

Autonomous Trajectory Control for Directional Drilling



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A thesis submitted for the degree of
Doctorate of Engineering

2013 Jan

Abstract

The demand on the worlds energy resources will intensify as the prediction of the global population rising by 2.3 billion by 2050 becomes a reality. With more than half of the worlds energy consumption being currently derived from oil and gas, there is a need to access challenging new reserves of which there is an estimated 5.5 trillion barrels remaining. The need, therefore, to develop and employ more sophisticated methods to extract these hard-to-reach reserves is clear. Rotary steerable drilling with near-bit sensors, actuators, and processors will play a significant role in accessing this oil, having already reduced the time and extended the reach of wellbores. In addition to this, trajectory control can further enhance the capability of a rotary steerable system, reducing costs and improving the hydrocarbon recovery volume in the oilfield development process.

This thesis presents a general system of trajectory control consisting of three subsystems: a subsystem for generating optimum geometric well-plan profiles through target volumes in the reservoir; a subsystem for stabilizing the propagating attitude of the centre-line of the borehole; and a subsystem for generating optimum attitude set-points for the downhole attitude control system for executing the well-plan.

The attitude control system, determines the attitude represented as a unit vector from on-board sensors located near the drill-bit. The control action is represented as an axis which rotates the attitude of the steering assembly towards the demand attitude along an arc of a great circle in the attitude state-space. This method is extended further to investigate the effect of spatially delaying the attitude sensors behind the drill-bit on the attitude tracking. This sensor-to-bit distance induces spiraling in the profile of borehole. This motivated an extension to the attitude control system whereby the feedback control law acts on the predicted attitude at the bit.

The well-planning methodology generates paths continuous in their position, direction, and curvature in a multi-objective optimization strategy. The solution is a b-spline space-curve minimizing a weighted sum of arclength, strain energy and torsion objective functions. The absolute curvature is constrained, ensuring that wells are planned with respect to the curvature capability of the steering assemblies. These well-plans are comprised of a concatenation of smaller well-plans between targets in a hydrocarbon reservoir. These targets are represented as a combination of equality and inequality constraints on the position and attitude.

The trajectory control strategy determines a correction well-plan coincident with the measured position and attitude of the steering assembly to guide the borehole towards the well-plan. This correction path is represented as a b-spline space curve which minimizes a weighted sum of wellbore tortuosity and well-plan deviation with respect to the curvature constraint of the steering assembly. The integrated system of the attitude and trajectory control is demonstrated in simulation to guide a modeled $8^{\circ}/100\text{ft}$ push-the-bit tool to converge towards and track a $2^{\circ}/100\text{ft}$ well-plan to an accuracy of 0.5m.

The commercial value of the system for the oilfield services industry is investigated through a market analysis for rotary steerable systems leading to a proposition of value. Reducing hole-spiraling through attitude control systems reduces the friction in the well-bore on the drill-string and casing liners extending the depth capability of well-bores. This allows greater coverage of hydrocarbon volumes to be accessed from a single platform, thereby reducing the number of platforms required in an oilfield development programme. Extending the reach of a well increases the production volume from a single well by increasing the footage contact with hydrocarbon saturated rocks. The value in introducing a trajectory control system, by minimizing well-plan deviation reduces costs associated with sidetracking a well. Furthermore the benefit of integrating a well-plan deviation minimizing trajectory control system with existing rate of penetration optimization technologies has scope to reduce rig time where it is estimated that there is a global annual benefit of USD490m.

This thesis is dedicated to the loving memory of my brother
Kunjai Panchal

Acknowledgements

The last four years have been an odyssey through the fascinating realms of control theory and directional drilling. I owe a great deal of gratitude to a vast number of people who have supported me with their kindness, help and inspiration.

Thanks to my supervisor James Whidborne, whose passion for control theory is inspiring and has been a role model. He always managed to find time for me and I thoroughly enjoyed all the creative discussions we had. Thanks to my supervisor Martin Bayliss for your encouragement, and all the help you have given. Without your vision, this project would not have existed or developed as it has. Thanks to Prof. Rick Mitchel for your guidance and expertise in the management research. Your direction has been very valuable, and created many fun research exercises.

Thanks to Maurice Ringer and Walt Aldred for welcoming me to the drilling group in SCR. I feel lucky to have been in the presence of such talented and enthusiastic people in the environment which you have created.

Thanks to Mark Frey, Geoff Downton, Minh Chou for the helpful discussions and for sharing your insight and passion. Thanks to everyone in Schlumberger who have helped me along the way: Jakub Jarebek, Jacques Orban, Saverio Messineo, Samba Ba, Sjoerd Brands, Devi Putra, Gokturk Tunc, Adam Bowler, Richard Ferguson, Junichi Sugura, Michalis Frangos, Murat Panayirci, John Cook, Chris Bogath, Ben Jeffries, Jonathon Dunlop.

Special thank my parents for all the continuing love and support, who have always given so much and asked very little in return.

Thanks to my darling Katie for all your love and care.

Thanks to the Engineering and Physical Sciences Research Council and Schlumberger Stonehouse Technology centre for the funding for the research.

Thanks to all my friends from Peterhouse and Cranfield. In particular Patrick Milton, Ramey Jamil, Leonardo Lopez for some unforgettable times over the last few years. I would also like to thank Nikhil Shah, Kwame Ofesu-Osei, Razzak Al-Gournawi, Isaac Kurira, Timm Faulwasser, Hiroyuki Fujiyoka. There are many other people I owe my thanks to, and please forgive me and accept my gratitude if I have forgotten to mention you.

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GLOSSARY

Glossary

ATCS	Autonomous Trajectory Control System
bbbl	barrels
BHA	Bottom Hole Assembly
bn	billion
boe	Barrels of oil equivalent
D&I	Direction and Inclination
DD	Directional Driller
DE	Drilling Engineer
DLS	Dog leg severity
DPV	Discounted Present Value

ERD	Extended Reach Drilling
IOC	International Oil Company
LWD	Logging While Drilling
mn	million
MPC	Model Predictive Control
MWD	Measurement While Drilling
NOC	National Oil Company
NPT	Non productive time
NPV	Net Present Value
ROP	Rate of Penetration
RPM	Revolutions per minute
RSS	Rotary Steerable System
SRACS	Spiral Reducing Attitude Control System
Tripping	The process of removing a string from the borehole
USD	United States Dollar
WOB	Weight on Bit

GLOSSARY

1

Introduction

Oil and gas resources supply over half the global energy demand. Since the 1980s production rates from existing oilfields are increasingly greater than the discovery rate of new reserves. With global economic development and rising demographics, challenging oil reserves, in hostile environments need exploiting to meet future energy demand. The practice of rotary drilling creates boreholes to extract oil and gas from deep beneath the Earth’s surface. Boreholes tailored to maximize contact with oil reservoirs increase the volume of production recovery. This can be achieved by directional drilling which is the process of creating geometric boreholes by steering a drilling tool along a well-plan. The well-plan is defined by a multidisciplinary team of reservoir engineers, drilling engineers, geosteerers, and geologists amongst others. Since their inception, Rotary Steerable Systems (RSS) have enabled steering automation with down-hole sensors, actuators, and processors close to the bit. This enables the drilling of longer reaching wells and complex well geometries to access challenging reserves. Automation adds capability to the drilling process in being a value driver with further potential to reduce cost per foot of a well, and maximize production which can be recovered in a reservoir. Since oil and gas resources are finite, reducing the cost per barrel is required to economically meet the energy demand for the near future.

This thesis presents a control system strategy to create boreholes along predefined well-plans with respect to geological targets. This control system comprises: (i) a subsystem for controlling the attitude of hole propagation for implementation on a down-hole rotary steerable system; (ii) a surface trajectory control system for generating steering

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commands for the down-hole attitude control system; and (iii) an optimization method for generating minimum energy well-plans with respect to geometric constraints to reduce borehole friction.

This chapter begins with Section 1.1 describing the anatomy of the drilling system. Section 1.2 describes the types of steering assemblies used in the industry to produce boreholes with emphasis on rotary steerable assemblies. Section 1.3 introduces the overall trajectory control system and its subsystems, in addition, providing a review of the current state of the art for well-planning, attitude control, and trajectory control. Section 1.4 details the aims and objective of the research embodied in the thesis, and finally section 1.5 outlines the thesis. This thesis is specific to drilling with rotary steerable tools. Other methods of producing boreholes involve the use of down-hole motors and turbines.

1.1 Anatomy of the Drilling System

For understanding the drilling process, it is imperative to understand the individual components of a drilling system and their interactions. The main components of a drilling system are the:

- drill-bit
- drill-string
- drilling-fluid
- top-drive
- bottom-hole-assembly (BHA)

Figure 1.1 shows a schematic representation of the main RSS directional drilling system components.

1.1.1 Drill-bit

Drilling boreholes involves the destruction of rocks, often sedimentary, in the Earth's crust, propagating from the surface to the target depth in a subsurface reservoir. The

1.1 Anatomy of the Drilling System

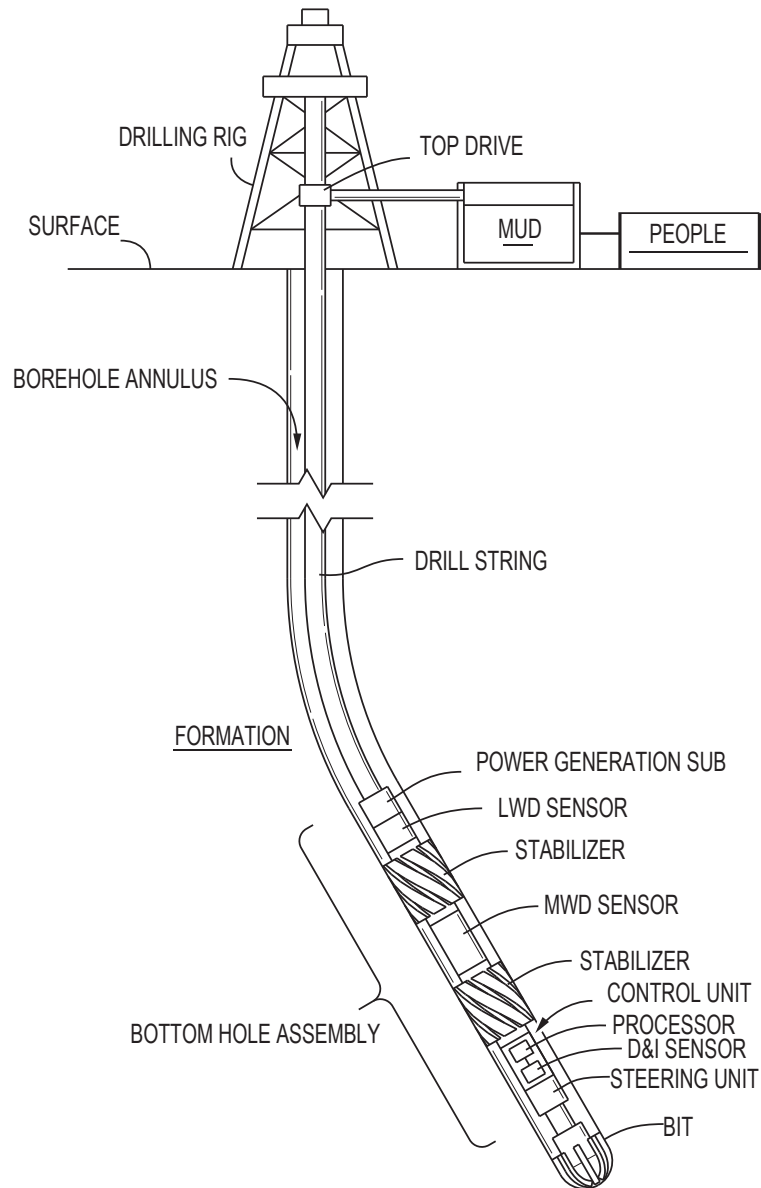


Figure 1.1: Schematic of main RSS directional drilling system components

rock destruction is achieved through a *drill-bit*, which interacts with the rock creating cuttings which are transported to the surface. The two main varieties of drill bits are roller-cone bits [Schumacher, 1987], and fixed-cutter bits [Bovenkerk, 1978]. Both fixed-cutter and roller-cone bits apply weight and torque to destroy a volume of rock.

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The combination of weight and torque to destroy this volume depend on the physical characteristics of the drill-bit and formation.

Roller-cone Bits

Roller-cone bits consist of three cones with teeth made from steel or tungsten carbide. The cones are mounted onto the body of the bit which is connected to the drill-string via the Bottom Hole Assembly (BHA). The cones rotate about its axis of symmetry, and the interaction of its teeth with the formation causes rock destruction. The cones are rotated passively as the drill-string is rotated actively from the surface. The body of the bit contains nozzles which jet drilling fluid transported from the drill-string to the formation, and the cuttings are transported up the annulus of the borehole. Roller-cone bits are sometimes called tri-cone bits. Variations which include two rotating cones exist.

Fixed Cutter Bits

Fixed cutter bits contain cutters mounted onto the body of the bit, in a geometry tailored to the drilling operation. The cutters have a circular face made from polycrystalline diamond compact (PDC), which shear the rock continuously generating cuttings which are transported to the surface via the drilling fluid.

1.1.2 Drill-string

The *drill-string* is the combination of the *drill-pipe* and the *BHA*. The drill-string is connected to the drill-bit at one end, and connected to the top-drive at the other end. The drill-string serves many purposes for the drilling process: circulation of *drilling-fluid* to the bit; transfer of weight to the bit, transfer of torque to the bit, medium of information transfer to the BHA via telemetry. The drill-pipe is cylindrical and hollow to allow the transportation of drilling fluid. During drilling, the drill-pipe from the surface to the BHA is under tension, with most of the BHA under compression. The drill-pipe tension is controlled by a hook suspended onto a cable, and this tension in the cable is known as the *hook-load*. The drill-pipes are constructed in 30ft (9.1m) standardized sections with threaded joints at each end. The drilling process is routinely interrupted to connect additional sections of drill-pipe to the drill-string at the surface

to increase the length. The drill-string is influenced by friction in the well-bore, which is undesirable for the transfer of weight and torque to the bit. Furthermore the curvature of the well-bore affects the failure of the threaded joints which can cause a *twist-off*. Because for RSS systems, the drill-string is always rotating, and a drill-pipe in a curved borehole will undergo tension-compression cycles by work done by the borehole in bending and twisting the pipe.

1.1.3 Drilling Fluid

Drilling-fluid, also known as *mud*, circulates through the drill-string to the bit, and up the annulus between the drill-pipe and the borehole to the surface. The drilling fluid transports cuttings from the bit to the surface. The drilling fluid from the surface flows to a *shale-shaker* where the cuttings are removed and the drilling fluid is recirculated to the bit, via surface pumps. A *mud-logger* analyzes cuttings in the shale-shaker to determine properties of the rock through which the bit is drilling. A *mud-engineer* designs the mud to be used depending on the operation and the environment. The pressure in the annulus between the drill-string and the borehole is designed by the mud-engineer to be sufficiently large to disallow an influx of geological fluids to enter the borehole, or to permit well-bore collapse. Additionally, the pressure in the borehole cannot be so large as to fracture the formation, which causes lost circulation of drilling fluid through the fracture cracks.

1.1.4 Top-drive

Electric or hydraulic motors power the top-drive, which connects to the suspension cable block-set via a hook. The top-drive connects directly to the drill-string and provides torque which transfers via the drill-string to the bit [Boyadjieff, 1983]. A hydraulic line in the top-drive carries the fluid flow from the pumps to the drill-string. The block which the top-drive is connected to is free to travel down during penetration and up when *tripping* the drill-pipe out of the hole.

1.1.5 Bottom Hole Assembly

The BHA is the active part of the directional drilling system and is constituted of subsystems or ‘subs’ from the bit back to the drill-pipe. The BHA consists of:

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- stabilizers;
- drill-collars;
- power generation module;
- Measurement While Drilling (MWD) subsystem;
- Logging While Drilling (LWD) subsystem;
- steering assembly.

The BHA lies entirely inside the borehole and the subs are of a lower maximum radial diameter than the bit to allow passage for fluid flow up the annulus to the surface.

Stabilizers

Stabilizers are mounted onto the BHA to mechanically stabilize the BHA in the borehole. Stabilizers and their contacts with the formation play a major role in determining the steering response of the BHA. The canonical steering BHA has a three point contact with the formation. The nature of this determines the curvature of the hole propagation. The three-point contact consists of the bit, a near bit actuating stabilizer, and a third stabilizer further down the BHA. The placement of stabilizers are designed by a *drilling-engineer* depending on the formation and the geometry of the intended well.

Drill collars

Drill collars are heavy sections of thick steel pipe mounted to the BHA in order to provide weight on bit. The inner and outer diameters of the drill-collars are designed for the intended flexibility of the BHA.

Power Generation Module

The PGM uses a turbine to convert the down-hole flow from the drilling fluid in the drill-string into electrical energy to power the BHA.

Measurement While Drilling (MWD)

MWD subs contain sensors which survey the drilling operation. The MWD surveying techniques for determining the orientation of the hole have particular relevance for the attitude and trajectory control systems studied in this thesis. Parameters which MWD tools record include:

- orientation inclination and azimuth;
- shock and vibration;
- collar angular velocity;
- flow rate;
- temperature;
- gauge (i.e. the shape and size of the hole).

Two sets of sensors in the BHA measure the borehole orientation: The MWD sub; and the D&I (Direction and Inclination) sub. The former is of a greater quality and placed further behind the bit than the latter. The D&I sub is an integral part of the steering assembly and interfaces directly with the actuators to form a closed loop for down-hole control systems.

Surveys are taken as static, or continuous. Static surveys are conducted during new drill-pipe connections. Continuous surveys are conducted during the drilling process. Because of the signal noise from vibrations while drilling, static surveys are always more accurate than continuous surveys. Furthermore the time elapsed for conducting a static survey is greater than for a continuous survey.

Directional Drillers (DD) monitor the attitude of the BHA by the inclination θ_{inc} and azimuth θ_{azi} angles (shown in Figure 1.2). The inclination angle $\theta_{inc} \in [0, 180]$ is the angular deviation from vertical with zero defined downward. The azimuthal angle $\theta_{azi} \in [0, 360]$ is measured relative to the projection of the Earths magnetic field in the horizontal plane. Zero corresponds to due North with 90° due East. The toolface angle θ_{tf} is the measure of the steering direction of the borehole measured with reference to

1. INTRODUCTION

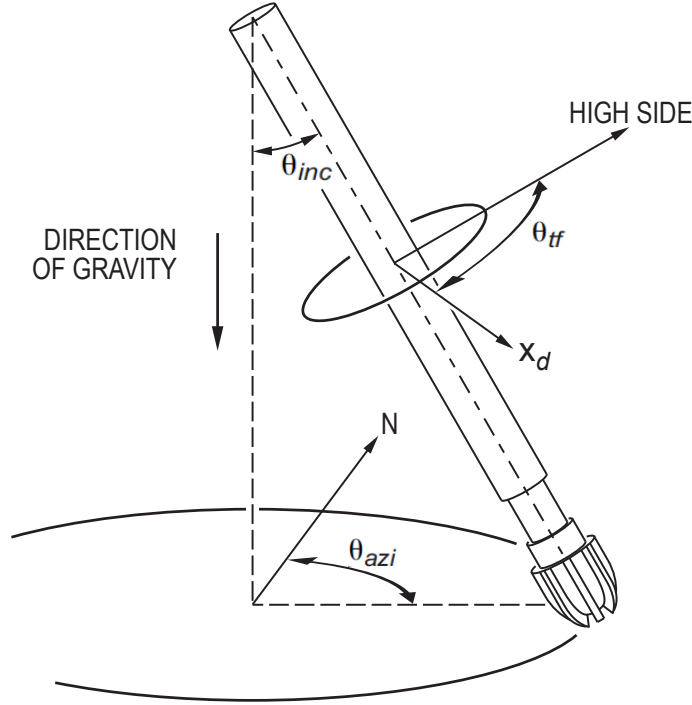


Figure 1.2: Conventional attitude and steering parameters for a Bottom Hole Assembly(BHA)

the highside (see Figure 1.2). The highside is defined to be a reference which is the highest point on a cross section of borehole. The normal to this cross section plane is a tangent to the centreline. This quantity is undefined for vertical holes since the highside of the hole is undefined. In this situation DD's steer relative to the *Magnetic Toolface* (MTF). The MTF is defined as the angle in the plane perpendicular to the tangent of the borehole, where the projection of the magnetic field line defines zero MTF.

The attitude sensor set consists of one triaxial set of accelerometers and one triaxial set of magnetometer transducers. All six transducer signals evaluate the orientation azimuth θ_{azi} and inclination θ_{inc} of the MWD and D&I. The continuous surveys use only the continuous axial accelerometer and magnetometer signals in combination with the previous static survey. Therefore continuous attitude measurements lose accuracy the further the tool propagates from the position where the static survey was taken. In

practice DD's periodically stop drilling to make accurate static surveys to improve the accuracy of the continuous surveys.

MWD sensors are positioned some distance (sometimes several tens of feet) behind the bit. This introduces a measurement delay in the attitude feedback measurement which causes oscillations in the borehole profile known as *hole-spiraling*. Additionally there is a dynamic response between the applied tool-face from the actuator and the response of the tool-face of the steering unit.

Logging While Drilling

Logging-while-drilling (LWD) subs are sophisticated surveying tools that are distinguishable from MWD by surveying petrophysical properties [Clark et al., 1993]. LWD subs sense:

- porosity;
- permeability;
- resistivity;
- gamma ray count;
- acoustic wave impedance;
- sampling of rocks;
- hydrocarbon saturation;
- and chemical properties

to characterize the surrounding formation during drilling operations to aid the geological placement of the well.

Telemetry

Data recorded down-hole is transmitted from the BHA to the surface via *telemetry*. The most commonly used method for telemetry for RSS drilling is *mud pulse* telemetry. Mud pulse telemetry involves modulating pressure waves in the drilling fluid generated

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by the BHA, which travel up-hole within the drill-string to be detected and interpreted at the surface [Kotlyar, 1987]. Mud pulse telemetry can be operated in real time during a drilling operation but the data transmission rate depends on the fluid properties and the depth of the BHA. Furthermore, for long wells, there is a communication latency which can be in the order of minutes between the sent and received signal. The *down-linking* of data from the surface to communicate steering instructions to the BHA uses pulses in the mud flow at a significantly slower rate. Recent developments for telemetry are *wired drill pipe* [Boyle et al., 2003], which allow higher data rates and lower delays in telemetry.

Steering Assembly

The steering assemblies are considered in more detail in Section 1.2.

1.2 Steering Assembly

The two main types of steering assemblies for drilling directional wells are steerable motors [Dellinger and Gravley, 1986], and rotary steerable systems [Griffiths, 2009]. Other methods to control the curvature of a hole include (i) *jetting* high pressure fluid to erode a preferred side of the borehole and (ii) placing a wedge down-hole to bias the weight on bit to a preferred side of the borehole. Prior to steerable motors, strategic placement of stabilizers, along the length of the BHA, influences the *fulcrum point* where the BHA effectively pivots. This fulcrum point along the BHA influences the dropping and the building of the inclination of the borehole. Modern steering assemblies are classified as either *push-the-bit* or *point-the-bit*. The former involves applying a side force on the formation to influence the change in borehole orientation. The latter is when the BHA has a bend whose orientation is nominally fixed in space even though the BHA is rotating. The first patented method of a push-the-bit drilling tool is from Page [1959], and the first patented point-the-bit steering system was from Giles and Wells [1955]. Details of these methods are described in Section 1.2.2.

1.2.1 Steerable Motors

Steerable motors (often called *mud motors*) consist of a fixed bend usually between $1 - 4^\circ$ which influences the curvature capability of the propagating borehole. Mud

motors are so called since mud or drilling fluid flowing down-hole through the drill pipe drives a motor to rotate the bit. For mud motor drilling the drill-string is often non-rotating and the steering direction is set when the tool is not penetrating. Rotation of the top-drive from the surface configures the steering orientation of the mud motor. Measurements from the down-hole MWD sensors are sent to the surface via mud pulse telemetry. Curved sections of the well are achieved by maintaining this orientation and penetrating by rotating the motor but not the drill-string; this is known as “slide drilling”. To drill a straight hole section, the motor is rotated by mud flow, and the drill-string is also rotated from surface. this is referred to as “rotate drilling”.

1.2.2 Rotary Steerable Systems

RSS have advantages over steerable motors, with the ability of changing the direction of propagation while drilling. In addition RSS involves a continuously rotating drill-string which provides an advantage over mud-motors, since drill-pipe rotation reduces axial friction, helps maintain the cuttings transport, and improves the circulation of drilling fluid. Control systems embedded in the downhole assembly stabilize the orientation of propagation, interfacing directly with sensors and actuators. RSS steering assemblies are positioned directly behind the bit in the drill-string. RSS tools have three main components: Power Generation Module (PGM); Control Unit; and Bias Unit [Barr et al., 1995]. The control unit contains: sensors in the form of accelerometers and magnetometers to determine the orientation of the propagating hole; processors, memory capacity, and embedded control systems. The *bias* unit is the actuation module of the steering assembly. The bias unit is categorized into: push-the-bit and point-the-bit actuators.

Push-the-Bit

Push the bit steering assemblies apply a sideforce to the formation through the intermediate stabilizer. This provides a lateral bias on the bit through bending the BHA in the hole. Push the bit steering assemblies are favourable in hard and consolidated formations. For softer formations point-the-bit assemblies are favourable.

Many designs of push-the-bit steering assemblies have been proposed. A system where the bias unit consists of three piston actuated pads separated by 120° is presented in

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Barr et al. [1995]. In this method a motor in the control unit rotates a rotary valve which directs a portion of the main flow into piston chambers. The differential pressure between the pressurized piston chambers and the formation applies a force across the area of the pads to the formation. The rotary valve directs the flow biased towards the direction in which the sideforce is desired into the piston chambers to actuate the corresponding pad. In this system, the tool is continuously steering, and sections of straight hole are achieved through rotating the direction of sideforce such that the net steering tendency is zero. Another push-the bit drilling system is presented by Poli et al. [1998]. This system actuates the sideforce through three independently actuated steering ribs in the bias unit. The advantage of the Poli et al. [1998] system is that the three actuators have their own hydraulic and electronics module. This allows greater redundancy in the control of the steering direction. The disadvantage is that the three ribs have residual pressure when they are off and this affects the steering mechanism. Furthermore these ribs are attached to a sleeve which is non-rotating relative to the formation. Hence the system presented by Barr et al. [1995] has hole cleaning and friction advantages since the entire BHA is rotating.

Point-the-Bit

A point-the-bit steering assembly is one where the rotation axis of the bit has an angular offset to the rotation axis of the BHA [Schaaf et al., 2000]. Since this system does not rely on pushing the surrounding formation, this method is better suited for softer, unconsolidated formations than push-the-bit systems. Moreover boreholes of higher curvature can be drilled with point-the-bit systems [Schaaf et al., 2000]. This is because the curvature is a property of the intrinsic geometry of the tool rather than the resultant forces on the formation. Because of this, more drill-bits are compatible with point-the-bit systems [Schaaf et al., 2000] since the bit is directed towards the steering direction.

An example of a point-the-bit BHA is detailed by Schaaf et al. [2000]. The BHA is an RSS tool where the outer housing is rotated from the surface which in turn rotates the drill-bit. The power-generating-turbine has an impeller which generates power from the mud flow, and drives a motor to counter-rotate to the rotation of the housing, keeping the BHA geometry geostationary. The control unit is located behind the motor, with

D&I sensors which measure the attitude to control the tool-face angle of the fixed bend.

A point-the-bit steering assembly described by Yonezawa et al. [2002] entails a drive shaft enclosed in two concentric rings which rotate relative to each other to orientate the toolface. The BHA housing in Yonezawa et al. [2002] is non-rotating relative to the formation, and consists of spring loaded rollers to aid the sliding of the BHA. The sensors are located on the outer-housing to sense the orientation. Since the outer-housing of the BHA is non-rotating, there is additional friction especially when back-reaming.

1.3 Control System Hierarchy

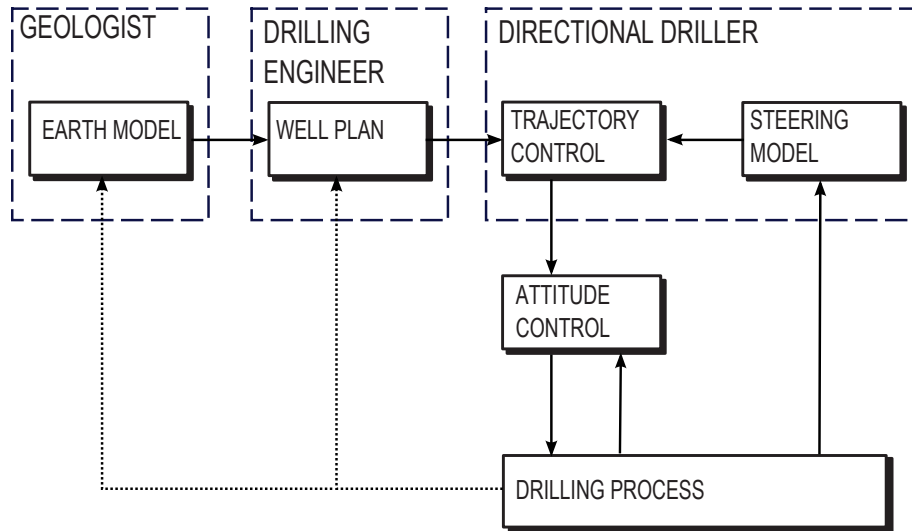


Figure 1.3: Directional drilling system workflow

Figure 1.3 shows the overall control system described in this thesis. From offset wells and geological survey data, an Earth model is created and used by a *geologist* to identify targets and obstacles in the substructure for the subsequent designing of a well. It is with respect to these targets and obstacles that a *Drilling Engineer* (DE) designs a *well-plan*. The DE is responsible for the design of the BHA and its constituent components for each well. In the real-time process, a Directional Driller (DD) located at the well site is responsible for steering the BHA along this well-plan. While drilling,

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The DD determines steering commands to send to the down-hole BHA. A Measurement While Drilling (MWD) Engineer and Logging While Drilling (LWD) Engineer are responsible for obtaining and interpreting survey data from the real-time process and communicating this information to the DD. The DD learns how the BHA responds to the steering commands, and incorporates this learning into determining the best commands for subsequent down-links. Down-hole attitude control systems stabilize the orientation of the BHA by continuously communicating steering commands to the actuator at a significantly higher frequency than is possible by the DD on the surface due to telemetry restrictions.

The control system presented in this thesis assumes the role of the DD to understand the steering model, and interact with a trajectory controller. The trajectory control system receives inputs from a DD and using the well-plan as a reference, determines steering commands to communicate to the inner-loop down-hole attitude control system. In this Section, well-planning, attitude control, and trajectory control are discussed with details of the current literature. Due to the proprietary nature of these commercial systems, the literature available is limited.

1.3.1 Well-Planning

Three categories classify the majority of oil and gas wells :

- vertical wells;
- deviated wells;
- horizontal wells.

Historically most wells drilled were *vertical wells* and *deviated wells*. Directional wells deviate from the vertical to hit strategic targets, and contain inclined sections which are rectilinear, and have a constant inclination angle. Often many directional wells are *side-tracked* from a vertical well known as a *mother-bore*. Most wells drilled nowadays are known as *horizontal wells*. These consist of: a vertical part, a curved part known as a build section, and a horizontal section which is steered with respect to geological features to maximize oil recovery from a reservoir [Williams, 2010].

High end wells which represent a small share of the market with respect to the numbers drilled include: Extended Reach Drilling (ERD) wells, and 3D wells. ERD wells contain long inclined sections known as *sail-sections* to reach reservoirs located large distances from the drilling rig. 3D wells are a non-standard geometry compared to horizontal wells. These are designed to intersect multiple pockets of hydrocarbons, and to avoid obstacles such as geopressures and existing wells nearby.

A family of well-plans are created once targets and obstacles are identified with respect to an Earth model and known features. The purpose of designing individual well-plans is for designing an optimal path to drill through targets and avoid obstacles. The well-plan is desired to be smooth with more exposure of the reservoir for maximizing recovery. Designing a smoother well-plan reduces friction, enabling a DD to transfer more weight and torque to the bit which reduces the difficulty. Techniques for generating a well-plan with respect to targets and smoothness are explored in Chapter 3.

Wellbore Tortuosity

Tortuosity is challenging to quantify with no industry standard definition [Samuel and Liu, 2009]. Tortuosity is used to describe the smoothness of a wellbore, and tortuosity measures are based on the intrinsic curvature of the wellbore profile. A review of tortuosity measures is presented by Samuel and Liu [2009]. An early measure for tortuosity is a method called the Drilling Difficulty Index (DDI) which quantifies the difficulty in drilling a wellbore [Oag and Mike, 2000]. The DDI is a ratio which is proportional to the product of reach, arclength and curvature and inversely proportional to the depth of the hole. This index is not based on any physical reasoning, but is an early metric for categorizing wells. Samuel and Liu [2009] present a physical measure in their review which they state to be their preferred measure. This measure is a combination of strain energy and torsion along a wellbore. The reasoning for this is to capture the elastic properties per unit well-bore length.

Brands and Lowden [2012] present a physical tortuosity index called the *Scaled Tortuosity Index* (STI) based on the strain energy and extends the work of Samuel and Liu [2009]. In addition to the strain energy across the well-bore, the difference in cross

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sectional diameters of a section of pipe being tripped into a hole with respect to the cross sectional diameter of the well-bore is captured in this index. The STI index is weighted such that a portion of the borehole closer to the surface contributes more to the index, than a portion of the borehole closer to the end of well. This is because the borehole closer to the end of the well interferes with less of a drill-pipe or section of casing when tripping that borehole closer to the surface. The measure presented by Brands and Lowden [2012] considers a two dimensional model of drill-string and the effect of torsion is neglected.

Current State of Well-planning

Most of the methods for well-planning in the literature are based on designing wells which are concatenations of circular and straight hole segments [Kikuchi, 1993, Sawaryn and Tulceanu, 2007, Xiushan et al., 2004, Sawaryn and Thorogood, 2003]. Samuel [2010] present the disadvantages of planning wells using a concatenation of circles and straight line segments. He argues that the discontinuity in the transition results in high stresses for the tubulars, increase of torque and drag, and poor hole cleaning.

Kikuchi [1993] presents a method for constructing two dimensional and three dimensional wells based on trigonometric calculations. The well-profiles are different combinations of build/hold and tangents and the geometry of the wells are constructed of circular sections and straight hole sections. Sawaryn and Thorogood [2003] presents multiple algorithms based on the *minimum curvature method* for planning well-paths. These are circle and straight line segments which have become the industry standard. Sawaryn and Tulceanu [2007] present 12 algorithms for calculating build and hold sections in addition to the minimum curvature method. The algorithms are based on arcs on the surfaces of tori and cylides for the curved parts of hole. Xiushan et al. [2004] present another method of well-planning based on circle and straight hole concatenations. Here emphasis is placed on a method of *soft-landing* where a direction is specified at the target location.

Samuel [2010] presents an improvement on planning wells using circle segments and straight hole concatenations by smoothing the transition discontinuity with a clothoid spiral section curve. Clothoid arcs have the property of linear variation of curvature

with arclength. It is shown that this method reduces the wellbore tortuosity and extends the reach of a well.

Drilling tools steer by toolface, and constant toolface curves are not sections of circles. Shengzong et al. [1999] presents a non-linear non-convex optimization method for constructing constrained well-paths based on modeling curves by the kinematic model (1.1). This method considers toolface angle as a parameter which allows for constant toolface curves. The application is for sidetrack wells. The optimization minimizes the arclength of the resultant curve.

1.3.2 Steering Model

From monitoring the response of the drilling tool through real-time albeit delayed measurements, a model for the steering response of the tool can be determined. The curvature capability of the drilling tool is the main parameter of interest. For example a drilling tool may have the capability to drill at say $8^\circ/100\text{ft}$ by design for a push the bit tool. If the formation is soft or unconsolidated then this curvature capability can be significantly reduced. Furthermore since the geological structures are in the form of stratified layers, this introduces an anisotropy in the steering capability and the direction a tool steers affects the curvature capability. Various methods have been proposed to estimate the curvature capability of the tool [Lesso Jr et al., 1999, Pastusek et al., 2005], based on post analysis of drilled wells, validated with finite element models for the relationship between steering commands and measured parameters. In this thesis, these models are assumed to be available and considered as constraints for the optimizations used in *well-planning* and *trajectory-control*.

1.3.3 Direction Controller

Rotary steerable tools contain embedded control systems in the steering assembly for stabilizing the propagating borehole attitude. Fundamentally, the steering direction of the desired toolface is controlled in a closed-loop manner. Attitude control systems serve as outer-loops to toolface control. These toolface control systems are specific to the steering assembly design and feature in all RSS tools. Since there is a limit to the bandwidth of data transmitted and received to the surface, there is an advantage in attitude control systems for a drilling smoother boreholes between downlinks. Attitude

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control systems are either in the form of inclination hold, or inclination with azimuth control systems. This subsection discusses these systems.

Inclination Hold

An early example of a control system for controlling the attitude of a downhole tool in a closed-loop manner which predates rotary steerable systems is presented in Underwood et al. [1994]. Only the inclination angle is controlled, through actuation of a near bit variable gauge stabilizer. The diameter of this stabilizer is increased if the measured inclination is greater than the planned inclination, and vice versa forming a proportional controller. A similar inclination hold control system is described by Yinghui [2002] for a push the bit rotary steerable tool of the kind similar to Poli et al. [1998] and Moody et al. [2004]. Steering forces onto the BHA drive shaft are applied by ribs or blades in contact with the formation. The control system follows the same principles as Underwood et al. [1994] to control the diameter of the steering stabilizer.

A method for controlling the orientation of a point the bit system Yonezawa et al. [2002] is presented by [Kuwana et al., 1994], where the angular positions of two eccentric stabilizers are controlled to generate the desired toolface angle of the BHA in a closed-loop manner. Details of the control algorithms used are omitted in the above publications, but the feedback is on the inclination angle derived from gravity measurements using an axial accelerometer. An elegant closed loop inclination hold control system is presented by Downton and Ignova [2011] and Downton [2007]. Here a two dimensional model of a BHA is modeled with respect to states which are the contact points with the borehole. These fixed contact points are spatially delayed from the bit and influence the direction of propagation. This method uses delay differential equations to model the propagation, where a transfer function is given in the wave-number domain. A proportional controller which acts on the inclination error to influence the force on the contact point closest to the bit is then derived. Sun et al. [2011] extends the use of the delay differential model presented by Downton [2007] for the design of a closed loop inclination hold controller based on L_1 adaptive control methods.

Inclination and Azimuth Hold

Genevois et al. [2003] discuss the need for ‘shoot and forget systems’ for closed loop trajectory control and state that the major challenge has been azimuth control. An architecture for a general closed-loop inclination and azimuth hold control system is presented by [Matheus and Naganathan, 2010b]. Implementation of the controller along with description of the parameters are given. Detailed information about the control law and plant dynamics are described later in Matheus et al. [2012]. The control inputs are stated to be toolface angle and steering ratio (defined to be the ratio over a fixed drilling interval of: straight section; and curved section). Panchal et al. [2010] present an inclination and azimuth hold controller with a similar architecture to Matheus and Naganathan [2010b] where the reference signals are target inclination and azimuth and the control inputs are on the steering ratio and toolface angle. The controller employed is in the form of a PI (proportional and integral) controller acting on a linearized kinematic model of the states (inclination and azimuth angle). The controller gains are designed by pole placement and the robust stability of the system is investigated by use of the small gain theorem on unstructured uncertainty in the curvature and rate of penetration of the tool. The kinematic model used by Panchal et al. [2010] and Matheus et al. [2012] are of the form

$$\begin{aligned}\dot{\theta}_{inc} &= V_{rop} K_{dls} \cos(\theta_{tf} - \bar{\theta}_{tf}) - v_{dr} \\ \dot{\theta}_{azi} &= \frac{V_{rop} K_{dls} \cos(\theta_{tf} - \bar{\theta}_{tf})}{\sin \theta_{inc}} - v_{tr},\end{aligned}\tag{1.1}$$

where V_{rop} is the rate of penetration, K_{dls} is the curvature, θ_{tf} is the toolface angle, $\bar{\theta}_{tf}$ is the toolface offset, θ_{inc} is the inclination angle, θ_{azi} is the azimuth angle, v_{dr} is the angular dropping rate and v_{tr} is the angular turning rate. Matheus and Naganathan [2010b] describe field test results where inclination and azimuth are held to within 2° . Matheus et al. [2012] extends the work of Matheus and Naganathan [2010b] where further details of sensor modeling and delays are given. The control law is stated as a proportional controller acting on the inclination and azimuth error designed from the kinematic model. Both [Panchal et al., 2010, Matheus et al., 2012] effectively have two independent single input, single output loops to control inclination and azimuth, but the kinematic plant model (1.1) has the azimuthal response θ_{azi} coupled in the inclination.

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1.3.4 Trajectory Controller

Given a *well-plan* as a reference trajectory, it is the role of the DD to steer a drilling tool in real-time along the well-plan within an allowed deviation which is specified *a-priori*. The directional driller plans the sequence of steering instructions to send downhole to track the well-plan. This is with respect to understanding the drilling tool behaviour by monitoring its response. In Chapter 4, a real time trajectory controller is presented which generates a continuous curve from the drilling tools current state to track the well-plan in a model-predictive-control (MPC) scheme. From this continuous curve, the required instructions to downlink to the drilling tool are derived.

The most comprehensive work on trajectory control for rotary steerable systems is presented by Pirovolou et al. [2011]. Here a model is used to estimate the position of the drill bit relative to the delayed sensor. The position is then compared to a well-plan, and a path is planned to meet a moving target along the well-plan which recedes during the drilling process. From the path which is planned to meet this target, a steering instruction of curvature and toolface angle is determined to steer the drilling tool open-loop along this path. The system also considers the scenario where the target is fixed in space. A general method for trajectory control of a drilling tool by sending steering commands from the surface to a down-hole control system is presented by Ignova et al. [2010]. A hybrid approach using a multi-loop trajectory control system is proposed by Matheus et al. [2012]. Here a framework for multiloop cascaded control system is presented where the design of the system is mindful of the interconnection with a geosteering loop. Results are presented for a method of *hold-the-line* where the objective is to maintain the true vertical depth of a horizontal well.

Another trajectory control system which calculates a correction path to a moving target on the well-plan is presented by Panchal et al. [2011b]. The position of the moving target is a function of the position and attitude of the drill. The correction path to the well-plan is described as two orthogonal sinusoids. The closest feasible point on the well-plan which can be steered to by this curve without violating a curvature constraint is chosen as the moving target point. The correction path is used to determine set-point signals for an inner attitude control system. A waypoint following trajectory control

system is presented by Panchal et al. [2011a]. Here a correction path to meet a target position and attitude is calculated by optimized Hermite polynomial splines. This path is used to determine the set-point attitude to communicate to an inner-loop attitude control system. A similar method of a closed loop controller to track fixed targets is presented by Schuh [2003]. Here the correction path to the targets is with respect to circular and straight hole segments.

1.4 Project Aims and Objectives

The work described in this thesis was sponsored by Schlumberger and the Engineering and Physical Sciences Research Council (EPSRC) through the Engineering Doctorate (EngD) programme at Cranfield University. For an EngD, both engineering and management research is conducted during the programme.

1.4.1 Engineering Component

Aim: To apply feedback control systems technology to autonomous navigation of a well-bore trajectory for directional drilling operations.

Objectives:

1. To design and analyse attitude hold control systems to interface the direction and inclination sensors and steering actuators for implementation on a downhole rotary steerable system.
2. To design and analyse a method for constructing minimum energy well-bore paths which are constrained for the curvature capability of a drilling tool.
3. To design and analyse a closed loop trajectory control system for navigating a rotary steerable tool along a well-bore trajectory
4. To test the attitude and trajectory control systems in the commercial drilling simulator *ST2*.

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1.4.2 Management Component

Aim: To assess the potential commercial impact of attitude and trajectory control on the business of Schlumberger.

Objectives:

1. To outline the business context of the directional drilling market in the oilfield services industry
2. To conduct a value proposition for an attitude control system for directional drilling.
3. To conduct a value proposition for automated trajectory control.

1.4.3 Contributions

This thesis presents the most relevant and important aspects to meet the objectives. Other methods have been investigated and proposed as part of the EngD study. Some of these are presented in the Appendices, and in other places as follows:

Journal Papers

Panchal, N., M. T. Bayliss, and J. F. Whidborne. Attitude control system for directional drilling bottom hole assemblies. *Control Theory & Applications*, IET 6.7(2012) : 884 – 892.

M. T. Bayliss, Panchal, N., and J. F. Whidborne. Path tracking for directional drilling as applied to attitude hold and trajectory following. *Schlumberger Journal of Modeling, Design, And Simulation*, (2011).

Conference Papers

Panchal, N. and Bayliss, MT and J.F. Whidborne, Robust linear feedback control of attitude for directional drilling tools, *Proc. 13th IFAC Symp. on Automation in Mining, Mineral and Metal Processing*, Cape Town, 2010.

Panchal, N. and Bayliss, M.T. and Whidborne, J.F., Recursive Variable Horizon Trajectory Control for Directional Drilling Using Elliptical Helixes, 18th IFAC World Congress, 2011, pages 10836–10841.

Panchal, N. and Bayliss, M.T. and Whidborne, J.F., Minimum strain energy waypoint-following controller for directional drilling using OGH curves, 2011 IEEE International Conference on Control Applications (CCA), pages 887–892.

Panchal, N. and Bayliss, M.T. and Whidborne, J.F., Vector based kinematic attitude control for drilling tools, IFAC Conference of Oil and Gas Automation, Trondheim, June 2012.

Bayliss, M.T. and Panchal, N. and Whidborne, J.F., Rotary Steerable Directional Drilling Stick/Slip Mitigation Control, IFAC Conference of Oil and Gas Automation, Trondheim, June 2012.

Patents Filed Non-Provisional

Panchal, N. and Bayliss, M.T. and Whidborne, J.F., Directional Drilling Attitude Hold Controller, Nov 2012, US Patent 13/682/528, UK Patent 12/208/419

Panchal, N. and Bayliss, M.T. and Whidborne, J.F., Minimum strain energy waypoint-following controller for directional drilling using OGH curves, AUG 2011, US Patent 13,222,983, WO Patent PCT/US12/49949

Patents Filed Provisional

Panchal, N. and Whidborne, J.F. and Bayliss, M.T. and Ringer, M., Optimal wellbore path planning, Dec 2012, US Patent 61/736/996

Panchal, N. and Whidborne, J.F. and Bayliss, M.T. and Ringer, M., Optimal trajectory control for directional drilling, Dec 2012, US Patent 61/737/051

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Bayliss, M.T. and Panchal, N. and Whidborne, JF, Path Tracking for Directional Drilling as Applied to Attitude Hold and Trajectory Following, Jul 2011, US Patent 61/510/592, WO Patent PCT/US/2012/047843

1.5 Outline of Thesis

The presentation of the research conducted is structured into two parts. Part I describes the engineering component with details of the trajectory control system and the attitude control, well-planning, and surface trajectory control system. Part II describes the oilfield services market and the value of the engineering component in oilfield development.

Part I: Autonomous Trajectory Control

- Chapter 2 presents a downhole attitude control system.
- Chapter 3 presents a method for constructing a well-plan.
- Chapter 4 presents a closed loop trajectory control. system

Part II: Management

- Chapter 5 presents a value proposition of autonomous trajectory control.

Appendix

As part of the EngD study, other methods have been investigated for trajectory control. These are provided in the appendix.

- Section A.1 describes a method for downhole attitude control for RSS as presented in Panchal et al. [2010].
- Section A.2 describes a method for path-following to generated attitude control set-points as presented in Panchal et al. [2011b].
- Section A.3 describes a method of trajectory control for way-point tracking as presented in Panchal et al. [2011a].

Part I

Autonomous Trajectory Control

2

Attitude Control for Rotary Steerable Tools

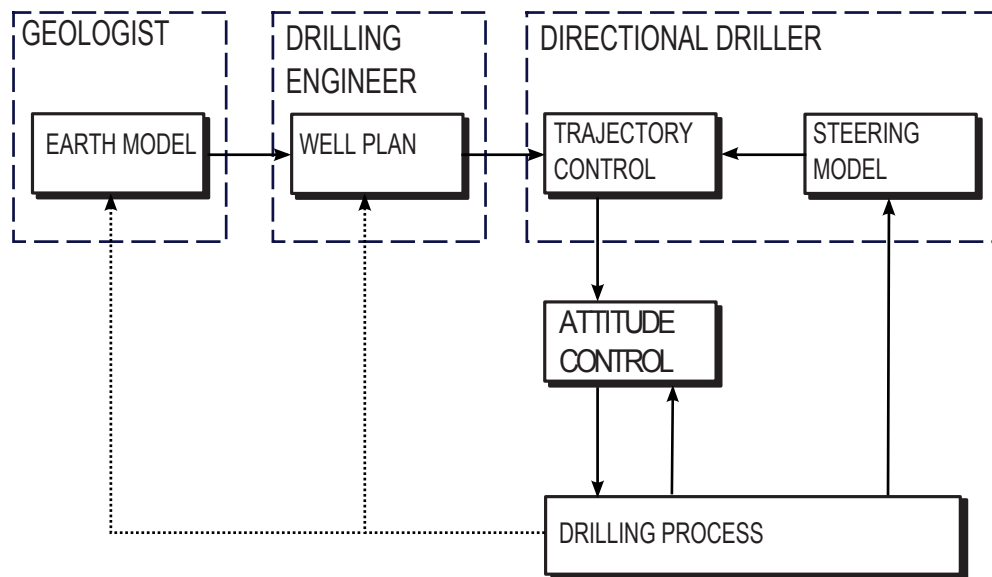


Figure 2.1: Trajectory Control Workflow

This chapter presents a general approach for the attitude control of rotary steerable systems for the oil and gas industry. The attitude is defined as the direction of propagation of the drilling tool. For this approach a unit vector represents the attitude for the down-hole control system, thus avoiding the non-linearities introduced by Euler angle representations. This chapter proposes three control laws, proves their stability

2. ATTITUDE CONTROL FOR ROTARY STEERABLE TOOLS

and tests their behaviour by numerical simulation. The merits of the laws from an engineering perspective are highlighted, and some details for the implementation of the laws on directional drilling tools are provided. Methods for determining the attitude from accelerometers and magnetometers are presented for angle-axis, quaternion and Euler angle methods. The attitude control laws are further investigated for the phenomenon of hole spiraling which occurs when the attitude sensors used in the feedback control are located behind the bit. An extension of the general system developed in this chapter to reduce the effect of this spiral is proposed.

2.1 Introduction

The objective for the directional-drilling attitude control system is to hold an attitude specified by inclination and azimuth angle set points [Genevois et al., 2003]. The data from the BHA is communicated to the surface via low bandwidth (2 to 20 bits per second) mud pulse telemetry [Allen et al., 2009]. The communication rate by a directional driller intending to follow a *well-plan* to the BHA is much slower and in the order of bits per minute [Allen et al., 2009]. The attitude control described in this chapter is general and applicable to RSS drilling tools for push or point the bit steering units with down-hole actuators and sensors. Attitude control systems provide an advantage for automating the drilling of long sections of rectilinear wellbores, providing consistency and accuracy. Moreover, a downhole embedded attitude control system reacts to changes in the attitude with much higher update rates compared to manual control from the surface due to the slow telemetry rates.

Wen and Kreutz-Delgado [1991] presents a general analytic framework for controlling rigid body attitudes in a coordinate independent way. Several control laws are proposed for the dynamics of a rigid body modeled by the Newton-Euler equations. Chaturvedi et al. [2011] provide a comprehensive review of attitude control methodologies. It is emphasised that Euler angle approaches (1.1) suffer from *gimbal-lock* for zero inclination $\theta_{inc} = 0$. The quaternion representation for attitude is not unique, with a double covering of the state space. This double covering causes *unwinding* for attitude control systems whereby feedback unnecessarily rotates the rigid body up to a full rotation [Mayhew et al., 2011]. Attitude control schemes are categorized into local and global

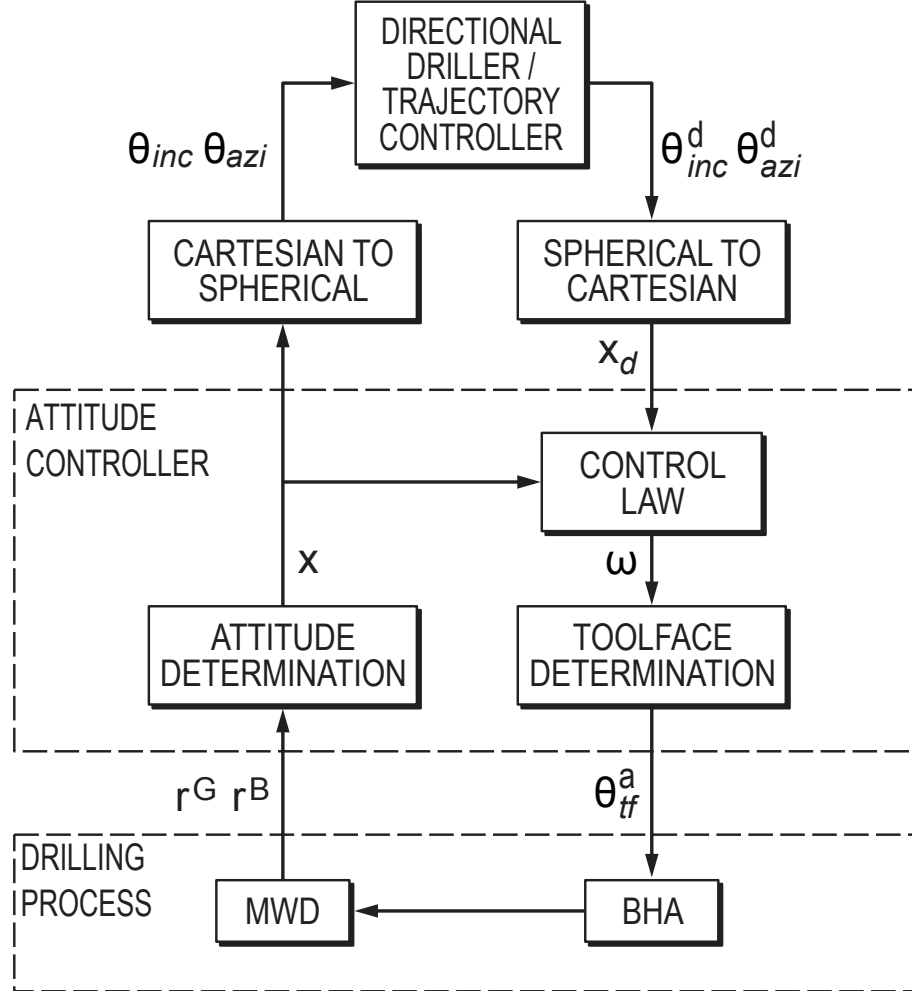


Figure 2.2: System structure for closed loop attitude control

schemes. Local schemes such as [Panchal et al., 2010, Matheus et al., 2012], stabilize an attitude in the neighbourhood of a desired attitude. Global schemes consider large variations in attitude errors and are widely used for highly maneuverable vehicles. Of the representations: Euler angles; angle-axis, quaternions, rotation matrix, only the rotation matrix representation gives a unique global representation [Chaturvedi et al., 2011].

The attitude control system proposed in this chapter is depicted in Figure 2.2. The directional driller communicates attitude reference signals to the drill, these being the

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inclination θ_{inc}^d and azimuth θ_{azi}^d set points (as per industry convention) of the desired tool orientation in the Earth reference frame. The Earth reference frame is fixed geologically and by convention is a right handed set with the x -axis pointing down-hole, and the y -axis north. These demanded inclination and azimuth signals are converted to a unit vector in the R^3 vector space, representing the drill's attitude. In the next section, the models used to represent the attitude response are presented and developed. The presentation begins with a continuous time model of a steering assembly with variable curvature. A model for a constant curvature steering assembly follows for both continuous time and discrete time representations.

Section 2.3 proposes control laws which stabilize and drive the attitude towards the demanded set point. The stability of the proposed laws are proven using the direct method of Lyapunov. Section 2.4 presents details about the coordinate transformations necessary for implementing the proposed controllers commensurate with existing convention for the drilling industry. This section presents methods for determining the attitude required for the control laws from downhole sensors. Section 2.5 presents simulation results of the attitude control laws with varying sensor to bit location to demonstrate the effect of hole spiraling. Here a system for reducing the spiraling based on predicting the attitude to the bit is presented. In the final section some conclusions are drawn and suggestions given for further work.

2.2 Kinematic model of the drill

Rotation matrices uniquely represent rigid body attitudes from a fixed frame to a local frame [Chaturvedi et al., 2011]. Transformations of this attitude are described by the kinematic relation

$$\dot{\mathbf{R}} = \mathbf{R}\omega^\times, \quad (2.1)$$

where $\mathbf{R}$ is a rotation matrix describing the attitude, and $\omega^\times$ is a skew symmetric matrix describing the control actions. The rotation matrix $\mathbf{R}$ can be interpreted as the transformation of three mutually orthogonal unit vectors in the local reference frame with respect to a global frame. The set of all attitudes $\mathbf{R}$, form a group under this control action $\omega^\times$. This chapter uses a reduced model of (2.1) to represent the kinematic attitude response of the BHA. The use of this reduced model involves representing the

attitude as a unit vector as opposed to a rotation matrix. This is because only the orientation in terms of inclination and azimuth is controlled and a rotational symmetry is assumed about the centre-line of the BHA.

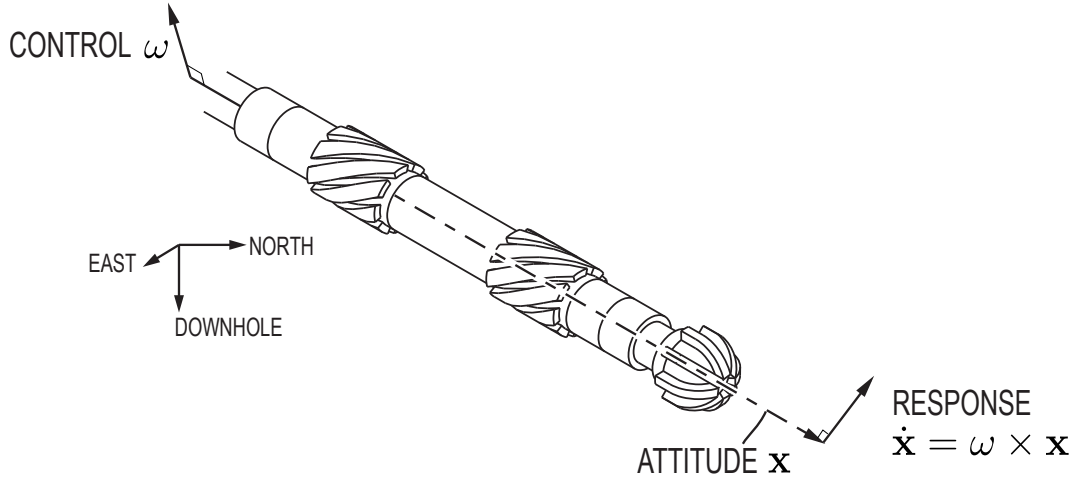


Figure 2.3: Kinematic response model of BHA

The BHA is modeled as a rigid rod hinged at one end that has only the rotational motion corresponding to what can be interpreted as pitch and yaw, with the roll motion ignored. The rotation rates are small and the translational motion is neglected because the drilling tool down-hole does not have a reliable estimate of the down-hole ROP. Furthermore, the motion of the BHA is constrained by the well and hence momentum terms are redundant.

2.2.1 Continuous Time Plant Model

The kinematic system representing the time varying response of the tool's attitude [Wen and Kreutz-Delgado, 1991] can be represented as

$$\dot{\mathbf{x}} = \omega \times \mathbf{x} \quad (2.2)$$

where $\mathbf{x} \in \mathbb{R}^3$ is a unit vector tangent to the BHA axis representing the tools attitude, $\omega \in \mathbb{R}^3$ is the angular velocity vector parameter and $\times$ denotes the vector product operator. The magnitude $\|\omega\|_2$ is referred to here as the *build rate* and is related to the

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ROP and curvature $0 \leq k_{dls} \leq \bar{k}_{dls}$ by

$$\|\omega\|_2 = \text{ROP } k_{dls}, \quad (2.3)$$

where $\bar{k}_{dls}$ is the maximum curvature capability of the steering assembly. The attitude $\mathbf{x}$ in the body frame of the BHA is one column of the rotation matrix $\mathbf{R}$, where $\mathbf{R}$ is the transformation of a mutually orthogonal triad of unit vectors fixed in the body frame. The two vectors perpendicular to $\mathbf{x}$ are ignored for the attitude control analysis which follows because of the roll symmetry of a BHA. Given an initial value, $\mathbf{x}(0) = x_0$, $\|\mathbf{x}_0\| = 1$, and a control ω , the resulting trajectory, $\mathbf{x}(t)$ lies on the surface of the unit sphere, that is $\|\mathbf{x}(t)\| = 1$ for all t and ω . The kinematic motion is controlled by varying ω via the toolface angle (see section 2.4.1 for details). Note that (2.2) can be expressed as:

$$\dot{\mathbf{x}} = M\mathbf{x} \quad (2.4)$$

where

$$M = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \quad \text{and} \quad \omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}. \quad (2.5)$$

The BHA is subject to disturbances due to varying rock formations. In addition there is a tendency for the BHA to drop towards a vertical orientation due to gravity, and a tendency for the BHA to drift horizontally. The disturbances are ignored in the model.

This model also ignores dynamics of the actuator where it has been assumed that the directional drilling tool can instantaneously change its steering direction. It is generally the case that the rate of change of toolface is much greater than the rate of change of attitude, that is $\frac{d\theta_{tf}}{dt} \gg \|\omega\|_2$. This is because changes in attitude require penetration through dense geological media and is dependent on ROP, bit properties and the curvature design capability of the steering assembly. Whereas toolface changes depend on the steering actuator mechanism, and a toolface change can be made when the BHA is not penetrating.

2.2.2 Continuous Time Model with Constant Build Rate

A constraint on the control requiring it to have a constant build rate is imposed by practical engineering considerations. It is far cheaper and simpler to engineer the

actuator to have a constant build rate. Drilling tools typically propagate through the ground at a ROP of 100 ft/hour or less with curvatures $\bar{k}_{dls}$ up to $15^\circ/100$ ft. Strictly speaking the curvature is not constant in drilling conditions and is dependent on a number of variables. The sideforce magnitude is not a control variable but is nominally constant in ideal drilling conditions (constant WOB, mud pressure, formation, BHA configuration).

2.2.3 Discrete Time Model with Constant Build Rate

Let us assume a constant build rate, K , and a constant sample time, δt . Continuous D&I attitude surveys are typically sampled in the order of every 100 seconds. Let us denote the state at the i^{th} sample time, $t = i \delta t$ by $\mathbf{x}(i)$. Define the constant $\gamma \in \mathbb{R}_+$ to be the cosine of the angle the tool builds in one sample time, that is $\gamma = \cos(K\delta t)$. The discrete-time system for the evolution of the attitude of the penetrating drill can be modeled by

$$\mathbf{x}(i+1) = \Gamma(\mathbf{x}(i), \tilde{\omega}(i)) = \cos(K\delta t) \mathbf{x}(i) + \sin(K\delta t) \tilde{\omega}(i) \times \mathbf{x}(i) \quad (2.6)$$

where $i \in \mathbb{Z}$ is the sample number and $\tilde{\omega}$ is the normalized unit angular velocity vector. The attitude must remain a unit vector and the term $\tilde{\omega}(i) \times \mathbf{x}(i+1)$ is restricted to be perpendicular to the tool's attitude, since the actuation to steer the drill does not contribute to the rate of penetration. The constant γ is the scalar advancement in the tools actuation in one time step, that is

$$\gamma = \mathbf{x}(i) \cdot \mathbf{x}(i+1). \quad (2.7)$$

The model (2.6) is interpreted as a rotation of the attitude $\mathbf{x}(i)$ at sample time i about the axis $\tilde{\omega}(i)$ by an angle $K\delta t$ to obtain the attitude $\mathbf{x}(i+1)$ at time $i+1$.

2.3 Attitude Control Laws and Stability

In this section, three control laws are proposed for the control of the tool attitude. The stability of the laws is proved by means of Lyapunov direct method using a lemma that is derived directly from the Lyapunov Theorem of Local Stability [Slotine and Li, 1991]. The following definition [Slotine and Li, 1991] is first required.

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Definition 1. An equilibrium point $\tilde{\mathbf{x}}$ is a state of the system (2.2) such that when $\mathbf{x}(t)$ is equal to $\tilde{\mathbf{x}}$, it remains so for all subsequent times. That is $\dot{\mathbf{x}} = 0$.

Lemma 1. If, for a dynamical system, there exists a scalar valued function $V(\mathbf{x})$ with continuous first partial derivatives in a neighbourhood B about the equilibrium point such that:

- $V(\mathbf{x})$ is positive definite,
- $\dot{V}(\mathbf{x})$ is negative semi-definite,

than the system is locally stable. In addition, if the derivative $\dot{V}(\mathbf{x})$ is negative definite in B , then the stability is asymptotic.

2.3.1 Variable build-rate controller

The variable build rate control law ω is the vector product of the attitude state $\mathbf{x} \in \mathbb{R}^3$ and a demand attitude signal $\mathbf{x}_d \in \mathbb{R}^3$ both given as unit vectors. This ensures that the control signal ω is orthogonal to both the state and demand. This control law is interpreted as steering the attitude towards the demand in the shortest possible arc in the attitude state space such that the attitude response $\dot{\mathbf{x}} = \omega \times \mathbf{x}$ lies in the plane spanned by $\mathbf{x}$ and $\mathbf{x}_d$. This is shown in Figure 2.4.

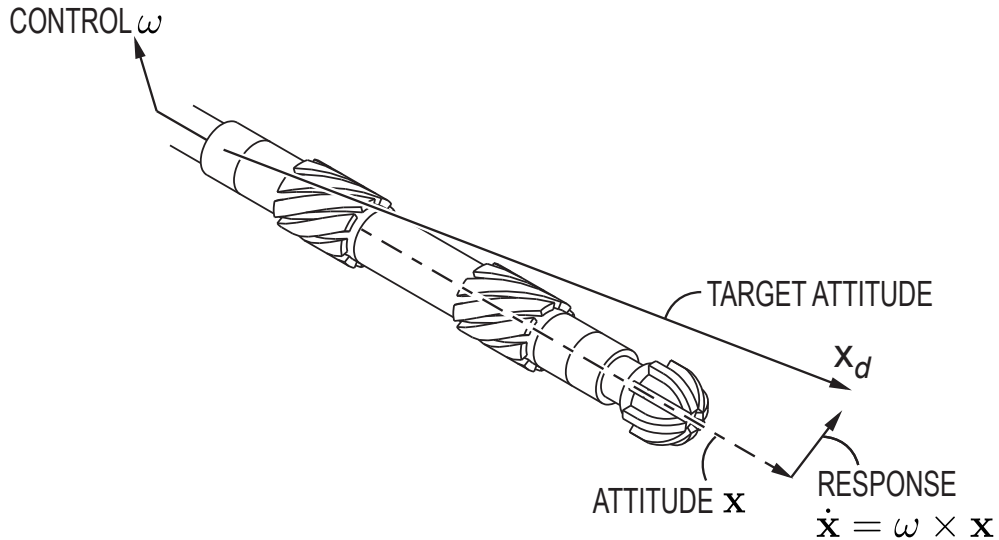


Figure 2.4: Feedback Control Law

Proposition 1. *The kinematic system given by (2.2) with the feedback control law*

$$\omega = k\mathbf{x} \times \mathbf{x}_d \quad (2.8)$$

is locally asymptotically stable at the equilibrium point $\mathbf{x} = \mathbf{x}_d$ for $\mathbf{x} \in B$, for all $k \in \mathbb{R}^+ \setminus \{0\}$ where $\mathbf{x}_d \in \mathbb{R}^3$ is the demand attitude of the tool, and where

$$B := \{\mathbf{x} \in \mathbb{R}^3 : \|\mathbf{x}\| = 1 \text{ and } \mathbf{x} \neq -\mathbf{x}_d\} \quad (2.9)$$

Proof. For the tool to converge towards its demand attitude $\mathbf{x}_d$, this must be an equilibrium point. The neighbourhood B around the equilibrium point is defined to be the set of all attitudes $\mathbf{x}$ such that $\mathbf{x} \neq -\mathbf{x}_d$. Substituting the feedback control law (2.8) into the attitude response (2.2) gives

$$\dot{\mathbf{x}} = k[\mathbf{x}_d - (\mathbf{x} \cdot \mathbf{x}_d)\mathbf{x}]. \quad (2.10)$$

Setting $\mathbf{x} = \mathbf{x}_d$ in (2.10) gives $\dot{\mathbf{x}} = 0$, and hence by Definition 1 this is an equilibrium point. The Lyapunov function candidate is chosen as

$$V(\mathbf{x}) = 1 - \mathbf{x} \cdot \mathbf{x}_d. \quad (2.11)$$

Here $V(\mathbf{x} = \mathbf{x}_d) = 0$ and for all $\mathbf{x} \in B$, $V(\mathbf{x}) > 0$, since $\mathbf{x}$ and $\mathbf{x}_d$ are unit vectors. The first derivative of the Lyapunov function is given by

$$\begin{aligned} \dot{V}(\mathbf{x}) &= -\dot{\mathbf{x}} \cdot \mathbf{x}_d \\ &= -(\omega \times \mathbf{x}) \cdot \mathbf{x}_d \\ &= -k[\mathbf{x}_d - (\mathbf{x}_d \cdot \mathbf{x})\mathbf{x}] \cdot \mathbf{x}_d \\ &= -k[1 - (\mathbf{x}_d \cdot \mathbf{x})^2], \end{aligned} \quad (2.12)$$

where the first step is the substitution of the attitude response (2.2) and the second step uses the vector triple product identity. Equation (2.12) is negative semi-definite and negative definite for $\mathbf{x} \in B$ and hence by Lemma 1 the system (2.2) with control law (2.8) is locally asymptotically stable. $\square$

The control law given by (2.8) is remarkably simple and elegant. Similar quaternion based control laws are proposed in Wen and Kreutz-Delgado [1991]. The control law is linear and avoids the non-linearities associated with Euler angle representations such as the controller proposed in Panchal et al. [2010] and Matheus et al. [2012].

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In the remainder of the chapter, the vector difference between the BHA attitude and the demand attitude $\mathbf{e} = \mathbf{x} - \mathbf{x}_d$ will be referred to as the *attitude error*. The *angle error* $\vartheta \in [0, \pi)$ is such that $\cos \vartheta = \mathbf{x} \cdot \mathbf{x}_d$ and $\sin \vartheta = |\mathbf{x} \times \mathbf{x}_d|$, and is related to the 2-norm of the attitude error by $\|\mathbf{e}\|_2 = 2 \sin\left(\frac{\vartheta}{2}\right)$.

Simulation

To demonstrate the effectiveness of the variable build rate controller, a simulation with the response of (2.2) is performed. The values of the control gain for the control law defined by (2.8) is set to $k = 5.4^\circ\text{hr}^{-1}$. The initial attitude in the Earth frame is $\mathbf{x}_0 = \text{unit}([1, 1, 0])$, and the attitude demand vector is $\mathbf{x}_d = [1, 0, 0]$.

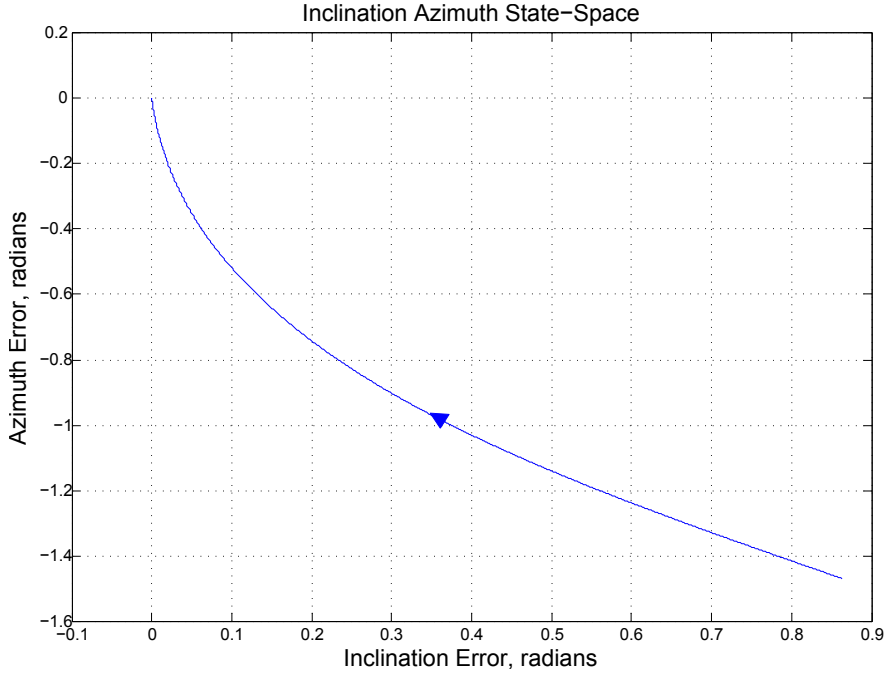


Figure 2.5: θ_{inc} and θ_{azi} angle error for the variable build rate controller

Figure 2.5 shows the attitude trajectory for control law (2.8) in terms of the inclination angle error and the azimuthal angle error for the unconstrained actuation model for the drill. The curve shown is a state-space trajectory which converges towards $(0, 0)$ and remains there.

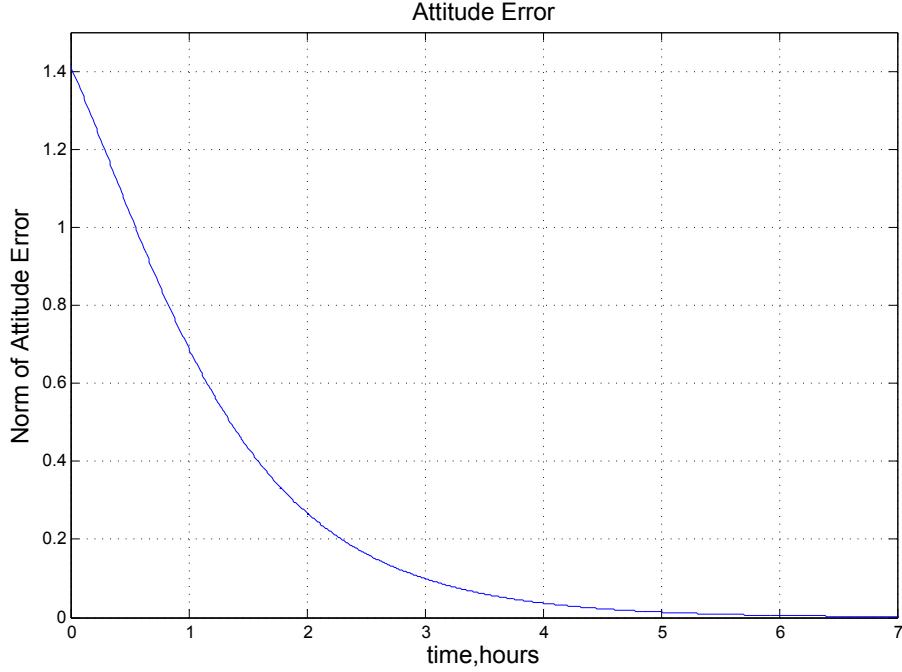


Figure 2.6: Norm of attitude error for the variable build rate controller

Figure 2.6 shows the norm of the attitude error given by $\|\mathbf{x} - \mathbf{x}_d\|$ as a function of time, for which the error converges asymptotically to zero.

2.3.2 Constant build-rate controller

The controller given by (2.8) assumes that the build rate magnitude $\|\omega\|_2$ varies with the attitude error $\mathbf{e}$. However, in practice, this is very difficult to engineer. For a real directional drilling BHA, the actuator build rate is generally nominally constant. Hence a second control law is proposed where the assumption of a continuously variable build rate is removed and it is assumed that the build rate is constant or zero.

Proposition 2. *The kinematic system given by (2.2) with the feedback control law*

$$\omega = \begin{cases} K \frac{\mathbf{x} \times \mathbf{x}_d}{\|\mathbf{x} \times \mathbf{x}_d\|} & \text{for } \mathbf{x} \neq \mathbf{x}_d \\ 0 & \text{for } \mathbf{x} = \mathbf{x}_d, \end{cases} \quad (2.13)$$

is locally asymptotically stable at the equilibrium point $\mathbf{x} = \mathbf{x}_d$ for $\mathbf{x} \in B$ where K is the constant build rate.

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Proof. From (2.13) by definition $\mathbf{x} = \mathbf{x}_d$ is an equilibrium point. We again use the Lyapunov function given by (2.11). The first derivative of the Lyapunov function is given by

$$\dot{V}(\mathbf{x}) = \begin{cases} \frac{-K[1-(\mathbf{x} \cdot \mathbf{x}_d)^2]}{\|\mathbf{x} \times \mathbf{x}_d\|} & \text{for } \mathbf{x} \neq \mathbf{x}_d \\ 0 & \text{for } \mathbf{x} = \mathbf{x}_d, \end{cases} \quad (2.14)$$

which is negative definite and hence by Lemma 1 the system (2.2) with control law (2.13) is locally asymptotically stable. $\square$

Corollary 1. *The control given by (2.13) solves the minimum time optimal control problem defined as*

$$\min_{\omega} \int_0^{t_f} dt \quad (2.15)$$

subject to (2.2), $\mathbf{x}(0) = \mathbf{x}_0$, $\|\mathbf{x}_0\| = 1$, $\mathbf{x}(t_f) = \mathbf{x}_d$, $\|\omega\|_2 \leq K$.

Proof. Let $\hat{\mathbf{x}}(t)$ be the trajectory that is the solution to the initial value problem for system (2.2) with the control given by (2.13) and $\mathbf{x}(0) = \mathbf{x}_0$, $\|\mathbf{x}_0\| = 1$. The tangent to the curve $\hat{\mathbf{x}}(t)$ is given by

$$\omega \times \mathbf{x} = K \frac{\mathbf{x}_d - \mathbf{x}(\mathbf{x} \cdot \mathbf{x}_d)}{\|\mathbf{x} \times \mathbf{x}_d\|} \quad (2.16)$$

Since the tangent vector is spanned by the vectors $\mathbf{x}_d$ and $\mathbf{x}$ which pass through the origin on the sphere, the state trajectory $\hat{\mathbf{x}}(t)$ is a geodesic. Geodesics on a sphere are arcs of great circles with the property that they are the intersection of the sphere and a plane passing through the origin, where the great circle lies on this plane. It is known that a geodesic gives the minimum distance over the surface between two points lying on that surface [Pressley, 2001, p. 218]. Clearly, the path of minimum length is the path of the minimal time trajectory if the speed along the path is always maximal. From (2.13), the build rate is maximum (except when the target is reached), hence the trajectory is minimum time. $\square$

Simulation

To demonstrate the effectiveness of the constant build rate controller, a time domain simulation with the response of (2.2) is performed. The value of the constant build rate is taken as $K = 15^\circ \text{hr}^{-1}$. The initial attitude in the Earth frame is $\mathbf{x}_0 = \text{unit}([1, 1, 0])$, and the attitude demand vector is $\mathbf{x}_d = [1, 0, 0]$.

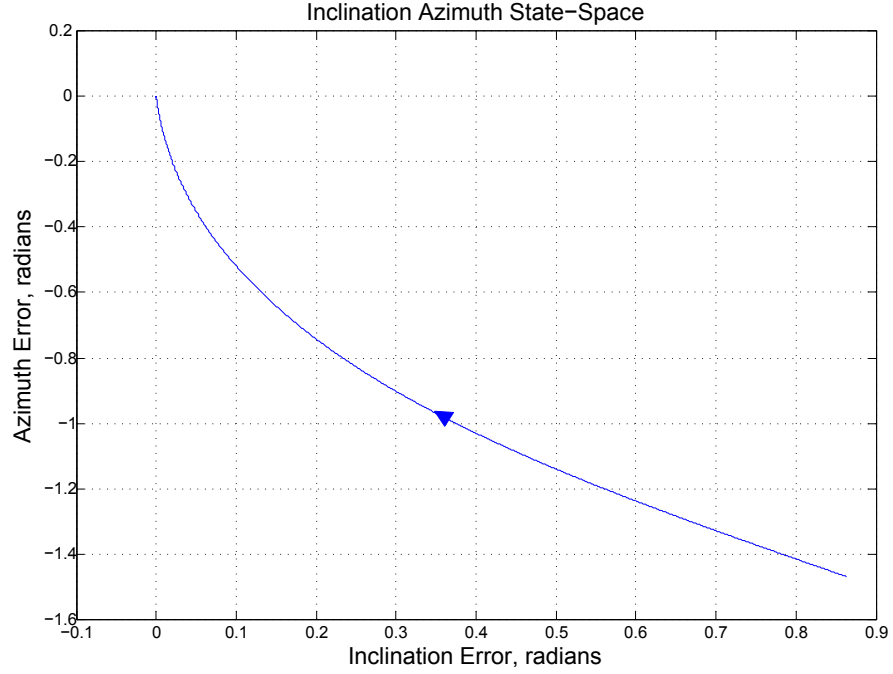


Figure 2.7: θ_{inc} and θ_{azi} angle error for the constant build rate controller

Figure 2.7 shows the attitude trajectory for control law (2.13). The curve appears identical to that of Figure 2.5, the trajectory for the variable build rate controller. The path taken by the two trajectories is actually identical, but the speeds along the path differ.

Figure 2.8 shows the norm of the attitude error given by $\|\mathbf{x} - \mathbf{x}_d\|$ as a function of time. The constant build rate controller converges directly to zero within 6 hours. Although the variable rate controller error converges in approximately the same time, the maximum build rate is much higher than the constant build rate.

The control law given by (2.13) assumes that build rate can be zero. In practice, the build rate can generally not be made zero and rectilinear boreholes are achieved through spinning ω about the $\mathbf{x}$ -axis. In addition, small perturbations on the system could cause the control to switch very rapidly about the desired attitude. This *chattering* behaviour is also well known to occur with sliding mode controllers switching about the sliding manifold [Edwards and Spurgeon, 1998]. Furthermore, the controllers are usually implemented with a digital processor. By implementing the control law in

2. ATTITUDE CONTROL FOR ROTARY STEERABLE TOOLS

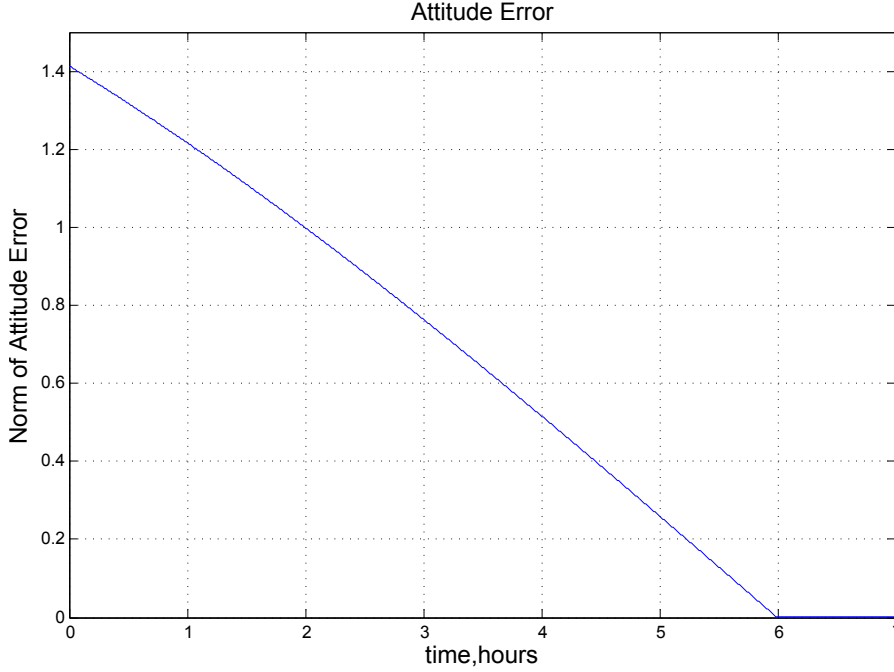


Figure 2.8: Norm of attitude error for the constant build rate controller

the discrete time domain, the chattering problem can be reduced. The final proposed controller is thus a discrete time controller with a similar control law to (2.13).

2.3.3 Discrete-time controller

The constant build rate constraint has implications for the stability of the closed loop system, because the system can no longer be at equilibrium when the error is zero. Hence we need to modify the stability requirements from standard asymptotic stability. Essentially, we need the error in the tool attitude to be bounded in a small region around the required attitude. Furthermore the attitude must converge to this region if it is outside it. The concept of (I, J) stability [Fagnani and Zampieri, 2003] allows us to proceed.

Definition 2. Given a discrete-time system, $x(i+1) = \Gamma(x(i))$, and two invariant sets $J \subseteq I$ with respect to the affine map $\Gamma(x(i))$, we say that $\Gamma(x(i))$ is (I, J) – stable if $\forall x(0) \in I, \exists i_0 \geq 0 : x_i \in J, \forall i \geq i_0$.

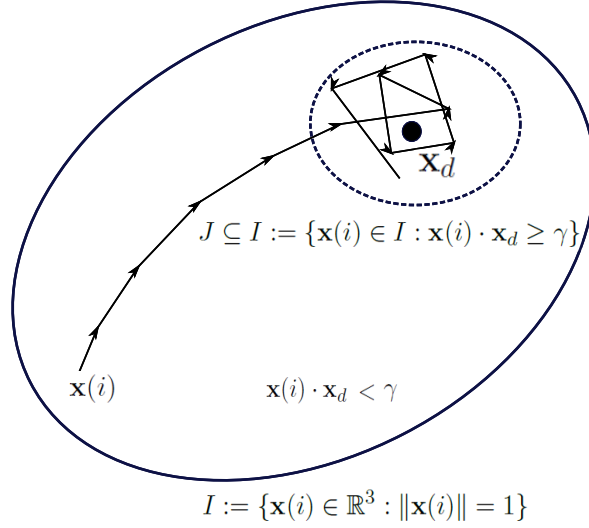


Figure 2.9: IJ stability

Proposition 3. The kinematic system in (2.6) with the controller given by

$$\tilde{\omega}(i) = \begin{cases} \frac{\mathbf{x}(i) \times \mathbf{x}_d}{\|\mathbf{x}(i) \times \mathbf{x}_d\|} & \text{for } \mathbf{x}(i) \neq \pm \mathbf{x}_d \\ \tilde{\omega}(i-1) & \text{for } \mathbf{x}(i) = \mathbf{x}_d, i \neq 0 \\ \tilde{\omega}_l & \text{for } \mathbf{x}(i) = -\mathbf{x}_d \text{ or } \mathbf{x}_d \text{ and } i = 0, \end{cases} \quad (2.17)$$

is (I, J) -stable, where $\tilde{\omega}_l$ is any unit vector satisfying $\tilde{\omega}_l \cdot \mathbf{x}(i) = 0$

Remark 1. The system response described by (2.6) advances the BHA attitude by $K\delta t$ in each time step. Hence the purpose of the control law 3 is to determine an axis in which to rotate the BHA attitude $\mathbf{x}(i)$ to converge toward $\mathbf{x}_d$.

Proof. The invariant set I is defined to be the set of all possible attitudes

$$I := \{\mathbf{x}(i) \in \mathbb{R}^3 : \|\mathbf{x}(i)\| = 1\}. \quad (2.18)$$

The set J is in the neighborhood of the desired attitude $\mathbf{x}_d$ and is

$$J \subseteq I := \{\mathbf{x}(i) \in I : \mathbf{x}(i) \cdot \mathbf{x}_d \geq \gamma\}, \quad (2.19)$$

where $\gamma = \cos(K\delta t)$, and $\sqrt{1 - \gamma^2} = \sin(K\delta t)$.

Substituting the control law from (2.17) into the kinematic system from (2.6) gives the closed-loop system

$$\mathbf{x}(i+1) = \mathbf{x}_d \frac{\sin(K\delta t)}{\sqrt{1 - (\mathbf{x}(i) \cdot \mathbf{x}_d)^2}} + \mathbf{x}(i) \left[\cos(K\delta t) - \sin(K\delta t) \frac{\mathbf{x}(i) \cdot \mathbf{x}_d}{\sqrt{1 - (\mathbf{x}(i) \cdot \mathbf{x}_d)^2}} \right]. \quad (2.20)$$

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To show that J is an invariant set, consider (i) the case when $\mathbf{x}(i) \cdot \mathbf{x}_d = \gamma$ and (ii) the case when $\mathbf{x}(i) \cdot \mathbf{x}_d > \gamma$.

(i) For the case $\mathbf{x}(i) \cdot \mathbf{x}_d = \gamma$, the proof is by induction, where it is to be shown that if $\mathbf{x}(i) \in J$ then $\mathbf{x}(i+1) \in J \forall i \in \mathbb{N}$. Taking the scalar product of (2.20) with $\mathbf{x}_d$ and substituting for $\mathbf{x}(i) \cdot \mathbf{x}_d = \gamma$ gives $\mathbf{x}(i+1) \cdot \mathbf{x}_d = 1$. This corresponds to the attitude being the desired attitude $\mathbf{x}(i+1) = \mathbf{x}_d$. The attitude at time $i+2$ is given using $\tilde{\omega}(i)$ as the control by

$$\mathbf{x}(i+2) = \cos(K\delta t)\mathbf{x}(i+1) - \frac{\sin(K\delta t)}{\sqrt{1 - (\mathbf{x}(i) \cdot \mathbf{x}_d)^2}} [(\mathbf{x}(i+1) \cdot \mathbf{x}_d)\mathbf{x}(i) - (\mathbf{x}(i+1) \cdot \mathbf{x}(i))\mathbf{x}_d]. \quad (2.21)$$

Taking the scalar product of (2.21) with $\mathbf{x}_d$ and substituting for $\mathbf{x}(i) \cdot \mathbf{x}_d = \gamma$ and $\mathbf{x}(i+1) \cdot \mathbf{x}_d = 1$ gives $\mathbf{x}(i+2) \cdot \mathbf{x}_d = \gamma$.

(ii) For the case when $\gamma < \mathbf{x}(i) \cdot \mathbf{x}_d < 1$ initially, take the scalar product of (2.20) with $\mathbf{x}_d$. This gives the expression

$$\mathbf{x}(i+1) \cdot \mathbf{x}_d = \sqrt{1 - (\mathbf{x}(i) \cdot \mathbf{x}_d)^2} \sin(K\delta t) + \mathbf{x}(i) \cdot \mathbf{x}_d \cos(K\delta t). \quad (2.22)$$

(2.22) can be written in terms of the double angle formula

$$\mathbf{x}(i+1) \cdot \mathbf{x}_d = A \cos(\vartheta - K\delta t) \quad (2.23)$$

where $A = 1$ and $\mathbf{x}(i) \cdot \mathbf{x}_d = \cos \vartheta$. Given the condition that $\gamma < \mathbf{x}(i) \cdot \mathbf{x}_d < 1$ implies that $0 < \vartheta < K\delta t$, then $0 < K\delta t - \vartheta < K\delta t$ and hence $\mathbf{x}(i+1) \cdot \mathbf{x}_d > \gamma$ and J is thus an invariant set.

Now consider $\mathbf{x}(i) \in I$, $\mathbf{x}(i) \neq \pm \mathbf{x}_d$ and $-1 < (\mathbf{x}(i) \cdot \mathbf{x}_d) < \gamma$. Here we chose a Lyapunov function $V(i)$ to be the angle subtended between $\mathbf{x}(i)$ and $\mathbf{x}_d$

$$V(i) = \cos^{-1}(\mathbf{x}(i) \cdot \mathbf{x}_d). \quad (2.24)$$

where $0 \leq V < \pi$. The Lyapunov function strictly decreases that is $V(i+1) < V(i)$ if the attitude converges towards the set J .

Rewriting equation 2.23 in terms of the Lyapunov function $V(i)$, and the angular advancement $K\delta t$ gives

$$\cos(V(i+1)) = \cos(V(i) - K\delta t). \quad (2.25)$$

2.3 Attitude Control Laws and Stability

Since $K\delta t > 0$, this implies the Lyapunov function decreases in each time step $V(i+1) < V(i)$.

Lastly consider an initial attitude being the antipodal point $\mathbf{x}_0 = -\mathbf{x}_d$. Here $\tilde{\omega}_l$ cannot be determined from the vector product of the current measured attitude and the desired attitude, however any direction $\tilde{\omega}_l$ would suffice since in the next time step, the attitude would move away from the antipodal point and a value of $\tilde{\omega}$ can be determined and the attitude will converge towards the set J . Furthermore, for an initial attitude beginning at the desired attitude $\mathbf{x}_0 = \mathbf{x}_d$, since there is no previous time step control to apply, any control $\tilde{\omega}_l$ would move the attitude to $\mathbf{x}_1 \cdot \mathbf{x}_d = \gamma$, since $\mathbf{x}(i) \cdot \mathbf{x}(i+1) = \gamma$, and the aforementioned argument for $\mathbf{x}(i) \cdot \mathbf{x}_d = \gamma$ implies (I, J) -stability. $\square$

Simulation

To demonstrate the effectiveness of the discrete time controller, a time domain simulation is performed. The values of the constant build rate is taken as $K = 15^\circ\text{hr}^{-1}$ corresponding to a BHA capable of steering at $15^\circ(100ft)^{-1}$ with an ROP of $100ft\text{hr}^{-1}$. The sampling time for the controller (2.17) is $\delta t = 100$ s. The initial attitude in the Earth frame is $\mathbf{x}_0 = \text{unit}([1, 1, 0])$, and the attitude demand vector is $\mathbf{x}_d = [1, 0, 0]$. Figures 2.10 and 2.11 show the attitude trajectory and error for the discrete-time control law (2.17). The curves are almost identical to those of Figures 2.7 and 2.8 except when the error approaches the bound. Detail of the norm of the attitude error plot is shown in Figure 2.12. The upper bound of the switching region given by $2\sin(KT_s/2)$ is also shown. Detail of the error curve near the origin is shown in Figure 2.13 which shows the discrete switching about the equilibrium point within the upper bound.

2. ATTITUDE CONTROL FOR ROTARY STEERABLE TOOLS

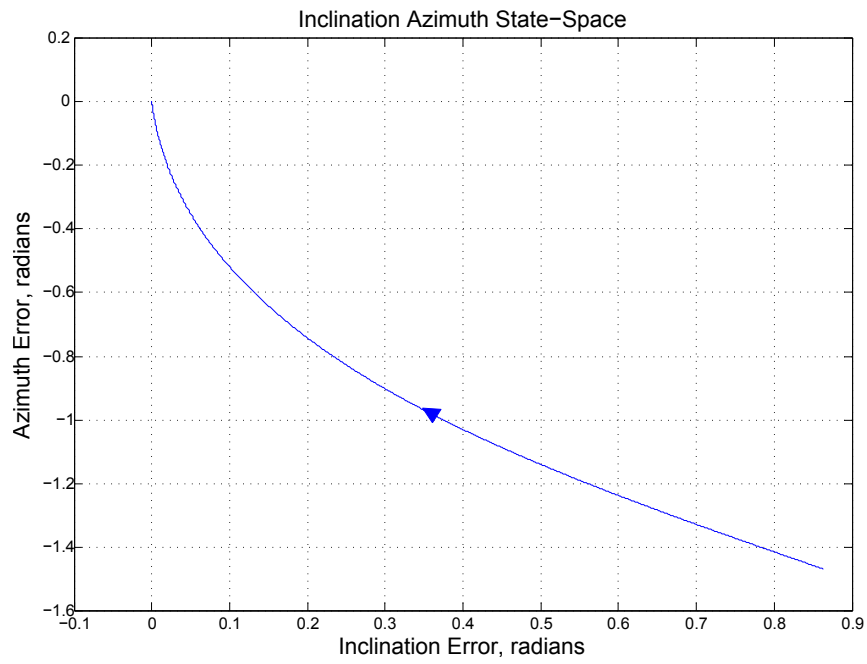


Figure 2.10: θ_{inc} and θ_{azi} angle error for the discrete-time controller

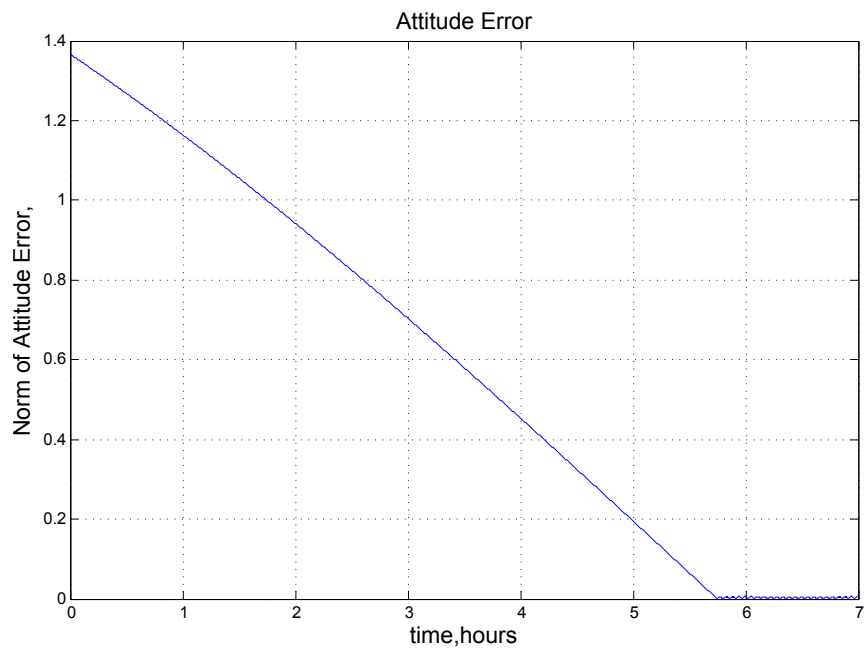


Figure 2.11: Norm of attitude error for the discrete-time controller

2.3 Attitude Control Laws and Stability

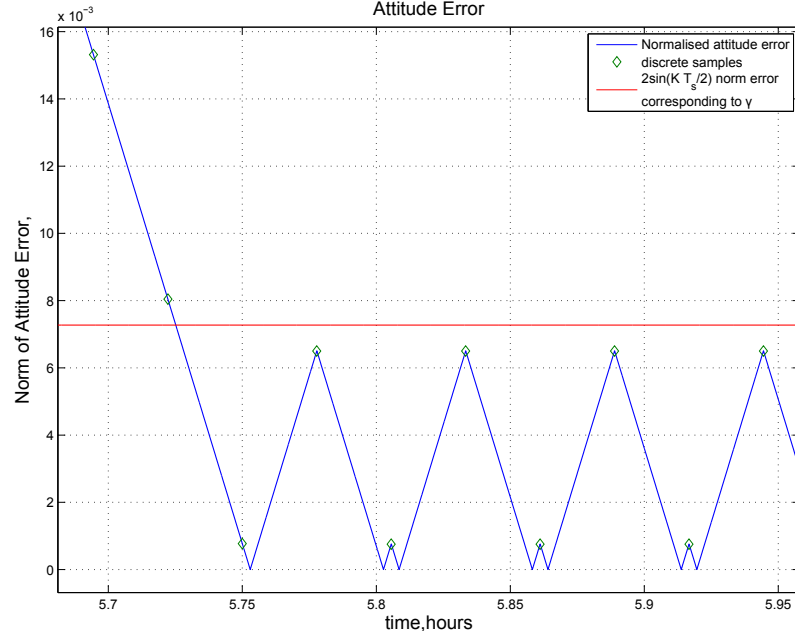


Figure 2.12: Norm of attitude error for the discrete-time controller – detail near the equilibrium point

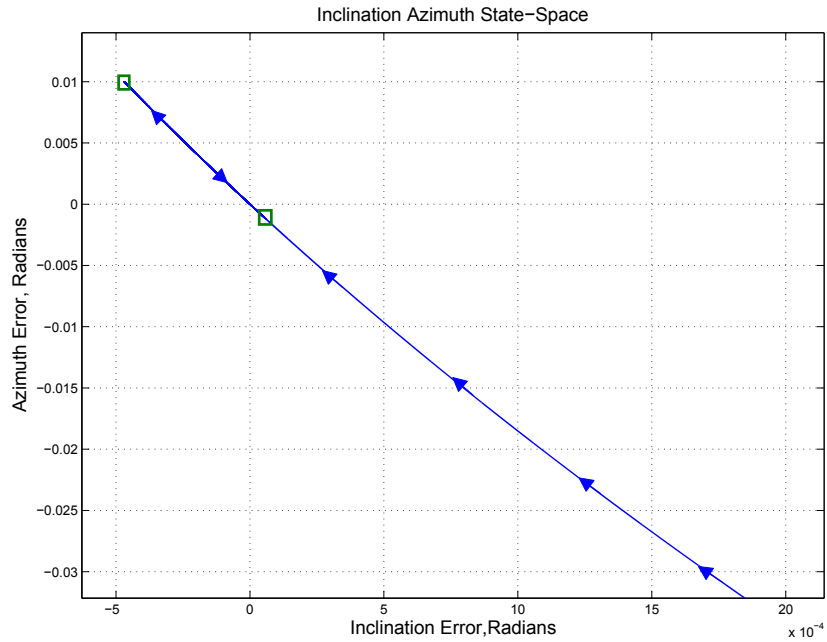


Figure 2.13: θ_{inc} and θ_{azi} angle error for the discrete-time controller – detail showing discrete switching near the equilibrium point

2.4 Controller Implementation

Although the control laws proposed in Section 2.3 are simple and can be analyzed using well-known methods, several coordinate transformations are required in order to implement the controller within the structure shown in Figure 2.2. The attitude of the BHA has to be determined from measurements of the accelerometers and magnetometers, the desired attitude, x_d , has to be calculated as a unit vector from desired Euler angles, and finally the toolface angle to enact the calculated control, ω , has to be computed. In this section, the calculations for these coordinate transformations are presented. This section begins with subsection 2.4.1 presenting the relationship to determine the toolface angle from knowledge of the control ω , and the Earth's gravitational and magnetic field direction. Subsection 2.4.2 describes methods for determining the attitude $\mathbf{x}$ from the sensor signals.

2.4.1 Toolface angle determination

RSS tools feature an internal toolface control system. The toolface angle (see Figure 2.14) is the clockwise difference in angle between the projection of $\mathbf{a}$ in the toolface plane and the steering direction in this plane. Where the unit vector $\mathbf{a}$ is defined to be either $-\mathbf{r}^G$ for the case of gravity toolface or $\mathbf{r}^B$ for magnetic toolface. The toolface angles are determined from the control, ω , and the field vector $\mathbf{a}$ by the relationship

$$\theta_{tf}^{\mathbf{a}} = \text{atan2}(\mathbf{a} \cdot \boldsymbol{\omega}, \mathbf{x}_d \cdot \mathbf{a} - (\mathbf{x} \cdot \mathbf{x}_d)(\mathbf{x} \cdot \mathbf{a})). \quad (2.26)$$

This expression for the toolface angle is determined from projecting the unit vectors $\mathbf{a}$, and $\boldsymbol{\omega}$ into the plane perpendicular to attitude $\mathbf{x}$. This projection is achieved through successive cross products with the attitude $\mathbf{x}$ (c.f. Figure 2.14).

2.4.2 Attitude Determination from Sensors

Here methods for determining the tool's attitude $\mathbf{x}$ in the Earth frame from accelerometers and magnetometers is presented. The Earth frame is the inertial frame which is fixed, and corresponds locally to the geology of the drilling operation. It is assumed that the variation in the Earth's magnetic and gravitational field over the region of an oil and gas well is negligible. In this frame there are two normalized reference vectors,

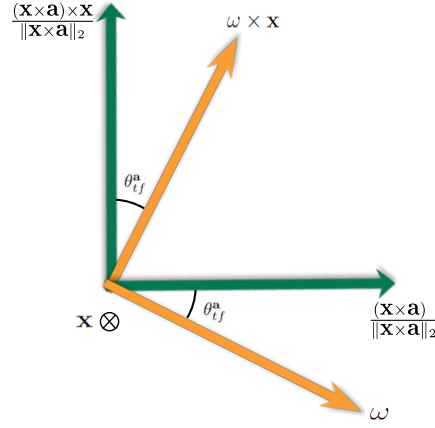


Figure 2.14: Kinematic response model of BHA

these being the magnetic field $\mathbf{r}^B$ and the gravitational field $\mathbf{r}^G$. These are given relative to a basis in the Earth frame which is easily measured offline and assumed known. On the BHA, the accelerometers $\mathbf{b}^G$ and magnetometers $\mathbf{b}^B$ provide another basis.

Quaternion Attitude Determination

We refer to a quaternion attitude determination algorithm by Markley [2002], which had been developed for the spacecraft attitude determination in an Earth orbit from sensing the Earth's magnetic field and the line of sight to the sun. The magnetic and gravitational field in the Earth frame is related to the body frame by the transformation

$$q\mathbf{r}^Gq^{-1} = \mathbf{b}^G, \quad (2.27)$$

$$q\mathbf{r}^Bq^{-1} = \mathbf{b}^B, \quad (2.28)$$

where the quaternion q can be determined from the Markley Algorithm [Markley, 2002]. From q , the attitude of the drill in the Earth frame is given by

$$\mathbf{x} = q^{-1}\mathbf{r}^Gq. \quad (2.29)$$

Given the target inclination and azimuth as Euler angles the corresponding quaternion is found from the relationship

$$q_{E2Q} = \left(\cos \frac{\theta_{inc}}{2} + \sin \frac{\theta_{inc}}{2} \mathbf{j} \right) \left(\cos \frac{\theta_{azi}}{2} + \sin \frac{\theta_{azi}}{2} \mathbf{i} \right). \quad (2.30)$$

2. ATTITUDE CONTROL FOR ROTARY STEERABLE TOOLS

This converts an azimuth and inclination set point attitude into a quaternion and hence a demand vector in the Earth's frame can be obtained.

$$\mathbf{x}_d = q_{E2Q}^{-1} \mathbf{r}^G q_{E2Q}. \quad (2.31)$$

Euler Angle Attitude Determination

The attitude can be determined through sensors by Euler angle rotations of θ_{inc} and θ_{azi} angles by the transformation

$$\mathbf{x} = R_y(\theta_{inc}) R_x(\theta_{azi}) \mathbf{r}^G, \quad (2.32)$$

where

$$\begin{aligned} \theta_{azi} &= \arctan\left(\frac{-\mathbf{b}_z^B}{\mathbf{b}_y^B}\right), \\ \theta_{inc} &= \arccos\left(\frac{\mathbf{b}_x^G}{\|\mathbf{b}^G\|}\right), \end{aligned} \quad (2.33)$$

and

$$R_x(\cdot) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\cdot) & \sin(\cdot) \\ 0 & -\sin(\cdot) & \cos(\cdot) \end{bmatrix}, \quad (2.34)$$

$$R_y(\cdot) := \begin{bmatrix} \cos(\cdot) & 0 & -\sin(\cdot) \\ 0 & 1 & 0 \\ \sin(\cdot) & 0 & \cos(\cdot) \end{bmatrix}. \quad (2.35)$$

The disadvantage of using Euler angles for attitude determination is that the radial magnetometer signals $\mathbf{b}_z^B$ and $\mathbf{b}_y^B$ are zero when drilling parallel to the Earth's magnetic field. Here the azimuth is undefined and the tool attitude cannot be evaluated.

Optimal Rotation Matrix from Sensors

For completeness a further method for attitude determination is referenced here [Markley, 1988] which uses the rotation matrix representation for the tools attitude. A rotation matrix $\mathbf{R}$ is chosen to minimize the following cost function

$$J(\mathbf{R}) = a_1 \|\mathbf{b}^G - \mathbf{R} \mathbf{r}^G\|_2 + a_2 \|\mathbf{b}^B - \mathbf{R} \mathbf{r}^B\|_2, \quad (2.36)$$

where a_i are positive scalars that satisfy $a_1 + a_2 = 1$. For continuous surveys, accelerometer sensors on the BHA experience vibrations. In this situation it is advantageous to weight the magnetometers a_2 more heavily than the accelerometers a_1 . For static surveys when the BHA is not penetrating and stationary, it is advantageous to weight the accelerometers $a_1 > a_2$ more heavily than the magnetometers, since for the Earth's magnetic field is non-uniform and attenuated by magnetic rocks. Other considerations influence the design of the weights, such as how far on-board sensors are from magnetic components on the BHA which induce interference.

A direct solution to (2.36) presented by Markley [1988] is stated as

$$\mathbf{R} = \mathbf{U} \text{diag}(1, 1, \det(\mathbf{U})\det(\mathbf{V})) \mathbf{V}^T, \quad (2.37)$$

where $\mathbf{U}$ and $\mathbf{V}$ are respectively the left and right singular vector matrices found by a singular value decomposition of a matrix $\mathbf{M}$ defined to be

$$\mathbf{M} = \mathbf{U} \mathbf{S} \mathbf{V}^T \quad (2.38)$$

$$= a_1 \mathbf{b}^G \otimes \mathbf{r}^G + a_2 \mathbf{b}^B \otimes \mathbf{r}^B, \quad (2.39)$$

where $\otimes$ is the vector outer product.

2.4.3 Output Signals

For effective steering, it is necessary for Directional Drillers to monitor the BHA attitude. The industry convention presents this attitude as the azimuth θ_{azi} and inclination θ_{inc} angles which can be calculated from the attitude unit vector $\mathbf{x}$ by the following transformation:

$$\theta_{inc} = \arccos \frac{\mathbf{x}_1}{\|\mathbf{x}\|}, \quad (2.40)$$

$$\theta_{azi} = \arctan \frac{-\mathbf{x}_3}{\mathbf{x}_2}. \quad (2.41)$$

2.5 Hole Spiraling

In this chapter control strategies for stabilizing the BHA attitude have been developed, and the determination of the required parameters with respect to attitude sensors discussed. In practice attitude sensors are located 1 – 4m behind the bit for D&I sensors in the steering assembly and a further 20m for MWD measuring subs. During the drilling process, the BHA is constrained to lie in the presently drilled borehole. The attitude of the section of hole between the sensors and the bit is unknown and cannot change. The sensors must pass through this section before the sensors reach the current bit location. The change in attitude because of the sensor-to-bit borehole is unavoidable. The passage of the D&I sensors through this section influences the feedback control signal, manifesting oscillations in the attitude of the centre-line of the propagating borehole. This phenomenon is known as *hole-spiraling*. Other factors which cause hole-spiraling include the ratio of the rate of change of toolface and ROP and the relationship between the stabilizer positions and stiffness in the BHA. Spiraling is undesirable because wellbore oscillations contribute to hole friction in the drill-string, which reduces energy transferred along the drill-string for the rock cutting process. In this section, the impact of the sensor-to-bit distance on hole-spiraling is investigated and a method for reducing this effect is developed.

Section 2.5.1 details simulation results which study the relationship between the spiral wavelength, amplitude and sensor-to-bit location. Section 2.5.2 describes a modified attitude control system for compensating the spatial delay, and Section 2.5.1 provides *ST2* simulation cases investigating this modification.

2.5.1 Effect of Sensor-to-bit Location on Spiraling

This section investigates the effect of the sensor-to-bit location μ on the amplitude A and wavelength λ of the steady state hole-spiral induced by the attitude control system presented in the preceding analysis. A commercial drilling simulator *ST2* simulates borehole propagation from the attitude control law (2.13) for a push-the-bit steering assembly [Barr et al., 1995], with a steering capability of $9^\circ/100ft$. The borehole is propagated for a 1000m run following a demand inclination of θ_{inc}^d of 30° and a demand

azimuth of θ_{azi}^d of 320° . The initial attitude is coincident with the demanded. Five sensor-to-bit locations cases $\mu \in \{2, 10, 20, 30, 40\}$ m are simulated, and the corresponding steady state average amplitudes and wavelengths are recorded.

The sensor-to-bit distance is plotted versus the steady-state average amplitude in Figure 2.15, and versus the steady state average wavelength in Figure 2.16. It can be seen that there is a clear linear relationship for both the amplitude and the wavelength of the attitude control induced hole-spiral in the steady state.

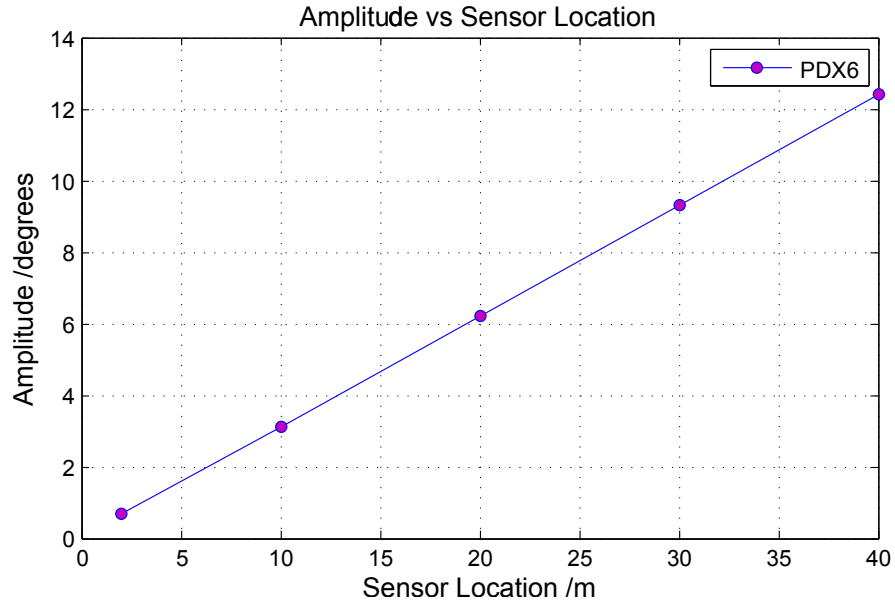


Figure 2.15: Spiral amplitude

The sensor-to-bit location μ and the curvature capability $\bar{k}_{dls}$ is nominally known by design of the BHA. However the latter may change during the drilling process due to disturbances. An estimate can be provided from the real time measurement data and from DD experience. It is postulated from Figure 2.15 that the wavelength of a spiral λ is related to the sensor to bit location μ by

$$\lambda = 4\mu. \quad (2.42)$$

Furthermore estimating the time for one period T of the spiral provides an estimate

2. ATTITUDE CONTROL FOR ROTARY STEERABLE TOOLS

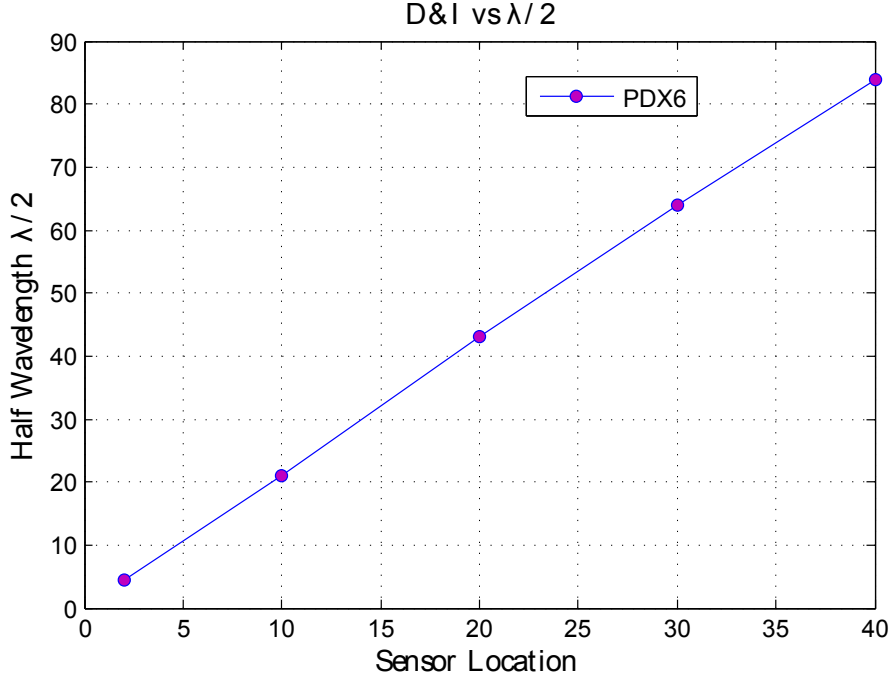


Figure 2.16: Azimuth spiral

for the downhole ROP of the drilling tool by

$$\text{ROP} = \frac{4\mu}{T}. \quad (2.43)$$

Moreover the amplitude of the spiral directly provides the angular difference $\bar{\theta}$ between the sensor and the bit.

2.5.2 Hole Spiraling Reduction

In this section, a method for mitigating hole-spiraling in the steady state is presented. The control signal is determined from a predicted attitude on the bit instead of the measured attitude signal. The predicted attitude is determined from parameters such as the amplitude and wavelength.

Figure 2.17 shows a modification to the attitude control system shown in Figure 2.2. The delayed time $\tau = \frac{T}{4}$ represents the transport along the borehole for the sensor to be coincident with the bit. From the history of the measured attitude at the sensor $\mathbf{x}(t - \tau)$ the spiral cycle properties of amplitude $\bar{\theta}$ and wavelength λ are interpreted.

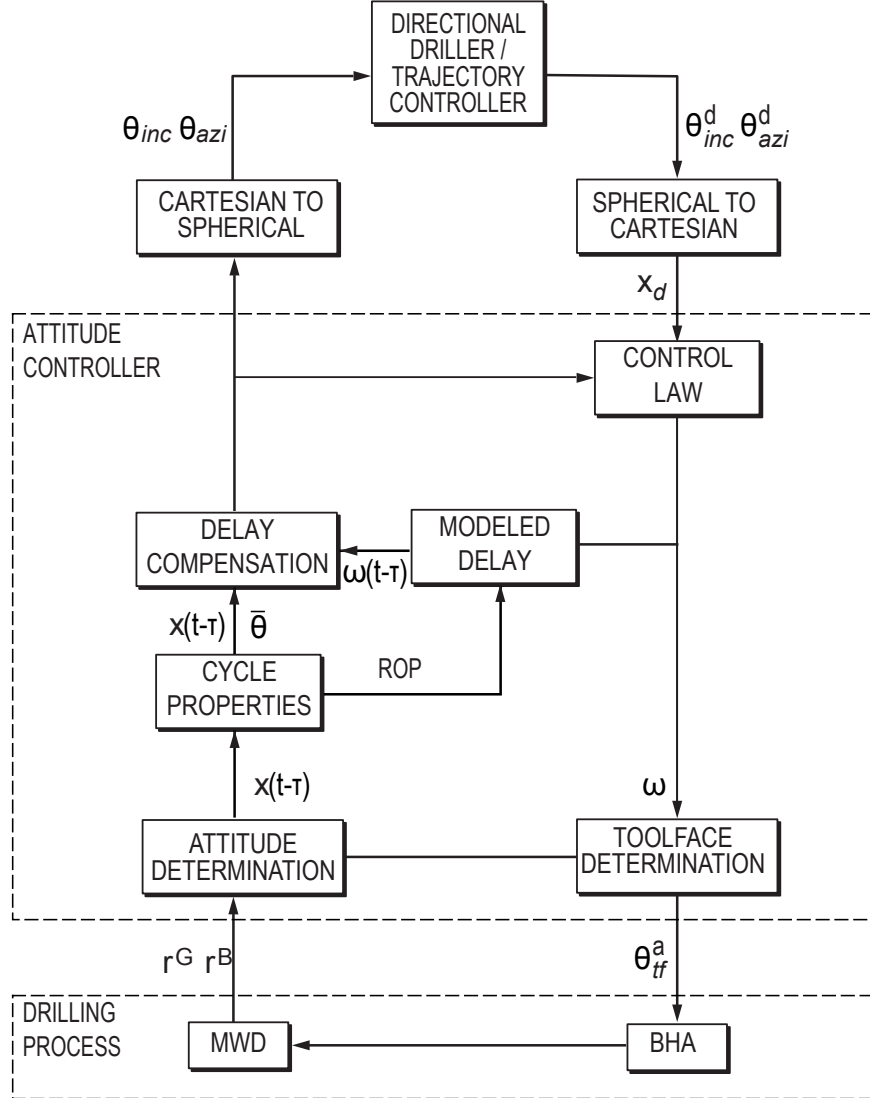


Figure 2.17: Spiral reduction control diagram

Estimating the ROP from equation (2.43) gives this delayed time τ . Knowledge of the control signal $\omega(t - \tau)$ applied at this previous delayed time τ allows us to predict the attitude at the bit by the relationship

$$\hat{\mathbf{x}}(t) = \mathbf{x}(t - \tau) \cos \bar{\theta} + \mathbf{x}(t - \tau) \times \omega(t - \tau) \sin \bar{\theta}. \quad (2.44)$$

This can be interpreted as rotating the attitude $\mathbf{x}(t - \tau)$ by an angle $\bar{\theta}$ about the axis $\omega(t - \tau)$.

2. ATTITUDE CONTROL FOR ROTARY STEERABLE TOOLS

2.5.3 Hole Spiraling Reduction Simulations

In this section, the sensor-to-bit delay compensated for by the correction (2.44), is tested in simulation. In practice the ROP and $\bar{\theta}$ are very uncertain due to noise in the drilling process. Furthermore when a correction for the attitude $\hat{\mathbf{x}}$ is applied, the cycle properties change presenting challenges for determining the wavelength λ and amplitude $\bar{\theta}$ in the real time process. For the sake of simplicity, the attitude correction (2.44), is tested with fixed values of $\bar{\theta}$ investigating also the effect of over- and under-compensating the projected attitude. This spiral compensation method is tested by ST2 simulations for a constant ROP with a push-the-bit steering assembly. The D&I sensors are placed 5m behind the bit. The BHA attitude initializes at an inclination of $\theta_{inc} = 45.2^\circ$ and an azimuth of $\theta_{azi} = 324.8^\circ$. The simulated wells are drilled to a depth of 800m with demand inclination of $\theta_{inc}^d = 45^\circ$ and azimuth of $\theta_{azi} = 326^\circ$.

Figure 2.18 displays the attitude response for three runs of varying $\bar{\theta}$. The steady state spiral with no delay $\bar{\theta} = 0$ compensation is shown in the black curve. There is a steady state spiral with an inclination amplitude of 2.7° , and an azimuth amplitude of 3.8° . A delay compensation of $\bar{\theta} = 0.03$ rad is simulated with the results depicted by the blue trajectory in Figure 2.18. The spiral is reduced with an amplitude of 1° inclination and 1.3° azimuth. The spiral is further reduced by increasing the compensation to $\bar{\theta} = 0.035$ rad. A steady state spiral depicted by the red trajectory has an inclination amplitude of 0.5° and an azimuth amplitude of 1° . The nature of the steady state spiral becomes more erratic the closer the sensors are projected to the bit.

Figure 2.19 shows two further runs to investigate the effect of over projecting the bit attitude. The black trajectory shows the steady state spiral with no delay compensation. Attitude state space trajectories for $\bar{\theta} = 0.08$ rad and $\bar{\theta} = 0.07$ rad are displayed in the green and the red trajectories curves respectively. Both these trajectories initially undershoot the target attitude and towards the end of the run are within the region in the state space enclosed by the steady state of the spiral induced when no compensation $\bar{\theta} = 0$ is applied. The attitude responses for these trajectories are not well behaved and do not exhibit steady state spirals like in the cases shown in Figure 2.18.

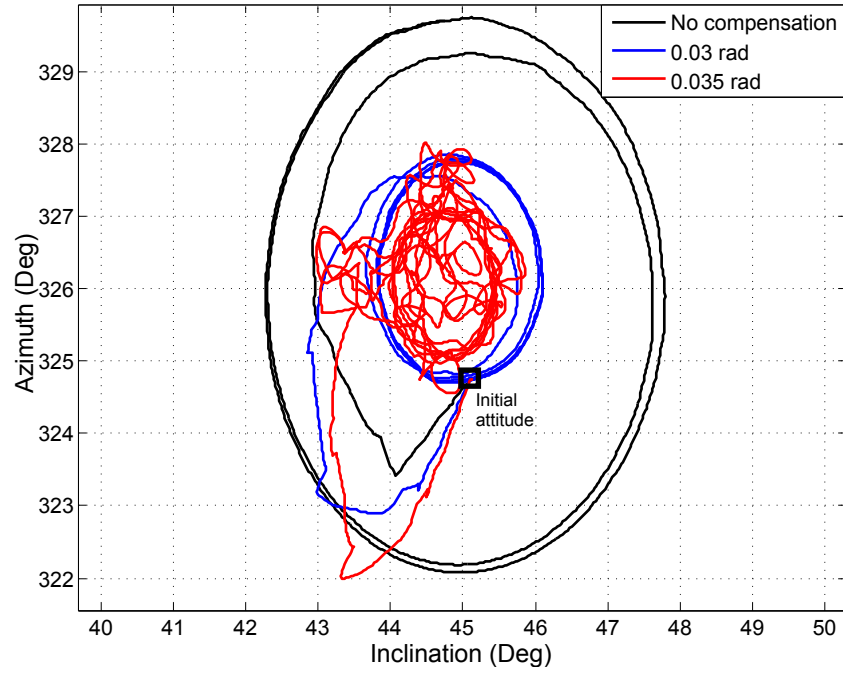


Figure 2.18: Phase plane spiral

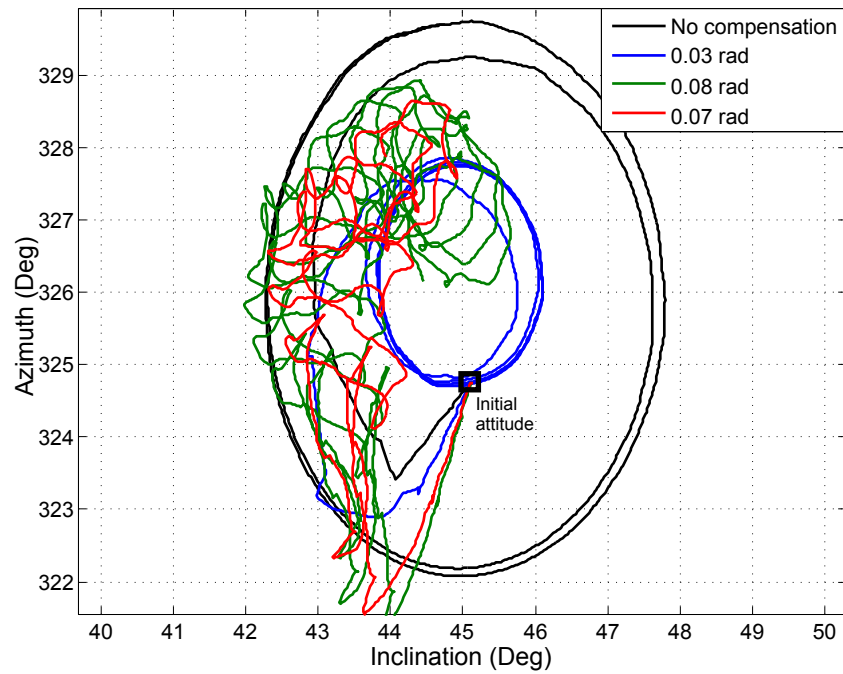


Figure 2.19: Phase plane spiral

2.6 Conclusions

This chapter proposes several control laws for the attitude control of the BHA for RSS tools suitable for directional drilling. Attitude control laws play an important role for drilling smooth straight hole sections and for attitude tracking for trajectory control systems. By representing the attitude as a unit vector, rather than in terms of the Euler angles, simple control laws are possible where the performance and stability is independent of the attitude state. The stability of the controlled system is proven using the Lyapunov direct method. The chapter summarizes the relevant quaternion algebra, rotation matrix and Euler angle based attitude determination. Simple simulations demonstrate the effectiveness of the proposed controllers. The effect of hole-spiraling due to the spatial delay of measurements because of the sensor being located behind the bit is investigated. A method for reducing this spiral is presented and tested in a drilling simulator. All the control laws are tested in simulation and in drilling simulators representing the push the bit rotary steerable tools. In further work, the control laws will also be tested by more complete simulation subjecting the system to disturbances, measurement errors actuator dynamics and uncertainty. Further work for spiral reduction includes determining a method for determining a delay compensation $\bar{\theta}$ online for an autonomous spiral reduction. Moreover further work on understanding the nature of the spiral to test the postulate of wavelength to sensor-to-bit distance is required.

3

Well-Planning

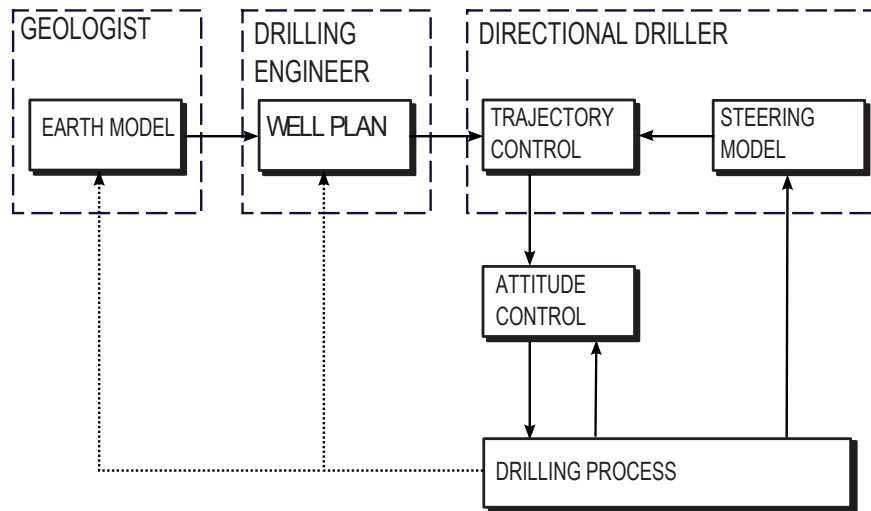


Figure 3.1: Well-planning interfaces the overall system defining the reference path for the trajectory control system to be described in Chapter 4, which guides the attitude control system in Chapter 2 to execute the well-plan

This chapter develops a strategy for 3D well-planning to navigate a directional drilling tool through a sequence of geological targets. For this strategy 3D b-spline space-curves represent the well-plan, which is optimized to minimize tortuosity with respect to strain energy and torsion. Curvature constraints representing the BHA steering capability ensure that the resultant plan is practical. The well-planning is synthesized in stages enabling the ability to tailor objectives and constraints between targets along the well-plan.

3. WELL-PLANNING

3.1 Introduction

Geological targets identified with respect to an Earth model are regions of interest in which to place a well. Wells are drilled for a variety of applications, where these regions of interest will depend on the application. For example:

- *producer* wells, for hydrocarbon extraction;
- *injector* wells, to inject fluids (usually water based) to drive production to producer wells;
- *geothermal* wells, to capture thermal energy from the geothermal gradient between the Earth's surface and crust;
- *relief* wells, to intersect wells which are inaccessible;
- *disposal* wells, for the safe disposal of rock cuttings;
- *water* wells, to produce water;
- *carbon-capture* wells, for the storage of carbon dioxide into porous rocks;
- *scientific* wells, for scientific research.

The well-plan curvature is influenced by the BHA and geological factors. For the former, the steering assembly, the drill-bit, stabilizers locations and drill collar stiffness affect the curvature capability. For the latter, the nature of the bedding planes, formation pressure, density and fluids properties affect the steering capability. In well-plan design, a drilling engineer estimates the BHA's curvature capability through experience of the BHA performances in nearby wells.

The objective of this well-planning strategy is to:

1. interpolate a sequence of predefined targets;
2. ensure feasibility of the BHA's steering capability;
3. minimize well-bore tortuosity;
4. allow user flexibility to customize drilling objectives and constraints along well-plan stages.

The economic motivation in drilling for oil and gas exploration is to maximize recoverable commodities in a reservoir whilst minimizing the costs (See Chapter 5 for a detailed business context). Operating costs are minimized by reducing the time taken to drill a well. This is achieved through minimizing the unnecessary length of a well, increasing the Rate of Penetration (ROP) and minimizing the downtime due to mistakes and failures. Maximizing the recoverable assets is achieved with accurate well-placement, where contact with volumes of high hydrocarbon saturation is maximized. Because of the high cost of setting up drilling rigs and equipment, it is desirable to be able to explore formations other than those located directly below the drilling rig, without having to move the rig or set up another rig. The reach of a well is limited if there is excess friction for the drill-string. Hence reducing friction increases the potential reservoir contact from a single well. Furthermore extending the reach of a well enables access to reserves which otherwise cannot be reached. For example, oil reservoirs located beneath protected regions like cities, national parks or forests need to be accessed from deviated wells located away from these areas.

The industry standard method for designing well-plans is the minimum curvature method. The principle of the minimum curvature method is based on fitting arcs of circles and straight line segments to targets and planes. A review of well-planning techniques based on the minimum curvature method which have been in use in the industry to plan wells is presented by Sawaryn and Thorogood [2003]. Samuel [2010] argues that the discontinuity in the transition between arcs of circles and straight line segments result in high stresses for the tubulars, increase of torque and drag, and poor hole cleaning. A number of authors have proposed well-planning methods where the interfaces between the circular arc and straight line segments are replaced with curves to smooth the curvature transition. Samuel [2010] replaces this transition with a clothoid curve, and Aadnoy et al. [2006] use a catenary profile. Both methods demonstrate improvements in the transfer of weight and torque to the bit, which has particular relevance for planning extended reach wells.

The drill-string is subject to bending moments from the borehole when being *tripped in* (run into the borehole), and when *tripped-out* (removed from the borehole). It is necessary for the well-plan to be smooth for minimizing friction due to bending on the

3. WELL-PLANNING

drill string. Friction reduces the potential length of the borehole reducing the energy transferred from the surface to the bit for the rock destruction process. Bending moments on the drill-string cause fatigue on the connections between drill-pipe stands and is a cause for mechanical failure. This is a cause for a twist-off where the BHA can be lost-in-hole and results in the well being abandoned. This smoothness criterion is also important for the casing operation. Casing is tripped-in with tubulars stiffer and of a larger diameter than the drill-string. Hence casing is subject to greater bending moments and friction. Methods for representing the smoothness of a well-plan in terms of its tortuosity is presented in [Samuel and Liu, 2009], where it is stated that there is no industry standard for defining tortuosity. A collection of methods are presented for representing tortuosity, where emphasis is placed on minimum strain energy. [Samuel and Liu, 2009] defines tortuosity for a general pipe in terms of the curvature and the torsion. A two dimensional extension for the use of strain energy for defining tortuosity is studied by Brands and Lowden [2012] where the strain energy is weighted for the borehole towards the surface. In addition factors such as the clearance of the hole are considered by Brands and Lowden [2012]. It is shown that minimizing tortuosity enhances the transfer of weight to the bit.

The well-planning work-flow is synthesized into stages. Each stage determines an optimum well-plan between an initial configuration and target destination. The properties of the optimum curve at the destination serves as an initial configuration for the next stage of the well-plan to the next destination. This process is repeated until all the destinations are exhausted. Curvature constraints and smoothness objectives are user-defined between each initial configuration and destination. The result is a continuous well-plan from the initial configuration through all the targets.

Using B-spline space curves to represent a well-plan in a hydrocarbon reservoir to define a path for steering a BHA is based on the work on variational curves by [Wesselink, 1996]. An extension to these methods is developed in this Chapter to specify the attitude along the well-plan, and handle curvature constraints. The optimum well-plan across each stage is constructed as a convex optimization problem [Boyd and Vandenberghe, 2004]. Interior point methods are used to exploit the benefits of the speed and

uniqueness of solutions based on the work by [Boyd and Vandenberghe, 2004].

The advantage of the strategy presented in this chapter over the minimum curvature method is the ability to plan wells with respect to constraints whilst simultaneously minimizing strain energy, torsion and arc length. Furthermore a variety of constraints can be defined and the result is a continuous curve where drilling parameters based on the derivative such as toolface, inclination and azimuth can be easily extracted.

The organization of the presentation in this chapter begins with section 3.2 explaining an overview of the well-planning strategy and the mathematical form of the non-linear programming methodology. Section 3.3 introduces b-spline space curves which are used to describe the well-plan. Section 3.4 describes the internal energy objective function in the optimization representing tortuosity and length in terms of b-splines. Section 3.5 develops the equality and inequality constraints to represent geometrical features of the well-plan and reservoir. Section 3.6 states the optimization problem with an outer-loop procedure to strictly constrain curvature. Section 3.7 demonstrates the well-planning strategy through an example, and finally conclusions are drawn in Section 3.8.

3.2 Optimization Structure

This Section describes the high level strategy for well-plan design through multiple geological targets. The well-plan is a concatenation of stages between adjacent targets. Section 3.2.1 describes the proposed strategy in terms of the stages, and Section 3.2.3 describes the optimization framework in which the subsequent analysis in this chapter is constructed for the solution of an individual stage. Each stage is constructed as a convex optimization problem, based on the work of [Boyd and Vandenberghe, 2004].

3.2.1 Proposed Scheme

Define the *pose* M to be a combination of a position and an orientation given for a point in space. Define the *target* D to be a position, orientation and a closed neighbourhood about this position, where this neighbourhood is a convex set. The solution workflow is shown in Algorithm. 3.1

3. WELL-PLANNING

Algorithm 3.1 Solution Workflow

- 1: Define initial pose M_0
 - 2: Define a sequence of targets D_i $i = 0 \dots i_n$
 - 3: Set steerability constraints
 - 4: Set smoothness objectives
 - 5: Set $i = 0$
 - 6: **while** $i < i_n$ **do**
 - 7: Solve for optimal well-plan $\mathbf{y}_i$ from M_i to D_i
 - 8: $i = i + 1$
 - 9: Set M_{i+1} equal the the pose of $\mathbf{y}$ in D_i .
 - 10: **end while**
 - 11: concatenate the vector well-plans $\mathbf{y} = \{\mathbf{y}_i\}$
-

3.2.2 Sketch of Well-plan Optimization

The structure of the optimization to solve for the well-plan $\mathbf{y}$ from an initial pose M to a destination D is

$$\begin{aligned}
& \underset{\mathbf{y}}{\text{minimize}} && \underbrace{E_{pipe}(\mathbf{y})}_{\text{tortuosity}} \\
& \text{subject to} && [\mathbf{y}(\lambda_0), \text{unit}(\mathbf{y}'(\lambda_0))] = M \quad (\text{initial pose}) \\
& && [\mathbf{y}(\lambda_f), \text{unit}(\mathbf{y}'(\lambda_f))] \in D \quad (\text{target destination}) \\
& && \forall \lambda, \kappa(\lambda) \leq \bar{k}_{dls} \quad (\text{curvature}) \\
& && \forall \lambda, \mathbf{y}(\lambda) \in F, \quad (\text{reservoir domain})
\end{aligned} \tag{3.1}$$

where the well-plan $\mathbf{y}(\lambda)$ is parametrized by $\lambda \in [\lambda_0, \lambda_f] \subset \mathbb{R}^+$, $\kappa(\lambda)$ is the curvature along $\mathbf{y}(\lambda)$, $\bar{k}_{dls}$ is a prescribed curvature constraint, $\text{unit}(\mathbf{y}'(\lambda))$ represents the unit tangent of $\mathbf{y}(\lambda)$ where $(\cdot)' = \frac{d(\cdot)}{d\lambda}$ is the tangent operator and $F \subset \mathbb{R}^3$ represents the constrained reservoir volume.

3.2.3 Convex Optimization

The material in the presentation of this section is taken from [Boyd and Vandenberghe, 2004]. The solution of the optimal well-plan stage from M_i to D_i is constructed to be in the form of a convex optimization problem. The advantage of forming the problem in this way is that for a convex optimization problem, if a solution exists, the local

3.2 Optimization Structure

minimum is the global minimum. In addition fast accurate methods such as interior point methods can be used to solve the problem. The form of the primal problem is:

$$\begin{aligned} & \text{Minimize } f_0(p) & (3.2) \\ & \text{subject to } f_i \leq 0 & i = 1, \dots, k \\ & h_j(p) = 0 & j = 1, \dots, l \end{aligned}$$

where $f_0(p)$ is the convex objective function to be minimized for a variable p , f_i are general convex inequality constraints, and $h_j(p)$ are equality constraints. Define the *Lagrangian* $L(p, \lambda, \nu)$ of the primal problem (3.2) to be

$$L(p, \lambda, \nu) = f_0(p) + \sum_{i=1}^k \lambda_i f_i(p) + \sum_{i=1}^l \nu_i h_i(p) \quad (3.3)$$

where $\lambda_i \geq 0$, and called the *dual variables*. Define the dual function to be

$$g(\lambda, \nu) = \inf_p L(p, \lambda, \nu). \quad (3.4)$$

The dual function $g(\lambda, \nu)$ has the property of being concave even if $L(p, \lambda, \nu)$ is not convex. Let the minimum value of the primal problem be $p = p^*$, and the maximum of the dual be $\lambda = \lambda^*$.

Theorem 1 (Slaters Condition). *If a primal problem (3.2) has a strictly feasible p and is convex with*

$$f_i \leq 0 \quad i = 0, \dots, k \quad (3.5)$$

$$h_j(p) = 0 \quad j = 1, \dots, l \quad (3.6)$$

then the minimum of the primal problem is equal to the maximum of the dual: $p^ = \lambda^*$*

Definition 3 (KKT Conditions).

$$f_i(p^*) \leq 0 \quad i = 1, \dots, k \quad (3.7)$$

$$h_j(p^*) = 0 \quad j = 1, \dots, l \quad (3.8)$$

$$\lambda_i^* \geq 0 \quad (3.9)$$

$$\lambda_i^* f_i(p^*) = 0 \quad (3.10)$$

$$\nabla f_0(p^*) + \sum_{i=1}^k \lambda_i^* \nabla f_i(p^*) + \sum_{i=1}^l \nu_i \nabla h_i(p^*) = 0 \quad (3.11)$$

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When Slater’s condition holds, the KKT conditions are necessary and sufficient conditions for optimality, and the optimization problem can be solved by interior point methods [Potra and Wright, 2000, Boyd and Vandenberghe, 2004].

Each stage is represented by a b-spline space-curve described in the next section. The objective function (section 3.4) will be constructed as a quadratic form to represent the well-plan tortuosity and length. The initial pose will be represented as an equality constraint. The curvature will be represented as pointwise quadratic inequality constraints along the well-plan, and the targets will be represented as a combination of convex inequality constraints. The representation of these constraints in terms of b-splines is described in section 3.5.

3.3 B-Spline Well-Plan Representation

This section introduces *b-spline* space curves [Wesselink, 1996]. The well-planning problem with its objectives and constraints will be developed with the representation of b-splines in a convex optimization problem.

3.3.1 Curve Representation

A piecewise continuous space-curve $\mathbf{y}(\lambda) \in \mathbb{R}^3$ with respect to a parameter $\lambda \in \mathbb{R}^+$ is represented as a uniform b-spline

$$\mathbf{y}(\lambda) = \sum_{i=0}^{m+d-1} \mathbf{P}_i N_i^d(\lambda), \quad (3.12)$$

as shown in Figure 3.2. The point-wise sequence of the *control points* $\mathbf{P}_i \in \mathbb{R}^3$ give

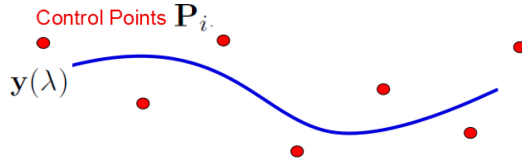


Figure 3.2: B-spline space curve

a polyhedron which is a convex hull whose volume encloses the resultant curve $\mathbf{y}(\lambda)$.

3.3 B-Spline Well-Plan Representation

The curve $\mathbf{y}(\lambda)$ does not interpolate the control points $\mathbf{P}_i$ but is an affine combination of its control points where the affine combination is determined by the choice of basis function $N_i^d(\lambda) \in \mathbb{R}^+$. For b-spline curves $N_i^d(\lambda) \in \mathbb{R}^+$ are polynomial basis functions of order d . The b-spline basis $N_i^d(\lambda)$ has a property of *local – support* that is to say that $N_i^d(\lambda)$ is non-zero for a fixed range of λ , where for a given order d , the curve $\mathbf{y}(\lambda)$ will be a combination of $d + 1$ adjacent control points. The b-spline basis is defined by the De-Boor recursion:

$$N_i^d(\lambda) = \frac{\lambda - i + 1}{d} N_i^{d-1}(\lambda) + \frac{i + d - \lambda}{d} N_{i+1}^{d-1}(\lambda) \quad (3.13)$$

$$N_i^0(\lambda) = \begin{cases} 1 & \text{if } i - 1 \leq \lambda < i \\ 0 & \text{otherwise} \end{cases} \quad (3.14)$$

The curve $\mathbf{y}(\lambda)$ contains $m \in \mathbb{Z}$ segments (regions between control points), and is defined in the domain $\lambda \in [d - 1, m + d - 1]$ where $d \in \mathbb{N}$ is the degree of the b-spline polynomial basis [Wesselink, 1996]. B-spline curves have the property of partition of unity, that is to say $\sum_{i=0}^{m+d-1} N_i^d(\lambda) = 1$.

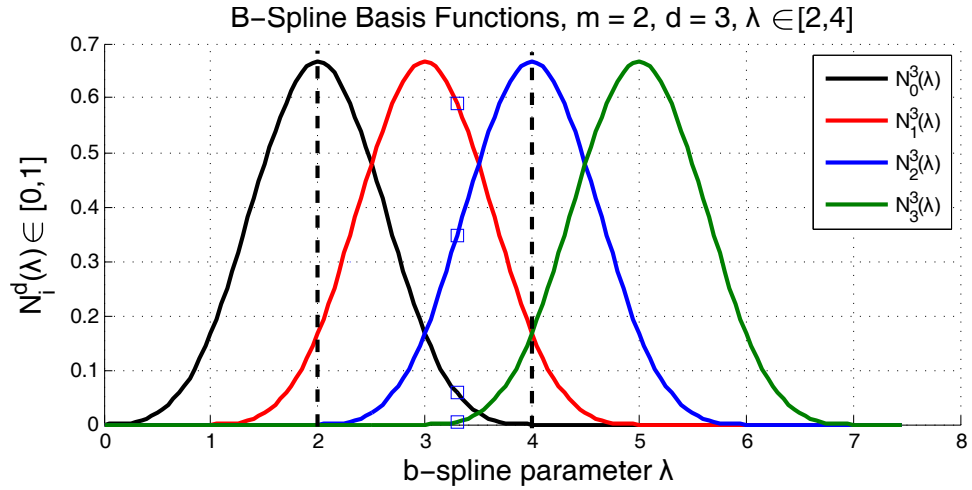


Figure 3.3: B-spline basis functions

The set of basis functions are shown in Figure 3.3 for a cubic b-spline of degree $d = 3$ for $m = 2$ segments for the range $\lambda \in [2, 4]$. For the case where $\lambda = 3.3$, the values for the corresponding bases are outlined by square markers, and it is the case that

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these sum to unity. The b-splines here are uniform with the spacing of each segment corresponding to an addition of λ by unity.

3.3.2 Tangent and Normal

A b-spline space curve is C^{d-1} continuous. Thus a cubic spline has continuous first and second derivatives. Furthermore the tangent and normal to a curve are also linear in their control points. The tangent to this curve $\mathbf{t}(\lambda) \in \mathbb{R}^3$ is given in terms of its control points by

$$\mathbf{t}(\lambda) = \sum_{i=0}^{m+d-1} \mathbf{P}_i \frac{dN_i^d(\lambda)}{d\lambda}. \quad (3.15)$$

Since the control points are fixed, only the derivative of the basis needs to be considered. The derivative is

$$\frac{dN_i^d(\lambda)}{d\lambda} = \frac{1}{d} \left(N_i^{d-1}(\lambda) - N_{i+1}^{d-1}(\lambda) \right), \quad (3.16)$$

where $\frac{dN_i^d(\lambda)}{d\lambda}$ is the basis for the tangent and is expressed in terms of the De-Boor relations equation (3.13). Similarly the normal to the curve $\mathbf{y}(\lambda)$ at λ can be found by replacing the first derivative term in (3.15) with a second derivative term and differentiating equation (3.16). For the remainder of this chapter, the well-plan will be represented as cubic b-splines, i.e. $d = 3$. This is the minimum order to have continuous second derivatives on the curve $\mathbf{y}(\lambda)$.

3.4 Minimum Energy Objective Function

The resultant well-plan must satisfy smoothness criteria. It is desired for the well-plan to minimize the bending and twisting by the borehole on tubulars. It is undesirable for well-plans to be unnecessarily large. The objective functions use approximations of stretch, strain, and torsion, for determining the control points [Wesselink, 1996]. The objective function is this multiobjective defined in terms of an affine combination of the arc length minimizer E_l , the bend energy E_c and the torsion E_t [Wesselink, 1996]. The proposed objective function is

$$E_{pipe} = \alpha_1 E_l + \alpha_2 E_c + \alpha_3 E_t, \quad (3.17)$$

3.4 Minimum Energy Objective Function

where

$$E_l = \int_{d-1}^{m+d-1} \left\| \frac{d\mathbf{y}(\lambda)}{d\lambda} \right\|_2^2 d\lambda \quad (3.18)$$

$$E_c = \int_{d-1}^{m+d-1} \left\| \frac{d^2\mathbf{y}(\lambda)}{d\lambda^2} \right\|_2^2 d\lambda \quad (3.19)$$

$$E_t = \int_{d-1}^{m+d-1} \left\| \frac{d^3\mathbf{y}(\lambda)}{d\lambda^3} \right\|_2^2 d\lambda. \quad (3.20)$$

The same energy representation without the arc-length minimizer is presented by Samuel and Liu [2009] for minimizing tortuosity.

Using the local support property, a curve $\mathbf{y}(\lambda)$ defined on the k^{th} segment $\lambda \in [2 + k, 3 + k]$ is written as

$$\mathbf{y}_k(\lambda) = \sum_{i=0}^3 \mathbf{P}_{i+k} N_{i+k}^3(\lambda). \quad (3.21)$$

Since the derivatives on $\mathbf{y}(\lambda)$ with respect to λ act only on the basis function $N(\lambda)$ and not the control points, the energy for the pipe E_{pipe} is written as a quadratic form in the control points P_{i+k} as (see Wesslink [1996] for details)

$$E_{pipe}^k = \sum_{i=0}^3 \sum_{j=0}^3 \mathbf{P}_i^T \mathbf{Q}_{ij}^k \mathbf{P}_j, \quad (3.22)$$

with

$$\mathbf{Q}^k = \alpha_1 \mathbf{L}^k + \alpha_2 \mathbf{C}^k + \alpha_3 \mathbf{T}^k. \quad (3.23)$$

The symmetric positive definite matrices $\mathbf{Q}^k$, $\mathbf{L}^k$, $\mathbf{C}^k$, $\mathbf{T}^k$ are

$$\mathbf{L}_{ij}^k = \int_{2+k}^{3+k} \frac{dN_{i+k}^3(\lambda)}{d\lambda} \frac{dN_{j+k}^3(\lambda)}{d\lambda} d\lambda \quad (3.24)$$

$$= \frac{1}{120} \begin{pmatrix} 6 & 7 & -12 & -1 \\ 7 & 34 & -29 & -12 \\ -12 & -29 & 34 & 7 \\ -1 & -12 & 7 & 6 \end{pmatrix}$$

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$$\mathbf{C}_{ij}^k = \int_{2+k}^{3+k} \frac{d^2 N_{i+k}^3(\lambda)}{d\lambda^2} \frac{d^2 N_{j+k}^3(\lambda)}{d\lambda^2} d\lambda \quad (3.25)$$

$$= \frac{1}{6} \begin{pmatrix} 2 & -3 & 0 & 1 \\ -3 & 6 & -3 & 0 \\ 0 & -3 & 6 & -3 \\ 1 & 0 & -3 & 2 \end{pmatrix}$$

$$\mathbf{T}_{ij}^k = \int_{2+k}^{3+k} \frac{d^3 N_{i+k}^3(\lambda^3)}{d\lambda^3} \frac{d^3 N_{j+k}^3(\lambda)}{d\lambda^3} d\lambda \quad (3.26)$$

$$= \begin{pmatrix} 1 & -3 & 3 & -1 \\ -3 & 9 & -9 & 3 \\ 3 & -9 & 9 & -3 \\ -1 & 3 & -3 & 1 \end{pmatrix}.$$

The total energy along the pipe E_{pipe} is the sum of E_{pipe}^k for all the segments k . The corresponding symmetric positive definite matrix $\mathbf{Q}$ can be constructed by representing the energy as [Wesselink, 1996]

$$E_{pipe} = \mathbf{p}^T \mathbf{Q} \mathbf{p}. \quad (3.27)$$

The $1 \times 3(m+d)$ vector $\mathbf{p}$ is the concatenation of the $3 \times (m+d)$ control points $\mathbf{P}_i$. The matrix $\mathbf{Q}$ is found from Algorithm 3.2.

Algorithm 3.2 Vectorizing $\mathbf{Q}$

- 1: Construct $\mathbf{Q}$ a $(m+d) \times (m+d)$ matrix with zero entries,
 - 2: add the matrix $\mathbf{Q}^k$ into $\mathbf{Q}$ such that $\mathbf{Q}_{1\dots 4, 1\dots 4} = \mathbf{Q}_{1\dots 4, 1\dots 4}^k$
 - 3: **for** $i = 2$ **to** $m+d-3$ **do**
 - 4: $\mathbf{Q}_{i\dots i+3, i\dots i+3} = \mathbf{Q}_{i\dots i+3, i\dots i+3} + \mathbf{Q}_{1\dots 4, 1\dots 4}^k$
 - 5: **end for**
 - 6: Replace each entry in $q_{i,j} \in \mathbf{Q}$ with $q_{i,j} I_{3 \times 3}$, where $I_{3 \times 3} \in \mathbb{R}^3$ is the identity matrix.
The result is a $3(m+d) \times 3(m+d)$ matrix $\mathbf{Q}$
-

3.5 Constraints

In this section the constraints for the well-planning optimization (3.2) are developed. The constraints consider the representation of the initial pose of a well-plan stage, tar-

gets (as described in Algorithm 3.1), steerability, lease-lines, and anti-collision.

Before proceeding, the expression for $\mathbf{y}(\lambda)$ needs to be modified so that the constraints are in terms of the concatenated control points $\mathbf{p}$.

Let $\hat{N} = \{N_i^d(\lambda_0)\}$ be the $1 \times (m + d)$ vector for the basis function N_i^d for a fixed λ_0 , where $i = 0 \dots m + d - 1$. Now replace each term $\hat{N}_i \rightarrow \hat{N}_i I_{3 \times 3}$ by the same term multiplied by the identity matrix. The result is a $3(m + d) \times 3$ matrix $\mathbf{N}(\lambda)$. Now the equation for the curve written in terms of the concatenated control points $\mathbf{p}$ is expressed as

$$\mathbf{y}(\lambda) = \mathbf{N}(\lambda)\mathbf{p}. \quad (3.28)$$

Similarly for the tangent and binormal

$$\mathbf{t}(\lambda) = \mathbf{N}'(\lambda)\mathbf{p}, \quad (3.29)$$

$$\mathbf{b}(\lambda) = \mathbf{N}''(\lambda)\mathbf{p}. \quad (3.30)$$

3.5.1 Position Constraint

Let $\mathbf{r} \in \mathbb{R}^3$ be a point in the reservoir which the resultant well-plan $\mathbf{y}(\lambda)$ must intersect for a specified parameter λ_0 along the well-plan as illustrated in Figure 3.4. Since λ_0 is specified, then the bases $N_i^d(\lambda)$ will have constant values and $\mathbf{r}$ can be written as a constant linear combination of the control points:

$$\mathbf{N}(\lambda_0)\mathbf{p} = \mathbf{r}, \quad (3.31)$$

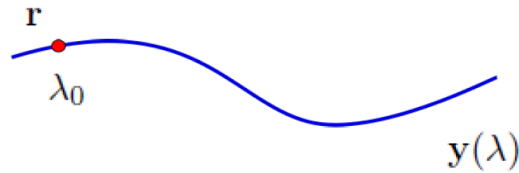


Figure 3.4: Position Constraint

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3.5.2 Direction Constraint

Let $\mathbf{s} \in \mathbb{R}^3$ be a tangent direction where the magnitude may be of interest at a specified parameter λ_0 along the resultant curve $\mathbf{y}(\lambda)$. The equation for the constraint is

$$\mathbf{N}'(\lambda_0)\mathbf{p} = \mathbf{s}. \quad (3.32)$$

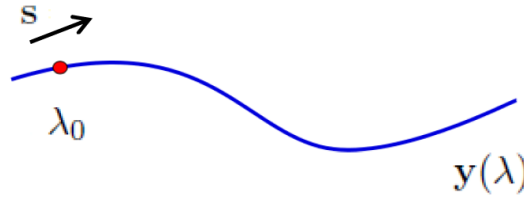


Figure 3.5: Direction Constraint

Specifying the magnitude of the direction may have undesirable effects as this imposes a spacing between the control points. A method to only consider the tangent direction, with the removal of the magnitude, is presented in Wesselink [1996] where the constraint is written in terms of the cross product, $\mathbf{s} \times \mathbf{y}'(\lambda_0) = 0$. Although for most cases this gives pleasing results, this method allows for the tangent to be in the opposite direction to $\mathbf{s}$. A solution to this problem is presented here by including a dummy scalar variable y in the objective function, with an additional constraint on $y > 0$ so that the modified constraint (3.32) is written as

$$\mathbf{N}'(\lambda_0)\mathbf{p} = \mathbf{s}y, \quad (3.33)$$

and the optimization problem containing this direction constraint appears with the following linear program

$$\begin{aligned} & \underset{\mathbf{p}}{\text{minimize}} && y \\ & \text{subject to} && \mathbf{N}'(\lambda_0)\mathbf{p} = \mathbf{s}y \\ & && y > 0. \end{aligned} \quad (3.34)$$

3.5.3 Target Constraints

In this section constraints on the target destination are considered [Wesselink, 1996]. When providing target constraints, an ellipsoid constraint represents volumes in the

reservoir desired for the well-plan to interpolate. By providing this target volume there is freedom for an optimization to determine the most optimum location in the volume to place the well. Point in plane constraints are useful for modeling geological faults. A generalized target is a combination of these constraints and these are discussed in this subsection.

Ellipsoid Constraint

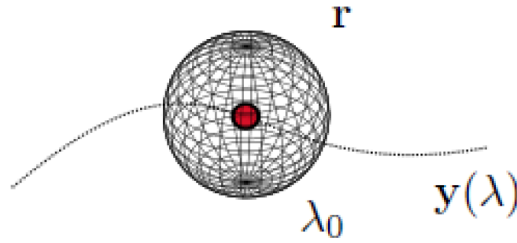


Figure 3.6: Ellipsoid Constraint

The general equation of an ellipsoid constraint [Wesselink, 1996] for a point on the well-plan $\mathbf{y}(\lambda_0)$ for a specified parameter λ_0 to be enclosed in an ellipsoid about a given position $\mathbf{r}$ is

$$(\mathbf{y}(\lambda_0) - \mathbf{r})^T \mathbf{R}^T \mathbf{\Pi}^{-1} \mathbf{R} (\mathbf{y}(\lambda_0) - \mathbf{r}) \leq 1, \quad (3.35)$$

where the rotation matrix $\mathbf{R}$ is an orthogonal matrix of eigenvectors of the principal axis of the ellipsoid, and the diagonal matrix $\mathbf{\Pi} = (a^2, b^2, c^2)$ contains the corresponding eigenvalues which represent the semi-major and semi-minor axis lengths. By substituting $\mathbf{y}(\lambda) = \mathbf{N}(\lambda)\mathbf{p}$ into (3.35), this constraint can be written as a convex quadratic inequality constraint

$$\mathbf{p}^T \mathbf{A} \mathbf{p} + \mathbf{B}^T \mathbf{p} + c \leq d^2. \quad (3.36)$$

In the special case of a sphere, $\mathbf{A} = \mathbf{I}_{3 \times 3}$, $\mathbf{B} = -2\mathbf{N}(\lambda + 0)\mathbf{p}$, $c = \mathbf{r}^T \mathbf{r}$ and d is the radius of the sphere.

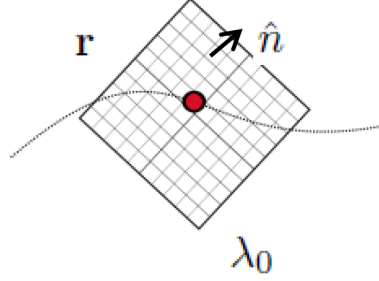


Figure 3.7: Plane Constraint

Plane Constraint

To constrain a well-plan point to lie in the plane described by its normal $\hat{n}$, and a fixed point $\mathbf{r}$ in the plane, a constraint of the form

$$\hat{n} \cdot \mathbf{N}(\lambda_0)\mathbf{p} = \hat{n} \cdot \mathbf{r} \quad (3.37)$$

is used [Wesselink, 1996].

Target Constraint

A *target* constraint is defined to be a combination of a point in an ellipsoid, a point in a plane and a tangent constraint. The combination of a point in plane and a point in ellipsoid give the intersection of an ellipsoid and a plane which is an ellipse. The addition of the tangent constraint for a specified λ_0 has the interpretation of requiring the end of a well-path subsection to hit a disk at a given attitude. This attitude constraint is useful when planning a well to drill through a known fault. Other forms of target constraints can be constructed in the framework presented by Wesselink [1996], provided they form convex sets.

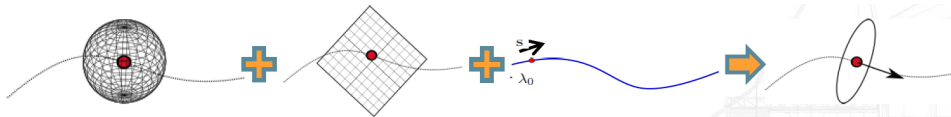


Figure 3.8: Target Constraint

3.5.4 Anti-collision and Lease-lines

Anti-collision and lease-lines are considered in this section. The convex hull property of b-splines states that the control points $\{\mathbf{P}_i\}$ define a polyhedron by which the resultant curve $\mathbf{y}(\lambda)$ is enclosed. The constraints for lease-lines and anti-collision are in the form of convex volumes enclosing the control points. These constraints are imposed over large sections of the well-plan. For Lease lines, these constraints are imposed over the entire well-plan. The constraints do not involve the basis functions and are imposed only on the control points.

Lease-Lines

Lease-lines are contractual boundaries of the area of land of the owner of the exploration rights. Lease-lines can be represented simply as inequality constraints on the control points $\mathbf{P}_i$. The inequality constraints are in the form of planes, and this is depicted in Figure 3.9.

Given an arbitrary known position $\mathbf{a} \in \mathbb{R}^3$ on a lease plane with normal vector $\hat{\mathbf{n}}$, the inequality constraint for each control point $\mathbf{P}_i$ is given by

$$\mathbf{P}_i \cdot \hat{\mathbf{n}} \geq \mathbf{a} \cdot \hat{\mathbf{n}}. \quad (3.38)$$

Lease line constraints are expressed for multiple boundaries provided that the enclosed volume is convex.

Anti-collision

It is undesirable for a borehole to intersect existing or other planned wells. These are generally known during the drilling process. Wells are planned to avoid this catastrophe. Inequality constraints on the control points to restrict the solution space is one method to ensure a resultant curve does not intersect an existing well. When solving for a well-plan between an initial pose and a destination, volume constraints in the form of multiple plane inequality constraints restrict the solution space of the resultant curve.

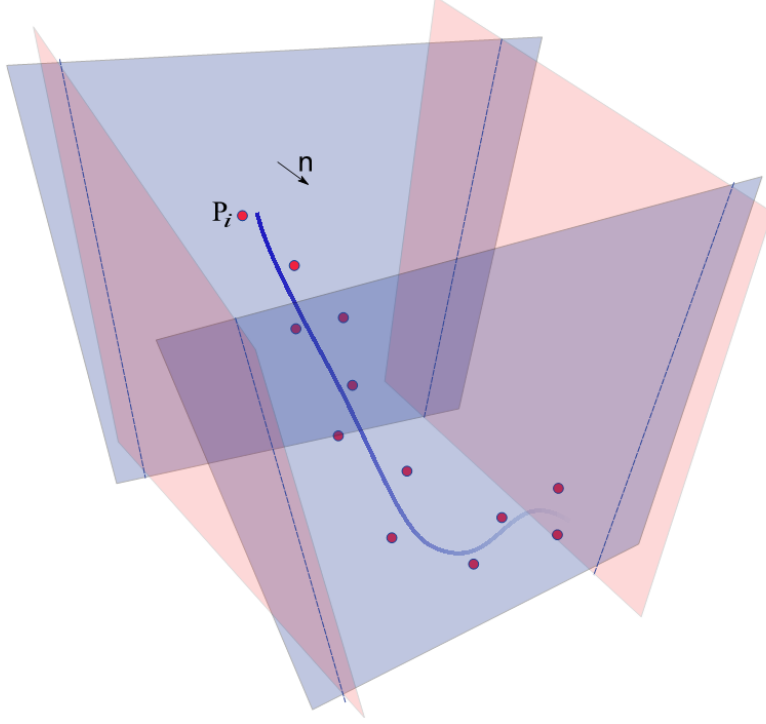


Figure 3.9: Lease line Constraint

3.5.5 Steerability Constraint

In this section the representation of the curvature capability as a constraint in the optimization for the well-planning is considered. The expression for the curvature $\kappa(\lambda)$ of a curve $\mathbf{y}(\lambda)$ is given by

$$\kappa(\lambda) = \frac{\|\mathbf{y}'(\lambda) \times \mathbf{y}''(\lambda)\|_2}{\|\mathbf{y}'(\lambda)\|_2^3}. \quad (3.39)$$

The curvature κ is approximated by $\|\frac{d^2\mathbf{y}(\lambda)}{d\lambda^2}\|_2$. For a unit speed $\|\mathbf{y}'(\lambda)\|_2 = 1$ arclength parametrized curve $\mathbf{y}(\lambda)$, the curvature is always equal to this value. For b-spline curves the parametrization will depend on the number of segments m , and the location of the control points¹. The relaxation of $\bar{\kappa} = \|\frac{d^2\mathbf{y}(\lambda)}{d\lambda^2}\|_2$ [Wesselink, 1996] will be useful here since the constraint can be expressed as a convex inequality constraint and can be

¹Spline curves of arc-length parameterization are considered in [Farouki, 2008] for Pythagorean Hodograph curves.

chosen to vary along the well-plan. This constraint will be expressed point-wise along a well-plan equally spaced along the parameter λ . The curvature constraint is given by

$$\left\| \frac{d^2 \mathbf{y}}{d\lambda^2}(\lambda) \right\|_2^2 \leq \bar{\kappa}^2, \quad (3.40)$$

where $\bar{\kappa}$ is a scalar upper bound which constrains the curvature measure. Written in terms of b-splines, the inequality constraint is expressed as

$$\mathbf{p}^T \mathbf{N}''(\lambda)^T \mathbf{N}''(\lambda) \mathbf{p} \leq \bar{\kappa}^2. \quad (3.41)$$

Since the expression (3.41) is a squared 2-norm, it is positive definite, and hence this forms a convex quadratic constraint. Equation (3.41) is a relaxation on the curvature capability of the drilling tool. In Section 3.6.2, an outer-loop is detailed in order to constrain the absolute curvature.

3.6 Well-Planning

This section states the non-linear well-planning program. Subsection 3.6.1 states the well-planning problem developed in this chapter as a quadratically constrained quadratic program (QCQP). The QCQP involves a relaxation on the curvature constraint. Section 3.6.2 describes an outerloop procedure on the parameter $\bar{\kappa}$ to give an absolute curvature constraint which is practical for drilling applications.

3.6.1 Convex Well-planning Program

The well-planning optimization for each stage is stated in (3.42). The resulting control points $\mathbf{p}$ define an optimal cubic b-spline which minimizes tortuosity, subject to initial

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pose, target, and relaxed curvature constraints. The optimization problem is:

$$\begin{aligned}
& \underset{\mathbf{p}}{\text{minimize}} && \beta_1 \underbrace{\mathbf{p}^T \mathbf{Q} \mathbf{p}}_{\text{tortuosity}} + \beta_2 (y_1 + y_2) \\
& \text{subject to} && \mathbf{N}(d-1)\mathbf{p} = \mathbf{p}_0 && (\text{initial position}) \\
& && \mathbf{N}'(d-1)\mathbf{p} = \mathbf{t}_0 y_1 && (\text{initial attitude}) \\
& && \mathbf{N}'(m)\mathbf{p} = \mathbf{t}_d y_2 && (\text{target attitude}) \\
& && \hat{\mathbf{n}} \cdot \mathbf{N}(m)\mathbf{p} = \hat{\mathbf{n}} \cdot \mathbf{r} && (\text{target plane}) \\
& && \mathbf{p}^T \mathbf{A} \mathbf{p} + \mathbf{B}^T \mathbf{p} + c \leq d^2 && (\text{target ellipsoid}) \\
& && \mathbf{p}^T \mathbf{N}'' \left(\frac{i}{d} \right)^T \mathbf{N}'' \left(\frac{i}{d} \right) \mathbf{p} \leq \bar{\kappa} && (\text{curvature}) \\
& && \mathbf{P}_j \cdot \hat{\mathbf{n}}_k \geq \mathbf{a}_k \cdot \hat{\mathbf{n}}_k && (\text{lease-lines}) \\
& && i = 2d, \dots, md \\
& && j = 1, \dots, m+d \\
& && k = 1, \dots, k_p
\end{aligned} \tag{3.42}$$

Since $\mathbf{Q}_i$ are real positive definite symmetric matrices, the cost is convex quadratic with linear equality constraints and convex quadratic inequality constraints. The structure of this problem is a quadratically constrained quadratic programme, where the inequality constraints are convex. If feasible, a unique solution can be found from interior point methods [Boyd and Vandenberghe, 2004], from tools such as CVX and SeDuMi. Here the weights $\beta_1 + \beta_2 = 1$. The purpose of the weighting coefficient β_2 serves to ensure the tangent constraints are met, and the spacing of the control points are well-behaved by not being unnecessarily large. The value of β_2 is recommended to be small such that the main emphasis of the optimization is in minimizing the energy term.

3.6.2 Procedure for Absolute Curvature Constrained Well-plans

In section 3.5.5 the curvature constraint used is a relaxation. An outer-loop algorithm to (3.42) is described here to constraint the absolute curvature $\bar{k}_{dls}$ for a well-plan. The well-planning (3.42) is iteratively solved modifying $\bar{\kappa}$ until the maximum curvature $\kappa_{max} = \max_{\lambda} \kappa(\lambda)$ along the curve $\mathbf{y}$ satisfies the absolute curvature constraint $\bar{k}_{dls}$. This is achieved by solving the unconstrained problem to find a value for κ_{max} . If $\kappa_{max} < \bar{k}_{dls}$ then the well-plan stage $\mathbf{y}(\lambda)$ is feasible. If $\kappa_{max} > \bar{k}_{dls}$, then a bisection

search on $\bar{\kappa}$ is performed until the constraint $\kappa_{max} < \bar{\kappa}_{dls}$ is met. This bisection search is summarized in Algorithm 3.3.

Algorithm 3.3 Absolute Curvature Constraint

```

1: Define curvature of tool  $\bar{\kappa}_{dls}$ 
2: Set tolerance  $\gamma$ 
3: Solve Optimization (3.42) without curvature constraint
4: Determine Well-Plan  $\mathbf{y}$ 
5: Determine maximum curvature  $\kappa_{max} = \max_{\lambda} \kappa(\lambda)$ 
6: if  $\kappa_{max} \leq \bar{\kappa}_{dls}$  then
7:   return  $\mathbf{y}$ 
8: else
9:   Set  $\bar{\kappa}$  numerically large and  $\kappa_{top} = \bar{\kappa}$ 
10:  Set  $\bar{\kappa}_{bottom} = 0$ 
11:  Solve Optimization (3.42)
12:  Determine Well-Plan  $\mathbf{y}$ 
13:   $\kappa_{max} = \max_{\lambda} \kappa(\lambda)$ 
14:  if  $\|\kappa_{max} - \bar{\kappa}_{dls}\|_2 \leq \gamma$  then
15:    return  $\mathbf{y}$ 
16:  else
17:    if  $\kappa_{max} < \bar{\kappa}_{dls}$  Or Infeasible then
18:       $\bar{\kappa} = \frac{1}{2}(\bar{\kappa}_{top} + \bar{\kappa})$ 
19:       $\kappa_{bottom} = \bar{\kappa}$  goto 11
20:    else
21:       $\bar{\kappa} = \frac{1}{2}(\bar{\kappa}_{bottom} + \bar{\kappa})$ 
22:       $\kappa_{top} = \bar{\kappa}$  goto 11
23:    end if
24:  end if
25: end if

```

3.7 Simulation Results

This section demonstrates by simulation the well-planning strategy developed in this chapter. The simulation implements Algorithm 3.1 for generating a well-plan to interpolate two targets. The coordinate system defines the z -axis negative down-hole, and

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the x -axis positive due north. These can be mapped to other conventions by simple coordinate transformations. The curvature is measured in degrees per 100 feet, and the distances are measured in meters. Table 3.1 shows the destination target sequence D_i and defines the initial pose as M_0 .

Horizontal wells begin with a vertical section drilled to a target depth. Often multiple horizontal and lateral wells are drilled from this vertical section. The point of deviation of a horizontal well from a vertical well is known as a *Kick-Off-Point* (KOP). From a KOP a well is deviated to a *landing-point* at the beginning of a reservoir. From the landing point, the remainder of the well-plan is designed to maximize contact with hydrocarbons.

Table 3.1: Destinations

	Position	Attitude	Azimuth	Inclination
M_0	(0, 0, 0)	(0, 0, -1)	0	0
D_1	(500, 500, -600)	(1, 1, 0.2)	45	111.3
D_2	(700, 700, -600)	(0.97, 0.23, 0)	-13.5	90

The target destination D_1 represents a landing point with a prescribed attitude chosen to have an extreme inclination of 111.3° . This is to investigate the ability to plan a well to a target with a large angular difference from the KOP.

The simulation of the well section from M_0 to D_1 is shown in Figure 3.10 and Figure 3.11. These wells demonstrate: the minimum tortuosity objective and the absolute curvature constraint. The minimum tortuosity objective has the following parameters for cases of minimizing arclength and strain energy:

- minimize arclength $a_1 = 0.9, a_2 = 0.1$ (Figure 3.10);
- minimize strain energy $a_1 = 0.1, a_2 = 0.9$ (Figure 3.11).

For both these cases multiple constraints of absolute curvature of $\bar{k}_{dls} = 4, 6, 8^\circ/100ft$ are simulated.

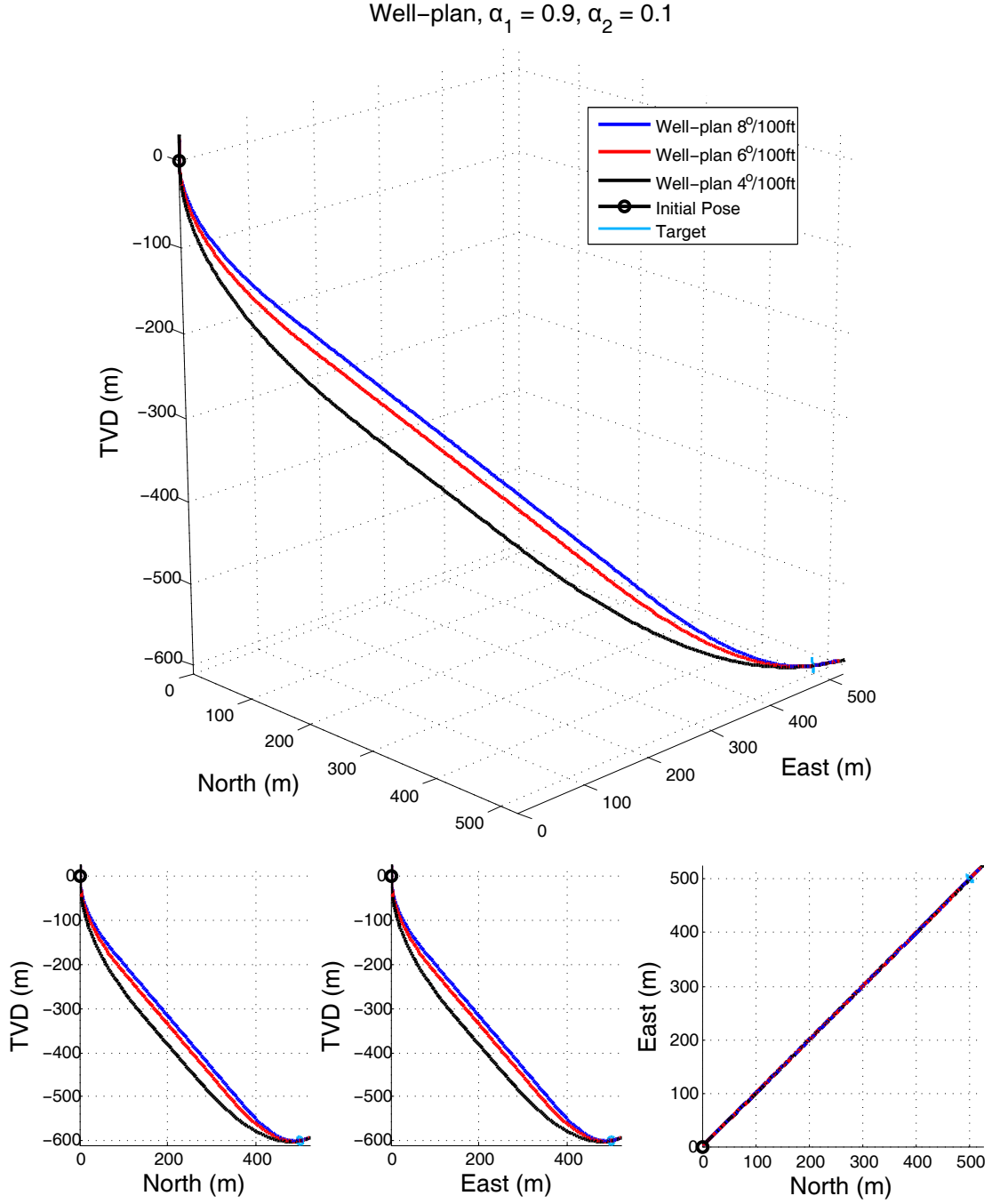


Figure 3.10: Well-plan for various dogleg constraints with a minimal arclength objective

Figure 3.12 shows the absolute curvature along the well-plan. The curvatures are higher at the start and end of the curves, and are continuous. The well-plans for minimizing

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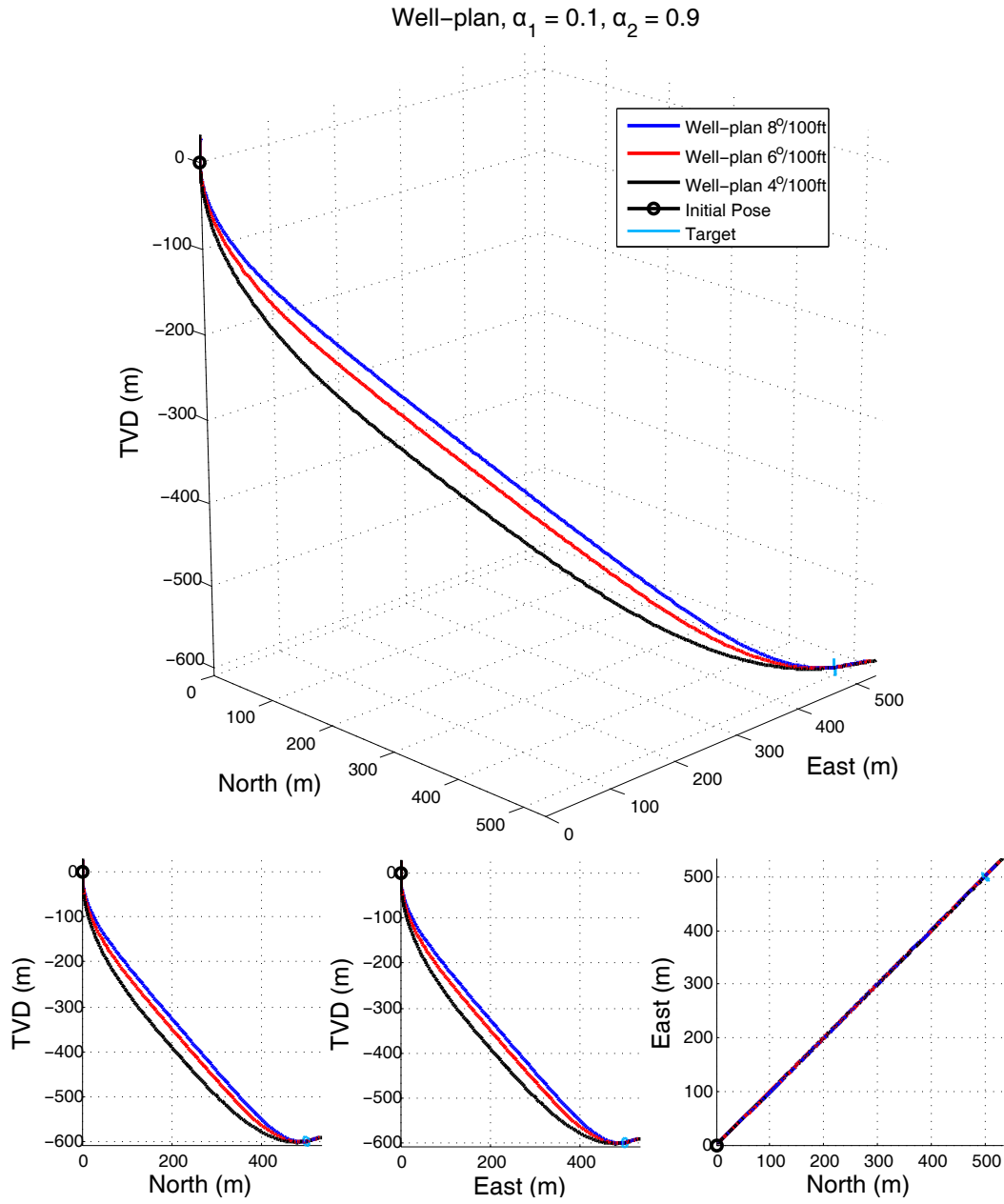


Figure 3.11: Well-plan for minimum strain energy objective for different curvature constraints

arc-length have straight sections and large changes in curvature. The well-plans for minimum strain energy have smooth variations in curvature and no straight sections of zero curvature. Designing well-plans with a lower curvature constraint will increase

the arclength of well-plan stage to its corresponding target.

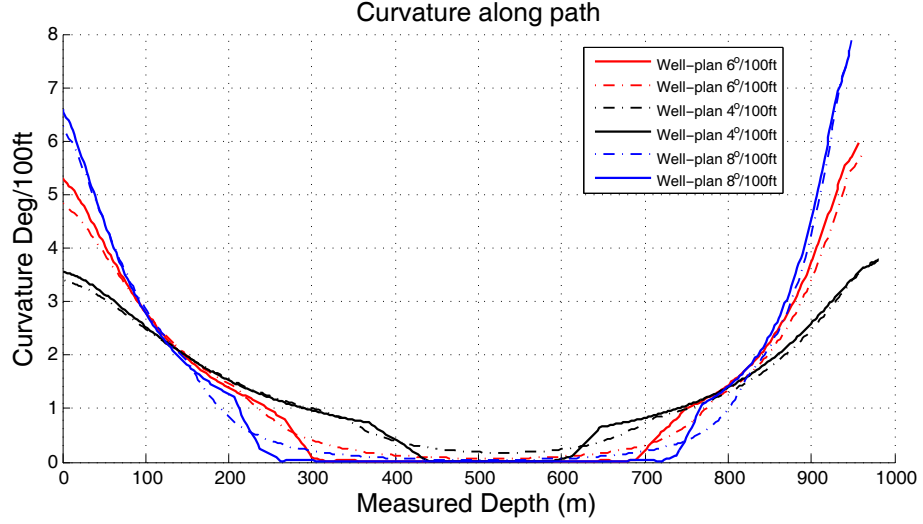


Figure 3.12: Dotted lines show minimum strain energy simulated curves

The pose of the well-plan at the end of the first stage coincident with M_1 is defined to be the initial pose M_2 for the second stage. The well-plan for the second stage is then solved to the target D_2 . Figure 3.13 displays the resultant well-plans with an absolute curvature constraint of $8^\circ/100ft$.

The target D_2 is a generalized target constraint consisting of a sphere constraint, a plane constraint and an attitude constraint. This sphere is centered about position D_2 in Table 3.1 with radii: 10, 20, 30 meters. The target plane is normal to the attitude of D_2 in Table 3.1.

Figure 3.14 shows the curvature along the well-plans for the second stage. Imposing a larger diameter target for the well-plan to hit, gives a shorter well-plan. There is a smooth continuous variation in the absolute curvature.

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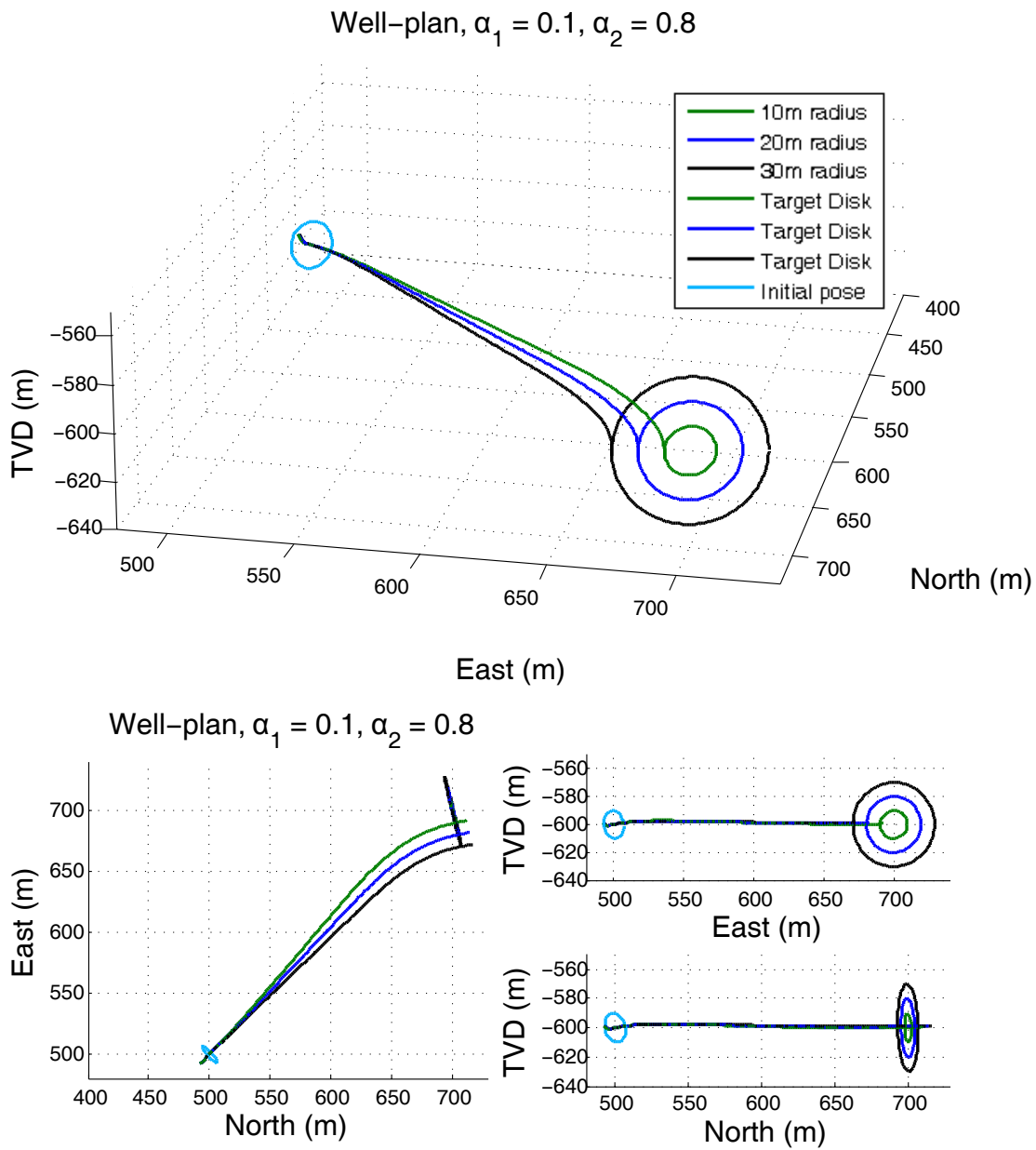


Figure 3.13: Second stage of well-plan for hitting a target with varying area to strike

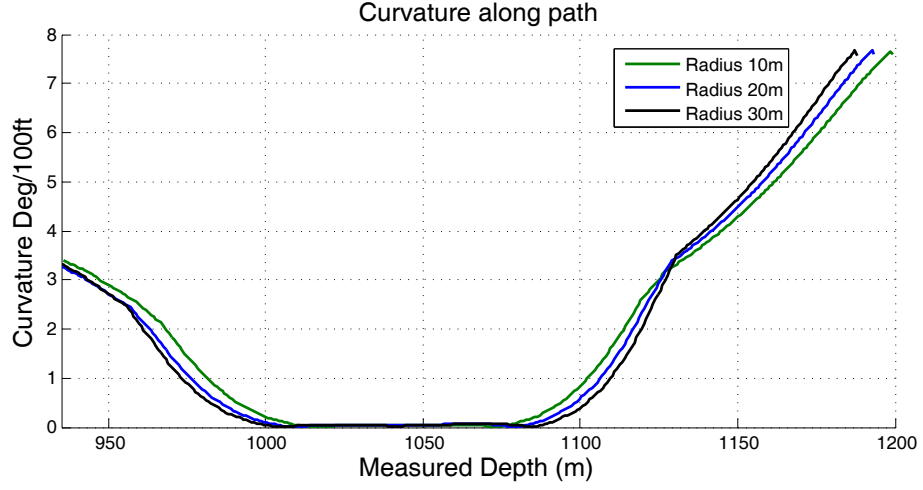


Figure 3.14: Curvature along the well-plan

3.8 Conclusion

A strategy for optimal well-planning is developed in this chapter for well-plans represented as b-splines. This strategy allows for rapid generation of continuous well-plans minimizing tortuosity and arc length. The tortuosity represents strain energy and torsion for downhole tubulars. The well-plans provide flexibility in their generation to modify for tortuosity objectives, target constraints and curvature constraints at each stage in the well-plan. A method for handling attitude constraints which is independent of magnitude and gives a unique attitude solution is shown. The well-plan handles constraints on absolute curvature commensurate with the maximum curvature capability of a steering assembly. The wells are planned to sequential targets which are treated as convex inequality constraints. An embodiment of this is given consisting of a combination of a point in ellipsoid, point in plane and prescribed direction. The well-planning optimization is demonstrated in simulation for interpolating two targets. Differences in the curvature profiles with minimum arclength and minimum tortuosity objective functions are demonstrated. The effects of varying the curvature constraints are demonstrated, in addition to examples of modifying the target constraints.

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Trajectory Control for RSS Tools

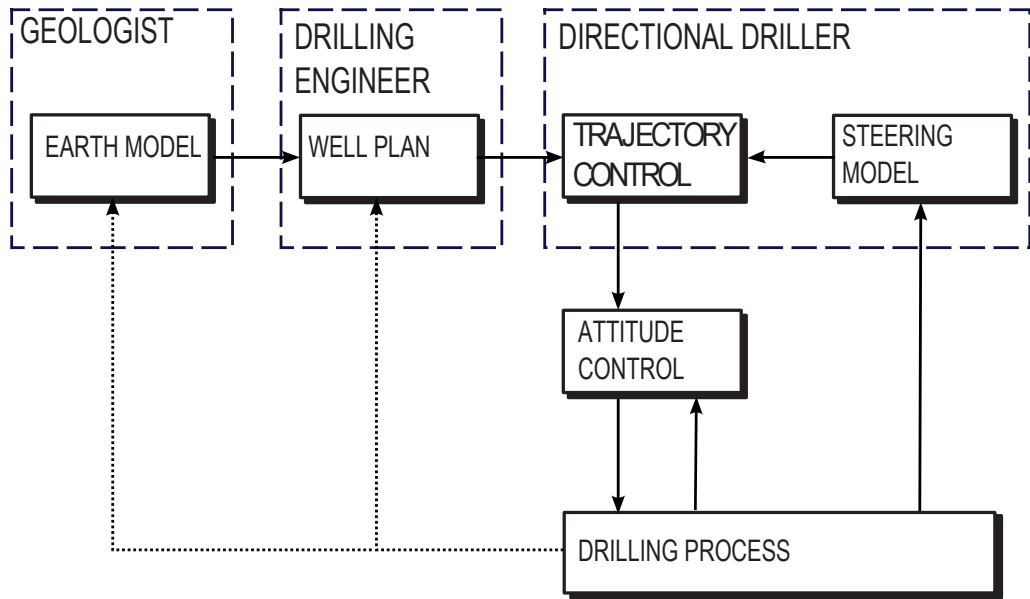


Figure 4.1: Trajectory Control Work-flow

This chapter presents a closed loop model predictive control system for generating optimal attitude set-points for executing a borehole trajectory along a predefined well-plan. A multi-objective finite horizon optimization strategy is developed which simultaneously minimizes tortuosity and well-plan deviation. The proposed strategy is implemented in a commercial drilling simulator *ST2* in conjunction with the attitude control system presented in Chapter 2 demonstrating well-plan tracking for the proposed integrated system.

4.1 Introduction

The emphasis of a trajectory control system for a DD is the dynamic re-planning of the well in the real-time drilling process, to react to unexpected events. For example when drilling from a hard formation into a soft formation, a push-the-bit steering assembly will lose steering capability and the borehole progresses off plan. There is a motivation for replanning a path to steer the borehole towards the well-plan where a supervisor to this system is able to input the desired curvature and factorize the loss of steering into the replanned path. The trajectory control strategy in this chapter dynamically generates an optimal trajectory coincident with the real-time measured pose of a drilling tool, in order to guide the drilling tool along the well-plan. This optimal trajectory is represented using b-splines similar to the optimization proposed in Chapter 3. The optimization used for the real-time trajectory control process differs from that of Chapter 3, by the removal of the target constraint, and the inclusion of a *well-plan-deviation* objective, for minimizing the deviation of the borehole trajectory from the well-plan.

The generated optimal path is a piecewise continuous curve and, for a cubic b-spline, the first and second derivatives exist, and hence the tangent and normal vectors at a given location along the path can be extracted. From the tangent and normal, expressions such as the tool-face, curvature and attitude can be calculated. This allows the control system in this chapter to be compatible with other embodiments of downhole control where the set-points can be generated in this way. Furthermore the trajectory control system in this chapter can be used in the absence of downhole attitude control for providing steering recommendations for a directional driller.

The accuracy of the placement of the well-bore trajectory along a well-plan has economic importance for maximizing hydrocarbons recovery. The requirement of minimizing well-plan deviation is vital for cost reduction. Water, oil and gas accumulate in impermeable *traps*, and because of the differences in density, gas accumulates above oil, and oil above water. The vertical displacement between the oil-water and oil-gas interfaces can be in the order of feet. Accurate drilling is especially important here for wells planned to steer entirely in oil saturated rocks, where large deviations from a well-plan will require the drilling of sidetrack wells to correct for these mistakes. This

process involves tripping the drillstring out of the borehole, sealing the unwanted section, and deviating a well to continue the original intended plan. The quality of the well-bore in terms of its smoothness affects the depth in which liners can be passed down-hole to case the well. Moreover equipment run into the hole such as down-hole pumps and logging tool experience bending and friction from the borehole.

The maturity of rotary steerable tools has enabled trajectory control allowing high bandwidth feedback control systems to be placed on the steering assembly down-hole. Improvements in telemetry such as the introduction of wired drill-pipe provide high bandwidth telemetry at a much higher update rate for communication to the surface, than is possible with mud-pulse telemetry. In the literature a trajectory control system for rotary steerable systems is presented by Pirovolou et al. [2011], where a correction-path is planned periodically to meet a moving target on the well-plan. The geometrical nature of this correction path is determined from the minimum curvature method. The proposed system relies on the MWD sensor set and includes a method to predict the location and attitude of the bit. The control system generates toolface angle and curvature commands to communicate to the BHA which between measurement samples runs open loop. In another example, a system for trajectory control by communicating steering commands from the surface to a down-hole control system is presented by Ignova et al. [2010]. In this patent the system describes a cascaded control system where multiple loops are nested for the steering process whereby the inner-loops are on board a rotary steerable system for attitude control. A multi-loop trajectory control system is proposed by Matheus et al. [2012], where a *hold-the-line* trajectory control system is detailed. This entails an outer-loop for an attitude control system for the purposes of steering an RSS for holding the vertical depth of a well for horizontal wells.

4.1.1 Problem Description

The aim of the trajectory control system is to generate attitude control set-points in the real-time drilling process. The requirements of the generated borehole are:

1. *accuracy* in minimizing deviation of a borehole with respect to a well-plan,
2. *quality* of the well in terms of minimizing tortuosity and

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3. *feasibility* of generating an achievable borehole for the steering assembly employed.

The *quality* requirement is to drill a smooth borehole by minimizing strain energy and torsion as described in section 3.4. Hence the trajectory control problem is a multiobjective problem where there is a compromise in achieving both objectives simultaneously. In the case of tracking a curved well-plan, the quality aim of minimizing curvature would necessitate a straight hole, whereas the accuracy aim would necessitate matching the curved profile of the well-plan. The ability to adjust the emphasis biased towards one of these objectives would be advantageous to a supervisor of the system.

4.1.2 Model Predictive Control

To solve the trajectory tracking problem, a Model Predictive Controller is designed. The concept of Model Predictive Control (MPC) is shown in Figure 4.2 [Maciejowski, 2002]. An MPC controller involves solving an optimization problem to minimize an objective function over a finite future horizon $k + H$, given measurements at time k in order to determine the optimal future output and control actions over that horizon. The optimization considers a model of the system dynamics along with state and input constraints. The optimum open-loop control inputs should, in the absence of disturbances, drive the system along the predicted output $\hat{\mathbf{y}}$. In an MPC controller the initial input is applied for the duration until the next measurement sample is received. The horizon is then shifted and the process is iterated generating a closed loop control system. In the iterated process, the solution for the finite horizon optimization in the previous sample may be used as an initial guess in order to reduce the computation time.

4.1.3 Chapter Organization

This chapter interfaces the attitude control system presented in Chapter 2 with the well-planing methodology from Chapter 3. This chapter is organized into two main sections: Section 4.2 develops the finite horizon optimization problem, allowing Section 4.3 to embed this optimization into a model predictive control scheme. The purpose of the optimization in Section 4.2 is to generate a smooth continuous correction path which is coincident with a measured position and attitude of the BHA, and plans a

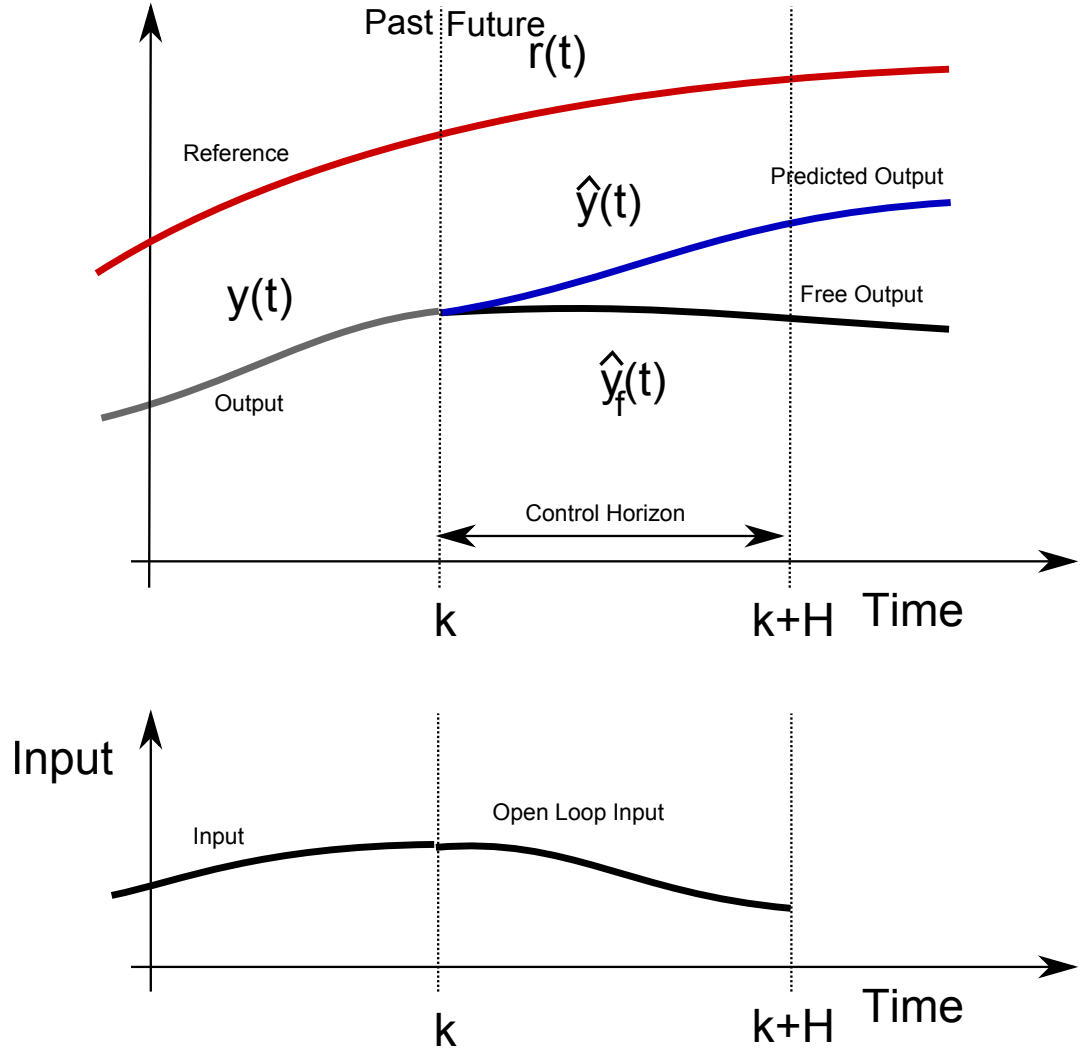


Figure 4.2: Model Predictive Control Paradigm [Maciejowski, 2002]

path to guide the BHA along the reference well-plan. From this correction path, the tangent provides the set-point for the attitude control system in Chapter 2. Section 4.3 describes the structure of the model predictive control method which is investigated in a commercial drilling simulator demonstrating the thesis' integrated system. Finally conclusions are made in Section 4.4.

4.2 Finite Horizon Optimization

This section develops the finite horizon optimization problem for the subsequent model predictive control scheme. This section begins with stating the optimal control problem with respect to the dynamics of the drilling tool developed in Chapter 2 in the time domain. As this section develops, the optimal control problem is reformulated as a convex optimization problem with respect to b-splines, where the time parametrization is replaced with the b-spline path parameter λ much in the same way the well-plan was parameterized in Chapter 3.

4.2.1 Optimal Control Problem

The optimal control problem over a finite time horizon h beginning at time t_0 to determine an optimum sequence of control inputs $\omega(t)$ with respect to a well-plan $\mathbf{r}(t)$ is stated as

$$\begin{aligned}
 & \underset{\omega(t)}{\text{minimize}} && J(\mathbf{x}, \mathbf{y}, \omega, \mathbf{r}, t_0, h) \\
 & \text{subject to} && \dot{\mathbf{y}} = ROP \mathbf{x} && \text{(propagation)} \\
 & && \dot{\mathbf{x}} = \omega \times \mathbf{x}, && \text{(attitude)} \\
 & && \|\omega\|_2 \leq ROP \bar{k}_{dls} && \text{(input constraint)} \\
 & && \mathbf{x} \in B, \mathbf{y} \in F && \text{(state constraints)} \\
 & && [\mathbf{y}(t_0), \mathbf{x}(t_0)] = [\mathbf{y}_0, \mathbf{x}_0], && \text{(initial conditions).}
 \end{aligned} \tag{4.1}$$

The objective function $J(\mathbf{x}, \mathbf{y}, \omega, \mathbf{r}, t_0, h)$ is a multi-objective combination of tortuosity E_{pipe} (see Section 3.4) and well-plan deviation E_{ca} (see Section 4.2.4). The position of the drilling tool at time t is represented by $\mathbf{y}(t) \in F \subset \mathbb{R}^3$, where F represents the oilfield subset, and the attitude $\mathbf{x}(t) \in B$ is represented as a unit vector such that

$$\dot{\mathbf{y}}(t) = ROP \mathbf{x}(t), \tag{4.2}$$

where ROP is the rate of penetration of the drilling tool and $B := \{\mathbf{x} \in \mathbb{R}^3 : \|\mathbf{x}\| = 1\}$. The initial position and attitude at time t_0 are represented as $[\mathbf{y}_0, \mathbf{x}_0]$. The kinematic response of the attitude $\mathbf{x}(t)$ is modeled as

$$\dot{\mathbf{x}} = \omega \times \mathbf{x}, \tag{4.3}$$

where ω represents an axis about which the attitude $\mathbf{x}$ rotates. The magnitude $\|\omega\|_2 = ROP k$ represents the build-rate of the drilling tool, where the curvature $0 \leq k \leq \bar{k}_{dls}$ is positive and bounded above by the maximum curvature $\bar{k}_{dls}$ the tool is capable of achieving.

4.2.2 Sketch of Well-plan Optimization

The structure of the optimal control problem (4.1), is difficult to solve due to the non-linear nature of the attitude dynamics and the hard constraint on the attitude $\mathbf{x} \in B$ being a unit vector. The finite horizon optimization will be reformulated as a convex optimization problem similar to the analysis in Chapter 3. The optimization solves for the predicted well-plan $\hat{\mathbf{y}}$ from an initial measured position $\mathbf{y}(t)$, and attitude $\mathbf{x}(t)$ with respect to a well-plan $\mathbf{r}$ from

$$\begin{aligned}
 & \underset{\hat{\mathbf{y}}}{\text{minimize}} && (1 - \beta_{att}) \underbrace{E_{pipe}}_{\text{tortuosity}} + \beta_{att} \underbrace{E_{ca}}_{\text{deviation}} \\
 & \text{subject to} && \hat{\mathbf{y}}(\lambda_0) = \mathbf{y}(t) && \text{(measured position)} \\
 & && \hat{\mathbf{y}}'(\lambda_0) = s\mathbf{x}(t) && \text{(measured attitude)} \\
 & && \forall \lambda, \kappa(\lambda) \leq \bar{k}_{dls} && \text{(curvature)} \\
 & && \forall \lambda, \hat{\mathbf{y}}(\lambda) \in F, && \text{(reservoir domain)} \\
 & && s \in \mathbb{R}^+, &&
 \end{aligned} \tag{4.4}$$

where the correction path $\hat{\mathbf{y}}(\lambda) = \mathbf{N}(\lambda)\mathbf{p}$ is a b-spline parametrized by $\lambda \in [\lambda_0, \lambda_f] \subset \mathbb{R}^+$, $\kappa(\lambda)$ is the curvature along $\hat{\mathbf{y}}(\lambda)$, $\bar{k}_{dls}$ is a prescribed positive real curvature constraint, $\hat{\mathbf{y}}'$ represents the tangent of $\hat{\mathbf{y}}(\lambda)$ where $(\cdot)' = \frac{d(\cdot)}{d\lambda}$ is the tangent operator and $F \subset \mathbb{R}^3$ represents the constrained reservoir volume. The coefficient $0 \leq \beta_{att} \leq 1$ represents the trade-off weight between the tortuosity and deviation from well-plan.

The optimization problem 4.4 will be formed in terms of b-spline control points $\mathbf{p}$ (stated in (4.18)), from which the output state trajectory $\hat{\mathbf{y}} = \mathbf{N}(\lambda)\mathbf{p}$ can be obtained. The control system is in the form of a reference-governor, where attitude set-points are determined from the tangent of $\hat{\mathbf{y}}$ for a stable inner-loop attitude control system such as those described in Chapter 2. In this formulation the plant dynamics (4.3) will

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be replaced by an inequality constraint in terms of the curvature $\kappa(\lambda)$ of $\hat{\mathbf{y}}$. This is explained in the next section.

4.2.3 Kinematic BHA Model

In this section, the representation of the modeling of the steering behaviour of the drilling tool for the optimization problem is explained. In Chapter 2 various attitude control systems were proposed of which the most general was the variable build rate controller where the closed loop dynamics are

$$\dot{\mathbf{x}} = k[\mathbf{x}_d - (\mathbf{x}_d \cdot \mathbf{x})\mathbf{x}]. \quad (4.5)$$

It is assumed in this chapter that the closed loop attitude control system is of a much higher bandwidth compared to the bandwidth of the trajectory tracking since the length scale of the well-plan, is much greater than the length scale of the hole drilled between measurement samples. Hence we can assume that $\mathbf{x} = \mathbf{x}_d$, and an approximation to the dynamics of the integrated closed loop system is expressed as

$$\dot{\mathbf{y}}(t) = ROP \mathbf{x}_d(t). \quad (4.6)$$

This implies a future sequence of control inputs $\mathbf{x}_d(t)$ can be recovered from the first derivative of the predicted correction path $\hat{\mathbf{y}}(t)$. When represented as a b-spline, the attitude demand signal is found by the unit tangent of $\hat{\mathbf{y}}(\lambda)$ by

$$\mathbf{x}_d(\lambda) = \frac{\frac{d\hat{\mathbf{y}}}{d\lambda}(\lambda)}{\|\frac{d\hat{\mathbf{y}}}{d\lambda}(\lambda)\|_2}. \quad (4.7)$$

The value of λ chosen will depend on the integration step related to the sampling communication rate between the surface trajectory controller and the down hole attitude controller.

Assuming that ROP is nominally constant, the control ω from the attitude response of the drilling tool (2.2) is related to its maximum curvature capability $\bar{k}_{dls}$ by

$$\|\omega\|_2 \leq ROP \bar{k}_{dls}, \quad (4.8)$$

The predicted correction path $\hat{\mathbf{y}}(\lambda)$, represented as a b-spline is feasible to track provided that its curvature $\kappa(\lambda)$ is less than the maximum tool curvature $\bar{k}_{dls}$.

$$\kappa(\lambda) = \frac{\|\hat{\mathbf{y}}'(\lambda) \times \hat{\mathbf{y}}''(\lambda)\|_2}{\|\hat{\mathbf{y}}'(\lambda)\|_2^3} \leq \bar{k}_{dls} \quad (4.9)$$

A relaxation of the curvature (c.f. Section 3.5.5) represented in terms of $\|\hat{\mathbf{y}}''(\lambda)\|_2$ will be enforced as an inequality constraint point-wise [Fujioka and Kano, 2012] along each segment of a b-spline representation of the predicted output trajectory $\hat{\mathbf{y}}$ by

$$\mathbf{p}^T \mathbf{N}''(\lambda)^T \mathbf{N}''(\lambda) \mathbf{p} \leq \bar{\kappa}, \quad (4.10)$$

The positive constant $\bar{\kappa}$ is related to the parametrization λ and the BHA steering capability $\bar{k}_{dls}$. Since the expression (4.10) is a 2-norm, it is positive definite, and hence this forms a convex quadratic constraint.

4.2.4 Well-plan Deviation Objective Function

Consider an offline well-plan $\mathbf{r}(\lambda)$ with respect to geological targets and BHA design generated from Algorithm 3.1 in Chapter 3

$$\mathbf{r}(\lambda) = \mathbf{N}(\lambda) \mathbf{q}, \quad (4.11)$$

where $\mathbf{N}(\lambda)$ are b-spline basis functions of degree d with respect to a parameter $\lambda \in [d-1, m_r + d-1]$ along the plan with m_r segments, and $\mathbf{q}$ represent the control points. The *accuracy* aim is to minimize the deviation

$$\int_{t_0}^{t_f} \|\mathbf{y}(t) - \mathbf{r}(\lambda^*(\mathbf{y}(t)))\|_2 dt \quad (4.12)$$

of the borehole trajectory $\mathbf{y}(t)$ from the well-plan $\mathbf{r}(\lambda^*(\mathbf{y}(t)))$ from some initial time t_0 to some final time t_f . In this problem the time taken to follow the plan is of less importance and the control of ROP is in an exo-system and not in the scope of this control system. Hence the reference $\mathbf{r}(\lambda^*(\mathbf{y}(t)))$ is not time dependent and instead dependent on the current position of the drilling tool $\mathbf{y}(t)$. The parameter $\lambda^*(\mathbf{y}(t))$ is defined to be

$$\lambda^*(\mathbf{y}(t)) = \arg \min_{\lambda} \|\mathbf{y}(t) - \mathbf{r}(\lambda)\|_2, \quad (4.13)$$

and represents the closest point on the well-plan to the drilling tool. It is assumed that this point is unique.

The structure of the well-plan deviation objective is taken from Wesselink [1996] to be of the form

$$E_{ca} = \int_{\lambda_0}^{\lambda_1} \|\hat{\mathbf{y}}(\lambda) - \mathbf{c}(\lambda)\|^2 d\lambda, \quad (4.14)$$

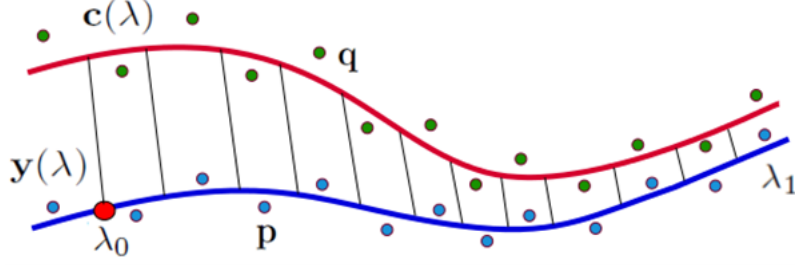


Figure 4.3: Well-plan deviation objective

where $\hat{\mathbf{y}}(\lambda) = \mathbf{N}(\lambda)\mathbf{p}$ is the correction path to be solved for, and $\mathbf{c}(\lambda) = \mathbf{N}(\lambda)\bar{\mathbf{q}}$ is a segment of the well-plan $\mathbf{r}(\lambda)$ for $\lambda^* \leq \lambda \leq (\lambda_1 - \lambda_0)$, and the control point sequence $\bar{\mathbf{q}}$ are a subset of $\mathbf{q}$. Without loss of generality $\mathbf{q}$ will be used to represent $\bar{\mathbf{q}}$ henceforth. The positive difference $\lambda_1 - \lambda_0$ defines the length of the optimization horizon in terms of the number of control points. This is shown graphically in Figure 4.3.

The well-plan deviation objective function (4.14) written in terms of the control points $\mathbf{q}$ and $\mathbf{p}$ is

$$E_{cl} = \mathbf{p}^T \mathbf{A} \mathbf{p} - 2\mathbf{p}^T \mathbf{A} \mathbf{q} + \mathbf{q}^T \mathbf{A} \mathbf{q}. \quad (4.15)$$

The matrix $\mathbf{A} = a_{ij}$ is found from evaluating the integral [Wesselink, 1996]

$$a_{ij} = \int N_i^d(\lambda) N_j^d(\lambda) d\lambda. \quad (4.16)$$

Due to the local support property of the splines for the cubic case, the basis $N_i^3(t)$ for a given t is non-zero for 4 segments i . So $\mathbf{A}$, can be found by constructing a square zero matrix depending on the number of control points required by $\mathbf{q}$, and then copying the matrix $\hat{\mathbf{A}}$

$$\hat{\mathbf{A}} = \begin{bmatrix} \frac{1}{252} & \frac{43}{1680} & \frac{1}{72} & \frac{1}{5040} \\ \frac{43}{1680} & \frac{140}{149} & \frac{720}{49} & \frac{84}{19} \\ \frac{1}{72} & \frac{720}{19} & \frac{180}{720} & \frac{720}{252} \\ \frac{1}{5040} & \frac{84}{19} & \frac{720}{252} & \frac{1}{252} \end{bmatrix} \quad (4.17)$$

along the diagonal starting from $(1, 1)$. This process is stated in Algorithm 4.1.

4.2.5 B-Spline Finite Horizon Optimization

In this section, the finite horizon optimization problem is stated. Given the measured position $\mathbf{y}(t)$ at time t , the measured attitude $\mathbf{x}(t)$, a section of the well-plan $\mathbf{c}(\lambda)$, and

Algorithm 4.1 Vectorizing $\mathbf{A}$

- 1: Construct $\mathbf{A}$ a $(m + d) \times (m + d)$ matrix with zero entries,
 - 2: add the matrix $\hat{\mathbf{A}}$ into $\mathbf{A}$ such that $\mathbf{A}_{1\dots 4, 1\dots 4} = \hat{\mathbf{A}}_{1\dots 4, 1\dots 4}^k$
 - 3: **for** $i = 2$ **to** $m + d - 3$ **do**
 - 4: $\mathbf{A}_{i\dots i+3, i\dots i+3} = \mathbf{A}_{i\dots i+3, i\dots i+3} + \hat{\mathbf{A}}_{1\dots 4, 1\dots 4}^k$
 - 5: **end for**
 - 6: Replace each entry in $a_{i,j} \in \mathbf{A}$ with $a_{ij}I_{3 \times 3}$, where $I_{3 \times 3} \in \mathbb{R}^3$ is the identity matrix.
The result is an $3(m + d) \times 3(m + d)$ matrix $\mathbf{A}$
-

$\bar{\kappa}$, the following optimization problem (4.18) solves for control points $\mathbf{p}$ to define the optimum correction path:

$$\begin{aligned}
 & \underset{\mathbf{p}}{\text{minimize}} && (1 - \beta_{att}) \underbrace{\mathbf{p}^T \mathbf{Q} \mathbf{p}}_{\text{tortuosity}} + \beta_{att} \underbrace{(\mathbf{p}^T \mathbf{A} \mathbf{p} - 2\mathbf{p}^T \mathbf{A} \mathbf{q} + \mathbf{q}^T \mathbf{A} \mathbf{q})}_{\text{curve attractor}} + \beta_2 y_1 \\
 & \text{subject to} && \mathbf{N}(d - 1)\mathbf{p} = \mathbf{y}(t) && \text{(initial position)} \\
 & && \mathbf{N}'(d - 1)\mathbf{p} = \mathbf{x}(t)y_1 && \text{(initial attitude)} \\
 & && \mathbf{p}^T \mathbf{N}'' \left(\frac{i}{d} \right)^T \mathbf{N}'' \left(\frac{i}{d} \right) \mathbf{p} \leq \bar{\kappa} && \text{(curvature)} \\
 & && y_1 > 0 \\
 & && i = 2d, \dots, md
 \end{aligned} \tag{4.18}$$

The terms in the objective function are the tortuosity defined in Section 3.4 and well-plan deviation defined in Section 4.2.4. The weighting coefficient $0 \leq \beta_{att} \leq 1$ determines the trade off between following the well-plan and drilling a smooth hole. The coefficient $\beta_2 > 0$ as in Section 3.6.1 is provided to ensure the attitude at the beginning of the correction path is aligned with $\mathbf{x}(t)$. Since the objective function and the inequality constraints are convex by construction, the finite horizon optimization problem (4.18) can be solved by interior point methods [Potra and Wright, 2000, Boyd and Vandenberghe, 2004] to determine the control points $\mathbf{p}$ for the correction path. From these control points $\mathbf{p}$, the optimal correction path $\hat{\mathbf{y}}(\lambda)$ is found by

$$\hat{\mathbf{y}}(\lambda) = \mathbf{N}(\lambda)\mathbf{p}. \tag{4.19}$$

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The attitude demand set point $\mathbf{x}_d$ is then determined for an appropriately chosen $\tilde{\lambda} > d - 1$ along the correction path $\hat{\mathbf{y}}$ ahead of the measured position of the BHA by

$$\mathbf{x}_d(\tilde{\lambda}) = \frac{\mathbf{N}'(\tilde{\lambda})\mathbf{p}}{\|\mathbf{p}^T\mathbf{N}'^T(\tilde{\lambda})\mathbf{N}'(\tilde{\lambda})\mathbf{p}\|_2}. \quad (4.20)$$

If required, multiple demand attitudes $\mathbf{x}_d$ may be sampled from the correction path $\hat{\mathbf{y}}(\lambda)$ and communicated to the attitude control system.

4.2.6 Absolute Curvature Constraint

The curvature constraint used in equation (4.18) is a quadratic relaxation to the curvature. In order to generate a feasible path which will not violate the true curvature capability $\bar{k}_{dls}$ of the drilling tool, the value for $\bar{\kappa}$ is modified using a bisection search by comparing the maximum curvature κ_{max} for the path $\hat{\mathbf{y}}$ with $\bar{k}_{dls}$. For details of this algorithm see (3.6.2).

4.2.7 Tuning Effects on Finite Horizon Path

A feature of the optimization problem (4.18) is the weighting coefficients in the objective function. This coefficient β_{att} represents the trade-off between following the well-plan and drilling a smooth borehole. A supervisor interacting with the trajectory control system can vary these parameters during the real time drilling process. The effects of these are shown in Figure 4.4. Moreover a curvature constraint can be imposed to be less than the curvature capability of the tool. The effect of this is shown in Figure 4.5.

Consider Figures 4.4 and 4.5 which display multiple paths $\hat{\mathbf{y}}$ produced as a result of the finite horizon optimization problem for a given well-plan $\mathbf{r}$. A well-plan is displayed in red and is generated from the optimization strategy from Chapter 3. The well-plan is defined with $m_r > m$ segments, where the subsequent trajectory controller will determine a correction path with m segments. This well-plan is a cubic spline $d = 3$ with $m_r = 30$ segments for starting pose with position $\mathbf{r}(\lambda = d) = [0, 0, 0]$ and attitude $\mathbf{r}'(\lambda = d) = [0, 0, 1]$ represented by the blue square, and ending at the pose with position $\mathbf{r}(\lambda = m - 1) = [\frac{600}{\sqrt{2}}, 0, \frac{600}{\sqrt{2}}]$ and attitude $\mathbf{r}'(\lambda = d) = [\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}]$ represented by the red square. The weighting coefficients of the tortuosity objective function in the well-plan are chosen to be $\alpha_1 = 0.8$, and $\alpha_2 = 0.1$, which is biased towards being shorter. The

curvature constraint is taken to be $7^\circ/100ft$.

Curvature Constraint

From a starting position $\hat{\mathbf{y}}(\lambda = d - 1) = [100, 0, 80]$ and attitude $\hat{\mathbf{y}}'(\lambda = d - 1) = [\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}]$ represented by the green circle in Figure 4.4, five predicted well-plans $\hat{\mathbf{y}}$ are generated with a horizon of $m = 15$ segments from the finite horizon optimization (4.18). The weights of the objective function are chosen to be $\alpha_1 = 0.8$, and $\alpha_2 = 0.1$. The curvature constraints are taken to be $\bar{k}_{dls} \in \{3, 5, 8, 10, 15\}^\circ/100ft$. The trade-off weight is taken to be $\beta_{att} = 0.9$. The results from Figure 4.4 demonstrate the generation of feasible smooth paths which are coincident with the pose of the drilling tool to track the well-plan. Tightening the curvature constraint from $15^\circ/100ft$ down to $3^\circ/100ft$ has the effect of generating a path which meets the well-plan much further away from the drilling tool with lower curvature. Much of the path is coincident with the well-plan towards the end of the horizon. For the path corresponding to the $3^\circ/100ft$ curvature constraint, there is an overshoot and the path does not coincide with the well-plan.

Deviation-Tortuosity Coefficient β_{att} Variation

Consider Figure 4.5 displaying results of the finite horizon optimization investigating the effect of changing the trade-off weight β_{att} on the correction path. As with Figure 4.4, the starting position depicted by the green circle in Figure 4.5 is $\hat{\mathbf{y}}(\lambda = d - 1) = [100, 0, 80]$ with attitude $\hat{\mathbf{y}}'(\lambda = d - 1) = [\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}]$. Correction paths $\hat{\mathbf{y}}$ are generated with a horizon of $m = 15$ segments from the finite horizon optimization (4.18). The weights of the objective function are chosen to be $\alpha_1 = 0.5$, and $\alpha_2 = 0.5$.

Here the same well-plan $\mathbf{r}$ as Figure 4.4 is taken as the reference. The trade-off weight β_{att} is varied from $\beta_{att} \in \{0.1, 0.2, 0.5, 0.9\}$. It is shown that when reducing β_{att} , the optimal correction path favours drilling a smoother straighter path. In addition, changing this parameter influences how aggressively the well-plan is tracked.

It must be emphasized that these weights and constraints can be tuned offline, where the values can be scheduled to vary along the drilling process. That is to say, for

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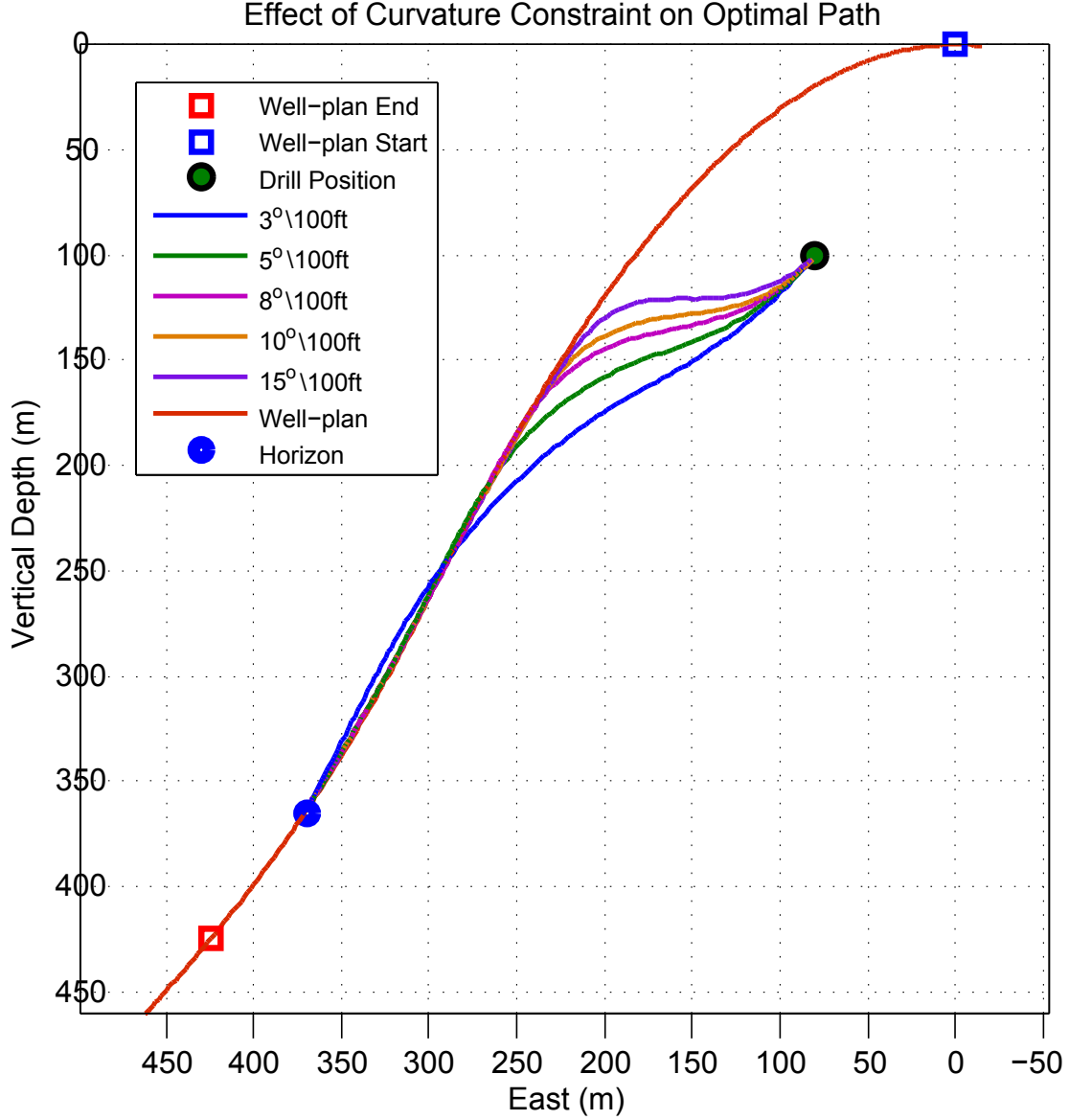


Figure 4.4: Finite horizon optimization with varying curvature constraints

sections of the well where speed and smoothness are more important, then the value of β_{att} can be set low. In well-plan sections in regions of high hydrocarbon saturation, the accuracy is of greater importance than smoothness for economic reasons regarding the subsequent recovery where β_{att} would be set high. Furthermore in a real-time drilling operation, the ability to change these weights and curvature constraints can prove to

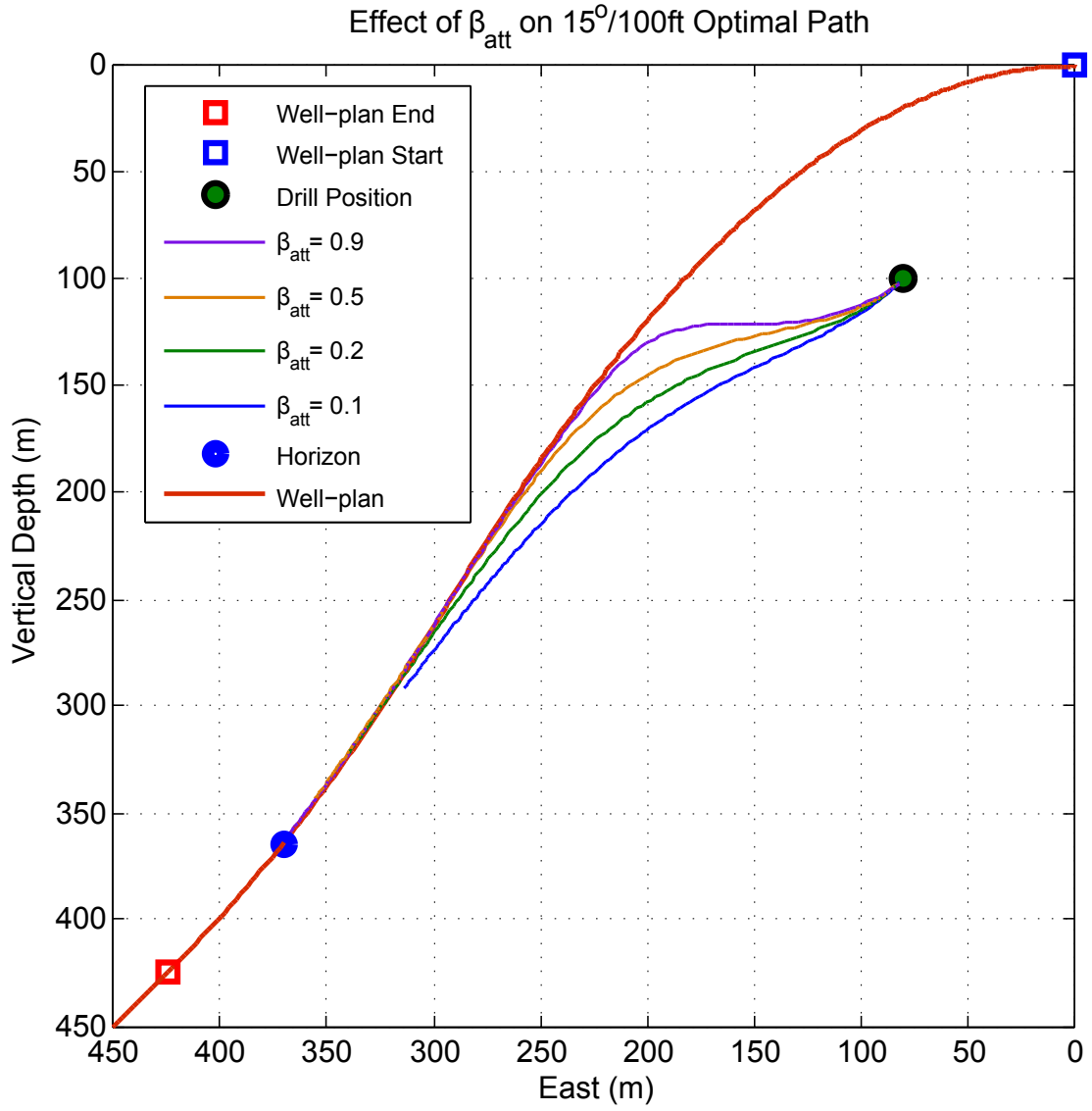


Figure 4.5: Finite horizon optimization with varying β_{att}

be valuable where a DD utilizes their experience to interact with this system.

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4.3 Trajectory Controller

Given the finite horizon optimization (4.18), the MPC trajectory controller is described in this section. The procedure is summarized in Algorithm 4.2.

Algorithm 4.2 MPC Control Loop

- 1: Set horizon length m
 - 2: Set drilling run time t_{max}
 - 3: **while** $t < t_{max}$ **do**
 - 4: Measure position $\mathbf{y}(t)$ and attitude $\mathbf{x}(t)$
 - 5: find $\mathbf{c}$ from Section 4.2.4
 - 6: Set $\beta_{att}, \bar{k}_{dls}$
 - 7: Solve $\hat{\mathbf{y}}$ from Algorithm 3.3
 - 8: Determine feedback value $\mathbf{x}_d$ from (4.20) and apply this to the attitude control system for the next sample period
 - 9: Increment t
 - 10: **end while**
-

A well-plan $\mathbf{r}$ is designed by the methods described in Chapter 3. A horizon length in terms of the number of segments of a b-spline is chosen to be sufficiently large such that the corresponding distance is larger than the distance that can be drilled between samples. A measurement is taken of the drilling states $\mathbf{x}(t), \mathbf{y}(t)$. From this the closest point to the well-plan is chosen and a corresponding section of the well-plan $\mathbf{c}(\lambda)$, where the number of segments match the horizon length is taken. If required, the values of the trade-off weight β_{att} are changed along with $\bar{k}_{dls}$. Now the finite horizon optimization problem is solved in order to determine the optimal control points $\mathbf{p}$. From these control points, the optimal state trajectory $\hat{\mathbf{y}}(\lambda)$ is chosen from which the closed loop control $\mathbf{x}_d$ is communicated to the attitude control system. The algorithm is then recursed until the end of the well-plan is reached.

4.3.1 Simulation Results

In this section, the MPC trajectory controller is tested in a time domain simulation. The model of the drilling process is taken from a commercial drilling simulator *ST2* which models the drill-string and the hole propagation process through finite element

analysis. The model of the drilling tool is chosen to be a push-the-bit drilling system. Two sets of simulations are produced to follow the same well-plan from the same pose. The first will demonstrate the effect of curvature constraints on the correction path to the tracking performance. The latter will consider the tracking performance against the variation in the weighting term β_{att} . This section begins by describing the well-plan used in the simulations.

Simulation Parameters

For the trajectory following, the correction path is chosen to be a cubic b-spline curve with $m = 12$ segments. The simulation will drill along a total depth of 190m. The minimum energy weights are chosen to be $\alpha_1 = 0.1$, and $\alpha_2 = 0.5$.

The *ST2* model parameters are chosen to be a push the bit drilling tool, drilling with an initial inclination of 85° , and an initial position of $[145, 500, 145]$. The attitude set-point is taken corresponding to a value of $\lambda = 0.4$ ahead of the bit. A new attitude demand signal $\mathbf{x}_d$ is downlinked every 2.5m, and the drilling tool is penetrating at a rate of $ROP = 30m/hr$. The sensors are located at a distance of 1m behind the bit.

Well-Plan

A well-plan is generated from the optimization presented in Chapter 3, with starting position $\mathbf{r}(\lambda = d) = [150, 0, 500]$ and attitude $\mathbf{r}'(\lambda = d) = [0, 0, 1]$ being due east. A cubic $d = 3$ b-spline with $m = 60$ segments is constructed to a final pose with position $\mathbf{r}(\lambda = m - 1) = [160, 15, 720]$ and attitude $\mathbf{r}'(\lambda = m - 1) = \frac{1}{5\sqrt{2}}[1, 2, 7]$. The energy weights of the optimization are chosen to be $\alpha_1 = 0.8$, and $\alpha_2 = 0.1$. The path is planned with a curvature constraint of $2^\circ/100ft$, and the curvature profile along the depth of the well-plan is shown in Figure 4.6.

MPC Controller with Varying Curvature Constraint

The MPC trajectory controller is run for four cases where the maximum allowed curvature $\bar{k}_{dls}$ of the correction path is varied. The cases run are for $\bar{k}_{dls} \in \{3, 5, 7, 9\}^\circ/100ft$. A visualization of the well-plan, drilled trajectory, and correction path can be seen for

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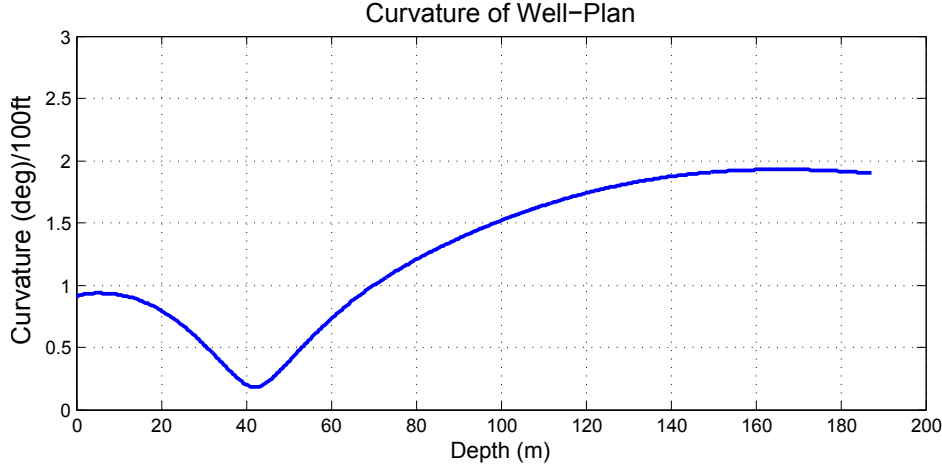


Figure 4.6: Curvature along the well-plan

the case of $9^\circ/100ft$ in Figure 4.7.

The deviation from the well-plan along the trajectory in simulation time measured from the bit to the closest point in the well-plan is shown for all the cases in Figure 4.8. Allowing the correction path to take larger curvatures results in the trajectory converging to the well-plan in shorter depths. The trajectory which converges to the well-plan at the lowest measured depth is the case with a curvature constraint of $9^\circ/100ft$. Decreasing the curvature constraint has the effect of meeting the well-plan at an increased measured depth. The deviation from the well-plan is less than 1m for all cases except the $3^\circ/100ft$, and for these three cases the tracking is less than 0.5m after the first 60m drilled. For the $3^\circ/100ft$ case, after 160m the deviation begins to diverge. This is because the well-plan curvature being $2^\circ/100ft$, and the control effort is significantly compromised with a strong constraint.

Figure 4.9 shows the toolface signals for the range of curvature constraints. These toolface signals are generated by the attitude control system presented in Chapter 2. The difference in the effect of the curvature constraint is clearer in the measured depth range of 40 – 60m. This corresponds to the depth along the trajectory at which the drilling tool converges with the well-plan from Figure 4.8. For the $3^\circ/100ft$ curvature constraint case, the toolface is held in much shorter bursts than for the $9^\circ/100ft$ case

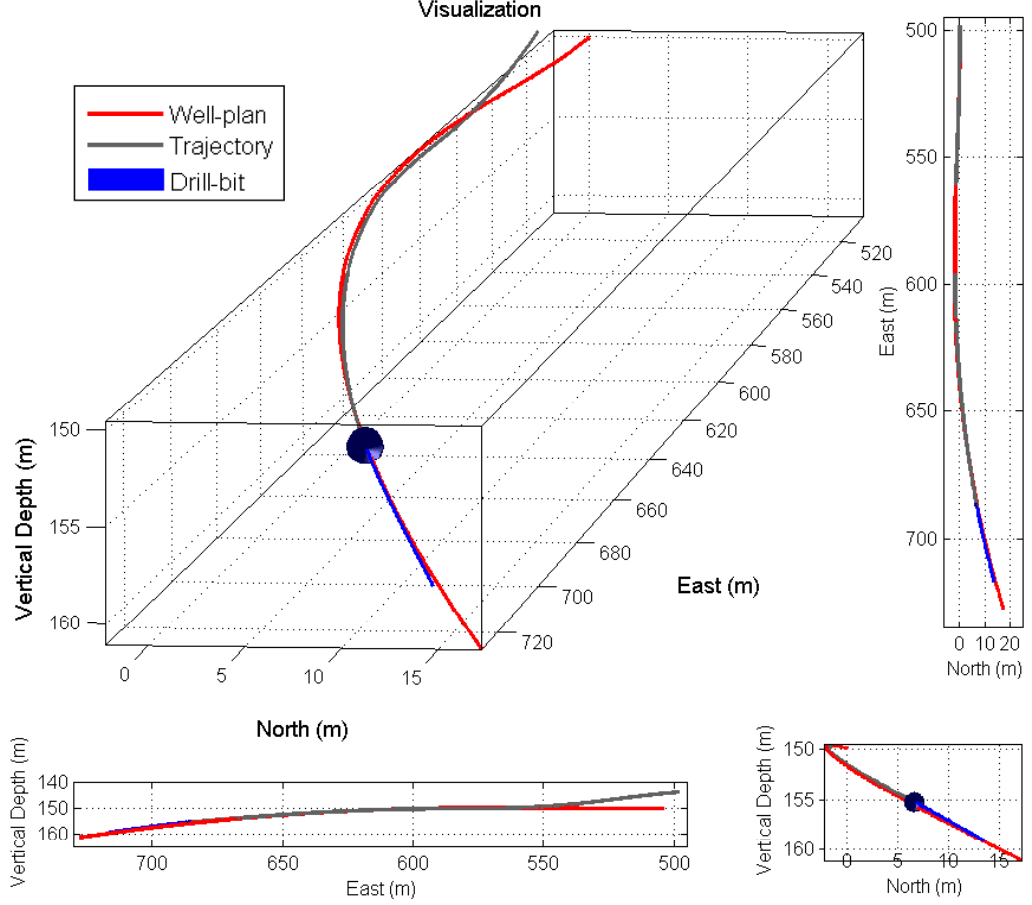


Figure 4.7: Result of trajectory control for $9^\circ/100ft$

in the 40 – 60m range in Figure 4.9. This is because to achieve a lower net curvature, the toolface must cycle a full revolution between the bursts of holding toolface in order to achieve steering at a lower toolface. The length of the bursts of holding toolface increases as the curvature constraint increases. In all these test cases the toolface signal is seen to frequently cycle a full rotation. This is because the well-plan is of a much lower curvature than the allowed curvature of the BHA, and for the BHA to lower its curvature, the toolface is cycled to achieve sections of rectilinear wellbores.

Figure 4.10 shows the attitude of the drilling tool versus measured depth along the borehole trajectory. The variation in attitude fluctuates every 5m by 0.5° in incli-

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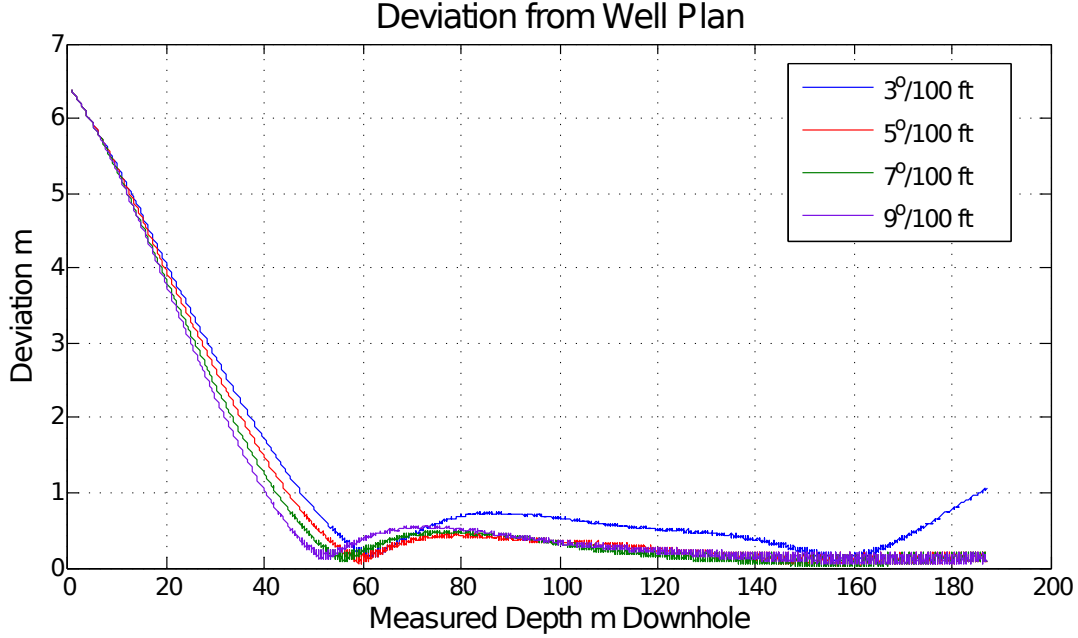


Figure 4.8: Deviation from well-plan for a range of curvature constraints

nation and in azimuth. This corresponds to the depth drilled between new attitude set points, where within this time the drilling tool reaches and holds an attitude by varying its toolface angle. The effect of the curvature constraints are clearly evident in the inclination, where the gradient equals the constraints. After 140m there is a clear difference in the tracking performance for the $3^\circ/100ft$ curvature constraint case.

The curvature of the correction path at the location coincident with the drill-bit as calculated in the finite horizon optimization is shown along the measured depth of the borehole trajectory in Figure 4.11. Since the correction path is only calculated as an outer-loop to the attitude control system it is shown in discrete values. The effect of the curvature constraint is shown to restrict the correction-path curvature in the early stages when the drilling tool is converging to the well-plan. The constraint for the $3^\circ/100ft$ case is active for much of the trajectory for the initial section of the correction-path.

The curvature of the drilled trajectory as measured by *ST2* is shown in Figure 4.12 for

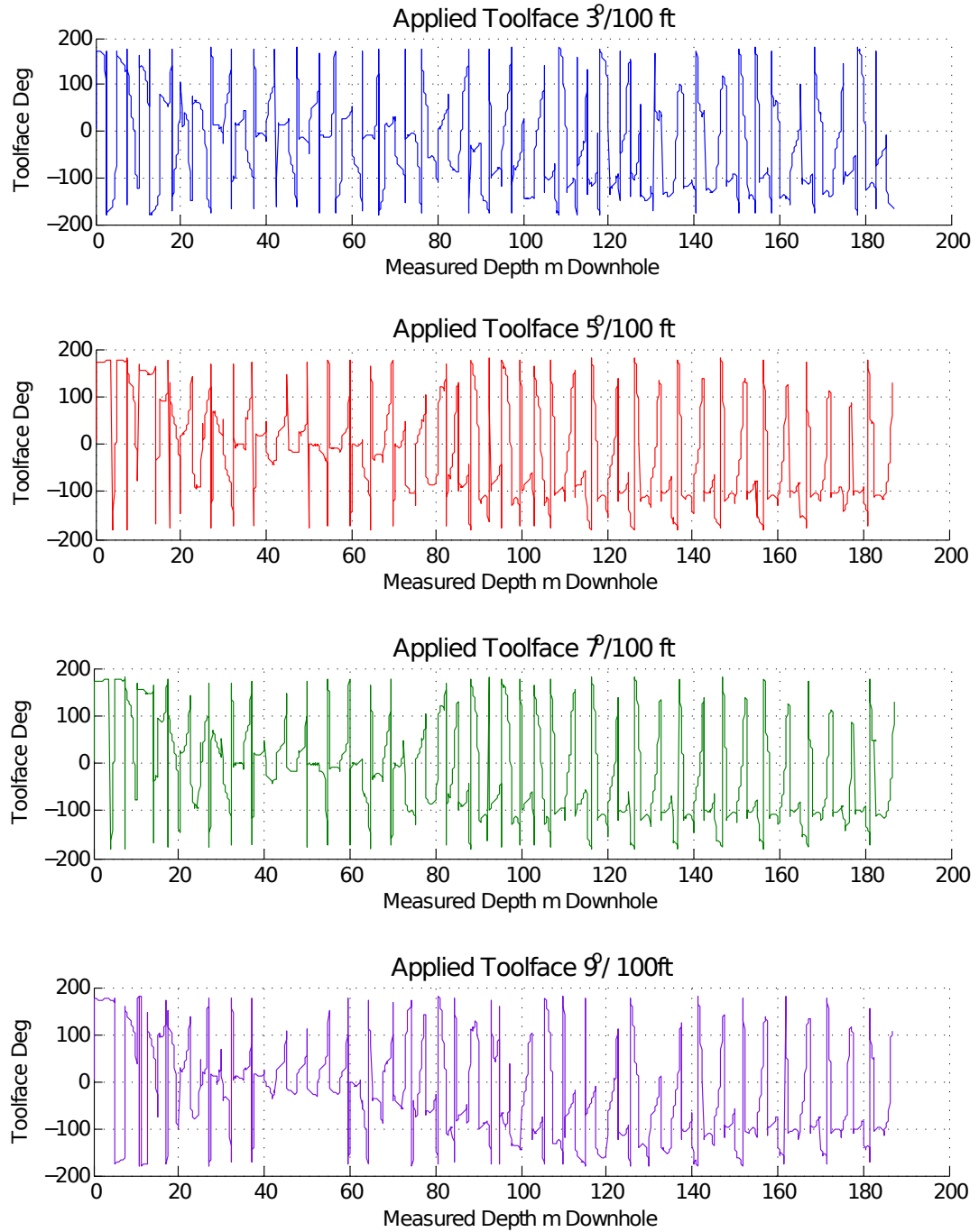


Figure 4.9: Toolface signals versus depth for different curvature constraints

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the absolute curvature, and the build-rate and turn-rate. These being the curvature components in the vertical and horizontal planes respectively. The difference between the absolute borehole curvature and the predicted borehole curvature from Figure 4.11 is because of transient effects in the response of curvature. Nevertheless there is a similarity in the shape of the absolute borehole curvature plots and the instantaneous predicted correction path curvatures. Furthermore the curvature constraint limits the curvature of the borehole and is seen to be effective.

MPC Controller with Varying Trade-off Weight

In this section the effect of varying the trade-off weight β_{att} on the resultant borehole trajectory is investigated for the closed loop trajectory controller. The simulation parameters are the same as in the previous section with the curvature constraint being fixed at $9^\circ/100ft$. The values for the trade-off weight are chosen to be $\beta_{att} \in \{0.2, 0.05, 0.01\}$.

The deviation of the borehole trajectory from the well-plan is displayed in Figure 4.13 versus measured depth. The effect of decreasing the weight β_{att} has the effect of reducing how quickly the borehole trajectory reaches the well-plan where for the case of $\beta_{att} = 0.2$, the borehole trajectory was first coincident with the well-plan at about 50m, whereas for the case of $\beta_{att} = 0.01$, this depth value was about 70m. Furthermore the tracking of the well-plan is compromised when reducing the weight to $\beta_{att} = 0.01$ which after reaching, tracks the well-plan after 100m with a larger deviation than the other two cases.

The toolface signals generated by the attitude control system are shown for the three cases in Figure 4.14. The bursts where the toolface is held are reduced in frequency and duration when the weight β_{att} is reduced. This implies that since there are more rotations of the toolface, then the curvature of the hole is reduced because the finite horizon optimization has emphasis on minimizing the energy of the correction path, in preference to the deviation from the well-plan.

The attitudes in terms of inclination and azimuth angles are shown in Figure 4.15. The difference in the attitudes can mainly be seen in the inclination where a higher weight

β_{att} implies a greater variation in inclination.

The instantaneous curvature on the predicted correction-path is shown in Figure 4.16. The finite horizon optimization does not affect the maximum curvature as all three cases have a $9^\circ/100ft$ section at the start of the trajectory, as this is still allowed in the optimization. The effect of changing β_{att} along the path is in a lower variation in curvature. This can be seen more clearly in Figure 4.17.

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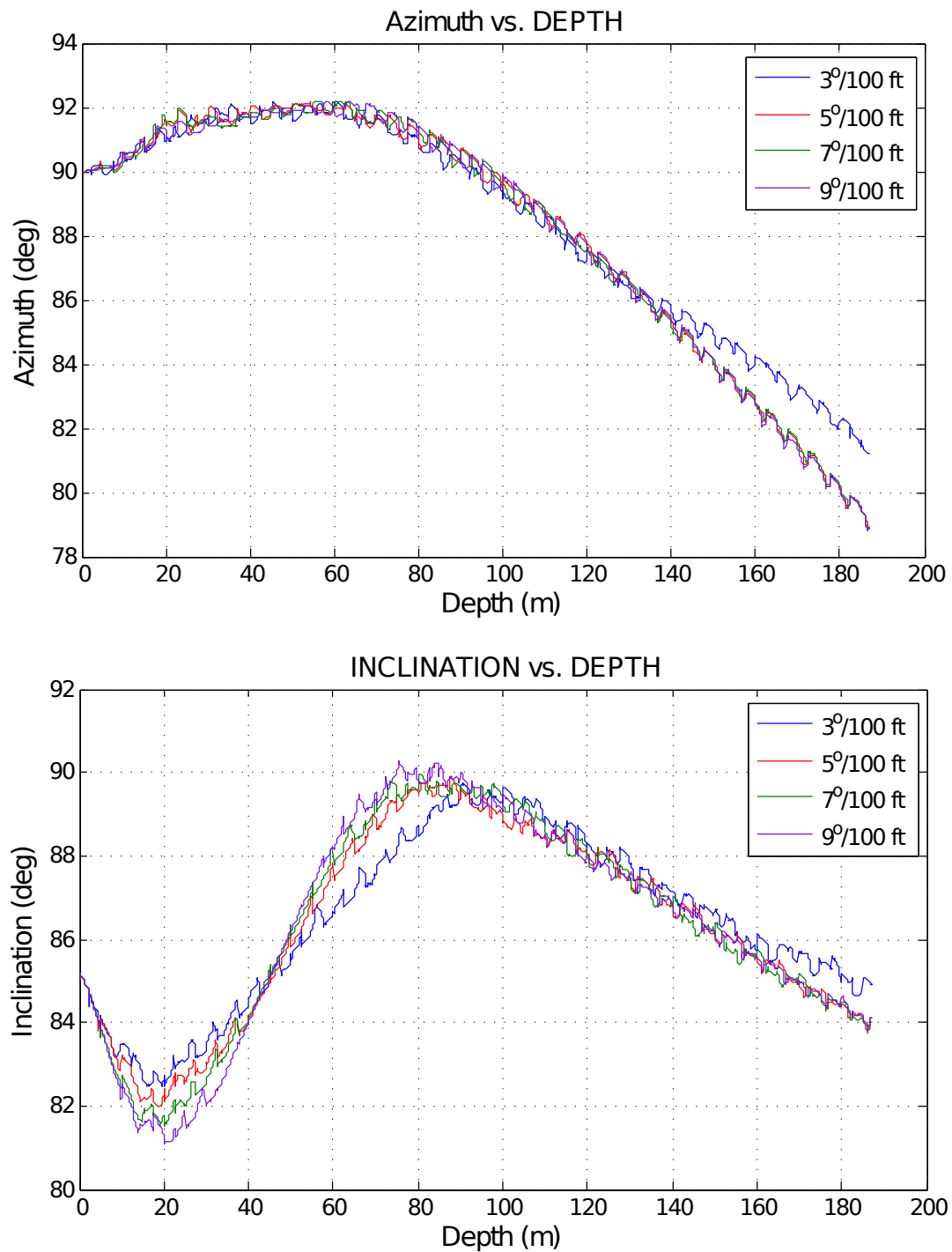


Figure 4.10: Inclination and azimuth signals versus depth for different curvature constraints

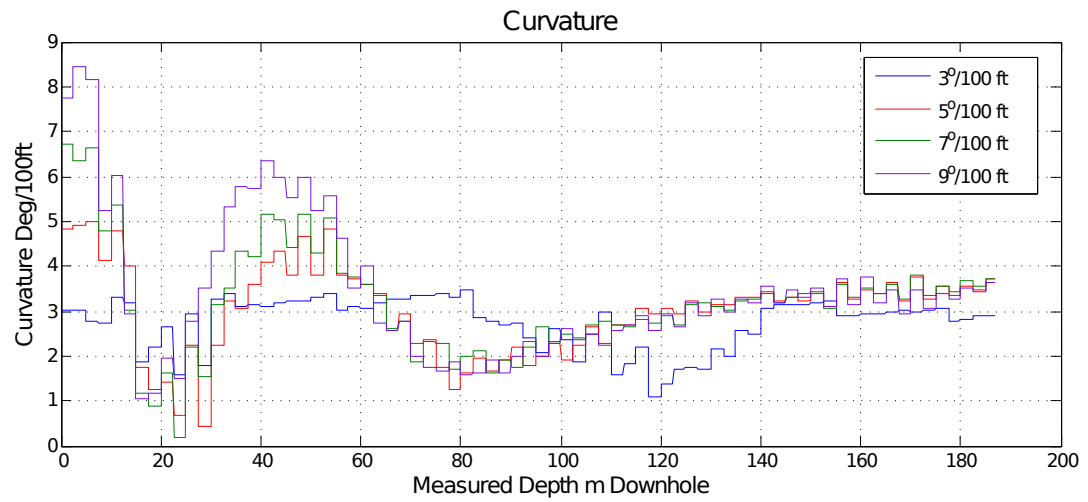


Figure 4.11: Instantaneous curvature of correction path versus depth for different curvature constraints

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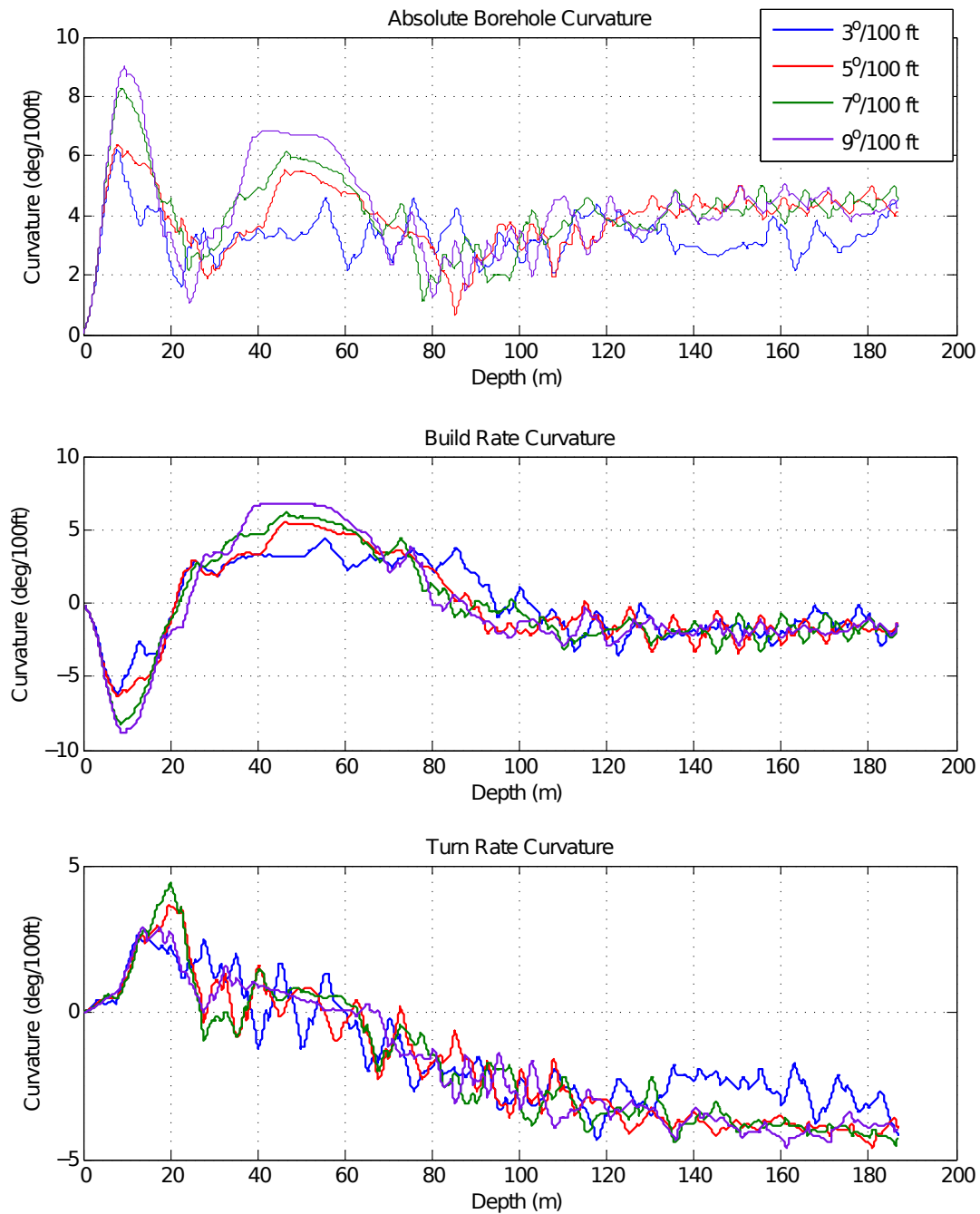


Figure 4.12: Borehole curvature from *ST2* simulation versus depth for different curvature constraints

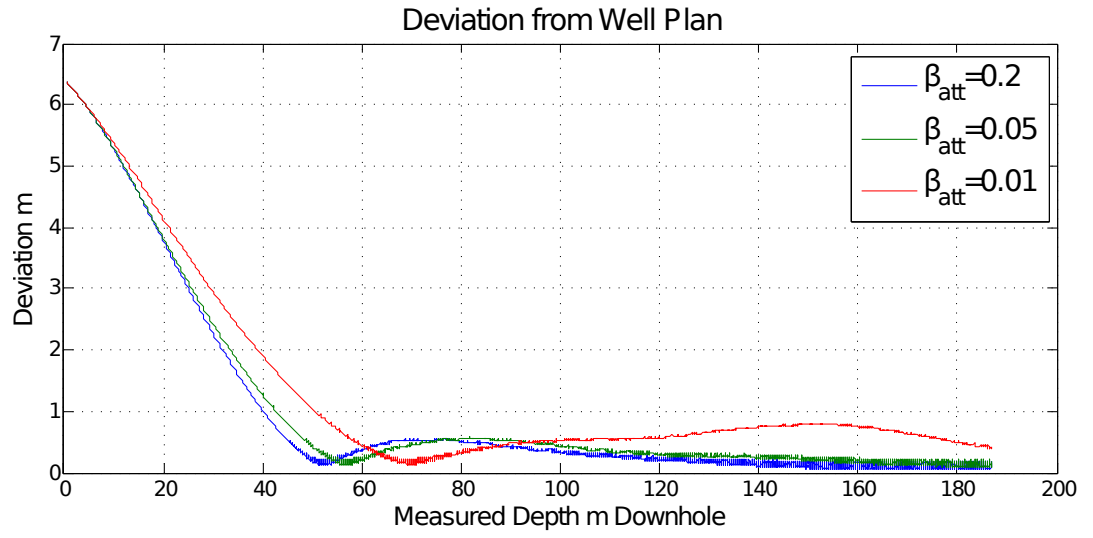


Figure 4.13: Deviation from well-plan versus depth for different β_{att}

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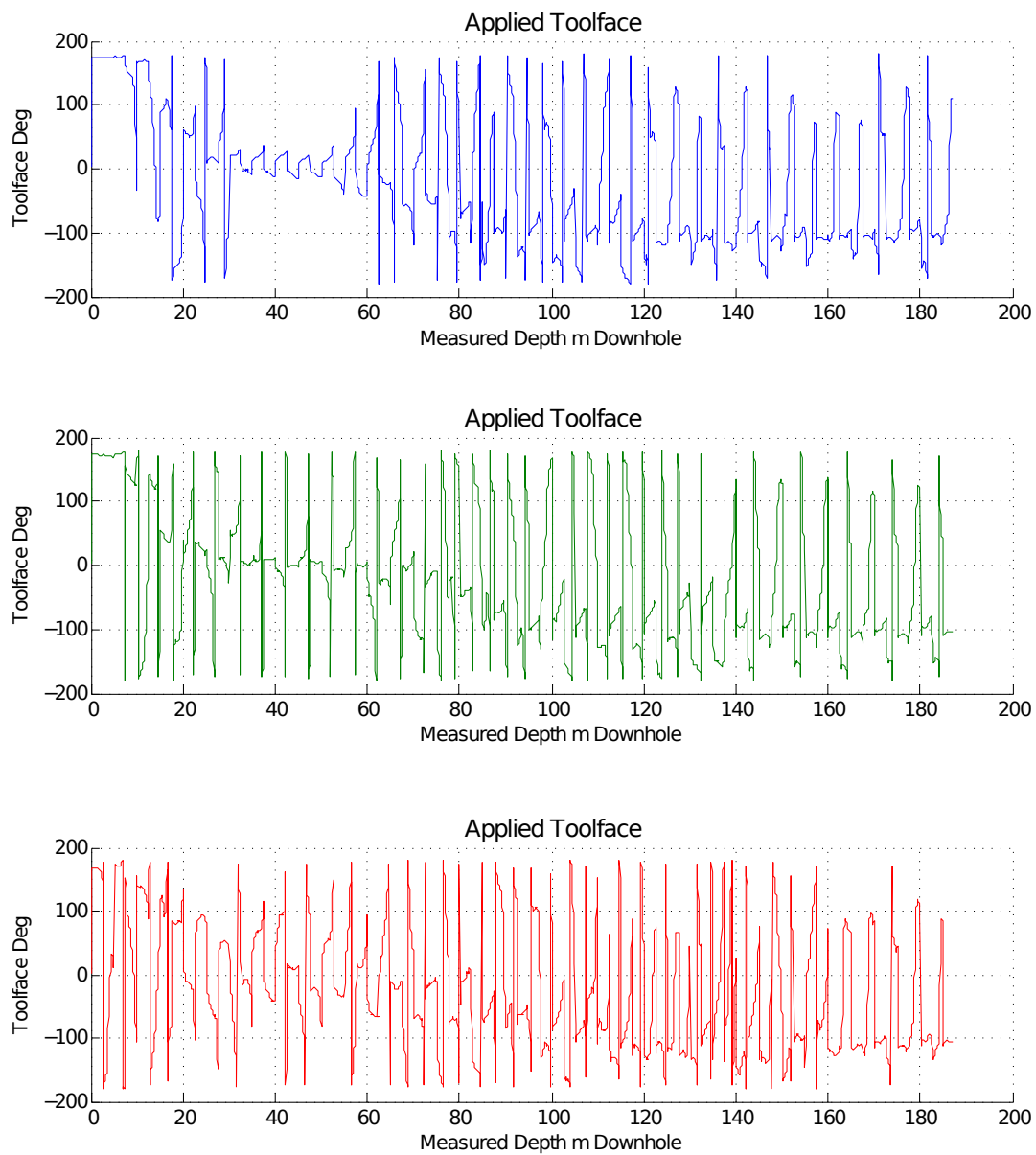


Figure 4.14: Toolface versus depth for different β_{att}

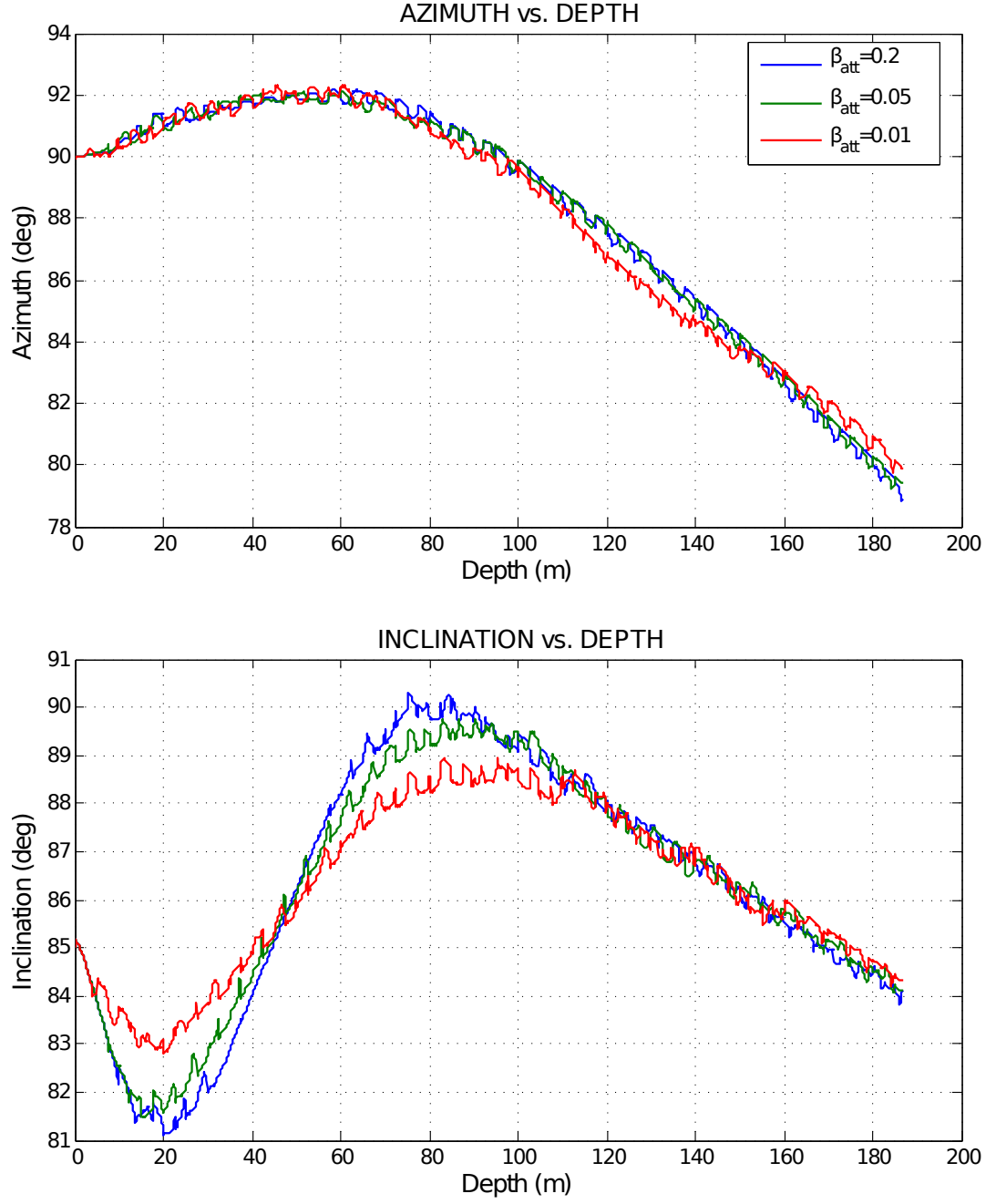


Figure 4.15: Inclination and azimuth versus depth for different β_{att}

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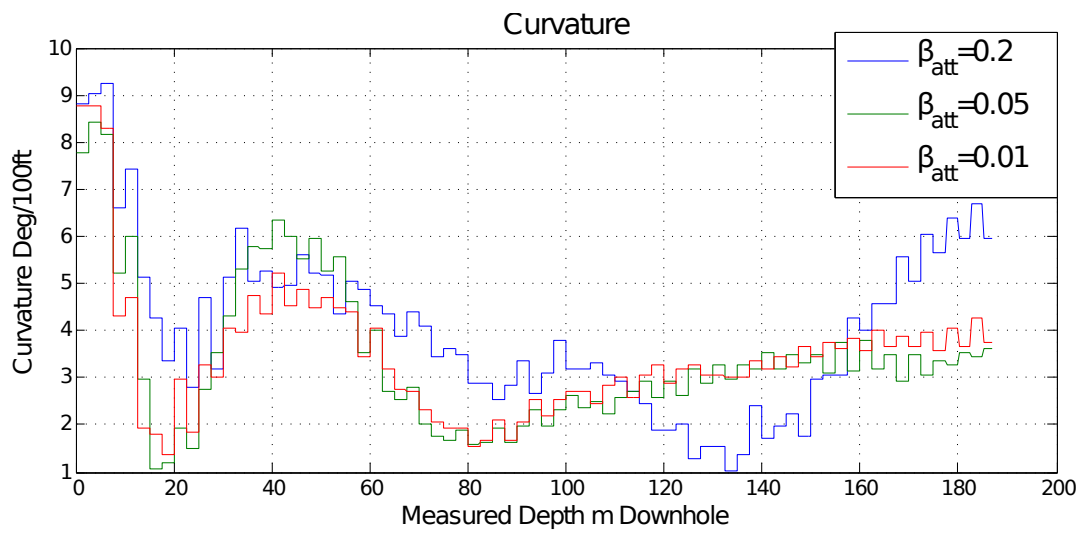


Figure 4.16: Instantaneous curvature of correction path versus depth for different β_{att}

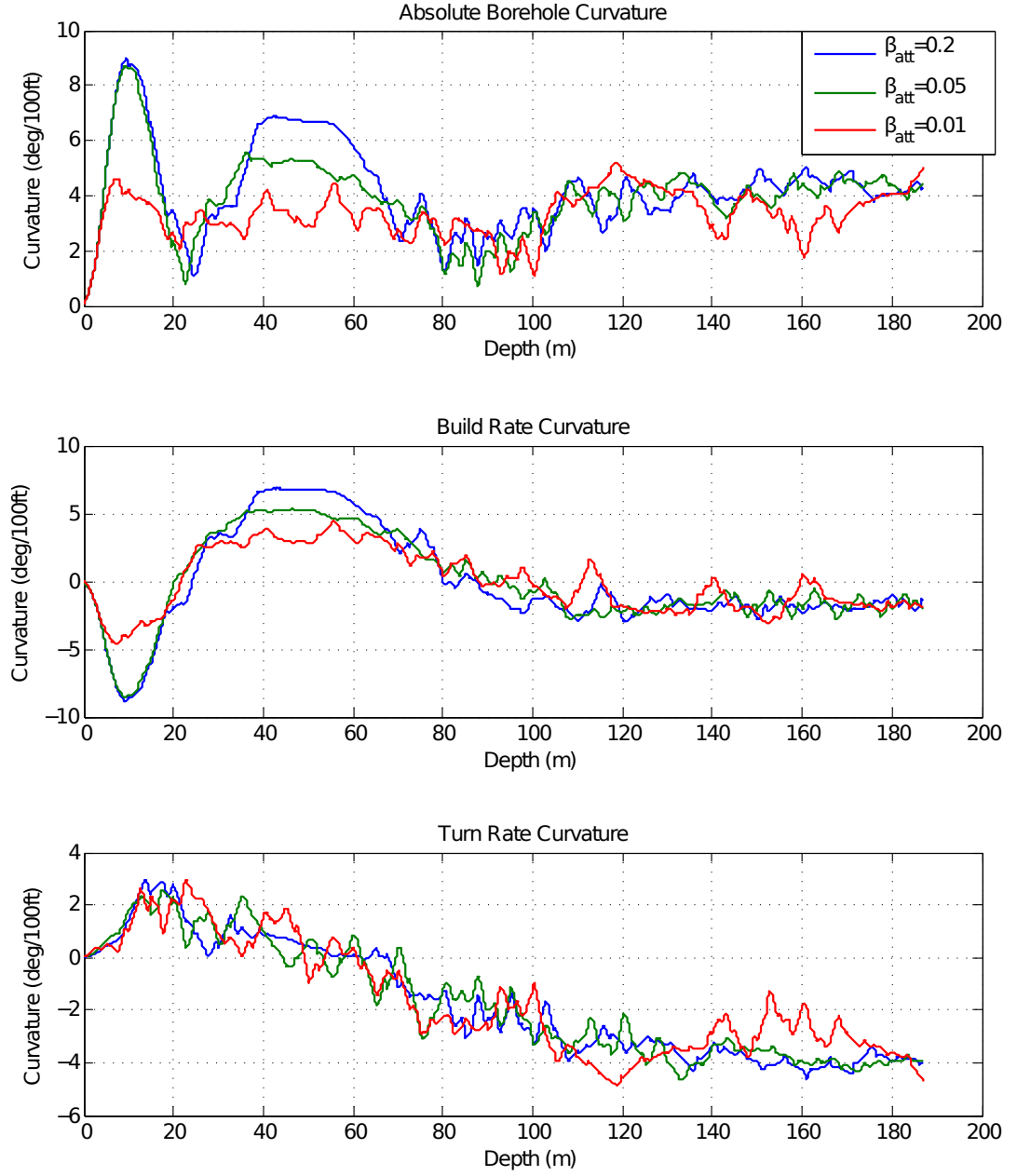


Figure 4.17: Curvature of correction path versus depth for different β_{att}

4.4 Conclusion

In this chapter a method for controlling the propagating trajectory of a borehole produced by a rotary steerable system is presented. The control system is based on a model predictive control strategy where a quadratically constrained quadratic program is repeatedly solved to generate optimal curves based on b-splines. A method for optimizing curvature constrained 3D splines is presented, based on a multi-objective optimization of minimizing tortuosity and deviation from a well-plan. The effect of changing the weights to reflect the importance of either objective functions is shown by simulation for the finite horizon optimization and the MPC trajectory following simulation. This has advantages for enhancing the capability of directional drillers, enabling them to tune the corresponding weighting coefficients in real time based on their experience and intuition for a drilling process. The trajectory control system is demonstrated with a commercial drilling simulator for following a $2^\circ/100ft$ well-path over 190m with varying curvature constraints and trade-off weights. The trajectory control system is shown to track the well-plan generated from the methods in Chapter 3, to within 0.5m demonstrating practical stability for these cases.

Part II

Management

5

Strategic Value of Directional Drilling Automation

As part of the wider EngD study, this chapter discusses the commercial aspects of directional drilling automation in the global economy. A strategic value proposition analysis of attitude and trajectory control in the oilfield services industry is conducted. The economic value is modeled through illustrative examples demonstrating quantitatively where these technologies reduce costs, increase production and become a platform for further technological development.

5.1 Introduction

Oil and gas resources supply over half the worlds energy demand. An increasing global population with increasing demand for transportation drives an increasing demand for oil and gas. However oil and gas resources are finite, where conventional resources are diminishing. The remaining unconventional resources cost comparatively more to extract. Directional drilling in the oilfield services industry is a key technology for hydrocarbon extraction within the oil and gas industry. Directional drilling automation has the potential to reduce the cost of drilling these challenging wells necessary to meet future energy demand. This chapter serves to emphasize the merits of directional drilling trajectory control systems presented in the thesis for meeting these challenges.

Section 5.2 provides a background of the oil industry. Factors which affect the supply

5. STRATEGIC VALUE OF DIRECTIONAL DRILLING AUTOMATION

and demand of crude oil are considered, discussing the predicted future role of crude oil as an energy source. A background of the field development process is presented, discussing the stages of development from exploration to abandonment of a field. The oilfield services industry provide specialist technology for oilfield development and the role of directional drilling within the oilfield services industry is discussed to provide an understanding of where value can be derived.

The latter half of the chapter from Section 5.3, develops a proposition of value of attitude and trajectory control. Arguments are developed investigating where these systems reduce the costs associated with the field development process and increase the volume of recoverable assets. Examples are given to quantify these benefits relating to realistic market situations, to demonstrate where there is a direct benefit for implementing these systems. Furthermore this chapter states the qualitative benefits of automating the said processes.

5.2 Business Background

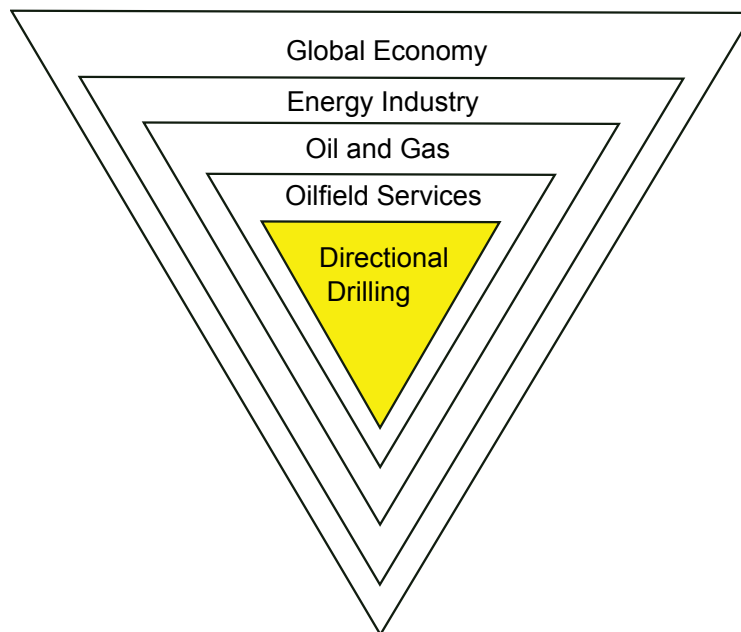


Figure 5.1: Industry hierarchies

To appreciate the implications of directional drilling automation, this Section explores the role of the directional drilling industry within the global economy. Figure 5.1 depicts this relationship, where the directional drilling market being part of the oilfield services industry provides engineering solutions to the oil and gas industry. Energy consumption for the global economy ultimately drives hydrocarbon production from the oil and gas industry. Section 5.2.1 discusses the hydrocarbon supply and demand relationship outlining its market share of the energy industry, in addition to the future role of hydrocarbons as an energy source.

The oil and gas industry develops oilfields to extract crude oil and natural gas. The hydrocarbon extraction process begins with identifying a reservoir, estimating its future production capacity, and determines a strategy for reservoir characterization and well construction. The oilfield life-cycle from appraisal to abandonment is discussed in Section 5.2.2.

The oilfield services industry provides engineering solutions for the well construction process. Section 5.2.3 describes the oilfield services industry with respect to the interaction of the operator, drilling contractor, service company and equipment manufacturer. The nature of contracts and the business models of these companies define their role in the oilfield services industry. Automation challenges the interaction of these companies, which in turn challenge the uptake of automation, since rig processes encapsulate a broad interconnected system, where an individual oilfield services company seldom provides its services in isolation.

The directional drilling services market is analyzed in Section 5.2.4. The market size and market growth potential are discussed, along with the historical growth attributed to rotary steerable systems and extended reach wells. This Section highlights the capacity of directional drilling automation, and allows us to proceed to a value demonstration in the sequel.

5. STRATEGIC VALUE OF DIRECTIONAL DRILLING AUTOMATION

5.2.1 Demand for Energy

Energy demand increased throughout the last century as economies developed and became more industrialized. The distribution of global energy consumption over the last two years is displayed in Table 5.1. The world consumed 12274 mtoe (million tons of oil equivalent) of energy in 2011, with more than half of this consumption 6964.7 mtoe derived from oil and gas sources [BPr]. In recent years there has been a drive towards finding alternative renewable energy sources in addition to consuming energy more efficiently. This is reflected in the growing share of non-oil and gas energy sources increasing from 42.6% in 2010 as a share of total energy consumption to 43.3% in 2011. Nevertheless the total global energy consumed increased from 11977.8 mtoe in 2010 to 12274.6 mtoe in 2011 as did the total consumption of oil and gas from 6875.0 mtoe in 2010 to 6964.7 mtoe in 2011. The International Energy Agency (IEA) predicts that global oil demand will increase with an average growth of 438 mbbl (million barrels of oil) per year over the 2010 – 2016 period [Schlumberger Business Consulting, 2011].

Table 5.1: Global consumption of energy in millions of tonnes of oil equivalent [BPr]

	2010			2011		
	Oil & Gas	Other	Total	Oil & Gas	Other	Total
N. America	1799.0	964.9	2763.9	1808.7	964.6	2773.3
S. &C. America	416.2	202.8	619.0	428.2	214.3	642.5
Europe	1915.3	1023.4	2938.7	1889.2	1034.2	2923.4
Middle East	703.8	12.6	716.5	733.8	13.7	747.5
Africa	256.8	125.4	382.2	257.2	127.4	384.5
Asia Pacific	1783.8	2773.7	4557.6	1847.7	2955.7	4803.3
Total	6875.0	5102.8	11977.8	6964.7	5309.9	12274.6

The demand for oil and gas energy is dominated by transportation fuels and chemical industries [Schlumberger Business Consulting, 2011]. Aviation, shipping and automobiles rely heavily on petroleum, diesel and kerosene as their energy source. The IEA estimates the proportion of oil consumption for transportation fuels will rise from 53% in 2009 to 60% in 2035 [Schlumberger Business Consulting, 2011].

The predicted rise in hydrocarbon demand is best explained by population growth. The global population has increased exponentially over the last century where in 1927 the global population is estimated to be 2bn people and in 2011 is estimated to be 6.97bn [U.N., 2011]. The United Nations predict this will continue to increase into 2050 where the population is estimated to reach 9.3bn [U.N., 2011]. The challenges of a rising global population is a growing energy requirement. Yet oil and gas are finite resources, and the extent to which they are finite is depicted in Figure.5.2.

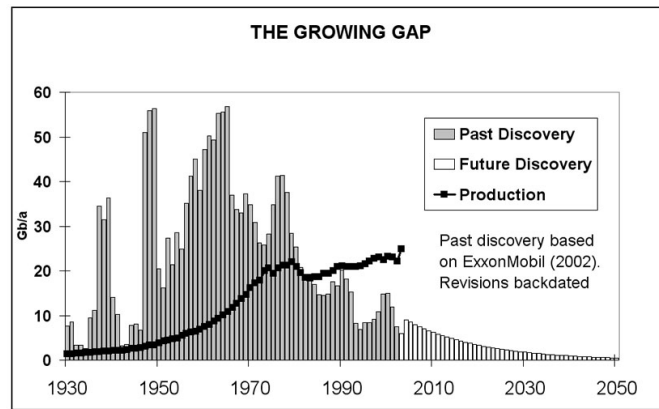


Figure 5.2: Evolution of the Drilling Envelope over time [Campbell, 2002]

The annual rate of discoveries of new recoverable reserves has declined since 1980, and is predicted to decline further [Campbell, 2002]. Furthermore the annual consumption rate has increased. Since the 1980s the annual rate of consumption is becoming increasingly greater than the annual discovery rate, and this phenomenon is known as the *Growing gap*. The oilfields in the future will become more challenging to develop, at greater cost with engineering challenges.

Figure 5.3 shows the production cost range on the vertical axis of energy resources verses the estimated volume available on the horizontal axis. The volume of hydrocarbons which have already been produced is shown to be just over 1000 bn bbl extracted at a cost range of USD (1–30)/bbl in 2008 value. The estimated volume of conventional and unconventional oil is estimated to be 6500bn barrels. Of the unconventional resources, much is heavy oil, oil shales, and viscous hydrocarbons which require enhanced

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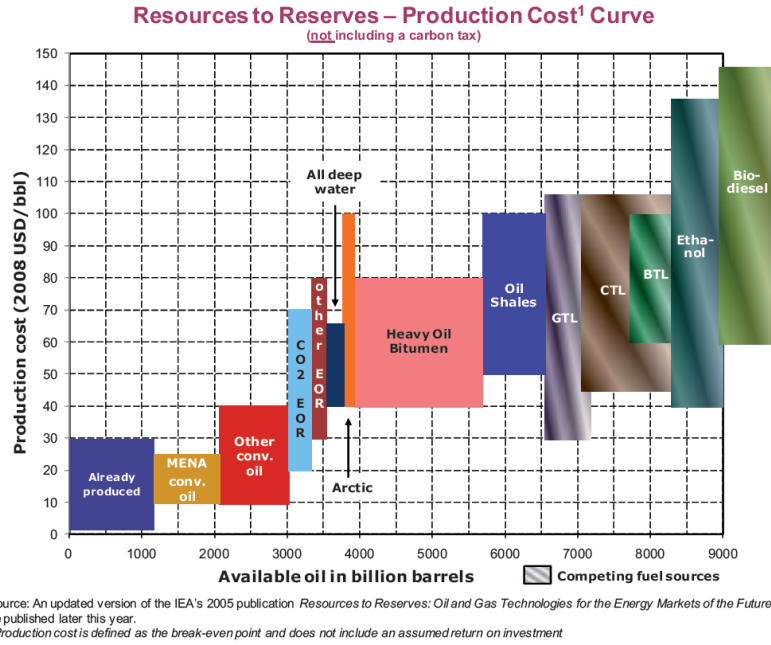


Figure 5.3: Cost of production [Babusiax et al., 2007]

oil recovery to extract, consequently the cost per barrel range of unexplored resources is much higher than conventional resources. This can be seen for the oil shales in Figure 5.3 where the estimated reserve volume is about 1000 bn bbl, but the production cost is in the range USD (50 – 100)/ bl.

As accessing reserves increases in difficulty, then so does the technological cost to supply the global demand. Hence the price per barrel of oil and gas will rise, unless technology enables the reduction in cost per barrel for unconventional resources. The process of developing an oilfield is discussed in the following subsection.

5.2.2 Oilfield lifecycle

Oilfield development is a lengthy and costly process typically lasting 20 – 50 years to develop a field [Braun et al., 2002]. Figure 5.4 displays the oilfield lifecycle stages of field development. At the beginning of the lifecycle, operators competitively bid for rights to explore for hydrocarbons from a land owner in a licencing round. The end of

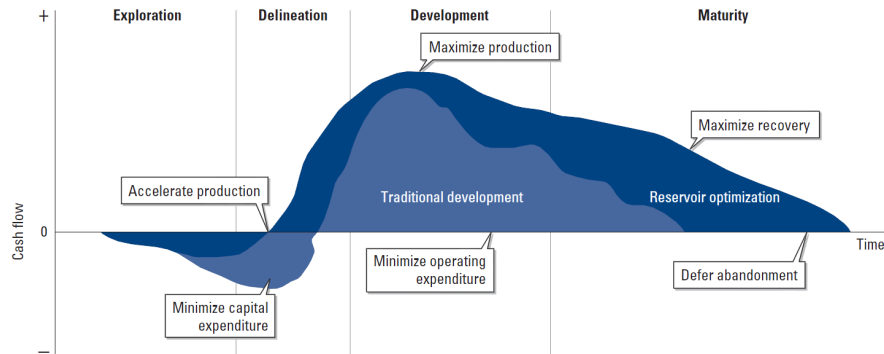


Figure 5.4: Oilfield lifecycle [Babusiax et al., 2007]

the lifecycle is abandonment of a reservoir, where production stops and the wells are safely shut in. This occurs when the marginal cost to extract exceeds the opportunity cost in developing another reserve and the field is said to be uneconomical. Nevertheless an abandoned field which had been uneconomical to develop in the past may become economical in the future if the market price of oil and/or gas rises to justify the cost of redeveloping a field. The oilfield lifecycle comprises of four key stages [Gluyas and Swarbrick, 2004]:

- Exploration,
- Delineation,
- Development
- Maturity.

Figure 5.4 graphically displays the cash flows associated with each stage in the lifecycle. Costs are committed to at the early stage in the field development where the risks are higher. Once the reservoir begins producing the field begins to generate revenue. For profit maximization over the lifetime of a reservoir the costs at the early stage of exploration and delineation must be reduced, and revenues are maximized by enhancing oil recovery volume.

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Petroleum *reserves* is defined to be hydrocarbons which are or will be recoverable and petroleum *resources* are hydrocarbons which are present in an oil or gas field without reference to the constraints and costs of accessing them [Babusiax et al., 2007]. Petroleum reserves are characterized by a 'Px' notation, where x denotes the probability likelihood that the true volume of a field exceeds the estimated figure. The Society of Petroleum Engineers (SPE) and the World Petroleum Congress (WPC) uses the terms: proven P90, probable P50 and possible oil P10 to quantify reserves [Babusiax et al., 2007].

Exploration

Exploration entails conducting geological and seismic surveys to search for new petroleum reserves. The cost of conducting exploration surveys depends largely on the environment and are in the order of USD5000/ km^2 [Babusiax et al., 2007]. The data from the surveys need to be processed and interpreted and the value is in identifying the size and location of hydrocarbon volumes and reducing the uncertainty in these estimates. Furthermore conducting surveys adds value to a field by changing the classification of a reserve from possible to probable. This allows oil companies to *farm out* their investment to other companies with expertise further down the lifecycle. At the exploration stage decisions are made on choices of reservoirs to exploit and future engineering costs of the entire lifecycle are considered. In the exploration phase simple *exploration wells* are drilled in order to cross as many rock boundaries as possible. The wells are meticulously logged for petrophysical data to determine the presence of hydrocarbon bearing formations. The assessment of the field at this stage will be classified as *probable oil*.

Delineation

If from the exploration stage there is an indication that hydrocarbons are present in the reservoir, drilling contractors and oilfield services companies are contracted to drill a set of delineation wells. Information such as fluid boundaries, stratigraphic layers, pressure distribution, geological faults, location and volume of porous and permeable rocks and thermodynamic characteristics of the entire reservoir is gathered from these wells [Lemouzy et al., 1991]. The main aim of this stage is to obtain a clear and accurate

geological picture of the field and assess its future value.

In the delineation phase the field development strategy is planned. The size and extent of wells to be drilled and the infrastructure required is decided at this stage. A cost benefit analysis is conducted, trading off the recoverable oil volume against the cost of engineering. In this stage a bidding round is conducted where oilfield services companies and drilling contractors bid for contracts for the development stage.

Development

In the development phase, the field development strategy is executed. Here a large number of wells are drilled to recover hydrocarbons from the reservoir. Producer and injector wells are drilled in accordance to well-plans. Once a well has been drilled, it is completed and ready for production. It is at this stage that the profits for an operator are made, and the vast majority of the oil wells are placed in the reservoir. Further seismic surveys may be conducted to obtain information relating to changes in fluid distributions in the reservoir.

Maturity

Over time an oil field enters a maturity phase where production declines as the oil reserves reduce. Here additional wells are drilled, but the marginal return on each new well will be diminishing. A point will be reached where drilling further is not economical. It would be more profitable for an oil company to divest the field or to focus efforts into another field. At this stage the wells are abandoned where environmental factors are considered. In wells with declining production, services such as artificial lift and enhanced oil recovery services increase the production of the wells.

5. STRATEGIC VALUE OF DIRECTIONAL DRILLING AUTOMATION

5.2.3 Oil and Gas industry stakeholders

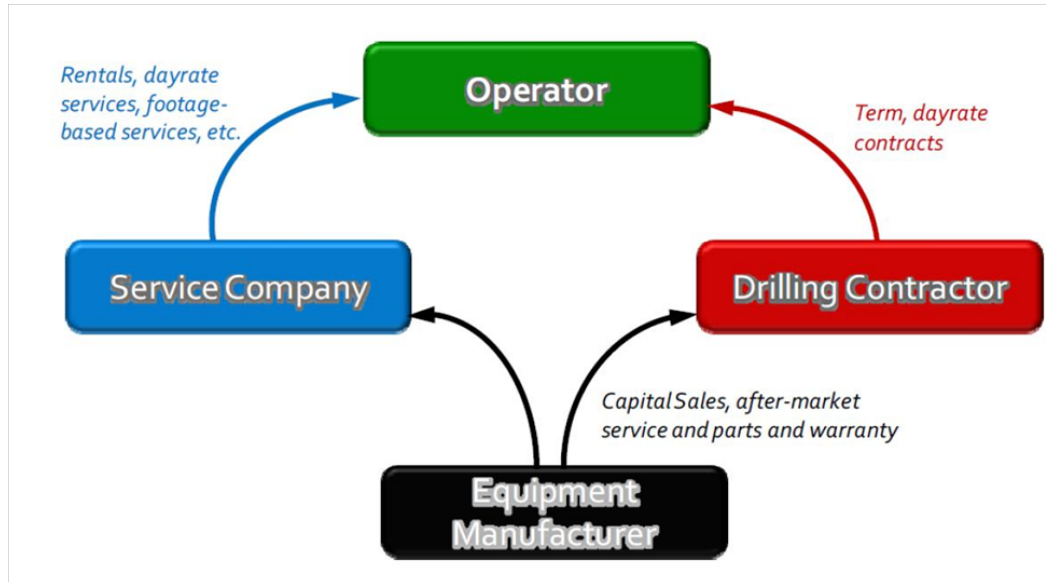


Figure 5.5: Supply Chain [Sadlier and Laing, 2011]

Figure 5.5 shows the relationship between the operator, drilling contractor, service company and equipment manufacturer in the oil and gas industry. Developing an oilfield successfully requires the effective collaboration of these entities, where each provides specialist equipment and services. The role of these entities in the oil and gas industry is described next.

Operator

The operator is either an international oil company (IOC) a national oil company (NOC) or an *independent* oil company. NOCs are state owned and usually own their own petroleum acreage which they exploit themselves or sell leases to other operators. Examples of NOC's include: Saudi Aramco, Statoil and Kuwait Oil Company. IOC's are large multinational companies sometimes called the *supermajors* which consist of: Total, Exxon Mobil, Chevron, BP, Royal Dutch Shell, Conoco Phillips and ENI. Globally 88% of reserves are controlled by NOC's and 6% by IOC's [Intelligence, 2004]. IOCs and NOCs are large vertically integrated companies which are involved in downstream activities such as the transportation, refining and marketing of refined products, in

addition to exploration and production. Independent oil companies are only involved in exploration activities. Operators bid for acreage where either a national oil company or a state land owner charges a fee for an exploration lease on a specified area of land. The entire revenue for: service companies, drilling contractors, and equipment manufacturers can be traced to the operator. The revenue for operators is related to the volume which can be produced from their reserves.

Service Company

The service company, contracted by an operator, provides services such as: seismic surveying, petrophysical logging, reservoir modelling and directional drilling which add value to the field development process. The service company often develop their own hardware such as steering assemblies and drill-bits as part of their offering. Typically service companies are rewarded on a day rate contract. This means that the service company is less sensitive to the price of oil or the risk associated with production rates. The service company would not generally take an equity stake in an oil prospect, such as receiving royalties in the final production of a well because this would align the interests of the service company with that of an operator. This would be advantageous for both parties since the risk to the operator is reduced and the service company can gain a greater share of the operators profit. Nevertheless this causes a conflict of interest between the service company and their other operator clients, where these operators are competing. In addition to day rates, service companies have turnkey contracts rewarded by key performance indicators (KPIs) intended to align their objectives with the operator's. Examples of KPIs are the footage of a well drilled, the deviation of a well from the plan and the percentage of the footage of a well in the oil reservoir for geosteered wells.

Service companies play similar roles to an operator company De Wardt [2010] in the case of *factory drilling* where a large volume of standardized wells are drilled in a field. Integrated project management teams in service companies manage the rig operations for national oil companies, which would otherwise be the role of operators. However when this occurs, unlike operators, the service companies are rewarded on turnkey based contracts.

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Drilling contractor

Drilling contractors are companies which manufacture and provide drilling rigs. These rigs are rented to an operator on a day rate or a termly rate similar to service companies. The rental cost of the drilling rigs is the most dominant operating cost to an operator for drilling a well. The selection of a drilling rig is very important to the type of drilling operation. For example drilling in deep sea environments requires specialist vessels to be constructed, whereas drilling offshore on continental shelves required rigs suspended on jacks. Land drilling rigs tend to be cheaper than offshore rigs. Moreover land rigs are easier to transport, quicker to set up, and require less space on board, as materials can be stored away from the rig. During a drilling campaign moving a rig from one location to another contributes to non-productive time. This cost is much larger for offshore wells where the logistics in moving a rig are more complex than moving a land rig. The preference for the contracts of the drilling contractor are in the day rate rentals. The type of rig employed has implications on the compatibility of the technology employed. Drilling contractors also provide rig personnel for the running and maintenance of the rigs.

Equipment manufacturer

An equipment manufacturer provides tools and technologies in the form of capital sold to a drilling contractor or a service company. Equipment manufacturers would charge maintenance costs and sell spare parts in the aftermarket. Examples of these products include: drill-pipes, stabilizers, top-drives and pumps. These products are mass produced products where the equipment manufacturers business model benefits from economies of scale.

5.2.4 Directional Drilling Market Analysis

To appreciate the impact automation can have on the directional drilling industry, the size of the market and its trends need to be assessed. In 2011 the oilfield services market size was USD 307bn and predicted to grow by 12% to USD343bn in 2012 [Spears and Associates, 2011]. This market grew year on year from 2003 with the exception of the global downturn in 2008 as is shown in Figure 5.6.

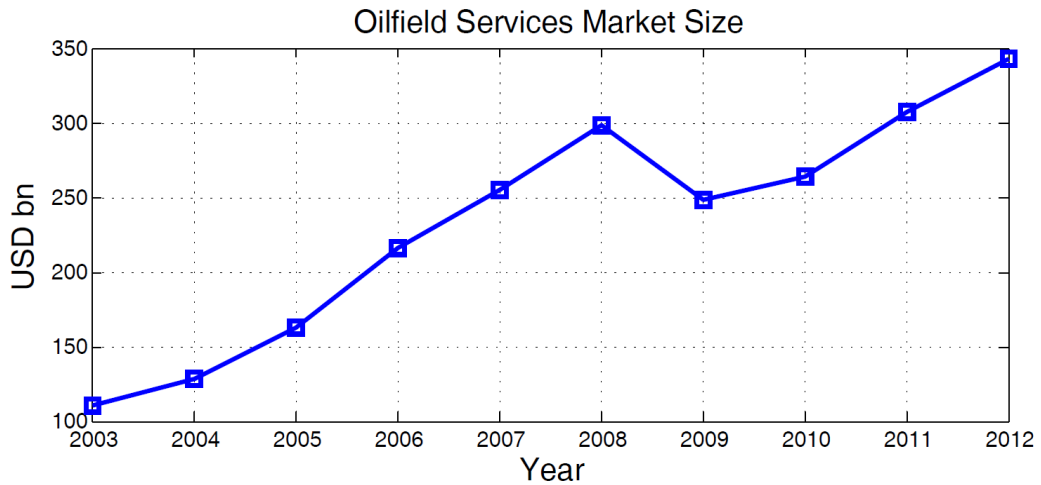


Figure 5.6: Oilfield services industry market size [Spears and Associates, 2011]

The directional drilling market size has grown with the oilfield services industry and has a value of USD11.3bn in 2011 and represents a share of 3.7% of the oilfield services market. The directional drilling market is dominated by rotary steerable and conventional mud-motors and turbine drilling tools as can be seen in Figure 5.7. Rotary steerable tools have gained market share from 12% in 2003 to 29% in 2011 to 3.2bn.

The market for directional drilling in 2011 is shown in Table 5.2 in terms of geographical share and of the share of companies which compete in this industry. The largest market for directional drilling services is the North American market which is worth USD5,000mn, representing a 43% share of the overall global market. The directional drilling market is dominated by four companies: Schlumberger, Halliburton, Baker

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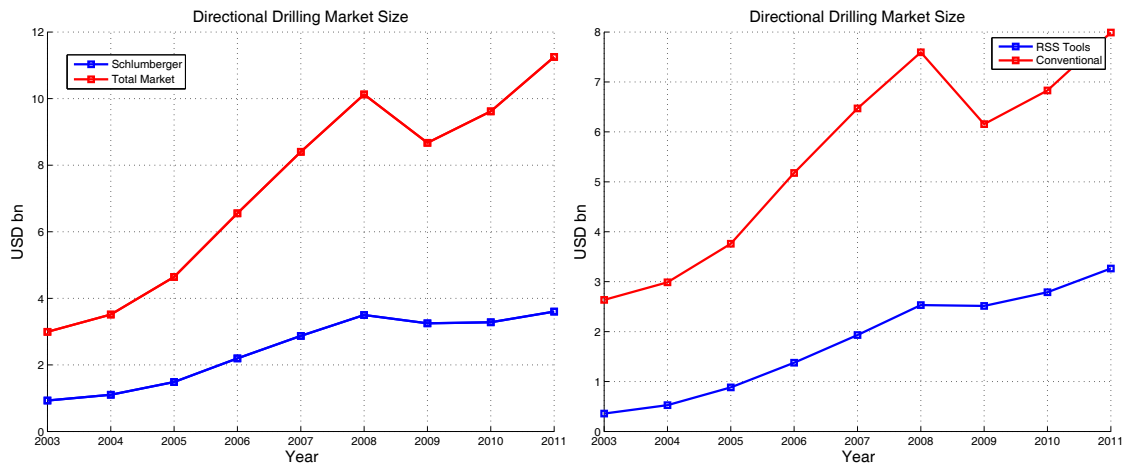


Figure 5.7: Directional Drilling Services market size [Spears and Associates, 2011]

Hughes and Weatherford, which compete for a 75% share of the global market. The largest of which is Schlumberger with a 32% share representing 3.6bn in 2011.

Table 5.2: Oilfield services industry market [Spears and Associates, 2011]

Directional Drilling Services	NaAm	LaAm	Eur Afr	ME Asia	Total	Split
Baker Hughes	900	300	450	250	1,900	17%
Halliburton	600	450	675	375	2,100	19%
Schlumberger	1,000	800	900	900	3,600	32%
Weatherford	400	150	300	150	1,000	9%
Others	2,100	100	300	200	2,700	24%
Total	5,000	1,800	2,625	1,875	11,300	100%

The major services constituting the directional drilling market are shown in Table 5.3 measured in USD m. The market for drilling with motors is 2.5bn and the market for RSS is 5bn. Rotary steerable tools have the largest market growth potential and the total market size for drilling with mud motors and rotary steerable tools combined is predicted to be USD7.5bn in 2012.

The largest market for directional drilling services is the North American market. The number of wells drilled and the length of feet drilled in this market over the period of 2003 – 2011 is shown in Figure 5.8 in terms of horizontal and vertical wells. Drilling

5.2 Business Background

Table 5.3: Market breakdown of directional drilling services [Spears and Associates, 2011]

	2005	2009	2012
Directional Svs	600	1,100	1,500
Motors	1,300	2,000	2,500
MWD	1,300	2,000	2,500
Rotary Steerable	900	2,500	5,000
Survey	200	200	300
Other	400	700	700
LWD	1,300	2,300	3,500

activity was significantly scaled down during the downturn of 2008 when the number of wells fell from 56,215 (by 38%) to 34,617. Nevertheless since 2008 the number of wells drilled has recovered to 50,306 in 2011. The reason for this increase is because of an increase in activity for unconventional resources such as shale gas drilling in North America [Schlumberger Business Consulting, 2011].

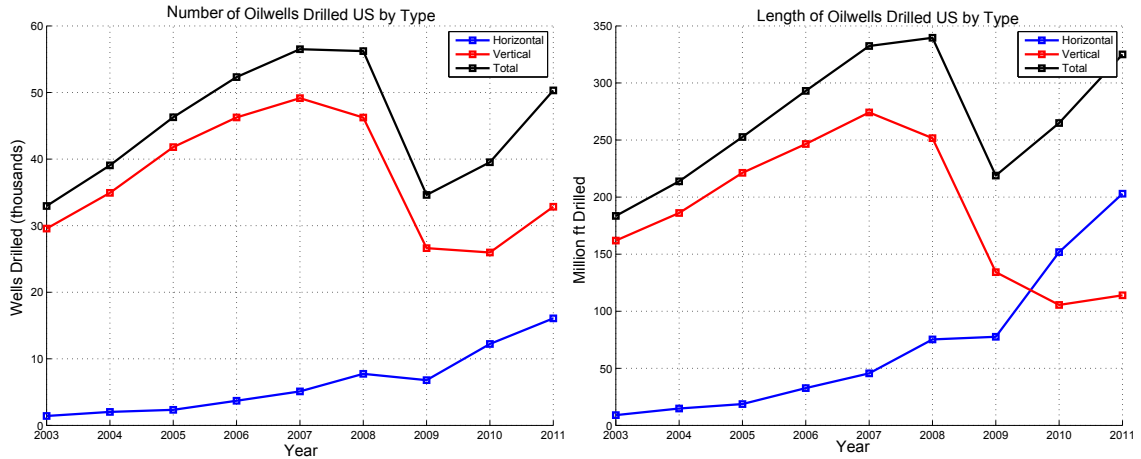


Figure 5.8: Directional Drilling Services market size [Spears and Associates, 2011]

The proportion of horizontal wells drilled in North America in 2003 was 4.3% and this has significantly gained market share, rising year on year to 32% in 2011. What is interesting is the proportion of feet drilled in horizontal wells in 2003 was 5% but in 2010 this proportion overtook vertical wells and in 2011 accounts for 62.4%. This proportion

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is likely to rise further as RSS tools gain more market share.

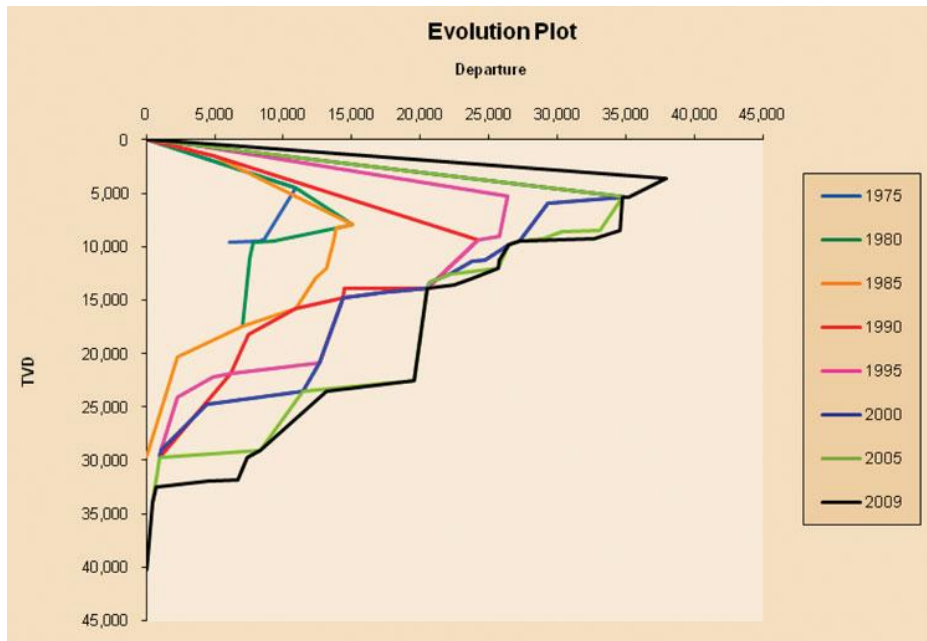


Figure 5.9: Evolution of the Drilling Envelope over time [Redlinger and McCormick, Mar], this graph is indicative of the advances in well construction technology

Wellbores can be categorized by their true vertical depth (TVD) and by their departure (which represents the horizontal distance from the platform to the furthest point in the well). A plot of TVD verses departure in feet is shown in Figure 5.9 from 1975 – 2009. An envelope in 5 year increments of all the world record wells at the time is shown. Figure 5.9 shows how technology over these years has enabled deeper and further reaching wells to be drilled and that there is a trend of the envelope being pushed out further. Enabling technology which allows the envelope to be pushed out creates a new market for that type of well. A company with the ability to drill to a record departure has a first mover advantage in that market and is far more likely to win a contract with an operator wishing to drill these wells. From 1995 to 2009 an extra 10,000ft of departure was added to the world record (see Figure 5.9). This can be explained with the emergence of RSS drilling tools.

5.3 Value Proposition

An oilfield services company (OSC) provides directional drilling services to IOC, NOC and independents. The service-provider-client relationship was discussed in Section 5.2.3. A service is of value to an OSC if it is of mutual value to its clients. For new technology to enter development, investment is needed to commercialize the products and services. For an OSC to invest in new technology, value must be demonstrated. These values can be tangible such as profit, and intangible such as improvements in safety. Value for an OSC propagates further downstream in the supply chain to an operator's production efficiency, which in turn reduces the cost of energy. This is the case when the cost of producing a unit barrel of oil can be reduced.

A value proposition is defined as having three components: increasing revenue; decreasing costs; being a platform for future value. The latter is less quantifiable than the other components as there is significant uncertainty in forecasting the future value. One way in which a new service can be a platform of future value is if it can create new markets and revenue streams which would not have otherwise existed.

The offering is for a general Spiral Reducing Attitude Control System (SRACS) and for a general Autonomous Trajectory Control System (ATCS). The values for each are explained below.

5.3.1 Spiral Reducing Attitude Control System

An attitude control system as described in Chapter 2 maintains the direction of propagation of a well. Direction and inclination sensors (D&I) spatially located behind the bit manifests a steady state spiral. A SRACS is defined to be a closed loop attitude hold control system which reduces its steady state spiral.

Hole spiraling induces tortuosity in the borehole contributing to undesirable friction. Brands and Lowden [2012] studies hole spiraling induced tortuosity, demonstrating that reducing the wavelength and amplitude of a spiral reduces borehole friction. This is important for casing liners and for tripping the drill-string into and out of the hole. Friction limits the depth a casing liner reaches into the wellbore under the influence of

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gravity. This subsequently limits the reach for horizontal wells. The main benefit for SRACS is for *extending the reach* for horizontal and deviated wells.

Reduced Costs

A SRACS reduces friction along a wellbore trajectory and reduces costs over a drilling operation by:

- Extending the lateral reach capability of individual wells, hence enabling greater coverage of oil and gas reservoirs which can be accessed from a single platform. This reduces capital infrastructure costs in an oil and gas reservoir since fewer platforms and pipelines would need to be constructed.
- Increasing the ability to run casing down-hole to the required depth due to friction in the well-bore. This friction causes further problems of adding difficulty to the subsequent cementing operation, and can limit the reach of the overall well.
- Reducing rig time by increasing the ROP, since reducing friction implies better transfer of weight to the bit.
- Automating the drilling of large sections of the well-bore which are at a constant orientation. This reduces the labour intensiveness of this part of the operation.
- Reducing the probability of a well being abandoned. When the drill-string cannot be recovered it is considered *lost in hole*, and the value of the tools and equipment on the drill-string are lost. Furthermore, if logging tools are lost in hole then there would be environmental levies if there are radioactive sources in the drill-string.

Increased Revenue

Drilling extended reach wells can increase the revenue through increasing production by the following:

- More feet of a well drilled in the hydrocarbon saturated reservoir rock. This would increase the production from a single well as there is more exposure to oil.
- Access to a greater volume of oil in the reservoir from a fixed platform.
- Reducing the labour intensiveness allows for labour to be distributed across other wells hence increasing the capacity per unit of labour.

New Markets

Drilling extended reach wells can be an enabler for creating new markets by:

- creating the ability to drill in regions which would not have been accessible otherwise. Areas where it is impractical to place a rig directly above a reservoir or nearby, can be accessed from extended reach wells.

5.3.2 Automated Trajectory Control

Automated Trajectory Control (ATS) is defined to be a system which is capable of guiding a directional drilling tool along a predefined well-plan by updating steering instructions in a closed loop manner. The main benefits of ATS considered here are in the *improved accuracy and consistency* and in allowing *ROP optimization* technologies to be used in horizontal and directional wells. Currently ROP optimization technology has been demonstrated only for vertical wells [Dunlop et al., 2011].

Reduced Costs

An ATS can reduce the costs of a drilling operation by:

- reducing nonproductive downtime by increasing the accuracy of well-placement and hence reducing the number of costly tripping out and sidetracks which would otherwise be the remedy for inaccurate drilling;
- enabling remote operations for drilling horizontal wells and hence reducing costs of having a directional driller on the rig.
- increasing automation and remote operations allowing for a skilled directional driller to oversee several wells at once.
- offloading routine tasks from the directional driller, thus allowing them to concentrate on more high value activities that require humans, such as handling contingencies. It may mean that a few highly skilled DDs overseeing several operations are needed rather than a lot of lower skilled DD's on each rig.
- allowing closed loop steering control to be used in conjunction with rate of penetration optimization in horizontal and directional wells and hence drilling faster means reduced rig time.

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Increased Revenue

An ATS can improve the revenue of a drilling operation by:

- increasing production through the accuracy of well placement in a reservoir and hence increasing the fraction of the well in more productive regions;
- increasing consistency in capital intensive factory drilling operations for repeatable wells and hence increasing labour capacity for difficult labour intensive high tier wells.

New Markets

An ATS can be an enabler for creating new markets by:

- allowing complex trajectory wells to be routinely planned. Presently the geometric profiles of many planned well-bores such as: 'S' wells, 'J' wells and vertical wells are standardized. Reducing the risk in drilling a wider class of geometric profiles redefines the types of wells which can be routinely planned.

5.4 Value Demonstration

This Section provides examples that quantitatively demonstrate some of the values described in Section 5.3 through the models described in Section 5.4.1. The examples given are synthetic and chosen to illustrate value applicable in general to real drilling situations which arise where these challenges are present. The following arguments are developed:

- The increase in production from reducing hole spiraling induced by an attitude control system for an individual extended reach well.
- The reduction in platform and infrastructure costs using extended reach wells to access multiple reservoirs.
- The benefit of increased well-bore accuracy for minimizing downtime operational expenditure.
- An estimate for the annual market size for integrating rate of penetration optimization and closed loop trajectory control.

5.4.1 Modeling the Value Proposition

To investigate the value of drilling automation in the field development process, synthetic case studies are developed to demonstrate the benefits. We consider four case studies related to drilling operations where the aforementioned technologies propose an increase in value. In this Section models used for the subsequent value demonstration analysis are introduced relating to the costs and revenues associated with directional drilling in well construction and the field development process.

Production Decline Curves

The production flow of hydrocarbons from oil wells is driven from a pressure gradient where reservoir pressure is greater than atmospheric pressure. Situations of highly viscous hydrocarbons, steam must be injected to heat the reservoir to lower the viscosity and stimulate production. When an operator has a producing oil-well as an asset it is important to know the production to estimate its economic value.

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The production $q(t)$ of a well in barrels at time t years is given by the relationship [Bourgoyne et al., 1986]

$$q(t) = q_0 e^{-\alpha t}, \quad (5.1)$$

where α represents the rate of decline. The total production is given by $Q = \frac{q_0}{\alpha}$. This model assumes an exponential decrease in production which is the case for many reservoirs. Other models need to be used for unconventional plays, where this model does not take into account when production can be stimulated.

Discounted Present Value

The discounted present value represents the value in today's money of a cash flow. This is based in the assumption that money at the present time is worth more than the same amount of money at a future date. The reason for this assumption is that cash which is available now can be invested into an investment A which will give a rate or return to the investor i_A . Hence when appraising investments one must *discount* the future values which feature in the cash flow of an investment B at a discount rate $i = i_A$ in order to determine if the value of investment B is greater than A . If the discounted present value of investment B discounted at i_A , is greater than 0, then this is the preferred investment. If this value is zero then both investments have the same value, and if negative, then investment A offers the greater rate of return.

For a continuous cash flow $F(t)$, the discounted present value is given by the Laplace transform

$$DPV(i) = \int_0^{\infty} F(t) \exp(-\lambda t) dt, \quad (5.2)$$

where $\lambda = \log(1 + i)$ for discount rate i . Considering the case the cash flow is proportional to the production rate, that is to say one obtains $USD F$ per barrel of oil q , then the net present value of production is given from substituting (5.1) into (5.2)

$$Q(i) = q_0 \int_0^{\infty} \exp[-(\alpha + \lambda)t] dt = \frac{q_0}{\alpha + \lambda}. \quad (5.3)$$

Wellbore Drilling Costs

A model for the total cost of drilling a well has been studied by Wilcox [2006]. The cost of a well as a function of the depth was found to be a non-linear relationship. The data in this study was for all US oil and gas wells between 1976 – 2004 from the Joint Association Survey (JAS). This study investigates the cost of geothermal wells using data for oil and gas wells in developing their model. The cost is a well USD C as a function of depth d in meters from Wilcox [2006] is

$$C(d) = \text{USD}1.5 \times e^{(\epsilon_1 d - \epsilon_2)} \text{mn}, \quad (5.4)$$

where the coefficients $\epsilon_1 = 7.2 \times 10^{-4}$ and $\epsilon_2 = 2.3$ with a correlation coefficient $R^2 = 0.984$. Since the study has been conducted in 2004 dollars, a multiplication factor of 1.5 is applied assuming a 5% rate of inflation.

The non-linear relationship between depth and cost implies that the marginal cost of the length of the well increases exponentially. This is because a well becomes increasingly more difficult to drill with depth since as tripping times increase. A similar exponential cost model is presented by Bourgoyne et al. [1986]. This model for the cost of a well is general, however this cost depends on the geographical location, geometry of well, and geological substructure.

Drilling Rig Day Rates

The cost to an operator for drilling a well is dominated by the cost of the drilling rig. The day rates for these are shown in Table 5.4. From this figure, non-productive time for a drilling operation can be quantified. For example, during tripping the drill-string out of the borehole, no drilling is taking place but the rig costs are still incurred. Furthermore the non-productive time in mooring a rig from one location to another can be measured at this rate.

When considering day rate contracts it is useful to know the proportion of the time on the rig which is productive for the drilling process. About 45% of the time on a rig is spent on the drilling and completion processes as shown in Table 5.5 Buehler and Charwinsky [2011].

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Table 5.4: Drilling Rig costs

Rig Type	Price '000 USD per day
Drillship	219 – 456
Semisub	252 – 413
Jackup	38 – 150
Barge	43 – 57
Land	27 – 24

Table 5.5: Drilling rig activities

Activity	Time Percentage
Drilling and Casing	45%
Rig Mobilisation	25%
Compliance and others	12%
Maintenance	13%
Idle time	5%

Platform Infrastructure Costs

In a field development programme, multiple *platforms* which are permanent constructions are installed connecting multiple wells in the field to pipelines. Offshore platforms often contain facilities to store the production before it can be transported away. The cost of constructing a platform and its infrastructure is the dominant cost for most large drilling operations for an operators expenditure. For example the Petronius platform in the Gulf of Mexico operated by Chevron and Marathon Oil was built in 2000 at a cost of USD500m [Texaco, 2000], and the Perdido platform built by Royal Dutch Shell in the Gulf of Mexico was built at a cost of USD1.8bn [Bogan, 2008]. Furthermore pipeline costs to transport production from the platform are significant where, for example, the Baku-Tbilisi-Ceyhan pipeline was built at a cost of about USD1.9mn per mile [Billmeier et al., 2004].

Research and Development Costs

When considering the benefits of a drilling automation project, the costs incurred in development and testing need to be assessed. In general there will be risks and

uncertainty with such an appraisal. The following model is used to capture the costs:

$$\text{DevCost} = \text{CapitalCosts} + \text{LabourCosts} + \text{TestingCosts} \quad (5.5)$$

The costs are calculated as an overestimate in order to gauge an order of magnitude figure. The capital costs are the costs of hardware required for the development of the automation system. Since the nature of the control systems are software based it is assumed these costs are of a lower order of magnitude compared to labour and testing costs. The labour costs are the costs of the personnel the corresponding overhead costs. The testing process incur the costs of each required field test, involving renting drilling rig and downhole equipment. Since the development process is iterative where lessons are continuously learned from each field test and the system is modified on each iteration, multiple field tests are required. For a more extensive estimate maintenance costs would need to be included in (5.5). For simplicity these costs are omitted in the following analysis. The costs are estimated based on a development time of a year as follows:

- cost of an electronic engineer at USD250k,
- cost of a control engineer at USD250k,
- cost of instrumented test wells drilled at 250k each.

The cost of each well is based on a 700m well drilled at a cost estimated from (5.4). Hence assuming 4 engineers and 8 field tests as an overestimate, the total cost of the development is estimated to be USD 3m. The implementation costs associated with the SRACS and ATS are trivial, since the SRACS requires only modification to the on-board electronics in the control unit of an RSS. The ATCS is software based on the surface which interprets the real time measurement data and provides a down link.

5.4.2 ERD Benefits from Spiral Reduction for Extra Production

The extended reach drilling example used is loosely based in the *OP – 11* well drilled in Sakhalin island, Russia in 2011 [Walker, 2012]. The purpose of this demonstration is to show that a SRACS extends the depth capability of drilling.

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Problem Description: A well is to be drilled using a push-the-bit RSS tool with a 8.5 inch bit. The well-bore geometry is shown in Figure 5.10. The entire horizontal section is in the reservoir rock in a soft unconsolidated formation which requires casing prior to production to stop wellbore collapse. The well is cased with a 7.5 inch liner string. Two scenarios for this well are drilled employing attitude control systems where one, of the scenarios uses a SRACS to investigate the NPV benefit of the algorithm in the subsequent production.

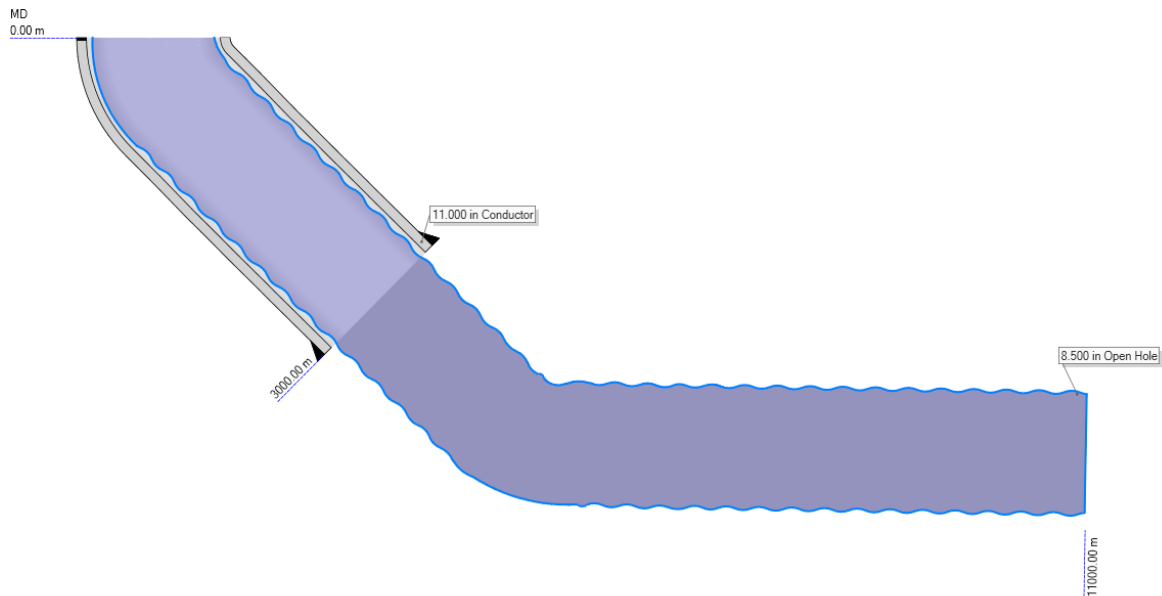


Figure 5.10: Wellbore geometry

Methodology:

1. **Trajectory Generation:** Using a push-the-bit steering assembly [Barr et al., 1995] simulated with *ST2* with a curvature capability of $8^\circ/100ft$ with D&I sensors located 10m behind the bit, two simulated paths are generated. The two paths differ as one is generated using a SRACS, and the other is using the attitude control system without the spiral reduction. The former created a spiral of amplitude 2° and a wavelength of 20m and the latter with an amplitude of 4° and the wavelength of 40m. Both paths initiate with an inclination of 45° , and

azimuth 326° . For both wells this attitude is held for 5000m, where the inclination set-point is changed to 89° and held to a total depth of 11,000m.

2. **Tripping-in Casing:** The first 3000m of the well was drilled using a 15 inch bit and cased with an 11 inch conductor pipe. The wellbore geometry is shown in Figure 5.10. A steel casing string of outer diameter 7.5 inches and an inner diameter of 6 inches is lowered into the well-bore under the influence of its own weight only. The block attached to the casing from the surface has a weight of 50kN. The tension in the cable (known as the *hookload*) that the casing suspends from as it is being lowered into the hole is measured against the depth of the casing. When the hookload is equal to the block weight, the casing can no longer fall into the wellbore under its own weight and this corresponding depth defines the reach of the hole. This analysis of hookload verses depth for this casing model and the two trajectories of interest was conducted using a drilling engineering software called Drilling Office.

3. **Demonstration of Benefit:** For each coefficient of friction, the difference in depth of the cased section in the horizontal well is assumed to produce oil at a rate of 90bbl per meter per day. The production of the well $Q(i)$ follows the decline model of equation (5.1) with $\alpha = 0.3$. The price of oil is taken at USD 100/bbl. From this it is assumed that 20% (USD20/bbl) contributes to the operators revenue and 2% ($p = \text{USD}2/\text{bbl}$) to the service company. The cost of the well $C(d)$ is taken to vary non-linearly with depth d and is modeled using equation (5.4). The production is discounted at a rate of $i = 0.1$, and it is assumed that there is a development cost $DevCost = \text{USD}3\text{m}$. From this the present value of the benefit minus the cost of the well and development of the SRACS is calculated. Fixed costs are not considered as it is assumed they are common to both wells. Hence the formula for the net present value for the service company of the wells as a function of the discount rate is:

$$NPV(i) = 365p Q(i)[d_2 - d_1] - [C(d_2) - C(d_1)] - DevCost \quad (5.6)$$

Results: The hook-load vs depth results for open-hole coefficient of friction of $\mu = 0.25, 0.3, 0.35$ for the trajectories generated with the SRACS and without are shown in Figure 5.11. The plot on the right shows the hook-load for tripping in the casing in the

5. STRATEGIC VALUE OF DIRECTIONAL DRILLING AUTOMATION

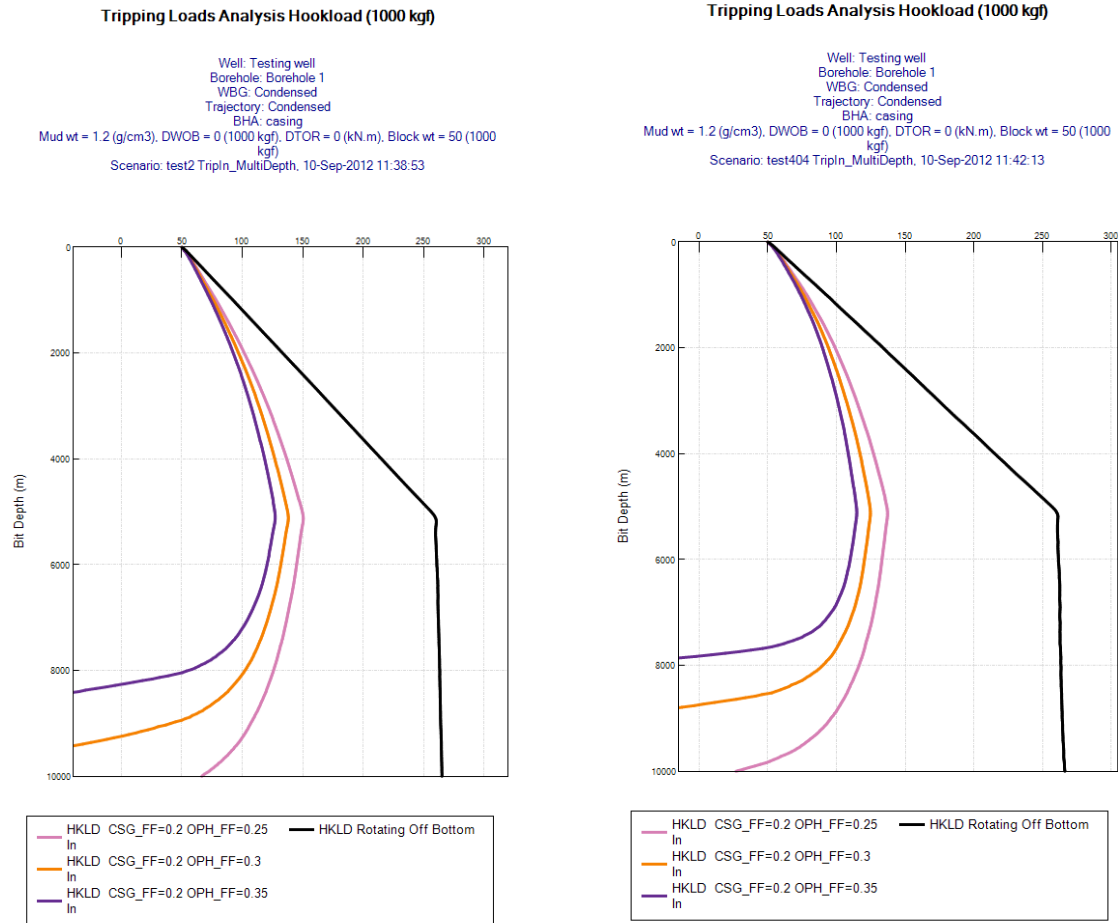


Figure 5.11: Hook-load vs depth

absence of spiral reduction for an attitude hold, and it can be seen that the depths at which the hook-loads are equal to the block weights are lower than the corresponding depths when the spiral reduction is applied.

A summary of the results is presented in Table 5.6. There is a value of USD 0.1 – 31.5m from the increase in production. It is also the case that the well that gives the greatest extended reach does not generate the most value since from equation (5.4) used to model the cost, the marginal cost increases with depth.

The main result in this example demonstrates that reducing spiral induced tortuosity,

Table 5.6: NPV for SRACS

μ	d_1	d_2	Δd (m)	Cost	Extra pmboe	NPV USDm
0.35	7800	8000	200	6.4	16.6	30.2
0.3	8400	8700	300	15.3	24.9	31.5
0.25	9900	10350	450	71.7	37.4	0.1

extends the reach of a well. In this example this is shown to increase the revenue for an operator and a service company. Nevertheless the production is proportional with the length of the well, and the cost of the well increases exponentially with depth. It must also be emphasized that once such a well has been drilled, many other wells can be sidetracked into the reservoir to obtain further production capacity. Moreover by reducing the friction, there is a greater transfer of weight to the bit, and hence a faster drilling time. This would reduce the cost per meter of a well and has not been considered in this model. In Table 5.6, the development cost of USD3mn has been considered in the NPV values, and for all three friction cases considered there is value demonstrated. It must be noted that if an SRACS is successful and scaled to more wells, the development cost will be further amortized. Extending the reach of a well allows reserves such as those in protected areas such as cities and national parks to be accessed from platforms placed away from these areas. Moreover multiple volumes of hydrocarbons can be accessed from a single platform. This is considered in the next example.

5.4.3 ERD Benefits from Reduced Infrastructure Costs

Extending the reach of individual wells generates value by reducing the infrastructure cost in a field development program.

Problem Description: Figure 5.12 displays two development scenarios of the same field. This example is typical of decisions made during the delineation stage of field development on where to place platforms, and representative of offshore fields. There are two equivalent reservoirs located 10km offshore each and separated by 20km. There is an existing pipeline as depicted in the figure which runs parallel to the shore. In the left image the technology for drilling a 10km well exists and in the right it does not. It

5. STRATEGIC VALUE OF DIRECTIONAL DRILLING AUTOMATION

is assumed that once a well is drilled into a reservoir, sidetrack wells can be drilled at the same production and cost for each case.

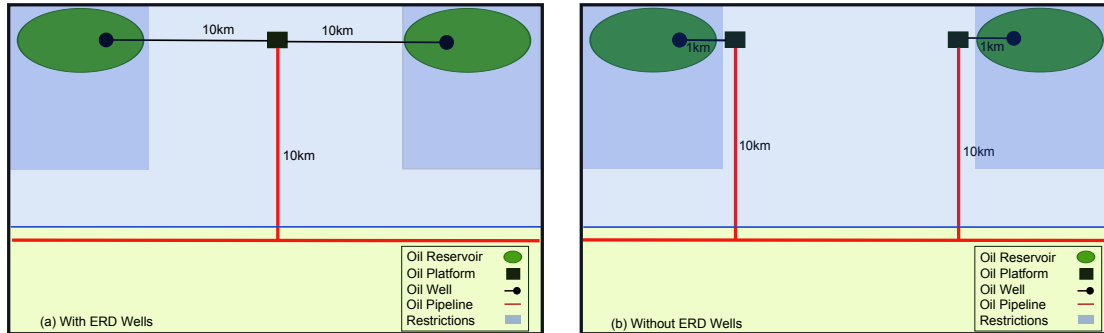


Figure 5.12: Placement of platform in field development

Methodology: The cost of the wells as a function of the depth is modeled by equation (5.4). It is assumed that the cost of a pipeline is USD2m per km and that the cost of installing a platform is USD1bn. The difference in cost of these two scenarios will demonstrate the value.

Results: From Table 5.7 there is a benefit of USD620m for minimizing the number of

Table 5.7: Value of SRACS for reduced infrastructure costs

Costs	Scenario 1	Scenario 2
Wells	400.06	0.62
Pipeline	20	40
Platforms	1000	2000
Total	1420.06	2040.62

platforms in a field. In general the cost of a platform for an offshore development is significantly higher than the drilling costs. The value which extending the reach of a well can bring is for increasing the coverage of each single platform.

5.4.4 Benefits of steering automation for increasing accuracy

The previous examples were concerned with the value of extending the reach of a well. In the following examples, the emphasis is on the benefits of automating the steering of

a borehole. In Chapter 4, a trajectory control system was presented which generated a sequence of commands to down-link to the steering assembly. The following example demonstrates the benefit of steering automation in terms of the accuracy in placing the well. Steering accuracy is required for avoiding collisions with existing wells, avoiding volumes of gas in a reservoir and for steering in the most productive zones. This example demonstrates the benefit of increasing the steering accuracy for minimizing the downtime in a rig process.

Problem Description: A hypothetical two-dimensional well of depth 2000m is to be drilled at a TVD of 100m with the aim of holding this depth. If the deviation from the path exceeds 10m then a sidetrack well would need to be drilled in order to correct.

Methodology: The two wells simulate automated trajectory control drilling in guiding the tool along the path. The deviation S_i from a path is modeled with initial deviation S_0 as a random walk

$$S_{i+1} \sim N\left(\frac{\mu - S_i}{\lambda}, \sigma\right), \quad (5.7)$$

normally distributed with mean $\frac{\mu - S_i}{\lambda}$ and standard deviation σ . The term $\frac{\mu - S_i}{\lambda}$ is the mean which has been shifted in order to revert the random walk towards the value μ , where λ is the reversion factor. The model of the automated trajectory control systems differ by the reversion factor to simulate a difference in accuracy. The allowed deviation from the well-plan is 10m and if this deviation is violated then drilling is stopped and the well is sidetracked. This simulation is repeated 1000 times and the number of sidetracks is recorded. When a sidetrack is needed, there is a cost associated where the drill-string would need to be tripped out resulting in non-productive downtime. The time taken to trip out a drill-string depends on the depth of the borehole. It is assumed that the cost of the rig time for a trip-out is a constant USD200k. This assumes a 12 hour time for tripping out, cementing, and tripping in again, for a rig which costs USD 400 per day to operate.

Results: Figure 5.13 shows graphically the trajectory for an individual simulation with parameters in Table 5.8. A total of 1000 runs are simulated and the results of the

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number of sidetracks given in Table 5.8.

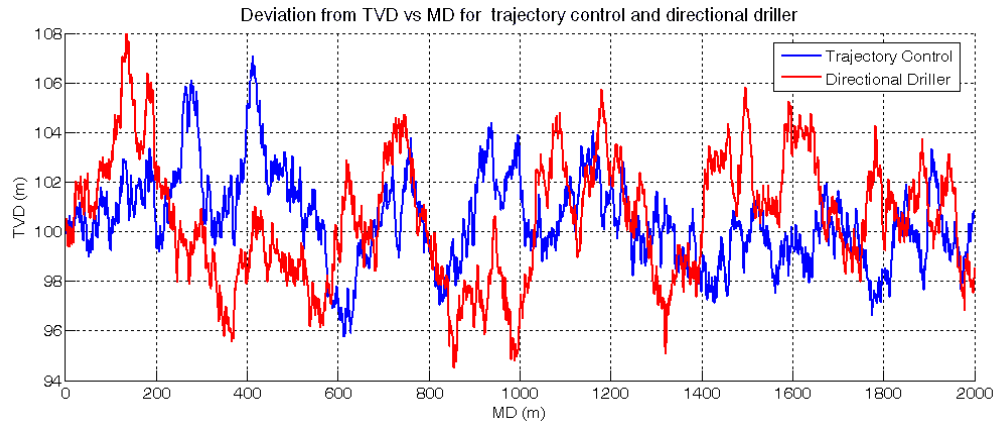


Figure 5.13: TVD hold random walk simulation

Table 5.8: ATCS simulation parameters and results

	Improved Accuracy ATCS	ATCS
Initial TVD (m)	100	100
Target TVD (m)	100	100
Depth (m)	2000	2000
Step Length (m)	1	1
Standard Deviation (m)	0.4	0.4
Reversion Factor (m)	50	75
Sidetracks	6	21
Cost (USDm)	1.2	4.2

It is shown that increasing the accuracy as measured by the reversion factor reduces the costs of sidetracking a well. Further benefits of improved accuracy accrue in the placement of a greater proportion of the wellbore in more productive regions of a reservoir hence increasing the production capacity.

5.4.5 Increased Rate of Penetration

Reducing the time taken to drill wells in the field development process reduces the costs. This is because much of the operational costs for the oilfield services company and drilling contractor are charged on day rates. A method for increasing the rate of

penetration of vertical wells is presented by Dunlop et al. [2011] by optimizing the hook-load and the surface RPM. This method is shown to increase the ROP by $\beta_{ROPO} = 32\%$, demonstrating a significant reduction in operational expenditure for the drilling process.

Problem Description:

ROP optimization is demonstrated for vertical wells saving 32% of rig time. For horizontal wells, this technique presents a challenge for directional drillers steering a bore-hole along a well-plan. This is because the changes in hook-load and RPM affect the steering capability of the drilling tool. Furthermore increasing the speed will increase the distance drilled between down-links and contribute to inaccuracy. In this example, the benefit of combining the use of ATCS and ROP optimization is estimated.

Methodology:

The most dominant operational cost in a drilling operation is the day rate to a drilling contractor for rig rentals. The annual saving from this rig time globally is estimated. The overall cost saving C is modeled by

$$C = 365n_{rig}\beta_{dr}\beta_mC_{rig}\beta_{ROPO}, \quad (5.8)$$

where n_{rig} is the number of active rigs, β_{dr} is the fraction of the time spent drilling, the share of the market for a typical oilfield services company is β_m , β_{ROPO} is the fractional saving of rig time from the use of ROP optimization and the weighted average day rate cost of a rig is C_{rig} .

- From Spears Oilfield Services Marketing Review Spears and Associates [2011] there were $n_{rig} = 921$ active directional and horizontal rigs in 2011.
- $\beta_{dr} = 45\%$ of the rig time is spent drilling activities.
- The largest 4 oilfield services companies which are involved in drilling activity have a combined 75% market share. This gives an average of a typical OFS company of $\beta_m = 18.75\%$ market share.

5. STRATEGIC VALUE OF DIRECTIONAL DRILLING AUTOMATION

- Taking the average land drilling day rate to be USD20K per day and a offshore rig rate to be USD150k per day. 77.9mft were drilled on land and 27.9mft offshore in 2011. This gives a weighted average rig day rate of $\text{USD}C_{rig} = 54\text{K}$ per day.
- The rig time saving is taken to be the demonstrated saving from ROP optimization [Dunlop et al., 2011], $\beta_{ROPO} = 0.32$.

Results:

- In 2011 there was 336,165rig days at a cost of USD 18.2bn.
- The fraction of this corresponding to drilling activity is USD7.2bn.
- The fraction of this corresponding to a typical OFS company is USD1.40bn.
- This gives a potential saving of $C = \text{USD}490.1\text{m}$ in a year

5.5 Conclusions

In this chapter the business context of the oilfield services industry is explained in the context of directional drilling automation. The demand for oil and gas is likely to grow along with the demand for horizontal directional drilling. The supply of oil is finite and as easy oil becomes depleted then more complex reservoirs need to be exploited. This comes at a much higher cost and requires more sophisticated technology.

A value proposition is made in the application of the attitude control system with spiral reduction for extended reach drilling. It has been demonstrated in a synthetic example for a Sakhalin type well that the depth in which a well can be cased is extended and this has implications for the amount of oil which can be produced from a reservoir. This example is shown to have a value to an OFS of between USD 0.1 – 30.2m per well from the extra production at a 10% discount rate for a reservoir which produces at 90bbl per foot per day, assuming a revenue of USD2/bbl. The value of SRACS for reducing costs in a field development programme is demonstrated from being able to extend the reach of the wells which can be drilled from a single platform and hence reduce the number of platforms needed to exploit a reservoir. This reduces the cost since the cost of offshore platforms are of a much higher order of magnitude to drilling

costs. Moreover, extending the reach capability of wells makes presently inaccessible fields possible to develop, allowing an operator to access these fields from platforms located away from these areas.

The argument for automated trajectory control is made for accurate drilling. Increasing the accuracy of a well reduces the number of sidetracks , thereby reducing the operational expenditure on rig time. A model for drilling a 2000m horizontal well to hold a vertical depth is demonstrated where the accuracy is measured by the mean reversion distance. Two systems characterized by their reversion distance are simulated for 1000 runs each for reversion distances of 50m and 75m. A sidetrack is needed when the borehole deviated beyond 10m of this target, and it is shown that there is a saving of USD3m in rig time based on a USD 400 per day rig.

Furthermore the benefit of steering automation has an annual global benefit of USD490.1m of rig time when integrated with rate of penetration optimization in horizontal wells.

5. STRATEGIC VALUE OF DIRECTIONAL DRILLING AUTOMATION

6

Conclusions and Discussion

This thesis presented a system for RSS tools to steer a borehole along a general curvilinear path. The design of the overall system incorporates the interaction between the directional driller, the drilling engineer and the geologist in oilfield operations. The geological targets determined by a geologist allow a drilling engineer to design a well-plan. This well-planning design process as described in Chapter 3, allows a drilling engineer to customize the shape of the well-plan. A directional driller executing this well-plan, chooses the appropriate commands to communicate to the down-hole steering assembly. A method for generating optimal commands was presented in Chapter 4. These optimal commands being attitude set points and determined from the tangent of a continuous correction path which coincides with the BHA's measured position and attitude. The attitude set-points are received by a down-hole attitude control system, presented in Chapter 2. The integrated system of the attitude and trajectory control is demonstrated in simulation to guide a modeled $8^\circ/100\text{ft}$ push-the-bit BHA to converge towards and track a $2^\circ/100\text{ft}$ well-plan to an accuracy of 0.5m.

Rotary steerable systems include navigation sensors, steering actuators and control systems in close proximity to the drill-bit. Attitude control laws perform an important role for drilling smooth straight borehole sections. In Chapter 2, simple attitude control laws were derived from representing the attitude as a unit vector, and proven to be stable. The tracking performance from this representation is independent of the attitude state. Several methods were summarized for determining the attitude from on-board sensors. The control laws were demonstrated in simple simulations, and later

6. CONCLUSIONS AND DISCUSSION

in a commercial drilling simulator *ST2*. Through these simulations the affect of hole spiraling caused by the spatial separation of the bit and the sensors was investigated where it was found that there is a linear relationship between the amplitude of this steady state spiral and the sensor-to-bit location. An extension to this attitude control system is provided whereby the attitude at the bit is predicted by knowledge of previous attitude states and control actions. It is shown in simulation that this reduced the steady-state spiral, however this does not eliminate it.

In further work, the control laws should be tested by more complete simulations subjecting the system to disturbances, measurement errors actuator dynamics and uncertainty. Further work for spiral reduction includes researching methods for determining a delay compensation online to achieve autonomous spiral reduction. Moreover further work on understanding the nature of the spiral to test the postulate of wavelength to sensor-to-bit distance is required.

Chapter 3 saw the development of an optimal well-planning strategy. Representing the well-plan as b-spline space curves allowed for the path planning problem to be formed as a convex optimization problem, allowing a global minimum solution to be found by interior point methods. Effects such as strain energy and torsion which contribute to tortuosity are represented as quadratic forms in terms of b-spline control points. Target positions and attitudes are represented as inequality constraints. A method for constraining an approximation of the curvature of the well-plan is presented as point-wise quadratic inequality constraints along the well-plan. An outer-loop to this optimization modifies the curvature constraint until the the well-plan meets the absolute curvature constraint which cannot be expressed as a convex function of the control points. The effectiveness of the well-planning is demonstrated in simulation, showing how paths can be created to sequential targets. Paths minimizing arc-length are shown and compared to paths minimizing strain energy. Furthermore the ability to constrain the absolute curvature of the well-plan is demonstrated.

In further work, the well planning problem should be extended to investigate including obstacle avoidance terms in the objective function. For the target constraints, the representation of a wider class of convex sets such as cones, and inequality constraints in

the direction of the well-plan in the target will be considered. The curvature capability of steering assemblies can be anisotropic in the reservoir, and dependent on the relative orientation to bedding planes. Further work will investigate capturing this anisotropy into the curvature constraint.

The constrained optimization methodology for well-planning is extended for use in a trajectory control system presented in Chapter 4. The model predictive control scheme recursively solves an optimal b-spline path, minimizing tortuosity and deviation from a reference b-spline represented well-plan, over a finite horizon defined by a finite number of control-points. The model for the attitude response is represented as an inequality constraint. The structure of the trajectory control scheme is a reference-governor system whereby optimal attitude set-points are determined for the stable inner-loop attitude control system presented in Chapter 3. The finite horizon optimization is multiobjective and weighted for tortuosity for generating smooth paths, and well-plan deviation. The effect of changing the weights to reflect the importance of either objective functions is shown by simulation for the finite horizon optimization and the MPC trajectory following. This has advantages for enhancing the capability of directional drillers, enabling them to tune the corresponding weighting coefficients in real time based on their experience and intuition for a drilling process. The trajectory control system is demonstrated with a commercial drilling simulator for following a $2^\circ/100ft$ well-path over 190m with varying curvature constraints and trade-off weights. The trajectory control system is shown to track the well-plan generated from the methods in Chapter 3, to within 0.5m.

In further work, the finite horizon optimization should be converted to a *slide-sheet* where data from the real-time drilling process is converted in table form to recommended steering commands which a directional driller can reference to assist the drilling process. This table is in the form of: position, direction, toolface, curvature, and attitude control setpoints which are indexed along the depth of the correction path. Moreover the objective function for well-plan deviation should be investigated for alternative penalty function. There is an advantage to having a deadzone in the cost in a neighbourhood about the well-plan. This is for situations where when the BHA is close to the well-plan, accuracy is less important than smoothness. In further work

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the stability of the trajectory control system should be investigated in addition to the effects of including objectives and constraints at the end of the finite horizon. Moreover the inclusion and tradeoff with other objective functions should be considered. For example with LWD tools, the local dip profile of the stratigraphic rock layers can be imaged. Often when steering a tool geologically it is desired to track the profile of the dip in preference to following a well-plan. In this situation the effect of including an objective function which penalizes the attitude deviation of the correction path from the projected dip profile should be considered.

Chapter 5 explained the oil and gas industry in which the directional drilling industry operates. Of the global energy consumption, a significantly large proportion (56.7%) is attributed to oil and gas. The global population is expected to rise by 2.3bn people over the next 40 years, and the proportion of oil and gas energy consumption attributed to transportation fuels is predicted to rise by 7% into 2035. Since the 1980s the production of oil and gas from existing fields is increasingly greater than the discovery rate of new reserves. Oil and gas reserves are finite, where there is estimated to be 6.5 trillion barrels of oil equivalent, of which 1 trillion barrels (the easy oil) has already been produced. Developing an oilfield takes 20 – 50 years, and requires the collaboration of Operators, Service Companies, Drilling Contractors and Equipment Manufacturers. The oilfield services industry is worth USD343bn if which the directional drilling industry has a value of USD11.3bn where the rotary steerable market size is USD5bn. In North America, 32% of wells drilled in 2011 are horizontal wells representing 62.4% of all the footage drilled. Wellbores are increasing in reach where where in 2009 wells had reached over 10km. The value of automation lies in enhancing the capability of oil-wells which is a necessity for meeting future energy demand.

Chapter 5 presents the value of trajectory control for rotary steerable systems. It is estimated that the cost for developing the attitude control system is USD3m where the dominant cost for development is the field testing experimentation. It is argued that the attitude control system by reducing hole-spiraling reduces friction and extends the reach capability of wellbores. Extending this reach enables access to otherwise inaccessible reserves where it is inconvenient or impossible to locate a drilling directly above

the reservoir. It is demonstrated that reducing the spiral amplitude of a $8^\circ/100\text{ft}$ push-the-bit RSS tool by half increases the depth in which a 7.5in casing liner can reach into a horizontal well of diameter 8.5in. This demonstration was conducted with a commercial drilling engineering software for a typical extended reach well for various friction coefficients. This extended reach allows a greater volume of hydrocarbons to be produced from a single well. Extending the reach of a well reduces the costs associated with a field development by reducing the number of platforms required to be installed, and the associated infrastructure such as pipelines.

The benefit of trajectory control for reducing costs from increasing the accuracy of placing a borehole is demonstrated. The drilling of a 2000m horizontal section held at a vertical depth is modeled as a mean reverted random walk. Two systems characterized by their reversion distance are simulated for 1000 runs each for reversion distances of 50m and 75m. In this example a sidetrack is required when the borehole is deviated beyond 10m of the target depth. Reducing the reversion distance from 75m to 50m provides a saving of USD3m in rig time based on a USD 400 per day rig. Moreover reservoirs where the oil-water and oil-gas contact are closer would require sidetracks at closer deviations than 10m. Furthermore accurate positioning of the wellbore in more productive regions of the reservoir increases the production capacity. In the final demonstration, an argument is made about the benefit of successfully integrating automated trajectory control with ROP optimization where an annual global benefit of USD490.1m of rig time cost savings is estimated for horizontal wells.

As the supply of oil and gas diminishes, the technological challenges in field development will increase. Today we consume more oil than ever before. Control systems in the oil and gas industry have huge potential to access difficult reservoirs, drill more accurately, and reduce the costs of field development.

6. CONCLUSIONS AND DISCUSSION

Appendix A

Selected Conference Papers

This appendix reproduces the following conference papers that contain work conducted during the Engineering Doctorate:

- Section A.1 presents, ‘Robust Linear Feedback Control of Attitude for Directional Drilling Tools’ [Panchal et al., 2010],
- Section A.2 presents, ‘Recursive Variable Horizon Trajectory Control for Directional Drilling Using Elliptical Helixes’ [Panchal et al., 2011b]
- Section A.3 presents, ‘Minimum Strain Energy Waypoint-following Controller for Directional Drilling using OGH Curves’ [Panchal et al., 2011a]

The content of these papers is not presented in the main body of the thesis, but is earlier work which forms part of the wider study.

A.1 Robust Linear Feedback Control of Attitude for Directional Drilling Tools

Abstract This paper describes the design of an inclination- and azimuth-hold controller for directional drilling tools typically used in the oil industry. A control input transformation that partially linearizes and decouples the plant dynamics is proposed. A pole-placement method is used to design the controller and an analysis of the stability robustness is performed using the small gain theorem. Results for a transient simulation of the proposed controller are also presented.

A. SELECTED CONFERENCE PAPERS

A.1.1 Introduction

In recent years the oil and gas industry has sought to extend the life of existing wells and to exploit smaller and hitherto difficult-to-commercialize wells by using directional drilling and rotary steerable drilling tools. An example is the current world record for extended reach wells set by a well that reached 40,320 ft of measured depth with a horizontal section of 35,770 ft in length using Schlumberger's PowerDrive Xceed[®] tool in Qatar's Al-Shaheen field [Pedersen et al., 2009].

Steerable tools enable the direction of the well propagation to be directed as required either by passive steering control from the surface using fixed-bend positive displacement motors [Yonezawa et al., 2002, Shoji et al., 1992] or by steering the wellbore propagation downhole using a rotary steerable system (RSS) drilling tool. Directional drilling, by either approach therefore is attitude control and is essentially concerned with controlling azimuth and inclination [Genevois et al., 2003]. Furthermore, attitude control with respect to measured depth leads to trajectory control for 3D wells. For example, the propagation of sidetracking wells from trajectory nudges is considered by Shengzong et al. [1999] and Li et al. [2009]; in the latter an open-loop optimal control strategy is proposed using the same kinematic plant model as stated in Zifeng and Jingyuan [2008].

This paper specifically addresses a generic (tool-independent) attitude control algorithm developed for use with directional drilling tools. The importance of attitude control is highlighted by Genevois et al. [2003] and Yonezawa et al. [2002] who propose control strategies for the direction of wellbore propagation based on holding tool-face angle. Genevois et al. [2003] discuss the need for closed loop 'shoot and forget systems' and state that the major challenge has been azimuth control. Many of the attitude control strategies presented in the literature are discussed in the context of specific tool architectures; Yonezawa et al. [2002], Genevois et al. [2003] and Yinghui [2002] are typical examples. Another interesting example is by Shoji et al. [1992] who describe a system for controlling attitude using two-way telemetry communication links with the surface where the steering correction, evaluated from the telemetry, is computed and then manually downlinked to the tool.

This paper presents an approach for designing a controller that has the control objective, once engaged, of automatically holding the inclination and azimuth of the

A.1 Robust Linear Feedback Control of Attitude for Directional Drilling Tools

tool at nominally constant values as the well is propagated. Usually the tool is manually controlled to the desired location and orientation, the azimuth and inclination are measured, and then these values are subsequently used as the demand azimuth and inclination for the control algorithm. The system is subject to considerable uncertainty and hence an analysis of the robust stability using the small gain theorem is performed. The paper also includes a description and results from a time domain simulation of the algorithm for a typical set of operating parameters.

The paper is structured so that the design methodology for the control algorithm is presented, starting with: a statement of the plant model; an analysis of its dynamics; use of the plant model for a subsequent frequency domain robust stability analysis; and finally its incorporation into a transient simulation using the plant model in non-linear form together with the necessary control architecture. Some discussion of standard directional drilling operating procedures are given where necessary.

A.1.2 Directional Drilling Tool

Tool Kinematics

The plant model is derived (see appendix) from kinematic considerations and ignores the lateral and torsional dynamics of the drillstring and bottom-hole-assembly(BHA). The resulting governing equations can be stated as follows:

$$\dot{\theta}_{inc} = V_{rop} (U_{dls} \cos U_{tf} - V_{dr}) \quad (\text{A.1})$$

$$\dot{\theta}_{azi} = \frac{V_{rop}}{\sin \theta_{inc}} (U_{dls} \sin U_{tf} - V_{tr}), \quad (\text{A.2})$$

where

- θ_{inc} is the inclination angle
- θ_{azi} is the azimuth angle
- U_{tf} is the tool-face angle control input
- U_{dls} is the ‘dog leg severity’ or curvature
- V_{dr} is the drop rate disturbance ($V_{dr} = \alpha \sin \theta_{inc}$)
- V_{tr} is the turn rate bias disturbance
- V_{rop} is the rate of penetration and is an uncontrolled parameter.

A. SELECTED CONFERENCE PAPERS

There is one further aspect of the tool dynamics that needs to be added to the plant model depending on whether the tool is what is known as a push- or point-the-bit tool. As stated the plant model is purely kinematic and takes no account of the dynamics of the BHA, as they are significantly faster than the dynamics of the borehole propagation. This is a reasonable assumption for a push-the-bit tool, in which the actuator on the tool pushes directly against the formation resulting in a high bandwidth for the toolface response. However, for a point-the-bit tool, the tool-face responds slowly because of the presence of significant dynamics (typically the bit has to machine laterally as the sleeve used to point the bit rotates into the desired pointing position), the bandwidth of the tool-face response can be of a similar order of magnitude to the band-width of the trajectory response. Therefore, for a point-the-bit tool the tool-face dynamics cannot be ignored. This paper assumes the tool-face dynamics are significant and discusses how to include their effect on the design and stability analysis (see Section A.1.3).

The Drilling Cycle

Many drilling tools will always respond with their maximum curvature response, K_{dls} , when drilling. Therefore, to generate curvatures less than K_{dls} , the tool is made to drill in drilling cycles (much the same idea as a duty cycle in power electronics or pulse-width-modulation) in which the drilling time is quantized into regularly spaced intervals which are further proportioned into neutral and bias periods. In the neutral phase the tool-face U_{tf} input is cycled at a constant rate such that the net trajectory response of the tool is approximately a tangent with zero net curvature, and in the bias phase the tool-face is held constant and the tool responds with a net curvature equal to K_{dls} . Consequently the average curvature over one drilling cycle can, in principle, be varied anywhere between zero and K_{dls} . The ratio of the neutral to bias phase in the drilling cycle is referred to as the % steering ratio, and therefore in all the subsequent equations in this paper the input U_{dls} implies the product of the steering ratio and K_{dls} . K_{dls} is therefore a plant parameter proportioned by the steering ratio to arrive at the control input U_{dls} .

On-Tool Instrumentation

It is assumed that the tool is equipped with its own attitude sensor (known in the industry as a D&I, direction and inclination sensor) instrumented with two three-axis

sets of accelerometer and magnetometer transducers. Using these globally referenced (Earth's gravity and magnetic fields) transducers, the sensor evaluates continuously the tool attitude in terms of azimuth and inclination. A factor to take into account is that the sensor datum and the inertial datum are not spatially coincident and therefore the attitude measurements have an inherent delay built into them, as elaborated in Section A.1.3.

A.1.3 System Modeling

Plant Model

The tool kinematics are non-linear with two state variables (azimuth and inclination) and two inputs (tool-face and steering ratio, see Section A.1.2). It can also be seen that the azimuth response is coupled to the inclination response by the sine of the inclination term in the denominator of the expression factoring the azimuth governing equation.

Consider (A.1) and (A.2) with the drop and turn disturbances removed:

$$\dot{\theta}_{inc} = V_{rop} U_{dls} \cos U_{tf} \quad (\text{A.3})$$

$$\dot{\theta}_{azi} = \frac{V_{rop}}{\sin \theta_{inc}} U_{dls} \sin U_{tf}. \quad (\text{A.4})$$

These represent the dominant dynamics of the tool response. The following transformation is proposed:

$$U_{tf} = \text{ATAN2}(U_{azi}, U_{inc}) \quad (\text{A.5})$$

$$U_{dls} = K_{dls} \sqrt{(U_{azi})^2 + (U_{inc})^2}, \quad (\text{A.6})$$

where U_{azi} and U_{inc} are virtual controls for the azimuth and inclination respectively. Substituting (A.5) and (A.6) into (A.3) and (A.4) gives the open loop dynamics as

$$\dot{\theta}_{inc} = V_{rop} K_{dls} U_{inc} \quad (\text{A.7})$$

$$\dot{\theta}_{azi} = \frac{V_{rop}}{\sin \theta_{inc}} K_{dls} U_{azi}. \quad (\text{A.8})$$

Therefore the control transformations, (A.5) and (A.6), partially linearize and decouple the governing equations.

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U_{tf} Actuator Dynamics

The model needs some modification to take into account a lag on the tool-face input response resulting from the actuator dynamics. It is assumed that the actuator dynamics can be modeled as a first order lag on the control inputs with a unity gain and a time constant of τ_d . The Laplace transfer function is thus

$$h_{lag} = \frac{1}{1 + \tau_d s} \quad (\text{A.9})$$

Measurement Delay

There are measurement delays on the feedback lines for the azimuth and inclination loops. These delays arise because the sensors which measure the change in azimuth and inclination are located a fixed distance behind the bit, and therefore there is a time delay dependent on V_{rop} , between the drilling of new hole by the tool and the measurement of the response azimuth and inclination. The delays are modeled by second order Padé approximants with the delay, τ_1 , corresponding to the chosen operating point V_{rop} :

$$h_1(s) = \frac{1 - s\tau_1/2 + s^2\tau_1^2/12}{1 + s\tau_1/2 + s^2\tau_1^2/12}. \quad (\text{A.10})$$

Actuation delay from U_{tf} quantized input

As explained in Section A.1.2, the demand tool-face U_{tf} is actually applied over a drilling cycle. This effectively introduces a sampling in the tool-face input. Hence a constant delay equal to half the drilling cycle period is introduced. This effect is again modeled using second order Padé approximants with the delay, τ_2 , equal to one half of the drilling cycle period, that is

$$h_2(s) = \frac{1 - s\tau_2/2 + s^2\tau_2^2/12}{1 + s\tau_2/2 + s^2\tau_2^2/12}. \quad (\text{A.11})$$

A.1.4 Control design

Linear Model

Linearizing (A.7) and (A.8) about a nominal inclination angle, $\hat{\theta}_{inc}$, gives the state-space relations

$$\dot{x}_{inc} = au_{inc} \quad (\text{A.12})$$

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$$\dot{x}_{azi} = cx_{inc} + bu_{azi}, \quad (\text{A.13})$$

where $a = V_{rop}K_{dls}$, $b = a \csc \hat{\theta}_{inc}$ and $c = -b \cot \hat{\theta}_{inc}$. Taking the output to be $y = [x_{inc}, x_{azi}]^T$, the nominal transfer function is then

$$G_0(s) = \begin{bmatrix} \frac{a}{s} & 0 \\ \frac{ac}{s^2} & \frac{b}{s} \end{bmatrix}. \quad (\text{A.14})$$

This can be factorized into:

$$G_0 = G_{i1}G'_0G_{i2} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{s} \end{bmatrix} \begin{bmatrix} a & 0 \\ ac & b \end{bmatrix} \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & 1 \end{bmatrix}. \quad (\text{A.15})$$

The Proportional plus Integral (PI) controller

It is proposed to use a PI controller. The gains are determined based on the nominal plant, G_0 , where the actuator and measurement dynamics are ignored. The proposed scheme is shown in Fig. A.1.

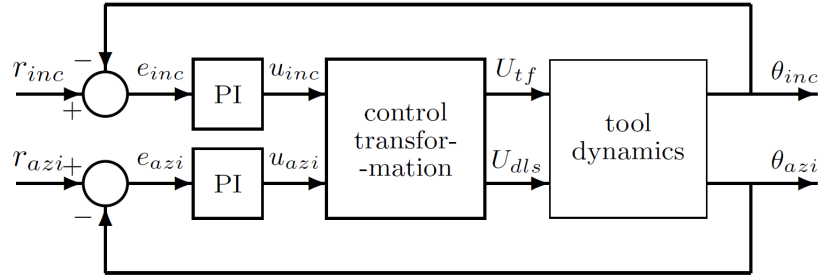


Figure A.1: PI Control Scheme

The PI controller for each channel is:

$$u_{inc} = k_{pi}e_{inc} + k_{ii} \int_0^t e_{inc} dt \quad (\text{A.16})$$

$$u_{azi} = k_{pa}e_{azi} + k_{ia} \int_0^t e_{azi} dt, \quad (\text{A.17})$$

where $e_{inc} = r_{inc} - x_{inc}$ and $e_{azi} = r_{azi} - x_{azi}$.

Closing the loop, in state space form, $\dot{x} = A_clx + B_clr$, with state vector

$$x = (x_{inc}, x_i, x_{azi}, x_a) \quad (\text{A.18})$$

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where $x_i = \int e_{inc} dt$ and $x_a = \int e_{azi} dt$ are the states of the two PI controllers, we have the nominal closed-loop system matrices:

$$A_{cl} = \begin{bmatrix} -ak_{pi} & ak_{ii} & 0 & 0 \\ -1 & 0 & 0 & 0 \\ c & 0 & -bk_{pa} & bk_{ia} \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad B_{cl} = \begin{bmatrix} ak_{pi} & 0 \\ 1 & 0 \\ 0 & bk_{pa} \\ 0 & 1 \end{bmatrix}. \quad (\text{A.19})$$

From A_{cl} the nominal closed-loop characteristic equation is independent of c and is as follows:

$$q(s) = (s^2 + aK_{pi}s + aK_{ii}) (s^2 + bK_{pa}s + bK_{ia}). \quad (\text{A.20})$$

If it is possible to operate at near-constant V_{rop} and θ_{inc} , then the PI controller gains can be chosen by pole placement. This method assumes an ideal fourth order characteristic equation of the form

$$q(s) = (s^2 + 2\zeta\omega_a s + \omega_a^2) (s^2 + 2\zeta\omega_i s + \omega_i^2) \quad (\text{A.21})$$

where ω_a and ω_i are the closed-loop azimuth and inclination natural frequencies respectively. If the damping ratio, ζ , is set equal to $1/\sqrt{2}$ to ensure a maximum rise time with no resonance and minimal overshoot (5%) then, by equating (A.20) and (A.21), the gains for the PI controllers in the SISO azimuth and inclination feedback loops as functions of the ω_a and ω_i natural frequencies and gain scheduled V_{rop} and K_{dls} operating point can be expressed as:

$$k_{ii} = \frac{\omega_i^2}{a}, \quad k_{pi} = \frac{\sqrt{2}\omega_i}{a}, \quad (\text{A.22})$$

$$k_{ia} = \frac{\omega_a^2}{b}, \quad k_{pa} = \frac{\sqrt{2}\omega_a}{b} \quad (\text{A.23})$$

Unmodeled Dynamics

Using (A.22) and (A.23) it is possible to design the four feedback gains using pole placement to ensure nominal stability at a given plant operating point. However, for each design operating point, for a given set of gains, it is also important to assess the robust stability of the design.

When the lags and delays are combined with the tool dynamics and the controller, the open-loop system becomes:

$$L(s) = G_1(s)K(s), \quad (\text{A.24})$$

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where $G_1 = H_2 G_0 H_1 H_{lag}$ is the model for the actual plant, and $H_i = h_i \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ where $i = 1, 2, lag$, and the h_i terms are defined in (A.9), (A.10) and (A.11) respectively. A block diagram representation of G_1 is shown in Fig. A.2.

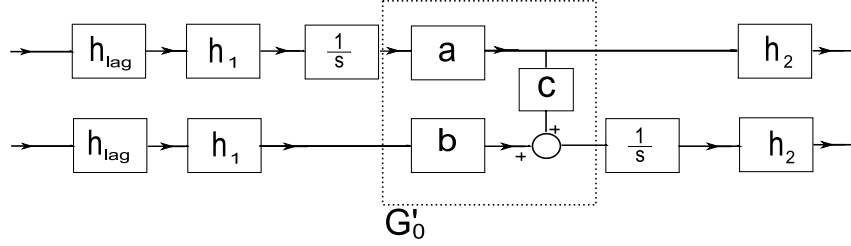


Figure A.2: Open loop plant G_1 with the integrators factored out of the nominal plant

Robust Stability Analysis

The closed-loop system, (A.19), is a function of the parameters V_{rop} and K_{dls} through the open-loop plant pole-position parameters a , b and c . It is the case that V_{rop} and K_{dls} are subject to significant uncertainty as V_{rop} is only indirectly controlled by the directional driller using weight on bit, and the K_{dls} can only be approximately estimated from off-line analysis and previous field experience for given drill bits and field locations.

As a working assumption, V_{rop} can be controlled crudely to within ± 50 ft/hr, and K_{dls} to within $\pm 2^\circ/100$ ft for a typical nominal K_{dls} of $8^\circ/100$ ft. Therefore, because of the large degree of uncertainty on the V_{rop} and K_{dls} parameters it is wise to perform a robust stability analysis.

Here the actual plant model is taken to be $G_1 = G_0 + \Delta$ where G_0 is the nominal plant given by (A.14) and Δ is the unstructured uncertainty arising from the delays, actuator lag, and uncertainty in V_{rop} and K_{dls} .

Because the delays and lag are scalar functions, the integrators in the plant can be factored outside the plant as shown in Fig. A.3 and this is represented by:

$$G_1 = G_{i1} G'_1 G_{i2} \quad (\text{A.25})$$

where $G_{i1} = \text{diag}(1, \frac{1}{s})$, $G'_1 = H_2 G'_0 H_1 H_{lag}$ and $G_{i2} = \text{diag}(\frac{1}{s}, 1)$.

Since G_{i1} and G_{i2} can be factored outside the plant, they can be extracted from the uncertainty, as shown in Fig. A.4. Hence the unstructured uncertainty block, Δ ,

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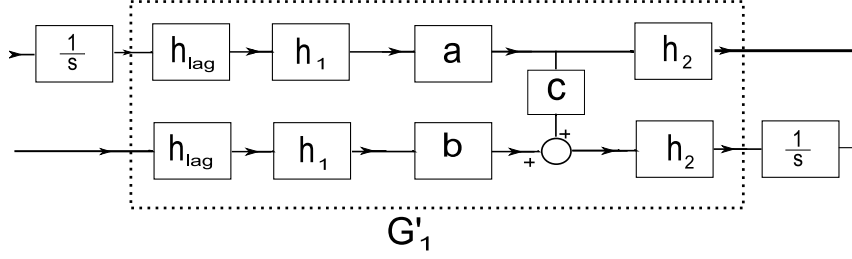


Figure A.3: Open loop plant with integrators factored outside

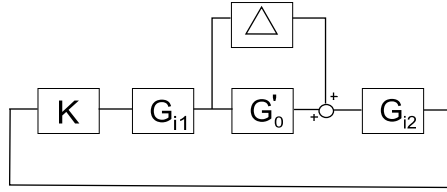


Figure A.4: Additive Uncertainty

can be redefined as:

$$\Delta = G'_1 - G'_0. \quad (\text{A.26})$$

With reference to Fig. A.4, let $K' = G_{i2}KG_{i1}$. From the small gain theorem [Skogestad and Postlethwaite, 2005, p.303], the system is robustly stable if and only if:

$$\bar{\sigma}(K'(j\omega)S(j\omega))\hat{\Delta}(\omega) < 1, \quad \forall \omega, \quad (\text{A.27})$$

where $S = (I + G'_0K')^{-1}$ is the closed-loop sensitivity function, $\bar{\sigma}(A) = \max \sqrt{\lambda(A^\dagger A)}$ denotes the maximum singular value, and

$$\hat{\Delta}(\omega) = \max_{\Delta \in \mathbf{\Delta}} \bar{\sigma}(\Delta(j\omega)) \quad (\text{A.28})$$

where $\mathbf{\Delta}$ denotes the set of all Δ .

To calculate the bound $\hat{\Delta}$ on $\mathbf{\Delta}$, the actual plant model G'_1 is perturbed in the parameters V_{rop} and K_{dls} :

$$\Delta = G'_1(V_{rop} + \delta V_{rop}, K_{dls} + \delta K_{dls}) - G'_0(V_{rop}, K_{dls}). \quad (\text{A.29})$$

The values of the bound were found by a numerical grid search over a range of $150 \text{ ft/hr} \leq V_{rop} \leq 250 \text{ ft/hr}$ and $6^\circ/100 \text{ ft} \leq K_{dls} \leq 10^\circ/100 \text{ ft}$, for the frequency range $10^{-5} \leq \omega \leq 10^2$.

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Table A.1: Design parameters and operating point values

Parameter	Value
θ_{inc}	90°
V_{rop}	200 ft/hr
K_{dls}	8°/100 ft
τ_1	corresponding to 10 ft @ V_{rop}
τ_d	120 s
τ_2	90 s
ω_a	$4\pi/5 \times 10^4$ rad/s
ω_i	$16\pi/25 \times 10^4$ rad/s

Consider the design parameters and operating point values given in Table A.1. These values imply, based on (A.22) and (A.23) the following design of gains:

$$\begin{aligned} K_{pa} &= 4.5821 & K_{ia} &= 8.14 \times 10^{-4} \\ K_{pi} &= 3.6657 & K_{ii} &= 5.21 \times 10^{-4}. \end{aligned}$$

The values of $\bar{\sigma}(K'S)\hat{\Delta}$ are found over the frequency range and are shown in Fig. A.5. In addition, the green plot in Fig. A.5 shows $\hat{\Delta}$ at each frequency. The red line in Fig. A.5 corresponds to $\bar{\sigma}(K'S)\hat{\Delta} = 1$.

From the figure,

$$\max_w \bar{\sigma}(K'(j\omega)S(j\omega))\hat{\Delta}(\omega) = 0.7358 \leq 1, \quad (\text{A.30})$$

and hence the small gain theorem is satisfied. Thus the IAH controller for this model is robustly stable for the given frequency and perturbation range.

Transient Simulation

Using the plant, control architecture and frequency domain analysis design previously described a Simulink[®] transient simulation was created to test directional drilling using the attitude control algorithm to hold azimuth and inclination at 270° and 90°, respectively. The simulation modeled the drilling cycle and accelerometers and magnetometers used to derive the azimuth and inclination measurements explicitly and also included all the system delays and lags previously discussed. The gain values used were those derived in the frequency analysis. The transient results given are for the

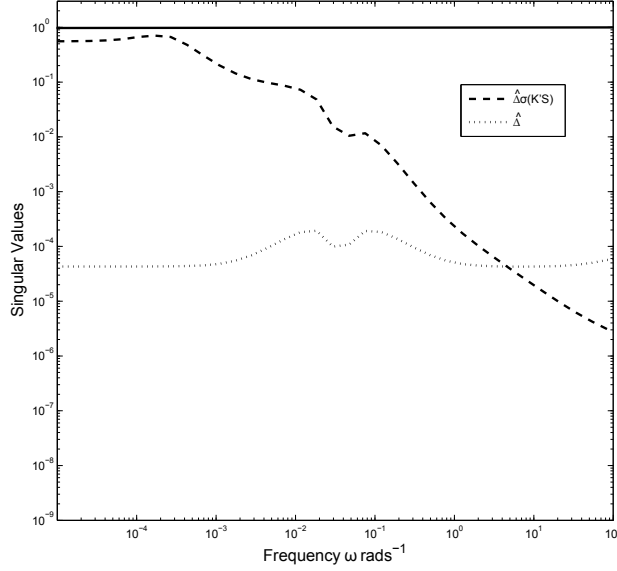


Figure A.5: Singular value plots for robust stability analysis

case where the tool has a drop bias (in the global reference frame) of $2^\circ/100$ ft and a turn bias (in the tool reference frame) of $0.5^\circ/100$ ft. Note that build generally refers to change of inclination with respect to measured depth, and turn refers to change of azimuth with respect to measured depth.

It can be seen from Fig. A.6 that the tool attitude converges to the target value as required. The typical objective of drilling lateral wells can be seen to have been achieved in Fig. A.7 where the true vertical depth (TVD) remains constant after 500 ft of drilling. Note that in Fig. A.8 the tool-face response is significantly different to the demand due to the lag dynamics of the response. In particular, when the steering ratio output from the controller is low, the rapid variation of the sleeve tool-face demand can be seen to be significantly different from the slower tool-face response.

A.1.5 Conclusion

This paper describes the design for a linear controller for attitude control of directional drilling tools. Since the gains are dependent on the operating conditions (see (A.22) and (A.23)), the controller can be easily gain scheduled. The robust stability of the

A.1 Robust Linear Feedback Control of Attitude for Directional Drilling Tools

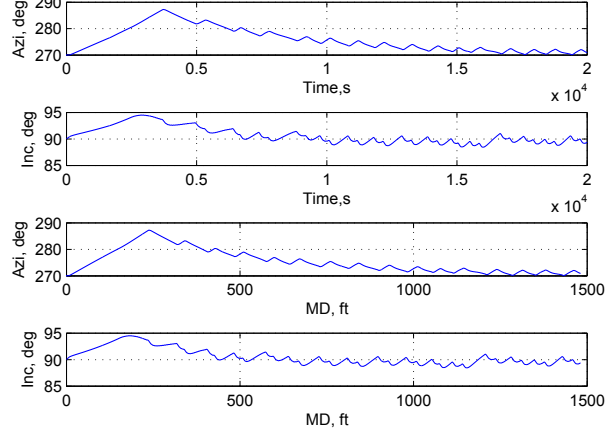


Figure A.6: Attitude response

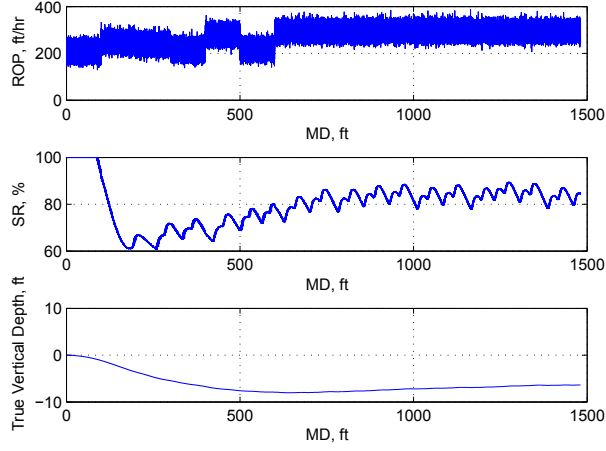


Figure A.7: ROP, % steering ratio, TVD

control scheme has been analyzed subject to the unmodeled uncertainties due to delays in the feedback signals, input quantization delays and a lag on the input tool face, plus parametric uncertainty in V_{rop} and K_{dls} . The scheme is tested by a transient simulation at a given operating point.

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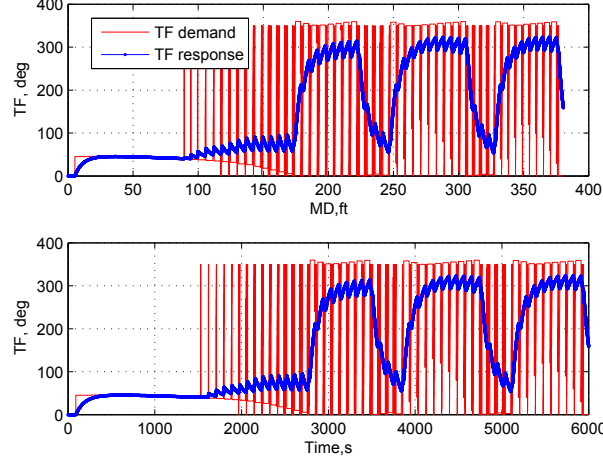


Figure A.8: Tool-face response and demand

Coordinate Systems

The coordinate systems used are defined below:

- Earth Frame (F_1) is a fixed frame and the azimuth and inclinations angles are with respect to this basis. This frame is uniquely defined by 2 vectors such as the magnetic field and the gravitational field. Here and in the following frames, $\mathbf{x}, \mathbf{y}, \mathbf{z}$ form an orthonormal basis. $\mathbf{x}$ points in the direction of gravity, $\mathbf{y}$ in the direction of maximum magnetic field perpendicular to $\mathbf{x}$, and $\mathbf{z}$ completes the right handed set.
- Sensor Frame (F_2) is co-moving with the drill, and represents the axis of the sensors with the $\mathbf{x}$ axis is aligned with the centreline of the drill and points downhole. $\mathbf{y}$ and $\mathbf{z}$ are perpendicular to $\mathbf{x}$ and corotate with the drill.
- Drill Body Frame (F_3) – the $\mathbf{y}$ axis here defines the 0° toolface. The $\mathbf{x}$ axis is the same as that of the F_2 frame, the $\mathbf{y}$ axis points in the direction perpendicular to $\mathbf{x}$ where an accelerometer would show minimum gravity, and $\mathbf{z}$ completes a right handed set.
- F_4 : Rotate F_3 by an angle of toolface around the F_{3x} axis. This frame has the $\mathbf{y}$ axis in the direction of lateral propagation confining the motion of the drill

A.1 Robust Linear Feedback Control of Attitude for Directional Drilling Tools

to 2D. Here the $\mathbf{x}$ axis is the same as that of the F_2 frame, and $\mathbf{z}$ completes a right-handed set.

- F_5 : Rotate the frame F_4 by an angle θ_{df} around the F_{4_z} axis. This frame is the new body axis.

Derivation

Consider the F_3 frame, this is achieved by the following sequence of rotations:.

$$F_3 = R_{inc}R_{azi}F_1. \quad (\text{A.31})$$

Here the rotation operators R_{inc} , and R_{azi} are defined by the matrices:

$$R_{inc} = \begin{bmatrix} \cos \theta_{inc} & \sin \theta_{inc} & 0 \\ -\sin \theta_{inc} & \cos \theta_{inc} & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_{azi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{azi} & \sin \theta_{azi} \\ 0 & -\sin \theta_{azi} & \cos \theta_{azi} \end{bmatrix}. \quad (\text{A.32})$$

Let the attitude be defined to be the unit vector aligned in the direction where the drill is pointing. The F_3 Frame has its $\mathbf{x}$ axis pointing in this direction. So:

$$\mathbf{x}_{F3} = R_{inc}R_{azi}\mathbf{x}_{F1} \quad (\text{A.33})$$

Now lets perturb the attitude to a new orientation. This will be done in two ways and both of these are equivalent.

Method 1

Consider two infinitesimal rotations acting on the basis, dR_{inc} and $R_{Inc}dR_{azi}R_{inc}^{-1}$, to achieve the new attitude.

$$\mathbf{x}_{F3} = dR_{inc}R_{Inc}dR_{azi}R_{inc}^{-1}\mathbf{x}'_{F3} \quad (\text{A.34})$$

The rotation matrices dR are defined by:

$$dR_{inc} = \begin{bmatrix} 1 & d\theta_{inc} & 0 \\ -d\theta_{inc} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, dR_{azi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & d\theta_{azi} \\ 0 & -d\theta_{azi} & 1 \end{bmatrix}. \quad (\text{A.35})$$

Since the azimuth angle is defined as a rotation about θ_{azi} in the F_1 axis then there is a need for a rotation of $R_{inc}^{-1}\mathbf{x}_{F3}$ before an azimuth perturbation can be applied. Hence

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the transformation from time t , to the attitude at time $t + \delta t$ is $dR_{inc}R_{inc}dR_{azi}R_{inc}^{-1} =$

$$\begin{bmatrix} 1 & d\theta_{inc} & -(s + d\theta_{inc}c)d\theta_{azi} \\ -d\theta_{inc} & 1 & -(c + d\theta_{inc}s)d\theta_{azi} \\ d\theta_{azi}s & d\theta_{azi}c & 1 \end{bmatrix} \quad (\text{A.36})$$

where $c = \cos \theta_{inc}$ and $s = \sin \theta_{inc}$. Now given that $d\theta_{azi}d\theta_{inc}$ is negligible and for a given $\mathbf{x}_{F3} = (1, 0, 0)^T$ we have

$$\mathbf{x}'_{F3} = \begin{bmatrix} 1 \\ d\theta_{inc} \\ -\sin \theta_{inc}d\theta_{azi} \end{bmatrix} \quad (\text{A.37})$$

This is the position of the new body axis with respect to the current body axis in terms of infinitesimal displacements of azimuth and inclination.

Method 2

One can arrive at the new attitude $\mathbf{x}'_{F3}$ from the original attitude $\mathbf{x}_{F3}$ from an infinitesimal rotation by a deflection angle θ_{df} . This is done by moving from frame F_3 to F_4 where the $\mathbf{y}$ axis is rotated clockwise about the $\mathbf{x}$ axis by an angle of θ_{tf} . Now from the F_4 frame, a rotation of the deflection angle about the $\mathbf{z}$ axis will give the required transformation. So:

$$\mathbf{x}_{F3} = R_{df}R_{tf}\mathbf{x}'_{F3} \quad (\text{A.38})$$

The rotation matrices are defined by:

$$R_{df} = \begin{bmatrix} 1 & d\theta_{df} & 0 \\ -d\theta_{df} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_{tf} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{tf} & \sin \theta_{tf} \\ 0 & -\sin \theta_{tf} & \cos \theta_{tf} \end{bmatrix} \quad (\text{A.39})$$

and for a given $\mathbf{x}_{F3} = (1, 0, 0)^T$

$$\mathbf{x}'_{F3} = \begin{bmatrix} 1 \\ d\theta_{df} \cos \theta_{tf} \\ -d\theta_{df} \sin \theta_{tf} \end{bmatrix}, \quad (\text{A.40})$$

and since the attitudes are equivalent in Method 1 and Method 2, we have

$$d\theta_{inc} = d\theta_{df} \cos \theta_{tf} \quad (\text{A.41})$$

$$\sin \theta_{inc}d\theta_{azi} = d\theta_{df} \sin \theta_{tf}. \quad (\text{A.42})$$

A.2 Recursive Variable Horizon Trajectory Control for Directional Drilling Using Elliptical Helixes

Here $d\theta_{df}$ is an angle deflection in the lateral direction in which the tool is propagating, where $d\theta_{df} = V_{rop}K_{dls}\delta t$. Taking the limit that δt is small gives:

$$\frac{d\theta_{inc}}{dt} = V_{rop}K_{dls} \cos \theta_{tf} \quad (\text{A.43})$$

$$\sin \theta_{inc} \frac{d\theta_{Azi}}{dt} = V_{rop}K_{dls} \sin \theta_{tf}. \quad (\text{A.44})$$

A.2 Recursive Variable Horizon Trajectory Control for Directional Drilling Using Elliptical Helixes

Abstract A modified Model Predictive Control (MPC) scheme is proposed, based on geometric construction, for the trajectory following of a directional drill used to produce well-bores in the oil and gas industry. Currently, trajectory control is done manually, but there is a drive to automate the process. The paper outlines the method and describes the construction of a helix that meets the boundary conditions as well as constraints on the curvature. The resulting curve provides a path that is followed to return the drill to the reference trajectory. The path is periodically recalculated to provide the basis for an MPC-type scheme. The scheme is tested in simulation and results indicate a good response.

A.2.1 Introduction

Directional drilling is becoming increasingly sophisticated and is of interest in the general drive to increase the levels of automation for the oil and gas industry. Typically, a well plan is produced from geological and commercial considerations. Then, to follow the path, an operator at the surface (known as a directional driller) sends steering commands to the down-hole drilling tool and the drill is manually steered along the desired path. The directional driller receives data from the sensors near the bit to decide on the commands. However, automation can improve the directional drilling process through more accurate following of the well plan in order to hit the designated targets and avoid collisions with existing wells. In addition, it is very important to reduce the well bore tortuosity to minimize the drag from the drill-string which inhibits weight transfer to the bit and to reduce the torque on drilling tubulars rotated from the surface through the hole.

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A toolface control system for a top drive is presented by Gillan et al. [2009] for slide drilling problems. A control system developed by Yinghui [2002] holds the inclination angle of a rotary steerable drilling tool. Furthermore, an Inclination and Azimuth Hold (IAH) algorithm is developed by Panchal et al. [2010] and by Matheus and Naganathan [2010a]. This paper builds on the work of the former. The path planning problem is studied in Guo and Feng [2008], in which the design of the path is determined through the solution of a multistage optimal control problem solved numerically by the Hooke–Jeeves algorithm. The path planning problem is also studied by [Gong and Liu, 2006] in which an optimal control problem is solved by using genetic algorithms where the fitness parameters are the accuracy of hitting the multistage targets.

In this work, it is assumed that the well plan (*reference trajectory* or *path*) is provided *a priori*. With a view to automating the path following process, this paper presents a trajectory following algorithm where the steering commands can be computed from the sensors in order for the drill to follow the path as closely as possible, and, for any given deviations, track back to the original path.

A.2.2 Overview

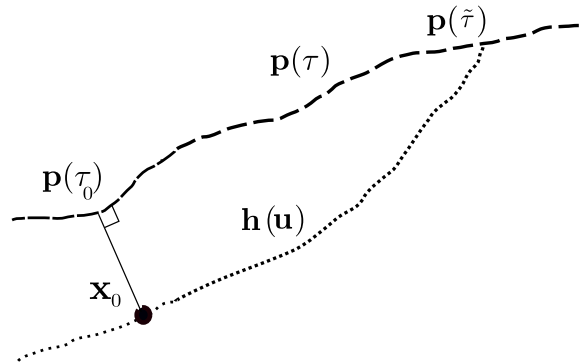


Figure A.9: Diagrammatic representation of receding horizon strategy

For the trajectory-control algorithm in this paper, we use a modified MPC strategy depicted in Fig. A.9. The parameters in the diagram are:

- The point $\mathbf{x}_0 \in \mathbb{R}^3$ is the drill's current position.

A.2 Recursive Variable Horizon Trajectory Control for Directional Drilling Using Elliptical Helixes

- The curve $\mathbf{p}(\tau) \in \mathbb{R}^3$ represents the preplanned reference trajectory which the drill is to follow.
- $\tau \in \mathbb{R}^+$ is the arclength which parameterizes $\mathbf{p}$ where $d\tau = \lim_{\delta \rightarrow 0} \|\mathbf{p}(s + \delta s) - \mathbf{p}(s)\|$.
- The curve $\mathbf{h}(\mathbf{u})$ represents the set-point trajectory which is a continuous function which takes the drill from its current position $\mathbf{x}_0$ to a position on the path $\mathbf{p}(\tilde{\tau})$; furthermore, the set-point trajectory begins at tangent to the current path and meets the reference trajectory at tangent.
- $\tilde{\tau}$ is the arclength which corresponds to the horizon solved for in Section A.2.3.
- $\mathbf{u}$ is the sequence of controls which take the drill from its current measured position $\mathbf{x}_0$, along $\mathbf{h}(\tau)$ to $\mathbf{p}(\tilde{\tau})$.

The sequence of controls, $\mathbf{u}$, is discretized where each control is for the duration of a drilling cycle (see Section A.2.5). After one input $\mathbf{u}$ has been applied, a new horizon $\tilde{\tau}$ is found, a new set-point trajectory $\mathbf{h}$ is calculated, and the algorithm is recursed.

It is assumed that the spatial position, $\mathbf{x}_0$, is known exactly from the D&I (Direction and Inclination) sensors and from the rate of penetration, V_{rop} , and is given by

$$\mathbf{x}_0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \int_{t_0}^{t_1} \begin{bmatrix} \cos \theta_{inc} \\ \sin \theta_{inc} \cos \theta_{azi} \\ \sin \theta_{inc} \sin \theta_{azi} \end{bmatrix} V_{rop} dt. \quad (\text{A.45})$$

The D&I sensors are typically three-axis magnetometers and three-axis accelerometers which measure the Earth's magnetic and gravitational fields, respectively to calculate a value of the drill's inclination $\theta_{inc} \in [0, \pi)$ and azimuth $\theta_{azi} \in [0, 2\pi)$. The Cartesian frame of reference in which the position vector $\mathbf{x}_0$ is measured is a right-handed set such that the x -axis points vertically down and parallel to the gravitational field, and the y -axis points in the direction of true North.

A.2.3 Variable Horizon

The horizon corresponds to a position along the reference trajectory $\mathbf{p}(\tilde{\tau})$ and represents the closest point which the drill can meet at tangent whilst meeting the curvature constraints. Here we choose a set-point trajectory $\mathbf{h}(\gamma)$, where $\gamma \in \mathbb{R}$ is a normalized

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parameter with $0 \leq \gamma \leq 1$ which meets the horizon $\mathbf{h}(1) = \mathbf{p}(\tilde{\tau})$, and $\tilde{\tau}$ is the solution to the optimal control problem:

$$\min \int_{\tau_0}^{\tilde{\tau}} d\tau \quad (\text{A.46})$$

subject to the constraints

$$\max_{\gamma} \kappa(\mathbf{h}(\gamma)) \leq \kappa_{max} \quad (\text{A.47a})$$

$$\mathbf{h}(0) = \mathbf{x}_0 \quad (\text{A.47b})$$

$$\mathbf{h}(1) = \mathbf{p}(\tilde{\tau}) \quad (\text{A.47c})$$

$$\mathbf{h}'(0) = \mathbf{x}'_0 \quad (\text{A.47d})$$

$$\mathbf{h}'(1) = \mathbf{p}'(\tilde{\tau}) \quad (\text{A.47e})$$

$$\tilde{\tau} \leq \tau_{max} \quad (\text{A.47f})$$

$$\tilde{\tau} \geq \tau_{min} \quad (\text{A.47g})$$

where

$$\kappa(\mathbf{h}(\gamma)) = \frac{\left\| \frac{d\mathbf{h}(\gamma)}{d\gamma} \times \frac{d^2\mathbf{h}(\gamma)}{d\gamma^2} \right\|}{\left\| \frac{d\mathbf{h}(\gamma)}{d\gamma} \right\|} \quad (\text{A.48})$$

is the curvature, and κ_{max} is the maximum curvature that the tool can drill. The position $\mathbf{p}(\tilde{\tau})$ on the reference trajectory is a distance $\tilde{\tau}$ along $\mathbf{p}(\tau)$ from $\mathbf{p}(\tau_0)$, where $\mathbf{p}(\tau_0)$ is the closest position on the reference trajectory to the drill. Since the communication rate of instructions sent to the drill are constrained to be once per drilling cycle (see Section A.2.5), the horizon hence must be greater than this corresponding distance (A.47g). A constraint is also placed on the maximum horizon distance (A.47f).

The value of $\tilde{\tau}$ is found by performing a one-dimensional line search from the constrained minimum (τ_{min}) value to the maximum (τ_{max}) allowed horizon value, for the smallest horizon value for a set-point trajectory $\mathbf{h}(\gamma)$ of the form described in Section A.2.4.

A.2.4 Helix Fitting

The set-point trajectory $\mathbf{h}(\gamma)$, which is given as a constraint in (A.47a), is dependent on using a method to fit a curve from the tool's current position to one on the path which satisfies the curvature constraints. Once this position is available, it is required to fit a

A.2 Recursive Variable Horizon Trajectory Control for Directional Drilling Using Elliptical Helixes

curve which joins these two points and matches the tangents. Methods such as Markov-Dubins curves [Sussmann, 1995], and polynomial splines are available; however, in this paper we present a method that uses an elliptical helix. This approach is chosen because it solves the problem of matching the boundary conditions and, since a helix is a smooth function, there are no discontinuities in the curvature and torsion. Furthermore, an analytical curve can be provided at each step in the algorithm.

Given the boundary conditions;

- $\mathbf{x}_0$ the current position of the tool,
- $\mathbf{p}(\tilde{\tau})$ the position of the horizon,
- $\mathbf{x}'_0$ the current attitude of the tool,
- $\mathbf{p}'(\tilde{\tau})$ the attitude at the horizon,

an elliptical helix can be constructed as follows.

An orthonormal basis $\{\mathbf{b}_i \in \mathbb{R}^3 | i = 1, 2, 3\}$ can be constructed such that

$$\mathbf{b}_1 = \frac{\mathbf{p}'(\tilde{\tau}) - \mathbf{x}'_0}{\|\mathbf{p}'(\tilde{\tau}) - \mathbf{x}'_0\|} \quad (\text{A.49})$$

is the difference between the initial and final tangents and, using the Gram-Schmidt process on $\mathbf{b}_1$ and $\mathbf{p}'(\tilde{\tau})$, gives

$$\mathbf{b}_2 = \frac{\mathbf{p}'(\tilde{\tau}) - (\mathbf{p}'(\tilde{\tau}) \cdot \mathbf{b}_1)\mathbf{b}_1}{\|\mathbf{p}'(\tilde{\tau}) - (\mathbf{p}'(\tilde{\tau}) \cdot \mathbf{b}_1)\mathbf{b}_1\|}. \quad (\text{A.50})$$

To complete the set, the cross product of $\mathbf{b}_1$ and $\mathbf{b}_2$ gives

$$\mathbf{b}_3 = \mathbf{b}_1 \times \mathbf{b}_2. \quad (\text{A.51})$$

The vector $\mathbf{b}_2$ is parallel to the centreline axis of the helix, and the vector $\mathbf{b}_3$ is orthogonal to both the initial and final tangents of the helix.

The equation of the elliptical helix is of the form

$$\mathbf{h}(\gamma) = \mathbf{h}_0 + h_1(\gamma)\mathbf{b}_1 + h_2(\gamma)\mathbf{b}_2 + h_3(\gamma)\mathbf{b}_3 \quad (\text{A.52})$$

in which $\{h_i \in \mathbb{R} | i = 1, 2, 3\}$ are scalar valued functions, and $\gamma = 0$ corresponds to the current position of the drill, and $\gamma = 1$ corresponds to the horizon. The functions are

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defined as:

$$h_1(\gamma) = A \cos \left(\pi\gamma - \frac{\pi\mu}{\bar{x}} \right), \quad (\text{A.53})$$

$$h_2(\gamma) = \gamma\bar{x}, \quad (\text{A.54})$$

$$h_3(\gamma) = \frac{\bar{z}}{2} \cos(\pi\gamma), \quad (\text{A.55})$$

where A , μ , $\bar{z}$, and $\bar{x}$ are parameters to be determined. If one thinks of the helix being propagated along its axis in the $\mathbf{b}_2$ direction, there are two amplitudes: A from (A.53), and $\bar{z}/2$ from (A.55).

The value of A is found by considering the projection of the helix in the plane perpendicular to $\mathbf{b}_3$. Here the projections of $\mathbf{p}'$ and $\mathbf{x}'_0$ maintain their magnitudes and make angles with the vector $\mathbf{b}_2$ (Fig. A.10) of equal magnitude denoted by θ .

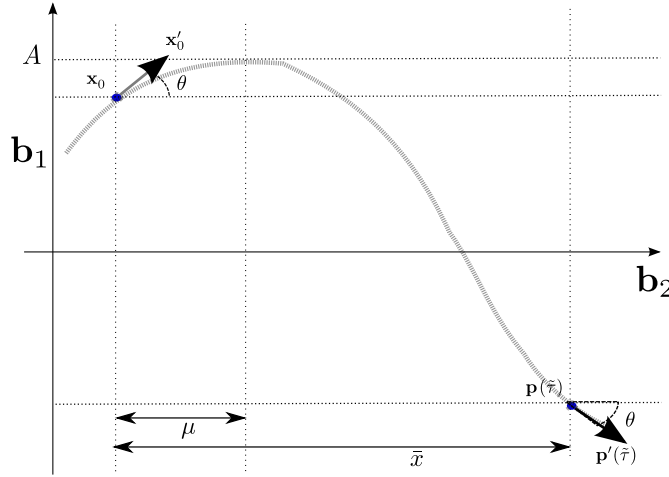


Figure A.10: Projection of helix onto plane perpendicular to $\mathbf{b}_3$

In the $\mathbf{b}_1$ - $\mathbf{b}_2$ plane, the equation of the curve takes the form of a sinusoid of the form given by (A.53) and (A.54) and is simplified to

$$h_1(h_2) = A \cos \left(\frac{\pi(h_2 - \mu)}{\bar{x}} \right). \quad (\text{A.56})$$

The equality of the angles made by the initial and final tangents with the centreline gives the expression

$$\tan \theta = \frac{\mathbf{x}'_0 \cdot \mathbf{b}_1}{\mathbf{x}'_0 \cdot \mathbf{b}_2} = \frac{\mathbf{p}'(\tilde{\tau}) \cdot \mathbf{b}_1}{\mathbf{p}'(\tilde{\tau}) \cdot \mathbf{b}_2} = \frac{dh_1}{dh_2}(0), \quad (\text{A.57})$$

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in which the gradient $dh_1(h_2)/dh_2$ found by differentiating equation (A.56) w.r.t h_2 is

$$\frac{dh_1}{dh_2}(h_2) = -\frac{A\pi}{\bar{x}} \sin\left(\frac{\pi(h_2 - \mu)}{\bar{x}}\right), \quad (\text{A.58})$$

and the initial value is

$$\frac{dh_1}{dh_2}(0) = \frac{A\pi}{\bar{x}} \sin\left(\frac{\pi\mu}{\bar{x}}\right). \quad (\text{A.59})$$

The initial value of the amplitude at $h_2 = 0$ is found from equating (A.56) to half the projection of the difference in the initial and final positions in this plane to the $\mathbf{b}_1$ axis

$$h_1(0) = \frac{\|(\mathbf{x}_0 - \mathbf{p}(\tilde{\tau})) \cdot \mathbf{b}_1\|}{2} = A \cos\left(\frac{\pi\mu}{\bar{x}}\right). \quad (\text{A.60})$$

Similarly, taking the projection of the difference in the initial and final positions to the $\mathbf{b}_2$ axis gives

$$\bar{x} = \|(\mathbf{x}_0 - \mathbf{p}(\tilde{\tau})) \cdot \mathbf{b}_2\|, \quad (\text{A.61})$$

which is also half the wavelength of the elliptical helix. The amplitude A and the phase μ can be determined by (A.60) and (A.59) and are given by

$$A = \sqrt{h_1^2(0) + \left(\frac{\bar{x}}{\pi} \frac{dh_1}{dh_2}(0)\right)^2} \quad (\text{A.62})$$

and

$$\mu = \frac{\bar{x}}{\pi} \arctan\left(\frac{x}{\pi h_1(0)} \frac{dh_1}{dh_2}(0)\right). \quad (\text{A.63})$$

In the plane containing $\mathbf{b}_3$ and $\mathbf{c} = \alpha_1 \mathbf{b}_2 + \alpha_2 \mathbf{b}_1$, the projections of $\mathbf{p}'$ and $\mathbf{x}'_0$ are always parallel or collinear, and the projection of the helix onto this plane is shown in Fig. A.11. The value for $\bar{z}$ is found from

$$\bar{z} = \|(\mathbf{x}_0 - \mathbf{p}(\tilde{\tau})) \cdot \mathbf{b}_3\|, \quad (\text{A.64})$$

and hence it is possible to obtain (A.52) purely from the boundary conditions.

A.2.5 Trajectory following algorithm

The structure of the trajectory following control system shown in Fig. A.12 consists of an inner loop and an outer loop. The former is in the form of an attitude control system which uses a PI controller and is detailed in Panchal et al. [2010]. The latter is a receding horizon controller form described in Section A.2.2, where an outline of the algorithm is:

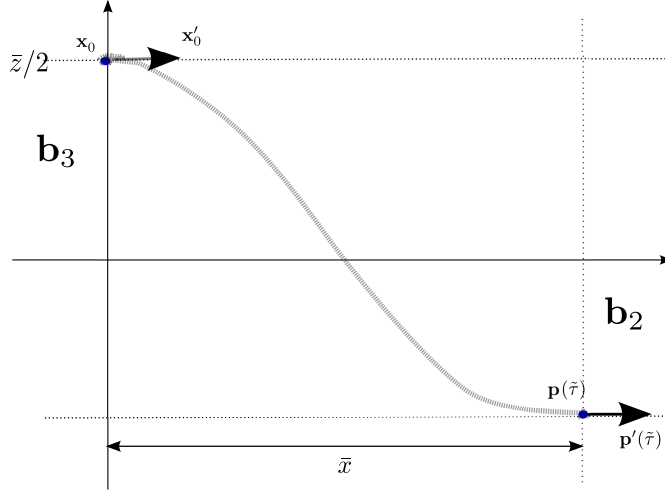


Figure A.11: Projection of helix onto plane containing $\mathbf{b}_3$

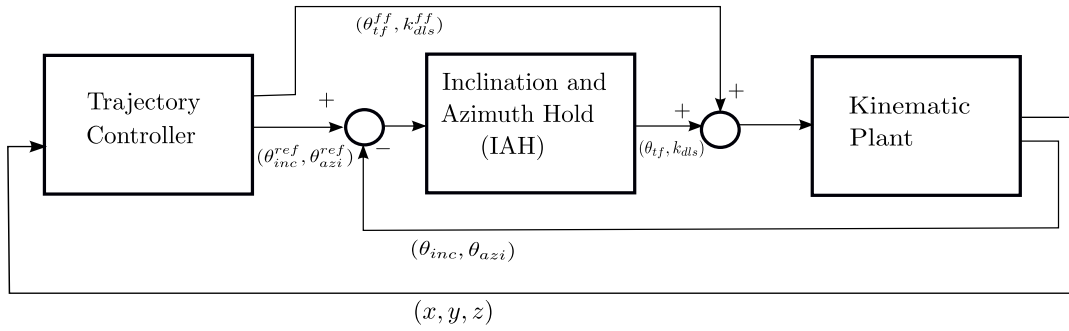


Figure A.12: Controller structure

1. Take readings of the tool's current position $\mathbf{x}_0$ and attitude $\mathbf{x}'_0$ from the sensors.
2. Solve for the closest point $\mathbf{p}(\tau_0)$.
3. Determine the helix which joins $\mathbf{x}_0$ to a position $\mathbf{p}(\tau_0 + \delta\tau)$, a small distance ahead.
4. Iterate step 3 until the curvature constraint from (A.47a) is met, and the corresponding horizon $\mathbf{p}(\tilde{\tau})$ is found.
5. Extrapolate the attitude states and controls which correspond to half an arclength of one drilling cycle along the helix.

A.2 Recursive Variable Horizon Trajectory Control for Directional Drilling Using Elliptical Helixes

Before iterating these steps, the attitude calculated half a drilling cycle ahead is fed into the IAH as the reference attitude, as shown in Fig. A.12, and the control is fed-forward to the plant. Here the IAH deals with disturbances in the attitude.

Drilling Cycle

Directional drilling tools in push- and point-the-bit varieties steer by applying a constant force against the formation or through having a fixed bend. Straight-hole sections are formed rotating the steering of the tool. Sections of boreholes with curvatures less than the maximum curvature are achieved through having a drilling cycle lasting t_{ds} , where each cycle is apportioned into a curved part t_c and a straight part, so that

$$\frac{\kappa}{\kappa_{max}} = \frac{t_c}{t_{ds}}. \quad (\text{A.65})$$

Curvatures which are a fraction of the maximum curvature are directly proportional to the ratio of the curved section to the total drilling cycle. The length of a drilling cycle is given by

$$\tau_{ds} = V_{rop} t_{ds}, \quad (\text{A.66})$$

where V_{rop} is the rate of penetration of the tool through the formation.

Feed Forward

Given that the position along the set-point elliptical helix trajectory $\mathbf{h}(\frac{\tau_{ds}}{2})$ is known half a drilling cycle ahead, it is required to find the normal vector

$$\mathbf{N}(\eta) = \frac{\mathbf{h}'(\eta) \times (\mathbf{h}''(\eta) \times \mathbf{h}'(\eta))}{\|\mathbf{h}'(\eta)\| \|\mathbf{h}''(\eta) \times \mathbf{h}'(\eta)\|} \quad (\text{A.67})$$

and the curvature

$$k_{ds}^{ff} = \frac{\|\mathbf{h}''(\eta) \times \mathbf{h}'(\eta)\|}{\|\mathbf{h}'(\eta)\|^3} \quad (\text{A.68})$$

to determine the toolface and steering ratio, where η is the fractional path half the drilling cycle projects onto the centreline axis and is the solution to

$$\frac{\tau_{ds}}{2} = \int_0^\eta \sqrt{\left(\frac{dh_1(\gamma)}{d\gamma}\right)^2 + \left(\frac{dh_2(\gamma)}{d\gamma}\right)^2 + \left(\frac{dh_3(\gamma)}{d\gamma}\right)^2} d\gamma, \quad (\text{A.69})$$

and can be solved for by numerical integration.

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Here the curvature can be interpreted directly to mean the steering ratio, and the toolface angle is determined by

$$\theta_{tf}^{ff} = \arctan \frac{\text{sgn}(\|\mathbf{N} \times \tilde{\mathbf{g}}\|) \|\mathbf{N} \times \tilde{\mathbf{g}}\|}{\mathbf{N} \cdot \tilde{\mathbf{g}}}. \quad (\text{A.70})$$

The vector $\tilde{\mathbf{g}}$ is the projection of the vector pointing in the opposite direction of the gravity vector $\mathbf{g}$ when projected onto the plane perpendicular to the direction in which the drill is penetrating in and is given by

$$\tilde{\mathbf{g}} = \frac{-\mathbf{g} + (\mathbf{g} \cdot \mathbf{x}'_0) \mathbf{x}'_0}{\|-\mathbf{g} + (\mathbf{g} \cdot \mathbf{x}'_0) \mathbf{x}'_0\|}. \quad (\text{A.71})$$

A.2.6 Model of the drill

To model the directional drill, a kinematic relationship rather than a dynamical model, based on the assumption that a slow-moving directional drill has its angular velocities of a greater order of magnitude to angular accelerations:

$$\dot{\theta}_{inc} = V_{rop} k_{dls} \cos \theta_{tf}, \quad (\text{A.72})$$

$$\dot{\theta}_{azi} = \frac{V_{rop}}{\sin \theta_{inc}} k_{dls} \sin \theta_{tf}. \quad (\text{A.73})$$

This relationship is derived in Panchal et al. [2010] and it is assumed that V_{rop} can be controlled separately.

A.2.7 Transient Simulation

The trajectory-following algorithm is tested here in two time domain simulations. Firstly, the simulation is tested for a drill following a straight horizontal section. This would correspond to commonly drilled wells, where the tool would reach a particular depth before initiating a curved section, until the tool is horizontal and placed within a pay zone. At this point, a horizontal section is held as much as possible within the pay zone. The path in this simulation is a straight line propagating 2500 ft in the y -axis and at a depth of 40 ft.

In the second simulation, the reference trajectory is generated by toolface and steering ratio instructions given to a drill which is penetrating at a rate of 100 ft/hr and a maximum curvature of $8^\circ/(100 \text{ ft})$. Here the instructions are indexed by measured depth and are given in Table A.2.

A.2 Recursive Variable Horizon Trajectory Control for Directional Drilling Using Elliptical Helixes

Table A.2: General Path Simulation

Measured Depth(ft)	Steering Ratio	Toolface °
0	1	0
200	0.7	90
400	0.5	110
600	1.0	210
800	0.5	80
1000	1.0	90
1200	0.2	50
1500	1.0	290
2000	1.0	90

The time domain simulations include a drop $\dot{\theta}_{dr}$ and turn rate $\dot{\theta}_{tr}$ bias, a position offset, and an attitude offset from the reference path. These values, along with simulation parameters used in the path following for both the simulations, are shown in Table A.3.

Table A.3: Simulation parameters

Parameter	Value
$\theta_{inc,0}$	95°
$\theta_{azi,0}$	275°
V_{rop}	50 ft/hr
K_{dls}	20°/(100 ft)
$\dot{\theta}_{dr}$	1°/(100 ft)
$\dot{\theta}_{tr}$	1°/(100 ft)
x_0	-25 ft
y_0	30 ft
z_0	30 ft

Horizontal Section Simulation

The visualization of the transient simulation for the path following of the horizontal section is shown in Fig. A.13, in which the blue line represents the centreline of a pay-zone region which is a horizontal rectilinear reference trajectory, and the red line is the path traced out by the drill.

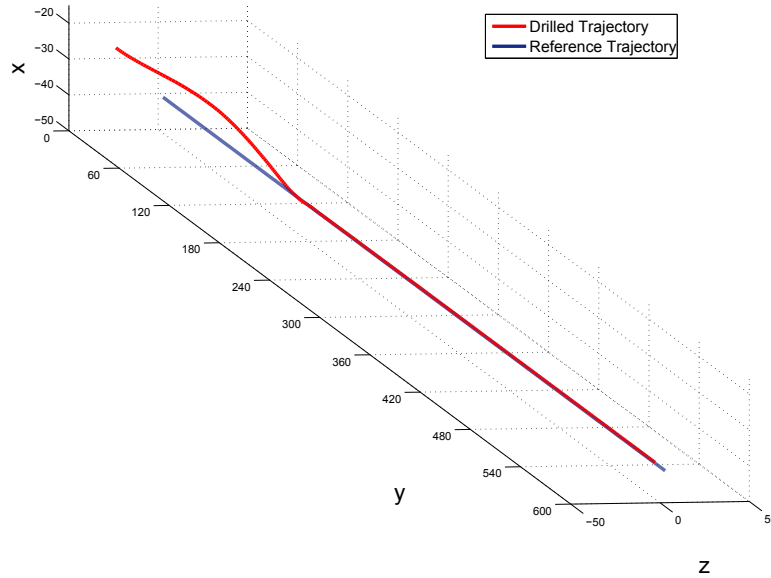


Figure A.13: 3D simulation results for horizontal section following

The norm of the closest distance between the drill and the reference trajectory is shown in Fig. A.14 in which it can be seen that the drilled trajectory converges towards the reference trajectory and remains along the path until the end of the reference trajectory is reached.

It can be seen in Fig. A.15, that initially the set-point trajectory joining the tool to the horizon is the form of a circular helix since the semi-minor and semi-major axes are equal. The set-point trajectory then becomes an elliptical helix with the length of the minor axis diminishing to zero at around 20 ft where the set-point trajectory becomes a two-dimensional sinusoidal wave. At a measured depth of around 150 ft, it can be seen that the set-point trajectory becomes a straight line when the tool is on the path which it is following. Here the distance of the longitudinal axis is never less than 20 ft as this is the constrained minimum horizon.

A.2 Recursive Variable Horizon Trajectory Control for Directional Drilling Using Elliptical Helixes

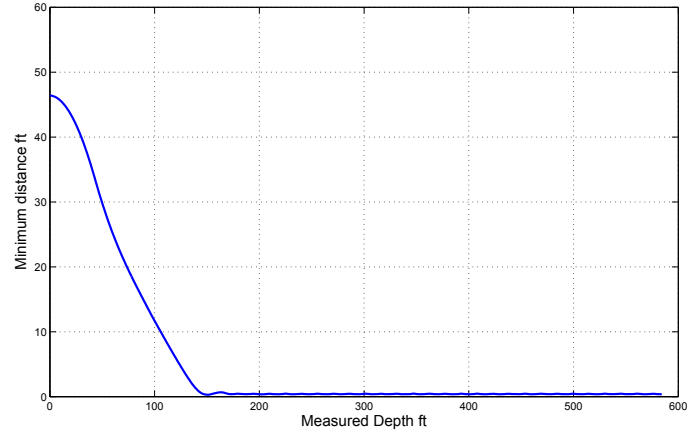


Figure A.14: Distance to closest approach converging with increasing measured depth

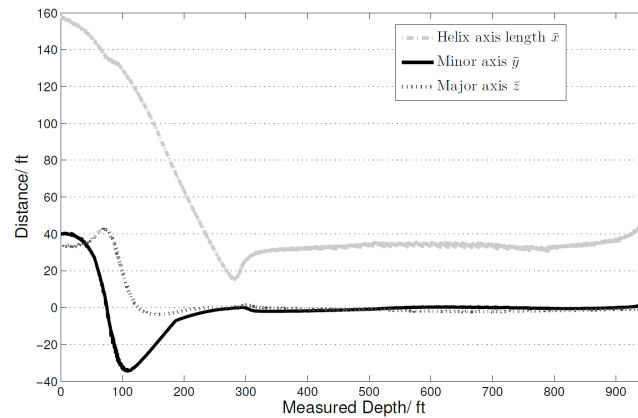


Figure A.15: Parameters of the set-point helix trajectory $(\bar{x}, \bar{y}, \bar{z})$ for horizontal path following

Arbitrary Path-following Simulation

The trajectory-following controller can be seen to converge towards an arbitrary path generated through the input parameters in Table A.2. The path following is displayed in Fig. A.16. Once converging towards the path, the drill remains within 10 ft of the reference trajectory and this is shown in Fig. A.17. The nature of the set-point trajectory in this simulation converges to a straight-line in a similar way to the horizontal section simulation. The set-point helix trajectory parameters are shown in Fig. A.18.

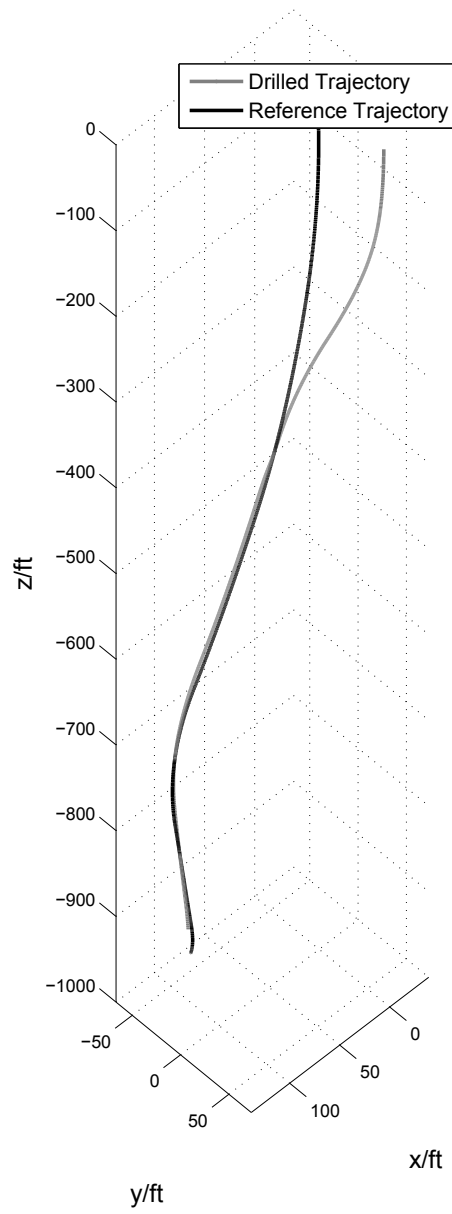


Figure A.16: 3D plot of path following for arbitrary path following

A.2 Recursive Variable Horizon Trajectory Control for Directional Drilling Using Elliptical Helixes

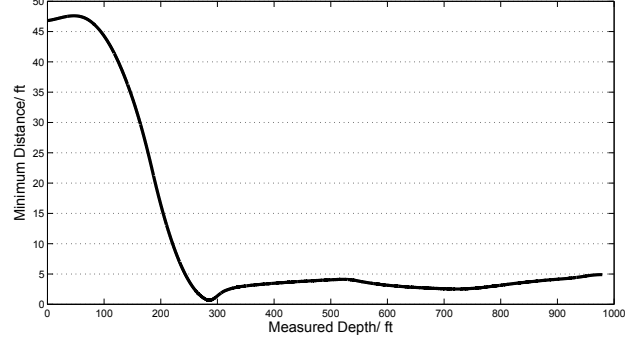


Figure A.17: Distance to closest approach converging with increasing measured depth

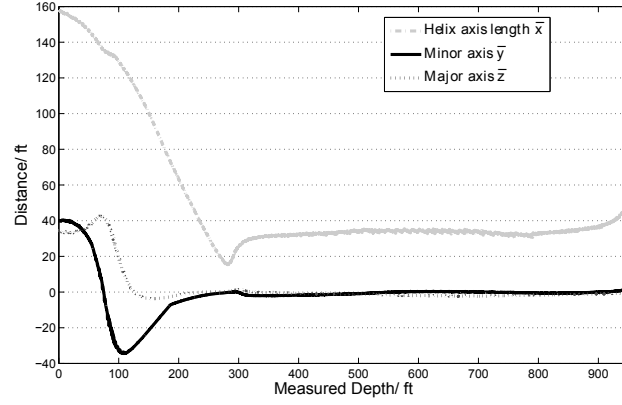


Figure A.18: Parameters of the set-point helix trajectory $(\bar{x}, \bar{y}, \bar{z})$ for arbitrary path following

A.2.8 Conclusion

This paper presents a variable horizon trajectory-following algorithm for directional drills. In addition, a method of elliptical helixes for meeting two positions at tangents in three dimensions is presented and used as the set-point trajectory. In simulation the tool is shown to converge to the reference trajectory in the presence of drop rate and turn rate biases for holding a horizontal section, and for an arbitrarily generated path. As the simulation progresses, the set-point trajectory tends to become a straight line. The advantages of this method are that the calculation of the set-point trajectory is straightforward and computationally fairly simple since the search for the optimal

solution over a finite range is in one dimension.

A.3 Minimum Strain Energy Waypoint-following Controller for Directional Drilling using OGH Curves

Abstract Directional drilling has become increasingly important as a means for better exploitation of oil and gas reservoirs as well as part of a drive to increase the level of automation in the industry. Typically, a well plan is created for a variety of reasons, such as maximizing productive drilling, minimizing well tortuosity, collision avoidance, or navigating the tool to strategic points within a pay zone. Following a nominal well plan created to satisfy these requirements whilst drilling leads to a clear need for automated trajectory-controlled drilling. This paper builds on the authors' earlier work on generalized attitude control of directional drilling to develop a path-following algorithm incorporating optimized geometric Hermite space curves as a means of generating the correction path from the instantaneous tool position to the well plan whilst minimizing the drillstring strain energy. The proposed scheme is tested in simulation and shown to perform satisfactorily for a typical set of drilling operating parameters for converging towards a target position and attitude.

A.3.1 Introduction

As the future of directional drilling moves towards the exploitation of increasingly complex reservoirs, there is a desire and a need for automating rig operations as much as possible. The implications and advantages of such a drive would enable rig teams to focus on higher levels of abstraction and decision making.

This paper presents the application of Optimized Geometric Hermite (OGH) curves Yong and Cheng [2003] as a method for real-time path planning and following for directional drilling tools which have the ability to hold inclination and azimuth. This method relies on a target position in space ahead of the bit with an associated target attitude. These targets, or waypoints, can be used as a sequence of points with attitudes corresponding to a well plan which is to be followed, or they can be a finite number of points in space which could correspond to strategic points in a pay-zone (a reservoir or portion of a reservoir that contains economically producible hydrocarbons is known as a pay or pay zone). Further scope for this method with the application of look-ahead

A.3 Minimum Strain Energy Waypoint-following Controller for Directional Drilling using OGH Curves

look-around geosensor technology would be to dynamically choose and assign these targets as an outer-loop to the method presented in this paper.

From the tool's instantaneous position measured from the tool sensor set (gyroscopes, accelerometers, and magnetometers), a path is determined to the next target. This path is periodically recalculated and provides a sequence of inclination and azimuth instructions which steer the tool towards that target.

There has been other work done on optimal path planning and trajectory generation for well drilling. For example, the path-planning problem is studied in Guo and Feng [2008], in which the design of the path is determined through the solution of a multistage optimal control problem solved numerically by the Hooke–Jeeves algorithm. The path-planning problem is also studied by Gong and Liu [2006] in which an optimal control problem is solved by using genetic algorithms in which the fitness parameters are the accuracy of hitting the multistage targets. Pirovolou *et al* Pirovolou et al. [2011] presents a system for trajectory following of a drill which identifies system parameters online and follows a fixed or moving target from the well-plan where the demand build and turn rates are given from the inclination and azimuth errors of the tools measured position and the target.

In this paper, the use of OGH Yong and Cheng [2003] space curves for generating the correction path from the tool's instantaneous position to the well-plan trajectory is described. Polynomial space curves have also been used in similar applications in which a path between two points in space with given start and end attitudes is required, such as multi-axis Computer Numerically Controlled (CNC) machines tracking curved paths at constant or variable speeds Farouki et al. [1998].

The paper begins by introducing the controller structure followed by summarizing the open-loop plant model and the authors' earlier work on Inclination and Azimuth Hold (IAH) control Panchal et al. [2010] which included a robust analysis of attitude hold for directional drilling. The paper then discusses the properties of OGH space curves. A path-following architecture is then presented which incorporates the OGH space curve construction as the correction path, set point attitude-generating outer loop, and the IAH for the attitude hold inner loop. Satisfactory algorithm operation is demonstrated by a transient simulation for a typical set of drilling operating parameters.

A.3.2 Controller Structure

A closed-loop scheme for following a trajectory for directional drilling operations is depicted in Fig. A.19. The reference trajectory is defined by a sequence of waypoints in space $\mathbf{x}_{wp}(i)$ which are positions which it is desired for the drill to penetrate through, where $1 \leq i \leq N$ is the index of a finite number of waypoints. Furthermore, the reference trajectory associated with these points would also have an associated attitude $\mathbf{x}'_{wp}(i)$ for which it is desired for the tool to meet at a tangent. These waypoints can be predetermined targets from geological and seismic information, and forthcoming waypoints could also be dynamically allocated from geosteering data.

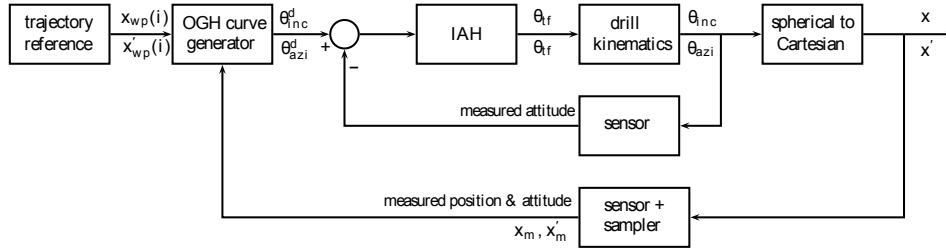


Figure A.19: Closed-loop block diagram for the trajectory controller. There are two feedback loops: the IAH controller is nested within the trajectory controller which tracks the drill towards its nearest reference trajectory waypoint.

The trajectory controller (see Section A.3.4) takes an estimate of the tool's current position and attitude $\mathbf{x}_m, \mathbf{x}'_m$ along with the position and attitude of the next waypoint target $\mathbf{x}_{wp}(i), \mathbf{x}'_{wp}(i)$ and interpolates for cubic polynomial space curve with the lowest strain energy. From this, the controller provides a new reference demand attitude for the IAH controller which is the inner loop and which controls the tool's direction.

A.3.3 Plant Dynamics and IAH

The plant model used to describe the changes in the direction of the drill is derived from kinematic considerations Panchal et al. [2010]. The angular azimuth and inclination responses are given in terms of tool face and curvature inputs as

$$\dot{\theta}_{inc} = V_{rop} (U_{dls} \cos U_{tf} - V_{dr}), \quad (\text{A.74})$$

$$\dot{\theta}_{azi} = \frac{V_{rop}}{\sin \theta_{inc}} (U_{dls} \sin U_{tf} - V_{tr}), \quad (\text{A.75})$$

A.3 Minimum Strain Energy Waypoint-following Controller for Directional Drilling using OGH Curves

where

- θ_{inc} is the inclination angle (rad),
- θ_{azi} is the azimuth angle (rad),
- U_{tf} is the tool face angle control input (rad),
- U_{dls} is the ‘dog leg severity’ or curvature (rad/m),
- V_{dr} is the drop rate bias disturbance (rad/m),
- V_{tr} is the turn rate bias disturbance (rad/m),
- V_{rop} is the rate of penetration and is an uncontrolled parameter (m/s).

Using the following transformations Panchal et al. [2010], Matheus and Naganathan [2010a] the control tool-face angle and steering ratio that guide the tool towards its target azimuth and inclination are found:

$$U_{tf} = \arctan\left(\frac{U_{azi}}{U_{inc}}\right) \quad (\text{A.76})$$

$$U_{dls} = K_{dls}\sqrt{(U_{azi})^2 + (U_{inc})^2} \quad (\text{A.77})$$

where U_{azi} and U_{inc} are transformed control inputs. Ignoring the disturbances, the plant model is thus transformed to

$$\dot{\theta}_{inc} = V_{rop}K_{dls}U_{inc}, \quad (\text{A.78})$$

$$\dot{\theta}_{azi} = \frac{V_{rop}}{\sin \theta_{inc}}K_{dls}U_{azi}, \quad (\text{A.79})$$

which is simpler than the model given by (A.74) and (A.75). The following PI controllers are proposed to control the tool attitude:

$$U_{inc} = k_{pi}e_{inc} + k_{ii} \int_0^t e_{inc}dt \quad (\text{A.80})$$

$$U_{azi} = k_{pa}e_{azi} + k_{ia} \int_0^t e_{azi}dt \quad (\text{A.81})$$

where $e_{inc} = \theta_{inc}^d - \theta_{inc}$ and $e_{azi} = \theta_{azi}^d - \theta_{azi}$ are the inclination and azimuth errors, respectively, and θ_{inc}^d and θ_{azi}^d are the inclination and azimuth reference demands,

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respectively. By linearizing the transformed plant model, the PI controller gains can be chosen to provide the closed-loop system with specified closed-loop poles. For details, see Panchal et al. [2010].

A robustness analysis of the proposed control system based on the small gain theorem was performed in order to guarantee the system stability subject to measurement feedback delays, input quantization delay, and parametric uncertainty on V_{rop} and K_{dls} . See Panchal et al. [2010] for details.

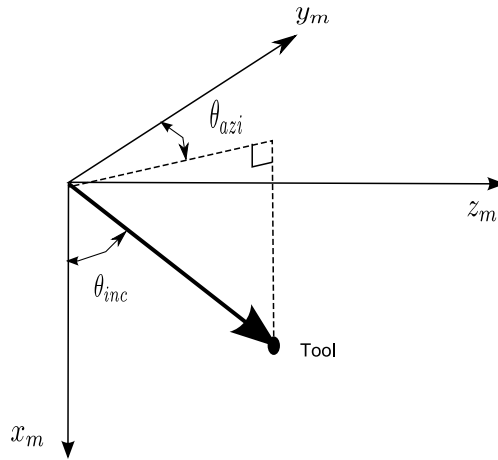


Figure A.20: Right-handed coordinate system used to define the reference frame of the Earth. The inclination angle θ_{inc} and the azimuthal angle θ_{azi} are defined with respect to this frame.

The outer trajectory-following loop in Fig. A.19 has three Cartesian position states $\mathbf{x}_m = x_m \mathbf{i} + y_m \mathbf{j} + z_m \mathbf{k}$ that are measured, either periodically or when some event occurs. Here x_m is defined to be parallel to the gravitational field, and y_m is in the direction of due North in the plane perpendicular to x_m . For this paper, the global coordinate system is a right-handed coordinate system with the x_m axis pointing down and azimuth and inclination θ_{azi} and θ_{inc} , respectively, are defined as shown in Fig. A.20. The attitude as a vector in this frame is given in terms of azimuth and inclination as

$$\mathbf{x}'_m = \begin{pmatrix} x'_m \\ y'_m \\ z'_m \end{pmatrix} = \begin{pmatrix} \cos \theta_{inc} \\ \sin \theta_{inc} \cos \theta_{azi} \\ \sin \theta_{inc} \sin \theta_{azi} \end{pmatrix}. \quad (\text{A.82})$$

In this paper, it is assumed that autonomous drill bit on/off bottom detection is available. This enables anti-windup to be implemented on the PI controllers of the

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IAH. Based on the surface measured depth measurement MD an estimate of the tool V_{rop} can be obtained. It is assumed that a reliable estimation of the tool position is subsequently available.

A.3.4 Path Following using Optimized Geometric Hermite Curves

In fitting a path between the tool's current position and the target, it is aimed to minimize the geometric strain energy of the interpolated spline. For an oil well, the shape of the curve has an impact on the drillstring leading up to the bottom hole assembly. Since axisymmetric cylindrical drillpipes are in an unstrained configuration prior to being used for drilling, work is done over the drilling operation by the borehole on the drillstring to bend the pipes to fit the shape. Similarly, in completing the well, a section of casing would experience the same forces.

Developing a borehole to minimize the strain energy from the tool to its target can improve fatigue life of threaded stand sections of the drillpipe. Also, reducing contact forces on the drillpipes and casing would reduce the sliding friction and would be beneficial for extended reach and increased V_{rop} drilling.

Strain Energy Cost Function

Using Euler-Bernoulli beam theory, assume an axisymmetric beam with a constant cross-sectional area and uniform mass distribution, as a general model for the drillpipe and section of casing.

The relationship between the local curvature κ to the bending moment M is given by [Farouki, 2008, p325]

$$\kappa = \frac{M}{EI}, \quad (\text{A.83})$$

where I is the second area moment and E is the modulus of elasticity of a section of casing or drillpipe and are both assumed constant.

When deflected into a space curve $f(t)$, the work done on an element $f(t+\delta t) - f(t)$ [Farouki, 2008, p325] is given by $\frac{1}{2}M d\theta$, where $t \in [0, 1]$ parameterizes the curve $f(t)$ from the start position to the end position, and M is the moment acting on the element dt deflecting it by an angle $d\theta$ where $\theta = \frac{df}{dt}(t)$. The curvature κ at $f(t)$ is given by

$$\kappa = \frac{d\theta}{dt} = \frac{d^2 f}{dt^2}(t). \quad (\text{A.84})$$

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The total strain energy over the length of the curve which we wish to minimize is given by

$$\Phi = \frac{1}{2}EI \int_t \left[\frac{d^2 f}{dt^2}(t) \right]^2 dt = \frac{1}{2}EI \int_t \kappa^2(t) dt. \quad (\text{A.85})$$

Optimized Cubic Splines

To fit a curve from the tool's current measured position to the target, a cubic spline is chosen as this is the lowest order polynomial curve that includes an inflection point, and, for fitting a feasible trajectory to a target position and attitude ahead of the bit, a smooth curve can be obtained.

Given the tool's current position $\mathbf{x}_m$ and current attitude $\mathbf{x}'_m$ a cubic spline is constructed to a target position $\mathbf{x}_{wp}$ and attitude $\mathbf{x}'_{wp}$, with respect to four Bezier control points $\mathbf{x}_m$, $\mathbf{x}_m + \frac{1}{3}\mathbf{x}'_m$, $\mathbf{x}_{wp} - \frac{1}{3}\mathbf{x}'_{wp}$, $\mathbf{x}_{wp}$ where the space curve given in terms of Bernstein polynomials is

$$f(t) = b_0\mathbf{x}_m + b_1(\mathbf{x}_m + \frac{1}{3}\mathbf{x}'_m) + b_2(\mathbf{x}_{wp} - \frac{1}{3}\mathbf{x}'_{wp}) + b_3\mathbf{x}_{wp}. \quad (\text{A.86})$$

The cubic Bernstein polynomial coefficients are given by the relationship

$$b_i = \binom{3}{i} t^i (1-t)^{(3-i)}. \quad (\text{A.87})$$

An OGH curve is a cubic Hermite curve (A.86) which has the smallest strain energy amongst all cubic Hermite curves that satisfy the endpoint conditions Yong and Cheng [2003].

Putting the cubic Hermite curve into the form given by (A.86), a minimal strain energy curve can be found by varying the two intermediate control points given the corresponding terms of the b_1 and b_2 Bernstein coefficients. Since the start and end points, $\mathbf{x}_m$ and $\mathbf{x}_{wp}$, are fixed in space, the free parameters are the magnitudes of the tangent vectors: $\mathbf{x}'_m$ and $\mathbf{x}'_{wp}$, and (A.86) is rewritten as:

$$f(t) = b_0\mathbf{x}_m + b_1(\mathbf{x}_m + \frac{1}{3}a_0\mathbf{x}'_m) + b_2(\mathbf{x}_{wp} - \frac{1}{3}a_1\mathbf{x}'_{wp}) + b_3\mathbf{x}_{wp}. \quad (\text{A.88})$$

The values for constants a_0 and a_1 that provide the OGH curve are found from the following theorem Yong and Cheng [2003].

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Theorem 2. *Given two endpoints $\mathbf{x}_m$ and $\mathbf{x}_{wp}$, and two endpoint tangent vectors $\mathbf{x}'_m$ and $\mathbf{x}'_{wp}$, an OGH curve $f(t)$, $t \in [0, 1]$ is obtained at $a_0 = a_0^*$ and $a_1 = a_1^*$ where*

$$a_0^* = \frac{6[(\mathbf{x}_{wp} - \mathbf{x}_m) \cdot \mathbf{x}'_m](\mathbf{x}'_{wp})^2 - 3[(\mathbf{x}_{wp} - \mathbf{x}_m) \cdot \mathbf{x}'_{wp}](\mathbf{x}'_m \cdot \mathbf{x}'_{wp})}{4\mathbf{x}_m^2 \mathbf{x}_{wp}^2 - (\mathbf{x}_m \cdot \mathbf{x}_{wp})^2} \quad (\text{A.89})$$

and

$$a_1^* = \frac{3[(\mathbf{x}_{wp} - \mathbf{x}_m) \cdot \mathbf{x}'_m](\mathbf{x}'_{wp} \cdot \mathbf{x}'_m) - 6[(\mathbf{x}_{wp} - \mathbf{x}_m) \cdot \mathbf{x}'_{wp}](\mathbf{x}'_m)^2}{(\mathbf{x}_m \cdot \mathbf{x}_{wp})^2 - 4\mathbf{x}_m^2 \mathbf{x}_{wp}^2}. \quad (\text{A.90})$$

Using the values for a_0 and a_1 from (A.89) and (A.90), the OGH curve from (A.88) gives a path for the tool to follow to get to its target.

Implementation

The scheme is implemented here in a similar manner to an MPC scheme, but with a fixed horizon rather than a receding horizon. Thus the position is periodically measured, and the attitude reference demand for the first portion of the OGH curve is passed to the inner-loop IAH controller shown in Fig. A.19.

To allow for the lags in the IAH controller, the gradient of the OGH curve a small arbitrary fixed distance ℓ ahead of the bit is calculated. The arc length L of the whole OGH curve from the tool's measured position to the target is calculated numerically using

$$L = \sum_{i=0}^{n-1} \left\| f\left(\frac{i+1}{n}\right) - f\left(\frac{i}{n}\right) \right\|_2, \quad (\text{A.91})$$

where n is the number of discretization points. The attitude demand vector can be found from

$$\mathbf{x}'_d = \frac{df}{dt}(t = \ell/L). \quad (\text{A.92})$$

Hence the inclination θ_{inc}^d and azimuth θ_{azi}^d reference demand values for the inner IAH loop can therefore be determined from

$$\theta_{inc}^d = \arctan \frac{[(\mathbf{x}'_d \cdot \mathbf{j})^2 + (\mathbf{x}'_d \cdot \mathbf{k})^2]^{\frac{1}{2}}}{\mathbf{x}'_d \cdot \mathbf{i}}, \quad (\text{A.93})$$

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and

$$\theta_{azi}^d = \arctan \frac{\mathbf{x}'_d \cdot \mathbf{k}}{\mathbf{x}'_d \cdot \mathbf{j}} \quad (\text{A.94})$$

and unit vectors, $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are defined in Section A.3.3.

Results

Waypoint-following Simulation

The trajectory-following architecture shown in Fig. A.19 incorporating OGH curves as outlined in Section A.3.4 and using (A.74) and (A.75) as the plant model was simulated in the time domain. The simulation parameters are shown in Table A.5 for an idealized initially horizontal directional drill capable of steering at $15^\circ/100$ ft drilling at a constant rate of penetration of 50 ft/hr. In this simulation, the tool started at a kick-off point (KOP) with two waypoint positions with associated attitudes as displayed in Table A.4.

Table A.4: General Path Simulation

Waypoint	x (m)	y (m)	z (m)	$\theta_{inc}^d(^{\circ})$	$\theta_{azi}^d(^{\circ})$
KOP	0	0	0	90	270
Intermediate	200	300	-40	90	308
End	208	800	-320	90	308

The resulting space curve from the simulated run is shown in Fig. A.21. In this diagram, the three triangular symbols show the waypoint positions listed in Table A.4. The black arrows in the diagram point in the direction of the applied tool-face angle, the relative magnitude of which indicates the curvature demanded from the tool by the algorithm.

It can be seen in Fig. A.21 and in Fig. A.24 that the trajectory-following controller does make the tool converge to the waypoints as the tool propagates. Furthermore, Fig. A.23 shows the attitude error magnitude of the tool computed as

$$\mathbf{x}'_{error} = \|\mathbf{x}'_m - \mathbf{x}'_{wp}\|, \quad (\text{A.95})$$

where $\mathbf{x}'_m$ is a unit vector representing the measured attitude of the drill at any instant in the simulation and $\mathbf{x}'_{wp}$ is the attitude of the nearest target.

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The value of the tool attitude error can be seen to diminish towards any given target waypoint, as can be seen from Fig. A.23. However, for the intermediate waypoint it does not diminish entirely. The reason for this is explained by Fig. A.22, where the curvature of the path shortly before reaching the first target is seen to saturate to the constrained maximum curvature of $15^\circ/100$ ft as the tool nears the target.

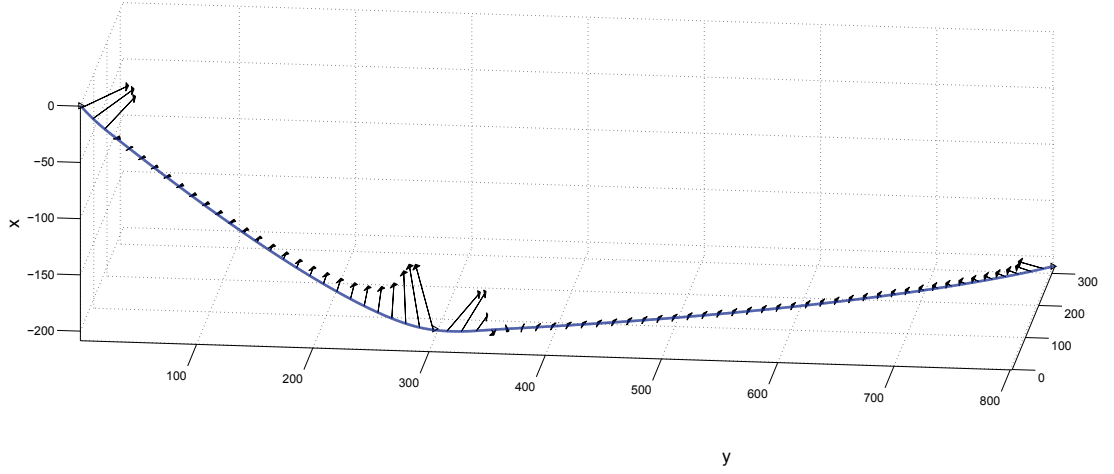


Figure A.21: Simulation results of online path-planning trajectory controller following two waypoints. The arrows signify the magnitude of curvature in the direction of the Frenet-Serret normal vector which displays the tool face and steering ratio signals determined by the controller to achieve the path.

Pay Zone Path-following Simulation

To demonstrate the use of OGH curves for reaching and following a well plan path in the pay zone that has been defined *a priori*, a transient simulation was run using (A.74) and (A.75) as the plant model, the operating point parameters listed in Table A.5, and the path following architecture shown in Fig. A.19. The KOP for the simulation was deliberately set to be at an arbitrary attitude and position before the start point of the *a priori* path. The *a priori* path was generated from a separately run simulation in which the tool's K_{dls} was set to $8^\circ/100$ ft and the tool steered open loop according to

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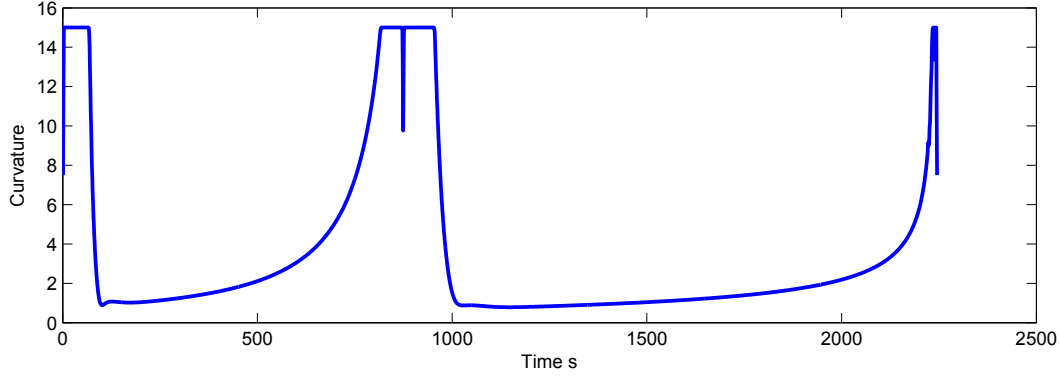


Figure A.22: Curvature in degrees per 100 feet of OGH path following response.

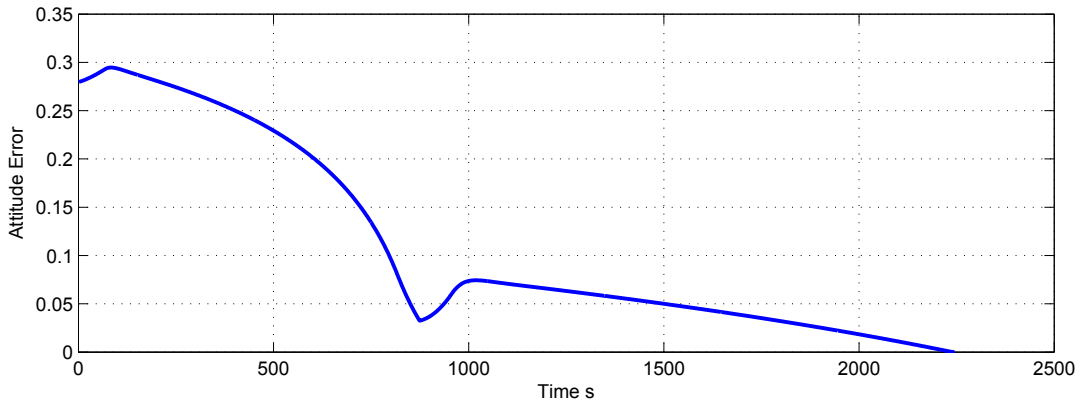


Figure A.23: Magnitude of error in attitude of the tool and its nearest target.

a well plan of predefined U_{tf} and U_{dls} down-links (commands) indexed by MD .

Table A.5: Transient simulation parameters

Parameter	Value & description
V_{rop}	50 ft/hr
ΔT	90 s, drilling cycle period
K_{dls}	$15^\circ/100$ ft tool capacity & $8^\circ/100$ ft well plan
ω_a	$2\pi/2.1 \times 10^4$ rad/s design θ_{azi} response natural frequency
ω_i	$2\pi/2.0 \times 10^4$ rad/s design θ_{inc} response natural frequency
T_z	Fixed step ode3 Bogaki-Shampine solver, 200s step size

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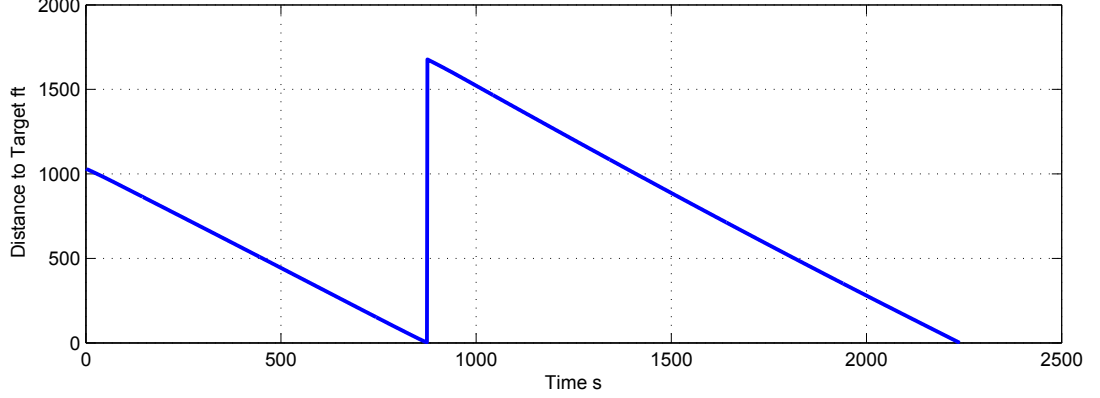


Figure A.24: Simulated magnitude of distance from the tool to the nearest target.

Fig. A.25 shows the trajectory response of the tool using the OGH curve algorithm. It can be seen that the tool not only successfully tracked the *a priori* path defined by a series of coarsely spaced waypoints, but also successfully reached the waypoints on the *a priori* path from the arbitrary KOP.

Fig. A.26 shows the control inputs, steering ratio, and tool face required to follow the waypoints shown in Fig. A.25. It can be seen from Fig. A.26 that generally the steering ratio drops as the tool nears the waypoint before increasing to saturation (100%) when the algorithm switches to the next waypoint. Plot 3 in Fig. A.26 shows the distance of the tool to the waypoint at any instant, with the distance decreasing linearly between waypoints due to the constant V_{rop} .

A.3.5 Conclusion

This paper describes the application of OGH interpolant space curve construction to path-following applications for directional drilling. Here OGH curves are used for online path planning to generate inclination and azimuth demand signals. The use of OGH space curves in this way enables a correction path in the form of a minimum strain energy cubic polynomial to be constructed between the instantaneous tool position and a succession of waypoints, each targeted in turn. The path-following architecture consists of a inner attitude-hold feedback loop based on the authors' earlier IAH work and an outer loop to generate IAH attitude reference demands. The OGH space curve fitting algorithm is implemented in the outer loop of the path-following architecture.

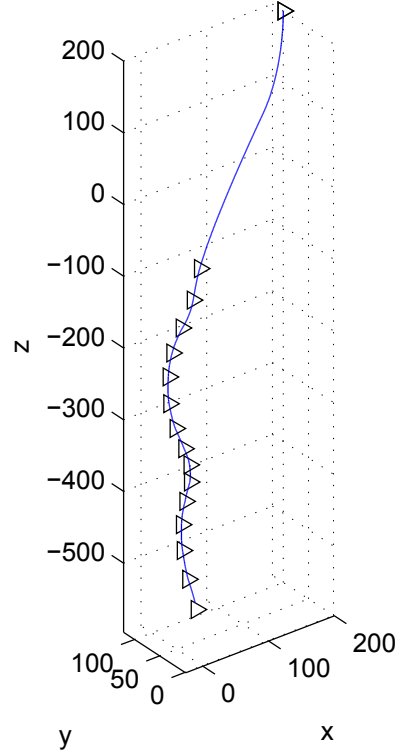


Figure A.25: OGH curve fit path following response

By simulation, using this algorithm, the drill is shown to converge toward the waypoint position and attitude satisfactorily for typical drilling operating conditions.

The advantage of using OGH space curves in this application is that for the generation of a correction path, the resulting curve is described by a readily differentiable polynomial that enables evaluation of the strain energy applied as the drillstring is bent along the projected correction path. The minimal strain energy OGH correction-path curve fitted in this way therefore promotes the reduction of wellbore friction and hence it is anticipated will be beneficial for extended reach drilling. Additionally, simulation shows that the minimal strain energy OGH space curve fit approach is able to reach and follow a path defined by a sequence of waypoints.

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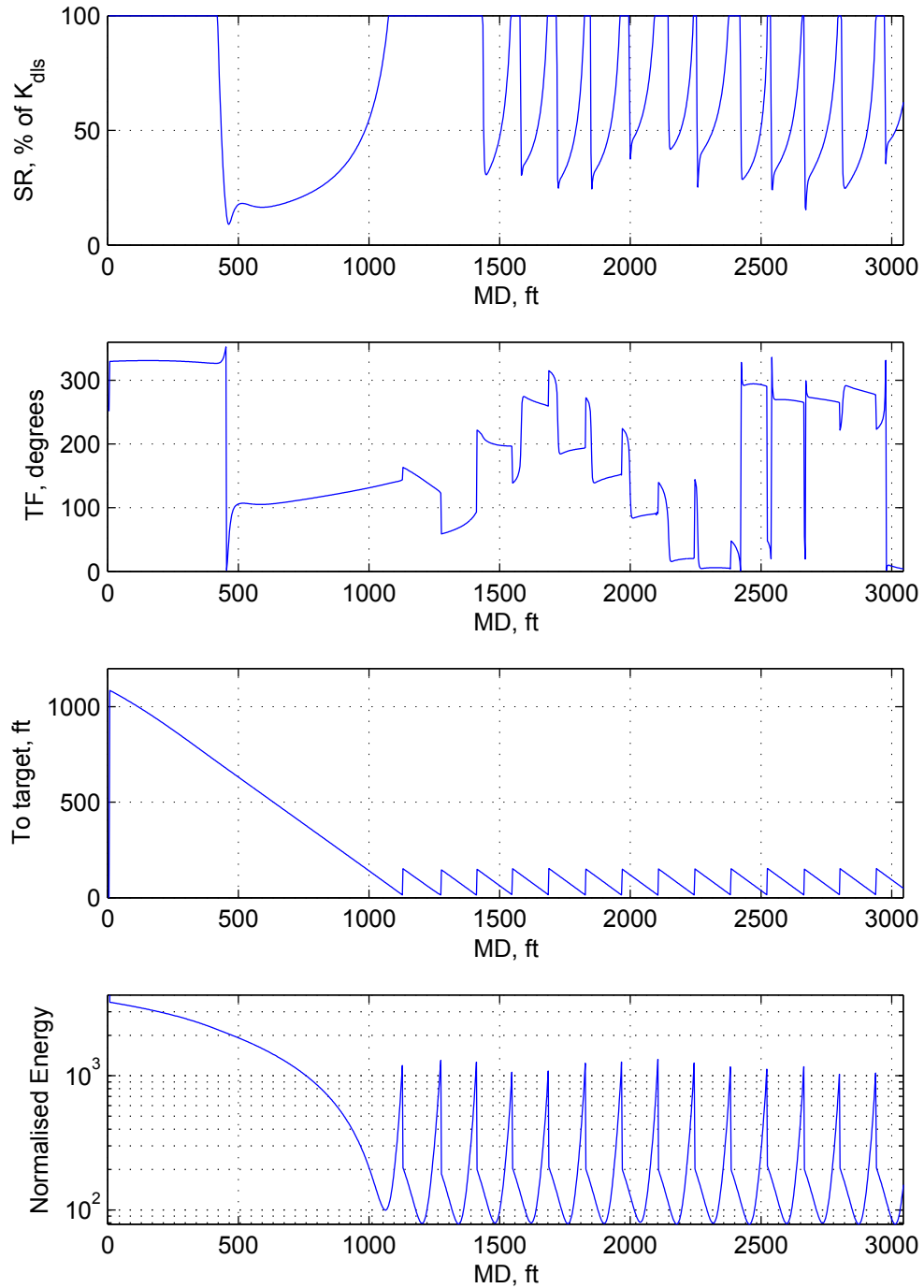


Figure A.26: Steering ratio, tool face and distance to way point

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