

College of Aeronautics Report No. 9008
July 1989



Third Quarterly Report on the Application of
Modified Stepwise Regression for the Estimation
of Aircraft Stability and Control Parameters

H A Hinds & M V Cook

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College of Aeronautics
Cranfield Institute of Technology
Cranfield, Bedford MK43 0AL, England



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*"The views expressed herein are those of the authors alone and do not
necessarily represent those of the Institute"*

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1.0 INTRODUCTION.

This report is intended to discuss the progress made during the past quarter, May - July, 1989, in accordance with the terms of MOD Agreement No.2082/192, (REF.1). This research programme is concerned with the use of a Modified Stepwise Regression Method for estimating the stability and control derivatives of a B.Ae Hawk aircraft from data obtained by the use of a scaled model on a dynamic wind tunnel test rig. At the last progress review meeting on the 16th May 1989, the following objectives were established for the present quarter.

1. The two Advanced Continuous Simulation Language (ACSL) computer programs LATYAW and LATROL will be combined to give a single simulation program for small perturbation lateral motion.
2. The sensitivity of the aircraft simulation programs to the actual value of particular derivatives will be investigated in order to establish the most significant derivatives in the various aircraft motions.
3. To have more flexibility and control over the inputs to the aircraft simulation models by improving the way in which control surface deflections are implemented. This will be done by having the control surface angle modelled by some function of time to simulate the stick-inputs of a real pilot. As a consequence of this, an improvement in the response of the aircraft model to "impulse" inputs is expected.
4. Initially it is planned to restrain the wind tunnel aircraft model to a vertical height translation of $\pm 3"$. Further tests to evaluate the new system for sensing the vertical position of the model are required before allowing the model full vertical freedom.
5. Various analogue and digital numerical methods will be looked at to find a suitable method for generating attitude angle rate data.
6. The full equations of motion of the aircraft are to be used for the present studies, rather than a set of equations which have been reduced to represent the experimental wind tunnel model.
7. The basic algorithms for the MSR process are to be established.

2.0 FULL SCALE EQUATIONS OF MOTION.

When the FORTRAN 77 modified stepwise regression program is written it will be necessary to test the program with data produced by the ACSL digital simulation programs. Previously, a reduced set of equations of motion for the Hawk model in the wind tunnel were manipulated into a form compatible with the MSR requirements, (REF.2). However, it was considered prudent that the MSR program should be initially written using the full equations of motion, as is the case with the ACSL simulation programs. Thus sections 2.1 and 2.2 give brief details of how the longitudinal and lateral equations of motion may be re-arranged in the form required for the MSR program.

2.1 LONGITUDINAL EQUATIONS OF MOTION.

The general dimensional equations of longitudinal symmetric motion for small disturbances (when referred to body axes) may be written as follows (REF.3):

$$m\dot{u} - \dot{X}_u \cdot u - \dot{X}_w \cdot w - \dot{X}_{\dot{w}} \cdot \dot{w} + (mW_e - \dot{X}_q) \cdot q + mg_1 \cdot \theta = \dot{X}_\eta \cdot \eta \quad (1)$$

$$-Z_u \cdot u - Z_w \cdot w + (m - Z_{\dot{w}}) \cdot \dot{w} - (mU_e + Z_q) \cdot q + mg_2 \cdot \theta = Z_\eta \cdot \eta \quad (2)$$

$$-\dot{M}_u \cdot u - \dot{M}_w \cdot w - \dot{M}_{\dot{w}} \cdot \dot{w} - \dot{M}_q \cdot q + I_y \cdot \dot{q} = \dot{M}_\eta \cdot \eta \quad (3)$$

where "." denotes a dimensional coefficient;

In the special case of wind axes and level flight, $\theta_e = 0$ hence

$$g_1 = g \cos \theta_e = g ; \quad g_2 = g \sin \theta_e = 0 ;$$

$$U_e = V \cos \alpha_e = V ; \quad W_e = V \sin \alpha_e = 0 ;$$

and since small perturbations are assumed $\dot{\theta} = q$.

Dividing equations 1 and 2 through by mass m , equation 3 by pitch inertia I_y and re-arranging all three equations into the form $M \dot{\tilde{x}} = A' \tilde{x} + B' \tilde{u}$, equation 4 (over leaf) is obtained.

$$\begin{pmatrix} 1 & -\dot{\hat{x}}_w & 0 & 0 \\ 0 & (1-\dot{\hat{z}}_w) & 0 & 0 \\ 0 & -\dot{\hat{m}}_w & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \dot{\hat{x}}_u & \dot{\hat{x}}_w & (\dot{\hat{x}}_q - W_e) & -g \\ \dot{\hat{z}}_u & \dot{\hat{z}}_w & (\dot{\hat{z}}_q + U_e) & 0 \\ \dot{\hat{m}}_u & \dot{\hat{m}}_w & \dot{\hat{m}}_q & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} \dot{\hat{x}}_\eta \\ \dot{\hat{z}}_\eta \\ \dot{\hat{m}}_\eta \\ 0 \end{pmatrix} \begin{pmatrix} \eta \end{pmatrix} \quad (4)$$

Pre-multiplying equation 4 through by the inverse mass matrix M^{-1} yields the required MSR format of $\dot{\hat{x}} = A\hat{x} + B\hat{u}$, as shown in equation 5:

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} x_u & x_w & x_q & -g \\ z_u & z_w & z_q & 0 \\ m_u & m_w & m_q & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{pmatrix} x_\eta \\ z_\eta \\ m_\eta \\ 0 \end{pmatrix} \begin{pmatrix} \eta \end{pmatrix} \quad (5)$$

where:

$$\dot{\hat{x}}_u = \dot{\hat{x}}_u / m ; \quad \dot{\hat{x}}_w = \dot{\hat{x}}_w / m ; \quad \dot{\hat{x}}_q = \dot{\hat{x}}_q / m ; \quad \dot{\hat{x}}_\eta = \dot{\hat{x}}_\eta / m .$$

$$\dot{\hat{z}}_u = \dot{\hat{z}}_u / m ; \quad \dot{\hat{z}}_w = \dot{\hat{z}}_w / m ; \quad \dot{\hat{z}}_q = \dot{\hat{z}}_q / m ; \quad \dot{\hat{z}}_\eta = \dot{\hat{z}}_\eta / m .$$

$$\dot{\hat{m}}_u = \dot{\hat{m}}_u / I_y ; \quad \dot{\hat{m}}_w = \dot{\hat{m}}_w / I_y ; \quad \dot{\hat{m}}_q = \dot{\hat{m}}_q / I_y ; \quad \dot{\hat{m}}_\eta = \dot{\hat{m}}_\eta / I_y .$$

$$x_u = \left(\frac{\dot{\hat{x}}_w \dot{\hat{z}}_u}{(1-\dot{\hat{z}}_w)} + \dot{\hat{x}}_u \right) ; \quad x_w = \left(\frac{\dot{\hat{x}}_w \dot{\hat{z}}_w}{(1-\dot{\hat{z}}_w)} + \dot{\hat{x}}_w \right) ; \quad x_q = \left(\frac{(U_e + \dot{\hat{z}}_q) \dot{\hat{x}}_w}{(1-\dot{\hat{z}}_w)} + (\dot{\hat{x}}_q - W_e) \right) ;$$

$$z_u = \left(\frac{\dot{\hat{z}}_u}{1-\dot{\hat{z}}_w} \right) ; \quad z_w = \left(\frac{\dot{\hat{z}}_w}{1-\dot{\hat{z}}_w} \right) ; \quad z_q = \left(\frac{U_e + \dot{\hat{z}}_q}{1-\dot{\hat{z}}_w} \right) ;$$

$$m_u = \left(\frac{\dot{\hat{m}}_w \dot{\hat{z}}_u}{(1-\dot{\hat{z}}_w)} + \dot{\hat{m}}_u \right) ; \quad m_w = \left(\frac{\dot{\hat{m}}_w \dot{\hat{z}}_w}{(1-\dot{\hat{z}}_w)} + \dot{\hat{m}}_w \right) ; \quad m_q = \left(\frac{(U_e + \dot{\hat{z}}_q) \dot{\hat{m}}_w}{(1-\dot{\hat{z}}_w)} + \dot{\hat{m}}_q \right) ;$$

$$x_\eta = \left(\frac{\dot{\hat{x}}_w \dot{\hat{z}}_\eta}{(1-\dot{\hat{z}}_w)} + \dot{\hat{x}}_\eta \right) ; \quad z_\eta = \left(\frac{\dot{\hat{z}}_\eta}{1-\dot{\hat{z}}_w} \right) ; \quad m_\eta = \left(\frac{\dot{\hat{m}}_w \dot{\hat{z}}_\eta}{(1-\dot{\hat{z}}_w)} + \dot{\hat{m}}_\eta \right)$$

2.2 LATERAL EQUATIONS OF MOTION.

The general dimensional equations of lateral asymmetric motion, referred to body axes, for small disturbances may be written as:

$$m\dot{v} - \dot{Y}_v.v - (mW_e + \dot{Y}_p)p + (mU_e - \dot{Y}_r)r - mg_1\phi - mg_2\Psi = \dot{Y}_\xi.\xi + \dot{Y}_\zeta.\zeta \quad (6)$$

$$-I_v.v + I_x.\dot{p} - I_p.p - I_{xz}.\dot{r} - I_r.r = I_\xi.\xi + I_\zeta.\zeta \quad (7)$$

$$-I_v.v + I_{xz}.\dot{p} - I_p.p + I_z.\dot{r} - I_r.r = I_\xi.\xi + I_\zeta.\zeta \quad (8)$$

In the special case of wind axes and level flight, $\theta_e = 0$ giving

$$g_1 = g\cos\theta_e = g; \quad g_2 = g\sin\theta_e = 0;$$

and since small perturbations are assumed the following relationship

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\Psi} \end{pmatrix} = \begin{pmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{pmatrix} \cdot \begin{pmatrix} p \\ q \\ r \end{pmatrix} \quad (9)$$

reduces to $\dot{\phi} = p$; $\dot{\theta} = q$; $\dot{\Psi} = r$.

Dividing equation 6 by m , equation 7 by I_x , equation 8 by I_z and re-arranging leads to equation (10) below which is of the form:

$$M.\ddot{\tilde{x}} = A'.\tilde{x} + B'.\tilde{u}.$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -e_x & 0 & 0 \\ 0 & e_z & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\Psi} \end{pmatrix} = \begin{pmatrix} \dot{Y}_v & (\dot{Y}_p + W_e) & (\dot{Y}_r - U_e) & g & 0 \\ I_v & I_p & I_r & 0 & 0 \\ \dot{N}_v & \dot{N}_p & \dot{N}_r & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} v \\ p \\ r \\ \phi \\ \Psi \end{pmatrix} + \begin{pmatrix} \dot{Y}_\xi & \dot{Y}_\zeta \\ I_\xi & I_\zeta \\ \dot{N}_\xi & \dot{N}_\zeta \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \xi \\ \zeta \end{pmatrix}$$

where $e_x = I_{xz}/I_x$; $e_z = I_{xz}/I_z$.

Thus pre-multiplying equation 10 by the inverse mass matrix M^{-1} yields the lateral equations of motion in the standard state variable form $\dot{\underline{x}} = A\underline{x} + B\underline{u}$, which is the form required by the MSR.

$$\begin{pmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} y_v & y_p & y_r & g & 0 \\ l_v & l_p & l_r & 0 & 0 \\ n_v & n_p & n_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} v \\ p \\ r \\ \phi \\ \psi \end{pmatrix} + \begin{pmatrix} y_\xi & y_\zeta \\ l_\xi & l_\zeta \\ n_\xi & n_\zeta \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \xi \\ \zeta \end{pmatrix} \quad (11)$$

WHERE:

$$\dot{y}_v = \dot{Y}_v/m; \quad \dot{y}_p = \dot{Y}_p/m; \quad \dot{y}_r = \dot{Y}_r/m; \quad \dot{y}_\xi = \dot{Y}_\xi/m; \quad \dot{y}_\zeta = \dot{Y}_\zeta/m.$$

$$l_v = \dot{L}_v/I_x; \quad l_p = \dot{L}_p/I_x; \quad l_r = \dot{L}_r/I_x; \quad l_\xi = \dot{L}_\xi/I_x; \quad l_\zeta = \dot{L}_\zeta/I_x.$$

$$\dot{n}_v = \dot{N}_v/I_z; \quad \dot{n}_p = \dot{N}_p/I_z; \quad \dot{n}_r = \dot{N}_r/I_z; \quad \dot{n}_\xi = \dot{N}_\xi/I_z; \quad \dot{n}_\zeta = \dot{N}_\zeta/I_z.$$

$$e_x = I_{xz}/I_x;$$

$$e_z = I_{xz}/I_z;$$

$$E_{xz} = 1 + e_x e_z$$

$$y_v = \dot{y}_v;$$

$$y_p = (\dot{y}_p + w_e);$$

$$y_r = (\dot{y}_r - u_e);$$

$$l_v = \left\{ \frac{\dot{l}_v}{E_{xz}} + \frac{e_x \dot{n}_v}{E_{xz}} \right\};$$

$$l_p = \left\{ \frac{\dot{l}_p}{E_{xz}} + \frac{e_x \dot{n}_p}{E_{xz}} \right\};$$

$$l_r = \left\{ \frac{\dot{l}_r}{E_{xz}} + \frac{e_x \dot{n}_r}{E_{xz}} \right\};$$

$$n_v = \left\{ \frac{-e_z \dot{l}_v}{E_{xz}} + \frac{\dot{n}_v}{E_{xz}} \right\};$$

$$n_p = \left\{ \frac{-e_z \dot{l}_p}{E_{xz}} + \frac{\dot{n}_p}{E_{xz}} \right\};$$

$$n_r = \left\{ \frac{-e_z \dot{l}_r}{E_{xz}} + \frac{\dot{n}_r}{E_{xz}} \right\};$$

$$y_\xi = \dot{y}_\xi;$$

$$l_\xi = \left\{ \frac{\dot{l}_\xi}{E_{xz}} + \frac{e_x \dot{n}_\xi}{E_{xz}} \right\};$$

$$l_\zeta = \left\{ \frac{\dot{l}_\zeta}{E_{xz}} + \frac{e_x \dot{n}_\zeta}{E_{xz}} \right\}$$

$$y_\zeta = \dot{y}_\zeta;$$

$$n_\xi = \left\{ \frac{-e_z \dot{l}_\xi}{E_{xz}} + \frac{\dot{n}_\xi}{E_{xz}} \right\};$$

$$n_\zeta = \left\{ \frac{-e_z \dot{l}_\zeta}{E_{xz}} + \frac{\dot{n}_\zeta}{E_{xz}} \right\}$$

3.0 ESTIMATION OF FULL SCALE HAWK DERIVATIVES.

To estimate a set of stability and control derivatives for the full scale Hawk aircraft a BAe document giving graphical details of various performance and stability and control data was used, (REF.4). A flight case was chosen which fell into the flight envelope which can be produced by the dynamic rig in the wind tunnel. Details of the flight case chosen and the estimation of the various dimensional derivatives now follows:

FLIGHT CASE DEFINITION and HAWK DESIGN DETAILS:

A/C SPEED	M = 0.31	V = 105.5 m/sec
A/C MASS	m = 9000 lb	m = 4082.4 kg
A/C HEIGHT	Sea Level	
A/C C.G. at	$h_g = 0.275 \bar{c}$	

WING AREA	$s = 179.635 \text{ ft}^2 = 16.6887 \text{ m}^2$
WING SPAN	$b = 30.808 \text{ ft} = 9.3903 \text{ m}$
HORIZONTAL TAIL ARM	$\bar{l}_T = 14.109 \text{ ft} = 4.299 \text{ m}$
INCLINATION OF FUSELAGE DATUM TO AIRSTREAM	$\alpha_f = 4^\circ$

MOMENT OF INERTIA ABOUT LONGITUDINAL, LATERAL, AND VERTICAL BODY AXES

	$I_x = 5346.7 \text{ kg/m}^2$
	$I_y = 19534.4 \text{ kg/m}^2$
	$I_z = 23786.5 \text{ kg/m}^2$
PRODUCT OF INERTIA	$I_{xz} = 816.74 \text{ kg/m}^2$

CONVERSION FACTORS:

1. $\rho V S = 2156.81 \text{ kg/sec}$
2. $\rho V S \bar{l}_T = 9272.11 \text{ kgm/sec}$
3. $\rho V^2 S = 227543.02 \text{ kgm/sec}^2$
4. $\rho S (\bar{l}_T)^2 = 377.83 \text{ kgm}$
5. $\rho V S (\bar{l}_T)^2 = 39860.79 \text{ kgm}^2/\text{sec}$
6. $\rho V^2 S \bar{l}_T = 978207.43 \text{ kgm}^2/\text{sec}^2$
7. $(1/2) \rho V S b = 10126.53 \text{ kgm/sec}$
8. $(1/2) \rho V^2 S b = 1068348.60 \text{ kgm}^2/\text{sec}^2$
9. $(1/4) \rho V S b^2 = 47545.56 \text{ kgm}^2/\text{sec}$

3.1 LONGITUDINAL DERIVATIVES AND MODES OF MOTION.

$$\dot{X}_u = X_u * \rho VS = -64.71 \text{ kg/sec} \Rightarrow x_u = -0.016$$

$$\dot{X}_w = X_w * \rho S \bar{l}_T = 0.0 \text{ kg} \Rightarrow x_w = 0.0$$

$$\dot{X}_w = X_w * \rho VS = +107.841 \text{ kg/sec} \Rightarrow x_w = 0.026$$

$$\dot{X}_q = X_q * \rho VS \bar{l}_T = 0.0 \text{ kgm/sec} \Rightarrow x_q = 0.0$$

$$\dot{X}_\eta = X_\eta * \rho V^2 S = 0.0 \text{ kgm/sec}^2 \Rightarrow x_\eta = 0.0$$

$$\dot{Z}_u = Z_u * \rho VS = -884.37 \text{ kg/sec} \Rightarrow z_u = -0.217$$

$$\dot{Z}_w = Z_w * \rho S \bar{l}_T = 0.0 \text{ kg} \Rightarrow z_w = 0.0$$

$$\dot{Z}_w = Z_w * \rho VS = -5478.297 \text{ kg/sec} \Rightarrow z_w = -1.342$$

$$\dot{Z}_q = Z_q * \rho VS \bar{l}_T = -5628.104 \text{ kgm/sec} \Rightarrow z_q = -1.379$$

$$\dot{Z}_\eta = Z_\eta * \rho V^2 S = -89196.856 \text{ kgm/sec}^2 \Rightarrow z_\eta = -21.849$$

$$\dot{M}_u = M_u * \rho VS \bar{l}_T = -120.536 \text{ kgm/sec} \Rightarrow m_u = -0.005$$

$$\dot{M}_w = M_w * \rho S (\bar{l}_T)^2 = -92.19 \text{ kgm} \Rightarrow m_w = -0.0047$$

$$\dot{M}_w = M_w * \rho VS \bar{l}_T = -1066.292 \text{ kgm/sec} \Rightarrow m_w = -0.048$$

$$\dot{M}_q = M_q * \rho VS (\bar{l}_T)^2 = -23039.658 \text{ kgm}^2/\text{sec} \Rightarrow m_q = -1.669$$

$$\dot{M}_\eta = M_\eta * \rho V^2 S \bar{l}_T = -383457.144 \text{ kgm}^2/\text{sec}^2 \Rightarrow m_\eta = -19.527$$

SHORT PERIOD PITCHING OSCILLATION:

$$(s^2 + 2\rho_{sp} \omega_{sp} s + \omega_{sp}^2)$$

$$\omega_{sp} = 2.8 \text{ rad/sec}$$

$$\rho_{sp} = 0.54$$

$$\Rightarrow s = (-1.512 \pm 2.357i)$$

PHUGOID:

$$(s^2 + 2\rho_p \omega_p s + \omega_p^2)$$

$$\omega_p = 0.077 \text{ rad/sec}$$

$$\rho_p = 0.065$$

$$\Rightarrow s = (-0.005 \pm 0.077i)$$

CHARACTERISTIC EQUATION:

$$\Delta(s) = s^4 + 3.034s^3 + 7.876s^2 + 0.096s + 0.046 = 0$$

3.2 LATERAL DERIVATIVES AND MODES OF MOTION.

$$\dot{Y}_v = Y_v * \rho V S = -864.879 \text{ kg/sec} \Rightarrow y_v = -0.212$$

$$\dot{Y}_p = Y_p * (1/2) \rho V S b = 0.0 \text{ kgm/sec} \Rightarrow y_p = 0.0$$

$$\dot{Y}_r = Y_r * (1/2) \rho V S b = 0.0 \text{ kgm/sec} \Rightarrow y_r = 0.0$$

$$\dot{Y}_\xi = Y_\xi * \rho V^2 S = 0.0 \text{ kgm/sec}^2 \Rightarrow y_\xi = 0.0$$

$$\dot{Y}_\zeta = Y_\zeta * \rho V^2 S = -31173.394 \text{ kgm/sec}^2 \Rightarrow y_\zeta = +7.636$$

$$\dot{L}_v = L_v * (1/2) \rho V S b = -486.073 \text{ kgm/sec} \Rightarrow l_v = -0.085$$

$$\dot{L}_p = L_p * (1/4) \rho V S b^2 = -20206.865 \text{ kgm}^2/\text{sec} \Rightarrow l_p = -3.780$$

$$\dot{L}_r = L_r * (1/4) \rho V S b^2 = +5943.196 \text{ kgm}^2/\text{sec} \Rightarrow l_r = +1.038$$

$$\dot{L}_\xi = L_\xi * (1/2) \rho V^2 S b = -188136.189 \text{ kgm}^2/\text{sec}^2 \Rightarrow l_\xi = -34.842$$

$$\dot{L}_\zeta = L_\zeta * (1/2) \rho V^2 S b = +30982.109 \text{ kgm}^2/\text{sec}^2 \Rightarrow l_\zeta = +5.075$$

$$\dot{N}_v = N_v * (1/2) \rho V S b = +875.945 \text{ kgm/sec} \Rightarrow n_v = +0.040$$

$$\dot{N}_p = N_p * (1/4) \rho V S b^2 = -3138.007 \text{ kgm}^2/\text{sec} \Rightarrow n_p = -0.002$$

$$\dot{N}_r = N_r * (1/4) \rho V S b^2 = -10550.361 \text{ kgm}^2/\text{sec} \Rightarrow n_r = -0.479$$

$$\dot{N}_\xi = N_\xi * (1/2) \rho V^2 S b = +25640.366 \text{ kgm}^2/\text{sec}^2 \Rightarrow n_\xi = +2.274$$

$$\dot{N}_\zeta = N_\zeta * (1/2) \rho V^2 S b = -107903.209 \text{ kgm}^2/\text{sec}^2 \Rightarrow n_\zeta = -4.711$$

ROLL SUBSIDENCE MODE: $(1 + sT_R); T_R = 0.33 \text{ sec}; s = -3.0 \text{ sec}^{-1}$

SPIRAL MODE: $(1 + sT_S); T_S = 91.74 \text{ sec}; s = -0.0109 \text{ sec}^{-1}$

DUTCH ROLL: $(s^2 + 2\rho_{dr}\omega_{dr}s + \omega_{dr}^2)$

$\omega_{dr} = 2.0 \text{ rad/sec} \quad \rho_{dr} = 0.178 \Rightarrow s = (-0.356 \pm 1.968i)$

CHARACTERISTIC EQUATION:

$$\Delta(s) = s(s^4 + 3.753s^3 + 6.198s^2 + 12.188s + 0.132) = 0$$

4.0 ADVANCED CONTINUOUS SIMULATION LANGUAGE PROGRAMS.

The stability and control derivatives of both the longitudinal and lateral ACSL programs have been changed to those of the full scale BAe Hawk aircraft. The details of the estimation of these derivatives are given in section 2. The relevant flight conditions were also inserted into the programs. (Previously the derivatives of the F-4 had been used to test the ACSL programs).

4.1 IMPROVEMENTS TO THE LATERAL SIMULATION PROGRAMS.

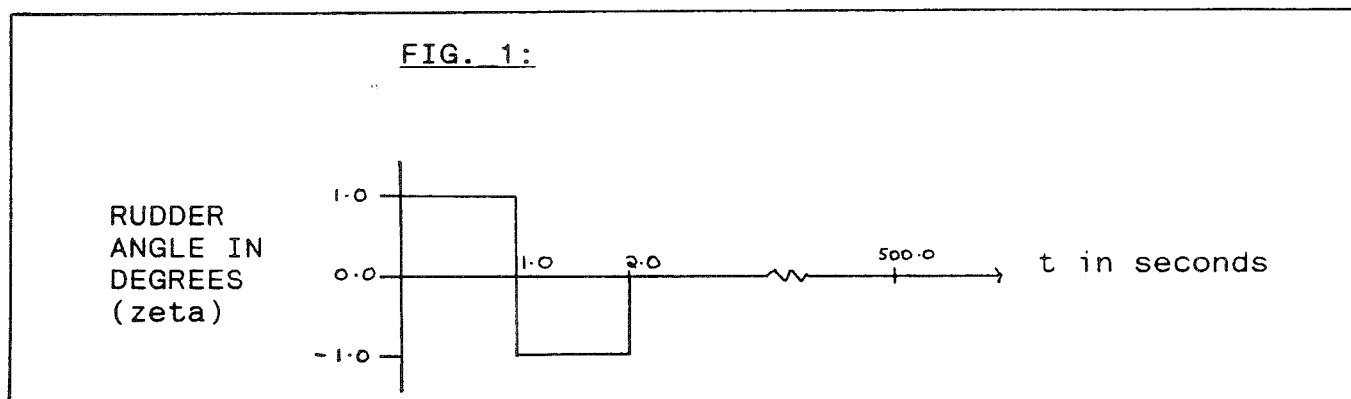
The two lateral ACSL programs LATROL and LATYAW have now been combined into a single lateral program HKLAT. The program has been written so that it produces the response of the "Hawk" to a step input of 1° of aileron by default. To change to an impulse input on the aileron control surface the constant TIMEZT is set to a very small value as appropriate. Further, to change to a step input of rudder the following is simply typed in when running the program:

```
SET RUDON = .T.  
SET AILON = .F.  
SET CMD=10      (which runs the model from a pre-defined command file)
```

The logical constant RUDON = .T. implies that a rudder input is required whilst the constant AILON = .F. implies that no aileron input is to be used. The ease of changing from aileron to rudder input in the same program gives a greater flexibility than was possible before with two programs.

4.2 MODELLING OF CONTROL SURFACE INPUTS.

In ACSL a TABLE function may be used to define a dependent variable which has up to three independent parameters. The control surface angle inputs to the aircraft model can thus be defined using the TABLE function with only one independent parameter, that of time. For example, the rudder doublet shown in FIG.1 may be defined in ACSL as shown below:



IN THE ACSL PROGRAM:

```
TABLE ZTDEG, 1, 7 ...  
  / 0.0, 0.99, 1.0, 1.01, 1.99, 2.0, 500.0 ...  
  / 1.0, 1.0, 0.0, -1.0, -1.0, 0.0, 0.0 ...
```

The first line shows that there is one independent variable, in this case time, and that there are 7 values of ZTDEG defined at 7 points in time. When the simulation is run ACSL will interpolate between the values defined in the above table to calculate the value of ZTDEG at each time step in the integration procedure. Thus between times $t = 0.99$ sec and $t = 1.01$ sec when zeta has been defined as changing from +1.0 to -1.0 a "smooth" change in zeta will be seen by the simulation rather than a sudden jump from +1.0 to -1.0 in a single time step.

It is also possible to define the TABLE function in a separate ACSL procedure which can be read from the main ACSL program; in the same way that a FORTRAN program accesses a subroutine. This facility will be useful when recorded control surface inputs from the dynamic rig are modeled in order to test the integrity of the control and stability derivatives estimated from the MSR procedure.

4.3 FULL SCALE HAWK SIMULATION RESULTS.

The longitudinal and lateral ACSL programs were run to compare the aircraft responses obtained with those predicted by the BAe report (REF.4) and those obtained using a control system design package on the BBC microcomputer. Figures 2 - 6 show the various longitudinal and lateral modes obtained using the ACSL simulation. TABLE 1 summarises the frequencies, damping ratios and time constants obtained from REF.4, the BBC package and measured from the ACSL responses, figures 2-6.

TABLE 1: MODES OF MOTION OF THE BAe HAWK.

SPPO:

BAe	$s = (-1.512 \pm 2.357i);$	$\omega_{sp} = 2.8 \text{ rad/sec};$	$\rho_{sp} = 0.54$
BBC	$s = (-1.512 \pm 2.231i);$	$\omega_{sp} = 2.7 \text{ rad/sec};$	$\rho_{sp} = 0.56$
ACSL GRAPH	$s = (-0.805 \pm 2.155i)$	$\omega_{sp} = 2.3 \text{ rad/sec}$	$\rho_{sp} = 0.68$

PHUGOID:

BAe	$s = (-0.005 \pm 0.077i);$	$\omega_p = 0.077 \text{ rad/sec};$	$\rho_p = 0.065$
BBC	$s = (-0.002 \pm 0.071i);$	$\omega_p = 0.069 \text{ rad/sec};$	$\rho_p = 0.073$
ACSL GRAPH	$s = (-0.001 \pm 0.071i);$	$\omega_p = 0.071 \text{ rad/sec};$	$\rho_p = 0.070$

ROLL SUBSIDENCE:

BAe	$s = (-3.0 \text{ sec}^{-1});$	$T_R = 0.33 \text{ sec}$
BBC	$s = (-3.8 \text{ sec}^{-1});$	$T_R = 0.26 \text{ sec}$
ACSL GRAPH	$s = (-2.3 \text{ sec}^{-1});$	$T_R = 0.43 \text{ sec}$

SPIRAL MODE:

BAe	$s = (-0.0109 \text{ sec}^{-1});$	$T_s = 91.73 \text{ sec}$
BBC	$s = (-0.0005 \text{ sec}^{-1});$	$T_s = 2000 \text{ sec}$
ACSL GRAPH	$s = (-0.0321 \text{ sec}^{-1});$	$T_s = 31.11 \text{ sec}$

DUTCH ROLL:

BAe	$s = (-0.356 \pm 1.968i);$	$\omega_{dr} = 2.0 \text{ rad/sec};$	$\rho_{dr} = 0.178$
BBC	$s = (-0.320 \pm 2.090i);$	$\omega_{dr} = 2.1 \text{ rad/sec};$	$\rho_{dr} = 0.153$
ACSL GRAPH	$s = (-0.152 \pm 2.044i)$	$\omega_{dr} = 2.05 \text{ rad/sec};$	$\rho_{dr} = 0.148$

FIGURE 2: LONGITUDINAL SHORT PERIOD PITCHING OSCILLATION.

INPUT: IMPULSE TO ELEVATOR.

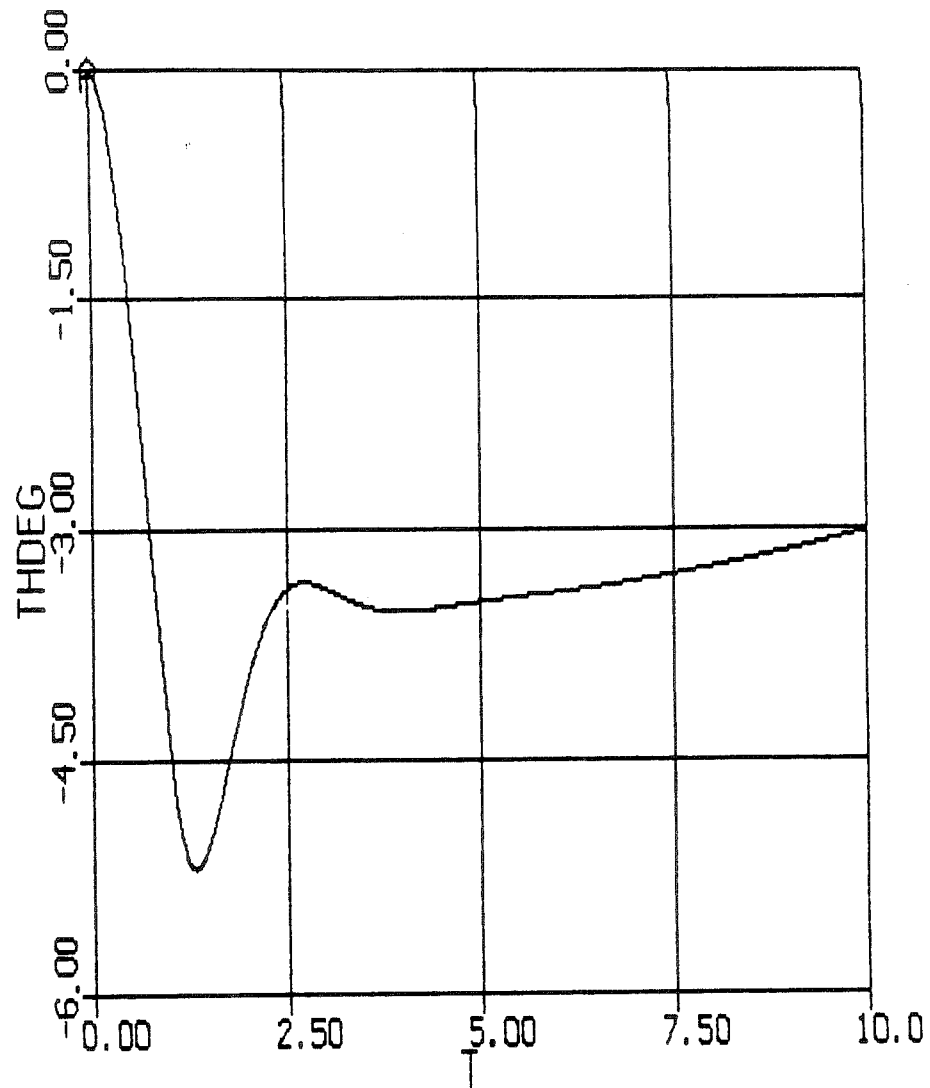


FIGURE 3: LONGITUDINAL PHUGOID OSCILLATION.

INPUT: IMPULSE TO ELEVATOR.

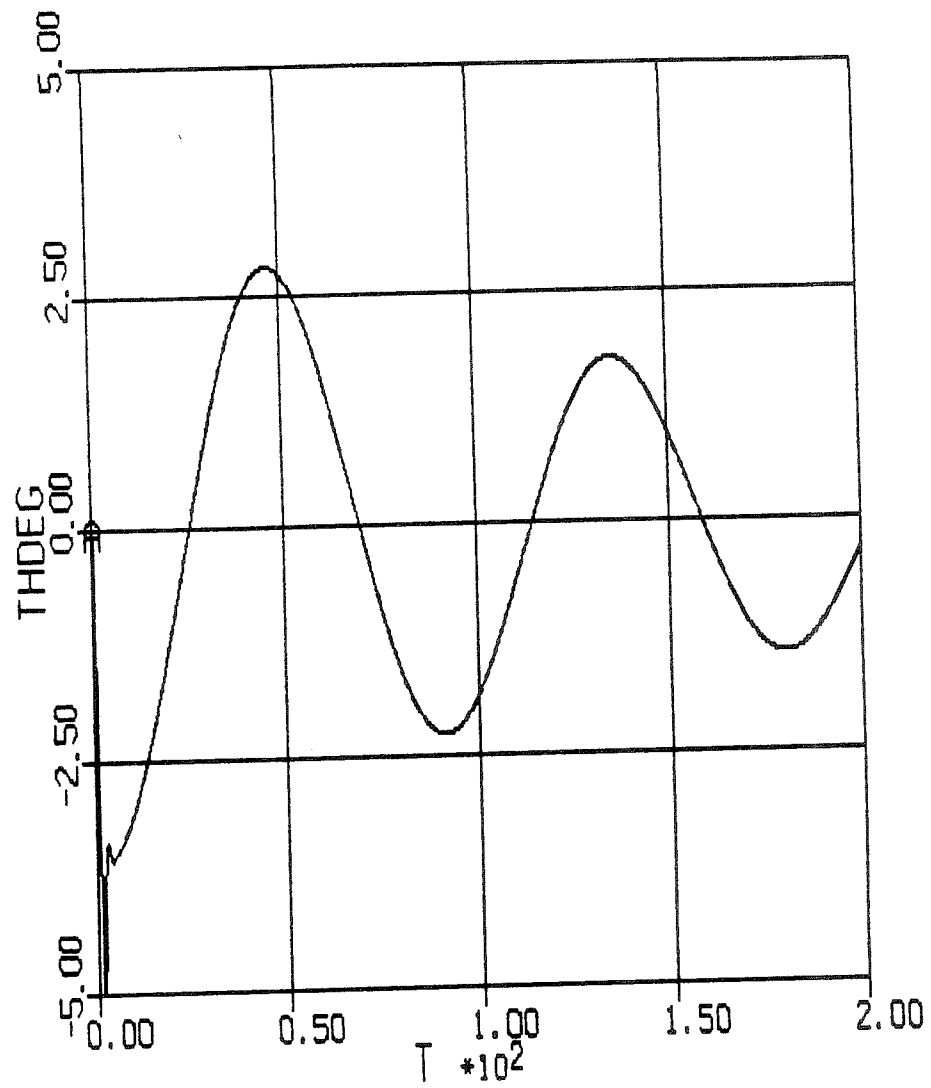


FIGURE 4: LATERAL ROLL SUBSIDENCE MODE.

INPUT: STEP TO AILERON.

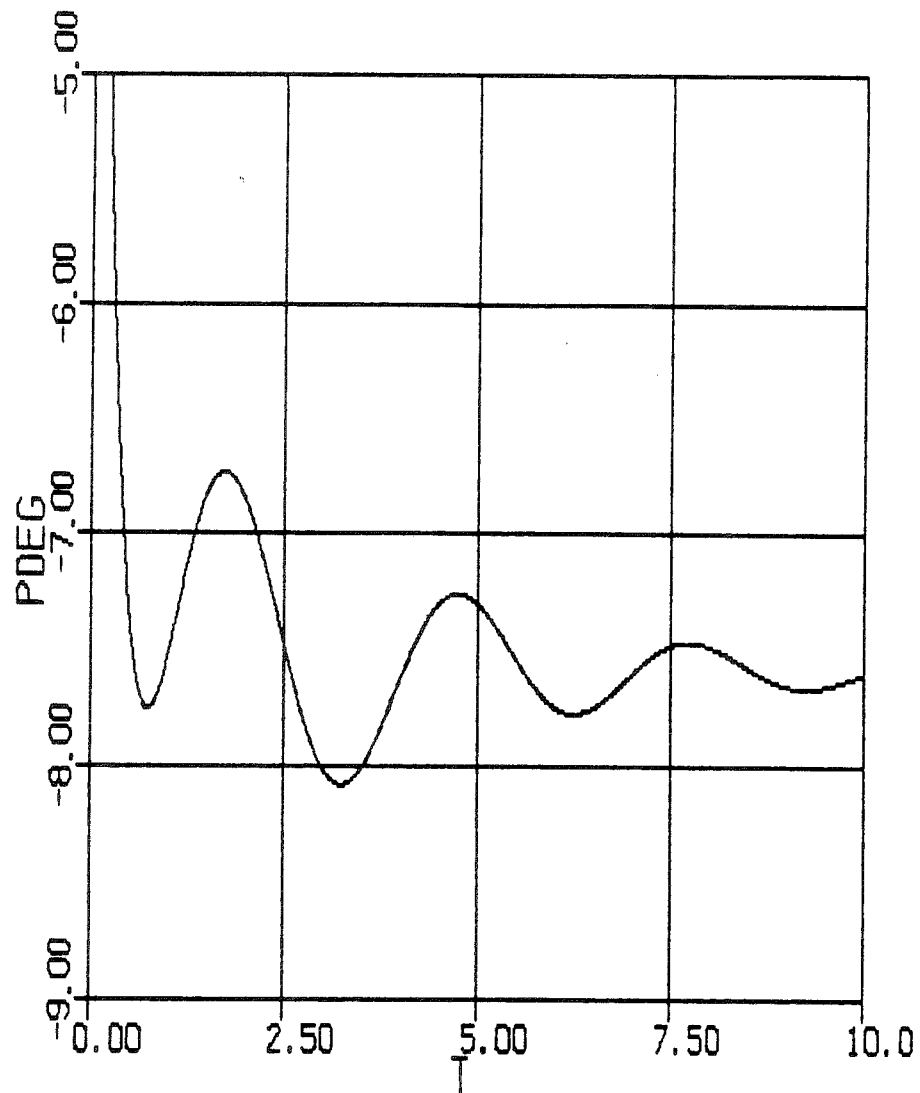


FIGURE 5: LATERAL SPIRAL MODE.

INPUT: STEP TO RUDDER.

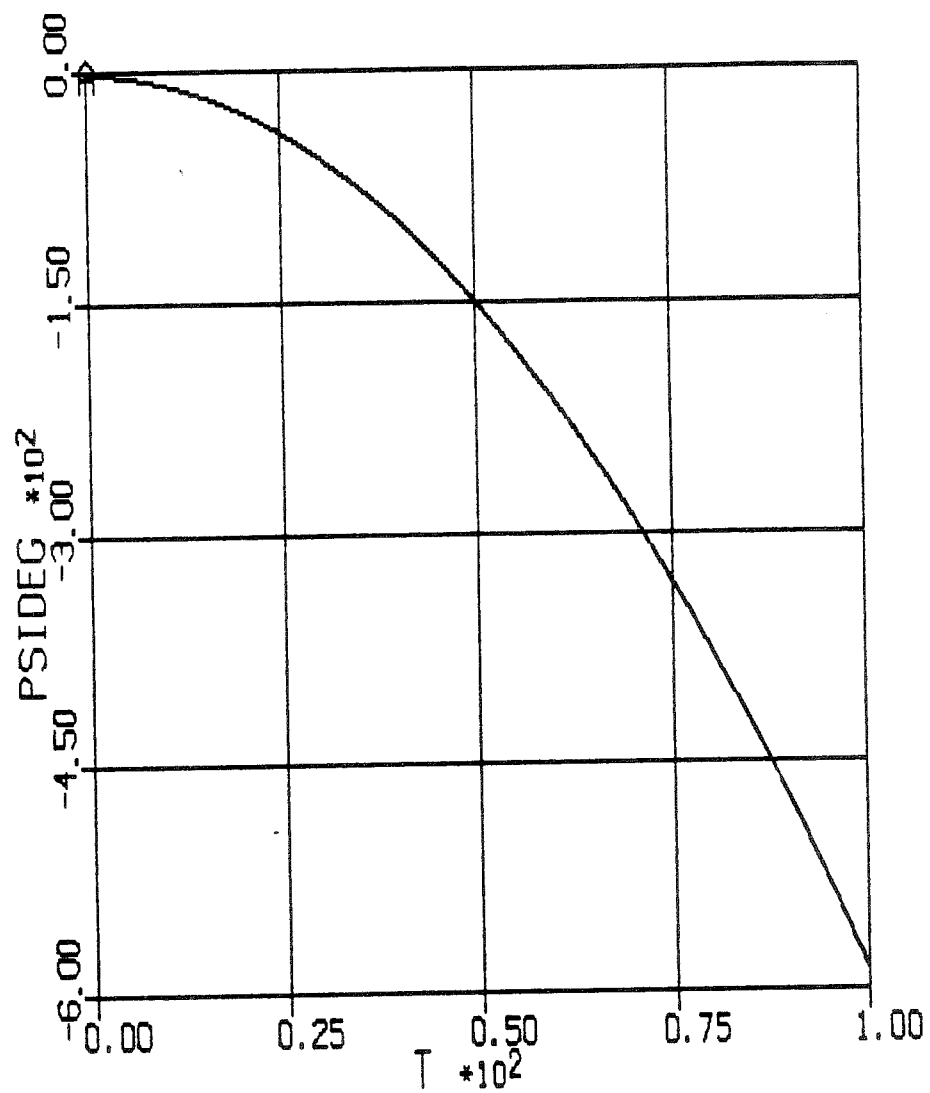
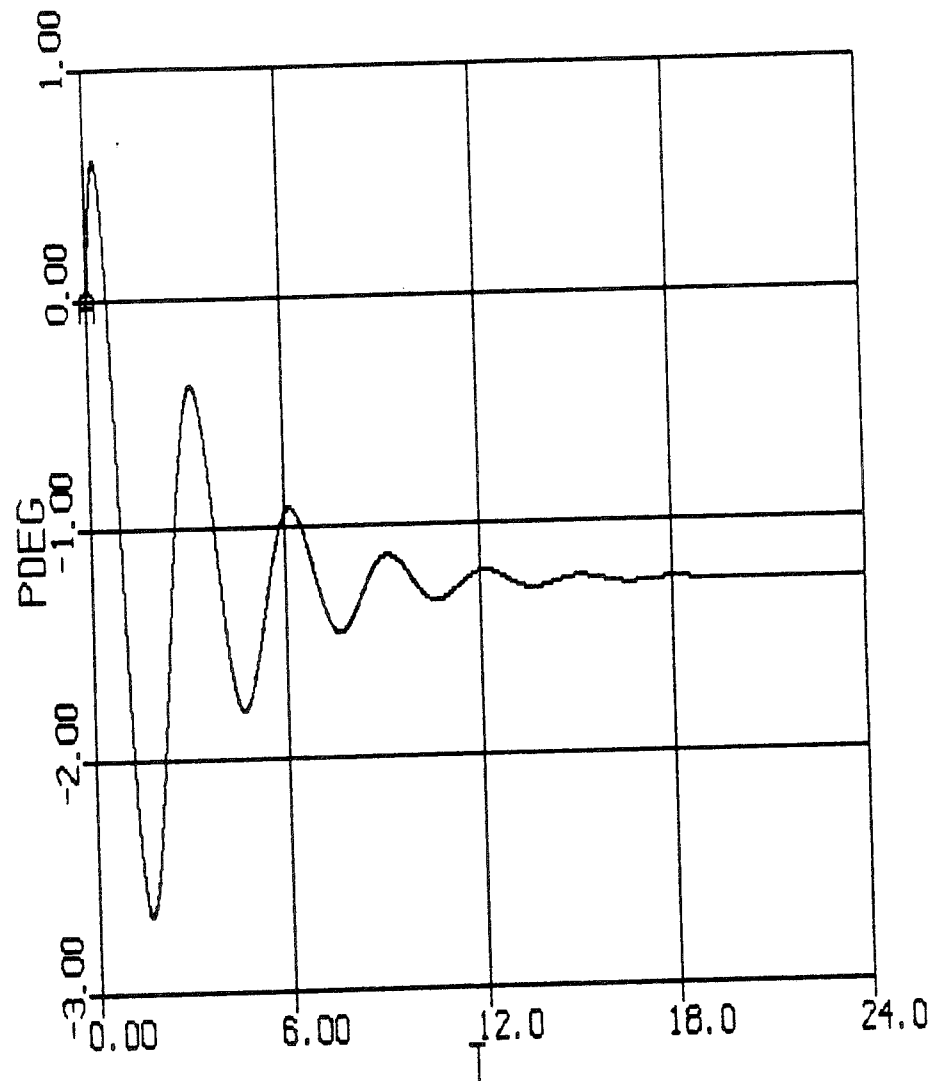


FIGURE 6: LATERAL DUTCH ROLL OSCILLATION.

INPUT: IMPULSE TO RUDDER.



5.0 HAWK MODEL AND DYNAMIC RIG STATUS.

The Hawk aircraft wind tunnel model and dynamic test rig, which have been the subject of a parallel development programme, are now fully operational and have been developed as far as that programme allowed. The development referred to and the current status of the experimental facility are described in an M.Sc thesis, (REF.5).

A servo system for measuring the vertical height, velocity and acceleration of the model on the vertical rod has been developed and demonstrated to work satisfactorily. Since the system controls the tension in the vertical cable attached to the model it is possible also to extend the scale flight envelope by artificially adjusting the weight or apparent "g" acting on the model over a limited range. Although the control system appears to work well in its basic form it may need adjustment or modification for the present research programme.

When flown in the wind tunnel the model is very lively, sufficiently so to make autostabilisation a prudent addition. Simple feedback loops have been demonstrated and shown to work well. However, the biggest remaining problem is to trim the model to a suitable vertical position in the wind tunnel whilst retaining adequate control over it. With this in mind some work has been undertaken to design and test a suitable "height hold autopilot". At the time of writing this work is not complete but sufficient progress has been made to indicate that it looks quite feasible in practice. A by-product of this activity is that the equation of motion in the wind tunnel model will need to be extended to include the feedback loops as appropriate.

The planned experiments to measure model inertias and suspension system friction have not yet taken place. It is hoped to complete these in the next quarter of the programme.

6.0 DERIVATION OF AIRCRAFT ATTITUDE RATES.

The dynamic rig was initially designed so that the attitude angles of the Hawk model may be accurately measured via various potentiometers. However, size constraints on the scaled model meant that it was not feasible to insert rate gyros into the model. Thus any attitude rate data required has to be generated using either analogue or digital methods.

The following two sections discuss analogue and digital/numerical methods for differentiating data. It is expected that both types of techniques will be evaluated to see which method produces the most reliable derivative estimates. A number of references are available on the design of an analogue differentiator and numerical differentiation of data. Some of these are given in the reference section at the end of this report.

6.1 ANALOGUE DIFFERENTIATION.

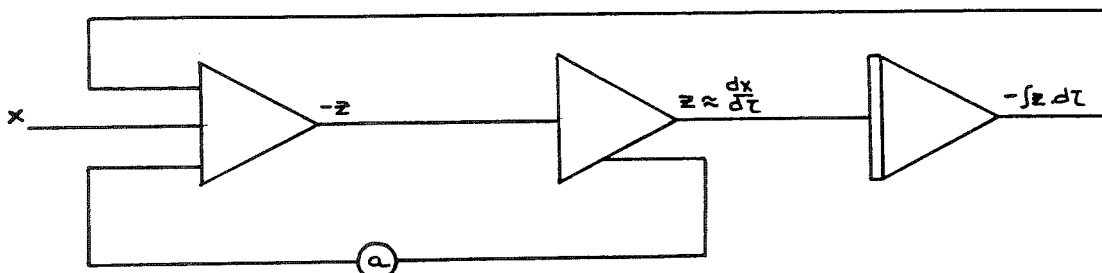
Figure 7 below shows a circuit for the design of an approximate differentiator using three amplifiers, (REF.6). This circuit is based upon the solution of the implicit differential equation:

$$(1-a) \cdot \frac{dx}{d\tau} + z = \frac{dx}{d\tau} \quad (12)$$

If $a = 1$, the above differential equation gives the desired relationship $z = dx/d\tau$. In practice, the magnitude of "a" which is used will depend on the frequency of x . If "a" is too large, a high frequency input will cause oscillations in the circuit. The value of "a" must therefore be determined by trial and error. It will typically be in the region of 0.96.

FIG. 7: CIRCUIT FOR APPROXIMATE DIFFERENTIATION.

As a is increased, z approaches the time derivative dx/dt .



DISCUSSION: Use of an analogue differentiator has three main drawbacks

1. It decreases the signal to noise ratio in the circuit.
2. An operational amplifier used as a differentiator may frequently be driven to saturation and overload.
3. Stability problems may be encountered, for some amplifiers are quite sensitive to capacitance loading.

6.2 DIGITAL DIFFERENTIATION.

The use of a digital computer to obtain attitude rates may be broken down into two main methods. The first method involves the substitution of $s\theta$ for q , $s\phi$ for p , and $s\psi$ for r in the mathematical definition of the equations of motion in the MSR procedure. Alternatively, numerical differentiation may be used and this method is now discussed more fully.

Tabulated data is generally not used directly in numerical differentiation because any scatter in the data can cause serious accuracy problems. Whenever possible, an analytical expression for a smooth curve which fits the data should be determined. Analytical differentiation can then be used, or numerical differentiation with ordinate values obtained from the fitted curve. The numerical method to be discussed utilizes Taylor-series expansions, REF.11.

The Taylor series for a function $y = f(x)$ at $(x_i + h)$ when expanded about x_i is

$$y(x_i + h) = y_i + y'_i \cdot h + \frac{y''_i h^2}{2!} + \frac{y'''_i h^3}{3!} + \dots \quad (13)$$

where $h = \Delta x$ and y_i is the ordinate corresponding to x_i and $(x_i + h)$ is in the region of convergence. The function at $(x_i - h)$ is similarly given by

$$y(x_i - h) = y_i - y'_i \cdot h + \frac{y''_i h^2}{2!} - \frac{y'''_i h^3}{3!} + \dots \quad (14)$$

Subtracting 14 from 13, we obtain

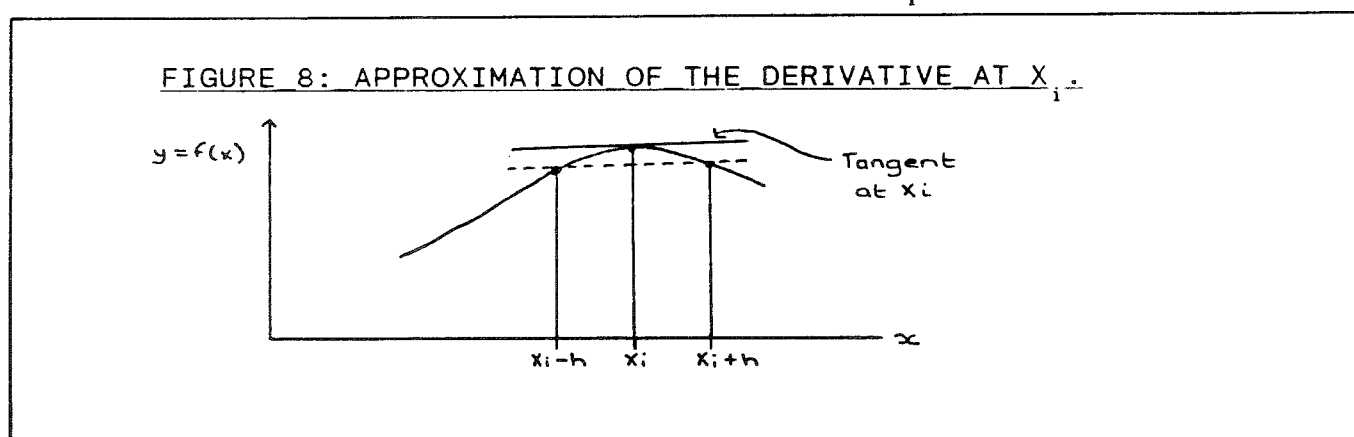
$$y'_i = \frac{y(x_i + h) - y(x_i - h)}{2h} - \left(\frac{1}{6} y'''_i h^2 + \dots \right) \quad (15)$$

Looking at FIG.8, it is seen that if we designate equally spaced points to the right of x_i as x_{i+1} , x_{i+2} , and so on, and those to the left of x_i as x_{i-1} , x_{i-2} , and identify the corresponding ordinates as y_{i+1} , y_{i+2} , y_{i-1} , and y_{i-2} , respectively, equation 15 may be written:

$$y'_i = \frac{y_{i+1} - y_{i-1}}{2h} \quad (16)$$

with error of order h^2 . Equation 16 is called the central-difference approximation of y' at x_i , with errors of order h^2 .

Graphically, the approximation represents the slope of the dashed line in FIG.8. The actual derivative is represented by the slope of the solid line drawn tangent to the curve at x_i .



It has been shown that the central-difference expressions for the various derivatives involve values of the function on both sides of the the x value at which the derivative of the function is desired. By utilizing the appropriate Taylor-series expansion, one can easily obtain expressions for the derivatives which are entirely in terms of values of the function at x_i and points to the right of x_i . These are known as forward-finite-difference expressions. In a similar manner, derivative expressions which are entirely in terms of the function x_i and points to the left of x_i can be found. These are known as backward-finite difference expressions. In numerical differentiation, forward-difference expressions are used when data to the left of a point at which a derivative is desired are not available, and backward-difference expressions are used when data to the right of the desired point are not available. Central-difference expressions, are more accurate than either forward- or backward- difference expressions. This can be seen by noting the order of the error in the summary of differentiation formulas which follows over leaf.

CENTRAL-DIFFERENCE EXPRESSIONS WITH ERROR OF ORDER h^2

$$y'_i = \frac{y_{i+1} - y_{i-1}}{2h}$$

$$y''_i = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

$$y'''_i = \frac{y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}}{3h^2}$$

$$y''''_i = \frac{y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}}{h^4}$$

CENTRAL-DIFFERENCE EXPRESSIONS WITH ERROR OF ORDER h^4

$$y'_i = \frac{-y_{i+2} + 8y_{i+1} - 8y_{i-1} + y_{i-2}}{12h}$$

$$y''_i = \frac{-y_{i+2} + 16y_{i+1} - 30y_i + 16y_{i-1} - y_{i-2}}{12h^2}$$

$$y'''_i = \frac{-y_{i+3} + 8y_{i+2} - 13y_{i+1} + 13y_{i-1} - 8y_{i-2} + y_{i-3}}{8h^3}$$

$$y''''_i = \frac{-y_{i+3} + 12y_{i+2} - 39y_{i+1} + 56y_i - 39y_{i-1} + 12y_{i-2} - y_{i-3}}{6h^4}$$

FORWARD-DIFFERENCE EXPRESSIONS WITH ERROR OF ORDER h

$$y'_i = \frac{y_{i+1} - y_i}{h}$$

$$y''_i = \frac{y_{i+2} - 2y_{i+1} + y_i}{h^2}$$

$$y'''_i = \frac{y_{i+3} - 3y_{i+2} + 3y_{i+1} - y_i}{h^3}$$

$$y''''_i = \frac{y_{i+4} - 4y_{i+3} + 6y_{i+2} - 4y_{i+1} + y_i}{h^4}$$

FORWARD-DIFFERENCE EXPRESSIONS WITH ERROR OF ORDER h^2 .

$$y'_i = \frac{-y_{i+2} + 4y_{i+1} - 3y_i}{2h}$$

$$y''_i = \frac{-y_{i+3} + 4y_{i+2} - 5y_{i+1} + 2y_i}{h^2}$$

$$y'''_i = \frac{-3y_{i+4} + 14y_{i+3} - 24y_{i+2} + 18y_{i+1} - 5y_i}{2h^3}$$

$$y''''_i = \frac{-2y_{i+5} + 11y_{i+4} - 24y_{i+3} + 26y_{i+2} - 14y_{i+1} + 3y_i}{h^4}$$

BACKWARD-DIFFERENCE EXPRESSIONS WITH ERROR OF ORDER h .

$$y'_i = \frac{y_i - y_{i-1}}{h}$$

$$y''_i = \frac{y_i - 2y_{i-1} + y_{i-2}}{h^2}$$

$$y'''_i = \frac{y_i - 3y_{i-1} + 3y_{i-2} - y_{i-3}}{h^3}$$

$$y''''_i = \frac{y_i - 4y_{i-1} + 6y_{i-2} - 4y_{i-3} + y_{i-4}}{h^4}$$

BACKWARD-DIFFERENCE EXPRESSIONS WITH ERROR OF ORDER h^2 .

$$y'_i = \frac{3y_i - 4y_{i-1} + y_{i-2}}{h}$$

$$y''_i = \frac{2y_i - 5y_{i-1} + 4y_{i-2} - y_{i-3}}{h^2}$$

$$y'''_i = \frac{5y_i - 18y_{i-1} + 24y_{i-2} - 14y_{i-3} + 3y_{i-4}}{2h^3}$$

$$y''''_i = \frac{3y_i - 14y_{i-1} + 26y_{i-2} - 24y_{i-3} + 11y_{i-4} - 2y_{i-5}}{h^4}$$

7.0 MODIFIED STEPWISE REGRESSION (MSR) PROCEDURE.

The mathematical format required for the MSR is best shown by the following example which is taken from the longitudinal equations of motion:

$$\dot{w} = z_0 + z_u \cdot u + z_w \cdot w + z_q \cdot q + z_\eta \cdot \eta \quad (17)$$

This is of the form:

$$y(t) = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + \dots + b_{n-1} \cdot x_{n-1} \quad (18)$$

In equation 18, b_1 to b_{n-1} are the stability and control derivatives to be estimated; b_0 is a constant dependent on the initial steady-state flight conditions; x_1 to x_{n-1} are independent aircraft state and control variables. Finally, y is the dependent variable and in the above example $y = \dot{w}$, where $\dot{w}$ is the rate of change of the perturbed vertical velocity (referred to body axes) which will have to be generated in some way using data from the experimental rig.

If a sequence of N readings of y and the x 's, (ie $\dot{w}$, u , w , q , η), are taken at times $t_1, t_2, \dots, t_N$ and denoted by $y(i), x_1(i), x_2(i), \dots, x_{n-1}(i)$ where $i = 1, 2, \dots, N$ then the data acquired can be related by the following set of N linear equations:

$$y(i) = b_0 + b_1 \cdot x_1(i) + b_2 \cdot x_2(i) + \dots + b_{n-1} \cdot x_{n-1}(i) + \epsilon(i) \quad (19)$$

$\epsilon(i)$ is the equation error which is introduced here as equation 19 is only an approximation of the actual aerodynamic relationship.

Further, equation 19 may be expressed in matrix form as $Y = \underline{b} \cdot X$ (20)

For $N \gg n$, the first estimate of the derivatives, $\underline{b}$, can be made using the method of least squares as shown below:

$$\underline{\beta} = \left(X^T X \right)^{-1} X^T Y \quad (21)$$

where $\underline{\beta}$ is the $n \times 1$ vector of parameter estimates, Y is the $N \times 1$ vector of measured variables of $Y(i)$, and X is the $N \times n$ matrix of measured independent variables.

The covariance matrix of parameter errors has the form:

$$E \left\{ (\beta - b) (\beta - b)^T \right\} = \sigma^2 \left(X^T X \right)^{-1} \quad (22)$$

For an estimate of this covariance matrix, σ^2 is replaced by its estimate

$$s^2 = \frac{1}{N - n} \sum_{i=1}^N \hat{\epsilon}^2(i) \quad \text{where} \quad \hat{\epsilon}(i) = y(i) - \hat{y}(i) \quad (23)$$

and
$$\hat{y}(i) = \beta_0 + \beta_1 \cdot x_1(i) + \beta_2 \cdot x_2(i) + \dots + \beta_{n-1} \cdot x_{n-1}(i) \quad (24)$$

A test is then carried out on this overall regression equation 24 by calculating the random variable F given by equation (25).

$$F = \frac{\beta^T X^T Y - N \bar{y}^2}{(n - 1) s^2} \quad (25)$$

This variable is represented by an F-distribution with $v_1 = n - 1$ and $v_2 = N - n$ degrees of freedom and a significance level of α_p . Tabulated values of the F-distribution, $F(v_1, v_2, \alpha_p)$, are found in statistical reference tables such as that given in REF.13. If the value of F (calculated from 25 above) is greater than $F(v_1, v_2, \alpha_p)$ then it is possible to say with a confidence of $(1 - \alpha_p) \times 100\%$ that not ALL of the derivatives $\beta_0, \beta_1, \beta_2, \dots, \beta_{n-1}$ are zero, although one or two may be zero.

If at least 100 sets of observations N have been recorded the effect of n (the number of independent variables) on the tabulated values of F is small and a critical value of 12 is selected, REF.14.

The following procedures are then carried out:

1. The significance of individual terms in the regression is examined next using a partial F-test. For each independent variable

the term
$$F_p = \frac{\beta_j^2}{s^2(\beta_j)^2} \quad \text{is calculated} \quad (26)$$

where $s^2(\beta_j)^2$ is the variance estimate of β obtained from 23.

If $F_p > F(v_1, v_2, \alpha_p)$ ie. $F_p > 12$, it may be assumed that the term being tested is not equal to zero and should be kept in the regression equation. If $F_p < 12$ there is a "chance" that $\beta_j = 0$ and the the term is rejected from the regression equation.

2. At this stage it is worth calculating the squared multiple correlation coefficient R^2 . This coefficient is used as an indication of how "well" the independent variables $x_1, x_2, \dots, x_{n-1}$ (which are in the regression equation) correlate with the dependent variable y . The closer the value of R^2 to 1, the better the correlation and the confidence in the regression model obtained. R^2 is given by 27:

$$R^2 = \frac{\sum_i [\hat{y}(i) - \bar{y}]^2}{\sum_i [y(i) - \bar{y}]^2} = \frac{\beta^T X^T Y - N \bar{y}^2}{Y^T Y - N \bar{y}^2} \quad (27)$$

and is related to the variable F by $F = \frac{N - n}{n - 1} \cdot \frac{R^2}{1 - R^2}$ (28)

3. A new set of derivatives β is estimated using equation 21. R^2 is calculated and there should be an improvement in its value having rejected a term from the regression equation and hopefully having improved the regression model.

4. The remaining independent variables have their new values of F_p calculated and examined to see if F_p is < 12 . Any variable with $F_p < 12$ is rejected from the regression equation.

5. The best of the variables not currently in the model (ie. the one whose partial correlation F_p with y , given the variables already in the equation, is greatest) is checked to see if it passes the partial F entry test. If F_p is > 12 this "best" variable is returned to the regression model.

Steps 3, 4 and 5 are repeated until no more variables are entered or are removed from the regression equation and no further improvement in R^2 is obtained. It is then considered that the "best fit model" for the observed data has been found. A computing scheme for the procedure described above may be found in REF.15.

8.0 FUTURE OBJECTIVES.

1. Work will continue on the "height hold" system for the model on the dynamic rig.
2. The model aircrafts moments and products of inertia will be measured via oscillatory methods. At the same time an attempt will be made to measure the mechanical friction damping terms of the model gimbal. This would enable the mechanical and aerodynamic damping terms to be separated in the mathematical model of the system.
3. The first version of the MSR program will be written. This program will be used to model a full set of equations using data produced by the ACSL simulations as input. Once the algorithms/method have been validated in this way the program can be extended to include the height hold control system and also take into account the reduced degrees of freedom of the model in the wind tunnel.

9.0 CONCLUSION.

Using the full scale Hawk stability and control derivatives which were estimated from REF.5 when running the ACSL simulation programs led to aircraft responses which were very close to those that were expected. The damping ratios and frequencies of the longitudinal SPP0 and phugoid oscillation were in good agreement, as were the lateral dutch roll and roll subsidence modes. The spiral mode was the only mode not clearly defined. The spiral time constant T_s is a difficult root to obtain accurately as it is a relatively small root close to zero. Overall, from these results it would appear that the ACSL simulation programs provide a satisfactory representation of the full scale Hawk aircraft.

LIST OF SYMBOLS

b	wing span
$\bar{c}$	aerodynamic mean chord
C_x	drag coefficient
C_y	sideforce coefficient
C_z	lift coefficient
C_l	rolling moment coefficient
C_m	pitching moment coefficient
C_n	yawing moment coefficient
I_x, I_y, I_z, I_{xz}	aircraft moment of inertias
$\bar{l}_T$	horizontal tail arm
$\hat{L}_v, \hat{L}_p, \hat{L}_r, \hat{L}_\xi, \hat{L}_\zeta$	dimensional rolling moment derivative due to sideslip, roll rate, yaw rate, etc.
$\hat{M}_u, \hat{M}_w, \hat{M}_\dot{w}, \hat{M}_q, \hat{M}_\eta$	dimensional pitching moment derivative due to forward velocity, side velocity, etc.
m	aircraft mass
M	mach no.
$\hat{N}_v, \hat{N}_p, \hat{N}_r, \hat{N}_\xi, \hat{N}_\zeta$	dimensional yawing moment derivative due to sideslip, roll rate, yaw rate, etc.
p, q, r	rate of roll pitch and yaw respectively
s	wing area
u, v, w	components of velocity
V	total velocity
$\hat{X}_u, \hat{X}_w, \hat{X}_{\dot{w}}, \hat{X}_q, \hat{X}_\eta$	dimensional drag force derivative due to forward velocity, side velocity, etc.
$\hat{Y}_v, \hat{Y}_p, \hat{Y}_r, \hat{Y}_\xi, \hat{Y}_\zeta$	dimensional sideforce derivative due to sideslip, roll rate, yaw rate, etc.
$\hat{Z}_u, \hat{Z}_w, \hat{Z}_{\dot{w}}, \hat{Z}_q, \hat{Z}_\eta$	dimensional lift force derivative due to forward velocity, side velocity, etc.
α_f	inclination of fuselage datum to air stream
α	angle of attack, $\tan^{-1}(w/u)$
β	angle of sideslip, $\sin^{-1}(v/V)$
θ, ϕ, ψ	attitude in pitch, bank and azimuth
η, ξ, ζ	control surface angle of elevator, aileron and rudder respectively

LIST OF SYMBOLS (continued)

Longitudinal Mass Matrices and Derivatives:

$$M = \begin{pmatrix} 1 & -\dot{\hat{x}}_w & 0 & 0 \\ 0 & (1-\dot{\hat{z}}_w) & 0 & 0 \\ 0 & -\dot{\hat{m}}_w & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad M^{-1} = \begin{pmatrix} 1 & \dot{\hat{x}}_w/(1-\dot{\hat{z}}_w) & 0 & 0 \\ 0 & 1/(1-\dot{\hat{z}}_w) & 0 & 0 \\ 0 & \dot{\hat{m}}_w/(1-\dot{\hat{z}}_w) & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix};$$

where:

$$\dot{\hat{x}}_u = \dot{\hat{x}}_u/m; \quad \dot{\hat{x}}_w = \dot{\hat{x}}_w/m; \quad \dot{\hat{x}}_q = \dot{\hat{x}}_q/m; \quad \dot{\hat{x}}_\eta = \dot{\hat{x}}_\eta/m.$$

$$\dot{\hat{z}}_u = \dot{\hat{z}}_u/m; \quad \dot{\hat{z}}_w = \dot{\hat{z}}_w/m; \quad \dot{\hat{z}}_q = \dot{\hat{z}}_q/m; \quad \dot{\hat{z}}_\eta = \dot{\hat{z}}_\eta/m.$$

$$\dot{\hat{m}}_u = \dot{\hat{M}}_u/I_y; \quad \dot{\hat{m}}_w = \dot{\hat{M}}_w/I_y; \quad \dot{\hat{m}}_q = \dot{\hat{M}}_q/I_y; \quad \dot{\hat{m}}_\eta = \dot{\hat{M}}_\eta/I_y.$$

$$x_u = \left(\frac{\dot{\hat{x}}_w \dot{\hat{z}}_u}{(1-\dot{\hat{z}}_w)} + \dot{\hat{x}}_u \right); \quad x_w = \left(\frac{\dot{\hat{x}}_w \dot{\hat{z}}_w}{(1-\dot{\hat{z}}_w)} + \dot{\hat{x}}_w \right); \quad x_q = \left(\frac{(U_e + \dot{\hat{z}}_q) \dot{\hat{x}}_w}{(1-\dot{\hat{z}}_w)} + (\dot{\hat{x}}_q - W_e) \right);$$

$$z_u = \left(\frac{\dot{\hat{z}}_u}{1-\dot{\hat{z}}_w} \right); \quad z_w = \left(\frac{\dot{\hat{z}}_w}{1-\dot{\hat{z}}_w} \right); \quad z_q = \left(\frac{U_e + \dot{\hat{z}}_q}{1-\dot{\hat{z}}_w} \right);$$

$$m_u = \left(\frac{\dot{\hat{m}}_w \dot{\hat{z}}_u}{(1-\dot{\hat{z}}_w)} + \dot{\hat{m}}_u \right); \quad m_w = \left(\frac{\dot{\hat{m}}_w \dot{\hat{z}}_w}{(1-\dot{\hat{z}}_w)} + \dot{\hat{m}}_w \right); \quad m_q = \left(\frac{(U_e + \dot{\hat{z}}_q) \dot{\hat{m}}_w}{(1-\dot{\hat{z}}_w)} + \dot{\hat{m}}_q \right);$$

$$x_\eta = \left(\frac{\dot{\hat{x}}_w \dot{\hat{z}}_\eta}{(1-\dot{\hat{z}}_w)} + \dot{\hat{x}}_\eta \right); \quad z_\eta = \left(\frac{\dot{\hat{z}}_\eta}{1-\dot{\hat{z}}_w} \right); \quad m_\eta = \left(\frac{\dot{\hat{m}}_w \dot{\hat{z}}_\eta}{(1-\dot{\hat{z}}_w)} + \dot{\hat{m}}_\eta \right)$$

LIST OF SYMBOLS (continued)

Lateral Mass Matrices and Derivatives:

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -e_x & 0 & 0 \\ 0 & e_z & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} ; \quad M^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1/E_{xz} & e_x/E_{xz} & 0 & 0 \\ 0 & -e_z/E_{xz} & 1/E_{xz} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} ;$$

$$\text{WHERE: } e_x = I_{xz}/I_x ; \quad e_z = I_{xz}/I_z ; \quad E_{xz} = 1 + e_x e_z$$

$$\dot{y}_v = \dot{Y}_v/m ; \quad \dot{y}_p = \dot{Y}_p/m ; \quad \dot{y}_r = \dot{Y}_r/m ; \quad \dot{y}_\xi = \dot{Y}_\xi/m ; \quad \dot{y}_\zeta = \dot{Y}_\zeta/m .$$

$$l_v = \dot{L}_v/I_x ; \quad l_p = \dot{L}_p/I_x ; \quad l_r = \dot{L}_r/I_x ; \quad l_\xi = \dot{L}_\xi/I_x ; \quad l_\zeta = \dot{L}_\zeta/I_x .$$

$$\dot{n}_v = \dot{N}_v/I_z ; \quad \dot{n}_p = \dot{N}_p/I_z ; \quad \dot{n}_r = \dot{N}_r/I_z ; \quad \dot{n}_\xi = \dot{N}_\xi/I_z ; \quad \dot{n}_\zeta = \dot{N}_\zeta/I_z .$$

$$y_v = \dot{y}_v ; \quad y_p = (\dot{y}_p + W_e) ; \quad y_r = (\dot{y}_r - U_e) ;$$

$$l_v = \left\{ \frac{\dot{l}_v}{E_{xz}} + \frac{e_x \dot{n}_v}{E_{xz}} \right\} ; \quad l_p = \left\{ \frac{\dot{l}_p}{E_{xz}} + \frac{e_x \dot{n}_p}{E_{xz}} \right\} ; \quad l_r = \left\{ \frac{\dot{l}_r}{E_{xz}} + \frac{e_x \dot{n}_r}{E_{xz}} \right\} ;$$

$$n_v = \left\{ \frac{-e_z \dot{l}_v}{E_{xz}} + \frac{\dot{n}_v}{E_{xz}} \right\} ; \quad n_p = \left\{ \frac{-e_z \dot{l}_p}{E_{xz}} + \frac{\dot{n}_p}{E_{xz}} \right\} ; \quad n_r = \left\{ \frac{-e_z \dot{l}_r}{E_{xz}} + \frac{\dot{n}_r}{E_{xz}} \right\} ;$$

$$y_\xi = \dot{y}_\xi ; \quad l_\xi = \left\{ \frac{\dot{l}_\xi}{E_{xz}} + \frac{e_x \dot{n}_\xi}{E_{xz}} \right\} ; \quad l_\zeta = \left\{ \frac{\dot{l}_\zeta}{E_{xz}} + \frac{e_x \dot{n}_\zeta}{E_{xz}} \right\}$$

$$y_\zeta = \dot{y}_\zeta ; \quad n_\xi = \left\{ \frac{-e_z \dot{l}_\xi}{E_{xz}} + \frac{\dot{n}_\xi}{E_{xz}} \right\} ; \quad n_\zeta = \left\{ \frac{-e_z \dot{l}_\zeta}{E_{xz}} + \frac{\dot{n}_\zeta}{E_{xz}} \right\}$$

REFERENCES.

1. MOD AGREEMENT No.2082/192: Use of a Stepwise Regression Method with Dynamic Rig Data. October 1988.
2. HINDS, H. A. and COOK, M. V.: Initial Review of Research Into the Application of Modified Stepwise Regression for the Estimation of Aircraft Stability and Control Parameters. January 1989, Cranfield Institute of Technology.
3. BABISTER, A.W.: Aircraft Dynamic Stability and Response. 1st Edition, Pergamon Press, 1980.
4. HAWKER SIDDELEY AVIATION LTD: Aerodynamic Data and Stability and Control Derivatives of the Hawker Siddeley H.S 1182. Airframe Engineering Note Aero 1880, July, 1972.
5. FILMER, S.W.: An Artificial "g" Control System for the Dynamic Wind Tunnel Facility. College of Aeronautics MSc Thesis, Cranfield Institute of Technology, 1989.
6. E.A.I. Handbook of Analogue Computation. Electronics Associates Inc., 1967.
7. FIFER, S.: Analogue Computation, VOL II, Theory, Techniques and Applications. McGraw-Hill Book Company, Inc., 1961.
8. JACKSON, A. S.: Analog Computation. McGraw-Hill Book Company, Inc., 1960.
9. SOROKA, W. A.: Analog Methods in Computation and Simulation. McGraw-Hill Book Company, Inc, 1954.
10. SMITH, G.N. and WOOD, R.C.: Principles of Analog Computations. McGraw-Hill Book Company Inc., 1959.
11. JAMES, M.L.; SMITH, G.M and WOLFORD, J.C.: Applied Numerical Methods For Digital Computation. 3RD Edition, Harper & Row Publishers Inc., 1985.

12. SCHEID, F: Numerical Analysis. Schaum Outline Series. McGraw-Hill Book company, 1968.
13. WHITE, J; YEATES, A. and SKIPWORTH, G.: Tables for Statisticians. 3rd Edition. Stanley Thornes (Publishers) Ltd., 1979
14. KLEIN, V.; BATTERSON, J.G. and MURPHY, P.C.: Determination of Airplane Model Structure from Flight Data by Using Modified Stepwise Regression. NASA TP-1916, 1981.
15. DRAPER, N.R., and SMITH, H.: Applied Regression Analysis. 2nd Edition. John Wiley and Sons, Inc., 1966.