



Effect of Embedded Shape Memory Alloy (SMA) on the Low-velocity Impact Behaviour of Stringer Stiffened Composite Plates

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Abstract

This study presents an analytical model of low-velocity impact on composite plates, possessing either flat or cylindrical shapes. The overall plate's stiffness properties are varied with an additionally assembled Ω -stringer and embedded shape memory alloy wires in the composite material, using only the shape memory effect. This analytical approach enables a low-effort but sufficiently accurate design of low-energy-impacted structures. Comprehensive analytical equations are developed based on the inclusion of the stringer effect on the curved panels. For both stringer-stiffened and non-stiffened plate models, parameter variations regarding the impact and shape memory alloy (SMA) wire integration are conducted. The effects can be classified as changes in essential and acquired stiffness. Our results indicate that SMAs improve the energy absorption capability of stiffened composite panels.

Keywords Smart material · Impact behaviour · Analytical modelling · Low-velocity impact · Shape memory alloy

1 Introduction

The effect of transverse low-velocity impact on plates has been researched for many decades. Consequently, accurate analytical modelling of these loading scenarios has been of interest. For the impactor and plate, an equivalent spring-mass system was defined; so that the contact force history can be determined [1–4]. The transverse shear deformation theory of shells and the plate's curvature were already included. Many later works (adaptions and extensions) are based on these publications, such as those from Khalili et al. [5, 6] or Arachchige et al. [7, 8].

In real-case scenarios, single composite plates are often stiffened with stringers to resist higher loading. Hence, adoptions for the analytical modelling were developed: The idealised spring-mass system was extended with extra stiffnesses and masses representing

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the additional components. Arachchige et al. [8, 9] presented an approach to the analytical modelling of curved composite plates with variable stiffness. The impacted plate was divided into several sections with different thicknesses or ply lay-ups, respectively. The natural frequencies and stiffnesses were computed separately following the methods of simple plate modelling, respectively. These variables were combined to a global, equivalent stiffness and natural frequency using connections in series and parallel. In [10] it is presented that this approach enabled also the inclusion of damaged sections by reducing individual section stiffnesses manually. Arachchige and Kodippili [11] conducted experimental tests and simulations of plates with varied thickness sections approving this analytical method. Arachchige and Ghasemnejad [9] also applied this variation of section thicknesses on sandwich composites by including extra core sections. Correias and Ghasemnejad [12] published an adapted analytical spring-mass model for impact in sandwich panels with damage to the face-sheet and core due to impactor indentation. In [13] Correias et al. used the same approach of spring-mass model adaption for plate assemblies with extra attached stringers. This analysis did not consider neither any damage to the base plate nor the failure of bonding. Due to the shape of the stringer, its sections were assumed as projected on the base plate. This approach was approved with FEM simulations and applied to laminate and thickness variations. Also, other authors investigated stringer stiffening: e.g. Song et al. [14], Rout and Karmakar [15] or Sun et al. [16].

Shape memory alloys (SMA) as multifunctional materials possess variable properties. They can be used to manufacture smart composites by embedding the composite in the shape of wire. After manufacturing the composite, these wires are activated, using the shape memory effect (SME), so that a pre-stress is generated internally. The stiffness against transverse impact is increased and the in-plane stressing during impact of the composite plies is reduced [17]. Rogers et al. [18] were authors who set the basics for SMA integration in composites. In particular, they gave a comprehensive overview and introduction to analytical modelling. To determine the correct macro-mechanical behaviour, they stated micromechanical constitutive equations. Birman et al. [19] were a few of the first authors who published analytical definitions of low-velocity impact on composite plates with embedded SMA wires. They also defined assumptions of the analytical modelling which are still valid in recent works: The impact problem is assumed as physically and geometrically linear because no plastic deformation and damage is considered. Only perfect bonding between composite and SMA wires exists. Independently of the extra embedded wires, the contact force history of the plate is based on a half sin-function dependent on time and with a amplitude proportional to the impact velocity. Further basics were determined by Khalili et al. [5]. Geometrical and material property variations were conducted. Results such as the contact force, deflection, in-plane strains and stresses were discussed. E.g. the volume fraction, orientation and through-the-thickness location of the SMA wires and the elastic modulus ratio were varied. They applied the contact model of Choi (references see in [5]). Different investigations were published by Khalili et al. in [20]. Besides further parameter variations, they defined that the stiffness change is based on two main effects: essential and acquired stiffnesses. The former mentioned includes all effects which are existent after the manufacturing of the composite. The latter-mentioned stiffness category is based on effects applied afterwards such as prestressing the SMA wires by application of external heat. The best combination of both categories has to be found to get the best total stiffness and plate behaviour. In addition, Khalili et al. [21] continued the previous research in which also impactor parameters were changed. In [5] the authors presented for composite plates with unidirectional lay-up an optimisation of SMA wire integration. Based on

the aforementioned, Khalili and Ardali [17] adapted their model to analytical formulations for double-curved panels. Several methods and manufacturing rules for optimised composites with embedded SMA wires are stated in the publications of Khalili et al. [22], Ozair et al. [23] and Guida et al. [24].

For both the stringers and SMA wire integration, approaches to analytical modelling of impacted composite plates already exist. However, both approaches have not been investigated in combination yet. It is necessary to identify their interaction effects and determine the best combination parameters to improve impact behaviour.

We have contributed to the combined SMA stiffening method with the stringer stiffening method and investigated a parameter variation. This study provides a comprehensive analytical model and further knowledge about combined parameter optimisation. The authors have obtained more relevant data of other parameter verifications for the SMA stiffening.

2 Development of Analytical Model

2.1 Introduction of Analytical Modelling

Both extensions, the SMA wire integration in the composite material and the assembling of the base plate and an addition omega stringer, can be combined without limitation as the first mentioned defines only the micro-mechanical behaviour whereas the latter mentioned affects the macro-mechanical system. The plates are analysed either with and without a shape of single curvature and with or without the stringer, respectively (see Fig. 1). The analytical model is mostly based on the works of Gong [4] (basic analytical model), Khalili et al. [17] (extension for SMA wires) and Correias et al. [13] (extension for stringer stiffening). All these publications do not consider any damage because the impact happens only under low-velocity conditions. Hence, also only the stiffness properties and no damage properties are researched.

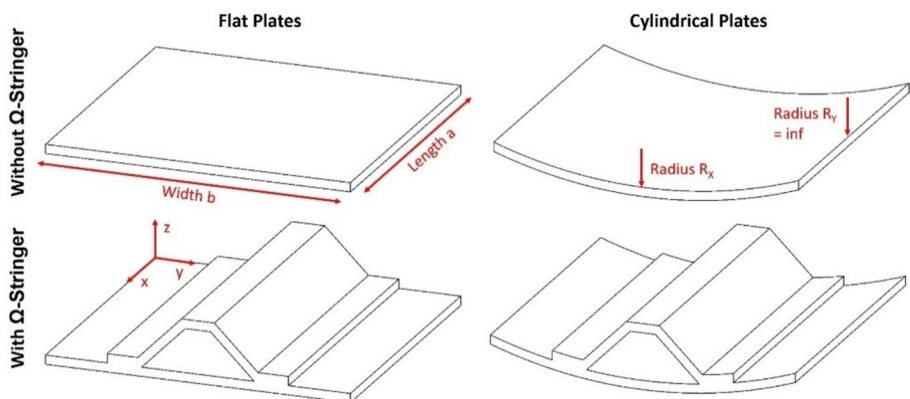


Fig. 1 Basic plate (assembly) shapes of analytical models, impactor striking on the bottom side

2.2 Constitutive Modelling of Shape Memory Effect

For the constitutive equations of SMA, a less complex macro-mechanical material model is used guaranteeing a more practical solving process for engineering problems. A basic one-dimensional model of this behaviour was defined by Fischer and Tanaka [25], which is based on exponential relations. Liang and Rogers [26] adapted this model by replacing the exponential behaviour with cosine relations. These relations idealise the phase transformation from austenite to martensite and reverse. The basic constitutive relation of a SMA material is defined following:

$$(\sigma - \sigma_0) = E(\xi) \cdot (\epsilon - \epsilon_0) + \Theta \cdot (T - T_0) + \Omega(\xi) \cdot (\xi - \xi_0) \quad (1)$$

The stress σ is dependent on the strain ϵ , Young's modulus E , thermo-elastic and phase transformation tensors Θ and Ω , the temperature T and the volume fraction of martensite ξ (between 0 and 100 percent). The index 0 defines the initial conditions of the SMA material which experiences a phase transformation. After the reverse transformation, the residual strain of the SMA material is restored. A fully restrained recovery means that no strain is allowed. So, the residual strain is converted into large internal stresses:

$$\epsilon - \epsilon_0 = 0 \quad (2)$$

More formula details and a iterative solution flowchart are given in [27] and [28].

2.3 Governing Equations of Displacement Field

The following analytical models are based on the first-order shear deformation theory (FSDT) [29] which describes the through-the-thickness shear effects accurate enough (see Witney and Pagano [3]). The displacement field of a composite plate is described by the following Eqs. 3, 4, 5. The variables u , v and w represent the displacement in the respective three directions along the x-, y- and z-axis of the plate. φ_x and φ_y represent the rotation of a normal to this surface, respectively. In addition, the position is dependent on the time t . The ratio of z/R can be ignored if the values of plate curvature are much greater in comparison with the plate thickness.

$$u(x, y, z, t) = u_0(x, y, t) + z \cdot \varphi_x(x, y, t); \quad (3)$$

$$v(x, y, z, t) = v_0(x, y, t) + z \cdot \varphi_y(x, y, t); \quad (4)$$

$$w(x, y, z, t) = w_0(x, y, t) \quad (5)$$

Khalili et al. (e.g. [17]) define the shell behaviour of transversely impacted plates with specially orthotropic laminates:

$$A_{16} = A_{26} = D_{16} = D_{26} = B_{ij} = 0; i, j = 1, 2, 6 \quad (6)$$

Coreas [30] argued that also non-specially orthotropic composites are represented accurately enough if the aforementioned variables are at least one order of magnitude smaller than other stiffness elements. The shell behaviour is represented by the following system of five differential Eqs. 7, 8, 9, 10, 11 a which include the uniform in-plane (pre-)stresses N_{xi} and N_{yi} .

$$\begin{aligned}
 0 = & A_{11} \cdot \left(\frac{\delta^2 u_0}{\delta x^2} + \frac{1}{R_x} \cdot \frac{\delta w_0}{\delta x} \right) + A_{12} \cdot \left(\frac{\delta^2 v_0}{\delta x \delta y} + \frac{1}{R_y} \cdot \frac{\delta w_0}{\delta x} \right) \\
 & + A_{66} \cdot \left(\frac{\delta v_0^2}{\delta x \delta y} + \frac{\delta^2 u_0}{\delta x^2} \right) + c_0 \cdot D_{66} \cdot \left(\frac{\delta^2 \varphi_x}{\delta y^2} + \frac{\delta^2 \varphi_y}{\delta x \delta y} \right) \\
 & + k_{sh} \cdot A_{55} \cdot \left(\frac{\varphi_x}{R_x} + \frac{1}{R_x} \cdot \frac{\delta w_0}{\delta x} - \frac{u_0}{R_x^2} \right) \\
 & + \frac{\delta N_x^i}{\delta x} + \frac{N_x^i}{R_x} \cdot \left(\frac{\delta w_0}{\delta x} - \frac{u_0}{R_x} \right) - I_0 \cdot \frac{\delta^2 u_0}{\delta t^2} - I_1 \frac{\delta^2 \varphi_x}{\delta t^2}
 \end{aligned} \quad (7)$$

$$\begin{aligned}
 0 = & A_{12} \cdot \left(\frac{\delta^2 u_0}{\delta x \delta y} + \frac{1}{R_x} \cdot \frac{\delta w_0}{\delta y} \right) + A_{22} \cdot \left(\frac{\delta^2 v_0}{\delta y^2} + \frac{1}{R_y} \cdot \frac{\delta w_0}{\delta y} \right) \\
 & + A_{66} \cdot \left(\frac{\delta v_0^2}{\delta x^2} + \frac{\delta^2 u_0}{\delta x \delta y} \right) - c_0 \cdot D_{66} \cdot \left(\frac{\delta^2 \varphi_x}{\delta x \delta y} + \frac{\delta^2 \varphi_y}{\delta x^2} \right) \\
 & + k_{sh} \cdot A_{44} \cdot \left(\frac{\varphi_y}{R_y} + \frac{1}{R_y} \cdot \frac{\delta w_0}{\delta y} - \frac{v_0}{R_y^2} \right) \\
 & + \frac{\delta N_y^i}{\delta y} + \frac{N_y^i}{R_y} \cdot \left(\frac{\delta w_0}{\delta y} - \frac{v_0}{R_y} \right) - I_0 \cdot \frac{\delta^2 v_0}{\delta t^2} - I_1 \frac{\delta^2 \varphi_y}{\delta t^2}
 \end{aligned} \quad (8)$$

$$\begin{aligned}
 0 = & k_{sh} \cdot A_{55} \cdot \left(\frac{\delta^2 w_0}{\delta x^2} + \frac{\delta \varphi_x}{\delta x} - \frac{1}{R_x} \cdot \frac{\delta u_0}{\delta x} \right) \\
 & + k_{sh} \cdot A_{44} \cdot \left(\frac{\delta^2 w_0}{\delta y^2} + \frac{\delta \varphi_y}{\delta y} - \frac{1}{R_y} \cdot \frac{\delta v_0}{\delta y} \right) - \frac{1}{R_x} \cdot \left(A_{11} \cdot \left(\frac{\delta u_0}{\delta x} + \frac{w_0}{R_x} \right) + A_{12} \cdot \left(\frac{\delta v_0}{\delta y} + \frac{w_0}{R_y} \right) \right) \\
 & + \frac{1}{R_y} \cdot \left(A_{12} \cdot \left(\frac{\delta u_0}{\delta x} + \frac{w_0}{R_x} \right) + A_{22} \cdot \left(\frac{\delta v_0}{\delta y} + \frac{w_0}{R_y} \right) \right) + \frac{\delta}{\delta x} \cdot \left(N_x^i \left(\frac{\delta w_0}{\delta x} - \frac{u_0}{R_x} \right) \right) \\
 & + \frac{\delta}{\delta y} \cdot \left(N_y^i \left(\frac{\delta w_0}{\delta y} - \frac{v_0}{R_y} \right) \right) + q - I_0 \cdot \frac{\delta^2 w_0}{\delta t^2}
 \end{aligned} \quad (9)$$

$$\begin{aligned}
 0 = & D_{11} \cdot \left(\frac{\delta^2 \varphi_x}{\delta x^2} \right) + D_{12} \cdot \left(\frac{\delta^2 \varphi_y}{\delta x \delta y} \right) + D_{66} \cdot \left(\frac{\delta^2 \varphi_y}{\delta x \delta y} + \frac{\delta^2 \varphi_x}{\delta y^2} \right) \\
 & - \left(k_{sh} \cdot A_{55} \cdot \left(\varphi_x + \frac{\delta w_0}{\delta x} - \frac{u_0}{R_x} \right) \right) - I_2 \cdot \frac{\delta^2 \varphi_x}{\delta t^2} - I_1 \cdot \frac{\delta^2 u_0}{\delta t^2}
 \end{aligned} \quad (10)$$

$$\begin{aligned}
 0 = & D_{22} \cdot \left(\frac{\delta^2 \varphi_y}{\delta y^2} \right) + D_{12} \cdot \left(\frac{\delta^2 \varphi_x}{\delta x \delta y} \right) + D_{66} \cdot \left(\frac{\delta^2 \varphi_x}{\delta x \delta y} + \frac{\delta^2 \varphi_y}{\delta x^2} \right) \\
 & - \left(k_{sh} \cdot A_{44} \cdot \left(\varphi_y + \frac{\delta w_0}{\delta y} - \frac{v_0}{R_y} \right) \right) - I_2 \cdot \frac{\delta^2 \varphi_y}{\delta t^2} - I_1 \cdot \frac{\delta^2 v_0}{\delta t^2}
 \end{aligned} \quad (11)$$

The variables ρ and h define the density and total thickness of the plate respectively. k_{sh} is the shear correction factor equal $\pi^2/12$, defined by Mindlin [31]. I_i are the mass inertias defined by the Eq. 12:

$$I_i = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho \left(1 + \frac{z}{R-1} \right) \cdot \left(1 + \frac{z}{R_y} \right) (z)^i dz \quad (12)$$

Following boundary conditions are valid for the displacement field of simply supported plates:

$$w(x, y, t) = \varphi_x(x, y, t) = 0 \text{ at } x = a; w(x, y, t) = \varphi_y(x, y, t) = 0 \text{ at } y = b \quad (13)$$

2.4 Constitutive Equations of Micromechanical Modelling

The constitutive equations (Eq. 14) describe the micromechanical relationship between stress σ and strain ϵ (principal directions) for a laminate with integrated SMA wires. The relation is defined by the reduced stiffness matrices for the composite with SMA wires (elements Q_{ij}) and without them (elements Q_{ij}^m). σ_r is the recovery stress of the activated SMA wires (calculation based on [5]). Stress, generated by the thermal expansion effect, is defined with the thermal expansion coefficients α of the composite only and the change of temperature ΔT . The respective influence of SMA effect and thermal expansion effect are regulated by the volume content of SMA wires k_s and its counterpart k_c for the remaining composite materials.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} + \begin{bmatrix} \sigma_r \\ 0 \\ 0 \end{bmatrix} \cdot k_s - \begin{bmatrix} Q_{11}^m & Q_{12}^m & 0 \\ Q_{12}^m & Q_{22}^m & 0 \\ 0 & 0 & Q_{66}^m \end{bmatrix} \begin{bmatrix} \alpha_1^c \\ \alpha_2^c \\ 0 \end{bmatrix} \cdot k_c \cdot \Delta T \quad (14)$$

Equivalent relations are defined for the extensional shear matrix in the next equation:

$$\begin{bmatrix} \tau_{23} \\ \tau_{13} \end{bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{bmatrix} \gamma_{23} \\ \gamma_{13} \end{bmatrix} \quad (15)$$

These relations can be integrated through the plate's thickness applying discontinuous functions on each layer (see Eq. 16). The outcome are constitutive equations defining the resulting elements of the force and couple moment vectors, respectively (N and M). The relations are defined i.a. by the extensional, coupling and bending stiffness matrices A , B , and D . ϵ_0 and ϵ_1 define the mid-plane strains and curvatures of the composite lay-up.

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{bmatrix} \epsilon^0 \\ \epsilon^1 \end{bmatrix} + \begin{bmatrix} N^i \\ M^i \end{bmatrix}; i, j = 1, 2, 6; \begin{bmatrix} N^i \\ M^i \end{bmatrix} = \begin{bmatrix} N^r - N^T \\ M^r - M^T \end{bmatrix}; i, j = 1, 2, 6 \quad (16)$$

N_r and M_r are the stress and moment vectors due to the recovery of the SMA wires in the activation process. N_t and M_t define the vector of internal stress and moment because of the thermal expansion effects. Equivalent integrated equations can be defined for the shear effects:

$$[S] = [k_{sh} A_{i,j}] [\epsilon^0]; i, j = 4, 5 \quad (17)$$

Because the plates always have a symmetrical lay-up, no bending moments occur: $M^i = 0$. So, the in-plane stresses can be defined as in Eq. 18.

$$N^i = \left[\begin{array}{l} \sigma_r \cdot k_{sx} \cdot h_x - (Q_{11}^m \cdot \alpha_l^c + Q_{12}^m \cdot \alpha_t^c) \cdot k_{cx} \cdot h_x \cdot \Delta T \\ \sigma_r \cdot k_{sy} \cdot h_y - (Q_{12}^m \cdot \alpha_l^c + Q_{22}^m \cdot \alpha_t^c) \cdot k_{cy} \cdot h_y \cdot \Delta T \end{array} \right] \quad (18)$$

h_x and h_y are the total thicknesses of the composite without SMA wire integration. With the SMA volume content in both x- and y-direction also the volume content of the remaining composite can be defined:

$$k_{cx} = 1 - k_{sx}; k_{cy} = 1 - k_{sy} \quad (19)$$

2.5 Deflection Analysis

For a rectangular plate with simply supported boundary conditions the displacement field can be defined with terms of double Fourier series. These expressions are applied on the load, displacement and rotation equations with a function of time and position only, respectively. In the following Eqs. 20, 21, 22, 23, 24 each displacement variable is defined based on the time dependent constants U_{mn} , V_{mn} , W_{mn} , φ_{mn} and φ_{mn} :

$$u_0(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{mn}(t) \cdot \cos\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \pi \cdot y}{b}\right); m, n = 1, 3, 5, \dots \quad (20)$$

$$v_0(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} V_{mn}(t) \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \cos\left(\frac{n \cdot \pi \cdot y}{b}\right); m, n = 1, 3, 5, \dots \quad (21)$$

$$w_0(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn}(t) \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \pi \cdot y}{b}\right); m, n = 1, 3, 5, \dots \quad (22)$$

$$\varphi_x(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} X_{mn}(t) \cdot \cos\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \pi \cdot y}{b}\right); m, n = 1, 3, 5, \dots \quad (23)$$

$$\varphi_y(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Y_{mn}(t) \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \cos\left(\frac{n \cdot \pi \cdot y}{b}\right); m, n = 1, 3, 5, \dots \quad (24)$$

The equivalent force distribution is based on a term of Fourier series with concentrated load located at the position (x_c, y_c) (in these studies always central):

$$q_n(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Q_{mn}(t) \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \pi \cdot y}{b}\right); m, n = 1, 3, 5, \dots \quad (25)$$

$$Q_{mn}(t) = \frac{4 \cdot F(t)}{a \cdot b} \cdot \sin\left(\frac{m \cdot \pi \cdot x^c}{a}\right) \cdot \sin\left(\frac{n \cdot \pi \cdot y^c}{b}\right); m, n = 1, 3, 5, \dots \quad (26)$$

The terms of Fourier series of the concentrated force $Q(t)$ include force the applied force $F(t)$ which is the contact force between the impactor and the plate. Because the rotary inertia effects can be neglected, the aforementioned system of Eqs. (7, 8, 9, 10, 11 and 20, 21, 22, 23, 24) can be transformed into a system of ordinary differential equations (ODE).

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} \\ c_{14} & c_{24} & c_{33} & c_{44} & c_{45} \\ c_{15} & c_{25} & c_{33} & c_{44} & c_{55} \end{bmatrix} \begin{bmatrix} U_{mn}(t) \\ V_{mn}(t) \\ E_{mn}(t) \\ X_{mn}(t) \\ Y_{mn}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \rho \cdot h \cdot \ddot{W}_{mn}(t) - Q_{mn}(t) \\ 0 \\ 0 \end{bmatrix} \quad (27)$$

All ODEs are second order of time regarding the transverse displacement direction:

$$U_{mn}(t) = K_U \cdot W_{mn}(t); \quad (28)$$

$$V_{mn}(t) = K_V \cdot W_{mn}(t); \quad (29)$$

$$X_{mn}(t) = K_X \cdot W_{mn}(t); \quad (30)$$

$$Y_{mn}(t) = K_Y \cdot W_{mn}(t) \quad (31)$$

The belonging coefficients for the equations consider pre-stress and are defined in the aforementioned literature. This system can be defined in one sole expression which represents a single-degree-of-freedom (sdf) spring-mass-system of the oscillating plate.

$$\ddot{W}_{mn}(t) + \omega_{mn}^2 \cdot W_{mn}(t) = \frac{Q_{mn}(t)}{\rho \cdot h} \quad (32)$$

The variable ω_{mn} represents the natural frequency of the plate. Cases of low-velocity impact and an impactor mass being at least two times greater than the plate's mass allow the assumption of considering only the first natural frequency. Also, no damping is considered as its effect would be minor regarding the displacements.

$$\omega_{mn}^2 = \frac{-(c_{13} \cdot K_U + c_{23} \cdot K_V + c_{33} + c_{34} \cdot K_X + c_{35} \cdot K_Y)}{\rho \cdot h} \quad (33)$$

Therefore, the final sdf-system can be defined as follows:

$$\ddot{W}_{mn}(t) + \omega_{mn}^2 \cdot W_{mn}(t) = \frac{4 \cdot F(t)}{a \cdot b \cdot \rho \cdot h} \cdot \sin\left(\frac{m \cdot \pi \cdot x^c}{a}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot y^c}{b}\right) \quad (34)$$

The solution, applying the initial displacement conditions (everywhere zero) and boundary conditions (simply supported edges), is determined to be:

$$W_{mn}(t) = \frac{4}{\omega_{mn} \cdot a \cdot b \cdot \rho \cdot h} \cdot \sin\left(\frac{m \cdot \pi \cdot x^c}{a}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot y^c}{b}\right) \int_0^t F(\tau) \sin(\omega_{mn}) \cdot (t - \tau) d\tau \quad (35)$$

The displacement of the plate due to transverse impact load can be expressed with following Eq. 36:

$$w_0(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\omega_{mn}} \cdot \frac{4}{a \cdot b \cdot \rho \cdot h} \cdot \sin\left(\frac{m \cdot \pi \cdot x^c}{a}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot y^c}{b}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot y}{b}\right) \cdot \int_0^t F(\tau) \cdot \sin(\omega_{mn}) \cdot (t - \tau) d\tau \quad (36)$$

Integrating over the impact time, the final solution is presented next:

$$w_0(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\omega_{mn}} \cdot \frac{4}{a \cdot b \cdot \rho \cdot h} \cdot \sin\left(\frac{m \cdot \pi \cdot x^c}{a}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot y^c}{b}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot y}{b}\right) \cdot \left[\frac{A_1 \cdot (C_p - 1)}{\omega_p^2 - \omega_{mn}^2} \cdot (\omega_p \cdot \sin(\omega_{mn} \cdot t) - \omega_{mn} \cdot \sin(\omega_p \cdot t)) + \frac{A_2 \cdot (C_i - 1)}{\omega_i^2 - \omega_{mn}^2} \cdot (\omega_i \cdot \sin(\omega_{mn} \cdot t) - \omega_{mn} \cdot \sin(\omega_i \cdot t)) \right] \quad (37)$$

2.6 Plate Stiffnesses

2.6.1 Single Plate

Based on the first natural frequency, the equivalent stiffness of the plate can be defined:

$$K_p = M^* \cdot \omega_{mn} \quad (38)$$

The equivalent Rayleigh–Ritz mass of the plate is belonging to the deformed-shape function of the plate.

$$M^* = \rho \cdot h \cdot \int_A \left(\sin\left(\frac{\pi \cdot x}{a}\right) \cdot \sin\left(\frac{\pi \cdot y}{b}\right) \right)^2 dA = \frac{M}{4} \quad (39)$$

2.6.2 Stringer Stiffened Plate

The analytical formulations of assembling an extra Ω -stringer to the base plate is presented in [13] and underlays some restrictions: A perfect bonding between both parts is idealised. Hence, no material failure is represented in the modelling. All the sections of the stringer are assumed as projected on the base plate so that the already presented formulations can be applied. The approach it is based on the division of the stringer into five subsections (the single plate region outside of the stringer (A), the stringer toes (B), the plate region

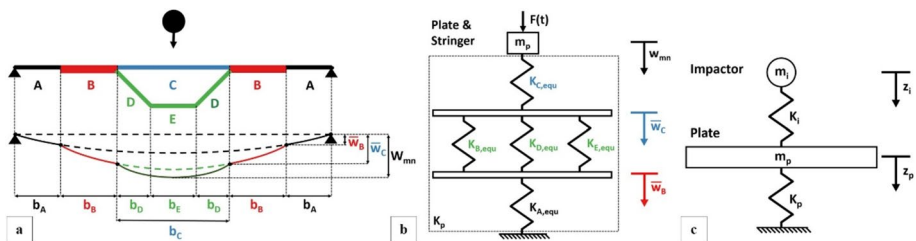


Fig. 2 **a** Idealisation of sections in the Ω -stringer stiffened assembly **(b)** equivalent spring-mass system. of Ω -stringer stiffened assembly **(c)** equivalent spring-mass system of impact system

within the springer toes (C), the stringer webs (D) and the stringer wing (E)) and combination of respective subsections' natural frequencies via connection of section stiffnesses in series and parallel (see Fig. 2b). The sections extend completely along the x-direction and are assumed as simply supported using extra mathematical correction factors: equivalent thickness, effective width and stiffness (see [13]). Similar to a non-stiffened plate, the impact force and displacement in the base plate centre are computed.

Based on the presented assumptions and more details of [13], the equivalent assembly stiffness can be calculated as presented in Eq. 40:

$$K_1 = \frac{1}{\left(\frac{\sin\left(\frac{\pi \cdot b_A}{a}\right)}{K_A} \right) + \left(\frac{\sin\left(\frac{\pi \cdot b_B}{2 \cdot b_B + b_C}\right)}{K_B + K_D + K_E} \right) + \left(\frac{1}{K_C} \right)} \quad (40)$$

The total plate mass is described on the mathematical expressions of deflection and thickness in [13]:

$$M_1 = 2 \cdot \rho \cdot \int_0^b \int_0^{\frac{a}{2}} t(y) \cdot \bar{w}(x, y)^2 dx dy \quad (41)$$

It is assumed that these equations are valid for curved panels as well based on following reasons: Most of the sections of the stringer are still kept flat. Only the base plate is curved and the projection method for the additional stringer is not affected. Furthermore, the definition of the parameters is independent of any curvature except the angle Θ between the base plate and stringer wings whose error is neglectable small.

2.7 Impact Modelling

The deflection history of the (base) plate is dependent on an external, transverse impact force. The analytical model defined by Gong [4] represents a two-dof spring-mass model as shown in Fig. 2c.

It is based on the highly nonlinear Hertzian contact model for the effect of local indentation in the plate.

$$F(t) = K_i \cdot \alpha(t)^p \rightarrow F(t) = K_i^* \cdot \alpha(t) \quad (42)$$

K_i defines the contact stiffness (dependent on geometries and materials of both parts) and p is an exponent equal to 1.5. The parameter α represents the local indentation of the plate. To avoid complex relations in the analytical, the Hertzian contact model is linearised. The effective contact stiffness K_i^* includes Euler's Gamma function Γ and the maximum indentation α_m :

$$K_i^* = \sqrt{\pi} \cdot \Gamma \cdot \left(\frac{p+1}{2} \right) \cdot \frac{2 \cdot \Gamma \cdot \left(\frac{p}{2} + 1 \right) + \sqrt{\pi} \cdot \Gamma \cdot \left(\frac{p+1}{2} \right)}{4 \cdot \Gamma^2 \cdot \left(\frac{p}{2} + 1 \right) + \pi \cdot \Gamma^2 \cdot \left(\frac{p+1}{2} \right)} \cdot \alpha_m^{(p-1)} \cdot K_i \quad (43)$$

The maximum indentation depends on the boundary conditions; for a simply support the mean value of clamped and free conditions is chosen. It depends on the masses, stiffnesses of impactor and plate and the impact velocity.

$$\alpha_m^c = \left(\frac{M_i \cdot M_p}{M_i + M_p} \right)^{0.4} \cdot \left(\frac{(p+1) \cdot v^2}{2 \cdot K_i} \right)^{\frac{1}{p+1}}; \alpha_m^f = (M_i)^{0.4} \cdot \left(\frac{(p+1)v^2}{2 \cdot K_i} \right)^{\frac{1}{p+1}}; \alpha_m^{ss} = \frac{\alpha_m^c + \alpha_m^f}{2} \quad (44)$$

In this study, for a flat and cylindrical shaped composite plate following contact stiffnesses, defined by Vaziri et al. [32], are applied:

$$K_{i,flat} = \frac{4}{3} \cdot \frac{E_{p,2}}{1 - \mu_{12}^2} \cdot \sqrt{R_i}; K_{i,cylindrical} = \frac{8}{3} \cdot \frac{G_{12}}{1 - \mu_{12}} \cdot \sqrt{\frac{R_i \cdot R_p}{R_i + R_p}} \quad (45)$$

The complete two spring-mass model can be defined with the displacements z_i and z_p by two ODEs and the initial conditions:

$$m_p \ddot{z}_p = -K_p z_p - K_i (z_p - z_i); m_i \ddot{z}_i = -K_i (z_i - z_p); \text{for } t = 0 : z_p = \dot{z}_i = z_i = 0; \dot{z}_i = V \quad (46)$$

Solving this system of equations gives the local indentation of impactor in the plate:

$$z_i(t) - z_p(t) = A_p \cdot (C_p - 1) \cdot \sin(\omega_p \cdot t) + A_i \cdot (C_i - 1) \cdot \sin(\omega_i \cdot t) \quad (47)$$

The natural frequencies of both the impactor and plate are identified as follows:

$$\omega_{p,i}^2 = \frac{1}{2} \cdot \left(\frac{K_p + K_i}{m_p} + \frac{K_i}{m_i} \right) \mp \sqrt{\frac{1}{4} \left(\frac{K_p + K_i}{m_p} + \frac{K_i}{m_i} \right)^2 - \frac{K_p \cdot K_i}{m_p \cdot m_i}} \quad (48)$$

The (linearised) contact force can be identified using the next Eq. 49:

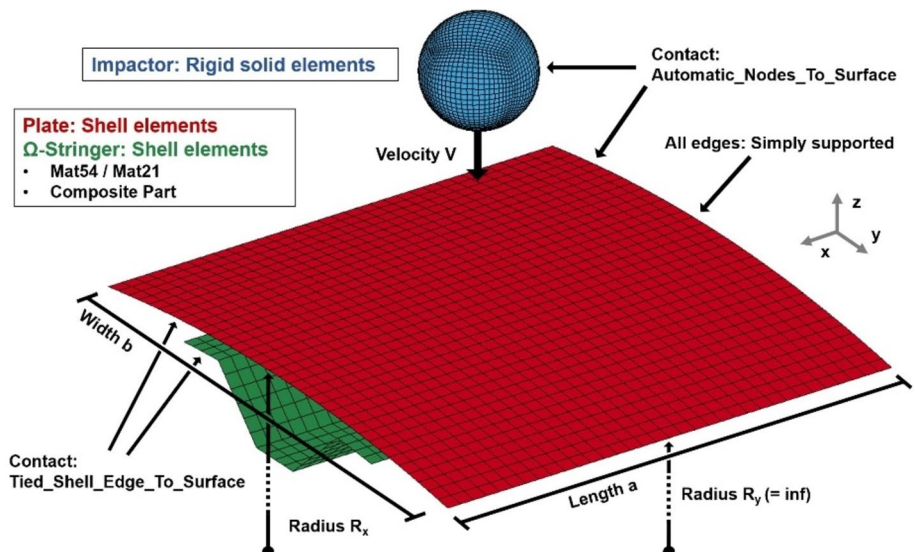


Fig. 3 FE model of the stiffened panel including SMA in LSDYNA

Table 1 material and geometrical properties of plates for CFRP and SMA (adapted to 1d behaviour of wires)

Material	E_1 GPa	$E_2=E_3$ GPa	$G_{12}=G_{13}$ GPa	G_{23} GPa	$\mu_{12}=\mu_{13}$ -	μ_{23} -	P kg/m ³	$\sigma_{r,1}$ MPa
SMA [5]	70.00	0.00	26.32	0.00	0.33	0.00	6500.0	220
T800H/3900-2 [32]	129.00	7.50	3.50	2.60	0.33	0.33	1540.0	0
Models	a mm	b mm	t mm	R₁ mm	R₂ mm	Lay-up		
Flat / Cylindrical	100.0	100.0	4.65	Inf / 300	inf	[45,90,90 SMA,-45,0,0 SMA] _{3s}		

$$F(t) = K_i^* \cdot (A_p \cdot (C_p - 1) \cdot \sin(\omega_p(t)) + A_i \cdot (C_i - 1) \cdot \sin(\omega_i(t))) \quad (49)$$

$$C_p = \frac{K_i}{K_i - \omega_p^2 \cdot m_i}; C_i = \frac{K_i}{K_i - \omega_i^2 \cdot m_i}; A_p = \frac{v}{\omega_p \cdot (C_p - C_i)}; A_i = \frac{v}{\omega_i \cdot (C_i - C_p)}$$

3 Model Validations

The analytical modelling is validated with FEM simulations implemented in LS-DYNA and published experimental data [13, 30, 32]. Based on that the analytical combination of both stiffening methods is verified.

3.1 FEM Simulation

Three published models (flat, cylindrical and with stringer [13, 30, 32]) are considered to verify the impact behaviour of a basic plate and the additional effects of curvature and stringer stiffening. The general setup of the FE model is illustrated in the Fig. 3.

An additional thermal expansion effect represents the internal pre-stressing. The CFRP material has a thermal expansion coefficient equal to zero assigned because it is neglectable small relatively (in comparison to the artificial thermal expansion of the SMA). Some SMA material properties in the transverse direction are adapted artificially to keep the one-dimensional mechanical behaviour of wires based on the original mechanical properties of Khalili et al. [21] (properties are stated in Table 1). The artificial thermal expansion coefficient of the SMA wires is calculated as:

$$\alpha = \frac{\sigma_r}{E \cdot \Delta T} \rightarrow \alpha = \frac{220 \text{ MPa}}{70 \text{ GPa} \cdot 39 \text{ K}} = 8.5861 \cdot 10^{-5} \frac{1}{K} \quad (50)$$

To generate pre-stress dynamic relaxation is activated and a *THERMAL_LOAD_CURVE* is defined. A mesh sensitivity is examined: For all researched models the most accurate results are generated with an impactor mesh size of max. 1 mm and plate mesh size of max. 3 mm. The overall element and node numbers have not been calculated/counted yet because alternatively the size of the parts and the element sizes have been presented. The contact was defined by using the automatic_nodes_to_surface contact in LS Dyna.

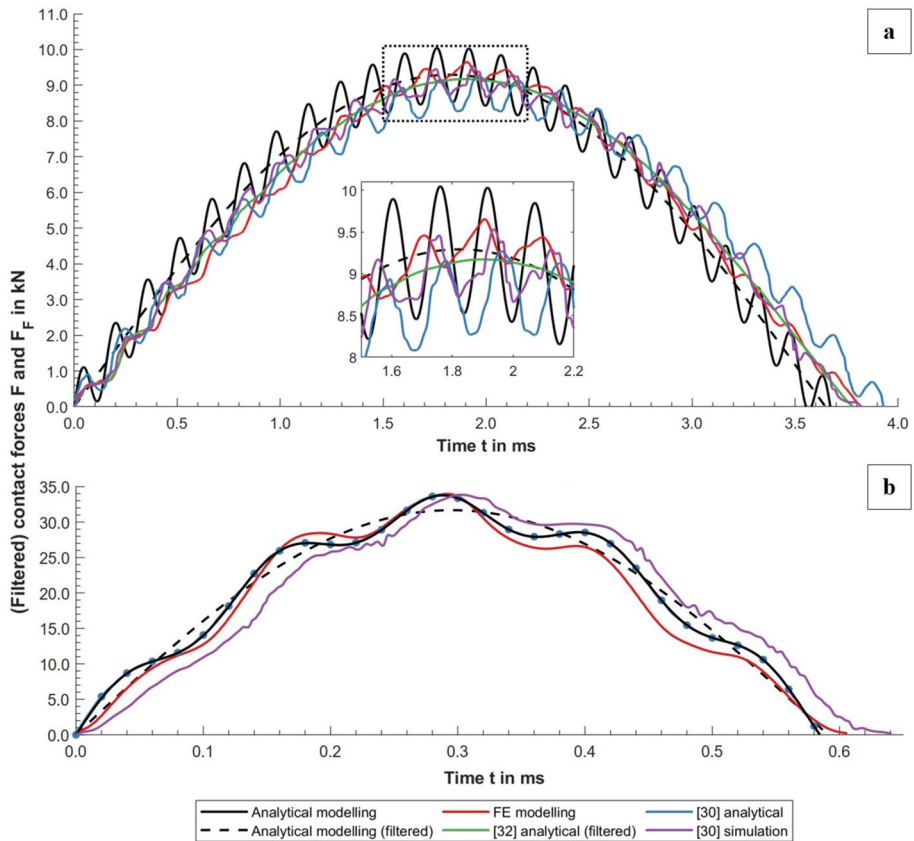


Fig. 4 Analytical and numerical validation of a) flat plate and b) cylindrical plate

3.2 Analytical Modelling

3.2.1 Basic Model Validation

In Fig. 4 the validation of the flat and cylindrical plate models regarding contact force is shown exemplary. The terms "filtered" and "non-filtered" specify the in- or exclusion of the secondary oscillations due to the local impactor indentation in the plate. Comparing all data, it is shown that only small variances exist due to the plate stiffness representations. Regarding the flat plate, the maximum differences in contact force are 0.9% (filtered) and 4.0 percent (non-filtered) between the analytical and FE models. The greatest variances to published data are 8.3% to [30]. All other relations show much smaller discrepancies. Equivalent results are defined referring to the impact time duration and in general for the curved (see Fig. 4b) and stringer-stiffened validations.

The influence of several terms regarding the Fourier series is also investigated. The minimum number of terms ($n=m$) is identified as 15 (error < 1%) which is equivalent to the results of Khalili et al. [20].

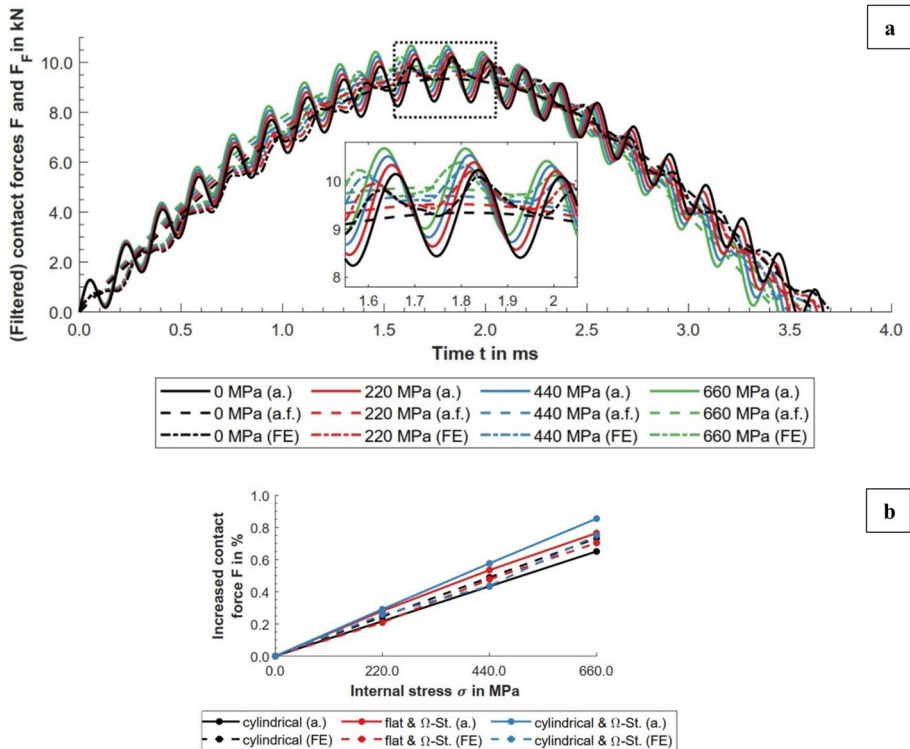


Fig. 5 Validation of SME for (filtered) analytical and FE models: **(a)** flat plate and **(b)** flat plate with stringer and cylindrical plates with and without stringer

3.2.2 Validation of Shape Memory Effect

The analytical modelling of the SME is validated by comparing the change of max. contact force. To exclude the change of essential effect, the lay-up is the same for all variations and only the stress level σ_r is changed. The pre-stresses σ_r are multiples of the stress defined by Khalili et al. [5]. The SMA volume content is set to 20 percent in the x- and y-direction for the flat plate and only in the y-direction for the curved and stringer-stiffened plates, respectively. The pre-stress affects a change of the force histories similar to a change of essential effect only (see Fig. 5a: flat plate). The greater the pre-stress values are the stiffer the plate becomes against transverse impact. The biggest change in maximum contact force is due to the first main plate oscillation whereas the secondary oscillation amplitudes are without great change. Therefore, it is legitimate to compare the change of maximum contact force of analytical modelling with FEM simulations. In Fig. 5b, the contact force changes for the other validation models are plotted. The relation between pre-stress and increased contact force is almost linear. The effect is about one-fifth smaller in comparison with the flat plate because of embedding in only one direction and other plate properties.

All in all, the results prove that the analytical modelling of the SMA effect is validated. Moreover, it is proved that the application of pre-stress by thermo-mechanical modelling in

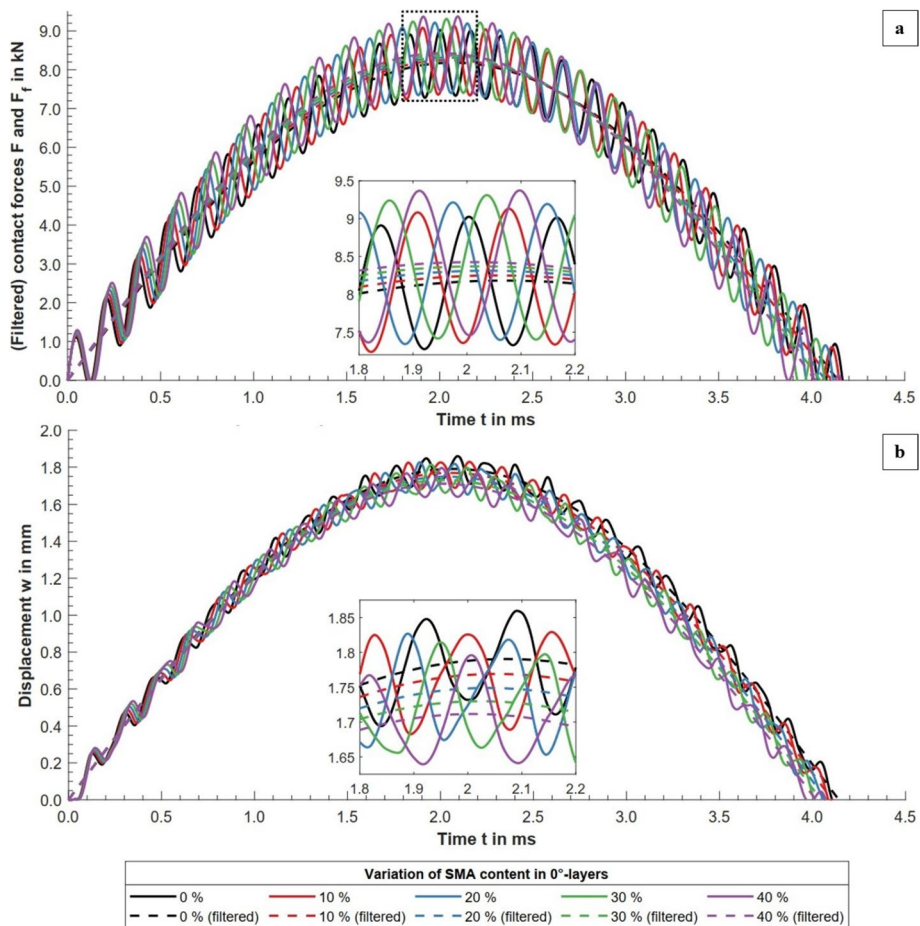


Fig. 6 Effect of varied SMA volume content on the (a) contact force (b) deflection

the FEM simulation is an applicable, easy solution for the correct representation of SMA pre-stressing behaviour.

4 Shape Memory Effect in Non-stiffened Composite Plates

Preliminary research on non-stiffened composite plates is conducted to investigate basic optimum parameter combinations against impact. The SMA content and lay-up are varied. In contrary to the previous section, the SMA includes both essential and acquired effects. All following analyses consider similar geometrical and material properties presented in Table 1 based on previous publications. If not described differently, the volume content is set equal for both the x- and y-directions. Impactor properties are similar to [32].

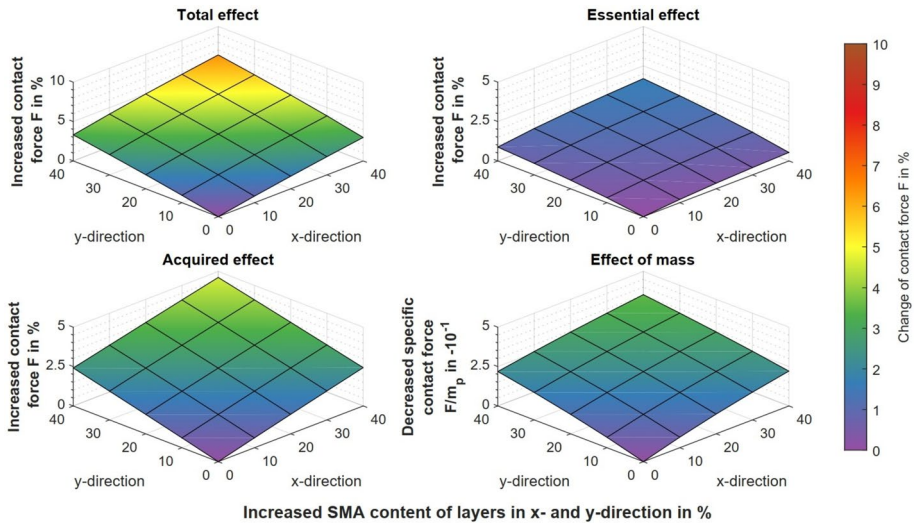


Fig. 7 Effect of varied SMA volume content in x- and y-direction (flat plate)

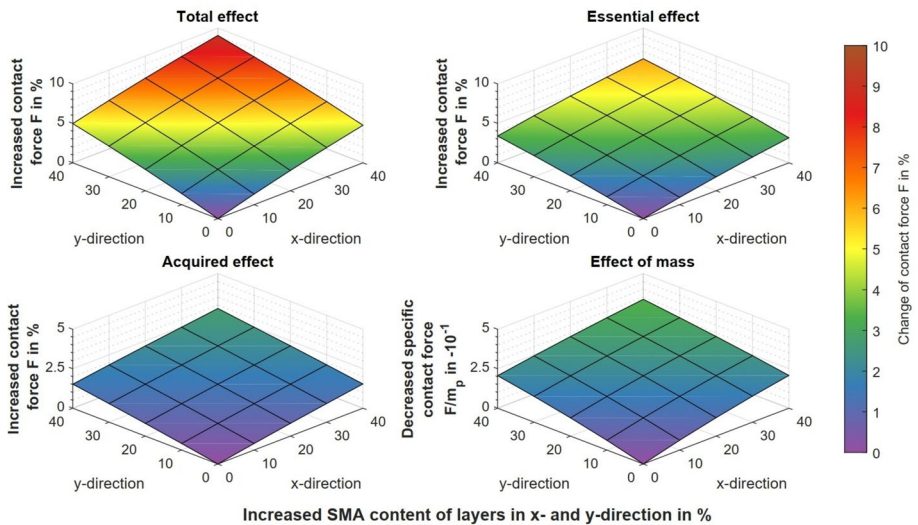


Fig. 8 Effect of varied SMA volume content in x- and y-direction (cylindrical plate)

4.1 Shape Memory Effect on Basic Impact Parameters

In Fig. 6a the dependency of variation of the SMA content (0 to 40%) on the contact force history for a flat plate is presented. It is proven that the more SMA wires are integrated the stiffer the plate results. Increasing the SMA volume content leads to a 3.9% reduction in impact time and a 4.95% increase in maximum contact force non-filtered and 3.1%

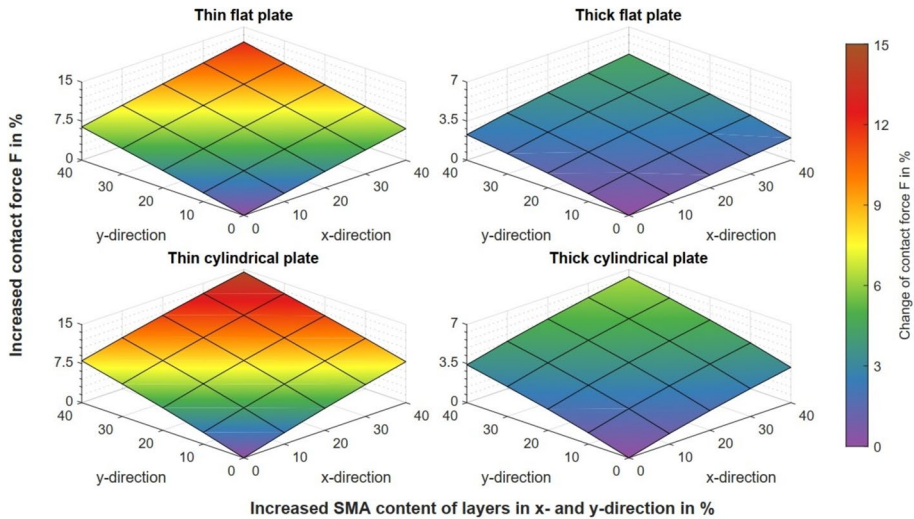


Fig. 9 Variation of plate thickness for varied SMA volume content

Table 2 Lay-up packs with varied percentage of total SMA volume content

Lay-up pack	1	2	3	4	5	6	7
[[45,90,-45,0]	-	-	-	33.30%	50%	50%	100%
[45,90,-45,0]	-	50%	100%	33.30%	-	50%	-
[45,90,-45,0] _s	100%	50%	-	33.30%	50%	-	-

and 2.9% filtered, respectively. The SMA activation mainly affects the global, main plate deflection oscillations and not the secondary due to the local indentation. So, in all following analyses, only the global (filtered) contact forces are investigated.

The increased stiffness can also be proven by the analysis of plate deflection (see Fig. 6b). The greater the SMA volume content is, the smaller the global deflections resulting in a maximum decrease of 1.55%.

4.2 Variation of SMA Volume Content

The influence of SMA volume content, both in x- and y-direction separately and combined, on the maximum contact force is presented in Figs. 7 and 8 (flat and cylindrical). The range of volume content is set between 0 to 40% for the belonging CFRP layers in both directions. Due to the complete lay-up, the maximum total SMA contents are only 10% separately and 20% combined (45° layers do not have embedded wires). Considering the total effect, for both possible wire directions the effect is almost equivalent. Combining embedding wires in both directions increases the force peak even more. Regarding the acquired effect, no difference between x- and y-directions is existent. The result is reasonable as the analytical model of the deflection analysis (chapter 2.5) does not consider the actual through-the-thickness position of SMA in the lay-up. The essential effect shows

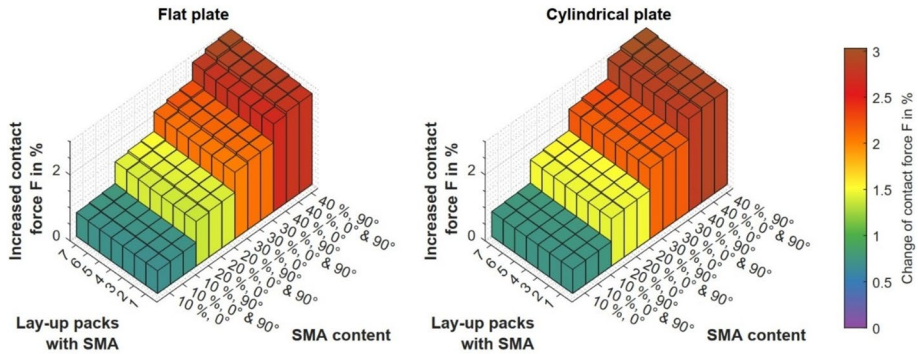


Fig. 10 Variation of SMA wire orientation and through-the-thickness location

small differences that are caused by different ABD matrices of the laminate. Because the 0° CFRP layers are closer to the mid-plane the contact force increase is smaller for embedding along the x-direction. Combining the maximum essential force increase of 2.4% and the acquired effect of 4.6% leads to a maximum total change of 6.4%. Due to the higher density of SMA in comparison with CFRP, the plate's mass is also affected so that the specific contact force (force per plate mass) is reduced (total reduction of this parameter equals 35.3%). However, a stiffness increase only with extra CFRP plies also reduces this property so that an optimum lay-up has to be found.

The effect of SMA wire integration leads to a greater total effect of contact force increase for cylindrical plates. This is caused by a stronger essential effect as the change of ply lay-up has a greater influence on already stiffer plates due to the curvature. Although the acquired stiffness is reduced the total stiffness results in a greater value.

4.3 Variation of Composite Lay-up

The composite lay-up can be changed as well: by through-the-thickness position of SMA and total thickness. The thickness is varied by laying the laminate code two or four times instead of three times originally. The thicker the laminate is the smaller the contact force increase results (see Fig. 9). The thin plates (both flat and cylindrical) have contact force peak values increased by about 5.4% and 4.9%, respectively. Likewise, the thicker plates show a reduction of about 7.9% and 8.0%, respectively. This increase of force values gets smaller for thicker plates because the influence of the extra stiffness due to SMA activation (acquired effect) becomes smaller in relation to the basis plate stiffness. The effect of different embedding orientation with its different force increase values is still valid for all plate configurations.

This small, secondary effect is analysed by defining different lay-up packs for the initial thickness. For pre-set total SMA volume contents the wires are concentrated in different z-positions (see Table 2).

As presented in Fig. 10, the greatest influence on the changed contact force level is still the SMA content. For each volume content the 90° embedding (y-direction) results in the maximum fore increase because of the distance to the laminate's mid-plane. The x-direction leads to the smallest force levels and an embedding in both directions shows an average effect. The more the SMA material is concentrated on the outside of the plate the stiffer

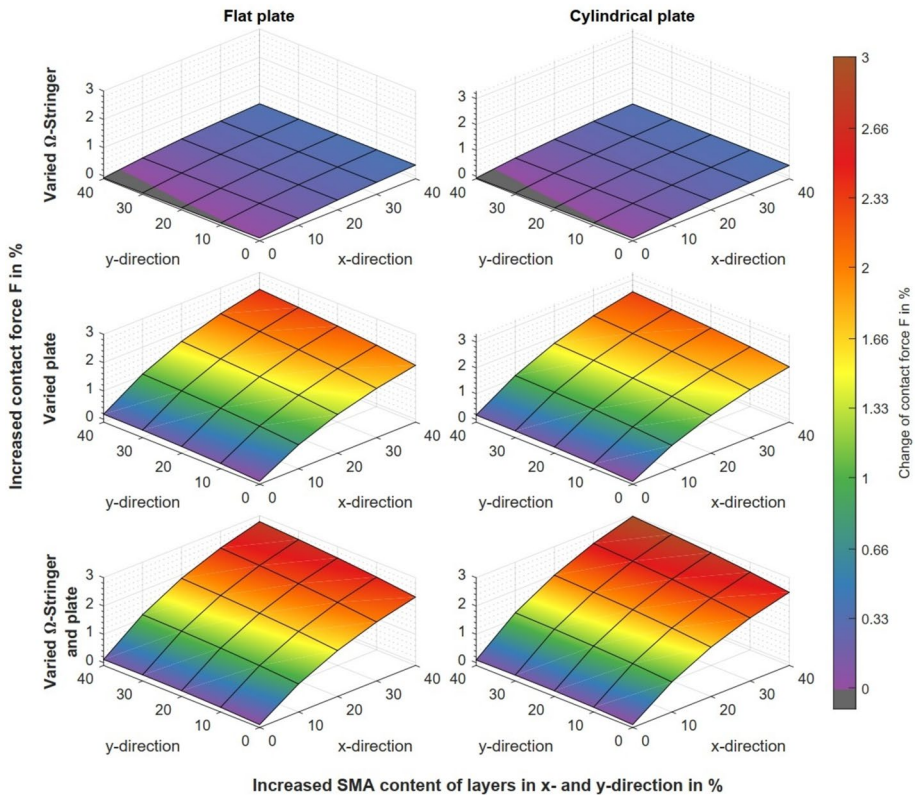


Fig. 11 Variation of SMA content in x- and y-direction (stiffened plate)

it becomes. This investigation shows that it makes most sense to integrate the SMA wires on the outside of the laminate. But this finding is dependent on the idealisation that the analytical model is not considering any change of local indentation stiffness.

5 Shape Memory Effect in Stiffened Composite Plates

The final research is concentrated on the change of impact behaviour for a combined stiffness increase by SMA wire integration and attaching an Ω -stringer on the backside.

5.1 Variation of SMA Volume Content

Figure 11 presents the variation of SMA volume content, separately for both base plate and Ω -Stringer and for both together equivalent to Figs. 7 and 8. The stringer model is reduced in size by 52.2% of the validation model (to 100 mm width) to guarantee comparability. The cylindrical base plate affects only the shape of the stringer toes as they follow the base plate's shape. All other stringer parts remain flat.

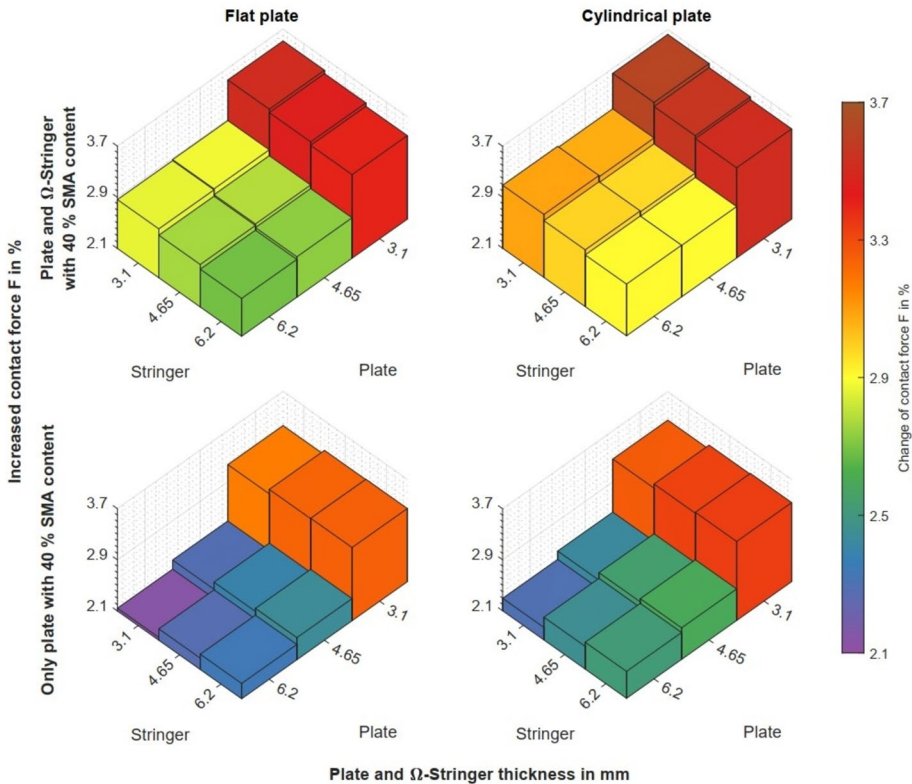


Fig. 12 Variation of plate and stringer thicknesses with constant SMA wire integration

If only wires are embedded in the base plate, similar trends as for the non-stiffened plates are existent. However, the levels of force increase are smaller because the initial stiffness of the assembly is greater than just for the single base plate. The contact force increase is much more significant for wire integration along the x-direction. This means that the secondary influence of the lay-up sequence is not that relevant anymore. The aforementioned stronger effect is due to the same orientation of wires and stringer. The effect of wire embedding in the y-direction is also increased by creating a higher SMA volume content along the stringer direction. The difference between flat and curved base plates is reduced as the initial total assembly stiffness does not differ that much anymore. If only the stringer possesses embedded wires, the stiffening effect changes. The stringer has an effect of contact force change about one order of magnitude smaller. The integration of SMA wires in y-direction shows a negative effect. Hence, a non-embedding is better because both the stiffness and specific stiffness are better. Combining the SMA integration of both base plate and stringer, the results follow the superposition principle. However, due to the different orders of magnitude, they are almost similar to the only base plate SMA integration. Correias et al. [13]

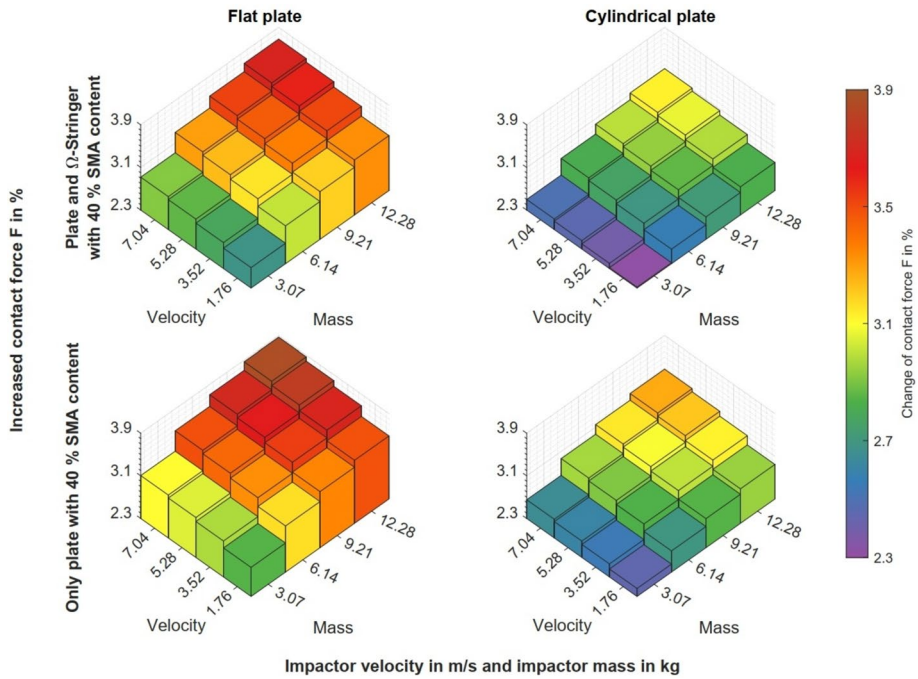


Fig. 13 Variation of impactor velocity and mass for Ω -stringer stiffened plate

identified related effects by changing the plate's and stringer's stiffnesses by variance of ply-lay-up and thickness. Also, the preferred embedding in the x-direction can be referred to in this publication.

5.2 Variation of Composite Lay-up

Similar to the non-stiffened model the lay-up thickness is varied by adding or taking a ply pack off (see Fig. 12). An integration in both stringer and base plate represents a universally applicable solution. The second option considers only an SMA wire integration in the base plate because of the results presented before. For all options, the SMA volume content is set to 40%. The change in base plate thickness has a bigger influence on the contact force increase than the change in stringer thickness. For the former mentioned variation, a regular trend is approved. The thicker the base plate or both base plate and stringer are the more the influence of SMA embedding is reduced. If the thickness of the stringer without embedded SMA wires is varied, this effect is reversed. In general, the influence of stringer thickness change is greater for thicker base plates. The difference between the base plate shapes is negligibly small.

5.3 Variation of Impact Energy

The low-velocity impact parameters are varied to research the influence of SMA wire integration for different impact situations (see Fig. 13). The impactor's velocity is up to four times

greater, and the mass is in the range of half to twice of the original value. The higher the impact energy gets the more significant the integration of SMA wires becomes. The stringer stiffened assembly with only SMA wires embedded in the plate only shows a higher dependency than the assembly with embedding in the stringer as well. Doubling the impact velocity leads to a smaller increase than doubling the impactor's mass although the impact energy is only linearly dependent on the mass but based on the squared value of velocity. For greater impact energies a force increase convergence is expected. However, the analytical model does not include any failure of the composite. Hence, modelling is limited to low-velocity as the equivalent of higher velocities in experiments will not stay undamaged.

6 Conclusions

This study is expanding the already well-known analytical modelling of low-velocity impact on composite plates with the combination effect of two stiffening approaches (SMA and Ω -stringer). A comprehensive overview of the analytical model and the FEM validation is given. Based on that, various parameter variations are presented for a single plate and in combination with the stringer. The results give an overview of which parameters are optimal to vary the stiffnesses. Especially for the assembly a dependency on SMA wire orientation, position, and volume content is existent. Because of the low-velocity, only linear relations and no damage are considered, and hence only stiffness change is analysed.

The results are valid for all types of monolithic composite plates. However, if they are combined with a stringer on the backside of the plate, the integration of SMA wires is affected. It is shown in the deflection analysis (FEM simulation) that the stringer deforms less than the impacted plate. The analytical model covers this behaviour as it is determined that the SMA integration is dominant for the base plate only. If the SMA wires are integrated into the stringer only, almost no stiffening effect can be seen. So, it does not make sense to include SMA wires in the stringer as it increases the weight drastically. In general, it is identified that the total effect of SMA wires is reduced because the essential stiffness of the complete assembly is already very high due to the attached stringer. Furthermore, it is found that the orientation of SMA wires should be only along the stringer axis. Wires in the other direction have almost no effect. Even more, for single-stringer stiffening, they reduce the contact force.

In real-case applications, the design process of stringer-stiffened parts can result in an optimised assembly of plate and stringer both with different thicknesses. However, it is identified that this optimisation of respective thicknesses possesses an influence on the stiffening due to SME. The influence of the impacted base plate thickness is dominant. Similar to the non-stiffened plate a reduced thickness increases the stiffness dependency on the SMA wires. For variations in the stringer thickness, the contact force does not change significantly. However, an interesting effect exists regarding the embedding of SMA wires in the stringer. If wires are embedded in this part, the resulting change of contact force is similar to the aforementioned variation of plate thickness. Though, if no wires are integrated into the stringer, then this effect is reversed. Furthermore, it was shown that the specific stiffness changes with the SMA content and the CFRP mass of extra plies so one has to make a compromise between both stiffening options.

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Declarations

Competing Interest The authors declare no competing interests.

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