

Tube-side mass transfer for hollow fibre membrane contactors operated in the low Graetz range



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ABSTRACT

Transformation of the tube-side mass transfer coefficient derived in hollow fibre membrane contactors (HFMC) of different characteristic length scales (equivalent diameter and fibre length) has been studied when operated in the low Graetz range ($Gz < 10$). Within the low Gz range, mass transfer is generally described by the Graetz problem ($Sh=3.67$) which assumes that the concentration profile comprises a constant shape over the fibre radius. In this study, it is experimentally evidenced that this assumption over predicts mass transfer within the low Graetz range. Furthermore, within the low Gz range (below 2), a proportional relationship between the experimentally determined mass transfer coefficient (K_{ov}) and the Graetz number has been identified. For Gz numbers below 2, the experimental Sh number approached unity, which suggests that mass transfer is strongly dependent upon diffusion. However, within this diffusion controlled region of mass transfer, tube-side fluid velocity remained important. For Gz numbers above 2, Sh could be satisfactorily described by extension to the Lévéque solution, which can be ascribed to the constrained growth of the concentration boundary layer adjacent to the fibre wall. Importantly this study demonstrates that whilst mass transfer in the low Graetz range does not explicitly conform to either the Graetz problem or classical Lévéque solution, it is possible to transform the experimentally derived overall mass transfer coefficient (K_{ov}) between characteristic length scales (d_h and L). This was corroborated by comparison of the empirical relationship determined in this study ($Sh=0.36Gz$) with previously published studies operated in the low Gz range. This analysis provides important insight for process design when slow tube-side flows, or low Schmidt numbers (coincident with gases) constrain operation of hollow fibre membrane contactors to the low Gz range.

1. Introduction

Hollow fibre membrane contactor (HFMC) technology has been demonstrated as a mass transfer process for numerous gas-liquid applications including oxygen desorption for industrial scale boilers [14], from ultrapure water [34], for absorption applications [5,43] and at smaller scale, for blood oxygenation in open heart surgery [9]. Their commercial advantage can be ascribed to the phase separation facilitated by the membrane, which promotes non-dispersive mass transfer between fluids, whilst simultaneously permitting considerable specific surface area to be incorporated into a module, thus providing large volumetric mass-transfer coefficients (Ka) to be achieved relative to conventional mass transfer technologies [3].

For industrial scale design of HFMC, an engineer needs to establish the overall separation, given a specific feed concentration, which can be correlated in terms of the average or overall mass transfer coefficient

(K_{ov}) [11]. The K_{ov} can then be used to establish the contactor length (L) necessary to achieve a stated treatment objective [32]:

$$L = \left[\frac{V_0}{K_{ov} a} \right] \left\{ \left(\frac{\frac{Q_s M}{Q_f}}{\frac{Q_s M}{Q_f} - 1} \right) \ln \left(\frac{y_0 - Mx_0}{y_1 - Mx_1} \right) \right\} \quad (1)$$

where V_0 is the superficial feed velocity, a is the surface area of membrane per volume of bed, Q is the solvent (s) and feed (f) flow, M is the partition coefficient, and x and y are the solute mole fractions in the feed and solvent respectively [32]. The mass transfer coefficient used in (1) can be adapted from the literature but requires correction when the characteristic length employed during determination of K_{ov} is different to that proposed in the design. In the case of tube-side flow, the characteristic length is the inner tube diameter (d_h) and translation of K_{ov} between length scales can be achieved using the dimensionless Sherwood number (Sh):

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Nomenclature		δ	membrane thickness (m)
<p>A mass transfer area provided by the membrane (m^2)</p> <p>a specific surface area (m^{-1})</p> <p>D diffusivity of water vapour in the gas phase ($\text{m}^2 \text{s}^{-1}$)</p> <p>d_h hydraulic diameter (lumen or inner fibre diameter) (m)</p> <p>k specific mass transfer coefficient (m s^{-1})</p> <p>K overall mass transfer coefficient (m s^{-1})</p> <p>L length of hollow fibres (m)</p> <p>P membrane permeability (barrer)</p> <p>$(1 \times 10^{-10} \text{ cm}^3 \text{ cm}^{-2} \text{ cm}^{-1} \text{ s}^{-1} \text{ cm Hg}^{-1})$</p> <p>Q flow rate ($\text{m}^3 \text{s}^{-1}$)</p> <p>R gas constant ($\text{J K}^{-1} \text{mol}^{-1}$)</p> <p>$\text{RH}_{\text{in}}$ relative humidity of inlet gas (%)</p> <p>RH_{out} relative humidity of outlet gas (%)</p> <p>T_1 water vapour concentration in the air phase (ppm)</p> <p>V water vapour concentration in the air phase (ppm)</p> <p>X_{Sat}^W water vapour concentration in the air phase (ppm)</p> <p>X_{Sat}^W saturated water vapour concentration (ppm)</p>		<p><i>Dimensionless numbers</i></p> <p>Gz Reynold number $[\rho d_h v / \mu]$ –</p> <p>Pe Reynold number $[\rho d_h v / \mu]$ –</p> <p>Re Reynold number $[\rho d_h v / \mu]$ –</p> <p>S dimensionless Raoult's law constant for water –</p> <p>Sc Schmidt number $[\mu / \rho D]$ –</p> <p>Sh Sherwood number $[k d_h / D]$ –</p>	
<p><i>Greek letters</i></p> <p>ρ density of nitrogen gas (kg m^{-3})</p> <p>μ dynamic viscosity of nitrogen gas ($\text{kg m}^{-1} \text{s}^{-1}$)</p>		<p><i>Subscript</i></p> <p>f feed</p> <p>g gas phase</p> <p>l liquid phase</p> <p>L liquid phase</p> <p>s solvent</p> <p>ov overall</p>	
		<p><i>Subscript</i></p> <p>a air</p> <p>w water</p>	

$$Sh = \frac{K_{ov} d_h}{D} \quad (2)$$

where D is mass diffusivity. As the Sherwood number represents the ratio of total rate of mass transfer to the rate of diffusive mass transport, it is implicitly governed by the fluid dynamics employed. Provided that tube-side flow is characteristically laminar, the Graetz number ($Gz = Re Sc d_h / L$) can be used to describe fluid behaviour through [28]:

$$Sh = a Re^b Sc^c \left(\frac{d_h}{L} \right)^e \quad (3)$$

where Re and Sc are Reynolds ($Re = \rho dV / \mu$) and Schmidt numbers ($Sc = \mu / \rho D$) respectively. Importantly, inclusion of the (d_h / L) term emphasises that K_{ov} is dependent on both the tube-side diameter and fibre length.

For gas-liquid applications with gas flowing on the tube-side, K_{ov}

represents contributions from the membrane (k_m), liquid phase (k_l) and gas phase (k_g) [17]. However, the overall resistance to mass transfer ($1/K_{ov}$) is often dominated by one or more phases. For example, during an air stripping study with air flow on the tube-side and water surrounding the external periphery of the fibre, Mahmud et al. [22] suggested that the liquid phase resistance to mass transfer was negligible, thus the residual resistance was accounted for by both the membrane resistance and air-phase resistance developing on the tube-side. For membranes with thin walls, where solutes exhibit high permeability [28] or for applications constrained to low flows, the membrane resistance can be considered negligible, and the concentration boundary layer developing in the fluid adjacent to the membrane wall, will generally dominate mass transfer resistance [10,28].

Numerous authors have demonstrated that the L  v  que solution can be usefully applied to estimate the tube side mass transfer coefficient (Table 1) [6,24]:

Table 1

A non-exhaustive list of studies that have undertaken tube-side mass/heat transfer analysis.

Process	Solute	Valid range ^a	Sc	d_h/L	Correlation	Ref.
Theoretical		$Gz > 20^b$	–	–	$Sh = 1.61 Gz^{1/3}$	[16]
Theoretical		$Gz < 10^c$	–	–	$Sh = 3.67$	[16]
Theoretical		1st term	–	–	$Sh = 1.61 Gz^{1/3} - 1.2$	[25]
Theoretical		1st and 2nd term	–	–	$Sh = 1.61 Gz^{1/3} - 1.2 - 0.28057 Gz^{-1/3}$	[25]
Hollow-fibre membrane	Dissolved methane	$Gz > 10$	671	1.95×10^{-3}	$Sh = 1.61 Gz^{1/3}$	[24]
Hollow-fibre membrane	Dissolved oxygen	$Re > 5$	507	4.2×10^{-4}	$Sh = 1.61 Gz^{1/3}$	[36]
Hollow fibre membrane	Protein	N/s	–	–	$Sh = 1.5 Gz^{1/3}$	[8]
Hollow fibre membrane	Dissolved oxygen	$Gz > 4$	476	$0.71 - 2.4 \times 10^{-3}$	$Sh = 1.61 Gz^{1/3}$	[41]
Tubular heat exchanger	Heat	$Re > 10$	–	–	$Sh = 1.86 Re^{1/3}$	[33]
Tubular heat exchanger/	Heat	$Re < 7$	–	–	$Sh = Re \left[0.5 \left(Sc \frac{d_h}{L} \right)^{2/3} \right]^d$	[33]
Packed bed	Various (gas and liquid)	$Pe < 10$	–	–	$Sh = 0.07 Pe^e$	[12]
Hollow-fibre membrane	Humidity	$Gz < 2$	0.55	$1.25 \times 10^{-4} - 4.8 \times 10^{-2}$	$Sh = 0.36 Gz$	This study

^a Evidenced by coherence of data to the relationship proposed.

^b The limit generally stated.

^c Not experimentally evidenced but often cited.

^d Provided a gradient of 0.5 when considering Sc for Glycerol (~ 1040).

^e Gradient estimated through data extrapolation.

$$Sh=1.6151Gz^{1/3} \quad (4)$$

This solution proposes that the concentration boundary layer is limited to a thin layer adjacent to the fibre wall, and so is only valid for relatively high fluid velocities (Gz generally exceeding 20) through short fibres operating in laminar flow [16]. However, for applications where only low driving pressures are available [2], or where pressure drop is constrained to promote energy conservation [6], often only low Graetz numbers (below Gz of 20) can be achieved. Operation in the low Graetz range is compounded when the tube-side fluid is gaseous, as the Schmidt number reduces to around 1 from a value of around 1000 for liquids. For small values of Gz , that are coincident with slow flows, low Sc and long tubes, Graetz proposed an approximate solution for the differential equation driven from the continuity equation, by which the average and local Sherwood number can be obtained for conditions below Gz of 10 [16]:

$$Sh=3.67 \quad (5)$$

This solution assumes that the concentration profile is completely developed, indicating a constant shape over the radius [18]. Through curve fitting of (4) and (5), Kreulen et al. [18] proposed the following solution to describe the Gz range:

$$Sh=\sqrt{3.67^3 + 1.62^3 Gz} \quad (6)$$

Analytical comparison of (4) and (6) with the available experimentally derived Sherwood data from the literature evidences several important points [23,24,30,41]:

- (i) the L  v  que solution provides general agreement for Gz numbers exceeding 20, and apparently satisfactory description of experimental Sh data above Gz around 5. The solution can therefore be used to provide an estimation of the mass transfer coefficient, when Gz exceeds 5;
- (ii) below Gz of around 5, experimental Sh data tends away from the available analytical solutions toward unity as the Gz number is reduced; and
- (iii) both the L  v  que and Graetz solutions grossly overestimate experimental Sh data reported within the low Graetz range ($Gz < 5$).

Such disparity has been previously reported in packed bed reactors operated at low P  clet numbers [19]. Numerous experimental membrane contactor studies operated below a Gz of 10, have similarly

evidenced such a phenomenon [7,24,31,15,41]. However, there is only limited knowledge of how the Sherwood number changes within the low Graetz range. Specifically, whether mass transfer in HFMC achieves a constant value or decreases as the Graetz number is reduced is currently unclear. Furthermore, as the existing analytical solutions do not appear to adequately describe the experimentally determined Sh within the low Graetz range, there is no existing knowledge as to whether the rate of mass transfer can be estimated through transformation of the Sherwood number using the characteristic length scales, L and d_h . The aim of this study is therefore to investigate tube-side mass transfer in hollow fibre membranes operated within the low Graetz range. Specific objectives are to: (i) identify whether the Sherwood number approaches a constant value within the low Graetz range; (ii) determine whether Sherwood analysis can describe mass transfer across a broad range of tube diameters within the low Graetz range; (iii) determine the impact of tube length on Sherwood analysis conducted in the low Graetz range; and (iv) compare this discrete Sherwood analysis conducted on single hollow-fibre membranes to the literature data produced from multi-fibre modules to determine whether analogous behaviour is observed.

2. Materials and methods

2.1. Experimental setup

To investigate the translatability of mass transfer data between characteristic length scales (d_h and L), hollow-fibre membranes with seven different inner fibre diameters and four different lengths were specified, all of which comprised identical wall thickness and membrane material (polydimethylsiloxane, PDMS). Wickramasinghe et al. [41] observed a decline in experimental Sh data with a reduction in Gz number, and attributed the behaviour to non-uniform flow caused by polydispersity in hollow fibre diameter between fibres in the bundle. In this study, mass transfer analysis was undertaken on single fibres to obviate such flow distribution effects, characteristic of multi-fibre bundles. Each PDMS membrane fibre was potted at both ends with individual end caps (of polypropylene construction) measuring 42 mm in length. This is greater than estimated entrance lengths which were below 36 mm (and were predominantly below 16 mm), indicating that flow was fully developed upon transition into the active length of the fibre. Each test comprised of a single-fibre which was held in place using a dual clamp retort stand (Fig. 1). The fibres were supported in

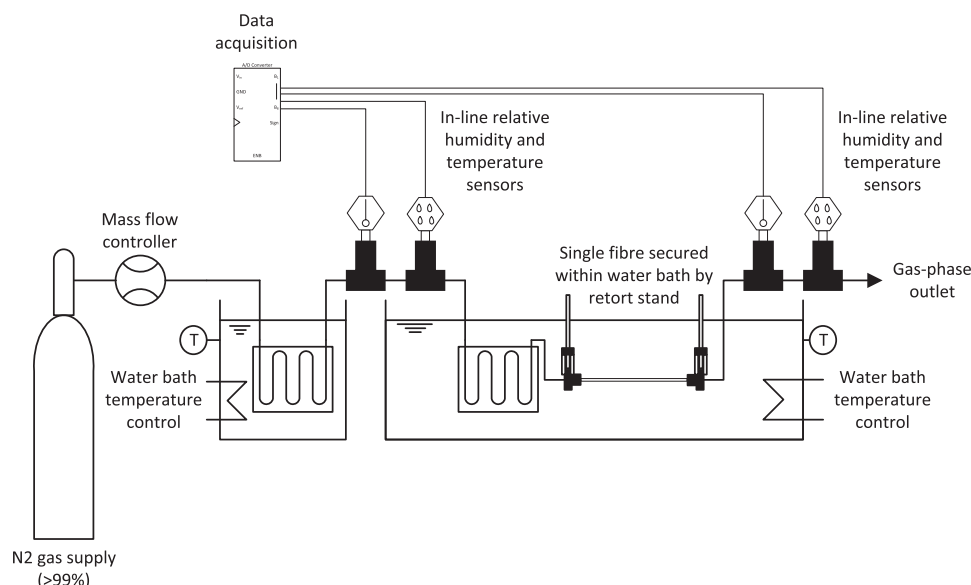


Fig. 1. Schematic of single fibre experimentation.

the dual clamp retort stand; the clamps were applied to the potted fibre ends. Both clamps were sited vertically downwards, with the fibre running parallel to the retort bar. The clamps were positioned to ensure the fibre was taut thereby producing a 'straight' (i.e. with respect to the retort bar) orientation without inducing unnecessary stretching. The fibre was immersed in a water bath to provide an excess of water to the fibre periphery. Whilst the PDMS fibre construction was of stable soft 'rubbery' material, once both fibre ends were clamped to ensure a solid fixing and the fibre installed within the water bath, no fibre movement was visually evident. Dry nitrogen gas (99% purity) was supplied from a cylinder (BOC Ltd., Manchester, England) to the lumen side of the hollow fibre. Nitrogen flow rate was regulated using mass flow controllers with a range of either 0.5–50 ml min⁻¹ (Cole-Parmer Instrument Co. Ltd., London, UK) or 0.005–1 l min⁻¹ (RM & C Ltd., Sheffield, UK), dependent upon which membrane fibre diameter was under investigation. Pressure drop was highest in the smallest diameter PDMS fibre. However, in all experiments, internal fibre pressure drop was below 0.74 bar(g). Throughout the trials there were no visual observations of fibre deformation following changes to hydrodynamic conditions. The mass flow controllers had an accuracy of $\pm 0.8\%$ of reading or $\pm 0.2\%$ full-scale. Volumetric flow rates downstream of the membrane were verified using by milli-gas counter (MGC, Ritter, North Marston, UK). The nitrogen gas supply passed through two columns in series, both of which were immersed in water baths (X12, Thermofisher, Suffolk, UK) to ensure that isothermal conditions were achieved ($\Delta T=0$) between liquid and gas phases. The gas and liquid phases were kept at a constant temperature of 21.5 °C, and the temperature difference between gas phase inlet and outlet was consistently below 0.25 °C. Non-immersed pipework was insulated using proprietary pre-insulated stainless steel pipework (Swagelok, London, UK). Temperature and humidity probes (Vaisala Ltd., Birmingham, UK) were sited upstream and downstream of the immersed fibre, and were supplied calibrated with a manufacturer stated range and accuracy of 0–100% and $\pm 1\%$ respectively. Temperature and relative humidity data were recorded using a 12-bit datalogger with a resolution of 15 s between readings (Picolog 1000 series, Pico Technology Ltd., St. Neots, UK).

To verify humidity sensor response, two nitrogen gas streams (dry nitrogen gas and a saturated nitrogen gas) were partially mixed to provide gases with 20%, 40%, 60%, and 80% humidity. The resultant humidity data was within 2% of the expected values. Prior to experimentation, a non-permeable pipe was installed in place of the membrane and a difference in relative humidity between the inlet and outlet of less than 1% recorded which evidenced the rig was gas-tight. As the membrane fibre was immersed in water, the resistance associated with the liquid phase ($1/k_L^W S$) can be neglected (see Appendix D for experimental verification) and only the resistances associated with the membrane and gas phase need be considered [22]:

$$\frac{1}{K_L} = \frac{1}{K_{OV}^W S} = \frac{1}{k_m^W S} + \frac{1}{k_g^W S} \quad (7)$$

The change in water vapour concentration in the air stream in the axial direction on the lumen side of a single hollow fibre under steady state conditions can be given by [22]:

$$\frac{v^a dX^W}{dZ} = K_{OV}^W a (X_{sat}^W - X^W) \quad (8)$$

on the assumption of the boundary conditions $X^W=0$ at $z=0$ and $X^W=X_L^W$ at $z=L$, then:

$$X_L^W = X_{sat}^W (1 - e^{-(K_{OV}^W a L / v^a)}) \quad (9)$$

from which the overall mass transfer coefficient can be determined. Whilst the L  v  que solution can provide description of the tube-side mass transfer coefficient close to the entrance region of the lumen (which is generally regarded as valid for Gz exceeding 20), Newman [25] offered an extension to the L  v  que solution for small values of L ,

which offers potential explanation for some of the lower Gz range:

$$Sh = 1.6151 \left(\frac{ReSc}{L/d_h} \right)^{1/3} - 1.2 - 0.28057 \left(\frac{L/d_h}{ReSc} \right)^{1/3} + \dots \quad (10)$$

For the full expansion series, refer to Appendix B. Before each experiment, dry nitrogen gas with a flow rate of 1 l min⁻¹ was used to dry the gas rig for at least 60 min before experiments began. In all experiments, the inlet relative humidity was below 3%. Based on experimental data, prediction intervals were calculated using:

$$Gz = Sh t_{0.95} \sqrt{1 + \frac{1}{n} + \frac{(Gz - Gz_m)^2}{SS_{xx}}} \quad (11)$$

where $t_{0.95}$ is the t-distribution fixed to 95% confidence, n is the number of observations, SE is standard error, SS_{xx} is the sum of squares of deviations from the sample mean, Gz_m is the average Graetz number. Each hollow fibre membrane experiment (both for variations in fibre length and for fibre inner diameter) was undertaken in triplicate. The triplicate comprised of testing three independent fibres with identical geometry, and the average of the three experiments used for mass transfer analysis (Appendix A).

2.2. Membrane material

The polydimethylsiloxane (PDMS) hollow fibre membranes were procured from a commercial supplier and comprised of lumen diameters: 100, 300, 800, 1300, 1800, 2800 and 4800 μm . The stated wall thickness was 100 μm . For comparison of lumen diameter, each fibre was prepared at a length of 0.2 m. To determine the impact of fibre length, three lumen diameters were selected (300, 1300 and 4800 μm) and prepared at four lengths: 0.1, 0.2, 0.4, and 0.8 m. Prior to experimentation, both fibre wall thickness and lumen diameter were assessed using environmental scanning electron microscopy (eSEM) at an acceleration voltage of 20 kV (XL30, FEI, Oregon, US) for the fibres with smaller lumen diameters (100–1800 μm) or optical microscopy for the larger lumen diameters (2800 and 4800 μm). Fibre samples for SEM were first coated with gold–palladium (Au–Pd) using a cool sputtering SEM coating unit (E5100, Polaron Equipment/Quorum Technologies Ltd., Lewes, UK). Measurements for both lumen (inner tube diameter) and wall thickness were undertaken until the standard error approached a constant value (Appendix A). Data on fibre lumen diameter and wall thickness were transformed into a size distribution using a log-normal distribution function [20].

3. Results

3.1. Membrane characterisation

A range of seven hollow fibre membrane lumen diameters were selected: 100, 300, 800, 1300, 1800, 2800 and 4800 μm (for illustration, see Fig. 2) to provide a broad data range and to ensure reasonable coverage of the lumen diameters ordinarily selected in the literature [24,41,17]. Accuracy of the specified inner diameter (lumen diameter) for each hollow fibre was ascertained using environmental scanning electron microscopy (ESEM) and optical microscopy (Fig. 2, Appendix A). The difference between the stated diameter and the measured diameter were within 6% with the exception of the smallest fibre (100 μm , Table 2). Fibre wall thickness was also measured for each lumen diameter, recording an overall average of 118.5 μm with a standard deviation of 10.8% across all seven fibre samples.

3.2. The impact of lumen diameter on mass transfer within the low Graetz range

Outlet relative humidity (RH) was measured directly within the gas phase downstream of the membrane to provide quantification of the

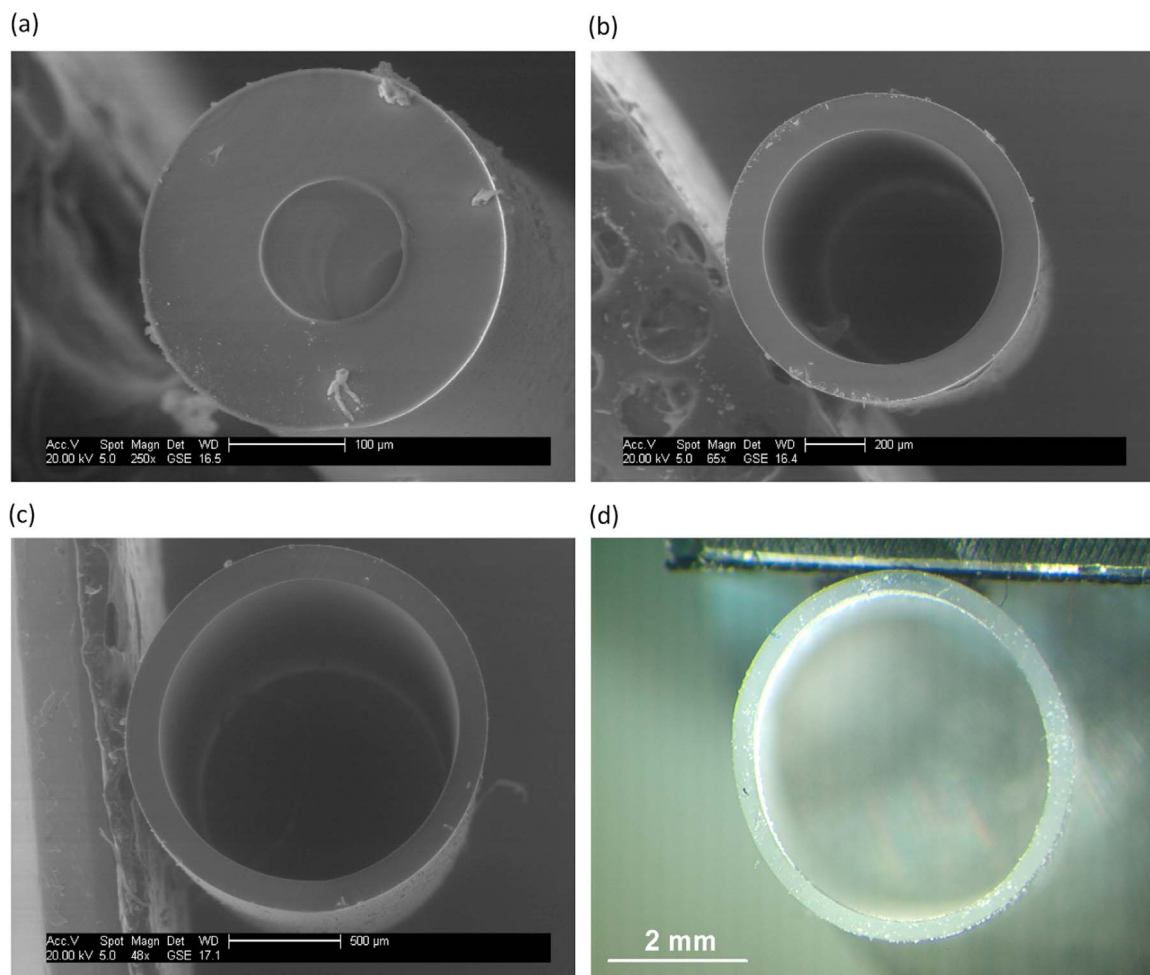


Fig. 2. Illustrative environmental scanning electron microscope (ESEM, a–c) and optical microscope images (d) of the hollow-fibre membranes used in this study with lumen diameters of: (a) 100 µm; (b) 800 µm; (c) 1300 µm; and (d) 4800 µm. All fibres comprised a wall thickness of around 120 µm.

Table 2

Estimated wall thickness and lumen diameter for each hollow fibre membrane lumen diameter studied ($n=25$).

Stated	Wall thickness		Lumen diameter	
	Measured	SE ^a	Measured	SE ^a
100	98.0	1.4	122.7	1.1
300	113.6	0.6	304.2	2.1
800	122.0	1.2	785.8	3
1300	138.0	2.0	1303.4	4.5
1800	115.5	2.7	1764.3	17.9
2800	113.2	2.5	2774.1	124.5
4800	128.9	4.7	4498.8	35.6
Average	118.5			
SD (%)	10.8			

^a SE – Standard error.

overall mass transfer coefficient (Fig. 3). For each lumen diameter, the outlet RH was highest at the lowest gas velocity (V_g) tested and declined upon increasing V_g . The highest outlet RH was achieved using the fibre with the smallest diameter (100 µm). Outlet RH for fibres with internal diameters of 100 and 300 µm was reasonably insensitive to an increase in V_g when compared to larger diameter fibres operated within the same velocity range. Transformation of the RH data into the overall mass transfer coefficient (K_{ov}) demonstrated that K_{ov} increased with an increase in gas flow rate (Q_G).

The Sherwood number was used to normalise the overall mass

transfer coefficient for the inner diameter (Eq. (2), Fig. 4) and the Graetz number also applied to enable normalisation of the fluid flow characteristics by inclusion of the hydraulic diameter and fibre length. The full dataset from seven fibres was observed to collapse into a linear slope that was characterised by a gradient of $Sh=0.36Gz$ (black dashed line, Fig. 4). Sherwood numbers of between 1×10^{-3} and 1 were obtained from transformation of the overall mass transfer coefficient, and was markedly below that estimated through extrapolation of the L  v  que solution. Membrane resistance was determined for each fibre through Wilson plot analysis with the inverse gas velocity raised to the power of one ($1/V_g$, Fig. 5, inset). Resistance in series analysis indicated that for Gz of below around 2, resistance to mass transfer was primarily associated with the gas phase on the tube-side. However, once Gz exceeded around 2, membrane resistance ($1/k_m$) dominated mass transfer. The membrane resistance ($1/k_m$) estimated at the intercept of the Wilson plot can be considered an estimate of the reciprocal of the permeance (or, δ/P) [17,26]. In this study, the estimated permeance was around 2550 GPU (1 GPU = $10^{-6} \text{ cm}^3 \text{ (STP) cm}^{-2} \text{ s}^{-1} \text{ cm Hg}^{-1}$) which compares to a permeance of 575 GPU estimated from literature permeability data ([35] after [21]). Both Bennett et al. [4] and Oliveira et al. [28] reported their experimentally determined PDMS permeability to be considerably higher (up to two orders of magnitude) than the corresponding literature permeability value. The authors explained the difference to arise from the use of different types of silicone membranes. The molecular properties of PDMS including the polymer backbone chemistry, crosslink density and fillers, which are used for promotion of mechanical integrity as well to introduce other unique properties into the polymer, are known to strongly affect water vapour

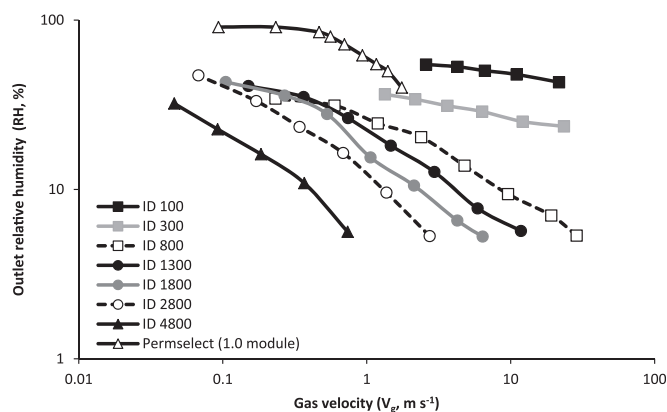


Fig. 3. Outlet relative humidity (RH, %) measured in the gas phase at the outlet of each of the hollow-fibre membrane diameters studied (100–4800 μm). Fixed fibre length, 0.2 m.

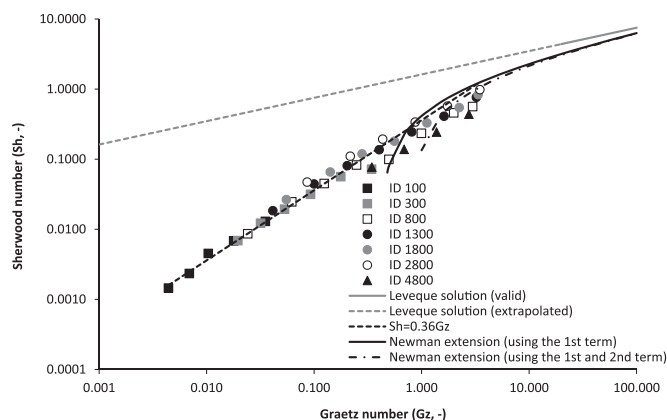


Fig. 4. Comparison of mass transfer data for the complete range of lumen diameters studied (100–4800 μm). Fixed fibre length, 0.2 m. Data transformed through Sherwood Graetz analysis to account for variation in hydraulic diameter (d_h , lumen diameter).

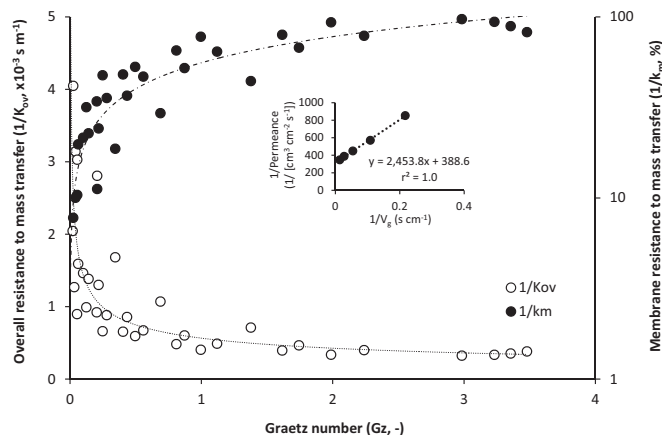


Fig. 5. Overall resistance to mass transfer identified for each fibre diameter tested. Inset shows an example Wilson plot (800 μm fibre ID) used to derive the membrane resistance ($1/k_m$, s m⁻¹) for each fibre diameter (100–4800 μm ID), which is plotted as a proportion of the total resistance (%).

permeability [38]. To illustrate, Van Reeth and Wilson [37] identified over two orders of magnitude in water vapour transmission rate through silicones due to changes in silicon chain length and substitution of the PDMS substrate. In this study, a water flux of around $12 \text{ g m}^{-2} \text{ h}^{-1}$ was determined in the Gz region where $1/k_m$ controlled mass transfer and is equivalent to estimated fluxes from several studies including that of a highly crosslinked PDMS film of equivalent thickness (100 μm ; [40,27]).

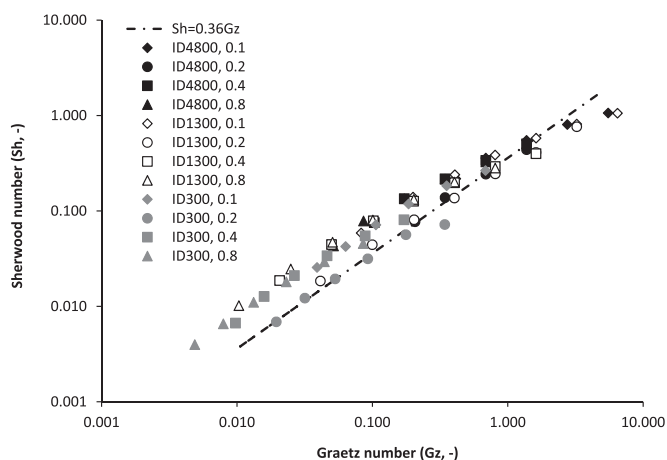


Fig. 6. Comparison of mass transfer data through a range of fibre lengths (0.1–0.8 m) for a selected number of fibres (300, 1300 and 4800 μm). Data transformed through Sherwood Graetz analysis to account for variation in fibre length and hydraulic diameter (d_h , lumen diameter).

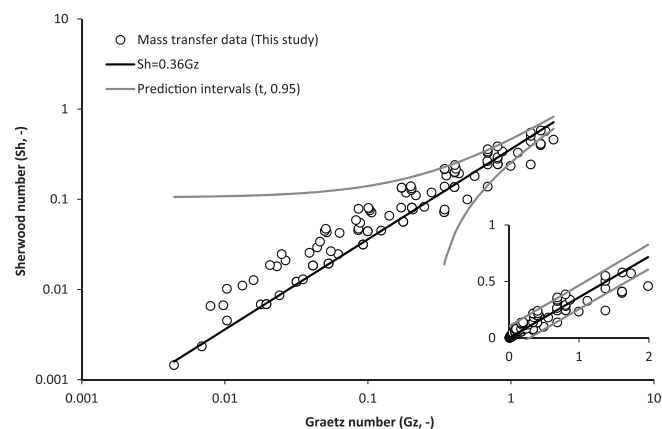


Fig. 7. Empirical Sherwood correlation ($Sh=0.36Gz$) derived from fibre diameter data is compared to both fibre diameter and fibre length data ($n=102$). Statistical analysis evidenced 93% of data was within prediction intervals ($t=0.95$), within Gz range 4×10^{-3} and 2. Inset: Linearised dataset.

3.3. The impact of fibre length on mass transfer within the low Graetz range

To determine whether mass transfer data can be extrapolated between membrane fibres of different geometries within the low Graetz range, variations in fibre length (0.1, 0.2, 0.4 and 0.8 m) were also trialled for three membranes of different internal diameter (300, 1300 and 4800 μm) (Fig. 6). Only mass transfer data from the region dominated by gas phase resistance was transformed into the Sherwood analysis (with use of the gas phase diffusion coefficient). Similar to the lumen diameter data, Sherwood data arising from fibres of different lengths collapsed into a linear trend coinciding with Graetz numbers between 5×10^{-3} and 1.7. The preliminary proportionate trend identified between Sh and Gz ($Sh=0.36Gz$) broadly described the data, and particularly for data below Gz of 2. To test this assertion, all Sherwood data produced below Gz of 2 were compared to the prediction when fitted with prediction intervals at $t_{0.95}$ (Fig. 7). For the total dataset ($n=102$), 93% of data was found to lie within the prediction intervals.

3.4. Comparison of experimental data with literature data from within the low Graetz range

Sherwood analysis of the experimentally determined mass transfer coefficient (which was determined in this study using single fibres) was compared to data from the literature (Fig. 8). The published data were

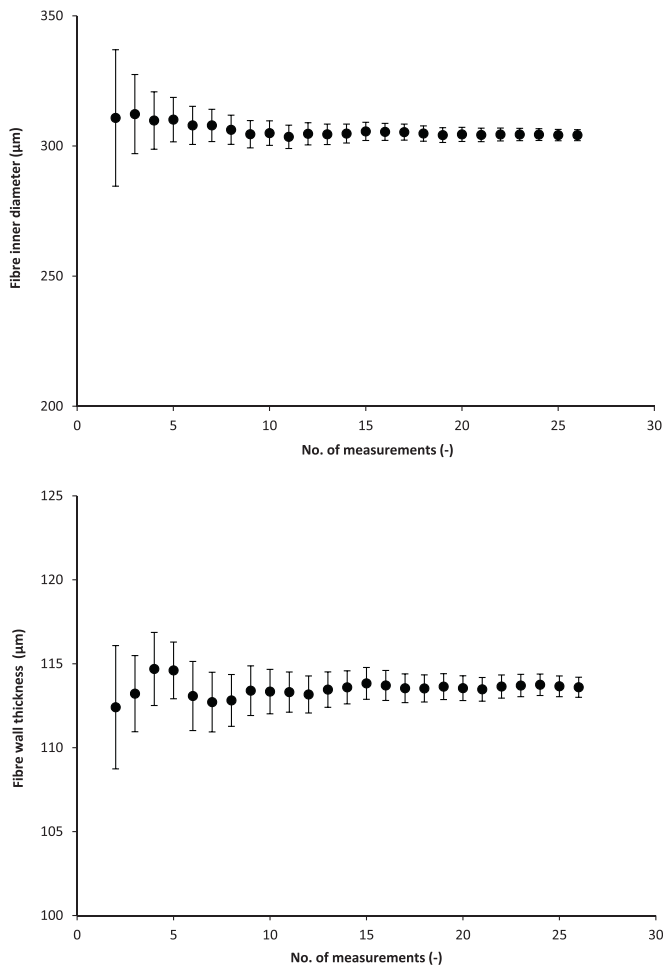


Fig. A1. Both fibre inner diameter (lumen) and wall thickness was quantified for each PDMS hollow fibre lumen diameter evaluated (range 100–4800 µm). The 300 µm fibre is used here for illustration.

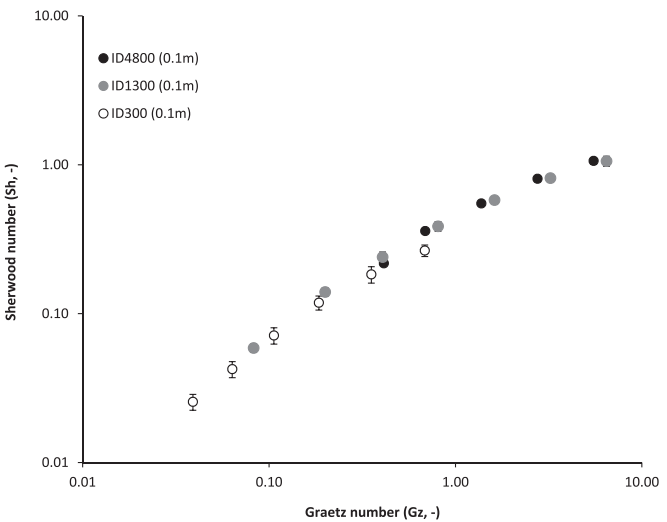


Fig. A2. Illustration of data reproducibility. Each data point represents an average value of three experiments undertaken with three fibre of equal dimensions. Error bars represent the standard deviation which in all cases was below 10% of the Sherwood number reported.

Sherwood analysis using the assumptions presented in Table A1 and using the method presented in Fig. A3.

Table A1
Data selected to provide mass transfer analysis from several applications.

Manufactured scale			Commercial	Commercial	Commercial
Application			O ₂ removal from water	Air humidification	O ₂ removal from water
Membrane type			PDMS	PDMS	Polypropylene ^a
			Dense, symmetric	Dense, symmetric	Microporous
Temperature		°C	22	22	22
Dynamic viscosity	μ	Pa s ⁻¹	0.954×10 ⁻³	1.847×10 ⁻⁵	0.954×10 ⁻³
Mass diffusivity	D	m ² s ⁻¹	1.97×10 ^{-9b}	2.54×10 ⁻⁵	1.97×10 ^{-9b}
Fluid density	ρ	kg m ⁻³	998 ^c	1.185	998 ^c
Schmidt number	Sc	–	485	0.61	485
Hydraulic diameter	d _h	m	190×10 ⁻⁶	190×10 ⁻⁶	210×10 ⁻⁶
Length	L	m	0.14	0.14	0.198
Diameter/length	d _h /L	–	1.36×10 ⁻³	1.36×10 ⁻³	1.11×10 ⁻³
Fibre number	n	–	12,600	12,600	7047
Single lumen area ^d	a _{SL}	m ²	2.84×10 ⁻⁸	2.84×10 ⁻⁸	3.80×10 ⁻⁸
Total lumen area ^d	a _{TL}	m ²	3.57×10 ⁻⁴	3.57×10 ⁻⁴	2.56×10 ⁻⁴
Membrane area ^e	a _m	m ²	1.0	1.0	0.9

^a Microporous.
^b 20 °C.
^c 21 °C.
^d Based on lumen diameter.
^e Available surface area for mass transfer.

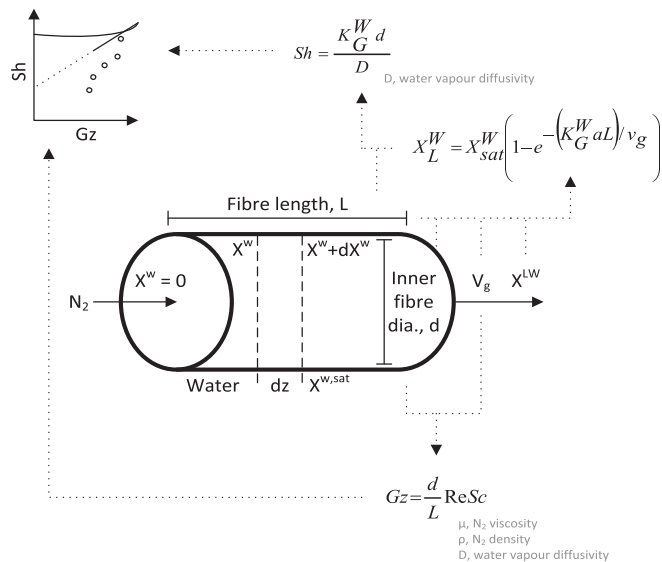


Fig. A3. Summary of calculation procedure following measurement of gas-side (tube-side) data.

Appendix B. Extension of the Lévêque solution

For a hollow fibre membrane operated with fully developed, laminar flow within the lumen, through which a constant wall concentration is achieved, the L  v  que solution can be considered valid near the entrance. For considerable flow rates, the axial diffusion term can be neglected arriving at the Graetz problem. Newman [25] set out an expansion of the L  v  que solution to small values of L , through inclusion of two variables, the L  v  que similarity variable ($B1$) and a dimensionless axial distance variable ($B2$), which are necessary for obtaining higher-order terms [25] (Fig. A4).

$$\xi = (R-r) \left(4[V_z]/9D \left[\frac{d_h}{2} \right] L \right)^{1/3} \quad (\text{B1})$$

$$\zeta=9DL/4[V_z]\left[\frac{d_h}{2}\right]^2 \quad (\text{B2})$$

From the dimensionless concentration $\theta = (c_i - c_0)/(c_b - c_0)$, the solution described as:

$$\frac{\delta^2 \theta}{\delta \xi^2} + 3\xi^2 \frac{\delta \theta}{\delta \xi} - 3\xi \zeta^{1/3} \frac{\delta \theta}{\delta \zeta^{1/3}} = \left(\frac{3}{2} \xi^3 \zeta^{1/3} + \frac{\zeta^{1/3}}{1 - \xi \zeta^{1/3}} \right) \frac{\delta \theta}{\delta \xi} - \frac{3}{2} \xi^2 \zeta^{2/3} \frac{\delta \theta}{\delta \zeta^{1/3}} \quad (\text{B3})$$

with the boundary conditions $\theta = 0$ at $\xi = 0$; $\theta = 1$ at $\xi = \infty$; $\theta = 1$ at $\zeta = 0$. The solution can then be set-out as a power series in $\zeta^{1/3}$:

$$\theta = \theta_0(\xi) + \zeta^{1/3} \theta_1(\xi) + \zeta^{2/3} \theta_2(\xi) + 0(\zeta) \quad (\text{B4})$$

in which $\theta_0(\xi)$ is the Lévêque solution. From this the ordinary differential equations for θ_0 , θ_1 , θ_2 are:

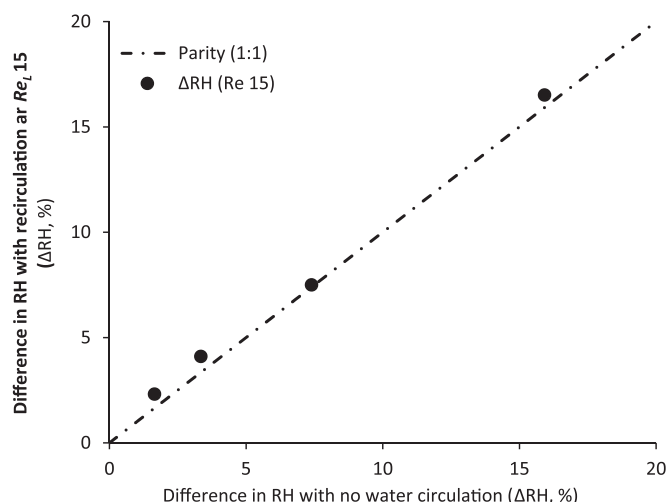


Fig. A4. Comparison of outlet relative humidity data recorded when no water is recirculated compared to when water is recirculated at a Reynold's number of 15.

$$\theta_0 + 3\xi^2\theta_0' = 0 \quad (\text{B5})$$

$$\theta_1 + 3\xi^2\theta_1' - 3\xi\theta_1 = \left(1 + \frac{3}{2}\xi^3\right)\theta_0' \quad (\text{B6})$$

$$\theta_2 + 3\xi^2\theta_2' - 6\xi\theta_2 = \left(1 + \frac{3}{2}\xi^3\right)\theta_1' + \xi\theta_0' - \frac{3}{2}\xi^2\theta_1 \quad (\text{B7})$$

with boundary conditions $\theta_0 = \theta_1 = \theta_2 = 0$ at $\xi = 0$, $\theta_0 = 1$, $\theta_1 = \theta_2 = 0$ at $\xi = \infty$. The solutions to the ordinary differential equations for θ_0 , θ_1 , θ_2 are as follows:

$$\theta_0 = \frac{1}{\Gamma(4/3)} \int_0^\xi e^{-x^3} dx \quad (\text{B8})$$

$$\theta_1 = -\frac{\xi^2 e^{-\xi^3}}{10\Gamma(4/3)} - \frac{(3/5)}{\Gamma(4/3)} \xi \int_\xi^\infty e^{-x^3} dx \quad (\text{B9})$$

$$\theta_2 = -\frac{3/10}{\Gamma(4/3)} \xi^2 \int_0^\infty e^{-x^3} dx - \left(\frac{11}{210} + \frac{1}{14}\xi^3 + \frac{3}{200}\xi^6\right) \frac{e^{-\xi^3}}{\Gamma(4/3)} + \frac{11}{630} \frac{\Gamma(5/3)}{[\Gamma(4/3)]^3} \int_0^1 \frac{x^{1/3}}{(1-x)^{2/3}} e^{-x^3/(1-x)} dx \quad (\text{B10})$$

The derivatives at the surface for θ_0 , θ_1 , θ_2 are as follows:

$$\theta_0'(0) = 1/\Gamma(4/3) = 1.11984652 \quad (\text{B11})$$

$$\theta_1'(0) = -0.6 \quad (\text{B12})$$

$$\theta_2'(0) = -\frac{11/140}{\Gamma(4/3)} \left[\frac{\Gamma(5/3)}{\Gamma(4/3)}\right]^2 = -0.0899230 \quad (\text{B13})$$

From this, the average Sherwood number relevant for the concentration difference at the inlet is:

$$Sh = 1.6151 \left(\frac{ReSc}{L/d_h}\right)^{1/3} - 1.2 - 0.28057 \left(\frac{L/d_h}{ReSc}\right)^{1/3} + \dots \quad (\text{B14})$$

Appendix C. Water vapour permeance of various membrane polymers

Literature data from previous studies evaluating water vapour transport in the gas phase (Table A2).

Table A2
Membrane water vapour permeance.

Membrane material	Structural type	Wall thickness (cm)	Permeance (P/l) (cm s ⁻¹)	(GPU ^h)	Reference
CTA ^a	TFC ^b	1.1 × 10 ⁻⁴	9 × 10 ⁻³	9000	[29]
SAC ^c	TFC ^d	5.0 × 10 ^{-3d}	1.0 × 10 ⁻²	10,000	[39]
PDMS ^e	Symmetric	1.0 × 10 ⁻²	5.2 × 10 ^{-4f}	520	[21]
PAN/PDMS ^g	TFC ^b	N/a	1.3 × 10 ⁻²	13,000	[42]

^a Cellulose triacetate.

^b Thin-film composite.

^c Siloxane-amide copolymer.

^d Estimated based on reported permeance and permeability.

^e Polydimethylsiloxane.

^f Estimated based on current fibre wall thickness in this study.

^g PDMS-coated PAN composite.

^h 1 GPU = 1 × 10⁻⁶ cm³(STP) cm² s⁻¹ cm Hg⁻¹.

Appendix D. Impact of liquid velocity on mass transfer

Within this paper, we postulate that as the membrane fibre was immersed in water, the resistance associated with the liquid phase ($1/k_L^w S$) can be neglected and only the resistances associated with the membrane and gas phase need be considered. This is analogous to observations of previous authors [22]. To verify this assumption, we undertook tests on a single fibre exposed to water fluid flow (Table A3) and compared the data to that produced without fluid recirculation (Fig. A4). The arising data confirmed the negligible impact of fluid flow on mass transfer.

Table A3

Module description used to determine the impact of liquid side hydrodynamics.

Module description		Units
<i>Fibre characteristics</i>		
Membrane material	PDMS	–
Wall thickness	100	μm
Lumen internal diameter	1800	μm
<i>Module characteristics</i>		
Shell volume	0.00184	m ³
Shell area (for liquid flow)	0.0144	m ²
No. of fibres	1	–
Total membrane area	0.00113	m ²
Packing fraction	NA	m ^{−1}
<i>Module operation</i>		
Temperature difference (ΔT_{gas})	0.07	°C
Temperature difference ($\Delta T_{\text{gas/liquid}}$)	0.15	°C
Liquid side <i>Re</i>	0 and 15	–
Gas side <i>Re</i>	up to 800	–

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