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Multistatic Frequency and Polarimetry Adaptive Autofocus for Synthetic Aperture Radar Imaging of a Moving Target

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Correspondence: Anmol Rattan (anmol.rattan@cranfield.ac.uk)**Received:** 27 March 2026 | **Revised:** 14 May 2026 | **Accepted:** 5 June 2026**Keywords:** inverse synthetic aperture radar (ISAR) | motion compensation | motion estimation | multistatic radar | radar imaging | synthetic aperture radar | target tracking

ABSTRACT

Synthetic Aperture Radar (SAR) images of moving targets are often displaced and defocused, making reliable localisation and recognition difficult. The problem becomes more pronounced when target behaviour departs from simple motion assumptions and is better described by more complex kinematics, such as six-degrees-of-freedom (6-DoF) motion. Although many autofocus methods rely on subaperture or full-aperture formulations, such approaches can become restrictive when motion varies nonlinearly over the aperture and finer compensation is required. Multistatic SAR provides additional receiver diversity and, when combined with frequency and polarimetric diversity, it helps constrain this problem; however, such information is not usually exploited explicitly in pulse-by-pulse autofocus. This paper introduces a multistatic, frequency- and polarimetry-adaptive autofocus algorithm (F-Pol) that extends Localised Threshold Sharpness (LTS) into the joint frequency–polarimetric domain by encouraging agreement across frequency sub-bands and polarisation channels. Experimental validation using the Cranfield University Ground-Based SAR system with emulated target motion shows that, across the tested signal-to-noise ratio (SNR) range of -5 to 20 dB, F-Pol gives improved image quality and target motion estimation relative to Intensity-Squared (ISQ) and LTS. Laboratory and supplementary simulation results indicate that joint frequency and polarimetric diversity enhances multistatic SAR autofocus performance.

1 | Introduction

Synthetic Aperture Radar (SAR) provides high-resolution imaging in all-weather, day–night conditions [1, 2] and is widely used in remote sensing, surveillance and environmental monitoring [3–9]. Its counterpart is inverse SAR (ISAR) [10–12]. SAR is limited by the assumption of a stationary scene during aperture synthesis [13]. When this assumption is violated, scene motion introduces phase errors that cause distortion and defocus [14–17], necessitating autofocus. The challenge is particularly acute for noncooperative targets with unknown motion. Realistic target dynamics may involve six degrees of freedom

(6-DoF)—three translations and roll, pitch and yaw—which generate geometry-dependent, nonlinear phase errors [18–23] that limit conventional autofocus.

Classical frequency-domain image formation methods, such as the Polar Format Algorithm (PFA), Range–Doppler Algorithm (RDA) and Chirp Scaling Algorithm (CSA) [24, 25], are efficient under far-field, simple-motion assumptions but are less suited to the space-variant errors in the near-field produced by complex 6-DoF motion. Time-domain back-projection (BPA) [26] offers greater geometric flexibility for arbitrary bistatic and multistatic

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geometries but still assumes a stationary scene; motion-induced pulse misalignment can produce smeared and displaced SAR imagery, requiring dedicated motion compensation. Motion correction approaches are generally divided into motion-aided (e.g., inertial-sensor-based) and data-driven autofocus methods [27–29].

Motion-aided approaches need access to the target, often unavailable in noncooperative scenarios. Data-driven autofocus infers corrections by optimising a sharpness measure, but many assume point scatterers or simplified motion and struggle with extended targets, particularly those in the near-field, where wavefront curvature produces strongly space-variant errors. SAR near-field is defined as the regime where the slant range $d < 2D_{cr}^2/\lambda_c$ for a scene with cross-range extent D_{cr} and centre wavelength λ_c ; analogous to the Fresnel diffraction limit [30, 31]. Existing autofocus methods such as Phase Gradient Autofocus (PGA) [32, 33], entropy minimisation [34, 35] and contrast maximisation [36, 37] address motion errors but are typically limited to low-order corrections and not designed for pulse-by-pulse operation. Extending autofocus to multistatic configurations remains challenging, although prior studies have shown that additional spatial or spectral diversity can improve tolerance to noise and motion ambiguity [38–41].

Multistatic SAR increases aspect coverage and sensitivity to motion-induced phase variations [19, 39, 42]. Polarimetric SAR (PolSAR) provides complementary information by encoding scattering behaviour across polarisations [43–45], yet the use of polarimetry in multistatic SAR autofocus remains limited. Joint exploitation of frequency, polarisation and receiver diversity therefore offers a promising basis for motion estimation [38, 46–48].

Localised Threshold Sharpness (LTS) is a recent multistatic autofocus metric capable of compensating arbitrary 6-DoF motion [19] but does not fully exploit frequency or polarimetric diversity. This paper therefore proposes Frequency- and Polarimetry-Adaptive Autofocus (F-Pol), which extends LTS by incorporating frequency segmentation and polarimetric weighting based on target scattering behaviour. The criterion jointly exploits amplitude and phase across frequency, polarisation and receiver dimensions to refine 6-DoF motion estimation in multistatic SAR.

The proposed framework is validated using experimental multistatic SAR data acquired with the Cranfield University Ground-Based SAR (GBSAR) system. Performance is compared with the intensity-squared (ISQ) and LTS metrics and assessed using correlation, image contrast and image entropy [49]. Section 2 introduces the signal model, followed by the target motion model in Section 3. The proposed F-Pol autofocus framework is presented in Section 4. Section 5 outlines the multistatic radar system and target scene, whereas Section 6 details the hybrid SAR/ISAR data acquisition methodology, experimental setup and supporting simulation framework. Autofocus evaluation metrics are defined in Section 7. Section 8 presents and discusses the experimental and simulation results. Section 9 concludes the paper with a summary of the main findings and directions for future work.

2 | Signal Model

For a given pulse $n \in \{1, \dots, N_{\text{pul}}\}$, let $\mathbf{p}_{t,n}$ and $\mathbf{p}_{r,n}$ denote positions of a single transmitter t and multiple receivers $r \in \{1, \dots, R\}$ in a global coordinate frame, respectively. The target centre $\mathbf{O}$ defines the scene reference point. Considering a target composed of discrete scatterers $\{k = 1, \dots, K\}$, the bistatic range b to each target scatterer k , using known transmitter and receiver positions, can be defined as:

$$b(r, n; k) = \|\mathbf{p}_{t,n} - \mathbf{a}_n^k\| + \|\mathbf{p}_{r,n} - \mathbf{a}_n^k\|, \quad (1)$$

where scatterer position $\mathbf{a}_n^k \equiv \mathbf{a}_0^k$ for a stationary target. For a given frequency f , phase ϕ_k of target scatterer k with reflectivity $\sigma(r, n; k)$ and its corresponding signal return s are:

$$\phi_k(f, r, n) = \frac{2\pi f}{c} b(r, n; k), \quad (2)$$

$$s(f, r, n; k) = \sigma(r, n; k) \exp[-i\phi_k(f, r, n)], \quad (3)$$

where c is the speed of light. Superposition of signal returns of all target scatterers results in the full signal as:

$$s(f, r, n) = \sum_k s(f, r, n; k). \quad (4)$$

Unknown target motion perturbs the initial bistatic range $b(r, n; k)$ to each scatterer, introducing a bistatic path-length error $\Delta b(r, n; k)$ such that the measured range becomes:

$$\tilde{b}(r, n; k) = b(r, n; k) + \Delta b(r, n; k) \quad (5)$$

The corresponding phase error at frequency f is:

$$\Delta\phi_k(f, r, n) = \frac{2\pi f}{c} \Delta b(r, n; k). \quad (6)$$

The phase error in Equation (6) is frequency-dependent and varies across scatterers indexed by k , with magnitude determined by the residual range error induced by target motion. Its effect on image quality also depends on imaging resolution: finer resolution requires tighter phase consistency during coherent integration, increasing sensitivity to residual phase errors. Consequently, perturbations that are negligible at coarse resolution can become a dominant source of blur or displacement as resolution increases. To mitigate these effects, back-projection imaging (BPA) [26] was employed. Its use of radar trajectory knowledge for pixel-based, pulse-by-pulse image formation enables localised phase correction, making it effective for addressing spatially and spectrally varying errors induced by target motion. This capability underpins the motion compensation and emulation strategies developed in the following sections.

3 | Target Motion Model

Accurate modelling of target motion is essential for autofocus in SAR imaging, particularly when dealing with noncooperative targets exhibiting complex dynamics. In this work, target motion is represented using a 6-DoF rigid-body model, which captures both translational and rotational components of motion. For each pulse n , $\mathbf{T}_n$ represents the translations ($T_{x,n}$, $T_{y,n}$, $T_{z,n}$) along all three orthogonal axes and $\mathbf{R}_n$ is a 3-by-3 rotation

matrix formed from the angles θ_n ($\theta_{x,n}$, $\theta_{y,n}$, $\theta_{z,n}$) about the x , y and z axes:

$$\mathbf{R}_n = \mathbf{R}_{x,n}(\theta_{x,n})\mathbf{R}_{y,n}(\theta_{y,n})\mathbf{R}_{z,n}(\theta_{z,n}), \quad (7)$$

where $\mathbf{R}_x$, $\mathbf{R}_y$ and $\mathbf{R}_z$ denote rotations about the x , y and z axes, respectively.

Under the rigid-body assumption, all scatterers maintain fixed relative positions in the target coordinate frame, so that the same translation and rotation are applied consistently across the entire target. The 6-DoF formulation provides a general representation of realistic target motion, accommodating arbitrary combinations of translation and rotation. Pure translation models cannot capture aspect-dependent Doppler effects introduced by target reorientation, whereas pure rotation models neglect bulk range variation arising from linear displacement. The combined rigid-body model therefore enables a unified treatment of these effects.

For a scatterer initially located at position $\mathbf{a}_n^k$, its new position $\mathbf{a}_n'^k$ following 6-DoF motion at pulse n is given by:

$$\mathbf{a}_n'^k = \mathbf{R}_n(\mathbf{a}_n^k - \mathbf{O}) + \mathbf{O} + \mathbf{T}_n. \quad (8)$$

The instantaneous bistatic range to this moving scatterer, for transmitter and receiver positions $\mathbf{p}_{t,n}$ and $\mathbf{p}_{r,n}$, is:

$$b'(r, n; k) = \|\mathbf{p}_{t,n} - \mathbf{a}_n'^k\| + \|\mathbf{p}_{r,n} - \mathbf{a}_n'^k\|. \quad (9)$$

Consistent with the signal model introduced in Section 2, target motion induces changes in bistatic range, leading to spatially varying phase errors across the target scene. These variations are generally nonlinear and can significantly degrade image focus if not compensated.

3.1 | Target Motion-Inverted SAR Trajectories

For a stationary target, the bistatic range measurements corresponding to target motion can be reproduced by applying the inverse of the target motion to the initial SAR trajectories $\mathbf{p}_{t,n}$ and $\mathbf{p}_{r,n}$. This yields the target motion-inverted SAR trajectories $\mathbf{p}_{t,n}^{\text{inv}}$ and $\mathbf{p}_{r,n}^{\text{inv}}$, which produce bistatic ranges equivalent to those obtained from a moving target observed using the initial radar geometry.

The inverse rotation matrix is given by:

$$\mathbf{R}_n^{-1} = \mathbf{R}_{z,n}(-\theta_{z,n})\mathbf{R}_{y,n}(-\theta_{y,n})\mathbf{R}_{x,n}(-\theta_{x,n}). \quad (10)$$

The corresponding transformed transmitter and receiver positions are:

$$\mathbf{p}_{t,n}^{\text{inv}} = \mathbf{R}_n^{-1}(\mathbf{p}_{t,n} - \mathbf{T}_n - \mathbf{O}) + \mathbf{O}, \quad (11)$$

$$\mathbf{p}_{r,n}^{\text{inv}} = \mathbf{R}_n^{-1}(\mathbf{p}_{r,n} - \mathbf{T}_n - \mathbf{O}) + \mathbf{O}. \quad (12)$$

This formulation enables motion compensation through trajectory transformation rather than direct global phase-error correction. For extended targets, motion-induced phase errors

vary spatially across scatterers, so a single global phase correction is generally insufficient and can lead to residual defocus.

3.1.1 | Implications for Autofocussing

The target motion-inverted formulation supports autofocus within BPA image formation. Since target motion is represented through the transformed radar trajectories, candidate motion parameters can be evaluated by applying the corresponding trajectory transformations during image reconstruction and assessing the resulting image focus.

This approach accommodates nonuniform and nonlinear motion (i.e., varying directions and magnitudes of translations and/or rotations between pulses). Unlike frequency-domain SAR processing approaches, where rotation-induced Doppler effects can produce cross-range scaling distortions requiring explicit correction [50], the time-domain back-projection framework accounts for such geometric effects through trajectory-based reconstruction [26]. By applying trajectory transformations based on the estimated 6-DoF motion, rotational motion is incorporated directly into image formation without requiring a separate scaling correction stage.

The trajectory inversion acts independently at each pulse, enabling arbitrary motion sequences, including acceleration, jerk or oscillatory behaviour. Consequently, autofocus can be formulated as an optimisation problem over the 6-DoF motion parameters, where the objective is to identify the motion yielding the most consistent and well-focused image.

4 | Frequency and Polarimetry Adaptive (F-Pol) Autofocus

The proposed F-Pol autofocus method builds on the Localised Threshold Sharpness (LTS) metric introduced in [19], which establishes a framework for pulse-by-pulse autofocus of multistatic SAR imagery. An overview of F-Pol autofocus is shown in Figure 1.

A single-pulse SAR image formed from measurements of a single transmitter pulse n , collected across all receivers and reconstructed via BPA, is denoted as L_n . Each L_n therefore represents a multistatic SAR image corresponding to one transmitter pulse. Coherent summation of these images over

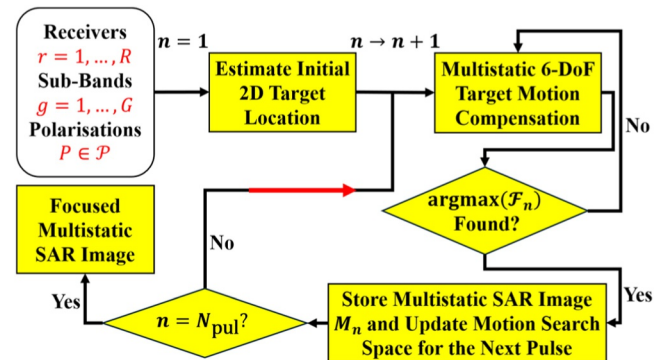


FIGURE 1 | Overview of frequency and polarimetry adaptive (F-Pol) autofocus.

pulses up to n produces the cumulative multistatic SAR image M_n . LTS quantifies the sharpness of a candidate subaperture image by measuring image-energy concentration around the peak pixel within a local neighbourhood. This metric guides the pulse-by-pulse autofocus search while enforcing temporal consistency in the evolution of the estimated target location. For a given pulse n , the metric LTS_n is defined as:

$$LTS_n = \frac{I_{n,\max}}{\sum_{(i,j) \in W} |I_{n(i,j)}| - I_{n,\max} + \zeta |\mathbf{a}_{n-1}^\Lambda - \mathbf{a}_n^\Lambda|}, \quad (13)$$

where $I_{n,\max}$ is the maximum pixel intensity in the subaperture image M_n , W is a fixed window centred on this peak, $I_{n(i,j)}$ is the intensity of a pixel located at row i and column j of the windowed image, $\mathbf{a}_n^\Lambda$ and $\mathbf{a}_{n-1}^\Lambda$ represent the estimated image-domain locations of candidate target peaks indexed by Λ for pulses n and $n - 1$, respectively. Using bistatic radar-to-target ranges estimated from range profiles of each receiver per pulse, target peak locations were determined via multilateration [51]. The denominator contains two terms: the residual energy within the local window excluding the peak, which penalises spatial spreading, and a localisation regularisation weight ζ , which discourages abrupt spatial jumps between pulses, as in an earlier work [19]. The weight ζ is data-dependent and was determined empirically in this study through convergence analysis across candidate values; a value of 0.07 provided stable localisation without over-constraining the motion search. Autofocus is achieved by maximising this metric across candidate 6-DoF target motions, with the optimal update corresponding to the motion estimate yielding the sharpest and most consistent localisation.

In multichannel SAR imaging, the target response can vary across frequency sub-bands and polarisation channels due to differences in scattering behaviour, signal strength and noise. Although this variation provides useful diversity, it also introduces challenges for autofocus. An incorrect motion estimate may produce channel-dependent focussing behaviour, where a candidate solution appears favourable in one channel while yielding weaker focus or displaced responses in others. When channels are treated independently, this can lead to ambiguous or channel-specific autofocus optima.

LTS is effective in single-polarimetric settings but does not fully exploit the full range of information available in a multistatic SAR setting, where multiple receivers, frequency sub-bands and polarisation channels provide additional information. F-Pol leverages this by ensuring that correct focus produces consistent sharpness and colocated peaks across frequency and polarisation. Defocus may yield sharpness in one channel, but rarely across all channels simultaneously. By extending LTS into the frequency-polarimetry domain, F-Pol enforces cross-channel consensus and suppresses spurious optima that may arise when sharpness is evaluated using a single channel alone. This reduces the likelihood of solutions that appear favourable in only a subset of the data but are not consistent with the full multichannel observations.

To achieve this, the local window W_n is redefined. The original LTS formulation used a fixed rectangle around the global peak, which at early pulses could span a large region. In F-Pol, the

window is derived through threshold-based support extraction: starting at the maximum-intensity pixel, the adaptive window is formed by grouping neighbouring pixels above a normalised threshold. A -3 dB threshold, conventionally used in SAR to mark the half-power point, captures the main lobe while minimising sidelobe and noise contributions. As a result, W_n corresponds directly to the energy peak rather than a geometric box and adapts to the focus structure over pulses—broad when energy is spread early and narrow as focus sharpens. Since the same rules apply across channels, the adaptive window promotes consistency while keeping the sharpness measure parameter-light and data-driven.

With this refinement in place, the LTS metric is evaluated independently for each polarisation and frequency channel. The resulting per-channel metric $LTS_n^{(P,g)}$ is defined as:

$$LTS_n^{(P,g)} = \frac{I_{n,\max}^{(P,g)}}{I_{W_n}^{(P,g)} - I_{n,\max}^{(P,g)} + \zeta |\mathbf{a}_{n-1}^{\Lambda(P,g)} - \mathbf{a}_n^{\Lambda(P,g)}|}, \quad (14)$$

where $P \in \mathcal{P} = \{HH, HV, VH, VV\}$ indexes the polarisation channel and $g \in \{1, \dots, G\}$ indexes the frequency sub-band. $M_n^{(P,g)}$ denotes the complex multistatic SAR image obtained by coherently combining all receiver measurements for pulses 1 to n and $I_n^{(P,g)}$ is its intensity image. The intensity image is then defined as:

$$I_n^{(P,g)} = |M_n^{(P,g)}|^2. \quad (15)$$

$I_{W_n}^{(P,g)}$ denotes the sum of intensities of pixels within the adaptive window $W_n^{(P,g)}$:

$$I_{W_n}^{(P,g)} = \sum_{(i,j) \in W_n^{(P,g)}} |I_{n(i,j)}^{(P,g)}|. \quad (16)$$

The support extraction is performed independently for each channel, so the adaptive window $W_n^{(P,g)}$ may differ between polarisations and frequency sub-bands. In this study, the full 8–12 GHz measurement band is partitioned into two contiguous sub-bands: 8–10 GHz and 10–12 GHz ($G = 2$). If only a single frequency band and polarisation are available, the formulation reduces exactly to the original LTS metric, preserving backward compatibility. Frequency diversity is incorporated by recognising that bistatic path-length errors produce phase errors that scale linearly with frequency:

$$\Delta\phi(f) = \frac{2\pi f \Delta b}{c}, \quad (17)$$

where Δb is the bistatic path-length error. In practice, a target motion estimate that is not fully correct may improve focus in one sub-band more than in another, if the bands are treated independently. To favour solutions that perform consistently across frequency, the sharpness scores for G sub-bands of each polarisation are fused using the geometric mean:

$$\Xi_n^{(P)} = \left(\prod_{g=1}^G LTS_n^{(P,g)} \right)^{1/G}. \quad (18)$$

A high value of $\Xi_n^{(P)}$ is obtained when all sub-bands contribute favourably, thereby reducing the influence of any single sub-band on the combined score. Sharpness agreement alone, however, does not always distinguish between genuinely consistent focusing and cases in which different sub-bands favour slightly different peak locations. To discourage such solutions, a Gaussian penalty is applied to the spread of the estimated peak locations across sub-bands. Let $\mathbf{a}_n^{\Lambda(P,g)}$ denote the 2D image-domain peak-location vector for sub-band g , polarisation P and pulse n :

$$\mathbf{a}_n^{\Lambda(P,g)} = \begin{bmatrix} x_n^{\Lambda(P,g)} \\ y_n^{\Lambda(P,g)} \end{bmatrix}. \quad (19)$$

Let $\bar{x}_n^{\Lambda(P)}$ and $\bar{y}_n^{\Lambda(P)}$ denote the corresponding centroid coordinates across the G sub-bands:

$$\bar{x}_n^{\Lambda(P)} = \frac{1}{G} \sum_{g=1}^G x_n^{\Lambda(P,g)}, \quad (20)$$

$$\bar{y}_n^{\Lambda(P)} = \frac{1}{G} \sum_{g=1}^G y_n^{\Lambda(P,g)}. \quad (21)$$

The cross-band peak spread is then defined as:

$$H_{x,n} = \frac{x_n^{\Lambda(P,g)} - \bar{x}_n^{\Lambda(P)}}{\delta_{rr}}, \quad (22)$$

$$H_{y,n} = \frac{y_n^{\Lambda(P,g)} - \bar{y}_n^{\Lambda(P)}}{\delta_{cr}}, \quad (23)$$

$$H_n^{(P)} = \frac{1}{G} \sum_{g=1}^G [H_{x,n}^2 + H_{y,n}^2], \quad (24)$$

where δ_{rr} and δ_{cr} are the range- and cross-range image sampling intervals, respectively. This normalisation yields a dimensionless measure of cross-band peak misalignment that accounts for the different spatial sampling along the two image axes. The corresponding frequency-consensus penalty is then defined as:

$$\Gamma_n^{(P)} = \exp\left(-\frac{H_n^{(P)}}{2Y^2}\right), \quad (25)$$

where Y is a dimensionless tolerance parameter controlling the permitted spread of cross-band peak locations. In this study, $Y = 0.75$ was selected empirically, allowing small cross-band peak offsets while penalising larger discrepancies associated with less consistent focussing. The resulting frequency-consensus (Z) score for polarisation P is then:

$$Z_n^{(P)} = \Xi_n^{(P)} \cdot \Gamma_n^{(P)}. \quad (26)$$

The use of two contiguous 2 GHz sub-bands is intended as a practical design choice for this dataset rather than a universal optimum. Partitioning the 8–12 GHz band in this way introduces an additional cross-band consistency check on the motion estimate while retaining sufficient bandwidth in each sub-band for stable sharpness evaluation. Using more sub-bands could provide additional diversity, but would also reduce the bandwidth, and typically the signal-to-noise ratio, available to

each sub-band, which may make the per-band autofocus metric less stable. The appropriate number of sub-bands is therefore application dependent. In the present study, two sub-bands were found to provide a useful balance between per-band discriminability and cross-band consistency. For the chosen partition, the sub-band centre frequencies are 9 and 11 GHz. Their separation introduces a measurable difference in phase sensitivity: a bistatic path-length error (Δb) of 2 cm produces an interband phase difference of approximately 48° :

$$\Delta\phi = \frac{2\pi\Delta f\Delta b}{c} \approx 48^\circ.$$

This difference means that an incorrect motion estimate may improve focus in one sub-band without producing comparable agreement in the other, whereas a more consistent estimate would be expected to yield both strong responses and close peak colocation across bands. This behaviour motivates the introduction of frequency diversity into the autofocus score.

The corresponding bistatic cross-range resolutions are slightly different: approximately 15.1 and 12.4 cm at 9 and 11 GHz, respectively, for Receiver 1 and 16.9 and 13.1 cm for Receiver 2. These values are sufficiently similar in the present configuration for the dominant image responses to be meaningfully compared across bands, whereas the difference in centre frequency still provides distinct phase sensitivity. Equal-width contiguous sub-bands were therefore adopted here to preserve comparable image-formation behaviour in each band while supporting a useful cross-band consistency check.

Alternative partitions illustrate the same trade-off. Strongly overlapping sub-bands would provide less additional phase diversity, whereas widely separated bands may reduce cross-band comparability because scattering behaviour can vary strongly with frequency. The selected 8–10 and 10–12 GHz partition was therefore adopted as a pragmatic compromise: the bands are sufficiently separated to provide useful differential phase sensitivity, yet close enough to support comparable peak localisation and joint autofocus evaluation.

Polarimetric diversity provides complementary multichannel information for autofocus because target responses often vary across polarisation channels. In a well-focused image, corresponding target features tend to remain spatially aligned across channels within a local image region, whereas defocus can reduce this consistency. F-Pol captures this behaviour by evaluating the local multichannel structure of the polarimetric responses within adaptive windows derived for each channel. To ensure consistent spatial sampling across channels, only image pixels retained by the adaptive windows of all considered polarisations are included. The resulting common image support for sub-band g is therefore defined as:

$$\Omega_n^{(g)} = \bigcap_P W_n^{(P,g)}. \quad (27)$$

At each pixel (i,j) within this common image support, the polarimetric scattering vector is written as:

$$\mathbf{S}_n(i,j) = [S_n^{\text{HH}}(i,j), S_n^{\text{HV}}(i,j), S_n^{\text{VH}}(i,j), S_n^{\text{VV}}(i,j)]^T, \quad (28)$$

where the entries denote the coregistered complex image values for the four polarisation channels used in this study. The following construction is developed for these four coregistered channels. A multichannel covariance matrix for sub-band g is then formed over the common image support as:

$$\mathbf{C}_n^{(g)} = \frac{1}{|\Omega_n^{(g)}|} \sum_{(i,j) \in \Omega_n^{(g)}} \mathbf{S}_n(i,j) \mathbf{S}_n(i,j)^\dagger, \quad (29)$$

here $|\Omega_n^{(g)}|$ denotes the number of pixels in the common image support and $(\cdot)^\dagger$ denotes the conjugate transpose. In this form, $\mathbf{C}_n^{(g)}$ summarises the local interchannel structure over the retained support.

Let $\{\lambda_{\mathbf{C}_n^{(g)},k}\}_{k=1}^4$ denote the eigenvalues of $\mathbf{C}_n^{(g)}$. These quantify how strongly the local multichannel response is concentrated into a small number of dominant channel-combination modes. For a well-focused rigid target, the response may be expected to be more strongly concentrated in a dominant mode, whereas defocus or inconsistent channel alignment can spread energy across multiple modes. Motivated by this behaviour, the following concentration score is defined:

$$U_n^{(g)} = \frac{\lambda_{\max}(\mathbf{C}_n^{(g)})}{\text{Tr}(\mathbf{C}_n^{(g)})}. \quad (30)$$

This score is evaluated for $\text{Tr}(\mathbf{C}_n^{(g)}) > 0$, where $\lambda_{\mathbf{C}_n^{(g)},1}$ is the largest eigenvalue. For the four-channel case used here, this score lies between 0.25 and 1, with larger values indicating stronger concentration into a dominant mode. To promote consistency across frequency, this concentration score is combined across the G sub-bands using a geometric mean:

$$U_n = \left(\prod_{g=1}^G U_n^{(g)} \right)^{1/G}. \quad (31)$$

This ensures the combined score remains high only when both sub-bands contribute favourably. The final F-Pol ($\mathcal{F}$) autofocus objective combines the frequency and polarimetric-consensus sharpness terms as:

$$\mathcal{F}_n = \sum_P Z_n^{(P)} \cdot (U_n)^\eta, \quad (32)$$

where $Z_n^{(P)}$ denotes the frequency-consensus sharpness score defined in Equation (26) for polarisation P and $\eta \in [0.5, 1]$ balances sharpness against multichannel concentration. Smaller η reduces the influence of the multichannel term when the interchannel structure is weak, whereas larger η enforces stronger cross-channel consistency. In practice, $\eta = 0.75$ is adopted as a conservative compromise: it is strong enough to suppress spurious sharpness that does not align across channels, but not so strong as to reject valid but weakly structured polarimetric responses. In this way, polarimetric diversity acts as a second consistency check: whereas the sharpness term favours spatial concentration, the multichannel concentration term favours agreement in the local polarimetric response structure.

Assuming no prior knowledge of target motion at this stage, motion compensation proceeds using the target motion-inverted trajectory method described in Section 3, whereby the SAR trajectories are transformed according to candidate 6-DoF target motions until the F-Pol objective is maximised:

$$(\widehat{\mathbf{T}}_n, \widehat{\boldsymbol{\theta}}_n) = \text{argmax}\{\mathcal{F}_n(\mathbf{T}_n, \boldsymbol{\theta}_n)\}. \quad (33)$$

This process is repeated pulse-by-pulse, progressively refining the target motion estimate while coherently combining successive single-pulse images. During autofocus, the precise target location is not assumed a priori. Instead, rotational motion (roll, pitch and yaw) is parameterised about the initial target peak location $\mathbf{a}_1$, identified in the image plane from the single-pulse SAR images formed across all receivers for the first transmitted pulse. This provides a practical reference for the rotational update; when the estimated target peak lies close to the target centre, the estimated rotations are expected to approximate the true target rotations about its own centre.

Optimisation of target peak positions and 6-DoF target motion parameters was performed using Particle Swarm Optimisation (PSO), which is well suited to multidimensional search problems [52]. For 6-DoF target motion estimation, a blind-search formulation was adopted: the algorithm does not assume prior knowledge of the true motion but instead explores the solution space using the autofocus objective. Search bounds were constrained to remain physically plausible, with translational limits of up to 0.5 m in each axis and rotational limits of $\pm 15^\circ$ in roll, pitch and yaw. These ranges were chosen to encompass plausible laboratory motions while excluding implausible platform-target geometries. PSO therefore remains exploratory, but within a physically plausible constrained search space. To improve computational efficiency, BPA image formation and evaluation of the F-Pol objective were parallelised within the PSO search.

F-Pol searches the same 6-DoF motion space as LTS but evaluates candidate solutions under stricter consistency requirements. Its formulation follows the local sharpness principle of LTS, which assesses focus through the compactness of image energy around a dominant target feature. F-Pol extends this by requiring sharpness to remain consistent across multiple frequency sub-bands and polarisation channels. In effect, a candidate motion estimate must not only yield sharp local focus, as in LTS, but must also remain consistent across all available channels. This additional constraint does not reduce the dimensionality of the unknown motion space, but it does narrow the set of acceptable solutions by rejecting candidates favourable in only a subset of channels. The resulting redundancy helps suppress spurious optima while still allowing PSO to explore the 6-DoF motion space effectively.

The design of F-Pol offers several practical advantages. Like LTS, it preserves interpretability in the single-channel limit. It avoids user-defined window sizes or arbitrary inter-band weighting, instead relying on adaptive threshold-based support extraction together with cross-band peak consistency and multichannel polarimetric concentration. By jointly enforcing sharpness and consistency across sub bands and channels, F-Pol

helps suppress spurious autofocus solutions. It operates directly in the image domain and on a pulse-by-pulse basis. As more pulses are accumulated, the cross-band and cross-channel consistency measures become more informative. These features make F-Pol a physically grounded and practically implementable autofocus framework for multistatic SAR.

5 | Multistatic Radar System and Scene

This study used both simulated and experimental data. The experimental work was conducted using the Cranfield University GBSAR laboratory facility, illustrated in Figure 2. This system is designed to capture bistatic SAR measurements through a Vector Network Analyser (VNA) linked to ultra-wideband horn antennas serving as transmitter (Tx) and receiver (Rx). The antennas are mounted on independent two-dimensional (2D) scanning mechanisms, enabling motion in separate vertical planes. Each scanner is installed on a precise rail system that defines the spatial aperture extent of each antenna. For each scanner window, the transmitter aperture spans 3.5 m in width and 1.45 m in height, whereas the receiver aperture covers 2.4 m by 1.45 m. Individually, each provides a 2D aperture; together, their coordinated operation enables bistatic four-dimensional (4D) spatial sampling.

The multistatic SAR geometry and target scene used in this study can be seen in Figure 3. As described in Section 3, the transformed version of this geometry is used to emulate the data collection of a moving target. Hence, it is also known as the initial SAR geometry. The setup comprised two bistatic configurations: a single transmitter moving along a 1 m horizontal path, and two stationary receivers. Although the laboratory houses only one physical receiver, data acquired sequentially at two receiver locations under identical, phase-coherent system settings yields a combined dataset equivalent to a simultaneous two-receiver multistatic acquisition. Full-polarisation (HH, HV, VH, VV) measurements were acquired over an 8–12 GHz frequency range (bandwidth $B = 4$ GHz, centre frequency $f_c = 10$ GHz). Simulation work employed a point-scatterer representation of a boat radar cross-section (RCS) model used in the laboratory trials, shown in Figure 4. Both the simulated and laboratory targets were dimensioned 1.37 m (length) $\times$ 0.23 m (width) $\times$ 0.36 m (height).

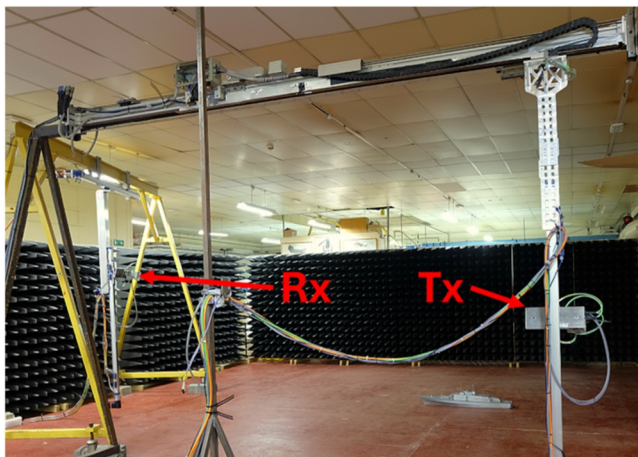


FIGURE 2 | Cranfield University GBSAR laboratory system.

To ensure consistent range referencing across channels, a bistatic-equivalent monostatic (BEM) framework was applied. For the initial multistatic SAR geometry in Figure 3, the resulting BEM trajectories (shown in magenta) yield azimuth angle apertures ϕ_w of 7.1° and 7.4° , and mean bistatic angles $\bar{\beta}$ of 54.8° and 72.2° for receivers 1 and 2 respectively. These correspond to theoretical bistatic range resolutions $\delta x_{rr,b}$ of approximately 4.2 and 4.6 cm, and cross-range resolutions $\delta y_{cr,b}$ of 13.6 and 14.4 cm at bistatic slant ranges of ~ 4.1 and ~ 4.2 m from the target centre for Receivers 1 and 2, respectively. For the geometry shown in Figure 3, the effective cross-range extent of the boat approximately corresponds to its broadside length. Substituting this extent into the near-field criterion $d < 2D_{cr}^2/\lambda_c$ yields a far-field transition distance of ~ 125 m. As the experimental slant ranges are ~ 4 m, the measurements are firmly within the SAR near-field regime. Note that this classification is aspect-dependent; bow-on aspects would reduce the effective cross-range extent and shift the transition distance accordingly.

Although the bandwidth used in this study exceeds that of typical operational X-band SAR systems, this reflects the constraints and purpose of the laboratory experiment rather than an attempt to achieve full electromagnetic scaling. The laboratory experiment uses a 1.37 m long boat model as a controlled target for evaluating autofocus and motion-estimation performance. The operating frequency band of the Cranfield GBSAR system is fixed by the available hardware, preventing direct frequency scaling as commonly performed in model-scale radar experiments. Instead, the experiment was designed to preserve comparable sampling of the target in resolution cells. For the experimental configuration shown in Figure 3, the boat model spans approximately 30–33 range resolution cells or 10–11 azimuth cells. For comparison, TerraSAR-X operates at 9.65 GHz with a 300 MHz bandwidth in

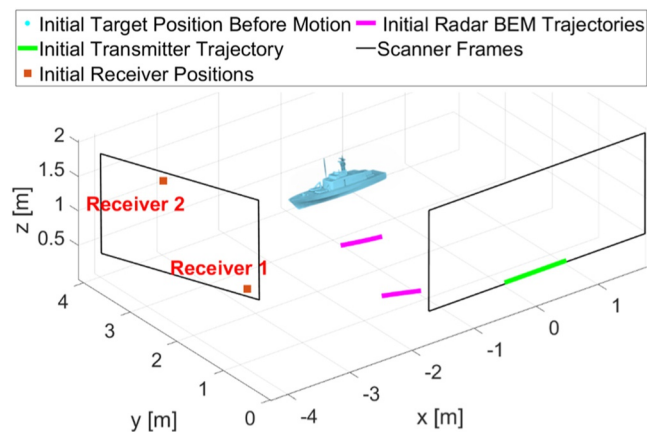


FIGURE 3 | Initial multistatic SAR geometry and target scene.

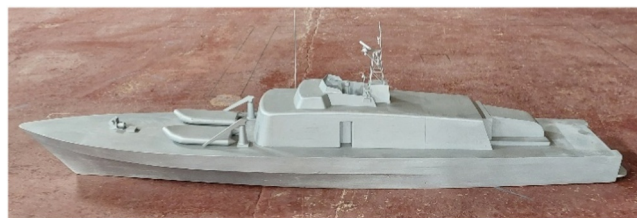


FIGURE 4 | Boat RCS model used in laboratory experimentation.

high-resolution spotlight mode [53], yielding range and azimuth resolutions of approximately 0.5 and 1 m, respectively. Under these conditions, a vessel of approximately 15 m length would span roughly 30 range resolution cells or about 15 azimuth resolution cells across the target.

This comparison is used solely to select an appropriate image grid resolution for autofocus evaluation and does not imply full electromagnetic equivalence between the scaled laboratory model and a full-scale maritime target. Bistatic and multistatic scattering responses depend on wavelength-to-target-size relationships, target geometry and observation configuration and cannot be fully recovered by increasing image resolution alone [54, 55]. Scattering centres and structural features may also vary with bistatic configuration and frequency. The experiment therefore uses the scaled model to obtain a controlled measured dataset for assessing whether the proposed autofocus method can produce focused and geometrically consistent imagery from multistatic measurements, rather than as a complete electromagnetic analogue of an operational full-scale vessel.

6 | Hybrid SAR/ISAR Data Acquisition

Hybrid SAR/ISAR motion emulation was used to generate SAR measurements representative of a moving target while keeping the physical target stationary during data collection. The principle is to acquire raw phase-history measurements of a stationary target by mathematically transforming the known radar platform trajectories to embed the inverse of a prescribed target motion. This produces virtual transmitter and receiver paths such that the resulting bistatic range measurements match those obtained from the multistatic SAR geometry in Figure 3 for an actual moving target. The dataset collected in the laboratory retains key characteristics of true radar acquisition—including hardware-induced imperfections and near-field propagation effects—while offering the controllability of a simulated motion model. However, because the physical target remains stationary, aspect-dependent multipath interactions with the surrounding environment are not reproduced.

6.1 | Target Motion Emulation

Radar experiments with mechanically actuated moving targets can lack repeatability. Driving large or complex trajectories with sub-wavelength accuracy across many pulses is mechanically demanding and expensive, whereas synchronisation with radar timing can also be difficult. Fully simulated data avoid these constraints but may sacrifice realism, often relying on idealised isotropic scatterers and simplified representations of multipath, shadowing, clutter, noise and timing jitter. Although higher-fidelity electromagnetic simulations can incorporate some of these effects, they typically incur significant computational cost and reduced flexibility when exploring complex motion models. Consequently, simulations may overestimate autofocus and motion compensation performance.

Laboratory hybrid SAR/ISAR emulation provides a practical compromise. Measurements of a stationary target are collected using real radar hardware and environment, whereas target motion is mathematically introduced through radar trajectory transformation. This enables realistic target dynamics without

mechanical actuation while preserving many nonidealised effects present in measured radar data. One limitation is that the stationary target fixes multipath interactions with surrounding objects. In true moving-target experiments, changing radar–target geometry would alter multipath strength and phase history, particularly near strong reflectors. The hybrid method therefore reproduces the direct-path component but not aspect-dependent multipath evolution. In the present study, this limitation is advantageous because it isolates autofocus behaviour from uncontrolled environmental variability.

Target motion was emulated using the target motion-inverted trajectory approach described in Section 3. In this study, motion was modelled as uniform increments between pulses. Over 100 pulses, the target simultaneously completed -0.2 , 0.15 , and 0.1 m translations along the x , y and z axes, and 5° roll, 2° pitch and 10° yaw rotations from a stationary position.

6.2 | Receiver Placement

Receiver placement plays an important role in the ability to localise a target accurately and infer its motion parameters required for autofocussing. As shown in Figure 3, a single transmitter illuminated the scene while the receivers remained fixed in distinct positions. One receiver was treated as a reference (Receiver 1 in this study) whereas the other receiver was positioned to maximise geometric diversity.

The first requirement is accurate localisation of scatterers. Using the bistatic range definition in (1), the Jacobian of the transmitter–receiver bistatic ranges from transmitter t and receivers r with respect to a scatterer k is:

$$\mathbf{J}_{\text{pos}}(k_n) = \begin{bmatrix} \frac{\partial b(r_1, n; k)}{\partial x} & \frac{\partial b(r_1, n; k)}{\partial y} & \frac{\partial b(r_1, n; k)}{\partial z} \\ \frac{\partial b(r_2, n; k)}{\partial x} & \frac{\partial b(r_2, n; k)}{\partial y} & \frac{\partial b(r_2, n; k)}{\partial z} \\ \vdots & \vdots & \vdots \end{bmatrix}, \quad (34)$$

with each row obtained by differentiating the bistatic range with respect to scatterer position:

$$\frac{\partial b(r, n; k)}{\partial \mathbf{a}_n^k} = \frac{\mathbf{a}_n^k - \mathbf{p}_{t,n}}{\|\mathbf{a}_n^k - \mathbf{p}_{t,n}\|} + \frac{\mathbf{a}_n^k - \mathbf{p}_{r,n}}{\|\mathbf{a}_n^k - \mathbf{p}_{r,n}\|}. \quad (35)$$

This corresponds to the sum of the transmitter-to-scatterer and receiver-to-scatterer line-of-sight unit vectors. Assuming Gaussian measurement errors with variance σ_b^2 , the Fisher Information Matrix (FIM) for scatterer localisation is:

$$\mathbf{F}_{\text{pos}}(k_n) = \frac{1}{\sigma_b^2} \mathbf{J}_{\text{pos}}^T \mathbf{J}_{\text{pos}}. \quad (36)$$

The Cramér–Rao Bound (CRB) then states that the covariance of any unbiased position estimator satisfies:

$$\text{Cov}(\hat{k}_n) \geq \mathbf{F}_{\text{pos}}^{-1}. \quad (37)$$

To scalarise this bound, we define the Position Dilution of Precision (PDOP) [56]:

$$\text{PDOP}(k_n) = \sqrt{\text{Tr}(\mathbf{F}_{\text{pos}}^{-1})}, \quad (38)$$

here Tr refers to the trace of the matrix. Small PDOP values indicate favourable geometry and low positional uncertainty, whereas large PDOP values—greater than 2—represent poor receiver geometry for reliable localisation of scatterers.

Accurate localisation alone does not ensure unambiguous motion estimation. Different 6-DoF trajectories can produce equally focused SAR images, leading to several issues. An ambiguous target motion estimation may yield sharp imagery for one receiver yet fail to maintain phase consistency with other receivers, causing coherence loss. Many autofocus pipelines update motion sequentially on a subaperture basis; an error at one stage can quickly propagate and corrupt later estimates. For tasks such as Automatic Target Recognition (ATR) or change detection, unique motion solutions are essential; otherwise, distinct kinematic behaviours may generate indistinguishable images, undermining reliable classification or comparison. Even basic interpretation—such as distinguishing translation from a turn—depends on unambiguous motion recovery. Let the motion state vector be:

$$\xi = [\Delta T_x, \Delta T_y, \Delta T_z, \Delta \theta_x, \Delta \theta_y, \Delta \theta_z]^T, \quad (39)$$

where the first three components represent translations of the target centre $\mathbf{O}$ and the final three correspond to small roll, pitch and yaw rotations. Differentiating the bistatic range with respect to these parameters yields the motion Jacobian $\mathbf{J}_{\text{mot}}(r, n)$. When stacked over receivers and pulses, this Jacobian describes the sensitivity of the measured ranges to all six motion parameters. The associated FIM is:

$$\mathbf{F}_{\text{mot}} = \frac{1}{\sigma_b^2} \sum_{r,n} \mathbf{J}_{\text{mot}}(r, n)^T \mathbf{J}_{\text{mot}}(r, n), \quad (40)$$

whose eigenvalue λ spectrum indicates how well different motion modes are constrained and distinguished by the measurements. A well-conditioned Fisher matrix, with eigenvalues of similar magnitude, implies that all six motion components are comparably well constrained; a poorly conditioned matrix suggests degeneracies, such as translations in the azimuth direction being confounded with rotations about the same axis. A useful scalar diagnostic is the condition number κ :

$$\kappa(\mathbf{F}_{\text{mot}}) = \frac{\lambda_{\max}(\mathbf{F}_{\text{mot}})}{\lambda_{\min}(\mathbf{F}_{\text{mot}})}, \quad (41)$$

where low values indicate favourable placement and high values indicate near-singular estimation geometry. The combined analysis of PDOP and Fisher conditioning can highlight receiver placements that enable both accurate scatterer localisation and target motion estimation.

Analytical criteria alone cannot fully capture practical ambiguity behaviour, so a Monte Carlo ambiguity analysis was performed. A region surrounding the boat model, consisting of 100 scatterers uniformly distributed throughout the target volume, was synthesised in simulation. For each possible receiver position on the laboratory receiver frame, target

motion-inverted trajectories were generated using both the true 6-DoF motion and randomly perturbed motion parameters over 100 trials. Bistatic range measurements to the target-region scatterers were then computed for both trajectory sets, and the absolute discrepancies between them were averaged across all pulses and trials. High mean range discrepancies indicate favourable receiver placement, as motions similar but distinct from the true motion produce distinguishable measurements, thereby supporting unambiguous estimation of the target motion parameters. The distribution of mean range discrepancies across the receiver frame therefore indicates whether a given receiver placement yields unique and stable solutions, or whether certain motions are frequently confused.

Figure 5 shows the ambiguity maps produced by this procedure, overlaid onto the laboratory receiver frame. The three perspectives—PDOP for localisation, Fisher conditioning for observability, and Monte Carlo analysis for global uniqueness—produce consistent preferred regions for Receiver 2. Final placement was selected from the intersection of low-PDOP and low- κ zones, restricted to areas exhibiting little global ambiguity under the Monte Carlo screen. Nonetheless, perfect global uniqueness for all possible motions cannot be guaranteed. Symmetries in geometry, finite apertures and limited look-angle diversity inevitably leave sets of motions that are nearly indistinguishable within tolerance.

Experimental feasibility further constrains receiver placement. Terrain, line-of-sight limits and the need for broad angular separation impose practical restrictions. Wider baselines can reduce coherence, whereas smaller separations reduce sensitivity to motion. The chosen configuration therefore reflects a compromise: receivers must remain deployable, particularly after target-motion inversion, while providing sufficient diversity. In the laboratory, the extent of the synthetic aperture frames meant only a subset of ideal placements were achievable; within these bounds, geometries that enabled emulation and accurate estimation of complicated 6-DoF target motion were prioritised.

6.3 | Experimental Radar Setup

Target motion-inverted SAR trajectories, although definable in simulation, can be impractical in the laboratory because they may extend beyond the limits of the 2D scanner frames. To address this, the trajectories were projected onto feasible scanner-frame paths. Measurements acquired along these paths were then range-corrected to reconstruct data equivalent to the original target motion-inverted trajectories. This section details the trajectory projection, range correction and implications for scatterer phase histories.

6.3.1 | Projected SAR Trajectories

To enable practical measurements, each target motion-inverted antenna coordinate was projected onto its respective 2D scanner frame using intersections of lines connecting the coordinate to the target centre $\mathbf{O}$. This projection converts the idealised motion-inverted radar trajectories into physically realisable paths compatible with the planar scanner geometry of the

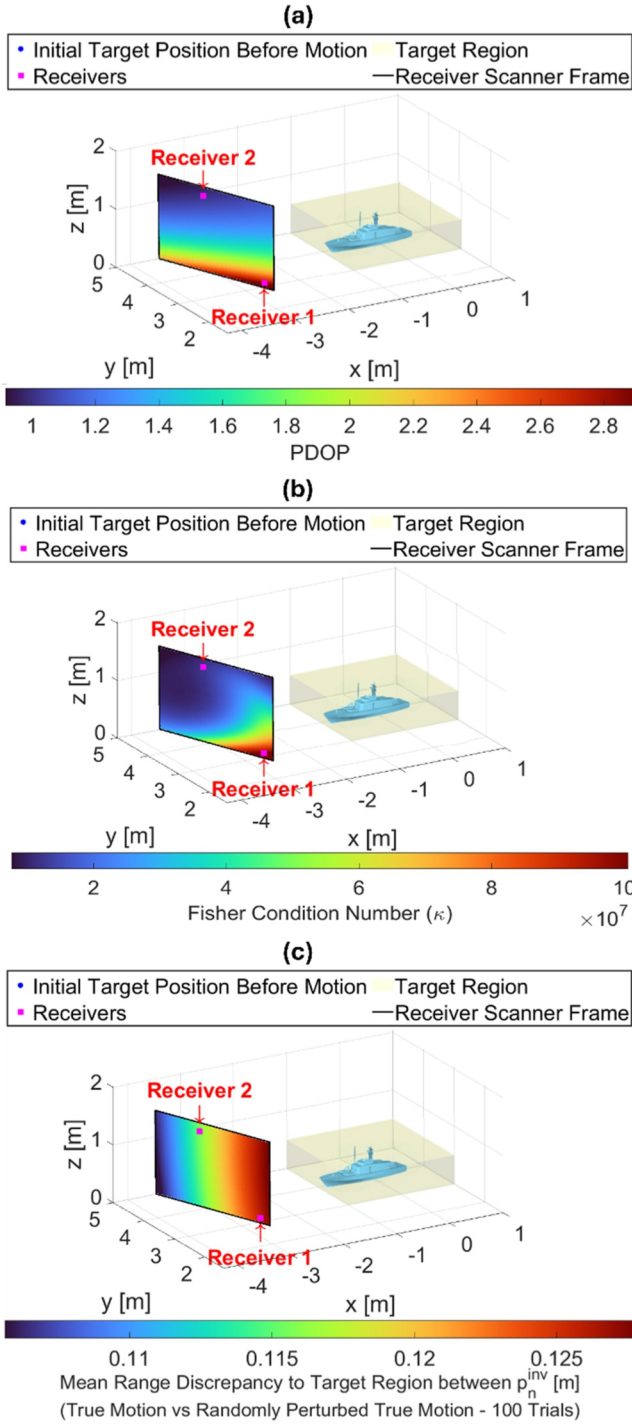


FIGURE 5 | Ambiguity maps to aid with the placement of receiver 2—(a) Position dilution of precision (PDOP), (b) Fisher conditioning κ and (c) Monte Carlo analysis of true target motion versus randomly perturbed true target motion.

laboratory system. This procedure follows standard ray-plane intersection methods widely applied in computer graphics and radar geometry modelling [57–59]. Formally, the projection requires defining a line:

$$\mathbf{p}_{t,n}^{\text{proj}} = \mathbf{p}_{t,n}^{\text{inv}} + \tau \mathbf{u}_{t,n}, \quad (42)$$

$$\mathbf{p}_{r,n}^{\text{proj}} = \mathbf{p}_{r,n}^{\text{inv}} + \tau \mathbf{u}_{r,n}, \quad (43)$$

$$\mathbf{u}_{t,n} = \frac{\mathbf{O} - \mathbf{p}_{t,n}^{\text{inv}}}{\|\mathbf{O} - \mathbf{p}_{t,n}^{\text{inv}}\|}, \quad (44)$$

$$\mathbf{u}_{r,n} = \frac{\mathbf{O} - \mathbf{p}_{r,n}^{\text{inv}}}{\|\mathbf{O} - \mathbf{p}_{r,n}^{\text{inv}}\|}, \quad (45)$$

where $\mathbf{u}$ is the unit direction vector from a target motion-inverted coordinate towards the target centre $\mathbf{O}$ and τ is a scalar parameter. The intersection with a scanner plane is determined by:

$$\tau_{t,n} = \frac{\chi_{t,n} \cdot (\mathbf{v}_{t,n} - \mathbf{p}_{t,n}^{\text{inv}})}{\chi_{t,n} \cdot \mathbf{u}_{t,n}}, \quad (46)$$

$$\tau_{r,n} = \frac{\chi_{r,n} \cdot (\mathbf{v}_{r,n} - \mathbf{p}_{r,n}^{\text{inv}})}{\chi_{r,n} \cdot \mathbf{u}_{r,n}}, \quad (47)$$

where χ is the outward normal vector of a scanner plane and $\mathbf{v}$ denotes a reference point on that plane. A geometric inclusion test is used to determine whether the intersection lies within the scanner aperture. If an intersection falls outside the aperture boundary, the corresponding motion-inverted trajectory cannot be realised by the scanner geometry and is therefore rejected. This procedure is repeated for all target motion-inverted transmitter and receiver coordinates over all pulses, ensuring that each coordinate is validly projected onto its respective scanner plane and yielding a complete set of projected coordinates executable by the laboratory scanner system.

These projected trajectories form the basis for subsequent data acquisition. Figure 6 shows the multistatic target motion-inverted SAR geometry alongside its projected counterpart, including both initial and displaced target positions. This complements the initial geometry of Figure 3 by presenting its evolved form, which accounts for target motion. The projected trajectories were transformed into local coordinates of each rail-mounted scanner to enable laboratory measurements. Subsequent processing addresses the range discrepancies between target motion-inverted and projected trajectories, so that the reconstructed measurements faithfully represent the intended target motion-inverted SAR geometry.

6.3.2 | Final Hybrid SAR/ISAR Measurements

Range discrepancies between the target motion-inverted and projected trajectories arise because the laboratory radar executes measurements using the projected coordinates, whereas the desired geometry is defined by the target motion-inverted coordinates. For each pulse n , the difference in bistatic range to the reference target centre $\mathbf{O}$ can be expressed as:

$$\delta b'(n) = \|\mathbf{p}_{t,n}^{\text{inv}} - \mathbf{O}\| - \|\mathbf{p}_{t,n}^{\text{proj}} - \mathbf{O}\|, \quad (48)$$

$$\delta b(r, n) = \|\mathbf{p}_{r,n}^{\text{inv}} - \mathbf{O}\| - \|\mathbf{p}_{r,n}^{\text{proj}} - \mathbf{O}\|, \quad (49)$$

$$\delta b^0(r, n) = \delta b'(n) + \delta b(r, n). \quad (50)$$

Measured data $s^{\text{proj}}(f, r, n)$ were corrected via pulse-by-pulse phase adjustments. The range discrepancy is identical across all frequency samples of a pulse; therefore, the correction can be implemented as a single-phase ramp:

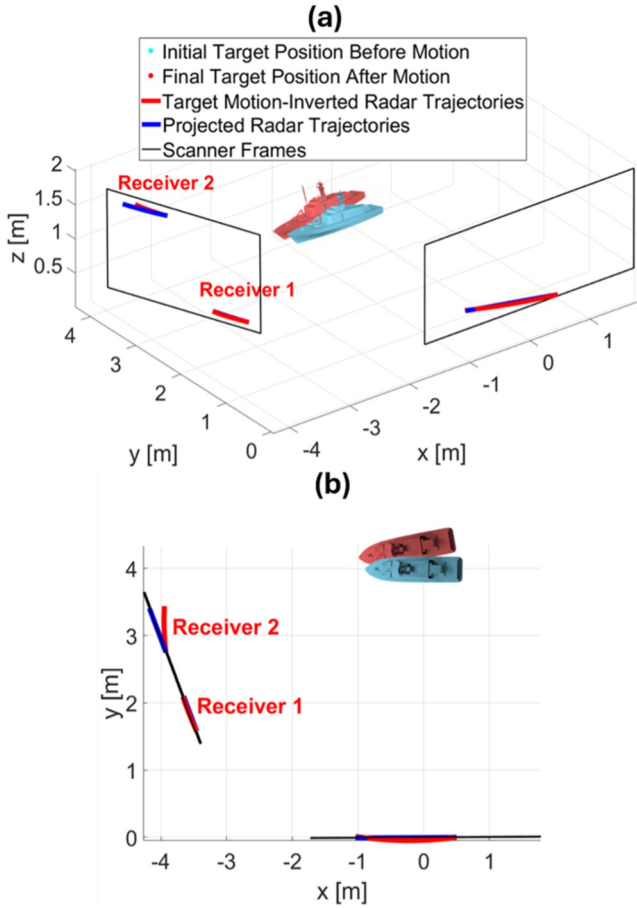


FIGURE 6 | (a) Perspective and (b) top views of target motion-inverted and projected radar geometries and target scene.

$$s^{\text{inv}}(f, r, n) = s^{\text{proj}}(f, r, n) \exp \left[-i \frac{2\pi f}{c} \delta b^{\text{O}}(r, n) \right]. \quad (51)$$

Although the correction above is exact for the reference target centre, scatterers located away from $\mathbf{O}$ experience residual phase errors because the difference between target motion-inverted and projected paths is not identical across the scene. For a scatterer at location $\mathbf{x}$, replacing the reference target centre $\mathbf{O}$ with $\mathbf{x}$ in Equations (48–50) gives $\delta b^{\mathbf{x}}(r, n)$, from which the estimated absolute residual phase error is:

$$|\Delta \phi_{\mathbf{x}}(r, n)| = \frac{2\pi f_c}{c} |\delta b^{\mathbf{x}}(r, n) - \delta b^{\text{O}}(r, n)|. \quad (52)$$

To ensure that the projected laboratory measurements remain a consistent approximation of the intended target motion-inverted geometry, the residual phase error must remain bounded across the target-scene volume. Following established SAR theory, image defocus is governed primarily by the magnitude of the phase error rather than its specific functional form. A commonly adopted requirement is that the phase deviation remains below approximately $\pi/4$ radians, corresponding to a two-way path error of $\lambda/8$, to preserve coherent integration and avoid significant degradation in image quality [32, 60–62]. Hence, the following condition is imposed:

$$|\Delta \phi_{\mathbf{x}}(r, n)| \leq \frac{\pi}{4} \quad \forall \mathbf{x} \in \mathbf{V}, \forall n, \quad (53)$$

where $\mathbf{V}$ denotes the scatterer volume considered.

In the present formulation, the pulse-by-pulse correction in Equation (51) removes the range discrepancy exactly at the reference target centre. The residual phase term in Equation (52) captures the remaining scene-dependent phase variation arising from the projected bistatic geometry, including higher-order contributions associated with path-length differences. Equation (53) therefore bounds the total residual phase variation across the scene, ensuring that all phase-error components—regardless of decomposition—remain within the tolerance required for coherent image formation.

This criterion is consistent with both classical and modern SAR autofocus formulations, where phase errors arising from motion uncertainty or propagation effects are treated as general perturbations whose impact is governed by their magnitude rather than a prescribed polynomial structure [32, 60–62]. Under the present formulation, maintaining the residual phase error below this threshold ensures that higher-order or nonquadratic contributions remain sufficiently small and do not introduce significant additional defocus.

6.4 | Simulation

Although this study is grounded primarily in laboratory measurements, simulation plays an important supporting role in validating the hybrid SAR/ISAR emulation methodology. Specifically, simulation was used to verify that the combined procedure of motion inversion, projection onto physical scanner rails and pulse-by-pulse range correction preserved the bistatic phase histories required to emulate moving-target measurements. For the laboratory target motion modelled in this study, Figure 7 shows that the stationary laboratory boat model is fully enclosed within the receiver-dependent tolerance volumes. These volumes represent the permissible target-scene extents within which residual errors between scatterer measurements made by target motion-inverted and projected radar trajectories, following single-pulse range corrections, remain within the adopted $\pi/4$ phase-error criterion. Hence, no notable artificial defocus was introduced by the emulation process itself.

The present approach employed simulation primarily to verify geometric transformations and range corrections. Autofocus performance was assessed principally using laboratory measurements, ensuring that the reported results reflected hardware-induced noise and real propagation effects. Simulation therefore served primarily as a verification tool, confirming that the motion-inverted acquisition strategy produced bistatic phase histories consistent with those of a true moving-target experiment.

Simulation was also used to perform a sensitivity study to assess whether autofocus performance depended strongly on the target motion realisation considered. Using the same multistatic acquisition geometry and point-scatterer boat model adopted in this study, four representative 6-DoF target motion cases, summarised in Table 1, were considered. The first corresponded to the laboratory target motion used throughout this work, namely -0.2 , 0.15 and 0.1 m translations along the x , y and z axes, and 5° roll, 2° pitch and 10° yaw over 100 pulses. The

remaining three cases were chosen to probe translation-dominant, rotation-dominant and mixed-motion behaviours while remaining comparable in overall motion magnitude. This supplementary study was not intended as an exhaustive analysis of the full motion space, but rather to test if main autofocus trends remained consistent across plausible motion variations under the same acquisition geometry.

7 | Autofocus Evaluation Metrics

Autofocus performance in this study was assessed using quantitative measures that capture both pulse-by-pulse focussing behaviour and final-image quality. Two complementary groups of measures were considered: intensity-squared (ISQ), used as a baseline pulse-by-pulse autofocus metric for comparison with LTS and F-Pol; and final-image quality measures, namely correlation (CC), image contrast (IC) and image entropy (IE), all widely used in SAR autofocus assessment. CC [63] was used to quantify similarity between each autofocused image and the corresponding ideally focused reference image, with values from 0 to 1, where larger values indicate closer agreement. By contrast, the reported IC and IE values are presented in normalised form, denoted hereafter as normalised image contrast (NIC) and normalised image entropy (NIE), respectively, obtained by dividing each metric by the largest value within the corresponding comparison set.

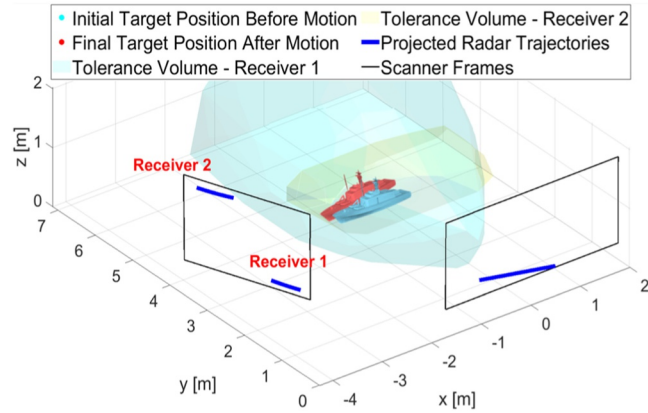


FIGURE 7 | Tolerance volumes for Receivers 1 and 2, within which a stationary target can be positioned to reproduce SAR measurements corresponding to the modelled target motion.

TABLE 1 | Representative simulated 6-DoF target motion cases used in the supplementary sensitivity study.

6-DoF motions	Simulated target motion cases			
	Laboratory target motion	Translation-dominant	Rotation-dominant	Mixed motion
ΔT_x (m)	-0.2	-0.25	-0.08	0.16
ΔT_y (m)	0.15	0.18	0.06	-0.12
ΔT_z (m)	0.1	0.12	0.04	0.09
$\Delta \theta_x$ (°)	5	2	8	-4
$\Delta \theta_y$ (°)	2	1	4	3
$\Delta \theta_z$ (°)	10	3	14	-8

7.1 | Intensity-Squared (ISQ)

ISQ [49] is a widely cited autofocus metric applicable directly in a pulse-by-pulse, image-domain setting comparable to LTS and F-Pol. Its appeal lies in the observation that well-focused images exhibit strong scatterers concentrated in a small number of pixels, whereas defocus spreads the same energy across a broader image region. Squaring the image intensity accentuates this difference, producing larger values as focus improves. Owing to its simplicity and efficiency, ISQ is used here as a baseline autofocus metric. The per-channel and global intensity-squared metrics, $Q_n^{(P,g)}$ and Q_n , are defined as:

$$Q_n^{(P,g)} = \frac{1}{|W_n^{(P,g)}|} \sum_{(i,j) \in W_n^{(P,g)}} |I_{n(i,j)}^{(P,g)}|^2, \quad (54)$$

$$Q_n = \left(\prod_{P \in \mathcal{P}} \prod_{g=1}^G Q_n^{(P,g)} \right)^{1/(N_{\text{pol}} G)}, \quad N_{\text{pol}} = 4, \quad (55)$$

where N_{pol} denotes the number of polarisations in the set $\mathcal{P}$ (here $N_{\text{pol}} = 4$). Autofocus then proceeds in the same way as for LTS and F-Pol:

$$(\hat{T}_n, \hat{\theta}_n) = \text{argmax}\{Q_n(T_n, \theta_n)\}. \quad (56)$$

This ensures a like-for-like comparison, as the same candidate motion search, image formation procedure and adaptive support are used for ISQ, LTS and F-Pol, with only the metric definition differing. The exponent $1/(N_{\text{pol}} G)$ improves comparability across different channel counts and prevents the global score from scaling disproportionately with the number of polarisations and sub-bands.

7.2 | Image Contrast (IC)

Image Contrast (IC) [49] is used in this study to assess the quality of the final, full-aperture autofocused multistatic SAR image. Let $I_{N_{\text{pul}}}$ denote the final cumulative multistatic intensity SAR image formed after coherent integration over all N_{pul} pulses. Contrast is defined as the ratio of the standard deviation of the image intensities to the mean intensity:

$$\mu_I = \frac{1}{N_{\text{pix}}} \sum_{i,j} I_{N_{\text{pul}}}(i,j), \quad (57)$$

$$\text{IC} = \frac{\sqrt{\frac{1}{N_{\text{pix}}} \sum_{i,j} (I_{N_{\text{pul}}}(i,j) - \mu_I)^2}}{\mu_I}, \quad (58)$$

where N_{pix} is the total number of image pixels. Well-focused images exhibit nonuniform intensity distributions dominated by bright scatterers, yielding high contrast. Defocused images spread energy more uniformly, resulting in lower values.

7.3 | Image Entropy (IE)

Image Entropy (IE) [49] complements contrast by quantifying the spread of intensity across the final image. For the final cumulative multistatic intensity SAR image $I_{N_{\text{pul}}}$, the normalised pixel intensities are defined as:

$$\rho(i, j) = \frac{I_{N_{\text{pul}}}(i, j)}{\sum_{i, j} I_{N_{\text{pul}}}(i, j)}. \quad (59)$$

The entropy is then defined as:

$$\text{IE} = -\sum_{i, j} \rho(i, j) \log(\rho(i, j)). \quad (60)$$

Focused images concentrate energy into a small number of dominant scatterers, leading to lower entropy. Defocused images spread energy more uniformly, increasing entropy. Thus, lower entropy corresponds to sharper imagery, with IE being particularly sensitive to subtle defocus effects.

8 | Results and Discussion

This section evaluates the performance of the ISQ, LTS and F-Pol metrics for pulse-by-pulse autofocus in multistatic SAR imaging. Emphasis is placed on full multistatic imagery rather than intermediate results from individual receivers, sub-bands or polarisation channels, as this most directly reflects the operational imaging outcome. Phase histories from both receivers and all polarisation channels were contaminated with additive white Gaussian noise. The signal-to-noise ratio (SNR) was defined in the phase-history domain prior to SAR image formation as the ratio of average signal power to noise power. The reported mean SNR, averaged across all receivers and polarisation channels, was varied between -5 and 20 dB to assess autofocus performance from strongly noise-corrupted to relatively high-SNR conditions.

Each autofocus result was obtained from 10 independent trials per mean SNR level to account for stochastic variation arising from both noise realisation and optimisation. The reported image-quality and motion-estimation measures stabilised after averaging over 10 trials, with further repetitions producing negligible change in the observed trends. Mean values are therefore reported throughout, whereas error bars represent one standard deviation across trials. Optimisation used PSO with 35 particles and a maximum of 50 iterations for each pulse-by-pulse autofocus update. These settings were selected following a convergence study, which showed negligible additional improvement for larger swarm sizes or iteration caps, and were therefore used throughout all scenarios and mean SNR levels.

Five data-combination scenarios were evaluated to investigate the contribution of polarimetric and frequency diversity to autofocus performance. S1 represents a single polarisation over the full frequency band (co-polarised VV, selected based on preliminary calibration results), whereas S2 uses the cross-

polarisations HV and VH over the full band, capturing orthogonal scattering components sensitive to depolarisation and geometric asymmetry. S3 employs a single polarisation (VV) over two sub-bands ($8\text{--}10$ and $10\text{--}12$ GHz), S4 uses cross-polarisations over the same two sub-bands, and S5 combines all four polarisations (HH, HV, VH, VV) across both sub-bands. These configurations span increasing levels of polarimetric and spectral diversity, enabling assessment of their effect on autofocus performance under noise.

The first set of results concerns the image-quality metrics CC, NIC and NIE, summarised in Tables 2–4, where NIC and NIE denote the reported normalised image contrast and entropy values, respectively. CC measures similarity with the ideally focused reference image, NIC reflects energy concentration and visual sharpness, and NIE quantifies energy dispersion. Accordingly, high CC and NIC together with low NIE indicate improved focus and reduced defocus artefacts.

Across all three metrics, a clear trend is observed. At low mean SNRs, single-channel configurations such as S1 exhibit weak CC, reduced contrast and elevated entropy, indicating strong defocus and degraded coherence. For example, at -5 dB, S1 yields CC values of 0.11, 0.12 and 0.15 for ISQ, LTS and F-Pol, respectively, with corresponding NIC values of 0.23–0.29 and NIE values close to unity. Introducing either polarimetric or frequency diversity individually, as in S2 and S3, improves performance, while combining both forms in S4 and S5 yields the highest CC and NIC and the lowest NIE across the tested SNR range. As SNR decreases, CC and NIC decrease while NIE increases, reflecting reduced similarity to the ideally focused image, lower energy concentration and greater image disorder, with the degradation most pronounced at 0 and -5 dB where noise forms a significant component of the measured return and target-feature localisation becomes less stable.

The relative behaviour of the autofocus metrics is consistent across scenarios. Performance degrades for all methods at lower SNRs (-5 to 10 dB), where noise increases phase uncertainty and reduces the reliability of target-feature localisation. At 5 dB mean SNR in S5, CC increases from 0.71 (ISQ) to 0.77 (LTS) and 0.79 (F-Pol), whereas NIC increases from 0.69 to 0.77 and 0.83 and NIE decreases from 0.71 to 0.65 and 0.58. These results indicate that the LTS-based approaches maintain better image quality under noise-dominated conditions, with F-Pol consistently outperforming LTS, although the gap narrows as SNR increases. At 20 dB, all methods approach high-quality reconstruction, but F-Pol retains a measurable advantage, for example a CC of 0.95 versus 0.92 and 0.90 for LTS and ISQ in S5. Overall, S5 provides the most favourable performance across the tested SNR range. Joint use of polarimetric and frequency diversity yields consistent gains in the low-to-moderate SNR regime and more closely grouped performance at higher SNRs by suppressing motion estimates only partially supported by the data. Consequently, the remaining analysis focuses on S5.

Multistatic SAR images for S5 are shown in Figures 8 and 9 for mean SNRs of 5 and 20 dB, respectively. These images were formed via coherent combination of the full-aperture SAR images from all receivers and polarisation channels. In each figure, panel (a) shows the unfocused image, panel (b) the ideally

TABLE 2 | Laboratory correlation (CC) results for five data scenarios (S1–S5).

CC															
Mean SNR (dB)	S1 (VV, full-band)			S2 (HV and VH, full-band)			S3 (VV, sub-bands)			S4 (HV and VH, sub-bands)			S5 (HH, HV, VH and VV, sub-bands)		
–5	0.11	0.12	0.15	0.14	0.16	0.20	0.16	0.20	0.24	0.19	0.24	0.29	0.24	0.31	0.33
0	0.27	0.28	0.35	0.32	0.36	0.42	0.37	0.41	0.46	0.43	0.45	0.51	0.47	0.51	0.56
5	0.49	0.57	0.62	0.58	0.61	0.66	0.64	0.67	0.73	0.67	0.73	0.75	0.71	0.77	0.79
10	0.66	0.67	0.72	0.70	0.74	0.77	0.74	0.78	0.79	0.79	0.81	0.84	0.80	0.82	0.85
15	0.71	0.77	0.79	0.79	0.80	0.83	0.82	0.84	0.86	0.85	0.87	0.89	0.89	0.89	0.91
20	0.79	0.81	0.84	0.82	0.85	0.88	0.86	0.87	0.91	0.90	0.90	0.94	0.90	0.92	0.95

Note: Red indicates ISQ; blue indicates LTS; black indicates F-Pol.

TABLE 3 | Laboratory normalised image contrast (NIC) results for five data scenarios (S1–S5).

NIC															
Mean SNR (dB)	S1 (VV, full-band)			S2 (HV and VH, full-band)			S3 (VV, sub-bands)			S4 (HV and VH, sub-bands)			S5 (HH, HV, VH and VV, sub-bands)		
–5	0.23	0.26	0.29	0.28	0.32	0.35	0.33	0.37	0.41	0.39	0.42	0.47	0.44	0.47	0.51
0	0.33	0.41	0.46	0.41	0.47	0.50	0.46	0.51	0.57	0.51	0.56	0.62	0.56	0.60	0.66
5	0.52	0.61	0.69	0.56	0.66	0.73	0.60	0.69	0.77	0.65	0.73	0.80	0.69	0.77	0.83
10	0.68	0.75	0.82	0.71	0.79	0.86	0.74	0.82	0.88	0.78	0.86	0.91	0.81	0.88	0.93
15	0.78	0.83	0.88	0.82	0.85	0.90	0.84	0.88	0.93	0.86	0.90	0.95	0.89	0.93	0.96
20	0.85	0.88	0.92	0.87	0.91	0.95	0.90	0.93	0.97	0.93	0.94	0.98	0.95	0.96	1.00

Note: Red indicates ISQ; blue indicates LTS; black indicates F-Pol.

TABLE 4 | Laboratory normalised image entropy (NIE) results for five data scenarios (S1–S5).

NIE															
Mean SNR (dB)	S1 (VV, full-band)			S2 (HV and VH, full-band)			S3 (VV, sub-bands)			S4 (HV and VH, sub-bands)			S5 (HH, HV, VH and VV, sub-bands)		
–5	1.00	0.98	0.97	0.98	0.97	0.96	0.97	0.96	0.94	0.96	0.95	0.91	0.95	0.93	0.90
0	0.95	0.93	0.92	0.93	0.91	0.89	0.92	0.88	0.87	0.91	0.87	0.86	0.90	0.86	0.83
5	0.91	0.90	0.85	0.88	0.83	0.82	0.82	0.78	0.83	0.79	0.72	0.67	0.71	0.65	0.58
10	0.84	0.78	0.63	0.76	0.76	0.61	0.72	0.68	0.60	0.66	0.62	0.57	0.62	0.54	0.46
15	0.71	0.67	0.42	0.64	0.66	0.39	0.61	0.59	0.37	0.57	0.53	0.33	0.48	0.41	0.29
20	0.55	0.48	0.35	0.52	0.46	0.34	0.48	0.43	0.33	0.44	0.38	0.29	0.39	0.33	0.25

Note: Red indicates ISQ; blue indicates LTS; black indicates F-Pol.

focused image obtained using true target motion and panels (c–e) the autofocus results obtained using ISQ, LTS and F-Pol. These SNRs were selected to illustrate both a noise-dominated regime and a higher-quality regime in which differences between the autofocus methods become less pronounced.

At 5 dB, the unfocused image exhibits substantial blur, loss of target structure and elevated background noise. ISQ recovers partial focus but retains noticeable distortion of prominent target features relative to the ideally focused image. LTS improves sharpness and suppresses energy spreading more effectively, whereas F-Pol produces the closest visual match to the ideally focused image, with clearer localisation of dominant

scattering features. This is consistent with Tables 2–4, where F-Pol gives the highest CC and NIC and the lowest NIE for S5 at 5 dB. At low SNRs, reduced phase coherence makes accurate autofocus more challenging.

At 20 dB, all three autofocus methods produce well-focused images with clear target structure, and the visual differences become subtle. F-Pol remains among the closest matches to the ideally focused image, consistent with its slight advantage in NIC and NIE at this higher SNR. These images therefore support the quantitative results while showing that the separation between methods decreases as SNR increases. At higher SNR, the reduced influence of noise leads to improved phase

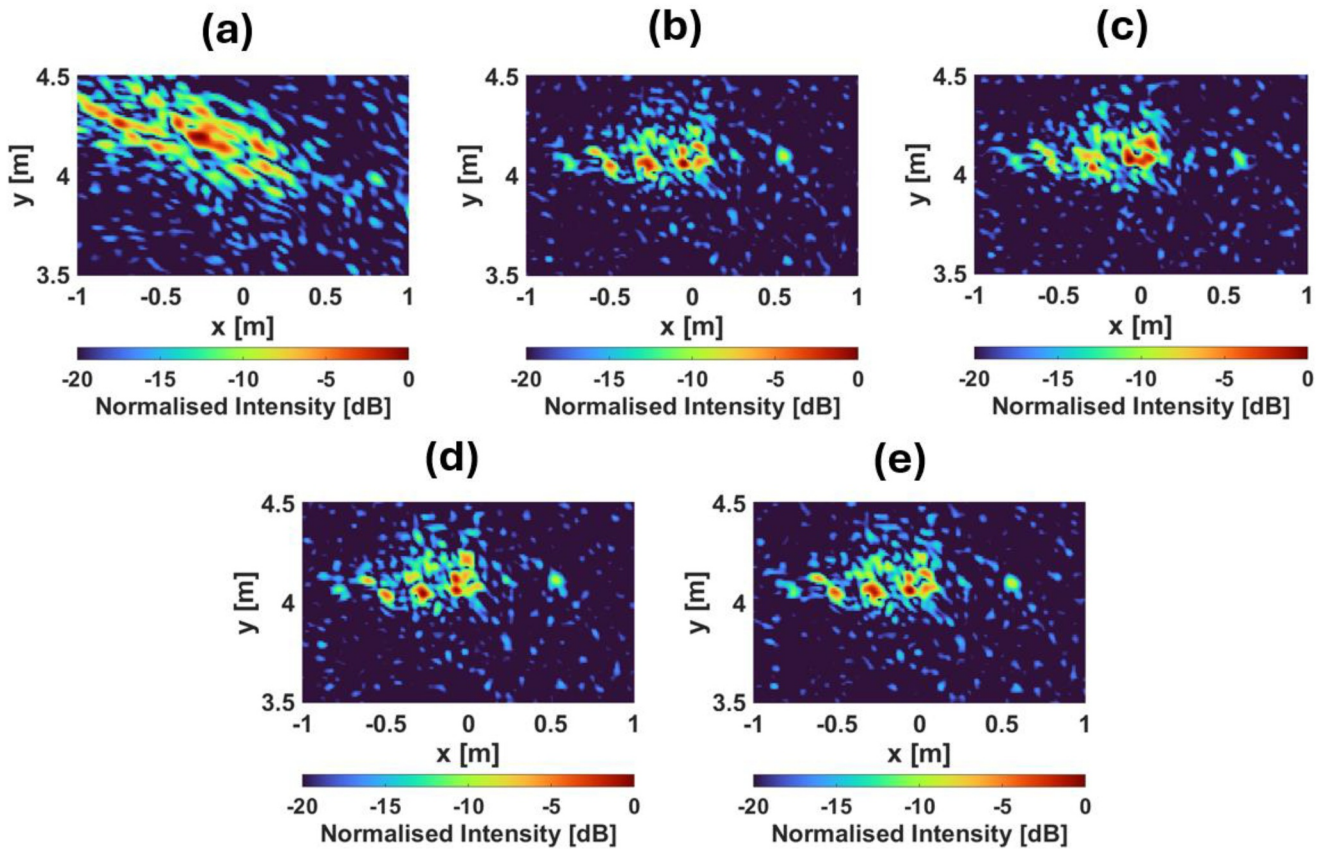


FIGURE 8 | Laboratory multistatic SAR images, for a mean SNR of 5 dB, (a) unfocused, (b) ideally focused, autofocused using (c) ISQ metric, (d) LTS metric and (e) F-Pol metric.

consistency, resulting in more uniform autofocus performance across all methods and a corresponding reduction in performance differences.

Further insight is obtained by examining the estimated target-motion parameters. Figures 10 and 11 show the absolute mean errors between the estimated and true target translations and rotations, respectively, for mean SNRs of 5 and 20 dB. Translation errors decrease substantially between the two SNR levels for all three autofocus methods. At 5 dB, the absolute mean translation errors lie approximately in the range of 2–6 cm/pulse, reducing to approximately 0.8–2.5 cm/pulse at 20 dB. F-Pol consistently yields the smallest errors across all three translation components, followed by LTS and ISQ.

A similar trend is observed for the rotational components. At 5 dB, the absolute mean rotation errors lie approximately between 0.5 and 3.5°/pulse, with yaw remaining the most difficult component to estimate. At 20 dB, these errors reduce to approximately 0.2–1.5°/pulse, again with F-Pol producing the smallest errors across roll, pitch and yaw. These results indicate that the improved image quality obtained using F-Pol is accompanied by more accurate recovery of the underlying 6-DoF target motion, particularly in the low-SNR regime where the additional diversity information is most beneficial.

To examine the physical consistency of the recovered target motion, Figure 12 compares the mean range discrepancy between bistatic SAR trajectories reconstructed from the estimated

and ideal target motions for Receivers 1 and 2, respectively. For both receivers, the discrepancy decreases steadily as mean SNR increases. At –5 dB, it remains on the order of 0.9–1.3 cm/pulse, depending on method and receiver, reducing to approximately 0.18–0.40 cm/pulse at 20 dB. Receiver 2 is consistently slightly worse than Receiver 1, but the same ordering is maintained throughout: F-Pol produces the smallest discrepancies, followed by LTS and ISQ.

These trajectory comparisons show that the image-domain gains are accompanied by improved geometric consistency of the estimated target motion. In the present experimental geometry, the lower image entropy, higher contrast and improved CC produced by F-Pol are matched by lower motion-parameter errors and smaller trajectory discrepancies. Although this does not establish general behaviour across all acquisition geometries, it indicates that, under the conditions tested here, the image-quality gains obtained by F-Pol are associated with more self-consistent motion recovery rather than sharper images alone.

A supplementary simulation study was performed to assess whether autofocus performance depended strongly on the target motion realisation considered. The representative simulated 6-DoF motion cases were defined earlier in Table 1, with the corresponding results shown in Table 5. CC was evaluated at mean SNRs of 5 and 20 dB using the full data-diversity configuration (S5), comprising both frequency sub-bands and all four polarisation channels. The target was represented using a point-scatterer model with polarisation-dependent scattering

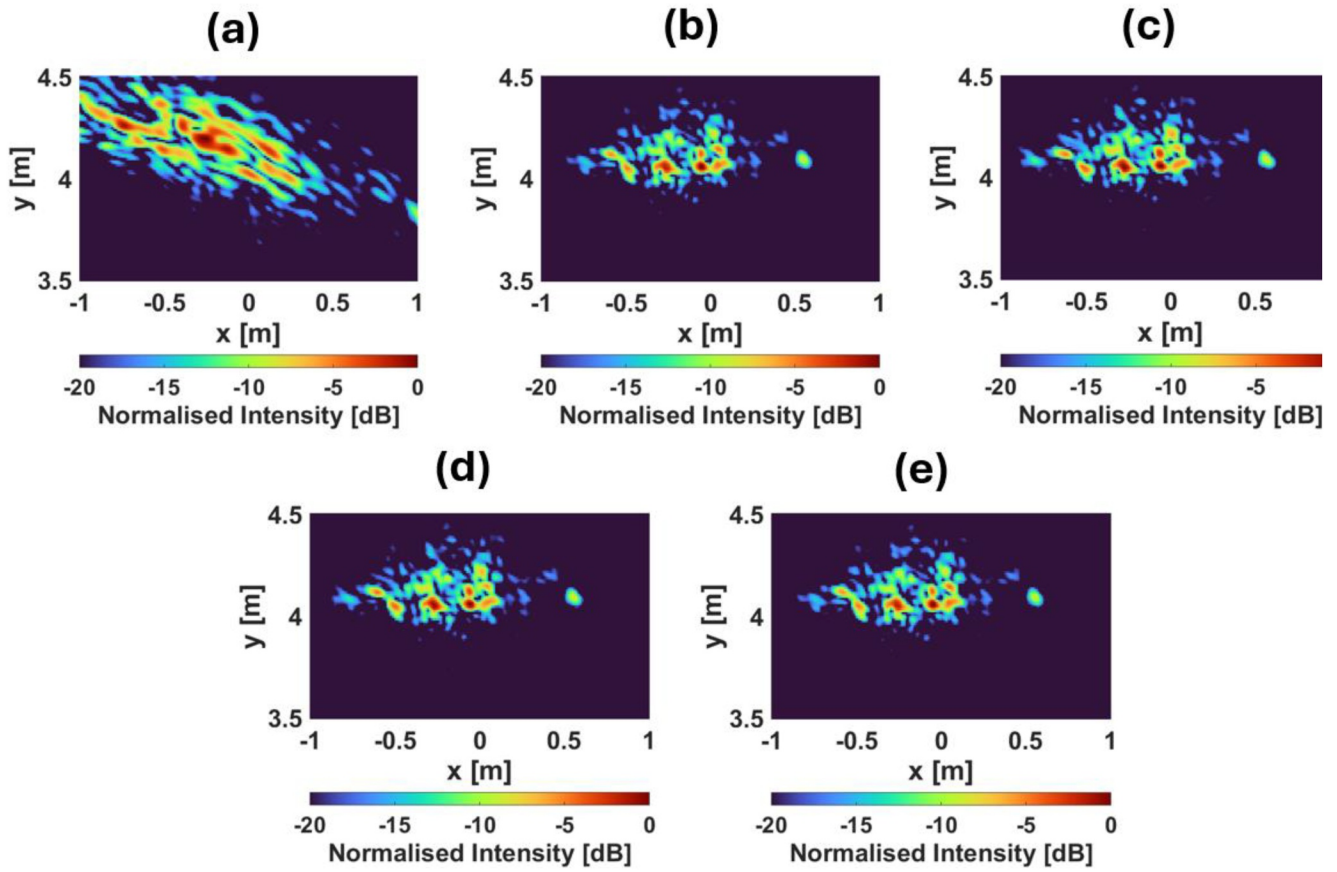


FIGURE 9 | Laboratory multistatic SAR images, for a mean SNR of 20 dB, (a) unfocused, (b) ideally focused, autofocused using (c) ISQ metric, (d) LTS metric and (e) F-Pol metric.

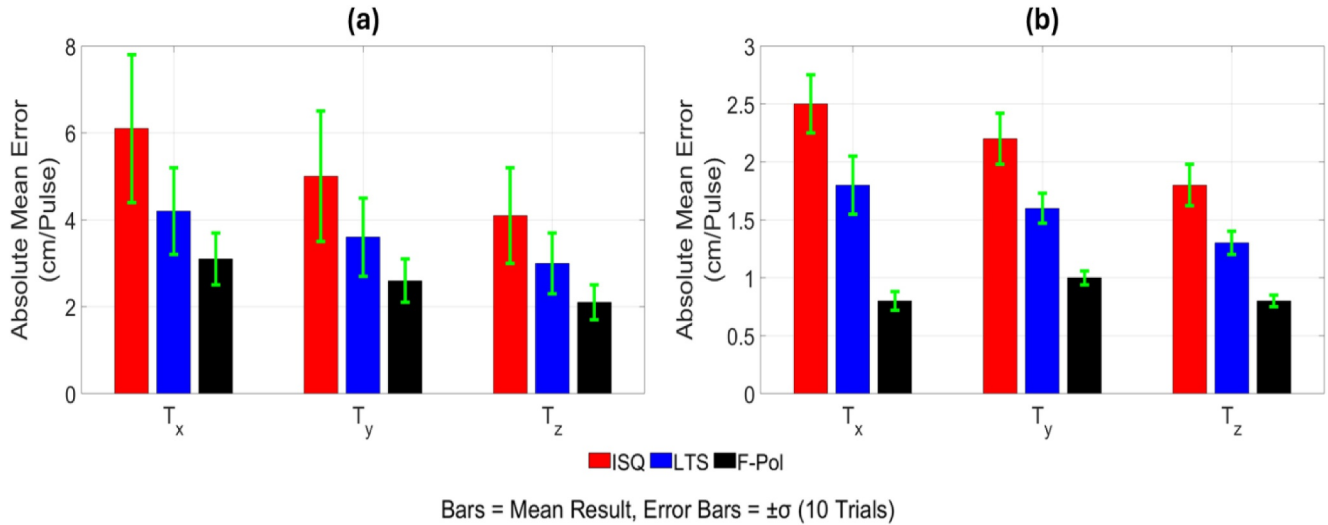


FIGURE 10 | Absolute mean errors between estimated and true target translations for mean SNRs (a) 5 dB and (b) 20 dB.

amplitudes assigned across the simulated channels, allowing the same frequency–polarimetric processing framework used in the laboratory study to be evaluated under alternative motion compositions.

The simulations used the same evaluation measures and processing chain as the laboratory results but served a different purpose. Although the laboratory measurements assessed

autofocus performance under hardware-induced noise and real propagation effects, the simulations examined sensitivity to alternative motion compositions. Since the simulated data did not include the full set of hardware-induced and propagation-related imperfections present in the laboratory measurements, exact numerical agreement was neither expected nor required. Some alternative motion compositions, particularly those with stronger rotational content, were also intrinsically more

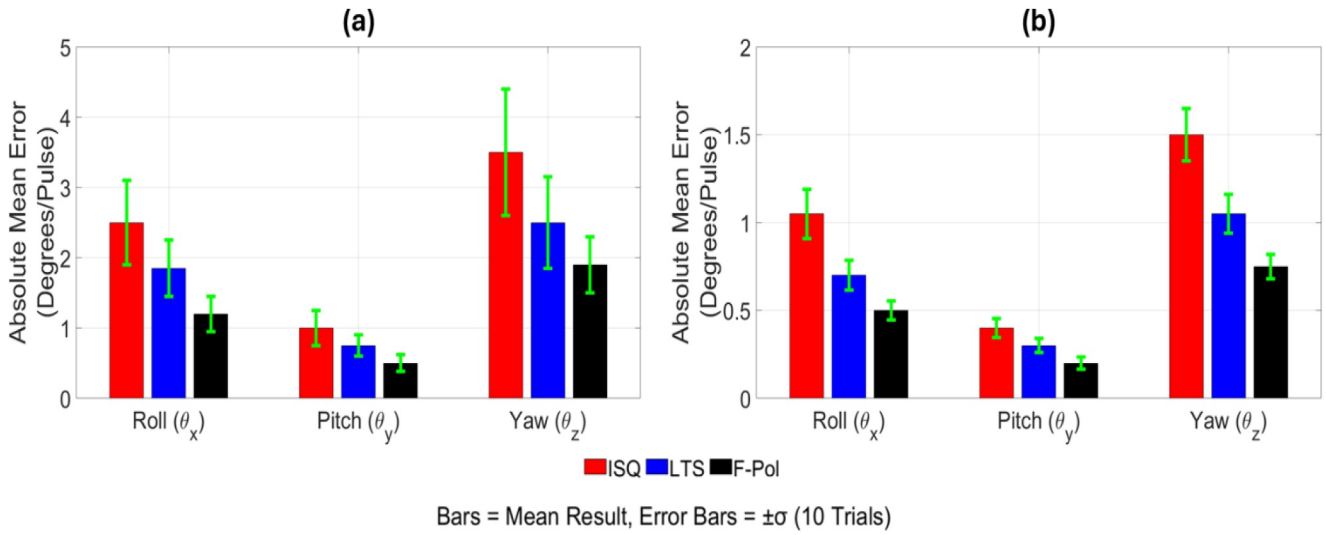


FIGURE 11 | Absolute mean errors between estimated and true target rotations for mean SNRs (a) 5 dB and (b) 20 dB.

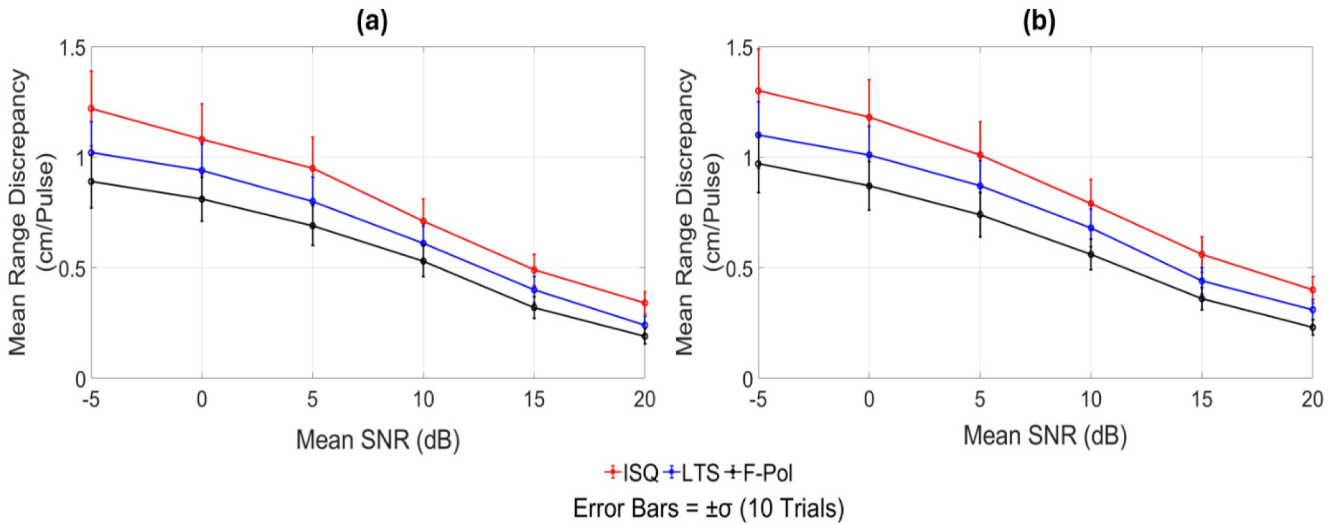


FIGURE 12 | Mean range discrepancy between target motion-inverted bistatic SAR trajectories, reconstructed using estimated target motion and ideal target motion, for (a) Receiver 1 and (b) Receiver 2.

TABLE 5 | Correlation (CC) results of simulated target motion cases for S5 at mean SNRs of 5 and 20 dB.

CC						
Simulated target motion cases	Mean SNR (dB)					
	5			20		
Laboratory target motion	0.69	0.77	0.80	0.89	0.92	0.96
Translation-dominant	0.75	0.81	0.84	0.93	0.95	0.97
Rotation-dominant	0.63	0.71	0.76	0.85	0.89	0.93
Mixed motion	0.68	0.74	0.78	0.88	0.91	0.94

Note: Red indicates ISQ; blue indicates LTS; black indicates F-Pol.

challenging. The simulation results therefore provide a complementary assessment of motion-case sensitivity rather than a repetition of the laboratory study.

Across the simulated motion cases, the relative ranking of the autofocus metrics is preserved, with F-Pol consistently producing the highest CC, followed by LTS and ISQ. Differences

between motion cases remain modest but visible, indicating sensitivity to motion composition. In particular, the rotation-dominant case remains the most challenging, whereas the laboratory-reference and translation-dominant cases are more favourable, consistent with the stronger aspect-dependent distortion introduced by increased rotational motion. Importantly, the relative advantage of F-Pol is not confined to the

single 6-DoF motion used in the experimental study. The simulation results support the view that the experimentally observed autofocus gains arise from the joint frequency–polarimetric formulation rather than dependence on a specific motion realisation.

9 | Conclusions

This paper presented a frequency- and polarimetry-adaptive autofocus algorithm (F-Pol) for multistatic SAR back-projection imaging of moving targets. Building on the LTS metric, F-Pol extends autofocus into the joint frequency–polarimetric domain by enforcing cross-channel consistency across sub-bands and polarisation channels, thereby exploiting the additional diversity available in multistatic measurements to improve the stability of pulse-by-pulse motion estimation.

Experimental validation using hybrid SAR/ISAR measurements of a target undergoing emulated 6-DoF motion demonstrated that F-Pol improves autofocus performance relative to both LTS and conventional ISQ. Across the tested SNR range, F-Pol consistently produced higher image correlation and contrast, lower entropy and reduced motion-estimation errors in both translation and rotation. The reconstructed bistatic SAR trajectories obtained using F-Pol also remained closer to those derived from the ideal target motion, indicating improved geometric consistency of the recovered motion.

The results further showed that increasing frequency and polarimetric diversity consistently improved autofocus performance. Among the tested configurations, the full multi-polarimetric dual-sub-band case (S5) gave the most favourable overall behaviour across all reported metrics. At 5 dB mean SNR, moving from the single-polarisation full-band case (S1) to S5 increased CC from 0.62 to 0.79 and NIC from 0.69 to 0.83, while reducing NIE from 0.85 to 0.58, demonstrating the benefit of joint diversity in noise-limited conditions. F-Pol also outperformed both ISQ and LTS most clearly in the low-to-moderate SNR regime (−5 to 10 dB), where the additional diversity provided the greatest benefit.

These image-quality gains were accompanied by improved motion-estimation accuracy. At 20 dB mean SNR, mean range discrepancies between reconstructed and ideal bistatic SAR trajectories were reduced by approximately 40%–45% relative to ISQ and by around 20%–25% relative to LTS, confirming that the image-quality improvements reflected more accurate recovery of the underlying 6-DoF target motion.

A supplementary simulation study further showed that these trends remained consistent across alternative target motion compositions. Using the full diversity configuration (S5), F-Pol maintained the highest correlation across all cases, including the more challenging rotation-dominant scenario.

The present study remains limited to one experimental geometry, one target class and a controlled set of target motions. Future work will therefore examine autofocus performance in more complex scenes, including multitarget and cluttered environments and extend the framework to nonrigid or multi-component motion. The influence of the adaptive-window

threshold on autofocus performance was also not explicitly investigated. Although a −3 dB threshold was adopted as a practical choice for capturing the main lobe region, future work will examine the sensitivity of the method to alternative threshold selections. Further work on computational acceleration will also be required to support higher-throughput implementations. Overall, this study demonstrates that joint frequency and polarimetric diversity can improve multistatic SAR autofocus performance.

Author Contributions

Anmol Rattan: conceptualisation, data curation, formal analysis, investigation, methodology, software, validation, visualisation, writing – original draft, writing – review and editing. **Daniel Andre:** conceptualisation, funding acquisition, investigation, methodology, project administration, software, supervision, writing – review and editing. **Mark Finnis:** resources, software.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

Data supporting this study are openly available from CORD at <https://doi.org/10.57996/cran.ceres-2858>.

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