

CRANFIELD UNIVERSITY

AYUSH NUWAL

EVALUATION OF OFF-ROAD UNINHABITED GROUND VEHICLE  
MOBILITY USING DISCRETE ELEMENT METHOD AND  
SCALABILITY INVESTIGATION

CRANFIELD DEFENCE AND SECURITY

THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY  
Academic Year: 2018 - 2022

Supervisor: Dr John T. Economou  
Associate Supervisor: Mr Ajay Kumar  
September 2022



CRANFIELD UNIVERSITY

CRANFIELD DEFENCE AND SECURITY

THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Academic Year: 2018 - 2022

AYUSH NUWAL

Evaluation of off-road uninhabited ground vehicle mobility using  
discrete element method and scalability investigation

Supervisor: Dr John T. Economou  
Associate Supervisor: Mr Ajay Kumar

This thesis is submitted in fulfilment of the requirements for the  
degree of Doctor of Philosophy

© Cranfield University 2022. All rights reserved. No part of this  
publication may be reproduced without the written permission of the  
copyright owner.

*To my Mom, Smt. Manju Nuwal,  
Who always encourages and motivates me to pursue my dreams.*



## ABSTRACT

Full-scale military vehicles are teaming up with uninhabited ground vehicles (UGVs) to improve the success rate of tactical operations on off-road terrains. UGV can perform initial mobility testing on soft soils during missions to evaluate the performance (e.g., go/ no-go) of full-scale military vehicles. Therefore, the concept of scale model testing is proposed.

The scale model testing can be divided into two parts, i.e. the scalability of soil and the scalability of the tyre-soil interaction. The scalability of soil is defined as a relationship between the mechanical properties of an in-situ terrain (heterogeneous) system and a laboratory (homogeneous) soil system while accounting for the differences in sand, silt and clay particle shapes and size distributions. Physical properties such as moisture content, bulk density, compaction, and interparticle forces are kept the same for laboratory and in-situ terrain conditions. The 2NS and fine-grained sands were modelled using the discrete element method with Edinburgh elastic-plastic adhesion contact model. It was found that the scalability depends on the testing conditions and the soil's nature. The heterogeneity of soil affects the cohesive and adhesive forces present in the soil system. The pressure-sinkage and shear stress vs shear displacement relationships are found scalable. The cone index vs depth relation is not scalable.

Further, the scalability of tyre-soil interaction is established using the dimensional analysis method to establish similarity in the full-scale and scaled systems. The developed non-dimensional parameters are kept the same in both systems. In this research, the lightweight Armoured Personnel Carriers such as the FED Alpha and Land Rover are considered as the full-scale systems (upper boundary) and UGVs for example, the Husky or Warthog as a scaled system (lower boundary). Consequently, the tyre-soil interaction behaviour is similar in this specific tyre size and loading range. The full-scale tyre modelled is FED Alpha tyre 335/65R22.5 and is scaled down by size to scales 0.7, 0.5 and 0.25. Six different terrain simulation models of both sands were prepared with cone indexes ranging from 14.79 kPa to 149 kPa. It was found that the drawbar pull and tractive efficiency vs slip relations are scalable. The mean error in drawbar pull prediction w.r.t. NATO experiments is 12% and 9% for 2NS and fine-grained sand, respectively. The drawbar pull varies from square to cubic power w.r.t. the scale of the system. The gross traction varies with square power w.r.t. the scale of the system. The tractive efficiency is constant w.r.t. the scale of the system. It is concluded that a 0.5 scale system can predict the full-scale system's mobility performance on sands. This key finding can be used to design and develop cost-effective and lighter UGVs to support full-scale military vehicles on the battlefield. The limitation of the DEM technique is that it is computationally expensive as the number of particles increases.

## **ACKNOWLEDGEMENTS**

I want to extend my thanks to Cranfield University, for providing financial support to the research, particularly Professor Amer Hameed, who supported my research interest. I wish to acknowledge and thank Dr Paramsothy Jayakumar (DEVCOM GVSC) and Dr David Gorsich (DEVCOM GVSC) for partial sponsorship of research work and studentship.

I would like to express my gratitude to my supervisor, Mr Ajay Kumar, for his enthusiasm and thought-provoking discussions, and also grateful for his time during the Christmas breaks and after-office hours. I am grateful to my supervisor, Dr John Economou, for his encouragement and motivation to drive and articulate the research work. I would also like to thank Dr Thiru Thirulogasingam for his support and guidance throughout the research. I would also like to thank Mr James Harber and Mr Rob Cooper for supporting the experimental work and guidance along the way.

Finally, and most of all, I would like to offer my heartfelt gratitude to my parents (Mrs Manju Nuwal and Mr Vishnu Swaroop Nuwal), Bade papa, wife (Mrs Aarti Nuwal) and younger sister (Ms Aditi Nuwal) for all their support and understanding throughout this journey. I would also like to thank my friend Sh. Mohamed Abdulrahman Ebrahim Al-Khalifa for his support and motivation.

# TABLE OF CONTENTS

|                                                      |       |
|------------------------------------------------------|-------|
| ABSTRACT .....                                       | ii    |
| ACKNOWLEDGEMENTS.....                                | iii   |
| LIST OF FIGURES .....                                | vii   |
| LIST OF TABLES .....                                 | xx    |
| LIST OF EQUATIONS.....                               | xxiii |
| LIST OF ABBREVIATIONS .....                          | xxxix |
| 1 INTRODUCTION.....                                  | 34    |
| 1.1 Graph theory and the energy estimation.....      | 38    |
| 1.2 Background.....                                  | 50    |
| 1.2.1 Mobility prediction methods.....               | 50    |
| 1.2.2 Basic soil theory .....                        | 52    |
| 1.2.3 Scale model testing .....                      | 54    |
| 1.2.4 Scaling of soil properties .....               | 54    |
| 1.3 Research Question .....                          | 61    |
| 1.4 Aim and Objectives .....                         | 61    |
| 1.5 Assumptions and constraints .....                | 62    |
| 2 LITERATURE REVIEW .....                            | 64    |
| 2.1 Introduction .....                               | 64    |
| 2.2 Mobility performance parameters.....             | 65    |
| 2.2.1 Longitudinal slip ratio (LSR) .....            | 66    |
| 2.2.2 Motion resistance .....                        | 67    |
| 2.2.3 Gross traction.....                            | 70    |
| 2.2.4 Drawbar pull .....                             | 73    |
| 2.2.5 Tractive efficiency .....                      | 76    |
| 2.3 Terramechanics based – Performance metrics ..... | 77    |
| 2.3.1 Vehicle cone index (VCI).....                  | 78    |
| 2.3.2 Wheel mobility number.....                     | 79    |
| 2.3.3 Nominal ground pressure .....                  | 82    |
| 2.3.4 Mean maximum pressure.....                     | 83    |
| 2.4 Soil system .....                                | 85    |
| 2.4.1 Cone index .....                               | 85    |
| 2.4.2 Bevameter technique .....                      | 88    |
| 2.5 Tyre construction .....                          | 99    |
| 2.5.1 Overall width (b) .....                        | 100   |
| 2.5.2 Overall diameter (d).....                      | 101   |
| 2.5.3 Section height (h) .....                       | 101   |
| 2.5.4 Aspect ratio .....                             | 101   |
| 2.5.5 Load index.....                                | 101   |
| 2.5.6 Tyre deflection.....                           | 101   |
| 2.5.7 Static loaded radius (slc) .....               | 102   |

|                                                              |     |
|--------------------------------------------------------------|-----|
| 2.5.8 Speed rating .....                                     | 102 |
| 2.5.9 Tyre tread .....                                       | 102 |
| 2.5.10 Mode of operation .....                               | 102 |
| 2.6 Tyre deflection test .....                               | 103 |
| 2.7 Tyre-surface contact area test .....                     | 107 |
| 2.8 Mobility prediction models .....                         | 111 |
| 2.8.1 Empirical method .....                                 | 112 |
| 2.8.2 Semi-empirical method .....                            | 116 |
| 2.8.3 Numerical methods .....                                | 118 |
| 2.9 Scalability testing .....                                | 129 |
| 2.10 Summary .....                                           | 132 |
| 3 THEORY OF SCALABILITY .....                                | 135 |
| 3.1 Introduction .....                                       | 135 |
| 3.2 Tyre deflection and contact area .....                   | 135 |
| 3.3 Dimensional analysis .....                               | 136 |
| 3.3.1 Tyre deflection .....                                  | 136 |
| 3.3.2 Tyre contact area .....                                | 138 |
| 3.3.3 Mobility performance parameters .....                  | 140 |
| 3.4 Experimental plan .....                                  | 145 |
| 3.4.1 Tyre selection .....                                   | 145 |
| 3.4.2 Research plan .....                                    | 146 |
| 3.4.3 Experimental setup .....                               | 147 |
| 3.4.4 Test procedure .....                                   | 150 |
| 3.4.5 Calibration of sensor .....                            | 152 |
| 3.5 Results and discussion .....                             | 154 |
| 3.5.1 Full-scale tyre .....                                  | 154 |
| 3.5.2 Scaled tyres .....                                     | 161 |
| 3.5.3 Similarity analysis .....                              | 166 |
| 3.6 Summary .....                                            | 172 |
| 4 SCALABILITY OF SOIL .....                                  | 175 |
| 4.1 Introduction .....                                       | 175 |
| 4.2 Problem formulation .....                                | 177 |
| 4.3 Soil characterization .....                              | 179 |
| 4.3.1 Bevameter testing .....                                | 181 |
| 4.3.2 Cone penetrometer test .....                           | 187 |
| 4.4 Discrete element method .....                            | 188 |
| 4.4.1 Algorithm .....                                        | 188 |
| 4.4.2 Equations of motion .....                              | 190 |
| 4.4.3 Edinburgh elastic-plastic adhesion contact model ..... | 191 |
| 4.4.4 Particle modelling .....                               | 199 |
| 4.4.5 Numerical integration method .....                     | 200 |
| 4.5 Numerical experiments .....                              | 202 |

|                                                                     |     |
|---------------------------------------------------------------------|-----|
| 4.5.1 Soil data .....                                               | 202 |
| 4.5.2 Laboratory model experiment.....                              | 204 |
| 4.5.3 In-situ soil model testing.....                               | 223 |
| 4.5.4 Discussion.....                                               | 261 |
| 4.6 Summary .....                                                   | 299 |
| 5 SCALABILITY OF TYRE-SOIL INTERACTION.....                         | 303 |
| 5.1 Introduction .....                                              | 303 |
| 5.2 Problem formulation.....                                        | 305 |
| 5.3 Theory of scalability .....                                     | 306 |
| 5.4 Soil models .....                                               | 310 |
| 5.5 Test equipment .....                                            | 312 |
| 5.6 Test procedure.....                                             | 315 |
| 5.7 Results & discussion.....                                       | 321 |
| 5.7.1 Model validation and comparison with NRMM .....               | 321 |
| 5.7.2 Homogeneous vs heterogeneous terrain model.....               | 326 |
| 5.7.3 Scalability analysis in the homogeneous terrain models .....  | 331 |
| 5.7.4 Scalability analysis in the heterogeneous terrain models..... | 338 |
| 5.7.5 Effect of cone-index on scalability .....                     | 344 |
| 5.7.6 Effect of normal load on scalability.....                     | 352 |
| 5.7.7 Effect of scaling on scalability .....                        | 358 |
| 5.8 Summary .....                                                   | 365 |
| 6 SUMMARY AND CONCLUSION.....                                       | 369 |
| 6.1 Summary .....                                                   | 369 |
| 6.2 Conclusions .....                                               | 371 |
| 6.3 Research contributions .....                                    | 373 |
| 6.4 Advantages and limitations of techniques.....                   | 376 |
| 7 FUTURE RECOMMENDATION .....                                       | 379 |
| REFERENCES.....                                                     | 380 |
| APPENDICES .....                                                    | 400 |
| Appendix A Specifications of instruments and accessories .....      | 401 |
| Appendix B Experimental data.....                                   | 403 |
| Appendix C Simulation data.....                                     | 414 |
| Appendix D Terms used in statistical analysis .....                 | 452 |

## LIST OF FIGURES

|                                                                                                                                                                                  |    |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| Figure 1-1 (a) REX UGV of Israel (Egozi, 2018); (b) TAROS V2 of Czech Republic (Army technology, 2014).....                                                                      | 35 |
| Figure 1-2 UGV conducting initial mobility testing for the full-scale military vehicle to decide go/no-go condition on off-road terrain (or soft soils) on the battlefield ..... | 36 |
| Figure 1-3 Ground UGVs sizing chart (Clearpath Robotics, no date).....                                                                                                           | 37 |
| Figure 1-4 An analogy between UGV and UAV based on G.P. Kladis et al. (2011) .....                                                                                               | 40 |
| Figure 1-5 UGV mission map showing starting point S and goal E (Picture Credit: Ted Wood, College of Media, Communication and Information, Colorado, Mexico) .....               | 40 |
| Figure 1-6 UGV mission map taken from Google Earth as per coordinates shared by Ted Wood, College of Media, Communication and Information, Colorado, Mexico .....                | 41 |
| Figure 1-7 UGV mission map marked with feasible paths .....                                                                                                                      | 41 |
| Figure 1-8 Graph G graphical representation of feasible paths of the present case .....                                                                                          | 43 |
| Figure 1-9 Adjacency matrix $A_{ij}$ of the present case .....                                                                                                                   | 43 |
| Figure 1-10 $S_{ij}$ matrix representing the distance between waypoints .....                                                                                                    | 45 |
| Figure 1-11 MR matrix representing the motion resistance coefficient.....                                                                                                        | 45 |
| Figure 1-12 $LE_{ij}$ matrix representing the energy lost to overcome motion resistance between waypoints.....                                                                   | 46 |
| Figure 1-13 D matrix representing the drawbar pull coefficient between waypoints .....                                                                                           | 46 |
| Figure 1-14 UE matric representing the useful energy due to drawbar pull between waypoints .....                                                                                 | 47 |
| Figure 1-15 Comparison graphs of the results obtained in the path planning problem.....                                                                                          | 49 |
| Figure 1-16 Flow chart of vehicle mobility .....                                                                                                                                 | 51 |
| Figure 1-17 Stress–strain relationship of an idealized elastoplastic material (Wong (2010)) .....                                                                                | 53 |
| Figure 1-18 Scalability of the soil flow chart .....                                                                                                                             | 55 |
| Figure 1-19 Scalability of the tyre-soil interaction flow chart.....                                                                                                             | 56 |

|                                                                                                                                                                                                                |     |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 1-20 Scales in the Scalability (Biswas and Si, 2014).....                                                                                                                                               | 57  |
| Figure 1-21 The scale triplet (Biswas and Si (2014)).....                                                                                                                                                      | 58  |
| Figure 1-22 NATO in-situ experiment site location of (a) 2NS sand, (b) Fine grain soil .....                                                                                                                   | 59  |
| Figure 1-23 In-situ field (heterogeneous) simulation model for soils .....                                                                                                                                     | 59  |
| Figure 1-24 The scalability of soil properties chart .....                                                                                                                                                     | 60  |
| Figure 2-1 Free body diagram of towing of a rigid wheel on a hard surface .....                                                                                                                                | 69  |
| Figure 2-2 Free body diagram showing the generation of gross traction by a powered wheel on a non-deforming surface.....                                                                                       | 71  |
| Figure 2-3 Gross traction vs slip curve based on Wong (1983) results .....                                                                                                                                     | 72  |
| Figure 2-4 Free body diagram showing the generation of drawbar pull.....                                                                                                                                       | 74  |
| Figure 2-5 Standard cone penetrometer (ASABE standard 2000) .....                                                                                                                                              | 86  |
| Figure 2-6 Bevameter set up (Wong (2010), reprinted by permission from Bekker (1969)).....                                                                                                                     | 88  |
| Figure 2-7 Grain size distribution of the used soils for the Bevameter tests (Apfelbeck et al. (2011)) .....                                                                                                   | 90  |
| Figure 2-8 Density variation in different soil samples (total 9 in number) for the Bevameter tests (Brunskill et al. (2011)).....                                                                              | 91  |
| Figure 2-9 Grain size distribution of the used soils for the Bevameter tests (Brunskill et al. (2011)).....                                                                                                    | 91  |
| Figure 2-10 Grain size distribution of the used soils for the Bevameter tests (Yang et al., 2018).....                                                                                                         | 93  |
| Figure 2-11 (a) Pressure-sinkage test device used to test width dependence (b) Experimental result of width dependence for a 114 mm diameter wheel on a clay/silt mix (Griffith and Spenko (2013, 2014)) ..... | 95  |
| Figure 2-12 Soil moisture and bulk density conditions of the soil samples (M.A. Sadek et al. (2011)) .....                                                                                                     | 98  |
| Figure 2-13 Representation of tyre parameter (R. He et al. (2020)) .....                                                                                                                                       | 100 |
| Figure 2-14 Load-deflection test apparatus (Cranfield University).....                                                                                                                                         | 106 |
| Figure 2-15 Tyre deflection a) on a hard surface, b) soft soil (Plackett (1984)) .....                                                                                                                         | 108 |
| Figure 2-16 General pull-torque-slip relation of tyre on soil (Wismer and Luth (1973)) .....                                                                                                                   | 114 |

|                                                                                                                                                                                                                                                                |     |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 2-17 Overview of tyre / soil interaction model using FEM in LS-DYNA (Wright (2012)) .....                                                                                                                                                               | 119 |
| Figure 2-18 Truck tyre model setup over gravelly soil with normal displacement contour (El-Sayegh et al. (2020)).....                                                                                                                                          | 121 |
| Figure 2-19 Rigid wheel of MER interaction with the Martian soil (a) Drawbar Pull coefficient w.r.t. wheel slip; (b) Wheel sinkage w.r.t. wheel slip (c) Tailing piles behind a MER rover in the JPL sand Rover Test Bed at NASA (Johnson et al. (2017)) ..... | 125 |
| Figure 2-20 An overview of typical Hertz-Mindlin Contact Algorithm (EDEM) .....                                                                                                                                                                                | 127 |
| Figure 3-1 Side view of the experimental setup.....                                                                                                                                                                                                            | 149 |
| Figure 3-2 Bottom view of the experimental setup .....                                                                                                                                                                                                         | 149 |
| Figure 3-3 a) Dial gauge; b) Load cell positioning .....                                                                                                                                                                                                       | 150 |
| Figure 3-4 Front view of the experimental setup .....                                                                                                                                                                                                          | 151 |
| Figure 3-5 Contact patch of 7.5R16 tyre at 310 kPa inflation pressure and 2 kN normal load.....                                                                                                                                                                | 152 |
| Figure 3-6 Calibration setup of load cell .....                                                                                                                                                                                                                | 153 |
| Figure 3-7 Calibration curve of load cell for normal load measurement .....                                                                                                                                                                                    | 153 |
| Figure 3-8 Relationship between normal load and tyre deflection at different inflation pressures of the full-scale tyre.....                                                                                                                                   | 155 |
| Figure 3-9 Relationship between inflation pressure and tyre deflection at different normal loads of full-scale tyre .....                                                                                                                                      | 155 |
| Figure 3-10 Relationship between inflation pressure and tyre contact area at different normal load conditions.....                                                                                                                                             | 156 |
| Figure 3-11 Comparison of developed deflection ratio model based on non-dimensional parameter with the test data .....                                                                                                                                         | 160 |
| Figure 3-12 Comparison of developed contact area ratio model based on non-dimensional parameter with the test data .....                                                                                                                                       | 161 |
| Figure 3-13 Relationship between normal load and tyre deflection at different inflation pressures of the scaled tyre (145/80R13) .....                                                                                                                         | 162 |
| Figure 3-14 Relationship between inflation pressure and tyre deflection at different normal loads of the scaled tyre (145/80R13) .....                                                                                                                         | 163 |
| Figure 3-15 Relationship between normal load and tyre deflection at different inflation pressures of the scaled tyre (135/80R13) .....                                                                                                                         | 163 |
| Figure 3-16 Relationship between inflation pressure and tyre deflection at different normal loads of the scaled tyre (135/80R13) .....                                                                                                                         | 164 |



|                                                                                                                                                                                                              |     |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 3-17 Relationship between inflation pressure and tyre contact area at different normal loads of the scaled tyre (145/80R13) .....                                                                     | 165 |
| Figure 3-18 Relationship between inflation pressure and tyre contact area at different normal loads of scaled tyre (135/80R13) .....                                                                         | 165 |
| Figure 3-19 The geometric similarity conditions holding same tyre deflection ratio ( $\delta h$ ) and tyre contact area ratio ( $A_h2$ ) between the full-scale and the scale-tyre on the test surface ..... | 167 |
| Figure 3-20 Comparison of scale model prediction of deflection ratio based on non-dimensional parameter with the test data of 145/80R13.....                                                                 | 168 |
| Figure 3-21 Relationship between experimental and predicted deflection ratio of the scaled tyre (145/80R13) .....                                                                                            | 168 |
| Figure 3-22 Comparison of scale model prediction of contact area ratio based on non-dimensional parameter with the test data of 145/80R13.....                                                               | 169 |
| Figure 3-23 Relationship between experimental and predicted contact area ratio of the scaled tyre (145/80R13) .....                                                                                          | 169 |
| Figure 3-24 Comparison of scale model prediction of deflection ratio based on non-dimensional parameter with the test data of 135/80R13.....                                                                 | 170 |
| Figure 3-25 Relationship between experimental and predicted deflection ratio of the scaled tyre (135/80R13) .....                                                                                            | 171 |
| Figure 3-26 Comparison of scale model prediction of contact area ratio based on non-dimensional parameter with the test data of 135/80R13.....                                                               | 171 |
| Figure 3-27 Relationship between experimental and predicted contact area ratio of the scaled tyre (135/80R13) .....                                                                                          | 172 |
| Figure 4-1 a) Grey colour – homogeneous soil sample; b) Real terrain model (Jayakumar (2016)) .....                                                                                                          | 179 |
| Figure 4-2 Soil texture triangle developed by United States Department of Agriculture (R. He et al. (2020)).....                                                                                             | 180 |
| Figure 4-3 Bevameter (NATO NRMM CDT (2018)).....                                                                                                                                                             | 182 |
| Figure 4-4 Shear ring used in bevameter (NATO NRMM CDT (2018)) .....                                                                                                                                         | 185 |
| Figure 4-5 Direct shear box test apparatus .....                                                                                                                                                             | 186 |
| Figure 4-6 Cone penetrometer test (NATO NRMM CDT (2018)) .....                                                                                                                                               | 188 |
| Figure 4-7 A schematic overview of the DEM Material model .....                                                                                                                                              | 189 |
| Figure 4-8 Calculation cycle for DEM .....                                                                                                                                                                   | 190 |
| Figure 4-9 Boundary condition of the box 0.5 m x 0.5 m x 0.8 m with red markings gets auto updated in Altair EDEM software .....                                                                             | 190 |

|                                                                                                                                                                             |     |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 4-10 EpAM normal force – displacement relationship (a) non-linear (b) linear (EDEM (2014)).....                                                                      | 192 |
| Figure 4-11 Grain size analysis (NATO NRMM CDT (2018)).....                                                                                                                 | 203 |
| Figure 4-12 1) 2NS sand sample 2) Fine grain soil on field (NATO CDT – 308 (2020)).....                                                                                     | 203 |
| Figure 4-13 3D 3-sphere particle model.....                                                                                                                                 | 205 |
| Figure 4-14 Direct shear box set up .....                                                                                                                                   | 206 |
| Figure 4-15 Pressure sinkage test set up.....                                                                                                                               | 207 |
| Figure 4-16 The total energy vs time plot of particle generation inside the box .....                                                                                       | 207 |
| Figure 4-17 Standard cone penetrometer model.....                                                                                                                           | 208 |
| Figure 4-18 Direct shear box simulation results in comparison with NATO CDT-308 (2020) experimental results for 2NS sand .....                                              | 212 |
| Figure 4-19 Direct shear box simulation results in comparison with NATO CDT-308 (2020) experimental results for FGS.....                                                    | 213 |
| Figure 4-20 Pressure sinkage simulation images of Dia 0.1 m plate size for 2NS sand in a normal loading condition to soil sample .....                                      | 215 |
| Figure 4-21 Pressure sinkage simulation results in comparison with NATO CDT-308 (2020) experimental results of Dia 0.1 m plate size for 2NS sand in loading condition.....  | 216 |
| Figure 4-22 Pressure sinkage simulation results in comparison with NATO CDT-308 (2020) experimental results of Dia 0.15 m plate size for 2NS sand in loading condition..... | 217 |
| Figure 4-23 Pressure sinkage simulation results in comparison with NATO CDT-308 (2020) experimental results of Dia 0.1 m plate size for FGS in loading condition .....      | 217 |
| Figure 4-24 Pressure sinkage simulation results in comparison with NATO CDT-308 (2020) experimental results of Dia 0.15 m plate size for FGS in loading condition .....     | 218 |
| Figure 4-25 Cone penetrometer laboratory simulation results in comparison with NATO CDT-308 (2020) experimental results for 2NS sand.....                                   | 221 |
| Figure 4-26 Cone penetrometer laboratory simulation results in comparison with NATO CDT-308 (2020) experimental results for FGS .....                                       | 221 |
| Figure 4-27 Average cone penetrometer laboratory simulation results with the polynomial fit for 2NS sand.....                                                               | 222 |

|                                                                                                                                                                                                              |     |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 4-28 Average cone penetrometer laboratory simulation results with the polynomial fit for FGS .....                                                                                                    | 222 |
| Figure 4-29: Heterogeneous terrain model created in six different patches in EDEM for both the soil types .....                                                                                              | 225 |
| Figure 4-30 (a) Particle percentage distribution at six different locations in Terrain model for both the soil models; (b) Particle size distribution of heterogeneous terrains of both the soil models..... | 226 |
| Figure 4-31 DEM simulation model of the shear ring test on the heterogeneous terrain .....                                                                                                                   | 227 |
| Figure 4-32 Pressure sinkage simulation model of Dia 0.15 m diameter plate size on the 2NS sand.....                                                                                                         | 228 |
| Figure 4-33 Shear ring simulation results at location – 1 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand .....                           | 233 |
| Figure 4-34 Shear ring simulation results at location – 2 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand .....                           | 233 |
| Figure 4-35 Shear ring simulation results at location – 3 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand .....                           | 234 |
| Figure 4-36 Shear ring simulation results at location – 4 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand .....                           | 234 |
| Figure 4-37 Shear ring simulation results at location – 5 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand .....                           | 235 |
| Figure 4-38 Shear ring simulation results at location – 6 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand .....                           | 235 |
| Figure 4-39 Shear ring simulation results at location – 1 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil .....                           | 236 |
| Figure 4-40 Shear ring simulation results at location – 2 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil .....                           | 236 |
| Figure 4-41 Shear ring simulation results at location – 3 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil .....                           | 237 |

|                                                                                                                                                                                    |     |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 4-42 Shear ring simulation results at location – 4 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil ..... | 237 |
| Figure 4-43 Shear ring simulation results at location – 5 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil ..... | 238 |
| Figure 4-44 Shear ring simulation results at location – 6 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil ..... | 238 |
| Figure 4-45 Pressure sinkage simulation results at location - 1 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand .....                        | 243 |
| Figure 4-46 Pressure sinkage simulation results at location - 2 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand .....                        | 244 |
| Figure 4-47 Pressure sinkage simulation results at location - 3 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand .....                        | 244 |
| Figure 4-48 Pressure sinkage simulation results at location - 4 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand .....                        | 245 |
| Figure 4-49 Pressure sinkage simulation results at location - 5 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand .....                        | 245 |
| Figure 4-50 Pressure sinkage simulation results at location - 6 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand .....                        | 246 |
| Figure 4-51 Pressure sinkage simulation results at location - 1 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand .....                       | 246 |
| Figure 4-52 Pressure sinkage simulation results at location - 2 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand .....                       | 247 |
| Figure 4-53 Pressure sinkage simulation results at location - 3 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand .....                       | 247 |
| Figure 4-54 Pressure sinkage simulation results at location - 4 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand .....                       | 248 |
| Figure 4-55 Pressure sinkage simulation results at location - 5 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand .....                       | 248 |
| Figure 4-56 Pressure sinkage simulation results at location - 6 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand .....                       | 249 |
| Figure 4-57 Pressure sinkage simulation results at location - 1 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS .....                             | 249 |
| Figure 4-58 Pressure sinkage simulation results at location - 2 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS .....                             | 250 |

|                                                                                                                                                                                                                                                                                     |     |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 4-59 Pressure sinkage simulation results at location - 3 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS .....                                                                                                                              | 250 |
| Figure 4-60 Pressure sinkage simulation results at location - 4 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS .....                                                                                                                              | 251 |
| Figure 4-61 Pressure sinkage simulation results at location - 5 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS .....                                                                                                                              | 251 |
| Figure 4-62 Pressure sinkage simulation results at location - 6 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS .....                                                                                                                              | 252 |
| Figure 4-63 Pressure sinkage simulation results at location - 1 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS .....                                                                                                                             | 252 |
| Figure 4-64 Pressure sinkage simulation results at location - 2 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS .....                                                                                                                             | 253 |
| Figure 4-65 Pressure sinkage simulation results at location - 3 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS .....                                                                                                                             | 253 |
| Figure 4-66 Pressure sinkage simulation results at location - 4 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS .....                                                                                                                             | 254 |
| Figure 4-67 Pressure sinkage simulation results at location - 5 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS .....                                                                                                                             | 254 |
| Figure 4-68 Pressure sinkage simulation results at location - 6 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS .....                                                                                                                             | 255 |
| Figure 4-69 Cone penetrometer terrain simulation results in comparison with NATO CDT-308 (2020) experimental and laboratory simulation results for 2NS sand.....                                                                                                                    | 258 |
| Figure 4-70 Average cone penetrometer terrain simulation results and laboratory simulation results with the polynomial fit for 2NS sand.....                                                                                                                                        | 258 |
| Figure 4-71 Cone penetrometer terrain simulation results in comparison with NATO CDT-308 (2020) experimental and laboratory simulation results for FGS .....                                                                                                                        | 259 |
| Figure 4-72 Average cone penetrometer terrain simulation results and laboratory simulation results with the polynomial fit for FGS .....                                                                                                                                            | 259 |
| Figure 4-73 Statistical analysis diagram.....                                                                                                                                                                                                                                       | 262 |
| Figure 4-74 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 10 kPa for 2NS sand.....                                                                                                 | 267 |
| Figure 4-75 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at normal stress of 10 kPa for 2NS sand ..... | 267 |

|                                                                                                                                                                                                                                                                                     |     |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 4-76 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 20 kPa for 2NS sand.....                                                                                                 | 270 |
| Figure 4-77 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at normal stress of 20 kPa for 2NS sand ..... | 270 |
| Figure 4-78 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 30 kPa for 2NS sand.....                                                                                                 | 273 |
| Figure 4-79 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at normal stress of 30 kPa for 2NS sand ..... | 273 |
| Figure 4-80 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 10 kPa for FGS.....                                                                                                      | 276 |
| Figure 4-81 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at a normal stress of 10 kPa for FGS .....    | 276 |
| Figure 4-82 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 20 kPa for FGS.....                                                                                                      | 279 |
| Figure 4-83 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at a normal stress of 20 kPa for FGS .....    | 279 |
| Figure 4-84 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 30 kPa for FGS.....                                                                                                      | 282 |
| Figure 4-85 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at a normal stress of 30 kPa for FGS .....    | 282 |
| Figure 4-86 Scalability analysis on the modulus of determination of cohesion, $K_c$ for 2NS sand.....                                                                                                                                                                               | 291 |
| Figure 4-87 F-test and t-test results on the modulus of determination of cohesion, $K_c$ for 2NS sand; (a) samples: Laboratory and actual terrain parameters, (b) samples: Laboratory and scaled terrain parameters.....                                                            | 292 |

|                                                                                                                                                                                                                                                  |     |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 4-88 Scalability analysis on the modulus of determination of friction, $K_\phi$ for 2NS sand.....                                                                                                                                         | 292 |
| Figure 4-89 F-test and t-test results on the modulus of determination of friction, $K_\phi$ for 2NS; (a) samples: Laboratory and actual terrain parameters, (b) samples: Laboratory and scaled terrain parameters.....                           | 293 |
| Figure 4-90 Scalability analysis on the modulus of determination of cohesion, $K_c$ for FGS .....                                                                                                                                                | 294 |
| Figure 4-91 F-test and t-test results on the modulus of determination of cohesion, $K_c$ for FGS; (a) samples: Laboratory and actual terrain parameters, (b) samples: Laboratory and scaled terrain parameters.....                              | 295 |
| Figure 4-92 Scalability analysis on the modulus of determination of friction, $K_\phi$ for FGS .....                                                                                                                                             | 295 |
| Figure 4-93 F-test and t-test results on the modulus of determination of friction, $K_\phi$ for FGS; (a) samples: Laboratory and actual terrain parameters, (b) samples: Laboratory and scaled terrain parameters.....                           | 296 |
| Figure 4-94 Scalability analysis on log p – log z domain of 0.1 m diameter sinkage plate for FGS .....                                                                                                                                           | 297 |
| Figure 4-95 Scalability analysis on log p – log z domain of 0.15 m diameter sinkage plate for FGS.....                                                                                                                                           | 297 |
| Figure 4-96 F-test and t-test results on log p – log z domain of 0.1 m diameter sinkage plate for FGS.....                                                                                                                                       | 298 |
| Figure 4-97 F-test and t-test results on log p – log z domain of 0.15 m diameter sinkage plate for FGS.....                                                                                                                                      | 298 |
| Figure 4-98 (a) F-test and (b) t-test on the cone index vs depth curve for 2NS sand.....                                                                                                                                                         | 299 |
| Figure 4-99 (a) F-test and (b) t-test on the cone index vs depth curve for FGS .....                                                                                                                                                             | 299 |
| Figure 5-1 (a) FED ALPHA military vehicle, (b) FED ALPHA military vehicle's 335/65R22.5 tyre model in 3D, illustrating tyre diameter, width, section height and tread pattern .....                                                              | 313 |
| Figure 5-2 Model construction: Full scale tyre- soil interaction on (a) 2NS Hetrogeneous and FGS Homogeneous soil model; (c) FGS Homogeneous soil model – 3D view (Model support disabled and different colours used to show rut formation)..... | 319 |
| Figure 5-3 Overview of DEM simulation model of tyre-soil interaction used to predict the tractive performance of tyres on the terrain .....                                                                                                      | 322 |
| Figure 5-4 Comparison of simulation, experimental and NRMM prediction results of full-scale tyre 335/65R22.5 on 2NS sand .....                                                                                                                   | 322 |

|                                                                                                                                                                     |     |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 5-5 Comparison of simulation, experimental and NRMM prediction results of full-scale tyre 335/65R22.5 on fine-grained sand .....                             | 323 |
| Figure 5-6 DEM simulation showing the tailing piles behind and side of the full-scale tyre when run at 100% slip condition for 5 seconds on fine-grained soil ..... | 323 |
| Figure 5-7 Wet fine-grained soil – Drawbar pull test and simulation results (NATO CDT-308 (2020)) .....                                                             | 325 |
| Figure 5-8 Dry 2NS sand – Drawbar pull test and simulation results (NATO CDT-308 (2020)) .....                                                                      | 326 |
| Figure 5-9 Comparison of the drawbar pull coefficient in the homogeneous and heterogeneous terrain conditions for 2NS sand.....                                     | 328 |
| Figure 5-10 Comparison of the drawbar pull coefficient in the homogeneous and heterogeneous terrain conditions for fine-grained sand.....                           | 328 |
| Figure 5-11 Comparison of the gross traction coefficient in the homogeneous and heterogeneous terrain conditions for 2NS sand.....                                  | 329 |
| Figure 5-12 Comparison of the gross traction coefficient in the homogeneous and heterogeneous terrain conditions for fine-grained soil .....                        | 329 |
| Figure 5-13 Comparison of the tractive efficiency in the homogeneous and heterogeneous terrain conditions for 2NS sand.....                                         | 330 |
| Figure 5-14 Comparison of the tractive efficiency in the homogeneous and heterogeneous terrain conditions for fine-grained sand.....                                | 330 |
| Figure 5-15 Scalability testing of drawbar pull coefficient of 2NS homogeneous terrain model.....                                                                   | 334 |
| Figure 5-16 Scalability testing of gross traction coefficient of 2NS homogeneous terrain model.....                                                                 | 334 |
| Figure 5-17 Scalability testing of tractive efficiency of 2NS homogeneous terrain model.....                                                                        | 335 |
| Figure 5-18 Scalability testing of drawbar pull coefficient of the fine-grained sand homogeneous terrain model .....                                                | 335 |
| Figure 5-19 Scalability testing of gross traction coefficient of the fine-grained sand homogeneous terrain model .....                                              | 336 |
| Figure 5-20 Scalability testing of tractive efficiency of the fine-grained sand homogeneous terrain model .....                                                     | 336 |
| Figure 5-21 Scalability testing of drawbar pull coefficient of 2NS heterogeneous terrain model.....                                                                 | 340 |
| Figure 5-22 Scalability testing of gross traction coefficient of 2NS heterogeneous terrain model.....                                                               | 340 |



|                                                                                                                                                                                       |     |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 5-23 Scalability testing of tractive efficiency of 2NS heterogeneous terrain model.....                                                                                        | 341 |
| Figure 5-24 Scalability testing of drawbar pull coefficient of the fine-grained sand heterogeneous terrain model.....                                                                 | 341 |
| Figure 5-25 Scalability testing of gross traction coefficient of the fine-grained sand heterogeneous terrain model.....                                                               | 342 |
| Figure 5-26 Scalability testing of tractive efficiency of the fine-grained sand heterogeneous terrain model.....                                                                      | 342 |
| Figure 5-27 Scalability testing of drawbar pull coefficient of 2NS heterogeneous terrain model at lower cone index.....                                                               | 346 |
| Figure 5-28 Scalability testing of gross traction coefficient of 2NS heterogeneous terrain model at lower cone index.....                                                             | 347 |
| Figure 5-29 Scalability testing of tractive efficiency of 2NS heterogeneous terrain model at lower cone index.....                                                                    | 347 |
| Figure 5-30 Scalability testing of drawbar pull coefficient of fine-grained sand heterogeneous terrain model at higher cone index.....                                                | 348 |
| Figure 5-31 Scalability testing of gross traction coefficient of fine-grained sand heterogeneous terrain model at higher cone index.....                                              | 348 |
| Figure 5-32 Scalability testing of tractive efficiency of fine-grained sand heterogeneous terrain model at higher cone index.....                                                     | 349 |
| Figure 5-33 % performance parameters data points of scale tyres in the one-standard-deviation band of full-scale tyre on 2NS sand at two different cone index (CI) values.....        | 349 |
| Figure 5-34 % performance parameters data points of scale tyres in the one-standard-deviation band of full-scale tyre on fine grain sand at two different cone index (CI) values..... | 350 |
| Figure 5-35 Scalability testing of drawbar pull coefficient of 2NS heterogeneous terrain model at half normal load condition.....                                                     | 354 |
| Figure 5-36 Scalability testing of gross traction coefficient of 2NS heterogeneous terrain model at half normal load condition.....                                                   | 354 |
| Figure 5-37 Scalability testing of tractive efficiency of 2NS heterogeneous terrain model at half normal load condition.....                                                          | 355 |
| Figure 5-38 Scalability testing of drawbar pull coefficient of the fine-grained sand heterogeneous terrain model at half normal load condition.....                                   | 355 |
| Figure 5-39 Scalability testing of gross traction coefficient of the fine-grained sand heterogeneous terrain model at half normal load condition.....                                 | 356 |

|                                                                                                                                                 |     |
|-------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure 5-40 Scalability testing of tractive efficiency of the fine-grained sand heterogeneous terrain model at half normal load condition ..... | 356 |
| Figure 5-41 Effect of the scale of similar systems on drawbar pull $D$ , [N] at different slip conditions on 2NS sand .....                     | 360 |
| Figure 5-42 Effect of the scale of similar systems on gross traction $T/r$ , (N) at different slip conditions on 2NS sand .....                 | 360 |
| Figure 5-43 Effect of the scale of similar systems on tractive efficiency $[\eta]$ at different slip conditions on 2NS sand .....               | 361 |
| Figure 5-44 Effect of the scale of similar systems on drawbar pull $D$ , [N] at different slip conditions on fine-grained sand.....             | 361 |
| Figure 5-45 Effect of the scale of similar systems on gross traction $T/r$ , (N) at different slip conditions on fine-grained sand.....         | 362 |
| Figure 5-46 Effect of the scale of similar systems on tractive efficiency $(\eta)$ at different slip conditions on fine-grained sand.....       | 362 |
| Figure 6-1 UGV conducting initial mobility testing with scaled payload to full-scale military vehicle's payload on off-road terrain.....        | 376 |

## LIST OF TABLES

|                                                                                                                |     |
|----------------------------------------------------------------------------------------------------------------|-----|
| Table 1-1 Path planning problem results .....                                                                  | 48  |
| Table 2-1 Wheel mobility number coefficients .....                                                             | 81  |
| Table 2-2 Experimentally derived values of constant k with respect to the number of axles m in a vehicle ..... | 84  |
| Table 2-3 Effect of soil density on Bekker's parameter .....                                                   | 92  |
| Table 2-4 Effect of moisture content and sinkage speed on Bekker's parameter .....                             | 93  |
| Table 2-5 Plate sinkage tests for small diameter wheel .....                                                   | 95  |
| Table 2-6 The mobility prediction models using the generalised Turnage (1972) model.....                       | 113 |
| Table 2-7 The mobility prediction models using generalised Brixius (1987) model .....                          | 116 |
| Table 2-8 Summary of selected DEM mobility prediction models reported in the literature.....                   | 128 |
| Table 3-1 Tyre deflection and contact area parameters .....                                                    | 137 |
| Table 3-2 Tyre – soil model-independent parameters .....                                                       | 141 |
| Table 3-3 Tyre – soil model-dependent parameters.....                                                          | 142 |
| Table 3-4 Tyre specifications used in the present study.....                                                   | 146 |
| Table 3-5 Research plan .....                                                                                  | 147 |
| Table 3-6 Statistical summary of the obtained deflection model for the full-scale tyre .....                   | 158 |
| Table 3-7 Statistical summary of the contact area model for the full-scale tyre .....                          | 159 |
| Table 3-8 Statistical summary of the developed models for the full-scale tyre.....                             | 161 |
| Table 3-9 Statistical summary of the scale model comparison for 145/80R13 .....                                | 170 |
| Table 3-10 Statistical summary of scale model comparison for 135/80R13 ...                                     | 172 |
| Table 4-1 Texture Soil Classification System, ISSS (ISTVS standards (2020)) .....                              | 180 |
| Table 4-2 Physical parameter of soil sample (NATO CDT – 308 (2020)).....                                       | 204 |
| Table 4-3 Soil material parameters in simulation models .....                                                  | 209 |

|                                                                                                                                |     |
|--------------------------------------------------------------------------------------------------------------------------------|-----|
| Table 4-4 Simulated soil model shear parameters of 2NS sand in comparison with NATO CDT – 308 (2020) experimental results..... | 213 |
| Table 4-5 Simulated soil model shear parameters of FGS in comparison with NATO CDT – 308 (2020) experimental results .....     | 214 |
| Table 4-6 Pressure-sinkage parameters derived for 2NS and FGS .....                                                            | 218 |
| Table 4-7 Average cone index gradient, G .....                                                                                 | 220 |
| Table 4-8 Particle shape properties (Akbar et al. (2020)).....                                                                 | 224 |
| Table 4-9 Average AI and SPH at different patches on the terrain model .....                                                   | 231 |
| Table 4-10 Summary of shear bevameter results for 2NS sand.....                                                                | 239 |
| Table 4-11 Summary of shear bevameter results for FGS soil.....                                                                | 240 |
| Table 4-12 Pressure-sinkage parameters summary for 2NS sand terrain model .....                                                | 255 |
| Table 4-13 Pressure-sinkage parameters summary for FGS terrain model ...                                                       | 256 |
| Table 4-14 Average cone index gradient, G of 2NS sand terrain model .....                                                      | 260 |
| Table 4-15 Average cone index gradient, G of FGS terrain model.....                                                            | 260 |
| Table 4-16 Bekker parameters after scaling for 2NS sand .....                                                                  | 286 |
| Table 4-17 Bekker parameters after scaling for FGS .....                                                                       | 288 |
| Table 5-1 Tyre – soil interaction system model-independent parameters used in the present study .....                          | 306 |
| Table 5-2 Tyre – soil model-dependent parameters used in the present study .....                                               | 307 |
| Table 5-3 Soil strength and EpAM model parameters.....                                                                         | 309 |
| Table 5-4 Soil compaction datasheet for the developed models .....                                                             | 311 |
| Table 5-5 Input simulation parameters for soil models .....                                                                    | 312 |
| Table 5-6 Specification of the test tyres models used in the present study ....                                                | 314 |
| Table 5-7 Tyre material properties used in the present study .....                                                             | 315 |
| Table 5-8 Test matrix of slip-time conditions for full-scale and scaled tyres...                                               | 320 |
| Table 5-9 Test matrix of full normal load conditions of full-scale and scaled tyre system .....                                | 320 |
| Table 5-10 Test matrix of half normal load conditions of full-scale and scaled tyre system .....                               | 321 |

|                                                                                                                                                                    |     |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Table 5-11 Quantitative comparison of simulation and experimental results for the positive slip of 335/65R22.5 (scale =1) tyre operating on 2NS sand and FGS ..... | 324 |
| Table 5-12 Quantitative comparison of scaled tyre systems performance parameters between the heterogeneous and homogeneous terrain conditions .....                | 331 |
| Table 5-13 Quantitative comparison of scaled tyre systems performance parameters in the homogeneous condition.....                                                 | 337 |
| Table 5-14 Quantitative comparison of scaled tyre systems performance parameters in heterogeneous condition.....                                                   | 343 |
| Table 5-15 Soil compaction datasheet for the developed models .....                                                                                                | 344 |
| Table 5-16 Quantitative comparison of scaled tyre systems performance parameters with variable cone index .....                                                    | 351 |
| Table 5-17 Comparison of test matrix of the full normal load and half normal load conditions of the full-scale and scaled tyre systems .....                       | 352 |
| Table 5-18 Effect of the normal load on the scalability of tyre-soil interaction                                                                                   | 357 |
| Table 5-19 Non-linear regression analysis between performance parameters vs scale for 2NS sand model.....                                                          | 363 |
| Table 5-20 Non-linear regression analysis between performance parameters vs scale for fine-grained sand model.....                                                 | 364 |

**LIST OF EQUATIONS**

(2-1)..... 66

(2-2)..... 67

(2-3)..... 70

(2-4)..... 70

(2-5)..... 70

(2-6)..... 70

(2-7)..... 70

(2-8)..... 72

(2-9)..... 72

(2-10)..... 72

(2-11)..... 72

(2-12)..... 73

(2-13)..... 73

(2-14)..... 73

(2-15)..... 73

(2-16)..... 73

(2-17)..... 75

(2-18)..... 75

(2-19)..... 75

(2-20)..... 75

(2-21)..... 75

(2-22)..... 76

(2-23)..... 77

(2-24)..... 77

(2-25)..... 77

(2-26)..... 77

(2-27)..... 79

(2-28)..... 79

|             |     |
|-------------|-----|
| (2-29)..... | 80  |
| (2-30)..... | 80  |
| (2-31)..... | 80  |
| (2-32)..... | 80  |
| (2-33)..... | 83  |
| (2-34)..... | 83  |
| (2-35)..... | 83  |
| (2-36)..... | 83  |
| (2-37)..... | 84  |
| (2-38)..... | 86  |
| (2-39)..... | 86  |
| (2-40)..... | 87  |
| (2-41)..... | 88  |
| (2-42)..... | 88  |
| (2-43)..... | 89  |
| (2-44)..... | 101 |
| (2-45)..... | 101 |
| (2-46)..... | 103 |
| (2-47)..... | 103 |
| (2-48)..... | 104 |
| (2-49)..... | 104 |
| (2-50)..... | 105 |
| (2-51)..... | 106 |
| (2-52)..... | 107 |
| (2-53)..... | 108 |
| (2-54)..... | 109 |
| (2-55)..... | 109 |
| (2-56)..... | 109 |
| (2-57)..... | 109 |

|             |     |
|-------------|-----|
| (2-58)..... | 110 |
| (2-59)..... | 110 |
| (2-60)..... | 110 |
| (2-61)..... | 111 |
| (2-62)..... | 111 |
| (2-63)..... | 112 |
| (2-64)..... | 112 |
| (2-65)..... | 112 |
| (2-66)..... | 112 |
| (2-67)..... | 113 |
| (2-68)..... | 113 |
| (2-69)..... | 113 |
| (2-70)..... | 114 |
| (2-71)..... | 114 |
| (2-72)..... | 115 |
| (2-73)..... | 115 |
| (2-74)..... | 115 |
| (2-75)..... | 117 |
| (2-76)..... | 117 |
| (2-77)..... | 117 |
| (2-78)..... | 118 |
| (2-79)..... | 118 |
| (2-80)..... | 118 |
| (2-81)..... | 118 |
| (2-82)..... | 122 |
| (2-83)..... | 122 |
| (2-84)..... | 123 |
| (3-1).....  | 136 |
| (3-2).....  | 137 |



|             |     |
|-------------|-----|
| (3-3).....  | 137 |
| (3-4).....  | 138 |
| (3-5).....  | 138 |
| (3-6).....  | 138 |
| (3-7).....  | 138 |
| (3-8).....  | 138 |
| (3-9).....  | 138 |
| (3-10)..... | 138 |
| (3-11)..... | 138 |
| (3-12)..... | 139 |
| (3-13)..... | 139 |
| (3-14)..... | 139 |
| (3-15)..... | 139 |
| (3-16)..... | 139 |
| (3-17)..... | 139 |
| (3-18)..... | 140 |
| (3-19)..... | 140 |
| (3-20)..... | 140 |
| (3-21)..... | 140 |
| (3-22)..... | 140 |
| (3-23)..... | 140 |
| (3-24)..... | 141 |
| (3-25)..... | 142 |
| (3-26)..... | 142 |
| (3-27)..... | 142 |
| (3-28)..... | 142 |
| (3-29)..... | 142 |
| (3-30)..... | 142 |
| (3-31)..... | 143 |

|             |     |
|-------------|-----|
| (3-32)..... | 143 |
| (3-33)..... | 143 |
| (3-34)..... | 143 |
| (3-35)..... | 143 |
| (3-36)..... | 143 |
| (3-37)..... | 144 |
| (3-38)..... | 144 |
| (3-39)..... | 144 |
| (3-40)..... | 157 |
| (3-41)..... | 157 |
| (3-42)..... | 158 |
| (3-43)..... | 159 |
| (4-1).....  | 182 |
| (4-2).....  | 183 |
| (4-3).....  | 183 |
| (4-4).....  | 183 |
| (4-5).....  | 183 |
| (4-6).....  | 184 |
| (4-7).....  | 184 |
| (4-8).....  | 184 |
| (4-9).....  | 184 |
| (4-10)..... | 184 |
| (4-11)..... | 184 |
| (4-12)..... | 185 |
| (4-13)..... | 186 |
| (4-14)..... | 186 |
| (4-15)..... | 187 |
| (4-16)..... | 187 |
| (4-17)..... | 191 |

|             |     |
|-------------|-----|
| (4-18)..... | 191 |
| (4-19)..... | 191 |
| (4-20)..... | 191 |
| (4-21)..... | 193 |
| (4-22)..... | 193 |
| (4-23)..... | 193 |
| (4-24)..... | 193 |
| (4-25)..... | 194 |
| (4-26)..... | 194 |
| (4-27)..... | 194 |
| (4-28)..... | 194 |
| (4-29)..... | 195 |
| (4-30)..... | 195 |
| (4-31)..... | 195 |
| (4-32)..... | 195 |
| (4-33)..... | 195 |
| (4-34)..... | 195 |
| (4-35)..... | 196 |
| (4-36)..... | 196 |
| (4-37)..... | 196 |
| (4-38)..... | 196 |
| (4-39)..... | 196 |
| (4-40)..... | 196 |
| (4-41)..... | 197 |
| (4-42)..... | 197 |
| (4-43)..... | 197 |
| (4-44)..... | 197 |
| (4-45)..... | 197 |
| (4-46)..... | 198 |

|             |     |
|-------------|-----|
| (4-47)..... | 198 |
| (4-48)..... | 198 |
| (4-49)..... | 199 |
| (4-50)..... | 200 |
| (4-51)..... | 201 |
| (4-52)..... | 201 |
| (4-53)..... | 201 |
| (4-54)..... | 201 |
| (4-55)..... | 201 |
| (4-56)..... | 219 |
| (4-57)..... | 223 |
| (4-58)..... | 223 |
| (4-59)..... | 229 |
| (4-60)..... | 229 |
| (4-61)..... | 230 |
| (4-62)..... | 230 |
| (4-63)..... | 263 |
| (4-64)..... | 285 |
| (4-65)..... | 290 |
| (5-1).....  | 307 |
| (5-2).....  | 307 |
| (5-3).....  | 307 |
| (5-4).....  | 310 |
| (5-5).....  | 310 |
| (5-6).....  | 310 |
| (5-7).....  | 310 |
| (5-8).....  | 310 |
| (5-9).....  | 310 |
| (5-10)..... | 310 |

|             |     |
|-------------|-----|
| (5-11)..... | 316 |
| (5-12)..... | 316 |
| (5-13)..... | 318 |
| (5-14)..... | 358 |

## LIST OF ABBREVIATIONS

|                |                                                          |
|----------------|----------------------------------------------------------|
| A              | Contact patch area, mm <sup>2</sup>                      |
| AI             | Angularity index                                         |
| AVT            | Applied Vehicle Technology                               |
| b              | Tyre's width, mm                                         |
| B <sub>n</sub> | Brixius number                                           |
| c              | Cohesion, Pa                                             |
| Cc             | Coefficient of curvature                                 |
| CI             | Cone index, Pa                                           |
| C <sub>n</sub> | Wheel numeric                                            |
| Cu             | Coefficient of uniformity                                |
| D              | Drawbar pull, N                                          |
| Dia            | Diameter                                                 |
| d              | Tyre's overall diameter, mm                              |
| DEM            | Discrete element method                                  |
| DPR            | Drawbar pull ratio                                       |
| e              | Restitution                                              |
| E              | Young's modulus, Pa                                      |
| f <sub>0</sub> | Constant pull-off force, N                               |
| FEM            | Finite Element Method                                    |
| G              | Shear modulus, Pa                                        |
| G              | Cone index gradient                                      |
| GT             | gross traction, N                                        |
| h              | Section height, mm                                       |
| j              | Shear displacement, mm                                   |
| K              | Soil deformation modulus, mm                             |
| k              | The modulus of inelastic deformation, N/m <sup>n+2</sup> |
| K <sub>t</sub> | Tyre stiffness, N/m                                      |
| k <sub>ø</sub> | Moduli of deformation to friction, N/m <sup>n+2</sup>    |
| k <sub>c</sub> | Moduli of deformation to cohesion, N/m <sup>n+1</sup>    |
| loc            | Location                                                 |
| LSR            | Longitudinal Slip Ratio                                  |
| m              | Number of axles in a vehicle                             |

|                |                                                 |
|----------------|-------------------------------------------------|
| MI             | Mobility index                                  |
| MMP            | Mean maximum pressure, Pa                       |
| MRR            | Motion resistance ratio                         |
| n              | Tractive efficiency                             |
| n              | Slope exponent                                  |
| N              | Normal reaction force                           |
| NATO           | North Atlantic Treaty Organization              |
| NGP            | Nominal ground pressure, Pa                     |
| NRMM           | NATO Reference Mobility Model                   |
| $\eta_m$       | The efficiency of motion                        |
| $\eta_s$       | The efficiency of slip                          |
| $\eta_t$       | The efficiency of the transmission system       |
| NS             | Natural sand                                    |
| P              | Pressure or normal stress, Pa                   |
| P              | Applied load, N                                 |
| $P_i$ or $p_i$ | Tyre's inflation pressure, Pa                   |
| $P_g$          | Ground contact pressure, Pa                     |
| $P_{cgr}$      | Critical ground pressure, Pa                    |
| R              | Wheel's radius, mm                              |
| S              | Wheel slip, %                                   |
| SPH            | Sphericity index                                |
| STO            | Science and Technology Organization             |
| T              | Torque, N-m                                     |
| $\tau$         | Shear stress of soil, Pa                        |
| UAV            | Uninhabited aerial vehicle                      |
| UGV            | Uninhabited ground vehicle                      |
| VCI            | Vehicle cone index                              |
| V              | The forward velocity of the wheel, m/s          |
| W              | Normal load on the tyre, N                      |
| w              | Water content, %                                |
| WES            | U.S. Army Engineer Waterways Experiment Station |
| Z              | Sinkage, mm                                     |
| $\Delta\gamma$ | Surface energy, J                               |

|             |                                     |
|-------------|-------------------------------------|
| $\Delta t$  | Timestep, s                         |
| $\lambda_p$ | Contact plasticity ratio            |
| $N$         | Sinkage exponent                    |
| $P$         | Particle density, kg/m <sup>3</sup> |
| $\nu$       | Poisson's ratio                     |
| $\Omega$    | Wheel's angular velocity, rad/s     |
| $\delta$    | Tyre deflection, mm                 |
| $\gamma$    | Specific weight, N/m <sup>3</sup>   |
| $\theta$    | Contact angle, rad                  |



# 1 INTRODUCTION

*“Speed is the essence of war. Take advantage of the enemy’s unpreparedness; travel by unexpected routes and strike him where he has taken no precautions”*

- SUN TZU (5<sup>th</sup> Century BC)

Speed and mobility offer key tactical advantages to the military on the battlefields. The rapid movement of mechanised forces concentrated at the enemy’s weak point often provides dominance on battlefields. This is one of the popular military tactics in the history of warfare. Mechanised warfare significantly depends on the use of infantry fighting vehicles, self-propelled artillery, and other combat vehicles, as well as other support units. However, climatic conditions and soil texture significantly affect vehicle mobility. Once the vehicle loses its ability to move and becomes immobile due to soil conditions or any other factors, then it becomes a sitting duck for an enemy. Therefore, vehicle mobility and survivability in off-road conditions need critical analysis.

With the advancements in modern military warfare, traditional mechanised vehicles are teaming up with autonomous systems. However, as reported by NATO TR SAS – 097 (2018), autonomous systems need significant development to quantitatively model the mobility performance metrics such as nominal ground pressure or vehicle cone index for autonomous systems, particularly in the scope of UGV. An uninhabited ground vehicle (UGV) is a military robot that mostly operates on off-road terrains and without an onboard human presence, as shown in Figure 1-1. The growing use of the UGVs in present-day military operations and their expected increased introduction over the coming decades raises questions such as how to correlate the UGVs mobility with the full-scale military vehicles in a range of terrains and different climatic conditions used on deployed NATO operations. With the advancements in Science and Technology (S&T), future operating environments such as high slippage areas, dense vegetation, or radioactive environment will be even more complex and uncertain, and future

UGVs will offer enormous potential to redefine the way modern warfare will be conducted.



**Figure 1-1 (a) REX UGV of Israel (Egozi, 2018); (b) TAROS V2 of Czech Republic (Army technology, 2014)**

There are different varieties of UGVs due to the greater number of options for propulsion and mobility designs, and a larger number of applications. The key military applications of UGVs include soldier UGVs, squad support systems, driverless vehicles and material handling. The military UGVs are better suited for performing 'dull, dirty or dangerous' tasks during strategic operations such as Improvised Explosive Device (IED) detections, 24 x 7 surveillance facility, collecting early samples of radioactive environment, breaching, clearing tunnels, entering buildings and trench lines etc (U.S. Marines Corps, 2017). Some currently deployed UGVs are SARGE (U.S.), Titan (U.K.), TAROS (Czech Republic), Dragon Runner (U.S.), Rex (Israel) and Daksh (India).

Lower downside risk and higher confidence in the success of missions are two strong motivators for the continuous expansion of UGVs across a broad spectrum of warfighting and peacetime missions. During such missions, UGVs can team up with full-scale military vehicles to ensure three critical tasks as mentioned below.

- UGVs can be used for initial mobility testing on soft soil during the missions to decide the go/no-go conditions for full-scale military vehicles. It will help the defence forces to stop full-scale military vehicles to become immobile during tactical operations on soft soils or off-road terrains as shown in

Figure 1-2. It will also contribute to the mobility analysis of terrain to find the most efficient path from starting point to the desired destination for a full-scale system to improve its fuel economy.

- The full-scale military vehicles often carry heavy weapon systems and other payloads during operations on off-road terrains. The payload is one of the critical factors, which affects the mobility performance on soft soils. UGVs can be used for initial mobility testing with scaled payload to full-scale vehicle payload. Therefore, it will help to decide the maximum payload carrying capacity of full-scale military vehicles on soft soils.
- Pre-planning the correct UGVs to team up with full-scale military vehicles for tactical operations is of utmost importance. The predictive capability of a UGV is needed to effectively monitor and guide the full-scale military vehicle, to keep the vehicle safe while mitigating the mission constraints (e.g., no go areas and communication limitations) and maximising performance metrics (e.g., speed and mobility). Therefore, the present study will help in deciding the correct supporting UGVs mobility specifications (e.g. tyre specifications and payload) in scale with full-scale military vehicles.

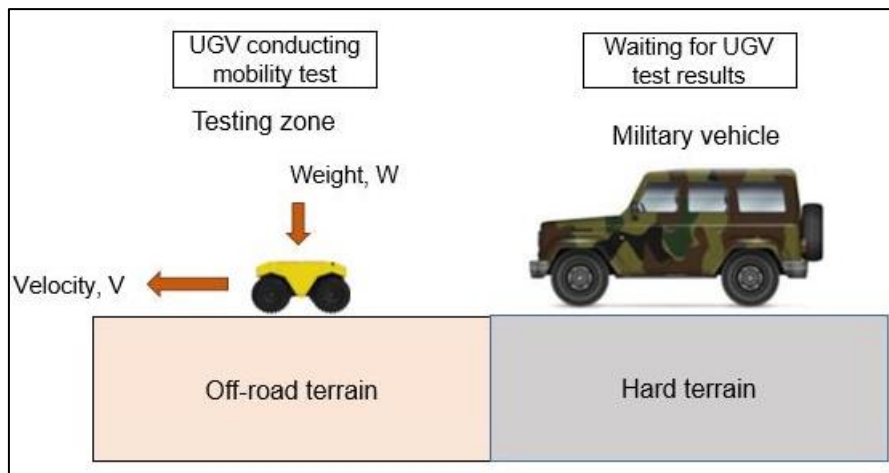


**Figure 1-2 UGV conducting initial mobility testing for the full-scale military vehicle to decide go/no-go condition on off-road terrain (or soft soils) on the battlefield**

Military UGV technology must accommodate the off-road terrains based on war-fighting areas in which current NATO militaries are involved. It should also be

able to handle a range of vehicle sizes (large to small) for addressing operational needs, as shown in Figure 1-3. Most UGVs have difficulty representing and analyzing off-road soil conditions and they tend to operate at slow speeds over relatively flat terrains.

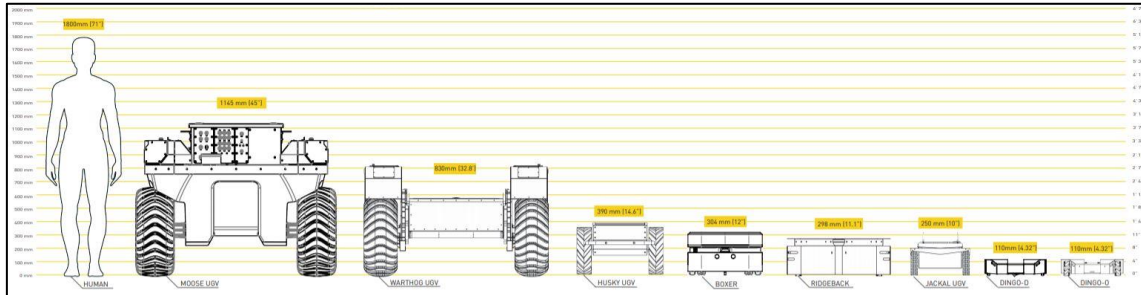


**Figure 1-3 Ground UGVs sizing chart (Clearpath Robotics, no date)**

The mobility performance of the ground vehicle depends on the vehicle-soil interaction and traction properties (e.g. soil shear) of the off-road terrains. Based on the mobility design, military UGVs can further be categorised into wheeled and tracked vehicles. However, the present research focuses on wheeled vehicles as they have better speed, stealth nature, reduced noise signature, low maintenance and easy to repair than tracked vehicles. More force is required to move a tracked vehicle due to its larger size, which creates more noise whereas wheeled vehicles lend themselves to a quieter approach (Paul, 1998).

Historically, the mobility of wheeled vehicles operating on off-road terrain has primarily been assessed through full-scale physical testing in natural and prepared soil conditions. Over the past 50 years, a substantial part of terramechanics research has primarily focused on lightweight military vehicles which led to the development of various mobility prediction methodologies. These methods are empirical with resource-intensive experimental testing and have been widely used by the defence research community (Jayakumar et al., 2013). However, as a consequence of their empirical nature, it remains an open question whether the mobility performance models obtained from these methods can be used to model the mobility performance of small, lightweight UGVs. Therefore, in the present study, the lightweight Armoured Personnel Carrier (APC) military vehicle tyre such as land rover (7.5R16) or FED Alpha tyre (335/65R22.5) are

considered as a full-scale system, whereas small, lightweight UGV tyres such as Warthog (24" Argo) as scale system.

Scale model testing can also provide economic benefits by conducting experiments with scale systems instead of a full-scale system and technical benefits by providing solutions to specific design and performance-related problems. It will also contribute to the mobility analysis of terrain to find the most efficient path from starting point to the desired destination for a full-scale system to improve its fuel economy and efficiency. Moreover, to address this, and to reduce the dependency on full-scale experimental testing, a novel simulation contact model (EDEM, 2014) has been used, to predict vehicle mobility based on vehicle and soil characteristics.

Further, in the year 2020, the North Atlantic Treaty Organization's (NATO) Science and Technology Organization (STO) formed the NATO's Applied Vehicle Technology (AVT) committee, which suggested the use of scale model testing in the field of vehicle research and development in its future course of action (NATO CDT – 308 (2020)). The AVT committee also suggested the scalability of soil models and the study of heterogeneity in soil including particle shape, size and moisture content in the soft soil simulation category. It also highlighted that the improvements are further required for simulation model calibration and validation. Therefore, this thesis contributes to some of these aspects of vehicle mobility technology.

## **1.1 Graph theory and the energy estimation**

The energy estimation of an off-road vehicle depends on several factors such as fuel consumption characteristics of the engine, transmission characteristics, mobility performance from the present location to the desired finish location, weight of the vehicle, driving cycle (conditions) and driver behaviour (Wong, 2001). Zoz and Grisso (2003) found that between 20 and 55% of tractive power losses are due to mobility resistance on off-road terrains. Therefore, mobility performance is a critical contributor to energy estimation.

The mobility performance significantly depends on the path planning between initial and terminal locations and the tractive efficiency of the vehicle on the desired path(s). Mathematically, path planning is a geometric evolution of the curve that contains the individual and jointly feasible paths between an initial and terminal configuration (Economou et al., 2007). By feasible path, it means that the terrain minimally allows a single pass of a vehicle (go condition/ positive drawbar pull). The tractive efficiency refers to a vehicle's capability to transform the input axle power from the engine to the usable power available at the drawbar (ISTVS standards, 2020). For optimum energy path and better tractive efficiency at low speed, minimization of the motion resistance and energy lost to overcome it by the vehicle on terrain is required. The motion resistance is often measured in terms of motion resistance coefficient i.e., the ratio of motion resistance (MR) to the vehicle's weight ( $W$ ). The energy lost to motion resistance is the work done by the motion resistance acting on a vehicle during its travel path.

The presented case study aims to determine the optimum energy path by minimizing the motion resistance coefficient and energy lost to overcome it by a single scaled UGV on a terrain. Further, the results are compared with the maximum useful energy path (i.e., the work done by drawbar pull). It is assumed that there is enough gross traction for vehicle mobility at 20% slip condition (as maximum tractive efficiency is generally between 10 and 30% slip). If tyre-soil interaction is scalable, then a unit of full-scale APCs can use this optimum path during tactical operations.

Consider the following possible scenario of the Colorado river in which, UGV is required to travel through an optimum path from starting point  $S \rightarrow$  goal  $E$  as shown in Figure 1-5 and 1-6. The feasible paths are then selected to determine the optimum mobility performance path as shown in Figure 1-7. However, the presented case study of the UGV path planning on off-road terrain is analogous to the UAV path-planning problem discussed in G.P. Kladis et al. (2011) as shown in Figure 1-4. The wind resistance of a UAV can be analogous to the off-road terrain motion resistance of a UGV. The wind profile is priori known in the UAV problem study similar to the terrain mobility profile is priori known in the UGV



problem study. Both studies aim to minimize the energy loss associated with both vehicles by utilizing the graph theory tools to find the optimum path in the path planning problem.

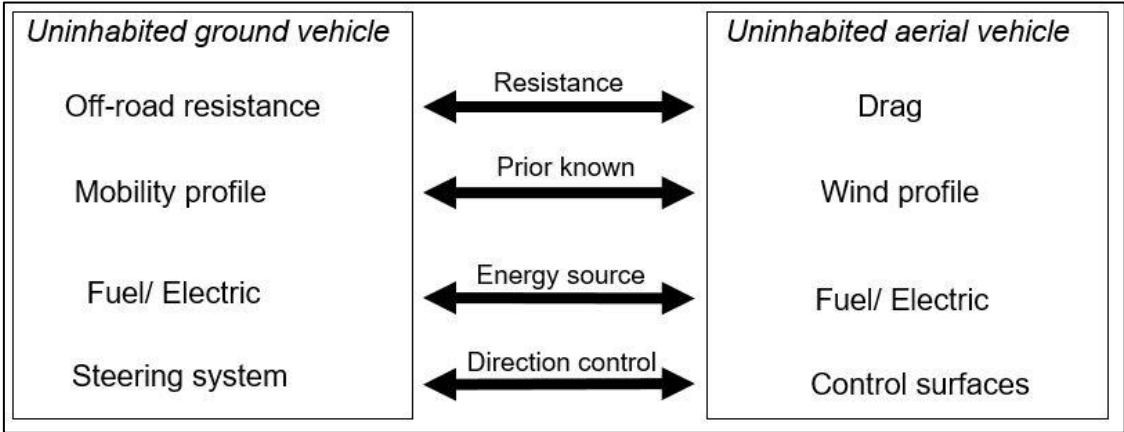


**Figure 1-4 An analogy between UGV and UAV based on G.P. Kladis et al. (2011)**



**Figure 1-5 UGV mission map showing starting point S and goal E (Picture Credit: Ted Wood, College of Media, Communication and Information, Colorado, Mexico)**

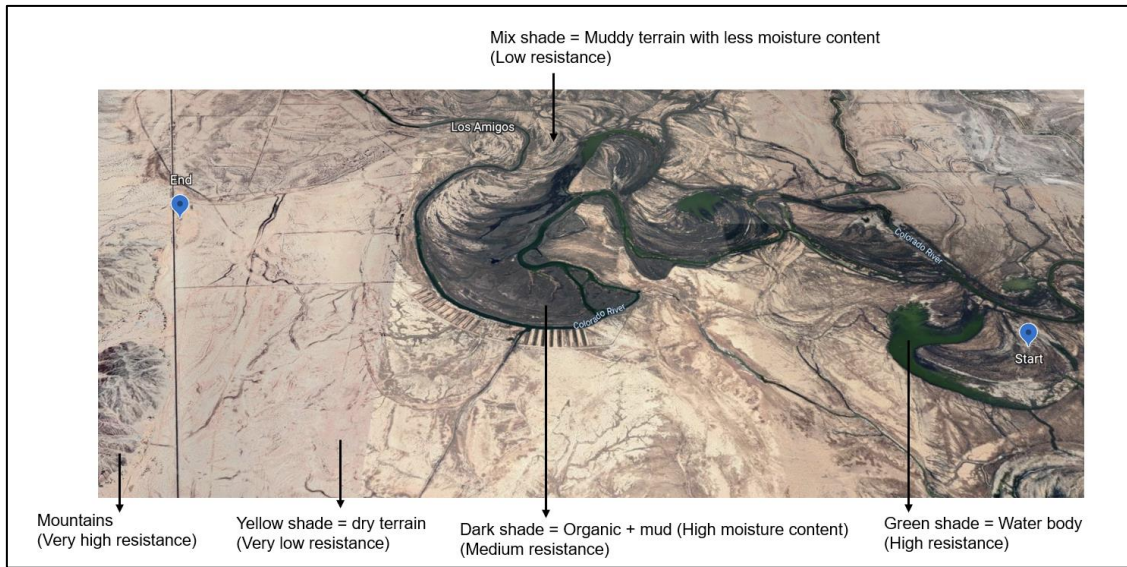


**Figure 1-6 UGV mission map taken from Google Earth as per coordinates shared by Ted Wood, College of Media, Communication and Information, Colorado, Mexico**

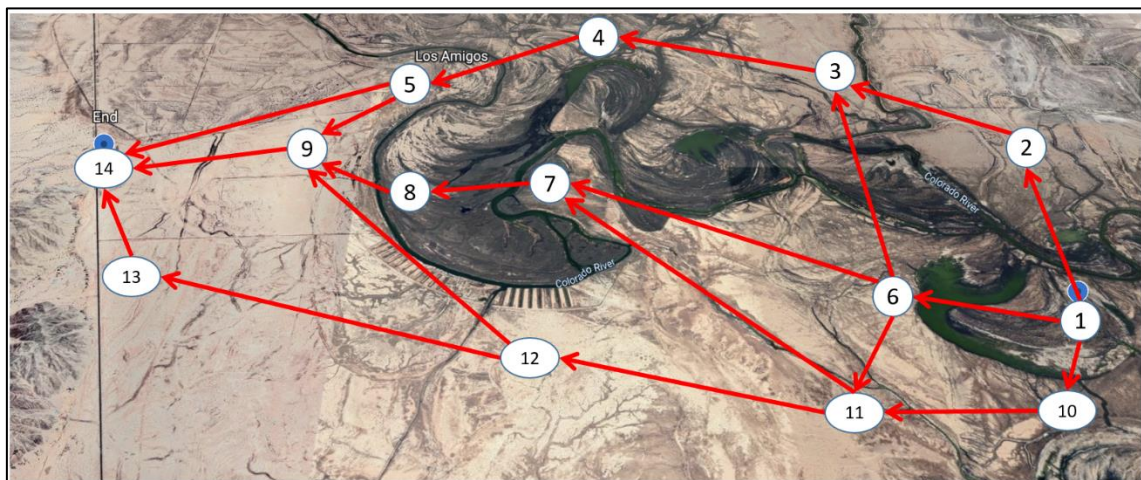


**Figure 1-7 UGV mission map marked with feasible paths**

Theoretically, the solution to the path planning problem involves three critical steps (Economou et al., 2007):

1. Search for the existence of paths on the map;
2. Design of paths between defined configurations or waypoints;



3. Describe the energy cost for the feasible paths and evaluate the optimum path.

Initially, the search for a path is conducted on a priori known map in which the starting and goal configurations are defined. During the initial phase of route planning, these polygonal paths are connected on a graph with a series of straight lines connecting the starting and goal positions through several locations or waypoints, avoiding the obstacles and critical terrain conditions such as deep-water bodies and mountains. The optimum paths for the UGV mission are evaluated using the Dijkstra algorithm for the following cases:

1. Shortest distance path,
2. Minimum motion resistance coefficient path
3. Path with the least energy loss due to motion resistance,
4. Path with maximum useful energy due to drawbar pull.

Similarly, to the above interpretation, a graph is obtained as shown in Figure 1-8 using the path planning algorithm in the MATLAB R2022a software. Other paths may exist and can be obtained from the existing method. In general, a graph  $G$  is an ordered triple  $(N, S, \emptyset)$ , where  $N$  is a set of vertices or nodes,  $S$  is the set of edges and  $\emptyset$  is an ordered pair of  $N$ . In the present case, the graph nodes  $N$  can be categorised into starting node (1), waypoints (2 to 13) and end node (14). The adjacency matrix  $A_{ij}$  for the oriented graph is shown in Figure 1-9. The adjacency matrix  $A_{ij}$  represents the existence of feasible paths on the map, which states the vehicle go/no-go condition. When  $A_{ij} = 1$ , it signifies go condition, where the vehicle is allowed to travel on a particular route. Similarly, when  $A_{ij} = 0$ , it signifies a no-go condition, where the vehicle is not allowed to travel on that particular route. It should be noted that the graph does not represent the Euclidian distances between the nodes; therefore, conventional geometry is not applicable in finding the optimum path.

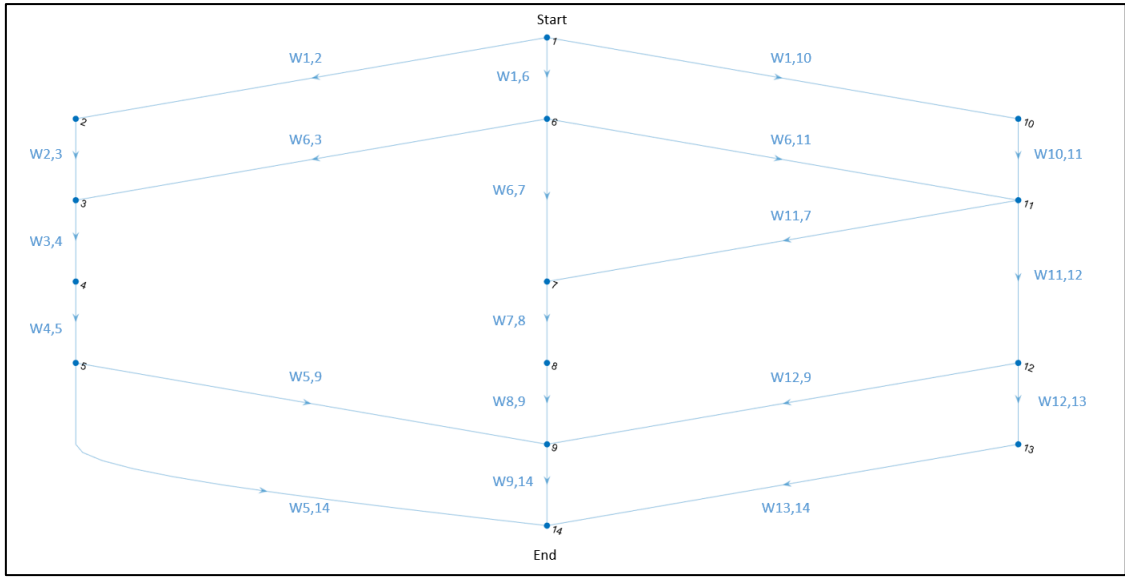


**Figure 1-8 Graph G graphical representation of feasible paths of the present case**

$$A_{ij} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**Figure 1-9 Adjacency matrix  $A_{ij}$  of the present case**

Ted Wood, College of Media, Communication and Information, Colorado, Mexico shared the coordinates of the location of the captured picture (31.752142, -114.958496), Figure 1.5. The actual distances between waypoints are measured using Google Earth Version 9.168.0.0. The distance matrix ( $S_{ij}$ ) is shown in Figure 1.10. Similarly, using Figure 1.6, the motion resistance matrix ( $MR_{ij}$ ) is created (Figure 1.11). The non-permissible routes such as mountains are given an infinite

motion resistance coefficient. The high resistance zone such as water bodies is represented with a green shade (Figure 1.6) and is given a 0.5 motion resistance coefficient. Similarly, the medium resistance zone such as high moisture content mud and organic waste is represented with a dark shade and is given a 0.4 motion resistance coefficient. The low resistance zone containing the muddy terrain (mix shade) with less moisture content is given a 0.2 motion resistance coefficient. The very low resistance zone such as dry terrain represented with a brown shade is given a 0.1 motion resistance coefficient. Therefore, the motion resistance coefficient matrix is shown in Figure 1.11. The constants of this matrix are assumed based on NATO CDT – 308 (2020) mobility experimental results on dry and wet soils. The wet terrain section is given relatively more weight than the dry terrain section. Generally, wet loose terrains offer more motion resistance and less drawbar pull to the vehicle's motion as compared to dry hard terrain. Further, the energy lost to overcome the motion resistance between waypoints is calculated using the work done by the motion resistance along the path (i.e., motion resistance\*displacement). Figure 1.12 shows the energy lost matrix ( $LE_{ij}$ ) of the present case.

The shortest distance path, minimum motion resistance path and minimum energy lost to the motion resistance path are compared with the maximum useful energy path. The useful energy ( $UE_{ij}$ ) between waypoints is calculated using the work done by the drawpull along the path (i.e., drawbar pull\*displacement). The drawbar pull coefficient ( $D_{ij}$ ) matrix is shown in Figure 1.13. Using Figure 1.6, it is considered that the high resistance path has a drawbar pull coefficient of 0.5, the medium resistance path has 0.6, the low resistance path has 0.8 and the very low resistance path has 0.9. It is assumed that the gross traction coefficient is equal to 1 i.e. (gross traction = vehicle weight,  $(D/W) = (GT/W) - (MR/W)$ , for more details, refer section 2.2.4) for a possible comparison in the trend. Therefore, the useful energy matrix ( $UE_{ij}$ ) is shown in Figure 1.14.



[illegible]

**Figure 1-12  $LE_{ij}$  matrix representing the energy lost to overcome motion resistance between waypoints**

[illegible]

**Figure 1-13 D matrix representing the drawbar pull coefficient between waypoints**

$$\begin{aligned}
 & \text{UE}_{ij} \\
 & = \begin{bmatrix}
 \infty & 1300 & \infty & \infty & \infty & 750 & \infty & \infty & \infty & 870 & \infty & \infty & \infty & \infty \\
 \infty & \infty & 1320 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
 \infty & \infty & \infty & 2440 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
 \infty & \infty & \infty & \infty & 1720 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
 \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1350 & \infty & \infty & \infty & \infty & 3420 \\
 \infty & \infty & 1860 & \infty & \infty & \infty & 3200 & \infty & \infty & \infty & 1320 & \infty & \infty & \infty \\
 \infty & \infty & \infty & \infty & \infty & \infty & \infty & 960 & \infty & \infty & \infty & \infty & \infty & \infty \\
 \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 840 & \infty & \infty & \infty & \infty & \infty \\
 \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 2340 \\
 \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1400 & \infty & \infty & \infty \\
 \infty & \infty & \infty & \infty & \infty & \infty & 3600 & \infty & \infty & \infty & \infty & 3150 & \infty & \infty \\
 \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 2880 & \infty & \infty & \infty & 3645 & \infty \\
 \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1440 \\
 \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty
 \end{bmatrix}
 \end{aligned}$$

**Figure 1-14 UE matric representing the useful energy due to drawbar pull between waypoints**

The Dijkstra algorithm is used to obtain the shortest distance path, minimum motion resistance path, minimum energy lost to motion resistance path and maximum useful energy path from starting point S → goal E using the MATLAB R2022a software. The Dijkstra algorithm is a graph search algorithm that is used to provide solutions to the single source shortest path problem for a graph with non-negative edge weights, producing a shortest path tree (Javaid, 2013). When given a source vertex in a graph, the algorithm finds the path with the lowest cost between that vertex and all other vertices. The algorithm can also be used to find the shortest path from a single vertex to a single destination vertex. It stops once the shortest path is determined. The pseudo-code of the algorithm is shown below.

1. **function** Dijkstra (Graph, Source)
2.     **for** each vertex V in Graph
3.         distance[V] <- infinite
4.         previous[V] <- NULL
5.         **if** V != Source, add V to Priority Queue Q
6.     distance [Source] <- 0
7.     **while** Q IS NOT EMPTY
8.         U <- Extract MIN from Q

```

9.      for each unvisited neighbour V of U
10.      tempDistance <- distance[U] + edge_weight(U, V)
11.      if tempDistance < distance
12.      distance[V] <- tempDistance
13.      previous[V] <- U
14.  return distance[], previous[]

```

The results obtained using this algorithm are presented in Table 1.1.

**Table 1-1 Path planning problem results**

| Parameter                                     | Results                                                                                                                                                           |
|-----------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Shortest distance path                        | 1→ 6→ 7→ 8→ 9→ 14<br>Total distance – 11100 m<br>Energy lost to MR – 3010 J<br>Useful energy – 8090 J<br>Total energy - 11100 J<br>Tractive efficiency – 72.88%   |
| Minimum motion resistance coefficient path    | 1→ 10→ 11→ 12→ 13→ 14<br>Total distance – 12350 m<br>Energy lost to MR – 1845 J<br>Useful energy – 10505 J<br>Total energy - 12350 J<br>Tractive efficiency – 85% |
| Minimum energy lost to motion resistance path | 1→ 10→ 11→ 12→ 13→ 14<br>Total distance – 12350 m<br>Energy lost to MR – 1845 J<br>Useful energy – 10505 J<br>Total energy - 12350 J<br>Tractive efficiency – 85% |

| Parameter                                               | Results                                                                                                                                                                                       |
|---------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Maximum useful energy with respect to drawbar pull path | <p>1→ 10→ 11→ 12→ 9→ 14</p> <p>Total distance – 12900 m</p> <p>Energy lost to MR – 2260 J</p> <p>Useful energy – 10640 J</p> <p>Total energy - 12900 J</p> <p>Tractive efficiency – 82.4%</p> |

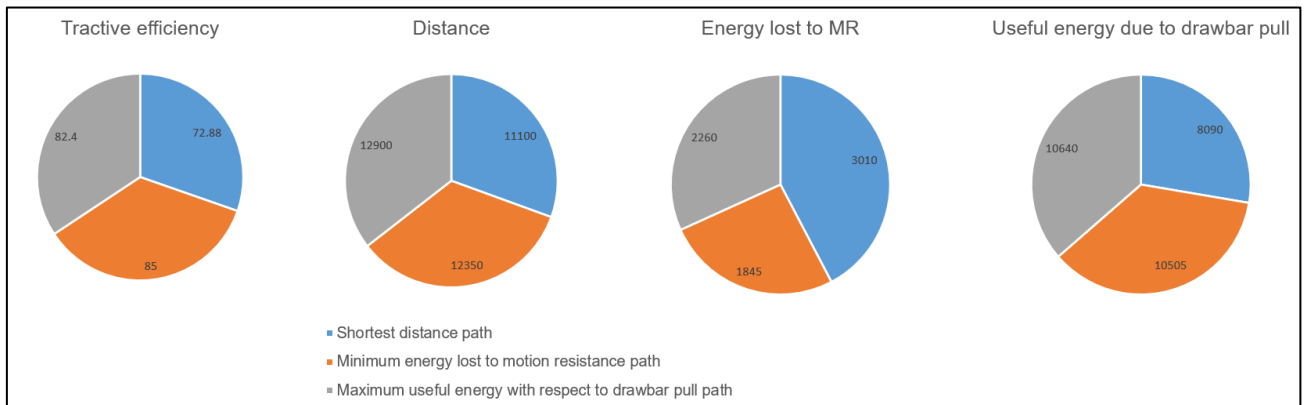


**Figure 1-15 Comparison graphs of the results obtained in the path planning problem**

The conclusions drawn from the presented case study are:

1. The minimum motion resistance coefficient path (1→ 10→ 11→ 12→ 13→ 14) and the minimal energy lost to the motion resistance path (1→ 10→ 11→ 12→ 13→ 14) are the same in the present case. This path is the optimum energy path.
2. The best tractive efficiency is found in the minimum motion resistance path and the minimum energy lost in the motion resistance path. Therefore, the vehicle performance is best on this path.
3. The shortest distance path is not the optimum mobility path. The energy lost to the motion resistance is maximum on this path. The useful energy is the least on this path. Therefore, tractive efficiency and vehicle performance are minimal on this path.



The optimum mobility path plays a critical role in mission planning at the tactical level with the full-scale lightweight Armoured Personnel Carrier (APC) military vehicles and scaled UGVs. It minimises the energy lost due to motion resistance on the mission map and therefore, the saved energy can be used to perform more useful work such as carrying payloads for longer time or travelling longer distances etc. The vehicles also achieve the best tractive efficiency on this path. Therefore, the optimum mobility path becomes important during the teaming operations of UGVs and APCs. This optimum path can be found using a single scaled UGV and can be used by a (or unit of) full-scale APCs for tactical operations if the scalability of tyre-soil interaction is established. Moreover, it will also avoid the immobility of full-scale vehicles during the operation.

## **1.2 Background**

### **1.2.1 Mobility prediction methods**

Ranging from entirely empirical to numerical, various methods have been developed to model tyre-soil interaction over the last century. Before recognising the tyre and the soil characteristics and the mechanism of tyre-soil interaction, many in-situ field tests (i.e. on undisturbed natural terrain) were carried out to measure soil properties, and tyre mobility and empirical methods were well developed to model tyre-soil interaction. With the advancement in the discipline of Terramechanics, the physical nature of tyres and soils were well understood, and analytical methods were applied to model the tyre-soil interaction by taking into account the major tyre and soil mechanical characteristics such as tyre stiffness, soil's pressure-sinkage and shear stress-displacement relationships. With the rapid development in computational capacity and efficiency and computation programs, numerical methods have been introduced to analyse the tyre-soil interaction.

Empirical models of the tyre-soil interaction can be employed to study vehicle mobility performance and soil trafficability. The vehicle's mobility performance is based on vehicle design aspects and soil properties, as shown in Figure 1-16. Simple numeric can be evaluated based on the tyre mobility performance and soil characteristics. These models can be used to estimate the performance of a

new tyre (or wheeled vehicle) when its design specifications are similar to the tested tyre (or wheeled vehicle) under similar experimental conditions. In any case, it is not certain that they can be extrapolated beyond the experimental conditions in which they were developed. Therefore, it seems unlikely that an empirical model could contribute significantly to the evaluation of new design concepts or the prediction of tyre (or wheeled vehicle) performance under new operating conditions (Wong, 2010).

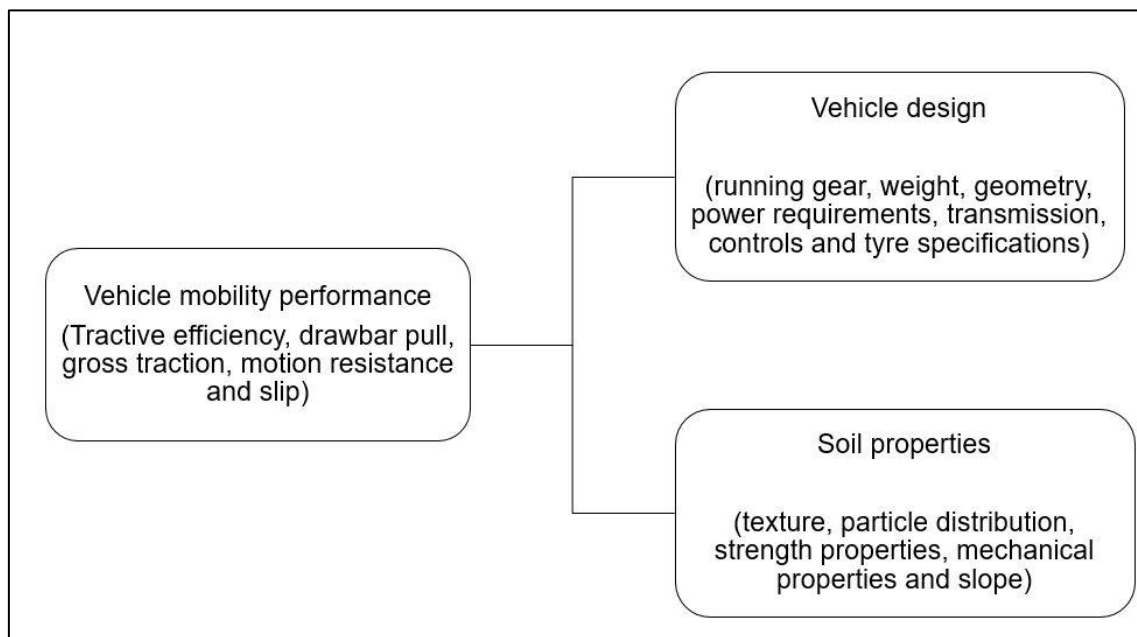


**Figure 1-16 Flow chart of vehicle mobility**

Analytical methods are developed based on the principles of Terramechanics and understanding of tyre-soil interaction mechanics. This method can evaluate the tyre performance on various soils but needs verification. The analytical method's fundamental features are the soil's pressure-sinkage and shear stress-shear displacement relationships. The developed models from this method can measure the forces and moments generated at the contact interface.

Numerical methods such as the finite element method (FEM), Smoothed Particle Hydrodynamics (SPH) and discrete element method (DEM) can evaluate the tyre mobility performance or soil compaction. The computation with these methods generally involves many finite or discrete elements to achieve better accuracy in simulations. These models need to be validated against the quantitative

experimental data obtained under similar operating conditions. Inter-particle mechanics and tyre-soil interaction can be better analysed using these methods.

### **1.2.2 Basic soil theory**

Soil affects wheeled vehicle mobility through contact with tyre surfaces and soil-embedded obstacles such as vegetation and rocks, which in turn contact the vehicle. Therefore, an understanding of soil behaviour under vehicle load is of importance to the study of tyre-soil interaction. In the past, the soil is widely modelled as an elastic medium or as a rigid, perfectly plastic material (Wong, 2010). Modelling the soil as an elastic medium (or with the theory of elasticity) has found applications in predicting the stress distribution in the soil and its compaction under vehicle load. Modelling the soil as a rigid, perfectly plastic material (with the theory of plastic equilibrium) has found applications in predicting the maximum traction developed by the vehicle or the maximum vehicle load that the soil can support without its failure.

While assuming soil medium as an elastic or a rigid, perfectly plastic medium may explain certain aspects of tyre-soil interaction, but there are limitations. For instance, the theory of elasticity may only be applied to dense terrain such as compact sand and in certain circumstances where its behaviour can be compared with the ideal elastoplastic medium stress-strain relationship and is under a certain load limit,  $A$  as shown in Figure 1.17. On the other hand, the theory of plastic equilibrium cannot be used to predict the vehicle sinkage due to the applied normal load or the slip of the vehicle. When the vehicle load exceeds a certain limit,  $A$ , then an infinitely small change in stress, produces a very large change in strain, which constitutes a plastic flow representing the failure of the terrain material.

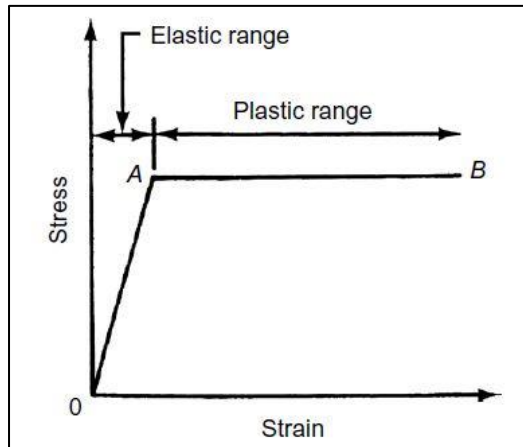


**Figure 1-17 Stress–strain relationship of an idealized elastoplastic material (Wong (2010))**

To overcome the limitations of both the above theories, the concept of critical state soil mechanics theory was later introduced. It can be used to predict the stress-strain behaviour of terrain in a wide range of conditions. However, due to its complexity and the variability in the in-situ field conditions, its application to the study of tyre-soil interaction is limited.

With the advancement in computer and computational technologies in recent years, numerical methods have emerged. Modelling the terrain using the Finite Element Method, Smoothed Particle Hydrodynamics or Discrete Element Method have the potential to capture certain aspects of tyre-soil interaction in great detail. However, the accuracy of soil modelling is dependent on its validation using sophisticated experimental data.

In this thesis, the discrete element method is used to model the 2NS sand and Fine-grained soil from the NATO CDT – 308 (2020). The experiments such as the sieve tests, the cone-penetrometer tests, the pressure-sinkage tests and the shear tests were conducted at the NATO site using sophisticated experimental equipments. These results were used to validate the soil models in the EDEM software. The pressure-sinkage, the shear tests and the cone penetrometer tests were simulated. The inter-particle interaction defined by the particle parameters (such as shape, size and density) and contact model parameters in the software were adjusted to get the approximate experimental bulk behaviour of the soil

models. Later, the developed soil models were used to study the scalability aspect of the mobility performance parameters of the tyre-soil interaction.

### **1.2.3 Scale model testing**

Full-scale model (e.g. Land rover 7.5R16 or FED Alpha 335/65R22.5 tyre) testing is the backbone of vehicle mechanics studies over the years. It is primarily because of their relatively small size and ease of testing compared to aircraft and ships. Most of the past research focused on studying vehicle tyre interaction with uniformly prepared homogeneous soil test sections that incur the same cost of scale model testing. Therefore, it was beneficial to perform full-scale model testing in the past. In the present scenario, with the desire to extend human operations to other planets and advance military capabilities on land, the present researcher's focus is shifting towards vehicle interaction with heterogeneous terrain systems. In 1970, the U.S. Army Engineer Waterways Experiment Station (WES) studied predicting full-scale vehicle mobility on dual-layered terrain (i.e. a terrain with two distinct strength of soil layers). The cost of preparing the layered soil test section was significantly higher than the preparation of homogeneous test sections. Swanson (1973) analysed that at the cost of two full-scale model tests on dual-layered terrain, five prototype scale model tests can be conducted. Therefore, it concluded that scale model testing is a viable method for reducing experimental costs.

The scalability of soil can be defined as a relationship between the mechanical properties of an in-situ terrain (heterogeneous) system and a laboratory (homogeneous) system while accounting for the differences in sand, silt and clay particle shapes and size distributions. Physical properties such as moisture content, bulk density, compaction, and interparticle forces are kept the same for laboratory and in-situ terrain simulation models by keeping the same contact model parameters. The laboratory system has a uniform particle shape and constant particle size distribution whereas the in-situ terrain has variable particle shapes and variable particle size distributions. The Bevameter technique such as the pressure-sinkage test and the shear test, and the cone penetrometer

technique are used to measure the mechanical properties in both the terrain simulation systems. The flow chart is shown in Figure 1.18.

The definition of scalability of tyre-soil interaction is based on the assumption that similar systems that behave similarly can be established (Bekker, 1956 and Freitag, 1966). Freitag (1966) states, “if the generalisations obtained by the systematic variation of the independent parameters are to be used with confidence, then it must be shown that the observed tyre-soil relations do not vary with the size of the system”. The selected scale tyre 4.00-7 was nearly one-half of the geometric size of the full-scale tyre 9.00-14 with the same tyre width/diameter ratio of 0.29. The tyres were tested under several different soil conditions (clay soil with different cone index values) at several different loads. Therefore, it becomes crucial to analyse the mobility performance parameters of tyres vs slip relations with respect to the size of the system. A similar system for full-scale and scale model tyres was established using two similarity criteria: geometric similarity and dynamic similarity. The geometric similarity is established by equating physical tyre parameters such as tyre diameter to tyre width (b/d) and the dynamic similarity is established between full-scale and scale systems when they experience similar net force. The flow chart is in Figure1-19.

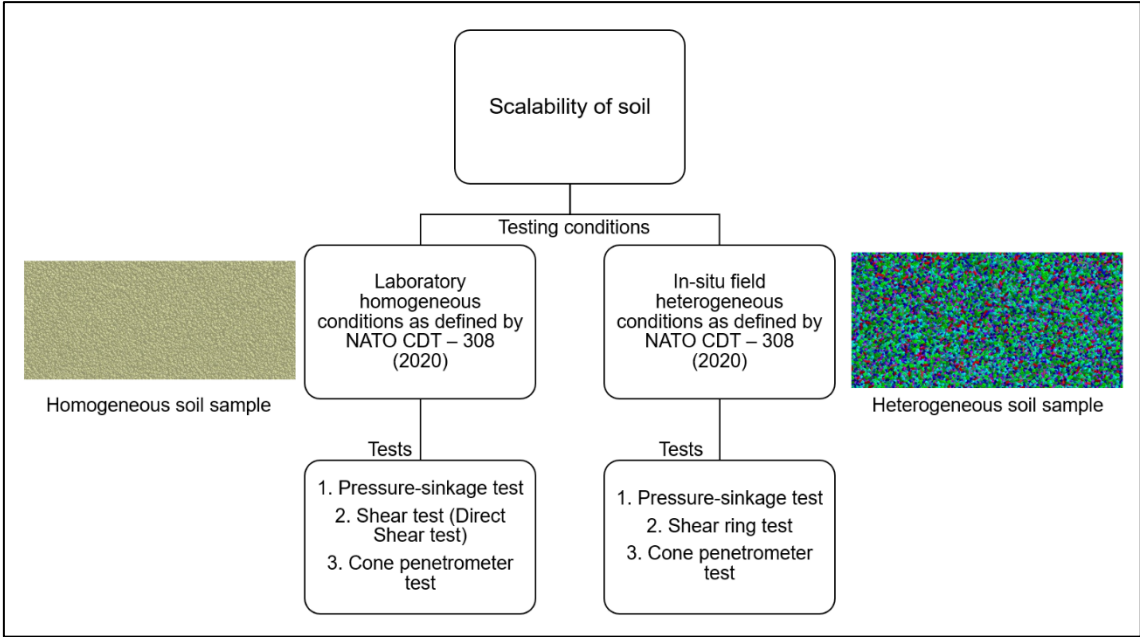


**Figure 1-18 Scalability of the soil flow chart**

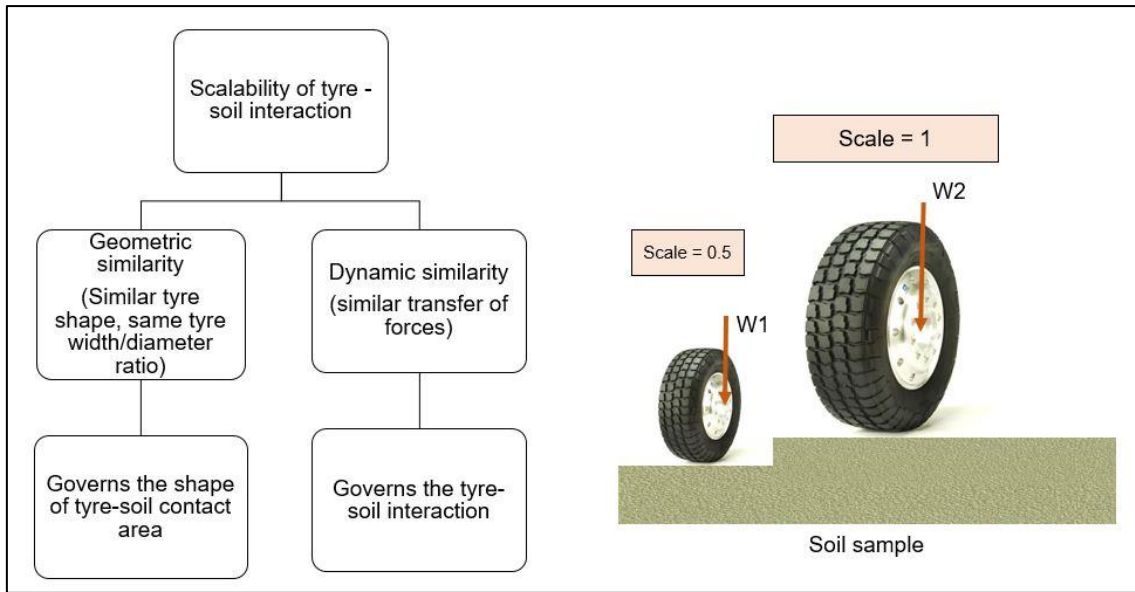


**Figure 1-19 Scalability of the tyre-soil interaction flow chart**

#### 1.2.4 Scaling of soil properties

The physical properties of soil are highly variable and show scale-dependent spatial variability (Biswas and Si, 2014). The soil spatial variability is defined as the soil properties measured at different locations exhibiting different values. The scale dependency is defined as the statistical measure of properties change with change in scale (e.g., length scale). The scale is the characteristic length or area over which the experiments are conducted (Biswas and Si, 2014). For environmental, agricultural and terramechanics applications, some questions are always being asked about soil properties: How can we interpret and correlate the experimental data or information over scales, given the data or information collected at a given scale? Can we interpolate or extrapolate the information from one measurement scale to others? Can we transfer the information from laboratory observations to large scales (e.g., in-situ field or simulation models)? (Jayakumar et al., 2016). These issues arise because the scale at which soil data is collected (e.g., laboratory experiments) is different from the scale at which the prediction is made or the data will be used (e.g., in-situ field or simulation models). To bridge this gap, we need to understand the scalability of soil properties.

We often need to use information, data, or mathematical relationships from one scale to another. This change in the scale is called the scalability (Figure 1.20). The scale at which the actual soil processes are going on (e.g., additions, losses, transformation and translocation) is called the process scale. However, the time scale for the process scale is too long compared to the time needed for the mobility experiments. Therefore, no change in the process scale is observed during the scalability investigation.

| Hierarchy | Scalability                                                | Remark                                                                                                                                                                     |
|-----------|------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1         | Process scale<br>a. Time dependent                         | It is constant in this research                                                                                                                                            |
| 2         | Measurement scale<br>a. Extent<br>b. Spacing<br>c. Support | Established between the NATO CDT – 308 (2020) experiments and simulation models in laboratory (homogeneous) conditions and in-situ (heterogeneous) conditions respectively |
| 3         | Model scale<br>a. Extent<br>b. Spacing<br>c. Support       | Established between the laboratory (homogeneous) simulation models and in-situ (heterogeneous) simulation models                                                           |

**Figure 1-20 Scales in the Scalability (Biswas and Si, 2014)**

Generally, there are two changes in scale, from the real process to the observation (i.e., measurement scale) and from the observation to the prediction (i.e., model scale). The scale at which measurements for any soil properties are taken is defined as the measurement scale or observation scale. According to Blöschl and Sivapalan (1995), the measurement scale consists of three critical characteristics: spacing, extent, and support, which are also known as the “scale triplet”. (Figure 1.21). Spacing is the distance between samples; extent refers to the maximum distance between two samples farthest locations, and support is the area over which the measurement is taken. The model scale is defined as the scale (or length scale) at which the model is established and used for validation and prediction. According to Blöschl and Sivapalan (1995), the model scale also consists of the “scale triplet” similar to the measurement scale.



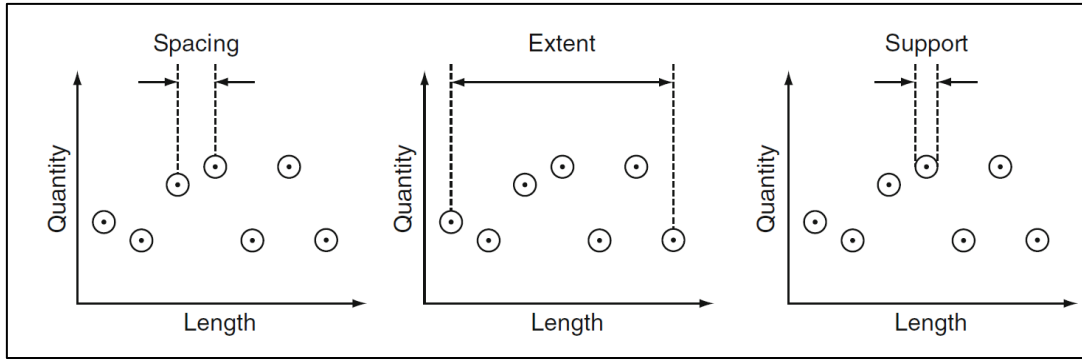


**Figure 1-21 The scale triplet (Biswas and Si (2014))**

In the present study, the experiments were performed at the Keweenaw Research Center (KRC), Michigan Technological University by the NATO research group (NATO CDT-308 (2020)). The pressure-sinkage, shear, cone penetrometer and vehicle mobility tests were performed on 2NS sand and Fine Grained soil (FGS). The measurement scale is established between the NATO experiments and the laboratory (homogeneous) simulation models keeping the same correlation range properties such as moisture content, bulk density, compaction, and interparticle forces. The correlation range is the representative spatial distance within which the soil's physical properties don't change. In the 'scale triplet' of the measurement scale, the support is kept the same e.g., the pressure-sinkage test plate area is the same in the experiments and simulations. The spacing and extent are not applicable at the measurement scale as the test observations are conducted at one location only i.e., in the laboratory.

The model scale is established with the laboratory (homogeneous) soil simulation model, and it is used to validate and compare the soil properties of the in-situ field (heterogeneous) terrain simulation model keeping the same correlation range properties such as moisture content, bulk density, compaction and interparticle forces. The heterogeneity is defined based on the differences in the sand, silt and clay particle shapes and sizes in the simulation models.

Further, the scale triplet is kept the same in the model scale. The support is the same as the same specification equipments used in the measurement area. The extent of the NATO in-situ field experiments on 2NS sand and FGS is approximately 6 m as shown in Figure 1.22. So, the same extent is used for the

in-situ field (heterogeneous) terrain simulation model. Similarly, the equidistant spacing at the test location of 6 m x 4 m is followed in the simulations (i.e., Figure 1.23). Therefore, the same scale triplets are used in the model scale.

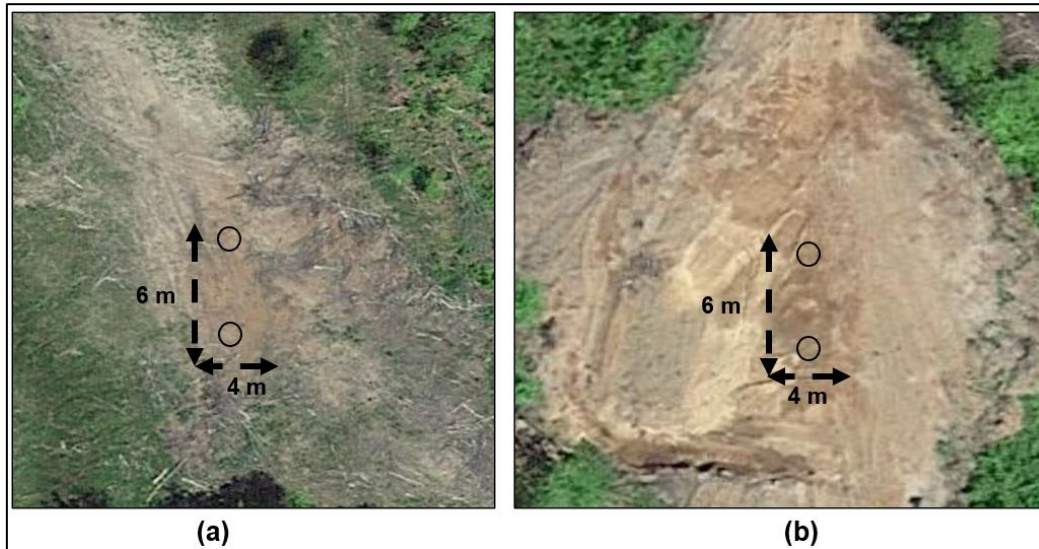


**Figure 1-22 NATO in-situ experiment site location of (a) 2NS sand, (b) Fine grain soil**

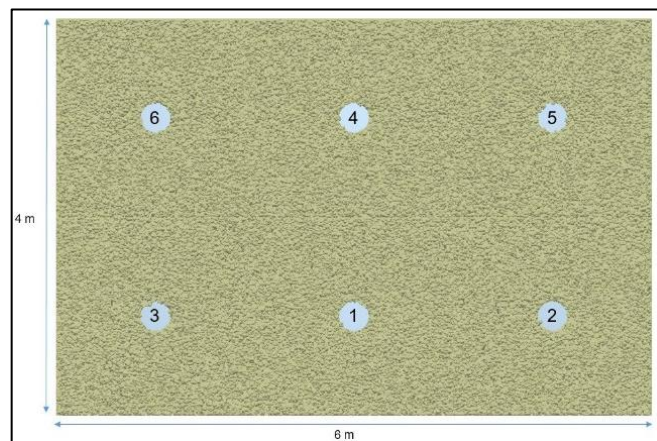


**Figure 1-23 In-situ field (heterogeneous) simulation model for soils**

Once the measurement scale and model scale are established, the scalability of soil can be defined as a relationship between the mechanical properties of an in-situ terrain (heterogeneous) system and a laboratory (homogeneous) system while accounting for the differences in sand, silt and clay particle shapes and size distributions. Physical properties such as moisture content, bulk density,

compaction, and interparticle forces are kept the same for laboratory and in-situ terrain conditions. It was found in this research that the heterogeneous terrain simulation model predicts better mobility performance parameters compared to the homogeneous terrain model (sections 5.7.1 and 5.7.2).

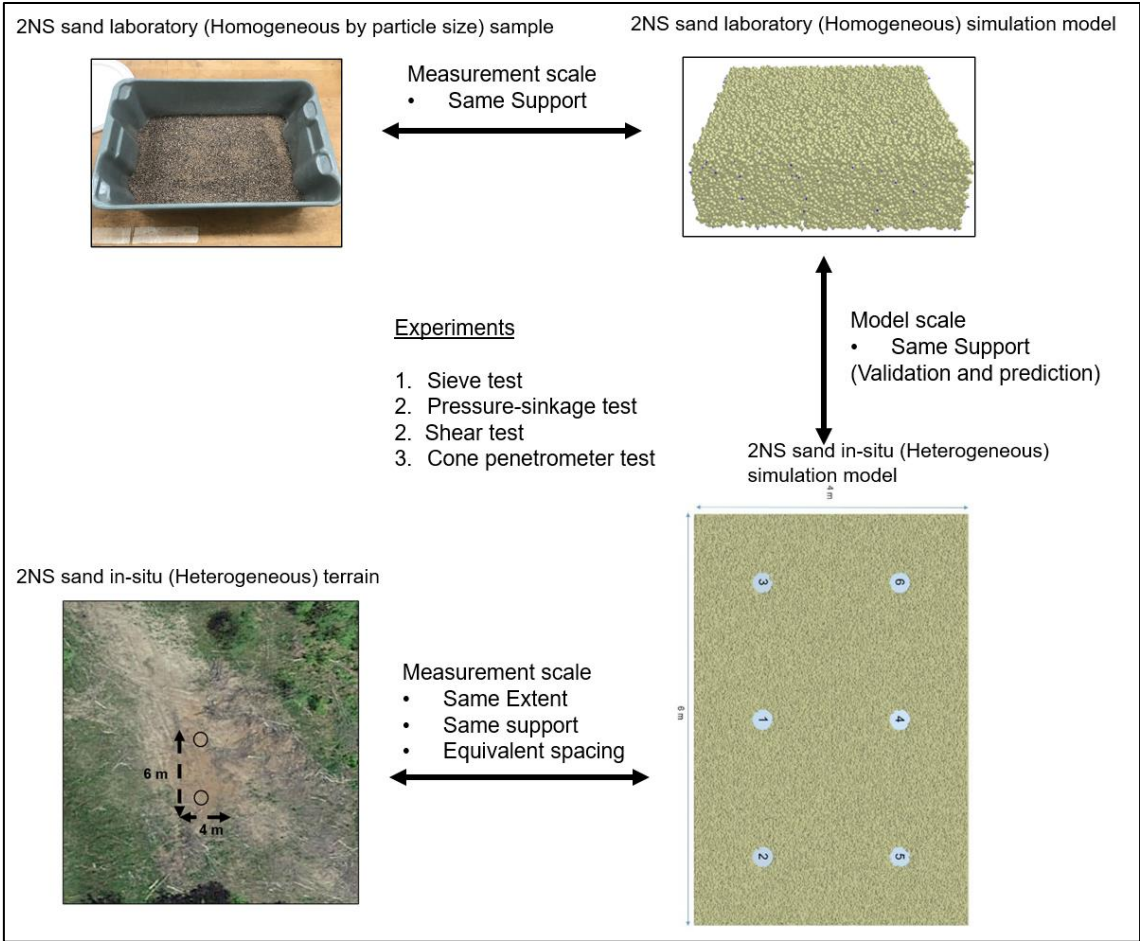


Figure 1-24 The scalability of soil properties chart

### **1.3 Research question**

Existing NATO's NRMM models for mobility performance prediction for lightweight vehicles are based on experimental-driven empirical modelling. To the contrary for small scale UGVs mobility performance prediction based on experimental data, is very limited.

Empirical models in general, are specific to the vehicle testing parametric conditions such as variability in the loading conditions. In these instances, the results are normally acceptable. However, scaling down from lightweight APC vehicle data to small vehicles (UGVs) is challenging. This is due to the complex nature of the transfer of forces in the tyre-soil interaction mechanics.

For example, when two vehicles are constructed/developed, one being a lightweight APC and another one being a small UGV, it would require extensive and expensive development of these (including testing). A non-empirical approach method could provide benefits for establishing the common thread of the tyre-soil interaction, similar to the concept of the Reynolds number which is highly used in aerodynamic airframe scaling.

As described in section 1.2 and narrowing the scope of an achievable subset of the overall problem of scalability of soil and scalability of tyre-soil interaction in vehicle research and development, the following research question is proposed:

*“How is the scalability of the soil models behaving with respect to soil heterogeneity? What is the lower bound of scalability while retaining consistent tyre soil relationship between the lightweight APC and smaller (UGV) vehicles?”*

To adequately answer the above questions, the aim and objectives are discussed in section 1.4.

### **1.4 Aim and objectives**

The work reported in this thesis aims to develop the method to test the scalability of soil models and ground vehicle mobility performance parameters on soft soil. Precisely, it is the scalability of (a) shear stress-shear displacement relationship, (b) pressure-sinkage relationship, (c) the cone index vs depth relationship, (d)

drawbar pull coefficient vs slip relationship, (e) tractive efficiency vs slip relationship.

Several objectives must be met to achieve the above aim. The key objectives are as follows:

- Development of a generic soil modelling approach, which can consider the effect of cohesive and adhesive forces present in the coarse-grained and fine-grained soil;
- Assessment of the cone penetrometer, shear and pressure-sinkage relationship of soil models, considering homogeneous and heterogeneous soil systems;
- Development of the theory of the scalability of the tyre-soil interaction system based on the dimensional analysis approach. Further, experimentally validating the obtained non-dimensional ratio with respect to full-scale and scale systems;
- Development of a generic tyre-soil interaction system, which can determine the drawbar pull coefficient and gross traction ratio of a ground vehicle's tyre over a range of longitudinal slip ratios;
- Validation of the developed soil models and tyre-soil interaction system with the experimental results of NATO CDT – 308 (2020);
- Assessment of mobility performance parameters vs slip relations for full-scale (larger military vehicle) and scale (UGV) tyres on developed soil models.

The aim and objectives set out in this section will be achieved by combining the novel simulation methodology put together.

## **1.5 Assumptions and constraints**

In the context of the present work, the problem is simplified, and assumptions are made to provide constraints that allow a solution to be developed within the given timeframe. The assumptions that have been made are as follows:

- It assumes that the soil's heterogeneity is defined using randomness in the size of six different particle shapes defined in the EDEM simulation software. These particle shapes represent the aggregate nature of soil particles. The adhesive and pull-off forces are defined to introduce soil conditions such as moisture content in the simulations. The homogeneous soil models are defined using a three-particle type in EDEM simulation software.
- Tyre models are assumed rigid for the full-scale and scaled tyres. However, both tyres' numerical stiffnesses are kept the same for computing the forces between soil particles and the tyre. It is done by keeping the same tyre material model parameters (such as tyre Elastic modulus or Poisson's ratio) in the simulations.
- The boundary of application is that lightweight Armoured Personnel Carriers such as the FED Alpha or Land Rover, are used as a full-scale system and UGVs, such as the Warthog, are considered a scaled system.

## **2 LITERATURE REVIEW**

### **2.1 Introduction**

Scale-model testing is an indispensable tool in the aerospace and shipbuilding industry. Small scale modelling of full-scale aircrafts and ships made it easy to address aerodynamics and hydrodynamics challenges over time. It has proven useful in checking analytical work and providing empirical and semi-empirical solutions to challenges, which withstand the analytical treatment. The high costs of constructing and testing full-scale aircraft and ship models also made scale-model testing an economic necessity in both industries. Scale-model testing also provides the opportunity to perform controlled condition experiments, which are hazardous to operators while performing full-scale model testing.

Aside from the economic benefits of small-scale model testing, it can solve immediate and specific technical problems. It is undoubtedly true in predicting changes in the full-scale model's mobility performance concerning design and environmental conditions using scale model testing. It can also yield particular design solutions to more complex challenges quickly and economically.

The tyre–soil mobility prediction methods are classified into empirical, semi-empirical, analytical and numerical models. The empirical models are developed from an experimental based approach. The experiments are conducted in controlled laboratory conditions and in-situ (undisturbed natural) field conditions with different tyres and soil parameters. The results are then correlated to obtain a mathematical model to evaluate the experimental tyre's mobility performance parameters on soft soil. This model's main drawback is that it does not explain the physical phenomenon between the tyre-soil interaction and considers a single parameter to characterise soil strength i.e. cone index (Tiwari et al., 2010).

The semi-empirical models improve the limited predictive capability and explain some of the physical processes involved in the tyre-soil interaction such as the sinkage of a tyre in soft soil. These models incorporate experimental results with empirical formulations and develop analytical solutions to model tyre-soil

interaction. One of the most commonly referenced work in this area is Bekker (1956), which is discussed in section 2.8.2.

The analytical models seek to improve the prediction capabilities of empirical mobility models by including the physics-based phenomenon involved in the tyre-soil interaction. Unlike empirical and semi-empirical models, these models represent the radial and tangential stress distribution during the tyre-soil interaction to determine the mobility performance parameters. Due to the complex nature of tyre-soil interaction, which involves dynamic contact, non-linear materials and large deformations, these models are generally simplified to reach a definite solution (Wright, 2012).

The numerical approach encompasses the Finite Element Method (FEM), Smoothed Particle Hydrodynamics (SPH) and the Discrete Element Method (DEM). In FEM, the soil is modelled as a continuum medium and broken down into finite elements to capture the tyre-soil interaction phenomenon. However, the real terrain is not a continuum medium, and therefore, FEM fails to capture the soil flowability during tyre-soil interaction. The SPH is a mesh-free method where particles are used to solve the governing equations of continuum mechanics (Cauchy equation of motion in the case of soils). A kernel smoothing function is used to discretize the continuum equations for each particle in order to evaluate each particle's properties and any fluxes or forces acting on it using neighbouring particles (NATO CDT – 308 (2020)). Similar to FEM, SPH also assumes the soil to be a continuum. The discrete element method has been used extensively (Nakashima et al., 2004, 2007, 2009; Smith et al., 2013; Zhao et al., 2014; Nishiyami et al., 2016, 2017; Johnson et al., 2015, 2017; Yonghao Du et al., 2017, 2018; Nakanishi et al., 2020; Ravula et al., 2021, Wu et al., 2022). This method treats the soil as a collection of discrete individual particles that interact with each other and tyres. In subsequent sections, this method will be discussed in more detail.

## **2.2 Mobility performance parameters**

In this section, critical parameters are introduced to analyse vehicle mobility performance.



### 2.2.1 Longitudinal slip ratio (LSR)

Slip can be defined as the reduction in distance travelled or speed because of flexing in the tractive device (tyre or wheel) or shearing within the soil (ISTVS standards, 2020). It generally occurs between the overlapping surfaces of the tractive device and terrain. The wheel slip ratio in the longitudinal direction of travel of the tractive device is considered as the longitudinal slip ratio (LSR). The angle formed between the direction of travel and the line of intersection of the longitudinal central plane of the tractive device is known as the slip angle (Wong, 2008). In this study, the wheel slip ratio is defined as the longitudinal slip ratio (LSR) at zero slip angle.

Mathematically, it can be defined as:

$$S = 1 - \frac{V_a}{\omega r} \quad (2-1)$$

where,  $S$  = Longitudinal slip ratio (LSR)

$\omega$  = angular velocity of the wheel, rad/s

$r$  = rolling radius of the wheel on a hard surface, m

$V_a$  = actual velocity of the wheel, m/s

As per ASAE standards, 2001b, zero-wheel slip can be defined using any of the four methods listed below:

- 1) a self-propelled condition of a wheel on a non-deforming surface
- 2) a self-propelling condition of a wheel on the test surface
- 3) a towed condition of a wheel on a non-deforming surface
- 4) a towed condition of a wheel on the test surface

There are many arguments for using any of the above methods in a particular single wheel testing. Generally, it is assumed that the towed condition occurs at negative slip and the self-propelled condition occurs at positive slip (Wismer and Luth, 1973). A zero-slip condition obtained using a non-deforming test surface (or

hard surface) in a self-propelled or towed condition makes the test data repeatable but it may not represent the actual zero slip condition on the test terrain (ASAE standards, 1998). A zero-slip condition obtained using a test surface (or experimental terrain) in a self-propelled or towed condition provides reliable test data on that terrain, but is not repeatable (ASAE standards, 1998). In any case, the zero condition should be stated in single wheel testing. It is because the zero condition is used to define the wheel's rolling radius (Upadhyaya et al., 1988)).

As per ISTVS standards (2020), the rolling radius can be defined as the distance travelled per revolution of the wheel when operating under zero slip condition, divided by  $2\pi$ . Generally, different zero conditions lead to different rolling radius; hence, it leads to different slip values for the same test. The rolling radius is a special case of the effective rolling radius under a zero condition (He et al., 2020). The effective rolling radius is defined as the quotient of the longitudinal forward speed of the tyre divided by the angular speed of the tyre (He et al., 2020). However, for the rigid tyre, the rolling radius and the effective rolling radius are the same.

### **2.2.2 Motion resistance**

In broad terms, motion resistance (MR) can be defined as the resistive force offered to the motion of a vehicle due to internal and external stimuli (ISTVS standards, 2020). The internal stimulus ( $R_{int}$ ) is primarily due to the hysteresis developed into the tyre material because of carcass deflection and its impact inside the wheel or track while rolling. The external stimulus is an aggregate of compact resistance ( $R_c$ ), bulldozing resistance ( $R_b$ ), aerodynamic motion resistance ( $R_a$ ), inertial resistance ( $R_i$ ), slope resistance ( $R_s$ ) and obstacle resistance ( $R_{ob}$ ).

$$MR = R_{int} + R_c + R_b + R_a + R_i + R_s + R_{ob} \quad (2-2)$$

Wong (2010) analysed that the significant proportion of motion resistance offered to a tyre is because of hysteresis developed inside the tyre at significant velocity and compact resistance. The tyre's hysteresis can be controlled by controlling the

vehicle's velocity. In low velocity regions i.e. 0.1 to 5.5 m/s, the loss of useful work due to hysteresis is small. Therefore, most of the work carried out in mobility performance studies is limited to the low-speed region.

The compact resistance is the resistive force developed due to the shearing in the soil during tyre-soil interaction. When vehicle weight increases, the tyre ground pressure on the soil also non-linearly increases due to non-linear tyre material stiffness, thus generating more sinkage to the vehicle. Therefore, the vehicle's weight is one of the critical parameter in controlling the compact resistance.

The bulldozing resistance is the horizontal terrain deformation resistance, which becomes significant at significant tyre sinkage. It is analogous to the force due to the passive earth pressure acting on the retaining wall (Wong, 2010). However, this analogue is not consistent with the tyre-soil interaction phenomenon and can be experimentally observed in the case of locked wheel and not for a driven or towed wheel (Wong, 2010). Therefore, performing the controlled experiments can simultaneously control bulldozing resistance and other resistive forces such as aerodynamic motion resistance and slope resistance.

Tyre parameters have a significant effect on motion resistance. Gee-Clough (1978) performed the single wheel experiments with five different wheels each of diameter 0.23 m and width ranging from 0.025 m to 0.23 m on the dry sand. The results showed that motion resistance increases rapidly at a given sinkage on the sand with the increase in the rigid wheel's width. Tyre motion leads to soil build-up formation in front of all wheels, thus increasing the bulldozing resistance. Elwaleed et al. (2006) experimented with studying the effect of the three inflation pressures (166, 193 and 221 kPa) of 5-12 tyre on sandy-clay-loam soil. The result showed that when the tyre nominal inflation pressure (221 kPa) was reduced to 193 kPa, motion resistance decreased by almost 4%, but when it was reduced to 163 kPa, MR increased by approximately 12%. Therefore, there exists an optimum tyre inflation pressure condition where MR is minimum on specific terrain. Lyasko (2010) studied the effect of tyre slip on soil's motion resistance. The experiments were performed with 18.4R38 tyre on sandy soil with a constant

velocity of 1.5 m/s. The results examined that for tyre, slip range between 0 to 30%, MR increases monotonically, but for slip greater than 30%, an increment of 50-60% was observed. It is because of the slip-sinkage effect. Within the slip range of 0 to 33%, sinkage increases continuously, and after that, a sudden increase in sinkage of up to 60% was observed.

Theoretically, motion resistance can be measured with the application of mechanics laws. On a terrain, a vehicle is towed and then moved with constant velocity and the force is measured. Due to dynamic equilibrium, the towed force is equal to MR. Since the vehicle is under constant vertical load, the tyre deformation and soil compaction would be producing MR.

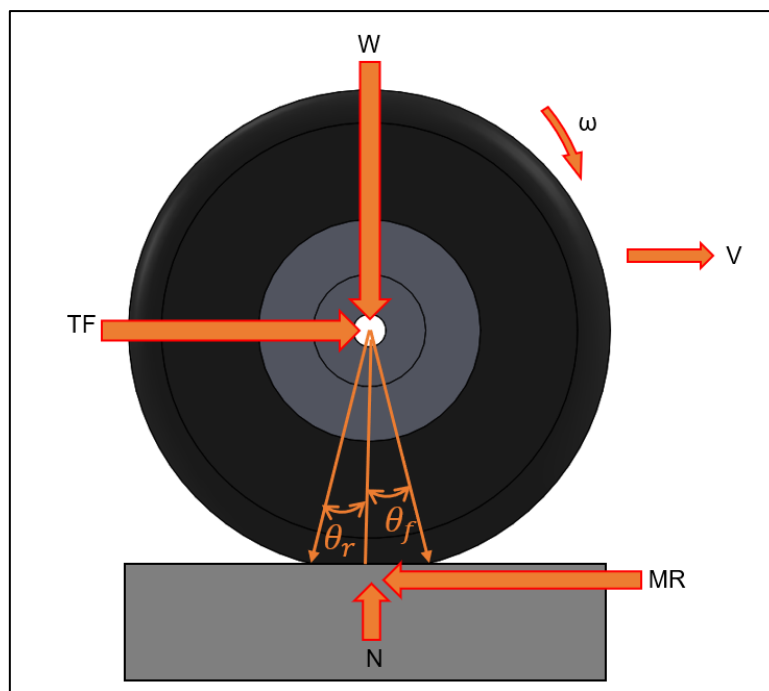


**Figure 2-1 Free body diagram of towing of a rigid wheel on a hard surface**

In the empirical approach, motion resistance cannot be measured directly. The applied torque,  $T$  (section 2.1.2) and drawbar pull,  $D$  (section 2.1.3) on the wheel of radius  $r$ , is measured. Motion resistance is then calculated using the theoretical approach shown in Equation 2-3. Generally, motion resistance is expressed as motion resistance ratio (MRR), i.e. the ratio of MR and normal load ( $W$ ) on a tyre. Extensively used MRR models are expressed as a function of slip ( $s$ ), wheel

numeric ( $C_n$ ), Brixius number ( $B_n$ ), mobility index (MI) as shown in Equation 2-4, 2-5 and 2-6 and covered in later sections of this chapter.

$$MRR = \frac{MR}{W} = \frac{\left(\frac{T}{r} - D\right)}{W} \quad (2-3)$$

$$MRR = \text{function}(s, C_n) \quad (2-4)$$

$$MRR = \text{function}(s, B_n) \quad (2-5)$$

$$MRR = \text{function}(MI) \quad (2-6)$$

In the semi-empirical approach, tyre-soil interaction works on a fundamental assumption for MR's calculation. Vertical deformation of the soil under normal tyre load is assumed similar to the soil deformation under a flat plate of the same tyre-soil contact area. Therefore, the distributed normal pressure,  $p$  applied on the terrain by vehicle, is measured using a plate sinkage test. Using the theoretical approach, motion resistance is calculated using Equation 2-7.  $\theta$  is an arbitrary angle between the tyre-soil contact arc.  $\theta_r$  and  $\theta_f$  are the rear and forward contact angles of the tyre-soil interface of width,  $b$  (Figure 2.1).

$$MR = r * b * \int_{\theta_r}^{\theta_f} p(\theta) \sin \theta \, d\theta \quad (2-7)$$

### 2.2.3 Gross traction

In the literature, gross traction tends to be a thrust force generated due to powered wheel interaction with terrain (ISTVS standards, 2020). The powered wheel signifies a tyre with an input torque from the engine or motor. During this interaction, the powered running wheel applies force on the soil, trying to break it. Therefore, as per Newton's third law of motion (every action has equal and opposite reaction force, and they act on different bodies), soil generates an equal and opposite force to wheel force. This opposite soil force generated is the thrust force or gross traction (GT). It helps the vehicle to overcome motion resistance and produce useful work.

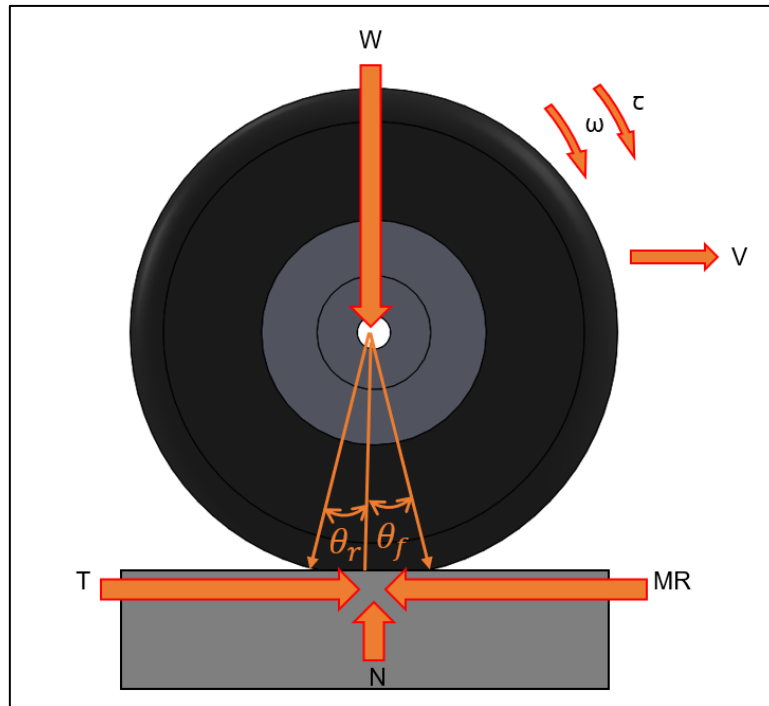


**Figure 2-2 Free body diagram showing the generation of gross traction by a powered wheel on a non-deforming surface**

Tyre characterisation and soil modelling influence the generation of traction. Micklethwaite (1944) analysed that traction increases with the tyre-soil contact area because of the increase in soil volume, which is in contact with the tyre and is responsible for the generation of traction. Upadhyaya et al. (1989) and Lyasko (2010) performed experiments with the wheeled vehicle to predict the traction under various soil and different loading conditions. The results showed that changes in soil conditions affect gross traction more than the change in tyre loading and geometry. Ravula et al. (2021) found that the gross traction increases with tyre load. However, for different soils but with the same tyre characteristics, the traction generated differs.

Upadhyaya et al. (1989)'s experimental results with three radial-ply tyres (16.9R38, 18.4R38 and 24.5R32) on Yoyo loam soil (plastic soil) showed that gross traction increases exponentially with the increase in slip and becomes steady at optimum slip condition. Wang et al. (2020) and Nakanishi et al. (2020) also obtained a similar trend. However, Wong (1983) performed the experiments with a tyre diameter of 30 cm and a tyre width of 25 cm operating on brittle soil (c

= 2.6 kPa and  $\phi = 29^\circ$ ). The results showed that gross traction increases with the longitudinal slip ratio (LSR) and reached its peak value at a longitudinal slip ratio (LSR) of 25% and then it decreases continually with the increase of the wheel's longitudinal slip ratio (LSR) (Figure 2.3).

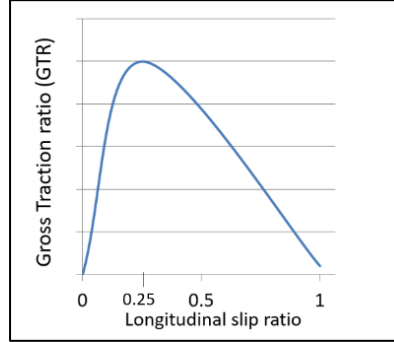


**Figure 2-3 Gross traction vs slip curve based on Wong (1983) results**

In the empirical approach, traction calculation is dependent on the input torque (T) and the radius of the wheel, as shown in 2.8. Generally, gross traction is calculated as gross traction ratio (GTR), i.e. a ratio of GT and normal load (W) on a tyre. Extensively used GTR models are expressed as a function of slip (s), wheel numeric ( $C_n$ ), Brixius number ( $B_n$ ), mobility index (MI) as shown in Equation 2-9, 2-10 and 2-11 and covered in later sections of this chapter.

$$GTR = \frac{T}{r \times W} \quad (2-8)$$

$$GTR = \text{function}(s, C_n) \quad (2-9)$$

$$GTR = \text{function}(s, B_n) \quad (2-10)$$

$$GTR = \text{function}(MI) \quad (2-11)$$

In the semi-empirical approach, soil-traction device interaction works on a basic assumption. The shear deformation of the soil under the wheel is assumed similar to the shear under the torsional shear device or grouser unit. Therefore, shear stress,  $\tau$  is calculated using the direct shear box. Micklethwaite (1944) based on the assumption, derived a correlation of maximum trust generated by a vehicle, which depends on the maximum shear strength of soil based on Mohr-Coulomb

theory (Equation 2-13). The theory correlates the maximum traction,  $GT_{\max}$  generated to the shear parameters of soil such as cohesion  $c$  & internal angle of friction  $\phi$ , wheel contact area  $A$ , normal load  $W$  and normal stress  $p$  as shown in Equation 2-12 and 2-13.

$$\tau = c + p \tan \phi \quad (2-12)$$

$$GT_{\max} = cA + W \tan \phi \quad (2-13)$$

Janosi and Hanamoto (1961) modified the Micklethwaite equation for deformable terrains, Equation 2-14. It includes the effect of soil deformation,  $j$  and soil deformation modulus,  $K$  on the gross tractive force. It can be analysed that with the increase in the soil displacement, the gross traction force increases to its maximum value and then becomes steady, as shown by Equation 2-14.

$$GT_{\max} = (cA + W \tan \phi) * (1 - e^{-\frac{j}{K}}) \quad (2-14)$$

Using the theoretical approach in the semi-empirical formulation, traction can be calculated using Equation 2-15. Here,  $\theta$  is an arbitrary angle between the wheel-soil contact arc.  $\theta_r$  and  $\theta_f$  are the rear and forward contact angles of the wheel-soil interface of width,  $b$  (Figure 2.2).

$$GT = r * b \int_{\theta_r}^{\theta_f} \tau(\theta) \cos \theta d\theta \quad (2-15)$$

#### 2.2.4 Drawbar pull

Previous studies mostly defined drawbar pull as the excessive force produced by the vehicle at the drawbar to do the external work in the direction parallel to vehicle motion (ISTVS standards, 2020). Theoretically, drawbar pull is defined as a difference between gross traction and motion resistance, as shown in Equation 2-16. Therefore, a negative drawbar pull means the vehicle is immobile.

$$D = GT - MR \quad (2-16)$$



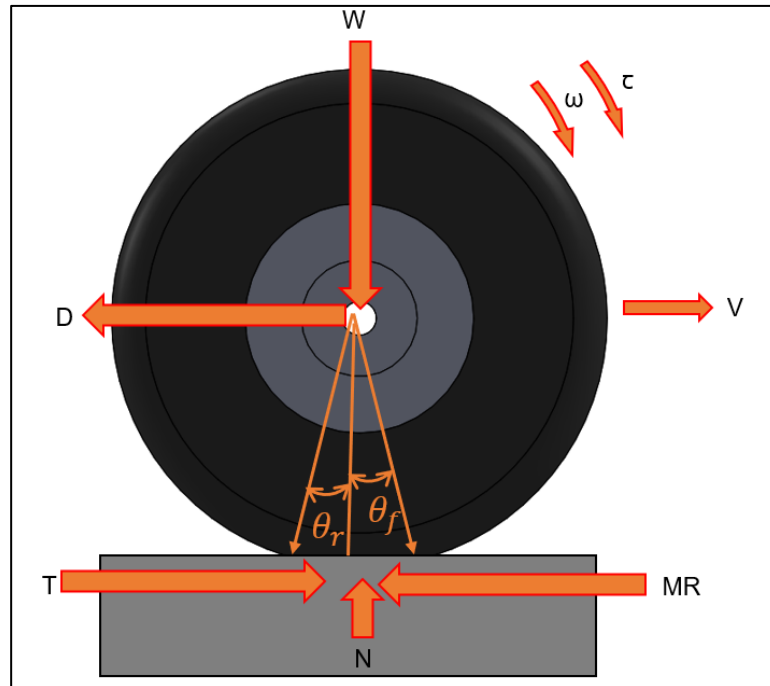


**Figure 2-4 Free body diagram showing the generation of drawbar pull**

Taghavifar et al. (2015) performed the experiments with 9.5L-14, 6 radial ply tyre on clay-loam soil at three different velocities (i.e. 0.7, 1.4 and 2 m/s), three slippage values (i.e. 10, 12 and 15%), three inflation pressure (i.e. 100, 200 and 300 kPa) and three vertical wheel loads (i.e. 2, 3, 4 kN) on the soil (%sand = 34.3, %silt = 22.2, %clay = 43.5, cone index (kPa) = 700, bulk density ( $\text{kg/m}^3$ ) = 2360,  $\phi = 32^\circ$ ). It analysed the effect of slip, velocity and wheel load on drawbar pull using a single wheel tester on the soil. The results showed that with the increase in wheel load, drawbar pull increases. It may be because, with the increase in the normal load, the bulk density locally increases which increases the shear stress, which increases the gross thrust or traction (Micklethwaite (1944) or equation 2.13). An increase in slippage increases the resistive tangential force of soil contact volume, thus increasing gross traction. The same effect of the slip was also observed in Wang et al. (2020), Nakanishi et al. (2020), Tiwari et al. (2009) and Zoz et al. (2003). However, no significant effect of velocity was observed on the drawbar pull. It may be because of invariability in the tyre-soil contact area at different velocities. Taghavifar et al. (2013) and Verschoore et al. (2001) recorded the same effect of velocity on traction. Elwaleed et al. (2006) observed the effect of tyre inflation pressure on traction on deformable surfaces like sand. The

analysis showed that reducing inflation pressure increases sand traction as stress under the wheel increases.

In the empirical approach, drawbar pull can be measured using the single wheel test experiment. This test consists of tyre test carriage, electronics (e.g. load cell) and a data acquisition system (DAS) (Tiwari et al., 2010). In the tyre test carriage, the applied normal load is transferred solely on the testing wheel (as shown in Figure 2.4) and the tyre is moved on a soil bed with slow and steady velocity (e.g., 0.1-5.5 m/s). The resultant drawbar pull is measured using the load cell and recorded by DAS. Generally, drawbar pull is calculated as drawbar pull ratio (DPR), i.e. the ratio of drawbar pull (D) and the normal load (W) on the test wheel, as shown in Equation 2-17. Extensively used DPR models are expressed as a function of slip (s), wheel numeric ( $C_n$ ), Brixius number ( $B_n$ ), mobility index (MI) as shown in Equation 2-18, 2-19 and 2-20 and covered in later sections of this chapter.

$$DPR = \frac{D}{W} \quad (2-17)$$

$$DPR = \text{function}(s, C_n) \quad (2-18)$$

$$DPR = \text{function}(s, B_n) \quad (2-19)$$

$$DPR = \text{function}(MI) \quad (2-20)$$

In the semi-empirical approach, drawbar pull can be calculated using Equations 2-7, 2-15 and 2-16. The general form of the drawbar pull is shown in Equation 2-21. Here,  $\theta$  is an arbitrary angle between the wheel of radius,  $r$  and soil contact arc.  $\theta_r$  and  $\theta_f$  are the rear and forward contact angles of the wheel-soil interface of width,  $b$  (Figure 2.4).

$$D = r * b \int_{\theta_r}^{\theta_f} \tau(\theta) \cos \theta \, d\theta - r * b \int_{\theta_r}^{\theta_f} \sigma(\theta) \sin \theta \, d\theta \quad (2-21)$$

### 2.2.5 Tractive efficiency

The term 'tractive efficiency' refers to the vehicle's capability in transforming the input axle power from the engine to the usable power available at the drawbar (ISTVS standards, 2020). Theoretically, it is defined as the drawbar power ( $P_d$ ) available to the input power ( $P_{in}$ ) by the vehicle engine with sprocket torque,  $T_{dw}$  and angular velocity of the drive wheel  $\omega_0$  as shown in Equation 2-22. The efficiency of motion depends on the transmission losses in power, motion resistance, sinkage and other vehicle & soil parameters. Research studies conducted by Zoz and Grisso (2003) indicates that between 20 and 55% of tractive power losses are due to tyre-soil interaction. Therefore, it becomes crucial to understand the variation of tractive efficiency concerning tyre characteristics and soil parameters.

$$\eta = \frac{P_d}{P_{in}} = \frac{D.V}{T_{dw}.\omega_0} \quad (2-22)$$

Upadhyaya et al. (1989), Elwaleed et al. (2006), Lyasko (2010), Taghavifar et al. (2014) and Ravula et al. (2021) studied the effect of slip on tractive efficiency. Results showed that with the increase in the slip, tractive efficiency increases monotonically to its optimum value and then decreases. It is because with the increase in the slip, drawbar pull increases exponentially and becomes steady at optimum slip condition, or the rate of increment decreases. On the other side, motion resistance keeps increasing with respect to slip, so tractive efficiency decreases after an optimum slip. Generally, the peak efficiency for a tyre moving on rigid or flexible terrains lies between 10% and 30% of slippage.

Taghavifar et al. (2014) studied the effect of wheel load and velocity on tractive efficiency. The experiments were performed with 220/65R21 radial ply tyre (Inflation pressure = 131 kPa) on clay-loam soil at three different velocities (i.e. 0.8, 1 and 1.2 m/s), three slippage values (i.e. 10, 12 and 15%) and three vertical wheel loads (i.e. 2, 3, 4 kN) on the soil (%sand = 34.3, %silt = 22.2, %clay = 43.5, cone index (kPa) = 700, bulk density (kg/m<sup>3</sup>) = 2360,  $\phi = 32^\circ$ ). The research found that tractive efficiency increases with the increase in wheel load and velocity. Keeping other system parameters constant, doubling the wheel load showed

growth in tractive efficiency of approximately three times. It may be because the increase in the wheel load increases the maximum shear stress under the tyre, which increases the drawbar pull and hence, improves the efficiency.

Senatore (2011) analysed the effect of tyre diameter and inflation pressure on lightweight vehicle's tractive efficiency. The simulations were performed with tyre diameter ranging from 400 to 450 mm and inflation pressure ranging from 168 kPa to 240 kPa on dry sand ( $c = 1.11$  kPa and  $\phi = 31.1$  deg) and moist loam (moisture content = 50%,  $c = 3.3$  kPa and  $\phi = 33.7$  deg). It was found that with the increase in wheel diameter and a decrease in inflation pressure, tractive efficiency increases. It may be because of the decrease in sinkage, which decreases compact resistance and hence increases the drawbar pull, which increases tractive efficiency under these conditions.

Wong (2010) categorised tractive efficiency into three sub-parts based on the vehicle-terrain system. These are the efficiency of motion  $\eta_m$ , the efficiency of slip  $\eta_s$  and the efficiency of the transmission system,  $\eta_t$ . The efficiency of motion measures the ability to produce useful work or drawbar pull from the input power. The slip's efficiency measures the amount of power lost due to the slip effect. The transmission system's efficiency shows the energy losses due to friction while transferring power to the wheel.

$$\eta = \frac{D \cdot R_w \cdot (1 - s)}{T_{dw}} \eta_t \quad (2-23)$$

$$\eta = \eta_m \cdot \eta_s \cdot \eta_t \quad (2-24)$$

$$\eta_m = \frac{D \cdot R_w}{T_{dw}} \quad (2-25)$$

$$\eta_s = (1 - s) \quad (2-26)$$

## 2.3 Terramechanics based – Performance metrics

The performance metrics are developed over the years to compare the relative mobility of different vehicles. These metrics are also used to specify a vehicle's

go/no-go mobility condition on a terrain. The performance metrics are often part of mobility prediction models. The critical metrics are discussed in this section.

### **2.3.1 Vehicle cone index (VCI)**

Rula (1971) proposed the VCI model in research studies conducted by the U.S. Army Corps of Engineers Waterways Experiment Station (WES). The primary objective of the VCI model was to give a measure of loss in soil strength with the number of vehicle's passes over the specific terrain. The International Society of Terrain- Vehicle System (ISTVS) defines VCI as "*minimum soil strength in the critical soil layer, in terms of rating cone index for fine-grained soils or cone index for coarse-grained soils, required for a specific number of passes of a vehicle, usually one pass ( $VCI_1$ ) or 50 passes ( $VCI_{50}$ )*". The critical soil layer is defined as the layer, which exerts maximum influence on a vehicle's mobility. The critical layer's depth depends on the wheel loading and contact area of tyre-soil interaction. Priddy et al. (2006) analysed that it is usually 6 to 12 in. below the ground for most vehicles. It cannot be expressed in terms of percent wheel diameter because VCI metric is used to compare the mobility of different vehicles irrespective of their traction element such as a wheeled vehicle can be compared with a tracked vehicle.

Wong et al. (2020) defines a standard technique to estimate VCI through inference from zero- and multi-pass test data. For zero-pass immobilisation tests, the self-propelled test vehicle (initially at zero velocity, no inertia) is operated at a slow and steady speed (in its lowest gear, usually 2-3 mph) in a straight line through the identified test area. The steady throttle is applied until the vehicle completely loses its forward movement or becomes immobilised. The vehicle is then reversed and an attempt is made to back out. If the vehicle does not move, then it is zero-immobilisation. For the multi-pass test, the self-propelled vehicle (initially at zero velocity, no inertia) moves at a slow and steady speed in its lowest gear through the lane. The vehicle will travel forward for the pass one and then travel in reverse for the second pass. This cycle of the vehicle continues until immobilisation occurs. If the vehicle makes six good passes and becomes immobilised during the seventh pass, this is considered as 6<sup>th</sup> pass run. The last

pass is recorded with the soil consistency data (e.g., moisture content and density). This test technique is repeated with different soil strengths to determine limited soil strength at the maximum number of passes.

Priddy et al. (2006) clarified that VCI is incorrectly regarded as the set of empirical equations for prediction, whereas it is a performance metric. It is a mobility index (MI) function, which relates the vehicle characteristics to terrain parameters. It has a linear relationship with it. The general relation between MI and vehicle parameters is shown in Equation 2-27. The vehicle characteristics such as ground contact pressure CPF, weight factor WF, wheel load factor WLF, traction element factor TEF, grouser factor GF, clearance factor CF, engine factor EF and transmission factor, TF are correlated, which influences the mobility performance. The go/no-go mobility conditions for a vehicle can also be predicted based on comparisons between MI and CI, as shown in 2.28.

$$MI = \left( \frac{CPF * WF}{TEF * GF} + WLF - CF \right) * EF * TF \quad (2-27)$$

$$\left\{ \begin{array}{l} MI > CI \mid \text{Vehicle immobile} \\ MI < CI \mid \text{Vehicle mobile} \end{array} \right\} \quad (2-28)$$

Maclaurin (2007) conducted a review to compare the different mobility models for tracked and wheeled vehicles. The research found that at the deep sinkage condition of a military vehicle, the multi-pass VCI test may be less reliable than other test methods. It may be because VCI methodology doesn't include the sinkage factor and tyre deflection factor of a vehicle in soft soil.

### 2.3.2 Wheel mobility number

A mobility number is a dimensionless number used to predict the tractive performance on a terrain. Generally, it is expressed as a function of cone index, wheel load and tyre parameters. Freitag (1965) and Smith and Freitag (1965) developed the first mobility number using the dimensional analysis technique on previously available experimental performance data of pneumatic tyres on sand and clay, respectively, incorporating both tyre and soil parameters. The clay number ( $\pi_c$ ) and sand number ( $\pi_s$ ) were obtained, as shown in 2.29 and 2.30.

The soil strength is represented by the gradient, G of cone index with depth within the sand number.

$$\pi_c = \frac{CI \cdot b \cdot d}{W} \cdot \left(\frac{\delta}{h}\right)^{0.5} \cdot \frac{1}{1 + \left(\frac{b}{2d}\right)} \quad (2-29)$$

$$\pi_s = \frac{G(bd)^{3/2}}{W} \cdot \frac{\delta}{h} \quad (2-30)$$

Turnage (1972) conducted experiments with pneumatic tyres moving on clay and dry sand at 20% slip condition. The broad range of tyres with (b/d) ratio ranging from 0.05 to 0.88, wheel load varying from 400 to 6000 N and tyre deflection of 0.15 to 0.35 for sand and 0.08-0.45 for clay was considered. The cone index varied from 55 to 469 kN/m<sup>2</sup> for both soil types. It was concluded that sand mobility number provides the right prediction of tyre performance for a wide range of soil conditions.

Wisner and Luth (1974) proposed a similar dimensional form of wheel mobility number. It is expressed as a wheel numeric product ( $C_n$ ) and a coefficient ( $k$ ), as shown in 2.31. The wheel numeric ( $C_n$ ) is a dimensionless number, which depends on soil strength CI, tyre load W, tyre width b and diameter d, as shown in 2.32. This term is used to model the mobility performance parameters of the tyre-soil interaction using the empirical methodology, discussed further in section 2.9.1. The widely used coefficients  $k$  to wheel mobility number are shown in Table 2-1.

$$M_i = C_n \cdot k_i \quad (2-31)$$

$$C_n = \frac{CIbd}{W} \quad (2-32)$$

Hegazy et al. (2013) studied the correlation between wheel mobility number and mobility performance parameters. Different researchers have proposed various mobility number expressions depending on the respective tyre-soil systems (Table-2.1). In this study, soil strength and tyre parameters on mobility number were investigated experimentally. It includes the effect of tyre inflation pressure

on tyre parameters such as tyre width, tyre diameter, tyre section height and tyre deflection at different tyre loading conditions. The results showed that the mobility number increases with soil strength. The wheel numeric increases non-linearly when the tyre pressure decreases, tyre diameter increases, tyre width decreases, and tyre load decreases. It was also found that the tyre load, tyre width and tyre diameter are inter-dependent parameters. With an increase in tyre load, tyre width increases, and tyre diameter decreases.

**Table 2-1 Wheel mobility number coefficients**

| References             | Coefficient, $k_i$                                               | Equation, $C_n$                                                                 |
|------------------------|------------------------------------------------------------------|---------------------------------------------------------------------------------|
| Freitag (1965)         | $k_F = \sqrt{\frac{\delta}{h}}$                                  | $C_n = \frac{Clbd}{W} \sqrt{\frac{\delta}{h}}$                                  |
| Turnage (1972)         | $k_T = \sqrt{\frac{\delta}{h}} \cdot \frac{1}{1 + \frac{b}{2d}}$ | $C_n = \frac{Clbd}{W} \sqrt{\frac{\delta}{h}} \cdot \frac{1}{1 + \frac{b}{2d}}$ |
| Wismer and Luth (1973) | $k_W = 1$                                                        | $C_n = \frac{Clbd}{W}$                                                          |
| Rowland et al. (1975)  | $k_T = \sqrt{\frac{\delta}{h}} \left(\frac{d}{b}\right)^{0.15}$  | $C_n = \frac{Clbd}{W} \sqrt{\frac{\delta}{h}} \left(\frac{d}{b}\right)^{0.15}$  |
| Maclaurin (1981)       | $k_M = \frac{\delta^{0.4}}{b^{0.2}d^{0.2}}$                      | $C_n = \frac{Clbd}{W} \frac{\delta^{0.4}}{b^{0.2}d^{0.2}}$                      |
| Brixius (1987)         | $k_B = \left(\frac{1 + 5\delta/h}{1 + 3b/d}\right)$              | $C_n = \frac{Clbd}{W} \left(\frac{1 + 5\delta/h}{1 + 3b/d}\right)$              |
| Hegazy et al. (2013)   | $k_{HS} = \sqrt{\frac{h - \delta}{d}}$                           | $C_n = \frac{Clbd}{W} \sqrt{\frac{h - \delta}{d}}$                              |



### 2.3.3 Nominal ground pressure

The Nominal ground pressure (NGP) is the actual pressure exerted by a vehicle on a terrain surface. NGP for a wheeled vehicle is defined as the ratio of weight on the tyre to the tyre-terrain contact area. The tyre-contact pressure and load transfer to the soil through the tyre play a critical role in vehicle trafficability (Misiewicz et al., 2015). The NGP system was also used to measure relative mobility between vehicles before the 1970s. Later on, its applicability decreased because of one of its critical disadvantages. Precise measurement of the ground contact pressure is challenging; hence, it was impossible to measure NGP even in the desired range of accuracy.

Hetherington and White (2002) measured the ground contact pressure under wheeled vehicles such as 8 Tonne Bedford Cargo (4x4) and Saxon Armoured Personnel Carrier (4x4) at three tyre inflation pressures (the standard, two-third of the standard, half of the standard) and three payloads (0, 3.2, 6.4 tonnes). The four pressure transducers were buried in the soil lane at a depth of 0.23 m in such a way that every wheel ran over two transducers. The vehicle speed was kept slow and steady to 0.7 m/s while traversing over the transducers. The major concern in the NGP measurement was the uncertainty of the appropriate factor in converting the measured pressure by the transducers at a certain depth to the actual pressure at the surface. Moreover, there was a significant difference between the measured pressure and the predicted pressure from the existing NGP models such as equation 2.33 and 2.34. Therefore, it was difficult to precisely measure the NGP from the underground measurements in vehicle moving conditions.

Theoretically, there are two main techniques by which NGP can be calculated (Hetherington and White, 2002). They are 1) the Radius method and 2) the French method. The radius method assumes that the contact area of tyre - soil is approximately rectangular, as shown in Equation 2-33. The rectangle's two dimensions are the tyre radius  $r$  and the tyre width  $b$ . The French method also assumes the rectangular contact area. One dimension is assumed to be the tyre width. The other dimension is calculated from the relationship between tyre

radius, tyre deflection, and sinkage. This methodology is termed as ‘Average pressure in soft ground (APSG)’ and can be seen in Equations 2-34 & 2-35 for m number of axles.

$$NGP = \frac{W}{2mbr} \quad (2-33)$$

$$APSG = NGP = \frac{W}{2m(EOAC)} \quad (2-34)$$

$$EOAC = 2b\sqrt{\{r^2 - [(r - \delta) - z]^2\}} \quad (2-35)$$

### 2.3.4 Mean maximum pressure

The term ‘mean maximum pressure’ defines the average of the peak pressure magnitude over the contact patch that occurs during vehicle-terrain interaction. The United Kingdom Ministry of Defence (UK MOD) conducted various experiments to measure the vertical pressure of vehicle running gear in the soil. Rowland (1972) found that only peak pressure values beneath the running wheels contribute significantly to traction generation. The UK MOD developed the ‘Mean Maximum Pressure criterion’. According to this criterion, vehicles with low MMP values can access a large portion of the terrain. Hence, they have better soft soil performance.

MMP was initially developed for tracked vehicles. However, Rowland (1972) developed an equivalent equation for wheeled vehicles. MMP of wheeled vehicles was computed using the data of MMP of tracked vehicles, which can generate approximately the same drawbar pull. Therefore, MMP for a wheeled vehicle is not the actual MMP beneath the wheel. Rowland developed Equation 2-36 using an empirical approach from full-scale trials. The value of constant k is experimentally derived with respect to the number of axles, m in a vehicle as shown in Table 2.2.

$$MMP = \frac{kW}{2mb^{0.85}d^{1.15}\left(\frac{\delta}{h}\right)^{0.5}} \quad (2-36)$$

**Table 2-2 Experimentally derived values of constant k with respect to the number of axles m in a vehicle**

|   |      |      |      |      |
|---|------|------|------|------|
| m | 2    | 3    | 4    | 5    |
| k | 3.66 | 3.90 | 4.10 | 4.32 |

Larminie (1992) analysed that Equation 2-36 overestimates MMP values to trafficability comparisons for low profile tyres. The research modified the tyre footprint size to  $\delta/d$  from  $\delta/h$  in the MMP model, which showed better results for low profile tyres.

Maclaurin (1997) modified the MMP model using the limiting cone index. It is the cone index of the weakest soil surface on which a vehicle can make a single pass. The research assumed that if the limiting cone index values for both wheeled and tracked vehicles are made equivalent, both vehicles have the same accessibility on the terrain. Barton (1998) analysed that Maclaurin Equation 2-37 is more favourable to wheeled vehicles than the Rowland equation. It was demonstrated that the formula of Maclaurin has 33 per cent less weight prediction compared to the formula of Rowland for wheeled vehicles, which gives it better accessibility on the terrain. However, both Rowland and Maclaurin do not claim that their predicted MMP values should match the experimental values. Instead, they used MMP as a performance matrix to compare the relative accessibility of vehicles on the terrain.

$$\text{MMP} = \frac{1.14W}{2mb^{0.85}d^{1.15}\left(\frac{\delta}{d}\right)^{0.5}} \quad (2-37)$$

Hetherington (2001) concluded that the physical significance of MMP for wheeled vehicles is not determined. Hetherington and white (2002) stated that the MMP values for a vehicle are soil dependent, but no physical soil parameter is included in the MMP equation. However, vehicle parameter's effects are critically modelled in the MMP equations. It was also found that there is a discrepancy between the predicted MMP values and experimental results, which shows no correlation.

Therefore, it makes MMP a not-so-useful performance matrix for wheeled vehicles.

## **2.4 Soil system**

Over the past few decades, substantial research has been conducted to understand the vehicle's interaction with soft soils. Numerous methodologies, resource-intensive experimental testing and critical analyses have been carried out to understand the soil parameters involved in tyre-soil mobility performance models. This section reviews previous work in this area, addressing the critical soil parameters used to quantify a vehicle's mobility performance.

### **2.4.1 Cone index**

ASABE (2006) declared the cone penetrometer as a standard instrument to measure soil penetration resistance. This soil penetration resistance is the soil cone index. A Cone penetrometer (Figure 2-5) is a device, which has a conical surface or probe at its head with a tip downward and a cylindrical shaft as its body. A load cell (or dial indicator) is present at the top of the shaft within a proving ring, which indicates the force applied axially to the penetrometer. There are circumferential bands around the shaft that indicate the depth of penetration (ISTVS Standards, 2020). New cone penetrometer models have pressure transducers just behind the cone tip to measure pore water pressure. The cone penetrometer's penetration velocity is kept slow and steady to maintain dynamic equilibrium to measure the cone index.

The cone index (CI) is the most widely used mechanical property to determine soil strength in empirical-based models. Theoretically, the force per unit base area is required to push a cone probe into soil at a steady rate. Cone index is mostly influenced by soil density, soil type and moisture content (Mason and Baylot, 2016, Quraishi et al., 2013, Santos et al., 2012, Dexter et al., 2007, Vaz and Hopmans, 2001). Oskoui et al. (1982) used the soil mechanics theory to determine the parameters, which CI represents. It was found that CI is a function of moisture content and bulk density,  $\gamma$  of soil, which jointly represents the cohesive and frictional behaviour of soil, as shown in 2.38. The soil's cohesive

strength is mostly influenced by moisture content compared to bulk density and angle of internal friction. Therefore, higher CI represents more relative compact soil, and lower CI represents tilled soil.

$$CI = f(c) + f(\gamma) \quad (2-38)$$

Mason and Baylot (2016) researched at the U.S. Army Engineer Research and Development Centre (ERDC) to adequately define the variation of soil strength with respect to the moisture content ( $m_g$ ) using Knight (1961) relation, as shown in Equation 2-39. The experiments were conducted for the twelve soil textures defined in the Unified Soil Classification System (USCS) with the residual moisture content varying from 1.6 to 7.1% and dry density from 840 to 1609 kg/m<sup>3</sup>. The experimental data followed the smooth curves satisfying Equation 2-39 with different model constants  $a$  and  $b$  ranging from  $3.987 < a < 13.641$  and  $-2.417 < b < 0.8150$  specified for each of the USCS soil types. Other experimental studies on moisture content and soil strength have also shown that cone index value decreases with an increase in moisture content (Turnage, 1970; Collins, 1971; Voorhees et al., 1977).

$$CI = e^{(a-b \ln(m_g))} \quad (2-39)$$

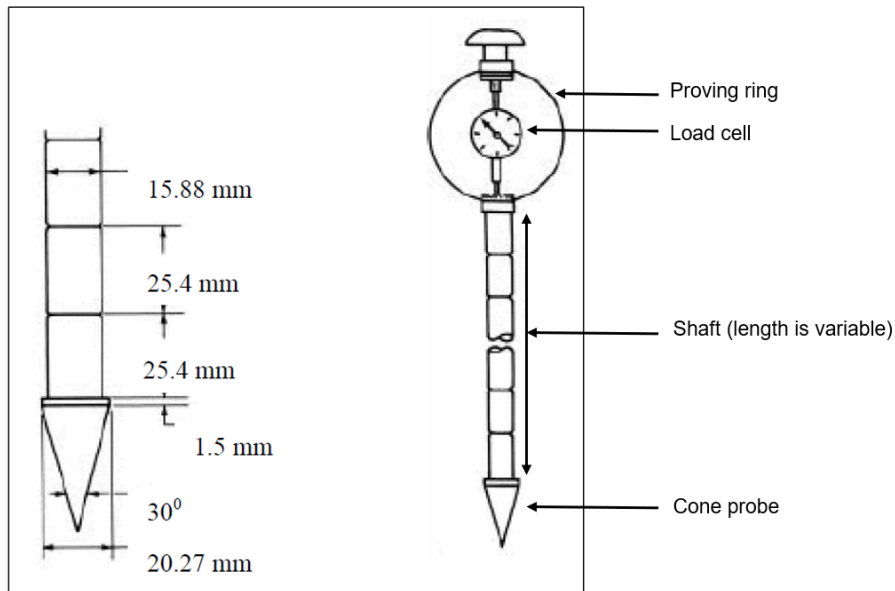


**Figure 2-5 Standard cone penetrometer (ASABE standard 2000)**

Wismer and Luth (1973) performed the cone penetrometer tests on silt loam, silt, silt clay and sandy loam soils in agricultural fields before the mobility tests of the vehicle. The mobility tests were performed with the bias-ply tyres with the conventional tread pattern with practical restrictions on tyre parameters such as  $b/d \approx 0.3$ ,  $\delta/h \approx 0.2$  and  $r/d \approx 0.475$  to measure tyre sinkage and performance parameters. The results showed a correlation between the penetration depth, the average cone index and the tyre sinkage. It was concluded that for the tyre sinkage less than 3 inches, the penetration depth is to be kept between 0-6 inches to calculate the average cone index. However, as tyre sinkage exceeds 3 inches, the cone index should be the average of a 6-inch band layer, including maximum tyre sinkage. The results of mobility performance parameters are further discussed in section 2.9.1.

Nam et al. (2010) mentioned a strong correlation between the rating cone index and tyre sinkage. Rating cone index (RCI) is a cone index function representing the soil strength of fine-grained soils such as clay and silt. RCI can be expressed as a function of wheel sinkage,  $z$ , tyre slip,  $s$  and wheel characteristics, as shown in 2.40. The research quoted that slip between vehicle and terrain should always be greater than zero and constant to predict RCI from Equation 2-40. If slip becomes zero or negative, then RCI also becomes zero or negative, which implies soil strength has become zero or negative, which is not possible in a real situation.

$$RCI = 2.6265 \frac{W(1-\frac{\delta}{h})^{1.5}}{bd^{0.4}} \cdot \left(\frac{s}{z^3}\right)^{0.2} \quad (2-40)$$

Pillinger et al. (2018) studied the correlation between soil density and cone index. The research used a theoretical Boussinesq formula for point load to determine the variation of cone index with the depth  $z$  and diameter of the cone probe,  $d$ . A mathematical correlation is developed between the soil density and the cone index of the soil as shown in equation 2.41. The drawback of this correlation is the dependency of the constants  $A$ ,  $B$  and  $CI_{max}$  on the moisture content. The cone penetrometer experiments were performed on the soil with six different soil densities ranging from 0.9 to 1.65 g/cc and three different moisture contents 9%,

12% and 15%. The cone index vs depth was analysed. The values of the constants A varied from 17.48 to 154.29, B varied from 1.97 to 5.57 and  $CI_{\max}$  varied from 3 Bar to 20 Bar. Therefore, at different moisture content, the values of the constants A, B and  $CI_{\max}$  are different for the same soil.

$$\rho = \frac{-B}{\ln\left(\frac{CI_{\max}}{A}\right)} \quad (2-41)$$

$$\text{where, } CI_{\max} = \frac{d(CI)}{dz} \cdot d \quad (2-42)$$

## 2.4.2 Bevameter technique

Bekker (1956) developed Bekker-value-meter or Bevameter to reproduce the forces acting during wheel soil interaction under loading conditions. It has the capabilities to determine the vertical and horizontal stress-strain relationship of soil beneath a running wheel. Wong (2010) describes that Bevameter facilitates two major types of tests to characterize the soil condition – plate penetration test (or pressure-sinkage test) and shear test.

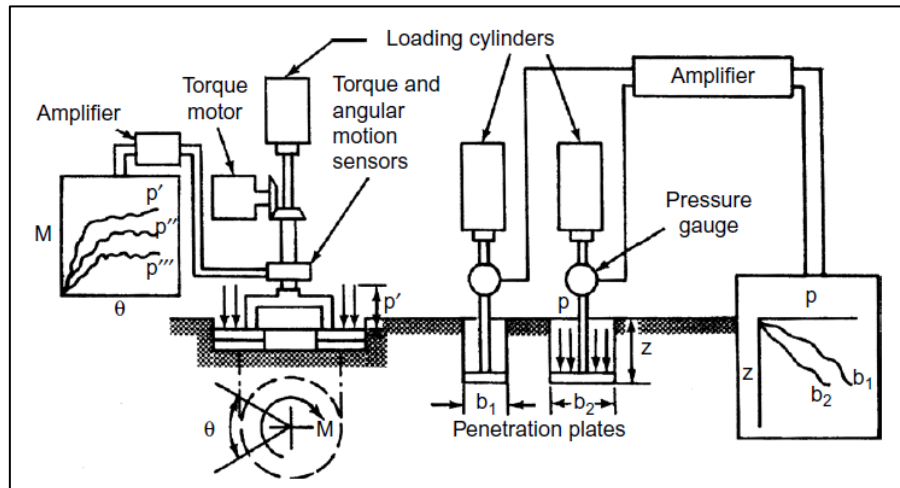


Figure 2-6 Bevameter set up (Wong (2010), reprinted by permission from Bekker (1969))

### 2.4.2.1 Pressure-sinkage test

The pressure sinkage test is used to determine the vertical stress-strain relationship of soil under normal loading conditions. In this test, a sinkage plate

(or flat plate), usually circular or rectangular cross-section and the same contact area as vehicle running gear, is pushed vertically into the terrain with constant penetration speed downwards. The input force on the sinkage plate is usually applied through hydraulics or manually (Wong, 2010). The applied force on the sinkage plate and sinkage are measured to determine the pressure-sinkage ( $p$ - $z$ ) relationship of a deforming soil. Bekker suggested Equation 2-43 to establish a pressure-sinkage mathematical model based on soil mechanics. The  $k_c [N/m^{n+1}]$  and  $k_\phi [N/m^{n+2}]$  are the moduli of deformation with respect to cohesion and friction, respectively, and  $n$  is an empirically determined model parameter that defines the shape of the load-displacement curve.

$$p = \left( \frac{k_c}{b} + k_\phi \right) z^n \quad (2-43)$$

The pressure-sinkage test is influenced by the plate shape, plate size (or aspect ratio), soil conditions and sinkage velocity. Therefore, it is crucial to understand the influence of critical model parameters. Van et al. (2007) conducted a plate sinkage test on dry Tottori sand in laboratory conditions. It is fine-medium sand with a bulk density of 1.5 g/cm<sup>3</sup>, moisture content of 0.35%, cohesion of 2 kPa and internal angle of friction of 38°. The experiment was performed with different aspect ratios (3 and 6) of sinkage plates (rectangular and circular cross-sections) with a constant penetration velocity of 13.3 mm/s through a depth of 120 mm. The results showed that the pressure sinkage curve slope for larger plates increases faster after a certain depth. The larger plates are often more affected by the bottom effect than smaller plates. As per the lateral earth pressure theory, the larger plate increases the stress at the bottom plate because of the increased surcharge load, which influences the total pressure applied by the larger plate after a certain depth. Therefore, the results were analysed up to 60 mm in depth to reduce this effect. The research concluded that a small sinkage plate with the ground contact area of an aspect ratio between 6 to 3 could be used to predict a vehicle's mobility performance parameters.

Apfelbeck et al. (2011) performed the plate penetration tests with three different circular plates at three different penetration velocities on three different soil types.



The radius of the plates used were 0.025, 0.05 and 0.075 m. The three different penetration velocities were  $0.48\text{e-}3$ ,  $2.4\text{e-}3$  and  $4.8\text{e-}3$  m/s. The soil samples used in the experiments are as follows: Soil 1 is dry quartz sand, Soil 2 is a mixture of 65% by weight of olivine and 35% by weight of dry quartz and Soil 8 is fine clay. The grain size distribution of the used soils is shown in figure 2.7. The experiment concluded that the smaller the grain size of soil particles, the steeper the pressure-sinkage curve at lower velocities.

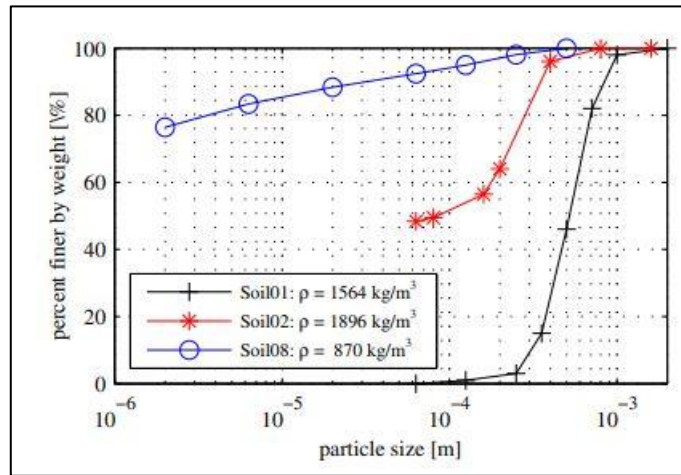


**Figure 2-7 Grain size distribution of the used soils for the Bevameter tests (Apfelbeck et al. (2011))**

Brunskill et al. (2011) studies the influences of soil density on the Bekker model parameters. The pressure-sinkage tests (plate sizes 5 cm and 12.5 cm) on two prepared samples of Martian soil simulants (ES- 1 (%w = 0.35,  $c = 0.5\text{-}1.5$  kPa,  $\phi = 16^{\circ}\text{-}21^{\circ}$ ) and ES-3 (%w = 0.34,  $c = 0\text{-}0.3$  kPa,  $\phi = 30^{\circ}\text{-}40^{\circ}$ )) at three different soil densities (low, medium and high) at constant moisture contents were performed, as shown in Figure 2.8. The particle size distribution of ES-I and ES-3 are shown in Figure 2.9. It was found that with the increase in soil density, the frictional modulus  $k_{\phi}$  showed an upward trend in both soil samples. It may be because the simulants were frictional materials in which soil strength depends on the internal angle of friction and normal stress. Therefore, with the increase in the soil density, the void ratio decreases, interlocking between the particles increases, soil strength increases thus friction increases. No general trend was followed for the cohesive modulus  $k_c$  and sinkage exponent  $n$  with the increase

in the soil densities. It may be because frictional soil is independent of the soil cohesion as per basic soil mechanics theory. Therefore, nothing concrete can be established about the change in Bekker parameters with the change in the soil density.

|                         | Particle density<br>(g/cm <sup>3</sup> ) | Low density<br>(g/cm <sup>3</sup> ) | Medium density<br>(g/cm <sup>3</sup> ) | High density<br>(g/cm <sup>3</sup> ) |
|-------------------------|------------------------------------------|-------------------------------------|----------------------------------------|--------------------------------------|
| ES-1                    |                                          |                                     |                                        |                                      |
| Density                 | 2.32                                     | 0.54                                | 0.79                                   | 1.21                                 |
| Std. dev.               | –                                        | 0.017                               | 0.018                                  | 0.015                                |
| Void ratio ( <i>e</i> ) | –                                        | 3.30                                | 1.93                                   | 0.91                                 |
| ES-2                    |                                          |                                     |                                        |                                      |
| Density                 | 2.56                                     | 1.24                                | 1.36                                   | 1.44                                 |
| Std. dev.               | –                                        | 0.004                               | 0.014                                  | 0.018                                |
| Void ratio ( <i>e</i> ) | –                                        | 1.06                                | 0.88                                   | 0.78                                 |
| ES-3                    |                                          |                                     |                                        |                                      |
| Density                 | 2.60                                     | 1.46                                | 1.58                                   | 1.64                                 |
| Std. dev.               | –                                        | 0.004                               | 0.006                                  | 0.029                                |
| Void ratio ( <i>e</i> ) | –                                        | 0.78                                | 0.64                                   | 0.58                                 |

**Figure 2-8 Density variation in different soil samples (total 9 in number) for the Bevameter tests (Brunskill et al. (2011))**

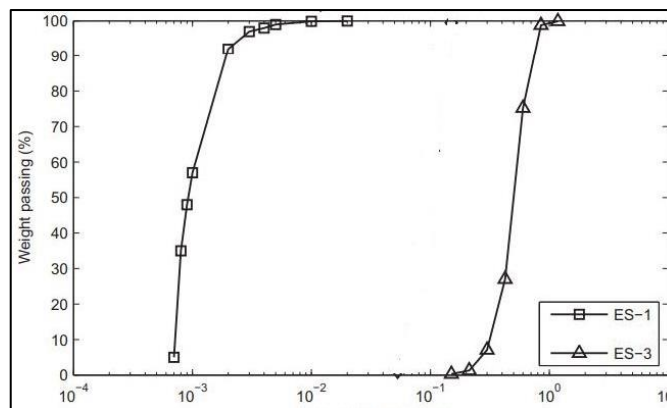


**Table 2-3 Effect of soil density on Bekker's parameter**

| Author                        | Soil type                                                            | Parameter       | Bekker parameter               | Trend    |
|-------------------------------|----------------------------------------------------------------------|-----------------|--------------------------------|----------|
| Brunskill<br>et al.<br>(2011) | ES-I<br><br>(%w = 0.35, c = 0.5-1.5 kPa, $\phi = 16^{\circ}$ - 21°)  | Soil<br>density | Cohesive modulus, $k_c$        | Upward   |
|                               |                                                                      |                 | Frictional modulus, $k_{\phi}$ | Upward   |
|                               |                                                                      |                 | Sinkage exponent, n            | Upward   |
|                               | ES-III<br><br>(%w = 0.34, c = 0- 0.3 kPa, $\phi = 30^{\circ}$ - 40°) |                 | Cohesive modulus, $k_c$        | Downward |
|                               |                                                                      |                 | Frictional modulus, $k_{\phi}$ | Upward   |
|                               |                                                                      |                 | Sinkage exponent, n            | Downward |

Yang et al. (2018) conducted the test on four common soil types found in Southwest China to study the effect of moisture content and sinkage speed on Bekker parameters. The test soils were named Soil SC, Soil GG, Soil YK and Soil CB as per their locations. Each soil sample was prepared with four different moisture contents 1%, 5%, 10% and 15%. The average particle size of soil samples was in the following increasing order: Soil CB, Soil YK, Soil SC and Soil GG. The particle size distribution of the above soils is shown in figure 2.10. The test was performed in a 500 mm x 500 mm x 1000mm soil bin. The test plates used for the experiments were circular of radii 45 mm and 70 mm. The research analysed the effect of moisture content and sinkage speed on cohesive modulus  $k_c$ , frictional modulus  $k_\phi$  and sinkage exponent n. The results are shown in Table 2-4. It was found that with the increase in the moisture content,  $k_\phi$  exponentially decreases and n linearly increases due to the reduction in inter-particle friction. With the increase in the sinkage speed,  $k_c$  linearly increases due to the viscosity effect and sinkage exponent n also increases due to the pore-pressure build-up in the soil.

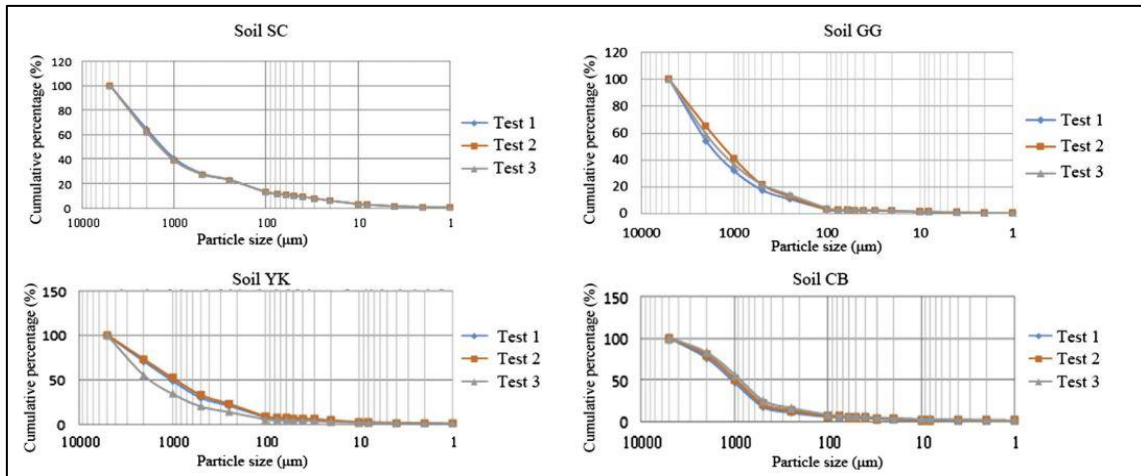


Figure 2-10 Grain size distribution of the used soils for the Bevameter tests (Yang et al., 2018)

Table 2-4 Effect of moisture content and sinkage speed on Bekker's parameter

| Author             | Parameter investigated | Bekker parameter             | Trend                   | Reason                                                                                          |
|--------------------|------------------------|------------------------------|-------------------------|-------------------------------------------------------------------------------------------------|
| Yang et al. (2018) | Moisture content       | Cohesive modulus, $k_c$      | Not clear               | 1) Influence of other interparticle forces<br>2) Moisture content is not uniform in soil layers |
|                    |                        | Frictional modulus, $k_\phi$ | Exponentially decreases | Reduction in friction caused by relative movement among particles                               |
|                    |                        | Sinkage exponent, $n$        | Linearly increases      | Soil becomes softer                                                                             |

|  | Parameter investigated | Bekker parameter             | Trend              | Reason                                   |
|--|------------------------|------------------------------|--------------------|------------------------------------------|
|  | Sinkage speed          | Cohesive modulus, $k_c$      | Linearly increases | Due to the viscosity effect              |
|  |                        | Frictional modulus, $k_\phi$ | Not clear          | Not clear                                |
|  |                        | Sinkage exponent, $n$        | Increases          | Influenced by the pore-pressure build-up |

Griffith and Spenko (2011, 2012, 2013, 2014) use the actual wheel section (as shown in figure 2.11 (a)) to deduce the ground contact pressure for a small diameter wheel (diameter < 50 cm) irrespective of Bekker's flat plate theory which works reasonably well for wheel diameter greater than 50 cm but fails for wheel diameter less than 50 cm.

Griffith and Spenko (2013) studied the plate sinkage test with five different wheel geometries ranging from 13 to 203 mm in width, and 114 to 265 mm in diameter. The experiment aimed to study the compactive soil's effect, such as clay/silt mixture on a small diameter vehicle wheel. The experimental results were compared with the new model for a lightweight vehicle wheel. It was found that wheel diameter and width are independent of each other on compact soil. It means that the increase in the wheel-soil contact area, whether by the diameter or width change, results in a similar response to the pressure-sinkage curve. Further, with the increase in wheel width for an applied normal pressure, wheel sinkage also increases (figure 2.11 (b)). This is because of the tendency of the contact area to increase at a faster rate (due to larger wheel curvature) than the normal force during the wheel sinkage for low bearing soils. With the increase in wheel width for a given sinkage, pressure decreases.

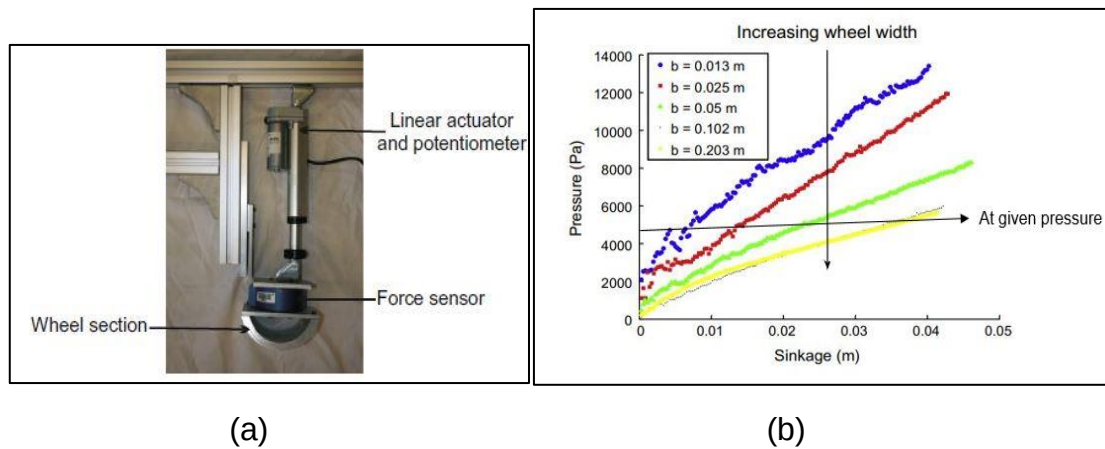


**Figure 2-11 (a) Pressure-sinkage test device used to test width dependence (b) Experimental result of width dependence for a 114 mm diameter wheel on a clay/silt mix (Griffith and Spenko (2013, 2014))**

**Table 2-5 Plate sinkage tests for small diameter wheel**

| Authors                    | Methodology/<br>Sample                                                                                                                                               | Experiment, model, output and results                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|----------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Griffith and Spenko (2013) | Semi-empirical<br><br>Soil - Clay/silt mixture<br><br>(80% clay by volume, $c = 10.6$ kPa and $\phi = 10.1^\circ$ , $\%w = 4\%$ and density = $783 \text{ kg/m}^3$ ) | Experiment- Plate sinkage test<br><br>Plates used- Sinkage plates at a constant diameter of 114 mm with widths 13, 25, 50, 102, 203 mm<br><br>Model used- $p(\Theta) = kr^n(\cos \Theta - \cos \Theta_s)^n(bl)^m$ , where<br>$l = \sqrt{Dz_0 - z_0^2}$<br><br>Output- Pressure (Pa) vs Sinkage (m)<br><br>Result- The model satisfies the experimental results for lightweight vehicle wheels of diameter less than 50 cm on compact soil. It also signifies that wheel diameter and wheel width are independent of each other on compact soil. |

| Authors                             | Methodology/<br>Sample                                                                                    | Experiment, model, output and results                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|-------------------------------------|-----------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Griffith<br>and<br>Spenko<br>(2014) | 1) Quartz ( $c = 2.05$ kPa, $\phi = 27.5$ deg.)<br><br>2) Ottawa sand ( $c = 1.40$ kPa, $\phi = 30$ deg.) | Soil type- Dilative soil<br><br>Experiment- Plate sinkage test<br><br>Wheels used- Diameter - 8 -38 cm & Width – 2 - 14 cm with downward velocity – 13 mm/s<br><br>Model used- $p(\Theta) = k r^n (\cos \Theta - \cos \Theta_s)^n D^m \alpha^p$<br><br>Output- Pressure (kPa) vs sinkage (m)<br><br>Result- The proposed model predicts more accurate results than Bekker's flat plate theory, which is over-estimating the experiment results for small scale wheels. |

Griffith and Spenko (2014) performed a plate sinkage test with five different wheel geometries ranging from 2 to 14 cm in width and 8 to 38 cm in diameter. The experiment aimed to study the dilative soil such as quartz and Ottawa sand on a small diameter vehicle wheel. The experimental results were compared with the new model for a lightweight vehicle wheel. It was found that wheel diameter and width are dependent on each other in dilative soil. With an increase in wheel diameter, the effect of wheel width increases and vice-versa. Increasing wheel width increases the pressure sinkage gradient but decreases the wheel soil contact path's curvature. When wheel width increases to large values, the pressure sinkage line converges for a test wheel section of a given stiffness.

#### 2.4.2.2 Shear test

The shear test is used to determine the horizontal stress-strain relationship of soil under a predetermined normal loading condition. A rotating shear head with known normal stress is used to reproduce the shearing action of wheel-soil interaction. The shear head is connected with a motor, which applies input torque

through a cylindrical shaft. The shear head's applied torque and angular displacement are measured to determine shear strength parameters such as cohesion and internal angle of friction (equation 2.12). When the input torque is applied to the shear ring, there is an additional sinkage of the shear ring, which is referred as the 'slip-sinkage' effect. The shear head may sink into the soil because of its rotary motion. However, this test is mostly influenced by soil conditions such as moisture content and bulk density.

Sadek et al. (2011) performed the direct shear test with sandy soil in controlled laboratory conditions. The sandy soil consists of 10% clay, 4% silt, and 86% sand particles, which was prepared under three different moisture content conditions named as dry, moist and wet and four different bulk densities such as loose, soft, firm and compact as shown in Figure 2.12. The normal load applied for the tests were 210 N, 480 N and 745 N with a shear speed of 0.52 mm/min. The result analysis showed that shear force rapidly increases as a horizontal displacement function before the soil sample's yield point. Further, yield stress was defined as a function of bulk density and moisture content, respectively, for different soil samples. The linear trend was found that the compact soil has more yield force followed by the firm, soft and loose soil samples at all normal loading conditions. Furthermore, the magnitude of yield force was recorded as highest for the dry soil followed by the medium and the wet soil at all soil bulk density levels. However, on comparing the trend between yield stress and normal stress, it was found that cohesion and internal angle of friction followed the same linear trend as yield force.



| Moisture level | Density level | Moisture content (%) | Bulk density (Mg/m <sup>3</sup> ) |
|----------------|---------------|----------------------|-----------------------------------|
| Dry            | Loose         | 0.023                | 1.06                              |
|                | Soft          | 0.022                | 1.28                              |
|                | Firm          | 0.016                | 1.39                              |
|                | Compact       | 0.021                | 1.40                              |
| Moist          | Loose         | 13.24                | 0.91                              |
|                | Soft          | 12.44                | 1.21                              |
|                | Firm          | 13.14                | 1.34                              |
|                | Compact       | 12.99                | 1.58                              |
| Wet            | Loose         | 21.07                | 1.01                              |
|                | Soft          | 21.30                | 1.34                              |
|                | Firm          | 21.09                | 1.35                              |
|                | Compact       | 22.46                | 1.53                              |

**Figure 2-12 Soil moisture and bulk density conditions of the soil samples (M.A. Sadek et al. (2011))**

Apfelbeck et al. (2011) performed the shear test on two different soils with three different shear velocities. The soil samples used in the investigation are as follows: Soil 1 is dry quartz sand, and Soil 2 is a mixture of 65% by weight of olivine and 35% by weight of dry quartz. The three different shear velocities are 0.1, 0.2 and 0.3 rpm. The tool used in the experiment has 12 grousers with 7.5 mm grouser height. The results showed no significant effect of shear velocity on soil shear parameters. The shear velocity refers to the speed at which the soil failure plane shears. It is kept slow and steady during experiments to determine the soil's shear strength, cohesion and internal angle of friction. Since the shear velocity is directly related to the shear displacement ( $j$ ), which also contributes to the traction force developed by the tyre during tyre-soil interaction (equation 2.14).

Wright (2012) performed the direct shear test (DST) and triaxial shear test (TST) experiments on the dry sand (%w = 0.86% and density = 1.39 g/cc) and the sandy loam (%w = 14.2% and density = 1.47 g/cc). The cohesion,  $c$  and internal angle of friction,  $\phi$  obtained from the DST and TST for the dry sand are (2.5 kPa, 32.35°) and (15 kPa, 32.35°) respectively. Similarly, the ( $c$ ,  $\phi$ ) obtained from the DST and TST for the sandy loam are (1.33 kPa, 13.13°) and (2.67 kPa, 32.35°) respectively. Further, both soils were modelled using the Lagrangian and Euler FE models in the LS-DYNA software using the parameters obtained from both

experiments. As a result, eight soil models were prepared in the simulation domain. Further, the pressure sinkage tests were simulated with a circular sinkage plate of diameter 75.2 mm in each case and compared with the experimental results respectively. It was found that the yield parameters obtained from the DST results produced a much better agreement with the experimental static or quasi-static plate sinkage profiles than those derived from the TST for the sandy loam soil. Using a fine mesh, the yield surface parameters obtained from the TST produced a mean percentage error between 70 and 88%, whereas the DST produced a mean percentage error between 5 and 7%.

## **2.5 Tyre construction**

Tyre construction plays a critical role in tractive performance. There are different types of tyre constructions: solid, radial-ply, and bias. The solid tyres are rolled into thin layers of rubber, mounted onto a metal frame, and pressed through a hydraulic system. In radial-ply tyres, the cord plies are extended to the beads at substantially 90° to the centreline of the tread. The plies network gives its strength, more flexibility and a larger contact patch as compared to bias ply tyres (Tyre standards, 2022). The radial-ply tyres are represented by the letter 'R' in their nomenclature. In bias-ply tyres, the cord plies are cross over each other at an angle usually between 30 to 45 degrees. The bias-ply is a relatively older design than the radial-ply.

This section covers the general tyre parameters in a loaded and unloaded condition, as shown in Figure 2-13.

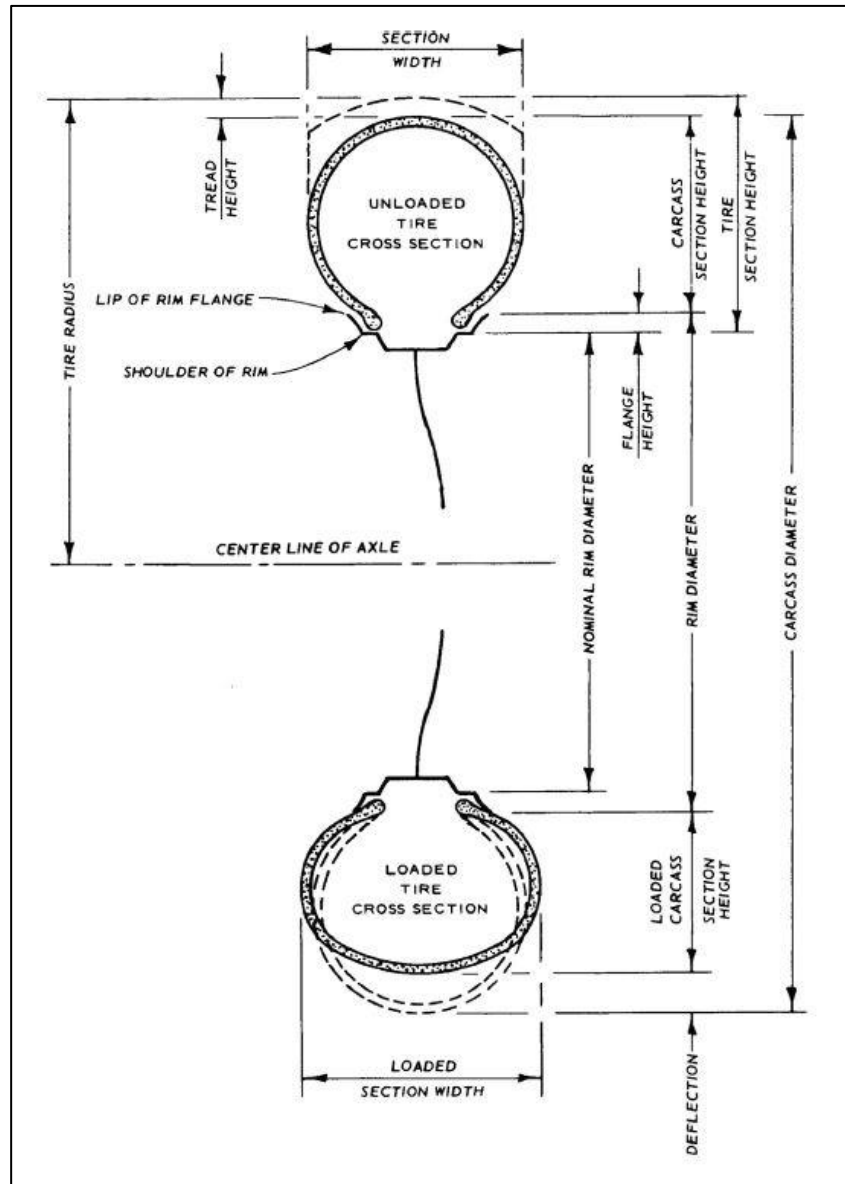


Figure 2-13 Representation of tyre parameter (R. He et al. (2020))

### 2.5.1 Overall width (b)

The overall width is the measured distance between the sidewall to the sidewall of the tyre, including the growth from rated maximum inflation pressure in unloaded condition for 24 hours (waiting period). It is the first number in the tyre size designation. For example, in a size 145/80 R13 tyre, the width is 145 millimetres.

### 2.5.2 Overall diameter (d)

The overall diameter is the radially measured distance between one side of the tyre to the other side passing through the centre of the wheel. It includes the growth from rated maximum inflation pressure in unloaded conditions for 24 hours. It can also be measured by dividing the tyre circumference by Pi. For example, a size 7.5 R16 tyre diameter is 787 mm.

### 2.5.3 Section height (h)

The section height of the tyre is represented by Equation 2-44.

$$\text{Section height (h)} = \frac{d - \text{nominal rim diameter}}{2} \quad (2-44)$$

### 2.5.4 Aspect ratio

The aspect ratio is the tyre section height ratio to the tyre width. It is the two-digit number after the slash mark in tyre size. For example, in a 135/80 R13 tyre, 80 means that the section height equals 80% of the tyre width. The bigger aspect ratio provides a bigger sidewall of a tyre.

### 2.5.5 Load index

The load index indicates the maximum load supported by a tyre at the recommended inflation pressure. The higher the load index, the greater is the load-carrying capacity of a tyre. The load index value is written on the tyre sidewall in terms of pounds or kilograms.

### 2.5.6 Tyre deflection

Tyre deflection is the difference between the section heights in loaded and unloaded conditions at a given load and inflation pressure (ISTVS standards, 2020). Mathematically, it can be calculated using Equation 2-45.

$$\text{Deflection } (\delta) = \frac{\text{Overall diameter (d)}}{2} - \text{static loaded radius (slc)} \quad (2-45)$$

### 2.5.7 Static loaded radius (slc)

A static loaded radius is a distance between the tyre rim's axle centreline to the tyre's hard supporting surface at a specific load and inflation pressure.

### 2.5.8 Speed rating

The tyre speed rating is an indicator of the tyre's maximum service speed. Therefore, a vehicle's speed should not exceed its tyre rating speed. The higher the speed rating, the better control and handling of the vehicle at higher speeds.

### 2.5.9 Tyre tread

Tyre treads are the extended body part from the tyre carcass (on the outer surface), which improves the vehicle's traction properties. Conventional vehicle tyres don't have very high treads on their surface, and the tread surface is sufficiently large compared to the tyre surface. Therefore, conventional Terramechanics has considered an assumption that the tyre with treads can be represented by a smooth tyre with an extended radius (Ding et al., 2014).

### 2.5.10 Mode of operation

Wong (2001) identified two types of mode of operation of a tyre on soft soil: rigid mode of operation and elastic mode of operation. In the rigid mode of operation, the tyre's average ground pressure is greater than critical ground pressure. In this case, a pneumatic tyre behaves like a rigid rim if the inflation pressure is sufficiently high and the terrain is relatively soft (Wong, 2010). In the elastic mode of operation, the tyre's average ground pressure is lower than critical ground pressure. In this case, some tyre circumference gets flattened, and the tyre operates like a flexible body.

Favaedi et al. (2011) calculated the average ground pressure ( $P_{gr}$ ) as shown in Equation 2-46.  $W$  is the load on the wheel,  $l_c$  is the contact length, and  $b$  is the wheel's width. Sharma et al. (2018) obtained the deformation characteristics of the flexible wheel from finite element analysis to evaluate the ground contact area and then average ground pressure. Wong (2010) defines the critical ground pressure ( $P_{cgr}$ ) as shown in 2.47.  $(k_c, k_\phi)$  are the Bekker constants,  $b_s$  is the

smaller dimension of contact area,  $d$  is the wheel's diameter, and  $n$  is the sinkage constant.

$$P_{gr} = \frac{W}{l_c \cdot b} \quad (2-46)$$

$$P_{cgr} = \left[ \frac{k_c}{b_s} + k_\phi \right] \frac{1}{2n+1} \left[ \frac{3W}{(3-n)b_s \sqrt{d}} \right] \frac{2n}{2n+1} \quad (2-47)$$

## 2.6 Tyre deflection test

Tyre deflection is a critical indicator of the distribution of forces transferred from the tyre to the operating surface. It is because of the change in the shape of a tyre with deflection. Tyre deflection is the difference between the undeformed and deformed radii of the tyre beneath its axis (Kirck, 1969)). It is also related to tyre performance and service life. When the tyre is over-deflected due to over-loading and low inflation pressure, its performance and service life reduces because it distorts the tyre footprint, which can drastically increase the tyre wear. Therefore, it is required that a tyre load should not exceed the recommended load and inflation pressure.

Freitag and Smith (1966) performed the deflection test on 9.00-14 and 11.0-20 tyre on Yuma sand and Mortar sand, respectively. A linear potentiometer was fitted within the tyre cavity to measure tyre deformation at a fixed point on tyre circumference. The test was conducted at 131 kPa and 310 kPa inflation pressure at a constant 3.8-3.9 kN load. The results showed that tyre deformation depends on inflation pressure, normal load and soil strength.

Komandi (1976) conducted the load-deflection test on tyres ranging from 9-24 to 15-30 on concrete payment. The tests were conducted for various inflation pressures ranging from 39.2 kPa to 156.9 kPa. An attempt was made to determine the empirical relation between tyre deflection ( $\delta$ ), wheel load ( $W$ ), inflation pressure ( $p_i$ ) and tyre parameters ( $b$ ,  $d$ ). The relationship developed is as follows.

$$\delta = C_1 \frac{W^{0.85}}{b^{0.7} d^{0.43} p_i^{0.6}} K \quad (2-48)$$

where  $C_1$  = constant depending on tyre design

$$K = 0.0015 b + 0.42$$

Lines and Murphy (1991) measured the dynamic stiffness of the seventeen rolling agricultural tractor tyres ranging from 7.50-18 to 18.4R38 in the radial direction. The results showed that inflation pressure, rolling speed, tyre size, and tyre age have a more considerable influence on tyre stiffness than tyre load, driving torque, and ply rating. The tyre stiffness model was developed, as shown below.

$$K_t = 172 - 1.77R + 5.6A + 0.34bRP \quad (2-49)$$

where,  $K_t$  = tyre stiffness, kN/m;

$R$  = rim diameter, m;

$A$  = tyre age, years;

$b$  = tyre section width, m;

$P$  = tyre pressure, Bar

Sharma and Pandey (1996) conducted the deflection characteristics test on three common agricultural tyres (11.2-28, 12.4-28 and 13.6-28) used in Indian tractors. The normal loads varied from 65 N to 102.5 N and inflation pressure from 50 kPa to 250 kPa. The results showed that the tyre deflection decreases with the increase in inflation pressure at constant load. It may be because of the decrease in load on the tyre's carcass. However, tyre deflection increases with the increase in load at constant inflation pressure. It may be because of the increase in the tyre carcass load. It was also analysed that tyre deflection decreases with the increase in the tyre's size. It may be because of the high load-carrying capacity of bigger tyres at the same inflation pressure and loading condition.

Rashidi et al. (2013) performed the tyre-deflection test on four radial-ply tyres (R13-165/65, R14-185/65, R15-185-65, R16-216/60 and R16-216/60) on a hard

surface. The inflation pressure varied from 30 kPa to 38 kPa, and the normal load varied from 5.87 kN to 13.69 kN. The multi-variable regression model derived from the deflection characteristics of the first three tyres and was found in good agreement with the fourth tyre's deflection characteristics. Further, the paired t-test was conducted on the sample data obtained from the experiments and predicted by equation 2.50 for the fourth tyre. It states how significant the difference between the two groups is measured in terms of the mean. This statistical test was conducted against the null hypothesis assuming there is no significant difference between the mean of sample groups. The statistical results indicated that the deflection values predicted by the model were less than the deflection values measured by the test apparatus. The average deflection difference between two methods was -1.60 mm (95% confidence interval for difference in means: -2.11 mm and -1.09 mm; p-value = 1.00). A similar statistical analysis is conducted in chapter 4 of this research.

$$\delta = 75.67 + 0.104b - 0.107d - 0.758P + 3.519W \quad (2-50)$$

Where,  $\delta$  = Deflection, mm,

$b$  = Width of the tyre, mm,

$d$  = Diameter of the tyre, mm,

$W$  = Normal load, kN and

$P$  = Inflation pressure, kPa



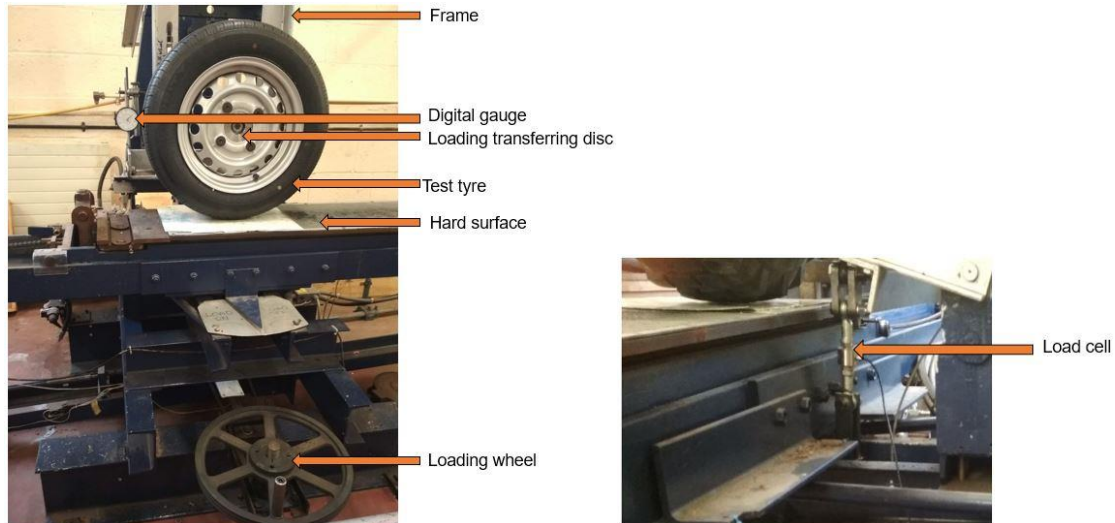


**Figure 2-14 Load-deflection test apparatus (Cranfield University)**

Krick (1969) developed the tyre deflection model using the dimensional analysis approach. The tyre deflection ratio was expressed as a function of tyre width ( $b$ ), tyre diameter ( $d$ ), ground pressure ( $p_g$ ) and tyre load ( $W$ ). The model (Equation 2-51) was tested for the numerous tyres at normal load between 1.96 kN and 15.7 kN and inflation pressure between 60.8 kPa and 253.3 kPa. It was found that the ply rating of the tyre has less influence on the tyre carcass.

$$\frac{\delta}{h} = 0.67 \left[ \frac{P_g}{W} (D \times B) \right]^{-0.8} \quad (2-51)$$

Kumar et al. (2018) determined four radial tyre's deflection characteristics (12.4 R 28, 13.6 R 28, 14.9 R 28, 16.9 R 28) on a hard surface. The inflation pressure varied from 41 kPa to 207 kPa and the normal load from 4.905 kN to 19.13 kN. The relationship between inflation pressure and normal load concerning tyre deflection was modelled using the dimensional analysis technique. A similar method was used for the tyre-soil interaction study. The experimental data were fitted and equation 2.52 was developed. The results showed that tyre deflection decreases non-linearly with the increase in inflation pressure and increases linearly with an increase in normal load. A similar trend was also observed in Rashidi et al. (2013) and Sharma and Pandey (1996). The analysis also showed that the rate of increase of tyre deflection with normal load is higher at lower inflation pressure than higher values. It may be because tyre stiffness changes

with inflation pressure (Lines and Murphy, 1991). A similar dimensional analysis approach is used to obtain tyre deflection and contact area model of full-scale Land Rover tyre (7.5R16) in chapter – 3.

$$\frac{\delta}{h} = 114.3 \left[ \frac{P_g}{W} (d \times b) \right]^{-1.07} \quad (2-52)$$

where,  $\frac{\delta}{h}$  = tyre deflection ratio

$b$  = tyre width, m

$d$  = tyre diameter, m

$W$  = normal load, kN

$P_g$  = Ground pressure ( $W/A$ ), kPa

## 2.7 Tyre-surface contact area test

The tyre-surface contact area is the loading area, which controls the transfer of forces developed in traction. Therefore, the shape and size of the contact area are critical, which directly influence a tyre's mobility performance parameters. Plackett (1984) measured the tyre's contact area by painting the probable tyre contact surface with black ink and loading it onto white paper. The hard surface supported the white paper in capturing the tyre's imprints. Upadhyaya and Wolfsohn (1988), Grecenko (1995), Romano et al. (2008) and Ekinici et al. (2011) also followed a similar technique. It concluded that the measurement of tyre contact area on hard ground is an accurate and reliable technique when the minimum compaction is caused by rolling tyre on the soil surface. Figure 2-15 analysed a comparison of a deflecting wheel's tyre contact area on a hard surface and soft soil and found it to be of approximately the same shape and size.

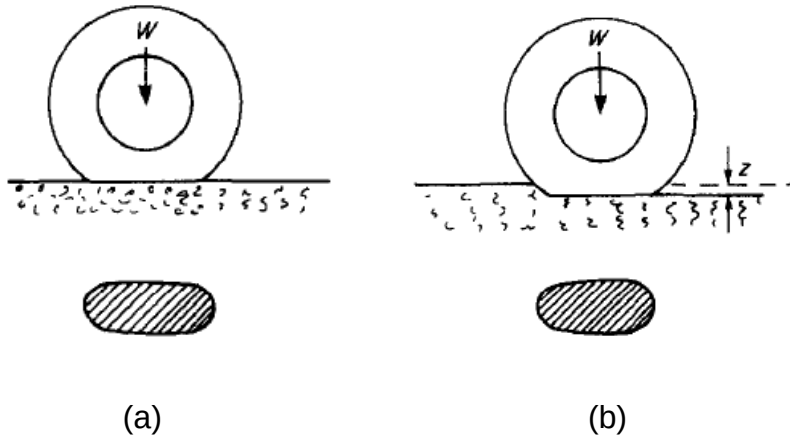


**Figure 2-15 Tyre deflection a) on a hard surface, b) soft soil (Plackett (1984))**

Upadhyaya et al. (1988) developed a mathematical model for the 2D contact area of pneumatic tyres (16.9 R 38, 18.4 R 38 and 24.5 R 32) on a hard surface. The mathematical model was based on tyre geometry and deflection characteristics. The tests were conducted using the multiple overlay technique where a steel plate (hard surface) was clamped with two white sheets and carbon paper between them. The tyre with a selected inflation pressure and the load was lifted and pressed on white paper and was lifted again. The same procedure was repeated, but this time the tyre was rotated a few degrees. This procedure was repeated to obtain a good tyre's imprint on white paper. The tyre contact area geometry is elliptical at small deflections and becomes rectangular with curved edges as deflection increases. The mathematical model (as shown in 2-53) was in good agreement with the experimental results.

$$A = \frac{\pi}{4} ab \left[ 1 - \frac{2\eta}{\pi} \right] \quad (2-53)$$

where,  $\eta = 0$ , if  $b < w$

$$\eta = \left[ 2 \left( 1 - \frac{w}{b} \right)^{0.5} - \frac{w}{b} \left[ 1 - \left( \frac{w}{b} \right)^2 \right]^{0.5} \right], \quad \text{if } b > w$$

$a$  = contact length of tyre, m

$b$  = contact width of tyre, m

$w$  = tyre width, m

Saarilahti (2002) proposed the general model of tyre contact area on hard ground, as shown in 2-54. The results showed that the large tyre diameters incur an elliptical contact area at high inflation pressure and a rounded contact area for wider tyres.

$$A = c. a. b \quad (2-54)$$

where,  $c$  = shape parameter,

$$c = \frac{\pi}{4} \text{ or } 0.785 \text{ for circle and ellipse, and}$$

$$c = 1 \text{ for square and rectangle}$$

Keller (2005) used the super-ellipse concept proposed by Hallonborg (1996) to model the tyre contact area. It was assumed that the length of the major axis ( $a$ ) and minor axis ( $b$ ) of the ellipse were the same as the longitudinal and transverse axes of the tyre contact area. The contact area was modelled as 2-55.

$$A = k. a. b \quad (2-55)$$

The  $k$  is a constant and is a function of  $n$ . It can be determined using a numerical integration technique on one quadrant's area function (2-56).

$$A_{\text{Quadrant}} = b \int_0^a \left(1 - \frac{x^n}{a^n}\right)^{1/n} dx = k. a. b \quad (2-56)$$

The shape parameter  $n$  can be found from Equation 2-57.

$$n = 2.10(w_{\text{tyre}}d_{\text{tyre}})^2 + 2.00 \quad (2-57)$$

where,  $a$  = contact length of tyre, m

$b$  = contact width of tyre, m

$w_{\text{tyre}}$  = tyre width, m

$d_{\text{tyre}}$  = tyre diameter, m

It was analysed that when  $w_{tyre}d_{tyre} = 0$ , the value of  $n = 2$ . It means that the curve is an ellipse, and when  $n \rightarrow \infty$ , the superellipse curve becomes a rectangle. Therefore, the above model was in good agreement with Diserens (2002).

Komandi (1976) gave the following formula for calculating the tyre contact area (A), which was experimentally verified with ten tyre's contact area properties.

$$A = (l - b)b + \frac{\pi b^2}{4} \quad (2-58)$$

$$l = 1.7\sqrt{(D - \delta)\delta} \quad (2-59)$$

$$b = C_2 B p_1^{-0.14} \left( 1 - e^{\frac{W}{0.4 W_{min}}} \right) \quad (2-60)$$

where,  $l$  = length of contact area, m;

$b$  = width of contact area, m;

$D$  = outer tyre diameter, m;

$\delta$  = tyre deflection, m;

$C_2$  = tyre rim constant;

$B$  = tyre width, m;

$p_1$  = inflation pressure;

$W$  = normal load, N;

$W_{min}$  = allowed tyre load at lowest inflation pressure, N;

Sharma and Pandey (1996) measured the tyre contact areas on a hard surface of three common agricultural tyres (11.2-28, 12.4-28 and 13.6-28) used in Indian tractors. The normal loads varied from 65 N to 102.5 N and inflation pressure from 50 kPa to 250 kPa. The tests were conducted using Upadhyaya et al. (1988). The results showed that the contact area decreases with the increase in the inflation pressure at constant load. It may be because of the increase in the tyre stiffness with the increase in the inflation pressure, which decreases the contact

area. However, the tyre contact area increases with the normal load at constant inflation pressure. It may be because of the flattening of the tyre carcass. It was analysed that the tyre contact area increases with tyre size.

Komandi (1990) suggested a contact area model for the pneumatic tyre deforming road surface. The model was experimentally tested for the ten tyres ranging from 9-24 to 15-30. The inflation pressure of the tyre varied from 0.4 bar to 1.6 bar.

$$A = CW^{0.7} \left( \frac{b}{d} \right)^{0.5} p_i^{-0.45} \quad (2-61)$$

$$\text{where, } C = \begin{cases} 0.30 - 0.32, \text{ Bearing soil} \\ 0.36 - 0.38, \text{ Sandy field} \\ 0.42 - 0.44, \text{ Loose sand} \end{cases}$$

Diserens (2009, 2011) measured the tyre contact area of 28 tyres in the field by powdering calcium carbonate around the tyre and then taking photographs of their footprints. The contact area was measured using image analysis. The multiple regression analysis of tyre contact area as a function of tyre size (TS = section width X outer diameter of the tyre), normal load (W), and tyre inflation pressure ( $P_i$ ). The derived models were in the form of Equation 2-62.

$$A = a \text{ TS} + b \text{ W} + c \text{ P}_i \quad (2-62)$$

Where a, b and c are model constants.

## 2.8 Mobility prediction models

Mobility prediction models were developed to quantify ground vehicle's mobility performance parameters on soft soils (Typical cone index, CI = 0 to 700 kN/m<sup>2</sup> (Brixius, 1987). These performance parameters are critical in the vehicle design and tyre manufacturing industry. Therefore, researchers tend to develop mobility prediction models, which provide cost-effective and time-optimising approaches in predicting ground vehicle performance on different terrains. There are different methods to predict mobility. However, they can be grouped into three main

categories: empirical method, semi-empirical method and numerical method. These methods are discussed in detail in this section.

### 2.8.1 Empirical method

Empirical models are developed from an experimental-based approach. The experiments are conducted in controlled laboratory conditions and in-situ field conditions with different tyres and soil parameters. The results are correlated to obtain a mathematical model (e.g., by curve fitting) to evaluate the experimental tyre's mobility performance parameters on soft soil. The empirical method's main drawback is that it does not explain the physical phenomenon between the tyre-soil interaction and considers a single parameter to characterise soil strength i.e., cone index (Tiwari et al., 2010).

Turnage (1972) developed the mobility model based on a single wheel test in an in-situ field and laboratory conditions at 20% slippage. The experiments aimed to determine minimum soil penetration resistance in no-go situations. The tests were performed with military tyres of older technology. Therefore, it may give lower mobility with the tyres of modern technology. The laboratory test and in-situ field test models are shown in Equations 2-63 – 2-66. Dwyer et al. (1976), Gee-Clough et al. (1978) and Dwyer et al. (1984) also used the Turnage equation to model the performance parameters.

The model derived from laboratory test:

$$\frac{D}{W} = 1.51 - \frac{12.37}{M_T + 5.94} \quad (2-63)$$

$$\frac{MR}{W} = 0.04 + \frac{0.2}{M_T - 1.5} \quad (2-64)$$

The model derived from in-situ field-testing:

$$\frac{D}{W} = 0.8 - \frac{1.31}{M_T - 2.45} \quad (2-65)$$

$$\frac{MR}{W} = 0.04 + \frac{0.2}{M_T - 2.5} \quad (2-66)$$

**Table 2-6 The mobility prediction models using the generalised Turnage (1972) model**

| Researcher              | Mobility performance model                                                                              |
|-------------------------|---------------------------------------------------------------------------------------------------------|
| Dwyer et al. (1975)     | $\left(\frac{D}{W}\right)_{20\%} = 0.56 - \frac{0.47}{M_T}$ $\frac{MR}{W} = 0.07 - \frac{0.2}{M_T}$     |
| Dwyer and Heigho (1984) | $\left(\frac{D}{W}\right)_{\max} = 0.796 - \frac{0.92}{M_T}$ $\frac{MR}{W} = 0.049 + \frac{0.287}{M_T}$ |

Further, to include different soil textures, Wismer and Luth (1974) defined the wheel numeric for cohesive-frictional soils, as shown in Equation 2-67. The experiments were performed with bias-ply tyres with conventional tread patterns with practical restrictions on tyre parameters such as  $b/d \approx 0.3$ ,  $\delta/h \approx 0.2$  and  $r/d \approx 0.475$ .

$$C_n = \frac{CI \cdot bd}{W} \quad (2-67)$$

$$\frac{D}{W} = 0.75(1 - e^{-0.3C_n s}) - \left(\frac{1.2}{C_n} + 0.04\right) \quad (2-68)$$

$$\frac{T}{rW} = 0.75(1 - e^{-0.3C_n s}) \quad (2-69)$$

The above equations are in good agreement with the test results of a wide range of tyres ranging from 14 to 33 in. (356- 838 mm) in width, 33 to 65 in. (838 – 1651 mm) diameter and wheel load ranging from 227 to 2950 kg. Soils used in testing are all cohesive-frictional soils with cone index varying from 19 to 500 psi. Wismer and Luth (1973) analysed a correlation between penetration depth and average cone index to be used precisely in the traction model. The research found that for the tyre sinkage of less than 3 in, the penetration depth is to be kept between 0-



6 in to calculate the average cone index. However, as tyre sinkage exceeds 3 in, the cone index should be the average of 6 in the band layer, including maximum tyre sinkage.

The simplicity of the Wismer-Luth equation coupled with the cone index as a single soil parameter has resulted in the widespread use of these equations. Clark (1985), Wulfsohn et al. (1988), Leviticus et al. (1983), Rummer et al. (1985), Ashmore et al. (1987), Upadhyaya et al. (1988, 1989) and Yu & Kushwaha (1994) also used the generalised Wismer and Luth traction prediction model to study the effect of tyre parameters and soil characteristics on vehicle mobility. The generalised Wismer-Luth equation is represented, as shown in Equation 2-70 and 2-71 and presented in the graph in Figure 2.16, and the constants of these equations should be found as per the tyre and soil conditions.

$$\frac{MR}{W} = \frac{C_1}{C_n} + C_2 \quad (2-70)$$

$$\frac{T}{rW} = C_3(1 - e^{(-C_4 C_n s)}) \quad (2-71)$$

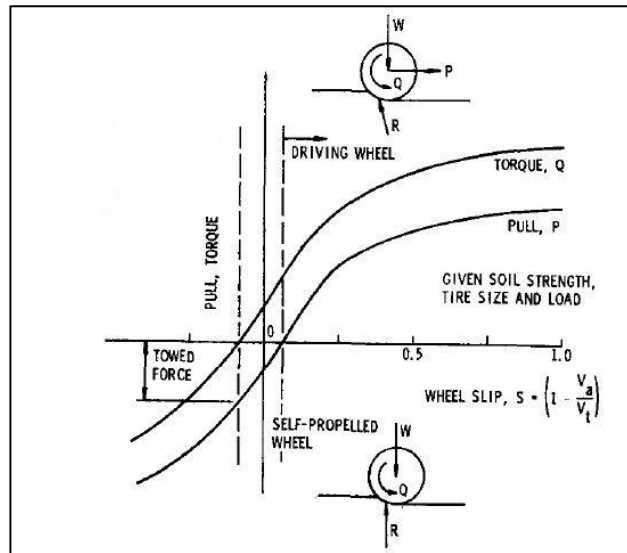


**Figure 2-16 General pull-torque-slip relation of tyre on soil (Wismer and Luth (1973))**

Brixius (1987) considered 2500 traction test results with 120 different tyre-soil conditions at Deere & Company, U.S. Waterways Experiment Station, the

National Institute of Agricultural Engineering and U.S. Army Tank Command. The tyre b/d ratio varied from 0.1 to 0.7, with static tyre deflection ranging from 10 to 30% of section height. The rolling radius ratio (r/d) was kept approximately equal to 0.475 for most agricultural and earthmoving tyres. The Brixius modified the mobility number to Brixius number ( $B_n$ ), as shown in Equation 2-72. Therefore, the Brixius model was developed to determine the tractive characteristics of bias-ply tyres, as shown in Equations 2-73 and 2-74.

$$B_n = \left( \frac{C I b d}{W} \right) \left( \frac{1 + 5 \frac{\delta}{h}}{1 + 3 \frac{b}{d}} \right) \quad (2-72)$$

$$\frac{MR}{W} = \frac{1}{B_n} + 0.04 + \frac{0.5s}{\sqrt{B_n}} \quad (2-73)$$

$$\frac{T}{rW} = 0.88(1 - e^{-0.1B_n})(1 - e^{-7.5s}) + 0.04 \quad (2-74)$$

Brixius (1987) analysed that with the increase in mobility number, the torque ratio increases. It may be because of the increase in soil strength and deflection ratio ( $\delta/h$ ) or decreased wheel load and width to diameter ratio ( $b/d$ ). It was also analysed that the motion resistance decreases with the increase in deflection ratio. It may be because of the larger contact area and low peak pressure at high deflection values, resulting in lower soil compaction and lower motion resistance. Motion resistance increases with an increase in the tyre width to diameter ( $b/d$ ) ratio. It may be because soil compaction is lowest for smaller ( $b/d$ ) ratio tyres, resulting in lower motion resistance. In Equation 2-74, the torque ratio approximates a constant value of 0.04 at zero slip for all conditions. A zero slip occurs when the tyre is in self-propelled operating condition on a hard surface. Under this state, torque is required only to overcome the tyre's motion resistance. Therefore, the motion resistance ratio equals 0.04 on a hard surface. It also represents a minimum value for any other surface.

The Brixius model is included in the ASABE standards as it is a widely used traction prediction model for radial-ply and bias-ply tyres. However, the general Brixius model's constants and coefficients should be adjusted as per the experimental conditions. Evans et al. (1991), Al-Hamed et al. (1994) and Tiwari

et al. (2010) also used the general form of the Brixius equation to develop the traction equations for radial and bias-ply tyres in controlled experimental conditions.

**Table 2-7 The mobility prediction models using generalised Brixius (1987) model**

| Researcher             | Mobility performance models                                                                                                         |
|------------------------|-------------------------------------------------------------------------------------------------------------------------------------|
| Evans et al. (1991)    | $\frac{T}{rW} = 0.88(1 - e^{-0.1B_n})(1 - e^{-4.15s}) + 0.04$ $\frac{MR}{W} = \frac{1}{B_n} + 0.04$                                 |
| Al-Hamed et al. (1994) | $\frac{T}{rW} = 0.88(1 - e^{-0.08B_n})(1 - e^{-9.5s}) + 0.032$ $\frac{MR}{W} = \frac{0.9}{B_n} + 0.032 + \frac{0.5s}{\sqrt{B_n}}$   |
| Tiwari et al. (2010)   | $\frac{T}{rW} = 0.66(1 - e^{-0.09B_n})(1 - e^{-5.25s}) + 0.035$ $\frac{MR}{W} = \frac{1.2}{B_n} + 0.035 + \frac{0.77s}{\sqrt{B_n}}$ |

### 2.8.2 Semi-empirical method

Semi-empirical models are based on the experimental results with empirical formulations and develop analytical methods to represent some of the physical processes involved in tyre-soil interaction. This method utilizes Bevameter testing to model the tyre-soil interaction. The vertical deformation of soil under tyre loading is analogous to soil deformation under the flat plate. This test is called the pressure-sinkage test. The shear deformation of soil under the traction device is analogous to shear due to torsional shear devices or rectangular grouser units. This test is called the shear test.

In the pressure sinkage test, Bernstein (Plackett, 1985) proposed a pressure sinkage (p-z) model (Equation 2-75) based on the semi-empirical approach to study the vehicle-soil contact dynamics based on the behaviour of penetration of

rectangular or circular plates into the soil. The  $k [N/m^{n+2}]$  is the modulus of inelastic deformation and  $n$  is the exponent of sinkage empirically determined. It was assumed that the soil is homogeneous from the surface to the depth of interest.

$$p = kz^n \quad (2-75)$$

Bekker (1956, 1969) modified the Bernstein pressure-sinkage mathematical model based on the soil mechanics and incorporated the width of the contact patch,  $b$  (m) (Equation 2-76). The  $k_c [N/m^{n+1}]$  and  $k_\phi [N/m^{n+2}]$  are the moduli of deformation with respect to cohesion and friction, respectively, and  $n$  is an empirically determined model parameter that defines the shape of the load-displacement curve. It also demonstrated that deformation's moduli are independent of plate geometry for homogeneous soil.

$$p = \left( \frac{k_c}{b} + k_\phi \right) z^n \quad (2-76)$$

In the shear test, Janoshi and Hanamoto (1961) equation (Equation 2-77) is the most widely used model to establish a relationship between shear stress and shear deformation of soil. It is a modified form of Mohr-Coulomb failure criteria as shown in equation 2.12.

$$\tau = (c + p \tan \phi) \left( 1 - e^{\frac{-j}{K_x}} \right) \quad (2-77)$$

where  $c$  is cohesion,  $\phi$  is the internal angle of friction,  $K_x$  is shear deformation modulus,  $j$  is shear deformation,  $p$  is applied normal stress, and  $\tau$  is shear stress.

Bekker (1956, 1960, 1969) and Wong (1989, 2001) determine the mobility performance parameters as a function of normal stress and shear stress generated during tyre-soil interaction. The total thrust generated during tyre-soil interaction is not utilized in useful work. Some energy is lost to overcome the motion resistance (MR), which is caused by soil compaction and bulldozing. Therefore, the compaction resistance is expressed, as shown below.

$$z_0 = \left( \frac{p_{gr}}{k_c/b + k_\phi} \right)^{1/n} \quad (2-78)$$

$$MR = \frac{bp_{gr}^{(n+1)/n}}{(n+1)(k_c/b + k_\phi)^{1/n}} \quad (2-79)$$

Bekker determined the critical ground pressure  $p_{cgr}$  above which a pneumatic tyre behaves as a rigid wheel. The expression for it is as shown below.

$$p_{cgr} = (k_c/b + k_\phi)^{1/(2n+1)} \left[ \frac{3W}{(3-n)b\sqrt{D}} \right]^{2n/(2n+1)} \quad (2-80)$$

Wong (2010) developed the gross traction (GT) for a flexible tyre obtained using Bekker's equation that relates it to the tyre and soil properties, where  $A$  is contact area,  $l$  is contact length, and  $s$  is slip. The expression for it is shown below.

$$GT = (cA + W \tan \phi) \left( 1 - \frac{K}{sl} (1 - e^{-\frac{j}{K}}) \right) \quad (2-81)$$

The drawbar pull can be determined as a difference between gross traction and motion resistance, as shown in Equation 2-16. Therefore, the semi-empirical approach presented in this section predicts the tractive performance for various types of pressure distribution of the tyre-soil interface.

### 2.8.3 Numerical methods

The numerical approach encompasses the Finite Element Method (FEM), the Smoothed Particle Hydrodynamics (SPH) and the Discrete Element Method (DEM). In FEM, the soil is modelled as a continuum medium and discretized into finite element mesh to capture the tyre-soil interaction phenomenon. However, the real terrain is not a continuum medium, and FEM fails to capture the soil flowability during the tyre-soil interaction. It is difficult to capture soil separation and reattachment phenomena even with remeshing.

Wright (2012) performed the tyre-soil interaction tests to predict the drawbar pull vs slip relation using the FEM in the LS-DYNA software on the dry sand (%w = 0.86% and density = 1.39g/cc) and the sandy loam (%w = 14.2% and density =

1.47 g/cc) soil models. The FE tyre models of 7.5R16 and 445/65R22.5 were designed and validated using the load deflection test data from the manufacturers. The mobility tests were simulated for both tyres at different inflation pressures (100 to 800 kPa for 445/65R22.5 and 50 to 275 kPa for 7.5R16) and different normal loads (29.43 kN for 445/65R22.5 and 3.78 to 9.07 kN for 7.5R16) on both soils, as shown in Figure 2.17. The translational velocity for both tyres on both soils was 3 m/s with slip ranging from 20 to 90%. The simulation results were compared with the experimental results under the same conditions to determine the accuracy of the FE mobility models.



**Figure 2-17 Overview of tyre / soil interaction model using FEM in LS-DYNA (Wright (2012))**

It was found that for the 7.5R16 tyre, the drawbar pull was significantly over-predicted for the dry sand and an error of 16.66% and 3.8% was found for the sandy loam soil model at low inflation pressure (50 kPa) and high inflation pressure (130 kPa) respectively. For the 445/65R22.5 tyre, the mean drawbar pull in the simulations over-predicted the experimental results in the range from 2.56% to 40.56% for both soils. It may be because the soil model relies on a continuum mechanics formulation.

The Smoothed Particle Hydrodynamics (SPH) is a mesh-free method where particles are used to solve the governing equations of continuum mechanics (Cauchy equation of motion in the case of soils). A kernel smoothing function is used to discretize the continuum equations for each particle in order to evaluate its properties and any fluxes or forces acting on it. Lescoe et al. (2010) and Dhillon et al. (2013) created a FEA tyre and SPH soil model PAM-CRASH software. This

setup was used to model the rolling of rigid and flexible tyres on the dry sand (density = 1.6 g/cc, shear modulus = 9 MPa, Bulk modulus = 15 MPa). The hydrodynamics elastic-plastic material model was used in the above research. It was concluded that SPH can be used to model the tyre-soil interaction, but further refinement of the material models is needed since it either showed excessive viscosity or incorrect material compressibility.

El-Sayegh et al. (2020) computed the truck tyre-gravelly soil (sand with gravel soil) interaction in PAM-CRASH software (Figure 2.18). The truck tyre 315/80R22.5 was modelled using the Finite-Element Analysis (FEA) technique and was validated using the manufacturer's datasheet of the tyre deflection test. The gravelly soil model (density = 1742 kg/m<sup>3</sup> and moisture content = 10%) has the particle size distribution of approximately 5% fines passing through 0.075 mm sieve. The hydrodynamics elastic plastic material model was used to model the tyre-soil interaction. The tyre is simulated with a longitudinal velocity of 2m/s, 45 kN normal load, 896 kPa (130 psi) inflation pressure and 45% slip. It was concluded that the simulations and experimental results are in good agreement in predicting the tyre rolling resistance characteristics. The maximum error of 13% and the minimum error of 0.8% were found in the rolling resistance coefficient.

As per NATO CDT – 308 (2020), the main advantage of SPH is that it can be used in large soil deformation including soil flow and soil adhesion to solid surfaces. However, it also states critical drawbacks such as it is based on a continuum mechanics formulation (i.e. assumes the soil is a continuum). Further, no constitutive soil material model is present which can accurately account for the combined effects of: large strain/flow, plasticity, elasticity, fracture, friction, compressibility, and cohesion. The SPH is also computationally slower than the Discrete Element Method (DEM) since a particle not only interacts with its immediate neighbours but with all the particles within the kernel radius.

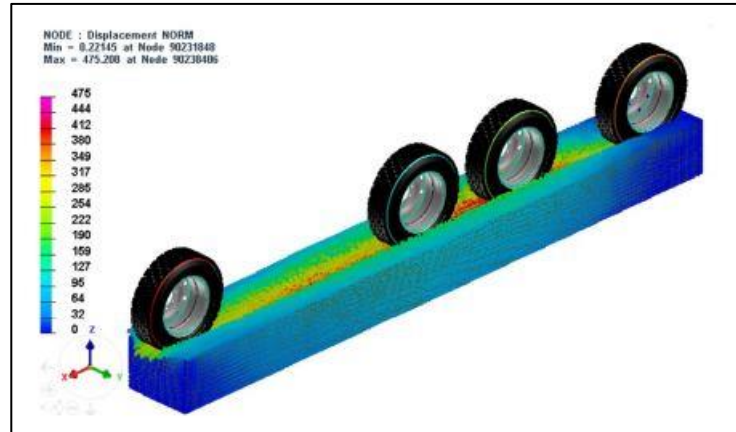


**Figure 2-18 Truck tyre model setup over gravelly soil with normal displacement contour (El-Sayegh et al. (2020))**

In the present study, the Discrete Element Method has been used extensively (Nakashima et al., 2004, 2007, 2009; Smith et al., 2013; Zhao et al., 2014; Nishiyami et al., 2016, 2017; Johnson et al., 2015, 2017; Yonghao Du et al., 2017, 2018; Nakanishi et al., 2020; Ravula et al., 2021, Wu et al., 2022). This method treats the soil as a collection of discrete individual particles that interact with each other and tyres. This method can describe the better physics behind the contact phenomenon of tyre-soil interaction and inter-particle mechanics. The particles act as independent entities and interact only at the particle's contacts and interfaces.

The DEM algorithm involves the application of Newton's 2<sup>nd</sup> law of motion to particles and the force-displacement law contact model at the interfaces. Newton's 2<sup>nd</sup> law of motion is used to determine the particle's motion arising from its contact and body forces. The force-displacement model is used to update the contact forces acting from each contact's relative motion. The calculation cycle requires the repeated application of both the laws at each particle and relative contact, respectively. Thus, making them computationally expensive. However, since the 1950's the computational power has improved, making them more feasible (Wong, 2009). These methods, when properly implemented, can provide several advantages over empirical and semi-empirical approaches.



Wu et al. (2022) calibrated the DEM material model parameters of clay (particle size = 0.05-0.075 mm, density = 1.45g/cc and moisture content = 91%) and sand (particle size = 0.5-1 mm, density = 1.97 g/cc and moisture content = 21 %) with the experimental shear test results using the trial-and-error approach. This method makes it feasible to change the model parameters and comparison results constantly until simulation results validate the experimental results. The initial soil model parameters were selected from the available published literature on similar soils. The mean absolute percentage error between the simulation models and experimental results are 7.5% and 11% for clay and sand respectively. Therefore, the simulation model is accurate and effective.

Nakashima et al. (2004) proposed an algorithm of the tyre-soil interaction theory based on the FE-DEM analysis. FEM cannot be effectively applied to the tyre-soil interaction because the contact point of the tyre lug tip behaves like a singular point of stress. The DEM can, however, predict the microscopic deformation of granular soil particles under the tyre tread. DEM simulation calculates inter-particle contact forces (normal and tangential forces) using Hertz Mindlin's contact algorithm, as shown in Equations 2-82 to 2-84, and Figures 2.20. The calibration of material model constants, such as material stiffness  $K$  and damping constant  $C$  is an integral part of the Discrete Element Method modelling methodology, but a commonly used trial and error calibration approach is both resource and time consuming (EDEM, 2014, Wu et al., 2022).

Later, Nakashima et al. (2007, 2009) analysed the tyre-soil interaction using the elastic tyres modelled using spring and damper characteristics. The elastic tyres were of two different tread patterns – smooth and grooved. It was found that the grooved tyre has better mobility performance as compared to the smooth tyre. The grooved tyre's simulation results were in good agreement with the experimental results.

$$F_n = -K_n \Delta u_n - C_n \Delta \dot{u}_n \quad (2-82)$$

$$F_t = -K_t \Delta u_t - f_t^v \quad (F_t < \mu F_n) \quad (2-83)$$

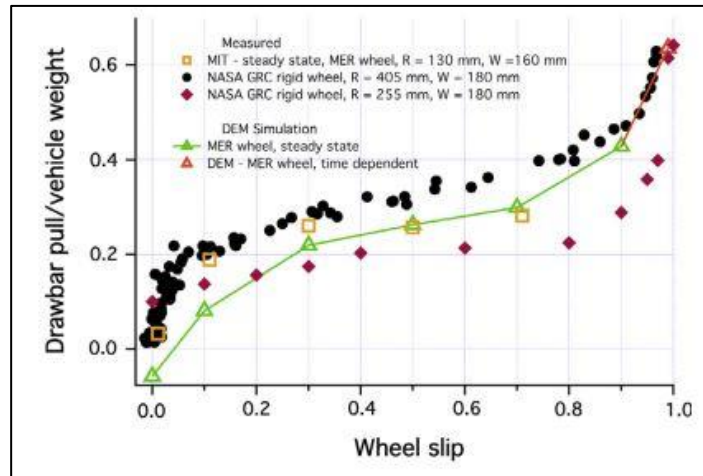
$$F_t = \mu F_n \quad (F_t \geq \mu F_n) \quad (2-84)$$

Zhao et al. (2014) developed a 3D simulation method based on the discrete element coupled with the finite element method to investigate the tyre-sand interaction. A rigid tyre was modelled and simulated on sandy soil ( $E = 7.5 \times 10^4$  MPa,  $\nu = 0.3$  and density  $= 2.4 \times 10^3$  kg/m<sup>3</sup>). The Hertz-Mindlin theory was used to calculate the contact forces between the elements. Single wheel test simulations were performed under different slip conditions ranging from 0 to 60%. It was found that the trend of the drawbar pull versus slip relation was in agreement with the soil-bin experimental results. Therefore, it shows the application of the DEM simulation of the tyre-soil interaction.

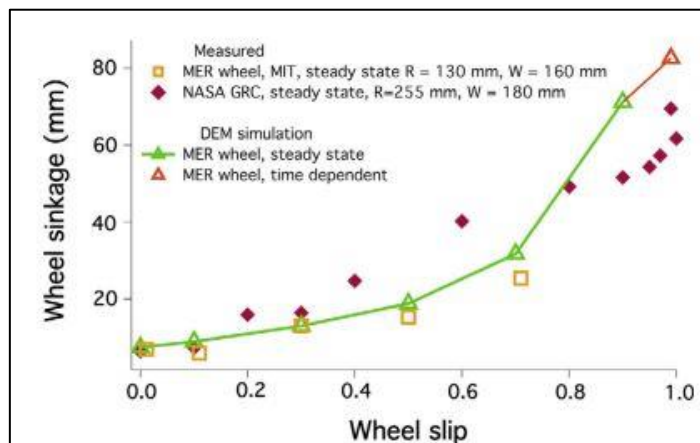
Smith et al. (2013) investigated the mobility performance and impacts of rough terrain operations on small, unmanned ground vehicles using the discrete element method (DEM). Flat, level soil terrain and 20 different sinusoidal profiles with varying frequency and amplitude were used to model the rough terrain. Two rigid wheels were modelled based on the experiments performed by Harbin Institute of Technology and Cornell University. The single wheel tests simulations were performed on rough terrains (dry sand,  $c = 5$  kPa &  $\phi = 36.9^\circ$ ), which showed decreases in mobility and efficiency on sinusoidal terrains compared to flat, level terrain. The average drawbar pull decreased by 15% and driving torque increased by 35% at low slip conditions. Moreover, the rough terrain frequency created oscillations in drawbar pull and driving torque, increasing the span between the minimum and maximum values.

Johnson et al. (2017) and Johnson et al. (2015) studied the DEM model of Mars Exploration Rover (rigid wheel) mobility performance on Mojave Martian simulant (density  $= 1750$  kg/m<sup>3</sup>,  $c = 600$  Pa and  $\phi = 35^\circ$ ) at different slip conditions (0 to 100%). The critical advantage of the DEM model is that once the initial conditions, control functions, particle parameters, and contact physics between soil and wheel are established, they become valid for the full range of slip conditions. The DEM simulated drawbar pull vs slip and wheel sinkage vs slip relations were validated with the experimental results as shown in Figure 2.19 (a) and (b)

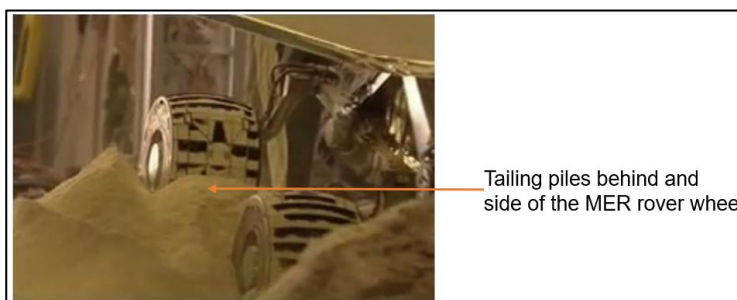
respectively. It was observed that the classical terramechanics models were inaccurate at high slip conditions as they don't consider the piling effect behind wheels as shown in Figure 2.19 (c). There is a non-linear increase in the drawbar pull and wheel sinkage at the high slip range above 70%. It may be because the tailing piles build-ups in height as the wheel erosion sinkage increases, resulting in the increase in the contact area with the regolith (Johnson et al., 2017). It may possibly exhibit increasing soil strength with depth.



(a)



(b)



(c)

**Figure 2-19 Rigid wheel of MER interaction with the Martian soil (a) Drawbar Pull coefficient w.r.t. wheel slip; (b) Wheel sinkage w.r.t. wheel slip (c) Tailing piles behind a MER rover in the JPL sand Rover Test Bed at NASA (Johnson et al. (2017))**

Yonghao Du et al. (2017) performed the numerical analysis of lug effects on the off-road wheel's tractive performance on dry sand by Discrete Element analysis software (PFC3D 4.0). In the simulation design, rigid lugged wheels (diameter = 940 mm and width = 320 mm) were used as the wheel's stiffness is much higher than the sands resulting in a relatively small deformation compared to the tyre diameter. Different tyre lug patterns were investigated. The soil test bin was prepared in volume 5.0 m x 1.0 m x 0.5 m with 19,242 particles the particle size distribution obtained from the grain test. The sand model (Porosity = 0.4, Young's modulus = 23.20 MPa, %w = 5%,  $c = 3.31$  kPa and  $\phi = 34.18^\circ$ ) was calibrated using the tri-axial shear test. The results concluded that with the increase in tyre lug height, drawbar pull and tractive efficiency increases at the lower slip range. The V-shaped lug design was the most effective in bad road conditions.

Yonghao Du et al. (2018) developed and validated an improved mobility prediction model for wheel-soil interaction under multiple operating conditions using the discrete element method. The simulations were conducted under straight driving and inclining conditions, both longitudinal and transverse. The soil mechanical parameters were found from the shear tests, which mainly include density ( $1650 \text{ kg/m}^3$ ), internal friction angle ( $34.18^\circ$ ) and cohesion ( $3310 \text{ Pa}$ ), which develops soil's mechanical behaviour in simulations. The wheels were modelled as rigid with three different dimensions and four different lug heights and numbers. They were driven at five different inclining angles and varied slip conditions ranging from 0 to 80%. The analysis results were compared with the previous models, and it was found that the Bekker model over-predicts the drawbar pull vs slip relation in straight driving conditions. An increase in drawbar pull and wheel torque was recorded with increased wheel radius and lug height. However, the simulated data also indicated that the wheel's traction properties were better at optimal lug number, which can be found using the developed predicted model.

Ravula et al. (2021) simulated the mobility performance of a rigid lugged wheel on a soil model similar to Nakashima et al. (2007). The wheel performance was evaluated based on two control modes: angular velocity-based control and slip

based – control. It was found that with increasing the wheel lug, its performance increases in angular velocity-based control, but no monotonous behaviour was recorded for slip-based control.

As shown in Table 2.8, the Hertz-Mindlin contact algorithm is widely used to model the dry, free-flowing and incompressible behavioural characteristics of the soil particles such as dry sand and Martian soil (widely used soils in DEM modelling). This model uses a spring and damper arrangement to represent the particle's elasticity and viscosity in the parallel combination and to calculate inter-particle contact forces (normal and tangential forces), as shown in figure 2.20. Mathematically, it is represented by equations 2.82, 2.83 and 2.84. The limitations of this contact algorithm are its inability to model the effect of moisture content, compressibility, resistance to motion of particles, elastic-plastic deformations and adhesive effects (NATO CDT – 308 (2020)).

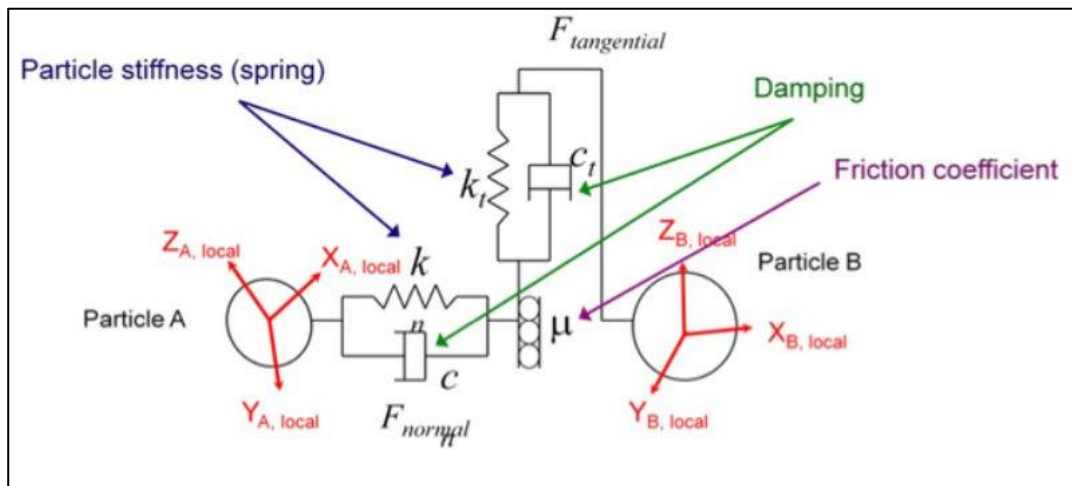


**Figure 2-20 An overview of typical Hertz-Mindlin Contact Algorithm (EDEM)**

**Table 2-8 Summary of selected DEM mobility prediction models reported in the literature**

| <b>Reference</b>            | <b>Tyre</b> | <b>Soil system</b> | <b>Soil type</b>        | <b>Contact</b>       |
|-----------------------------|-------------|--------------------|-------------------------|----------------------|
| Nakashima et al. (2004)     | Rigid       | Homogeneous        | Dry & Granular sand     | Hertz contact theory |
| Nakashima et al. (2008)     | Elastic     | Homogeneous        | Dry & Granular sand     | Hertz Mindlin theory |
| Nakashima et al. (2009)     | Elastic     | Homogeneous        | Dry & Granular sand     | Hertz Mindlin theory |
| Smith et al. (2013)         | Rigid       | Homogeneous        | Dry sand                | Hertz contact theory |
| Zhao et al. (2014)          | Rigid       | Homogeneous        | Dry sand                | Hertz Mindlin theory |
| Nishiyami et al. (2017)     | Rigid       | Homogeneous        | Dry sand                | Hertz Mindlin theory |
| Yonghao Du et al. (2017)    | Rigid       | Homogeneous        | Sand                    | Hertz contact theory |
| Johnson et al. (2015, 2017) | Rigid       | Homogeneous        | Mojave Martian simulant | Hertz contact theory |
| Yonghao Du et al. (2018)    | Rigid       | Homogeneous        | Sand                    | Hertz contact theory |
| Ravula et al. (2021)        | Rigid       | Homogeneous        | Dry granular media      | Hertz contact theory |

## 2.9 Scalability testing

Swanson (1973) conducted scale model testing at U.S. Army Engineer Waterways Experiment Station (WES) in 1970 to evaluate various scale tyre's tractive performance on dual-layered terrain. It was analysed that at the cost of two full-scale model testing on dual-layered terrain, five prototype scale model testing could have been conducted. Therefore, it was concluded that scale model testing is a viable method for reducing experimental costs.

The 2.50-4 tyre was selected as a scale tyre which was approximately a  $\frac{1}{5}$  – scale model of 11.00-20 tyre. Tests were conducted on the clay soil at two different cone index values, 160 kPa and 55 kPa. The testing's first objective was to establish a relation between the wheel load and tyre deflection for tyres. Further, traction tests were conducted at different wheel loads ranging from 111 to 667 N and different wheel deflections from 15% to 35%. The 20% slip was maintained constant for all the tyre tests. Fifteen different combinations of wheel load, tyre deflection and soil strength were tested with three replications. Drawbar pull, torque and sinkage were the key tractive parameters measured during the tests with respect to clay mobility number.

The results concluded that sixty-eight per cent of the experimental data between traction coefficients (P/W, T/W, z/d) and WES clay mobility number were in the scatter band (one standard deviation from the predictive curve). This indicates that a scale tyre can be used to predict full-scale tyre performance. However, more tests should be performed with different tyres and soil conditions.

Hetherington (2005) investigated a one-tenth scale model of a main battle tank's running gear. The mobility performance relationship of full scale tracked vehicles was adopted from Turnage (1971). Therefore, the model scale experiments were conducted on the same soil conditions as a full-scale model. The scaling was carried out based on a non-dimensional quantity  $\left(\frac{F}{\gamma z^3}\right)$  developed by Schuring and Emori (1965). This quantity represents that the angle of friction on sandy soil is kept the same for both model scale and full scale. It was observed that the mobility numeric for both the model scale and full scale is identical provided the



same soil conditions are used. It was found that the experimental results of the model scale are the same as predicted from the Turnage equation. Therefore, it was concluded that the test's one-tenth scale model of a tracked vehicle correlates well with the performance predictions of full-scale vehicles.

Lankenau et al. (2012) investigated the effect of a bucket wheel scale on excavation forces. Three scale models were constructed based on the full-scale model's geometric dimensions. The bucket wheel's diameter, width, blade tangent extension, cutting depth, and advance speed were scaled, whereas angular velocity and blade thickness were kept constant. All tests were done on the same soil, i.e. lunar simulant. The excavation forces were measured using a 6 DOF Force/Torque load cell. The nonlinear regression in power-law form was used to fit the excavation force vs scale data set. The results showed that the longitudinal excavation force is proportional to the excavation dimension's cube.

Hetherington (1992) performed a scale model testing on the one-eighth scale model of Challenger and Warrior's earth anchor design. Reece (1973) developed a theory that relates the maximum force developed in earth anchor to the soil's properties, anchor design, weight, and recovery vehicle geometry. Further, a dimensional analysis approach was followed to develop six Pi terms which were kept identical at full and model scale. The experimental results were in good correlation with full-scale theoretical prediction based on Reece theory and full-scale anchor forces deduced by scaling results from model trials. Therefore, it was concluded that one-eighth scale model tests could validate earth anchor designs for both vehicles.

Freitag (1965) performed the scale model testing on an approximately one-half scale model of a full-scale tyre. The paper quotes, "*The application of dimensional analysis to the experimental study of the vehicle-soil system is based on the fundamental assumption that similar systems that behave in a similar manner can be established. Thus, if the generalisations obtained by a systematic variation of the independent parameters are to be used with confidence, it must be shown that the observed tyre – soil relations do not vary with the size of the system.*"

A similar system for full scale and scale model tyres was established using two similarity criteria: geometric similarity and dynamic similarity. The geometric similarity was established between the full scale and scale model tyre using deflection number ( $\delta/h$ ) and shape number (tyre width (b)/ tyre diameter (d)). The deflection number and shape number were kept the same for both tyres to assure that both tyres have similar geometry while interacting with the soil surface. The dynamic similarity was established using clay loading number ( $\frac{CI \cdot d^2}{W}$ ). Therefore, the clay loading number was kept the same for both tyres to ensure that both tyres interact with the soil in the same manner. It was recommended by Freitag (1965) to describe the tyre deflection and contact area model accurately between the full-scale and scale tyre models to improve the prediction of the mobility performance parameters.

The tests were conducted on clay because it is purely cohesive for practical consideration. The cohesion and density of soil were interpreted as a function of the average cone index. Therefore, the clay properties were reflected in the average cone index. Further, the effect of velocity was also neglected, and the same forward speed was used for both tyres. Later, the dynamic similarity criterion was verified experimentally, and the individual curves between mobility performance parameter to clay number were found close together.

Like Freitag (1966), Bekker (1956) proposed the dimensional analysis method to develop non-dimensional parameters of the tyre-soil interaction system. These non-dimensional parameters can be used to develop similar systems. He also defines the concept of geometric and dynamic similarity of tyre-soil interaction. The geometric similarity is established by equating physical tyre parameters such as tyre diameter to tyre width (b/d) between full-scale and scale systems. The dynamic similarity is established between full-scale and scale systems when they experience similar net force. However, apart from performance parameters, the separable force components interacting with the tyre-soil interaction system are stated as:

- Forces generated due to soil cohesion

- Forces arising from acceleration of the soil particles
- Forces generated by moving soil with or against gravity
- Forces from elastic deformation of the soil
- Forces resulted from the applied force and input torque
- Forces generated from soil friction
- Forces generated by soil to wheel friction
- Forces arising from viscous shearing of soil

Bekker derived all the above separable force components into the non-dimensional group at the safe load criterion, as shown below.

$$(\phi, \frac{cl^2}{F})$$

Therefore, if the full-scale and scale tyres are tested on the same soil model and the same soil conditions, these non-dimensional parameters should be balanced in both systems.

## 2.10 Summary

The main findings of the literature survey can be summarised as follows:

- The tyre-soil interaction system's scalability can be verified after establishing geometric and dynamic similarities in the full-scale and scale systems. The geometric similarity controls the tyre's shape and curvature in a similar system. It is established by keeping the same deflection number ( $\delta/h$ ) between the full scale and scale tyre models. It assures that both tyres have the same geometry while interacting with the soil surface. The dynamic similarity controls the nature of transferrable forces in the tyre-soil interaction in similar systems. It is established by keeping the same wheel numeric in the full scale and scale systems.
- In mobility performance studies, most of the work is limited to the low-speed region (0.1 – 5.5 m/s) to keep the tyre speed slow and steady. This is due to the inherently dynamic nature of the tyre-soil interface. It is found that there was no significant effect of the tyre velocity on the performance

prediction of the system. It is because there is no significant change in the tyre-soil contact pressure and contact patch during testing.

- Bekker's flat plate theory assumes that a vehicle's wheel's ground contact area can be replicated using a flat rectangular or circular plate of the same area. However, this assumption works reasonably for a large diameter wheel (diameter > 50 cm) but fails for a small diameter wheel (diameter < 50 cm). Griffith and Spenko (2011, 2012, 2013, 2014) use the actual wheel section to deduce the ground contact pressure for a small diameter wheel (diameter < 50 cm). It is because the rate of change of wheel curvature is significant in the small diameter wheel. This theory works reasonably well for small diameter wheels, but no concrete evidence has been found for its application on the large-diameter wheel. However, no evidence has been found which relates both theories.
- A dimensional analysis approach was found reliable to reduce the parameters involved in the tyre-soil interaction system using the empirical methodology. The advantage is that there is no need to establish absolute numerical equality when quantities are similar qualitatively. Therefore, the non-dimensional ratios can establish similarity in the systems.
- Numerical methods based on the discrete element method have been developed recently. This method needs results from the analytical models or experiments to determine and validate the bulk DEM soil model since all the inter-particle properties (e.g. forces and elasticity) at the microscopic level can be measured. The contact model parameters are optimised using the trial-and-error method with initial guess points, which are either derived using the analytical method or taken from the existing literature database. It is also interesting to note that most of the research carried out using the DEM is done in homogeneous conditions and using a rigid tyre due to the limitations of the existing technology.
- Wide ranges of methods are available to quantify a vehicle's performance, such as VCI and MMP, which are frequently used in the empirical theory. However, the direct measurements of the mobility performance parameters such as the drawbar pull and the gross traction are more

useful to access vehicle performance on soil systems. Therefore, it becomes crucial to understand the drawbar pull vs slip relations of a vehicle on the test terrains.

- The tyre's tractive performance depends on the physical tyre parameters, tyre loading and soft soil conditions. If the inflation pressure of a pneumatic tyre is sufficiently high and the terrain is relatively soft, then the tyre may behave like a rigid wheel (referred to as the rigid mode of operation). When the inflation pressure of the pneumatic tyre is relatively low and the terrain is relatively firm, then the tyre deflects and is considered an elastic mode of operation (Wong (2010)).

## **3 THEORY OF SCALABILITY**

### **3.1 Introduction**

The tyre soil interaction is a complex mechanism and is under investigation quantitatively. The mobility performance depends on the stress-strain relationships developed in deformable terrain due to vehicle loading conditions. However, these relationships are not well understood because the soil system is not entirely defined. In other disciplines such as fluid dynamics and aerodynamics, satisfactory solutions to equally complex problems have been obtained through empirical relationships between the measured area of interest and dimensionless ratios (e.g. Reynolds number). Therefore, the dimensional analysis technique is followed in the tyre-soil system's empirical parameters. Further, the derived non-dimensional terms are used to establish similar tyre-soil interactions for different scales of the system.

Many of the existing empirical models use the cone index as a soil parameter to study soil's influence on vehicle mobility. This view is supported by the fact that the cone index profile is a composite property, reflecting both the soil's cohesive and frictional nature. However, it is difficult to separate the contribution of cohesion and friction between soil particles to soil strength. Therefore, the cone index is used to analyse soil strength's effect on vehicle mobility in the present study.

This chapter discusses the tyre deflection, tyre contact area characteristics and the dimensional analysis of tyre and tyre-soil system. The non-dimensional models developed were validated using experimental evidence.

### **3.2 Tyre deflection and contact area**

The study of tyre deflection and contact area at different inflation pressure and different loading conditions is the first objective in understanding a vehicle's mobility performance on soil (Swanson, 1973). It is important because the tyre deflection and contact area contribute to the ground pressure acting between the tyre and the soil. It results in a change in contact forces acting in tyre soil interaction, which affects a tyre's mobility performance.

The pneumatic tyres show two types of operational behaviour when loaded against the soil: (i) if the average ground pressure is greater than critical ground pressure, then it behaves as a rigid wheel; (ii) if the average ground pressure is lower than critical ground pressure, then it behaves as an elastic wheel. In both operational behaviours, soil deforms due to the tyre's sinkage and forms a rut on the surface. However, if the soil's sinkage decreases, then the pneumatic tyre running on soft soil can be approximated as the tyre running on a hard surface (Plackett, 1984).

The tyre deflection and contact area tests are conducted on a hard surface. It has the significant advantage of establishing a lower limit for the contact area in soft soil. A hard surface is readily available and standard. Therefore, it provides a basis for reliable and repeatable data on tyre deflection and tyre contact area. Hence, it is used as a reference in determining the tyre deflection and tyre contact area characteristics.

### **3.3 Dimensional analysis**

Dimensional analysis is a powerful tool to reduce complex physical problems to the simplest form. In this approach, a non-dimensional relationship is developed by analysing the fundamental dimensions of the physical parameters involved in the process. In the present study, dimensional analysis is conducted on the tyre deflection model, contact area model and mobility performance parameters.

#### **3.3.1 Tyre deflection**

Tyre deflection can be expressed as a function of tyre diameter, tyre width, tyre section height, inflation pressure and normal load (Freitag and Smith, 1966; Krick, 1969; Komandi, 1976; Sharma and Pandey, 1996; Rashidi et al., 2013 and Kumar et al., 2018). Therefore, it can be expressed, as shown in 3-1.

$$\delta = f(d, b, h, P_i, W) \quad (3-1)$$

**Table 3-1 Tyre deflection and contact area parameters**

| Parameter               | Symbol   | Dimensions      |
|-------------------------|----------|-----------------|
| Tyre diameter           | $d$      | L               |
| Tyre width              | $b$      | L               |
| Tyre section height     | $h$      | L               |
| Tyre deflection         | $\delta$ | L               |
| Tyre inflation pressure | $P_i$    | $ML^{-1}T^{-2}$ |
| Vertical wheel load     | $W$      | $MLT^{-2}$      |
| Tyre contact area       | $A$      | $L^2$           |

According to Lines and Murphy (1991), the tyre's stiffness varies with the tyre age, affecting the tyre deflection model. Therefore, in our present model, it is assumed that tyre stiffness is constant with the tyre age and uniform throughout the tyre. Since the tyres were procured from single supplier source, these tyres were manufactured in the same batch.

According to the Buckingham Pi theorem, four pi terms are needed to develop a non-dimensional model. Therefore, assuming the product formulation, Equation 3-1 can be expressed, as shown below:

$$\delta = C \cdot d^x \cdot b^y \cdot h^z \cdot P_i^w \cdot W^r \quad (3-2)$$

where  $C$ ,  $r$ ,  $w$ ,  $x$ ,  $y$ ,  $z$  are non-dimensional constants.

The fundamental dimensions of the left-hand side (L.H.S) and right-hand side (R.H.S) should be the same. Therefore, substituting Table 3-1 leads to the following equation:

$$L = L^x \cdot L^y \cdot L^z \cdot [ML^{-1}T^{-2}]^w \cdot [MLT^{-2}]^r \quad (3-3)$$

Now, equating the dimensions on L.H.S and R.H.S, which provides the following equations:



$$0 = w + r \quad (3-4)$$

$$1 = x + y + z - w + r \quad (3-5)$$

$$0 = -2w - 2r \quad (3-6)$$

On analysing the above equations, it was found that two equations are identical, i.e. Equation 3-4 and 3-6. Therefore, the above equations are simplified to Equations 3-7 and 3-8. On rearranging the terms, the following equations were found:

$$w = -r \quad (3-7)$$

$$z = 1 - x - y + 2w \quad (3-8)$$

Therefore, Equation 3-2 can be re-written as:

$$\delta = C \cdot d^x \cdot b^y \cdot h^{1-x-y+2w} \cdot P_i^w \cdot W^{-w} \quad (3-9)$$

On collecting the like exponential terms to produce the Pi terms, the following equation is developed:

$$\frac{\delta}{h} = C \left( \frac{d}{h} \right)^x \cdot \left( \frac{b}{h} \right)^y \cdot \left( \frac{P_i h^2}{W} \right)^w \quad (3-10)$$

However, the two Pi terms,  $\frac{d}{h}$  and  $\frac{b}{h}$  are constant for a given tyre. Therefore, on multiplying two Pi terms,  $\frac{d}{h}$  and  $\frac{b}{h}$  with third Pi term,  $\frac{P_i h^2}{W}$ , a more simplified equation can be formed, as shown below.

$$\frac{\delta}{h} = C_1 \cdot \left( \frac{P_i b \cdot d}{W} \right)^{C_2} \quad (3-11)$$

where  $C_1$  and  $C_2$  are constants.

### 3.3.2 Tyre contact area

Tyre contact area can be expressed as a function of tyre diameter, tyre width, tyre section width, inflation pressure and normal load for a given test tyre

construction (Komandi, 1990; Sharma and Pandey, 1996 and Diserens, 2009, 2011). Therefore, it can be expressed, as shown below:

$$A = f(d, b, h, P_i, W) \quad (3-12)$$

The contact area model was developed on the hard supporting surface to make the model repeatable and accurate, and it applies to the rolling tyre with minimum compaction on the soil surface (Plackett, 1984). The tyre stiffness is kept independent of tyre age and uniform throughout the tyre (Lines and Murphy, 1991). A technique similar to Plackett (1984), Upadhyaya and Wolfson (1988), Grecenko (1995), Romano et al. (2008) and Ekinici et al. (2011) is followed to measure contact area during the experiments.

According to the Buckingham Pi theorem, four pi terms are needed to develop a non-dimensional model. Therefore, assuming the product formulation, Equation 3-12 can be expressed, as shown below:

$$A = C \cdot d^x \cdot b^y \cdot h^z \cdot P_i^w \cdot W^r \quad (3-13)$$

where C, r, w, x, y, z are non-dimensional constants.

Using Table 3-1, the parameters in Equation 3-13 can be replaced with their fundamental dimensions. Therefore, on substitution, the following equation is developed.

$$L^2 = L^x \cdot L^y \cdot L^z \cdot [ML^{-1}T^{-2}]^w \cdot [MLT^{-2}]^r \quad (3-14)$$

On equating L.H.S and R.H.S's fundamental dimensions, the following equations were obtained.

$$0 = w + r \quad (3-15)$$

$$2 = x + y + z - w + r \quad (3-16)$$

$$0 = -2w - 2r \quad (3-17)$$

On analysing the above equations, it was found that two equations are identical, i.e. Equation 3-15 and 3-17. Therefore, the above equations are simplified to

Equations 3-18 and 3-19. On rearranging the terms, the following equations are found:

$$w = -r \quad (3-18)$$

$$z = 2 - x - y + 2w \quad (3-19)$$

Therefore, equation 3-13 can be written as:

$$A = C \cdot d^x \cdot b^y \cdot h^{2-x-y+2w} \cdot P_i^w \cdot W^{-w} \quad (3-20)$$

On collecting the like exponential terms to produce the Pi terms, the following equation is developed:

$$\frac{A}{h^2} = C \left(\frac{d}{h}\right)^x \cdot \left(\frac{b}{h}\right)^y \cdot \left(\frac{P_i h^2}{W}\right)^w \quad (3-21)$$

However, the two Pi terms,  $\frac{d}{h}$  and  $\frac{b}{h}$  are constant for a given tyre. Therefore, on multiplying two Pi terms,  $\frac{d}{h}$  and  $\frac{b}{h}$  with a third Pi term,  $\frac{P_i h^2}{W}$  a more simplified equation can be formed, as shown below.

$$\frac{A}{h^2} = C_3 \cdot \left(\frac{P_i b \cdot d}{W}\right)^{C_4} \quad (3-22)$$

Where  $C_3$  and  $C_4$  are constants.

On analysing Equations 3-11 and 3-22, it was found that the tyre deflection ratio and contact area ratio depends on the same Pi term i.e.  $\frac{P_i b \cdot d}{W}$ . Therefore, it can state that the contact area ratio is a deflection ratio function. Therefore, an empirical relation can be formed, as shown below:

$$\frac{A}{h^2} = C_5 \cdot \left(\frac{\delta}{h}\right)^{C_6} \quad (3-23)$$

### 3.3.3 Mobility performance parameters

Tyre – soil interaction can be expressed as a function of nine independent variables as shown in Table 3-2 and three dependent variables as shown in Table

3-3 (Bekker, 1958; Freitag, 1966 and Swanson, 1973). However, the effect of the tyre's velocity is neglected by keeping it the same for both the full-scale and scale model testing (Freitag, 1966). Therefore, wheel torque can be expressed as a function of pertinent parameters shown in Tables 3-2 and 3-3.

$$T = f(d, b, r, \delta, h, W, S, MR, D, CI) \quad (3-24)$$

**Table 3-2 Tyre – soil model-independent parameters**

| Parameter        | Symbol   | Dimension       |
|------------------|----------|-----------------|
| <b>Tyre</b>      |          |                 |
| Overall diameter | D        | L               |
| Section width    | B        | L               |
| Section height   | H        | L               |
| Rolling radius   | R        | L               |
| Deflection       | $\delta$ | L               |
| <b>Soil</b>      |          |                 |
| Cone index       | CI       | $ML^{-1}T^{-2}$ |
| <b>System</b>    |          |                 |
| Normal load      | W        | $MLT^{-2}$      |
| Slip             | S        | -               |
| Velocity         | V        | $LT^{-1}$       |

**Table 3-3 Tyre – soil model-dependent parameters**

| Parameter         | Symbol | Dimension    |
|-------------------|--------|--------------|
| Torque            | T      | $ML^2T^{-2}$ |
| Motion resistance | MR     | $MLT^{-2}$   |
| Drawbar pull      | D      | $MLT^{-2}$   |

According to the Buckingham Pi theorem, nine pi terms need to develop a non-dimensional model. Therefore, assuming the product formulation, Equation 3-24 can be expressed, as shown below:

$$T = C \cdot d^x \cdot b^y \cdot r^z \cdot \delta^w \cdot h^r \cdot W^\alpha \cdot S^\beta \cdot MR^\gamma \cdot D^\theta \cdot CI^\varphi \quad (3-25)$$

where C is a non-dimensional constant.

Using Table 3-2 and 3-3, the parameters in Equation 3-25 can be replaced with their fundamental dimensions. Therefore, on substitution, the following equation is developed.

$$ML^2T^{-2} = C \cdot L^x \cdot L^y \cdot L^z \cdot L^w \cdot L^r \cdot [MLT^{-2}]^\alpha \cdot [MLT^{-2}]^\beta \cdot [MLT^{-2}]^\gamma [ML^{-1}T^{-2}]^\varphi \quad (3-26)$$

On equating L.H.S and R.H.S's fundamental dimensions, the following equations are obtained.

$$1 = \alpha + \gamma + \theta + \varphi \quad (3-27)$$

$$2 = x + y + z + w + r + \alpha + \gamma + \theta - \varphi \quad (3-28)$$

$$-2 = -2\alpha - 2\gamma - 2\theta - 2\varphi \quad (3-29)$$

On analysis of the above equations, it was found that two equations are identical, i.e. Equation 3-27 and 3-29. Therefore, the above equations are simplified to Equations 3-30 and 3-31. On rearranging the terms, the following equations are found:

$$z = 1 - x - y - w - r + 2\varphi \quad (3-30)$$

$$\alpha = 1 - \gamma - \theta - \varphi \quad (3-31)$$

Therefore, Equation 3-25 can be written as:

$$T = C \cdot d^x \cdot b^y \cdot r^{1-x-y-w-r+2\varphi} \cdot \delta^w \cdot h^r \cdot W^{1-\gamma-\theta-\varphi} \cdot S^\beta \cdot MR^\gamma \cdot D^\theta \cdot CI^\varphi \quad (3-32)$$

Associating like exponential terms to produce the Pi terms, the above equation is simplified:

$$\frac{T}{rW} = C \left(\frac{d}{r}\right)^x \cdot \left(\frac{b}{r}\right)^y \cdot \left(\frac{\delta}{r}\right)^w \cdot \left(\frac{h}{r}\right)^r \cdot (S)^\beta \cdot \left(\frac{MR}{W}\right)^\gamma \cdot \left(\frac{D}{W}\right)^\theta \cdot \left(\frac{CI \cdot r^2}{W}\right)^\varphi \quad (3-33)$$

The above equation can be written in its functional Pi form.

$$\frac{T}{rW} = \left\{ \left(\frac{d}{r}\right), \left(\frac{b}{r}\right), \left(\frac{\delta}{r}\right), \left(\frac{h}{r}\right), (S), \left(\frac{MR}{W}\right), \left(\frac{D}{W}\right), \left(\frac{CI \cdot r^2}{W}\right) \right\} \quad (3-34)$$

The last Pi term  $\left(\frac{CI \cdot r^2}{W}\right)$  can be multiplied with  $\left(\frac{d}{r}\right)$  and  $\left(\frac{b}{r}\right)$  Pi terms to form more convenient Pi term and later Pi terms are constant for any specified tyre at constant load. Therefore, a more appropriate Pi term can be derived i.e.  $\left(\frac{CI \cdot b \cdot d}{W}\right)$  which helps to understand the relationship between tyre soil interactions. Later, Equation 3-34 can be simplified in more appropriate Pi terms, as shown below (Freitag (1966), Wismer and Luth (1973), Brixius (1987)).

$$\frac{T}{rW} = \left\{ \left(\frac{b}{d}\right), \left(\frac{r}{d}\right), \left(\frac{\delta}{h}\right), \left(\frac{h}{d}\right), (S), \left(\frac{MR}{W}\right), \left(\frac{D}{W}\right), \left(\frac{CI \cdot b \cdot d}{W}\right) \right\} \quad (3-35)$$

However, the relationship between dependent parameters is as shown below.

$$\frac{MR}{W} = \frac{T}{rW} - \frac{D}{W} \quad (3-36)$$

The  $\left(\frac{r}{d}\right)$  ratio is constant for most of the tyres, and thus, this term can be neglected in the tyre – soil interaction analysis. The relationship is shown in Equation 3-37 between tyre parameters also holds.

$$\frac{h}{d} = \frac{1 - \frac{2r}{d}}{\frac{\delta}{h}} \quad (3-37)$$

Therefore, the dependent parameters can be re-written, as shown below.

$$\frac{T}{rW} = f\left(\frac{b}{d}, \frac{\delta}{h}, S, \frac{CI \cdot bd}{W}\right) \quad (3-38)$$

$$\frac{D}{W} = g\left(\frac{b}{d}, \frac{\delta}{h}, S, \frac{CI \cdot bd}{W}\right) \quad (3-39)$$

where  $f$  and  $g$  are two different functions.

All four Pi terms should be the same to govern the similar tyre-soil interaction for both the full-scale and scaled tyres. The Pi terms  $b/d$  and  $\delta/h$  were used as principal independent tyre variables. These parameters kept the same during selecting scaled tyres to full-scale tyre. In the operational load and inflation pressure range of scale tyres, the stiffness of full-scale tyres (7.5R16) and scale tyres (135/80R13 and 145/80R13) is approximately the same in the above range of tyres specification. In this analysis, the dependent parameters are evaluated as a slip function. Therefore, the slip Pi term is not considered a variable. The last Pi term  $\frac{CI \cdot bd}{W}$  contained the significant soil parameter  $CI$  and the load  $W$ . Both these parameters are independent variables and involves the concept of force. Therefore, for a constant tyre geometry, only changes in the cone index  $CI$  and load  $W$  can cause changes in the dependent variables. Also, a change in one of the two independent parameters can be compensated by the change in the other. Therefore, if the cone index is kept the same to develop the same soil condition for the full-scale and scaled tyres, an equivalent loading condition needs to be developed to govern similar tyre-soil interaction. So, all the variations of soil parameter  $CI$  and load  $W$  can be calculated from the last Pi term. Freitag (1966) also developed a similar term called the clay-loading number  $\frac{CI \cdot d^2}{W}$  to compare the dynamic behaviour of tyre– soil interaction of the full-scale and scaled tyres.

### **3.4 Experimental plan**

#### **3.4.1 Tyre selection**

In the present study, the tyres are categorized as full-scaled and scaled tyres. Full-scaled tyres are military-scale tyres used in different Armoured Personnel carriers (APCs)/ Armoured Fighting Vehicles (AFVs). The full-scale tyre selected in this chapter is 7.5R16, commonly used in Land Rover vehicles (upper boundary of application). In the tyre market, the nearest scale tyres found are 145/80R13 and 135/80R13, which are used to test the scalability theory. The scale tyres are similar to Husky and Warthog (24" Argo tyre) UGVs (lower boundary of application). These tyres have the same radial-ply construction and approximately the same tyre width/diameter ratio of 0.25. It was assumed that the tyres are made of the same rubber compound and are of the same age. Therefore, the scaled tyres were purchased from the same supplier.

Tyre lugs are approximated based on conventional Terramechanics theory. It assumed that the tyres with lugs could be replaced with a tyre with an extended radius (Ding et al., 2014). Therefore, in the present study, the tyre diameter includes the lug height by default. The tyre specification summary is given in Table 3-4.

The selected tyre sizes with their specified rims are as follows.

Full-scaled tyre: 1) 7.5 R 16 tyre mounted on rim size, W – 16

Scaled tyre: 1) 145/80 R 13 tyre mounted on rim size, W – 13

2) 135/80 R 13 tyre mounted on rim size, W – 13



**Table 3-4 Tyre specifications used in the present study**

| <b>Parameters</b> | <b>Ply</b> | <b>Diameter,<br/>d, mm</b> | <b>Width,<br/>b, mm</b> | <b>Section<br/>height, h,<br/>mm</b> | <b>Rim<br/>size</b> | <b>Width/<br/>Diameter<br/>(b/d)</b> |
|-------------------|------------|----------------------------|-------------------------|--------------------------------------|---------------------|--------------------------------------|
| 7.5R16            | Radial     | 787                        | 190                     | 708                                  | W-16                | 0.24                                 |
| 145/80R13         | Radial     | 560                        | 145                     | 116                                  | W-13                | 0.25                                 |
| 135/80R13         | Radial     | 540                        | 135                     | 108                                  | W-13                | 0.25                                 |

### **3.4.2 Research plan**

This study aims to achieve the tyre deflection and contact area characteristics of radial-ply tyres at different inflation pressures and normal loads. Three different radial tyres were tested at five inflation pressures and six normal loads on a hard surface to accomplish the objective. A hard surface provides a repeatable test condition, which can be reproduced at other locations. The detailed test plan is shown in Table 3-5.

**Table 3-5 Research plan**

| Independent parameters                  |   |                                                                                                                                                                       |      |
|-----------------------------------------|---|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| Tyre                                    | 3 | 1) 7.5 R16<br><br>2) 135/80 R 13<br><br>3) 145/80 R13                                                                                                                 |      |
| Inflation pressure, kPa (or psi)        | 5 | 1) 448 (65), 379 (55), 310 (45), 241 (35), 172 (25)<br><br>2) 310 (45), 276 (40), 241(35), 207 (30),172 (25)<br><br>3) 310 (45), 276 (40), 241(35), 207 (30),172 (25) |      |
| Normal load                             | 6 | 1, 2, 3, 4, 5, 5.45 (in kN)                                                                                                                                           | - T1 |
|                                         | 6 | 0.5, 1.5, 2.5, 3, 3.5, 4 (in kN)                                                                                                                                      | - T2 |
|                                         | 6 | 0.5,1.5, 2.5, 3, 3.5, 4 (in kN)                                                                                                                                       | - T3 |
| Supporting surface                      | 1 | Solid plate (Hard surface)                                                                                                                                            |      |
| Replication                             | 4 |                                                                                                                                                                       |      |
| Dependent parameters:                   |   |                                                                                                                                                                       |      |
| 1) Vertical tyre deflection, mm         |   |                                                                                                                                                                       |      |
| 2) Ground contact area, mm <sup>2</sup> |   |                                                                                                                                                                       |      |

### 3.4.3 Experimental setup

The experimental set-up consists of tyre testing carriage and instrumentations. The tyre testing carriage could accommodate different sizes of tyres. It can also provide free vertical movement to the test tyre under static positions, which helped transfer the normal load solely on the wheel.

The experimental setup is shown in Figure 3-1. It consists of loading wheels, loading arm, load actuator, hard surface (or base plate), drive chains, dial gauge, load cell and test tyres. The procedure is as follows. The input load is transferred to set-up by rotating the outer loading wheel manually. It then rotates the inner

loading wheel, pulling the loading arm downwards through the actuator (Figure 3-2). It results in the vertically downward force and vertical displacement of the test tyre. The limitation of the setup is that it can transfer a maximum load of 5.45 kN before the load actuator begins to twist. This setup allows testing tyres up to 230 mm wide contact patch.

The step-by-step procedure is as follows:

Step 1: Rotate the outer loading wheel manually.

Step 2: The outer loading wheel rotates the inner loading wheel.

Step 3: The inner loading wheel applies tension on the load cell.

Step 4: The inner loading wheel pulls the loading arm downwards.

The tyre's vertical displacement was measured using a dial gauge (Figure 3-3a), and data was recorded manually. The long-stroke dial gauge had a standard range of 50 mm with an accuracy of 0.01 mm. The dial gauge is fixed on the tyre testing carriage frame, and its contact point touches the tyre loading arm's frame. The dial gauge reading was zero at zero displacements. This measures the vertical displacement as the loading arm applies a vertical load on the tyre and compresses.

The tyre's vertical load was measured using a cylindrical load cell, and data was displayed in the Data Acquisition System (DAS). The cylindrical load cell had a standard range of 10 kN with nominal sensitivity of 2.0mV/V. It was connected to the input power supply of 10 V. The load cell was installed in a vertical position between the loading wheel and the loading arm (Figure 3-3b). Therefore, when the load was applied through-loading wheel, the load cell experiences a tension force equal to the applied load. The detailed specifications of the load cell are given in Appendix – A.

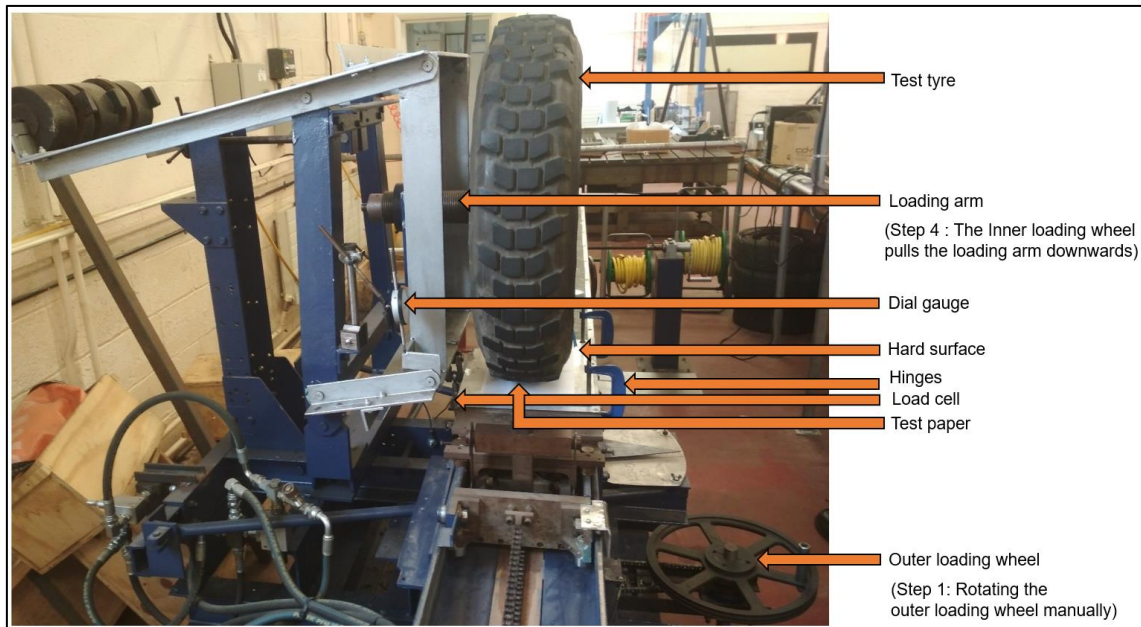


**Figure 3-1 Side view of the experimental setup**

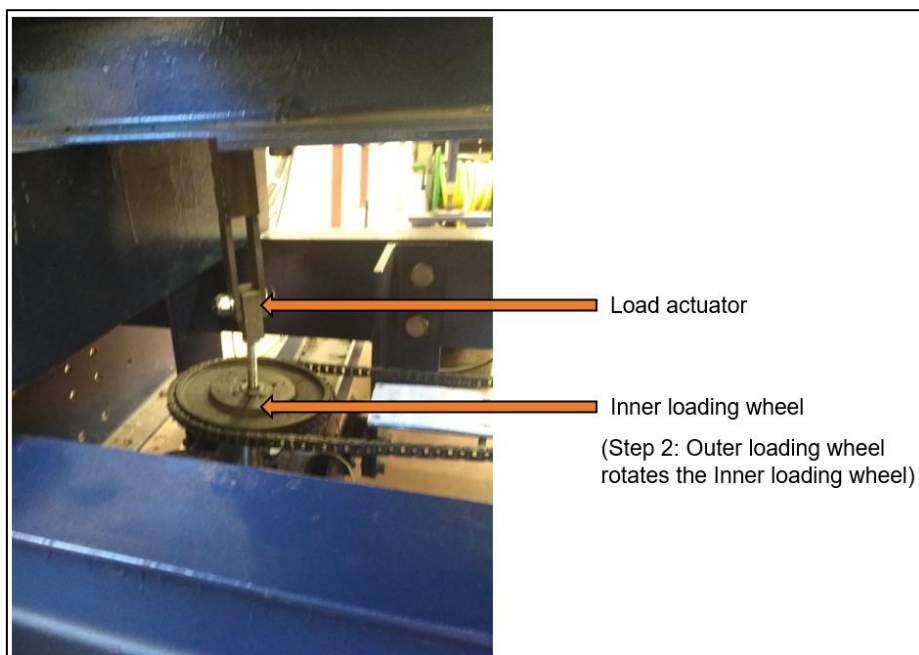


**Figure 3-2 Bottom view of the experimental setup**

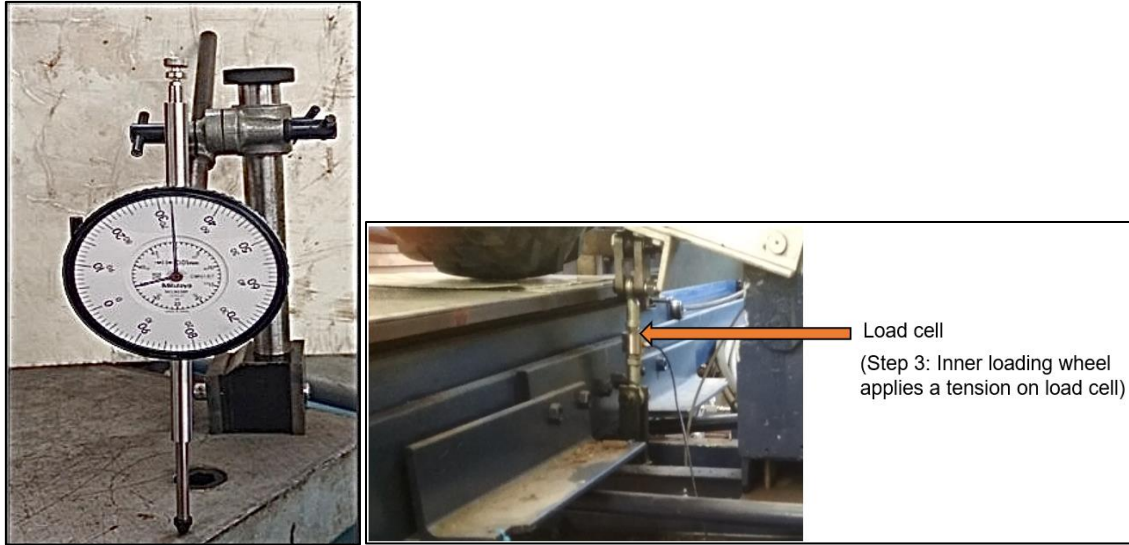


**Figure 3-3 a) Dial gauge; b) Load cell positioning**

The load cell was calibrated before conducting the tests (section 3.4.5). First, DAS's initial reading was recorded when the test tyre touches the hard surface. Then the dial gauge was set to zero on the loading arm frame. When the load was applied, the dial gauge displayed the vertical displacement, which was equal to the tyre deflection. The difference between the initial and final reading of DAS is the tyre's load.

### **3.4.4 Test procedure**

Tyre deflection and contact area were measured using the overlay technique to get consistent results (Plackett (1984)). The test procedure is as follows (Figure 3.4). Firstly, the expected tyre contact patch was inked blue to take the contact patch's imprints on a hard surface. The centre of the expected contact patch was marked using a marking strip. A base sheet was clamped on the hard surface to protect the experimental white sheet from damage due to excessive force. The white sheet was fixed on the base sheet using a tape roll.

The test tyre with the given inflation pressure was slowly displaced down and allowed to rest on the white sheet while keeping the marking strip vertical to the hard surface. The load cell reading was recorded at zero tyre deflection. Further, the tyre was loaded to the desired vertical load with a controlled outer loading wheel manually. The wheel's load was recorded using DAS. The dial gauge

output was recorded manually for deflection measurement. The procedure obtained an excellent imprint of the tyre contact patch on the white sheet (Figure 3.5). The contact area imprint was measured using similar techniques followed by Komandi (1976), Upadhyaya et al. (1988), Saarilahti (2002) and Keller (2005).

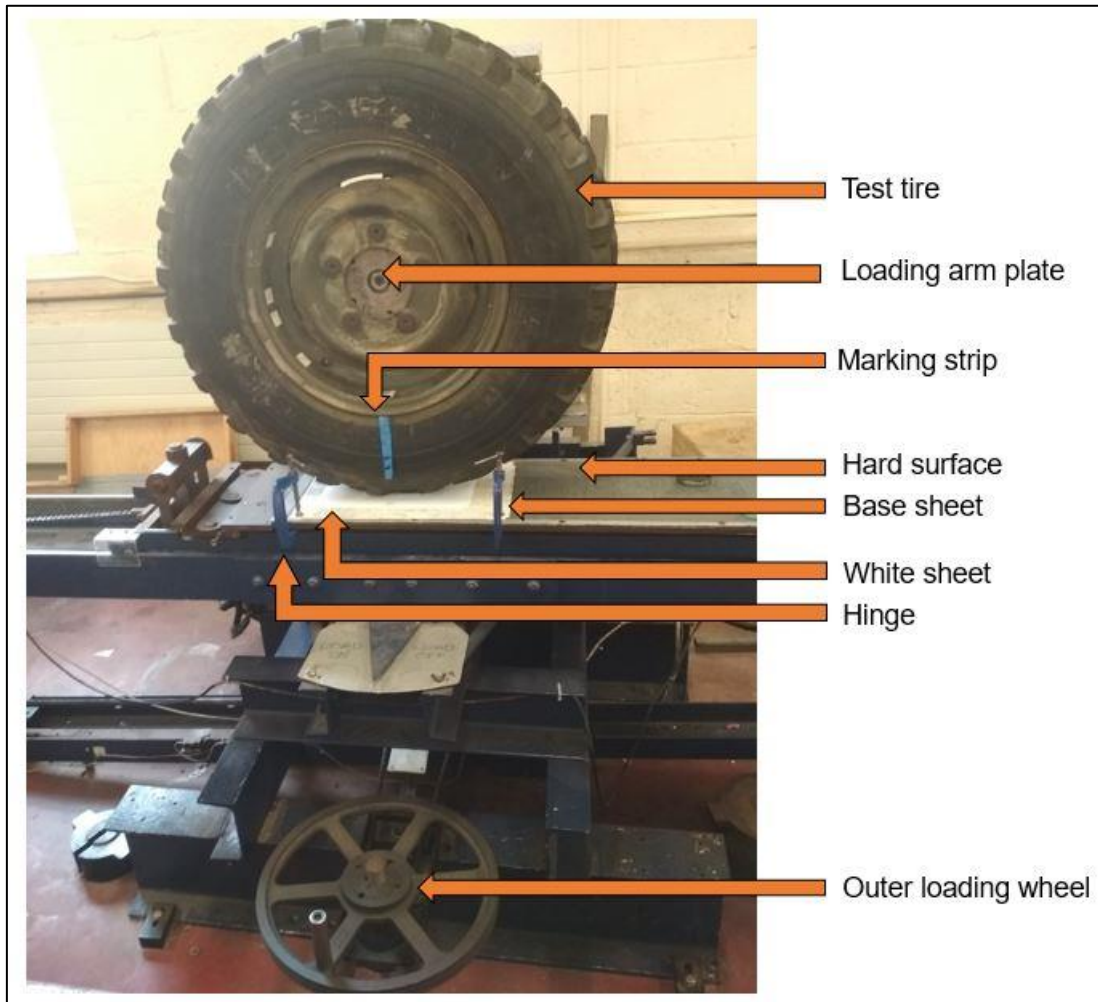


**Figure 3-4 Front view of the experimental setup**

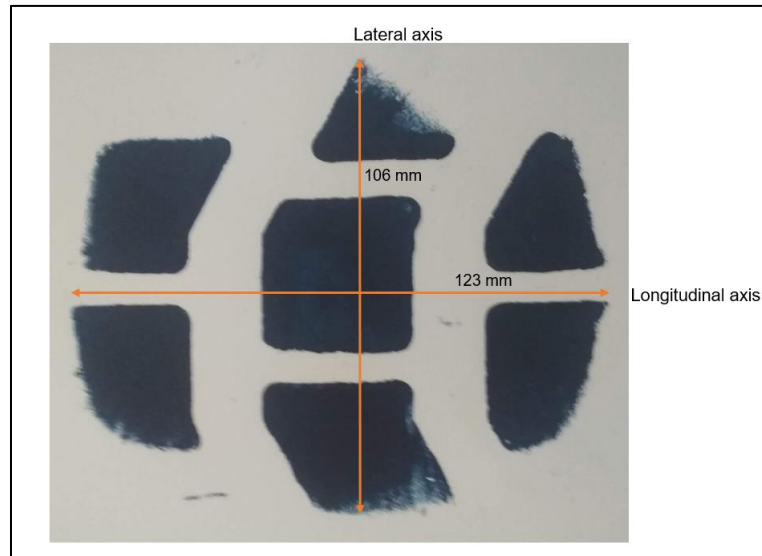


**Figure 3-5 Contact patch of 7.5R16 tyre at 310 kPa inflation pressure and 2 kN normal load**

#### **3.4.5 Calibration of sensor**

The load cell was calibrated to measure applied normal load in deflection and contact area measurement. The load cell selected for these tests is the tension load cell. The calibration method is as follows. One end of the load cell was fixed using a rod and hinge. The known weights were put at their free end, as shown in Figure 3-5. The load cell reading was recorded with different known weights using Data Acquisition System (DAS). The calibration curve (Figure 3-6) shows a linear relationship between input weights and load cell reading.



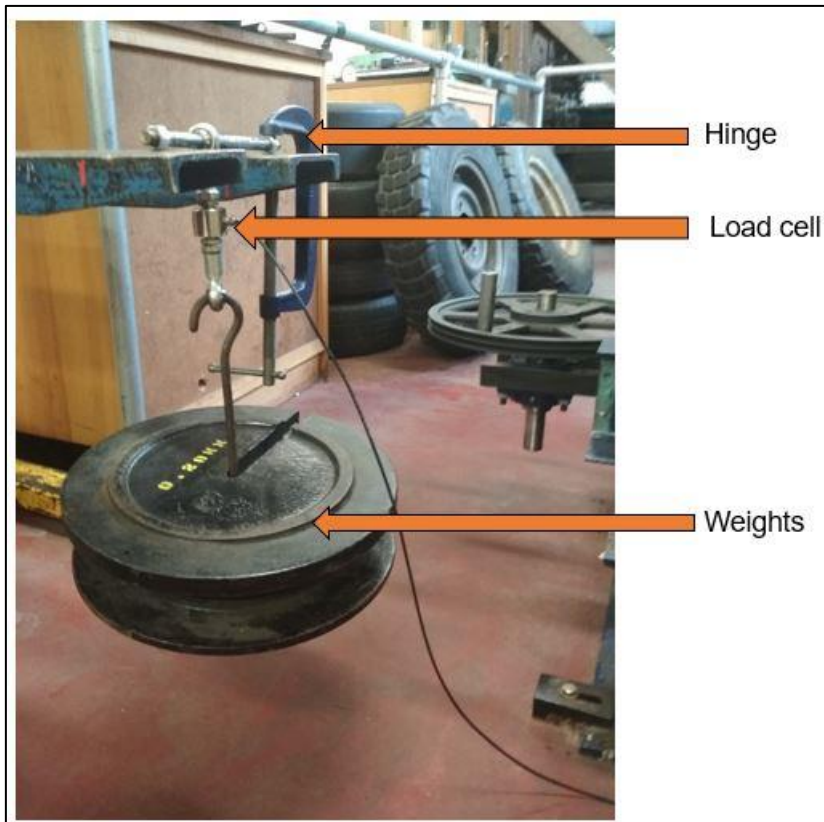


**Figure 3-6 Calibration setup of load cell**

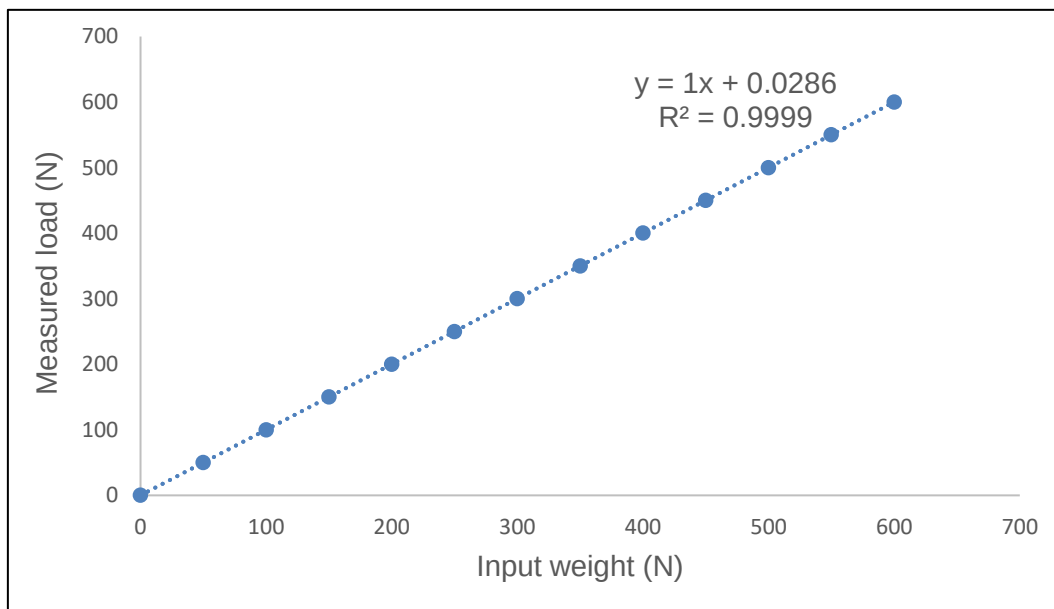


**Figure 3-7 Calibration curve of load cell for normal load measurement**



## **3.5 Results and discussion**

### **3.5.1 Full-scale tyre**

#### **3.5.1.1 Deflection characteristics**

The tyre deflection test was conducted on the full-scale tyre (7.5R16) at different normal loads and inflation pressure. The normal load varied from 1 kN to 5.45 kN. The maximum normal load which the test set up can apply was 5.45 kN. After the maximum load, the test setup's load actuator started to twist. The maximum load carrying capacity of 7.5R16 tyre on mud and sand is 7.84 kN (Michelin off-road tyre handbook X-Force (2016-2017)) and the maximum recommended inflation pressure is 450 kPa. There is a maximum load carrying capacity of 4.6kN and 4 kN for the scaled tyres selected for the study 145/80R13 and 135/80R13 respectively, and 310 kPa (45 psi) is the maximum recommended inflation pressure for both tyres. Therefore, the selected load range of the full-scale tyre covers the entire band of the load range of scale tyres.

Further, the inflation pressure varied from 172 kPa to 448 kPa. This range of inflation pressure includes the normal operational range of full-scale tyre under different soil conditions. The tests were repeated four times at each normal load and inflation pressure to keep the standard error lower than 10% (NATO CDT – 308 (2020)).

Figure 3-8 shows the relationship between normal load and tyre deflection of the full-scale tyre at different inflation pressures. Figure 3-9 shows the relationship between inflation pressure and tyre deflection of the full-scale tyre under different loading conditions. The general trend shows that the tyre deflection increases linearly with the increase of normal load and decreases non-linearly with the increase in inflation pressure. Kumar et al. (2018), Rashidi et al. (2013), Sharma and Pandey (1996) and Plackett (1983) obtained a similar trend.

It is observed that the slope of deflection with respect to the normal load curve increases with the decrease in inflation pressure. It is higher for lower inflation pressure. It is because carcass stiffness is non-linear and changes with the inflation pressure (Kumar et al., 2018; Lines and Murphy, 1991; Karafaith and

Nawatzki, 1978). It was observed that carcass stiffness reduces with inflation pressure.

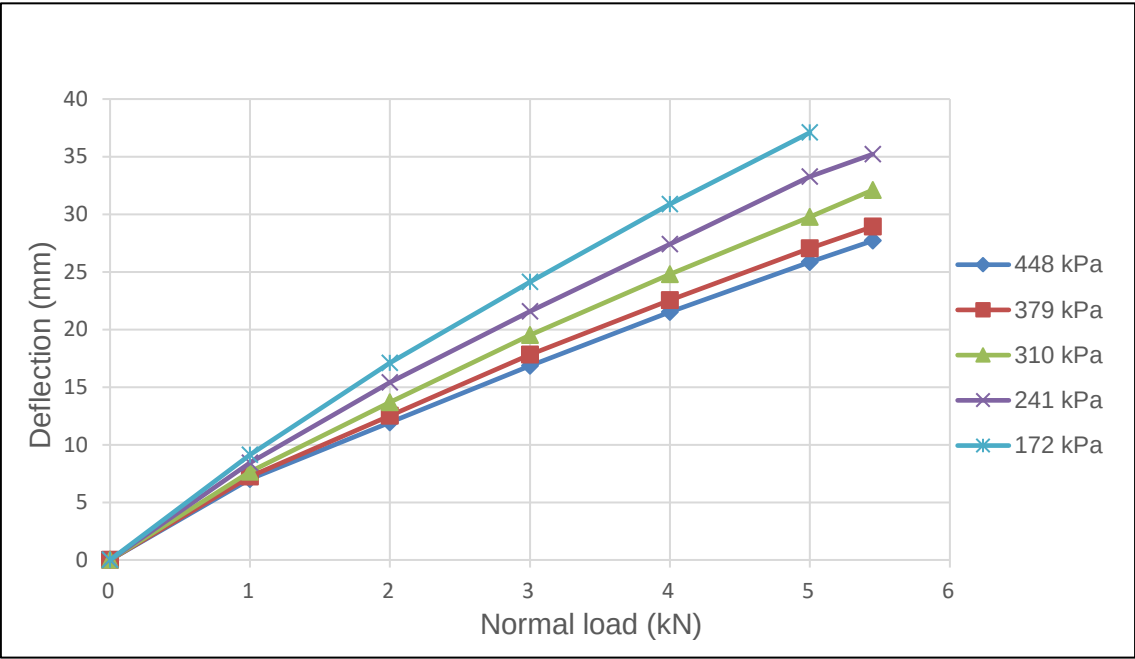


**Figure 3-8 Relationship between normal load and tyre deflection at different inflation pressures of the full-scale tyre**

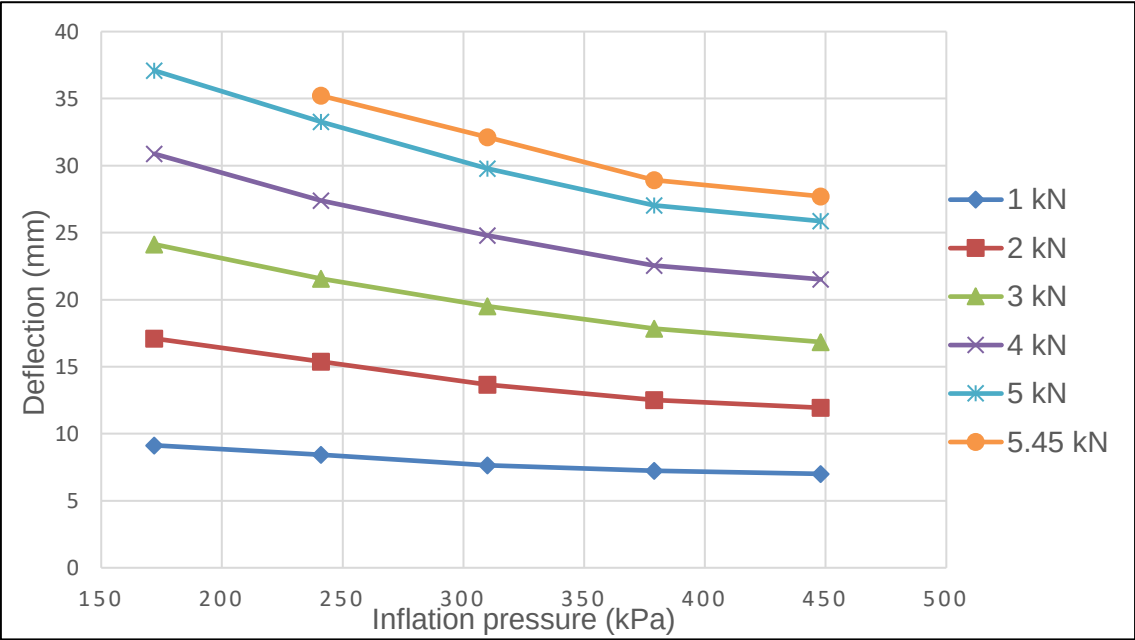


**Figure 3-9 Relationship between inflation pressure and tyre deflection at different normal loads of full-scale tyre**

### 3.5.1.2 Contact area characteristics

The contact area test was conducted on the full-scale tyre at different normal loads and different inflation pressures. The normal loading conditions varied from 1 kN to 5.45 kN, and inflation pressure varied from 172 kPa to 448 kPa. The tests were repeated four times at each inflation pressure and normal loading conditions to keep the standard error less than 10% (NATO CDT – 308 (2020)).

Figure 3-10 shows the relationship between inflation pressure and contact area under different loading conditions. The general trend is that the tyre contact area decreases non-linearly with increased inflation pressure (Sharma and Pandey, 1996). It may be because of a non-linear increase in tyre stiffness with the increase in inflation pressure, which non-linearly decreases the contact area.

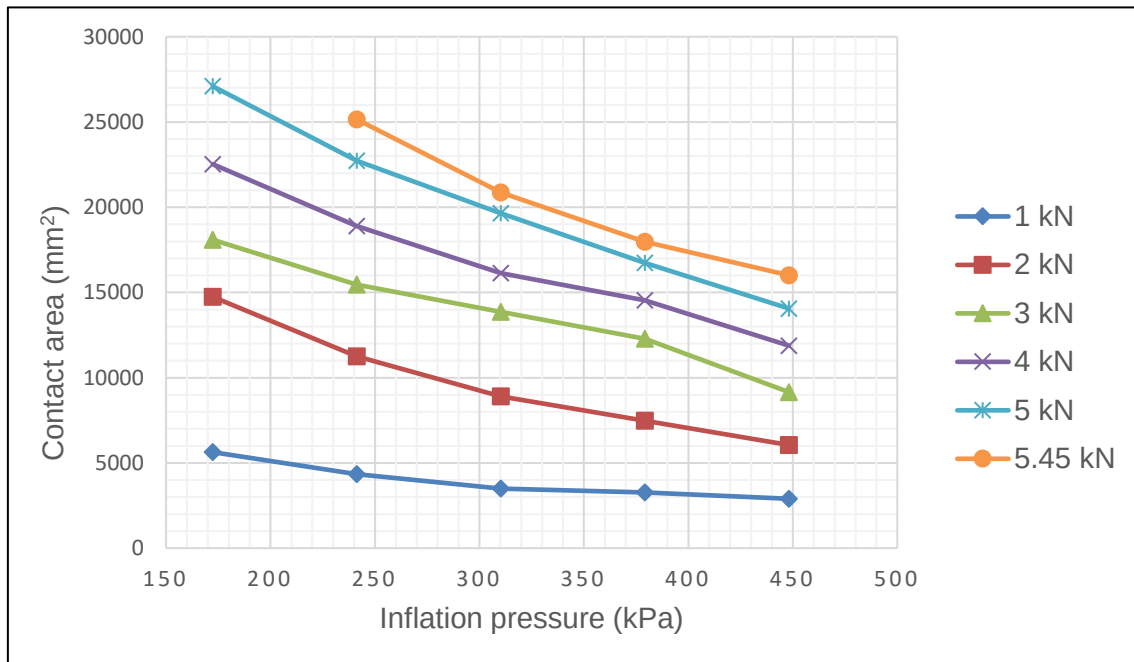


**Figure 3-10 Relationship between inflation pressure and tyre contact area at different normal load conditions**

### 3.5.1.3 Deflection model development

The deflection of tyres depends on their inflation pressure, normal load and tyre width/diameter (b/d) ratio, tyre construction (radial-ply) and tyre material. The experimental data were analysed to develop a tyre deflection model based on dimensional analysis (section 3.3.1) for the full-scale tyre at different normal loads

and inflation pressures. The experimental data of tyre deflection of the full-scale tyre was fitted into this model, and the value of constants ( $C_1$  and  $C_2$ ) were determined. The generalised deflection model is shown below.

$$\frac{\delta}{h} = C_1 \times \left( \frac{P_i(b \times d)}{W} \right)^{C_2} \quad (3-40)$$

Where,  $(\delta/h)$  = deflection ratio,

$P_i$  = Inflation pressure, kPa,

$W$  = Normal load, kN,

$b$  = tyre width, m

$d$  = tyre diameter, m

$C_1$  and  $C_2$  = constants

This model, Equation 3-40, can be used to predict the tyre deflection in terms of normal load and inflation pressure of full-scale and scale tyres for the tyre width/diameter ratio ( $b/d$ ) of 0.24 and 0.25 (Table 3.4). The model is valid in this range as the  $b/d$  ratio of the full-scale tyre is 0.25 and the scale tyre is 0.24. The non-linear least square method with the Trust-Region algorithm in MATLAB was used to curve fit the above model with experimental data. The obtained model with a statistical summary is shown below.

$$\frac{\delta}{h} = 0.7126 \times \left( \frac{P_i(b \times d)}{W} \right)^{-0.6516} \quad (3-41)$$

**Table 3-6 Statistical summary of the obtained deflection model for the full-scale tyre**

| Statistical parameter        | Value    |                          |         |
|------------------------------|----------|--------------------------|---------|
| Sum of squared error, SSE    | 0.007957 |                          |         |
| R-square                     | 0.9053   |                          |         |
| Adjusted R-square            | 0.9018   |                          |         |
| Root mean square error, RMSE | 0.01717  |                          |         |
| Constants                    | Estimate | 95 % confidence interval |         |
|                              |          | Lower                    | Upper   |
| C <sub>1</sub>               | 0.7126   | 0.5488                   | 0.8764  |
| C <sub>2</sub>               | -0.6516  | -0.7484                  | -0.5548 |

#### 3.5.1.4 Contact area model development

The tyre's contact area depends on its inflation pressure, normal load and tyre width/diameter (b/d) ratio, tyre construction (radial ply) and tyre material. The experimental data were analysed to develop a contact area model based on a dimensional analysis approach (section 3.3.2) for the full-scale tyre at different normal loads and inflation pressures. The experimental data of the full-scale tyre's contact area were fitted into this model, and the value of constants (C<sub>3</sub> and C<sub>4</sub>) were determined. The generalised contact area model is shown below.

$$\frac{A}{h^2} = C_3 \cdot \left( \frac{P_i b \cdot d}{W} \right)^{C_4} \quad (3-42)$$

where (A/h<sup>2</sup>) = contact area ratio,

P<sub>i</sub> = Inflation pressure, kPa,

W = Normal load, kN,

b = tyre width, m

d = tyre diameter, m

h = section height, m

C<sub>3</sub> and C<sub>4</sub> = constants.

This model, Equation 3-42, can be used to predict the contact area in terms of normal load and inflation pressure of full-scale tyre for the tyre width/diameter ratio (b/d) of 0.24 and 0.25 (Table 3.4). The model is valid in this range as the b/d ratio of the full-scale tyre is 0.25 and the scale tyre is 0.24. The non-linear least square method with the Trust-Region algorithm in MATLAB was used to curve fit the above model with experimental data. The obtained model with the statistical summary is shown below.

$$\frac{A}{h^2} = 3.841 \left( \frac{P_i (b \times d)}{W} \right)^{-0.7747} \quad (3-43)$$

**Table 3-7 Statistical summary of the contact area model for the full-scale tyre**

| Statistical parameter        | Value    |                          |         |
|------------------------------|----------|--------------------------|---------|
| Sum of squared error, SSE    | 0.0587   |                          |         |
| R-square                     | 0.9659   |                          |         |
| Adjusted R-square            | 0.9647   |                          |         |
| Root mean square error, RMSE | 0.0466   |                          |         |
| Constants                    | Estimate | 95 % confidence interval |         |
|                              |          | Lower                    | Upper   |
| C <sub>1</sub>               | 3.8410   | 3.231                    | 4.449   |
| C <sub>2</sub>               | -0.7747  | -0.8431                  | -0.7062 |

### 3.5.1.5 Validation and comparison of the developed model

The developed deflection ratio model and contact area model were validated using the experimental data (shown in Appendix B.3), which were not included in developing the full-scale tyre model. The experiments were performed at different inflation pressures (172 kPa to 448 kPa) and different deflection ratios (0.03 to 0.12). The normal loads and contact areas (in mm<sup>2</sup>) were measured. The experimental and predicted deflection ratios and contact area ratios were plotted against the developed non-dimensional term, respectively, as shown in Figures 3-11 and 3-12. It is found that the models predict the deflection ratio and contact area ratio in the acceptable error band of  $\pm 10\%$  (NATO CDT – 308 (2020)). The coefficient of determination for the non-dimensional models was found acceptable for both models i.e. 0.93 and 0.96, respectively. Therefore, it was recommended to predict the deflection ratio and contact area ratio for full-scaled and scaled tyres using the non-dimensional parameter. The statistical analysis for the validation and comparison of the developed model is shown in Table 3-8.

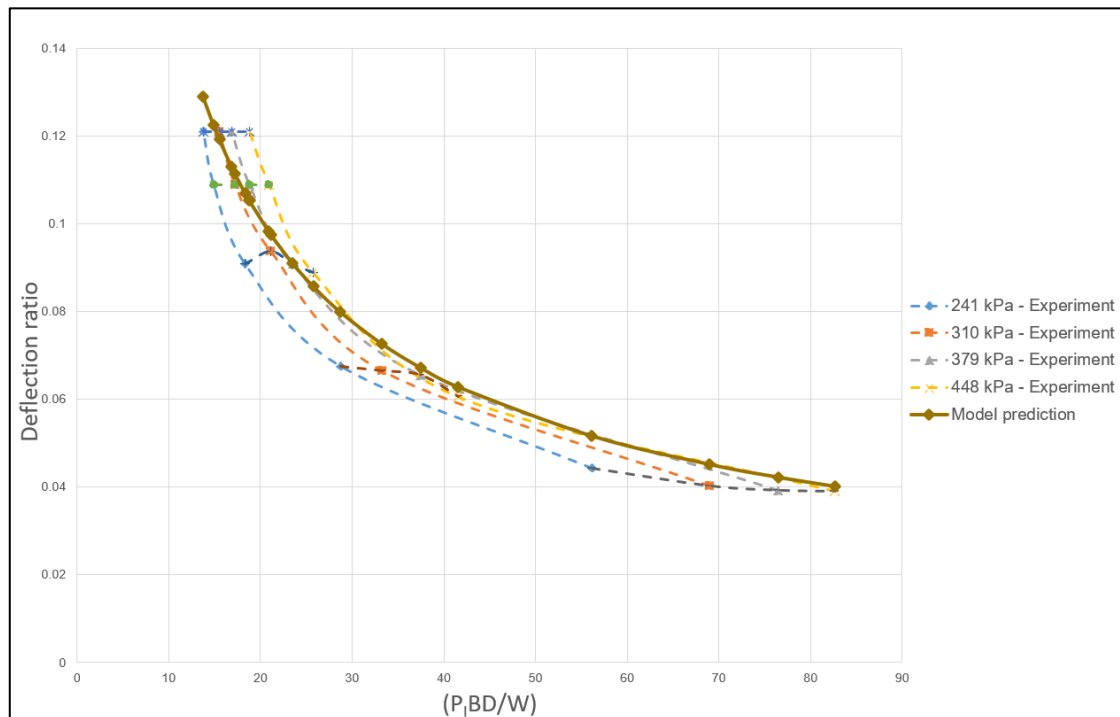


**Figure 3-11 Comparison of developed deflection ratio model based on non-dimensional parameter with the test data**

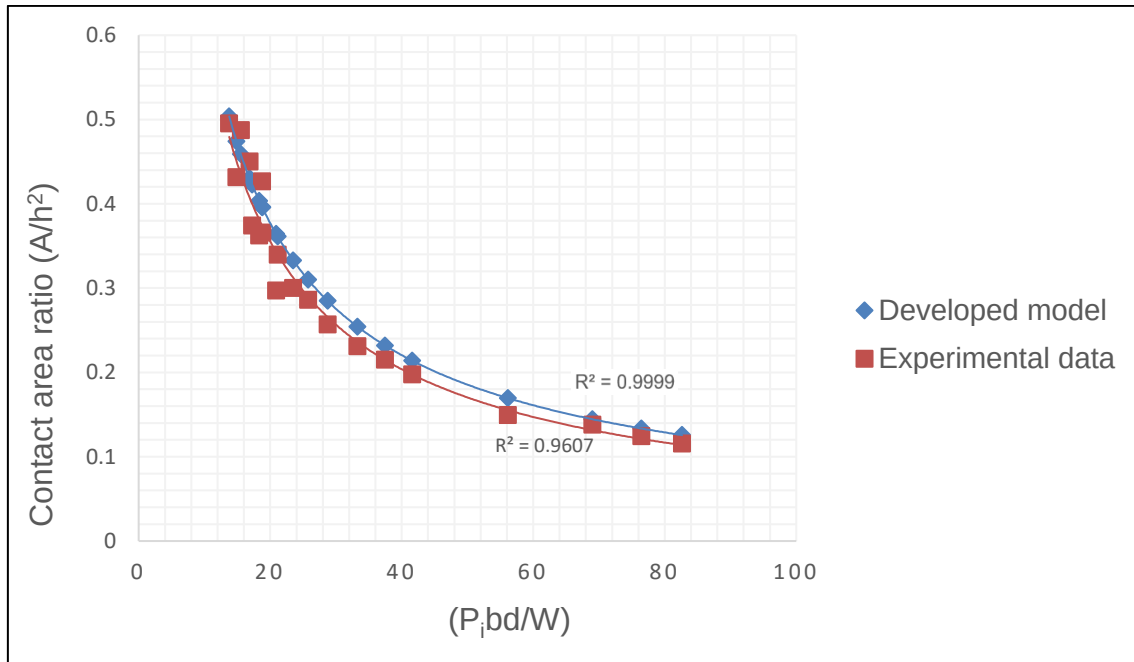


**Figure 3-12 Comparison of developed contact area ratio model based on non-dimensional parameter with the test data**

**Table 3-8 Statistical summary of the developed models for the full-scale tyre**

| Parameters             | Mean error | Standard deviation, % | RMSE, % | SSE, % | R <sup>2</sup> |
|------------------------|------------|-----------------------|---------|--------|----------------|
| Deflection ratio model | 0.0017     | 0.7384                | 0.7424  | 0.1268 | 0.9314         |
| Contact area model     | 0.0159     | 2.3535                | 2.8021  | 1.8059 | 0.96           |

### 3.5.2 Scaled tyres

#### 3.5.2.1 Deflection characteristics

The tyre deflection tests were conducted on the scaled tyres 145/80R13 and 135/80R13 at different normal loads and inflation pressures. Both tyres have a load index of 4 kN and maximum inflation pressure of 310 kPa (or 45 Psi). The scaled tyre's normal loads varied from 0.5 kN to 4 kN. The scaled tyre's inflation pressures varied from 172 kPa (or 25 Psi) to 310 kPa (or 45 Psi). The



experimental results obtained for tyre deflections at different inflation pressures and normal loads are as follows. Figure 3-13 and 3-14 shows the relationships between tyre deflection, normal load and inflation pressure for 145/80R13 tyre. Figure 3-15 and 3-16 shows the relationships between tyre deflection, normal load and inflation pressure for 135/80R13 tyre. The interpretation of the results is the same as in section 3.5.1.1.

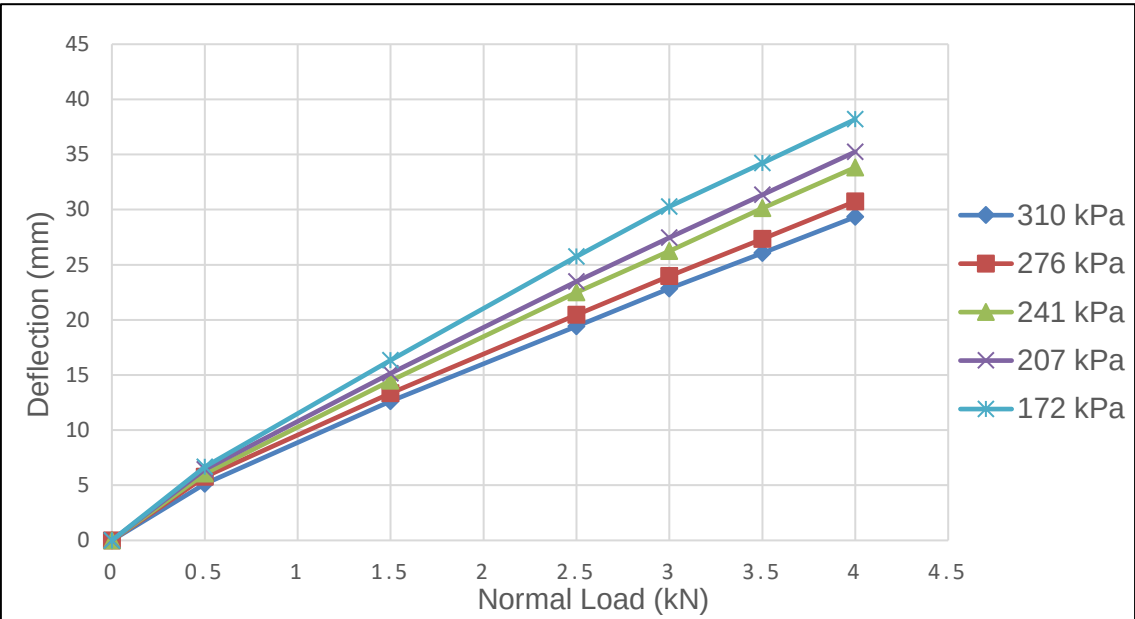


**Figure 3-13 Relationship between normal load and tyre deflection at different inflation pressures of the scaled tyre (145/80R13)**

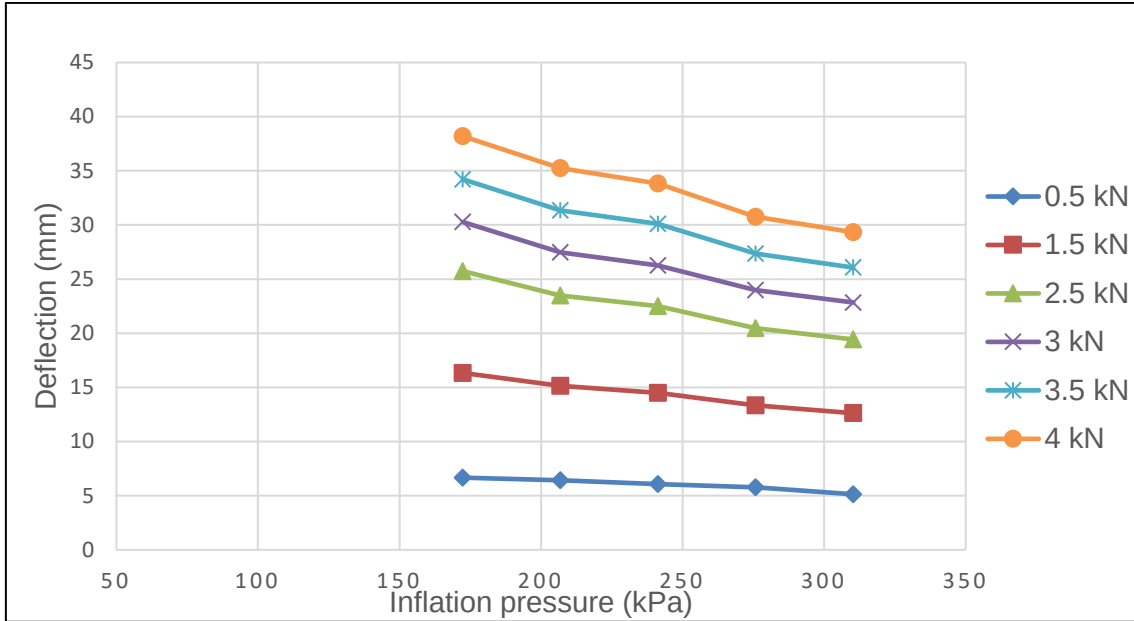


**Figure 3-14 Relationship between inflation pressure and tyre deflection at different normal loads of the scaled tyre (145/80R13)**

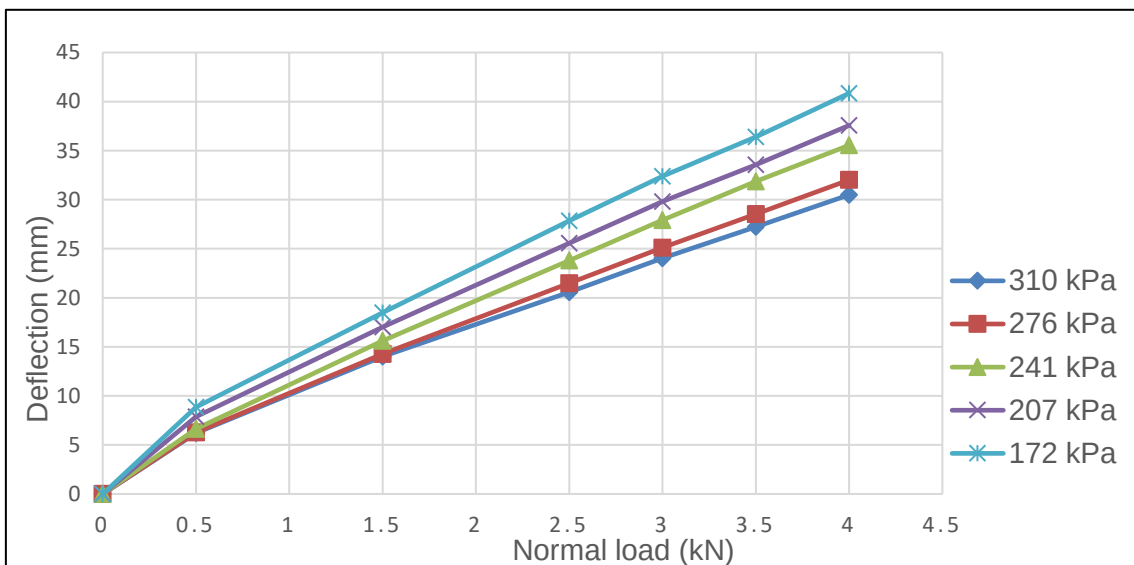


**Figure 3-15 Relationship between normal load and tyre deflection at different inflation pressures of the scaled tyre (135/80R13)**

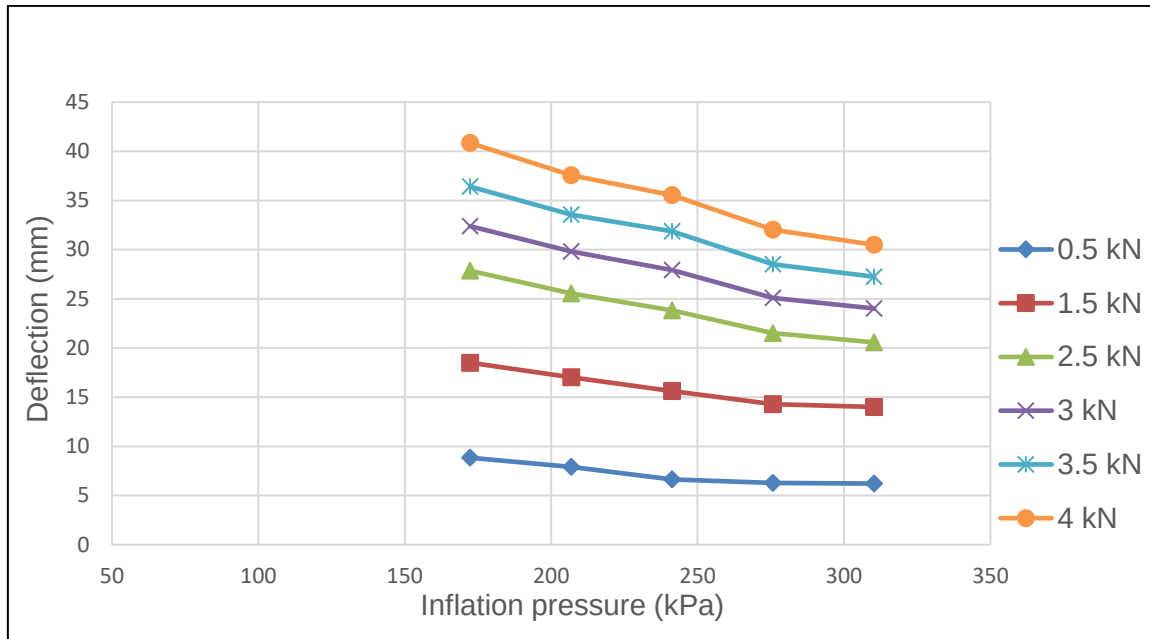


**Figure 3-16 Relationship between inflation pressure and tyre deflection at different normal loads of the scaled tyre (135/80R13)**

### 3.5.2.2 Contact area characteristics

The contact area tests were conducted on the scaled tyres 145/80R13 and 135/80R13 at different normal loads and inflation pressures on a hard surface (section 3.4.4). The normal loads for both the scaled tyres varied from 0.5 kN to 4 kN. The inflation pressures for both the scaled tyres varied from 172 kPa (or 25 Psi) to 310 kPa (or 45 Psi). The experimental results obtained for the tyre contact area at different inflation pressures and normal loads are as follows. Figure 3-17 shows the relationship between inflation pressure and contact area at different normal loads for 145/80R13 tyre. Figure 3-18 shows the relationship between inflation pressure and contact area at different normal loads for 135/80R13 tyre. The interpretation of the results is the same as in section 3.5.1.2.

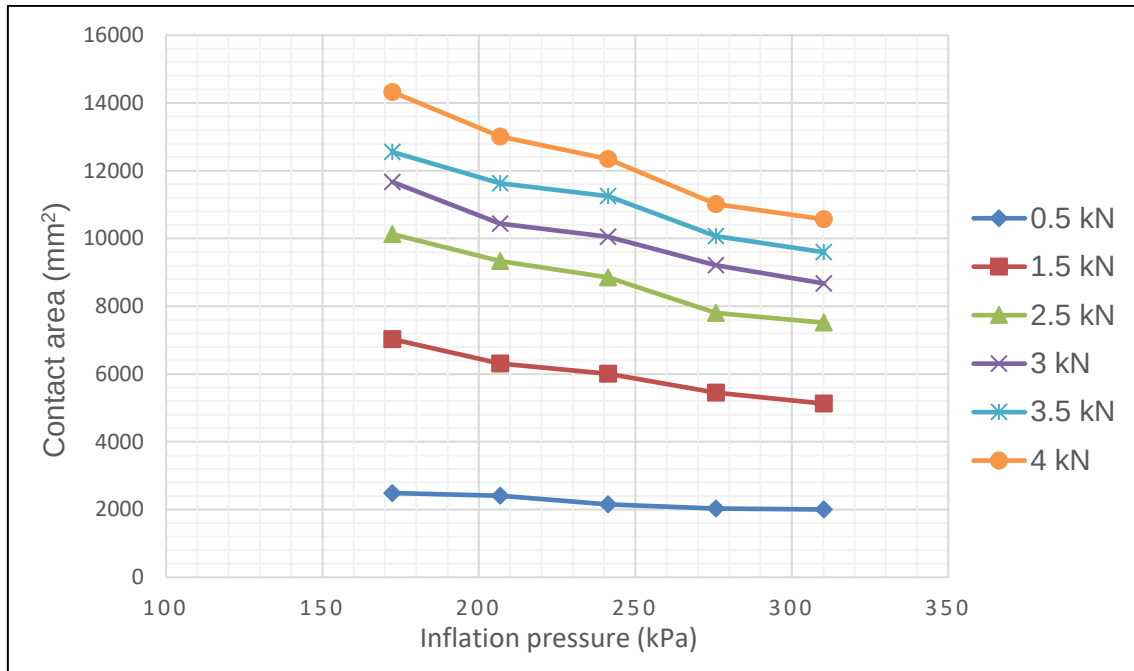


**Figure 3-17 Relationship between inflation pressure and tyre contact area at different normal loads of the scaled tyre (145/80R13)**

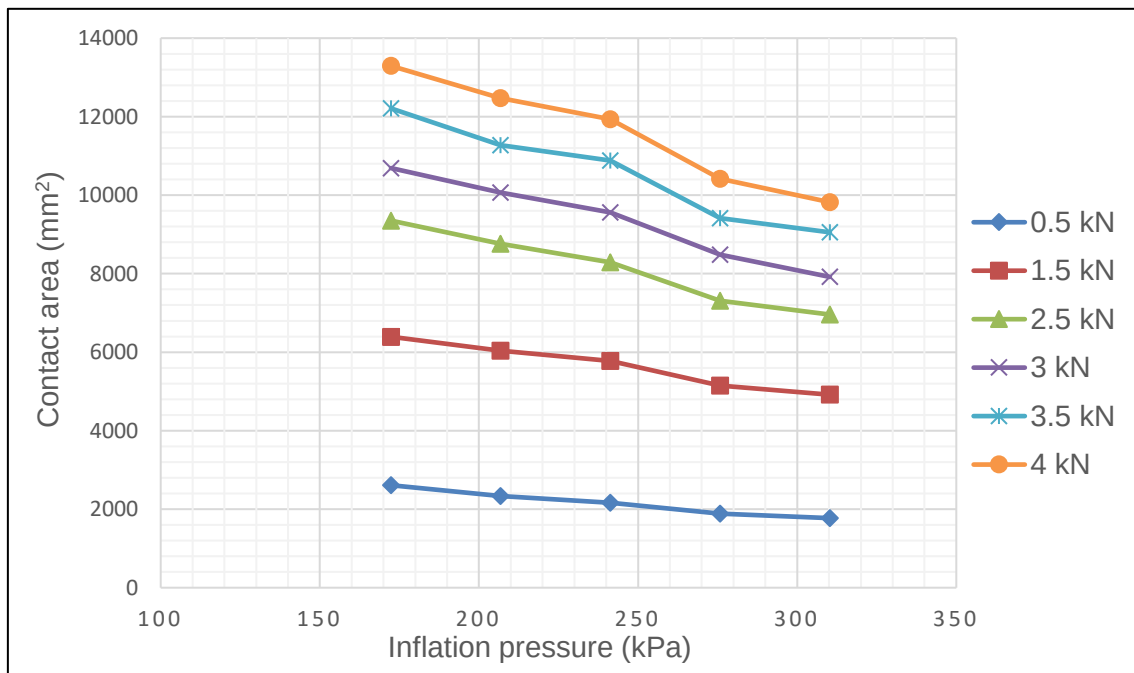


**Figure 3-18 Relationship between inflation pressure and tyre contact area at different normal loads of scaled tyre (135/80R13)**

### 3.5.3 Similarity analysis

The scalability theory is based on applying dimensional analysis to the vehicle-soil system's experimental study. It is based on the fundamental assumption that similar systems behave similarly. Thus, if generalisation obtained by a systematic variation of independent parameters is to be used with confidence, then it must be shown that a similar tyre-soil system does not vary with the system's size. Therefore, there is a need to establish similarity conditions between two different sizes of systems. In tyre-soil interaction, the two similarity criteria are needed to establish similar systems. They are geometric similarity and dynamic similarity. The geometric similarity predicts the distorted shape and curvature of the tyre, which controls the tyre-soil contact area, contact pressure and load transfer to the soil surface as shown in Figure 3.19. The dynamic similarity controls the nature of the transfer of forces in the tyre-soil system. The verification of similarity criteria is needed with the assumption that the tyre stiffness is similar for the selected full-scale 7.5R16 and scale tyres 145/80R13 and 135/80R13.

In the dimensional analysis of tyre deflection and contact area, the inflation pressure  $P_i$  and load  $W$  are the independent variables that change the tyre shape and curvature. Both these parameters are associated with the same  $P_i$  term,  $\frac{P_i b.d}{W}$ . The change in one parameter can be compensated with the change in another parameter. Therefore, it becomes important to understand this  $P_i$  term's effect on the deflection ratio and the contact area ratio of the full-scale and scaled tyres.

The full-scale models of dependent variables developed in Equations 3-41 and 3-43 are used to predict the deflection ratios and contact area ratios of scaled tyres, respectively. The scaled tyres were tested at different inflation pressure and normal loads. Since the dependable variables are also considered part of tyre geometry, a comparison of these variables was employed to determine the similarity between the full-scale and scaled tyre geometries.

In Figures 3-20 to 3-23, a comparison between the scale model prediction from the developed full-scale tyre models (Equations 3-41 and 3-43) is shown graphically with the experimental results of scaled tyre 145/80R13 previously

discussed in section 3.5.2. Figures 3-24 to 3-27 show a comparison between the scale model prediction from the developed full-scale tyre models (Equations 3-41 and 3-43) and experimental results of scaled tyre 135/80R13 previously discussed in section 3.5.2. Here, the deflection ratio and contact area ratio data points are plotted with respect to the obtained non-dimensional term  $\frac{P_i b d}{W}$ . It is evident that the full-scale model and scaled models are effectively similar systems. The coefficient of determination ( $R^2$ ) is found acceptable i.e. 0.97 between predicted and experimental values for the tyre deflection model and contact area model (Table 3.9 and 3.10). The mean per cent deviation is less than 10% for tyre deflection tests for both the scaled tyres. However, it is slightly more than 10% for the tyre contact area model for both scaled tyres. This slight increase in mean per cent deviation in tyre contact area prediction might be because of the measurement errors. As the tyre contact area is also a function of tyre deflection, the established Pi term can hold similarities between the systems confidently.

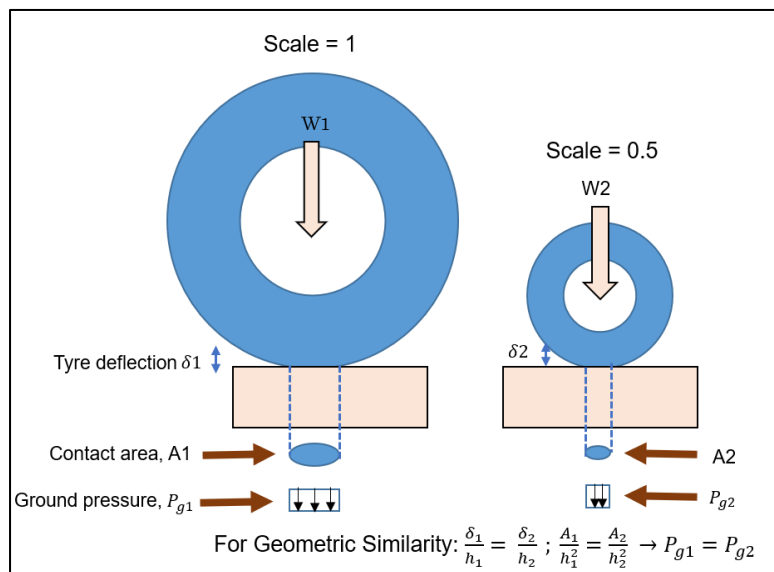


**Figure 3-19 The geometric similarity conditions holding same tyre deflection ratio ( $\frac{\delta}{h}$ ) and tyre contact area ratio ( $\frac{A}{h^2}$ ) between the full-scale and the scale-tyre on the test surface**

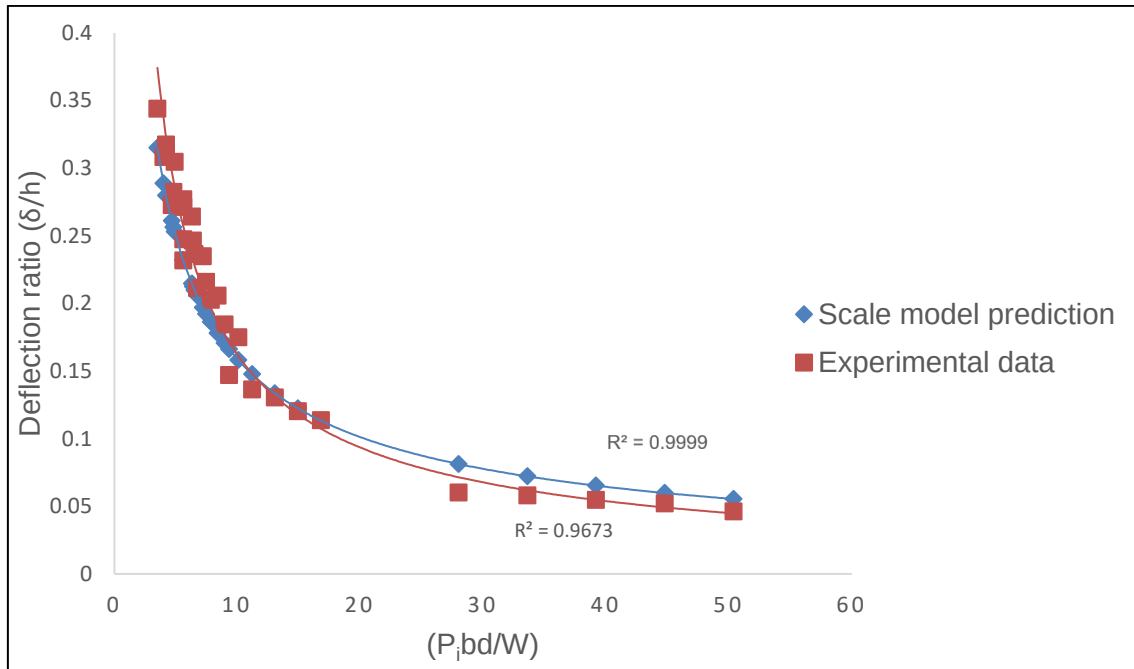


**Figure 3-20 Comparison of scale model prediction of deflection ratio based on non-dimensional parameter with the test data of 145/80R13**

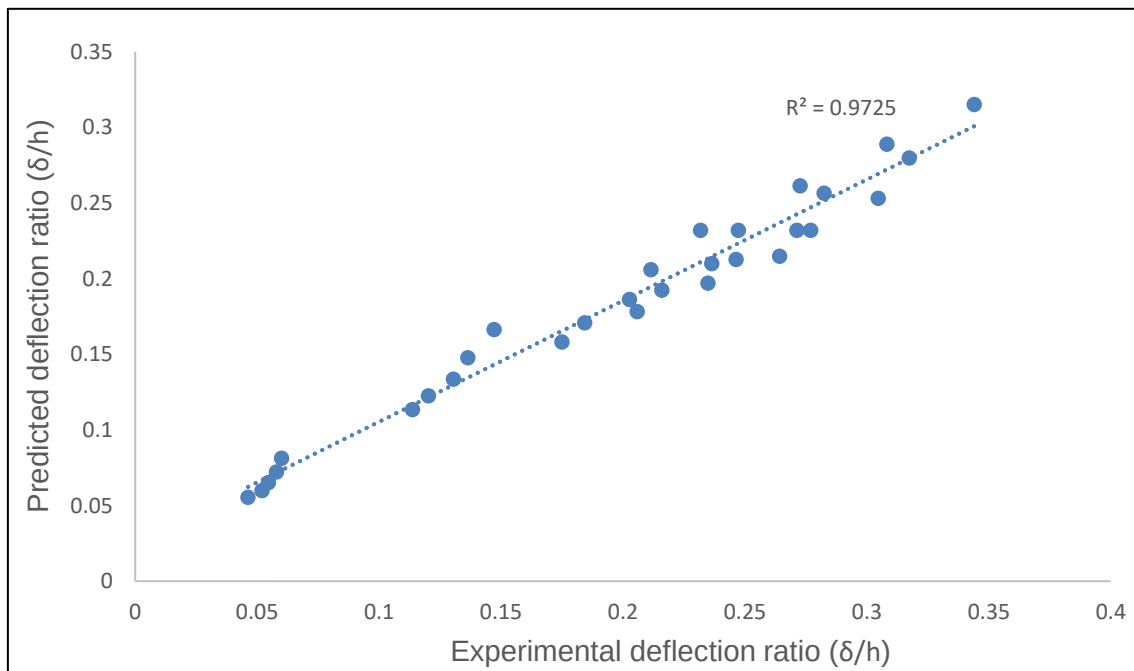


**Figure 3-21 Relationship between experimental and predicted deflection ratio of the scaled tyre (145/80R13)**

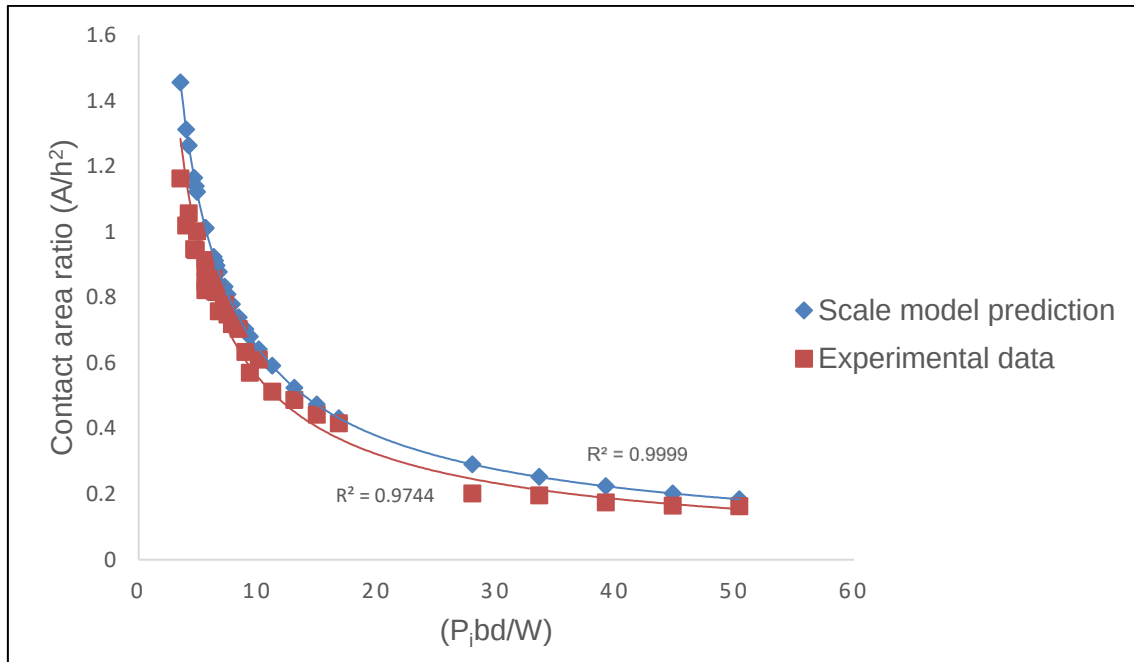


**Figure 3-22 Comparison of scale model prediction of contact area ratio based on non-dimensional parameter with the test data of 145/80R13**

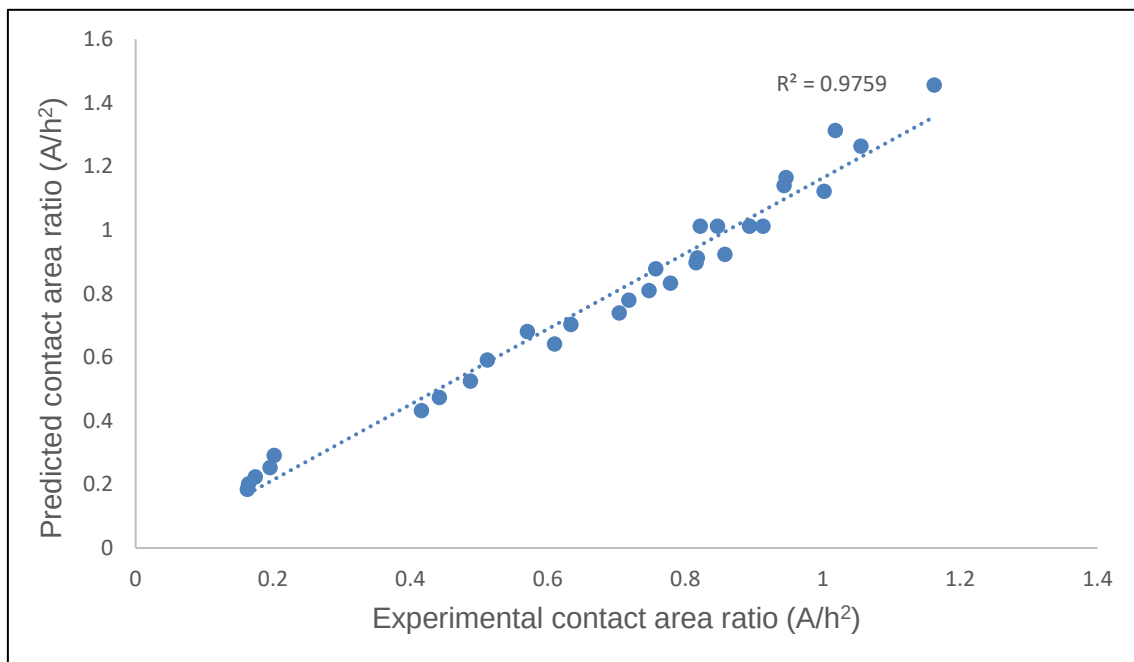


**Figure 3-23 Relationship between experimental and predicted contact area ratio of the scaled tyre (145/80R13)**



**Table 3-9 Statistical summary of the scale model comparison for 145/80R13**

| Parameters  | Mean, % deviation | Standard deviation, % | SSE, % | RMSE, % | R <sup>2</sup> |
|-------------|-------------------|-----------------------|--------|---------|----------------|
| Figure 3-19 | 2.22              | 0.41                  | 0.41   | 1.17    | 0.9725         |
| Figure 3-21 | 13.19             | 5.4                   | 8.48   | 5.31    | 0.9759         |

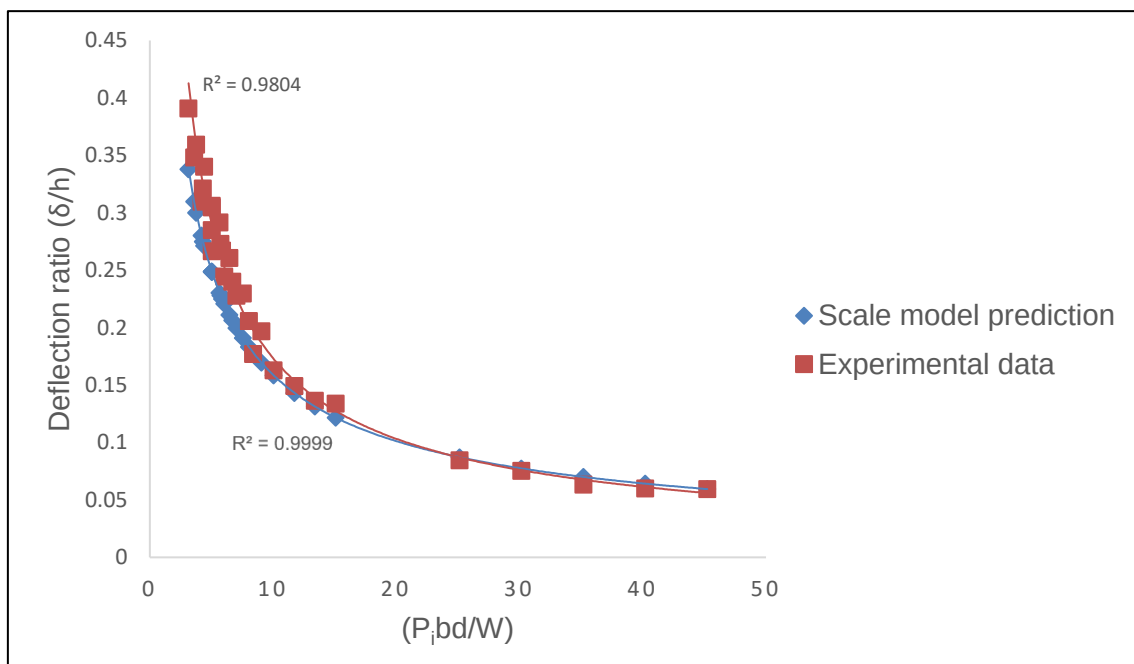


**Figure 3-24 Comparison of scale model prediction of deflection ratio based on non-dimensional parameter with the test data of 135/80R13**

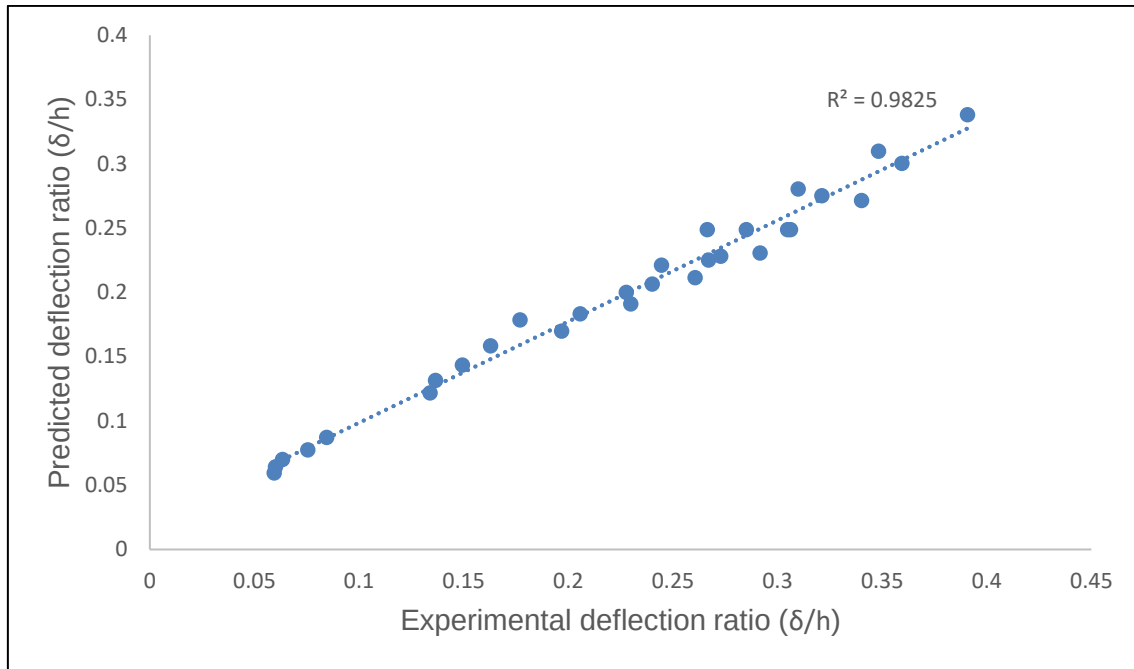


**Figure 3-25 Relationship between experimental and predicted deflection ratio of the scaled tyre (135/80R13)**

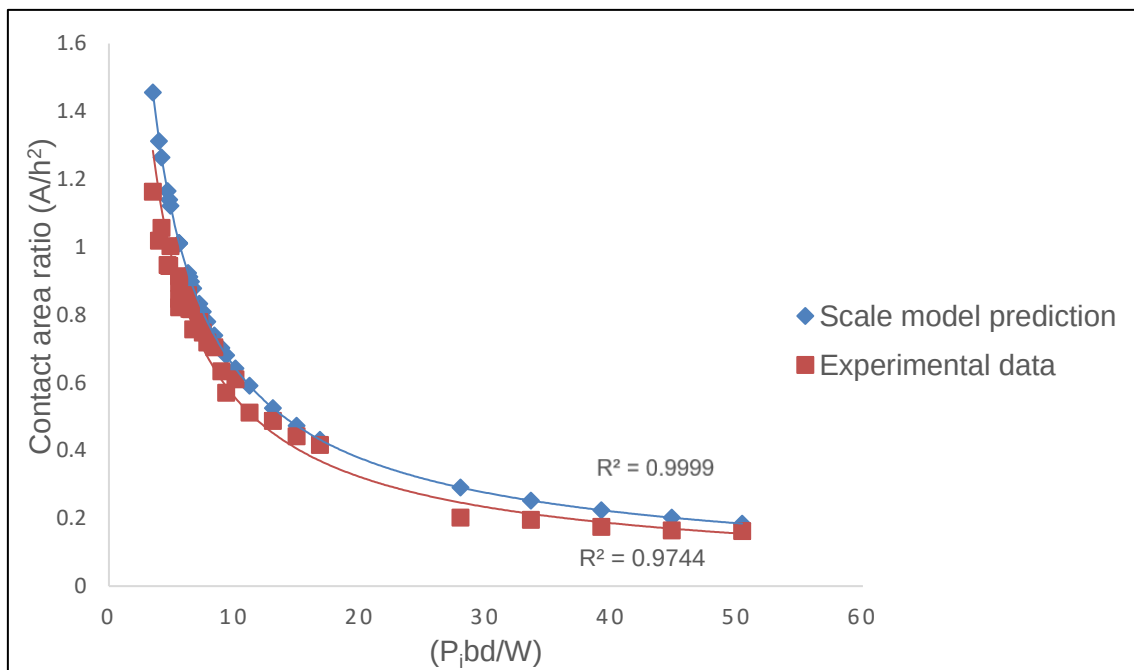


**Figure 3-26 Comparison of scale model prediction of contact area ratio based on non-dimensional parameter with the test data of 135/80R13**

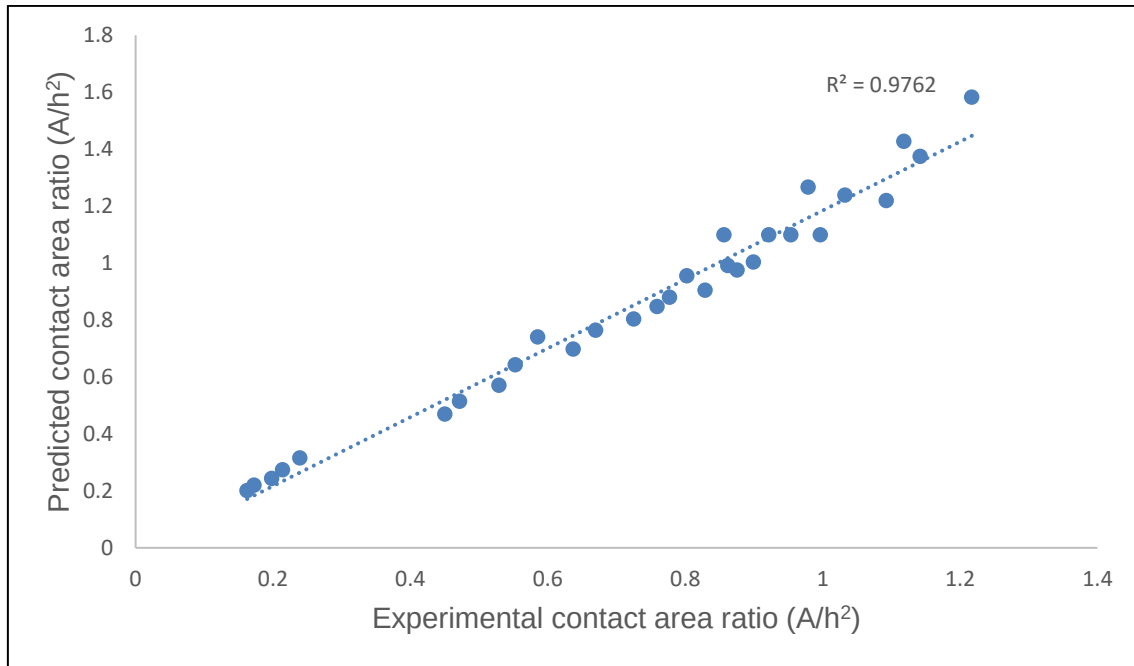


**Figure 3-27 Relationship between experimental and predicted contact area ratio of the scaled tyre (135/80R13)**

**Table 3-10 Statistical summary of scale model comparison for 135/80R13**

| Parameters  | Mean, %<br>deviation, | Standard<br>deviation,<br>% | RMSE, % | SSE, % | R <sup>2</sup> |
|-------------|-----------------------|-----------------------------|---------|--------|----------------|
| Figure 3-23 | 9.6                   | 1.02                        | 1.31    | 0.51   | 0.9825         |
| Figure 3-25 | 14.77                 | 5.84                        | 5.74    | 9.90   | 0.9762         |

### 3.6 Summary

The work reported above has investigated the methods used to measure the deflection ratio and the contact area ratio of the tyre as a function of the non-dimensional parameter to represent and compare their geometrical aspects. The main findings of this work can be summarised as follows:

- In the scalability theory, the geometric similarity predicts the tyre's distorted shape and curvature in a similar system. It is known that the tyre-terrain contact pressure through normal load and contact area influences

the transfer of force. Therefore, the tyre deflection and contact area can be expressed in a single non-dimensional Pi term. Therefore, the Pi term establishes the geometric similarity between the full-scale and scaled tyre systems.

$$\left(\frac{P_i b d}{W}\right)_{FS} = \left(\frac{P_i b d}{W}\right)_{SM}$$

- The developed models of deflection ratio and contact area ratio of the full-scale tyre (7.5R16) were used to predict the respective ratios for two scaled tyres, 145/80R13 and 135/80R13. The full-scale tyre model predictions for scaled tyres were compared with scaled tyre's experimental results. There exists a good correlation between the predicted results and experimental results for scaled tyres.
- A mathematical relationship between deflection ratio and contact area ratio is expressed as a function of independent tyre parameters, inflation pressure and normal load using a dimensional analysis approach. It was found that both ratios can be expressed as a function of a single Pi term  $\frac{P_i b d}{W}$ . The deflection and contact area ratios are critical in deciding tyre geometry. Therefore, a generalised mathematical relationship is developed to predict the deflection ratio and the contact area ratio for the full-scale tyre, 7.5R16.

$$\frac{\delta}{h} = 0.7126 \times \left(\frac{P_i (b \times d)}{W}\right)^{-0.6516}$$

$$\frac{A}{h^2} = 3.841 I^{-0.7747}$$

- The dynamic similarity controls the nature of tyre-soil interaction in similar systems. A dimensional relationship is expressed between the mobility performance parameters and tyre-soil system parameters. It was found that the non-dimensional term  $\frac{C I b d}{W}$  considers the effect of soil strength in the tyre-soil system. The effect of this non-dimensional term on the scalability of tyre-soil interaction is studied by Freitag (1966) and

discussed in section 2.9. Therefore, this non-dimensional term can be used as shown.

$$\left(\frac{\text{Cl. bd}}{W}\right)_{\text{FS}} = \left(\frac{\text{Cl. bd}}{W}\right)_{\text{SM}}$$

## 4 SCALABILITY OF SOIL

### 4.1 Introduction

The Discrete Element Method (DEM) is a numerical technique used in computing the multi-body dynamics interactions between wheeled and tracked vehicles and soft soils. The dynamics of vehicle-terrain interaction are too complex because of the involvement of complicated soil parameters. Over the past 50 years, the mobility of vehicles operating on soft soil is accessed through full-scale physical testing in controlled laboratory conditions. However, in naturally occurring soft soils, where the spatial and temporal variations in soil properties are expected, the difference in a vehicle's performance can be concealed by the inherent variability of real terrain. Therefore, an effort has been put to address the difference in soil properties considering particle shape and size distribution of the simulated laboratory soil and the naturally occurring terrain models.

In the Terramechanics, the soil system's mechanical behaviour is divided into stress-strain soil relationships in horizontal and vertical directions. Bernstein (Plackett, 1985) proposed a pressure sinkage model based on the semi-empirical approach to study the vehicle-soil contact dynamics based on the behaviour of penetration of rectangular or circular plates into the soil. Later, Bekker (1956, 1969) introduced an experimental technique called the Bevameter test (Wong, 2010) to study the effect of the size of the contact patch on soil parameters. More recent studies conducted by Ishigami et al. (2007) and Griffith and Spenko (2011, 2013, 2014) considered the effect of wheel geometry and curvature.

In the empirical approach, cone index (CI) is used to determine soil strength. Theoretically, it is the force per unit base area required to push a cone probe into soil at a steady rate. Cone index is mostly influenced by soil density, soil type, moisture content and soil metal friction (Mason and Baylot, 2016; Quraishi et al., 2013; Santos et al., 2012; Dexter et al., 2007; Vaz and Hopmans, 2001). Setkei et al. (2018) reviewed that the various mechanical properties such as load-bearing capacity, shear strength, hardness and cone index of soil are subject to the combination of various stresses, friction and deformations and are less

accurately defined. The main difficulties in their systematic measurement are extreme variability of soil texture and variable distribution of moisture content, hardness, and soil density (Mason et al., 2016 and Pillinger et al., 2018).

Besides empirical and semi-empirical models, discrete element method has also been used extensively (Nakashima et al., 2004, 2007, 2009; Smith et al., 2013; Zhao et al., 2014; Nishiyami et al., 2016, 2017; Johnson et al., 2015, 2017; Yonghao Du et al., 2017, 2018; Nakanishi et al., 2020; Ravula et al., 2021, Wu et al., 2022). This method can describe the better physics behind the contact phenomenon of wheel-soil interaction and inter-particle mechanics. DEM models were used for studying the mechanical behaviour of particles in soil mechanics studies generally consisting of linear springs, in the tangential and normal direction and a damper in between the two soil particles (Asaf et al., 2006). The spring stiffness, damping constants and related properties of soils in DEM are generally determined by analytical or Trial and Error method using either the DEM software material library (e.g., EDEM material library) or the existing literature base. The simulation model's validation is a crucial step to determine its applicability. Therefore, the previous studies showed a good correlation between DEM simulations and experimental validation results.

In the present study, the DEM simulations were carried out for the determination of soil mechanical properties in a controlled homogeneous environment (Kobayakawa et al., 2020; Nakashima et al., 2007). The DEM provides an advantage to control the extreme variability of soil conditions by preparing the soil bed with uniform particle distribution, same particle shape and constant particle density. This homogeneous environment can be regarded as a controlled laboratory experiment environment. However, NATO CDT-308 (2020) states that these soil parameters are never verified with respect to real terrain parameters. Therefore, an in-situ terrain model is prepared using the randomised algorithm in the EDEM with varying soil particle parameters. The physical properties such as moisture content, bulk density and compaction were kept the same for both models. Therefore, there is a need to investigate soil parameter's scaling scope between homogeneous laboratory and in-situ terrain conditions.

Hence, the scalability of soil is defined as a relationship between the mechanical properties of an in-situ terrain (heterogeneous) system and a laboratory (homogeneous) system while accounting for the differences in sand, silt and clay particle shapes and size distributions. Physical properties such as moisture content, bulk density, compaction, and interparticle forces are kept the same for laboratory and in-situ terrain conditions. The laboratory system has a uniform particle shape and constant particle size whereas the in-situ terrain has variable particle shapes and variable particle size distributions.

In scalability, the inter-particle forces are kept the same (or uniform) between laboratory (homogeneous) and in-situ (heterogeneous) systems, whereas, in variability, the inter-particle forces may change (or variable). In scalability analysis, soil conditions such as moisture content, bulk density, and compaction are kept the same, whereas they can be different in variability analysis. Therefore, the scalability of soil can be considered a subset of the variability of soil.

This section is organised as follows. Section 4.2 describes the problem formulation. Section 4.3 describes the soil characterization tests. Section 4.4 describes the discrete element method modelling, and 4.5 describes the numerical experiments and results, followed by Section 4.6 to incur the summary.

## **4.2 Problem formulation**

The scalability of soil is defined as a relationship between the mechanical properties of an in-situ terrain (heterogeneous) system and a laboratory (homogeneous) system while accounting for the differences in sand, silt and clay particle shapes and size distributions. Physical properties such as moisture content, bulk density, compaction, and interparticle forces are kept the same for laboratory and in-situ terrain conditions. In the present study, seven soil parameters are investigated. They are moduli of deformation w.r.t cohesion  $k_c$ , moduli of deformation w.r.t friction  $k_\phi$ , sinkage exponent  $n$ , cohesion  $c$ , internal angle of friction  $\phi$ , shear modulus  $k_s$  and cone index  $CI$ . NATO CDT – 308 (2020) highlights the research challenges/ gap in understanding the difference between the soil behaviour in homogeneous and heterogeneous terrain systems while



considering different sand, silt and clay particle shapes and sizes. The particle size distribution is determined using the soil grading curve. This curve is plotted on a horizontal logarithmic axis of grain size against percentage on an arithmetical vertical axis (e.g. Figure 4.9). Further, the report also mentions a need to improve the scalability of the Terramechanics models for heterogeneous terrain systems.

NATO CDT – 308 (2020) and Jayakumar (2016) at Tank Automotive Research Development and Engineering Center (TARDEC), United States Army, presented the challenging nature of Terramechanics. Generally, the in-situ terrain has random particle distribution and is considered heterogeneous and anisotropic, making it less confident to use soil parameters determined from laboratory experiments in mobility prediction models. The laboratory soil samples are assumed homogeneous and isotropic. Therefore, the concept of soil scalability establishes a relationship between the soil properties determined between the laboratory and in-situ terrain conditions.

In the present study, two different soils are modelled and validated in EDEM simulation software using the soil datasheet from NG NRMM CDT (2018) report. The soils selected are 2NS sand and fine-grained sand. The laboratory experiments were modelled in the simulation software to test homogeneous soil sample's mechanical properties. The experiments include pressure-sinkage, shear and cone penetrometer tests. The simulated soil conditions are assumed the same as experimental conditions. The in-situ terrain model is prepared using random algorithms to keep the same soil conditions (e.g., moisture content) to generate random particle properties (e.g., shape and size) to prepare heterogeneous terrain models.

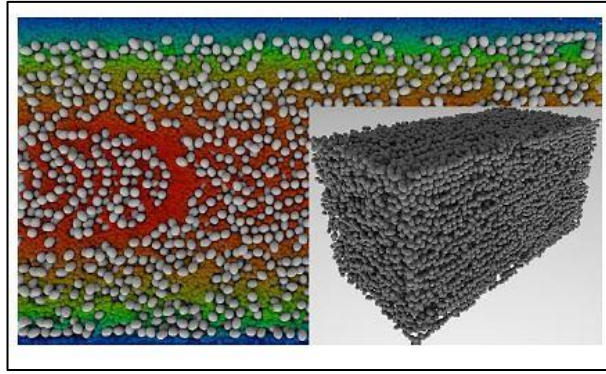


**Figure 4-1 a) Grey colour – homogeneous soil sample; b) Real terrain model (Jayakumar (2016))**

### **4.3 Soil characterization**

A soil system can be described as a multiphase system consisting of solid, liquid and gas. Theoretically, the soil system is very complicated because of its non-homogeneous, anisotropic and variable stress-strain relationship. However, the soil's response to applied loading is controlled by the movement and deformation of these constituents. Therefore, it depends on the size and shape of soil particles, moisture content, and bulk density.

Soils are primarily classified based on their particle size distribution, which relatively defined the percentage of sand, clay and silt present in its solid phase. Based on the relative percentage of these three constituents, the soil is identified as one of its generic soil types based on the soil texture triangle developed by the United States Department of Agriculture (Figure 4-2). The International Society of Soil Science, Textural Soil Classification System, ISSS (ISTVS standards (2020)) categorized these three constituents based on their particle sizes, as shown below.

**Table 4-1 Texture Soil Classification System, ISSS (ISTVS standards (2020))**

| Soil type    | Particle diameter (in mm) |
|--------------|---------------------------|
| Clay         | < 0.002                   |
| Silt         | 0.002–0.02                |
| Sand, fine   | 0.02–0.2                  |
| Sand, coarse | 0.2–2                     |
| Gravel       | > 2                       |

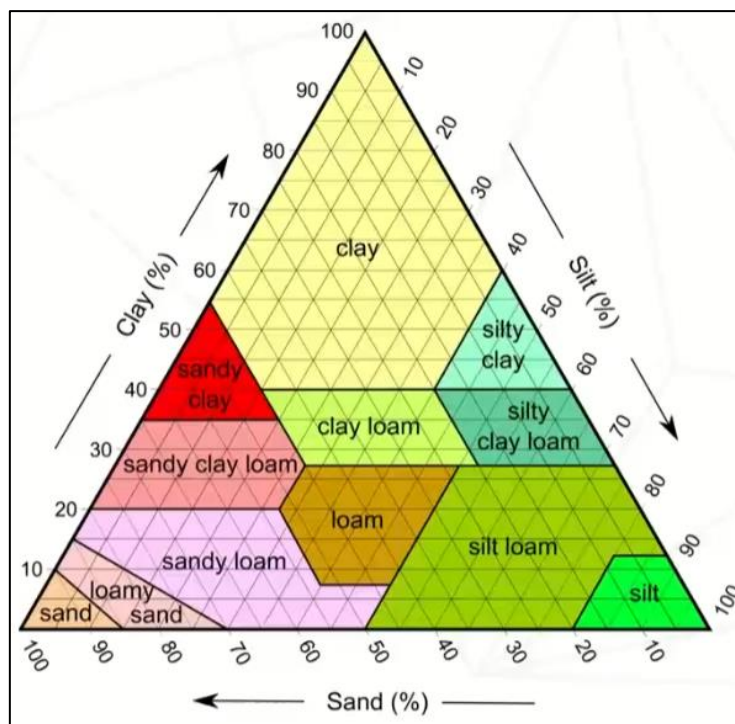


**Figure 4-2 Soil texture triangle developed by United States Department of Agriculture (R. He et al. (2020))**

Particle size distribution of soil is determined using the sieve test. In this test, a soil sample is allowed to pass through a series of sieves that are arranged with progressively smaller mesh size from top to bottom. The cumulative percentage mass of particles passing through each sieve is used to plot the grading curve for

the soil, as shown in figure 4.9. Therefore, the grading curve determined the particle size distribution of the soil.

In addition to the particle size distribution, soil can be further classified in terms of bulk density and moisture content. As the bulk density of soil increases with an increase in compaction, the friction between the soil particles increases due to an increase in the interlocking, which increases the soil's shear strength. Similarly, the soil's moisture content leads to a change in soil response to applied loading. The effect of moisture content on fine grained soils can be explained in terms of Atterberg limits as solid, semi-solid, plastic or liquid (Liu et al., 1964). As moisture is added to the soil in the solid or semi-solid state, it increases the level of adhesion between soil particles, which increases the level of stress that the soil can support before yield. In this condition, soil response changes from a brittle to a plastic state. In clay or silt, the minimum moisture content at which soil is considered to behave in a plastic manner is known as the plastic limit (PL). As the moisture is increased further so that soil approaches the liquid state, there is a reduction in the soil strength from inter-particle friction. The maximum water content at which silt or clay will become liquid, i.e. turning into mud is known as the liquid limit (LL). Therefore, the range of moisture content at which clay or silt remains plastic is known as the plasticity index (PI).

#### **4.3.1 Bevameter testing**

Bekker (1956) developed Bekker-value-meter or Bevameter to reproduce the forces acting during wheel soil interaction under loading conditions. It has the capabilities to determine the vertical and horizontal stress-strain relationship of soil beneath a wheel. Wong (2010) described that bevameter measurements facilitate two major types of tests to characterize the soil behaviour – plate penetration test (or pressure-sinkage test) and shear test, as shown in Figure 4-3.



**Figure 4-3 Bevameter (NATO NRMM CDT (2018))**

#### **4.3.1.1 Pressure sinkage test**

The pressure sinkage (p-z) test is used to determine the vertical stress-strain relationship of soil under normal loading conditions. In this test, a uniform flat plate (or sinkage plate) is usually a circular or rectangular cross-section of the same contact area as vehicle running gear (Wong, 2010). It is pushed vertically into the terrain with constant penetration speed downwards. The input force on the sinkage plate is usually applied through hydraulics. The applied force on the flat plate and sinkage were measured to determine the pressure-sinkage (p-z) relationship of a deforming soil. Bernstein (Plackett, 1985) proposed a pressure sinkage (p-z) model (Equation 4-1) based on the semi-empirical approach to study the vehicle-soil contact dynamics based on the behaviour of penetration of circular plate into the soil. The  $k$  [ $N/m^{n+2}$ ] is the modulus of inelastic deformation, and  $n$  is the exponent of sinkage empirically determined.

$$p = kz^n \quad (4-1)$$

Bekker (1956, 1969) modified the Bernstein pressure-sinkage mathematical formulation based on soil mechanics and incorporated the width of the contact patch,  $b$  (m) (Equation 4-2). The  $k_c$  [ $N/m^{n+1}$ ] and  $k_\phi$  [ $N/m^{n+2}$ ] are the moduli of

deformation with respect to cohesion and friction, respectively. Both parameters are curve fit constants in the Bekker model and have no physical significance.  $n$  is an empirically determined model parameter that defines the curvature of the load-displacement curve. It also demonstrated that deformation moduli are independent of plate geometry for homogeneous soil.

$$p = \left( \frac{k_c}{b} + k_\phi \right) z^n \quad (4-2)$$

Determination of Bekker parameters was initially performed by Wong (1980, 2006) using the weighted least square method (WLS). Jayakumar et al. (2014) analysed the alternative methods to estimate Bekker parameters and found Wong's method reliable for estimation.

Wong's method requires a minimum of two pressure-sinkage tests to perform with two different plate widths  $b_1$  &  $b_2$ . Then, convert the  $p - z$  domain into a log-log domain to obtain a straight-line relation of the Bekker equation, as shown in Equation 4-3. The conventional least square method cannot be used, as the measured quantities are  $p - z$ , not  $\ln p - \ln z$ . It gives more weight to the datasets measured at low-pressure points, generally insignificant.

$$\ln p = \ln \left( \frac{k_c}{b} + k_\phi \right) + n \ln z \quad (4-3)$$

An error function,  $F$ , is defined to obtain the above equation's optimum values, as shown in Equation 4-4.

$$F = \sum w_r - \ln p - \ln \left( \frac{k_c}{b} + k_\phi \right) - n \ln z]^2 \quad (4-4)$$

The weighting factor,  $w_r$ , is taken as  $p^2$  because the error function is biased towards low-pressure values to give equal weight to all data points in the log-log domain.

$$F = \sum p^2 - \ln p - \ln \left( \frac{k_c}{b} + k_\phi \right) - n \ln z]^2 \quad (4-5)$$

Minimization of the above equation involves taking the partial derivatives of  $F$  – function with respect to  $n$  and  $k_{eq}$ , where  $k_{eq} = (\frac{k_c}{b} + k_\phi)$  and equating them to zero. The obtained equations are:

$$\ln k_{eq} \sum p^2 \ln z + n \sum p^2 (\ln z)^2 = \sum p^2 \ln p \ln z \quad (4-6)$$

$$\ln k_{eq} \sum p^2 + n \sum p^2 \ln z = \sum p^2 \ln p \quad (4-7)$$

Solving the above equations simultaneously gives Equation 4-8 and 4-9 for  $n$  and  $k_{eq}$ .

$$n = \frac{\sum p^2 \sum p^2 \ln p \ln z - \sum p^2 \ln p \sum p^2 \ln z}{\sum p^2 \sum p^2 (\ln z)^2 - (\sum p^2 \ln z)^2} \quad (4-8)$$

$$\ln k_{eq} = \frac{\sum p^2 \ln p - n \sum p^2 \ln z}{\sum p^2} \quad (4-9)$$

The value of  $n$  is obtained for each of the two different plate sizes using Equation 4-8. Therefore, it is required to take the average of the  $n$  – values to obtain a unique  $n$  value for a particular terrain. This unique  $n$  value is used in Equation 4-9 to calculate the natural logarithm of  $k_{eq}$ . However, since  $k_{eq} = (\frac{k_c}{b} + k_\phi)$ , there are two resulting  $k_{eq}$  values, each corresponding to two different plate sizes,  $b_1$  and  $b_2$ . Accordingly, there are two following equations to determine  $k_c$  and  $k_\phi$ , is shown in Equation 4-10 and 4-11.

$$K_c = \frac{(k_{eq})_{b_1} - (k_{eq})_{b_2}}{b_1 - b_2} b_1 b_2 \quad (4-10)$$

$$k_\phi = (k_{eq})_{b_1} - \left[ \frac{(k_{eq})_{b_1} - (k_{eq})_{b_2}}{b_1 - b_2} \right] b_2 \quad (4-11)$$

Wong (1980) also developed the “goodness-of-fit” Equation to define the error between the experimental data and theoretical data. The ratio of the root mean square error to the mean pressure, is shown in Equation 4-12.  $p_m$  is measured pressure,  $p_c$  is estimated pressure, and  $N$  is the number of data points.

$$E = \frac{\sqrt{\frac{\sum (p_m - p_c)^2}{N - 2}}}{\sum p_m / N} \quad (4-12)$$

#### 4.3.1.2 Shear test

In the shear test, the shear stress – shear displacement relationship is measured at various normal pressures. This test provides the required input parameters such as cohesion and internal angle of friction, for predicting the shear stress distribution of vehicle running gear (tyre) interface with terrains (Wong, 2010). In the Bevameter described above, a shear ring of an outer diameter of 22.70 cm and an inner diameter of 13.33 cm was used for in-situ soil testing. Shear plate with a grouser height of 0.35 cm and 4.56 cm in length with 20 degrees of grouser spacing is used as shown in Figure 4-4. Therefore, a CAD geometry was developed for testing shear behaviour in the in-situ testing model in simulations.



**Figure 4-4 Shear ring used in bevameter (NATO NRMM CDT (2018))**

In laboratory conditions, soil shear properties can be defined using the direct shear test technique (Figure 4-5). In this test, the soil sample is constrained within two rigid rectangular containers, one above the other. The containers are



disconnected to have relative lateral movement, which results in the shearing of the soil sample. Varying weights can be applied on the shear box's top surface, allowing the effect of confinement on shear strength. Wright (2012) also found that the yield parameters obtained from the direct shear tests were in much better agreement with plate sinkage tests than those derived from other shear tests. Therefore, the direct shear test was simulated under laboratory conditions.

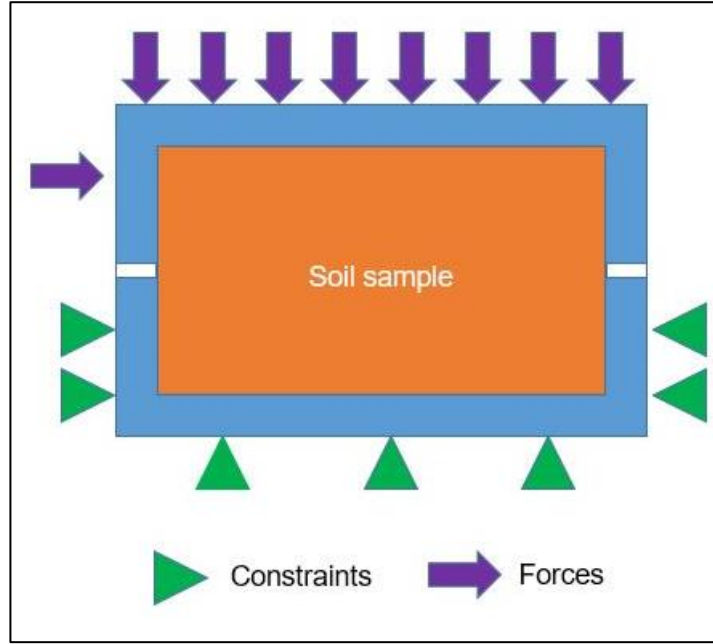


**Figure 4-5 Direct shear box test apparatus**

The soil parameters determined from shear tests are cohesion ( $c$ ), internal angle of friction ( $\phi$ ) and shear modulus ( $K_x$ ). These parameters can be deduced from Janosi and Hanamoto equation (Janosi and Hanamoto, 1961) which proposes the shear stress ( $\tau$ )-shear displacement ( $j$ ) relationship and Mohr-Coulomb failure criterion as shown below in Equations 4-13 and 4-14, respectively.

$$\tau = \tau_{\text{res}} \left( 1 - e^{\frac{-j}{K_x}} \right) \quad (4-13)$$

$$\tau_{\text{res}} = c + p \tan \phi \quad (4-14)$$

a) Determination of shear modulus ( $K$ )

The shear modulus can be determined by minimizing Equation 4-13 using Wong's weighted least square method described earlier. Note that the weighting factor is  $(1 - \tau/\tau_{\max})^2$ . To obtain the optimum value from the above equation, an error function E shown in Equation 4-15 is defined by following the least square approach.

$$E(K, \tau_{\text{res}}) = \sum_{i=1}^n \left( \tau_i - \tau_{\text{res}} \left( 1 - e^{-\frac{j}{K}} \right) \right)^2 \quad (4-15)$$

Further, the partial derivative of the error function is taken with respect to K and set equal to zero. To minimize the error function from the curve fit, K's optimum value can be computed, as shown in Equation 4-16.

$$K = - \frac{\sum (1 - \tau/\tau_{\max})^2 j^2}{\sum (1 - \tau/\tau_{\max})^2 j [\ln (1 - \tau/\tau_{\max})]} \quad (4-16)$$

#### 4.3.2 Cone penetrometer test

ASABE (2006) declared the cone penetrometer as a standard instrument to measure soil penetration resistance. This soil penetration resistance is the soil cone index. ASABE set the standards of cone penetrometer used to measure the soil's strength. The standard cone penetrometer (ISTVS standards (2020)) consists of a cone probe with a vertex angle of 30° and a base area of 3.23 cm<sup>2</sup>. The diameter of the cone probe is 3.5 cm. The cone probe is attached to a cylindrical shaft of diameter 1.5 cm and a length of 30 cm. The penetration velocity should be kept constant and usually kept around 0.01 m/s to maintain the experiment's steady nature. The penetration depth is usually 20 cm with a standardised cone penetrometer depending on the tyre- soil contact pressure and tyre sinkage (Wismer and Luth, 1973).



**Figure 4-6 Cone penetrometer test (NATO NRMM CDT (2018))**

## **4.4 Discrete element method**

### **4.4.1 Algorithm**

DEM simulates the mechanical behaviour of a collection of arbitrarily shaped and arranged particles comprising in a system. A typical DEM material model schematic overview is shown in Figure 4.7. It contains a choice of particle shape, particle size distribution, particle parameters (e.g., particle density) and contact model. The particles act as independent entities and interact only at the contacts and interfaces between them. The algorithm involves the application of Newton's 2<sup>nd</sup> law of motion to particles and force-displacement law contact models at the interfaces. Newton's 2<sup>nd</sup> law of motion is used to determine the particle's motion arising from the contact and body forces on it. The force-displacement model is used to update the contact forces acting from each contact's relative motion. The calculation cycle requires the repeated application of both the laws at each particle and relative contact, respectively, as shown in Figure 4-8. At the start of the simulation, the geometry (e.g. the pressure-sinkage test set-up box) should be created or imported in the EDEM software. Then, the boundary condition is auto updated with the boundary of the box as shown in Figure 4.9. The particles are generated and are allowed to settle under gravity. At the start of each cycle,

the contacts are determined from the known particle positions. The force-displacement contact model is further applied to each contact to update the contact forces based on the relative motion, damping coefficient, and overlapping between the two entities at each time step, typically less than 1% of particle diameter. Later, the law of motion is applied to each particle to update its position and velocity based on the resultant forces and moments from the constitutive contact model and body forces acting on the particle. Once the simulation time is completed, the results are available for post-processing. Further, the simulations were performed in the Altair EDEM software purchased under the academic licence.

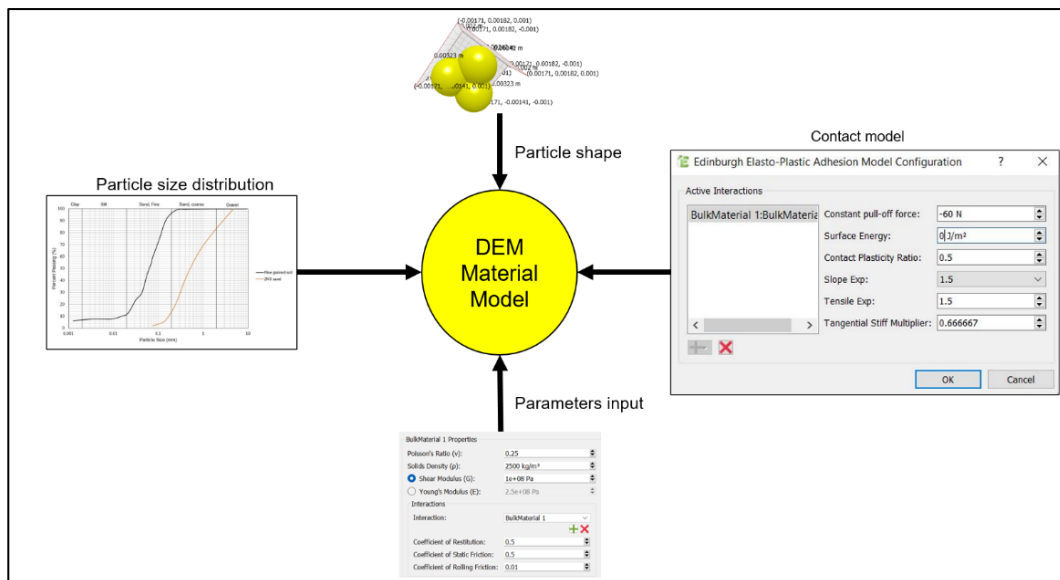


**Figure 4-7 A schematic overview of the DEM Material model**

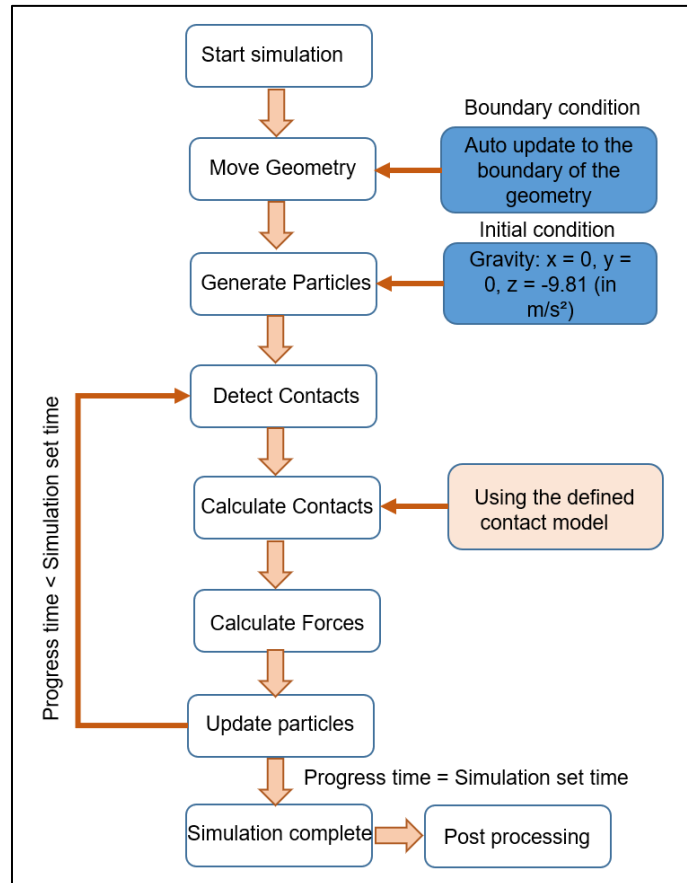


**Figure 4-8 Calculation cycle for DEM**

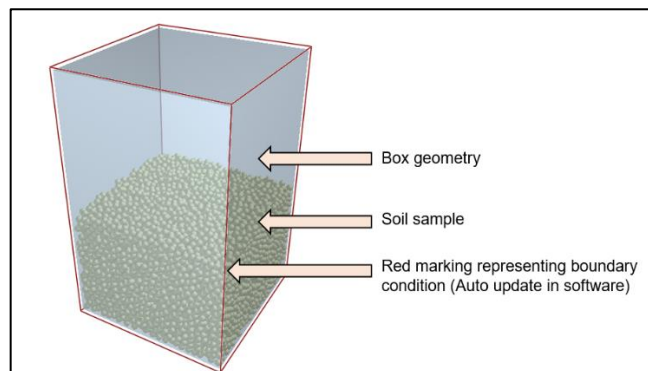


**Figure 4-9 Boundary condition of the box 0.5 mx 0.5 mx 0.8 m with red markings gets auto updated in Altair EDEM software**

#### 4.4.2 Equations of motion

The motion of a single rigid particle under the influence of contact and gravitational forces is calculated from the numerical integration of Newton's motion equations. The translational motion equation for particle  $i$  is as Follows.

$$m_i \frac{d^2}{dt^2} x_i = \sum_c f_i^c + \sum_{nc} f_i^{nc} + m_i g \quad (4-17)$$

where,  $m_i$  is the mass of the particle,  $x_i$  is the position,  $t$  is time,  $\sum_c f_i^c$  is the summation of all the contact forces acting on the particle,  $\sum_{nc} f_i^{nc}$  is the summation of all the non-contact forces acting on the particle, and  $g$  is the acceleration due to gravity. The rotational motion equation of particle  $i$  is as follows.

$$I_i \frac{d^2}{dt^2} \theta_i = M \quad (4-18)$$

where,  $I_i$  is the moment of inertia of particle  $i$ ,  $\theta_i$  is the angular position and  $M$  is the total moment acting on it. In DEM, accelerations are numerically integrated over a time step to update particle velocity and positions.

$$x(t + \Delta t) = x(t) + v(t) \Delta t \quad (4-19)$$

$$v(t + \Delta t) = v(t) + a(t) \Delta t \quad (4-20)$$

where  $v(t)$  is the velocity of a particle at time  $t$ ,  $a(t)$  is acceleration, and  $\Delta t$  is the time step. The rotational velocity and particle orientation are also updated using the same technique. However, the selection of time step ( $\Delta t$ ) is of critical importance and should be sufficiently small to prevent excessive overlapping between particles, resulting in unrealistically high forces and making simulation unstable. Moreover, it also avoids the effect of the disturbance waves. Therefore, the time step is usually approximated from the Rayleigh surface wave propagation speed. Typically, a fraction of this time step is taken to ensure realistic force transmission rates and prevent instability.

#### 4.4.3 Edinburgh elastic-plastic adhesion contact model

The DEM contact model proposed here is based on the physics phenomenon observed in adhesive contacts between different natures and sizes of particles and entities. When two particles or entities are pressed together, they experience elastic and plastic deformation. It is assumed that the adhesive (or pull-off)

strength increases with an increase in the plastic contact area. Therefore, a non-linear contact model that accounts for both elastic and plastic deformation and contact area dependent adhesion is proposed. A schematic diagram of particle contact and normal force overlap ( $f_n$ - $\delta$ ) curve for this model are in Figure 4-10 (a) and (b).

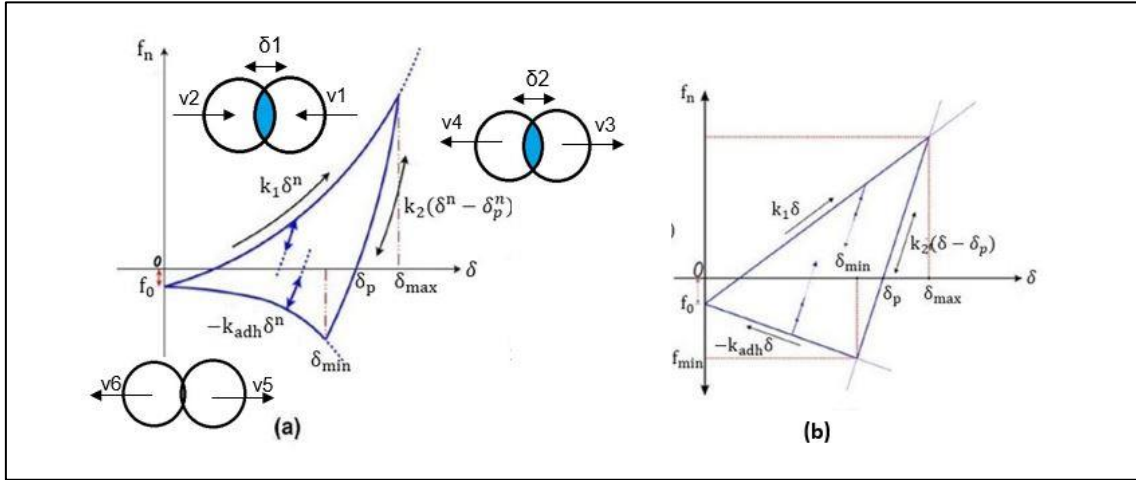


**Figure 4-10 EpAM normal force – displacement relationship (a) non-linear (b) linear (EDEM (2014))**

The loading, unloading/reloading, and adhesive branches are characterised by seven parameters used in the model: the virgin loading stiffness parameter  $k_1$ , the unloading and re-loading stiffness parameter  $k_2$ , the contact plasticity ratio  $\lambda_p$ , the constant adhesion force  $f_0$ , the adhesion energy parameter  $\Delta\gamma$ , the adhesion exponent  $\chi$  and the stiffness exponent  $n$ . The linearity and non-linearity of the loading and unloading branches are controlled by parameter  $n$ , and all become linear when  $n$  is equal to 1. The adhesion exponent  $\chi$  controls the rate at which adhesion forces drop during contact separation; the higher the value, the sharper the drop when plastically flattened contacts separately.

During the initial loading of particles with velocity  $v_1$  &  $v_2$ , the contact is determined using virgin loading path,  $k_1$  and later, upon unloading of particles with velocity  $v_3$  &  $v_4$ ; the contact switches to unloading/reloading path  $k_2$ , reaching zero force at a specific overlap which is also described as the plastic overlap,  $\delta_p$ . If reloading occurs, then initially, the contact follows the unloading

path  $k_2$  but later switches to the virgin loading path  $k_1$  when the previous maximum loading force is reached. Unloading along the  $k_2$  path below the plastic overlap  $\delta_p$  results in the maximum adhesive force development,  $f_{min}$  at  $-f_0 - k_{adh}\delta_{min}$ . Further, unloading past this point reduces both the normal overlap and the attractive forces until the separation occurs at  $\delta = 0$ . If reloading of the contact occurs while on the adhesion branch, the contact follows the  $k_2$  path (there are an infinite number of  $k_2$  paths depending on the point of initial unloading) parallel to the initial unloading/reloading path until reaching the  $k_1$  path. Further, loading takes place on the virgin loading path,  $k_1$ . When the particles are separated with velocities  $v_5$  &  $v_6$ , the contact information is lost.

In the contact model relationship, the virgin loading stiffness  $k_1$  is calculated from the specified particle properties in the simulation, as shown in Equation 4-23. The stiffness is dependent on the equivalent Young's modulus  $E$  and equivalent particle radius  $R$  in all cases, as shown in Equation 4-21 and 4-22. The linear and non-linear virgin loading stiffness is based on the Hertz contact theory.

$$\frac{1}{E} = \frac{(1 - \nu_i^2)}{E_i} + \frac{(1 - \nu_j^2)}{E_j} \quad (4-21)$$

$$\frac{1}{R} = \frac{1}{R_i} + \frac{1}{R_j} \quad (4-22)$$

where  $E_i$ ,  $\nu_i$ ,  $R_i$  and  $E_j$ ,  $\nu_j$ ,  $R_j$  are Young's modulus, Poisson ratio and Radius of each sphere in Contact.

$$k_1 = \begin{cases} 2ER & \text{if } n = 1 \\ \frac{4}{3}\sqrt{RE} & \text{if } n > 1 \end{cases} \quad (4-23)$$

The contact plasticity ratio  $\lambda_p$  relates the magnitude of the virgin loading stiffness  $k_1$  to unloading/reloading stiffness  $k_2$ , as shown in Equation 4-24.

$$\lambda_p = 1 - \frac{k_1}{k_2} \quad (4-24)$$



Therefore, when the virgin loading stiffness  $k_1$  becomes equal to unloading/reloading stiffness  $k_2$ , then the contact plasticity ratio  $\lambda_p$  is equal to 0, leading to elastic particle contact.

Unloading occurs along the  $k_2$  path where force becomes 0 at plastic overlap  $\delta_p$ , as shown in Equation 4-25. This maximum plastic overlap is then tracked and updated in the contact model as a history parameter. The plastic deformation only exists during contact; therefore, the contact information is lost when the particles are separated.

$$\delta_p = \lambda_p^{\frac{1}{n}} \delta \quad (4-25)$$

The limiting adhesion force for the contact  $f_{\min}$  is defined as the maximum adhesion (or pull-off) force that contact can sustain before separation occurs, as defined in Equation 4-26.

$$f_{\min} = \pi \Delta\gamma \psi a \quad (4-26)$$

where  $\Delta\gamma$  is the adhesion energy of the material (J/m<sup>2</sup>),  $\psi$  is the adhesion constant, and  $a$  is the contact patch radius.

The adhesion constant can be found using the JKR theory. The total energy,  $U_T$  is defined as the summation of elastic energy  $U_E$ , mechanical potential energy  $U_M$  and surface energy  $U_s$  of two elastic bodies in collision at a particular condition. The  $f_o$  is the initial loading force and  $f_1$  is the force at the  $k_1$  path. The total energy can be defined as below.

$$U_T = U_E + U_M + U_s \quad (4-27)$$

$$U_T = \frac{1}{K^{\frac{2}{3}} R^{\frac{1}{3}}} \left( \frac{1}{15} f_1^{\frac{5}{3}} + \frac{1}{3} f_0^2 f_1^{-\frac{1}{3}} \right) - \frac{1}{K^{\frac{2}{3}} R^{\frac{1}{3}}} \left( \frac{f_0 f_1^{\frac{2}{3}}}{3} + \frac{2}{3} f_0^2 f_1^{-\frac{1}{3}} \right) - \gamma \pi \frac{R^{\frac{2}{3}} f_1^{\frac{2}{3}}}{K^{\frac{2}{3}}} \quad (4-28)$$

To find the equilibrium condition  $\frac{dU_T}{da_1} = 0$ . However, the load-displacement relationship is given by  $P = \frac{3}{2}Ka_1\delta$ . Therefore, the equilibrium condition is modified to  $\frac{dU_T}{dP_1} = 0$ .

$$\frac{dU_T}{dP_1} = \frac{f_1^{\frac{-4}{3}}}{9K^{\frac{2}{3}}R^{\frac{1}{3}}}(f_1^2 - f_0^2 - 2f_1f_0 + 2f_0^2 - 6\gamma\pi Rf_1) = 0 \quad (4-29)$$

Therefore, at a stable equilibrium condition,

$$f_1 = f_0 + 3\gamma\pi R + \sqrt{6\gamma\pi Rf_0 + (3\gamma\pi R)^2} \quad (4-30)$$

Using the Hertz equation, the force  $f$  is corresponding to the contact radius  $a$ , which can be defined at point 1, is  $a^3 = Rf/K$ . Using the Hertz equation, Equation 4-30 is modified as given below.

$$a^3 = \frac{R}{K}(f + 3\gamma\pi R + \sqrt{(6\gamma\pi Rf) + (3\gamma\pi R)^2}) \quad (4-31)$$

When the applied load  $f$  is negative, then the contact radius decreases. Therefore to get a real solution to Equation 4-31.

$$6\gamma\pi Rf \leq (3\gamma\pi R)^2 \quad (4-32)$$

$$f \geq \frac{-3}{2}\gamma\pi R \quad (4-33)$$

Therefore, separation of the spheres occurs when

$$f_{\min} = \frac{-3}{2}\gamma\pi a \quad (4-34)$$

Therefore on comparing, Equation 4-26 and 4-34, the adhesive constant  $\psi = \frac{-3}{2}$ .

The total contact force during particle interaction can be categorised into total normal contact force  $F_n$  and the contact tangential force  $F_t$ . The total contact force  $F_n$  is the summation of the hysteretic spring force,  $f_{hys}$  and the normal damping force  $f_n^d$ , as shown in Equation 4-35.

$$F_n = (f_{hys} + f_n^d)u \quad (4-35)$$

where  $u$  is the unit normal vector pointing from the contact point to the particle's centre. The force overlap relationship for normal contact  $f_{hys}$  is mathematically shown in Equation 4-36.

$$f_{hys} = \begin{cases} f_0 + k_1 \delta^n & \text{if } k_2(\delta^n - \delta_p^n) \geq k_1 \delta^n \\ f_0 + k_2(\delta^n - \delta_p^n) & \text{if } k_1 \delta^n > k_2(\delta^n - \delta_p^n) > -k_{adh} \delta^n \\ f_0 - k_{adh} \delta^n & \text{if } -k_{adh} \delta^n > k_2(\delta^n - \delta_p^n) \end{cases} \quad (4-36)$$

The normal damping force  $f_n^d$  is given by Equation 4-37.

$$f_n^d = \begin{cases} \beta_l v_n & \text{if } n = 1 \\ -2 \sqrt{\frac{5}{6}} \beta_{nl} \sqrt{k_n m} v_n & \text{if } n > 1 \end{cases} \quad (4-37)$$

where  $v_n$  is the relative normal velocity between two particles at their contact, and  $k_n$  is the Hertzian normal stiffness which depends on the equivalent radius, Young's modulus and normal overlap  $\delta_p$  and can be calculated from Equation 4-38.

$$k_n = 2E\sqrt{R\delta_n} \quad (4-38)$$

The normal dashpot coefficient for the linear  $\beta_l$  and non-linear  $\beta_{nl}$  model is given by Equation 4-39 and 4-40, respectively.

$$\beta_l = \sqrt{\frac{4m^*k_1}{1 + \left(\frac{\pi}{\ln e}\right)^2}} \quad (4-39)$$

$$\beta_{nl} = \frac{\ln e}{\sqrt{\ln^2 e + \pi^2}} \quad (4-40)$$

where the equivalent mass  $m^*$  of the particles is equal to  $m_i m_j / (m_i + m_j)$ , and  $e$  is the coefficient of restitution given to the simulation code as an input parameter.

The contact tangential force  $F_t$  can be defined as the summation of tangential spring force  $f_{ts}$  and tangential damping force  $f_{td}$ , as given by Equation 4-41.

$$F_t = f_{ts} + f_{td} \quad (4-41)$$

The tangential spring force is expressed in incremental terms, as shown below.

$$F_{ts} = f_{ts(n-1)} + \Delta f_{ts} \quad (4-42)$$

where  $f_{ts(n-1)}$  is the tangential spring force at the previous time step and  $\Delta f_{ts}$  is the increment in the tangential spring force and is given by Equation 4-43.

$$\Delta f_{ts} = k_t \delta_t \quad (4-43)$$

where  $k_t$  is the tangential stiffness and  $\delta_t$  is the tangential displacement. The tangential stiffness  $k_t$  can be used for both linear and non-linear cases and is given by Equation 4-44, where  $\zeta_{tm}$  is the tangential stiffness multiplier.

$$k_t = \zeta_{tm} \begin{cases} k_1 & n = 1 \\ 8G^* \sqrt{R\delta_n} & n > 1 \end{cases} \quad (4-44)$$

The tangential stiffness multiplier  $\zeta_{tm}$  serves as a link between tangential stiffness  $k_t$  and normal stiffness  $k_1$  in the linear case. When  $\zeta_{tm}$  equal to 1, then tangential stiffness  $k_t$  is equal to normal stiffness  $k_1$ . The tangential stiffness multiplier ratio lies in the range of 2/3 to 1 for real elastic materials depending upon their Poisson's ratio of the material. While varying values of tangential stiffness multiplier  $\zeta_{tm}$  is used in literature; in this study, it is set to 2/3 (Walton et al.,1986).

The tangential damping force can be expressed as Equation 4-45, depending on the linear or non-linear case.

$$f_t^d = \begin{cases} -\beta_t v_t & n = 1 \\ -2 \sqrt{\frac{5}{6}} \beta_{nl} \sqrt{k_t m^*} v_t & n > 1 \end{cases} \quad (4-45)$$

where  $v_t$  is the relative tangential velocity between the particles at their contact,  $k_t$  is the tangential stiffness which is calculated by Equation 4-44. The linear tangential dashpot coefficient  $\beta_t$  is dependent on tangential stiffness and can be given by Equation 4-46.

$$\beta_t = \sqrt{\frac{4m^*k_t}{1 + \left(\frac{\pi}{\ln e}\right)^2}} \quad (4-46)$$

The limiting tangential friction force  $f_{ct}$  is calculated by Coulomb friction criteria with the normal force replaced by adhesive forces, as shown in Equation 4-47.

$$f_{ct} = \mu |(f_{hys} + k_{adh}\delta^n - f_0)| \quad (4-47)$$

where  $\mu$  is the friction coefficient. The above equation has some limited experimental studies in support conducted by Skinner and Gane (1972), who conducted micro friction experiments between a soft metal stylus and a hard smooth surface of graphite or diamond in a scanning electron microscope. The result showed that the attractive force can be considered as an additive term to the normal force while calculating limiting friction. Based on contact mechanics theory, Savkoor et al. (1977) and Thornton et al. (1991) also theoretically derived a similar equation. The limiting friction criteria is also consistent with existing tangential force models such as Walton et al. (2009), Luding (2008), Tomas (2003) and Thornton et al. (1991).

The tangential force  $F_{tt}$  is coupled with the normal force satisfying the Coulombic friction criteria. It should be such that the tangential force does not exceed the maximum allowed friction value of the shear slider  $f_{ct}$ .

$$F_{tt} = \begin{cases} f_t + f_t^d & f_t \leq \mu_s f_{hys} \\ f_{ct} & f_t > \mu_s f_{hys} \end{cases} \quad (4-48)$$

where,  $\mu_s$  is the coefficient of sliding friction.

Although both linear and non-linear relationships have been found between load and friction in microscopic friction experiments (Skinner et al., 1972; Berman et al., 1998; Briscoe et al., 1979; Ecke et al., 2001; Ruths et al., 2003; Jones et al., 2004). In this study, a non-linear relationship is selected for two main reasons. Firstly, real materials have multi-asperity contacts, and literature suggested that multi-asperity contacts or plastic contacts share a non-linear relationship. Secondly, the normal load response of fine adhesive particles at inter-particle

contacts is non-linear elastic-plastic in most cases with averaged normal macroscopic stresses above 1 kPa.

The default EDEM rolling friction model is adopted in the present study, which was proposed by Ai et al. (2011). It is a contact independent directional constant torque model. The total applied torque  $\tau_i$  is given by Equation 4-49.

$$\tau_i = -\mu_r f_{hys} R_i \omega_i \quad (4-49)$$

where,  $\mu_r$  is the coefficient of rolling friction between two particles, and a particle and a wall.  $\mu_r$  is the ratio of the rolling resistance force (F) and the normal reaction force (N).  $R_i$  is the distance between the contact point and particle centre of mass and  $\omega_i$  is the angular velocity at the contact point.

#### 4.4.4 Particle modelling

Previous studies have shown that the particle shape plays a crucial role in the bulk behaviour of particulate systems such as porosity (Hidalgo et al., 2009) and compaction, and computational time. The spherical particle is widely used as it is computationally less expensive than non-spherical particle shapes. Moreover, the arbitrary shapes are derived from the combination of spherical particles to calculate the interference volume which is required to calculate the inter-particle forces by the simulation solver. However, Walton (1994) found that the effect of particle shape could not be approximated by adjusting inter-particle friction. Non-spherical particle shape is required to achieve the angle of repose greater than 31 degrees in the simulation. Lin and Ng (1997) analysed the enhanced shear strength and reduced particle rotation for non-spherical particles compression than spherical particles. The formation of high-density inter-particle contacts is attributed to the increased strength. Hartle and Ooi (2008) found the increased peak friction angle for non-spherical particles as they provide a higher degree of particles interlocking than spherical particles. Therefore, in the present thesis, non-spherical particles define the soil medium.

#### 4.4.5 Numerical integration method

Calculation of particle interaction is the most time-consuming step in DEM. Therefore, the time step ( $\Delta t$ ) becomes a critically important choice in simulations. It has to be sufficiently small for two reasons: to prevent excessive overlaps, which result in unrealistic high forces and avoid disturbance waves (or Rayleigh waves). The movement of particles within the granular flow is affected by immediate neighbouring contacts and disturbance propagated from particles far away. By choosing a small enough time step in DEM, disturbance waves from each particle are prevented from propagating further than its neighbouring partners. Therefore, an appropriate time step (Rayleigh time step) is approximated from the Rayleigh surface wave propagation speed. Generally, a fraction of this time step is taken to ensure realistic force transmission rates and prevent numerical instability. A typical time step is in the range of  $1e-4$  to  $1e-6$  s.

$$T_R = \frac{\pi R \sqrt{\rho/G}}{0.1631\vartheta + 0.8766} \quad (4-50)$$

where,  $T_R$  is Rayleigh time step;

$R$  is the equivalent particle radius;

$\rho$  is the density;

$G$  is the shear modulus;

$\vartheta$  is the Poisson's ratio of the particle.

Various integration schemes are used to calculate the particle trajectory and rotations in DEM. The positions and velocities of particles are known at time  $t$ . Then, the sum of each particle's forces is calculated, yielding each particle's net acceleration. This acceleration is numerically integrated to determine the particle's positions and velocities at the next time step ( $t + \Delta t$ ). An excellent numerical scheme must be stable, should not amplify errors, achieve the required accuracy, and satisfy the energy and momentum conservation. Furthermore, it should not use excessive memory or be computationally time-consuming. Two

critical numerical schemes derived from the Taylor series expansion used in EDEM simulations are the Euler scheme and the Verlet scheme.

The Euler scheme is a one-step scheme derived from a truncated Taylor expansion after the first-order term. It is regarded as a straightforward integration scheme and is widely used in DEM simulations due to its simplicity and feasibility and requires minimal memory. The forces are calculated once per time step:

$$x(t + \Delta t) = x(t) + v(t)\Delta t \quad (4-51)$$

$$v(t + \Delta t) = v(t) + a(t)\Delta t \quad (4-52)$$

The Verlet scheme is an integration method of second order. Positions and velocities are calculated for time steps, which are  $\Delta t/2$  apart. It is better for particle flow where damping is considered. This method comes at a small additional computational cost (up to 10%) but with increased computational accuracy. It is calculated as:

$$v\left(t + \frac{1}{2}\Delta t\right) = v\left(t - \frac{1}{2}\Delta t\right) + a(t)\Delta t \quad (4-53)$$

$$x(t + \Delta t) = x(t) + v\left(t + \frac{1}{2}\Delta t\right)\Delta t \quad (4-54)$$

$$v(t + \Delta t) = v\left(t + \frac{1}{2}\Delta t\right) + \frac{1}{2}a(t)\Delta t \quad (4-55)$$

Both the Euler and Verlet scheme are used to conduct the simulation based on the computational efficiency and accuracy required. The auto-time step box was ticked in EDEM, which automatically adjusts the Rayleigh Time Step to the smallest radius currently in the simulation rather than the smallest radius to be created. It uses 20% for this particle size. For example, a simulation with 10 mm particles from  $t=0s$  and 3 mm particles from  $t=3s$  will use 20% of the Rayleigh time step obtained with 10mm until time 3s, when it would be updated to the time step value obtained with 3mm.



## **4.5 Numerical experiments**

### **4.5.1 Soil data**

Soil data represents the soil texture and strength parameters, which differentiate between types of soils. In the present study, two soil samples were selected from the NATO NRMM CDT (2018). They are 2NS sand and fine-grained soil. Both sands are selected because of the availability of the soil test data (e.g., pressure-sinkage) and mobility performance test data (drawbar pull test data). 2NS sand is a poorly graded medium sand but contains a fraction closer to the coarse sand than the coarse pit (NATO CDT-308 (2020)). It also has more angular sand particles than the coarse pit. There are gravel particles in the sand, which makes it a little stiffer (Table 4.1). The fine-grained sand has cohesiveness due to the presence of silt and clay. It is sandy silt soil of low plasticity. It was selected since it is highly susceptible to moisture content such as 6% moisture is very stable whereas 18% moisture has nearly no strength. Based on the soil texture triangle (Figure 4.2), 2NS sand is a gravelly sand and Fine-grained soil is a sandy silt soil.

The soil samples were taken from two different soil areas near MKU/KRC test centre for laboratory testing. The sieve analysis, moisture content and bulk density were conducted to determine the soil conditions. Figure 4-11 shows the grain size distribution for both soils. Table 4-2 shows the soil condition results for both soil samples. There was no organic content found in both soil samples.

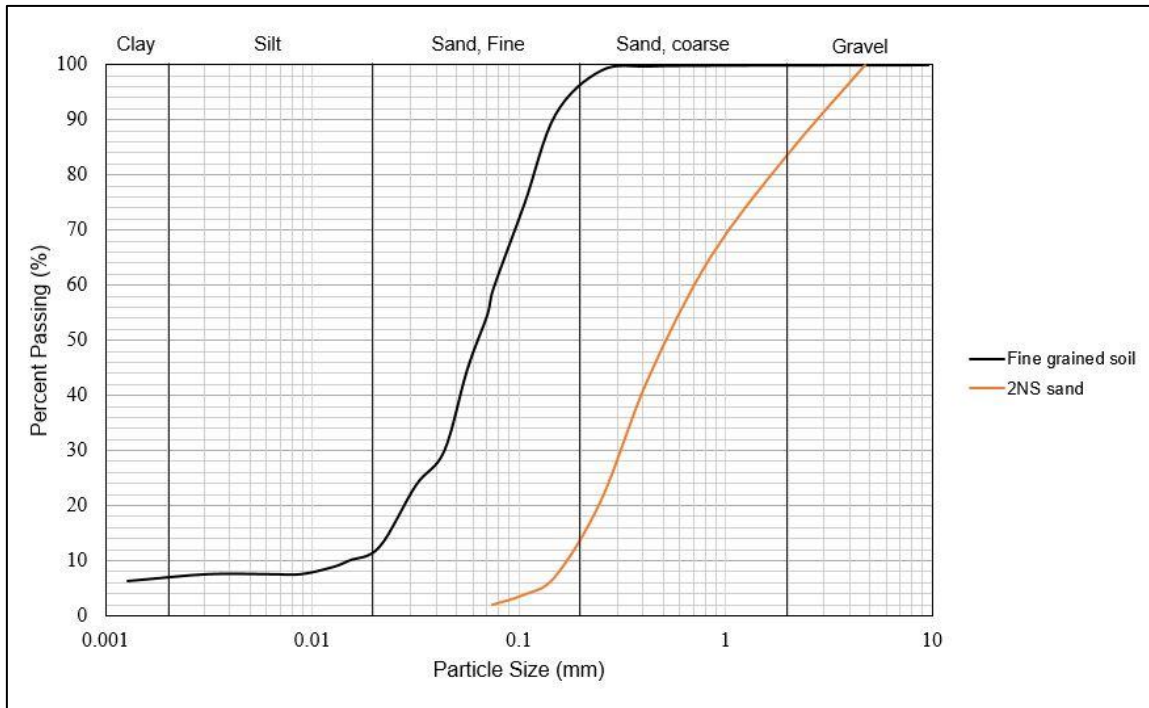


**Figure 4-11 Grain size analysis (NATO NRMM CDT (2018))**



**Figure 4-12 1) 2NS sand sample 2) Fine grain soil on field (NATO CDT – 308 (2020))**

**Table 4-2 Physical parameter of soil sample (NATO CDT – 308 (2020))**

| <b>Parameters</b>                | <b>2NS sand</b> | <b>Fine-grained soil</b> |
|----------------------------------|-----------------|--------------------------|
| D10/mm                           | 0.17            | 0.015                    |
| D30/mm                           | 0.3             | 0.042                    |
| D60/mm                           | 0.68            | 0.074                    |
| %gravel                          | 16.0            | 0.0                      |
| %sand, coarse                    | 70.0            | 4.0                      |
| %sand, fine                      | 14.0            | 84.0                     |
| %silt                            | 0.0             | 5.5                      |
| %clay                            | 0.0             | 6.5                      |
| Coefficient of uniformity,<br>Cu | 4               | 4.9                      |
| Coefficient of curvature,<br>Cc  | 0.7             | 1.58                     |
| Porosity                         | 0.5             | 0.7                      |
| Bulk density/ kg/m <sup>3</sup>  | 2722            | 1488                     |
| Water content/ w%                | 1.13            | 18.55                    |
| Organic content/ %               | 0               | 0.3                      |

#### **4.5.2 Laboratory model experiment**

In laboratory experiments, the prepared soil sample follows the simplifying assumptions used in the complex Terramechanics models. NATO CDT-308 (2020) states that the soil sample can be assumed isotropic (same response in any direction), homogeneous/ uniform (same properties in any location), constant moisture content (change in moisture content takes much longer time than vehicle loading) and constant temperature. Therefore, constant particle shape

and size distribution were maintained to develop the laboratory soil conditions in simulations. 3D 3- sphere particle model (as shown in figure 4.13) was selected as discussed in section 4.4.4 because it limits rolling and encourages particle interlocking in the consolidated bed of material. Particle-particle contact was modelled using the EpAM contact model as discussed in section 4.4.3 due to its ability to capture physical soft soil macro-mechanical behaviour and computationally efficient mesoscopic modelling approach.

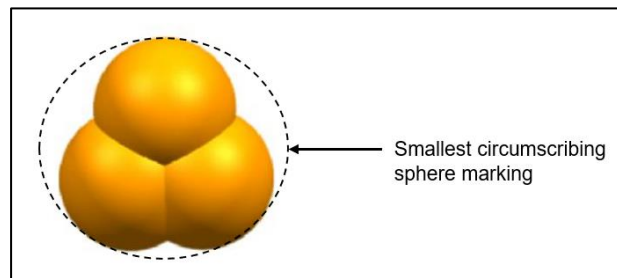


**Figure 4-13 3D 3-sphere particle model**

The mechanical behaviour of the soil model is determined through horizontal and vertical stress-strain relationships. The soil model's shear properties are determined using the direct shear test in simulations. Wright (2012) found that the yield surface parameters derived from the direct shear tests were in good agreement with the experimental sinkage profiles and produces a lower mean percentage error. The vertical soil parameters are determined using the pressure-sinkage test simulations using two different plate sizes. The soil strength of the model is determined using cone penetrometer test simulations. The simulation procedure for each test is discussed below in the following sequence: 1) The direct shear test, 2) The pressure-sinkage test, and 3) the cone penetrometer test

In the direct shear test, the particles generated in a shear box model of dimensions 0.5 m x 0.5 m x 0.8 m is shown in Figure 4-14. A vertical force is applied to the top of the shear box through a compression plate, which remains constant throughout the simulation time. The soil sample undergoes static shearing with the bottom shear box moving horizontally with a velocity of 3 mm/min. The simulations were run till the shearing is completed or up to 5 cm of horizontal displacement. The simulation methodology was similar to ASTM

International (2004) D3080-04, the standard procedure code for the Direct Shear Test.

Further, the simulation time step was set to  $5.59 \times 10^{-5}$  s, and data was saved at every 1 s interval. Euler time integration scheme was used to simulate to improve computational efficiency. The tests were performed at four normal stress conditions 13.79 kPa (2 psi), 27.58 kPa (4 psi), 41.37 kPa (6 psi) and 55.16 kPa (8 psi), and shear stress vs shear displacement curves were obtained. The normal stress vs shear stress curves were plotted to determine soil shear properties. A similar simulation methodology was followed for both soil samples.

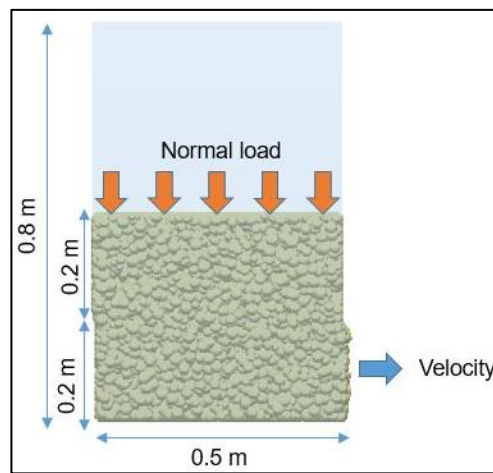


**Figure 4-14 Direct shear box set up**

In the pressure-sinkage test, the particles were generated in a closed enclosed box of dimensions 1.2 m x 1.2 m x 0.8 m, as shown in Figure 4-15. NATO NRMM CDT (2018) conducted experiments using circular bevameter plates of dimensions 0.1 m and 0.15 m, respectively. Therefore, the same plate sizes were used in the present simulations. The simulations were run for 20 s for 2NS sand and 4.50 s for FGS in the following steps. Initial 1 s was used to generate and set up particles in the box. The successful particle setup in the box was checked by the total internal energy plot as shown in Figure 4.16. After the curve becomes constant, there is no further motion inside the particle model and the soil model is stabilized. Further, followed by the next 6.50 s were used to compress the particles using a compression plate for 2NS sand. However, no pre-compression was required for FGS. Next 12 s and 4 s of sinkage plate penetrating the soil bed

for 2NS sand and FGS, respectively. The sinkage velocity of plates was maintained at 0.025 m/s. The simulation time step was set to  $5.595 \times 10^{-5}$  s, and data was saved at every 0.10 s interval. Verlet scheme simulator was used for increased computational accuracy but at an additional computational cost. The pressure vs sinkage curve was obtained. The data set was further processed to determine Bernstein and Bekker's model parameters.

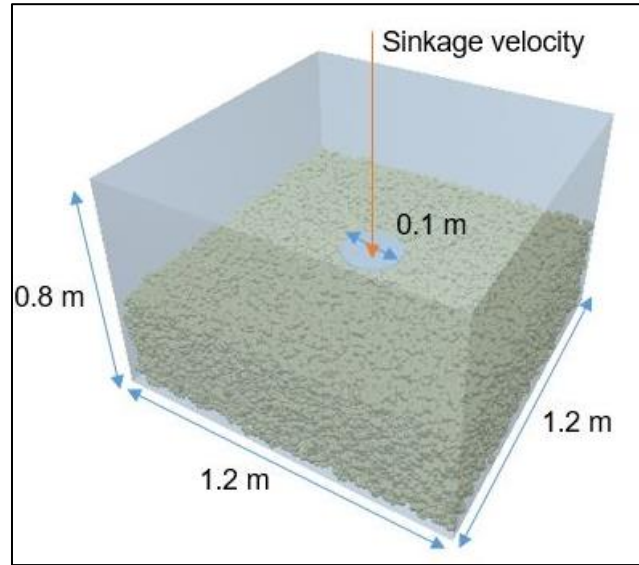


**Figure 4-15 Pressure sinkage test set up**

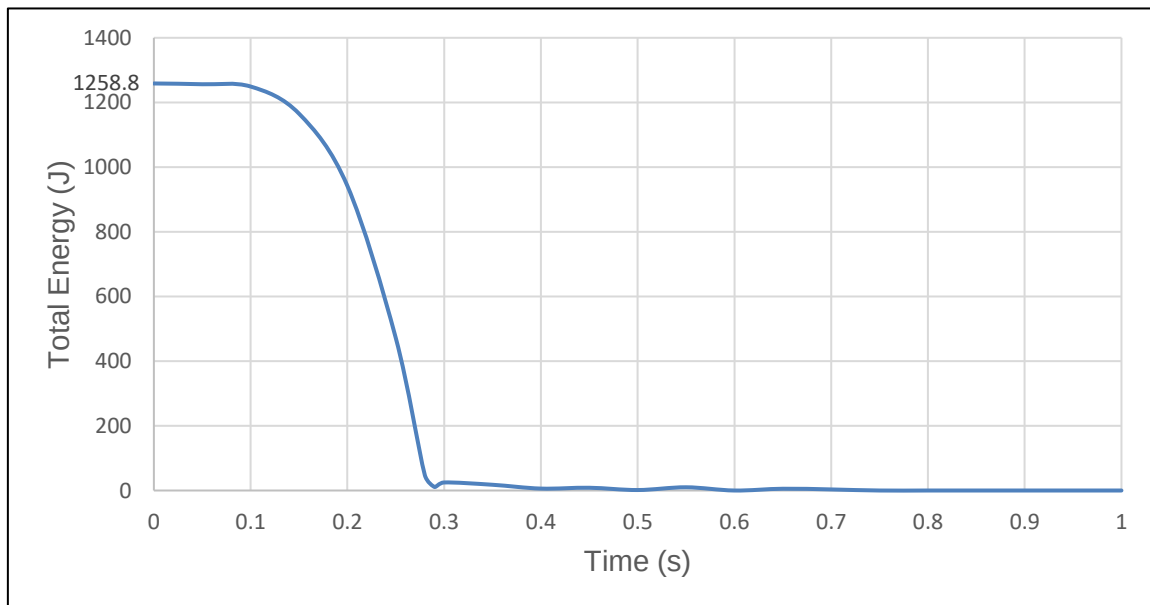


**Figure 4-16 The total energy vs time plot of particle generation inside the box**

In the cone penetrometer test, the particles were generated in an open box of dimensions 6 m x 4 m x 2 m. The soil bed preparation method was similar, as described above for 2NS sand and FGS. The cone penetrometer's CAD geometry was developed as per the standard cone penetrometer (ASABE, 2006), as shown in Figure 4-17. The sinkage velocity of the cone penetrometer was maintained at 0.01 m/s. The simulations were run for 25 s to attain a depth of 0.25 m (as per Wismer-Luth (1974) and NATO NRMM CDT (2018)). The simulation time step was 5.594e-5 s, and data was saved at every 0.1 s interval. The pressure acting at the cone penetrometer's circular base was obtained as a function of time. This measured pressure is the cone index (CI) of the soil sample. The test was repeated three times at different locations on the testbed.

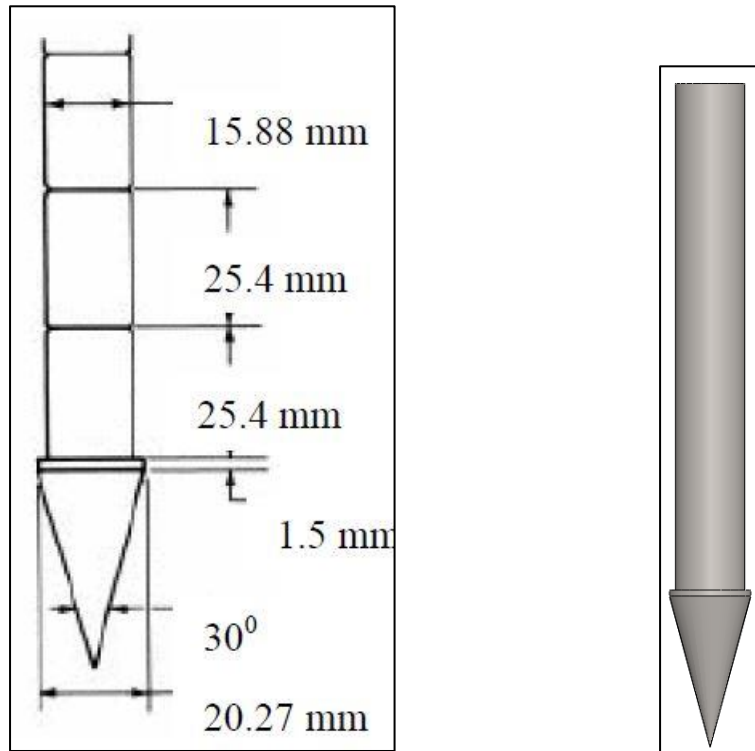


**Figure 4-17 Standard cone penetrometer model**

The custom EDEM Coupling Interface application controlled the top layer height, pre-compression height, test velocity, overall simulation time and parameterized inputs. The input simulation parameters are shown in Table 4-3 for both soil samples. The initial parameter values were chosen from the recommended material library from the EDEM. Later, contact model parameters were iterated to

validate with the experimental (direct shear box and pressure-sinkage tests) results (Wu et al., 2022; Morrissey, 2013; Nakashima et al., 2004, 2007, 2009). The simulated soil was also further compressed to a defined pre-compression height using a compression plate to control the soil packing minimally. Therefore, the dimensions of test setup geometry have been chosen such that there is no boundary effect during the simulations. Further, the static and rolling friction coefficients are taken from the EDEM material model library.

**Table 4-3 Soil material parameters in simulation models**

| <b>Input parameter</b>                            | <b>2NS sand</b> | <b>Fine-grained soil</b> |
|---------------------------------------------------|-----------------|--------------------------|
| Particle size (mm)                                | 10              | 10                       |
| Particle density $\rho$ (kg/m <sup>3</sup> )      | 2500            | 2500                     |
| Shear modulus, G (Pa)                             | 5E+6            | 5E+8                     |
| Poisson's ratio, $\nu$                            | 0.4             | 0.25                     |
| Restitution, e                                    | 0.6             | 0.6                      |
| Static friction                                   | 0.5             | 0.5                      |
| Rolling friction                                  | 0.01            | 0.01                     |
| Constant pull off force $f_0$ (N)                 | -60             | -2                       |
| Surface energy $\Delta\gamma$ (J/m <sup>2</sup> ) | 0               | 7                        |
| Contact plasticity ratio $\lambda_p$              | 0.5             | 0.9                      |
| Slope exponent n                                  | 1.5             | 1.5                      |
| Tensile exponent                                  | 1.5             | 1.5                      |
| Tangential stiffness multiplier                   | 0.67            | 0.67                     |
| Pre-compaction height/ m                          | 0.150           | 0                        |



| Input parameter                         | 2NS sand | Fine-grained soil |
|-----------------------------------------|----------|-------------------|
| Average bulk density/ kg/m <sup>3</sup> | 2716     | 1503              |
| Moisture content/%                      | 1.13     | 18.55             |

#### 4.5.2.1 Results and validation

The fundamental assumptions used in simulating the Terramechanics experimental methods and their associated analytical constructs are: the particle size has been up-scaled to improve the simulation time (Smith et al., 2013 and Yonghao Du et al., 2017, 2018). The soil model's bulk behaviour is adjusted to get the desired experimental soil response by iterating the pull-off force and surface energy in the contact model parameters. However, there is a limit to the particle upscaling, so that the discrete particle sizes should not be significantly affecting the simulation results. Further, the prepared soil model is homogeneous and deep enough to be affected by boundary wall effects. In the pressure-sinkage simulations, the soil is assumed to be unconfined that is without any lateral support. Therefore, the lateral surface is free from any stress. In the direct shear test simulations, the soil is confined in upper and lower rigid boxes. However, the failure plane in this test is assumed to be horizontal with a uniform stress state. Further, the single-layer soil response is considered throughout the study. Moreover, the penetration plates and cone penetrometer models are good stress state surrogates (NATO CDT- 308 (2020)).

Figure 4-13 to 4-22 shows the simulation results in terms of macroscopic behaviour for the 2NS sand and FGS under laboratory assumptions. To validate the macro-mechanical properties between the laboratory soil samples and the DEM model, the direct shear test, the pressure-sinkage test and the cone penetrometer test are carried out in EDEM under the same test conditions as the experimental and followed the same procedure as described in section 4.5.2.

In direct shear test simulations, the normal and shear forces on the shear plane (or split plane) are deduced from the upper half sample's static equilibrium by summing the vertical and horizontal forces acting on its boundary, respectively.

The vertical displacement of the compression plate is obtained, representing the overall volume change. However, no significant vertical displacement of the compression plate was found during shearing. It might be because the system was simulated to achieve a static equilibrium condition under a specific normal load before shearing. For each normal stress, the simulations were terminated when the horizontal displacement reached 5 cm or the model had already passed the peak shear resistance and levelled off to the critical state hence the peak shear resistance can be identified easily.

The interpretation of all types of shear tests is ordinarily based on the Mohr-Coulomb failure criterion. According to this criterion, the failure planes (shear bands) are planes of maximum stress obliquity. In the present study, constant normal stress simulations were performed, which showed the evolution of the vertical and horizontal normal stress acting in the central part of the specimen approximating the shear band. However, the average vertical normal stress over the complete specimen was controlled to remain constant throughout the simulation. As the relative displacement between the upper and lower parts increases, the shear resistance acting at the shear band increases, reaching the maximum value.

The maximum shear stress vs applied normal stress curves were plotted to obtain the shear properties of the soil model, as shown in Figures 4-18 and 4-19. The cohesion and internal angle of friction of soil are determined using Mohr-Coulomb failure criteria. The cohesion is the intercept of the Mohr-Coulomb failure criteria (equation 4.14), and the angle of friction is its slope. Therefore, the cohesion  $c$  of the 2NS sand model is 4.18 kPa and the internal angle of friction is 28.68 degrees. Similarly, the cohesion  $c$  of the FGS soil is 0.51 kPa and the internal angle of friction is 35.63 degrees. Tables 4-4 and 4-5 show a comparison between obtained simulated shear parameters and NATO CDT-308 (2020) experimental results for 2NS sand and FGS. The NATO experiments were performed at Keweenaw Research Center (KRC), Michigan Technological University. It can be seen that the shear properties of DEM models validate well with the NATO CDT – 308 (2020) experimental results for different moisture conditions. The shear

strength parameters ( $c$  and  $\phi$ ) depend on the soil particle arrangement at the shear plane during the DST. Since 2NS is gravelly sand. Therefore, the particle arrangements at the shear plane vary with the change in the moisture content and bulk density for gravelly sands. Therefore, the trend is inconsistent in Table 4.4. It can be verified that the soil's modelling by DEM represents both sands reasonably accurate.

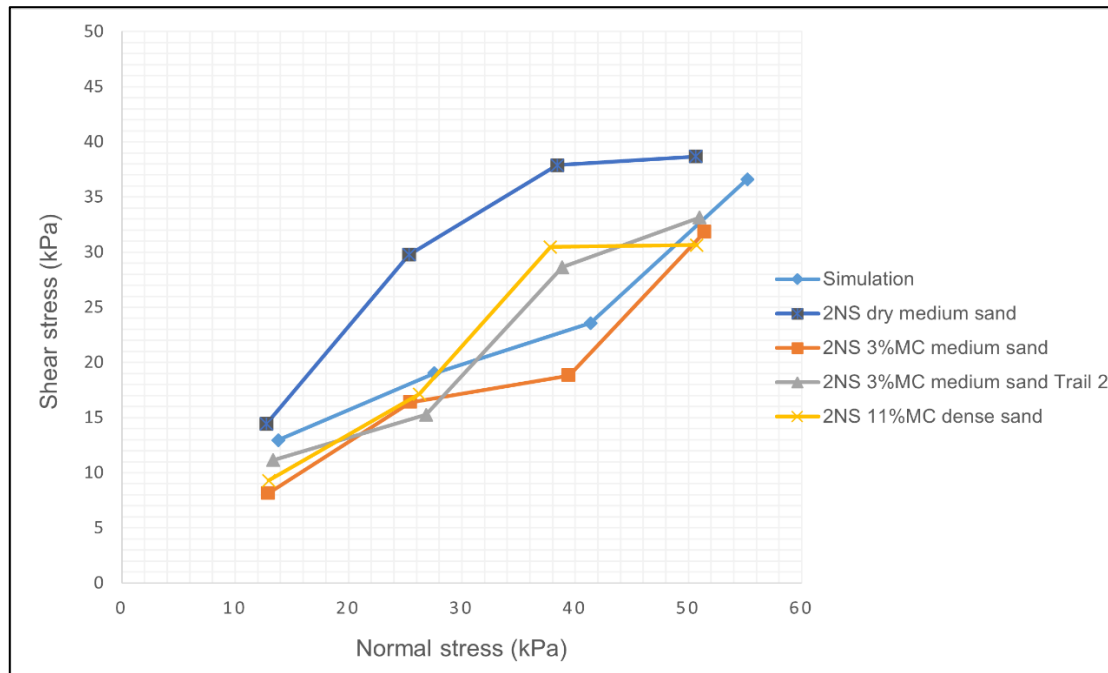


**Figure 4-18 Direct shear box simulation results in comparison with NATO CDT-308 (2020) experimental results for 2NS sand**

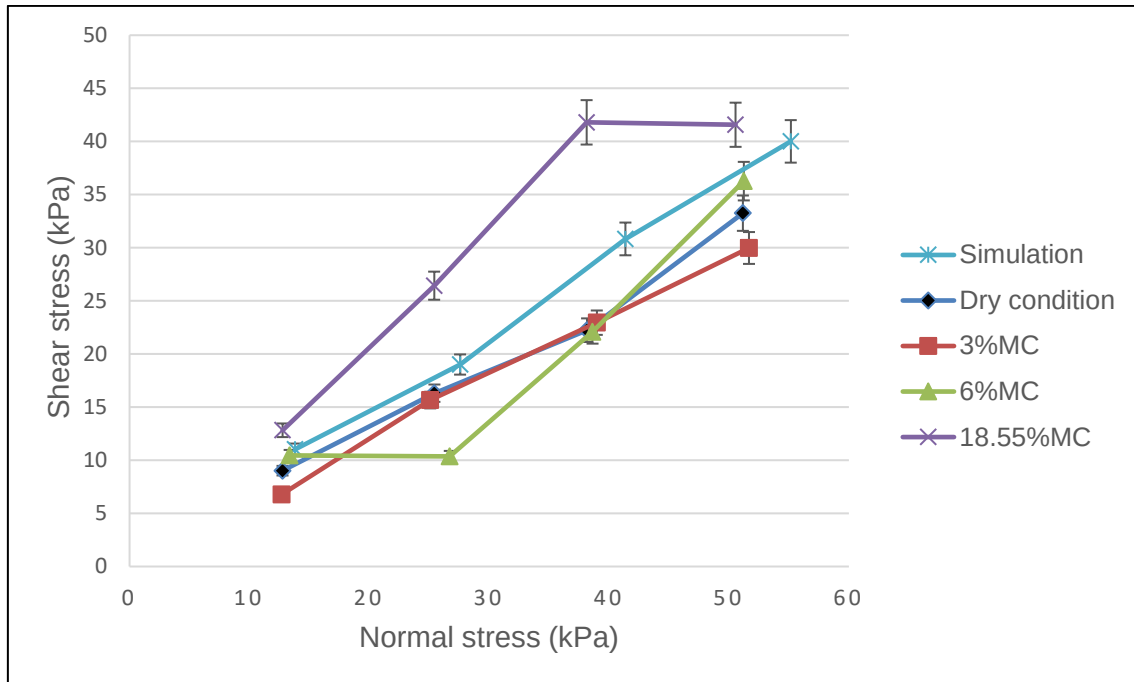


**Figure 4-19 Direct shear box simulation results in comparison with NATO CDT-308 (2020) experimental results for FGS**

**Table 4-4 Simulated soil model shear parameters of 2NS sand in comparison with NATO CDT – 308 (2020) experimental results**

| Soil condition                                    | Cohesion, $c$ (kPa) | Internal angle of friction, $\phi$ (°) |
|---------------------------------------------------|---------------------|----------------------------------------|
| 2NS – dry medium sand                             | 9.86                | 32.59                                  |
| 2NS – 3% moisture content (Trail - 1) medium sand | 0.59                | 29.39                                  |
| 2NS – 3% moisture content (Trail - 2) medium sand | 1.48                | 32.32                                  |
| 2NS – 11% moisture content – dense sand           | 2.19                | 31.67                                  |
| 2NS – simulation model                            | 4.18                | 28.68                                  |

**Table 4-5 Simulated soil model shear parameters of FGS in comparison with NATO CDT – 308 (2020) experimental results**

| Soil condition                             | Cohesion, $c$ (kPa) | Internal angle of friction, $\phi$ (degrees) |
|--------------------------------------------|---------------------|----------------------------------------------|
| FGS – dry condition                        | 0.60                | 35.19                                        |
| FGS – 3% moisture content                  | 0                   | 30.19                                        |
| FGS – 6% moisture content                  | 3.12                | 35.19                                        |
| FGS – 18.55% moisture content – dense sand | 5.05                | 38.92                                        |
| FGS – simulation model                     | 0.51                | 35.63                                        |

Initial pressure-sinkage simulations for the determination of Bekker soil parameters of 2NS sand and FGS were performed using two plate sizes as previously stated; 0.1 m and 0.15 m as shown in Figure 4.20. These tests were simulated under laboratory assumptions on 2NS sand and FGS, and the results are shown in Figures 4-21 to 4-24. It is important to note that the standard Bevameter simulation procedure was used to control soil packing. The pre-compression height was varied from 0 to 0.2 m in increments of 0.025 m to establish the optimum pressure-sinkage relationship in simulations to achieve real soil behaviour. It was found that with the pre-compaction height of 0.15 m, the 2NS simulation model shows more specific real behaviour. However, no pre-compaction was required in the case of FGS.

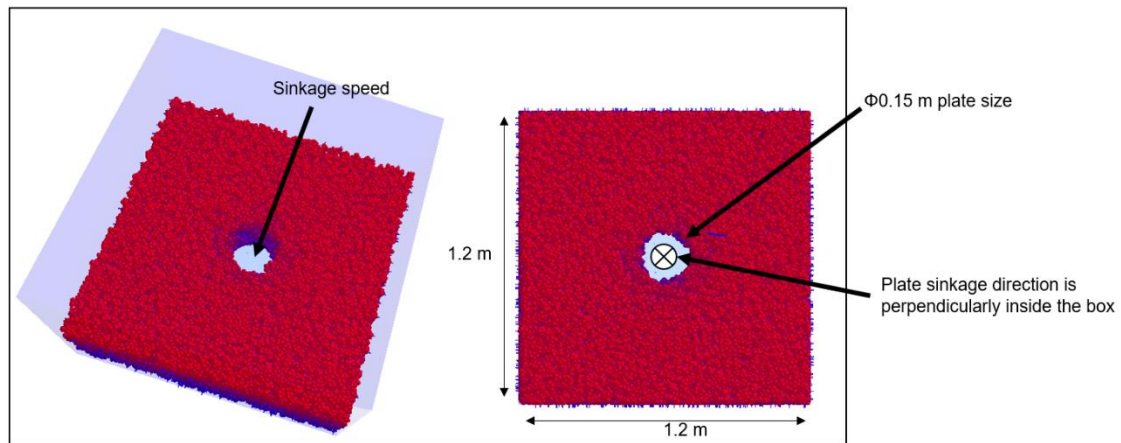


**Figure 4-20 Pressure sinkage simulation images of Dia 0.1 m plate size for 2NS sand in a normal loading condition to soil sample**

Figures 4-21 and 4-22 show the raw simulation data for 0.1 m and 0.15 m plates for 2NS sand. Figures 4-23 and 4-24 show the raw simulation data for 0.1 m and 0.15 m plates for FGS, respectively. The simulation plots were found in good agreement with the experimental data of two runs at two different locations, respectively. Further, the Bernstein model was curve fit in each of the data sets to obtain  $k$  and  $n$  values with respect to each plate run. The R-square (Appendix -D) value (or 'goodness of fit') of each curve is shown in the figures. The method (section 4.3.1.1) does not fit the low sinkage data points well, but it matches well with the rest of the data points (Jayakumar, 2013). The procedure to obtain the model parameters is discussed in section 4.3.1.1. Table 4-6 shows the obtained model parameters. It can be seen that Bernstein curve-fit model corresponds well to the simulated and experimental data. The coefficient of determination ( $R^2$ ) is 0.97 for 0.1 m plate and 0.98 for 0.15 m plate for 2NS sand. Similarly, it is 0.90 for 0.1 m plate and 0.95 for 0.15 m plate for FGS. The closer these values are to 1, the better the fit. In all the cases, the curve fits were reasonably accurate.

Initial observation reveals that for a small change in plate size between 0.1 m and 0.15 m, the pressure-sinkage curves are close to each other for 2NS sand and FGS. It means the size effect is minimal in both soil types.

The prepared 2NS sand model has an average sinkage exponent,  $n_{avg}$  approximately equal to 0.63. Since this value is less than one, it suggests the 'soil

weakening' behaviour, which means that the soil model has reached compaction limits and is failing due to internal shearing in their far-field. Similarly, the prepared FGS model has  $n_{avg}$  approximately equal to 1.26. This value is greater than one, suggesting the 'soil strengthening' behaviour. It means the soil is the loose type and is under compaction. However, it should be noted that a thin hard top crust also behaves like 'soil weakening' initially, but then subsequently under further sinkage, it transits into 'soil strengthening' behaviour as the deeper lower layer is loose and not compacted.

It can be seen in Table 4-6 that there are negative values calculated for  $k_c$  and  $k_\phi$  for FGS. However, these negative values are not an issue because  $k_c$  and  $k_\phi$  do not explicitly represent physical soil parameters and correlate with  $n$ . Therefore, as the value of  $n$  changes,  $k_c$  and  $k_\phi$  changes correspondingly. As per Wong (2010) and Jayakumar (2013), there are no generally accepted guidelines for obtaining the  $k_c$ ,  $k_\phi$  and  $n$  values, therefore, it depends on the method used by the investigator. Therefore, the values of  $k_c$ ,  $k_\phi$  and  $n$  change, based on the method used to calculate them (Oravec, 2009).

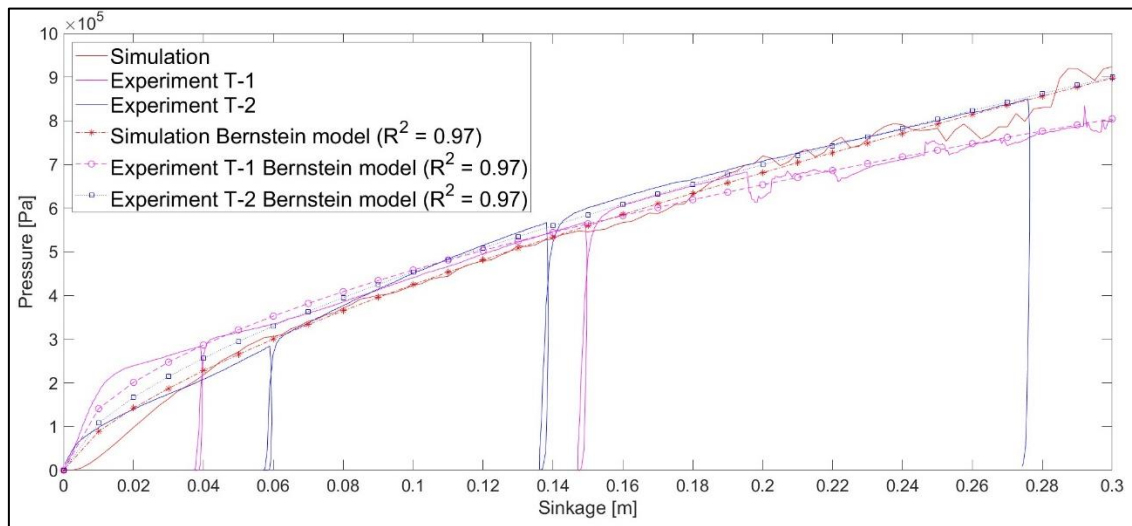


**Figure 4-21 Pressure sinkage simulation results in comparison with NATO CDT-308 (2020) experimental results of Dia 0.1 m plate size for 2NS sand in loading condition**

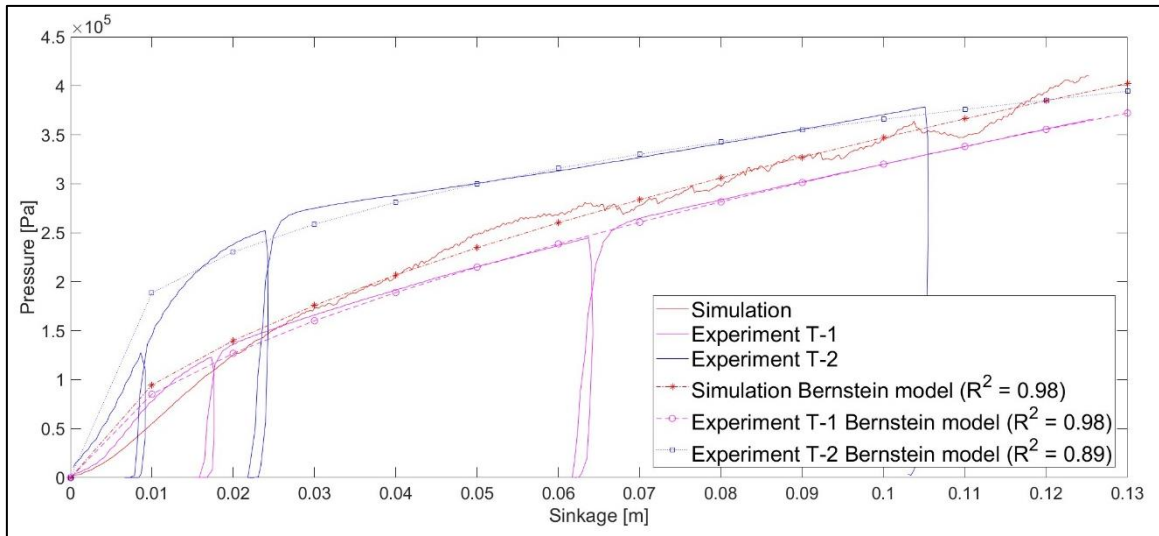


**Figure 4-22 Pressure sinkage simulation results in comparison with NATO CDT-308 (2020) experimental results of Dia 0.15 m plate size for 2NS sand in loading condition**

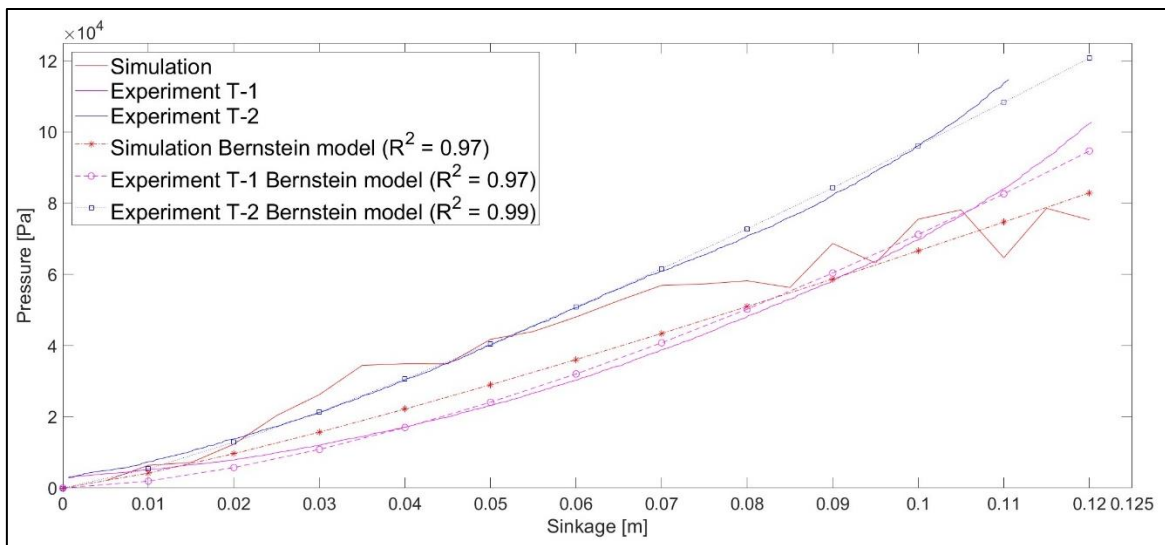


**Figure 4-23 Pressure sinkage simulation results in comparison with NATO CDT-308 (2020) experimental results of Dia 0.1 m plate size for FGS in loading condition**



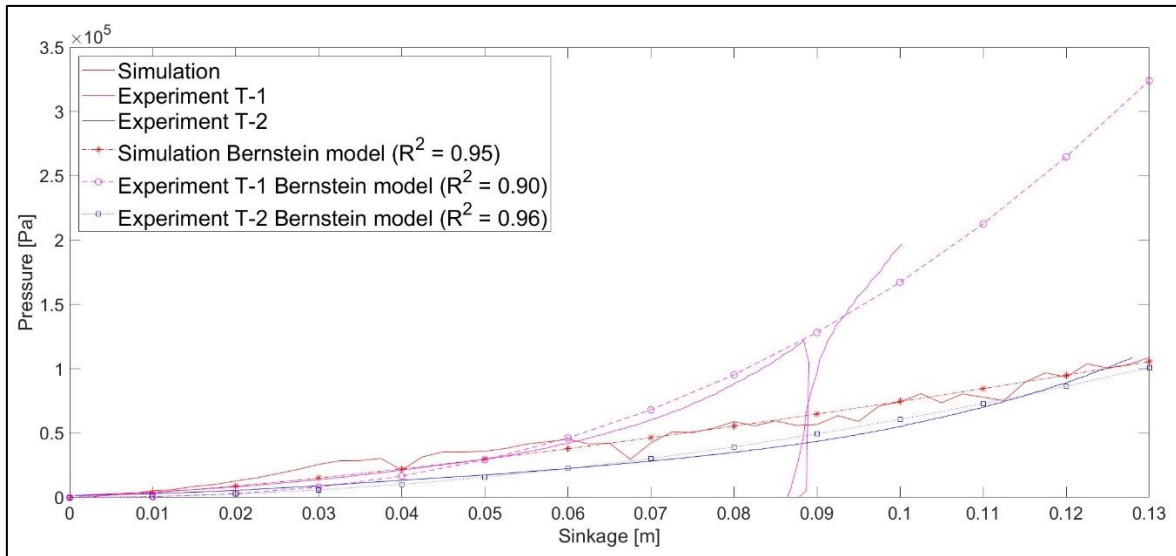


**Figure 4-24 Pressure sinkage simulation results in comparison with NATO CDT-308 (2020) experimental results of Dia 0.15 m plate size for FGS in loading condition**

**Table 4-6 Pressure-sinkage parameters derived for 2NS and FGS**

| Plate diameter | 0.1 m |         | 0.15 m |         | Bekker parameters |          |          |
|----------------|-------|---------|--------|---------|-------------------|----------|----------|
|                | n     | k       | n      | K       | $n_{avg}$         | $k_c$    | $k_\phi$ |
| 2NS            |       |         |        |         |                   |          |          |
| Experiment T-1 | 0.5   | 1.53E+6 | 0.56   | 1.07E+6 | 0.53              | 1.38E+5  | 1.49E+5  |
| Experiment T-2 | 0.67  | 1.51E+6 | 0.32   | 1.26E+6 | 0.49              | 7.61E+4  | 7.49E+5  |
| Simulation     | 0.68  | 1.79E+6 | 0.59   | 1.39E+6 | 0.63              | 1.19E+5  | 5.96E+5  |
| FGS            |       |         |        |         |                   |          |          |
| Experiment T-1 | 1.56  | 8.03E+6 | 2.52   | 5.54E+7 | 2.04              | -2.71E+6 | 3.52 E+7 |
| Experiment T-2 | 1.25  | 4.07E+6 | 1.94   | 5.39E+6 | 1.5               | 4.73E+5  | -6.65E+5 |
| Simulation     | 1.2   | 1.40E+6 | 1.32   | 1.39E+6 | 1.26              | 4.50E+3  | 1.36E+6  |

Further, the cone penetrometer tests were simulated at three locations on the homogeneous soil models for 2NS sand and FGS. Each simulation was performed with the standard cone penetrometer discussed in section 4.3.2. This test can quickly and efficiently provide the cone penetration resistance (CPR) as a function of depth for soil systems while creating little disturbances. This test is conducted before the vehicle mobility testing to ensure controlled preparation of the soil system. Each test was run upto a depth of 0.2 m (7.9 in) based on the available experimental data from NATO CDT (2020) and the recommendation of Wismer and Luth (1973). For each test, a plot of cone index value (kPa) vs depth (m) was created.

The average cone index value is 149.07 kPa (coefficient of variance, CV = 9.38%) for the homogeneous laboratory soil model of 2NS sand compared to 140.65 kPa experimentally. Similarly, the average cone index value of 17.75 kPa (CV = 7.63%) was found for the FGS model compared to 29.29 kPa experimentally. The error in the average cone index values for 2NS sand and FGS models are 5.68% and 39.39% with respect to experimental values.

Further, the corresponding cone index gradient (G) of each test was determined. The gradient (G) was calculated using the slope of the linear regression line through the data points of each simulation set, as shown in Equation 4-56.

$$G = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad (4-56)$$

where,  $y_i$  is the cone penetration resistance (kPa),  $\bar{y}$  is the arithmetical mean of the penetration resistance (kPa),  $x_i$  is the corresponding depth,  $\bar{x}$  is the arithmetical mean of the depth (m), and n is the total number of observations. The obtained values of G of each run for both soils are shown in Table 4-7. The average cone index gradient (G) of the homogeneous 2NS sand model is 1118.19 kPa/m with a standard deviation of  $\pm 16.11$  kPa/m for the 2NS sand model. Similarly, the average cone index gradient (G) of the homogeneous FGS model is 194.60 kPa/m with a standard deviation of  $\pm 14.86$  kPa/m. The coefficient of variance (CV) for the gradient is 1.42% for 2NS and 7.62 % for FGS obtained in simulations.

It can be seen in Figure 4-25 and 4-26 that the curves of cone penetration resistance generally increases linearly with increasing depth. The linearity of this curve reflects the homogeneity of the soil sample with respect to depth. It means the more linear the curve, the more homogeneous is the soil model. The homogeneity can be evaluated by a second-order polynomial curve fit for the average cone index vs depth plot, as shown in Figures 4-27 and 4-28. The linearity can be determined by inspecting the coefficient of the second-order term. The closer this coefficient is to zero, the more linear the curve is, which means the soil model is relatively homogeneous. If this coefficient is slightly positive, then generally, soil strength increases with depth. If this coefficient is slightly negative, then generally, soil strength decreases with depth. As shown in Figures 4-27 and 4-28, the coefficient of the second-order polynomial is close to zero for both soils, which shows nearly linear behaviour. Therefore, both soil models can be considered homogeneous.

**Table 4-7 Average cone index gradient, G**

| <b>Category</b>               | <b>Average cone index gradient, G<br/>(kPa/m)</b> |
|-------------------------------|---------------------------------------------------|
| 2NS Laboratory simulation T-1 | 1153.88                                           |
| 2NS Laboratory simulation T-2 | 1140.92                                           |
| 2NS Laboratory simulation T-3 | 1115.11                                           |
| 2NS Experiment                | 3637.393                                          |
| FGS Laboratory simulation T-1 | 216.32                                            |
| FGS Laboratory simulation T-2 | 174.34                                            |
| FGS Laboratory simulation T-3 | 193.14                                            |
| FGS Experiment                | 200.35                                            |

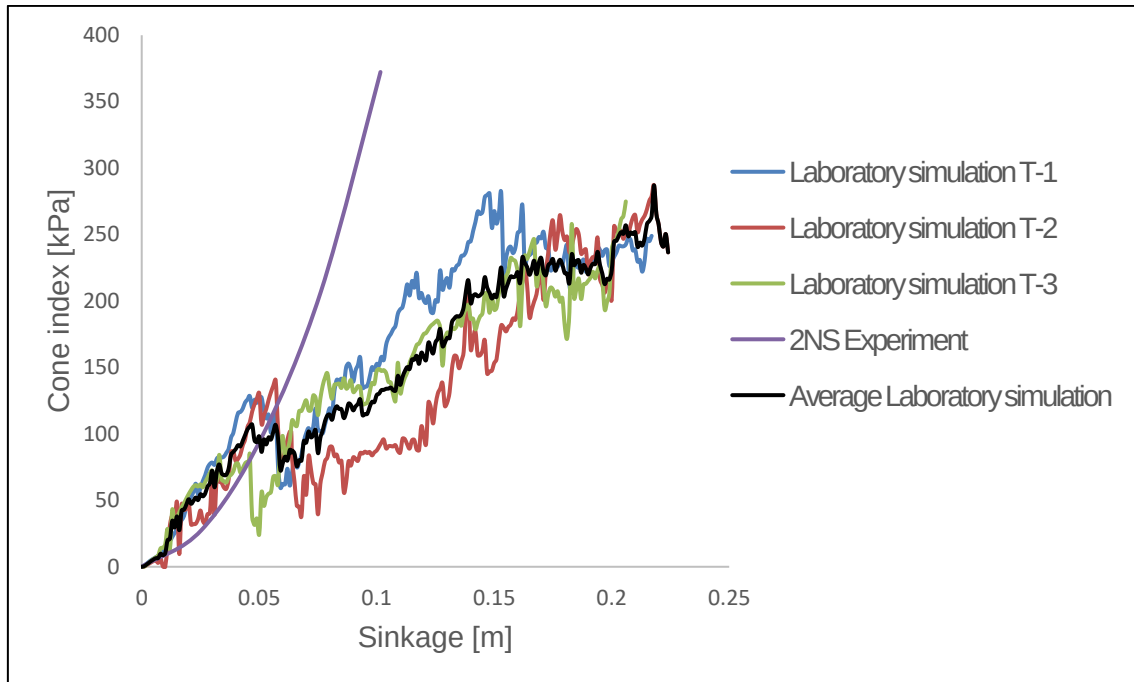


**Figure 4-25 Cone penetrometer laboratory simulation results in comparison with NATO CDT-308 (2020) experimental results for 2NS sand**

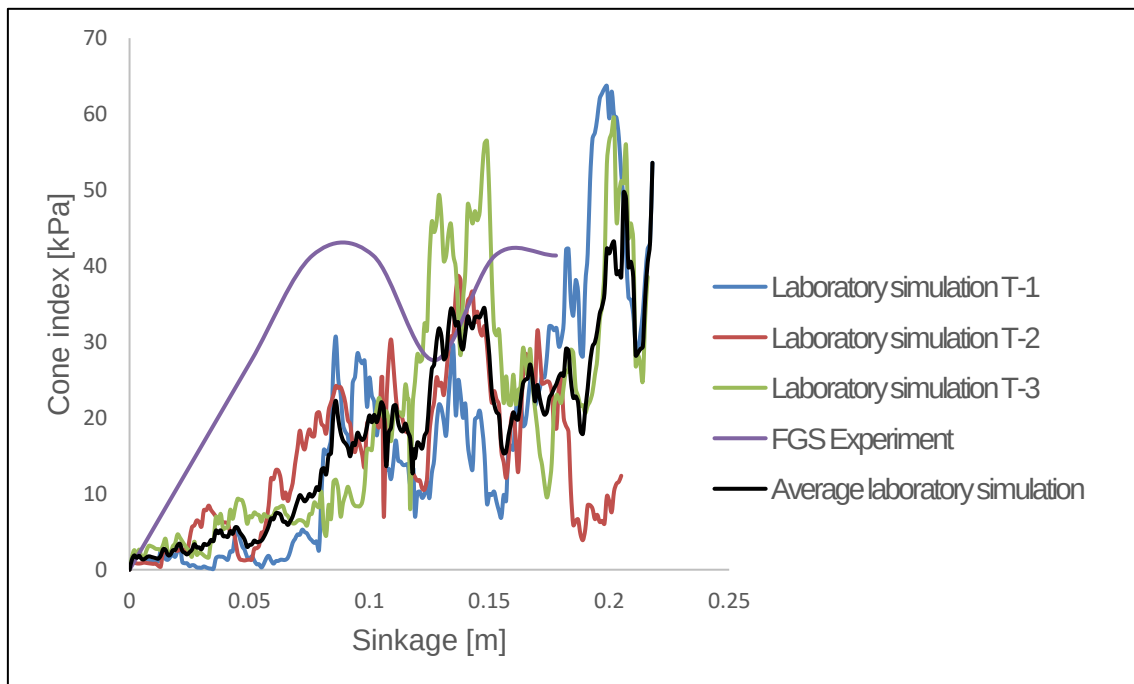


**Figure 4-26 Cone penetrometer laboratory simulation results in comparison with NATO CDT-308 (2020) experimental results for FGS**

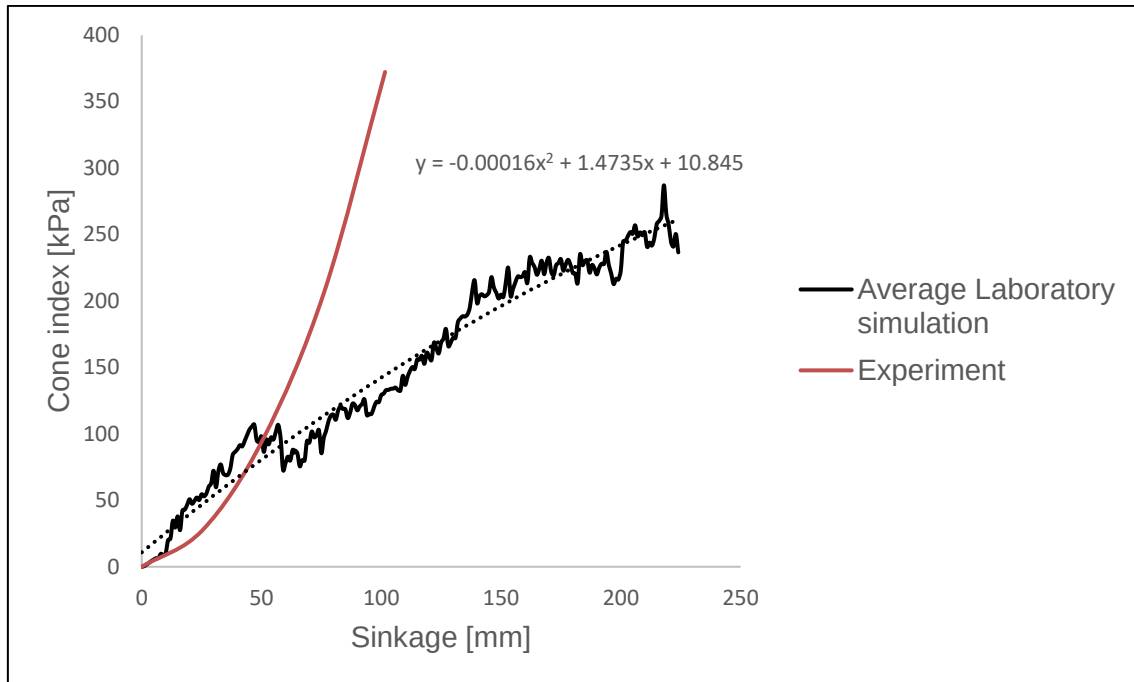


**Figure 4-27 Average cone penetrometer laboratory simulation results with the polynomial fit for 2NS sand**

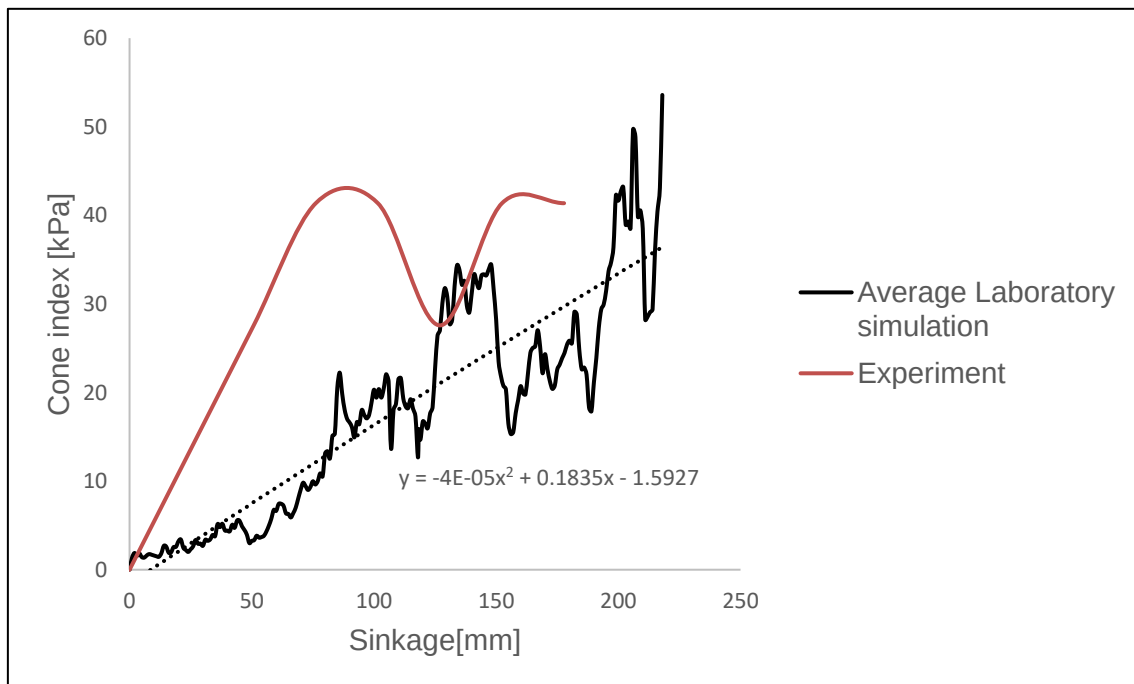


**Figure 4-28 Average cone penetrometer laboratory simulation results with the polynomial fit for FGS**

### 4.5.3 In-situ soil model testing

In the in-situ soil model, the prepared soil sample follows the simplifying assumptions used in the complex terramechanics models. NATO CDT-308 (2020) states that the terrain should have discrete patches of different particle shapes and size distribution. This variable distribution type changes the particle packing in the terrain model. The in-situ terrain may be embedded with rocks, debris and organic content. However, there is approximately zero organic content in both the real soils. Therefore, variable particle size, shape and space distributions are used to develop the in-situ soil models.

The variable particle shapes include the single sphere, double sphere, triple sphere, four-particle flat, four-particle straight and tetrahedron. These polygonal surfaces can be defined by Equation 4-57.

$$\left(\frac{x_1}{r_1}\right)^{p_1} + \left(\frac{x_2}{r_2}\right)^{p_2} + \left(\frac{x_3}{r_3}\right)^{p_3} = 1 \quad (4-57)$$

where,  $x_i$  are the spatial coordinates on the surface of the particles;  $r_i$  and  $p_i$  are adjustable constants used to define the particle shapes ( $i = 1, 2, 3$ ). In general, any particle shape geometries can be used to define the shape of the simulated particles provided the normal vector and signed distances between any point and then particle's surface is determined. The variable particle shapes can be quantified using the angular index (AI) and sphericity index (SPH) table developed by Akbar et al. (2020) based on Ashmawy et al. (2001) and Krumbein (1941). The angular index (AI) is the weighted average of all three views of particles (top, front and side) as shown in Equation 4-58. The sphericity index (SPH) is the ratio of particle volume to that of the smallest circumscribing sphere (Figure 4.13). The particle shape properties are shown in Table 4-8.

$$AI = \frac{\text{Ang(front)} \times \text{area(front)} + \text{Ang(top)} \times \text{area(top)} + \text{Ang(side)} \times \text{area(side)}}{\text{area(front)} + \text{area(top)} + \text{area(side)}} \quad (4-58)$$

**Table 4-8 Particle shape properties (Akbar et al. (2020))**

| <b>Particle shape</b>                                                               | <b>Angular index (AI)</b> | <b>Sphericity index (SPH)</b> | <b>Number of spheres</b> |
|-------------------------------------------------------------------------------------|---------------------------|-------------------------------|--------------------------|
|    | 0                         | 1                             | 1                        |
|    | 3                         | 0.94                          | 2                        |
|    | 7                         | 0.92                          | 3                        |
|    | 3                         | 0.75                          | 4                        |
|   | 7                         | 0.90                          | 4                        |
|  | 13                        | 0.81                          | 4                        |

These variable particle shapes are generated using a random static factory in the EDEM. The particle sizes were randomised in software; however, the lower bound of 0.5 scale and upper bound of 1.5 scale was fixed for both the terrain models. Following the variable distributions, a terrain model of dimensions 6 m x 4 m x 2 m was generated, as shown in Figure 4-29. The randomisation of the terrain model was increased with the random percentage of particle distribution generated in six different patches of dimensions 2 m x 2 m x 2 m. These six patches are marked from locations 1 to 6 in Figure 4-29. This randomised percentage distribution of particle shapes was created using the random numbers generated using MATLAB, as shown in Figure 4-30. These distributions exhibit spatial variation throughout the model.

It should be noted that the soil conditions of terrain models were kept the same as laboratory soil models. It was done by keeping the same EpAM model parameters for both soil models. For 2NS sand, the terrain model was pre-

compacted with a height of 0.150 m, as same as the laboratory (homogeneous) model to achieve the same compaction. The average bulk density of the heterogeneous terrain model of 2NS sand is reduced to 2687.66 kg/m<sup>3</sup> as compared to 2716 kg/m<sup>3</sup> of laboratory (homogeneous) model. The moisture content is 1.13%. No pre-compaction was needed for FGS. The average bulk density of the heterogeneous terrain model is 1466.64 kg/m<sup>3</sup> and moisture content is 18.55%.

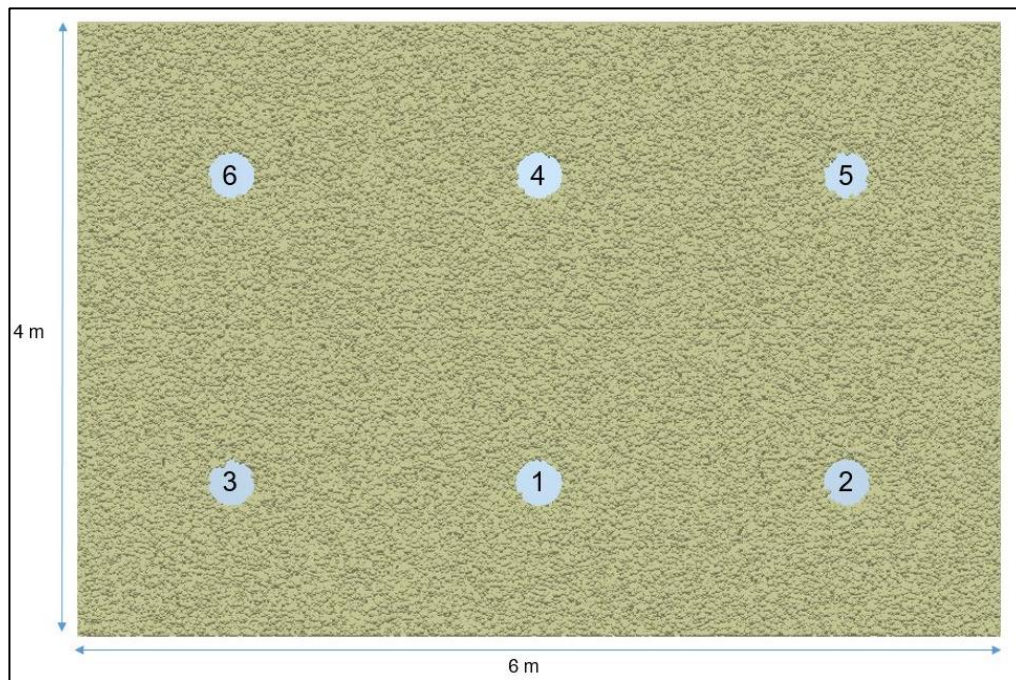
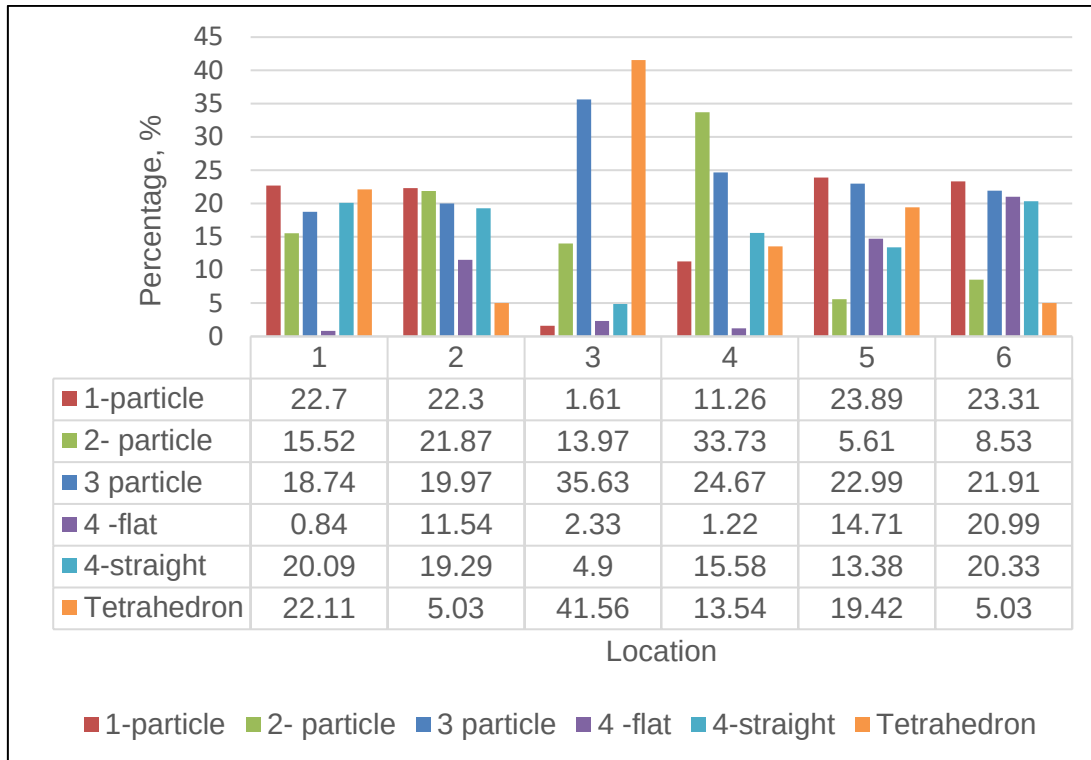
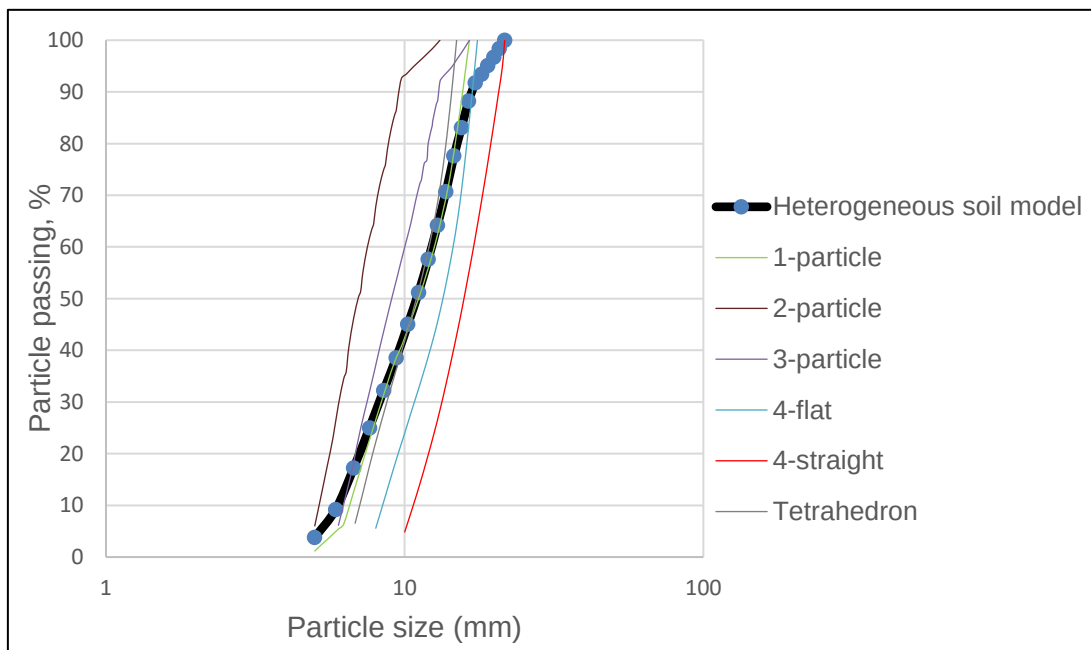


**Figure 4-29: Heterogeneous terrain model created in six different patches in EDEM for both the soil types**





(a)



(b)

**Figure 4-30 (a) Particle percentage distribution at six different locations in Terrain model for both the soil models; (b) Particle size distribution of heterogeneous terrains of both the soil models**

The shear ring, pressure-sinkage and cone penetrometer tests were simulated at six different locations on the terrain model, as marked in Figure 4-23. The pressure-sinkage and cone penetrometer tests followed the same simulating procedures mentioned in laboratory soil testing, section 4.5.2.

The shear test was simulated using the shear ring mentioned in section 4.3.1.2. A vertical force is applied to the top of the shear ring, which remains constant throughout the simulation time as shown in Figure 4.31. The shear ring was kept free for sinkage in the soil model to attain a static equilibrium condition in the initial 2 s. A constant angular velocity of 0.0174 rad/s was then applied to rotate the shear ring for the next 20 s. The soil sample undergoes static shearing. Verlet scheme simulator was used for increased computational accuracy but at an additional computational cost. The tests were simulated at three different normal stress conditions, i.e. 8.62 kPa (1.25 psi), 17.24 kPa (2.5 psi) and 25.85 kPa (3.75 psi). The simulation testing conditions were the same as the NATO experimental testing conditions. The measured torque vs angular displacement curve was obtained. The normal stress vs shear stress curve was plotted to determine soil shear properties obtained from the shear ring test.

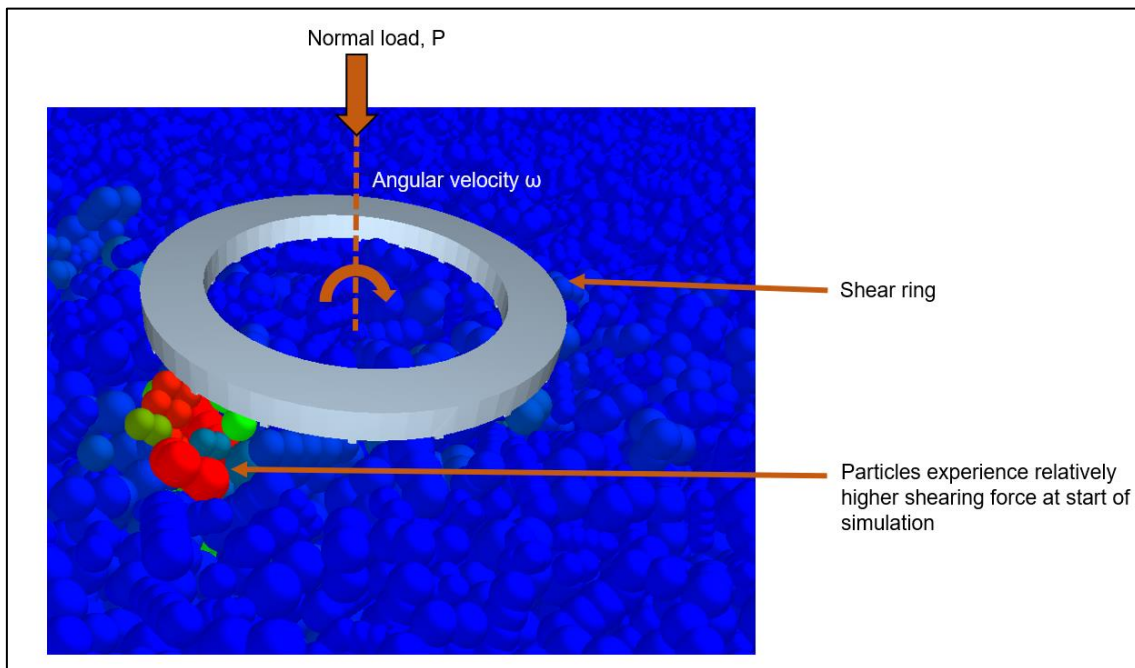


**Figure 4-31 DEM simulation model of the shear ring test on the heterogeneous terrain**

In the pressure-sinkage test, the particles were generated in a closed enclosed box of dimensions 6 m x 4 m x 2 m, as shown in Figure 4-32. NATO NRMM CDT (2018) conducted experiments using circular bevameter plates of dimensions 0.1 m and 0.15 m, respectively. Therefore, the same plate sizes were used in the present simulations. The simulations were run for 20 s for 2NS sand and 4.50 s for FGS in the following steps. Initial 1 s was used to generate and set up particles in the box. Next 6.50 s were used to compress the particles using a compression plate for 2NS sand. However, no pre-compression was required for FGS. Next 12 s and 4 s of sinkage plate penetrating the soil bed for 2NS sand and FGS, respectively. The sinkage velocity of plates was maintained at 0.025 m/s. The simulation time step was set to 5.595e-5 s, and data was saved at every 0.10 s interval. Verlet scheme simulator was used for increased computational accuracy but at an additional computational cost. The pressure vs sinkage curve was obtained. The data set was further processed to determine Bernstein and Bekker's model parameters.

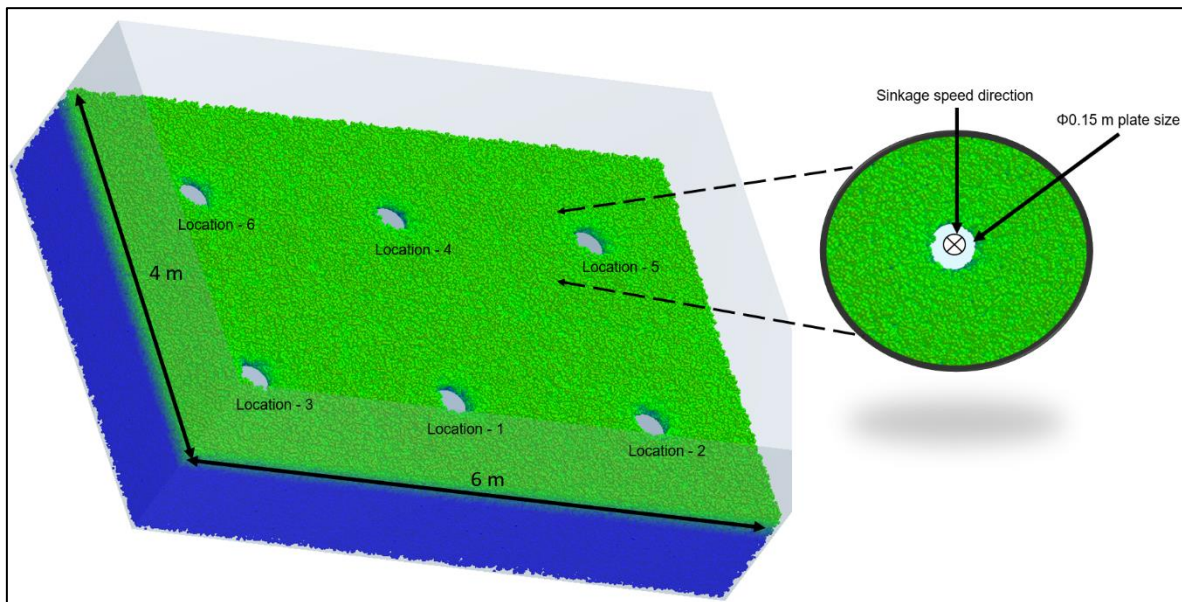


**Figure 4-32 Pressure sinkage simulation model of Dia 0.15 m plate size on the 2NS sand**

#### **4.5.3.1 Results**

The Bevameter shear test is designated to simulate the vehicle's shear loads on the terrain surface. The shear test results provide the basis for predicting vehicle

traction on specific terrain. The fundamental assumptions of Bevameter experimental methods and their associated analytical constructs are: Bevameter plates and shear rings are good stress substitutes to predict the tyre's pressure-sinkage and shear stress-shear displacement relationship respectively; in-situ bevameter measurements are sensitive to soil substrate and heterogeneities at geospatial points in specific terrain; the soil is heterogeneous and deep enough to be unaffected by boundary effects.

The laboratory shear tests such as the Direct shear test or Tri-axial shear tests have always been interrogated as they don't operate on the actual soil condition on which vehicle mobility performance is measured. According to Bekker (1969), "since one of the basic conditions for predicting vehicle performance is that the vehicle's shear action be simulated by instrument-predictor, the value of devices like the tri-axial shear apparatus has been questioned, in as much as their operation is too dissimilar to actual conditions for developing vehicle thrust." Therefore, the shear ring test becomes critical to measure the internal shear strength of the in-situ soil on which the vehicle is moving.

The shear ring tests were simulated at three different normal stress conditions as shown in Figure 4.31. The normal loads applied to the shear ring were calculated from Equation 4-59. During each test, the torque was measured at each angular displacement or shear displacement travelled by the shear ring. NATO CDT – 308 (2020) obtained the shear stress of soil, as shown in Equation 4-60. It is obtained by calculating the moment by integrating shear stress over a shear ring area.

$$\sigma = \frac{P}{A} = \frac{P}{\pi(r_o^2 - r_i^2)} \quad (4-59)$$

$$\tau = \frac{3T_m}{2\pi(r_o^3 - r_i^3)} \quad (4-60)$$

where  $P$  is the applied normal load (N),  $\sigma$  is the normal stress (N/m<sup>2</sup>),  $T_m$  is the measured torque (N-m),  $\tau$  is the shear stress (N/m<sup>2</sup>),  $r_o$  and  $r_i$  are the external and internal radius of the shear ring, respectively.

The shear strength (or shear stress) of soil depends on soil's bulk properties, such as particle sizes and shapes distribution, hardness, cohesion, angle of friction, and bulk density. The particle's shape affects its ability to interlock, increasing friction and shear strength. Importantly, spherical particles typically flow smoothly, while angular soil particles such as 3-particle shape or tetrahedron typically interlock and obstruct soil flow. The shear strength of soil under load is directly related to friction between soil particles and provides a measure of aggregate interlock and soil structure. And also, shear tests are less sensitive to density variation than pressure-sinkage tests. Therefore, the change in soil density due to the change in soil particle properties doesn't have much effect on the change in shear strength.

Figure 4-33 to 4-38 shows the results of shear ring tests simulated at six different locations on the 2NS sand terrain model. Further, a comparison was made between the laboratory simulation (or homogeneous condition) and experimental results obtained in NATO CDT (2020). Similarly, Figure 4-39 to 4-44 shows the results for FGS soil. Using Table 4-9 and Figure 4-24, the average angular index (AI) and average sphericity index (SPH) at each terrain model location is calculated using Equation 4-61 and 4-62, respectively, where  $p_i$  is the per cent distribution of  $i^{\text{th}}$  particle at the  $j^{\text{th}}$  location.

$$AI_{loc=j} = \frac{\sum_{i=1}^6 AI_i \times p_i}{\sum p_i} \quad (4-61)$$

$$SPH_{loc=j} = \frac{\sum_{i=1}^6 SPH_i \times p_i}{\sum p_i} \quad (4-62)$$

**Table 4-9 Average AI and SPH at different patches on the terrain model**

| <b>Location on the terrain model</b> | <b>Average angular index (AI)</b> | <b>Average sphericity index (SPH)</b> |
|--------------------------------------|-----------------------------------|---------------------------------------|
| 1                                    | 6.08                              | 0.911                                 |
| 2                                    | 4.40                              | 0.913                                 |
| 3                                    | 8.73                              | 0.873                                 |
| 4                                    | 5.63                              | 0.916                                 |
| 5                                    | 5.68                              | 0.891                                 |
| 6                                    | 4.50                              | 0.896                                 |

The resulting shear parameters for 2NS sand and FGS simulations performed in this thesis is summarised in Table 4-10 and 4-11, respectively. As shown, the average value of shear displacement modulus  $K$  is minimum at location 3, which has the highest average angular index (AI) value and the lowest average sphericity value (SPH) for 2NS sand. However, there is not much difference between average  $K$  values at different locations for the FGS model. Still, location 1 has a minimum average  $K$  value with the second-highest average angular index in the FGS model. It makes physical sense because when the average angularity index value is higher, there is better interlocking between the particles. Therefore, it takes a shorter shear displacement for the soil particles to rearrange themselves and fill in the voids to ultimately reach a point where the soil reaches its maximum shear strength.

Additionally, location 3 and location 1 have the highest angle of friction for 2NS sand and FGS, respectively. This finding is similar to Mvelase et al. (2017), who found that the higher angularity crushed particles produced a higher internal angle of friction than the particles with less angularity. However, no significant AI and SPH trend with cohesion and internal angle of friction is found. It might be because the effect of angularity in the particle sample is more apparent at high

normal stresses (Danesh et al., 2020). This phenomenon is the result of increased particle interlocking under higher normal stress. However, the normal stresses considered in this study are not that high.

Tables 4-10 and 4-11 also compare the shear properties obtained from terrain simulation models and experimental results for 2NS sand and FGS, respectively. The cohesion values varied from 0.48 to 3.84 kPa and the internal angle of friction varied from  $33.3^\circ$  to  $40.49^\circ$  for the 2NS heterogeneous terrain model. The cohesion value of 1.38 kPa and the internal angle of friction of  $32^\circ$  was obtained for 2NS sand experimentally by NATO CDT (2020). However, the cohesion value of 4.18 kPa and the internal angle of friction of  $28.68^\circ$  was obtained in laboratory test simulations of 2NS sand. Therefore, it can be found that the heterogeneous terrain model follows more closely the response behaviour of actual 2NS experimental terrain as compared to homogeneous laboratory tests.

Similarly, the cohesion values varied from 0.71 to 2.62 kPa and the internal angle of friction varied from  $15.03^\circ$  to  $35.81^\circ$  for FGS heterogeneous terrain model. The cohesion value of 3.45 kPa and the internal angle of friction of  $35.2^\circ$  was obtained for FGS experimentally by NATO CDT (2020). However, the cohesion value of 0.51 kPa and the internal angle of friction of  $35.63^\circ$  was obtained in laboratory test simulations. The internal angle of friction was found approximately the same for both conditions. However, the FGS heterogeneous terrain model captures a better cohesion value than the homogeneous laboratory model. Therefore, the experimental soil sample's shear properties are better represented with heterogeneous terrain models than homogeneous laboratory models.

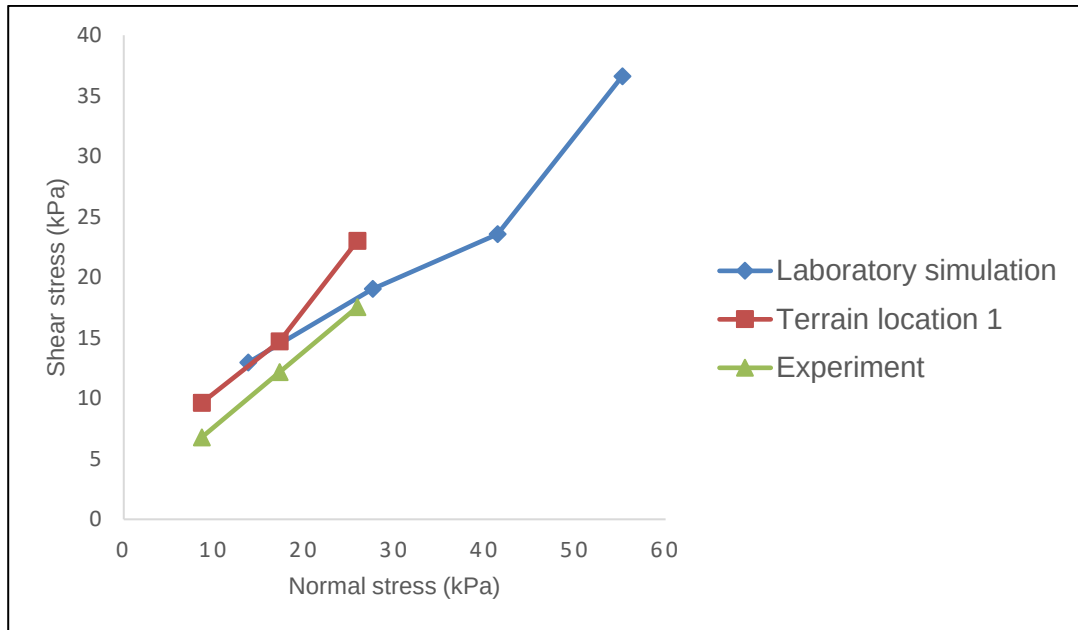


**Figure 4-33 Shear ring simulation results at location – 1 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand**

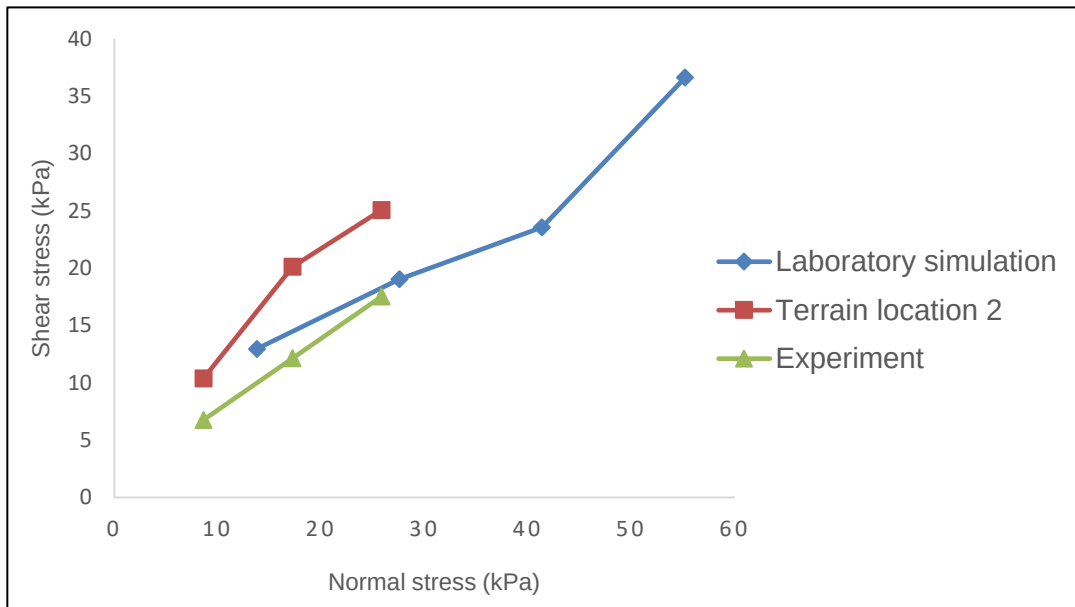


**Figure 4-34 Shear ring simulation results at location – 2 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand**



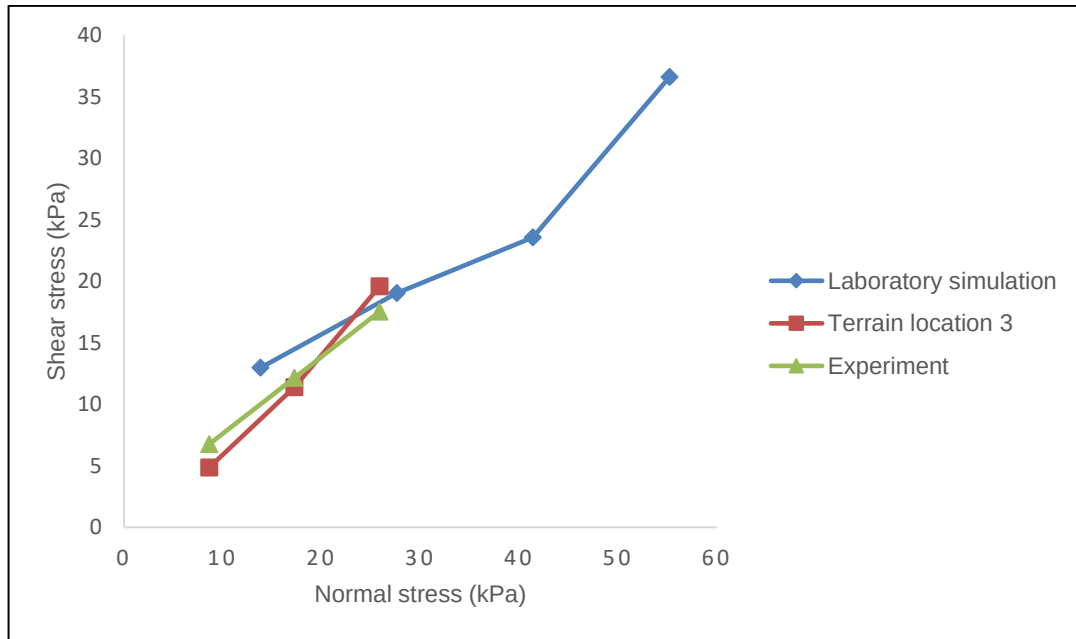


**Figure 4-35 Shear ring simulation results at location – 3 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand**

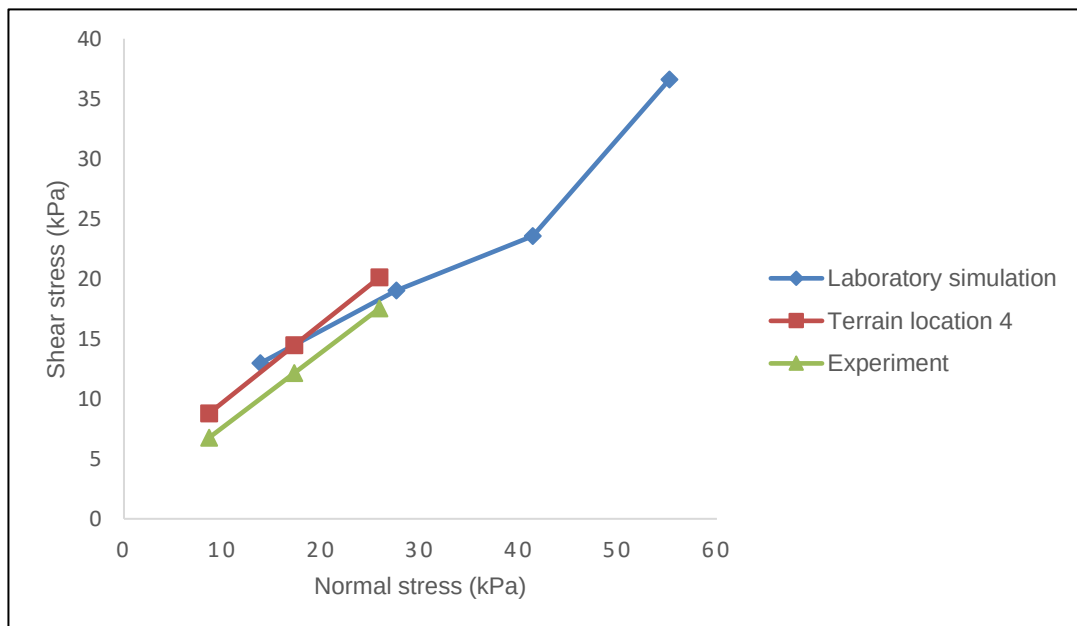


**Figure 4-36 Shear ring simulation results at location – 4 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand**

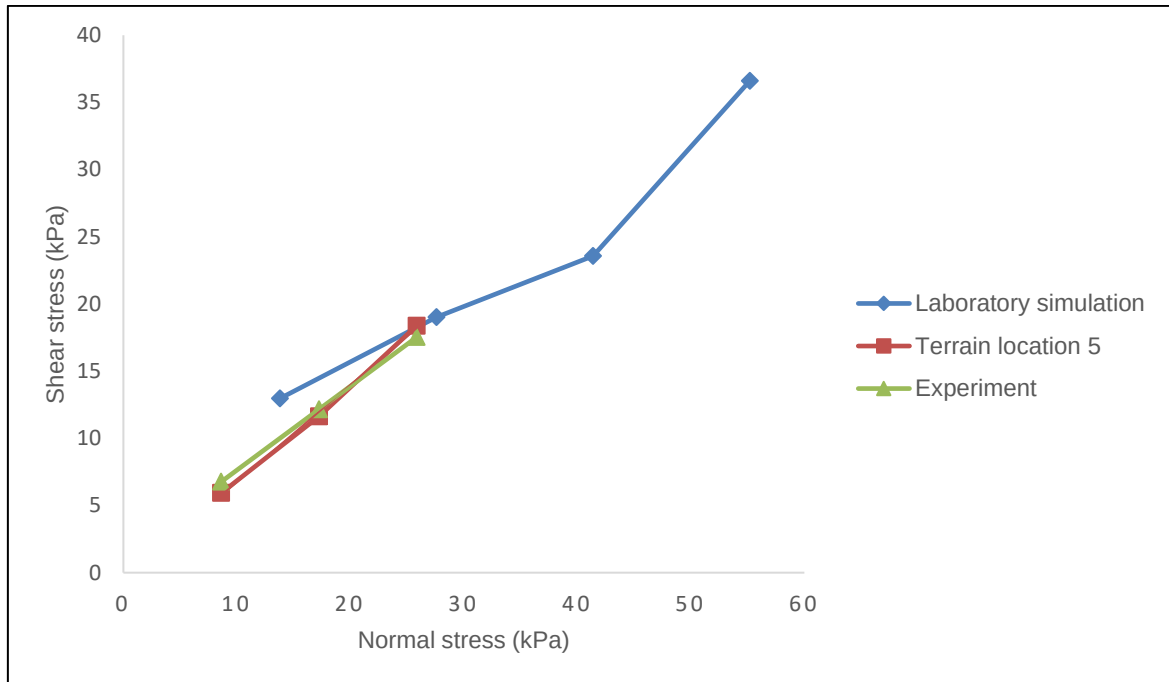


**Figure 4-37 Shear ring simulation results at location – 5 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand**

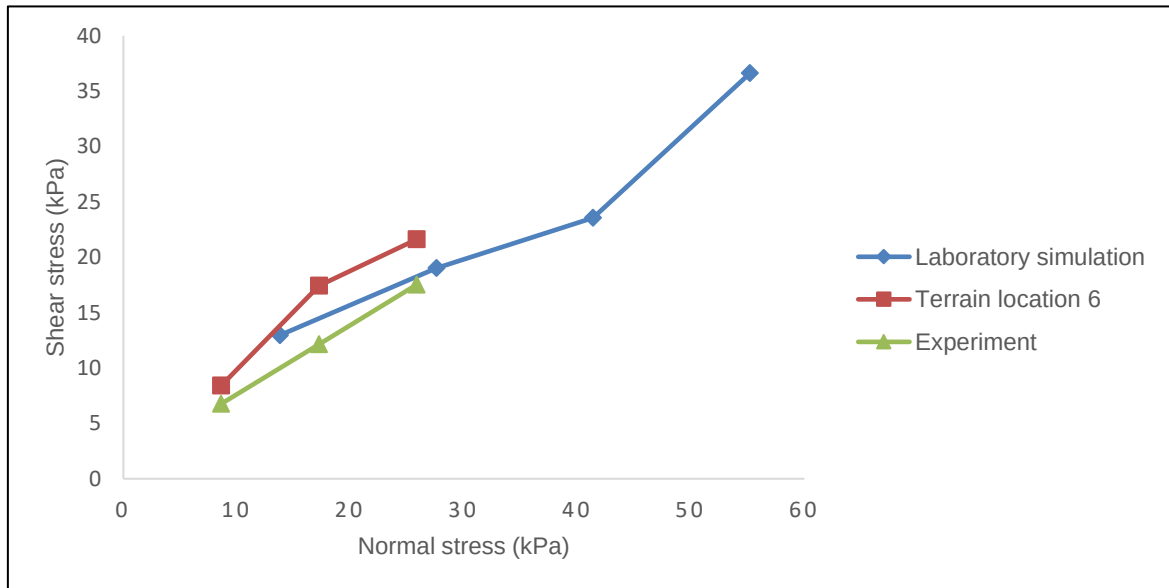


**Figure 4-38 Shear ring simulation results at location – 6 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for 2NS sand**

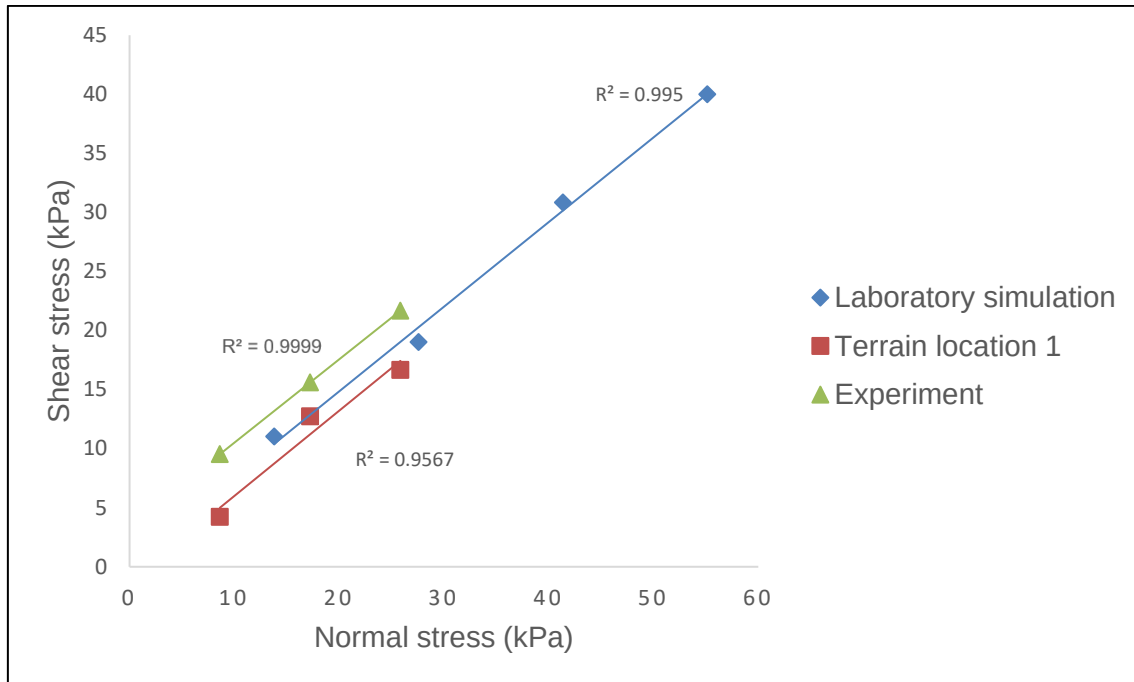


**Figure 4-39 Shear ring simulation results at location – 1 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil**

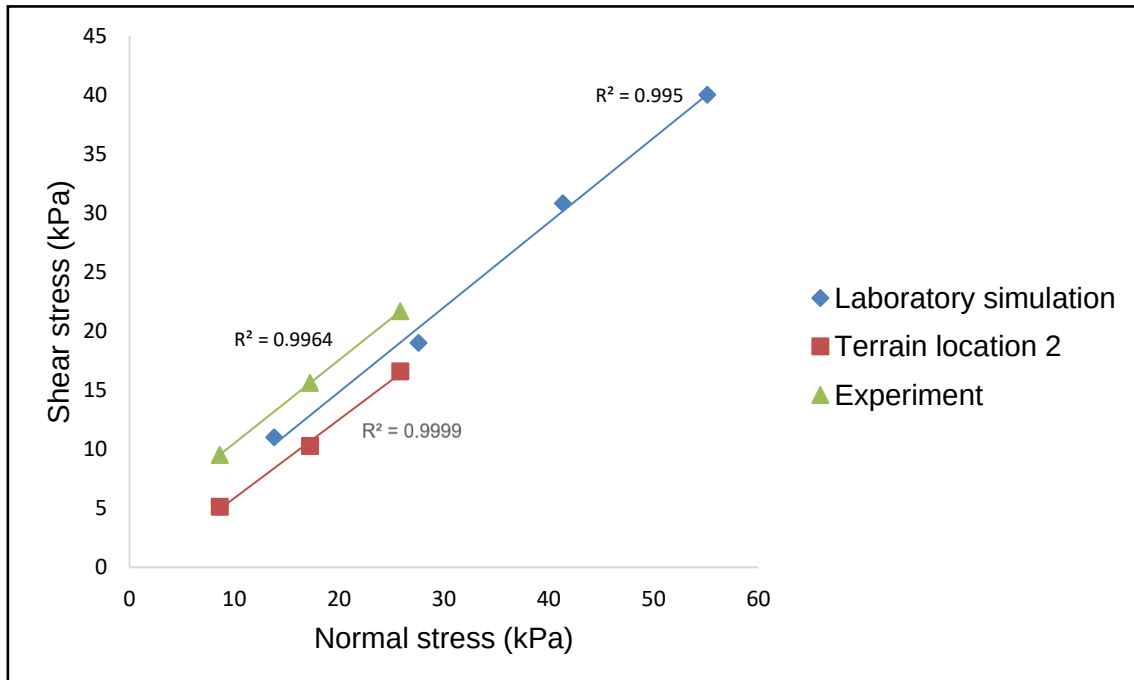


**Figure 4-40 Shear ring simulation results at location – 2 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil**

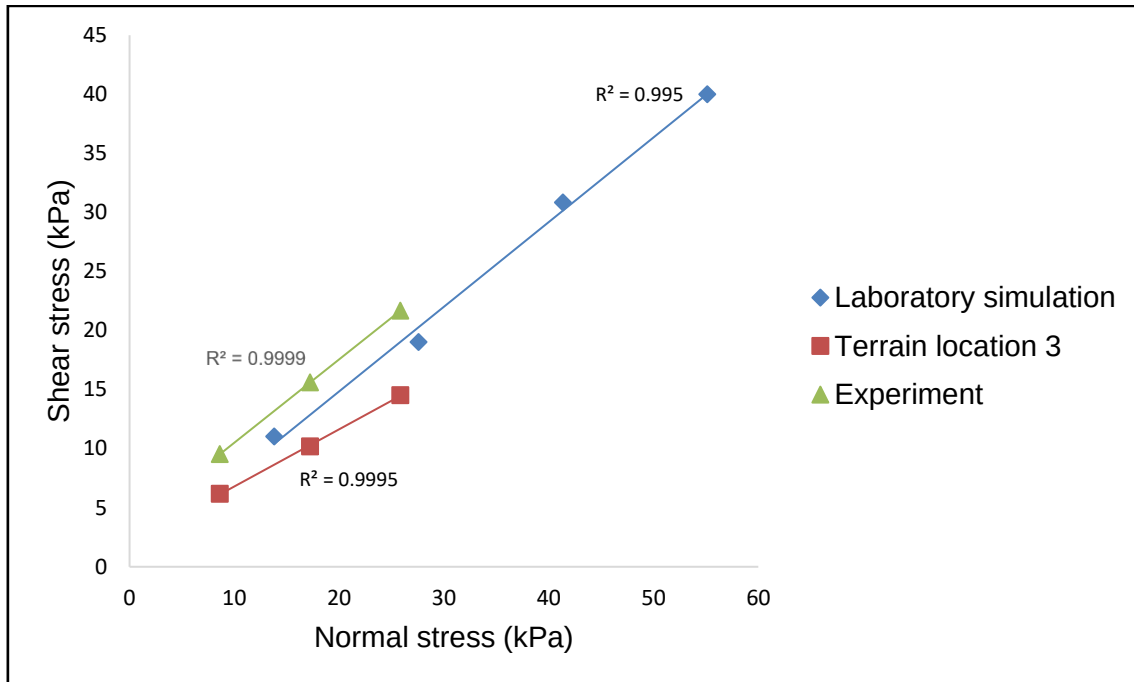


**Figure 4-41 Shear ring simulation results at location – 3 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil**

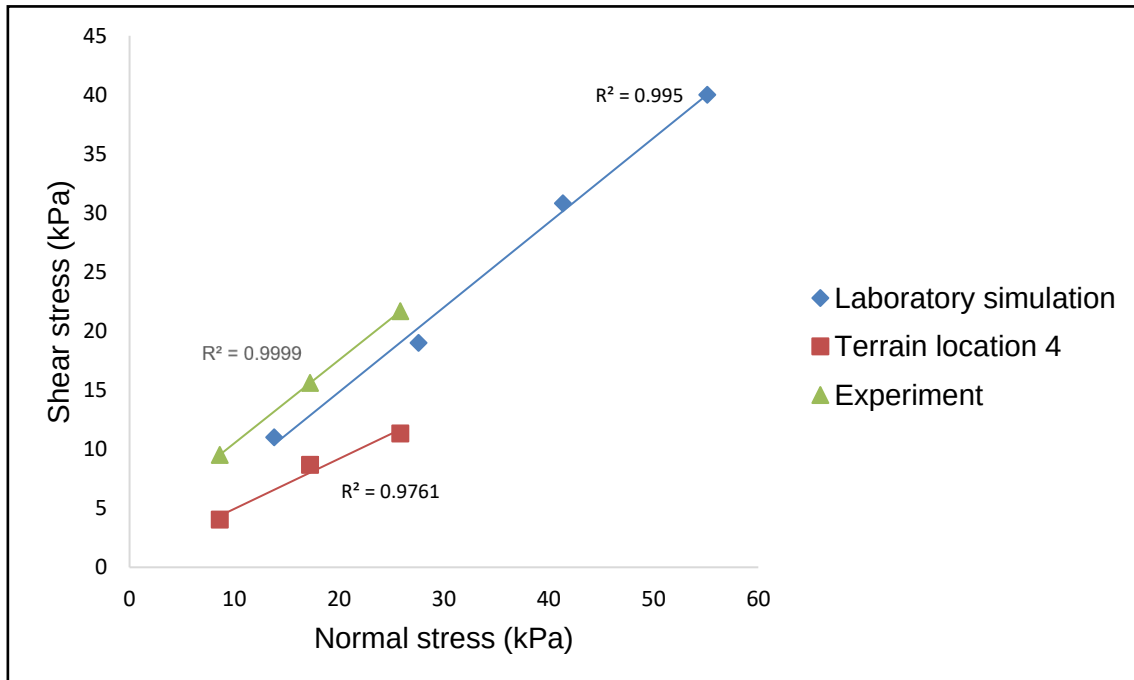


**Figure 4-42 Shear ring simulation results at location – 4 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil**

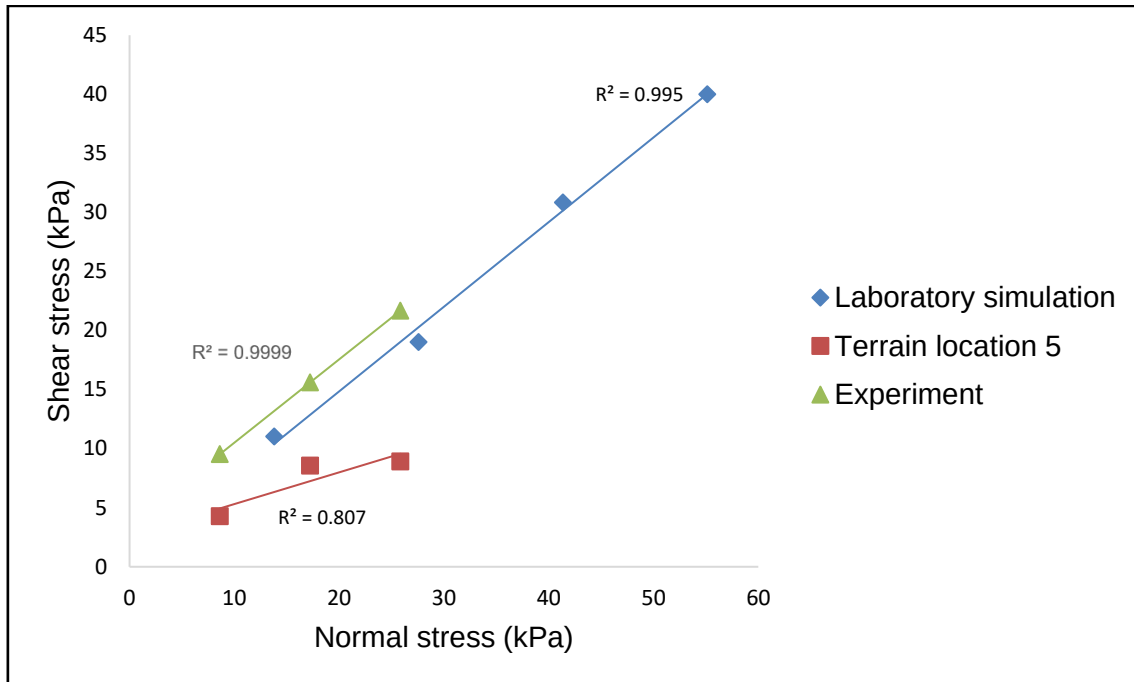


**Figure 4-43 Shear ring simulation results at location – 5 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil**

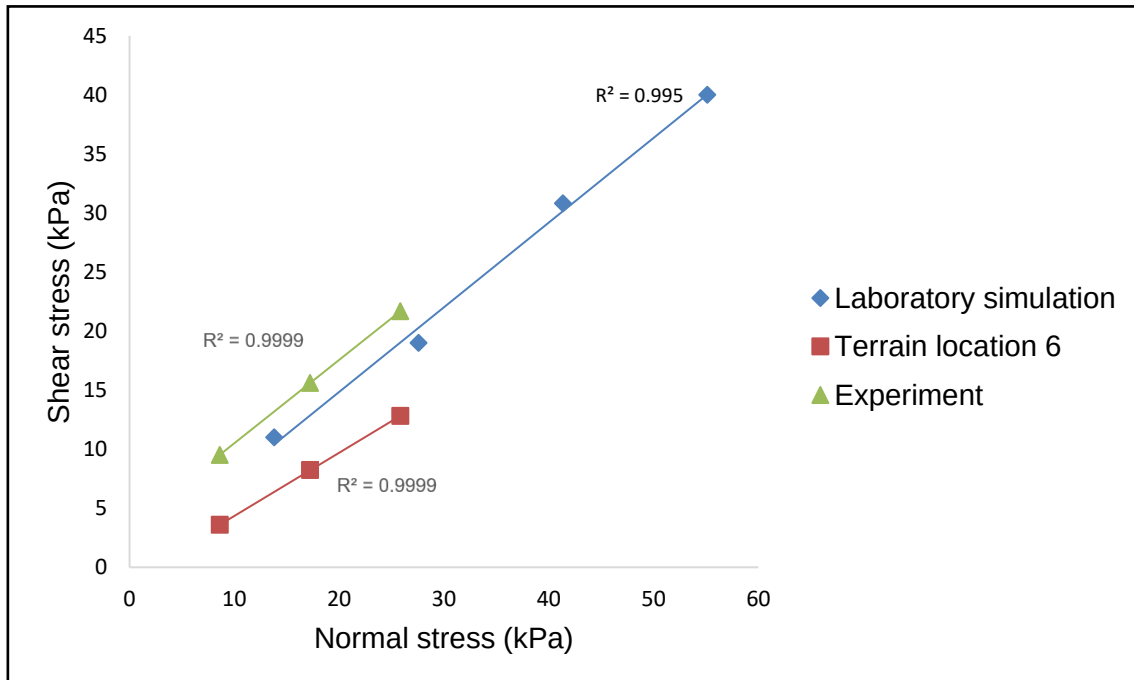


**Figure 4-44 Shear ring simulation results at location – 6 in comparison with laboratory model simulation and experimental shear ring results of NATO CDT (2020) for FGS soil**

**Table 4-10 Summary of shear bevameter results for 2NS sand**

| <b>2NS sand</b>                     | <b>Cohesion, c<br/>(kPa)</b> | <b>Internal<br/>angle of<br/>friction, <math>\phi</math> (°)</b> | <b>Average K<br/>(m)</b> | <b>Standard<br/>deviation<br/>(m)</b> |
|-------------------------------------|------------------------------|------------------------------------------------------------------|--------------------------|---------------------------------------|
| Terrain model<br>location – 1       | 2.39                         | 37.82                                                            | 0.048                    | $\mp 0.011$                           |
| Terrain model<br>location – 2       | 3.84                         | 40.43                                                            | 0.055                    | $\mp 0.005$                           |
| Terrain model<br>location – 3       | 2.77                         | 40.49                                                            | 0.03                     | $\mp 0.001$                           |
| Terrain model<br>location – 4       | 3.13                         | 33.30                                                            | 0.058                    | $\mp 0.014$                           |
| Terrain model<br>location – 5       | 0.48                         | 35.87                                                            | 0.081                    | $\mp 0.024$                           |
| Terrain model<br>location – 6       | 2.61                         | 37.82                                                            | 0.070                    | $\mp 0.032$                           |
| In-situ shear<br>ring<br>experiment | 1.38                         | 32.0                                                             | 0.020                    | $\mp 0.002$                           |
| Laboratory<br>simulation            | 4.18                         | 28.68                                                            | 0.043                    | $\mp 0.012$                           |

**Table 4-11 Summary of shear bevameter results for FGS soil**

| <b>FGS</b>                          | <b>Cohesion, c<br/>(kPa)</b> | <b>Internal<br/>angle of<br/>friction, <math>\phi</math> (°)</b> | <b>Average K<br/>(m)</b> | <b>Standard<br/>deviation<br/>(m)</b> |
|-------------------------------------|------------------------------|------------------------------------------------------------------|--------------------------|---------------------------------------|
| Terrain model<br>location – 1       | 1.22                         | 35.81                                                            | 0.027                    | $\mp 0.005$                           |
| Terrain model<br>location – 2       | 0.79                         | 33.63                                                            | 0.031                    | $\mp 0.001$                           |
| Terrain model<br>location – 3       | 1.96                         | 25.81                                                            | 0.036                    | $\mp 0.019$                           |
| Terrain model<br>location – 4       | 0.71                         | 22.99                                                            | 0.033                    | $\mp 0.014$                           |
| Terrain model<br>location – 5       | 2.62                         | 15.03                                                            | 0.065                    | $\mp 0.005$                           |
| Terrain model<br>location – 6       | 1.01                         | 28.15                                                            | 0.127                    | $\mp 0.114$                           |
| In-situ shear<br>ring<br>experiment | 3.45                         | 35.2                                                             | 0.030                    | $\mp 0.002$                           |
| Laboratory<br>simulation            | 0.51                         | 35.63                                                            | 0.003                    | $\mp 0.0005$                          |

The pressure sinkage tests for the determination of Bekker's soil parameters for 2NS sand (the average bulk density is 2687.66 kg/m<sup>3</sup> and moisture content is 1.18%) and FGS (the average bulk density is 1466.64 kg/m<sup>3</sup> and moisture content is 18.55%) were simulated on the developed heterogeneous terrain model at six different locations as shown in Figure 4.32. As already stated, two different plate sizes of diameter 0.1 m and 0.15 m were used in simulations. It is important to

note that the pressure range over which data was evaluated was assumed to represent the range of expected ground pressures from the vehicle on both soils, respectively. Figures 4-45 to 4-50 and Figures 4-51 to 4-56 represent the pressure-sinkage plots of 0.1 m and 0.15 m plate diameter for 2NS sand. Similarly, Figures 4-57 to 4-62 and Figures 4-63 to 4-68 represent the pressure sinkage plots of 0.1 m and 0.15 m plate diameter for FGS soil.

Generally, increased pressure results in increased soil sinkage for fixed dimensions of Bevameter plates (Upadhyaya et al., 1997 and Wong et al., 1992). However, Figures for 2NS sand and FGS indicate some degree of yielding, i.e. characteristic peak followed by a stable zone, which shows necking and further followed by a decrease in the load value, reflecting rupture. The maximum pressure corresponds to the critical sinkage, beyond which the load-bearing top layer is expected to fail. The observation indicates that the heterogeneous terrain model has lower maximum pressure than the homogeneous laboratory model. It may be because of the change in vertical stiffness of terrain bed originating because of the increase in void spaces due to different particle shapes and sizes and heterogeneity in terrain bed. It can also be seen that the slope of the pressure-sinkage curves for heterogeneous terrain is less than that of the homogeneous laboratory model. The weaker slope represents the loose part of the soil. Therefore, it can be observed that the heterogeneity makes the terrain weaker, which also affects its traction generation properties.

For 2NS sand, the value of  $n$  varies from 0.48 to 0.58 at six different locations on the terrain model (Table 4.12). Since these values are less than one, it suggests 'soil weakening' behaviour. It means that the soils have reached compaction limits and are failing due to internal shearing in their far-field. However, the  $n$  value obtained from terrain models is less than that obtained from the laboratory model, i.e. 0.63. It means heterogeneous soils shows better soil weakening behaviour than homogeneous soil sample.

Similarly, for FGS soil, the value of  $n$  varies from 0.79 to 1.01 at six different locations on the terrain model (Table 4.13). At three locations with the lowest sphericity index, the  $n$  values are closer to one, which suggests the approximately



linear behaviour of the heterogeneous soil sample. It means the terrain behaviour doesn't change much with respect to sinkage. At the other three locations with the highest sphericity index, the terrain shows soil weakening behaviour. Under laboratory conditions for FGS, the obtained  $n$  value is 1.2, which shows some soil strengthening effect. However, it can be seen that the  $n$  value decreases for heterogeneous terrain models as compared to homogeneous soil models.

For the 2NS terrain model, the values of Bekker's coefficient for cohesion ( $k_c$ ) vary from  $2.07\text{E}+04$  to  $4.77\text{E}+04$  and the coefficient for friction ( $k_\phi$ ) vary from  $1.76\text{E}+05$  to  $4.13\text{E}+05$  at six different locations of the heterogeneous terrain model (Table 4.12). It was found that the highest  $k_c$  value corresponded to the maximum average angular index value at location 3, and the minimum  $k_c$  value corresponded to the minimum average angular index at location 2. However, no significant trend was followed concerning  $k_c$ . It may be because of non-uniform force distribution in a heterogeneous terrain system (Yang et al., 2018). The maximum  $k_\phi$  value corresponded to the minimum average angular index at location 2. No significant trend was found concerning  $k_\phi$ . It might be because of the irregular reduction in friction by relative movement among heterogeneous particles system (Yang et al., 2018). Moreover,  $k_c$  and  $k_\phi$  do not explicitly represent physical soil parameters and correlate with  $n$ .

It can also be seen from corresponding Figures that Bernstein curve-fit model corresponds well to the simulated terrain data at different locations. The coefficient of determination ( $R^2$ ) ranges from 0.88 to 0.98 for a 0.1 m plate and 0.94 to 0.98 for a 0.15 m plate on 2NS sand. The closer these values are to 1, the better the fit. In all the cases, the curve fits were reasonably accurate.

Similarly, for the FGS terrain model, the values of Bekker's coefficient for cohesion ( $k_c$ ) vary from  $-6.56\text{E}+04$  to  $1.71\text{E}+04$  and the coefficient for friction ( $k_\phi$ ) vary from  $2.32\text{E}+05$  to  $9.84\text{E}+05$  at six different locations of the heterogeneous terrain model (Table 4.13). It was found that the highest  $k_c$  value corresponded to the maximum average angular index value at location 3. However, no significant trend was found for  $k_c$  and  $k_\phi$  concerning the angularity index. It should be noted that there are negative values calculated for  $k_c$  and  $k_\phi$  for FGS. However, these

negative values are not an issue because  $k_c$  and  $k_\phi$  do not explicitly represent physical soil parameters and correlate with  $n$ . Therefore, as the value of  $n$  changes,  $k_c$  and  $k_\phi$  changes correspondingly. Moreover, it can also be seen from corresponding figures that Bernstein curve-fit model corresponds well to the simulated terrain data at different locations. The coefficient of determination ( $R^2$ ) ranges from 0.81 to 0.91 for 0.1 m plate and 0.81 to 0.95 for 0.15 m plate on FGS.

It can be concluded that no significant trend of  $k_c$  and  $k_\phi$  concerning angularity index was found for both the soil types. However, these parameters are difficult to analyse as they don't represent soil physical parameters and are dependent on pressure, sinkage, and  $n$  (Oravec, 2009).

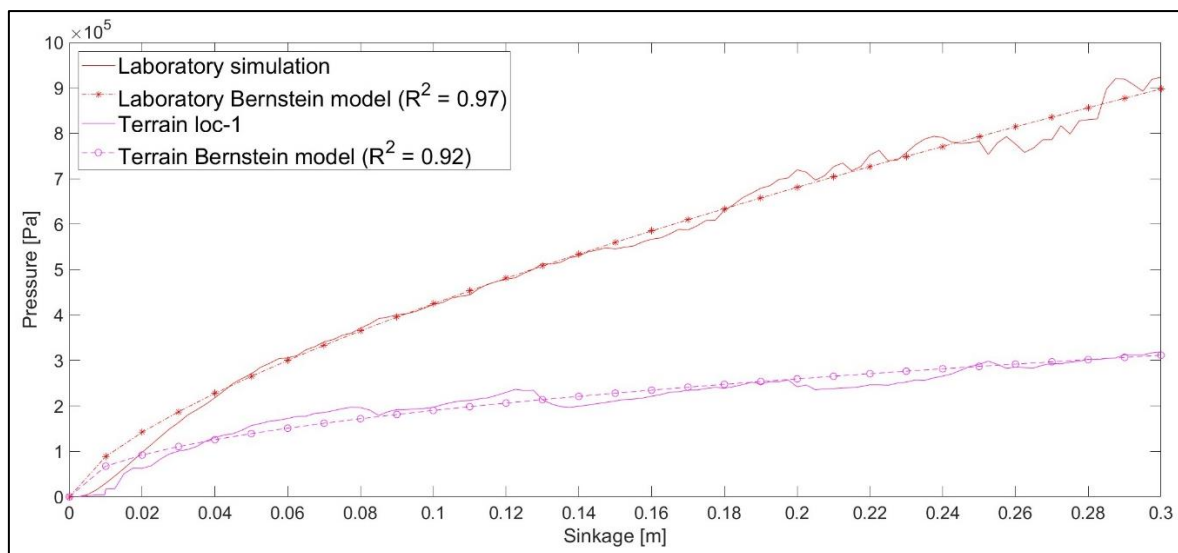


**Figure 4-45 Pressure sinkage simulation results at location - 1 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand**

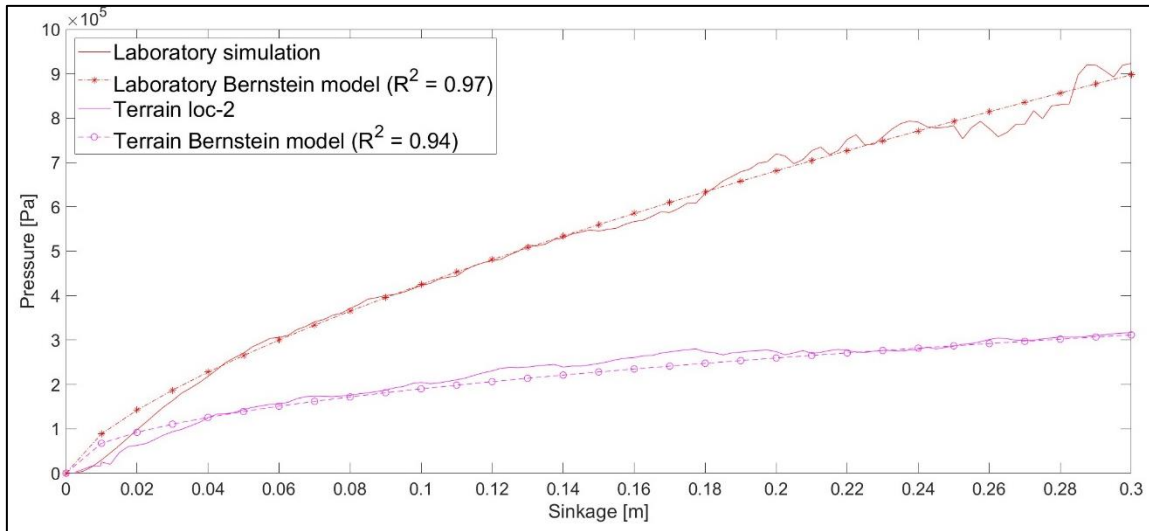


**Figure 4-46 Pressure sinkage simulation results at location - 2 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand**

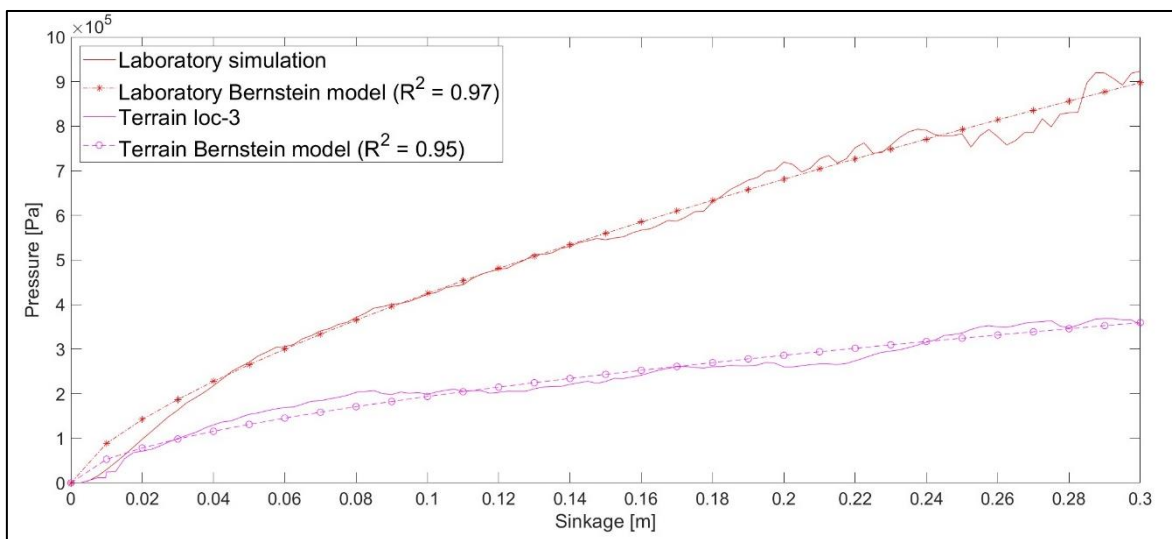


**Figure 4-47 Pressure sinkage simulation results at location - 3 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand**

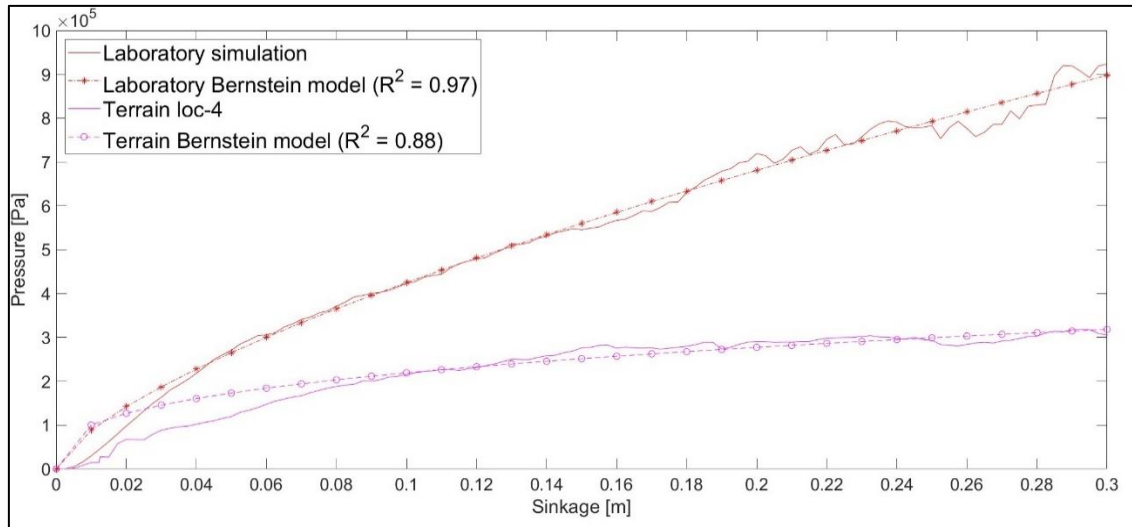


**Figure 4-48 Pressure sinkage simulation results at location - 4 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand**

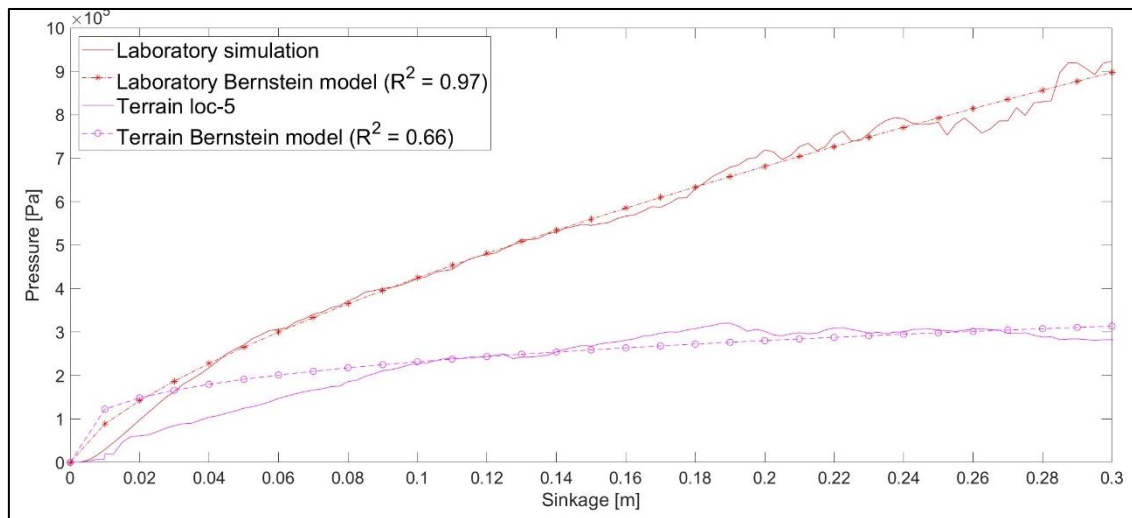


**Figure 4-49 Pressure sinkage simulation results at location - 5 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand**

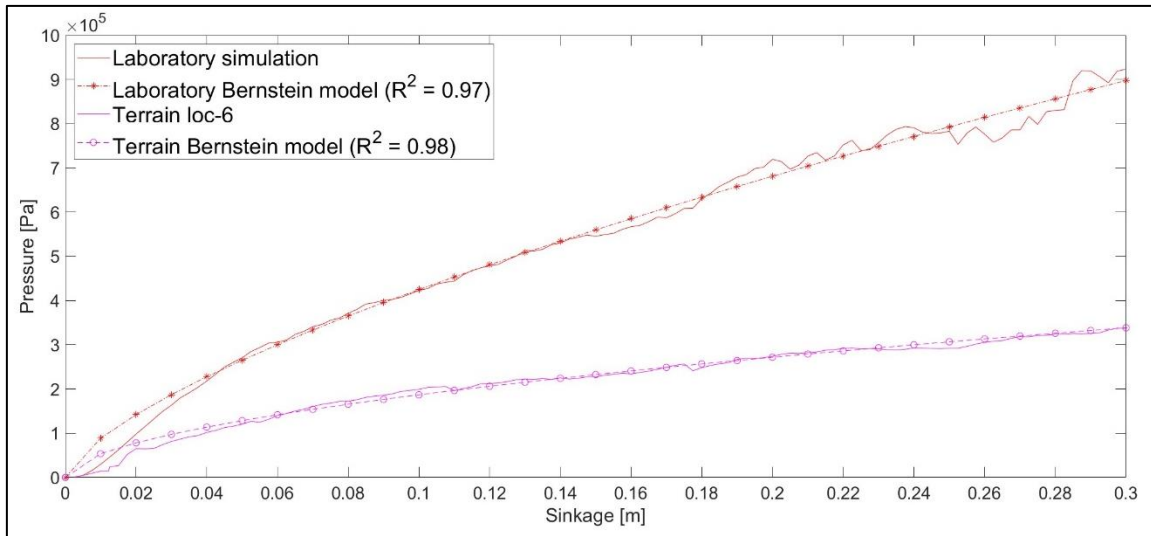


**Figure 4-50 Pressure sinkage simulation results at location - 6 in comparison with laboratory simulation results of Dia 0.1 m plate size for 2NS sand**

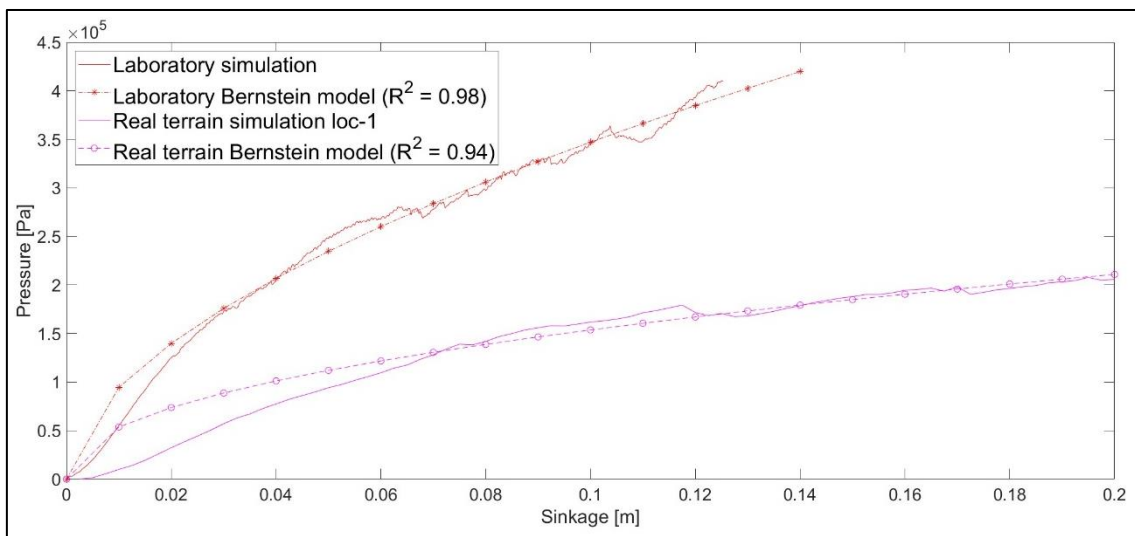


**Figure 4-51 Pressure sinkage simulation results at location - 1 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand**

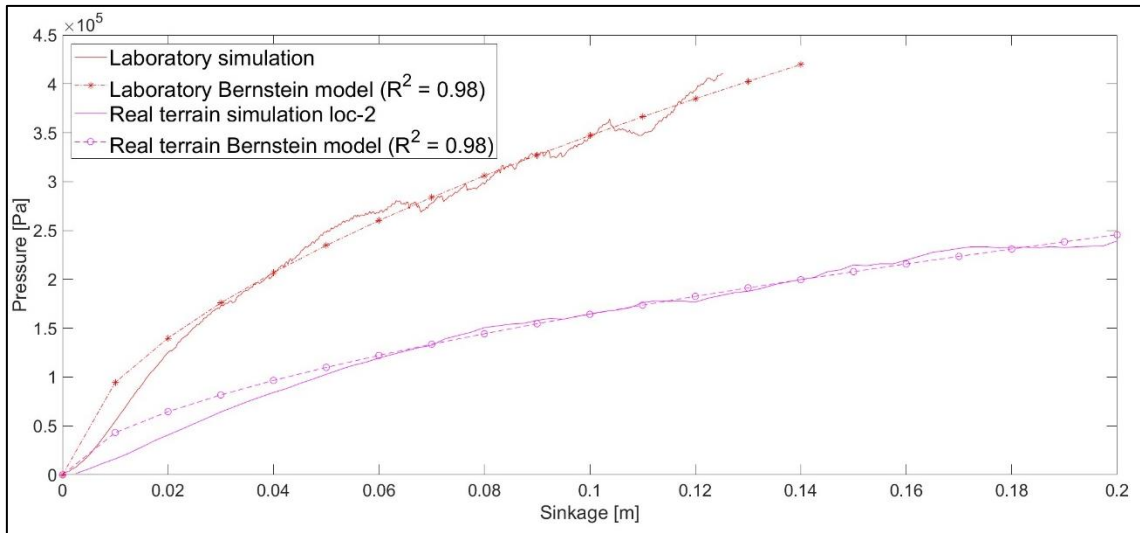


**Figure 4-52 Pressure sinkage simulation results at location - 2 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand**

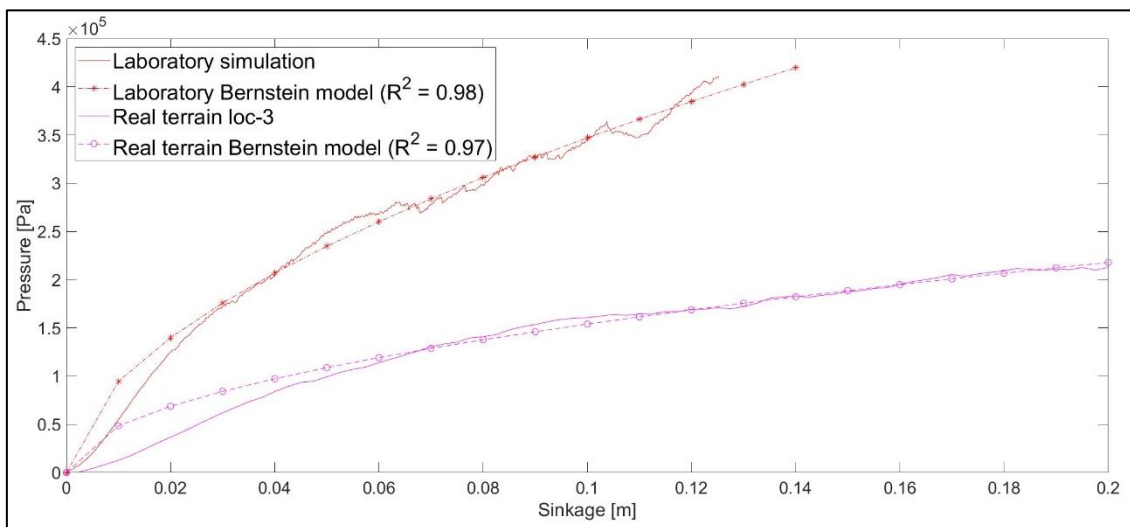


**Figure 4-53 Pressure sinkage simulation results at location - 3 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand**

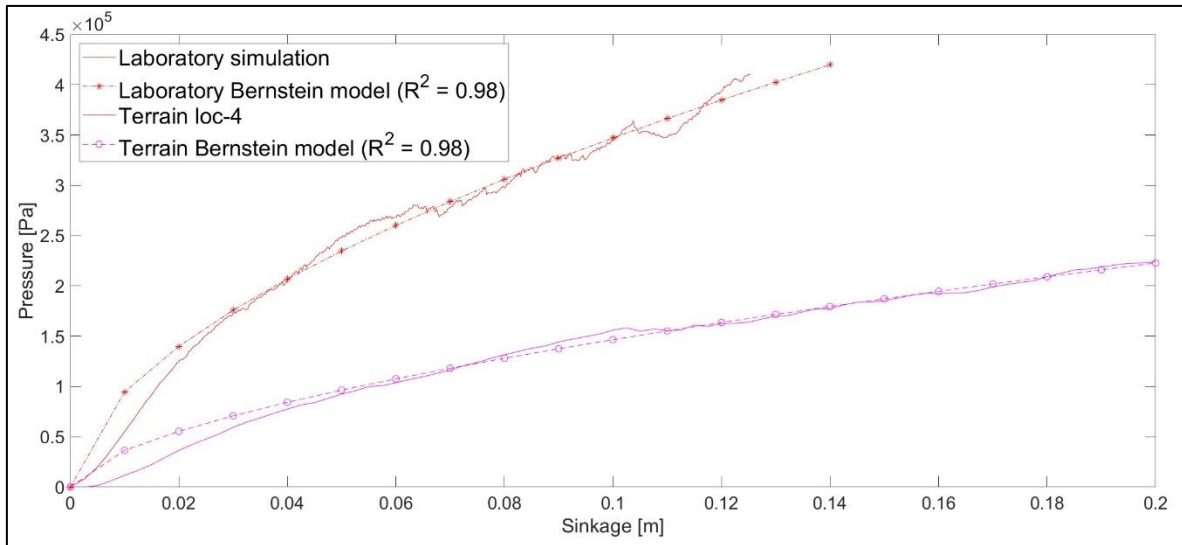


**Figure 4-54 Pressure sinkage simulation results at location - 4 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand**

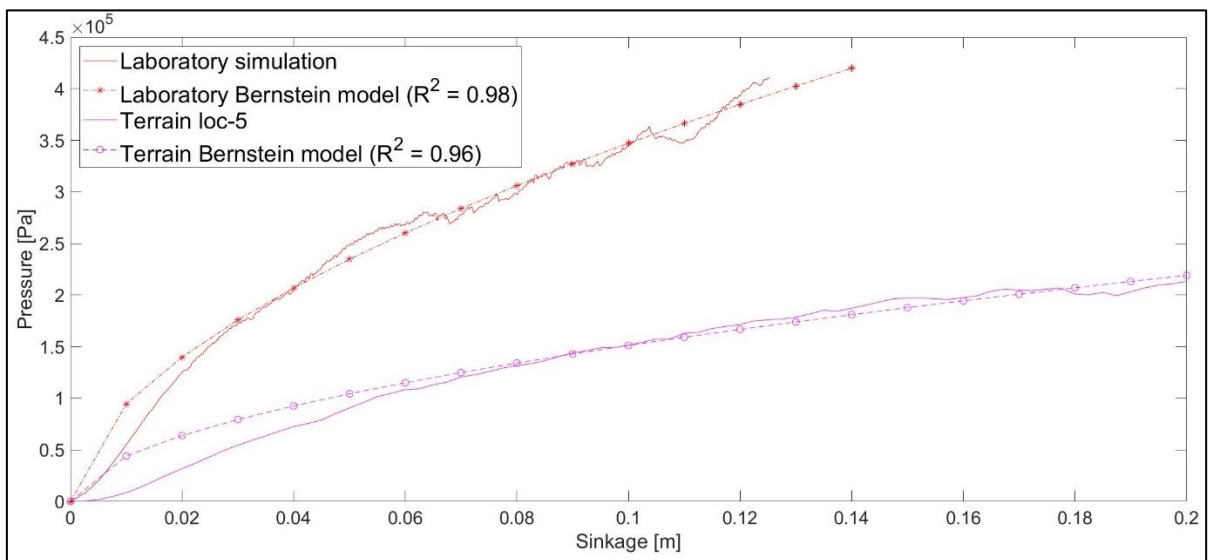


**Figure 4-55 Pressure sinkage simulation results at location - 5 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand**

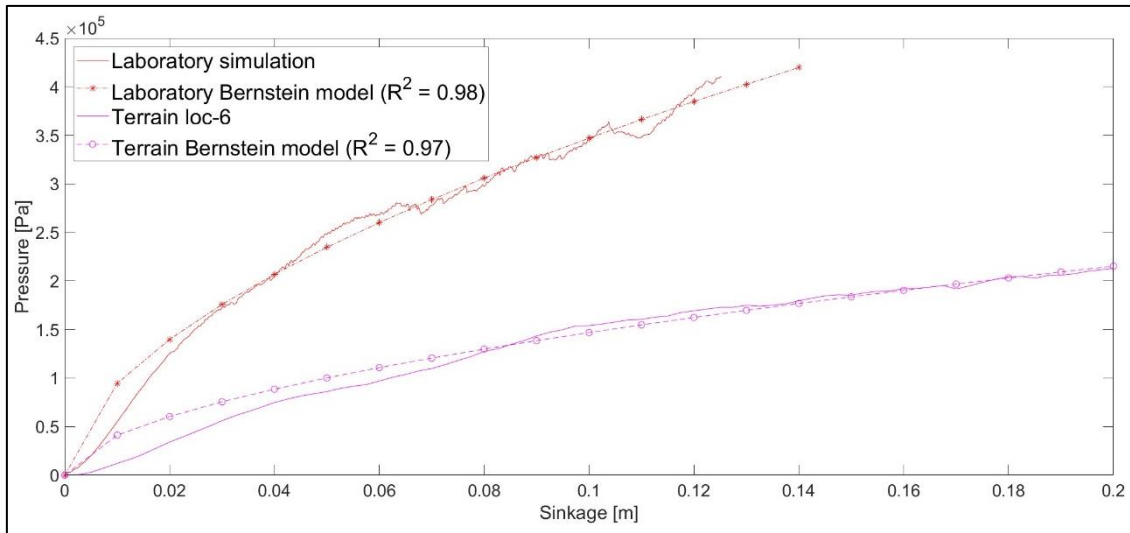


**Figure 4-56 Pressure sinkage simulation results at location - 6 in comparison with laboratory simulation results of Dia 0.15 m plate size for 2NS sand**

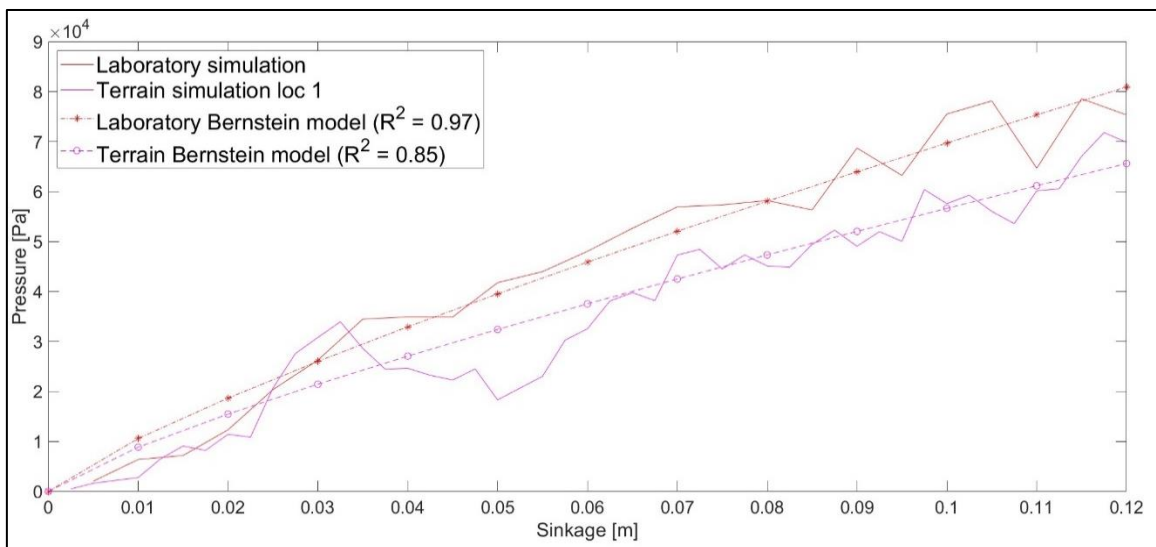


**Figure 4-57 Pressure sinkage simulation results at location - 1 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS**



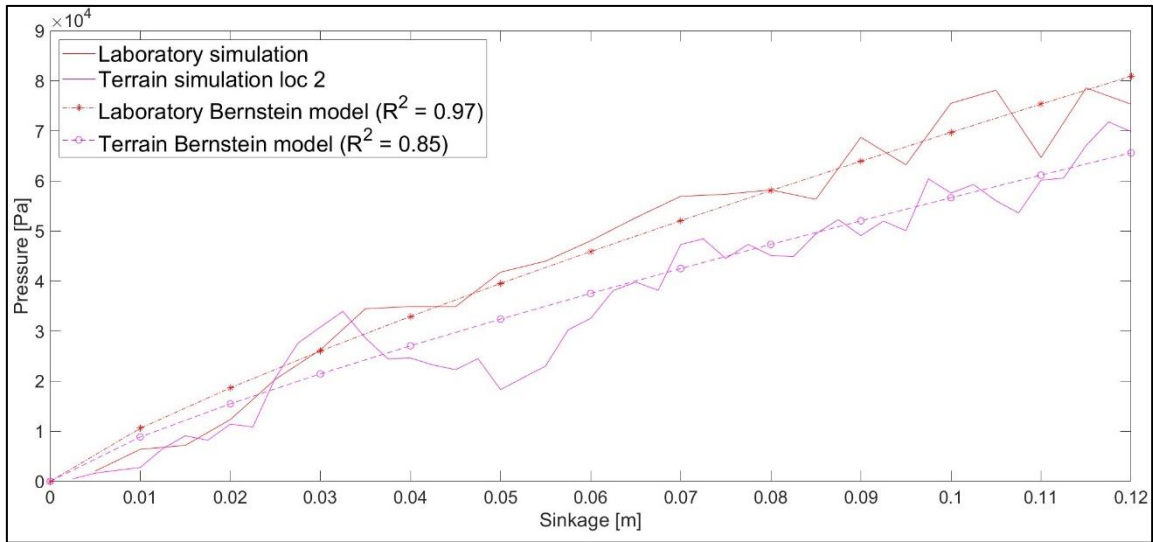


**Figure 4-58 Pressure sinkage simulation results at location - 2 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS**

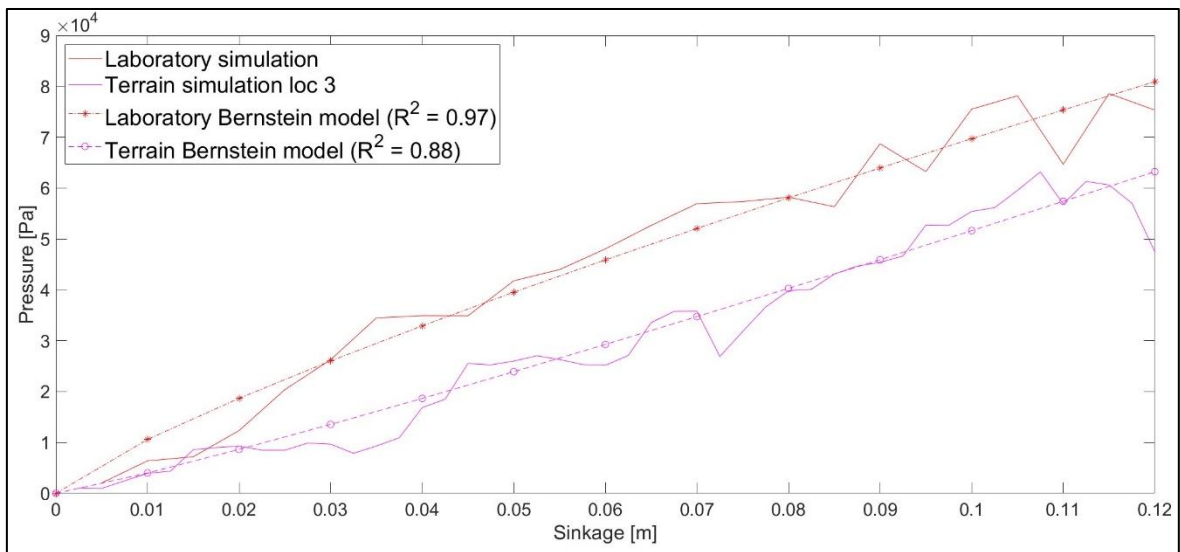


**Figure 4-59 Pressure sinkage simulation results at location - 3 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS**

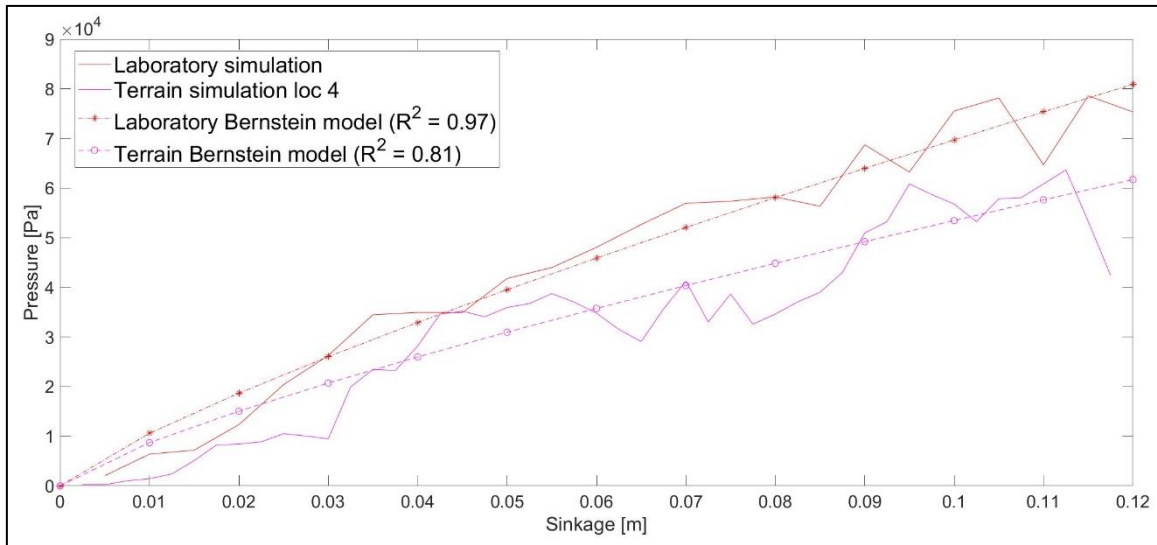


**Figure 4-60 Pressure sinkage simulation results at location - 4 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS**

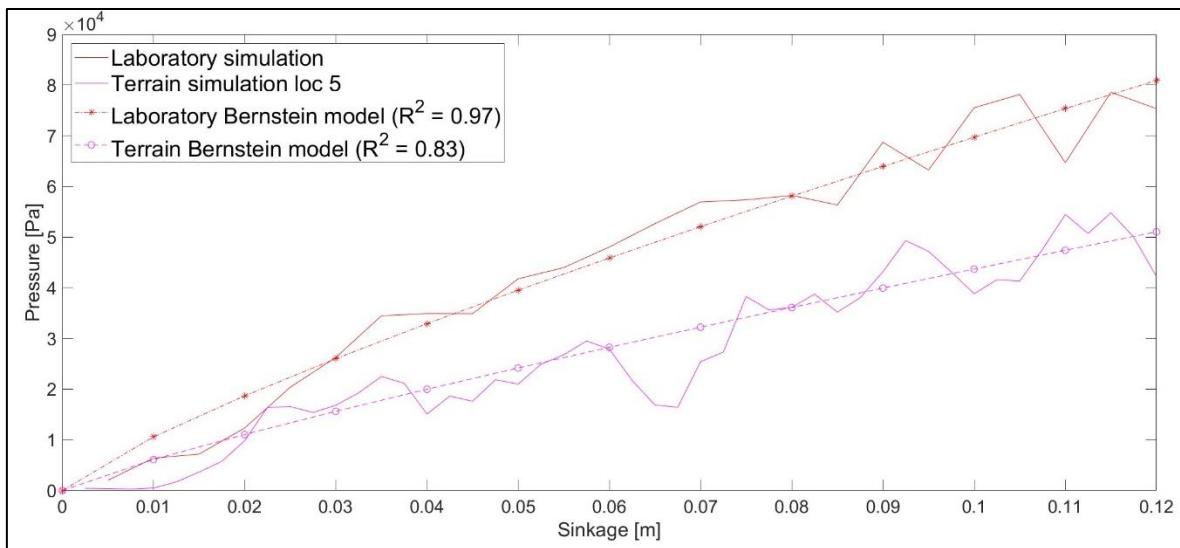


**Figure 4-61 Pressure sinkage simulation results at location - 5 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS**

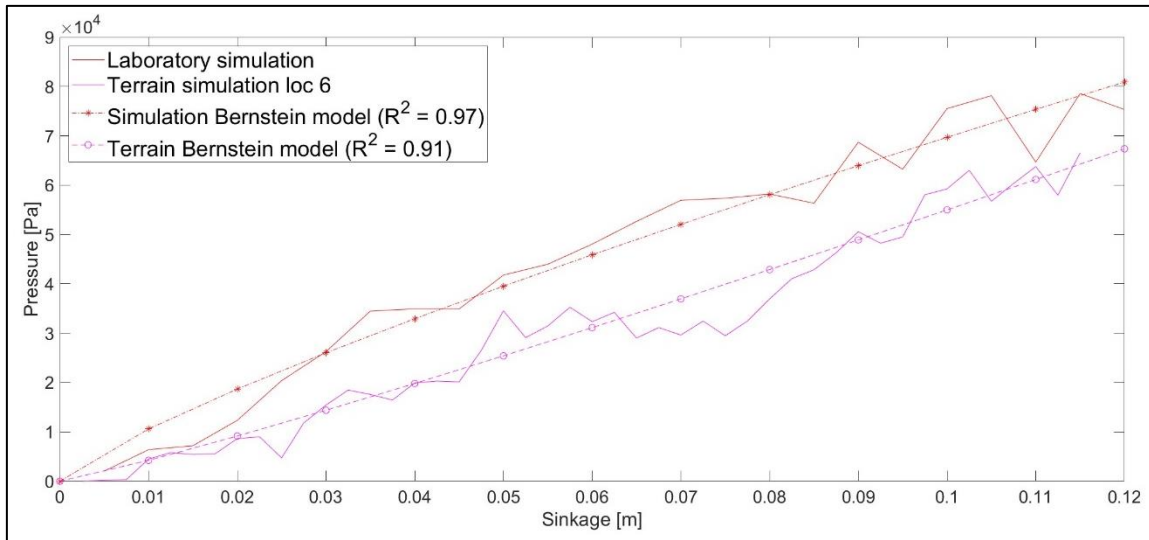


**Figure 4-62 Pressure sinkage simulation results at location - 6 in comparison with laboratory simulation results of Dia 0.1 m plate size for FGS**

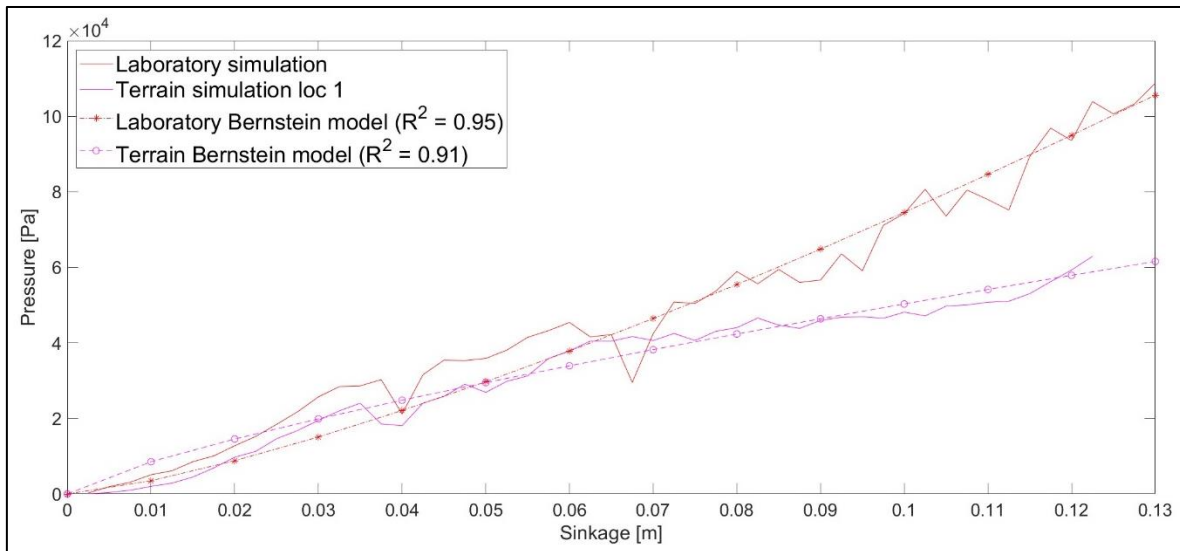


**Figure 4-63 Pressure sinkage simulation results at location - 1 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS**

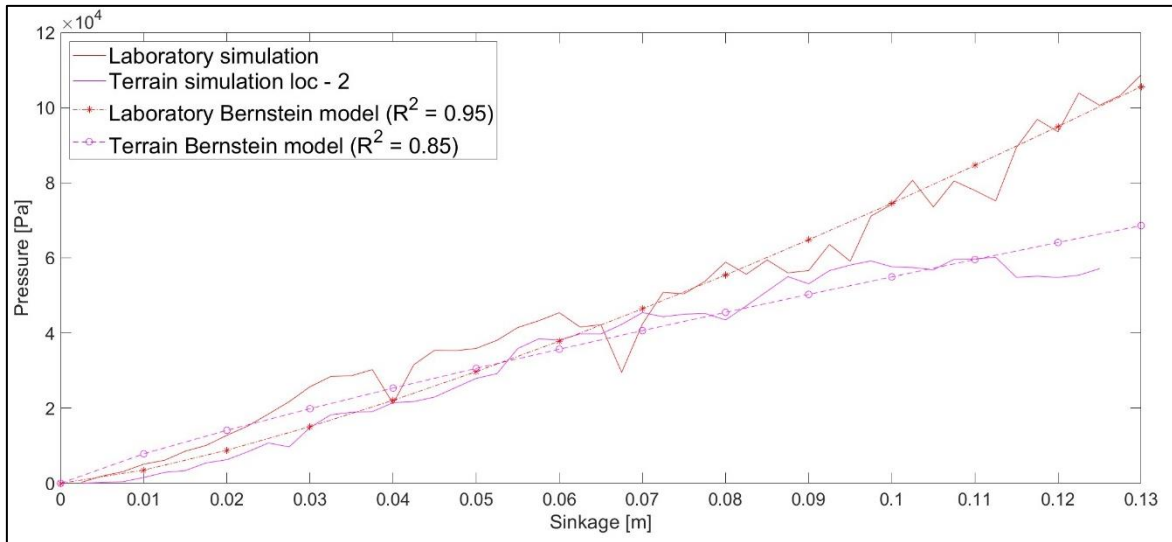


**Figure 4-64 Pressure sinkage simulation results at location - 2 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS**

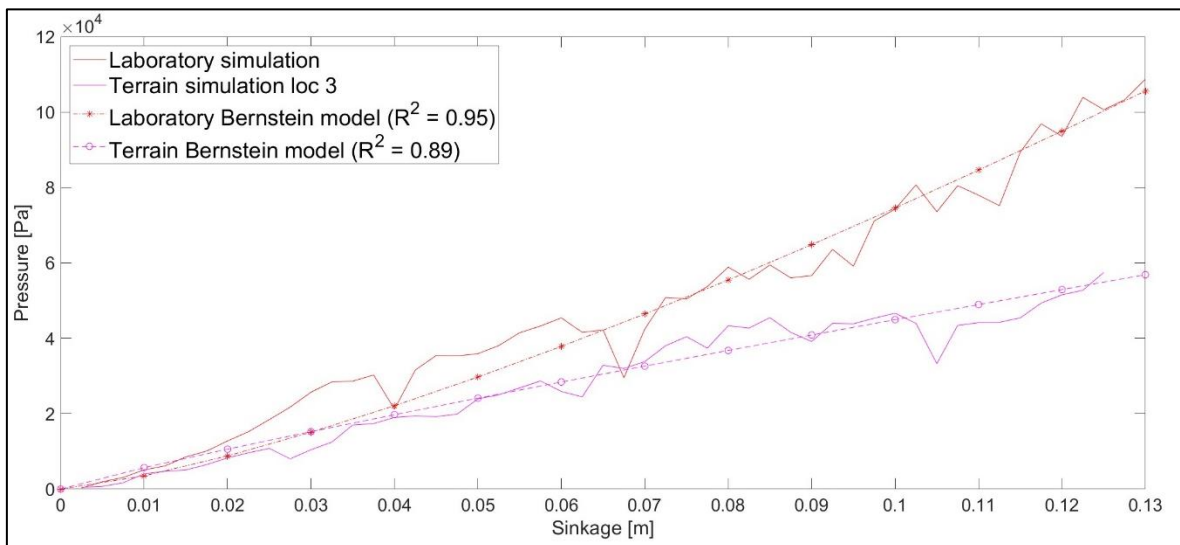


**Figure 4-65 Pressure sinkage simulation results at location - 3 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS**

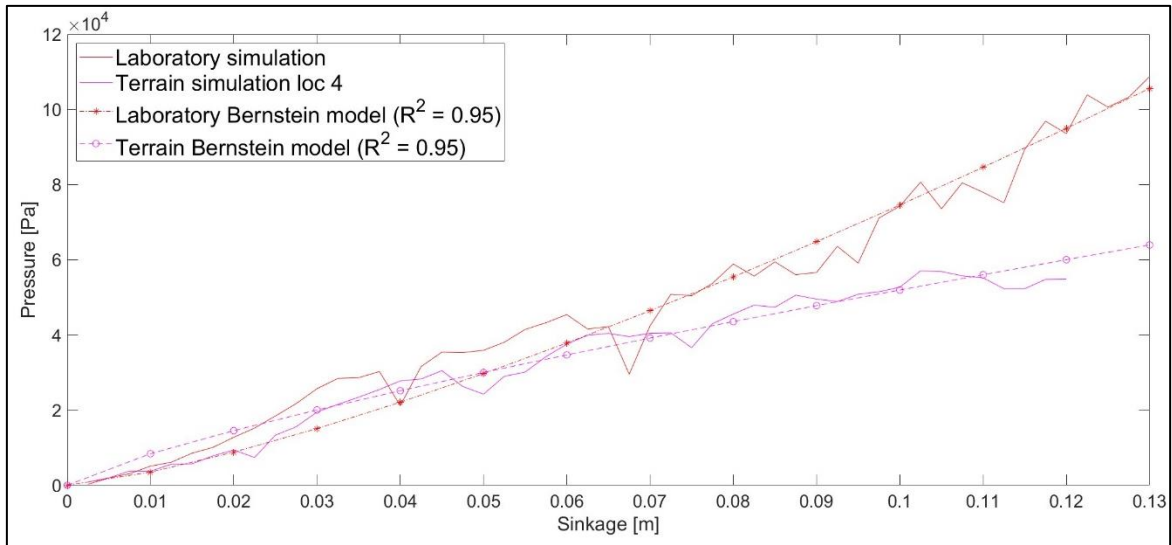


**Figure 4-66 Pressure sinkage simulation results at location - 4 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS**

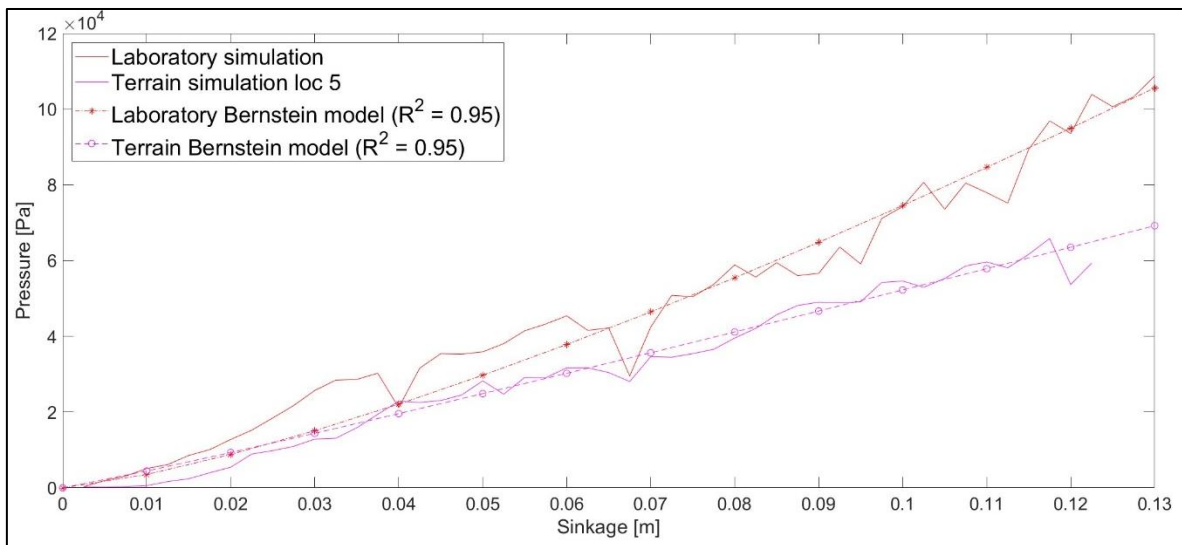


**Figure 4-67 Pressure sinkage simulation results at location - 5 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS**

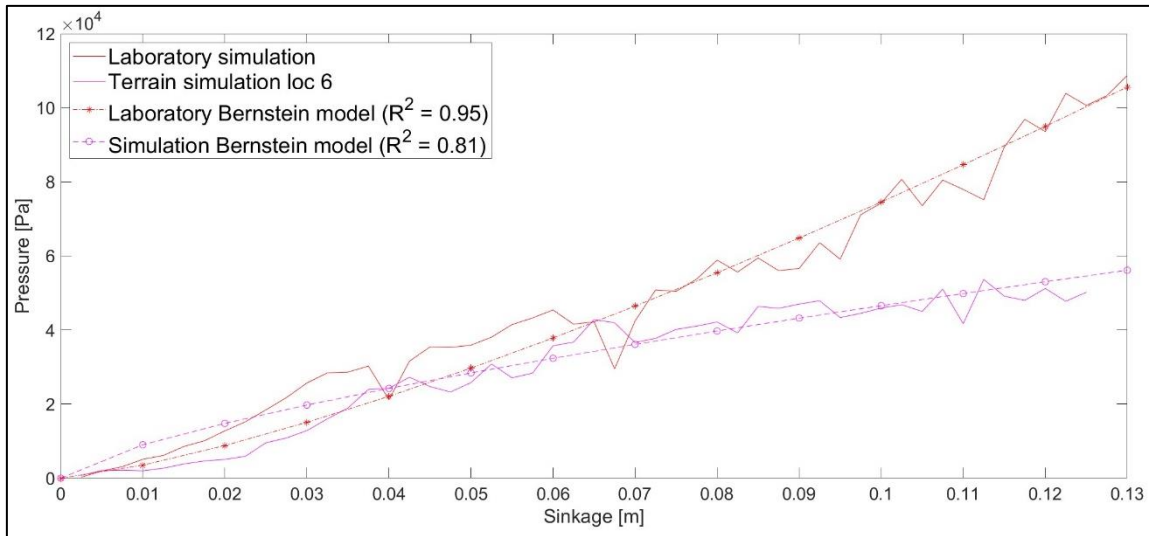


**Figure 4-68 Pressure sinkage simulation results at location - 6 in comparison with laboratory simulation results of Dia 0.15 m plate size for FGS**

**Table 4-12 Pressure-sinkage parameters summary for 2NS sand terrain model**

| Plate diameter       | 0.1  |          | 0.15 |          | Bekker parameter |                |                |
|----------------------|------|----------|------|----------|------------------|----------------|----------------|
|                      | n    | K        | n    | k        | n <sub>avg</sub> | k <sub>c</sub> | k <sub>φ</sub> |
| Terrain location – 1 | 0.42 | 5.63E+05 | 0.54 | 4.58E+05 | 0.48             | 3.14E+04       | 2.49E+05       |
| Terrain location – 2 | 0.42 | 6.20E+05 | 0.63 | 5.51E+05 | 0.52             | 2.07E+04       | 4.13E+05       |
| Terrain location – 3 | 0.53 | 6.85E+05 | 0.56 | 5.26E+05 | 0.54             | 4.77E+04       | 2.08E+05       |
| Terrain location – 4 | 0.41 | 6.38E+05 | 0.64 | 4.95E+05 | 0.52             | 4.30E+04       | 2.07E+05       |
| Terrain location – 5 | 0.37 | 6.17E+05 | 0.62 | 4.70E+05 | 0.49             | 4.41E+04       | 1.76E+05       |
| Terrain location – 6 | 0.54 | 6.90E+05 | 0.63 | 5.50E+05 | 0.58             | 4.18E+04       | 2.71E+05       |

**Table 4-13 Pressure-sinkage parameters summary for FGS terrain model**

| Plate diameter       | 0.1  |          | 0.15 |          | Bekker parameter |                |                |
|----------------------|------|----------|------|----------|------------------|----------------|----------------|
|                      | n    | K        | n    | k        | n <sub>avg</sub> | k <sub>c</sub> | k <sub>φ</sub> |
| Terrain location – 1 | 0.80 | 3.50E+05 | 0.77 | 3.11E+05 | 0.79             | 1.17E+04       | 2.32E+05       |
| Terrain location – 2 | 0.81 | 3.12E+05 | 0.84 | 3.70E+05 | 0.82             | -1.74E+04      | 4.85E+05       |
| Terrain location –3  | 1.11 | 5.14E+05 | 0.90 | 4.57E+05 | 1.01             | 1.71E+04       | 3.43E+05       |
| Terrain location – 4 | 0.79 | 3.29E+05 | 0.79 | 5.47E+05 | 0.79             | -6.56E+04      | 9.84E+05       |
| Terrain location – 5 | 0.85 | 4.13E+05 | 1.1  | 4.78E+05 | 0.96             | -1.94E+04      | 6.07E+05       |
| Terrain location – 6 | 1.11 | 4.37E+05 | 0.71 | 4.04E+05 | 0.91             | 1.01E+04       | 3.36E+05       |

The cone penetrometer tests simulated at six different patches of heterogeneous terrain model for both the soils. The tests were performed at the terrain model's marked centre locations, as shown in Figure 4-29. Figures 4-69 and 4-70 show the raw data of the cone penetration resistance or cone index (kPa) vs depth plot for the 2NS terrain model at six different locations compared with the average laboratory results of 2NS sand (section 4.5.2.1, Figure 4.27). Similarly, Figures 4-71 and 4-72 show the cone penetration resistance or cone index (kPa) vs depth plot for the FGS terrain model at six different locations compared with the average laboratory results of FGS (section 4.5.2.1, Figure 4.28). On analysing Figures 4-70 and 4-72, it was found that the cone penetration resistance decreases in a heterogeneous terrain system as compared to a similar homogeneous terrain system at the same depth of penetration. The average cone index value was

found to be 149.07 kPa (Coefficient of variance, CV = 9.38%) for the homogeneous laboratory soil model (average of 3 locations), which decreased to 114.22 kPa (CV = 11.1%) for the heterogeneous terrain model for 2NS sand at same soil conditions. This decrease in cone index value is because of the increase in void spaces in the heterogeneous terrain model. Therefore, it needs more compaction in a heterogeneous terrain model than in a homogeneous model to rearrange the particles to generate the same penetration resistance. Similar behaviour was observed in the FGS terrain model, where the average cone index value of 17.75 kPa (CV = 7.63%) in the homogeneous soil model was reduced to 14.79 kPa (CV = 31.64%) in the heterogeneous terrain model. The average cone index gradient (G) of the heterogeneous terrain model is 500.82 kPa/m with a standard deviation of  $\pm 274.72$  kPa/m for the 2NS terrain model. Similarly, the average cone index gradient (G) of the heterogeneous terrain model is 111.09 kPa/m with a standard deviation of  $\pm 87.78$  kPa/m.

It can be seen from Figure 4-70 and 4-72 that the curves of cone penetration resistance generally increases linearly with increasing depth. The linearity of this curve reflects the homogeneity of the soil sample with respect to depth. It means more linear the curve, homogeneous is the soil model, and similarly, the non-linear curve represents heterogeneity in soil models. The homogeneity can be evaluated by a second-order polynomial curve fit for the average cone index vs depth plot shown in both Figures. The linearity and non-linearity can be determined by inspecting the second-order term's coefficient. If the cone index (kPa) vs depth curve is not linear as determined by the second-order term's coefficient, then the cone index gradient varies at different penetration depths. The magnitude of the variation is dependent on the absolute value of the second-order term's coefficient.

The larger the second-order term's coefficient, the more significant the cone index gradient variation. However, the second-order term's coefficient for the 2NS sand heterogeneous terrain model is -0.0035 compared to -0.00016 for the homogeneous model. The magnitude of the coefficient of the heterogeneous terrain model is more than the homogeneous model, and the negative value



signifies that the soil strength decreases with the increasing penetration depth. Similarly, the second-order term's coefficient for the heterogeneous terrain system for the FGS model is 0.0002 compared to -4E-05 for the homogeneous model. The positive value signifies that the soil strength increases with the increasing penetration depth.

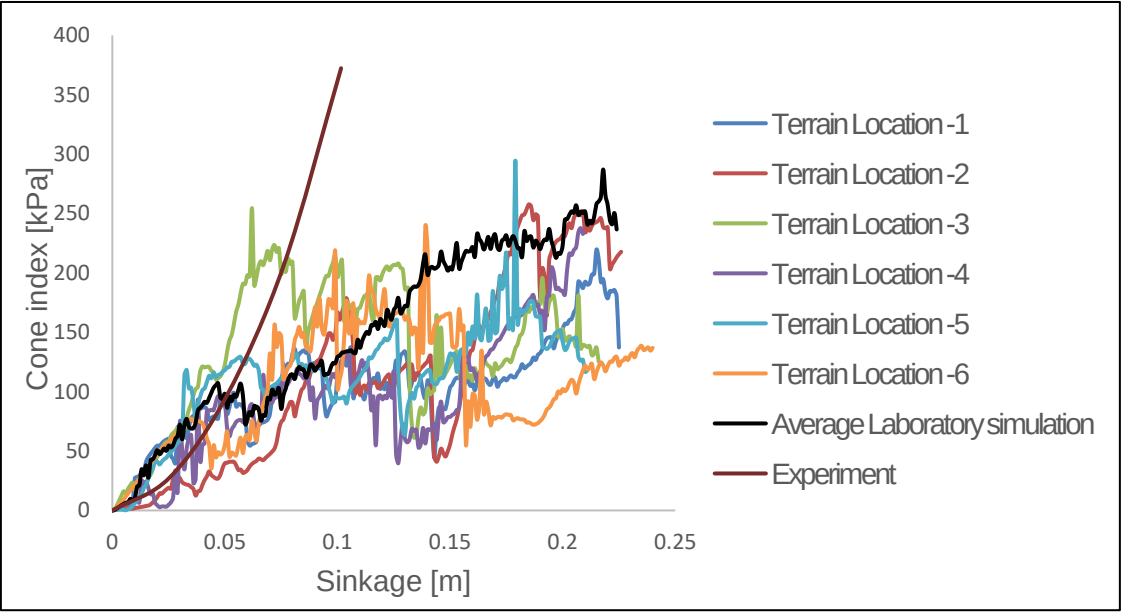


**Figure 4-69 Cone penetrometer terrain simulation results in comparison with NATO CDT-308 (2020) experimental and laboratory simulation results for 2NS sand**

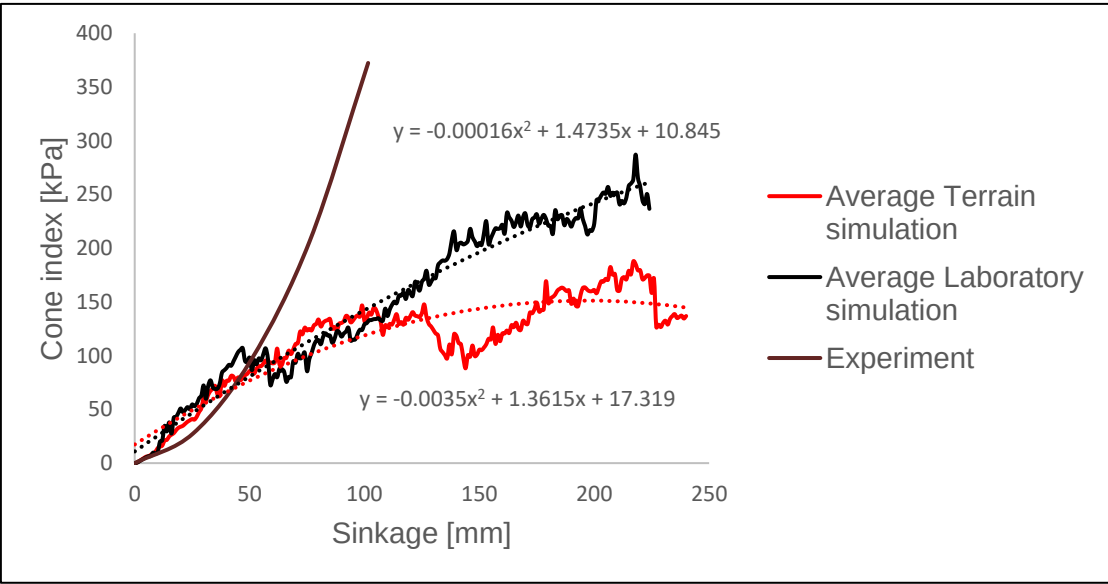


**Figure 4-70 Average cone penetrometer terrain simulation results and laboratory simulation results with the polynomial fit for 2NS sand**

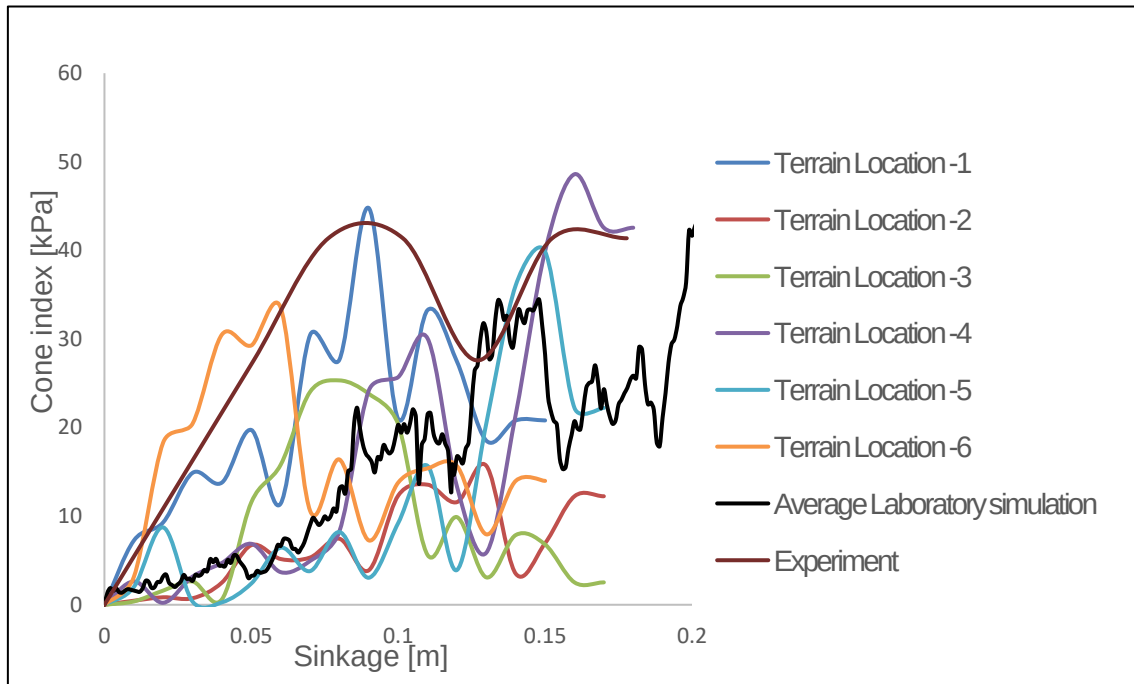


**Figure 4-71 Cone penetrometer terrain simulation results in comparison with NATO CDT-308 (2020) experimental and laboratory simulation results for FGS**

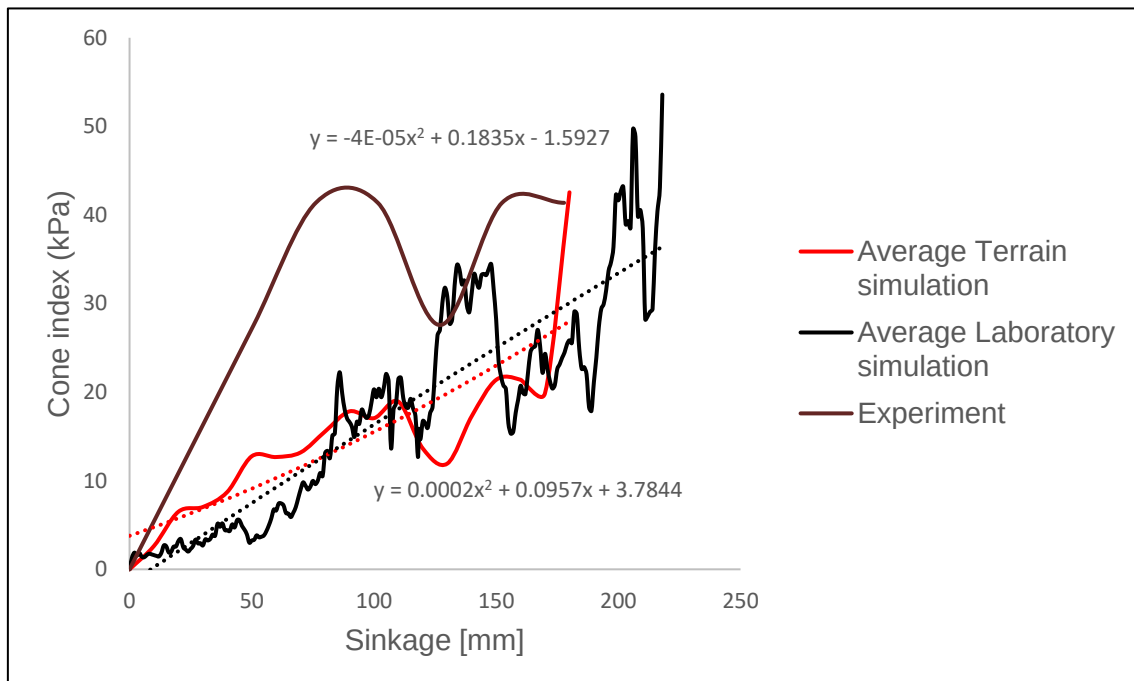


**Figure 4-72 Average cone penetrometer terrain simulation results and laboratory simulation results with the polynomial fit for FGS**

**Table 4-14 Average cone index gradient, G of 2NS sand terrain model**

| <b>Category</b>       | <b>Average cone index gradient, G<br/>(kPa/m)</b> |
|-----------------------|---------------------------------------------------|
| Terrain location -1   | 545.13                                            |
| Terrain location -2   | 1120.43                                           |
| Terrain location -3   | 342.39                                            |
| Terrain location -4   | 752.52                                            |
| Terrain location -5   | 560.51                                            |
| Terrain location -6   | 303.52                                            |
| Laboratory simulation | 1118.19                                           |
| Experiment            | 3637.39                                           |

**Table 4-15 Average cone index gradient, G of FGS terrain model**

| <b>Category</b>       | <b>Average cone index gradient, G<br/>(kPa/m)</b> |
|-----------------------|---------------------------------------------------|
| Terrain location -1   | 138.59                                            |
| Terrain location -2   | 72.95                                             |
| Terrain location -3   | 15.22                                             |
| Terrain location -4   | 252.25                                            |
| Terrain location -5   | 178.36                                            |
| Terrain location -6   | 9.23                                              |
| Laboratory simulation | 194.60                                            |
| Experiment            | 200.35                                            |

#### 4.5.4 Discussion

Scalability of soil is defined as establishing a relationship of soil mechanical properties of in-situ (or real) terrain models from laboratory models keeping the same soil conditions. In the present study, seven soil parameters are investigated. They are moduli of deformation w.r.t cohesion  $k_c$ , moduli of deformation w.r.t friction  $k_\phi$ , sinkage exponent  $n$ , cohesion  $c$ , internal angle of friction  $\phi$ , shear modulus  $k_x$  and cone index CI. The physical properties such as moisture content (Table 4.2), bulk density and compaction kept the same for the laboratory soil and real terrain models in simulations.

NATO CDT – 308 (2020) states the laboratory and in-situ experiments assumptions. In laboratory experiments, the soil sample can be assumed isotropic (same response in any direction), homogeneous/ uniform (same properties in any location), constant moisture content (change in moisture content takes much longer time than vehicle loading) and constant temperature. Therefore, constant particle shape and size distributions were maintained to develop the laboratory soil conditions in simulations. In in-situ experiments, the terrain should have discrete patches of different particle shapes and size distribution. The soil model can be assumed as anisotropic (different responses in all directions), heterogeneous/ non-uniform (different properties in all locations), constant moisture content and constant temperature. This variable distribution type changes the particle packing in the terrain model. Therefore, variable particle size, shape and space distributions are used to develop the in-situ soil models.

Three tests, i.e. the pressure-sinkage, the shear and the cone penetrometer tests, were simulated to investigate the seven different soil parameters. The tests were simulated on two different soils, i.e. 2NS sand and FGS, on four different soil models, i.e. two homogeneous (representing laboratory conditions) and two heterogeneous (representing in-situ soil model). The results obtained from laboratory conditions and in-situ conditions were analysed.

The statistical t-tests, F-tests and Cohen's  $d$  were performed to compare the results obtained from both the different conditions. The F-test states how

significant is the difference between the two group's variances. The t-test states how significant is the difference between the two groups measured in terms of means. Both these tests hold against the assumed hypothesis. The Cohen's d value is significant in statistical analysis as it is the ratio of the difference between the sample mean's to the pooled standard deviation. This value is independent of the group size, unlike the t-test and F-test. The statistical diagram is shown in Figure 4-73.

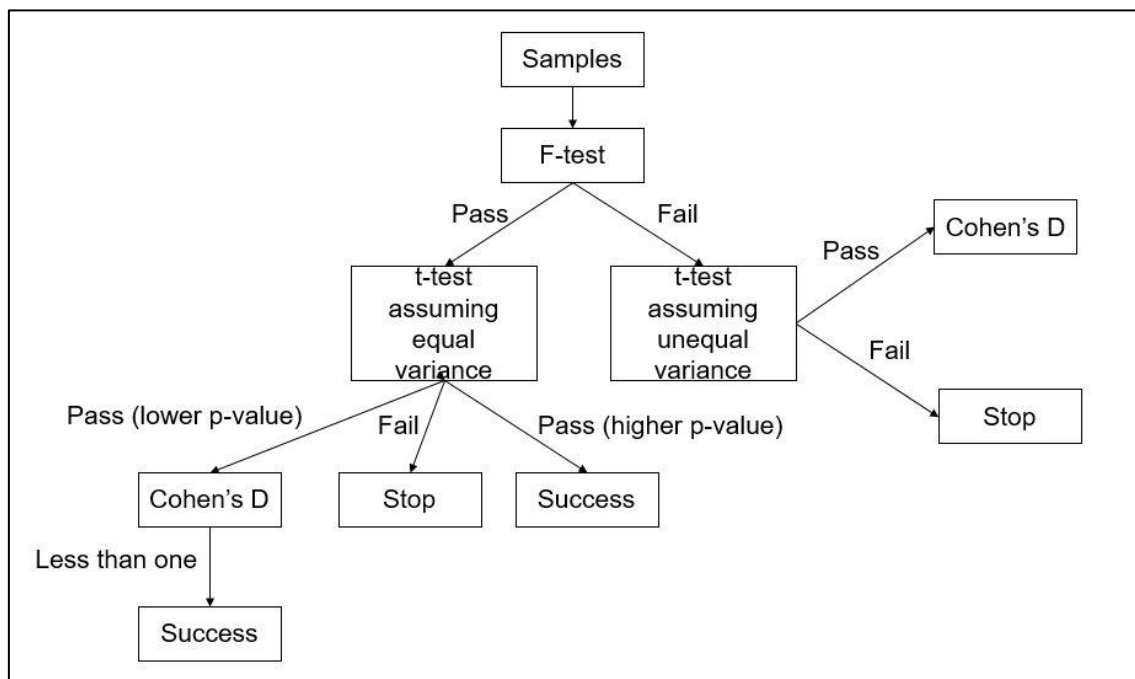


**Figure 4-73 Statistical analysis diagram**

#### 4.5.4.1 Shear tests

In the present study, two different shear tests were simulated based on their testing conditions. Wright (2012) found that the yield parameters obtained from the direct shear tests were in much better agreement with plate sinkage tests than those derived from other shear tests. Therefore, the direct shear test was simulated under laboratory conditions. The measurement of terrain properties in the field has certain advantages over laboratory conditions such as the measurements are taken when the terrain is in its natural state. Therefore, it eliminates the possibility of disturbing the terrain samples during the sampling

process. Therefore, the in-situ field shear test such as the shear ring test is simulated in the field conditions.

Three different soil parameters specifically, cohesion  $c$  (kPa), internal angle of friction  $\phi$  ( $^\circ$ ) and shear displacement modulus  $K$  (m) obtained from the direct shear test on laboratory conditions and shear ring test on terrain conditions were compared for both the soils, 2NS sand and FGS. Soils data are shown in Tables 4-9 and 4-10, respectively. Mathematically, the soil's shear behaviour is expressed using Janosi and Hanamoto (1961) equation considering soil shear failure based on Mohr-Coulomb failure criteria. The final expression is as follows.

$$\tau = (c + \sigma * \tan\phi) * \left(1 - e^{\frac{-j}{K}}\right) \quad (4-63)$$

where  $\tau$  is the shear stress,  $\sigma$  is the normal stress and  $j$  is the shear displacement. Using data from Tables 4-9 and 4-10 for both the soils, the shear stress-shear displacement curves were computed for both the conditions, laboratory and in-situ, under the same normal stresses. The normal stresses selected for the study are 10 kPa, 20 kPa and 30 kPa for both the soil types. These values are in the common band of normal stresses under which both the laboratory and in-situ experiments were conducted. The shear stresses were calculated for the entire range of shear displacement, i.e. 0 to 0.5 m as per the simulation model's dimensions.

Swanson (1973) analysed the scale model tests performed at the U.S. Army Engineer Waterways Experiment Station (WES) by developing a scatter band of one standard deviation from the prediction curve. It was found that sixty-eight per cent of the scaled system data points fell within the developed scatter band. Therefore, in the present study, a similar approach is followed. One standard deviation band is developed around the laboratory simulation plots to compare the obtained shear stress-shear displacement relations of terrain models at six different locations for both soils, as shown in Figures 4-74 to 4-85.

It was found that on increasing the normal stress from 10 kPa to 30 kPa, the deviation of shear strength behaviour between laboratory homogeneous soil

model and in-situ heterogeneous soil model increases. The 2NS sand is more sensitive to the change in normal stress than FGS soil. It might be because coarse-grained soil strength is governed by the grain size of particles and structural arrangement. Therefore, when the normal load is increased on 2NS soil, the structural arrangement of particles changes, which changes the soil strength. However, for fine-grained soils, the strength is mostly controlled by mineralogical factors and moisture content. When the soil is wet, it poses little or no load-bearing strength. Therefore, not much deviation is observed with normal stress for FGS.

It is interesting to note that the shear strength properties such as cohesion and internal angle of friction vary differently for both soil types (Table 4-9 and 4-10). On average, for coarse-grained 2NS sand (Table 4-9), the in-situ heterogeneous soil model's cohesion is significantly lower than the corresponding value for the homogeneous laboratory soil. Similarly, the in-situ soil model's internal angle of friction is significantly higher than the corresponding value for the laboratory soil. Since soil cohesion is a function of mineralogy, moisture content, particle orientation and density. Because of the heterogeneity of the sandy soil model, there is an inherent spatial variation that reduces the ultimate bearing capacity (in an average sense) of soil (Popescu et al., 2005). However, this is not a universal phenomenon and it also depends on particle size distribution. Therefore, the soil strength decreases, thus decreasing the soil cohesion in the selected soils. The angle of internal friction is also a function of particle roundness or angularity and particle size. Therefore, with the increase of heterogeneity, the participating particle's angularity increases with the larger particle size. The individual larger soil grains can interlock, much like the teeth of two highly irregular gears. The individual grain particle must be lifted against the normal load for sliding to occur. Therefore, the friction generated between such individual particles increases, which increases the internal friction of soil.

The contrasting effect of heterogeneity was observed on the FGS. On average, for FGS (Table 4-10), the cohesion of in-situ heterogeneous soil is slightly higher than the corresponding value for the laboratory homogeneous soil model. The

average internal angle of friction decreases for in-situ heterogeneous soil compared to the laboratory homogeneous soil model. Unlike coarse-grained soil, fine-grained soil depends on dry strength, dilatancy, dispersion and plasticity. As the soil's moisture content increases and the soil becomes wet, it loses much or near zero of its strength. Therefore, there is a slight difference of approximately 0.8 kPa between the FGS cohesion value for both soil models. It is also interesting to note that the angle of internal friction is higher for the laboratory homogeneous soil model than for the in-situ soil model. It might be because of the effect of dilatancy, which means that the soil becomes more viscous under high pressures. Since the laboratory condition tests were performed at high normal stress values (12-52 kPa), which results in increased friction between the soil layers compared to in-situ shear ring tests performed at lower normal stress values (8.6-25.8 kPa).

Further, the statistical tests were conducted between the laboratory simulation results and the in-situ shear test at three normal stress values, i.e. 10kPa, 20 kPa and 30 kPa, considered as the samples mentioned in Figure 4-73. Initially, F-tests were conducted at a 5% significant level between the laboratory soil model and the respective in-situ terrain location to compare the sample's variance. The F-value was calculated as a ratio of both the variance's square. This obtained F-value is compared with the F-critical obtained from F-distribution at  $\alpha = 0.05$ . If the obtained F value is less than the F-critical, then there is no significant difference between the sample's variance. The F tests were successful for both the 2NS sand and FGS except for FGS at 30kPa normal stress. Therefore, there is no significant difference between the sample's variances.

Based on the results obtained from F tests, t-tests assuming equal variances between the laboratory soil model and respective in-situ terrain locations were performed. The t-tests were conducted at a 5% significant level between the laboratory soil model and in-situ terrain location to compare the mean between the samples. The t-test is performed against the hypothesized mean difference of  $\pm 10\%$  of the laboratory mean soil samples. As per the NATO CDT – 308 (2020), the required maximum error is  $\pm 10\%$  as per guidelines. The t value was



calculated as the ratio of the difference of means between the samples to each sample set's variances. The obtained t-value was calculated with the t-distribution table for one tail and two tails at  $\alpha = 0.05$ . If the obtained t value is less than the t-critical for both one tail and two tails, there is no significant difference between the sample's mean. The probability level (alpha level, p-value) to accept the criterion is also greater than 0.05, except in one case. Therefore, t-tests were successful for both the 2NS and FGS soil except for FGS at 30kPa normal stress. Therefore, there is no significant difference between the means of the samples.

Cohen's d value for 2NS sand is less than one for all the three normal stress values. Therefore, it can be concluded that there is no significant difference between the stress-shear displacement relations between the laboratory conditions and the in-situ terrain model except for location 2. However, Cohen's d value for FGS is greater than one for 20 kPa and 30 kPa normal stress values at location 4 and location 5. Therefore, the hypothesis is rejected in these two cases. There is no significant difference between the shear stress – shear displacement relations between the laboratory conditions and the in-situ terrain model except at location 4 and location 5.

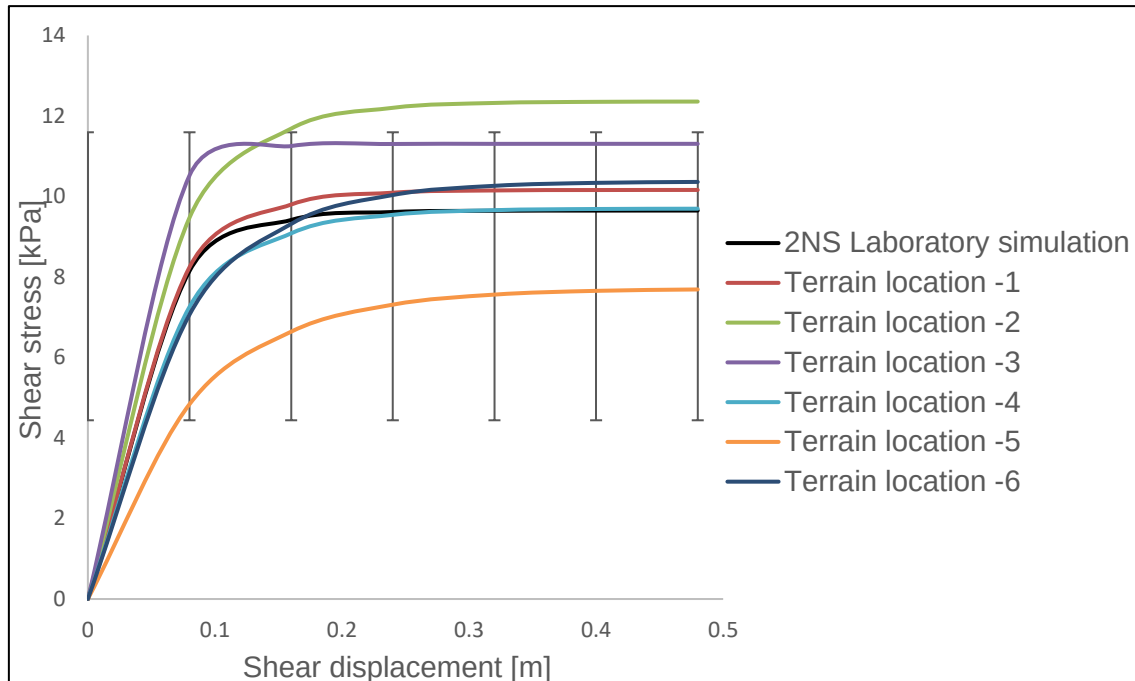


**Figure 4-74 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 10 kPa for 2NS sand**

| F-Test Two-Sample for Variances |            |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|------------|------------|---------------------------------------------|------------|------------|
|                                 | Location-1 | Laboratory |                                             | Location-1 | Laboratory |
| Mean                            | 8.372579   | 8.017651   | Mean                                        | 8.3725792  | 8.017651   |
| Variance                        | 14.11179   | 12.7967    | Variance                                    | 14.111786  | 12.7967    |
| Observations                    | 7          | 7          | Observations                                | 7          | 7          |
| df                              | 6          | 6          | Pooled Variance                             | 13.454242  |            |
| F                               | 1.102768   |            | Hypothesized Mean Difference                | 0.801      |            |
| P(F<=f) one-tail                | 0.454255   |            | df                                          | 12         |            |
| F Critical one-tail             | 4.283866   |            | t Stat                                      | -0.227515  |            |
|                                 |            |            | P(T<=t) one-tail                            | 0.4119269  |            |
|                                 |            |            | t Critical one-tail                         | 1.7822876  |            |
|                                 |            |            | P(T<=t) two-tail                            | 0.8238538  |            |
|                                 |            |            | t Critical two-tail                         | 2.1788128  |            |
|                                 |            |            | Cohen's d                                   | 0.0967634  |            |

(a)

**Figure 4-75 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at normal stress of 10 kPa for 2NS sand**

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Location-2</i> | <i>Laboratory</i> |
| Mean                            | 10.056217         | 8.017651          |
| Variance                        | 20.738284         | 12.7967           |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.6205964         |                   |
| P(F<=f) one-tail                | 0.2861469         |                   |
| F Critical one-tail             | 4.2838657         |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Location-2</i> | <i>Laboratory</i> |
| Mean                                        | 10.056217         | 8.017651          |
| Variance                                    | 20.738284         | 12.7967           |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 16.767491         |                   |
| Hypothesized Mean Difference                | 0.801             |                   |
| df                                          | 12                |                   |
| t Stat                                      | 0.5654163         |                   |
| P(T<=t) one-tail                            | 0.2911041         |                   |
| t Critical one-tail                         | 1.7822876         |                   |
| P(T<=t) two-tail                            | 0.5822083         |                   |
| t Critical two-tail                         | 2.1788128         |                   |
| Cohen's d                                   | 0.4978411         |                   |

(b)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Location-3</i> | <i>Laboratory</i> |
| Mean                            | 9.5717656         | 8.017651          |
| Variance                        | 17.898329         | 12.796699         |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.3986677         |                   |
| P(F<=f) one-tail                | 0.3470296         |                   |
| F Critical one-tail             | 4.2838657         |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Location-3</i> | <i>Laboratory</i> |
| Mean                                        | 9.5717656         | 8.017651          |
| Variance                                    | 17.898329         | 12.796699         |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 15.347514         |                   |
| Hypothesized Mean Difference                | 0.801             |                   |
| df                                          | 12                |                   |
| t Stat                                      | 0.3596467         |                   |
| P(T<=t) one-tail                            | 0.3626805         |                   |
| t Critical one-tail                         | 1.7822876         |                   |
| P(T<=t) two-tail                            | 0.725361          |                   |
| t Critical two-tail                         | 2.1788128         |                   |
| Cohen's d                                   | 0.3967017         |                   |

(c)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Laboratory</i> | <i>Location-4</i> |
| Mean                            | 8.017651          | 7.84718           |
| Variance                        | 12.796699         | 12.73878          |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.0045467         |                   |
| P(F<=f) one-tail                | 0.4978736         |                   |
| F Critical one-tail             | 4.2838657         |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Laboratory</i> | <i>Location-4</i> |
| Mean                                        | 8.017650988       | 7.84718           |
| Variance                                    | 12.79669891       | 12.73878          |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 12.76773935       |                   |
| Hypothesized Mean Difference                | 0.801             |                   |
| df                                          | 12                |                   |
| t Stat                                      | -0.330127712      |                   |
| P(T<=t) one-tail                            | 0.37349771        |                   |
| t Critical one-tail                         | 1.782287556       |                   |
| P(T<=t) two-tail                            | 0.74699542        |                   |
| t Critical two-tail                         | 2.17881283        |                   |
| Cohen's d                                   | 0.047708294       |                   |

(d)

| F-Test Two-Sample for Variances |             |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|-------------|------------|---------------------------------------------|------------|------------|
|                                 |             |            |                                             |            |            |
|                                 | Laboratory  | Location-5 |                                             | Laboratory | Location-5 |
| Mean                            | 8.017650988 | 5.957253   | Mean                                        | 8.017651   | 5.957253   |
| Variance                        | 12.79669891 | 7.917975   | Variance                                    | 12.796699  | 7.917975   |
| Observations                    | 7           | 7          | Observations                                | 7          | 7          |
| df                              | 6           | 6          | Pooled Variance                             | 10.357337  |            |
| F                               | 1.616158073 |            | Hypothesized Mean Difference                | 0.801      |            |
| P(F<=f) one-tail                | 0.287229093 |            | df                                          | 12         |            |
| F Critical one-tail             | 4.283865714 |            | t Stat                                      | 0.7321046  |            |
|                                 |             |            | P(T<=t) one-tail                            | 0.2390801  |            |
|                                 |             |            | t Critical one-tail                         | 1.7822876  |            |
|                                 |             |            | P(T<=t) two-tail                            | 0.4781602  |            |
|                                 |             |            | t Critical two-tail                         | 2.1788128  |            |
|                                 |             |            | Cohen's d                                   | 0.6402169  |            |

(e)

| F-Test Two-Sample for Variances |            |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|------------|------------|---------------------------------------------|------------|------------|
|                                 |            |            |                                             |            |            |
|                                 | Laboratory | Location-6 |                                             | Laboratory | Location-6 |
| Mean                            | 8.197543   | 8.017651   | Mean                                        | 8.017651   | 8.197543   |
| Variance                        | 14.44287   | 12.7967    | Variance                                    | 12.7967    | 14.44287   |
| Observations                    | 7          | 7          | Observations                                | 7          | 7          |
| df                              | 6          | 6          | Pooled Variance                             | 13.61978   |            |
| F                               | 1.12864    |            | Hypothesized Mean Difference                | 0.801      |            |
| P(F<=f) one-tail                | 0.443482   |            | df                                          | 12         |            |
| F Critical one-tail             | 4.283866   |            | t Stat                                      | -0.497244  |            |
|                                 |            |            | P(T<=t) one-tail                            | 0.314001   |            |
|                                 |            |            | t Critical one-tail                         | 1.782288   |            |
|                                 |            |            | P(T<=t) two-tail                            | 0.628002   |            |
|                                 |            |            | t Critical two-tail                         | 2.178813   |            |
|                                 |            |            | Cohen's d                                   | -0.048745  |            |

(f)

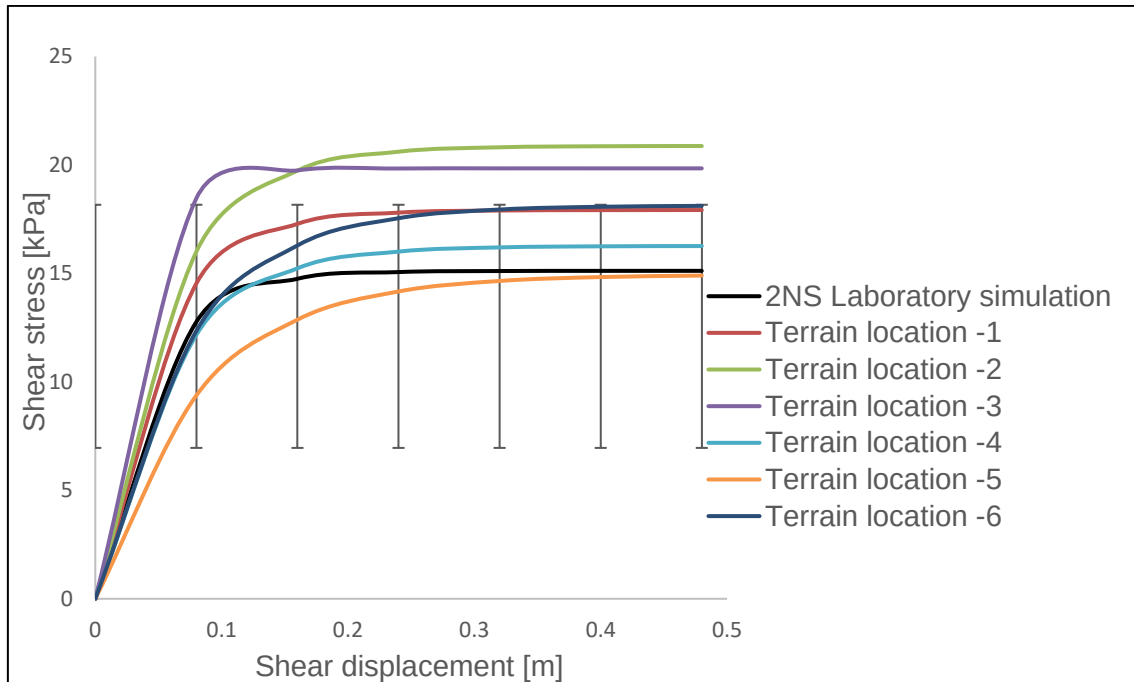


**Figure 4-76 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 20 kPa for 2NS sand**

| F-Test Two-Sample for Variances |            |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|------------|------------|---------------------------------------------|------------|------------|
|                                 | Location-1 | Laboratory |                                             | Location-1 | Laboratory |
| Mean                            | 14.76785   | 12.56248   | Mean                                        | 14.767848  | 12.56248   |
| Variance                        | 43.90337   | 31.41626   | Variance                                    | 43.903375  | 31.41626   |
| Observations                    | 7          | 7          | Observations                                | 7          | 7          |
| df                              | 6          | 6          | Pooled Variance                             | 37.659817  |            |
| F                               | 1.397473   |            | Hypothesized Mean Difference                | 1.256      |            |
| P(F<=f) one-tail                | 0.347398   |            | df                                          | 12         |            |
| F Critical one-tail             | 4.283866   |            | t Stat                                      | 0.2894206  |            |
|                                 |            |            | P(T<=t) one-tail                            | 0.3885996  |            |
|                                 |            |            | t Critical one-tail                         | 1.7822876  |            |
|                                 |            |            | P(T<=t) two-tail                            | 0.7771992  |            |
|                                 |            |            | t Critical two-tail                         | 2.1788128  |            |
|                                 |            |            | Cohen's d                                   | 0.3593701  |            |

(a)

**Figure 4-77 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at normal stress of 20 kPa for 2NS sand**

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Location-2</i> | <i>Laboratory</i> |
| Mean                            | 16.988096         | 12.56248          |
| Variance                        | 59.182455         | 31.41626          |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.8838161         |                   |
| P(F<=f) one-tail                | 0.2301645         |                   |
| F Critical one-tail             | 4.2838657         |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Location-2</i> | <i>Laboratory</i> |
| Mean                                        | 16.988096         | 12.56248          |
| Variance                                    | 59.182455         | 31.41626          |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 45.299357         |                   |
| Hypothesized Mean Difference                | 1.256             |                   |
| df                                          | 12                |                   |
| t Stat                                      | 0.8810379         |                   |
| P(T<=t) one-tail                            | 0.1978005         |                   |
| t Critical one-tail                         | 1.7822876         |                   |
| P(T<=t) two-tail                            | 0.395601          |                   |
| t Critical two-tail                         | 2.1788128         |                   |
| Cohen's d                                   | 0.6575483         |                   |

(b)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Location-3</i> | <i>Laboratory</i> |
| Mean                            | 16.798794         | 12.562481         |
| Variance                        | 55.12957          | 31.416259         |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.7548101         |                   |
| P(F<=f) one-tail                | 0.2556945         |                   |
| F Critical one-tail             | 4.2838657         |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Location-3</i> | <i>Laboratory</i> |
| Mean                                        | 16.798794         | 12.562481         |
| Variance                                    | 55.12957          | 31.416259         |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 43.272915         |                   |
| Hypothesized Mean Difference                | 1.256             |                   |
| df                                          | 12                |                   |
| t Stat                                      | 0.847594          |                   |
| P(T<=t) one-tail                            | 0.2066253         |                   |
| t Critical one-tail                         | 1.7822876         |                   |
| P(T<=t) two-tail                            | 0.4132505         |                   |
| t Critical two-tail                         | 2.1788128         |                   |
| Cohen's d                                   | 0.6439913         |                   |

(c)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Location-4</i> | <i>Laboratory</i> |
| Mean                            | 13.161908         | 12.56248          |
| Variance                        | 35.83755          | 31.41626          |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.1407326         |                   |
| P(F<=f) one-tail                | 0.4385457         |                   |
| F Critical one-tail             | 4.2838657         |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Location-4</i> | <i>Laboratory</i> |
| Mean                                        | 13.16190791       | 12.56248          |
| Variance                                    | 35.83755019       | 31.41626          |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 33.62690471       |                   |
| Hypothesized Mean Difference                | 1.256             |                   |
| df                                          | 12                |                   |
| t Stat                                      | -0.211823165      |                   |
| P(T<=t) one-tail                            | 0.417900027       |                   |
| t Critical one-tail                         | 1.782287556       |                   |
| P(T<=t) two-tail                            | 0.835800053       |                   |
| t Critical two-tail                         | 2.17881283        |                   |
| Cohen's d                                   | 0.103369641       |                   |

(d)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Laboratory</i> | <i>Location-5</i> |
| Mean                            | 12.56248069       | 11.54366          |
| Variance                        | 31.41625922       | 29.731            |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.056683591       |                   |
| P(F<=f) one-tail                | 0.474174947       |                   |
| F Critical one-tail             | 4.283865714       |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Laboratory</i> | <i>Location-5</i> |
| Mean                                        | 12.562481         | 11.54366          |
| Variance                                    | 31.416259         | 29.731            |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 30.573629         |                   |
| Hypothesized Mean Difference                | 1.256             |                   |
| df                                          | 12                |                   |
| t Stat                                      | -0.0802502        |                   |
| P(T<=t) one-tail                            | 0.4686805         |                   |
| t Critical one-tail                         | 1.7822876         |                   |
| P(T<=t) two-tail                            | 0.9373611         |                   |
| t Critical two-tail                         | 2.1788128         |                   |
| Cohen's d                                   | 0.1842562         |                   |

(e)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Location-6</i> | <i>Laboratory</i> |
| Mean                            | 14.33234          | 12.56248          |
| Variance                        | 44.14895          | 31.41626          |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.40529           |                   |
| P(F<=f) one-tail                | 0.344996          |                   |
| F Critical one-tail             | 4.283866          |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Location-6</i> | <i>Laboratory</i> |
| Mean                                        | 14.33234          | 12.56248          |
| Variance                                    | 44.14895          | 31.41626          |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 37.7826           |                   |
| Hypothesized Mean Difference                | 1.256             |                   |
| df                                          | 12                |                   |
| t Stat                                      | 0.156399          |                   |
| P(T<=t) one-tail                            | 0.43916           |                   |
| t Critical one-tail                         | 1.782288          |                   |
| P(T<=t) two-tail                            | 0.878319          |                   |
| t Critical two-tail                         | 2.178813          |                   |
| Cohen's d                                   | 0.287934          |                   |

(f)

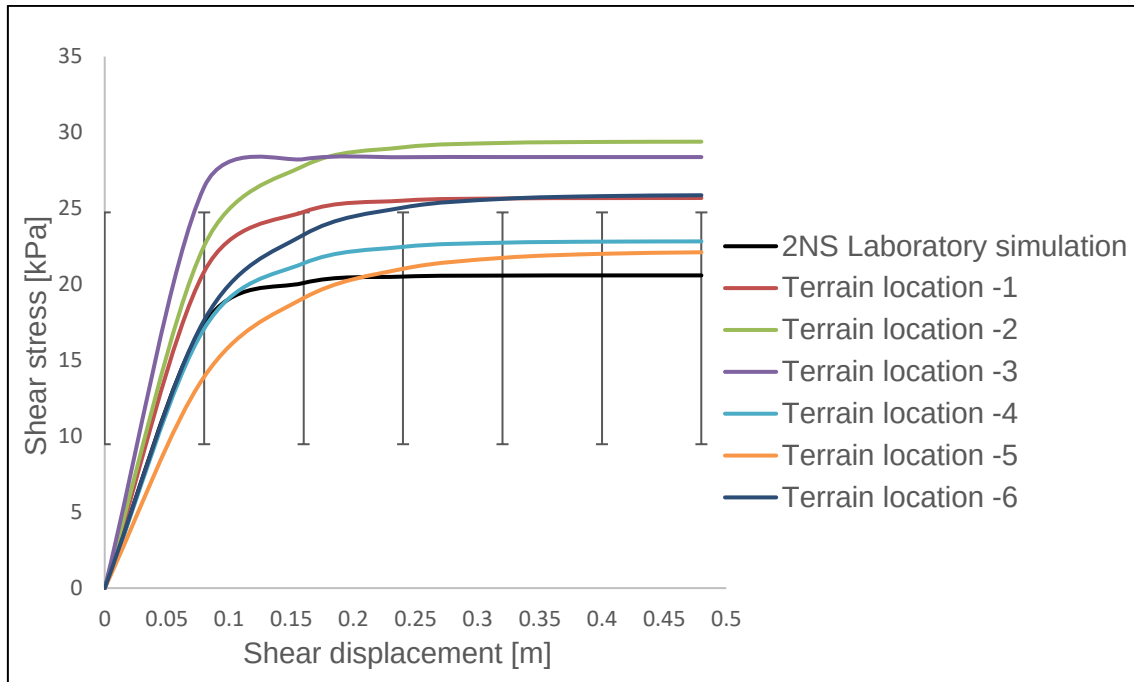


**Figure 4-78 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 30 kPa for 2NS sand**

| F-Test Two-Sample for Variances |            |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|------------|------------|---------------------------------------------|------------|------------|
|                                 | Location-1 | Laboratory |                                             | Location-1 | Laboratory |
| Mean                            | 21.16312   | 17.10731   | Mean                                        | 21.163118  | 17.10731   |
| Variance                        | 90.16184   | 58.25955   | Variance                                    | 90.161836  | 58.25955   |
| Observations                    | 7          | 7          | Observations                                | 7          | 7          |
| df                              | 6          | 6          | Pooled Variance                             | 74.210693  |            |
| F                               | 1.547589   |            | Hypothesized Mean Difference                | 2.116      |            |
| P(F<=f) one-tail                | 0.304611   |            | df                                          | 12         |            |
| F Critical one-tail             | 4.283866   |            | t Stat                                      | 0.4212689  |            |
|                                 |            |            | P(T<=t) one-tail                            | 0.340503   |            |
|                                 |            |            | t Critical one-tail                         | 1.7822876  |            |
|                                 |            |            | P(T<=t) two-tail                            | 0.6810061  |            |
|                                 |            |            | t Critical two-tail                         | 2.1788128  |            |
|                                 |            |            | Cohen's d                                   | 0.4708083  |            |

(a)

**Figure 4-79 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at normal stress of 30 kPa for 2NS sand**



| F-Test Two-Sample for Variances |            |            |
|---------------------------------|------------|------------|
|                                 | Location-2 | Laboratory |
| Mean                            | 23.9199758 | 17.10731   |
| Variance                        | 117.334307 | 58.25955   |
| Observations                    | 7          | 7          |
| df                              | 6          | 6          |
| F                               | 2.01399267 |            |
| P(F<=f) one-tail                | 0.20758925 |            |
| F Critical one-tail             | 4.28386571 |            |

| t-Test: Two-Sample Assuming Equal Variances |             |            |
|---------------------------------------------|-------------|------------|
|                                             | Location-2  | Laboratory |
| Mean                                        | 23.91997581 | 17.10731   |
| Variance                                    | 117.3343071 | 58.25955   |
| Observations                                | 7           | 7          |
| Pooled Variance                             | 87.7969287  |            |
| Hypothesized Mean Difference                | 2.116       |            |
| df                                          | 12          |            |
| t Stat                                      | 0.937743327 |            |
| P(T<=t) one-tail                            | 0.183433614 |            |
| t Critical one-tail                         | 1.782287556 |            |
| P(T<=t) two-tail                            | 0.366867229 |            |
| t Critical two-tail                         | 2.17881283  |            |
| Cohen's d                                   | 0.727071963 |            |

(b)

| F-Test Two-Sample for Variances |            |            |
|---------------------------------|------------|------------|
|                                 | Location-3 | Laboratory |
| Mean                            | 24.025823  | 17.10731   |
| Variance                        | 112.76775  | 58.25955   |
| Observations                    | 7          | 7          |
| df                              | 6          | 6          |
| F                               | 1.9356097  |            |
| P(F<=f) one-tail                | 0.2208257  |            |
| F Critical one-tail             | 4.2838657  |            |

| t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------------------|------------|------------|
|                                             | Location-3 | Laboratory |
| Mean                                        | 24.025823  | 17.10731   |
| Variance                                    | 112.76775  | 58.25955   |
| Observations                                | 7          | 7          |
| Pooled Variance                             | 85.513652  |            |
| Hypothesized Mean Difference                | 2.116      |            |
| df                                          | 12         |            |
| t Stat                                      | 0.9715939  |            |
| P(T<=t) one-tail                            | 0.1752151  |            |
| t Critical one-tail                         | 1.7822876  |            |
| P(T<=t) two-tail                            | 0.3504302  |            |
| t Critical two-tail                         | 2.1788128  |            |
| Cohen's d                                   | 0.7481609  |            |

(c)

| F-Test Two-Sample for Variances |            |            |
|---------------------------------|------------|------------|
|                                 | Location-4 | Laboratory |
| Mean                            | 18.47664   | 17.10731   |
| Variance                        | 70.62304   | 58.25955   |
| Observations                    | 7          | 7          |
| df                              | 6          | 6          |
| F                               | 1.212214   |            |
| P(F<=f) one-tail                | 0.410617   |            |
| F Critical one-tail             | 4.283866   |            |

| t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------------------|------------|------------|
|                                             | Location-4 | Laboratory |
| Mean                                        | 18.47664   | 17.10731   |
| Variance                                    | 70.62304   | 58.25955   |
| Observations                                | 7          | 7          |
| Pooled Variance                             | 64.4413    |            |
| Hypothesized Mean Difference                | 2.116      |            |
| df                                          | 12         |            |
| t Stat                                      | -0.174014  |            |
| P(T<=t) one-tail                            | 0.432378   |            |
| t Critical one-tail                         | 1.782288   |            |
| P(T<=t) two-tail                            | 0.864756   |            |
| t Critical two-tail                         | 2.178813   |            |
| Cohen's d                                   | 0.170579   |            |

(d)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Location-5</i> | <i>Laboratory</i> |
| Mean                            | 17.13008          | 17.10731          |
| Variance                        | 65.46976          | 58.25955          |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.12376           |                   |
| P(F<=f) one-tail                | 0.445492          |                   |
| F Critical one-tail             | 4.283866          |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Location-5</i> | <i>Laboratory</i> |
| Mean                                        | 17.13008          | 17.10731          |
| Variance                                    | 65.46976          | 58.25955          |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 61.86465          |                   |
| Hypothesized Mean Difference                | 2.116             |                   |
| df                                          | 12                |                   |
| t Stat                                      | -0.497886         |                   |
| P(T<=t) one-tail                            | 0.313781          |                   |
| t Critical one-tail                         | 1.782288          |                   |
| P(T<=t) two-tail                            | 0.627563          |                   |
| t Critical two-tail                         | 2.178813          |                   |
| Cohen's d                                   | 0.002895          |                   |

(e)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Location-6</i> | <i>Laboratory</i> |
| Mean                            | 20.46714          | 17.10731          |
| Variance                        | 90.03272          | 58.25955          |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.545373          |                   |
| P(F<=f) one-tail                | 0.305194          |                   |
| F Critical one-tail             | 4.283866          |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Location-6</i> | <i>Laboratory</i> |
| Mean                                        | 20.46714          | 17.10731          |
| Variance                                    | 90.03272          | 58.25955          |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 74.14614          |                   |
| Hypothesized Mean Difference                | 2.116             |                   |
| df                                          | 12                |                   |
| t Stat                                      | 0.27024           |                   |
| P(T<=t) one-tail                            | 0.395783          |                   |
| t Critical one-tail                         | 1.782288          |                   |
| P(T<=t) two-tail                            | 0.791567          |                   |
| t Critical two-tail                         | 2.178813          |                   |
| Cohen's d                                   | 0.390187          |                   |

(f)

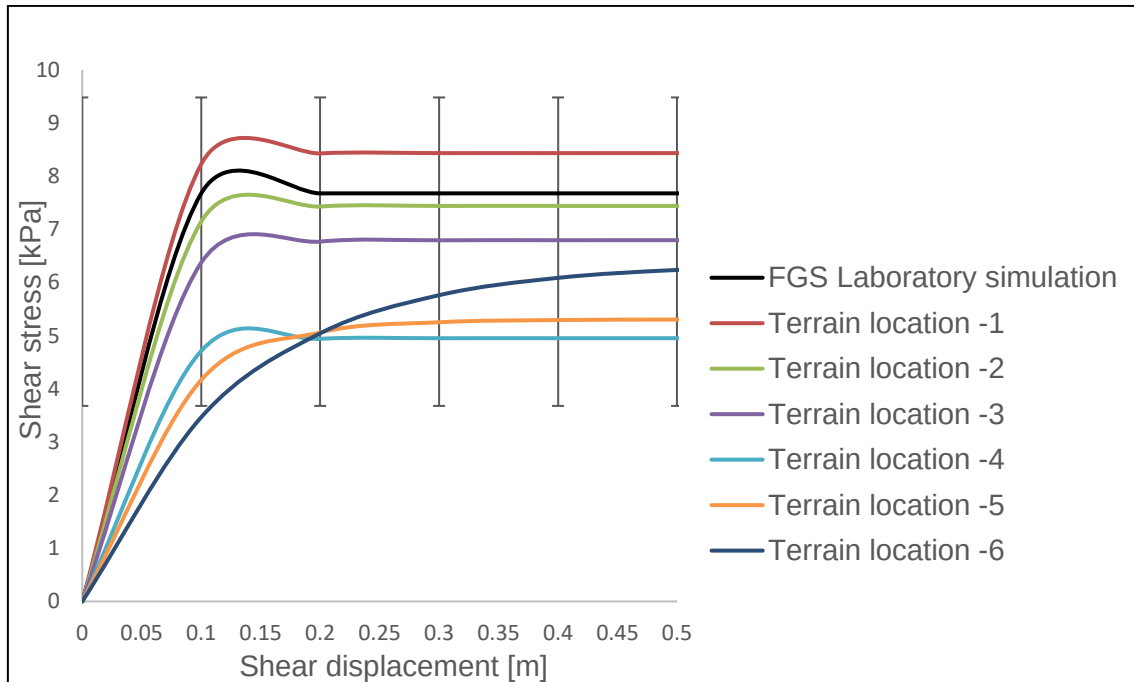


**Figure 4-80 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 10 kPa for FGS**

| F-Test Two-Sample for Variances |            |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|------------|------------|---------------------------------------------|------------|------------|
|                                 | Location-1 | Laboratory |                                             | Location-1 | Laboratory |
| Mean                            | 7.199467   | 6.580474   | Mean                                        | 7.199467   | 6.580474   |
| Variance                        | 10.08445   | 8.419957   | Variance                                    | 10.08445   | 8.419957   |
| Observations                    | 7          | 7          | Observations                                | 7          | 7          |
| df                              | 6          | 6          | Pooled Variance                             | 9.252202   |            |
| F                               | 1.197684   |            | Hypothesized Mean Difference                | 0.658      |            |
| P(F<=f) one-tail                | 0.416125   |            | df                                          | 12         |            |
| F Critical one-tail             | 4.283866   |            | t Stat                                      | -0.023991  |            |
|                                 |            |            | P(T<=t) one-tail                            | 0.490627   |            |
|                                 |            |            | t Critical one-tail                         | 1.782288   |            |
|                                 |            |            | P(T<=t) two-tail                            | 0.981254   |            |
|                                 |            |            | t Critical two-tail                         | 2.178813   |            |
|                                 |            |            | Cohen's d                                   | 0.2035     |            |

(a)

**Figure 4-81 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at a normal stress of 10 kPa for FGS**

| F-Test Two-Sample for Variances |            |            |
|---------------------------------|------------|------------|
|                                 |            |            |
|                                 | Laboratory | Location-2 |
| Mean                            | 6.5804739  | 6.334481   |
| Variance                        | 8.4199571  | 7.814165   |
| Observations                    | 7          | 7          |
| df                              | 6          | 6          |
| F                               | 1.0775248  |            |
| P(F<=f) one-tail                | 0.4650488  |            |
| F Critical one-tail             | 4.2838657  |            |

| t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------------------|------------|------------|
|                                             |            |            |
|                                             | Laboratory | Location-2 |
| Mean                                        | 6.5804739  | 6.334481   |
| Variance                                    | 8.4199571  | 7.814165   |
| Observations                                | 7          | 7          |
| Pooled Variance                             | 8.1170612  |            |
| Hypothesized Mean Difference                | 0.658      |            |
| df                                          | 12         |            |
| t Stat                                      | -0.270545  |            |
| P(T<=t) one-tail                            | 0.395669   |            |
| t Critical one-tail                         | 1.7822876  |            |
| P(T<=t) two-tail                            | 0.7913379  |            |
| t Critical two-tail                         | 2.1788128  |            |
| Cohen's d                                   | 0.086342   |            |

(b)

| F-Test Two-Sample for Variances |            |            |
|---------------------------------|------------|------------|
|                                 |            |            |
|                                 | Laboratory | Location-3 |
| Mean                            | 6.580474   | 5.761066   |
| Variance                        | 8.419957   | 6.477826   |
| Observations                    | 7          | 7          |
| df                              | 6          | 6          |
| F                               | 1.299812   |            |
| P(F<=f) one-tail                | 0.379162   |            |
| F Critical one-tail             | 4.283866   |            |

| t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------------------|------------|------------|
|                                             |            |            |
|                                             | Laboratory | Location-3 |
| Mean                                        | 6.580474   | 5.761066   |
| Variance                                    | 8.419957   | 6.477826   |
| Observations                                | 7          | 7          |
| Pooled Variance                             | 7.448892   |            |
| Hypothesized Mean Difference                | 0.658      |            |
| df                                          | 12         |            |
| t Stat                                      | 0.11064    |            |
| P(T<=t) one-tail                            | 0.456865   |            |
| t Critical one-tail                         | 1.782288   |            |
| P(T<=t) two-tail                            | 0.913731   |            |
| t Critical two-tail                         | 2.178813   |            |
| Cohen's d                                   | 0.30023    |            |

(c)

| F-Test Two-Sample for Variances |            |            |
|---------------------------------|------------|------------|
|                                 |            |            |
|                                 | Laboratory | Location-4 |
| Mean                            | 6.5804739  | 4.209253   |
| Variance                        | 8.4199571  | 3.452935   |
| Observations                    | 7          | 7          |
| df                              | 6          | 6          |
| F                               | 2.438493   |            |
| P(F<=f) one-tail                | 0.1511557  |            |
| F Critical one-tail             | 4.2838657  |            |

| t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------------------|------------|------------|
|                                             |            |            |
|                                             | Laboratory | Location-4 |
| Mean                                        | 6.5804739  | 4.209253   |
| Variance                                    | 8.4199571  | 3.452935   |
| Observations                                | 7          | 7          |
| Pooled Variance                             | 5.9364459  |            |
| Hypothesized Mean Difference                | 0.658      |            |
| df                                          | 12         |            |
| t Stat                                      | 1.3154797  |            |
| P(T<=t) one-tail                            | 0.1064645  |            |
| t Critical one-tail                         | 1.7822876  |            |
| P(T<=t) two-tail                            | 0.212929   |            |
| t Critical two-tail                         | 2.1788128  |            |
| Cohen's d                                   | 0.973215   |            |

(d)

| F-Test Two-Sample for Variances |            |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|------------|------------|---------------------------------------------|------------|------------|
|                                 |            |            |                                             |            |            |
|                                 | Laboratory | Location-5 |                                             | Laboratory | Location-5 |
| Mean                            | 6.5804739  | 4.340038   | Mean                                        | 6.5804739  | 4.340038   |
| Variance                        | 8.4199571  | 3.830805   | Variance                                    | 8.4199571  | 3.830805   |
| Observations                    | 7          | 7          | Observations                                | 7          | 7          |
| df                              | 6          | 6          | Pooled Variance                             | 6.1253813  |            |
| F                               | 2.19796    |            | Hypothesized Mean Difference                | 0.658      |            |
| P(F<=f) one-tail                | 0.1802822  |            | df                                          | 12         |            |
| F Critical one-tail             | 4.2838657  |            | t Stat                                      | 1.196172   |            |
|                                 |            |            | P(T<=t) one-tail                            | 0.127365   |            |
|                                 |            |            | t Critical one-tail                         | 1.7822876  |            |
|                                 |            |            | P(T<=t) two-tail                            | 0.25473    |            |
|                                 |            |            | t Critical two-tail                         | 2.1788128  |            |
|                                 |            |            | Cohen's d                                   | 0.9052447  |            |

(e)

| F-Test Two-Sample for Variances |            |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|------------|------------|---------------------------------------------|------------|------------|
|                                 |            |            |                                             |            |            |
|                                 | Laboratory | Location-6 |                                             | Laboratory | Location-6 |
| Mean                            | 6.5804739  | 4.700085   | Mean                                        | 6.5804739  | 4.700085   |
| Variance                        | 8.4199571  | 5.286363   | Variance                                    | 8.4199571  | 5.286363   |
| Observations                    | 7          | 7          | Observations                                | 7          | 7          |
| df                              | 6          | 6          | Pooled Variance                             | 6.8531602  |            |
| F                               | 1.5927693  |            | Hypothesized Mean Difference                | 0.658      |            |
| P(F<=f) one-tail                | 0.2930166  |            | df                                          | 12         |            |
| F Critical one-tail             | 4.2838657  |            | t Stat                                      | 0.8735702  |            |
|                                 |            |            | P(T<=t) one-tail                            | 0.1997484  |            |
|                                 |            |            | t Critical one-tail                         | 1.7822876  |            |
|                                 |            |            | P(T<=t) two-tail                            | 0.3994968  |            |
|                                 |            |            | t Critical two-tail                         | 2.1788128  |            |
|                                 |            |            | Cohen's d                                   | 0.7182938  |            |

(f)

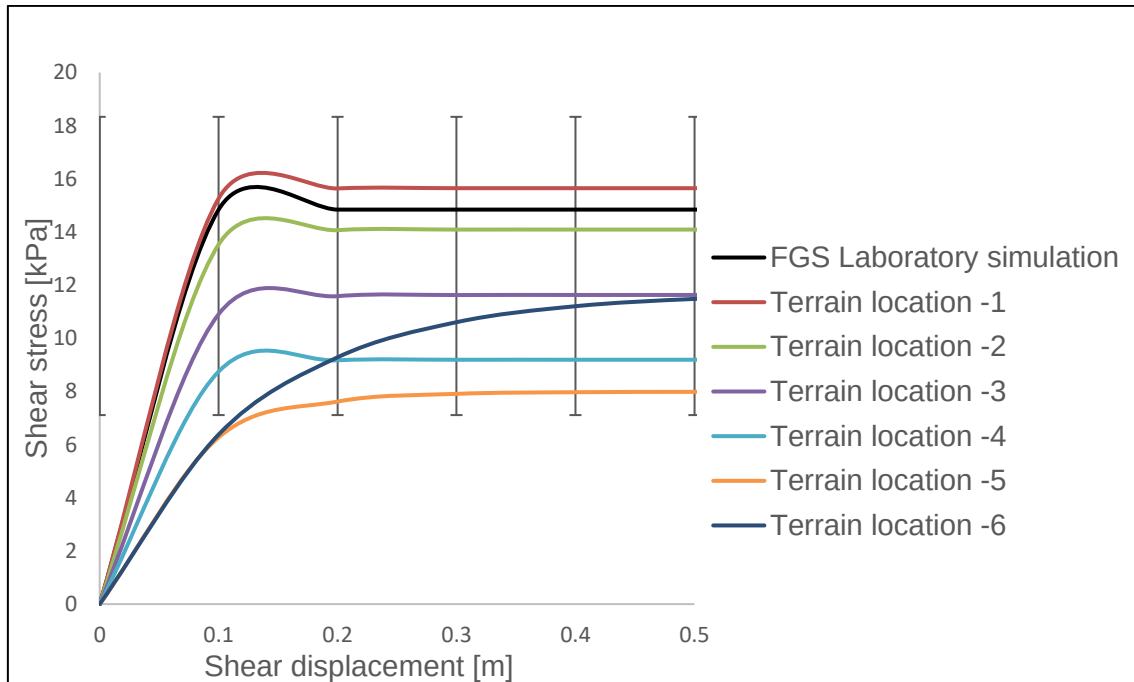


**Figure 4-82 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 20 kPa for FGS**

| F-Test Two-Sample for Variances |            |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|------------|------------|---------------------------------------------|------------|------------|
|                                 | Location-1 | Laboratory |                                             | Location-1 | Laboratory |
| Mean                            | 13.35762   | 12.7238    | Mean                                        | 13.35762   | 12.7238    |
| Variance                        | 34.7144    | 31.47962   | Variance                                    | 34.7144    | 31.47962   |
| Observations                    | 7          | 7          | Observations                                | 7          | 7          |
| df                              | 6          | 6          | Pooled Variance                             | 33.09701   |            |
| F                               | 1.102758   |            | Hypothesized Mean Difference                | 1.272      |            |
| P(F<=f) one-tail                | 0.454259   |            | df                                          | 12         |            |
| F Critical one-tail             | 4.283866   |            | t Stat                                      | -0.207532  |            |
|                                 |            |            | P(T<=t) one-tail                            | 0.419537   |            |
|                                 |            |            | t Critical one-tail                         | 1.782288   |            |
|                                 |            |            | P(T<=t) two-tail                            | 0.839075   |            |
|                                 |            |            | t Critical two-tail                         | 2.178813   |            |
|                                 |            |            | Cohen's d                                   | 0.110171   |            |

(a)

**Figure 4-83 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at a normal stress of 20 kPa for FGS**

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Laboratory</i> | <i>Location-2</i> |
| Mean                            | 12.723805         | 11.99649          |
| Variance                        | 31.479624         | 28.02649          |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.1232096         |                   |
| P(F<=f) one-tail                | 0.445719          |                   |
| F Critical one-tail             | 4.2838657         |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Laboratory</i> | <i>Location-2</i> |
| Mean                                        | 12.723805         | 11.99649          |
| Variance                                    | 31.479624         | 28.02649          |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 29.753058         |                   |
| Hypothesized Mean Difference                | 1.272             |                   |
| df                                          | 12                |                   |
| t Stat                                      | -0.186815         |                   |
| P(T<=t) one-tail                            | 0.4274633         |                   |
| t Critical one-tail                         | 1.7822876         |                   |
| P(T<=t) two-tail                            | 0.8549266         |                   |
| t Critical two-tail                         | 2.1788128         |                   |
| Cohen's d                                   | 0.1333391         |                   |

(b)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Laboratory</i> | <i>Location-3</i> |
| Mean                            | 12.7238           | 9.860696          |
| Variance                        | 31.47962          | 18.97749          |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.658788          |                   |
| P(F<=f) one-tail                | 0.277041          |                   |
| F Critical one-tail             | 4.283866          |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Laboratory</i> | <i>Location-3</i> |
| Mean                                        | 12.7238           | 9.860696          |
| Variance                                    | 31.47962          | 18.97749          |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 25.22856          |                   |
| Hypothesized Mean Difference                | 1.272             |                   |
| df                                          | 12                |                   |
| t Stat                                      | 0.592635          |                   |
| P(T<=t) one-tail                            | 0.282213          |                   |
| t Critical one-tail                         | 1.782288          |                   |
| P(T<=t) two-tail                            | 0.564426          |                   |
| t Critical two-tail                         | 2.178813          |                   |
| Cohen's d                                   | 0.570022          |                   |

(c)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Laboratory</i> | <i>Location-4</i> |
| Mean                            | 12.723805         | 7.815082          |
| Variance                        | 31.479624         | 11.9027           |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 2.644747          |                   |
| P(F<=f) one-tail                | 0.130865          |                   |
| F Critical one-tail             | 4.2838657         |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Laboratory</i> | <i>Location-4</i> |
| Mean                                        | 12.723805         | 7.815082          |
| Variance                                    | 31.479624         | 11.9027           |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 21.691161         |                   |
| Hypothesized Mean Difference                | 1.272             |                   |
| df                                          | 12                |                   |
| t Stat                                      | 1.4608406         |                   |
| P(T<=t) one-tail                            | 0.0848717         |                   |
| t Critical one-tail                         | 1.7822876         |                   |
| P(T<=t) two-tail                            | 0.1697433         |                   |
| t Critical two-tail                         | 2.1788128         |                   |
| Cohen's d                                   | 1.0539672         |                   |

(d)

| F-Test Two-Sample for Variances |            |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|------------|------------|---------------------------------------------|------------|------------|
|                                 |            |            |                                             |            |            |
|                                 | Laboratory | Location-5 |                                             | Laboratory | Location-5 |
| Mean                            | 12.723805  | 6.536687   | Mean                                        | 12.723805  | 6.536687   |
| Variance                        | 31.479624  | 8.689974   | Variance                                    | 31.479624  | 8.689974   |
| Observations                    | 7          | 7          | Observations                                | 7          | 7          |
| df                              | 6          | 6          | Pooled Variance                             | 20.084799  |            |
| F                               | 3.6225221  |            | Hypothesized Mean Difference                | 1.272      |            |
| P(F<=f) one-tail                | 0.0712324  |            | df                                          | 12         |            |
| F Critical one-tail             | 4.2838657  |            | t Stat                                      | 2.051796   |            |
|                                 |            |            | P(T<=t) one-tail                            | 0.0313363  |            |
|                                 |            |            | t Critical one-tail                         | 1.7822876  |            |
|                                 |            |            | P(T<=t) two-tail                            | 0.0626727  |            |
|                                 |            |            | t Critical two-tail                         | 2.1788128  |            |
|                                 |            |            | Cohen's d                                   | 1.3805579  |            |

(e)

| F-Test Two-Sample for Variances |            |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|------------|------------|---------------------------------------------|------------|------------|
|                                 |            |            |                                             |            |            |
|                                 | Laboratory | Location-6 |                                             | Laboratory | Location-6 |
| Mean                            | 12.723805  | 8.653858   | Mean                                        | 12.723805  | 8.653858   |
| Variance                        | 31.479624  | 17.92112   | Variance                                    | 31.479624  | 17.92112   |
| Observations                    | 7          | 7          | Observations                                | 7          | 7          |
| df                              | 6          | 6          | Pooled Variance                             | 24.700371  |            |
| F                               | 1.756566   |            | Hypothesized Mean Difference                | 1.272      |            |
| P(F<=f) one-tail                | 0.2553237  |            | df                                          | 12         |            |
| F Critical one-tail             | 4.2838657  |            | t Stat                                      | 1.0532263  |            |
|                                 |            |            | P(T<=t) one-tail                            | 0.1564917  |            |
|                                 |            |            | t Critical one-tail                         | 1.7822876  |            |
|                                 |            |            | P(T<=t) two-tail                            | 0.3129834  |            |
|                                 |            |            | t Critical two-tail                         | 2.1788128  |            |
|                                 |            |            | Cohen's d                                   | 0.8189115  |            |

(f)



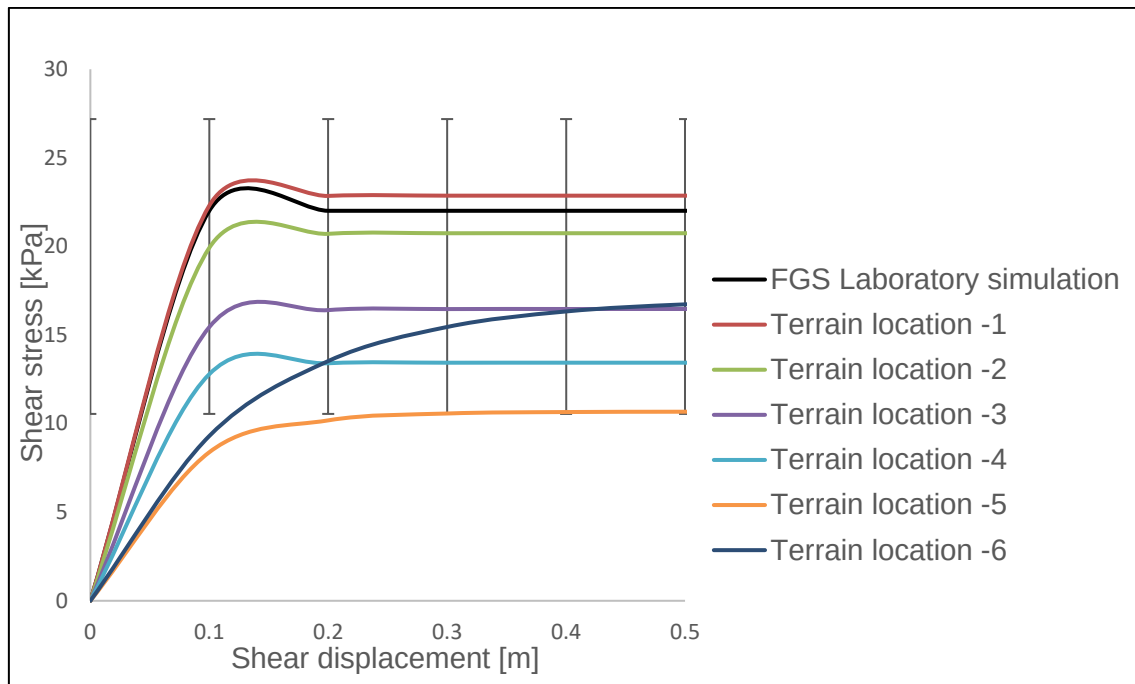


**Figure 4-84 Shear stress-shear displacement plot for laboratory (with one standard deviation band) and in-situ simulation conditions at a normal stress of 30 kPa for FGS**

| F-Test Two-Sample for Variances |            |            | t-Test: Two-Sample Assuming Equal Variances |            |            |
|---------------------------------|------------|------------|---------------------------------------------|------------|------------|
|                                 | Location-1 | Laboratory |                                             | Location-1 | Laboratory |
| Mean                            | 19.51578   | 18.86714   | Mean                                        | 19.51578   | 18.86714   |
| Variance                        | 74.10082   | 69.21616   | Variance                                    | 74.10082   | 69.21616   |
| Observations                    | 7          | 7          | Observations                                | 7          | 7          |
| df                              | 6          | 6          | Pooled Variance                             | 71.65849   |            |
| F                               | 1.070571   |            | Hypothesized Mean Difference                | 1.887      |            |
| P(F<=f) one-tail                | 0.468072   |            | df                                          | 12         |            |
| F Critical one-tail             | 4.283866   |            | t Stat                                      | -0.273683  |            |
|                                 |            |            | P(T<=t) one-tail                            | 0.394491   |            |
|                                 |            |            | t Critical one-tail                         | 1.782288   |            |
|                                 |            |            | P(T<=t) two-tail                            | 0.788982   |            |
|                                 |            |            | t Critical two-tail                         | 2.178813   |            |
|                                 |            |            | Cohen's d                                   | 0.076625   |            |

(a)

**Figure 4-85 (a-f) F-test and t-test assuming equal variances between shear stress-shear displacement results grouped into the laboratory and in-situ simulation conditions at six different locations (a-f representing location 1-6) at a normal stress of 30 kPa for FGS**

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Laboratory</i> | <i>Location-2</i> |
| Mean                            | 18.867136         | 17.6585           |
| Variance                        | 69.216159         | 60.72505          |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.1398288         |                   |
| P(F<=f) one-tail                | 0.4389125         |                   |
| F Critical one-tail             | 4.2838657         |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Laboratory</i> | <i>Location-2</i> |
| Mean                                        | 18.867136         | 17.6585           |
| Variance                                    | 69.216159         | 60.72505          |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 64.970604         |                   |
| Hypothesized Mean Difference                | 1.887             |                   |
| df                                          | 12                |                   |
| t Stat                                      | -0.157447         |                   |
| P(T<=t) one-tail                            | 0.4387552         |                   |
| t Critical one-tail                         | 1.7822876         |                   |
| P(T<=t) two-tail                            | 0.8775104         |                   |
| t Critical two-tail                         | 2.1788128         |                   |
| Cohen's d                                   | 0.1499473         |                   |

(b)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Laboratory</i> | <i>Location-3</i> |
| Mean                            | 18.86714          | 13.96033          |
| Variance                        | 69.21616          | 38.03774          |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 1.819671          |                   |
| P(F<=f) one-tail                | 0.242435          |                   |
| F Critical one-tail             | 4.283866          |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Laboratory</i> | <i>Location-3</i> |
| Mean                                        | 18.86714          | 13.96033          |
| Variance                                    | 69.21616          | 38.03774          |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 53.62695          |                   |
| Hypothesized Mean Difference                | 1.887             |                   |
| df                                          | 12                |                   |
| t Stat                                      | 0.771475          |                   |
| P(T<=t) one-tail                            | 0.227675          |                   |
| t Critical one-tail                         | 1.782288          |                   |
| P(T<=t) two-tail                            | 0.455349          |                   |
| t Critical two-tail                         | 2.178813          |                   |
| Cohen's d                                   | 0.670051          |                   |

(c)

| F-Test Two-Sample for Variances |                   |                   |
|---------------------------------|-------------------|-------------------|
|                                 |                   |                   |
|                                 | <i>Laboratory</i> | <i>Location-4</i> |
| Mean                            | 18.867136         | 11.42091          |
| Variance                        | 69.216159         | 25.42025          |
| Observations                    | 7                 | 7                 |
| df                              | 6                 | 6                 |
| F                               | 2.7228747         |                   |
| P(F<=f) one-tail                | 0.124108          |                   |
| F Critical one-tail             | 4.2838657         |                   |

| t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------------------|-------------------|-------------------|
|                                             |                   |                   |
|                                             | <i>Laboratory</i> | <i>Location-4</i> |
| Mean                                        | 18.867136         | 11.42091          |
| Variance                                    | 69.216159         | 25.42025          |
| Observations                                | 7                 | 7                 |
| Pooled Variance                             | 47.318205         |                   |
| Hypothesized Mean Difference                | 1.887             |                   |
| df                                          | 12                |                   |
| t Stat                                      | 1.5119384         |                   |
| P(T<=t) one-tail                            | 0.0782147         |                   |
| t Critical one-tail                         | 1.7822876         |                   |
| P(T<=t) two-tail                            | 0.1564293         |                   |
| t Critical two-tail                         | 2.1788128         |                   |
| Cohen's d                                   | 1.0824853         |                   |

(d)

| F-Test Two-Sample for Variances |                   |                   | t-Test: Two-Sample Assuming Unequal Variances |                   |                    |
|---------------------------------|-------------------|-------------------|-----------------------------------------------|-------------------|--------------------|
|                                 |                   |                   |                                               |                   |                    |
|                                 | <i>Laboratory</i> | <i>Location-5</i> |                                               | <i>Laboratory</i> | <i>Location -5</i> |
| Mean                            | 18.867136         | 8.733337          | Mean                                          | 18.867136         | 8.733337           |
| Variance                        | 69.216159         | 15.51185          | Variance                                      | 69.216159         | 15.51185           |
| Observations                    | 7                 | 7                 | Observations                                  | 7                 | 7                  |
| df                              | 6                 | 6                 | Hypothesized Mean Difference                  | 1.887             |                    |
| F                               | 4.4621482         |                   | df                                            | 9                 |                    |
| P(F<=f) one-tail                | 0.045746          |                   | t Stat                                        | 2.3703962         |                    |
| F Critical one-tail             | 4.2838657         |                   | P(T<=t) one-tail                              | 0.0209409         |                    |
|                                 |                   |                   | t Critical one-tail                           | 1.8331129         |                    |
|                                 |                   |                   | P(T<=t) two-tail                              | 0.0418818         |                    |
|                                 |                   |                   | t Critical two-tail                           | 2.2621572         |                    |
|                                 |                   |                   | Cohen's d                                     | 1.5569469         |                    |

(e)

| F-Test Two-Sample for Variances |                   |                   | t-Test: Two-Sample Assuming Equal Variances |                   |                   |
|---------------------------------|-------------------|-------------------|---------------------------------------------|-------------------|-------------------|
|                                 |                   |                   |                                             |                   |                   |
|                                 | <i>Laboratory</i> | <i>Location-6</i> |                                             | <i>Laboratory</i> | <i>Location-6</i> |
| Mean                            | 18.867136         | 12.60763          | Mean                                        | 18.867136         | 12.60763          |
| Variance                        | 69.216159         | 38.03755          | Variance                                    | 69.216159         | 38.03755          |
| Observations                    | 7                 | 7                 | Observations                                | 7                 | 7                 |
| df                              | 6                 | 6                 | Pooled Variance                             | 53.626854         |                   |
| F                               | 1.8196798         |                   | Hypothesized Mean Difference                | 1.887             |                   |
| P(F<=f) one-tail                | 0.2424336         |                   | df                                          | 12                |                   |
| F Critical one-tail             | 4.2838657         |                   | t Stat                                      | 1.1170513         |                   |
|                                 |                   |                   | P(T<=t) one-tail                            | 0.1429196         |                   |
|                                 |                   |                   | t Critical one-tail                         | 1.7822876         |                   |
|                                 |                   |                   | P(T<=t) two-tail                            | 0.2858392         |                   |
|                                 |                   |                   | t Critical two-tail                         | 2.1788128         |                   |
|                                 |                   |                   | Cohen's d                                   | 0.8547691         |                   |

(f)

#### 4.5.4.2 Pressure-sinkage tests

The bevameter is designated to simulate the normal loads exerted by the vehicle on the terrain surface is the pressure-sinkage test. These tests are useful in predicting vehicle sinkage and compaction resistance against vehicle mobility. In the present study, pressure-sinkage tests were simulated in laboratory conditions and in-situ terrain locations using 0.1 m and 0.15 m diameter sinkage plates for 2NS sand and FGS. Six multiple measurements are needed over a large area of the terrain model to ensure representative results of the entire terrain. The terrain model area (6 m x 4 m) designates the required vehicle mobility testing area and

consists of six different heterogeneous soil patches. Therefore, pressure-sinkage tests were performed at six different locations on the terrain model. Moreover, it was also found that the coefficient of variation (CV) at these locations is less than  $\pm 10\%$  of the mean value, which is following the guidelines of NATO CDT-308 (2020).

NATO CDT -308 (2020) states that pressure sinkage relationships consist of at least two scaling parameters and a power-law exponent in the data set from which the model parameters are derived. These parameters describe the load-deformation characteristics such as linear stiffness and maximum bearing pressure loading. Therefore, in the present study, the scaling ratio,  $\lambda$  is defined as the ratio of equivalent stiffness parameters between the laboratory model and in-situ terrain model, as shown in Equation 4-64 concerning section 4.3.1.1. This approach is similar to the definition of Bekker's scaling parameter (Bekker, 1969), i.e. the ratio of the critical parameters of the full-scale system to the scale system.

$$\lambda = \frac{(k_{eq})_{lab}}{(k_{eq})_{ter}} \quad (4-64)$$

where,  $(k_{eq})_{ter}$  is the average equivalent stiffness obtained from six different locations of the in-situ terrain model, [Pa/m<sup>n</sup>] and  $(k_{eq})_{lab}$  is the equivalent stiffness from the laboratory model simulation, [Pa/m<sup>n</sup>].

For coarse-grained 2NS sand, the average  $(k_{eq})_{ter}$  value is found to be 6.36E+05 Pa/m<sup>n</sup> and 5.09E+05 Pa/m<sup>n</sup> with respect to 0.1 m and 0.15 m sinkage plate, respectively. The  $(k_{eq})_{lab}$  is 1.79E+06 Pa/m<sup>n</sup> and 1.39E+06 Pa/m<sup>n</sup> with respect to 0.1 m and 0.15 m sinkage plates respectively. Interestingly, the average sinkage exponent  $n$  is approximately the same in laboratory condition and the in-situ terrain model. Both the soil models are showing the 'soil weakening' effect. Therefore, there is not much effect of sinkage exponent on the scalability of coarse-grained soil. The scaling ratio,  $\lambda$  obtained are 2.81 and 2.73 for 0.1 m and 0.15 m plate diameters. Thus, the average scaling ratio  $\lambda$  for 2NS sand is 2.77. Further, the terrain stiffness at each location is multiplied by the scaling ratio. The

obtained scaled parameters are shown in Table 4-16. Then scaled  $k_c [N/m^{n+1}]$  and  $k_\phi [N/m^{n+2}]$  are determined and compared with the laboratory condition and original terrain  $k_c [N/m^{n+1}]$  and  $k_\phi [N/m^{n+2}]$  parameters, as shown in Figures 4-86 & 4-88, respectively. The method of obtaining  $k_c [N/m^{n+1}]$  and  $k_\phi [N/m^{n+2}]$  from  $k_{eq}$  values for 0.1 m and 0.15 m plate diameters is discussed in detail in section 4.3.1.1.

**Table 4-16 Bekker parameters after scaling for 2NS sand**

| Plate diameter        | 0.1  |          | 0.15 |          | Bekker parameter |                |                |
|-----------------------|------|----------|------|----------|------------------|----------------|----------------|
|                       | n    | K        | n    | K        | n <sub>avg</sub> | k <sub>c</sub> | k <sub>φ</sub> |
| Terrain location – 1  | 0.42 | 1.56E+06 | 0.54 | 1.27E+06 | 0.48             | 8.69E+04       | 6.90E+05       |
| Terrain location – 2  | 0.42 | 1.72E+06 | 0.63 | 1.53E+06 | 0.52             | 5.73E+04       | 1.14E+06       |
| Terrain location – 3  | 0.53 | 1.90E+06 | 0.56 | 1.46E+06 | 0.54             | 1.32E+05       | 5.76E+05       |
| Terrain location – 4  | 0.41 | 1.77E+06 | 0.64 | 1.37E+06 | 0.52             | 1.19E+05       | 5.75E+05       |
| Terrain location – 5  | 0.37 | 1.71E+06 | 0.62 | 1.30E+06 | 0.49             | 1.22E+05       | 4.89E+05       |
| Terrain location – 6  | 0.54 | 1.91E+06 | 0.63 | 1.52E+06 | 0.58             | 1.16E+05       | 7.52E+05       |
| Laboratory parameters | 0.68 | 1.79E+06 | 0.59 | 1.39E+06 | 0.63             | 1.19E+05       | 5.95E+05       |

The statistical analysis is conducted between the laboratory condition, original terrain and scaled terrain parameters,  $k_c [N/m^{n+1}]$  and  $k_\phi [N/m^{n+2}]$  as discussed in Figure 4-87 and 4-89, respectively, considered as samples in the analysis as mentioned in Figure 4-73. For  $k_c$ , the F-test failed when conducted between the original terrain parameters and laboratory parameters. The obtained F value is more than the F-critical value at a 5% significant level. It concludes that there is a significant difference between the variance of both data sets. Further, a t-test assuming unequal variance was conducted on the sample data sets, which failed. The obtained t-value is not in the desired band of t-critical at one tail value. There is a significant difference between the mean values of the original terrain parameter and the laboratory parameter. Moreover, the obtained Cohen's d value is 4.1. It means that the difference between the means is approximately four times the pooled standard deviation. For  $k_\phi$ , the F-test failed, but the t-test assuming unequal variance passed with a lower p-value, which shows inconclusive results. The obtained Cohen's d value is 1.43, which is not accepted as per the guidelines of NATO CDT-308 (2020). Therefore, it can be concluded that there is a significant difference between the data sets of original terrain parameters and laboratory parameters.

Further, the statistical F-test and t-tests were conducted between the scaled terrain parameters and laboratory parameters. For  $k_c$  and  $k_\phi$ , the F-test passed as the obtained F value is less than the F-critical value at a 5% significant level. It means that there is no significant difference between the variance of both data sets. Therefore, a t-test assuming equal variance was conducted on the data sets. The obtained t-value is in the expected band of t-critical one tail and two tails. The p-value for the t-test is also greater than the significant value, 0.05. The Cohen's d values are 0.2 and -0.85 for  $k_c$  and  $k_\phi$ , respectively. Therefore, the difference between the mean is less than one standard deviation. Therefore, it can be concluded that there is no significant difference between the data sets of scaled terrain parameters and laboratory parameters.

For fine-grained sand, the average  $(k_{eq})_{ter}$  value is found to be  $3.92E+05 \text{ Pa/m}^n$  and  $4.28E+05 \text{ Pa/m}^n$  with respect to 0.1 m and 0.15 m sinkage plate, respectively.

The  $(k_{eq})_{lab}$  is  $1.40E+06 \text{ Pa/m}^n$  and  $1.39E+06 \text{ Pa/m}^n$  with respect to 0.1 m and 0.15 m sinkage plate, respectively. It should be noted that there is a difference in the average sinkage exponent,  $n_{avg}$  between the original terrain parameters and laboratory parameters. The  $n_{avg}$  obtained for laboratory conditions is 1.2, and that of actual terrain condition is 0.9. Initially, the same scaling procedure of coarse-grained soil is followed to check its applicability to fine-grained sand. The scaling ratio,  $\lambda$  obtained are 3.57 and 3.24 with respect to 0.1 m and 0.15 m plate diameter. Thus, the average scaling ratio  $\lambda$  for FGS is 3.4. Further, the terrain stiffness at each location is multiplied by the scaling ratio. The obtained scaled parameters are shown in Table 4-17. Then scaled  $k_c [N/m^{n+1}]$  and  $k_\phi [N/m^{n+2}]$  are determined and compared with the laboratory condition and original terrain  $k_c [N/m^{n+1}]$  and  $k_\phi [N/m^{n+2}]$  parameters, as shown in Figures 4-90 & 4-92, respectively. The method of obtaining  $k_c [N/m^{n+1}]$  and  $k_\phi [N/m^{n+2}]$  from  $k_{eq}$  values for 0.1 m and 0.15 m plate diameter is discussed in detail in section 4.3.1.1.

**Table 4-17 Bekker parameters after scaling for FGS**

| Plate diameter       | 0.1  |            | 0.15 |            | Bekker parameter |            |            |
|----------------------|------|------------|------|------------|------------------|------------|------------|
|                      | n    | K          | n    | K          | $n_{avg}$        | $k_c$      | $k_\phi$   |
| Terrain location – 1 | 0.80 | $1.19E+06$ | 0.77 | $1.06E+06$ | 0.79             | $3.98E+04$ | $7.91E+05$ |
| Terrain location - 2 | 0.81 | $1.06E+06$ | 0.84 | $1.26E+06$ | 0.82             | $-5.9E+04$ | $1.65E+06$ |
| Terrain location– 3  | 1.11 | $1.75E+06$ | 0.90 | $1.56E+06$ | 1.01             | $5.81E+04$ | $1.17E+06$ |

| Plate diameter       | 0.1  |          | 0.15 |          | Bekker parameter |           |          |
|----------------------|------|----------|------|----------|------------------|-----------|----------|
| Terrain location – 4 | 0.79 | 1.12E+06 | 0.79 | 1.86E+06 | 0.79             | -2.2E+05  | 3.34E+06 |
| Terrain location – 5 | 0.85 | 1.40E+06 | 1.1  | 1.62E+06 | 0.96             | -6.61E+04 | 2.06E+06 |
| Terrain location – 6 | 1.11 | 1.49E+06 | 0.71 | 1.37E+06 | 0.91             | 3.4E+04   | 1.14E+06 |

Initially, the statistical analysis between laboratory condition parameters and actual terrain parameters was conducted, as shown in Figures 4-91 & 4-93. For  $k_c$  and  $k_\phi$ , F-test failed. The F-value is higher than the F-critical at a 5% significant level. It means that there is a significant difference between the sample's variances. Further, a t-test assuming unequal variance was conducted on both parameters. It was found that the obtained t value is in the band of the t-critical two-tail, but the p-value doesn't provide strong evidence to support the test. Therefore, it remains inconclusive. Cohen's d value was found to be greater than one. It means that the mean difference of data sets is greater than one standard deviation of the pool, which contradicts NATO's guidelines. Therefore, it can be concluded that the data sets of both parameters have a significant difference between them.

The statistical tests were conducted between the laboratory condition parameters and scaled terrain parameters. For  $k_c$  and  $k_\phi$ , F-tests failed. The F-value is higher than the F-critical at a 5% significant level. It means that there is a significant difference between the sample's variances. Further, t-tests assuming unequal variance were conducted on both the parameters. It was found that the obtained t value is in the band of t-critical two-tail, but the p-value doesn't provide strong



evidence to support the t-test. Therefore, it remains inconclusive. Cohen's d value was found to be less than one. It means that the mean difference of data sets is less than one standard deviation of the pool. With the F test's failure and significant differences between the data set's variances, no confident conclusions can be derived for both parameters.

No concrete evidence is found to support the scalability of soil for FGS. It might be because of neglecting the effect of sinkage exponents in the previous scaling procedure. Therefore, to include the effect of  $n_{avg}$ , following Wong's method, the exponential form of Bekker's equation is converted into the straight-line form by changing its domain to log-log, as shown in Equation 4-65. This equation requires a minimum of two plate tests results.

$$\ln p = \ln(k_{eq}) + n \ln z \quad (4-65)$$

Using the scaled parameters,  $n$  and  $k_{eq}$  from Table 4-16. The  $\log p - \log z$  curves are plotted with sinkage of plates varying from 0.01 to 0.12 m for each of the terrain locations and laboratory simulations (Figures 2-94 and 2-95). Interestingly, the variation of  $\log p - \log z$  straight lines at each of the six terrain model locations is in the one standard deviation band of the laboratory results for 0.1 m and 0.15 m diameter plate. However, at lower  $\log z$  values for 0.15 m plate, the deviation slightly increases. The results of F tests and t-tests, as shown in Figure 4-96 and 4-97, remains inconclusive because of lower p-values. However, Cohen's d value is 0.41 for 0.1 m plate diameter and -0.04 for 0.15 m plate diameter. The mean difference between the straight line's obtained from laboratory simulations and average terrain locations is less than one standard deviation of pool variance, accepted as per NATO guidelines. Therefore, it can be concluded that for fine-grained sand, the  $\log p$ - $\log z$  domain is scalable for 0.1 m and 0.15 m plate sizes.

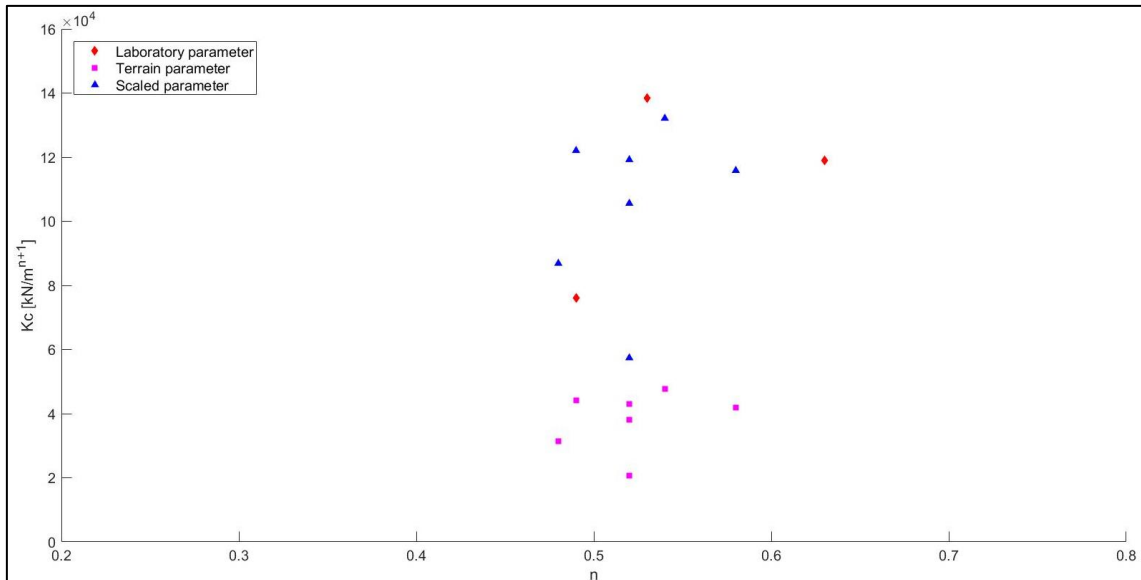


**Figure 4-86 Scalability analysis on the modulus of determination of cohesion,  $K_c$  for 2NS sand**

| F-Test Two-Sample for Variances |             |          | t-Test: Two-Sample Assuming Unequal Variances |             |            |
|---------------------------------|-------------|----------|-----------------------------------------------|-------------|------------|
|                                 |             |          |                                               | Laboratory  | Terrain    |
| Mean                            | 111220      | 38115    | Mean                                          | 38115       | 111220     |
| Variance                        | 1016850900  | 85570311 | Variance                                      | 102684373.2 | 1016850900 |
| Observations                    | 3           | 7        | Observations                                  | 6           | 3          |
| Hypothesized Mean Difference    | 0           |          | Hypothesized Mean Difference                  | 0           |            |
| df                              | 2           | 6        | df                                            | 2           |            |
| t Stat                          | -3.87420705 |          | t Stat                                        | -3.87420705 |            |
| P(T<=t) one-tail                | 0.030314331 |          | P(T<=t) one-tail                              | 0.030314331 |            |
| t Critical one-tail             | 2.91998558  |          | t Critical one-tail                           | 2.91998558  |            |
| P(T<=t) two-tail                | 0.060628661 |          | P(T<=t) two-tail                              | 0.060628661 |            |
| t Critical two-tail             | 4.30265273  |          | t Critical two-tail                           | 4.30265273  |            |
| Cohen's d                       | 4.097010334 |          | Cohen's d                                     | 4.097010334 |            |

(a)

| F-Test Two-Sample for Variances |            |             | t-Test: Two-Sample Assuming Equal Variances |             |                |
|---------------------------------|------------|-------------|---------------------------------------------|-------------|----------------|
|                                 |            |             |                                             | Laboratory  | Scaled terrain |
| Mean                            | 111220     | 105578.55   | Mean                                        | 111220      | 105578.55      |
| Variance                        | 1016850900 | 656572439.3 | Variance                                    | 1016850900  | 656572439.3    |
| Observations                    | 3          | 7           | Observations                                | 3           | 7              |
| df                              | 2          | 6           | Pooled Variance                             | 746642054.5 |                |
| F                               | 1.54872614 |             | Hypothesized Mean Difference                | 0           |                |
| P(F<=f) one-tail                | 0.28687612 |             | df                                          | 8           |                |
| F Critical one-tail             | 5.14325285 |             | t Stat                                      | 0.299187997 |                |
|                                 |            |             | P(T<=t) one-tail                            | 0.386210866 |                |
|                                 |            |             | t Critical one-tail                         | 1.859548038 |                |
|                                 |            |             | P(T<=t) two-tail                            | 0.772421731 |                |
|                                 |            |             | t Critical two-tail                         | 2.306004135 |                |
|                                 |            |             | Cohen's d                                   | 0.206459333 |                |

(b)

**Figure 4-87 F-test and t-test results on the modulus of determination of cohesion,  $K_c$  for 2NS sand; (a) samples: Laboratory and actual terrain parameters, (b) samples: Laboratory and scaled terrain parameters**

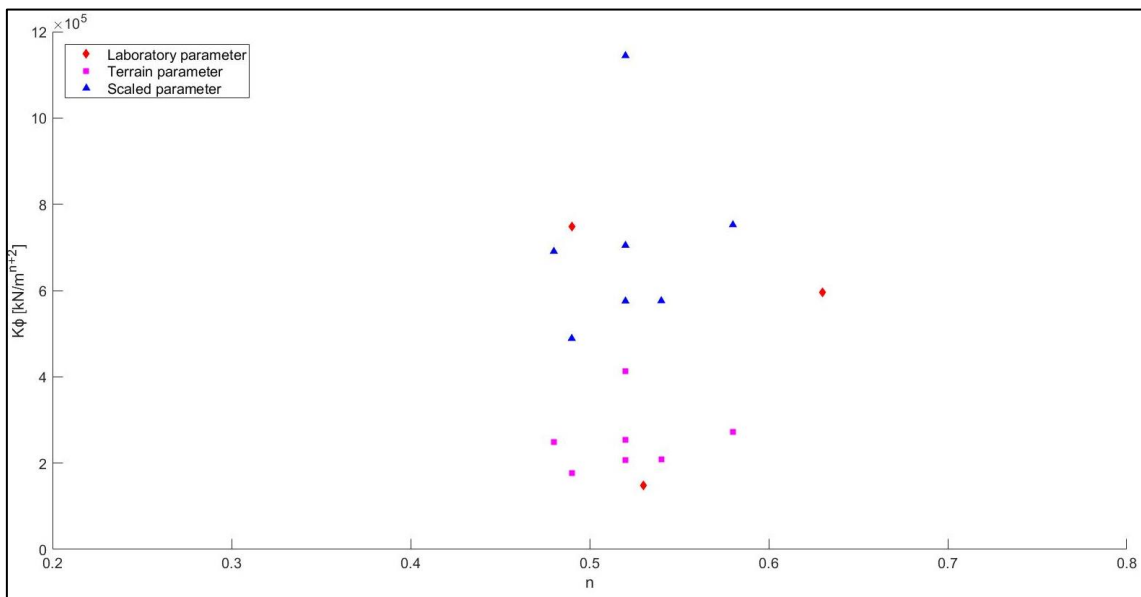


**Figure 4-88 Scalability analysis on the modulus of determination of friction,  $K_\phi$  for 2NS sand**

| F-Test Two-Sample for Variances |                   |                | t-Test: Two-Sample Assuming Unequal Variances |                   |                |
|---------------------------------|-------------------|----------------|-----------------------------------------------|-------------------|----------------|
|                                 |                   |                |                                               |                   |                |
|                                 | <i>Laboratory</i> | <i>Terrain</i> |                                               | <i>Laboratory</i> | <i>Terrain</i> |
| Mean                            | 497600            | 254420         | Mean                                          | 254420            | 497600         |
| Variance                        | 96964410000       | 5984912333     | Variance                                      | 7.182E+09         | 96964410000    |
| Observations                    | 3                 | 7              | Observations                                  | 6                 | 3              |
| df                              | 2                 | 6              | Hypothesized Mean Difference                  | 0                 |                |
| F                               | 16.20147541       |                | df                                            | 2                 |                |
| P(F<=f) one-tail                | 0.003813818       |                | t Stat                                        | -1.3282685        |                |
| F Critical one-tail             | 5.14325285        |                | P(T<=t) one-tail                              | 0.1576945         |                |
|                                 |                   |                | t Critical one-tail                           | 2.9199856         |                |
|                                 |                   |                | P(T<=t) two-tail                              | 0.3153889         |                |
|                                 |                   |                | t Critical two-tail                           | 4.3026527         |                |
|                                 |                   |                | Cohen's d                                     | 1.4347018         |                |

(a)

| F-Test Two-Sample for Variances |                   |                       | t-Test: Two-Sample Assuming Equal Variances |                   |                       |
|---------------------------------|-------------------|-----------------------|---------------------------------------------|-------------------|-----------------------|
|                                 |                   |                       |                                             |                   |                       |
|                                 | <i>Laboratory</i> | <i>Scaled terrain</i> |                                             | <i>Laboratory</i> | <i>Scaled terrain</i> |
| Mean                            | 497600            | 704743.4              | Mean                                        | 497600            | 704743.4              |
| Variance                        | 96964410000       | 45921633842           | Variance                                    | 96964410000       | 45921633842           |
| Observations                    | 3                 | 7                     | Observations                                | 3                 | 7                     |
| df                              | 2                 | 6                     | Pooled Variance                             | 58682327882       |                       |
| F                               | 2.111519166       |                       | Hypothesized Mean Difference                | 0                 |                       |
| P(F<=f) one-tail                | 0.202168637       |                       | df                                          | 8                 |                       |
| F Critical one-tail             | 5.14325285        |                       | t Stat                                      | -1.23915913       |                       |
|                                 |                   |                       | P(T<=t) one-tail                            | 0.125203267       |                       |
|                                 |                   |                       | t Critical one-tail                         | 1.859548038       |                       |
|                                 |                   |                       | P(T<=t) two-tail                            | 0.250406535       |                       |
|                                 |                   |                       | t Critical two-tail                         | 2.306004135       |                       |
|                                 |                   |                       | Cohen's d                                   | -0.855101038      |                       |

(b)

**Figure 4-89 F-test and t-test results on the modulus of determination of friction,  $K_0$  for 2NS; (a) samples: Laboratory and actual terrain parameters, (b) samples: Laboratory and scaled terrain parameters**

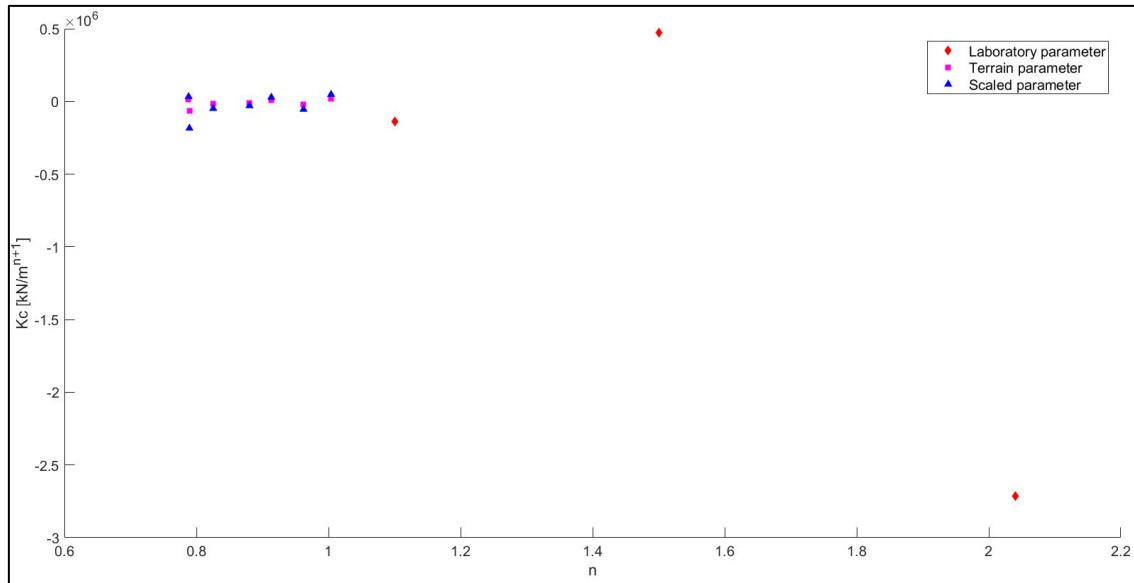


**Figure 4-90 Scalability analysis on the modulus of determination of cohesion,  $K_c$  for FGS**

| F-Test Two-Sample for Variances |            |         | t-Test: Two-Sample Assuming Unequal Variances |            |             |
|---------------------------------|------------|---------|-----------------------------------------------|------------|-------------|
|                                 | Laboratory | Terrain |                                               | Laboratory | Terrain     |
| Mean                            | -791967    | -10579  | Mean                                          | -791967    | -10579      |
| Variance                        | 2.861E+12  | 9.7E+08 | Variance                                      | 2.861E+12  | 967993162.8 |
| Observations                    | 3          | 6       | Observations                                  | 3          | 6           |
| df                              | 2          | 5       | Hypothesized Mean Difference                  | 0          |             |
| F                               | 2955.9815  |         | df                                            | 2          |             |
| P(F<=f) one-tail                | 2.076E-08  |         | t Stat                                        | -0.800025  |             |
| F Critical one-tail             | 5.786135   |         | P(T<=t) one-tail                              | 0.2538112  |             |
|                                 |            |         | t Critical one-tail                           | 2.9199856  |             |
|                                 |            |         | P(T<=t) two-tail                              | 0.5076223  |             |
|                                 |            |         | t Critical two-tail                           | 4.3026527  |             |
|                                 |            |         | Cohen's d                                     | -0.863834  |             |

(a)

| F-Test Two-Sample for Variances |            |                | t-Test: Two-Sample Assuming Unequal Variances |            |                |
|---------------------------------|------------|----------------|-----------------------------------------------|------------|----------------|
|                                 | Laboratory | Scaled terrain |                                               | Laboratory | Scaled terrain |
| Mean                            | -791967    | -35968.6       | Mean                                          | -791967    | -35968.6       |
| Variance                        | 2.861E+12  | 11190000962    | Variance                                      | 2.861E+12  | 11190000962    |
| Observations                    | 3          | 6              | Observations                                  | 3          | 6              |
| Hypothesized Mean Difference    |            |                | Hypothesized Mean Difference                  | 0          |                |
| df                              | 2          | 5              | df                                            | 2          |                |
| F                               | 255.70774  |                | t Stat                                        | -0.77334   |                |
| P(F<=f) one-tail                | 9.224E-06  |                | P(T<=t) one-tail                              | 0.2601078  |                |
| F Critical one-tail             | 5.786135   |                | t Critical one-tail                           | 2.9199856  |                |
|                                 |            |                | P(T<=t) two-tail                              | 0.5202155  |                |
|                                 |            |                | t Critical two-tail                           | 4.3026527  |                |
|                                 |            |                | Cohen's d                                     | -0.8321    |                |

(b)

**Figure 4-91 F-test and t-test results on the modulus of determination of cohesion,  $K_c$  for FGS; (a) samples: Laboratory and actual terrain parameters, (b) samples: Laboratory and scaled terrain parameters**

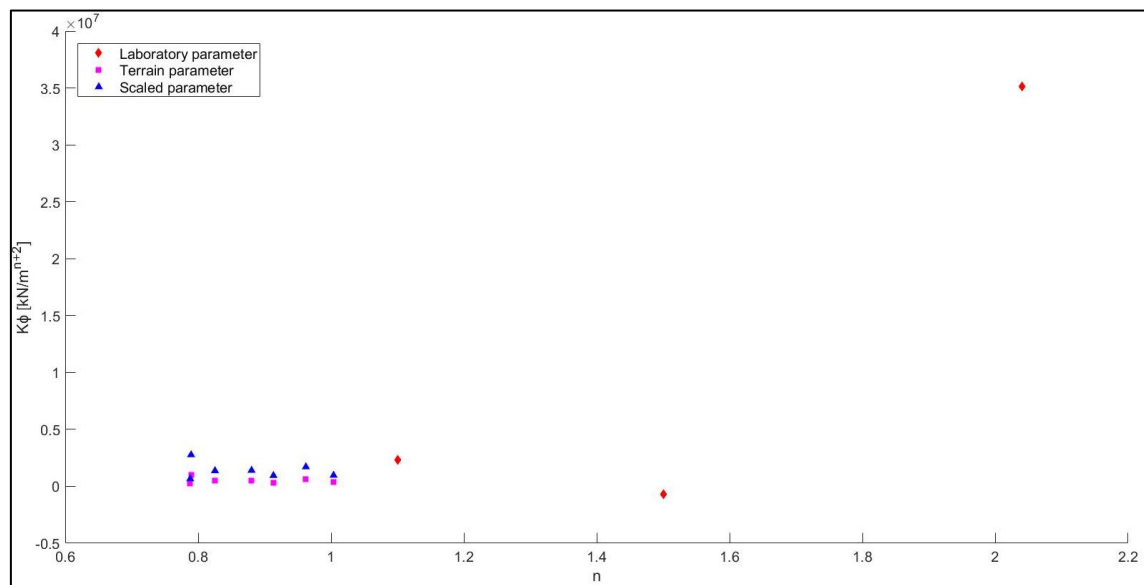


**Figure 4-92 Scalability analysis on the modulus of determination of friction,  $K_\phi$  for FGS**

| F-Test Two-Sample for Variances |                   |                |
|---------------------------------|-------------------|----------------|
|                                 | <i>Laboratory</i> | <i>Terrain</i> |
| Mean                            | 12261446.67       | 498231.6667    |
| Variance                        | 3.954E+14         | 61545055581    |
| Observations                    | 3                 | 7              |
| df                              | 2                 | 6              |
| F                               | 6424.568029       |                |
| P(F<=f) one-tail                | 1.01677E-10       |                |
| F Critical one-tail             | 5.14325285        |                |

| t-Test: Two-Sample Assuming Unequal Variances |                   |                |
|-----------------------------------------------|-------------------|----------------|
|                                               | <i>Laboratory</i> | <i>Terrain</i> |
| Mean                                          | 11949866.67       | 498231.6667    |
| Variance                                      | 4.05012E+14       | 73854066697    |
| Observations                                  | 3                 | 6              |
| Hypothesized Mean Difference                  | 0                 |                |
| df                                            | 2                 |                |
| t Stat                                        | 0.985540471       |                |
| P(T<=t) one-tail                              | 0.214127786       |                |
| t Critical one-tail                           | 2.91998558        |                |
| P(T<=t) two-tail                              | 0.428255572       |                |
| t Critical two-tail                           | 4.30265273        |                |
| Cohen's d                                     | 1.182867516       |                |

(a)

| F-Test Two-Sample for Variances |                   |                       |
|---------------------------------|-------------------|-----------------------|
|                                 | <i>Laboratory</i> | <i>Scaled terrain</i> |
| Mean                            | 12261446.67       | 1693987.667           |
| Variance                        | 3.954E+14         | 8.53753E+11           |
| Observations                    | 3                 | 6                     |
| df                              | 2                 | 5                     |
| F                               | 463.1320667       |                       |
| P(F<=f) one-tail                | 2.11224E-06       |                       |
| F Critical one-tail             | 5.786135043       |                       |

| t-Test: Two-Sample Assuming Unequal Variances |                   |                       |
|-----------------------------------------------|-------------------|-----------------------|
|                                               | <i>Laboratory</i> | <i>Scaled terrain</i> |
| Mean                                          | 12261446.67       | 1693987.667           |
| Variance                                      | 3.954E+14         | 8.53753E+11           |
| Observations                                  | 3                 | 6                     |
| Hypothesized Mean Difference                  | 0                 |                       |
| df                                            | 2                 |                       |
| t Stat                                        | 0.919979906       |                       |
| P(T<=t) one-tail                              | 0.227351444       |                       |
| t Critical one-tail                           | 2.91998558        |                       |
| P(T<=t) two-tail                              | 0.454702889       |                       |
| t Critical two-tail                           | 4.30265273        |                       |
| Cohen's d                                     | 0.991             |                       |

(b)

**Figure 4-93 F-test and t-test results on the modulus of determination of friction,  $K_0$  for FGS; (a) samples: Laboratory and actual terrain parameters, (b) samples: Laboratory and scaled terrain parameters**

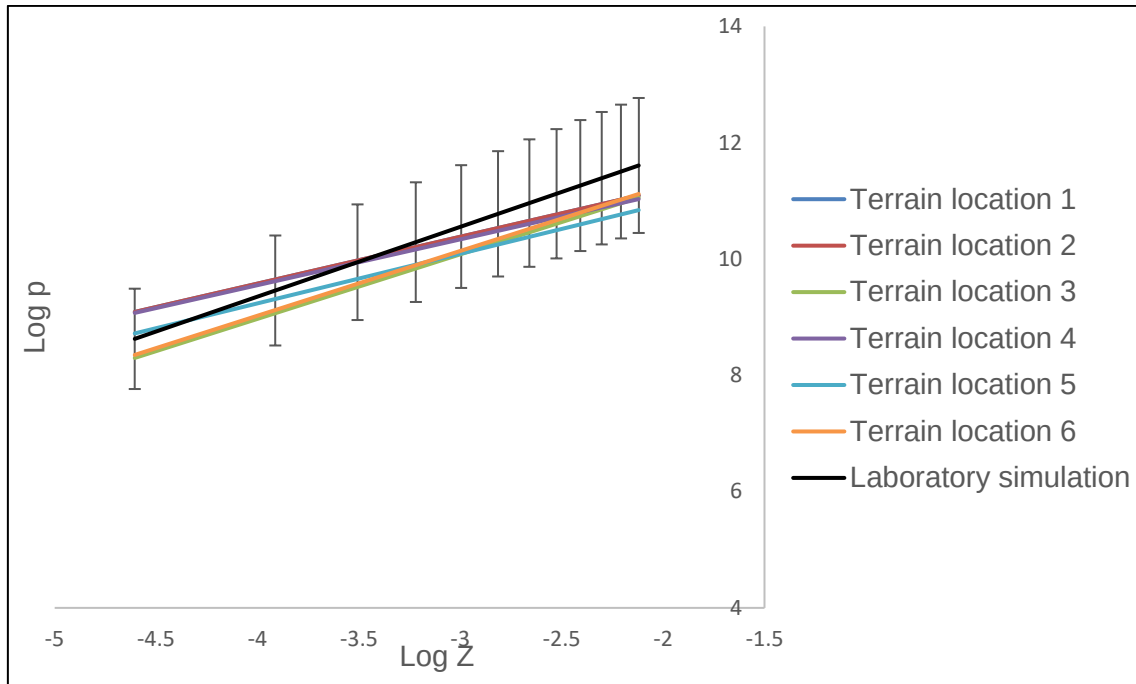


**Figure 4-94 Scalability analysis on log p – log z domain of 0.1 m diameter sinkage plate for FGS**

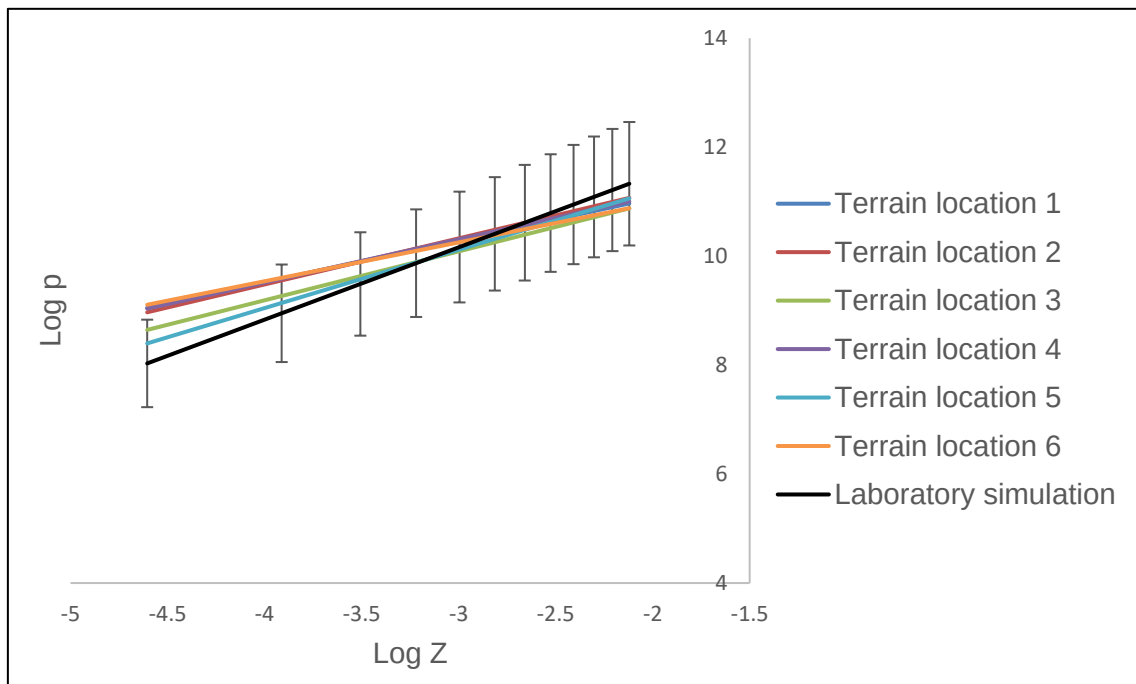


**Figure 4-95 Scalability analysis on log p – log z domain of 0.15 m diameter sinkage plate for FGS**



| F-Test Two-Sample for Variances |            |                     | t-Test: Two-Sample Assuming Equal Variances |            |                     |
|---------------------------------|------------|---------------------|---------------------------------------------|------------|---------------------|
|                                 | Laboratory | Avg. scaled terrain |                                             | Laboratory | Avg. scaled terrain |
| Mean                            | 10.62507   | 10.29019276         | Mean                                        | 10.62507   | 10.29019276         |
| Variance                        | 0.822656   | 0.475965348         | Variance                                    | 0.822656   | 0.475965348         |
| Observations                    | 12         | 12                  | Observations                                | 12         | 12                  |
| df                              | 11         | 11                  | Pooled Variance                             | 0.649311   |                     |
| F                               | 1.728395   |                     | Hypothesized Mean Difference                | 0          |                     |
| P(F<=f) one-tail                | 0.188959   |                     | df                                          | 22         |                     |
| F Critical one-tail             | 2.81793    |                     | t Stat                                      | 1.017974   |                     |
|                                 |            |                     | P(T<=t) one-tail                            | 0.159878   |                     |
|                                 |            |                     | t Critical one-tail                         | 1.717144   |                     |
|                                 |            |                     | P(T<=t) two-tail                            | 0.319755   |                     |
|                                 |            |                     | t Critical two-tail                         | 2.073873   |                     |
|                                 |            |                     | Cohen's d                                   | 0.415586   |                     |

**Figure 4-96 F-test and t-test results on log p – log z domain of 0.1 m diameter sinkage plate for FGS**

| F-Test Two-Sample for Variances |            |                     | t-Test: Two-Sample Assuming Equal Variances |            |                     |
|---------------------------------|------------|---------------------|---------------------------------------------|------------|---------------------|
|                                 | Laboratory | Avg. scaled terrain |                                             | Laboratory | Avg. scaled terrain |
| Mean                            | 10.24098   | 10.28043186         | Mean                                        | 10.24098   | 10.28043186         |
| Variance                        | 1.005999   | 0.410864586         | Variance                                    | 1.005999   | 0.410864586         |
| Observations                    | 12         | 12                  | Observations                                | 12         | 12                  |
| df                              | 11         | 11                  | Pooled Variance                             | 0.708432   |                     |
| F                               | 2.448493   |                     | Hypothesized Mean Difference                | 0          |                     |
| P(F<=f) one-tail                | 0.076505   |                     | df                                          | 22         |                     |
| F Critical one-tail             | 2.81793    |                     | t Stat                                      | -0.11481   |                     |
|                                 |            |                     | P(T<=t) one-tail                            | 0.454817   |                     |
|                                 |            |                     | t Critical one-tail                         | 1.717144   |                     |
|                                 |            |                     | P(T<=t) two-tail                            | 0.909634   |                     |
|                                 |            |                     | t Critical two-tail                         | 2.073873   |                     |
|                                 |            |                     | Cohen's d                                   | -0.04687   |                     |

**Figure 4-97 F-test and t-test results on log p – log z domain of 0.15 m diameter sinkage plate for FGS**

#### 4.5.4.3 Cone penetrometer tests

Figure 4-70 and 4-72 shows the average cone index vs depth plot for the 2NS sand and the FGS model. Figures 4-98 and 4-99 show the F-test results and t-test between the laboratory condition model and the terrain model for 2NS sand and FGS. It can be seen that for 2NS sand, F-test and t-test both failed as the calculated values exceeded the critical values at a 5% significant value. Therefore, there is a significant variation in the cone index vs depth relation between both models. For FGS, both the F-test and t-test pass. However, the p-value is not significant to derive the conclusion. Cohen's d is 0.24 for the FGS. The mean difference is less than one standard deviation of the pool, which falls

within the acceptable limit of the NATO guideline. Therefore, it can be concluded that there is no significant difference between the laboratory condition and terrain model of FGS in terms of cone index variation.

| F-Test Two-Sample for Variances |                   |                | t-Test: Two-Sample Assuming Unequal Variances |                   |                |
|---------------------------------|-------------------|----------------|-----------------------------------------------|-------------------|----------------|
|                                 |                   |                |                                               |                   |                |
|                                 | <i>Laboratory</i> | <i>Terrain</i> |                                               | <i>Laboratory</i> | <i>Terrain</i> |
| Mean                            | 149.07202         | 114.227        | Mean                                          | 149.07202         | 114.227        |
| Variance                        | 5500.0941         | 1952.55        | Variance                                      | 5500.0941         | 1952.55        |
| Observations                    | 226               | 242            | Observations                                  | 226               | 242            |
| df                              | 225               | 241            | Hypothesized Mean Difference                  | 0                 |                |
| F                               | 2.8168703         |                | df                                            | 362               |                |
| P(F<=f) one-tail                | 3.932E-15         |                | t Stat                                        | 6.1211358         |                |
| F Critical one-tail             | 1.240631          |                | P(T<=t) one-tail                              | 1.209E-09         |                |
|                                 |                   |                | t Critical one-tail                           | 1.6490738         |                |
|                                 |                   |                | P(T<=t) two-tail                              | 2.417E-09         |                |
|                                 |                   |                | t Critical two-tail                           | 1.9665388         |                |
|                                 |                   |                | Cohen's d                                     | 0.5755416         |                |

**Figure 4-98 (a) F-test and (b) t-test on the cone index vs depth curve for 2NS sand**

| F-Test Two-Sample for Variances |                   |                | t-Test: Two-Sample Assuming Equal Variances |                   |                |
|---------------------------------|-------------------|----------------|---------------------------------------------|-------------------|----------------|
|                                 |                   |                |                                             |                   |                |
|                                 | <i>Laboratory</i> | <i>Terrain</i> |                                             | <i>Laboratory</i> | <i>Terrain</i> |
| Mean                            | 17.750514         | 14.7892        | Mean                                        | 17.750514         | 14.7892        |
| Variance                        | 150.09269         | 81.9396        | Variance                                    | 150.09269         | 81.9396        |
| Observations                    | 220               | 19             | Observations                                | 220               | 19             |
| df                              | 219               | 18             | Pooled Variance                             | 144.9165          |                |
| F                               | 1.8317482         |                | Hypothesized Mean Difference                | 0                 |                |
| P(F<=f) one-tail                | 0.0667919         |                | df                                          | 237               |                |
| F Critical one-tail             | 1.9452934         |                | t Stat                                      | 1.0287562         |                |
|                                 |                   |                | P(T<=t) one-tail                            | 0.1523215         |                |
|                                 |                   |                | t Critical one-tail                         | 1.6513084         |                |
|                                 |                   |                | P(T<=t) two-tail                            | 0.304643          |                |
|                                 |                   |                | t Critical two-tail                         | 1.970024          |                |
|                                 |                   |                | Cohen's d                                   | 0.2459933         |                |

**Figure 4-99 (a) F-test and (b) t-test on the cone index vs depth curve for FGS**

## 4.6 Summary

The work reported above has established the scalability of soil based on assumptions associated with the laboratory condition experiments and in-situ terrain systems as per the guidelines of NATO CDT – 308 (2020). The laboratory models are assumed to be homogeneous, whereas the in-situ terrain models can be categorised into different patches of heterogeneous soil. In the present study,

two different soil types are simulated: 2NS (coarse-grained sand) and FGS (fine-grained sand). The pressure-sinkage tests, shear tests and cone penetrometer tests were simulated on both the soils in laboratory conditions and in-situ terrain conditions. Direct shear tests were conducted in laboratory conditions, whereas Bevameter's shear ring tests were conducted in in-situ terrain conditions. Seven different soil parameters are investigated in this study. They are moduli of deformation w.r.t cohesion  $k_c$ , moduli of deformation w.r.t friction  $k_\phi$ , sinkage exponent  $n$ , cohesion  $c$ , internal angle of friction  $\phi$ , shear modulus  $k_x$  and cone index  $CI$ . The physical properties such as moisture content, bulk density and compaction kept the same for the laboratory soil and in-situ terrain models in simulations. The main findings of this work can be summarised as follows:

- Janosi and Hanamoto (1961) govern the scalability of shear soil behaviour in a laboratory (homogeneous) and an in-situ (heterogeneous) terrain conditions. The final expression considering soil shear failure based on Mohr-Coulomb failure criteria is as follows.

$$\tau = (c + \sigma * \tan\phi) * \left(1 - e^{\frac{-j}{k_x}}\right)$$

It was found that at lower normal stress values, there is no significant deviation between the shear stress-shear displacement relationship for 2NS and FGS sand. It can be concluded that soil's scalability at lower normal stresses holds for both soils. However, at higher normal stress, the scalability of soil holds for 2NS sand, but there are deviations (more than one standard deviation) found at location 4 and location 5 of the heterogeneous terrain patch of FGS. Significant evidence has been found which holds for scalability of 2NS sand and FGS at higher normal stresses. Therefore, the scalability of the soil shear relationship holds.

- In in-situ (heterogeneous) terrain, the shear strength properties such as cohesion and internal angle of friction vary differently with respect to heterogeneity for 2NS sand and FGS. On average, for coarse-grained 2NS sand, the in-situ heterogeneous soil model's cohesion is significantly lower than the corresponding value for the homogeneous laboratory soil. Similarly, the in-situ soil model's internal angle of friction is significantly

higher than the corresponding value for the laboratory soil. The main reason behind such behaviour of 2NS sand is the particle orientation, grain size and void ratio.

The contrasting effect of heterogeneity was found on fine-grained soil. On average, for FGS, the cohesion of in-situ heterogeneous soil is slightly higher than the corresponding value for the laboratory homogeneous soil model. The average internal angle of friction decreases for in-situ heterogeneous soil compared to the laboratory homogeneous soil model. The main reason behind such behaviour of FGS is the moisture content, dilatancy, dispersion and plasticity.

- In the present study, the scaling ratio,  $\lambda$  is defined as the ratio of equivalent stiffness parameters between the laboratory model and terrain model.

$$\lambda = \frac{(k_{eq})_{lab}}{(k_{eq})_{ter}}$$

where,  $(k_{eq})_{ter}$  is the average equivalent stiffness obtained from six different locations of the terrain model, [Pa/m<sup>n</sup>] and  $(k_{eq})_{lab}$  is the equivalent stiffness from the laboratory model simulation, [Pa/m<sup>n</sup>].

- For 2NS sand, the average scaling ratio  $\lambda$  found is 2.77. There is evidence to support the scalability of soil parameters moduli of deformation w.r.t cohesion  $k_c$  and moduli of deformation w.r.t friction  $k_\phi$  with respect to sinkage exponent  $n$  obtained from pressure-sinkage tests. It was found that there is no significant difference in the  $n_{avg}$  value of the homogeneous laboratory model and heterogeneous terrain model. Also, there was no significant difference between the scaled Bekker's soil parameters ( $k_c$  and  $k_\phi$ ) of terrain models and laboratory conditions. Cohen's  $d$  value is also less than one, supporting the results.
- For FGS sand, the average scaling ratio  $\lambda$  found is 3.4. There is evidence to reject the scalability of soil parameters moduli of deformation w.r.t cohesion  $k_c$  and moduli of deformation w.r.t friction  $k_\phi$  with respect to sinkage exponent  $n$  obtained from pressure-sinkage tests. The main reason for rejection was the significant difference in the  $n_{avg}$  value of the

homogeneous laboratory model and the heterogeneous terrain model. Therefore, to include the effect of  $n_{avg}$ , following Wong's method, the exponential form of Bekker's equation is converted into the straight-line form by changing the p-z domain to log-log.

$$\ln p = \ln(k_{eq}) + n \ln z$$

Further investigation of scalability aspects in the log-log domain was carried out. Significant evidence supports the scalable aspect of the  $\ln p$ - $\ln z$  equation for FGS sand. The Cohen's d value is also less than one for a 0.1 m and a 0.15 m diameter sinkage plate, supporting this evidence.

- It was found that the average cone index value of the heterogeneous terrain model is less than the homogeneous laboratory model for both 2NS and FGS sand. It might be because of the increase in void spaces in the heterogeneous terrain model. It needs more compaction in the heterogeneous terrain model than a homogeneous model to rearrange the particles to generate the same penetration resistance as a homogeneous laboratory model.
- The cone index vs depth relation has a significant difference between the homogeneous laboratory conditions and heterogeneous terrain conditions for 2NS sand. However, no significant difference was found between the homogeneous laboratory conditions and heterogeneous terrain conditions for FGS.

## 5 SCALABILITY OF TYRE-SOIL INTERACTION

### 5.1 Introduction

This chapter presents the test procedure and analysis of the proposed scalability of tyre-soil interaction studies (discussed in chapter -3). The theory of scalability of tyre-soil interaction is developed based on a dimensional analysis approach to the system parameters. Freitag (1966), Bekker (1969) and Swanson (1973) also proposed a similar approach to non-dimensionalise the system parameters. In the present study, as discussed in chapter – 3, section 3.3.3, the obtained non-dimensional parameters are kept the same for full-scale and scale systems for scalability investigation. The full-scale system has a large diameter tyre (scale = 1), while the scale systems have scaled-down tyres to scale 0.7, 0.5 and 0.25 as per the critical theories in Terramechanics. However, a research gap has been found in the critical tyre-soil interaction theories such as Bekker's flat plate theory (1956, 1960, 1969) and Griffith and Spenko (2011, 2012, 2013, 2014), as discussed in chapter 2, which makes scalability demanding.

- Bekker's flat plate theory assumes that the vehicle's wheel's ground contact area can be replicated using a flat rectangular or circular plate of the same area. However, this assumption works reasonably for a wheel diameter greater than 50 cm but fails for a wheel diameter less than 50 cm (Griffith and Spenko, 2011, 2012, 2013, 2014).
- Griffith and Spenko (2011, 2012, 2013, 2014) use the actual wheel section to deduce the ground contact pressure for a small diameter wheel (diameter < 50 cm). This theory works reasonably well for a small diameter wheel, but no concrete evidence has been found on its large-diameter wheel.

Therefore, it becomes important to cover both theories application domains in scalability investigation. Wright (2012) summarised the other complexities involved in the nominal tyre-soil interaction studies regardless of the methods used are discussed below:

- Dynamic variations in wheel load, translational velocity and rotational velocity;
- Non-linear tyre and soil properties;
- Three-dimensional soil deformation and displacement;
- Contact between two deformable bodies.

In the present study, empirical and analytical methods are followed, simplifying some of the associated assumptions. The Engineering Design Handbook, Automotive series, United States Army Material Command (1967) and Wong (2001) proposes a common approach that if the tyre inflation pressure is greater than the soil strength, then the tyre-soil contact area is round, and the tyre behaves like a rigid wheel. The cone index of sandy soil ranges between 0 to 700 kN/m<sup>2</sup> (Brixius, 1987), therefore, if the tyre pressure is greater than the soil's cone index value then the deformations in the terrain will be much greater than the tyre deformation. Therefore, in this case, a pneumatic tyre can be regarded as a rigid wheel (Wong, 2001). Similarly, if the pneumatic tyre operates at low inflation pressures, then the terrain can be assumed as rigid, while pneumatic tyres operating at their recommended pressure on deformable terrains can be represented by a rigid wheel.

The discrete element method has been used extensively (Nakashima et al., 2004, 2007, 2009; Smith et al., 2013; Zhao et al., 2014; Nishiyami et al., 2016, 2017; Johnson et al., 2015, 2017; Yonghao Du et al., 2017, 2018; Nakanishi et al., 2020; Ravula et al., 2021, Wu et al., 2022). This method treats the soil as a collection of discrete individual particles that interact with each other and tyres. This method can describe the better physics behind the contact phenomenon of tyre-soil interaction and inter-particle mechanics. The particles act as independent entities and interact only at the particle's contacts and interfaces. This method provides better soil particle flowability during tyre-soil interaction in 3D space.

This chapter is organised as follows. Section 5.2 summarised the problem formulation. Section 5.3 summarised the scalability theory, followed by sections 5.4, 5.5 and 5.6 about the terrain models, test equipment and testing procedure,

respectively. Sections 5.7 and 5.8 shows the results & discussion and summary, respectively.

## **5.2 Problem formulation**

The definition of scalability of tyre-soil interaction is based on the assumption that similar systems that behave similarly can be established (Bekker, 1956 and Freitag, 1966). Freitag (1966) states, “if the generalizations obtained by the systematic variation of the independent parameters are to be used with confidence, then it must be shown that the observed tyre-soil relations do not vary with the size of the system”. Therefore, it becomes important to analyse the mobility performance parameters of tyres vs slip relations with respect to the system size.

This study has a dual purpose. Firstly, it was intended to find relevant evidence to support the scalability theory based on dimensional analysis. Secondly, to study the effect of homogeneous and heterogeneous terrain systems on mobility performance parameters.

In the present study, critical mobility performance parameters are investigated, such as drawbar pull and gross traction, as discussed in section 2.2. However, in Terramechanics, both these parameters are expressed as the drawbar pull coefficient ( $D/W$ ) and gross traction coefficient ( $GT/W$ ). These performance coefficients are expressed as a function of wheel slip. NATO CDT-308 (2020) performs mobility performance experiments on 2NS sand and fine-grained sand (FGS) using FED alpha military vehicle. The tyre used in this vehicle is 335/65R22.5, which is considered the full-scale system tyre in this chapter. Both 335/65R22.5 and 7.5R16 (used in chapter 3) tyres are similar class lightweight tactical vehicle tyres with radial ply construction. This tyre is modelled in the 3D modelling software- SOLIDWORKS using an engineering datasheet from Goodyear Technical Centre as shown in Appendix B5. The tyre material properties were taken from Baranawoski (2012). Later, the full-scale system was scaled down to scales 0.7, 0.5 and 0.25. In the EDEM, the soil models were kept the same for 2NS sand and FGS, as discussed in section 4.5.1.



### 5.3 Theory of scalability

The independent parameters considered in the tyre-soil interaction system is shown in Table 5-1.

**Table 5-1 Tyre – soil interaction system model-independent parameters used in the present study**

| Parameter        | Symbol   | Dimension       |
|------------------|----------|-----------------|
| <b>Tyre</b>      |          |                 |
| Overall diameter | D        | L               |
| Section width    | B        | L               |
| Section height   | H        | L               |
| Rolling radius   | R        | L               |
| Deflection       | $\delta$ | L               |
| <b>Soil</b>      |          |                 |
| Cone index       | CI       | $ML^{-1}T^{-2}$ |
| <b>System</b>    |          |                 |
| Normal load      | W        | $MLT^{-2}$      |
| Slip             | S        | -               |
| Velocity         | V        | $LT^{-1}$       |

The dependent parameters of the tyre-soil interaction system are shown in Table 5-2.

**Table 5-2 Tyre – soil model-dependent parameters used in the present study**

| Parameter         | Symbol | Dimension    |
|-------------------|--------|--------------|
| Torque            | T      | $ML^2T^{-2}$ |
| Motion resistance | MR     | $MLT^{-2}$   |
| Drawbar pull      | D      | $MLT^{-2}$   |

The non-dimensional analysis was conducted in section 3.3.3. The dependent parameters can be written, as shown below.

$$\frac{T}{rW} = f\left(\frac{b}{d}, \frac{\delta}{h}, S, \frac{CI \cdot bd}{W}\right) \quad (5-1)$$

$$\frac{D}{W} = g\left(\frac{b}{d}, \frac{\delta}{h}, S, \frac{CI \cdot bd}{W}\right) \quad (5-2)$$

$$\frac{MR}{W} = h\left(\frac{b}{d}, \frac{\delta}{h}, S, \frac{CI \cdot bd}{W}\right) \quad (5-3)$$

Therefore, all four Pi terms should be kept the same for similar systems. It governs the similar tyre-soil interaction for both full-scale and scale tyres. In this thesis, the lightweight APC military vehicle tyre such as land rover (7.5R16) or FED Alpha tyre (335/65R22.5) are considered a full-scale system, whereas small, lightweight UGV tyres such as Warthog (24" Argo) as a scaled system. The full-scale tyre 335/65R22.5 was scaled down by size to scale 0.7, 0.5 and 0.25. Since scaling was done geometrically; therefore, the b/d ratio was kept the same for all the tyres to 0.33. Moreover, in the simulations, the full-scale and scale tyres numerical stiffnesses are kept the same for computing the forces between soil particles and tyres. The deflection ratio  $\delta/h$  can be expressed as a function of  $\frac{P_i \cdot b \cdot d}{W}$  as discussed in section 3.1.1. As per the Engineering Design Handbook, automotive series, United States Army Material Command (1967) and Wong (2010) propose that if the tyre inflation pressure is greater than the soil strength, then the tyre-soil contact area is round, and the tyre behaves like a rigid wheel. The cone index of sandy soil ranges between 0 to 700 kN/m<sup>2</sup> (Brixius, 1987), therefore, if the tyre pressure is greater than the soil's cone index value then the

terrain deformations will be much greater than the tyre deformation. Therefore, in this case, a pneumatic tyre can be regarded as a rigid wheel (Wong, 2001), it becomes independent of inflation pressure. Moreover, when the pneumatic tyre operates at recommended inflation pressure, it can be assumed as a rigid wheel on deformable terrain. The third Pi term is wheel slip. The mobility performance parameters are expressed as a function of wheel slip. Therefore, it is not considered a variable. The fourth Pi term  $\frac{CI \cdot bd}{W}$  contained the significant soil parameter CI and the load W. Both these parameters are independent variables and involves the concept of force. Therefore, for a constant tyre geometry, only changes in the cone index CI and load W can cause changes in the dependent variables. Therefore, CI was kept the same as both full-scale and scaled tyres were driving on the same simulated soil lane.

Bekker (1956) identified the separable force components, which can interact with the tyre-soil interaction system. The stated force components are:

- Forces generated due to soil cohesion
- Forces arising from acceleration of the soil particles
- Forces generated by moving soil with or against gravity
- Forces from elastic deformation of the soil
- Forces resulted from the applied force and input torque
- Forces generated from soil friction
- Forces generated by soil to wheel friction
- Forces arising from viscous shearing of soil

Bekker derived all the above separable force components into the non-dimensional group at the safe load criterion, as shown below.

$$\left(\phi, \frac{cl^2}{F}\right)$$

Therefore, it is proposed that keeping the same soil conditions and holding geometric similarity can keep these parameters the same for full-scale and scaled tyres. Further, the concept of dynamic similarity is proposed in the present study. In the same soil texture and condition, dynamic similarity exists when the total

force acting in tyre-soil interaction is similar in full-scale and scale systems. Therefore, the fourth Pi term becomes the dynamic similarity criterion in Equations 5-1 & 5-2.

Therefore, considering the tyre-soil interaction system based on tyre behaviour as a rigid wheel and similar soil penetration consistency (or same soil conditions), the geometric and dynamic similarity criterion can be reduced to the term  $\frac{bd}{W}$  or  $\frac{W}{bd}$  which is a pressure term. However, the only pressure term acting in the above conditions is the average ground pressure. It is important to note that the average ground pressure is generated due to the transfer of forces from the tyre to the soil system. On equating the above parameter for full-scale and scale systems in the above conditions, the average ground pressure is the same for both systems. Therefore, similar full-scale and scale systems can be achieved.

Further, the cone index is expressed in terms of soil properties and EpAM model parameters to study the effect of these parameters on soil strength. Table 5-3 shows the model parameters with their respective dimensions.

**Table 5-3 Soil strength and EpAM model parameters**

| Parameter                       | Symbol      | Dimension       |
|---------------------------------|-------------|-----------------|
| Cone index                      | CI          | $ML^{-1}T^{-2}$ |
| Pull-off force                  | $f_0$       | $MLT^{-2}$      |
| Surface energy                  | $\gamma$    | $MT^{-2}$       |
| Contact plasticity ratio        | $\lambda_p$ | -               |
| Slope constant                  | N           | -               |
| Tensile exponent                | X           | -               |
| Tangential stiffness multiplier | $\Xi$       | -               |

According to the Buckingham Pi theorem, the cone index (CI) can be expressed as a function of pull-off force  $f_0$  and surface energy  $\gamma$ , as shown in Equation 5-4 and 5-5. Further using Table 5-3, the dimensional equation can be written as Equation 5-6.

$$CI = f(f_0, \gamma) \quad (5-4)$$

$$CI = C. f_0^x \gamma^y \quad (5-5)$$

where C, x and y are constants.

$$ML^{-1}T^{-2} = C. [MLT^{-2}]^x [MT^{-2}]^y \quad (5-6)$$

The fundamental dimensions of the left-hand side (L.H.S) and right-hand side (R.H.S) should be the same. Therefore, equating the dimensions, which provides the following equations:

$$1 = x + y \quad (5-7)$$

$$-1 = x \quad (5-8)$$

$$-2 = -2x - 2y \quad (5-9)$$

On analysing the above equations,  $x = -1$  and  $y = 2$ . Therefore, Equation 5-4 can be re-written, as shown in Equation 5-10.

$$CI = C. \frac{\gamma^2}{f_0} \quad (5-10)$$

It is interesting to note that the cone index depends on the square of surface energy and inversely proportional to the particle's pull-off force.

## 5.4 Soil models

In the present study, six different soil systems of 6m x 4m x 2m are developed to study the scalability of tyre-soil interaction. These dimensions are based on the available experimental data of NATO CDT – 308 (2020) and also such that there is no boundary wall effect during the simulations. These soil systems consist of two different soil types and textures, i.e. 2NS sand and fine-grained sand (FGS).

The soil compaction was different in each of the six cases, as shown in Table 5-4. The moisture content for 2NS sand is 1.13%, and FGS is 18.55%. A detailed description of soil data is shown in Table 5-4. The model preparation method is discussed in section 4.5.2.

**Table 5-4 Soil compaction datasheet for the developed models**

| <b>Terrain model type</b> | <b>Pre-compaction</b> | <b>Average cone index (kPa)</b> | <b>Gradient, G (kPa/m)</b> |
|---------------------------|-----------------------|---------------------------------|----------------------------|
| 2NS – Homogeneous soil    | 0.150 m               | 149.07 $\pm$ 13.99              | 1118.19 $\pm$ 16.11        |
| 2NS – Heterogeneous soil  | 0.150 m               | 114.22 $\pm$ 12.68              | 500.82 $\pm$ 274.72        |
| 2NS – Heterogeneous soil  | 0                     | 109.88 $\pm$ 16.02              | 796.61 $\pm$ 225.39        |
| FGS – Homogeneous soil    | 0                     | 17.75 $\pm$ 1.98                | 194.60 $\pm$ 14.86         |
| FGS – Heterogeneous soil  | 0                     | 14.79 $\pm$ 4.68                | 111.09 $\pm$ 87.78         |
| FGS – Heterogeneous soil  | 0.150 m               | 22.44 $\pm$ 6.40                | 192.44 $\pm$ 154.22        |

In EDEM, Edinburgh elastic-plastic adhesion contact model (EpAM) is used to develop the soil models in both homogeneous and heterogeneous conditions for both the soils, 2NS and FGS. The EpAM model is discussed in section 4.4. The input simulation parameters for the 2NS sand and fine-grained sand model are shown in Table 5-5. The in-situ heterogeneous terrain system consists of random particle shape and size distribution. Its detailed description is given in section 4.5.3.

**Table 5-5 Input simulation parameters for soil models**

| <b>Input parameter</b>                | <b>2NS sand</b> | <b>Fine-grained sand</b> |
|---------------------------------------|-----------------|--------------------------|
| Particle density (kg/m <sup>3</sup> ) | 2500            | 2500                     |
| Shear modulus (Pa)                    | 5E+6            | 5E+8                     |
| Poisson's ratio                       | 0.4             | 0.25                     |
| Restitution                           | 0.6             | 0.6                      |
| Static friction                       | 0.5             | 0.5                      |
| Rolling friction                      | 0.01            | 0.01                     |
| Constant pull off force (N)           | -60             | -2                       |
| Surface energy (J/m <sup>2</sup> )    | 0               | 7                        |
| Contact plasticity ratio              | 0.5             | 0.9                      |
| Slope exponent                        | 1.5             | 1.5                      |
| Tensile exponent                      | 1.5             | 1.5                      |
| Tangential stiffness multiplier       | 0.67            | 0.67                     |

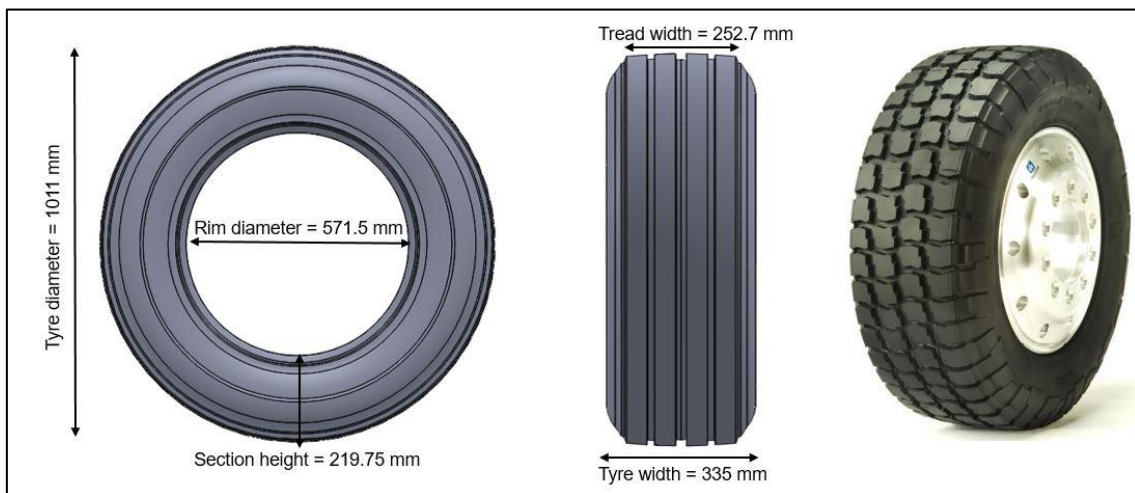
## 5.5 Test equipment

Four similar tyres of different scales were developed based on the critical tyre-soil interaction studies, i.e. Bekker's flat plate theory (1956, 1960, 1969) and Griffith and Spenko (2011, 2012, 2013, 2014) as discussed in chapter 2. In the present study, the full-scale tyre model (scale = 1) is developed from the U.S. Army military vehicle FED ALPHA tyre, i.e. 335/65R22.5, which is also part of the research programs at Tank Automotive Research, Development and Engineering Center (TARDEC), U.S. Army and NATO CDT-308 (2020). However, as per conventional terramechanics, the tyre with treads can be represented by a smooth tyre with an extended radius (Ding et al., 2014). Therefore, the tyre tread

in the 3D model is replaced with smooth extended grooves. Figure 5-1 shows the 3D CAD model of the exterior of the full-scale tyre developed in Solidworks – 2018 using an engineering datasheet from Goodyear Technical Centre as shown in Appendix B5.



(a)



(b)

**Figure 5-1 (a) FED ALPHA military vehicle, (b) FED ALPHA military vehicle's 335/65R22.5 tyre model in 3D, illustrating tyre diameter, width, section height and tread pattern**

Further, three scaled prototypes of the full-scale tyre of scale 0.7, 0.5 and 0.25 were constructed in 3D modelling software (SOLIDWORKS). The scaling is done based on critical geometrical dimensions such as tyre diameter and width. For narrow tyres, the (b/d) ratio is less than 1. The tyre width/ diameter (b/d) ratio for



scale tyres is 0.33, same as the full-scale tyre. A detailed description of test tyres is given in Table 5-6.

**Table 5-6 Specification of the test tyres models used in the present study**

| <b>Scale</b> | <b>Diameter d,<br/>mm</b> | <b>Width b,<br/>mm</b> | <b>Tyre<br/>radius, mm</b> | <b>Rim size</b> | <b>Width/<br/>Diameter</b> |
|--------------|---------------------------|------------------------|----------------------------|-----------------|----------------------------|
| 1            | 1011                      | 335                    | 505                        | 22.5            | 0.33                       |
| 0.7          | 708                       | 234                    | 354                        | 16              | 0.33                       |
| 0.5          | 505                       | 167                    | 253                        | 11              | 0.33                       |
| 0.25         | 253                       | 84                     | 126                        | 5.5             | 0.33                       |

Baranawoski et al. (2012) defined the tyre material properties validated against the tyre testing used in the present study. The material parameters were derived using special fatigue machines to assist modern optical methods for strain measuring. These material parameters were implemented into the present DEM modelling, which provided high accuracy to the provided numerical solutions. The material parameters are shown in Table 5-7. These material parameters are kept the same for all four tyres to keep the same material stiffness. The parameters such as coefficient of restitution, coefficient of static friction and coefficient of rolling friction are taken from the material library of EDEM representing the tyre material interaction with the soil particles.

**Table 5-7 Tyre material properties used in the present study**

| Material parameter                                 | Value     |
|----------------------------------------------------|-----------|
| Poisson's ratio, $\nu$                             | 0.45      |
| Density, $\rho$ [kg/m <sup>3</sup> ]               | 1173      |
| Young's modulus, E [Pa]                            | 14 E+06   |
| Shear modulus, G [Pa]                              | 4.828E+06 |
| Coefficient of restitution, e                      | 0.5       |
| Coefficient of static friction                     | 0.6       |
| Coefficient of rolling friction                    | 0.01      |
| Constant pull-off force, $f_0$ (N)                 | 0         |
| Surface energy, $\Delta\gamma$ (J/m <sup>2</sup> ) | 0.0045    |
| Contact plasticity ratio, $\lambda_p$              | 0.5       |
| Slope exponent, n                                  | 1.5       |
| Tensile exponent                                   | 1.5       |
| Tangential stiffness multiplier                    | 0.67      |

## 5.6 Test procedure

Bekker had defined the semi-empirical theory defining drawbar pull, gross traction, and tractive efficiency to evaluate off-road vehicle's tractive performance. While operating on terrain, positive slip drawbar pull (D) is used to measure the wheeled vehicle's ability to accelerate, climb gradients, and pull loads as it measures the net tractive force that driven wheels can generate. The wheels travelled in a straight line in the present study, with no applied forces. The term 'slip' throughout relates to the longitudinal slip ratio (LSR) and should not be confused with lateral slip. Independent control over the wheel's translational and rotational speed allowed a range of slip conditions to be achieved. As per this

definition, a 100% slip condition means that the vehicle is immobilized and a 0% slip means no drawbar pull and pure rolling of a wheel is taking place.

Mathematically, drawbar pull (D) is the difference between the gross tractive force (GT) applied by the soil under the wheel and the resistance (MR) from the soil piled in front of the wheel, as shown in Equation 5-11. The drawbar pull (D) is positively correlated with the wheel slip. If  $D > 0$ , then the wheel is driven by the positive force and if  $D < 0$ , then the wheel's tractive force cannot exceed the soil's resistance under the current slip ratio. The tractive efficiency is the vehicle's capability to transform the input axle power from the engine to the useful power available at the drawbar. Mathematically, it is calculated using input torque ( $T_{dw}$ ), the rolling radius of the tyre at zero slip condition ( $R_w$ ) on the test surface and normal load (W) on the wheel, as shown in Equation 5-12.

$$D = GT - MR \quad (5-11)$$

$$\eta = \frac{(D/W)}{(T_{dw}/R_w \cdot W)} (1 - s) \quad (5-12)$$

The soil bin was defined as a rectangular box of dimension 6 m long, 4 m wide and 2 m deep, and 2 m high void region above which the soil could flow. The same dimensions were followed for all six soil systems, as discussed in Figure 4-29. The soil bin's length was varied based on the wheel's longitudinal distance to minimise model runtime for small-scaled models.

NATO CDT-308 (2020) performed the drawbar pull test following the guidelines outlined in the Test Operations Procedure 2-2-604, US Army Development Test Command (2007). The test procedure states, "This test is conducted with the test vehicle connected at the tow points to another low-friction load, usually dynamometer or another vehicle. A load cell or other force measuring instrumentation is installed at the test vehicle tow point to measure the amount of force the test vehicle is exerting on the load vehicle. The available power at the test vehicle tow point is measured in as many gear combinations as possible over

the speed range of the vehicle at full throttle and full load, at discrete points in the normal operating speed range of the engine.”

NATO CDT-308 (2020) performed full-scale vehicle tests with FED ALPHA for positive slip ratios. The vehicle was driven into the in-situ terrain conditions of 2NS sand and fine-grained sand at a set translational speed of 2.76 m/s. The constant throttle setting was maintained to achieve a constant rotational speed of 52.26 RPM (or 5.47 rad/s). The average normal load on each vehicle's tyre is 13438 N. The recommended inflation pressure of all the tyres is 517 kPa (75 psi). Since the tyre inflation pressure is much greater than all the six soil strengths (Table 5-3), therefore, the tyre is assumed to behave like a rigid wheel. Further, the 'smart' winch applied a steady-state operating condition, increasing the restraining force to the rear of the vehicle to gradually reduce the translational speed from the initial value to zero in approximately 3-4 seconds of run time. Under steady-state operating conditions, the inertia force is zero (Wong, 2010). In this manner, a range of positive slip values was obtained, and the positive slip drawbar pull was measured as the force developed at the tyre's centre.

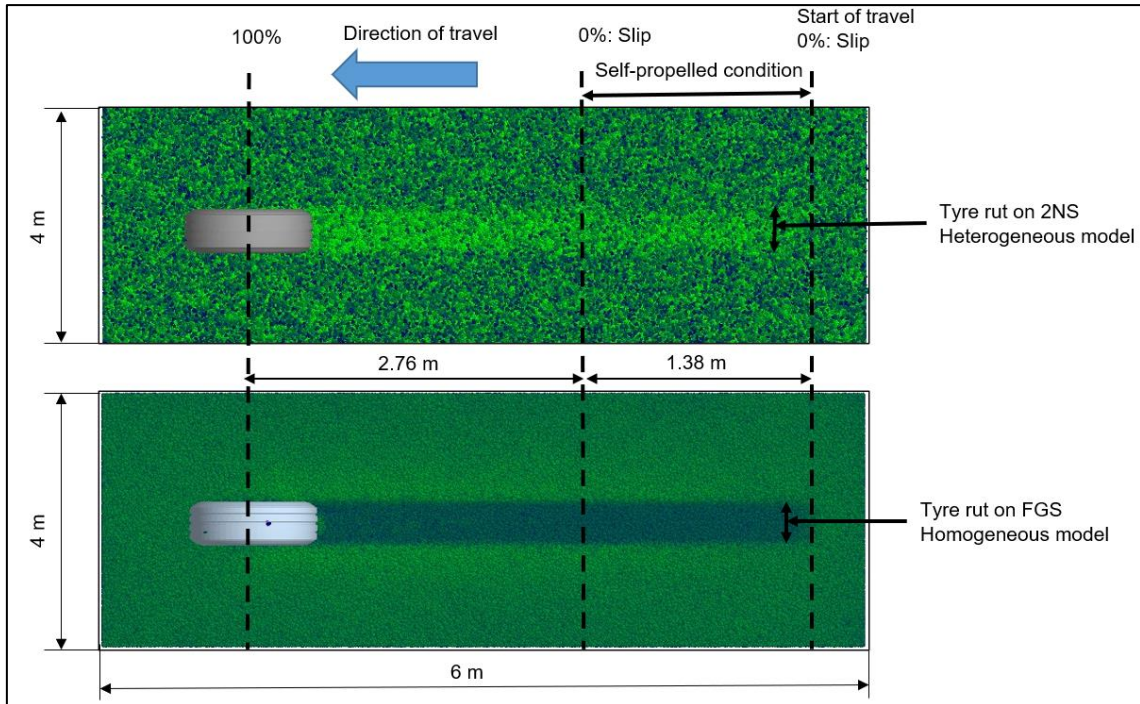
A similar approach, as discussed above, was followed in simulation model construction. Initially, the wheel was driven at a self-propelled condition where the longitudinal slip ratio is zero, with the circumferential velocity equal to the wheel's translational velocity. The wheel's self-propelled condition was maintained for an initial 0.5 s, where no additional force was required to maintain the vehicle's current speed. The translational speed of 2.76 m/s and rotational speed of 52.26 RPM (or 5.47 rad/s) was achieved in this condition. The normal load of 13438 N was applied at the wheel's centre.

After the self-propelled condition was established, the rotational speed of the full-scale tyre was maintained at 52.26 RPM (or 5.47 rad/s), but the translational speed was gradually reduced from 2.76 m/s to zero in two seconds. Simulation time is comparable to NATO's experimental time, and it reduces the computational time for each full-scale tyre test run by 18 hours. The positive slip varied from 0 % to 100 % in 2 seconds, as shown in Figure 5.2 and Table 5-8. This approach was similar to NATO's experimental procedure. However, apart

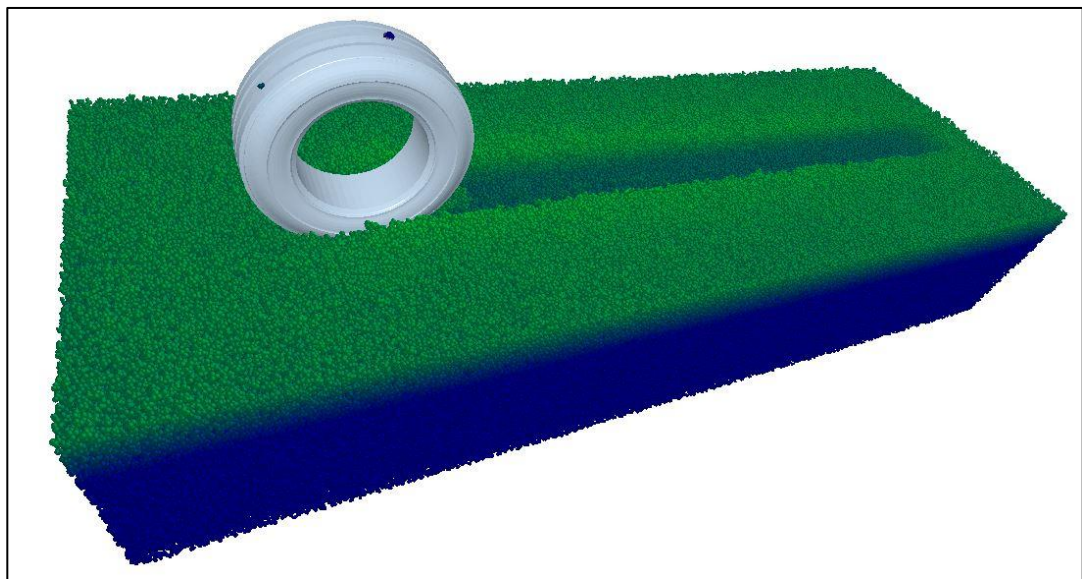
from it, this approach also provides other benefits, i.e. maintaining constant rotational speed and reducing translation speed, which reduces forward distance travelled, also reducing computational time.

A similar approach was followed for the other three-scaled tyres 0.7, 0.5 and 0.25. The rotational speed of 52.26 RPM (or 5.47 rad/s) was kept the same for all tyres. However, the initial translation speed was different for all four tyres to achieve self-propelled conditions. The reduction of translational speed for tyres in 2 seconds is shown in Table 5-8. The applied normal load was calculated by equating the scaling parameter,  $\frac{bd}{W}$  for the full-scale tyre to a scaled tyre, as shown in Equation 5-13. This parameter is independent of the soil system in the present study. The test matrix of the full normal load condition is shown in Table 5-9. Further, to investigate the effect of normal load on scalability, simulations were performed at half normal loading conditions, as shown in Table 5-10.

$$\left(\frac{bd}{W}\right)_{\text{full-scale}} = \left(\frac{bd}{W}\right)_{\text{scaled}} \quad (5-13)$$



(a)



(b)

Figure 5-2 Model construction: Full scale tyre- soil interaction on (a) 2NS Heterogeneous and FGS Homogeneous soil model; (c) FGS Homogeneous soil model – 3D view (Model support disabled and different colours used to show rut formation)

**Table 5-8 Test matrix of slip-time conditions for full-scale and scaled tyres**

| <b>Tyres</b> | <b>Scale = 1</b> | <b>Scale =<br/>0.7</b> | <b>Scale =<br/>0.5</b> | <b>Scale =<br/>0.25</b> | <b>Time</b> |
|--------------|------------------|------------------------|------------------------|-------------------------|-------------|
| <b>Slip</b>  | V (t), m/s       | V (t), m/s             | V (t), m/s             | V (t), m/s              | (Second)    |
| 0            | 2.76             | 1.93                   | 1.38                   | 0.69                    | 0           |
| 10           | 2.48             | 1.74                   | 1.24                   | 0.62                    | 0.2         |
| 20           | 2.21             | 1.54                   | 1.10                   | 0.55                    | 0.4         |
| 30           | 1.93             | 1.35                   | 0.97                   | 0.48                    | 0.6         |
| 40           | 1.66             | 1.16                   | 0.83                   | 0.41                    | 0.8         |
| 50           | 1.38             | 0.96                   | 0.69                   | 0.34                    | 1.0         |
| 60           | 1.10             | 0.77                   | 0.55                   | 0.28                    | 1.2         |
| 70           | 0.83             | 0.58                   | 0.41                   | 0.21                    | 1.4         |
| 80           | 0.55             | 0.39                   | 0.28                   | 0.14                    | 1.6         |
| 90           | 0.28             | 0.19                   | 0.14                   | 0.07                    | 1.8         |
| 100          | 0                | 0                      | 0                      | 0                       | 2           |

**Table 5-9 Test matrix of full normal load conditions of full-scale and scaled tyre system**

| <b>Tyre</b>  | <b>Diameter,<br/>mm</b> | <b>Width, mm</b> | <b>Normal load,<br/>N</b> | <b><math>\left(\frac{bd}{W}\right)</math></b> |
|--------------|-------------------------|------------------|---------------------------|-----------------------------------------------|
| Scale = 1    | 1011                    | 335              | 13438                     | 25.20                                         |
| Scale = 0.7  | 708                     | 234              | 6584.62                   | 25.20                                         |
| Scale = 0.5  | 505                     | 167              | 3359.20                   | 25.20                                         |
| Scale = 0.25 | 253                     | 84               | 839.87                    | 25.20                                         |

**Table 5-10 Test matrix of half normal load conditions of full-scale and scaled tyre system**

| <b>Tyre</b>  | <b>Diameter, mm</b> | <b>Width, mm</b> | <b>Normal load, N</b> | <b><math>\left(\frac{bd}{W}\right)</math></b> |
|--------------|---------------------|------------------|-----------------------|-----------------------------------------------|
| Scale = 1    | 1011                | 335              | 6719                  | 50.40                                         |
| Scale = 0.7  | 708                 | 234              | 3292.31               | 50.40                                         |
| Scale = 0.5  | 505                 | 167              | 1679.75               | 50.40                                         |
| Scale = 0.25 | 253                 | 84               | 419.94                | 50.40                                         |

## 5.7 Results & discussion

### 5.7.1 Model validation and comparison with NRMM

Figure 5.3 shows the DEM simulation model of tyre-soil interaction used in the present study. The accuracy of the modelling prediction for positive slip drawbar pull of full-scale tyre (scale = 1), 335/65R22.5 operating on 2NS sand and fine-grained soil (FGS) is shown in Figures 5-4 and 5-5, respectively. NATO CDT – 308 (2020) performed the in-situ drawbar pull tests using the FED ALPHA vehicle on both soils. Since the in-situ testing conditions are considered heterogeneous, therefore, the heterogeneous simulation conditions are validated against the in-situ experimental results for both the soil models. Figure 5.6 shows the DEM simulation model of the full-scale tyre at 100 % slip condition which shows the non-linear increase in the drawbar pull. It may be because the tailing piles build-ups in height, resulting in the increased contact area between tyre and sand. The trend of the drawbar pull results is similar to Johnson et al. (2017) and Johnson et al. (2015) i.e., solid wheel on soft soil. A quantitative comparison between the simulation results and experimental results is shown in Table 5-11. The mean error in drawbar pull prediction is 12% and 9% for the 2NS sand and fine-grained sand model, respectively.



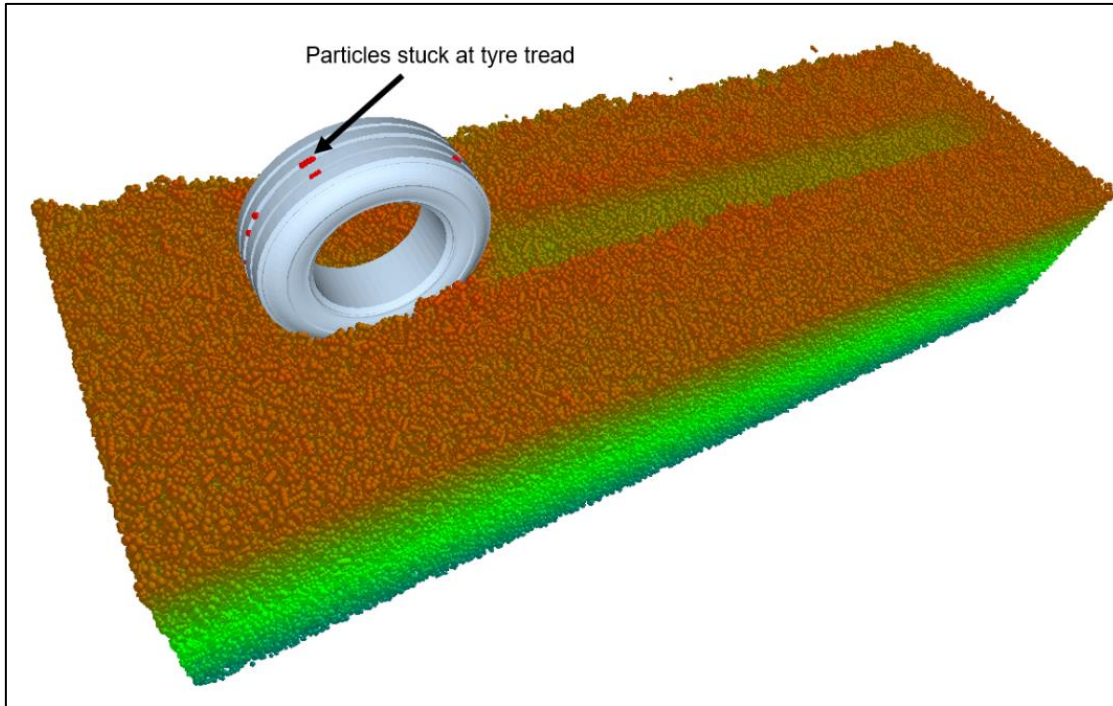


**Figure 5-3 Overview of DEM simulation model of tyre-soil interaction used to predict the tractive performance of tyres on the terrain**

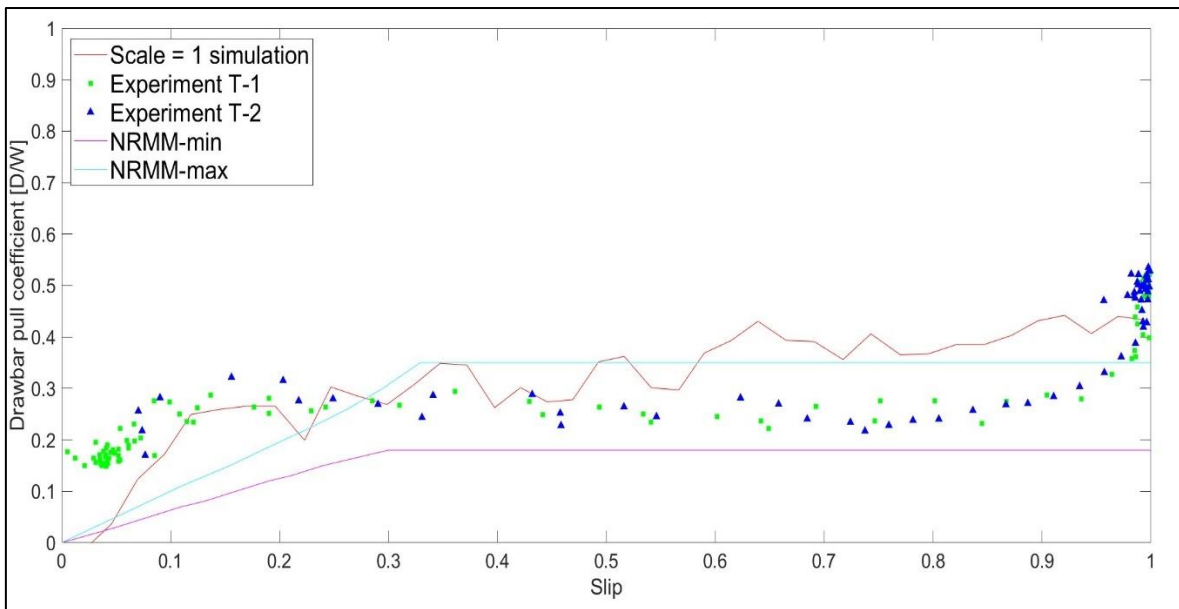


**Figure 5-4 Comparison of simulation, experimental and NRMM prediction results of full-scale tyre 335/65R22.5 on 2NS sand**

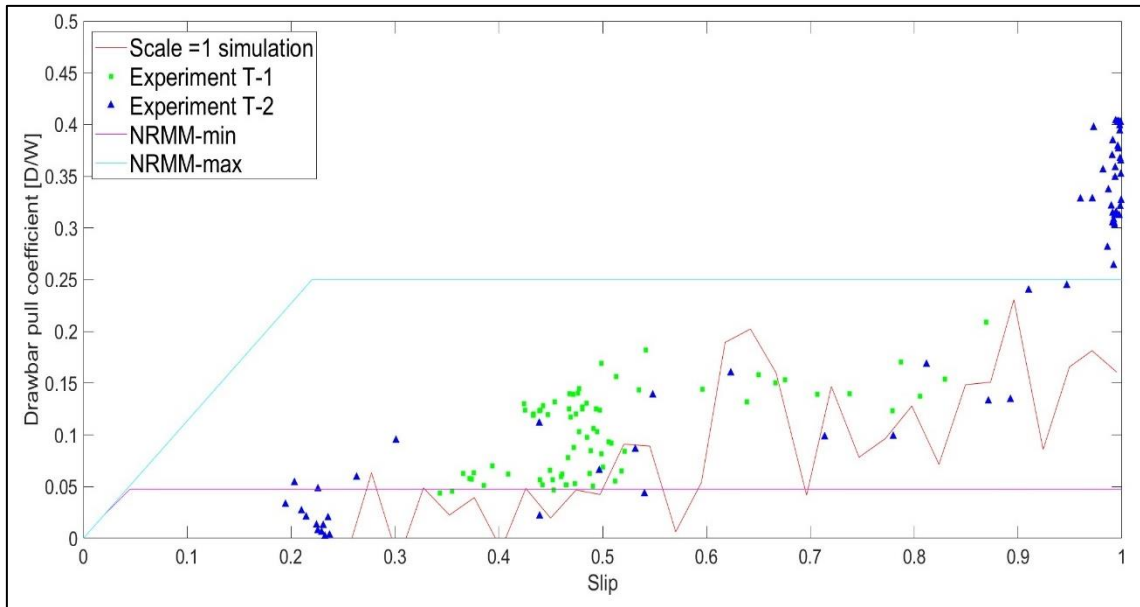


**Figure 5-5 Comparison of simulation, experimental and NRMM prediction results of full-scale tyre 335/65R22.5 on fine-grained sand**

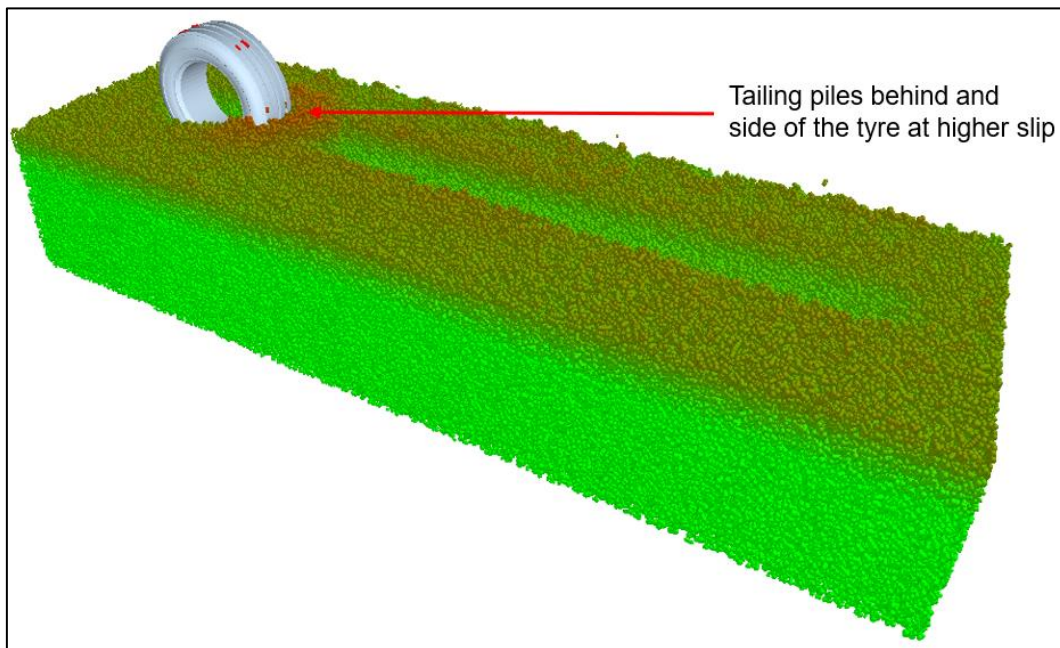


**Figure 5-6 DEM simulation showing the tailing piles behind and side of the full-scale tyre when run at 100% slip condition for 5 seconds on fine-grained soil**

**Table 5-11 Quantitative comparison of simulation and experimental results for the positive slip of 335/65R22.5 (scale =1) tyre operating on 2NS sand and FGS**

| <b>Tyre: 335/65R22.5 (Scale = 1)</b> |                                   |                         |                                   |                         |
|--------------------------------------|-----------------------------------|-------------------------|-----------------------------------|-------------------------|
|                                      | <b>2NS sand</b>                   |                         | <b>Fine-grained sand (FGS)</b>    |                         |
| <b>Slip</b>                          | <b>Average experimental (D/W)</b> | <b>Simulation (D/W)</b> | <b>Average experimental (D/W)</b> | <b>Simulation (D/W)</b> |
| +10                                  | 0.26                              | 0.22                    | -                                 | -0.09                   |
| +20                                  | 0.27                              | 0.23                    | -                                 | -0.06                   |
| +30                                  | 0.27                              | 0.28                    | 0.09                              | 0.04                    |
| +40                                  | 0.28                              | 0.26                    | 0.07                              | 0.04                    |
| +50                                  | 0.26                              | 0.35                    | 0.06                              | 0.04                    |
| +60                                  | 0.24                              | 0.39                    | 0.14                              | 0.17                    |
| +70                                  | 0.24                              | 0.35                    | 0.14                              | 0.15                    |
| +80                                  | 0.27                              | 0.37                    | 0.14                              | 0.12                    |
| +90                                  | 0.29                              | 0.43                    | 0.14                              | 0.15                    |
| +100                                 | 0.49                              | 0.43                    | 0.31                              | 0.16                    |
| <b>Mean</b>                          | 0.29                              | 0.33                    | 0.12                              | 0.11                    |

Further, the experimental and simulation results are compared with the NATO Reference Mobility Model (NRMM), as shown in the above Figures. NRMM is an empirical-based model that considers a limited number of vehicle and terrain parameters measured with a cone penetrometer.

It is interesting to note that NATO CDT -308 (2020) stated for fine-grained sand models, “Simple Terramechanics models over predict the drawbar coefficient throughout the entire slip range, while NRMM over predicts at low slip and under

predicts with its average value at large slip. All simulations have difficulties capturing the test results with reasonable accuracy throughout the entire slip range.” It was found that other simulation methods such as MSC-CT and CM Labs over-predicted the drawbar pull coefficient throughout the slip range (Figure 5-7). However, it can be seen that the present simulation model of wet fine-grained sand captures better experimental data of the drawbar pull coefficient (D/W) of the full-scale tyre. The present simulation technique's obtained results fall in the prediction band of NRMM with an error of less than 10%. Therefore, it is recommended to use the present simulation technique consisting of a discrete element method using the Edinburgh elastic-plastic adhesion contact model to represent experimental conditions in software.

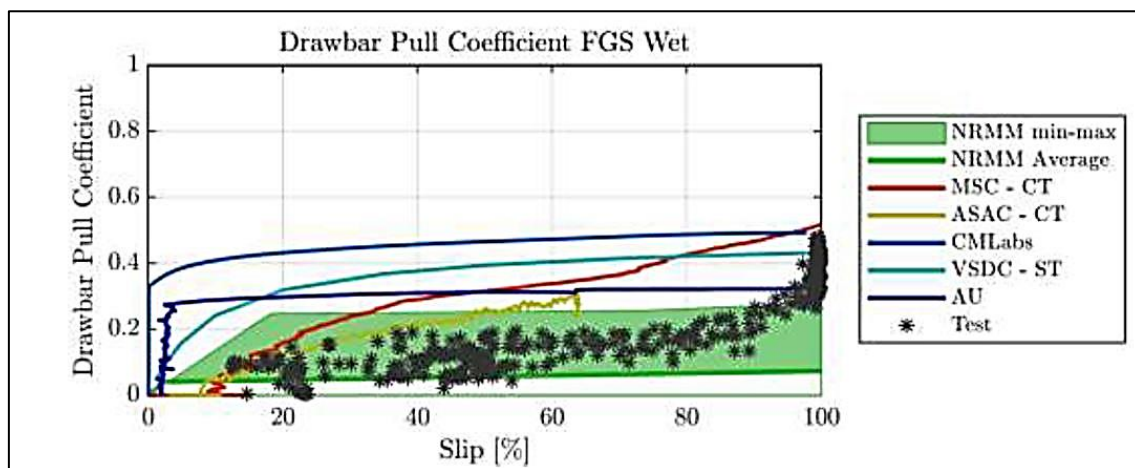


**Figure 5-7 Wet fine-grained soil – Drawbar pull test and simulation results (NATO CDT-308 (2020))**

However, NATO CDT-308 (2020) also stated for 2NS sand (coarse-grained sand), “In conclusion, the coarse grain soil appears to be outside of the capabilities of the simulation models. None of the models captures the drawbar pull coefficient minimum around 60% slip.” It can be seen in Figure 5-8 that other simulation methods over-predicted the experimental results at higher slip values, and similarly, NRMM under-predicted experimental results at lower slip range and over-predicted at higher slip range. However, the present simulation technique under-predicted the drawbar pull coefficient at lower slip, less than 10%, but predicted fairly accurate at up to 60% and later over-predicted at a higher slip range. It should be noted that at 100% slip, the simulation model accuracy improves again.

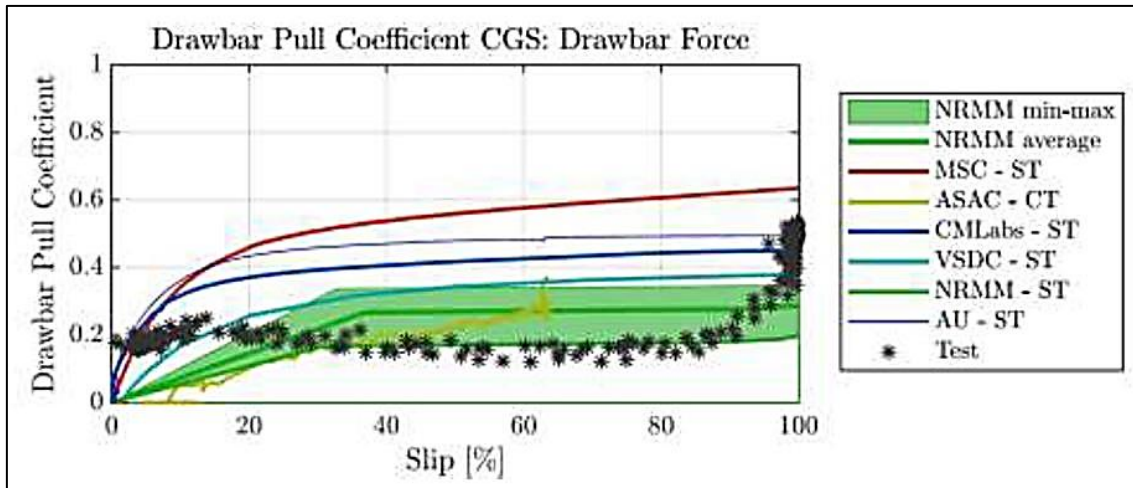


**Figure 5-8 Dry 2NS sand – Drawbar pull test and simulation results (NATO CDT-308 (2020))**

### 5.7.2 Homogeneous vs heterogeneous terrain model

NATO CDT – 308 (2020) assumes that the laboratory testing conditions are assumed to be homogeneous and isotropic, whereas the in-situ terrain conditions are assumed to be heterogeneous and anisotropic. Therefore, it becomes crucial to understand the relation between mobility performance predictions in homogeneous laboratory conditions and in-situ heterogeneous terrain conditions, keeping the same soil conditions. The EpAM model parameters were kept the same for both soil models to keep the same soil conditions. In homogeneous laboratory condition, the 3-sphere particle model with uniform size distribution was generated in the soil bin. In in-situ heterogeneous terrain conditions, six different particle shapes each having a random size distribution were generated in six different patches, as discussed in section 5.4.

From Table 5-4, it can be seen that in similar conditions, homogenous terrains of sandy soils have more soil strength (higher cone index) than heterogeneous terrains for both soil models. It might be because of a decrease in the bearing capacity of sandy soils with an increase in heterogeneity which reduces soil strength (Popescu et al., 2005). However, this is not a universal phenomenon and it also depends on particle size distribution. Therefore, when the tyre interacts with the heterogeneous terrain, less drawbar pull is generated than the homogeneous terrain because of lower soil strength (Figures 5-9 and 5-10). This

effect is more vigilant in 2NS sand than in fine-grained sand as the cone index difference between the homogeneous and heterogeneous terrain model of 2NS sand is significantly higher than in fine-grained sand. Moreover, fine-grained sand has significantly less soil strength in wet conditions. Table 5-12, 80.4% and 93.3% drawbar pull coefficients obtained from homogeneous terrain models fall in the one-standard-deviation prediction band of heterogeneous terrain models for 2NS sand and FGS, respectively (Swanson, 1973 and NATO CDT-308 (2020)). The full-scale and scale tyres sinkages w.r.t. slip are more in heterogeneous terrain models than homogeneous terrain models for both sands as shown in Appendix C.3.9 and C.3.10 because of the lower soil strength of heterogeneous models under similar normal loads than in homogeneous models.

No significant difference was found in the tractive efficiency between homogeneous and heterogeneous terrain models for both the soils, 2NS sand and FGS (Figures 5-13 and 5-14, and Table 5-12). The gross traction coefficient has similar changes as of drawbar pull coefficient between homogeneous and heterogeneous terrain models (Figures 5-11 and 5-12). Table 5-12, 80.4% and 63.4% gross traction coefficients obtained from homogeneous terrain models fall in the 1-standard deviation prediction band of heterogeneous terrain models for 2NS sand and FGS, respectively.

It can be concluded that there is no significant difference between the traction properties of the homogeneous and heterogeneous terrain models of the 2NS sand and fine-grained sand. However, heterogeneous terrain models better represented the in-situ field test's experimental results, whereas homogeneous terrain models over-predicted it.



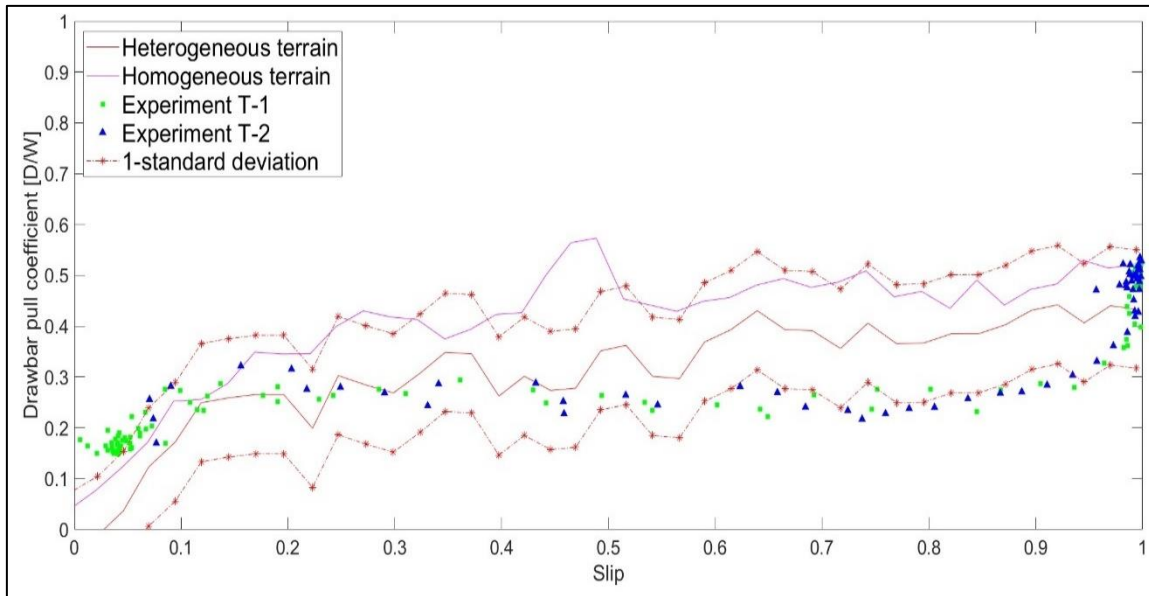


**Figure 5-9 Comparison of the drawbar pull coefficient in the homogeneous and heterogeneous terrain conditions for 2NS sand**

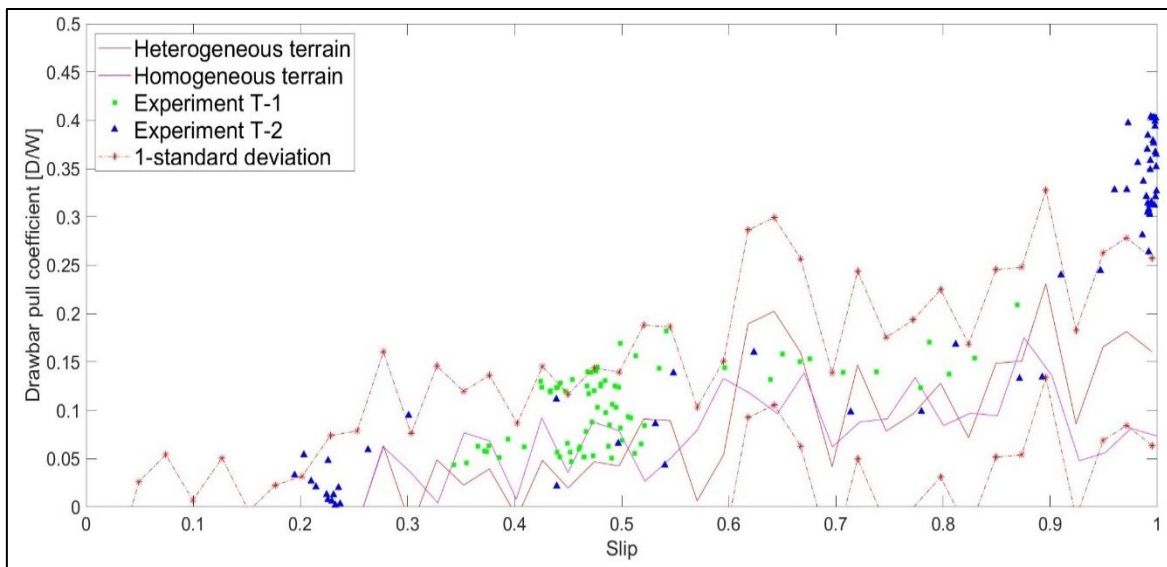


**Figure 5-10 Comparison of the drawbar pull coefficient in the homogeneous and heterogeneous terrain conditions for fine-grained sand**

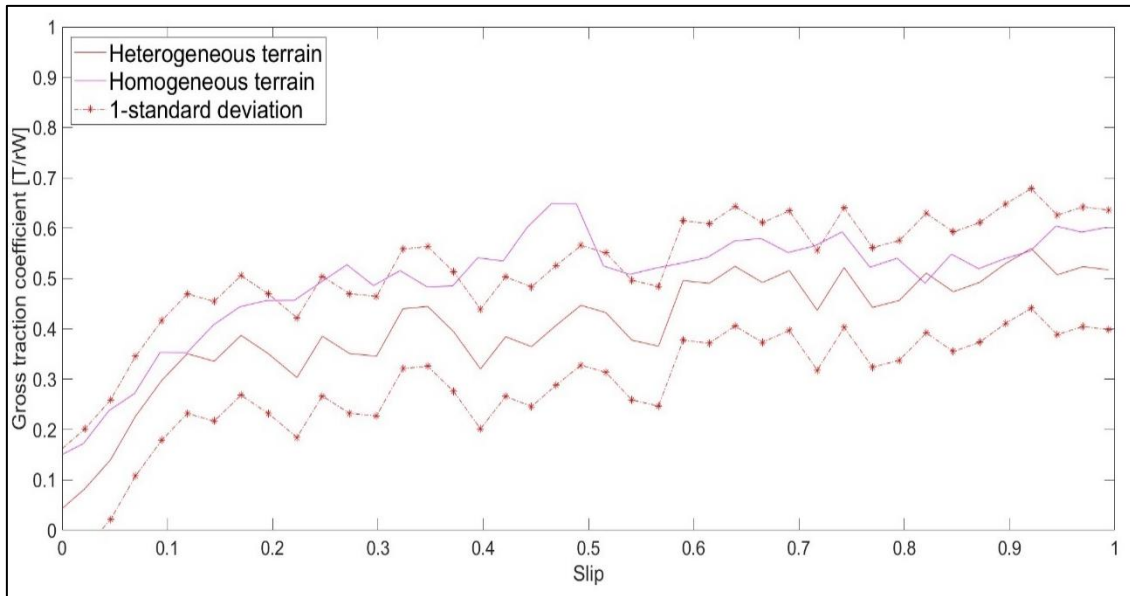


**Figure 5-11 Comparison of the gross traction coefficient in the homogeneous and heterogeneous terrain conditions for 2NS sand**

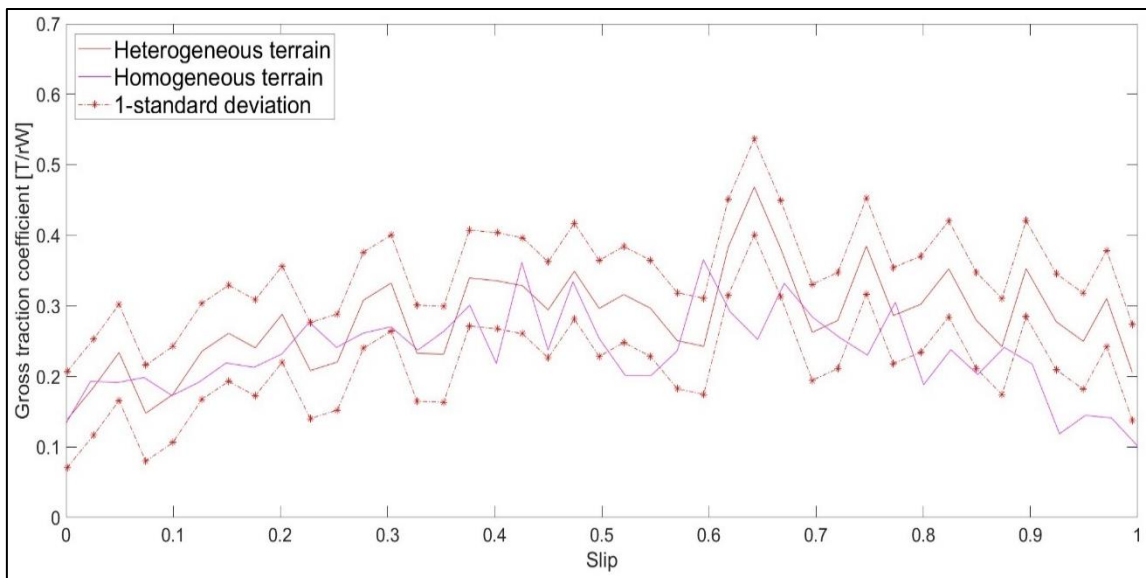


**Figure 5-12 Comparison of the gross traction coefficient in the homogeneous and heterogeneous terrain conditions for fine-grained soil**



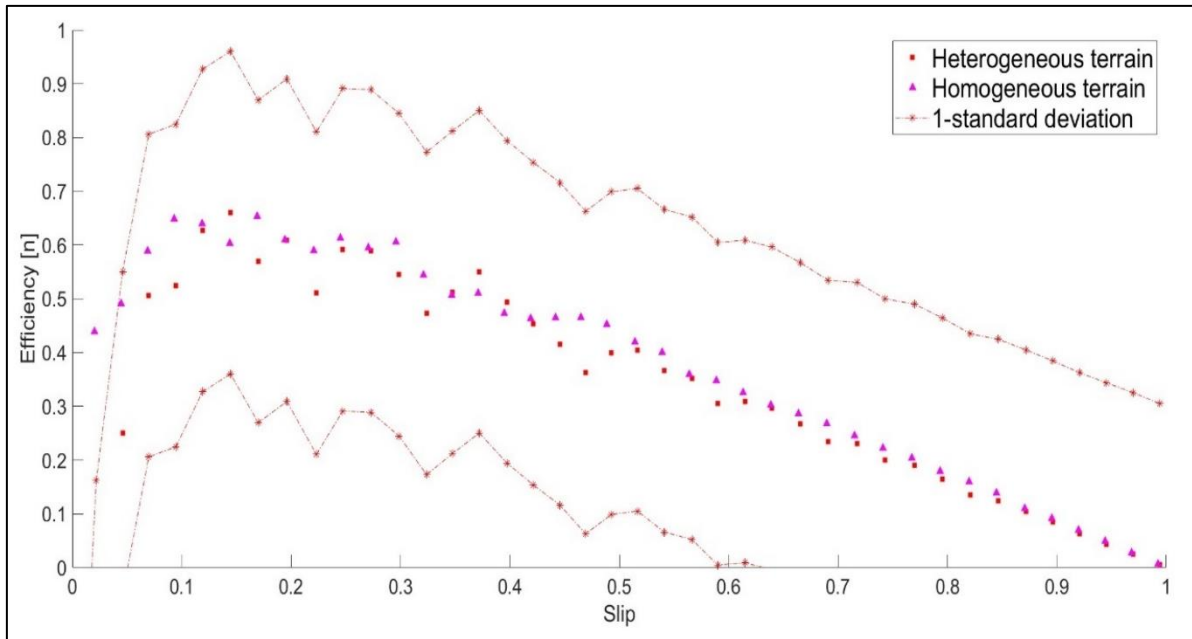


**Figure 5-13 Comparison of the tractive efficiency in the homogeneous and heterogeneous terrain conditions for 2NS sand**

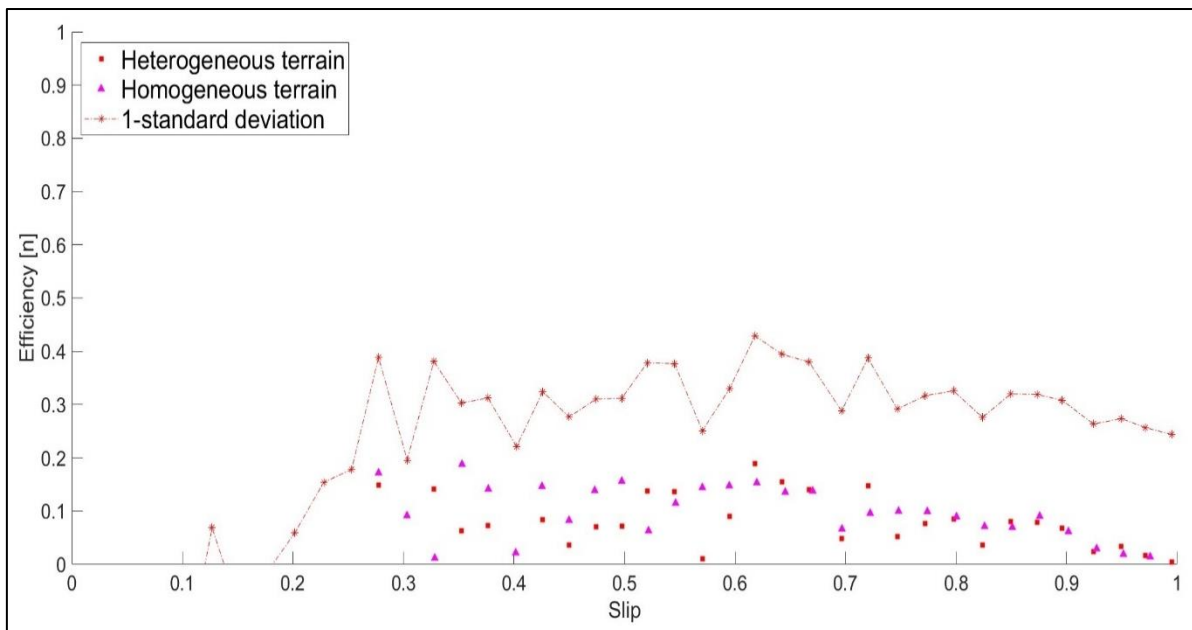


**Figure 5-14 Comparison of the tractive efficiency in the homogeneous and heterogeneous terrain conditions for fine-grained sand**

**Table 5-12 Quantitative comparison of scaled tyre systems performance parameters between the heterogeneous and homogeneous terrain conditions**

| Homogeneous soil model         | Full-scale tyre system | Performance parameter | % data points in the one-standard-deviation band of the heterogeneous terrain model |
|--------------------------------|------------------------|-----------------------|-------------------------------------------------------------------------------------|
| 2NS sand (coarse-grained sand) | Scale = 1              | P/W                   | 80.4                                                                                |
|                                |                        | T/rW                  | 80.4                                                                                |
|                                |                        | n                     | 97.6                                                                                |
| Fine-grained sand (FGS)        | Scale = 1              | P/W                   | 93.3                                                                                |
|                                |                        | T/rW                  | 63.4                                                                                |
|                                |                        | n                     | 100                                                                                 |

### 5.7.3 Scalability analysis in the homogeneous terrain models

The primary purpose of these tests was to determine whether the scaled prototypes (scale = 0.7, 0.5 and 0.25) can predict the performance of the full-scaled tyre in homogeneous conditions. Two different homogeneous soil models of 2NS sand (coarse-grained sand) and fine-grained sand were developed at uniform moisture content and bulk density, as discussed in section 5.4.

Three independent parameters were incorporated to evaluate the performance of powered-driven wheels. They are drawbar pull, input torque and performance efficiency. Drawbar pull was expressed as the drawbar pull coefficient [D/W]. The input torque was expressed as the gross traction ratio [T/rW]. The performance efficiency [n] is the ratio of drawbar pull coefficient to gross traction ratio, as discussed in section 5.6. A comparison between scaled tyres (scale = 0.7, 0.5 and 0.25) performance parameters was made graphically with the full-scale tyre (scale = 1) results, as shown in Figures 5-15 to 5-20. As the limit tends to zero slip (self-propelled condition), the curves of the drawbar pull coefficient (D/W) and

gross traction ratio ( $T/rW$ ) are in agreement with Figure 2.15 (Wismer and Luth, 1973). As the limit tends to zero slip (self-propelled condition), the gross traction should be equal to the motion resistance, since the drawbar pull is zero. As a result, gross traction is positive (force in the direction).

Further, the spikes in Figures 5.15 and 5.16 may be because of numerical error in the Euler scheme used in the simulations. It can be reduced by selecting even smaller time-step but at an expense of increased computational time (computational system limitation to the present research). The spikes are similar to Wright (2012). However, it doesn't change the general trend of the performance parameters vs slip relation. Further, full-scale tyre results were plotted with a one-standard-deviation scatter band indicated by red dash and star lines which represents the accepted error band, similar to Swanson (1973) and NATO CDT – 308 (2020). The percentage of scaled tyre systems (scale = 0.7, 0.5 and 0.25) data points that fall in the scatter band of one- standard deviation of the full-scale tyre (scale = 1) performance data points are shown in Table 5-13.

It was found that the accuracy of scalability prediction of the full-scale system relatively decreases as the size of the system reduces. The prediction accuracy also decreases with the decrease in the scaled system's size from 0.7 to 0.5 to 0.25. According to Freitag (1966), the accuracy could decrease for two possible reasons. There is a change in the physics of tyre-soil interaction because of the changes in tyre shape and size (i.e., the change in tyre curvature) and the limiting condition of traction could also involve tyre-soil friction in combination with soil-soil shear for soft soils.

For 2NS sand, the drawbar pull coefficient  $P/W$  for scale = 0.7 was found maximum with 100% fell into the scatter band of the full-scale system followed by 93% of scale = 0.5 and 75.6% of scale = 0.25. Similar trend was followed for gross-traction coefficient with maximum of 97.1% of scale = 0.7, followed by 86.1% of scale = 0.5 and 80.5% of scale = 0.25. The tractive efficiency  $[n]$  also followed a similar trend with a maximum percentage for scale = 0.7 with 100% data points lying in its scatter band of the full-scale system followed by 97.7% of

scale = 0.5 and 87.8% of scale = 0.25. These data points suggest that the scaled systems (scale = 0.7, 0.5 and 0.25) can be used to predict the performance of the full-scale system in 2NS sand (coarse-grained sand).

For fine-grained sand, the drawbar pull coefficient  $P/W$  for scale = 0.7 was found to be maximum with 85.7% fell into the scatter band of the full-scale system followed by 70.7% of scale = 0.5 and 17.1% of scale = 0.25. Similar trend was followed for gross-traction coefficient with maximum of 78% of scale = 0.7, followed by 41.4% of scale = 0.5 and 9.8% of scale = 0.25. The tractive efficiency  $[n]$  also followed similar trend with maximum percentage for scale = 0.7 with 63.4% data points lied in its scatter band of full-scale system followed by 56.1% of scale = 0.5 and 41.5% of scale = 0.25. These data points suggest that the scaled systems (scale = 0.7 and 0.5) can be used to predict the performance of the full-scale system on fine-grained soil. However, scale = 0.25 can't be used in prediction as the maximum of its data points fell outside the scatter band.

Therefore, in both the homogeneous soil models, scale = 0.7 predicted better performance of full-scale tyre system followed by scale = 0.5 and then by scale = 0.25. It might be because of the increase in the rate of change of tyre curvature from 0.7 to 0.5 to 0.25 scaled tyre systems. As per Bekker's flat plate theory (1956, 1960, 1969), for tyres with a diameter greater than 50 cm, the ground contact area can be assumed similar to a circular or rectangular flat plate. It is because the rate of change of tyre curvature is relatively very low for larger diameter tyres. However, the theory fails for a tyre diameter of less than 50 cm. As per Griffith and Spenko (2011, 2012, 2013, 2014) theory, the tyre curvature effect becomes significant in predicting tyre-soil ground contact pressure for tyre diameter less than 50 cm. The rate of change of tyre curvature is relatively higher than the larger diameter tyres. So, as the size of the system reduces, the effect of tyre curvature can't be ignored in predicting a tyre's mobility performance. Therefore, the small-scaled system (scale = 0.25) predicted the full-scale system's mobility performance poorly.

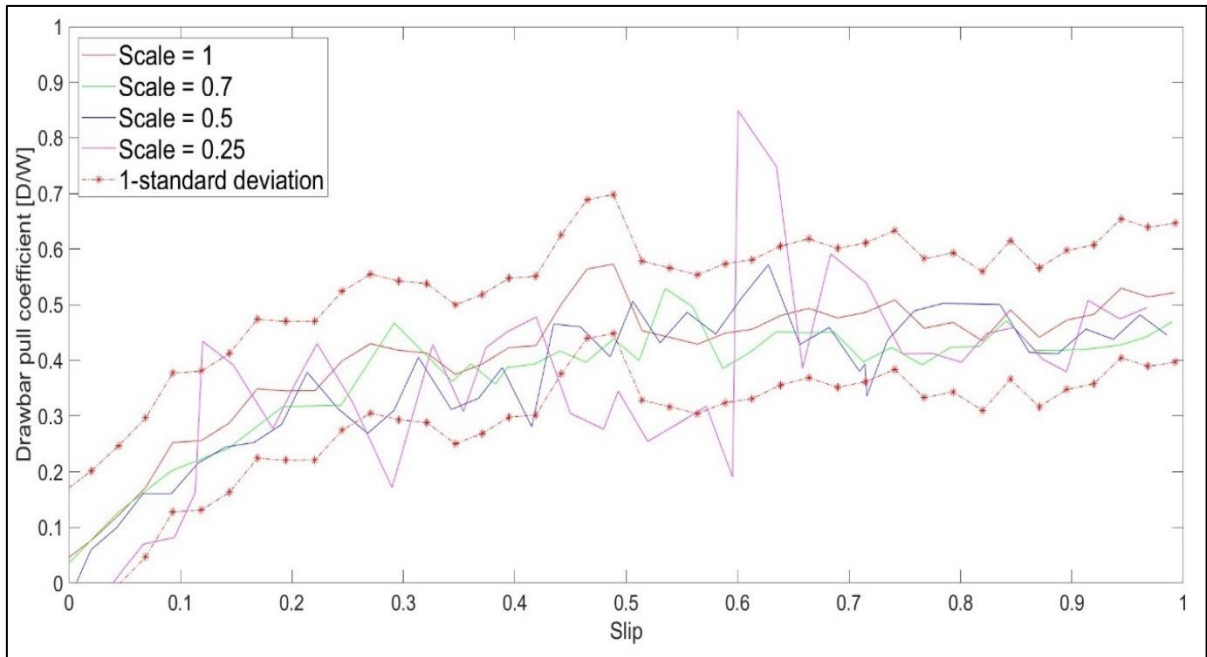


**Figure 5-15 Scalability testing of drawbar pull coefficient of 2NS homogeneous terrain model**

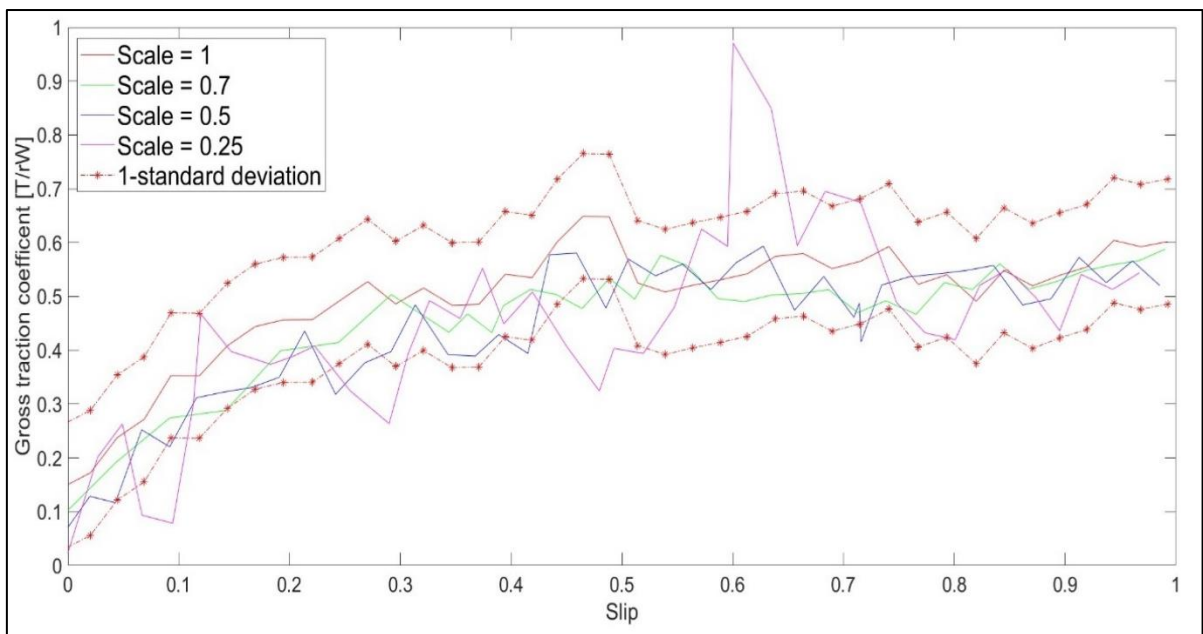


**Figure 5-16 Scalability testing of gross traction coefficient of 2NS homogeneous terrain model**

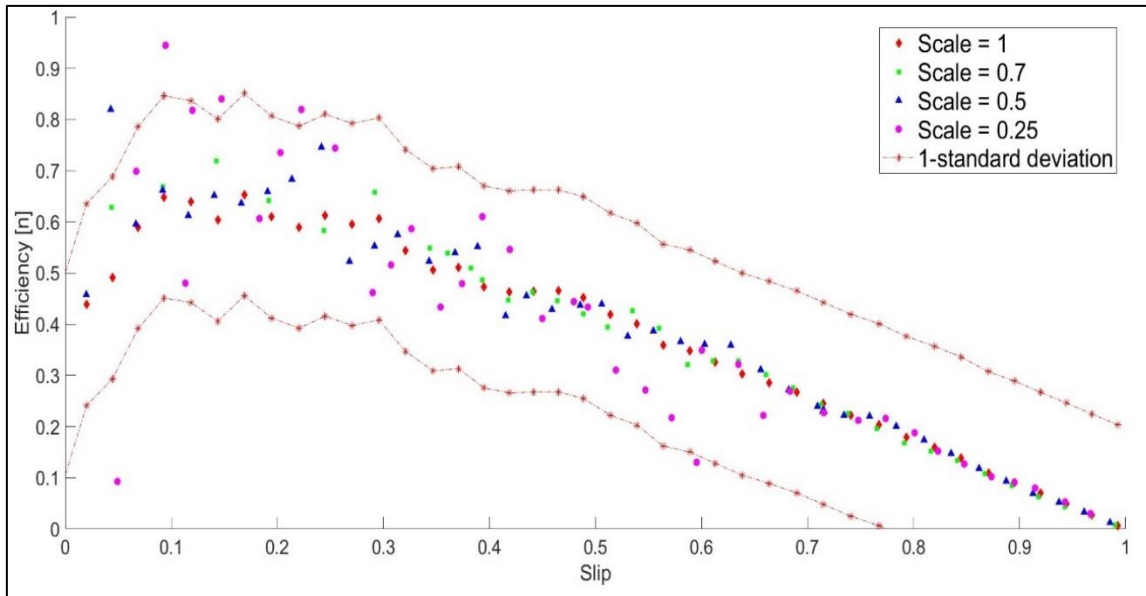


**Figure 5-17 Scalability testing of tractive efficiency of 2NS homogeneous terrain model**

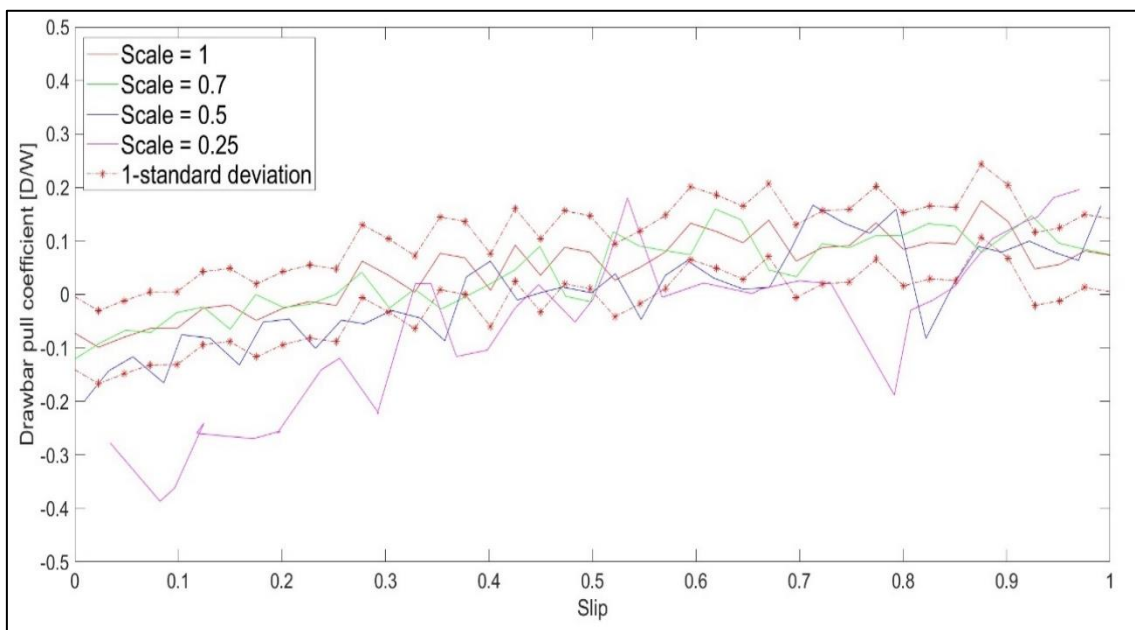


**Figure 5-18 Scalability testing of drawbar pull coefficient of the fine-grained sand homogeneous terrain model**

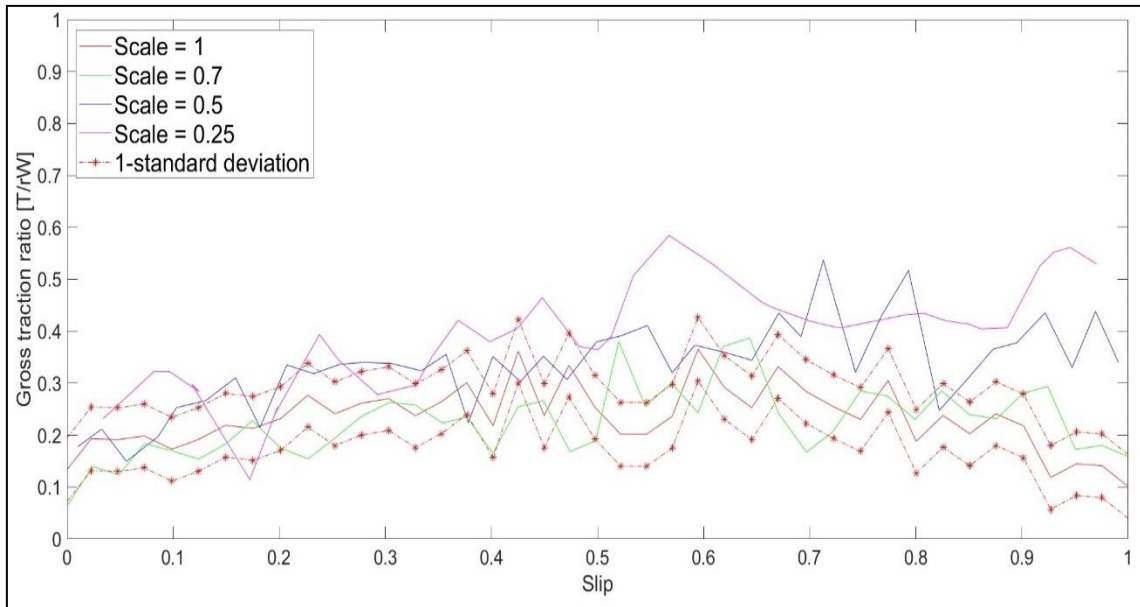


**Figure 5-19 Scalability testing of gross traction coefficient of the fine-grained sand homogeneous terrain model**

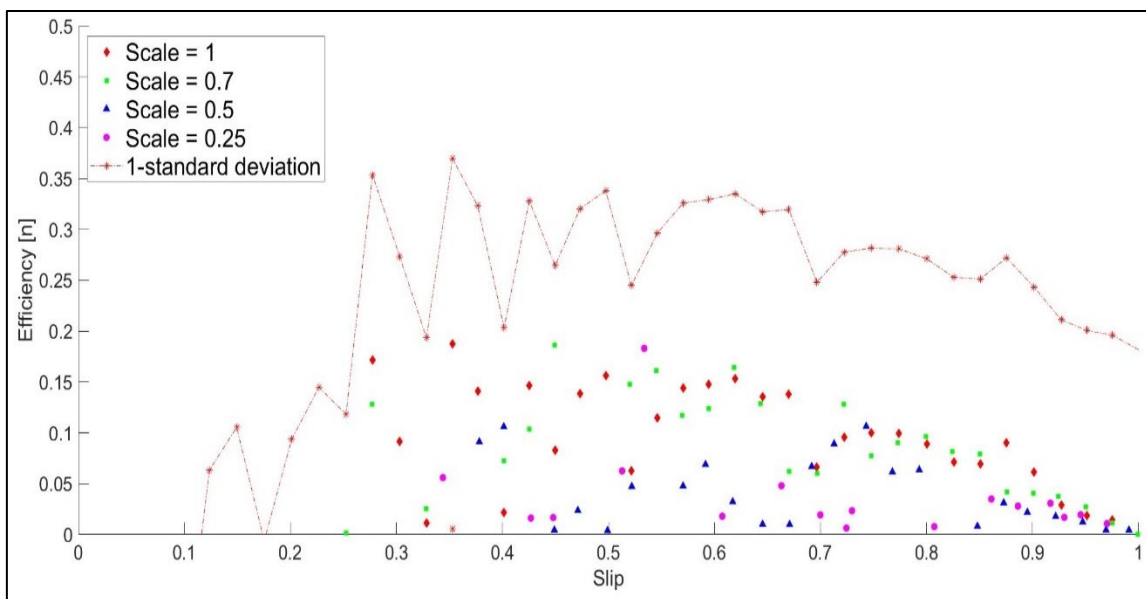


**Figure 5-20 Scalability testing of tractive efficiency of the fine-grained sand homogeneous terrain model**

**Table 5-13 Quantitative comparison of scaled tyre systems performance parameters in the homogeneous condition**

| <b>Homogeneous soil model</b>  | <b>Scaled tyre system</b> | <b>Performance parameter</b> | <b>% of data points in the one-standard-deviation band of full-scale tyre</b> |
|--------------------------------|---------------------------|------------------------------|-------------------------------------------------------------------------------|
| 2NS sand (coarse-grained sand) | Scale = 0.7               | P/W                          | 100                                                                           |
|                                |                           | T/rW                         | 97.1                                                                          |
|                                |                           | n                            | 100                                                                           |
|                                | Scale = 0.5               | P/W                          | 93.0                                                                          |
|                                |                           | T/rW                         | 86.1                                                                          |
|                                |                           | n                            | 97.7                                                                          |
|                                | Scale = 0.25              | P/W                          | 75.6                                                                          |
|                                |                           | T/rW                         | 80.5                                                                          |
|                                |                           | n                            | 87.8                                                                          |
| Fine-grained sand (FGS)        | Scale = 0.7               | P/W                          | 85.7                                                                          |
|                                |                           | T/rW                         | 78.0                                                                          |
|                                |                           | n                            | 63.4                                                                          |
|                                | Scale = 0.5               | P/W                          | 70.7                                                                          |
|                                |                           | T/rW                         | 41.4                                                                          |
|                                |                           | n                            | 56.1                                                                          |
|                                | Scale = 0.25              | P/W                          | 17.1                                                                          |
|                                |                           | T/rW                         | 9.8                                                                           |
|                                |                           | n                            | 41.5                                                                          |



#### **5.7.4 Scalability analysis in the heterogeneous terrain models**

The primary purpose of these tests was to determine whether the scaled prototypes (scale = 0.7, 0.5 and 0.25) can be used to predict the performance of the full-scaled tyre in heterogeneous terrain conditions. Two different heterogeneous soil models of 2NS sand (coarse-grained sand) and fine-grained sand were developed at uniform moisture content and bulk density, as discussed in section 5.4.

Three independent parameters were incorporated to evaluate the positive torque powered-driven wheel's performance, as discussed in the earlier section. A comparison between scaled tyres (scale = 0.7, 0.5 and 0.25) was made graphically with the full-scale tyre (scale = 1) results as shown in Figures 5-18 to 5-22. Further, full-scale tyre results were plotted with a one-standard-deviation scatter prediction band indicated by red dash and star lines, similar to Swanson (1973) and NATO CDT – 308 (2020). The percentage of scaled tyre systems (scale = 0.7, 0.5 and 0.25) data points fall in between the scatter band of the full-scale tyre (scale = 1) is shown in Table 5-14.

It was found that the accuracy of scalability prediction of the full-scale system relatively decreases as the size of the system reduces. The prediction accuracy also decreases with the decrease in the scaled system's size from 0.7 to 0.5 to 0.25. According to Freitag (1966), the accuracy could decrease for two possible reasons. There is a change in the physics of tyre-soil interaction because of the changes in tyre shape and size (i.e., the change in tyre curvature), and the limiting condition of traction could also involve tyre-soil friction in combination with soil-soil shear for soft soils.

For 2NS sand, the drawbar pull coefficient  $P/W$  for scale = 0.7 was found maximum with 90.2% fell into the scatter band of the full-scale system followed by 80.5% of scale = 0.5 and 61.8% of scale = 0.25. Similar trend was followed for gross-traction coefficient with maximum of 85.4% of scale = 0.7, followed by 80.5% of scale = 0.5 and 61.8% of scale = 0.25. The tractive efficiency  $[n]$  also followed a similar trend with maximum percentage for scale = 0.7 with 90.2% data points lied in its scatter band of full-scale system followed by 90.2% of scale =

0.5 and 85.3% of scale = 0.25. These data points suggest that the scaled systems (scale = 0.7, 0.5 and 0.25) can be used to predict the performance of the full-scale system on 2NS (coarse-grained sand).

For fine-grained sand, the drawbar pull coefficient  $P/W$  for scale = 0.7 was found to be maximum with 95.1% fell into the scatter band of the full-scale system followed by 74.4% of scale = 0.5 and 39.5% of scale = 0.25. Similar trend was followed for gross-traction coefficient with maximum of 87.8% of scale = 0.7, followed by 76.7% of scale = 0.5 and 67.4% of scale = 0.25. The tractive efficiency  $[n]$  also followed similar trend with maximum percentage for scale = 0.7 with 46.3% data points lied in its scatter band of full-scale system followed by 44.2% of scale = 0.5 and 23.2% of scale = 0.25. These data points suggest that the scaled systems (scale = 0.7 and 0.5) can predict the performance of the full-scale system on fine-grained soil. However, scale = 0.25 can't be used in prediction as the maximum of its data points fell outside the scatter band.

Therefore, in both the heterogeneous soil models, scale = 0.7 predicted better performance of full-scale tyre system followed by scale = 0.5 and then by scale = 0.25. It might be because of the increase in the rate of change of tyre curvature from 0.7 to 0.5 to 0.25 scaled tyre systems. As per Bekker's flat plate theory (1956, 1960, 1969), for tyres with a diameter greater than 50 cm, the ground contact area can be assumed similar to a circular or rectangular flat plate. It is because the rate of change of tyre curvature is relatively very low for larger diameter tyres. However, the theory fails for a tyre diameter of less than 50 cm. As per Griffith and Spenko (2011, 2012, 2013, 2014) theory, the tyre curvature can't be ignored in predicting tyre-soil contact pressure for tyre diameter less than 50 cm. As the rate of change of tyre curvature is relatively higher than larger diameter tyres. So, as the size of the system reduces, the effect of tyre curvature change can't be ignored in predicting a tyre's mobility performance. Therefore, the scaled system (scale = 0.25) predicted the full-scale system's mobility performance poorly.

It is also interesting to note that the heterogeneous fine-grained sand model predicted better performance parameters of the full-scale system than the

homogeneous fine-grained sand model. However, for 2NS sand, the homogeneous terrain model predicted better performance parameters than the heterogeneous terrain model.

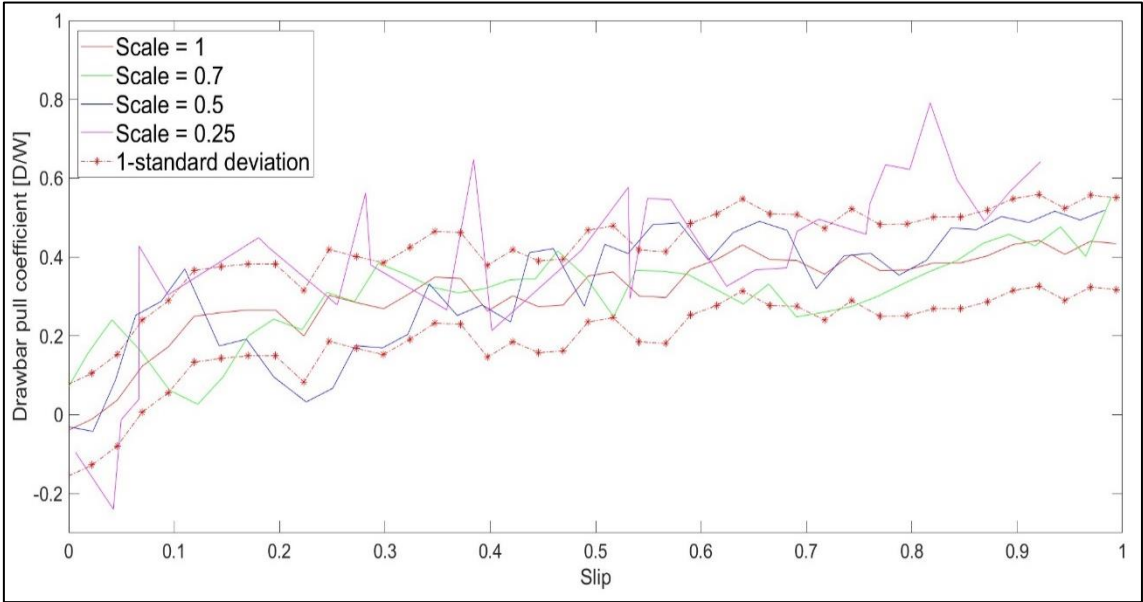


**Figure 5-21 Scalability testing of drawbar pull coefficient of 2NS heterogeneous terrain model**

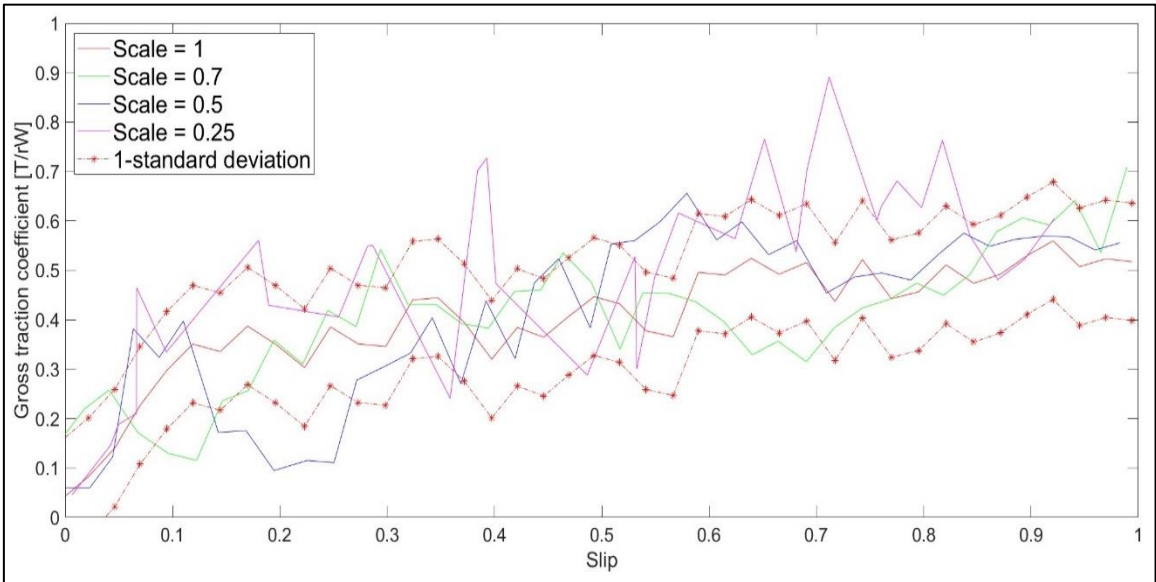


**Figure 5-22 Scalability testing of gross traction coefficient of 2NS heterogeneous terrain model**

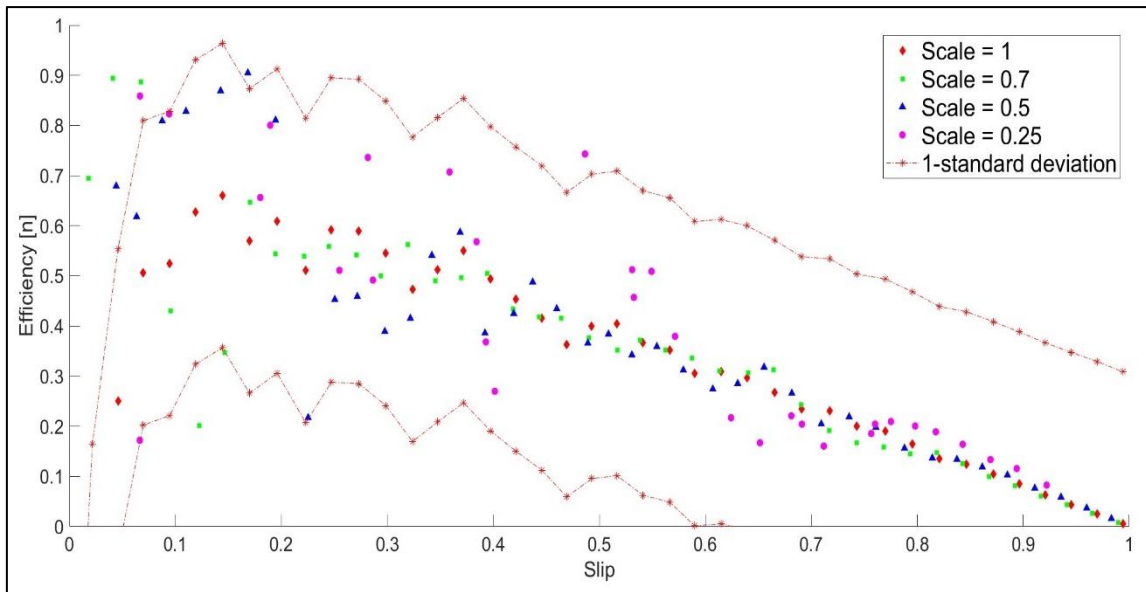


**Figure 5-23 Scalability testing of tractive efficiency of 2NS heterogeneous terrain model**

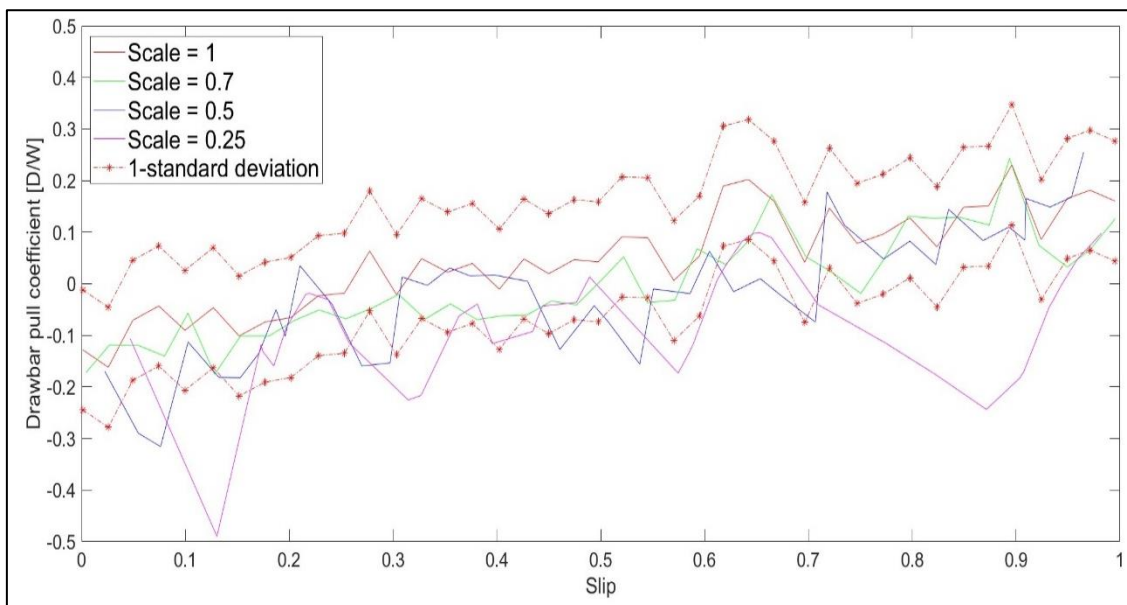


**Figure 5-24 Scalability testing of drawbar pull coefficient of the fine-grained sand heterogeneous terrain model**

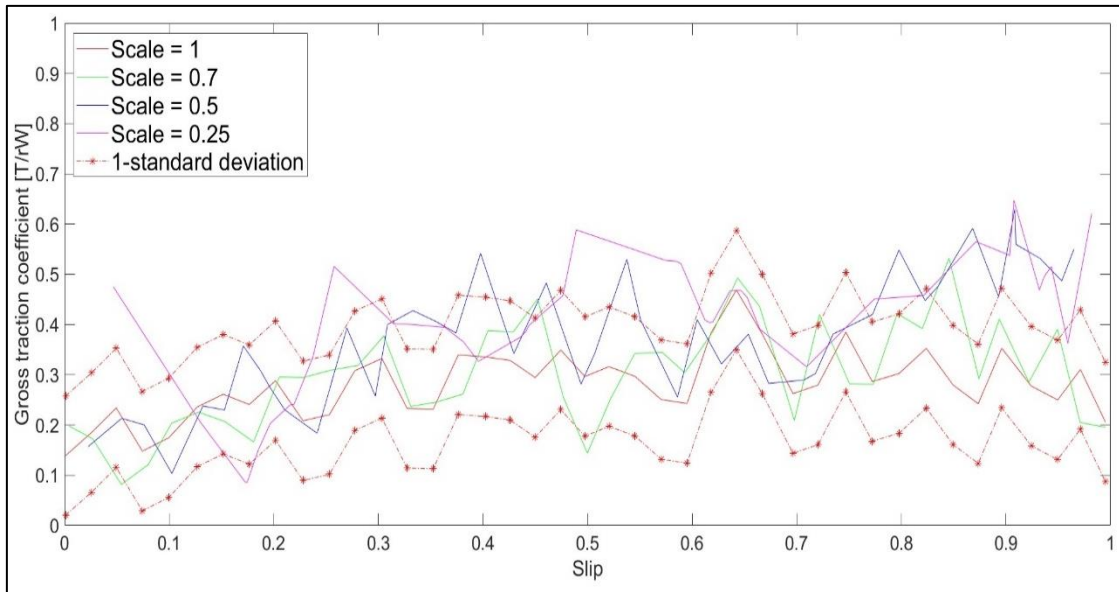


**Figure 5-25 Scalability testing of gross traction coefficient of the fine-grained sand heterogeneous terrain model**

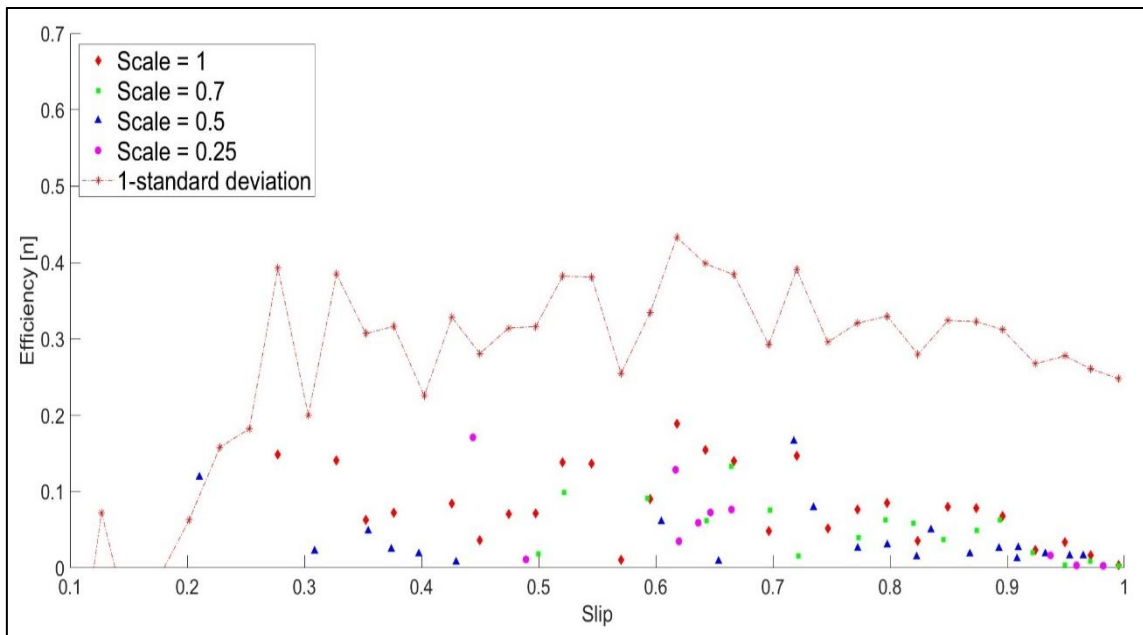


**Figure 5-26 Scalability testing of tractive efficiency of the fine-grained sand heterogeneous terrain model**

**Table 5-14 Quantitative comparison of scaled tyre systems performance parameters in heterogeneous condition**

| <b>Heterogeneous soil model</b> | <b>Scaled tyre system</b> | <b>Performance parameter</b> | <b>% data points in a one-standard-deviation band of full-scale tyre</b> |
|---------------------------------|---------------------------|------------------------------|--------------------------------------------------------------------------|
| 2NS sand (coarse-grained sand)  | Scale = 0.7               | P/W                          | 90.2                                                                     |
|                                 |                           | T/rW                         | 85.4                                                                     |
|                                 |                           | n                            | 90.2                                                                     |
|                                 | Scale = 0.5               | P/W                          | 80.5                                                                     |
|                                 |                           | T/rW                         | 80.5                                                                     |
|                                 |                           | n                            | 90.2                                                                     |
|                                 | Scale = 0.25              | P/W                          | 61.8                                                                     |
|                                 |                           | T/rW                         | 58.8                                                                     |
|                                 |                           | n                            | 85.3                                                                     |
| Fine-grained sand (FGS)         | Scale = 0.7               | P/W                          | 95.1                                                                     |
|                                 |                           | T/rW                         | 87.8                                                                     |
|                                 |                           | n                            | 46.3                                                                     |
|                                 | Scale = 0.5               | P/W                          | 74.4                                                                     |
|                                 |                           | T/rW                         | 76.7                                                                     |
|                                 |                           | n                            | 44.2                                                                     |
|                                 | Scale = 0.25              | P/W                          | 39.5                                                                     |
|                                 |                           | T/rW                         | 67.4                                                                     |
|                                 |                           | n                            | 23.2                                                                     |

### 5.7.5 Effect of cone-index on scalability

The primary purpose of these tests was to study the effect of the soil cone index on tyre-soil interaction's scalability. Two different heterogeneous soil models of 2NS sand (coarse-grained sand) and fine-grained sand were developed at uniform moisture content but with different soil cone indexes, as shown in Table 5.15. The moisture content for 2NS sand is 1.13%, and FGS is 18.55%.

**Table 5-15 Soil compaction datasheet for the developed models**

| <b>Terrain model type</b> | <b>Pre-compaction</b> | <b>Average cone index (kPa)</b> | <b>Gradient, G (kPa/m)</b> |
|---------------------------|-----------------------|---------------------------------|----------------------------|
| 2NS – Heterogeneous soil  | 0.150 m               | 114.22 $\pm$ 12.68              | 500.82 $\pm$ 274.72        |
| 2NS – Heterogeneous soil  | 0                     | 109.88 $\pm$ 16.02              | 796.61 $\pm$ 225.39        |
| FGS – Heterogeneous soil  | 0                     | 14.79 $\pm$ 4.68                | 111.09 $\pm$ 87.78         |
| FGS – Heterogeneous soil  | 0.150 m               | 22.44 $\pm$ 6.40                | 192.44 $\pm$ 154.22        |

Three independent parameters were incorporated to evaluate the powered-driven wheel's performance, as discussed in earlier sections. A comparison between scaled tyres (scale = 0.7, 0.5 and 0.25) was made graphically with the full-scale tyre (scale = 1) results as shown in Figures 5-27 to 5-32. Further, full-scale tyre results were plotted with a one-standard-deviation scatter prediction band indicated by red dash and star lines, similar to Swanson (1973) and NATO CDT – 308 (2020). The percentage of scaled tyre systems (scale = 0.7, 0.5 and 0.25) data points fell in the scatter band of the full-scale tyre (scale = 1) is shown in Table 5-16. Later a comparison was made between Table 5-14 and 5.16 to study the effect of cone index on the scalability of tyre-soil interaction.

For 2NS sand, the drawbar pull coefficients  $P/W$  for scale = 0.7 and scale = 0.5 with 81.8% and 85.7% respectively fell into the scatter band of one-standard deviation of the full-scale system followed by scale = 0.25 with 70.9%. The prediction accuracy of scale = 0.7 and scale = 0.5 was found close enough. However, no trend was obtained for the gross-traction ratio. The gross traction coefficients for scale = 0.5 was found maximum with 66.7% data points lied into the prediction scatter band followed by scale = 0.25 with 35.5% and scale = 0.7 with 27.3%. This disruption in trend might be because of the non-uniform settling of particles under gravity in the simulation model with no pre- compaction, which generates inconsistent soil strength. However, the tractive efficiency  $[\eta]$  follows the similar trend for scale = 0.7 and scale = 0.5 with 81.8% and 85.7% data points respectively fell into the prediction scatter band followed by scale = 0.25 with 74.2%.

For fine-grained sand, the drawbar pull coefficient  $P/W$  for scale = 0.7 was found maximum with 85.7% to fell into the scatter band of the full-scale system followed by 82.6% of scale = 0.5 and 70.8% of scale = 0.25. However, no trend was obtained for the gross-traction ratio. The gross traction coefficients for scale = 0.5 was found maximum with 30.4% data points lied into the prediction scatter band followed by scale = 0.25 with 29.2% and scale = 0.7 with 14.3%. It might be because of lower constant pull-off forces between the soil particles, which re-arranges randomly when a pre-compression of 0.150 m is released. Therefore, it generates inconsistent soil strength. The tractive efficiency  $[\eta]$  also followed similar trend with maximum percentage for scale = 0.7 with 61.9% data points lied in its scatter band of full-scale system followed by 60.8% of scale = 0.5 and 29.2% of scale = 0.25.

It is evident that for 2NS sand, the drawbar pull coefficient and tractive efficiency of the scaled system (scale = 0.7, 0.5 and 0.25) can predict the respective parameters of the full-scale system. However, no scaled system can predict the gross traction coefficient of the full-scale system at a lower cone index value. It might be because of inconsistent soil strength. For FGS, the drawbar pull coefficient and tractive efficiency of the scaled system (scale = 0.7 and 0.5) can



be used to predict the performance parameters of the full-scale system. However, scale = 0.25 can only predict the drawbar pull coefficient of the full-scale system.

On analysing Figures 5.33 and 5.34, it is interesting to note that at higher cone index values for both soils, the scaled system's drawbar pull coefficient and tractive efficiency predicted the full-scale system's more efficiently in the one-standard deviation band (i.e., more datapoints lies in this band). Therefore, the prediction accuracy increases with the increase in the cone index. It is because of the increase in tractive performance at a higher soil cone index, decreasing the variability in the data points of performance parameters. Therefore, the prediction accuracy is better for 2NS sand and FGS at a higher cone index, and accuracy decreases with the decrease in cone index. The data also indicated that tractive efficiency increases with the decrease in wheel slip with increased soil cone index from soft to hard soil conditions. Upadhyaya et al. (1989), Elwaleed et al. (2006), Lyasko (2010) and Taghavifar et al. (2014) obtained a similar trend.

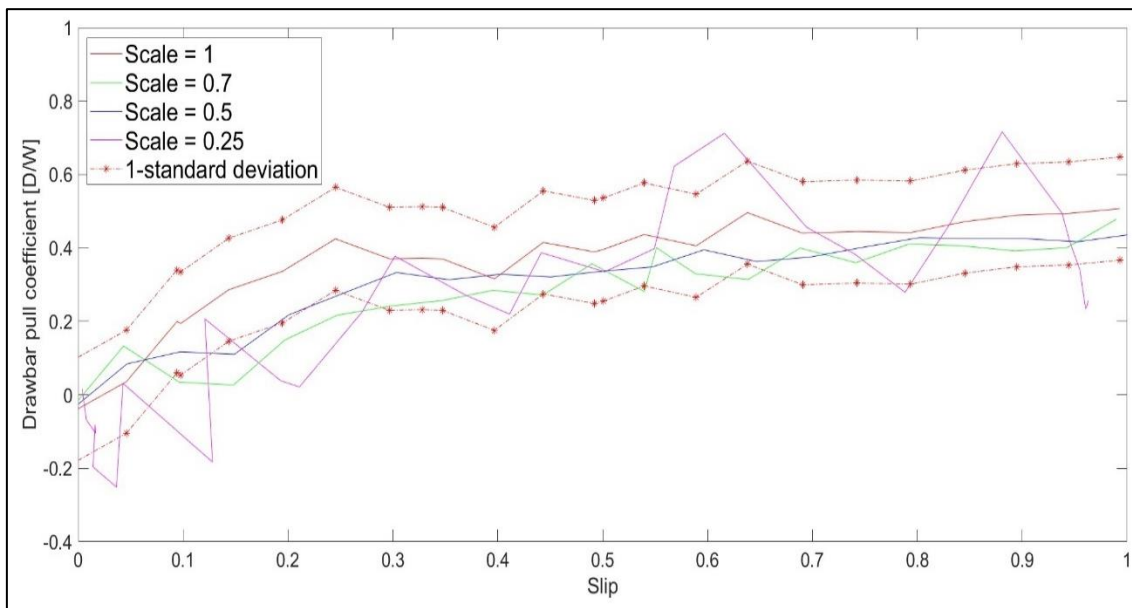


**Figure 5-27 Scalability testing of drawbar pull coefficient of 2NS heterogeneous terrain model at lower cone index**

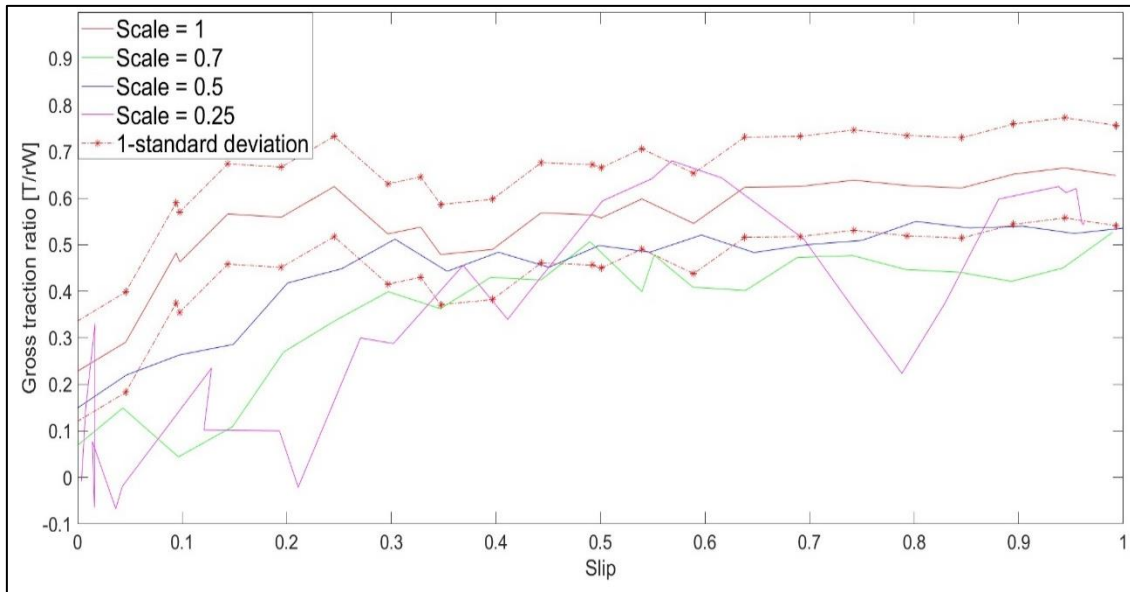


**Figure 5-28 Scalability testing of gross traction coefficient of 2NS heterogeneous terrain model at lower cone index**

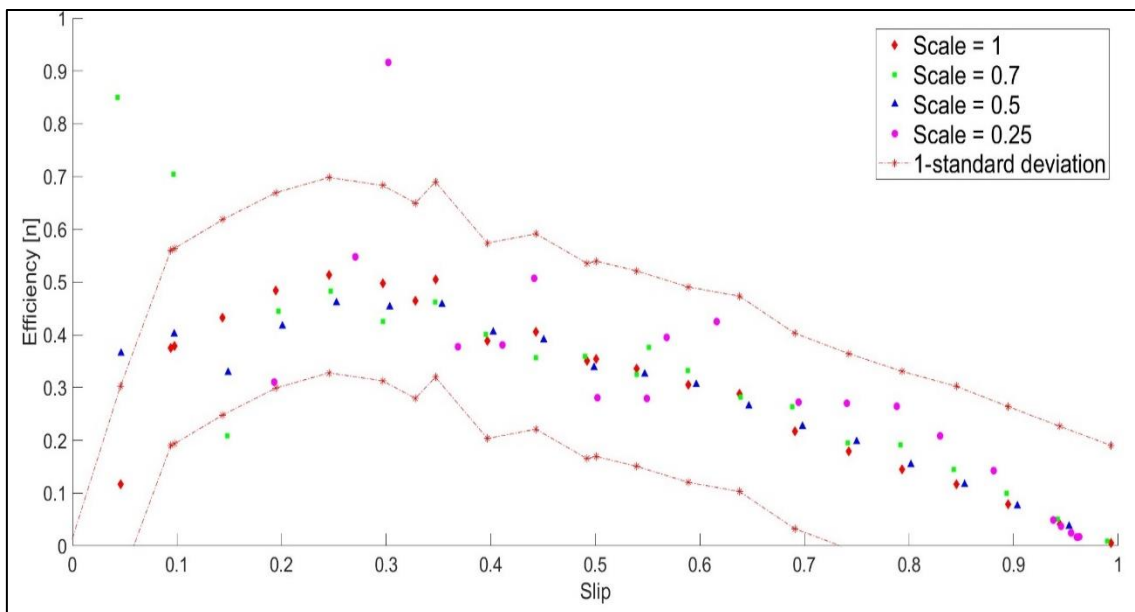


**Figure 5-29 Scalability testing of tractive efficiency of 2NS heterogeneous terrain model at lower cone index**

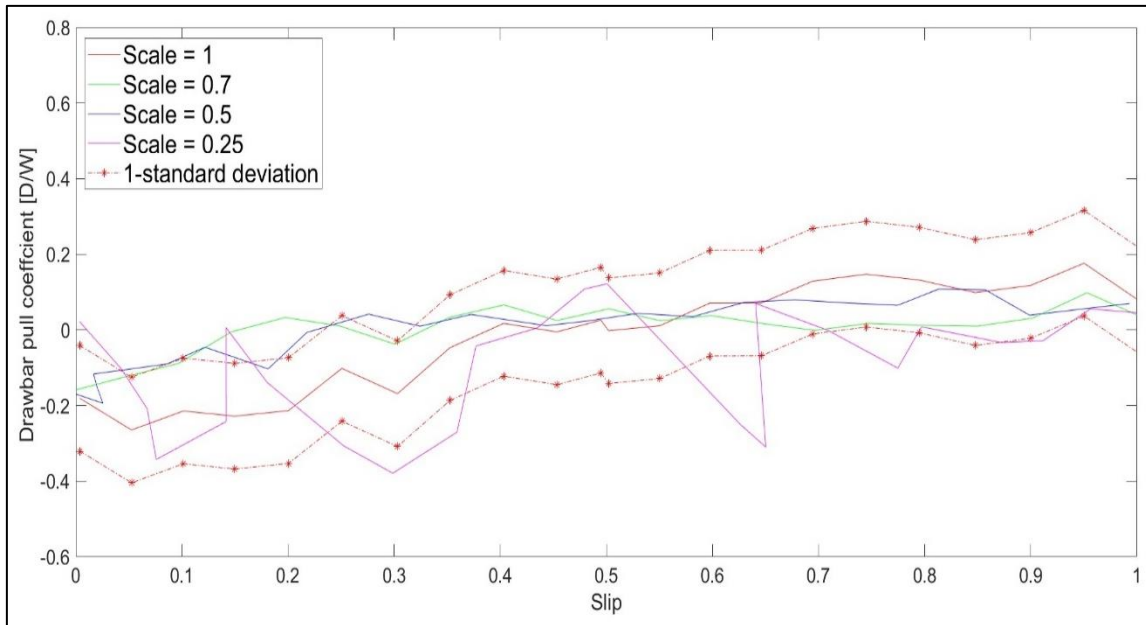


**Figure 5-30 Scalability testing of drawbar pull coefficient of fine-grained sand heterogeneous terrain model at higher cone index**

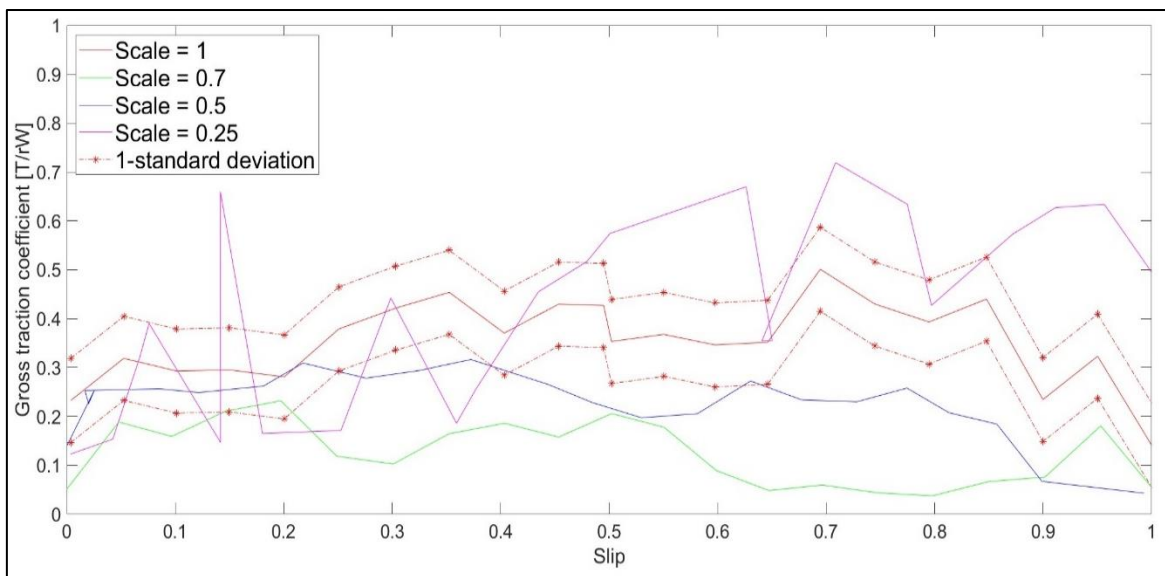


**Figure 5-31 Scalability testing of gross traction coefficient of fine-grained sand heterogeneous terrain model at higher cone index**

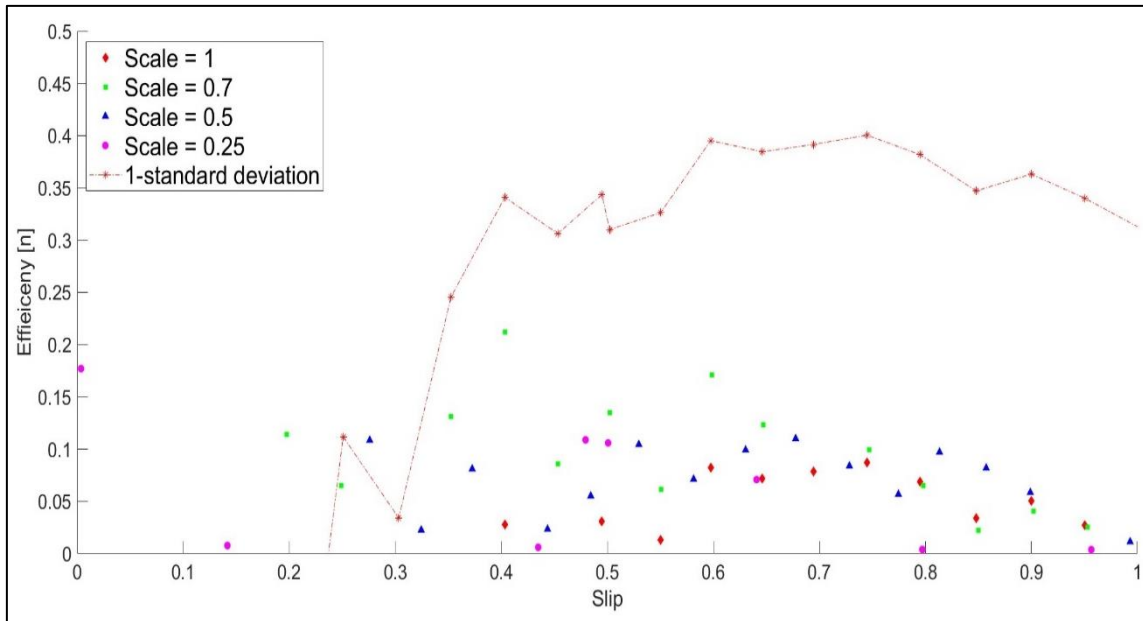


**Figure 5-32 Scalability testing of tractive efficiency of fine-grained sand heterogeneous terrain model at higher cone index**

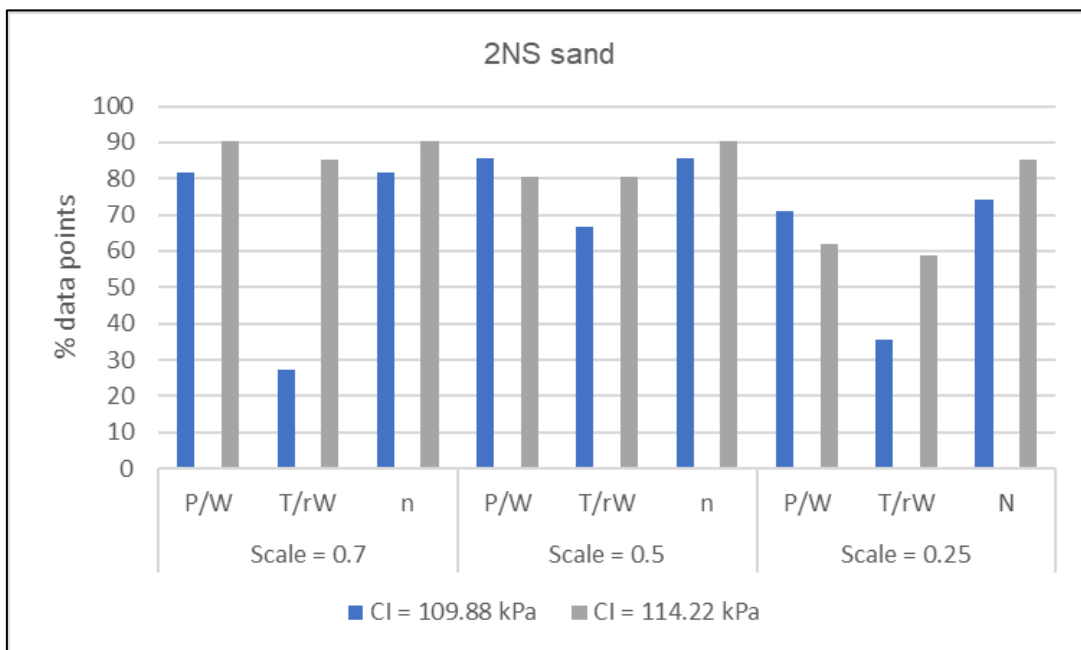


**Figure 5-33 % performance parameters data points of scale tyres in the one-standard-deviation band of full-scale tyre on 2NS sand at two different cone index (CI) values**

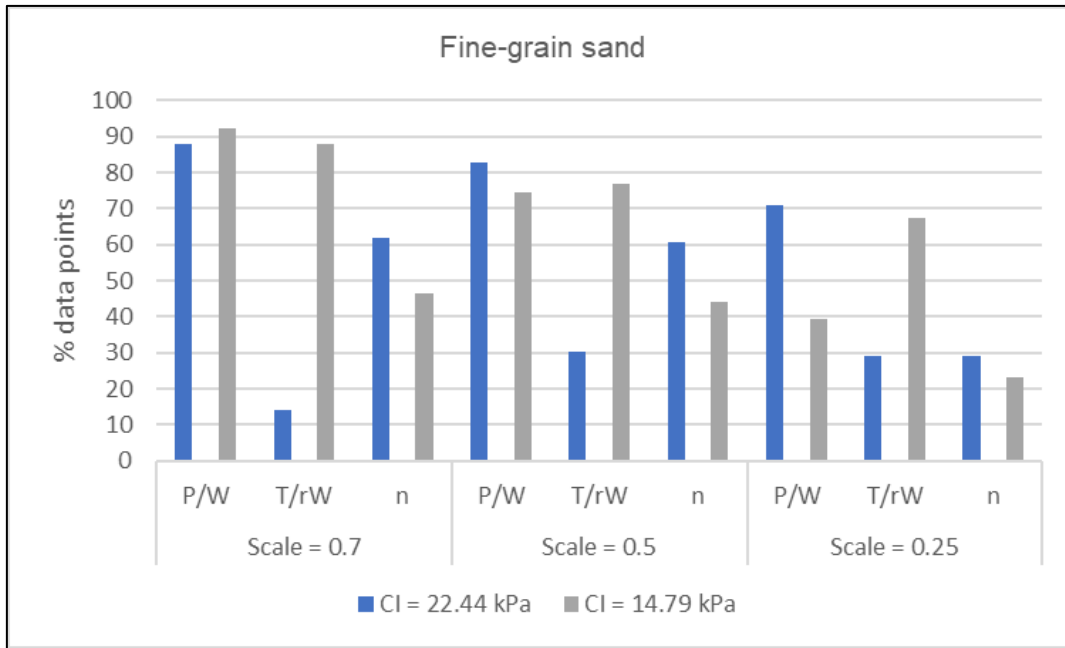


**Figure 5-34 % performance parameters data points of scale tyres in the one-standard-deviation band of full-scale tyre on fine grain sand at two different cone index (CI) values**

**Table 5-16 Quantitative comparison of scaled tyre systems performance parameters with variable cone index**

| <b>Heterogeneous soil model</b>   | <b>Scaled tyre system</b> | <b>Performance parameter</b> | <b>% data points in the one-standard-deviation band of full-scale tyre</b> |
|-----------------------------------|---------------------------|------------------------------|----------------------------------------------------------------------------|
| 2NS sand<br>(coarse-grained sand) | Scale = 0.7               | P/W                          | 81.8                                                                       |
|                                   |                           | T/rW                         | 27.3                                                                       |
|                                   |                           | n                            | 81.8                                                                       |
|                                   | Scale = 0.5               | P/W                          | 85.7                                                                       |
|                                   |                           | T/rW                         | 66.7                                                                       |
|                                   |                           | n                            | 85.7                                                                       |
|                                   | Scale = 0.25              | P/W                          | 70.9                                                                       |
|                                   |                           | T/rW                         | 35.5                                                                       |
|                                   |                           | n                            | 74.2                                                                       |
| Fine-grained sand (FGS)           | Scale = 0.7               | P/W                          | 85.7                                                                       |
|                                   |                           | T/rW                         | 14.3                                                                       |
|                                   |                           | n                            | 61.9                                                                       |
|                                   | Scale = 0.5               | P/W                          | 82.6                                                                       |
|                                   |                           | T/rW                         | 30.4                                                                       |
|                                   |                           | n                            | 60.8                                                                       |
|                                   | Scale = 0.25              | P/W                          | 70.8                                                                       |
|                                   |                           | T/rW                         | 29.2                                                                       |
|                                   |                           | n                            | 29.2                                                                       |

### 5.7.6 Effect of normal load on scalability

The primary purpose of these tests was to study the effect of the normal load on tyre-soil interaction's scalability. Two different heterogeneous soil models of the 2NS sand (coarse-grained sand) and fine-grained sand were developed at uniform moisture content and bulk density, as discussed in section 5.4. In this section, the normal load on each of the four similar systems (i.e. three scaled systems and one full-scale system) was reduced to half, as shown in Table 5-17. Full normal load test results were discussed in section 5.7.4.

**Table 5-17 Comparison of test matrix of the full normal load and half normal load conditions of the full-scale and scaled tyre systems**

| Tyre         | Full-normal load (W), N | $\left(\frac{bd}{W}\right)_w$ | Half-normal load (W/2), N | $\left(\frac{bd}{W}\right)_{w/2}$ |
|--------------|-------------------------|-------------------------------|---------------------------|-----------------------------------|
| Scale = 1    | 13438                   | 25.20                         | 6719                      | 50.40                             |
| Scale = 0.7  | 6584.62                 | 25.20                         | 3292.31                   | 50.40                             |
| Scale = 0.5  | 3359.20                 | 25.20                         | 1679.75                   | 50.40                             |
| Scale = 0.25 | 839.87                  | 25.20                         | 419.94                    | 50.40                             |

Three independent parameters were incorporated to evaluate the powered-driven wheel's performance, as discussed in section 5.7.3. A comparison between scaled tyres (scale = 0.7, 0.5 and 0.25) was made graphically with the full-scale tyre (scale = 1) results as shown in Figures 5-35 to 5-40. Further, full-scale tyre results were plotted with a one-standard-deviation prediction band indicated by red dash and star lines, similar to Swanson (1973) and NATO CDT – 308 (2020). The percentage of scaled tyre systems (scale = 0.7, 0.5 and 0.25) data points fell in the scatter band of the full-scale tyre (scale = 1) is shown in Table 5-18.

It was found that the accuracy of scalability prediction of the full-scale system relatively decreases as we decrease the size of the system. The prediction accuracy also decreases with the scaled system's size from 0.7 to 0.5 to 0.25. For 2NS sand, the drawbar pull coefficient P/W for scale = 0.7 was found

maximum with 87.8% fell into the scatter band of the full-scale system followed by 83.3% of scale = 0.5 and 51.4% of scale = 0.25. Similar trend was followed for gross-traction coefficient with maximum of 85.4% of scale = 0.7, followed by 88.1% of scale = 0.5 and 51.4% of scale = 0.25. The tractive efficiency [ $\eta$ ] also followed similar trend with maximum percentage for scale = 0.7 with 95.1% data points lied in its scatter band of full-scale system followed by 90.5% of scale = 0.5 and 86.5% of scale = 0.25. These data points suggest that the scaled systems (scale = 0.7, 0.5 and 0.25) can be used to predict the performance of the full-scale system on 2NS (coarse-grained sand).

For fine-grained sand, the drawbar pull coefficient  $P/W$  for scale = 0.7 was found maximum with 87.8% to fell into the scatter band of the full-scale system followed by 74.4% of scale = 0.5 and 74.0% of scale = 0.25. Similar trend was followed for gross-traction coefficient with maximum of 65.9% of scale = 0.7, followed by 60.5% of scale = 0.5 and 60.0% of scale = 0.25. The tractive efficiency [ $\eta$ ] also followed similar trend with maximum percentage for scale = 0.7 with 68.3% data points lied in its scatter band of full-scale system followed by 58.1% of scale = 0.5 and 46.0% of scale = 0.25. These data points suggest that the scaled systems (scale = 0.7, 0.5 and 0.25) can be used to predict the performance of the full-scale system on fine-grained soil.

On comparing Tables 5-14 and 5-18 for 2NS sand, there was no significant effect of the normal load found on the scalability analysis of tyre-soil interaction. It might be because the mobility performance is evaluated in terms of the drawbar pull coefficient, which is expressed in terms of normal load on the tyre. Similarly, for fine-grained sand, no significant effect of the normal load was found for scale = 0.7 and scale = 0.5 on the scalability of tyre-soil interaction. It might be because of the same contact pressure. However, on halving the normal load, scale = 0.25 also showed better scalable characteristics than the full normal load condition. It might be because better tyre stability is achieved at a lower normal load as contact pressure is reduced, which increases the tyre's tractive efficiency at lower slip conditions.



The data also indicated that the tyre's tractive efficiency increases while the decrease in wheel slip with the decrease in normal load for all four tyres. The decrease in wheel slip with the decrease in normal load for a constant drawbar pull coefficient may be due to increased pull in proportion to the dynamic weight. Tiwari et al. (2010), Zoz et al. (2003) and Charles (1994) obtained similar results.

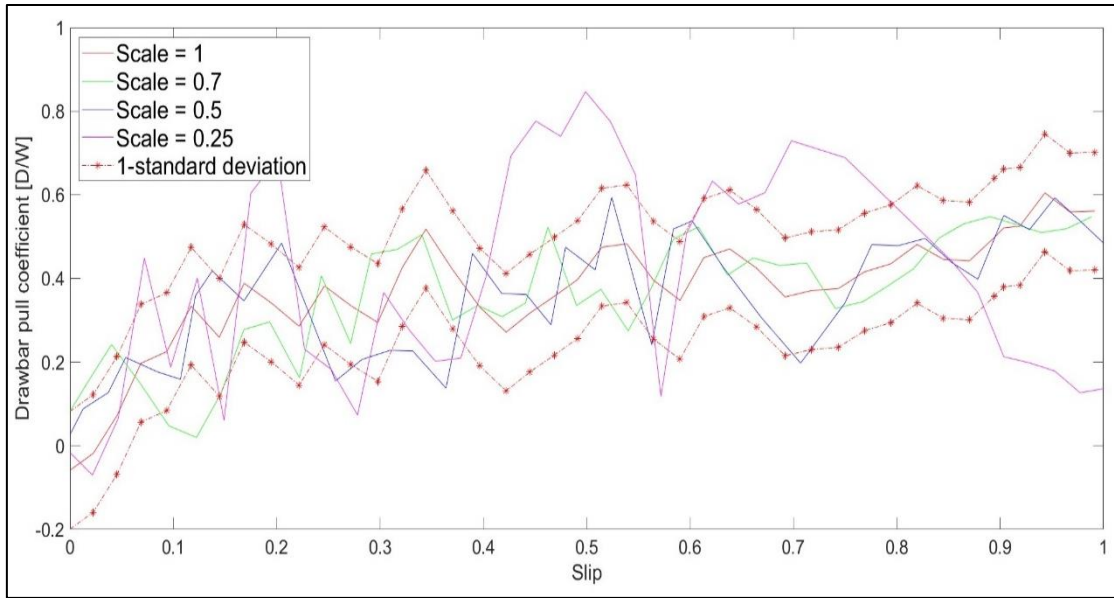


**Figure 5-35 Scalability testing of drawbar pull coefficient of 2NS heterogeneous terrain model at half normal load condition**

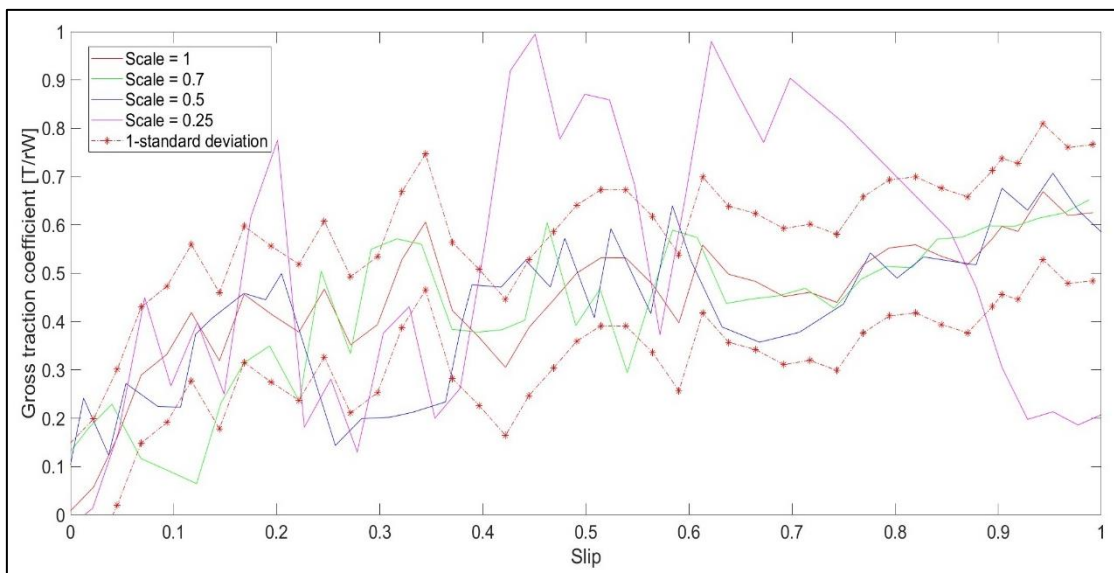


**Figure 5-36 Scalability testing of gross traction coefficient of 2NS heterogeneous terrain model at half normal load condition**

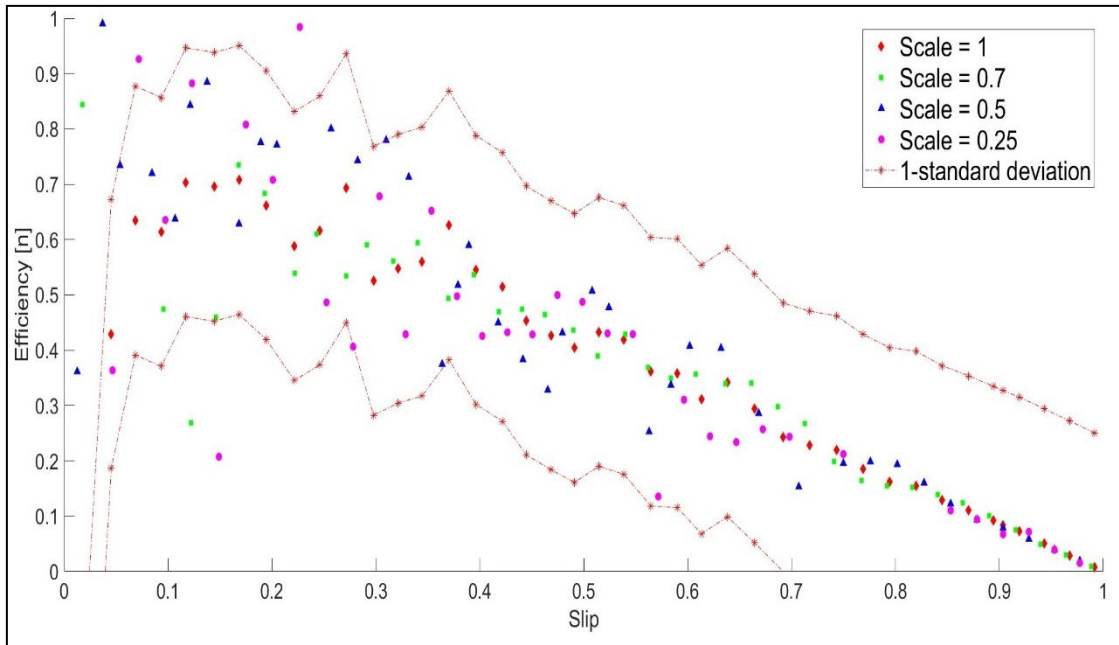


**Figure 5-37 Scalability testing of tractive efficiency of 2NS heterogeneous terrain model at half normal load condition**

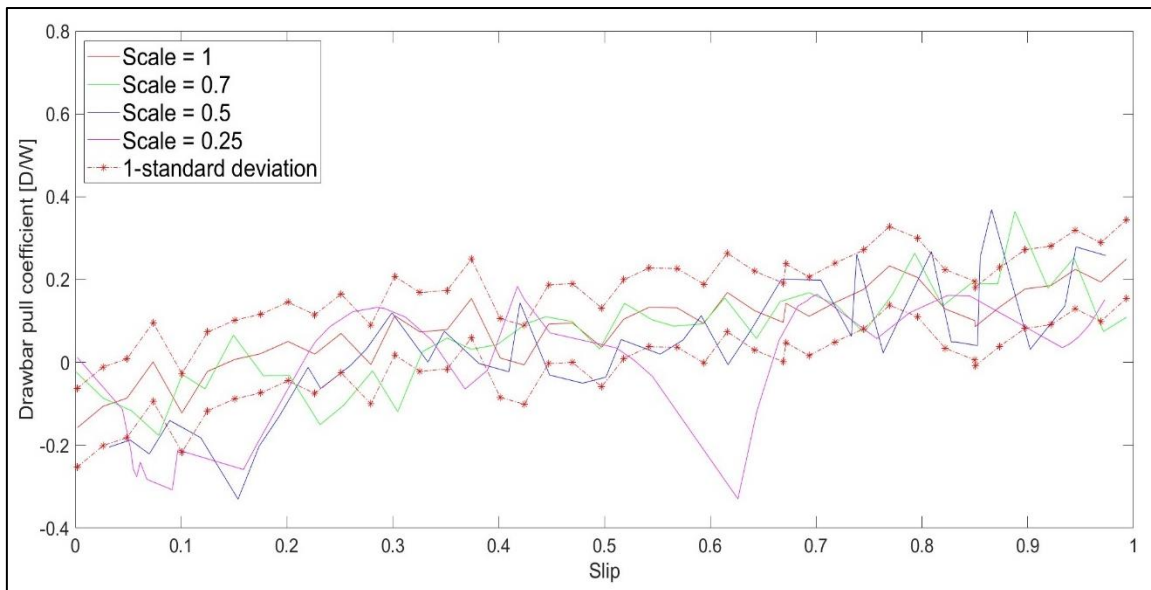


**Figure 5-38 Scalability testing of drawbar pull coefficient of the fine-grained sand heterogeneous terrain model at half normal load condition**

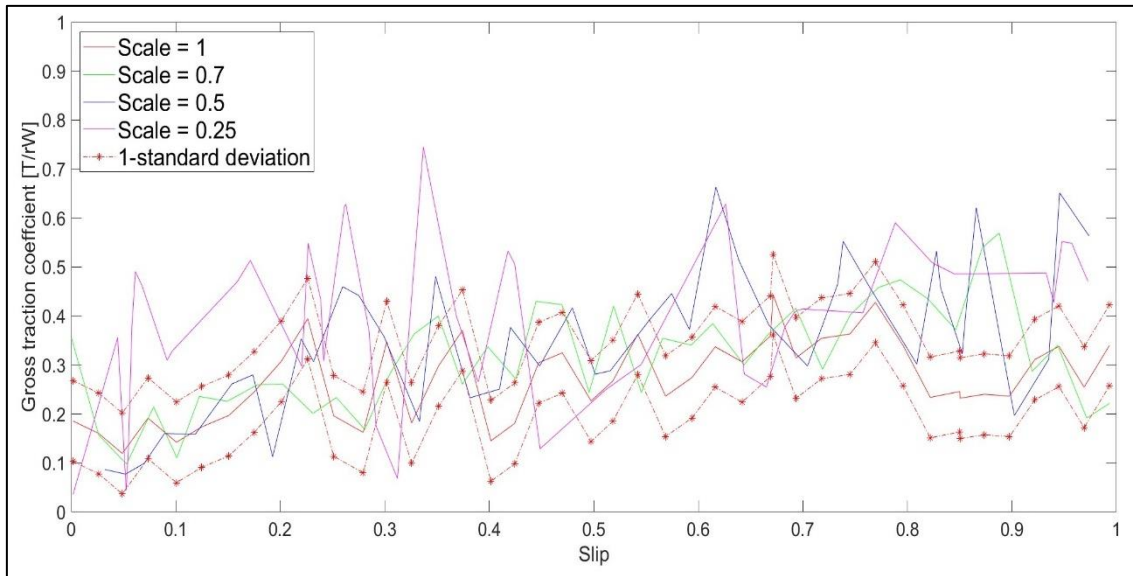


**Figure 5-39 Scalability testing of gross traction coefficient of the fine-grained sand heterogeneous terrain model at half normal load condition**

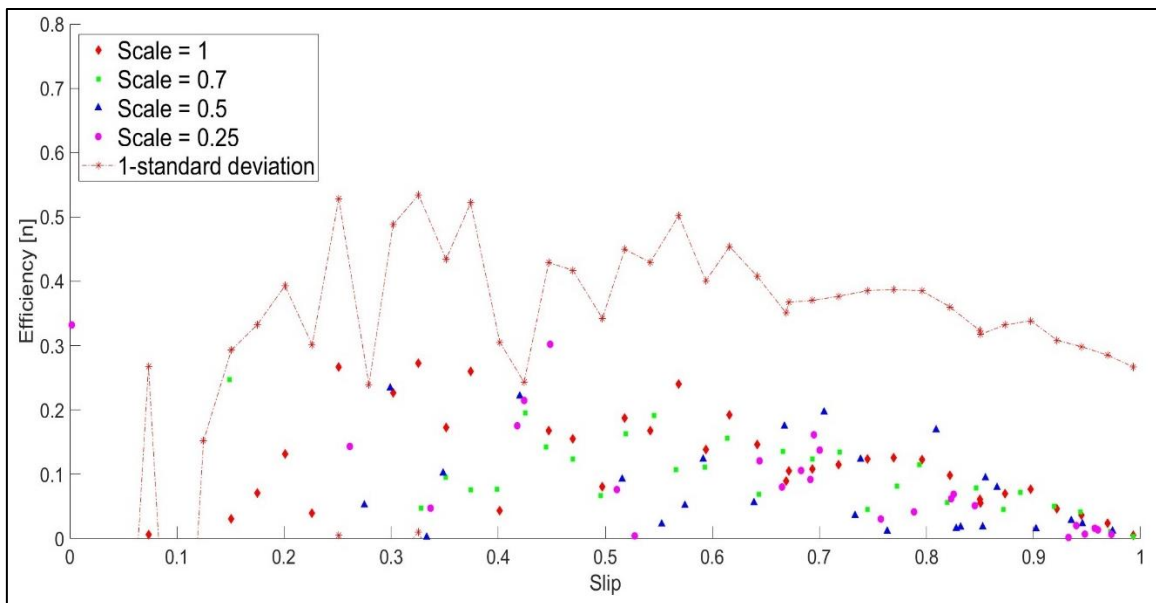


**Figure 5-40 Scalability testing of tractive efficiency of the fine-grained sand heterogeneous terrain model at half normal load condition**

**Table 5-18 Effect of the normal load on the scalability of tyre-soil interaction**

| <b>Heterogeneous soil model</b> | <b>Scaled tyre system</b> | <b>Performance parameter</b> | <b>% data points in the one-standard-deviation band of full-scale tyre</b> |
|---------------------------------|---------------------------|------------------------------|----------------------------------------------------------------------------|
| 2NS sand (coarse-grained sand)  | Scale = 0.7               | P/W                          | 87.8                                                                       |
|                                 |                           | T/rW                         | 85.4                                                                       |
|                                 |                           | n                            | 95.1                                                                       |
|                                 | Scale = 0.5               | P/W                          | 83.3                                                                       |
|                                 |                           | T/rW                         | 88.1                                                                       |
|                                 |                           | n                            | 90.5                                                                       |
|                                 | Scale = 0.25              | P/W                          | 51.4                                                                       |
|                                 |                           | T/rW                         | 51.4                                                                       |
|                                 |                           | n                            | 86.5                                                                       |
| Fine-grained sand (FGS)         | Scale = 0.7               | P/W                          | 87.8                                                                       |
|                                 |                           | T/rW                         | 65.9                                                                       |
|                                 |                           | n                            | 68.3                                                                       |
|                                 | Scale = 0.5               | P/W                          | 74.4                                                                       |
|                                 |                           | T/rW                         | 60.5                                                                       |
|                                 |                           | n                            | 58.1                                                                       |
|                                 | Scale = 0.25              | P/W                          | 74.0                                                                       |
|                                 |                           | T/rW                         | 60.0                                                                       |
|                                 |                           | n                            | 46.0                                                                       |

### 5.7.7 Effect of scaling on scalability

The primary purpose of these tests was to study the effect of the scale (or size) of the system on mobility performance parameters such as drawbar pull and tractive efficiency. Two different heterogeneous soil models of 2NS sand (coarse-grained sand) and fine-grained sand were developed at uniform moisture content and bulk density, as discussed in section 5.4. Four different scaled tyre models of scale = 1, 0.7, 0.5 and 0.25 were used to study the effect of scaling similar systems to full-scale systems. All four scaled tyres were having a constant tyre width/ diameter (b/d) ratio of 0.33. The scale analysis was conducted at six different wheel slip conditions 10%, 20%, 30%, 50 %, 70% and 90% as per the guidance of ASABE standard (2000). It states that the maximum tractive efficiency is obtained at an optimum slip range of 11-16% for soft soils and sands. Therefore, based on this recommendation, scaling was analysed on the above-mentioned slip range to ensure the optimum slip and high slip conditions.

Three independent parameters were incorporated to evaluate the powered-driven wheel's performance, i.e. drawbar pull  $D$ , gross traction ( $T/r$ ) and tractive efficiency ( $\eta$ ). A comparison between performance parameters with respect to the system's scale (scale =1, 0.7, 0.5 and 0.25) was made, as shown in Figures 5-35 to 5-40. Non-linear regression was used to fit the performance parameters vs scale data to a power law of the form (Lankenau et al., 2012):

$$F = c * s^{\alpha} \quad (5-14)$$

where  $F$  is the performance parameter,  $s$  is the scale and  $c$  and  $\alpha$  are the regressed coefficient and exponent, respectively. The parameter values are shown in Tables 5-18 and 5-19 for 2NS sand and fine-grained sand.

It was found that the average regression exponent of drawbar pull for 2NS sand and fine-grained sand is 2.28 and 2.66, respectively, and the average regression coefficient is 4221.67 and 1895.8, respectively. It means that drawbar pull increases between square to cubic power with respect to the system's scale for 2NS sand and fine-grained sand. Negative drawbar pull slip conditions were neglected in the fine-grained sand. Similarly, the average regression exponent of

gross traction (gross tractive force) for 2NS sand and fine-grained sand is 2.27 and 1.86, respectively, and the average regression coefficient is 5576.16 and 3814.66, respectively. Similarly, the average regression exponent of tractive efficiency ( $n$ ) for 2NS sand and fine-grained sand is -0.96 and 1.96, respectively, and the average regression coefficient is 0.36 and 0.09, respectively. The tractive efficiency is approximately constant with respect to the system's size for 2NS sand. However, it varies approximately as a square with respect to the system's size for fine-grained sand. The regression coefficients are much smaller than one in both cases; therefore, a minor variation is observed.

Even though no previous work on the scaling of performance parameters of tyre-soil interaction system was found, the growth between square to cubic power was expected for drawbar pull, based on scaling of soil penetration pressure and tyre-soil contact area. Penetration pressure increases with sinkage exponent ( $n$ ) of 0.63 and 1.26 for 2NS sand and fine-grained sand, respectively (as discussed in section 4.5). The drawbar pull is the difference between gross traction (or gross tractive force) and motion resistance. The gross traction is approximately increasing with the square with respect to the system's size. The motion resistance is estimated using the Bekker model, which depends on the sinkage exponent ( $\frac{n+1}{n}$ ). Therefore, the estimation for motion resistance was found to increase with the power of 2.58 for 2NS sand and 1.79 for fine-grained sand with respect to depth. Therefore, it can be estimated that the drawbar pull should vary between square to cubic power with respect to the size of the system for both soils.

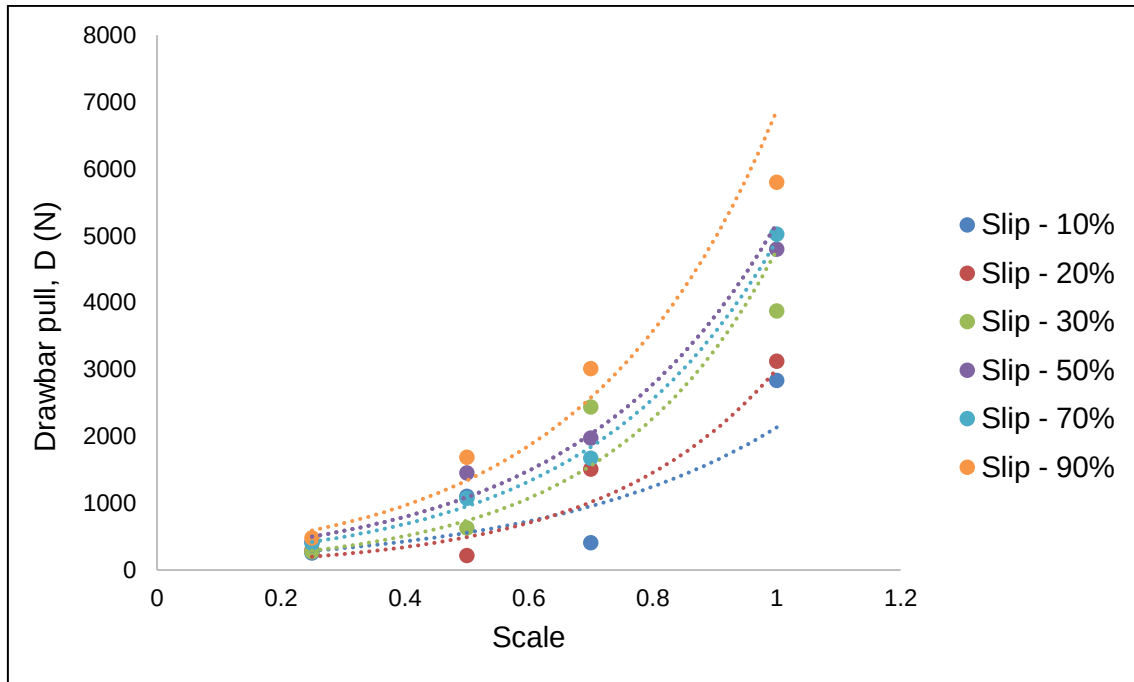


**Figure 5-41 Effect of the scale of similar systems on drawbar pull  $D$ , [N] at different slip conditions on 2NS sand**

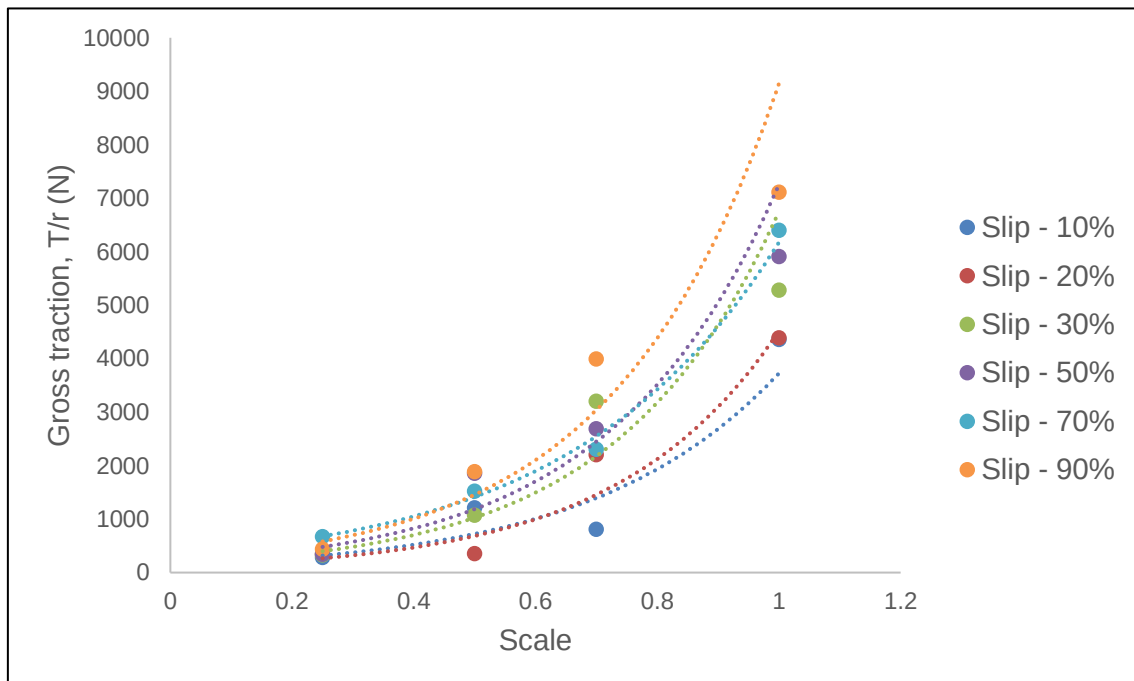


**Figure 5-42 Effect of the scale of similar systems on gross traction  $T/r$ , (N) at different slip conditions on 2NS sand**

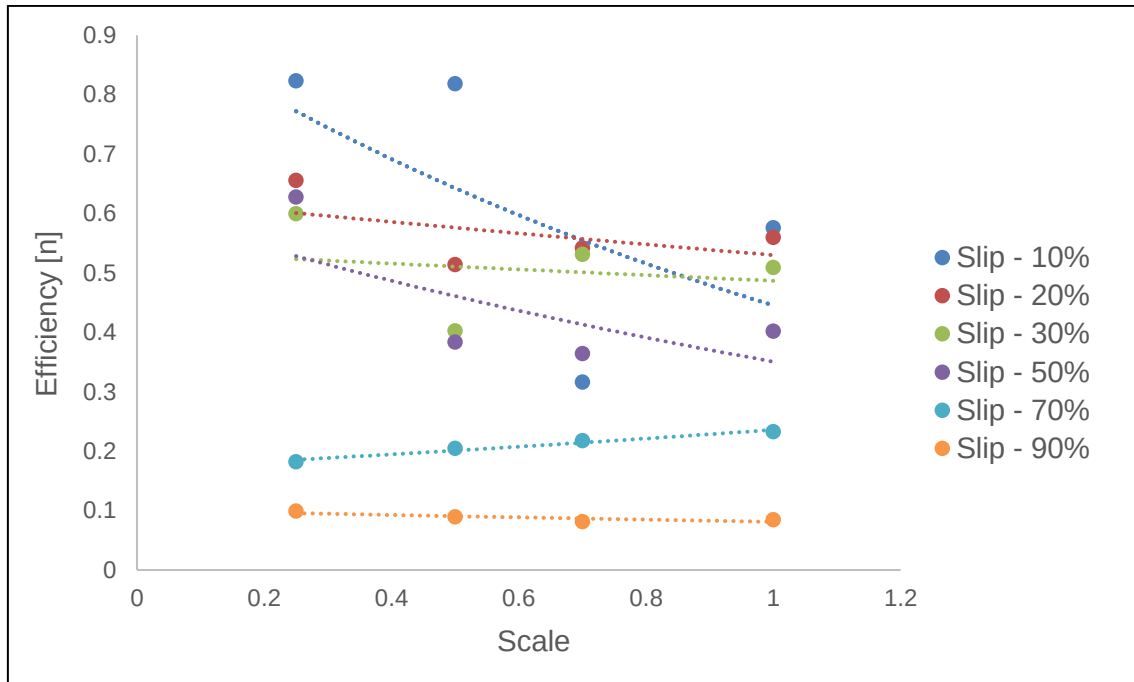


**Figure 5-43 Effect of the scale of similar systems on tractive efficiency [n] at different slip conditions on 2NS sand**

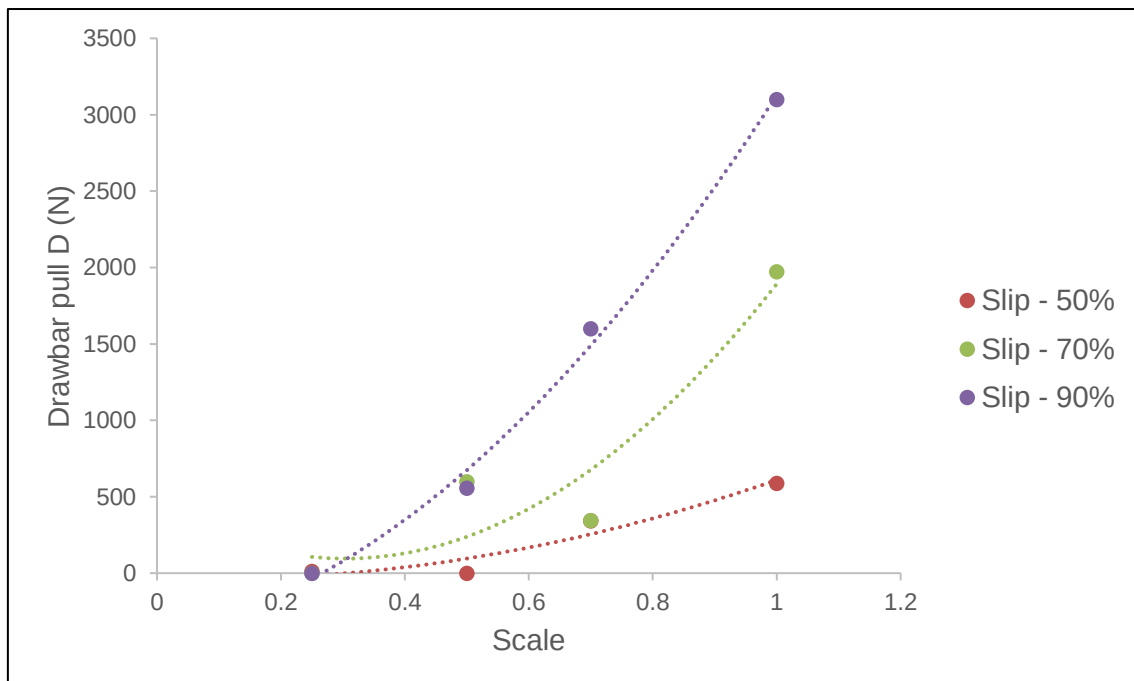


**Figure 5-44 Effect of the scale of similar systems on drawbar pull D, [N] at different slip conditions on fine-grained sand**



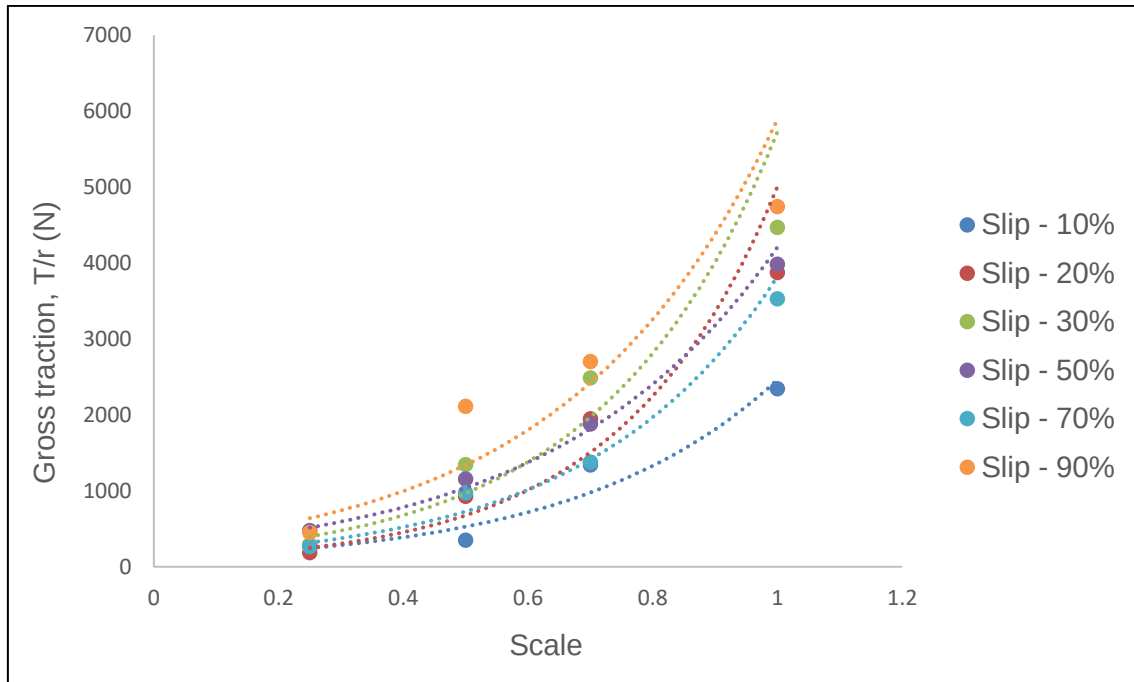


**Figure 5-45 Effect of the scale of similar systems on gross traction  $T/r$ , (N) at different slip conditions on fine-grained sand**

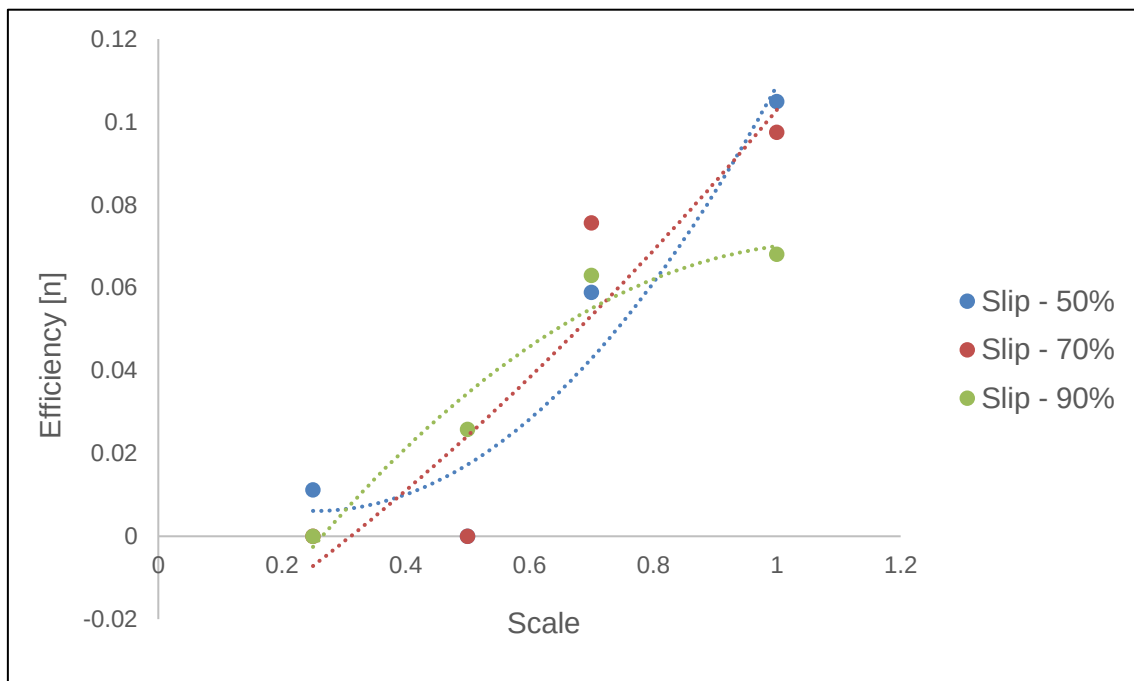


**Figure 5-46 Effect of the scale of similar systems on tractive efficiency (n) at different slip conditions on fine-grained sand**

**Table 5-19 Non-linear regression analysis between performance parameters vs scale for 2NS sand model**

| <b>2NS<br/>Heterogeneous<br/>soil model</b> | <b>Slip</b> | <b>Regression<br/>coefficient<br/>c</b> | <b>Order of–<br/>scale <math>\alpha</math></b> | <b>R<sup>2</sup> - value</b> |
|---------------------------------------------|-------------|-----------------------------------------|------------------------------------------------|------------------------------|
| Drawbar pull D<br>(N)                       | 10          | 2730                                    | 2.85                                           | 0.78                         |
|                                             | 20          | 3161                                    | 2.52                                           | 0.96                         |
|                                             | 30          | 3979                                    | 1.92                                           | 0.95                         |
|                                             | 50          | 4710                                    | 2.01                                           | 0.98                         |
|                                             | 70          | 4961                                    | 2.61                                           | 0.99                         |
|                                             | 90          | 5789                                    | 1.80                                           | 0.99                         |
| Gross traction<br>(T/r)                     | 10          | 4274                                    | 3.26                                           | 0.84                         |
|                                             | 20          | 4454                                    | 2.43                                           | 0.96                         |
|                                             | 30          | 5396                                    | 1.87                                           | 0.97                         |
|                                             | 50          | 5846                                    | 1.91                                           | 0.99                         |
|                                             | 70          | 6300                                    | 2.36                                           | 0.97                         |
|                                             | 90          | 7187                                    | 1.81                                           | 0.99                         |
| Tractive<br>efficiency [n]                  | 10          | 0.48                                    | -0.4                                           | 0.42                         |
|                                             | 20          | 0.52                                    | -0.1                                           | 0.50                         |
|                                             | 30          | 0.53                                    | -0.1                                           | 0.39                         |
|                                             | 50          | 0.33                                    | -0.4                                           | 0.77                         |
|                                             | 70          | 0.23                                    | 0.17                                           | 0.99                         |
|                                             | 90          | 0.08                                    | -0.13                                          | 0.84                         |

**Table 5-20 Non-linear regression analysis between performance parameters vs scale for fine-grained sand model**

| <b>FGS<br/>Heterogeneous<br/>soil model</b> | <b>Slip</b> | <b>Regression<br/>coefficient<br/><b>c</b></b> | <b>Order of–<br/>scale <math>\alpha</math></b> | <b>R<sup>2</sup> - value</b> |
|---------------------------------------------|-------------|------------------------------------------------|------------------------------------------------|------------------------------|
| Drawbar pull D<br>(N)                       | 10          | -                                              | -                                              | -                            |
|                                             | 20          | -                                              | -                                              | -                            |
|                                             | 30          | -                                              | -                                              | -                            |
|                                             | 50          | 607.5                                          | 2.49                                           | 0.92                         |
|                                             | 70          | 1934                                           | 3.28                                           | 0.89                         |
|                                             | 90          | 3146                                           | 2.21                                           | 0.99                         |
| Gross traction<br>(T/r)                     | 10          | 2377                                           | 2                                              | 0.96                         |
|                                             | 20          | 3889                                           | 2                                              | 0.99                         |
|                                             | 30          | 4496                                           | 1.74                                           | 0.99                         |
|                                             | 50          | 3930                                           | 1.84                                           | 0.99                         |
|                                             | 70          | 3472                                           | 2.2                                            | 0.98                         |
|                                             | 90          | 4724                                           | 1.39                                           | 0.98                         |
| Tractive<br>efficiency [n]                  | 10          | -                                              | -                                              | -                            |
|                                             | 20          | -                                              | -                                              | -                            |
|                                             | 30          | -                                              | -                                              | -                            |
|                                             | 50          | 0.1                                            | 2.46                                           | 0.90                         |
|                                             | 70          | 0.1                                            | 2.1                                            | 0.82                         |
|                                             | 90          | 0.07                                           | 1.34                                           | 0.84                         |

## 5.8 Summary

The work reported above has established the scalability of tyre-soil interaction, which is based on the assumption that similar systems that behave in a similar manner can be established (Bekker, 1956 and Freitag, 1966). Six terrain models consisting of homogeneous and heterogeneous conditions were developed for 2NS sand and fine-grained sand with different cone index values, as discussed in section 5.4. U.S. Army's FED ALPHA military vehicle tyre 335/65R22.5 was selected as the full-scale tyre (scale = 1). Three different scaled tyres of scale = 0.7, 0.5 and 0.25 were modelled to investigate the scalability. The cone index was kept the same for all the tyre tests on specific terrain. In the experimental testing conducted by NATO CDT – 308 (2020), the tyre inflation pressure is sufficiently high and the terrain is relatively soft. Therefore, a pneumatic tyre may behave like a rigid rim without affecting the accuracy of results (Wong, 2010). Further, to determine the testing conditions of similar systems,  $\frac{bd}{W}$  ratio was developed based on the dimensional analysis approach to the tyre-soil interaction system. The normal load on scaled tyres were calculated based on this parameter to develop similar scaled systems to the full-scale system. The key findings of this work can be summarised as follows:

- The present simulation technique, i.e. the discrete element method using Edinburgh elastic-plastic adhesion contact model, predicts better experimental results of drawbar pull vs slip relation of the full-scale tyre than the simulation techniques used by NATO CDT -308 (2020) for both 2NS sand (coarse-grained sand) and fine-grained sand. It also fell in the prediction band of NRMM.
- Heterogeneous terrain models predicted better in-situ experimental testing results of drawbar pull vs slip relation of the full-scale tyre compared to homogeneous terrain models for 2NS sand. The homogeneous terrain model over-predicted the drawbar pull coefficient vs slip and gross traction coefficient vs slip relation for 2NS sand in single wheel testing. However, no significant difference was found between the homogeneous and heterogeneous terrain model for both the performance parameters

relationships with respect to slip for fine-grained sand. Interestingly, there was no significant difference between the tractive efficiency of homogeneous and heterogeneous terrain models for both sands.

- In homogeneous terrain conditions, the accuracy of the scalability of tyre-soil interaction reduces with the system's size. However, there was no significant difference in the drawbar pull coefficient, gross traction coefficient, and tractive efficiency vs slip relationships of the full-scale system (scale = 1) and scaled systems (scale = 0.7, 0.5 and 0.25) for 2NS sand and fine-grained sand, except scale = 0.25 poorly predicted for fine-grained sand. Therefore, it can be concluded that the proposed theory of scalability of tyre-soil interaction works well in the homogeneous terrain conditions for both soils and tyres ranging from lightweight military vehicle (e.g. Land Rover and FED Alpha) to UGV (Warthog).
- In heterogeneous terrain conditions, the accuracy of the scalability of tyre-soil interaction reduces with the system's size. However, there was no significant difference in the drawbar pull coefficient, gross traction coefficient, and tractive efficiency vs slip relationships of the full-scale system (scale = 1) and scaled systems (scale = 0.7, 0.5 and 0.25) for 2NS sand and fine-grained sand, except scale = 0.25 poorly predicted for fine-grained sand. Therefore, it can be said that the proposed theory of scalability of tyre-soil interaction works well in heterogeneous terrain conditions for both soils and tyres ranging from lightweight military vehicles (e.g. Land Rover and FED Alpha) to UGVs (Warthog).
- The accuracy of the scalability of the tyre-soil interaction is proportional to the cone index of soil models. The higher the cone index, the better the prediction accuracy of drawbar pull coefficient and tractive efficiency vs slip of full-scale system (scale = 1) from similar scaled systems (scale = 0.7, 0.5 and 0.25) for 2NS sand and fine-grained sand except for scale = 0.25 poorly predicted the tractive efficiency for fine-grained sand.
- No significant effect of the normal load was found on the accuracy of the scalability of the tyre-soil interaction. There is no significant difference in the prediction of drawbar pull coefficient, gross traction coefficient and

tractive efficiency vs slip relationships of the full-scale system (scale = 1) and scaled systems (scale = 0.7, 0.5 and 0.25) for 2NS sand and fine-grained sand. However, the prediction accuracy of scale = 0.25 increases at lower normal loads as the tyre achieves better stability as contact pressure is reduced.

- The drawbar pull varies with the exponent of 2.28 and 2.66 with respect to the system's size for 2NS sand and fine-grained sand, respectively. Therefore, it can be concluded that drawbar pull varies between square to cubic power with respect to the system's size for coarse-grained sand and fine-grained sand.
- The gross traction varies with an exponent of 2.27 and 1.86 with respect to the system's size for 2NS sand and fine-grained sand, respectively. Therefore, it can be concluded that gross traction varies approximately with square power with respect to the size of systems for coarse-grained sand and fine-grained sand.
- The tractive efficiency is approximately constant with respect to the system's size for 2NS sand. However, it varies approximately as a square with respect to the system's size for fine-grained sand. However, the regression coefficients are smaller than one. Therefore, a minor variation is observed.



## 6 SUMMARY AND CONCLUSION

### 6.1 Summary

This chapter summarizes the work presented in this thesis in the context of the conclusions drawn and contributions made. The work presented in this thesis was carried out to achieve the aim laid out in chapter 1: To understand the scalability aspects of soil testing and tyre-soil interaction system, which can provide economic benefits by conducting experiments with scaled systems instead of the full-scale systems and technical benefits by providing solutions to specific design and performance-related problems. Moreover, during the teaming operations of UGVs and full-scale military vehicles on the battlefield, it can also provide an estimate of the mobility of UGVs in correlation to full-scale vehicle system in deciding go/no-go conditions on off-road terrains and energy cost optimisation on the feasible paths. It can also estimate the maximum payload capacity of UGVs and full-scale military vehicles on soft soils during such military operations.

The objective specified in Chapter 1 was to establish the theory of scalability of the tyre-soil interaction system, which was developed in Chapter 3. It was carried out using the dimensional analysis approach to derive non-dimensional parameters to define the tyre-soil interaction system. The resulting parameters were qualitatively validated with tyre deflection and contact area tests to establish geometrical similarities between the tyres. The dynamic similarity parameter establishes the nature of the transfer of force in similar tyre-soil interaction systems. The resulting outputs of prototype tyre systems were found in good agreement with the full-scale tyre system. Therefore,  $\frac{P_i bd}{W}$  and  $\frac{Clbd}{W}$  can be used to establish geometric similarity and dynamic similarity of the tyre-soil interaction system.

The second objective specified in Chapter 1 was to develop coarse-grained sand (2NS sand) and fine-grained sand models using the discrete element method, which is discussed in chapter 4. The Edinburgh elastic-plastic adhesion (EpAM) contact model was used to define the particle's inter-particle mechanics and tyre-soil interaction phenomenon. The advantage of EpAM over the previously used



Hertz-Mindlin model in literature is that it can define the adhesive pull-off forces among the particles, which can be used to derive the effect of soil conditions such as moisture content. Further, both sand models were validated against the experimental results conducted by NATO CDT – 308 (2020) in chapter 3. The shear properties and pressure-sinkage relations of both the soil models were in good agreement with the experimental results.

The third objective specified in chapter 1 was to establish soil scalability using 2NS (coarse-grained) sand and fine-grained sand. The scalability of soil is defined as the determination of mechanical properties of in-situ heterogeneous soil from the homogeneous laboratory soil models as defined by NATO CDT – 308 (2020), keeping the same soil conditions. It is important as most of the Terramechanics models developed in laboratory conditions are used to predict vehicle performance in in-situ conditions. The shear tests, pressure-sinkage tests and cone penetrometer tests were conducted on both the soil models. The direct shear tests were simulated in the laboratory conditions, whereas shear ring tests were simulated in the in-situ terrain conditions. The effect of heterogeneity was also analysed on the soil parameters. The second and third objectives were discussed in chapter – 4.

The scalability of the shear stress-shear displacement relationship depends on the applied normal stress on the soil. The lower the normal stress, the better the accuracy of scalability and vice versa. Therefore, it was concluded that the scalability of soil's shear relationship holds. Bekker's parameter of the pressure-sinkage relationship had different scaling relationships for both soils. The coarse-grained sand model parameters were found scalable by the ratio between the equivalent stiffness of the laboratory soil sample and the in-situ soil sample. The fine-grained sand model parameters were scalable in Bekker's pressure-sinkage model's log-log domain. Further, there was a significant difference between the average cone index of the laboratory soil model and the in-situ soil model.

The fourth objective specified in Chapter 1 was to verify the scalability of mobility performance parameters with respect to slip relations for the tyre-soil interaction system. The full-scale tyre (scale = 1) system was 335/65R22.5 used in the

present study to conduct the single wheel testing on homogeneous and heterogeneous soil systems of the coarse-grained sand (2NS sand) and fine-grained sand. Three similar scaled tyres (scale = 0.7, 0.5 and 0.25) were constructed. The testing conditions were derived from the proposed theory of scalability in chapter – 3. The cone index and normal load effect were also investigated on the scalability of the drawbar pull coefficient, gross traction coefficient, and tractive efficiency of similar systems.

Further, the relations of drawbar pull, gross traction and tractive efficiency were developed with respect to the system's scale for both the soil models. It was found that the accuracy of scalability of mobility performance parameters for the tyre-soil interaction system decreases with the size of the system in both homogeneous and heterogeneous soil systems. The drawbar pull varies from square to cubic power with respect to the system's size. The gross traction varies with approximately square power with respect to the system's size. Tractive efficiency depends on the soil type. It was found constant with respect to scale for coarse-grained sand but varied with approximately square power for fine-grained sand. However, the regression coefficients are much smaller than one. Therefore, a minor variation is observed.

## **6.2 Conclusions**

The following specific conclusions can be drawn for the scalability of the tyre-soil interaction system based on the work carried out in this thesis.

- The accuracy of the scalability of the shear relationship, i.e. shear stress vs shear displacement of the soil, decreases with the increase in the normal stress. Significant evidence has been found which holds for scalability of 2NS sand and FGS at higher normal stress too. Therefore, the scalability of the soil shear relationship holds.

The effect of heterogeneity on soil shear properties such as cohesion and internal angle of friction depends on the soil system. The effect was contrasting in 2NS sand and fine-grained sand. It depends on the particle orientation, grain size, moisture content, plasticity and other soil properties.

- In pressure-sinkage (p-z) tests, the Bekker model parameter, i.e. moduli of deformation w.r.t cohesion  $k_c$  and moduli of deformation w.r.t friction  $k_\phi$  with respect to sinkage exponent  $n$  were found scalable with the scaling ratio,  $\lambda$  in the p-z domain for the coarse-grained sand.  $\lambda$  is defined as the ratio of equivalent stiffness between the homogeneous laboratory model and the heterogeneous terrain model. There was no significant difference between the  $n_{avg}$  of both the models of 2NS sand. However, for fine-grained sand, the Bekker model was found scalable in the  $\log p - \log z$  domain, including the effect of sinkage exponent  $n$ .
- The average cone index is higher for the homogenous soil models than for the heterogeneous soil models keeping the same soil contact model parameters for 2NS sand and fine-grained sand models in simulations (Table 5.4). The cone index vs depth relation has a significant difference between the homogeneous laboratory conditions and heterogeneous in-situ conditions for 2NS sand. However, statistically, no significant difference was found between the homogeneous laboratory conditions and heterogeneous terrain conditions for FGS.
- As per the developed theory of scalability, the geometric similarity controls the tyre's distorted shape and curvature in a similar system. The shape of the tyre-soil contact area controls the pressure exerted by the tyre on the soil. The dynamic similarity controls the forces in the tyre-soil interaction in similar systems. Similar scalable systems can be established on a specific soil model by equating the pressure term (as shown below) when tyre inflation pressure is more than the soil cone index.

$$\left(\frac{bd}{W}\right)_{Full-scale} = \left(\frac{bd}{W}\right)_{Scaled}$$

- The drawbar pull varies from square to cubic power with respect to the size of systems for coarse-grained sand and fine-grained sand. Gross traction varies approximately with square power with respect to the system's size. The tractive efficiency is approximately constant with respect to the system's size for 2NS sand and fine-grained sand.

- In homogeneous and heterogeneous sand models, the accuracy of scalability of mobility performance parameters of the tyre-soil interaction system decreases with the size of the system for 2NS sand and fine-grained sand. There was no significant difference in the drawbar pull coefficient, gross traction coefficient and tractive efficiency vs slip relationships of the full-scale system (scale = 1) and scaled systems (scale = 0.7, 0.5 and 0.25) for 2NS sand and fine-grained sand, except scale = 0.25 poorly predicted and had the significant difference for fine-grained sand. Therefore, the proposed theory of scalability of tyre-soil interaction works well for both soils and tyres ranging from lightweight military vehicles (e.g., Land Rover and FED Alpha) to UGVs (Warthog).
- There was no significant effect of the normal load on the tyre was found on the accuracy of the scalability of the tyre-soil interaction system due to the same contact pressure. However, the prediction accuracy of scale = 0.25 increases at lower normal loads as the tyre achieves better stability as contact pressure is reduced.
- The present simulation technique, i.e. the discrete element method using Edinburgh elastic-plastic adhesion contact model, predicts better experimental results of drawbar pull vs slip relation of the full-scale tyre than the simulation techniques used in NATO CDT -308 (2020) for both 2NS sand (coarse-grained sand) and fine-grained sand. It also fell in the prediction band of NRMM.

### 6.3 Research contributions

The research carried out in this thesis is novel because it helps to understand the scalability aspects of soil testing and the tyre-soil interaction system. Over the last few decades, the resource-intensive experimental testing on lightweight military vehicles on different terrains has led to the development of various mobility prediction methodologies including the NATO Reference Mobility Model (NRMM) (Jayakumar et al., 2013). However, with the recent developments of small, lightweight ground vehicles, there is a lack of mobility performance empirical database and simulation methods for this class of vehicles (Jayakumar

et al., 2013)). This research establishes that the mobility performance data of full-scale lightweight military vehicles such as FED Alpha and small, lightweight ground vehicles such as Warthog are in the one-standard deviation band in the simulation domain. Therefore, it can be suggested that the mobility prediction methodologies such as NRMM for a lightweight military vehicle can be used to predict the mobility performance of small, lightweight ground vehicles on similar terrains. Moreover, during the teaming operations of UGVs and full-scale military vehicles on the battlefield, it can also provide an estimate of the mobility of UGVs in correlation to full-scale vehicle system in deciding go/no-go conditions on off-road terrains and energy cost optimisation on the feasible paths. It can also estimate the maximum load-carrying capacity of UGVs and full-scale military vehicles on soft soils during such military operations.

NATO CDT – 308 (2020) specifies the assumptions associated with laboratory condition testing and in-situ soil testing. The laboratory simulation conditions and in-situ simulation conditions were developed based on these assumptions. Therefore, the scalability of soil is defined as a relationship between the mechanical properties of an in-situ terrain (heterogeneous) system and a laboratory (homogeneous) system while accounting for the differences in sand, silt and clay particle shapes and size distributions. Physical properties such as moisture content, bulk density, compaction, and interparticle forces are kept the same for laboratory and in-situ terrain conditions. The shear tests (direct shear tests and shear ring tests), pressure-sinkage tests and cone penetrometer tests were simulated in the 2NS sand and FGS soil models. It was found that the shear properties of soil, such as cohesion and internal angle of friction have a significant effect on heterogeneity in the model. This effect is more evident at higher normal stresses than lower normal stresses. However, the shear stress-shear displacement relationships of both the sands models were scalable in the one-standard-deviation band of laboratory models.

The pressure-sinkage model parameters were found scalable based on the ratio of the equivalent stiffness of homogenous and heterogeneous soil models for coarse-grained sand in the  $p$ - $z$  domain and  $\log p$  –  $\log z$  domain for fine-grained

sand. The cone index was higher for homogeneous models than for heterogeneous models for both soils. Therefore, it can be suggested that there is a significant difference in the mechanical properties of the laboratory testing and in-situ testing of soil depending on the sand, silt and clay particle's shape and size distribution.

Further, the scalability of tyre-soil interaction is based on the assumption that similar systems that behave in a similar manner can be established (Bekker, 1956 and Freitag, 1966). It helps to predict the mobility performance of the full-scale tyre system from the prototype tyre systems. The theory of scalability was developed based on a dimensional analysis of the tyre-soil interaction system. The geometric similarity controls the tyre's distorted shape and curvature in the system. The shape of the tyre-soil contact area controls the pressure exerted by the tyre on the soil. The dynamic similarity controls the forces of tyre-soil interaction in the system. It is evident that on a specific soil model, similar scalable systems can be established by equating the pressure term  $\frac{bd}{W}$ . The accuracy of the mobility performance prediction of the full-scale system decreases with the system's size. Scale = 0.5 and scale = 0.7 can predict the full-scale tyre performance (scale = 1) in 2NS sand (coarse-grained sand) and fine-grained sand, respectively. Therefore, it is recommended to develop the single wheel tester/ UGVs to 0.5 scale to predict the mobility performance of the full-scale tyre system. It makes the single wheel tester/ UGVs lightweight and reduces operational costs. It also becomes easy to operate in difficult environmental conditions.

The other key application of the present study can be seen during the teaming operations in battlefield and peacetime missions with UGVs and full-scale military vehicles. The mobility performance of the full-scale military vehicle is the same as similar UGVs up to a reduced scale of 0.5 on off-road terrains. The maximum load on the full-scale lightweight vehicle on terrain can be found using

$$\left(\frac{bd}{W}\right)_{UGV} = \left(\frac{bd}{W}\right)_{\text{Military vehicle}}$$

i.e. the maximum load on a UGV while conducting a mobility test on the same terrain (Figure 6.1). Therefore, this research can help in decision making of go/no-go conditions and the maximum payload capacity of UGVs and full-scale military vehicles during operations on off-road terrains.

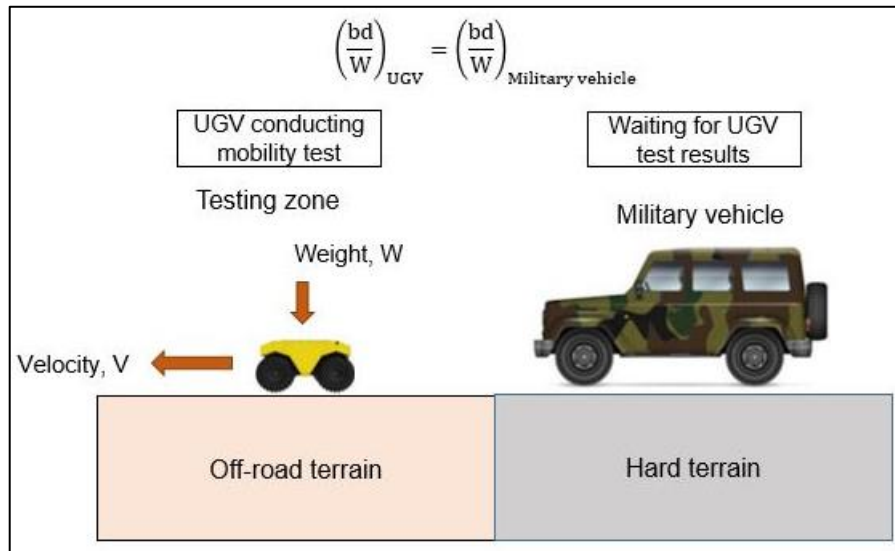


**Figure 6-1 UGV conducting initial mobility testing with scaled payload to full-scale military vehicle's payload on off-road terrain**

The other critical contribution to simulation techniques in UGV mobility technology and Terramechanics is the developed methodology based on the discrete element method using Edinburgh elastic-plastic adhesion contact model to simulate the inter-particle mechanics and the tyre-soil interaction in EDEM. This methodology has simulated better experimental results of the tyre-soil interaction system than simulation techniques used in NATO CDT – 308 (2020). The drawbar pull coefficient vs slip relation of full-scale tyre 335/65R22.5 in simulations follows the predicted band of the NATO Reference Mobility Model (NRMM) for 2NS sand and fine-grained sand.

#### 6.4 Advantages and limitations of techniques

The advantage of the theory of scalability of the tyre-soil interaction developed in Chapter -3 is based on the dimensional analysis technique, which reduces the physical parameters of the tyre-soil interaction system into non-dimensional quantity. These non-dimensional quantities can be equated to develop similar

tyre-soil interaction systems as the ratios of the magnitude of two like physical quantities are independent of the units used in their measurement, provided the same units are used in their evaluation. The other advantage this approach provides is that there is no need to establish absolute numerical equality when quantities are similar qualitatively. Therefore, the obtained non-dimensional ratios can establish similarity in the systems.

The other advantage of the proposed theory of scalability is that it accounts for a single soil parameter, i.e. cone index. Over the past 50 years, the resource intensive experimental testing on lightweight military vehicles mobility on different terrains has generated a vast amount of database of cone index values in the laboratory and in-situ field conditions (Huang et al., 2020). This database is used in the development of mobility prediction methodologies such as the NATO Reference Mobility Model (NRMM). Therefore, this research intends to use the existing available database for mobility prediction of small lightweight ground vehicles. However, it can be considered its limitations as the other complex soil parameters such as soil cohesion, internal angle of friction and bulk modulus of elasticity are not considered as the system parameters. It should be noted that the cone index is the simplified function of complex soil properties. Therefore, the combined effect of these complex soil parameters can be analysed together in terms of cone index (Oskoui et al., 1982).

The other technique used in this work is based on the discrete element method (DEM) using Edinburgh elastic-plastic adhesion (EpAM) model. The DEM provides the advantage of analysing the soil system as a group of discrete particles rather than a continuum medium. It facilitates to define inter particle mechanics better than any other simulation technique. Further, DEM supported by EpAM can define the cohesive and adhesive forces in the soil system, which better represented the experimental soil conditions. Once the expected shear properties are obtained for a soil model, pressure-sinkage curves can also be adjusted by changing its pre-compaction.

The limitation of this technique is that it is computationally expensive with a larger number of particles and bigger geometries. It took 10 days for a single run of full-



scale tyre test on a sand model with a laptop of RAM 6 GB. However, this limitation was resolved by using a 64 GB RAM CPU. Therefore, it took approximately 36 hours to run a single full-scale tyre test on a sand model with this CPU. Moreover, it is based on the trial & error method to derive initial model parameters, which requires quite a large number of trial runs to get the optimised model parameters. This limitation was resolved to some extent by selecting initial guess points from the literature.

## 7 FUTURE RECOMMENDATION

This chapter outlines the work that can be carried out in the future based on the technique and investigation carried out in this thesis. This future work can build up on the contributions to the field of UGV technology and Terramechanics presented here. Further work that can be carried out in this area involves:

- To design and develop a half-scale single wheel tester/ UGVs for a full-scale military system and verify the performance parameters experimentally. Moreover, the costs of performing the experimental trials from both systems should be compared.
- The study of soil complex parameters such as cohesion, internal angle of friction, the bulk modulus of elasticity and viscous properties affect the system's scalability. Since the developed methodology is based on a dimensional analysis approach, these parameters can be represented as a combination of non-dimensional terms. Therefore, these complex soil parameters can be tested with the developed scalability of soil and tyre-soil interaction system.
- It is proposed to extend the scalability of soil and tyre-soil interaction system to a broader soil domain. Since the United States Department of Agriculture developed a soil texture triangle containing twelve different soil texture types, these concepts can be further advanced. The discrete element method using Edinburgh elastic-plastic adhesion model can further be validated against the experimental results of soil's mechanical properties.
- It is further proposed that soil heterogeneous models can be developed through a hybrid method of coupling discrete and continuum approaches. It introduces the constraints and boundary conditions in the model to simulate a more realistic behaviour of soil. However, this methodology is still under development in the industry (YUE et al. (2018)).
- The use of flexible tyres in discrete element method simulations is recommended. The technology is under development at ALTAIR-EDEM.

## REFERENCES

Ai, J., Chen J-F., Rotter J.M. and Ooi J.Y. (2011) 'Assessment of rolling resistance models in discrete element simulations', *Powder Technology*, 206 (3), pp.269-282. Available at: <https://doi.org/10.1016/j.powtec.2010.09.030>.

Akbar, D., Asghar, M. A. and Palassi, M. (2020) 'Evaluation of particle shape on direct shear mechanical behaviour of ballast assembly using discrete element method (DEM)', *Transportation Geotechnics*, 23, 100357. <https://doi.org/10.1016/j.trgeo.2020.100357>.

Apfelbeck, M., Sebastian, K., Bernhard, R. and Bernd S. (2011) 'A systematic approach to reliably characterize soils based on Bevameter testing', *Journal of Terramechanics*, 48, pp.360–371. <https://doi.org/10.1016/j.jterra.2011.04.001>.

ASABE (2006a) *ASAE S313.3 FEB04: Soil cone penetrometer*. St. Joseph, Michigan: ASABE.

ASABE (2006b) *ASAE EP542 FEB99: Procedures for using and reporting data obtained with the soil cone penetrometer*. St. Joseph, Michigan: ASABE.

ASAE (1998) *ASAE S296.4 DEC95: General terminology for traction of agricultural tractors, self-propelled implements, and traction and transport devices*. St. Joseph, Michigan: ASABE.

Asaf, Z., Rubinstein, D. and Shmulevich, I. (2006) 'Evaluation of link-track performances using DEM', *Journal of Terramechanics*, 43 (2), pp. 141-161.

Ashmore, C., Burt, C. and Turner, J. (1987) 'An empirical Equation for predicting tractive performance of log-skidder tyres', *Transactions of the ASAE*, 30(5), pp.1231-1236.

ASTM International (2004) *D3080-04: Standard test method for direct shear test of soils under consolidated drained conditions*. West Conshohocken, PA: ASTM International.

Baranowski, P., Bogusz, P., Gotowicki, P., Małachows, J. (2012) 'Assessment of mechanical properties of offroad vehicle tire: coupons testing and -FE model development', *Acta Mechanica et Automatica*, 6(2), pp.17-22.

Barton, P.C. (1998) 'The influence of prediction formulae on wheeled AFV design', *Wheels & Tracks Symposium*. RMCS, Shrivenham, Wiltshire, September 1998.

Bekker, M.G. (1969) *Introduction to terrain-vehicle systems, vol. I*. Ann Arbor (MI): University of Michigan Press.

Bekker, M. G. (1956) *Theory of land locomotion*. The University of Michigan Press.

Berman, A., Drummond, C. and Israelachvili, J. (1998) 'Amontons' law at the molecular level', *Tribology Letters*, 4, pp.95–101.

Biswas, A. and Si, B.C. (2011) 'Scaling of soil physical properties', In: Gliński, J., Horabik, J. and Lipiec, J. (eds.). *Encyclopedia of Agrophysics*. Dordrecht: Springer. (Encyclopedia of Earth Sciences Series). [https://doi.org/10.1007/978-90-481-3585-1\\_243](https://doi.org/10.1007/978-90-481-3585-1_243).

Blöschl, G. and Sivapalan, M. (1995) 'Scale issues in hydrological modelling: a review', *Hydrological Processes*, 9, pp.251–290.

Brixius, W.W. (1987) 'Traction prediction Equations for bias ply tires', *American Society of Agricultural Engineers*, Paper No. 87-1622.

Brunskill, C., Patel, N., Gouache, T. P., Scott, G. P., Chakravarthini, M. S., Marcus, M. and Liang, C. (2011) 'Characterisation of martian soil simulants for the ExoMars rover testbed', *Journal of Terramechanics*, 48(6), pp. 419-438. <https://doi.org/10.1016/j.jterra.2011.10.001>.

Burt, E. C., Lyne, P. W. L. and Keen J. F. (1982) 'Ballast and inflation pressure effects on tractive efficiency', *ASAE*, pp. 82-1567.

Chong, B.H., Park, Y.J., Park, H.K., Park, S.B. and Kim, K.U. (2005) 'Mapping of cone index for precision tillage', *Journal of Biosystem Engineering*, 30, pp.127–133.

Clark, R.L. (1985) 'Tractive modelling with the modified Wismer-Luth model', *ASAE*, pp. 85-1049.

Collins, J. G. (1971) *Technical Memorandum no. 3-331: Forecasting trafficability of soils: Report 10: Relations of strength to other properties of fine-grained soils and sands with fines*. Vicksburg, MS: U.S. Army Engineer Waterways Experiment Station.

Cruz, Z., Maza, D. and Ignacio, P. (2009) 'Role of particle shape on the stress propagation in granular packings', *Article in Physical Review Letters*.  
<https://doi.org/10.1103/PhysRevLett.103.118001>.

Cutini, R. and Bisaglia, C. (2008) 'Agricultural tyre footprint shape-correlation to treads penetration in different soils', *International Conference*. 15-17 September 2008, Ragusa – Italy.

DEM Solutions Ltd.: *EDEM 2.4 Theory Reference Guide*, Edinburgh, Scotland, UK (2011).

Dexter, A. R. , Keller, T. and Arvidsson J. (2007) 'Soil structures produced by tillage as affected by soil water content and the physical quality of soil', *Soil and Tillage Research*, 92 (1–2), pp. 45-52.

Dhillon, R., Ali R., El-Gindy M., Philipps D., Öijer F. and Johansson I. (2013) 'Development of Truck Tire-Soil Interaction Model using FEA and SPH', in *Proceedings of SAE, World Congress*.

Ding, L., Gao H., Zongquan D., Li Y. and Liu G. (2014) 'New perspective on characterizing pressure–sinkage relationship of terrains for estimating interaction mechanics', *Journal of Terramechanics*, 52, pp.57-76.  
<https://doi.org/10.1016/j.itterra.2014.03.001>.

Diserens, E. (2009) 'Calculating the contact area of trailer tyres in the field', *Soil and Tillage Research*, 103(2), pp. 302-309.

Diserens, E., Defossez, P., Duboisset A. and Alaoui A. (2011) 'Prediction of the contact area of agricultural traction tyres on firm soil', *Biosystems Engineering*, 110(2): pp.73-82.

Dwyer, M.J. and Heigho, D.P. (1984) 'The tractive performance of some large tractor tyres compared with dual tyres', *Journal of Agricultural Engineering Research*, 29(1): pp.43-50.

Dwyer, M.J. (1984) 'The tractive performance of wheeled vehicles', *Journal of Terramechanics*, 21(1): pp.19-34.

Dwyer, M.J., Evernden, D.W. and McAllister M. (1976): *Handbook of agricultural tyre performance*, NIAE report No.18, Edition: 2.

Ecke, S. (2001) 'Friction between individual microcontacts', *Journal of Colloid Interface Science*, 244, pp. 432–435.

Economou, J.T., Kladis, G., Tsourdos, A., White, B.A. (2007) 'UAV Optimum Energy Assignment using Dijkstra's Algorithm', Proceedings of the European Control Conference, July 2-5, Kos, Greece.

Economou, J.T., Kladis, G., Tsourdos, A., White, B.A. (2007) 'A node-to-node composite graph and pseudo-Boolean modelling: an unmanned aerial vehicle energy application', *Journal of Aerospace Engineering*, JAERO191: IMechE 2007, 221, Part: G. <https://doi.org/10.1243/09544100JAERO191>.

Ekinci, S. and Carman, K. (2011) 'Effects on tyre contact area of tyre structural and operational parameters', 6th International Advanced Technologies Symposium (IATS'11), 16-18 May, Elazığ, Turkey.

Elwaleed, A.K., Yahya, A., Zohadie, M., Ahmad, D. and Kheiralla, A.F. (2006) 'Effect of inflation pressure on motion resistance ratio of a high-lug agricultural tyre', *Journal of Terramechanics*, 43(2), pp.69-84.

Engineering Design Handbook (1967): *Automotive series*, United States Army Material Command.

Evans, M.D., Clark, R.L. and Manor, G. (1991) 'An improved traction model for ballast selection', *Transactions of the ASAE*, 34(3), pp.773-780.

Favaedi, Y., Pechev, A., Scharringhausen, M and Richter, L (2011) 'Prediction of tractive response for flexible wheels with application to planetary rovers', *Journal of Terramechanics*, 48(3), pp.199-213.

Federal Motor Vehicle Safety Standards Consumer Information (2022): *Standard Reference Test Tire*. A Rule by the National Highway Traffic Safety Administration on 06/08/2022.

Forrest, P. J., Reed, F. and Constantakis, G. V. (1962) 'Tractive characteristics of radial ply tires', *Transactions of the ASAE*, 5(2), pp.108 -115.

Freitag, D.R and Smith M.E (1966) 'Center-line deflection of pneumatic tires moving in dry sand', *Journal of Terramechanics*, 3(1), pp.31-46.

Freitag, D.R. (1962): *The performance of transport wheels in sand*. Auburn University.

Freitag, D.R. (1965): *A dimensional analysis of the performance of pneumatic tyres on soft soils*. Technical report number: 3-688, USAE Waterways Experiment Station.

Gee-clough, D., Mcallister, M. and Evernden, D. W. (1977) 'Tractive performance of tractor drive tires - A comparison of radial and cross-ply carcass construction', *Journal of Agriculture Engineering Research*, 22(4), pp.385-395.

Gee-Clough, D. (1978) 'The effect of wheel width on the rolling resistance of rigid wheels in sand', *Journal of Terramechanics*, 15 (4), pp.161-184.

Gee-Clough, D., McAllister, M., Pearson, G. and Evernden, D. W. (1978) 'The empirical prediction of tractor-implement field performance', *Journal of Terramechanics*, 15(2), pp.81-94.

Mason, G.L. and Baylot, E. A. (2016): *Predicting soil strength in terms of Cone Index and California Bearing Ratio of Trafficability*. Technical report: 16-1, ERDC/GSL.

Goodman, J. and Lee, N. (1963) 'Effect of remolding on soil values related to vehicle mobility', Proceedings of 1st International Conference on the Mechanics of Soil-Vehicle Systems, pp.63-75.

Grechenko, A. (1995) 'Tyre footprint area on hard ground computed from catalogue value', *Journal of Terramechanics*, 32(6), pp.325-333.

Griffith, M. G. and Spenko, M. (2014) 'Development and experimental validation of an improved pressure-sinkage model for small-wheeled vehicles on dilative, deformable terrain', *Journal of Terramechanics*, 51, pp.19-29.

Griffith, M. G. and Spenko, M. (2013) 'A pressure-sinkage model for small-diameter wheels on compactive, deformable terrain', *Journal of Terramechanics*, 50(1), pp. 37-44. <https://doi.org/10.1016/j.jterra.2012.05.003>.

Griffith, M. G. and Spenko, M. (2011) 'A modified pressure-sinkage model for small, rigid wheels on deformable terrains', *Journal of Terramechanics*, 48 (2), pp. 149-155.

Grisso, R., Perumpral, J. and Zoz, F. (2006) 'An empirical model for tractive performance of rubber-tracks in agricultural soils', *Journal of Terramechanics*, 43(2), pp. 225-236.

Hallonborg, U. (1996) 'Super ellipse as tyre-ground contact area', *Journal of Terramechanics*, 33(3), pp.125-132.

Hamed, Al, Grisso, S.A., Zoz, R.D. and Barga K.V. (1994) 'Tractor performance spread sheet for radial tyres', *Computers and Electronics in Agriculture*, 10(1), pp.45-62.

Hetherington, J.G and White, J.N (2002) 'An investigation of pressure under wheeled vehicles', *Journal of Terramechanics*, 39 (2), pp. 85-93.

Hetherington, J.G. (2001) 'The applicability of the MMP concept in specifying off-road mobility for wheeled and tracked vehicles', *Journal of Terramechanics*, 38 (2), pp. 63-70.



Hetherington, J.G. (1992) 'The use of scale model tests to verify earth anchor design predictions', *Journal of Terramechanics*, 29(2), pp. 257-263.

Hetherington, J.G. (2005) 'Tracked vehicle operations on sand – investigations at model scale', *Journal of Terramechanics*, 42 (1), pp. 65-70.

Hetherington, J.G. and Littleton, I. (1978) 'The rolling resistance of towed, rigid wheels in sand', *Journal of terramechanics*, 15(2), pp. 95 – 105.

Hornback Paul (1998): *The Wheel Versus Track Dilemma*, US Report, ARMOR.

Egozi, A. (2018) 'US Army pursues Israeli robots', *Breaking Defense*, November 12. Available at: <https://breakingdefense.com/2018/11/us-army-pursues-israeli-robots/> (Accessed: 24 May 2021).

Clearpath Robotics (no date) *Husky*. Available at: <https://clearpathrobotics.com/husky-unmanned-ground-vehicle-robot/> (Accessed: 27 May 2021).

Wood, Ted (2019) 'Restoring the Colorado: Bringing New Life to a Stressed River', Published: Yale School of the Environment, February 14. Available at: <https://e360.yale.edu/features/restoring-the-colorado-bringing-new-life-to-a-stressed-river> (Accessed: 27/05/2021).

Army Technology (2014) 'TAROS V2 Unmanned Ground Vehicle (UGV)', Available at: <https://www.army-technology.com/projects/taros-v2-unmanned-ground-vehicle-ugv/> (Accessed: 24/05/2021).

Huang, W., Wong, J.Y., Preston-Thomas, J., Jayakumar, P. (2020) 'Predicting terrain parameters for physics-based vehicle mobility models from cone index data', *Journal of Terramechanics*, 88, pp. 29-40.

Janosi, Z. and Hanamoto, B. (1961): *An analysis of the drawbar pull vs. slip relationship for track laying vehicles*. Technical report: RR-47. Land locomotion laboratory.

Janosi, Z. and Hanamoto, B (1961) '*The analytical determination of drawbar pull as a function of slip, for tracked vehicles in deformable soils*', Proceedings of the 1st International Conference Mechanics Soil-Vehicle Systems, Turin, Italy.

Javaid, A. (2013) 'Understanding Dijkstra Algorithm', *SSRN Electronic Journal*. DOI: <https://doi.org/10.2139/ssrn.2340905>.

Jayakumar, P. (2016): *STE, Analytics, Mobility Research & Development*, Tank Automotive Research, Development and Engineering Center Research, Development and Engineering Command Army Materiel Command United States Army – presentation

Jayakumar, P, Melanz, D, MacLennan, J, Gorsich, D, Senatore, C and Iagnemma, K (2014) 'Scalability of classical terramechanics models for lightweight vehicle applications incorporating stochastic modeling and uncertainty propagation', *Journal of Terramechanics*, 54, pp. 37-57.

Johnson, B., Duvoy, X., Kulchitsky, V., Creager, C. and Moore, J. (2017) 'Analysis of Mars Exploration Rover wheel mobility processes and the limitations of classical terramechanics models using discrete element method simulations', *Journal of Terramechanics*, 73, pp. 61-71.

Johnson, B., Kulchitsky, V., Duvoy, P, Iagnemma, K, Senatore, C, Raymond, E. A. and Jeffery, M. (2015) 'Discrete element method simulations of Mars Exploration Rover wheel performance', *Journal of Terramechanics*, 62, pp. 31-40.

Keller, T. (2005) 'A Model for the Prediction of the Contact Area and the Distribution of Vertical Stress below Agricultural Tyres Readily Available Tyre Parameters', *Biosystems Engineering*, 92(1): pp.85-96.

Kladis, P., Economou, J., Knowles, K., Lauber, J., Guerra, T. (2011) 'Energy conservation based fuzzy tracking for unmanned aerial vehicle missions under a priori known wind information', *Engineering Applications of Artificial Intelligence*, 24, pp.278-294.

Knight, S. J. (1961) '*Some factors affecting moisture content-density-cone index relations*', Miscellaneous Paper No. 4- 457. Vicksburg, MS: U.S. Army Engineer Waterways Experiment Station.

Kobayakawa, M, Miyai, S, Tsuji, T, Tanaka, T (2020) 'Interaction between dry granular materials and an inclined plate (comparison between large-scale DEM simulation and three-dimensional wedge model)', *Journal of Terramechanics*, 90, pp. 3-10.

Komandi, G. (1976) 'The determination of the deflection, contact area dimensions and load carrying capacity for driven pneumatic tyres operating on concrete pavement', *Journal of Terramechanics*, 13(1), pp.15-21.

Komandi, G. (1990) 'Establishment of soil-mechanical parameters which determine traction on deforming soil', *Journal of Terramechanics*, 27(2), pp.115-124.

Krick, G. (1969) 'Radial and shear stress distribution under rigid wheels and pneumatic tyres operating on yielding soils with consideration of tyre deformation. *Journal of Terramechanics*, 6(3), pp.73-98.

Krumbein, WC (1941) 'Measurement and geological significance of shape and roundness of sedimentary particles', *Journal of Sediment Petrol*, 11, pp. 64–72.

Kumar, R, Pandey, KP, Gupta, DK, Tewari, VK, Prakash, S and Kumar, S (2018) 'Deflection characteristics for radial-ply tractor tyres', *Journal of Pharmacognosy and Phytochemistry*; SP1, pp. 2016-2021.

Lai, J (2004): *Direct Shear Testing*. Department of Construction Engineering Advanced Geotechnical Laboratory Chaoyang University of Technology.

Lankenau, G.F., Diaz, Skonieczny, K., Whittaker, W.L. and Wettergreen D.S. (2012) 'Effect of bucket-wheel scale on excavation forces and soil motion', *Journal of Terramechanics*, 49 (6), pp. 341-348.

Larminie, J.C. (1992) 'Modifications to the mean maximum pressure system', *Journal of Terramechanics*, 29 (2), pp. 239 – 255.

Lescoe, R., M. El-Gindy, K., Koudela, F., Öijer, M. Trivedi and Johansson I. (2010) '*Tire-Soil Modeling Using Finite Element Analysis and Smooth Particle Hydrodynamics Techniques*', ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference.

Leviticus, L.I. and Reyes, J.F. (1983) 'Traction on concrete-I, dynamic ratio and traction quotient', *ASAE*, pp. 83-1558.

Lin, X and Ng, T. (1997) 'A three-dimensional discrete element model using arrays of ellipsoids', *Géotechnique*, 47(2), pp. 319-329.

Lines, J.A. and Murphy, K. (1991) 'The stiffness of agricultural tractor tyres', *Journal of Terramechanics*, 28 (1), pp.49-64.

Liu, T, Liang, L, Zhao, Y, Cao, D (2020) 'Equivalent boundary model of lunar soil drilling simulation by DEM', *Journal of Terramechanics*, 91, pp. 85-95.

Liu, T, Thornburn, K, Thomas, M (1964): Study of the reproducibility of Atterberg Limits. National Academy of Sciences. Available at: <http://onlinepubs.trb.org/Onlinepubs/hrr/1964/63/63-003.pdf>

Luding, S (2008) 'Cohesive, frictional powders: contact models for tension', *Granular Matter*, 10, pp.235–246.

Lyasko, M (2010) 'How to calculate the effect of soil conditions on tractive performance', *Journal of Terramechanics*, 47 (6), pp. 423-445.

Lyasko, M (2010) 'LSA model for sinkage predictions', *Journal of Terramechanics*, 47(1), pp. 1-19.

Maclaurin, EB (1997): *Proposed revisions to MMP based on the results of tractive performance trials with single pneumatic tyres and a modular track system*, DERA/LS4/TR9701222/1.0.

Maclaurin, B. (2007) 'Comparing the NRMM (VCI), MMP and VLCI traction models', *Journal of terramechanics*, 44 (1), pp. 43 – 51.

Micklethwaite, E.W.E. (1944) '*Soil mechanics in relation to fighting vehicles*', Military college of science, Chertsey.

Misiewicz, PA, Blackburn, K, Richards, TE, Brighton, JL and Godwin, RJ (2015) 'The evaluation and calibration of pressure mapping system for the measurement of the pressure distribution of agricultural tyres', *Biosystems Engineering*, 130, pp. 81-91.

Morrissey, J. (2013): *Discrete Element Modelling of Iron Ore Pellets to Include the Effects of Moisture and Fines*, Institute for Infrastructure and Environment, School of Engineering, University of Edinburgh.

Morrissey, J., Thakur, S. C. and Ooi, J. Y. (2014): *EDEM contact model: Adhesive Elasto-Plastic Model*, v1, School of Engineering, University of Edinburgh.

Mvelase, GM, Gräbe, PJ and Anochie-Boateng, JK (2017) 'The use of laser technology to investigate the effect of railway ballast roundness on shear strength', *Transportation Geotechnology*, 11, pp.97–106.

Nakanishi, R, Nakashima, H, Miyasaka, J and Ohdoi K (2020) 'Tractive performance analysis of a lugged wheel by open-source 3D DEM software', *Journal of Terramechanics*, 92, pp. 51-65.

Nakashima, H and Oida A (2004) 'Algorithm and implementation of soil–tire contact analysis code based on dynamic FE–DE method', *Journal of Terramechanics*, 41(2–3), pp. 127-137.

Nakashima, H., Fujii, H., Oida, A., Momozu, M., Kawase, Y., Kanamori, H., Aoki, S., Yokoyama, T. and Corrigendum, T (2008) 'Parametric analysis of lugged wheel performance for a lunar microrover by means of DEM', *Journal of Terramechanics*, 45 (1–2), pp. 63.

Nakashima, H., Khot, R., Vilas, S. and Jayasuriya, H.P.W. (2007) 'Experimental validation of distinct element simulation for dynamic wheel–soil interaction', *Journal of Terramechanics*, 44(6), pp.429-437.

Nakashima, H, Takatsu, Y, Shinone, H, Matsukawa, H and Kasetani T (2009) 'Fe-DEM Analysis of the Effect of Tread Pattern on Tractive Performance of Tires operating on Sand', *Journal of Mechanical Systems For Transportation And Logistics*, 2 (1), pp. 09- 0086.

Nam, Ju S., Young, J. P. and Kyeong, U. K. (2010) 'Determination of rating cone index using wheel sinkage and slip', *Journal of Terramechanics*, 47, pp. 243–248.

National Swedish Testing Institute for Agricultural Machinery (1963): *Traktordack*, Report No. 1663, Uppsala, Sweden

Nishiyama, K, Nakashima, H, Shimizu, H, Miyasaka, J, Ohdoi, K (2017) '2D FE–DEM analysis of contact stress and tractive performance of a tire driven on dry sand', *Journal of Terramechanics*, 74, pp. 25-33.

Nishiyama, K, Nakashima, H, Yoshida, T, Ono, T, Shimizu, H, Miyasaka, J and Ohdoi, K (2016) '2D FE–DEM analysis of tractive performance of an elastic wheel for planetary rovers', *Journal of Terramechanics*, 64, pp.23-35.

NATO TR SAS – 097 (2018): North Atlantic Treaty Organization *STO Technical report: AC/323 (SAS-097) TP/776*, Research and Technology Organization.

NATO CDT – 308 (2020): North Atlantic Treaty Organization *STO Technical report Cooperative Demonstration of Technology (CDT- 308): Next-Generation NATO Reference Mobility Model (NG-NRMM)*, Research and Technology Organization.

NATO NRMM CDT (2018): North Atlantic Treaty Organization *STO Technical report AVT-248: Next-Generation NATO Reference Mobility Model (NRMM) Development*, Research and Technology Organization.

Ooi, J.Y. and Hartl, J. (2008) 'Experiments and simulations of direct shear tests: Porosity, contact friction and bulk friction', *Granular Matter*, 10(4), pp.263-271,[10.1007/s10035-008-0085-3](https://doi.org/10.1007/s10035-008-0085-3).

Oravec, H A (2009): *Understanding mechanical behavior of Lunar soils for the study of vehicle mobility*, PhD Thesis, Department of Civil Engineering, Case Western Reserve University, USA.

Oskoui, K. E. and Witney, B.D. (1982) 'The determination of plough draught—Part I. prediction from soil and meteorological data with cone index as the soil strength parameter', *Journal of Terramechanics*, 19 (2), pp.97-106. [https://doi.org/10.1016/0022-4898\(82\)90014-3](https://doi.org/10.1016/0022-4898(82)90014-3).

Pillinger, G, Attila, G, Zoltán, H and Péter, K (2018) 'Determination of soil density by cone index data', *Journal of Terramechanics* 77, pp.69–74.

Plackett, C.W. (1984) 'The ground pressure of some agricultural tyres at low load with zero sinkage', *Journal of Agricultural Engineering Research*, 29(2): pp.159-166.

Pollock, J R., Geldart, H.M. and Verlinden-Luts, D. (2004) 'Frictional forces between cohesive powder particles studied by AFM', *Ultramicroscopy*, 100, pp.59–78.

Popescu, R., Deodatis, G., and Nobahar, A. (2005) 'Effects of Random Heterogeneity of Soil Properties on Bearing Capacity', *Probability Engineering Mechanics*, 20, pp. 324–341.

Priddy, J.D. and Willoughby, W.E. (2006) 'Clarification of vehicle cone index with reference to mean maximum pressure', *Journal of terramechanics*, 43 (2), pp. 85 – 96.

Qskoui, K., Eradat and Witney, B. D. (1982) 'The determination of plough draught—Part I. prediction from soil and meteorological data with cone index as the soil strength parameter', *Journal of Terramechanics*, 19(2), pp. 97-106.

Quraishi, M. Z. and Mouazen A. M (2013) 'Development of a methodology for in situ assessment of topsoil dry bulk density', *Soil and Tillage Research*, 126, pp. 229-237, <https://doi.org/10.1016/j.still.2012.08.009>.

Rashidi, S., Razavi, S., Niyazadeh, M. and Arkian, M. (2013) 'Prediction of radial-ply tire deflection based on section width, overall unloaded diameter,

inflation pressure and vertical load' *World Applied Sciences Journal*, 21(12), pp. 1804-1811.

Ravula, P., Gizem, A. and Balakumar, B. (2021) 'Discrete element method-based studies on dynamic interactions of a lugged wheel with granular media', *Journal of Terramechanics*, 94, pp.49-62.

Reece, A.R. (1964) 'Curve fitting technique in soil vehicle mechanics', *Journal of Terramechanics*, 1, pp.44-55.

Rowland, D. (1972) 'Tracked vehicle ground pressure and its effect on soft ground performance', Proceedings of the 4th International Society for Terrain Vehicle Systems (ISTVS) conference, Stockholm, pp. 353 –384.

He, R., Corina, S., Mousavi, H., Shenvi, M. N., Braun, K., Kruger, R. and Schalk P. (2020) 'Updated Standards of the International Society for Terrain-Vehicle Systems', *Journal of Terramechanics*, 91, pp.185-231.

Rula, A. A, and Nuttall, C. J. (1971): An analysis of ground mobility models (ANAMOB), Technical report M-71-4, U.S. Army Engineers Waterways Experiment Station. Available at: <http://hdl.handle.net/11681/30624>.

Rummer, R. and Ashmore, C. (1985) 'Factors affecting the rolling resistance of rubber tyred skidders', *ASAE*, pp. 85-1611.

Ruths, M., Alcantar, N.A. and Israelachvili, J.N. (2003) 'Boundary Friction of Aromatic Silane Self-Assembled Monolayers Measured with the Surface Forces Apparatus and Friction Force Microscopy', *Journal of Physical Chemistry B*, 107 (40), pp. 11149–11157.

Saarilahti, M. (2002): *Soil Interaction Model*, Technical Report 5, Quality of Life and Management of Living Resources.

Sadek, S., Najjar, S.S. and Freiha, F. (2010) 'Shear strength of fiber-reinforced sands', *Journal of Geotechnical and Geoenvironmental Engineering*, 136(3), pp. 490–499.



Savkoor, A. and Briggs, G. (1977) 'The effect of tangential force on the contact of elastic solids in adhesion', *Proceedings of Royal Society A*, 356 (1684), pp. 103–114.

Schuring, DJ and Emori RI (1965) 'Feasibility of model study in earthworking equipment', *Transactions of the ASAE*, 8(2).

Senatore, C. and Sandu, C. (2011) 'Torque distribution influence on tractive efficiency and mobility of off-road wheeled vehicles', *Journal of Terramechanics*, 48 (5), pp. 372-383.

Sharma, G., Tiwary, S., Kumar, A., Kumar, S. and Murthy, K. (2018) 'Systematic design and development of a flexible wheel for low mass lunar rover', *Journal of Terramechanics*, 76, pp. 39-52.

Sharma, A.K. and Pandey, K.P. (1996) 'The deflection and contact characteristics of some agricultural tyres with zero sinkage', *Journal of Terramechanics*, 33(6), pp. 293-299.

Hegazy, S. and Corina, S. (2013) 'Experimental investigation of vehicle mobility using a novel wheel mobility number', *Journal of Terramechanics*, 50 (5–6), pp. 303-310.

Sitkei, G., Pillinger, G., Máthé, L., Gurmai, L. and Kiss, P. (2018) 'Methods for generalization of experimental results in terramechanics', *Journal of Terramechanics*, 81, pp. 23-34.

Skinner, J. and Gane, N. (1972) 'Sliding friction under a negative load', *Journal of Physics D: Applied Physics*, 5, pp. 2087.

Smith W. and Huei P. (2013) 'Modeling of wheel–soil interaction over rough terrain using the discrete element method', *Journal of Terramechanics*, 50(5–6), pp. 277-287.

Sukumaran, B and Ashmawy AK (2001) 'Quantitative characterization of the geometry of discrete particles', *Geotechnique*, 51(7), pp.619–627.

Swanson, G D (1973) 'Scale-model Tire Tests in clay', *Journal of Terramechanics*, 10 (3), pp. 21-27.

Taghavifar Hamid, Aref Mardani, Investigating the effect of velocity, inflation pressure, and vertical load on rolling resistance of a radial ply tire, *Journal of Terramechanics*, Volume 50, Issue 2, 2013, Pages 99-106, ISSN 0022-4898.

Taghavifar, H. and Mardani, A. (2015) 'Net traction of a driven wheel as affected by slippage, velocity and wheel load', *Journal of the Saudi Society of Agricultural Sciences*, 14 (2), pp. 167-171.  
<https://doi.org/10.1016/j.jssas.2013.11.002>.

Taylor J. H., E. C. Burt and A. C. Bailey (1976) 'Radial tire performance in firm and soft soils', *Transactions of the ASAE*, 19(6), pp.1062-1064.

Thornton, C. and Yin, K. (1991) 'Impact of elastic spheres with and without adhesion', *Powder Technology*, 65, pp.153–166.

Tiwari V.K., Pandey K.P. and Pranav P.K. (2010) 'A review on traction prediction equations', *Journal of Terramechanics*, 47 (3), pp.191-199.

Tomas, J. (2003) 'Mechanics of nanoparticle adhesion—a continuum approach', *Particle Surfaces*, 8, pp.183–229.

Tsuji, T., Nakagawa, Y., Matsumoto, N., Kadono, Y., Takayama, T. and Tanaka, T. (2012) '3-D DEM simulation of cohesive soil-pushing behaviour by bulldozer blade', *Journal of Terramechanics*, 49 (1), pp.37-47.

Turnage, GW (1971) 'Performance of soils under track loads', Technical Report No M-71-5. US Army Waterways Experiment Station, USA.

Turnage, G.W. (1972) 'Tyre selection and performance prediction for off-road wheeled vehicle operations', Proceedings of the fourth international conference on soil terrain-vehicle systems, Stockholm.

Upadhyaya, S. K., Wulfsohn, D. and Jubbal, G. (1989) 'Traction prediction Equations for radial ply tyres', *Journal of Terramechanics*, 26(2), pp. 149-175.

Upadhyaya, S.K. and Wulfsohn, D. (1988) 'Relationship between tyre deflection characteristics and soil tyre contact area', *ASAE*, pp. 88-1005.

Upadhyaya, S.K., Chancellor, W.J., Wulfsohn, D. and Glancey, J.L. (1988) 'Sources of variability in traction data', *Journal of Terramechanics*, 25(4), pp. 249-272.

USMC: Information Paper (2011). Technical report, Marine Corps Warfighting Laboratory in Quantico, VA.

Van, N. N., Takaaki, M., Tatsuya, K. and Shigeki, I. (2007) 'Experimental Device for Measuring Sandy Soil Sinkage Parameters', *Bull. Fac. Agr., Saga University*, 93, pp. 91- 99.

Vaz, C. M.P. and Hopmans, J.W. (2001) 'Simultaneous Measurement of Soil penetration resistance and water content with a combined Penetrometer–TDR Moisture Probe', *Soil Science Society of America Journal*, 65(1), pp. 4-12.  
<https://doi.org/10.2136/sssaj2001.6514>.

Verschoore, R. and Duquesne, F. (2001) 'Simulation of the influence of differential control on tractor work rate', *Journal of Terramechanics*, 38(4), pp. 221-233.

Voorhees, W.B. (1977) 'Soil compaction: how it influences moisture, temperature, yield, root growth', *Crops Soils*, 29, pp. 7-10.

Walton, O. R. and Braun, R. L. (1986) 'Stress calculations for Assemblies of Inelastic Spheres in Uniform Shear', *Acta Mechanica*, 63, pp. 73-86.

Walton, O.R. and Johnson, S.M. (2009) '*Simulating the effects of interparticle cohesion in micron-scale powders*', AIP Conference on Proceedings, Golden, CO, pp. 897–900.

Wang Z., Yang, H., Ding, L., Yuan, B., Fengtian, L., Gao, H. and Deng, Z. (2020) 'Wheel's performance of Mars exploration rovers: Experimental study from the perspective of terramechanics and structural mechanics', *Journal of Terramechanics*, 92, pp. 23-42.

Wismer, RD and Luth, HJ (1973) 'Off-road traction prediction for wheeled vehicles', *Journal of Terramechanics*, 10, pp.49–61.

Wismer, R.D. and Luth, H.J. (1974) 'Off-road traction prediction for wheeled vehicles', *Transactions of the American Society of Agricultural and Biological Engineers*, 17 (1), pp. 8 – 10.

Wong, J. (2010) *Terramechanics and Off-Road Vehicle Engineering*. Edition- 2, Wiley.

Wong J. (2008) *Theory of ground vehicles*. 4th ed. Wiley.

Wong, J.Y. and Preston-Thomas, J. (1983) 'On the characterization of the pressure-sinkage relationship of snow covers containing an ice layer', *Journal of Terramechanics*, 20(1), pp.1-12.

Wong, J. Y. (1980) 'Data processing methodology in the characterization of the mechanical properties of terrain', *Journal of Terramechanics*, 17, pp.13-41.

Wong, J. and Reece, A.R. (1967) 'Prediction of rigid wheel performance based on the analysis of soil-wheel stresses. Part II. Performance of driven rigid wheels', *Journal of Terramechanics*, 4 (1), pp. 81 – 98.

Wong, J. and Reece, A.R. (1967) 'Prediction of rigid wheel performance based on the analysis of soil-wheel stresses. Part II. Performance of towed rigid wheels', *Journal of Terramechanics*, 4 (2), pp. 7 – 25.

Worthington, W. H. (1962) 'A discussion of performance characteristics of radial-ply tires', *Transactions of the ASAE*, 5(2), pp.113.

Wright A (2012): Tyre / Soil Interaction Modelling within a Virtual Proving Ground Environment, PhD Thesis, Cranfield University, School of Applied Sciences, Academic year: 2009 – 2012.

Wu, J., Shen, Y., Yang, S. and Feng, Z. (2022) 'Simulation of Track-Soft Soil Interactions Using a Discrete Element Method', *Applied Science*, 12, pp. 2524. <https://doi.org/10.3390/app12052524>

Wulfsohn, D., Upadhyaya, S. K. and Chancellor, W. J. (1988) 'Tractive characteristics of radial ply and bias ply tires in a California soil', *Journal of Terramechanics*, 25 (2), pp. 111-134.

Yang, C., Yang, G., Liu, Z., Huaxiong, C., Yongsheng, Z. (2018) 'A method for deducing pressure–sinkage of tracked vehicle in rough terrain considering moisture and sinkage speed', *Journal of Terramechanics*, 79, pp. 99-113.

Yonghao, D., Jingwei G., Lehua, J., Yuanchao, Z. (2018) 'Development and numerical validation of an improved prediction model for wheel-soil interaction under multiple operating conditions', *Journal of Terramechanics*, 79, pp. 1-21.

Yonghao, D., Jingwei G., Lehua, J., Yuanchao, Z. (2017) 'Numerical analysis on tractive performance of off-road wheel steering on sand using discrete element method', *Journal of Terramechanics*, 71, pp. 25-43.

Yonghao, Y., Breannan, S., Chen, P.Y., Chantharayukhonthorn, M., Kamrin, K., Grinspun, E. (2018) 'Hybrid Grains: Adaptive Coupling of Discrete and Continuum Simulations of Granular Media', Article no. 283, ACM Transactions on Graphics.

El-Sayegh, Z., El-Gindy, M., Johansson, I., Öijer, F. (2020) 'Development and validation of off-road tire-gravelly soil interaction using advanced computational techniques', *Journal of Terramechanics*, 91, pp. 45-51.  
<https://doi.org/10.1016/j.itera.2020.05.004>.

Zhao, C. and Mengyan, Z. (2014) 'Analysis of rigid tire traction performance on a sandy soil by 3D finite element–discrete element method', *Journal of Terramechanics*, 55, pp. 29-37.

Zoz, F.M. and Grisso, R.D. (2003): *Traction and tractor performance*. ASAE Distinguished Lecture No. 27. Agricultural Equipment Technology Conference, 9-11 February, Louisville, Kentucky, U.S.A.



## **APPENDICES**

## Appendix A Specifications of instruments and accessories

### A.1 Load cell specifications

- i. Type: Cylindrical load cell
- ii. Operating mode: Tension
- iii. Capacity: 10 kN
- iv. Nominal sensitivity: 2mV/V
- v. Input power supply: 10 V
- vi. Operating temperature range: -20 to +80 °C
- vii. Accuracy:  $\leq \pm 0.03\%$
- viii. Weight: 25 gm
- ix. Construction material: Aluminium alloy
- x. Manufacturer: Applied Measurements Ltd

#### A.1.1 Calibration data of load cell

| Input weights (kN) | Reading of load cell (kN) |
|--------------------|---------------------------|
| 0                  | 0                         |
| 50                 | 50                        |
| 100                | 99.9                      |
| 150                | 150.1                     |
| 200                | 200.2                     |
| 250                | 250                       |
| 300                | 300.1                     |
| 350                | 350                       |
| 400                | 400.1                     |
| 450                | 450                       |
| 500                | 500.1                     |
| 550                | 550                       |
| 600                | 600                       |



## **A.2 Cranfield University computing system**

### **A.2.1 Device specification**

Device name: DESKTOP-STG23CE

Processor: AMD Ryzen 7 2700 Eight-core Processor 3.20 GHz

Installed RAM: 64 GB

System type: 64-bit operating system, x64-based processor

### **A.2.2 Windows specification**

Edition: Windows 10 Enterprise

Version: 20H2

OS Build: 19042.804

Experience: Windows Feature Experience Pack 120.2212.551.0

## Appendix B Experimental data

### B.1 Tyre deflection at different normal loads and inflation pressures for test tyres

| Tyre – 7.5 R 16          |                  |                    |       |       |       |       |
|--------------------------|------------------|--------------------|-------|-------|-------|-------|
| Inflation pressure (kPa) | Normal load (kN) | Deflection (in mm) |       |       |       |       |
|                          |                  | Rep 1              | Rep 2 | Rep 3 | Rep 4 | Mean  |
| 448                      | 1                | 6.51               | 7.15  | 6.77  | 7.6   | 7.00  |
|                          | 2                | 10.99              | 11.59 | 12.74 | 12.43 | 11.93 |
|                          | 3                | 16.16              | 16.3  | 17.42 | 17.5  | 16.84 |
|                          | 4                | 21                 | 21.03 | 22.19 | 21.86 | 21.52 |
|                          | 5                | 25.57              | 25.4  | 26.25 | 26.22 | 25.86 |
|                          | 5.45             | 27.3               | 27.11 | 28.18 | 28.25 | 27.71 |
| 379                      | 1                | 7.97               | 6.95  | 7.2   | 6.88  | 7.25  |
|                          | 2                | 12.48              | 12.5  | 12.6  | 12.45 | 12.50 |
|                          | 3                | 17.75              | 17.94 | 17.8  | 17.83 | 17.83 |
|                          | 4                | 22.65              | 22.57 | 22.55 | 22.45 | 22.55 |
|                          | 5                | 26.99              | 27.14 | 26.84 | 27.21 | 27.04 |
|                          | 5.45             | 29.01              | 28.85 | 28.92 | 28.95 | 28.93 |
| 310                      | 1                | 7.7                | 7.33  | 7.78  | 7.78  | 7.64  |
|                          | 2                | 13.84              | 13.55 | 13.6  | 13.7  | 13.67 |
|                          | 3                | 19.67              | 19.18 | 19.55 | 19.64 | 19.51 |
|                          | 4                | 24.76              | 24.6  | 24.86 | 24.95 | 24.79 |
|                          | 5                | 29.73              | 29.9  | 29.75 | 29.7  | 29.77 |
|                          | 5.45             | 32.28              | 32.07 | 32.06 | 32.04 | 32.11 |
| 241                      | 1                | 8.32               | 8.45  | 8.3   | 8.65  | 8.43  |
|                          | 2                | 15.2               | 15.42 | 15.3  | 15.65 | 15.39 |
|                          | 3                | 21.78              | 21.46 | 21.5  | 21.51 | 21.56 |
|                          | 4                | 27.5               | 27.37 | 27.33 | 27.42 | 27.40 |
|                          | 5                | 33.3               | 33.01 | 33.96 | 32.8  | 33.26 |
|                          | 5.45             | 35.2               | 35.05 | 35.22 | 35.42 | 35.22 |
| 172                      | 1                | 9.1                | 9.06  | 8.83  | 9.55  | 9.13  |
|                          | 2                | 17.36              | 16.75 | 16.81 | 17.5  | 17.10 |
|                          | 3                | 24.56              | 23.9  | 23.69 | 24.38 | 24.13 |
|                          | 4                | 31.27              | 30.22 | 30.94 | 31.1  | 30.88 |
|                          | 5                | 37.37              | 36.5  | 36.6  | 37.9  | 37.09 |

| Tyre – 135/80R13         |                  |                    |       |       |       |       |
|--------------------------|------------------|--------------------|-------|-------|-------|-------|
| Inflation pressure (kPa) | Normal load (kN) | Deflection (in mm) |       |       |       |       |
|                          |                  | Rep 1              | Rep 2 | Rep 3 | Rep 4 | Mean  |
| 310                      | 0.5              | 6.24               | 6.13  | 6.45  | 6.04  | 6.21  |
|                          | 1.5              | 13.65              | 14.62 | 13.82 | 13.92 | 14.00 |
|                          | 2.5              | 20.36              | 20.35 | 20.76 | 20.82 | 20.57 |
|                          | 3                | 24.01              | 23.87 | 24.06 | 24.13 | 24.01 |
|                          | 3.5              | 26.95              | 27.31 | 27.4  | 27.32 | 27.24 |
|                          | 4                | 29.81              | 30.69 | 30.72 | 30.74 | 30.49 |
| 275                      | 0.5              | 6.24               | 6.25  | 6.5   | 6.25  | 6.31  |
|                          | 1.5              | 14.25              | 14.29 | 14.25 | 14.31 | 14.27 |
|                          | 2.5              | 21.49              | 21.47 | 21.51 | 21.53 | 21.5  |
|                          | 3                | 24.95              | 25.08 | 25.06 | 25.29 | 25.09 |
|                          | 3.5              | 28.37              | 28.52 | 28.55 | 28.64 | 28.52 |
|                          | 4                | 31.85              | 31.82 | 32.25 | 32.12 | 32.01 |
| 241                      | 0.5              | 6.92               | 6.42  | 6.51  | 6.67  | 6.63  |
|                          | 1.5              | 16.08              | 15.29 | 15.47 | 15.61 | 15.61 |
|                          | 2.5              | 24.24              | 23.54 | 23.7  | 23.75 | 23.80 |
|                          | 3                | 28.25              | 27.6  | 27.93 | 27.86 | 27.91 |
|                          | 3.5              | 32.28              | 31.59 | 31.72 | 31.82 | 31.85 |
|                          | 4                | 36                 | 35.32 | 35.36 | 35.5  | 35.54 |
| 206                      | 0.5              | 7.59               | 7.96  | 7.99  | 8.02  | 7.89  |
|                          | 1.5              | 16.79              | 17.09 | 17.13 | 17.1  | 17.02 |
|                          | 2.5              | 25.39              | 25.59 | 25.69 | 25.57 | 25.56 |
|                          | 3                | 29.47              | 29.92 | 30.11 | 29.68 | 29.79 |
|                          | 3.5              | 33.41              | 33.62 | 33.64 | 33.59 | 33.56 |
|                          | 4                | 37.42              | 37.71 | 37.6  | 37.54 | 37.56 |
| 172                      | 0.5              | 8.57               | 9.4   | 8.7   | 8.7   | 8.84  |
|                          | 1.5              | 18.25              | 19.01 | 18.44 | 18.3  | 18.5  |
|                          | 2.5              | 27.75              | 28.54 | 27.45 | 27.66 | 27.85 |
|                          | 3                | 32.39              | 32.91 | 32.04 | 32.19 | 32.38 |
|                          | 3.5              | 36.67              | 36.3  | 36.3  | 36.35 | 36.40 |
|                          | 4                | 40.99              | 40.58 | 40.88 | 40.92 | 40.84 |

| Tyre – 145/80R13         |                  |                    |       |       |       |       |
|--------------------------|------------------|--------------------|-------|-------|-------|-------|
| Inflation pressure (kPa) | Normal load (kN) | Deflection (in mm) |       |       |       |       |
|                          |                  | Rep 1              | Rep 2 | Rep 3 | Rep 4 | Mean  |
| 310                      | 0.5              | 4.8                | 5.09  | 5.16  | 5.49  | 5.13  |
|                          | 1.5              | 12.28              | 12.68 | 12.74 | 12.78 | 12.62 |
|                          | 2.5              | 18.91              | 19.5  | 19.66 | 19.63 | 19.42 |
|                          | 3                | 22.63              | 22.89 | 22.86 | 22.99 | 22.84 |
|                          | 3.5              | 25.88              | 26.08 | 26.15 | 26.16 | 26.06 |
|                          | 4                | 29.18              | 29.28 | 29.45 | 29.41 | 29.33 |
| 275                      | 0.5              | 5.69               | 5.82  | 5.79  | 5.8   | 5.77  |
|                          | 1.5              | 13.32              | 13.36 | 13.36 | 13.36 | 13.35 |
|                          | 2.5              | 20.41              | 20.49 | 20.49 | 20.46 | 20.46 |
|                          | 3                | 23.98              | 23.97 | 23.98 | 23.95 | 23.97 |
|                          | 3.5              | 27.31              | 27.4  | 27.34 | 27.36 | 27.35 |
|                          | 4                | 30.78              | 30.72 | 30.77 | 30.69 | 30.74 |
| 241                      | 0.5              | 5.84               | 6.05  | 6.12  | 6.28  | 6.07  |
|                          | 1.5              | 14.49              | 14.45 | 14.5  | 14.5  | 14.48 |
|                          | 2.5              | 22.25              | 22.53 | 22.38 | 22.85 | 22.50 |
|                          | 3                | 26.2               | 26.37 | 26.2  | 26.21 | 26.24 |
|                          | 3.5              | 30.11              | 30.17 | 30.1  | 30.08 | 30.11 |
|                          | 4                | 33.8               | 33.84 | 33.8  | 33.86 | 33.82 |
| 206                      | 0.5              | 6.4                | 6.55  | 6.43  | 6.35  | 6.43  |
|                          | 1.5              | 15                 | 15.21 | 15.21 | 15.14 | 15.14 |
|                          | 2.5              | 23.45              | 23.5  | 23.42 | 23.51 | 23.47 |
|                          | 3                | 27.58              | 27.46 | 27.31 | 27.49 | 27.46 |
|                          | 3.5              | 31.39              | 31.39 | 31.28 | 31.35 | 31.35 |
|                          | 4                | 35.32              | 35.24 | 35.2  | 35.19 | 35.23 |
| 172                      | 0.5              | 6.7                | 6.6   | 6.69  | 6.68  | 6.66  |
|                          | 1.5              | 16.29              | 16.38 | 16.35 | 16.32 | 16.33 |
|                          | 2.5              | 25.64              | 25.87 | 25.74 | 25.7  | 25.73 |
|                          | 3                | 30.05              | 30.37 | 30.36 | 30.3  | 30.27 |
|                          | 3.5              | 34.19              | 34.33 | 34.19 | 34.15 | 34.21 |
|                          | 4                | 38.2               | 38.1  | 38.3  | 38.15 | 38.18 |

## B.2 Tyre contact area at different normal loads and inflation pressures for test tyres

| Tyre – 7.5 R 16          |                  |                                    |        |        |        |        |
|--------------------------|------------------|------------------------------------|--------|--------|--------|--------|
| Inflation pressure (kPa) | Normal load (kN) | Contact area (in cm <sup>2</sup> ) |        |        |        |        |
|                          |                  | Rep 1                              | Rep 2  | Rep 3  | Rep 4  | Mean   |
| 448                      | 1                | 24.59                              | 30.06  | 29.44  | 29.78  | 28.97  |
|                          | 2                | 36.80                              | 41.72  | 47.15  | 56.31  | 45.50  |
|                          | 3                | 80.20                              | 97.62  | 88.04  | 100.25 | 91.52  |
|                          | 4                | 106.57                             | 113.80 | 133.04 | 121.68 | 118.77 |
|                          | 5                | 125.51                             | 137.15 | 150.32 | 149.22 | 140.55 |
|                          | 5.45             | 155.11                             | 161.85 | 160.49 | 162.85 | 160.07 |
| 379                      | 1                | 31.43                              | 31.88  | 37.36  | 30.11  | 32.69  |
|                          | 2                | 68.21                              | 76.90  | 77.73  | 78.34  | 75.29  |
|                          | 3                | 137.79                             | 137.44 | 144.62 | 131.77 | 137.90 |
|                          | 4                | 152.98                             | 149.94 | 151.79 | 147.05 | 150.44 |
|                          | 5                | 161.85                             | 173.56 | 169.75 | 163.86 | 167.25 |
|                          | 5.45             | 183.49                             | 175.54 | 178.67 | 181.07 | 179.69 |
| 310                      | 1                | 35.18                              | 31.88  | 37.35  | 35.05  | 34.87  |
|                          | 2                | 94.49                              | 102.39 | 105.17 | 94.07  | 99.03  |
|                          | 3                | 152.80                             | 147.77 | 136.38 | 157.57 | 148.63 |
|                          | 4                | 172.84                             | 190.10 | 172.18 | 169.83 | 176.24 |
|                          | 5                | 192.91                             | 203.71 | 193.32 | 196.02 | 196.49 |
|                          | 5.45             | 206.74                             | 208.28 | 209.95 | 209.34 | 208.58 |
| 241                      | 1                | 42.55                              | 46.015 | 33.25  | 51.45  | 43.31  |
|                          | 2                | 115.60                             | 128.45 | 115.60 | 124.24 | 120.97 |
|                          | 3                | 134.89                             | 159.05 | 161.98 | 162.55 | 154.62 |
|                          | 4                | 190.84                             | 184.53 | 190.33 | 190.10 | 188.95 |
|                          | 5                | 230.85                             | 222.16 | 224.79 | 230.90 | 227.17 |
|                          | 5.45             | 257.36                             | 253.81 | 249.93 | 244.84 | 251.49 |
| 172                      | 1                | 41.04                              | 62.57  | 61.82  | 59.79  | 56.31  |
|                          | 2                | 142.34                             | 138.93 | 148.90 | 139.40 | 142.39 |
|                          | 3                | 180.04                             | 181.07 | 186.60 | 175.92 | 180.91 |
|                          | 4                | 215.75                             | 234.76 | 213.11 | 225.40 | 222.25 |
|                          | 5                | 276.72                             | 265.77 | 274.44 | 266.90 | 270.96 |

| Tyre – 135/80R13         |                  |                                    |        |        |        |        |
|--------------------------|------------------|------------------------------------|--------|--------|--------|--------|
| Inflation pressure (kPa) | Normal load (kN) | Contact area (in cm <sup>2</sup> ) |        |        |        |        |
|                          |                  | Rep 1                              | Rep 2  | Rep 3  | Rep 4  | Mean   |
| 310                      | 0.5              | 18.09                              | 17.34  | 18.09  | 17.34  | 17.72  |
|                          | 1.5              | 49.47                              | 48.81  | 49.47  | 48.88  | 49.16  |
|                          | 2.5              | 68.90                              | 69.09  | 70.58  | 69.74  | 69.58  |
|                          | 3                | 80.37                              | 79.69  | 79.44  | 77.18  | 79.17  |
|                          | 3.5              | 90.87                              | 91.55  | 90.19  | 89.50  | 90.53  |
|                          | 4                | 97.44                              | 98.95  | 98.55  | 97.85  | 98.20  |
| 275                      | 0.5              | 18.85                              | 18.84  | 19.22  | 18.45  | 18.84  |
|                          | 1.5              | 51.33                              | 51.33  | 50.73  | 52.68  | 51.52  |
|                          | 2.5              | 72.35                              | 73.22  | 73.88  | 73.00  | 73.12  |
|                          | 3                | 83.44                              | 85.10  | 85.77  | 85.10  | 84.85  |
|                          | 3.5              | 92.92                              | 94.29  | 95.06  | 94.36  | 94.16  |
|                          | 4                | 104.34                             | 104.34 | 104.34 | 103.63 | 104.16 |
| 241                      | 0.5              | 21.59                              | 21.22  | 21.59  | 22.05  | 21.61  |
|                          | 1.5              | 58.65                              | 58.05  | 56.84  | 57.45  | 57.75  |
|                          | 2.5              | 83.78                              | 81.80  | 84.11  | 81.80  | 82.87  |
|                          | 3                | 97.44                              | 95.65  | 94.97  | 94.29  | 95.59  |
|                          | 3.5              | 108.85                             | 109.55 | 108.14 | 108.85 | 108.85 |
|                          | 4                | 119.35                             | 119.35 | 118.63 | 119.94 | 119.32 |
| 206                      | 0.5              | 23.74                              | 23.27  | 22.32  | 24.03  | 23.34  |
|                          | 1.5              | 60.64                              | 61.68  | 60.47  | 58.65  | 60.36  |
|                          | 2.5              | 88.78                              | 88.11  | 86.42  | 87.08  | 87.60  |
|                          | 3                | 100.90                             | 101.59 | 99.75  | 100.44 | 100.67 |
|                          | 3.5              | 113.09                             | 111.68 | 112.38 | 113.80 | 112.74 |
|                          | 4                | 124.35                             | 125.78 | 125.07 | 123.64 | 124.71 |
| 172                      | 0.5              | 25.32                              | 25.80  | 26.22  | 27.05  | 26.10  |
|                          | 1.5              | 63.70                              | 60.44  | 64.52  | 67.00  | 63.92  |
|                          | 2.5              | 94.99                              | 94.33  | 93.01  | 91.70  | 93.51  |
|                          | 3                | 107.81                             | 105.90 | 106.59 | 107.27 | 106.89 |
|                          | 3.5              | 121.57                             | 122.28 | 121.57 | 122.98 | 122.10 |
|                          | 4                | 133.64                             | 132.93 | 132.93 | 132.21 | 132.93 |

| Tyre – 145/80R13         |                  |                                    |        |        |        |        |
|--------------------------|------------------|------------------------------------|--------|--------|--------|--------|
| Inflation pressure (kPa) | Normal load (kN) | Contact area (in cm <sup>2</sup> ) |        |        |        |        |
|                          |                  | Rep 1                              | Rep 2  | Rep 3  | Rep 4  | Mean   |
| 310                      | 0.5              | 19.62                              | 20.02  | 20.02  | 20.41  | 20.02  |
|                          | 1.5              | 50.45                              | 51.02  | 51.24  | 52.30  | 51.25  |
|                          | 2.5              | 74.31                              | 75.86  | 75.64  | 74.76  | 75.14  |
|                          | 3                | 86.77                              | 88.07  | 85.70  | 86.39  | 86.73  |
|                          | 3.5              | 96.12                              | 96.12  | 96.12  | 95.42  | 95.95  |
|                          | 4                | 105.49                             | 105.49 | 105.49 | 106.21 | 105.67 |
| 275                      | 0.5              | 19.62                              | 21.16  | 20.30  | 20.01  | 20.27  |
|                          | 1.5              | 55.03                              | 53.71  | 54.19  | 54.91  | 54.46  |
|                          | 2.5              | 78.34                              | 78.34  | 78.10  | 77.43  | 78.06  |
|                          | 3                | 91.56                              | 91.56  | 92.26  | 92.96  | 92.09  |
|                          | 3.5              | 100.37                             | 100.05 | 100.77 | 101.78 | 100.74 |
|                          | 4                | 110.29                             | 109.55 | 110.29 | 110.29 | 110.10 |
| 241                      | 0.5              | 21.20                              | 21.59  | 20.39  | 22.83  | 21.50  |
|                          | 1.5              | 60.80                              | 59.26  | 60.64  | 59.42  | 60.03  |
|                          | 2.5              | 88.82                              | 88.14  | 88.14  | 88.82  | 88.48  |
|                          | 3                | 101.78                             | 101.78 | 101.48 | 97.15  | 100.55 |
|                          | 3.5              | 111.99                             | 112.71 | 113.43 | 111.99 | 112.53 |
|                          | 4                | 123.28                             | 123.28 | 124.02 | 123.28 | 123.47 |
| 206                      | 0.5              | 23.63                              | 23.95  | 24.83  | 23.95  | 24.09  |
|                          | 1.5              | 64.08                              | 63.70  | 62.66  | 61.87  | 63.08  |
|                          | 2.5              | 93.99                              | 92.92  | 93.60  | 92.61  | 93.28  |
|                          | 3                | 103.90                             | 104.61 | 102.49 | 106.48 | 104.37 |
|                          | 3.5              | 116.86                             | 115.60 | 112.92 | 119.78 | 116.29 |
|                          | 4                | 131.40                             | 129.93 | 129.19 | 129.93 | 130.11 |
| 172                      | 0.5              | 25.32                              | 24.83  | 24.91  | 24.43  | 24.87  |
|                          | 1.5              | 70.40                              | 71.24  | 71.24  | 68.24  | 70.28  |
|                          | 2.5              | 103.63                             | 100.21 | 101.78 | 99.52  | 101.28 |
|                          | 3                | 119.21                             | 119.21 | 115.77 | 112.53 | 116.68 |
|                          | 3.5              | 128.55                             | 127.08 | 123.55 | 122.92 | 125.53 |
|                          | 4                | 142.49                             | 143.25 | 144.76 | 142.50 | 143.25 |

### B.3 Full scale tyre deflection and contact area data used for model validation

| Tyre - 7.5R15         |                                     |                                       |                     |                                |               |
|-----------------------|-------------------------------------|---------------------------------------|---------------------|--------------------------------|---------------|
| Deflection<br>(in mm) | Deflection<br>Ratio ( $\delta/h$ )% | Contact area<br>(in cm <sup>2</sup> ) | Normal<br>load (kN) | Inflation<br>pressure<br>(kPa) | $P_i b d / W$ |
| 6.5                   | 0.0390                              | 31.5                                  | 0.82                | 448.1594                       | 82.6413       |
| 10                    | 0.0605                              | 53.9                                  | 1.63                |                                | 41.5741       |
| 14.6                  | 0.0888                              | 78                                    | 2.63                |                                | 25.7665       |
| 18                    | 0.1090                              | 81.03                                 | 3.24                |                                | 20.9154       |
| 20                    | 0.1211                              | 116.26                                | 3.61                |                                | 18.7717       |
| 6.5                   | 0.0392                              | 33.9                                  | 0.75                | 379.2118                       | 76.458        |
| 10.8                  | 0.06556                             | 58.6                                  | 1.53                |                                | 37.473        |
| 15                    | 0.0908                              | 81.8                                  | 2.44                |                                | 23.501        |
| 18                    | 0.109                               | 99.8                                  | 3.05                |                                | 18.801        |
| 20                    | 0.1211                              | 122.66                                | 3.4                 |                                | 16.868        |
| 6.6                   | 0.0402                              | 37.65                                 | 0.68                | 310.2642                       | 68.994        |
| 11                    | 0.0665                              | 63                                    | 1.41                |                                | 33.279        |
| 15.5                  | 0.0938                              | 92.56                                 | 2.22                |                                | 21.138        |
| 18                    | 0.1090                              | 102                                   | 2.72                |                                | 17.241        |
| 20                    | 0.1211                              | 132.9                                 | 3.02                |                                | 15.537        |
| 7.3                   | 0.0442                              | 40.7                                  | 0.65                | 241.3166                       | 56.134        |
| 11.1                  | 0.0675                              | 70.01                                 | 1.27                |                                | 28.737        |
| 15                    | 0.0908                              | 98.9                                  | 1.99                |                                | 18.333        |
| 18                    | 0.1090                              | 117.6                                 | 2.45                |                                | 14.896        |
| 20                    | 0.1211                              | 135                                   | 2.65                |                                | 13.765        |



## B.4 NATO Experimental Data used in present study

### B.4.1 Direct Shear test

| 2NS dry medium sand              |                                |
|----------------------------------|--------------------------------|
| Final Failure Shear Stress (psi) | Normal Stress at Failure (psi) |
| 2.094526466                      | 1.852711569                    |
| 4.316428181                      | 3.675512736                    |
| 5.494542326                      | 5.571543591                    |
| 5.607680552                      | 7.344320169                    |
| 49.75460363                      | 22.18628103                    |

| 2NS 3%MC medium sand             |                                |
|----------------------------------|--------------------------------|
| Final Failure Shear Stress (psi) | Normal Stress at Failure (psi) |
| 1.179782915                      | 1.877392904                    |
| 2.376489194                      | 3.693335307                    |
| 2.729804517                      | 5.722001832                    |
| 4.615093154                      | 7.458209062                    |
| 7.673738591                      | 22.41700337                    |

| 2NS 3% MC medium sand - Trail 2  |                                |
|----------------------------------|--------------------------------|
| Final Failure Shear Stress (psi) | Normal Stress at Failure (psi) |
| 1.617382641                      | 1.935301515                    |
| 2.211654842                      | 3.893945494                    |
| 4.153051752                      | 5.636754403                    |
| 4.804227797                      | 7.391126148                    |
| 9.087460298                      | 23.24310347                    |

| 2NS 11%MC medium sand            |                                |
|----------------------------------|--------------------------------|
| Final Failure Shear Stress (psi) | Normal Stress at Failure (psi) |
| 1.347800097                      | 1.880392399                    |
| 2.485634412                      | 3.807150392                    |
| 4.418076932                      | 5.484227347                    |
| 4.444719239                      | 7.356398474                    |
| 15.10223652                      | 22.08562581                    |

| Fine-grained sand-dry            |                                |
|----------------------------------|--------------------------------|
| Final Failure Shear Stress (psi) | Normal Stress at Failure (psi) |
| 1.307981858                      | 1.85025621                     |
| 2.365835681                      | 3.687922298                    |
| 3.225661121                      | 5.534635635                    |
| 4.822692748                      | 7.41953379                     |
| 16.13060091                      | 22.80550796                    |

| Fine-grained sand 3MC            |                                |
|----------------------------------|--------------------------------|
| Final Failure Shear Stress (psi) | Normal Stress at Failure (psi) |
| 0.981661083                      | 1.837804308                    |
| 2.272005604                      | 3.637649498                    |
| 3.328309987                      | 5.652122241                    |
| 4.347647028                      | 7.493498662                    |
| 11.18685328                      | 22.44075308                    |

| Fine-grained sand - 6.5% MC      |                                |
|----------------------------------|--------------------------------|
| Final Failure Shear Stress (psi) | Normal Stress at Failure (psi) |
| 1.515416211                      | 1.936766329                    |
| 1.502784281                      | 3.87374532                     |
| 3.202064008                      | 5.60076973                     |
| 5.259970215                      | 7.430487338                    |
| 14.93704343                      | 22.02933307                    |

| Fine-grained sand - 18.55%MC     |                                |
|----------------------------------|--------------------------------|
| Final Failure Shear Stress (psi) | Normal Stress at Failure (psi) |
| 1.859422722                      | 1.851277103                    |
| 3.832972644                      | 3.684932572                    |
| 6.062645713                      | 5.528989338                    |
| 6.029535001                      | 7.331136661                    |
| 22.36189876                      | 22.11849911                    |

#### B.4.2 Cone penetrometer test

| 2NS sand    |          |
|-------------|----------|
| Sinkage [m] | CI (kPa) |
| 0           | 0        |
| 0.0254      | 27.579   |
| 0.0508      | 96.527   |
| 0.0762      | 206.84   |
| 0.1016      | 372.32   |

| Fine-grained sand |              |
|-------------------|--------------|
| CI (kPa)          | Sinkage [mm] |
| 0                 | 0            |
| 13.78952          | 25.4         |
| 27.57904          | 50.8         |
| 41.36856          | 76.2         |
| 41.36856          | 101.6        |
| 27.57904          | 127          |
| 41.36856          | 152.4        |
| 41.36856          | 177.8        |

## B.5 Engineering Technical Datasheet, 335/65R22.5

| Confidential                                                                       |             | ENG DATA SHEET        |                   | Page 1 |      |      |      |      |
|------------------------------------------------------------------------------------|-------------|-----------------------|-------------------|--------|------|------|------|------|
| <b>GOODYEAR TECHNICAL CENTER - AKRON</b>                                           |             |                       |                   |        |      |      |      |      |
| AKRON, OH 44316-0001                                                               |             |                       |                   |        |      |      |      |      |
| <b>** RADIAL TRUCK - ENGINEERING DATA SHEET **</b>                                 |             |                       |                   |        |      |      |      |      |
| TIRE SIZE                                                                          | 335/65R22.5 |                       |                   |        |      |      |      |      |
| LI                                                                                 | 147 M       |                       |                   |        |      |      |      |      |
| TIRE DESCRIPTION                                                                   | G275 MSA    |                       |                   |        |      |      |      |      |
| MAX SPEED - MPH                                                                    | 81          |                       |                   |        |      |      |      |      |
| LOAD RANGE / PLY RATING                                                            | G / 14      |                       |                   |        |      |      |      |      |
|                                                                                    | KPA         | 450                   | 480               | 520    | 550  | 590  | 620  | 660  |
|                                                                                    | KGS         |                       |                   |        |      |      |      |      |
| LOAD CAPACITY: SINGLE                                                              |             | 2360                  | 2460              | 2590   | 2725 | 2820 | 2940 | 3075 |
| DUAL                                                                               |             |                       |                   |        |      |      |      |      |
|                                                                                    | PSI         | 65                    | 70                | 75     | 80   | 85   | 90   | 95   |
|                                                                                    | LBS         |                       |                   |        |      |      |      |      |
| LOAD CAPACITY: SINGLE                                                              |             | 5205                  | 5430              | 5700   | 6005 | 6220 | 6475 | 6780 |
| DUAL                                                                               |             |                       |                   |        |      |      |      |      |
| TIRE WEIGHT                                                                        | 122 LBS     |                       |                   |        |      |      |      |      |
| WIDTHS-MEASURING RIM                                                               | 9.0 IN      |                       |                   |        |      |      |      |      |
| -NEW TIRE                                                                          | 13.15 IN    |                       | 334 mm            |        |      |      |      |      |
| -LOADED TIRE                                                                       | 14.10 IN    |                       | 358 mm            |        |      |      |      |      |
| OVERALL DIAMETER                                                                   | 39.8 IN     |                       |                   |        |      |      |      |      |
| STATIC LOADED RADIUS                                                               | 18.3 IN     |                       | RIM 22.5          |        |      |      |      |      |
| REVOLUTIONS PER MILE                                                               | 522 RPM     |                       |                   |        |      |      |      |      |
| TREAD-CONTACT WIDTH                                                                | 9.95 IN     |                       | 6780 lbs @ 95 psi |        |      |      |      |      |
| TREAD-CONTACT LENGTH                                                               | 9.14 IN     |                       | 6780 lbs @ 95 psi |        |      |      |      |      |
| -TREAD RADIUS                                                                      | 35.3 IN     |                       |                   |        |      |      |      |      |
| -DEPTH                                                                             | 20/32       |                       | or 0.630 IN       |        |      |      |      |      |
| FOOTPRINT AREA (GROSS)                                                             | 88.3 SQ IN  |                       | 6780 lbs @ 95 psi |        |      |      |      |      |
| NET TO GROSS                                                                       | 63.2%       |                       |                   |        |      |      |      |      |
| BELT MATERIAL                                                                      | 3 Ply Steel |                       |                   |        |      |      |      |      |
| CARCASS MATERIAL                                                                   | 1 Ply Steel |                       |                   |        |      |      |      |      |
|  |             |                       |                   |        |      |      |      |      |
| Confidential                                                                       |             | Issued 09/25/08 Rev 1 |                   |        |      |      |      |      |

## Appendix C Simulation data

### C.1 Direct shear test

| 2NS sand                         |                                |
|----------------------------------|--------------------------------|
| Final Failure Shear Stress (kPa) | Normal Stress at Failure (kPa) |
| 36.6111756                       | 55.19944856                    |
| 23.55939492                      | 41.36856                       |
| 19.0295376                       | 27.57904                       |
| 12.9621488                       | 13.78952                       |

| Fine-grained sand                |                                |
|----------------------------------|--------------------------------|
| Final Failure Shear Stress (kPa) | Normal Stress at Failure (kPa) |
| 40.01                            | 55.15                          |
| 30.83                            | 41.36                          |
| 19.01                            | 27.57                          |
| 11.02                            | 13.8                           |

### C.2 Shear ring test

| 2NS sand Loc-1 |                     |                    |
|----------------|---------------------|--------------------|
| Moment (N-m)   | Normal stress (kPa) | Shear stress (kPa) |
| 23.48          | 8.62                | 9.615066349        |
| 35.91          | 17.24               | 14.70515471        |
| 56.14          | 25.85               | 22.98934518        |

| 2NS sand Loc-2 |                     |                    |
|----------------|---------------------|--------------------|
| Moment (N-m)   | Normal stress (kPa) | Shear stress (kPa) |
| 25.38          | 8.62                | 10.39311686        |
| 49.15          | 17.24               | 20.12693829        |
| 61.23          | 25.85               | 25.07370156        |

| 2NS sand Loc-3 |                     |                    |
|----------------|---------------------|--------------------|
| Moment (N-m)   | Normal stress (kPa) | Shear stress (kPa) |
| 11.88          | 8.62                | 4.864863212        |
| 27.81          | 17.24               | 11.38820252        |
| 47.8           | 25.85               | 19.57411292        |

| 2NS sand Loc-4 |                     |                    |
|----------------|---------------------|--------------------|
| Moment (N-m)   | Normal stress (kPa) | Shear stress (kPa) |
| 21.47          | 8.62                | 8.791970805        |
| 35.3           | 17.24               | 14.45535954        |
| 49.11          | 25.85               | 20.11055828        |

| 2NS sand Loc-5 |                     |                    |
|----------------|---------------------|--------------------|
| Moment (N-m)   | Normal stress (kPa) | Shear stress (kPa) |
| 14.48          | 8.62                | 5.929563915        |
| 28.38          | 17.24               | 11.62161767        |
| 44.9           | 25.85               | 18.38656214        |

| 2NS sand Loc-6 |                     |                    |
|----------------|---------------------|--------------------|
| Moment (N-m)   | Normal stress (kPa) | Shear stress (kPa) |
| 20.55          | 8.62                | 8.415230556        |
| 42.54          | 17.24               | 17.4201415         |
| 52.8           | 25.85               | 21.62161428        |

| Fine-grained sand Loc-1 |                     |                    |
|-------------------------|---------------------|--------------------|
| Moment (N-m)            | Normal stress (kPa) | Shear stress (kPa) |
| 10.34                   | 8.62                | 4.234232796        |
| 31.12                   | 17.24               | 12.74364841        |
| 40.7                    | 25.85               | 16.666661          |

| Fine-grained sand Loc-2 |                     |                    |
|-------------------------|---------------------|--------------------|
| Moment (N-m)            | Normal stress (kPa) | Shear stress (kPa) |
| 12.55                   | 8.62                | 5.139228393        |
| 25.1                    | 17.24               | 10.27845679        |
| 40.54                   | 25.85               | 16.60114096        |

| Fine-grained sand Loc-3 |                     |                    |
|-------------------------|---------------------|--------------------|
| Moment (N-m)            | Normal stress (kPa) | Shear stress (kPa) |
| 15.106                  | 8.62                | 6.185911084        |
| 24.9                    | 17.24               | 10.19655673        |
| 35.46                   | 25.85               | 14.52087959        |

| Fine-grained sand Loc-4 |                     |                    |
|-------------------------|---------------------|--------------------|
| Moment (N-m)            | Normal stress (kPa) | Shear stress (kPa) |
| 9.85                    | 8.62                | 4.033577663        |
| 21.2                    | 17.24               | 8.681405732        |
| 27.7                    | 25.85               | 11.34315749        |

| Fine-grained sand Loc-5 |                     |                    |
|-------------------------|---------------------|--------------------|
| Moment (N-m)            | Normal stress (kPa) | Shear stress (kPa) |
| 10.46                   | 8.62                | 4.283372828        |
| 20.9                    | 17.24               | 8.558555651        |
| 21.76                   | 25.85               | 8.910725884        |

| Fine-grained sand Loc-6 |                     |                    |
|-------------------------|---------------------|--------------------|
| Moment (N-m)            | Normal stress (kPa) | Shear stress (Kpa) |
| 8.8                     | 8.62                | 3.603602379        |
| 20.1                    | 17.24               | 8.230955435        |
| 31.31                   | 25.85               | 12.82145347        |

### C.3 Mobility performance test based on soil conditions

#### C.3.1 2NS sand – Homogeneous model at full load and 0.15 m pre-compaction

| Scale =1 |        |        |            |
|----------|--------|--------|------------|
| Slip     | D/W    | T/rW   | Efficiency |
| -0.0041  | 0.0402 | 0.1462 | 0.27587    |
| 0.0202   | 0.0771 | 0.1722 | 0.43892    |
| 0.0445   | 0.1221 | 0.2376 | 0.49103    |
| 0.0688   | 0.1719 | 0.2717 | 0.58911    |
| 0.093    | 0.2526 | 0.3532 | 0.6486     |
| 0.1186   | 0.2559 | 0.3526 | 0.63973    |
| 0.1439   | 0.2878 | 0.4083 | 0.60346    |
| 0.1689   | 0.349  | 0.4439 | 0.65344    |
| 0.1942   | 0.3453 | 0.4564 | 0.60969    |
| 0.2206   | 0.3456 | 0.4568 | 0.58973    |
| 0.245    | 0.3992 | 0.4917 | 0.61299    |
| 0.2705   | 0.4302 | 0.5275 | 0.595      |
| 0.2958   | 0.4182 | 0.4861 | 0.60588    |
| 0.3209   | 0.4133 | 0.5157 | 0.54429    |
| 0.3468   | 0.375  | 0.4835 | 0.50658    |
| 0.3709   | 0.3939 | 0.4853 | 0.51059    |
| 0.3948   | 0.4231 | 0.5412 | 0.47307    |
| 0.4189   | 0.4265 | 0.5349 | 0.4634     |
| 0.4416   | 0.5009 | 0.6015 | 0.465      |
| 0.4648   | 0.5642 | 0.6489 | 0.46532    |

|        |        |        |         |
|--------|--------|--------|---------|
| 0.4885 | 0.5732 | 0.6481 | 0.45237 |
| 0.5143 | 0.4535 | 0.5249 | 0.41964 |
| 0.5391 | 0.4415 | 0.5083 | 0.40038 |
| 0.5639 | 0.4292 | 0.5207 | 0.35944 |
| 0.5887 | 0.449  | 0.5308 | 0.34795 |
| 0.6131 | 0.4561 | 0.542  | 0.32559 |
| 0.6385 | 0.4807 | 0.5745 | 0.30253 |
| 0.6637 | 0.4936 | 0.5797 | 0.28636 |
| 0.6896 | 0.4764 | 0.5517 | 0.26803 |
| 0.715  | 0.4866 | 0.565  | 0.24546 |
| 0.741  | 0.5087 | 0.5928 | 0.22229 |
| 0.7674 | 0.458  | 0.5225 | 0.20389 |
| 0.7932 | 0.4684 | 0.5403 | 0.17923 |
| 0.8197 | 0.435  | 0.491  | 0.15971 |
| 0.8447 | 0.4908 | 0.5484 | 0.13895 |
| 0.8706 | 0.4416 | 0.5196 | 0.10995 |
| 0.8954 | 0.4725 | 0.5394 | 0.09166 |
| 0.9198 | 0.4831 | 0.5548 | 0.06985 |
| 0.9441 | 0.5298 | 0.6042 | 0.04904 |
| 0.9683 | 0.5142 | 0.5923 | 0.02752 |
| 0.9924 | 0.5219 | 0.6018 | 0.00658 |

| Scale = 0.7 |        |        |            |
|-------------|--------|--------|------------|
| Slip        | D/W    | T/rW   | Efficiency |
| -0.0039     | 0.0282 | 0.0948 | 0.298452   |
| 0.0436      | 0.1263 | 0.1922 | 0.628609   |
| 0.0919      | 0.2019 | 0.2743 | 0.66832    |
| 0.1426      | 0.2416 | 0.2881 | 0.718966   |
| 0.192       | 0.3172 | 0.3991 | 0.642086   |
| 0.2441      | 0.3194 | 0.4142 | 0.582947   |
| 0.2918      | 0.4673 | 0.5034 | 0.657458   |
| 0.3439      | 0.3624 | 0.4333 | 0.54871    |
| 0.3608      | 0.394  | 0.4675 | 0.538745   |
| 0.3826      | 0.3572 | 0.4328 | 0.509601   |
| 0.3931      | 0.3871 | 0.4829 | 0.486466   |
| 0.4176      | 0.3935 | 0.513  | 0.446775   |
| 0.4404      | 0.4169 | 0.5037 | 0.463129   |
| 0.464       | 0.397  | 0.4774 | 0.445658   |
| 0.4887      | 0.4388 | 0.5336 | 0.42052    |



|        |        |        |          |
|--------|--------|--------|----------|
| 0.5116 | 0.3999 | 0.4949 | 0.394667 |
| 0.5349 | 0.5294 | 0.5763 | 0.427154 |
| 0.5598 | 0.4961 | 0.5565 | 0.392408 |
| 0.5867 | 0.3858 | 0.496  | 0.321516 |
| 0.6106 | 0.4141 | 0.4906 | 0.328589 |
| 0.6349 | 0.4517 | 0.5025 | 0.328176 |
| 0.6609 | 0.4497 | 0.505  | 0.302015 |
| 0.6865 | 0.4511 | 0.5126 | 0.275859 |
| 0.7129 | 0.3976 | 0.4698 | 0.243019 |
| 0.7381 | 0.4233 | 0.492  | 0.225346 |
| 0.7655 | 0.3921 | 0.4667 | 0.19701  |
| 0.7911 | 0.4236 | 0.5256 | 0.168407 |
| 0.8164 | 0.4252 | 0.5128 | 0.152234 |
| 0.841  | 0.4714 | 0.5607 | 0.133664 |
| 0.8674 | 0.418  | 0.5136 | 0.107925 |
| 0.8927 | 0.4184 | 0.5279 | 0.085014 |
| 0.9179 | 0.4212 | 0.5478 | 0.063156 |
| 0.9426 | 0.4275 | 0.5587 | 0.043911 |
| 0.9668 | 0.4423 | 0.5665 | 0.025939 |
| 0.9902 | 0.4699 | 0.5877 | 0.007818 |

| Scale = 0.5 |        |        |            |
|-------------|--------|--------|------------|
| Slip        | D/W    | T/rW   | Efficiency |
| -0.0016     | -0.036 | 0.0666 | -0.54224   |
| 0.0198      | 0.0601 | 0.1287 | 0.45787    |
| 0.0429      | 0.0998 | 0.1166 | 0.81979    |
| 0.0665      | 0.161  | 0.2523 | 0.59567    |
| 0.0918      | 0.1607 | 0.2205 | 0.66176    |
| 0.116       | 0.2159 | 0.3117 | 0.61239    |
| 0.1404      | 0.2443 | 0.3222 | 0.65166    |
| 0.1659      | 0.2525 | 0.3309 | 0.63638    |
| 0.1908      | 0.285  | 0.3499 | 0.65913    |
| 0.2137      | 0.3786 | 0.4357 | 0.68336    |
| 0.2415      | 0.313  | 0.3182 | 0.74605    |
| 0.268       | 0.2692 | 0.3768 | 0.52301    |
| 0.2916      | 0.3102 | 0.3975 | 0.55282    |
| 0.3135      | 0.4057 | 0.4843 | 0.57512    |
| 0.3431      | 0.312  | 0.3918 | 0.52306    |
| 0.3676      | 0.3321 | 0.3892 | 0.53966    |

|        |        |        |         |
|--------|--------|--------|---------|
| 0.3888 | 0.3873 | 0.4291 | 0.55163 |
| 0.4152 | 0.2808 | 0.3941 | 0.41671 |
| 0.4349 | 0.4656 | 0.5775 | 0.45557 |
| 0.4589 | 0.4602 | 0.5806 | 0.42889 |
| 0.4856 | 0.4066 | 0.4784 | 0.43726 |
| 0.5059 | 0.5063 | 0.5694 | 0.43939 |
| 0.5304 | 0.4317 | 0.5383 | 0.37662 |
| 0.5546 | 0.4865 | 0.56   | 0.38696 |
| 0.5803 | 0.4474 | 0.5128 | 0.36617 |
| 0.6029 | 0.511  | 0.5624 | 0.36071 |
| 0.6276 | 0.5724 | 0.5935 | 0.35914 |
| 0.6558 | 0.4288 | 0.4743 | 0.31115 |
| 0.6822 | 0.4593 | 0.537  | 0.27185 |
| 0.7094 | 0.3802 | 0.4612 | 0.23963 |
| 0.7144 | 0.3938 | 0.4878 | 0.23058 |
| 0.7158 | 0.3362 | 0.4154 | 0.23    |
| 0.7344 | 0.4362 | 0.5211 | 0.22234 |
| 0.7585 | 0.4891 | 0.5361 | 0.22034 |
| 0.7836 | 0.5024 | 0.5416 | 0.20069 |
| 0.8099 | 0.5018 | 0.5479 | 0.17413 |
| 0.8354 | 0.5006 | 0.5575 | 0.14782 |
| 0.8617 | 0.4143 | 0.4841 | 0.11835 |
| 0.8873 | 0.412  | 0.4958 | 0.0936  |
| 0.9126 | 0.4563 | 0.5726 | 0.06967 |
| 0.9372 | 0.4379 | 0.5256 | 0.05235 |
| 0.9609 | 0.4821 | 0.5661 | 0.03331 |
| 0.9852 | 0.4458 | 0.5206 | 0.01264 |

| Scale = 0.25 |         |        |            |
|--------------|---------|--------|------------|
| Slip         | D/W     | T/rW   | Efficiency |
| -0.0033      | -0.0144 | 0.0022 | -6.46938   |
| 0.0267       | -0.0332 | 0.2009 | -0.16096   |
| 0.0492       | 0.0257  | 0.2631 | 0.09298    |
| 0.0669       | 0.0701  | 0.0936 | 0.69886    |
| 0.0945       | 0.0819  | 0.0785 | 0.94489    |
| 0.1132       | 0.1628  | 0.3005 | 0.48035    |
| 0.1199       | 0.4344  | 0.4674 | 0.81796    |
| 0.1473       | 0.3918  | 0.3976 | 0.84027    |
| 0.183        | 0.2776  | 0.3739 | 0.6065     |

|        |        |        |         |
|--------|--------|--------|---------|
| 0.2029 | 0.3583 | 0.3884 | 0.73528 |
| 0.2226 | 0.4301 | 0.4079 | 0.81967 |
| 0.2545 | 0.3245 | 0.3251 | 0.74408 |
| 0.2899 | 0.1715 | 0.2637 | 0.46184 |
| 0.3072 | 0.2936 | 0.3942 | 0.51601 |
| 0.3264 | 0.4287 | 0.4922 | 0.58669 |
| 0.3539 | 0.3081 | 0.459  | 0.43373 |
| 0.3742 | 0.4232 | 0.5524 | 0.47948 |
| 0.3934 | 0.4518 | 0.4491 | 0.61028 |
| 0.4192 | 0.4781 | 0.5084 | 0.54616 |
| 0.4499 | 0.3049 | 0.4078 | 0.4113  |
| 0.4796 | 0.2767 | 0.3242 | 0.44424 |
| 0.4928 | 0.345  | 0.4035 | 0.43374 |
| 0.5192 | 0.2546 | 0.394  | 0.31064 |
| 0.5471 | 0.2873 | 0.4792 | 0.27157 |
| 0.5718 | 0.3176 | 0.6251 | 0.21755 |
| 0.5954 | 0.1911 | 0.5929 | 0.13043 |
| 0.6003 | 0.8487 | 0.9709 | 0.34936 |
| 0.6348 | 0.7485 | 0.8494 | 0.32185 |
| 0.6583 | 0.3862 | 0.5944 | 0.22202 |
| 0.6834 | 0.5915 | 0.6951 | 0.26937 |
| 0.7154 | 0.5397 | 0.675  | 0.22759 |
| 0.7478 | 0.4116 | 0.4887 | 0.21239 |
| 0.7737 | 0.4131 | 0.4322 | 0.21628 |
| 0.8009 | 0.3968 | 0.4198 | 0.1882  |
| 0.823  | 0.4481 | 0.5207 | 0.1523  |
| 0.8478 | 0.4593 | 0.5502 | 0.12709 |
| 0.8734 | 0.4024 | 0.4982 | 0.10223 |
| 0.8951 | 0.3797 | 0.4361 | 0.09132 |
| 0.9145 | 0.508  | 0.5409 | 0.0803  |
| 0.9429 | 0.4746 | 0.513  | 0.05285 |
| 0.9669 | 0.4945 | 0.5437 | 0.03013 |

### C.3.2 2NS sand – Heterogeneous model at full load and 0.15 m pre-compaction

| Scale =1 |         |       |            |
|----------|---------|-------|------------|
| Slip     | D/W     | T/rW  | Efficiency |
| -0.0028  | -0.0422 | 0.038 | -1.1149    |

|        |         |        |         |
|--------|---------|--------|---------|
| 0.0218 | -0.0117 | 0.0829 | -0.1384 |
| 0.0459 | 0.0367  | 0.1398 | 0.25046 |
| 0.0698 | 0.1233  | 0.2267 | 0.50597 |
| 0.0946 | 0.1726  | 0.2977 | 0.52485 |
| 0.1189 | 0.2495  | 0.3507 | 0.62685 |
| 0.1444 | 0.2591  | 0.3357 | 0.66036 |
| 0.1699 | 0.2657  | 0.3872 | 0.56947 |
| 0.1961 | 0.2656  | 0.3504 | 0.60928 |
| 0.2231 | 0.1993  | 0.3033 | 0.5106  |
| 0.2472 | 0.3028  | 0.3855 | 0.59125 |
| 0.273  | 0.2843  | 0.3511 | 0.58877 |
| 0.2987 | 0.2687  | 0.346  | 0.54473 |
| 0.3237 | 0.3077  | 0.44   | 0.47296 |
| 0.3473 | 0.3487  | 0.4446 | 0.51188 |
| 0.372  | 0.3455  | 0.3943 | 0.55041 |
| 0.3974 | 0.2626  | 0.3203 | 0.49405 |
| 0.4213 | 0.3015  | 0.3844 | 0.4539  |
| 0.4458 | 0.2737  | 0.3647 | 0.41594 |
| 0.4692 | 0.2781  | 0.4069 | 0.36267 |
| 0.4928 | 0.3516  | 0.4468 | 0.39911 |
| 0.5165 | 0.3623  | 0.4324 | 0.40514 |
| 0.5413 | 0.3012  | 0.3776 | 0.36602 |
| 0.5665 | 0.297   | 0.3656 | 0.35217 |
| 0.5899 | 0.3687  | 0.4959 | 0.30488 |
| 0.6148 | 0.3933  | 0.4905 | 0.30891 |
| 0.6392 | 0.4305  | 0.5244 | 0.29622 |
| 0.6651 | 0.3934  | 0.4921 | 0.26768 |
| 0.6908 | 0.3912  | 0.5158 | 0.23453 |
| 0.7173 | 0.3562  | 0.4369 | 0.23046 |
| 0.7428 | 0.4061  | 0.5218 | 0.20014 |
| 0.7698 | 0.3655  | 0.4427 | 0.19008 |
| 0.7952 | 0.367   | 0.4565 | 0.16461 |
| 0.8208 | 0.385   | 0.5109 | 0.135   |
| 0.8466 | 0.3852  | 0.4738 | 0.12474 |
| 0.8716 | 0.4027  | 0.4924 | 0.10505 |
| 0.8961 | 0.4314  | 0.5295 | 0.08468 |
| 0.9205 | 0.4421  | 0.5597 | 0.06276 |
| 0.9453 | 0.4066  | 0.5075 | 0.04387 |
| 0.9698 | 0.4401  | 0.5236 | 0.02538 |
| 0.994  | 0.4337  | 0.5174 | 0.00507 |

| Scale = 0.7 |        |        |            |
|-------------|--------|--------|------------|
| Slip        | D/W    | T/rW   | Efficiency |
| -0.0043     | 0.0518 | 0.1566 | 0.33199    |
| 0.0181      | 0.1557 | 0.2199 | 0.69522    |
| 0.0408      | 0.2405 | 0.2579 | 0.89446    |
| 0.0674      | 0.1637 | 0.1722 | 0.88679    |
| 0.0958      | 0.0618 | 0.1298 | 0.43052    |
| 0.1224      | 0.0265 | 0.1154 | 0.20175    |
| 0.1465      | 0.0961 | 0.2359 | 0.34755    |
| 0.1706      | 0.2005 | 0.257  | 0.64702    |
| 0.1944      | 0.2424 | 0.3589 | 0.544      |
| 0.2213      | 0.2152 | 0.3106 | 0.5394     |
| 0.2448      | 0.3102 | 0.4189 | 0.55911    |
| 0.2711      | 0.2868 | 0.3861 | 0.54137    |
| 0.2938      | 0.384  | 0.5424 | 0.5        |
| 0.3194      | 0.3564 | 0.4312 | 0.56268    |
| 0.3454      | 0.3231 | 0.431  | 0.49068    |
| 0.3697      | 0.309  | 0.3919 | 0.49693    |
| 0.3943      | 0.3195 | 0.3829 | 0.5053     |
| 0.4187      | 0.342  | 0.4575 | 0.43446    |
| 0.443       | 0.3454 | 0.46   | 0.41828    |
| 0.4639      | 0.4153 | 0.5354 | 0.41584    |
| 0.49        | 0.3512 | 0.4765 | 0.37587    |
| 0.517       | 0.2485 | 0.3407 | 0.35231    |
| 0.5383      | 0.3662 | 0.4549 | 0.37163    |
| 0.5622      | 0.3641 | 0.4536 | 0.35137    |
| 0.5875      | 0.3557 | 0.4364 | 0.33624    |
| 0.6135      | 0.3187 | 0.3969 | 0.31032    |
| 0.6402      | 0.2808 | 0.3291 | 0.30697    |
| 0.6642      | 0.3315 | 0.3566 | 0.31216    |
| 0.6906      | 0.248  | 0.3152 | 0.24344    |
| 0.717       | 0.2601 | 0.3846 | 0.19138    |
| 0.7426      | 0.2741 | 0.4237 | 0.16647    |
| 0.7683      | 0.3011 | 0.4415 | 0.15798    |
| 0.7934      | 0.3341 | 0.4741 | 0.1456     |
| 0.8184      | 0.3648 | 0.4498 | 0.14729    |
| 0.8429      | 0.3909 | 0.4913 | 0.12498    |
| 0.8679      | 0.4348 | 0.5783 | 0.0993     |
| 0.8922      | 0.4576 | 0.6063 | 0.08134    |

|        |        |        |         |
|--------|--------|--------|---------|
| 0.9168 | 0.4279 | 0.5917 | 0.06016 |
| 0.9411 | 0.4762 | 0.6414 | 0.04375 |
| 0.9651 | 0.4015 | 0.536  | 0.02616 |
| 0.9893 | 0.5515 | 0.7082 | 0.00833 |

| Scale = 0.5 |         |        |            |
|-------------|---------|--------|------------|
| Slip        | D/W     | T/rW   | Efficiency |
| -0.0021     | -0.0297 | 0.0599 | -0.4974    |
| 0.0229      | -0.0423 | 0.06   | -0.6888    |
| 0.0442      | 0.0871  | 0.1226 | 0.67911    |
| 0.0635      | 0.252   | 0.3819 | 0.61804    |
| 0.0875      | 0.2872  | 0.3239 | 0.80902    |
| 0.11        | 0.3698  | 0.3974 | 0.82817    |
| 0.1427      | 0.1743  | 0.172  | 0.86908    |
| 0.1684      | 0.1915  | 0.176  | 0.9048     |
| 0.1945      | 0.0956  | 0.0949 | 0.81061    |
| 0.2253      | 0.0323  | 0.1153 | 0.21714    |
| 0.2504      | 0.067   | 0.1109 | 0.45293    |
| 0.2717      | 0.1751  | 0.2779 | 0.45887    |
| 0.2976      | 0.1693  | 0.3057 | 0.38909    |
| 0.3217      | 0.2036  | 0.3326 | 0.41527    |
| 0.342       | 0.332   | 0.404  | 0.54075    |
| 0.3686      | 0.2514  | 0.2705 | 0.58693    |
| 0.3921      | 0.2782  | 0.4381 | 0.38602    |
| 0.4192      | 0.2352  | 0.3214 | 0.42493    |
| 0.4371      | 0.411   | 0.4745 | 0.48763    |
| 0.4598      | 0.4212  | 0.5238 | 0.43442    |
| 0.4892      | 0.2748  | 0.3839 | 0.36576    |
| 0.5086      | 0.432   | 0.553  | 0.38384    |
| 0.5307      | 0.4087  | 0.5604 | 0.34228    |
| 0.5543      | 0.4819  | 0.5985 | 0.35892    |
| 0.5793      | 0.4866  | 0.656  | 0.31213    |
| 0.6071      | 0.3918  | 0.5614 | 0.27423    |
| 0.6305      | 0.4618  | 0.5982 | 0.28523    |
| 0.6554      | 0.4904  | 0.5317 | 0.31779    |
| 0.6816      | 0.4676  | 0.5601 | 0.26584    |
| 0.7094      | 0.3196  | 0.4539 | 0.20464    |
| 0.7357      | 0.4028  | 0.4866 | 0.2188     |
| 0.7609      | 0.4095  | 0.4948 | 0.19785    |

|        |        |        |         |
|--------|--------|--------|---------|
| 0.7879 | 0.3538 | 0.4805 | 0.15615 |
| 0.8142 | 0.3921 | 0.5333 | 0.13664 |
| 0.8374 | 0.4736 | 0.5752 | 0.1339  |
| 0.8613 | 0.4693 | 0.5487 | 0.11862 |
| 0.885  | 0.5025 | 0.5627 | 0.10269 |
| 0.9108 | 0.4875 | 0.5693 | 0.07635 |
| 0.9356 | 0.516  | 0.5677 | 0.05858 |
| 0.9598 | 0.4936 | 0.5412 | 0.03667 |
| 0.9831 | 0.5186 | 0.5558 | 0.01573 |

| Scale = 0.25 |         |        |            |
|--------------|---------|--------|------------|
| Slip         | D/W     | T/rW   | Efficiency |
| 0.0064       | -0.0958 | 0.0457 | -2.08182   |
| 0.0423       | -0.2398 | 0.1464 | -1.56863   |
| 0.0496       | -0.0129 | 0.1883 | -0.06507   |
| 0.0664       | 0.0388  | 0.2106 | 0.17199    |
| 0.0667       | 0.4274  | 0.4645 | 0.85872    |
| 0.0941       | 0.3038  | 0.3344 | 0.82299    |
| 0.1802       | 0.449   | 0.5609 | 0.65633    |
| 0.1895       | 0.4242  | 0.4295 | 0.8005     |
| 0.2548       | 0.2782  | 0.4058 | 0.51091    |
| 0.2816       | 0.5621  | 0.5487 | 0.73597    |
| 0.2864       | 0.3795  | 0.5509 | 0.49162    |
| 0.3587       | 0.2657  | 0.241  | 0.70722    |
| 0.3842       | 0.6471  | 0.7016 | 0.568      |
| 0.3929       | 0.4414  | 0.7276 | 0.36828    |
| 0.4014       | 0.2132  | 0.473  | 0.26983    |
| 0.4865       | 0.417   | 0.2882 | 0.74318    |
| 0.531        | 0.5766  | 0.5279 | 0.51232    |
| 0.5326       | 0.2941  | 0.3008 | 0.457      |
| 0.5493       | 0.5484  | 0.4856 | 0.50904    |
| 0.5714       | 0.5452  | 0.6159 | 0.37931    |
| 0.6243       | 0.3256  | 0.5641 | 0.21687    |
| 0.6516       | 0.3672  | 0.7656 | 0.16712    |
| 0.6812       | 0.3723  | 0.5375 | 0.22085    |
| 0.6912       | 0.4648  | 0.703  | 0.20421    |
| 0.7118       | 0.4958  | 0.8912 | 0.16035    |
| 0.7565       | 0.4576  | 0.6009 | 0.1854     |
| 0.7602       | 0.5351  | 0.6282 | 0.20429    |

|        |        |        |         |
|--------|--------|--------|---------|
| 0.7751 | 0.6338 | 0.6809 | 0.20933 |
| 0.798  | 0.622  | 0.6268 | 0.20048 |
| 0.8175 | 0.7902 | 0.7635 | 0.18888 |
| 0.8428 | 0.5959 | 0.571  | 0.16403 |
| 0.869  | 0.4904 | 0.4813 | 0.13349 |
| 0.8938 | 0.5678 | 0.5219 | 0.11554 |
| 0.922  | 0.6411 | 0.6058 | 0.08251 |

### C.3.3 Fine-grained sand – Homogeneous model at full load and no pre-compaction

| Scale =1 |          |         |            |
|----------|----------|---------|------------|
| Slip     | D/W      | T/rW    | Efficiency |
| -0.00215 | -0.07016 | 0.12812 | -0.54879   |
| 0.02283  | -0.09879 | 0.19314 | -0.49983   |
| 0.04773  | -0.07984 | 0.19137 | -0.39728   |
| 0.07267  | -0.06373 | 0.19841 | -0.29784   |
| 0.0988   | -0.06317 | 0.17295 | -0.32915   |
| 0.12366  | -0.02591 | 0.19158 | -0.11849   |
| 0.14934  | -0.01968 | 0.21907 | -0.07642   |
| 0.17527  | -0.04856 | 0.21299 | -0.18805   |
| 0.20125  | -0.02558 | 0.23173 | -0.08817   |
| 0.22689  | -0.01324 | 0.27685 | -0.03698   |
| 0.25246  | -0.02053 | 0.24117 | -0.06362   |
| 0.27745  | 0.06215  | 0.26144 | 0.17175    |
| 0.30317  | 0.03546  | 0.27047 | 0.09136    |
| 0.32828  | 0.00411  | 0.23736 | 0.01162    |
| 0.3529   | 0.07676  | 0.26444 | 0.18784    |
| 0.37674  | 0.06825  | 0.30106 | 0.1413     |
| 0.40156  | 0.00783  | 0.21829 | 0.02145    |
| 0.42532  | 0.09211  | 0.36158 | 0.14639    |
| 0.44978  | 0.03566  | 0.23741 | 0.08266    |
| 0.47303  | 0.08795  | 0.33435 | 0.13862    |
| 0.49749  | 0.07886  | 0.25392 | 0.15607    |
| 0.52168  | 0.02657  | 0.20182 | 0.06296    |
| 0.5462   | 0.05097  | 0.20167 | 0.1147     |
| 0.57039  | 0.0793   | 0.23638 | 0.14413    |
| 0.59474  | 0.13301  | 0.36539 | 0.14752    |



|         |         |         |          |
|---------|---------|---------|----------|
| 0.61967 | 0.11752 | 0.29176 | 0.1532   |
| 0.64537 | 0.09657 | 0.25255 | 0.1356   |
| 0.67023 | 0.13863 | 0.33189 | 0.13775  |
| 0.69669 | 0.06206 | 0.28398 | 0.06629  |
| 0.72224 | 0.08791 | 0.25483 | 0.09582  |
| 0.74789 | 0.0911  | 0.23021 | 0.09976  |
| 0.77394 | 0.13391 | 0.30516 | 0.09919  |
| 0.80035 | 0.08419 | 0.18829 | 0.08927  |
| 0.82571 | 0.09712 | 0.23796 | 0.07113  |
| 0.85081 | 0.09432 | 0.20267 | 0.06943  |
| 0.8756  | 0.17499 | 0.24115 | 0.09027  |
| 0.90159 | 0.13578 | 0.21777 | 0.06136  |
| 0.92712 | 0.04773 | 0.11867 | 0.02931  |
| 0.95133 | 0.05605 | 0.1448  | 0.01884  |
| 0.97552 | 0.0813  | 0.14126 | 0.01409  |
| 1.00031 | 0.07316 | 0.10091 | -0.00023 |

| Scale = 0.7 |          |         |            |
|-------------|----------|---------|------------|
| Slip        | D/W      | T/rW    | Efficiency |
| -0.00016    | -0.12079 | 0.06308 | -1.91501   |
| 0.02407     | -0.09089 | 0.13977 | -0.63465   |
| 0.04876     | -0.06717 | 0.12356 | -0.51708   |
| 0.07326     | -0.07104 | 0.18378 | -0.35824   |
| 0.09825     | -0.03424 | 0.17034 | -0.18124   |
| 0.12389     | -0.0234  | 0.1538  | -0.13331   |
| 0.14989     | -0.06512 | 0.1836  | -0.30151   |
| 0.1747      | -0.00013 | 0.22811 | -0.00048   |
| 0.20133     | -0.02438 | 0.17492 | -0.11132   |
| 0.22708     | -0.01829 | 0.15437 | -0.09156   |
| 0.25234     | 0.00034  | 0.19408 | 0.00131    |
| 0.277       | 0.04169  | 0.23547 | 0.12801    |
| 0.30381     | -0.02543 | 0.26323 | -0.06725   |
| 0.32804     | 0.00982  | 0.25818 | 0.02555    |
| 0.35393     | -0.0272  | 0.22338 | -0.07868   |
| 0.37674     | -0.00489 | 0.23508 | -0.01297   |
| 0.4015      | 0.02007  | 0.16517 | 0.07272    |
| 0.42518     | 0.046    | 0.25497 | 0.10371    |
| 0.44908     | 0.08991  | 0.26585 | 0.18631    |
| 0.47411     | -0.00374 | 0.16828 | -0.01169   |

|         |          |         |          |
|---------|----------|---------|----------|
| 0.4982  | -0.01357 | 0.18913 | -0.03601 |
| 0.51998 | 0.117    | 0.38025 | 0.1477   |
| 0.54499 | 0.09064  | 0.25565 | 0.16132  |
| 0.56981 | 0.08173  | 0.30047 | 0.11701  |
| 0.59471 | 0.07425  | 0.24308 | 0.1238   |
| 0.61849 | 0.15935  | 0.37062 | 0.16403  |
| 0.64349 | 0.13955  | 0.38709 | 0.12852  |
| 0.67047 | 0.04556  | 0.24231 | 0.06196  |
| 0.69706 | 0.03314  | 0.16729 | 0.06002  |
| 0.72185 | 0.09504  | 0.20665 | 0.12793  |
| 0.74777 | 0.08724  | 0.28469 | 0.07729  |
| 0.77308 | 0.10964  | 0.27551 | 0.0903   |
| 0.79935 | 0.11051  | 0.23029 | 0.09629  |
| 0.82429 | 0.13223  | 0.28533 | 0.08143  |
| 0.8503  | 0.12745  | 0.23983 | 0.07955  |
| 0.87609 | 0.07817  | 0.23155 | 0.04183  |
| 0.90106 | 0.11464  | 0.28044 | 0.04044  |
| 0.92432 | 0.14704  | 0.29377 | 0.03788  |
| 0.95046 | 0.09561  | 0.17246 | 0.02746  |
| 0.97516 | 0.08437  | 0.18038 | 0.01162  |
| 0.99948 | 0.07438  | 0.15948 | 0.00024  |

| Scale = 0.5 |          |         |            |
|-------------|----------|---------|------------|
| Slip        | D/W      | T/rW    | Efficiency |
| 0.00974     | -0.19766 | 0.17724 | -1.10437   |
| 0.03262     | -0.14281 | 0.21164 | -0.65274   |
| 0.056       | -0.11654 | 0.14959 | -0.73542   |
| 0.08581     | -0.16542 | 0.19729 | -0.76648   |
| 0.10306     | -0.07541 | 0.25288 | -0.26745   |
| 0.1306      | -0.08174 | 0.26656 | -0.26661   |
| 0.15884     | -0.13197 | 0.31029 | -0.35776   |
| 0.18183     | -0.05185 | 0.21499 | -0.19732   |
| 0.20706     | -0.046   | 0.33507 | -0.10886   |
| 0.23237     | -0.10083 | 0.31834 | -0.24314   |
| 0.25735     | -0.04815 | 0.33654 | -0.10625   |
| 0.27944     | -0.05505 | 0.34007 | -0.11664   |
| 0.30573     | -0.02989 | 0.33801 | -0.06139   |
| 0.33374     | -0.04414 | 0.32385 | -0.09082   |
| 0.35726     | -0.08683 | 0.35536 | -0.15705   |

|         |          |         |          |
|---------|----------|---------|----------|
| 0.37819 | 0.03283  | 0.22371 | 0.09126  |
| 0.40129 | 0.06224  | 0.35132 | 0.10607  |
| 0.42731 | -0.01047 | 0.30319 | -0.01978 |
| 0.449   | 0.0028   | 0.35171 | 0.00439  |
| 0.47118 | 0.01377  | 0.30711 | 0.02371  |
| 0.49916 | 0.00297  | 0.37974 | 0.00392  |
| 0.52204 | 0.03871  | 0.39121 | 0.04729  |
| 0.54694 | -0.0467  | 0.41107 | -0.05147 |
| 0.5704  | 0.0356   | 0.32001 | 0.04779  |
| 0.59156 | 0.06299  | 0.37315 | 0.06895  |
| 0.61722 | 0.03043  | 0.36076 | 0.03228  |
| 0.64542 | 0.0099   | 0.34429 | 0.01019  |
| 0.67088 | 0.01312  | 0.43513 | 0.00992  |
| 0.69175 | 0.08458  | 0.38956 | 0.06693  |
| 0.71286 | 0.16666  | 0.53666 | 0.08917  |
| 0.74313 | 0.13299  | 0.32101 | 0.10642  |
| 0.76786 | 0.11435  | 0.43049 | 0.06166  |
| 0.79317 | 0.15913  | 0.51678 | 0.06369  |
| 0.82206 | -0.08236 | 0.24775 | -0.05915 |
| 0.84807 | 0.01648  | 0.30648 | 0.00817  |
| 0.87286 | 0.08965  | 0.36556 | 0.03118  |
| 0.89516 | 0.07891  | 0.37781 | 0.0219   |
| 0.92188 | 0.09961  | 0.43558 | 0.01787  |
| 0.94738 | 0.07782  | 0.33025 | 0.0124   |
| 0.9694  | 0.06356  | 0.43814 | 0.00444  |
| 0.99102 | 0.16517  | 0.33998 | 0.00436  |

| Scale = 0.25 |          |         |            |
|--------------|----------|---------|------------|
| Slip         | D/W      | T/rW    | Efficiency |
| 0.03416      | -0.27997 | 0.23197 | -1.16568   |
| 0.08219      | -0.35441 | 0.19085 | -1.70437   |
| 0.09631      | -0.40429 | 0.32356 | -1.12915   |
| 0.12463      | -0.21462 | 0.28424 | -0.66097   |
| 0.11753      | -0.09554 | 0.38512 | -0.21893   |
| 0.1722       | -0.27615 | 0.11419 | -2.00181   |
| 0.19873      | -0.25095 | 0.2596  | -0.77457   |
| 0.19553      | -0.07304 | 0.23441 | -0.25069   |
| 0.23788      | -0.32883 | 0.39368 | -0.63658   |
| 0.25581      | -0.11867 | 0.34483 | -0.25611   |

|         |          |         |          |
|---------|----------|---------|----------|
| 0.29342 | -0.22136 | 0.27931 | -0.55999 |
| 0.29212 | -0.01202 | 0.2757  | -0.03086 |
| 0.32895 | -0.18159 | 0.29664 | -0.41079 |
| 0.34384 | 0.02996  | 0.3508  | 0.05603  |
| 0.36859 | -0.11985 | 0.4597  | -0.16462 |
| 0.39844 | -0.10088 | 0.37853 | -0.16032 |
| 0.42378 | -0.05976 | 0.40505 | -0.08501 |
| 0.4269  | 0.00922  | 0.32518 | 0.01626  |
| 0.44791 | 0.01411  | 0.46586 | 0.01672  |
| 0.4831  | -0.05082 | 0.40106 | -0.0655  |
| 0.50042 | -0.01145 | 0.3445  | -0.01661 |
| 0.51291 | 0.05088  | 0.39581 | 0.06262  |
| 0.53378 | 0.19905  | 0.50664 | 0.18317  |
| 0.56731 | -0.0506  | 0.37226 | -0.05882 |
| 0.60738 | 0.02436  | 0.53085 | 0.01802  |
| 0.65365 | -0.30751 | 0.45329 | -0.23497 |
| 0.6549  | -0.02955 | 0.40707 | -0.02505 |
| 0.66319 | 0.07185  | 0.50339 | 0.04807  |
| 0.69989 | 0.02708  | 0.41992 | 0.01936  |
| 0.72447 | 0.01258  | 0.54228 | 0.00639  |
| 0.72975 | 0.03502  | 0.40434 | 0.0234   |
| 0.79154 | -0.18976 | 0.4313  | -0.09172 |
| 0.80735 | 0.01613  | 0.39543 | 0.00786  |
| 0.82844 | -0.00903 | 0.42851 | -0.00361 |
| 0.85007 | -0.01361 | 0.40951 | -0.00498 |
| 0.86122 | 0.07963  | 0.31579 | 0.035    |
| 0.88627 | 0.10011  | 0.40671 | 0.02799  |
| 0.91692 | 0.15398  | 0.41578 | 0.03077  |
| 0.93    | 0.13249  | 0.54906 | 0.01689  |
| 0.94547 | 0.20175  | 0.5653  | 0.01946  |
| 0.97031 | 0.19276  | 0.52872 | 0.01082  |

#### C.3.4 Fine-grained sand – Heterogeneous model at full load and no pre-compaction

| Scale =1 |          |         |            |
|----------|----------|---------|------------|
| Slip     | D/W      | T/rW    | Efficiency |
| 0.00107  | -0.12807 | 0.13915 | -0.91934   |

|         |          |         |          |
|---------|----------|---------|----------|
| 0.02573 | -0.16195 | 0.18486 | -0.85353 |
| 0.04938 | -0.07072 | 0.23387 | -0.28746 |
| 0.07433 | -0.04304 | 0.14786 | -0.26946 |
| 0.09956 | -0.09019 | 0.17438 | -0.46573 |
| 0.12686 | -0.04629 | 0.23584 | -0.17138 |
| 0.15174 | -0.10118 | 0.26118 | -0.32861 |
| 0.17651 | -0.07441 | 0.24064 | -0.25462 |
| 0.20164 | -0.06538 | 0.28817 | -0.18114 |
| 0.22791 | -0.0232  | 0.20842 | -0.08593 |
| 0.25312 | -0.01818 | 0.2204  | -0.06161 |
| 0.27745 | 0.06337  | 0.30801 | 0.14865  |
| 0.30335 | -0.0211  | 0.33245 | -0.04422 |
| 0.32747 | 0.04875  | 0.23283 | 0.14082  |
| 0.35243 | 0.02253  | 0.2316  | 0.06299  |
| 0.37641 | 0.03937  | 0.33962 | 0.07229  |
| 0.40236 | -0.01051 | 0.33553 | -0.01871 |
| 0.42598 | 0.04822  | 0.32857 | 0.08425  |
| 0.44986 | 0.0196   | 0.29433 | 0.03664  |
| 0.47439 | 0.04672  | 0.34913 | 0.07034  |
| 0.49749 | 0.04241  | 0.29643 | 0.0719   |
| 0.52043 | 0.091    | 0.31612 | 0.13805  |
| 0.54534 | 0.08923  | 0.29673 | 0.13673  |
| 0.57035 | 0.00628  | 0.25062 | 0.01076  |
| 0.59511 | 0.05403  | 0.24274 | 0.09013  |
| 0.61794 | 0.18943  | 0.38323 | 0.18885  |
| 0.64227 | 0.20237  | 0.46862 | 0.15449  |
| 0.66697 | 0.15998  | 0.38118 | 0.13977  |
| 0.69637 | 0.04157  | 0.26252 | 0.04808  |
| 0.7202  | 0.14678  | 0.27939 | 0.14699  |
| 0.74685 | 0.07838  | 0.38478 | 0.05157  |
| 0.77216 | 0.09643  | 0.28639 | 0.07672  |
| 0.79767 | 0.12789  | 0.3023  | 0.0856   |
| 0.82382 | 0.07157  | 0.35256 | 0.03576  |
| 0.8494  | 0.14856  | 0.2797  | 0.07999  |
| 0.87347 | 0.15089  | 0.24228 | 0.0788   |
| 0.89596 | 0.23076  | 0.35274 | 0.06806  |
| 0.9241  | 0.08585  | 0.27763 | 0.02347  |
| 0.94942 | 0.16564  | 0.24978 | 0.03354  |
| 0.97118 | 0.18141  | 0.31042 | 0.01684  |
| 0.99506 | 0.1606   | 0.2058  | 0.00385  |

| Scale = 0.7 |          |         |            |
|-------------|----------|---------|------------|
| Slip        | D/W      | T/rW    | Efficiency |
| 0.00439     | -0.17197 | 0.1979  | -0.86516   |
| 0.02694     | -0.11844 | 0.17225 | -0.66909   |
| 0.05425     | -0.11914 | 0.08144 | -1.38358   |
| 0.07988     | -0.14026 | 0.12074 | -1.06885   |
| 0.10245     | -0.0569  | 0.20377 | -0.25062   |
| 0.12813     | -0.17548 | 0.22543 | -0.67868   |
| 0.15318     | -0.1012  | 0.2068  | -0.4144    |
| 0.18069     | -0.10112 | 0.16609 | -0.49882   |
| 0.20531     | -0.07077 | 0.2954  | -0.19039   |
| 0.22866     | -0.05084 | 0.29464 | -0.13311   |
| 0.25455     | -0.06802 | 0.30902 | -0.16409   |
| 0.28062     | -0.04632 | 0.31863 | -0.10458   |
| 0.30526     | -0.02055 | 0.37753 | -0.03782   |
| 0.33097     | -0.06834 | 0.23692 | -0.19299   |
| 0.35519     | -0.03903 | 0.2453  | -0.1026    |
| 0.38083     | -0.06961 | 0.26151 | -0.16482   |
| 0.40476     | -0.06203 | 0.38769 | -0.09524   |
| 0.42901     | -0.06052 | 0.38539 | -0.08966   |
| 0.45235     | -0.03359 | 0.4511  | -0.04078   |
| 0.47676     | -0.04069 | 0.25537 | -0.08338   |
| 0.49977     | 0.00541  | 0.14383 | 0.01883    |
| 0.522       | 0.05222  | 0.25237 | 0.0989     |
| 0.54507     | -0.03641 | 0.34215 | -0.04841   |
| 0.57137     | -0.03182 | 0.34464 | -0.03957   |
| 0.59281     | 0.06819  | 0.30411 | 0.0913     |
| 0.62005     | 0.03721  | 0.38656 | 0.03657    |
| 0.64339     | 0.08535  | 0.49274 | 0.06177    |
| 0.66457     | 0.17272  | 0.43549 | 0.13303    |
| 0.6977      | 0.0523   | 0.20897 | 0.07565    |
| 0.72174     | 0.02332  | 0.41961 | 0.01547    |
| 0.75043     | -0.01876 | 0.28196 | -0.01661   |
| 0.77314     | 0.04895  | 0.28106 | 0.03951    |
| 0.79605     | 0.1309   | 0.42182 | 0.06329    |
| 0.82004     | 0.12707  | 0.39208 | 0.05832    |
| 0.84553     | 0.12915  | 0.53195 | 0.0375     |
| 0.87412     | 0.11374  | 0.29188 | 0.04905    |
| 0.89365     | 0.24308  | 0.41061 | 0.06296    |

|         |         |         |         |
|---------|---------|---------|---------|
| 0.92189 | 0.07489 | 0.28521 | 0.02051 |
| 0.94928 | 0.03239 | 0.39045 | 0.00421 |
| 0.97083 | 0.06529 | 0.20493 | 0.00929 |
| 0.99525 | 0.12557 | 0.19548 | 0.00305 |

| Scale = 0.5 |          |         |            |
|-------------|----------|---------|------------|
| Slip        | D/W      | T/rW    | Efficiency |
| 0.02245     | -0.17024 | 0.15675 | -1.06167   |
| 0.05465     | -0.28977 | 0.21327 | -1.28442   |
| 0.07607     | -0.31585 | 0.20012 | -1.45825   |
| 0.10258     | -0.113   | 0.10335 | -0.9812    |
| 0.13226     | -0.18157 | 0.23778 | -0.66261   |
| 0.15288     | -0.18207 | 0.22978 | -0.67124   |
| 0.17093     | -0.13414 | 0.35718 | -0.31137   |
| 0.18728     | -0.0501  | 0.30832 | -0.13206   |
| 0.19617     | -0.10131 | 0.27547 | -0.29563   |
| 0.21039     | 0.03477  | 0.23027 | 0.11924    |
| 0.2414      | -0.04039 | 0.18359 | -0.1669    |
| 0.26996     | -0.15946 | 0.39275 | -0.29641   |
| 0.29709     | -0.15345 | 0.25718 | -0.41939   |
| 0.30874     | 0.01298  | 0.40019 | 0.02242    |
| 0.33308     | -0.00308 | 0.42766 | -0.0048    |
| 0.35451     | 0.03064  | 0.4066  | 0.04865    |
| 0.37422     | 0.01522  | 0.38308 | 0.02486    |
| 0.3977      | 0.01689  | 0.54185 | 0.01878    |
| 0.42953     | 0.00477  | 0.34206 | 0.00796    |
| 0.46056     | -0.12748 | 0.48279 | -0.14244   |
| 0.49373     | -0.04266 | 0.28131 | -0.07678   |
| 0.50699     | -0.07327 | 0.34332 | -0.10521   |
| 0.53756     | -0.15591 | 0.52999 | -0.13604   |
| 0.55071     | -0.01008 | 0.40899 | -0.01107   |
| 0.58599     | -0.01907 | 0.25529 | -0.03093   |
| 0.60475     | 0.06309  | 0.40983 | 0.06085    |
| 0.62797     | -0.01537 | 0.32152 | -0.01778   |
| 0.65363     | 0.00986  | 0.38062 | 0.00897    |
| 0.6731      | -0.02201 | 0.28268 | -0.02545   |
| 0.70671     | -0.07448 | 0.28942 | -0.07547   |
| 0.71788     | 0.17819  | 0.30248 | 0.1662     |
| 0.73467     | 0.11382  | 0.3809  | 0.07929    |

|         |         |         |         |
|---------|---------|---------|---------|
| 0.77242 | 0.04796 | 0.41998 | 0.02599 |
| 0.79764 | 0.08267 | 0.5482  | 0.03051 |
| 0.82273 | 0.03742 | 0.44769 | 0.01482 |
| 0.83495 | 0.14439 | 0.47567 | 0.0501  |
| 0.86815 | 0.08381 | 0.59164 | 0.01868 |
| 0.89302 | 0.11003 | 0.45515 | 0.02586 |
| 0.90842 | 0.08495 | 0.62916 | 0.01237 |
| 0.9095  | 0.16548 | 0.55965 | 0.02676 |
| 0.93259 | 0.14883 | 0.5316  | 0.01887 |
| 0.95336 | 0.16797 | 0.48692 | 0.01609 |
| 0.96497 | 0.25435 | 0.55014 | 0.0162  |

| Scale = 0.25 |          |         |            |
|--------------|----------|---------|------------|
| Slip         | D/W      | T/rW    | Efficiency |
| 0.04675      | -0.10635 | 0.47472 | -0.21355   |
| 0.13035      | -0.48991 | 0.20399 | -2.08859   |
| 0.17284      | -0.11874 | 0.50624 | -0.19401   |
| 0.17457      | -0.13043 | 0.08813 | -1.22163   |
| 0.1851       | -0.15958 | 0.12565 | -1.03491   |
| 0.19672      | -0.08657 | 0.21775 | -0.31937   |
| 0.21581      | -0.52528 | 0.22638 | -1.81962   |
| 0.21931      | -0.19021 | 0.37958 | -0.39121   |
| 0.2375       | -0.40034 | 0.33818 | -0.90265   |
| 0.25762      | -0.11536 | 0.51542 | -0.16615   |
| 0.31464      | -0.22561 | 0.17936 | -0.8621    |
| 0.32698      | -0.27983 | 0.39715 | -0.47422   |
| 0.36341      | -0.36586 | 0.24605 | -0.94657   |
| 0.38121      | -0.03935 | 0.47696 | -0.05105   |
| 0.39567      | -0.24263 | 0.3199  | -0.45836   |
| 0.43465      | -0.08562 | 0.36    | -0.13447   |
| 0.43731      | -0.09961 | 0.68176 | -0.08221   |
| 0.44107      | -0.05082 | 0.49479 | -0.05741   |
| 0.44395      | 0.12334  | 0.40066 | 0.17117    |
| 0.47648      | -0.03619 | 0.41996 | -0.04512   |
| 0.4892       | 0.01321  | 0.60025 | 0.01124    |
| 0.5746       | -0.17064 | 0.53088 | -0.13673   |
| 0.58539      | -0.05979 | 0.5069  | -0.04891   |
| 0.5893       | -0.1247  | 0.53549 | -0.09564   |
| 0.61216      | -0.2544  | 0.67017 | -0.14722   |



|         |          |         |          |
|---------|----------|---------|----------|
| 0.61692 | 0.28187  | 0.83938 | 0.12864  |
| 0.61977 | 0.03724  | 0.40438 | 0.03502  |
| 0.63627 | 0.07507  | 0.46095 | 0.05924  |
| 0.64666 | 0.10497  | 0.50989 | 0.07274  |
| 0.65265 | -0.09631 | 0.36979 | -0.09047 |
| 0.66461 | 0.08926  | 0.39084 | 0.0766   |
| 0.70939 | -0.04217 | 0.31607 | -0.03877 |
| 0.77408 | -0.11458 | 0.46121 | -0.05612 |
| 0.82128 | -0.16979 | 0.45723 | -0.06637 |
| 0.87137 | -0.22749 | 0.57098 | -0.05125 |
| 0.90356 | -0.2208  | 0.53784 | -0.03959 |
| 0.90758 | -0.1411  | 0.64766 | -0.02013 |
| 0.93198 | -0.2606  | 0.47198 | -0.03756 |
| 0.93485 | -0.02337 | 0.45909 | -0.00332 |
| 0.93696 | 0.1484   | 0.56542 | 0.01655  |
| 0.94331 | -0.09532 | 0.51469 | -0.0105  |
| 0.95918 | 0.03054  | 0.36263 | 0.00344  |
| 0.98202 | 0.10327  | 0.62142 | 0.00299  |

### C.3.5 2NS sand – Heterogeneous model at half load and 0.15 m pre-compaction

| Scale =1 |          |         |            |
|----------|----------|---------|------------|
| Slip     | D/W      | T/rW    | Efficiency |
| -0.00243 | -0.06259 | 0.00314 | -19.9688   |
| 0.02236  | -0.01821 | 0.05785 | -0.30769   |
| 0.04534  | 0.07229  | 0.16074 | 0.42935    |
| 0.0685   | 0.19718  | 0.2895  | 0.63446    |
| 0.09337  | 0.22487  | 0.33213 | 0.61383    |
| 0.11717  | 0.33357  | 0.41859 | 0.70351    |
| 0.14436  | 0.25934  | 0.31906 | 0.69549    |
| 0.16835  | 0.38844  | 0.45653 | 0.70761    |
| 0.19469  | 0.34139  | 0.41536 | 0.6619     |
| 0.22152  | 0.28572  | 0.37816 | 0.58819    |
| 0.24616  | 0.38211  | 0.46711 | 0.61667    |
| 0.27162  | 0.33494  | 0.35192 | 0.69322    |
| 0.29773  | 0.29465  | 0.3938  | 0.52546    |
| 0.3214   | 0.42598  | 0.52801 | 0.54747    |
| 0.34435  | 0.5178   | 0.60605 | 0.56018    |

|         |         |         |         |
|---------|---------|---------|---------|
| 0.37032 | 0.4203  | 0.42312 | 0.62549 |
| 0.39639 | 0.3316  | 0.36719 | 0.5451  |
| 0.42196 | 0.27159 | 0.30532 | 0.51418 |
| 0.44474 | 0.31688 | 0.38791 | 0.45359 |
| 0.46869 | 0.35761 | 0.44515 | 0.42683 |
| 0.49108 | 0.39734 | 0.50047 | 0.40405 |
| 0.51458 | 0.4748  | 0.5322  | 0.43307 |
| 0.5387  | 0.48272 | 0.53164 | 0.41885 |
| 0.5646  | 0.39566 | 0.47692 | 0.36122 |
| 0.5902  | 0.34762 | 0.3975  | 0.35837 |
| 0.61318 | 0.44953 | 0.55861 | 0.31129 |
| 0.6383  | 0.47065 | 0.49804 | 0.3418  |
| 0.66432 | 0.42454 | 0.4832  | 0.29492 |
| 0.69199 | 0.3558  | 0.45182 | 0.24256 |
| 0.71721 | 0.37088 | 0.46066 | 0.22767 |
| 0.74336 | 0.37566 | 0.43975 | 0.21923 |
| 0.76867 | 0.41565 | 0.51705 | 0.18597 |
| 0.79415 | 0.43493 | 0.55251 | 0.16204 |
| 0.81962 | 0.48125 | 0.55888 | 0.15533 |
| 0.84502 | 0.4458  | 0.53487 | 0.12917 |
| 0.87019 | 0.44189 | 0.51765 | 0.11081 |
| 0.89447 | 0.49802 | 0.57221 | 0.09184 |
| 0.90352 | 0.52086 | 0.59701 | 0.08418 |
| 0.91906 | 0.52495 | 0.58659 | 0.07243 |
| 0.94336 | 0.60455 | 0.6689  | 0.05119 |
| 0.96763 | 0.5591  | 0.61958 | 0.02921 |
| 0.99176 | 0.56114 | 0.62542 | 0.00739 |

| Scale = 0.7 |         |         |            |
|-------------|---------|---------|------------|
| Slip        | D/W     | T/rW    | Efficiency |
| -0.00436    | 0.06497 | 0.11962 | 0.54546    |
| 0.01772     | 0.155   | 0.18027 | 0.84454    |
| 0.04008     | 0.24197 | 0.22915 | 1.01359    |
| 0.06816     | 0.14785 | 0.11673 | 1.18029    |
| 0.09547     | 0.04769 | 0.09086 | 0.47477    |
| 0.12226     | 0.01977 | 0.06467 | 0.26835    |
| 0.14594     | 0.12482 | 0.23204 | 0.45944    |
| 0.16814     | 0.27764 | 0.31433 | 0.73477    |
| 0.19303     | 0.2963  | 0.34968 | 0.6838     |

|         |         |         |         |
|---------|---------|---------|---------|
| 0.22197 | 0.16238 | 0.2345  | 0.53876 |
| 0.24309 | 0.40631 | 0.50454 | 0.60955 |
| 0.2714  | 0.24458 | 0.33365 | 0.53409 |
| 0.29154 | 0.45852 | 0.54975 | 0.59091 |
| 0.31659 | 0.46932 | 0.57112 | 0.5616  |
| 0.34037 | 0.50478 | 0.56014 | 0.59444 |
| 0.37002 | 0.30077 | 0.38401 | 0.49342 |
| 0.39444 | 0.33569 | 0.37844 | 0.53716 |
| 0.41802 | 0.30852 | 0.38287 | 0.46896 |
| 0.44064 | 0.34129 | 0.4028  | 0.47394 |
| 0.4624  | 0.52251 | 0.60487 | 0.46439 |
| 0.4901  | 0.33521 | 0.39176 | 0.4363  |
| 0.51359 | 0.37411 | 0.46739 | 0.38934 |
| 0.54005 | 0.27455 | 0.29459 | 0.42866 |
| 0.56188 | 0.37423 | 0.44394 | 0.36932 |
| 0.58401 | 0.49433 | 0.58918 | 0.34902 |
| 0.60801 | 0.52294 | 0.57389 | 0.35719 |
| 0.63622 | 0.40786 | 0.43756 | 0.33909 |
| 0.66118 | 0.44899 | 0.44706 | 0.34028 |
| 0.68675 | 0.43087 | 0.45334 | 0.29772 |
| 0.7127  | 0.43686 | 0.46885 | 0.26769 |
| 0.74064 | 0.32821 | 0.42773 | 0.19902 |
| 0.76711 | 0.34431 | 0.48807 | 0.16429 |
| 0.79158 | 0.38243 | 0.51415 | 0.15503 |
| 0.81628 | 0.42334 | 0.51166 | 0.152   |
| 0.8408  | 0.49631 | 0.5705  | 0.1385  |
| 0.86489 | 0.53064 | 0.57489 | 0.12471 |
| 0.89037 | 0.54768 | 0.59783 | 0.10044 |
| 0.91545 | 0.52919 | 0.59696 | 0.07495 |
| 0.93982 | 0.50956 | 0.61431 | 0.04991 |
| 0.96416 | 0.51886 | 0.6252  | 0.02974 |
| 0.98835 | 0.54697 | 0.6523  | 0.00977 |

| Scale = 0.5 |         |         |            |
|-------------|---------|---------|------------|
| Slip        | D/W     | T/rW    | Efficiency |
| -0.00443    | 0.00564 | 0.05695 | 0.09943    |
| 0.01258     | 0.08838 | 0.24142 | 0.36146    |
| 0.03696     | 0.1273  | 0.12381 | 0.99017    |
| 0.05374     | 0.21099 | 0.27195 | 0.73416    |

|         |         |         |          |
|---------|---------|---------|----------|
| 0.0845  | 0.17666 | 0.22465 | 0.71992  |
| 0.10672 | 0.15895 | 0.22275 | 0.63743  |
| 0.12127 | 0.35874 | 0.37398 | 0.84292  |
| 0.1375  | 0.418   | 0.40749 | 0.88474  |
| 0.16821 | 0.34647 | 0.45858 | 0.62845  |
| 0.18916 | 0.42584 | 0.44509 | 0.77577  |
| 0.20462 | 0.48433 | 0.49945 | 0.7713   |
| 0.25684 | 0.15483 | 0.14377 | 0.80033  |
| 0.28243 | 0.20628 | 0.19924 | 0.74293  |
| 0.30982 | 0.22827 | 0.20209 | 0.77961  |
| 0.33163 | 0.22641 | 0.21225 | 0.71298  |
| 0.36373 | 0.13779 | 0.23388 | 0.37485  |
| 0.37897 | 0.32682 | 0.3921  | 0.51764  |
| 0.38941 | 0.45974 | 0.47611 | 0.5896   |
| 0.41766 | 0.36405 | 0.47131 | 0.44982  |
| 0.44148 | 0.36149 | 0.52678 | 0.38326  |
| 0.46536 | 0.28929 | 0.47131 | 0.32816  |
| 0.47942 | 0.47475 | 0.57269 | 0.43155  |
| 0.50807 | 0.42069 | 0.4081  | 0.5071   |
| 0.52416 | 0.59413 | 0.59227 | 0.47733  |
| 0.56289 | 0.24111 | 0.41677 | 0.25287  |
| 0.58385 | 0.51818 | 0.63965 | 0.33712  |
| 0.60196 | 0.53757 | 0.52525 | 0.40737  |
| 0.63207 | 0.42662 | 0.38884 | 0.40368  |
| 0.66834 | 0.30831 | 0.35739 | 0.28612  |
| 0.70696 | 0.1976  | 0.37789 | 0.15323  |
| 0.74986 | 0.34149 | 0.43651 | 0.19569  |
| 0.77582 | 0.48075 | 0.54237 | 0.19871  |
| 0.80172 | 0.47856 | 0.48987 | 0.1937   |
| 0.82749 | 0.49571 | 0.53376 | 0.16021  |
| 0.85308 | 0.43836 | 0.52541 | 0.12258  |
| 0.87844 | 0.39818 | 0.5174  | 0.09355  |
| 0.90355 | 0.55055 | 0.67588 | 0.07857  |
| 0.92839 | 0.51661 | 0.63035 | 0.05869  |
| 0.95298 | 0.59297 | 0.70699 | 0.03943  |
| 0.97736 | 0.53776 | 0.63079 | 0.0193   |
| 1.00046 | 0.48358 | 0.58442 | -0.00038 |
| 1.00153 | 0.48305 | 0.57073 | -0.00129 |

| Scale = 0.25 |          |          |            |
|--------------|----------|----------|------------|
| Slip         | D/W      | T/rW     | Efficiency |
| -0.00323     | -0.00856 | -0.02879 | 0.29835    |
| 0.02153      | -0.07016 | 0.01415  | -4.85298   |
| 0.04655      | 0.06527  | 0.1711   | 0.3637     |
| 0.07183      | 0.44873  | 0.44967  | 0.92624    |
| 0.09735      | 0.18785  | 0.26687  | 0.63539    |
| 0.12306      | 0.40061  | 0.39811  | 0.88244    |
| 0.14893      | 0.06096  | 0.25005  | 0.20749    |
| 0.17488      | 0.60397  | 0.61686  | 0.80788    |
| 0.20084      | 0.68737  | 0.77587  | 0.70801    |
| 0.22674      | 0.23104  | 0.18152  | 0.98424    |
| 0.25251      | 0.18297  | 0.28112  | 0.48651    |
| 0.27811      | 0.07322  | 0.13004  | 0.40649    |
| 0.30347      | 0.36632  | 0.37618  | 0.67827    |
| 0.32858      | 0.27533  | 0.43102  | 0.4289     |
| 0.35343      | 0.20191  | 0.20022  | 0.65205    |
| 0.37803      | 0.20971  | 0.26212  | 0.49759    |
| 0.40235      | 0.40304  | 0.56571  | 0.4258     |
| 0.42655      | 0.69307  | 0.91902  | 0.43246    |
| 0.45064      | 0.77611  | 0.99474  | 0.42861    |
| 0.47468      | 0.73978  | 0.77765  | 0.49974    |
| 0.49875      | 0.84646  | 0.87049  | 0.48742    |
| 0.5229       | 0.77499  | 0.85868  | 0.4306     |
| 0.54722      | 0.647    | 0.68284  | 0.42901    |
| 0.57174      | 0.11802  | 0.37253  | 0.13568    |
| 0.59651      | 0.50718  | 0.65912  | 0.31048    |
| 0.62153      | 0.63274  | 0.97995  | 0.24437    |
| 0.64682      | 0.57773  | 0.87221  | 0.23394    |
| 0.67234      | 0.60481  | 0.7706   | 0.25717    |
| 0.69806      | 0.72909  | 0.90358  | 0.24363    |
| 0.74988      | 0.68882  | 0.81109  | 0.21241    |
| 0.8531       | 0.44156  | 0.58776  | 0.11036    |
| 0.87846      | 0.36609  | 0.47162  | 0.09435    |
| 0.90356      | 0.21289  | 0.30524  | 0.06726    |
| 0.9284       | 0.19818  | 0.19774  | 0.07176    |
| 0.953        | 0.17854  | 0.21343  | 0.03932    |
| 0.97737      | 0.12679  | 0.18596  | 0.01543    |
| 1.00153      | 0.13687  | 0.20889  | -0.001     |

### C.3.6 Fine-grained sand – Heterogeneous model at half load and no pre-compaction

| Scale =1 |          |         |            |
|----------|----------|---------|------------|
| Slip     | D/W      | T/rW    | Efficiency |
| 0.00189  | -0.15761 | 0.18546 | -0.84819   |
| 0.02617  | -0.10602 | 0.16061 | -0.64283   |
| 0.04882  | -0.08624 | 0.11947 | -0.68659   |
| 0.07342  | 0.00126  | 0.19161 | 0.0061     |
| 0.10066  | -0.122   | 0.1424  | -0.77047   |
| 0.12482  | -0.02183 | 0.17379 | -0.10994   |
| 0.15078  | 0.00722  | 0.19715 | 0.03109    |
| 0.17507  | 0.0211   | 0.24526 | 0.07098    |
| 0.20087  | 0.0505   | 0.3074  | 0.13129    |
| 0.22629  | 0.02006  | 0.39464 | 0.03933    |
| 0.25111  | 0.06997  | 0.1963  | 0.26694    |
| 0.27929  | -0.00515 | 0.16257 | -0.02285   |
| 0.30184  | 0.1127   | 0.34754 | 0.22639    |
| 0.32568  | 0.07345  | 0.18181 | 0.27243    |
| 0.35133  | 0.07917  | 0.29799 | 0.17234    |
| 0.37427  | 0.15424  | 0.37086 | 0.26024    |
| 0.40149  | 0.01059  | 0.1453  | 0.04362    |
| 0.42414  | -0.00589 | 0.18078 | -0.01877   |
| 0.4472   | 0.09236  | 0.30481 | 0.16751    |
| 0.46979  | 0.0951   | 0.32496 | 0.15517    |
| 0.49735  | 0.03634  | 0.22679 | 0.08053    |
| 0.51827  | 0.10447  | 0.26779 | 0.18794    |
| 0.54198  | 0.13289  | 0.36297 | 0.16768    |
| 0.56859  | 0.13178  | 0.23644 | 0.24044    |
| 0.59385  | 0.09387  | 0.27455 | 0.13886    |
| 0.6161   | 0.16888  | 0.33726 | 0.19224    |
| 0.64178  | 0.12503  | 0.30653 | 0.14612    |
| 0.66887  | 0.09678  | 0.3593  | 0.0892     |
| 0.6715   | 0.14261  | 0.44376 | 0.10557    |
| 0.69338  | 0.11125  | 0.31447 | 0.10847    |
| 0.71771  | 0.14435  | 0.35457 | 0.11492    |
| 0.74501  | 0.17641  | 0.36351 | 0.12375    |
| 0.76927  | 0.23303  | 0.42793 | 0.12564    |
| 0.79575  | 0.2052   | 0.3401  | 0.12323    |
| 0.82184  | 0.12919  | 0.23395 | 0.09838    |
| 0.85004  | 0.10076  | 0.24605 | 0.06141    |

|         |         |         |         |
|---------|---------|---------|---------|
| 0.8503  | 0.08638 | 0.23243 | 0.05563 |
| 0.87355 | 0.13387 | 0.24027 | 0.07046 |
| 0.89755 | 0.17739 | 0.23645 | 0.07686 |
| 0.92195 | 0.18562 | 0.31149 | 0.04651 |
| 0.94494 | 0.22453 | 0.33832 | 0.03654 |
| 0.969   | 0.19399 | 0.25491 | 0.02359 |
| 0.99323 | 0.24954 | 0.34004 | 0.00497 |

| Scale = 0.7 |          |         |            |
|-------------|----------|---------|------------|
| Slip        | D/W      | T/rW    | Efficiency |
| 0.00029     | -0.02225 | 0.35494 | -0.06266   |
| 0.02632     | -0.08637 | 0.15706 | -0.53544   |
| 0.05295     | -0.11644 | 0.09799 | -1.12538   |
| 0.07877     | -0.17641 | 0.21453 | -0.75751   |
| 0.10104     | -0.02918 | 0.11067 | -0.23706   |
| 0.12256     | -0.06372 | 0.23592 | -0.23697   |
| 0.14932     | 0.06573  | 0.22614 | 0.24726    |
| 0.17783     | -0.03263 | 0.25943 | -0.1034    |
| 0.20201     | -0.03051 | 0.26166 | -0.09303   |
| 0.23118     | -0.1503  | 0.20135 | -0.5739    |
| 0.25364     | -0.10341 | 0.23354 | -0.3305    |
| 0.28084     | -0.02021 | 0.16866 | -0.08618   |
| 0.30461     | -0.11929 | 0.28112 | -0.29508   |
| 0.32786     | 0.02558  | 0.36307 | 0.04736    |
| 0.35116     | 0.05874  | 0.40002 | 0.09528    |
| 0.37419     | 0.03169  | 0.2609  | 0.07602    |
| 0.39858     | 0.04286  | 0.33733 | 0.07642    |
| 0.4253      | 0.09253  | 0.27157 | 0.19581    |
| 0.44469     | 0.11025  | 0.43007 | 0.14236    |
| 0.46946     | 0.09861  | 0.42325 | 0.12361    |
| 0.49543     | 0.03235  | 0.24404 | 0.06689    |
| 0.51892     | 0.1425   | 0.42038 | 0.16308    |
| 0.54538     | 0.1025   | 0.24384 | 0.1911     |
| 0.56581     | 0.08742  | 0.35463 | 0.10704    |
| 0.59303     | 0.09312  | 0.34073 | 0.11122    |
| 0.61372     | 0.15575  | 0.3846  | 0.15643    |
| 0.64361     | 0.0578   | 0.30018 | 0.06862    |
| 0.66596     | 0.14663  | 0.36067 | 0.1358     |
| 0.69326     | 0.16835  | 0.4159  | 0.12416    |

|         |         |         |         |
|---------|---------|---------|---------|
| 0.71871 | 0.13928 | 0.29131 | 0.13449 |
| 0.74515 | 0.07165 | 0.40208 | 0.04541 |
| 0.77241 | 0.16521 | 0.45861 | 0.08199 |
| 0.79318 | 0.26361 | 0.47398 | 0.11502 |
| 0.81954 | 0.13614 | 0.43512 | 0.05646 |
| 0.84622 | 0.19072 | 0.37204 | 0.07883 |
| 0.87164 | 0.18945 | 0.53962 | 0.04507 |
| 0.88777 | 0.36439 | 0.56953 | 0.07181 |
| 0.91921 | 0.17911 | 0.2872  | 0.05038 |
| 0.94367 | 0.25244 | 0.34026 | 0.04179 |
| 0.97149 | 0.07459 | 0.19186 | 0.01108 |
| 0.99336 | 0.10908 | 0.22194 | 0.00326 |

| Scale = 0.5 |          |         |            |
|-------------|----------|---------|------------|
| Slip        | D/W      | T/rW    | Efficiency |
| 0.03198     | -0.2043  | 0.08687 | -2.27671   |
| 0.05186     | -0.18755 | 0.07741 | -2.29703   |
| 0.06985     | -0.22106 | 0.09942 | -2.06826   |
| 0.08923     | -0.14027 | 0.16048 | -0.79607   |
| 0.11857     | -0.18214 | 0.15857 | -1.01239   |
| 0.15375     | -0.33025 | 0.26159 | -1.06837   |
| 0.17379     | -0.20178 | 0.28003 | -0.59536   |
| 0.19268     | -0.13001 | 0.11276 | -0.93081   |
| 0.21994     | -0.01188 | 0.35322 | -0.02623   |
| 0.2318      | -0.06304 | 0.3069  | -0.1578    |
| 0.25967     | -0.00808 | 0.45985 | -0.01301   |
| 0.27502     | 0.03204  | 0.44213 | 0.05253    |
| 0.29918     | 0.11979  | 0.35846 | 0.23421    |
| 0.3332      | 0.00064  | 0.18483 | 0.0023     |
| 0.34858     | 0.07532  | 0.48089 | 0.10203    |
| 0.38141     | -0.00264 | 0.23314 | -0.00702   |
| 0.40972     | -0.02264 | 0.25096 | -0.05324   |
| 0.42017     | 0.14412  | 0.37683 | 0.22176    |
| 0.44813     | -0.03049 | 0.29809 | -0.05645   |
| 0.47938     | -0.05013 | 0.41663 | -0.06264   |
| 0.50132     | -0.0356  | 0.28125 | -0.06312   |
| 0.51578     | 0.05519  | 0.28845 | 0.09264    |
| 0.5527      | 0.01995  | 0.39074 | 0.02284    |
| 0.57438     | 0.05432  | 0.44596 | 0.05184    |



|         |         |         |          |
|---------|---------|---------|----------|
| 0.59142 | 0.11278 | 0.37255 | 0.12368  |
| 0.61671 | -0.0056 | 0.66338 | -0.00324 |
| 0.63887 | 0.0797  | 0.51291 | 0.05611  |
| 0.66732 | 0.20083 | 0.38197 | 0.17491  |
| 0.70436 | 0.1986  | 0.29806 | 0.19699  |
| 0.73327 | 0.06319 | 0.46541 | 0.03622  |
| 0.73846 | 0.26119 | 0.55262 | 0.12362  |
| 0.76334 | 0.02284 | 0.46063 | 0.01173  |
| 0.80897 | 0.26752 | 0.3018  | 0.16933  |
| 0.82772 | 0.04935 | 0.53263 | 0.01596  |
| 0.83185 | 0.04935 | 0.45679 | 0.01816  |
| 0.8525  | 0.04049 | 0.32283 | 0.0185   |
| 0.85502 | 0.25404 | 0.38911 | 0.09465  |
| 0.86586 | 0.36892 | 0.62092 | 0.0797   |
| 0.90228 | 0.03131 | 0.19697 | 0.01553  |
| 0.93517 | 0.13662 | 0.31197 | 0.02839  |
| 0.94557 | 0.27882 | 0.65148 | 0.02329  |
| 0.97376 | 0.25807 | 0.56328 | 0.01202  |

| Scale = 0.25 |          |         |            |
|--------------|----------|---------|------------|
| Slip         | D/W      | T/rW    | Efficiency |
| 0.00178      | 0.01207  | 0.03628 | 0.33223    |
| 0.04443      | -0.1136  | 0.3556  | -0.30527   |
| 0.05259      | -0.2057  | 0.05308 | -3.67166   |
| 0.05461      | -0.25829 | 0.11415 | -2.13913   |
| 0.05803      | -0.27512 | 0.40834 | -0.63466   |
| 0.0613       | -0.21858 | 0.01574 | -13.0359   |
| 0.0676       | -0.28339 | 0.46107 | -0.5731    |
| 0.09166      | -0.30892 | 0.3011  | -0.93193   |
| 0.09681      | -0.21082 | 0.3336  | -0.57079   |
| 0.15884      | -0.26682 | 0.45112 | -0.49751   |
| 0.17134      | -0.19757 | 0.51641 | -0.31703   |
| 0.22058      | -0.15741 | 0.29681 | -0.41335   |
| 0.2267       | -0.24764 | 0.55906 | -0.34254   |
| 0.23873      | -0.20976 | 0.24327 | -0.6564    |
| 0.2414       | -0.75904 | 0.30933 | -1.86145   |
| 0.26142      | 0.12124  | 0.6245  | 0.14339    |
| 0.26268      | -0.15973 | 0.46107 | -0.25543   |
| 0.28491      | -0.16978 | 0.38208 | -0.31774   |

|         |          |         |          |
|---------|----------|---------|----------|
| 0.29161 | -0.49365 | 0.04967 | -7.04001 |
| 0.31207 | -0.09299 | 0.41231 | -0.15516 |
| 0.33686 | 0.05336  | 0.74514 | 0.04749  |
| 0.36827 | -0.06471 | 0.40988 | -0.09974 |
| 0.38894 | -0.01254 | 0.25997 | -0.02947 |
| 0.38969 | -0.02426 | 0.13101 | -0.113   |
| 0.41778 | 0.18363  | 0.60914 | 0.17552  |
| 0.42426 | 0.15114  | 0.40523 | 0.21473  |
| 0.44853 | 0.07131  | 0.13015 | 0.30216  |
| 0.51084 | 0.03722  | 0.23835 | 0.07638  |
| 0.52753 | 0.00341  | 0.37309 | 0.00432  |
| 0.54525 | -0.03202 | 0.29078 | -0.05007 |
| 0.62614 | -0.3297  | 0.62911 | -0.19593 |
| 0.64361 | -0.05954 | 0.23186 | -0.09152 |
| 0.64407 | 0.11294  | 0.33244 | 0.12092  |
| 0.66506 | 0.06099  | 0.25437 | 0.0803   |
| 0.68287 | 0.12624  | 0.378   | 0.10591  |
| 0.69157 | 0.07073  | 0.23706 | 0.09203  |
| 0.69483 | 0.45774  | 0.86522 | 0.16145  |
| 0.70027 | 0.16552  | 0.36031 | 0.13768  |
| 0.70072 | -0.39909 | 0.41443 | -0.2882  |
| 0.75734 | 0.05111  | 0.40558 | 0.03058  |
| 0.78835 | 0.11674  | 0.59256 | 0.0417   |
| 0.823   | 0.16487  | 0.46971 | 0.06213  |
| 0.82534 | 0.21911  | 0.5529  | 0.06921  |
| 0.84515 | 0.16049  | 0.48373 | 0.05138  |
| 0.93265 | 0.01416  | 0.49176 | 0.00194  |
| 0.93989 | 0.13826  | 0.4079  | 0.02037  |
| 0.94791 | 0.07458  | 0.54818 | 0.00709  |
| 0.95719 | 0.23663  | 0.62349 | 0.01625  |
| 0.9601  | 0.16585  | 0.48246 | 0.01372  |
| 0.97265 | 0.11661  | 0.47314 | 0.00674  |

### C.3.7 2NS sand – Heterogeneous model at full load and no pre-compaction

| Scale =1 |          |         |            |
|----------|----------|---------|------------|
| Slip     | D/W      | T/rW    | Efficiency |
| -0.00256 | -0.04191 | 0.22496 | -0.18675   |

|         |         |         |         |
|---------|---------|---------|---------|
| 0.0459  | 0.03574 | 0.29055 | 0.11737 |
| 0.09416 | 0.19976 | 0.48204 | 0.37539 |
| 0.09767 | 0.19424 | 0.46276 | 0.37874 |
| 0.14358 | 0.2863  | 0.566   | 0.4332  |
| 0.19467 | 0.33618 | 0.55896 | 0.48436 |
| 0.24529 | 0.42475 | 0.62484 | 0.51304 |
| 0.29673 | 0.37036 | 0.52311 | 0.49791 |
| 0.32801 | 0.37194 | 0.5378  | 0.46474 |
| 0.34711 | 0.37033 | 0.47884 | 0.50494 |
| 0.39676 | 0.31571 | 0.48984 | 0.3888  |
| 0.44332 | 0.41501 | 0.56855 | 0.40634 |
| 0.49208 | 0.38903 | 0.5639  | 0.35041 |
| 0.50079 | 0.39594 | 0.55753 | 0.35453 |
| 0.53955 | 0.43654 | 0.59832 | 0.33594 |
| 0.58911 | 0.40572 | 0.54558 | 0.30556 |
| 0.63798 | 0.49615 | 0.62349 | 0.28809 |
| 0.69066 | 0.44004 | 0.62497 | 0.2178  |
| 0.74226 | 0.44463 | 0.63871 | 0.17942 |
| 0.7935  | 0.44178 | 0.62696 | 0.14551 |
| 0.8452  | 0.47158 | 0.62172 | 0.11742 |
| 0.89492 | 0.48913 | 0.65138 | 0.0789  |
| 0.94415 | 0.49389 | 0.66496 | 0.04148 |
| 0.99288 | 0.50688 | 0.64857 | 0.00557 |

| Scale = 0.7 |          |         |            |
|-------------|----------|---------|------------|
| Slip        | D/W      | T/rW    | Efficiency |
| -0.00262    | -0.02399 | 0.06426 | -0.37427   |
| 0.04328     | 0.13279  | 0.14935 | 0.85064    |
| 0.09636     | 0.03452  | 0.04429 | 0.70431    |
| 0.14801     | 0.02675  | 0.10895 | 0.20922    |
| 0.19701     | 0.14936  | 0.26958 | 0.4449     |
| 0.24684     | 0.21672  | 0.33767 | 0.48337    |
| 0.29699     | 0.24141  | 0.39902 | 0.42533    |
| 0.34672     | 0.25669  | 0.36253 | 0.46256    |
| 0.39506     | 0.28445  | 0.42986 | 0.4003     |
| 0.44303     | 0.27173  | 0.42416 | 0.35681    |
| 0.48986     | 0.35687  | 0.50657 | 0.35939    |
| 0.53965     | 0.28138  | 0.39911 | 0.32455    |
| 0.55131     | 0.40076  | 0.47765 | 0.37646    |

|         |         |         |         |
|---------|---------|---------|---------|
| 0.58831 | 0.32978 | 0.40884 | 0.33208 |
| 0.63874 | 0.31408 | 0.40175 | 0.28242 |
| 0.68824 | 0.39952 | 0.47259 | 0.26355 |
| 0.74134 | 0.3598  | 0.47659 | 0.19528 |
| 0.79197 | 0.41058 | 0.44663 | 0.19124 |
| 0.84286 | 0.40569 | 0.44089 | 0.14459 |
| 0.89313 | 0.39184 | 0.42127 | 0.09941 |
| 0.94239 | 0.40054 | 0.45015 | 0.05126 |
| 0.9898  | 0.47828 | 0.52669 | 0.00926 |

| Scale = 0.5 |          |         |            |
|-------------|----------|---------|------------|
| Slip        | D/W      | T/rW    | Efficiency |
| -0.0032     | -0.03295 | 0.14461 | -0.22856   |
| 0.04651     | 0.08426  | 0.21995 | 0.36529    |
| 0.09731     | 0.11714  | 0.26317 | 0.40181    |
| 0.14889     | 0.1105   | 0.28586 | 0.32899    |
| 0.2008      | 0.21783  | 0.41779 | 0.41669    |
| 0.25247     | 0.27645  | 0.44831 | 0.46095    |
| 0.30343     | 0.33308  | 0.51193 | 0.45322    |
| 0.35339     | 0.31353  | 0.44282 | 0.45781    |
| 0.40237     | 0.32819  | 0.48383 | 0.40538    |
| 0.45066     | 0.32103  | 0.4515  | 0.3906     |
| 0.49876     | 0.33629  | 0.49821 | 0.33834    |
| 0.54724     | 0.34819  | 0.48383 | 0.32583    |
| 0.59652     | 0.39489  | 0.52078 | 0.30595    |
| 0.64683     | 0.36286  | 0.48318 | 0.26522    |
| 0.69808     | 0.37531  | 0.50004 | 0.22661    |
| 0.74989     | 0.40262  | 0.50909 | 0.1978     |
| 0.80175     | 0.42798  | 0.55004 | 0.15425    |
| 0.85311     | 0.42531  | 0.5358  | 0.1166     |
| 0.90357     | 0.42516  | 0.5398  | 0.07595    |
| 0.95301     | 0.41681  | 0.52412 | 0.03737    |
| 1.00153     | 0.43606  | 0.53594 | -0.00124   |

| Scale = 0.25 |          |          |            |
|--------------|----------|----------|------------|
| Slip         | D/W      | T/rW     | Efficiency |
| 0.00369      | 0.01634  | -0.00815 | -1.99819   |
| 0.00771      | -0.06785 | 0.14993  | -0.44905   |

|         |          |          |          |
|---------|----------|----------|----------|
| 0.01666 | -0.10451 | 0.3303   | -0.31113 |
| 0.01601 | -0.08076 | -0.06379 | 1.24578  |
| 0.01409 | -0.19618 | 0.07686  | -2.51651 |
| 0.03646 | -0.2512  | -0.06719 | 3.60234  |
| 0.04272 | 0.03118  | -0.01904 | -1.56808 |
| 0.12808 | -0.18305 | 0.23502  | -0.67911 |
| 0.12073 | 0.2068   | 0.10202  | 1.78238  |
| 0.19294 | 0.03859  | 0.10027  | 0.31061  |
| 0.2109  | 0.02148  | -0.02074 | -0.81715 |
| 0.2705  | 0.22507  | 0.29966  | 0.54791  |
| 0.30203 | 0.37766  | 0.28767  | 0.9163   |
| 0.36862 | 0.27247  | 0.45569  | 0.37752  |
| 0.41119 | 0.21967  | 0.33963  | 0.38084  |
| 0.44151 | 0.38705  | 0.42612  | 0.50727  |
| 0.502   | 0.33508  | 0.59409  | 0.28088  |
| 0.54928 | 0.39803  | 0.64189  | 0.27949  |
| 0.5682  | 0.62247  | 0.68011  | 0.39521  |
| 0.61605 | 0.71231  | 0.64331  | 0.42513  |
| 0.69439 | 0.45637  | 0.51207  | 0.27237  |
| 0.74051 | 0.38296  | 0.36747  | 0.27043  |
| 0.78825 | 0.27923  | 0.22342  | 0.26464  |
| 0.82967 | 0.4595   | 0.37529  | 0.20856  |
| 0.88088 | 0.71615  | 0.59804  | 0.14265  |
| 0.93792 | 0.49453  | 0.62482  | 0.04914  |
| 0.9453  | 0.41713  | 0.6117   | 0.0373   |
| 0.95495 | 0.34011  | 0.62081  | 0.02468  |
| 0.96036 | 0.23447  | 0.54604  | 0.01702  |
| 0.96185 | 0.2425   | 0.54323  | 0.01703  |
| 0.96274 | 0.25776  | 0.54952  | 0.01748  |

### C.3.8 Fine-grained sand – Heterogeneous model at full load and 0.15 m pre-compaction

| Scale =1 |          |         |            |
|----------|----------|---------|------------|
| Slip     | D/W      | T/rW    | Efficiency |
| 0.00384  | -0.18104 | 0.23282 | -0.77459   |
| 0.05251  | -0.26469 | 0.31875 | -0.78682   |
| 0.10131  | -0.21447 | 0.29281 | -0.65825   |
| 0.14971  | -0.22812 | 0.29529 | -0.65688   |

|         |          |         |          |
|---------|----------|---------|----------|
| 0.20041 | -0.21355 | 0.28073 | -0.60824 |
| 0.25077 | -0.10174 | 0.37892 | -0.20117 |
| 0.3032  | -0.16867 | 0.42152 | -0.27882 |
| 0.35252 | -0.0472  | 0.45379 | -0.06734 |
| 0.40327 | 0.01736  | 0.3705  | 0.02796  |
| 0.45332 | -0.00524 | 0.42965 | -0.00666 |
| 0.49484 | 0.0259   | 0.42705 | 0.03064  |
| 0.5021  | -0.00198 | 0.35329 | -0.0028  |
| 0.55027 | 0.01094  | 0.36775 | 0.01338  |
| 0.59742 | 0.07076  | 0.34628 | 0.08227  |
| 0.64605 | 0.07119  | 0.35161 | 0.07167  |
| 0.69461 | 0.12898  | 0.50094 | 0.07863  |
| 0.74482 | 0.14745  | 0.42996 | 0.08751  |
| 0.79478 | 0.13201  | 0.3932  | 0.0689   |
| 0.84779 | 0.09879  | 0.44003 | 0.03417  |
| 0.89978 | 0.11752  | 0.23443 | 0.05024  |
| 0.95022 | 0.17644  | 0.32312 | 0.02718  |
| 1.00183 | 0.0779   | 0.13441 | -0.00106 |

| Scale = 0.7 |          |         |            |
|-------------|----------|---------|------------|
| Slip        | D/W      | T/rW    | Efficiency |
| -0.00029    | -0.15891 | 0.0503  | -3.16001   |
| 0.04791     | -0.12314 | 0.18859 | -0.62165   |
| 0.09674     | -0.08863 | 0.15905 | -0.50333   |
| 0.14651     | -0.00516 | 0.21077 | -0.02089   |
| 0.1973      | 0.03294  | 0.23196 | 0.11398    |
| 0.24888     | 0.01026  | 0.11875 | 0.06492    |
| 0.30079     | -0.03773 | 0.10263 | -0.25702   |
| 0.35247     | 0.03342  | 0.16455 | 0.1315     |
| 0.40343     | 0.06605  | 0.18606 | 0.21176    |
| 0.45339     | 0.02487  | 0.15773 | 0.08618    |
| 0.50235     | 0.05588  | 0.20577 | 0.13513    |
| 0.55064     | 0.02436  | 0.17772 | 0.06161    |
| 0.59874     | 0.03802  | 0.08923 | 0.17098    |
| 0.64721     | 0.01691  | 0.04833 | 0.12343    |
| 0.69649     | -0.00119 | 0.05961 | -0.00603   |
| 0.7468      | 0.01724  | 0.04386 | 0.09955    |
| 0.79804     | 0.01214  | 0.03761 | 0.06518    |
| 0.84986     | 0.01003  | 0.06669 | 0.02257    |

|         |         |         |          |
|---------|---------|---------|----------|
| 0.90171 | 0.03121 | 0.07591 | 0.04041  |
| 0.95307 | 0.09841 | 0.18069 | 0.02556  |
| 1.00353 | 0.03525 | 0.04547 | -0.00274 |

| Scale = 0.5 |          |         |            |
|-------------|----------|---------|------------|
| Slip        | D/W      | T/rW    | Efficiency |
| 0.00045     | -0.16997 | 0.14156 | -1.20015   |
| 0.02546     | -0.19392 | 0.25367 | -0.74498   |
| 0.0201      | -0.15155 | 0.22593 | -0.65729   |
| 0.01666     | -0.11664 | 0.25303 | -0.4533    |
| 0.08578     | -0.0903  | 0.25635 | -0.32206   |
| 0.12172     | -0.04574 | 0.24883 | -0.16144   |
| 0.18122     | -0.1032  | 0.26208 | -0.32241   |
| 0.2177      | -0.00689 | 0.30917 | -0.01743   |
| 0.27585     | 0.0417   | 0.27828 | 0.10852    |
| 0.32442     | 0.00982  | 0.29369 | 0.02258    |
| 0.37252     | 0.04089  | 0.31641 | 0.08109    |
| 0.44354     | 0.01119  | 0.26551 | 0.02345    |
| 0.48431     | 0.02448  | 0.22849 | 0.05526    |
| 0.52966     | 0.04384  | 0.19731 | 0.1045     |
| 0.58132     | 0.035    | 0.20561 | 0.07128    |
| 0.63034     | 0.07311  | 0.2724  | 0.09922    |
| 0.67755     | 0.07979  | 0.23379 | 0.11006    |
| 0.72827     | 0.07102  | 0.22994 | 0.08392    |
| 0.77446     | 0.065    | 0.25797 | 0.05683    |
| 0.81324     | 0.10796  | 0.20756 | 0.09714    |
| 0.85726     | 0.10585  | 0.1843  | 0.08198    |
| 0.89889     | 0.03895  | 0.06721 | 0.0586     |
| 0.993       | 0.06983  | 0.04295 | 0.01138    |

| Scale = 0.25 |          |         |            |
|--------------|----------|---------|------------|
| Slip         | D/W      | T/rW    | Efficiency |
| 0.00358      | 0.02185  | 0.12299 | 0.17705    |
| 0.04242      | -0.10153 | 0.15354 | -0.63325   |
| 0.06723      | -0.20828 | 0.32064 | -0.6059    |
| 0.07586      | -0.34262 | 0.39018 | -0.81149   |

|         |          |         |          |
|---------|----------|---------|----------|
| 0.14168 | -0.24231 | 0.14685 | -1.41632 |
| 0.14161 | 0.00595  | 0.6593  | 0.00774  |
| 0.18038 | -0.13915 | 0.16513 | -0.69067 |
| 0.25277 | -0.30728 | 0.17135 | -1.34    |
| 0.29873 | -0.37907 | 0.44228 | -0.60104 |
| 0.35922 | -0.27045 | 0.18591 | -0.93218 |
| 0.37713 | -0.04246 | 0.25578 | -0.1034  |
| 0.43474 | 0.00491  | 0.45499 | 0.0061   |
| 0.47941 | 0.10811  | 0.51691 | 0.10888  |
| 0.50067 | 0.12189  | 0.5742  | 0.10599  |
| 0.62622 | -0.24978 | 0.66974 | -0.1394  |
| 0.65021 | -0.30989 | 0.35471 | -0.30559 |
| 0.64071 | 0.06987  | 0.35421 | 0.07088  |
| 0.70872 | -0.00097 | 0.71915 | -0.00039 |
| 0.77471 | -0.10104 | 0.63401 | -0.0359  |
| 0.79705 | 0.00836  | 0.42713 | 0.00397  |
| 0.87204 | -0.03335 | 0.57305 | -0.00745 |
| 0.91155 | -0.02846 | 0.62715 | -0.00401 |
| 0.95634 | 0.05572  | 0.63386 | 0.00384  |
| 1.00302 | 0.04465  | 0.48529 | -0.00028 |



**C.3.9 2NS sand tyre slip-sinkage data on homogeneous (Hom.) and heterogeneous (Hetero.) soil models at 0.150 m soil pre-compaction**

| Slip | Tyre sinkage (mm) |        |             |        |             |        |              |        |
|------|-------------------|--------|-------------|--------|-------------|--------|--------------|--------|
|      | Scale = 1         |        | Scale = 0.7 |        | Scale = 0.5 |        | Scale = 0.25 |        |
|      | Hom               | Hetero | Hom         | Hetero | Hom         | Hetero | Hom          | Hetero |
| 0    | 0                 | 0      | 0           | 0      | 0           | 0      | 0            | 0      |
| 0.1  | 1.12              | 1.19   | 0.29        | 2.26   | 0.28        | 2.28   | 0.21         | 0.29   |
| 0.2  | 1.51              | 1.94   | 1.42        | 4.23   | 0.70        | 3.75   | 0.6          | 0.63   |
| 0.3  | 1.77              | 3.55   | 1.98        | 4.68   | 2.07        | 6.55   | 0.9          | 1.44   |
| 0.4  | 2.02              | 5.83   | 3.61        | 5.91   | 3.63        | 7.78   | 1.14         | 2.14   |
| 0.5  | 4.62              | 8.26   | 4.41        | 7.60   | 3.99        | 8.02   | 1.32         | 3.54   |
| 0.6  | 6.74              | 10.80  | 5.21        | 9.32   | 4.42        | 8.20   | 1.51         | 4.08   |
| 0.7  | 7.52              | 12.35  | 6.87        | 11.99  | 5.66        | 8.40   | 1.80         | 4.83   |
| 0.8  | 8.59              | 13.93  | 8.38        | 14.32  | 6.11        | 8.54   | 2.01         | 5.45   |
| 0.9  | 9.4               | 15.42  | 9.68        | 15.72  | 7.38        | 9.2    | 2.44         | 6.62   |
| 1    | 11.40             | 16.28  | 11.66       | 16.41  | 9.46        | 10.57  | -            | -      |

**C.3.10 Fine-grained sand tyre slip-sinkage data on homogeneous (Hom.) and heterogeneous (Hetero.) soil models at 0 m pre-compaction**

| Slip | Tyre sinkage (mm) |        |             |        |             |        |              |        |
|------|-------------------|--------|-------------|--------|-------------|--------|--------------|--------|
|      | Scale = 1         |        | Scale = 0.7 |        | Scale = 0.5 |        | Scale = 0.25 |        |
|      | Hom               | Hetero | Hom         | Hetero | Hom         | Hetero | Hom          | Hetero |
| 0    | 0                 | 0      | 0           | 0      | 0           | 0      | 0            | 0      |
| 0.1  | 4.19              | 7.28   | 4.58        | 7.91   | 7.26        | 6.67   | 10.76        | 12.11  |
| 0.2  | 6.84              | 12.41  | 7.65        | 15.22  | 12.42       | 13.38  | 13.24        | 13.78  |
| 0.3  | 10.06             | 19.23  | 11.8        | 19.36  | 16          | 21.65  | 18.26        | 19.12  |
| 0.4  | 13.6              | 25.85  | 15.48       | 25.09  | 20.23       | 27.67  | 22.73        | 25.15  |
| 0.5  | 18.8              | 32.78  | 20.58       | 30.23  | 26.42       | 31.27  | 26.93        | 29.67  |
| 0.6  | 23.26             | 39.82  | 24.66       | 34.20  | 30.71       | 31.96  | 32.01        | 28.17  |
| 0.7  | 27.19             | 46.35  | 29.49       | 39.23  | 35.96       | 38.38  | 34.56        | 34.92  |
| 0.8  | 31.60             | 54.34  | 34.16       | 45.90  | 41.56       | 48.55  | 42.83        | 42.72  |
| 0.9  | 37.30             | 62.25  | 38.84       | 52.42  | 47.75       | 51.52  | 48.41        | 49.45  |
| 1    | 44.74             | 71.82  | 46.35       | 60.53  | 52.49       | 56.54  | 53.10        | 55.62  |

## Appendix D Terms used in statistical analysis

- Percent deviation measures the percentage difference between the predicted value and the observed value.

$$D = \frac{s - o}{o} \times 100$$

- Root mean squared error measures the error magnitude of the error. It is the squared difference between the observed value and corresponding predicted value and then averaged over the sample. Further, the square root of the average is taken. Since the errors are squared before averaged, RMSE gives relatively high weight to large errors. It is useful when large errors are particularly undesirable.

$$RMSE = \frac{\sqrt{\frac{\sum(o_i - s_i)^2}{N}}}{o_{avg}} \times 100$$

- Coefficient of determination ( $R^2$ ) describes the collinearity between the simulated data and measured data. It describes the proportion of variance in measured data explained by the model. It ranges from 0 to 1, with higher values indicate less error variance. It is computed as

$$R^2 = \left( \frac{\sum[o_i - o_{avg}][s_i - s_{avg}]}{[\sum(o_i - o_{avg})^2]^{0.5} [\sum(s_i - s_{avg})^2]^{0.5}} \right)^2$$

- Standard deviation is the measure of the dataset's dispersion relative to the mean. It is calculated as the square root of the variance. If the data points are further from the mean, there is a higher standard deviation, which means more spread out of data.

$$\sigma = \sqrt{\frac{\sum(o_i - o_{avg})^2}{N}}$$

- The coefficient of variation (CV) is the ratio of the standard deviation to the mean. It helps to compare the degree of variation from one data set to another, even if the means are drastically different. The lower the CV, the lower is the dispersion in the data.

$$CV = \frac{\sigma}{\mu}$$

- F – test is used to determine whether the variability between group means is larger than the variability of the groups' observations. If this ratio is large, it can be concluded that not all means are equal. Statistically, the F-test compares the ratio of two variances  $s_1$  and  $s_2$  against the null hypothesis. The null hypothesis assumes that variances are equal.

$$F = \frac{s_1^2}{s_2^2}$$

- T-test calculates the t-value, which signifies the difference between the groups and the difference within the groups. The larger the t-value, the more is the differences. The smaller the t-value, the groups are similar. A two-sample t-test determines any significant difference between the means ( $\mu$ ) of the two groups available in one-tail and two-tail formats. The t-value is computed using the formula against the null hypothesis. Therefore, the null hypothesis assumes that means are equal.

$$T = \frac{(\mu_1 - \mu_2)}{\sigma/\sqrt{N}}$$