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Enhancing Aircraft Design: Assessing Adverse Yaw and Vertical Tail Plane Reduction using Prandtl Wing Principles in Saiplanes

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High aspect ratio wings have garnered increasing attention due to their potential to improve aerodynamic efficiency. At the same time, the flight of birds without vertical tail structures has inspired aerodynamicists to explore alternatives to traditional lift distributions. Challenging the elliptical lift distribution could yield minimum-drag configurations while also addressing structural constraints, such as the wing root bending moment. This research examines the effects of Prandtl's bell-shaped lift distribution on the aerodynamic performance and stability of sailplanes. By modifying the wing configuration of the Slingsby T51 Dart 15, the bell-shaped lift distribution is compared to traditional elliptical and baseline designs. The goal is to assess potential improvements in handling, particularly through proverse yaw behavior, and to evaluate whether the reduction of the vertical tailplane size compromises critical flight performance metrics such as crosswind landing and Dutch roll stability. Findings indicate that this alternative distribution alters aerodynamic behavior, and although reducing the vertical tailplane size does not compromise crosswind landing performance but it raises challenges in Dutch roll stability.

I. Nomenclature

$C_{n\delta a}$	=	yawing moment coefficient due to aileron deflection
$C_{l\delta a}$	=	roll moment coefficient due to aileron deflection
$C_{n_{\delta_r}}$	=	rudder yaw control power
$C_{L_{\alpha_F}}$	=	fin lift curve slope
$\overline{V}_F$	=	fin volume coefficient
η_F	=	fin dynamic pressure ratio
τ_R	=	fin angle of attack effectiveness parameter
b_R	=	rudder span
b_F	=	fin span
ω_d	=	damped natural frequency
ζ_d	=	damping ratio
C_n	=	yaw moment coefficient
δ_r	=	rudder deflection
V_0	=	Flight velocity
S_w	=	wing area
b	=	wingspan
$\dot{N}_v$	=	yawing moment due to sideslip
$\dot{N}_r$	=	yawing moment due to yaw rate
$\dot{Y}_v$	=	sideforce due to sideslip
m	=	aircraft mass
I_{zz}	=	body moment of inertia along Z-axis

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II. Introduction

Advancements in aerodynamic efficiency often rely on optimizing lift distributions across the wingspan, a principle well-established in classical aerodynamics. While elliptical lift distributions are traditionally valued for minimizing induced drag, a deviation towards the bell-shaped distribution offers structural advantages. Recent studies have highlighted that Prandtl's bell-shaped spanload, envisioned for structural benefits as well as induced drag reduction, also improves the aircraft flight dynamics. This change in distribution is such that the lift acts closer to the wing root to potentially reduce adverse yaw and allow for a reduction in vertical tailplane (VTP) size. For sailplanes, which prioritize a maximum glide ratio, incorporating a non-elliptic lift distribution presents an opportunity to improve lateral-directional stability while potentially reducing sizing requirements for the VTP.

Despite this potential, existing research leaves critical gaps in understanding the impact of non-elliptic distributions on flight dynamics, particularly under conditions that stress lateral-directional stability, such as crosswinds and Dutch roll modes. The study presented here addresses this gap by assessing the aerodynamic and stability implications derived from using a bell-shaped lift distribution on a Slingsby T51 Dart 15 sailplane as a case study. Three lift distributions are compared: the original baseline, an equivalent-elliptic, and Prandtl's bell-shaped distribution, as elaborated in the methodology section. The research specifically examines the feasibility of VTP reduction, by implementing the bell-shaped lift distribution and focusing on its impact on proverse yaw characteristics, crosswind stability, and Dutch roll damping.

The following sections provide a brief overview of Prandtl's lift distribution approach, describe the methodology adopted in this study to evaluate the effects of the bell-shaped span-load on sailplane stability and dynamics, and discuss the findings and results.

III. Technical Background

The classical lifting line theory proposed by Prandtl in the early 20th century [1] established an elliptical lift distribution as the ideal shape to minimize induced drag across a wingspan. This approach, based on a uniform distribution of lift, remains a foundational principle in aerodynamics, especially for high-aspect-ratio wings, where induced drag is a critical consideration for efficiency. However, subsequent research revealed limitations of the elliptical distribution, particularly with regard to stability and control characteristics such as adverse yaw. Recognizing these limitations, Prandtl proposed an alternative bell-shaped lift distribution in 1933, which more closely resembles the spanwise loading observed in bird flight and mitigates adverse yaw by redistributing lift closer to the wing root. [2] explains this alternate lift distribution that could result in the same lift and integrated bending moment at the wing root, while reducing the induced drag compared to the elliptical lift distribution. A drag reduction of 11% was observed for a 22% increase in the wing span. It also suggests that at the wing tips, this new distribution results in an upwash and an 'induced negative drag'. Jones [3] studied this theory independently of Prandtl by fixing the lift and wing root bending moment, and reached the same conclusion that a bell shaped lift curve provides a reduced induced drag, and a similar downwash/upwash profile.

Recent studies have explored the potential of non-elliptic, bell-shaped lift distributions to improve lateral-directional stability and reduce control surface requirements. Bowers et al. [4] at NASA conducted extensive research on Prandtl's principles, developing the Prandtl-D wing configuration. Their experimental work demonstrated proverse yaw characteristics in a flying wing configured with bell span-load, suggesting that these designs may not require traditional vertical tailplanes for stability under certain flight conditions, mimicking the bird flight structure. This design of an aircraft without a rudder is patented in [5]

This was further corroborated by Bragado et al. [6] that analyzed the feasibility of applying non-elliptic lift distributions to sailplane Silent 2 Targa Pure, highlighting significant improvements in efficiency. Their work, based on high-aspect-ratio wings, indicated that VTP size reductions might be feasible with non-elliptic designs but underscored the increased sensitivity to crosswind and Dutch roll stability that these configurations introduce. In addition, [7] presents a method for re-designing ailerons to maintain roll authority and achieve proverse yaw characteristics, using the same configurations as [6].

Further, [8] re-examined the effect of transverse span loads on leading-edge suction and induced drag, and concluded that while Prandtl's bell span-load might be preferable under span limitations or when considering structural benefits of the root-bending-moment, the classical elliptical distribution remains ideal for minimizing induced drag in the absence of such constraints. Their CFD and Vortex-Lattice code studies on AR=12 wings showed that a bell span-load had a poorer span efficiency factor compared to an elliptical load for similar lift coefficients. [9] also reinforces that for a given wingspan, bell span-loads result in higher induced drag for the same lift generation, as compared to the elliptical

distribution. Experimental research conducted by [10] investigated a straight tapered flying wing designed with a bell span-load for a Mars glider, highlighting its potential for proverse yaw, eliminating the need for vertical surfaces, and its increased efficiency compared to the elliptical distribution under certain constrained conditions like structural weight.

Beyond flying wing applications, simulations conducted on the Cirrus SF50, a GA aircraft demonstrated that bell span-loads yield a 1% improvement in the lift-to-drag (L/D) ratio with a 4% reduction in vertical tail size, albeit with some destabilizing effects on Dutch roll behavior. Their findings emphasize a trade-off: while adverse yaw can be reduced, lateral-directional stability requirements under airworthiness criteria must be rigorously tested. [11]

Building on this foundation, the present study applies Prandtl's bell-shaped lift distribution to a sailplane configuration, the Slingsby T51 Dart 15, with a focus on understanding how the lift redistribution impacts adverse yaw, crosswind stability, and Dutch roll dynamics. While prior research has established the theoretical basis for reduced VTP sizing, limited studies have examined these configurations in full-scale aircraft or within the stringent lateral-directional stability criteria required by modern airworthiness standards. The unique properties of this bell span-load offer the possibility of designing aircraft with reduced or modified vertical tailplanes (VTP), as the distribution can induce a natural proverse yaw characteristic. While promising, this approach requires rigorous assessment to confirm its practicality in different flight conditions, especially under lateral-directional dynamic modes. By implementing Prandtl's bell-shaped lift distribution on a baseline sailplane, this study investigates its impact on adverse yaw reduction and the potential for VTP resizing.

IV. Methodology

A. Framework and Analysis Approach

To evaluate the effects of the bell-shaped lift distribution on vertical tailplane sizing and stability, this study builds upon a pre-existing framework that integrates MATLAB and Athena Vortex Lattice (AVL) software - an aerodynamics solver based on vortex lattice methods. The Slingsby T51 Dart 15, a single-seater sailplane with a high-aspect-ratio wing (HARW), is shown in Figure 1 and was selected as the baseline aircraft due to its minimal drag, absence of propulsion (eliminating airframe-engine integration effects), and the availability of comprehensive data as presented in Table 1 , making it ideal for testing modified lift distributions.

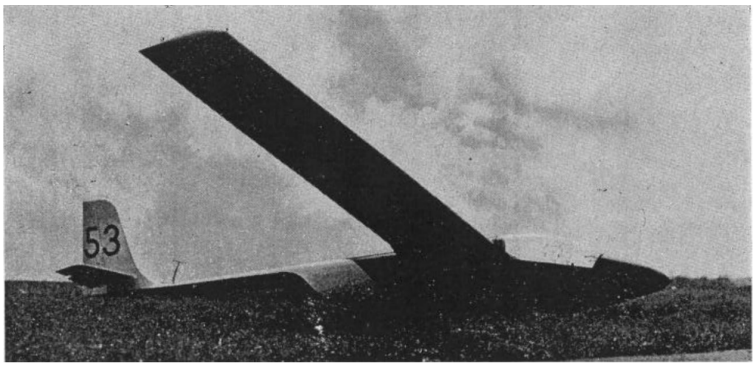


Fig. 1 Sailplane - Slingsby T51 Dart 15 [12]

The pre-existing framework, which is rooted in Prandtl's lifting line theory, was adapted in MATLAB to accommodate three specific lift distributions:

1) Original baseline distribution: This span-load represents the lift configuration derived from the original wing geometry of the baseline Slingsby T51 Dart 15 wing. It serves as a reference to benchmark the aerodynamic and stability characteristics of the original sailplane design without modifications.
2) Equivalent elliptic distribution: This span-load follows the approach of Prandtl's 1933 work, where the planform is redefined to match the total structural and aerodynamic loads of the baseline wing. The span of the modified wing is recalculated as 1.046 times the original baseline span in order to maintain this aero-structural equivalence of constant lift and integrated bending moment with the baseline sailplane, and to provide the minimum induced drag configuration. Once the required elliptic lift distribution is known, the local twist distribution spanwise variation that would yield such a span-load, is achieved by application of the inverse lifting line theory. The

Table 1 Reference data - Slingsby T51 Dart 15 [13],[12], [14]

Wing - naca64$_3$618		Fuselage	
Wingspan	15.0 m	Length	7.47 m
Wing area	12.63 m^2	Diameter	0.78 m
Aspect Ratio	17.8		
Stall speed	35 kts	**VTP - naca63015**	
c/4 sweep	-0.8°	VS Span	1.25 m
MAC	0.835 m	Sweep	16°
Dihedral	2°	VS LE	4.617 m
Incidence	9°	VS Croot	0.878 m
Root chord	1.04 m	VS Ctip	0.405 m
Tip chord	0.63 m	VS Area	0.96 m^2
Mass Data		**HTP - naca65006**	
Mass	318 Kg	HS Span	2.61 m
Ixx	1368 Kg-m^2	Sweep	0°
Iyy	432 Kg-m^2	HS LE	4.693 m
Izz	1778 Kg-m^2	HS Croot	0.574 m
Ixz	-4.1 Kg-m^2	HS Ctip	0.304 m

camber is fixed as the airfoils are not changed from the original. This configuration, known for optimal induced drag efficiency, offers an idealized reference point against which to compare the behavior of Prandtl's bell span-load.

3) Prandtl's bell span-load distribution: The bell-shaped lift distribution, also based on Prandtl's 1933 work, redistributes the lift to act closer to the wing root to alleviate bending loads. This design shows inherent proverse yaw effect that can reduce reliance on the VTP for yaw stability. However, the bell span-load configuration results in the span efficiency factor to fall below 1, as the span-load is non-elliptic. Therefore, it is necessary to extend the wing span to reduce the induced drag. In the family of non-elliptic lift distributions, the ideal bell span-load is given by a 22% span extension and is characterized by a reduction of about 11% in induced drag as compared to the equivalent elliptic distribution.

For all modified configurations, the chord distribution is changed to account for the span variation required to satisfy the design constraints whilst the wing area is fixed.

The workflow adopted in this framework is detailed in Figure 2. Numerical application of the lifting line theory provides the foundational equations within the MATLAB code, enabling the analysis of spanwise lift distribution and induced drag under various configurations. The theoretical approach underlying the framework coded in MATLAB is extended to include P33 assumptions in order to modify the circulation distribution according to the aerodynamic and structural constraints, therefore shifting the lift inboard as the span is extended. For the bell-shaped distribution, the spanwise circulation is modified to apply Prandtl's 1933 principles and shift the lift vector inboard towards the wing root. This configuration, designed for aero-structural benefits, also reduces adverse yaw tendencies, allowing for possible VTP size reduction while meeting stability criteria.

The MATLAB code serves as a gateway to AVL. The input files containing the modified aircraft geometry derived as described above and airfoil characteristics for each lift distribution, generated in MATLAB, are used by AVL to model the complete aircraft and estimate the aerodynamic behavior of each configuration. Through this iterative parametric process, AVL computes key stability derivatives, such as yawing and rolling moment derivatives, which are essential for evaluating proverse yaw characteristics, crosswind stability, and Dutch roll damping behaviour. AVL results are run through MATLAB to extract and analyze the stability derivatives to determine the impact of different VTP sizes and lift distributions on lateral-directional stability.

This MATLAB-AVL framework thus enables efficient simulation and comparison of various configurations, providing a systematic approach to explore non-elliptic lift distributions and their influence on VTP sizing and sailplane stability.

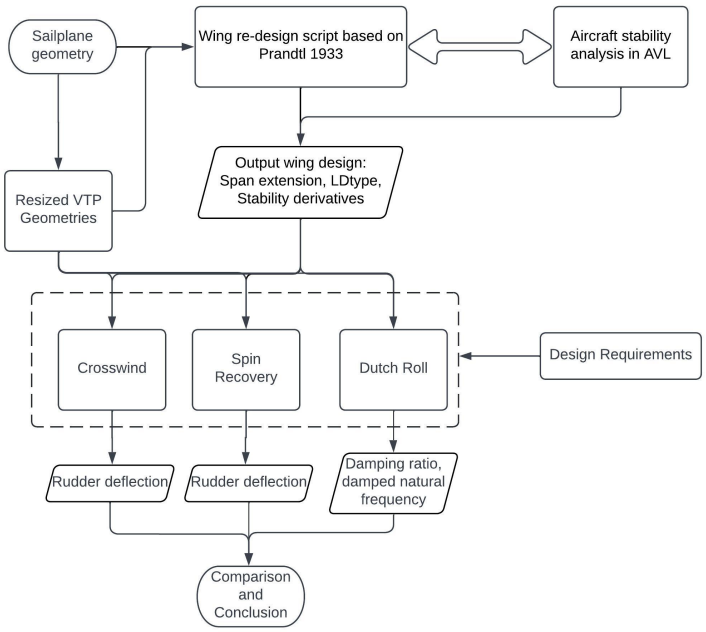


Fig. 2 Methodology flowchart

The reader should note that for the purpose of this study the reference axes are defined as originating from the aircraft's centre of gravity, such that the logitudinal X-axis points positive towards the nose of the aircraft, the lateral Y-axis is positively directed from the centerline of the aircraft outward through the right wing, and then as per the standard right hand convention, the positive Z-axis points vertically downwards.

B. Key Criteria for VTP Reduction

The following criteria were adopted to assess the feasibility of VTP resizing:

1) Proverse yaw characteristics: One of the indicators of a viable VTP reduction is the existence of proverse yaw. For each configuration, the yawing moment coefficient due to aileron deflection, $C_{n_{\delta_a}}$, was analyzed. A negative $C_{n_{\delta_a}}$, aligning with the rolling moment coefficient, indicates proverse yaw and supports potential VTP reduction.
2) Crosswind handling: Compliance with crosswind landing criteria is critical for safety in sailplane design. For this study, crosswind stability was evaluated under a perpendicular wind condition of 15 km/h (per CS-22 requirements) at 1.3 times the stall speed [15]. Rudder deflection requirements were examined for each lift distribution across varied VTP sizes.
3) Dutch roll stability: The Dutch roll mode—an oscillatory motion in response to sideslip—is a critical criterion for VTP resizing because the vertical tailplane significantly contributes to damping this motion. To assess this, the damped natural frequency, ω_d, and damping ratio, ζ_d, were calculated for each configuration. A higher damping frequency and damping ratio indicate faster, more controlled recovery from disturbances, which is essential for maintaining acceptable handling characteristics, especially in turbulent conditions. Evaluating Dutch roll stability thus ensures that VTP reduction does not compromise lateral-directional stability.

C. Simulation Setup and Data Collection

For each configuration, simulations were conducted using the MATLAB-AVL interface to extract control forces, moments, and stability derivatives. The parametric study examined VTP resizing in incremental steps while monitoring the impact on proverse yaw, crosswind stability, and Dutch roll damping. These data served as the basis for evaluating the minimum viable VTP size across the tested lift distributions.

The vertical tail size reduction factor, f, , is the fractional representation of the new fin geometries against the original baseline fin planform, as represented in Figure 3, and can be calculated as the ratio of new VTP area to the

original VTP area:

$$f = \frac{S_t'}{S_t} \tag{1}$$

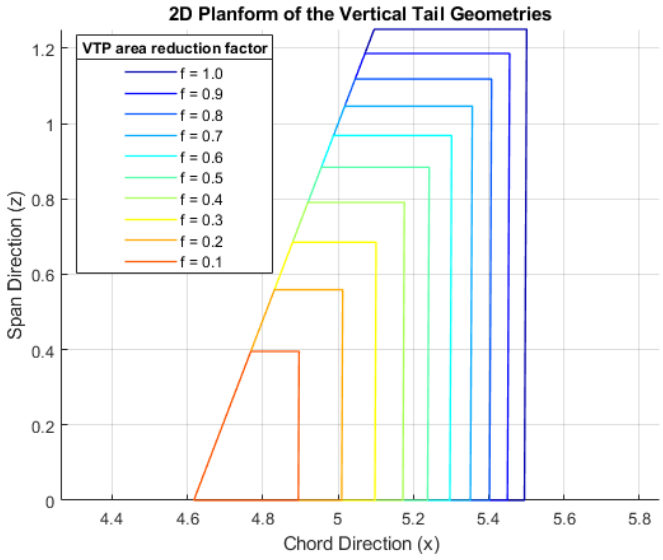


Fig. 3 Visualisation of varying VTP sizes

The deflection required by the rudder can help analyse its control authority. One of the ways to measure rudder effectiveness is the yaw control power [16]:

$$C_{n_{\delta_r}} = \frac{\partial C_n}{\partial \delta_r} \tag{2}$$

$$C_{n_{\delta_r}} = C_{L_{\alpha_F}} \overline{V}_F \eta_F \tau_R \frac{b_R}{b_F} \tag{3}$$

If the control authority is greater, less rudder deflection is needed to counteract the same moment. Therefore, for a given fin-rudder size, the effectiveness of rudder authority, as span and lift distribution vary, is assessed by the equivalent control surface deflection required. This provides a useful metric for comparing how much the fin can be scaled down while still meeting the requirements to keep the aircraft controllable under critical conditions.

The characteristics of Dutch roll mode can be assessed through calculations of damped natural frequency ω_d and damping factor ζ_d.

$$\omega_d^2 = \frac{\dot{N}_v V_0}{I_{zz}} = \frac{1}{2}\rho V_o^2 S_w b \frac{N_v}{I_{zz}} \tag{4}$$

$$\zeta_d \omega_d \cong -\frac{\dot{N}_r}{I_{zz}} - \frac{Y_v}{m} = -\frac{1}{2}\rho V_o^2 S_w \left[b^2 \frac{N_r}{I_{zz}} + \frac{Y_v}{m}\right] \tag{5}$$

D. AVL Validation

To validate the results from the MATLAB-AVL interface for this study, the baseline configuration of the Slingsby T51 Dart 15 was first assessed. The initial validation involved running simulations for eleven sets of lift coefficients under longitudinal trim conditions. These results were compared with those reported in [1] for the same sailplane. The comparison confirms the fidelity of the interface and is illustrated in the following figure 4:

As shown, the maximum deviation observed was 1 deg for the angle of attack and 1.5 deg for the elevator deflection angle. The close agreement between the AVL-generated data and the reference results demonstrates the reliability of the MATLAB-AVL simulation framework. This validation step was essential before extending the analysis to modified lift distributions and varying vertical tailplane sizes. Ensuring consistency with established performance data provided a strong foundation for the subsequent evaluations of lateral-directional stability and control.

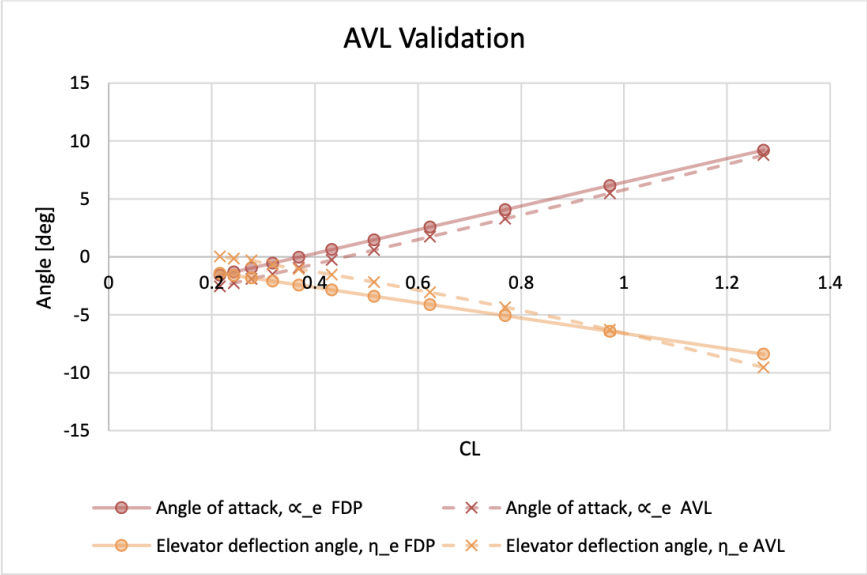


Fig. 4 AVL validation methodology

V. Results and Discussion

This section presents the results of applying Prandtl's alternative approach to wing lift distribution to the Slingsby T51 Dart 15 and discusses its impact on proverse yaw characteristics, crosswind handling, and Dutch roll stability. The three lift distributions are labelled as LDtype 1 (original baseline), LDtype 2 (baseline elliptic), and LDtype 3 (bell span-load, NELD). The analysis framework explores five different span lengths, ranging from the original baseline to Prandtl's theoretical 22.4% span extension for a bell-shaped span load. This extension is adjusted by the span extension factor rs (ratio of modified span to the original baseline span) to maintain structural equivalence. For each lift distribution type, the ideal cases are defined based on the ideal span for the corresponding circulation - ideal case for LDtype 1 considers the ideal span as is (rs=1.000), LDtype 2 considers the second span extension (rs=1.046 representing the ideal span for the equivalent elliptic lift distribution), and LDtype 3 considers the the last span extension (rs=1.282 representing Prandtl's proposal).

Additionally, fin sizes are denoted either by the vertical tailplane reduction factor, f_c, or by the VTP index, where index 1 represents the baseline size and higher indices correspond to smaller VTP sizes. By implementing and comparing these three lift distributions, the study assesses the feasibility of VTP reduction without sacrificing stability.

A. Proverse Yaw Characteristics

The bell span-load produced notable proverse yaw characteristics, in contrast to the baseline and elliptic configurations. The effectiveness of the bell span-load in producing proverse yaw was evident across all wingspans tested. As the lift distribution over the wing is modified, the aileron control derivative, $C_{n\delta a}$, becomes negative, aligning with the sign of roll moment coefficient due to aileron deflection, $C_{l\delta a}$. This indicates that the aircraft experiences yaw in the same direction as roll, a hallmark of proverse yaw. This characteristic is significant for VTP reduction since it reduces the adverse yaw that typically necessitates larger VTP surfaces for correction. Figures 5 and 6, demonstrate this behaviour effectively across the different wingspans and lift distributions for original VTP planform, highlighting the consistent proverse yaw achieved with the bell span-load and the increased sensitivity of elliptic configurations to adverse yaw.

Figure 5 shows this tendency for the bell-shaped lift distribution with the most negative yawing coefficient $C_{n\delta a}$. As $C_{l\delta a}$ is already negative, this represents the proverse yaw behaviour.

Similar pattern is observed along all the spans across the three types of lift distributions studied and is shown in Figure 6.

These results support the potential for VTP downsizing with a bell span-load, as the inherent proverse yaw reduces reliance on the vertical tail for directional control. With adverse yaw minimized, the VTP size could be reduced incrementally while maintaining effective lateral-directional control, offering a structural advantage for high-aspect-ratio designs where weight reduction is critical.

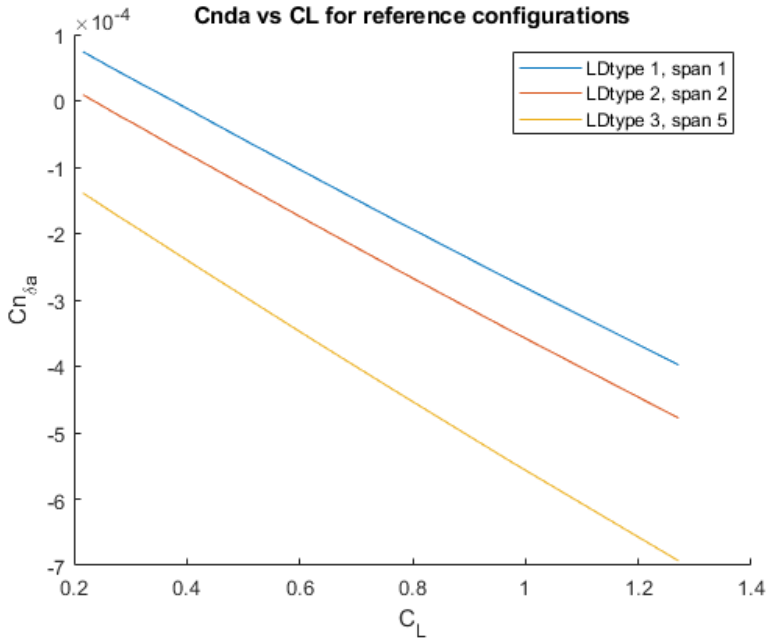


Fig. 5 Yawing moment due to aileron deflection for ideal cases

B. Crosswind handling

Crosswind performance was evaluated in accordance with the airworthiness requirements, which specify the ability to land in a perpendicular crosswind of 15 km/h at 1.3 times stall speed. This analysis measured the rudder deflection required to maintain runway alignment and evaluated the impact of VTP downsizing. The ailerons are assumed neutral in this analysis to isolate the effect of rudder control and the changes in aerodynamic forces and moments, that will have a direct impact on the inherent stability of the configurations.

The contour plots shown in Figures 7, 8 and 9 illustrate that as the wingspan is increased, the required rudder deflection decreases for a given lift distribution and VTP size. Although the increase in wingspan does not directly affect the rudder deflection in a perpendicular crosswind scenario, it enhances other aerodynamic effects. These include a greater dihedral effect, improved roll stability, and reduced sideslip, all of which contribute to a decreased need for rudder deflection. The side force and yaw moment due to the crosswind are influenced by the vertical tail area and fuselage cross section. The former is discussed below, and the latter has not been modeled and remains unchanged in this analysis. Without the aileron deflection, an increased wingspan results in higher roll inertia and therefore, a higher roll stability. Further, the sailplane's 2° dihedral helps it naturally roll into the wind, creating a yawing moment that aligns the nose with the free stream. Overall, the inherent roll stability and dihedral effect can reduce the need for rudder deflection, and this is the trend observed in the plots for a given lift distribution and VTP size – focusing solely on the impact of an increasing span.

It can also be observed that as the vertical tailplane size is reduced, the required rudder deflection also decreases due to the smaller tail area opposing the crosswind force. However, the rudder deflection per unit area increases as the VTP size decreases. This trend, presented in Figures 10, 11 and 12, occurs because the reduced tail area diminishes the rudder's yaw authority, necessitating a higher rudder deflection per unit area to achieve the same control effectiveness. Although reducing VTP size increases rudder deflection per unit area, the absolute deflection required remains within acceptable crosswind landing criteria, so VTP resizing does not pose a limitation under this criteria.

Changing lift distributions shifts the lift vector inward, closer to the fuselage, which might typically reduce the yawing moment and rudder deflection needed for crosswind control. However, by maintaining the moment of inertia, this reduction is offset, resulting in minimal changes in rudder deflection across different distributions. Simulations revealed that Prandtl's 1933 bell span-load distribution, with its specific span extension, demands the least rudder deflection compared to the baseline elliptic distribution to maintain runway alignment under crosswind conditions, making it advantageous for directional control, as shown in Figure 13.

The longer wingspans in the bell-shaped configuration amplified dihedral effects, naturally counteracting lateral

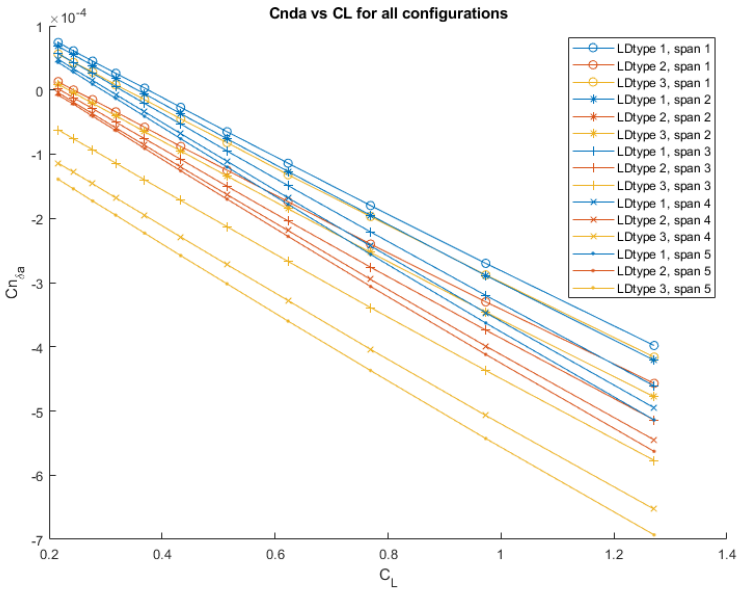


Fig. 6 Yawing moment due to aileron deflection for all configurations

drift. This effect reduced the need for rudder intervention, especially for maintaining heading during crosswind landings. The bell span-load, therefore, offers dual benefits in crosswind performance: decreased rudder deflection requirements and effective stabilization from the lift distribution itself, making it an optimal configuration for sailplanes requiring minimal control input under crosswind conditions. Further, as the VTP size decreases incrementally, the bell span-load continues to meet the crosswind stability requirements.

C. Dutch Roll Stability

The Dutch roll mode was analyzed by calculating the damped natural frequency, ω_d, and damping ratio, ζ_d across the lift distributions and VTP sizes, providing insight into how each configuration affects lateral-directional oscillations.

1) Damped natural frequency

The damping frequency of the Dutch roll mode depends on both roll and yaw damping, with roll damping mainly influenced by lift distribution and wingspan, while yaw damping is affected by VTP size. Extending the wingspan increases roll inertia, which lowers the natural frequency of roll oscillations, thus reducing the overall damping frequency, as shown in Figures 14., 15, and 16. Meanwhile, reducing VTP size decreases yaw stiffness, which further reduces the damping frequency as the tailplane's aerodynamic contribution diminishes. Comparison of ideal cases suggests that, as illustrated in Figure 17, extending wingspan, reducing VTP size, and using the bell span-load collectively reduce Dutch roll frequency, indicating a slower, less responsive recovery from yaw disturbances. This is consistent with previous studies indicating that bell-shaped distributions inherently decrease yaw damping due to their concentrated lift near the wing root. Although a higher damping frequency supports faster recovery from disturbances, the bell span-load configuration with a reduced tail size and extended span yields smoother, slower Dutch roll oscillations, which can improve flight comfort in mild conditions but may be less effective in turbulence.

2) Damping ratio

A high damping ratio is generally preferred in aircraft to effectively dampen oscillations in flight dynamics, ensuring smoother handling without exceeding a unit value, as overdamping can make the system sluggish and reduce maneuverability. Figures 18 and 19 illustrate that for the baseline and elliptic lift distributions, increasing the wingspan raises the damping ratio. This is because a larger wingspan increases roll inertia, which slows the roll rate and thereby enhances lateral stability. This increased roll inertia, combined with the stabilizing influence of aerodynamic forces, supports a more controlled yaw-roll coupling, resulting in smoother oscillations with quicker damping in Dutch roll.

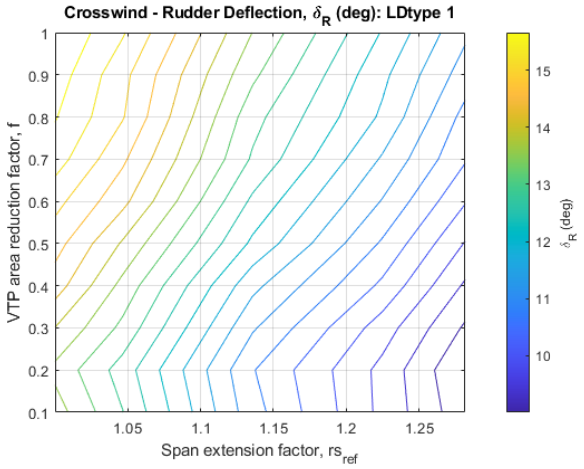


Fig. 7 Crosswind rudder deflection for LDtype 1

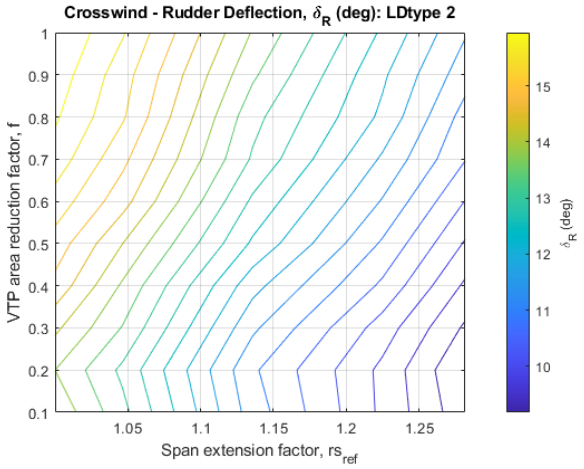


Fig. 8 Crosswind rudder deflection for LDtype 2

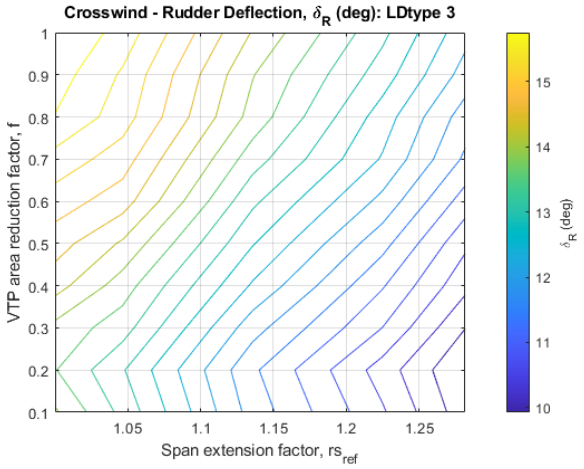


Fig. 9 Crosswind rudder deflection for LDtype 3

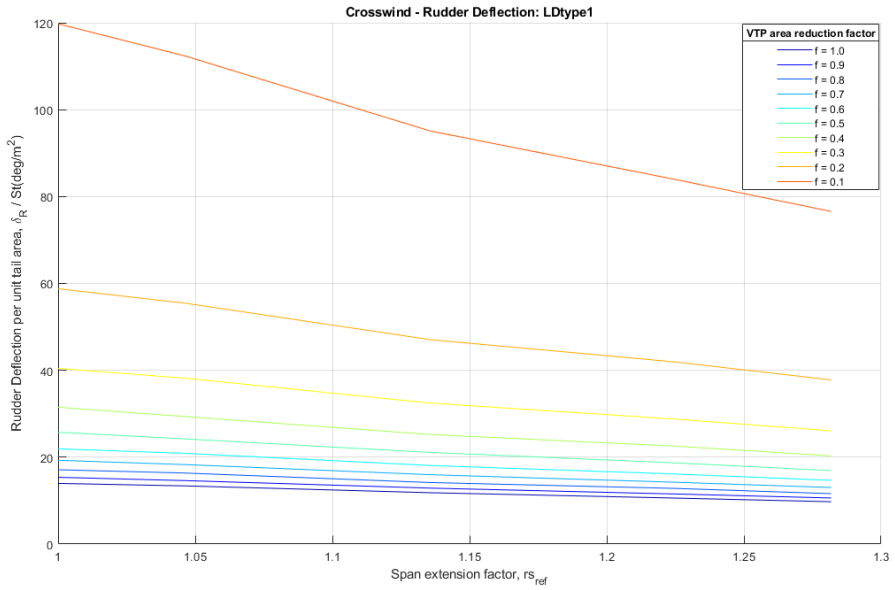


Fig. 10 Crosswind: Rudder deflection per unit area for LDtype 1

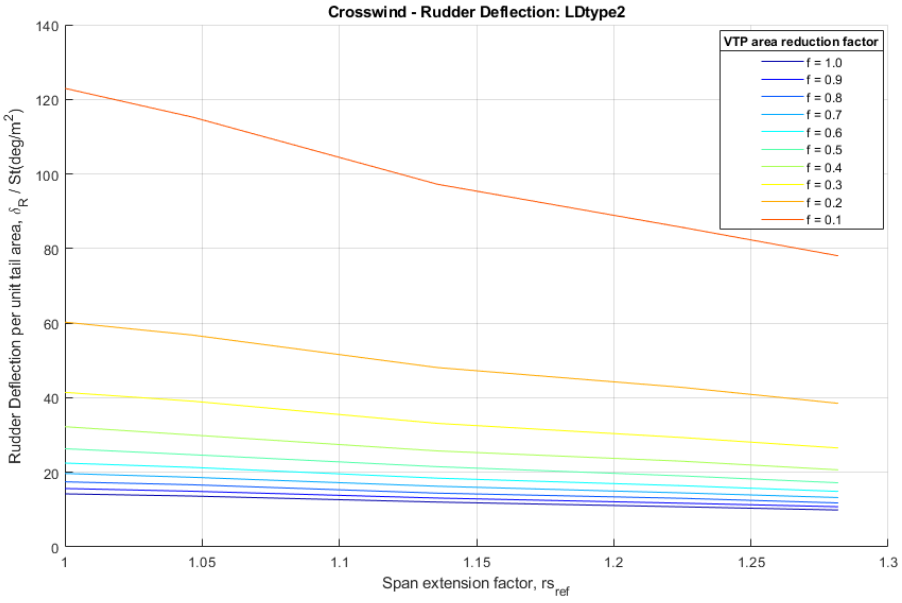


Fig. 11 Crosswind: Rudder deflection per unit area for LDtype 2

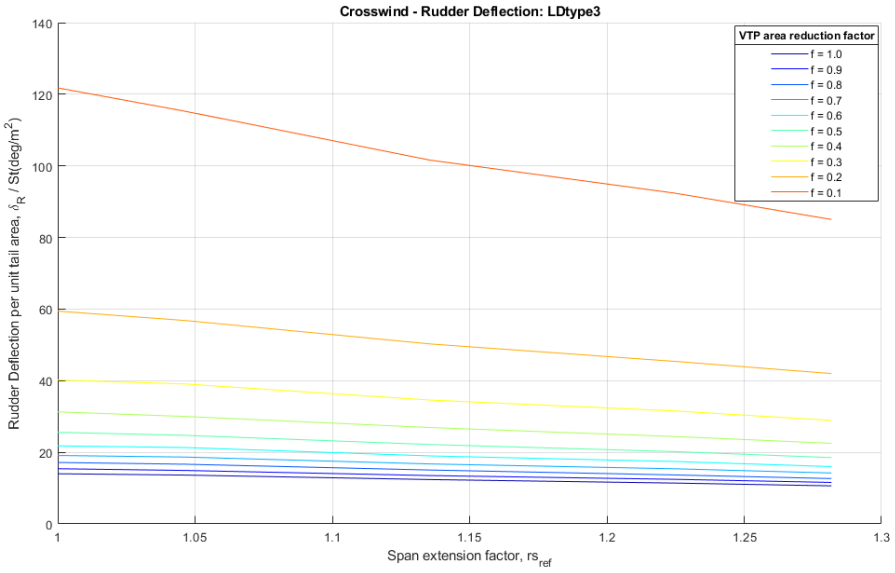


Fig. 12 Crosswind: Rudder deflection per unit area for LDtype 3

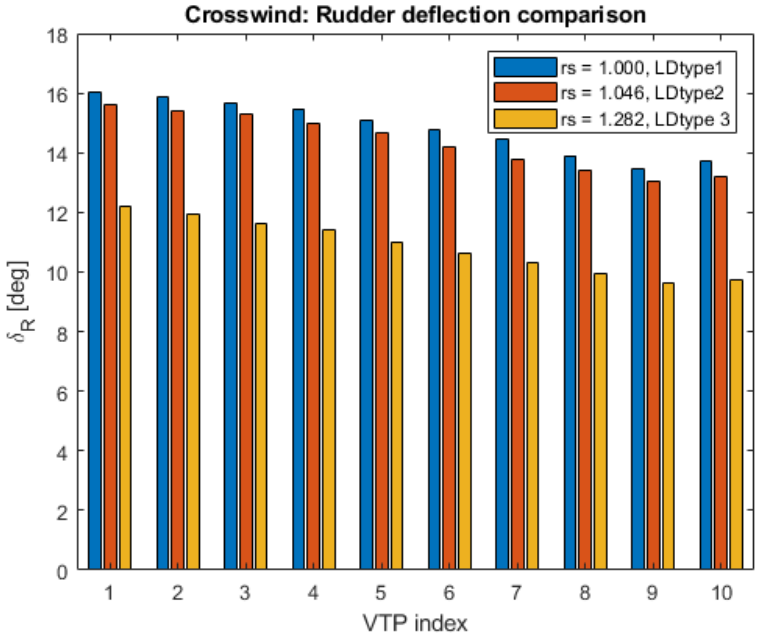


Fig. 13 Crosswind: Comparison of ideal cases

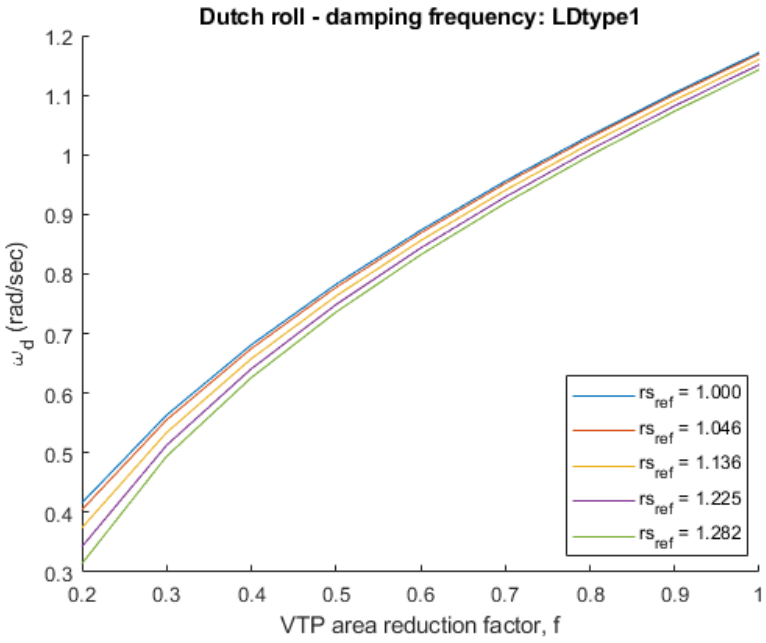


Fig. 14 Dutch roll: damping frequency for LDtype 1

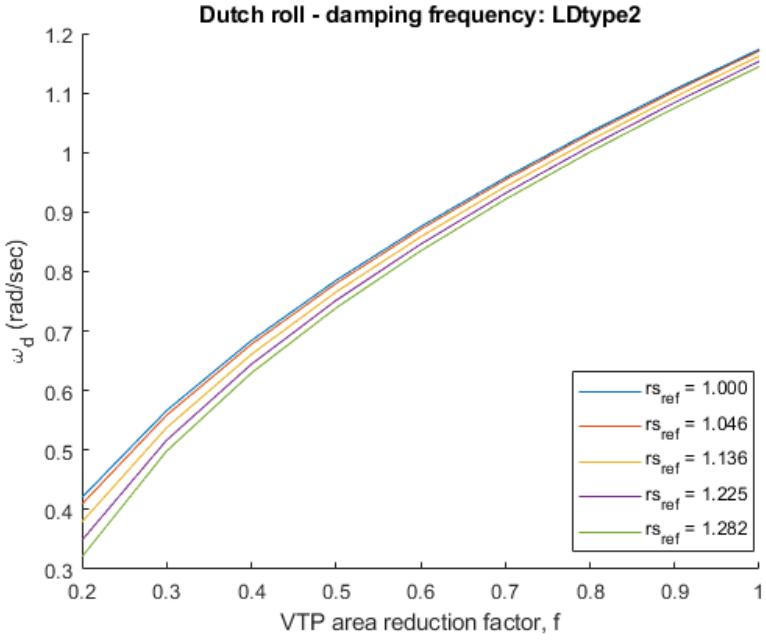


Fig. 15 Dutch roll: damping frequency for LDtype 2

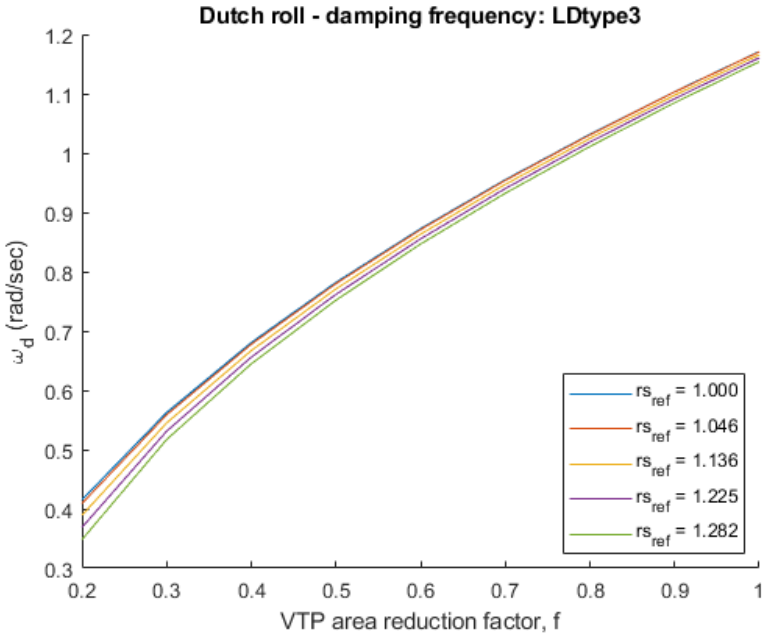


Fig. 16 Dutch roll: damping frequency for LDtype 3

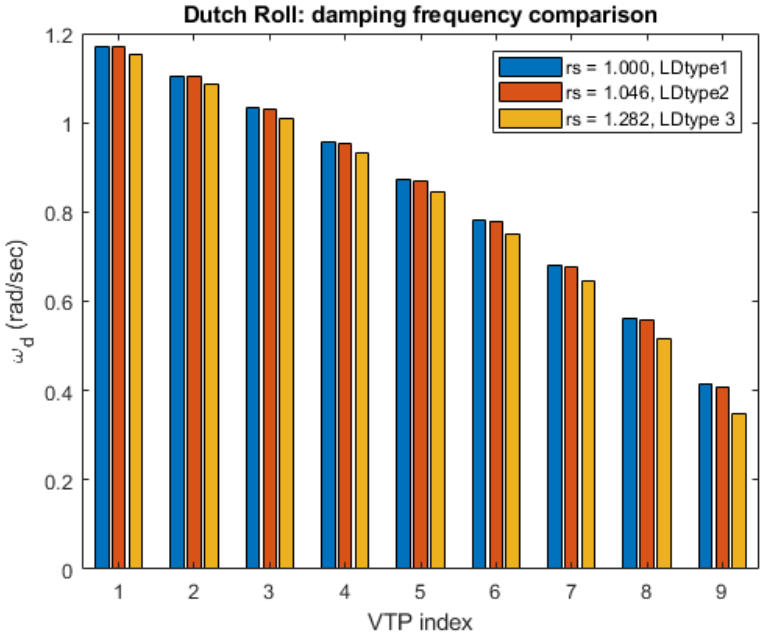


Fig. 17 Dutch roll: comparison of ideal cases - damping frequency

However, for the bell span-load distribution (LDtype 3), increasing the wingspan unexpectedly leads to a decreased damping ratio, as shown in Figure 20. Unlike the more uniform lift distributions, the bell span-load concentrates lift near the wing root, shortening the effective moment arm for roll forces. This reduction in the moment arm heightens roll sensitivity, making the aircraft more responsive to minor roll disturbances. Additionally, because lift is more concentrated near the root, the roll rate increases even with small disturbances, which reduces the damping of roll oscillations. Thus, while the increased wingspan generally stabilizes the roll for elliptic distributions, it has the opposite effect for the bell span-load, which becomes more sensitive and less well-damped.

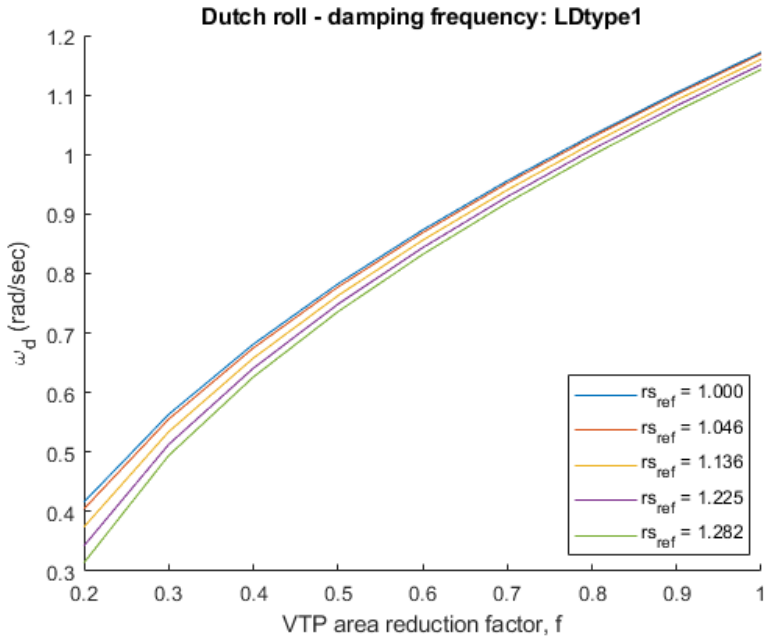


Fig. 18 Dutch roll: damping ratio for LDtype 1

The size of the vertical tailplane (VTP) also plays a significant role in Dutch roll damping. As the VTP size decreases, yaw damping is reduced because the yawing moment coefficient, which depends on the VTP area, diminishes. This reduced yaw damping means that sideslip disturbances in yaw are less effectively countered, resulting in prolonged yaw-roll oscillations and a lowered overall damping ratio for the Dutch roll mode, as shown in Figure 21.

In the final comparison through Figure 21, the ideal cases—original baseline, extended elliptic span, and the bell span-load with extended span—show that while baseline and elliptic configurations benefit from enhanced damping ratios with greater wingspans, the bell span-load combined with extended wingspan and reduced VTP area demonstrates lower Dutch roll damping. This is because the non-uniform lift distribution amplifies discrepancies in force distribution across the span, resulting in higher roll sensitivity and less effective yaw stabilization. Consequently, the bell span-load configuration struggles to achieve desirable Dutch roll damping, posing challenges for stability and comfort in dynamic flight conditions.

The bell span-load configuration with extended wingspan showed mixed results for stability. While the increased span contributed to roll stability through dihedral effects, it simultaneously reduced yaw damping, leading to prolonged Dutch roll oscillations that could undermine handling predictability in turbulent conditions. This configuration could limit the practical extent of VTP reduction, as handling characteristics become less predictable under turbulent conditions. Although larger VTPs generally improved Dutch roll damping across configurations, the bell-shaped distribution exhibited a decrease in damping as wingspan increased, indicating challenges for maintaining stable handling with VTP reduction alone. These findings suggest that practical applications of the bell span-load may require a balanced approach, incorporating moderate VTP downsizing along with careful tuning of other stability parameters, such as spanwise twist, to ensure acceptable Dutch roll stability and control.

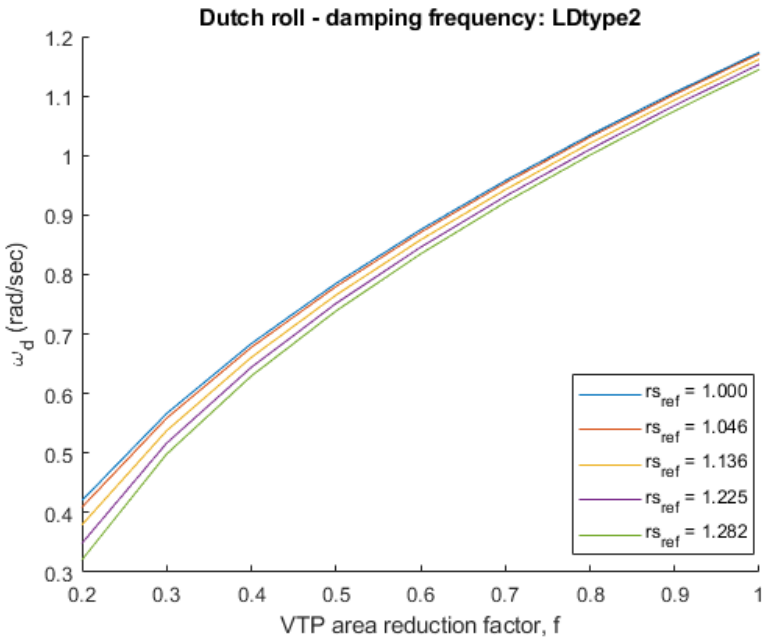


Fig. 19 Dutch roll: damping ratio for LDtype 2

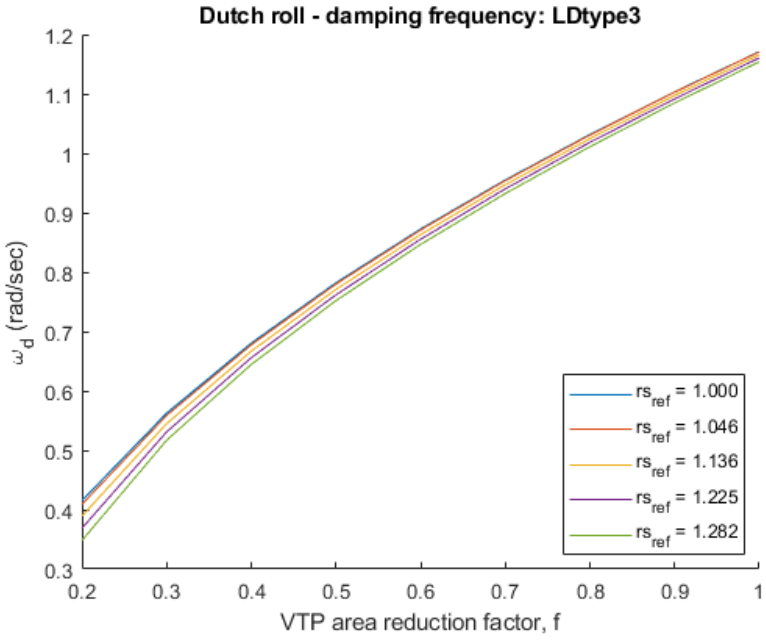


Fig. 20 Dutch roll: damping ratio for LDtype 3

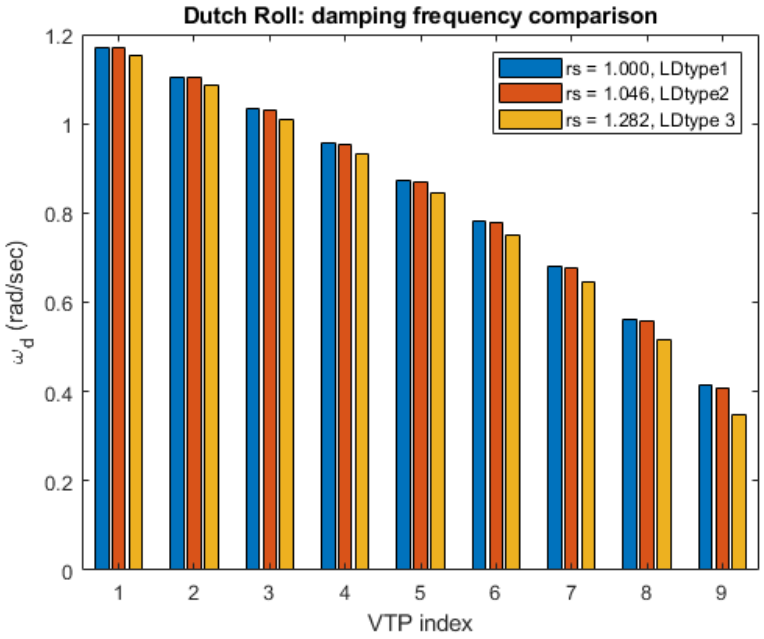


Fig. 21 Dutch roll: comparison of ideal cases - damping ratio

VI. Conclusion

The study presented in this paper applied Prandtl's bell-shaped lift distribution to assess the feasibility of vertical tailplane (VTP) reduction in a sailplane configuration, using the Slingsby T51 Dart 15 as a baseline. By comparing the original baseline, an equivalent elliptic distribution, and Prandtl's bell span-load, the analysis explored how lift distribution influences stability and control, specifically focusing on proverse yaw, crosswind handling, and Dutch roll stability.

The results indicate that the bell-shaped span-load produces a definitive proverse yaw effect, reducing the need for a larger VTP to counteract adverse yaw. This feature supports the possibility of downsizing the VTP, achieving weight and drag reductions, which is advantageous for high-aspect-ratio designs aimed at aerodynamic efficiency. In crosswind scenarios, the bell span-load demonstrated improved stability by requiring less rudder deflection to maintain alignment, thus achieving stability in conditions where traditional configurations would require more substantial control surface intervention. These findings confirm the bell span-load's suitability in configurations where lateral stability and reduced control input are desired, particularly under controlled flight conditions.

However, the Dutch roll analysis showed that the bell span-load, while beneficial for proverse yaw, exhibited reduced damping ratios and frequencies, indicating that increased wingspan and reduced VTP size might compromise Dutch roll stability. Thus, while VTP reduction is feasible, practical applications of the bell-shaped lift distribution may require additional measures—such as spanwise twist adjustments or moderate VTP sizing—to maintain Dutch roll damping in more turbulent or dynamic flight conditions.

Overall, this study supports Prandtl's bell-shaped lift distribution as a viable solution for adverse yaw mitigation and partial VTP reduction in sailplane configurations. Future work should examine the interactions between spanwise twist and lift distribution to optimize stability, while also exploring how the bell span-load performs under varying flight conditions, ensuring that the stability improvements are robust across a broader operational envelope.

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