



Destination and time-series inference of moving objects via conditionally Markov process

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Abstract

This paper presents a destination and time-series inference algorithm for tracking moving targets. The destination of the object is considered the intent, and inference and state estimation are performed in the Bayesian framework. To describe the destination-aware target motion, we construct the state transition model using a conditionally Markov process. We introduce a multiple model to achieve simultaneous intent and time-series inferences. Given finite destination candidates, the maximum a posteriori hypothesis is chosen as the destination. For time-series inference, local estimates obtained from Kalman filters are fused to yield target state estimates. To address unspecified terminal conditions, the proposed algorithm incorporates parameter correction techniques based on relative geometry. Numerical simulations are performed to validate the proposed inference algorithm.

Keywords Intent inference · Conditionally Markov process · Target tracking · Multiple model adaptive estimation · Bayesian inference · Kalman filtering

1 Introduction

Destination inference has garnered considerable attention in recent years as it can enable better understanding of the object behavior and enhance pattern recognition, thereby supporting informed decision-making. The destination helps clarify the object intent and expected future behavior, thereby enabling more accurate state estimations. In general, destination inference involves selecting the most probable intent within a finite set of destination candidates collected a priori. The intent inference problem has been investigated using probabilistic approaches based on estimation theory [1–4], the Bayesian inference framework [5–7], pattern matching [8, 9], and machine learning techniques [10,

11]. In addition, intent inference has been extended to many applications, such as surveillance [9], air-traffic control [2, 3, 10], human–machine interaction [4, 11, 12], and intelligent interactive displays [5, 6, 13].

Considering the inferred destination, time-series inference requires sophisticated modeling of the target motion. This type of inference is aimed at estimating the object's current and future trajectory over time. The inferred destination provides information regarding the future goal point, which can be used to infer the time-series of the target more accurately. To incorporate destination information into time-series inference, destination-driven dynamic modeling is necessary. Unlike the Markov process (MP), the destination-driven dynamic model generally consists of the destination component in addition to the initial distribution and evolution law, enabling more accurate tracking by incorporating the expected future information into the current estimates. However, the destination-driven dynamic model requires a generalized stochastic process that exhibits non-causal properties, as the destination implies the future position of the object. Several stochastic processes, such as reciprocal process (RP) [14–17], Markov bridge [6], and conditionally Markov process (CMP) [15, 18–21], have been developed to address these non-causal properties.

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Intent inference and state estimation have been extensively studied. In [1, 2], hybrid state estimators using an interactive multiple model (IMM) filtering were proposed. Moreover, state estimation has been achieved using Markov process models for finite flight modes, with destination inference and state estimation simultaneously performed by computing the likelihood between the target's velocity vector and destination. However, these studies primarily focused on the flight mode inference for state estimation, and the destination information was not directly incorporated into the inference. Other researchers [5, 6] incorporated the destination information into the dynamic model for Bayesian intent inference. Reference [5] used an Ornstein–Uhlenbeck (OU) process to build the state transition probability, with destination inference performed by sequentially calculating the likelihoods via the multiple model adaptive estimator (MMAE). Reference [6] derived the RP model as the state transition probability using the Markov bridge. MMAE-based filters capture the influence of the endpoint on the dynamic model and have been noted to be effective in human–machine interaction tasks. However, the Markov-bridging model in [6] assumes that the initial distribution is independent of the terminal distribution, which may be inconsistent with the Markov nature of the transition density. To apply a more generic model that includes such a correlation, it is necessary to describe the dynamic model with a general and constructive framework.

In this context, CMP can suitably address the destination and time-series inference in a constructive and generalized manner. Stochastic sequences based on CMP consist of three components: initial distribution, terminal distribution, and transition probability. The principle of CMP is that the state evolution can be described as the state transition probability conditioned on the current state and endpoint. Several studies [15, 20] have demonstrated that CMP is a more general process than MP and RP. Moreover, a Markov-induced dynamic model can properly describe destination-oriented motion [19], and a Kalman filter can be implemented for state estimation based on CMP models [21]. Although CMP models have been recently extended to a series of waypoint tracking target estimation tasks [20], intent inference based on CMP has not been extensively explored.

Building upon the work of [19–21], this study is aimed at extending CMP-based time-series inference to simultaneously address the intent and time-series inference problems. Specifically, we propose a destination and time-series inference algorithm based on CMP, with the effective management of unspecified parameters in the modeling process. For intent inference, a destination is considered the intent of the object, and the Bayesian inference framework is used to estimate the intended destination by choosing the maximum posterior probability density from a finite set of candidates. For each candidate destination, the destination-driven

state transition model is formulated using the CMP. Thus, the state transition model based on the CMP can accurately describe more general destination-driven target motion in time-series inference, outperforming the Markov-bridging model outlined in Ref. [6]. To incorporate the arrival time and terminal velocity as critical components of the CMP model, we introduce a recursive estimation process for terminal values before the time-series inference step, thereby easing the dependence on the prior distribution for state estimation. Consequently, the proposed intent inference process yields reliable destination and state estimates.

The main contributions of this study can be summarized as follows. First, we develop intent and time-series inference algorithms using a general class of destination-aware models via CMP. The CMP-based intent inference can capture more general destination-aware motion than the existing approaches based on MP and RP. Second, the recursive parameter update strategy in time-series inference can suitably manage undetermined parameters present in terminal states by considering the instantaneous relative geometry. This treatment can alleviate the dependency on the accuracy of prior distributions, thereby improving destination inference as well as time-series estimation.

The remaining paper is organized as follows. Section 2 provides preliminary results for CMP along with MP and RP. Based on this mathematical background, the intent inference algorithm is proposed, as outlined in Sect. 3. To demonstrate the effectiveness of the proposed algorithm, a numerical study is performed, and the results are discussed in Sect. 4. Section 5 presents the concluding remarks and provides recommendations for future research.

2 Preliminaries

This section provides a preliminary and mathematical basis for describing destination-aware motion based on stochastic processes. Definitions of key stochastic processes, such as MP, RP, and CMP, are introduced, and several useful properties, such as the inclusive relations between these processes, are presented. The destination-aware dynamical model is introduced based on the stochastic process presented in [19].

2.1 Definitions and preliminaries

Throughout this study, the following notation is used to describe the sequences of state variables:

$$x_i^j = \{x_i, x_{i+1}, \dots, x_j\}, \quad [x_k] = [x_k]_0^N.$$

Various stochastic processes can be used to represent state sequences. In this section, we briefly review several stochastic processes with important properties.

Lemma 1 [19] $[x_k]$ is Markov iff $F(\xi_i|[x_k]_0^j) = F(\xi_i|x_j) \forall j < i, \forall \xi_i \in \mathbb{R}^n$, where $F(\cdot|\cdot)$ denotes the conditional cumulative distribution function.

Lemma 2 [16] $[x_k]$ is RP iff $F(\xi_i|[x_k]_0^j, [x_k]_l^N) = F(\xi_i|x_j, x_l) \forall j < i < l \in [0, N], \forall \xi_i \in \mathbb{R}^n$.

Lemma 3 [19] $[x_k]$ is CM_c if and only if $F(\xi_i|[x_k]_0^j, x_c) = F(\xi_i|x_j, x_c) \forall j < i \in [0, N], \forall \xi_i \in \mathbb{R}^n$.

In particular, the concept of CM_L is applied when c in CM_c is designated as the endpoint N . Unlike MP, RP and CMP exhibit non-causal properties suitable for describing destination-aware motions. RP can be characterized by a three-point distribution, where the conditional probability density function (PDF) depends on its adjacent points. In contrast, CMP, especially in the context of CM_L , realizes stochastic sequences based on both the endpoint and previous point. The PDF in CM_L satisfies $p(x_k|[x_k]_0^{k-1}, x_N) = p(x_k|x_{k-1}, x_N)$. Based on this relationship, the CM_L sequence can be generated through the following steps: (i) generate endpoint states from the density $p(x_0, x_N)$ and (ii) sequentially update the states using $p(x_k|x_{k-1}, x_N)$

$$\begin{aligned} p([x_k]) &= p([x_k]_0^{N-1}|x_N)p(x_N) \\ &= p([x_k]_1^{N-1}|x_0, x_N)p(x_0|x_N)p(x_N) \\ &= \left(\prod_{i=1}^{N-1} p(x_i|x_{i-1}, x_N) \right) p(x_0|x_N)p(x_N). \end{aligned} \quad (1)$$

The inclusive relationships between MP, RP, and CMP have been previously investigated [14, 15, 19]. Rezaie and Li [20] identified the properties of stochastic processes in the context of CM_L . As indicated in these studies, suppose sequence $[x_k]$ is a zero-mean nonsingular Gaussian (ZMNG). The dynamic model based on (1) can be expressed as

Theorem 1 [20] A ZMNG $[x_k]$ is CM_L if and only if it satisfies

$$x_k = G_{k,k-1}x_{k-1} + G_{k,N}x_N + e_k \quad (2)$$

with boundary condition

$$x_N = e_N, \quad x_0 = G_{0,N}x_N + e_0, \quad (3)$$

where $e_k \sim \mathcal{N}(0, Q_k)$ and $(G_{k,k-1}, G_{k,N})$ represent state transition matrices.

Based on (2) and (3), the following theorems clarify how the CM_L model becomes a reciprocal (Markov) process as

Theorem 2 [20] A ZMNG $[x_k]$ is reciprocal if and only if it satisfies (2) with (3) and

$$G_k^{-1}G_{k,c} = G'_{k+1,k}G_{k+1}^{-1}G_{k+1,c}. \quad (4)$$

Moreover, a ZMNG $[x_k]$ is Markov if and only if it satisfies (2) – (4) and

$$G_0^{-1}G_{0,N} = G'_{1,0}G_1^{-1}G_{1,N}. \quad (5)$$

These theorems indicate that CM_L is a more general process than RP and MP, i.e., $MP \subset RP \subset CM_L$.

2.2 Markov-induced CM_L model

This section describes the destination-aware dynamic model introduced in [21]. The modeling process requires the following assumption.

Assumption 1 The initial and terminal state probability densities are assumed to be known in prior as $x_0 \sim \mathcal{N}(\mu_0, C_0)$ to $x_N \sim \mathcal{N}(\mu_N, C_N)$.

Using this assumption, consider a (nearly) constant velocity model in two-dimensional space $x_k = [p_x, p_y, v_x, v_y]_k^T$

$$x_k = Fx_{k-1} + w_{k-1}, \quad (6)$$

where $w_{k-1} \sim \mathcal{N}(0, Q)$, and state transition matrix F is given by

$$F = \begin{bmatrix} I & \Delta t I \\ \mathbf{0} & I \end{bmatrix}, Q = q \begin{bmatrix} \frac{\Delta t^3}{3} I & \frac{\Delta t^2}{2} I \\ \frac{\Delta t^2}{2} I & \Delta t I \end{bmatrix}, \quad (7)$$

where I is an identical 2-by-2 matrix, and $\mathbf{0}$ is a 2-by-2 zero matrix. The covariance matrix is established considering the cross-correlation between the origin and destination as $C = \begin{bmatrix} C_0 & C_{0,N} \\ C_{N,0} & C_N \end{bmatrix}$. Then, the dynamic model evolving from x_0 to x_N with evolution law (6) can be re-expressed in terms of a Markov-induced CM_L model [19] as

$$\begin{aligned} x_k &= G_{k,k-1}x_{k-1} + G_{k,N}x_N + e_k \\ x_0 &= \mu_0 + G_{0,N}(x_N - \mu_N) + e_0 \\ x_N &= \mu_N + e_N, \end{aligned} \quad (8)$$

where the matrices are calculated to represent the evolution law (7) in terms of CM_L as

$$\begin{aligned} G_{0,N} &= C_{0,N}C_N^{-1}, \quad G_N = C_N, \quad G_0 = C_0 - C_{0,N}C_N^{-1}C'_{0,N} \\ G_{k,k-1} &= (I_4 - G_{k,N}F^{N-k})F \\ G_{k,N} &= G_k(F^{N-k})'C_{N,k}^{-1} \\ G_k &= (Q^{-1} + (F^{N-k})'C_{N,k}^{-1}F^{N-k})^{-1} \\ C_{N|k} &= \sum_{n=k}^{N-1} F^{N-n-1}Q(F^{N-n-1})', \end{aligned} \quad (9)$$

where I_4 denotes the 4-by-4 identity matrix. Detailed derivations of the state-transition matrices can be found in [19].

Figure 1 shows the sample paths generated by different stochastic sequences, where parameters for the initial and terminal distributions are set as $\mu_0 = [2000, 70, 5000, 0]^T$, $\mu_N = [80000, 70, 7000, 0]^T$ and

$$C_0 = C_N = \text{diag}\left(\begin{bmatrix} 100 & 1 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 100 & 1 \\ 1 & 5 \end{bmatrix}\right).$$

The MP paths expand from the initial location and rarely reach the designated goal point. In contrast, sample paths using the CMP disperse in the space after around (30,000, 6000) to satisfy the terminal condition and eventually reach the designated goal location. The CMP incorporates terminal information and approaches the goal point more effectively. Notably, the final time N significantly influences the trajectories of RP and CMP sequences. In Fig. 1b, considerable speed variations can be observed in CMP sequences, aimed at satisfying the boundary conditions. Trajectories driven by the CMP are shaped, while the target reaches the designated destination in accordance with the speed variation. CM_L paths were generated with two specified terminal times. It is observed that sample paths with identical terminal times exhibit similar patterns, while different terminal times result in distinct velocity profiles. This result indicates that the terminal incident N is properly determined to accurately estimate the target behavior during the entire tracking period, and accurate terminal time estimation is crucial, particularly when such information is not provided in advance. If the terminal time is not known in prior, approximation methods must be used to appropriately estimate or adjust the terminal parameter.

3 Bayesian approach for destination and time-series inference

This section describes the design of our intent and time-series inference process for moving objects. To address the inference problem, we use the multiple model approach based on MMAE [22–24]. The following assumptions are introduced.

Assumption 2 a set of destinations $g \in \mathcal{G} = \{g^{(1)}, g^{(2)}, \dots, g^{(l)}\}$ and the corresponding location information set $\mathcal{D} = \{\mathbf{d}^{(1)}, \mathbf{d}^{(2)}, \dots, \mathbf{d}^{(l)}\}$, $\mathbf{d}^{(i)} \in \mathcal{R}^2$ are given, where l is the number of destination candidates.

Assumption 3 The true intent g is invariant and has a mode space S that is time-invariant and identical to the time-invariant finite model set $\mathcal{G}$.

Assumptions 3 and 4 indicate that the true intent exists within the candidate set and does not change during maneuvers. Using these assumptions, the intent inference problem can be interpreted as the selection of the maximum posterior probability $\rho_k^{(i)} = p(g^{(i)}; Z_k)$ among the candidate set, given the observed data $Z_k = [z_1, z_2, \dots, z_k]^T$

$$\hat{g}_k = \arg \max_{g \in \mathcal{G}} \rho_k^{(i)}, \quad (10)$$

where superscript (i) corresponds to the mode variables. Note that the estimated intent $\hat{g}$ has the argument k in its subscript, because the maximum mode may change depending on the observed data Z_k . Furthermore, state estimation is regarded as a means of time-series inference

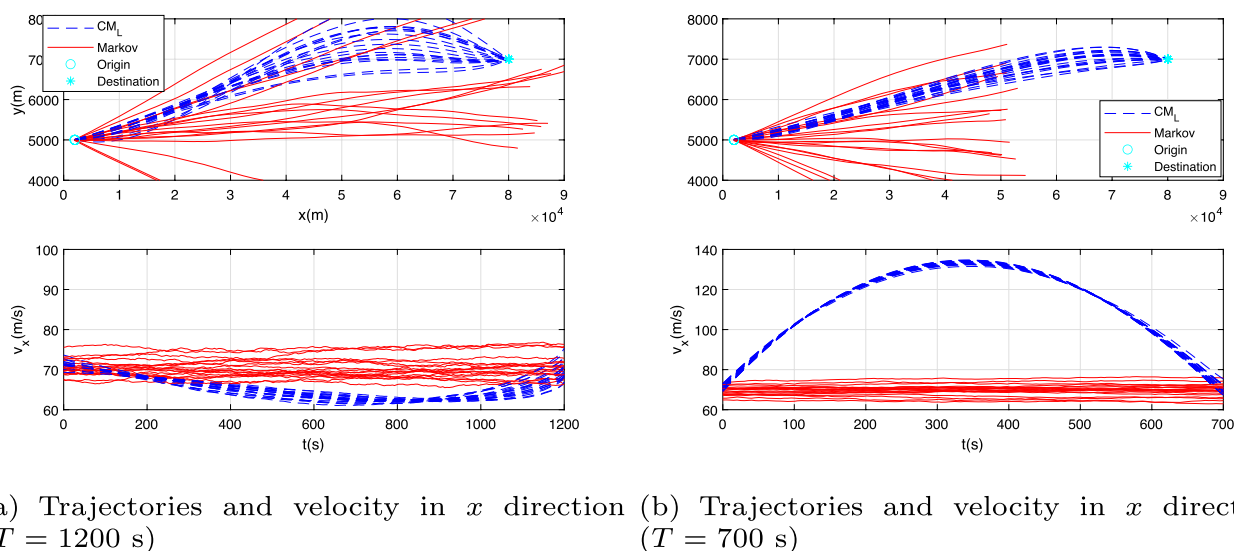


Fig. 1 Sample paths based on different stochastic sequences (blue: Markov-induced CM_L ; red: CV model (MP))

$$\hat{x}_k = \sum_i \rho_k^{(i)} \hat{x}_k^{(i)}. \quad (11)$$

The MMAE-based filter is designed to obtain the posterior probability $\rho_k^{(i)}$ and state estimates $\hat{x}_k^{(i)}$ for each destination $g^{(i)}$. We construct multiple models for each destination. State estimation is performed using the MMAE filter as a means of time-series inference. Section 3.1 presents an approach to address the undetermined terminal conditions using the relative geometry. Section 3.2 describes the filtering strategy for time-series inference, and Sect. 3.3 discusses destination inference using the Bayesian framework.

3.1 Recursive correction of undetermined terminal parameters

In general, the final time instance t_f and N should be known or estimated through a prior distribution. As mentioned in Sect. 2.2, the CMP model requires terminal conditions. Considering Assumptions 1 and 3, the terminal mean $\mu_N^{(i)}$ for each destination mode can be represented as $\mu_N^{(i)} = (\mathbf{d}^{(i)}, \mathbf{v}_f^{(i)})$, where $\mathbf{v}_f^{(i)} = (v_{x,f}^{(i)}, v_{y,f}^{(i)})$ and N_i are parameters to be determined. To address the undetermined parameters, Ref. [6] adopted a quadrature rule to marginalize out t_f on the conditional PDF, which requires parallel calculation of multiple Kalman filters (KFs) for each quadrature point $t_f^j, j = 1, \dots, N_p$. In addition, $\mathbf{v}_f^{(i)}$ is assumed to be known along with the terminal position for each destination. However, the discrepancy of these parameters at the destination may lead to performance degradation of the intent inference.

To address this limitation, this study proposes a recursive correction of terminal constraints to update the endpoint distributions. Given destination $\mathbf{d}^{(i)} = [d_x^{(i)}, d_y^{(i)}]$, the relative distance r , line-of-sight angle λ , and current speed V can be calculated as

$$\begin{aligned} r^{(i)} &= \sqrt{(d_x^{(i)} - p_{x,k})^2 + (d_y^{(i)} - p_{y,k})^2}, \\ \lambda^{(i)} &= \tan^{-1} \left(\frac{d_y^{(i)} - p_{y,k}}{d_x^{(i)} - p_{x,k}} \right) \\ V &= \sqrt{v_{x,y}^2 + v_{y,k}^2}. \end{aligned} \quad (12)$$

Note that state values $p_{x,k}$, $p_{y,k}$, $v_{x,k}$, and $v_{y,k}$ are used to calculate values in (12) when these are available. Considering that the true states are unavailable, we can substitute the state estimates $\hat{x}_k$ into (12) to obtain the parameters. When the moving target has a constant speed, the desired motion of the CV model is to move toward the destination by aligning its heading with the destination, which coincides with the line-of-sight (LOS). In this respect, the ideal heading

angle $\bar{\mathbf{v}}_{f,\text{ideal}}^{(i)}$ is considered in terms of the current speed V and the instantaneous geometry with the destination as

$$\begin{aligned} \bar{\mathbf{v}}_{f,\text{ideal}}^{(i)} &= \begin{bmatrix} \bar{v}_{x,f,\text{ideal}}^{(i)} \\ \bar{v}_{y,f,\text{ideal}}^{(i)} \end{bmatrix} \\ &= \begin{bmatrix} V \cos \lambda^{(i)} \\ V \sin \lambda^{(i)} \end{bmatrix}. \end{aligned} \quad (13)$$

Using (13), the terminal velocity vector $[\bar{v}_{x,f}^{(i)}, \bar{v}_{y,f}^{(i)}]$ can be recursively corrected as

$$\begin{aligned} \bar{v}_{x,f,k}^{(i)} &= \bar{v}_{x,f,k-1}^{(i)} + \alpha (\bar{v}_{x,f,\text{ideal}}^{(i)} - \bar{v}_{x,f,k-1}^{(i)}) \\ \bar{v}_{y,f,k}^{(i)} &= \bar{v}_{y,f,k-1}^{(i)} + \alpha (\bar{v}_{y,f,\text{ideal}}^{(i)} - \bar{v}_{y,f,k-1}^{(i)}), \end{aligned} \quad (14)$$

where $(\bar{v}_{x,f}^{(i)}, \bar{v}_{y,f}^{(i)})_k$ represent the terminal velocity parameters recursively updated at the current step k , and $(\bar{v}_{x,f}^{(i)}, \bar{v}_{y,f}^{(i)})_{k-1}$ denote the parameters from the previous step $k-1$. $\alpha \in [0, 1]$ is the design parameter for the recursive update. The bar notation implies that the updated values will be used as the mean, and the terminal velocity $\mathbf{v}_f^{(i)}$ is selected at every time instant k by the probability distribution, which has the mean and covariance

$$\mathbf{v}_{f,k}^{(i)} \sim \mathcal{N}((\bar{v}_{x,f,k}^{(i)}, \bar{v}_{y,f,k}^{(i)}), \Sigma_{v,f}), \quad (15)$$

where $\Sigma_{v,f}$ is the covariance matrix.

For determining the terminal time, the ideal time-to-go is considered as the desired motion of the target, described by a constant velocity model directly heading toward the destination. By the definition $\bar{t}_f = \bar{t}_{go} + t$, and t is the current time instant, the ideal terminal time can be expressed as

$$\bar{t}_{f,\text{ideal}}^{(i)} = \frac{r^{(i)}}{V} + t. \quad (16)$$

Finally, the recursive update for the terminal time can be expressed in the similar form shown in (14)

$$\bar{t}_{f,k}^{(i)} = \bar{t}_{f,k-1}^{(i)} + \alpha (\bar{t}_{f,\text{ideal}}^{(i)} - \bar{t}_{f,k-1}^{(i)}). \quad (17)$$

Similar to (15), the terminal time $t_{f,k}^{(i)}$ and terminal incident in the discrete time domain N_f are selected by considering uncertainty around the updated value $\bar{t}_{f,k}^{(i)}$ with the following probability distribution:

$$t_{f,k}^{(i)} \sim \mathcal{N}(\bar{t}_{f,k}^{(i)}, \sigma_1^2), \quad N_{f,k}^{(i)} = \text{round} \left(\frac{t_{f,k}^{(i)}}{\Delta t} \right), \quad (18)$$

where σ_1 is the covariance, and Δt is the time step for discretization.

Remark 1 The recursive update with a non-zero value for α enables a seamless transition from the previous parameters to the current ones, because of its mathematical formulation. When α is set to 0, the parameter estimate relies entirely on the initial setting, and the update revolves around this mean due to the influence of covariance (Eq. (14)). This approach may result in inaccurate estimation, particularly when the initial guess is incorrect. On the other hand, when α is set to 1, the update is primarily driven by the instantaneous geometry. Setting a value between 0 and 1 for α effectively acts as a low-pass filter, facilitating a smooth shift from the previous parameters to the input derived from the current geometry. This adjustment offers robust performance, mitigating errors associated with the initial setting and estimation.

Remark 2 Among the two unknown parameters, inaccurate temporal information has a greater impact on estimation accuracy. When the actual terminal time deviates from the predefined range, the estimation filter tends to over- or under-estimate the target state, resulting in significant estimation errors near the destination. Compared to the temporal parameter, inaccurate terminal velocity may show marginal impact on estimation performance. In scenarios where the terminal velocity or speed is unknown, one approach is to configure a relatively large covariance in the initial distribution of the terminal velocity and accommodate uncertainty. Bayesian filtering can adapt the initial error covariance. However, this adaptation may result in slower convergence and a reduced utilization of the CM property. The proposed recursive correction of terminal parameters based on instantaneous geometry can be beneficial, especially when the target abruptly changes its heading toward the destination. This alignment of the terminal velocity direction with the current velocity direction quickly adjusts the heading intent in the destination-directed motion, leading to enhanced estimation accuracy.

3.2 Filtering for time-series inference

State-augmentation based filtering is used for time-series inference of destination-aware object motion [21]. An augmented state variable $y_k = [x_k, x_N]^T$ is introduced. Then, the state transition model can be expressed in the linear form as

$$y_k = G_{k,k-1}^y y_{k-1} + e_k^y, \quad (19)$$

where $\text{cov}(e_k^y) = G_k^y$ and

$$G_{k,k-1}^y = \begin{bmatrix} G_{k,k-1} & G_{k,N} \\ \mathbf{0}_4 & I_4 \end{bmatrix}, \quad (20)$$

$$G_k^y = \begin{bmatrix} G_{k+1} & \mathbf{0}_4 \\ \mathbf{0}_4 & \mathbf{0}_4 \end{bmatrix},$$

where $\mathbf{0}_4$ denote the zero matrix, and $(G_{k,k-1}, G_{k,N}, G_{k+1})$ defined in (9) are used to build the state transition model. Because the evolution law is described in a linear form, the state predictive density and its priori estimates (for $i = 1, \dots, l$) are obtained as

$$p(y_k^{(i)} | Z_{k-1}) = \mathcal{N}(y_k^{(i)} | \hat{y}_{k|k-1}^{(i)}, \Sigma_{k|k-1}^{(i)})$$

$$\hat{y}_{k|k-1}^{(i)} = G_{k,k-1}^{y,(i)} \hat{y}_{k-1|k-1}^{(i)}, \quad (21)$$

$$\Sigma_{k|k-1}^{(i)} = G_{k,k-1}^{y,(i)} \Sigma_{k-1}^{(i)} (G_{k,k-1}^{y,(i)})^T + G_k^y.$$

Moreover, the observation $z_k = [p_x, p_y]^T$ can be expressed in the state space form $z_k = H_k y_k + v_k$, where

$$H_k = [I \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}]. \quad (22)$$

Then, the observed predictive density and its estimates can be described as

$$p(z_k | Z_{k-1}, g^{(i)}) \sim \mathcal{N}(\hat{z}_{k|k-1}^{(i)}, \Sigma_{k|k-1}^{mm,(i)})$$

$$\hat{z}_{k|k-1}^{(i)} = H_k \hat{y}_{k|k-1}^{(i)}, \quad (23)$$

$$\Sigma_{k|k-1}^{mm,(i)} = H_k \Sigma_{k|k-1}^{(i)} H_k^T + R_k.$$

The state posteriori for each mode i can be calculated as

$$K_k^{(i)} = \Sigma_{k|k-1}^{(i)} H_k^T (\Sigma_{k|k-1}^{mm,(i)})^{-1}$$

$$\hat{y}_{k|k}^{(i)} = \hat{y}_{k|k-1}^{(i)} + K_k^{(i)} (z_k - \hat{z}_{k|k-1}^{(i)}), \quad (24)$$

$$\Sigma_k^{(i)} = \Sigma_{k|k-1}^{(i)} - K_k^{(i)} H_k \Sigma_{k|k-1}^{(i)}.$$

Finally, the state estimates can be obtained as

$$\hat{x}_k = \sum_i \rho_k^{(i)} \hat{x}_k^{(i)} = \sum_i \rho_k^{(i)} [I_4 \ \mathbf{0}_4] \hat{y}_{k|k}^{(i)}, \quad (25)$$

where $\rho_k^{(i)}$ is the mode probability to be determined as described in the next section.

3.3 Destination inference

The key principle of intent inference is to calculate the mode probabilities based on Bayesian inference

$$\rho_k^{(i)} \triangleq p(g^{(i)} | Z_k) \propto p(Z_k | g^{(i)}) p(g^{(i)}), \quad (26)$$

where $p(Z_k | g^{(i)})$ is the likelihood, and $p(g^{(i)})$ is the prior distribution of the destination. Without knowledge of any prior information, the prior function can be intuitively set as the uniform distribution $p(g^{(i)}) = 1/l$. In this case, the MAP is equivalent to the maximum likelihood problem. Prediction error decomposition yields the likelihood expressed in the recursive form

$$p(Z_k | g^{(i)}) = p(z_k | Z_{k-1}, g^{(i)}) p(Z_{k-1} | g^{(i)}). \quad (27)$$

Using Eq. (26), the likelihood can be recursively calculated. After updating the likelihood, the posterior probability is normalized to evaluate the mode probability

$$\rho_k^{(i)} = \frac{p(Z_k|g^{(i)})p(g^{(i)})}{\sum_{g^{(j)} \in \mathcal{G}} p(Z_k|g^{(j)})p(g^{(j)})}. \quad (28)$$

The overall flow of the destination and time-series inference using MMAE is presented as Algorithm 1.

Algorithm 1 Intent and time-series inference via MMAE

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procedure MAP( $\hat{\cdot}$ ,  $\hat{\mathbf{g}}$ ,  $\mathbf{y}$ )
    Input:  $p(g^{(i)})$  (priors),  $\mathbf{d}^{(i)}$  (intended goal point)  $i = 1, \dots, l$ 
    Initialize  $\hat{\mathbf{y}}_0^{(i)}, \Sigma_0^{(i)}, (\bar{v}_{x,f,0}^{(i)}, \bar{v}_{y,f,0}^{(i)}, \bar{r}_{f,0}^{(i)})$ 
    for each observation at time instants  $k = 1, \dots, N$  do
        for all Intent candidate  $i = 1, \dots, l$  do
            Update mean values of terminal parameters  $(\bar{v}_{x,f,k}^{(i)}, \bar{v}_{y,f,k}^{(i)}, \bar{r}_{f,k}^{(i)})$ 
            using Eqs. (14) and (17)
            Calculate parameters for terminal conditions  $\mathbf{v}_{f,k}^{(i)}$  and
             $(t_{f,k}^{(i)}, N_{f,k}^{(i)})$  using Eqs. (15), and (18)
            Calculate state matrices  $G_{k,k-1}^{y,(i)}, G_k^{y,(i)}$  using Eq. (20)
            Calculate  $\hat{\mathbf{y}}_{k|k-1}^{(i)}$  and  $\Sigma_{k|k-1}^{(i)}$  using Eq. (21)
            Calculate  $\hat{z}_{k|k-1}^{(i)}$  and  $\Sigma_{k|k-1}^{mm,(i)}$  using Eq. (23)
            Calculate likelihood  $l_k^{(i)} = p(z_k|Z_{k-1}, g^{(i)})$  and  $L_k^{(i)} = p(Z_k|g^{(i)})$ 
            Calculate unnormalized posterior probability  $\tilde{p}(g^{(i)}|Z_k)$ 
            Calculate  $\hat{\mathbf{y}}_{k|k}^{(i)}$  and  $\Sigma_k^{(i)}$  using Eq. (24)
        end for
        Calculate normalized posterior probability  $\rho_k^{(i)} = \frac{\tilde{p}(g^{(i)}|Z_k)}{\sum_j \tilde{p}(g^{(j)}|Z_k)}$ 
        Store estimated intent  $\hat{g}_k = \arg \max_{g^{(i)} \in \mathcal{G}} \rho_k^{(i)}$ 
        Store estimated states  $\hat{\mathbf{y}}_k = \sum_i \rho_k^{(i)} \hat{\mathbf{y}}_{k|k}^{(i)}$ 
    end for
    return  $\hat{\mathbf{g}} = \{\hat{g}_1, \dots, \hat{g}_N\} \wedge \hat{\mathbf{y}} = \{\hat{\mathbf{y}}_1, \dots, \hat{\mathbf{y}}_N\}$ 
end procedure
    
```

4 Numerical simulation

Numerical simulations are performed to validate the proposed inference algorithm. In the simulation scenario, a single target is moving from an origin (0, 0) with constant speed $V_0 = 10$ m/s toward a destination. The target motion is described as

$$\begin{aligned} \dot{r} &= -V_0 \cos(\lambda - \gamma) \\ \dot{\lambda} &= \frac{V_0}{r} \sin(\lambda - \gamma) \\ \dot{\gamma} &= \frac{a_{\text{cmd}}}{V_0}, \end{aligned} \quad (29)$$

where r and λ represent the relative distance and LOS angle from the target position to a destination $\mathbf{d}$ with known g , respectively. γ is the flight path angle of the target, and a_{cmd} is the lateral acceleration. The target executes a homing maneuver until it reaches the destination

$$a_{\text{cmd}} = 3V\dot{\lambda} + \mathcal{U}(0, 2) \sin((r/r_0)^3).$$

For tracking the single target, the inference algorithm is required to generate inferred state estimates and the expected destination. In the first scenario, we focus on the time-series inference performance assuming that the destination is given in prior. The second scenario considers both the destination and time-series inferences. The following simulation parameters are selected for the proposed inference algorithm

$$C_0 = \text{diag}\{1, 1, 0.1, 0.1\},$$

$$C_N = \text{diag}\{1, 1, 100, 100\}$$

$$C_{0,N} = \text{diag}\{0.5, 0.5, 0.01, 0.01\},$$

$$C = \begin{bmatrix} C_0 & C_{0,N} \\ C_{0,N}^T & C_N \end{bmatrix}$$

$$\Delta t = 0.1, q = 1, Q = q \begin{bmatrix} \frac{\Delta t^3}{2} I & \frac{\Delta t^2}{2} I \\ \frac{3}{2} I & \Delta t I \end{bmatrix}$$

$$= \begin{bmatrix} 0.0003 & 0 & 0.0050 & 0 \\ 0 & 0.0003 & 0 & 0.0050 \\ 0.0050 & 0 & 0.1000 & 0 \\ 0 & 0.0050 & 0 & 0.1000 \end{bmatrix}$$

$$R_k = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \Sigma_{v,f} = \text{diag}\{3, 3\}, \sigma_1 = 0.04.$$

The performance of the proposed algorithm is compared with the method outlined in Ref. [6]. The simulation parameters are identical for both algorithms, and the prior distribution of the final time is chosen as $t_f \sim \mathcal{U}(15, 35)$, where $\mathcal{U}$ represents the uniform random distribution. Two performance measures, i.e., the root-mean-square error (RMSE) and success rate, are considered to statistically assess the performances

$$\text{RMSE} = \sum_{i=0}^N \sqrt{(p_x - \hat{p}_x)^2 + (p_y - \hat{p}_y)^2} \quad (30)$$

$$\text{SR} = \sum_{i=1}^N \frac{S(i)}{N}, \quad S(i) = \begin{cases} 1 & \text{if } \hat{g} = g \\ 0 & \text{otherwise} \end{cases}. \quad (31)$$

4.1 Scenario I: time-series inference

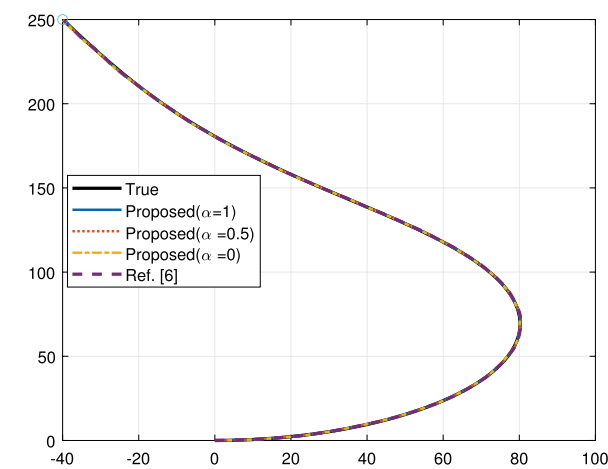
We consider two simulation cases with different locations of the destination

- Case I-a: $\mathbf{d} = [-40, 250]$, $[v_{x,f}, v_{y,f}]$: unspecified, t_f : unspecified
- Case I-b: $\mathbf{d} = [-40, 350]$, $[v_{x,f}, v_{y,f}]$: unspecified, t_f : unspecified.

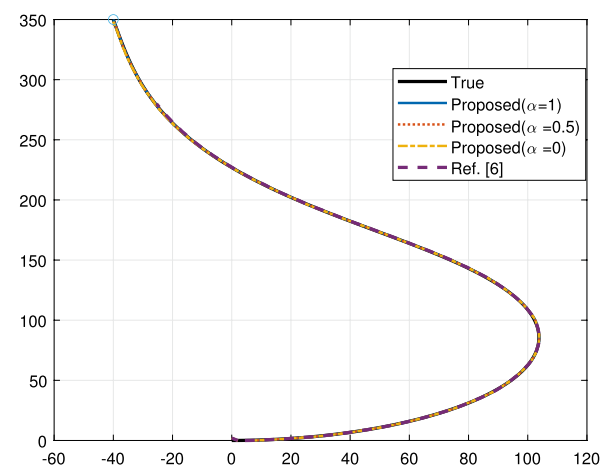
In Case I-b, the destination is farther than in Case I-a, resulting in a longer expected time of arrival. The selection of this case aims to highlight the impact of inaccurate prior information of the terminal time on time-series inference and to demonstrate the benefits of the recursive update of the terminal time parameter proposed in this study, particularly in mitigating performance deterioration caused by inaccurate temporal information.

Figure 2 shows the simulation results. In case I-a, both time-series inference algorithms can successfully estimate the target trajectory during target tracking. Using the benchmark approach, i.e., Kalman filtering with state augmentation, it was observed that enlarging the covariance improved the estimation error. However, substantial state errors persist,

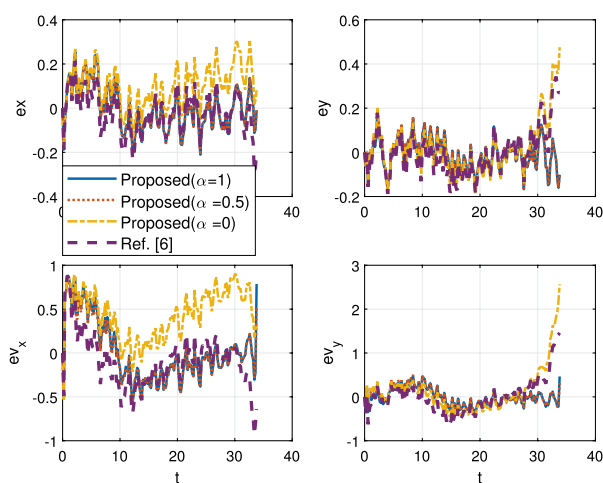
particularly as the target nears its arrival. This response is expected as larger values of covariances are used. On the other hand, near the destination, the proposed algorithm demonstrates better tracking accuracy. Using the proposed recursive update approach, the unspecified mean values of the velocity are appropriately corrected. Thus, the state estimation performance of the proposed method is improved in terms of accuracy and consistency. To investigate the effect of the recursive parameter on the inference performance, we conducted simulations with different choices of α . The case where α is set to zero exhibits poor performance. This outcome can be attributed to the inaccuracies in the initial settings. In contrast, when a non-zero value of α is used, it effectively corrects the terminal parameters based on the



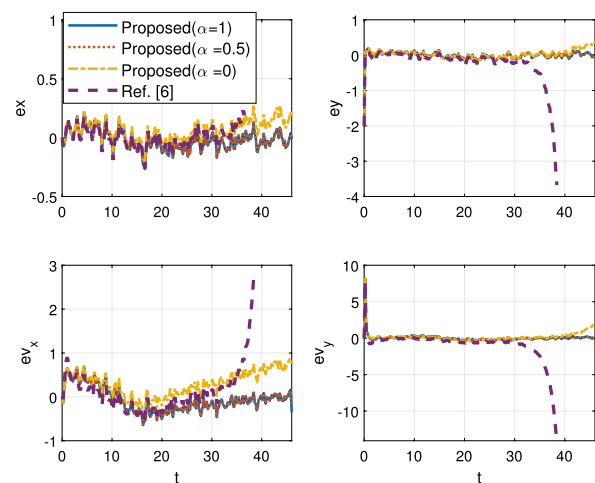
(a) Trajectory (Case I-a)



(b) Trajectory (Case I-b)



(c) Estimates error (Case I-a)



(d) Estimates error (Case I-b)

Fig. 2 Simulation results: time-series inference (Scenario I)

new information derived from the current geometry. In case I-b, the benchmark approach tends to diverge and no longer produces reliable state estimates. This phenomenon occurs, because the actual final time exceeds the prior distribution of the final time $t_f \in (15, 35)$. This result highlights the strong influence of the prior distribution when addressing undetermined parameters using the conventional Bayesian framework. In contrast, using the proposed parameter updates, the expected final time is corrected by the instantaneous relative geometry. Consequently, the corrective step can address the incorrect prior information and improve consistency for the time-series inference. The RMSE values for Scenario 1 are summarized in Table 1.

4.2 Scenario II: destination inference

In this scenario, we consider the destination inference problem along with time-series inference. We are presented with a finite number of destinations as the following set:

$$\mathcal{D} = \{[-40, 150], [-40, 250], [50, 350], [100, 150]\}.$$

Two cases of inference scenario are considered, where the actual destination is selected from the above-mentioned candidates

- Case II-a: $g = g^{(2)}$, $\mathbf{d}^{(2)} = [-40, 250]$, $[v_{xf}, v_{yf}]$: unspecified, t_f : unspecified
- Case II-b: $g = g^{(3)}$, $\mathbf{d}^{(3)} = [50, 350]$, $[v_{xf}, v_{yf}]$: unspecified, t_f : unspecified.

Figure 3 shows the simulation result for case II-a. The target initially tends to move toward $g^{(4)}$ and gradually alters its heading counter-clockwise to reach the actual destination $g^{(2)}$, as shown in Fig. 3a. Both inference algorithms produce trajectories that closely resemble the true trajectory, indicating that the time-series inference is satisfactory for both approaches. In terms of the destination inference performance, the existing method [6] cannot clearly determine the destination around the 13-s mark, as shown in Fig. 3c. During this period, the target is equally close to $g^{(1)}$ and $g^{(2)}$, and the corresponding probabilities remain similar until the target approaches the true destination. In contrast, the proposed inference algorithm increases ρ_4 at the beginning but promptly captures the correct inference probability ρ_2 . This

behavior may be attributable to the fact that the posterior probability for mode 1 is computed to be extremely small owing to the terminal conditions determined in the initial phase. Consequently, the proposed intent algorithm captures the true destination while easily distinguishing the actual intention (mode 2) from other possibilities.

In case II-b, the target trajectory exhibits an S-shaped curve toward destination 3, as shown in Fig. 3b. Because the destination is the farthest from the origin, the sinusoidal random maneuver leads to an actual terminal time of approximately 37 s, which lies beyond the prior distribution. Consequently, the existing inference algorithm [6] does not yield state estimates after 35 s and fails the time-series inference task. For the same reason, the existing algorithm no longer provides an inferred destination after this temporal mark. In contrast, the proposed algorithm consistently produces state estimates. The terminal conditions (v_x, v_y, t_f) are recursively updated in the CM_L model while compensating for unmatched terminal time. As shown in Fig. 3d, the intent inference probability increases as the state estimates approach $g^{(4)}$ and $g^{(2)}$ but remains consistent after 20 s. Table 2 summarizes the statistical comparison results for both cases. The inference using the proposed algorithm outperforms that achieved using the existing method [6] in terms of both the RMSE and success rate. Overall, the inference performance of the proposed algorithm remains satisfactory even under variations in the destination and terminal conditions.

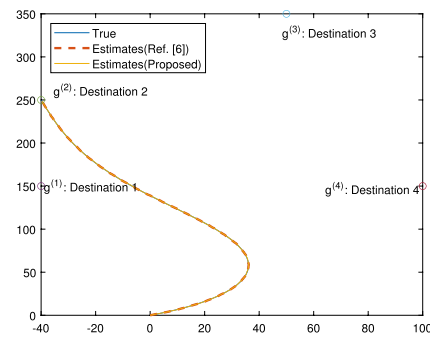
5 Conclusion

We address the destination and time-series inference problem for moving target tracking. The MMAE is used as the inference algorithm. The constructed CM model suitably represents the destination-driven time-series and can be incorporated in intent inference under the Bayesian framework. The formulated CM model can describe more general stochastic motion. The proposed approach effectively alleviates the dependency of the prior distribution setting by repeatedly updating the terminal conditions based on the relative geometry.

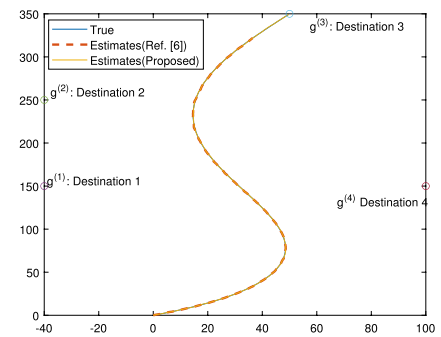
In future work, the proposed inference algorithm can be extended to other applications, such as guided target tracking and inference of group behaviors. Furthermore, the inference

Table 1 Simulation results: Scenario I

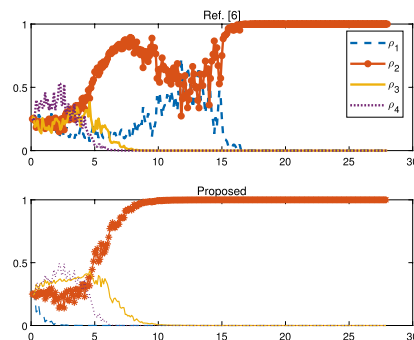
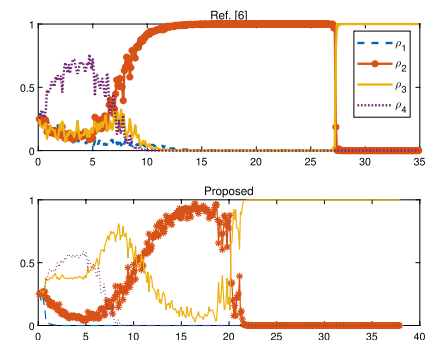
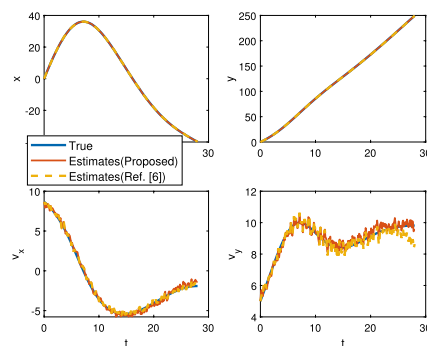
	Case I-a				Case I-b			
	Ref. [6]	Proposed			Ref. [6]	Proposed		
		$\alpha = 1$	$\alpha = 0.5$	$\alpha = 0$		$\alpha = 1$	$\alpha = 0.5$	$\alpha = 0$
RMSE	0.6210	0.4154	0.4042	0.7540	NaN	0.6636	0.6642	0.8407

Fig. 3 Simulation results: destination inference (Scenario II)

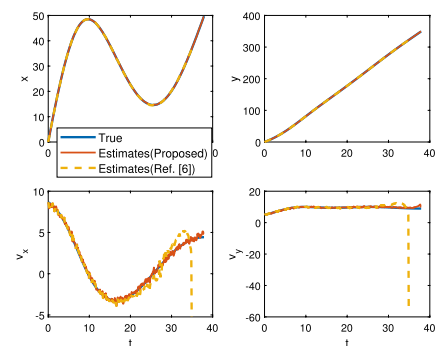
(a) Trajectory (Case II-a)



(b) Trajectory (Case II-b)

(c) Inference probability $\mu_k^{(i)}$ (Case II-a)(d) Inference probability $\mu_k^{(i)}$ (Case II-b)

(e) State responses (Case II-a)



(f) State responses (Case II-b)

algorithm can be validated under more complex conditions involving changes in the destination property, such as moving or variable destinations. Interactive multiple model estimation, data-driven, and learning-based inference schemes

can be designed to capture more general destination-driven dynamic motion of the target for intent inference. Additionally, intent inference can also be extended to meta-level intent inference applications.

Table 2 Simulation results: Scenario II

	RMSE		Success rate	
	Ref. [6]	Proposed	Ref. [6]	Proposed
Case II-a	0.1083	0.1050	0.8429	0.8571
Case II-b	NaN	0.1060	0.2032	0.5567

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Data availability The data sets generated during this study are available from the first author upon reasonable request.

Declarations

Conflict of interest The authors have no conflict of interest to declare related to the research presented in this article.

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