

Modified Dynamic Programming for Asteroids Belt Exploration

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Abstract

In the past, space trajectory optimisation was limited to the design of space transfers to single destinations. More recent mission concepts however present the challenge to target multiple destinations; be it for science, exploration or even exploitation. This paper focuses on space trajectories that aim to visit multiple main-belt asteroids with one single spacecraft. The trajectory design of these missions is complicated by the fact that the asteroid sequence is not known a priori, but it is the objective of the optimisation problem. Usually, these problems are tackled as global optimisation problems under the formulation of Mixed-Integer Non-Linear Programming, on which the design variables assume both continuous and discrete values. However, beyond the aim of finding the global optimum, mission designers are usually interested in providing a wide range of mission design options reflecting the multi-modality of the problem. The paper describes a multi-fidelity design pipeline that enables a consistent exploration of all feasible asteroid tours with a given set of mission boundary conditions (e.g., launch window, mission duration, Δv). The consistency of the exploration is ensured by a deterministic search scheme which retains the n -best paths at each stage of a dynamic programming process. Bellman's principle of optimality is made applicable by transcribing the search space into a series of discretized events defined by asteroid index and fly-by epoch, as well as ensuring optimal substructure property with the choice of transfer model and design variables. The method is deployed to explore asteroid tour opportunities visiting 10 or more main belt objects and mission boundary conditions compatible with medium or discovery class missions. 79 million such trajectories are found with launch date in 2037.

1. Introduction

New technologies, as well as more daring approaches to space operations, are giving rise to the challenge of missions that target several or, potentially, many different destinations; be it celestial objects, such as different moons or asteroids, or simply different orbital configurations. NASA's Dawn mission, which visited two different minor planets of the main asteroid belt [1], is one such example. Nevertheless, many other more ambitious multi-target mission are being studied, planned or even launched, such as ESA's JUICE [2] with more than twenty close passages to Jovian moons; commercial concepts for On Orbit Servicing [3], or asteroid tour missions such as Lucy [4], CASTAway [5] or MANTIS [6], which plan to visit many asteroids in one single spacecraft.

Designing the spacecraft trajectory that visits multiple orbital waypoints (planets or specific orbits) is a notoriously challenging problem. This is due to the potentially very large search space of the design problem, as well as its multimodal nature. This complexity spawns from the fact that the selection and visiting order of the waypoints is not known a priori, but it is instead part of the optimisation itself, leading to a mixed-integer optimisation problem [7, 8]. In such problems, a set of integer parameters encode the sequence of orbital waypoints to visit, while continuous variables define all the necessary operational aspects to course the path visiting the given set of waypoints (e.g., departure times, time of flights, thrusting arcs, fly-by configurations, etc).

This paper focuses on the spacecraft trajectory design of a tour within the main asteroid belt (MAB). The term tour refers here to a mission aiming to visit not one but multiple objects. There is a clear scientific interest in exploring the MAB to improve our understanding of the evolution of the Solar System. This could be achieved by establishing a detailed map of asteroid composition in the main belt region [5]. The mission concept explored in this paper is thus that of a multiple fly-by mission, similar to those of Lucy [9], MANTIS [6] and CASTAway [10]. The aim is to pass-by as many asteroids as viable, to obtain key information about composition and geomorphology of the

visited objects [5]. A mission that could fly-by 10 asteroids, for example, would already double the number of main belt bodies explored so far. Moreover, such a mission could possibly be done within a short mission lifespan and a relatively constrained budget [10, 11].

There have already been several propositions of asteroid tour mission problems in the literature, such as the multiple asteroid rendezvous problem proposed for the 2nd Global Trajectory optimisation Competition (GTOC) [12] or mission design-oriented studies such as those in [10, 13, 14]. Any of these examples describe constraints and boundaries that are specific to the problem at hand, but all require dealing with comparable complexities in a similar dynamical framework. It is certainly the enormity of the search space and its highly multimodal nature that has made asteroid tour trajectory design a rather ubiquitous challenge in optimisation competitions, such as those of GTOC [15] and CTOC [16]. Most of these previous problem propositions tackle the asteroid tour problem as a single-objective optimisation problem [17], with fitness functions ranging from spacecraft performance criteria (such as Δv , propellant mass, time of flight, etc), to the number of visited objects. Multi-objective optimisation approaches are also a logic extension, coming from the fact that the *goodness* of a realistic trajectory would rarely be influenced by one single criteria [17], but by multiple and conflicting objectives, such as Δv and time of flight [18].

Nevertheless, performing a mission analysis study is a substantially different challenge than that of tackling a pure optimisation problem. For a start, it is often the scientific outcome that really defines the *goodness* of a mission proposal [11]. One may still attempt to quantify the potential science return of an asteroid tour, based on, for example, the diversity of asteroid families visited or their spectral type [10, 19]. Yet such heuristic fitness criteria will certainly provide only a highly uncertain measure of the science claimed by a given mission proposal, which ultimately will anyway be judged by a panel of expert science evaluators [11]. Moreover, a true exploration of the search space is still required during feasibility studies, in contrast to the localization of only some isolated notable solutions.

Therefore, a realistic mission study may be more akin to a constraint satisfaction problem (CSP) [20], where one wants to find one, multiple or all design solutions that are compliant with a set of mission-driven constraints; and among those also an optimal, or at least good, solution with respect to an objective function [17]. In the context of asteroids missions, a CSP is interesting because the available range of different feasible asteroid tours is relevant for an accurate scientific assessment of the mission concept. A CSP solution of an asteroid tour problem is the set of design variables describing a meaningful fraction or, if possible, all the feasible asteroid tours. All of these tours would thus represent different asteroid sequences satisfying the same mission boundary conditions (e.g., launch date, mission duration) and spacecraft operational requirements (e.g., Δv , encounter conditions, etc). Ultimately, there are clearly two needs that should be addressed when in the process of a preliminary design of asteroid tour mission. Firstly, one may wish to find the optimum of the mixed-integer problem at hand with respect to an objective function (e.g., the Δv cost of the tour), such that the minimum necessary spacecraft performance that satisfies some mission objectives is established. Secondly, one wants to solve a CSP to provide adequate exploration of mission design options, such that science objectives can be adequately studied and traded-off, as well as the design flexibility can be analysed (i.e., extent of launch windows, spacecraft requirements, de-scoping options, etc).

There is an abundant literature of proposed algorithmic processes to tackle path-planning mixed-integer optimisation problems such as that of an asteroid tour design [7, 17, 21, 22]. All the proposed techniques can generally be categorised as either deterministic or stochastics. Deterministic approaches take advantage of discretization of the search space and the consequent tree-graph data structure of the problem. Many deterministic strategies are also *complete*, such as depth first (DF) and breadth first (BF) enumeration [23], meaning that they guarantee to find the optimal, as well as all feasible solutions, by systematically exploring all search space. Branch & Bound [24], and other variants, allow pruning the search space by ruling out expansion of branches that are either known not to be optimal, or have some unfeasibility on them. In Backtracking Branch & Bound [25-27], for

example, a backtracking heuristic corrects the latest decision that led to an unfeasible solution, improving the runtime for full exploration of CSP formulations.

However, as problem complexity increases and dimensionality of the parameter space grows, the computational costs associated with complete problems becomes difficult to handle and incomplete searches become the obvious alternative. While many deterministic incomplete algorithms have been proposed [28], Beam Search (BS) is a notable example in this branch of the taxonomy. BS [29] follows a BF exploration strategy, however it bounds its computational effort by ranking branches at any one level and capping the number that can be further expanded. Nevertheless, stochastic approaches such as Genetic Algorithms [30], Particle Swarm Optimization [31] and Ant Colony Optimization [32], are most often the techniques of choice to tackle the most arduous path-planning mixed-integer optimisation problems [33]. Lastly, one can also distinguish two broad strategies at solving the mixed-integer optimisation; either the problem is tackled in its full form (i.e., black-box) [8], via, for example, nested-loop optimisation [34], or in a multi-level approach, on which, after a suitable graph transcription, the integer optimisation is tackled first, and the full problem is refined a posteriori [35].

Despite tackling a highly complex problem, this paper presents a novel deterministic search scheme based on dynamic programming principles [36], which retains and expands the n -best solutions reaching each unique and separable node of a tree-graph. The process, referred thereafter as Modified Dynamic Programming or n DP, is complete for an optimisation formulation, i.e., guarantees the localization of the global optima, and also renders a good approximation of the CSP solution, by providing a large diverse population of feasible trajectories. n DP is embedded within a multi-fidelity pipeline (i.e., multi-level approach) that allows exploring large search spaces of asteroid tour trajectories and returning millions of long sequences of asteroid fly-bys for different launch dates and visiting different regions of the MAB. Section 2 presents the asteroid tour design problem as a black box, featuring the top-level fidelity of the trajectory design proposed. Section 3 provides further description on the tree-graph structure of this kind of problems and how these can be defined in such a way that Bellman’s optimality principle applies. Section 4 presents the multi-fidelity pipeline, composed of three consecutive optimisation problems; (1) a multiple-gravity assist trajectory to reach the MAB satisfying constraints in mission duration and total Δv , (2) a combinatorial only problem where sequences of asteroids are inserted into trajectories computed in step (1) and, finally, (3) a refinement optimisation of the continuous design variables back into the highest fidelity dynamical framework. Section 5 presents the n DP approach used to solve step (2) of the multifidelity pipeline. Lastly, Section 6 showcases the overall process by providing an overview the CSP solution of feasible asteroid tours compliant with the mission boundary conditions of the ESA’s latest science programme call for medium class missions [11].

2. Main Asteroid Belt Exploration Design Blackbox

Asteroid tours thus refer to a mission that performs a long sequence of asteroid encounters with one single spacecraft. Specifically, this paper tackles the problem of designing an asteroid tour to explore the main asteroid belt [5]. The design of the tours presented here are also sought to be broadly compatible with the mission boundary conditions typically imposed by ESA for its medium class mission calls [11, 37]. Figure 1 shows an example of such main asteroid belt tour missions. The tour begins with a direct escape from Earth, defined through $\vec{v}_\infty$ vector, which is then followed by 12 fly-bys. Two of these fly-bys are gravitational assists with Mars and Earth, which increases the orbital energy of the spacecraft and allows it to obtain a better reach within the MAB. Two deep-space manoeuvres (DSMs) are used after Mars and Earth fly-bys. The remaining fly-bys, or encounters, are all asteroids; all of which provide negligible gravitational perturbations.

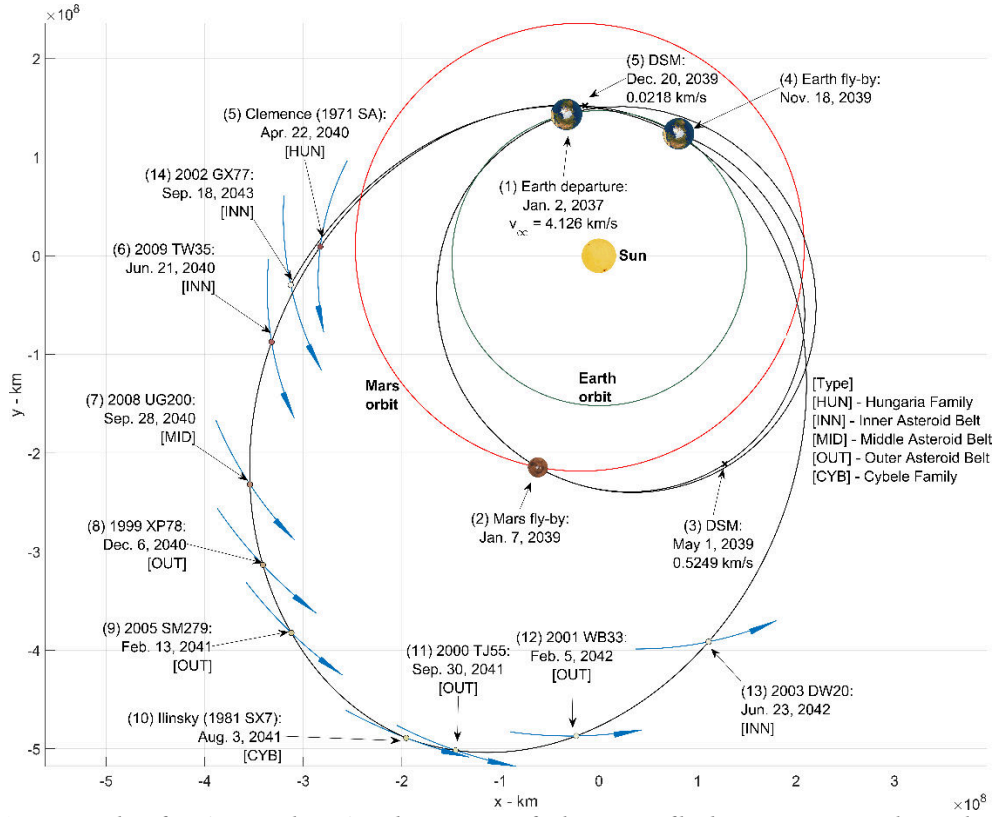


Figure 1. Example of MAB exploration by means of planetary fly-bys at Mars and Earth. Arrowed lines represent asteroids paths.

The dynamic framework used to compute the asteroid tour in Figure 1 is the so-called multiple-gravity assist (MGA) with DSMs, also known as MGA-DSM model [38, 39], which considers a coast arc followed by a Lambert arc for each individual leg of the transfer. The initial coast arc is defined either by the escape $\vec{v}_\infty$ vector at Earth departure, or by 2 fly-by parameters for any other planetary body thereafter (see also Table 1). The DSM is then computed as discontinuity in the velocity at the end of the coast-arc and the velocity necessary to reach the following asteroid or planet at a given time. Table 1 summarises all the design variables necessary to define the asteroid tour as proposed here and the associated values for the specific example in Figure 1. For this example, vector X encodes a number $n_{obj} = 13$ objects (accounting for Earth departure), while vector y encodes a number $n_{cont} = 19$ continuous-varying variables, defining all the events necessary to characterize the trajectory between each object. The total Δv (i.e., sum of v_∞ and DSMs) is 6.68 km/s, while total Time of Flight (ToF) is 6.70 years. One should notice that if either more complex transfer options or more complex dynamical frameworks are implemented, such as low thrust or n -DSMs, the number of optimisable parameters would rise sharply. However, for the purposes of the present paper, the MGA-DSM model is deemed of a sufficient accuracy to provide good representative trajectories of asteroid tour missions for preliminary mission design.

Table 1. Input variables to describe an asteroid tour in the MGA-DSM model with reference to Figure 1 scenario. Total Δv and overall ToF for the mission are reported.

Integer variables	Description
$X_i, i = 1, \dots, n_{obj}$	Asteroids and planet integer identifier in the sequence
$X = [3, 4, 3, 1661, 64569, 64512, 8793, 35171, 1972, 27671, 55773, 29576, 48449]$	Earth (assumed departure from Earth) – Mars – Earth – Clemence (1971 SA) – 2009 TW35 – 2008 UG200 – 1999 XP78 – 2005 SM279 – Ilinsky (1981 SX7) – 2000 TJ55 – 2001 WB33 – 2002 GX77

Continuous variables	Description
$y_1 = t_0$	Earth launch date – Jan. 2, 2037
$y_{2,...,13} = ToF_{2,...,13}$	ToF to reach each of the objects encountered along the trajectory: $ToF = [735.4, 314.9, 155.5, 59.5, 100.1, 68.9, 69.3, 169.4, 57.1, 129.5, 137.5, 451.4]$ days
$y_{14,...,19} = \gamma_M, r_{p,M}, \alpha_M, \gamma_E, r_{p,E}, \alpha_E$	For each of the planetary gravity assist, Mars (M) and Earth (E) in the case of Figure 1, (γ, r_p) are the inclination and periapsis of the fly-by plane, respectively; α is the fraction of ToF at which a DSM is performed after a fly-by with the planet: $[\gamma_M, r_{p,M}, \alpha_M] = [4.1242 \text{ rad } 4.4786 \cdot R_M 0.3579]$ for Mars fly-by (R_M is Mars radius) $[\gamma_E, r_{p,E}, \alpha_E] = [0.06064 \text{ rad } 1.1953 \cdot R_E 0.1988]$ for Earth fly-by (R_E is Earth radius)
$\Delta v = 6.68 \text{ km/s}$ $ToF = 6.70 \text{ years}$	

The design variables in Table 1 represent thus the input variables of a function $f(X, y)$ that defines a full asteroid tour. The design problem would typically be defined as a mixed-integer non-linear programming (MINLP) problem, where function $f(X, y)$ may output a single or multiple objective quantitative criteria (i.e., fitness criteria) to be optimised, such as total Δv or total time of flight.

However, as argued earlier, beyond localising some isolated good solutions, here we are interested in also providing an extensive exploration of the feasible search space. Hence, a CSP formulation such as:

$$\begin{aligned}
&\text{Given a set of variables:} && (X, y) \text{ where } X \in \mathbb{Z}^{n_{\text{obj}}}, y \in \mathbb{R}^{n_{\text{cont}}}, \\
&\text{And their domains:} && X_{lb} \leq X \leq X_{ub} \\
& && y_{lb} \leq y \leq y_{ub} \\
&\text{And a set of constraints:} && \text{e.g., } f(X, y) \leq 0 \\
&\text{Find all, or as many as possible, sets of } (X, y) \text{ that satisfy all defined constraints,}
\end{aligned}$$

is argued to be also relevant.

3. Graph Structure and Separability of the Search Space.

Regardless the complexity of the search space for the continuous variables in Table 1, the catalogue of interesting objects in the MAB can already be very large, with more than 100,000 objects of potential interest to choose from [10]. Hence, the discrete combinatorial optimisation problem that arises from selecting the subset of asteroids to visit leads to a complex NP-hard problem, which is computationally intensive to solve [19, 23, 40].

To achieve a thorough and efficient exploration of these type of search spaces, this paper proposes an approach based on the application of dynamic programming. Firstly, for dynamic programming to be applicable, the search space must exhibit an optimal substructure [36], that is, the optimal solution to the problem must incorporate optimal solutions to related subproblems which can be solved independently [23]. The concepts of subproblem and solving independently refer here to the capacity to compute the fitness of a given part of the trajectory (e.g., Δv) without the need to know what the path was prior to arriving to that point. One should note that an object-to-object low thrust transfer, for example, would not respect the optimal substructure property because it would exhibit a substantial sensitivity to the path taken up to the relevant leg, since the acceleration of the spacecraft would depend on the propellant mass consumed up to that point.

Secondly, to apply dynamic programming the search space must be entirely composed of discrete variables. The MGA-DSM model [38, 39] proposed in Section 2 requires a minimum of 5 independent continuous design variables to fully define a transfer between two celestial objects (or 6 at the starting planet of a complete trajectory). Instead, as argued in Bellome et al[35], the Δv -defects model requires only 2 independent variables and thus efficiently reduces the dimensionality of the search space without much loss of search space information. In the Δv -defects model, a Lambert arc is considered for each object-to-object transfer in the sequence and hence only the times at each object are necessary. Velocity discontinuities between arriving and departing arcs, at a given object, are assumed equivalent to a DSM occurring immediately after the encounter. Further details on the model are provided in Appendix A. The point to highlight is that the Δv -defects model allows to transcribe efficiently the complex search space of the MGA-DSM model by discretizing only the time at which each encounter may occur [40]. Doing so makes the asteroid tour problem separable, as each discretized event P is defined by a fixed epoch t and celestial body ID such that $P_i = (t_i, ID_i)$, so that each event is unique. Note that if time of flight was used as independent design variable, rather than a fixed epoch t , then one would still need the full path to compute the ephemeris of the object to be visited. Furthermore, the convenience of employing Δv -defects is highlighted by the fact that each transfer is a solution in the MGA-DSM model, and therefore a simple refinement process allows to reoptimise a sequence into the full MGA-DSM, which may obtain at most some Δv reduction (generally small) [35] (see further discussion in Section 4.c).

The transcribed search space S can therefore be expressed as $S = \{P_1, P_2, P_3, \dots, P_m\}$, where m is the total number of discretized events, and its exploration can be carried out by noting that its structure corresponds to that of a graph. In graph theory, a graph G is defined by two finite sets V and E , such that $G = (V, E)$. The elements of V are the *vertices* of the graph, while the elements of E are the *edges* and correspond to the connection between a pair of vertices [41]. As the objective of an asteroid tour is to flyby the bodies, a triplet of events must be considered [42]. This is because the Δv required to connect any two asteroids is not unique but depends both on the current and previous trajectories linking successive asteroids through the departure and arrival velocity $\vec{v}_{departure}$ and $\vec{v}_{arrival}$, respectively, as shown in Figure 2. Note that, since each event has an associated epoch t , the time of flight is known. Hence, for the asteroid tour problem the structure of the graph must be defined in terms of vertices which are formed by two events, such that a vertex is (P_i, P_j) , with $t_i < t_j$. Doing so defines the possible connections between pairs of vertices as those where the first event of the second vertex coincides with the second event of the first vertex, thus respecting the tri-dependency and yielding a unique cost for each connection. The set of edges E of the graph is further defined by the fact that feasible connections will exist only for those pairs of vertices where the event dates respect a chronological order, that is, where the triplet of events fulfils $t_i < t_j < t_k$. Since every one of these pairs can be joined, the graph is *connected* [41]. In addition, the pairs of vertices are *ordered* (with respect to time) and the resulting graph is *directed* [41].

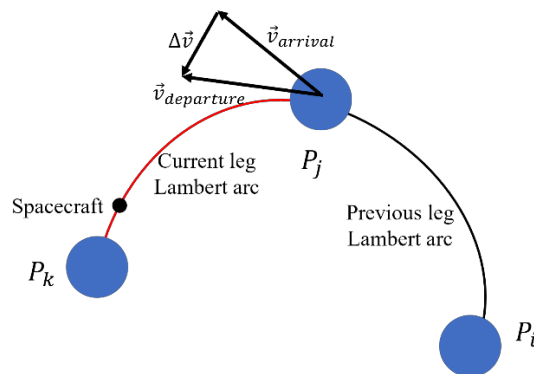


Figure 2. Graphical representation of the tri-dependency between events P_i , P_j and P_k . Spacecraft is moving from P_j to P_k having previously visited P_i .

The graph can be represented by means of the graph adjacency matrix $A = (a_{ij})$, which is a square matrix of size $|V| \times |V|$ where the rows and columns are the vertices of the graph and the information regarding the feasible edges is defined in the elements of the matrix, so that $a_{ij} = 1$ if two vertices are *adjacent*, that is, if the edge is feasible and 0 otherwise [43]. Furthermore, since the cost (e.g., Δv) of each edge is unique, a *weight function* W mapping each edge to the associated cost can be defined and thus the graph is *weighted* [41]. Consequently, the elements of the weight function (which can be defined by a matrix with the same dimensions as A) correspond to the cost (ΔV) of the respective edge and therefore $w_{ij} = \Delta V_{ij}$ or $w_{ij} = NaN$, enabling the definition of a “score” for each edge. Although for the case of the asteroid tour problem being considered in this paper the driving parameter is the ΔV , the applicability of the weight function can be extended to any mission design parameter that is deemed interesting, since the feasibility of the edges and their associated cost can be defined in terms of, for example, the sun phase angle or the hyperbolic excess velocities of the flybys. In addition, this “score matrix” can also be used to prune the feasible search space by enabling to identify edges which are feasible but not interesting from a mission design perspective such that, for instance, one may want to retain only the edges with a ΔV cost lower than a certain bound, thus reducing the dimensions of the search space to explore.

The directed graph, or *digraph*, G defined previously is acyclic, since edges exist only for vertices respecting chronological order and thus no cycle can be formed. This property, combined with the fact that the graph is *connected* implies that G is a *tree* [43]. Therefore, single- or multi-objective tree-exploration strategies can be applied to traverse the tree and scan the search space to obtain the set of discrete variables (i.e., sequence of bodies, flyby epochs etc.) characterizing feasible asteroid tours. The results presented in this paper were obtained by applying a modified dynamic programming approach, explained in Section 5.

4. Asteroid Belt Tour Design Pipeline

Solving the back-box asteroid belt tour problem as described in Section 2, with no a-priori knowledge of the design variables, would only be feasible for problem formulations with very small search domains for both the integer and the continuous-varying variables (X, y). Since the purpose of the paper is instead to explore large search space domains, the paper implements a multi-level approach process, on which the mixed-integer formulation of the full asteroid tour is solved via a multi-fidelity pipeline refinement that manages this complexity more efficiently. This pipeline is composed of three consecutive optimisation problems, which are described in detail in the following subsections. Step 1 tackles the problem of a multiple gravity assist trajectory design to maximise the staying time within the MAB with constrained maximum mission duration and total Δv . Step 2 implements a combinatorial only optimization where sequences of asteroid fly-bys are inserted into each solution found in Step 1. The possible asteroids to fly-by are identified with a scan of their minimum orbit intersection distance (MOID) and the epoch at which the asteroid passes through its MOID points. Finally, Step 3 refines Step 2 trajectories by re-optimising the continuous design variables only.

a. Step 1: MGA Trajectories to Explore the Main Asteroid Belt

It should be noted that the (X, y) vector described in Section 2 also contains the information necessary to describe a multiple planetary gravity assist trajectory. The few design variables (both integer X and continuous varying y) defining the planetary encounters will disproportionately contribute to the chances of the spacecraft to encounter main-belt asteroids at a low Δv expenditures [44]. This is mostly because a planetary encounter will greatly affect the overall energy and orientation of the spacecraft trajectory, hence affecting its capability to reach the MAB and therefore exploring it. Ultimately, different MGA sequences will allow the spacecraft to enter the MAB with different trajectories and spend more or less time within its bounds or reaching deeper into it. Figure 3 shows three specific examples of planetary encounters to access the MAB. It showcases a simple Earth (E) direct escape with no planetary encounters, as well as an Earth-Venus-Earth (EVE), an

Earth-Mars (EM) and an Earth-Mars-Mars (EMM). The dotted lines represent the MAB limits (assumed to be 1.8 AU minimum and 3.5 AU maximum), and so it can be observed that each transfer opportunity features different paths within it.

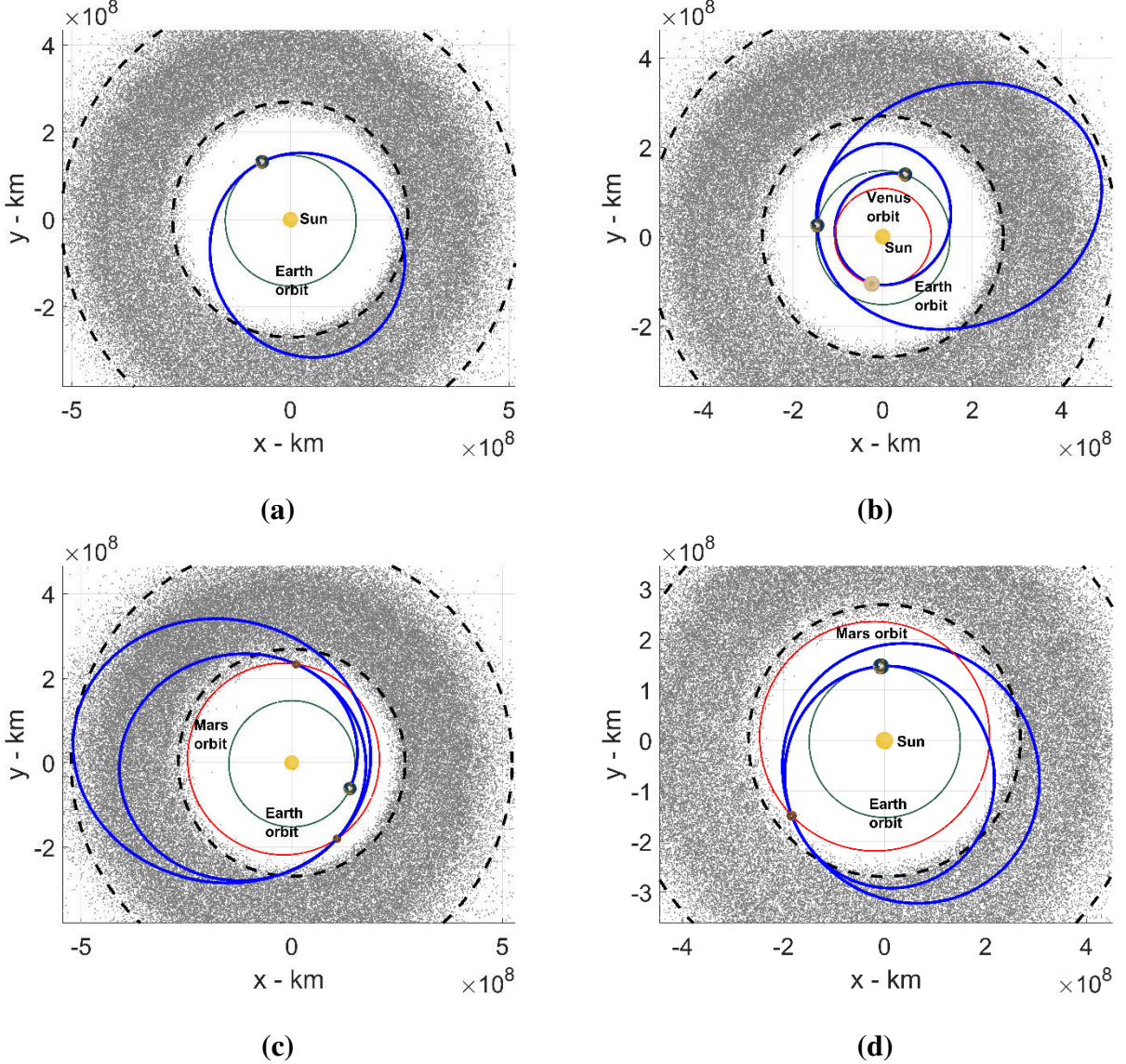


Figure 3 Four MGA trajectory options with E alone (a), EVE (b), EMM (c) and EM (d). Grey points represent asteroids in the dataset, and dotted lines are the limits of the MAB.

It should be recalled that since we are purposely targeting asteroid tour designs potentially compatible with medium-size class missions [11], launch capabilities compatible with medium-lift launchers and relatively short overall mission durations (< 7 years) are both to be expected. Hence, while circularization within the MAB cannot be feasibly considered, a range of elliptic orbit options are shown to allow a fast and decent reach within the MAB [10, 45].

To generate baseline MGA trajectories to access the MAB region, planetary sequences are analysed that include all permutations of Earth, Venus, and Mars, with up to 2 planetary swing-bys. The search for trajectories is performed by applying the pipeline presented by authors in [17, 35]. A multi-start search [46] is used to scan launch year 2037 (step size of 5 days, thus 74 unique launch dates are analysed), compatible with latest ESA M-class mission call [11]. For each date scanned, a tree-based multi-objective dynamic programming (MODP) algorithm [35] is applied to look for trajectories that maximise the time spent in the MAB (Δt_{MAB}) while minimizing the Δv consumption. The outcome of this process is a set of 1148 different trajectories, whose performances are summarized in Figure 4. Figure 4 shows the $\Delta t_{MAB}/\Delta v$ ratio (y axis) for each of these trajectories

along the 2037-2038 launch window (x axis). Such ratio allows to identify promising MGA strategies that spend more time in the MAB, thus likely allowing a sustained exploration of the region, while having reduced Δv cost. As a general understanding of Figure 4, one observes that using a single planetary fly-by, i.e., EV or EM, allows to reduce the overall Δv consumption, as no phasing manoeuvres are needed to encounter a third planet, with EM having higher $\Delta t_{MAB}/\Delta v$ ratio than EV, as the spacecraft is launched directly towards high-apoapsis regions. Among strategies that use two planetary fly-bys, EVE provides the highest $\Delta t_{MAB}/\Delta v$ ratio, although not consistently throughout the full launch window analysed.

Each trajectory crosses therefore the MAB at different times and depths and it is impossible to a priori known which one would enable the *best* asteroid tour. Thus, this full set of 1148 MGA sequences is retained for further analysis in Step 2, as described in the following subsection. Results of the pipeline process on these 1148 MGA sequences are described in Section 6.

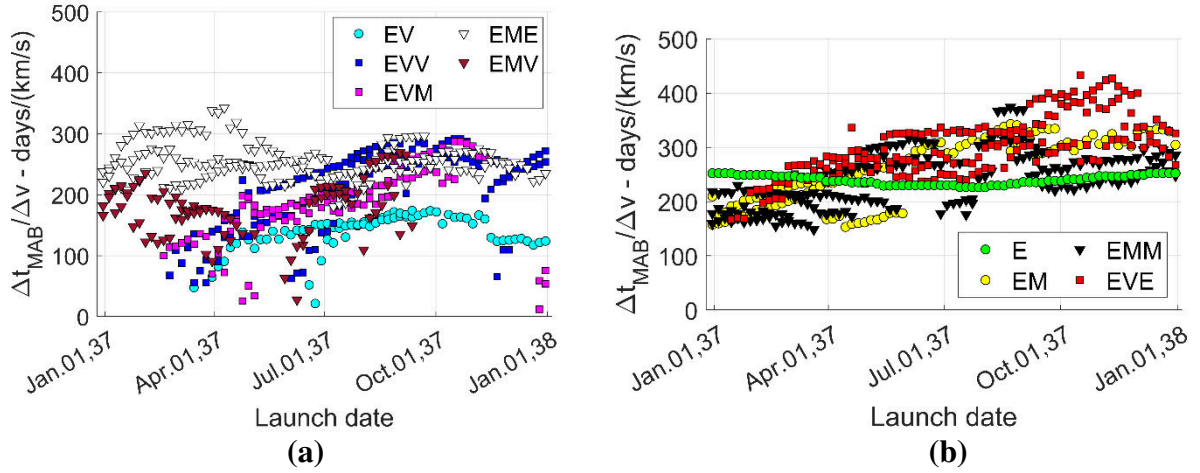


Figure 4. Time accumulated in the MAB per total Δv with respect to the launch date for different strategies: EV, EVV, EVM, EME, EMV (a), and E, EM, EMM, EVE (b).

b. Step 2: Asteroid-belt Tour Transcribed Problem

The outcome of Step 1 is a catalogue of MAB-reaching trajectories that will act as a substrate for the definition of a catalogue of asteroid belt tours. Each transfer identified in Step 1 is thus defined by a planetary sequence and the epochs for each of these planetary encounters. Each of these transfers can now be *doped* with as many asteroids as can possibly be added in the path, while still satisfying a set of user-defined mission constraints (e.g., Δv , mission duration, etc). This subsection provides an overview on the Step 2, the asteroid fly-by path-planning. This overview first discusses the asteroid data set used, to then focus on the identification of a minimum size set of discrete asteroid encounter epochs, which is here defined based on the minimum distance encounters.

As of March 2023, slightly over 1 million asteroids in the MAB are known, according to the Jet Propulsion Laboratory Small-Body Database¹. Exploring all the potential, for example, 10-asteroids long sequences within the known population would require computing trajectories for more than 10^{53} different sequences. Computing this number of trajectories would take many orders of magnitude more than the age of the universe and, therefore, there is an obvious need to prune out the overall catalogue and target only a sensible set of interesting MAB objects. Here, a pruned database of $\sim 102,000$ main belt asteroids is used to search for main-belt asteroid sequences [10, 47]. This database provides a pre-filtered population of MAB objects: all asteroids larger than 10 km in diameter are retained in the database, while smaller objects are pruned out maintaining a representative diversity of asteroids in size and orbital distribution. The database can be accessed in

¹ https://ssd.jpl.nasa.gov/tools/sbdb_query.html, last accessed March 2023

[48], and further information on the filtering process can be found in [49]. The orbital elements and other physical information of the asteroids in the database is downloaded from Jet Propulsion Laboratory Small-Body Database. It should be noted that an asteroid set of $\sim 102,000$ objects is already larger than any GTOC-related asteroid set [16] and the addition of one or multiple gravity assists with planets adds further complexity to the problem.

The transcription, or discretization process, as described in Section 3, can now be initiated. Recall that, at this stage, two different sub-problems need to be solved to design an asteroids tour. Firstly, the right sequence of asteroids (i.e., completing the integer vector variable X), among the $\sim 102,000$ targets, need to be appropriately chosen, which requires solving a discrete combinatorial problem. However, the quality of a given asteroid tour can only be assessed after identifying the actual dates for each asteroid encounter (i.e., continuous-varying vector variable y). It should be noted that the Δv cost of an asteroid tour will be highly sensitive to the actual dates of the asteroid encounters. A priori, the possible dates for each asteroid fly-by (t_{fb}) could be any date that satisfies $y_1 < t_{fb} < y_1 + TOF$ (TOF being the overall mission time of flight). The fact that the two sub-problems are tightly associated defines the crux of the asteroid tour problem and differentiates it from other classic combinatorial problems, such as the Travelling Salesman Problem (TSP).

At this point, and following discussion in Section 3, one may define a set of unique nodes as each of the asteroids in the set (i.e., $\sim 102,000$ targets) and an appropriate discretization of the epoch time vector t , for example $[y_1: 5 \text{ days}: y_1 + TOF]$. However, given that asteroid belt tours are built upon specific trajectories reaching the MAB, a much more efficient approach would consider only the asteroids that are close to the spacecraft at any one point during the trajectory, since these encounter opportunities will define the set of feasible objects to visit and the encounter times. Therefore, an analytical algorithm identifying the minimum orbital intersection distance (MOID), combining both [50] and [51] algorithms, is implemented to identify asteroids within a MOID range, as well as their true anomaly at the epoch of the MOID point crossing. The distance between the spacecraft and the asteroid at the epoch at which the asteroid crosses its MOID point is here used as first estimate of the asteroid minimum distance (note multiple asteroid epochs may exist within the spacecraft TOF).

Algorithm 1 summarizes the main steps of the transcription process. For any given asteroid in the catalogue, the algorithm computes the MOID i.e., the minimum distance between the asteroid and the spacecraft reference orbit on the leg. Then, the distance between the position of the asteroid and the spacecraft at the asteroid's MOID point crossing epoch is computed, which will be used as a discriminator of the actual close approach distance. If the latter is less than a pre-defined threshold d_{thr} , then the given asteroid is considered as a candidate object to be potentially reached. The procedure is repeated for all the asteroids in the $\sim 102,000$ -object catalogue. Algorithm 1 is thus used to compute a sub-set of asteroids whose orbits intersect the spacecraft trajectory close enough to be suitable targets, with respect to the distance threshold.

Algorithm 1. Pseudo-code for computing potentially reachable asteroids and their MOID epochs with respect to a reference trajectory.

```

1:   Load reference MGA trajectory, the population of MAB objects and  $d_{thr}$ 
2:   for each leg on the MGA trajectory do:
3:     for each asteroid in the dataset do:
4:       Compute the MOID between the leg and the asteroid orbit
5:       Compute spacecraft-asteroid distance at asteroid's MOID-point epoch
6:       if the distance is less than threshold  $d_{thr}$  then
7:         Save the asteroid and its MOID-point crossing epoch
8:       end if
9:     end for
10:  end for

```

Figure 5 shows an EM example trajectory and its close-by asteroids at their MOID point epochs with varying values of distance threshold d_{thr} . The set of distance threshold represented in the figure

includes d_{thr} of 0.03 AU, 0.04 AU, 0.05 AU, 0.10 AU and 0.15 AU. One can thus prune out all asteroids that do not satisfy a given distance threshold d_{thr} , which for the five thresholds in Figure 5 will leave an asteroid dataset containing 49, 98, 158, 562 and 1026 asteroids, for d_{thr} of 0.03 AU, 0.04 AU, 0.05 AU, 0.10 AU and 0.15 AU, respectively. Recall that, from Algorithm 1, each asteroid in these sets has also an epoch associated with it, which identifies the epoch at which the asteroid crosses its MOID point. This uniquely identifies each asteroid with the time of its possible fly-by. The approach implemented by Algorithm 1 thus not only allows reducing the asteroid data set enormously, but also identifies one single discrete date of encounter for each feasible asteroid.

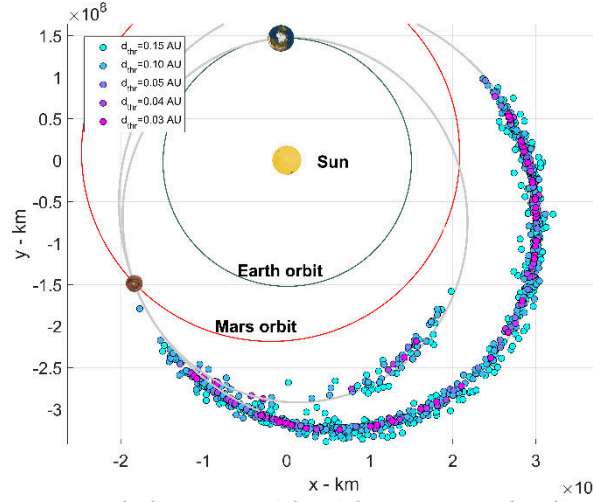


Figure 5. Reference trajectory and close asteroids with respect to the distance threshold. Spacecraft trajectory is in grey.

As discussed in Section 3, even if the problem can be posed in a separable manner, the enormity of the search space may still prevent the use of complete search algorithms, such as a dynamic programming-based search. The application of Algorithm 1 effectively reduces the number of unique events P (i.e., pairing of object ID and epoch) to very manageable numbers onto which dynamic programming can be easily applied. Further discussion on this point, as well as details on the dynamic programming-based search implemented can be found in Section 5. The following and final subsections are dedicated to the Step 3 of the pipeline, i.e., the refinement of the asteroid tours identified by exploring the asteroid-belt tour transcribed problem.

c. Step 3: Final Refinement

Step 3, or the refinement step, takes all or a defined subset of promising solutions encountered at Step 2 and optimises the continuous design variables, to further improve the overall trajectory and deliver a fully-fledged solution. It is important to recall that Step 2 does not only provides a fixed sequence of objects to visit, but also a fully detailed trajectory (i.e., a continuous varying y is also identified). The visiting epochs and planetary fly-by parameters, variables encoded in the vector y , identified during the Step 2 process only need to be refined in case a lower Δv solution exist in the same neighbourhood. For this process to be efficient, Step 2 solutions must be close enough to the real minimum solution.

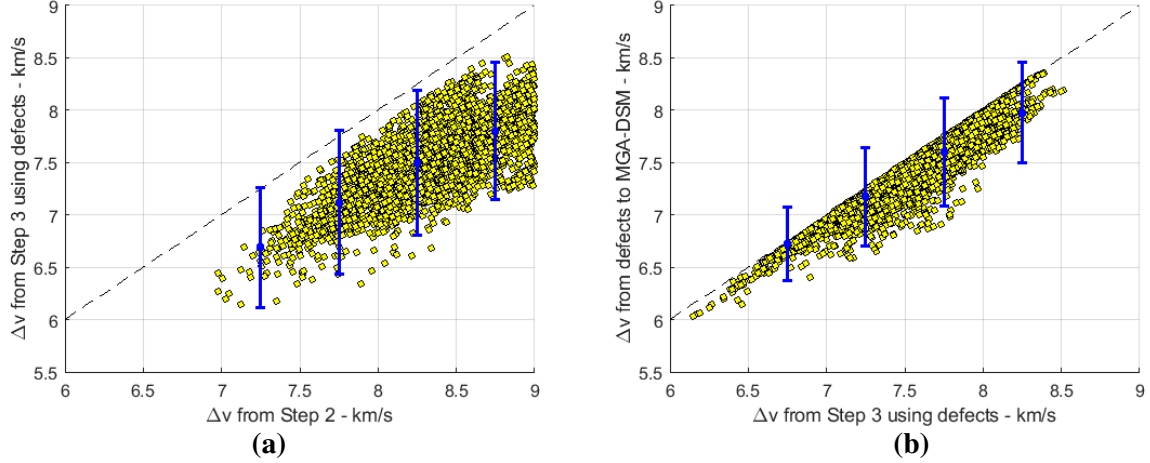


Figure 6. Summary of Step 3 solutions using defects model (a) and from defects model to MGA-DSM model (b). Blue error bars indicate 3σ distributions at 6.75, 7.25, 7.75, 8.25 and 8.75 km/s.

Figure 6 reports the total Δv costs of 5000 unique 12-asteroids fly-by tours as identified in Step 2 and Step 3, all of which generated from one single Step 1 baseline EM trajectory. Table 2 describes the EM baseline trajectory that is used as substrata in Step 2 to incorporate asteroids within its path. Each point in Figure 6 thus represents a trajectory with 12 asteroids plus Mars with total Δv defined as the sum of the departing v_∞ and all remaining DSMs to follow the entire tour. The specifics of the search process used in Step 2 will be discussed in Section 5. Instead, this subsection focuses on the appropriateness of the transcription used in Step 2, as well as the refinement process of Step 3.

Table 2. Details for the Earth-Mars trajectory solution identified in Step 1.

Event	Value
Earth departure	Dec. 24, 2030
Departing v_∞	4.23 km/s
Mars fly-by	Mar. 3, 2033
Time in the MAB	1250 days

It should be recalled that the solution presented in Section 2 featured an MGA-DSM as dynamical framework. Instead, the transcription implemented in Step 2 assumes a Δv -defects model, which reduces search space dimensionality and features optimal substructure property. Thus, a potential refinement process would see solutions from Step 2 being re-optimised first using the Δv -defects models for subsequently re-optimise into the MGA-DSM model. Figure 6.a maps the Δv of each unique solution of Step 2 (x-axis) into the Δv of the same solution refined into Δv -defects model (y-axis). Figure 6.b instead maps the Δv of each refined solution in the Δv -defects model (x-axis) into Δv of the same solution refined into MGA-DSM model (y-axis). The visiting epochs in vector y are allowed to vary on a ± 30 days-range with respect to the corresponding Step 2 values. MATLAB Genetic algorithm (with a population of 1500 individuals and 600 maximum generation) is used to refine Step 2 solutions.

Figure 6.a does show that the refinement of Step 2 solutions may achieve some improvement of the total Δv . However, the balance between reduction of the search space achieved through the transcription process described in Step 2 together with the accuracy of the Δv costs is still solidly advantageous. This confirms that Step 2 already provides good sub-optimal solutions, likely converging to lower Δv within 3σ distribution. Moreover, refining using defects model as from [35] already provides the main benefit in Δv cost reduction, compared to more detailed models. Recall that the a-priori knowledge of the asteroid fly-by time before the transcription is $y_1 < t_{fb} < y_1 + ToF$ (i.e., any date within the maximum mission timespan ToF_{max}). Indeed, a 10^6 -evaluations

random search of this non-transcribed search space would typically yield transfer costs in the order of 100 km/s. Finally, Figure 6.b suggests also that Δv -defects model is indeed sufficient to approximate any manoeuvre required during the transfer in a simpler search space and in lighter computational effort [35]. Thus, the complex search space defined by MGA-DSM model is accurately represented by the Δv -defects model, as shown in Figure 6.b where a median Δv cost improvement of only 80 m/s is achieved after refinement.

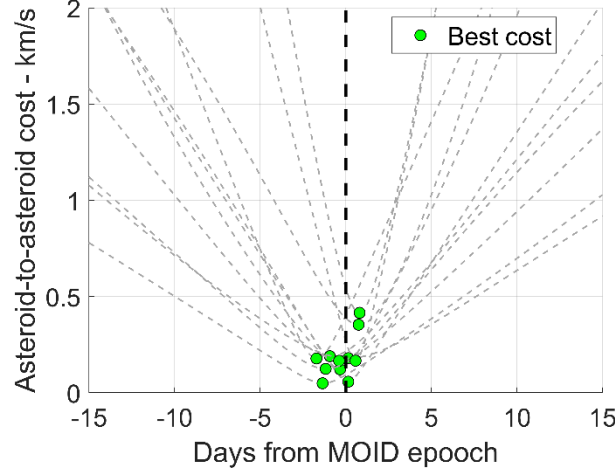


Figure 7. Δv variation on the different legs of the CASTAway EM reference trajectory with respect to the MOID epochs for each of the visited asteroids.

Instead, as shown in Figure 7, Step 2 transcription automatically yields a set of fly-by epochs within a few days of its optimal configuration. Figure 7 shows the Δv costs of each asteroid-to-asteroid legs within one specific 12-asteroid tours (within the solutions in Figure 6). The x-axis in Figure 7 describes the sensitivity of each asteroid-to-asteroid leg to changes on the asteroid fly-by epoch, as centred around each asteroid's MOID point crossing epoch. Circles highlight the actual minimum Δv possible for one single leg (i.e., disregarding potential increases in other legs). Figure 7 illustrates that the Asteroid-belt Tour Transcribed Problem proposed in Step 2 results in a very efficient process to find a fixed time of flight between two consecutive objects.

5. Modified Dynamic Programming Exploration

This section revisits the Step 2 asteroid-belt tour transcribed problem, whose search space was identified as a tree of connected vertices (Section 3). The efficient transcription process described in Section 4 (Step 2) saw also a potentially unattainable search space to be reduced to a few hundred events, depending on distance threshold d_{thr} of choice, where each event P_i is defined by a fixed epoch t and celestial body ID. Given the seemingly manageable number of events to explore, it may be tempting to consider complete tree traverse algorithms to solve the CSP problem, by finding all possible feasible routes linking a fixed number of vertices in the tree.

Depth first (DF) and breadth first (BF) strategies would typically come to mind as paradigms of complete enumerative search algorithms [23]. However, by means of the binomial coefficient representing the number of combinations without repetition, one can quickly see that the total number of possible paths in a tree quickly grows to unfeasible values. Table 3 shows, for example, all the possible sequences of 12-asteroid-long paths that exist for each of the sets identified for different distance threshold as in Section 4.b.

Table 3. Number of combinations without repetitions of 12 asteroids sequences

Number of asteroids	Number of sequences with 12 asteroids
49	9×10^{10}
98	8×10^{14}
158	3×10^{17}
562	2×10^{24}
1026	2×10^{27}

Given the quick growth of the CP search space, one may then want to consider incomplete searches like Beam Search (BS), on which the computational effort associated with a BF tree exploration is bounded by capping the number of branches that can be expanded at any level [29]. More specifically, from all the branches generated at one level, only a limited set of them, referred as the beam, is selected to be expanded at successive levels. The dimension of the beam (i.e., the number of sequences to be kept in memory for further expansion) is referred as the beam width (BW). The sequence selection is usually done by prioritizing those that minimize a cost function, e.g., total Δv up to current node. Note that the Beam Search strategy ultimately sacrifices the guarantee of global optimality, provided by complete algorithms, in favour of computational efficiency.

Dynamic programming provides an alternative to the above trade-off. As discussed in Section 3, the search space features an optimal substructure property and, therefore Bellman's principle of optimality becomes applicable. Figure 8 provides an intuitive representation of Bellman's principle applied to the asteroid tour graph exploration. Because each individual linkage of two events in the graph has its associated cost Δv_i (refer to Figure 2 for clarification on this point), the optimal path leading to any node must minimize the partial sum of Δv that is leading to it. This implies that when exploring the tree one can discard all paths arriving to (or departing from) a specific node having less than the optimal partial cost. Hence, while exploring the graph, one can apply pruning based on Bellman's principle to each individual node, limiting the polynomial growth of the exploration, while still guaranteeing global optimality.

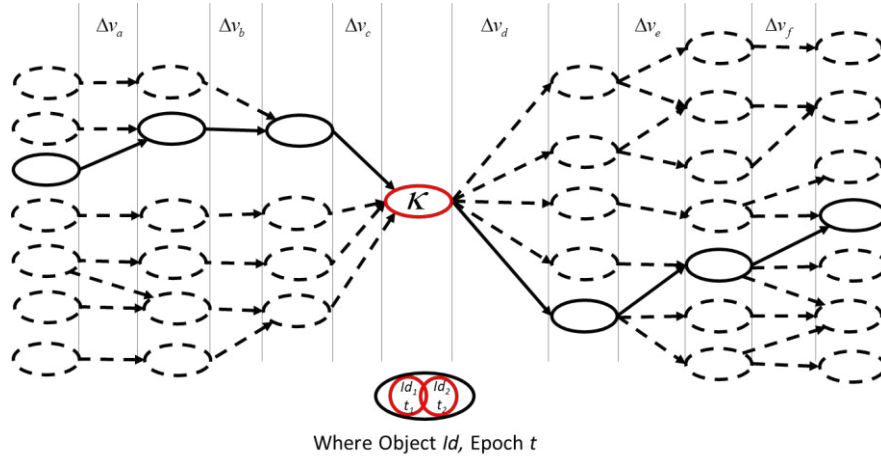


Figure 8. Optimal paths leading or departing event κ must satisfy $\min[\sum \Delta v]$.

In more practical terms, as the search algorithm expands the tree following a BF strategy, all paths arriving at each specific node visited at the latest level are analysed and filtered/pruned accordingly to Bellman's optimality principle. Following the example in Figure 8, at a given event κ , one may choose a single objective to rank and filter all but the optimal solution of the intermediate tours reaching event κ , which is typically referred as single-objective dynamic programming (SODP) [17]. Multi-objective dynamics programming (MODP), on which only the Pareto optimal set of solutions arriving to event κ is retained, is also a viable alternative if competing criteria are considered [35, 52]. It should be noted that both dynamic programming paradigms, SODP and MODP, solve an optimisation problem with well-defined quantitative fitness functions.

As discussed previously, here we aim at solving the CSP; this is having as many feasible solutions as possible, among other reasons, because the fitness function (e.g., the scientific goodness of a tour) is not easily evaluated. Hence, a new filtering paradigm is proposed, on which not only the optimal solution as ranked by a single objective criterion (i.e., Δv) is retained, but instead a larger set of the n top ranked solutions is kept for further expansion. This process is thereafter referred as modified dynamic programming (n DP).

This modified approach lies somewhat in the middle between the SODP (one path saved per node evaluation) and MODP (multiple paths saved per node evaluation according to multiple criteria) approaches. One also notices that $n = 1$ corresponds to the SODP optimisation. In this way, the global optimality is still ensured for crucial and quantifiable functions (like the overall Δv) and diversity of solutions is preserved. The interest in obtaining a set n of such solutions is that is diversified in their properties (the degree of diversification being dependent on how large n is).

For example, one can explore a set of 158 asteroids as presented in Section 4.b ($d_{thr} = 0.05 AU$) with nDP at different n values. The number of routes stored at each tree level expansion is provided in Table 4 for sequences of 12 asteroids. A BS is also run with BW equal to the maximum number of trajectories stored for each nDP case considered. In this way, the proposed modified dynamic programming scheme is benchmarked against beam search for different parameters n . From Table 4, one notices that, for the cases considered, the BS does not find the global optimum solution (in terms of Δv) and always finds more than one order of magnitude less solutions than the modified dynamic programming. In other words, while not ensuring global optimality on the transcribed space, BS needs to evaluate more asteroids sequences than nDP to obtain the same order of magnitude of solutions (one notices that BW to obtain global optimum Δv is 115000 from Table 4).

Table 4. Number of solutions per tree level retained with nDP at different n values. BS results are included for comparison.

Number of solutions per tree-level			
Tree level	$n=1$	$n=2$	$n=5$
1	14	14	14
2	755	755	755
3	4137	8211	20197
4	5800	11600	28842
5	5583	11166	27740
6	5007	10014	25030
7	4437	8845	21915
8	3893	7774	19320
9	3119	6203	15274
10	2266	4504	11006
11	1644	3273	8117
12	6783	13266	31504
Global optimum Δv		6.977 km/s	
BW	5800	11600	28842
Number of solutions	82	301	626
Best Δv	7.583 km/s	7.583 km/s	7.583 km/s
BW to obtain global optimum		115000	

In addition, nDP approach manages the information in a way that the solutions quality is better (in terms of best Δv) compared to the BS simulations considered here. This can be seen from Figure 9.a, where the solutions from BS and nDP (at tree level 12) are classified in different Δv -bands. One notices that nDP finds again many more order of magnitude of solutions compared to the BS. For example, dynamic programming finds overall 260 solutions in the 6.7-7 km/s and 7-7.5 km/s bands (one of which is the globally optimal one), while BS finds none. This ultimately implies that the solutions quality is also improved overall and not only their number.

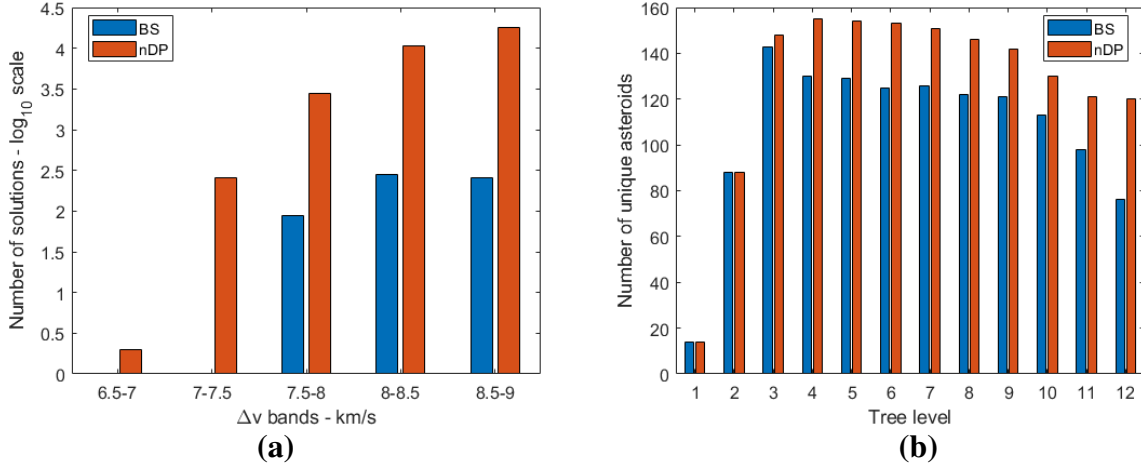


Figure 9. Number of solutions with respect to different Δv bands (a) and number of unique asteroids in all routes up to a given tree level (b) for BS (BW=28842) and nDP ($n=5$).

Finally, Figure 9.b shows that the nDP approach provides higher asteroids' diversity along the tree expansion compared to BS. Diversity here is measured as the number of unique asteroids in the set of solutions in all routes stored up to a given tree level. One notices here that, similarly to well-known strategies like BS, the tuning of the parameter n is a compromise between solution number and computational effort and depends upon the specific working environment (e.g., if one has high RAM availability, then $n \gg 1$ can be selected, similarly to the tuning of the BW). Higher n thus corresponds to higher diversity in the solutions' set. However, as opposed to the tuning of BW, the choice of n does not affect the ability of the search scheme to find the global optimum of the set, since this is ensured in the proposed scheme.

6. Complete MAB exploration.

Up to this point, the entire asteroid tour design pipeline has been presented and partial results discussed for only a single baseline trajectory. This same pipeline, and its specific search schemes, was employed to perform a full trajectory design exploration for the submission of CASTAway [53], a candidate mission proposal for ESA M7 mission call [11]. This section reports on those results to provide further insight on the efficiency and performance of the exploration.

Analysing possible transfer scenarios relevant for MAB tour missions like CASTAway requires a complete exploration of the search space, in terms of launch window analysis and transfer strategies. This implies a high number of MINLP/CSP problems to be solved, with associated challenges as described in previous sections. The process based on transcription (Section 4) and deterministic exploration (Section 5) is tested on CASTAway trajectory design. Table 5 summarizes the boundary conditions for the mission derived from ESA's reports [11, 37] and past CASTAway designs [10, 54]. In particular, the boundaries of Earth escape velocity are set to be compatible with Ariane 6 launcher [55], while those on ToF , N_{asts} and Δv (after Step 3) are set to reduce operations cost and to be compatible with ESA's Cost at Completion [11]. A maximum Δv is accepted at Step 2 (9 km/s) that is higher compared to Step 3 (6.5 km/s) as one expects solutions to reduce their cost in successive stages of the design (see also Section 4.c).

Table 5. Boundary conditions for CASTAway mission compatible with ESA's M-class calls.

Value	Description	Constraints
t_0	Launch epoch	Jan. 1, 2037 – Dec. 31, 2037
v_∞	Earth escape velocity	$v_\infty \leq 5 \text{ km/s}$
ToF	Overall mission duration	$ToF \leq 7 \text{ years}$
N_{asts}	Number of visited asteroids	$N_{asts} \geq 10$
Δv	Overall cost (sum of v_∞ and DSMs)	$\Delta v \leq 9 \text{ km/s}$ (Step 2) $\Delta v \leq 6.5 \text{ km/s}$ (Step 3)

As from Section 4, Step 1 starts with a multi-start launch window exploration, on which MODP is used to find trajectories that maximise the time in the MAB, while minimising the Δv (i.e., the sum of v_∞ and planetary defects). The launch window is discretized and explored in steps of 5 days, which yields a good compromise between computational costs and discretization of the search space [35]. Overall, 1148 individual MGA trajectories to access the MAB region are obtained from Step 1, all compatible with the boundary conditions in Table 5. Each of these trajectories are then fed into Step 2. The d_{thr} for the MOID-informed transcription process described in Section 4b (Step 2) is set at 0.05 AU. One notices that such choice (alongside the launch date discretization step) ensures a sustained sweep of the MAB, exploring consistently all the reachable space. In theory, larger step size could also be allowed in launch date discretization (e.g., up to 10 days), as shown in Figure 10, with the results of having smaller number of options to be passed to Step 2.

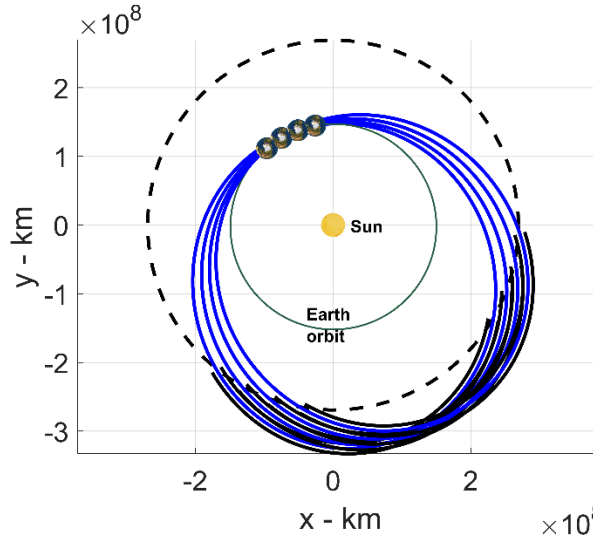


Figure 10. Reference trajectories for a direct launch from Earth (blue lines). Black lines, centred in reference trajectories and distant 0.05 AU from them, represent the space within which asteroid catalogues are built to explore asteroid tour opportunities. Earth departures are spaced by 10 days. Dotted circle is the MAB lower limit (1.8 AU).

The Step 2 asteroid-belt tour transcription process (Section 4.b) assigns a MOID crossing epoch to each potentially reachable asteroid. Modified dynamic programming with $n=5$ is applied to the search yielding a total of approximately 79 million trajectories visiting 10 or more asteroids in the MAB. One notices again that such choice of n is user-specific and results from a compromise between solutions number, quality, and computational effort, that depends upon the specific user setup (in this case a standard 4 GHz laptop with 16 GB of RAM). Recall that contrary to beam search, $n=1$ already enables a global optimum guaranteed search, and that the choice of $n=5$ simply adds a larger diversity of tours to the output solution. The database of trajectories as from Step 2 are represented in Figure 11, for each different MGA sequence analysed. Such trajectories differ in terms of Δv , asteroids encountered, and regions visited. Figure 11 highlights the diversity of asteroids regions explored by different strategies, showing the performance of MGA sequences in visiting asteroids from different MAB regions: Hungaria (from 1.5 to 2 AU), Inner Belt (from 2 to 2.5 AU), Middle Belt (from 2.5 to 2.8 AU), Outer Belt (from 2.8 to 3.3 AU), Cybele (from 3.3 to 3.7 AU), and Hylda (from 3.7 to 5 AU). In strategies like EVE or EME, having the Earth as last planetary fly-by permits high-apoapsis to be reached, thus allowing farther regions to be explored sustainedly (e.g., Cybele and Hylda), compared to strategies with Venus, as EVV or EMV. Lower apoapsis are reached with Mars strategies (e.g., EM, EMM or EVM) due to lower Mars gravity field.

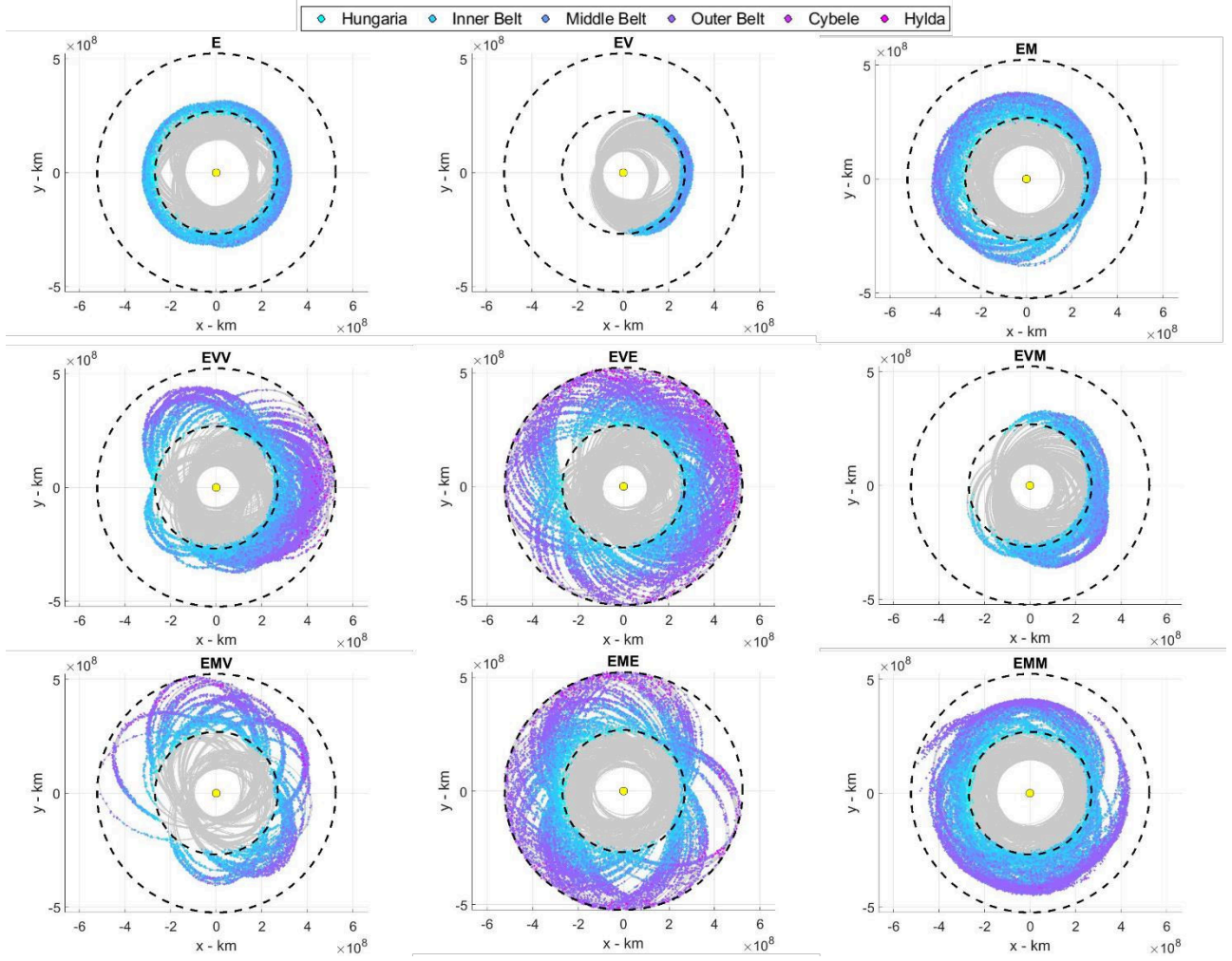
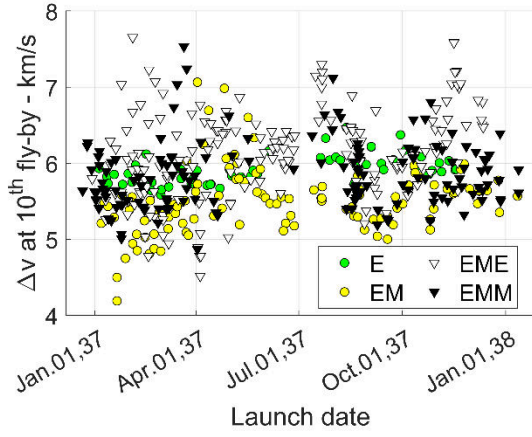
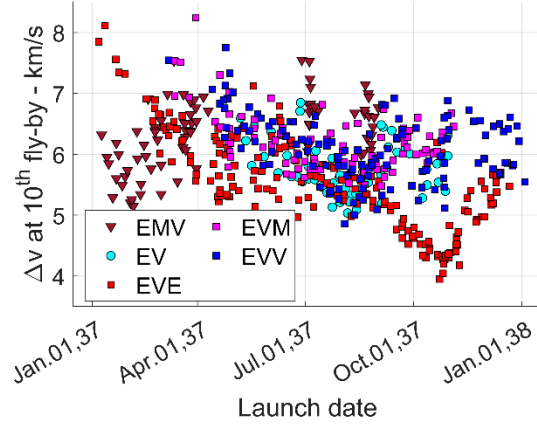


Figure 11. Database of trajectories visiting MAB asteroids as from Step 2 with different MGA strategies. Dotted lines represent MAB limits. Spacecraft trajectories out of MAB region are depicted in grey.

Step 3 finally takes a subset of Step 2 trajectories and refines the continuous-varying variables in vector y . For example, Figure 12 shows an overall launch window exploration (x axis) for different planetary sequences and the Δv performances of such strategies at the 10th asteroid (y axis), where only the best Δv solutions are shown for each of the 1148 baseline MGA trajectories. Recall the total Δv includes v_{∞} and the successive manoeuvres to visit all asteroids and planets. One notices from Figure 12 that visiting at least 10 asteroids is feasible within ESA's M-class boundary constraints throughout the whole launch window, where both EM and EVE strategies provide the lowest options in terms of Δv consumption, with 4.19 km/s and 3.95 km/s, respectively. Figure 13 shows the best Δv solutions in the set for a sequence of 10 asteroids (Figure 13.a) and for a sequence that maximizes the number of encounters, i.e., 16 in this case (Figure 13.b). Both example trajectories follow an EVE strategy, which confirmed by Figure 11 and Figure 12 allows a consistent reach within the MAB. The overall Δv and ToF are 3.95 km/s and 6.99 years for the option in Figure 13.a, respectively, and 6.48 km/s and 6.98 years for the option in Figure 13.b, respectively.

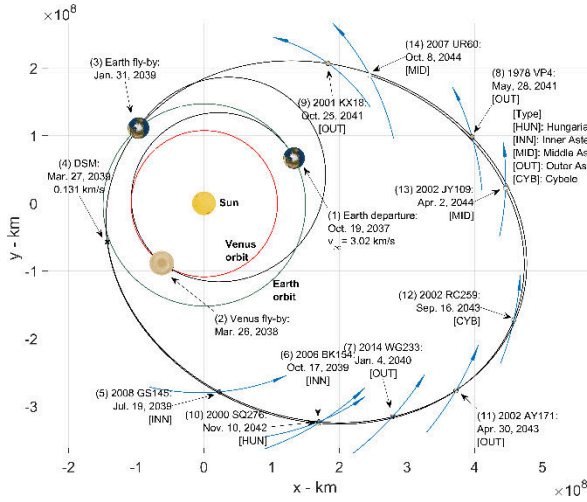


(a)

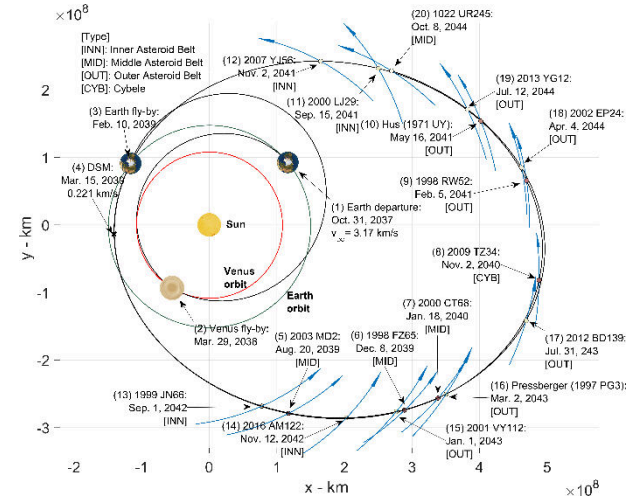


(b)

Figure 12. Launch window analysis for 2037 launch opportunities for CASTAway M7. Different gravity assist options are shown: E, EM, EME, EMM (a) and EMV, EV, EVE, EVM, EVV (b).



(a)



(b)

Figure 13. Minimum Δv EVE strategies visiting 10 asteroids (a) and maximum number of asteroids (16 in this case) (b). The overall Δv is 3.95 km/s for (a) and 6.48 km/s for (b).

The design pipeline for multi-target missions presented in this paper is thus extremely useful to perform consistent and comprehensive exploration of the search space of complex multi-target missions. The test case presented here required a detailed exploration of asteroid tours over an entire 1-year launch window. The results provide an accurate detail of all possible and feasible trajectories within this launch window, rather than identifying only some notable solutions. The entire analysis for this test case was run in a standard personal laptop (4 GHz laptop with 16 GB of RAM) in approximately 12 h of runtime. Step 2 being the most time consuming and requiring approximately 8 to 10 hours. Step 1 requires a negligible time as compared with steps 2 and 3. The runtime of Step 3 depends on the number of trajectories that are refined, but the current set up is capable to refine approximately 500 asteroid tour trajectories in Δv -defects model per hour.

7. Conclusions

This paper tackles the challenge of finding trajectories that fly-by as many main belt asteroids as possible with mission boundary conditions relevant to medium-size or discovery class missions. The design problem itself is modelled as a complex mixed-integer optimisation problem, featuring both a vast and highly multimodal search space. Moreover, preliminary mission design (i.e., feasibility

analyses of a new mission concept) requires not only to find some notable solutions, but also to ensure a systematic exploration of the topography of the feasible search space.

A three step multi-fidelity design pipeline is proposed to deal with this problem. A key aspect of the methodology is the ability of each design step to provide solutions that map accurately into near optimal solutions at the subsequent step: Particularly between Step 2 and Step 3 and, for the latter, two different dynamical frameworks are also considered, namely, Δv -defects and MGA-DSM models. Within this pipeline, a key aspect is the transcription of the mixed-integer problem into a fully combinatorial one, which considers only visiting asteroids that dwell near some preliminary spacecraft trajectory. This transcription is shown to allow for a very efficient pruning of the less interesting search space. However, even in relatively limited integer search spaces complete systematic search schemes are still a challenge to be implemented. A modified dynamic programming (n DP) scheme is thus proposed, which retains the n -best paths at each unique and separable node. Retaining the n -best, rather than a single optimum, in a dynamic programming process yields a numerous and diverse representation of all existing asteroid tours within the considered launch window. The computational cost of the proposed scheme is also very manageable and possible with a single standard laptop computer.

This approach is obviously not only applicable to asteroid tours, but also to other missions considering visiting several different orbital waypoints; hence, presents a useful approach to tackle mission design analysis of novel multi-target missions. The paper provides a rigorous description of how multiple fly-by trajectory problems can be defined such that the problem exhibits optimal substructure property. Such representation offers opportunities for an effective pruning of the search space by, first, eliminating all non-feasible building blocks and, then, applying Bellman's principle of optimality to the pruning criteria of the graph exploration.

Acknowledgements

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Appendices

A. DV defect model

The Δv defects model can be seen as a particular case of the MGA-DSM model on which the midcourse manoeuvres on each planet-to-planet leg occur immediately after planetary fly-bys (i.e., effectively after leaving the planet sphere of influence). In this case, the variable $\alpha = 0$ for each leg in the transfer (see also Table 1). In this way, each leg can be modelled as a Lambert arc linking two successive planetary encounters. The cost of the leg is the velocity discontinuity Δv occurring at the fly-by epoch between incoming and outgoing spacecraft relative velocities with respect to the planet, which are solutions of Lambert problem for the given leg. Such infinity velocity defects are computed as:

$$\Delta v = \begin{cases} ||\vec{v}_{\infty}^+|| - ||\vec{v}_{\infty}^-|| & \text{if } \delta \leq \delta_{max} \\ \sqrt{||\vec{v}_{\infty}^+||^2 + ||\vec{v}_{\infty}^-||^2 - 2||\vec{v}_{\infty}^+||||\vec{v}_{\infty}^-||\cos(\delta_{max} - \delta)} & \text{otherwise} \end{cases}$$

where: $\vec{v}_{\infty}^-$ and $\vec{v}_{\infty}^+$ are the spacecraft velocities relative to the fly-by planet before and after the encounter, respectively; δ is the angle between $\vec{v}_{\infty}^-$ and $\vec{v}_{\infty}^+$, positive in the 180 degree-range counter-clockwise, and represents the change of direction between the incoming and outgoing legs of the fly-by; δ_{max} is the maximum possible deflection at the fly-by for the incoming relative velocity $\vec{v}_{\infty}^-$ and reads as:

$$\delta_{max} = 2 \operatorname{asin} \left(\left(1 + \frac{r_{p,min} |\vec{v}_{\infty}^-|^2}{\mu_{pl}} \right)^{-1} \right)$$

with $r_{p,min}$ being the minimum periapsis of the fly-by hyperbola as in [56] and μ_{pl} the gravitational constant of the fly-by planet.

In other words, the velocity discontinuity between $\vec{v}_{\infty}^-$ and $\vec{v}_{\infty}^+$ implied by Lambert's problem solution can in part be compensated by a gravity assist at the planet and if required an additional spacecraft manoeuvre is added at the fly-by. Since $\vec{v}_{\infty}^-$ and $\vec{v}_{\infty}^+$ are solutions of Lambert's problem, they only depend upon the transfer time ToF between two fly-bys, and thus the overall Δv consumption accumulated along the MGA mission ultimately depends upon the ephemerides of the objects through their heliocentric velocities at the encounter epochs which define $\vec{v}_{\infty}^-$ and $\vec{v}_{\infty}^+$. In this way, the y vector from Table 1 only encodes the departing date from the first object and the transfer times between successive objects, reducing the dimension of the problem and the computational effort [17, 35].

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