

85

CRANFIELD INSTITUTE OF TECHNOLOGY

AERODYNAMICS DEPARTMENT

PhD THESIS

I.A.MALIK

The Design Development and
Evaluation of an Active Control
Aircraft Model Wind Tunnel Facility

Supervisor:

Mr.M.V.Cook

September 1982

ProQuest Number: 10820911

All rights reserved

INFORMATION TO ALL USERS

The quality of this reproduction is dependent upon the quality of the copy submitted.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if material had to be removed, a note will indicate the deletion.



ProQuest 10820911

Published by ProQuest LLC (2019). Copyright of the Dissertation is held by Cranfield University.

All rights reserved.

This work is protected against unauthorized copying under Title 17, United States Code
Microform Edition © ProQuest LLC.

ProQuest LLC.
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106 – 1346

CONTENTS

Chapter		Page
	SYNOPSIS	
	ACKNOWLEDGEMENTS	
	NOTATION	
1	INTRODUCTION	1
	1.1 Background	1
	1.2 Aims of Research Programme	3
2	LITERATURE REVIEW	5
	2.1 Introduction	5
	2.2 Free-Flight Research Techniques	5
	2.3 Semi-Free-Flight Research Techniques	17
	2.4 Facility Design Considerations	24
3	DESIGN AND CONSTRUCTION	34
	3.1 Introduction	34
	3.2 Model Suspension System	34
	3.3 Aircraft Model	38
	3.4 Electronic Control Unit	41
	3.5 Comments on the Design and Construction	48
4	THEORETICAL BACKGROUND	49
	4.1 Longitudinal Dynamics	49
	4.2 Lateral Dynamics	60
	4.3 Feedback Techniques	70
	4.4 Scaling Limitations	78
	4.5 Stability Derivatives	80
	4.6 Static Stability	81

Chapter		Page
5	EXPERIMENTAL INVESTIGATION	84
5.1	Introduction	84
5.2	Preliminary Experiments	85
5.3	Dynamic Stability Test Techniques	91
5.4	Model Dynamic Response Test Conditions	96
6.	RESULTS AND DISCUSSION	99
6.1	Introduction	99
6.2	Longitudinal Static Stability Tests	99
6.3	Open-Loop Model Dynamics	101
6.4	Closed-Loop Model Dynamics	112
7	COMMENTS, RECOMMENDATIONS AND CONCLUSIONS	117
7.1	Comments and Recommendations	117
7.2	Conclusions	129
	REFERENCES	132
Appendix		
A	DERIVATION OF SIMILARITY REQUIREMENTS	140
B		146
C	EQUATIONS OF MOTION	152
D	ESTIMATION OF DYNAMIC STABILITY CHARACTERISTICS	159
E	DATA REDUCTION	189
Figure		
1a	Miniaturisation of Representative Flight Control Hardware	199

Figure		Page
1b	Computer System Reliability	200
2	Active Control Technology Design Approach	201
3	Free Flight Model Testing Techniques	202
4	Remotely Piloted Model Control Loops	203
5	NASA Langley 20ft Free-Spinning Tunnel	204
6	Cross-Sectional View of Free-Spinning Tunnel at the NAE, Bedford	205
7	Setup for Free-Flight Tests	206
8	Magnetic Suspension	207
9	The NASA Langley Control-Line Facility	208
10	Diagrammatic Sketches of Funicular Suspension Systems used by ONERA	209
11	Two Cable Mount	210
12	Vertical Rigid Rod Mounting	210
13	Scale Factor to Achieve a Given Model Reynolds Number	211
14	Model Span to Achieve a Reynolds Number of 0.5×10^6	212
15	'Weybridge' Wind Tunnel	213

Figure		Page
16	Frequency Regimes of Vertical Rod and Cable Suspensions	214
17	Gimbal System for Model Suspension	215
18	The Model Suspension System	216
19	$\frac{1}{12}$ Scale Hawk Aircraft Model	217
20	Details of Control Surfaces Actuation Mechanisms	218
21	Designed Panel Layout of the Electronic Control Unit	219
22	Systematic Block Diagram of the Electronic Control Unit and Model Interface	220
23a	A Typical Summing Amplifier	221
23b	A Typical Low Drift Integrator	221
24	Input Control Unit Card	222
25	Model Output Card	223
26	Analogue Element Card	224
27	Design of ± 5 Volts Power Supply	225
28a	Airplane Axis System and Plane of Symmetry	226
28b	Airplane Axis System and Planes of Asymmetry	226

Figure		Page
29	Position of the Axis in the Vertical Plane in Steady Flight	227
30a	Longitudinal Stability Augmentation of Model with Pitch Attitude Feedback to Elevator	228
30b	Root Locus Plot. Effect of Pitch Attitude Feedback to Elevator on Longitudinal Modes of Model	228
31a	Longitudinal Stability Augmentation of Model with Pitch Rate Feedback to Elevator	229
31b	Root Locus Plot. Effect of Pitch Rate Feedback to Elevator on Longitudinal Modes of Model	229
32a	Lateral Stability Augmentation of Model with Roll Attitude Feedback to Aileron	230
32b	Root Locus Plot. Effect of Roll Attitude Feedback to Ailerons on Lateral Modes of Model	230
33a	Lateral Stability Augmentation of Model with Roll Rate Feedback to Ailerons	231
33b	Root Locus Plot. Effect of Roll Rate Feedback to Ailerons on Lateral Modes of Model	231
34a	Lateral Stability Augmentation of Model with Yaw Attitude Feedback to Rudder	232
34b	Root Locus Plot. Effect of Yaw Attitude Feedback to Rudder on Lateral Modes of Model	232

Figure		Page
35a	Lateral Stability Augmentation of Model with Yaw Rate Feedback to Rudder	233
35b	Root Locus Plot. Effect of Yaw Rate Feedback to Rudder on Lateral Modes of Model	233
36	Estimated Longitudinal Stability Derivatives	234
37	Estimated Longitudinal Stability Derivatives	235
38	Estimated Longitudinal Stability Derivatives	236
39	Estimated Lateral Stability Derivatives	237
40	Estimated Lateral Stability Derivatives	238
41	Estimated Lateral Stability Derivatives	239
42	Elevator Deflection vs Input Voltage	240
43	Aileron Deflection vs Input Voltage	241
44	Rudder Deflection vs Input Voltage	242
45	Pitch Angle vs Voltage Output	243
46	Bank Angle vs Voltage Output	244
47	Yaw Angle vs Voltage Output	245
48	Pseudo Rate Signal Circuit	91
49	Input Forcing Functions	93
50	Incidence for Steady State Conditions	246

Figure		Page
51	Tail Angle to Trim for Steady State Conditions	247
52	Short Period Mode - Frequency (c.g.=0.29 $\bar{c}$)	248
53	Short Period Mode - Frequency (c.g.=0.325 $\bar{c}$)	249
54	Short Period Mode - Damping (c.g. at 0.29 $\bar{c}$)	250
55	Short Period Mode - Damping (c.g. at 0.325 $\bar{c}$)	251
56	Short Period Mode Frequency Correlation	252
57	Short Period Mode Damping Ratio Correlation	253
58	Roll Mode - Time($t_{\frac{1}{2}}$) to Half Amplitude	254
59	Model Roll Response to Pulse Aileron Input	107
60	Dutch Roll Mode - Frequency (ω_{DR})	255
61	Dutch Roll Mode - Damping Ratio (ζ_{DR})	256
62	Influence of Adding Positive Dihedral Effect	111
63a	Model Short Period Mode - Pitch Rate Feedback to Elevator	257
63b	Model Short Period Mode - Pitch Rate Feedback to Elevator	257
64a	Model Short Period Mode - Pitch Rate Feedback to Elevator	258
64b	Model Short Period Mode - Pitch Rate Feedback to Elevator	258

Figure		Page
65	Short Period Mode Damping - Pitch Rate Feedback to Elevator	259
66	Short Period Mode Frequency - Pitch Rate Feedback to Elevator	260
67	Short Period Mode - Pitch Rate Feedback to Elevator (Pitch Freedom Only)	261
68	Roll Mode:- Roll Rate Feedback to Aileron at V=70 ft/sec	262
69	Roll Mode:- Roll Rate Feedback to Aileron at V=100 ft/sec	263
70	Dutch Roll Mode:- Yaw Rate Feedback to Rudder	264
71	Dutch Roll Mode - Yaw Rate Feedback to Rudder	265
Plate		
1	The Gimbal System and Vertical Rod	266
2	The Model Suspension System	267
3	Model Fuselage Construction Technique	268
4	Component Layout of the Model with Construction Detail	269
5	Wing and Gimbal System Attachment Points on Model Fuselage	270
6	The Completed Model with Detachable Parts	271
7	The Electronic Control Unit	272

8	Interior View of the Electronic Control Unit	273
9	Experimental Set-up	274
10	Model Flying in the Wind Tunnel	275

SYNOPSIS

Recent progress in the field of Active Controls Technology (ACT) has resulted in increased interest in dynamic wind tunnel testing for basic research. The present work reports the findings of a three year research and development programme to build a dynamic wind tunnel testing facility. The task included the design and construction of a controllable dynamically scaled aircraft model, a suspension system to give the model four degrees of freedom, and an electronic control unit to interface with the model for operating the primary controls, for stability augmentation and for providing output signals for measurement purposes.

The dynamic characteristics of the model have been recorded for some simulated representative flight conditions and are compared with theoretical predictions and the expected characteristics derived from full size aircraft data. The use of various stability augmentation functions has also been investigated to assess the usefulness of the electronic control unit as a means for providing stability augmentation. The results show the system to have considerable potential as an ACT Simulator. Modifications are suggested for further development of the facility to achieve a higher degree of accuracy and versatility.

ACKNOWLEDGEMENTS

Acknowledgement is made gratefully to:-

1. Mr.M.V.Cook for his valuable guidance, immense help and continual assistance for the last three years, without which this project would not have seen this day.
2. Pakistan Air Force for sponsorship and financial support.
3. The staff of the Department of Aerodynamics workshop. Particular thanks are due to Mr.Malcolm Goodridge, Mr.J.F.Churchill, Mr.T.Atkins and Mr.W.T.Munn for their help in the development of the test rig and their support during the experimental work.
4. British Aerospace, Kingston, in particular Mr.R.Balmer for providing Hawk dimensional data.
5. R.Gregory for his expertise and assistance in wiring the electronic control unit and help during fault diagnosis.
6. Design drawing office, in particular Mr.Geoff Sewell for help in panel layout and manufacture.
7. Mrs.Mo Vernon for undertaking the tough task of deciphering the manuscript and typing the thesis.
8. My wife Shamim for encouragement and support and my sons Kashif and Hassan for cheerfully accepting lack of attention on my part.

NOTATION

The following is a list of notations used in the thesis. The aeronormalized and dynamic normalized concise stability derivatives are presented in Table C1 and C2 of Appendix C.

Notation	Meaning
A	Aspect ratio, b^2/S
$A_i (i=1,2,3\dots)$	Coefficients of λ^4 in stability quartic
a	Aircraft lift curve slope, $dC_L/d\alpha$
a	Speed of sound
a_1	Tailplane (or fin) lift curve slope
a_2	Lift curve slope for control
$B_i (i=1,2,3\dots)$	Coefficient of λ^3 in stability quartic
b	Wing span (tip to tip)
$C_i (i=1,2,3\dots)$	Coefficient of λ^2 in stability quartic
C_D	Drag/ $\frac{1}{2}\rho V^2 S$
C_L	Lift/ $\frac{1}{2}\rho V^2 S$
C_{L_T}	Tailplane lift/ $\frac{1}{2}\rho V^2 S_T$
C_m	Pitching moment about O_y / $\frac{1}{2}\rho V^2 S \bar{c}$
c	Wing chord
$\bar{c}$	Mean aerodynamic chord of wing
D	Drag
D	Differential operator, d/dt
$\hat{D}$	Differential operator, $d/\hat{d}t$
$D_i (i=1,2,3\dots)$	Coefficient of λ in stability quartic
$E_i (i=1,2,3\dots)$	Term independent of λ in stability quartic
e_x	$-I_{zx}/I_x$
e_z	$-I_{zx}/I_z$

Notation	Meaning
F_r	Froude Number
g	Acceleration due to gravity
$\hat{g}$	$mg/\frac{1}{2}\rho_e V_e^2 S = C_L \sec \Theta_e$
g_1	$g \cos \Theta_e$
$\hat{g}_1$	$\hat{g} \cos \Theta_e = C_L$
g_2	$g \sin \Theta_e$
$\hat{g}_2$	$\hat{g} \sin \Theta_e = C_L \tan \Theta_e$
h	Height above a given datum
h_F	Distance of aerodynamic centre of fin above Ox
$h_{\bar{c}}$	Distance of c.g. aft of leading edge of mean aerodynamic chord of wing
$h_m^{\bar{c}}$	Distance of manoeuvre point, stick fixed, aft of leading edge of mean aerodynamic chord of wing
$h_n^{\bar{c}}$	Distance of aerodynamic centre of aircraft aft of leading edge of mean aerodynamic chord of wing
$h_0^{\bar{c}}$	Distance of aerodynamic centre of aircraft without tail aft of leading edge of mean aerodynamic chord of wing
I_x	Moment of inertia about longitudinal (rolling) axis Ox
I_y	Moment of inertia about lateral (pitching) axis Oy
I_z	Moment of inertia about yawing axis Oz

Notation

Meaning

I_{xy}	Product of inertia about Ox and Oy
I_{yz}	Product of inertia about Oy and Oz
I_{zx}	Product of inertia about Oz and Ox
i_x	I_x/mb^2
i_y	$I_y/m\bar{c}^2$
i_z	I_z/mb^2
i_{zx}	I_{zx}/mb^2
K_n	Static margin, stick fixed
L	Lift
L_W, L_T	Wing lift, tail lift
L	Rolling moment about Ox
$\dot{L}_p \dot{L}_r \dot{L}_v \dot{L}_\xi \dot{L}_\zeta$	Rolling moment derivatives, $\partial L/\partial p, \partial L/\partial r,$ $\partial L/\partial v, \partial L/\partial \xi, \partial L/\partial \zeta$
L_F	Distance of aerodynamic centre of fin aft of c.g. of aircraft
L_T	Distance of aerodynamic centre of tailplane aft of c.g. of aircraft
l_μ	Characteristic length
M	Mach Number
M	Pitching moment about Oy
$\dot{M}_q \dot{M}_u \dot{M}_w \dot{M}_{\dot{w}} \dot{M}_\eta$	Pitching moment derivatives, $\partial M/\partial q, \partial M/\partial u,$ $\partial M/\partial w, \partial M/\partial \dot{w}, \partial M/\partial \eta$
m	Aircraft mass
N	Yawing moment about Oz
$\dot{N}_p \dot{N}_r \dot{N}_v \dot{N}_\xi \dot{N}_\zeta$	Yawing moment derivatives, $\partial N/\partial p, \partial N/\partial r,$ $\partial N/\partial v, \partial N/\partial \xi, \partial N/\partial \zeta$

Notation	Meaning
$N_{\eta}^{\theta}, N_{\xi}^{\phi}, N_{\zeta}^{\psi}$	Transfer functions
p	Aircraft angular velocity in roll
$\hat{p}$	τ_p
P	Atmospheric Pressure
q	Aircraft angular velocity in pitch
$\hat{q}$	τ_q
R_e	Raynolds Number
r	Aircraft angular velocity in yaw
$\hat{r}$	τ_r
r	Non-dimensional damping(negative real part of λ)
r_y	Radius of inertia
S	Wing area
S_F	Fin area (vertical tail area)
S_T	Tailplane area
s	Non-dimensional frequency (imaginary part of λ)
T	Periodic time of oscillation
t	Time
$\hat{t}$	Non-dimensional time, t/τ
$t_{\frac{1}{2}}$	Time to half amplitude
t/c	Thickness/chord ratio of aerofoil
u	Perturbation velocity along Ox
$\hat{u}$	u/V_e
V_e	Resultant velocity of aircraft c.g. in datum steady flight
$\bar{V}$	Volume ratio, $S_T l / s \bar{c}$

Notation	Meaning
$\bar{V}_F$	Fin volume ratio, S_{F1_F}/S_b
$\bar{V}_T$	Tailplane volume ratio, $S_{T1_T}/S_{\bar{c}}$
v	Component of velocity increment of c.g. along Oy in disturbed flight
$\bar{v}$	$v/V_e (= \beta, \text{ for small angles of sideslip})$
W	Aircraft weight, mg
w	Perturbation velocity along Oz
$\hat{w}$	w/V_e
X	Force component along Ox
$\ddot{X}_q \ddot{X}_u \ddot{X}_w \ddot{X}_{\dot{w}} \ddot{X}_\eta$	Force component derivatives, $\partial X/\partial q, \partial X/\partial u, \partial X/\partial w, \partial X/\partial \dot{w}, \partial X/\partial \eta$
Y	Force component along Oy
$\ddot{Y}_p \ddot{Y}_r \ddot{Y}_v \ddot{Y}_\xi \ddot{Y}_\zeta$	Force component derivatives, $\partial Y/\partial p, \partial Y/\partial r, \partial Y/\partial u, \partial Y/\partial \xi, \partial Y/\partial \zeta$
Z	Force component along Oz
$\ddot{Z}_q \ddot{Z}_u \ddot{Z}_w \ddot{Z}_{\dot{w}} \ddot{Z}_\eta$	Force component derivatives, $\partial Z/\partial q, \partial Z/\partial u, \partial Z/\partial w, \partial Z/\partial \dot{w}, \partial Z/\partial \eta$
α	Incidence (angle of attack) of mean aerodynamic chord of wing
α_T	Tailplane incidence
β	Angle of sideslip (the angle the direction of motion of the aircraft c.g. makes with the plane of symmetry Oxz)
Γ	Dihedral angle
γ	Angle of bank
Δ	Stability polynomial
ϵ	Angle of downwash at tailplane

Notation	Meaning
ξ, η, ζ	Angular displacements of ailerons, elevator and rudder, respectively
η	Increment in elevator angle from trimmed position
θ_e	Inclination of Ox to the horizontal in the datum steady flight (positive upwards)
θ	Angle of pitch
Λ	Angle of sweepback
λ	Root of stability quartic($=-r+is$)
λ	Geometric scale factor $\frac{l_m}{l_a}$
μ_1	Longitudinal relative density parameter $m/\frac{1}{2}\rho_e S \bar{C}$
μ_2	Lateral relative density parameter $m/\frac{1}{2}\rho_e S b$
ρ	Air density
τ	Magnitude of time unit, $m/\frac{1}{2}\rho_e V_e S$
ϕ	Angle of bank
ψ	Angle of yaw

CHAPTER 1

INTRODUCTION

1.1 Background

Over recent decades, aircraft have been designed with increasing dependence on flight control systems for control and stability augmentation. The utilisation of such systems was initially limited to correcting deficiencies like poor handling characteristics or poor stability which could not be corrected through conventional aerodynamic balancing or configuration changes. The revolution in solid state electronics has permitted a considerable reduction in control computer sizes, power and cost¹. Furthermore, in the 70's, due to integrated circuit design concepts, electronics equipment has become much smaller (Fig.1a) cheaper and more reliable (Fig. 1b). The airborne computer thus developed is efficient, safe and reliable. Aircraft designers have capitalised on these gains primarily by devising new improved flight control system functions aimed at increasing aircraft performance and flying and handling qualities improvements. In fact flight control technology has advanced to the stage where stability and control augmentation systems are now vital to virtually all high performance aircraft.

We are now in an era in which control systems are a central element in optimizing aircraft design to meet their mission and performance objectives. This design philosophy has become known as the Control-Configured-Vehicle Concept (CCV). In a conventional aircraft design process trade off's

are made in aerodynamics, structures, and propulsion to meet the mission requirements. Once all the compromises have been made that lead to the definition of an aircraft configuration, the vehicle subsystems, including the flight control system, are designed. The Basic Active Control or Control-Configured-Vehicle Concept² aims at optimising aircraft geometry for each flight condition by considering flight controls in the preliminary design process as indicated in figure 2. Trade-off's are still made, but it is possible to relax the constraints in the aerodynamics, structures and propulsion system areas and to rely on the control system to provide the same effective margins artificially. For example, it is possible to reduce the static aerodynamic stability of the aircraft to neutral or even negative to gain a performance advantage and rely on the control system to stabilise the aircraft artificially³. Inherent in this approach is the use of a full time, full authority fly by wire control system^{4,5,6}.

Research into the potential benefits arising from the use of ACT has been carried out in the following ways.

- (a) Flight testing purpose built experimental aircraft or modified existing aircraft^{7,8}.
- (b) Use of large scale free flying models⁷ for simulation.
- (c) Use of a wind tunnel facility for dynamically testing suitable aircraft models^{9,10}.

Although there is no doubt that full scale flight testing is a must in the aircraft development phase, due to the increased cost, complexity and risk associated with flight testing new aircraft

with advanced flight control features, wind tunnel tests of suitably representative aircraft models became an increasingly effective method for carrying out fundamental research into the application of active control technology. The main advantages of this kind of investigation are that the risks are negligible, it is more realistic than computer simulation because of the limitations of mathematical modelling¹¹, it permits systematic testing under reproducible conditions and it is relatively inexpensive. The main disadvantages are the uncertainty associated with the interpretation of small scale model tests in terms of the full scale equivalent aircraft and the limitations on the number of degrees of freedom that a restrained model can have.

1.2 Aims of Research Programme

In view of this increased interest in Dynamic Wind Tunnel testing due to ACT, it was felt that a modest facility for demonstrating ACT concepts should be established. The initial feasibility study was done in 1979-80 and was the subject of an M.Sc. Thesis¹².

The primary objective was to design and build a facility that would allow a dynamically representative aircraft model to "fly" in the wind tunnel with as many of the natural degrees of freedom as reasonably possible. Due to limited resources and the constraints of available departmental workshop support, the design of this facility is limited in terms of complexity and size. Resource limitations necessitated the choice of the least complex technique and the department's 42" "Open Jet Wind Tunnel" dictated the size of the facility.

The technique selected allows four degrees of freedom to the model, which for this investigation is $\frac{1}{12}$ th scale British Aerospace Hawk aircraft.

In order to achieve the above mentioned objectives the following subsystems were designed and constructed.

- (i) A controllable dynamically scaled Hawk aircraft model.
- (ii) A suspension system for giving four degrees of freedom to the model.
- (iii) An electronic control unit to interface with the model. The electronic control unit provides the means of operating the primary controls of the model, provides feedback loops for stability augmentation and provides output signals for recording and display.

Open loop and closed loop response experiments pertaining to the dynamic mode analysis of the model were conducted to evaluate the completed facility. The wind tunnel simulation results are compared with the theoretical predictions and the actual full scale Hawk data for correlation to discover how closely the dynamics of the model represent those of the full size Hawk. Consequently the fidelity of the simulation could be evaluated. The closed loop experiments were carried out to evaluate the usefulness of the electronic control unit as a means for artificially augmenting the stability of the model.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Stability and controllability as quantitative measures of handling and ride qualities have for many years constituted a major area of interest in the field of flight mechanics. Full scale and scaled model free-flight testing have played a vital role in the research in this field. In the case of fundamental research, free-flight model testing has obvious attractions for testing new concepts before risking full scale construction and flight test. The main advantages of free-flight model testing techniques are that the model has complete dynamic freedom, potentially hazardous manoeuvres can be investigated since crew safety need not be considered and the cost involved is relatively small as compared with full scale testing.

It is the purpose of this literature review to review the various techniques which have been used in free-flight model testing. The techniques discussed are limited to those used in low-speed model studies as high speed model testing is beyond the scope of the present investigation. The techniques are grouped into two general categories; free-flight techniques in which the model flies with six degrees of freedom, and semi-free-flight techniques in which one or more degrees of freedom are suppressed. The free-flight techniques are broken down as shown in figure 3.

2.2 Free-Flight Research Techniques

Free-flight techniques are those in which there is no

appreciable restraint of the model under normal flying conditions. As indicated earlier, the free flight techniques are subdivided into two types: those used either outdoors or in a large enclosed area other than a wind tunnel and those used in wind tunnels.

2.2.1 Outdoor Free-Flight Research Techniques

The free-flight techniques which have been used outdoors are divided into three categories as follows:

(a) Air Launched Models

Bisgood¹³ reported research conducted at the RAE, in the early sixties using expendable simple unpowered models of slender wing configuration. These uncontrolled, fibre glass-plastic models were launched from balloons and helicopters. To investigate model behaviour, the pitch response of the model was measured by two normal accelerometers fitted with a scratch recording clock-work driven mechanism. Pulses were applied to the elevator by an electro-pneumatic control system. Kinetheodolite and high speed cine camera facilities on the ground were also used. The results of the investigation indicated that the pitch response of the model varied markedly with the magnitude of the initial elevator pulse. For small and medium pulses the motion was very heavily damped and the response to large pulses was oscillatory, though still heavily damped. The investigation also determined that the use of free flight models for overall dynamics research appeared to be practicable and could, with improved instruments, offer distinct advantages over other methods.

Recent free-flight tests at the RAE are reported by Fail^{14,15} and form part of an investigation of the stall and post-stall dynamics of aircraft at low speeds and altitudes. For these tests a $\frac{1}{4}$ scale model of a fighter type aircraft was chosen, since the models are fairly large and have a mass of about 450 pounds, special handling and retrieval systems have been developed. The model is recovered by a parachute system.

Considerable research has been performed by NASA using radio controlled models. A report by Libbey et al¹⁶ gives a complete description of the technique and associated equipment. Although this technique was developed primarily to study the incipient and developed spin characteristics of an aeroplane, it had also been used for other research such as flight characteristics of aircraft and re-entry vehicles.

The models were dropped from a helicopter and controlled from ground stations usually by two pilots. The models were constructed of fibre glass cloth and plastic, the fuselage being a $\frac{1}{4}$ inch thick hollow shell and the wing and tail surfaces having solid balsa covers. The model weight varied from about 25-200 lbs and the largest model tested had a length of 8 feet and a wing span of 6 feet. Radio receivers and electric motor powered control actuators were installed in the model to provide simultaneous operation of all control surfaces and to release the recovery parachute. Both "bang-bang" (full on or off) and proportional type controls were used. The data recording in the model was

accomplished photographically by an electrically driven movie camera which photographed the view of the horizon and recorded the position of flow-direction and airspeed indication vanes, control position indicators and a timing light. Magnetic tape recorders on the ground were used to record control signals and voice communication between the helicopter and ground control station.

Holleman ¹⁷ reported on tests carried out by NASA on a large scale ($3/8$) F-15 fighter airplane model to investigate the stability and controllability of the configuration at high angle of attack. The motivation to use a $3/8$ scale model was to provide a model large enough such that it could be controlled 'normally' by a pilot and to obtain data near the full scale Reynold's number condition by correlation with other model test results. The model was built primarily of fibre glass with metal load carrying members. The model was unpowered and a remote control piloting technique was used. The pilot controlled the model from a ground-based simulator-like isolated cockpit with complete displays and controls. The instrumentation on the model was very extensive and the quantities recorded such as attitude angles, rates, acceleration, airspeed, altimeter and control surface position etc., were transmitted to the ground station for display to the pilot and as inputs to the flight control system computer. The control system used was very sophisticated and figure 4 gives some indication of the model control loop. The model was launched from a B-52 airplane and was recovered by parachute

with mid-air retrieval by helicopter.

The large scale model spin characteristics were compared with the smaller scale ($\frac{1}{8}$) model results and showed good agreement at low to medium high angles of attack and reasonable agreement at high angles of attack. The data from a simulated flight spin also agreed satisfactorily with the model flight data.

(b) Catapulted Models

Use of this technique was recorded as early as the 1930's for spin research studies in both the United Kingdom and the United States^{18,19}. These studies were not very extensive and were discontinued after the free spinning wind tunnels were put into operation. Later, in the 1950's, the NACA Langley Laboratory used unpowered, catapult launched models for studies of stall and incipient spin as reported by Stone et al²⁰. In the earlier tests, a building about 70 feet square by 60 feet high was used. The launching apparatus was located near one wall of the building about 55 feet above the floor. The model, made from balsa wood, was launched at a speed slightly higher than the stalling speed and at an angle of attack slightly below the stall angle. The elevator control was preset to a position which would cause the model to pitch up through the stall and, in some cases, the rudder was preset to initiate a yawing motion thus giving roll-off at the stall.

The model was retrieved by a large net hung

above the floor and up to the wall opposite the catapult. Later this facility was not considered satisfactory because of space limitations and a much larger Navy airship hangar was used. In these tests the catapult was located 137 feet above the floor and there was sufficient space to accommodate all possible model flight paths. Models up to 5 lbs in weight could be launched at speeds up to 50 ft/sec. The model control surfaces were operated in flight by radio control and data was obtained from filmed records.

(c) Powered Radio Controlled Models

Hewes²¹ reported some flight investigations with powered radio controlled models by NASA. The propeller driven aeroplane model equipped with a parawing was taken off from the ground and was controlled by conventional elevator and rudder surfaces. This technique for testing power radio controlled models was very simple and is essentially the same as that used by model aircraft hobbyists. The technique has also been used in a number of instances where exploratory flight of some new features were desired. For example, the Lockheed Aircraft Corporation used this technique on studies of their non-rigid rotor helicopter and the Consolidated Vultee Aircraft Company tested radio controlled dynamically scaled powered models of a sea plane for hydrodynamics research²².

2.2.2 Free-Flight Research Techniques in Wind Tunnels

The techniques used in wind tunnels are grouped into

four categories based on the type of wind tunnel in which they are used, i.e. vertical, tilting test section, conventional and magnetic suspension tunnels.

(a) Techniques Used in Vertical Tunnels

RAE in 1932²³ and NACA in 1935²⁴ built their first vertical wind tunnels, (at Farnborough and Langley respectively), especially for conducting free spinning tests of dynamic models. The construction of these tunnels was preceded in both establishments by spin research studies in which dynamic scale models were launched in spins from a high platform as discussed in 2.2.1(b).

The vertical spin tunnel at the RAE was of the closed throat type with a working section 30 feet in height and of circular cross-section with diameter 12 feet. Tunnel speeds up to 56 ft/sec were achieved. Dynamically scaled balsa models of span up to 30 inches were used in the tests. The test models were launched in the wind tunnel by first letting it rotate in a spinning attitude on a spindle mounted on a retractable arm. When the proper wind tunnel speed was reached, the model would lift off and the arm would be retracted. Data was obtained by visual observation and motion picture records. To attempt spin recovery a delayed action mechanism was used to reverse the controls of the model.

The first NACA tunnel was generally similar to the RAE tunnel in construction and operating technique except that after initial tests, the use of a launching spindle was discontinued and the model was merely launched by hand into the airstream

with an initial spinning motion. The model was retrieved from a net at the bottom of the test section. The test section was 12 sided and measured 15 feet across the flat.

Later NACA replaced the 15-foot tunnel with a 20-foot tunnel (figure 5) in 1941 and used it extensively for spin research²⁵. The maximum tunnel speed was 97 ft/sec. Very rapid changes in speed could be obtained with a special speed control on the fan motor. The model control actuation could be continuously operated instead of using a preset timing device as was used earlier. This was achieved by placing copper coils around the periphery of the tunnel which set up magnetic field in the tunnel when energized and this magnetic field actuated a magnetic device in the model to operate the controls. Motion picture records which included model motion, records of time and tunnel velocity were the primary source of data.

Another major facility for spinning tests was set up at the N.A.E.(Bedford) in 1954²⁶. The tunnel, shown in figure 6, has several unique design features which enhance its research capabilities over that of earlier models. It was designed for pressurisation at up to four atmospheres in order to obtain high Reynolds' numbers. The test section is 15 feet in diameter and 30 feet high and protective netting is arranged around the tunnel wall to keep the damage to the test model to a minimum. The launching and retrieval of the model is accomplished by means of a nylon cord suspended from above by a winch and attached to the model a little behind its centre of gravity. For tests carried out under pressurised condition a periscope is provided for observation of the

model. Model control is achieved by radio-control.

The above mentioned facilities have been used for most of the work done in vertical tunnels. Other prominent free spinning tunnels which have been used in spin research include the University of Lille tunnel with test section of 2 metres in diameter, the University of Sydney 3-foot tunnel and 12-foot diameter USAF wind tunnel²².

(b) Techniques Used in Tilting Test Section
Wind Tunnels

This type of wind tunnel was used in early days of free flight model testing and are no longer in use. However, the technique developed on this type of tunnel by NASA was the forerunner of some of the more advanced flying model techniques now being used.

The first NACA 5-foot diameter tunnel was completed in 1937 and served as a pilot model for a larger 15-foot diameter tunnel to be built later in 1939²⁷. Only a few investigations were carried out in the small tunnel but they served to prove that the free-flight technique employed was feasible and warranted further development in a larger tunnel.

The 12-foot tunnel was a simple closed throat, open return type and was mounted on pivots to permit its longitudinal axis to be tilted to give a suitable flight path angle to the free-flying model. The test section length was 15 feet. Prior to each flight the model was placed on the floor of the tunnel with the tunnel tilted to an angle slightly higher than that required for

equilibrium flight at the predetermined flight condition. When the tunnel speed was brought up to the flight speed the model would rise from the floor and fly under the control of the pilot, who stood behind the test section and flew the model by means of a small control stick. The control stick operated electric switches to energize the electro-magnetic control actuators in the model. A range of tunnel angles from 40° glide to 15° climb could be covered and the tunnel speed could be varied rapidly and smoothly over a range from 0-90 ft/sec. The floor and lower walls of the tunnel were lined with sponge rubber to protect the model from excessive damage in crashes.

The models used in the 12-foot tunnel had a wing span in the range of 3 to 4 feet. Originally they were constructed with solid balsa wings and hollow balsa fuselages²⁸ but later models had wings of built-up construction with spruce spars and fuselages of much stronger construction. The earlier model control actuators were a simple spring-centred electro-magnetic mechanism, but later, more powerful pneumatic actuators controlled by solenoid-operated air valves were used. Both types of actuators provided a "flicker" or "bang bang" type of control which proved to be more satisfactory than proportional actuators for manual control of small scale models.

The use of the 12 foot tunnel for model flight testing was discontinued after the improved version of the free flight technique had been developed for use in the Langley Full Scale Tunnel.

(c) Techniques Used in Conventional Tunnels

A unique free-flight testing capability exists in the NASA Langley full-scale tunnel. Development started in 1949²⁹ and the technique was essentially a refinement of the technique previously deployed in the 12-foot free-flight tunnel.

The tunnel is extremely useful for model flight testing because of its large 30 feet x 60 feet test section and open throat. A relatively large (typically around 4 feet span) powered model can be flown without restraint. It has a top speed of about 180 ft/sec but is usually operated below 100 ft/sec due to turbulence at high speeds. The model is controlled by three operators (pitch, roll and yaw control) who can transmit pneumatic and electric power and control signals to the model via a flexible trailing cable (figure 7). This flexible cable also incorporates a thin steel safety cable which is passed through a pulley above the test section and which can be used to catch the model if an uncontrolled motion or mechanical failure occurs. The entire flight cable is kept slack during the flight and is controlled by a separate safety cable operator.

Models used for flight testing in this facility are in the 25-80 lbs weight range. The models are designed to be true dynamic models but the construction is much more durable than that of the balsa models used in earlier free flight testing work. Liberal use is made of fibre glass, plastic, hardwood and steel in place of balsa and aluminium²⁹.

The techniques for determining low speed dynamic stability and control characteristics with free-flying models have reached a high degree of development in the above mentioned facility. Although highly developed, the technique is still basically simple and considered primarily as a qualitative rather than a quantitative research tool³⁰. The results obtained with this technique have been generally in the form of pilot opinion of flight characteristics and motion-picture records obtained with 16 mm cameras located at three or four different positions around the test section.

(d) Technique Used in Magnetic Suspension Tunnels

Dynamic testing with magnetic suspension models has been pursued for a number of years, mainly at MIT, at the University of Southampton and the University of Virginia. Reference 31 gives a useful review of magnetic suspension techniques used in these tunnels.

An experimental arrangement used at the University of Virginia in an 11 cm x 11 cm vertical subsonic wind tunnel is shown in figure 8. A large pair of Helmholtz coils produce a uniform field to magnetize the model and the second pair of opposed coils create a vertical gradient which opposes the weight and drag of the model. The remaining coils create horizontal gradients which introduce sideforces on the model. The test section of the wind tunnel is located inside the coils. Model position is sensed optically and a feedback control system, which drives power amplifiers, supplies power to the magnet coils used to keep the model in the centre.

The magnetic suspension system provides interference free motion and has the added attraction of the possibility of simulation of artificial gravity. But, due to cost and complexity of equipment, such facilities tend to be very small and consequently of limited rather specialised application.

2.3 Semi-Free-Flight Research Techniques

Semi-free-flight techniques are those which involve elimination of one or more of the degree's of freedom. These techniques are subdivided into two types. Those used outdoors and those used in wind tunnels.

2.3.1 Outdoor Semi-Free-Flight Research Techniques

These techniques are grouped into two classes: The control line techniques in which the model flies in a circle at the end of a tethering line and the track technique in which the model is mounted on a servo-controlled carriage.

(a) Control Line Technique

In this technique the model is flown in circular flight at the end of a tethering line or, in some cases, a rotating boom. The models are restrained in roll and yaw attitude and lateral displacement but experience only minor restraints in pitch attitude, vertical and fore and aft displacement. Braun³² reported the early research using the control line technique by the USAF but most extensive research has been carried out by NASA on its control line facility ^{29,33}. The facility was built primarily for VTOL research. Figure 9 is a sketch of this facility which is located in a wooded area that serves as a wind break and

permits testing even when it is fairly windy elsewhere. The testing rig consists of a standard crane with its circular track mounted on concrete pillars placed in the centre of a 130-foot diameter concrete circle. Four operators control the functioning of the facility. The arrangement of the overhead safety cable and the power and control cables is the same as that used in the Langley full-scale tunnel free-flying model technique. In this case, the support of the overhead cable is provided by a special jib attached to the vertical boom.

The VTOL models used are usually powered by compressed air but, in some cases rocket motors have been used. Before starting on a transition test, the model takes off vertically and is trimmed for steady hovering flight. Then the pitch pilot operates the model controls to perform the transition to forward flight at any desired rate while the power operator adjusts the model power to maintain the desired attitude. The crane operator rotates the crane so that the end of the jib is above the model at all times, so that the crane rotation has no effect on the model motion.

Test data from the control line facility are obtained in the form of motion picture records obtained from a camera mounted on the top of the cab of the crane. Also included in the field of view of the camera are indicators of model velocity and control position.

(b) Track Technique

The Princeton University free-flight facility³⁴ is the only successful application of this

technique which was developed primarily for testing VTOL models in hovering and low speed flight. The facility involves the use of a servo-controlled carriage which runs along a straight horizontal track 750 feet long. Mounted on this carriage is a vertical track on which runs a second servo-controlled carriage which carries the model support boom. The model is attached to this boom with ± 9 inches of fore and aft freedom, ± 3 inches vertical freedom and angular freedom in pitch. During the test, the model propulsion system provides the lift to support the model weight and the thrust to overcome the drag in forward flight. The model support system is located horizontally and vertically by the two servo-controlled carriages in response to signals from position indicators at the model so that the model stays at the centre of its small range of horizontal and vertical freedom, thus providing no restraint to the model in the horizontal and vertical direction. Since the model is restrained in lateral displacement and in bank and yaw attitudes, only studies of the longitudinal characteristics in hover and forward flight are possible. Lateral characteristics in hover only are possible by making tests with the model turned through 90° about its vertical axis.

The models can be tested at speeds up to 40 ft/sec and can weigh up to 40 lbs. Unlike the models tested with the control-line facility, models tested in this facility are not equipped with controls for flying, so the flight data obtained are limited to longitudinal stability information and response to pulse disturbances. The data are transmitted from the model to the recording equipment by telemetry. The results of the

tests indicate good correlation between the longitudinal dynamic stability characteristics measured from the models and corresponding full scale aircraft.

2.3.2 Semi-Free-Flight Research Technique in Wind Tunnels

These techniques are grouped into three classes. The ONERA funicular Technique, cable mount techniques and the rigid-rod mounting technique.

(a) ONERA Funicular Technique

This technique has been developed and used by ONERA in its large open-throat tunnel at Chalais-Meudon in Paris³⁵. Figure 10 shows sketches of the two versions of the suspension system used in this technique. An overhead cable system with counterweight is used to support a proportion of the model weight and a longitudinal towline is used to supplement or replace the thrust of model. The system shown in figure 10 a is without servo controls. The overhead and towline cables are attached to the model by means of bridles attached to points on the wing that lie on a line passing through the centre of gravity and perpendicular to the plane of symmetry. In the servo-controlled system (figure 10 b) no bridles are used but the vertical and longitudinal cables are servo-controlled so that they remain essentially perpendicular and parallel to the airstream respectively. Thus there are four degrees of freedom: three degrees of rotational freedom and freedom of lateral translation.

The powered models used in this facility weigh up to 125 lbs. The models were equipped with

control actuators that were operated by two pilots located in a control room at the side of the test section. Typically, one of the pilots controls longitudinal motion while directly observing the model. The second pilot controls the model laterally by observing it on a television screen. The television camera which provides this view is located at the rear of the test section. Records of model motion and control deflections are obtained by photographing instruments mounted on the model, motion picture records of the model in flight are also obtained. In one of the investigations involving an experimental aeroplane model a pilot sat in the cockpit of the model to study the low speed stability and control characteristics.

Although the system of cable restraints used in this technique produces flight characteristics that are not directly applicable to a complete free-flying aircraft, the research group at ONERA conducting the tests has found that reliable information can be obtained by careful analysis and interpretation of results.

(b) Cable Mount Technique

The NASA Langley transonic wind tunnel facility utilizes this technique and the facility was originally developed for flutter research³⁶. With some modification the same set-up has been used for the analysis of rigid body motion of the aircraft as described in reference 36. Figure 11 shows the general arrangement of the model mount system which allows lateral and vertical translation of the model as well as angular rotation about all three axes, thus giving the model five degrees of freedom.

The unique feature which makes this facility highly suitable for the study of dynamic and aeroelastic problems of aerospace vehicles is the use of a variable density Freon-12 (or air) test medium. Freon-12 has several characteristics which make it a very attractive test medium for scaled dynamic model studies. The most advantageous characteristics are the high density of, and low speed of sound in Freon-12 relative to air at the same pressure and temperature. The relative low speed of sound is significant for several reasons:-

For dynamic model tests in which the reduced frequency scaling parameter must be satisfied, the lower tunnel speed for a given Mach number reduces directly all of the pertinent frequencies and consequently simplifies the instrumentation required. The lower velocity also reduces the inertia loads. For dynamic tests (including flutter) where the ratio of structural density to test medium density must be the same for the model as for the full size aircraft, the denser Freon-12 permits heavier models to be constructed. This is a distinct advantage considering the difficulties of fabricating models light enough to simulate mass characteristics of aircraft at high speed and low altitudes. Another benefit is that for a given model size, the test condition of equal Mach No. and stagnation pressure produces a Reynold's number in Freon-12 approximately three times that of air. The principal uncertainty associated with the use of Freon-12 as a test medium is the fact that its specific heat ratio γ is not the same as for air (1.13 as compared with 1.4 for air). There have been numerous studies to relate the quantitative difference which exists between the compressibility factor for air and Freon-12 so that data obtained in Freon-12 can be

utilized to predict the corresponding characteristics in air.

Reference 10 has attempted to assess the validity of predictions obtained from dynamically scaled models in the Langley facility. The test results relate to gust and buffet response, control surface effectiveness, flutter and active control of aeroelastic effects. The results in general indicate that predictions from the wind tunnel studies were substantiated by full scale flight measurements.

(c) Rigid Rod Mounting Technique

This technique is used by DFVLR at the Institute für Flugmechanik wind tunnel facility for free-flight research.^{9,10,37} Reference 36 gives the details of the facility used for flutter research. The wind tunnel used for this facility has a 4 metre test section with a maximum speed of 70 metres/sec. The wind tunnel mounting fixture consists of a tube frame with a vertical rod on which the model can slide (figure 12). The model possesses three degrees of freedom, i.e. pitch, yaw and vertical translation; a weight (gravity) compensation system is also being used. Earlier, a spring and cam arrangement was used to provide the vertical force but later the system was modified and an electric torque motor was used to provide the vertical force, via a suspension cable.

The tests were performed on a $\frac{1}{8}$ scale model of a light, twin engine transport aircraft (Dornier DO 28 TNT). The model was built from carbon fibre reinforced plastic. Electric motors provided control actuation of elevator, landing

flaps and aileron. The measurement system consisted mainly of angle of attack vane, accelerometers, and a pitch rate gyro. The measurement signals and control command were connected to the control centre via a cable. The control centre consists of a PDP 11/45 digital computer, TR48 analogue computer and a registration and evaluation compartment.

2.4 Facility Design Considerations

2.4.1 General Constraints

(a) Similarity Laws

In a study where scaled models are to be used for investigation, one must consider how scale model tests may be conducted so that the results are applicable to the corresponding full scale problem. In order to use a model to simulate the conditions of a prototype, the model system must satisfy certain similarity parameters. These similarity parameters, which one tries to find in dimensionless form, can be derived either by considering relevant equations describing the motion or by the method of dimensional analysis. Some of the required similarity relationships in the study of dynamic behaviour of an aircraft have been derived in the literature. Scherburg and Rhode³⁸ derived similarity requirements for free flight spin models but they neglected the variation of air density between the test condition and corresponding flight attitude. Neihouse and Pepoon³⁹ restricted their similitude requirement to the study of full-scale Mach number simulation. Bisplinghoff⁴⁰ derived the similarity laws for

an unrestricted flutter model using the technique of dimensional analysis and pointed out how these laws could be extended to a dynamic stability model. The derivation of similarity requirements for rigid body dynamics are presented in Appendix A. Table A.1 in the appendix lists a summary of similitude laws for model design and interpretation of model motion.

For a complete dynamic simulation the scaled model must be geometrically, kinematically and dynamically similar to the full scale aircraft.

For geometric similarity, the model must have the same shape as the full scale aircraft, that is, the model must be a scaled down version of prototype. Then the ratio between the corresponding linear dimensions of the model and of the full scale aircraft will be uniformly similar. When aerodynamically important regions of the aircraft are geometrically scaled, then one can achieve the same aerodynamic characteristics on a model as exist on a full scale aircraft (kinematic similarity) provided Mach No. and Reynold's No. similarity is also satisfied.

The following non-dimensional parameters must be equal for dynamic similarity of the scaled model and aircraft.

(i) Mach Number: The Mach Number takes into account the compressibility effects of the airflow and is defined as:

$$\text{Mach Number } M = V/a \quad (2.1)$$

The discrepancy in Mach Number between the model

and full scale flight is unimportant for low speed investigations as compressibility effects at low Mach number are not very significant. As long as the model speed remains below a value corresponding to the full scale speed at which compressibility effects begin to appear, Mach number effects can be neglected. In practice if the Mach number of the aircraft is below $M \approx 0.4$, the Mach number effects are neglected. At higher speeds compressibility effects become dominant and Mach number equality must be satisfied for dynamic similarity. In transonic and supersonic dynamic testing this requirement poses even greater difficulty in model design as compared to subsonic testing. Use of Freon-12 instead of air is one way of overcoming this problem as discussed earlier in this chapter.

(ii) Mass And Mass Distribution: To ensure that a scaled model exhibits similar dynamic behaviour to the full scale aircraft the relative mass parameter and the relative radius of inertia must be correct for both model and aircraft. The above parameters are defined as follows.

$$\text{Relative mass parameter: } \mu_L = \frac{2m}{\rho s l_\mu} \quad (2.2a)$$

$$\text{Relative radius of Inertia: } K_y = \left(\frac{r_y}{l_\mu} \right)^2 \quad (2.2b)$$

The compliance of the above two parameters results in the correct weight and inertia relationship between the model and full scale aircraft.

(iii) Strouhal Number: The Strouhal number corresponds to reduced frequency and is defined as follows.

$$\text{Strouhal number } S = \frac{\omega l_{\mu}}{V} \quad (2.3)$$

When it is equal for model and aircraft then the dynamic angles of attack are equal. The time scale factor τ can be determined from Strouhal number equality for a given geometric scaling.

(iv) Froude Number: Froude number scales the gravitational effect and is defined as:

$$\text{Froude number } F_r = \frac{V}{\sqrt{gl}} \quad (2.4)$$

In free-flight dynamics gravity is of great importance because the ratio between motion acceleration and gravity acceleration must be constant. The significance of Froude number as applied to an aeroplane is shown by the combination of the Froude number with the relative mass parameter μ_L as follows:^{4 1}

$$\frac{\mu_L}{F_r^2} = \frac{2mg}{\rho s V^2} = C_{Lo} \quad (2.5)$$

where C_{Lo} is the lift coefficient in the initial steady flight condition. This shows that the Froude number determines the steady flight condition and the equality of Froude number ascertains that a model flies at the same reference angle of attack as the full scale aircraft. Implicit in the above statement is the fact that the model produces the lift necessary to support its own weight.

The Froude number equality results in the following expression for velocity ratio:

$$F_{r_m} = F_{r_a}$$

$$\frac{V_m}{V_a} = \sqrt{\lambda} \frac{g_m}{g_a} \quad (2.6)$$

Now if acceleration due to gravity on the model and aircraft is assumed the same then

$$\frac{V_m}{V_a} = \sqrt{\lambda} \quad (2.7)$$

It is clear from the above expression that where the model flies under natural gravity i.e. model lift supports model weight, the velocity ratio is dependent on geometric ratio of the model and full scale aircraft. It is also evident that if geometric and velocity scale factors are established independently, then the requirement of Froude number equality is difficult to achieve without changing the apparent gravitational effect.

In order to carry out similar wind tunnel tests, it is often desirable to have the velocity ratio independent of geometric scale factor. Let us consider a case where $\frac{V_m}{V_a} < \sqrt{\lambda}$ i.e. model is flying at a lower velocity than demanded by Froude number equality. Then from equation (2.6), we get

$$g_m / g_a < 1$$

which implies that for proper simulation the acceleration due to gravity acting on the model should be less than that on the aircraft. In other words the model should produce less lift than weight in this configuration, and in order to fly the model in the wind tunnel, its weight has to be compensated. This technique is used at

DVLR³⁷ as discussed earlier.

(v) Reynold's Number. Theoretically for dynamic simulation Reynold's number or ratio of inertia to viscous forces is required to be the same for the model and the aircraft. It can be seen from the following consideration that the use of a scaled down model makes it impossible to simulate the Reynold's number correctly in the dynamic wind tunnel test using air only.

$$R_{e_m} = \frac{V_m l_m}{\gamma_m}, \quad R_{e_a} = \frac{V_a l_a}{\gamma_a}$$

Let us consider the case where gravitational equality is maintained.

Then $\frac{V_m}{V_a} = \sqrt{\lambda}$, giving

$$\frac{R_{e_m}}{R_{e_a}} = \frac{\gamma_a}{\gamma_m} \lambda^{3/2} \quad (2.8)$$

It is clear from the above expression that in the case of dynamically similar models, there is a scaled down relationship between the full scale aircraft and model Reynold's number in terms of the fixed parameters.

Figure 13 illustrates the relationship between scale factor and the dimension on which Reynold's number is based for three values of model Reynold's number. It is assumed that both aircraft and model are operating at sea level and full scale Mach No. is 0.3. Based on the same assumption figure 14 shows the model span necessary to achieve a Reynold's number of 0.5 million (based on mean chord) plotted

against the full scale span for a number of aspect ratios. It is evident that to achieve both dynamic similarity and a reasonable test Reynold's number would entail the use of very large models when the representative aircraft has large span and aspect ratio. Conversely, models of economic size can be employed only when the aspect ratio is low.

It is a general practice in dynamic wind tunnel testing to achieve a Reynold's number in the wind tunnel higher than the critical value ($Re = 0.4 \times 10^6$) at which the flow becomes turbulent.

(b) Wind Tunnel

The test section size and maximum speed of the wind tunnel selected for dynamic simulation impose direct constraint on geometric scale ratio and velocity ratio. If the size of the test section is large, larger models can be accommodated thus giving more room in the model for instrumentation and controls. Higher wind tunnel speeds can be used to simulate the corresponding full scale flight condition of higher Mach number.

There are a number of low speed wind tunnels in the College of Aeronautics but only two are large enough and have the desired speed range to be considered suitable for dynamic wind tunnel testing. They are the 8' x 6' closed test section tunnel and the 42" diameter open jet tunnel. Finally, the 42" diameter 'Weybridge' tunnel was preferred over the 8' x 6' tunnel for the following reasons:-

(i) Availability Being the largest tunnel in the

Department, the 8' x 6' tunnel is heavily committed to various other research activities apart from scheduled MSc course laboratory exercises. The Weybridge tunnel is usually comparatively less committed and can accommodate this additional activity.

(ii)Open-Jet Test Section The 'Weybridge' tunnel has an open jet test section which gives greater flexibility as far as the installation of the model support system rig is concerned. Thus the idea of a model suspension frame which can be rolled in and out of the tunnel as a complete unit was conceived.

The 'Weybridge' tunnel is an open jet return flow wind tunnel, figure 15 gives the general layout and leading dimensions of the tunnel. The tunnel has a maximum speed of 130 ft/sec. The nozzle has a circular cross-section of 42" diameter. A clear length of 5 feet is available between the nozzle and return pipe face.

Since the 'Weybridge' tunnel calibration was not available, it was decided to re-calibrate the wind tunnel. The calibration was carried out in three planes, at the jet efflux face, at 28" and 44" from the jet efflux face. At each plane, a pitot and static pressure survey of the complete jet was carried out, and values were recorded in a grid pattern of 2" vertical by 4" horizontal distances. From these readings variations in local velocity from the mean velocity were calculated. The complete calibration is reported in reference 42. The calibration was carried out to ascertain the useful area of working section where jet velocity variation is low so that the geometric scale ratio of the model can be determined.

(c) Cost and Complexity The resources available for this investigation were limited and the project was to be supported by the departmental funds with maximum use of local workshop facilities to keep the costs within a limit. The above considerations contributed to the choice of the least complex technique and the subsequent compromises in the design of sub-systems. However, the design is sufficiently flexible to allow for ready improvement, modification and expansion as and when necessary.

2.4.2 Technique Selected

From the literature research of free-flight techniques used in various aeronautical research establishments, it is evident that a variety of techniques have been used for free-flight dynamic testing. However, free and semi-free flight techniques using conventional wind tunnels are limited in number. The wind tunnel free-flight techniques require large test sections so that the models have sufficient space for the desired test manoeuvres or flight paths. The models currently in use for free-flight techniques are non-expendable because of cost and complexity in making these highly sophisticated models and requires elaborate safety features to protect against accidental damage. As these models have up to six degrees of freedom, a number of controllers or pilots are required to control the model effectively and precisely and add further to the cost and complexity. The limited size of the 'Weybridge' tunnel test section and the necessity to keep the technique simple restricted the choice to a semi-free flight technique.

Among the semi-free-flight techniques discussed

earlier in this chapter, the rigid rod type of model suspension technique shown in figure 12 was selected for the following reasons.

(a) The technique is relatively simple and can be employed at a small scale to which the present facility is restricted.

(b) The rod and the supporting frame can be made quite rigid and suitably braced so that the model dynamics can be made free from support influence.

(c) It allows unpowered models to be used thus avoiding the complexity of a powered model. Unpowered models are relatively inexpensive, light and simple to construct. The absence of a power plant allows more internal volume for instrumentation and controls.

(d) Although the rigid rod suppresses the longitudinal and lateral translation of the model, it does not effect the short-period dynamics of the model (figure 16).

CHAPTER 3

DESIGN AND CONSTRUCTION

3.1 Introduction

As mentioned earlier, the overall design of the facility was determined by the available wind tunnel and the necessity for simplicity and low cost. Hence the rigid rod technique was selected to be used in the 42" Weybridge wind tunnel. The primary objective was to build a facility that would allow a dynamically representative aircraft model to fly in the wind tunnel with as many of the natural degrees of freedom as reasonably possible. In order to achieve this objective the design and construction process was required to produce a controllable dynamically scaled aircraft model, a suspension system for the model, and an electronic control unit to interface with the model.

3.2 Model Suspension System

3.2.1 Introduction

The heart of the suspension system, designed for the facility is a vertical rod mounted in bearings at its upper and lower ends so that it may rotate about the vertical axis. The rod is carried by means of its upper and lower bearings mounted in plates which are suspended in a large transportable "DEXION" framework by means of rigid wire bracing. A sleeve is keyed into the vertical rod such that it may slide freely but is constrained to rotate with the rod. The sleeve thus forms part of the gimbal system which is mounted in the model. Thus the model is free to rotate in pitch and roll about the

sleeve piece, rotation in yaw is about the rod vertical axis and vertical translation involves the sleeve sliding on the rod.

3.2.2 Gimbal System

The vertical rod mounting technique suppresses longitudinal and lateral translation. The model suspension was designed to retain the remaining four degrees of freedom with as much movement as was reasonably possible. The gimbal system thus designed was to provide pitch and roll freedom and provision to measure these attitudes accurately. It was also to be designed to allow the model to fly freely along the longitudinal axis of the vertical rod. In order to minimise the external airflow interference on the model, the gimbal system attachment to the model had to be housed inside the model fuselage which produced the constraint on overall size of the gimbal arrangement, dictated primarily by the model fuselage width. Since the scaled model weight for a given flight condition (altitude) is fixed, the weight of the gimbal arrangement, which forms a part of the model weight, had to be kept to a minimum whilst not sacrificing the desired strength of the gimbal to withstand the aerodynamic and shock loads caused by the model impacting the vertical motion end stops.

The final design of the gimbal arrangement is presented in figure 17 and plate 1. The gimbal arrangement consists of two frames comprising the outer and the inner gimbal. The outer gimbal provides roll freedom to the model. It houses a roll attitude measuring potentiometer and $\pm 30^\circ$ roll angle safety lock to restrain the model to controllable limits whilst remaining sufficiently practical. The

potentiometers used for measuring attitude angles are Penny and Giles rotary conductive plastic potentiometers with virtually infinite resolution. These high quality potentiometers were selected for their accuracy, sensitivity and resolution, the characteristics required for accurate measurement of model attitude. The outer gimbal also provides the attachment between the suspension system and the model. The gimbal is bolted to the model, about its c.g. point, the securing bolts passing through elongated holes in the gimbal frame to allow for adjustment of model pivot point and c.g.. The inner gimbal provides the pitching freedom of the model, it houses the pitch attitude measuring potentiometer and the $\pm 30^\circ$ pitch attitude lock. Although the above mentioned high resolution potentiometers were of the smallest size available, the casing of the pitch potentiometer had to be cut down to allow the potentiometer to fit into the small space available between the two gimbal frames, a constraint due to the limited model fuselage width. The inner gimbal is attached to the vertical sleeve which slides along the vertical rod of the suspension thus providing the vertical freedom to the model. To reduce the effect of friction, the shaft attachments between the outer and the inner gimbal frames and the sleeve are carried in precision bearings with small ball races, whereas the sliding vertical sleeve incorporates precision linear ball bearings.

3.2.3 Vertical Rod

The vertical rod, by the very name of the technique selected, had to be very rigid for the rigid body dynamics simulation. It had also to provide yaw

and vertical freedom to the model and provision for the measurement of yaw attitude.

The salient parameters of the designed rod are given in table B.1 of Appendix B. The length of the rod represents the maximum vertical travel of the model in the wind tunnel, and hence the wind tunnel test section size imposes the maximum length constraint on the rod design. In the absence of large compression loads on the rod, which could cause buckling, the bending deformation due to the aerodynamic loads (drag) on the vertical rod, was the main design criterion in determining the rod length, diameter and material. Like any design process, the design variables have a contradictory influence on the design requirement, e.g. larger diameter size had favourable effect on the rigidity and hence bending criteria, but it had an adverse effect on the design constraint on the gimbal sleeve diameter. A larger rod diameter is also undesirable as it would have an adverse interference effect on the airflow around the model. The moment of inertia of the rod also increases with an increase in diameter, which is undesirable as it contributes to the model yaw inertia.

The vertical and yaw freedom are separated by the following design arrangements. The rod is grooved (plate 1) along its length and a nylon key is installed in the vertical sleeve of the gimbal system (figure 17) which runs in the groove. This enables the gimbal (and hence the model) to move freely in vertical direction whilst constraining it to rotate about the yaw axis with the rod. The rod is mounted on the upper and lower end plates (figure 18) in precision ball bearings thus allowing it to rotate in yaw. Since the vertical rod moves with the model in yaw,

the yaw moment of inertia of the model includes contribution from the vertical rod but it is very small, as seen from table B.1, by virtue of the small rod diameter.

3.2.4 Mounting Frame

The upper and lower end plates & vertical rod assembly is rigidly wire braced to a large "DEXION" frame as shown in figure 18 and plate 2. The "DEXION" frame was constructed primarily to avoid rigging the model in the wind tunnel thus saving installation and removal time and avoiding excessive handling of the model and support system. It also provides a very rigid platform for the facility. The main advantage is that the model can be prepared for test outside the wind tunnel and the whole facility moved into the wind tunnel as required.

3.3 Aircraft Model

3.3.1 Introduction

A dynamically scaled model of the British Aerospace Hawk was chosen as the subject for the development of the facility. The reasons for the choice were that when scaled to suit, the model has sufficient volume to incorporate the controls and suspension mechanism, dimensional and other data was readily available and it was of topical interest. To suit the wind tunnel, the model was geometrically scaled in the ratio 1:12. A three view drawing of the model is shown in figure 19 and its leading geometric particulars are given in table B.2 of Appendix B. Dynamic scaling of the aircraft imposed a maximum model weight limit of 5.2 pounds

for sea level flight conditions (table B.3). As this weight would include the contained suspension mechanism, sensors, servo actuators and weight allowance for mass and inertia balancing, it was necessary to reduce the basic air frame weight to a minimum. To meet this requirement, the model was constructed from balsa wood with lightweight hardwood for the wings and tail surfaces. The model is controlled by the usual surfaces, ailerons, all moving tailplane and rudder.

3.3.2 Design Description

The model fuselage was built using conventional "keel and former" aeromodelling techniques^{4 3} as shown in plates 3 and 4. $\frac{1}{8}$ " thick balsa sheet was used for the construction of the keel and bulkheads which were traced and cut to shape from $\frac{1}{12}$ scale model drawings, these drawings having been prepared from geometric data provided by BAe. Bulkheads were chosen such that an exact fuselage profile could be obtained. In all 25 bulkheads were used to ensure the desired contours and were cut away inside to give a hollow structure to the fuselage to provide sufficient space for sensors, servo actuators and mass and inertia balancing. The fuselage structure was strengthened by using balsa longerons following the contours of the fuselage as seen in plate 4. The fuselage structure also housed a strong central box-like structure for mounting the suspension gimbal and wing attachment (plate 5). Removeable access panel structure, vertical rod cut-out and empennage attachment areas were also reinforced using hardwood. The model fuselage was skinned with $\frac{1}{8}$ " thick balsa sheet which gave a very strong and rigid structure.

The wing was constructed from lightweight hardwood for accuracy and strength as it was found from the preliminary experiments with a $1/15$ scale model that balsa wood was not adequate for this purpose for both accuracy and strength reasons. The wing airfoil section geometry was approximated to from inspection of the Hawk drawings provided by BAe. The model wing has a scaled span of 30.8 inches, leading edge sweep of 26° , taper ratio of 0.34 and 2° dihedral. The outer trailing edges of the wing carry scaled down ailerons. The two wing halves were joined together to form a one-piece assembly (plate 6) and a round hole cut to provide passage for the vertical rod of the suspension system and allowance for $\pm 30^\circ$ freedom in pitch and roll. The wing-fuselage attachment is by three bolts which engage with captive nuts secured in the fuselage structure thus permitting easy removal for internal access.

The ailerons are actuated through control rods by a small electrically driven precision D.C. servo actuator installed in the centre section of the wing (plate 6). The aileron control hinge extends along the span of the wing and is embedded in it, the details are shown in figure 20.

The fin and rudder and all moving horizontal tail are also made of light hardwood. The fin is permanently fixed to the fuselage. Metal inserts were embedded into the horizontal tail to provide secure hinge attachments and 10° anhedral angle was provided by drilling the hinge axle hole at the appropriate angle in the metal inserts. Both surfaces were joined together by a common hinge rod which was mounted through the fuselage in alloy bearing blocks as shown in figure 20. The rudder

and the horizontal tail are actuated by the D.C. servo actuators mounted in the front fuselage bay accessible through a detachable access panel (plate 6), comprising the cockpit canopy.

The front fuselage bay also houses two miniature 'D' type sockets which provide electric connections to all servo actuators and sensors installed in the model. The model is linked to the electronic control unit through an umbilical cord which plugs into these sockets.

The overall weight of the assembled model including the weight of the servo actuators, linkage and gimbal system came out to be about two pounds less than the required scaled weight of the model and provided sufficient margin for mass and inertia scaling with respect to the c.g. position. The method adopted to measure moment of inertia was essentially the same as reported in reference 44. A procedure for calculating the weight, its dimension and location with appropriate inclination to the three axes for satisfying the moment of inertia requirements is given in reference 38. As the model is small and access is very limited it was found quite impractical to apply the procedure in this case. A trial and error procedure of shifting weights to bring the model inertia to scaled inertia values was adopted but was found to be very time consuming and tiresome. The final values of model weight and inertia are given in table B.4 (Appendix B)

3.4 Electronic Control Unit

3.4.1 Introduction

The electronic control unit is a relatively small

transportable console and interfaces with the model by means of simple electrical connections. The unit is constructed in modular fashion to ease fault diagnosis and further functional expansion. The following functions are provided by the unit.

- (i) Primary control of the model
- (ii) Programmable analogue computer elements for feedback purposes
- (iii) Output signals for recording and display
- (iv) Input and Output signal interfaces
- (v) Electrical power supplies

Figure 21 and plate 7 show the panels of the electronic control unit. Figure 22 is a detailed functional block diagram of the electronic control unit and its interfaces with the model.

A major design requirement from the outset was that all electronic control unit functions should be selectable and hence programmable by means of external patch cord connections as used in a general purpose analogue computer.

The control panel incorporates a three axis control stick for manual control of the model and three control switches to input known signal forms to the control surfaces for example a step or a pulse input.

The model input panel displays external inputs to the model which are obtained from the signal conditioning amplifiers for elevator, aileron, rudder and an auxilliary control channel reserved for future expansion. The input to each amplifier includes a trim potentiometer for trimming the

corresponding flight control for steady flight condition and two sockets with gains of 1 and 10 are also provided for each control surface for feedback loop and other external inputs from the control panel. The conditioned demand signals are monitored by the level indicators. Two sockets are also provided for quantitative monitoring of the model input signal.

The model output signals are fed to the model output panel via signal conditioning amplifiers. Ten output amplifiers are provided of which six condition pitch, roll and yaw attitudes and elevator, aileron and rudder angles respectively. The remaining four auxilliary amplifiers are reserved for further instrumentation expansion. A gain selection switch is provided to scale each amplifier sensitivity factors of 1 or 10. The output sockets thus provide the conditioned signals for use in feedback loops and also for monitoring and recording the model response.

The analogue feedback elements panel provides the input and output terminals of 6 integrators and 12 summers. Two input points of unity gain, one input point for gain 10 and three output points are available for each integrator. The summers also have three output points but have four input points, i.e. two of unity gain and two of gain 10. These analogue computer elements are designed to be used for feedback or other signal shaping purposes to provide the means for experimental investigation of stability and control augmentation systems.

The potentiometer panel houses 24 high stability, single turn, cermet track potentiometers laid on the panel in six groups of four. The potentiometers

in groups A and B have free earth connections whilst the remaining are earthed at one end. The input and output arms of the potentiometers are connected to their corresponding sockets on this panel for use by means of external connections.

The monitor panel houses a 0 - ± 10 volts scaled voltmeter for monitoring potentiometer settings or reading signals from any part of the analogue circuitry. Six potentiometer selection switches, one for each group of potentiometers, are also located on this panel. When a switch is depressed, it disconnects the corresponding potentiometers from the external connections, connects their inputs to ± 10 volts of reference supply and their outputs to the rotary selection switch. The rotary selection switch on selection connects each potentiometer to the meter for monitoring its setting against the ± 10 v reference. The selection of rotary switch to EXT. setting connects the meter to the external sockets for monitoring external signals.

The bus-bar panel houses two groups of 12 sockets each arranged four in a row. Each row of sockets is internally connected for providing multiple outlets for common connections.

The signal trunks panel houses two groups (A and B) of sixteen sockets each. Each group is internally wired to the trunks socket at the back of the electronic control unit for external connections to monitoring, recording or other equipment.

The mode logic panel contains the master OFF/ON switch for the electronic control unit. The compute mode switch on this panel, when depressed puts all the integrators of the feedback analogue elements panel into the compute mode. The

normal position of the switch corresponds to the reset mode of the integrators.

The power supply requirements of the electronic control unit for its various functions are as follows:-

- (i) ± 15 volts, 1 amp D.C. power supply for the 32 operational amplifiers being used in the unit for signal conditioning, summer and integrator circuits.
- (ii) ± 10 volts, 100 mA D.C. reference power supply for the various potentiometers in the unit, i.e. stick unit, trim pots and attitude sensors etc and for monitoring potentiometer settings.
- (iii) ± 5 volts, 1 amp D.C. power supply for the servo actuators used for actuating control surfaces of the model.
- (iv) 6 volts, 200 mA D.C. power supply for the indicator lights used on the mode logic panel.

High quality commercial power supplies were used except for the ± 5 volts 1 amp power supply. The design of this power supply is discussed later in the chapter.

3.4.2 Circuit Design

The functional circuits of the input control panel, output control panel and the feedback analogue elements panel are designed to utilise general purpose printed circuit boards stacked in a card

rack mounted inside the electronic control unit (plate 8). The electronic circuit cards for each panel function are designed to be identical for interchangeability. The modular nature of the design also helps in quick fault diagnosis.

The following two types of high gain operational amplifiers have been used in the circuits.

- (i) RS 741 (Equivalent - Texas Instruments μ A 741)
This is an inexpensive general purpose operational amplifier satisfactory for low frequency applications. These amplifiers were used in all summers including input and output signal conditioning amplifier circuits. In each case the input offset voltage is nulled out using a trim pot between offset adjustment terminals. A typical summer circuit is shown in figure 23a.

- (ii) RS J FET 355 (Equivalent μ AF 355)
This high performance amplifier with very low input offset current (10 pA) was found suitable for the integrator circuits after some experimental trials. The input offset voltage of this amplifier is also low and is further reduced using an offset adjustment circuit. A typical integrator circuit^{4,5} is shown in figure 23b.

Each model input card contains three input signal conditioning amplifiers for the corresponding three control surfaces. The card circuitry is given in figure 24. The command signals from the stick unit, trim pot, and the two external connection points on

the input panel are connected appropriately to each amplifier. The signals are conditioned to proper signal levels through various gains and are fed to the inputs of the model control surface servo actuators. The sensitivity of the control surface with the stick unit and the trim pot can be varied to suit by changing the corresponding input resistance to the amplifier.

Each model output card also contains three signal conditioning amplifiers for the corresponding three outputs from the model sensors. The circuit is shown in figure 25. The outputs from the model sensors are fed to the signal conditioning amplifiers through a gain selection switch (gain 1 or 10) on the output control panel. The signal is amplified to suitable values by adjusting the variable feedback resistor (0 - 100 K) of each amplifier. The output of the amplifiers is then fed to the output control panel.

Each analogue element card contains one integrator and two summers. The circuit is given in figure 26. Each integrator is designed for three inputs, two of which have unity gain and the third has a gain of 10. The feedback capacitor of the integrator is shorted out by means of the Reset/Compute switch. Each summer is designed for four inputs, two of unity gain and two of gain 10. The output is fed back to the analogue element panel.

The design of ± 5 volts, 1 amp power supply for the servo actuators is presented in figure 27. The design procedure is taken from reference 46.

3.5 Comments on the Design and Construction

It is felt that the objectives of the design and construction of the facility are achieved, keeping in view the limited size, low cost of the equipment and overall simplicity of operation.

However, suggestions for improvement in both the designed facility and operating procedures are given in Chapter 7.

CHAPTER 4

THEORETICAL BACKGROUND

4.1 Longitudinal Dynamics

The investigation of longitudinal stability and control involves the study of the characteristics exhibited by the aircraft in the plane of symmetry (figure 28a). This plane of symmetry divides the aircraft into two symmetrical halves and contains components of motion only along the X (drag) and Z (lift) axes and about the Y (pitch) axis. Aircraft motion in the plane of symmetry, i.e. longitudinal motion, generally results in insignificant motion in the plane of asymmetry, i.e. lateral and directional motion. Therefore, longitudinal stability and control can be investigated independently from lateral-directional stability and control.

4.1.1 Equations of Motion

(i) Free Flight

The linearised dimensionless longitudinal equations of motion (Appendix C) are obtained from references 47 and 48. The equations are developed using the small perturbation theory for an aircraft in the three degrees of longitudinal freedom. These non-dimensional equations referring to wind axes (stability axes) are:-

$$(\hat{D} + x_u) \hat{u} + (x_w \hat{D} + x_w) \hat{w} + x_q \hat{q} + \hat{g}_1 \theta + x_\eta \dot{\eta} = 0$$

$$z_u \hat{u} + \left[(1 + z_w) \hat{D} + z_w \right] \hat{w} + (z_q - 1) \hat{q} + \hat{g}_2 \theta + z_\eta \dot{\eta} = 0$$

$$m_u \hat{u} + (m_w \hat{D} + m_w) \hat{w} + (\hat{D} + m_q) \hat{q} + m_\eta \dot{\eta} = 0$$

(4.1)

where

$$\hat{q} = \hat{D} \theta$$

$$\hat{g}_1 = \hat{g} \cos \Theta_e = C_L$$

$$\hat{g}_2 = \hat{g} \sin \Theta_e = C_L \tan \Theta_e$$

$$\hat{g} = \frac{mg}{\frac{1}{2} \rho_e V_e^2 S} = C_L \sec \Theta_e$$

and the differential operator is

$$\hat{D} = \frac{d}{dt}$$

The equations (4.1) can be further simplified by noting that for most aircraft, the dimensionless derivatives, x_q , z_q , x_w and z_w are insignificantly small and are found to be of secondary importance. Also, in case of steady level flight $\tan \Theta_e = 0$ for wind axes. (Figure 29)

The resulting simplified equations are:-

$$\begin{array}{l} \begin{array}{c} 1 \\ \hline \hat{D} + x_u \end{array} \hat{u} + \begin{array}{c} 2 \\ \hline x_w \hat{D} + x_w \end{array} \hat{w} + \begin{array}{c} 3 \\ \hline C_L \theta \end{array} = \begin{array}{c} 4 \\ \hline -x_\eta \dot{\eta} \end{array} \\ \begin{array}{c} 2 \\ \hline z_u \hat{u} \end{array} + \begin{array}{c} 2 \\ \hline (\hat{D} + z_w) \hat{w} \end{array} - \begin{array}{c} 3 \\ \hline \hat{q} \end{array} = \begin{array}{c} 4 \\ \hline -z_\eta \dot{\eta} \end{array} \\ \begin{array}{c} 3 \\ \hline m_u \hat{u} \end{array} + \begin{array}{c} 2 \\ \hline (m_w \hat{D} + m_w) \hat{w} \end{array} + \begin{array}{c} 3 \\ \hline (\hat{D} + m_q) \hat{q} \end{array} = \begin{array}{c} 4 \\ \hline -m_\eta \dot{\eta} \end{array} \end{array} \quad (4.2)$$

Where row 1 represents Drag Characteristics
 row 2 represents Lift Characteristics
 row 3 represents Pitching Moment
 Characteristics^{ics}

and

column 1 represents terms generated by changes
 in horizontal velocity
 column 2 represents terms generated by changes
 in angle of attack
 column 3 represents terms generated by changes
 in pitch altitude
 column 4 represents terms generated by changes
 in elevator angle

Where $\hat{q} = \hat{D}\theta$

and $\hat{D} = \frac{d}{dt}$

(a) Longitudinal Dynamic Modes

The longitudinal dynamic modes may be determined from the characteristic equation obtained from the dimensionless longitudinal equation (4.2). This is achieved by considering free motion ($\eta=0$) and assuming solution of equation (4.2) of the form

$$\hat{u} = \rho_1 e^{\lambda \hat{t}}, \quad \hat{w} = \rho_2 e^{\lambda \hat{t}}, \quad \hat{\theta} = \rho_3 e^{\lambda \hat{t}} \quad (4.3)$$

where ρ_1, ρ_2, ρ_3 and λ are constants, and $\hat{t} = t/\tau$

The characteristic equation thus obtained is

$$\begin{vmatrix} \lambda + x_u & x_w & C_L \\ z_u & \lambda + z_w & -\lambda \\ m_u & \lambda m_w + m_w & \lambda^2 + \lambda m_q \end{vmatrix} = 0 \quad (4.4)$$

Equation (4.4) yields the longitudinal quartic characteristic equation in terms of dimensionless operator λ

$$A_1 \lambda^4 + B_1 \lambda^3 + C_1 \lambda^2 + D_1 \lambda + E_1 = 0 \quad (4.5)$$

The coefficients in the above equation in terms of the aerodynamic stability derivatives are

$$\begin{aligned} A_1 &= 1 \\ B_1 &= x_u + z_w + m_q + m_{\dot{w}} \\ C_1 &= (x_u z_w - x_w z_u) + m_q (x_u + z_w) + m_{\dot{w}} x_u + m_{\dot{w}} z_w \\ D_1 &= m_q (x_u z_w - x_w z_u) + m_{\dot{w}} z_u C_L + m_{\dot{w}} x_u \\ E_1 &= m_{\dot{w}} z_u C_L \end{aligned} \quad (4.6)$$

For a conventional aircraft the coefficients D_1 and E_1 in the characteristic equation are usually small as compared to C_1 and equation (4.5) can approximately factorise into the form

$$(\lambda^2 + B_1 \lambda + C_1) \left(\lambda^2 + \frac{C_1 D_1 - B_1 E_1}{C_1^2} \lambda + \frac{E_1}{C_1} \right) = 0 \quad (4.7)$$

The longitudinal motion represented by equation (4.7) exhibits two different types of longitudinal oscillation.

- (i) One of short period with relatively heavy damping, called the "Short Period" mode.
- (ii) Another of long period with very light damping, called the "Phugoid" mode.

(b) Short Period Oscillation

From equation (4.7), the short period pitching oscillation is described by the complex conjugate pair of roots,

$$(\lambda^2 + B_1\lambda + C_1) = 0 \quad (4.8)$$

describing the mode characteristics, undamped natural frequency, $\omega_{sp} = \sqrt{C_1}$

$$\text{damping ratio, } \zeta_{sp} = \frac{B_1}{2\sqrt{C_1}} \quad (4.9)$$

where

$$B_1 = x_u + z_w + m_q + m_{\dot{w}}$$

and

$$C_1 = (x_u z_w - x_w z_u) + m_q (x_u + z_w) + m_{\dot{w}} x_u + m_w$$

The Short Period mode is characterised primarily by variation in angle of attack and pitch angle with very little change in forward speed. Relative to the Phugoid, the Short Period has a high frequency and heavy damping. Typical values for its damped period are in the range of 2-5 seconds. Generally, the short period motion is very important in the handling qualities context since it provides a dominant contribution to aircraft response at frequencies similar to human pilot frequency.

(c) Phugoid Oscillation

From equation (4.7) the Phugoid oscillation is

described by the complex conjugate pair of roots

$$\lambda^2 + \frac{(C_1 D_1 - B_1 E_1)}{C_1^2} \lambda + \frac{E_1}{C_1} = 0 \quad (4.10)$$

resulting in
undamped natural frequency $\omega_{ph} = \sqrt{\frac{E_1}{C_1}}$ (4.11)

$$\text{damping ratio } \zeta_{ph} = \frac{C_1 D_1 - B_1 E_1}{2 \sqrt{E_1 C_1^3}}$$

where

$$B_1 = x_u + z_w + m_q + m_w$$

$$C_1 = (x_u z_w - x_w z_u) + m_q (x_u + z_w) + m_w x_u + m_w$$

$$D_1 = m_q (x_u z_w - x_w z_u) + m_w z_u C_L + m_w x_u$$

$$E_1 = m_w z_u C_L$$

The Phugoid mode is characterised mainly by variation in speed u and pitch attitude θ with incidence α nearly constant. This long period oscillation can be thought of as a constant total energy situation with cyclic exchange of potential and kinetic energy.

Because of low damping, the motion may decay over many cycles. However, the long period combined with the low damping results in an oscillation that is easily controlled by the pilot even when marginally divergent. When the pilot is in the loop, he is frequently not aware that the Phugoid mode exists as he continuously corrects the motion with control inputs. Typical values for the

period are in the order of 45 to 90 seconds.

(ii) Wind Tunnel Simulation

The model is suspended in the wind tunnel so that the x-axis is suppressed thereby allowing the drag force equation to be removed from equations (4.2). Also, the air velocity in the tunnel is constant during a test run and hence the terms describing the perturbation in velocity u can also be removed from equations (4.2). Thus the longitudinal equations reduce to

$$\begin{aligned} (\hat{D} + z_w) \hat{w} - \hat{q} &= -z_\eta' \eta \\ (m_w \hat{D} + m_w) \hat{w} + (\hat{D} + m_q) \hat{q} &= -m_\eta' \eta \end{aligned} \quad (4.12)$$

and the corresponding characteristic equation

$$\begin{vmatrix} \lambda + z_w & -1 \\ \lambda m_w + m_w & \lambda + m_q \end{vmatrix} = 0 \quad (4.13)$$

or

$$\lambda^2 + B_2 \lambda + C_2 = 0 \quad (4.14)$$

resulting in

undamped natural frequency $\omega_{sp} = \sqrt{C_2}$

and

damping ratio $\zeta_{sp} = \frac{B_2}{2\sqrt{C_2}}$

where

$$\begin{aligned} B_2 &= z_w + m_q + m_w \\ C_2 &= z_w m_q + m_w \end{aligned} \quad (4.15)$$

Equations (4.14) and (4.15) describe the Short Period pitching oscillation mode of model motion in wind tunnel simulation. It is a modified mode due to above mentioned restrictions but its frequency and damping terms contain all the important stability derivatives which contribute to the Short Period mode in free flight.

(iii) Comparison

The comparison of equations (4.7), (4.9) and (4.14) highlight the following characteristics of the wind tunnel simulation.

- (a) The wind tunnel simulation technique has suppressed the Phugoid mode in the longitudinal dynamics. This is due to the fact that the Phugoid mode comprises airspeed and attitude variation and by suppressing longitudinal freedom and by keeping the tunnel speed constant, the variables essential to the Phugoid mode are removed.
- (b) The Short Period mode can be effectively demonstrated by the simulation technique selected. The undamped short period frequency, ω_{sp} , and damping ratio ζ_{sp} terms in equations (4.9) and (4.14) are similar except for terms due to the perturbation in forward velocity. But, as the short period response is very quick and highly damped, the short time interval does not allow significant speed changes in either the free flight or the simulated aircraft. Hence, terms due to perturbation in the forward velocity can be safely omitted.

4.1.2 Effects of Various Parameters on the Longitudinal Simulated Short Period Pitching Oscillation

The characteristics of this motion to be investigated are defined by,

- (a) Frequency or period of motion
- (b) Damping of the motion.

(i) Frequency

The undamped natural frequency, ω_{sp} , is given by the equation (4.15). Through several mathematical manipulations, the frequency equation can be written in terms of various parameters in the following two forms

$$(i) \quad \omega_{sp} = \frac{1}{(k_y / \bar{c})} \sqrt{-\mu_1 (M_w + a \frac{M_q}{\mu})} \quad (4.16)$$

and

$$(ii) \quad \omega_{sp} = \sqrt{\frac{\gamma/2 P_a M^2}{I_y} \bar{s} \bar{c} a (\bar{x}_{c.g.} - N_m)} \quad (4.17)$$

where equations (4.16) and (4.17) represents undamped Short Period mode frequency in terms of stability derivatives and flight condition respectively.

Several important relationships can be gathered from the study of equations (4.16) and (4.17).

- (a) The undamped natural frequency of the short period motion increases as Mach number increases. In the subsonic flight regime, the airspeed variation has the same effect.

- (b) The undamped natural frequency of the short period motion decreases with increasing pressure altitude at a constant Mach number.
- (c) The undamped natural frequency of the short period motion decreases as the airplane cg is moved aft toward the stick-fixed manoeuvre point. When the cg is at the stick fixed manoeuvre point, the frequency becomes zero, i.e. the motion becomes nonoscillatory.
- (d) The undamped natural frequency of the short period motion decreases with an increase in moment of inertia in pitch. This is analogous to increasing the mass in a spring-mass-damper system.
- (e) The undamped natural frequency of the short period motion increases with M_w , the pitch stiffness aerodynamic stability derivative. The contribution to frequency of the pitch damping derivative M_q is negligible for high performance aircraft having large μ_1 .

(ii) Damping Ratio

The damping ratio of the Short Period mode, ζ_{sp} , can be expressed as follows:-

$$\zeta_{sp} = \frac{\frac{a}{2} \left(\frac{k_y}{c} \right) - \frac{M_q}{2 \left(\frac{k_y}{c} \right)} \left(1 + \frac{d\epsilon}{d\alpha} \right)}{\sqrt{-\mu_1 (M_w + a M_q / \mu_1)}} \quad (4.18)$$

and

$$\zeta_{sp} = \frac{\sqrt{\rho S_{\perp} / 2}}{2 \sqrt{-\frac{\bar{c}}{I_y} a(x_{cg} - N_m)}} \left[\frac{a}{W/g} - \frac{M_{\dot{q}} \bar{c}^2}{I_y} - \frac{M_{\dot{w}} \bar{c}^2}{I_y} \right] \quad (4.19)$$

The following important observations can be made from equations (4.18) and (4.19).

- a) Pitch rate damping increases with lift curve slope and with the "downwash lag" term.
- b) Increasing the 'pitch stiffness stability derivative, $M_{\dot{w}}$, decreases short period damping.
- c) Moving the cg forward decreases short period damping.
- d) Damping of the Short Period mode is not a direct function of airspeed or Mach number.
- e) Damping of the Short Period mode decreases with increasing μ_1 , the aircraft longitudinal relative density parameter.
- f) The pitch rate damping stability derivative $M_{\dot{q}}$, because of its magnitude is the most dominant term contributing to short period damping.
- g) The vertical damping term $\frac{a}{W/g}$ also contributes significantly to total damping and in a conventional configuration its contribution can be in the range of 20-50%.

4.2 Lateral Dynamics

The investigation of lateral stability and control involves the study of characteristic motion exhibited in the aircraft's planes of asymmetry. Asymmetric motion comprises translation along the Y-axis (side force) and rotation about the X (roll) and Z(yaw) axes (Figure 28b). Lateral motion generally results in some motion in the plane of symmetry but as this is of very small amplitude it will be considered insignificant in this study for the flight conditions of interest.

4.2.1 Equations of Motion

(i) Free Flight

As for the longitudinal case, the linearised dimensionless lateral equations of motion (Appendix C) are obtained from references 47 and 48. These dimensionless equations are:-

$$(\hat{D}+y_v)\hat{v}+y_p\hat{p}+(1+y_r)\hat{r}-g_1\hat{\phi}-g_2\hat{\psi}+y_\xi\hat{\xi}+y_\zeta\hat{\zeta}=0$$

$$l_v\hat{v}+(\hat{D}+l_p)\hat{p}+(e_x\hat{D}+l_r)\hat{r}+l_\xi\hat{\xi}+l_\zeta\hat{\zeta}=0$$

$$n_v\hat{v}+(e_z\hat{D}+n_p)\hat{p}+(\hat{D}+n_r)\hat{r}+n_\xi\hat{\xi}+n_\zeta\hat{\zeta}=0$$

where

$$\begin{aligned}\hat{p} &= \hat{D}\phi \\ \hat{r} &= \hat{D}\psi \\ e_x &= -i_{zx}/i_x = -I_{zx}/I_x \\ e_z &= -i_{zx}/i_z = -I_{zx}/I_z\end{aligned}\quad (4.20)$$

$$\begin{aligned}\hat{g}_1 &= g\cos\Theta_e = C_L \\ \hat{g}_2 &= g\sin\Theta_e = C_L \tan\Theta_e\end{aligned}$$

and

$$\hat{D} = d/\hat{dt}$$

In general the stability derivatives y_p and y_r are very small and can be neglected. The resulting simplified equations are:-

$$\begin{array}{l} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \left| \begin{array}{c} \hat{D} + y_v \end{array} \right| \hat{v} \\ \left| \begin{array}{c} l_v \end{array} \right| \hat{v} \\ \left| \begin{array}{c} n_v \end{array} \right| \hat{v} \end{array} \begin{array}{c} \left| \begin{array}{c} -C_L \phi \end{array} \right| \\ \left| \begin{array}{c} +(\hat{D}^2 + l_p \hat{D}) \phi \end{array} \right| \\ \left| \begin{array}{c} + (e_z \hat{D}^2 + n_p \hat{D}) \phi \end{array} \right| \end{array} \begin{array}{c} \left| \begin{array}{c} + \hat{r} \end{array} \right| \\ \left| \begin{array}{c} + (e_x \hat{D} + l_r) \hat{r} \end{array} \right| \\ \left| \begin{array}{c} + (\hat{D} + n_r) \hat{r} \end{array} \right| \end{array} \begin{array}{c} \left| \begin{array}{c} = -y_{\xi}' \zeta - y_{\zeta}' \xi \end{array} \right| \\ \left| \begin{array}{c} = -l_{\xi}' \xi - l_{\zeta}' \zeta \end{array} \right| \\ \left| \begin{array}{c} = -n_{\xi}' \xi - n_{\zeta}' \zeta \end{array} \right| \end{array} \end{array} \quad (4.21)$$

Where row 1 represents side force characteristics
row 2 represents rolling moment characteristics
row 3 represents yawing moment characteristics
and

column 1 represents terms generated by changes in
side slip angle
column 2 represents terms generated by changes in
bank angle
column 3 represents terms generated by changes in
yaw rate
column 4 represents terms generated by changes in
control angle.

(a) Lateral Dynamic Modes

The lateral dynamic modes may be determined from the characteristic equation obtained from equation (4.21). This is achieved by considering free motion $\dot{\xi} = \dot{\zeta} = 0$ and assuming that there is a solution of the form

$$\hat{v} = \rho_1 e^{\lambda t}, \quad \hat{\phi} = \rho_2 e^{\lambda t}, \quad \hat{\psi} = \rho_3 e^{\lambda t} \quad (4.22)$$

where ρ_1, ρ_2, ρ_3 and λ are constant. The lateral characteristic equation thus obtained is

$$\begin{vmatrix} \lambda + y_v & -C_L & 1 \\ l_v & \lambda^2 + l_p \lambda & e_x \lambda + l_r \\ n_v & e_z \lambda^2 + n_p \lambda & \lambda + n_r \end{vmatrix} = 0 \quad (4.23)$$

the determinant (4.23) yields the quartic characteristic equation in terms of the dimensionless operator λ

$$A_3 \lambda^4 + B_3 \lambda^3 + C_3 \lambda^2 + D_3 \lambda + E_3 = 0 \quad (4.24)$$

where

$$A_3 = 1 - e_x e_z$$

$$B_3 = l_p + n_r - e_x n_p - e_z l_r + (1 - e_x e_z) y_v$$

$$C_3 = l_p n_r - l_r n_p - n_v + e_z l_v + (l_p + n_r - e_x n_p - e_z l_r) y_v$$

$$D_3 = l_v n_p - l_p n_v + C_L l_v - e_x C_L n_v + (l_p n_r - l_r n_p) y_v$$

$$E_3 = (n_r C_L) l_v - (l_r C_L) n_v$$

The roots of equation (4.24) determine the lateral stability characteristics, the roots correspond to the Roll Subsidence mode, the Spiral mode and the Dutch Roll mode.

(b) Roll Subsidence Mode

This mode is characterized by a large negative real root. The root is usually sufficiently large that it may be predicted approximately by retaining the λ^4 and λ^3 terms of the equation (4.24) only, whence

$$\lambda_{rm} = -\frac{B_3}{A_3} - \left(\frac{C_3}{\lambda} + \frac{D_3}{\lambda^2} + \frac{E_3}{\lambda^3} \right) \quad (4.25)$$

where terms in parenthesis are added to provide iterations to find λ_{rm} to any required accuracy. A very good approximation is

$$\lambda_{rm} \doteq -\frac{B_3}{A_3} \doteq l_p + n_r - e_x n_p - e_z l_r + y_v \quad (4.26)$$

where $e_x e_z \doteq 0$

The classic Roll mode is a heavily damped, first order, non-oscillatory mode of motion. The motion is largely rolling, corresponding to the rapid build-up in roll rate to a steady state value for a given lateral control input. This rolling motion is utilised by the pilot to vary and control bank angle. Thus, the characteristics of this mode have a major influence on the pilot's opinion of the manoeuvring capabilities of the airplane.

(c) Spiral Mode

This mode is characterised by a small real root and is usually small enough to be given accurately by the $D_3\lambda + E_3$ terms of equation (4.24), whence,

$$\lambda_{sm} = -\frac{E_3}{D_3} - \left(\frac{C_3}{D_3}\lambda^2 + \frac{B_3}{D_3}\lambda^3 + \frac{A_3}{D_3}\lambda^4 \right) \quad (4.27)$$

where again the extra terms suggest iteration which usually converges very rapidly. If E_3 is positive, then the Spiral mode is stable and the motion following an initial displacement, say in sideslip v , would consist of a slow convergence to a level altitude and straight flight path. If, on the other hand, E_3 is negative, then the mode will be unstable and after an initial displacement, the aircraft will enter an ever-steepening spiral descent.

The pilot does not normally excite the Spiral mode intentionally; however, it is excited whenever the bank angle of the aircraft is disturbed from a trimmed wings level condition. Therefore it has considerable influence on the aircraft lateral flying qualities. However, this mode is similar to the Phugoid mode in that a pilot can easily control it even if it is dynamically unstable provided the time constant is large.

(d) Dutch Roll Mode

The remaining two roots of equation (4.24) are a complex pair describing the Dutch Roll mode. There is no simple approximation for these roots. However, a procedure often used is to reduce the quartic to a quadratic by synthetic division, when the real roots λ_{rm} and λ_{sm} are known. To get better accuracy, λ_{rm} must be determined to high accuracy using the iterative process suggested by equation (4.25). The Dutch Roll mode is a tightly coupled yawing and rolling motion with a relatively high frequency. It is sometimes referred to as a "nuisance" or "annoyance" mode

since the motion is not normally deliberately excited in normal flying. However, it is inadvertently excited almost continually by pilot control inputs or by external disturbances. Therefore, the characteristics of this mode of motion greatly influence the pilot's opinion of the aircraft.

(ii) Wind Tunnel Simulation

As discussed earlier, the wind tunnel model is restricted in lateral translation and hence the side force equation in equation (4.21) is disregarded in the model motion. The rate of change of the sideslip angle, $\hat{v}$, is the negative of the yaw rate, $\hat{r}$ ($\beta = -\psi$) as the model is suspended in the wind tunnel.

Applying the above conditions to the equation (4.21) result in the following model lateral equation of motion.

$$\begin{aligned} l_v \hat{v} + (\hat{D}^2 + l_p \hat{D}) \phi + (e_x \hat{D} + l_r) \hat{r} &= -l_\xi' \xi - l_\zeta' \zeta \\ n_v \hat{v} + (e_z \hat{D}^2 + n_p \hat{D}) \phi + (\hat{D} + n_r) \hat{r} &= -n_\xi' \xi - n_\zeta' \zeta \\ -\hat{r} &= \hat{D} \hat{v} \\ \hat{p} &= \hat{D} \phi \end{aligned} \quad (4.28)$$

and the corresponding characteristic equation is

$$\begin{vmatrix} l_v - (e_x \lambda + l_r) \lambda & \lambda^2 + l_p \lambda \\ n_v - (\lambda + n_r) \lambda & e_z \lambda^2 + n_p \lambda \end{vmatrix} = 0 \quad (4.29)$$

or

$$A_4 \lambda^4 + B_4 \lambda^3 + C_4 \lambda^2 + D_4 \lambda = 0 \quad (4.30)$$

where

$$A_4 = 1 - e_x e_z$$

$$B_4 = l_p + n_r - e_x n_p - e_z l_r$$

$$C_4 = l_p n_r - l_r n_p - n_v + e_z l_v$$

$$D_4 = l_v n_p - l_p n_v$$

The characteristic equation is a quartic, but one of the roots is zero, representing neutral stability in yaw (Reference 47), so the order is reduced to third. The roots are a large negative real one and a complex pair.

The real root of equation (4.30) is given by

$$\lambda_{rm} = - \frac{B_4}{A_4} - \frac{C_4}{\lambda} - \frac{D_4}{\lambda^2} + R \quad (4.31)$$

where R is zero when $F(\lambda) = 0$.

The remaining two roots of equation (4.30) are a complex pair and are obtained by reducing the quartic to a quadratic by synthetic division when λ_{rm} is known.

(iii) Comparison

The comparison of equation (4.24) and (4.30) results in the following observations about the lateral

modes of the model in wind tunnel simulation.

- (a) The wind tunnel simulation technique suppresses the Spiral mode. This is due to the fact that lateral translation is suppressed which is an essential component of the Spiral mode.
- (b) The Roll subsidence mode characteristic is exhibited by the model but the time constant obtained in simulation $\frac{1}{B} \left(-\frac{A_4}{A_4} \right)$ does not contain a side force contribution due to side slip (y_v). Although the sideslip derivative y_v contribution is generally not a crucial factor in design but its magnitude can contribute up to 10% to the damping of this lateral motion.
- (c) The wind tunnel simulation technique demonstrates the Dutch Roll mode approximately, again because lateral translation is suppressed. The undamped natural frequency and damping ratio do not contain a sideslip contribution y_v and weight (C_L term), and hence correlation with free flight may be doubtful especially in the low speed regime where the required C_L is high.

4.2.2 Effect of Various Parameters on Lateral Modes of Model Motion

(i) Roll Subsidence Mode

Roll acceleration is governed by the Roll mode time constant, τ_r , which is defined as the time

required for the roll rate to reach 63.2 percent of the steady state roll rate following a step input of lateral control. The roll rate time constant can be calculated from equation (4.31) and is given approximately by

$$\tau_r = - \frac{1}{l_p + n_r - e_x n_p - e_z l_r} \quad (4.32)$$

The most important contribution in equation (4.32) is the derivative l_p , known as the roll damping parameter. The Roll mode time constant is approximately inversely proportional to the roll damping parameter.

(ii) Dutch Roll Mode

There is no simple approximation for the frequency and damping ratio of the Dutch Roll motion. However, rough approximation may be made to show the effect of the most important stability derivatives contributing to this mode of motion. Assume that the model is suppressed in roll. This results in the single degree of freedom described by the yawing equation in equation (4.21)

$$(\hat{D}^2 \hat{v} + n_r \hat{D} \hat{v} - n_v \hat{v}) = 0 \quad (4.33)$$

with corresponding characteristic equation

$$\lambda^2 + n_r \lambda - n_v = 0 \quad (4.34)$$

$$\text{where } \omega_{DR} = \sqrt{\mu_2 N_v / i_z} \quad \text{and } \zeta_{DR} = \frac{-N_r / i_z}{2 \sqrt{\mu_2 N_v / i_z}}$$

which can be reduced to

$$\omega_{DR} = M \sqrt{N_V \frac{\gamma^P a S b}{2 I_z}} \quad (4.35)$$

and

$$\zeta_{DR} = -N_r \sqrt{\frac{\rho S b^3}{2 N_V I_z}} \quad (4.36)$$

From equation (4.35), the following important relationship can be gathered.

- (a) The undamped natural frequency of the Dutch Roll motion increases as Mach number increases or the period decreases as the Mach number increases.
- (b) The undamped natural frequency increases with increasing N_V , the directional stability derivative and decreases with decreasing N_V .
- (c) An increase in pressure attitude, decreases the undamped natural frequency.
- (d) The frequency decreases with an increase in the moment of inertia in yaw.

The damping ratio of the Dutch Roll mode (equation 4.36) highlights the following important relationships

- (a) Damping of the Dutch Roll motion is largely dependent on yaw rate damping.
- (b) Damping of the Dutch Roll motion decreases

with increasing altitude.

- (c) Increased directional stability decreases Dutch Roll damping.
- (d) Increasing the yawing moment of inertia decreases the Dutch Roll damping.
- (e) Damping of the Dutch Roll mode of motion is not a direct function of airspeed or Mach number.

4.3 Feedback Techniques

If an aircraft has deficiencies in its basic stability characteristics, for this aircraft to adequately perform in its intended role some sort of stability augmentation would be required. In fact, the present design trend is to incorporate stability augmentation from the outset to gain additional performance benefits.

The augmentation is usually implemented by completing the appropriate feedback loops around the aircraft. In practice the output variable is sensed, the electrical signal from the sensor is conditioned and compensated electronically and the resultant signal is used to actuate the control surface through a power control actuator.

In the present study of model dynamics the model outputs, pitch, roll and yaw angles, are measured by potentiometers. The rate signals are derived by differentiating the attitude signals electronically using analogue computer elements in the electronic control unit. The details of

this procedure will be discussed in the next chapter. Since the time constants of the servo actuators used in the model were around 0.025 sec the servo actuator dynamics are ignored, because its influence on the model dynamics ($T \approx 1$ sec) was expected to be negligible.

4.3.1 Longitudinal Dynamics

(i) Equations of Motion

The dimensional longitudinal equations of the model motion are (see Appendix C)

$$\begin{aligned} m\dot{w} - \dot{Z}_w w - mV_e \dot{q} &= \dot{Z}(t) \\ -\dot{M}_w \dot{w} - \dot{M}_w w + I_y \dot{q} - \dot{M}_q q &= \dot{M}(t) \end{aligned} \quad (4.37)$$

where $\dot{Z}(t)$ and $\dot{M}(t)$ are input parameters.

Taking the Laplace transform of equation (4.37) and assuming zero initial conditions, the equation may be written as follows:-

$$\begin{aligned} (sm - \dot{Z}_w)w - smV_e \theta &= \dot{Z}_\eta \eta \\ -(s\dot{M}_w + \dot{M}_w)w + (s^2 I_y - s\dot{M}_q)\theta &= \dot{M}_\eta \eta \end{aligned} \quad (4.38)$$

where $\eta \sim$ elevator input.

The stability polynomial $\Delta(s)$ is given by

$$\Delta(s) = \begin{vmatrix} (sm - \dot{Z}_w) & -smV_e \\ -(s\dot{M}_w + \dot{M}_w) & (s^2 I_y - s\dot{M}_q) \end{vmatrix}$$

or

$$\Delta(s) = s (As^2 + Bs + C) \quad (4.39)$$

where

$$A = mI_y$$

$$B = (-mV_e \dot{M}_w - m\dot{M}_q - \dot{Z}_w I_y)$$

$$C = (\dot{Z}_w \dot{M}_q - mV_e \dot{M}_w)$$

(ii) Pitch Attitude Feedback to Elevator

Figure 30a gives the block diagram of the system.

The open loop transfer function relating pitch attitude θ to elevator angle η is obtained by applying Cramers rule to equation (4.38)

$$\therefore \frac{\theta}{\eta} = \frac{N_{\eta}^{\theta}(s)}{\Delta(s)} \quad (4.40)$$

where $N_{\eta}^{\theta}(s)$ is given by

$$N_{\eta}^{\theta}(s) = (m\dot{M}_{\eta} + \dot{Z}_{\eta} \dot{M}_w)s + (\dot{M}_w \dot{Z}_{\eta} - \dot{Z}_w \dot{M}_{\eta}) \quad (4.41)$$

The closed loop transfer function is given by

$$\frac{\theta}{\eta} = \frac{N_{\eta}^{\theta}(s)}{\Delta(s) + KN_{\eta}^{\theta}(s)} \quad (4.42)$$

Thus the characteristic equation of the closed loop model configuration is given by

$$\Delta(s) + KN_{\eta}^{\theta}(s) = 0 \quad (4.43)$$

The root locus plot of this equation showing variation of closed loop roots on s-plane with variation in gain K is given in figure 30b, and it is clear that as the gain K increases, the Short Period mode frequency is increased and damping

ratio is reduced. Neither of these two results are desirable.

(iii) Pitch Rate Feedback to Elevator

Figure 31a shows the block diagram of the system when pitch rate feedback is employed to augment the stability characteristics of the model.

The open loop transfer function is the same as given by equation (4.40).

The closed loop transfer function is

$$\frac{\theta}{\eta} = \frac{N_{\eta}^{\theta}(s)}{\Delta(s) + KsN_{\eta}^{\theta}(s)} \quad (4.44)$$

where the characteristic equation is

$$\Delta(s) + KsN_{\eta}^{\theta}(s) = 0 \quad (4.45)$$

The root locus plot of equation (4.45) is given in figure 31b.

The root locus plot shows that

- a) As the feedback gain K increases, the Short Period mode frequency remains almost constant.
- b) As the feedback gain K increases, the Short Period mode damping ratio increases rapidly.
- c) The pitch rate feedback to elevator is equivalent to increasing the pitch rate damping derivative M_q
- d) The effect of pitch rate feedback is highly desirable as it improves the Short Period mode stability.

4.3.2 Lateral Dynamics

(i) Equations of Motion

The dimensional lateral equations of the model motion are (see Appendix C)

$$\begin{aligned} -\dot{L}_v v + I_x \dot{p} - \dot{L}_p p - I_{zx} \dot{r} - \dot{L}_r r &= \dot{L}(t) \\ -\dot{N}_v v - I_{zx} \dot{p} - \dot{N}_p p + I_z r - \dot{N}_r r &= \dot{N}(t) \end{aligned} \quad (4.46)$$

where $p = \dot{\phi}$, $r = \dot{\psi}$ and $v = -\psi V_e$

also $\dot{L}(t)$ and $\dot{N}(t)$ are input parameters.

Taking the Laplace transform of equation (4.46) and assuming zero initial conditions the equations may be written as follows:-

$$\begin{aligned} (V_e \dot{L}_v - s^2 I_{zx} - s \dot{L}_r) \psi + (s^2 I_x - s \dot{L}_p) \phi &= \dot{L}'_\xi \xi + \dot{L}'_\zeta \zeta \\ (V_e \dot{N}_v + s^2 I_z - s \dot{N}_r) \psi + (-s^2 I_{zx} - s \dot{N}_p) \phi &= \dot{N}'_\xi \xi + \dot{N}'_\zeta \zeta \end{aligned} \quad (4.47)$$

where $\xi \sim$ aileron input
 $\zeta \sim$ rudder input

The stability polynomial $\Delta(s)$ is given by

$$\Delta(s) = \begin{vmatrix} (V_e \dot{L}_v - s^2 I_{zx} - s \dot{L}_r) & (s^2 I_x - s \dot{L}_p) \\ (V_e \dot{N}_v + s^2 I_z - s \dot{N}_r) & (-s^2 I_{zx} - s \dot{N}_p) \end{vmatrix} \quad (4.48)$$

or

$$\Delta(s) = s(As^3 + Bs^2 + Cs + D)$$

where $A = (I_{zx}^2 - I_x I_z)$

$$B = [I_x \dot{N}_r + I_z \dot{L}_p + I_{zx} (\dot{N}_p + \dot{L}_r)]$$

$$C = (-I_{zx} \dot{V}_e \dot{L}_v - I_{xz} \dot{V}_e \dot{N}_v + \dot{N}_p \dot{L}_r - \dot{N}_r \dot{L}_p)$$

$$D = (\dot{L}_p \dot{N}_v - \dot{L}_v \dot{N}_p) \dot{V}_e$$

(ii) Roll Attitude Feedback to Aileron

The block diagram of the system is shown in figure 32a. The open-loop transfer function relating bank angle ϕ to aileron angle ξ is obtained by applying Cramers rule to equation (4.47)

$$\therefore \frac{\phi}{\xi} = \frac{N_{\xi}^{\phi}(s)}{\Delta(s)} \quad (4.49)$$

where the numerator $N_{\xi}^{\phi}(s)$ is given by

$$N_{\xi}^{\phi}(s) = As^2 + Bs + C$$

where

$$A = (-I_{zx} \dot{N}_{\xi} - I_{xz} \dot{L}_{\xi})$$

$$B = (\dot{N}_r \dot{L}_{\xi} - \dot{L}_r \dot{N}_{\xi}) \quad (4.50)$$

$$C = (\dot{L}_v \dot{N}_{\xi} - \dot{N}_v \dot{L}_{\xi}) \dot{V}_e$$

The denominator $\Delta(s)$ is given by equation (4.48)

The closed loop transfer function is

$$\frac{\phi}{\xi} = \frac{N_{\xi}^{\phi}(s)}{\Delta(s) + KN_{\xi}^{\phi}(s)} \quad (4.51)$$

Characteristic equation of the closed loop configuration is given by

$$\Delta(s) + KN_{\xi}^{\phi}(s) = 0 \quad (4.52)$$

Typical root locus plot of this equation showing variations of closed loop roots on the s-plane with

variation in gain K is given in figure 32b. The effect of bank angle feedback on the Dutch Roll mode is very small. The Roll mode time constant increases with an increase in gain K and the Roll subsidence mode tends to couple with the Spiral mode at higher gains.

(iii) Roll Rate Feedback to Aileron

The block diagram in figure 33a represents the model with roll rate feedback. The open loop transfer function is given by equation (4.49). The closed loop transfer function is given by

$$\frac{\phi}{\xi} = \frac{N_{\xi}^{\phi}(s)}{\Delta(s) + KsN_{\xi}^{\phi}(s)} \quad (4.53)$$

and the characteristic equation is

$$\Delta(s) + KsN_{\xi}^{\phi}(s) = 0 \quad (4.54)$$

Typical root locus plot (figure 33b) of equation (4.54) demonstrate the following points.

- a) As the gain increases, the Roll mode increases in stability.
- b) The Dutch Roll mode is not affected appreciably.
- c) Roll rate feedback to aileron is equivalent to increasing the roll damping parameter l_p and is a very effective method for modifying roll damping.

(iv) Yaw Angle Feedback to Rudder

Figure 34a gives the block diagram of the system

with yaw angle feedback. The open loop transfer function relating yaw angle ψ and rudder angle ζ is obtained by applying Cramers rule to equation (4.47).

$$\frac{\psi}{\zeta} = \frac{N_{\zeta}^{\psi}(s)}{\Delta(s)} \quad (4.55)$$

where $N_{\zeta}^{\psi}(s)$ is given as follows:-

$$N_{\zeta}^{\psi}(s) = (-I_{zx} \dot{L}_{\zeta} - I_x \dot{N}_{\zeta}) s^2 + (\dot{L}_p \dot{N}_{\zeta} - \dot{N}_p \dot{L}_{\zeta}) s \quad (4.56)$$

and $\Delta(s)$ is given by equation (4.48).

The closed loop transfer function is

$$\frac{\psi}{\zeta} = \frac{N_{\zeta}^{\psi}(s)}{\Delta(s) + K N_{\zeta}^{\psi}(s)} \quad (4.57)$$

where the characteristic equation is modified to

$$\Delta(s) + K N_{\zeta}^{\psi}(s) = 0 \quad (4.58)$$

The root locus plot of equation (4.58) (figure 34b) shows that

- a) The Roll mode remains unchanged due to yaw angle feedback to the rudder.
- b) The Dutch Roll mode frequency increases rapidly as the gain increases whilst the damping ratio decreases.

(v) Yaw Rate Feedback to Rudder

The block diagram in figure 35a represents the system with yaw rate feedback to rudder.

The open-loop transfer function is the same as that of yaw angle feedback to rudder (equation 4.55).

The closed loop transfer function is given by

$$\frac{\psi}{\zeta} = \frac{N_{\zeta}^{\psi}(s)}{\Delta(s) + KsN_{\zeta}^{\psi}(s)} \quad (4.59)$$

The modified characteristic equation is

$$\Delta(s) + KsN_{\zeta}^{\psi}(s) = 0 \quad (4.60)$$

Typical root locus plot of equation (4.60) is shown in figure 35b. The following important trends can be deduced from the figure:-

- a) The Roll mode characteristics remain unchanged with the increasing feedback gain K.
- b) The Dutch Roll mode damping ratio increases rapidly with increasing feedback gain K.
- c) The Dutch Roll mode frequency remains almost constant with increasing feedback gain K.
- d) Yaw rate feedback to rudder is equivalent to an increase in the stability derivative N_r .

4.4

Scaling Limitations

In chapter 2, different similarity law constraints for dynamic model testing were discussed in detail. The design and development process of the facility, in which these similarity requirements were the

major consideration, has resulted in the scaling constraints as shown in table B3 of Appendix B.

As the present simulation facility is limited to low Mach number simulations, the compressibility effects can be neglected ($M \leq 0.4$) and hence Mach number equality can be argued to be unimportant in this region.

The mass and inertia criteria for the model to exhibit similar behaviour to that of the full scale aircraft has also been satisfied in the simulation by bringing the model weight and moment of inertia to the required scaled values as discussed in chapter 4.

At present, tests are conducted under natural gravity conditions only. When Froude number equality is satisfied, the model and full scale speeds have a unique scale relationship as discussed in chapter 2. Hence the model dynamic simulation is conducted at scaled velocities ($V_m = \sqrt{\lambda} V_a$).

With the $\frac{1}{12}$ scale Hawk model, the scale being dictated by the wind tunnel test section size the maximum usable tunnel speed is 100 ft/sec, the Reynold's number range for the simulation is 2.2×10^5 to 3.1×10^5 .

The Strouhal number equality is not a design constraint for the dynamic simulation but provides a relationship (time scale factor) between the model and full scale aircraft frequency to ensure that the model dynamic angle of attack is the same as in the full scale. The relationship is derived in Appendix A and the time scale factor for the present simulation is given in table B3 of

Appendix B .

It can be seen that all the required similarity laws are satisfied except for Reynold's number. As discussed earlier in chapter 2, the use of a small scale model makes it impossible to simulate the Reynold's number in this particular situation. Generally, dynamic tests would be conducted at a Reynold's number of 0.4×10^6 or higher. The discrepancy arising from the Reynold's number constraint will be further discussed in chapter 7.

4.5 Stability Derivatives

For the comparison of the model simulation results with the theoretical predictions, the stability derivative of the Hawk aircraft and the scaled model were estimated. Reference 49 and 50 are the most comprehensive sources of estimating stability derivatives. Although these methods have inherent deficiencies, they are extensively used as they are, in most cases, the only sources of information available. In the present work references 49, 51 and 47, in order of priority, were used for estimation techniques or other data. Table D14 of Appendix D gives the summary of the stability derivative prediction accuracy and their relative importance. A sample calculation of the stability derivatives and the tabulated values of estimated stability derivatives of the full scale aircraft and the model at the corresponding simulation condition are presented in Appendix D.

The estimated dimensionless stability derivative for the model, full scale aircraft and those obtained from the Hawk wind tunnel test data (reference 52) are plotted in figures 36 to 41 for comparison

purposes so that their resulting contribution to discrepancies between the dynamic simulation tests and theoretical predictions can be identified. It is pointed out that the prediction technique assumes attached flow conditions. Due to the inherent limitation of the estimation method, the stability derivatives of the model do not account for the low Reynold's number effects. The differences between the estimated full scale and the model stability derivatives is due to the scaled speed effects (Mach No.) and can be seen to be small because of lower full scale Mach number. The estimated full scale derivatives in most cases compare well with those derived from the Hawk wind tunnel data at $M = 0.3$. Only one dominant derivative, L_v , has poor correlation and the estimated value is about 50% higher than the Hawk wind tunnel data suggests, although the prediction accuracy is of the order of 20%. If the Hawk wind tunnel data pertains to the aircraft with ventral fins, then the discrepancy, to a large extent, can be attributed to this fact as the model used in the simulation does not incorporate ventral fins. But, then it would be expected that the ventral fin contribution to N_v would show a similar disparity, which is not the case.

4.6 Static Stability

(a) Longitudinal Static Stability

The static longitudinal stability of the model is ensured by keeping the model c.g. ahead of the stick fixed neutral point (aerodynamic centre of the model). The location of the neutral point of the model was estimated to be $0.365\bar{c}$ as derived

in Appendix D and agrees well with the Hawk wind tunnel data. The centre of gravity position of the model was adjusted to be coincident with the model pivot point by mass and inertia balancing. Two c.g. positions were used during the simulation tests and were $0.29\bar{c}$ and $0.325\bar{c}$ respectively.

In order to test experimentally the static stability of the model, model trim angles were calculated, see Appendix D, for different tunnel velocities. The comparison with experiment is discussed in chapter 6.

Locating the c.g. at the model pivot point although not essential was preferred for dynamic stability considerations. The position of the pivot point does not influence the usual static stability characteristics or its dependence on model c.g. and aerodynamic centre.

(b) Lateral Static Stability

The change of c.g. position has insignificant effect on the lateral motion and hence in general the lateral static stability characteristics of the model would be expected to be similar to the full scale. The directional static stability is ensured by a positive value of the stability derivative N_v , which is the case for the Hawk model (Figure 39).

The lateral static stability also has a dominant contribution from the stability derivative L_v , which must be negative for stability. Figure 40 shows that the model has adequate lateral static stability in this respect also, in fact as

mentioned earlier, it has been theoretically determined to be greater than the full scale.

CHAPTER 5

EXPERIMENTAL INVESTIGATION

5.1 Introduction

The primary aim of the design and construction of the facility is for Active Control Technology demonstrations. To achieve this objective the facility must demonstrate the following:

- i) The model dynamic stability characteristics (modes) should represent the corresponding full size (scale) aircraft dynamic stability characteristics.
- ii) The capability to augment the dynamic stability of the aircraft model so that Active Control Technology concepts can be implemented.

The experimental investigations were carried out to specifically evaluate the above two features. The open-loop experiments were conducted to determine the basic dynamic stability of the model and to assess whether the semi-free flight scaled model dynamics in the wind tunnel are representative of the full scale aircraft dynamics. The closed loop experiments aim at evaluating the electronic control unit as a means of artificially augmenting the dynamic stability of the model through the use of its analogue circuitry. Rate feedback was chosen for closed loop experiments for its apparent effectiveness (chapter 4) in stability augmentation and for evaluating the electronic control unit elements and their use for generating rate signals for feedback loops.

The experimental investigation is broadly divided into three sections.

- (a) Preliminary Experiments.
- (b) Dynamic Stability Test Technique.
- (c) Model Dynamic Response Test Conditions.

5.2 Preliminary Experiments

In this section, calibration of the control surfaces of the model and attitude measuring potentiometers, experimental procedure for flying the model in the the tunnel at trim conditions and the data recording techniques are discussed.

5.2.1 Calibration Tests

(i) Control Surfaces

The model control surfaces are actuated by electrically driven control servo's mounted inside the model fuselage. In order to establish the control surface angle to the applied voltage scaling and in order to interpret and monitor control surface angles for different input voltage signals derived from the electronic control unit, calibration of the control surfaces was carried out. The model was rigged in a straight and level reference attitude on a true flat surface to ensure that measured control angles against the calibrated scale were true and that there was no error due to model misalignment. The control surface deflections were recorded for the corresponding voltage signals to the control servo's and are presented in figures 42, 43 and 44.

(ii) Attitude Sensors

As discussed earlier, the model attitude angles (pitch, roll and yaw) are measured by three potentiometers mounted on the model support system (gimbal and vertical rod). The potentiometers are supplied with ± 10 volts reference voltage from the electronic control unit. The output voltage signals from the potentiometers are fed directly to corresponding signal scaling and conditioning amplifiers. The scaled output signals from these amplifiers are available for data recording and feedback purposes at the electronic control unit.

The calibration was conducted to relate attitude angles in terms of the output voltage from the signal conditioning amplifiers. It was performed in the wind tunnel (wind off) with the model on its support system. The roll and pitch potentiometers were calibrated using the tunnel mounted clinometer and the yaw potentiometer was calibrated using a suitably graduated scale and pointer arrangement. The resulting calibration curves are presented in figures 45, 46 and 47.

5.2.2 Experimental Procedure for Flying the Model in Wind Tunnel at Trim Conditions

Before commencing the dynamic response experiments, the model must be flying at that flight condition in trimmed equilibrium. This section describes the procedure for flying the model at trim conditions.

The layout of the experimental set-up is illustrated in plate 9.

An essential aspect of any rig should be simplicity of operation, without requiring any special skill on the part of the operator. To a certain extent this has been achieved in this design. Once the model, support system and mounting frame is properly rigged up in the wind tunnel and the electronic control unit suitably connected, the operation of the equipment is fairly simple and does not require more than one operator/controller.

For the proper and safe functioning of the facility the following precautionary pre-test checks are necessary before starting the tunnel.

- 1) The wind tunnel test section and nozzle face should be clear of foreign objects otherwise the model and the rig are liable to damage when the tunnel is turned on.
- 2) The vertical rod should be rigidly braced to the mounting frame and the bracing wires tension should be correctly set.
- 3) Using a clinometer, the vertical rod should be checked for true vertical. Only a small offset is enough to prevent the model flying freely.
- 4) The top safety spring retainer (see figure 18) should be properly and securely keyed to the vertical rod. An inadvertent operation of the model controls switch or tunnel speed regulator can force the model to climb very rapidly and impact the mounting frame which can result in serious damage to the model.

- 5) The electronic control unit connections to the model should be checked for correctness, i.e. input and output cable plugs are connected to their respective sockets. Also the umbilical cord which carries signals to and from the model should have sufficient slackness to let the model fly freely through the vertical rod length. The umbilical cord is made up from a number of very fine wires (dia. 0.49 mm) and a small amount of tension is enough to cause damage.

After the completion of the above mentioned checks, the procedure to fly the model in the wind tunnel at a trim condition is as follows:-

- 1) The "Power On" switch of the electronic control unit is switched on.
- 2) The model control surfaces are checked for proper functioning. Using the control stick each control surface is checked for smooth operation.
- 3) To ensure that all the three model attitude measuring potentiometers are alive and functioning properly, the model is disturbed in pitch, roll and yaw attitude and corresponding output voltage signals are checked.
- 4) Using the trim pots on the input control panel of the electronic control unit, rudder and ailerons are trimmed to neutral position. The elevator is set at an angle

slightly higher than that required for the desired flight condition so that if the wind tunnel speed is coarsely increased to its desired value, the model does not fly up in an out of trim condition.

- 5) The wind tunnel is switched "on". Using "coarse control", the wind tunnel speed is gradually increased to a value just lower than that required by the flight condition and then "fine control" is used to bring it to desired value.
- 6) The model is trimmed more nose down than demanded by the chosen flight condition as the model at this stage is very sensitive to trim change and wind tunnel speed changes. Speed should be continuously monitored and trim changes should be made gradually and patiently, until the model lifts from the bottom safety spring stop and flies steadily in the wind tunnel (plate 10). The response experiments can be conducted on the model at this flight condition.

In order to change the flight speed of the model, the following procedure should be followed:

- 1) If the test speed is to be increased, first set the elevator angle as discussed in step 4, earlier and then increase the speed to the desired value as per step 5 and adjust the elevator as given in step 6.
- 2) If the test speed is to be decreased, first

bring the speed down to desired value using coarse and fine speed controls and then trim the elevator as per step 6.

It should be noted that the model trim should not be allowed to go excessively nose up at any time in order to avoid an uncontrollable condition. This is due to the fact that in actual free flight speed changes with the change in trim condition and consequently climb or descent is gradual, but in the case of the present simulation where tunnel speed is constant, the increase or decrease of attitude (and lift) due to trim changes, forces the model to climb up or down uncontrollably.

5.2.3 Data Recording

The output signals from the sensors are (pitch, roll and yaw) model attitude angles. These signals are routed through the signal conditioning amplifiers to the signal output interface panel of the electronic control unit where they become available for data recording and feedback purposes. All the three attitude signals were connected to a six-channel chart recorder to record time history responses of the model (plate 9). The control input used to excite the model is also fed to a channel of the chart recorder.

5.2.4 Generation of Rate Signal

In the closed loop investigation attitude rate signals were used to augment the basic dynamic stability of the model. The analogue elements

of the electronic control unit were used to generate pseudo rate signals electronically as only attitude signals are available from the sensors currently installed in the model. The analogue differentiating circuit was derived from the equation⁵³

$$(1-f)\frac{dy}{dx} + y = \frac{dx}{dt} \quad (5.1)$$

The analogue differentiating circuit used for deriving a pseudo rate signal and which represents equation(5.1) is given in figure 48 below.

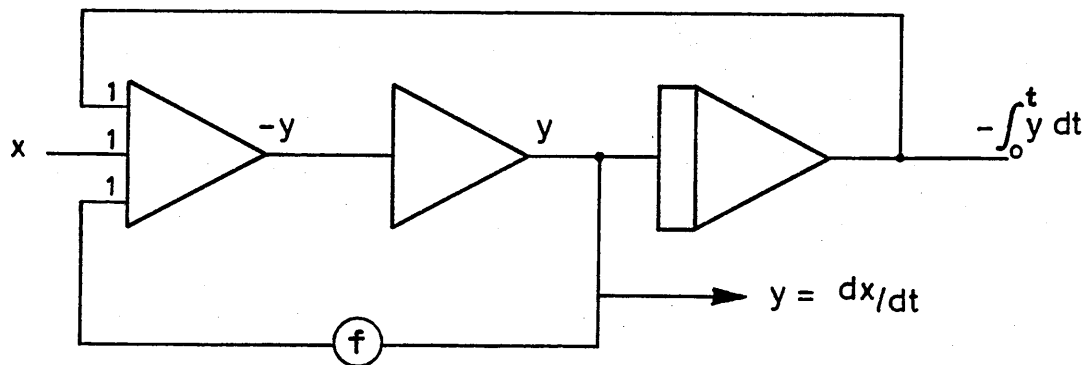


Figure 48 Pseudo Rate Signal Circuit

As f approaches unity, y approaches $\frac{dx}{dt}$.

The magnitude of " f " depends on the frequency of x . If " f " is too large, a high frequency input will cause oscillation in the circuit but in the case of low frequency input, as is the case in question, values of " f " up to 0.96 were found to give satisfactory results.

5.3 Dynamic Stability Test Techniques

5.3.1 Introduction

The purpose of the dynamic stability test was to investigate the primary Short Period modes of

motion of the model, since the Phugoid and Spiral modes of motion are suppressed due to the suppression of longitudinal and lateral translation respectively. These investigations result in the determination of the frequency, damping and the time constant of the modes. For a dynamically scaled model these characteristics can be scaled to represent those of the actual aircraft in terms of the similarity parameters at the corresponding flight condition of full scale aircraft and hence can be used to evaluate some of the aspects of handling qualities.

The aircraft with six degrees of freedom is a rather complicated dynamic system and will move in a manner that is a simultaneous combination of the various modes of motion. One problem of flight testing is to excite the various modes independently. To a certain extent this problem is inherently overcome in the present technique due to restrictions on the model degrees of freedom. The complete restriction on longitudinal translation of the model suppresses the Phugoid mode which can couple with the Short Period mode dynamics in the free flight case. The denial of the lateral translation to the model suppresses the Spiral mode of the model motion which generally couples with the Dutch Roll mode in free flight dynamics.

5.3.2 Control Inputs

Several different control inputs were available to excite the dynamic modes of motion of the model. To obtain the free response of the model, the appropriate control input is applied and the resulting model motion is recorded for evaluation.

Three input forcing functions that are frequently used in stability and control investigations are the step, the pulse and the doublet⁵⁴ as shown in figure 49 below.

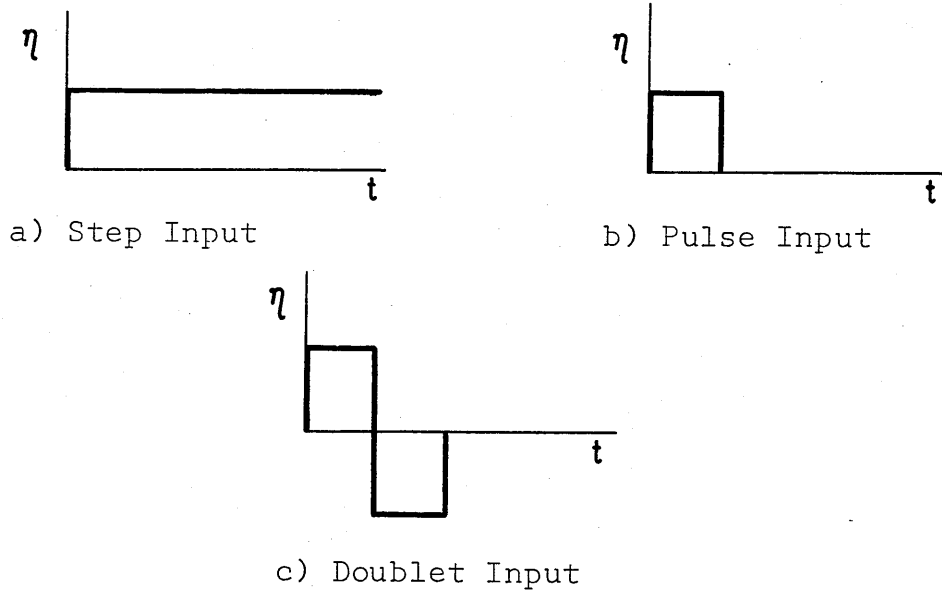


Figure 49 Input Forcing Functions

In the present technique, the pulse input was used to excite the natural modes of model motion as the following difficulties were encountered when the step input was used.

a) Elevator Input

In the free flight case when a step elevator input is applied to excite the longitudinal modes of motion, the new steady state equilibrium condition is achieved at a higher or lower speed depending upon the sign of the input applied. Since in this technique the wind tunnel speed remains constant, in response to an elevator step input, the model either flies up or down instantaneously and impacts the vertical motion

limit safety spring stops. This can result in serious damage to the model if the input is inadvertently large.

b) Aileron Input

Although a step aileron input is very useful in evaluating Roll mode dynamics, it is found unsuitable in this wind tunnel technique as even a small magnitude step aileron input results in a substantial roll rate thus requiring larger bank angle freedom than $\pm 30^\circ$ attained on this rig, or, alternatively severely limiting observation time.

c) Rudder Input

A step or pulse rudder input can be used to excite the Dutch Roll mode. In the case of a step input of rudder, the steady state equilibrium condition of the model is changed and a new equilibrium condition is achieved at a side-slip angle. This position of the model results in a decrease of tunnel speed due to the increased blockage effect and hence pulse rudder input is used.

5.3.3 Test Techniques

The test techniques discussed in this section apply to both the open loop and closed loop response experiments.

(i) Longitudinal Mode (Short Period)

To examine the Short Period pitching mode, the model was trimmed at the desired flight condition tunnel speed (Section 5.2.2). Using a pulse generator, a disturbance was applied to the

elevator in a nose down sense. The magnitude of the control input must be carefully limited. It was found necessary to start with a small magnitude which was gradually increased to the desired level of excitation to avoid violent response.

The recorded data is in the form of time histories of pitch, roll and yaw angles and control input. The pitch angle trace was used to determine the Short Period response frequency and damping ratio (Appendix E).

ii) Lateral Modes

a) Roll Mode

The model was trimmed at the desired flight condition. It was then re-trimmed at 20° bank angle and stabilised. This was done to allow 50° of bank angle freedom to the model in one direction. Using a pulse generator, an aileron pulse input was applied and its magnitude was kept sufficiently small to allow not more than 50° of bank angle variation in the transient. The data was recorded until the transient had died out. The time to half amplitude $t_{\frac{1}{2}}$ of the Roll mode was determined from the bank angle trace. (Appendix E).

b) Dutch Roll Mode

The model was trimmed at the chosen flight test condition. Using the pulse signal generator a rudder pulse input was applied to the model. Once, again, the

magnitude of the input must be approached with care as the model has $\pm 30^\circ$ yaw angle freedom only and an excessive input magnitude can damage the model or its support.

The data recorded was in the form of time histories of pitch, roll and yaw angles and control input. The yaw angle response time histories were used to determine Dutch Roll frequency and damping ratio⁵⁵.

5.4 Model Dynamic Response Test Conditions

The model dynamic response experiments were carried out at various test conditions which are described below.

5.4.1 Open-Loop Experiments

The information defining the test conditions includes the test velocity, the center of gravity position, the type of control input used to excite the model and the recorded response data.

(i) Longitudinal Mode (Short Period)

Tunnel Velocity:-	70 - 100 ft/sec
c.g. position:-	$x_{c.g.} = 0.29\bar{c}$ & $0.325\bar{c}$
Control Input	Elevator pulse
Data	Time history records of pitch, roll, yaw angle and control input.

(ii) Lateral Modes

a) Roll Mode

Tunnel Velocity	70 - 100 ft/sec
c.g. position	$x_{c.g.} = 0.29\bar{c} \text{ \& } 0.325\bar{c}$
Control Input	Aileron pulse
Data	Time history records of pitch, roll, yaw angle and control input.

b) Dutch Roll Mode

Tunnel Velocity	70-100 ft/sec
c.g. position	$x_{c.g.} = 0.29\bar{c} \text{ \& } 0.325\bar{c}$
Control Input	Rudder pulse
Data	Time history records of pitch, roll, yaw angle and control input.

5.4.2 Closed-Loop Experiments

All closed loop tests were conducted at the tunnel speeds of 70 ft/sec and 100 ft/sec. The relevant control inputs and the recorded response data was the same as in open loop experiments.

(i) Longitudinal Mode (Short Period)

c.g. position	$x_{c.g.} = 0.29\bar{c} \text{ \& } 0.325\bar{c}$
Feedback	Pitch rate feedback to elevator (fig.31)

(ii) Lateral Modes

a) Roll Mode

c.g. position

$$x_{c.g.} = 0.29\bar{c}$$

Feedback

Roll rate feedback to
aileron (fig.33)

b) Dutch Roll Mode

c.g. position

$$x_{c.g.} = 0.29\bar{c}$$

Feedback

Yaw rate feedback to
rudder (fig.35)

CHAPTER 6

RESULTS AND DISCUSSION

6.1 Introduction

The results are discussed under the following headings:

1. Static Stability Tests
2. Open-Loop Model Dynamic Simulation
3. Closed-Loop Model Dynamic Simulation

The model open-loop and closed-loop dynamic simulation is centred on the analysis of the dynamic modes of motion which can be readily simulated on the test rig. It is recalled that due to the suppression of longitudinal and lateral translation of the model, the Phugoid and Spiral modes of motion are suppressed. The closed-loop simulation was also intended to evaluate the electronic control unit as a means of artificially augmenting the stability of the model although the present model of the Hawk aircraft which is used for this evaluation possesses adequate stability and does not warrant stability augmentation.

6.2 Longitudinal Static Stability Tests

Before conducting the dynamic tests on the model, it was essential that the model was trimmed to fly steadily at that flight condition. Also important is the fact that the model and the corresponding full scale aircraft should have the same reference angles if the model is required to simulate full scale characteristics. This is achieved when the full

scale aircraft and the scaled model Froude Number equality is satisfied along with other similarity parameters. The model then attains the same reference angle of attack as the full scale aircraft with a unique relationship between the full scale velocity V_a , and the model velocity V_m (i.e. $V_m = \sqrt{\lambda} V_a$) as discussed in chapter 2.

The theoretical and experimental values of model incidence are plotted in figure 50. The correlation between the two is good at higher velocities but deteriorates with decreasing velocity, the experimental values being lower than that predicted by theory. This disparity at lower velocities can be attributed to the decrease in Reynold's Number to a value around 1×10^5 .

Reference 56 reports these trends at lower Reynold's Numbers (10^5 to 5×10^5) to be due to the change in flow characteristics on the lifting surface. The flow, when the Reynold's number decreases to this range, becomes part laminar and part turbulent, thus resulting in lower zero lift angle of attack (maximum 3°) although the lift curve slope remains essentially unchanged. However, the shift is expected to be very small (about 1°) in the Reynold's number range (2.2×10^5 to 3.1×10^5) of the present simulation. From reference 56 it is also noted that the attainable $C_{L_{\max}}$ is reduced at high angles

of incidence as lower Reynold's number effects reduce the attainable $C_{L_{\max}}$. Since, in the dynamic

simulation, the required incidence angle is relatively small, the above effect can be assumed to be absent. This conclusion is further supported by good correlation of Hawk aircraft incidence at Mach number of 0.3 ($Re \approx 4 \times 10^6$) with the theoretical

value at the corresponding scaled model velocity as shown in figure 50 .

The model tail angle to trim is presented in figure 51 . The experimental values of trim angle show agreement with the theoretical trend but are scattered because of problems in control resolution due to backlash in the model flying controls. The Hawk aircraft data also shows good agreement with that predicted by the theory at the same c.g. position ($M = 0.3, c.g. = 0.245\bar{C}$). It can be seen from the plot that the control angles required to trim are very small as an all moving horizontal tail is employed for longitudinal control. This resulted in the model being very sensitive to control setting requiring very fine control for trimming the model.

6.3 Open-Loop Model Dynamics

6.3.1 Longitudinal Dynamics Short Period Mode

(a) Frequency

The frequency of the Short Period mode for c.g. positions at $0.29\bar{C}$ and $0.325\bar{C}$ is plotted in figures 52 and 53 respectively. It is evident from the figures that theoretically, the suppression of forward translation in the wind tunnel simulation has no significant effect on the frequency of the Short Period mode. The theoretical values of the short period frequency of the full scale aircraft at different Mach numbers are about the same as those of a free-flight scaled model and semi-free model as used in the present simulation technique at the corresponding scaled velocities. As mentioned earlier in Chapter 4, this is due to the fact that all the important stability derivatives which effect

the short period frequency retain their contribution in the modified short period motion. The effect of compressibility, depicted by the difference in correlation between full scale and scaled model frequency, is not significant due to low values of simulation Mach numbers. The experimental values of model short period frequency exhibit the theoretical trend of increasing with the speed but are lower (maximum of about 10%) in magnitude.

The comparison of figures 52 and 53 shows that the simulation also demonstrates the decrease in the value of short period frequency with aft shift of c.g. and again the trend is the same as predicted theoretically.

(b) Damping Ratio

The damping ratio of the Short Period mode for the two c.g. positions investigated is presented in figures 54 and 55. Again as in the case of the Short Period mode frequency, the theoretical values of damping ratio for full scale, free-flight scaled model and the semi-free model show close agreement due to the fact that the suppressed Short Period mode equations retain all the important stability derivatives which influence the mode. The damping ratio, as expected, does not change with speed in this regime of lower Mach numbers. The damping ratio obtained from the wind tunnel simulation does not compare well with the theoretical values although the trend is the same. The disagreement on the average is about 20%. The increase in short-period damping ratio due to rearward shift in the position of center of gravity is also exhibited as seen in figures 54 and 55 .

(c) Correlation Analysis

Although some disagreement between the theoretical and experimental results was expected due to scale effects and estimation accuracy of longitudinal stability derivatives, but 20% disagreement in damping ratio was too large to be attributed to these factors. However, a critical review of the short-period simulation procedure and short-period damping and frequency equation (4-15) brought out a limitation in the simulation technique which seems to be the major contributor to the above mentioned disagreement.

In the case of free-flight simulation and semi-free flight simulation with vertical and pitching freedom, the short period damping and natural frequency terms contain a contribution z_w due to vertical freedom. The effect of this derivative on coefficient C_2 of the longitudinal characteristics equation (4-14) is additive to the m_w term, thus having the effect of increasing the static stability. The effective increase amounts to a few percent chord of manoeuvre margin for aircraft with low longitudinal relative density μ_1 and negligible for high performance aircraft with high μ_1 . However, in such cases when the c.g. is aft, the contribution of the z_w term can be significant even for medium-high μ_1 aircraft since at this flight condition, m_w the most dominant derivative for short period frequency is also very small. Also, the effect of the stability derivative z_w on the short period damping ratio is additive to the contribution of the stability derivative m_q , the pitch damping coefficient and m_w . This contribution due to z_w can be quite significant (up to 50%) and is often described as a 'vertical damping' term.

In the present simulation technique, the Short Period mode was excited by a short pulse elevator input. The vertical response of the model is not totally free as the length of the vertical rod (36") (dictated by wind tunnel test section size (42" dia) is limited and the model can impact the safety springs. This denies the model full vertical freedom and hence the contribution of the z_w term is partial only. In the case of the Hawk model, the contribution of the z_w term to short period damping is about 35% (see Appendix D) and due to its partial effect the discrepancy of about 20% in damping ratio could reasonably be expected.

In order to further identify the disagreement due to incorrect or reduced z_w , the model was restrained in vertical freedom and its response was evaluated. Since in this case, only pitch freedom was allowed, the contribution due to the stability derivative z_w in the short period frequency and damping ratio is absent (see Appendix C). The theoretical and experimental values of frequency and damping ratio are also presented in figures 52 to 55. It can be seen that agreement between the theoretical and experimental values of frequency and damping ratio is good at model c.g. = $0.29\bar{c}$. For the test condition of model c.g. = $0.325\bar{c}$, the frequency appears to correlate more closely than at $0.29\bar{c}$ but the damping ratio correlation is comparatively not so good and the experimental values are about 15% less. This is not surprising, but it could be argued that since the Short Period mode is so sensitive to c.g. position, a fractional forward shift in model c.g. can result in the above differences. However, it is clear from these results that when the model is restricted in vertical freedom, the correlation between the theoretical

and experimental values of frequency and damping ratio improves considerably. This confirms the fact that when the model is allowed vertical freedom in the simulation, the limitation of vertical rod length results in reduced short period damping ratio and, to a lesser extent (square-root relationship), reduced short period frequency due to partial contribution of the stability derivative z_w .

The direct comparison of model simulation results with Hawk aircraft data was not possible because the relevant aircraft data at $M = 0.3$ was only available for the c.g. position at $0.245\bar{c}$. As the experimental and theoretical correlation is already discussed, an alternative way to correlate model data with Hawk aircraft data is to extend the theoretical prediction to $c.g. = 0.245\bar{c}$ and match it with the aircraft data. The result is given in figures 56 and 57. It can be seen that the theoretical prediction of short period damping ratio and frequency relate well with that deduced from Hawk aircraft data. Hence it can be argued that the correlation discrepancy between theoretical prediction and the experimental results and their analysis discussed earlier are pertinent to this case as well.

6.3.2 Lateral Dynamics

(a) Roll Mode

The Roll mode time to half amplitude ($t_{\frac{1}{2}}$) is presented in figure 58. The theoretical values of $t_{\frac{1}{2}}$ for the full scale aircraft and that of the free-flight model at corresponding scaled velocity match extremely well as compressibility effects are negligible at these low Mach numbers. The

theoretically predicted values of $t_{\frac{1}{2}}$ for semi-free model of present simulation techniques, where lateral translation is suppressed, also show good agreement with those of free-flight model and full scale aircraft and show a slightly higher trend (maximum 8%). This is due to the fact that the Roll mode response can be safely approximated by a single degree of freedom about the longitudinal axis and hence suppression of lateral translation has very little effect on its characteristics. The stability derivative which contributes dominantly (up to 90%) to the Roll mode time constant is l_p and is retained unchanged in the semi-free flight simulation equations.

The experimental values of $t_{\frac{1}{2}}$ are also plotted in figure 58 . The simulation $t_{\frac{1}{2}}$ values show the same trend as that of the theoretical predictions but are scattered in a band of higher values (10 - 30%). This discrepancy in results is thought to be due to the following;

- (i) As discussed earlier, generally the Roll mode response in free-flight is best obtained by applying a step aileron input. The step aileron input, when applied to a scaled model, generates higher roll rates than full scale (Table B3) thus demanding greater bank angle freedom than provided by the gimbal arrangement ($\pm 30^\circ$) in the simulation. Hence, pulse aileron input was used and $t_{\frac{1}{2}}$ was obtained from the bank angle response. A typical response is sketched in the following figure 59.

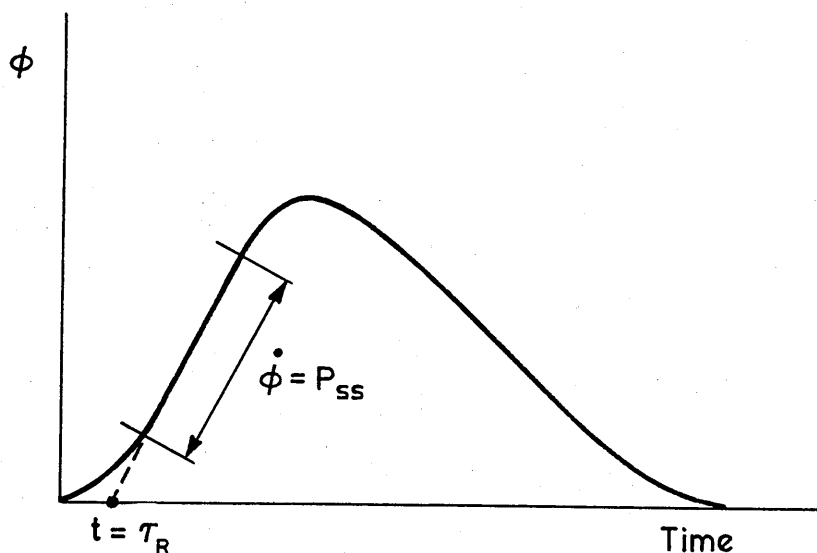


Figure 59 Model Roll Response to
Pulse Aileron Input

Only the initial response (roll rate build up) is used to evaluate $t_{\frac{1}{2}}$ as it represents the initial response to a step input and therefore it can be argued that the slope of this portion should be the same as that of a step input. The decaying half of the response indicates a less steep slope due to 'carryover' inertia effect. It can therefore be assumed that the above method of Roll mode response evaluation is an approximation of an actual case and can be a cause of discrepancy in correlation.

- (ii) During simulation, before applying the input pulse, the model's wings could not be kept very steady at the level position due to wind tunnel flow perturbations as reported in reference 42. The bank angle fluctuations at times

were as high as $\pm 6^\circ$. This obviously made the roll response analysis more difficult and is also suspected to be the main contributor to scatter of the experimental $t_{\frac{1}{2}}$ data.

- (iii) The theoretical predictions are greatly influenced by the stability derivative l_p , which has a prediction accuracy of $\pm 10\%$ (reference 49). The lower values of theoretically predicted $t_{\frac{1}{2}}$ can be due to higher theoretical values of l_p .

As expected, the effect of c.g. shift on Roll mode response was found to be insignificant as $t_{\frac{1}{2}}$ values for both c.g. positions lie in the same band of experimental data.

The time to half amplitude ($t_{\frac{1}{2}}$) deduced from Hawk aircraft data at $M = 0.3$ is also plotted in figure 58. The value is about 12% higher than that of the semi-free simulation results, which is within a reasonable limit considering the scale factor involved. The correlation is very poor (about 50% high) with the theoretical predictions of $t_{\frac{1}{2}}$ value. This result is more surprising because the values of the stability derivative l_p deduced from Hawk data and theoretical estimates using reference 49, correlate very well and are within 10% (figure 40). Considering the dominance of the stability derivative l_p in its contribution to $t_{\frac{1}{2}}$, 50% disagreement was unexpected.

(b) Dutch Roll Mode

(i) Frequency

The Dutch Roll mode frequency for different flight

conditions is plotted in figure 60 . The theoretically predicted values of Dutch Roll mode frequency for full scale aircraft and free-flight scaled models match very closely due to negligible compressibility effect at these low Mach number flight conditions. The Dutch Roll mode frequency of the semi-free scaled model of the wind tunnel simulation also displays good correlation with full scale and free-flight model, showing the trend of 5 - 10% lower magnitude. The trend of increase of the frequency with increase in speed is also the same. It can therefore be argued that the suppression of lateral translation has very little effect on the Dutch Roll mode frequency for the flight conditions under consideration. The suppression of lateral translation results in omission of terms containing the stability derivative y_v and lift coefficient C_L in the stability quartic of the lateral motion (equation 4-24 and 4-30). The stability derivative which contributes dominantly to the Dutch Roll frequency is the directional stability derivative n_v , and the contribution of the y_v term, due to its small magnitude, is very small as compared to n_v . Hence the theoretical correlation between the free and semi-free model is good. The values of frequency obtained from the wind tunnel simulation also match to within 8% of the theoretical values. The Hawk aircraft data at $M = 0.3$ also compares well with the simulation and theoretical values and therefore it can be said that the present technique of simulation brings good overall correlation of Dutch Roll mode frequency over the range of flight conditions considered.

(ii) Damping Ratio

The Dutch Roll mode damping ratio for various simulated

velocities is given in figure 61 . The predicted values of Dutch Roll damping for full scale and free-flight scaled models are more or less the same due to low flight speeds ($M < 0.4$) under consideration. The unusual trend of increase in damping ratio with increasing flight speed is due to low speeds of simulation. The higher values of lift coefficient C_L , at these speeds have a considerable effect on the coefficient D_3 (specially when value of l_v is large) and E_3 of lateral stability quartic (equation 4.24) which in turn influences the Dutch Roll damping. At these low speeds, lift coefficient C_L is very sensitive to speed changes, and a decrease in speed increases C_L considerably thus having overall effect of decreasing the Dutch Roll damping. However, at higher speeds, where the contribution due to C_L is smaller, the damping of Dutch Roll mode would be expected to remain less sensitive to speed changes. The theoretical values of damping ratio for semi-free model of the wind tunnel simulation also shows the same trend but as the contribution of the C_L term is omitted due to suppression of lateral translation, the trend is less severe. This gradual trend is also influenced by the contribution of the yaw damping derivative l_r (equation 4-30) which decreases considerably with increasing speed in the speed range under consideration (figure 40). The difference between the semi-free and the free-flight model damping ratios at higher speeds can be attributed to the absence of the contribution due to the stability derivative y_v in the semi-free flight simulation because of the suppression of lateral translation. The stability derivative y_v is analogous to the longitudinal stability derivative z_w , except that the reference surface area is smaller and is also non-lifting in lateral motion, thus having an overall smaller effect.

The values of the Dutch Roll mode damping obtained from the wind tunnel simulation show the same sort of trend and order of magnitude as predicted by theory. The simulation gave higher values of Dutch Roll damping ratio and the correlation difference varies to a maximum of 20%.

The agreement between the Hawk data and the corresponding theoretical and simulation Dutch Roll damping is very poor, the Hawk data showing a 65% greater damping ratio. The disagreement can be attributed mainly to the discrepancy in the stability derivative l_v . It is recalled that the value of stability derivative l_v predicted using reference 49 is about 50% higher than that given in the Hawk data (figure 40) whereas estimated prediction accuracy of this derivative is given as $\pm 20\%$. The above mentioned difference can be explained if the Hawk data is for the final version of aircraft which employs ventral fins, as the effect of the ventral fin is to reduce the stability derivative l_v . The effect of this derivative on the Dutch roll damping is quite significant as shown by root locus sketch below.

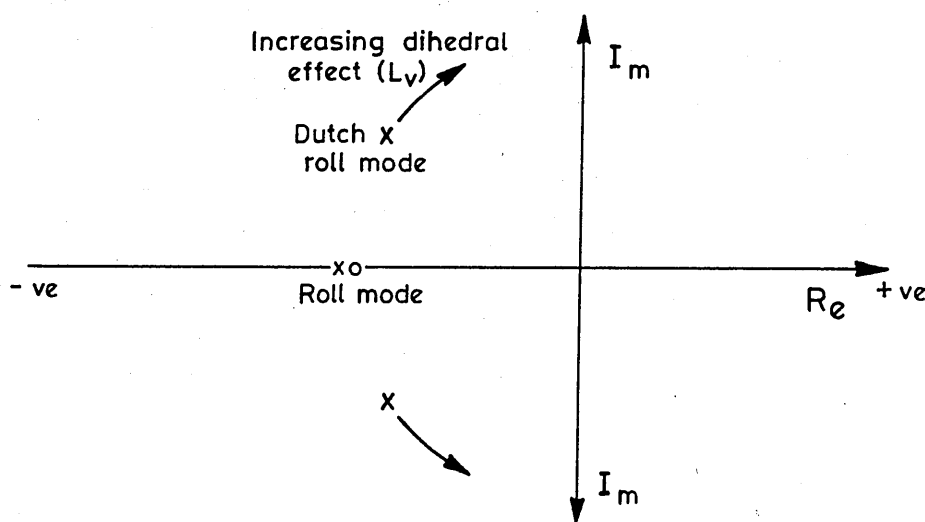


Figure 62 Influence of Adding Positive Dihedral Effect

If the dihedral effect is decreased, using ventral fins, then the damping ratio of the Dutch Roll mode will increase with very small influence in its frequency. Thus, if the estimated value of l_v is decreased to that obtained from the Hawk data, the estimated value of damping ratio will increase to 0.14 and the correlation will improve to within 20% thus showing better agreement with the simulation results. Note that the decrease of dihedral effect also has a small influence on the Roll mode time to half amplitude ($t_{\frac{1}{2}}$).

6.4 Closed-Loop Model Dynamics

6.4.1 Longitudinal Dynamics-Short Period Mode

The closed-loop Short Period mode root locus plot for theoretical and experimental results at model velocities of 70 ft/sec and 100 ft/sec are given in figures 63a and 64a respectively. It is recalled that pitch rate feedback to elevator was used in these experiments. The experimental values exhibit the same trend as theoretically expected, i.e. increase of damping with the increase of pitch rate feedback gain. However, the experimental plot shows some compression along the real axis. This is due to the fact that the real part of the short period root corresponds to $-\zeta\omega_n$ and as discussed in the open-loop case, the experimental values of damping ζ_{sp} and frequency ω_n are lower than the theoretical or actual values because of the limited vertical freedom of the model. To demonstrate the closed-loop behaviour more clearly the short period damping ratio and frequency are plotted against the pitch rate feedback gain K in figures 65 and 66 respectively. From figure 65 it can be seen that the experimental and theoretical values of damping

ratio vary linearly with the pitch rate feedback gain and a small increase in the feedback gain improves the short period damping considerably. Similarly, small values of negative gain degrade the short period damping to a great extent. It can also be observed that for a given open loop ($K = 0$) difference in the damping ratio between the theoretical and experimental results, the correlation difference does not change much when feedback is applied. The higher slope at a model velocity of 100 ft/sec as compared to that at 70 ft/sec indicates that for a given increment in damping ratio, higher feedback gains are required at lower speeds. This is due to the increase in control effectiveness with increase in speed and is theoretically indicated by the variation of high-frequency gain factor with speed ($\propto v^2$).

It can also be observed from figure 66 that the natural frequency of the Short Period mode changes with the pitch rate feedback gain factor K . This can be attributed to the low stability margins of the model at the flight conditions under consideration. At higher stability margins, the open-loop natural frequency has greater value and correspondingly its sensitivity to the effect of feedback gain change decreases. This trend is shown in figure 66, where the slope of the frequency plot is less in the case of model at c.g. = 0.29 $\bar{c}$ than at 0.325 $\bar{c}$. It can therefore be argued that at higher stability margins, the slope will flatten and hence the Short Period mode natural frequency will not change appreciably with pitch rate feedback as generally reported.^{4,5,7} The correlation between experimental and theoretical values of short period frequency for various feedback gains is better than the short period damping ratio and the difference remains about the

same as that at $K = 0$ (open loop).

The closed loop response of the model with pitch freedom (vertical translation suppressed) only is also presented in figures 63b, 64b and 67. As in the open-loop model dynamics, the correlation between experimental and theoretical results is improved considerably. In this case frequency does not change with feedback gain as seen in figure 67 because the vertical freedom of the model is suppressed which essentially eliminates the effect of pitch damping on the frequency (z_w is zero in equation 4-15). Figure 67 also shows that the damping ratio sensitivity with closed loop pitch rate gain increases with the decrease in static margin. This trend is not visible in the simulation with vertical freedom as, in that case, the frequency does not remain constant with gain K .

Since the theoretical values of open-loop short period frequency and damping ratio for full-scale and semi-free model simulations show good agreement, it can be assumed that the same agreement will be exhibited in the closed-loop simulation. Therefore, figures 65 and 66 are also scaled to the full scale aircraft parameters. The relationship between the pitch rate feedback gain K for the model and full scale is found to be a function of geometric scale factor of the model i.e. $K_{\text{full scale}} = \sqrt{\lambda} K_{\text{model}}$.

6.4.2 Lateral Dynamics

(a) Roll Mode

The theoretical and experimental time to half amplitude ($t_{\frac{1}{2}}$) of the Roll mode for different gains K of roll

rate feedback to aileron are presented in figures 68 and 69 for model speeds of 70 ft/sec and 100 ft/sec respectively. The experimental results show less stability than predicted by the theory but the trend is essentially the same. It is clear from these figures that the Roll mode time constant decreases rapidly, showing a rapid increase in roll stability, with initial modest roll rate feedback gain K . At higher gains, the rate at which the stability is increased is lower. Similarly it can be seen that the stability of the Roll mode is appreciably degraded with very small values of negative gain. A comparison of figures 68 and 69 shows that the sensitivity of the Roll mode stability decreases with decreasing model velocity and therefore at lower velocities higher feedback gains are required for the same increase in roll stability. The difference between experimental and theoretical results are about the same for all feedback gains and are of the same order of magnitude as with the open-loop ($K = 0$) case.

Since, in the open-loop case, the theoretical predictions of time to half amplitude ($t_{\frac{1}{2}}$) for semi-free wind tunnel simulation and full scale show good agreement, it could be argued that the same theoretical trend will be expected in the closed loop case. However, it is recalled that the theoretical predictions of full scale do not match well with the Hawk data, the reason for this having been discussed earlier in the chapter.

(b) Dutch Roll Mode

The Dutch Roll mode root locus plots for yaw rate feedback to rudder at model velocities of 70 ft/sec

and 100 ft/sec are presented in figure 70 . The simulation root locus matches very well with the theoretical trend for positive gains. The Dutch Roll mode natural frequency remains nearly constant while the damping ratio improves considerably with the increase of positive gain. But for the negative gains, although the Dutch Roll mode damping decreases, the frequency also decreases which is unlike the theoretical trend of constant frequency. It is, however, realised that the corresponding damping ratios at these negative gains are very low and would warrant positive gains for stability augmentation.

The Dutch Roll mode frequency and damping ratio plots for various feedback gains are also given in figure 71 and show the fore-mentioned trends more clearly. It can also be seen that the feedback loop has a more favourable effect at higher speeds (higher damping plot slope) due to the increase of control sensitivity with speed.

Although the plots are also scaled to show full-scale characteristics, only frequency can be interpreted confidently since its correlation with theory and the Hawk data is good in the open-loop analysis. In the case of the Dutch Roll mode damping only the trend and approximate interpretation of values can be guaranteed due to large discrepancies in correlation of open loop results.

CHAPTER 7

COMMENTS, RECOMMENDATIONS AND CONCLUSIONS

7.1 Comments and Recommendations

7.1.1 Design and Construction

(i) Model Suspension System

(a) Gimbal System

The gimbal assembly performance was found to be satisfactory in providing the model with $\pm 30^\circ$ of pitch and roll freedom and the included potentiometers provided adequate attitude signals. The pitch and roll motion has very low friction levels due to the high quality bearings used and the high quality potentiometers gave excellent resolution in attitude measurement.

The sliding sleeve bearing for vertical motion was satisfactory in operation with one minor exception due to the use of a nylon key to constrain the sleeve bearing to rotate with the vertical rod. The linear ball bearings installed in the sleeve provide very free vertical movement of the gimbal but the nylon key had the tendency to deform with use thus introducing friction into the vertical motion.

In the initial design a sub-miniature bearing was considered instead of the nylon key, but due to the small size of the sleeve, the smallest sub-miniature bearing available could not be accommodated in the sleeve. However, the design

of the sleeve allows for easy replacement of the key and this short-coming can be considered to be of a minor nature.

The outer gimbal frame also provides the means for shifting the model c.g. through an adjustable attachment to the model and during the simulation it was discovered that with the present outer gimbal frame length, the range of c.g. shift is rather too limited. An extension in the outer gimbal frame length and rearrangement of the attachment points would provide a larger range of c.g. shift.

(b) Vertical Rod

The vertical freedom of the model is very sensitive to vertical alignment accuracy since the vertical rod is held in position by wire bracing and a slight loosening of the tensioning screws can alter the vertical alignment of the rod. It was observed that even a small offset angle introduces appreciable frictional effects. Thus the vertical alignment requires careful setting up before and continuous monitoring of the position during the simulation tests. A more rigid attachment of the vertical rod to the frame could be a possible solution to this problem.

The vertical rod geometry was a compromise resulting from a number of design constraints (chapter 4); bending deformation under aerodynamic loads and the requirement for a small diameter to suit the gimbal system being the most critical. This led to a compromise on the bending deformation (notably at higher tunnel speeds) in order to keep the diameter

of the rod down to a reasonable size for the gimbal sleeve. This is one of the reasons why the present simulation tests are limited to a maximum practical tunnel velocity of 100 ft/sec. The other factors being the jet flow oscillation or turbulence in the tunnel near its maximum speed and the violent response of the model to very small control changes due to greater control sensitivity at higher speeds.

The yaw freedom of the model, provided by the rotation of the vertical rod in the end plate bearings, is not as free of friction as the pitch and roll freedom. This is specially so at higher speeds and when tension in the bracing wire is too great.

Bearing in mind the above points, if the vertical rod is properly set up and its bearings properly lubricated, the frictional effects can be minimised. Overall the performance of the suspension system has been found to be satisfactory and the mobility of the mounting frame has proved to be a useful feature of the support system.

(ii) Model

The model design and construction technique used provided a reasonably satisfactory aircraft model. The model as constructed has adequate strength and rigidity and is light enough to allow for mass and inertia scaling by the addition of mass. The model central box structure and the gimbal system which is attached to it, have withstood unreasonably violent impacts against the end plates during the course of development with little real damage.

Through experience, operating techniques have been evolved which avoid over-large inputs being applied to the model and precautionary measures have been taken by installing safety springs at both ends of the rod to absorb the impact. The wing and the tailplane attachments are secure and rigid and the control surface movement is relatively free of static friction and backlash. The canopy panel provides adequate access to the servo actuators and electrical circuitry.

However, the adjustment of the model inertia by the selective addition of mass was unsatisfactory primarily due to the limited access to the model. It is suggested that in future more consideration should be given to this aspect and provision in the initial stages of fuselage construction should be made for better access so that a systematic procedure for mass balancing could be adopted such as described in reference 38.

The servo actuators used for the control surfaces have not been entirely free of problems. Their response and performance on the test bench was satisfactory but when connected to the electronic control unit through the umbilical cord connection for calibration tests, they showed oscillatory tendencies. Initially, to overcome this problem, servo amplifier feedback gain was decreased, which stopped the oscillations but made the servo response very sluggish and hence too slow. Eventually, the problem was traced to the voltage fluctuations in the power supplies to the servo amplifiers due to the voltage drop in the long connection line particularly when the motors were drawing current. The problem was solved by

stabilizing the power supply in the model prior to supplying the servo actuators. The response of the servos was also enhanced by making improvements in the feedback circuitry of the servo amplifiers.

It is, however, pointed out that the servo amplifier circuit is included within the miniature servo housing and its miniature size prevented any sensible circuit modifications. It is therefore suggested that in future applications the servo amplifier circuit should be removed from the servo housing, redesigned and redrawn on a larger P.C. board for easy access and modifications. The circuit could be mounted in the fuselage without any problem, and a much better amplifier design would probably result.

The presence of backlash in the model control linkages made precise trimming of the model very difficult. The major contribution to the backlash is in the servo actuator gear train although PTFE tubing control linkages and their end connectors are also inadequate for high resolution precision control. It is therefore suggested that the backlash problem should be looked into and should be rectified even at the expense of better quality servo actuators. The improved trimming capability thus obtained would make the task of flying the model in the tunnel much easier. It could also be confidently expected that with better servo actuator performance, by virtue of reduced backlash, small more realistic inputs could be used in the model mode excitation techniques.

(iii) Electronic Control Unit

A great deal of careful thought was applied to the design of the electronic control unit panels which are self explanatory in their functions and display adequate information for the ease of the user. The provision of external patch connections, like an analogue computer, has provided great flexibility in the way in which the control unit functions may be used. The overall performance of all the panel functions is satisfactory and it was encouraging to see it perform as intended at the very first time it was switched "on" for testing.

During subsequent system evaluation experiments the following shortcomings in the panel components were observed:

- (a) In some instances single turn potentiometers were found to be inadequate for accurate adjustment when low feedback gains were required. It is suggested that a few high stability multi-turn potentiometers should be installed to replace some of the single turn potentiometers on the potentiometer panel.
- (b) A digital voltmeter should be installed to replace the present analogue meter which cannot be used to monitor low voltage signals to the required accuracy.
- (c) The pulse/doublet generation circuit, which was built later and used for mode excitation was an addition to control functions already provided. It is suggested that its circuit

card should be properly housed in the card rack in the electronic control unit and the switch installed on the control panel appropriately.

The performance of the signal conditioning amplifiers, summers and integrators was found to be extremely satisfactory. The testing of various operational amplifiers in purpose-built circuits helped in the correct selection of suitable cost effective operational amplifiers for the above mentioned circuits. The use of low tolerance (1%) and high stability resistors for input and feedback resistance has resulted in circuits with accurate and stable open loop gains. In all cases the offset adjustment circuit has been used to null offset voltage most effectively.

The modular nature of the design has contributed to the simplicity and flexibility in design and has made fault diagnosis very easy during the development stage.

7.1.2 Operational Aspects

- (i) As the Hawk model uses an all moving horizontal tail for longitudinal control the trim angles required for straight level flight are very small. Due to the presence of backlash in the model controls, as discussed earlier, the precise trimming of the model in the wind tunnel was difficult. The systematic procedure explained in chapter 5 has to be strictly adhered to to avoid too violent a response in the event of inadvertant over trimming. Monitoring

model attitude (incidence) angles whilst flying the model at velocity as a guide to trimming has proved to be helpful.

As suggested earlier if the backlash problem of the control is minimised, model trimming will improve considerably.

- (ii) When flying in the wind tunnel the model experiences fluctuations in bank angle due to disturbances in the wind tunnel jet. It is suggested that the wind tunnel flow characteristics could usefully be improved for more consistent trimming in roll; this would also improve the simulation results of roll response.
- (iii) The operation of the electronic control unit was found to be quite satisfactory, the flexibility provided by the external patch cord connection proving to be a valuable feature. The recommendations for improvement to the control panel components made earlier would further improve the operational performance.

7.1.3 Performance of the System

- (i) Open Loop Experiments
- (a) Short Period Mode

The suppression of forward translation, due to the technique selected, has suppressed the Phugoid mode of the model but the effect on Short Period mode, which is of primary

interest, is negligible. The theoretical values of the Short Period mode frequency and damping ratio show good correlation between the semi-free wind tunnel model, a true free flight model and the full scale cases.

The model simulation results show satisfactory short period frequency correlation to within 10% with the theoretically predicted full scale results and Hawk data. However, the short period damping is not so good and only agrees to within 20% with the theoretical prediction and Hawk data and is influenced by the limitation of the vertical freedom of the model as discussed in chapter 6. When the model was completely restrained in the vertical freedom, the simulation and theoretical results showed better correlation in both short period frequency and damping. This result reaffirms the belief that an alternative mode excitation technique requiring limited vertical freedom is expected to give better simulation results. This could possibly be achieved by reducing the control backlash as suggested earlier. This, in turn, would allow for smaller pulse inputs to be used to excite the Short Period mode thus requiring less vertical freedom in any event.

It can therefore be argued that with an improved mode excitation technique it would be possible to predict full scale Short Period mode characteristics from the

wind tunnel simulation to within the accuracy (10%) expected from this limited facility.

(b) Roll Mode

Theoretical results show that suppression of lateral translation has a very small effect on the Hawk model roll mode characteristics. The Roll mode simulation results show satisfactory trends but satisfactory degrading to poor correlation (10-30%) with the full scale theoretical predictions. The correlation with the Hawk data is better. The above discrepancy can be attributed to the approximate mode analysis technique (i.e. pulse input) used due to limitation in bank angle freedom and the tunnel flow characteristics as discussed in chapter 6. A better mode excitation technique would help to improve the fidelity of the simulation. Improved tunnel flow characteristics, and improvement of the control backlash might allow smaller amplitude impulse inputs to aileron to excite the Roll mode within the constraint.

(c) Dutch Roll Mode

The theoretical results show that the suppression of lateral translation has a limited effect on the Dutch Roll mode frequency but the damping is appreciably changed due to the low velocity flight regime of the simulation.

The Dutch Roll mode simulation results show good frequency correlation with the predicted full scale values and the Hawk data. The damping ratio correlation and trends are reasonable considering the low magnitudes of damping involved. The correlation of theoretical and simulation results with the Hawk data is very poor and the Hawk data shows a better Dutch Roll mode damping ratio. As discussed in chapter 6, the presence of ventral fins on the Hawk aircraft could explain the difference in the correlation.

(ii) Closed-Loop Experiments

The closed loop experiments in which the effect of rate feedback was studied, showed the expected trends and allowing for the discrepancies in the open loop results gave good correlation with theoretical predictions. Since the rate signals were obtained by electronically differentiating attitude signals using the analogue elements of the electronic control unit, it could be said that the electronic control unit has provided satisfactory results and can be used effectively for feedback loop circuits to augment the stability and control characteristics of the model. If the open loop simulation results of the model could be improved to bear better correlation with the full scale, the closed loop simulation could be used to predict full scale performance with a reasonable degree of confidence.

(iii) General Performance

The scale effects due to low operating Reynold's number cannot be directly assessed from the results due to the presence of other more dominant effects. In cases where the results are satisfactory, correlation between the full scale and model simulation is as close as 10%. This trend shows that the scale effects are within 10% and are not seriously affecting the model performance at the low levels of model incidence used in the simulation.

In order to get much closer insight into the scale effects, it is recommended that static wind tunnel tests on the model should be conducted to find its lift and drag characteristics. This would involve some modification to the model suspension system.

The present evaluation of the facility was restricted to dynamic stability mode analysis only which provides a partial indication of the complete response. The use of a complimentary computer simulation would provide a means for comparing the complete response thus giving a better insight into the aircraft dynamics and hence facility performance.

Once the capability of the facility is further improved as per suggestions and the reported shortcomings are overcome, the facility could be used for Active Controls Technology research for example. Optimisation of airframe geometry using augmented controls, the use of unconventional control techniques and the study of unconventional

aircraft configuration in which airframe is unstable are some of the possible applications.

7.2 Conclusions

1. The performance of the designed suspension system is quite satisfactory and successfully provides the model with the intended four degrees of freedom.
2. The constructional technique adopted for the model, utilising a hollow balsa wood fuselage structure with solid light hard wood wing and control surfaces provides a strong, lightweight and reasonably accurate wind tunnel model. However, with improved internal access to the model, mass and inertia adjustment could be made very much easier and along with minor modifications to the gimbal mounting arrangement for provision of a larger c.g. shift would provide greater flexibility in choice of test flight condition.
3. In any future model it would be necessary to incorporate better quality servo actuators. This would alleviate some of the problems arising from the limited control system resolution and would make it easier to set the extremely precise trimming required.
4. The electronic control unit meets all the initial design requirements adequately. It provides effective control of the model, its input and output interfaces with the model and the analogue computing elements provide excellent flexibility in its functional capabilities.

5. The model is extremely sensitive to trim changes arising, in part, from the suppression of longitudinal translation and the constant wind tunnel speed. A slight over-trim results in an uncontrollable oscillatory vertical motion due to the change in lift and the presence of vertical motion end stops.
6. The suppression of longitudinal translation implicit in the adoption of the rigid rod suspension technique suppresses the Phugoid mode although the Short Period model remains almost unchanged, as would be expected.
7. The suppression of lateral translation has eliminated the Spiral mode but the Roll mode is not changed significantly. However, the Dutch Roll mode frequency does not seem to be changed in any substantial way but the damping ratio is considerably changed especially at the lower wind tunnel velocities.
8. The simulation shows that it is possible to predict the Short Period mode characteristics of the full scale aircraft from the scaled model responses with reasonable accuracy and confidence.
9. The Roll mode characteristic in particular could be predicted to a reasonable level of confidence if the wind tunnel flow qualities were improved.
10. The Dutch Roll mode can be predicted only approximately. In this exercise the Dutch

Roll mode frequency predictions were satisfactory but the damping ratio prediction was poor. The situation could be improved if the model was given the addition of lateral translation.

11. The stability of the model can be augmented effectively by artificial means using selected feedback loops and hence the full scale closed loop characteristics could be predicted but only to a level corresponding to the validity of the open loop characteristics.
12. Overall, the initial evaluation of the facility shows promising results, the initial objectives are achieved and with the suggested improvements it has a viable potential for ACT research.

REFERENCES

1. Osder, S.S. Chronological Overview of Fast Avionic Flight Control System Reliability in Military and Commercial System Operations AGARDOGRAPH AG-224, May, 1977.
2. Duc, J.M. Control-Configured-Vehicle Design Philosophy" AGARDOGRAPH GCP-514, Oct. 1978.
3. Charles A Anderson F-16 Multinational Fighter AGARD AG 234, Nov. 1978.
4. Kurzvals, P.R. System Implications of Active Controls AGARD CP 260, 1978.
5. Warner, J.C. The CCV-Concept AGARD HIGHLIGHTS 72/2, 1976.
6. Harris, R.B. Active-Control Design Criteria AGARD AG 234, Nov. 1978.
7. Kurzhal, P.R. Active Control in Aircraft Design AGARD AG 234, Nov. 1978
8. Herman A. Status and Trends in Active Control
Reddiness & Technology
Kenneth J.Szalai NASA/UNIVERSITY CONFERENCE ON
AERONAUTICS, Oct. 1974.

9. Subke H. and Krag, B. Dynamic Simulation in Wind Tunnels
Part II
Proceedings of AGARD Flight
Mechanics Panel Symposium on Flight/
Ground Testing Facilities Correlation,
Valloire, France, 1975.
10. Wilmar, H. Reed III Comparisons of Flight Measurements
with Predictions from Aeroelastic
Models in the NASA Langley Transonic
Dynamic Tunnel
Proceedings of AGARD Flight Mechanics
Panel Symposium on Flight/Ground
Testing Facilities Correlation,
Valloire, France 1975.
11. Lee S. Wasserman Computer Vs Model
RTD-TDR-63-4197 Part I, Proceedings
of Symposium on Aeroelastic and
Dynamic Modeling Technology, Dayton,
Ohio, Air Force Systems Command, 1963.
12. Kumar, H. A Preliminary Study into the Design
of a Free-Flight Wind Tunnel Model
for Demonstrating the Dynamic
Characteristics of Aircraft
MSc. Thesis, College of Aeronautics,
C.I.T., Sept. 1980.
13. Bisgood, P.L. Low-Speed Investigations Using
Free-Flight Models.
RAE Tech. Note 2713, Sept. 1960
AGARD CP NO.17, 1966.

14. Fail, R.A. The Role of Free-Flight Models in Aircraft Research and Development. AGARD CP NO. 119, 1972
15. Fail, R.A. Experimental Determination of Stability and Control Aerodynamic Characteristics
VKI Lecture Series on "Aircraft Stability and Control", May 1975
16. Libbey, C.E. & A Technique Utilizing Free-Flying
Burk, S.M. Radio Controlled Models to Study the Incipient and Developed Spin Characteristics of Airplanes.
NASA MEMO 2-6-59L, 1959.
17. Holleman, E.C. Summary of Flight Tests to Determine the Spin and Controllability Characteristics of a Remotely Piloted Large-Scale ($3/8$) Fighter Airplane Model
NASA TN D-8052, 1976
18. Stephens, A.V. Free-Flight Spinning Experiments with Several Models
R & M NO.1404, British ARC, 1931.
19. Soule, H.A. A Method of Flight Measurement of
Scudder, N.F. Spins
NACA Rep. 377, 1931
20. Stone, R.W. Study of Motion of Model of Personal-
et al Owner or Liaison Airplane through the Stall and into the Incipient Spin by Means of a Free-Flight Testing Technique.
NACA TN 2923, 1953.

21. Hewes, D.E. Free Flight Investigation of
Radio-Controlled Models with
Parawings.
NASA TN D-927, 1961
22. Campbell, J.P. Free and Semi-Free Model Flight-
Testing Techniques used in Low-
Speed Studies of Dynamic Stability
and Control.
AGARDOGRAPH 76, 1963
23. Finn, E. The R.A.E. Free-Spinning Tunnel
R.A.E. TECH.NOTE NO.AERO 637, 1939.
24. Zimmermann, C.H. Preliminary Tests in the NACA Free-
Spinning Wind Tunnel.
NACA REP 557, 1936
25. Neihouse, A.I. Status of Spin Research for Recent
et al Airplane Designs.
NASA TR R57, 1960.
26. Clarke, A.E. The Vertical Spinning Tunnel at the
Maltby, R.L. National Aeronautical Establishment,
Bedford.
RAE TECH.NOTE NO.AERO 2339, 1954
27. Shortal, J.A. Preliminary Stability and Control
Osterhout, C.J. Tests in the NACA Free-Flight Wind
Tunnel and Correlation with Full
Scale Flight Tests.
NACA TN 810, 1941.
28. Campbell, J.P. The Effect of Mass Distribution on
Seacord, C.L.Jr. the Lateral Stability of Control
Characteristics of an Airplane as
Determined by Tests of a Model in the
Free-Flight Tunnel.
NACA Rep.769, 1943

29. Campbell, J.P. Techniques for Testing Models of VTOL
and STOL Airplanes.
AGARD REPORT 61, 1956.
30. Grafton, S.B. Wind Tunnel Free-Flight Investigations
et al of a Model of a Spin-Resistant
Fighter Configuration.
NASA TN D-7716, 1974.
31. Covert, E.E. Magnetic Balance and Suspension System
et al for Use with Wind Tunnel.
Progress in Aerospace Sciences, Vol.
14, pp. 27.107, Pergamon Press, 1974.
32. Braun, G.W. Testing Longitudinal Stability on
Models in Harnessed Circling Flight.
AF TECH. REP.NO. 5979, 1949.
33. Schade, R.O. Flight Test Investigation on the
Langley Control-Line Facility of a
Model of Propeller-Driven Tail-Sitter-
Type Vertical Take-Off Airplane with
Delta Wing During Rapid Transitions.
NACA TN 4070, 1957.
34. Martinez, E. A New Facility for the Study of
Aircraft Dynamics.
Department of Aeronautical
Engineering, Princeton University,
Rep. No.532, 1961.
35. Maurice, R. Means and Examples of Aeronautical
Research in France at ONERA.
Journal of Aero-space Sciences, Vol.
26, No.4, April, 1959.

44. Bradley, W.G. Measurement of the Moment of Inertia
of the Morane-Saulnier MS760 'PARIS
AIRCRAFT'
C.O.A. Note No.160, Nov. 1963
45. Stout, D.F. Handbook of Operational Amplifier
and Circuit Design
Kaufman McGraw-Hill Book Company, 1976
46. Richards, J.C.S. Simplified Design of d.c. Power
Supplies .
Wireless World, August 1981
47. Babister, A.w. Aircraft Dynamics Stability and
Response
Pergamon Press, 1980.
48. Cook, M.V. Stability and Control Lecture Notes
College of Aeronautics, Cranfield
Institute of Technology, 1980-81.
49. Engineering Science Data Unit, London
50. DATCOM (USAF Stability and Control
DATCOM)
Flight Control Division, Air Force
Flight Dynamics Laboratory.
51. Young, A.B. A Review of Methods for Obtaining
Subsonic Longitudinal Aerodynamic
Derivatives .
Von Karman Institute for Fluid
Dynamics, Lecture Series 99, 1977.
52. Aerodynamic Data and Stability and
Control Derivatives of the Hawker
Siddeley H.S. 1182
Hawker Siddeley Aviation Ltd., 1972.

- 53. Alan Carlson et al Editors Handbook of Analog Computation
Electronics Associate, Inc.
Princeton, New Jersey, 1967.

- 54. Perkins, C.D. AGARD Flight Test Manual
Volume II, Stability and Control
New York, Pergamon Press, Inc., 1962

- 55. Anon Flying Qualities of Piloted Airplanes
MIL-F-8785B(ASG), Sep.1974.

- 56. Hoerner, S.F. Fluid Dynamic Lift
Borst, H.V. Hoerner Fluid Dynamics
Brick Town, N.J., U.S.A., 1975

- 57. McRuer, D. Aircraft Dynamics and Automatic Control
Ashkenas, I. Princeton University Press, 1973.
Graham, D.

APPENDIX A

DERIVATION OF SIMILARITY REQUIREMENTS

The following assumptions are made in deriving the similitude requirements for the dynamic wind tunnel model.

- (i) Compressibility effects can be neglected. This implies that the model speed is limited to a value corresponding to the full scale aircraft speed at which compressibility effects begin to appear. Therefore, the requirement to maintain full scale Mach number can be ignored.
- (ii) Although it is difficult to satisfy Reynold's number equality between the model and full scale aircraft as discussed in chapter 2, the difference between the relevant model and full scale Reynold's numbers is such that the Reynold's number dependence is not a serious limitation. To avoid Reynold's number dependence, this number is generally kept higher than the critical value ($\approx 0.4 \times 10^6$) where the airflow becomes turbulent.

In the following relationships, quantities with subscript (m) and (a) correspond to the model and full scale aircraft respectively.

1. Linear Scale Factor

This factor is chosen as per tunnel, practical and economic constraints. Necessity to limit the model size below a specified limit may arise due to practical or economic considerations, but in case of small tunnels the maximum span of the model which can be tested in the wind tunnel and the requirements of model freedom determine this factor.

$$\text{Linear scale ratio, } \lambda = \frac{l_m}{l_a} \quad (\text{A.1})$$

2. Velocity Ratio

This ratio may sometimes be fixed by the necessity to keep acceleration scale factor at unity i.e. testing the model in natural gravity conditions (refer to Froude number discussion in chapter 2). For a low speed wind tunnel it may be determined by the maximum usable tunnel speed to satisfy Reynold's number requirement.

$$\text{Velocity ratio} = \frac{V_m}{V_a} \quad (\text{A.2})$$

3. Density Ratio

This ratio accounts for differences between altitude of the aircraft and the model. If ρ_a is the density of air at the flight altitude of the aircraft and ρ_m is the density of air in the wind tunnel, then

$$\text{Density ratio} = \frac{\rho_m}{\rho_a} \quad (\text{A.3})$$

4. Angular Velocity

For dynamic similarity between the model and the aircraft, the Strouhal number must be equal, i.e. the helix angles of the corresponding points of any rotary motion must be equal, expressed thus,

$$\frac{\omega_m l_m}{V_m} = \frac{\omega_a l_a}{V_a}$$

$$\therefore \frac{\omega_m}{\omega_a} = \frac{1}{\lambda} \frac{V_m}{V_a} \quad (\text{A.4})$$

5. Time Scale Factor

Time scale factor τ is determined from (A.4).

$$\frac{\omega_m}{\omega_a} = \frac{1}{\lambda} \frac{V_m}{V_a} = \frac{1}{\tau}$$

$$\therefore \tau = \lambda \frac{V_a}{V_m} \quad (\text{A.5})$$

6. Mass

Equilibrium is to be maintained between aerodynamic, centripetal and gravitational forces. For dynamic similarity the vector diagrams of these forces are similar triangles. Equating centripetal and aerodynamic forces.

$$\frac{m_m \omega_m^2 l_m}{m_a \omega_a^2 l_a} = \frac{C_m \frac{1}{2} \rho_m V_m^2 l_m^2}{C_a \frac{1}{2} \rho_a V_a^2 l_a^2}$$

Assuming $C_m = C_a$ (aerodynamic force coefficients)

$$\frac{m_m}{m_a} = \frac{\rho_m}{\rho_a} \lambda^3 \quad (\text{A.6})$$

7. Linear Acceleration

Similarly equating inertia and aerodynamic forces

$$\frac{m_m a_m}{m_a a_a} = \frac{C_m \frac{1}{2} \rho_m V_m^2 l_m^2}{C_a \frac{1}{2} \rho_a V_a^2 l_a^2}$$

assuming $C_m = C_a$

$$\frac{a_m}{a_a} = \frac{1}{\lambda} \left(\frac{V_m}{V_a} \right)^2 \quad (\text{A.7})$$

8. Angular Accelerations

The relationship between the linear and angular acceleration is

$$a = \alpha l$$

where a is linear acceleration and
 α is angular acceleration.

$$\therefore \frac{\alpha_m}{\alpha_a} = \frac{a_m}{a_a} \frac{l_a}{l_m}$$

or

$$\frac{\alpha_m}{\alpha_a} = \frac{1}{\tau^2} \quad (\text{A.8})$$

9. Moment of Inertia

The inertia relationship can be determined from the consideration of moments

$$\frac{I_m \alpha_m}{I_a \alpha_a} = \frac{C_m \frac{1}{2} \rho_m V_m^2 l_m^3}{C_a \frac{1}{2} \rho_a V_a^2 l_a^3}$$

for $C_m = C_a$

$$\therefore \frac{I_m}{I_a} = \frac{\rho_m}{\rho_a} \lambda^5 \quad (A.9)$$

Table A.1 Summary of Similarity Laws

A. Optional Parameters

Scale

1. Model Scale	l_m/l_a
2. Speed	V_m/V_a
3. Density	ρ_m/ρ_a

B. Scaling Constraint

4. Mass	$\frac{m_m}{m_a} = \frac{\rho_m}{\rho_a} \lambda^3$
5. Inertia	$I_m/I_a = \frac{\rho_m}{\rho_a} \lambda^5$
6. Gravity	$g_m/g_a = \frac{1}{\lambda} \left(\frac{V_m}{V_a} \right)^2$

C. Analysis Relationship

7. Time	$\frac{t_m}{t_a} = \tau = \frac{V_a}{V_m} \lambda$
8. Linear Displacement	$\frac{x_m}{x_a} = \lambda$
9. Angular Displacement	$\theta_m/\theta_a = 1$
10. Linear Velocity	$\dot{x}_m/\dot{x}_a = \frac{\lambda}{\tau}$
11. Angular Velocity	$\dot{\theta}_m/\dot{\theta}_a = \frac{1}{\tau}$
12. Linear Acceleration	$\ddot{x}_m/\ddot{x}_a = \frac{\lambda}{\tau^2}$
13. Angular Acceleration	$\ddot{\theta}_m/\ddot{\theta}_a = 1/\tau^2$
14. Load Factor	$n_m/n_a = \frac{1}{\lambda} \left(\frac{V_m}{V_a} \right)^2$

APPENDIX B

DESIGN INFORMATION DATA

Table B.1 Features of the Vertical Rod

1.	Material	Silver Steel
	1a. Modulus of Elasticity	2.9×10^6
	1b. Density	0.28 lb/inch^3
2.	Length	40 inches
	2a. Maximum usable length	36 inches
3.	Diameter	$\frac{3}{8}$ inch
4.	Weight	0.94 lbs.
5.	Inertia	$1 \times 10^{-2} \text{ lb-inch}^2$
6.	Maximum Lateral Deflection centre of rod, at 90 ft/sec	0.008 inches

Table B.2 Summary of Aircraft and Model Dimensions

Wing:-

	Aircraft	Model
Area (gross)	179.63 ft ²	179.00 inch ²
Span	30.8 ft	30.8"
L.E.Sweep	26 ^o	26 ^o
Root Chord	8.7 ft	8.7"
Tip Chord	2.96 ft	2.96"
Mean Aerodynamic Chord	5.83 ft	5.83"
Aspect Ratio	5.284	5.28
Airfoil Section	-	-
Dihedral	2.0 ^o	2.0 ^o

Tailplane:-

Area (gross)	45.0 ft ²	45.0inch ²
Span	13.91 ft	13.90"
L.E.Sweep	35.3 ^o	35.3 ^o
Root Chord	4.86 ft	4.86"
Tip Chord	1.61"	1.60"
Taper Ratio	0.33	0.33
Mean Aerodynamic Chord	3.24 ft	3.24"
Aspect Ratio	4.30	4.30
Airfoil Section	64 series 8.5% thick	same
L _T	14.1 ft	14.1"
$\bar{V}_T$, Tail Volume Ratio	0.6058	0.605

Fin:-

Area (gross)	27 ft ²	27 inch ²
L.E.Sweep	45 ^o	45 ^o
Airfoil Section	64 series	64009
Thickness to Chord Ratio	9%	-
Aspect Ratio	2.5	2.5
Taper Ratio	0.35	0.35
L _F	11.39 ft	11.4"
$\bar{V}_F$, Fin Volume Ratio	0.055	0.055

Table B.2 Cont.

Fuselage:-

	Aircraft	Model
Overall Length	35.9 ft	35.9"
d _{effective}	4.5 ft	4.5"

Table B.3 Important Similarity Parameters of $\frac{1}{12}$ Scale Model and Hawk Aircraft

<u>Parameter</u>	<u>Aircraft</u>	<u>$\frac{1}{12}$ Scale Model</u>	<u>Support</u>
1. Mass (lbs)	9000	5.21	0.41
2. Flight Altitude (ft)	Sea Level	Sea Level	-
3. Speed (ft/sec)	V_a	$\frac{V_a}{3.47}$	-
4. Gravity (ft/sec ²)	32.17	32.17	-
5. Time Scale	1	0.287	-
6. Frequency	ω_a	$3.47\omega_a$	-
7. Moment of Inertia			
I_x (slug.ft ²)	3500	0.016	3.1×10^{-5}
I_y " "	14700	0.059	1.2×10^{-5}
I_z " "	17140	0.069	5×10^{-5}
I_{xz} " "	330	0.0024	-

Table B.4 Model Weight And Moments of Inertia

Flight condition:- Sea Level, Mach No. = 0.31

Aircraft Weight = 9000 lbs.

<u>Parameter</u>	<u>Scaled Value Required</u>	<u>Measured</u>
1. Weight (lbs)	5.21	5.25
2. Moment of Inertia		
$I_x(\text{slug-ft}^2)$	0.016	0.015
$I_y(\text{slug-ft}^2)$	0.059	0.057
$I_z(\text{slug-ft}^2)$	0.071	0.073

APPENDIX C

EQUATIONS OF MOTION

C.1 The Dimensional Longitudinal Equations

(i) Dimensional equations - general (free flight)

The general equations of longitudinal symmetric motion for small disturbance are^{47, 48}.

$$\begin{aligned} m\ddot{u} - \dot{X}_u \dot{u} - \dot{X}_w \dot{w} - \dot{X}_{\dot{w}} \dot{\dot{w}} + (mW_e - \dot{X}_q)q + mg_1\theta &= \ddot{X}(t) \\ -\dot{Z}_u \dot{u} + (m - \dot{Z}_{\dot{w}})\dot{\dot{w}} - \dot{Z}_w \dot{w} - (mV_e + \dot{Z}_q)q + mg_2\theta &= \ddot{Z}(t) \\ -\dot{M}_u \dot{u} - \dot{M}_{\dot{w}} \dot{\dot{w}} - \dot{M}_w \dot{w} + I_y \dot{q} - \dot{M}_q q &= \ddot{M}(t) \end{aligned} \quad (C.1)$$

where $q = \dot{\theta}$

These equations involve only symmetric disturbances.

(ii) Dimensional Equations - Semi-Free Flight

In the case of semi-free flight wind tunnel simulation in which longitudinal translation is suppressed and the tunnel speed remains constant, ($u=0$) Equation C.1 is reduced to

$$\begin{aligned} (m - \dot{Z}_{\dot{w}})\dot{\dot{w}} - \dot{Z}_w \dot{w} - (mV_e + \dot{Z}_q)q + mg_2\theta &= \ddot{Z}(t) \\ -\dot{M}_{\dot{w}} \dot{\dot{w}} - \dot{M}_w \dot{w} + I_y \dot{q} - \dot{M}_q q &= \ddot{M}(t) \end{aligned} \quad (C.2)$$

In the above equation, generally the stability derivatives $\dot{Z}_w$ and $\dot{Z}_q$ are small, and can be neglected, also in the case of steady flight $g_2=0$,

hence equation C.2 is further simplified to the form

$$\begin{aligned} m\ddot{w} - \ddot{Z}_w \dot{w} - mV_e \dot{q} &= \ddot{Z}(t) \\ -\ddot{M}_w \dot{w} - \ddot{M}_w w + I_y \dot{q} - \ddot{M}_q q &= \ddot{M}(t) \end{aligned} \quad (C.3)$$

where $\ddot{Z}(t)$ and $\ddot{M}(t)$ are input parameters.

(iii) Dimensional Equation-Pitch Freedom Only

When only pitching freedom is permitted in longitudinal motion, then the equation C.3 is reduced to,

$$-\ddot{M}_w \dot{w} - \ddot{M}_w w + I_y \dot{q} - \ddot{M}_q q = \ddot{M}(t)$$

also in this special case, $\frac{w}{V_e} = \alpha = \theta$.

Hence the above equation can be simplified to the form

$$I_y \ddot{\theta} + (-\ddot{M}_q - V_e \ddot{M}_w) \dot{\theta} + (-\ddot{M}_w) \theta = \ddot{M}(t) \quad (C.4)$$

C.2 The Dimensional Lateral Equations

(i) Dimensional Equation-General

The general equations of lateral asymmetric motion for small disturbance are ^{47, 48}

$$\begin{aligned} m\dot{v} - \ddot{Y}_v v - (mW_e + \ddot{Y}_p) \dot{p} + (mV_e - \ddot{Y}_r) \dot{r} - mg_1 \phi - mg_2 \psi &= \ddot{Y}(t) \\ -\ddot{L}_v v + I_x \dot{p} - \ddot{L}_p \dot{p} - I_{zx} \dot{r} - \ddot{L}_r \dot{r} &= \ddot{L}(t) \\ -\ddot{N}_v v - I_{zx} \dot{p} - \ddot{N}_p \dot{p} + I_z \dot{r} - \ddot{N}_r \dot{r} &= \ddot{N}(t) \end{aligned} \quad (C.5)$$

$$\begin{aligned} \text{where } p &= \dot{\phi} \\ r &= \dot{\psi} \end{aligned}$$

These equations involve only asymmetric disturbances.

(ii) Dimensional Equations - Semi-Free Flight

In the case of semi-free flight wind tunnel simulation, in which lateral translation is suppressed, equation (C.5) is reduced to

$$\begin{aligned} -\overset{\circ}{L}_v v + \overset{\circ}{I}_x \dot{p} - \overset{\circ}{L}_p p - \overset{\circ}{I}_{zx} \dot{r} - \overset{\circ}{L}_r r &= \overset{\circ}{L}(t) \\ -\overset{\circ}{N}_v v - \overset{\circ}{I}_{zx} \dot{p} - \overset{\circ}{N}_p p + \overset{\circ}{I}_z \dot{r} - \overset{\circ}{N}_r r &= \overset{\circ}{N}(t) \end{aligned} \quad (C.6)$$

where $p = \dot{\phi}$, $r = \dot{\psi}$ and $v = \psi V_e$

C.3 The Dimensionless Longitudinal Equations

(i) Dimensionless Equation - General

The dimensionless longitudinal equation of motion can be obtained from equation (C.1), through several steps given in references 47, 48. The resulting equation is

$$\begin{aligned} (\hat{D} + x_u) \hat{u} + (x_w \hat{D} + x_w) \hat{w} + x_q \hat{q} + g_1 \hat{\theta} &= -x_\eta' \hat{\eta} \\ Z_u \hat{u} + [(1 + Z_w) \hat{D} + Z_w] \hat{w} + (Z_q - 1) \hat{q} + g_2 \hat{\theta} &= -z_\eta' \hat{\eta} \end{aligned} \quad (C.7)$$

$$m_u \hat{u} + (m_w \hat{D} + m_w) \hat{w} + (\hat{D} + m_q) \hat{q} = -m_\eta' \hat{\eta}$$

where

$$\hat{q} = \hat{D}\theta$$

$$\hat{g}_1 = \hat{g} \cos \theta_e = C_L$$

$$\hat{g}_2 = \hat{g} \sin \theta_e = C_L \tan \theta_e$$

$$\hat{g} = \frac{mg}{\frac{1}{2} \rho_e V_e^2 S} = C_L \sec \theta_e$$

(ii) Dimensionless Equation - Pitch Freedom Only

When only pitching freedom is permitted in longitudinal motion, then equation (C.7) is reduced to the form

$$(m_w \hat{D} + m_w) \hat{w} + (\hat{D} + m_q) \hat{q} = -m_\eta' \eta$$

Now, in this case $\hat{w} = \theta$

Therefore the above equation can be written as

$$\hat{D}^2 \theta + (m_q + m_w) \hat{D} \theta + m_w \theta = -m_\eta' \eta \quad (C.8)$$

C.4 The Dimensionless Lateral Equations

The dimensionless lateral equations of motion can be obtained from the equation (C.5)^{47, 48}. The equation thus obtained is

$$(\hat{D} + y_v) \hat{v} + y_p \hat{p} + (1 + y_r) \hat{r} - g_1 \hat{\phi} - g_2 \psi = -y_\xi' \xi - y_\zeta' \zeta$$

$$l_v \hat{v} + (\hat{D} + l_p) \hat{p} + (e_x \hat{D} + l_r) \hat{r} = -l_\xi' \xi - l_\zeta' \zeta$$

$$n_v \hat{v} + (e_z \hat{D} + n_p) \hat{p} + (\hat{D} + n_r) \hat{r} = -n_\xi' \xi - n_\zeta' \zeta$$

where

$$\hat{p} = \hat{D} \phi$$

$$\hat{r} = \hat{D} \psi$$

$$e_x = -i_{zx} / i_x = -I_{zx} / I_x$$

$$e_z = -\frac{i_{zx}}{i_z} = -\frac{I_{zx}}{I_z}$$

$$\hat{g}_1 = \hat{g} \cos \theta_e = C_L$$

$$\hat{g}_2 = \hat{g} \sin \theta_e = C_L \tan \theta_e$$

Table C.1 Longitudinal Stability Derivatives

Dimensional Derivative	Divisor for Obtaining Aeronormalized Derivative(d)	Aeronormalized Derivative	Multiplying Factor(m)	Dynamic Normalized Concise Derivative
$\dot{X}_u$	$\frac{1}{2}\rho_e V_e S$	X_u	-1	x_u
$\dot{X}_w$	$\frac{1}{2}\rho_e V_e S$	X_w	-1	x_w
$\dot{X}_{\dot{w}}$	$\frac{1}{2}\rho_e S \bar{c}$	$X_{\dot{w}}$	$-\frac{1}{\mu_1}$	$x_{\dot{w}}$
$\dot{X}_q$	$\frac{1}{2}\rho_e V_e S \bar{c}$	X_q	$-\frac{1}{\mu_1}$	x_q
$\dot{Z}_u$	$\frac{1}{2}\rho_e V_e S$	Z_u	-1	z_u
$\dot{Z}_w$	$\frac{1}{2}\rho_e V_e S$	Z_w	-1	z_w
$\dot{Z}_{\dot{w}}$	$\frac{1}{2}\rho_e S \bar{c}$	$Z_{\dot{w}}$	$-\frac{1}{\mu_1}$	$z_{\dot{w}}$
$\dot{Z}_q$	$\frac{1}{2}\rho_e V_e S \bar{c}$	Z_q	$-\frac{1}{\mu_1}$	z_q
$\dot{M}_u$	$\frac{1}{2}\rho_e V_e S \bar{c}$	M_u	$-\mu_1/i_y$	m_u
$\dot{M}_w$	$\frac{1}{2}\rho_e V_e S \bar{c}$	M_w	$-\mu_1/i_y$	m_w
$\dot{M}_{\dot{w}}$	$\frac{1}{2}\rho_e S \bar{c}^2$	$M_{\dot{w}}$	$-1/i_y$	$m_{\dot{w}}$
$\dot{M}_q$	$\frac{1}{2}\rho_e V_e S \bar{c}^2$	M_q	$-1/i_y$	m_q
$\dot{X}_\eta$	$\frac{1}{2}\rho_e V_e^2 S$	X_η	-1	x_η
$\dot{Z}_\eta$	$\frac{1}{2}\rho_e V_e^2 S$	Z_η	-1	z_η
$\dot{M}_\eta$	$\frac{1}{2}\rho_e V_e^2 S \bar{c}$	M_η	$-\mu_1/i_y$	m_η

Table C.2 Lateral Stability Derivatives

Dimensional Derivative	Divisor for Obtaining Aeronormalized Derivative(d)	Aeronormalized Derivative	Multiplying Factor(m)	Dynamic Normalized Concise Derivative
$\dot{Y}_v$	$\frac{1}{2}\rho_e V_e S$	Y_v	-1	y_v
$\dot{Y}_p$	$\frac{1}{2}\rho_e V_e S b$	Y_p	-1	y_p
$\dot{Y}_r$	$\frac{1}{2}\rho_e V_e S b$	Y_r	$-1/\mu_2$	y_r
$\dot{L}_v$	$\frac{1}{2}\rho_e V_e S b$	L_v	$-\mu_2/i_x$	l_v
$\dot{L}_p$	$\frac{1}{2}\rho_e V_e S b^2$	L_p	$-1/i_x$	l_p
$\dot{L}_r$	$\frac{1}{2}\rho_e V_e S b^2$	L_r	$-1/i_x$	l_r
$\dot{N}_v$	$\frac{1}{2}\rho_e V_e S b$	N_v	$-\mu_2/i_z$	n_v
$\dot{N}_p$	$\frac{1}{2}\rho_e V_e S b^2$	N_p	$-1/i_z$	n_p
$\dot{N}_r$	$\frac{1}{2}\rho_e V_e S b^2$	N_r	$-1/i_z$	n_r
$\dot{Y}_\xi$	$\frac{1}{2}\rho_e V_e^2 S$	Y_ξ	-1	y_ξ
$\dot{Y}_\zeta$	$\frac{1}{2}\rho_e V_e^2 S$	Y_ζ	-1	y_ζ
$\dot{L}_\xi$	$\frac{1}{2}\rho_e V_e^2 S b$	L_ξ	$-\mu_2/i_x$	l_ξ
$\dot{L}_\zeta$	$\frac{1}{2}\rho_e V_e^2 S b$	L_ζ	$-\mu_2/i_x$	l_ζ
$\dot{N}_\xi$	$\frac{1}{2}\rho_e V_e^2 S b$	N_ξ	$-\mu_2/i_z$	N_ξ
$\dot{N}_\zeta$	$\frac{1}{2}\rho_e V_e^2 S b$	N_ζ	$-\mu_2/i_z$	N_ζ

APPENDIX D

ESTIMATION OF DYNAMIC STABILITY CHARACTERISTICS

D.1 Introduction

The dynamic stability characteristics of the full scale Hawk aircraft and $\frac{1}{12}$ scale model are presented in this Appendix. The estimation methods are taken from references 49, 50 and 51, unless otherwise stated. The Appendix contains a sample calculation of the characteristics for full scale Hawk aircraft at sea level, $M = 0.31$ flight condition. The remaining estimated values for full scale and the model are presented in tabulated form.

Wherever a particular ESDU data sheet is referred to for the estimation method, the symbols used are those of the ESDU data sheets.

The detailed particulars of the Hawk aircraft and $\frac{1}{12}$ scale model used for estimation are given in Appendix B.

D.2 Aerodynamic Data

(i) Lift Curve Slope (ESDU Data Sheet 70011)

(a) Wing

$$A = 5.284, \lambda = 0.34, \Lambda_{\frac{1}{2}} = 16.77^\circ \text{ and}$$

$$A \tan \Lambda_{\frac{1}{2}} = 1.59$$

$$\text{At } M = 0.31, \beta = 0.95, \beta A = 5.02.$$

From figure 1,

$$\frac{1}{A} \frac{dC_L}{d\alpha} = 0.795$$

$$\therefore C_{L_{\alpha W}} = 4.2$$

(b) Fin

Following the same procedure as in (a)

$$A = 2.5, \lambda = 0.35, \Lambda_{\frac{1}{2}} = 31.58^\circ \text{ and}$$

$$\text{Atan} \Lambda_{\frac{1}{2}} = 1.54$$

$$C_{L_{\alpha F}} = 2.85$$

(c) Horizontal Tail

$$A = 4.30, \lambda = 0.33, \Lambda_{\frac{1}{2}} = 25.4^\circ \text{ and}$$

$$\text{Atan} \Lambda_{\frac{1}{2}} = 2.04$$

$$C_{L_{\alpha T}} = 3.74$$

(ii) Downwash Estimate (ESDU 80020)

The downwash angle is estimated from the following equation

$$\frac{d\bar{\epsilon}}{d\alpha} = B \left(\frac{d\bar{\epsilon}}{d\alpha} \right)_0^* - \left| \zeta_T \right| \left| \frac{d}{d\zeta_T} \left(\frac{d\bar{\epsilon}}{d\alpha} \right) \right| \quad (1)$$

where

$\bar{\epsilon}$ = Average downwash angle across the tailplane
and

$$\left(\frac{d\bar{\epsilon}}{d\alpha} \right)_0^* = \frac{2}{\pi} \frac{a_1}{A} (H_P - \Delta H_P) F \quad (2)$$

$$\text{where } H_P = H_{P_e} - (1 - H_{P_e})$$

At $M = 0.3$, using figures 1, 2, 3, 4, 5 and 8

$$\left(\frac{d\bar{\epsilon}}{d\alpha} \right)_0^* = \frac{2}{\pi} \frac{4.2}{5.284} (1.07 - 0.1)(0.987)$$

$$\therefore \left(\frac{d\bar{\epsilon}}{d\alpha} \right)_0^* = 0.48$$

$$\frac{d\bar{\epsilon}}{d\alpha} \frac{a_1 F}{A} = \frac{0.48}{\frac{4.2 \times 0.987}{5.287}} = 0.62$$

$$\left| \frac{d}{d\rho_T} \left(\frac{d\bar{\epsilon}}{d\alpha} \right) \right| \frac{a_1}{A} F = 0.54 \text{ (figure 9) and}$$

$$\therefore \frac{d}{d\rho_T} \left(\frac{d\bar{\epsilon}}{d\alpha} \right) = 0.42$$

substituting in (1)

$$\therefore \frac{d\bar{\epsilon}}{d\alpha} = 0.52$$

(iii) Aircraft C_{L_α} (lift curve slope) and Incidence
Angle

The total lift of the aircraft is the sum of the tail lift and that of the wing-body contribution.

$$L = L_{WB} + L_T \quad (3)$$

Hence the total lift coefficient is

$$C_L = C_{L_{WB}} + \frac{C_{LT} \frac{1}{2} \rho V^2 S_T}{\frac{1}{2} \rho V^2 S_W}$$

or

$$C_L = C_{L_{WB}} + \frac{S_T}{S_W} C_{LT}, \text{ which can be reduced to}$$

$$C_{L_{\alpha}} = C_{L_{\alpha_{WB}}} \left[1 + \frac{C_{L_{\alpha T}}}{C_{L_{WB}}} \frac{S_T}{S_W} (1 - \frac{d\epsilon}{d\alpha}) \right] \quad (4)$$

where $C_{L_{\alpha_{WB}}} = 1.05 C_{L_{\alpha W}}$ (reference D1)

At $M = 0.31$, for full scale aircraft

$$C_{L_{\alpha}} = 1.05(4.2) \left(1 + \frac{3.78}{4.2 \times 1.05} \frac{45}{179} (1 - 0.52) \right)$$

$$C_{L_{\alpha}} = 4.89$$

Now

Incidence angle is given by

$$\alpha = \frac{C_L}{C_{L_{\alpha}}} = \frac{W / \frac{1}{2} \rho V^2 S}{C_{L_{\alpha}}} \quad (5)$$

$$\therefore \alpha = \frac{0.35}{4.89} = 4.13^\circ$$

(iv) Aircraft Drag

The total drag coefficient of the aircraft is given by

$$C_D = C_{D_0} + \frac{1}{\pi A e} C_L^2 \quad (6)$$

or

$$C_D = 0.02 + \frac{1}{\pi \times 5.284 \times 0.8} C_L^2$$

where $C_{D_0} = 0.02$ (reference 52)

$$\therefore C_D = 0.020 + 0.075 C_L^2$$

$$\text{At } M = 0.31, C_L = 0.35$$

$$\therefore C_D = 0.029$$

(v) Summary of Aerodynamic Data

The summary of estimated aerodynamic data for full scale aircraft and model is given in Table D.1.

D.3 Stick Fixed Static Longitudinal Stability

(i) Stick Fixed Neutral Point

From reference 47,

$$h_n = h_o + \bar{V} \frac{a_1}{a} \left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad (7)$$

at $M = 0.31$, substituting the above values

$$h_n - h_o = 0.224$$

from reference 52, $h_o = 0.14$.

Therefore $h_n = 0.364$.

(ii) Static Margin

The c.g. positions under investigation are $0.29\bar{c}$ and $0.325\bar{c}$ (model c.g. positions)

$$\therefore \text{for } h = .29\bar{c}$$

$$\text{Static margin } K_n = h_n - h = 0.074$$

$$\text{for } h = 0.325\bar{c}$$

$$K_n = 0.04$$

Longitudinal Stability Derivatives(a) Dimensionless Derivatives

The flight condition under consideration is,
 $M = 0.31$ at sea level, $C_L = 0.35$.

X-Force Component Derivatives

$$(i) \quad X_u = 2 \left[-C_D + \frac{1}{S\rho V} \frac{dT}{dV} - \frac{V}{2} \frac{\partial C_D}{\partial V} - \frac{M}{2} \frac{\partial C_D}{\partial M} \right] \quad (8)$$

for low speed and neglecting thrust effects,

$$X_u = -2C_D = -0.058$$

$$(ii) \quad X_w = C_L - \frac{dC_D}{d\alpha} \quad (9)$$

$$X_w = C_L - \frac{dC_D}{dC_L} \frac{dC_L}{d\alpha}$$

$$X_w = 0.095$$

Z-Force Component Derivatives

$$(iii) \quad Z_u = 2 \left[-C_L - \frac{M}{2} \frac{\partial C_L}{\partial M} - W \frac{\partial C_{LW}}{\partial L_W} - \frac{\partial C_{LW}}{\partial u} \right] \quad (10)$$

In our case of low Mach numbers

$$Z_u = -2C_L = -0.70$$

$$(iv) \quad Z_w = -\left(C_D + \frac{dC_L}{d\alpha}\right) \quad (11)$$

$$Z_w = -4.89$$

$$(v) \quad Z_q = 1.2 Z_q(\text{tail}) = 1.2(-a_1 \bar{V}_T) \quad (12)$$

$$Z_q = -2.49$$

$$(vi) \quad Z_w = Z_q(\text{tail}) \frac{d\epsilon}{d\alpha} \quad (13)$$

$$Z_w = 1.18$$

Pitching Moment Derivatives

$$(vii) \quad M_u = 0 \text{ at low speeds}$$

$$(viii) \quad M_w = -\frac{dC_L}{d\alpha} K_n \quad (14)$$

therefore

$$M_w = 0.365 \text{ for c.g.} = 0.29\bar{c}$$

$$M_w = 0.194 \text{ for c.g.} = 0.325\bar{c}$$

$$(ix) \quad M_q = 1.2M_q(\text{tail}) = 1.2 \frac{L_T}{C} Z_q(\text{tail}) \quad (15)$$

$$M_q = 6.30$$

$$(x) \quad M_{\dot{w}} = M_q(\text{tail}) \frac{d\epsilon}{d\alpha} \quad (16)$$

$$M_w = 2.73$$

The estimated dimensionless longitudinal stability derivatives of full scale aircraft and the model for various flight conditions are given in table D.2.

(b) Concise Derivatives

For horizontal flight,

$$\tau = \frac{m}{\frac{1}{2}\rho V S} = \frac{1131}{V} \quad i_y = \frac{I_y}{m\bar{c}^2} = 1.54$$

$$\mu_1 = \frac{m}{\frac{1}{2}\rho S \bar{c}} = 224$$

$$\hat{g}_1 = C_L$$

$$(i) \quad x_u = -X_u = 0.058 \quad (vi) \quad z'_w = \frac{-Z_w}{\mu_1} = 0.005$$

$$(ii) \quad x_w = -X_w = -0.095 \quad (vii) \quad m_w = -\mu_1 \frac{M_w}{i_y} = 53.1$$

at $0.29\bar{c}$

$$(iii) \quad z_u = -Z_u = 0.70 \quad (viii) \quad m_q = -\frac{M_q}{i_y} = 4.09$$

$$(iv) \quad z_w = -Z_w = 4.89 \quad (ix) \quad m_{\dot{w}} = \frac{-M_w}{i_y} = 1.77$$

$$(v) \quad z_q = \frac{-Z_q}{\mu_1} = 0.011 \quad (x) \quad m_w = 28.2$$

at $0.325\bar{c}$

The estimated concise stability derivatives of full scale aircraft and the model for various flight conditions are given in table D.3.

D.5 Lateral Stability Derivatives

(a) Dimensionless Derivatives

The flight condition under consideration is
 $M = 0.31$, sea level, $C_L = 0.35$.

(i) Rolling Moment Derivative Due To Sideslip L_v

1. Contribution due to Wing Dihedral (ESDU 06 01 03)

$$\Gamma = 2^\circ, \Lambda_{\frac{1}{4}} = 21.5^\circ, \lambda = 0.34, a_{10M} = 5.5 \text{ rad}^{-1}$$

from the data sheet

$$\Lambda_{\epsilon} = \tan^{-1} \frac{\tan \Lambda_1}{(1-M^2)^{\frac{1}{2}}} = 22.51^\circ$$

also

$$K = \frac{a_{10} M}{2\pi} (1-M^2)^{\frac{1}{2}} = 0.83$$

$$\frac{A(1-M^2)^{\frac{1}{2}}}{K} = 6.05$$

from figure 1(B) and 1(C)

$$\frac{(L_V)_{\Gamma} (1-M^2)^{\frac{1}{2}}}{K\Gamma} = -0.0117$$

$$\therefore (L_V)_{\Gamma_{\text{wing}}} = -0.02$$

$$\text{Similarly } (L_V)_{\Gamma_{\text{tail}}} = +0.01$$

Contribution due to Wing Sweepback (ESDU 80033)

$$L_V = -0.11 C_L$$

$$\therefore L_V = -0.039$$

Contribution due to Fin (ESDU 82010)

$$h_f = 5.813, \quad z_{\text{crf}} = 1.5, \quad h_{BF} = 4.25, \quad m_F = 11.395$$

$$C_{rF} = 6.88, \quad C_{tf} = 2.408, \quad \frac{b_T}{2} = 6.956, \quad S_F = 27, \quad S_W = 179.6$$

$$A_F = 2.5, \quad \lambda_F = 0.35, \quad \frac{h_{BF}}{(h_{BF} + h_F)} = 0.422$$

$$Y_{VF} = -J_B J_T J_W (C_{L_{\alpha_F}}) \frac{S_F}{S_W} \quad (16)$$

From figures 1, 2a and 3a

$$J_B=1.33, J_T=1.08 \text{ and } J_W=0.93$$

$$\therefore Y_{VF} = -0.201 C_{L_{\alpha_F}} = -0.57$$

From figure 4, for body mounted tailplane,

$$\bar{z}_F/h_F = 0.4$$

$$\therefore z_{cr_F} + 0.85\bar{z}_F = 3.476$$

$$m_F + 0.7\bar{z}_F \tan \Lambda_{\frac{1}{4}F} = 12.713$$

Now

$$(L_V)_F = +(Y_V)_P \left[(z_{cr_F} + 0.85\bar{z}_F) \cos \alpha - (m_F + 0.7\bar{z}_F \tan \Lambda_{\frac{1}{4}F}) \sin \alpha \right] / b \quad (17)$$

$$= -0.57 \quad 3.476 \cos \alpha - 12.713 \sin \alpha \quad / 30.8$$

Now for $\alpha = 4.13^\circ$

$$(L_V)_F = -0.048$$

$$\text{Therefore } L_V = (L_V)_{r_{wing}} + (L_V)_{r_{Tail}} + L_{V_{w.s}} + (L_V)_F$$

$$\therefore L_V = -0.022 - 0.039 + 0.01 - 0.048$$

Hence

$$L_V = -0.099 \text{ for complete aircraft.}$$

(ii) Rolling Moment Derivative due to Rate of Roll L_P

Contribution due to Wing (ESDU Aircraft 060101)

$$A=5.284, \Lambda_{\frac{1}{4}}=21.5, \lambda=0.34, M=0.31, (a_{10})_M=5.5 \text{ rad}^{-1}$$

$$K = 0.83, \frac{\beta A}{K} = 6.05, \Lambda_E = 22.5^\circ$$

from figure 1(b) and 1(c)

$$-\beta \frac{L_P}{K} = 0.195$$

$$\text{Therefore } L_P = 0.177$$

Similarly Contribution due to Tailplane can be estimated

$$L_{P_{\text{Tail}}} = 0.01$$

Contribution from the fin and fuselage are negligible. Hence for complete aircraft,

$$L_P = 0.187.$$

(iii) Rolling Moment Derivative due to Rate of Yaw L_r

Contribution due to Wing (ESDU 72021)

$$A = 5.284, \lambda = 0.34, \Lambda_{\frac{1}{4}} = 21.5^\circ$$

$$\frac{(L_{r_o})}{C_L} = \frac{1}{g(\Lambda_{\frac{1}{4}})} \frac{(L_{r_o})}{C_L} . g(\Lambda_{\frac{1}{4}}) \quad (18)$$

Using figure 1a and 1b

$$L_{r_o} = 0.133 C_L = 0.047$$

Dihedral Contribution

From figure 3

$$(L_{r_o})_{\Gamma} = 0.0008\Gamma = 0.0016$$

$$\therefore L_{r_o} = 0.049$$

Contribution due to Fin (ESDU 82017)

$$(L_r)_F = -(L_v)_F \left[m_F + 0.7 \bar{z}_F \tan \Lambda_{1/4} \cos \alpha + (Z_{crF} + 0.85 \bar{z}_F) \sin \alpha \right] / b$$

(19)

$$\therefore (L_r)_F = 0.020$$

Hence for complete aircraft,
 $L_r = 0.049 + 0.02 = 0.069$.

(iv) Yawing Moment Derivative due to Sideslip N_v

Wing-body Contribution (ESDU 79006)

$$l_b = 35.9, S_b = 161.5, h_1/h_2 = 1.2, l_b^2/S_b = 7.98$$

from figure 1,

$$-N_{v_{mid}} = 0.132 \frac{161.5 \times 35.9}{179.6 \times 30.8} = 0.138.$$

Now

$$-Y_v = \left[0.0714 + 0.674 \frac{h^2}{S_b} + \frac{h_b F_{FW}}{S_b} \left(4.95 \frac{|z|}{h} - 0.12 \right) \right] \frac{S_b}{S} + 0.006 |\Gamma|$$

(20)

from figure 2, for $\frac{z}{h} = 0.35, \frac{b}{d} = 6.84$
 $F = 0.079$

from figure 3, for $A = 5.284, \lambda = 0.34 F_w = 1.09$

$$\text{Hence } -Y_v = 0.262$$

$$\therefore N_v = N_{v_{mid}} + \frac{(x_{c.g.} - x_{mid})}{b} Y_v$$

$$= -0.138 + \frac{2.03}{30.8} (-0.262)$$

$$\text{Hence, } N_v = -0.155$$

Contribution due to Fin (ESDU 82010)

$$(N_V)_F = -(Y_V)_F \left[(m_F + 0.7 \bar{z}_F \tan \Lambda_{1/4F}) \cos \alpha + (\bar{z}_{crF} + 0.85 \bar{z}_F) \sin \alpha \right] / b$$
$$\therefore (N_V)_F = 0.239 \quad (21)$$

Hence for the complete aircraft

$$N_V = 0.239 - 0.155 = 0.084$$

(v) Yawing Moment due to Rate of Roll, N_P

Contribution due to Wing

From reference D2, for $\Lambda = 26^\circ$

$$\frac{\Delta C_{n_P}}{C_L} = -0.09, \text{ or } C_{n_P} = 0.032$$

$$\therefore N_P = \frac{-C_{n_P}}{2} = -0.016$$

Contribution due to Tail

$$C_{n_{P_{\text{tail}}}} = 2 \left(\frac{z}{b} \right) C_{n_{\beta_{\text{tail}}}}$$

$$\therefore C_{n_{P_{\text{tail}}}} = -0.031$$

$$N_{P_{\text{tail}}} = -0.016$$

$$\text{Hence, } N_P = -0.032$$

(vi) Yawing Moment due to Rate of Yaw N_r

Contribution due to Wing (ESDU 71017)

$$A = 5.284, \lambda = 0.34, \Lambda_{\frac{1}{4}} = 21.5^\circ,$$

from figure 1a and 1b

$$N_{r_o} = -0.0014$$

from figure 2b and 2c

$$N_{r_u} = -0.0012$$

$$\text{Hence, } N_{r_{\text{wing}}} = -0.003$$

Contribution due to Fin (ESDU 82017)

$$(N_r)_F = -(N_v)_F \left\{ (m_F + 0.7 \bar{z}_F \tan \Lambda_{\frac{1}{4}F}) \cos \alpha + (z_{crF} + 0.85 \bar{z}_F) \sin \alpha \right\} / b \quad (22)$$

$$\therefore N_{r_F} = -0.100$$

Hence for the complete aircraft

$$N_r = -0.103$$

(vii) Side Force Derivative due to Sideslip Y_v

Contribution due to Fuselage (ESDU 79006)

$$Y_v = -0.262$$

Contribution due to Fin (ESDU 82010)

$$Y_{v_{\text{Fin}}} = -0.570, \text{ (see } L_v \text{)}$$

Therefore for the complete aircraft,

$$Y_v = -0.262 - 0.570 = -0.832$$

The estimated dimensionless lateral stability derivatives of the full scale aircraft and

$\frac{1}{12}$ scale model at various flight conditions are presented in table D.4.

(b) Concise Lateral Dynamic Stability Derivatives

$$\mu_2 = \frac{m}{\frac{1}{2}\rho_e S b} = 42.4 \quad i_z = \frac{I_z}{mb^2} = 0.066$$

$$i_x = \frac{I_x}{mb^2} = 0.015 \quad i_{xz} = \frac{I_{zx}}{mb^2} = 0.0023$$

$$(i) \quad l_v = -\mu_2 L_v / i_x = 283 \quad (iv) \quad n_v = -\mu_2 N_v / i_z = -54.0$$

$$(ii) \quad l_p = -\frac{L_p}{i_x} = 12.5 \quad (v) \quad n_p = -\frac{N_p}{i_z} = 0.49$$

$$(iii) \quad l_r = -\frac{L_r}{i_x} = -4.6 \quad (vi) \quad n_r = -\frac{N_r}{i_z} = 1.56$$

$$(vii) \quad y_v = -Y_v = 0.832$$

The estimated concise lateral stability derivative of the full scale aircraft and $\frac{1}{12}$ scale model are given in table D.4.

Table D.1 Summary of Estimated Aerodynamic Data

	Full Scale Aircraft					Scaled Model $\frac{1}{12}$				
	Mach. No.					Velocity ft/sec				
	0.34	0.31	0.28	0.25	0.22	110	100	90	80	70
$C_{L_{\alpha_W}}$	4.23	4.20	4.16	4.12	4.09	4.02	4.02	4.02	4.02	4.02
$C_{L_{\alpha_{WB}}}$	4.44	4.41	4.37	4.33	4.29	4.22	4.22	4.22	4.22	4.22
$C_{L_{\alpha_T}}$	3.78	3.74	3.72	3.70	3.68	3.66	3.66	3.66	3.66	3.66
$C_{L_{\alpha}}$	4.89	4.86	4.82	4.78	4.74	4.69	4.69	4.69	4.69	4.69
$\frac{de}{d\alpha}$	0.53	0.52	0.52	0.51	0.51	0.49	0.49	0.49	0.49	0.49
α	3.4	4.13	5.11	6.59	8.70	3.54	4.28	5.25	6.72	8.80

Table D.2 Estimated Dimensionless Longitudinal Stability Derivatives

	Full Scale Aircraft Mach. No.					Scaled Model $\frac{1}{12}$ Velocity ft/sec				
	0.34	0.31	0.28	0.25	0.22	110	100	90	80	70
X_u	-0.053	-0.058	-0.068	-0.085	-0.118	-0.053	-0.058	-0.068	-0.085	-0.118
X_w	0.077	0.095	0.119	0.156	0.208	0.077	0.095	0.119	0.156	0.208
Z_u	-0.58	-0.70	-0.86	-1.10	-1.44	-0.58	-0.70	-0.86	-1.10	-1.44
Z_w	-4.92	-4.89	-4.85	-4.82	-4.80	-4.72	-4.72	-4.72	-4.72	-4.72
Z_q	-2.52	-2.49	-2.48	-2.47	-2.45	-2.44	-2.44	-2.44	-2.44	-2.44
$Z_{\dot{w}}$	-1.21	-1.18	-1.17	-1.15	-1.14	-1.09	-1.09	-1.09	-1.09	-1.09
M_w at 0.29c	0.367	0.365	0.362	0.359	0.356	0.352	0.352	0.352	0.352	0.352
M_q	-6.38	-6.30	-6.28	-6.25	-6.20	-6.17	-6.17	-6.17	-6.17	-6.17
$M_{\dot{w}}$	-2.82	-2.73	-2.72	-2.66	-2.64	-2.52	-2.52	-2.52	-2.52	-2.52
M_w at 0.325c	0.196	0.194	0.193	0.191	0.190	0.188	0.188	0.188	0.188	0.188

Table D.3 Estimated Concise Longitudinal Derivatives

	Full Scale Aircraft					Scaled Model $\frac{1}{12}$				
	Mach. No.					Velocity ft/sec				
	0.34	0.31	0.28	0.25	0.22	110	100	90	80	70
x_u	0.053	0.058	0.068	0.085	0.118	0.053	0.058	0.068	0.085	0.118
x_w	-0.077	-0.095	-0.119	-0.156	-0.208	-0.077	-0.095	-0.119	-0.156	-0.208
z_u	0.58	0.70	0.86	1.10	1.44	0.58	0.70	0.86	1.10	1.44
z_w	4.90	4.89	4.85	4.83	4.80	4.72	4.72	4.72	4.72	4.72
z_q	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
$z_{\dot{w}}$	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
m_w at 0.29c	53.4	53.1	52.7	52.2	51.8	51.2	51.2	51.2	51.2	51.2
m_q	4.14	4.09	4.08	4.06	4.03	4.01	4.01	4.01	4.01	4.01
$m_{\dot{w}}$	1.83	1.77	1.77	1.73	1.71	1.64	1.64	1.64	1.64	1.64
m_w at 0.325c	28.6	28.2	28.1	27.8	27.6	27.4	27.4	27.4	27.4	27.4

Table D.4 Estimated Dimensionless Lateral Stability Derivatives

	Full Scale Aircraft					Scaled Model $\frac{1}{12}$				
	Mach. No.					Velocity ft/sec				
	0.34	0.31	0.28	0.25	0.21	110	100	90	80	70
L_v	-0.105	-0.107	-0.112	-0.119	-0.127	-0.096	-0.097	-0.099	-0.103	-0.107
L_p	-0.188	-0.187	-0.186	-0.185	-0.183	-0.181	-0.181	-0.181	-0.181	-0.181
L_r	0.062	0.069	0.078	0.091	0.110	0.060	0.067	0.075	0.087	0.107
N_v	0.088	0.084	0.085	0.082	0.078	0.080	0.082	0.080	0.080	0.078
N_p	-0.029	-0.032	-0.035	-0.041	-0.047	-0.027	-0.029	-0.032	-0.037	-0.044
N_r	-0.104	-0.103	-0.104	-0.105	-0.108	-0.101	-0.101	-0.102	-0.102	-0.105
Y_v	-0.843	-0.832	-0.832	-0.821	-0.820	-0.810	-0.810	-0.810	-0.810	-0.810

Table D.5 Estimated Concise Lateral Stability Derivatives

	Full Scale Aircraft					Scaled Model $\frac{1}{12}$				
	Mach. No.					Velocity ft/sec				
	0.34	0.31	0.28	0.25	0.22	110	100	90	80	70
l_v	280	283	291	305	319	271	274	280	291	302
l_p	12.5	12.5	12.4	12.3	12.2	12.10	12.10	12.10	12.10	12.10
l_r	-4.10	-4.60	-5.20	-6.10	-7.30	-4.0	-4.50	-5.00	-5.80	-7.10
n_v	-56.5	-54.0	-54.6	-52.7	-50.1	-51.4	-52.7	-52.0	-52.0	-50.10
n_p	0.44	0.49	0.53	0.63	0.71	0.41	0.44	0.49	0.57	0.67
n_r	1.58	1.56	1.58	1.59	1.64	1.53	1.53	1.53	1.53	1.53
y_v	0.84	0.83	0.83	0.82	0.82	0.81	0.81	0.81	0.81	0.81

Table D.6 Coefficients of the Longitudinal Characteristic Equation

Characteristic equation, $A\lambda^4+B\lambda^3+C\lambda^2+D\lambda +E = 0$ where
 $A=1$, $B=x_u^z+z_w^m+m_q^m$, $C=(x_u^z-z_w^x-x_w^zy)^+m_q(x_u^z+z_w^x)^+m_w^x^u+m_w^m$

$D=m_q(x_u^z-z_w^x-z_w^zu)^+m_w^z^uC_L+m_w^xu$, $E=m_w^z^uC_L$

	Full Scale Aircraft					Scaled Free Flight Model				
	Mach. No.					Velocity ft/sec				
	0.34	0.31	0.28	0.25	0.22	110	100	90	80	70
c.g.=0.29c̄	A	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	B	10.9	10.8	10.8	10.7	10.4	10.4	10.4	10.5	10.5
	C	74.4	73.8	73.3	72.8	70.7	70.8	70.9	71.2	71.7
	D	4.40	4.95	6.00	7.84	4.17	4.33	5.79	7.64	11.20
	E	8.98	13.01	19.50	31.60	8.61	12.54	18.30	30.98	53.08
c.g.=0.325c̄	A	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	B	10.9	10.8	10.8	10.7	10.4	10.4	10.4	10.5	10.5
	C	49.6	48.9	48.9	48.4	46.9	47.0	47.1	47.3	47.8
	D	3.09	3.50	4.33	5.77	2.91	3.13	4.16	5.61	8.36
	E	4.80	6.90	10.40	16.80	4.60	6.70	10.10	16.55	28.40

Table D.7 Coefficients of the Longitudinal Characteristic Equation

Semi-Free Model

Characteristic equation, $A\lambda^2 + B\lambda + C = 0$
where
 $A=1.0$, $B=z_w + m_q + m_w^*$, $C=m_w + m_q z_w$

Model with Pitch Freedom Only

Characteristic equation, $A\lambda^2 + B\lambda + C = 0$
where
 $A=1$, $B=m_q + m_w$, $C=m_w$

Semi-free Model Simulation						Model with Pitch Freedom Only					
Velocity ft/sec											
110						110					
100						100					
90						90					
80						80					
70						70					
c.g.=0.29c	A	1.0	1.0	1.0	1.0	1.0	-1.0	1.0	1.0	1.0	1.0
	B	10.37	10.37	10.37	10.37	10.37	5.65	5.65	5.65	5.65	5.65
	C	70.13	70.13	70.13	70.13	70.13	51.2	51.2	51.2	51.2	51.2
c.g.=0.325c	A	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	B	10.37	10.37	10.37	10.37	10.37	5.65	5.65	5.65	5.65	5.65
	C	46.28	46.28	46.28	46.28	46.28	27.35	27.35	27.35	27.35	27.35

Table D.8 Coefficients of the Lateral Characteristic Equation

Characteristic equation, $A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$

Full Scale & Free Model

$$A = 1 - e_x e_z$$

$$B = l_p + n_r - e_x n_p - e_z l_r + (1 - e_x e_z) Y_v$$

$$C = l_p n_r - l_r n_p - n_v + e_z l_v + (l_p + n_r - e_x n_p - e_z l_r) Y_v$$

$$D = l_v n_p - l_p n_v + C_L l_v - e_x C_L n_v + (l_p n_r - l_r n_p) Y_v$$

$$E = (n_r l_v - l_r n_v) C_L$$

Semi-Free Model

$$A = 1 - e_x e_z$$

$$B = l_p + n_r - e_x n_p - e_z l_r$$

$$C = l_p n_r - l_r n_p - n_v + e_z l_v$$

$$D = l_v n_p - l_p n_v$$

$$E = 0$$

	Full Scale Aircraft					Free-Flight Model							Semi-Free Model					
	Mach. No.					Velocity ft/sec							Velocity ft/sec					
	0.34	0.31	0.28	0.25	0.22	110	100		90	80	70	110	100	90	80	70		
A	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
B	15.0	15.0	14.9	14.8	14.8	14.5	14.5	14.5	14.5	14.6	14.6	13.7	13.7	13.7	13.7	13.8		
C	99.5	97.1	98.5	9.8.0	97.6	91.9	93.5	93.6	94.9	94.8	80.8	80.8	82.4	82.5	83.7	83.6		
D	931	934	978	1028	1093	830	870	907	977	1050	733	733	755	766	795	809		
E	61.1	67.6	75.6	89.9	113.0	60.6	64.3	72.4	79.0	77.0	0.00	0.00	0.00	0.00	0.00	0.00		

Table D.9 Estimated Longitudinal Short Period Characteristics

		c.g. = 0.29c̄					c.g. = 0.325c̄				
		Velocity ft/sec (Mach.No.)					Velocity ft/sec (Mach.No.)				
		110 (0.34)	100 (0.31)	90 (0.28)	80 (0.25)	70 (0.22)	110 (0.34)	100 (0.31)	90 (0.28)	80 (0.25)	70 (0.22)
Full Scale	ω_n	2.50	2.26	2.03	1.78	1.56	2.03	1.84	1.64	1.45	1.27
	ζ	0.64	0.63	0.63	0.63	0.63	0.78	0.78	0.77	0.77	0.77
Free-Flight Model	ω_n	8.46	7.69	6.93	6.12	5.38	6.88	6.25	5.63	4.98	4.38
	ζ	0.62	0.62	0.62	0.62	0.62	0.76	0.76	0.76	0.76	0.76
Semi-Free Model (W.T. Simulation)	ω_n	8.46	7.68	6.92	6.11	5.37	6.87	6.24	5.26	4.97	4.36
	ζ	0.62	0.62	0.62	0.62	0.62	0.76	0.76	0.76	0.76	0.76
Model with Pitch Freedom Only	ω_n	7.23	6.56	5.91	5.22	4.59	5.28	4.80	4.32	3.82	3.35
	ζ	0.39	0.39	0.39	0.39	0.39	0.54	0.54	0.54	0.54	0.54

Table D.10 Estimated Lateral Mode Characteristics
(Roll Mode & Dutch Roll Mode)

		Velocity ft/sec (Mach.No.)				
		110 (0.34)	100 (0.31)	90 (0.28)	80 (0.25)	70 (0.22)
Full Scale Aircraft	$t_{R_{\frac{1}{2}}}$	0.185	0.202	0.223	0.25	0.278
	ω_{DR}	2.46	2.22	2.05	1.84	1.65
	ζ_{DR}	0.12	0.113	0.103	0.085	0.066
Free-Flight Model	$t_{R_{\frac{1}{2}}}$	0.055	0.060	0.066	0.073	0.081
	ω_{DR}	8.20	7.61	6.95	6.29	5.66
	ζ_{DR}	0.123	0.113	0.102	0.086	0.066
Semi-Free Flight Model (W.T. Simulation)	$t_{R_{\frac{1}{2}}}$	0.057	0.063	0.069	0.078	0.088
	ω_{DR}	7.88	7.26	6.58	5.89	5.19
	ζ_{DR}	0.106	0.104	0.100	0.094	0.090

Table D.11 Estimated Control Derivatives (Model)

	Velocity (ft/sec)	
	70	100
$\dot{Z}_\eta$	-5.68	-11.59
$\dot{M}_\eta$	-6.66	-13.57
$\dot{L}_\xi$	-3.08	-6.29
$\dot{L}_\zeta$	0.56	1.14
$\dot{N}_\xi$	0.37	0.76
$\dot{N}_\zeta$	-1.76	-3.59

Table D.11 Estimated Longitudinal Transfer Functions - Model Short Period Mode

Semi Free Model

$$\frac{\theta}{\eta} = \frac{N_{\eta}^{\theta}(s)}{\Delta(s)}$$

where $\Delta(s) = s^2 + Bs + C$

$$A = m \ddot{I}_y$$

$$B = (-m V \dot{M}_w - m \dot{M}_q - \dot{Z}_w \ddot{I}_y)$$

$$C = (\dot{Z}_w \dot{M}_q - m V \dot{M}_w)$$

and

$$N_{\eta}^{\theta} = A s + B, \quad A = m \ddot{M}_{\eta} + \dot{Z}_{\eta} \dot{M}_w$$

$$B = \dot{M}_w \dot{Z}_{\eta} - \dot{Z}_w \dot{M}_{\eta}$$

Model with Pitch Freedom Only

$$\frac{\theta}{\eta} = \frac{N_{\eta}^{\theta}(s)}{\Delta(s)}$$

where $\Delta(s) = A s^2 + B s + C$

$$A = I_y$$

$$B = \dot{M}_q - V \dot{M}_w$$

$$C = -\dot{M}_w V$$

and $N_{\eta}^{\theta} = \dot{M}_{\eta}$

Semi-Free Model					Model with Pitch Freedom Only					
A(s)			N_{η}^{θ}			A(s)			N_{η}^{θ}	
A	B	C	A	B	A	B	C	N_{η}^{θ}		
ft/sec										
c.g.=0.29c	70	9.54×10^{-3}	6.34×10^{-2}	2.79×10^{-1}	1.07	3.16	5.9×10^{-2}	2.14×10^{-1}	1.26	6.66
	100	9.54×10^{-3}	9.02×10^{-2}	5.60×10^{-1}	2.19	9.22	5.9×10^{-2}	3.04×10^{-1}	2.53	13.57
	70	9.54×10^{-3}	6.34×10^{-2}	1.83×10^{-1}	1.07	3.48	5.9×10^{-2}	2.14×10^{-1}	6.72×10^{-1}	6.66
c.g.=0.325c	100	9.54×10^{-3}	9.02×10^{-2}	3.69×10^{-1}	2.19	9.33	5.9×10^{-2}	3.04×10^{-1}	1.35	13.57

Table D.13 Estimated Lateral Transfer Functions

$$\frac{\phi}{\zeta} = \frac{N_{\zeta}^{\phi}(s)}{\Delta(s)} \qquad \text{and} \qquad \frac{\psi}{\zeta} = \frac{N_{\zeta}^{\psi}(s)}{\Delta(s)}$$

where $\Delta(s) = s(A s^3 + B s^2 + C s + D)$

$$A = I_{zx}^2 - I_x I_z$$

$$B = I_z \dot{N}_r + I_z \dot{L}_p + I_{zx} (\dot{N}_p + \dot{L}_r)$$

$$C = -I_{zx} V \dot{L}_v - I_x V \dot{N}_p + \dot{N}_p \dot{L}_r - \dot{N}_r \dot{L}_p$$

$$D = (\dot{L}_p \dot{N}_v - \dot{L}_v \dot{N}_p) V_e$$

$$N_{\zeta}^{\phi}(s) = A s^2 + B s + C$$

$$A = (-I_{zx} \dot{N}_{\zeta} - I_z \dot{L}_{\zeta})$$

$$B = (\dot{N}_r \dot{L}_{\zeta} - \dot{L}_r \dot{N}_{\zeta})$$

$$C = (\dot{L}_v \dot{N}_{\zeta} - \dot{N}_v \dot{L}_{\zeta}) V_e$$

$$N_{\zeta}^{\psi}(s) = s(A s + B)$$

$$A = I_{zx} \dot{L}_{\zeta} - I_x \dot{N}_{\zeta}$$

$$B = \dot{L}_{\zeta} \dot{N}_{\zeta} - \dot{N}_{\zeta} \dot{L}_{\zeta}$$

	$\Delta(s)$				N_{ζ}^{ϕ}			N_{ζ}^{ψ}	
	A	B	C	D	A	B	C	A	B
70ft/sec	-1.13x10 ⁻³	-9.85x10 ⁻³	-3.96x10 ⁻²	-2.43x10 ⁻¹	0.22	0.195	3.78	3x10 ⁻²	2.35x10 ⁻¹
100ft/sec	-1.13x10 ⁻³	-1.41x10 ⁻²	-7.79x10 ⁻²	-6.52x10 ⁻¹	0.448	0.573	16.69	6x10 ⁻²	6.67x10 ⁻¹

Table D.14 Relative Importance and Prediction Accuracy of
Stability Derivatives

LONGITUDINAL (Short Period)			LATERAL (Roll and Dutch Roll Mode)		
Derivative	Relative* Importance	Estimated Prediction Accuracy**	Derivative	Relative* Importance	Estimated Prediction Accuracy
X_u	5	$\pm 20\%$	L_v	10	$\pm 20\%$
X_w	5	$\pm 10\%$	L_p	10	$\pm 10\%$
Z_u	5	$\pm 20\%$	L_r	7	$\pm 10\%$ to $\pm 20\%$
Z_w	10	$\pm 5\%$	N_v	10	$\pm 20\%$
Z_q	3	$\pm 20\%$	N_p	8	$\pm 50\%$
$Z_{\dot{w}}$	4	$\pm 40\%$	N_r	9	$\pm 15\%$
M_w	10	$\pm 10\%$	Y_v	7	$\pm 20\%$
M_q	9	$\pm 15\%$			
$M_{\dot{w}}$	7	40%			

* 10 = Major, 5 = Minor, 0 = Negligible

** Using Theoretical Methods such as those of references 49 and 50.

REFERENCES

- D1 "AEROSPACE HANDBOOK"
Second Edition, 1973,
General Dynamics, Convair Aerospace
Division.
- D2 "Summary of Methods for Calculating
Dynamic Lateral Stability and Response
and for Estimating Lateral Stability
Derivatives".
NACA Report 1098, 1952.

APPENDIX E

DATA REDUCTION

1. Short Period Data Reduction

The short period frequency and damping ratio are determined from a trace of the time history of the model in response to an elevator input pulse as shown in figure E-1. Pitch angle response is used to determine the short period response frequency and damping ratio

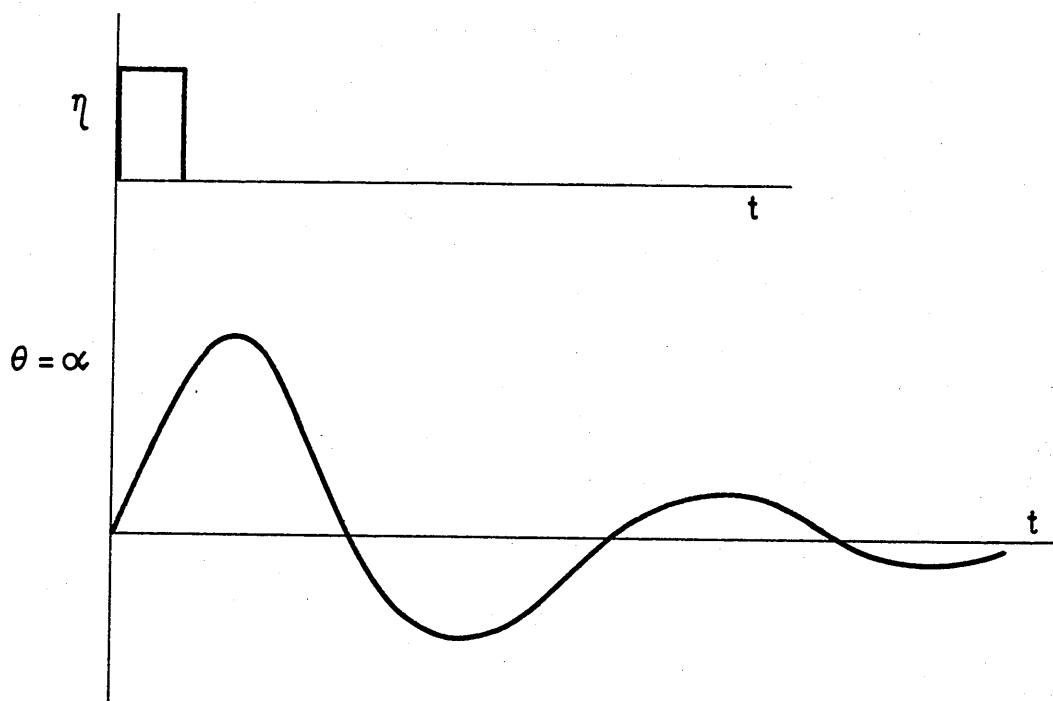


Figure E-1. Short Period Response

Either the Log Decrement or Time Ratio data reduction method can be applied to the short period response trace.

(a) Log Decrement Method

If the short period response is oscillatory and the damping ratio is 0.5 or less (3 or more overshoots) proceed with the log decrement method^{E1} of data reduction. This method is also called the subsidence ratio or the transient peak ratio method.

Using the pitch angle trace, measure the values of each peak deviation from the trim value for ΔX_1 , ΔX_2 , ΔX_3 , etc., as shown in figure E-2.

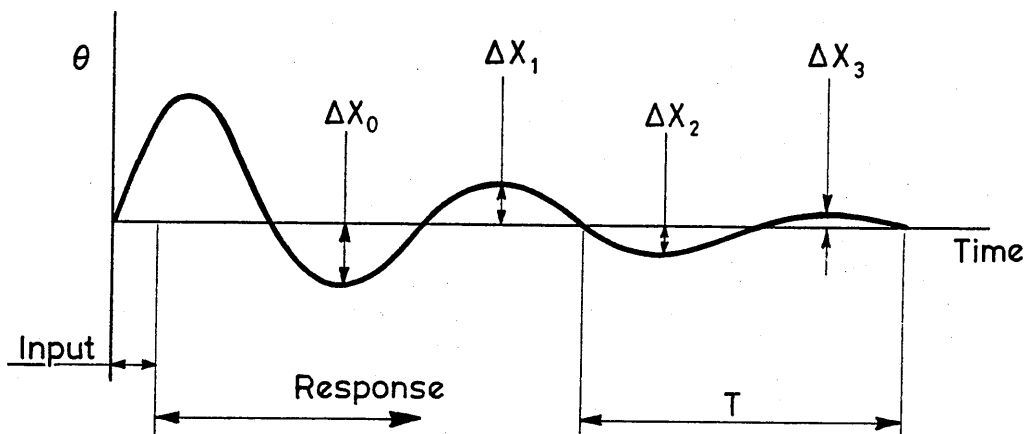


Figure E.2 Transient Peak or Subsidence Ratio Analysis

Determine the transient peak ratios,

$\frac{\Delta X_1}{\Delta X_0}$, $\frac{\Delta X_2}{\Delta X_1}$, $\frac{\Delta X_3}{\Delta X_2}$; Enter figure E-5 with the

transient peak ratio values and determine the corresponding values for damping ratio

ζ_1 , ζ_2 , ζ_3 . The average of the damping

ratio will yield the overall short period damping ratio. For a very lightly damped oscillation figure E-6 can be used to determine the damping ratio. The $m = 1$ line is used when comparing peak ratios $\frac{\Delta X_1}{\Delta X_0}$, $\frac{\Delta X_2}{\Delta X_1}$; the $m = 2$ line is used for peak ratios of $\frac{\Delta X_2}{\Delta X_0}$, $\frac{\Delta X_3}{\Delta X_1}$, etc.

The period, T , of the short period response can be determined by measuring the time between peaks as shown in figure E-2.

The short period damped frequency is then calculated by

$$\omega_d = \frac{2\pi}{T} \text{ rad/sec} \quad (\text{E.1})$$

The short period undamped natural frequency is computed using

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} \quad (\text{E.2})$$

b) Time Ratio Method

If the damping ratio is greater than 0.5 (2 or less overshoots), then the time ratio method of data reduction can be used to determine short period response frequency and damping ratio.

Select a peak on the pitch angle trace where the response is free, i.e. free of influence of forcing function. Divide the

amplitude of the peak into the values of 0.736, 0.406 and 0.199 according to reference E.1 . Measure time values t_1 , t_2 , t_3 as shown in figure E-3.

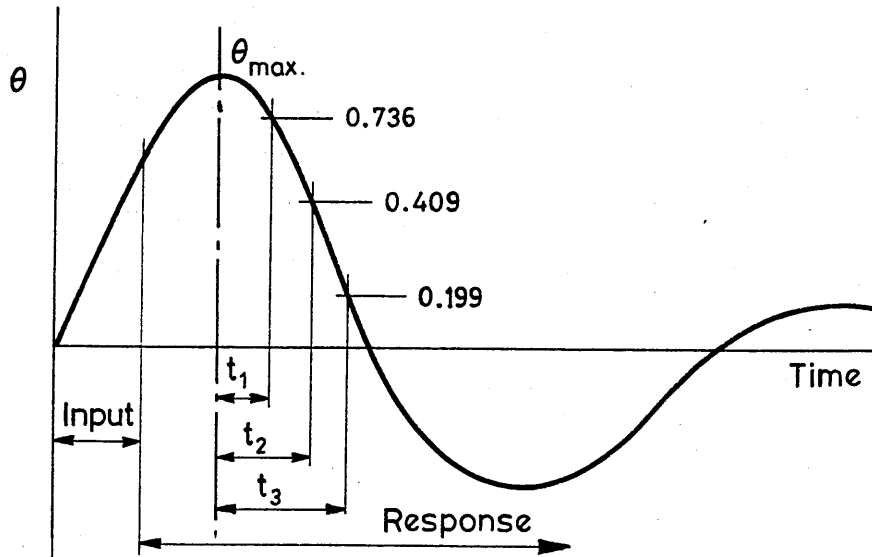


Figure E.3 Time Ratio Analysis

From the time ratios t_2/t_1 , t_3/t_1 and

$(t_3 - t_2)/(t_2 - t_1)$ enter figure E-7 at the time

ratio axis and find the corresponding damping ratio. Average these damping ratios to determine the short period damping ratio ζ_{sp} .

Re-enter E.7 with the average short period damping ratio and find the frequency-time products $\omega_n t_1$, $\omega_n t_2$ and $\omega_n t_3$. To determine the natural frequency, ω_n , compute

$$\omega_n = \frac{\omega_n t_1}{t_1}; \quad \omega_n = \frac{\omega_n t_2}{t_2}; \quad \omega_n = \frac{\omega_n t_3}{t_3}$$

Average these natural frequencies to determine the overall undamped short period natural frequency.

2. Roll Mode Data Reduction

The Roll mode time constant, τ_R is generally measured from a time history trace of the roll rate response (figure E-4a) or bank angle response (figure E-4b) to a step aileron input. From the roll rate, p , trace the time constant, τ_R , can be measured as the time for the roll rate to achieve 63.2% of the steady-state roll rate (figure E-4b). τ_R can also be determined ^{E1} from the bank angle, ϕ , trace as the time for the extension of the linear slope of the ϕ trace to intersect the initial ϕ axis as represented in figure E-4b. It is important to note that the Roll mode time constant is independent of the size of the step aileron input.

In the present wind tunnel simulation, a step aileron input cannot be applied due to bank angle limitation ($\pm 30^\circ$) and hence a pulse input was used and the bank angle response was recorded (figure E-4c). The initial response is similar to that obtained with a step input and the steady roll rate is then achieved (linear slope). Therefore the data reduction technique of analysing the bank angle trace as for a step input was adopted.

3. Dutch Roll Mode Data Reduction

The Dutch Roll mode frequency and damping ratio are

determined from the yaw angle time history response to a rudder pulse input. The methods for determining Dutch Roll frequency and damping ratio are the same as used for short period data reduction.

REFERENCES

- E1 Langdon, S.D. Naval Test Pilot School Flight
 and Test Manual
 Cross, W.V. Naval Air Test Center, Maryland,
 U.S.A., 1975.

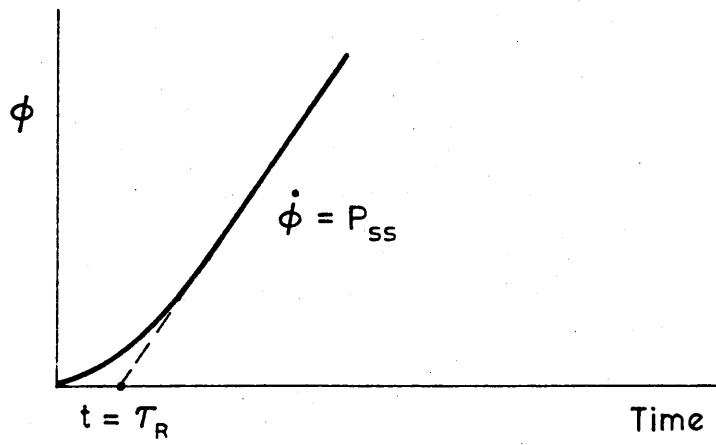


Figure E.4a Bank Angle Trace (Step Input)

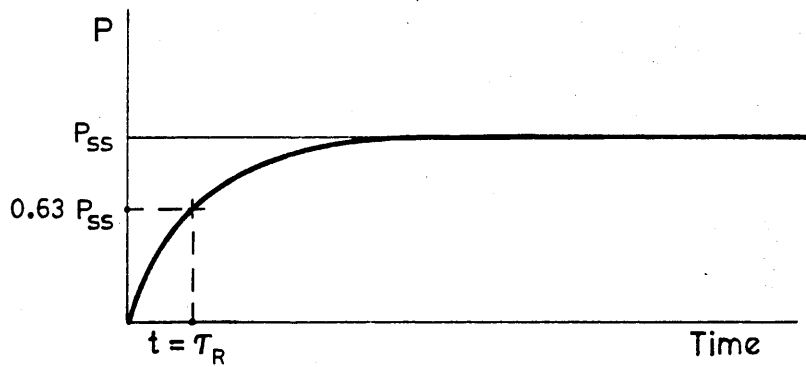


Figure E.4b Roll Rate Trace (Step Input)

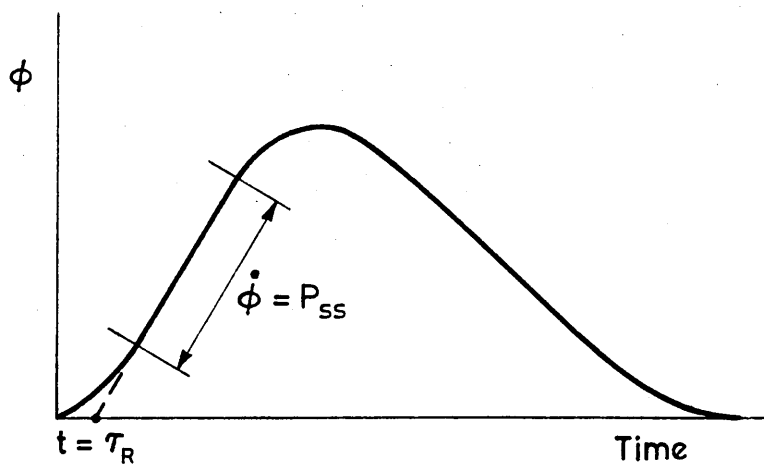


Figure E.4c Bank Angle Trace (Pulse Input)

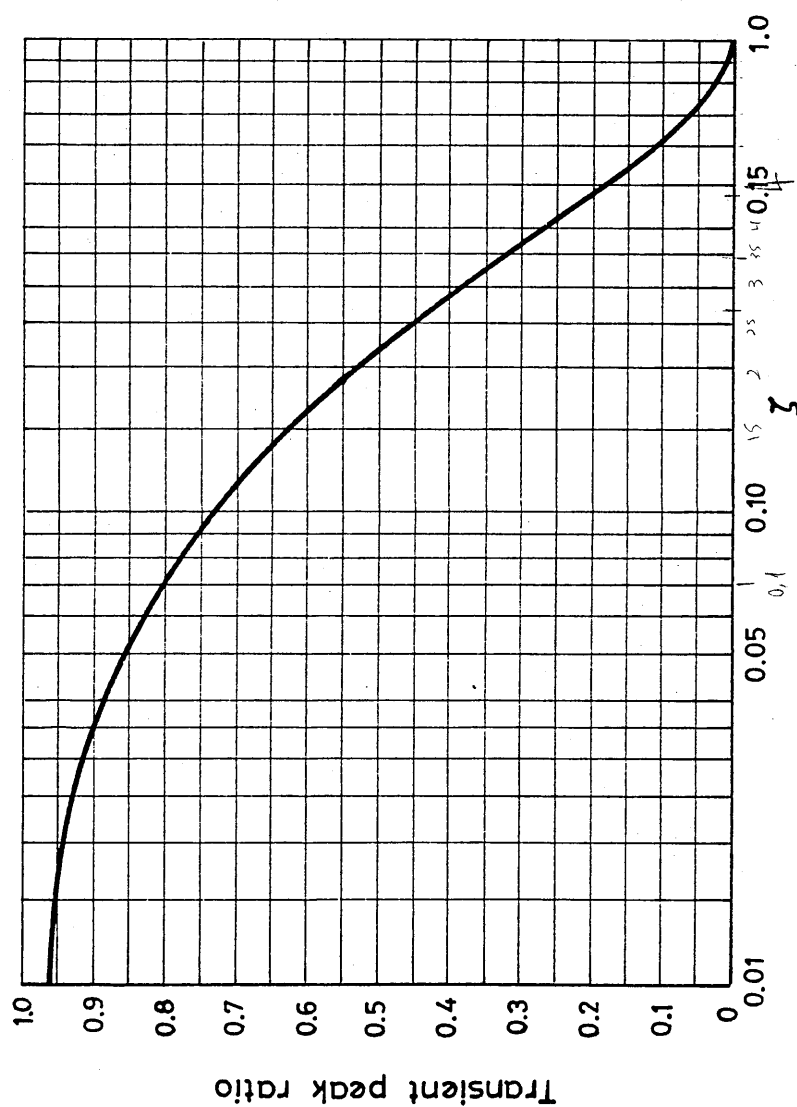


Figure E.5. Determining z by transient peak ratio method.

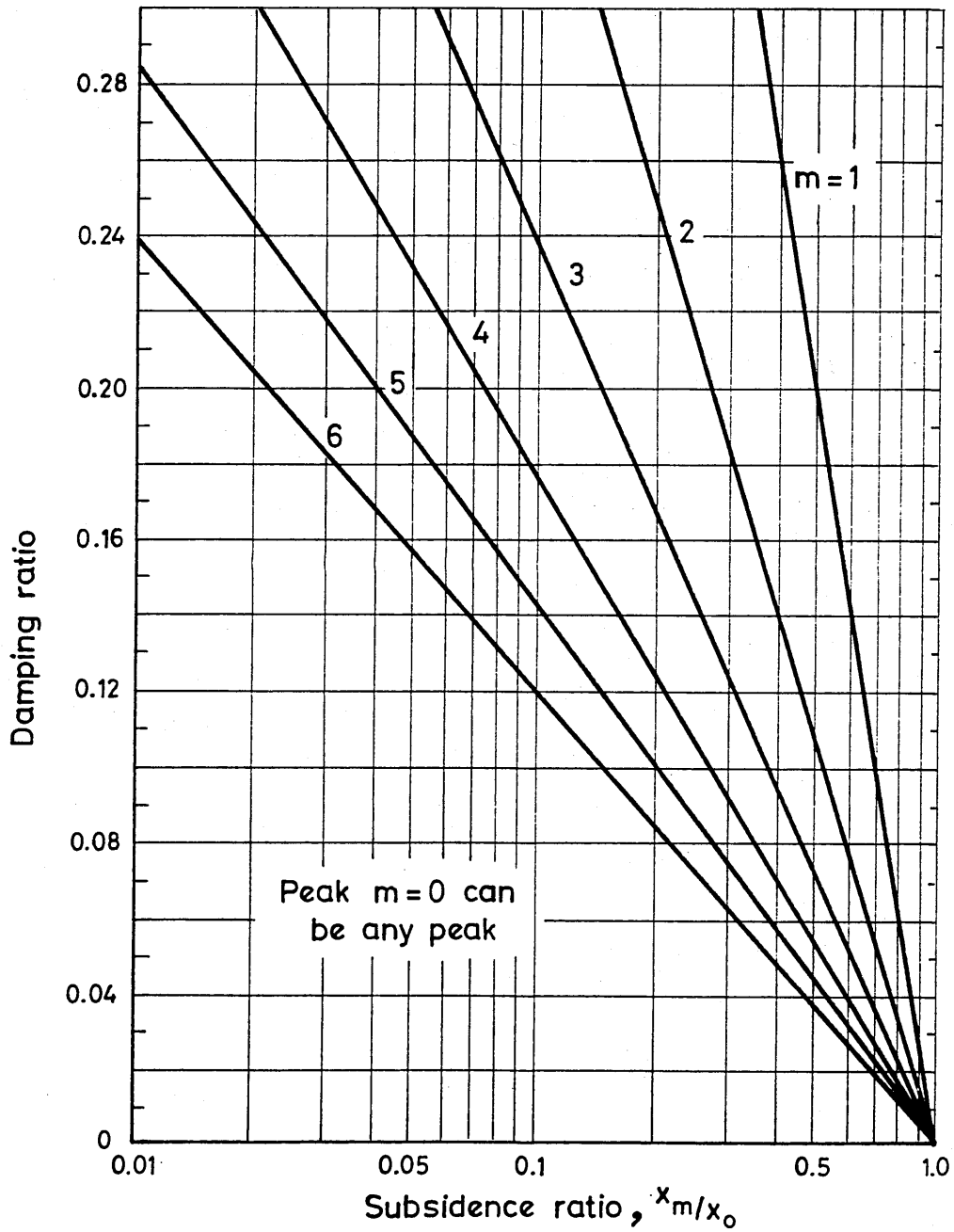


Figure E.6. Damping ratio as a function of subsidence ratio.

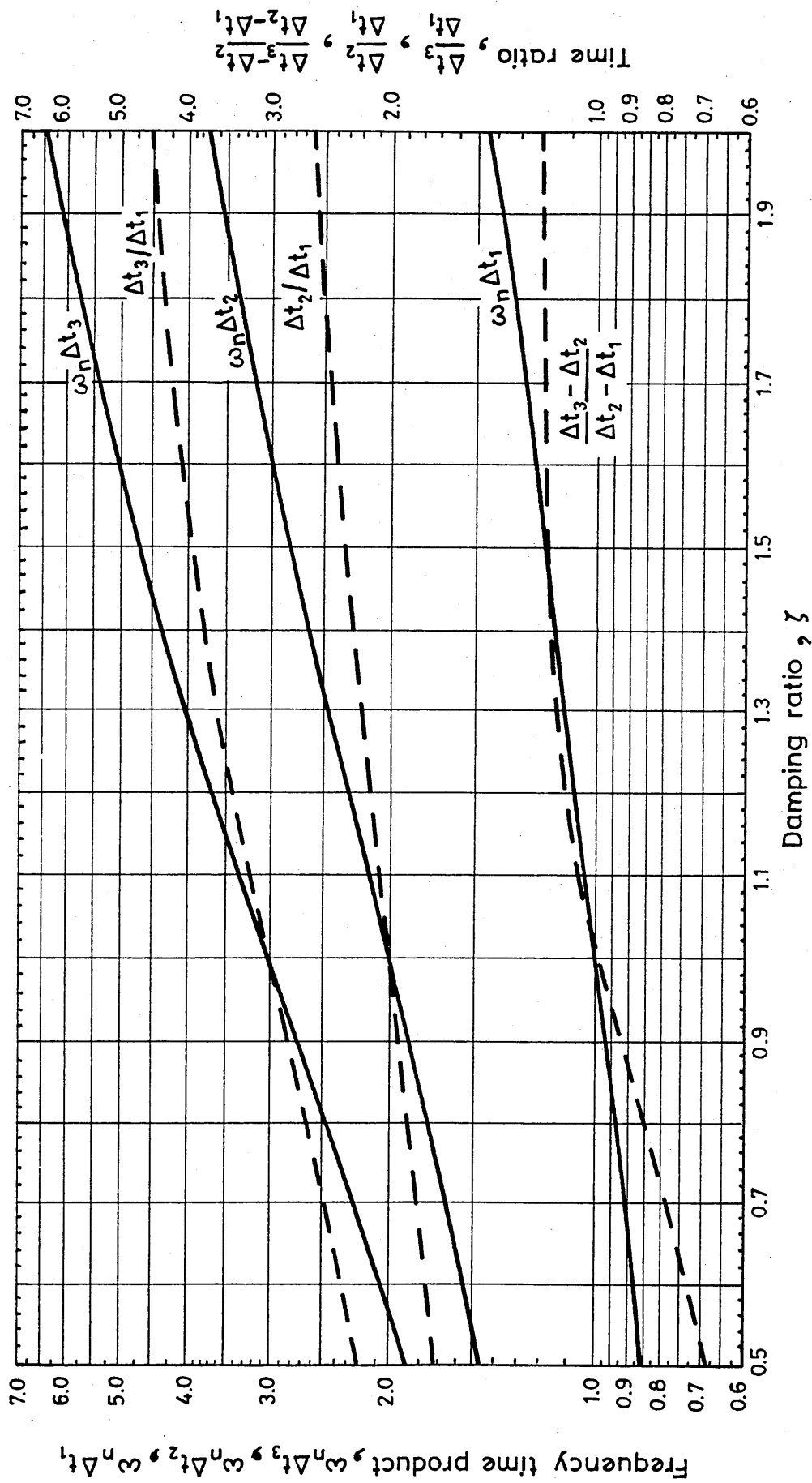


Figure E.7. Determination of ζ and ω_n by time ratio method.

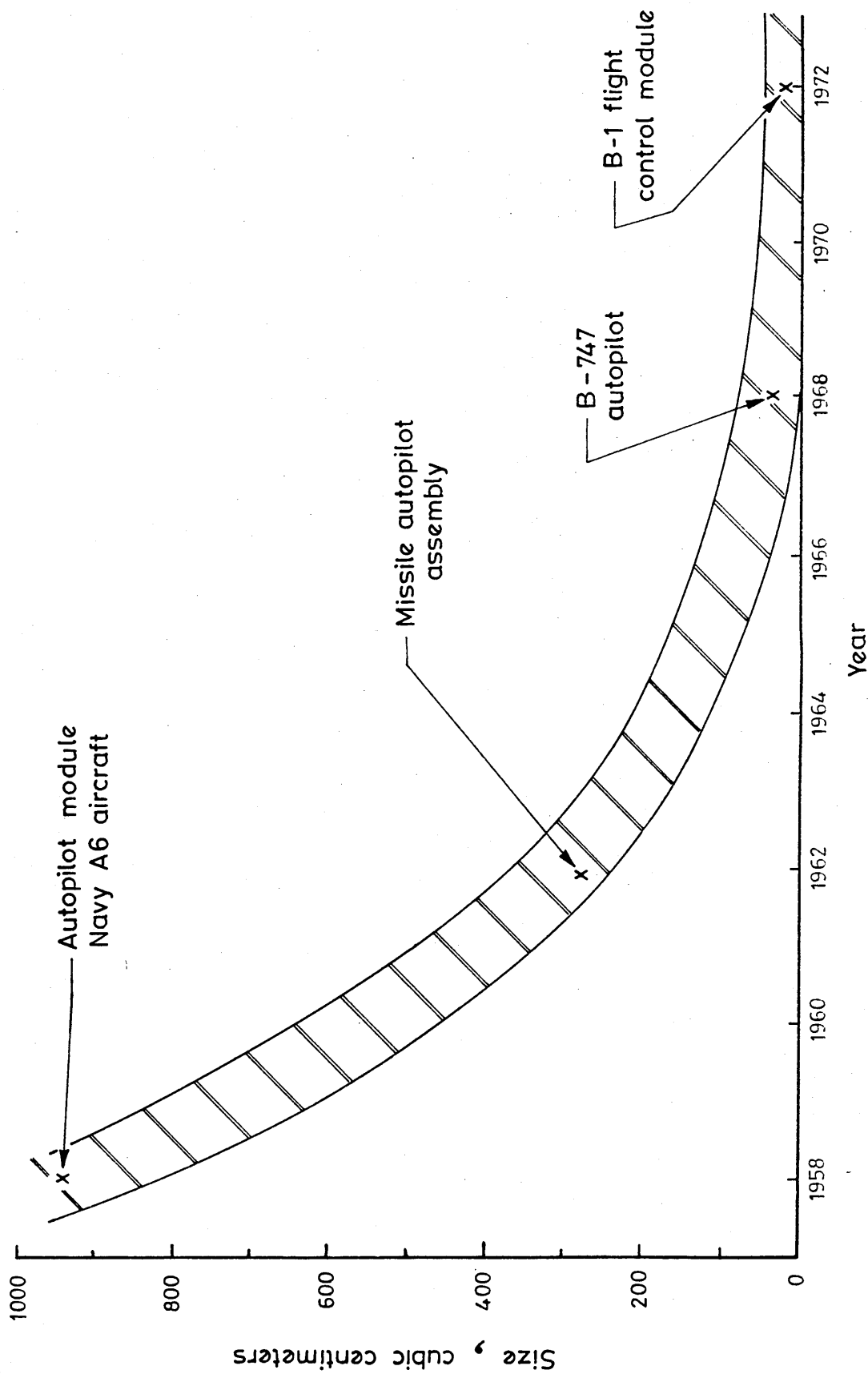


Figure 1.a. Miniaturisation of representative flight control hardware.

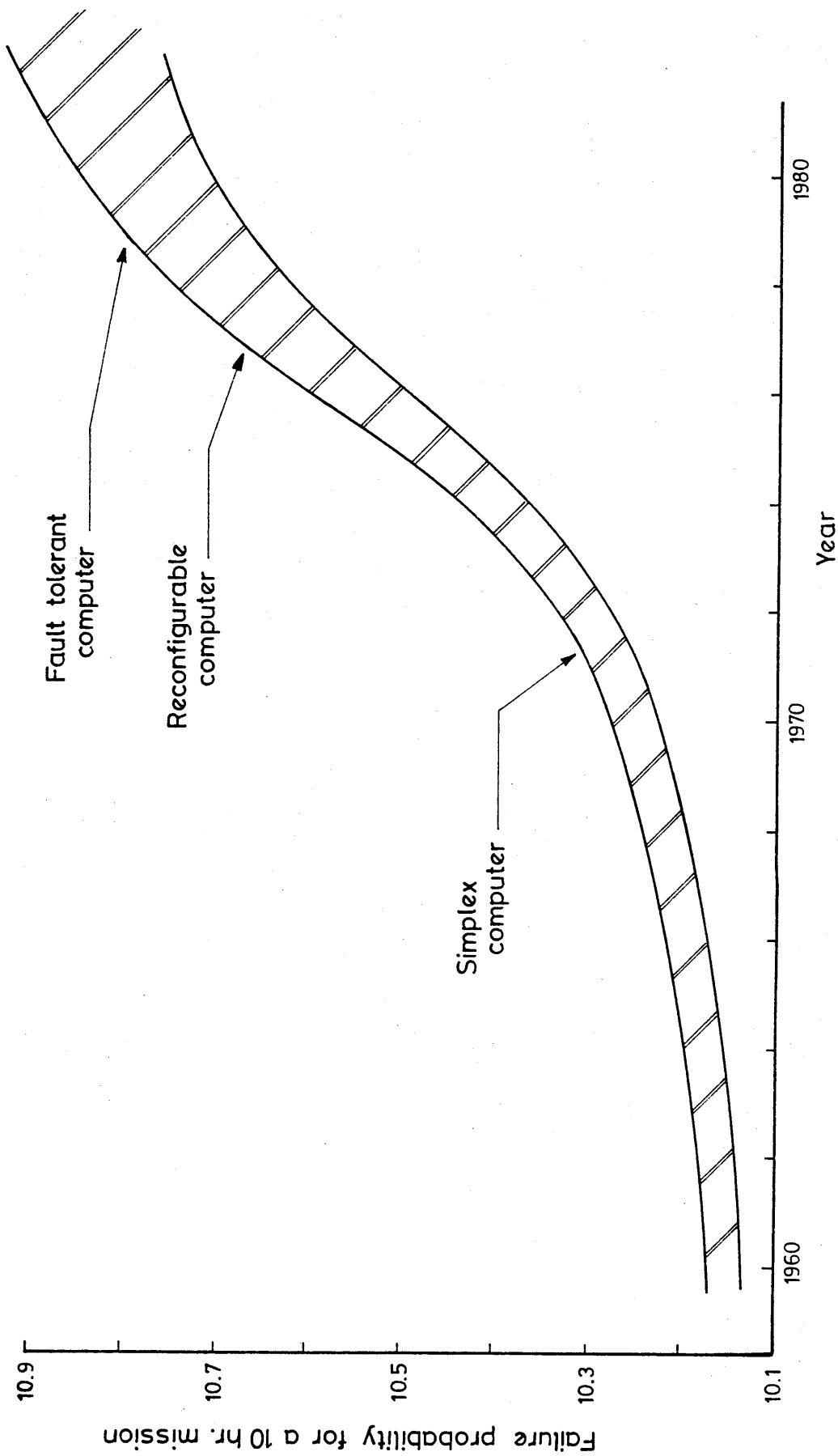


Figure 1.b. Computer system reliability.

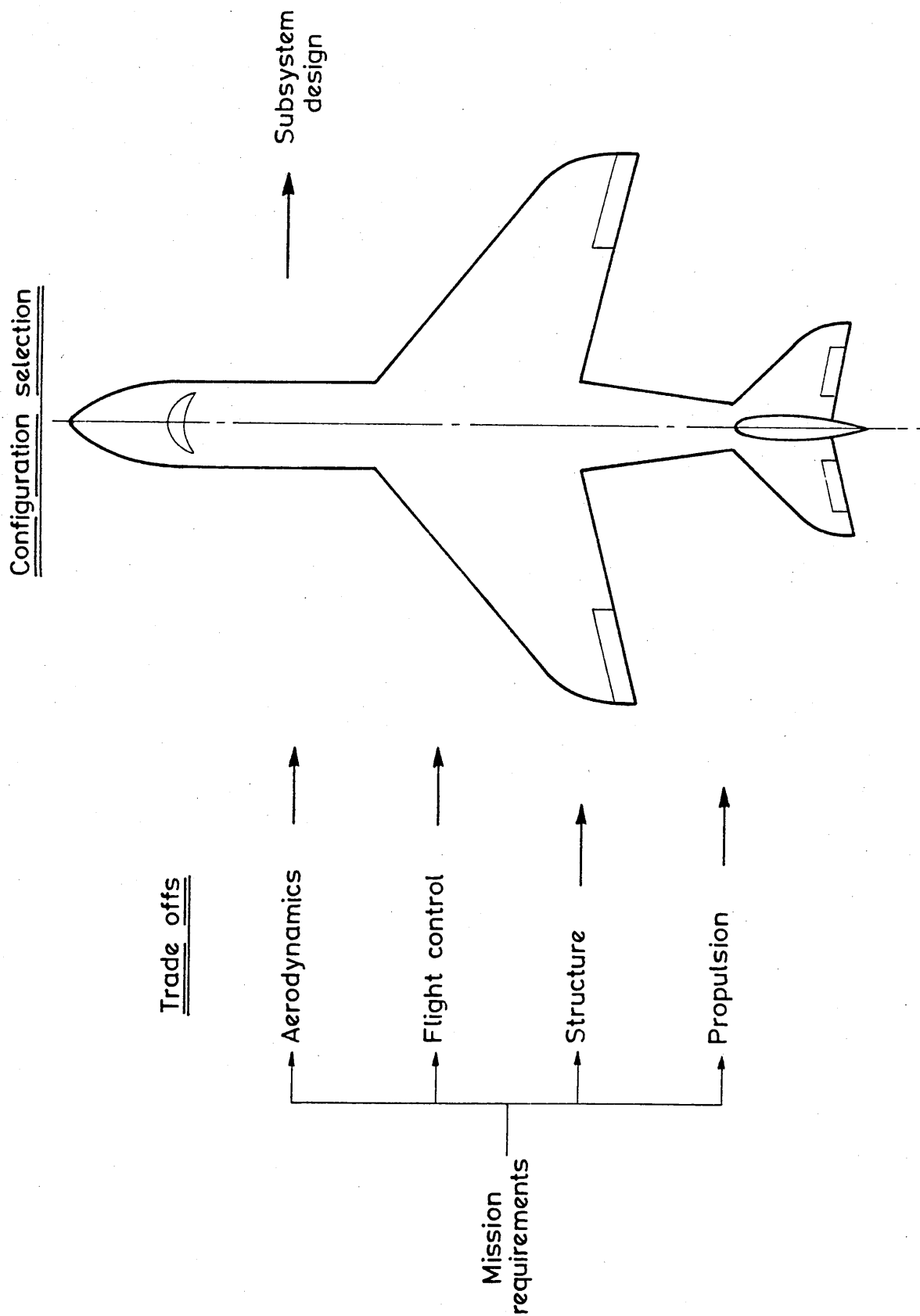


Figure 2. Active Control Technology design approach.

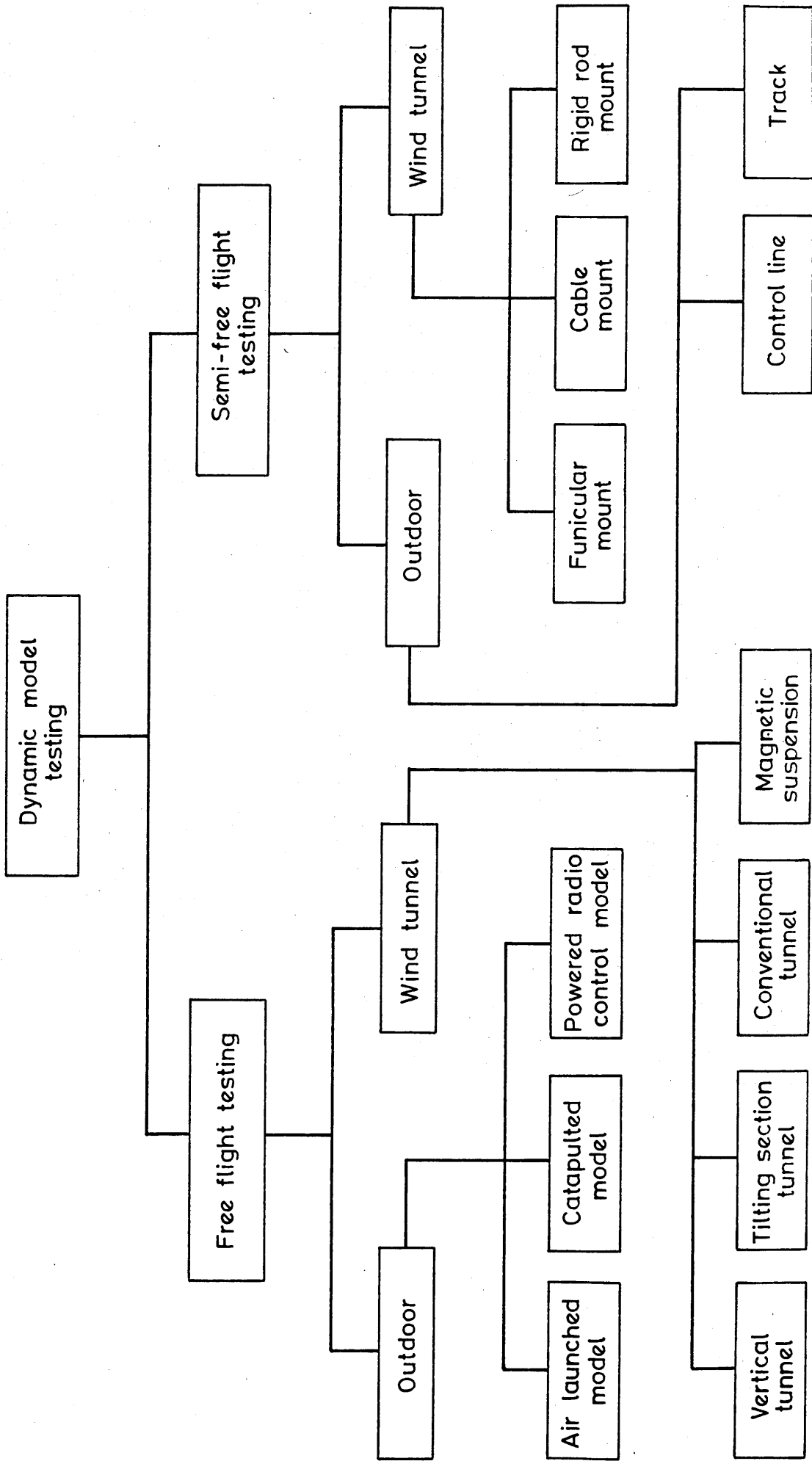


Figure 3. Free flight model testing techniques.

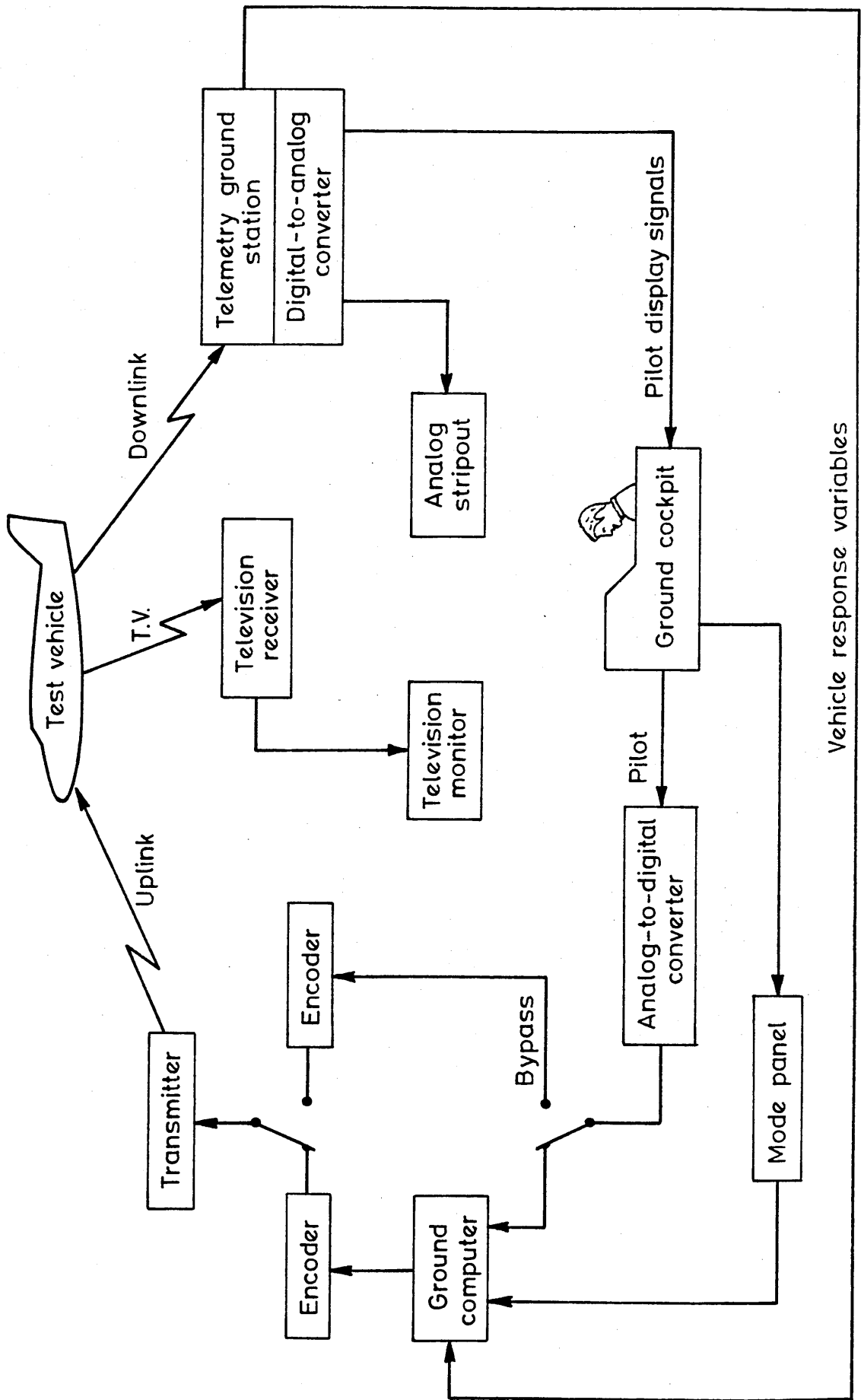
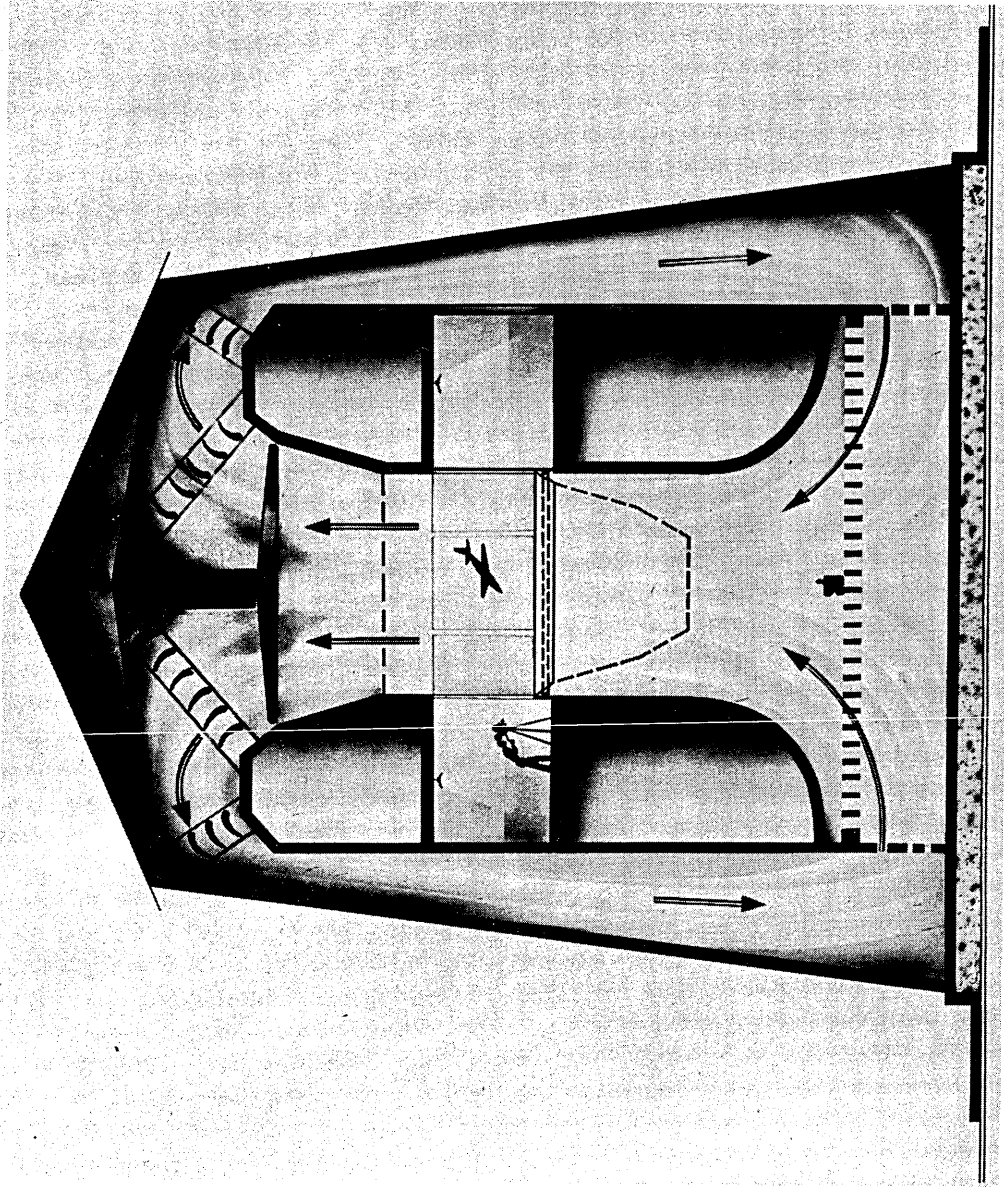


Figure 4. Remotely piloted model control loops.



(b) Cross-sectional view

Figure 5. NASA Langley 20 ft free-spinning tunnel.

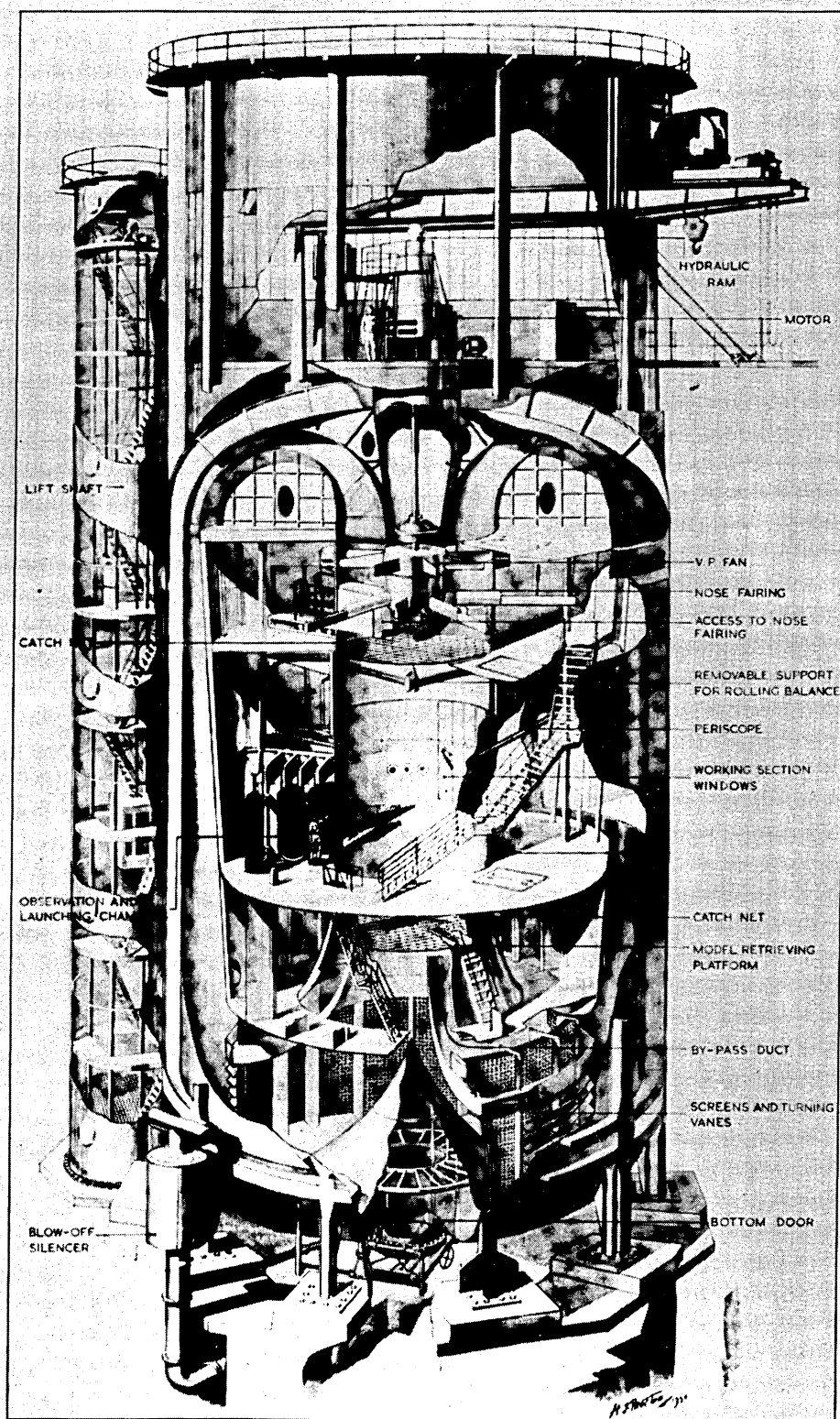


Figure 6. Cross-sectional view of free-spinning tunnel at the NAE, Bedford.

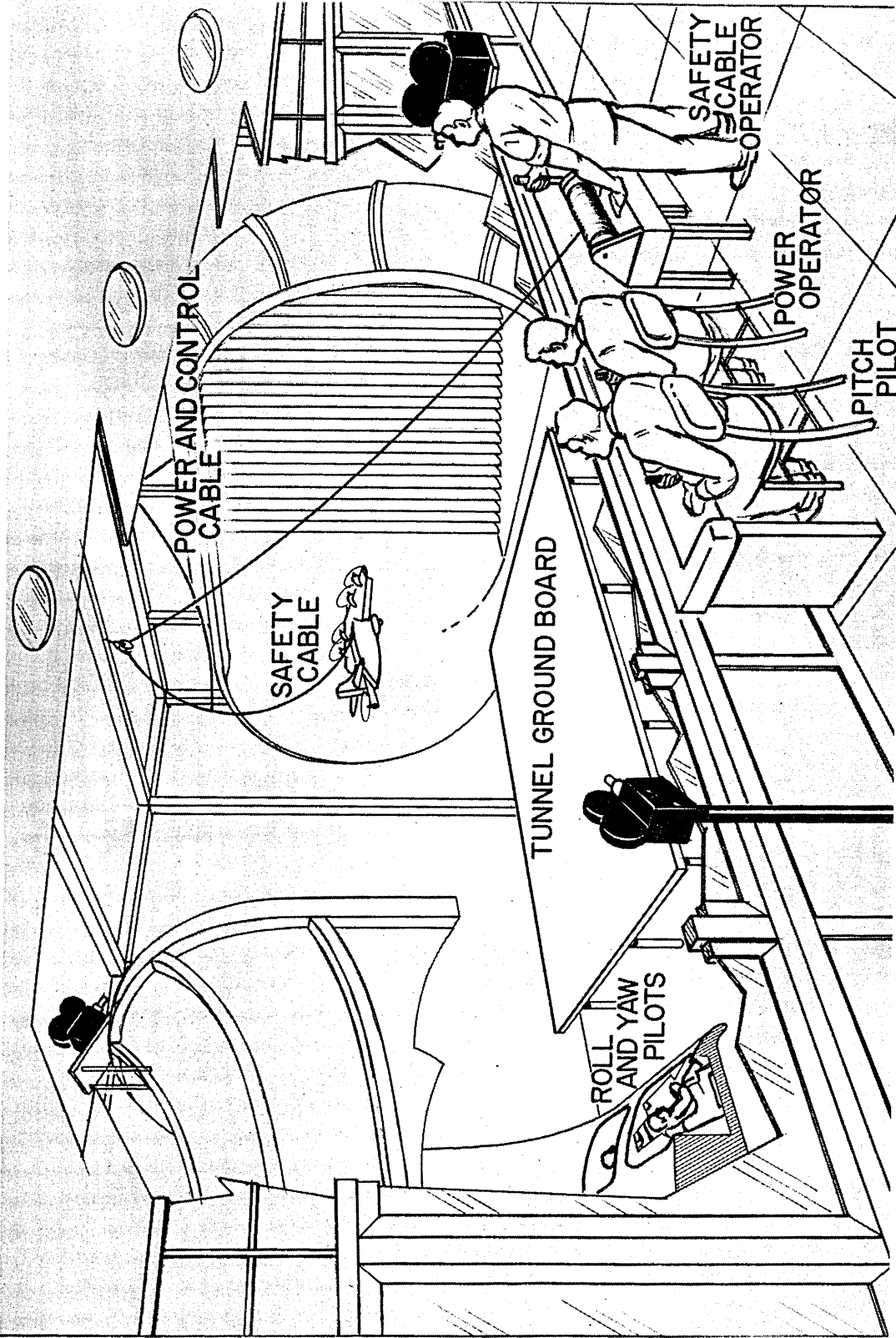
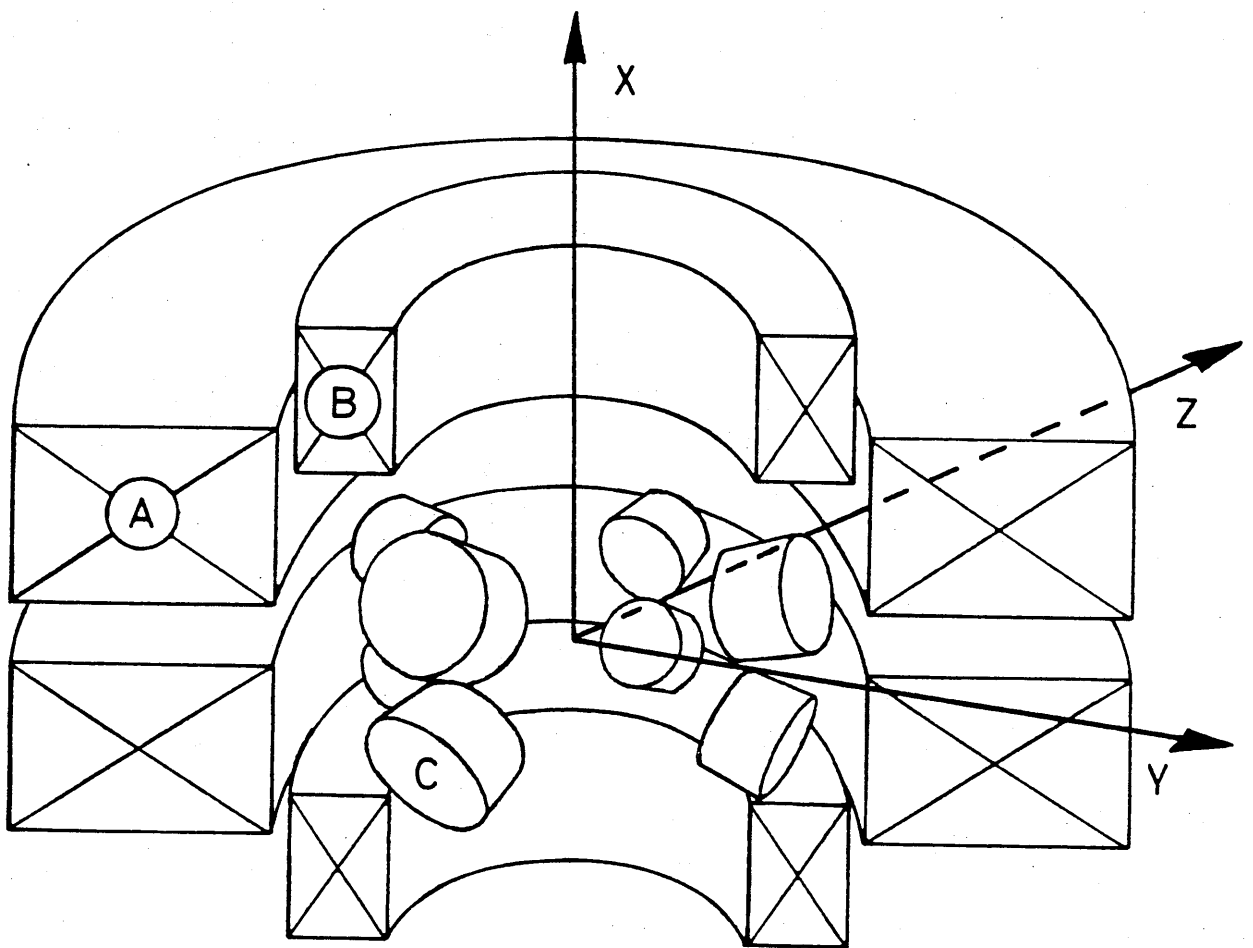


Figure 7. Setup for free-flight tests.



- A. Magnetizing coil.
- B. Vertical gradient coil.
- C. Horizontal gradient coil.

Figure 8. Magnetic suspension. University of Virginia.

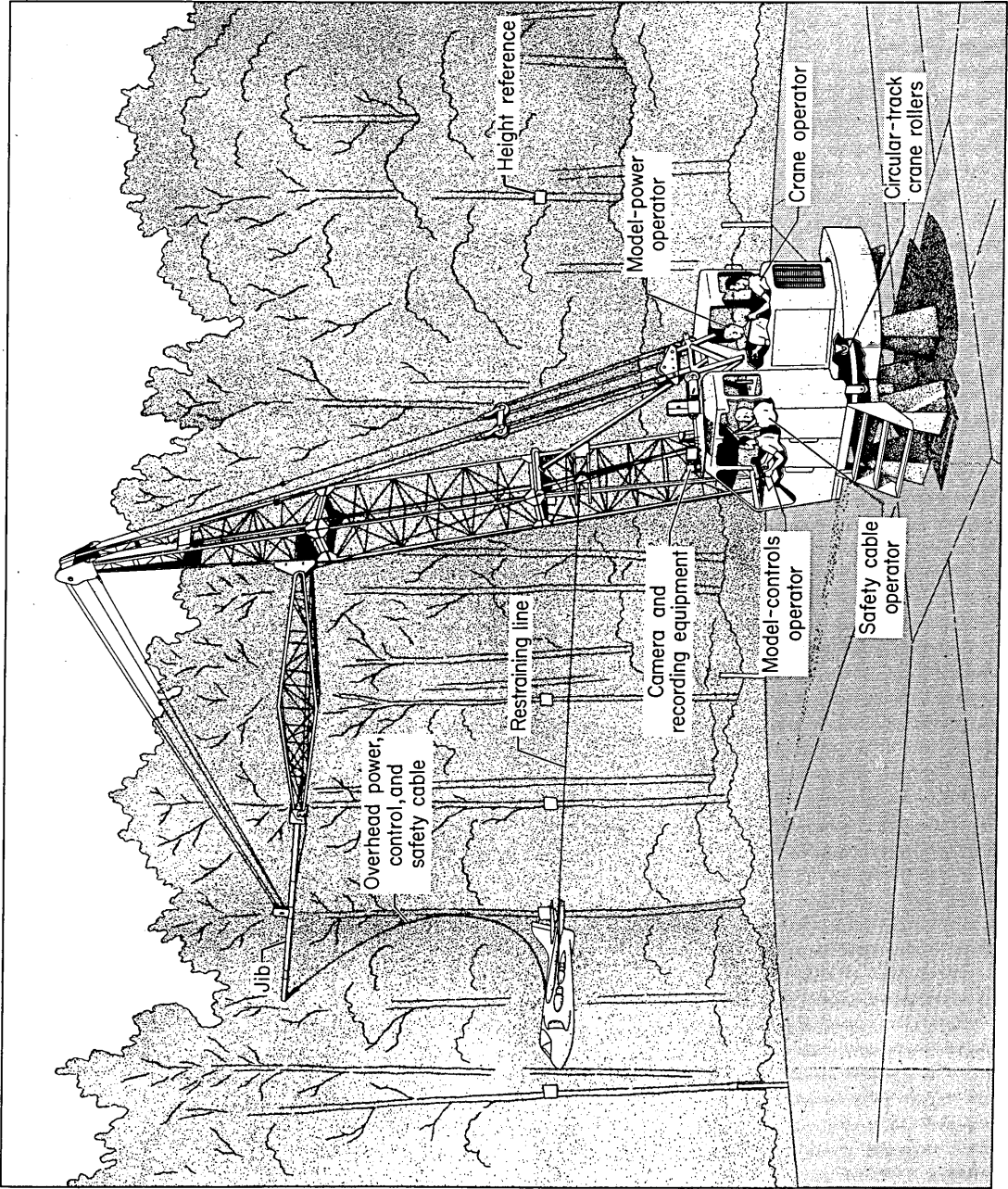


Figure 9. The NASA Langley control-line facility.

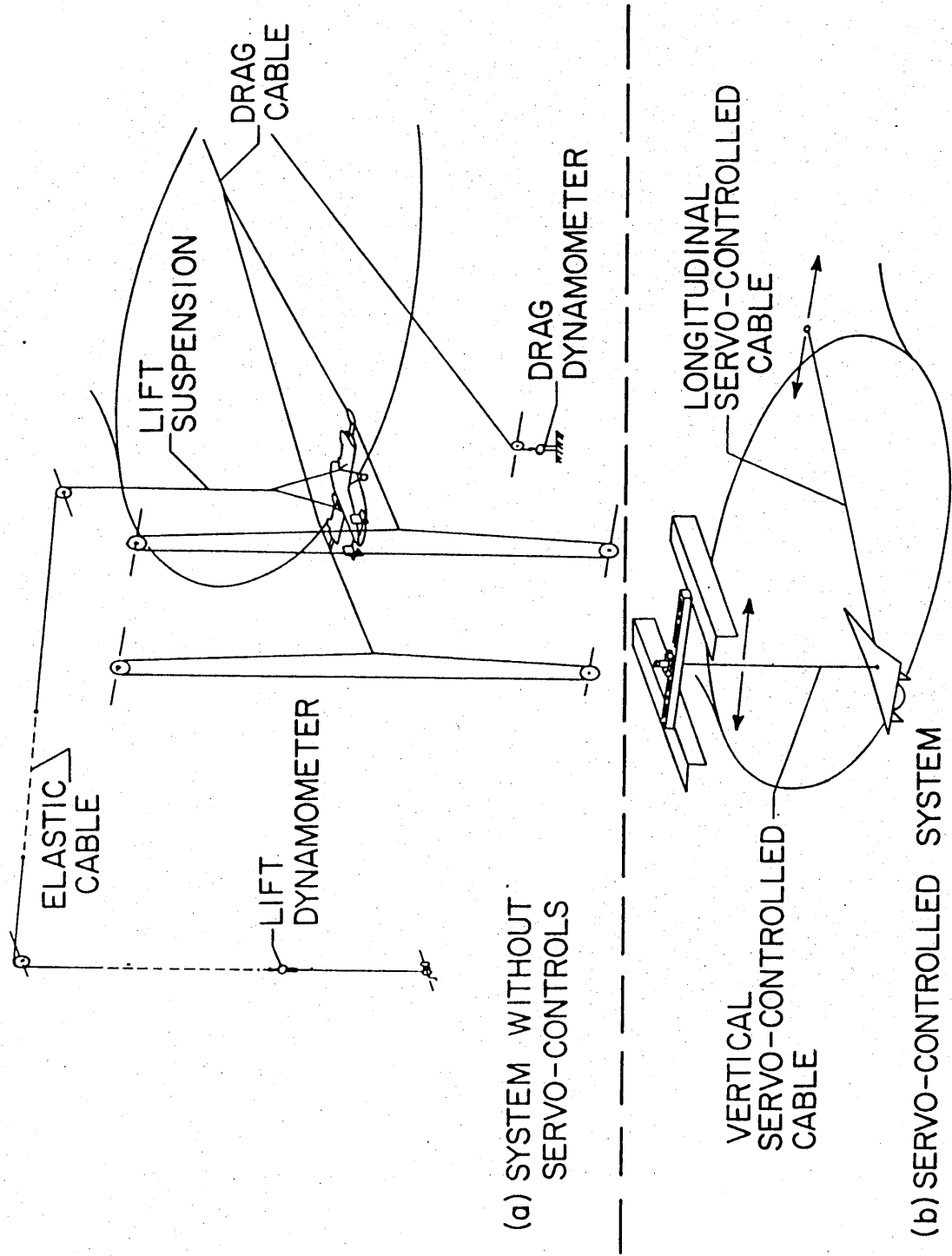


Figure 10. Diagrammatic sketches of funicular suspension systems used by ONERA.

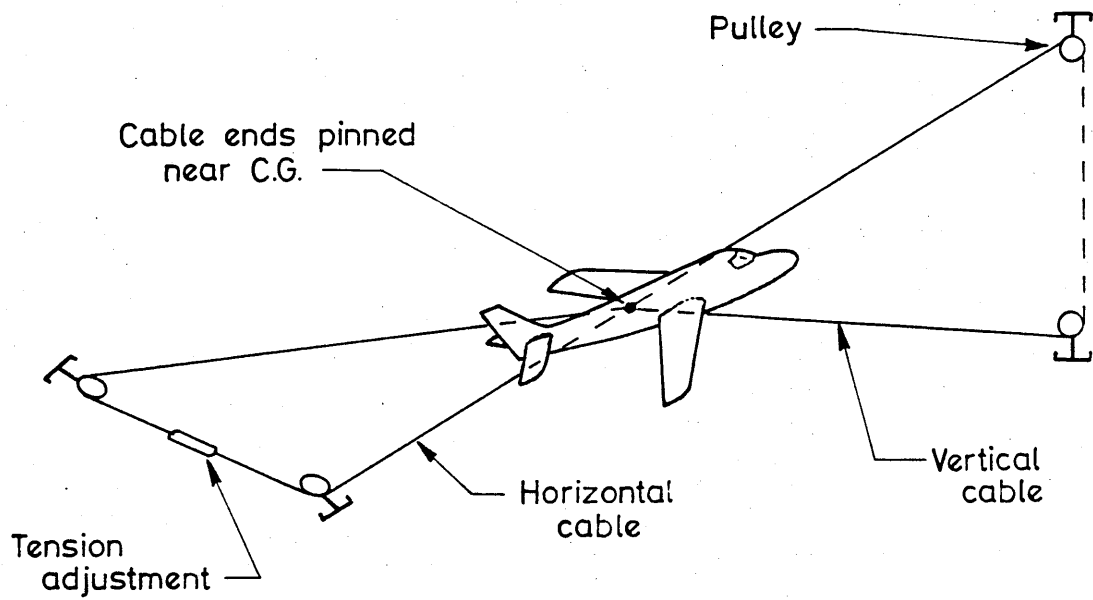


Figure 11. Two cable mount.

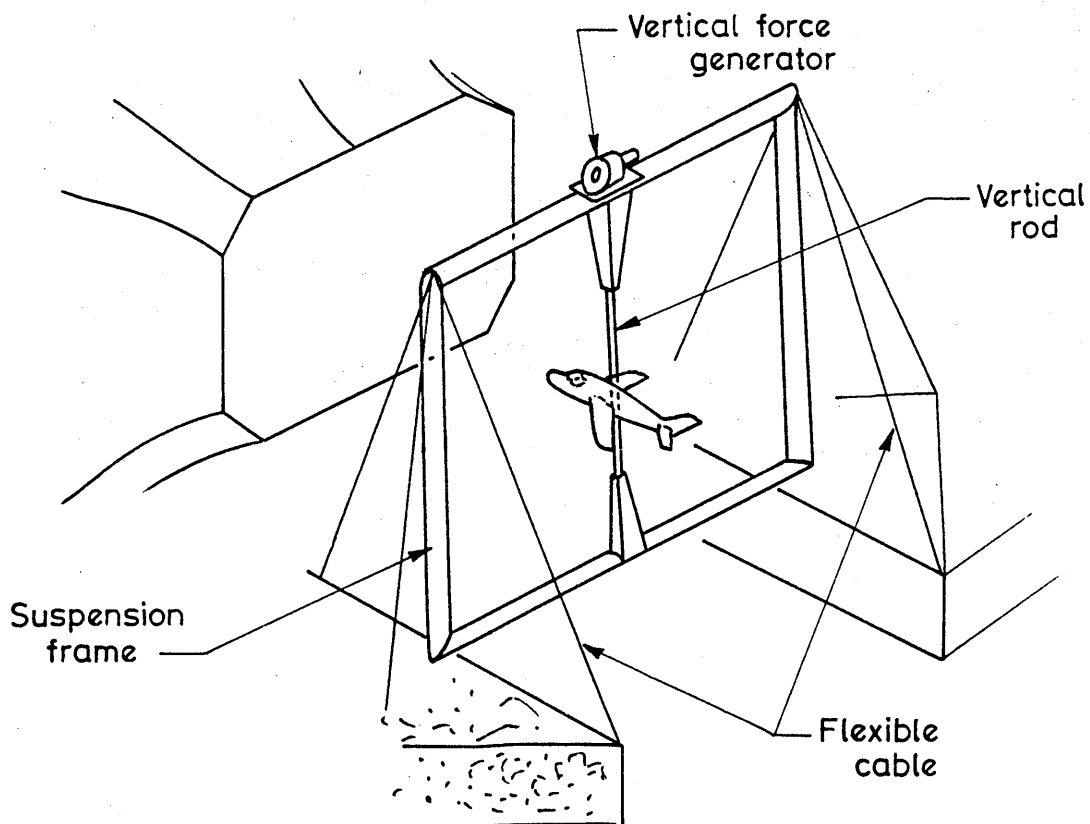


Figure 12. Vertical rigid rod mounting.

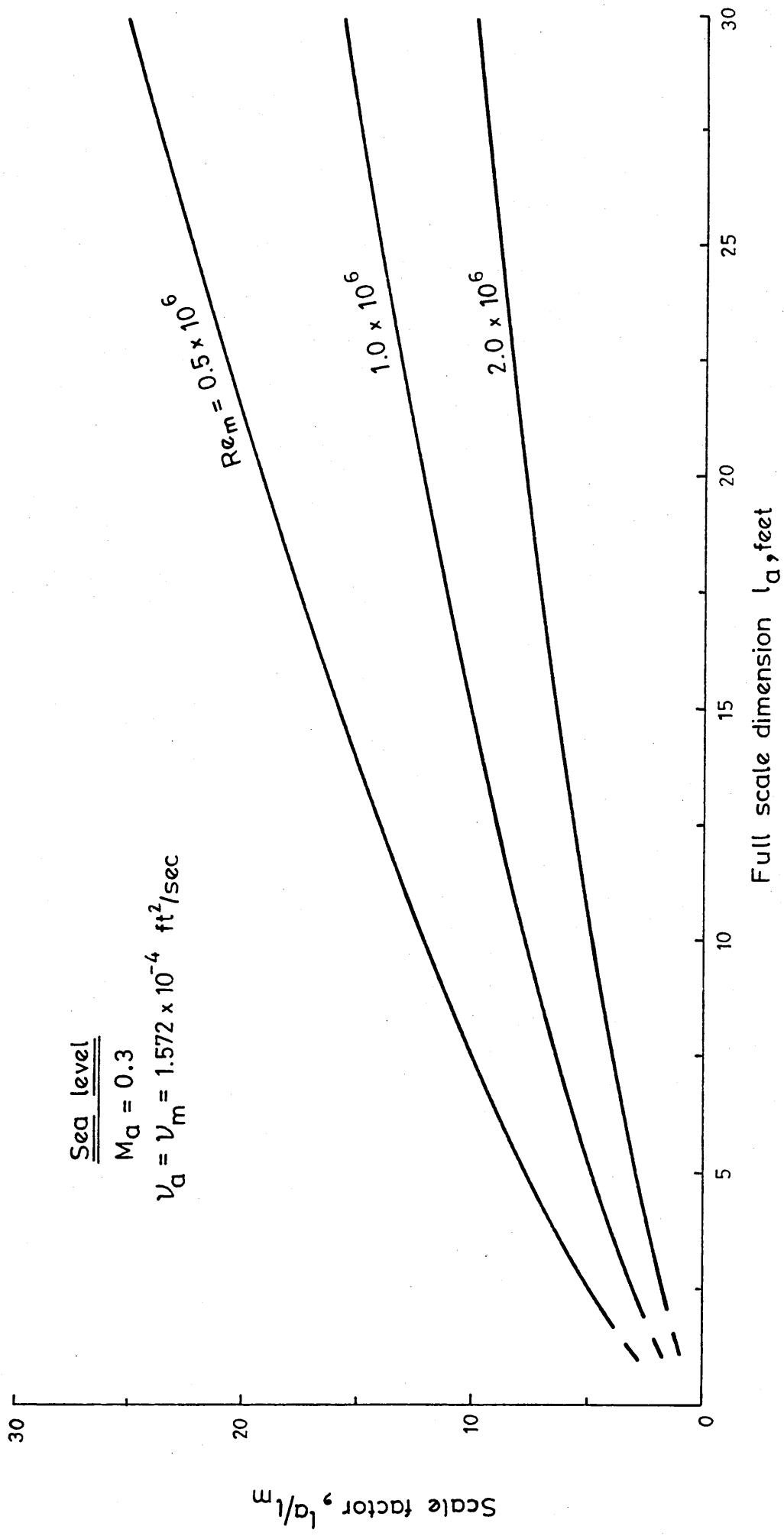


Figure 13. Scale factor to achieve a given model Reynolds number.

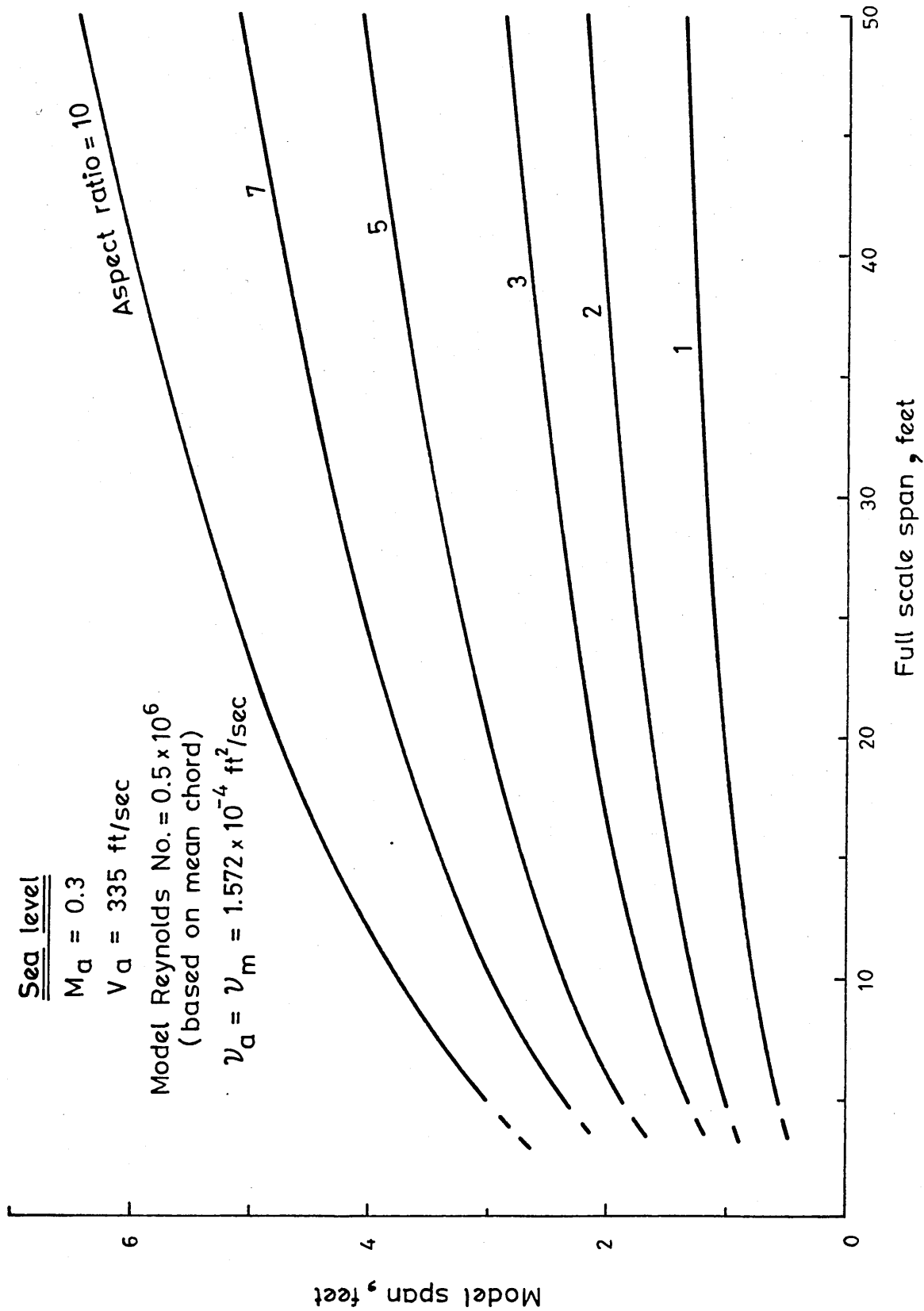


Figure 14. Model span to achieve a Reynolds number of 0.5×10^6

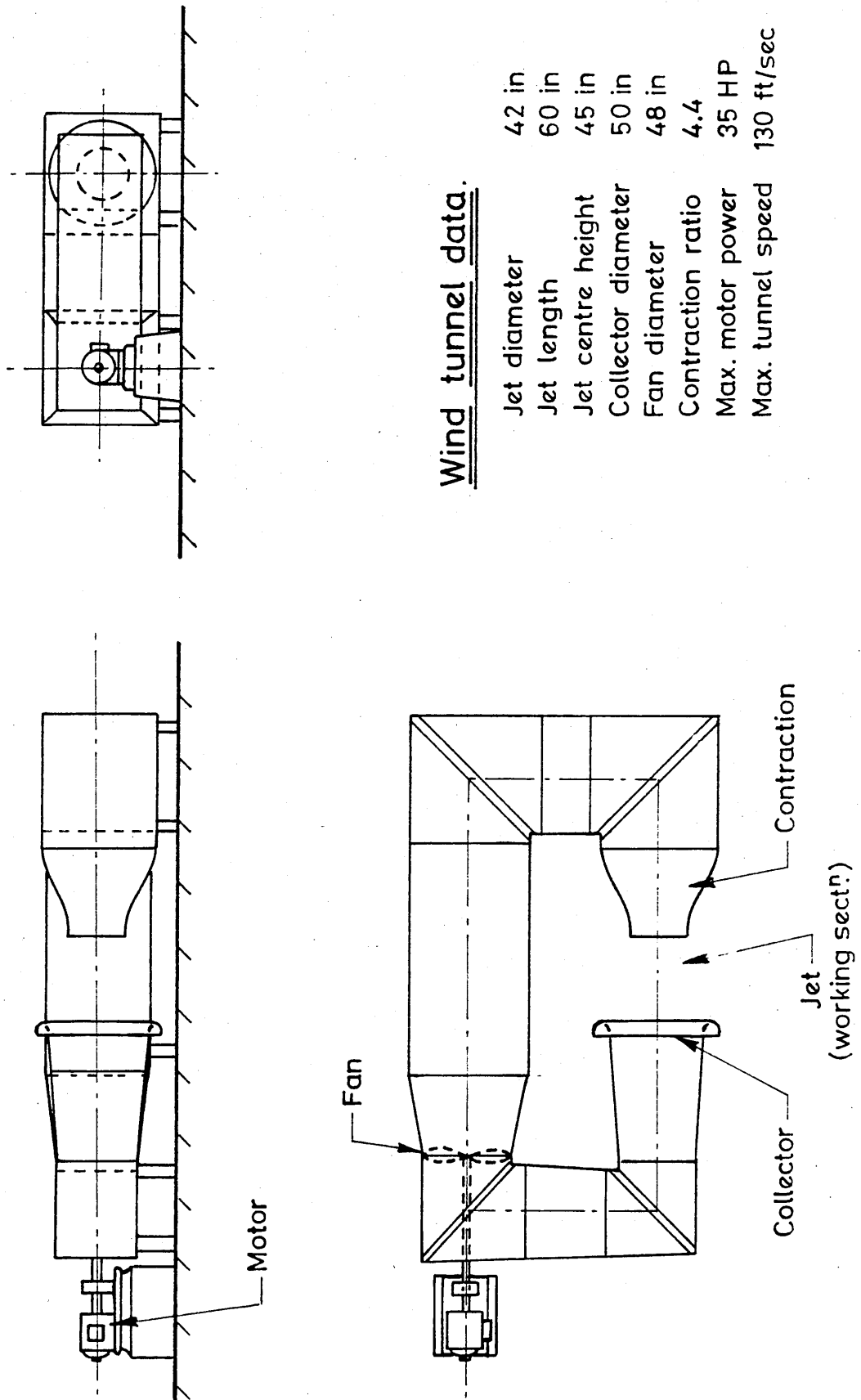


Figure 15. 'Weybridge' wind tunnel.

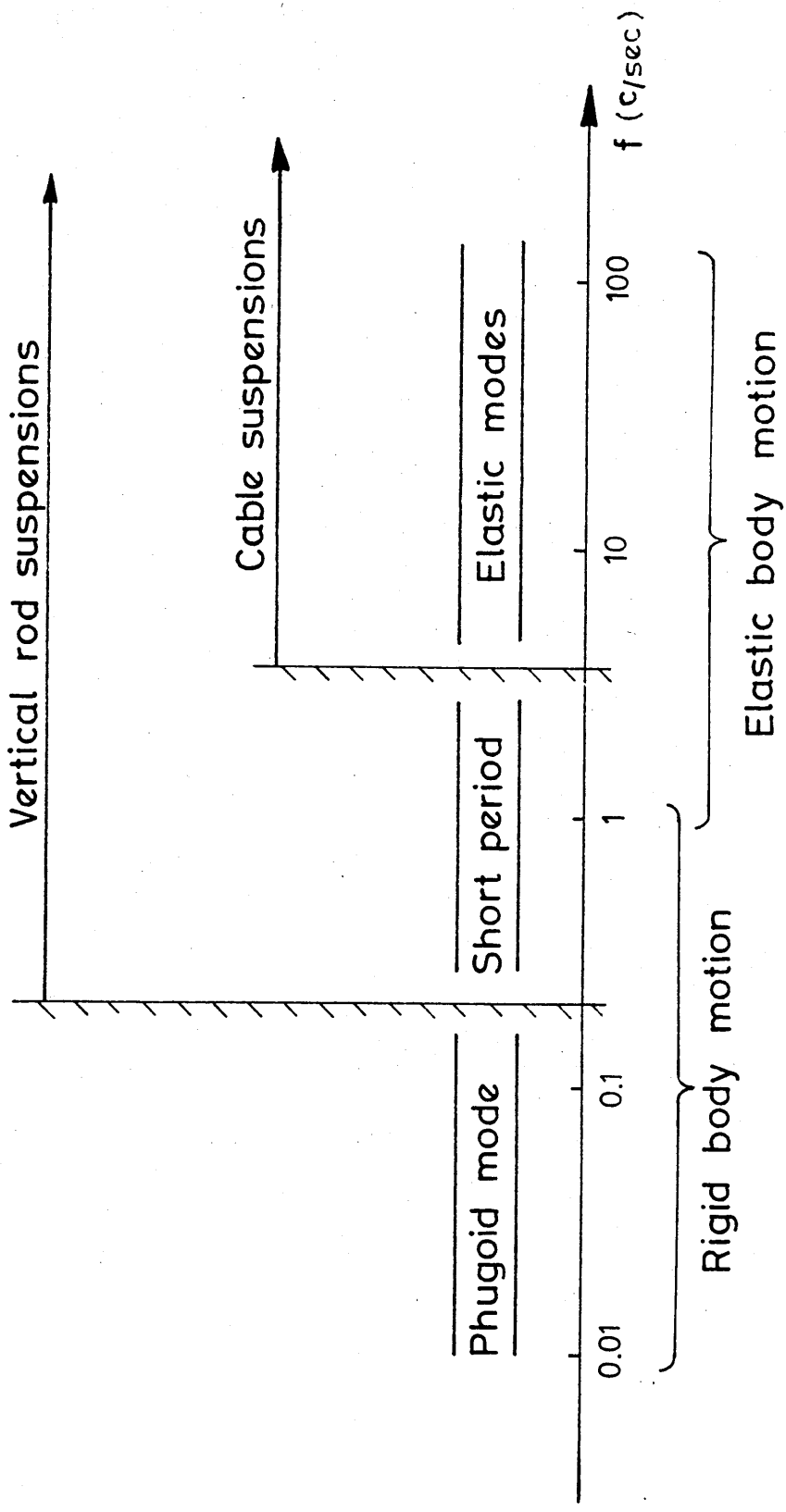


Figure 16. Frequency regimes of vertical rod and cable suspensions.

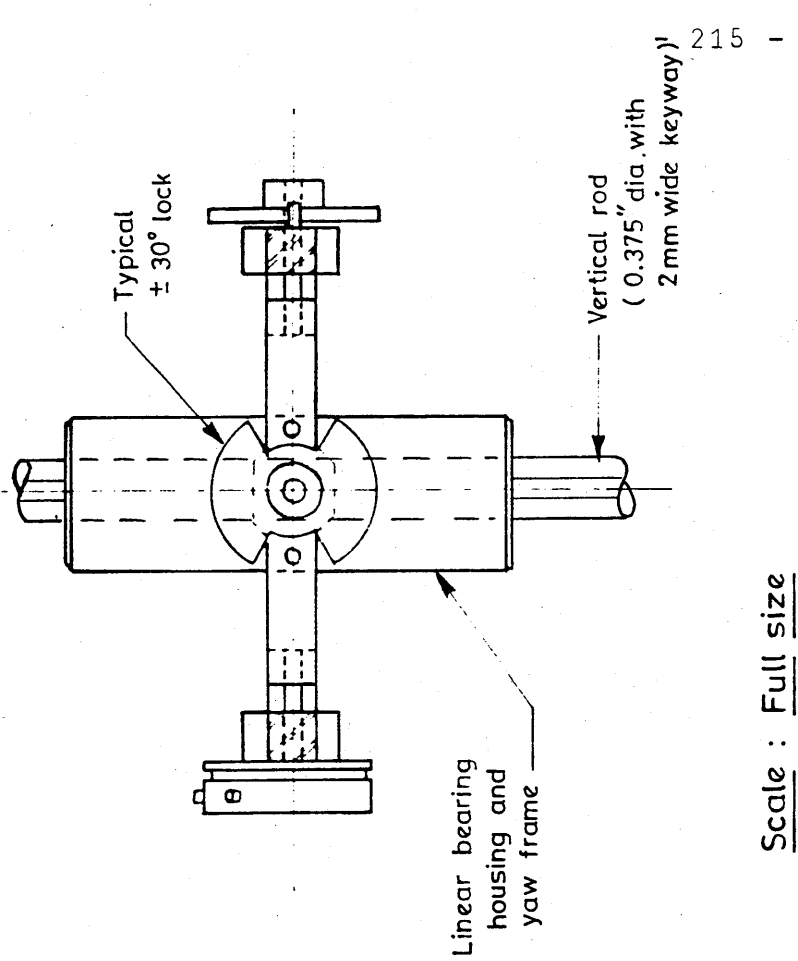
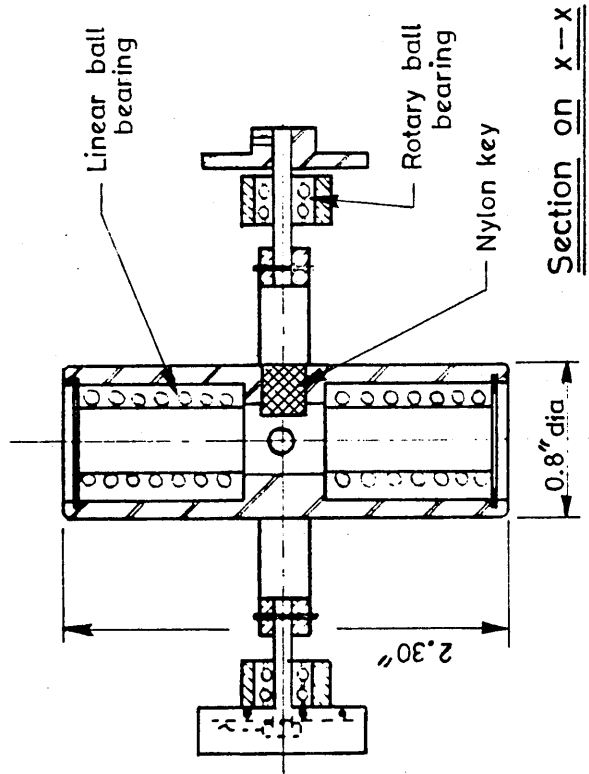
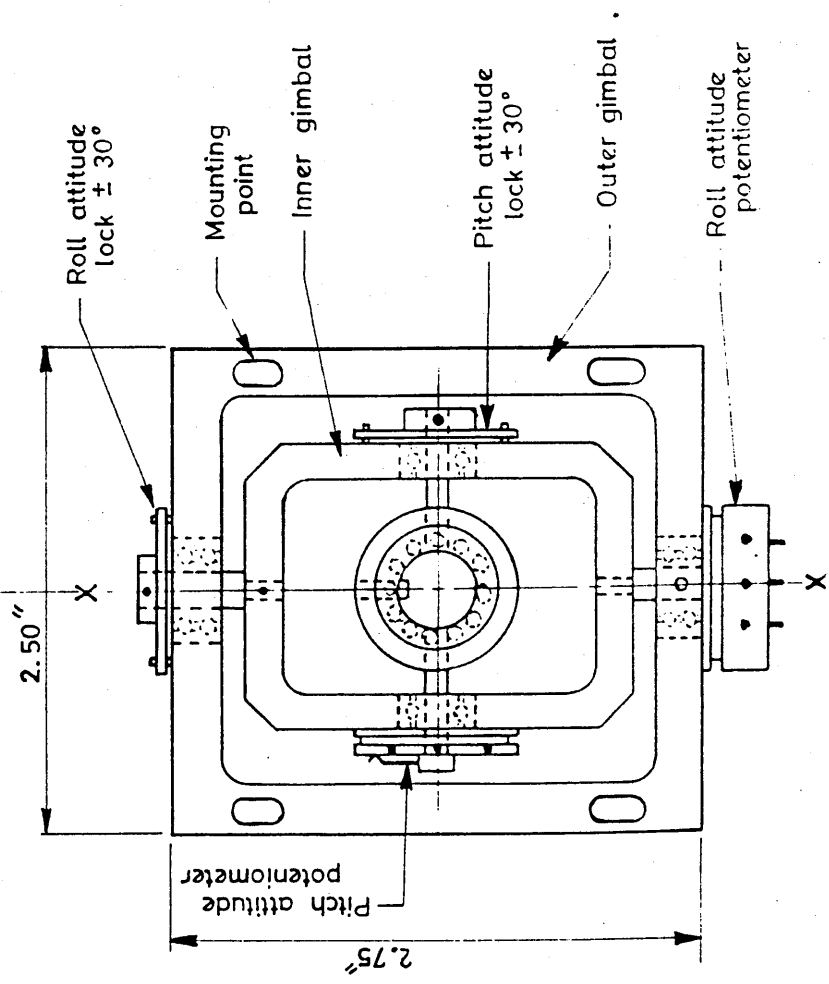


Figure 17. Gimbal system for model suspension.



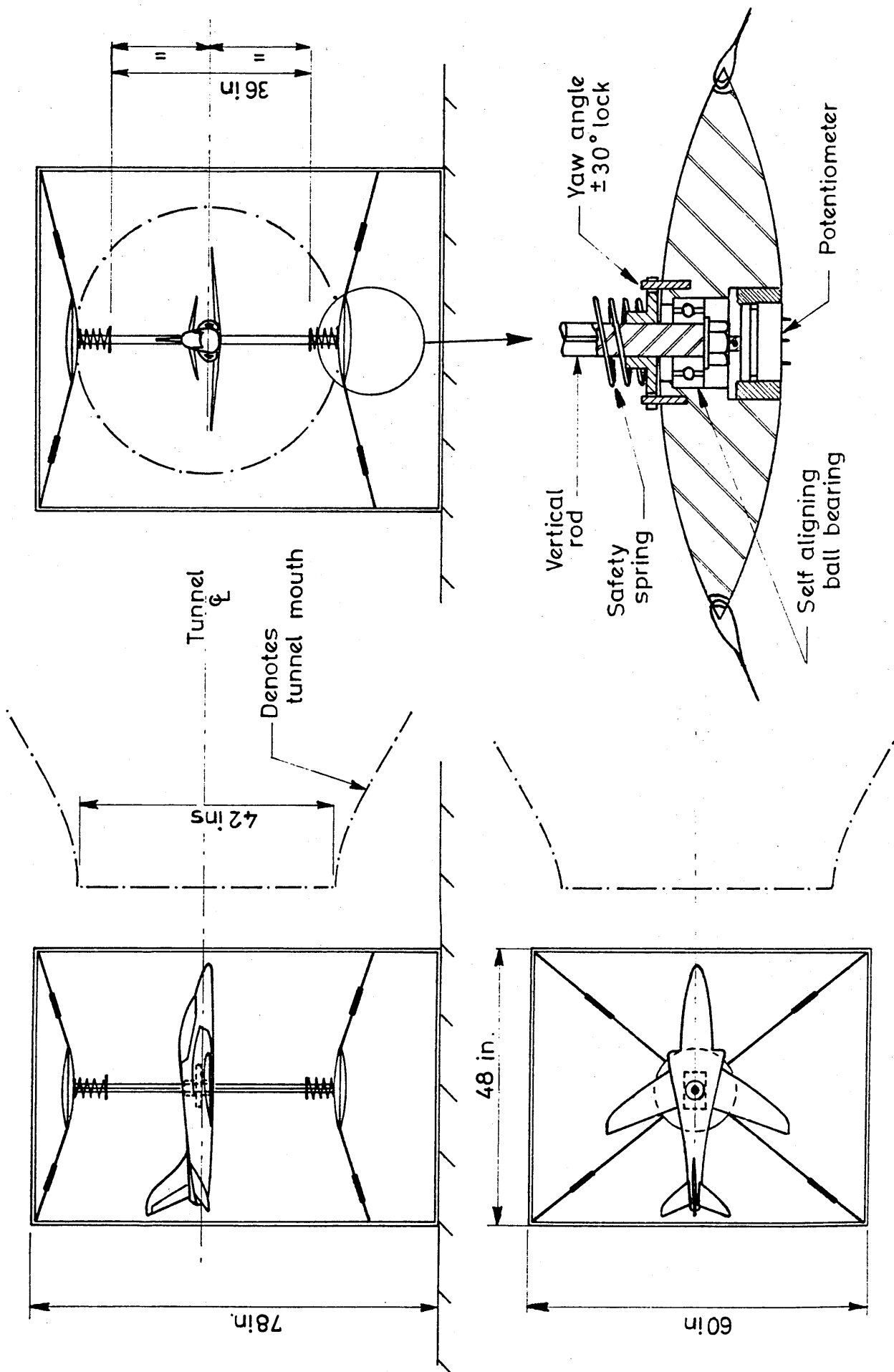
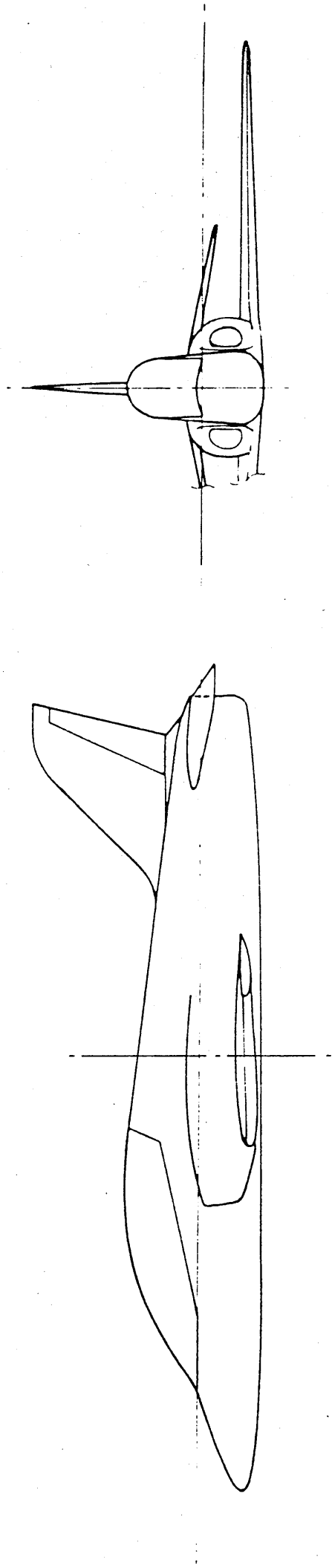


Figure 18. The model suspension system.



Hawk model data (1/12 scale)

Overall length	35.9 in
<u>Wing :-</u>	
Span	30.8 in
Area	179.6 in ²
Aspect ratio	5.2
Sweep (C/4)	21.5°
Taper ratio	0.34
Dihedral	2°
<u>Vertical tail :-</u>	
Area	27.0 in ²
Sweep (C/4)	39°
Aspect ratio	2.5
<u>Horizontal tail :-</u>	
Area	45 in ²
Aspect ratio	4.3
Sweep (C/4)	29.5°
Taper ratio	0.35
Dihedral	-10°

Figure 19. 1/12 scale Hawk aircraft model.

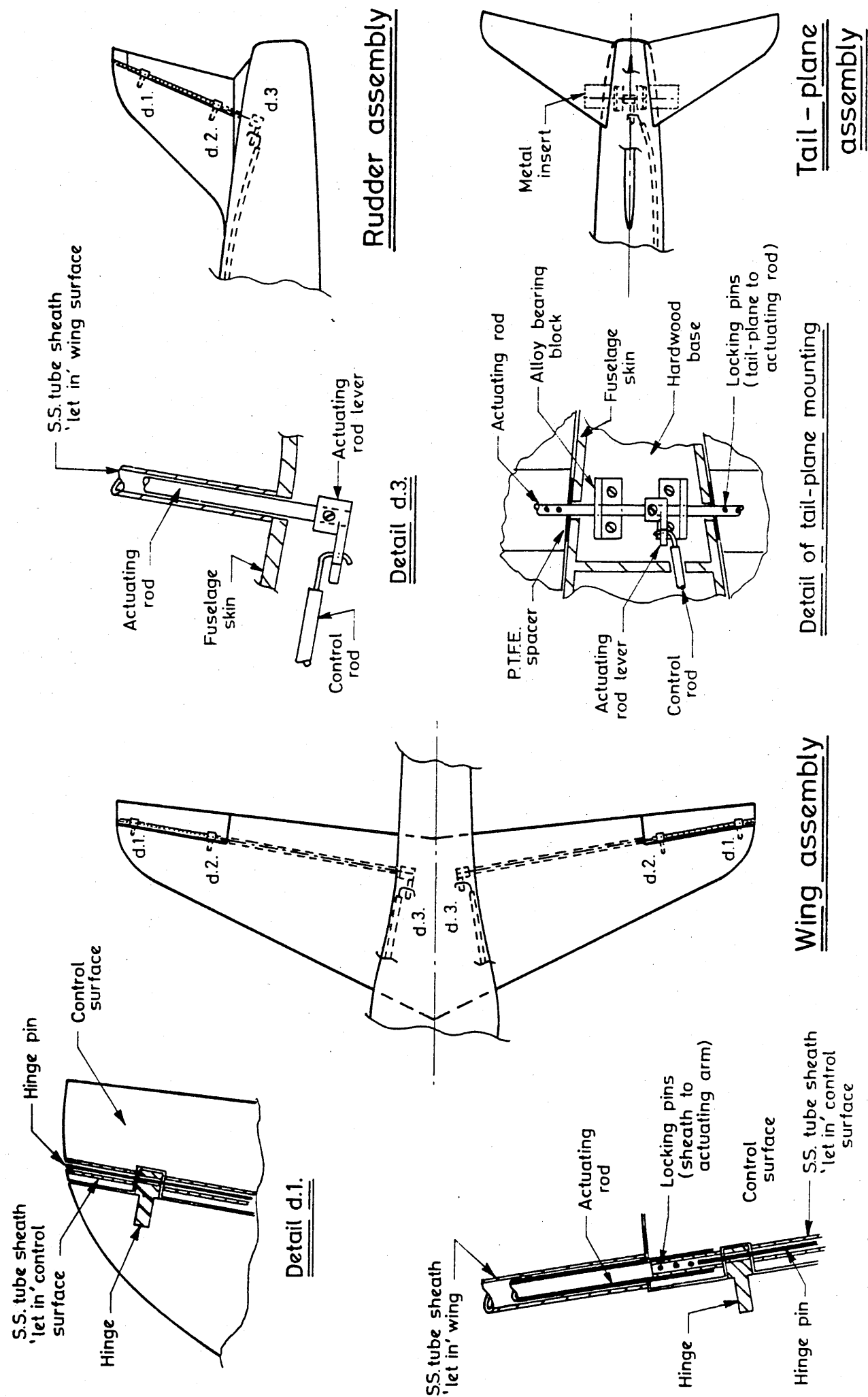


Figure 20. Details of control surfaces actuation mechanisms.

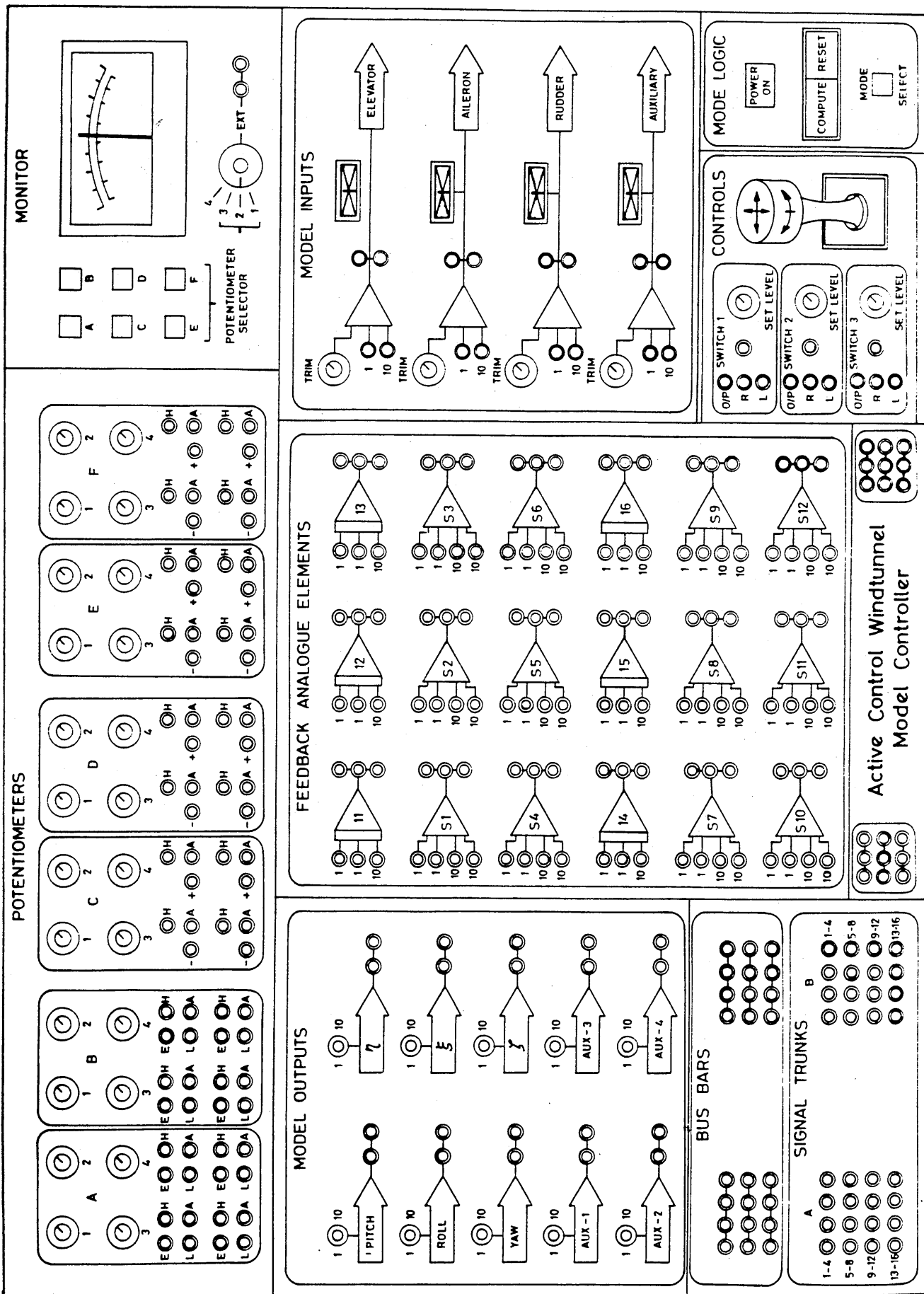


Figure 21. Designed panel layout of the electronic control unit.

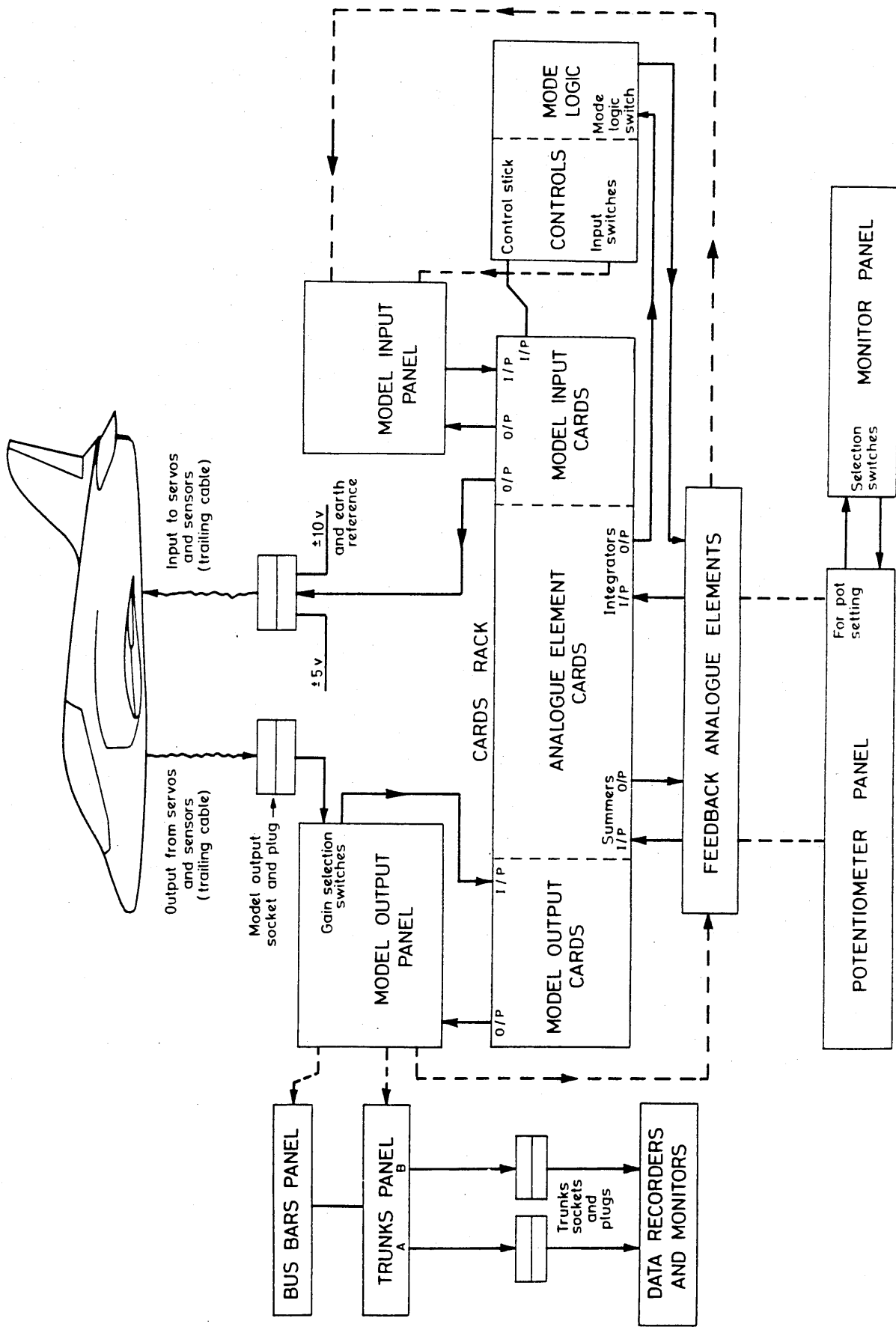
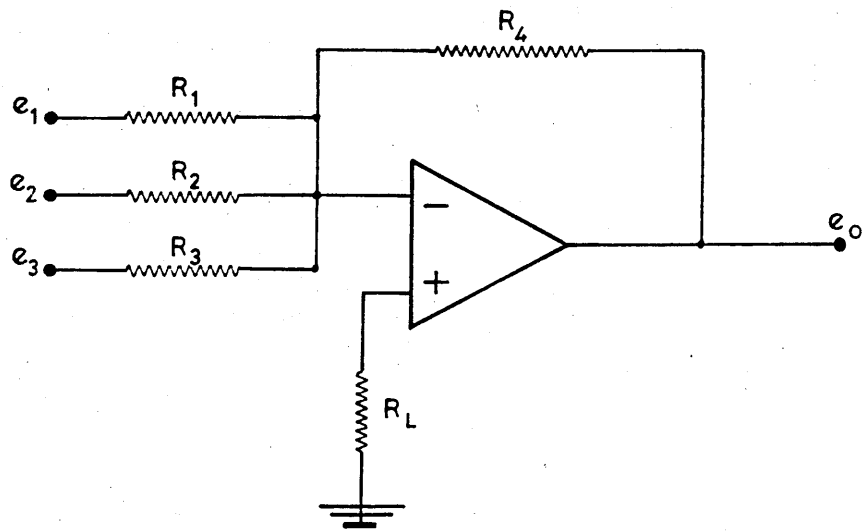
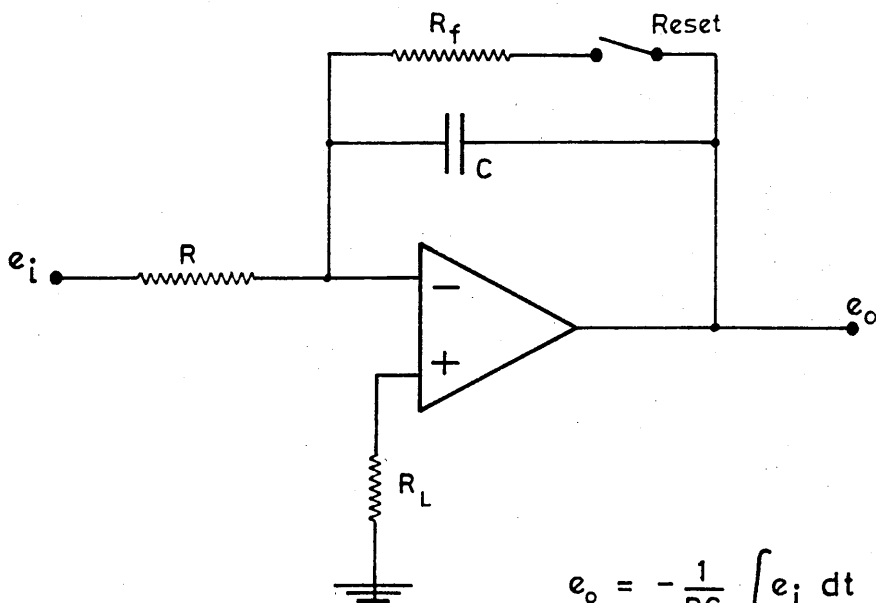


Figure 22. Systematic block diagram of the electronic control unit and model interface.



$$e_o = - \left[e_1 \frac{R_4}{R_1} + e_2 \frac{R_4}{R_2} + e_3 \frac{R_4}{R_3} \right]$$

Figure 23.a. A typical Summer circuit.



$$e_o = - \frac{1}{RC} \int e_i \, dt$$

$$\text{Circuit gain} = - \frac{1}{RC}$$

Figure 23.b. A typical low drift Integrator.

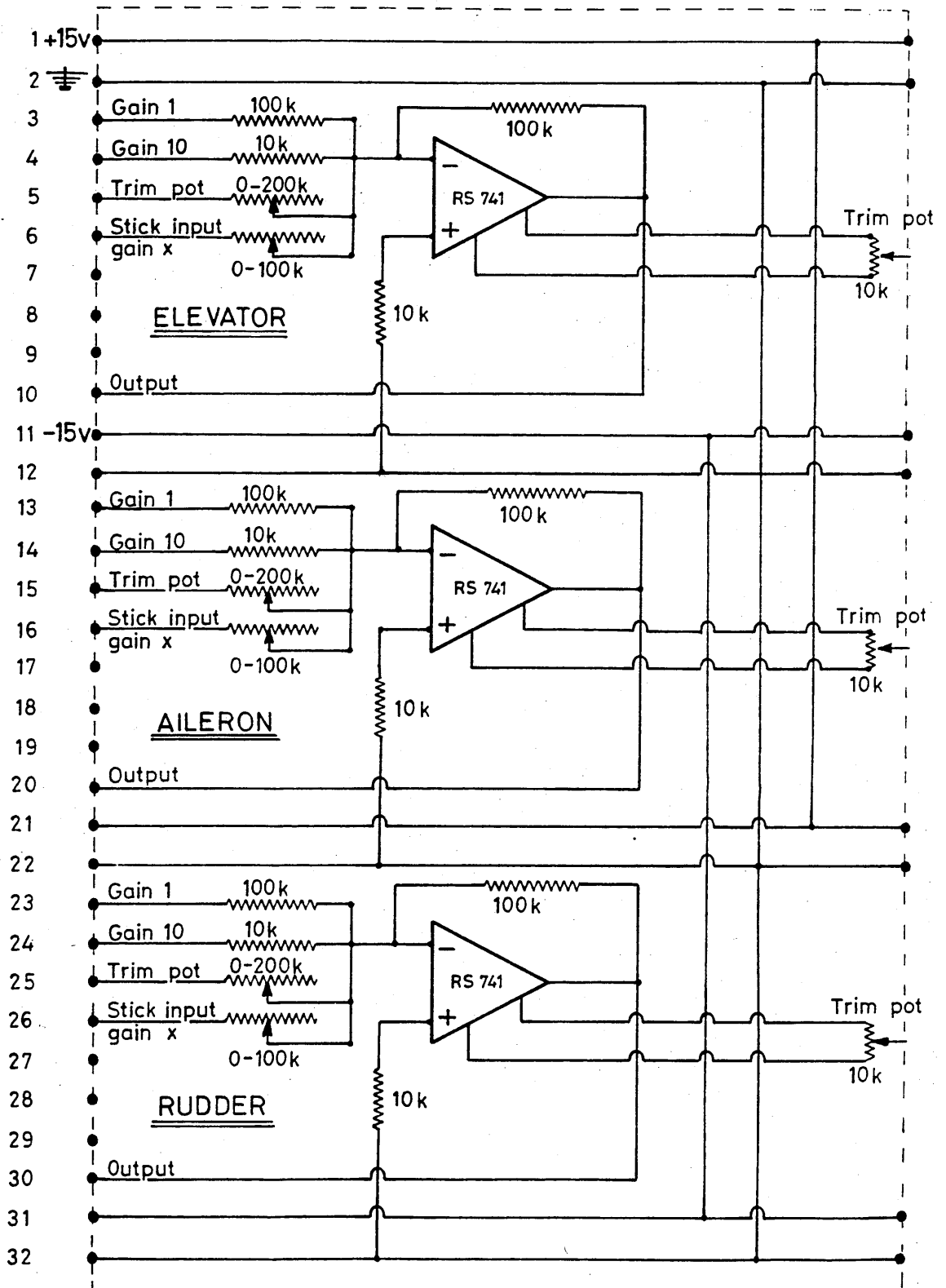


Figure 24. Input control unit card.

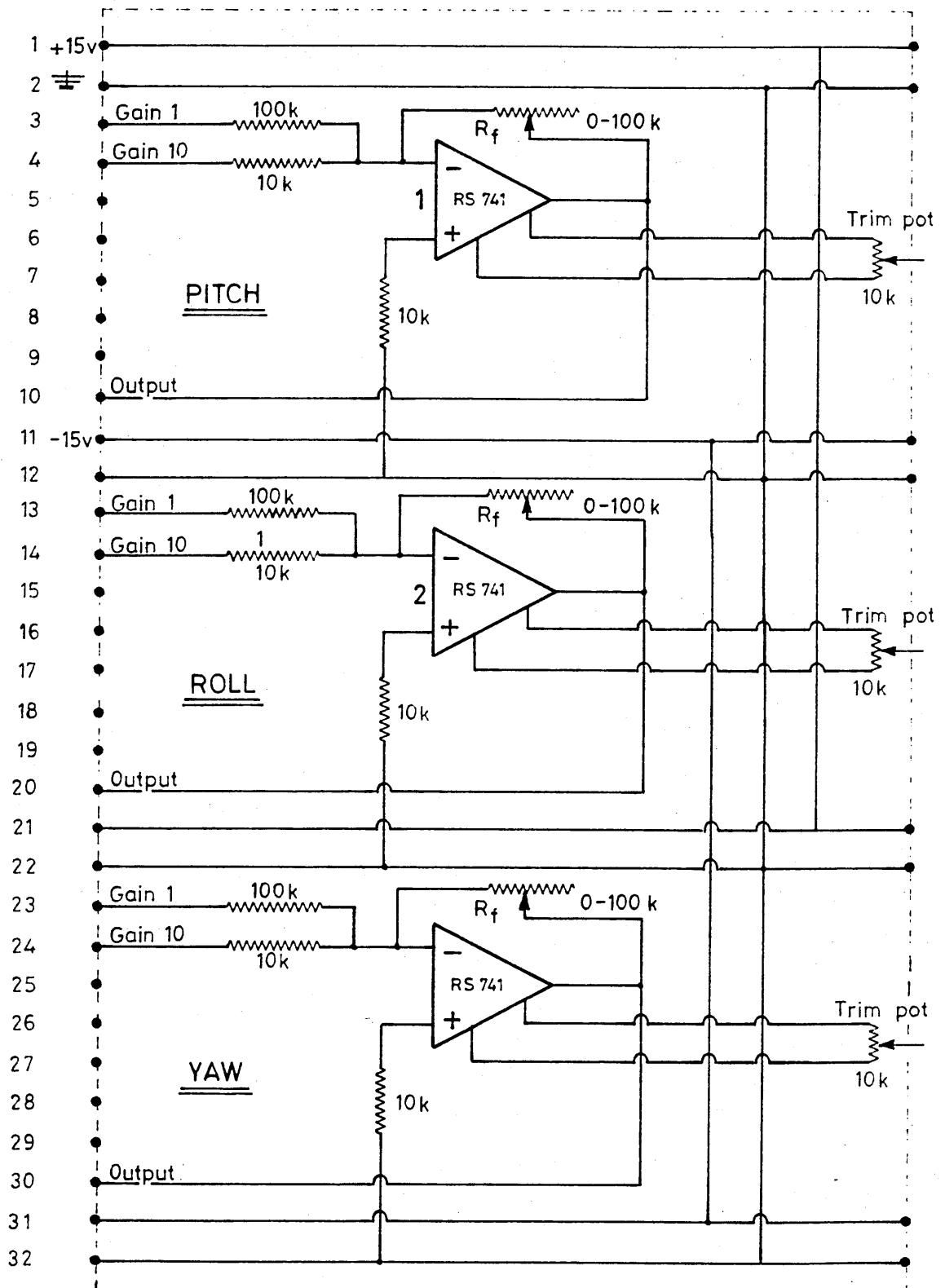


Figure 25. Model output card.

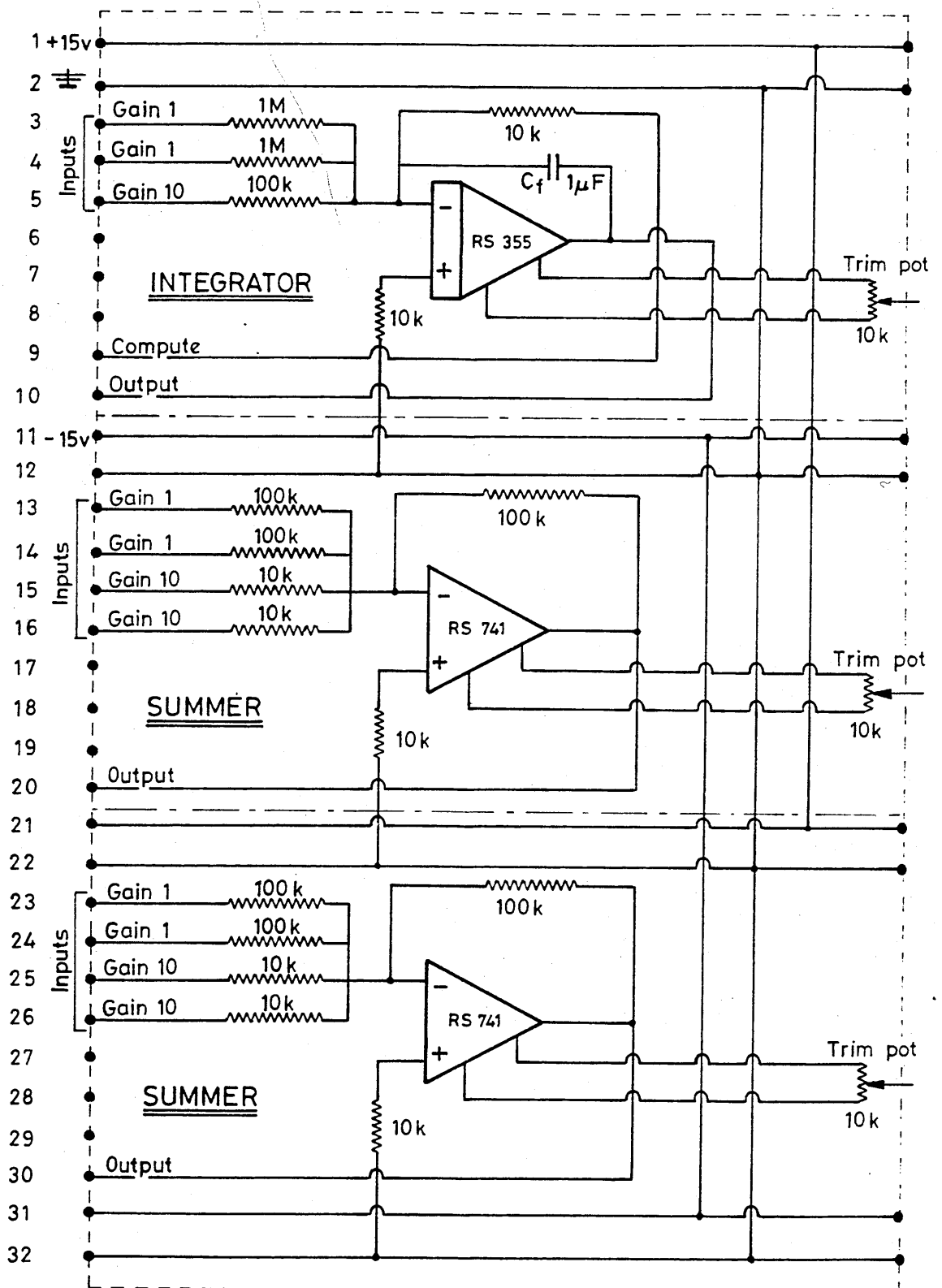
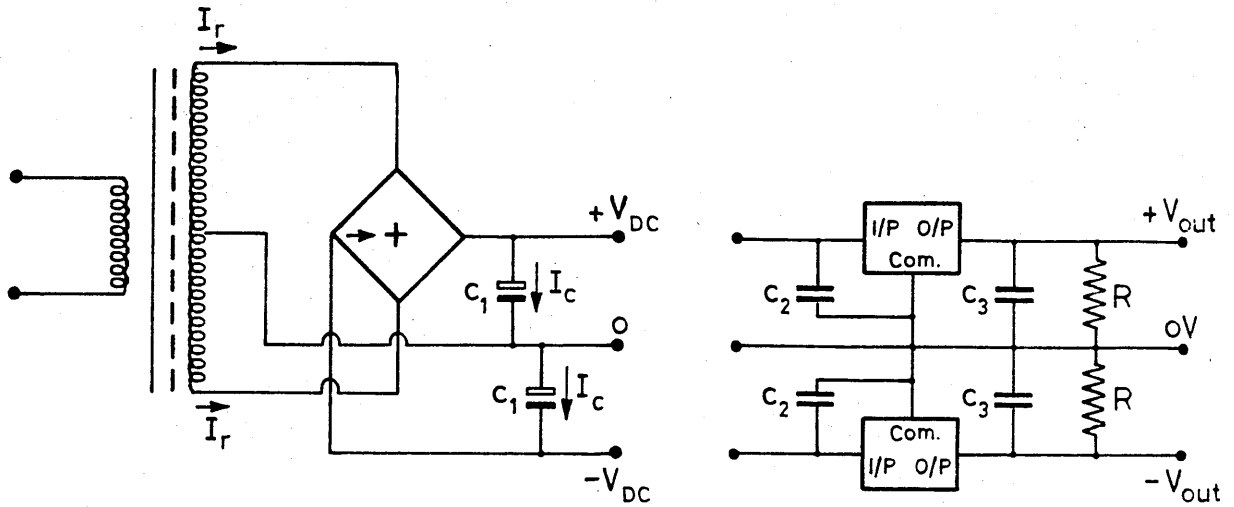


Figure 26. Analogue element card.

1. Circuit



2. Components

- a) Transformer
9 volts, 1.5 Amps, Two Secondaries
- b) Bridge Rectifier
4A, 200 Volts
- c) Voltage Regulators
 - (i) +5 volts, 1A
 - (ii) -5 volts, 1A
- d) Capacitors
 - (i) $C_1 = 10,000\mu\text{F}$, 40V, single ended can electrolytic
 - (ii) $C_2 = 0.1\mu\text{F}$, 35V, solid tantalum ($\pm 20\%$)
 - (iii) $C_3 = 0.47\mu\text{F}$, 35V, solid tantalum ($\pm 20\%$)
- e) Resistors
4.7K Ω , carbon composition ($\pm 10\%$)

Figure 27. Design of ± 5 volts power supply.

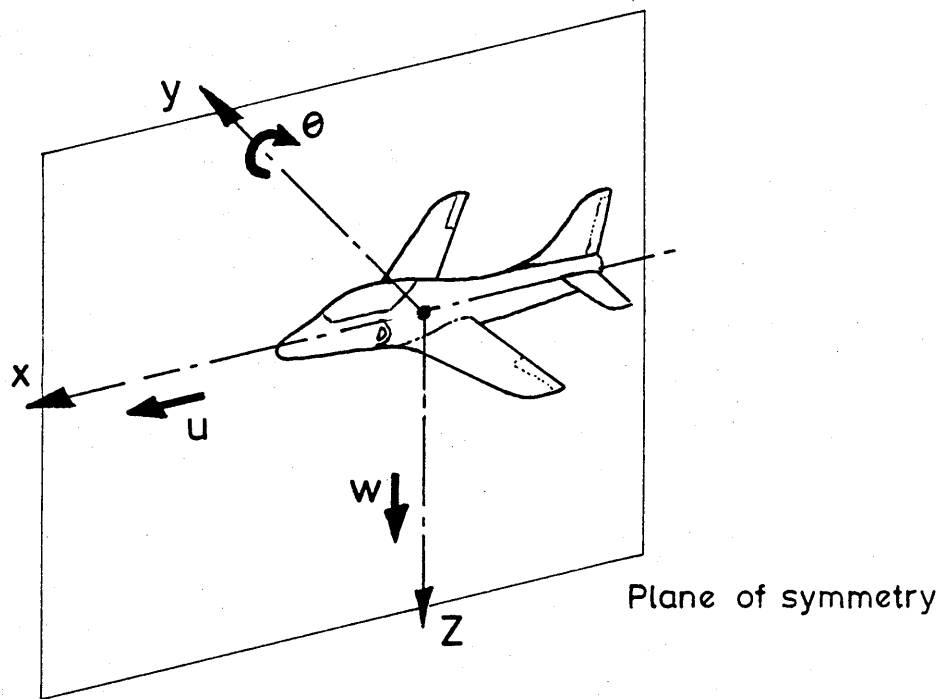


Figure 28.a. Airplane axis system and plane of symmetry.

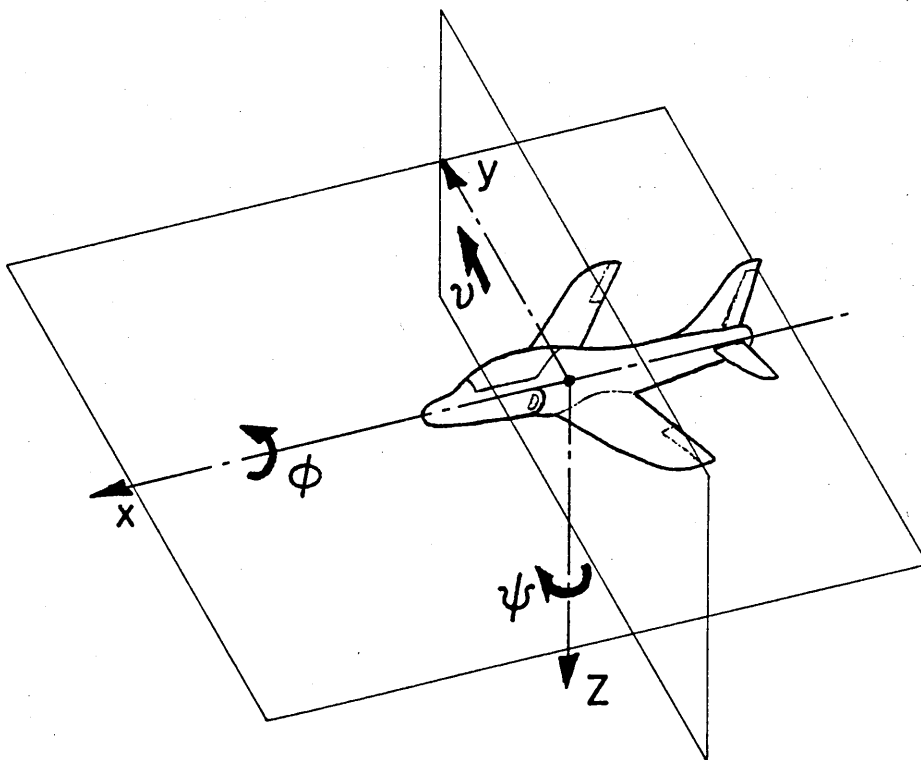


Figure 28.b. Airplane axis system and planes of asymmetry .

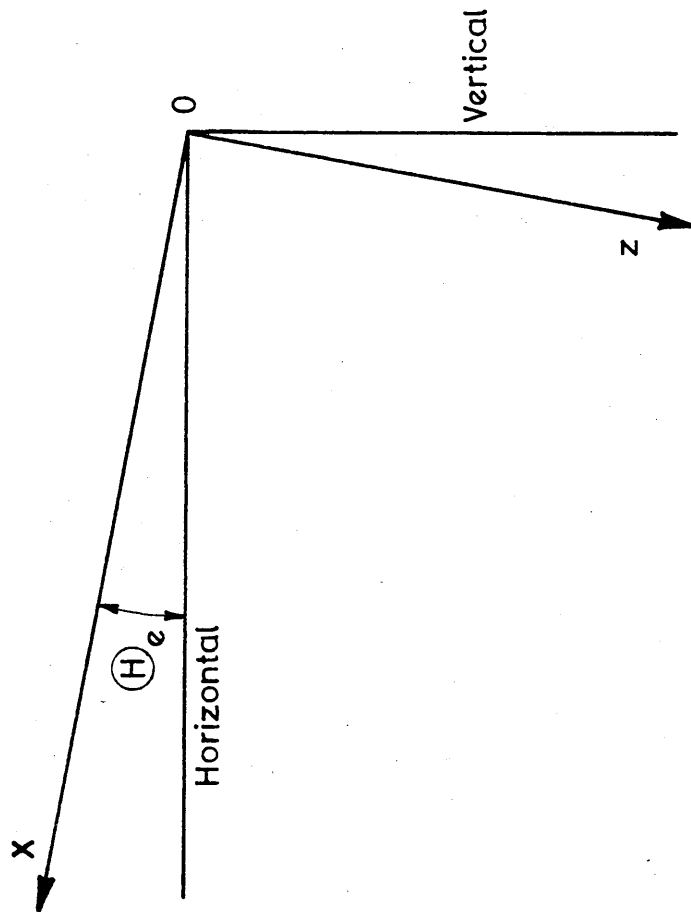


Figure 29. Position of the axis in the vertical plane in steady flight.

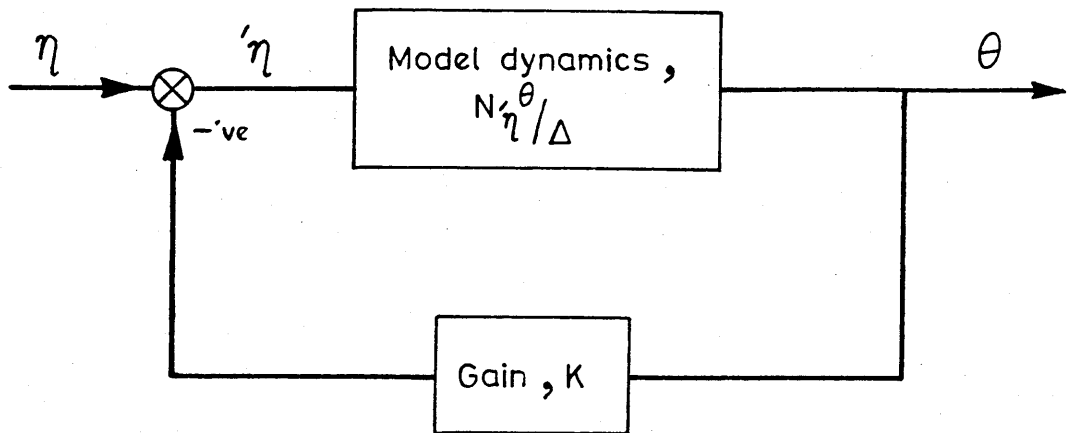


Figure 30.a. Longitudinal stability augmentation of model with pitch attitude feedback to elevator.

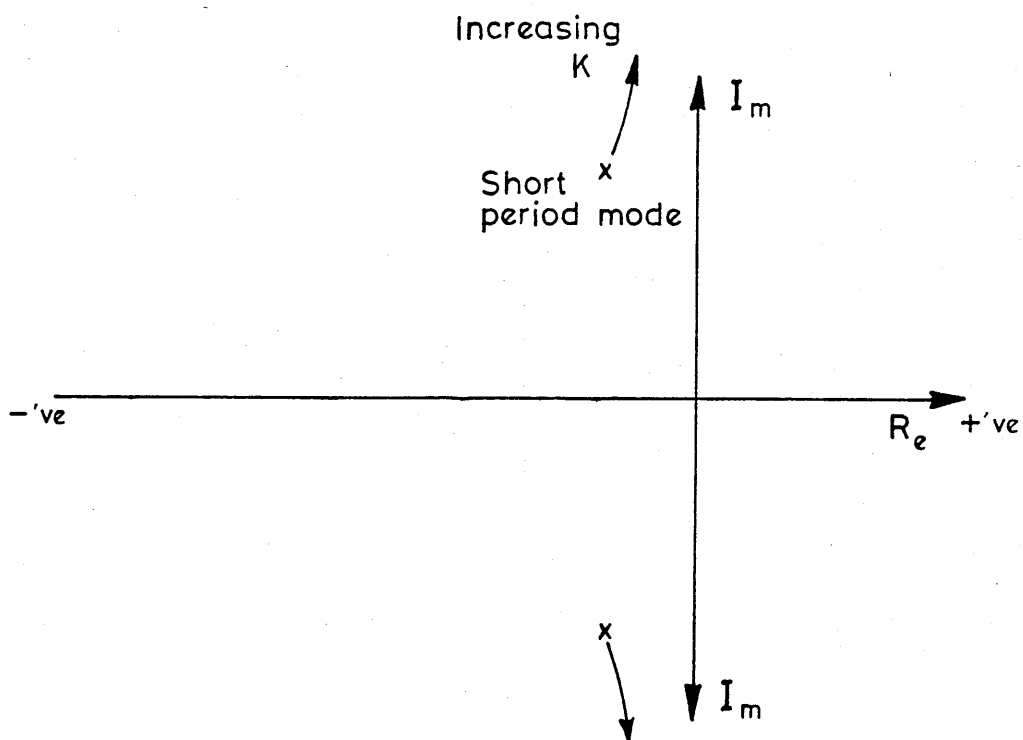


Figure 30.b. Root locus plot. Effect of pitch attitude feedback to elevator on longitudinal modes of model.

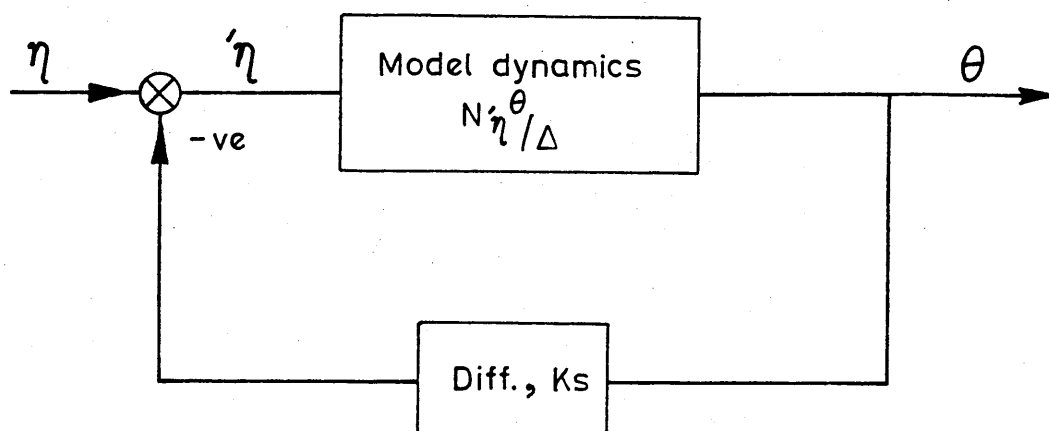


Figure 31.a. Longitudinal stability augmentation of model with pitch rate feedback to elevator.

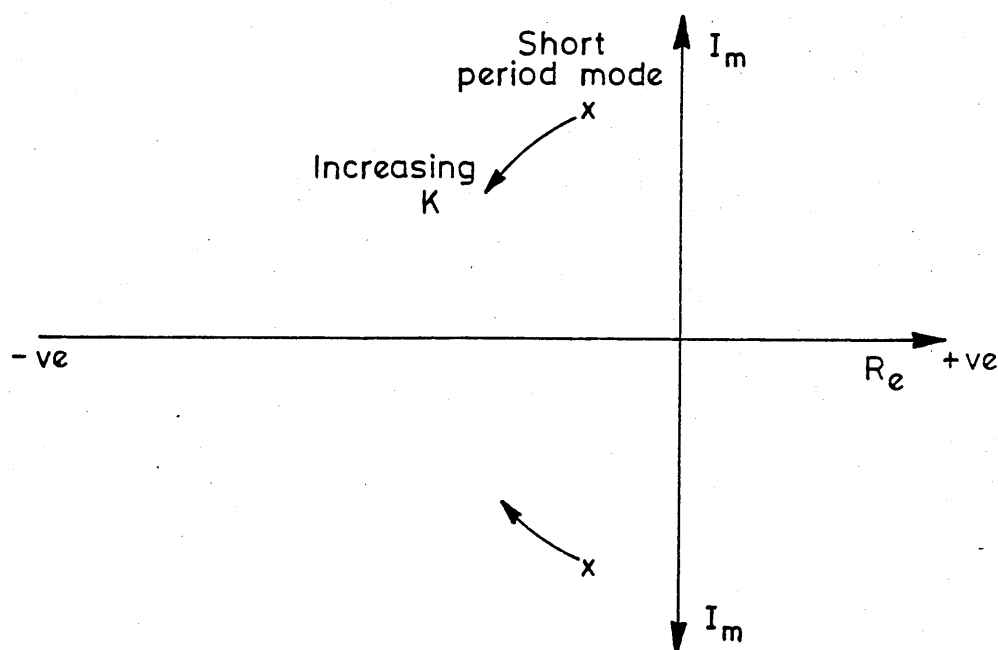


Figure 31.b. Root locus plot. Effect of pitch rate feedback to elevator on longitudinal modes of model.

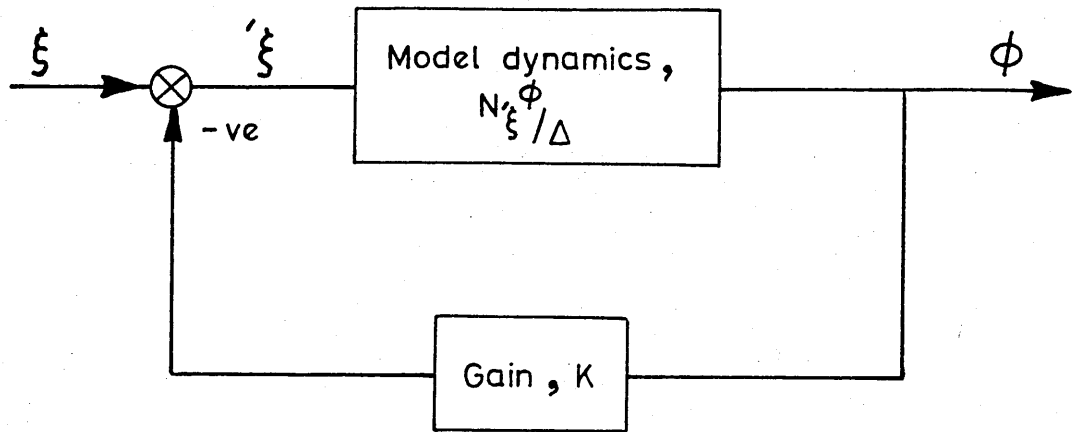


Figure 32.a. Lateral stability augmentation of model with roll attitude feedback to aileron.

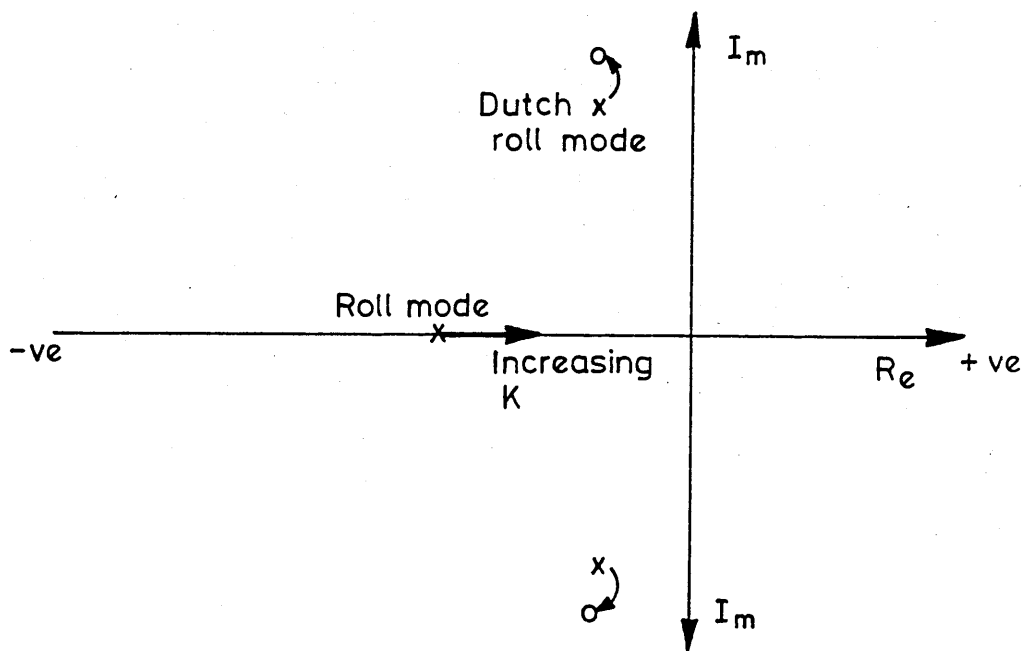


Figure 32.b. Root locus plot. Effect of roll attitude feedback to ailerons on lateral modes of model.

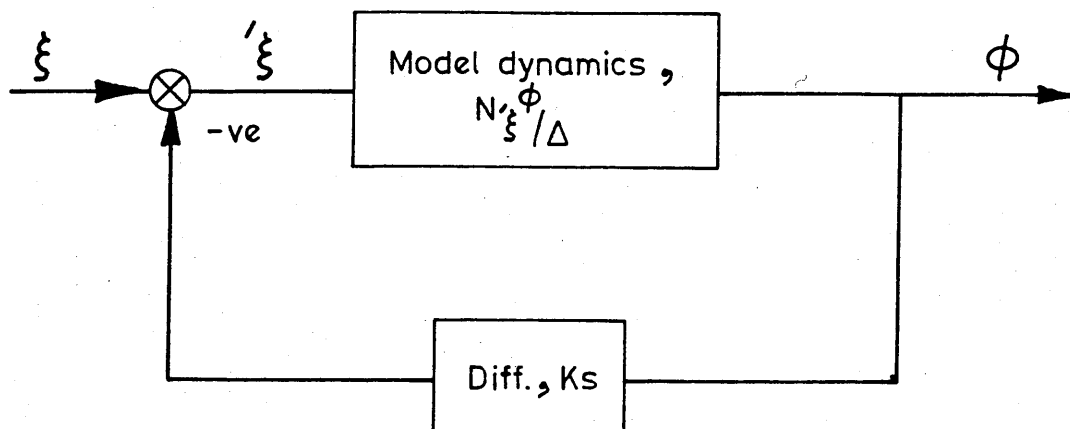


Figure 33.a. Lateral stability augmentation of model with roll rate feedback to ailerons.

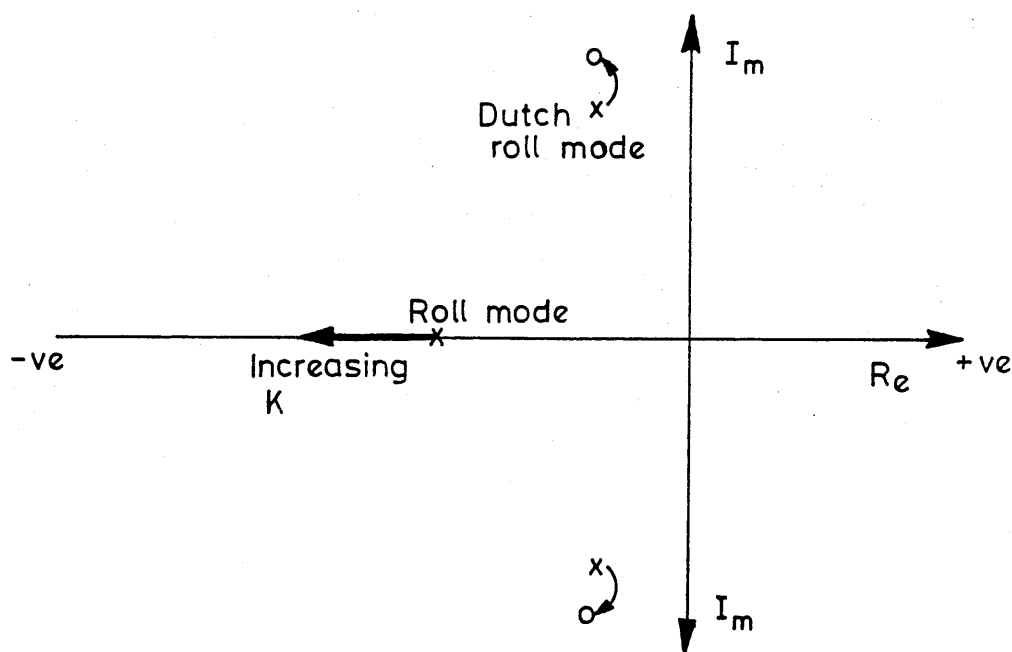


Figure 33.b. Root locus plot. Effect of roll rate feedback to ailerons on lateral modes of model.

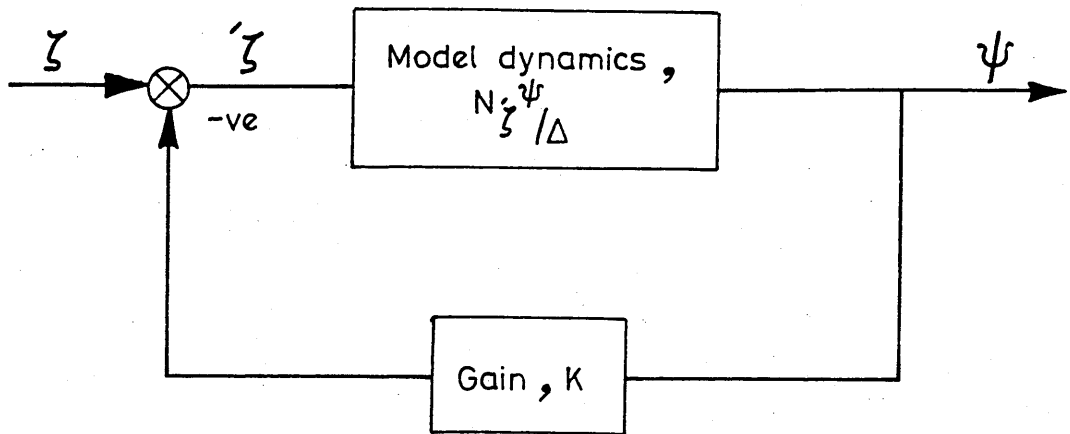


Figure 34.a. Lateral stability augmentation of model with yaw attitude feedback to rudder.

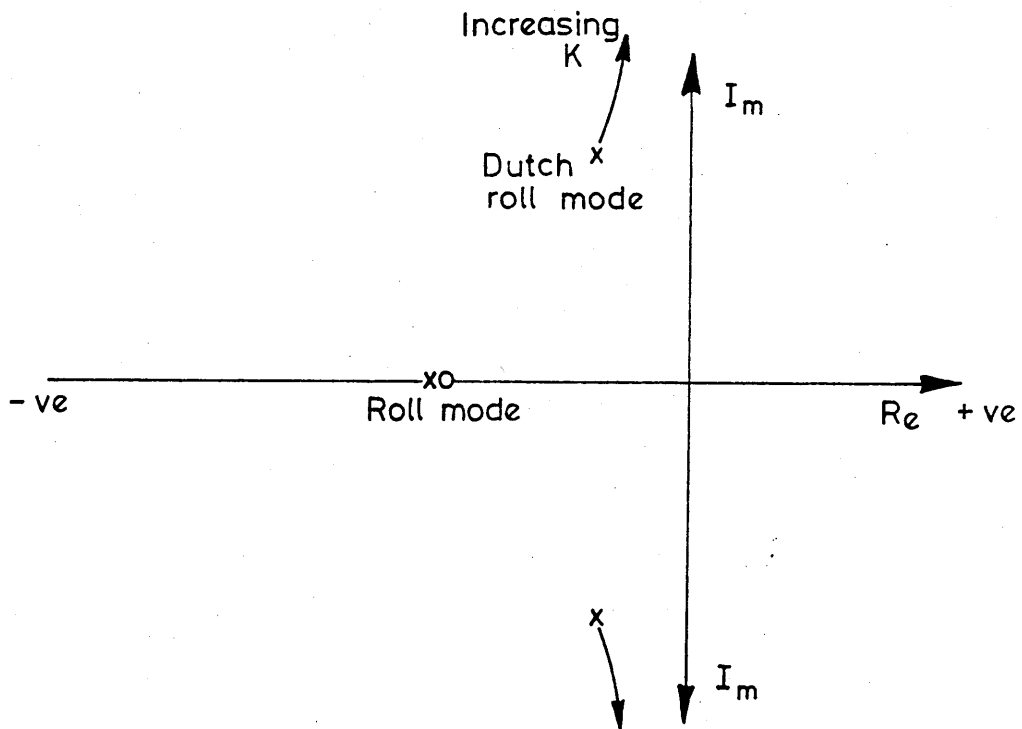


Figure 34.b. Root locus plot. Effect of yaw attitude feedback to rudder on lateral modes of model.

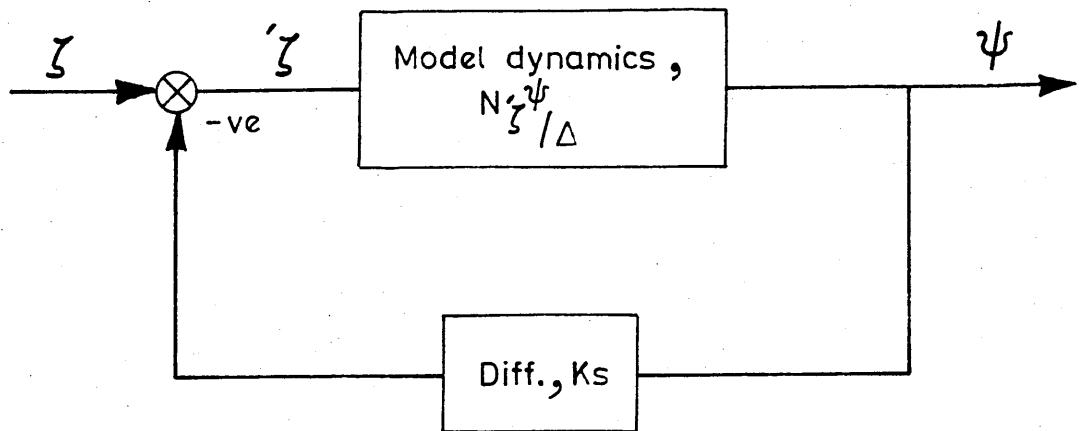


Figure 35.a. Lateral stability augmentation of model with yaw rate feedback to rudder.

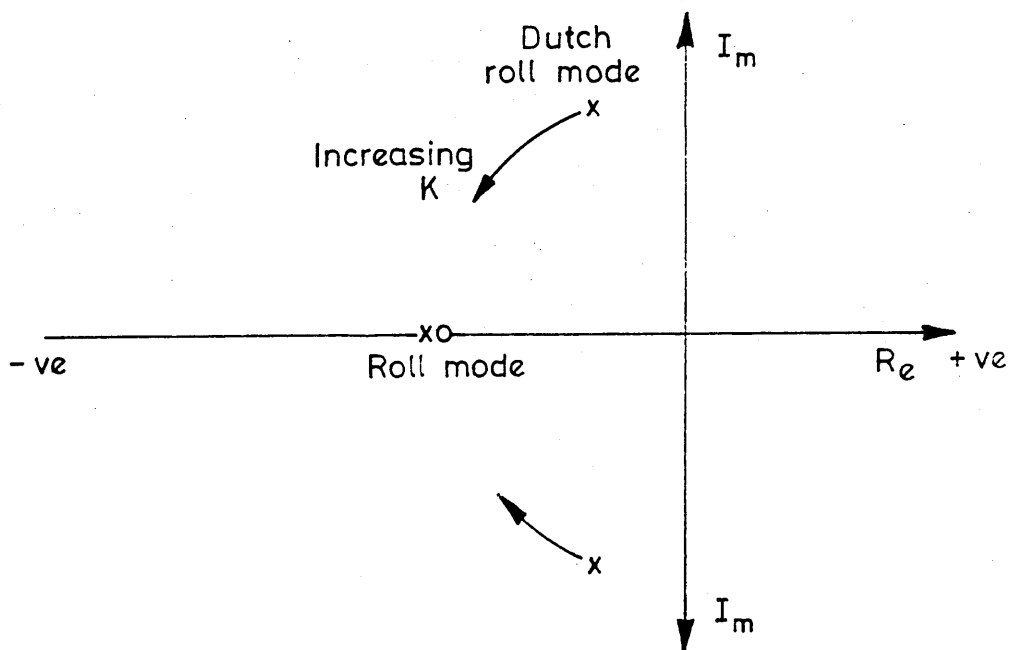


Figure 35.b. Root locus plot. Effect of yaw rate feedback to rudder on lateral modes of model.

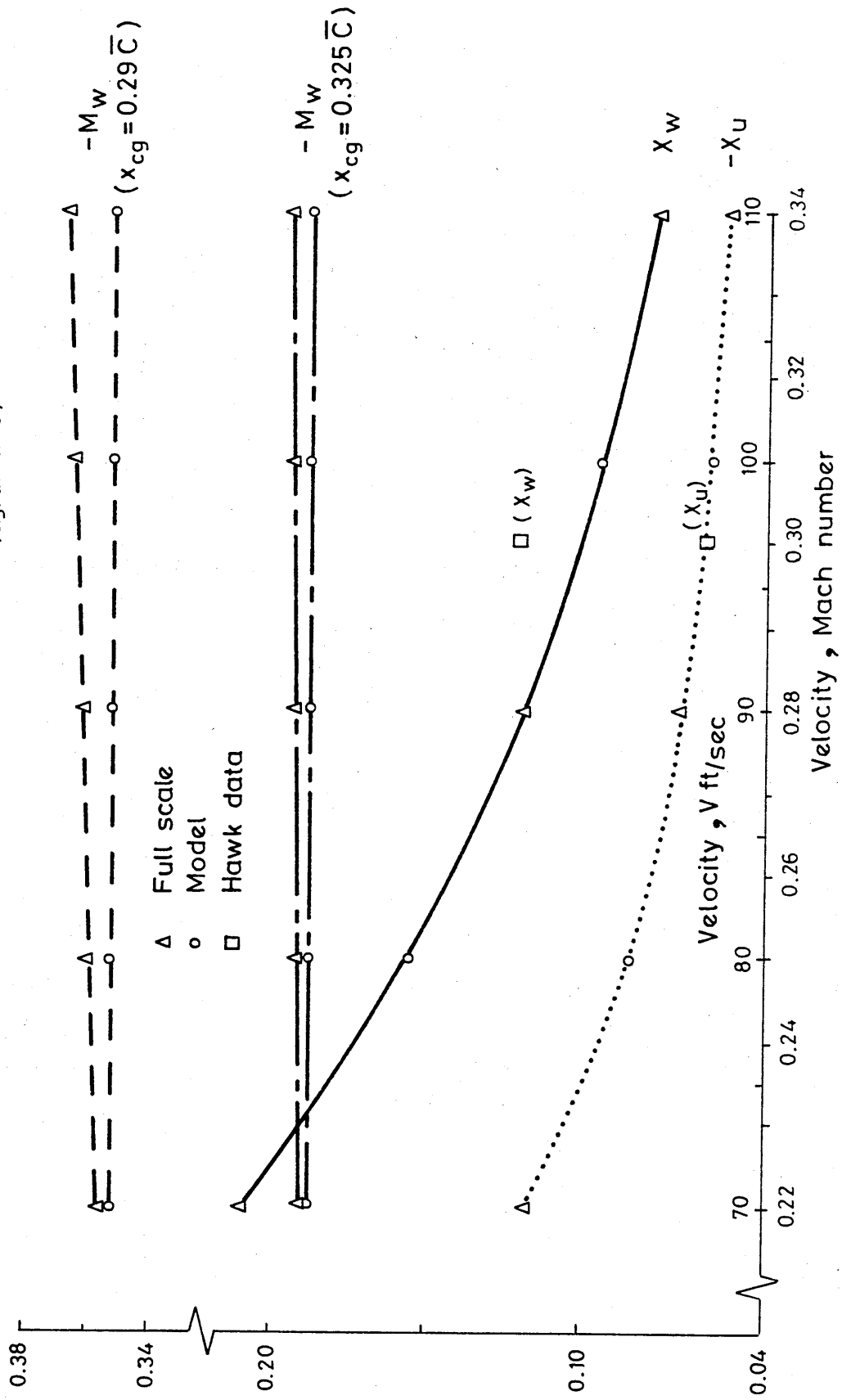


Figure 36. Estimated longitudinal stability derivatives.

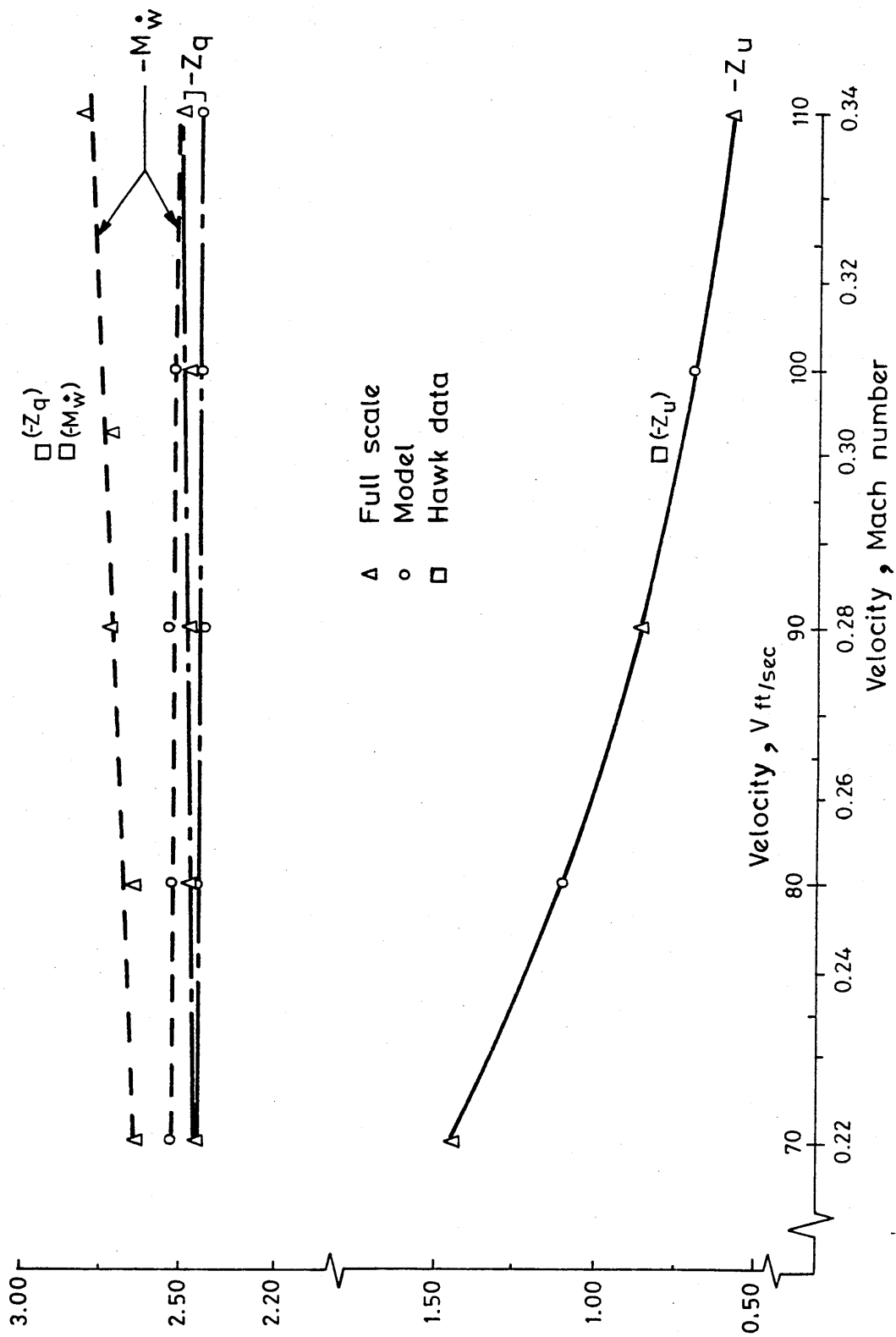


Figure 37. Estimated longitudinal stability derivatives.

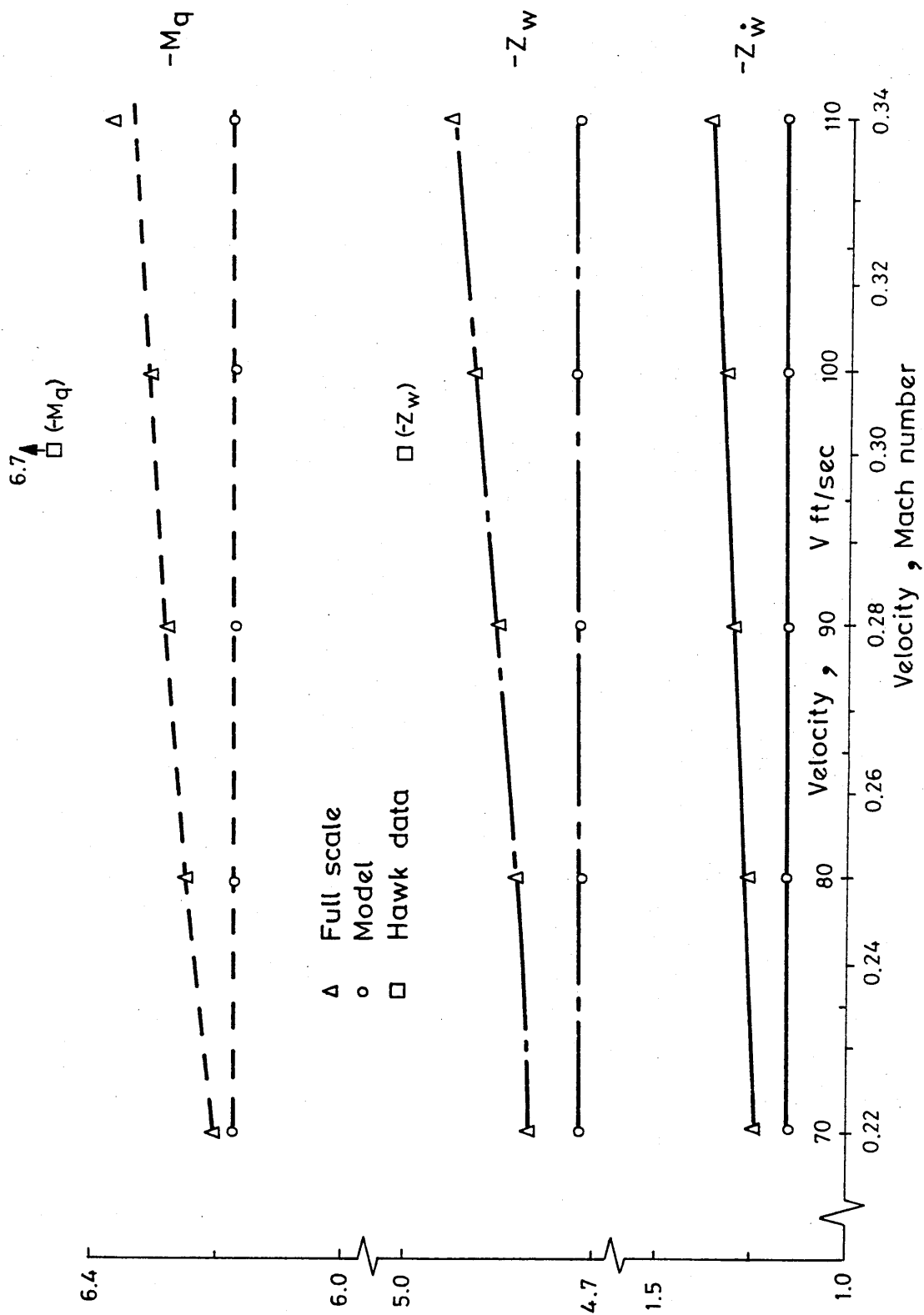


Figure 38. Estimated longitudinal stability derivatives.

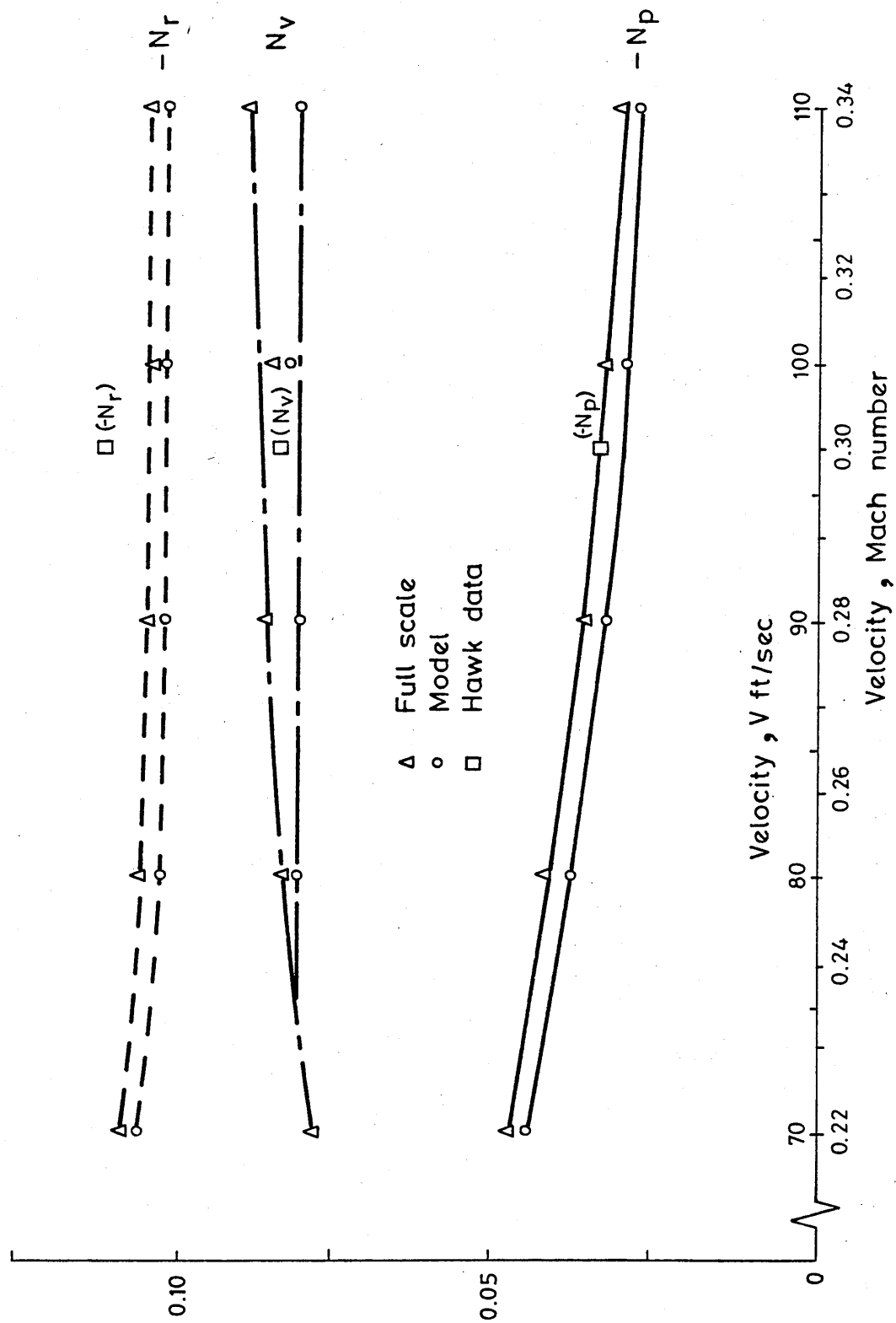


Figure 39. Estimated lateral stability derivatives.

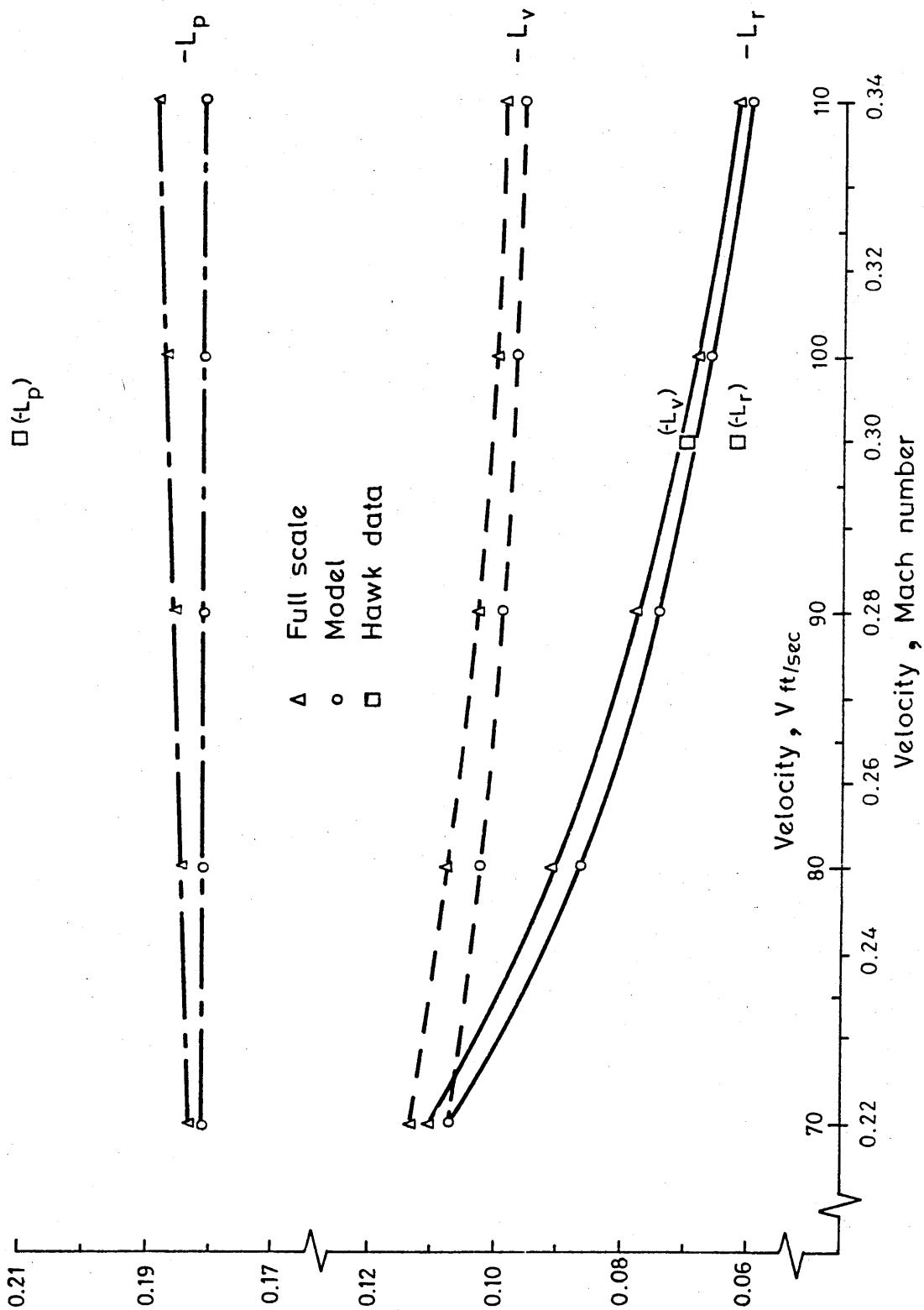


Figure 40. Estimated lateral stability derivatives.

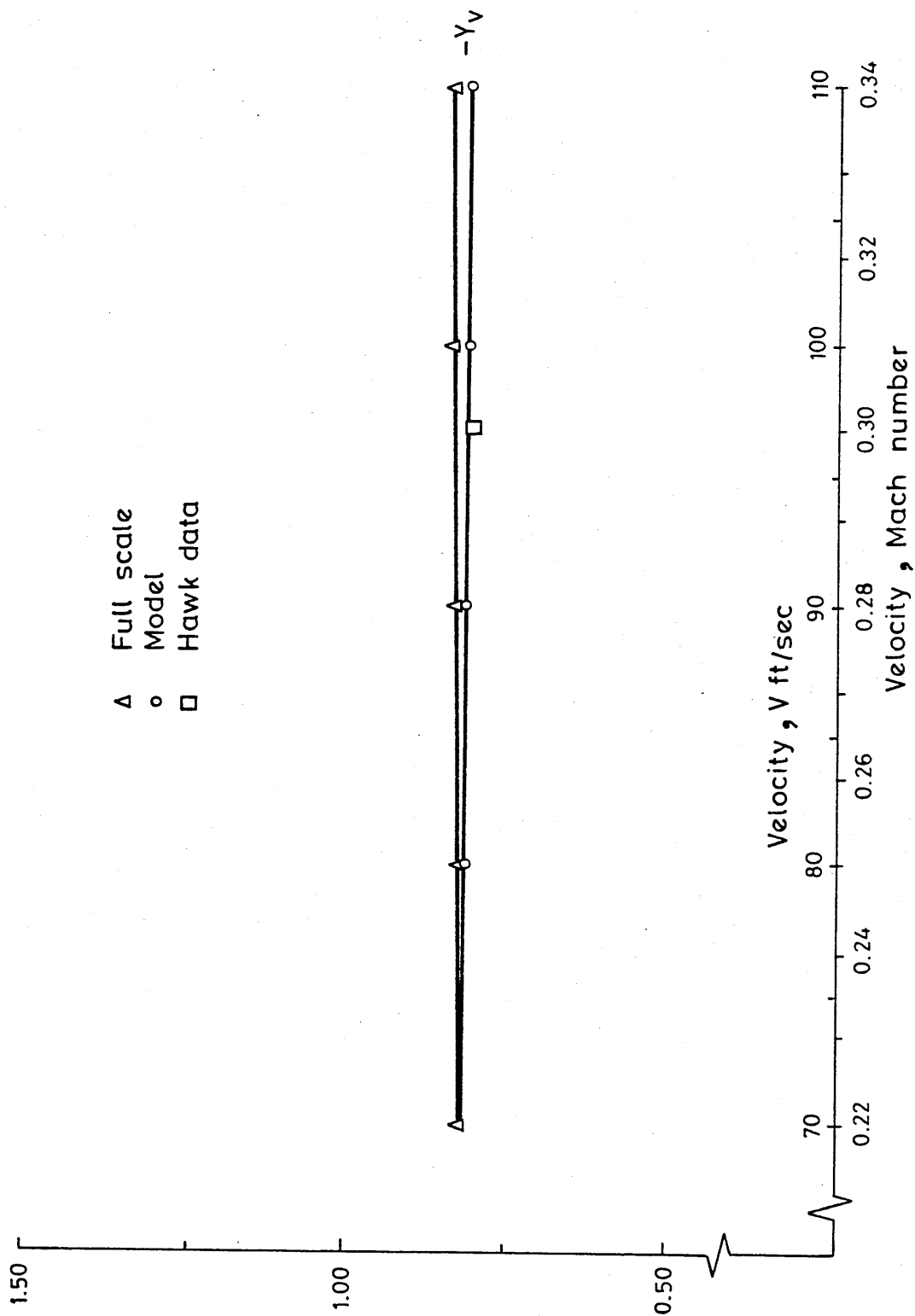


Figure 41. Estimated lateral stability derivative, Y_v .

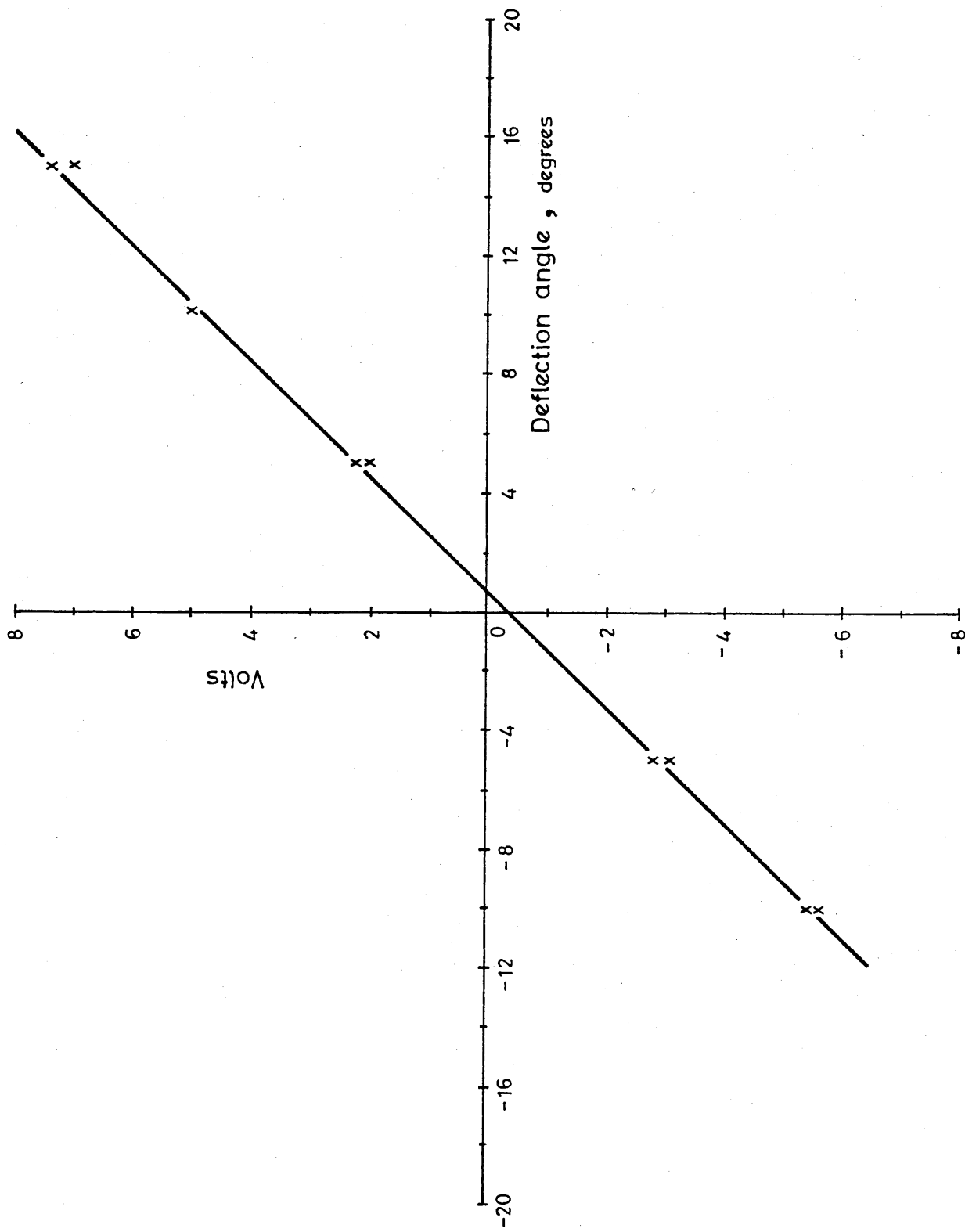


Figure 42. Elevator deflection vs input voltage .

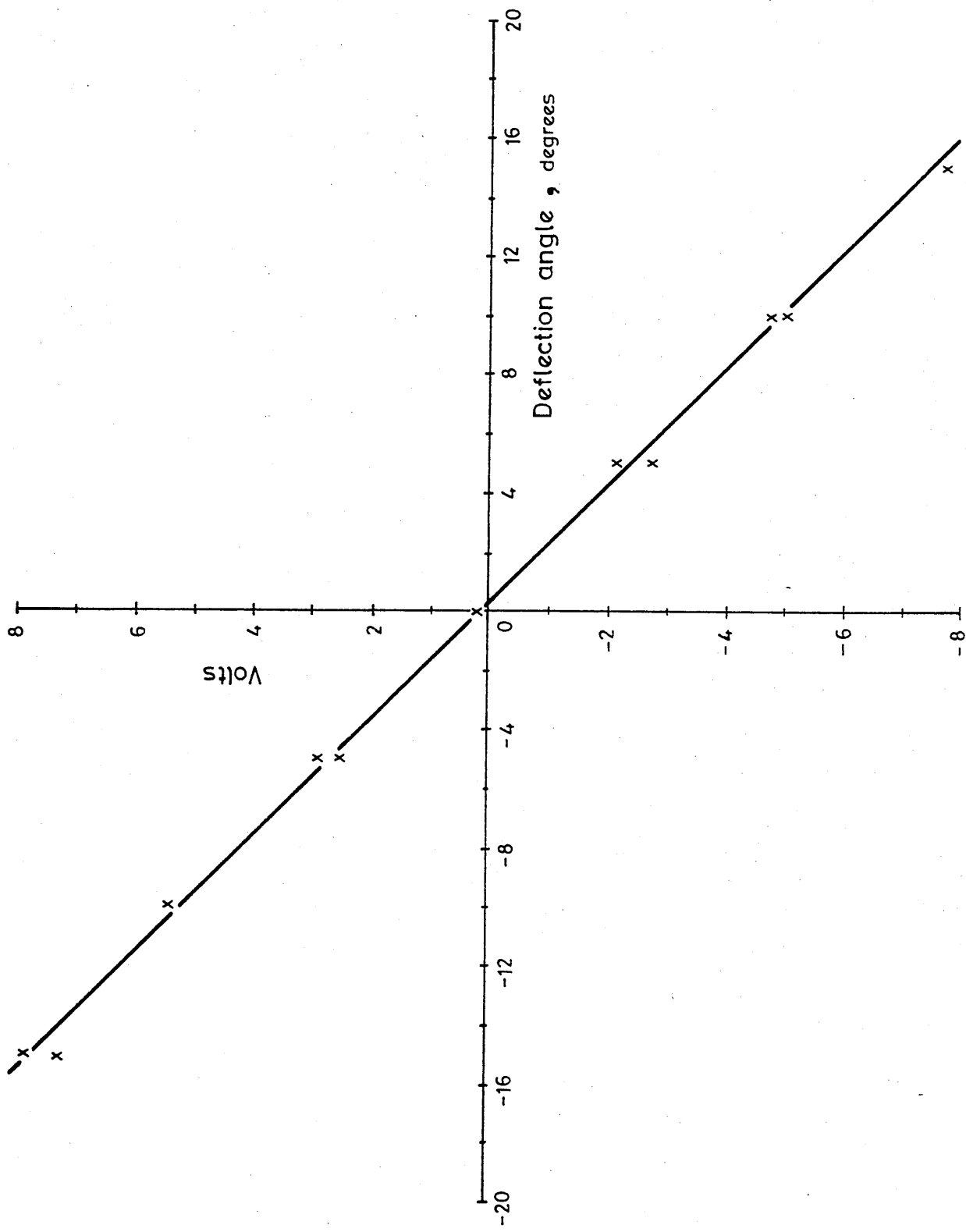


Figure 43. Aileron deflection vs input voltage.

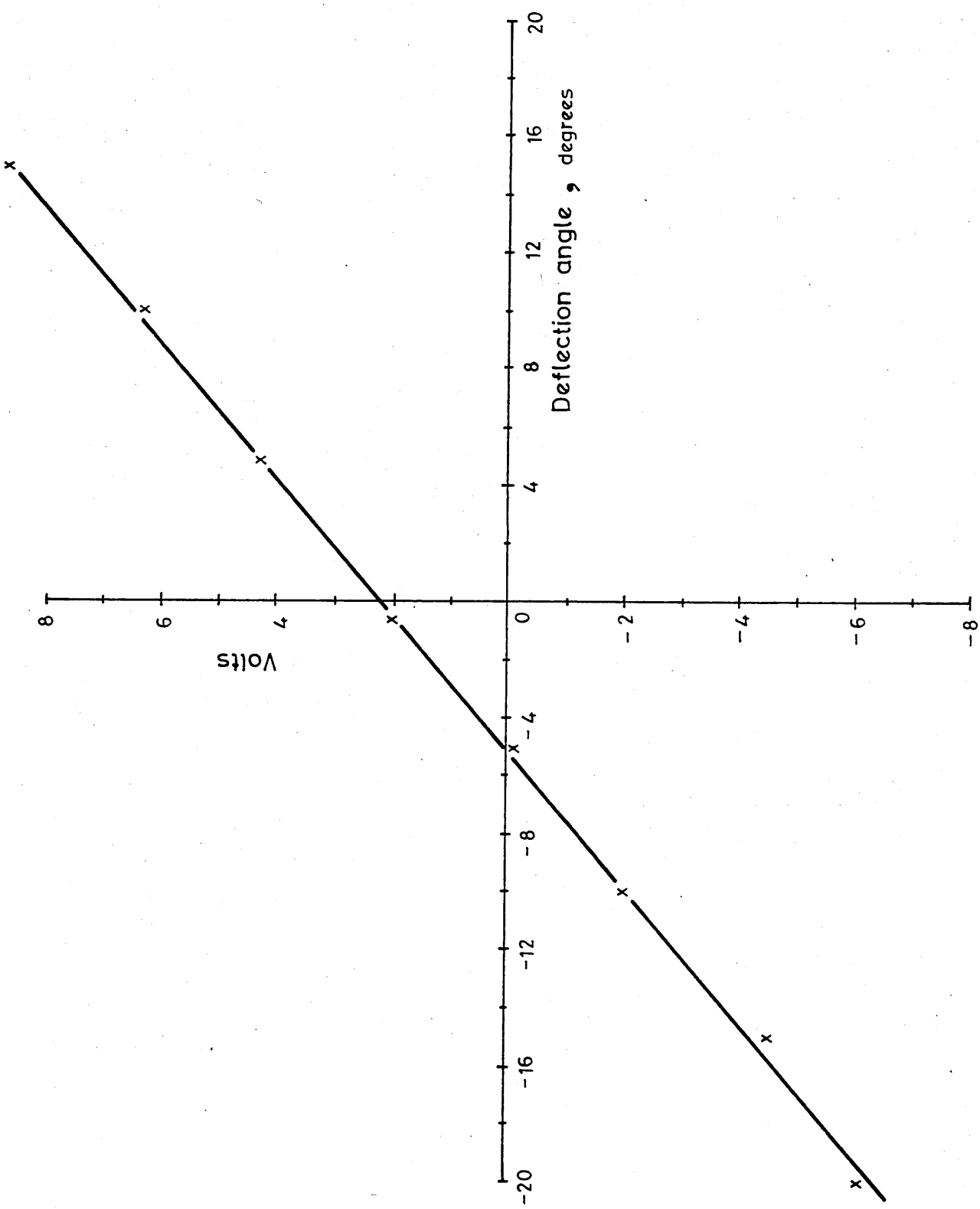


Figure 44. Rudder deflection vs input voltage.

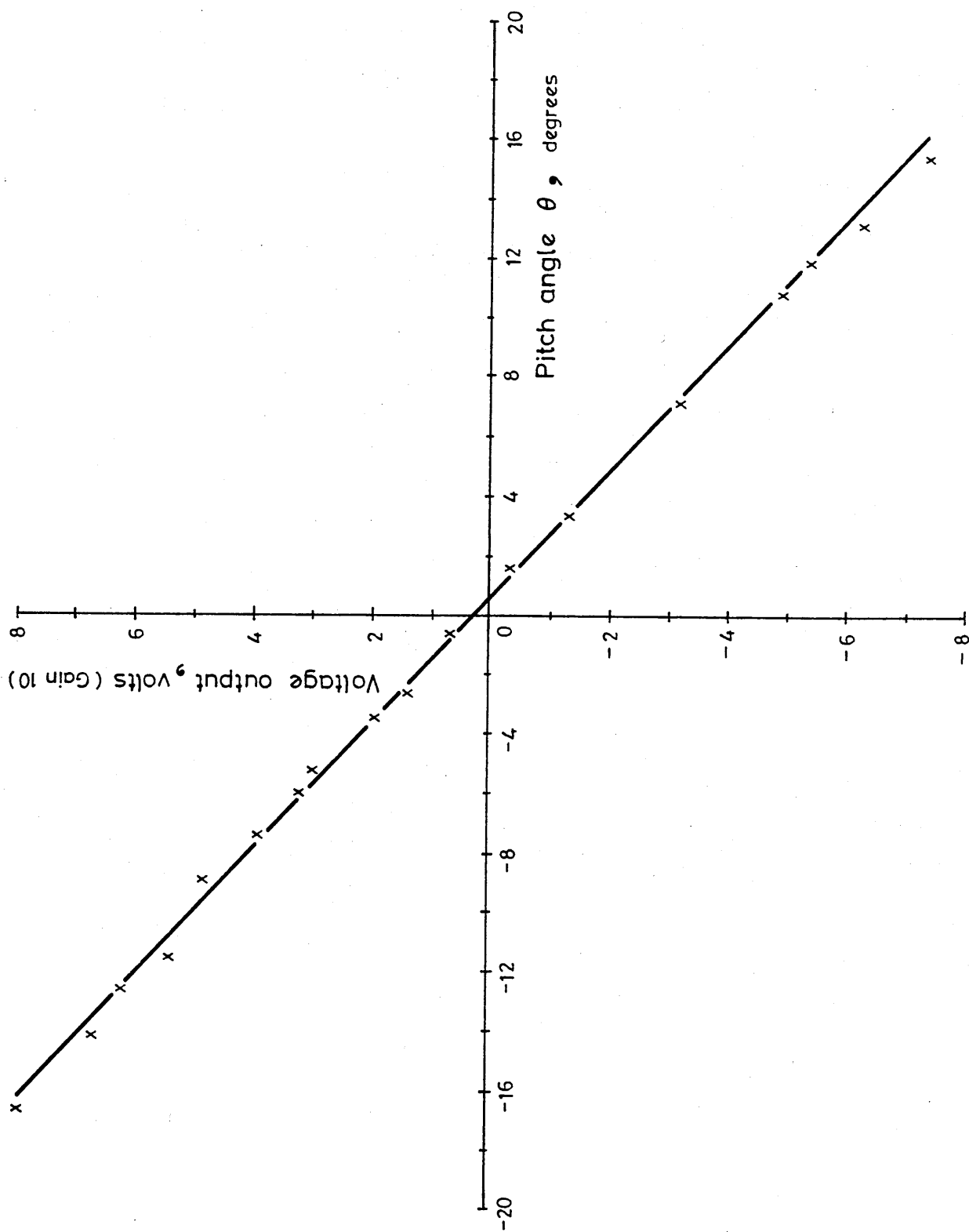


Figure 45. Pitch angle vs voltage output.

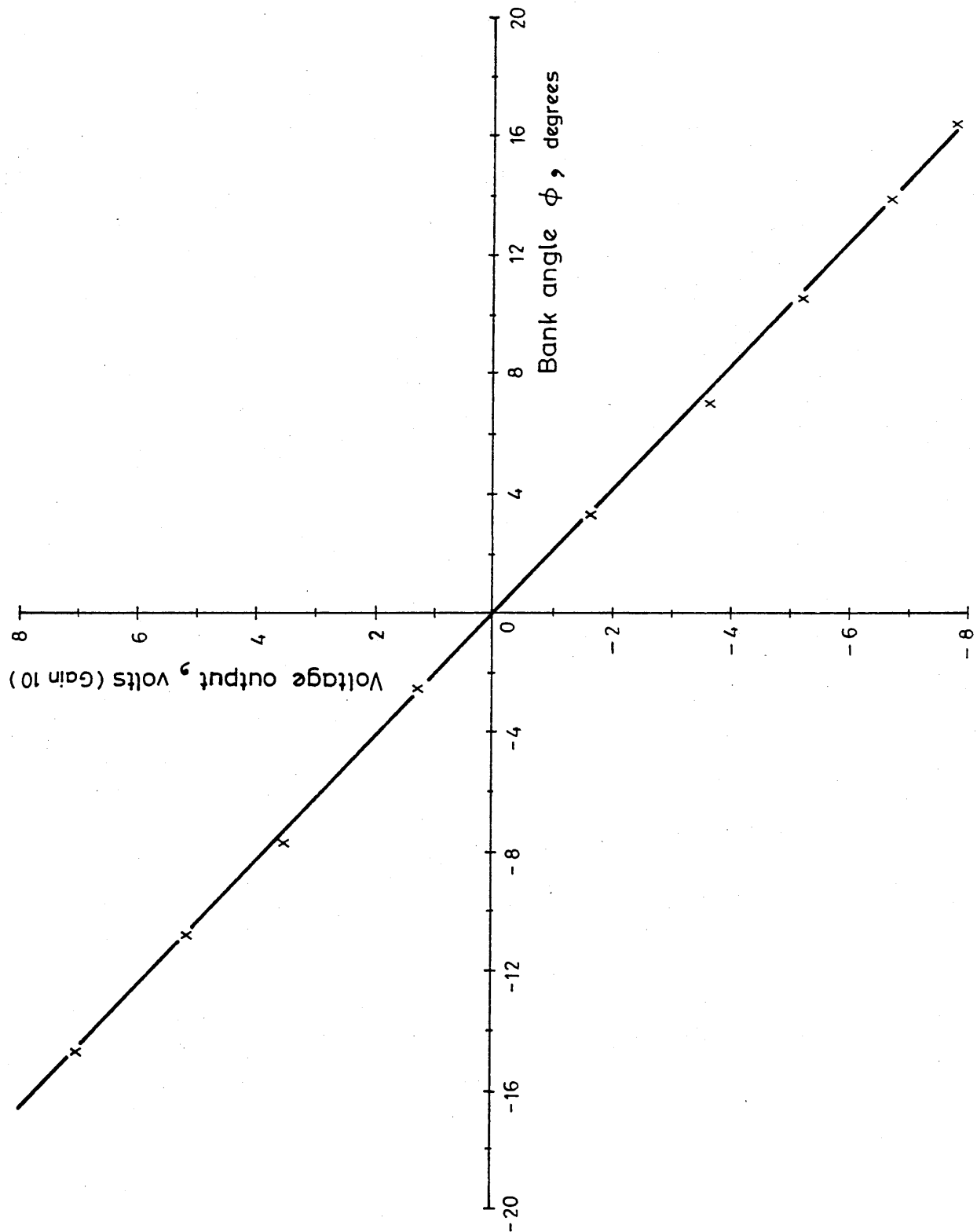


Figure 46. Bank angle vs voltage output.

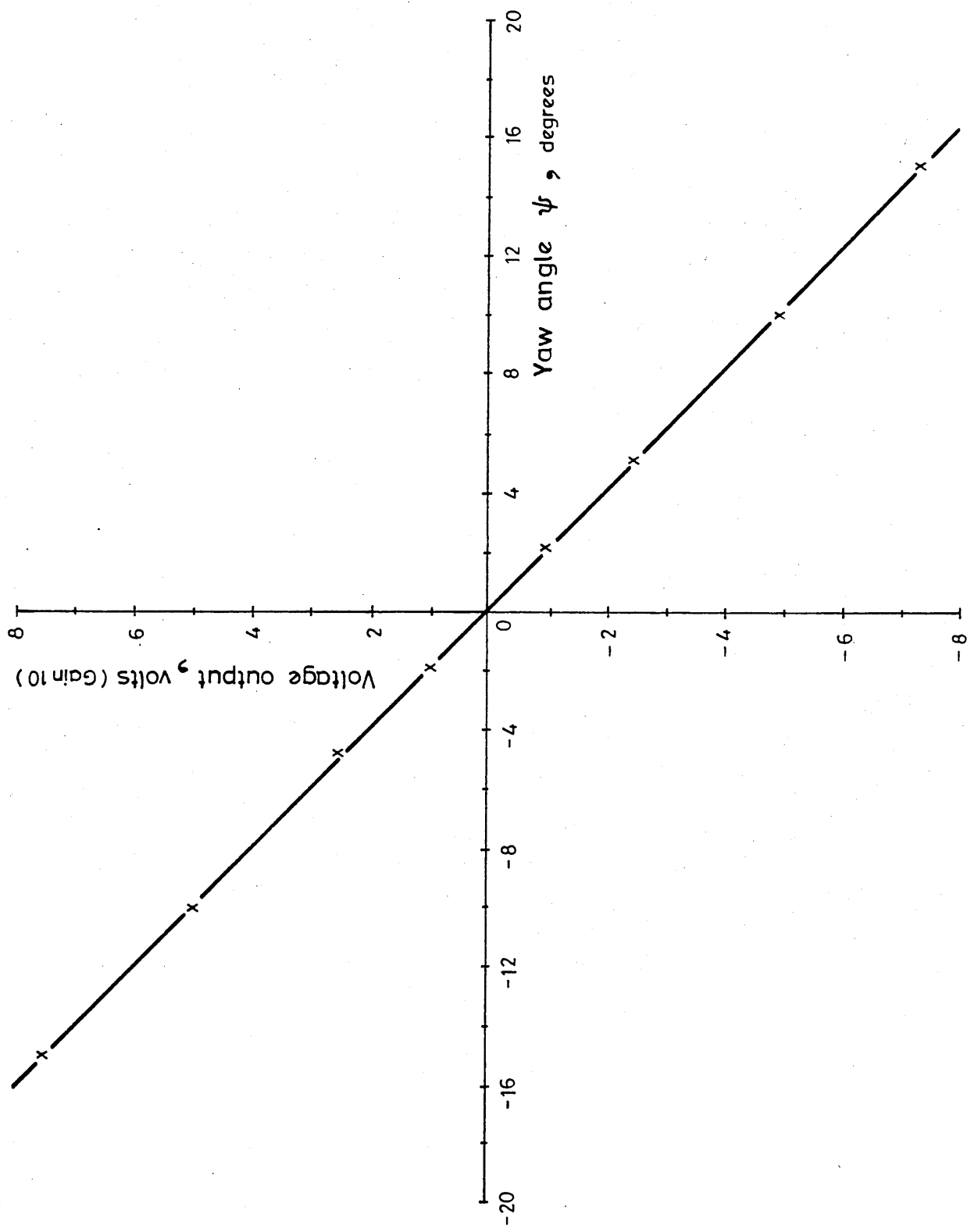


Figure 47. Yaw angle vs voltage output.

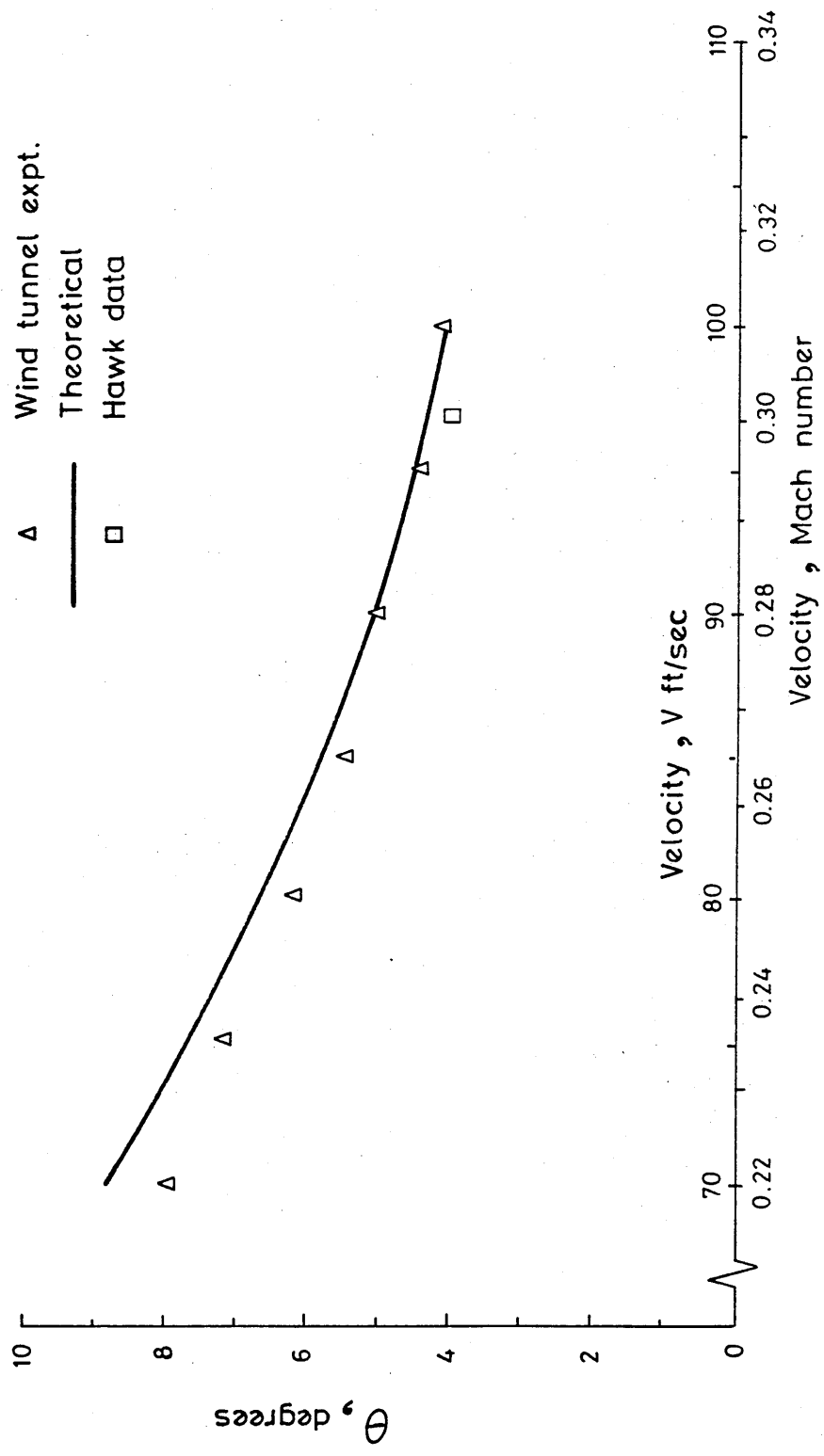


Figure 50. Incidence for steady state conditions.

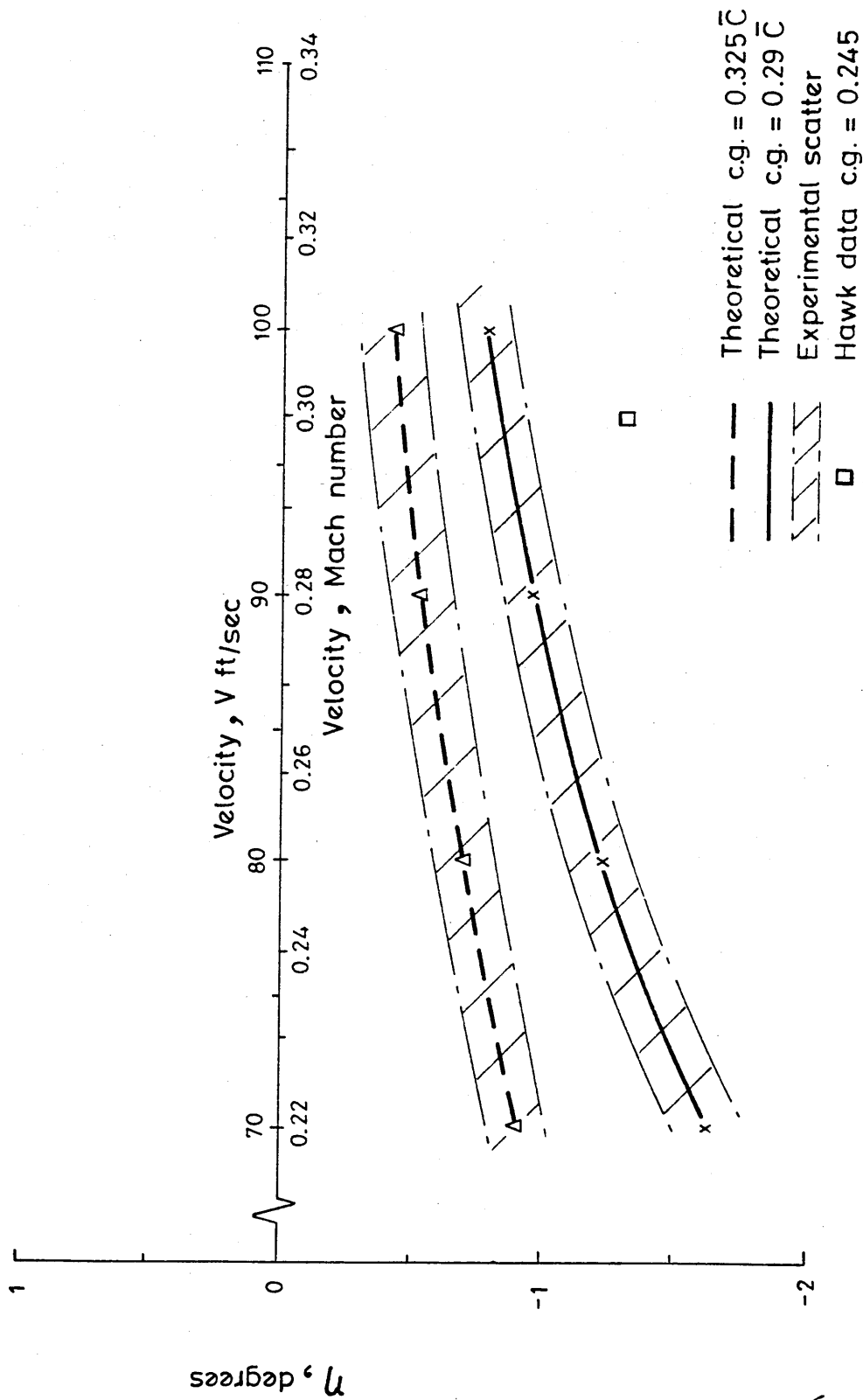


Figure 51. Tail angle to trim for steady state conditions.

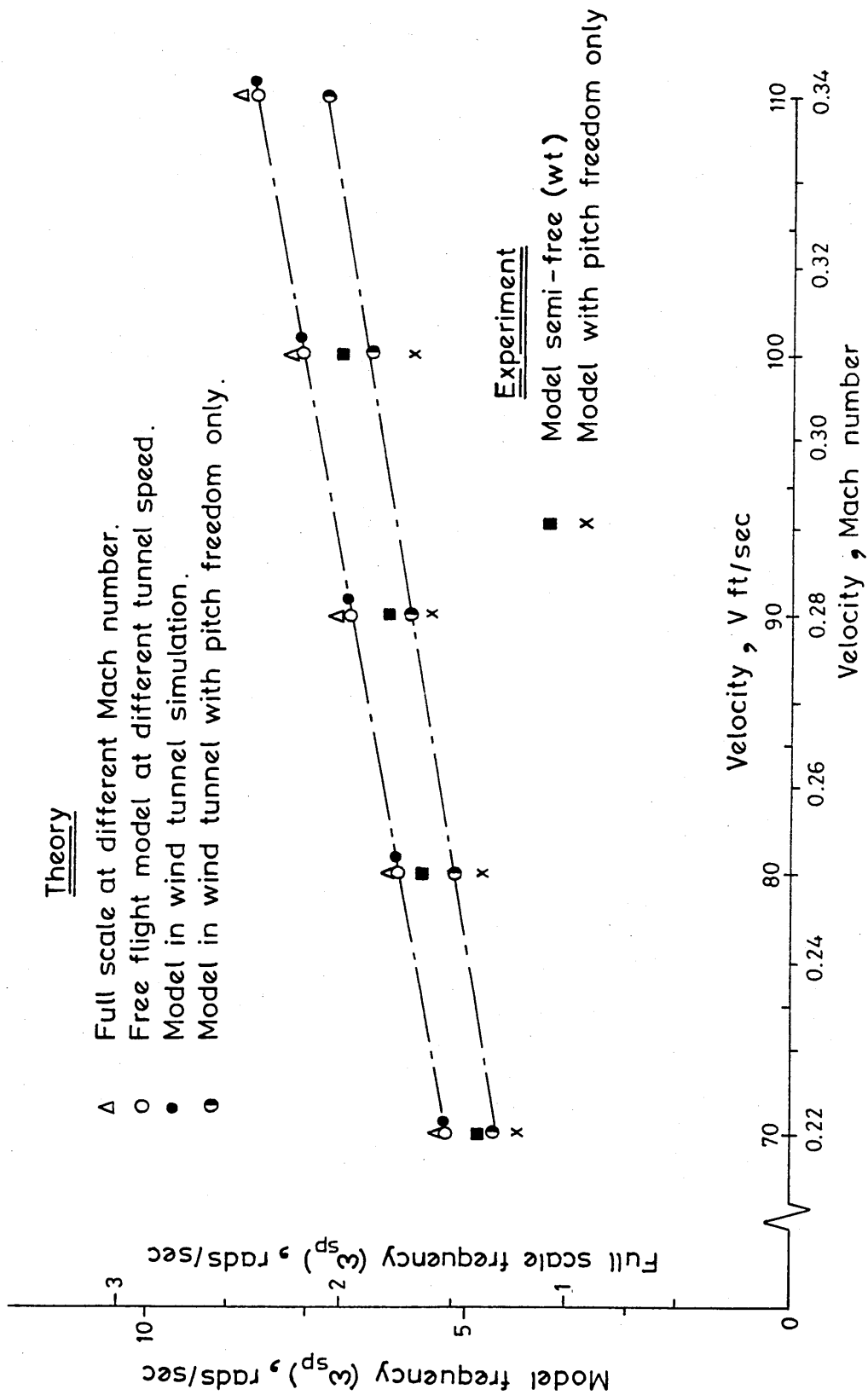


Figure 52. Short period mode - frequency (c.g.=0.29 $\bar{C}$)

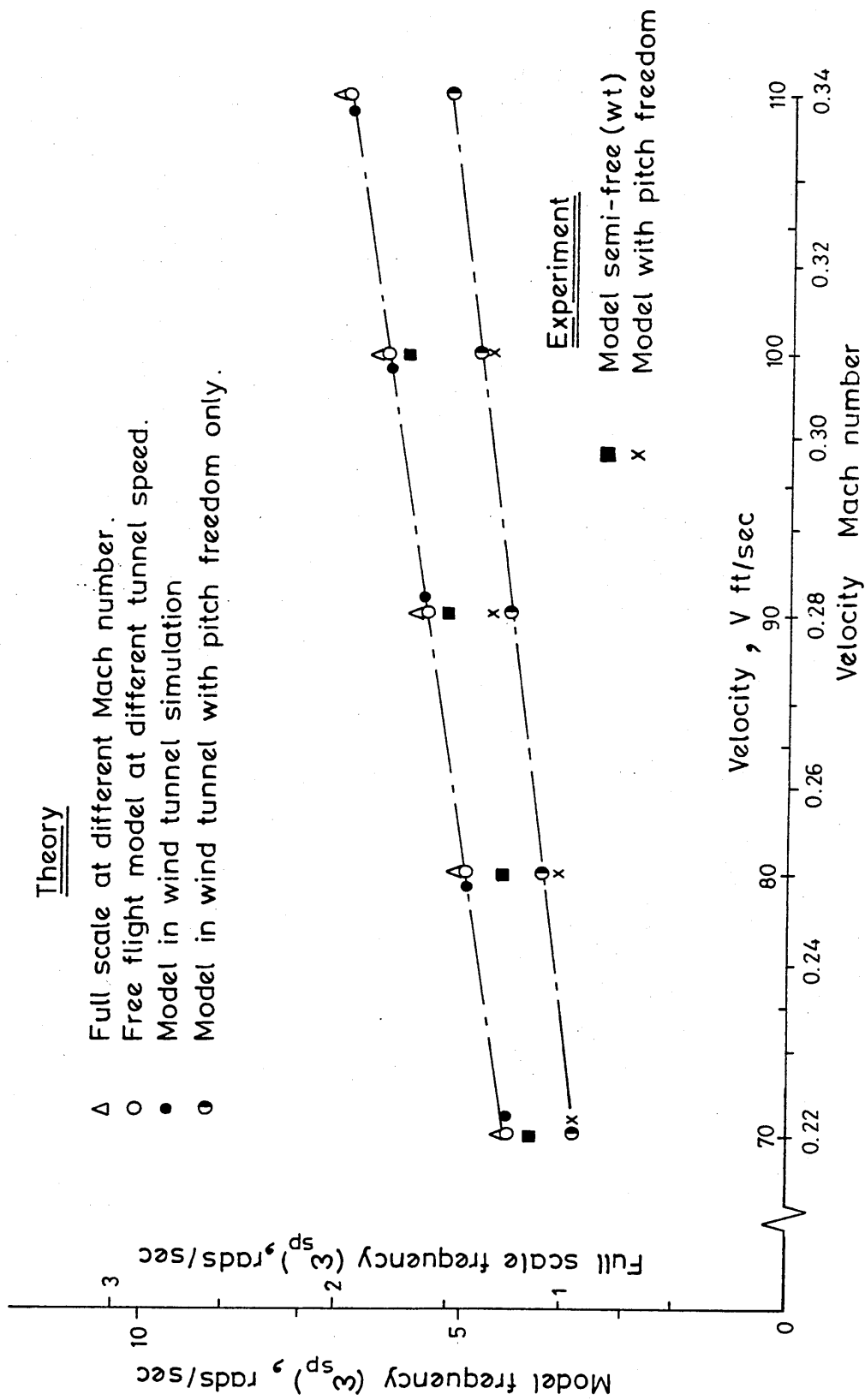


Figure 53. Short period mode - frequency (c.g. = 0.325 $\bar{c}$)

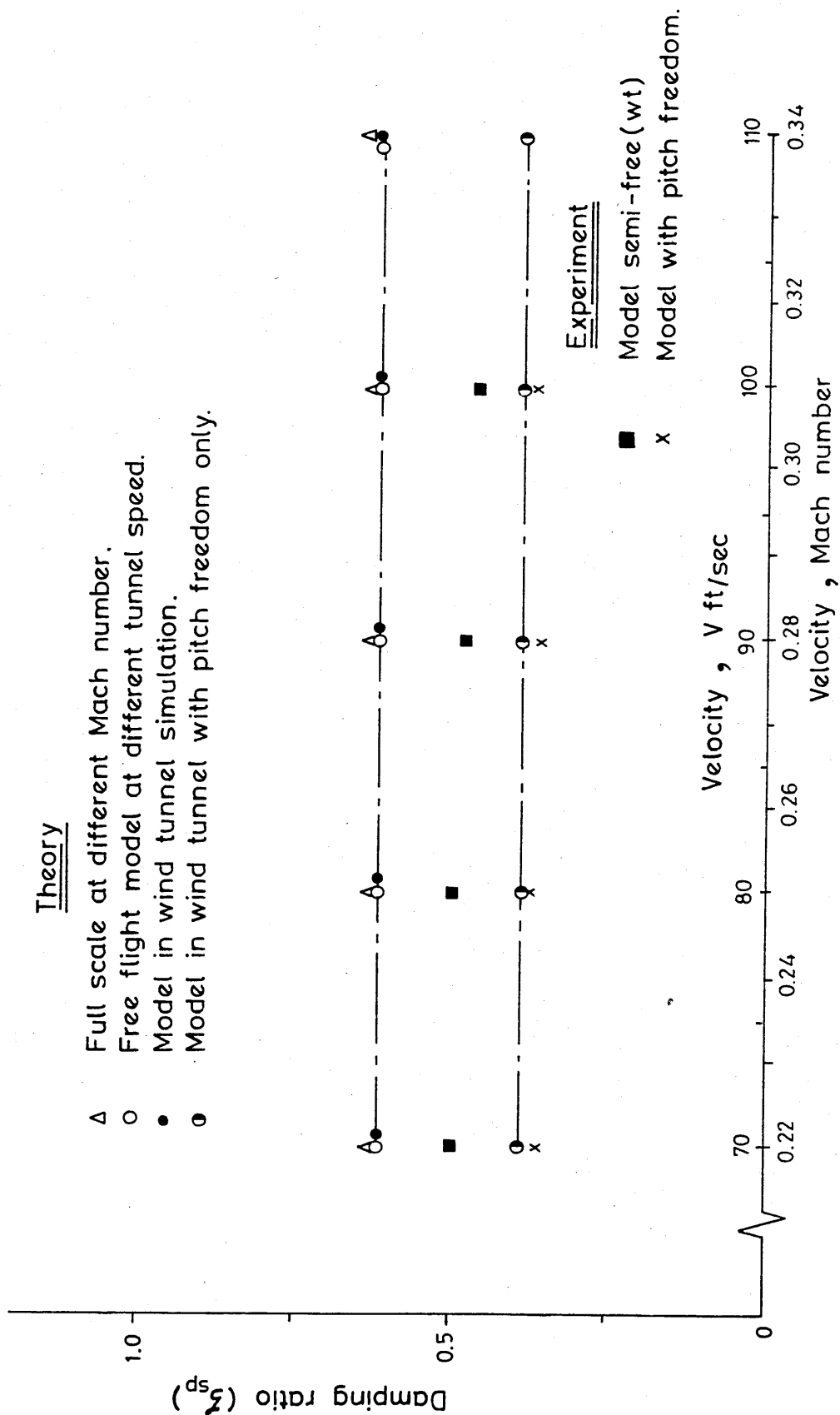


Figure 54. Short period mode - damping (c.g. at $0.29\bar{C}$)

Theory

- Δ Full scale at different Mach number.
- $\circ$ Free flight model at different tunnel speed.
- $\bullet$ Model in wind tunnel simulation.
- $\circ$ Model in wind tunnel with pitch freedom only.

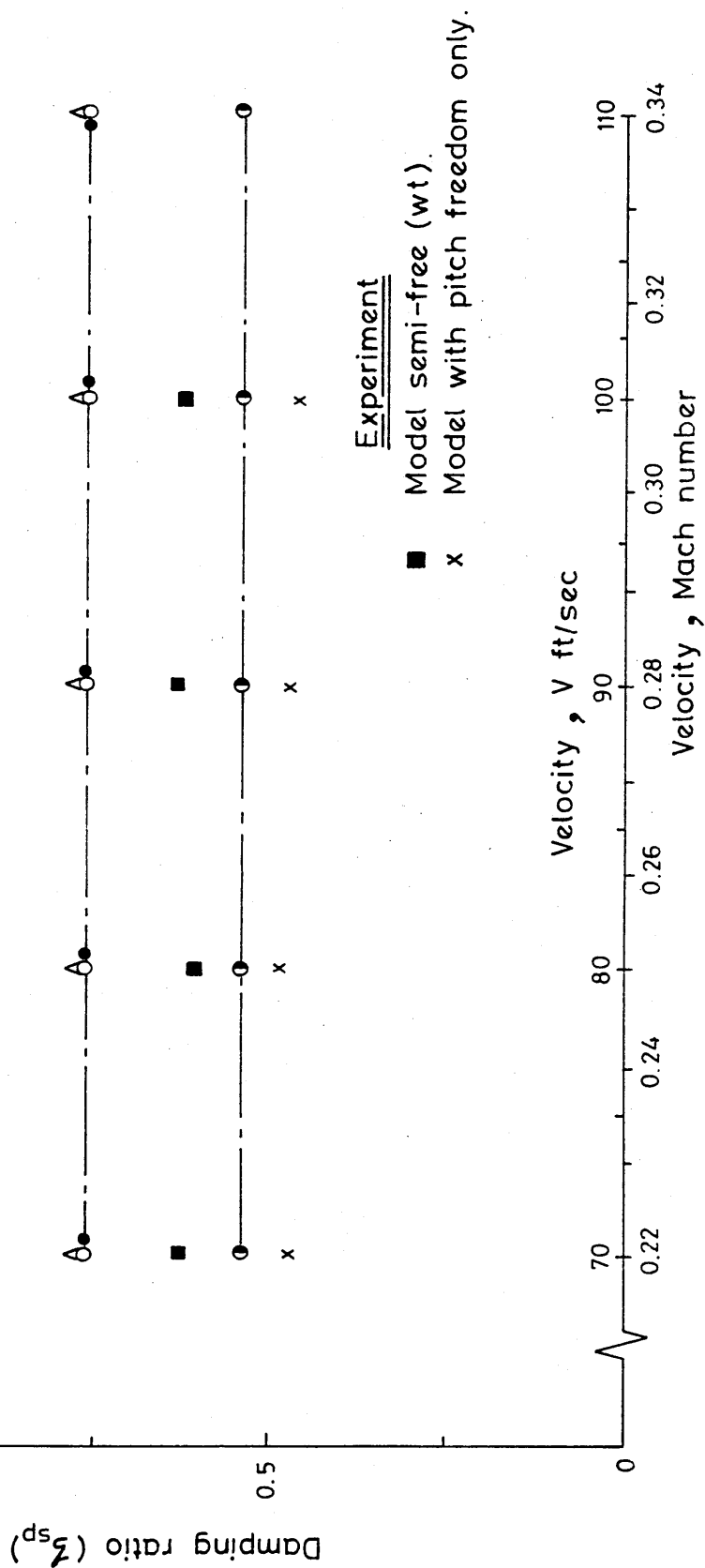


Figure 55. Short period mode - damping (c.g. at 0.325C)

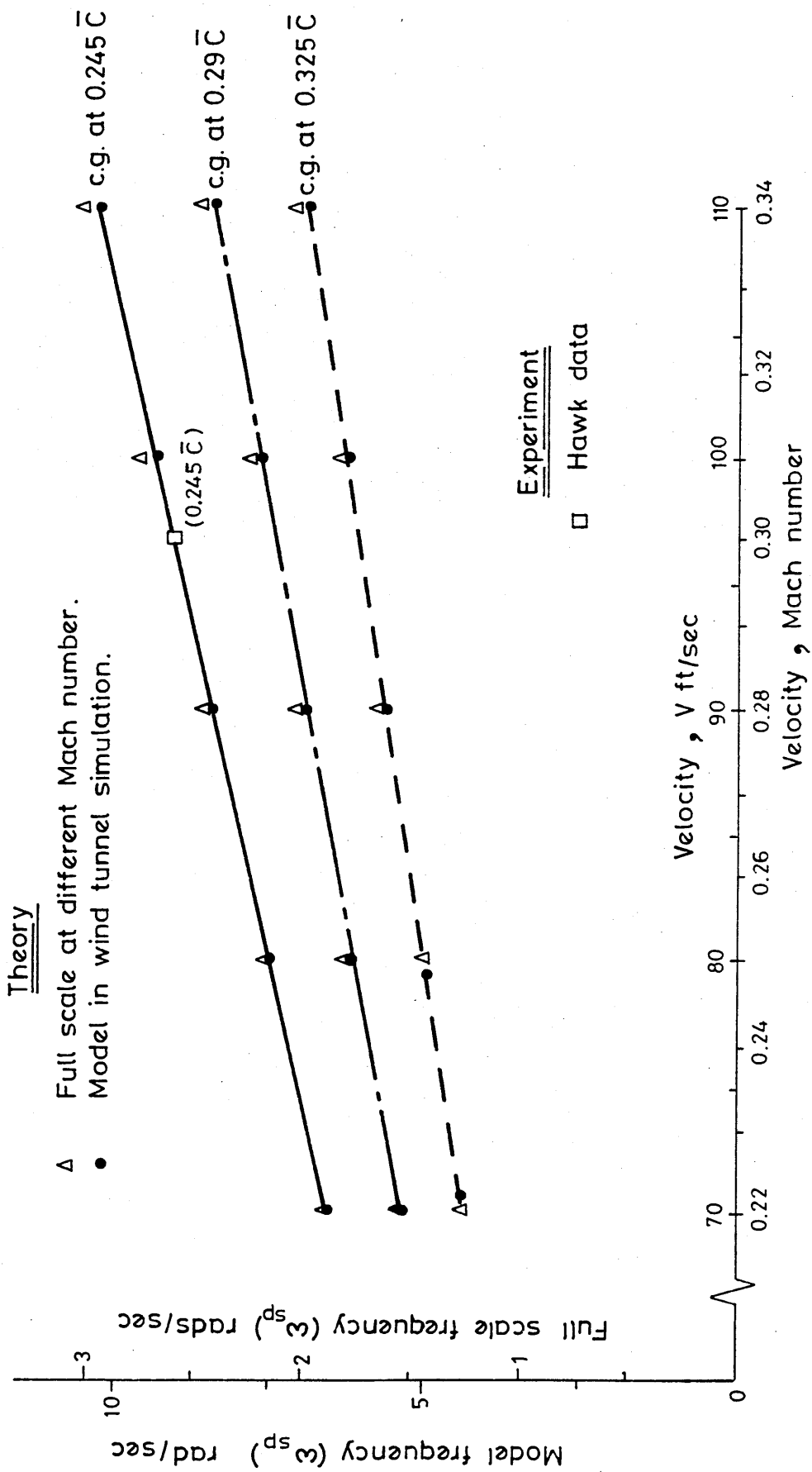


Figure 56. Short period mode frequency correlation.

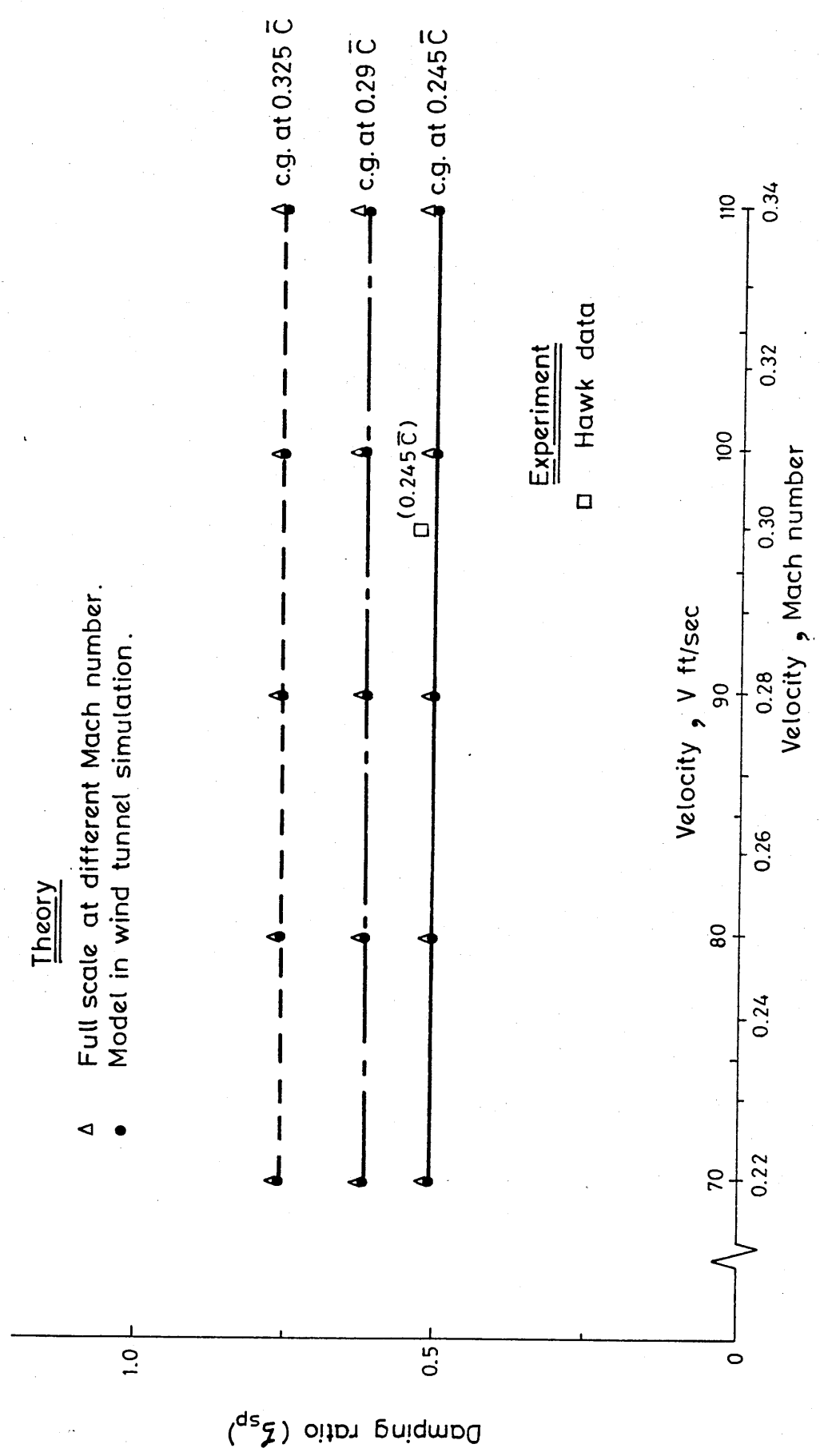


Figure 57. Short period mode damping ratio correlation.

Theory

- Δ Full scale of different Mach number.
- Free flight model at different tunnel speed.
- Model in wind tunnel simulation.

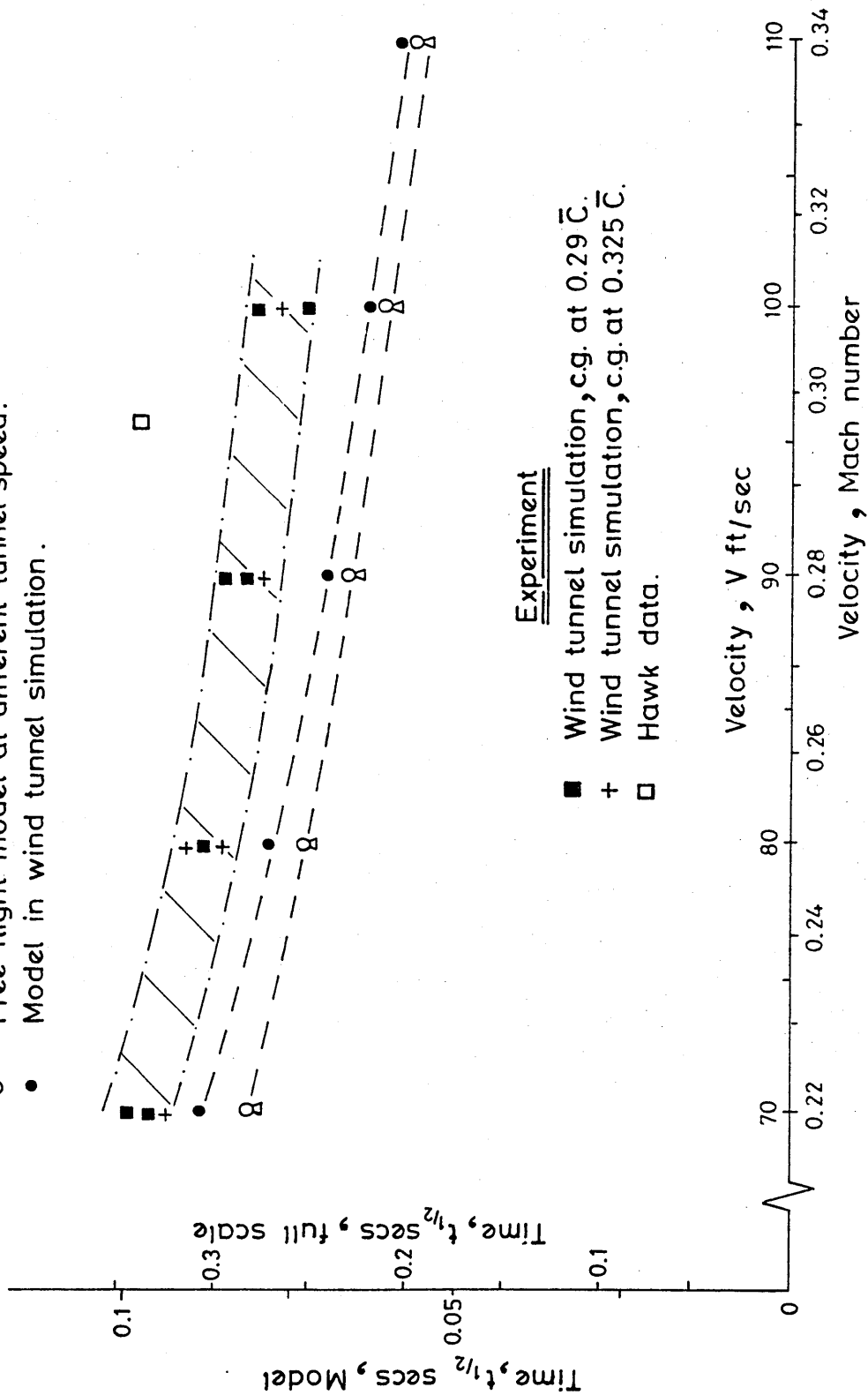


Figure 58. Roll mode - time $t_{1/2}$ to half amplitude.

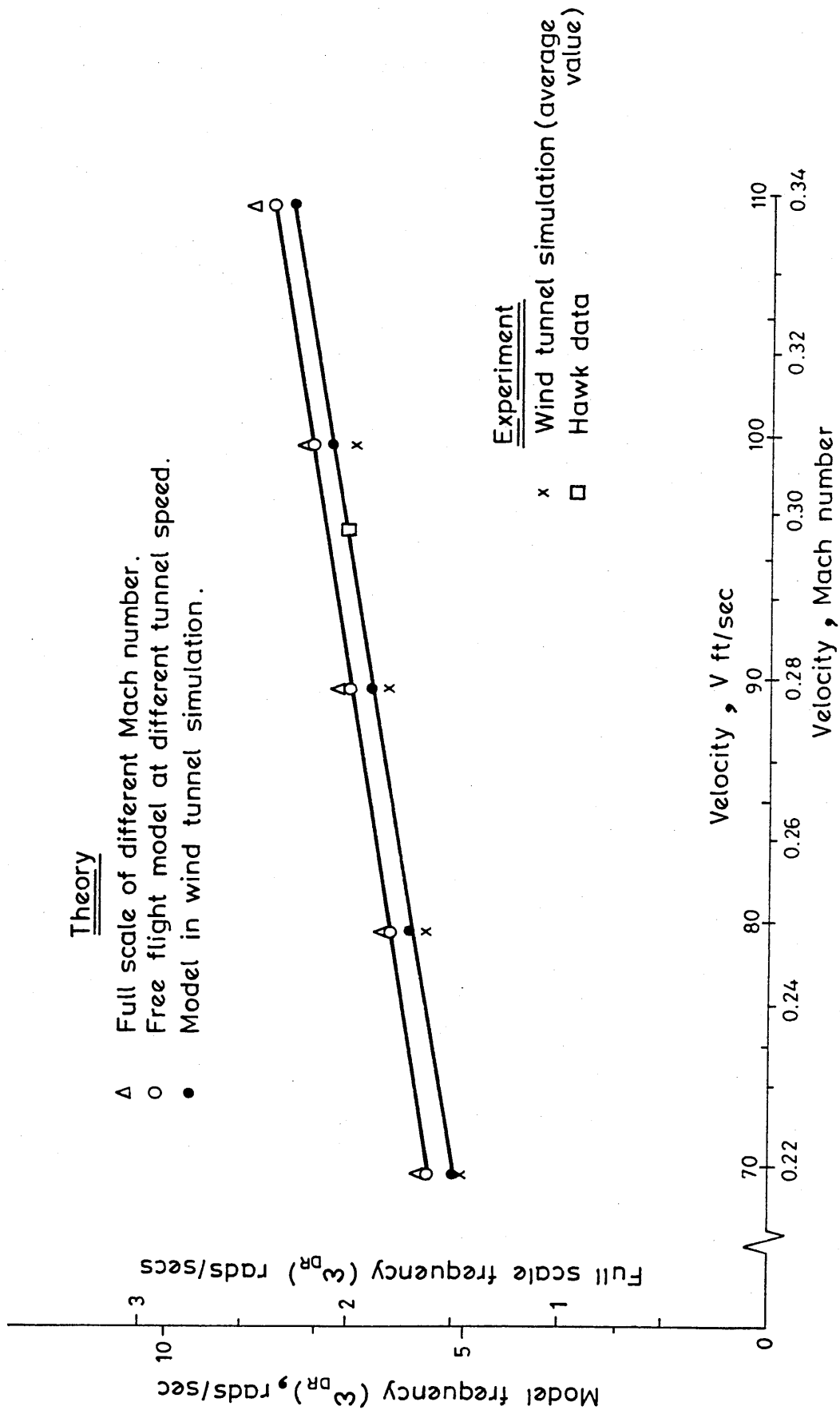


Figure 60. Dutch roll mode - frequency (ω_{DR}).

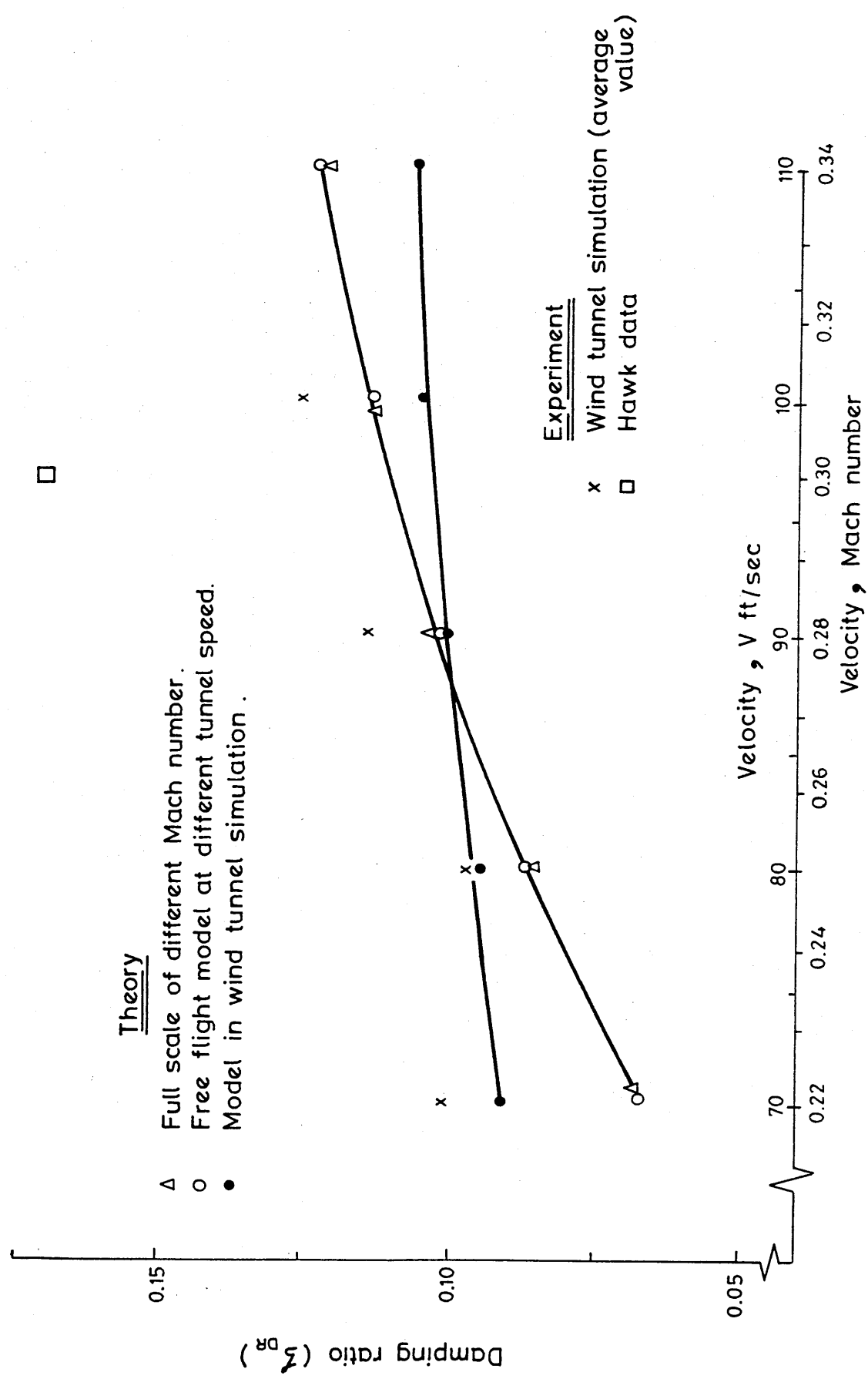


Figure 61. Dutch roll mode - damping ratio (ζ_{DR}).

Wind tunnel simulation,
V = 70 ft/sec

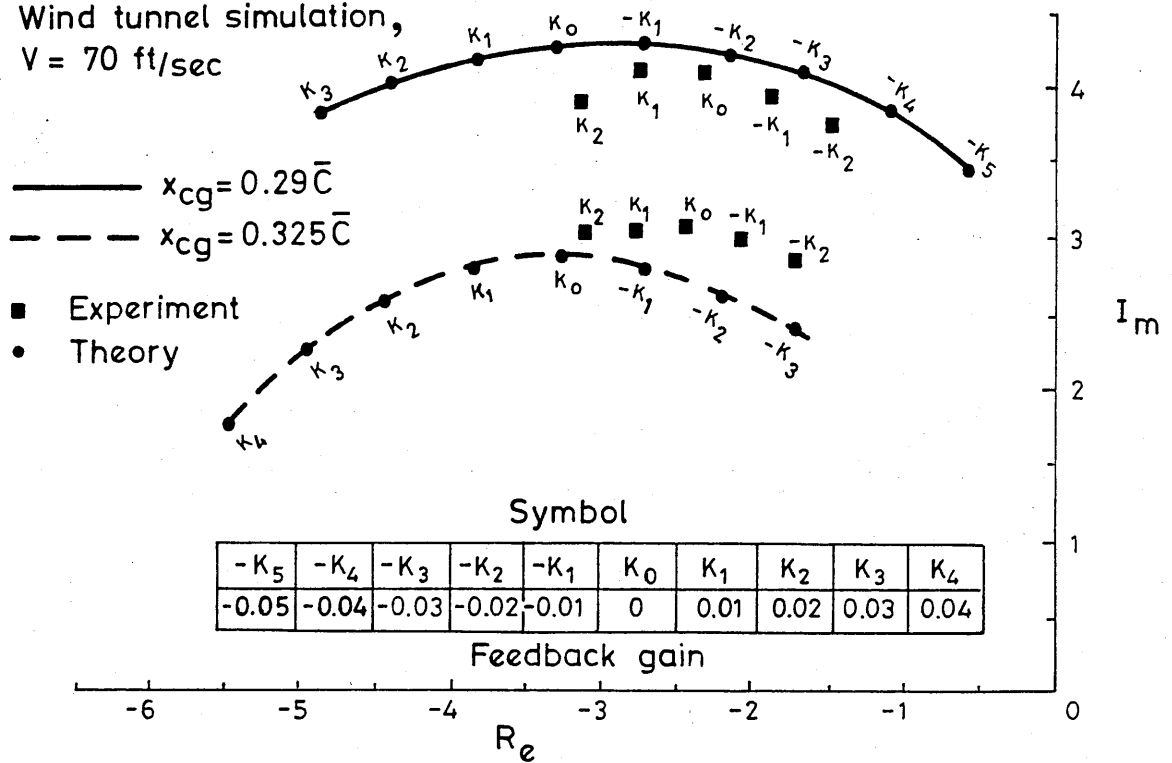


Figure 63.a. Model short period mode - pitch rate feedback to elevator.

Wind tunnel simulation, V = 70 ft/sec
(pitch freedom only)

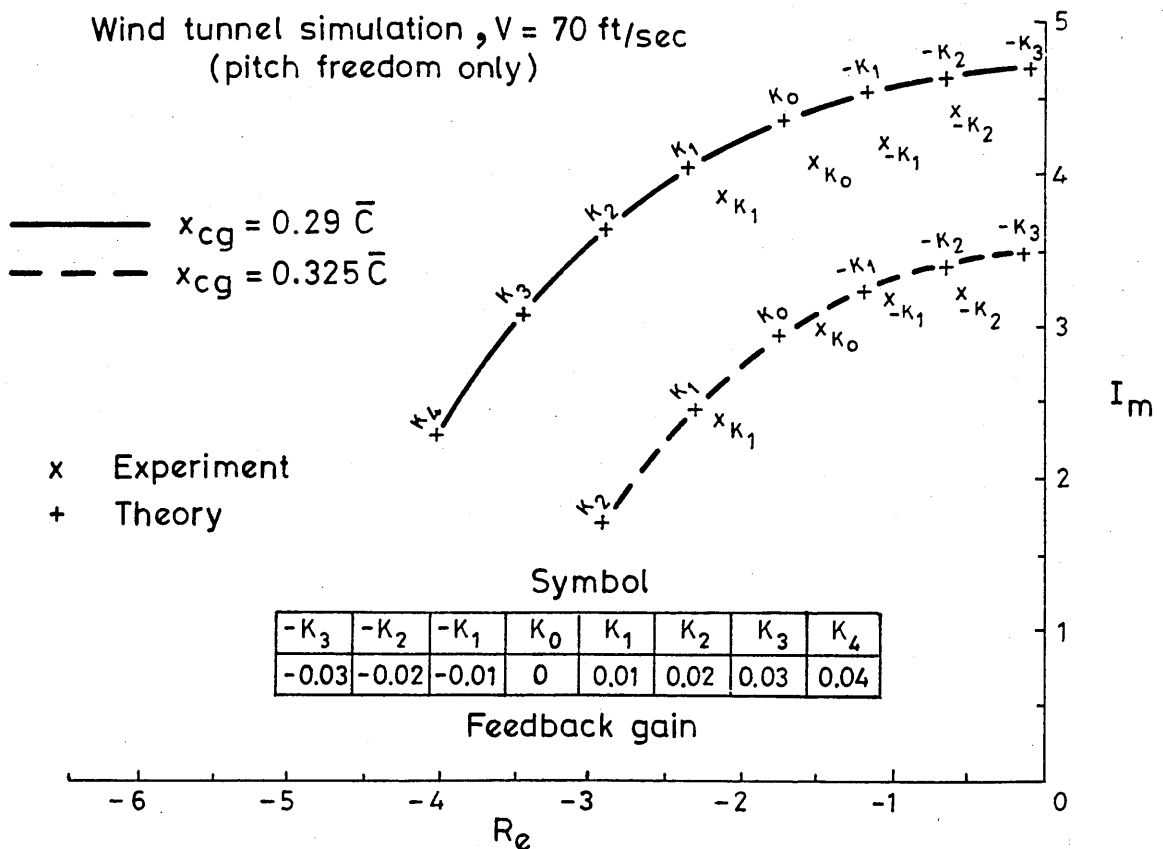


Figure 63.b. Model short period mode - pitch rate feedback to elevator.

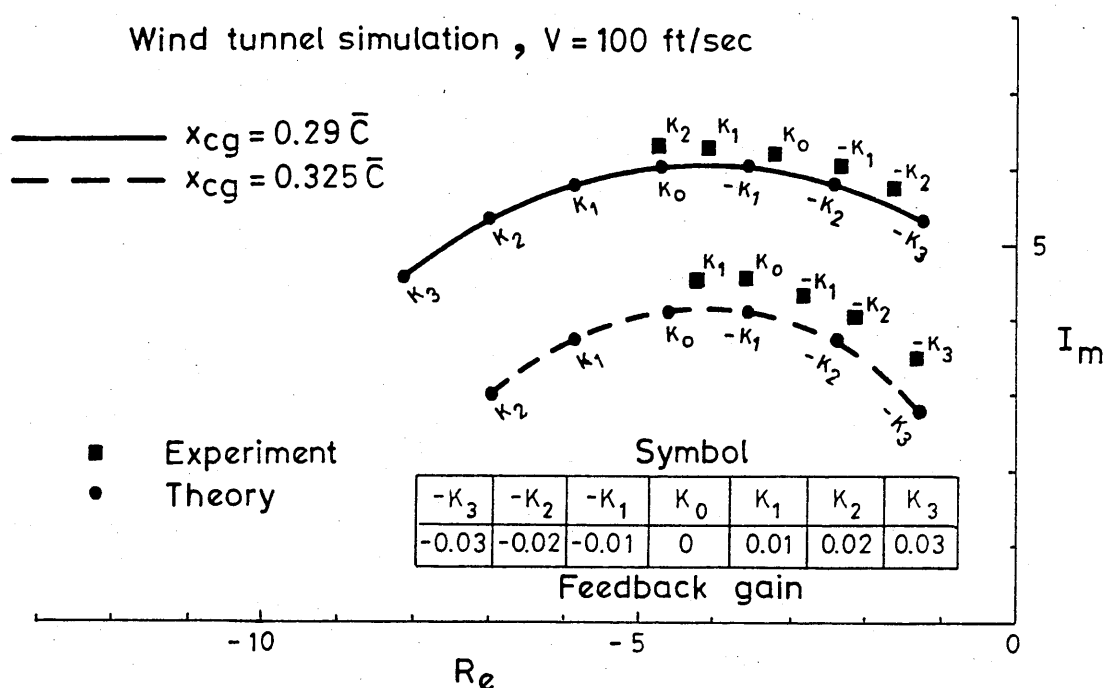


Figure 64.a. Model short period mode - pitch rate feedback to elevator.

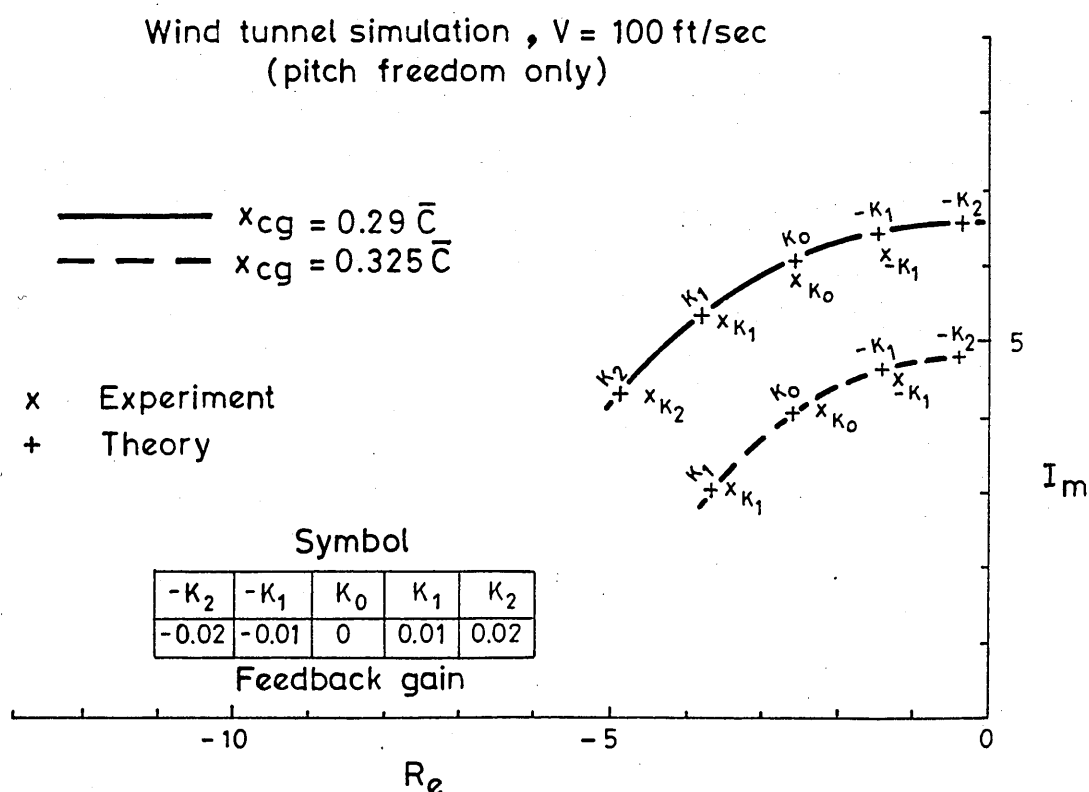


Figure 64.b. Model short period mode - pitch rate feedback to elevator.

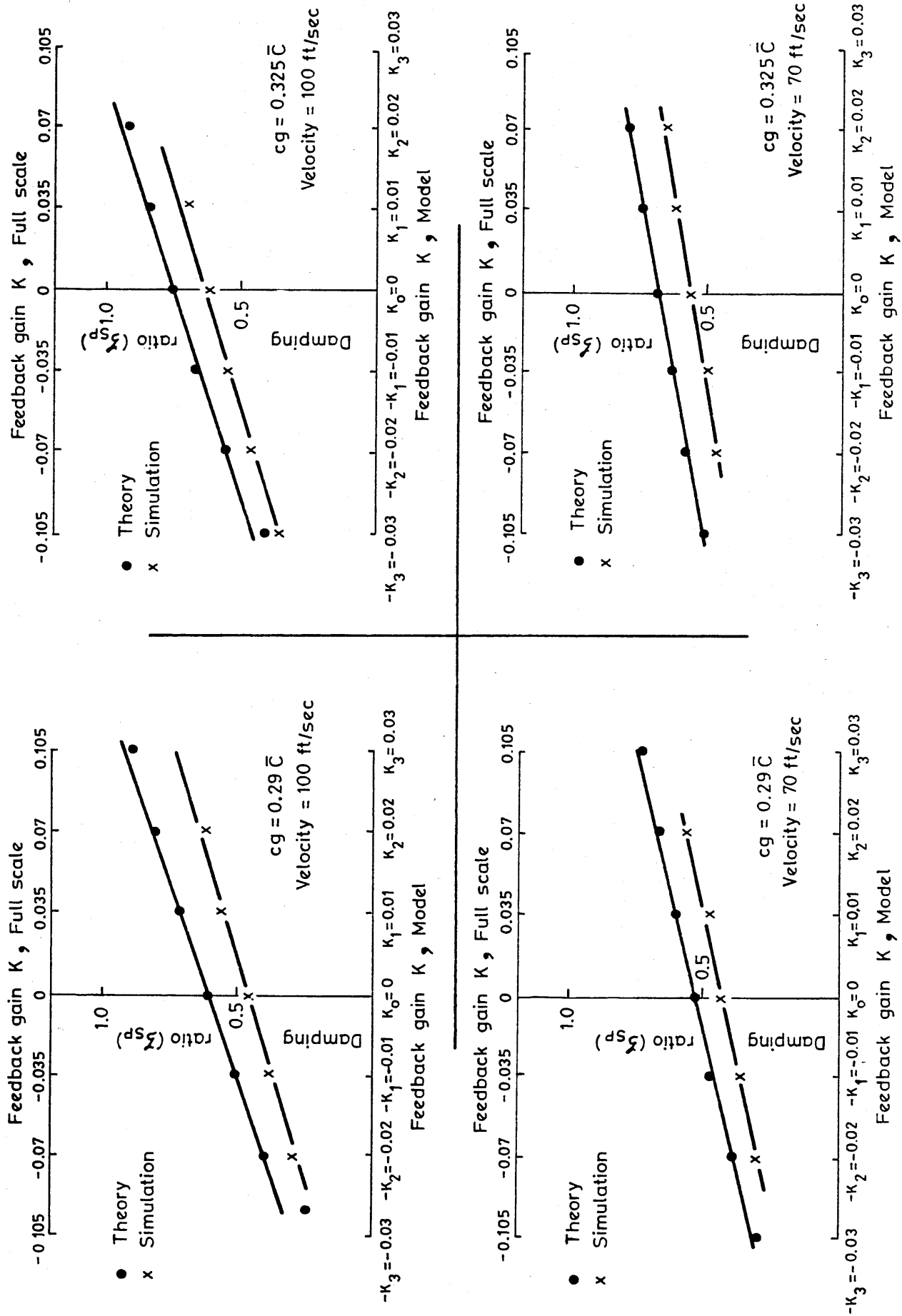


Figure 65. Short period mode damping - pitch rate feedback to elevator.

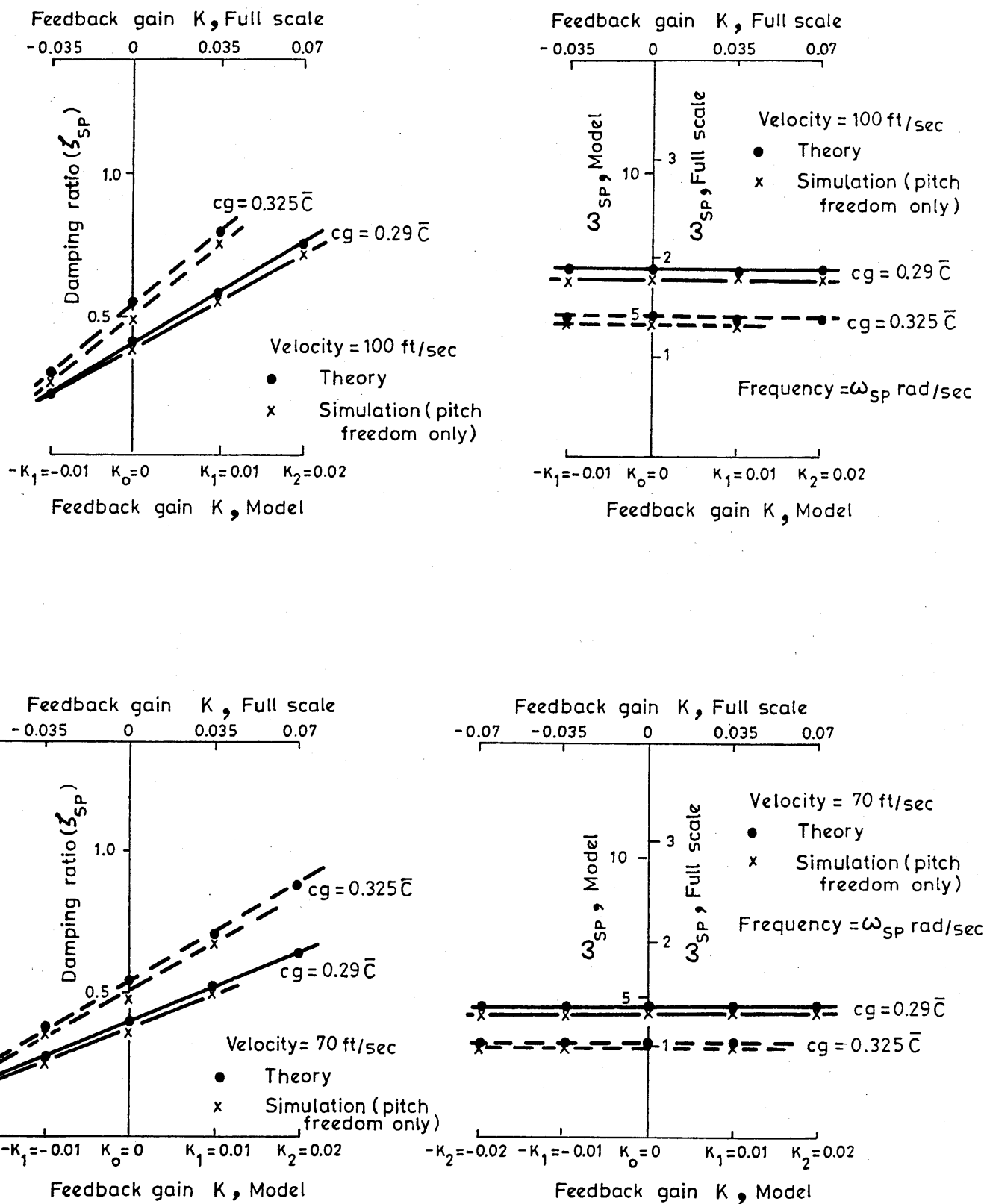


Figure 67. Short period mode - pitch rate feedback to elevator (pitch freedom only).

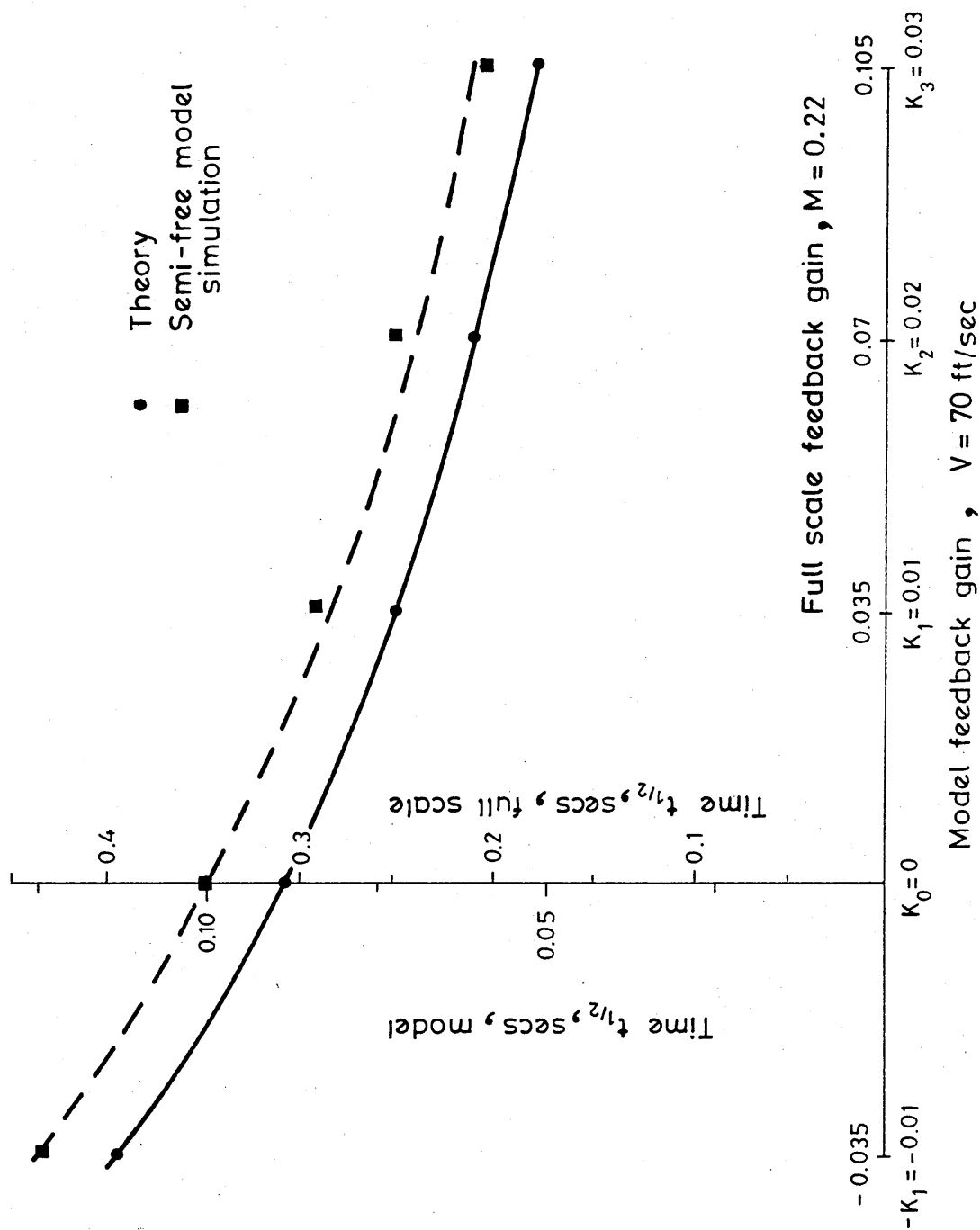


Figure 68. Roll mode :- roll rate feedback to aileron.

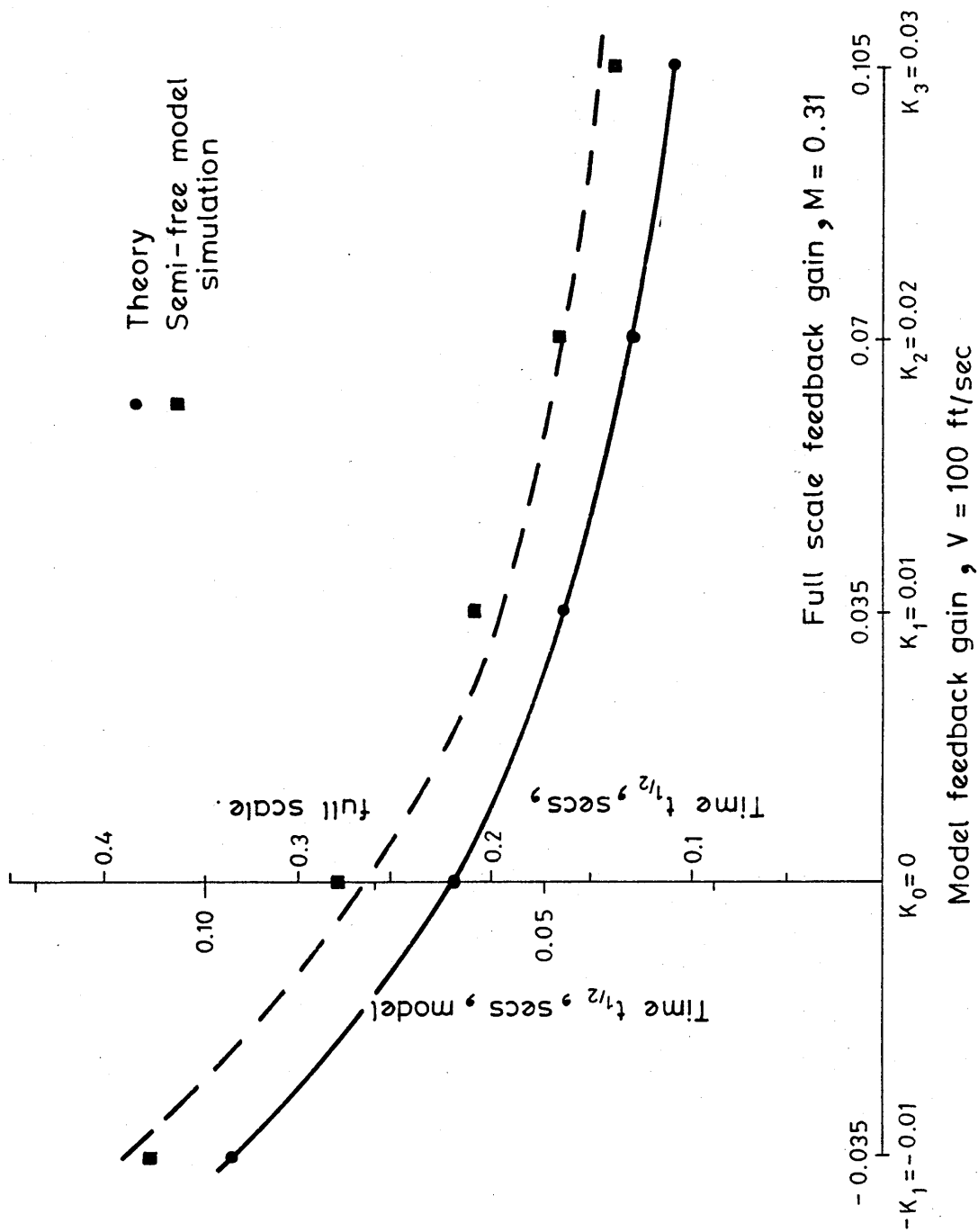


Figure 69. Roll mode :- roll rate feedback to aileron.

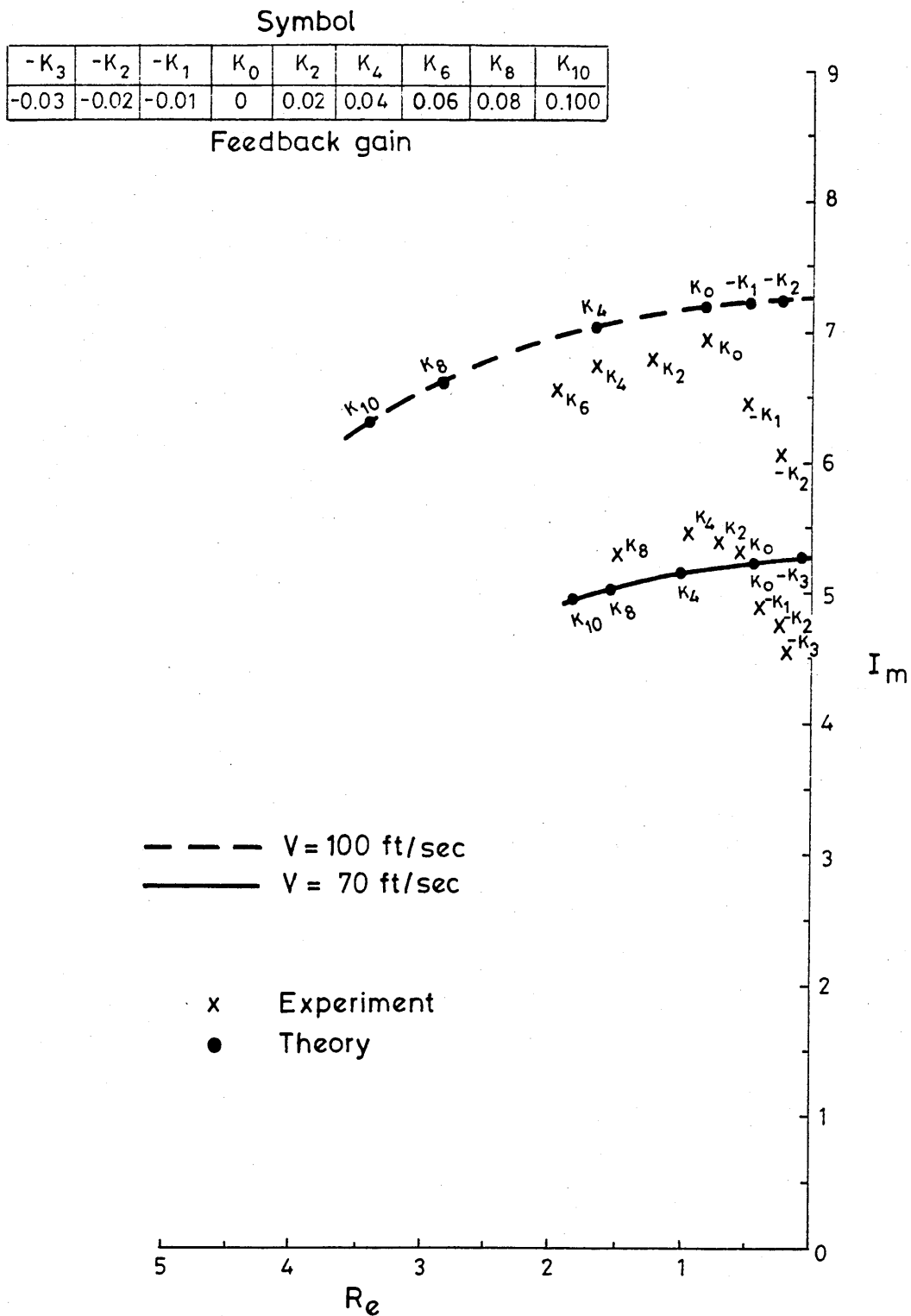


Figure 70. Dutch roll mode :- yaw rate feedback to rudder.

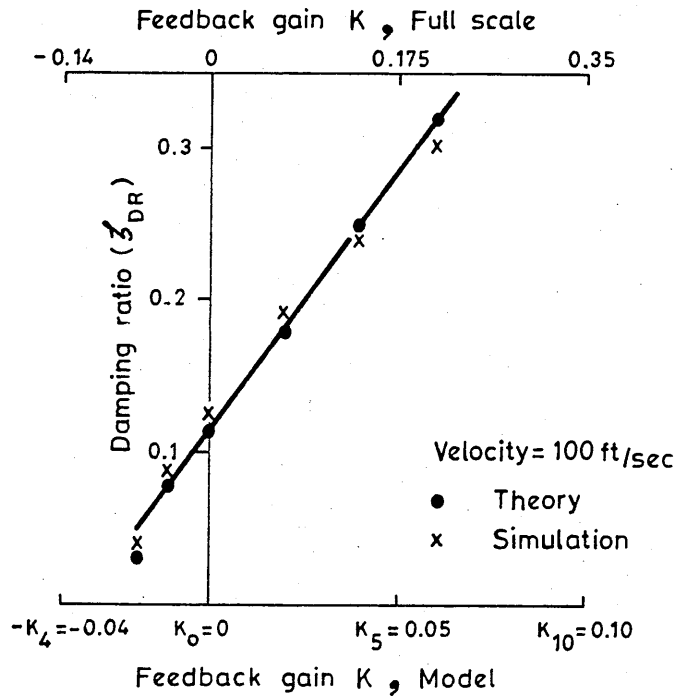
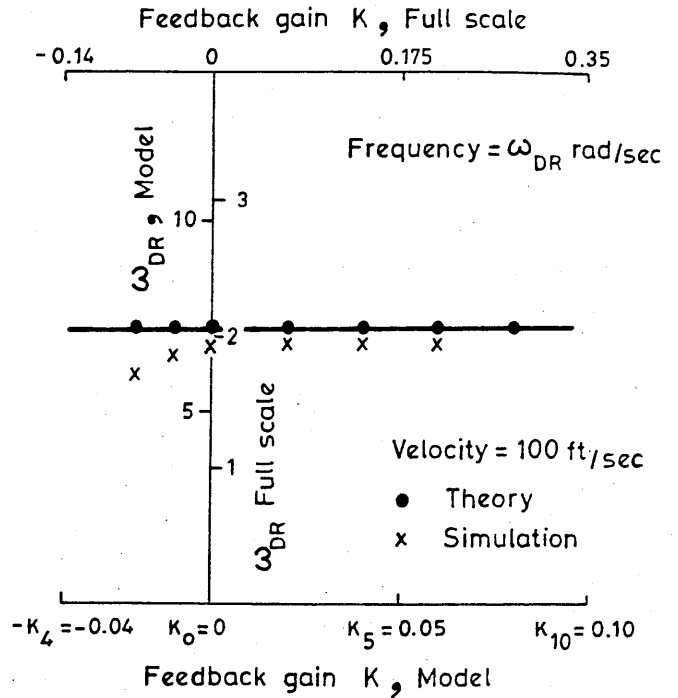
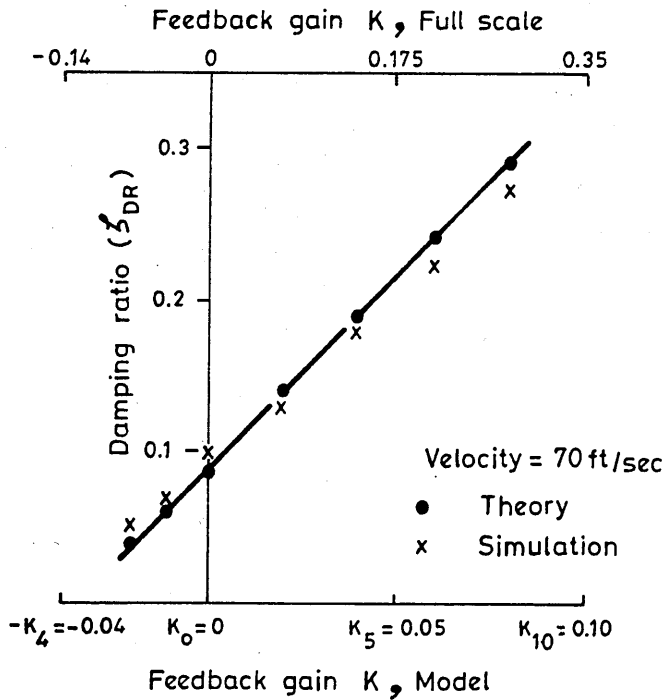
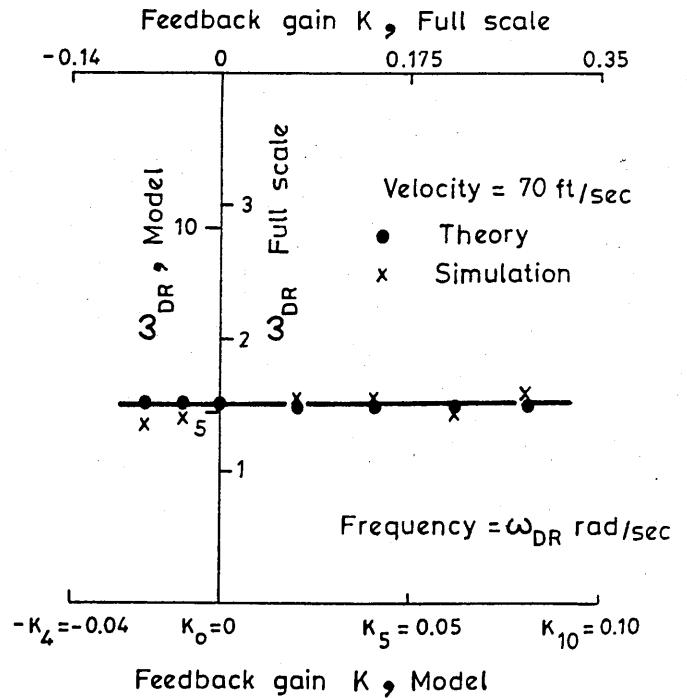


Figure 71. Dutch roll mode - yaw rate feedback to rudder.

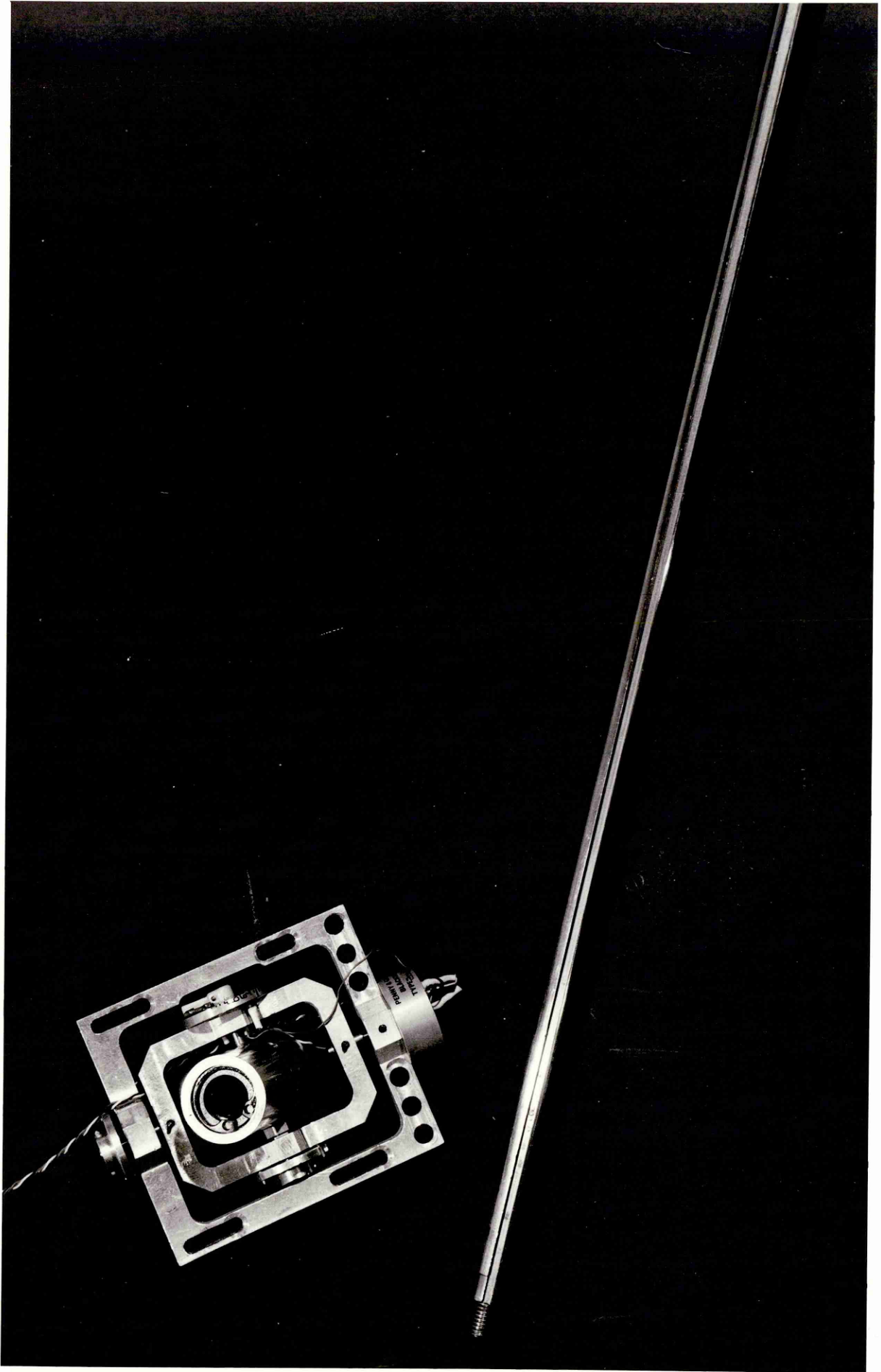


Plate 1. The gimbol system and vertical rod.

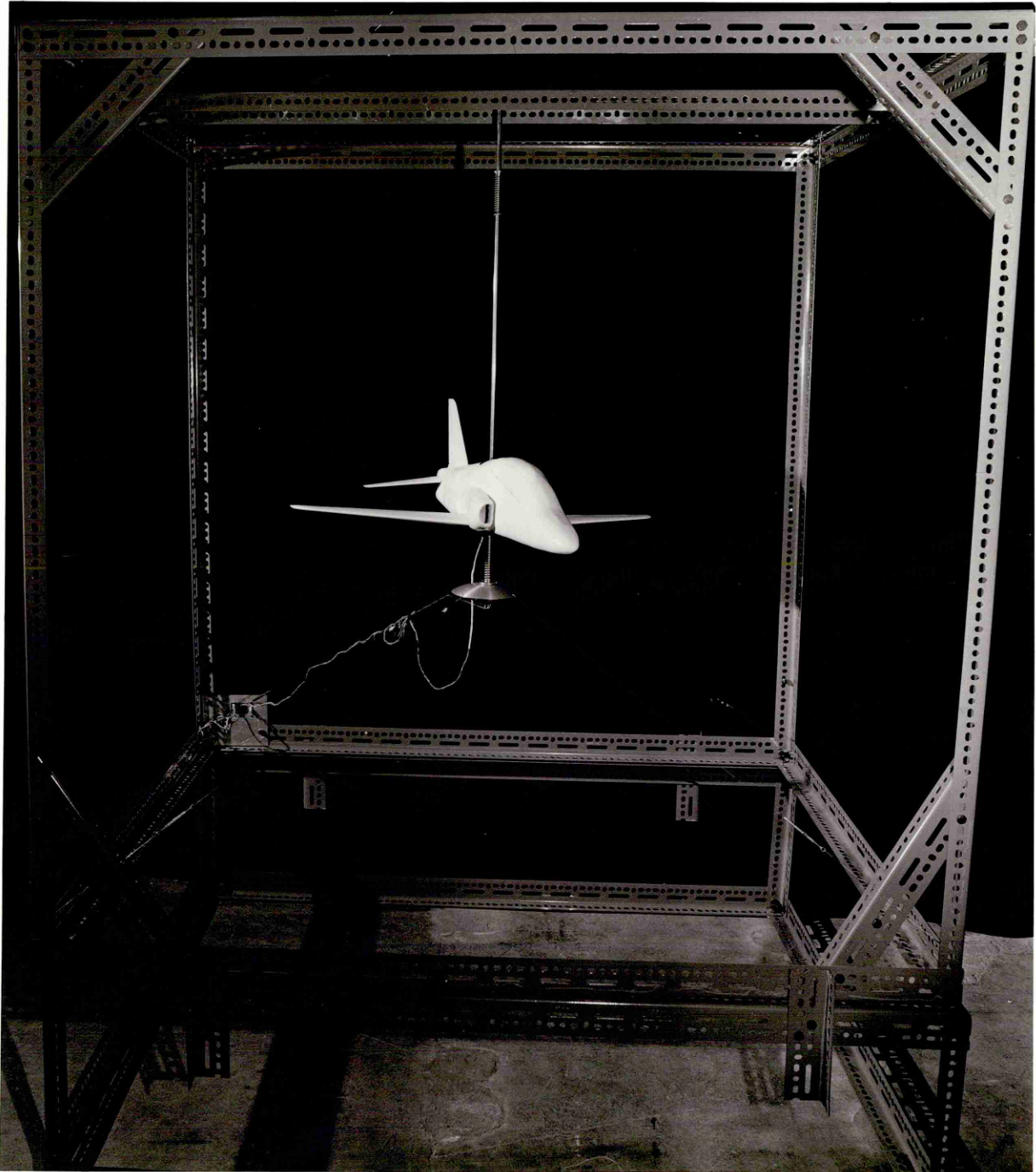


Plate 2. The model suspension system.

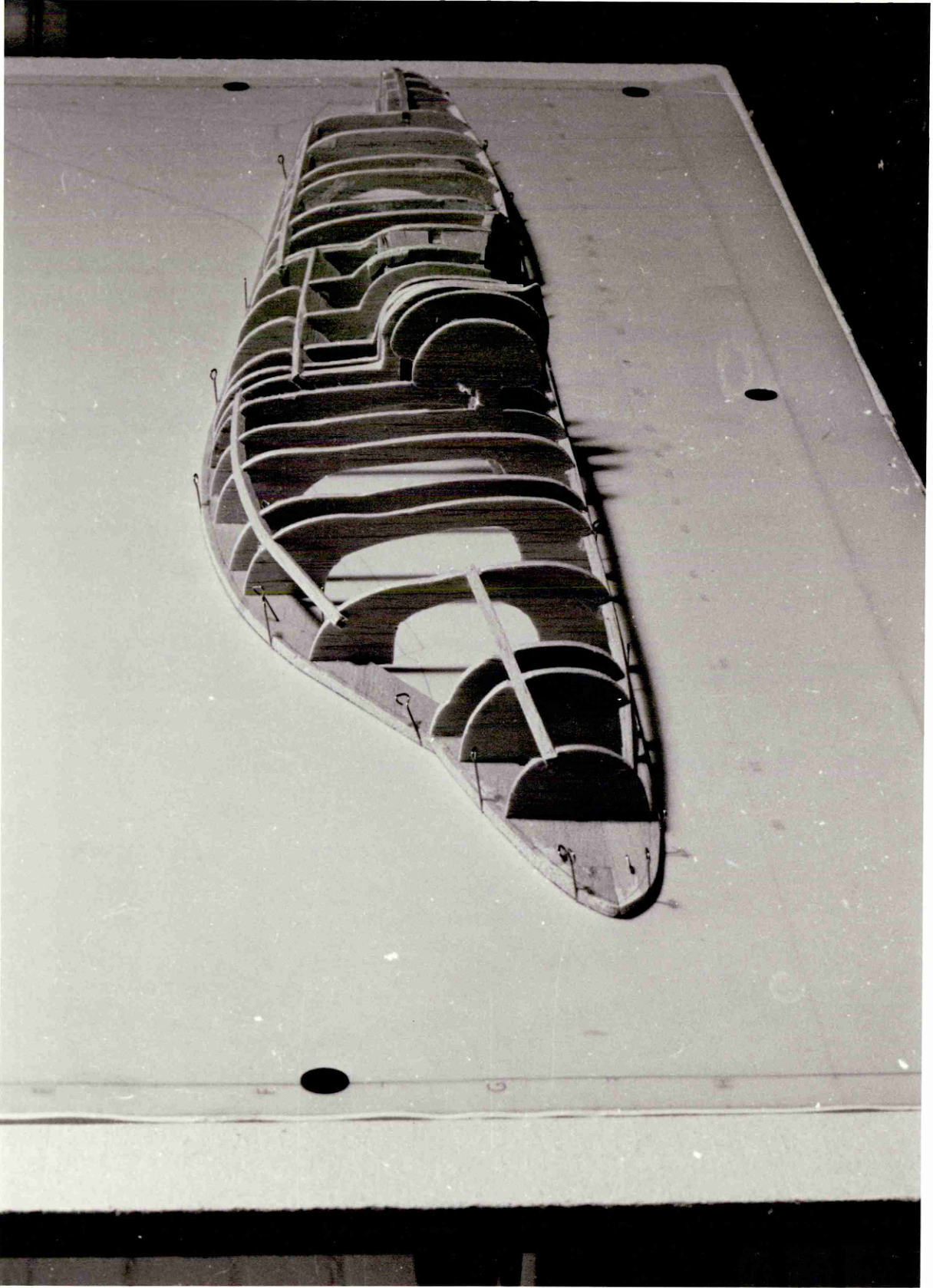


Plate 3. Model fuselage construction technique.

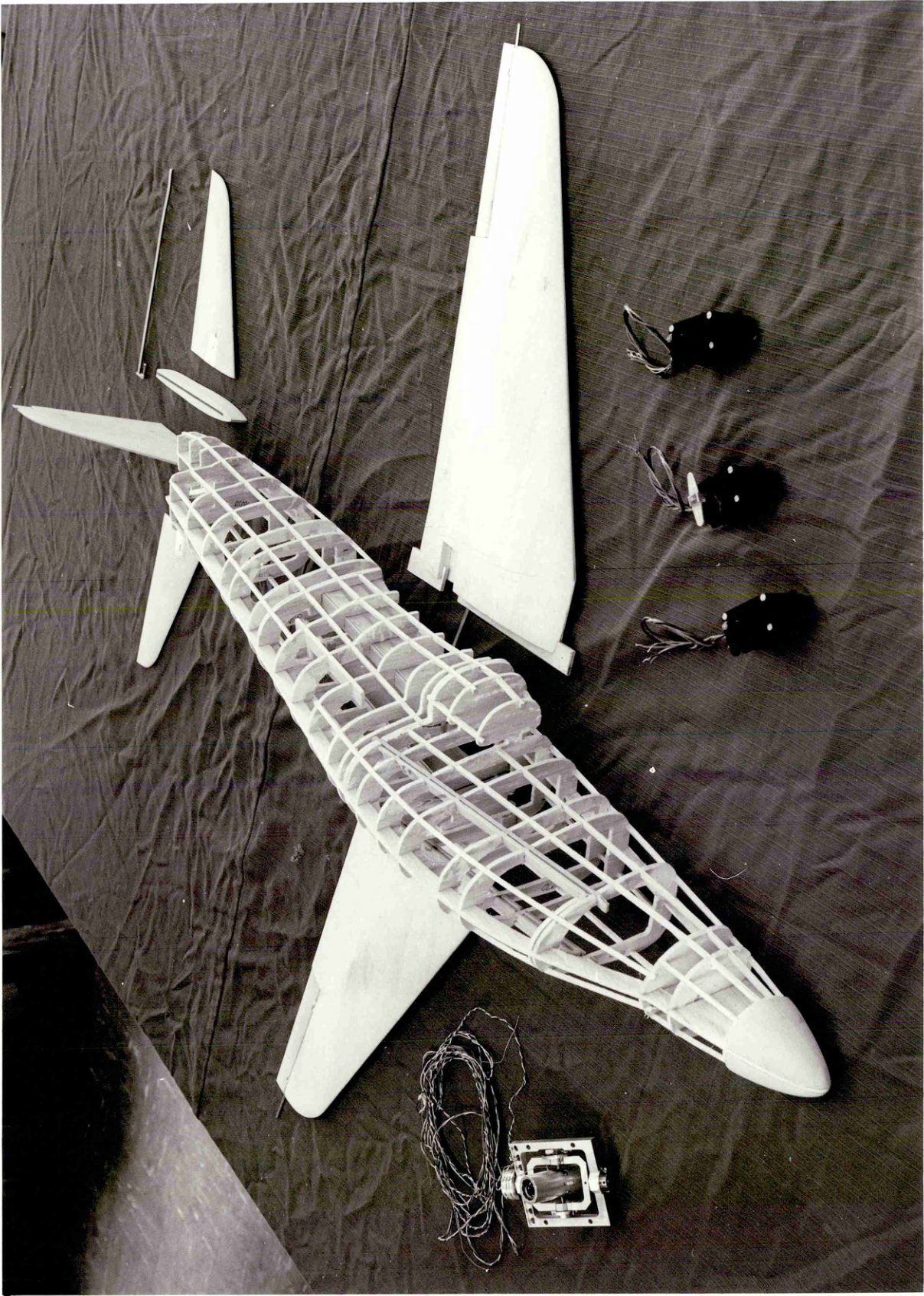
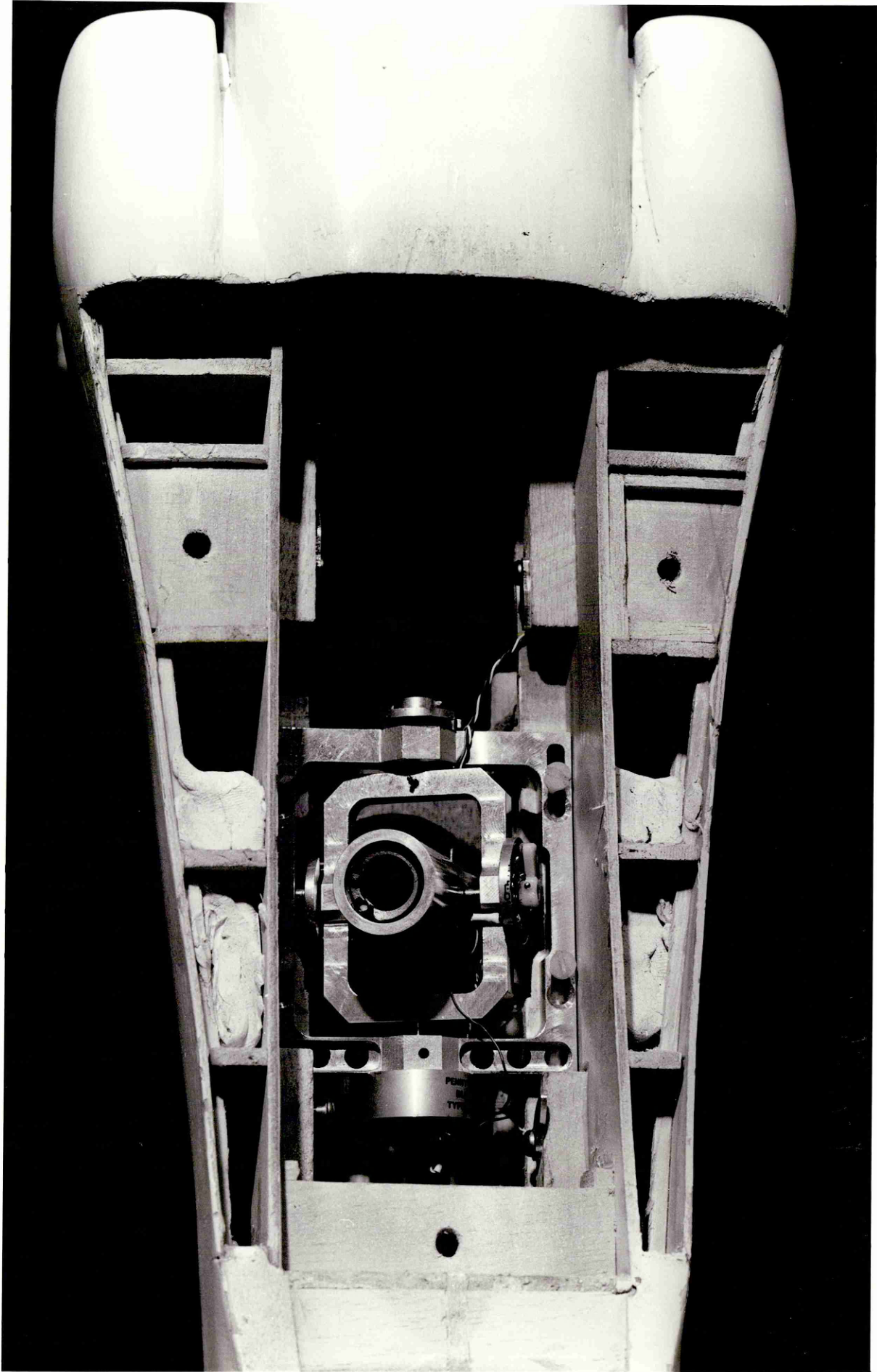


Plate 4. Component layout of the model with construction detail.



- Plate 5. Wing and gimbal system attachment points on model fuselage.

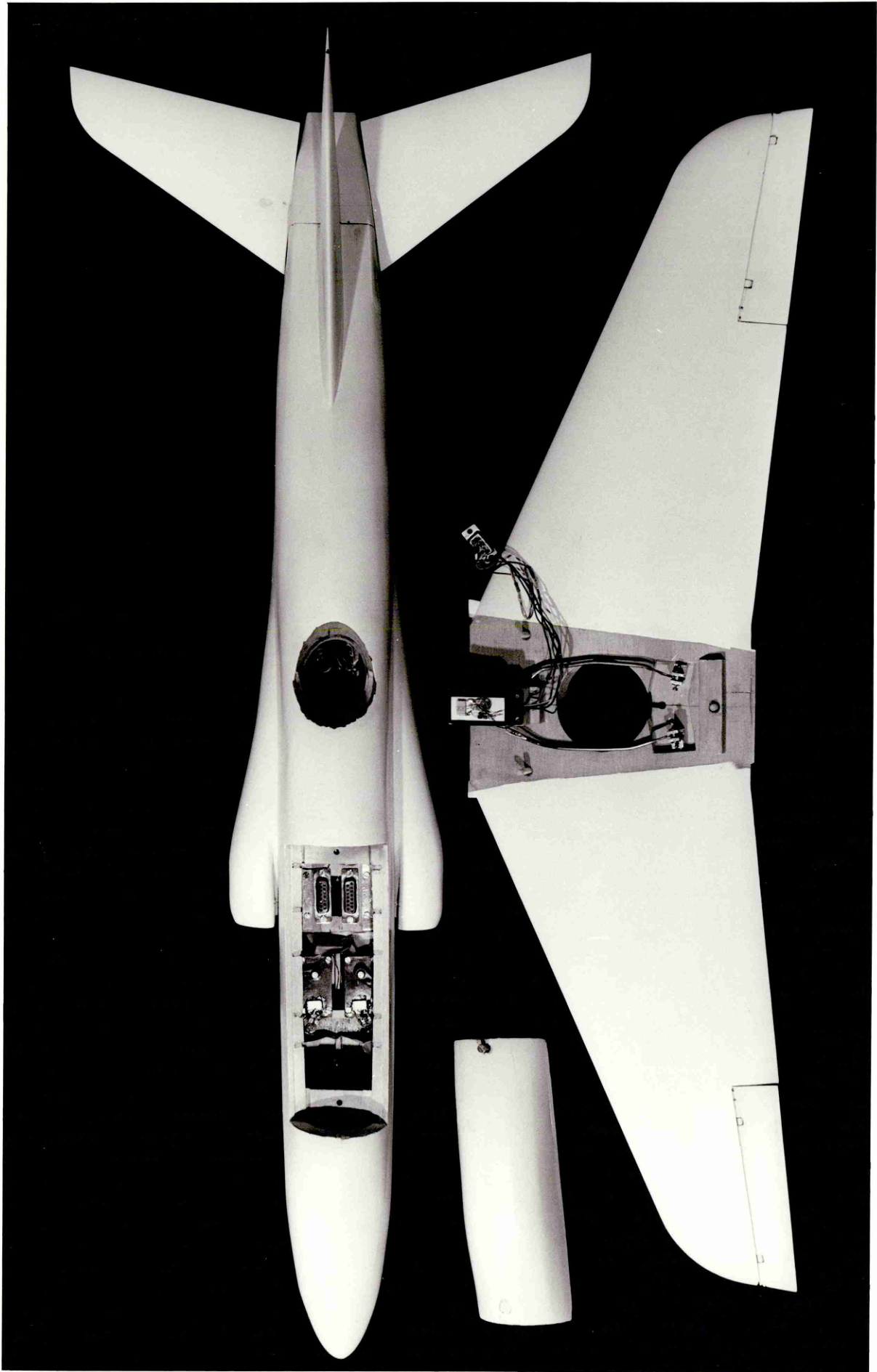


Plate 6. The completed model with detachable parts.

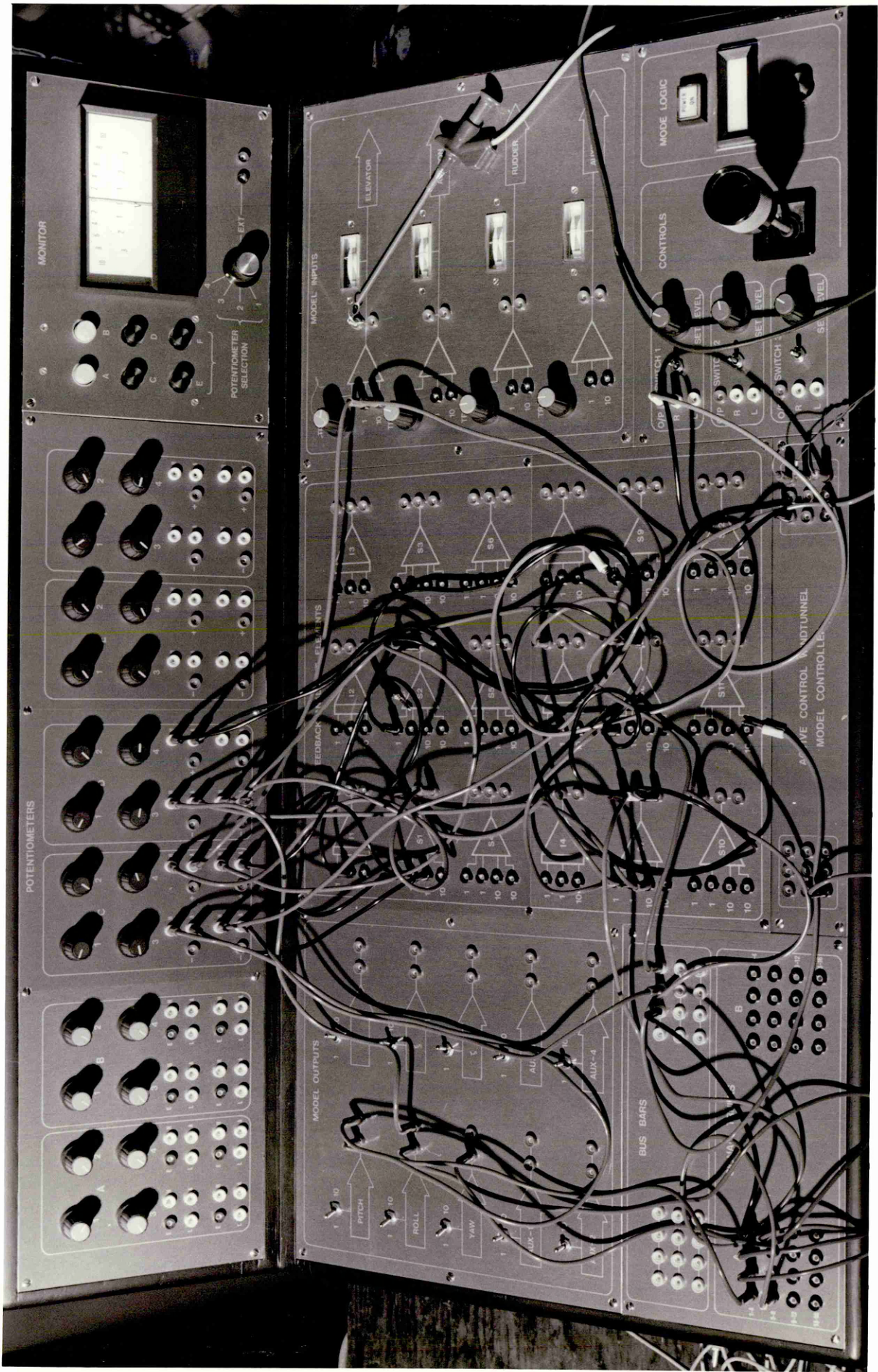


Plate 7. The electronic control unit.

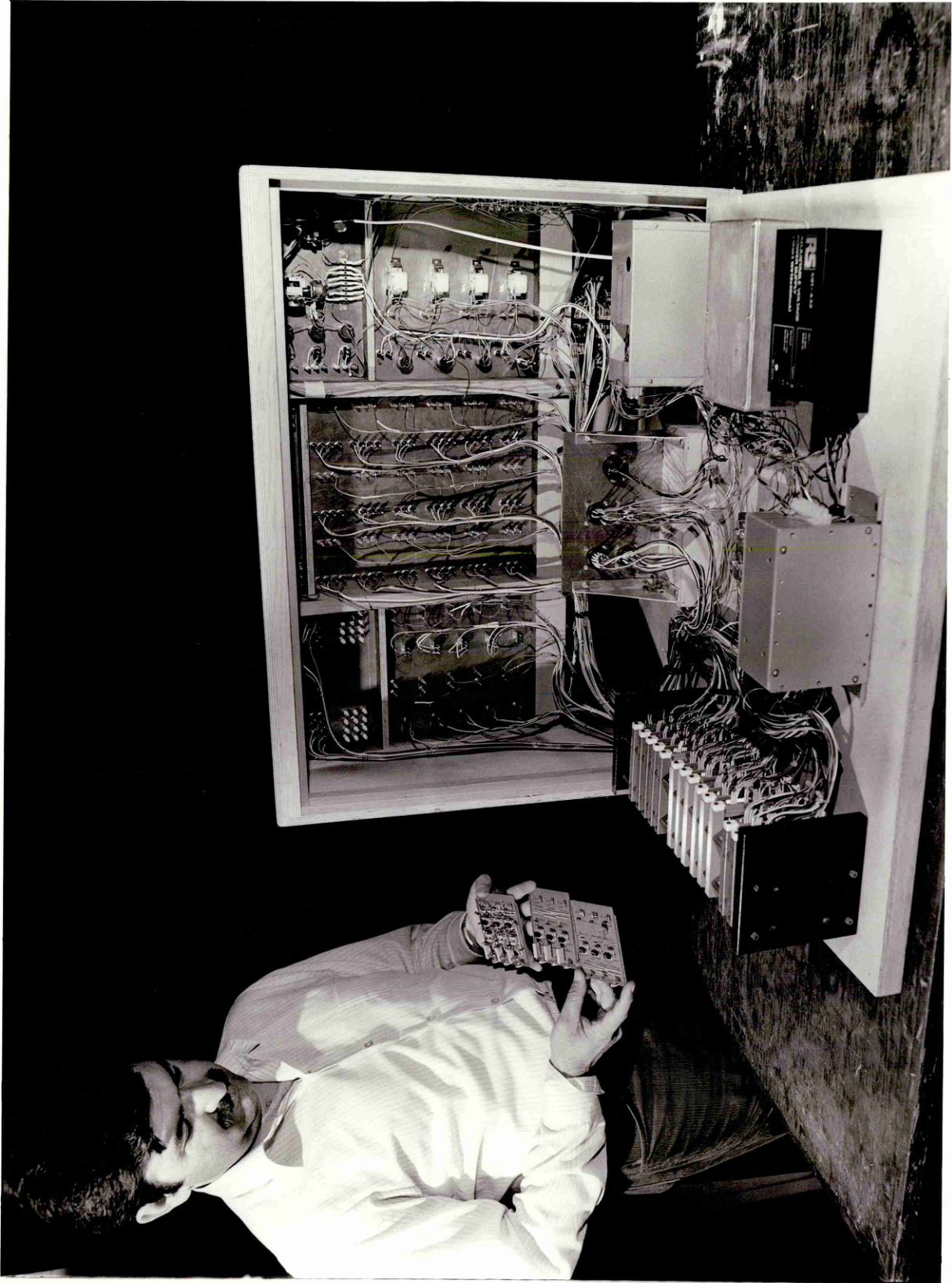


Plate 8. Interior view of the electronic control unit.

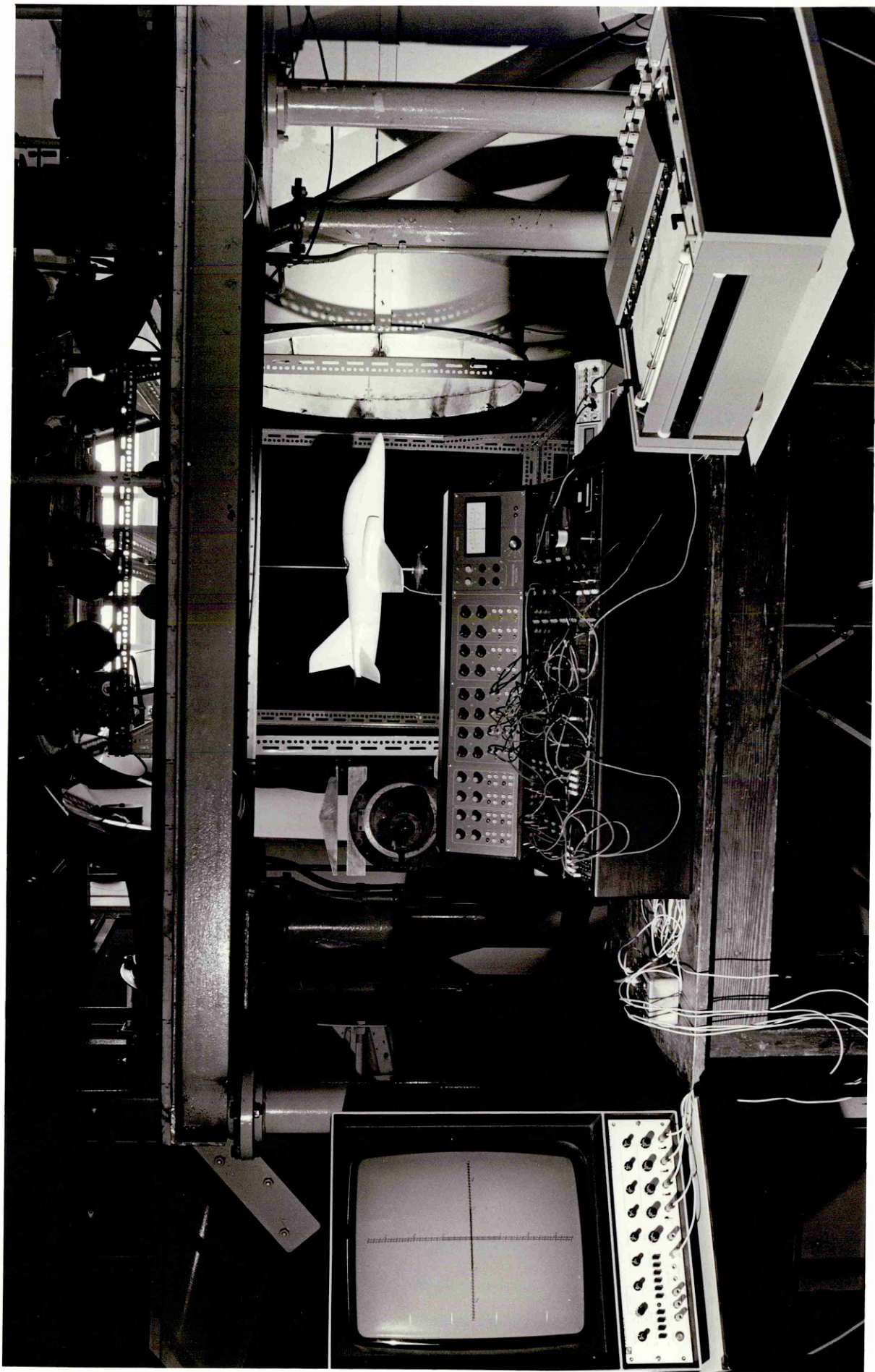


Plate 9. Experimental set-up.

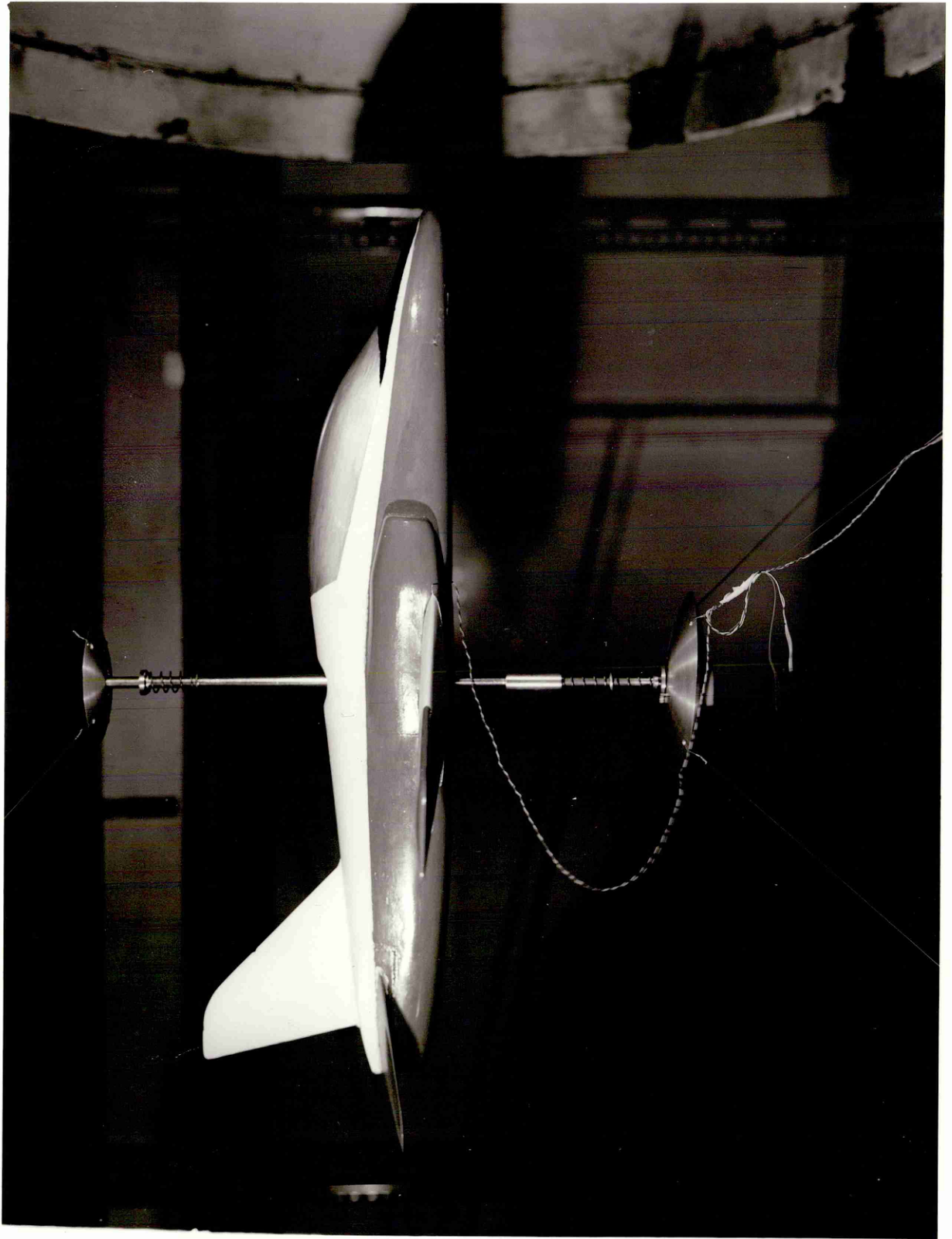


Plate 10. Model flying in the wind tunnel.