

Absolute angle measurement using dual-wavelength laser speckle: Theory and method

Sam J. Gibson, Thomas O.H. Charrett*, Ralph P. Tatam

Centre for Engineering Photonics, Cranfield University, Cranfield, Bedford, MK43 0AL, United Kingdom

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ABSTRACT

This paper presents a method utilising the speckle pattern formed by dual-wavelength illumination for the measurement of the two out-of-plane surface angles with respect to the sensor frame. Theoretical expressions are derived relating the observed speckle shift between patterns formed by two wavelengths for tilted surfaces with both on-axis and off-axis detector positions. These expressions are verified experimentally, showing RMS errors of between 0.5–1.0 μm . Finally, an on-axis implementation of the concept is presented using dual-wavelength illumination generated from two modes of a standard FP diode laser. Simplified expressions for the calculation of surface angles from measured speckle shift using this arrangement are presented, given in terms of three sensor constants; the responsivity or sensitivity of the sensor, C , and the zero surface tilt speckle shifts, A_{x0} and A_{y0} . Experimental results using this sensor for a range of surface tilts between 0.0° and 7.5° showed an RMS error of 0.10° in θ_x and 0.19° in θ_y .

1. Introduction

The relationship between objective speckle and surface deformations such as displacement, rotation, and strain has been widely explored [1,2]. Broadly speaking, these surface deformations will cause a translation and decorrelation of the speckle pattern, which can be measured using techniques such as fringe measurements [3] or correlation [4]. Other factors that can cause translation of a speckle pattern are changes in the wavelength of the light source, or changes to its angle of illumination [1]. These basic properties of speckle patterns have been used in various fields for measuring vibration [5], depth [6], topology [7], and odometry [8,9].

More particularly, these properties have been used to measure angular displacement, which is the measurand of interest in this paper. For example, an early paper from Tiziani [3] reported a method using defocused speckle photography to measure the tilt applied to a surface between two exposures, while Gregory [10] described the existence of a geometry sensitive only to surface tilt (i.e. insensitive to translation) utilising diverging illumination. More recently, Smíd et al. [11] used two detectors to measure the speckle displacement caused by a small (40 μm) translation of a tilted surface in a known direction, and thus determine the surface slope over that region. The technique could be

applied to either a flat or shaped object. Kelly et al. [12] used optical fractional Fourier transforms (OFRTs) to measure surface tilt and translation, both individually [12] and combined [13]. In each case, a before and after speckle image was used to determine the relative surface deformation. An alternative approach from Molina Prado et al. [14] used modulated speckle patterns generated with a double aperture pupil to again measure relative tilts of a surface.

The approaches outlined above are all for the measurement of relative angular displacement of the surface, rather than absolute angle with respect to the sensor or reference frame. Here we describe an approach utilising dual-wavelength illumination for the measurement of the two out-of-plane absolute surface angles with respect to the sensor frame. The concept requires two wavelengths λ_1 and λ_2 , separated by $\Delta\lambda$, simultaneously and collinearly illuminating the optically rough surface whose surface angles are to be determined. Each wavelength of light produces a distinct speckle pattern which, due to the wavelength difference, will be correlated but have a relative displacement. The resulting image recorded by the detector will therefore be the superposition of the two distinct speckle patterns, from which their displacement can be found by autocorrelation and detection of the side peak. The direction and magnitude of this displacement, referred to as the speckle shift, is directly related to the absolute tilt between the detector plane

* Corresponding author.

E-mail address: t.charrett@cranfield.ac.uk (T.O.H. Charrett).

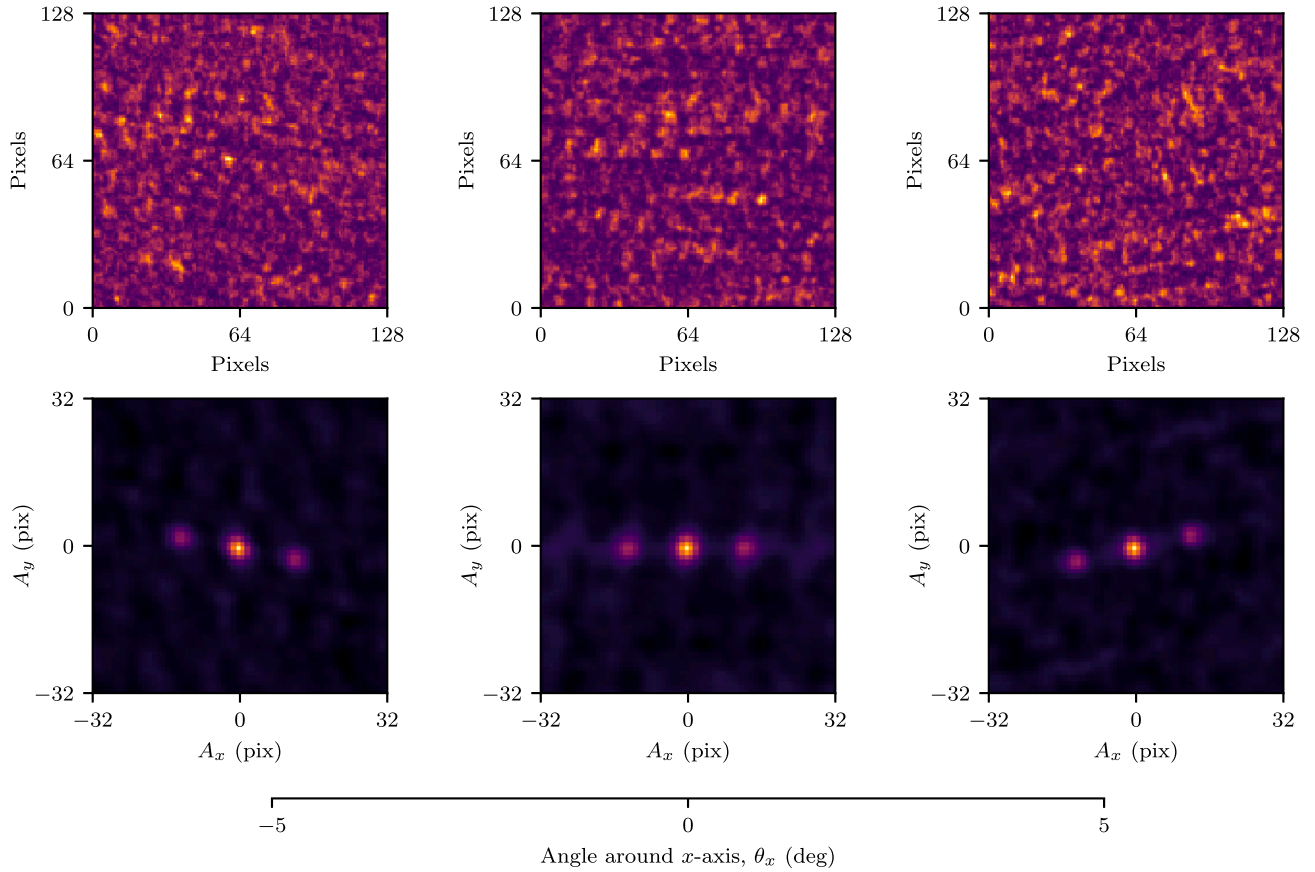


Fig. 1. Dual speckle patterns and corresponding correlation images for a surface at various tilts around the x axis and illuminated with two wavelengths simultaneously. As the surface moves, the magnitude and direction of the shift between the two speckle patterns changes accordingly.

and the surface. This principle is demonstrated in Fig. 1, showing combined dual-wavelength speckle patterns and the resulting (normalised) autocorrelation images for three different surface tilts.

The mathematical underpinnings of the relationship between speckle displacement and surface deformations have been described previously by Jacquot and Rastogi [15], Yamaguchi [4], and Horvath et al. [16]. Jacquot's approach is based on homology, considering points in the diffraction field with equal amplitudes before and after the object displacement. The paper presents equations to predict the speckle displacement for a given object or source translation, or wavelength change. Alternatively, Yamaguchi's formulation considers the cross-correlation of speckle pattern intensities before and after deformation, taking into account the same object and source factors as Jacquot in addition to considering object strain. A more recent derivation from Horvath et al. [16], based on Yamaguchi's approach, extends the theory to include more complex optical systems. However, these formulations are restricted to cases where the surface is planar and parallel to the detector plane. A form of the theory that explicitly includes the gradient of the object surface was proposed by Charrett and Tatam [17] for applications where the measurement of object translations is of interest. This derivation broadly follows the approach used by Yamaguchi [4] and Horvath [16], with some changes in the path length approximations that make the theory applicable for off-axis detector positions.

This work aims to expand on the theory presented by Charrett and Tatam [17] by considering a dual-wavelength approach for absolute angle measurement. The intended application for this work is robotic manufacturing, particularly areas such as drilling and riveting [18] or coating application [19] where perpendicularity to the work surface is required. The aerospace industry, which widely employs robotic drilling and riveting, requires a 0.5° accuracy [18,20] to reduce fatigue on the rivets and riveting holes. However, this level of accuracy is not

achievable with standard commercial robots [21], leading to demand for external sensors.

We have previously reported a 1D angle sensor using a partial theory only applicable for on-axis detectors [22], however a full description of the speckle behaviour for the proposed technique of absolute angle measurement has not been presented. As such, in Section 2 we derive the full theoretical expressions for the speckle shift between the speckle patterns resulting from illumination by two wavelengths, described as a function of the object surface angles and the illumination and observation geometries. These expressions are then experimentally verified in Section 3. Lastly in Section 4 we demonstrate the technique for 2D angle measurement using dual-wavelength illumination generated using two modes of a standard Fabry-Perot (FP) diode laser, and discuss potential issues and challenges.

2. Description and derivation of theory

To derive expressions governing the speckle shift between two wavelengths for given surface angles, we follow a method similar to Charrett and Tatam [17]. Fig. 2 shows a surface illuminated by two collinear wavelengths, λ_1 and λ_2 , from a source described by the position vector $\mathbf{S} = (S_x, S_y, S_z)$. The detector position is described by the vector $\mathbf{D} = (D_x, D_y, D_z)$ and is assumed to be parallel to the xy plane. Both illuminating wavelengths will have an associated speckle pattern at the detector, which will be offset by some shift $\mathbf{A} = (A_x, A_y, 0)$. Note that the shift in z , A_z , is assumed to be zero, as the speckle patterns are measured in the xy plane.

We now consider the paths followed by the light between the source, $\mathbf{S}$, an arbitrary point $\mathbf{P} = (x, y, z)$ on the surface within the illuminated area, and an arbitrary speckle in each pattern at the detector. The path between the source $\mathbf{S}$ and the surface point $\mathbf{P}$ will be the same for both

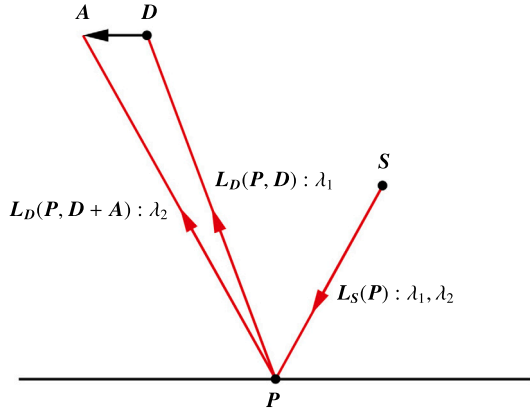


Fig. 2. Schematic showing the vector paths followed by light at two wavelengths, λ_1 and λ_2 , between a source $S = (S_x, S_y, S_z)$, an arbitrary surface scattering point $P = (x, y, z)$, and a speckle formed by λ_1 at D along with the corresponding shifted speckle formed by λ_2 at $D + A$. $A = (A_x, A_y, 0)$ describes the speckle shift between the patterns at the detector.

wavelengths as the illumination is collinear, and is given by $L_S(P)$. However, the paths between the surface point P and the same speckle in the two patterns will be different. For the first wavelength, λ_1 , the light from P to the speckle at point D , will have a path given by the vector $L_D(P, D)$. For λ_2 , the light from the same point to the same speckle follows a slightly different path, $L_D(P, D + A)$, where a relative speckle shift of A is included.

The full expressions for the three paths $L_S(P)$, $L_D(P, D)$, and $L_D(P, D + A)$ are given below:

$$L_S(P) = (S_x - x, S_y - y, S_z - z) \quad (1)$$

$$L_D(P, D) = (D_x - x, D_y - y, D_z - z) \quad (2)$$

$$L_D(P, D + A) = ((D_x + A_x) - x, (D_y + A_y) - y, D_z - z) \quad (3)$$

The tilt of the surface is considered in terms of surface gradient, where a gradient of $m_x = \tan \theta_y$ is the change in z over the change in x . This is equivalent to a rotation around the y axis (θ_y) for a planar surface laying in the xy plane. Correspondingly, the gradient $m_y = \tan \theta_x$ is the change in z over the change in y , or a rotation of θ_x about the x axis.

We now define the unit vector from P to the source, equal to $\hat{s} = (s_x, s_y, s_z)$, and from P to the detector, $\hat{d} = (d_x, d_y, d_z)$. The surface gradients can be incorporated into $\hat{s}$ and $\hat{d}$ using the expression $z = m_x x + m_y y$ [17].

$$\hat{s} = \left(\frac{S_x - x}{L_S}, \frac{S_y - y}{L_S}, \frac{S_z - (m_x x + m_y y)}{L_S} \right) \quad (4)$$

$$\hat{d} = \left(\frac{D_x - x}{L_D}, \frac{D_y - y}{L_D}, \frac{D_z - (m_x x + m_y y)}{L_D} \right) \quad (5)$$

L_S and L_D are the magnitudes of the vectors $L_S(P)$ and $L_D(P, D)$ respectively.

The aim of the derivation is to find a mathematical expression for the separation at the detector of the speckle patterns generated by each wavelength, described by $A = (A_x, A_y, 0)$. We initially follow the approach used by Yamaguchi [4], considering the complex amplitudes of the two light sources, U_1 and U_2 , and the cross correlation of their intensities, I_1 and I_2 .

$$\langle I_1 I_2 \rangle = \langle |U_1|^2 \rangle \langle |U_2|^2 \rangle + |\langle U_1 U_2^* \rangle|^2 \quad (6)$$

The first term is the cross correlation of the mean intensities, and thus constant. The second term, however, is equal to the cross correlation of the change in the intensities, i.e. $\langle \Delta I_1 \Delta I_2 \rangle = |\langle U_1 U_2^* \rangle|^2$, where $\Delta I_1 = I_1(P) - \langle I_1(P) \rangle$ and $\Delta I_2 = I_2(P) - \langle I_2(P) \rangle$.

The complex amplitude at the detector D is equal to the integral of the light scattered from all illuminated points P . For each wavelength λ_1 and λ_2 , the complex amplitude is given by the following, where $n \in [1, 2]$.

$$U_n(P, D) = \iint K_n(P, D) \sqrt{I_n(P)} \exp(i\varphi) \exp(ik_n L_S(P)) d^2 P \quad (7)$$

The intensity at the detector from P is given by $I_n(P)$. φ is the phase of the illuminating light, common to both wavelengths, and k_n is the wavenumber, where $k_n = 2\pi/\lambda_n$. $L_S(P)$ is the magnitude of $L_S(P)$.

$K_n(P, D)$ is the transmission function from P to the detector. For objective speckle, K_1 and K_2 are equal to:

$$K_1(P, D) = \exp(ik_1 L_D(P, D)) \quad (8)$$

$$K_2(P, D + A) = \exp(ik_2 L_D(P, D + A)) \quad (9)$$

This derivation, unlike that outlined by Yamaguchi [4], considers only the speckle shift caused by the wavelength difference; as the two patterns are recorded simultaneously, there will be no difference in the patterns resulting from translation or strain.

Now, the value of $\langle U_1 U_2^* \rangle$ can be found using the expressions for K_1 and K_2 . The intensities of the two beams are assumed to be equal, giving $I_1(P) = I_2(P) = I(P)$.

$$\langle U_1 U_2^* \rangle = \iint I(P) \exp \left(ik_1 \left(L_D(P, D) + L_S(P) - \frac{k_2}{k_1} L_D(P, D + A) - \frac{k_2}{k_1} L_S(P) \right) \right) d^2 P \quad (10)$$

The next stage of the derivation is to express $L_D(P, D + A)$ in terms of $L_D(P, D)$. Yamaguchi [4] uses a Fresnel, or parabolic, approximation to achieve this. However, this version of the theory was found to be inaccurate for off-axis detector positions. An alternative method, proposed by Charrett and Tatam [17], uses a Taylor expansion directly around $P = (x, y, z)$ to find an expression for $L_D(P, D + A)$. This approach utilises the fact that the speckle shift A at the detector is equivalent to a shift of $-A$ at the surface, giving:

$$L_D(P, D + A) = L_D(P - A, D) \approx L_D(P, D) + \left(\frac{\partial L_D}{\partial x} A_x + \frac{\partial L_D}{\partial y} A_y + \frac{\partial L_D}{\partial z} A_z \right) \quad (11)$$

Substituting the values of the partial derivatives, this expression can be rewritten using the unit vector $\hat{d} = (d_x, d_y, d_z)$.

$$L_D(P - A, D) \approx L_D(P, D) + \hat{d}(P, D) \cdot A \quad (12)$$

This can now be substituted into the cross correlation $\langle U_1 U_2^* \rangle$ (equation (10)).

$$\langle U_1 U_2^* \rangle = \iint I(P) \exp \left(ik_1 \left(\left(1 - \frac{k_2}{k_1} \right) (L_D(P, D) + L_S(P)) - \frac{k_2}{k_1} \hat{d}(P, D) \cdot A \right) \right) d^2 P \quad (13)$$

The next step once more follows Yamaguchi's method, applying a Taylor expansion around $P = 0$. Substituting this and taking the constants outside of the integral gives the following result:

$$\langle U_1 U_2^* \rangle = \exp \left(ik_1 \left(\left(1 - \frac{k_2}{k_1} \right) (L_D(0, D) + L_S(0)) - \frac{k_2}{k_1} \hat{d}(0, D) \cdot A \right) \right) \times \iint I(P) \exp \left(ik_1 \left(\left(1 - \frac{k_2}{k_1} \right) \nabla [L_D(P, D) + L_S(P)]_0 - \frac{k_2}{k_1} A_x \nabla [d_x(P, D)]_0 - \frac{k_2}{k_1} A_y \nabla [d_y(P, D)]_0 \right) \cdot P \right) d^2 P \quad (14)$$

The peak in the cross-correlation $\langle U_1 U_2^* \rangle$ will occur when the integral term is maximised, which corresponds to:

$$\left(1 - \frac{k_2}{k_1}\right) \nabla [L_D(\mathbf{P}, \mathbf{D}) + L_S(\mathbf{P})]_0 - \frac{k_2}{k_1} A_x \nabla [d_x(\mathbf{P}, \mathbf{D})]_0 - \frac{k_2}{k_1} A_y \nabla [d_y(\mathbf{P}, \mathbf{D})]_0 = 0 \quad (15)$$

Splitting this equation into x and y components gives a set of two simultaneous equations describing A_x and A_y .

$$\frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial L_D}{\partial x} + \frac{\partial L_S}{\partial x} \right) - A_x \frac{\partial d_x}{\partial x} - A_y \frac{\partial d_y}{\partial x} = 0 \quad (16)$$

$$\frac{\Delta\lambda}{\lambda_1} \left(\frac{\partial L_D}{\partial y} + \frac{\partial L_S}{\partial y} \right) - A_x \frac{\partial d_x}{\partial y} - A_y \frac{\partial d_y}{\partial y} = 0 \quad (17)$$

After solving the simultaneous equations and substituting the partial derivatives, the complete expressions describing A_x and A_y are now as follows:

$$A_x = \frac{\Delta\lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z - m_y d_y d_z} ((d_x + s_x + d_x d_y s_y - d_y^2 s_x) + m_x (d_z + s_z + d_y d_z s_y - d_y^2 s_z) + m_y (d_x d_y s_z - d_y d_z s_x)) \quad (18)$$

$$A_y = \frac{\Delta\lambda}{\lambda_1} \frac{L_D}{d_z^2 - m_x d_x d_z - m_y d_y d_z} ((d_y + s_y + d_x d_y s_x - d_x^2 s_y) + m_x (d_x d_y s_z - d_x d_z s_y) + m_y (d_z + s_z + d_x d_z s_x - d_x^2 s_z)) \quad (19)$$

The geometry terms in these equations come from the unit vectors to the source and the detector, as well as the working distance to the detector, L_D . The equations also have a cross-dependency between the two components of the surface gradient, and so must be solved simultaneously if used to calculate a surface tilt from a measured speckle shift. This theory will be referred to here as the ‘Taylor expansion’ theory.

For comparison, the final equations resulting from the derivation using the Fresnel approximation, i.e. Yamaguchi’s approach [4], are as follows:

$$A_x = L_D \frac{\Delta\lambda}{\lambda_1} (d_x + s_x + m_x (d_z + s_z)) \quad (20)$$

$$A_y = L_D \frac{\Delta\lambda}{\lambda_1} (d_y + s_y + m_y (d_z + s_z)) \quad (21)$$

These equations are notably simpler than those above, and have no cross-dependency between the x and y gradients. However, as noted previously, these equations were found experimentally (see Section 3) to be accurate only for on-axis detector positions. We refer to this version as the ‘Fresnel approximation’ theory.

It can also be seen that (18) and (19) are equivalent to equations (20) and (21) for on-axis detector positions, i.e. when $d_x = d_y = 0$ and $d_z = 1$. In other words, the two formulations of the theory are the same for an on-axis detector.

3. Experimental verification of theory

In order to verify the above derivation, two experimental geometries were constructed to measure the speckle shifts at different surface angles. The first version, shown in Fig. 3a, used an on-axis detector placed directly opposite the laser spot along the z axis. The second version (Fig. 3b) placed the detector off-axis, at a distance of -75.0 mm along the x axis. In both cases, the source and detector were at 162.0 mm in z , and the source was positioned at an x distance of 152.0 mm.

The source was a FiberTecII FTEC2-658-30 FP laser diode, which could be tuned to two different wavelengths, $\lambda_1 = 657.42$ nm and $\lambda_2 = 657.94$ nm ($\Delta\lambda = 0.52$ nm), by changing the applied drive current. An example spectrum of each is shown in Fig. 4. The two separate tunings allowed for acquisition of the two speckle patterns individually. The light was decoupled from the fibre with a 15 mm focal length lens placed 16 mm from the fibre tip, giving a slightly diverging beam. The

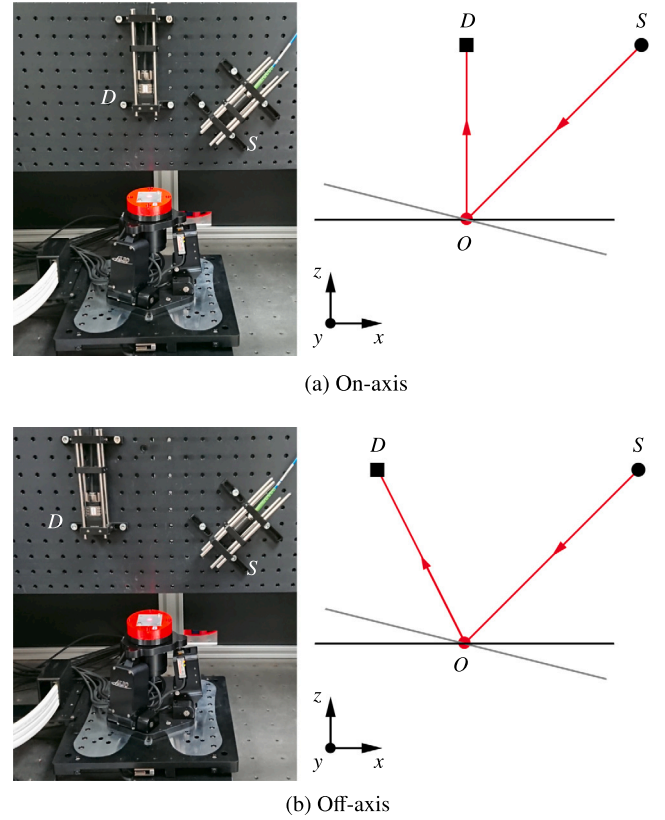


Fig. 3. Photos and schematics of the experimental setup for both on-axis, (a), and off-axis, (b), detector positions. The source (fibre output and lens tube) and detector (CMOS camera) are mounted above a Hexapod stage, with an aluminium plate attached via a fixed angle 3D printed mount. In terms of the geometry, the source is placed at $S = (152.0, 0.0, 162.0)$ mm, and the detector is at $D = (0.0, 0.0, 162.0)$ mm on-axis and $D = (-75.0, 0.0, 162.0)$ mm off-axis, all in reference to the laser spot at $O = (0.0, 0.0, 0.0)$ mm.

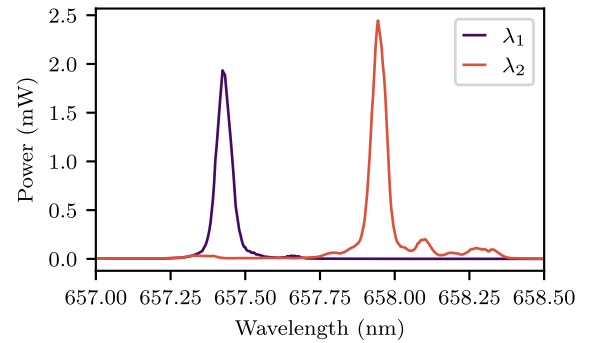


Fig. 4. Example laser spectra for two different injection currents of the FiberTecII laser diode used throughout experiments. The wavelengths are centred at 657.42 nm and 657.94 nm.

spot size on the surface was 6.8 mm, resulting in a transverse speckle size of 10 μ m.

A Ximea MQ013MG-ON 8-bit monochrome CMOS camera was used for the detector, which has 1280x1024 pixels and a pixel size of 4.8x4.8 μ m. Here a 512x512 pixel region of interest was used, which balanced the correlation processing speed with the reduced background noise from a larger image. Additionally, a laser line filter centred at 655 nm with a bandwidth of 50 nm was placed in front of the camera to reduce background light.

An Alio Hybrid Hexapod stage (model AI-HH-200XY-15Z-56R-ACS, pitch and roll accuracy 5.0 arcsec, repeatability 0.6 arcsec) was used to

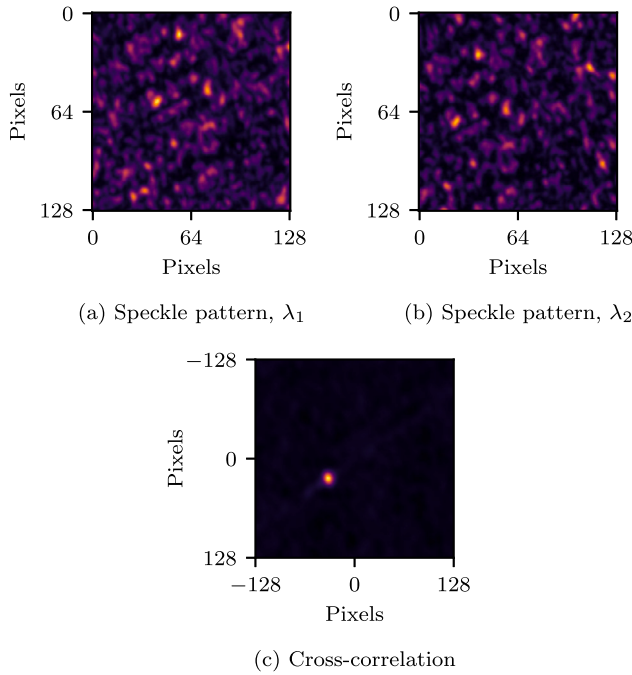


Fig. 5. Example speckle images, (a) and (b), and their cross-correlation, (c). Each speckle pattern is captured for a different wavelength, at the same surface tilt. The cross-correlation image shows the speckle shift between the two patterns, in this case -17.9 pix in A_x and 12.6 pix in A_y .

apply a series of known angles to an aluminium plate attached to the stage via a 3D printed mount. The multi-axis stage facilitated the testing of the theory for both θ_x and θ_y surface angles, as well as combinations of angles. The 3D printed mount applied a fixed tilt of 8° to the metal plate, which increased the possible surface tilts that could be applied by the stage from $\pm 8^\circ$ to $\pm 16^\circ$. For reference, a tilt of 8° is equivalent to a surface gradient of 0.14, and 16° is equal to 0.29. To test combinations of θ_x and θ_y angles, a fixed magnitude mount of 4.0° was rotated around θ_z . This applied a continuously varying angle in both θ_x and θ_y , while keeping the overall magnitude of tilt constant.

To validate the theory, speckle images were recorded for each geometry for three sets of angles: for θ_x and θ_y varying between -16° and $+16^\circ$ in 1° steps with the surface tilt around the perpendicular axis remaining at 0° , and for θ_z a fixed magnitude mount of 4° rotating between 0° and 360° in 10° steps around the z axis. Each set of angles was applied twice, first with the laser illuminating at λ_1 and then again with λ_2 .

The two speckle images at each angle were then processed using a normalised 2D circular cross-correlation, and the sub-pixel speckle shift between them determined from the peak location using a 3-point Gaussian fit [23]. The correlation process is illustrated in Fig. 5, where 5a and 5b show the two speckle images recorded at λ_1 and λ_2 , and 5c is the correlation image showing the speckle shift.

From the literature on particle imaging velocimetry (PIV) [23], the accuracy of this peak fitting is maximised for a particle (or speckle) with a 2–3 pixel diameter, hence the $10 \mu\text{m}/2.1 \text{ pix}$ speckles used here. A larger image size also increases the peak fitting accuracy due to the decrease in background noise, though PIV generally uses images of up to 64×64 pixels. As such, the accuracy of the peak fitting was estimated as 0.05–0.10 pix [23], or 0.24–0.48 μm .

The measured speckle shifts for the on-axis geometry are shown in Fig. 6. Here, each column shows the results for varying θ_x , θ_y , and θ_z consecutively, with the top row showing the measured speckle shift between λ_1 and λ_2 as data points, and the theoretical values found using the expressions above as solid lines. For the on-axis detector, the expected speckle shifts for each applied surface angle are the same re-

Table 1

RMS errors for the measured speckle shifts in Fig. 6, for the on-axis detector position $D = (0.0, 0.0, 162.0)$.

	θ_x		θ_y		θ_z	
	A_x	A_y	A_x	A_y	A_x	A_y
(μm)	0.98	1.23	0.51	0.64	0.97	0.92

gardless of the theory derivation used, i.e. the Fresnel approximation version and the Taylor expansion version both predict the same result. As a result, the graphs in Fig. 6 only show one theory line.

The measured values of A_x and A_y are the mean of 20 repeat measurements taken at each angle, and the standard deviations on these values are too small to be shown on the plot; the average standard deviation across all angles is $0.176 \mu\text{m}$ in A_x and $0.180 \mu\text{m}$ in A_y . The lower two rows of Fig. 6 show the error residuals by taking the measured speckle shifts minus the theoretical ones, and demonstrate a good agreement with the theory. The average root mean square (RMS) errors for each series of angles are given in Table 1. In each case, the magnitude of the residuals is in the region of $1 \mu\text{m}$, with the average RMS errors across all measurements being $0.85 \mu\text{m}$ for A_x and $0.96 \mu\text{m}$ for A_y .

For an off-axis detector geometry, the predictions of the two formulations of the theory deviate. This can be seen in Fig. 7, which plots the theoretically calculated speckle shifts, A_x and A_y , for sensor geometries with different values of D_x . As described above, the theory derived using the Fresnel approximation and that using the Taylor expansion both predict the same outcome for an on-axis detector. However, the Taylor expansion version predicts that as D_x moves off-axis, A_x will no longer have a linear relationship with the applied angles. For $D_x = -50.0 \text{ mm}$ the line shows a slight curve, becoming more pronounced when the detector is located at $D_x = -100.0 \text{ mm}$. The theory line still crosses zero at the same point as the Fresnel approximation theory, but noticeably curves away at negative angles. As θ_x (and thus A_y) is at zero, this remains the same throughout for this geometry.

The experimental verification of this theory used a geometry with $D_x = -75.0 \text{ mm}$, as shown in Fig. 3b. The measured results are plotted in Fig. 8, along with the two versions of the theoretical predictions. The results clearly show the measured speckle shifts agreeing more closely with the solid line of the Taylor expansion theory, with the lower axes showing the residuals calculated from this version. Here, the RMS errors have an average magnitude of $0.50 \mu\text{m}$ for both directions across all measurements. The full breakdown of RMS errors is given in Table 2. For comparison, the RMS errors calculated using the Fresnel approximation theory (also in Table 2) result in values of around $7.8 \mu\text{m}$ in A_x , and range from $0.56 \mu\text{m}$ to $2.94 \mu\text{m}$ in A_y .

4. Application as angle sensor

To demonstrate the dual-wavelength speckle absolute angle sensing concept, a sensor was constructed and tested using simultaneous illumination with both wavelengths. The same FiberTecII FTEC2-658-30 laser diode was used for this sensor, now with an applied injection current such that there were two output modes present at once, shown in Fig. 9, with wavelengths of 656.93 nm and 657.39 nm , giving a separation of 0.46 nm .

The theoretical expressions derived using the Taylor expansion contain a considerable number of terms, plus a cross-dependency between the x and y terms of the speckle shift and surface gradient. As a result, an on-axis geometry was chosen for this demonstration, which allows for the use of the simplified version equivalent to that derived using the Fresnel expansion.

Taking the final on-axis equations (20) and (21), we now substitute the surface angles in place of surface gradients ($m_x = \tan \theta_y$ and $m_y = \tan \theta_x$). The resulting equations for use as an angle sensor are then as follows:

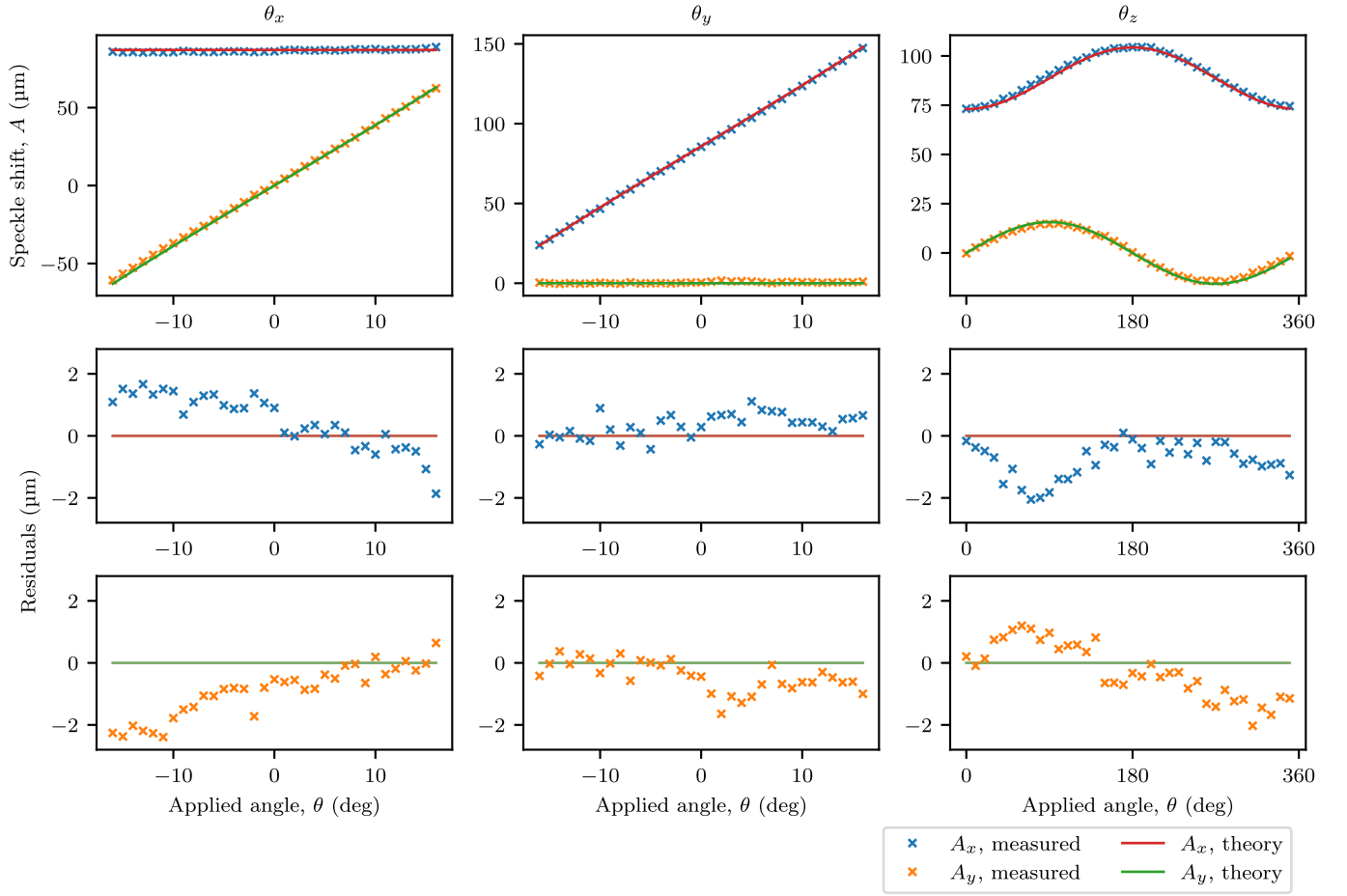


Fig. 6. Measured speckle shifts from the on-axis sensor, as compared to the predicted results from theory. The lower two rows of graphs show the residuals (measured minus theoretical angles) of the results.

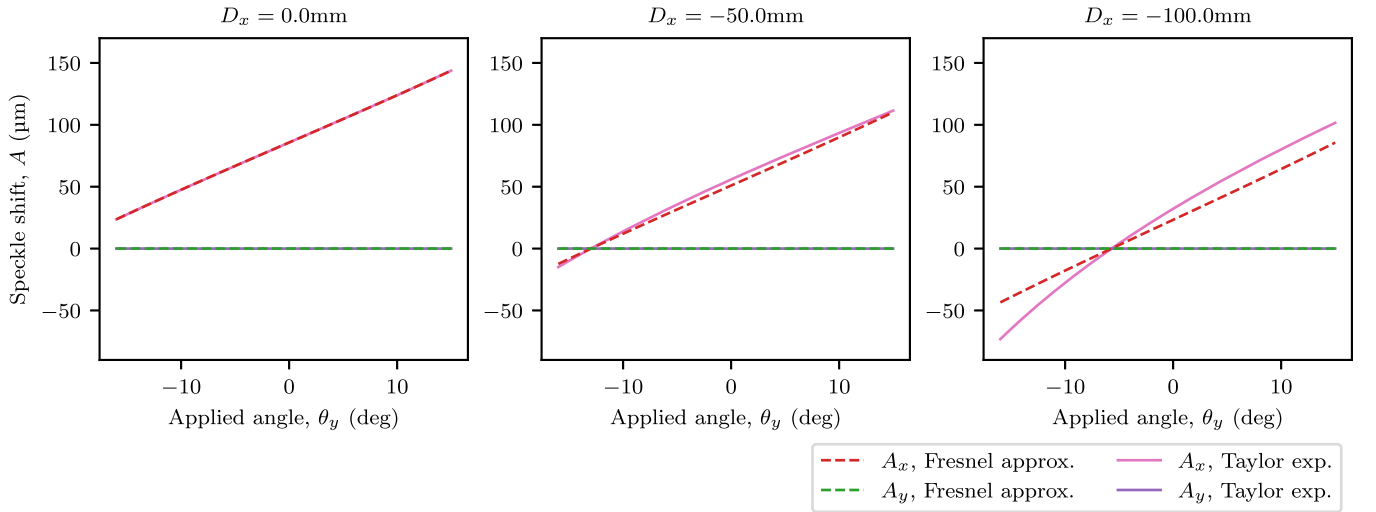


Fig. 7. Comparison of the Fresnel approximation and Taylor expansion versions of the theory for detector positions in x of $D_x = 0.0, -50.0, -100.0$ mm, where $D_y = 0.0$ mm and $D_z = 162.0$ mm. The applied angle is in θ_y , with the angle in θ_x equal to zero.

$$A_x = \frac{\Delta\lambda}{\lambda_1} L_D (s_z + 1) \tan \theta_y + \frac{\Delta\lambda}{\lambda_1} L_D s_x \quad (22) \quad C = \frac{\Delta\lambda}{\lambda_1} L_D (s_z + 1) \quad (24)$$

$$A_y = \frac{\Delta\lambda}{\lambda_1} L_D (s_z + 1) \tan \theta_x + \frac{\Delta\lambda}{\lambda_1} L_D s_y \quad (23) \quad A_{x0} = \frac{\Delta\lambda}{\lambda_1} L_D s_x \quad (25)$$

In this form it can be seen that a further simplification can be made by defining the following terms:

$$A_{y0} = \frac{\Delta\lambda}{\lambda_1} L_D s_y \quad (26)$$

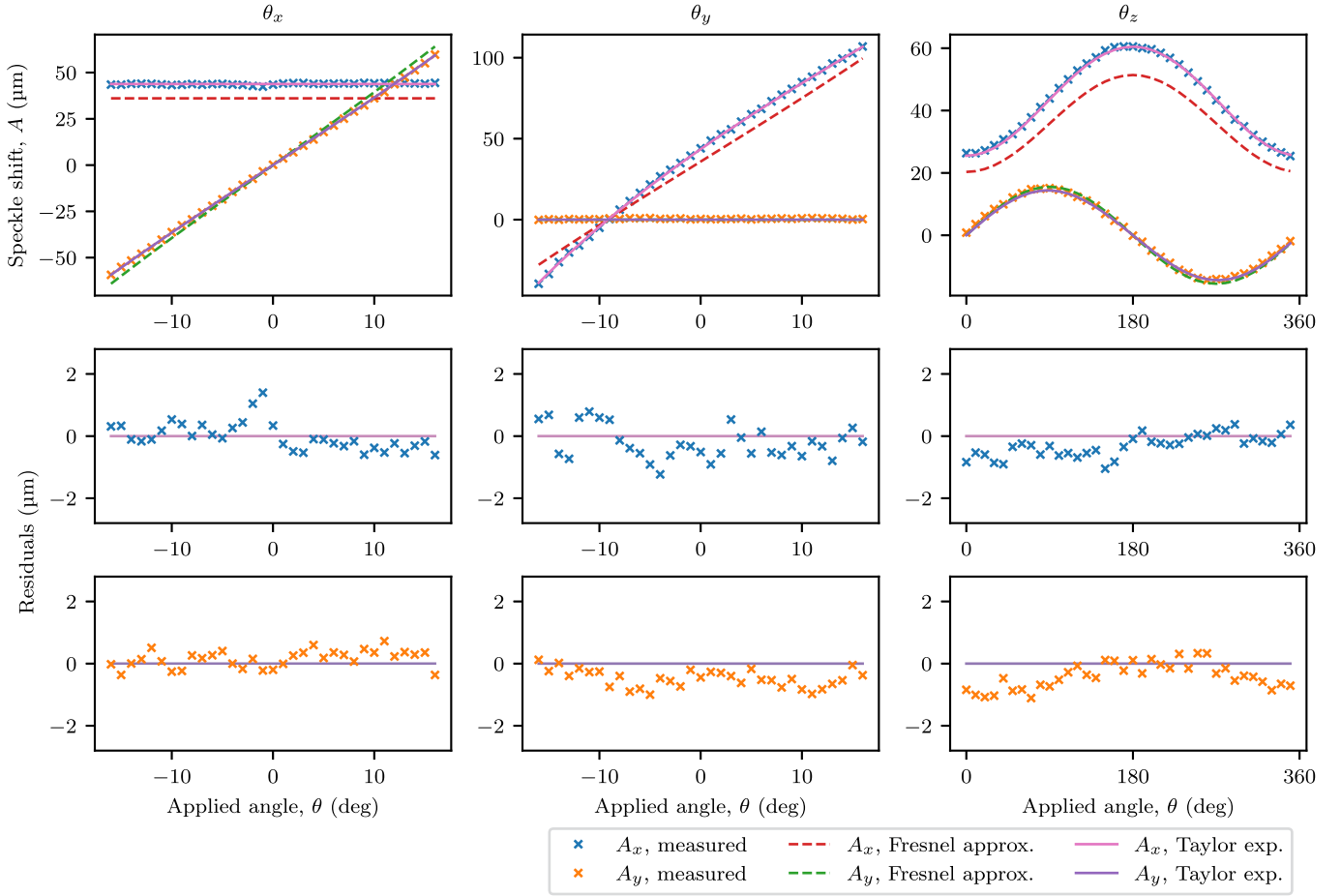


Fig. 8. Measured speckle shift for the off-axis detector setup ($D_x = -75.0$ mm), as compared to both the Fresnel approximation and Taylor expansion versions of the theory. The lower two rows show the residuals as calculated using the Taylor expansion version.

Table 2

RMS errors for the measured speckle shifts in Fig. 8, for the off-axis detector position $D = (-75.0, 0.0, 162.0)$. The two formulations of the theory (Fresnel approximation and Taylor expansion) are compared.

(μm)	θ_x		θ_y		θ_z	
	A_x	A_y	A_x	A_y	A_x	A_y
Fresnel approx.	7.76	2.94	7.70	0.56	7.84	0.87
Taylor exp.	0.45	0.31	0.57	0.56	0.47	0.57

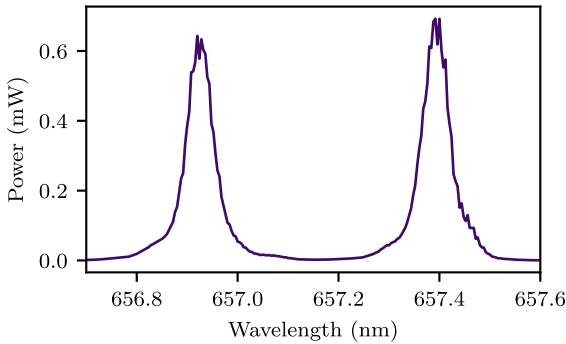


Fig. 9. An example laser spectra for the FiberTecII laser diode running with a dual-wavelength output. The wavelength modes are at 656.93 nm and 657.39 nm.

C can be considered as the responsivity or sensitivity of the sensor. A_{x0} and A_{y0} represent the speckle shift at zero surface tilt, which is non-zero for any geometry where the source is off-axis. Using these sensor constants, the equations used to measure surface angle can be written simply as:

$$\tan \theta_x = \frac{A_y - A_{y0}}{C} \quad (27)$$

$$\tan \theta_y = \frac{A_x - A_{x0}}{C} \quad (28)$$

The schematic and photo of the sensor used in this experiment are shown in Fig. 10. The geometry values needed for the theory equations are the working distance to the detector, L_D , and the unit vector to the source, $\hat{s} = (s_x, s_y, s_z)$. In this case, these are equal to 104.0 mm for L_D and (0.71, 0.00, 0.71) for $\hat{s}$, giving theoretical sensor constants of $A_{x0} = 0.051$ mm, $A_{y0} = 0.000$ mm, and $C = 0.124$ mm. This particular geometry was chosen to maximise the sensitivity while keeping a practical size for the sensor head. Although increasing the working distance will increase the sensitivity, decreasing the angle of incidence of the source will decrease the sensitivity. As such, 45° was chosen as the incident angle of the source, with a working distance of 104 mm. The total width of the sensor head, which was 3D printed and can be seen in Fig. 11, was 150 mm.

The camera and filter were also the same as those used for the previous experiments. The image processing, illustrated in Fig. 12, was slightly different however; an autocorrelation was applied to a single image acquired from the detector, resulting in a correlation image with three peaks instead of the previously described one peak. The three peaks consist of a central peak produced by the correlation of the

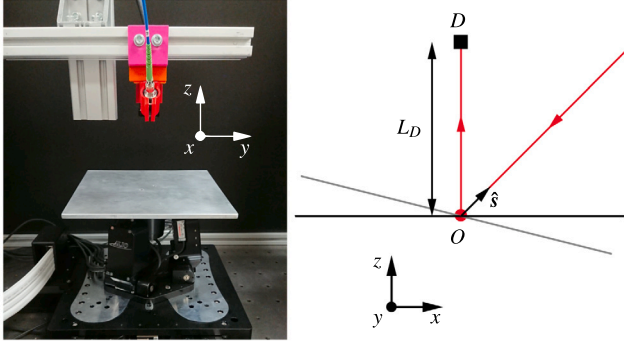


Fig. 10. Photo of experimental setup and schematic of the sensor used take angle measurements with dual-wavelength illumination. The source is at $S = (104.0, 0.0, 104.0)$ mm, and the detector at $D = (0.0, 0.0, 104.0)$ mm, with the sensor head mounted above the Hexapod stage. Note that the photo and schematic are from different orientations.

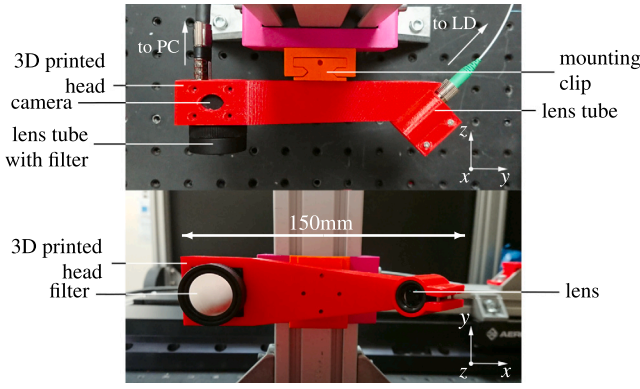


Fig. 11. Photos of the 3D printed sensor head, from above (top) and from the front (bottom). The total width of the head is 150 mm. [PC: personal computer, LD: laser diode].

whole image with itself, and two side peaks, one from the correlation of speckle pattern λ_1 with pattern λ_2 , and the other from the reverse correlation. In this case, the central peak and one side peak are masked, and then the remaining peak and speckle shift are found using a Gaussian 3-point fit as before.

The sensor constants can be used to estimate the range of the sensor, also using information from the signal processing. In θ_y , the negative range limit is dictated by how close the centre of the correlation peak can be to the masked central peak while still being measurable. With a 5 pixel mask, the angular limit of this sensor is -12.4° . For the positive direction, the range limit can be estimated using the ‘quarter image rule’ for the peak finding [24], which gives a value of $+77.6^\circ$ for the 512 pixel width image used here. In θ_x , the range is only limited by the quarter image rule, which gives range of $\pm 78.6^\circ$. Practically, however, the latter three values are more likely to be limited by the scattering cone of the spot, which has not been investigated here. We have chosen to define the sensor range using the -12.4° value limited by the central correlation peak, giving a maximum range of $\pm 12.42^\circ$ in both θ_x and θ_y .

Before being used to measure angles, this version of the sensor was calibrated using an Aerotech AGR-50 rotation stage, in order to refine the values of the sensor constants A_{x0} , A_{y0} , and C to account for any geometrical misalignment. A series of 3D printed mounts with fixed magnitude angles, from 1° to 7° , were rotated continuously around 180° (five full circles) and a speckle image recorded every 1° . This took around two hours to complete, including changing over the 3D printed mounts. The measured speckle shifts at each point can be seen in Fig. 13, along with circles fitted to each measurement set using a

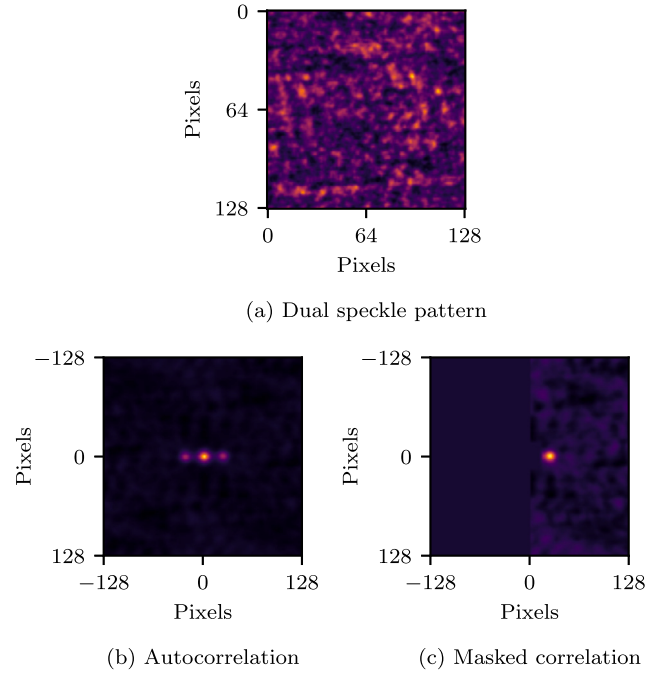


Fig. 12. Example dual speckle image, (a), and its autocorrelation, (b) and (c). The speckle image contains the two patterns from wavelengths λ_1 and λ_2 . The autocorrelation shows correlation peaks for the full image (centre), the correlation of pattern λ_1 with λ_2 , and that of pattern λ_2 with λ_1 (two side peaks). The final image shows a masked correlation image, isolating the desired side peak.

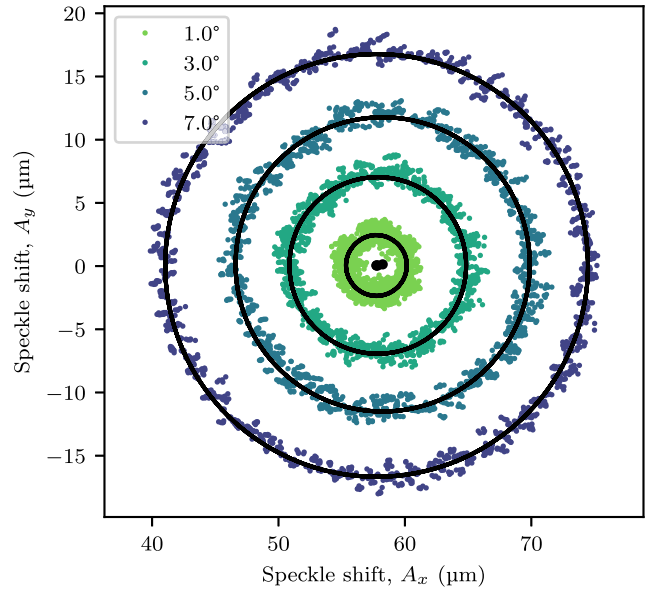


Fig. 13. Calibration data used to verify the sensor constants A_{x0} and A_{y0} , given by the centre of the circles, and C , proportional to the radius.

least squares fitting. The centres of these circles are equal to the speckle shift at zero surface tilt, or (A_{x0}, A_{y0}) , and their radii are proportional to the sensitivity constant C . Following the calibration, the values of the three sensor constants were determined to be $A_{x0} = 0.058$ mm, $A_{y0} = 0.000$ mm, and $C = 0.135$ mm.

After calibration, the Alio Hexapod stage was used to measure a series of angles with fixed direction and varying magnitude, shown in the results graph in Fig. 14. For each direction of tilt a series of angles was applied by the stage moving from a magnitude of 0.0° to 7.5° in 0.5°

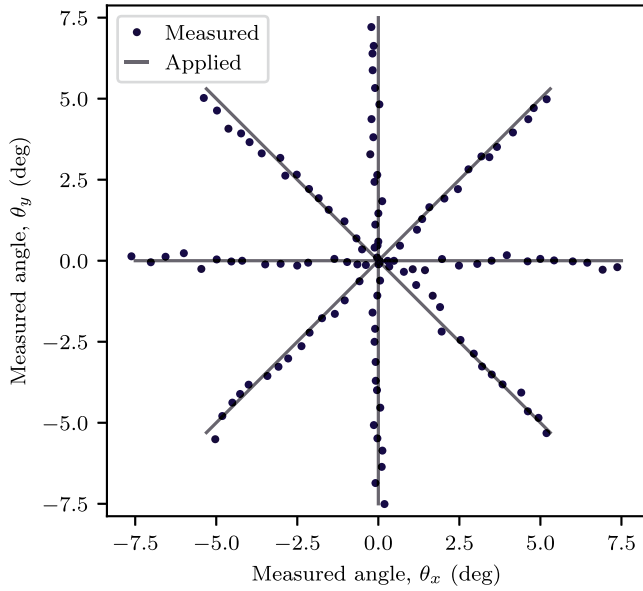


Fig. 14. Measured angles from the dual-wavelength speckle sensor, as compared to the angles applied by the Hexapod stage.

Table 3

Expected uncertainty and average RMS error in angle measurements for the dual-wavelength angle sensor (full results in Fig. 14).

	θ_x	θ_y
Expected uncertainty	0.11° – 0.16°	0.21° – 0.30°
Average RMS error	0.10°	0.19°

steps. This was repeated for directional steps of 45° . Each point in the figure represents the mean angle over 50 repeats at that stage position.

The figure shows a reasonable agreement between the measured and applied angles at all points, with an RMS error of 0.10° in θ_x and 0.19° in θ_y . This also aligns with the lower end of the anticipated uncertainty in the peak fitting, which is 0.05 pixels and has a corresponding angular uncertainty of 0.11° [22]. The other main source of uncertainty is the laser instability, which is expected to cause an additional uncertainty of up to 0.05° in θ_x and between 0.10° and 0.19° in θ_y depending on the tilt angle [22]. These values were determined from a long term stability analysis of the laser using an optical spectrum analyser, which found that the wavelength stability over around 6 hours was between 2–3 pm. Combining the uncertainty predictions from both signal processing and laser instability (see Table 3), the average RMS error of the measured data is in good agreement, being slightly below the minimum expected errors with 0.10° vs. 0.16° for θ_x and 0.19° vs. 0.21° for θ_y . This is likely due to slightly better than anticipated laser stability during angle measurements.

An advantage of this technique over other speckle-based approaches that rely on measuring speckle translation is that this technique is insensitive to lateral surface translations. However care must be taken to ensure that the working distance is fixed and known, and that the spot is located on the centre of rotation of the surface, otherwise systematic errors will be introduced. This is common to other optical angle measurement techniques, including speckle-based approaches and optical autocollimators. As an example for the intended use-case of measuring angles close to zero, a 1% change in working distance for the geometry presented here will have a resulting bias error of 0.23° .

The range of $\pm 12.42^\circ$ for this sensor is also comparable to other speckle-based approaches to angle measurement. For instance, the technique presented by Smíd et al. [11] using a known translation of a surface to measure its tilt has an angle range of 10° – 30° . Autocollima-

tors, on the other hand, are more limited and have angular ranges of up to 3° [25].

A higher-resolution implementation of a dual-wavelength speckle angle sensor would require a laser source with a greater wavelength separation to reduce the effect of uncertainties from both peak finding and laser instability. For example, an uncertainty of less than 0.01° could be achieved for this geometry and $\lambda_1 \approx 657$ nm by using an increased wavelength separation of $\Delta\lambda = 10$ nm and a long term laser stability of below 1 pm. This would likely require two lasers, rather than the dual-mode single laser used here, but demonstrates the potential flexibility of the technique.

5. Conclusion

This paper has presented a method of measuring the absolute angle of a surface relative to the detector plane in a single exposure using dual-wavelength laser speckle. This is in contrast to other speckle based techniques which measure the change in angle relative to an initial reference image.

To support this, theoretical expressions relating the shift between speckle patterns from different wavelengths and at different surface angles have been derived. The theory derivation broadly followed the method outlined by Yamaguchi [4], with a change in the approximation of the path length to the detector in order to expand the applicability of the theory to off-axis detector positions, similar to that presented by Charrett and Tatam [17] for the case of object translation. This revised version used a Taylor expansion, where Yamaguchi's approach used a Fresnel approximation. The simpler form of the expressions, valid only for on-axis detectors, was presented and tested alongside the revised version.

To verify the accuracy of the theoretical expressions, experimental results were presented that measured the speckle shift at various surface angles using both an on-axis and off-axis detector. For the on-axis detector geometry, the measured speckle shifts showed RMS errors of less than $0.98 \mu\text{m}$ in A_x and up to $1.23 \mu\text{m}$ in A_y , when compared to the theoretically calculated values. In this case, the two versions of the theory (Fresnel approximation and Taylor expansion) predicted the same results.

The experimental data for an off-axis detector geometry clearly showed the improved accuracy of the Taylor expansion version, with average RMS errors between measured and expected speckle shifts of less than $0.57 \mu\text{m}$ in both A_x and A_y . In contrast, the expressions using the Fresnel approximation resulted in RMS errors of up to $7.84 \mu\text{m}$ in A_x and $2.94 \mu\text{m}$ in A_y .

Finally, an implementation of the technique as a complete angle sensor was presented using simultaneous dual-wavelength illumination, which was generated by tuning a FP laser diode to output two longitudinal modes with a spacing of 0.46 nm. Simplified expressions for the calculation of surface angles from measured speckle shift were presented for an on-axis detector geometry, given in terms of three sensor constants: the responsivity or sensitivity of the sensor, C , and the zero surface tilt speckle shifts, A_{x0} and A_{y0} . An approach to the practical calibration of a sensor was also presented using fixed magnitude slopes rotated around the z axis, with the resulting speckle shifts plotting a circle in $A_x A_y$ space. The three sensor constants are found via least square fitting to these circles, determining (A_{x0}, A_{y0}) from the circle centre and the sensitivity C from their radii.

Experimental testing of this sensor applied a range of surface tilts between 0.0° and 7.5° in magnitude, and results showed an RMS error of 0.10° in θ_x and 0.19° in θ_y . These were in line with the predicted uncertainties calculated from the expected peak fitting error and measured laser instability.

We foresee this technology being applied in robotic manufacturing, and as such used the aerospace requirement of 0.5° accuracy as an aim for the sensor demonstrated here. With RMS errors below 0.2° , we have

achieved this benchmark. The maximum range of $\pm 12.42^\circ$ also makes our sensor suitable for this and similar applications.

One option for future work to improve performance would be to employ a laser source with a greater wavelength separation to increase the sensor responsivity, as well as to reduce uncertainties resulting from the peak finding and laser instability. As an example, an increased wavelength separation of $\Delta\lambda = 10$ nm and a long term laser stability of below 1 pm is predicted to give an uncertainty of less than 0.01° in angle measurements.

CRedit authorship contribution statement

Sam J. Gibson: Formal analysis, Investigation, Methodology, Software, Visualization, Writing – original draft. **Thomas O.H. Charrett:** Conceptualization, Funding acquisition, Methodology, Software, Supervision, Writing – review & editing. **Ralph P. Tatam:** Funding acquisition, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data supporting this study are openly available from the Cranfield Online Research Data Repository (CORD) at <https://doi.org/10.17862/cranfield.rd.24088686>.

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Gibson, Sam J.; Charrett, Thomas O. H.; Tatam, Ralph P.

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