

Diversity-based Heuristic Search for Multiple-asteroid Tours

Jan Grabowski
Centre for Autonomous and Cyberphysical Systems
Cranfield University
Cranfield, UK, MK43 0AL
jan.grabowski.257@cranfield.ac.uk

Andrea Bellome
Department of Aerospace Vehicles Design and Control
ISAE Supaero
Toulouse, FR, 31400
andrea.bellome@isae-supero.fr

Leonard Felicetti
Centre for Autonomous and Cyberphysical Systems
Cranfield University
Cranfield, UK, MK43 0AL
leonard.felicetti@cranfield.ac.uk

Abstract—An asteroid tour is a sequence of asteroid flybys performed by a spacecraft in a single mission. Visiting different types of asteroids would be of a great scientific return, but planning such type of mission is highly complex. This is because the combinatorial problem of selecting sequences of asteroids is coupled with optimal control problem of finding viable trajectories in terms of transfer times and manoeuvre locations. The combinatorial problem can be tackled with heuristic algorithms that explore the search space to find optimized flyby sequences of asteroids. This paper proposes a search strategy to increase the diversity of the set of flyby sequences found from the resolution of the optimization problem. The goal is to find a tree exploration method that will improve the quality of the solutions by increasing the number of targets encountered and eliminating duplicated sequences while preserving the average Δv cost. A diversity measure for a set of solutions is defined based on the Δv cost of the transfer and the nature of the asteroids in the sequences. A variant of the Beam Search (BS) algorithm, referred to as Diversity Search (DS), is tested. Unlike BS, the DS algorithm eliminates duplicated sequences from the final set of solutions. New asteroid tours are found, allowing to reach targets with a higher inclination or eccentricity, but the average Δv of a flyby sequence is higher than solutions found with BS. A hybrid search strategy using BS and DS is explored. The goal is to reduce the search space with respect to two criteria: Δv and diversity score. By pruning the search space according to Δv first, the hybrid search algorithm is able to generate a diverse set of asteroid tours while preserving the total Δv of the solutions. Compared to DS, less new asteroids are encountered in the solutions but the average Δv remains similar to results obtained with BS.

TABLE OF CONTENTS

1. INTRODUCTION.....	1
2. METHODOLOGY	2
3. RESULTS AND ANALYSIS	4
4. CONCLUSION	8
REFERENCES	8
BIOGRAPHY	9

1. INTRODUCTION

Asteroids and cometary objects are targets of growing interest among the scientific community. Since the first discovery of an asteroid Ceres, in 1801, more than one million small bodies have been found, with significant variations in shape,

composition, orbit and spectral types. They are spread in different regions of the Solar System, most of them orbiting between Mars and Jupiter in the Main Asteroid Belt (MAB). Space agencies and industries are interested in collecting and processing asteroid material, what is referred to as In-Situ Resource Utilization (ISRU). Extracting and sending asteroid samples back to Earth remains a difficult task but could pave the way for future asteroid mining missions. Asteroids carry many valuable metals such as nickel, cobalt, gold or platinum, and large amounts of water, carbon and silicon compounds. There is also a strong scientific interest in these bodies as they could give valuable insights on the early formation of the Solar System and how they brought chemical compounds that helped life appear on Earth.

Given all the key drivers for a mission to asteroids, there is a need to study efficiently the small body population in our Solar System. A way to prospect a large number of asteroids is to design an asteroid tour, in other words a sequence of asteroids successively visited by a single spacecraft.

In the space trajectory design community, the concept of an asteroid tour became popular with the second Global Trajectory Optimization Competition (GTOC), organized by JPL in 2006. GTOC is a competition held every one to two years, in which aerospace engineers and mathematicians around the world compete to find the best solutions to complex interplanetary trajectory optimization problems. The goal of the 2006 edition was to design a trajectory that allows a spacecraft to perform multiple rendezvous with asteroids from different regions of the Solar System [1] [2].

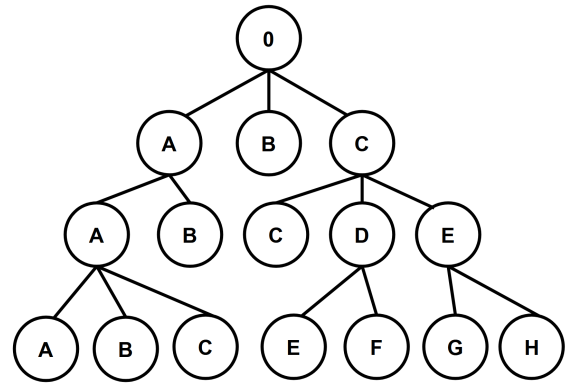
The optimization of multi body missions is a difficult task that requires to define an appropriate mathematical model. Di Carlo [3], Sanchez et al. [4] and Bellome et al. [5] divide the problems in two coupled sub-problems. First, finding the right flyby sequence in a set of asteroids can be modeled as a discrete combinatorial problem. Then, given a sequence of asteroids to visit, finding the best trajectory is assimilated to a continuous optimization problem. Several approaches have been studied in the literature to tackle the asteroid selection problem modeled by combinatorial optimization. In particular, the complexity of the resolution of a combinatorial problem relies in the exploration of the search space. Heuristic search algorithms have been extensively used [5] [6] [7] in order to find local optima, corresponding to optimised asteroid tours. Vasile et al. [8] reviews the performance of global search algorithms on space trajectory design problems with multiple transfers. Vroom et al. [9] and Johnson [10] propose an application of heuristic methods to the asteroid tour problem. However, most of these search algorithms do not guarantee a minimum level of diversity in the set of solutions.

Search algorithms such as BS tend to find sequences that visit the same asteroids but at different epochs. Therefore BS generates duplicated flyby solutions that do not participate to the diversity of the solutions. The quality of the solutions depends on how much of the search space is explored and how asteroids are selected when building the flyby sequences. The level of diversity in the asteroids encountered in the solutions is assessed by Carrillo [11], who introduces a similarity measure to evaluate the quality of the solutions as a post-process in the resolution of the combinatorial problem. Other approaches improve the selection of asteroids by assigning them a score based on their scientific interest [12], [13], [14], but this score is assigned manually.

The study is organized as follows: the concept of diversity in multi-body missions is first reviewed, with a focus on the generation of distinct solutions in combinatorial problems. A diversity score is then defined for a flyby sequence by comparing all solutions together in the set, and is implemented in a search algorithm. Results between Δv -based and diversity-based search algorithms are compared, and the benefits and limits of the diversity-based heuristic search is discussed in the conclusion.

In this section, a method is developed to adapt heuristic methods in order to preserve diversity of the solutions. A definition of diversity for the asteroid tour optimization problem is proposed. This definition is then implemented in a search algorithm inspired from BS, and a tree exploration procedure is described.

The goal of combinatorial optimization is to search for global or local optima of an objective function in a discrete search space. This search space can be represented as a tree graph with different levels, shown in Figure 1. The branches of the tree are called edges and connect two nodes together. One path in the tree corresponds to a specific solution, represented by a succession of nodes. There exist as many paths as possible solutions. The value of the objective function for a given solution is the sum of the cost for each branch.



epochs. Therefore, two consecutive nodes share the same object and encounter time. Two objects visited at different epochs are two distinct nodes. In that way, the cost of the branch between each node is unique.

Beam Search is a widely known tree search algorithm to tackle combinatorial optimization problems [7]. When exploring the tree, the Beam Search algorithm will sort all the branches with respect to a criterion (Δv for example) at a given level of the tree, and keep only the branches with the best objective values. The number of branches kept correspond to the Beam Width (BW). Figure 2 illustrates how the tree can be reduced by BS.

Figure 2. Beam Search diagram with BW=2. Dotted nodes are not kept for further exploration.

combinatorial problem for an asteroid tour design should generate flyby sequences with a sufficient level of variety in the bodies encountered to widen the choice available to mission designers.

Searching for diverse asteroid tours

Mathematically, a solution from a combinatorial problem is an array of elements. In the case of the asteroid tour problem, each element is a pair made of an asteroid index and an encounter epoch. Several approaches in the literature have been studied to define diversity and to generate a diverse set of solutions. Diversity is often measured as a distance between solutions [15] [16] [17]. The greater the diversity distance is, the more different two solutions are. This distance is characterized by the elements contained in each solution.

In order to optimize diversity, solutions need to be selected in a way that the relative diversity distance between them is maximized. All pairwise diversity distances can be combined in a single measure by applying an aggregator function [15]. For example, all the distances can be added together and solutions that maximize the sum are kept.

In combinatorial optimization, this approach is known as the Maximum Diversity Problem (MDP). Methods to tackle this problem have been studied in the literature [18] [19] [20]. The MDP is formulated as follows:

Let N be a set of n elements. A subset M containing m elements is selected such that:

$$z_{MDP}(x) = \sum_{i < j} d_{ij} x_i x_j \text{ is maximised} \quad (1)$$

$$\text{Subject to } \sum_{i=1}^n x_i = m \quad (2)$$

Where $x_i = 0, 1, i = 1, \dots, n$. x_i is equal to 1 when the element with index i is selected and 0 otherwise.

In the case of an asteroid tour optimization, where solutions are sequences of asteroids, the diversity metric relies on the definition of a distance between asteroids.

This distance used here is called the Improved Orbital Indicator (IOI), defined in Equation 3. This parameter is defined from the relative position and velocities of the departure and arrival bodies. It is an approximation of the Δv necessary to perform a transfer between the two asteroids [21]. The IOI has previously been used in the context of asteroid tour search [22].

$$d'_0 = |x_2 - x_1| \quad (3)$$

Where x is defined as follows:

$$x = [\frac{1}{\Delta T} \Delta r(t_0) + \Delta v(t_0), \frac{1}{\Delta T} \Delta r(t_0), \quad (4)$$

$$\frac{1}{\Delta T} \Delta r(t_0 + \Delta T) - \Delta v(t_0 + \Delta T), \frac{1}{\Delta T} \Delta r(t_0 + \Delta T)]$$

Now that a distance between consecutive asteroids in a sequence has been defined, a diversity measure between two

flyby sequences can be created, with the following considerations:

- The diversity distance includes a term that depends on both solutions, which is the number of common asteroids in the two sequences n . This term allows to measure the similarity of two solutions in terms of objects visited.
- A term that is unique to each sequence is included. In this study, the sum of the IOI for all the transfers in the sequence is used.
- When considering two sequences with asteroids visited in the same order (but not necessarily at the same time), the diversity distance should be zero.

The diversity distance between two flyby sequences A and B is given by Equation 5, where l is the length of the sequence and n the number of common asteroids.

$$Div(A, B) = \frac{d'_0(A) + d'_0(B)}{2} \frac{l - n}{l} \quad (5)$$

This diversity measure relies on the assumption that sequences with a high IOI value correspond to longer transfers between asteroids, likely to result in more possibilities to explore different regions of the Solar System and increase the diversity of visited bodies. The distance is scaled by the $(l - n)$ factor, corresponding to the number of different asteroids when comparing the two sequences. The diversity distance rewards sequences that are the most different from each other, thus increasing the chance of finding new asteroids to add to the sequence during the search. Using this definition, the diversity scores obtained should be in the same order of magnitude as the total Δv value.

For a given set of solutions of size N , the final distance is computed between all pairs of sequences and the corresponding distance matrix is created. For each sequence A , the mean diversity distance with respect to all other solutions can be calculated from this matrix. This distance, called the diversity score, is defined by Equation 6.

$$J_{div}(A_i) = \frac{\sum_{k=1, k \neq i}^N Div(A_i, A_k)}{N - 1} \quad (6)$$

Diversity Search algorithm

The DS algorithm is a variant of BS, as it ranks all the legs at a given level of the tree according to the diversity score and keeps a set of solutions which size corresponds to the Beam Width (BW). A flowchart of the DS algorithm is presented in Figure 3. The diversity search algorithm first reduces the set to unique sequences of asteroids. When two identical flyby sequences are found, the sequence with the best total Δv is kept. Relative diversity distances are then computed for all pairs of solutions and a distance matrix is created. By summing each row and dividing by the number of diversity distances computed (which is $N - 1$), the average diversity distance of a sequence A relative to all other sequences is obtained. Now that all solutions are assigned a diversity measure, they can be ranked in descend order, from most to least diverse sequence. The most diverse sequence is the one that is the furthest from all other sequences. In other words, it is the one with the greatest number of different asteroids compared to all other flyby sequences, on average. The best solutions are kept until the BW is full. Thus, the selection

of the solutions is based on maximizing the average relative diversity distance with other solutions in the same set. It can be noted that if two sequences are found that visit the same asteroid but in a different order, then the two solutions are kept as they are considered to bring the same level of diversity by the DS algorithm. DS differs from BS as the diversity measure assigned to each branch at a specific level of the tree depends on all other branches. In other words, the same sequence found in two different sets of solutions will have two distinct diversity scores.

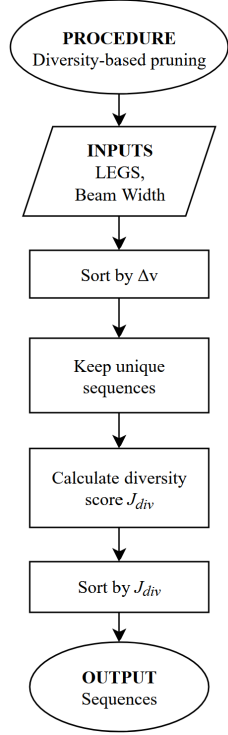


Figure 3. Diversity search algorithm flowchart

Tree exploration algorithm

Now that different ways of selecting diverse solutions have been presented, they are implemented in the tree exploration algorithm. The script builds sequences of asteroids iteratively, from the database of GTOC 7 asteroids.

The principle of the tree search algorithm is illustrated in Figure 4. It is organized as follows:

1. First, a pool of bodies containing asteroids and/or Earth is initialized as departing points, from the database of all known asteroids. The initial and final epochs, minimum and maximum TOF and the constraints on maximum Δv values for the transfer are also given.
2. From the initial pool, trajectories to all other asteroids in the database are computed using Lambert arcs, in order to build the first leg of each sequence with the information about TOF and Δv for the transfer. The legs kept are the one below the velocity requirement for the transfer.
3. Then, the Δv of the next transfer is assessed from asteroids in the departing pool to all other asteroids, using the IOI. The new pool of reachable asteroids is defined by transfers that comply with constraints on Δv at departure and arrival on the next body. If the new pool is not empty, Lambert arcs are computed to all asteroids in the pool, giving the next leg

of the sequence.

4. At the current state of the search, the tree is explored and pruned using DP and BS, to select the most promising nodes (thus the asteroids) that will be kept for further expansion.

The process is iterated from step 3. to compute new transfers from the nodes kept at step 4., until the desired length is reached for the sequence. The output is a matrix which rows are sequences of asteroids with information on asteroid index id, encounter epoch t in Modified Julian Day (MJD) and transfer cost Δv .

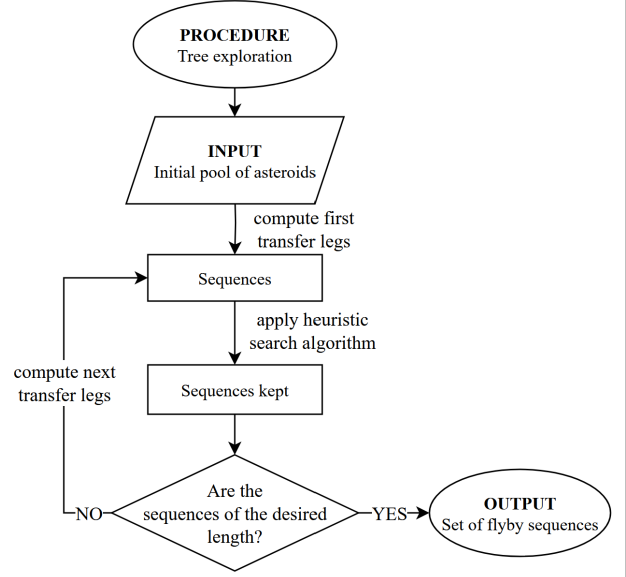


Figure 4. Tree exploration algorithm flowchart

3. RESULTS AND ANALYSIS

The following results section is an assessment of the performance of DS, and analyzes different search strategies using BS and DS to find diverse asteroid tours. The goal is to assess the diversity of the output solutions in order to find a set of flyby sequences with a minimized Δv and a maximized diversity score.

The search algorithms are tested on a data set of 16,256 asteroids from the MAB, used in GTOC7 problem. The position of all the asteroids at an arbitrary epoch is plotted in Figure 5 for visualization purposes. All asteroids tours start from the Earth.

The search algorithms are compared on the same scenario given in Table 1. The transfer time between each asteroid in the sequence is fixed to 250 days, and sequences of 9 asteroid flybys are considered.

The analysis of the performance of diversity-based algorithms is done in three steps.

1. First, diversity performance is evaluated by comparing the Diversity Search algorithm to Beam Search. Nodes kept for further exploration at a specific level of the tree are selected based on Δv or diversity, respectively. The set of flyby sequences obtained are then compared in terms of unique solutions, asteroids encountered and average Δv and diversity score.

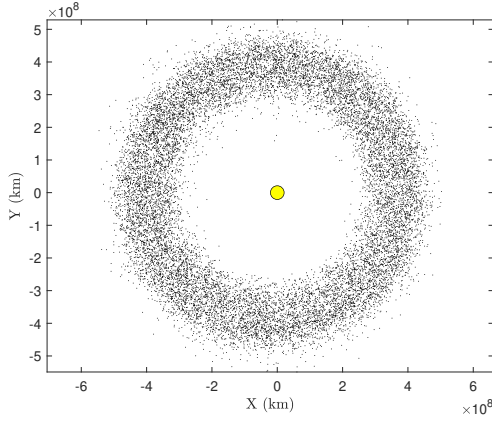


Figure 5. Main Asteroid Belt

Scenario	
Departure body	Earth
Δv_1 range (km/s)	[0 5]
Δv_2 max (km/s)	5
Transfer time between targets (days)	250
Search window (years)	5
Δv_1 range (km/s)	[0 2]
Δv_2 max (km/s)	2
Length of astreoid flyby sequence	9
Complete revolutions of the Lambert Arc	0

Table 1. Scenario parameters of the tree exploration

2. From the results of DS and BS, the accessibility of the asteroids in the two sets is discussed by comparing the average and maximum semi-major axis, eccentricity and inclination.

3. It has been demonstrated with DS that more diverse flyby sequences can be found. A hybrid search strategy including BS and DS is then tested with the aim of finding better Δv sequences that still preserve the diversity performance obtained with DS.

The results analysis shows the advantages and drawbacks of the implementation of a diversity score in search algorithms:

1. The diversity-based search algorithm generates more unique flyby sequences, with a greater number of asteroids encountered.
2. Sequences with a high diversity score often include targets with higher inclination or eccentricity compared to average values of the set. These sequences are often found at the cost of a significantly higher Δv .
3. The diversity-based search strategy needs to implement optimization of both Δv and diversity in order to find diverse sequences with optimal Δv , as in the solutions found through BS.

Performance of the Diversity Search algorithm

Diversity performance is assessed through the comparison of BS and DS. The parameters used to compare output solutions are the number of unique sequences in the set, the number of unique asteroids, the average number of encounters of an asteroid as well as the average and best Δv and diversity score values. Both search algorithms have a fixed BW of 1,000 and generates tours of 9 asteroids from the dataset, departing from the Earth. The two sets made of 1,000 flyby sequences are

compared and results are shown in Table 2.

BW = 1,000	BS	DS
Number of solutions	1,000	1,000
Number of unique solutions	711	1,000
Number of unique asteroids	418	1,527
Average encounter occurrences	23.92	6.55
Average Δv (km/s)	21.40	31.70
Minimum Δv (km/s)	20.20	26.98
Average diversity score (km/s)	46.14	59.15
Maximum diversity score (km/s)	52.46	64.25

Table 2. Solution characteristics for BS and DS

Unlike BS that finds many duplicated flyby sequences, all solutions found with DS are unique. This allows to reach a greater number of different asteroids and increases the diversity of the set.

The diversity score is defined in a way that favors the nodes leading to new asteroids being encountered at each level of the tree, thus generating new flyby sequences. The greater number of asteroids encountered in the set is explained by this new tree exploration strategy. On average, a single asteroid is encountered in 6 to 7 flyby sequences with DS, compared to almost 24 with BS. The diversity-based optimization in DS is well illustrated by a higher diversity and worse Δv score compared to BS, due to new flyby sequences found.

The evolution of the number of different solutions (Figure 6) and different asteroids (Figure 7) at each step of the search demonstrates that new paths are explored in the tree compared to BS. In DS, all solutions in the BW are unique from the 6th search iteration, while BS finds between 500 and 700 unique sequences throughout the search procedure.

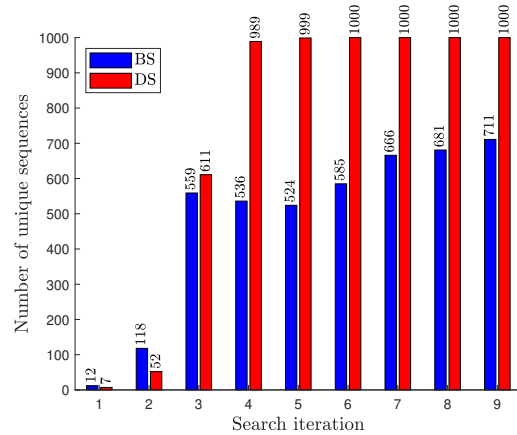


Figure 6. Number of different flyby sequences at each step of the search

DS finds an increasing number of different asteroids during the search. On the opposite, BS keeps a smaller number of asteroids in the set, because it finds better Δv values in identical sequences with different encounter epochs. Therefore BS generates more duplicated solutions (Figure 6) and discovers less new asteroids in the search space compared to DS.

The diversity score confirms that DS performs better in terms of diversity. The drawback is that optimal Δv sequences are not found with DS. This high Δv cost can be explained by the greater number of asteroids explored, some of them requiring more Δv to be reached.

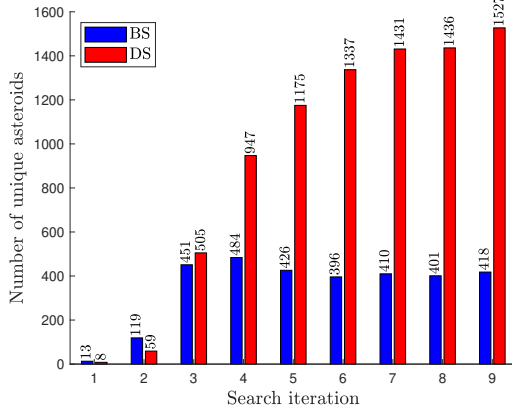


Figure 7. Number of unique asteroids encountered at each step of the search

	BS		DS	
Avg a	2.7640		2.6495	
Avg e	0.0774		0.0950	
Avg i ($^{\circ}$)	2.7330		3.7987	
	Value	Asteroid	Value	Asteroid
Max a	2.9979	2000 DN	2.9990	1999 XA1
Max e	0.2466	Couteau	0.3120	Tito
Max i ($^{\circ}$)	8.7109	1999 JT57	13.6844	1992 UU4

Table 3. Characterization of asteroid populations found with BS and DS

Accessibility of asteroids encountered with BS and DS

The performance of DS is also assessed by characterizing the populations of asteroids encountered in the search space with the two search algorithms. In particular, the accessibility of the two asteroid populations can be evaluated thanks to their orbital elements. Semi-major axis a , eccentricity e and inclination i are considered. The average and maximum values for these orbital elements are given in Table 3, and the position of the asteroids encountered in the two sets is shown in Figure 8. Flyby sequences with the best diversity score from the two sets of solutions are also compared.

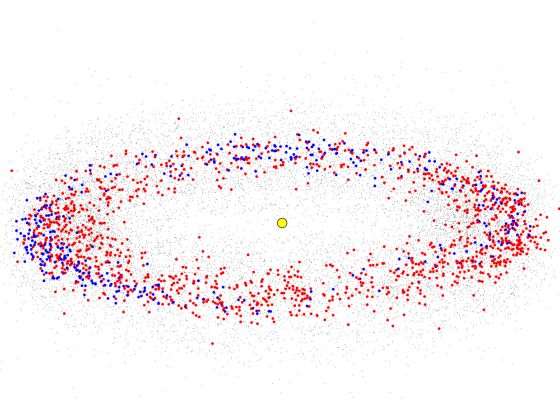


Figure 8. Position of asteroids encountered in the solutions found with BS and DS

More bodies can be visited from the sequences generated by the DS algorithm. Unlike BS, DS finds asteroids spread all around the Main Belt, some of them even have orbits far from the core of the Belt.

Average values are similar, but the maximum eccentricity and inclination are higher in the case of DS. This means that the tree search based on diversity found transfers to reach asteroids with more elliptical and inclined orbits.

The flyby sequences with the best diversity score, containing the most diverse visited bodies, are plotted in Figure 9 and characterized in Table 4.

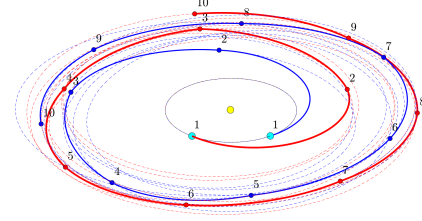


Figure 9. Most diverse flyby trajectories (3D plot) found with BS (blue) and DS (red)

	BS	DS
1	Earth	Earth
2	Honkasalo	Qwerty
3	1992 ML	Silentium
4	Chanal	Konradbenes
5	Ondrejov	1995 WL13
6	1997 GT36	Barnowl
7	Robinwilliams	1998 XZ9
8	Corradolamberti	2000 LX8
9	Garyross	Tetsujiyamada
10	Gembloux	1990 QQ4
Total Δv (km/s)	21.60	35.56
Diversity (km/s)	52.46	64.25

Table 4. characterization of most diverse flyby sequence from BS and DS

The most diverse flyby sequence found with DS has a higher diversity score than BS because it visits asteroids that are less encountered in other solutions. Asteroids with greater eccentricity and inclination are reached.

The search for more diverse asteroid tours is impacted by a high Δv cost, as illustrated in Table 4. A modified search strategy is discussed in the next section in order to improve the Δv cost of the flyby sequences found.

Hybrid search strategy with DS and BS

Now that the performance of DS has been demonstrated, two search strategies using BS and DS are compared. The objective is to set up a diversity-based search such that the flyby sequences kept in the final set of solutions are the best ones both in terms of Δv and diversity score.

At each level of the tree, the nodes kept are selected using both Δv and diversity criteria. The Diversity First (DF) search method selects the best nodes according to their di-

versity score ($BW = 5,000$) and from these nodes only the 1,000 branches with the best Δv are kept. On the opposite, Δv First (DVF) prunes the search space with respect to Δv before selecting the most diverse flyby sequences. The two search strategies are compared to the performance of DS. Results are given in Table 5.

	DS	DF	DVF
Number of solutions	1,000	1,000	1,000
Number of unique solutions	1,000	1,000	1,000
Number of unique asteroids	1,527	923	1,012
Average Δv (km/s)	31.70	24.81	26.09
Minimum Δv (km/s)	26.98	23.27	23.63
Average diversity score (km/s)	59.15	51.19	53.44
Maximum diversity score (km/s)	64.25	57.20	59.16

Table 5. Solution characteristics for DS, DF and DVF

By first selecting nodes with the Δv criterion, the DVF hybrid search guarantees that optimal Δv sequences are found at the current level of the tree. Then the diversity score is assessed for the 5,000 sequences in the set and the 1,000 most diverse ones are kept. Unlike BS, the lowest Δv solution is not found as pruning with respect to the diversity criterion does not guarantee that the minimum Δv solutions are kept to be further explored.

As seen in Table 5, both hybrid search strategies give better Δv results than DS, with a better diversity performance in the case of DVF. Like DS, DF and DVF eliminate all duplicated sequences for 9 asteroid-long tours (Figure 10) and generates more encounters with new asteroids (Figure 11).

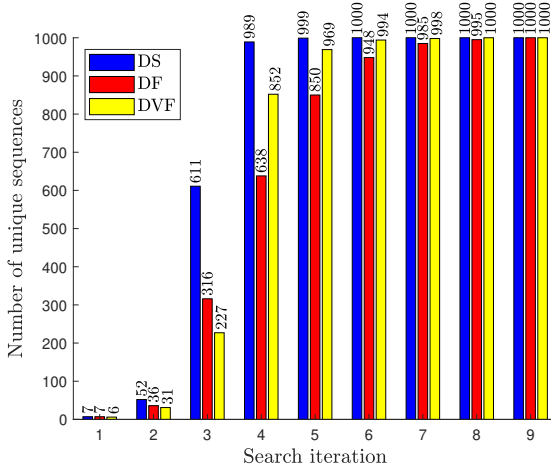


Figure 10. Number of different flyby sequences at each step of the search

DF gives lower diversity solutions compared to DS and DVF. Despite pruning the search space based on diversity score, the selection based on Δv keeps similar nodes - with the same asteroid visited at different epochs. This selection prevents DF from finding new asteroids as illustrated by Figure 11.

On the other hand, the diversity performance of DVF is better than DF. Selecting the best nodes according to Δv first does not impact the global diversity of the solutions. Nodes with optimal Δv cost are selected first, and from this subset only the most diverse ones are kept. In that way, diversity of the solutions is only limited by the first Δv -based selection.

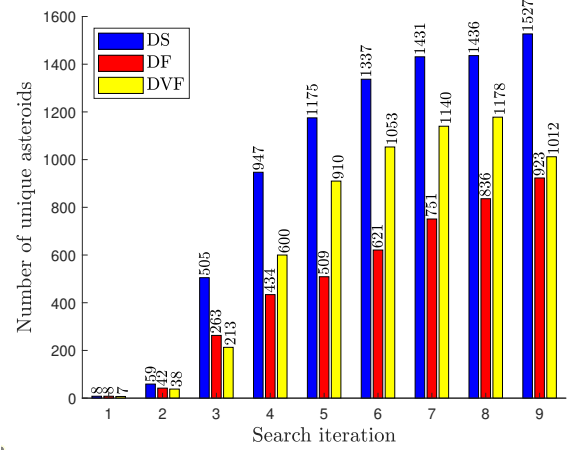


Figure 11. Number of unique asteroids encountered at each step of the search

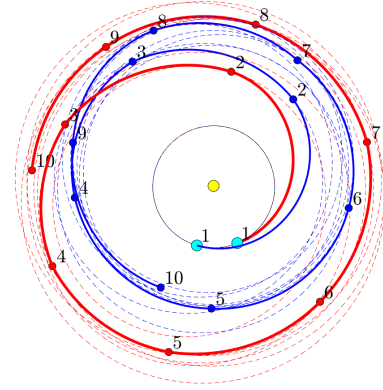


Figure 12. Most diverse flyby trajectories (2D plot) found with DVF (blue) and DF (red)

Compared to DS, less new asteroids are encountered, but all flyby sequences found remain unique.

	DF	DVF
Total Δv (km/s)	25.28	26.29
Diversity score (km/s)	57.20	59.16

Table 6. characterization of the most diverse flyby sequence from BS and DS

The most diverse trajectories obtained with DF and DVF, plotted in Figure 12, show that DVF finds better solutions than DF in terms of diversity score. Unlike DS, the Δv score of DVF and DF remain close to the results obtained with BS (Table 6).

DVF finds more balanced solutions in terms of Δv and diversity, however the optimum Δv solution from BS and the optimum diversity solution from DS are not found.

4. CONCLUSION

This study focused on a search strategy to find optimal multiple-asteroid tours with increased diversity in the bodies encountered. A Diversity Search algorithm based on Beam Search was implemented. Unlike BS that finds duplicated sequences, all solutions found by DS in the set are unique and a greater number of asteroids is reached.

A hybrid search combining DS and BS is tested and its performance is compared to single BS and DS search strategies. Compared to DS, better Δv solutions are found with a similar diversity score.

Generally, the search strategy based on diversity allow for less accessible targets to be reached compared to standard Δv optimization, with an overall higher eccentricity or inclination. Unlike Δv , the calculation of the diversity score that guides the search depends on other sequences in the solution set. The diversity score of a sequence can only be compared with other sequences in the same set.

Computing the diversity score also requires a high computational effort, as the relative diversity distance is calculated for all pairs of sequences in a set. This draw a limitation on the size of the initial pool of asteroids. Therefore the exploration of the search space is limited by the diversity score, as a memory issue appears when too many new legs are computed.

REFERENCES

- [1] D. Izzo, D. Hennes, L. F. Simões, and M. Märten, “Designing complex interplanetary trajectories for the global trajectory optimization competitions,” *Space Engineering: Modeling and Optimization with Case Studies*, pp. 151–176, 2016, https://doi.org/10.1007/978-3-319-41508-6_6.
- [2] D. Izzo, T. Vinkó, C. Bombardelli, S. Brendelberger, and S. Centuori, “Automated asteroid selection for a ‘grand tour’ mission,” in *58th International Astronautical Congress*, Hyderabad, India, 2007.
- [3] M. Di Carlo, M. Vasile, and J. Dunlop, “Low-thrust tour of the main belt asteroids,” *Advances in Space Research*, vol. 62, no. 8, pp. 2026–2045, 2018, <https://doi.org/10.1016/j.asr.2017.12.033>.
- [4] J. P. Sánchez, A. Gibbings, C. Snodgrass, S. Green, and N. Bowles, “Asteroid belt multiple fly-by options for M-class missions,” in *Proceedings of the International Astronautical Congress*, 2016.
- [5] A. Bellome, J.-P. Sánchez, L. Felicetti, and S. Kemble, “Multiobjective design of gravity-assist trajectories via graph transcription and dynamic programming,” *Journal of Spacecraft and Rockets*, pp. 1–19, 2023, <https://doi.org/10.2514/1.A35472>.
- [6] C. Blum and A. Roli, “Metaheuristics in combinatorial optimization: Overview and conceptual comparison,” *ACM computing surveys (CSUR)*, vol. 35, no. 3, pp. 268–308, 2003, <https://doi.org/10.1145/937503.937505>.
- [7] L. F. Simões, D. Izzo, E. Haasdijk, and A. E. Eiben, “Multi- rendezvous spacecraft trajectory optimization with beam P-ACO,” in *European Conference on Evolutionary Computation in Combinatorial Optimization*. Springer, 2017, pp. 141–156, https://doi.org/10.1007/978-3-319-55453-2_10.
- [8] M. Vasile, E. Minisci, and M. Locatelli, “Analysis of some global optimization algorithms for space trajectory design,” *Journal of Spacecraft and Rockets*, vol. 47, no. 2, pp. 334–344, 2010, <https://doi.org/10.2514/1.45742>.
- [9] A. Vroom, M. Di Carlo, J. M. R. Martin, and M. Vasile, “Optimal trajectory planning for multiple asteroid tour mission by means of an incremental bio-inspired tree search algorithm,” in *2016 IEEE Symposium Series on Computational Intelligence (SSCI)*. IEEE, 2016, pp. 1–8.
- [10] G. P. Johnson, “A tabu search methodology for spacecraft tour trajectory optimization,” Master Thesis, The University of Texas at Austin, 2014.
- [11] M. Carrillo Barrenechea, “Stochastic Optimisation for Complex Mixed-Integer Programming Problems in Asteroid Tour Missions,” Master Thesis, Cranfield University, 2021.
- [12] J. A. Englander, M. A. Vavrina, L. F. Lim, L. A. McFadden, A. R. Rhoden, and K. S. Noll, “Trajectory optimization for missions to small bodies with a focus on scientific merit,” *Computing in Science & Engineering*, vol. 19, no. 4, pp. 18–28, 2017, <https://doi.org/10.1109/MCSE.2017.3151246>.
- [13] J. R. Stuart, K. C. Howell, and R. S. Wilson, “Design of end-to-end trojan asteroid rendezvous tours incorporating scientific value,” *Journal of Spacecraft and Rockets*, vol. 53, no. 2, pp. 278–288, 2016, <https://doi.org/10.2514/1.A33396>.
- [14] P. Cui, D. Qiao, H. Cui, and E. Luan, “Target selection and transfer trajectories design for exploring asteroid mission,” *Science China Technological Sciences*, vol. 53, pp. 1150–1158, 2010, <https://doi.org/10.1007/s11431-010-0007-6>.
- [15] L. Ingmar, M. G. de la Banda, P. J. Stuckey, and G. Tack, “Modelling diversity of solutions,” in *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 34, no. 02, 2020, pp. 1528–1535, <https://doi.org/10.1609/aaai.v34i02.5512>.
- [16] F. Casas, C. E. Torres, and I. Araya, “A heuristic search based on diversity for solving combinatorial problems,” *Journal of Heuristics*, vol. 28, no. 3, pp. 287–328, 2022, <https://doi.org/10.1007/s10732-022-09494-4>.
- [17] E. G. Carrano, F. Campelo, and R. H. Takahashi, “Diversity-driven selection operator for combinatorial optimization,” in *International Conference on Evolutionary Multi-Criterion Optimization*. Springer, 2021, pp. 178–190, <https://doi.org/10.1007/s10732-022-09494-4>.
- [18] F. Glover, C.-C. Kuo, and K. S. Dhir, “Heuristic algorithms for the maximum diversity problem,” *Journal of information and Optimization Sciences*, vol. 19, no. 1, pp. 109–132, 1998, <https://doi.org/10.1080/02522667.1998.10699366>.
- [19] D. C. Porumbel, J.-K. Hao, and F. Glover, “A simple and effective algorithm for the maxmin diversity problem,” *Annals of Operations Research*, vol. 186, pp. 275–293, 2011, <https://doi.org/10.1007/s10479-011-0898-z>.
- [20] M. G. Resende, R. Martí, M. Gallego, and A. Duarte, “Grasp and path relinking for the max-min diversity problem,” *Computers & Operations Research*, vol. 37, no. 3, pp. 498–508, 2010, <https://doi.org/10.1016/j.cor.2008.05.011>.
- [21] D. Hennes, D. Izzo, and D. Landau, “Fast approx-

imators for optimal low-thrust hops between main belt asteroids,” in *2016 IEEE Symposium Series on Computational Intelligence (SSCI)*, 2016, pp. 1–7, <https://doi.org/10.1109/SSCI.2016.7850107>.

- [22] H. Li and H. Baoyin, “Sequence optimization for multiple asteroids rendezvous via cluster analysis and probability-based beam search,” *Science China Technological Sciences*, vol. 64, no. 1, pp. 122–130, 2021, <https://doi.org/10.1007/s11431-020-1560-9>.

BIOGRAPHY



Jan Grabowski graduated from Aerospace Engineering at ENSMA, France in 2022 and received his M.Sc. in Astronautics and Space Engineering from Cranfield University in 2023. His research interests include mission analysis, astrodynamics and trajectory design for deep space missions.



Andrea Bellome received his B.S. degree Aerospace Engineering from La Sapienza – University of Rome, It., in 2018 and the M.Sc. in Space Engineering from the University of Surrey, Guildford, UK., in 2019. In 2023, he obtained a Ph.D. in Aerospace Engineering in collaboration with Airbus Defence & Space at Cranfield University, Cranfield, U.K. From 2023, he is working as Research Fellow at ISAE-Supaero, Toulouse, Fr., for the ESA funded project GIGANTES. His research interests include spacecraft trajectory optimization, mission analysis, and mixed-integer space trajectory design.



Leonard Felicetti Leonard Felicetti is a Senior Lecturer in Space Robotics and Guidance Navigation and Control at Cranfield University (UK). He has active roles in teaching and research activities in the fields of Guidance, Navigation and Control of Space Systems, Spacecraft Attitude and Orbital Control Systems, Space Robotics and Space Systems and Missions Design.



DSPACE

<https://dspace.org/>

Diversity-based heuristic search for multiple-asteroid tours

Grabowski, Jan; Bellome, Andrea; Felicetti, Leonard

2024-03-02

Institute of Electrical and Electronics Engineers (IEEE)

<https://doi.org/10.1109/AERO58975.2024.10521350>

Grabowski J, Bellome A, Felicetti L. (2024) Diversity-based heuristic search for multiple-asteroid tours. In: 2024 IEEE Aerospace Conference, 2-9 March 2024, Big Sky, MT, USA