

Control Allocation for Electric Vehicles with Brake-by-Wire to Optimise Safety and Energy Efficiency Across Full Envelope of Operation*

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Abstract—To enhance vehicle stability and maximise energy efficiency, this paper presents a novel control allocation strategy for electric vehicles (EVs) equipped with a brake-by-wire system, which can independently allocate tractive and brake torques on 4 in-wheel motors and friction brakes. The proposed approach integrates a real-time optimisation algorithm that dynamically allocates torques to individual motors/brakes by considering the tyre, motor and friction brake dynamic limits. It also employs an asymmetrical cost function, penalising positive and negative motor torques differently to maximise regenerative braking energy recovery. Simulation results using a high-fidelity vehicle model demonstrate significant improvements in vehicle handling and energy efficiency performances compared to the traditional control allocation method. This paper therefore showcases how to construct a global optimal wheel torque strategy considering both performance-oriented and energy-efficient EV applications.

Index Terms—optimal control allocation, safety and energy efficiency, tyre and actuator dynamic limits, asymmetrical cost function.

I. INTRODUCTION

The rapid advancement of electric vehicle (EV) technology has accelerated the evolution of vehicle control strategies, particularly with the adoption of in-wheel motor and brake-by-wire systems. These systems enable independently positive and negative torque control at each wheel, offering significant flexibility in vehicle dynamics and energy efficiency optimisation. However, the integration of multiple actuators presents a complex control challenge [1] for advanced torque allocation and energy management strategies that simultaneously target vehicle stability and energy efficiency.

Recent research has highlighted the importance of coordinated torque distribution in multi-motor EVs. For instance, to enhance vehicle stability, Zhai et al. [2] and Guo et al. [3] proposed optimal control allocation systems for four in-wheel motor EVs. Zhao et al. [4] proposed a model predictive control allocation strategy to improve vehicle stability during critical driving conditions. However, these works assume

motors provide sufficient braking torques without considering friction brakes. While [5]–[7] proposed cooperative control strategies of regenerative and friction braking torques, they neglected motor or friction brake dynamic limits. Shuai et al. [8] proposed an optimal control allocation strategy using dynamic message priority scheduling, but applied limits only to total wheel torque without considering regenerative/friction brake distribution.

Several studies have also investigated energy efficiency aspects. Chen et al. [9], Yuan et al. [10], and Pennycott et al. [11], [12] minimized motor power losses through control allocation, demonstrating energy savings for specific motor types using static efficiency maps. However, the energy efficiency is based on the off-line optimised allocation strategy using motor's efficiency map. Ma et al. [13] considered both lateral stability and energy efficiency by reducing tyre dissipation energy. Jafari et al. [14] introduced a reinforcement learning-based torque allocation method for all-wheel-drive EVs with real-time optimisation, but they failed to consider the actuator dynamic limits, either.

This paper addresses these gaps by introducing a global optimal control allocation strategy that dynamically distributes torques across four in-wheel motors and friction brakes. The proposed method employs a real-time optimisation algorithm with an asymmetrical cost function to prioritize regenerative braking, thereby maximizing energy recovery without compromising vehicle stability. Key contributions include:

- Integrated tyre & actuator limits: Tyre, motor, and friction brake constraints are explicitly modelled to ensure safe and efficient torque distribution.
- Asymmetrical cost function: Negative motor torques (regenerative braking) are penalized less than positive torques, promoting energy recovery.
- Real-time optimisation: A nonlinear solver dynamically allocates torques under varying driving conditions, from aggressive maneuvers to steady-state cruising.

Simulation results validate the strategy's superiority over baseline methods in scenarios such as split- μ braking, single-

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lane change, and closed-loop racetrack driving, demonstrating significant improvements in driving safety while maintaining high levels of energy efficiency.

II. VEHICLE MOTION CONTROL STRATEGY

Schematic of the proposed vehicle motion control algorithm is shown in Fig. 1. A reference generator (includes both longitudinal force and yaw rate reference generators), a high-level controller, and a control allocation algorithm are constructed.

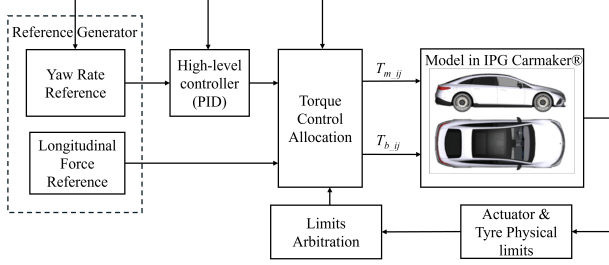


Fig. 1. Schematic of the Overall Control Strategy.

A. Reference Generator

The longitudinal force reference ($\hat{F}_{tot}$) is derived from the driver's acceleration and brake pedal inputs. For lateral dynamics, a 2-DOF bicycle model is used to generate the yaw rate reference signal. The target yaw rate ($\hat{\psi}$) is derived from the steering wheel angle (δ), longitudinal vehicle velocity (v_x), and the understeer gradient (K_V) [15]:

$$\hat{\psi} = \frac{v_x \delta}{l_f + l_r + K_V v_x^2} \quad (1)$$

The understeer gradient K_V is defined as:

$$K_V = \frac{m(l_r C_r - l_f C_f)}{2C_f C_r (l_f + l_r)} \quad (2)$$

Here, C_f and C_r denote the cornering stiffness of the front and rear tyres, respectively, while l_f and l_r represent the distances from the vehicle's center of gravity to the front and rear axles. m corresponds to the vehicle mass.

B. High-Level Controller

The high-level controller computes the requested yaw moment ($\hat{M}_z$) using a combination of PID and PI controllers. The first part consists a PID controller that reduces the yaw rate error ($e_{\dot{\psi}}$) between the actual yaw rate and the driver-provided reference from the yaw rate generator. The second part consists a PI controller that minimizes the sideslip angle error (e_{β}), ensuring the vehicle sideslip angle remains close to zero for stability.

$$\begin{aligned} \hat{M}_z(t) = & K_{P1} e_{\dot{\psi}}(t) + K_{I1} \int_0^t e_{\dot{\psi}}(\tau) d\tau + K_{D1} \frac{de_{\dot{\psi}}(t)}{dt} \\ & + K_{P2} e_{\beta}(t) + K_{I2} \int_0^t e_{\beta}(\tau) d\tau \end{aligned} \quad (3)$$

C. Torque Control Allocation

The torque control allocation algorithm is designed to obtain tractive and braking torques at each wheel to track desired vehicle longitudinal movement and yaw moment determined by the high-level controller. A diagram of the vehicle in its xy plane is shown in Fig 2. F_{mij} and F_{bij} denote longitudinal tyre forces delivered by the motor and the friction brake, respectively, at the ij th wheel, where ij represents $[fl, fr, rl, rr]$. Here $F_{ij} = F_{mij} + F_{bij}$ represents the total longitudinal force per wheel. w_f and w_r are the vehicle front and rear track-widths, respectively. Assuming a static longitudinal force distribution and neglecting front wheel steering angles [11], the net longitudinal force and yaw moment are derived as:

$$\begin{aligned} F_{tot} &= F_{fl} + F_{fr} + F_{rl} + F_{rr} \\ M_z &= \frac{1}{2}(-F_{fl}w_f + F_{fr}w_f - F_{rl}w_r + F_{rr}w_r) \end{aligned} \quad (4)$$

In the next section, detailed control allocation strategies will be introduced to demonstrate how F_{ij} are allocated to achieve the desired longitudinal and yaw requirements.

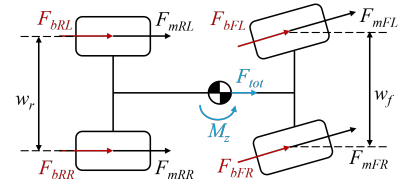


Fig. 2. Configuration of the vehicle model in xy plane view.

III. DETAILS OF CONTROL ALLOCATION STRATEGIES

A. Baseline control allocation

1) *Torque Allocation per Wheel*: The baseline control allocation strategy distributes the desired total longitudinal force $\hat{F}_{tot}$ and the desired yaw moment $\hat{M}_z$ evenly across all four wheels, under the assumption of balanced load distribution. To generate the required yaw moment, a differential force $\Delta\hat{F}$ is defined as:

$$\Delta\hat{F} = \frac{2\hat{M}_z}{w} \quad (5)$$

where w is the average of the front and rear track widths, used to minimize imbalance within the system.

Assuming equal distribution of the total longitudinal force and symmetric application of the differential force across the left and right wheels, the resulting wheel forces are given by:

$$\begin{aligned} F_{fl} &= \frac{\hat{F}_{tot}}{4} - \frac{\Delta\hat{F}}{4}, & F_{fr} &= \frac{\hat{F}_{tot}}{4} + \frac{\Delta\hat{F}}{4} \\ F_{rl} &= \frac{\hat{F}_{tot}}{4} - \frac{\Delta\hat{F}}{4}, & F_{rr} &= \frac{\hat{F}_{tot}}{4} + \frac{\Delta\hat{F}}{4} \end{aligned} \quad (6)$$

The desired torque applied to the ij -th wheel is then calculated as $T_{ij} = rF_{ij}$, where r is the effective radius of the tyre.

2) *Regenerative and Friction Brake Distribution*: In this strategy, regenerative braking torque from the electric motor is blended with conventional friction braking. The primary braking effort is delivered through the electric motor to maximize energy recovery. When the required braking torque exceeds the motors' regenerative braking capability, the excess braking demand is fulfilled by the friction braking system.

B. Optimal control allocation

In order to more accurately distribute the regenerative and friction braking torque, an optimal control allocation strategy is proposed by taking into consider the dynamic limits of tyres and actuators. Under this strategy, motor torques and friction braking torques will be treated separately through out the optimisation process to simulate the break-by-wire system. Thus, Eq.4 will be rewritten in matrix form as:

$$\mathbf{v} = \mathbf{B}\mathbf{u} \quad (7)$$

where $\mathbf{u} = [F_{mfl}, F_{mfr}, F_{mrl}, F_{mrr}, F_{bfl}, F_{bfr}, F_{brl}, F_{brr}]^T$, $\mathbf{v} = [F_{tot}, M_z]^T$, and

$$\mathbf{B} = \begin{bmatrix} 1, & 1, & 1, & 1, & 1, & 1, & 1, & 1 \\ -\frac{w}{2}, & \frac{w}{2}, & -\frac{w}{2}, & \frac{w}{2}, & -\frac{w}{2}, & \frac{w}{2}, & -\frac{w}{2}, & \frac{w}{2} \end{bmatrix}$$

1) *Cost Function and Constraints Formulation*: Let the target force and yaw moment be represented as $\hat{\mathbf{v}} = [\hat{F}_{tot}, \hat{M}_z]^T$. The control allocation problem is formulated as the minimization of the following cost function:

$$J = \mathbf{Q}\|\mathbf{v} - \hat{\mathbf{v}}\|^2 + \mathbf{R}\|\mathbf{u}\|^4 \quad (8)$$

The first term penalizes tracking error to ensure accurate force and moment allocation, while the second term minimizes control effort for energy efficiency. A quartic norm is used here as it fits the exponential term, which will be introduced in the extended method in the following section, than the traditional quadratic term. Weighting matrices are defined as $\mathbf{Q} = \gamma \mathbf{W}_v^T \mathbf{W}_v$ with $\mathbf{W}_v = \text{diag}(1, W_{Mz})$, $\gamma = 1000$, and $W_{Mz} = 10$. The matrix $\mathbf{R} = 2 \times 10^{-7} \mathbf{I}_8$ where $\mathbf{I}_8$ is the identity matrix.

Eq.(8) is subject to the following constraints:

$$\begin{cases} \mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max}, \\ \mathbf{F}_{t\min} \leq \mathbf{G}\mathbf{u} \leq \mathbf{F}_{t\max} \end{cases} \quad (9)$$

where $\mathbf{G} = [\mathbf{I}_4, \mathbf{I}_4]$.

2) *Limits Arbitration* : Under this strategy, $\mathbf{u}_{\max}$ and $\mathbf{u}_{\min}$ are determined based on motors and friction brakes dynamic limits where both the torque and torque rate limits are considered. $\mathbf{F}_{t\min}$ and $\mathbf{F}_{t\max}$ are determined by the slip-based tyre force limits. At each time step k , they are calculated as:

$$\begin{aligned} \mathbf{u}_{\max}^k &= \min \left\{ \frac{\mathbf{T}_{\max}^k}{r}, \quad \mathbf{u}^{k-1} + \frac{d\mathbf{T}_{\max}}{r} \Delta t \right\}, \\ \mathbf{u}_{\min}^k &= \max \left\{ \frac{\mathbf{T}_{\min}^k}{r}, \quad \mathbf{u}^{k-1} + \frac{d\mathbf{T}_{\min}}{r} \Delta t \right\}, \\ \mathbf{F}_{t\max} &= \mu_{\max}^k \mathbf{F}_z^k, \\ \mathbf{F}_{t\min} &= -\mu_{\max}^k \mathbf{F}_z^k. \end{aligned}$$

Here, $\mathbf{F}_z^k = [F_{zfl}^k, F_{zfr}^k, F_{zrl}^k, F_{zrr}^k]$ are the tyre vertical loads on each wheel, and $\mu_{\max}^k$ is the tyre maximum friction coefficient at the k th time step. In this study, we assume that they can be estimated using a tyre friction coefficient estimator.

$$\begin{aligned} \mathbf{T}_{\max}^k &= [T_{mfl+}, \quad T_{mfr+}, \quad T_{mrl+}, \quad T_{mrr+}, \\ &\quad 0, \quad 0, \quad 0, \quad 0]^T \\ \mathbf{T}_{\min}^k &= [T_{mfl-}, \quad T_{mfr-}, \quad T_{mrl-}, \quad T_{mrr-}, \\ &\quad T_{bfl-}, \quad T_{bfr-}, \quad T_{brl-}, \quad T_{brr-}]^T \\ d\mathbf{T}_{\max} &= [dT_{mfl}, \quad dT_{mfr}, \quad dT_{mrl}, \quad dT_{mrr}, \\ &\quad dT_{bfl}, \quad dT_{bfr}, \quad dT_{brl}, \quad dT_{brr}]^T \\ d\mathbf{T}_{\min} &= [-dT_{mfl}, \quad -dT_{mfr}, \quad -dT_{mrl}, \quad -dT_{mrr}, \\ &\quad -dT_{bfl}, \quad -dT_{bfr}, \quad -dT_{brl}, \quad -dT_{brr}]^T \end{aligned}$$

T_{mij+} , T_{mij-} , dT_{mij} , T_{bij-} and dT_{bij} are the motor maximum, minimum, rate, and friction brake minimum and rate torque limits, respectively.

C. Optimal Control Allocation Considering Energy Efficiency

While the quartic cost term in Eq. (8) penalizes braking effort, it treats positive and negative torques equally, offering no incentive for regenerative braking. To promote energy recovery, an asymmetric exponential cost function is introduced for motor torque:

$$L_u = ce^{au}, \quad \text{where } c, a \neq 0 \quad (10)$$

Here, u is the actual control input. This formulation penalizes less on negative torque, encouraging the use of regenerative torque for improved energy efficiency.

1) *Cost Function and Constraints Formulation*: With Eq.(10), the cost function is rewritten as:

$$J = \mathbf{Q}\|\mathbf{v} - \hat{\mathbf{v}}\|^2 + f(\mathbf{u}) \quad (11)$$

where $f(\mathbf{u}) = (p \sum_{ij} L_{u_{mij}} + q \sum_{ij} u_{bij}^4)$. Here u_{mij} and u_{bij} represent the motor and friction brake torque inputs, respectively. Constraints in this formulation is same as Eq.(9). p and q are weighting parameters where $p = 1$ and $q = 2 \times 10^{-7}$. $c = 31730$ and $a = 0.0023$, which are obtained by fitting the positive side of $f(\mathbf{u})$ with $\mathbf{R}\|\mathbf{u}\|^4$, as shown in Figure 3.

2) *Limits Arbitration*: Same as Section III-B2.

IV. SIMULATION RESULTS

To evaluate the effectiveness of the proposed optimal control allocation strategies, three driving scenarios are simulated: braking on a split- μ surface, single-lane change, and closed-loop racetrack driving.

A. Braking on Split- μ Surface

A straight-line braking scenario on a split- μ surface is simulated to demonstrate the importance of tyre force constraints for maintaining vehicle stability. The vehicle accelerates to 70 km/h before entering a surface with asymmetric friction coefficients: 0.4 on the left and 0.8 on the right. A full braking request is applied upon reaching this surface. The simulation results are shown in Fig. 4. Since the vehicle

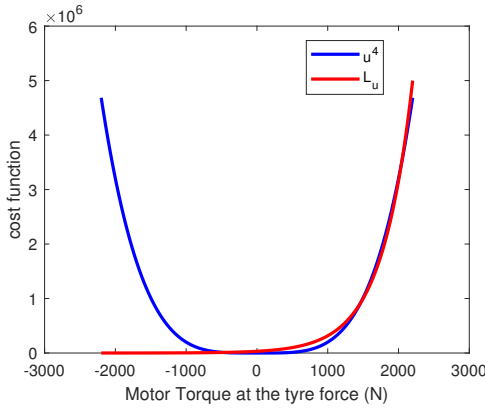


Fig. 3. Comparison of LINEX and Quadratic terms.

responses under both the optimal control allocation (CA) strategy (Section III-B) and its energy-efficient variant (CA-EXP, Section III-C) are nearly identical in terms of stability, only the CA results are presented in Fig. 4 for clarity. This is demonstrated in Fig. 5, where front-left wheel torques are taken as an example to showcase motor, friction brake and total torques differences between the two strategies. It can be seen that the total torques are nearly identical. The main difference lies in torque distribution: CA-EXP favors regenerative braking over friction braking, with these differences discussed further in Section IV-C.

Fig. 4(a) shows the yaw rate error and sideslip angle, where the baseline strategy fails to maintain stability—both metrics exceed safe thresholds. Figs. 4(b) and (c) present motor and brake torque commands (denoted as CMD in dashed lines) versus actual delivered torques (solid lines), revealing actuator saturation. The baseline strategy commands torques beyond the available tyre force limits, resulting in excessive slip ratios and reduced stability. In contrast, the CA strategy automatically balances left and right torques, minimizing undesired yaw moments and enabling better tracking of the zero yaw rate command from the driver in this scenario.

B. Single-Lane Change

A single-lane change scenario is used to evaluate the proposed optimal control allocation strategies. The steering input is governed by a Carmaker (CM) driver model, and 70% of full brake demand is applied during the maneuver at a coasting speed of 90 km/h. Simulation results are shown in Fig. 6. As seen in Figs. 6(a) and (d), the optimal strategy significantly improves vehicle stability, with reduced yaw rate error, sideslip angle, and wheel slip ratios. Additionally, the baseline strategy exhibits large discrepancies between commanded and actual brake torque due to the absence of brake rate constraints. Without total brake torque limits, it also requests more torque than the friction brake can deliver, resulting in saturation, particularly evident in the front-left wheel.

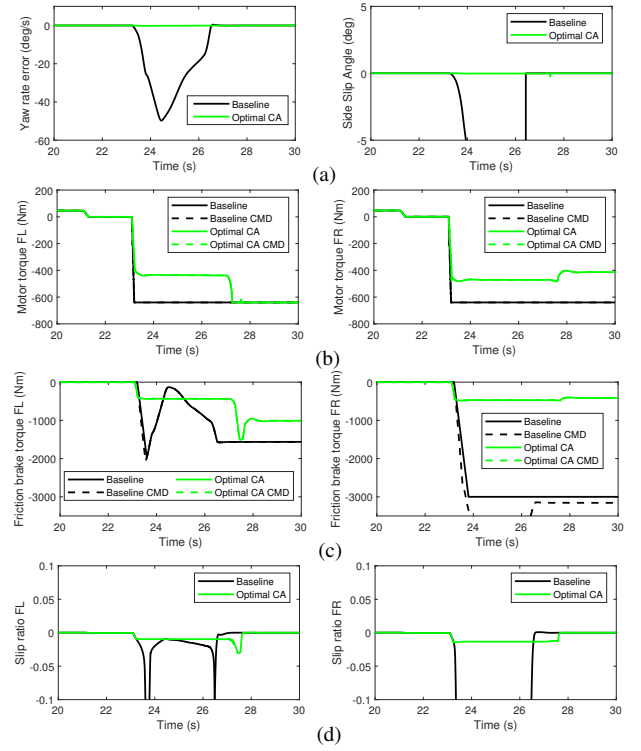


Fig. 4. Simulation results of the open-loop brake steering scenario

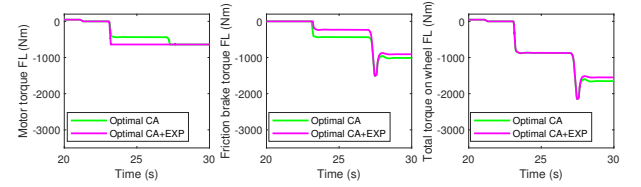


Fig. 5. Example front-left wheel motor, friction brake, and total torques comparison between the CA and CA+EXP strategies.

C. Hockenheim Racetrack

A closed-loop simulation on the Hockenheim racetrack is conducted to evaluate the energy efficiency of the proposed control allocation strategies. A CM driver model is used to maintain a cruising speed of 100 km/h. The simulation results are shown in Fig. 7. While the optimal control allocation improves stability in extreme conditions, it results in higher energy consumption compared to the baseline. However, by incorporating the exponential-based cost function, the energy-efficient variant achieves same energy consumption to the baseline under normal driving conditions, thus maximising energy recuperation.

V. CONCLUSION

This paper presented a novel optimal control allocation strategy for electric vehicles equipped with in-wheel motors and a brake-by-wire system, focusing on enhancing vehicle stability and maximizing energy recovery through regenerative braking. The proposed strategy integrates real-time optimisation to dynamically allocate tractive and braking torques, while

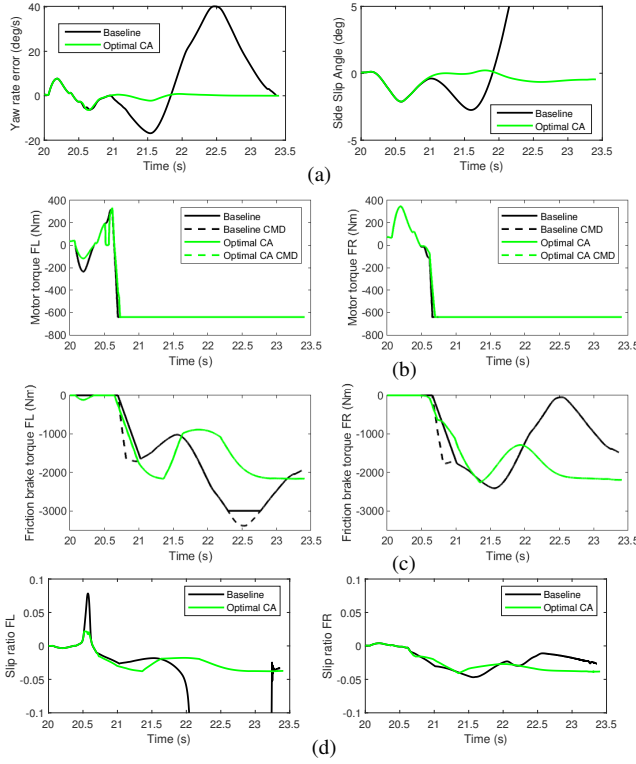


Fig. 6. Simulation results of the single-lane change scenario

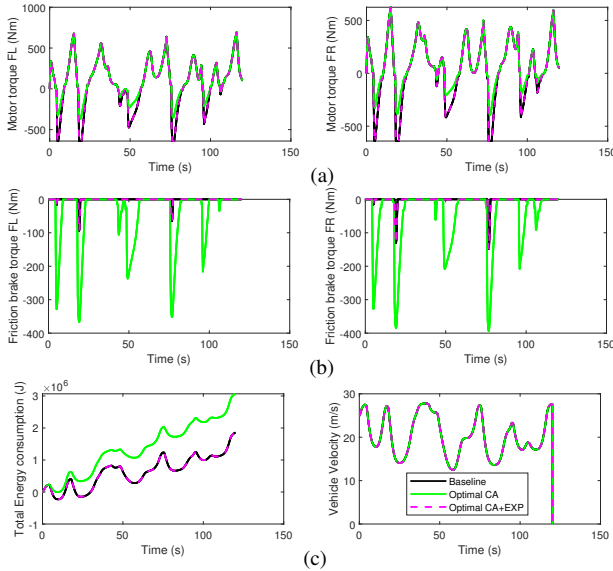


Fig. 7. Simulation results of the closed-loop Hockenheim racetrack

accounting for the dynamic constraints of tyres, motors, and friction brakes. An asymmetrical cost function is employed to prioritize regenerative braking torque, thereby improving energy efficiency without compromising vehicle performance. Simulation results across various scenarios—including split- μ braking, single-lane change, and closed-loop racetrack driving—demonstrate the effectiveness of the proposed approach. Compared to a baseline strategy, the optimal control allocation significantly enhances vehicle stability under extreme ma-

noeuvres and maintains comparable energy efficiency during normal driving conditions. Compared to the optimal control allocation strategy, the extended asymmetrical cost function strategy further increases energy recovery by distributing more regenerative braking over friction braking. The final strategy is therefore able to handle any scenario across the full envelope of vehicle operation without compromising on safety or energy efficiency. For future work, the control allocation framework can be extended to handle motor failure cases by simply applying zero constraints to the affected actuators. Additionally, the flexibility of the formulation allows for straightforward adaptation to various powertrain topologies through modifications to the B matrix. Incorporating battery capacity and C-rate limitations into the control allocation problem will also be explored to ensure energy-efficient operation without violating battery constraints.

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