad (3.18)\end{aligned}$$

where $h_o = h_{o_1} + k \beta_o$; $\Delta h = k \beta_1$ and $\nu \equiv \frac{\omega c_f}{U_\infty}$.

Note that an equivalent linear equation may be used to represent equation (3.14) as:

$$\ddot{\beta} - 2\lambda \dot{\beta} + \omega_n^2 \beta = 0$$

where $\omega_n^2 = \omega^2 + \lambda^2$ and $-\lambda = \zeta \omega_n$.

The set of equations (3.16) to (3.18) was evaluated for an "ideal" coupled system. (That is, dry friction was not included.) The system is taken to be initially at rest at the steady-flow "equilibrium" position. Initially then $\beta_1 = \Delta h = 0$. Parameter values matched the experimental configuration with a few minor exceptions. Moment of inertia, A , was taken as constant for the coupled system. It is actually a weak function of the coupling parameter k . Also mechanical damping was taken as constant. It is also a weak function of k . Values of $C_{H\beta}$ and $C_{H\dot{\beta}}$ result from the series of flap-forced testing with zero spoiler height. The parameter values are:

$$\begin{aligned}A &= .0099 \text{ sl-ft}^2; B = .0115 \frac{\text{ft-lb-sec}}{\text{rad}} \\ C &= .477 \frac{\text{ft-lb}}{\text{rad}}; C_{H\beta} = -.518 \\ C_{H\dot{\beta}} &= -1.29; q_\infty b_f c_f^2 = 10.20 \text{ ft-lb} \\ (\text{i.e., } U_\infty &= 150 \frac{\text{ft}}{\text{sec}})\end{aligned}$$

Figure 3.6 presents the system stability parameters over a wide range of coupling parameters. The lines of constant β_o are seen to be straight, which is to be expected since $h_{o_1} = h_o - k \beta_o$. The curve of $\lambda = 0$ defines the system stability boundary. Regions in the upper right side are most unstable, while the stable region is confined roughly to the lower left. We find too that at $\beta_o \cong -4^\circ$, which corresponds to $h_o \cong .065$, the system is most unstable for any given k . This coincides with the peak value of $\hat{N}_2$. The constant frequency curves nearly follow the curves of constant λ due primarily to the similarity

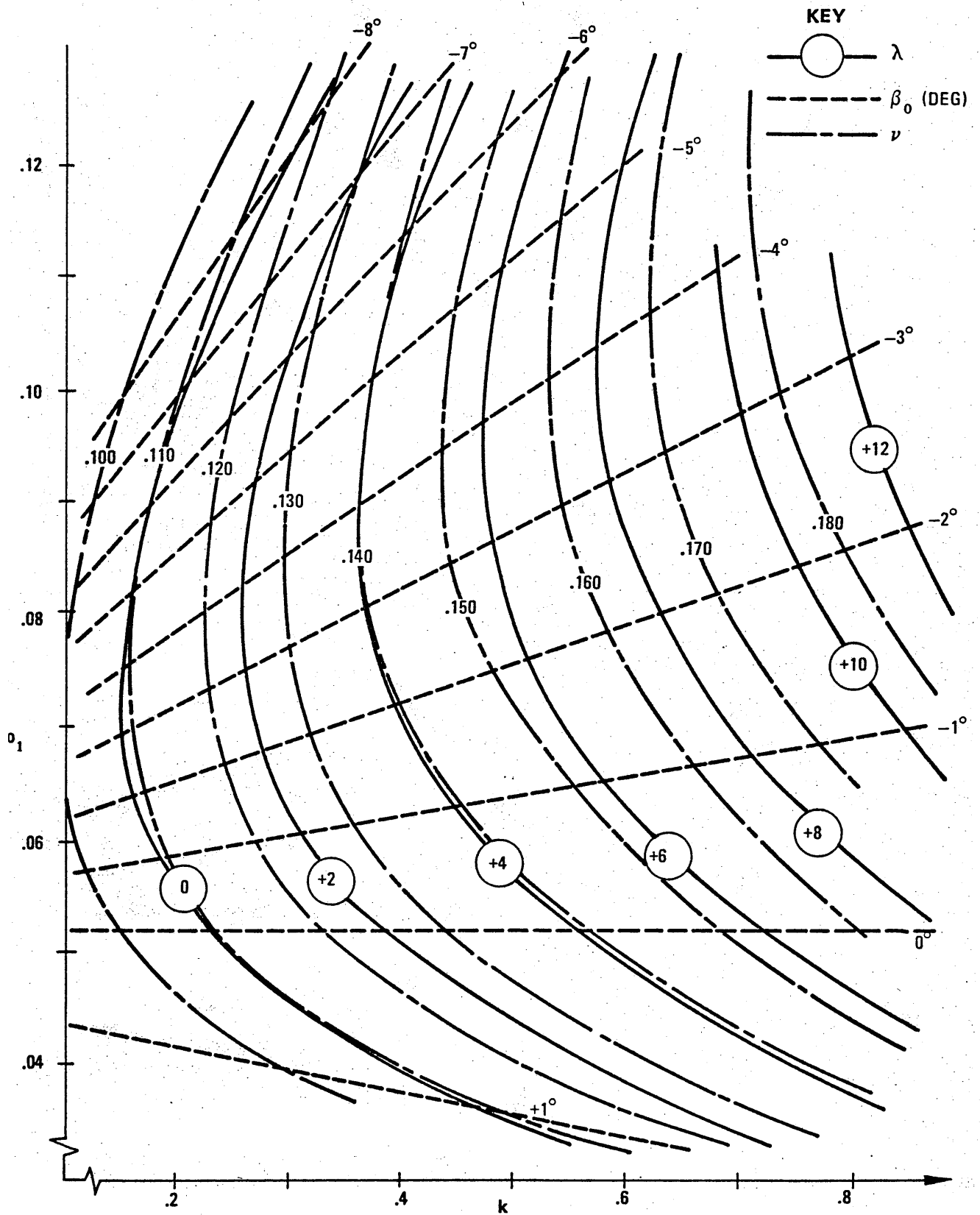


FIG. 3.6 SYSTEM STABILITY PARAMETERS

in h_0 dependence of $|N_1|$ and N_2 . An equivalent damping ratio, ζ , takes on a value of approximately $-.07$ at $h_{01} = .08$; $k = .3$. For $h_{01} = .10$; $k = .6$, then $\zeta \cong -.21$.

It is found that unstable combinations of coupling parameters will lead to sustained oscillations, or limit-cycle behavior. The system of equations (3.16) to (3.18) was included in an algorithm which searched through permissible values of β_0 , β_1 and ω in order to satisfy the set of equations with $\lambda = 0$. A high-speed digital computer was used. At the limit cycle we have:

$$\Delta \hat{C}_{H_0} = \Delta C_{H_0}; \quad \hat{N}_1 = N_1 \quad \text{and} \quad \hat{N}_2 = N_2$$

and

$$\Delta C_H \cong \Delta C_{H_0} + N_1 k \beta_1 \cos \omega t + \frac{N_2}{\omega} (-\omega k \beta_1 \sin \omega t)$$

Figure 3.7 contains curves of constant parameter values; β_0 , β_1 and ν . Note that there is evidence of the "rectification" effect on ΔC_{H_0} . A comparison of Figures 3.3, 3.6 and 3.7 shows that the $\beta_0 \cong -4^\circ$ curve is relatively unaffected, which follows from the unsteady theory (see Figure 3.3). For low values of h_{01} (hence, low values of h_0) we find that $\beta_{0LC} < \beta_{0SS}$; where LC and SS stand for limit-cycle and steady-state "equilibrium," respectively. Conversely, for high values of h_0 ; $\beta_{0LC} > \beta_{0SS}$. This effect is more pronounced for large β_1 . Limit-cycle frequencies are less than for $\beta_1 \cong 0$. This is due to reduced magnitudes of N_1 as β_1 goes from zero to limit-cycle amplitude. The frequency change is more pronounced for large β_1 . Maximum limit-cycle amplitudes occur near $h_{01} = .08$; $k = .26$. This corresponds to $h_0 \cong .065$ as expected. Limit-cycle amplitude decreases as k increases past about .26 because of the combined frequency effect and the nonlinear dependence of N_2 on Δh . Also, $\beta_1 = \frac{\Delta h}{k}$. Note also that for low values of h_{01} and k , limit-cycle behavior is predicted even though the system is predicted to be stable at "equilibrium." This apparent anomaly is due to the rectification effect on ΔC_{H_0} , hence β_0 . In this case "hard oscillator" behavior exists; that is, the system needs an initial disturbance of sufficient magnitude to provoke limit-cycle behavior. Limitations of the present model could also account for this phenomenon.

The theory was modified to allow for the presence of dry friction since it was discovered that this was a significant damping contribution to the actual coupled system. Several tests were performed in order to verify that dry friction was, in fact, present and to measure its contribution. An equivalent linear representation was calculated and is included in the

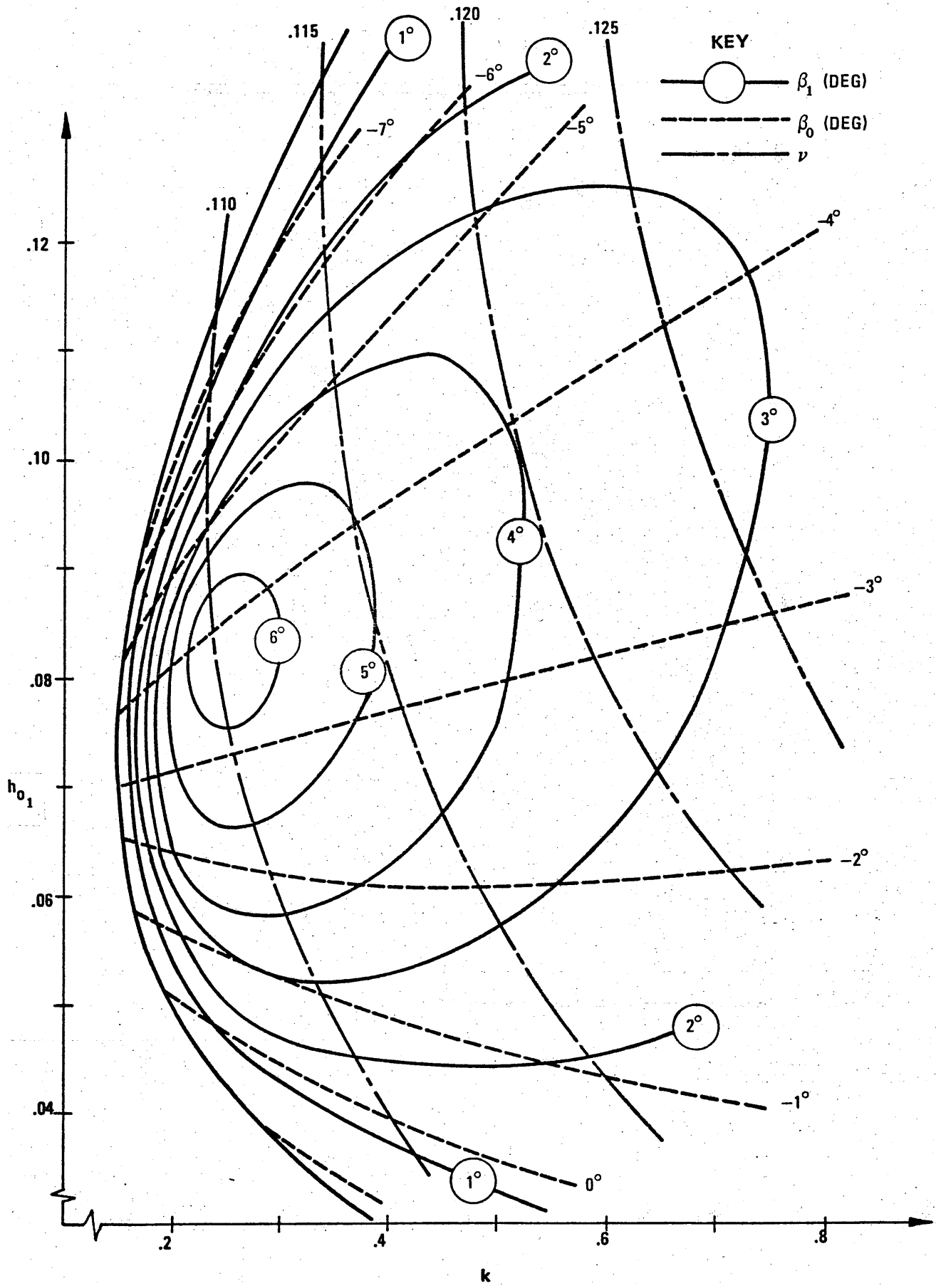


FIG. 3.7 LIMIT-CYCLE PARAMETERS (IDEAL COUPLING)

system equation of motion as an additional damping term of the form:

$$B' = k_s k^2 \frac{h_o}{\Delta h \omega} q_\infty b_f c_f^2$$

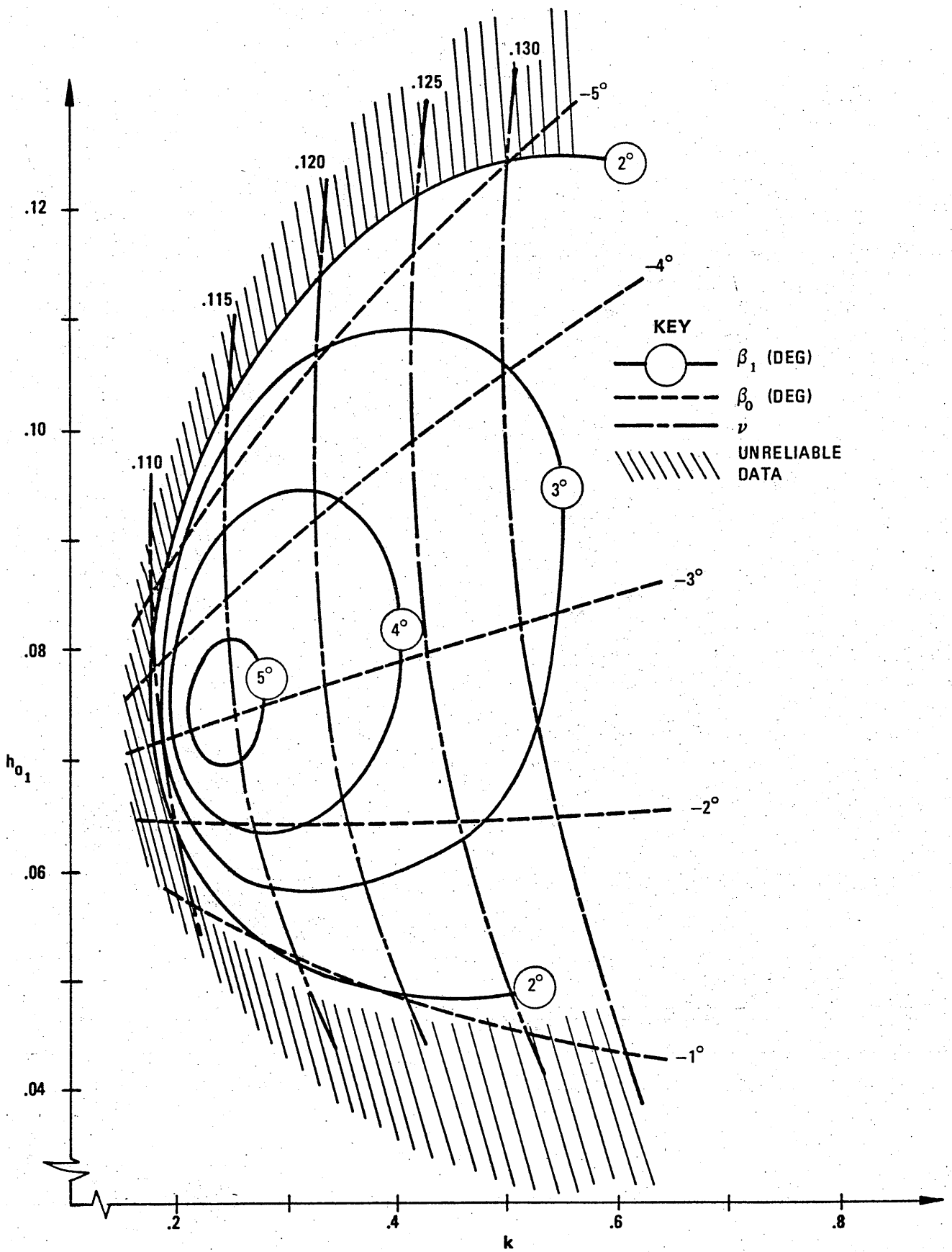
The dependence on $h_o q_\infty b_f$ occurs because the normal force on the spoiler face is a function of dynamic pressure and spoiler wetted area. The k^2 dependence represents the effect of gear ratio on moment and provides a best fit to the test data. The constant value is $k_s \cong .232$.

Thus, at the limit-cycle, equation (3.17) becomes:

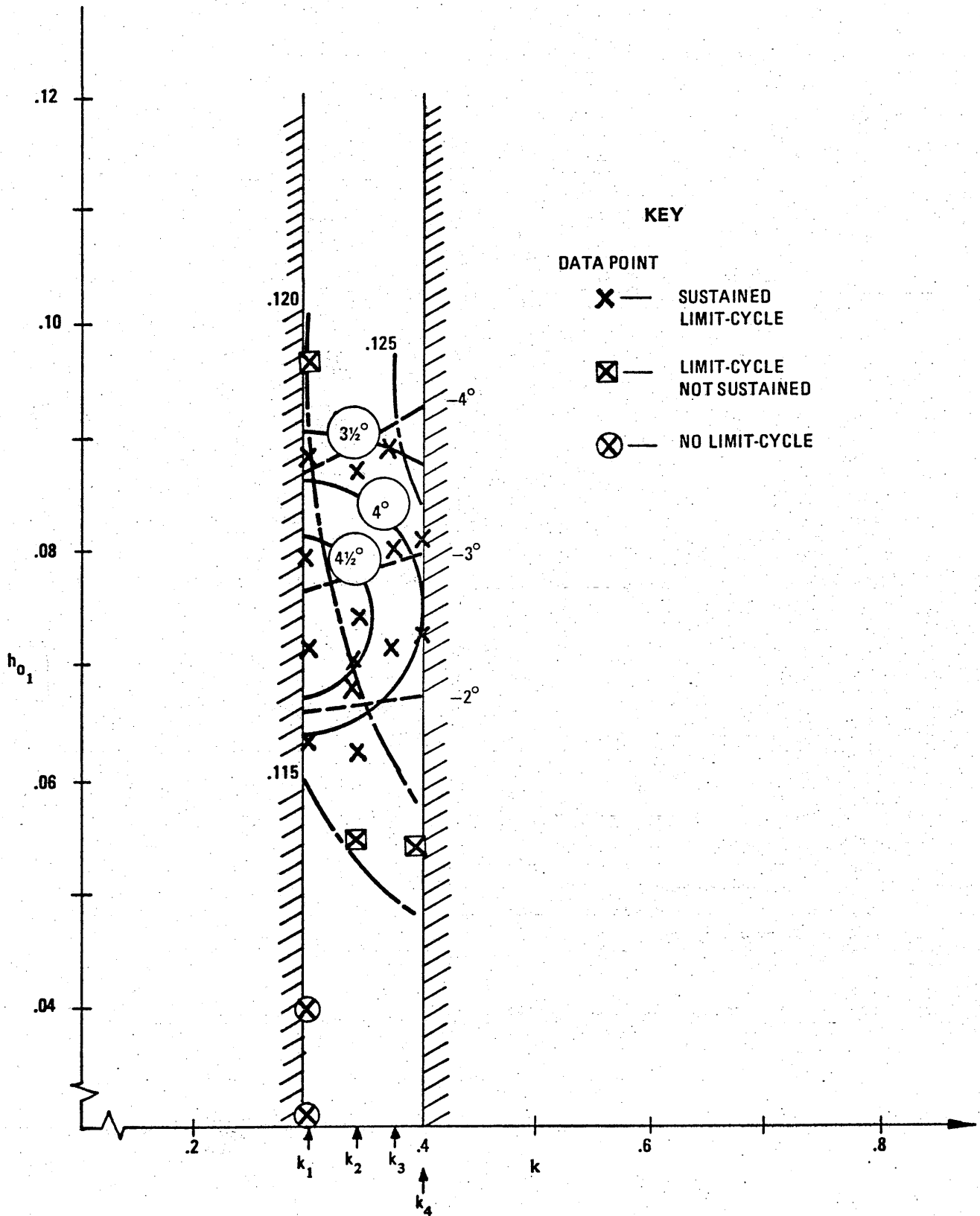
$$B = \left[\frac{c_f}{U_\infty} C_H \beta + k \frac{N_o}{\omega} - k_s k^2 \frac{h_o}{\Delta h \omega} \right] q_\infty b_f c_f^2$$

Once again a search algorithm was run and results are shown in Figure 3.8. In this case, however, computer data was considered unreliable in the region shown. Suitable convergence in the shaded region could not be achieved unless parameter increments were made small at the cost of large amounts of computer time. The presence of dry friction, which is inversely proportional to Δh , precludes self-excited oscillations as verified in the experiment; however, stable limit-cycle behavior is predicted as was observed. The effect of dry friction is not extremely significant except that lower amplitude limit-cycles are reached, with the maximum occurring near $h_{o1} = .073$ and $k = .25$. The contours of constant β_1 are shifted downward and to the left due to dependence of dry friction on h_o and k . The curves of constant β_o are not significantly altered; however, frequencies are uniformly increased due to the fact that the lower amplitudes, β_1 , relate to lower values of Δh at limit-cycle, hence to higher values of $|N_1|$ and frequency.

Finally, in Figure 3.9 data points taken from several coupled system tests are shown. Curves of constant frequency, β_o and β_1 are shown interpolated between the points. In all cases amplitude data was within 20% of the final results shown. Frequency results are within 10%. Only data taken for a free-stream speed of $U_\infty = 150$ ft/sec was used. Some of the tests at extreme values of h_{o1} did not demonstrate long-term sustained limit-cycle behavior. It is probable that three-dimensional effects could account for this. Although the range of parameter variations was small for the series of tests (due primarily to model limitations) the agreement between experiment and theory is surprisingly good. Frequencies, amplitudes and mean values of flap angle appear to be well predicted. Note that largest amplitudes occur near $\beta_o = -3^\circ$ which corresponds to $h_o \cong .06$. This is the region where the unsteady theory is expected to be most accurate.



**FIG. 3.8 LIMIT-CYCLE PARAMETERS
(ACTUAL COUPLING, INCLUDES DRY FRICTION)**



**FIG. 3.9 EXPERIMENTAL RESULTS
LIMIT-CYCLE PARAMETERS**

The concept of relative stability of the limit-cycle oscillations is worthy of mention. Consider the function λ ,

$$\lambda = f \{h_o, \Delta h, \nu\}$$

and its total derivative:

$$d\lambda = \frac{\partial \lambda}{\partial h_o} dh_o + \frac{\partial \lambda}{\partial \Delta h} d\Delta h + \frac{\partial \lambda}{\partial \nu} d\nu \quad (3.19)$$

The value of $\frac{d\lambda}{d\Delta h}$ can be taken as a measure of the relative stability of sustained nonlinear oscillations. Also we can evaluate sensitivities; e.g.,

$$\frac{\partial \lambda}{\partial h_o} ; \frac{\partial \lambda}{\partial \Delta h} \text{ and } \frac{\partial \lambda}{\partial \nu}$$

The relative stability can be investigated approximately by reference to equations (3.16) to (3.18) and Figures 3.3 and 3.5. From Figure 3.5 and equation (3.17) we have:

$$\frac{\partial \lambda}{\partial \Delta h} < 0 \quad \text{for} \quad .045 < h_o < .085$$

and

$$\frac{\partial \lambda}{\partial h_o} \text{ is } \begin{cases} > 0 \text{ for } h_o < .065 \\ < 0 \text{ for } h_o > .065 \end{cases}$$

also,

$$\frac{\partial \lambda}{\partial \nu} > 0$$

We also find from Figure 3.3, equation (3.16) and the relation $h_o = h_{o1} + k \beta_o$:

$$\frac{\partial h_o}{\partial \Delta h} \text{ is } \begin{cases} < 0 \text{ for } h_o < .065 \\ > 0 \text{ for } h_o > .065 \end{cases}$$

and from equation (3.18) and Figure 3.5

$$\frac{\partial \nu}{\partial \Delta h} < 0 \quad (\text{approximately})$$

Equation (3.19) is approximately:

$$\frac{d\lambda}{d\Delta h} \cong \left[\frac{\partial \lambda}{\partial h_0} \frac{\partial h_0}{\partial \Delta h} + \frac{\partial \lambda}{\partial \Delta h} + \frac{\partial \lambda}{\partial \nu} \frac{\partial \nu}{\partial \Delta h} \right] < 0$$

for $.045 < h_0 < .085$. Thus, relative stability of the limit-cycle oscillations in Figures 3.7 and 3.8 is predicted. A more rigorous demonstration of stability will not be attempted here.

In conclusion, it has been shown via small perturbation analysis, that significant unsteady effects alter the steady-flow bubble growth relations. Of course, a complete solution of the problem must involve a matching of the bubble dynamics to those of the external, inviscid flow field. The significant dynamic effects, however, are shown to primarily involve the viscous bubble region. The unsteady mass flux and continuity relation applied to the bubble and the momentum relations that are assumed to prevail near reattachment yield a dynamic behavior that has the following characteristics:

1. Bubble length lags spoiler motion due to mass flux which is convected into the bubble near reattachment and which is assumed to travel upstream at a mean backflow velocity.
2. The pressure rise to reattachment, hence pressure in the dead-air region, is altered in the unsteady case in order to satisfy momentum relations.
3. The combined effect of 1 and 2 above results in dynamic bubble behavior that is in good qualitative and fair quantitative agreement with experimental observations.
4. Negative damping, for small amplitudes of oscillation, and subsequent limit-cycle behavior is predicted for the coupled system. This predicted behavior is confirmed by experimental observations.

The unsteady bubble model is expected to be valid when reattachment occurs on the flap surface. For larger bubbles, the model must be modified in order to account for mass flux from the shear layer on the flap lower surface. Further experimental and analytical work is needed in order to properly model large bubble dynamics. Also, the unsteady energy relation within the bubble needs to be better understood. It is probable that additional integral effects exist and they are expected to be highly dependent on the frequency of spoiler motion. Also, three-dimensional effects are not included. Theisen (Ref 19) is one among many who recognize that spanwise flow in the bubble can alter the dynamics of the separated region. The present model, however, contains two important dynamic effects which have not previously been modeled. This is thought to be the

most significant analytical result of the investigation, although some of the important assumptions must be regarded as somewhat speculative at present.

It will be shown that these methods may be applied to other similar problems in order to obtain useful results.

4. FURTHER ANALYSIS OF RELATED PROBLEMS

In the study of interactions between an oscillating body and a separating flow it is not always possible to consider the separating region as in the previous section. For a type of flow perturbation such as a spoiler, it is expected that in some important cases the perturbation may be considered overwhelming and consequent analysis may neglect, as a first approximation, the influence of the external flow field. In the case of a laminar separation bubble at the leading-edge of an airfoil, for example, it has been demonstrated (Ref. 44 and 45) that the external inviscid flow field may dominate the behavior of the separated region. It will be shown, however, that results of the previous section are useful.

4.1. Initiation of Dynamic Stall

As an airfoil pitches upward through its static stall angle it is known that an "overshoot" of this stall angle usually occurs. At a higher incidence "dynamic stall" is initiated with resulting forces and moments that differ in character and intensity from the steady flow stall case.^{46,47} Similarly, on the downstroke, reattachment may be delayed; or in other words, the static reattachment angle is "undershot." Even if hysteresis in separation and reattachment did not exist in steady flow, the overshoot and undershoot phenomena result in apparent hysteresis in all cases of unsteady flow.

For an airfoil that is free to oscillate in pitch, the loading due to dynamic stall can lead to a self-excited oscillation that builds in amplitude until a limit-cycle is reached. Such a motion is termed "stall flutter" and is known to be applicable to helicopter rotor blades, engine compressor blades, space shuttle (and lifting body) configurations, certain V/STOL aircraft types, and to the study of aircraft response to severe gusts.

Motion of a control-surface can also initiate dynamic stall.^{2,48} Furthermore, "stall flutter" of a flap-type control-surface has been observed on an airfoil at incidence.⁴⁹ This type of instability involved the single-degree-of-freedom of rotation of the flap about the hinge-line.

This section is concerned with the prediction of the overshoot and undershoot angles of dynamic stall and it deals with the type of dynamic stall that involves a leading-edge separation bubble. Pertinent elements of the flow field are modeled so that their interactions in unsteady flow can be studied. It is found that the resulting model for flow over an airfoil is useful for predicting when the leading-edge bubble bursts (hence initiating dynamic stall) and for studying the conditions necessary for flow to reattach. The following development is, in part, a summary of material contained in reference 37.

4.1.1. Flow Elements

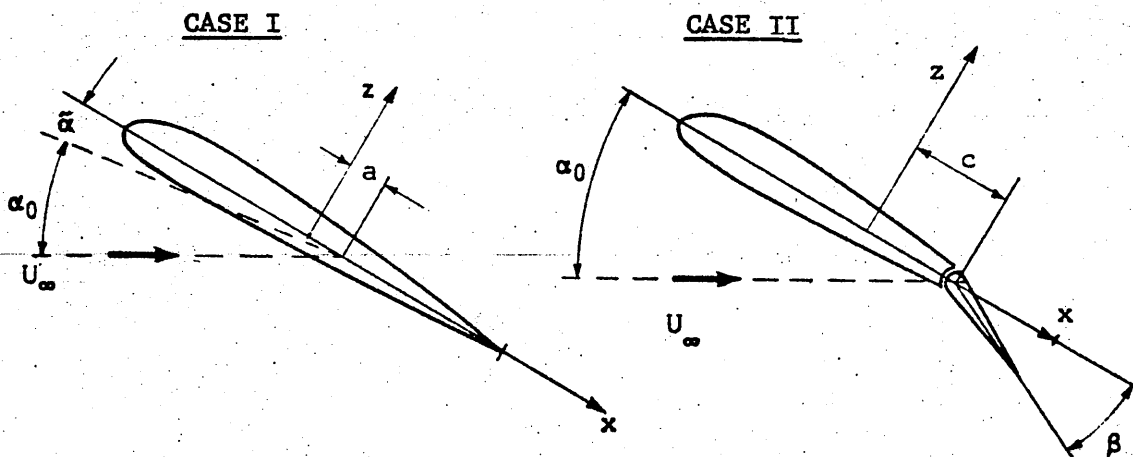
When flow is attached to an airfoil that is oscillating about a moderate value of mean incidence, or for an airfoil at moderate incidence with an oscillating flap, the flow elements are taken to be:⁶

1. A potential flow field that acts over the surface of airfoil with mean properties that are related to the airfoil at its mean incidence.
2. The effects of airfoil or flap motion on the potential flow field to include the effect of vorticity which is shed into the wake.
3. A laminar boundary layer from the leading-edge stagnation point to the point of laminar separation on the upper surface.
4. A constant pressure laminar shear layer to the transition point.
5. The turbulent mixing and reattachment region.
6. The turbulent shear layer from the reattachment point to the trailing edge.

These elements are associated with a leading-edge separation bubble, and are only considered during the portion(s) of a cycle of motion when a bubble exists. The laminar separation and turbulent reattachment points define the fore and aft limits of the bubble which is shown schematically in Figure 4.1.

The effect of the turbulent shear layer aft of reattachment will be neglected in this section, thus precluding the influence of trailing-edge separation.

Airfoil or flap motion is prescribed. We consider two cases of forced motion, with reference to the diagram below.



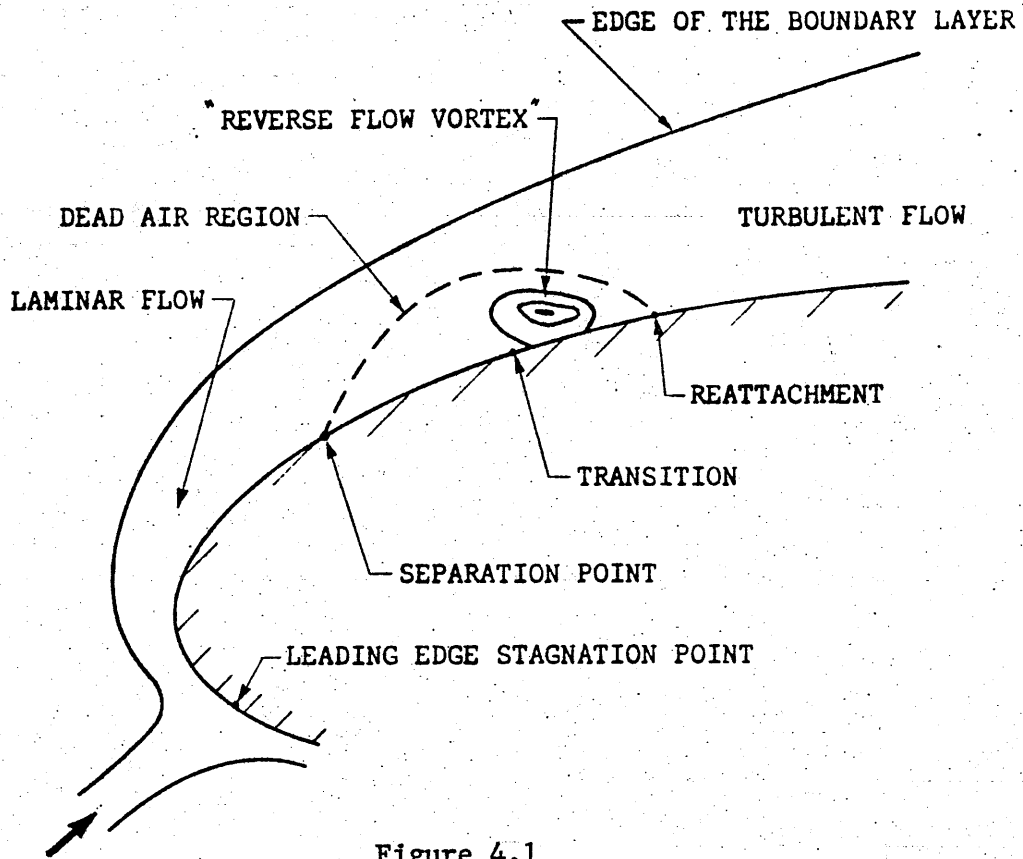


Figure 4.1
Leading-Edge Bubble Definitions

CASE I. The airfoil, at a mean incidence α_0 , is forced to pitch about an axis $x = a$, with amplitude $\Delta\alpha$ such that

$$\alpha(t) = \alpha_0 + \tilde{\alpha} = \alpha_0 + \Delta\alpha \cos \omega t$$

where α_0 , $\Delta\alpha$ and ω are constant prescribed values.

CASE II. The airfoil is fixed at an incidence α_0 , and the flap is forced to oscillate with amplitude $\Delta\beta$. The flap is a plain flap with hinge-line at $x = c$.

$$\alpha = \alpha_0$$

$$\beta(t) = \Delta\beta \cos \omega t$$

We now define non-dimensional time and reduced frequency parameters.

CASE I

$$\tau_a \equiv \frac{U_\infty t}{b} \quad \text{and} \quad \nu_a \equiv \frac{\omega b}{U_\infty}$$

also

$$\alpha' \equiv \frac{d\alpha}{d\tau_a} = \left(\frac{b}{U_\infty}\right) \dot{\alpha} \quad \text{and} \quad \alpha'' \equiv \left(\frac{b}{U_\infty}\right)^2 \ddot{\alpha}$$

where b is airfoil semi-chord.

CASE II

$$\tau \equiv \frac{U_\infty t}{c_f} \quad \text{and} \quad \nu \equiv \frac{\omega c_f}{U_\infty}$$

so that

$$\beta' \equiv \frac{d\beta}{d\tau} = \left(\frac{c_f}{U_\infty}\right) \frac{d\beta}{dt}, \text{ etc.}$$

also we have

$$\frac{c_f}{b} = (1 - c)$$

so that

$$\nu = (1 - c) \nu_a$$

Also, we have a non-dimensional chordwise distance x based on b with origin at mid-chord:

$$x \equiv \frac{X}{b} - 1$$

Pressures and velocities at the edge of the boundary and shear layers are found by considering an appropriate potential flow model. The model used here involves a non-linear pressure-velocity relation and considers the instantaneous value of velocity to be a superposition of a mean, steady flow value and a time varying perturbation. McCroskey (Ref. 45) recently and independently considered a similar inviscid flow field.

We consider the non-linear Bernoulli equation for unsteady flow.

$$C_p \equiv \frac{p - p_\infty}{\frac{1}{2} \rho U_\infty^2} = 1 - \left(\frac{U}{U_\infty}\right)^2 - \frac{2}{U_\infty^2} \frac{\partial \phi}{\partial t} \quad (4.1)$$

with U , the velocity evaluated at the airfoil surface from the potential flow solution, assumed equal to the velocity at the edge of the boundary layer in attached flow.

Now

$$\frac{U}{U_{\infty}} = \frac{U_0}{U_{\infty}} + \frac{\tilde{u}}{U_{\infty}} ; \quad \phi = \phi_0 + \tilde{\phi} \quad (4.2)$$

and

$$\frac{\partial \phi}{\partial t} = \frac{\partial \tilde{\phi}}{\partial t}$$

U_0/U_{∞} will be found from the NACA method and $\tilde{u}/U_{\infty}$ will be found from "classical" unsteady thin airfoil theory.

The unsteady thin airfoil results are found to contain the usual singular terms with corresponding infinite velocities at the leading edge and flap hinge-line. Since the leading edge region is critical in this analysis we chose to correct the unsteady results by employing Lighthill's correction factor, ³¹ which is applicable to the thin airfoil approximation. The factor makes the first order thin airfoil solution valid over the entire airfoil surface.

Thus, in equation (4.2) where U_0/U_{∞} is known from the NACA method, we find $\tilde{u}/U_{\infty}$ from:

$$\frac{\tilde{u}}{U_{\infty}}(x,t) = \left(\frac{1+x}{1+x+\rho_N} \right)^{\frac{1}{2}} \left(\frac{\tilde{u}}{U_{\infty}} \right)_{th} \quad (4.3)$$

where

$$\left(\frac{\tilde{u}}{U_{\infty}} \right)_{th}$$

is the thin airfoil result and the correction factor;

$$\left(\frac{1+x}{1+x+\rho_N} \right)^{\frac{1}{2}}$$

due to Lighthill includes the parameter ρ_N which is the non-dimensional leading-edge radius based on chord length, i.e.,

$$\rho_N = .0158$$

for a NACA 0012 profile.

$(\frac{\tilde{u}}{u_\infty})_{th}$ is found, as explained in reference 37, from the relation:

$$(\frac{\tilde{u}}{u_\infty})_{th} = -\frac{1}{2} (\tilde{C}_{pu} + \frac{2}{u_\infty^2} \frac{\partial \tilde{\phi}}{\partial t}) \quad (4.4)$$

where

$$\tilde{C}_{pu} = \begin{cases} C_{p\alpha} \tilde{\alpha} + C_{p\dot{\alpha}} \tilde{\alpha}' + C_{p\ddot{\alpha}} \tilde{\alpha}'' + C_{p\Gamma(\alpha)} & \text{CASE I} \\ C_{p\beta} \beta + C_{p\dot{\beta}} \beta' + C_{p\ddot{\beta}} \beta'' + C_{p\Gamma(\beta)} & \text{CASE II} \end{cases}$$

The non-dimensional derivatives are for Case I:

$$C_{p\alpha} = 0$$

$$C_{p\dot{\alpha}} = - \left[(1 + 2x) \sqrt{\frac{1-x}{1+x}} + 2 \sqrt{1-x^2} \right]$$

$$C_{p\ddot{\alpha}} = (2a - x) \sqrt{1-x^2}$$

and for Case II:

$$C_{p\beta} = \frac{2}{\pi} \left[\ln|N| + \sqrt{1-c^2} \sqrt{\frac{1-x}{1+x}} \right]$$

$$C_{p\dot{\beta}} = \frac{-2}{\pi (1-c)} \left\{ \cos^{-1} c \sqrt{1-x^2} - 2 (x-c) \ln|N| \right. \\ \left. + \frac{1}{2} \left[\cos^{-1} c (1+2x) + c \sqrt{1-c^2} \right] \sqrt{\frac{1-x}{1+x}} \right\}$$

$$C_{p\ddot{\beta}} = \frac{-1}{\pi (1-c)^2} \left\{ [\sqrt{1-c^2} + (x-2c) \cos^{-1} c] \sqrt{1-x^2} \right. \\ \left. - (x-c)^2 \ln|N| \right\}$$

where

$$N = \frac{(1 - cx - \sqrt{1-c^2} \sqrt{1-x^2})}{(x-c)}$$

The above derivatives are valid for attached flow and general motion, but with the Kutta condition satisfied.

The perturbation quantities $C_{p\Gamma(\alpha)}$ and $C_{p\Gamma(\beta)}$ are sensitive to the distribution of shed vorticity in the wake, and we have the approximations:

$$C_{p\Gamma(\alpha)} = -\frac{2}{U_\infty} C \cdot Q(\alpha) \quad \text{and} \quad C_{p\Gamma(\beta)} = -\frac{2}{U_\infty} C \cdot Q(\beta)$$

where

$C \equiv C(\nu_a)$; the Theodorsen Function

and Q is found from the Kutta condition to be:

$$Q \equiv \frac{1}{2\pi} \int_1^\infty \frac{x_o + 1}{\sqrt{x_o^2 - 1}} \gamma(x_o, t) dx_o$$

$$= \begin{cases} U_\infty [\tilde{\alpha} + (\frac{1}{2} - a) \tilde{\alpha}'] & \text{CASE I} \\ \frac{U_\infty}{\pi} [T_{10}\beta + \frac{T_{11}}{2(1-c)} \beta'] & \text{CASE II} \end{cases}$$

where

$$T_{10} = \sqrt{1 - c^2} + \cos^{-1} c$$

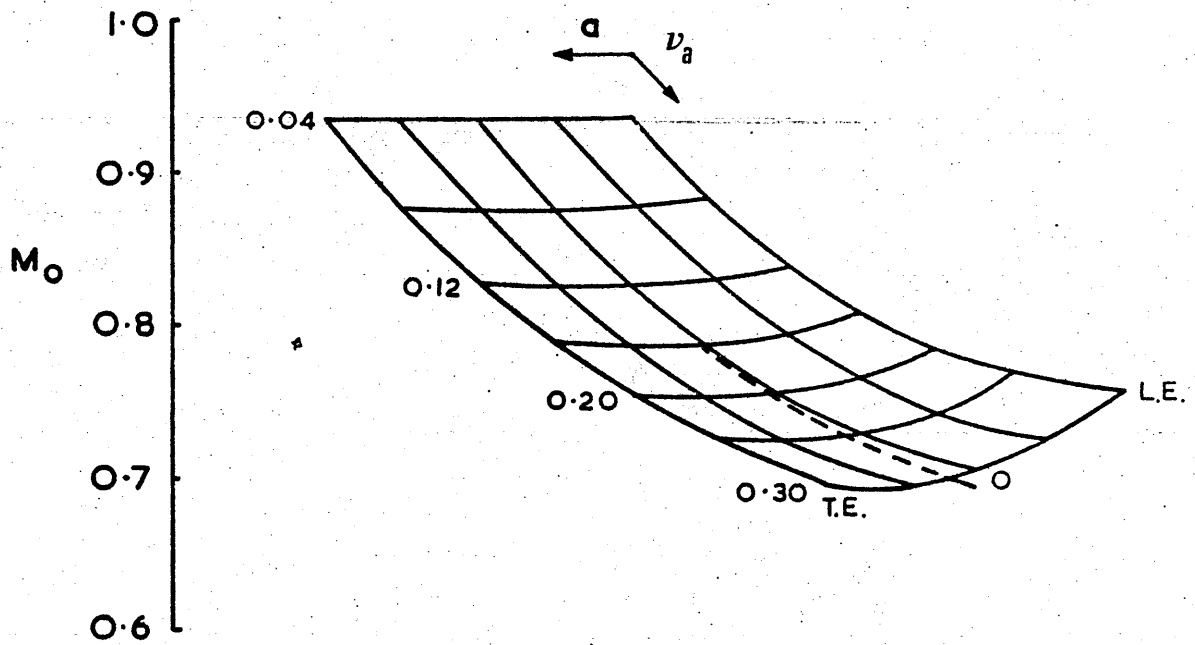
$$T_{11} = \cos^{-1} c (1 - 2c) + \sqrt{1 - c^2} (2 - c)$$

It can be seen that dynamic effects modify the magnitude and phase relation of pressures due to circulation. The relation:

$$M = M_o e^{-i\psi} \equiv \frac{C_{p\Gamma \text{ unsteady}}}{C_{p\Gamma \text{ steady}}}$$

is plotted in Figure 4.2 and shows the dependence of circulation effects on frequency and pitch-axis location. A combination of high frequency and aft location of the pitch axis contributes to reduced circulation effects that lag airfoil or flap motion. At low frequencies, pitch axis location is of lesser importance. This "circulation lag" effect due to shed vorticity has been referred to by Ericsson and Reding⁵⁰ as a significant factor in stall overshoot. Flap effects are given by the dashed curves.

Figures 4.3 and 4.4 indicate the chordwise distribution of the components of pressure coefficient. Note that the "apparent



$$M_0 e^{-i\psi} = \frac{C_{p\Gamma \text{ UNSTEADY}}}{C_{p\Gamma \text{ STEADY}}}$$

— AIRFOIL MOTION
 --- FLAP MOTION

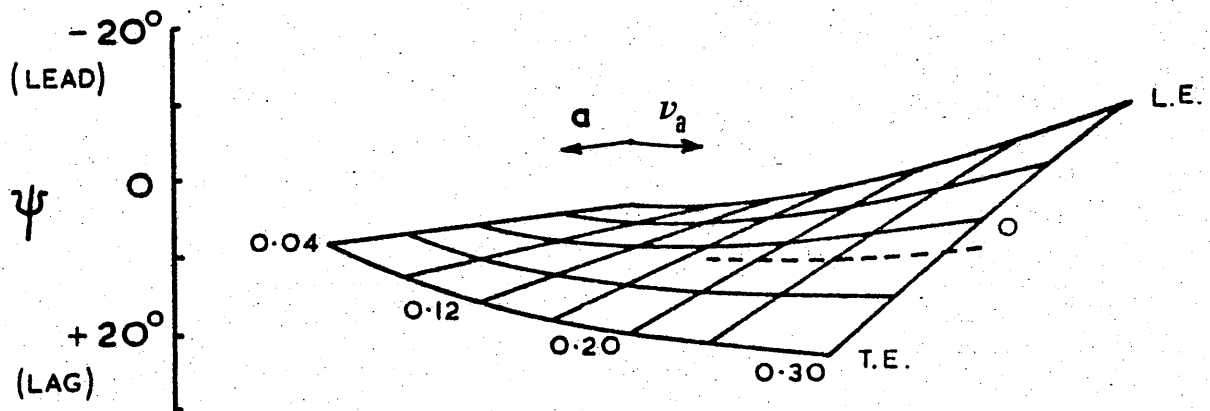


FIG. 4.2 EFFECT OF PITCH AXIS AND FREQUENCY ON CIRCULATION.

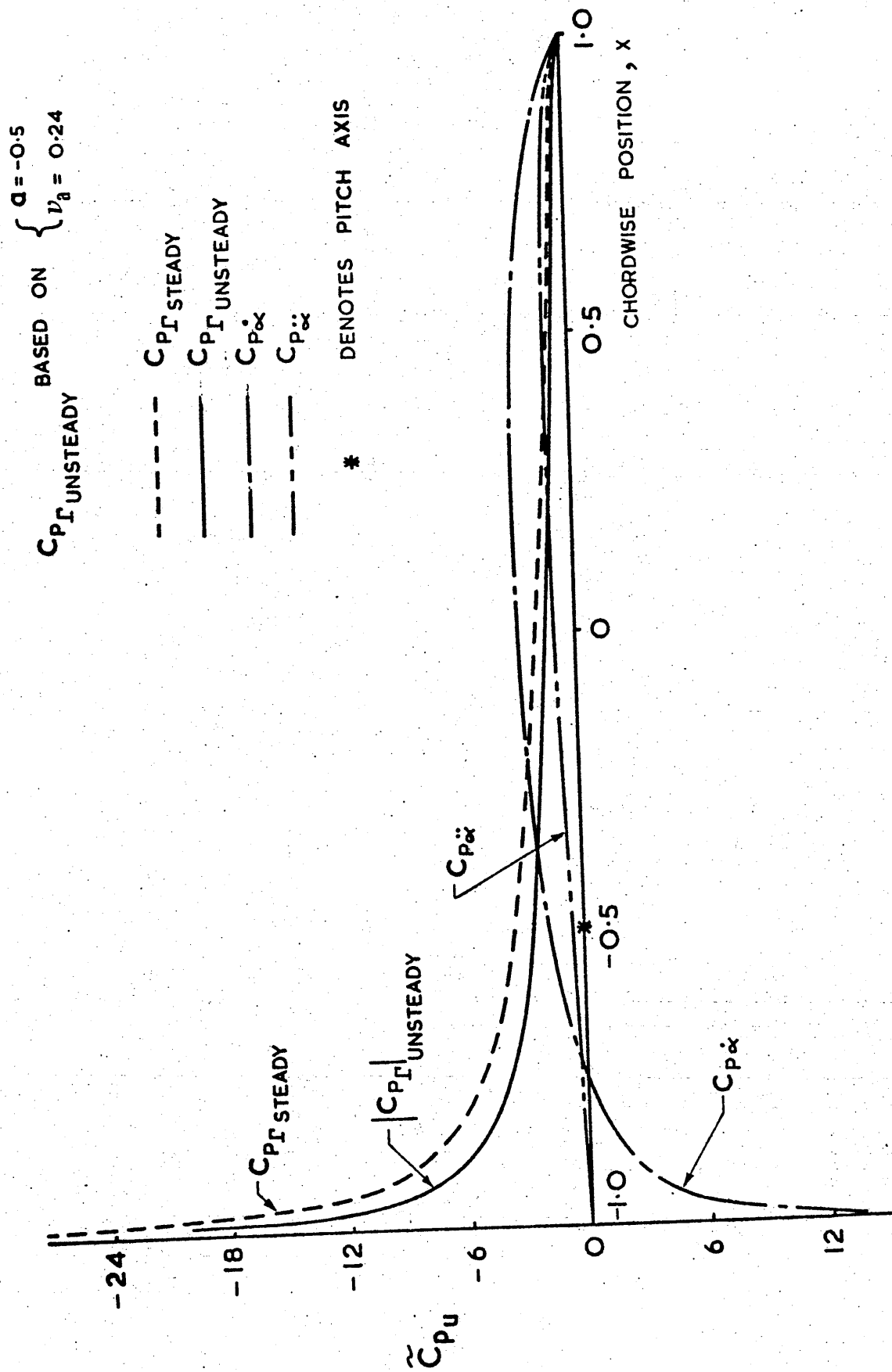


FIG. 4.3 COMPONENT PARTS OF $\tilde{C}_{pu}$ DUE TO AIRFOIL MOTION.

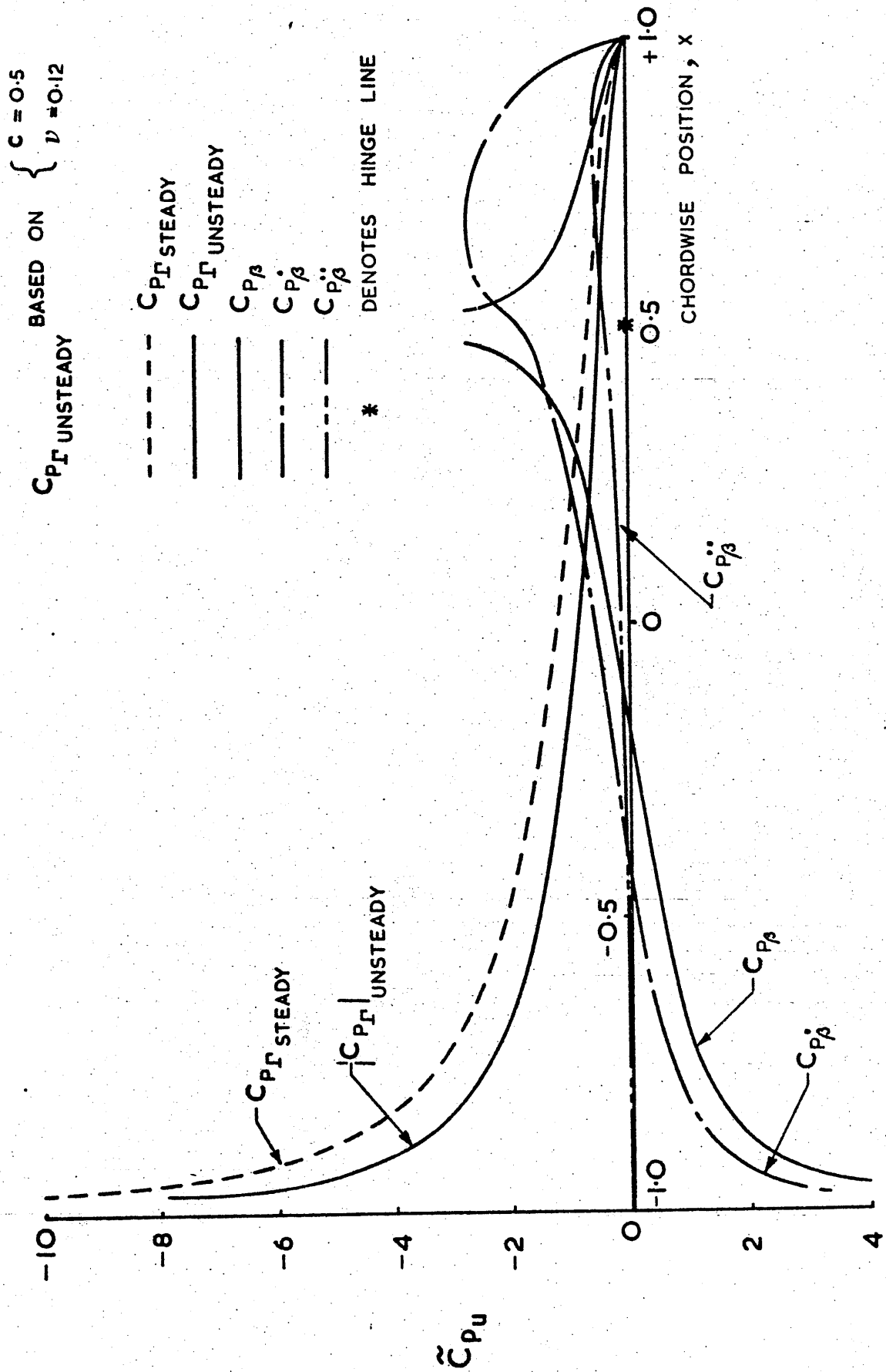


FIG. 4.4 COMPONENT PARTS OF $\tilde{C}_{p_u}$ DUE TO FLAP MOTION.

mass" terms $C_{p\ddot{\alpha}}$ and $C_{p\ddot{\beta}}$ are small. Their overall contributions ($C_{p\ddot{\alpha}} \ddot{\alpha}'$ and $C_{p\ddot{\beta}} \ddot{\beta}''$) are usually small except for high frequencies. The $C_{p\ddot{\alpha}}$ term is highly dependent upon pitch axis location. The damping terms $C_{p\dot{\alpha}}$ and $C_{p\dot{\beta}}$ are definitely significant, especially near the leading edge. The curves of Figures 4.3 and 4.4 do not include the influence of the correction factor due to Lighthill (equation 4.3) which is significant near the leading edge. The correction factor, however, would not cause a noticeable modification to the curves due to the scale of the x axis. The circulation dependent components, $C_{p\Gamma}$, in the figures are based on unit values of $\tilde{\alpha}$ and β , and are dependent on frequency and pitch axis location.

The combined effect of pressure components is shown in Figures 4.5 and 4.6. In Figure 4.5 we see a comparison between steady flow and unsteady flow perturbations. The motion involved is represented by:

$$\tilde{\alpha} = 5^\circ \cos (0.16 \tau_a)$$

and the five pictures represent the instantaneous pressure distributions at time intervals that are equally spaced on the airfoil upstroke (i.e., $\dot{\alpha} > 0$). This type of representation (again excluding Lighthill's leading-edge correction) has been offered by Carta⁴⁴ as indicative of the important role unsteady flow effects play in dynamic stall. This additional, unsteady distribution involves favorable pressure gradients along the entire airfoil chord as the airfoil pitches upward past $\tilde{\alpha} = 0$. This effect helps not only in a delay of bubble bursting, but in a "suppression" of trailing-edge stall as well. See reference 37 for more details on this.

Figure 4.6 involves motion of the form:

$$\beta = 5.73^\circ \cos (0.12 \tau)$$

and indicates trends, near the leading-edge especially, that are qualitatively similar to those for airfoil pitching.

Let us now solve for the unsteady velocity perturbations. We may represent the last term in equation (4.4) as:

$$\frac{2}{U_\infty^2} \frac{\partial \tilde{\phi}}{\partial t} = \frac{2}{U_\infty^2} \left[\frac{\partial \tilde{\phi}_\epsilon}{\partial t} + \frac{\partial \tilde{\phi}_\Gamma}{\partial t} \right]$$

where:

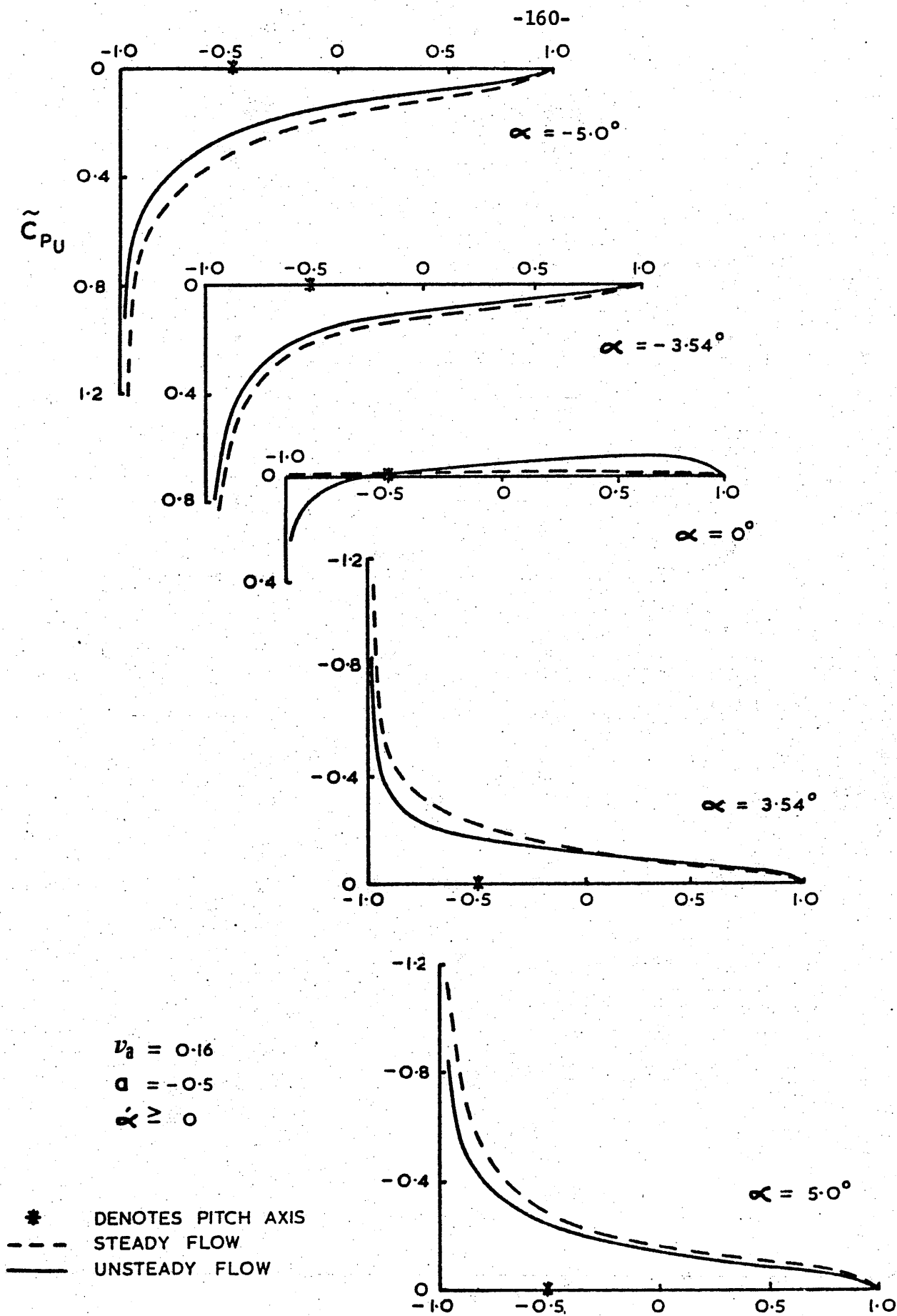


FIG. 4.5 INSTANTANEOUS PRESSURE DISTRIBUTION DUE TO AIRFOIL MOTION.

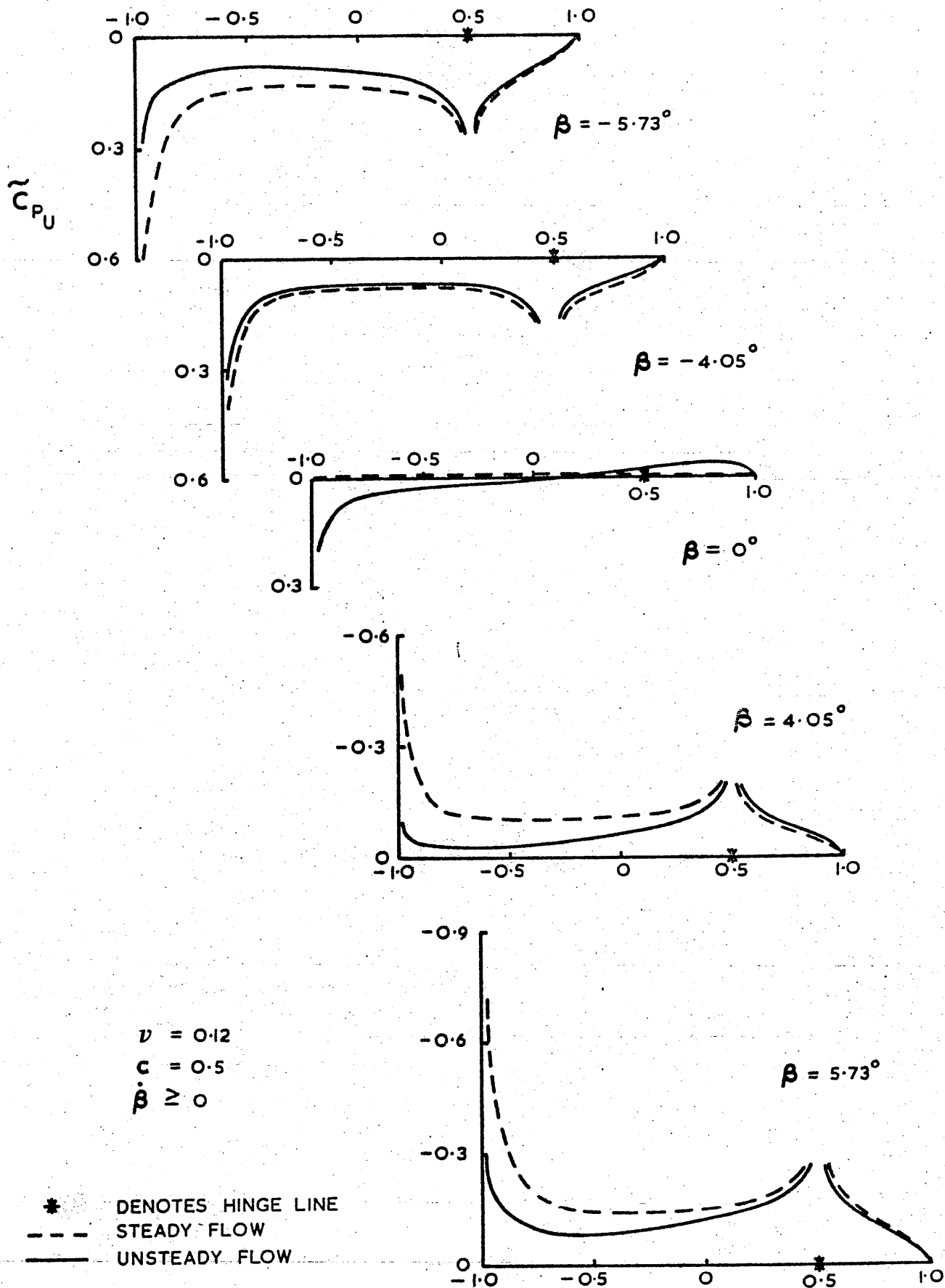


FIG. 4.6 INSTANTANEOUS PRESSURE DISTRIBUTION DUE TO FLAP MOTION.

$$\frac{2}{U_{\infty}} \frac{\partial \tilde{\phi}_{\epsilon}}{\partial t} = \begin{cases} 2 \sqrt{1-x^2} \tilde{\alpha}' + (x-2a) \sqrt{1-x^2} \tilde{\alpha}' & \text{CASE I} \\ \frac{2}{\pi (1-c)} [\cos^{-1} c \sqrt{1-x^2} - (x-c) \ln|N|] \beta' \\ + \frac{1}{\pi (1-c)^2} [\sqrt{1-c^2} \sqrt{1-x^2} + \cos^{-1} c \cdot \\ (x-2c) \sqrt{1-x^2} - (x-c)^2 \ln|N|] \beta'' & \text{CASE II} \end{cases}$$

and

$$\frac{2}{U_{\infty}^2} \frac{\partial \tilde{\phi}_{\Gamma}}{\partial t} = - \frac{4}{\pi U_{\infty}} \sqrt{1-x^2} \frac{I(\nu_a, x)}{[H_1^{(2)}(\nu_a) + i H_0^{(2)}(\nu_a)]} \cdot Q$$

where

$$I(\nu_a, x) \equiv \int_1^{\infty} \frac{e^{-i\nu_a x_0} dx_0}{\sqrt{x_0^2 - 1} (x_0 - x)}$$

and I is evaluated numerically.

In Figures 4.7 and 4.8 we see the difference in the unsteady quantities:

$$\tilde{C}_{p_u} \text{ and } -2 \frac{\tilde{u}}{U_{\infty}}$$

It is apparent that for either airfoil or flap motion, these two quantities differ in magnitude and phase by very small amounts near the leading edge. The largest magnitude discrepancy occurs near mid-chord and the largest phase difference occurs at the trailing-edge. In all cases, as one would expect, the velocity perturbations - $\tilde{u}/U_{\infty}$ lag the pressure perturbations $\tilde{C}_{p_u}$. These magnitude and phase relations are very much frequency and pitch axis location dependent, and the curves of Figures 4.7 and 4.8 must be treated as such.

The laminar boundary layer, separation point and shear layer are approximately accounted for by the quasi-steady assumption that they react instantaneously to changes in the outer flow. This assumption has been justified for example in references 37 and 45. Figure 4.9 will serve to define some parameters in the assumed bubble model.

The laminar separation point is found by use of an empirical result found by Owen and Klanfer, Gault and Györgyfalvy,⁵ that is,

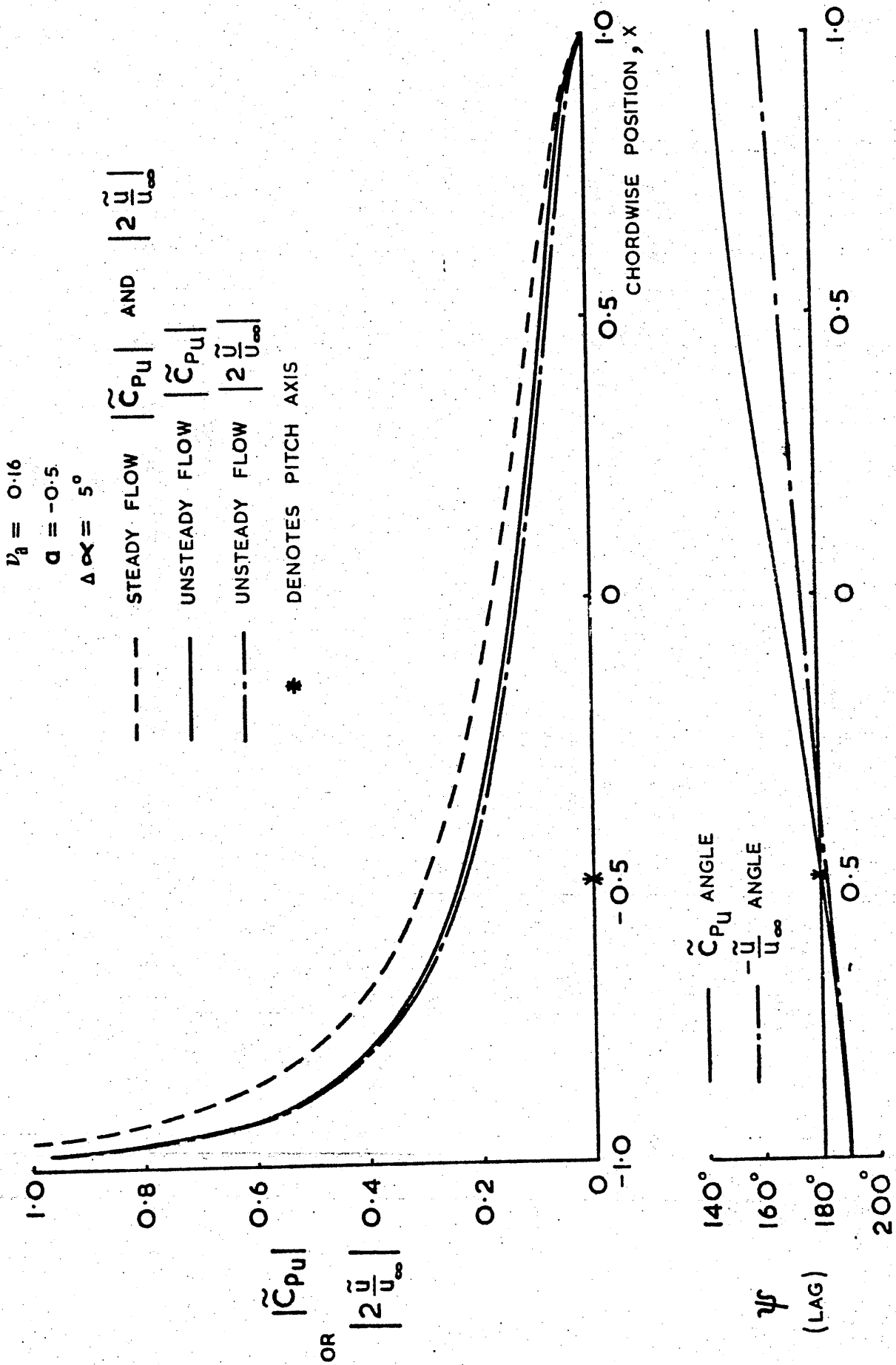


FIG. 4.7 MAGNITUDE AND PHASE OF $\tilde{C}_{pu}$ AND $\frac{\tilde{u}}{u_\infty}$ FOR AIRFOIL MOTION.

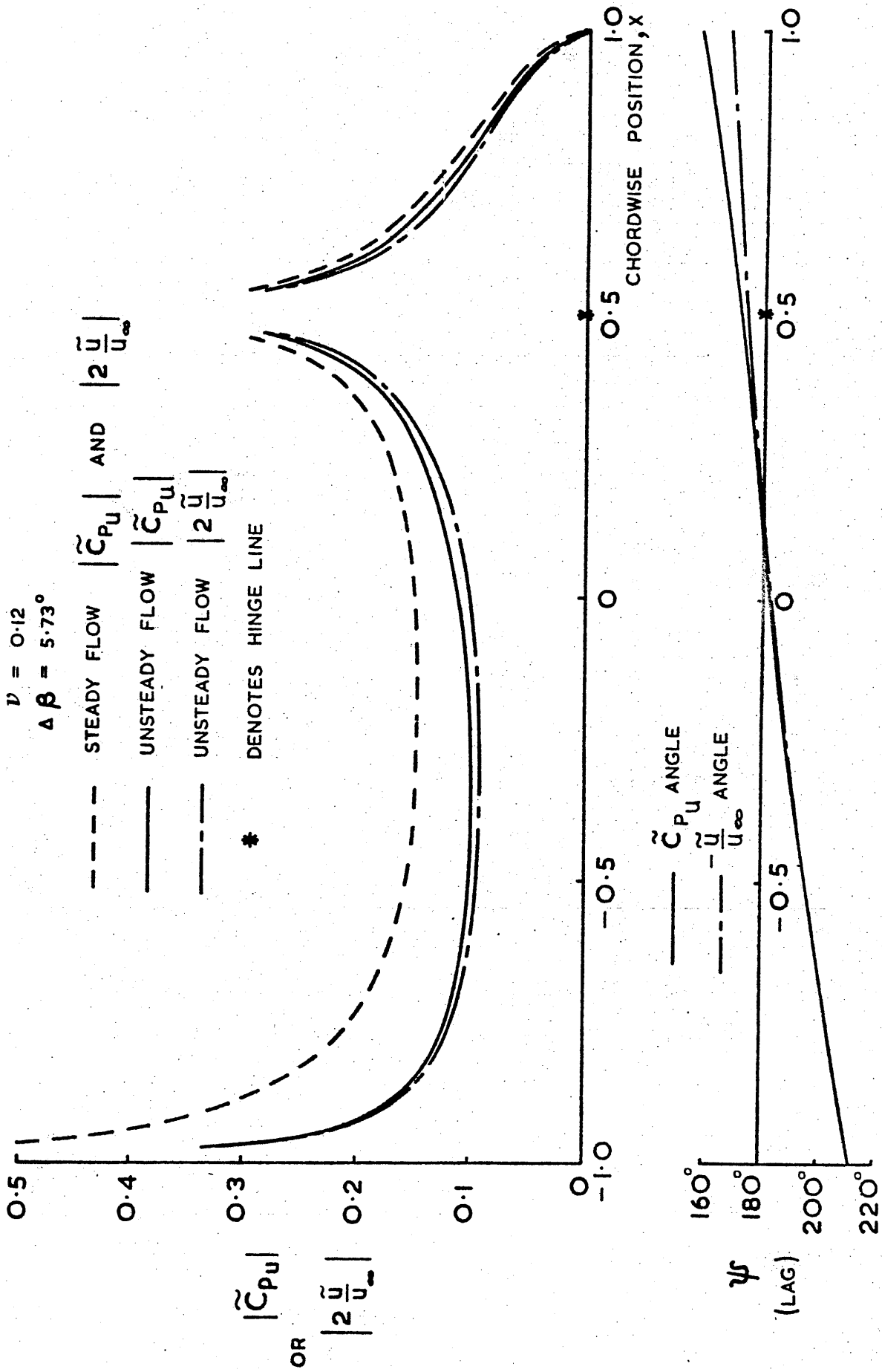


FIG.4.8 MAGNITUDE AND PHASE OF $\tilde{C}_{P_u}$ AND $\frac{\tilde{u}}{u_\infty}$ FOR FLAP MOTION.

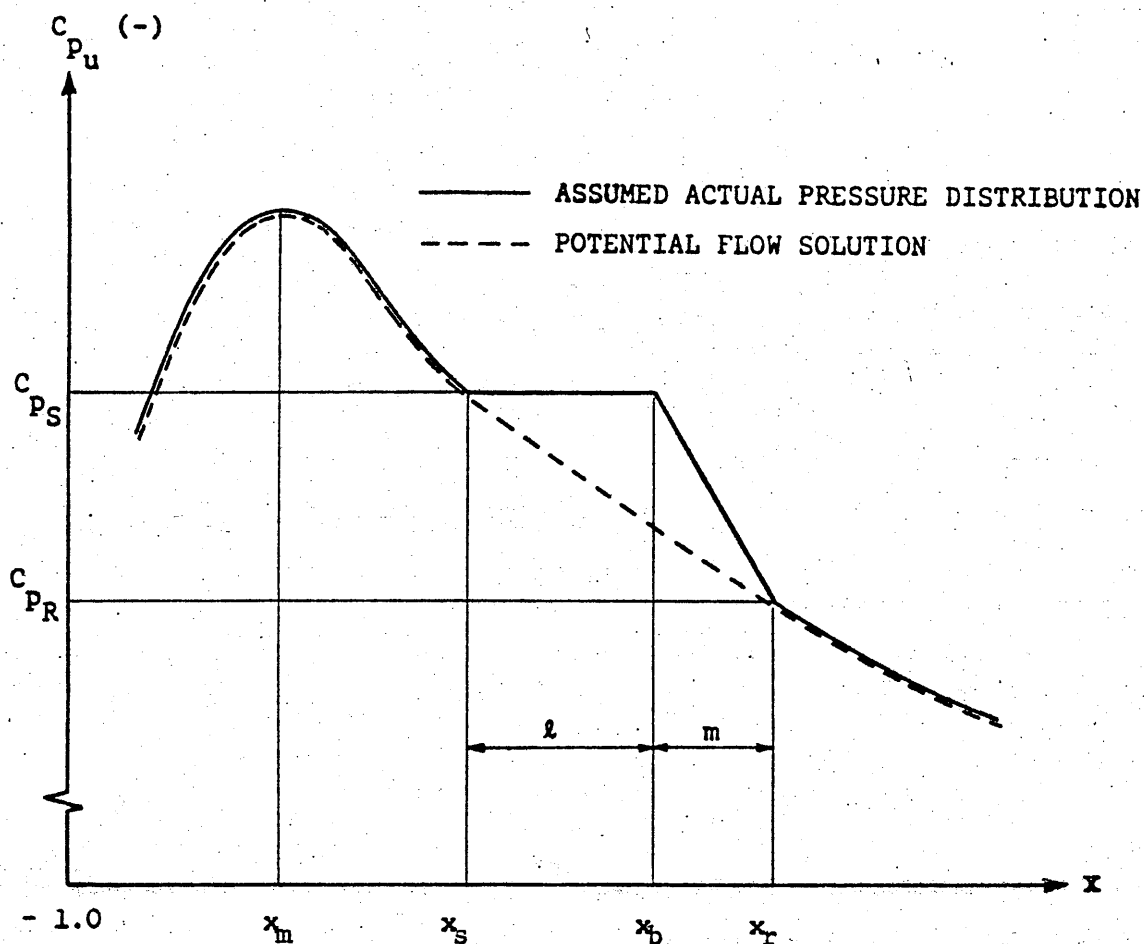


Figure 4.9
Leading-Edge Bubble Parameters

$$\frac{U_s}{U_\infty} \approx .95 \frac{U_m}{U_\infty}$$

where U_s/U_∞ and U_m/U_∞ are velocity ratios at separation and the point of maximum velocity.

If we consider the mean steady flow then from the NACA method:

$$\begin{aligned} \frac{U_o}{U_\infty} &= \frac{U_1}{U_\infty} + \frac{U_2}{U_\infty} C_{L\alpha} \cdot \alpha_o \\ &= \frac{U_1}{U_\infty} (x) + \frac{U_2}{U_\infty} (x) C_{L\alpha} \cdot \alpha_o \end{aligned}$$

and we find

$$\frac{U_m}{U_\infty} = \left(\frac{U_o}{U_\infty}\right)_{\max}$$

from:

$$\frac{\partial \left(\frac{U_1}{U_\infty}\right)}{\partial x} + \frac{\partial \left(\frac{U_2}{U_\infty}\right)}{\partial x} C_{L\alpha} \cdot \alpha_o = 0$$

which yields the magnitude and location of

$$\left(\frac{U_o}{U_\infty}\right)_{\max}$$

as a function of α_o .

From the above relations we find the predicted separation point.

Velocity perturbations are accounted for by considering the following:

$$x_s = x_{s0} + \Delta x_s = x_{s0} + \frac{\partial x_s}{\partial \left(\frac{U}{U_\infty}\right)} \cdot \frac{\tilde{u}}{U_\infty}$$

where x_{s0} is the predicted steady flow value of x_s , the laminar separation point. Although

$$\left| \frac{\Delta x_s}{x_{s0}} \right| \ll 1$$

this perturbation quantity is found to yield a significant effect.

The quantity $\partial x_s / \partial (U/U_\infty)$ is evaluated for $x_s = x_{s0}$; $\tilde{u}/U_\infty = 0$, as discussed in reference 37.

The relation which is used to determine the instantaneous location of the laminar separation point is then:

$$x_s(t) = x_{s0}(\alpha_o) + \left[\frac{\partial x_s}{\partial \left(\frac{U}{U_\infty}\right)} \right]_{\alpha = \alpha_o} \cdot \frac{\tilde{u}}{U_\infty}(t)$$

$x_s = x_{s0}$

On Figure 4.9 the locations x_m and x_s refer to maximum velocity and separation points. x_r is the point of turbulent reattachment. Distance ℓ defines the laminar shear layer length and m defines the turbulent mixing and reattachment region. Note that transition and the beginning of reattachment are taken to coincide at a point (x_b). Also note that at separation and reattachment, pressures (and velocities) are assumed equal to their potential flow values at the same point. These assumptions have been shown to be reasonably accurate.

Now we are interested in finding the lengths ℓ and m so that we can determine the pressure (or velocity) rise across the assumed bubble.

At present no analytical solution or numerical method is found to be adequate in predicting the length ℓ . Even with knowledge of the laminar boundary layer parameters at separation, free-stream turbulence level, surface roughness and Reynolds No., there is no known method that allows for a determination of transition point in a laminar shear layer that is also influenced by unsteady pressures, velocities and surface curvature.

We, therefore, choose to fall back on an empirical model that is due to Von Doenhoff.⁵ The Von Doenhoff model is:

$$Re_{x_b} \equiv \frac{\rho U_s (x_b - x_s) b}{\mu} = \frac{\rho U_s \ell b}{\mu} \cong 5 \times 10^4$$

where U_s is the velocity at the edge of the boundary layer at the separation point, x_s . Therefore, we have:

$$\ell(t) \cong \frac{5 \times 10^4 \mu}{U_s(t) b \rho}$$

It is shown in reference 37 that an appropriate quasi-steady flow model for the turbulent mixing and reattachment region is:

$$m(t) \cong \frac{k_1 \mu}{U_s(t) b \rho}$$

where k_1 is a function of $Re_{\delta_s}^*$; that is, the Reynolds number based on shear-layer displacement thickness at separation.

Based on experimental results, the relations above are combined and a constant added so that we have approximately:

$$\ell(t) + m(t) = \frac{k_2}{U_s(t)} + k_3$$

Thus, the bubble length parameters can be found for quasi-steady bubble growth. Since the potential flow solution is known for all time, the pressures and velocities at separation and reattachment can also be found.

An example of steady flow predictions for the location of separation, transition and reattachment points is shown in Figure 4.10 for the NACA 0012 airfoil. Parameter values are found in reference 37.

The separation point results for the NACA 0012 are in good agreement with values that were interpolated from NACA 0010 (mod) and NACA 66₃ - 018 experimental measurements for $Re_c = 2 \times 10^6$ which were taken from reference 51. Results for bubble length at $\alpha = 14^\circ$ which are found in reference 52 are in very good agreement with present results; however, data from reference 51 indicates that the laminar shear layer length of the model is consistently overestimated by about 0.75% of chord, and the turbulent region length is overestimated by about 0.5% chord. The rates of movement of these points as incidence is varied are found to be in good agreement with experiment. It must be noted, however, that the present model provides solutions at incidence above the static stall so the high incidence values found in Figure 4.10 should not be taken to imply that bubble bursting is not achieved. In the case of the NACA 0012 (for $Re_c = 2 \times 10^6$) it appears that static stall is achieved when the leading-edge bubble bursts at an incidence of around 13° - 16° .

The pressure recovery coefficient, σ , or Crabtree's parameter can be evaluated also. It is defined:

$$\sigma \equiv \frac{C_{pr} - C_{ps}}{1 - C_{ps}}$$

It has been evaluated in steady-flow for the NACA 0012 and results are plotted in Figures 4.11 and 4.12.

Figure 4.11 involves the NACA 0012 with various combinations of α_0 and $\tilde{\alpha}$. The trends appear at least qualitatively correct except that most combinations of $\alpha_0 + \tilde{\alpha}$ do not yield exactly the same result for an equivalent "non-perturbed" incidence. For example, for $\alpha_0 = 14^\circ$, $\tilde{\alpha} = 0^\circ$ we have $\sigma = .273$ whereas for $\alpha_0 = 10^\circ$; $\tilde{\alpha} = 4^\circ$ we have $\sigma = .276$. This discrepancy is expected, however, and thought to be acceptable in light of the fact that the pressure distribution imposed by perturbations, α , is of the thin airfoil form (i.e., $\sim \sqrt{(1-x)/(1+x)}$) and includes Lighthill's correction factor. Similarly, the results of Figure 4.12 show the same effect. The irregular curvature to the $\alpha = 0$ curve at high incidence is related to the assumed polynomial representation of velocity ratios from the NACA method and to the limitations of the NACA method.

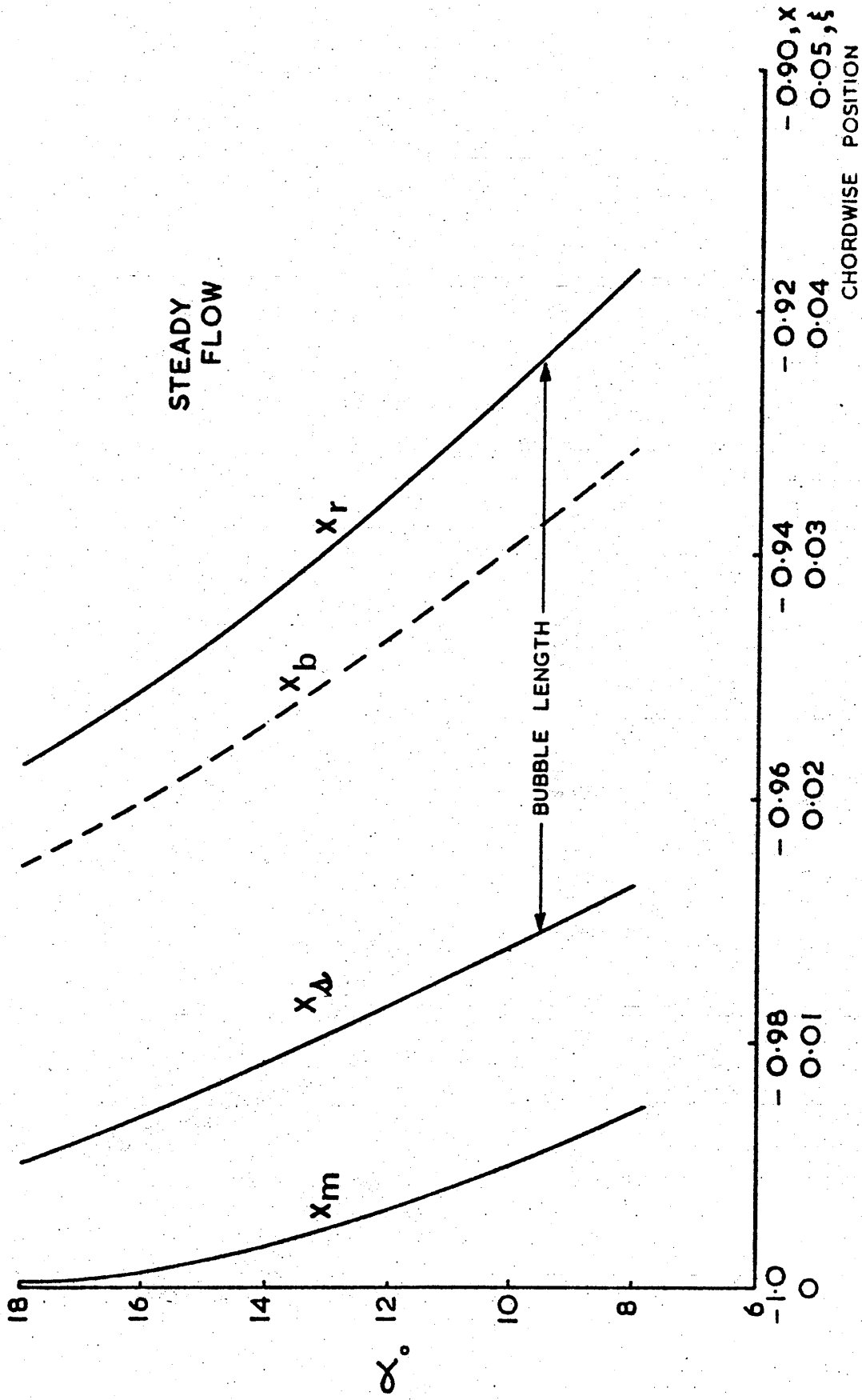


FIG. 4.10 NACA. 0012 BUBBLE LENGTH PARAMETERS.

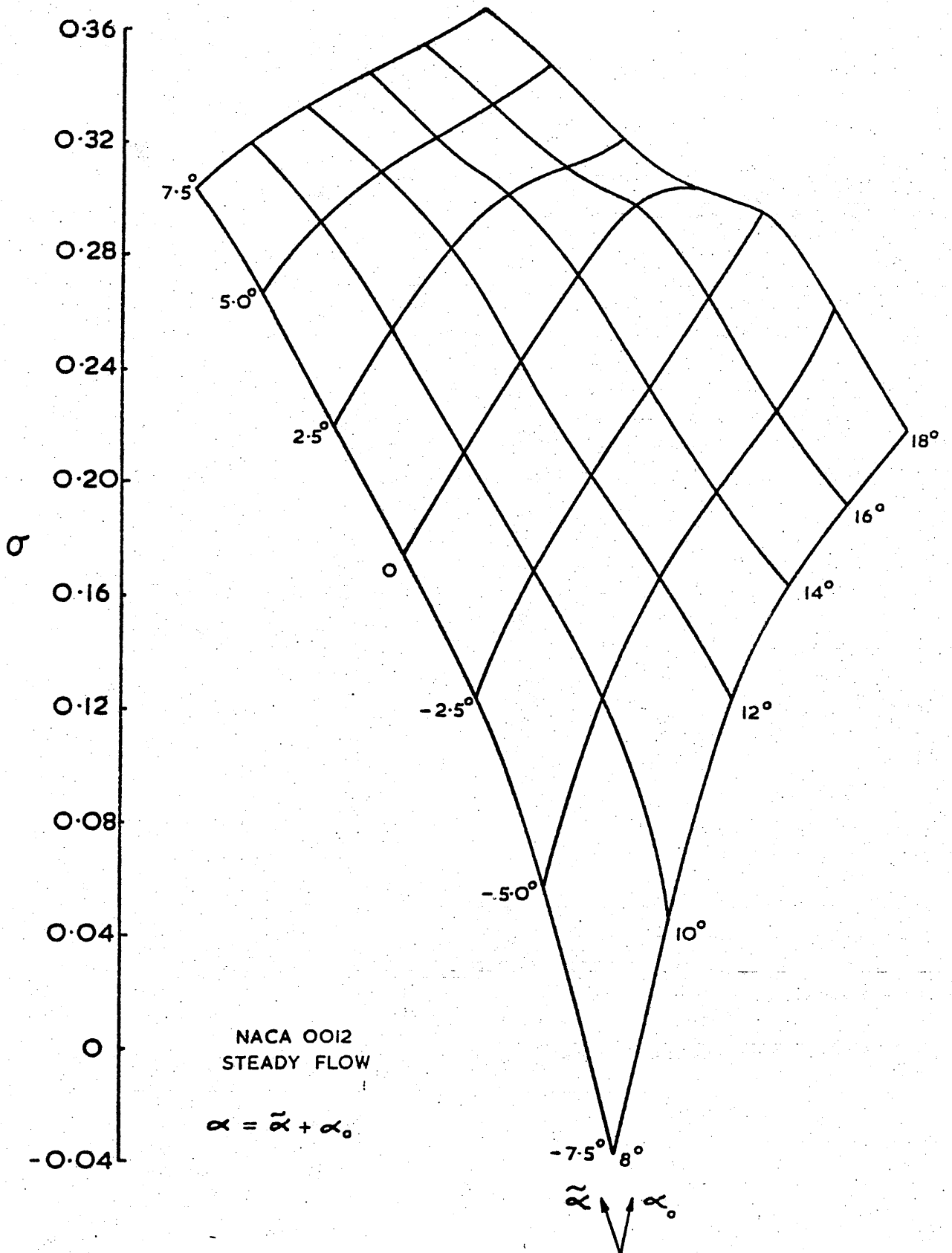


FIG.4.11 CRABTREE'S PARAMETER FOR NACA 0012 INCIDENCE EFFECT.

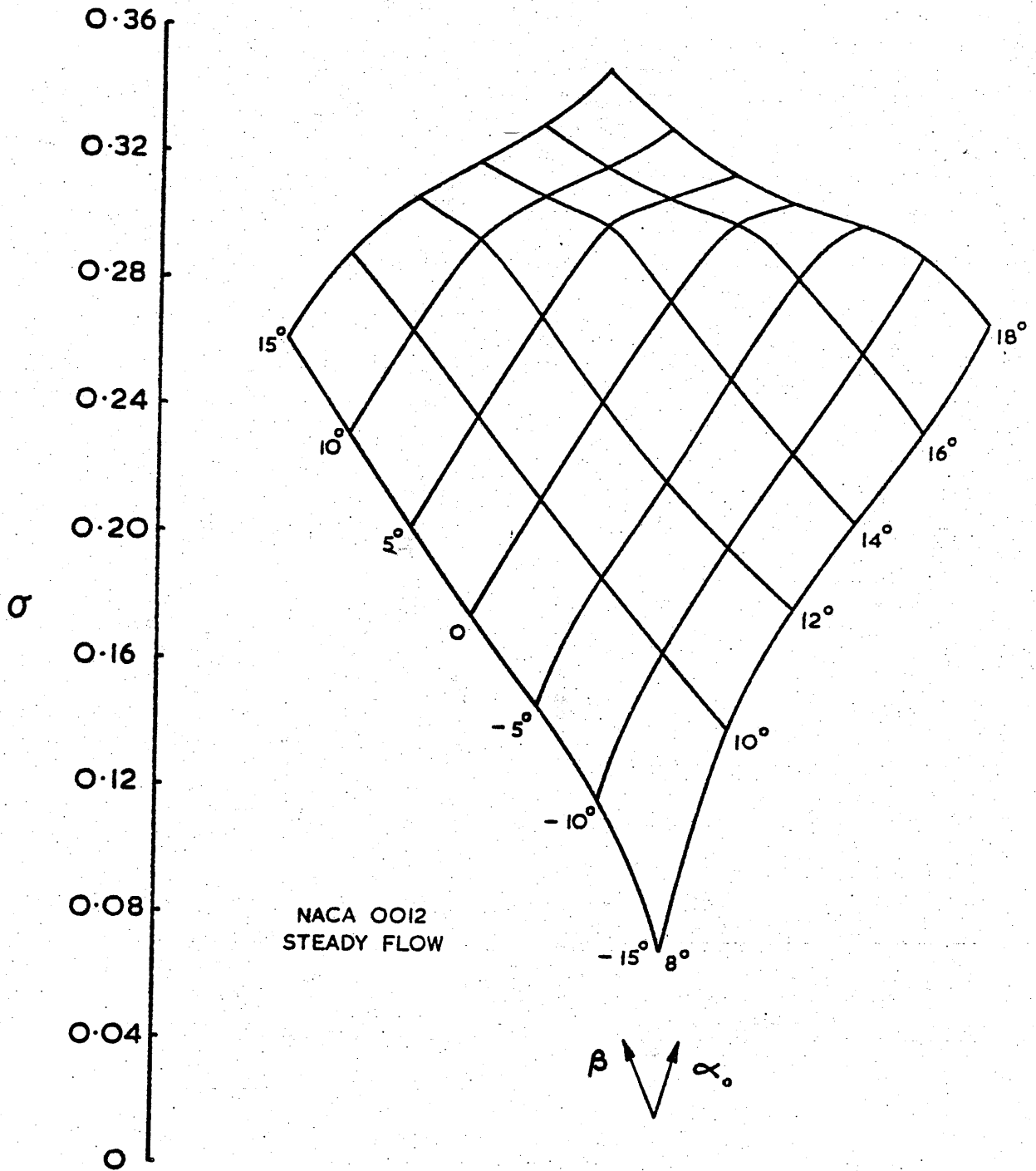


FIG.4.12 CRABTREE'S PARAMETER FOR NACA 0012 FLAP EFFECT.

The present model was not derived in order to predict static stalling; however, it appears that the method does compare favorably with steady flow results concerning Crabtree's parameter.

A possible use of these results is best illustrated by considering Figure 4.12. It is suggested that a critical value of Crabtree's parameter can be found from rigid airfoil tests and applied to the prediction of airfoil stall which is induced by flap deflection. If pertinent parameters are not varied for the flap test (e.g., Re , free-stream turbulence, M , surface roughness) then it is suggested that stall will occur when $\sigma = \sigma_{CR}$. For example, if $\alpha_{stall} = 13^\circ$ then we take $\sigma_{CR} = .26$ and can find combinations of α_0 and β that will cause bubble bursting and consequent airfoil stall. At present this is only a suggestion, and one that lacks complete experimental verification.

4.1.2. Prediction of Bubble Bursting

In reference 37 it was argued that a critical value of velocity drop across the bubble exists which relates to bubble bursting in unsteady flow. In steady flow, Crabtree's parameter is seen to be an appropriate measure of velocity drop since,

$$\sigma \equiv \frac{C_{pr} - C_{ps}}{1 - C_{ps}} = 1 - \left(\frac{U_r}{U_s}\right)^2$$

Now define a parameter, Young and Horton's parameter,¹⁴ which is:

$$\frac{\Delta U}{U_s} = \frac{U_r - U_s}{U_s}$$

Then in steady-flow:

$$\sigma = - \left[\frac{2\Delta U}{U_s} + \left(\frac{\Delta U}{U_s}\right)^2 \right]$$

A typical variation of Young and Horton's parameter during a cycle of airfoil pitching is shown in Figure 4.13 for two cases. NACA 0012 pitching motion is considered with:

$$\alpha = 12^\circ + 5^\circ \cos \nu_a \tau_a ; \quad \nu_a = .04 \text{ and } .30$$

The steady-flow variation, represented by the dashed curve, follows from the related variation of Crabtree's parameter in Figure 4.11. The hysteresis effect due to unsteady motion is clearly evident in Figure 4.13. If we assume that a critical

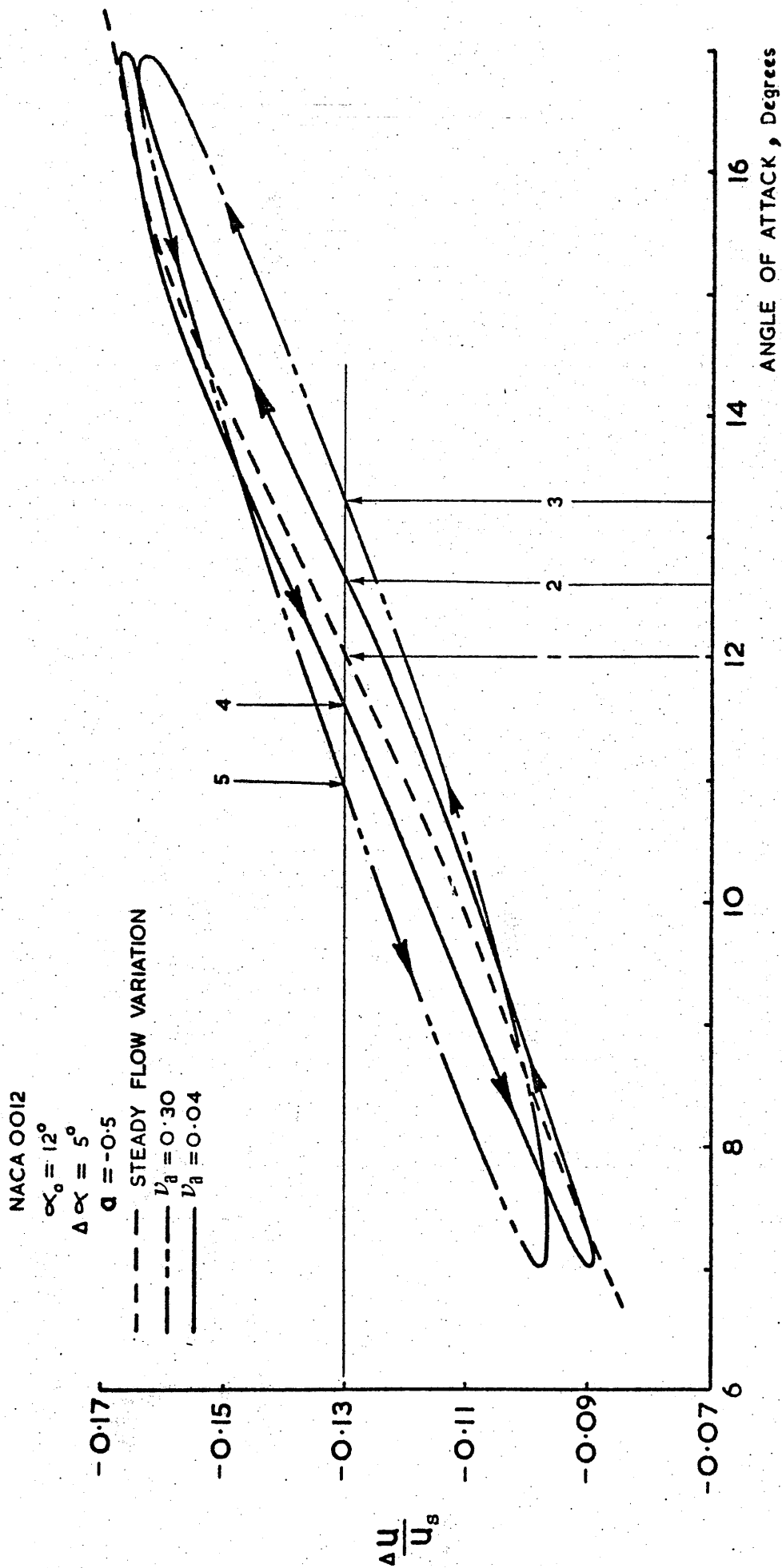


FIG.4.13 TYPICAL VARIATION OF YOUNG & HORTON'S PARAMETER IN UNSTEADY FLOW.

value of $\Delta U/U_s$ exists and is not dependent on unsteady effects, then we could proceed as follows to evaluate static stall angle overshoot. We first find the value of

$$\left(\frac{\Delta U}{U_c}\right)_{CR}$$

where at present, it appears that static stall experiments must be relied upon to yield this value. For example, if

$$\alpha_{\text{static stall}} = 12.0^\circ$$

then the critical value of $\Delta U/U_s$ follows from point (1) on Figure 4.13 and we have:

$$\left(\frac{\Delta U}{U_s}\right)_{CR} = - .13 \text{ for } \alpha_{\text{static stall}} = 12.0^\circ$$

Then, in the unsteady case we assume that the bubble does not burst for:

$$\left(\frac{\Delta U}{U_s}\right) > - .13$$

Thus, on the upstroke, bubble bursting (and the initiation of dynamic stall) is assumed to occur at

$$\frac{\Delta U}{U_s} = \left(\frac{\Delta U}{U_s}\right)_{CR}$$

From Figure 4.13 we find that for

$$\nu_a = 0.04 \text{ (i.e., (2)); } \alpha_{\text{dynamic stall}} = 12.6^\circ$$

or for

$$\nu_a = 0.30 \text{ (i.e., (3)); } \alpha_{\text{dynamic stall}} = 13.2^\circ$$

If we define a stall angle overshoot, $\Delta \alpha_s$ where:

$$\Delta \alpha_s \equiv \alpha_{\text{dynamic stall}} - \alpha_{\text{static stall}}$$

then we have:

$$\Delta \alpha_s = \begin{cases} .6^\circ & \text{for } \nu_a = 0.04 \\ 1.2^\circ & \text{for } \nu_a = 0.30 \end{cases}$$

Points (4) and (5) on Figure 4.13 can be assumed to yield the reattachment angles as discussed previously.

The difference between using a velocity-based criterion $\Delta U/U_s$, and a pressure-based criterion σ , has been investigated over a range of pertinent parameters. It appears that the difference is mostly sensitive to frequency effects, and if we define an error parameter $\epsilon(\nu)$, where:

$$\epsilon(\nu) \equiv \frac{\Delta \alpha_{sU} - \Delta \alpha_{sp}}{\Delta \alpha_{sU}}$$

and the U and p subscripts refer to unsteady overshoot predictions based on velocities and pressures respectively, we find that at $\nu_a = 0.30$ an error of 8% exists. The approximate relation shown below is characteristic of the frequency effect:

$$\epsilon(\nu) \sim .267 \nu_a$$

Thus, at low frequencies, predictions based solely on pressures and Crabtree's criterion will yield reasonable results.

Results based on the velocity method of determining bubble bursting will now be presented as parameters are varied for the NACA 0012 and as airfoil and flap motion is considered. Computational procedures are discussed in reference 37.

Figure 4.14 contains results for the NACA 0012 pitching case where stall angle overshoot angle ($\Delta \alpha_s$) is plotted versus pitch rate at bubble burst (α'_s). The effect of frequency and amplitude of oscillation is clearly evident and we find that for a given frequency, $\Delta \alpha_s$ is roughly proportional to the amplitude of oscillation ($\Delta \alpha$) which is a more-or-less expected result since a linear unsteady flow model is used. Notice, however, that for a given value of $\Delta \alpha$ the effect of frequency is decidedly nonlinear. Vortical wake effects primarily account for this trend. Also, it must be mentioned that for Figure 4.14 a critical value of ($\Delta U/U_s$) was determined from the somewhat arbitrary selection of steady-flow stall angle where

$\alpha_{\text{static stall}}$

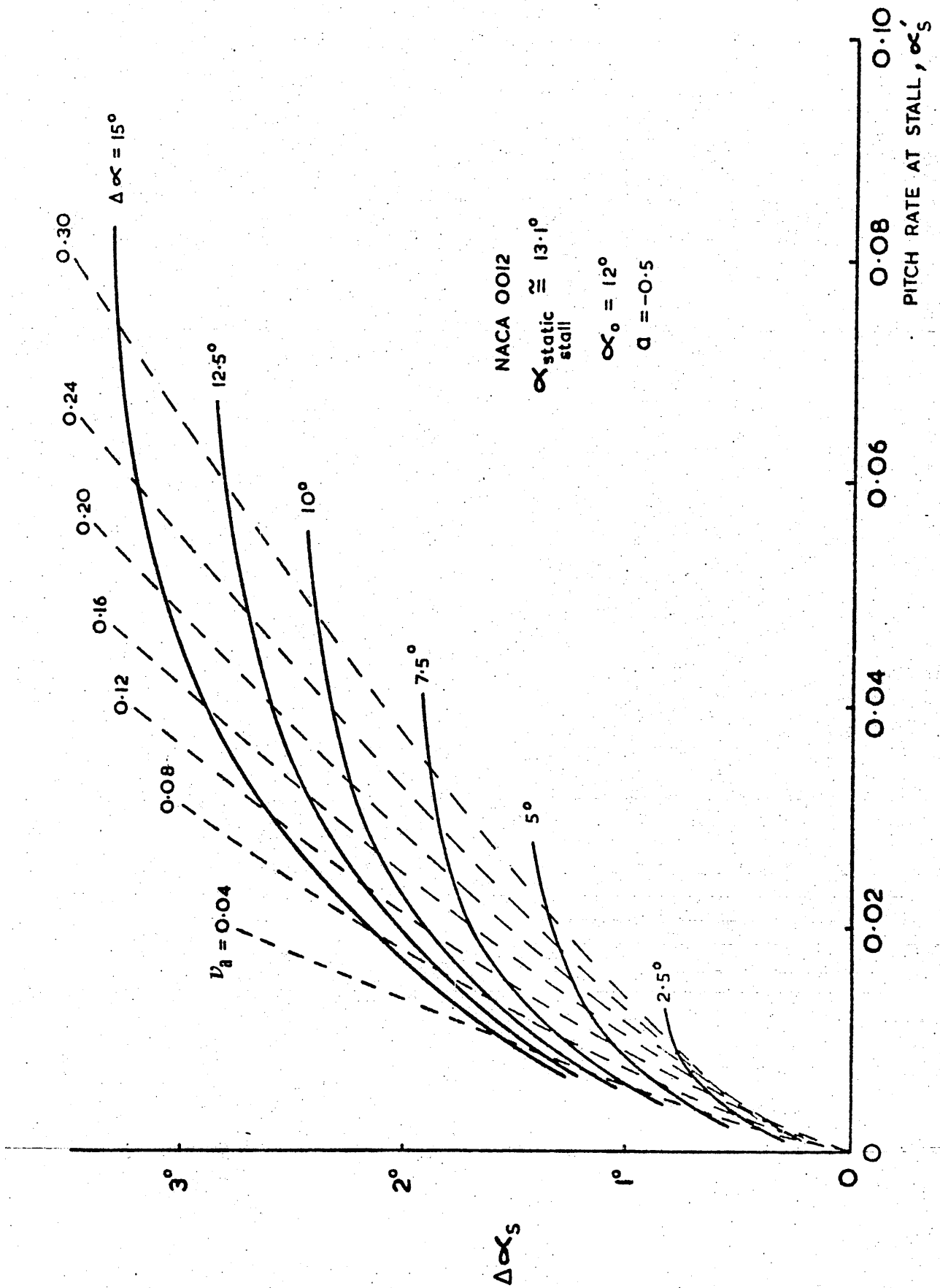


FIG.4.14 PITCH RATES AT STALL AND OVERSHOOT ANGLES FOR NACA 0012

was assumed to be 13.1° . (This corresponds to $\alpha_{CR} = .26$ and

$$\left(\frac{\Delta U}{U_s}\right)_{CR} = - .14)$$

The results relate to airfoil pitching about the quarter-chord ($a = -.5$) at a mean incidence of 12° .

In Figure 4.15 we see a seemingly irregular result that can fortunately be explained. Flap motion effects on the NACA 0012 leading-edge bubble are considered where $\alpha_0 = 12^\circ$ and

$$\left(\frac{\Delta U}{U_s}\right)_{CR} = - .14$$

which yields

$$\beta_{\text{static stall}} = 3.4^\circ$$

Also,

$$\Delta\beta_s \equiv \beta_{\text{dynamic stall}} - \beta_{\text{static stall}}$$

For the largest amplitude considered (i.e., $\Delta\beta = 20^\circ$) the effect of frequency on $\Delta\beta_s$ and β'_s generally follows trends which were seen previously; however, the lower amplitudes, $\Delta\beta = 10^\circ, 15^\circ$ exhibit a different effect. Consider now the constant amplitude curve $\Delta\beta = 10^\circ$. At very high frequencies it appears that dynamic stall occurs at lower flap rates. This effect simply relates to the fact that at higher frequencies bubble bursting is delayed to such an extent that it occurs close to the maximum flap deflection angle, which for harmonic motion coincides with zero flap rate.

Notice also that bubble bursting does not occur during an oscillation for $\Delta\beta = 5^\circ$ (and for $\nu \geq .08$). This effect relates to the so called "stall-free regime" mentioned by McCroskey in reference 45.

Reference 37 contains further results of the application of the present theory to the cases of airfoil pitching and flap motion. Details are given of the effects of amplitude and frequency of oscillation, pitch axis location and mean incidence on the overshoot angle and interval (during a cycle) of overshoot. Furthermore, discussion is offered concerning the effects of wake vortical distribution and the speed of propagation of vortex elements into the wake. At high frequencies it appears that the assumption of constancy of circulation is justified when one considers loading due to growth of the separated region.⁵³

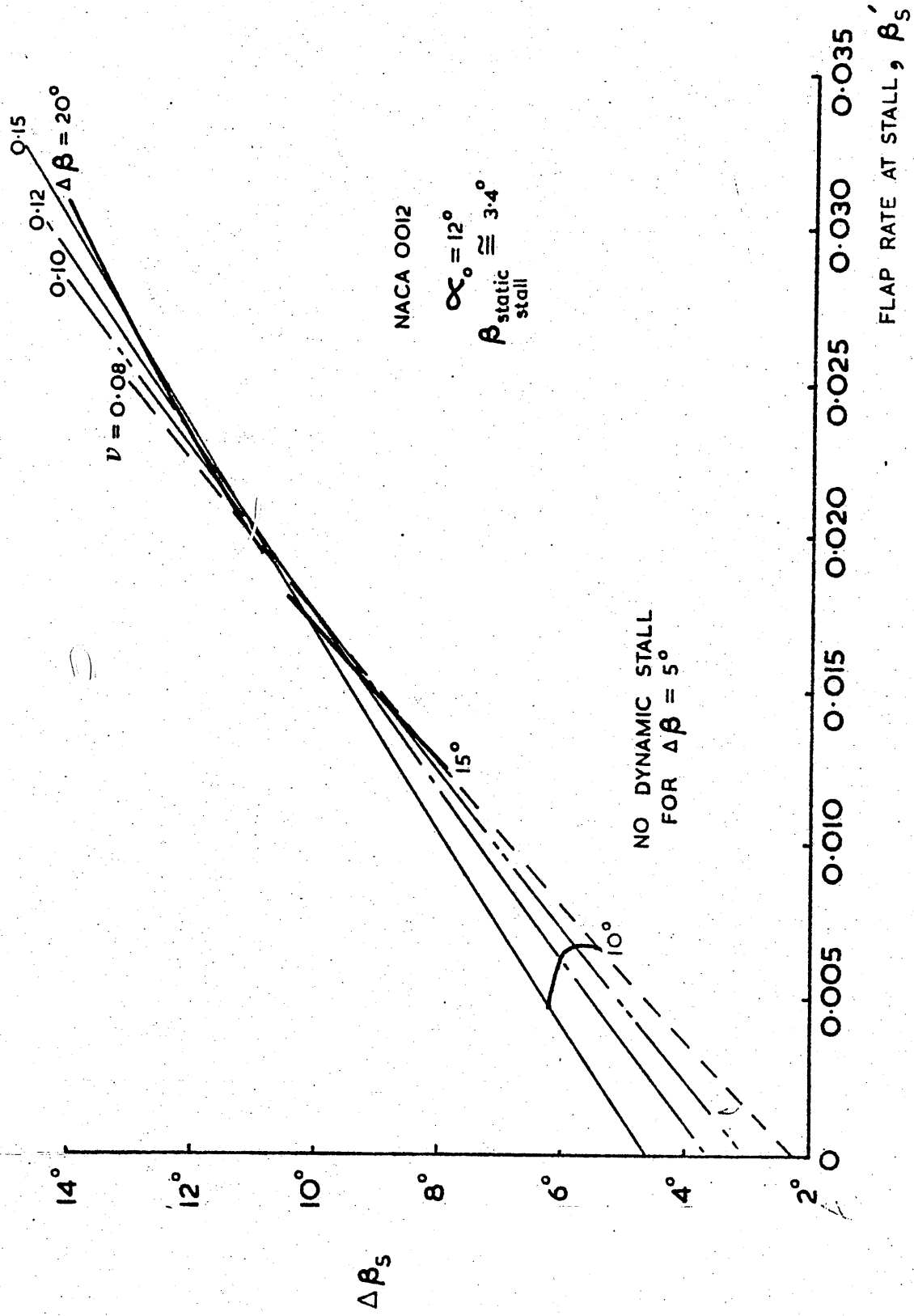


FIG. 4.15 FLAP RATES AT STALL AND OVERSHOOT ANGLES FOR NACA 0012.

However, for low frequencies dissipation of energy in the growing bubble is significant as it moves across the airfoil surface, thus, the assumption of constancy of circulation does not appear justified.

Reference 37 also contains an approximate analysis of the effect of unsteady pressures on turbulent trailing-edge separation. Stratford's criterion⁵⁴ is used to show that on the airfoil upstroke (or flap downstroke) trailing-edge separation is likely to be suppressed so that leading-edge separation and bubble bursting will occur in unsteady cases.

Finally, an additional element must be considered. The present analysis is expected to be useful in order to predict the onset of bubble instability. The term bursting, however, is probably misleading in the unsteady case since a finite amount of time is required for the bubble to grow once it does become unstable. Thus, the present quasi-steady model is limited in that it does not account for the dynamics of bubble growth once instability is achieved. McCroskey⁴⁵ has recently recognized this also and has compared a simple burst prediction (based on the instantaneous pressure gradient at 10% chord) to experimental observations which involve both the initial breakdown of the potential flow and data based on moment stall of an airfoil.

Consider now an extension of the previous quasi-steady model to allow for the dynamics of bubble growth. The model of separation bubble dynamics of section 3 serves as a starting point.

Crimi in Reference 55 has concluded that growth of the leading-edge separation bubble is dependent primarily on mass flux into the bubble from near the reattachment point. An incremental mass flux in the unsteady case is essential so that the bubble may grow, and we have:

$$\dot{m}_\epsilon \cong \rho \frac{d}{dt} (A_{\text{bubble}})$$

From the analysis of section 3.2.2 we find that the pressure rise near reattachment is expected to alter in proportion to the additional mass flux. So we have a change to the pressure drop across the bubble:

$$\Delta \tilde{C}_{p_{rb}} \equiv \frac{\Delta \tilde{p}_{rb}}{\frac{1}{2} \rho U_\infty^2} = \left[\frac{1}{3} \left(4 + \frac{1}{a} + \frac{1}{a^2} \right) \right] \frac{\dot{m}_\epsilon u_B}{\frac{1}{2} \rho U_\infty^2 \delta_r}$$

For quasi-steady bubble growth we have approximately:

$$A_{\text{bubble}} \cong k_1 b^2 (\alpha - \alpha_{\text{stall}})^2$$

or

$$\dot{m}_e \cong 2 \rho k_1 b^2 (\alpha - \alpha_{\text{stall}}) \dot{\alpha}$$

and

$$\delta_r \cong k_2 b (\alpha - \alpha_{\text{stall}})$$

The above relations are appropriate for thin airfoil-type stall which involves the growth of a long bubble type. See also Wood's theory of bubble growth in reference 56.

The quasi-steady mass flux is assumed valid as for the bubble behind a spoiler. This appears well justified since the bubble is small and the shear layer is expected to have a small characteristic time. So we have for the pressure perturbation:

$$\begin{aligned} \Delta \tilde{C}_{p_{rb}} &\cong \left[\frac{1}{3} \left(4 + \frac{1}{a} + \frac{1}{a^2} \right) \right] \frac{4k_1}{k_2} \frac{u_B b}{U_\infty^2} \dot{\alpha} \\ &\cong k_3 \frac{u_B}{U_b} \left(\frac{U_b}{U_\infty} \right) \frac{\dot{\alpha} b}{U_\infty} \end{aligned}$$

but

$$\frac{\dot{\alpha} b}{U_\infty} = -\nu_a \Delta \alpha \sin \omega t = \alpha'$$

for

$$\alpha = \alpha_0 + \Delta \alpha \cos \omega t$$

and

$$\frac{U_b}{U_\infty} = (1 - C_{p_s})^{\frac{1}{2}}$$

Also, assume $\frac{u_B}{U_b} = \text{constant}$ so that

$$\Delta \tilde{C}_{p_{rb}} \cong k_4 (1 - C_{p_s})^{\frac{1}{2}} \alpha'$$

In the unsteady case then, the short leading-edge bubble will grow when sufficient mass is returned to the bubble. In effect, this means that the critical pressure recovery coefficient is modified in the unsteady case, so we have:

$$\sigma_{\text{growth}} = \sigma_{\text{CR}} + \Delta\sigma$$

where

$$\Delta\sigma = \frac{\Delta \tilde{C}_{p_{rb}}}{(1 - C_{p_s})} \cong k_4 \frac{\alpha'}{(1 - C_{p_s})^{\frac{1}{2}}}$$

and

$$k_4 = \left[\frac{1}{3} \left(4 + \frac{1}{a} + \frac{1}{a^2} \right) \right] \frac{4k_1}{k_2} \frac{u_B}{U_B}$$

Computations were performed in order to find the overshoot angle-of-attack for a pitching airfoil. Overshoot angle in this case is defined to be:

$$\frac{\Delta\alpha_s}{\Delta\alpha} = \frac{\alpha_{\text{bubble growth}} - \alpha_{\text{static stall}}}{\Delta\alpha}$$

A single case was chosen in order to compare with experimental data. Motion is of the form:

$$\alpha = 12^\circ + 5^\circ \cos \omega t$$

σ_{CR} was chosen so that $\alpha_{\text{static}} = 12^\circ$. In this case, a pressure stall

criterion was used in order to predict overshoot angle, thus a small discrepancy is expected with overshoot angle being slightly underestimated.

Results are plotted in Figure 4.16, as curves for bubble growth. Two values of the shear layer parameter, k_4 , were used (i.e., $k_4 = .25$ and $k_4 = .15$) and results show that overshoot is not strongly dependent on this parameter. Experimental data is taken from reference 45. Also shown in Figure 4.16 are results from the unmodified quasi-steady theory. The curve labeled "bubble burst based on pressures" is nearly coincident with results if either the pressure gradient at 1.25% chord or σ_{CR} is evaluated with bubble length fixed. The curve labeled "bubble burst based on velocities" follows directly from the original theory, which is based on Young and Horton's parameter, and is found to be nearly coincident with a prediction of McCroskey⁴⁵ based on the pressure gradient at 10% chord.

The onset of bubble growth, as predicted from the present theory, is seen to correspond well to observations of airfoil moment stall. Thus, it appears that the instant of dynamic stall initiation is associated not with bubble instability but

MOMENT STALL DATA

- CRIMI AND REEVES THEORY
- ◇ WINDSOR EXPERIMENT
- CARTA AND NIEBANCK
- ▽ LIIVA, et al
- △ HALFMAN, et al
- THEORY, BUBBLE GROWTH
- + INITIAL BREAK-DOWN OF FLOW, McCROSKEY
- } THEORY, BUBBLE INSTABILITY

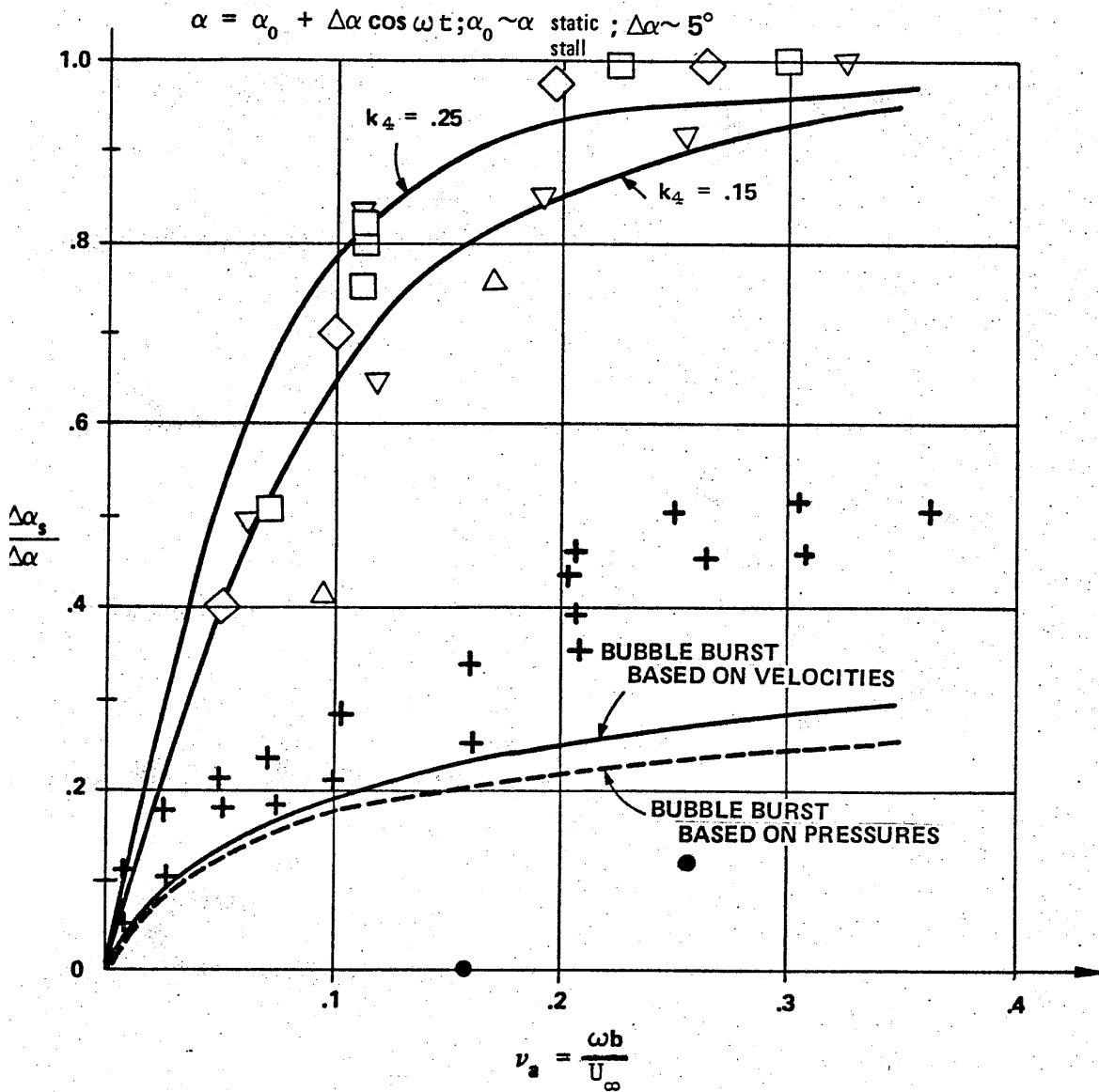


FIG. 4.16 THEORETICAL AND EXPERIMENTAL STALL DELAY ON OSCILLATING NACA 0012 AIRFOILS

with actual growth of the separation bubble. McCroskey's data for the initial breakdown of potential flow lies, as expected, between the instant for bubble bursting (or instability) and bubble growth.

In conclusion regarding dynamic stall, it is seen that the unsteady inviscid flowfield dominates bubble behavior and that a critical pressure or velocity drop across the bubble may be determined by considering that the small bubble reacts in a quasi-steady manner to changes in the external flowfield. Growth of the bubble, however, depends on dynamic effects in that it does not begin until mass is forced into the bubble at a sufficient rate to cause growth.

4.2. A Type of Control-Surface Buzz

The dynamics of a separating region are known to be an element in the problem of control-surface buzz. Lambourne in references 1,49 & 57 discusses this example of flutter in one degree-of-freedom which involves the interaction of a moving flap, shock wave(s) and a separating flow. A type of control-surface buzz (CSB), which is termed type A, involves shock wave(s) which lie on the airfoil surface ahead of the flap hinge-line. The flap is in subsonic flow and boundary-layer separation is induced by the shock wave(s).

Some conclusions concerning type A buzz are:¹

1. The critical Mach number for buzz correlates with a criterion for the onset of shock-induced separation, e.g., trailing-edge pressure divergence.
2. The critical Mach number falls with increasing airfoil incidence.
3. The critical Mach number is not greatly altered by a change in the elastic hinge stiffness.
4. At constant Mach number, the buzz frequency is given approximately by:

$$\omega = \left[\frac{C - H}{A} \right]^{\frac{1}{2}}$$

The theoretical work of Eckhaus^{58,59} included shock waves at the wing surface in an inviscid flow theory and has predicted the existence of a Mach number and frequency range in which the flap is negatively damped. Other analyses which included a time lag in the motion of the shock and subsequent lag in the aerodynamic hinge moment were offered.^{60,61} At present, however, the exact mechanisms and interactions remain unknown.

Based on the results and theory for the oscillating spoiler case and some recent information concerning viscous-inviscid interactions at transonic speeds one can further speculate as to the mechanisms and interactions in type A control-surface buzz.

Consider now a typical type A flow which involves an airfoil at incidence, Mach number near critical, and a single shock-wave ahead of the flap on the upper surface. A separated region exists aft of the shock wave with reattachment occurring on the flap surface and no divergence in trailing edge pressure. The flap is fixed at a "steady flow equilibrium" angle with $C_H = 0$.

In this case, which borders between the so-called "weak" and "strong" interactions of the shock/shear-layer system and the overall flow⁶² we expect that coupling (via shear layer displacement thickness effects) between the shear layer and pressure field is strong locally (in the region of the separation bubble) but relatively weak in the overall sense, since trailing-edge pressure has not diverged.

Now if the flap were to be oscillated harmonically with very small amplitude, as an assumption, we neglect a direct effect on the separation bubble and shear layer and instead consider that the outer inviscid flow responds as in the weak interaction, or attached flow, case. This assumption follows from results of the spoiler experiment where, for moderate spoiler heights, it was found that aerodynamic hinge-moment was approximately independent of spoiler height and bubble length.

As a result of flap motion, pressure perturbations are realized at the shock. If the region of supersonic flow ahead of the shock is sufficiently large, then propagation of pressure perturbations to the shock from upstream may be neglected. The flow may be considered "frozen" ahead of the shock. Tijdeman and Zwaan in reference 63 show that this type of pressure disturbance travels upstream as a receding wave and that the time taken to reach the aft face of the shock is as discussed in reference 61 and is approximately:

$$\Delta t = \frac{d}{\bar{a}_2 (1 - M_2)}$$

where d is the distance between T.E. and shock location, $\bar{a}_2$ and M_2 are the mean speed of sound and local Mach number along d . We can also write:

$$\Delta \tau \equiv \frac{\Delta t U_\infty}{c_f} = \frac{a_\infty M_\infty}{\bar{a}_2 (1 - M_2)} \frac{d}{c_f}$$

A phase shift may be found:

$$\phi = 2\pi f \Delta t = \nu \Delta \tau$$

Thus, the pressure perturbations at the aft face of the shock, δp_2 , result from flap motion and a time lag. Lambourne in reference 49 shows that this change in shock strength can be related to shock motion by means of the 1-D flow jump relations for a normal shock.

$$\frac{\delta p_2}{p_1} \approx \frac{4\gamma}{\gamma + 1} M_1 \left(\frac{U_s}{a_1} \right) \quad \text{for} \quad \left| \frac{U_s}{a_1} \right| \ll M_1$$

where U_s is shock velocity upstream relative to the airfoil surface and subscript 1 denotes conditions immediately upstream of the shock. Also, we have the pressure jump across the shock:

$$\frac{p_2 - p_1}{\frac{1}{2} \gamma p_1 M_1^2} = \frac{4}{\gamma + 1} \left(1 - \frac{1}{M_1^2} \right) + \frac{8}{\gamma + 1} \left(\frac{U_s}{U_1} \right)$$

For a normal shock, then, we find that the flap can "drive" the shock across the surface of the airfoil via unsteady pressure perturbations. This result is also found in Eckhaus' work^{58,59} along with an equation that may be solved for the pressure perturbations due to flap motion.

Forced oscillation tests confirm that the flap/shock interaction as described above is qualitatively correct in the absence of significant separation. In reference 63 it is shown that for sufficiently large perturbations the shock does not exist during part of its downstream motion. We find in that case that $p_2 \approx p_1$. This is very strong evidence to support the hypothesis that prior to T.E. pressure divergence, flap motion serves to force the shock along the airfoil surface as described above, thus we expect that at the onset of buzz, coupling between the flap and shock motion may be represented by a parameter which is primarily a function of freestream Mach number, frequency and airfoil geometry. It is further probable, but remains to be verified, that for larger degrees of initial separation (for example those which involve recompression on the flap upper surface) the flap may also control the outer flow to a significant degree or may be able to force the shock by affecting mass flow in the separated region. It is expected, therefore, that the coupling relation between flap and shock depends on the amplitude of flap motion as well as on the initial degree of shock-induced separation. This non-linear effect is not simulated in the coupled spoiler/flap experiment where coupling parameters are held constant.

As the shock wave moves across the airfoil surface it affects the boundary layer. A forward moving shock causes severe separation; a rearward moving shock is weaker and leads to reduced separation. Measurements by Nakamura⁶⁴ and Lambourne, at both fixed chordwise positions and fixed distances aft of the moving shock, have shown that shear layer thickness is related to shock strength. In references 17 and 65 it is shown that the angle at which the boundary-layer is forced to separate from the airfoil surface is a function of shock strength. In the steady flow case for the region near the foot of a normal shock where an oblique shock emanates from the edge of the shear layer, we have an approximation from reference 66.

$$\frac{p_2 - p_1}{p_1} \approx \frac{\gamma M_1^2}{\sqrt{M_1^2 - 1}} \theta$$

where θ is the deflection angle of the outer flow and can be taken as equal to the angle at which the shear layer separates from the surface.

The above results show that shock strength corresponds to shear layer deflection angle at the foot of the shock. Trilling also shows this to be the case in unsteady flow, where the preceding equation yields approximately:

$$\frac{\delta p_2}{p_1} \approx \frac{\gamma M_1^2}{\sqrt{M_1^2 - 1}} \delta \theta \quad \left(\text{for } \left| \frac{U_s}{a_1} \right| \ll M_1 \right)$$

Entrainment of fluid from the "dead air" region under the shear-layer into the shear-layer (as discussed in sections 2 and 3) will result in a transverse pressure gradient and can cause shear layer curvature and reattachment.

Separation of the shear layer, as in the case of the oscillating spoiler, involves vorticity being convected into or past the separation "bubble." As the shock moves forward, vorticity is figuratively "peeled away" from the airfoil surface and may be convected downstream so that bubble circulation and size increase. Pressure in the region of the bubble is significantly reduced. Thus, in a manner analogous to spoiler induced effects, one would expect that shock motion induces alterations to separation bubble size and pressures.

As the shock moves forward, the shear layer (which has a small characteristic time) will separate at an angle which relates to shock strength (i.e., rate). The mechanism of bubble growth, however, involves mass flux and a time lag. In this case also, movement of the separation point and bubble is a factor. Bubble growth is expected to lag shock rate whereas pressure perturbations are assumed to depend on shock rate. (See section 3 for a discussion of possible integral effects.)

The net result is in a first harmonic component of incremental hinge-moment which depends on shock rate. For small perturbations the effect of shock and separation point location may be neglected as an approximation. The effect of shock position may also cancel the effect of lag in bubble growth. An incremental hinge-moment is nevertheless produced by shock motion. If the flap is free to respond, then we have the possibility of self-excited oscillations; that is, control-surface-buzz.

The preceding discussion involves a considerable amount of speculation concerning the exact interactions between flap motion, shock movement and incremental hinge-moment. For example, the displacement thickness effect of the shear-layer on the outer flow is neglected for small perturbations. The important assumptions, which are only appropriate at the onset of instability, are that the shock is primarily driven by flap motion through the outer flow and that shock strength, hence incremental hinge-moment, is altered due to shock motion. This scheme of interactions can be approximately modeled in order to study the onset of buzz.

As in the coupled spoiler/flap experiment, we take as the equation of motion:

$$\ddot{\beta} + \frac{(B - H_{\beta}^2)}{A} \dot{\beta} + \frac{(C - H_{\beta})}{A} \beta = \frac{\Delta H}{A} \quad (4.5)$$

where H_{β}^2 and H_{β} are aerodynamic terms which may be found from tests at moderate values of freestream Mach no. in the absence of significant shock-induced separation.

The incremental hinge-moment is related to the coefficient, ΔC_H , which can be determined in steady flow with the flap fixed by varying freestream Mach number, M_{∞} , over a range from that which involves the onset of shock-induced separation effects to that which corresponds to large-scale separation (e.g., when the shock wave(s) move onto the flap surface). Ideally ΔC_H should be evaluated by extrapolation with total hinge-moment on the flap equal to zero at a fixed Mach number. This will insure that ΔC_H is known near the steady-flow "equilibrium" flap position. The incremental hinge-moment coefficient as a function of M_{∞} is expected to qualitatively resemble the corresponding term in the spoiler experiment as a function of spoiler height.

A solution for the coupling relation between flap and shock motion can possibly be found by extending Eckhaus' work. For flap motion of the form:

$$\beta = \beta_0 + \beta_1 \cos \omega t$$

shock position can be represented from the linear theory of Eckhaus as:

$$x_{sh} \equiv \frac{x_{sh}}{c} = x_{sh0} + x_1 \cos (\omega t - \gamma)$$

where $x_1 = \frac{\partial x_1}{\partial \beta_1} \beta_1$, $\frac{\partial x_1}{\partial \beta_1}$ and γ (the lag angle) are functions of x_{sh0} , M_1 , ν and $\frac{c_f}{c}$. x_{sh0} can be found as a function of M_∞ from flap fixed experiments with $\beta = \beta_0$. β_0 should be the steady flow equilibrium value for a given M_∞ . It will be expected in the present asymmetric flow example (i.e., a single shock exists at the onset) that a "rectification effect" will exist so that $\beta_0 = \beta_0(t)$ as in the spoiler/flap experiment; therefore, x_{sh0} will vary during the transient growth of oscillations in a buzz situation. We may take:

$$x_{sh0} \cong (x_{sh0})_{\beta=0} + \frac{dx_{sh}}{d\beta} \beta_0$$

where $\frac{dx_{sh}}{d\beta}$ is found from experiment or theory, or as an approximation:

$$x_{sh0} \cong (x_{sh0})_{\beta=0} + \frac{\partial x_1}{\partial \beta_1} \beta_0$$

The ratio $\frac{M_\infty}{M_1}$ should also be found as M_∞ is varied. Experimental results found by Nakamura (Ref 64) indicate that $\gamma = 0$ for $\nu \cong .09$, however, other tests and the time lag theory previously discussed indicate that γ may be nearer 80° or so.⁶³ γ is found to be primarily a function of frequency, ν .

The effect of shock motion on ΔC_H is complex. An equivalent change to the freestream Mach number is approximately:

$$\tilde{M}_\infty = \frac{M_\infty}{M_1} \tilde{M}_1 = \frac{M_\infty}{M_1 a_1} U_s$$

where $U_s = - \frac{dx_{sh}}{dt} = - c \dot{x}_{sh}$ and for transient motion:

$$U_s \cong - c \left[\dot{x}_{sh0} + \frac{\partial x_1}{\partial \beta_1} \dot{\beta}_1 \cos (\omega t - \gamma) - \omega \frac{\partial x_1}{\partial \beta_1} \beta_1 \sin (\omega t - \gamma) \right]$$

where Eckhaus' theory could be used to find values of $\frac{\partial x_1}{\partial \beta_1}$ and γ which are based on average values of ν and x_{sh0} over a cycle of motion.

We then have:

$$\tilde{M}_\infty \cong - \frac{M_\infty c}{M_1 a_1} [\dot{x}_{sh0} + \frac{\partial x_1}{\partial \beta_1} \dot{\beta}_1 \cos (\omega t - \gamma) - \omega \frac{\partial x_1}{\partial \beta_1} \beta_1 \sin (\omega t - \gamma)]$$

or with the approximation $\dot{x}_{sh0} \cong \frac{\partial x_1}{\partial \beta_1} \dot{\beta}_0$ we have:

$$\tilde{M}_\infty \cong - \frac{M_\infty c}{M_1 a_1} \frac{\partial x_1}{\partial \beta_1} [\dot{\beta}_0 + \dot{\beta}_1 \cos (\omega t - \gamma) - \omega \beta_1 \sin (\omega t - \gamma)]$$

In an actual buzz case the coupling terms $\frac{\partial x_1}{\partial \beta_1}$ and γ are expected to be functions of flap amplitude as well as shear-layer displacement thickness. The present relation can be written for the limit-cycle case as:

$$\tilde{M}_\infty \cong \frac{M_\infty c}{M_1 a_1} \frac{\partial x_1}{\partial \beta_1} [\omega \beta_1 \sin (\omega t - \gamma)]$$

or as:

$$\tilde{M}_\infty \cong k_1 \beta_1 \cos (\omega t - \frac{\pi}{2} - \gamma)$$

which shows that the equivalent Mach number perturbation lags flap motion by $90^\circ + \gamma$, where k_1 and γ depend on frequency and M_1 . This relation may be compared to the spoiler height perturbation in the coupled spoiler/flap experiment.

$$\tilde{h} = k \beta_1 \cos \omega t$$

where

$$h = h_0 + \tilde{h}$$

The Mach number perturbation may be modified, as in the spoiler case, to account for a lag in bubble growth which is also expected in the compressible flow behind a moving shock. A modified theory would yield an "effective" Mach number perturbation:

$$\tilde{M}_{\infty \text{eff}} \cong A k_1 \beta_1 \cos \left(\omega t - \frac{\pi}{2} - \gamma - \alpha \right)$$

For a critical range of γ it is probable that effects due to lag in bubble growth are countered by the effect of instantaneous shock location; that is, the shear-layer reattachment point may not move significantly across the flap surface. Also, a correction to bubble dead air pressure is required due to unsteady momentum flux to the bubble and alterations to bubble circulation. These corrections would involve evaluation of shear layer parameters and a steady-flow bubble growth law. The correction would be applied to steady-flow ΔC_H results as in the spoiler case.

The net result is that unsteady incremental hinge moment may be approximately found as a function of β_0 , β_1 , ν and M_∞ . Under the apparently reasonable assumption that flap motion is approximately of the form:

$$\beta = \beta_0 + \beta_1 \cos \omega t$$

We can proceed to find ΔC_H in describing function form:

$$\Delta C_H \cong \Delta C_{H_0} + N_1 k_1 \beta_1 \cos \omega t + \frac{N_2 k_1}{\omega} (-\omega \beta_1 \sin \omega t)$$

and for transient motion we have:

$$\begin{aligned} \Delta C_H \cong & \Delta \hat{C}_{H_0} + \left[\hat{N}_1 + \frac{\hat{N}_2}{\omega} \lambda \right] k_1 \beta_1 \cos \omega t \\ & + \frac{\hat{N}_2 k_1}{\omega} (-\omega \beta_1 \sin \omega t) \end{aligned}$$

where $\lambda \equiv \frac{\dot{\beta}_1}{\beta_1}$ and terms in the series are found in the obvious way, with $\frac{\Delta C_{H_1}}{\Delta M_\infty} = N_1 + i N_2$ and $\frac{\Delta \hat{C}_{H_1}}{\Delta M_\infty} = \hat{N}_1 + i \hat{N}_2$.

Consider now a qualitative discussion of transient and limit cycle behavior for the buzz problem based on results for the coupled spoiler/flap problem and other experimental data.

In the experiment with spoiler and flap coupled we have the coupling relation:

$$\begin{aligned} h &= h_{o_1} + k \beta = (h_{o_1} + k \beta_0) + k \beta_1 \cos \omega t \\ &= h_o + \Delta h \cos \omega t \end{aligned}$$

for flap motion of the form:

$$\beta = \beta_0 + \beta_1 \cos \omega t$$

Thus, the mean spoiler height is $h_0 = h_{01} + k \beta_0$ and spoiler amplitude is $\Delta h = k \beta_1$. The spoiler oscillates in phase with the flap.

In the buzz problem for flap motion of the form $\beta = \beta_0 + \beta_1 \cos \omega t$ we have an approximate mean effective freestream Mach number.

$$(M_\infty)_{\text{eff}} \cong M_\infty + \frac{\beta_0}{\frac{d \beta_0}{d M_\infty}}$$

where $\frac{d \beta_0}{d M_\infty}$ may be found from experiment or we may use as an approximation:

$$\frac{d \beta_0}{d M_\infty} \cong \frac{M_1 \frac{dx_{sh}}{d M_1}}{M_\infty \frac{dx_{sh}}{d \beta}}$$

with the derivatives on the RHS found by a suitable transonic flow theory. The mean effective freestream Mach number is analogous to mean spoiler height.

We also have a shock relation:

$$\begin{aligned} x_{sh} &= x_{sh0} + x_1 \cos (\omega t - \gamma) \\ &= x_{sh0} + \frac{\partial x_1}{\partial \beta_1} \beta_1 \cos (\omega t - \gamma) \end{aligned}$$

and an effective freestream Mach number perturbation that is approximately:

$$\begin{aligned} (\tilde{M}_\infty)_{\text{eff}} &\cong A k_1 \beta_1 \cos [\omega t - (\frac{\pi}{2} + \gamma + \alpha)] \\ &= (\Delta M_\infty)_{\text{eff}} \cos [\omega t - (\frac{\pi}{2} + \gamma + \alpha)] \end{aligned}$$

with

$$A \approx 1.0; \quad k_1 = \frac{M_\infty c}{M_1 a_1} \frac{\partial x_1}{\partial \beta_1} \omega$$

This relation is valid for $|\dot{\beta}_0| \ll |\dot{\beta}_1|$ and $|\lambda| \ll 1$.

We may tentatively assume that the lag in bubble growth is negligible and we have:

$$(\tilde{M}_\omega)_{\text{eff}} \approx k_1 \beta_1 \cos [\omega t - (\frac{\pi}{2} + \gamma)]$$

The above result is essentially a quasi-steady model of shock effects.

At the onset of instability we find from Nakamura's results⁶⁴ that the perturbations act nearly in quadrature with flap motion for $\nu \approx .09$. Results based on the time lag theory and flap forced testing indicate that the perturbations would lag flap motion by a greater amount. Perhaps this anomaly relates to the increasing shear layer displacement thickness effect as oscillations build in amplitude. In order to qualitatively study the stability of a coupled system, therefore, it appears appropriate to tentatively take $\gamma \approx 0$. In this case, perturbations are in-quadrature with flap deflection and we have a result which also includes the pressure perturbations to the separated region, which are assumed to act in phase with $(\tilde{M}_\omega)_{\text{eff}}$:

$$\Delta C_H \approx \Delta \hat{C}_{H_0} + \frac{\hat{N}_2}{\omega} \lambda k_1 \beta_1 \cos \omega t + \frac{\hat{N}_2 k_1}{\omega} (-\omega \beta_1 \sin \omega t)$$

When the above expression is substituted into the equation of motion (4.5) we find as in section 3:

$$\beta_0 \approx \frac{\Delta \hat{C}_{H_0}}{\frac{C}{(q_\infty b_f c_f^2 - C_{H\beta})}}$$

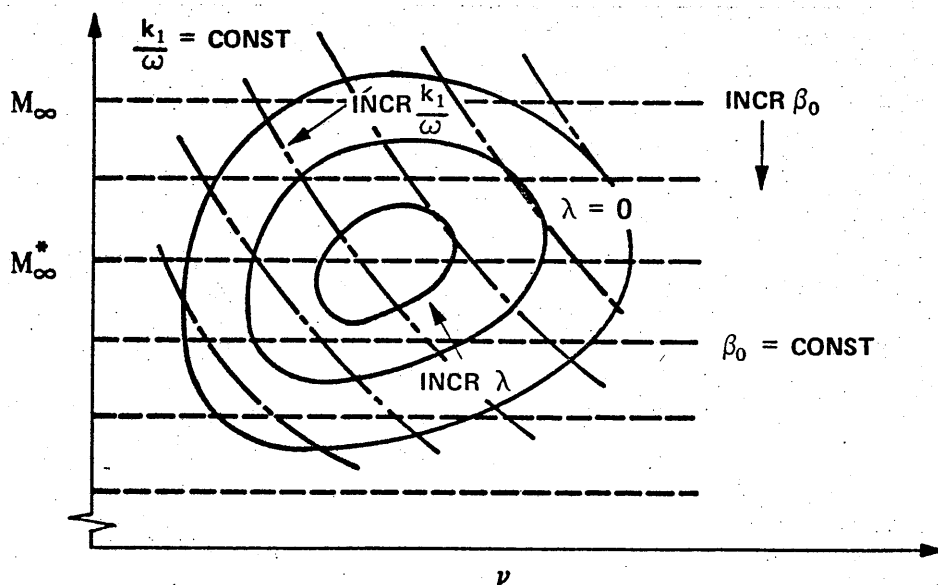
$$\lambda \approx \frac{1}{2A} \left[\left(\frac{c_f}{U_\infty} C_{H\beta} + \frac{\hat{N}_2 k_1}{\omega} \right) q_\infty b_f c_f^2 - B \right]$$

$$\omega^2 \approx -\lambda^2 + \left[\frac{C - C_{H\beta} q_\infty b_f c_f^2}{A} \right]$$

Since the effect of freestream Mach number on ΔC_H in steady flow is expected to be qualitatively similar to the spoiler height effect, it follows that the describing function $\frac{\Delta \hat{C}_{H1}}{\Delta M_\infty}$ will be of similar form. Refer to Figure 3.4 and note that a maximum value of $\left| \frac{\Delta C_{H1}}{\Delta h} \right|$ occurs at about $h = .065$. In the buzz case we expect a similar dependence on M_∞ . The effect of Δh on $\frac{\Delta C_{H1}}{\Delta h}$ is expected to reflect the dependence of $\frac{\Delta C_{H1}}{\Delta M_\infty}$ on $(\Delta M_\infty)_{\text{eff}}$ where $(\Delta M_\infty)_{\text{eff}} \cong A k_1 \beta_1 \cong \frac{M_\infty c}{M_1 a_1} \frac{\partial x_1}{\partial \beta_1} \omega \beta_1$. Thus, we can study system stability with the notion that M_∞ relates to h_{01} and k_1 relates to k for the spoiler and flap system.

Unlike the spoiler and flap system, however, k_1 is a function of M_∞ , β_0 , β_1 , $\frac{c_f}{c}$ and ν . As M_∞ and ν increase we expect $\frac{\partial x_1}{\partial \beta_1}$ to decrease. This follows because in transonic flow as $\frac{\omega b}{U_\infty}$ increases, receding waves which are propagated upstream tend to cancel and for $\frac{\omega b}{U_\infty} \gg 1$ upstream effects can be neglected. The parameter $\frac{k_1}{\omega}$ then will decrease as ν or M_∞ increase, and $\frac{\hat{N}_2 k_1}{\omega}$ will decrease as ν increases.

The solution for stability parameters in the buzz case may be represented as in the sketch below:

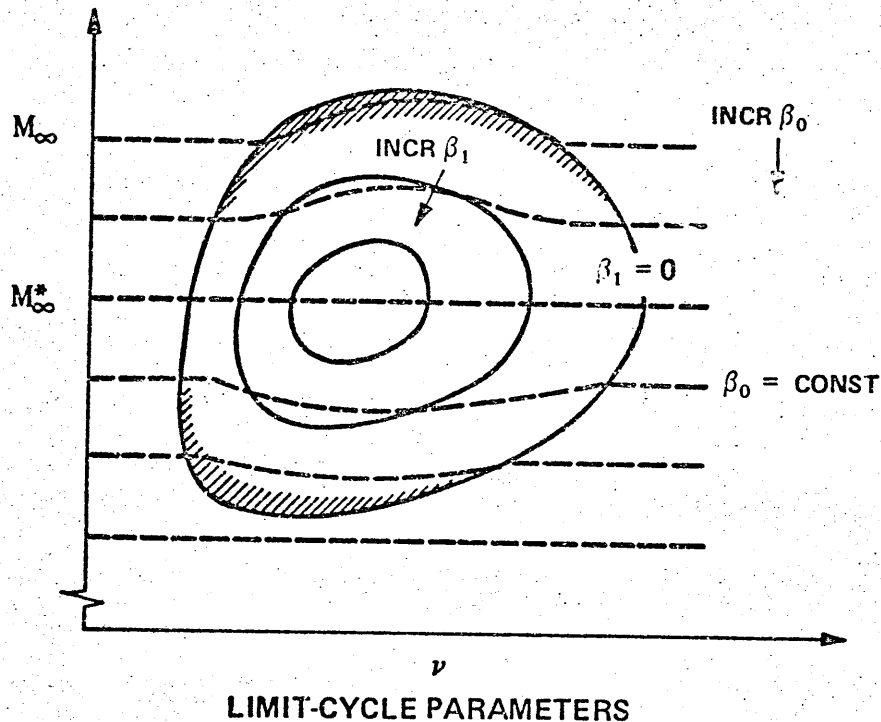


SYSTEM STABILITY PARAMETERS

The $\lambda = 0$ curve is a closed curve due to the effect of ν at low and high frequencies and because $\frac{N_2 k_1}{\omega} \rightarrow 0$ at high frequencies. The horizontal line labeled M_∞^* relates to the M_∞ value for $\left| \frac{\Delta \hat{C}_{H_1}}{\Delta M_\infty} \right|_{\max}$. The horizontal lines labeled $\beta_0 = \text{const}$ represent "steady flow equilibrium" flap angles. The coupling parameter $\frac{k_1}{\omega}$, being a function of M_∞ and ν , acts to distort the curves of constant λ . A point within the $\lambda = 0$ contour would relate to a self-excited oscillation.

Limit-cycle behavior may also be predicted even in the absence of amplitude effects on $\frac{\partial x_1}{\partial \beta_1}$ and γ since $\frac{\Delta \hat{C}_{H_1}}{\Delta M_\infty} \rightarrow 0$ as $(\Delta M_\infty)_{\text{eff}}$ increases as in the spoiler/flap case.

A sketch of limit cycle parameters, as in Figure 3.7, would likely appear as follows:



Maximum amplitude limit-cycle oscillations relate to M_∞^* and a critical value of ν (somewhere around $\nu = .10$). For lower frequencies the coupling parameter $\frac{k_1}{\omega}$ is increased and in the actual case γ , the shock lag angle, would increase as β_1 increases, thus allowing for large amplitude limit cycles at low frequencies. Curves of constant amplitude, β_1 , are therefore skewed to the left. At high frequencies the coupling

parameter $\frac{k_1}{\omega}$ decreases. At $M_\infty = M_\infty^*$ the $\beta_1 = 0$ locations will likely correspond to the $\lambda = 0$ points in the preceding sketch. For values of M_∞ above or below M_∞^* we expect that a "rectification effect" (as in the spoiler experiment) would serve to modify β_0 , with the degree of rectification depending on amplitude, β_1 .

The shaded regions in the sketch lie outside the previous $\lambda = 0$ curve, and result primarily from the rectification effect. These regions would relate to "hard oscillator" behavior.

In conclusion, it can be stated that some similarities appear to exist between the coupled spoiler/flap problem and control-surface-buzz. Significant additional effects, such as the lag in shock motion, exist and of course mechanisms of the shock/shear layer interaction are not well understood. It appears, however, that the clue to Mach number effect on stability can be found by study of the steady flow hinge-moment dependence on Mach number. Frequency effect relates primarily to coupling strength and a phase relation that is associated with shock motion. It is obvious that further tests are necessary in order to gain insight into the many unknown aspects of this problem. Also, the effect of three-dimensional flow needs to be studied since it is likely that it is significant. It is probably very important in other related cases of shock/shear-layer/body interactions such as the problem of wing rock, for example.

5. CONCLUSIONS

Specific conclusions have previously been discussed in appropriate sections. What follows is a brief summary and final discussion.

It has been shown in the experiments with an oscillating spoiler that significant dynamic effects modify the behavior of an unsteady separating flow. Quasi-steady models are not appropriate even at low frequencies of oscillation. It appears, however, that a useful analytical model can be derived if mass flow in a separated region is considered. A quasi-steady model for mass flux in the reattachment region of a separated bubble and the assumption of propagation of mass upstream at a mean backflow velocity is used in order to predict the experimentally observed lag in bubble growth. The quasi-steady assumption can be used in conjunction with the momentum equation in order to find the perturbation to "dead air" pressure that accompanies unsteady mass flux. The resultant analytical model of bubble behavior is then used to predict experimental results with good qualitative and fair quantitative agreement.

In the study of a laminar leading-edge separation bubble on an airfoil, it is shown that a model based on an unsteady inviscid flowfield acting on a small quasi-steady bubble is useful in order to predict bubble instability. Instability is taken to occur when the turbulent mixing and reattachment process can not take place due to an extreme pressure rise or velocity drop across the bubble. In order for the bubble to grow, mass must be forced into the bubble at a sufficient rate to cause growth. In unsteady flow then we find that an overshoot of static stall angle may occur. This may be due to airfoil or flap motion. As the static stall angle is overshoot, bubble instability first occurs and is followed by a larger rate dependent overshoot which is necessary in order for the bubble to grow. As the bubble continues to grow, consideration must be given to the effect of viscosity on energy dissipation within the bubble. At very high frequencies it may be reasonable to assume (as Ham did in Ref 53) constancy of circulation over the flowfield.

In the case of control-surface-buzz (or other cases which involve shock-induced separation), it is argued that prior to trailing-edge pressure divergence the motion of an airfoil or flap will control the inviscid flowfield (as in attached flow) so that unsteady pressure perturbations are realized at the shock. These perturbations can be related to shock motion across the airfoil surface and subsequent unsteady boundary-layer separation. Dynamics of the separated region are expected to follow the predictions of the separation bubble model when compressibility corrections are made. The assumption of quasi-steady shear layer behavior also appears justified (where the shear-layer above is taken to be external to a separation bubble).

The phenomenon of self-excited oscillations and subsequent limit-cycle motion was investigated for the coupled spoiler/flap configuration and discussed in the case of control-surface-buzz. It appears that the nonlinear nature of these problems is strongly related to the steady-flow functional dependence of incremental hinge-moment coefficient on either spoiler height or freestream Mach number. Additional nonlinear effects in unsteady flow, such as amplitude dependent phase lags and coupling, may not be necessary additions to an analytical model which is used to describe the coupled system motion.

More information is needed in order to better model the fluid dynamics of unsteady separating flows. In particular, it is desirable to study dissipation of energy in separation bubbles in the unsteady case, also the character and role of convection, diffusion and creation of vorticity needs to be studied, and perhaps most important are the effects of spanwise flow in the bubble interior, end plates, and other three-dimensional flow considerations.

The most significant results of the present study relate to better understanding of and a useful analytical model for the dynamics of an unsteady separation bubble. It is expected that these results are appropriate when considering most, if not all, of the aerodynamic problem areas mentioned in the introduction.

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