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On the Effect of Apophis on Earth-Orbiting Satellites

Faculty of Engineering and Applied Sciences
School of Aerospace, Transport Systems and Manufacturing

MSc in Astronautics and Space Engineering
Academic Year: 2024–2025

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requirements for the degree of MSc in Astronautics and
Space Engineering.

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Abstract

While most planetary defence efforts focus on impact prevention, even a non-collisional encounter has the potential to perturb satellite, with consequences for communications, navigation and space safety. However, little attention has been given to the effect of close asteroid approaches on Earth-orbiting satellites. The 2029 flyby of asteroid Apophis (99942), which will pass closer than the geostationary belt, offers a unique opportunity to study this overlooked aspect of planetary defence.

This thesis develops a framework to estimate the perturbations induced by asteroid flybys on satellites. A numerical propagator was implemented in Python to simulate the Sun-Earth-Moon-Asteroid system, including major gravitational perturbations, Earth's oblateness and triaxiality, Solar Radiation Pressure and Apophis's influence.

The methodology was first applied to geostationary satellites, given the proximity of Apophis's trajectory. Six representative GEO spacecraft and debris objects were analysed. Results indicate that perturbations remain very limited, producing longitudinal ground-track drifts of a few metres over several months, while Keplerian elements such as semimajor axis, eccentricity, inclination and RAAN remain essentially unchanged. The study was then extended to tailored artificial orbital configurations dynamically close to Apophis during the encounter. In these extreme cases, perturbations reached 30–37 metres in longitude and induced permanent but small shifts in semimajor axis (1 m) and eccentricity (10^{-8}).

Finally, a search for real satellites near these sensitive regions identified objects such as PROBA-3 CSC (62256) and PSLV R/B (62257). These showed much larger perturb-

ations, with kilometre-scale changes in semimajor axis and hundreds of kilometres in ground-track displacement.

In conclusion, while Apophis's 2029 encounter will not cause catastrophic disruptions, it can significantly affect certain Earth-orbiting satellites. This study demonstrates that asteroid-induced perturbations are a genuine hazard for space operations and provides a novel framework to assess their impact, complementing impact-focused planetary defence strategies with the evaluation of orbital stability risks.

Keywords:

Orbital propagator, RK45 integrator, asteroid-induced perturbations, dynamical modelling, close encounter dynamics, planetary defence.

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List of Abbreviations

CR2BP	Circular Restricted Two-Body Problem
ECEF	Earth-Centered Earth-Fixed
ECI	Earth-Centered Inertial
ESA	European Space Agency
FEAS	Faculty of Engineering and Applied Sciences
GEO	Geostationary Earth Orbit
HEO	Highly Elliptical Orbit
ICRF	International Celestial Reference Frame
JPL	Jet Propulsion Laboratory
LEO	Low Earth Orbit
MEO	Medium Earth Orbit
NASA	National Aeronautics and Space Administration
NEO	Near-Earth Object
ODE	Ordinary Differential Equation
PHA	Potentially Hazardous Asteroid
RAAN	Right Ascension of the Ascending Node
SDP	Simplified Deep Space Perturbations
SGP	Simplified General Perturbation
STEM	Science, Technology, Engineering and Mathematics
STK	Systems Tool Kit
SRP	Solar Radiation Pressure
TDB	Barycentric Dynamical Time
TLEs	Two-Line Elements set
TT	Terrestrial Time
UT	Universal Time
UTC	Coordinated Universal Time

Chapter 1

Introduction

Earth is constantly exposed to space debris and other material, receiving an estimated 100 tons of cosmic dust daily. Whilst most of it is harmless and burns up in the atmosphere, the potential threat of a significant asteroid impact remains a real concern. As of July 2025, a total of 38,612 asteroids, classified as Near-Earth Objects (NEO), have been identified in orbits that bring them in close proximity with our planet, and this number continues to grow as more objects are detected [12].

On average, every year between two and three asteroids closely approach Earth getting into the geostationary ring [13]. Larger bodies tend to pose less immediate risk, as Davide Farnocchia engineer at NASA's Jet Propulsion Laboratory (JPL) and scientist for the Center for Near Earth Object Studies states: "The bigger asteroids are the less tendency they have to impact with Earth" [14]. Nevertheless, the possibility of hazardous encounters makes the study and prediction of near-Earth object trajectories a critical area of research.

To address this challenge, space agencies such as ESA and NASA have established dedicated planetary defence offices. Their work focuses primarily on analysing the probability of collisions, since a direct impact represents the most severe threat. Activities such as monitoring NEOs, calculating and predicting their orbits, and issuing warnings are all oriented towards anticipating and mitigating this specific risk [13, 14, 15].

While the catastrophic potential of an impact dominates public concern, even a non-collisional close approach could pose risks to modern society. If a NEO passes close enough to Earth, it may interfere with satellites or orbital debris. Though the consequences would be milder than a direct impact, the disruption or destruction of Earth-orbiting spacecraft could have serious implications for telecommunications, navigation and scientific monitoring. This specific scenario, asteroid-induced perturbations on satellites, has received relatively little academic attention.

On April 13th 2029, Apophis (99942), a 375-meter and 20-million-ton NEO, will pass at only 32,000 kilometers from Earth's surface, a distance closer than the orbit of geostationary satellites. This will be the closest approach ever recorded for an asteroid of comparable size, making it a landmark event in planetary science.

The encounter will provide a rare "natural experiment" in celestial mechanics. For the first time, scientists will be able to observe in real time how a large NEO responds to Earth's

gravity during such a close flyby. The sheer scale of Apophis compared to previously observed meteors underscores its uniqueness.

From a planetary defence perspective, most efforts have traditionally focused on predicting and mitigating potential collisions. However, the unprecedented proximity of Apophis also raises another overlooked question: how the passage of a massive asteroid might perturb Earth-orbiting satellites. Addressing this issue is essential for a complete understanding of the risks posed by near-Earth objects.

This research project will investigate the potential orbital disturbances caused by Apophis on Earth-orbiting spacecraft. By simulating and analysing these effects, the study seeks to shed light on an unexplored hazard of close asteroid encounters. The findings could then provide valuable insights for satellite operators, mission planners and planetary defence strategists, supporting future preparedness for NEOs that come close, even without impact.

1.1 Aim

This research project aims to set the basis to develop a computational framework to estimate the possible effects of asteroid close approaches on Earth-orbiting satellites.

The goal is to start to delineate a methodology to determine which orbital regions are most vulnerable to gravitational disturbance and to provide a framework that can be applied to future asteroid encounters.

The method is supported by the development of a modular orbit propagator that incorporates asteroid-induced perturbations and simulates multiple satellite configurations.

The approach is demonstrated through the case study of asteroid Apophis (99942) and its close flyby of Earth in 2029.

1.2 Objectives

To achieve this aim, the following objectives are defined:

- Model the dynamics of Sun-Earth-Moon-Asteroid system by implementing a numerical orbit propagator that includes relevant perturbations and uses real ephemerides of celestial bodies, with accuracy comparable to high-fidelity references such as JPL Horizons [4].
- Compare propagators and their sensibility to identify the most suitable one for the case in object.
- Preliminary design of a pipeline to identify the most perturbed orbital configurations for satellites around Earth during a generic asteroid close approach, based on encounter geometry and relative velocity.
- Apply the methodology to the 2029 Apophis flyby by quantifying orbital perturbations on the identified configurations.

1.3 Thesis structure

This thesis is organised into seven chapters, each addressing specific aspects of the study and guiding the reader towards the fulfillment of the stated objectives.

- **Chapter 1. Introduction:** It introduces the context of planetary defence and the motivation for investigating asteroid-induced perturbations on Earth-orbiting satellites. It defines the aim, outlines the objectives and highlights the novelty of focusing on satellite effects.
- **Chapter 2. Literature Review:** Provides the theoretical background necessary to frame the study. It presents the main characteristics of asteroid Apophis, the orbital description of Earth satellites, the types of orbital propagators and existing analytical methods for close approaches. This chapter clarifies the knowledge gap, showing that very limited literature addresses asteroid effects on satellites.
- **Chapter 3. Methodology:** Details the development of the framework. It describes the reference frames, the dynamical model including perturbations, and the implementation of the numerical propagator. Validation and sensitivity analyses are presented, followed by the definition of the perturbation region and the selection of candidate satellites. This chapter directly addresses the first two objectives: the implementation and verification of the propagator, and the identification of the most suitable integration scheme.
- **Chapter 4. Apophis influence on GEO satellites:** Applies the methodology to real geostationary satellites and debris. The analysis quantifies ground-track drifts and variations in Keplerian elements, thus addressing the third objective of assessing Apophis's perturbations on existing orbital configurations.
- **Chapter 5. Apophis maximal effect:** Extends the analysis to hypothetical orbits and identifies the most perturbed configurations. It also connects these findings to real satellites located near such regions, thereby fulfilling the fourth objective of the thesis.
- **Chapter 6. Discussion and Conclusions:** synthesises the results obtained across the study. It revisits the objectives and demonstrates how they were achieved, evaluates the relative importance of encounter geometry in amplifying perturbations, and places the findings in the broader context of planetary defence.
- **Chapter 7. Limitations and Future Work:** Highlights the simplifications and assumptions adopted, and proposes possible extensions to improve the accuracy and applicability of the methodology.

The scripts developed for this project can be accessed on GitHub [here](#).

Finally, a note on notation: throughout this thesis, bold symbols in equations represent vectors, while a dot placed above a variable indicates time differentiation.

Chapter 2

Literature review

2.1 Apophis

Since its discovery in June 2004, asteroid (99942) Apophis has attracted significant scientific attention due to its orbital characteristics and potential risk to Earth. Initially, Apophis was classified as a potentially hazardous asteroid (PHA) with an alarming 2% probability of impacting Earth in 2029. Although later observations ruled out the possibility of an impact in 2029 and 2036, a low-probability scenario for a future impact in 2068 still remains under consideration.

The closest approach in April 2029 is remarkable because of its proximity, approximately 32,000 kilometers from Earth's surface, closer than the geostationary orbit.

Although the asteroid will remain outside Earth's Roche limit (the distance to Earth within which it would be torn apart by tidal forces), measurable tidal torques are expected, these may alter its spin and potentially its internal structure. The precise outcome of this interaction is uncertain, which motivates real-time observations during the flyby [16]. The encounter is also expected to alter Apophis's heliocentric orbit, with predictions indicating its transition from an Aten-class asteroid (solar orbital period of 0.9 Earth years), to an Apollo class (period of 1.2 Earth years) [16].

From a planetary-defence perspective, the event offers a rare opportunity to characterise the physical and dynamical properties of PHAs, information essential for developing effective mitigation strategies and future risk assessments [1].

The singularity of the 2029 flyby is illustrated in Figure 2.1, which indicates that no other asteroid of comparable size is expected to pass as close within two centuries. For context, Apophis is estimated to be roughly 5,000 times more massive than the Chelyabinsk meteor that impacted over Russia in 2013.

Taken together, these facts frame the 2029 encounter as a “natural experiment” in celestial mechanics since it gives the chance to observe how a large NEO responds to Earth's gravity during a close approach. Given the expected effects on the asteroid, it is reasonable to consider potential consequences for Earth's orbital environment as well. In particular, Apophis could induce perturbations in trajectories of satellites or debris, making the 13th April 2029 flyby a suitable scenario to study possible influences on Earth-orbiting spacecraft.

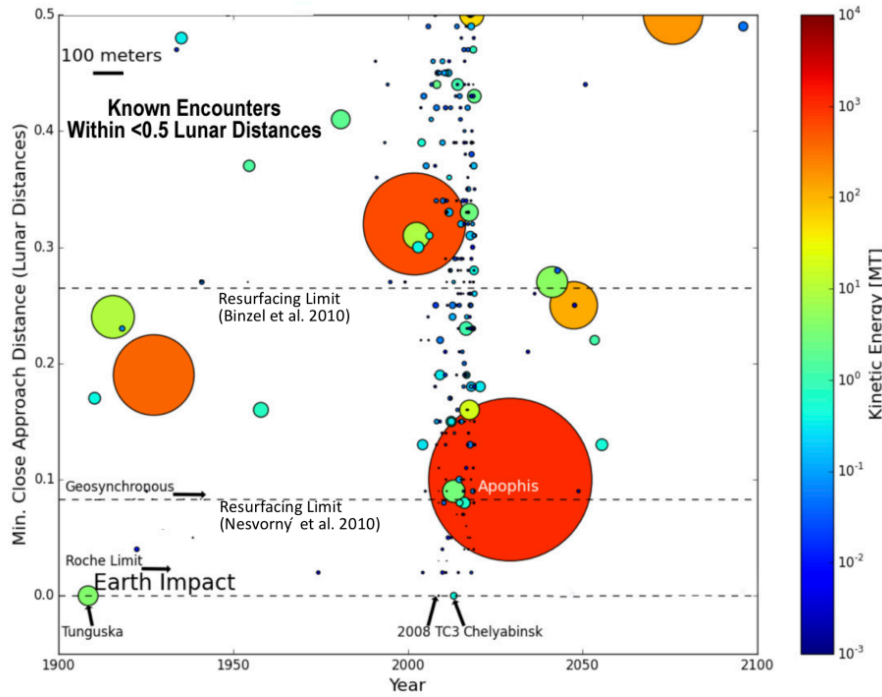


Figure 2.1: Earth approaches within 0.5 lunar distance. Source:[1].

2.1.1 Apophis orbital data

In order to analyse the 2029 encounter, accurate orbital data are required. For this purpose, the trajectory of Apophis is obtained from NASA's JPL Horizons system [4]. This is an online database that provides regularly updated ephemerides, based on all available ground and space-based observations, and is therefore considered the reference source for Apophis's orbital evolution. All the information described here can also be found in the System's online manual.

Horizons allows users to generate ephemerides by specifying a set of configuration parameters. The main options are:

- **Ephemeris Type:** The user can request different output formats:
 - Observer table: apparent quantities seen by an observer (topocentric or from the center of a planet), applying corrections such as perspective.
 - Vector table: cartesian position and velocity state vectors of a body with respect to a reference body.
 - Osculating orbital element table: instantaneous osculating orbital elements with respect to a planet or a barycenter.
 - Small-Body SPK file: binary file with interpolable cartesian position and velocity state vector of small-bodies.

For this project, the Vector Table output was selected, as it provides the state vectors needed for dynamical calculations.

- **Target Body:** The object for which the ephemeris is generated. Horizons uses an internal catalogue of identifiers for Solar System bodies.
- **Observer Location:** Defines the origin of the coordinate system. Options include:
 - Predefined locations (e.g., planetary centers).
 - A list of Earth-based locations.
 - User-specified coordinates (geodetic or cylindrical).
 - Two-Line Elements (TLEs) for Earth satellites.
- **Time Specification:** Defines the epochs for which ephemerides are produced. This can be set as:
 - A continuous time span (Gregorian or Julian dates) with a specified timestep.
 - A discrete list of epochs (Gregorian, Julian or Modified Julian dates), in Universal Time (UT), Terrestrial Time (TT), or Barycentric Dynamical Time (TDB).
- **Table Settings:** The output can be further customised, including the state vector components, their uncertainties, and additional options:
 - Reference frame (e.g, ICRF).
 - Reference plane (e.g, equatorial).
 - Vector correction: geometric states for no correction, astrometric states for light-time correction and apparent states for stellar correction.
 - Calendar Type: Gregorian or Gregorian and Julian day.
 - Output units: specify the units in which the obtained values will be.

Access to Horizons was automated through a Python script. The following inputs are used to request the data:

- | | |
|--|---------------------------------|
| • Type of Ephemeris: V (vector table). | • TLIST: Julian Date |
| • Output units: km-s | • Reference system: ICRF |
| • Command: code number of the target body | • Reference Plane: frame |
| • Center: 500@0 | • CSV Format: Yes |
| • Time Type: TDB | • Vector table: 3 |

The Horizons identifiers relevant for this research are summarised in the next Table 2.1:

Table 2.1: Celestial body codes used in NASA's JPL Horizons System [4].

Code	Definition
399	Earth's center
10	Sun's center of mass
301	Moon's center
99942	Apophis
500@0	Solar System Barycenter

It is important to note that the ephemerides provided by Horizons includes associated uncertainty, derived from the covariance of the orbital fit to observational data. This uncertainties decrease as new observations are made. Regarding their temporal evolution, the further the propagation extends from the last observation, the larger the uncertainties become. Table 2.2 summarises the maximum, minimum, mean and standard deviation of the uncertainties associated with the cartesian state components.

Table 2.2: Apophis's trajectory uncertainties [4].

State	Maximum	Minimum	Mean	Standard deviation
X	1220.646 km	0.184 km	159.620 km	284.655
Y	1714.737 km	1.112 km	477.700 km	555.568
Z	153.679 km/s	0.217 km/s	72.687 km/s	57.980
V_x	$3.85 \cdot 10^{-4}$ km/s	$7.480 \cdot 10^{-8}$ km/s	$7.210 \cdot 10^{-5}$ km/s	$1.13 \cdot 10^{-4}$
V_y	$2.31 \cdot 10^{-4}$ km/s	$3.8 \cdot 10^{-8}$ km/s	$9.53 \cdot 10^{-5}$ km/s	$8.24 \cdot 10^{-5}$
V_z	$3.87 \cdot 10^{-5}$ km/s	$3.53 \cdot 10^{-8}$ km/s	$1.81 \cdot 10^{-5}$ km/s	$1.46 \cdot 10^{-5}$

The largest position uncertainty reaches 1700 km, which corresponds to about 5% of the 32,000 km closest approach distance. It should be noted, however, that this value is only indicative, since the 32,000 km refer to the full distance magnitude, whereas the uncertainty is expressed along a single direction. Mean values are significantly smaller, indicating that the trajectory is well constrained. Velocity uncertainties are on the order of 10^{-4} km/s or less, which is negligible compared to Apophis's orbital velocity (around 30 km/s).

Although these uncertainties are small, they should be acknowledged when interpreting the results. Nevertheless, the current ephemerides provided by Horizons are considered sufficiently reliable for the purposes of this study and will be adopted as the reference data.

2.2 Satellites' orbits

The orbital description of Earth-orbiting satellites is also required in order to analyse the potential perturbations caused by 2029 Apophis flyby. This section introduces the sources of satellite orbital data, the use of TLEs, and the propagation methods considered.

2.2.1 Two-Line Element Sets (TLEs)

The most widely used format for publicly available satellite orbital data is the Two-Line Element set. It provides a compact representation of a satellite's osculating orbital elements at a given epoch, derived from observations maintained by the United States Space Command (USSPACECOM). TLEs include parameters such as the inclination, right ascension of the ascending node, eccentricity, argument of perigee, mean anomaly and mean motion. Figure 2.2 shows the data contained in a TLE and their actual position.

Card #	Satellite Number	Class	International Designator			Yr	Epoch					Mean motion derivative (rev/day /2)					Mean motion second derivative (rev/day2 /6)					Bstar (/ER)					Eph	Elem num	Chk Sum					
			Year	Lch#	Piece							S				S					S													
1	1 6 6 0 9	U	8 6	0 1	7 A		9 3	3 5	2 .	5 3	5 0	2 9	3 4		. 0	0 0	0 0	0 7	8 8	9		0	0	0 0	0 0		1 0	5 2	9 - 3	0		3 4	2	
			Inclination (deg)			Right Ascension of the Node (deg)			Eccentricity			Arg of Perigee (deg)			Mean Anomaly (deg)			Mean Motion (rev/day)			Epoch Rev			Chk										
2	1 6 6 0 9		5 1	. 6	1 9	0	1 3	. 3	3 4	0	0	0 0	5 7	7 0		1 0	2 .	5 6	8 0		2 5	7 .	5 9	5 0		1 5	. 5	9 1	1 4	0 7	0 4	4 7	8 6	9

Figure 2.2: Two-Line Element Set [2].

The satellite catalog number and the international designator are two ways of identifying an object. The former corresponds to the number assigned by the US Space Force, while the other one is a combination of the year it was launched, the launch number of the year and the piece of the launch (e.g. 'A' for payloads) [17].

In practice, TLEs are more suitable for short-term propagations since they become progressively unreliable for longer timescales.

The initial approach was to query space-track.org each time the TLE of a satellite was required. However, retrieving large numbers of TLEs in order to identify suitable candidates quickly exceeded the access limits permitted by the platform. To overcome this issue, all available historical TLE data were downloaded and consolidated into a single file containing the most recent epoch for the satellite. This local database enables efficient access to TLEs without repeatedly querying the space-track.org server.

2.2.2 TLEs propagation

The propagation of TLEs must be carried out with compatible analytical models, since the orbital elements provided in this format are not purely osculating but have been adjusted using specific perturbation theories. Using different type of propagator would introduce inconsistencies and rapidly degrade prediction accuracy [18]. The Simplified General Perturbation (SGP) models are widely used semi-analytical propagators, primarily for processing Two-Line Element (TLE) datasets. They propagate mean orbital elements using precomputed perturbation models.

The first model was developed by the U.S. Air Force in the 1960s, and the SGP family has evolved to include more refined models such as SGP4 and Simplified Deep Space Perturbations (SDP4). While SGP4 is optimised for orbits with periods below 225 minutes, SDP4 is suited for longer-period orbits [19].

The most commonly used model is SGP4, which includes atmospheric drag modelling, Earth's oblateness and third-body perturbations from major celestial bodies such as the Moon and the Sun. It typically achieves 1 kilometer accuracy at epoch, with growing errors by 1 and 3 kilometers per day [20]. This systematic error further limits its applicability for precise or long-term studies.

In response to the need for higher accuracy, the U.S. Space Force released SGP4-XP in 2020, the advanced version of the SGP4 algorithm. It introduces several major improvements:

- Enhanced lunar perturbation modelling.
- Solar Radiation Pressure modelling.
- Inclusion of the J5 zonal harmonic.
- Updated geopotential model (WGS-72 is replaced by EGM-96).
- Jacchia-70 atmospheric density model for drag estimation.

It introduces a new parameter, Area over Gamma over Mass (AGOM) to distinguish SRP from drag effects in the dynamic model. The TLE produced are marked with Ephemeris Type 4. TLEs generated with SGP4-XP (Ephemeris Type 4) are not backward-compatible with the classical SGP4 propagator. They must always be propagated using the same SGP4-XP algorithm in order to retain consistency [21].

According to [21], SGP-XP shows a significant improvement in position accuracy for MEO and GEO satellites and better velocity accuracy in LEO. Its enhanced modelling improves post-fit propagation, allowing TLEs to maintain reliability for longer than the SGP4. Despite its modest increase in runtime, the benefits in accuracy are significant [21].

Despite their limited long-term accuracy, TLEs retrieved from space-track.org will be propagated with SGP4 to obtain satellites' initial conditions. From that point onward, the dedicated orbital propagator developed in this work is employed to model the dynamics over the desired time span. Although the inherent uncertainties of TLE-based states are acknowledged, using SGP4 for the initialisation step is expected to minimise inconsistencies, since TLEs are specifically structured to be propagated with this model.

2.3 Orbital Propagators

In astrodynamics, orbital propagation refers to the prediction of a satellite's future trajectory based on its current state and the forces acting on it. Accurate propagation is essential for mission design, orbit determination and payload data analysis. As missions grow in complexity, so does the demand for precision in orbit modeling and prediction [22].

The core inputs for propagating include the initial state vector (position and velocity), time and the physical models describing the perturbing forces. Depending on the mathematical approach and fidelity of the model, orbital propagators are categorised into three classes: analytical, semi-analytical and numerical [23, 20]:

2.3.1 Analytic propagators

Analytical (or general perturbation) use analytic approximations to compute the satellites motion over time. These models are based on simplifying assumptions that allow the analytical integration of the equations of motion. Offering, therefore, fast computation but limited accuracy.

Given their characteristics, they are typically suitable for early mission design, or scenarios where computational efficiency is prioritised over precision since smaller perturbations tend to be omitted, only considering major gravitational effects.

Classic examples include the two-body problem, which models motion under mutual gravitational attraction of two point masses, and The J2 propagator, which incorporates Earth's oblateness. [20, 23, 22].

Despite their efficiency, analytical propagators rely on series expansions whose truncation introduces error, and they are generally not appropriate for operations requiring high-accuracy or extended time spans.

2.3.2 Semi-analytic propagators

Semi-analytic propagators offer a compromise between speed and fidelity by treating long- and short-period effects differently: long-term variations are modeled numerically, while short-period terms are treated analytically. These methods can incorporate time-dependent perturbations with greater accuracy than purely analytical models, while maintaining reasonable computational efficiency [22].

2.3.3 Numerical propagators

Numerical (or special perturbation) propagators directly integrate the full equations of motion using high-fidelity force models. These include gravitational effects from multiple celestial bodies, non-spherical geopotential fields, atmospheric drag, SRP and more [20, 22].

This class of propagators is the most accurate, making it suitable for more complex scenarios. However, they are computationally intensive and require careful selection of numerical integration schemes and time-step controls.

Numerical integration plays a central role in high-accuracy orbit propagation by approximating the solution to nonlinear ordinary differential equations (ODEs) given an initial state. In orbital mechanics, the primary equation to be solved is the two-body equation of motion, derived from Newtonian mechanics, often extended with perturbation models. These solvers estimate the trajectory by integrating this ODE over time, and their accuracy depends on three main factors: the fidelity of the force model, the choice of integrator and the properties of the orbital elements [24].

Numerical integrators can be broadly characterised along the following dimensions:

- **Explicit or Implicit:** Explicit methods compute the next state directly from the current state, while implicit methods require solving equations that depend on current and future states. While implicit methods offer enhanced stability, they do not inherently guarantee greater accuracy.

- **Single-step or Multi-step:** Single-step methods rely only on the current state to estimate the next one, whereas multi-step incorporate several past states. Multi-steps schemes often achieve higher accuracy through improved local error control.
- **Fixed-step or variable-step:** Fixed-step integrators use uniform time instants, while variable-steps methods dynamically adjust the timestep to maintain accuracy, based on user-defined error tolerances. Variable-step regulate accuracy via user-defined tolerances.

Based on these characteristics, the following integrators are widely used in orbital propagation:

- **Runge-Kutta Integrator**

A family of explicit, single-step methods that compute the next state using only the current value. They can be in both fixed and variable step configuration. The updated rule is given by:

$$y_{k+1} = y_k + h\phi(t_k, y_k, h) \quad (2.1)$$

Where ϕ is an increment function computed by averaging the derivatives across multiple evaluation stages. Graphically represented in Figure 2.3.

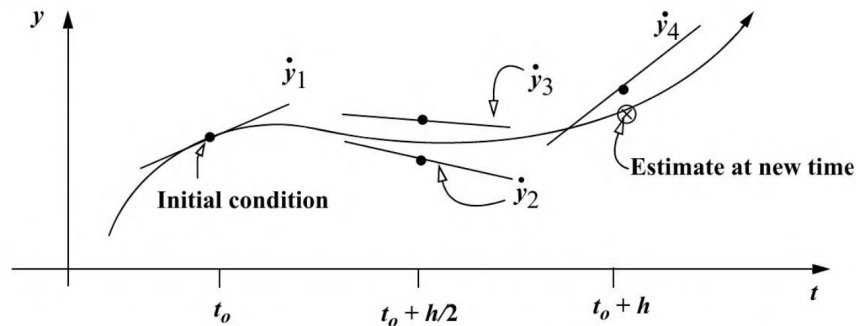


Figure 2.3: Runge-Kutta graphical representation. Source: [2].

- **Dormand-Prince Integrator**

A high-order, explicit single, variable-step method. It uses a high-order estimate for integration and a lower-order estimate to control error and adapt step size. This dual-estimate approach improves computational efficiency.

- **Adams-Bashforth-Moulton Integrator**

A predictor-corrector method that combines the explicit Adams-Bashforth step with Adams-Moulton for implicit correction. It is multi-step, variable-order, and requires a starting method. The method adapts its order based on local error estimation.

Overall, the literature shows a diverse use of propagators depending on the scope of the study: analytical formulations and ephemeris-based approaches are often employed for long-term trajectory analysis, while high-order numerical schemes are preferred for investigating particle dynamics.

In the case of propagating perturbed orbits of small bodies, studies frequently adopt adaptive step-size Runge-Kutta methods, which provide a reliable compromise between accuracy and efficiency.

2.3.3.1 Tolerances and Error Control

Tolerances are crucial in regulating integration accuracy. Two types of tolerances can be defined:

- **Relative Tolerances:** specifies the allowable percentage error per integration step, ensuring solution convergence across a wide dynamic range.
- **Absolute Tolerances:** defines a fixed error threshold for state variables approaching zero, where relative errors lose significance.

Given the sensitivity of orbital dynamics, propagation algorithms typically require tight tolerances to maintain long-term accuracy and stability.

2.3.3.2 Integrator Performance Comparison

An evaluation was conducted using JPL Horizons ephemerides over a 25-day span across four orbit types and several physical models. Based on this analysis, the following integrator recommendations are made:

- **LEO:** High-order methods (ODE113 or DOPRI87) are recommended due to the short periods and rapid dynamics increase the computational cost.
- **GEO:** Integrators like RK45 or ODE45 perform well under two-body dynamics when paired with tight tolerances.
- **HEO:** RKF methods handle moderate tolerances effectively, though long-term accuracy may degrade due to error accumulation. All integrators exhibit higher computational cost as orbital eccentricity increases.

From the study, three major conclusions were drawn:

- Simulations failed to converge when relative tolerances exceed $1 \cdot 10^{-10}$. Tolerances selection should be based on orbital characteristics such as period and semi-major axis.
- The choice of numerical method significantly affects performance. High-order integrators reduce computational time while maintaining accuracy.
- The computational cost is primarily driven by orbital parameters, such as shape, size and period (e, a, τ). Lower altitudes and high eccentricities generally lead to increased computational demands.

Based on these findings, a relative tolerance of $1 \cdot 10^{-12}$ is recommended for Earth-orbiting satellites. Nevertheless, different tolerance settings will be tested in order to determine the most appropriate configuration for the present study.

2.4 Effects of a Close Approach

To characterise the perturbation induced by Apophis during its 2029 flyby, this study focuses on identifying the worst-case orbital configuration, which is the orbit most strongly affected by the asteroid's gravitational influence. This requires determining the set of orbital parameters for which the satellite experiences the maximum disturbance.

The characteristics of the close approach must therefore be taken into account, in particular the encounter geometry and the relative dynamics between the asteroid and the satellite. According to [25], the three most relevant factors in close encounters are:

- The minimum distance between the two bodies.
- Their relative velocity.
- The relative direction of motion.

It is important to note that there is very limited literature directly addressing the impact of asteroids on Earth-orbiting satellites. Consequently, existing theoretical frameworks developed for flybys between small bodies and planets have been adopted as references. Although not specifically designed for satellite-asteroid interactions, these models provide the analytical foundation to characterise the encounter geometry and assess the resulting dynamical effects.

Three different used methods for characterisation of close approaches between small bodies are: Opik's theory, the Patched Conics approximation and the Keplerian Map. A concise description of each method, including its assumptions, limitations and the main effects is provided in the following subsections.

2.4.1 Opik's theory

One classical method to describe close encounters between a small body and a planet is the theory developed by Opik's. This framework approximates the gravitational interaction as a two-body hyperbolic encounter and provides a way to estimate collision probabilities and the changes induced in the heliocentric orbital elements of the small body.

The theory relies on several simplifying assumptions [26]:

- The small body is within the Hill radius of the planet, so the Sun's perturbation can be neglected and the interaction is reduced to a two-body problem.
- The planetocentric velocity of the body is considered constant before and after the encounter.
- The relative velocity of the bodies during the encounter is constant.

- The planet moves on a circular orbit of unit radius in a reference plane.
- The encounter takes place at one of the orbital nodes of the small body.

The geometry of the encounter is described by the deflection angle (γ), which measures the angular deviation of the incoming planetocentric velocity vector during the flyby [3]. The angle depends on the impact parameter (σ), the relative velocity, and the planetary mass. Consequently, closer approaches or smaller relative velocities lead to larger deflections. The mass ratio between the planet and the small body also plays a role, with larger mass differences enhancing the deflection. In practice, both the distance and mass factors contribute comparably to the outcome of the encounter [26].

Although the velocity magnitude relative to the planet remains constant, larger deflection angle implies a greater modification of the heliocentric orbit. Figure 2.4 illustrates the dependence of the deflection angle on the encounter geometry, while Figure 2.5 shows how the corresponding change in heliocentric orbital energy depends on the encounter geometry. The most deflected orbits correspond to the ones with a higher energy change which, considering orbital energy conservation, means significant variations in the asteroid's trajectory, specifically the semimajor axis.

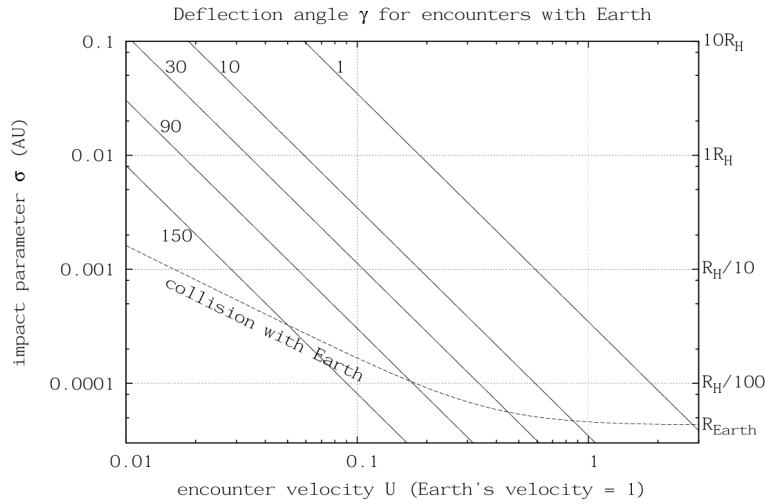


Figure 2.4: Deflection angle as a function of the impact parameter and relative velocity. Source: [3].

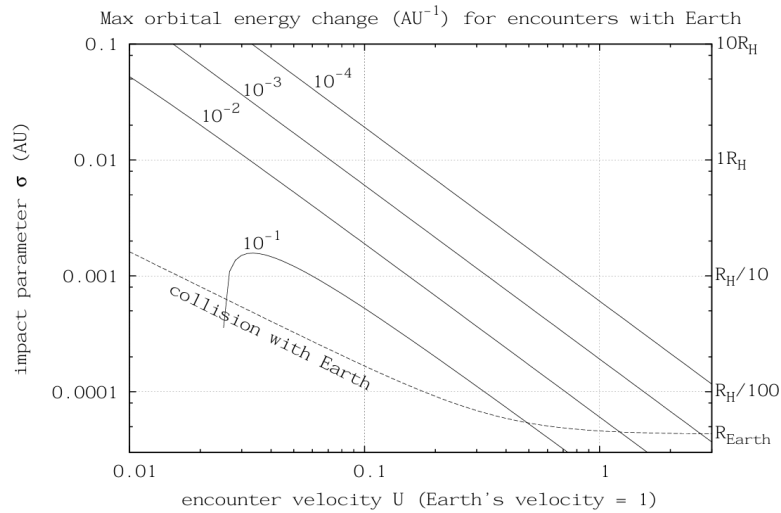


Figure 2.5: Maximum orbital energy variation as a function of the impact parameter and relative velocity. Source: [3].

Despite its usefulness and simplicity, Opik's theory presents some limitations. It becomes less reliable for very low relative velocities, where temporary captures may occur, and it cannot properly describe resonant or nearly-tangent encounters [3]. Nevertheless, comparisons with numerical integrations have shown that it provides a satisfactory approximation for a wide range of encounter conditions and remains a fundamental analytical tool to model planetary close approaches [26].

2.4.2 Patched Conics

The Patched Conics approximation is a widely used analytical method for modelling planetary flybys. It replaces the complexity of the three-body problem by a sequence of simpler two-body trajectories, each one governed by a single dominant gravitational attraction [25, 27]. The trajectory is split into three arcs. Far from the planet, the small body follows a heliocentric Keplerian orbit. Once it enters the sphere of influence of the secondary body, approximated by the Hill radius, its motion is described as a hyperbolic passage relative to that body. After leaving this region, the trajectory is again an hyperbolic heliocentric orbit. The arcs are “patched” at the boundaries of the sphere of influence, ensuring continuity in position and velocity.

The method relies on several simplifying hypotheses:

- The trajectory can be separated into two-body problems, with the Sun acting alone outside the SOI and the planet acting alone inside it.
- The planet follows a circular orbit around the Sun.
- The effect of the Sun during the short hyperbolic passage is neglected.
- The deflection is assumed to occur instantaneously at the periapsis of the hyperbola.

Within the Patched Conics framework, a flyby has the strongest perturbation effect on the small body's trajectory in planar flybys, where the energy exchange is maximised. In particular, a flyby with an α of 270 degrees (behind the secondary body) results in the largest energy gain, whereas a flyby with α being 90 degrees (in front of the secondary body) leads to the maximum energy loss. For encounters of values between 0 and 180 degrees, the small body experiences no change in inclination change and the energy variation is neither maximal nor minimal. Furthermore, the overall effect decreases if the periapsis distance or the periapsis velocity increase [27].

While the method provides a simple and fast analytical description of flybys, its accuracy deteriorates for distant or slow encounters, where the gravitational influence of both the planet and the Sun must be considered simultaneously. Moreover, because the interaction is assumed to occur at a single point, the method cannot reproduce long-duration perturbations [25].

2.4.3 Keplerian Map

The Keplerian Map is a semi-analytical designed to model long-term influence of a secondary body on the orbital elements of a third massless particle. Unlike Opik's theory or the Patched Conics approximation, which focus on encounters inside the sphere of influence, this method describes distant interactions that occur outside this boundary and accumulate their effects over multiple orbital periods [28, 29].

The method is based on the Circular Restricted Three-Body Problem (CR3BP) and is derived through a perturbative approach using the Lagrange planetary equations. Its main assumptions are:

- The small body is massless and does not influence the motion of the primaries.
- The gravitational parameter of the secondary body is smaller than the primary body one.
- The osculating Keplerian orbital elements are constant during the integration interval.
- The secondary body follows a circular orbit around the primary.

The gravitational perturbation depends on the relative angular phasing α between the small body and the secondary. The largest variations in orbital elements are observed when α is small. The major effect occurs when the two bodies are geometrically aligned and pass at their minimum separation distance.

The accuracy of the method decreases as the second body's gravitational parameter increases. Errors also arise from the approximation of constant orbital elements during one period, which may not hold in strongly perturbed cases. Moreover, this method is not suitable for encounters inside the Hill sphere. Lastly, mathematical singularities appear in the computation of some variables such as true anomaly or periapsis.

Chapter 3

Methodology

This chapter outlines the methodology developed to study the potential perturbations that Earth-orbiting satellites may experience during Apophis's close approach in 2029.

The study was focused on the effect of Apophis's on satellites in geostationary orbit, as this region was considered the most likely to be influenced due to this altitude. This first stage was essential to develop, implement and validate the numerical propagator required for the analysis.

Furthermore, it provided valuable insight into the limited impact expected in GEO, highlighting the need to expand the analysis beyond this orbit. The methodology was thus refined to systematically identify the orbital configurations around Earth most susceptible to asteroid-induced perturbations, and it is structured around two key components:

- Development of a modular numerical orbit propagator incorporating asteroid-induced gravitational effects.
- Identification of the orbital configurations around Earth expected to be most susceptible to asteroid-induced gravitational effects.

Once both components are developed, the identified high-risk orbital regions are tested using the propagator to quantify the magnitude of disturbances and to determine whether any real or plausible satellite currently resides within those configurations.

Overall, this methodology provides a novel and adaptable framework for assessing asteroid-satellite interactions, a topic that has received little attention in the field of planetary defence.

The first two sections (Section 3.1 and 3.2), which introduce the reference frames and the dynamical model, provide the necessary foundation for the subsequent development of the project.

3.1 Systems of Reference

The selection of the system of reference is essential when describing orbits [2]. Since the same physical position can be represented differently depending on the selected frame, special care must be taken in order to ensure consistency and accuracy. In this study,

the challenge arises from the fact that the two bodies of interest orbit different central bodies: Apophis orbits the Sun, whereas the satellites orbit Earth. To provide a common framework for comparison, the International Celestial Reference Frame (ICRF) is adopted as the primary reference system.

Nevertheless, satellite states are often expressed in the Earth-Centered Inertial (ECI) frame, which requires appropriate transformations between both systems. In addition, results will also be evaluated in the Earth-Centered Earth-Fixed (ECEF) frame, as this geodetic system provides a practical perspective directly related to Earth's surface.

Figure 3.1 illustrates the heliocentric orbits of Earth and Apophis after the close encounter. This representation highlights not only their trajectories around the Sun, but also the need to transform between multiple reference frames when analysing the problem, underlining the inherent complexity of the dynamical interactions.

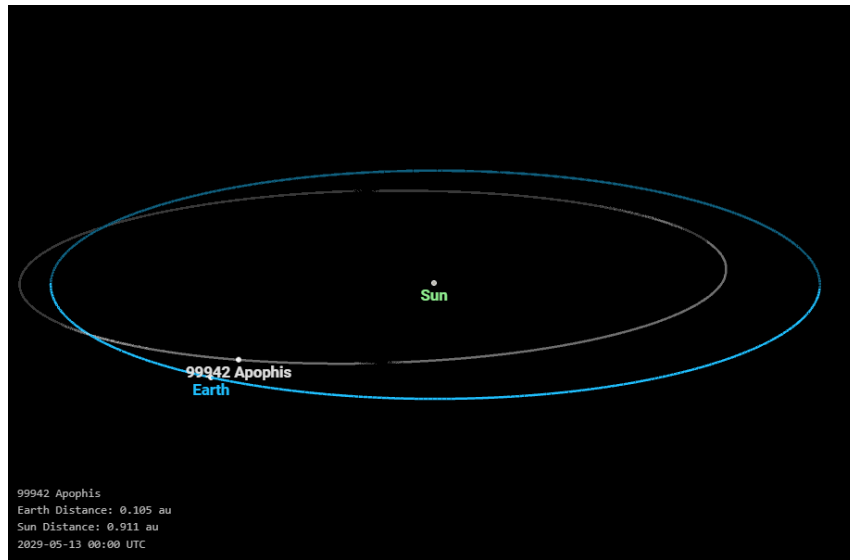


Figure 3.1: Earth's and Apophis's orbit around the Sun.

International Celestial Reference Frame (ICRF)

The International Celestial Reference Frame (ICRF) has its origin at the solar system barycenter, and its three basis axes are defined relative to extragalactic radio sources, primarily quasars. These objects are so distant that their angular motion is negligible, making them excellent fixed reference points. Ephemerides of solar system bodies are typically described in this coordinate system [30].

Geocentric Equatorial Coordinate System

This coordinate system, also known as Earth-Centered Inertial (ECI), is a quasi-inertial reference frame fixed relative to distant stars. It does not rotate with Earth, which means that from its perspective, the equator and equinox shift slowly over time due to precession and nutation.

- **Origin:** Center of Earth.

- **X-axis:** Points towards the vernal equinox.
- **Z-axis:** Aligned with Earth's spin axis, pointing toward the North Pole.
- **Y-axis:** Completes the right-handed orthonormal set in the equatorial plane, 90 degrees east of the X-axis.

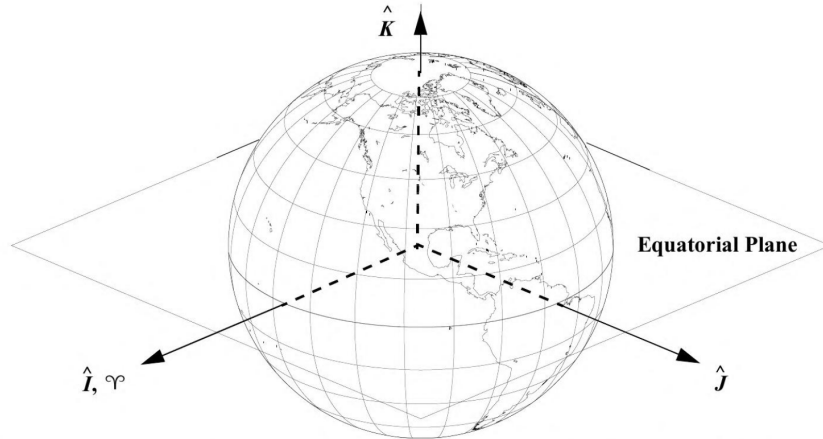


Figure 3.2: Geocentric Equatorial Coordinate System, with $\mathbf{I}$, $\mathbf{J}$, and $\mathbf{K}$ denoting the unit vectors of the Cartesian frame, corresponding to the $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ axes, respectively. Source: [2].

Earth-Centered Earth-Fixed (ECEF) System of Reference

The ECEF frame rotates with Earth, meaning that the coordinate axes are fixed relative to Earth's surface. It is commonly used for positions expressed in terms of latitude, longitude and altitude.

- **Origin:** Center of Earth.
- **X-axis:** Points towards the intersection of the equator and the prime meridian (0 degrees latitude ,0 degrees longitude).
- **Z-axis:** Lies along Earth's spin axis.
- **Y-axis:** Completes the orthonormal set and lies in the equatorial plane.

3.1.1 Changes of reference frames

Once the reference frames are introduced, the methodology for transforming from one system to another can be outlined.

From ICRF to Geodetic

A coordinate transformation essential to this project involves converting ICRF positions into geodetic coordinates (latitude and longitude):

$$\text{ICRF} \xrightarrow{\text{Translation}} \text{ECI} \xrightarrow{\text{Rotation}} \text{ECEF} \xrightarrow{\text{Geodetic conversion}} \lambda, \delta$$

Equations (3.1) to (3.4) describe the procedure used to convert ICRF coordinates into geodetic form. The first step involves a translation from ICRF to ECI, as:

$$\mathbf{r}_{ECI} = \mathbf{r}_{ICRF} - \mathbf{r}_{ICRF}^{Earth} \quad (3.1)$$

Here, $\mathbf{r}_{frame}$ denotes the position vector in the given reference frame, and $\mathbf{r}_{frame}^{Earth}$ represents the position of Earth in that frame.

To rotate from ECI to ECEF, a rotation matrix is applied to account for Earth's rotation over time. The rotation angle θ represents the angular displacement of Earth since a fixed reference time, known as the J2000 epoch, which corresponds to 12:00 TT (Terrestrial Time) on March 21st, 2000, and serves as a standard baseline for celestial coordinate systems. The angle is computed as [31]:

$$\theta = \omega_E \Delta t = \omega_E (t - t_{J2000}) \quad (3.2)$$

Where ω_E is Earth's angular velocity, and t_{J2000} is the reference epoch. Then, the rotation matrix for this transformation is defined as [31]:

$$\mathbf{r}_{ECEF} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{r}_{ECI} \quad (3.3)$$

Finally, ECEF coordinates are converted to geodetic coordinates using the trigonometric relations expressed in Equation (3.4) (λ as latitude and δ as longitude). Figure 3.3 depicts the conversion between the reference frames.

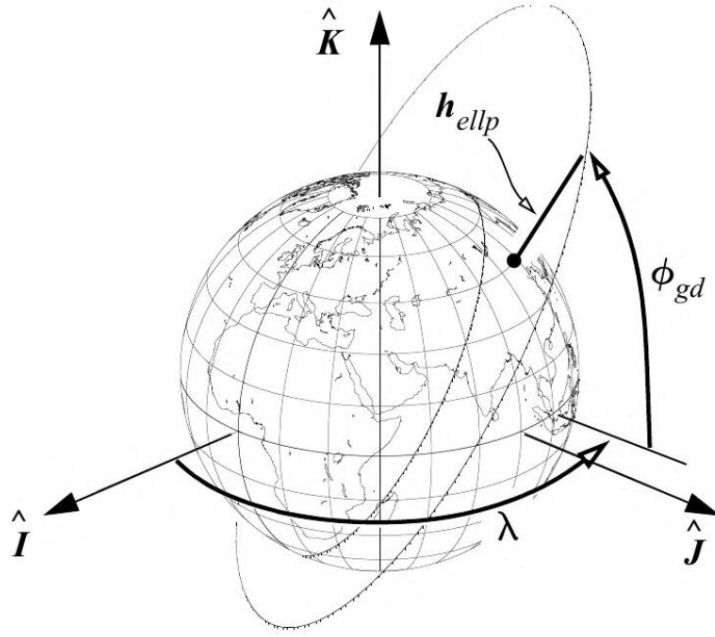


Figure 3.3: Conversion from ECEF to geodetic coordinates, with $\mathbf{I}$, $\mathbf{J}$, and $\mathbf{K}$ denoting the unit vectors of the Cartesian frame, corresponding to the $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ axes, respectively. Source: [2].

$$\lambda = \arcsin \left(\frac{z_{ECEF}}{\sqrt{x_{ECEF}^2 + y_{ECEF}^2 + z_{ECEF}^2}} \right) \quad (3.4)$$

$$\delta = \text{atan2}(y_{ECEF}, x_{ECEF})$$

However, when converting velocity vectors between reference frames, the procedure differs slightly from that used for position vectors. In particular, when transforming from ECI to ECEF, the rotation matrix is time-dependent and the transformation must account for the rotational velocity of the frame itself.

$$\mathbf{v}_{ECEF} = \mathbf{R} \mathbf{v}_{ECI} + \dot{\mathbf{R}} \mathbf{r}_{ECI} \quad (3.5)$$

In this expression, the term $\dot{\mathbf{R}} \mathbf{r}_{ECI}$ introduces an additional velocity component due to the rotation of the reference frame. This term accounts for the apparent motion observed in the rotating ECEF frame and is derived from the time derivative of the rotation matrix, which can be expressed as a function of Earth's angular velocity vector.

From ECI to Keplerian Elements

In orbital mechanics, it is often convenient to describe a satellite's trajectory in terms of Keplerian elements rather than Cartesian coordinates. These classical parameters provide a compact and intuitive representation of the orbit, capturing its size, shape and orientation in space.

Starting from the state vectors expressed in the ECI frame, the corresponding Keplerian elements can be obtained through standard analytical relations, as outlined hereafter [32].

$$\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}} \quad (3.6)$$

$$e = \frac{\dot{\mathbf{r}} \times \mathbf{h}}{\mu} - \frac{\mathbf{r}}{\|\mathbf{r}\|} \quad (3.7)$$

$$i = \arccos\left(\frac{h_z}{\|\mathbf{h}\|}\right) \quad (3.8)$$

$$\Omega = \begin{cases} \arccos\left(\frac{n_x}{\|\mathbf{n}\|}\right) & \text{if } n_y \geq 0 \\ 2\pi - \arccos\left(\frac{n_x}{\|\mathbf{n}\|}\right) & \text{if } n_y < 0 \end{cases} \quad (3.9)$$

3.2 Dynamical Model

The equations of motion of the Circular Restricted Two-Body Problem (CR2BP) around a primary body, can be written as [31]:

$$\begin{cases} \dot{x} = v_x \\ \dot{y} = v_y \\ \dot{z} = v_z \\ \dot{v}_x = -\frac{\mu}{|\mathbf{r}|^3}x \\ \dot{v}_y = -\frac{\mu}{|\mathbf{r}|^3}y \\ \dot{v}_z = -\frac{\mu}{|\mathbf{r}|^3}z \end{cases} \quad (3.10)$$

Where μ is the gravitational constant of the central body. Moreover, the equations of motion for the two-body problem (Equation (3.10)) can also be expressed as the gradient of a scalar potential ($U_{unperturbed}$), such that $\ddot{\mathbf{r}} = -\nabla U_{unperturbed}$.

In reality, bodies are subjected to perturbations, described by Vallado as deviations from normal, idealised or undisturbed motion [2]. Understanding these perturbations is essential for mission design, long-term orbital prediction and station-keeping.

The effect of perturbations can be modeled in two primary ways: by modifying the potential function (U^*) or by adding an external term to the acceleration equation, such that ($\ddot{\mathbf{r}} = U + \mathbf{b}$). This section outlines the main perturbative forces acting on Earth-orbiting satellites, which include:

- Additional gravitational perturbation caused by other massive celestial bodies such as the Sun or Moon.
- Earth's non-sphericity, which influences Earth's gravitational field.
- Solar Radiation Pressure.

Other perturbations include atmospheric drag, which becomes relevant for altitudes lower than 600 km and can lead to orbital decay and re-entry. Albedo forces, resulting from Earth-reflected sunlight, also act on satellites but are typically less significant than the

previously-mentioned perturbations [5, 33]. Given their small influence this kind of perturbations will not be included in this study.

Typical magnitudes for the perturbative accelerations are summarized in the following Table 3.1:

Table 3.1: Typical perturbation acceleration magnitudes for GEO satellites [5].

Perturbation Source	Acceleration [km/s ²]	Notes
Earth's gravity	10^{-4}	Dominant central force
Earth's oblateness	10^{-8}	RAAN precession and regression of the argument of perigee
Earth's triaxiality	10^{-10}	Causes long-term orbital drift
Lunar/Solar gravity	10^{-8}	Third-body perturbations
Solar Radiation Pressure (SRP)	10^{-10}	Depends on area-to-mass ratio
Albedo and other effects	$< 10^{-10}$	Negligible in simplified models

3.2.1 Additional Gravitational Perturbations

Luni-solar perturbations, originating from the gravitational influence of the Moon and Sun, are particularly relevant for high-altitude orbits. Over time, they induce oscillations in eccentricity, inclination, RAAN and argument of perigee. In orbits with inclinations greater than 55 degrees, the Lidov-Kozai mechanism can cause a significant growth in eccentricity, potentially leading to atmospheric re-entry in approximately 120 years for GEO satellites [5, 33].

In the dynamical model, the gravitational influence of external bodies is modelled as a perturbation in the potential, following the expression (3.11) [31]:

$$U_{body} = -\frac{\mu_{body}}{\sqrt{(x-x_{body})^2 + (y-y_{body})^2 + (z-z_{body})^2}} \quad (3.11)$$

3.2.2 Earth's non-sphericity

Earth's non-sphericity and mass distribution irregularities cause deviations from a purely central gravitational field. Two primary effects, represented as zonal harmonics, are:

1. **Oblateness (J2 effect):** Caused by Earth's equatorial bulge, this perturbation causes the precession of the RAAN and regression of the argument of perigee. Its influence in the potential is given by the following equation (3.12):

$$U_{J_2} = -\frac{\mu}{r} \left(\frac{R_E}{r}\right)^2 J_2 \left(\frac{3}{2} \sin^2(\delta) - \frac{1}{2}\right) \quad (3.12)$$

Here, r is the geocentric distance of the satellite, R_E is Earth's radius, and J_2 is the second zonal harmonic coefficient representing Earth's oblateness. Its typical value is $-0.108 \cdot 10^{-2}$ [31].

2. **Triaxiality (Tesseral Harmonics):** This perturbation arises from Earth's deviation from perfect axisymmetry, known as triaxiality. It introduces periodic variations in the semi-major axis and can lead to long-term kilometric drifts and the formation of stable and unstable equilibrium points. Its contribution to the gravitational potential is given by (3.13):

$$U_{\text{triaxiality}} = -\frac{\mu}{r} \left(\frac{R_E}{r} \right)^2 3 (1 - \sin(\delta)^2) (C_{2,2} \cos(2\lambda) + S_{2,2} \sin(2\lambda)) \quad (3.13)$$

Where $C_{2,2}$ and $S_{2,2}$ are the Tesseral harmonic coefficients associated with Earth's triaxiality, with respective value of $0.157 \cdot 10^{-5}$ and $-0.904 \cdot 10^{-6}$ [31].

3.2.3 Solar Radiation Pressure

Solar radiation pressure results from the momentum transfer of solar photons impacting or reflecting from a satellite's surface. Its magnitude depends on the area-to-mass ratio, and it amplifies eccentricity and inclination oscillations induced by luni-solar perturbations. In some cases, SRP can be exploited for orbital maneuvers to reduce fuel consumption [5, 33].

$$\mathbf{a}_{SRP} = S \frac{A}{m} P \left(\frac{a_s}{r_s} \right)^2 \frac{\mathbf{r}}{|\mathbf{r}|} \quad (3.14)$$

Where s is the reflective parameter, $\frac{A}{m}$ is the cross sectional area to mass ratio, P is the solar pressure, a_s is the distance between the Earth and the Sun, r_s is the distance between the spacecraft and the Sun and r is the position vector.

3.3 Propagator definition

The propagator is the central tool used to compute the motion of satellites and celestial bodies during the encounter. A numerical propagator is chosen due to its flexibility in modelling perturbations and its higher fidelity compared with analytical approaches. A Python script will be developed to simulate the response of satellites under a dynamic model that accounts for the asteroid's perturbation.

3.3.1 Scenario Definition

The propagator study is conducted with the following features:

- **Reference frame:** International Celestial Reference Frame (ICRF).
- **Celestial bodies ephemerides:** Extracted directly from NASA's JPL Horizons service [4].
- **Satellites TLR:** Extracted from space-track.org historical TLEs.

- **Orbits of interest:** High-altitude orbits are prioritised, while LEO satellites are discarded due to negligible expected influence.
- **Timespan of the study:** 6 days, 3 before close-approach and 3 after.
- **Maximum timespan of integration:** 100 seconds.
- **Solar Radiation Pressure parameters:** The coefficients used are in Table 3.2.

Table 3.2: Parameters used for Solar Radiation Pressure modeling.

Parameter	Value
Solar Influx (P_{SRP})	$4.57 \cdot 10^{-3} \text{ km/s}^2$
Reflectivity Coefficient (C_R)	1.8
Area-to-Mass Ratio (A/m)	Depends on the satellite

3.3.2 Dynamic model

While the position of the asteroid is obtained by the Horizons System in the ICRF frame, satellite positions are derived from space-track.org TLEs. The latter can be easily converted into Keplerian Elements, which can then be derived to the Earth-Centered Inertial (ECI) system of reference. The transformation between these reference systems is described in Section 3.1.1.

The equation of motion includes Earth's gravity and the following perturbations, described in the previous Section 3.2:

- Gravitational Perturbations of the Moon and the Sun, using Equation (3.11).
- Non-Spherical Earth (J_2 and triaxiality terms), Equations (3.12) and (3.13).
- Solar Radiation Pressure (Equation (3.14)).
- Gravitational Perturbation of Apophis, which is modelled as the Moon and the Sun, using Equation (3.11).

The general form of the perturbed equation of motion is shown in Equation (3.15).

Special consideration is required when evaluating the non-spherical Earth perturbations. As shown in equations (3.12) and (3.13), both J_2 and triaxiality effects are formulated in terms of geodetic coordinates, which are expressed in the Earth-Centered Earth-Fixed (ECEF) frame. However, since the numerical integrator operates in the ICRF, a transformation of coordinates, described in Section 3.1.1 is required.

$$\ddot{\mathbf{r}} = -\nabla U_{unperturbed} - \nabla U_{moon} - \nabla U_{sun} - \nabla U_{J_2} - \nabla U_{triaxiality} + \mathbf{a}_{SRP} - \nabla U_{Apophis} \quad (3.15)$$

3.3.3 Numerical integration

Given that the initial study focused on GEO satellites, and following the approach described in Section 2.3.3, a high-order Runge-Kutta integrator is used. The `solve_ivp` function in Python [34] provides two suitable explicit methods: RK45 (Runge-Kutta of order 5) and DOP853 (Runge-Kutta of order 8).

Both integrators are assessed in terms of computational efficiency (execution time and number of integration steps) and step-size behaviour (mean and standard deviation of the timesteps). Larger timesteps can lead to reduced accuracy during close encounters, making this analysis critical. Figures 3.4 and 3.5 summarise the results for RK45 and DOP853, respectively.

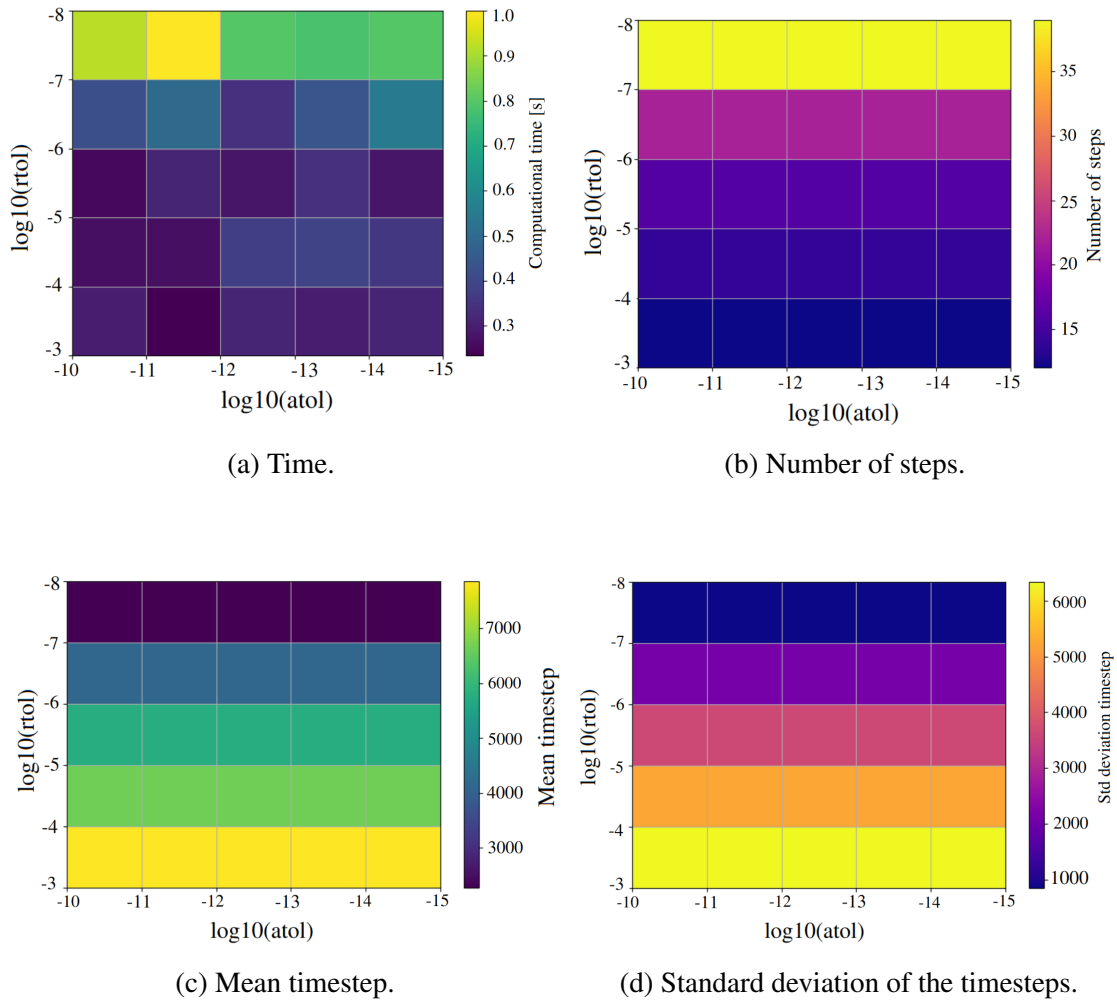


Figure 3.4: Study on the response of RK45 integrator.

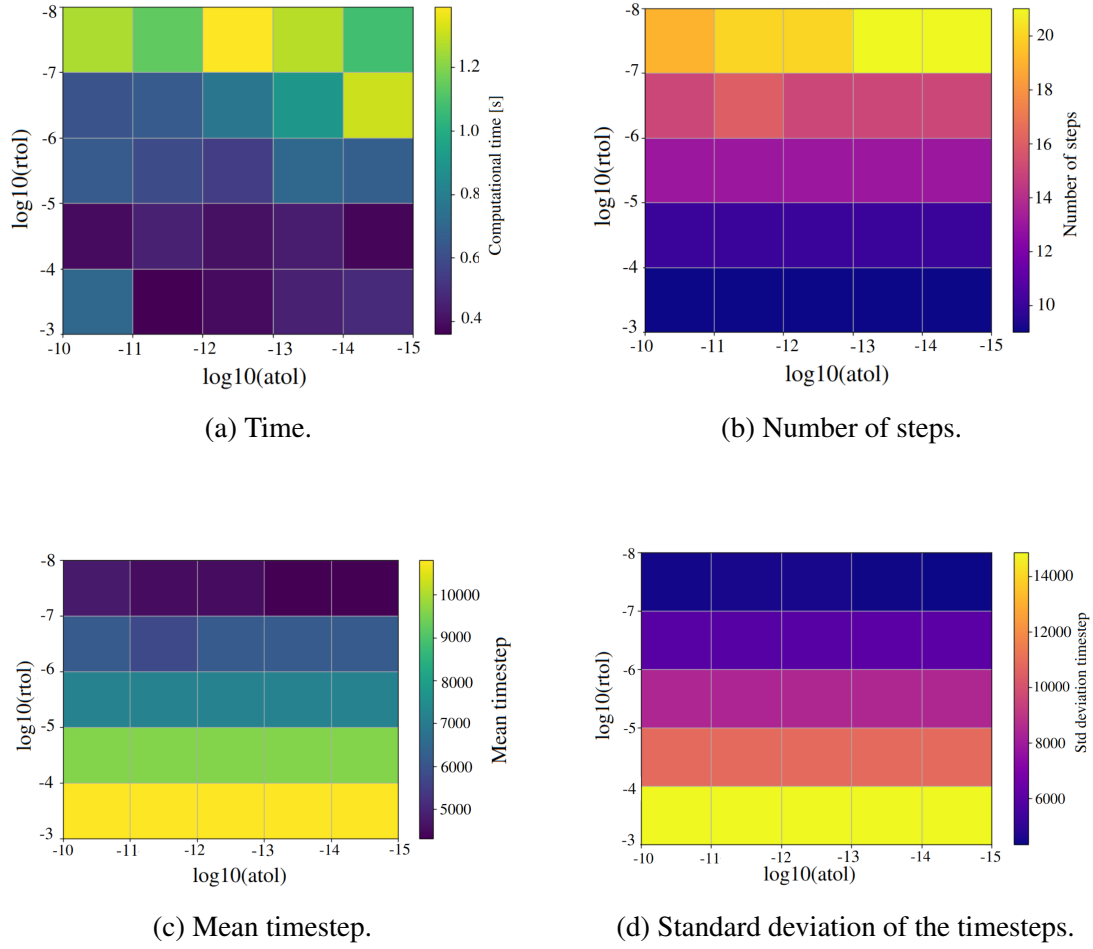


Figure 3.5: Study on the response of DOPRI87 integrator.

RK45 proves to be faster than DOP853, while DOP853 requires fewer steps to complete the same propagation interval, which reflects its larger adaptive timesteps. Nevertheless, since the two methods differ in order, a comparison of their accuracy should also be evaluated. For this purpose, the Moon's orbit is propagated and compared against high-fidelity ephemerides from NASA JPL's Horizons system. Figures 3.6 and 3.7 show the normalised position error for both methods, using Horizons initial states as reference.

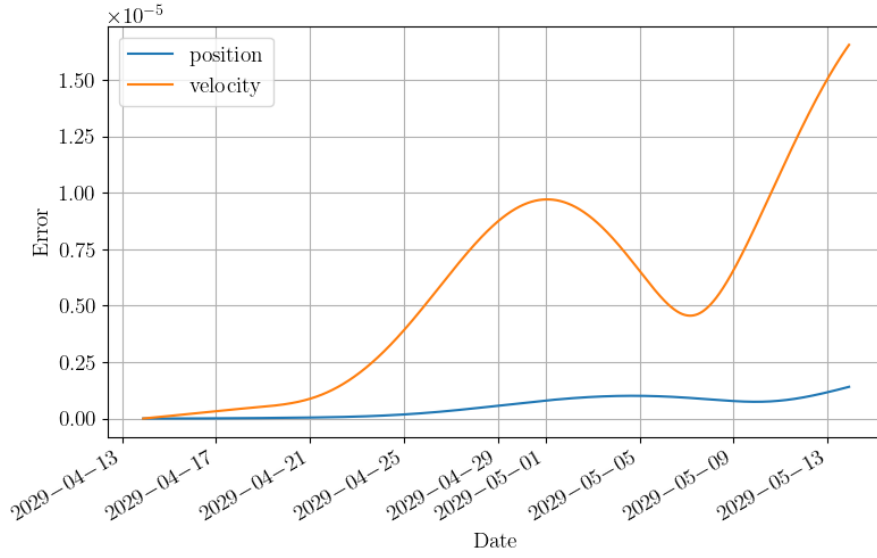


Figure 3.6: Error using method RK45.

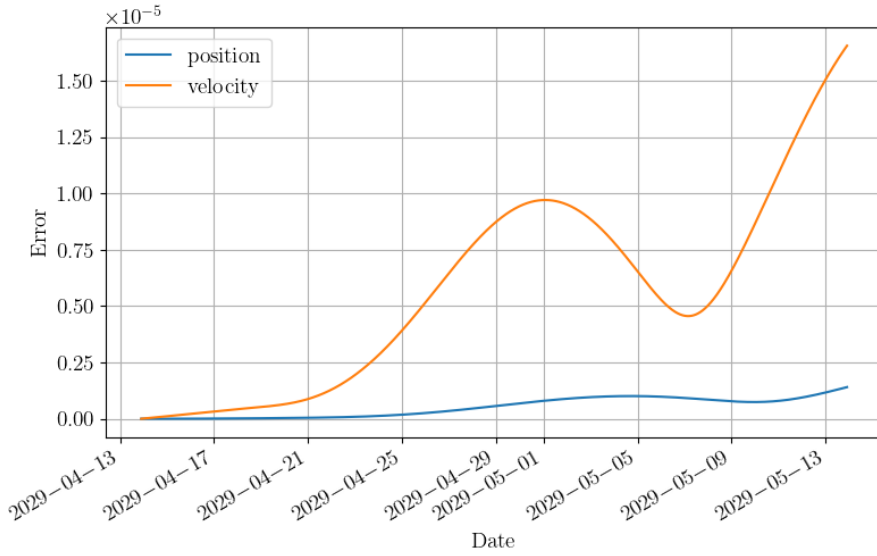


Figure 3.7: Error using method DOP853.

Both integrators produced comparable accuracy. As expected, velocity errors are larger than position errors, since velocity results from the first integration of acceleration and is therefore more sensitive to variations in the perturbative forces. Moreover, the difference in magnitude between states leads to a relative reduction of position errors after normalisation. Accumulation of error over time is also observed, though within acceptable levels for the purpose of this project.

Given that both methods achieve similar accuracy, the selection criterion is based on computational efficiency. RK45 is therefore adopted as the integration method for the propagator, with the tolerance settings listed in Table 3.3.

Table 3.3: Tolerance settings used in the propagation.

Tolerance Type	Value
Relative Tolerance	1×10^{-5}
Absolute Tolerance	1×10^{-10}

3.3.4 Sensibility analysis

Integration accuracy does not only depend on the chosen tolerance, other factors, such as the maximum allowed timestep or the precision of the initial state, also influence the results. This section outlines a series of sensibility analysis conducted on the GEO satellite Intelsat 603, whose parameters are given in Tables 4.4. The ground track drift in longitude and latitude, induced by the asteroid, is used as the main comparative metric.

The first case investigates the influence of Apophis over a symmetric interval around the time of closest approach (t_{c0}), from three days before to three days after, as shown in Table 3.4.

Table 3.4: Intelsat 603 simulation times.

Instant	Date
t_0	10 April 2029 at 21:46
t_f	16 April 2029 at 21:46

With an initial maximum timestep of 1000 seconds, some drift appears in both latitude and longitude (Figure 3.8). However, given its small magnitude, this deviation is likely due to numerical or computational errors. To test this, the integration was refined to a maximum timestep of 300 seconds. Under this condition the deviation disappears, and the asteroid's effect on Intelsat 603 becomes negligible in both latitude and longitude (Figures 3.9).

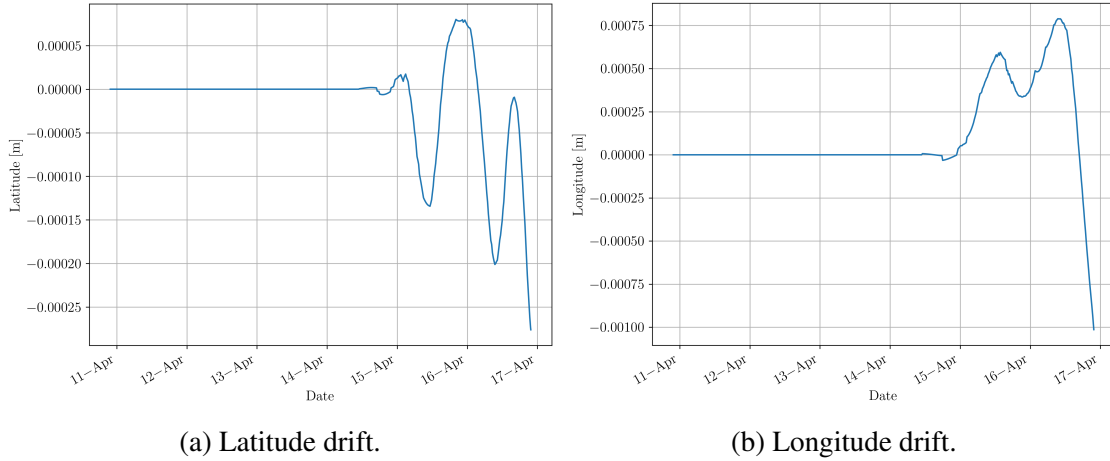


Figure 3.8: Ground Track drift of Intelsat 603 with 1000 seconds as timestep.

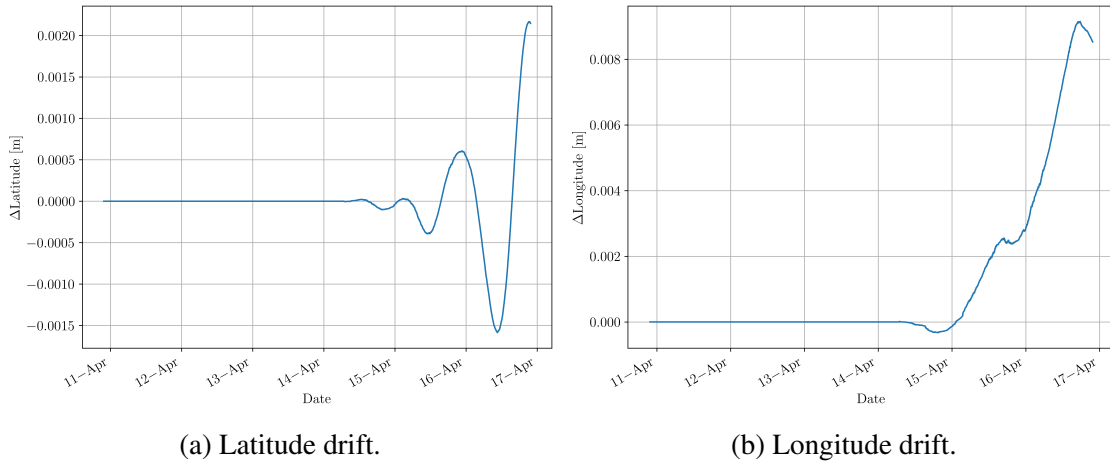


Figure 3.9: Ground Track drift of Intelsat 603 with 300 seconds as timestep.

To confirm the trend, a larger maximum timestep of 1300 seconds was also tested. As expected, increasing the timestep produces a larger drift, showing that the maximum integration step strongly affects the results by introducing numerical errors. The maximum timestep is set to 100 seconds to avoid such errors.

Not only does the maximum integration timestep play a role, but also does the precision of the initial state. During the propagator development, it was observed that reducing the number of significant digits (for instance, storing values in 32-bit instead of 64-bit) increased the apparent drift attributed to Apophis. Re-running the 300-second timestep case with 32-bit precision produced the deviations shown in Figure 3.10. Hence, all subsequent analyses are performed using 64-bit precision to minimise round-off errors.

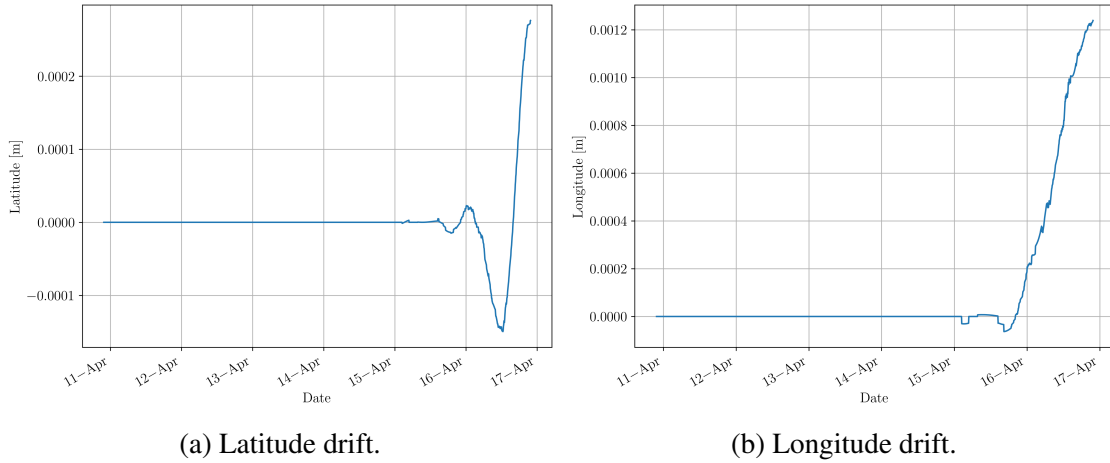


Figure 3.10: Ground Track drift of Intelsat 603 with 300 seconds as timestep and 32 bytes.

A second scenario considers a longer integration span. Instead of six days, the propagation was extended to seven months, using the dates in Table 4.4. Comparing the seven-month drift (Figure 3.11) with the six-day case (Figure 3.9) shows that the magnitude of the drift grows with time. However, part of this increase may be due to the accumulation of numerical errors, which makes the long-term results less reliable.

Table 3.5: Intelsat 603 simulation times.

Instant	Date
t_0	13 th February 2029 at 21:46
t_f	9 th September 2029 at 21:46

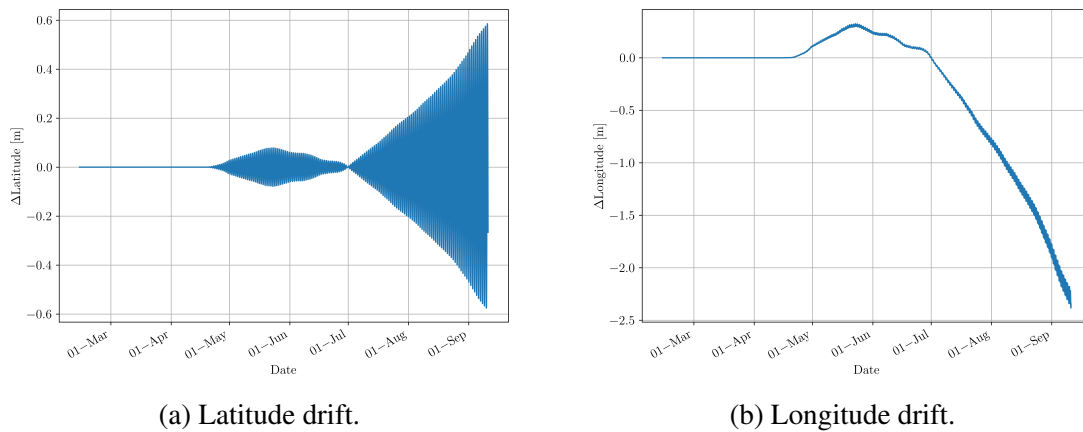


Figure 3.11: Ground Track drift of Intelsat 603 in 7 months.

3.4 Validation and verification of the propagator

Proving that the model is capable of reproducing known orbital behaviours with sufficient accuracy is a fundamental step in the development of the propagator. A reliable validation provides confidence that the numerical methods, dynamical model, and integration schemes are correctly implemented and suitable for the problem under study. Validation is performed in three stages:

- **Qualitative validation:** Verification of the overall trajectory of a GEO satellite after one orbital period.
- **Reference-frame validation:** Comparison of the satellite's ground track generated by the propagator with that obtained from STK.
- **Quantitative validation:** Assessment of the accelerations produced by each perturbation and comparison with reference values from the literature.

In the first stage, the three-dimensional trajectory of Intelsat 603 satellite is propagated for one orbital period. Given the initial conditions, a closed circular orbit is expected and successfully reproduced, as shown in Figure 3.12.

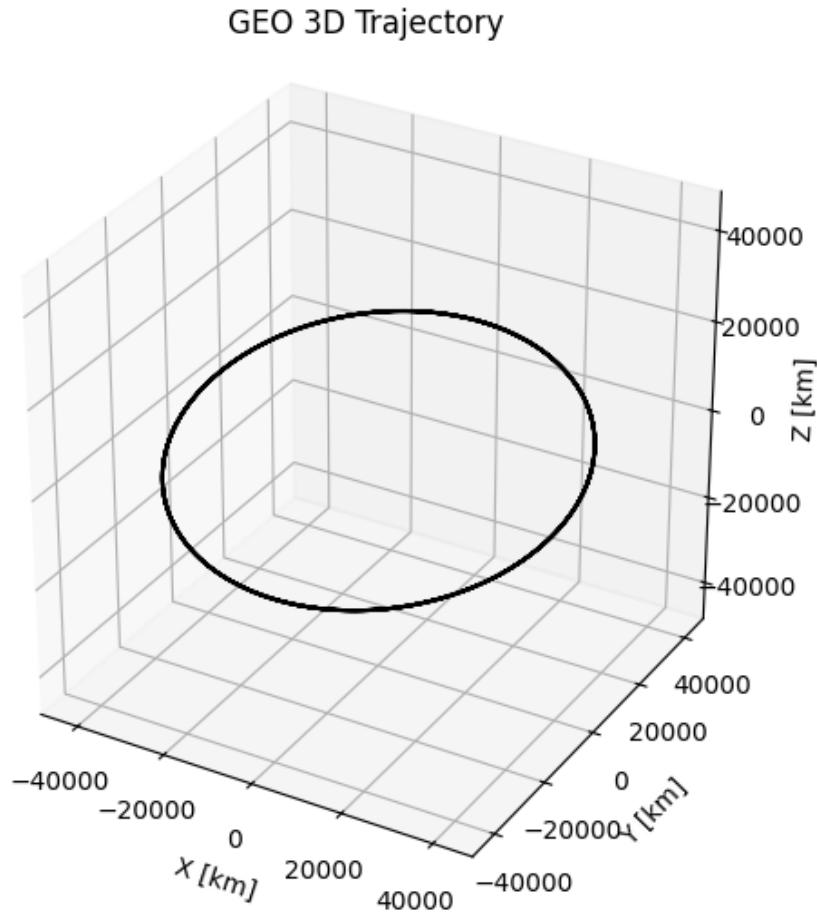


Figure 3.12: Three-dimension trajectory of a GEO satellite.

As for the second stage, the ground track of Intelsat 603 is propagated and compared with the output from STK over a time span of three days before to three days after the close approach. Figures 3.13 and 3.14 illustrate the close agreement between the two results.

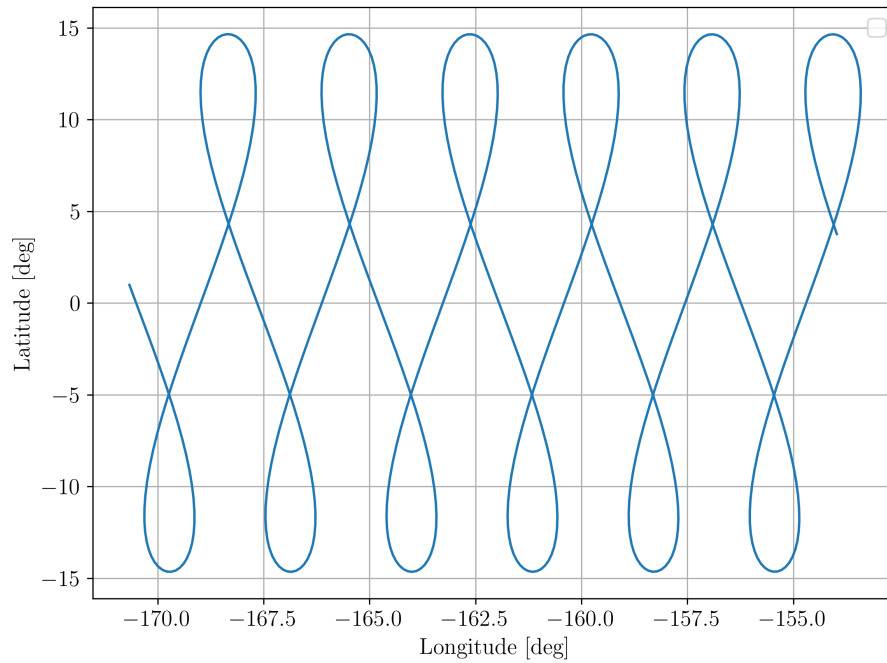


Figure 3.13: Propagator ground track of satellite Intelsat 603.



Figure 3.14: STK Ground track of satellite Intelsat 603.

Finally, a quantitative validation is performed by evaluating the perturbation accelerations acting on a GEO satellite. The results are compared with expected orders of magnitude reported in the literature [5]. As shown in Figure 3.15, the accelerations obtained from the propagator are consistent with the reference values, confirming the reliability of the implemented force models.

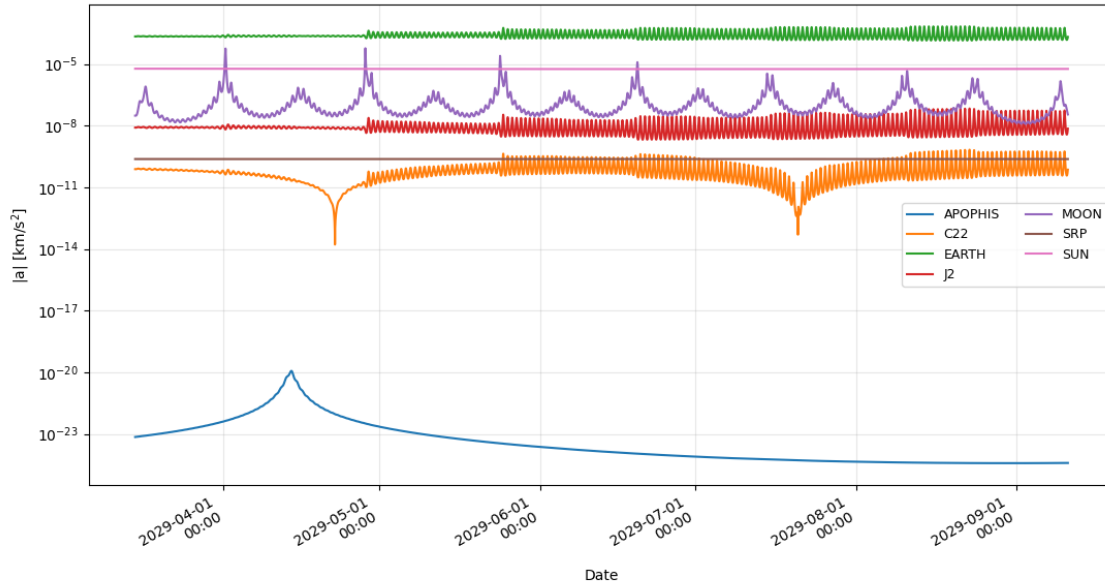


Figure 3.15: Perturbation accelerations for a GEO satellite.

3.5 Definition on the perturbation region

Estimating the orbital region most likely to experience strong gravitational influence from the asteroid is vital for defining a path and avoiding the sampling of random orbital configurations.

The hypothesis is built based on Apophis's geometry during the close approach, including the distance, relative velocity and orientation with respect to Earth-orbiting satellites.

The hypothesised region will be validated through simulation using the propagator developed in the previous Section 3.3. The goal of this step is not only to predict where the highest sensitivity occurs, but also enable the identification of existing or plausible satellites around that region.

Recalling Section 2.4, three general conditions define which is the strongest gravitational influence between the bodies during a close approach:

- Orientation should be the most parallel possible.
- Relative velocity should be the smallest possible.
- Proximity between the bodies, it should be the closest.

Therefore, the most similar the trajectory is, the most perturbed it would be. However, special attention should be taken since Apophis's orbit is hyperbolic, meaning no Earth-orbiting satellite would be capable of having the same trajectory. The goal would then be finding the orbital configuration that resembles as much as possible, always inside the feasible-orbit features, which include:

- The satellite is always located inside Earth's Earth Sphere of Influence.
- The spacecraft's velocity is always smaller than the escape velocity.
- The trajectory has negative energy, meaning it is bounded to Earth.
- The eccentricity of the orbit is between 0 and 1.

3.5.1 Encounter position and velocity

Given that the degree of similarity between a satellite's and Apophis's state vectors largely determines the magnitude of the perturbation, the asteroid's position and velocity vectors at the instant of closest approach are taken as the starting reference. For the position, the satellite location is defined as a displacement from Apophis's position vector, as in Equation (3.16):

$$\mathbf{r}_s = \mathbf{r}_{A,t_{c0}} + d \cdot \hat{\mathbf{u}} \quad (3.16)$$

where $\mathbf{r}_{A,t_{c0}}$ is Apophis's position vector (in the ECI reference frame) at the time of closest approach, $\hat{\mathbf{u}}$ is the unit direction vector of the closed offset and d is the offset distance between the asteroid and the satellite.

For the velocity, the reference is Apophis's velocity vector at closest approach. However, to ensure that the satellite remains gravitationally bound to Earth, its speed must be below the local escape velocity. Therefore, the satellite's velocity is defined as Equation 3.17.

$$\mathbf{v}_s = v_{esc} (1 - \delta) \hat{\mathbf{v}}_{A,t_{c0}} \quad (3.17)$$

where v_{esc} is the escape velocity at the satellite's position, $\hat{\mathbf{v}}_{A,t_{c0}}$ is the unit vector of Apophis's velocity (in the ECI reference frame), and δ is a reduction factor ensuring the velocity magnitude is below escape conditions. This formulation guarantees that the satellite shares the same velocity orientation as the asteroid, while remaining on a bound trajectory around Earth.

3.5.2 Analysis Description

Different offset values are assigned to the displacement distance d . However, in order to simplify the analysis, the unit vectors $\hat{\mathbf{u}}$ are limited to the coordinate axes. The velocity reduction factor δ , in contrast, is varied between 0.01 and 0.4. This produces a set of feasible orbits that could potentially be influenced by Apophis. To identify the most-perturbed orbit and to establish a consistent trend, all of these cases are evaluated in terms of longitude and latitude drift. The tailored orbits with the higher drift in longitude, latitude and in the combination of both are further analysed.

Since the only state known corresponds to the closest approach epoch, the tailored satellites must be propagated backwards until the chosen initial epoch of integration. The resulting state vectors, obtained in the ICRF frame, are then used as initial conditions for the satellite propagation in the whole integration interval. Standard satellite parameters are adopted for the simulations, as summarised in Table 4.4.

Once the most-perturbed orbit is identified, its Keplerian Elements are obtained as described in Section 3.1.1. Knowing the Keplerian Elements, they are compared to the existing historical data of Earth-orbiting satellites to determine the most-perturbed existing satellite.

3.6 Satellite assessment

To facilitate the identification of potentially affected satellites, a dedicated search tool was developed. This tool queries historical TLEs data from both space-track [17] and CelesTrak archives [35]. It is capable of filtering the satellite catalog based on specific orbital parameters, including semimajor axis, inclination, eccentricity and Right Ascension of the Ascending Node (RAAN). This initial screening narrows down the number of candidate satellites whose orbits could be subject to perturbation. To confirm the physical proximity between a satellite and the asteroid, the filtered list is then visualised using STK. This visualisation is crucial for a preliminary assessment, as it determines whether the relative positions of the satellite and the asteroid are close enough for a perturbation to be plausible.

A separate functionality was then developed to propagate the orbits of selected satellites for further analysis. This functionality, a key part of the developed tool, retrieves a satellite's latest TLE data using its unique NORAD ID. It then uses the Python library Skyfield to propagate these TLEs with the SGP4 model. This process accurately calculates the satellite's position and velocity vectors at any specified time. This precise state vector data is essential for quantitatively evaluating the extent to which the asteroid affects a satellite's trajectory, thus forming the basis for the final analysis.

3.7 Results analysis

This section is intended to detail the results that will be obtained from the study. To characterise the asteroid's influence, each propagation is performed twice, with and without the gravitational contribution of Apophis. This subtraction of both solutions isolates the perturbation induced on the satellite's motion. This analysis is carried using two indicators:

- **Ground track drift:** By analysing both longitude and latitude.
- **Keplerian elements perturbations:** Semimajor axis, eccentricity and inclination drift.

While the ground track drift reflect displacements directly observable from Earth and provide a practical measure of orbital stability, the drift in the Keplerian elements quantifies the changes in orbital energy, shape and orientation.

By the combination of these perspectives, the analysis evaluates both short-term effects during the encounter and the long-term cumulative deviations. This framework enables the identification of orbital configurations most vulnerable to asteroid-induced perturbations and ensures consistency with the objectives of this study, namely to assess Apophis's

effect on real satellites and to establish a methodology applicable to future close approaches.

Chapter 4

Apophis influence on GEO satellites

This chapter details the potential gravitational influence of the asteroid Apophis on Earth-orbiting satellites, both active and inactive within the Geostationary Orbit (GEO). The close approach of Apophis to the GEO belt suggest a possible, though limited, perturbative effect on satellites in this region.

To identify the satellites most susceptible to this influence, a search was conducted based on the criteria for strong gravitational interaction, as discussed in Section 2.4. The core principle is that the satellite's trajectory should resemble that of the asteroid as closely as possible. For this reason, Apophis's orbital parameters at its GEO-crossing distance were converted into Keplerian elements to serve as a reference for the search.

The initial target parameters were set as:

$$i = 140^\circ \quad \Omega = 160^\circ$$

However, a review of the GEO satellite catalog revealed that no satellites match these values. Therefore, the complementary configuration was considered, using the inclination and RAAN corresponding to the opposite orbital sense of Apophis, which is:

$$i = 50^\circ \quad \Omega = 347^\circ$$

Within this new configuration, the satellite with NORAD ID 4296 was identified as having the most similar orbit to Apophis's trajectory. Its orbital elements are listed in Table 4.1. Nevertheless, Figure 4.1 indicates that the satellite will remain far from the asteroid during the close approach, making any significant perturbation highly unlikely.

Table 4.1: Satellite 42965 keplerian elements.

a [km]	e	i [°]	Ω [°]
42161.1	0.0749	40.29	347.29

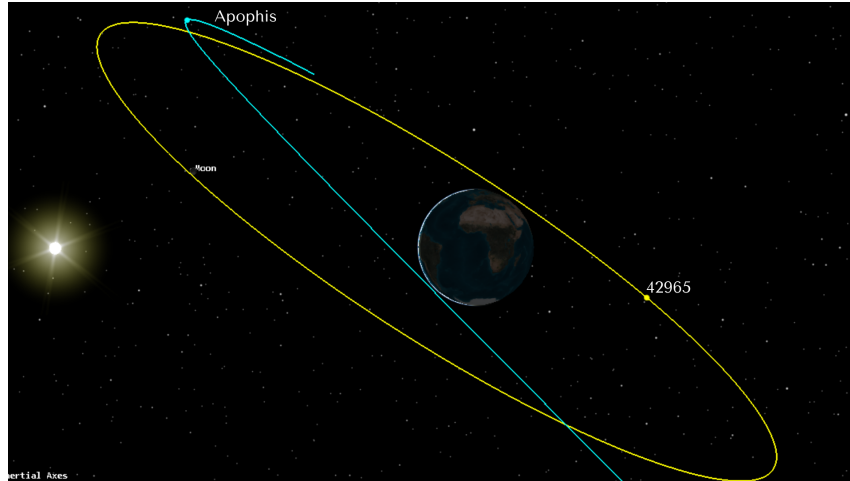


Figure 4.1: Satellite 42965 during Apophis's close approach.

Although no GEO satellites were found to have a strong orbital resemblance to Apophis, TLEs of other objects in the region were filtered by inclination and RAAN to identify the closest possible matches. These candidates were then visually inspected using STK to confirm their proximity to Apophis at the time of encounter. It should be emphasised that each satellite experiences its own specific close approach with the asteroid, which does not necessarily coincide with the moment of minimum Earth-Apophis distance.

The active and inactive GEO objects that most closely approximate Apophis's trajectory are summarised in Table 4.2. The status of each satellite is indicated as A for active and I for inactive. These satellites are further assessed in terms of ground-track deviations and drift in Keplerian elements over the encounter timespan.

Table 4.2: Summary of the GEO satellites analysed.

Name	NORAD ID	Status	a [km]	e	i [°]	Ω [°]
TDRS 3	19548	A	42163.0	0.0036	12.83	342.97
Intelsat 603	20523	I	42387	-	13.80	-
Debris SL-12 R/B	22883	I	42104.5	0.00024	14.0	348.9
IUS R/B (2)	23615	I	42101.2	0.0011	15.87	344.3
LES 9	08747	I	42163.0	0.0025	21.81	40.84
Titan 4 Centaur R/B	22989	I	42071.9	0.0014	16.46	25.74

4.1 TDRS 3

TDRS 3 is an active spacecraft that belongs to the Tracking & Data Relay Satellite system, which provides continuous tracking of Earth-orbiting spacecrafts. The satellite, initially designed as TDRS-C, has the parameters listed in Table 4.3 [6]. The relative geometry of both TDRS 3 and Apophis during their close encounter is illustrated in Figure 4.2.

Table 4.3: TDRS 3 satellite parameters. Source: [6].

NORAD ID	Height [m]	Width [m]	Mass [kg]
19548	17.3	14.2	2268

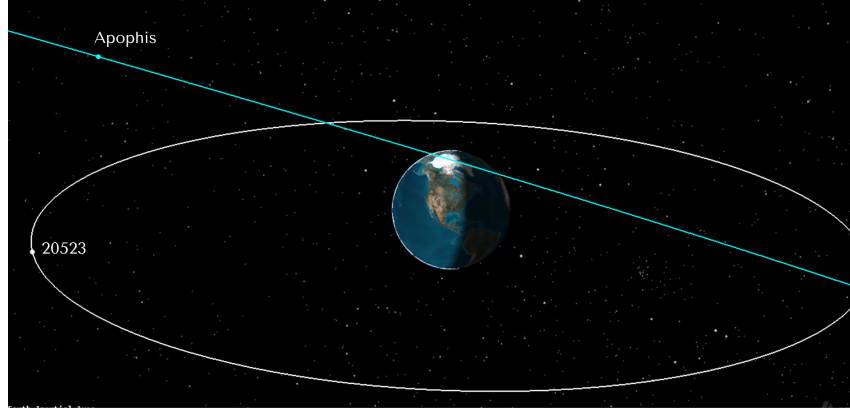


Figure 4.2: Satellite TDRS 3 during Apophis's close approach.

Using the developed propagator, the perturbative effect of Apophis on TDRS 3 was quantified. The resulting ground-track drift over a period of seven months shows that latitude exhibits oscillatory growth, while longitude displays a cumulative drift, reaching several meters over the analysed span, as illustrated in Figure 4.3.

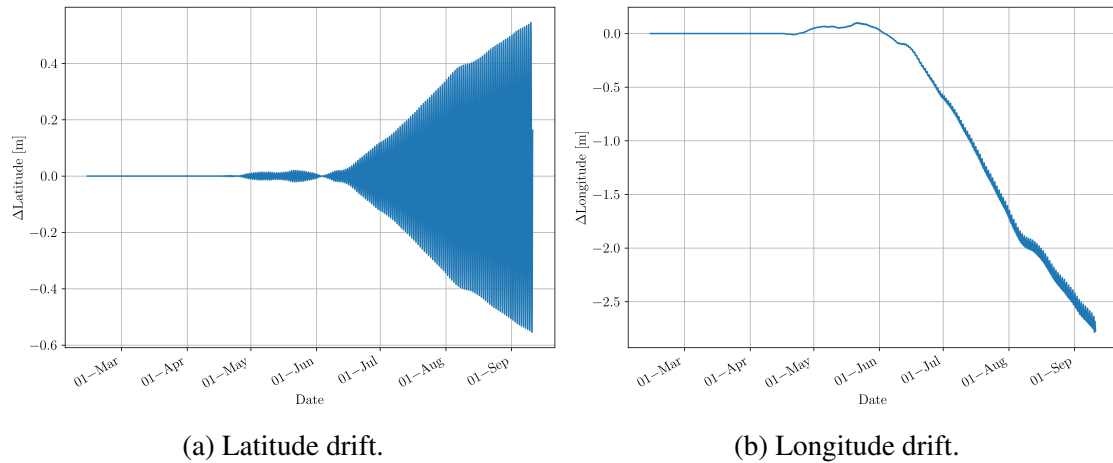
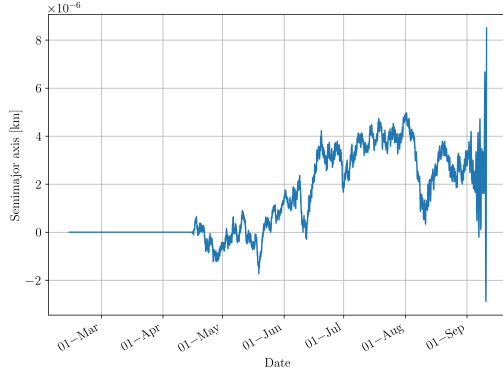


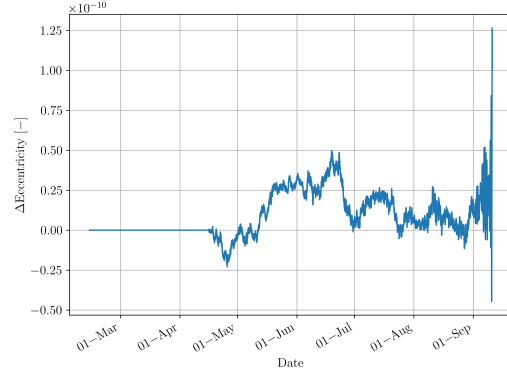
Figure 4.3: Ground Track drift of TDRS 3 in 7 months.

The evolution of the satellite's Keplerian elements during the same period is shown in Figure 4.4. Variations in the semimajor axis and eccentricity remain at the level of 10^{-3}

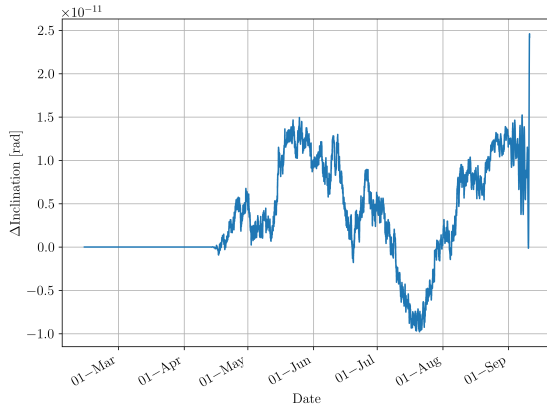
m and 10^{-10} , respectively, indicating negligible changes in orbital energy and shape. Similarly, the perturbations in inclination and RAAN are on the order of 10^{-11} m and 10^{-10} rad, confirming the orbital plane orientation remains unaffected.



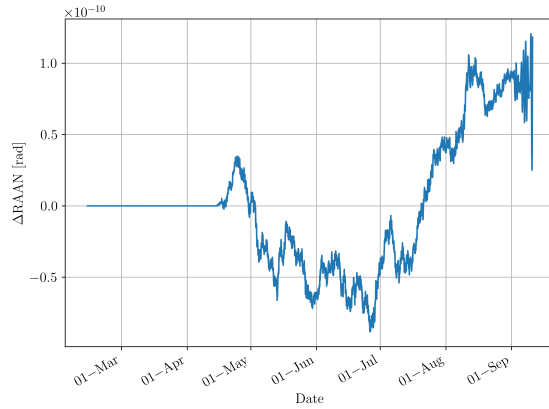
(a) Semimajor axis drift.



(b) Eccentricity drift.



(c) Inclination drift.



(d) RAAN drift.

Figure 4.4: Keplerian elements drift of TDRS 3 in 7 months.

Therefore, although the orbital elements experience only minimal deviations, the satellite's geocentric position may progressively shift due to the accumulation of small differences in the trajectory. Overall, the results indicate that the asteroid's flyby does not significantly alter the fundamental orbital characteristics of TDRS 3, but can induce a measurable positional drift, particularly in longitude, over time. However, given the negligible magnitude of these effects, no operational impact is expected on currently active satellites.

4.2 Intelsat 603

Intelsat 603 is decommissioned communication satellite launched in 1990 and relocated to a graveyard orbit above the GEO belt in 2015. Its main parameters are listed in Table 4.4,

while the relative geometry during Apophis's close approach is shown in Figure 4.5.

Table 4.4: Intelsat 603 satellite parameters. Source: [7].

NORAD ID	Diameter [m]	Height [m]	Mass [kg]
20523	$3.6 \cdot 10^{-3}$	$5.3 \cdot 10^{-3}$	1910

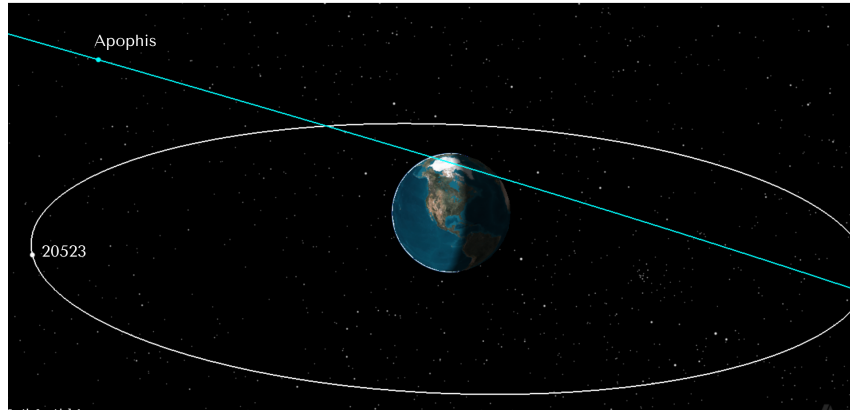
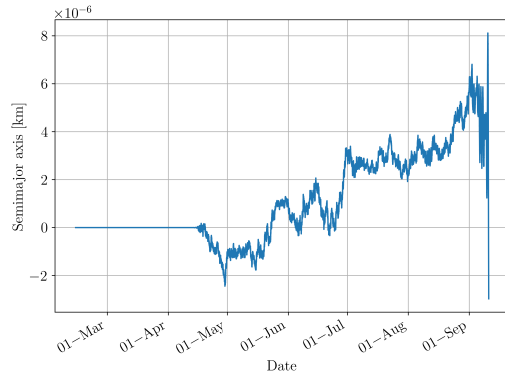


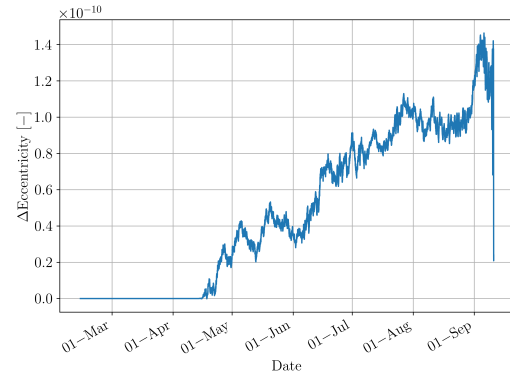
Figure 4.5: Satellite Intelsat 603 during Apophis's close approach.

The ground-track drift of this satellite was used as part of the sensitivity tests of the propagator, with the results illustrated in Figure 3.11. The latitude grows into oscillations that extend to about 0.6 meters, while the longitude exhibits a clear cumulative displacement of nearly 2.5 meters.

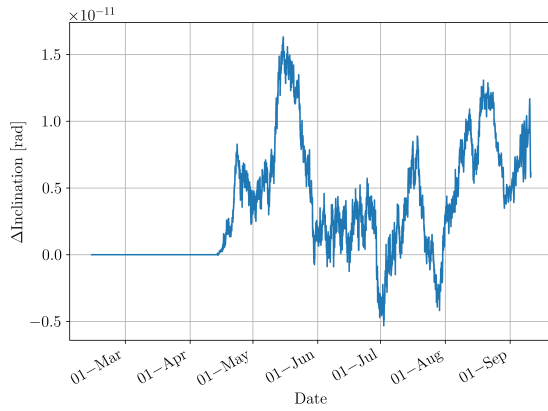
The orbital elements show only minor adjustments in Figure 4.6. The semimajor axis fluctuates around 10^{-3} meters level, with alternating increases and decreases but no destabilising trend. The eccentricity rises steadily, reaching values of order 10^{-10} , while the inclination experiences small-amplitude oscillations around 10^{-11} rad. The RAAN shows a modest upward drift to the same order. Collectively, these variations remain negligible in dynamical terms, leaving the orbit's fundamental geometry and orientation essentially intact.



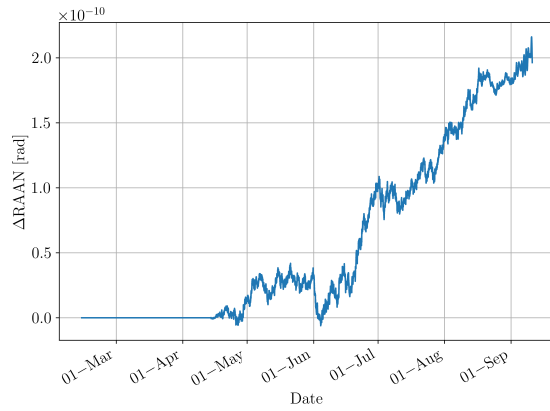
(a) Semimajor axis drift.



(b) Eccentricity drift.



(c) Inclination drift.



(d) RAAN drift.

Figure 4.6: Keplerian elements drift of Intelsat 603 in 7 months.

4.3 Debris SL-12 R/B (2)

This Earth-orbiting object is not an active satellite but the upper stage of a rocket body. It was launched in 1993 by the Russian Strategic Rocket Forces [36]. Its main parameters are listed in Table 4.5, and the geometry of the encounter with Apophis is shown in Figure 4.7.

Table 4.5: SL-12 R/B (2) parameters. Source: [8].

NORAD ID	Diameter [m]	Height [m]	Mass [kg]
22883	3.7	7.1	2560

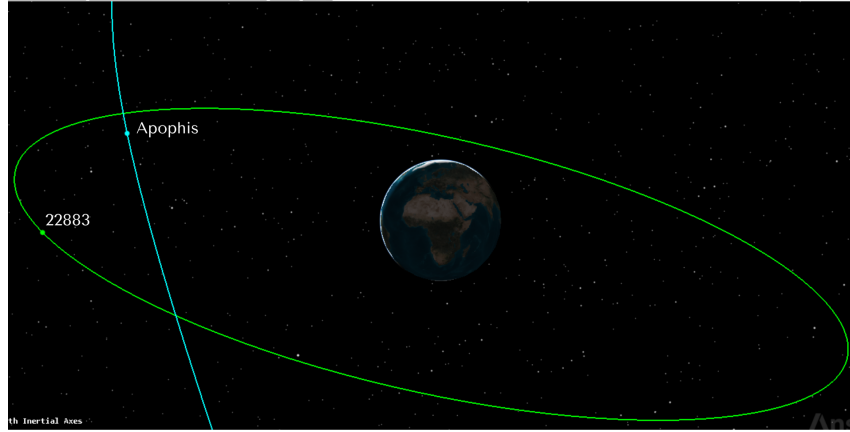


Figure 4.7: Satellite SL-12 R/B (2) during Apophis's close approach.

The perturbative influence of Apophis on SL-12 R/B (2) was assessed over a seven-month period. The ground-track analysis, depicted in Figure 4.8, reveals that the latitude undergoes oscillatory deviations that gradually increase in amplitude, reaching approximately 0.4 meters. The longitude shows a clearer cumulative trend, with a progressive drift exceeding 1.5 meters. These results indicate that the accumulated effect of the asteroid can lead to measurable shift in the satellite's geocentric position, particularly in longitude.

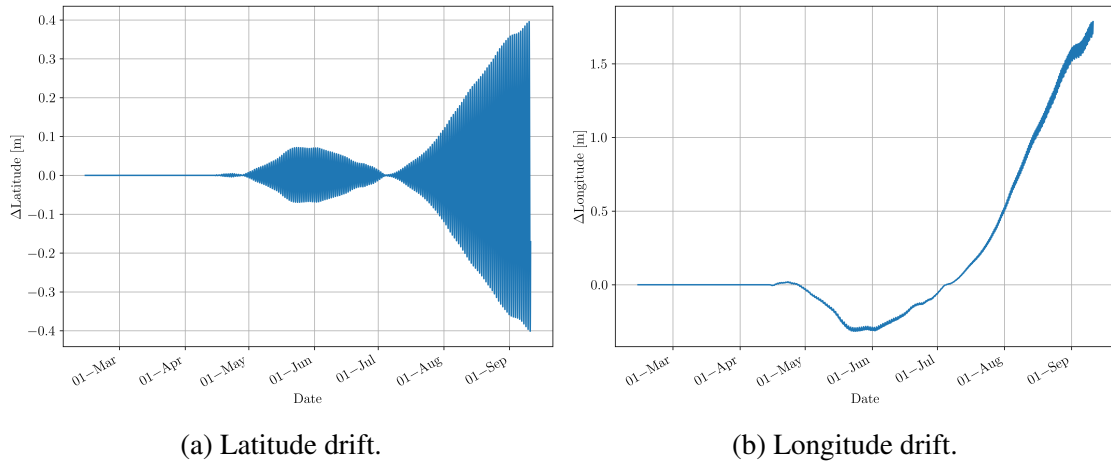
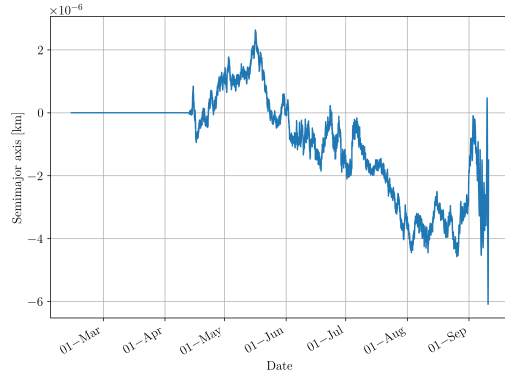
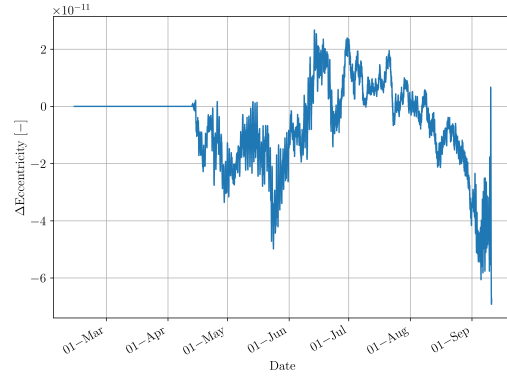


Figure 4.8: Ground Track drift of SL-12 R/B (2) in 7 months.

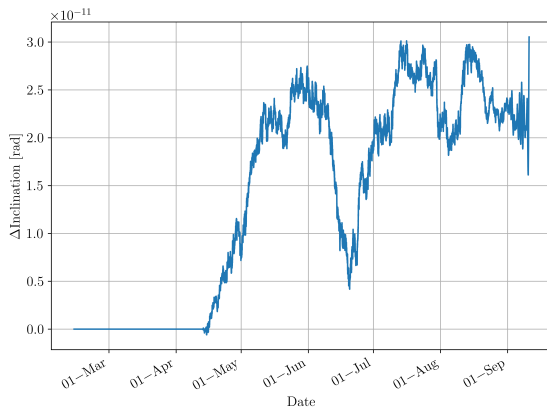
Analysis of the Keplerian elements over the same period, Figure 4.9 confirms the very limited dynamical impact of the encounter. The semimajor axis oscillates at the level of 10^{-3} m, and the eccentricity remains confined to variations of order 10^{-11} , leaving both orbital energy and shape essentially unchanged. The inclination shows a weak upward trend of about 10^{-11} rad, while the RAAN undergoes fluctuations on the order of 10^{-10} rad. These values demonstrate that the orientation of the orbital plane remains practically stable despite the flyby.



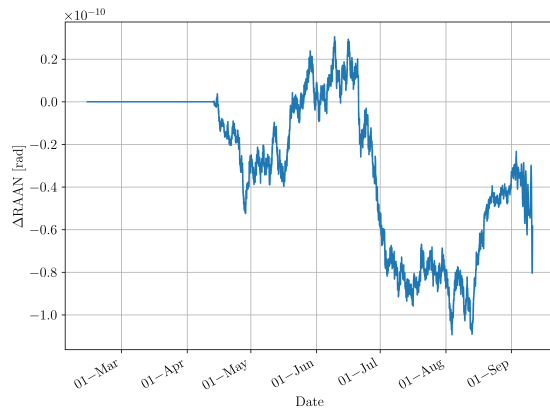
(a) Semimajor axis drift.



(b) Eccentricity drift.



(c) Inclination drift.



(d) RAAN drift.

Figure 4.9: Keplerian elements drift of SL-12 R/B (2) in 7 months.

Apophis, therefore, produces only negligible modifications in the satellite's orbital elements. The most evident outcome is a progressive shift in the ground-track, dominated by longitude drift.

4.4 IUS R/B (2)

IUS R/B (2) is another Earth-orbiting rocket body currently located in the GEO graveyard. It was launched on 1995 by NASA [9]. Since its physical dimensions are not available, the parameters of Intelsat 603 (Table 4.4) were adopted as a reference, as shown in Table 4.6 indicated with *. The geometry of its close encounter with Apophis is presented in Figure 4.10.

Table 4.6: IUS R/B (2) parameters. Source: [9].

NORAD ID	Diameter [m]	Height [m]	Mass [kg]
23615	3.6*	5.3*	1100

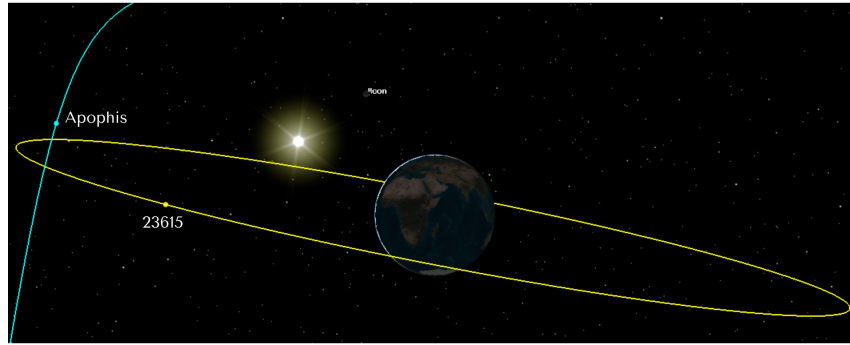


Figure 4.10: Satellite IUS R/B (2) during Apophis's close approach.

The simulations for IUS R/B (2) show that the asteroid's perturbation mainly affects its ground-track (Figure 4.11) rather than its orbital structure. The latitude remains centered around zero but exhibits a progressive widening of oscillations, eventually reaching amplitudes of about 0.15 meters. The longitude displays a smoother drift, marked by alternating periods of increase and decrease, but ultimately accumulates a displacement of roughly 0.6 meters over seven months.

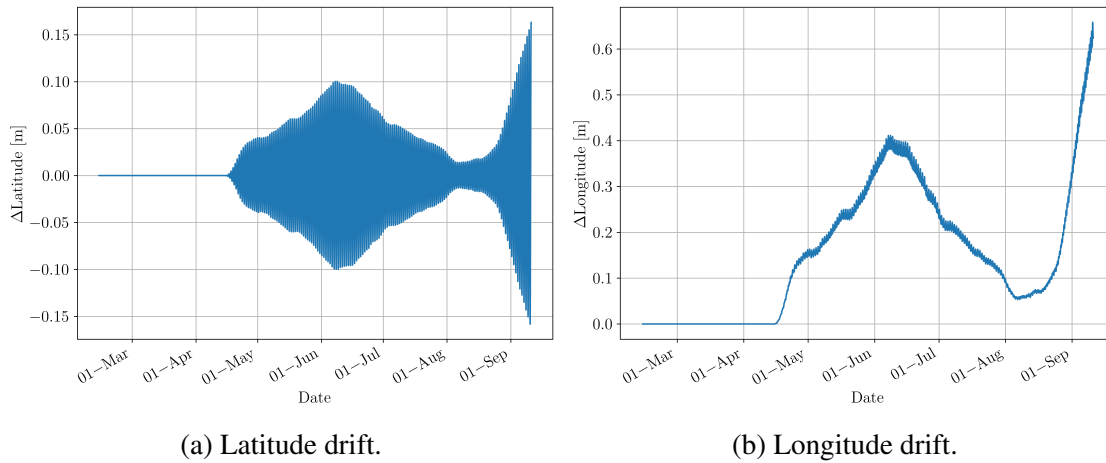


Figure 4.11: Ground Track drift of IUS R/B (2) in 7 months.

With respect to the orbital elements, the variations remain extremely small. The semimajor axis fluctuated within the order of 10^{-3} m, without any persistent long-term trend, while the eccentricity oscillates around values of order 10^{-11} . The inclinations remains close to its initial value, showing only marginal deviations of 10^{-11} rad, whereas the RAAN experiences more noticeable oscillations at the 10^{-10} rad level. Taken together, these results confirm that the fundamental orbital configuration of the satellite is unaffected.

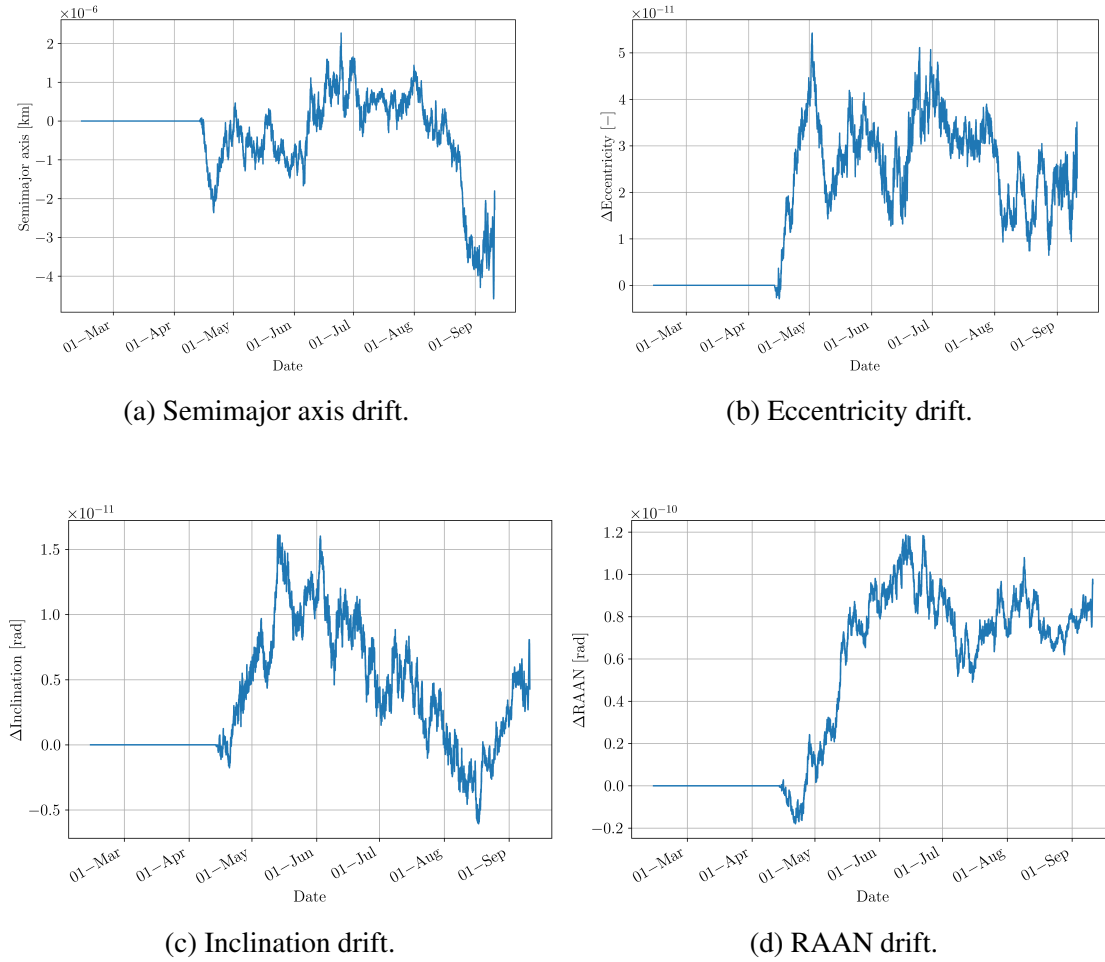


Figure 4.12: Keplerian elements drift of IUS R/B (2) in 7 months.

4.5 LES 9

LES 9 (Lincoln Experimental Satellite 9) was a geostationary communication satellite launched in 1976 and decommissioned in 2020. For the purposes of the simulation, the parameters of Intelsat 603 were adopted whenever specific values were unable (indicated with *). The geometry of its closest approach to Apophis is shown in Figure 4.13.

Table 4.7: LES 9. Source: [10].

NORAD ID	Diameter [m]	Height [m]	Mass [kg]
08747	3.6*	5.3*	454

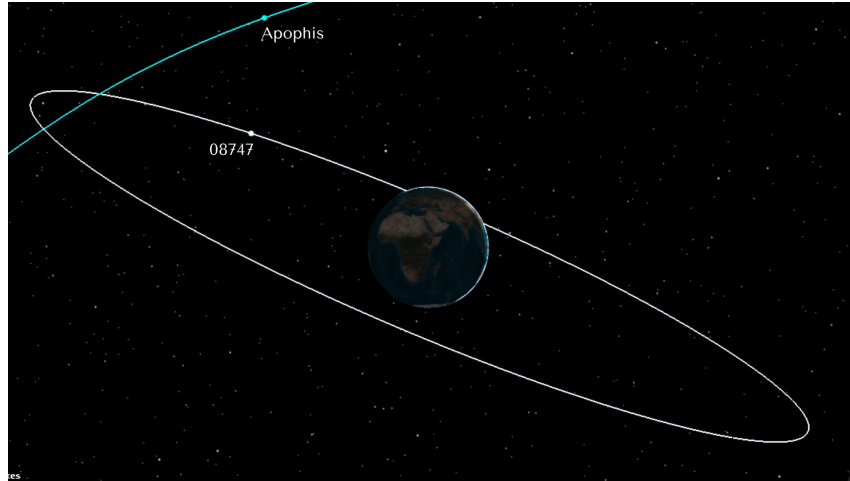


Figure 4.13: Satellite LES 9 during Apophis's close approach.

The ground-track evolution of LES 9, Figure 4.14 reveals the strongest perturbative signature amongst the analysed cases. Latitude variations grow steadily in amplitude, ultimately surpassing 1 m, while longitude exhibits a pronounced cumulative drift of more than 2.5 meters. These results highlight that the most visible consequence of Apophis's passage is a progressive displacement in the satellite's apparent geocentric position, with longitude being particularly sensitive.

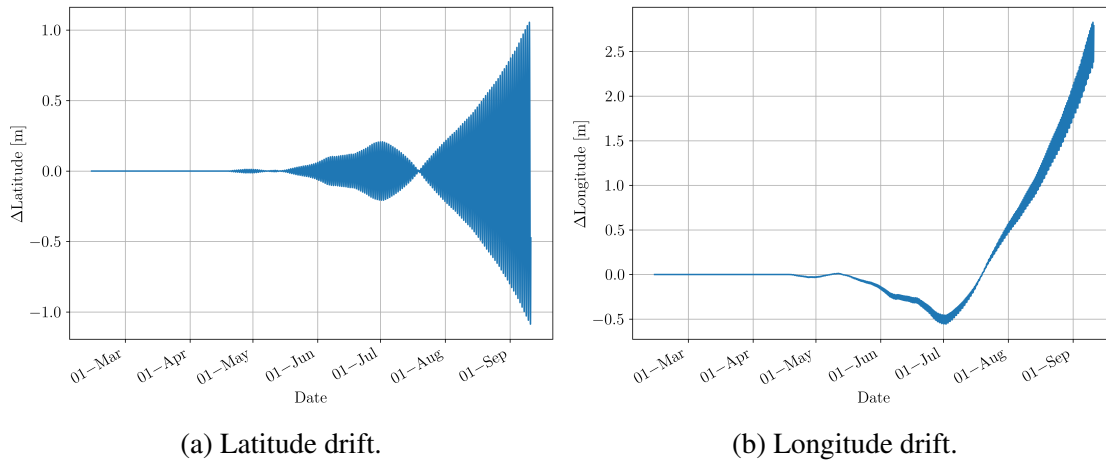
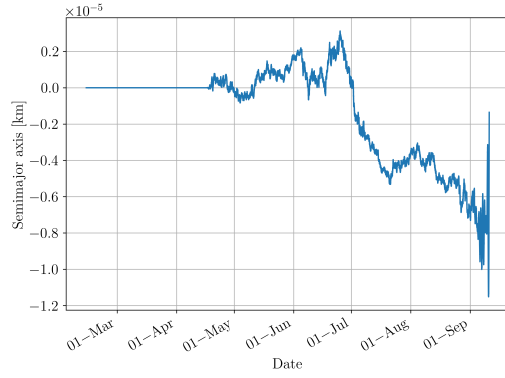
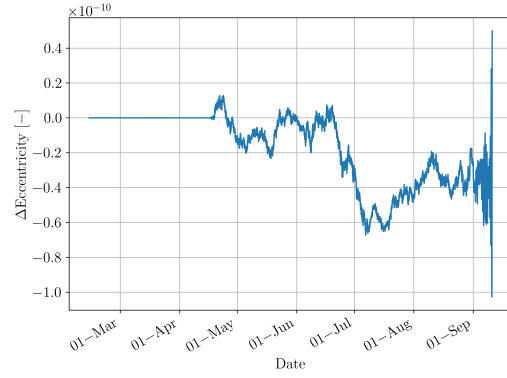


Figure 4.14: Ground Track drift of LES 9 in 7 months.

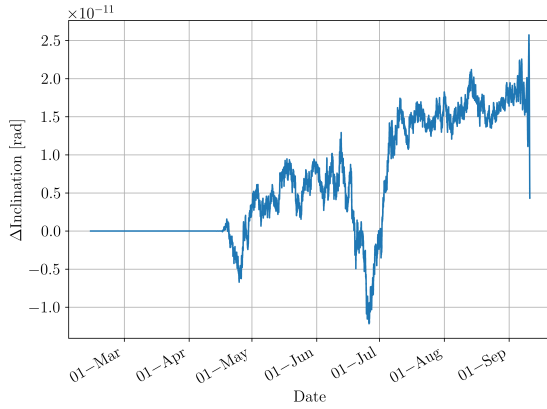
Keplerian elements, Figure 4.15, remain only marginally affected. The semimajor axis shows fluctuations of order 10^{-2} m, indicating negligible impact on orbital energy, while the eccentricity oscillates within 10^{-10} . The inclinations registers small excursions of the order of 10^{-11} rad, and the RAAN undergoes oscillations of comparable magnitude. Despite these variations, the orbital shape and orientation remain essentially preserved.



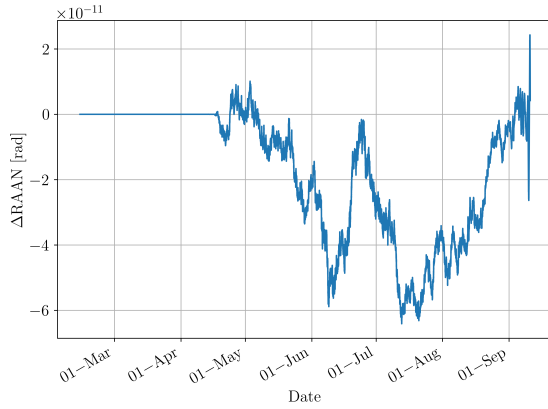
(a) Semimajor axis drift.



(b) Eccentricity drift.



(c) Inclination drift.



(d) RAAN drift.

Figure 4.15: Keplerian elements drift of LES 9 in 7 months.

In summary, this satellite demonstrates that, although Apophis's flyby does not induce significant modifications in the classical orbital elements, it can still lead to measurable drifts in ground-track position.

4.6 Titan 4 Centaur R/B

Titan 4 Centaur R/B is the upper stage of a launch vehicle that remains in Earth orbit. It was launched in 1994 by the United States Space Force [11]. Its propagation parameters are listed in Table 4.8, with the missing values adopted from Intelsat 603 (indicated with *).

It should be noted that Titan 4 Centaur R/B does not strictly meet the Keplerian filter criteria. However, it was included in the analysis due to its close proximity to Apophis during the encounter, as illustrated in Figure 4.16.

Table 4.8: Titan 4 Centaur R/B. Source: [11].

NORAD ID	Diameter [m]	Height [m]	Mass [kg]
22989	4.5	5.3*	2775

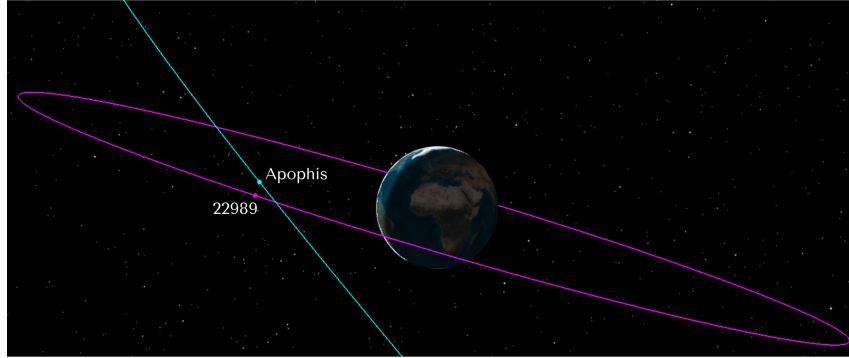


Figure 4.16: Satellite Titan 4 Centaur R/B during Apophis's close approach.

For Titan 4 Centaur R/B, the simulated ground-track perturbations shown in Figure 4.17 show a gradual accumulation of displacement during the seven-month integration. The latitude grows into oscillations of increasing amplitude, reaching around 0.6 meters. Longitude, in turn, experiences a more monotonic drift, after an initial rise the displacement drops until 2 meters.

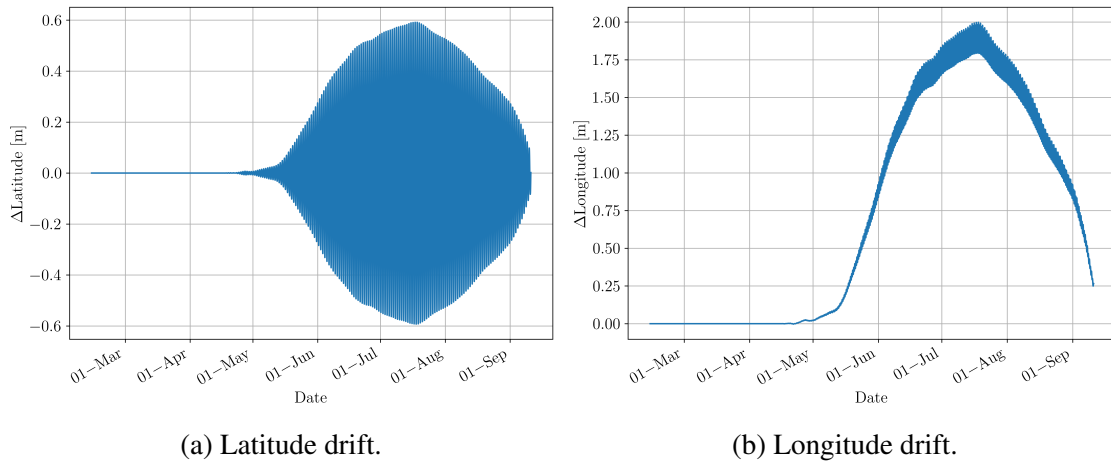
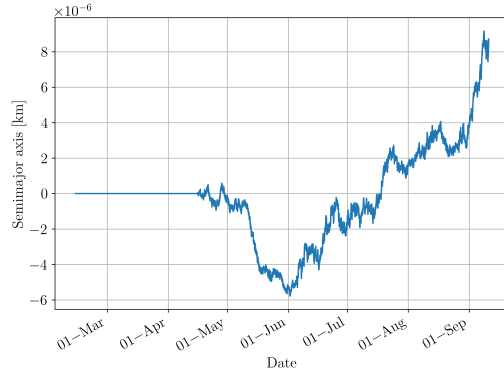


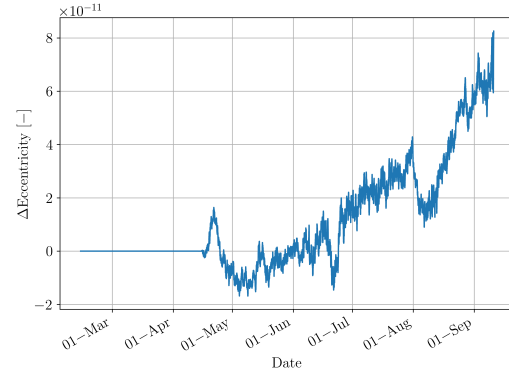
Figure 4.17: Ground Track drift of Titan 4 Centaur R/B in 7 months.

The orbital elements, Figure 4.18, corroborate the limited dynamical impact. The semimajor axis undergoes fluctuations of order 10^{-3} m, with alternating increases and de-

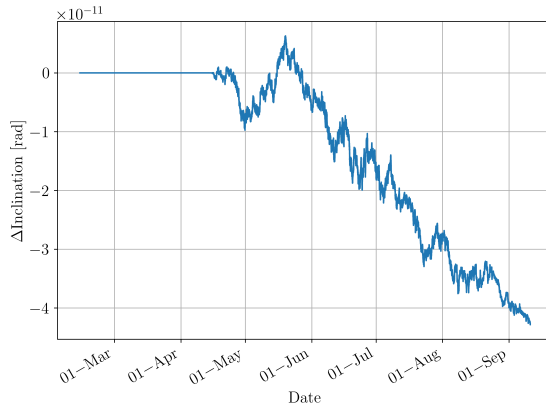
creases that cancel out any long-term divergence. The eccentricity remains at the 10^{-11} scale, while the inclination shows a gradual negative trend of a few 10^{-11} rad. The RAAN oscillates within the same order of magnitude, without sustained drift. Together, this results confirm that the essential geometry and orientation of the orbit remain practically intact.



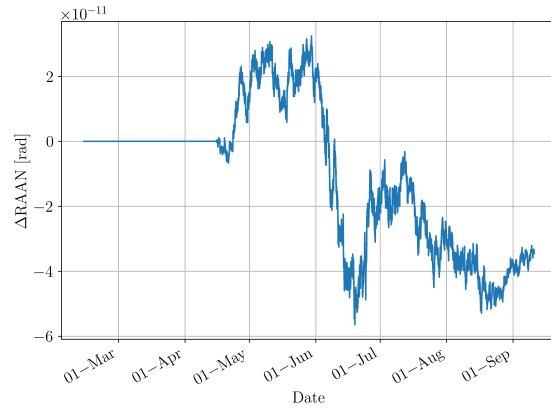
(a) Semimajor axis drift.



(b) Eccentricity drift.



(c) Inclination drift.



(d) RAAN drift.

Figure 4.18: Keplerian elements drift of Titan 4 Centaur R/B in 7 months.

4.7 Overall Assessment

In all the analysed satellites, the perturbation in longitude is consistently greater than in latitude. The largest drift is observed for satellite 08747, whereas the smallest corresponds to satellite 23615. This ordering also matches the behaviour of the semimajor axis, satellites that display stronger ground-track deviations also show larger fluctuations in their semimajor axis. For other Keplerian elements, however, the ranking does not follow the same order. Table 4.9 summarises the absolute drifts of each space object for each parameter. To isolate the close-approach conditions, the simulations were performed over the interval from 13th April at 20:00 UTC to 14th April at 00:46 UTC.

Table 4.9: Absolute maximum drifts for each satellite.

Satellite ID	Δ Latitude [m]	Δ Longitude [m]	Δa [m]	Δe	Δi [rad]	$\Delta \Omega$ [rad]
19548	0.58	2.7	$8.3 \cdot 10^{-3}$	$1.25 \cdot 10^{-10}$	$2.5 \cdot 10^{-11}$	$1.25 \cdot 10^{-10}$
20523	0.6	2.4	$8 \cdot 10^{-3}$	$1.4 \cdot 10^{-10}$	$1.52 \cdot 10^{-11}$	$2.2 \cdot 10^{-10}$
22883	0.4	1.7	$6 \cdot 10^{-3}$	$6 \cdot 10^{-11}$	$3 \cdot 10^{-11}$	$1 \cdot 10^{-10}$
23615	0.15	0.65	$4.3 \cdot 10^{-3}$	$5.2 \cdot 10^{-11}$	$1.6 \cdot 10^{-11}$	$1.2 \cdot 10^{-10}$
08747	1	2.7	$1.15 \cdot 10^{-2}$	$1 \cdot 10^{-10}$	$2.5 \cdot 10^{-11}$	$6 \cdot 10^{-11}$
22989	0.6	2	$9 \cdot 10^{-3}$	$8 \cdot 10^{-11}$	$4 \cdot 10^{-11}$	$5.8 \cdot 10^{-11}$

The ground-track evolution highlights longitude as the most sensitive parameter to Apophis's perturbation, reaching up to a few meters in the most affected cases. Latitude also shows measurable deviations, though at smaller amplitudes, typically below one meter. Eccentricity, inclination and RAAN remain several orders of magnitude smaller, with values on the order of 10^{-10} - 10^{-11} , confirming that orbital shape and orientation are essentially preserved.

To better understand the reasons behind the different perturbation levels, Table 4.10 summarises the conditions of closest distance and velocity similarity between each satellite and Apophis.

- ΔR_{min} corresponds to the minimum distance between the two bodies.
- $\alpha_{V,d}$ is the angle between velocities during the closest approach.
- ΔV_d is the velocity difference at that same instant.
- ΔV_{min} represents the absolute minimum velocity difference reached between two bodies during the analysed interval.
- $\alpha_{V,V}$ is the angle between the velocity vectors at the moment when that minimum velocity difference occurs.
- **First** indicates which condition takes place first: the closest physical distance or the closest velocity.

Table 4.10: GEO approaches.

Satellite ID	ΔR_{min} [km]	$\alpha_{V,d}$ [°]	ΔV_d [km/s]	ΔV_{min} [km/s]	$\alpha_{V,V}$ [°]	ΔR_V [km]	First
19548	18802.3	19.93	10.36	9.024	18.26	77416.61	Velocity
20523	33043.8	14.91	8.59	5.18	10.20	99047.27	Velocity
22883	11492.3	19.39	10.28	8.60	15.44	85156.34	Distance
23615	10028.1	16.55	9.41	6.29	11.00	99684.85	Distance
08747	14967.8	18.39	9.76	7.45	14.85	91220.82	Velocity
22989	912.03	18.49	10.02	7.91	13.95	90606.08	Distance

Chapter 5

Apophis maximal effect

This chapter investigates which orbital configurations around Earth are most strongly perturbed by Apophis during its 2029 flyby. The analysis begins with an examination of the close-approach geometry and the asteroid's dynamical behaviour relative to Earth.

5.1 Understanding the close approach

The encounter is defined using ephemerides from NASA's JPL Horizons system. Apophis will reach its closest approach to Earth on April 13th 2029, at 21:46 UTC, with a minimum distance between the two bodies of 32,000 km, as shown in Figure 5.1.

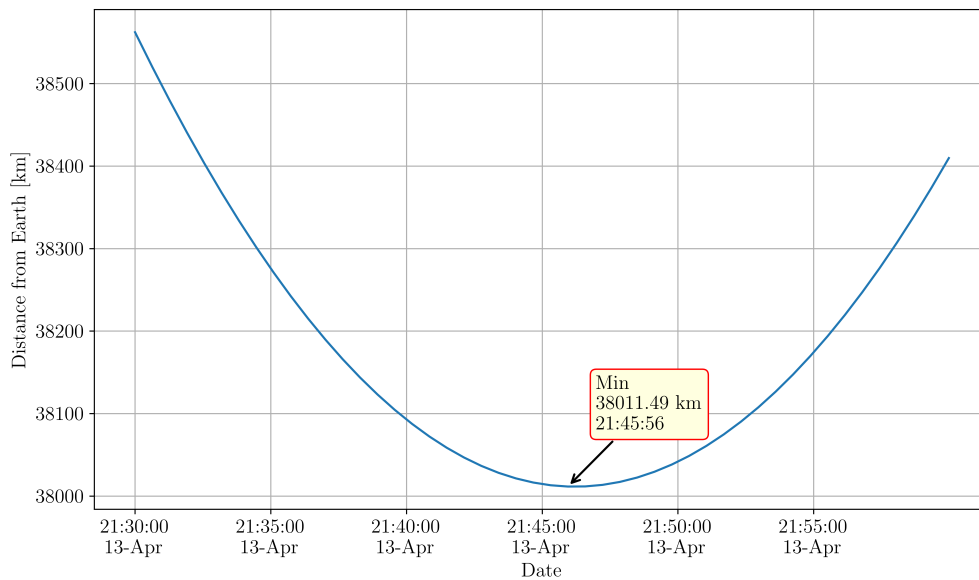


Figure 5.1: Apophis close approach date and distance.

To further describe the encounter, the asteroid's velocity and specific orbital energy relative to Earth are plotted in Figures 5.2 and 5.3. The results confirm that Apophis follows a hyperbolic trajectory, as expected, indicated by its positive energy. This contrasts with

Earth-orbiting satellites, which remain bound on elliptical or circular orbits with negative specific energy. This confirms that Apophis will not be gravitationally captured by Earth, but will instead continue on a hyperbolic escape trajectory.

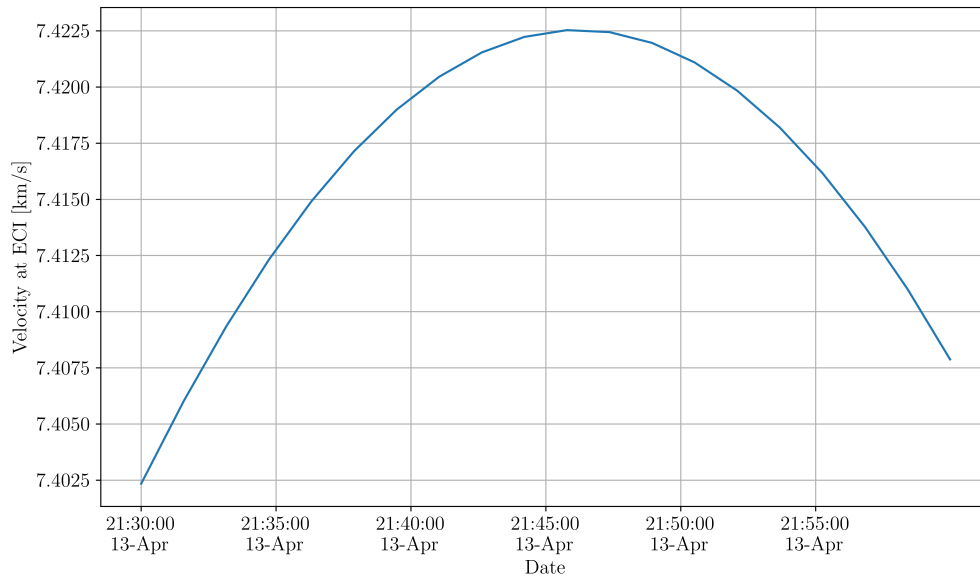


Figure 5.2: Apophis velocity at ECI.

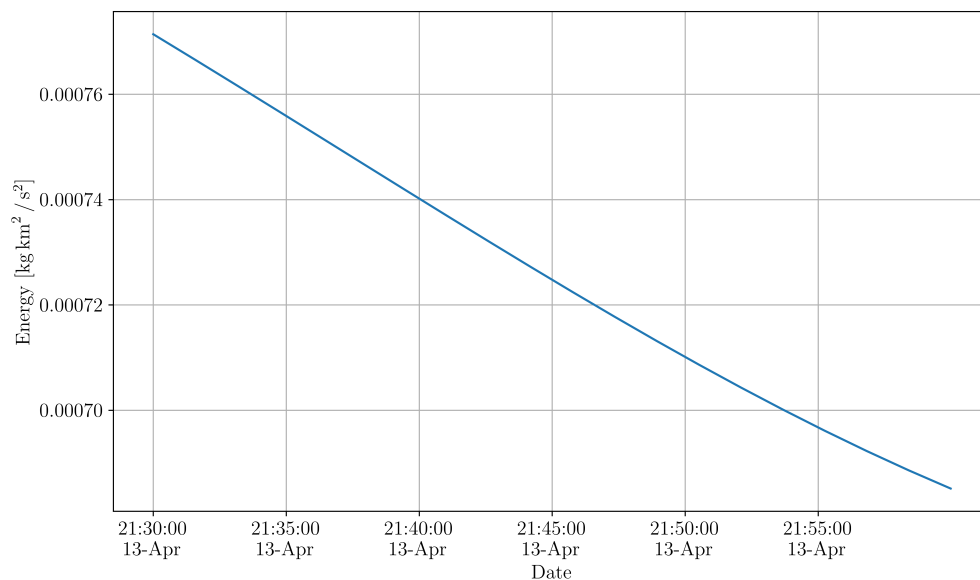


Figure 5.3: Apophis energy evolution during close approach.

Unlike the constant energy expected from a pure two-body problem, the curve in Figure 5.3 shows a slight variation. This is explained by third-body perturbations (considered

in Horizons System ephemeris) and by the fact that the Earth-Centered Inertial (ECI) frame is not strictly inertial due to Earth's acceleration around the Sun. Furthermore, additional deviations arise from numerical errors introduced by coordinate transformations.

The changes observed in velocity are in agreement with energy conservation, since the maximum velocity occurs at closest approach, which is the perigee of the hyperbolic trajectory.

Although no Earth-bounded orbit can exactly reproduce Apophis's hyperbolic motion, the most-perturbed satellite orbit is expected to be one whose position and velocity are closest to those of the asteroid during the encounter. To estimate this, Figure 5.4 illustrates the closest feasible combinations of position and velocity. Apophis's state is represented as a black point above the escape-velocity curve (red line), confirming its unbound trajectory, since only orbits below the red curve correspond to satellites gravitationally bound to Earth. The green and yellow points mark the closest feasible matches in position and velocity, while the purple point represents the configuration that best balances both.

These configurations, located near the escape boundary and dynamically close to Apophis, are expected to be the most sensitive to its gravitational influence. However, this comparison is only preliminary, since it considers only the magnitudes of position and velocity. A more complete assessment requires evaluating the full state vectors, which are addressed in the next Section 5.2.

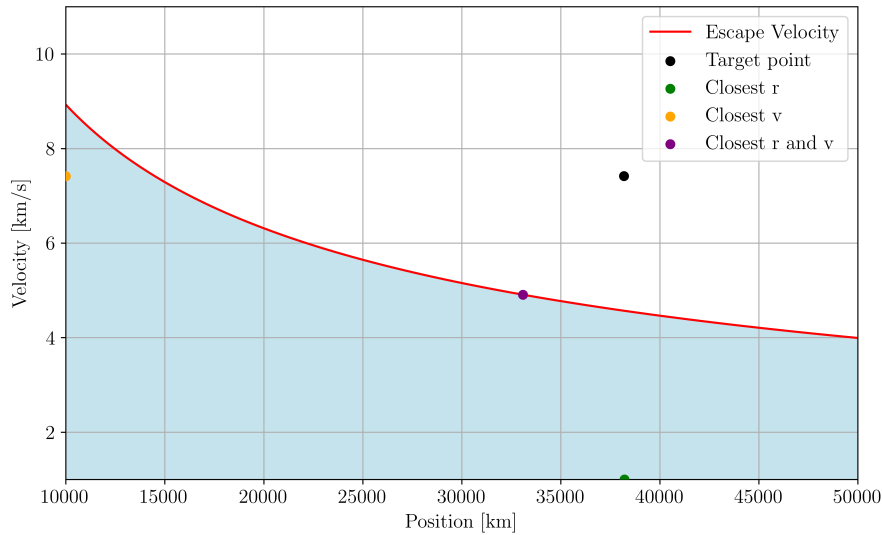


Figure 5.4: Possible position-velocity satellite combinations.

5.2 Possible influenced orbits

Since the dynamical model implemented excludes satellites within Apophis's gravitational domain, the minimum offset distance has to be larger than the asteroid's Hill Radius, which for the 2029 encounter is 0.436 km. Accordingly, the minimum distance offset is of 0.5 km. Regarding the velocity reduction factor (δ), it ranges from 0.01 to 0.4. An example of the orbits studied for one offset is shown in Figure 5.5.

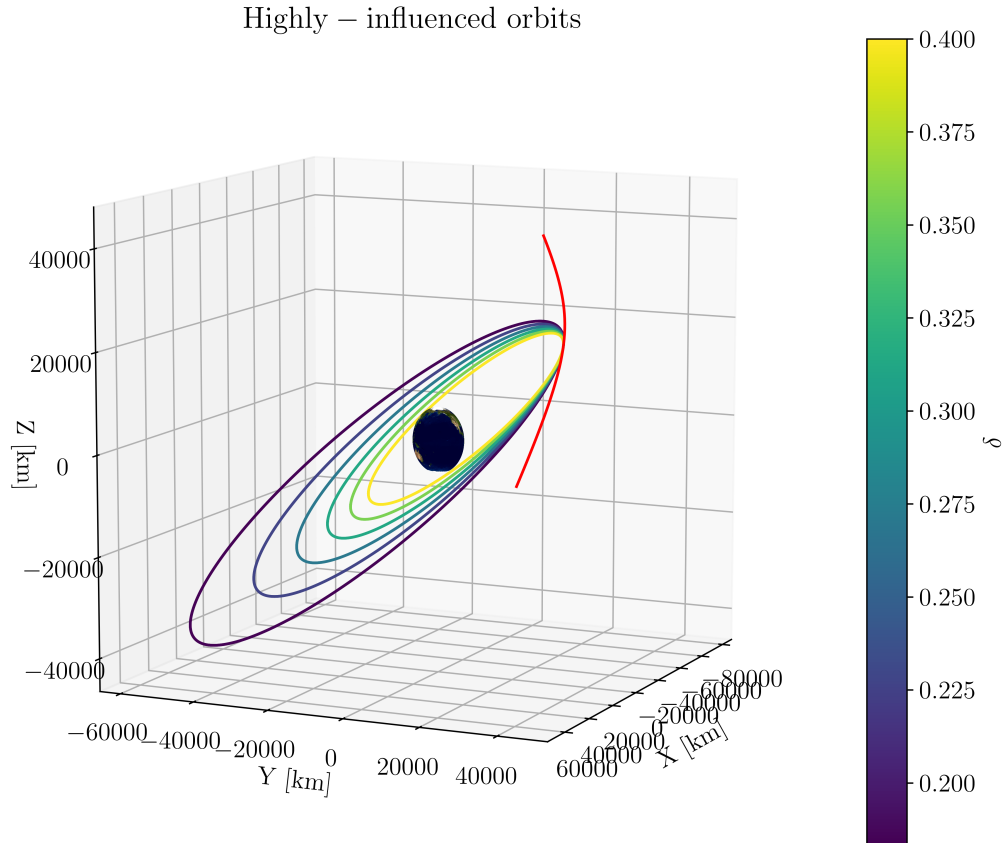


Figure 5.5: 3D representation of possible perturbed orbits with a distance offset 0.5 km.

The orbits are studied in terms of maximum drift produced in longitude, latitude and the combination of both. However, depending on the velocity orientation, two different encounter geometries arise: parallel and antiparallel.

5.2.1 Parallel encounter

In this configuration, the satellite moves approximately parallel to Apophis, prolonging the duration of the close approach. Results indicate that, except for the -0.5 km offset, the largest ground-track drifts occur for offsets applied in the x-direction (or y-direction for the -0.5 km case). As expected, the perturbation magnitude increases as the satellite-asteroid distance decreases. Except for the farthest offset, each case exhibits a single orbit

that dominates the perturbation, producing the maximum drift in longitude, latitude or both.

Table 5.1 summarises the most perturbed orbit for each offset, listing the drift magnitude, energy, semimajor axis and eccentricity.

Table 5.1: Most perturbed orbits as a function of the offset for parallel velocity.

Offset [km]	δ	Maximum	Value [m]	Energy [kg km ² /s ²]	a [km]	e
-10	0.313	Longitude	1.513	-5.541	35967.159	0.0570
-10	0.313	Latitude	0.691	-5.541	35967.159	0.0570
-10	0.313	Both	1.513	-5.541	35967.159	0.0570
-5	0.270	Longitude	4.114	-4.898	40691.461	0.0658
-5	0.270	Latitude	2.017	-4.898	40691.461	0.0658
-5	0.270	Both	4.116	-4.898	40691.461	0.0658
-0.5	0.313	Longitude	14.536	-5.542	35962.044	0.0570
-0.5	0.313	Latitude	6.485	-5.542	35962.044	0.0570
-0.5	0.313	Both	14.536	-5.542	35962.044	0.0570
0.5	0.313	Longitude	31.824	-5.542	35962.143	0.0570
0.5	0.313	Latitude	14.307	-5.542	35962.143	0.0570
0.5	0.313	Both	31.825	-5.542	35962.143	0.0570
5	0.227	Longitude	3.688	-4.215	47280.00	0.196
5	0.227	Latitude	1.799	-4.215	47280.00	0.196
5	0.227	Both	3.704	-4.215	47280.00	0.196
10	0.4	Longitude	0.972	-6.7121	29692.494	0.280
10	0.183	Latitude	0.556	-3.493	57057.136	0.334
10	0.4	Both	0.977	-6.7121	29692.494	0.280

The orbit with the smallest offset (0.5 km) is taken as reference to investigate which orbits are most affected by the asteroid's influence. Figure 5.6 shows the drift Apophis would produce on a satellite placed in this reference orbit. The asteroid's influence is most visible in longitude. After the closest approach, the longitude drift becomes clearly negative, reaching more than 30 metres by April 17th, and showing oscillatory behaviour. Latitude drifts also displays noticeable oscillations, with amplitudes of about 10-15 meters in the days following the flyby. Although these changes are smaller than those in longitude, they confirm that the encounter introduces measurable displacements in both ground-track coordinates.

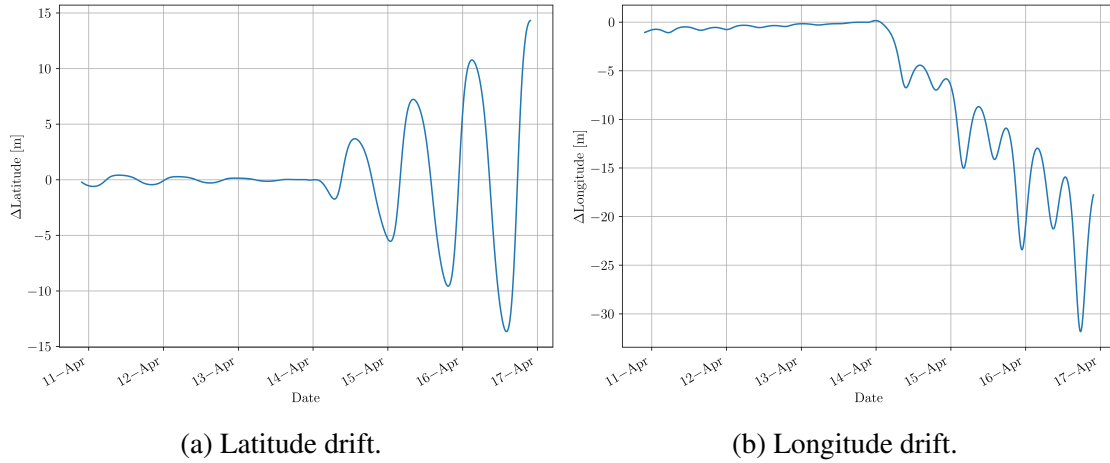
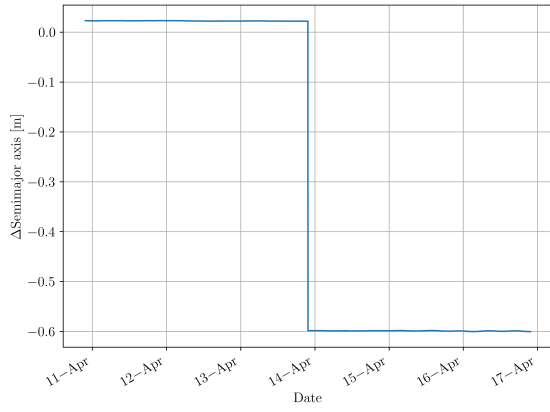
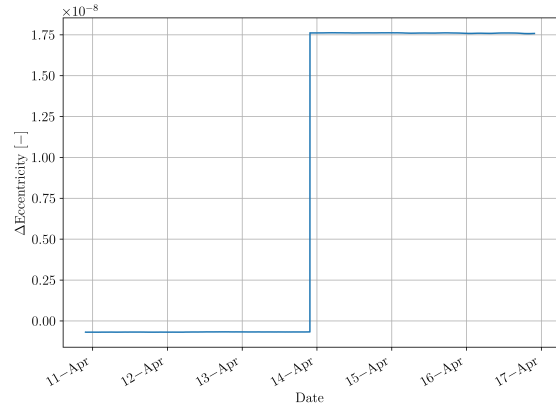


Figure 5.6: Ground-track drift of a satellite orbiting the most parallel perturbed orbit.

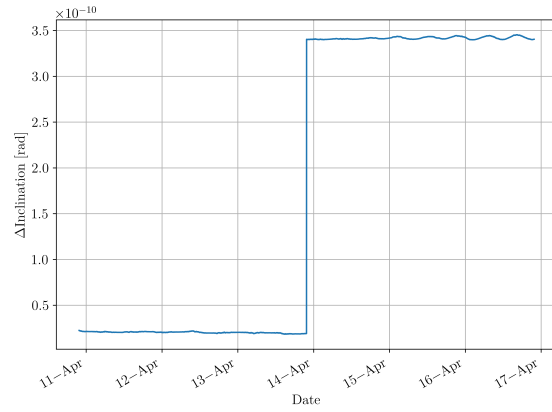
Turning to the Keplerian elements displayed in Figure 5.7, the semimajor axis shows a step-like decrease of about 0.6 metres at the time of closest approach, pointing to a small reduction in orbital energy. Eccentricity also increases slightly, with a jump of the order of 10^{-8} , reflecting a minimal reshaping of the orbit. The RAAN exhibits a very small positive shift (10^{-10} rad), stabilising afterwards, while inclination variations are negligible. These results show that the encounter induces permanent, though very small, modifications to the orbital parameters, with the most relevant dynamical signature being the ground-track drift, especially in longitude.



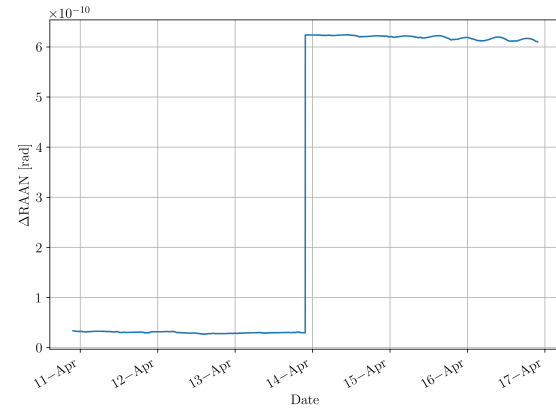
(a) Semimajor axis drift.



(b) Eccentricity drift.



(c) Inclination drift.



(d) RAAN drift.

Figure 5.7: Keplerian elements drift of a satellite orbiting the most parallel perturbed orbit.

5.2.2 Antiparallel encounter

Conversely, when the satellite moves antiparallel to Apophis, the drift behaviour changes. The results show again that the largest drifts occur when the offset is applied in the x-direction (except for the -0.5 km offset), but in this case the effect is stronger for positive offsets. The most extreme case corresponds to the 0.5 km offset, with longitude drifts approaching 37 meters in a 6-day simulation, the largest perturbation observed in this study.

Table 5.2: Most perturbed orbits as a function of the offset for antiparallel velocity.

Offset [km]	δ	Maximum	Value [m]	Energy [kg/km ² /s ²]	a [km]	e
-10	0.270	Longitude	0.842	-4.898	40694.164	0.0658
-10	0.270	Latitude	0.406	-4.898	40694.164	0.0658
-10	0.270	Both	0.842	-4.898	40694.164	0.0658
-5	0.270	Longitude	3.377	-4.898	40691.461	0.0658
-5	0.270	Latitude	1.599	-4.898	40691.461	0.0658
-5	0.270	Both	3.382	-4.898	40691.461	0.0658
-0.5	0.183	Longitude	17.870	-3.493	57064.175	0.334
-0.5	0.183	Latitude	5.931	-3.493	57064.175	0.334
-0.5	0.183	Both	18.392	-3.493	57064.175	0.334
0.5	0.183	Longitude	36.993	-3.493	57064.333	0.334
0.5	0.183	Latitude	12.227	-3.493	57064.333	0.334
0.5	0.183	Both	38.061	-3.493	57064.333	0.334
5	0.183	Longitude	4.729	-3.493	57060.924	0.334
5	0.183	Latitude	1.589	-3.493	57060.924	0.334
5	0.183	Both	4.866	-3.493	57060.924	0.334
10	0.4	Longitude	2.172	-6.712	29692.494	0.280
10	0.4	Latitude	1.097	-6.712	29692.494	0.280
10	0.4	Both	2.197	-6.712	29692.494	0.280

The ground-track results, Figure 5.8, show that the asteroid's flyby affects longitude more strongly than latitude. After the closest approach, the longitude drift grows to values larger than 35 metres, with clear negative excursions around 16th-17th April. By contrast, latitude drift remains limited, with oscillations of only a few meters (up to about 10 m) and no lasting trend. Overall, the ground-track analysis points to a measurable but localised displacement, dominated by longitude.

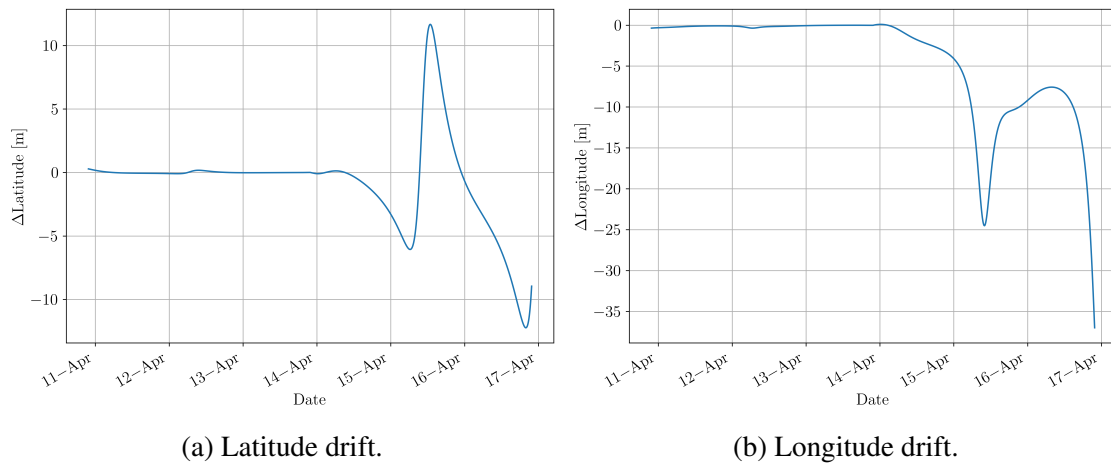
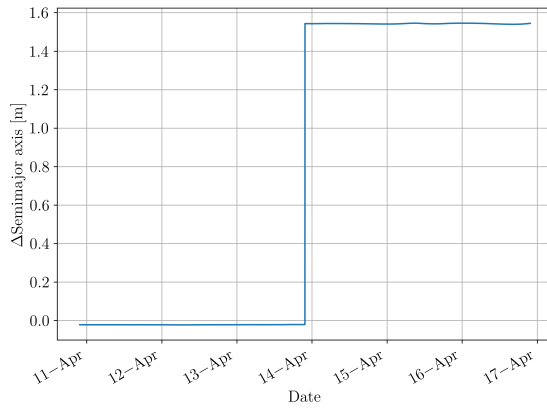
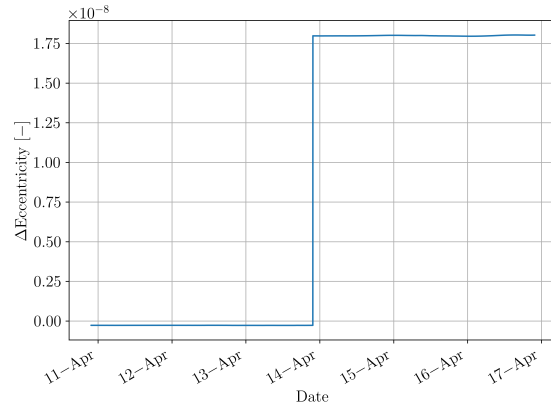


Figure 5.8: Ground-track drift of a satellite orbiting the most antiparallel perturbed orbit.

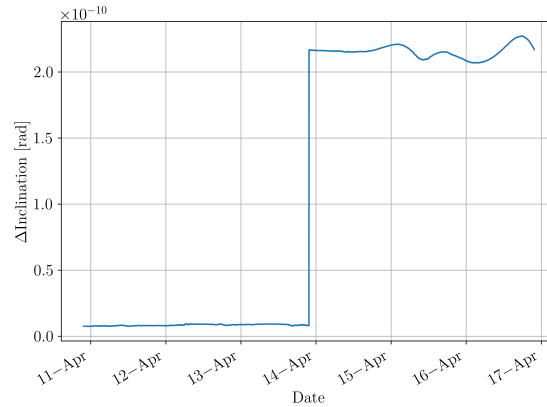
Looking at the Keplerian elements (Figure 5.9), the perturbation appears mainly as step-like permanent shifts that occur at the closest approach. The semimajor axis increases by 1.5m and stays at this level afterwards, showing a small but lasting change in orbital energy. The eccentricity also raises slightly, by about 10^{-8} , which corresponds to a very small modification of the orbital shape. The orbital plane is essentially unaffected: both inclination and RAAN show variations of only 10^{-10} rad, which are negligible in practice. In summary, the flyby produces small but permanent changes in the elements, mostly in semimajor axis and eccentricity, while the main visible effect on the satellite is the longitudinal ground-track drift.



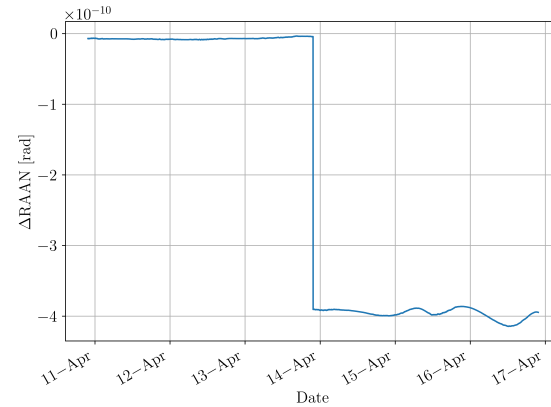
(a) Semimajor axis drift.



(b) Eccentricity drift.



(c) Inclination drift.



(d) RAAN drift.

Figure 5.9: Keplerian elements drift of a satellite orbiting the most antiparallel perturbed orbit.

5.3 Most influenced satellite

Once the most sensitive orbital region has been identified, the corresponding Keplerian elements can be computed as described in Section 3.1.1. For the parallel configuration,

the derived orbital parameters are:

$$a = 35962.1434 \text{ km} \quad e = 0.057 \quad i = 140.6409^\circ \quad \Omega = 167.40^\circ$$

These parameters are then used to filter the historical TLE database. As no satellite with such characteristics exists, the search is broadened adding more variation to the keplerian elements.

However, given the rarity of satellites at inclinations near 140° , the filter was relaxed to include prograde orbits with inclinations around 50° , which can be considered the complementary configuration. This search identified four satellites, among which satellites 62256 and 62257 were selected as the most relevant cases (Table 5.3)

Table 5.3: Most perturbed satellites considering parallel motion.

Satellite	NORAD ID	Status	a [km]	e	i [°]	Ω [°]
PROBA-3 CSC	62256	Active	36982.586	0.811	59.2564	144.1426
PSLV R/B	62257	Inactive	37315.923	0.813	59.3460	144.2857

For the antiparallel geometry, the reference orbital elements of the most perturbed case are:

$$a = 57064.33 \text{ km} \quad e = 0.33 \quad i = 39.36^\circ \quad \Omega = 347.4^\circ$$

Although no satellites were found with exactly these parameters, several with similar conditions are gathered in Table 5.4. Among them are satellites 34112, 11057 and 11896, which are analysed despite their difference in semimajor axis.

Table 5.4: Most perturbed satellites considering parallel motion.

Satellite	NORAD ID	Status	a[km]	e	i [°]	Ω [°]
QZS-4 (MICHIBIKI-4)	42965	Active	42161.1	0.075	40.29	347.29
SL-23 R/B	34112	Inactive	24938.4	0.69	35.19	343.65
MOLNIYA 3-10	11057	Inactive	16492.1	0.56	63.67	347.59

All these objects were analysed in the same way as the GEO satellites in Section 4: first verified in STK, then compared in terms of closest approach and velocity similarity, and finally numerically propagated.

Table 5.5: Most-perturbed satellites' approaches.

Satellite ID	ΔR_{min} [km]	$\alpha_{V,d}$ [°]	ΔV_d [km/s]	ΔV_{min} [km/s]	$\alpha_{V,V}$ [°]	ΔR_V [km]	First
62256	22571.2	12.23	6.83	4.42	8.84	94001.61	Velocity
62257	18848.5	16.36	8.72	7.63	15.16	103300.90	Velocity
42965	62450.0	8.70	5.74	4.013	7.68	80344.73	Velocity
34112	55867.2	10.32	5.98	4.23	7.75	68649.73	Velocity
11057	34739.2	21.99	12.84	4.33	6.047	47976.61	Distance

From this set, satellites 62256 and 62257 clearly stand out as the most affected, while the others are studied in the Appendix B.

Satellite 62257

The geometry of the close approach is shown in Figure 5.10.

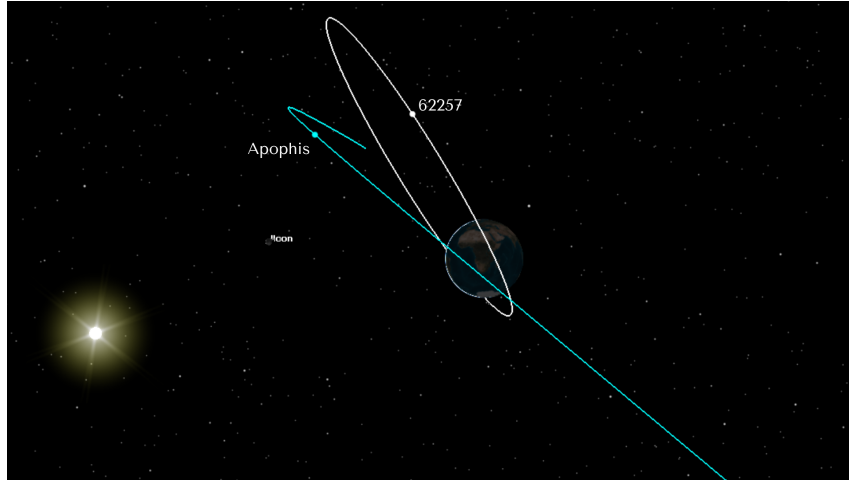


Figure 5.10: Satellite 62257 during Apophis's close approach.

The ground-track effect on one of the most influenced satellite, which is satellite 62257, is depicted in Figure 5.11. The evolution shows a divergence in both latitude and longitude. Latitude variations expand progressively, ultimately surpassing hundreds of kilometers in amplitude. The longitude, however, exhibits abrupt jumps of the order of 10^8 meters, indicating strong cumulative divergence in the propagation. These results suggest that the satellite's geocentric position is highly unstable under the influence of Apophis, with longitude being the most severely affected parameter.

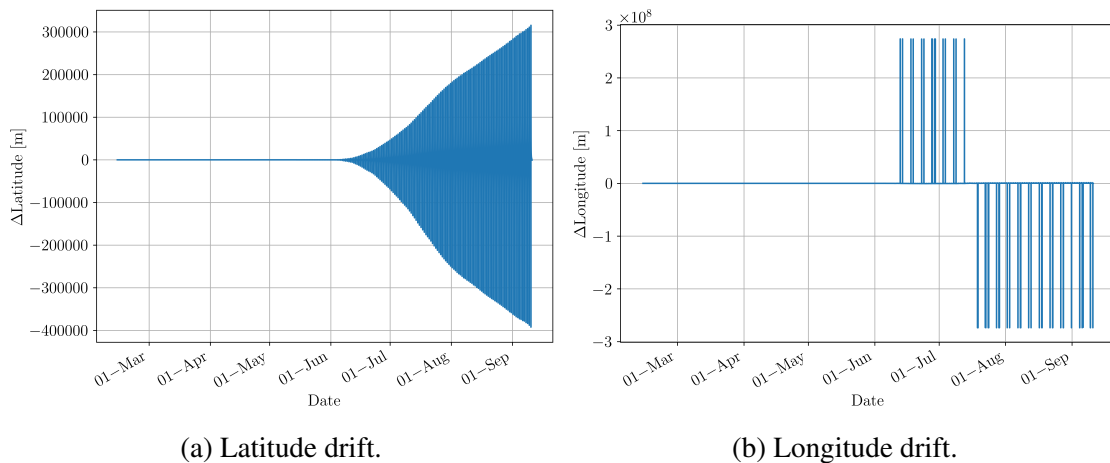


Figure 5.11: Ground-track drift of satellite 62257.

The evolution of the orbital elements, Figure 5.12, reinforces the instability of the orbit. The semimajor axis oscillates with increasing amplitude, reaching variations of the order of kilometers, which signals significant perturbation in orbital energy. The eccentricity shows fluctuations up to 10^{-6} , many orders of magnitude larger than those of the other GEO cases studied, suggesting that the orbit's shape is no longer preserved. The inclination remains closer to its initial value, though it still displays perturbations on the order of 10^{-6} rad. The RAAN also exhibits noticeable oscillations of comparable magnitude.

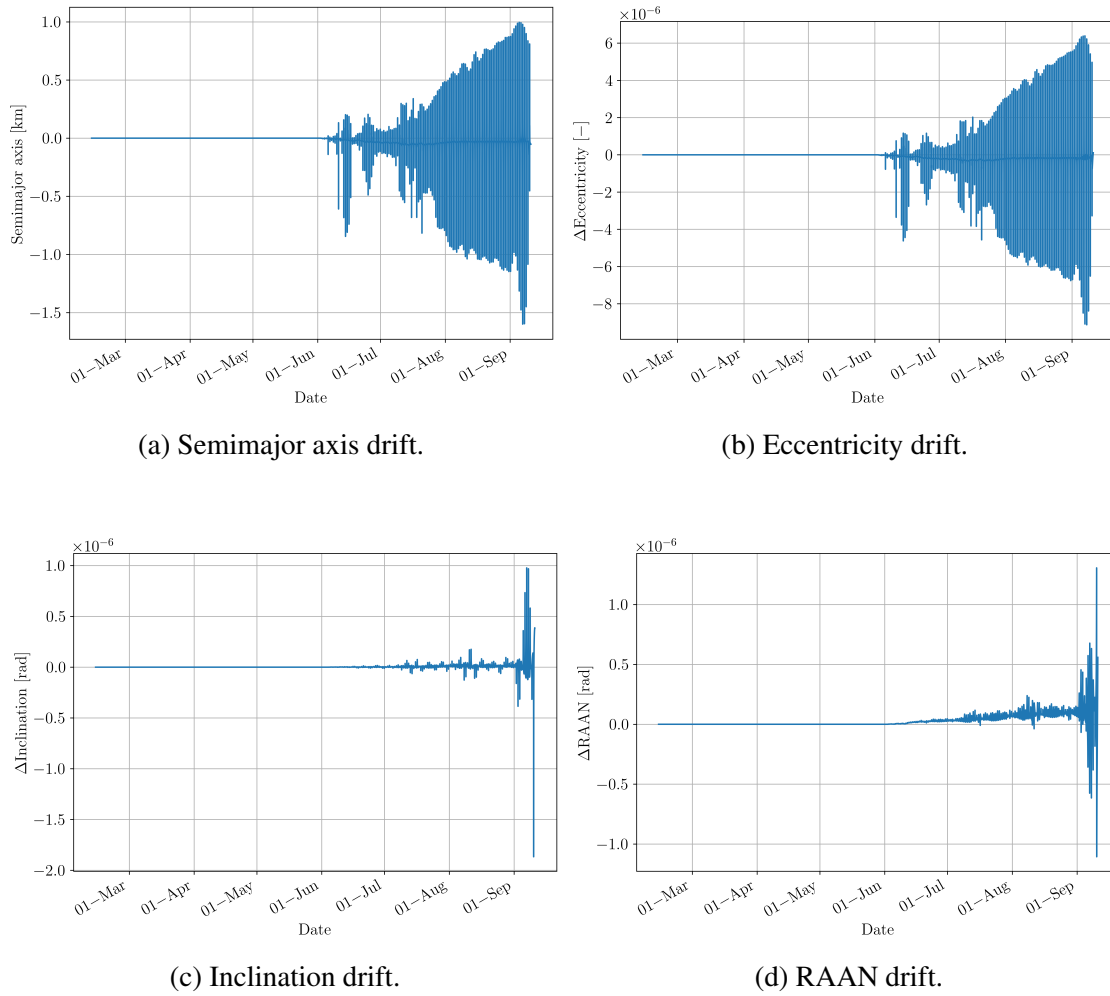


Figure 5.12: Keplerian elements drift of a satellite 62257.

Satellite 62256

The geometry of the close approach is shown in Figure 5.13.

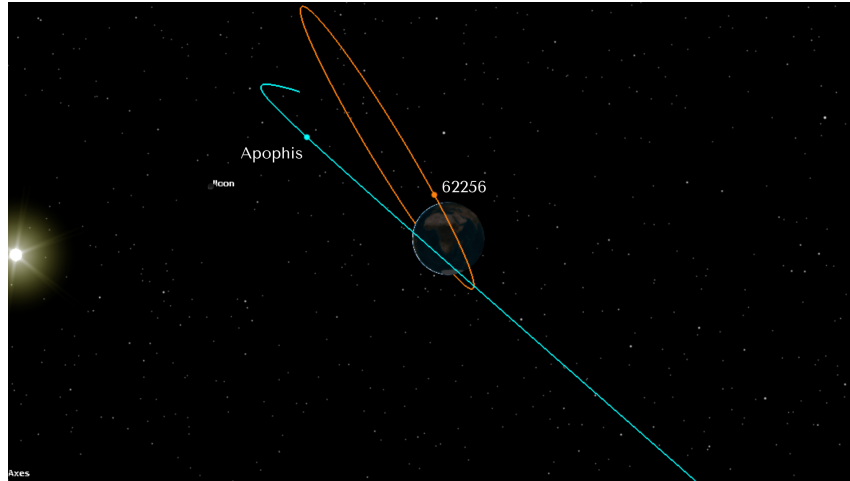


Figure 5.13: Satellite 62256 during Apophis's close approach.

For satellite 62256, the ground-track perturbations shown in Figure 5.14 are highly pronounced. The latitude exhibits oscillatory growth that gradually widens, surpassing 150 km in amplitude. Longitude, on the other hand, shows even more extreme behaviour, characterised by sudden jumps reaching values of the order of 10^8 m. Such discontinuities reflect strong numerical divergence and indicate that the accumulated effect of Apophis produces substantial displacement in geocentric position, especially along longitude.

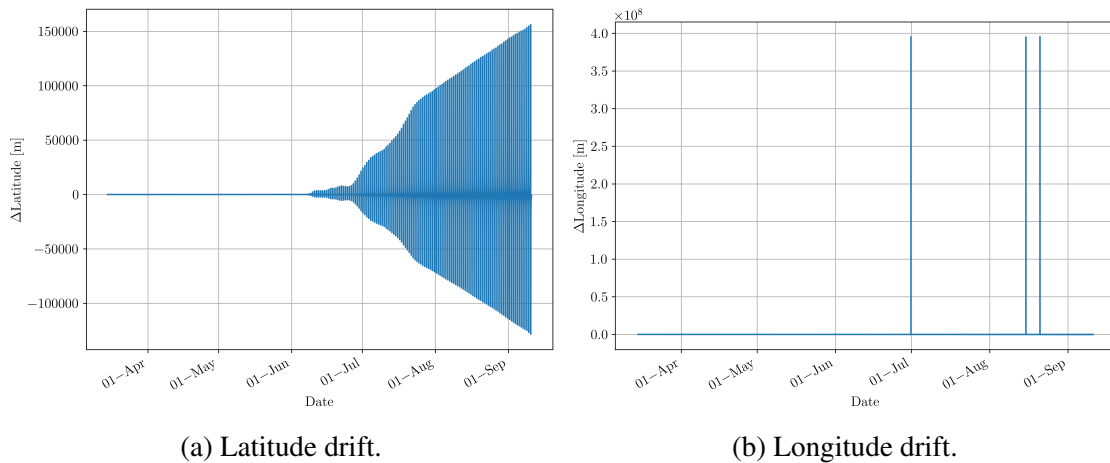
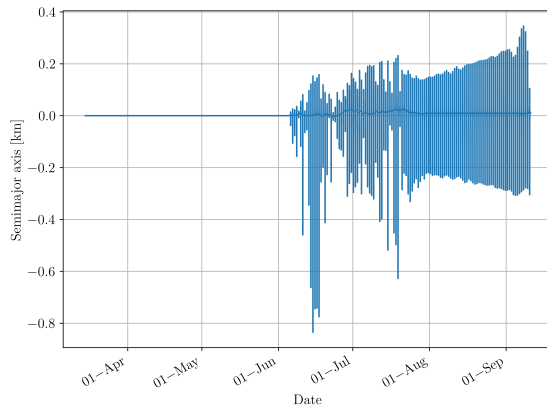


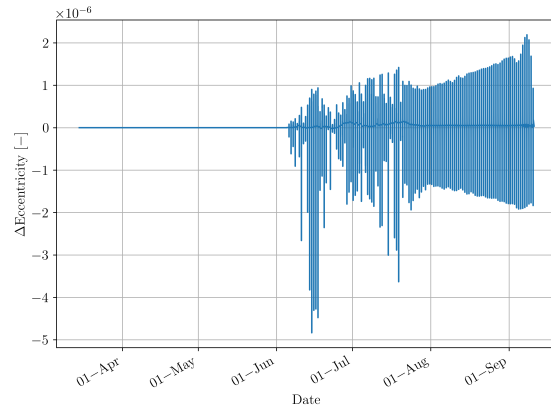
Figure 5.14: Ground-track drift of satellite 62256.

Looking at the orbital elements of Figure 5.15, the effect of Apophis is also visible. The semimajor axis shows oscillations that reach up to several hundred meters, which already indicates noticeable changes in orbital energy. The eccentricity also varies on the order of 10^{-6} , meaning that orbit's shape is slightly more distorted than usual. The inclination and RAAN remain more stable overall, but both show small oscillations, around 10^{-7} .

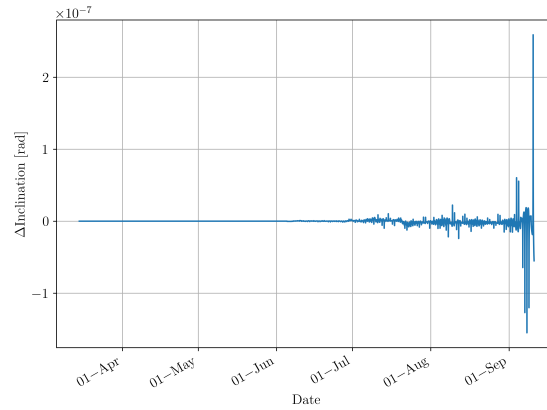
rad. Although the strongest impact is seen in the ground-track, the orbital elements also confirm that this satellite is significantly perturbed.



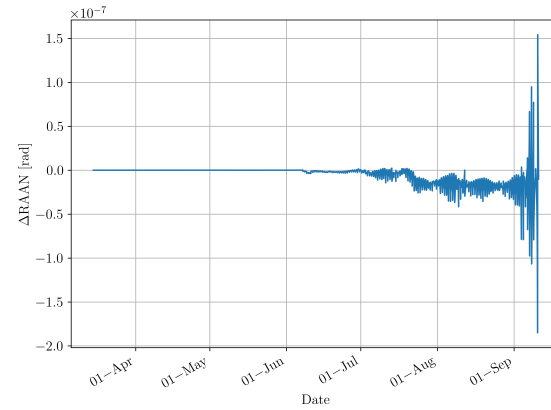
(a) Semimajor axis drift.



(b) Eccentricity drift.



(c) Inclination drift.



(d) RAAN drift.

Figure 5.15: Keplerian elements drift of a satellite 62257.

Chapter 6

Discussion and Conclusions

The aim of this research was to establish a computational framework to evaluate the effects of asteroid close approaches on Earth-orbiting satellites. To achieve this, the objectives were to implement and validate a numerical propagator including relevant perturbations, apply the framework to quantify Apophis's effect on geostationary satellites, and extend the methodology to identify the most perturbed orbital configurations. The following sections discuss the results obtained for each of these objectives.

6.1 Numerical Integrator

The first objective was to implement and validate a numerical orbit propagator for the Sun–Earth–Moon–Asteroid system. The propagator achieved the required accuracy when validated against JPL Horizons ephemerides, with relative errors of order 10^{-5} for the Moon's position. Each perturbation included in the model produced effects consistent with their expected order of magnitude, confirming the soundness of the implementation.

The assessment of numerical integration methods showed that both RK45 and DOP853 provide comparable levels of accuracy when benchmarked against high-fidelity ephemerides. As expected, velocity errors were larger than position errors, but remained within acceptable limits. Since RK45 proved more efficient in terms of execution time, it was selected as the default propagator for this project, using a relative tolerance of 10^{-5} and an absolute tolerance of 10^{-10} .

The sensitivity analysis confirmed that integration accuracy is not only determined by tolerance settings but also by factors such as the maximum timestep and the numerical precision of the initial state. Large timesteps artificially introduced drifts in the ground track, while reducing the step size suppressed these spurious deviations. Likewise, using 32-bit instead of 64-bit precision led to amplified apparent perturbations. Extending the propagation interval to several months further increased the drift, although part of this growth can be attributed to the accumulation of numerical errors.

Overall, these studies demonstrate that accurate modelling of Apophis's influence requires careful numerical treatment. A high-order integrator with conservative tolerance settings, a sufficiently small timestep, and 64-bit precision are essential to prevent the overestimation of the perturbations induced by the asteroid.

6.2 Apophis influence on GEO

The comparison of close-approach geometry and velocity similarity detailed in Table 4.10 further clarifies the difference between satellites. Satellite 08747 exhibits the strongest perturbation despite not being the closest in distance or the most similar velocity. It is one of the three cases where the closest velocity similarity occurs before the minimum distance. This configuration implies that the relative velocities remain comparable throughout the approach, prolonging the effective duration of the gravitational interaction. In contrast, when the minimum distance is reached before the velocity similarity, the encounter is sharper, since the bodies are already diverging in velocity at closest approach, reducing the interaction time.

Among the three velocity-first cases, LES 9 combines a relatively small minimum distance (14,968 km) with a moderate velocity difference (9.76 km/s). Intelsat 603, by contrast, reaches the smallest velocity difference at a much larger distance (33,043 km), which substantially weakens the gravitational interaction. TDRS 3 gets closer to the Apophis (18,802 km) but with a higher velocity difference (10.36 km/s), resulting in a shorter effective interaction. Satellite 08747 therefore represents the most effective balance, the encounter occurs close enough to experience significant gravitational pull while maintaining sufficiently low relative velocity to prolong interaction. This combination explains why 08747 accumulates the largest perturbation among the analysed GEO satellites and it is consistent with Opik's theory, the patched conics method and Keplerian Map's framework discussed in Section.

6.3 Apophis maximal influence

The analysis on the maximal effect shows that Apophis's flyby produces measurable but limited perturbations on nearby orbits. The main effect appears in the ground track, with longitude drifts up to 30–37 metres, especially in antiparallel encounters. Keplerian elements experience only minor permanent shifts—about one metre in semimajor axis and 10^{-8} in eccentricity—while inclination and RAAN remain essentially unaffected. Overall, the perturbations are small, though detectable in specific orbital configurations.

Once the most perturbed region is defined, satellites near such region are identified. The comparison of close-approach geometry and velocity similarity in Table 5.5 highlights why the satellites are affected differently. Satellite 62257 shows the strongest perturbation because it combines a relatively small closest distance (18,848 km) with a moderate velocity difference (8.72 km/s), and—importantly—its velocity similarity occurs before the minimum distance. This prolongs the effective interaction, amplifying the perturbation. Satellite 62256 shares the same velocity-first configuration but at a larger distance, which weakens the effect, while 42965 and 34112, despite their low velocity differences, remain too far from Apophis for significant influence. Finally, 11057, the only distance-first case, undergoes the weakest perturbation since the encounter is sharper and the relative velocity grows rapidly at closest approach.

The behaviour observed in these satellites is consistent with the pattern already identified for GEO objects. Velocity-first encounters produce stronger perturbations because the relative velocities remain comparable for longer, while distance-first cases result in shorter,

weaker interactions. Nevertheless, it should be noted that the perturbations obtained for the satellites analysed in this set are significantly larger than for the GEO cases. These unusually high values suggest that their interactions with Apophis may not be fully captured by the present modelling and therefore deserve further investigation.

6.4 General conclusion

Overall, the discussion confirms that all the objectives of the project were achieved. A validated propagator was developed, Apophis's negligible effect on geostationary orbits was demonstrated, and the most vulnerable orbital configurations were identified.

While Apophis's 2029 encounter is not expected to cause operational risks to GEO Earth satellites, the analysis shows that encounter geometry, particularly antiparallel trajectories, plays a key role in amplifying perturbations. The framework developed here thus provides a foundation for systematically evaluating asteroid-induced perturbations in future encounters, with the potential to support both scientific studies and space situational awareness. Nevertheless, the large perturbations found in some satellites suggest that these cases should be carefully reviewed, as they may reflect modelling limitations or highlight configurations where asteroid-induced effects become operationally significant.

Chapter 7

Limitations and Future Work

This work has also revealed unexpectedly large perturbations for some satellites, particularly in cases close to the most sensitive orbital configurations. While these results suggest that Apophis may induce significant deviations, they could also reflect limitations of the propagator or the assumptions made in the dynamical model. For this reason, such cases should be carefully reviewed in future studies, with more refined propagation techniques and independent validation against alternative software or higher-fidelity models. Addressing this issue is a priority, as it will help distinguish genuine dynamical effects.

This study has certain limitations that should be acknowledged. First, the orbital propagation relies on the SGP4 model, which becomes increasingly inaccurate as the prediction extends far from the epoch of the TLE data. This limitation can introduce uncertainties in long-term analyses, particularly when assessing subtle perturbative effects.

Moreover, the dynamical model implemented does not include all relevant perturbations. While the main gravitational effects have been considered, further refinements could be achieved by incorporating non-gravitational forces, such as additional third-body interactions or atmospheric drag for lower orbits.

Another aspect not addressed in this work is the role of uncertainty in the initial conditions. In practice, both position and velocity are known only within certain error bounds, and the way these uncertainties propagate during the encounter can strongly affect the predicted outcome. Considering this effect in future studies would make the analysis more robust and provide confidence intervals for the perturbations reported here.

In the longer term, it would also be valuable to investigate whether inactive satellites-objects without manoeuvring capability could experience perturbations large enough to alter their trajectories significantly, so that might create potential collision risks with operational spacecraft. If that was to happen, this type of analysis could gain more relevance to space debris management and traffic safety.

Future work should therefore focus on adopting more accurate propagation techniques combined with precise force models. Extending the analysis to account for a broader range of perturbations would provide a more complete picture of Apophis' potential impact on different orbital regimes. Finally, applying the methodology to other planetary encounters or different asteroid trajectories could help generalise the results and strengthen their relevance for mission planning and space situational awareness.

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Appendix A

Ethical Approval Letter

The Ethical approval letter is attached hereafter.



15 May 2025

Dear Miss Martínez Ambrosio ,

Reference: CURES/25154/2025

Project ID: 28125

Title: APOPHIS and SATELLITES STATIONKEEPING

We are pleased to inform you that you have successfully declared that your research project is a **Literature Review – based solely on openly available literature which is in the public domain, and you are undertaking desk-based research not involving any other form of data or information.**

You have also confirmed that your project **does not meet** any of the literature review specified exceptions, listed both within the relevant section of the CURES form and below:

1) Your supervisor has requested that you apply for approval through CURES because:

- The journals or data are in a sensitive area (please discuss this with your supervisor)
- The project will be embargoed **i.e. will not be publicly available via the Cranfield library immediately or longer term**

2) Approval is/will be specifically required by another external body e.g. journal publishers

Therefore, **you do not require ethical approval** and your CURES application will be automatically closed.

Please keep a copy of this letter safe, if this exception is in relation to your thesis project, you will need to include a copy with your final thesis submission.

If you have any queries, please contact CURES Support.

We wish you every success with your project.

Regards,

CURES Team

Appendix B

Satellites with maximal Apophis's effect

This section describes the results obtained for the satellites assessed in Apophis maximal effect.

B.1 Satellite 34112

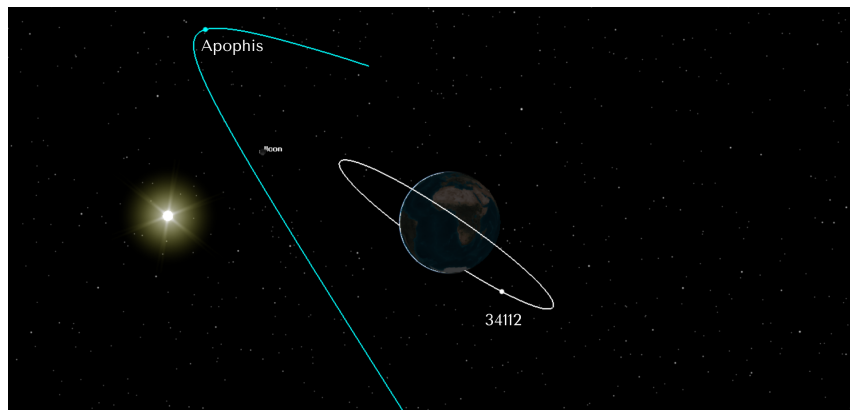


Figure B.1: Satellite 34112 during Apophis's close approach.

The ground-track variations of satellite 34112, in Figure B.2, show a strong divergence. Latitude begins to deviate progressively and reaches several thousand metres, indicating a significant departure from nominal geostationary stability. Longitude, however, is even more severely affected: instead of a smooth drift, it presents abrupt jumps of tens of millions of metres, suggesting strong numerical divergence and a cumulative instability in its apparent geocentric position. As in other perturbed cases, longitude clearly emerges as the most sensitive parameter.

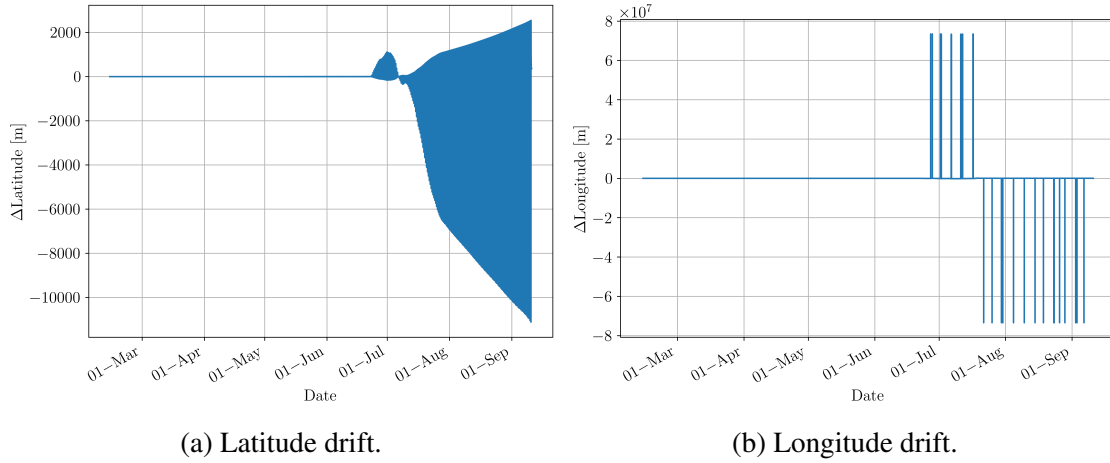
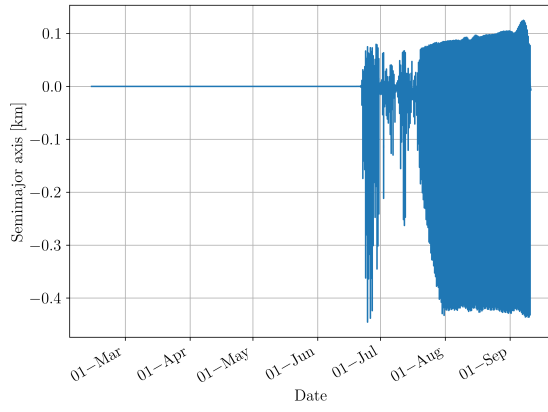
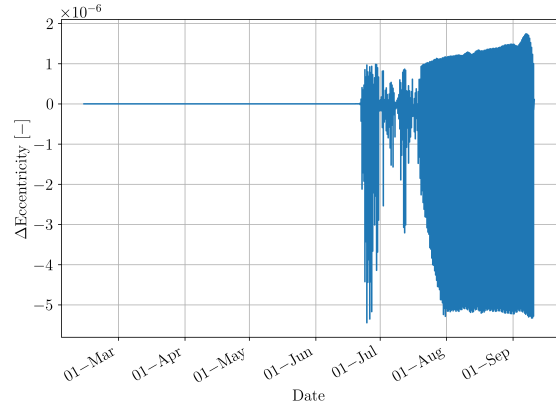


Figure B.2: Ground-track drift of satellite 34112.

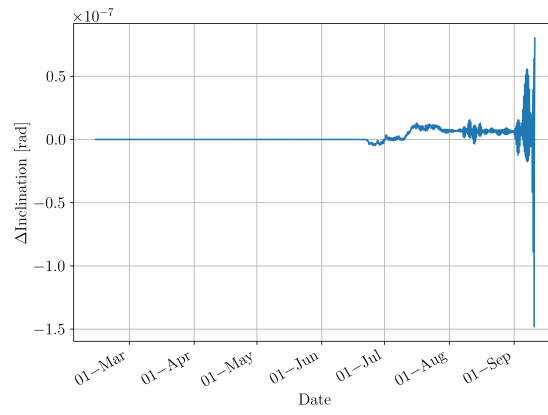
The Keplerian elements confirm that the encounter with Apophis produces substantial dynamical effects. The semimajor axis oscillates with variations of several hundred metres, reflecting clear perturbations in orbital energy. The eccentricity also shows changes on the order of 10^{-6} , pointing to a deformation of the orbit's shape. The inclination and RAAN remain closer to their initial values but still show oscillations in the 10^{-7} rad range, which although small, confirm that the orbital plane is not completely stable. Altogether, these results demonstrate that satellite 34112 experiences a significant level of disturbance.



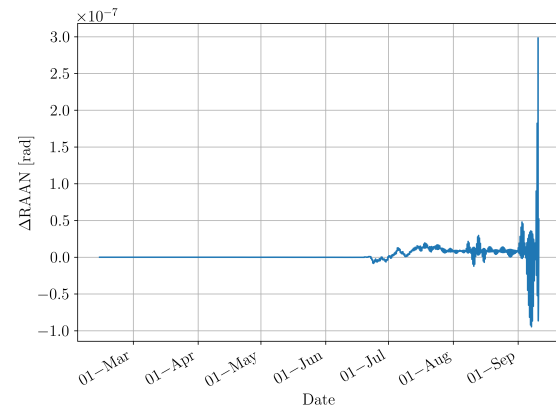
(a) Semimajor axis drift.



(b) Eccentricity drift.



(c) Inclination drift.



(d) RAAN drift.

Figure B.3: Keplerian elements drift of a satellite 34112.

B.1.1 Satellite 11057

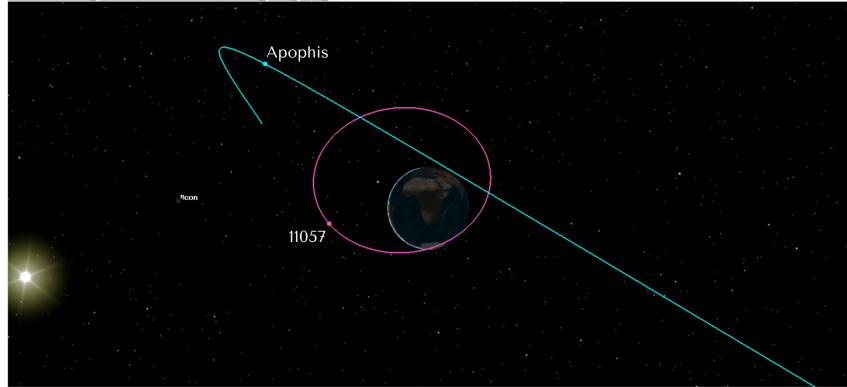


Figure B.4: Satellite 11057 during Apophis's close approach.

For satellite 11057, the ground-track in Figure B.5 shows a clear loss of stability. Latitude begins with small deviations but soon develops into wide swings that reach tens of kilometres. Longitude, on the other hand, behaves in a very irregular way, with sudden jumps and discontinuities rather than a smooth accumulation. These changes, on the order of 10^7 – 10^8 m, make longitude the parameter where the effect of Apophis is most evident and underline the strong perturbative character of the encounter.

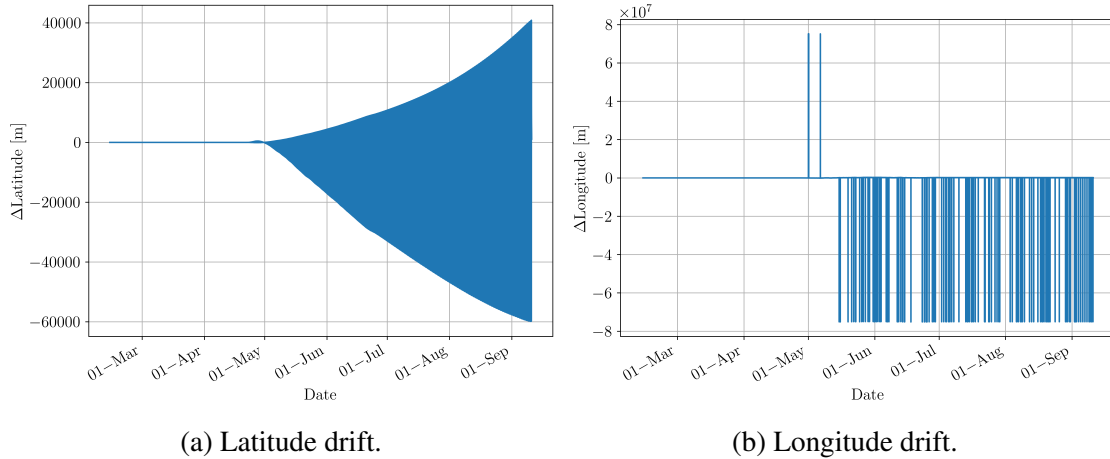
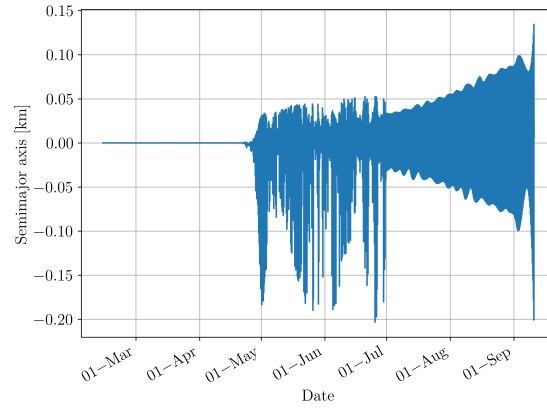


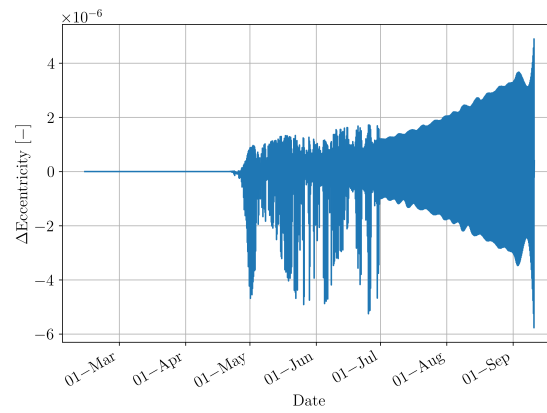
Figure B.5: Ground-track drift of satellite 11057.

The orbital elements (Figure B.6) also suggest that the flyby has a lasting effect on this satellite. The semimajor axis fluctuates with variations of several hundred meters, showing that orbital energy is being noticeably altered. The eccentricity grows to values around

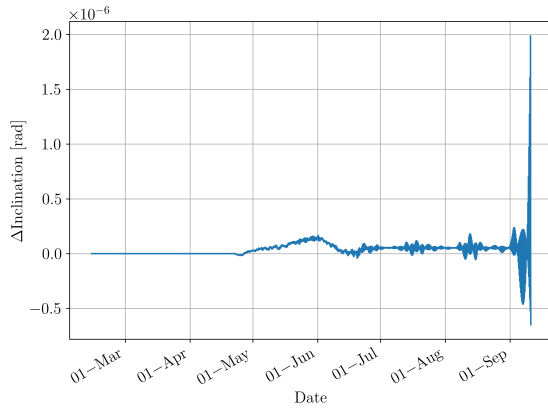
10^{-6} , which implies that the shape of the orbit is no longer preserved as in the more stable cases. The inclination and RAAN stay closer to their initial values but still display visible oscillations in the 10^{-7} rad range. In simple terms, the data indicate that 11057 undergoes one of the strongest perturbations among the analysed satellites, with both its ground-track and orbital elements showing signs of instability.



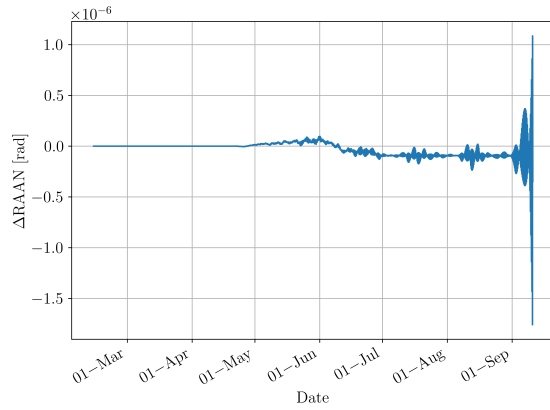
(a) Semimajor axis drift.



(b) Eccentricity drift.



(c) Inclination drift.



(d) RAAN drift.

Figure B.6: Keplerian elements drift of a satellite 11057.

B.2 Satellite 42965

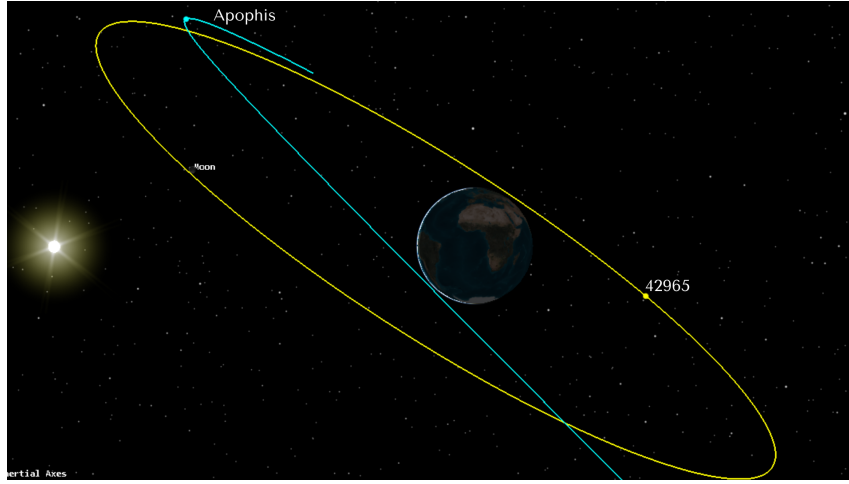


Figure B.7: Satellite 42965 during Apophis's close approach.

In the case of satellite 42965, the ground-track represented in Figure B.8 barely shows any sign of disturbance. Latitude variations stay almost flat, with tiny oscillations that never reach more than a couple of meters. Longitude grows slowly and smoothly, ending with only a slight drift of a few meters over the whole period. Compared to the rest of the satellites, this trajectory is clearly the most stable, with Apophis leaving only a faint trace in its ground position.

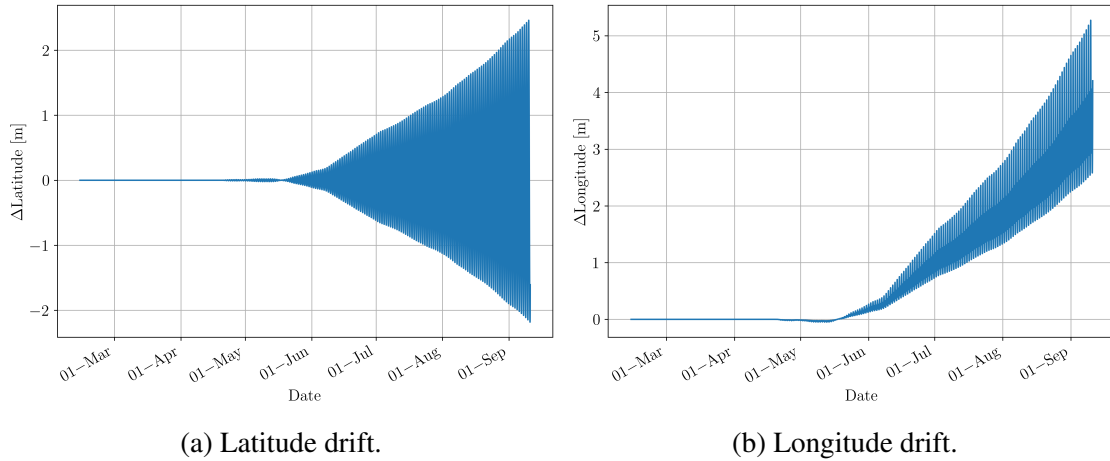
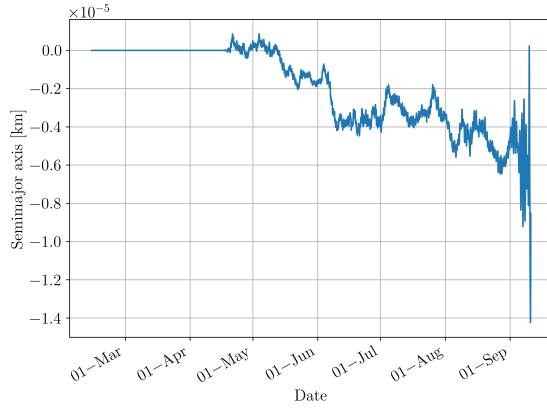


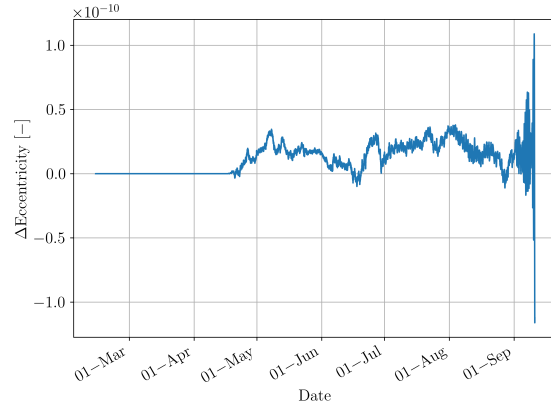
Figure B.8: Ground-track drift of satellite 42965.

The orbital elements in Figure B.9 tell the same story. The semimajor axis moves within a range of just a few centimeters, too small to suggest any meaningful change in orbital energy. The eccentricity stays practically constant, while inclination and RAAN show

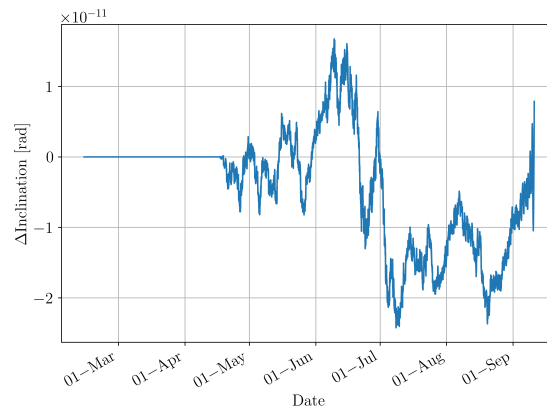
only tiny wiggles that remain at the limit of detectability. Overall, 42965 is by far the least affected object in the group, making it a good reference point to measure how much stronger the perturbations are in the other satellites.



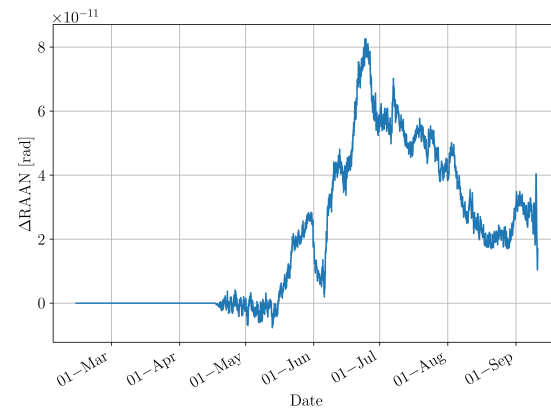
(a) Semimajor axis drift.



(b) Eccentricity drift.



(c) Inclination drift.



(d) RAAN drift.

Figure B.9: Keplerian elements drift of a satellite 42965.