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Stall and Surge in Axial Flow Compressors

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*"The man who thinks he knows something
does not yet know as he ought to know."*

(1 Cor 8:2, NIV)

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Except where stated to the contrary the research described in this dissertation is the original work of the author, with the exception of parts of the work described in chapters 3 and 4. The early stages of the work described in each of these chapters was carried out in partnership with C. Freeman of Rolls-Royce plc. In the former case the collaborative part of the work covered the experimental work itself and the initial analysis of the results reported in Wilson and Freeman [1994], which corresponds broadly to section 3.4.2. In the latter case the collaborative part covered the development of the model and the comparison with experiment of the predicted overall stalling behaviour as reported in Freeman and Wilson [1993], which corresponds broadly to section 4.3.1.

A G Wilson

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ABSTRACT

The objective of the work described in this thesis is twofold; to elucidate the nature of stall and surge in an axial flow aeroengine compressor, and to improve on current computational stall modelling techniques. Particular attention is paid to the initial stages of the stall/surge transient, and to the possibility of using active control techniques to prevent or delay the onset of stall/surge.

A detailed analysis is presented of measurements of the stalling behaviour of a Rolls-Royce VIPER jet engine, showing a wide variety of stall inception and post-stall behaviour. Stall transients are traced from initial disturbances through to stable rotating stall or axisymmetric surge. The stall inception pattern at nearly all speeds is shown to conform to the short circumferential lengthscale pattern described by Day [1993a].

A multiple compressors in parallel stall model is developed using conventional stall modelling techniques, but extended to include the effects of the jet engine environment. The model is shown to give a good representation of the overall stalling behaviour of the engine, although the details of the stall inception period are not accurately predicted. A system identification technique is applied to the results of the model in order to develop a method of active control of stall/surge.

A new stall model is introduced and developed, based on a time-accurate three dimensional (but pitchwise averaged) solution of the viscous flow equations, with bladerow performance represented by body forces. The flow in the annulus boundary layers is calculated directly, and hence this new method is sufficiently complex to model the initial localised disturbances that lead to stall/surge. At the same time the computational power required is compatible with application to long multistage compressors.

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NOTATION

General

a	Speed of Sound	κ	Heat Transfer Coefficient
A	Area	λ	Second Coefficient of Viscosity
c_p	Specific Heat Capacity at Constant Pressure	μ	Dynamic Viscosity
c_v	Specific Heat Capacity at Constant Volume	ν	Kinematic Viscosity
e	Internal Energy	ξ	Stagger Angle Relative to Axial
m	Mass Flow	ρ	Density
n	Unit Normal Vector	Φ	Dissipation
p	Static Pressure	Ω	Shaft Speed (Angular)
P	Total Pressure	$\bar{\quad}$ (Overscore)	Average Quantity
Pr	Prandtl Number		
r	Radius		
$\hat{r}$	Unit Vector in Radial Direction		
t	Time		
T	Total Temperature		
u	Speed		
u	Velocity		
z	Axial Distance		
$\hat{z}$	Unit Vector in Axial Direction		
θ	Circumferential Angle		
$\hat{\theta}$	Unit Vector in Circumferential Direction		

Subscripts

r	Radial Direction
θ	Circumferential Direction
z	Axial Direction
lam	Laminar
t	Turbulent (Excluding Laminar Contribution)
τ	Turbulent (Including Laminar Contribution)
ave	Average
atm	Atmospheric

Chapter 2, on Moore and Greitzer [1986a,b]

a	Reciprocal Time-Lag Parameter of Blade Passage	Φ	Annulus Averaged Axial Flow Coefficient; Axial Velocity Divided by Wheel Speed
l_c	Total Aerodynamic Length of Compressor in Wheel Radii	Ψ	Total to Static Pressure Rise Coefficient
m	Compressor Duct Flow Parameter	ψ_c	Axisymmetric Pressure Rise Coefficient
Y	Disturbance Potential at Compressor Entrance	Subscripts	
ξ	Time in Radians of Wheel Rotation	θ	Derivative with respect to θ
		ξ	Derivative with respect to ξ

Chapter 2, on Dunham [1965]

n	Circumferential Mode Number	z	Gap Length Normalised by Tip Radius
U	Axial Velocity Disturbance	γ	Mean Flow Angle
V	Circumferential Velocity Disturbance	Subscripts	
P	Total Pressure Disturbance Normalised by Density and Mean Axial Velocity	0	Condition at Blade Exit/Gap Inlet

Chapter 4

$a_1, a_2, a_3,$ a_4, a_5	Constants in Axisym. Sys. Id. Equ 4.17	$\alpha, \beta, \gamma, \delta,$ $\bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\delta},$ ϵ	Constants in Non- Axisym. Sys. Id. Equ 4.22
b	Puffer Jet Bleed Input (§ 4.4)	$\alpha_x, \beta_x, \bar{\alpha}_x,$ $\bar{\beta}_x, \epsilon_x$	Input Parameters for Non-Axisym Control System (Equ 4.25)
F_B	Axial Blade Force (Equ 4.1)	ϕ	Velocity Potential (Equ 4.3)
H	Enthalpy Flux (Equ 4.14)	$\omega=2\pi m/32$	Phase Shift between Circumferential Nodes
l	Compressor Length	'(Dash)	Derivative with respect to Time
l_{in}	Inlet Duct Length	Subscripts	
m	Circumferential Mode Number	eff	Effective
M	Total Mass of Fluid in the Compressor (Equ 4.2)	c	Combustor Cold Chamber
q	Local Flow Rate Scaled to Entire Annulus Area (§ 4.4)	h	Combustor Hot Chamber
α, β	Input Parameters for Axisym Control System (§ 4.4.1.1)	x	Input Parameter for Control System

Chapter 5

A	Constant in van Driest Expression (Equ 5.38)	α	Time Lag Coefficient (Equ 5.46)
$\underline{e}_i$	Vector (Equ 5.28)	$\underline{\underline{\beta}}_i$	Matrix Defined Prior to Equ 5.29
$\underline{E}$	Matrix (Equ 5.28)	$\underline{\underline{\Gamma}}$	Matrix Defined in Equ 5.30
$\underline{F}$	Body Force (Equ 5.1 and § 5.2.6)	λ	Oscillating Boundary Layer Parameter (Equ 5.56)
$\underline{f}$	Vector (Equ 5.28)	χ	Constant in van Driest Expression (Equ 5.38)
$\underline{g}$	Vector (Equ 5.28)	ϕ	Arbitrary Function of Position and Time
l	Mixing Length (Equ 5.38)	$\omega = 1/\rho r$	Inverse Density Parameter
m	Circumferential Mode Number	ω	Frequency (Equ 5.51)
$\underline{M}$	Matrix (Equ 5.28)	$\underline{\underline{\Omega}}_i$	Matrix Defined in Equ 5.36
$\underline{P}$	Matrix (Equ 5.28)	$\underline{\underline{\nabla}}$	Gradient in r-z Plane
R_1	Radius of Inner Wall (Equ 5.47)	Superscripts	
R_2	Radius of Outer Wall (Equ 5.47)	+	Relating to Sweep from Exit to Inlet Plane (Equ 5.34)
s	Weighting Function (§ 5.2.2.2)	-	Relating to Sweep from Inlet to Exit Plane (Equ 5.35)
v_*	Friction Velocity (Equ 5.38)		
y	Distance from Wall		
y^+	Normalised Distance from Wall (Equ 5.38)		
$\alpha, \beta, \gamma, \delta, \varepsilon$	Coefficients in 5-Point Circumferential Discretisation (Equ 5.21)		

1. INTRODUCTION

In May 1992 high pressure air was pumped into the combustion chamber of a single shaft Rolls-Royce VIPER Mk 522 jet engine running at 13500 RPM. For about a second the engine operated normally as the pressure in the combustion chamber increased. Then, within the space of twenty milliseconds, normal compressor performance had collapsed entirely. Flames leapt from the rear of the engine as unburned fuel made its way downstream, and from the front of the engine as the pressure in the combustion chamber forced air backwards through the compressor. The flame in the combustor went out, the fuel was cut and the engine ran to a standstill.

This spectacular event, one of a series of controlled surges reported in chapter 3, was typical of a surge in a high speed aeroengine. In this case surge was induced artificially, but on an aeroplane crosswind effects, high inlet temperatures, over-fast engine acceleration and compressor deterioration all make an engine liable to surge.

Great care is taken during compressor design and development to allow a big enough safety margin that the events described only rarely lead to surge. The safety margin is achieved by compressor blading changes, by adding compressor bleed ports, by adding variable stator vanes or by limiting aircraft operability, for instance by restricting engine acceleration rates. How these work in improving the safety margin will be considered later, but the important thing to note here is that there is a penalty associated with each one. In the case of blading changes the penalty is efficiency related; if the blading is designed for optimum safety margin, it cannot also be designed for optimum efficiency. Likewise if the compressor puts work into a proportion of the flow that is then dumped overboard through a bleed port, the overall efficiency is not going to be high. In the case of variable stator vanes, the penalty is one of increased complexity. Restricting the operability of the aircraft is clearly undesirable.

More recently, attempts have been made to improve the safety margin by 'passive stall control'. Here geometrical changes are made, generally to the casing near rotor tip, to change the flow pattern to one more resilient to surge. Engines with passive stall control are now operating in service. A great deal of work is being done on producing a passive stall control configuration that has no associated efficiency penalty during normal engine operation.

Engine surge starts as a small amplitude disturbance that grows very quickly, and an alternative to all of the above methods of increasing safety margin is to detect this

disturbance while it is still small, and apply a counter-disturbance to damp it out. In this way the same conditions may be present as would normally make an engine liable to surge, but disturbances in the flow are kept at a very low amplitude, and the engine operates stably. This technique is called active stall control (the distinction between 'stall' and 'surge' is drawn in the next section), and has been demonstrated in laboratory tests. The major advantage of such a method is that there is no associated efficiency penalty during normal engine operation; if there are no disturbances to control, then the control system has no effect on the engine.

The next three sections of this introductory chapter are devoted to explaining some of the fundamental ideas of stall and surge in axial flow compressors. The scope of the work described in this thesis is discussed in the final section. The extensive literature on stall, surge and active stall control is reviewed in chapter 2.

1.1 STALL AND SURGE

The fundamental building block for an understanding of stall and surge is the compressor operating map (fig 1-1). The constant speed lines are a function of compressor design, and relate the flow through the compressor to the pressure ratio across it during steady operation at a given compressor speed. The working line is defined by the environment in which the compressor is operating. In the case of a single shaft engine such as the VIPER, it is controlled by the flow area of the nozzle guide vanes at the exit of the combustor. In steady state operation the compressor always operates on this line. During a transient manoeuvre, such as an engine acceleration, the operating point can move from the working line. The surge line is primarily a function of compressor design, and defines the limit of stable operation. If the pressure ratio across the compressor rises above this line at a given inlet flow, then the compressor will fall into stall/surge, often with catastrophic consequences as described at the start of the chapter. The vertical distance between the working line and the surge line is called the surge margin.

A stall/surge event can be divided into two periods; 'stall inception' and 'post-stall'. The stall inception period concerns the initial breakdown of the flowfield, just as the operating point is crossing the surge line. It will be seen from the literature (chapter 2) and the measurements presented in chapter 3 that this period is typified by non-axisymmetric flow disturbances rotating around the compressor. These disturbances can be either be low circumferential harmonic flow disturbance patterns or well defined (often circumferentially small) patches of low momentum fluid (stall cells). Early modelling work concentrated on the harmonic disturbances, but recent measurements

suggest that the latter type of inception pattern is more common in high speed compressors.

Detailed post-stall behaviour varies considerably from engine to engine, leading to a bewildering variety of terms; 'deep surge', 'classic surge', 'hung stall', 'big stall'. In general, however, at least for those cases where the engine performance is seriously affected, two types of behaviour are observed, loosely called 'rotating stall' and 'surge'. In the former case the non-axisymmetric disturbances in the stall inception period grow into a stable rotating pattern, with part of the annulus operating just above design flow, and part in reversed flow. In a surge event the disturbances grow circumferentially to fill the annulus with reversed flow. The pressure in the compressor exit volume then typically falls to an extent where the flow reaccelerates to just above the design flow, and the stalling process repeats (a 'surge cycle'). In this thesis only the terms rotating stall (or 'stall') and 'surge' are used to describe post-stall behaviour. Some confusion can still arise out of the fact that the terms are used, somewhat interchangeably, to describe the whole of the event including the stall inception period. The context, however, usually makes the meaning clear.

There are important differences between the stall inception and post-stall periods. The disturbance in the former case is localised within the compressor, and can therefore be modelled without reference to the surrounding components. The post-stall behaviour, however, is critically dependent on the surrounding components, particularly the rate at which the downstream pressure changes in response to flow changes out of the compressor. Also, whereas the stall inception disturbance typically starts off localised radially and axially as well as circumferentially, post-stall disturbances are often fairly uniform in the axial and radial directions, and can be modelled using simpler techniques where it is assumed that the flow is uniform in these directions.

1.2 COMPRESSOR STABILITY

The compressor working line (fig 1-1) can be fairly simply determined from the geometry of the engine (or compressor rig) without experiment. In this chapter the basic concepts of compressor stability are introduced, starting with how one might attempt to predict the surge line in the same manner.

A simple criterion for compressor stability can be derived from the constant speed characteristics (fig 1-1). Imagine a compressor operating at an arbitrary point (m, p) on the operating map (fig 1-2). This point may not lie on the working line, as the engine may be undergoing a transient manoeuvre. Generally speaking, the flow

through a compressor adjusts very quickly to changes in speed and exit pressure, and so it can be assumed that the constant speed characteristic passing through x corresponds to the engine speed. Consider next the effect of a small transitory disturbance in the flow through the compressor.¹ Say the flow dropped momentarily by a small amount Δm . At this lower flow the compressor is capable of achieving a higher pressure rise $p + \Delta p$, but the actual pressure rise from inlet to exit is still only p . The force supplied by the compressor to the air is thus greater than the pressure force opposing it and the air accelerates. This moves the level of flow back towards the original flow m . Likewise, if a disturbance temporarily increases the flow above m , then the compressor is capable of less work than that required to match the opposing pressure force, and the flow decelerates. This argument shows that the point (m, p) is stable to small disturbances in flow. The same argument can be used for all such points, provided that the pressure rise capability goes up as the flow falls (that is, the slope of the constant speed characteristic is negative). If this were not true, and the flow were to fall slightly, then the pressure rise capability would decrease, and the flow would decelerate further (leading to stall/surge). Thus the stability point is likely to lie around the peak of the constant speed characteristic. There will be a peak to the characteristic, because as the flow through the compressor falls, the air approaches the blades at higher and higher angles of attack, leading to progressively lower efficiency and higher deviation from the design blade exit air angles. Both of these reduce the pressure rise capability of the compressor.

The stability argument laid out above is simple, but not always accurate. No account is taken of unsteady effects (if, for example, the upstream flow is accelerating then the inlet pressure will change), or of any stabilising effect of components upstream and downstream. No account either is taken of tip clearance effects (beyond their effect on the constant speed characteristic) or any three-dimensional effects. It is assumed too that all of the flow in the compressor moves together. This is not necessarily true, however. It is possible for one end of a compressor to experience rotating stall, with large changes in throughflow velocity, whilst the flow at the other end is virtually constant².

Attempts to deal with the limitations laid out above have led to successively more complex models, used not only for surge point prediction but also for modelling what

¹ For the sake of the argument the disturbance is assumed fast compared to both the transient moving the operating point from the working line and the rate at which the engine speed changes. The latter is true for most flow disturbances due to the inertia of the spool.

² This is often the case in high speed compressors operating at very low speeds. The experimental results presented in chapter 3 show an example of this behaviour.

happens during and after the surge line is crossed. Many of these models are reviewed in chapter 2. The model developed in chapter 5 is one of the most complex attempted, and could be used to predict the position of the surge line. All models of the type considered in this thesis (that is 'pitchwise average' - see section 2.2 for a definition of this term) suffer from a fundamental weakness with respect to surge point prediction, however, in that they require performance characteristics of the whole compressor or of compressor bladerows or of compressor stages as input. These are hard to predict, even for steady flow, because close to the surge point the blades operate a long way from design incidence, boundary layers are large, and the flow can be highly three dimensional.

Although the simple stability argument outlined above is not comprehensive, it can be used to explain how the conventional methods of increasing surge margin work. Taking bleed from the compressor, for instance, increases the flow through part of the compressor. As the flow increases, however, the slope of the constant speed characteristic becomes more negative (fig 1-1), and therefore the compressor becomes more stable. Variable stator vanes reduce air incidence angles onto the blades (rotors and stators) at low flow levels such that the pressure rise capability of the bladerow does not fall away if the flow falls further.

The objective of active control of stall/surge, like the conventional methods of surge margin improvement, is to make the compressor stable to small flow disturbances. The approach is more direct, however, in that disturbances are first of all measured and then forcibly damped out by some mechanical device (or 'actuator'). By way of example, in one such control system [Day, 1993b, described in section 2.3] hot wire anemometers are used to detect regions of low flow around the annulus and locally reaccelerate the flow by injecting high pressure air through 'puffer jets' arrayed around the casing at compressor inlet.

1.3 NATURE OF STALL CELLS

It was stated earlier that the stall inception period is often characterised by well defined patches of low momentum fluid called 'stall cells' rotating around the compressor. It has already been noted that stall cells often start from a localised disturbance, but grow very quickly axially. This can be simply explained from considerations of continuity, and is most easily understood for the case of low speed compressors where the effects of compressibility can be neglected. In this case, the flow entering each blade passage must be equal to the flow leaving the passage at each moment of time. The axial gaps between the blades are sufficiently small that there is little flow redistribution before

the air enters the next bladerow, and so the mass flow disturbance in the succeeding bladerow must again be similar. There is initially opportunity for radial redistribution of the flow; the results from the model developed in chapter 5 predict flow 'ducking under' a stalled region at rotor tip. Once the cell becomes radially large, however, radial redistribution too becomes negligible and then the flow deficit must be the same from stage to stage right along the axial length of the compressor.

One corollary of the above argument is that stall cells extend axially through the compressor, and do not, as is sometimes assumed, follow the helical path of the airflow from inlet to exit. If the rate at which the stall cell rotates is equal to the average rate at which the clean flow rotates around the annulus (that is, around half shaft speed), then the stalled flow and the clean flow areas do not 'collide', but rather rotate together.

An easy way to visualise an axial stall cell is shown in fig 1-3. This figure represents a two dimensional (high hub/tip ratio) compressor with a very large number of stages (such that the effects of the inlet and exit flowfields are negligible compared to those of the bladed region). The blading is assumed to be 'symmetrical' (that is, the stators and rotors are assumed to be reflections of each other). The compressor is viewed in a frame rotating at exactly half the speed of the rotor, such that the rotors move one way and the stators the other at the same speed. Viewed in this frame the flow is broadly axial. The same continuity argument as before can be used to show that a stall cell in such a compressor will be distributed axially along the compressor, and it is clear that if the disturbance is fixed in a circumferential position (in this frame) there is no 'collision' between the stalled and clean flow areas.

In many ways the rotating frame of reference described above is the natural one in which to consider stall disturbances; the concept of a disturbance extending axially along the compressor in the direction of the flow is entirely intuitive. If the disturbance in this frame is fixed in a circumferential position, then by moving back to the stationary frame the answer can be found to the question of why stall cells rotate (in a multistage compressor). Moving from one frame to the other requires the addition or subtraction of the average circumferential velocity, and it is this velocity component that makes the stall cells rotate - the rotors effectively sweep the low momentum fluid in the stall cell around the annulus (this process is defined more precisely during the development of the 'compressors in parallel' model in chapter 4).

1.4 SCOPE OF WORK DESCRIBED IN THE THESIS

The objective of the work described in this thesis is to increase understanding of the phenomenon of stall/surge in multistage axial flow compressors operating in an aeroengine environment, and to improve on current computational stall modelling techniques. Particular attention is paid to the stall inception period and the possibility of preventing or delaying the onset of stall/surge using an active control technique.

Although the work is directed primarily towards aeroengines, the stall inception (as opposed to post-stall) behaviour of compressors is likely to be independent of the operating environment, and many of the results can be applied immediately to multistage axial flow compressors in other applications. The stalling behaviour of aeroengine fans and centrifugal compressors can be markedly different from that of multistage axial flow compressors and is not considered in detail. No consideration either is given to spool coupling effects in two and three shaft engines, although the stall modelling work described in chapters 4 and 5 could be extended to cover this area.

In order to apply active stall control (or any novel method of improving stall margin) to a compressor it is first necessary to understand the nature of stall, and particularly the small amplitude disturbances that precipitate it. A large amount of test data has been published on the subject, but mostly from low speed compressors and/or with very sparse instrumentation. In order to identify the stalling behaviour of a compressor at high speed extensive tests were performed on a single shaft Rolls-Royce VIPER jet engine, described in chapter 3. This work represents the first detailed analysis of stall inception in a compressor operating in an engine environment based on a good definition of high response instrumentation, and one of the first such tests on any high speed compressor.

Two computational models of compressor stall/surge are developed in chapters 4 and 5. The first of these is a multiple compressors in parallel model, based on similar principles to those of previous authors, but extended to include the effects of the engine environment. Whilst it is anticipated that an improved understanding of stall/surge will be of benefit in a wide range of related areas, the particular impetus of the work is towards the application of active stall control. To this end a control system was devised and tested numerically using the model.

The multiple compressors in parallel model is validated against the experimental data described in chapter 3, and is shown to agree well with the overall stall transient. The agreement with respect to the stall inception period is less good, due to the simplicity

of the model. This period of time is critical, however, for numerical testing of active control of stall/surge. In order to overcome this obstacle a completely new stall model is introduced and developed in chapter 5. This method combines a throughflow (axial by radial) description of the flow, including viscous effects, with a simple circumferential discretisation. Preliminary evaluation against stall inception data from a single stage compressor rig suggests that this new model is sufficiently complex to model the initial localised disturbances that lead to stall/surge, and yet it is compatible (in terms of computational time) with application to long multistage compressors.

The mechanical and control theoretical design of specific active stall control systems is a broad subject that is only briefly considered with respect to the application of the compressors in parallel model in chapter 4. The stall model developed in chapter 5 is designed specifically for numerical testing of active control schemes, but the application of the model in this area is beyond the scope of the present work.

2. BACKGROUND

There is a tremendous amount of literature available on rotating stall and surge, and a comprehensive survey is beyond the scope of this thesis. A very thorough survey of publications related to stall and surge has been undertaken by Pampreen [1993]. Good introductory material can be found in Cumpsty's [1989] book or in Greitzer's [1980, 1981] papers. The former is broad in scope, whereas the latter are more technical, concentrating on the dynamics of stall and surge.

The review presented in this chapter is designed to give an overview of the major advances in the field, whilst paying particular attention to those papers directly related to stall inception, stall modelling and active control of stall. For convenience the review has been split into four sections. The first of these contains papers based primarily on experiment. Particular attention is paid to experimenters examining stall inception behaviour in multistage compressors with a good definition of high response instrumentation. The second section reviews stall inception modelling, looking first at a number of approaches to modelling a compression system, and then categorising the various published models of stall/surge. In the third section a number of papers are analysed which relate to active control of stall/surge. A final section is devoted to a discussion of the background literature.

2.1 EXPERIMENTAL MEASUREMENTS OF STALL/SURGE

2.1.1 Early Work

One of the most widely quoted papers on stall inception is that of Emmons et al [1955]. In this paper two compressor tests were discussed, one centrifugal (with an axial inducer) and one single stage axial. The axial compressor stage has a low hub/tip ratio (0.57). It also has a large gap between rotor and stator (greater than one tip radius), and so the stalling behaviour of the rotor is likely to be closer to that of an isolated bladerow than half of a compressor stage. Based on the results a clear distinction was drawn between a 'Helmholz' type of disturbance [that is, axisymmetric surge], and 'the stalling of the flow through the bladerows ... in propagating groups' [that is, rotating stall]. It is noted that the frequency of the rotating stall is proportional to the wheel speed. A model of rotating stall is presented that is the clear fore-runner of many of the more recent modelling attempts described in section 2.2. A surge model is also presented.

The results of the axial stage are somewhat unusual, in that the compressor falls into rotating stall at all speeds with no discontinuity in the overall characteristic. The first stall regime encountered on closing the throttle is five cell rotating stall of the rotor only, biased heavily towards the tip region (the low hub/tip ratio makes such radially non-uniform disturbances more likely). On closing the throttle further this changes to single cell full span stall. This behaviour is similar to that of the Rolls-Royce VIPER engine compressor at low speed, as described in chapter 3.

The linear incompressible model of rotating stall presented by the authors has many of the features of the more recent models described in section 2.2, although formulated for two dimensional flow through a cascade. The basis of the model is the coupling of the bladerow performance with the inlet flowfield (the static pressure at exit to the cascade is assumed constant). The bladerow performance is derived from loss and deviation (or rather just loss, as the deviation is assumed to be zero for convenience). The inlet flowfield is represented by a Fourier series in exactly the same way as, for instance, the model described in chapter 4. The time dependency (and hence the rotation relative to the bladerow) comes from the assumption that the response of the bladerow to a given set of inlet conditions is not immediate. Instead, the rate of change of performance is assumed to be proportional to the difference between the current and ideal level as calculated from the inlet conditions.

In terms of stability, the constant of proportionality in the delayed response is immaterial, and Emmons et al derive a condition for stable operation that is equivalent to the peak inlet total to exit static pressure ratio condition quoted by Dunham [1965, described in section 2.2.3]. In terms of rotation rate, however, the constant is significant, and here the modelling breaks down slightly in that the authors, while giving no indication as to a value of the constant, indicate that it depends primarily on the time taken for the blade surface boundary layers to react. Moore [1984b, described in section 2.2.3.2] showed that the main cause of delayed response was the inertia of the fluid in the blade passages.

Iura and Rannie [1954] made a series of measurements of rotating stall, primarily on a three stage low speed axial compressor of free-vortex design. As with Emmons et al [1955, described above], the main objects of study were the 'stable' forms of rotating stall rather than the stall inception period. On throttling the three stage compressor four distinct types of rotating stall regime were observed. The first type of disturbance encountered was single cell part span rotating stall in the hub to mid-radius region. The second and third types of disturbance corresponded to two and three cell part span stall, with the cells equispaced and each one looking similar to the single cell variant.

As the throttle was closed further, the three cell part span rotating stall gave way to single cell full span rotating stall, with similar magnitudes of velocity fluctuation across the span, but covering a greater circumferential distance at the hub than the tip.

In addition to the three stage compressor the authors tested one of the stages on its own, and found that in this case the first type of instability encountered was again part span stall, but in this case limited to the tip region of the compressor. Thus the radial position of part span stall inception (hub or tip) cannot be a simple function of blade design.

Various compressor configurations were tested by the authors. One of the most interesting was a test with every alternate stator removed. This showed only a small change in the rate of rotation, which led the authors to question the then common assumption that the time response of a bladerow to given inlet conditions was dependent on the time required for the blade boundary layer to readjust (cf the final part of the discussion of Emmons et al [1955], above).

2.1.2 Low Speed Measurements and Resulting Theory

Some of the first really detailed measurements of compressor operation during and after stall initiation were taken by Day [1976], on a wide range of compressor builds. The resulting database was used by Day, Greitzer and Cumpsty [1978], to formulate a correlation based prediction of compressor performance in rotating stall. Whilst not directly related to stall inception, the concepts in the paper are significant, most notably the prediction of whether part span or full span stall will develop in a particular compressor. The underlying principle (a development of an original idea attributed by the authors to A. McKenzie) is that compressor stages in stall operate at one of two fixed levels of inlet total to exit static pressure rise, depending on whether the stall pattern is part-span or full span. The two fixed levels ($(P - p)/\rho U^2 = 0.17$ for part span stall, 0.11 for full span stall) are more or less constant between compressors. This constrains the operating point of the compressor at a given throttle setting to lie either on the unstalled characteristic, or one of these two lines of constant pressure ratio (fig 2-1, based on fig 6 of the paper). The authors suggest that the correct choice between these lines can be made on considerations of blockage alone. As the throttle is closed beyond the stall point the compressor will fall into part span stall if the blockage so produced is less than a critical value of 30%, where the blockage is simply calculated from the mass flow in part span stall and the mass flow at the same pressure ratio on the unstalled characteristic (bc/ac on fig 2-1). If the blockage in part span stall would be over 30% then the compressor falls immediately into full span stall. A similar

argument is followed to determine the size of the hysteresis loop¹. Once in rotating stall the compressor will remain stalled until the throttle is opened sufficiently to reduce the blockage to below 30%. Simple calculations are added to demonstrate the effects of stage numbers and design axial velocity ratio (a high value of either leads to a propensity for full span stall and a large hysteresis loop).

The results of the paper are supported by a large amount of experimental data, and are of immediate use to the aeroengine designer. No aerodynamic reasons are supplied, however, for the constant pressure ratio and blockage assumptions. Heuristic arguments can be found for most of the assumptions. It is not unreasonable, for instance, that the pressure ratio in rotating stall should be set by the stalled part of the flow (although it is puzzling as to why the pressure ratio in part span stall should be independent of the blading details, even though the majority of the span sees a relatively normal level of throughflow). The main interest to the stall modeller, however, is that the correlations work, and therefore similar results should be obtained from a good model of post-stall transients.

A second paper based on the detailed measurements of Day is that by Cumpsty and Greitzer [1982]. Here a simple model is proposed for stall cell propagation. The model is based on the observation that the trailing edge of stall cells is often very sharp, and that any fluid entering or leaving the cell across that boundary has to be accelerated/decelerated axially by a local increase or decrease in static pressure gradient. The precise formulation of the model is dependent on the unknown lengthscale over which the acceleration/deceleration occurs, and in this way is inferior to subsequent models such as are described in section 2.2, where the shape of the stall cell is a result of the calculation. The two significant observations of Cumpsty and Greitzer, however, are that the pressure differences along the compressor which give rise to flow acceleration/deceleration must be consistent with the pressure ratio as set by the compressor inlet and exit boundary conditions, and that this condition is inter-related with the stall cell rotation speed. These two considerations form the basis of many of the later models.

Further experimental work on a four stage low speed compressor was published by Day [1994a]. The main emphasis of the paper is to make a detailed examination of a compression system surge cycle, and to provide quantitative measurements of the boundary between rotating stall and surge type behaviour. The measurements were

¹ That is, the region where the compressor will operate either normally or in rotating stall depending on its operating condition when it crossed the boundary.

not directed toward the small timescale of the stall inception period. Nonetheless, some important conclusions were reached about stall inception, namely that rotating stall was observed to be a precursor of axisymmetric surge (this is confirmed for a jet engine compressor in the results described in chapter 3), that Helmholtz perturbations, though present, played little part in determining the stalling behaviour, and that long circumferential “modal” waves were only sometimes present prior to stall (Day [1993a], discussed below, deals with this last point in greater detail).

An analysis similar to that used by Day, Greitzer and Cumpsty [1978], based on the overall compressor characteristic, is used to suggest which compressors will exhibit “deep surge” (where the flow becomes fully reversed and axisymmetric for part of the surge cycle), and which “classic surge” (where the flow remains non-axisymmetric, but the annulus average operating point still follows a surge cycle). A parametric study is used to show that the Greitzer B parameter (defined in section 2.2.3.1 under Greitzer [1976a,b]) gives a reliable indication of whether the compressor fails in rotating stall or surge, although the point is forcibly made that the value of the B parameter is dependent on the compressor design parameters. An appendix by E M Greitzer presents a modified B parameter which accounts for some of these effects.

McDougall, Cumpsty and Hynes [1990, also McDougall, 1988] examined McDougall’s detailed stall inception measurements in a low speed single stage compressor at three different tip clearances. They identified the endwall blockage as critical for determining the stall inception pattern. At small tip clearance the hub blockage was large, and stall disturbances could first be seen at the hub. At large tip clearances the tip blockage was large, and the stall disturbances first became visible there.

At the large tip clearance clear pre-stall waves were observed, rotating at almost exactly half rotor speed. These waves were identified as a variation in rotor tip endwall blockage which rotated around the annulus with time. The strongest velocity fluctuations were accordingly found to be within the rotor passage just outside the annulus and suction surface boundary layers. In phase with the tip blockage variation velocity measurements upstream of the rotor showed a rotating variation in axial velocity. This upstream variation was found to be almost constant radially, and was viewed as a “mode of the system”, responding to the blockage variations in the rotor passages. No experimental or theoretical support was given for this viewpoint, however.

The amplitude of the pre-stall waves was shown to rise and fall in time, but to be generally strongest as stall was approached. When stall did occur the stall cell was shown to grow out of the pre-stall waves, in the sense that there was no discontinuity in the phase of the disturbance. The influence of the pre-stall waves on the inception process was proved by the fact that even with a blade removed the stall inception was not fixed to a circumferential position, but still grew out of the pre-stall waves.

Day [1993a] used data from the same compressor as McDougall et al [1990, described above] to draw quite different conclusions. Day looked primarily at the stall cell that grew out of the pre-stall waves observed by McDougall et al at 3% tip clearance and marked the point of *irretrievable* breakdown in the flow structure. Whilst confirming the existence of the pre-stall waves, and that the start of the stall cell coincided with the low flow point of the pre-stall wave, he also showed that the result was not general, in the sense that at the two lower tip clearances irretrievable flow breakdown was caused by stall cells of a similar nature to the 3% tip clearance case, but without any pre-stall waves visible.

Day's conclusion, that stall cells and the pre-stall waves observed by McDougall et al are fundamentally different phenomena, was strongly supported by his measurements from a four stage low speed compressor fitted with an active control system. Using the control system, Day was able to measure the forced response of the pre-stall waves and confirm that the damping was large except for a small region around the stall point. He was also able to actively prevent the formation of stall cells, whilst throttling the compressor to a flow well below the natural stall point, and demonstrate that the pre-stall wave disturbances continued to rise in amplitude without causing an irretrievable breakdown of the flow structure.

Day also noted that the formation of the stall cells tended to be fixed circumferentially in the absolute frame and often in the rotating frame too. This is contrary to McDougall's findings, that the position in the absolute frame was not fixed even with one blade missing. Day suggested that the underlying cause was that at 3% tip clearance the stall cells were circumferentially larger when they grew out of the pre-stall waves, and therefore rotated more slowly, in fact at the same speed as the pre-stall waves. Under these conditions a high degree of coupling between the two disturbances could be expected. In the four stage rig the stall cells when they first appeared rotated at around 70% of shaft speed, whereas the pre-stall waves of the type observed by McDougall et al, when they were present, rotated at only 18% speed, and there was less coupling.

Day observed that in both compressors stalling disturbances started in the tip region and grew radially inwards. Both he and McDougall et al (for the high tip clearance case) were agreed that stall initiated in the annulus boundary layer / tip clearance region, which accords with the common observation that tip clearance in particular plays a large part in determining the stall point of a compressor.

The question of when stall is initiated by modal behaviour and when by 'finite stall cells' is further investigated in Camp [1995]. Camp used the same four stage low speed compressor as Day [1993a]. By altering the vane settings he was able to demonstrate both types of stall inception (following Day). He was also able to demonstrate that the 'finite stall cells' appeared whenever a certain incidence onto the first rotor was exceeded. That is, if the inlet guide vane was staggered such that the first rotor experienced high incidence, then 'finite stall cells' would appear without any pre-stall modal waves. If the stages were better matched, with similar incidences at each stage, then 'finite stall cells' did not appear and stall inception was of the 'modal' type.

Garnier [1989, also Garnier, Epstein and Greitzer, 1991] made stall inception measurements in a single stage low speed compressor, a three stage low speed compressor, and a three stage high speed compressor. All three compressors showed small amplitude pre-stall waves. In the low speed compressors these were shown to be first order circumferential harmonic disturbances. In the case of the high speed machine, however, the nature of these rotating disturbances is not clear. Only the first three circumferential harmonics are considered in detail. Of these the third showed no significant period of coherent rotation. It was also notable that the first two harmonics were observed to rotate together at around 18% of rotor speed, suggesting the possibility of modal coupling. The disturbance therefore could well be quite different in nature to the single harmonic disturbances observed in the low speed machine, and quite different to the dissociated harmonics predicted by linear theory.

Garnier presented some stall inception data for the high speed compressor taken with a fast throttle closure rate. Again, small amplitude pre-stall disturbances were present, but showed some quite different and unexpected behaviour, with circumferential harmonics even travelling in different directions at different axial stations. This dependency on the throttle closure rate is significant, as it represents the speed with which the stall line is approached by an engine on the wing, which will vary widely depending on what causes the stalling behaviour.

Some results with inlet distortion were also presented by Garnier, but are not considered here.

Mathioudakis and Breugelmas (1984) and Breugelmans and Mathioudakis (1984) presented some very detailed measurements of rotating stall in a single stage low speed compressor. The work is primarily aimed at instrumentation and 'stable' rotating stall (that is, rotating stall at a constant mean operating point), neither of which are the main theme here. Two types of stall are observed, 'small stall', and 'big stall'.

The 'small stall' identified by the authors occurs first, as the compressor is throttled from steady operating conditions. It consists of eight cells, and is heavily biased toward the outer annulus. It is termed 'small stall' because none of the flow parameters change vary a long way from their original values. Upstream velocity fluctuations, for instance, increase from 5% at the hub to 13% at the tip. The overall flow structure is therefore similar to that before stall.

The classification into 'small' and 'big' stall is put forward by the authors as an alternative to distinction according to performance ('progressive' or 'abrupt' stall) or extent ('part span', or 'full span'). In practice none of these classifications is sufficient in itself, as a number of combinations of these types have been observed. The Rolls-Royce VIPER engine, for instance, shows at part speed a 'progressive' form of stall that is neither 'part span' in extent nor 'small' in character, although it is in fact 'part length', in the sense that it affects only the front stages of the compressor (chapter 3).

Single cell 'big stall' occurs when the compressor is throttled beyond the 'small stall' flow region, causing a big drop in pressure ratio. The flow field in the 'big stall' regime is measured by the authors in the stationary and rotating frame. Large variations in flow velocity and angle are observed, with a strong recirculation at rotor tip, and reversed flow upstream of the rotor towards the outer wall. Large radial velocities are observed and the flow is very three dimensional.

Some of the most detailed measurements of fully developed rotating stall were presented by **Das and Jiang [1984]**, from a three stage low speed compressor. Ensemble average circumferential profiles of velocities (axial and circumferential, but not radial) and pressures (total and static) at mid-height are presented for each interrow gap as well as inlet and exit. Circumferential profiles are also presented for a number of radial positions ahead of rotor 2 for two builds with difference sizes of axial gap.

The overall shape of the stall cell is well defined by the measurements. It extends very nearly axially through the compressor, with width dependent on the position of the throttle. It is about half as big again at the tip than the hub for this compressor (a free vortex design). The radial profile is demonstrated to be a function of the axial gap, with more of a bias toward the outer regions at the larger axial gap tested.

Das and Jiang demonstrate that the flow in a stall cell is very active, with high circumferential velocities (greater than rotor speed) upstream of the rotors, areas of low axial velocity (forward and reversed), and large radial velocities. From the two-dimensional measured velocities Das and Jiang build up a picture of the internal structure of the stall cell as shown in fig 2-2, reproduced from fig 13 of their paper. This figure shows schematically the axial and radial velocities at the centre of the stall cell. The flow at rotor inlet is seen to be reversed over the whole span, giving rise to the high circumferential velocities in that plane. In the rotor itself the trend is for the fluid to be centrifuged outwards. Downstream of the rotor the flow is reversed over much of the span, but directed forwards at the tip.

Giannissis et al [1988] followed Das and Jiang, using the same probe to measure rotating stall in different builds of the same three stage low speed compressor. Again the theme was stable rotating stall examined using an ensemble average technique. Three builds were tested, with stages mismatched in an attempt to model the mismatching that occurs in high speed compressors when running off-design.

Different stall regimes were observed depending on the average inlet flow. In all builds the first instability met on closing the throttle was part span rotating stall (single or multiple cell), which changed suddenly into full span stall at a lower flow setting. No simple correlation was found between the onset of the part span stall and the individual stage characteristics. In particular it was noted that stages could operate axisymmetrically on a rising characteristic in the presence of other unstalled stages. Full span stall, on the other hand, was observed to occur in each build at around the peak of the overall total to static pressure rise characteristic.

Most details are presented for the third build, with the blades restaggered such that the rear bladerows operated closer to stall than the front. Measurements were taken at an operating point where the instability is shown to be part span part length stall, limited to the hub at the back of the compressor. The flow disturbance seen at the front stage was considered by the authors to represent merely a reaction to the distortion imposed by the rotating stall at the rear. The authors suggest that the free vortex design of the blading is a likely cause for the rotating stall to appear at the hub rather than the tip

region, but it should be noted that the compressor build tested by Das and Jiang [1984] was also a free vortex design.

2.1.3 High Speed Measurements

Some of the first very detailed measurements of stall inception at high speed were made by Schlamann et al, [1985]². The authors note that in two test compressors rotating stall was always seen to precede surge. The measurements presented in chapter 3 of this thesis accord with this observation. The authors observed an initial stall disturbance limited to the tips of one or more rotors, before the point of irretrievable flow breakdown, which was marked by the appearance of a single full span stall cell.

Similar measurements to those of Schlamann et al were presented by Riess and Walbaum [1996] for a six stage research compressor. The exit volume was made intentionally small in order to separate the (system) surge instability from the rotating stall inception. Again, the initial stalling disturbances were seen near the outer annulus, this time limited to the first stage. These disturbances are distinct from rotating stall in that they are less coherent, lasting in general for less than one revolution. The authors state that these disturbances travel with the rotor. No conclusive evidence was presented to support this statement, however, and at a different guide vane setting the authors show similar disturbances rotating at 50-60% of rotor speed.

The stall inception itself is shown to occur in the same region as the pre-stall disturbances. It is stated that the disturbances which grow into the stall cell rotate initially at rotor speed, in line with the observations of Jackson [1987], but again conclusive evidence is not provided. The incipient stall cell grows from the tip of the first rotor almost axially to the back of the machine, and also radially, such that at the back of the machine the whole span is affected whilst at the front the cell is limited to the outer half of the blades. This period of growth lasts for three to four revolutions of the cell, by which time it has developed into full span stall. In full span stall, the 'clean' flow between occurrences of the cell show less fluctuation than before stall inception. This is attributed to the stall cell blockage giving rise to increased axial velocity in the rest of the annulus. No evidence was found of stall being initiated by long circumferential lengthscale 'pre-stall waves' such as observed by McDougall et al [1990, described earlier].

² At the time of writing only the summary of this paper was available to the author

The observations of Riess and Walbaum, of initial very short circumferential lengthscale largely incoherent disturbances at the tip of the first rotor, followed by stall inception in the first stage, with no sign of modal pre-stall waves, shows remarkable agreement with the measurements reported in chapter 3 at middle speeds.

Wilson and Freeman [1994] presented detailed measurements of stall inception on an eight stage compressor operating in a jet engine as opposed to a rig environment. Chapter 3 gives a more detailed description of the results of this and a subsequent experiment. Day and Freeman [1994] compared the results with those of Day's previous low speed compressor tests, concluding that the stall inception patterns were broadly similar in high and low speed machines.

Borys and Moffatt [1996] presented some data on rotating stall from a turbojet engine with a ten stage compressor. Without any means of artificially inducing stall/surge, the authors limited their investigation to the 'benign' rotating stall that was present on the engine working line at low speed. This rotating stall was found to consist of generally four or five stall cells, extending more or less axially through the compressor and rotating at around half rotor speed. Each cell affected the whole span in the first stage, although heavily biased towards the tip region. Reversed or nearly reversed flow is seen downstream of rotors 1 and 2 in the tip region. The stall cells were seen to be strongly attenuated in the third and fourth stages, where only the tip region was affected. This behaviour is strongly reminiscent of the 'benign' stall in the Rolls-Royce VIPER engine described in sections 3.4.2.1 and 3.4.3.1.

A variety of inlet distortion gauzes were applied to the engine. These were found to have little effect on the stall pattern.

Hoying [1993] presented stall inception data from high frequency pressure measurements in a four stage high speed compressor. The data acquisition rate was such that only the first two circumferential modes could be analysed at all speeds. At low speed clear pre-stall waves were found for up to 1000 rotor revolutions before stall, with the second mode dominating. The first and second mode were observed to rotate at different speeds (60% and 68% respectively), leading to the conclusion that they behave as dissociated harmonics such as predicted by linear theory. Phase plots of the first two modes were drawn, and the authors state that in general there is no discontinuity in the phase of one of the modes (that is, either the first or the second mode) as the compressor stalls. This fact is cited as evidence that the compressor stalls as a result of the growth of the modal pre-stall waves. Only one phase plot is presented, however, on such a small scale that even a large discontinuity would be

difficult to see, and in as far as can be made out given the scale of the graph it would appear that there is a discontinuity in both modes.

Between 70 and 90% speed there was considerably less pre-stall activity, especially at the front of the machine. The exit transducers, however, still showed pre-stall modal waves for around 200 rotor revolutions before stall. At 100% speed no pre-stall waves were observed prior to stall. Here a small disturbance is seen rotating at 95% rotor speed around the time and circumferential position of stall inception. This disturbance is tentatively classified as a 'finite' stall cell as described by Day [1993a, described above].

A system identification procedure was applied to the pre-stall waves, and information deduced about modal speeds and damping. In the 80-90% speed range this system was found to be more effective in finding pre-stall waves than tracking the phase or amplitude of harmonics using a windowed Fourier analysis. At lower speeds, this method reacted at the same point as increased activity is observed using the windowed Fourier technique.

Inlet distortion was found by Hoying to have relatively little effect on the stalling behaviour.

Escuret & Garnier [1995] took some detailed stall inception measurements in a four stage high speed compressor using high response casing static pressure measurements. They found that rotating stall was initiated by very short circumferential lengthscale disturbances. No evidence of pre-stall waves was found; the analysis method of Tryfonidis et al [1995 - this paper is described below] gave no indication of impending stall. No sign of impending stall was discovered in the signals until at best five rotor revolutions before stall. Because short circumferential lengthscale disturbances are not amenable to the method used by Tryfonidis et al (this fact is considered further in section 3.4.5.3), Escuret and Garnier presented a simple method of tracking them around the annulus by considering only disturbances from each transducer signal that reached a given amplitude over a given timescale.

The short circumferential lengthscale disturbances leading to stall were seen to start either from the front of the compressor or the rear, depending on the stage matching. The variation in loading between the stages was varied systematically by stalling at different speeds (such that the compressibility effect alters the matching) and by altering the inlet guide vane setting angle. The authors state that under all conditions tested the highest loading was to be found in the first rotor or last stator, and hence link the axial point of stall initiation to the point of highest stage loading. It has been

suggested by Day [1994b], that the modal type of stall inception is often found in machines which are axially matched, and the authors suggest this may be a reason why this type of inception is not seen, in that the loading of the first rotor and last stator mean that the compressor is never evenly matched axially.

The circumferential position at which the short circumferential lengthscale disturbances start is shown to be fixed in both the rotating and stationary frames. That is, there is a preferred point for stall initiation in the stator set and the rotor set.

Fast throttling of the compressor (from choke to surge in one second) was shown to have no effect on the stalling pattern. It should be remembered that no pre-stall warning was found, and so this result cannot be compared with that of Garnier [1989, described above], for instance, who showed considerable differences in pre-stall disturbance behaviour with a fast throttle transient.

Owen [1995] and Owen et al [1996] have made extensive stall measurements in a T55-L-712 turboshaft engine compressor, with seven axial stages and one centrifugal. The referenced papers compare the stalling behaviour of the same compressor in rig and engine up to ground idle (60% speed). The post-stall behaviour was markedly different in the two cases. This is as expected; the environment in which a compressor is operating has in general a much greater effect on post-stall transients than on stall inception. In the second paper the authors use the same method applied by Tryfonidis et al [1995, described below] to compare pre-stall disturbances in the rig and the engine. In both cases rotating waves could be tracked for short time intervals prior to stall but little change in the rotating wave energy spectrum could be observed as stall was approached. Significantly also, the rotating wave energy spectrum was quite different for rig and engine. Neither showed a single frequency dominating, but whereas the rig data showed a broad peak in forward travelling wave energy, the engine data showed wave energy at a number of distinct frequencies (similar, in fact, to the analysis of the VIPER engine results described in section 3.4.5.3).

One other interesting feature of the measurements made by Owen is that as the engine is accelerated from stationary to ground idle it operates in rotating stall limited to the front stage, with the number of cells dropping from four to three, and then to one before dropping out completely. This behaviour is strikingly similar to that of the Rolls-Royce VIPER engine described in section 3.4.6.

Tryfonidis et al [1995] use data from seven different compressors to investigate pre-stall behaviour, and in particular to evaluate the use of 'travelling wave energy' as a method of estimating proximity to stall. At design speed four of the seven

compressors showed clear first or second circumferential order disturbances for a significant time period prior to stall with the phase changing linearly with time, whereas the other three compressors did not. All of the compressors, however, showed an increase in 'travelling wave energy' with time prior to stall (in the latter cases this was broad-band, and could not be related to circumferential harmonics rotating at constant speed). The method used by Tryfonidis to calculate this parameter is described in section 3.4.5.3, where it is applied, with mixed results, to experimental data from a Rolls-Royce VIPER engine. At 70% speed all of the compressors showed significant levels of travelling waves, though the detailed behaviour of these waves was different for the different compressors.

One of the possible reasons cited by the authors for the differences between the compressors is the high levels of excitation, coupled with non-linear effects. This would be consistent with the observations of Day [1993a, described earlier] of 'modal' and 'finite stall cell' types of stall inception. The authors base their study on a circumferential fourier transform, and so concentrate on the low circumferential harmonics of disturbances. The results from the VIPER engine presented in chapter 3 show (at least at some speeds and axial locations) stall cells growing and dying away prior to stall that are of short circumferential lengthscale. It is shown in section 3.4.5.3 that such disturbances are not ideal for this type of analysis, and their existence may account for some of the differences between compressors seen by Tryfonidis et al.

2.2 MODELLING STALL/SURGE

2.2.1 What to Model

Many models of compression system stall and surge have been put forward in the literature. Before analysing these in detail it is important first to consider what parts of the system need to be modelled, and in what detail. A schematic view of an axial flow compressor rig in a test cell is given in fig 2-3, showing the various components that might be included in a model of a stall transient. Early stall models tended to involve just the compressor and perhaps the inlet and exit duct. This is sufficient for modelling non-axisymmetric disturbances whilst they are small. More recent work, notably Moore & Greitzer's [1986a,b] papers, has shown that a non-axisymmetric compressor model combined with a simple plenum and throttle model can be used successfully to track large flow disturbances such as are encountered in surge cycles and fully developed rotating stall.

This section describes how the various components labelled in fig 2-3 (apart from the compressor itself) might be treated in a stall model. Consideration too is given as to the consequences of modelling a compressor in an engine as opposed to a rig test (most of the experimental and theoretical work developed in this thesis is based on a compressor in an engine environment).

2.2.1.1 External Flowfield

The external flow upstream of the inlet duct is almost always neglected, leading to an upstream boundary condition of constant total pressure and temperature. The inlet duct is usually long enough to ensure that any circumferential flow variations are negligible at its upstream face. Axisymmetric fluctuations will cause an unsteady disturbance to the external flowfield resulting in changes to the inlet conditions, but these changes are small. Lord Rayleigh [1896] calculated the effect of the external flowfield on a simple Helmholtz resonator, finding that the effect (for small amplitude disturbances) could be accounted for by adjusting the duct length by $0.8R$, where R is the radius of the (circular) aperture at inlet. In a compressor rig environment this correction is generally small compared to the inlet duct length. In an engine environment the extra length could be significant, depending on the geometry of any upstream ducting.

The external flowfield downstream of the throttle is likewise neglected. A small correction could be applied similar to that described for the inlet plane.

2.2.1.2 Plenum and Throttle

Rig compressors are usually tested with a considerable exhaust volume downstream. The most significant effect of this volume with respect to stall/surge modelling is to store energy in the form of pressurised gas. Greitzer [1976a,b] showed that the size of the volume has a critical role in determining whether a compression system falls into axisymmetric surge or rotating stall once the steady state stability line is crossed. There are two cases, however, where the effect of the plenum and throttle can be ignored. The first is the very start of the stall event, for which the static pressure in the plenum is approximately constant (the static pressure only changes with the time-integrated mass flow into the plenum). The second is during stable rotating stall (or where just rotating stall is to be modelled). Here the integrated flow into the plenum is constant, and again the static pressure is constant (neglecting circumferential variations within the plenum). For this reason some modellers (for instance Dunham [1965]) use a simple constant static pressure downstream boundary condition.

Where the effect of the plenum and throttle are considered important, they are usually modelled in a mixed-out sense (that is, the flow in the plenum is assumed uniform). The determining equations are then conservation of mass and energy. The exit flow is determined from the pressure drop across the throttle using the throttle area and a discharge coefficient as appropriate.

Escuret and Garnier [1994] took an alternative approach. They used a slightly more complex model, where the exit plenum is modelled axially (but not circumferentially or radially), with the objective of accounting for axial waves in the downstream volume. This is a useful approach if the geometry is regular, but the flow path downstream of a test compressor can be highly irregular, and in these cases the reflection of waves produced by such a model could be misleading.

2.2.1.3 Inlet and Exit Ducts

If the plenum and throttle can be treated in a simple manner assuming uniform flow, the remaining components (inlet duct, compressor and exit duct) certainly cannot. During a stall transient these components see unsteady disturbances that can be localised axially, circumferentially, and radially. The type and complexity of model used for the inlet and exit ducts generally reflects that used for the compressor, which is considered in detail below (section 2.2.3). Where the geometry is sufficiently simple the flow in the inlet duct is often solved analytically, as it can be assumed irrotational and inviscid, leading to a potential solution of the Euler equations. This approach is valid even for large flow disturbances, though not for reversed flow (section 2.2.2.1).

An example of this in two dimensions (axial and circumferential) can be found in chapter 4. Dunham [1965] solves the three-dimensional inlet flow using Fourier-Bessel harmonics. Modellers using a time-marching technique to calculate the compressor flow (for instance Escuret and Garnier [1994]) usually use the same method for the inlet duct, as it adds nothing to the complexity of the model.

The exit flow is more problematical as it is in general rotational. If the compressor model is based on a solution of the Euler equations, then the exit duct can be included in the compressor calculation (provided it is not too long). Otherwise some simplifying assumption is usually made, such as small perturbations (allowing the use of linearised equations), or high-solidity outlet guide vanes, such that the exit flow is uni-directional.

2.2.1.4 Effect of Engine Environment

It was noted above that an engine compressor can be expected to exhibit a different stalling behaviour from a rig compressor, due to the different operating environment. A schematic diagram of an engine configuration (for ground testing) is shown in fig 2-4. It can be seen by comparison with fig 2-3 that the environment is quite different. The inlet duct is usually much shorter, taking flow from within the test cell rather than from outside. The exit duct from the compressor is also much smaller, and strongly diffuses the flow into the combustor. The plenum at exit is a combustion chamber, with associated heat input. The throttle is now the inlet guide vane to the turbine(s). The turbine itself responds to stalling disturbances, changing the shaft power driving the compressor, but the inertia of the rotating system is large enough that the compressor speed can usually be assumed constant for the initial stall transient (the measurements in chapter 3 show that for the Rolls-Royce VIPER engine the change in compressor speed is not that great even over a surge cycle).

The external flowfield, inlet duct and compressor can be treated in identical fashion to the test rig case described above. The combustor is similar in principle to the plenum in the test rig, but with heat addition, which has an effect on the dimensional flow rate through the nozzle. The precise nature of the modelling must depend on the type of combustor. For the annular combustor modelled in chapter 4 and reported by Freeman and Wilson [1993], it is insufficient to model the combustor as a single volume, because the heat addition would then be spread over the entire mass of air in the chamber. Instead, a two chamber model was employed, using a 'hot' chamber and a 'cold' chamber with the flow between the two determined by the pressure drop across the cooling holes. The combustor exit nozzle is choked at high speed, and almost

choked at lower speeds, and can be treated in the same way as the throttle on the compressor rig.

The diffuser is hard to model accurately, as it entails calculation of complex unsteady flows in a high adverse static pressure gradient with separations present. The model in chapter 4 assumes that the diffuser is short enough to give approximately constant static pressure at compressor exit. Moore [1984a,b,c] in his stall model derives a more accurate representation using an empirical constant representing the length of the exit duct between 'short' (circumferentially uniform static pressure at compressor exit) and 'long' (inertia in the duct fully accounted for).

2.2.2 General Comments

2.2.2.1 Reverse Flow

If the model is to cover post-stall transients, then each of the components must be modelled in such a way as to handle reversed flow, which occurs very shortly after the stall line is crossed either in part of the annulus (rotating stall) or the entire annulus (surge). The objective here is usually more to make the model robust than to precisely match reality. There is very little information available as to the behaviour of compressors in reversed flow. The modelling is also much more complex: bladerow performance can no longer be determined from the inlet conditions, and the flow in the inlet duct is no longer irrotational.

Fortunately for stall modellers, the precise behaviour does not appear to be as important as it seems at first sight as far as the overall behaviour of the compressor is concerned. If it is accepted that a very large pressure rise would be required to push air very fast backwards through a rotating compressor, then the reverse flow at all points in a stall/surge transient must be relatively small (compared to the forward flow at the surge point). If the reverse flow is small, then the errors in the momentum transfer and unsteady flow terms of the axial momentum equation will also be small. The biggest effect might be thought to be the calculation of the surge cycle period, as the amount of reversed flow determines the rate at which the plenum empties. Here too the error will not be large, because by the argument above more flow leaves the plenum forwards through the throttle than backwards through the compressor.

2.2.2.2 Linear and Non-Linear Models

The evidence from chapter 3 indicates that in high speed compressors stall disturbances grow so quickly that linear modelling is inadequate after only a small number of rotor revolutions. Escuret and Elder [1993, described below in section 2.2.3.1] obtain a

similar result (though for purely axisymmetric disturbances) from a theoretical comparison of a linear with a non-linear stall model. For this reason recent models, including the two presented in this thesis, have tended to be non-linear. There are two cases, however, where linear modelling is appropriate. The first is stability analysis, where the disturbances are by definition infinitesimal. Corbett and Elder [1974] and Escuret and Elder [1993], both described below in section 2.2.3.1, show the effects of non-linearity to be small with respect to surge point prediction for axisymmetric disturbances. Even here, though, care must be taken, because a linear analysis can show whether a system is stable, but not *how* stable. A system that is linearly stable may still be unstable when subject to large forcing disturbances, and there is considerable noise in a compression system, both deterministic (blade wakes, for instance) and random (turbulent fluctuations in velocity are typically 5% of mean flow [Camp, 1995]). These disturbances are not accounted for in either of the papers quoted above.

The second area to which linear analysis is ideally suited is in increasing the understanding of the physics underlying a model. Even if a linear model does not precisely match the growth of stall disturbances, the prime movers will be the same as the non-linear counterparts. In his review of stall modelling, Longley [1994] makes effective use of linear analysis to aid understanding, whilst noting that the model he develops would usually be run in a non-linear manner.

Linear analysis of post-stall transients is impossible, as the gross fluctuations in flow are heavily dependent on the non-linear nature of the compressor pumping capability [Moore and Greitzer, 1986a,b]. The model described in chapter 4, which is capable of predicting post-stall transients, uses a non-linear formulation. Moore and Greitzer themselves adopt an in-between approach, where the non-linear behaviour is simplified using a 'Galerkin' approach. In these papers they adopt only a first order approximation, and the 'Galerkin' procedure reduces to looking at the zeroth and first circumferential harmonic of flow disturbances (including the non-linear interaction between the two) and ignoring higher order terms. The results are necessarily only approximate, but are used very effectively by the authors to describe post-stall behaviour. Higher order approximations have since been applied (Ishii and Kashiwabara [1989]).

2.2.2.3 Pitchwise Average and Quasi-Steady Assumptions

In this section two assumptions will be considered that underlie all pitchwise average stall models. The first concerns the pitchwise averaging itself; the assumption is made

that the variation of blade inlet and exit conditions across each individual pitch can be ignored to some order of accuracy. The second 'quasi-steady' assumption, is that the reaction time of the boundary layers inside a bladerow is sufficiently small compared to time variations in the flow to be ignored or modelled in a fairly simple manner. If the reaction time can be ignored, then the bladerow performance (in terms of loss and deviation or equivalent measures) equates to the steady state performance with inlet flow as the instantaneous conditions in the unsteady model.

The pitchwise average and quasi-steady assumptions are related, in that stall disturbances rotate relative to rotor and stator, so big gradients in the circumferential direction also give big time derivatives in the bladerow relative frame. The question of the validity of the assumptions has been given greater importance by recent high speed compressor tests, including the one described in chapter 3. Until recently there was some debate as to the circumferential lengthscale of disturbances leading to stall in high speed compressors (section 2.1). A number of low speed compressors had shown harmonic type disturbances of low order around the circumference. The recent tests, however, indicate that in practical high speed compressors stall disturbances generally have short circumferential lengthscales, down to the order of a few blade pitches (chapter 3).

Pitchwise Averaging

There are numerous examples of experiment and calculation concerning the effect on bladerow performance of upstream and downstream disturbances. The exact question of the validity of the pitchwise average assumption has rarely been addressed however. Longley [1994] reviews the sparse information available and gives a 'rough constraint' for the pitchwise average assumption to be valid, namely that the wavelength of any flow disturbance should be at least eight blade pitches.

Strictly speaking, pitchwise averaging refers to the question of to what extent the average bladerow performance over a single pitch is dependent on local variation of flow conditions. In practice compressors are usually broken down circumferentially into segments of a few blade pitches, and the related and perhaps more important question arises as to what extent 'crossflow' between the segments attenuates circumferential flow variations. The subject of crossflow is considered in the discussions below on Dunham [1965] (crossflow in gaps), Mazzawy [1977] (crossflow in attached volumes such as bleed chambers), and Plourde and Stenning [1968] (crossflow generally).

Quasi-Steadiness

The usual method of assessing the magnitude of unsteady effects is to calculate a 'reduced frequency'. This is the disturbance frequency in the blade relative frame multiplied by the time taken for a particle to traverse a particular distance. Care has to be taken in applying this idea to multistage compressors. For example, if the VIPER compressor described in chapter 3 were subject to an inlet distortion covering half the annulus, then the reduced frequency seen by the first rotor based on axial chord would be roughly 0.08, which would normally be taken as implying that unsteady effects are relatively unimportant. In the case of multistage compressors, however, axial velocity variations tend to be fairly constant axially, so the relevant lengthscale for reduced frequency calculations is the entire length of the compressor, and unsteady effects are seen to be significant.

The reduced frequency based on the blade chord can be used as a measure of the significance of the time taken for the boundary layers in the blade passage to react to changed inlet conditions. In this case the absolute value of the reduced frequency has less meaning: a value of unity no longer implies a balance in the magnitudes of unsteady and steady effects. Nonetheless it provides a qualitatively useful guide, and certainly a value a long way from 1 would be conclusive.

One of the first modellers to tackle the quasi-steady assumption was Mazzawy [1977, described below]. He presented experimental data on bladerow loss and deviation under the effect of unsteady (in the blade relative frame) distortion. Based on this data he used a time-lag in his inlet distortion model to represent the time taken for the rotor blades to react as they moved through the distorted region. This method is easy to implement and has been adopted by numerous subsequent modellers.

It is significant that the data presented by Mazzawy is derived from an inlet distortion test (indeed, his model was aimed at modelling inlet distortion), where the frequency of the disturbance as seen by the rotor is much lower than that seen during a short circumferential lengthscale stall transient. In the initial stages of rotating stall the disturbance may cover only 5% of the annulus (chapter 3), and rotate at around half shaft speed. Under these circumstances the reduced frequency is around 0.4 (compared to the value of 0.08 for the inlet distortion case quoted above), and the unsteady effects are relatively more important. More work is required to assess the validity of the quasi-steady assumption with regard to this type of flow disturbance. This could be done experimentally or more easily using an unsteady viscous flow computation.

Moore [1984a,b,c] follows Mazzawy by introducing a constant 'time lag' into his bladerow performance calculations. He then, though, combines it with a second lag representing the inertia of the fluid in the blade passages as the axial flow changes, and shows that the latter dominates. The 'parallel compressors' model described in chapter 4 neglects the second order approximation, but includes the first order effect of inertia in the blade passages.

2.2.2.4 Axisymmetric Pressure Rise Characteristic

The 'axisymmetric pressure rise' is an important concept in pitchwise averaged stall models. It can be defined as the pressure rise that can be supported by the compressor at a given flow in the absence of non-axisymmetric disturbances. A clear distinction must be drawn between this and the measured pressure rise. Whilst the compressor is operating stably and axisymmetrically the two are identical. Whenever rotating stall is present, however, the measured flow and pressure ratio are averages of the operating point around the circumference. As an example, fig 2-5 shows both the measured and axisymmetric pressure rise characteristics for a hypothetical compressor. When the compressor is operating in stable rotating stall at point B on the measured characteristic in fig 2-5, the stalled sector is in fact operating in reverse flow at point A on the axisymmetric characteristic, and the unstalled sector at point C, both exhausting to approximately the same static pressure, and therefore giving approximately the same pressure rise. Between the stalled and unstalled sectors there is a transition region, where the flow lies between points A and C. In this region the 'axisymmetric pressure rise,' that is the pressure rise that would be obtained if the compressor were operating axisymmetrically, is substantially different from the measured pressure rise. The difference between the two is balanced by the acceleration of the flow, including momentum transfer terms (ie the material derivative of the axial velocity). In this way the axisymmetric pressure rise characteristic can be considered as a circumferentially local force characteristic, which during axisymmetric operation of the compressor is equivalent to the pressure rise across it.

The axisymmetric pressure rise characteristic is often the only compressor-specific input to a lumped parameter stall model. It is, however, difficult to derive at flows where normal compressor operation is unstable. Some useful information can be obtained if the compressor, or part of the compressor, can be stabilised such that it operates axisymmetrically beyond its normal stall point, for instance by guide vane adjustment (Camp [1995], Giannisis et al [1988]) or as naturally happens in the front stages of a long multistage compressor at low speed. This will only provide information in the neighbourhood of the natural stall point, but it is this region which is

most important for stall inception modelling. One important feature observed in such cases is that stabilised stage performance characteristics, and therefore the overall axisymmetric pressure rise characteristic, tend to roll over gradually, rather than dip sharply at stall. This supports the smooth characteristic shape used by Moore and Greitzer [1986a,b], over a sharp change of gradient at stall. At still lower flows, where it is impossible to run stably, the characteristic can only be obtained from extrapolation of correlations (presumably beyond their bounds of applicability), or from transient measured behaviour, having corrected for local inertia changes [Koff and Greitzer, 1986].

2.2.2.5 'CFD' Type Calculations

There are two approaches to modelling stall in axial flow compressors. The 'CFD' approach, not considered in detail in this thesis, is to use blade-to-blade or 3D unsteady calculations to model the interaction between a stall disturbance and bladerow performance. The other approach, considered in this thesis, is to use a pitchwise averaged description of bladerow performance, either analytically or computationally.

'CFD' type calculations over a meaningful timescale (at least 40ms for a stall transient, even at high speed) are usually limited by the size of the problem to one or two bladerows at a limited number of conditions. They also require calculations to be performed at well off-design conditions close to stall, where such calculations are generally considered to be least accurate: fully three-dimensional multistage CFD calculations are only now beginning to be used for routine design of axial flow compressors *at design conditions*. For this reason they are unlikely to trace a circumferentially broad disturbance any more accurately than the pitchwise-averaged models, which use measurements or correlations to predict bladerow performance close to and during stall.

Although CFD type calculations are not (yet) the ideal tool for tracking stall development in multistage compressors they remain the only tool to model the *detailed* response of the flow in the blade passages to the stall disturbances. One such use would be to generate information such as the time lag parameters described earlier for input to a pitchwise averaged model. In addition, they can be used to model the very start of the stall event. It is likely that many stall disturbances start with a circumferential lengthscale that is of the order of a few blade pitches (the results described in chapter 3, for instance, point toward such a conclusion). A pitchwise

averaged model, however, can only properly track disturbances that are circumferentially large compared to the blade pitch.

The more detailed CFD type models will assume greater significance with time. For the present, however, pitchwise averaged techniques represent the only practical way to trace the development of stall disturbances in long multistage compressors over significant intervals of time. The two models developed in this thesis are both of this type: the Compressors in Parallel model described in chapter 4 effectively treats the entire compressor as a semi-actuator disk, and the 3D Time Marching model (chapter 5) uses pitchwise averaged bladerow performance data in a finite element format.

2.2.2.6 Inlet Distortion and Stability

Inlet distortion is not the main theme of this thesis. Historically, however, the requirement to predict compressor behaviour with distorted inflow has driven much of the work that forms the basis for recent attempts at stall modelling. A number of the models described in the next section were originally formulated to calculate the attenuation of distortion through a compressor or compressors. Some of these are suitable for applying in an unsteady manner for stability boundary prediction in the presence of distortion. Mazzawy's [1977] model in particular is designed to be used in this manner. Likewise the stall models developed in chapters 4 and 5 of this thesis could easily be modified for surge point prediction with distorted inflow.

Beyond noting the close link between inlet distortion and stall modelling the subject is not considered in detail in this thesis. There is one approach however, specifically devoted to prediction of the instability boundary with distorted inflow, which is discussed in this section, because it offers valuable insight into how the nature of stall inception changes when inlet distortion is present. The approach was first put forward by Hynes and Greitzer [1987]. The essential observation is that with distorted inflow the operating point on the compressor characteristic can vary substantially around the annulus. Under these circumstances the slope of the characteristic cannot be considered constant, and consequently linear stability models cannot fully capture the effect of the distortion.

Unsteady time-marching models can easily be made to incorporate a non-linear compressor characteristic. Hynes and Greitzer's approach is different in that they derive a stability analysis analytically, which gives more insight into the fundamental fluid dynamics. The basic equations used in all of the calculations described below are based on those of Moore [1984a,b,c] and Moore and Greitzer [1986a,b]. Both of these papers are discussed in the course of the next section.

The first step in Hynes and Greitzer's method is to calculate a steady 'background' flow based on the specified inlet distortion. A schematic version of this steady background flow is shown in fig 2-6, for an average flow around the peak pressure rise of the undistorted characteristic. The pressure rise capability of the spoiled sector is reduced, the difference being made up by the circumferential transfer of momentum due to the passage of the rotors (in the phraseology of chapter 4: Hynes and Greitzer refer to 'the unsteady correction to the pressure rise as balancing the acceleration of the fluid in the bladerow'. This is discussed further under Moore and Greitzer [1986a,b] in the next section). Thus the flow rate *falls* in the direction of rotor motion throughout the distorted section, and *rises* throughout the undistorted section.

Having calculated the steady distorted flow solution the unsteady equations are linearised with respect to small perturbations about this background flow. Taking a Fourier transform with respect to time (that is, considering disturbances of the form $f(\theta)e^{i\omega t}$) defines an eigensystem, with the point of instability being defined by the first unstable eigenvalue $\tilde{\omega}$ reached as the flow is reduced. The eigenvectors (representing the circumferential variation in flow) are not simple circumferential harmonics as with clean inlet flow, but reflect the difference in underlying stability as the perturbation travels around the circumference. The disturbance is found to grow strongly in areas where the slope of the axisymmetric pressure rise characteristic is most positive which, referring to fig 2-6 and assuming a convex characteristic, can be seen to be the area around the end of the spoiled sector in the direction of the rotor.

Hynes and Greitzer present a number of calculations using their model, showing in particular good agreement with the frequently used $DC(\theta_{crit})$ criterion for compressor stability. From their initial calculations they put forward a 'rough rule of thumb' that instability will occur approximately where the annulus average of the compressor characteristic slope (that is the slope of the undistorted compressor characteristic calculated at the local flow coefficient in the steady 'background' flow, and including the effect of any change in the level of distortion with flow coefficient) is zero. They also note that this point is near the peak pressure rise of the *measured* flow characteristic, and suggest this ('tentatively') as 'an operational criterion for onset of instability in distorted flow'. The theory put forward in Hynes and Greitzer [1987] has been refined, notably by Hynes, Chue, Greitzer and Tan [1987], who consider the characteristic slope stability criterion in more detail and consider instances where it is not applicable.

2.2.3 Overview of Pitchwise Averaged Compressor Stall Models

It is convenient to categorise the various types of pitchwise averaged models in order of complexity with respect to the spatial dimensions described. The order is quite different to steady state compressor modelling. In steady state modelling the simplest models beyond a simple overall characteristic have only an axial dimension ('mean line models'). Next in order of complexity are the throughflow models which add a radial dimension. More complex still are the 3 dimensional models with some account of blade-to-blade effects. In stall modelling, however, non-axisymmetric disturbances are important, and conversely axial variations in flow are less important. For this reason the circumferential dimension is frequently the first to be given any definition, and a sensible order is

1. Surge Type Models (No circumferential definition, either lumped parameter or with axial definition)
2. θ -type Models (Axially lumped parameter, but with circumferential definition)
3. θz -type Models (Circumferential and axial definition)
4. $r\theta z$ -type Models (Circumferential, axial and radial definition)

Models of each type can be formulated in a linear or non-linear manner (section 2.2.2.2). In linear models of the θ -type and θz -type the circumferential dimension usually reduces to considering the circumferential harmonics of flow disturbances independently.

The following sections give an overview of published stall models according to the categories listed above.

2.2.3.1 Surge Type Models (No Circumferential Definition)

Models in this category are axisymmetric throughout the compressor and any upstream and downstream components. The reason for calling these 'surge type' models is that the lack of a circumferential dimension limits analysis to surge-type (that is, essentially axisymmetric) disturbances. Non-axisymmetric disturbances such as rotating stall can only be modelled inasmuch as they affect the circumferentially averaged performance. Nonetheless, such models can track the mean operating flow and pressure ratio during a stall or surge transient with some accuracy given only the geometry of the compression system and a measured pressure rise characteristic (as distinct from the more difficult to derive axisymmetric pressure rise characteristic described earlier).

It has been noted previously that a period of rotating stall generally precedes surge in experimentally tested compressors (certainly in the high speed compressor tests described in chapter 3). This period of growth of the stall disturbance is impossible to match accurately with a surge type model: there is the inevitable assumption that there is a size of circumferential non-uniformity associated with each operating point (flow and pressure ratio). This is clearly not always the case, otherwise there would be no hysteresis as observed between the flow at rotating stall onset and drop-out. It is unlikely to be the case either around the stall point. It can be seen from the formulation of the more complex models in subsequent sections that the instantaneous operating conditions (pressure ratio and flow) broadly govern the rate of change in the size of the disturbance, rather than the size itself. Under these circumstances there will inevitably be hysteresis (path-dependence) in the size of circumferential flow variations at a given operating condition.

The point of the above discussion is that the period of growth of non-axisymmetric disturbances is of first importance with respect to the objective of the work described in this thesis. In particular it will be shown in the discussion of the experimental results in chapter 3 that for modelling of active control of stall/surge this is precisely the critical period. For this reason only a few of the large number of such models are covered in this section.

Greitzer [1976a,b] presents a non-linear surge type model which can effectively track the operating point of a compression system through large fluctuations in flow and pressure ratio such as are observed during a surge cycle. He uses an actuator disk for the entire compressor, with the measured circumferentially averaged pressure rise characteristic as input to the model. A time lag is applied to the compressor performance near the stall point to account for the gradual growth of rotating stall disturbances. A simple procedure is followed to model the observed hysteresis between stall onset and stall drop-out. In part 2 of the paper [1976b], results are presented showing remarkably good agreement between the predicted and observed locus of the operating point for a variety of geometries tested.

The major theoretical advance made by Greitzer in this work was to recognise of the importance of the 'Greitzer B' parameter in determining the type of flow disturbance observed. This is a non-dimensional quantity based on the Helmholtz frequency of the system, and is defined as

$$B = \frac{U}{2a} \sqrt{\frac{V_p}{A_c L_c}} .$$

In this equation U is the mean rotor speed, a the speed of sound, V_p the volume of the downstream plenum, A_c the compressor annulus area, and L_c the effective length of the compressor. Although the detailed form of flow instabilities as calculated by Greitzer's model is heavily dependent on the shape of the pressure rise characteristic, it is shown that for a given compressor, the B parameter can be used to determine whether the final operating condition of the compression system will be surge or 'stable' rotating stall. Surge is predicted to occur at high values of the parameter (for instance if the volume V_p at compressor exit is very large). This is a remarkable result from a model which has no circumferential definition (although strictly, the result is merely related to whether the operating point stabilises at a low pressure rise on the measured characteristic or oscillates as a limit cycle. It is an observation that the low pressure rise part of the characteristic is related to rotating stall). In an appendix to Day's paper [1994a, described in section 2.1], Greitzer extends the B parameter to include in an approximate manner the effects of compressor design parameters, namely the pressure rise ψ_p and flow coefficient ϕ_p at the peak of the characteristic. The new parameter is defined as

$$B' = B \psi_p / \phi_p .$$

Escuret and Elder [1993] develop a linear and a non-linear surge type model with the compressor split into elements consisting of one stage each, with an axial force term to represent stage performance. An active control scheme is developed based on high response controlled bleed off. The results of the linear and non-linear models are compared for stall control on the Olympus 593 LP compressor. At low speed the two models gave similar results. At middle and high speeds, however, the two were very different, with the non-linear model predicting relatively little surge margin improvement with active control. The cause was found to be high growth rates of the linear instabilities close to the surge line at these speeds, which is attributed to stage mismatching.

Numerous authors, including Escuret and Elder, have used surge type models to predict the point of onset of compressor instability. Corbett and Elder [1974] review and develop a number of surge type models and demonstrate reasonable agreement with test data for a seven stage compressor. Early theoretical analysis [Dunham 1965, discussed below] suggests that both axisymmetric and non-axisymmetric modes of stall would be expected to occur somewhere near the peak of the total to static pressure rise characteristic, and this is only slightly modified in more complex models. To a first order, therefore, surge type models will successfully predict the surge line of

compressors even if the underlying process involves rotating stall. There are compressors, however, where the axisymmetric instability mode ('surge') occurs at a substantially lower flow than the non-axisymmetric modes. Haynes, Hendricks and Epstein [1994], for instance, used active stabilisation to successfully control the non-axisymmetric modes (only), thus extending the range of operation of the compressor. It is difficult to see how the different modes could be distinguished by a surge type model.

Many industrial compressor models, particularly of whole engine configurations, use a zero dimensional compressor model of this type, and it is useful if such a model can be made to work through stalled operation. Merriman [1993] has developed one such model, using similar principles to those of Greitzer [1976a,b].

2.2.3.2 θ -type (Axially Lumped with Circumferential Definition)

In this type of model the compressor performance is lumped into a single function of flow (or occasionally flow and inlet angle). Now, however, the operating point on this characteristic is allowed to vary around the circumference. Typically some account is taken too of the unsteady effects as blades pass through flow disturbances.

Moore [1984a,b,c] developed a model of rotating stall that has served as a basis for models by numerous subsequent authors. The basic theme was that of finding steady rotating disturbances (that is, steady in a rotating frame of reference) that could exist within an inlet duct/compressor/exit duct system between an inlet boundary at constant total pressure, and an exit boundary at constant static pressure. The basic non-linear equations derived in these papers were further developed in conjunction with Greitzer (Moore and Greitzer [1986a,b]); a detailed analysis of the equations is given under the review of that paper).

The three papers [Moore 1984a,b,c] refer in order to a linearised analysis, a non-linear analysis, and limit cycles for rotating stall disturbances. The linearised analysis is useful for assessing the relative importance of various design features, such as exit duct length, and bladerow response times. The second paper is almost unique in that Moore finds an analytic solution for a non-linear equation set. The analysis assumes a locally parabolic axisymmetric pressure rise characteristic (see section 2.2.2.4 for a definition of this term). This is more relevant to stall inception than fully developed rotating stall, where the characteristic cannot be assumed parabolic [Moore and Greitzer, 1986a,b]. The analytic solution described in the second paper is of limited value, partly because it is limited to steady (that is, not growing) disturbances, and partly because any change to the fundamental equation, for instance to model an active control strategy, would

invalidate the non-linear analysis and perhaps even make an analytic solution impossible. At the same time it offers little in the way of physical insight beyond that provided by the linearised analysis. The theory on which the non-linear analysis is based, however, forms an excellent starting point for stall modelling, and has been used by Moore and Greitzer [1986a,b], Longley [1994], and in simplified form, also by Freeman and Wilson [1993]. The third paper, on limit cycles, is related to fully developed rotating stall rather than stall inception. The most significant results relate recoverability from stall to design parameters, and in particular the shape of the axisymmetric pressure rise characteristic.

The benchmark paper for rotating stall type models is **Moore and Greitzer [1986a,b]**. This work was aimed primarily at modelling post-stall transients: the assumptions implicit in models of this type do not allow the initial stages of stall inception to be accurately traced, where the flow disturbance is localised axially and circumferentially. A detailed description of Moore and Greitzer's model is given here, partly as a basis for discussion of other models, including the one described in chapter 4 and summarised in Freeman and Wilson [1993].

The Moore and Greitzer model can be split into three parts; the compressor, the inlet and exit ducts, and a downstream plenum (with attached throttle). The simplifying assumption of incompressibility is made for the compressor and associated ducting. The same assumption could not be made for the plenum, however, as the compressed air in the plenum acts as an energy store, driving the flow backwards through the compressor during a surge cycle. The flow in the plenum is considered axisymmetric, however (in line with the discussion of the components surrounding the compressor in section 2.2.1 above), whereas non-axisymmetry is allowed for in the compressor and the inlet and exit ducts.

Moore and Greitzer [1986a,b] - Compressor Model

The compressor part of the Moore and Greitzer model is essentially that of Moore [1984a,b,c, summarised briefly above]. The essential observation is that stall disturbances, once grown to an extent where they affect the whole compressor, extend more or less axially through the compressor. Moore and Greitzer quote 'a continuity argument' and low speed experimental results (Day, Greitzer and Cumpsty [1978]) to support this. The experimental work described in chapter 3 is the first confirmation of this for a high speed compressor in an engine environment. This observation, coupled with an assumption of incompressible type flow in the compressor, allows for the use of a 'lumped compressor' model, where for any given circumferential position the

performance of the individual bladerows can be added together to give an overall pressure rise governed solely by the flow through that sector (apart from certain 'lag' terms described in section 2.2.2.3). This overall characteristic of 'axisymmetric pressure rise' (defined in section 2.2.2.4) versus flow is the only compressor-specific performance information required for the model.

Moore and Greitzer [1986a,b] - Mathematics

The assumptions described in the previous sections give rise to the following equation governing flow disturbances in the compression system [Moore and Greitzer, 1986a, equ 42]. The symbols are defined in the notation section of this thesis.

$$\Psi(\xi) + I_c \frac{d\Phi}{d\xi} = \psi_c(\Phi - Y_{\theta\theta}) - mY_\xi + \frac{1}{2a}(2Y_{\xi\theta\theta} + Y_{\theta\theta\theta})$$

This is a momentum balance equation: the terms on the left represent the instantaneous overall pressure rise of the compression system and the circumferentially averaged rate of change of momentum in the system. The first term on the right is the force generated by the compressor as calculated from the axisymmetric pressure rise characteristic as described in section 2.2.2.4. The middle term represents the unsteady potential effects of the inlet and exit ducts. The final term adjusts the rate of change of momentum for local velocity gradients. The two terms on the left are constant circumferentially. The system is closed by two further equations in time (ξ) only. One fixes the rate of change of pressure in the plenum based on the flow into and out of it. The second sets the rate of change of overall flow (in effect the annulus average of the equation given above).

The major source of difficulty in solving the above equation is its non-linearity. The main source of the non-linearity is the axisymmetric characteristic of the compressor (there is also an equivalent non-linear term for the throttle characteristic in one of the closing equations). For the sort of case tackled by Moore and Greitzer, taking a small initial disturbance right through to fully developed rotating stall, this is extremely important. For the very start of the transient (a fraction of a rotor revolution), however, the pressure rise characteristic (and certainly the throttle setting) could be linearly approximated, and in this case the system becomes entirely linear.

Starting from the equations described above, Moore and Greitzer proceed to remove the circumferential dependency by adopting a first order 'Galerkin' procedure, essentially approximating the axial velocity disturbance around the annulus by an axisymmetric component coupled with a first harmonic sine wave of varying phase and amplitude. The resulting equations are then applied to a number of cases, particularly

in part II of the paper. Whilst this first order approximation does help to understand the balance between surge-like and rotating stall-like disturbances, we know from the measurements described in chapter 3 that this is not a generally good approximation in the initial phases of rotating stall in high-speed compressors. A good model of stall inception therefore requires either more harmonics in the Galerkin procedure such as used by Ishii and Kashiwabara [1989] (reviewed briefly below), or a numerical solution of the equation set.

Moore and Greitzer [1986a,b] - Conclusions

Moore and Greitzer [1986a,b] is an extremely informative paper, bringing together as it does a great deal of work by the authors and their associates. The model predicts well the fundamental aspects of compressor stall. The sudden drop in pressure rise as the surge line is reached is correctly ascribed to the inception of rotating stall, rather than a sharp change in the axisymmetric pressure rise characteristic. The advantage over earlier models is that the plenum and throttle terms allow the trajectory of the operating point to be traced. Disturbances can then be tracked over longer time periods than previously possible. The simplified 'Galerkin' analysis of the resulting equations gives reasonable agreement for the overall post-stall transient (the objective of the paper), but is unlikely to be accurate enough to model the short circumferential lengthscale disturbances measured in high speed compressors (chapter 3).

Ishii and Kashiwabara [1989] consider the effect of taking more terms in the Galerkin procedure used in Moore and Greitzer's model, and conclude that between 6 and 9 harmonics are required to reasonably match the non-linear aspects of compressor stall. They present a single comparison of their model with a three stage compressor in a multiple surge cycle with repeated periods of rotating stall. They show a single harmonic calculation predicting pure surge, whilst the multiple harmonic prediction correctly captures the periods of rotating stall.

In addition to increasing the number of terms in the Galerkin procedure, Ishii and Kashiwabara make a number of changes to the modelling, of which the most important is the inclusion of an inlet distortion capability, although there is little discussion of the results with inlet distortion, and no comparison with test data. The modelling of the inlet duct is changed to a solution of Laplace's equation as described in section 2.2.1.3. One final point of interest is that as well as looking at the effect of the 'Greitzer B' parameter (see under Greitzer [1976a,b], in section 2.2.3.1, for a definition of this term) on the trajectory of the operating point, they also consider the effect of the overall pressure ratio of the compressor.

Freeman and Wilson [1993] present a θ -type model which is described in detail in chapter 4. The underlying equations are similar to those of Moore and Greitzer [1986a,b], although simplified slightly and changed to account for an engine rather than a rig environment for the compressor. In this model, however, a numerical solution is obtained, allowing shorter circumferential lengthscale disturbances to be modelled. Further details can be found in chapter 4.

It should be noted that the more complex models described in the following sections can sometimes be used in an axially integrated manner. One example is Mazzawy [1977], who includes details of this approach in his paper. Mazzawy's model is described in the next section.

Longley [1994] provides a useful review of models both of this type and the θz -type. His comments are summarised in the course of the next section.

2.2.3.3 θz -type (*Circumferential and Axial Definition*)

This type of model is differentiated from the previous type by the inclusion of an axial dimension. This is typically bladerow by bladerow using an actuator disk or semi-actuator disk type approach, but could equally well be stage by stage or even a geometric discretisation, which would allow compressibility effects to be more readily accounted for.

One of the earliest, most careful, and most quoted contributions to models of this type is Dunham's monograph [Dunham 1965]. In common with most practice at that time, whilst computers were still small and expensive, the analysis uses linearised flow equations to derive analytical results. Whilst somewhat approximate, this type of analysis often affords greater insight into the physical flow phenomena. The model developed by Dunham is suited to steady distortion and also stable rotating disturbances, which he treats as steady in a rotating frame.

There are three parts to Dunham's monograph. The first concerns high hub/tip ratio compressors (two dimensional flow). The second part concerns low hub/tip ratio compressors and is discussed in the next section on 3 dimensional models. The final section of the monograph is devoted to rotating stall, primarily based on the two-dimensional analysis of the first section.

The assumptions underlying the high hub/tip ratio model are incompressible flow, small amplitude disturbances (such that the flow equations can be linearised), and disturbances of sufficiently large lengthscale and timescale that the bladerow performance can be treated as circumferentially continuous. The bladerow

performance is assumed to be quasi-steady, in this case defined by loss only, as deviation is assumed to be constant in view of the small amplitude of the disturbances under consideration. The "chief original feature of the theory...is the linearising assumption that the gaps are small enough to take

$$e^{nz} = 1 + nz \quad [z \text{ is the length of the gap normalised by tip radius, } n \text{ is the circumferential mode number}].$$

The Moore & Greitzer model described earlier assumed that "any circumferential nonuniformity of axial velocity within the compressor must, by a continuity argument and by evidence cited in [Day, Greitzer and Cumpsty, 1978], go straight through the machine." Implicit in this statement is not only incompressible flow but also that the effect of the gaps between the bladerows in redistributing the flow is small. The major conclusion of Dunham's work is that the effect of gaps is indeed small, justifying the 'compressors in parallel' models such as that used by Moore and Greitzer and the one described in chapter 4. While Dunham correctly derives this important result, he does not detail the physical reasons behind it, which are discussed here.

The linearised analysis of Dunham leads to the result that circumferential harmonics of disturbances develop independently. The small gap assumption specified earlier gives the following equations (Dunham's equations 4) governing the development of the n^{th} harmonic coefficients of axial velocity, whirl velocity, and stagnation pressure (normalised by density and mean axial velocity) respectively in a gap of length z (normalised by the tip radius), at mean flow angle γ ;

$$\begin{aligned} U &= U_0 - inzV_0 \\ V &= V_0 + inzU_0 - inzP_0 \\ P &= P_0 (1 - inz \tan \gamma) \end{aligned}$$

where the subscript 0 refers to the respective value at the inlet to the gap (ie blade exit). The symbols are defined in the notation section of this thesis. Note that the velocities and pressures are all harmonic variations on top of a mean level (hence for example $\tan \gamma \neq V_0/U_0$), and that circumferential velocity is measured in a frame such that any rotating disturbance is steady.

From these equations the fundamental reason behind the smallness of the gap terms can be seen to be the (quite reasonable) assumption of constant deviation. If the air angle is fixed at the exit of a stationary upstream bladerow, then U_0 and V_0 are forced to move together, and in particular V_0/U_0 is real. A little work with velocity triangles shows that this latter statement is true (although with reversed sign), for rotating

blades also - a peak in axial velocity corresponds to a peak (or a trough) in tangential velocity. Hence U_0 , for instance, and the correcting term $inzV_0$ are perpendicular in the complex plane, and any decay in the axial velocity profile is of order z^2 , rather than z as might be inferred from the equations. Physically this means that the flow out of the preceding blade is constrained to follow the blade direction plus some constant deviation. Immediately on leaving the blade, therefore, the fluid is not moving tangentially in such a way as to change the axial velocity profile. It takes time (that is, axial distance) for such tangential flow to develop, and so the decay rate of the axial velocity profile is $O(z^2)$. This is in juxtaposition to the case at inlet to the compressor, where there is a high rate of decay. Here the tangential velocity is not constrained (mathematically, V_0 and U_0 tend to be perpendicular in the complex plane), and the decay rate upstream is $O(z)$.

The stagnation pressure equation constrains the profile merely to rotate around the annulus with axial distance (to first order in z). The only equation of the three, therefore, which may include a first order decay term is the tangential velocity equation (V), which includes a stagnation pressure term ($inzP_0$). In practical disturbances, however, (rotating stall and inlet distortion) the total pressure defect tends to be in phase circumferentially with the velocity defect, and so any $O(z)$ decay term is also small. Also, for cases where the static pressure is nearly uniform circumferentially, for instance towards the back of the compressor, $\rho\bar{U}P - \rho\bar{U}U - \rho\bar{V}V \approx 0$. In this case, the circumferential velocity equation reduces to

$$V = V_0(1 - inz \tan \gamma), \quad \text{that is, identical form to the total pressure equation.}$$

It can be seen, then, that to first order there is no decay of flow disturbances (of moderate circumferential harmonic number n) in the gap regions. Rather, the effect is to rotate the three fluid properties around the annulus, circumferential velocity and total pressure broadly according to the mean flow angle, and axial velocity according to $\tan^{-1}(V_0/U_0)$, which for constant deviation equates to the exit air angle in a rotating or stationary frame for rotors and stators respectively. Further, for rotating stall in particular, which tends to rotate at around half rotor speed, and near 50% reaction blading, the rotation in the rotors and stators will cancel exactly.

Whilst there is no first order decay of disturbances in the gaps, there may be first order terms in the effect that the rotation has on subsequent bladerows. The argument above, however, shows that there are at least some circumstances in which the actual effect of the gaps is $O(z^2)$, which is no bigger than the terms in the expansion of e^{nz} which have been neglected. Dunham presents a sample calculation including the

effects of the gaps on subsequent bladerows, and points out that in this case too his calculated gap terms often cancel, and so the second order terms in the exponential expansion may well prove to be more significant.

In the light of the above discussion it is important to check the size of the neglected terms in the exponential expansion. Dunham quotes a typical gap as being $z=0.014$ tip radii in a multistage compressor. One might consider the second order terms in the exponential expansion to be cumulative (this would also be a worst case). Then a measure of their effect is $e^{Nnz} / (1 + Nnz)$, where N is the number of gaps. For an eight stage (sixteen gap) compressor the value is 1.02 for the first mode, 1.08 for the second, 1.17 for the third, and 1.29 for the fourth. Thus the approximation starts to break down in such a compressor even for quite low harmonics. Given the recent emphasis on short circumferential lengthscale disturbances (high values of n), this is perhaps a field for further research.

The third part of Dunham's monograph develops the simple stability criterion that rotating stall will develop at the peak of the overall total to static pressure characteristic. This has been found to be in general agreement with experimental data, and has been modified only slightly as a result of later more complex models.

Plourde and Stenning [1968] present an interesting axial by circumferential type compressor model designed primarily to predict the attenuation of inlet distortion through a compressor. The analysis is again incompressible and linearised. Even so, they present a good match to an experimental three stage compressor with substantial inlet distortion (80% of inlet dynamic head over 180 degrees of circumference, albeit one radius upstream of the compressor face).

There are two particularly interesting aspects of Plourde and Stenning's work. The first is that the compressor performance was modelled by a distributed axial blade force, in effect averaging rotor and stator performance. This allowed the mean circumferential velocity to be neglected even within the compressor, simplifying the analysis. The second feature was an empirical 'k' factor, representing "the increased resistance to flow in the tangential direction imposed by the presence of the blading within the compressor." The value of k was allowed to vary from 1 (no resistance) to infinity (no crossflow) within the compressor, but was set to 1 at compressor inlet and exit.

The experimental results from their 3 stage compressor were best matched by a value of $k=\text{infinity}$, supporting the conclusion of Dunham [1965] that crossflow can be ignored. The main reason for this, judging from the evidence in Plourde and

Stenning's paper, is that with crossflow in the compressor, the model underpredicted the static pressure disturbance at compressor inlet. This may, however, be due to the fact that the stagger of the blades was not accounted for in the momentum calculations. With the $\cos^2 \xi$ correction described in section 4.2.3, the change in momentum flux across the compressor for a given change in axial velocity would rise, and so the static pressure perturbation at inlet would increase. With this correction it may be that the best match would allow for some crossflow within the compressor.

Mathematically, Plourde and Stenning eliminated the pressure term from the axial and circumferential Euler equations in much the same way as is done when deriving the vorticity equation, but with the inclusion of their empirical k factor. They then used a Fourier decomposition in the circumferential direction, leading to a third order ordinary differential equation for the amplitude of the harmonics (with respect to length along the compressor: it will be remembered that the objective of the work was to model the attenuation of inlet distortion along the compressor, and thus they have no time dependence). If $k=\infty$ (no crossflow), the equation reduces to first order, and a simplified version of the model is given for this case.

Plourde and Stenning's model could fairly easily be reconstrued into a form suitable for modelling low amplitude stall disturbances. It is unsuitable for large disturbances, however, even in non-linear form, as the simplification of ignoring the mean circumferential velocity within the compressor prevents any account being taken of air inlet angle to each bladerow. That is, the axial blade force per unit length at any axial position can only be a function of the axial flow. This is a reasonable assumption for small flow perturbations, but not sufficiently general to track the growth of stall disturbances for more than a few rotor revolutions.

One interesting result of Plourde and Stenning's work is that for steady inlet total pressure distortion the overall attenuation through the compressor is heavily dependent on the slope of the constant speed characteristic, but is broadly independent of the resistance to crossflow (k), and the mode number of the circumferential harmonic. The reason can be inferred from their handling of total pressure. Total pressure perturbations remain constant in the inlet duct (steady flow), and are shown in the paper to decrease along the compressor at a rate proportional to the characteristic slope (taken to be constant) and the amplitude of the axial flow perturbation at that axial plane. At the rear of the compressor, where the flow is axial and static pressure is assumed constant around the annulus, the amplitudes of the total pressure and velocity perturbations must match. If the flow perturbation is constant along the compressor ($k=\infty$), it is easy to see that there is only one level of flow perturbation which

gives the desired gradient of total pressure perturbation amplitude from the prescribed inlet distortion to the matching exit level (independent of mode number). Even if some crossflow is allowed within the compressor ($k < \infty$), it is shown in the paper that a great deal of the flow redistribution occurs upstream of the compressor, and so the velocity perturbation is fairly constant through the compressor, and the calculated attenuation of the total pressure distortion is not much changed.

Mazzawy [1977] develops an axial by circumferential non-linear model for circumferential flow distortion. Although the model concerns only steady distortion, it contains many of the features required for a stall inception model. Notably the paper contains some rare experimental data on how unsteady flow affects blade row loss and deviation. Even in steady distortion a passing rotor sees an unsteady inlet flow, which Mazzawy found to be significant. This unsteady effect was approximated using a simple time lag, as described in section 2.2.2.3, an approach which has frequently been followed by subsequent authors.

Mazzawy calls his model a "multiple segment parallel compressor model". Like conventional parallel compressor models, crossflow between the segments within the compressor is ignored, although Mazzawy makes the interesting observation that crossflow can occur in attached volumes such as stator shroud boxes or a bleed plenum, and accounts for such in the model.

The effects of compressibility are modelled in an approximate manner: stage matching is accounted for, and an effect is calculated on the circumferential rotation of the flow disturbance as it moves along the compressor. These steady effects are of course completely different to the pressure waves generated by compressibility in an unsteady model. Unlike many similar models the circumferential profile of temperature distortion is tracked along with the other parameters, allowing corrected speed effects to be accommodated.

The stability criterion for inlet distortion proposed by Mazzawy is to assume instability when the pressure rise capability of the worst segment (including the unsteady effects in the model) reaches a maximum. This however neglects the fact that stall disturbances leading to surge rotate around the compressor *through* the different parallel segments. This topic is covered further in section 2.2.2.6.

In addition to the fully axial by circumferential model, Mazzawy notes that it can be used with a lumped compressor characteristic if blade row performance data is unavailable. The model then becomes similar to those described in the previous section, though without any time dependency.

Breuer and Servaty [1996] compare the results of a circumferential by axial stall inception model (that is, a model of the initial stages of rotating stall/surge) with experiments on a three stage high speed axial flow compressor. A full set of Euler equations is used (mass, axial momentum, tangential momentum, and energy, in conservation form), with extra terms for bladerow loss and deviation, for which a simple lag parameter is introduced, such as described in section 2.2.2.3. The authors state that "No special effort concerning the discretization has been made to distinguish between bladed and unbladed elements of the duct." In this sense, then, the model reflects that of Plourde and Stenning with their "k"-parameter equal to 1. It is perhaps surprising, therefore, that the agreement with the experimental data is good. Data is only presented for 100% speed, however, where a fairly broad stall disturbance starts at the back and grows very rapidly. There is insufficient information to make detailed comparisons: in particular, it would be interesting to see if such a model with complete communication circumferentially could successfully generate a short circumferential lengthscale disturbance at the front of the machine, as might be expected at a lower shaft speed.

Ham and Williams [1983] present an axial by circumferential actuator disk-based compression system model that exists in two versions. Most of the paper is devoted to the effects of spool coupling on inlet distortion, and this problem is tackled using an actuator disk representation of each compressor. As far as modelling stall (or at least the stall inception process) is concerned, the other version of the model is more interesting, in which each bladerow is represented by a semi-actuator disk (that is, an actuator disk but accounting for inertial effects in the blade passages). There is relatively little about this model in the paper. Like Dunham [1965] and Plourde and Stenning [1968] it is based on linearised flow equations, and so is ideally suited to small amplitude disturbances. Unlike Plourde and Stenning's model, however, swirl is accounted for within the compressor, and so incidence effects on the bladerows could be accounted for. Unlike either of the above authors, Ham and Williams allow for compressible flow, and a useful comparison of compressible and incompressible duct flow is given in the paper.

Longley [1994] gives a useful summary of stall models of the type described in this section and the one dimensional type described in the previous section. His discussion is limited to incompressible models, although some reference is made to the effects of compressibility. After first considering the assumptions underlying pitchwise averaged models (some of his comments are referenced in section 2.2.2.3 above), he proceeds to derive a two dimensional model "in essence, the one presented by Moore [1984a]."

The model is used in linearised (where possible) and non-linear form to investigate the effects of various simplifying assumptions. He then presents an enhanced model, which accounts for unsteady variations in bladerow loss and deviation by the use of time lags.

2.2.3.4 $r\theta z$ -type (Circumferential, Axial and Radial Definition)

Longley [1994], in his review of stall models, says "...here, as in virtually all treatments to date, attention is limited to flow variations that are primarily in the axial and circumferential directions,..." and there are indeed few published models which include radial effects. Dunham [1965] bravely provides an analytical linear analysis using Bessel-Fourier harmonics. Escuret and Garnier [1994] obtain a numerical solution of Euler's equations. These papers are described below. In both cases the flow is assumed inviscid, and the only difference from the two dimensional case is the change in radial matching brought about by the stall disturbance or inlet distortion.

The model described in chapter 5, which is also three dimensional, has a motivation that is altogether different to the two described below. One advantage of a three dimensional model is that it gives the opportunity to get closer to the actual breakdown in the flowfield that initiates stall, which is localised radially, as well as axially and circumferentially, typically at rotor tips. If this breakdown is assumed to depend on the balance between the viscous and pressure forces acting in the annulus boundary layer, it cannot be modelled by a purely inviscid technique. In the three dimensional method described in chapter 5 the boundary layer is modelled directly by including viscous terms in the flow equations.

Parts 1 and 3 of Dunham's monograph [Dunham 1965], concerning two dimensional compressor modelling, were considered in the previous section. Part 2 concerns compressors of low hub/tip ratio and is considered here. As with the two dimensional models, Part 2 is a linear analytical treatment of the passage of a disturbance through a compressor, but this time the radial dimension is included. The complexity of the mathematics limits the range of application of the approach, and for this reason results are only presented for an isolated blade-row of free vortex design, and the results compared with a cascade experiment.

Although the full analytical method cannot easily be applied to multiple bladerows, Dunham does suggest a simpler alternative model for low hub/tip ratio multistage compressors. Based on the high hub/tip calculations which suggested that the effect of crossflow in the gap regions is negligible, Dunham suggests, in essence, that the compressor be modelled by a throughflow (axial by radial) solution at each

circumferential position, ignoring the effects of the gaps completely, but allowing radial non-uniformities to attenuate through radial redistribution of the flow. This is perhaps a good idea for steady inlet distortion, or 'stable' rotating stall which can be made steady by choosing a rotating frame of reference, but the solution, involving Bessel-fourier harmonics, is almost as complex as a numerical solution of the Euler equations such as presented by Escuret and Garnier (see below), which allows for compressibility as well as large scale disturbances, and also allows the inclusion of gap effects for little extra complexity.

Escuret and Garnier [1994] present two compression system models. The first models primarily axisymmetric flow instabilities and is not considered here. The second is based on "a 3D unsteady Euler solution in blade-free volumes [that is, ducts and inter-row gaps] which has been dynamically coupled with a multiple throughflow solution within bladerows." A finite volume time-marching technique is used with a structured grid in each of the blade-free volumes. Time lags are introduced to account for the unsteady response of both compressor bladerows and throttle. The model has the capability of being linked to an axisymmetric model of a downstream duct/plenum (section 2.2.1.2).

The only results given in the paper are for a single stage compressor with no inlet guide vanes. The model predicts that the different circumferential harmonics of disturbances have the same rate of propagation, but different growth rates, with the lowest harmonics growing fastest (indeed, the authors present the "puzzling result" that all of the circumferential harmonics become unstable at a lower flow than the axisymmetric instability). These results appear to be preliminary ones: there is little discussion and no comparison with experiment.

2.3 ACTIVE CONTROL OF STALL/SURGE

2.3.1 Early Ideas

Ludwig and Nenni [1978] demonstrated a stall control system on a low speed research compressor and on a turbojet engine with an eight stage compressor. Pressure transducers were used to detect the presence of rotating stall, and subsequent control action changed the variable stator vanes or, in the case of the engine, the inlet guide vanes and the positions of compressor bleed doors. As with Riess and Bloecker [1987, described below] the control action was (approximately) axisymmetric, and the objective was to change the compressor configuration such that operation was stable at the required operating point. In particular, no attempt was made to stabilise the compressor in a geometrical configuration that would otherwise be unstable.

The position of the blocker doors part-way along the compressor made the control system suitable only up to 80% of design speed. With this proviso, successful control was demonstrated using the same actuators already in place for standard engine control. Care was taken to integrate the stall control scheme into the existing engine control system.

The main drawback to the system devised by Ludwig and Nenni is that the relatively slow speed of the sensor and actuator systems (only two sensors circumferentially, standard engine control actuators) limited the system to stopping rotating stall that had already started, so that rotating stall existed for periods of up to 325ms before dropping out under the influence of the control scheme. In some circumstances the authors report successful control with no rotating stall present. In these cases the control system is presumably responding to pre-stall disturbances in much the same way as the system devised by Riess and Bloecker [1987, see below]. The existence of rotating stall even for short periods has obvious implications with regard to blade stresses: this is not a system intended for regular operation at high speed. The data presented showed relatively little effect in terms of overall engine operation (engine speed, compressor exit pressure). This is surprising, given the speed of stall inception in the VIPER engine for instance (chapter 3). It may be a function of the particular compressor and the type of stall it exhibits: the VIPER engine at low speed, for instance, shows 'benign' stall, which has little effect on normal compressor operation.

Epstein, Ffowcs Williams and Greitzer [1986] were the first to present a detailed analysis of how active control might be applied to extend the stable operating range of a compressor, including control of incipient rotating stall as well as the axisymmetric

component of stalling disturbances. Starting with the modelling of Moore [1984a,b,c, reviewed in section 2.2.3.2], the authors added a controlling disturbance based on the (circumferentially) local disturbance to the axial flow. By way of example the authors perform calculations to simulate an upstream disturbance generated by oscillating blades, and show that the compressor can be made to operate in a linearly stable manner with respect to rotating stall disturbances beyond the uncontrolled stall point (which according to the modelling technique used lies at the peak of the total to static pressure rise characteristic). The authors also note that the same device could be used to destabilise a compressor in an undesirable 'stable' state such as unrecoverable stall.

In addition to control of the non-axisymmetric component of stall disturbances Epstein et al consider the possibility of controlling axisymmetric surge-type disturbances, using either a movable wall in the downstream plenum or a variable exit nozzle area, finding that the spatial uniformity, low frequency and relatively slow growth rate of this type of instability make it easier to control than rotating stall type disturbances.

For both types of control, the authors find that active control is feasible, and that the required controller power input is of the order of the general level of 'ambient' fluctuations, which is small compared to the overall power consumption of the compressor.

Ffowcs Williams and Huang [1989] applied the work of Epstein et al [1986, described above] to actively control surge in a small centrifugal compressor. In centrifugal compressors axisymmetric surge is generally a more significant problem than rotating stall, and the authors limit their objectives to controlling axisymmetric disturbances. Equations representing the dynamics of the system are taken from a combination of theory [Greitzer, 1976a, reviewed in section 2.2.3.1] and previous experimental work. The movable plenum wall in the analysis of Epstein et al was supplied by way of a loudspeaker in the wall of the downstream plenum. Control was successfully demonstrated, with operation beyond the uncontrolled surge point. Beyond this, the authors found that the control system was able to stabilise the compression system even if it was already in a multiple surge cycle, even though the overall energy changes through the cycle were much greater than the integrated controller power.

Riess and Bloecker [1987] report initial tests of a surge control device for a six stage axial flow compressor. Each stage was fitted with a set of fast acting guide vanes. The vanes in each row moved together, whereas each row could be controlled independently of the others. Pressure transducers were used for stall detection. The

authors observed that on their compressor aperiodic pressure fluctuations occurred for a considerable period of time prior to stall/surge, and the principle of their control device was to detect these signals, and to use the fast guide vane adjustments to change the configuration of the compressor to one with more stable operation. This type of control is sometimes termed "avoidance of stall" as opposed to "control of stall", as the compressor never operates at a condition that would normally be unstable. Initial test results are presented to demonstrate the effectiveness of the guide vanes in moving the compressor to a new configuration with greater surge margin.

The objective of the control system proposed by Riess and Bloecker is to allow normal compressor operation to be optimised for efficiency without having to keep a safe distance from the surge line. There are applications where this could give a significant improvement in operating costs. The main drawback is that the system relies on the presence of pre-stall disturbances for a significant time prior to stall. Already this causes the same difficulties encountered with all active control techniques: do such pre-stall disturbances exist for a given compressor, where are they located axially, radially, and circumferentially, are they consistently detectable at all speeds and operating conditions, are there any causes of stall that would not give these signals. The problem with the system used by Riess and Bloecker (as it stands) is made greater by the relatively slow response of the guide vanes. In some compressors pre-stall disturbances do exist for a considerable period prior to stall at low speed, but at high speeds such disturbances last only a short time if at all. In the VIPER experiments reported in chapter 3, for instance, the complete stalling event at design speed from initial disturbance to significant drop in combustion pressure lasted for only around 20ms, which is too fast for the guide vanes used by Riess and Bloecker, which are quoted as moving through 90 degrees in one second.

2.3.2 Recent Experiments

The first experimental demonstration of active control of rotating stall, in the sense of demonstrating fundamentally axisymmetric operation beyond the natural stall limit, was performed by Day [1993b] on a four stage low speed compressor. Earlier experiments on the same compressor (Day [1993a], see section 2.1 for details), had shown two different stall inception patterns, depending on tip clearance over the first rotor. The first of these was a long circumferential lengthscale 'modal' type of disturbance as predicted by linear theory. The second consisted of a short circumferential lengthscale 'finite stall cell' originating at the tip of the first rotor. In accordance with this observation Day used two different control schemes, the first a simple first circumferential harmonic sinusoidal feedback for control of 'modal' disturbances, and

the second an 'on and hold for a period' response to localised disturbances timed to coincide with a passing stall cell and eliminate it, thereby preventing its growth into fully developed stall or surge. For each type of control 8 hot wires were used as input devices, and twelve high speed on/off valves were used to inject high speed air into the flow at specific circumferential locations. In order to facilitate the second type of control scheme, these valves were located upstream of the tip of the first rotor, which is where the 'finite stall cells' were observed to originate.

Day found that when modal waves were present, using the sinusoidal feedback (or as close as could be managed with 12 discrete on/off actuators), the stable operating range could be extended by four percent of flow, using an actuator inbleed of less than one percent of compressor flow. When the throttle was closed further, short circumferential lengthscale stall cells appeared which stalled the compressor. Thus the useful operating range was found to be limited not by the effectiveness of the actuator system, but by the appearance of a fundamentally different type of stall inception behaviour which could not be controlled with this type of low circumferential harmonic algorithm.

If the phase of the actuator response was set to its least favourable position, stall could be initiated prematurely by amplifying modal type waves. The stall margin decrease in this case is much less than the improvement with optimal phase setting, which may be related to the one-sided nature of the control system (that is, the control system only ever *added* energised air to the rotor tip region).

At low tip clearance levels in the first rotor, circumferentially short 'finite stall cells' led to compressor stall/surge with no pre-stall modal waves, and Day used the second type of actuation system. This was found to be successful either operating all valves simultaneously when a cell appeared or operating the nearest valve (defined using a pre-defined stall rotation speed) and the one opposite it ('because it balanced out the one-sided effect of using just one valve' - no further clarification is offered in the paper). Operated in this way, it was found that the valves were only required to be on intermittently (emphasising the role played by blade interactions or random unsteadiness in the formation of such stall cells). This type of control yielded a six percent drop in flow at the stall point.

Virtually at the same time as Day was carrying out his experiments, Paduano et al [1993] were performing active control experiments on a single stage low speed rig. This rig showed very clear 'modal' stall inception behaviour, with a smooth transition from first circumferential harmonic disturbances into fully developed rotating stall. In

place of the 'puffer jet' actuation devices used by Day, the authors used twelve 'wiggly VIGVs', that is, inlet guide vanes with fast response stagger adjustment. Input signals were taken from eight hot wires placed at mid-span upstream of the first stage. A proportional feedback system was used to control first and second circumferential harmonics of measured flow disturbances. With control applied to the first harmonic only a flow reduction of $\pm 11\%$ was obtained at the stall point, depending on the phase relationship between measurement and actuation. Note that here, with more clearly linear behaviour and a two-sided control system (the vanes could be opened *and* closed), the improvement with optimal phase matches the deterioration with the worst phase relationship. With control applied also to the second circumferential harmonic, the compressor could be stably operated at flows up to 20% lower than the uncontrolled stall point.

Paduano et al used a system identification technique to check their modelling of the effects of the moving IGVs. The form of the model (effectively a second order linear equation) was shown to give a good representation of the unsteady flow pattern, although the authors state that they 'have yet to make the quantitative prediction of the Bode plot [viz, the coefficients in the second order equation] from the compressor geometry.'

Haynes, Hendricks, and Epstein [1994] applied the control system devised by Paduano et al to a low speed three stage rig (in fact, the same rig analysed by Garnier [1989] as discussed in section 2.1). The results are similar in nature to those of Paduano et al. Control of the first circumferential harmonic gave a 3% decrease in stall point flow coefficient. Control of the first two harmonics simultaneously gave an increase of 8%. The stall transient in this case still appeared to be dominated by the first two modes, suggesting that the limiting factor was the 'strength' of the control system rather than the appearance of a new flow phenomenon such as observed by Day [1993b, discussed above]. Accordingly, control of the third harmonic as well as the first two produced no further benefit. A system identification procedure was followed in much the same way as by Paduano et al. This time, however, the Moore and Greitzer [1986a,b] stall model was matched to the results by the inclusion of blade row response time lags. The improved model matched both the uncontrolled and controlled dynamics. The stall margin improvement predicted by the model was twice that achieved experimentally, which is attributed by the authors to uncertainty in fitting the control parameters due either to modelling inaccuracies or experimental noise.

One significant point made by the authors is that they found the introduction of controls methodology (as opposed to fluid dynamic theory) extremely useful.

Eveker et al [1995] present a novel system for controlling rotating stall and surge simultaneously. Theoretical work is presented starting with the model created by Moore and Greitzer [1986, described in section 2.2.3.2]. The resulting control system is then successfully demonstrated on a low speed three stage compressor. A circumferential array of pressure transducers is used as input to the control algorithm. Actuation is supplied by a single bleed-off valve in the downstream plenum. In this respect, the control system is similar to that of Ludwig and Nenni [1978, described above]. The objective of Ludwig and Nenni, however, was stall avoidance. Here the objective is to allow operation at flow rates that would normally be associated with surge or large amplitude rotating stall.

The control system devised by Eveker et al does not change the 'normal operating range' of the compressor. What it does do is change the behaviour once throttled beyond the normal stability point, using the non-linear coupling of the non-axisymmetric and axisymmetric components of rotating stall/surge as predicted by Moore and Greitzer. Without control the compressor falls into surge or large amplitude rotating stall (with a hysteresis effect as the compressor is unthrottled), depending on the volume downstream of the compressor. With control, the compressor falls into much lower amplitude rotating stall, regardless of the downstream volume, and with no hysteresis effect. The theoretical model predicts a smooth transition from zero rotating stall amplitude at the normal stability point, whereas in practice there is a sharp drop in pressure rise coefficient as that point is reached. Although this drop is considerably less than in the uncontrolled case, it may limit the applicability of the control scheme. The reason for the sharp drop may lie with the short length scale stall inception pattern observed by the authors in their compressor. That is, the control system is not designed for the type of disturbance seen during the inception period.

The above paper is one example of a number of linear and non-linear control strategies that have been developed from the Moore and Greitzer model of stall/surge. Many of these fall into the realm of control theory and are beyond the scope of this survey. Some caution has to be applied in approaching these strategies, as they are naturally dependent on the assumptions used by Moore and Greitzer to develop their model. Many, including Eveker et al, use the same cubic characteristic as Moore and Greitzer. Whereas the shape of the characteristic was arbitrary for Moore and Greitzer, for active control strategies it is extremely important, as it defines the rate at which stall disturbances grow. More importantly, Moore and Greitzer's model was developed for

'post-stall transients', and for many compressors does not accurately represent the stall *inception* period (see sections 2.1 and 2.4.1).

2.4 DISCUSSION

2.4.1 Experimental Measurements of Stall/Surge

Stall inception measurements have been made for almost as long as the jet engine has been in existence. Early measurements characterised the nature of rotating stall and surge. Different types of stall/surge events were recorded ('part-span stall', 'full-span stall', 'deep surge', 'classic surge'). Correlations were produced to determine which type of behaviour a given compressor would exhibit; part or full span stall, the size of any hysteresis loop, the overall compressor performance in stall (Day, Greitzer and Cumpsty [1978]), and the speed of rotating stall propagation (Cumpsty and Greitzer [1982]).

More recently, use has been made of modern digital data reduction capability to make very detailed measurements of the structure of rotating stall cells, concentrating mainly on 'stable' rotating stall (Das and Jiang [1984], Gianassis et al [1988], Mathioudakis and Breugelmans [1984], Breugelmans and Mathioudakis [1984], Borys and Moffat [1996]). There seems to be general agreement amongst the authors in a number of areas. Stall cells extend approximately axially along the compressor. Although characterised by low axial flow they are not 'dead flow' areas such as wakes behind bluff bodies. Rather, tangential velocities in particular can be as high or higher than in the mainstream. This is particularly true in gaps ahead of rotors, although there is open debate as to whether or not this implies that fluid is passing through the stall cell from the 'clean flow' area. The flow in stall cells is observed to be highly three-dimensional, typically with fluid being centrifuged out in the rotors and with reverse flow at rotor tip.

Another group of experiments has been performed with the intention of validating models of stall/surge such as those described in section 2.2. Day [1994a] for instance, confirms the role of the 'Greitzer B' parameter in determining whether the final operating condition of a stall/surge event will be rotating stall or surge. Many of the modellers themselves supply data for validating their own codes (as described in section 2.2). Generally speaking, good agreement is found in the overall transient. That is, once rotating stall is established the movement of the compressor operating point in time can be reasonably predicted using, for instance, Moore and Greitzer's [1986] model. The differentiation between events leading to stable rotating stall or multiple surge is well captured. The time period of multiple surge events in particular can be accurately calculated as the filling time of the compressor exit plenum. The

model presented in chapter 4 is validated by the measurements reported in chapter 3, and demonstrates the capability of such a model.

Although the 'overall transient' as predicted by stall modellers has been borne out by experiment, the sequence of events predicted for the stall inception period have not. Linear theory (such as described by Longley [1994]) predicts that non-axisymmetric disturbances should be unstable at higher compressor flows than axisymmetric disturbances, and this has been demonstrated for low and high speed compressors (Day [1994a], Schlamann et al [1985] and the measurements described in chapter 3). It also suggests a 'modal' type of stall inception. The modes correspond to fourier harmonics in the circumferential direction. These modes are predicted to behave independently, with the surge point defined by the neutral stability point of the most unstable mode, usually a low order harmonic. This possibility has been the subject of a number of experiments.

McDougall et al [1990] were the first to observe low amplitude modal waves that grew smoothly and directly into fully developed stall (of course linear theory, limited to small amplitude disturbances, cannot be used to calculate the disturbance growth once it has become large). Other experimenters have also found such stall precursors (Garnier [1989], Hoving [1993] among others). The 'modal' view of stall inception has been strongly challenged, however, particularly by Day [1993a], who demonstrated a 'finite cell' stall inception pattern, where the initial stall disturbance occurred over a circumferentially limited portion of the annulus, often a small number of blade passages wide, and grew circumferentially with no discernible period of 'modal' behaviour. He also used data collected on the same rig as used by McDougall to demonstrate that the pre-stall waves observed by McDougall et al were only sometimes influential in determining the stall inception behaviour. Other experimenters have also found 'finite cell' stall inception with no modal behaviour beforehand (Riess and Walbaum [1996], Escuret and Garnier [1995] and the measurements described in chapter 3). Camp [1995] provides results from a low speed four stage rig showing that either a 'modal' or a 'finite cell' type of inception can be obtained depending on the stage matching. In particular, a finite cell pattern was obtained whenever a critical incidence onto the first rotor was exceeded.

The experiments show that a linear pattern of stall inception with dissociated harmonics is at best only sometimes applicable. Most of the later models, however, are non-linear in nature (for instance Moore and Greitzer [1986]). There are two results of the experiments that limit the applicability of such models. The first is that the initial stall disturbance is sometimes very sharp circumferentially, which violates the

'pitchwise average' assumptions of some of the models. This is discussed in detail in section 2.2. The second result is that initial stall disturbances are not in general radially uniform, but tend to be initiated at hub or tip. McDougall et al [1990], for instance, found that the endwall blockage was critical in determining the radial position of stall inception (hub or tip), and Day [1993a] reports initial stall disturbances growing radially inward from the tip region. This feature is consistent with the commonly observed fact that the size of the tip gap is critical in determining the compressor surge margin. A model which takes account of such an effect would have to be three dimensional and also viscous. The model described in chapter 5 includes a basic model of the endwall boundary layer in order to account for this effect.

Apart from the two types of stall inception described above, there are other areas of conflict amongst stall inception results from various experimenters. Camp [1995] shows cells generated at the front of the compressor. Escuret and Garnier [1995], on the other hand, show that they can be generated at the back of the compressor. Riess and Walbaum [1996] show stall inception at the tip of the first rotor. As stall progresses the effect grows axially along the compressor and also radially, in such a way that at the rear of the compressor the whole annulus is affected. Giannissis et al [1988], however, observe a stable part span part length rotating stall pattern that affects only the hub region at the back of the compressor. Even allowing for some mis-interpretation of results, it has to be conceded that there is a great variety of stall inception behaviour. This variety entails considerable problems for both stall modelling and active control of stall, as discussed in sections 2.2 and 2.3.

In view of the variety of stall inception behaviour observed, it is worth emphasising certain points of agreement. Rotating stall is always seen as a precursor to axisymmetric surge. The axial position of stall inception is dependent on stage matching (for modal type stall inception, this is seen rather as a change in amplitude of the pressure response to the velocity disturbance with axial position). Stall disturbances usually start in the endwall regions and grow radially.

One further point of agreement between experimenters concerns pre-stall activity. Many experiments show rotating disturbances prior to stall, particularly at low to middle speeds, often with little or no discernible pre-stall activity at high speed. Tryfonidis et al [1995] present one method of tracking such activity, demonstrating its existence in a number of compressors. Their method has been tried by a number of authors (Hoying [1993], Owen [1996] and the results described in chapter 3) with mixed success. The method was designed around modal type stall inception patterns, but will register also finite cell type disturbances, which is significant: the results

described in chapter 3 at middle speeds do show rotating disturbances prior to stall, but these are of limited circumferential extent. Thus an indication of pre-stall disturbances using this (or any other low harmonic Fourier based method) does not imply a modal type of stall inception pattern, as has sometimes been inferred.

2.4.2 Modelling Stall/Surge

The stall models described in section 2.2 are largely complementary. How complex a model is required depends which part of the stall event is of primary interest. If the interest lies in the event as a whole, that is a period of one or more surge cycles, then Greitzer [1976a,b] in particular showed that given a measured characteristic of flow against pressure ratio a relatively simple surge type model can accurately track the locus of the operating point with time. Such a model can answer questions such as whether a compressor will end up in a surge cycle or stable rotating stall, and the period of time for which a compressor operates in reverse flow (which may be important for mechanical reasons). Simple correlations such as that put forward by Day, Greitzer and Cumpsty [1978] can be added to tackle issues connected with recoverability from rotating stall. If the measured characteristics are not available, or if greater accuracy is required, a more fundamental approach would be to use a θ -type model such as or with an axisymmetric pressure rise characteristic generated from correlations. This has the advantage that the circumferential extent of any stall cells is calculated directly. Whichever type of geometric representation is used a non-linear element is essential to take account of the characteristic shape. Many simplifications can be made, however. Moore [1984a,b,c] found that bladerow unsteady response time lags can be neglected when looking at the event as a whole. Likewise compressibility and the reaction of individual bladerows are not greatly significant: Greitzer [1976a,b], Moore and Greitzer [1986] and the model developed in chapter 4 all assume incompressible flow with the compressor performance derived from a single pressure rise characteristic.

If the main area of interest lies in the accurate prediction of the stability boundary, then the models considered above would be entirely inappropriate. It is no longer sufficient to model the compressor by a single characteristic, although this may be sufficient for compressors operating at a well matched condition, where each stage is operating at a similar point on its pressure rise characteristic. Under these conditions Dunham's peak inlet total to exit static pressure rise criterion has been shown to be accurate in a number of cases.

In the case of a long multistage compressor much of the speed range involves operation at a mismatched condition, where either the front stages (at low speed) or the back stages (at very high speed) operate at lower flow points on their individual characteristics. In these cases 'unstable' stages can be stabilised by 'stable' ones. There is here a requirement of compressibility in a stall prediction model. The ability of a 'stable' stage to stabilise an 'unstable' one must depend on the period of time required for a signal to pass from one to the other and back: if this period is long, then a flow disturbance at the unstable stage may have grown to an extent where it cannot be stabilised before the 'stable' stage has had time to respond.

Stability boundary prediction models are frequently linear. This is attractive mathematically, and is warranted by the fact that stability boundary predictions are by definition related to the growth of small disturbances. Some caution is required, however, as there are large random disturbances in all practical compressors. Camp [1995] reports a typical level of turbulent velocity fluctuation of 5% of mainstream flow. Small scale turbulent fluctuations will have little effect on the stability of the compressor, of course, but large scale ones might well, and a linearly stable compressor may well be unstable in the presence of such random disturbances. The size of the effect will clearly be dependent on the non-linearity of the compressor (or stage) characteristic. If the slope remains roughly constant over a large flow range (which tends to be true at low speeds), a linear prediction will be sufficient. If the characteristic turns over very sharply (as can be the case at high speed), then some account of these non-linear effects should be taken. One approach would be to impose random disturbances on a time accurate model, in a manner similar to that described for the model in chapter 4.

A model aimed at numerical testing of active control of stall/surge falls somewhere between these two types of model. A modern high-speed actuation device should be capable of responding to a stimulus in a few milliseconds. Depending on the positioning of the sensor/actuator system a similar time will be required for detection/processing, and again for the actuator response signal to reach the affected area of the compressor. In this time period (say ten milliseconds), stall disturbances in a high speed compressor can grow significantly; this period of time represents most of the initial stall transient of the VIPER engine compressor at high speed (chapter 3). Thus the simplification of a linear model as often used for stability boundary prediction cannot be used. At the same time, it is important to accurately predict how a disturbance would grow in that time period. Hence the detailed effects required for

accurate stability boundary prediction, such as compressibility and unsteady bladerow response, are still required here.

One further complication in modelling active control of stall/surge is that the radial dimension can become important, for two reasons. Firstly, the period of time being modelled covers the very start of the stall event. Often this represents the three dimensional growth of an initial disturbance near the annulus wall of the compressor. The precise nature of this disturbance is significant. Different types or amplitudes of actuation would be required for different types of disturbance. For instance, fast guide-vane adjustment affects the flow across the entire span; this may be unnecessary and detrimental if the initial stall disturbance is confined to the tip region. Likewise the growth rate of a tip stall could be markedly different from a full-span stall. The second reason that the radial dimension is important is that in practical compressors sensors and actuators are often most easily installed in the compressor casing. The inclusion of a radial dimension in the model thus allows the control system to be more accurately represented.

Two models are developed in chapters 4 and 5 of this thesis. The first is a θ -type model which represents the compressor performance by a single axisymmetric pressure rise characteristic. In accordance with the above discussion, this is most suitable for mapping the overall stall transient, and in this respect it will be seen in chapter 4 that the model proved very successful. The precise details of the initial stall transient were not captured so well, however: the 'benign' front end stall of the VIPER compressor at low speed (section 3.4.2.1), for instance, could not be modelled at all using a single pressure rise characteristic. This model is insufficient, therefore, for an accurate representation of active control schemes. Nonetheless, useful preliminary calculations were performed to test a puffer jet control scheme similar to Day [1993b], described in section 4.4. A control algorithm was deduced which successfully controlled instabilities in the model. Although the model is not sufficiently accurate to expect this algorithm to carry over to a real compressor without modification, it gives an indication as to the type of control algorithm required, and is a useful prototype for the methodology by which it was deduced.

The second model developed in this thesis, described in chapter 5, is an $r\theta z$ -type model designed to match the requirements laid out above for active control of stall/surge. It includes the effects of compressibility, unsteady bladerow response and attenuation of disturbances in the gaps. The annulus boundary layer is modelled in a simple way to capture to first order the balance between viscous and pressure forces that is often thought to control the initiation of stall/surge.

2.4.3 Active Control of Stall/Surge

To date relatively few experiments have been performed in the field of active control. Two types of control system have been attempted, termed here 'stall avoidance', and 'stall control'.

The objective of stall avoidance systems is to detect the onset of a stall/surge event, and to change the geometry of the compressor to one which is stable under the given operating conditions. If substantial periods of low amplitude disturbances exist prior to stall, these can be used to trigger the geometry changes without a dramatic change in compressor performance. Ludwig and Nenni [1978] and Riess and Bloecker [1987], described above, both present stall avoidance systems on high speed machines.

Stall control differs from stall avoidance in that the objective is to extend the stable operating range of the compression system, rather than to move away from unstable areas. The concept was first elucidated by Epstein et al [1986], starting with the work of Moore [1984a,b,c], and showing how a circumferentially distributed actuation system could be used to change the dynamic behaviour of a compression system to stabilise it with respect to rotating stall disturbances. In the same paper, the authors considered the complementary concept of controlling axisymmetric surge-type disturbances, this time using a single actuation device.

In centrifugal compressors, surge-type disturbances are often of more significance than non-axisymmetric ones. Ffowcs Williams and Huang [1989] applied the concept of Epstein et al to a small high speed centrifugal compressor, successfully demonstrating stable operation beyond the normal surge limit.

In axial flow compressors, rotating stall generally precedes surge (section 2.1, 3.4.2), and control of axisymmetric disturbances is ineffective without first controlling rotating stall. There are three components to any active control system: detection, computation, and actuation. Control of non-axisymmetric disturbances is more difficult than axisymmetric control, generally requiring higher response rates in all three components and circumferentially distributed detection and actuation systems (but see on Eveker et al [1995, described in section 2.3]).

Central to the issue of stall control is the type of stall inception displayed by the compressor. In particular two types of compressor stall are discussed in sections 2.1 and 2.4.1. The first is 'modal' behaviour, where the first sign of instability is the growth of low circumferential harmonics rotating around the annulus. Because the low harmonics are generally less stable than the higher harmonics, control of this type of disturbance requires relatively slow actuation devices (response rate of the order of

the compressor rotation rate). Furthermore the dynamics often remain linear for a substantial period of time, with the disturbance amplitude growing slowly, and can be controlled using a linear control algorithm. Paduano et al [1993] demonstrated stall control on a low speed single stage compressor with this type of behaviour, showing a considerable improvement in the stable operating range (11% of flow on controlling the first harmonic, 20% on controlling the first and second harmonics). Haynes et al [1994] continued the work on a three stage compressor, showing a smaller but still significant improvement.

The second type of stall inception generally occurs over a short circumferential lengthscale. If this short lengthscale is to be matched by the actuation device a considerably higher response rate (several times the compressor rotation rate) is required. In addition, short circumferential lengthscale disturbances are often three dimensional, and grow in a non-linear fashion (that is, with strongly coupled circumferential harmonics). Consequently this type of disturbance is harder to control effectively. Day [1993b] demonstrated active stall control in a compressor showing both modal and short circumferential lengthscale stall inception patterns. In the former case a simple first harmonic proportional feedback control is used in the same way as Paduano et al. In the latter case a non-linear control technique is employed, 'aiming' the control action at nascent stall cells. Less improvement (roughly 3% of stalling flow) was demonstrated than in the compressors with long length scale stall inception patterns.

Stall avoidance is easier than active control. Generally, the geometry changes required to move the compressor to stable operation can be performed with a single actuator (bleed or variable guide vanes). If pre-stall disturbances can be detected for a sufficient time period prior to stall (or if a degree of small amplitude rotating stall can be tolerated) the required response rate of the actuators can be greatly reduced, and the same actuators can be used as are used for steady state engine control, as demonstrated by Ludwig and Nenni. As against these benefits, stall avoidance by definition does not allow operation in regions that would normally be unstable; it is only worthwhile in allowing operation closer to the surge line than would otherwise be safe. Even here care has to be applied, in that the new geometry to which the control system moves the compression system will generally be inferior in some way (otherwise it would be the geometry adopted for steady state running). This is no problem if the issue is one of efficiency. If the issue were thrust, for instance in a vertical landing aircraft, then the reduced thrust associated with the new geometry could be catastrophic.

For the reasons explained above active control of stall is harder than stall avoidance, and to date no results have been published demonstrating this type of system on a high speed machine. The benefits are potentially large, however, as has been demonstrated at low speed. Unlike stall avoidance, active control allows operation under conditions that would otherwise be unstable. In a military application this could represent tighter aircraft turns or faster engine acceleration. In a civil engine the benefit could come from increased efficiency due to relaxing the design constraint imposed by ensuring sufficient surge margin.

If active control of stall is to be put into practice, for instance in a jet engine environment, the question of risk must be addressed. There are two elements of risk. The first concerns the mechanics of the system; is it possible to reproduce a laboratory experiment as a flightworthy control system? The second concerns the stall inception pattern itself. Numerous experiments have been performed showing an immense range of behaviour (section 2.1). Even in a single compressor large variations in stall behaviour have been observed (Day [1993a], also chapter 3). The rotating stall control systems so far demonstrated have been tuned to the particular type of disturbance shown by the compressor under study. In order to apply this technology to a jet engine, one would have to be sure that the system was capable of controlling *all* possible types of stall inception, whatever the cause (hot gas ingestion, compressor wear, inlet pressure distortion etc). To some extent this risk can be mitigated by collecting data on stall inception, both on the ground and in flight. The control system itself will change the stalling characteristics, however, and hence some form of modelling would have to form part of the validation procedure.

Control system design can be helped significantly by the use of stall modelling. Epstein et al, Ffowcs Williams and Huang, Paduano et al and Eveker et al all base their work on the modelling of Moore [1984a,b,c] and Moore and Greitzer [1986a,b], using the stall model to give an indication of the type of control algorithm required. A model can also be used to test control strategies numerically and perform 'what if' calculations. In chapter 4 a stall model is derived for a compressor in an engine environment, and a control algorithm is devised based upon it. At high speed the growth rate of the disturbance is such that accurate modelling of the initial stages of stall inception is essential. The stall inception model developed in chapter 5 suggests a route by which this requirement could be met.

3. STALL INCEPTION MEASUREMENTS

3.1 OVERVIEW

The experimental data presented in this chapter was taken from two tests of a Rolls-Royce VIPER Mk 522 engine. The first preliminary test was performed with a reduced instrumentation set. A summary of the results was published by Wilson and Freeman [1994]. A comparison of the results with low speed data was also published by Day and Freeman [1994].

The two experiments are described together in section 3.3 below. The results are presented in section 3.4 and discussed in the following section, with special reference to their implications with regard to stall modelling and active control of stall.

3.2 OBJECTIVES

The primary objective of the experiments reviewed in this chapter was to provide a detailed analysis of the stall inception behaviour of a compressor in a jet engine environment. As well as providing insight into the fluid dynamics of stall inception the analysed data was to be used to check the generality of the results of low speed experimenters (chapter 2), and also as a test case for stall inception models such as those described in chapters 4 and 5.

3.3 DESCRIPTION OF EXPERIMENT

3.3.1 Engine and Instrumentation

The vehicle used for the test was a Rolls-Royce VIPER Mk 522 engine. This is a single-shaft turbojet with an 8 stage compressor producing a pressure ratio of 5:1 at a design speed of 13800 rpm. The Mach number varies from 0.9 at rotor 1 tip to 0.4 at the compressor exit. The engine has been used for research purposes since 1990. The general arrangement of the compressor and combustor is shown in fig 3-1. The airmeter, which extended 1.2m in front of the engine, is not shown. For historical reasons the stages are numbered from zero to seven and not from one to eight.

In order to trace the growth of the rotating disturbances leading to stall/surge the engine was instrumented with 40 high response Kulite pressure transducers spread circumferentially and axially along the outer wall of the compressor. Fig 3-2 shows their positions on an 'unwrapped' compressor layout. Table 3-1 gives the circumferential position of each instrument. Those transducers with names starting 'N'

were available only for the second experiment. The positions are not exact due to the complexity of the engine geometry. Where the analysis required more exact spacings, these were derived from the phase relationship between instruments for signals rotating at shaft speed. In addition to the high response instruments, a number of Druck pressure transducers were added at various points along the machine including the outer (cold) part of the combustion chamber.

In addition to the above, two Druck transducers were used to measure the depression in the airmeter, and a further transducer was installed to measure the pressure of the combustor inbleed used to induce stall/surge. At the end of the second experiment six miniature Kulite pressure transducers were glued to the inlet guide vanes to gauge the radial extent of the measured stall cells.

3.3.2 Test Procedure

3.3.2.1 Data Acquisition Hardware and Procedures

The data from the first experiment was digitised using a 32 channel data acquisition unit ('unit A') with a fixed acquisition rate of 4kHz per channel. The data from the second experiment was digitised with the same unit and a further 48 channel device ('unit B') which had a variable data acquisition rate, initially set to the same rate as unit A. In the last part of the second experiment unit B was set to sample at 25kHz in order to broaden the measured frequency band. Each data acquisition unit was controlled by an IBM compatible PC.

Both data acquisition units contained a rolling buffer which was frozen by a keypress. Thus for each stall event the buffer could be frozen on hearing the surge. Unit A would then preserve the previous 16000 data points for each channel (equivalent to 4s at 4kHz sample frequency), and unit B the previous 40000 points (equivalent to 10s at 4kHz).

Both units had integral anti-aliasing filtering (set to 1.5kHz for the 4kHz sampling rate and 12kHz for the 25kHz sampling rate), variable gain settings for each channel, and the ability to record data from each channel either DC coupled (unfiltered) or AC coupled (highpass filtered to about 5Hz).

3.3.2.2 Data Storage

The data collected by the data acquisition units was stored temporarily on the hard disks of the two controlling PCs. From there it was downloaded to digital tape and/or magneto-optical disks. Each recorded event had four associated files; one data file and one log file from each of the two acquisition units. The log files recorded information

regarding the type of event, the instrumentation list (specifying which channel is attached to which instrument), the gain settings for each channel, and whether the channel was AC or DC coupled.

Each event is identified uniquely by a letter (R for the first test, T for the second) followed by a six digit number.

3.3.2.3 *Methods of Inducing Stall/Surge*

In an engine as opposed to a rig environment there is no easy way to change the compressor working line in such a way as to induce stall/surge across the entire speed range.

At low speeds the compressor was stalled by overfuelling at a rate much higher than is permitted in normal engine operation, causing the transient working line to cross the surge line.

In order to stall the machine at higher speeds compressed air was introduced into the combustion chamber through a number of pipes from a manifold around the outside of the engine. The manifold was fed from a large (approx 30m³) vessel via a high speed valve. This system generated an air flow rate into the combustor that was non-uniform in time, with an initial transient as illustrated (for a 100% speed stall event) by the compressor exit pressure trace shown in fig 3-3.

At middle speeds the surge line was approached gradually by moving to a higher speed, inbleeding air at too slow a rate to stall the engine, and then decelerating slowly until the compressor could no longer support the increased pressure rise caused by the inbleed.

At high speeds (over 90%) the above procedure caused prolonged high temperatures and could not be used. Instead, a large amount of air was bled into the combustion chamber, causing a relatively fast stall.

The precise speed at which stall occurred could not be controlled as accurately as in a rig test, particularly at middle speeds, where the amount of inbleed was specified by a fairly coarse control.

3.3.2.4 *Test Schedule/Log*

In each experiment sufficient runs were included to map the stalling characteristics of the engine over the full speed range, with repeat runs with the data acquisition units in AC coupled mode with high gains. In the second experiment further runs were

performed with the adhesive Kulites added, and finally with the data acquisition unit B set to a high sampling frequency (25kHz).

3.3.2.5 Basic Signal Processing Methods

Calibrations were applied to the raw data at the time of analysis. An offset was then added to DC coupled data, calculated from a calibration run performed at the start of each day with the engine stationary. This removed any day-to-day drift in transducer calibration and effectively reduced each measurement to a "gauge" level. This procedure is only partially accurate, in that transducer drift is taken out in calibrated units only, but the errors are small given the type of analysis that is being undertaken.

Three techniques were often used to process the signals prior to more detailed analysis. The first was to remove harmonics of the shaft frequency from the signal in a predictive fashion. The method adopted was similar to a feed-forward adaptive filter, but was applied in the time domain. The speed was first calculated, either from the speed signal or often from a Fourier analysis of a pressure signal using a high harmonic of the shaft frequency. The calculated speed was then used to interpolate each signal onto a 1000 point grid on a rev by rev basis. The interpolated values were filtered at each gridpoint to effectively ensemble average the once per rev signal over about 25 cycles on a rolling basis. The resulting signal was then subtracted from the original data.

The second technique was a simple normalisation of the data to similar amplitudes using a standard deviation calculated prior to stall inception. The transducers in an axial plane can show markedly different levels of behaviour (presumably due to minor variations in geometry). The most active transducers are then overweighted in subsequent calculations if the signals are not normalised.

The third technique was used to produce the maps of static pressure around the compressor casing (eg fig 3-9), reproduced from Wilson and Freeman [1994]. The data for these graphs was interpolated from the rings of Kulite measurements at inlet, stage 2, stage 3 and stage 6 and the single Kulite at compressor exit that was available in the first experiment. The interpolation technique used relies on the fact that the stall inception process is often related to flow disturbances which rotate fast around the compressor, but change only slowly in time (relative to the time taken for the disturbance to get from one measurement position to the next). In these circumstances the type of interpolation used can be significant.

Fig 3-4 demonstrates the interpolation procedure used to calculate the static pressure maps such as fig 3-9. Fig 3-4a shows the raw pressure traces from the 5 measurements at compressor inlet at 78% speed, spaced evenly on the y axis with the first one repeated at the top. Because the initial disturbance is so sharp, interpolating linearly between the traces at any one time would give a totally erroneous picture, spreading out the disturbance or even showing two peaks instead of one. However, once the rate of rotation of the disturbance is known or has been deduced, the time traces can be corrected for rotation as in fig 3-4b. This procedure allows a linear interpolation to be performed (fig 3-4c) along a direction in which the disturbance is changing only gradually. Fig 3-4d shows the raw and interpolated traces with the rotational speed restored. The static pressure maps such as fig 3-9 use data interpolated in this way at 5 degree intervals around the compressor at each of the 5 measurement planes.

It is easier to trace the development of the disturbances if they are kept stationary on the x axis for successive time steps. This was achieved by moving the origin of the x axis between plots around the annulus at the same (constant) rate of rotation as used in the interpolation procedure. A circumferential offset was then chosen for each speed to position the disturbance in the centre of the picture, although the changes in the rotational speed of a stall cell as it develops mean that over a period of time it does move off centre. Changes in the rotational speed also affect the quality of the interpolation. Once the disturbance gets circumferentially large, however, the effect of the poorer circumferential interpolation is quite small.

3.4 RESULTS

3.4.1 Overall Performance

Fig 3-5 shows the change in overall operating point (mass flow and pressure ratio) prior to stall during events T180806, T180802, and T180803 stalling at 66.7%, 80.5% and 99.7% speed respectively. No account has been taken of unsteady effects during the transients; prior to stall the movement in operating point is relatively slow and the errors will not be large. These are not constant speed characteristics: in an engine environment there is no mechanism with which to control compressor speed during a transient. An approximate stall line has been added to the figure. The stall points show reasonable agreement with the mean-line prediction shown as a background map in fig 3-31.

3.4.2 Stall/Surge in the Viper Engine

In this section the basic stalling behaviour of the VIPER engine compressor is described and related to the compressor stage characteristics. The stalling process is also demonstrated to be highly repeatable. Particular aspects of the experiments that warranted a fuller investigation are covered in later sections.

3.4.2.1 Low Speed

At low speeds (up to around 69%, or 75% if stage 4 bleed is switched off - see section 3.4.2.2) the compression system moves through various regimes of rotating stall both before and after reaching the surge line (defined conventionally as the point of irretrievable breakdown of the flow structure). The rotating stall on the stable side of the surge line is 'benign' in the sense that it does not inhibit normal operation of the engine.

Figs 3-6 and 3-7 show time traces from the Kulites in the front and rear stages respectively for a stall event at 67% speed. At the start of the time window the compressor is operating in 'benign' stall, shown later to consist of four cells rotating around the annulus. In figure 3-6 the stall cells can be seen to extend through the 'zero' stage, but the level of activity at stage 2 exit is considerably reduced, and by stage 3 it is very small.

During the stall transient the rear stages show a growing disturbance at around 57% of shaft frequency. The underlying cause is the growth of one of the benign stall cells axially to the back of the machine, and then circumferentially. The leading edge of this growing cell continues initially at around the same speed as the other cells. The trailing edge, however, lags further and further behind. As it grows the cell slows down (eventually to about 40% of rotor speed), swallowing the other three benign stall cells.

Fig 3-8 shows in more detail the period around transition from the benign stall to the single cell leading to irreversible stall. The top graph shows pressure traces from five transducers in the inlet plane. The cell that eventually grows into single cell stall has been marked, and can be seen to be bigger than the other cells in circumferential extent throughout the 80ms (18.4 rotor rev) time period shown on the graph. The 'swallowing' of one of the smaller cells can be seen on the top trace at 60ms.

The lower graph of fig 3-8 takes data from the first compressor inlet transducer along with one from the exit plane of stages 0, 2 and 3. These instruments are all around top dead centre: a small correction has been applied to the time scale to account for the

discrepancy in circumferential position, using the measured stall cell rotation speed. The graph confirms that the sharp upward spikes representing stall cells at compressor inlet are related to *downward* pressure fluctuations at stage 0 exit. This is consistent with a region of low axial momentum fluid in the first stage causing an upstream blockage effect. The stage 2 and stage 3 Kulites initially show an ill-defined response to the passing stall cells, although where it is distinct the response is in phase with the stage 0 pressure. This is also true of the later stages (see fig 3-7) where the response is better defined and relates more clearly to the cell that leads to irreversible stall. From around 39ms in the figure the growing cell causes an increase in stage 2 and 3 exit pressure, slightly lagging the high pressure region at the inlet plane. The change in sign is assumed to relate to the stalling of downstream bladerows under the influence of the passing stall cell.

Fig 3-9 shows a graphical representation of the stalling process for a similar low speed event from the first experiment using the interpolation procedure described in section 3.3.2.5. Fig 3-9a maps out the static pressure around the outer annulus wall during the overfuelling, but prior to reaching the surge line at 60% speed. The five axial planes correspond to the Kulite measurement planes at compressor inlet, above stator 2, above stator 3, and in the exit duct. The pressure at each stage is shown relative to a datum level at the start of the event. The compressor turns in a clockwise direction viewed from the front, which is from left to right in the figure. The four patches of high static pressure at the front of the compressor represent the four areas of low axial momentum fluid, that is, stall cells. Each cell has a 'tail' of low pressure reaching part way along the length of the compressor. This picture is consistent with the existence of a number of stalled blade passages, limited circumferentially to the region of each individual stall cell, and axially to a region between the measurements at compressor entry and those above stator 2 (fig 3-6 shows the axial position to be limited to just the first stage).

Fig 3-9b shows the same map 50ms (7 rotor revolutions) later. Note that the origin of the x axis has been shifted around the compressor relative to fig 3-9a, according to the procedure in section 3.3.2.5. This keeps the disturbance from moving across the page on successive plots, and allows its development to be traced more easily. One of the stall cells has disappeared almost entirely, leaving three irregularly spaced cells with the one on the right much the strongest, reaching almost axially to the back of the compressor. The next figure shows it to have deepened further 125ms (17 rotor revolutions) later. The high static pressure area in the middle of the compressor

indicates more stalled blades towards the back of the machine, left in the wake of the stall cell as it rotates around the annulus.

By fig 3-9d, just 12.5ms (1.7 rotor revolutions) later, the strongest cell has grown significantly. The leading edge of the cell still proceeds at the same rate, but the trailing edge lags further and further behind, until in this frame it is about to swallow one of the other two cells. This large cell slows to about 40% rotor speed, swallowing both of the smaller cells. Fig 3-10 shows the pressure map (on a new scale), having achieved a stable rotating stall condition 12.5ms (1.7 rotor revolutions) after fig 3-9d, with what was the original stall cell occupying over half the circumference. The pressure ratio has dropped from about 2 to 1.5; hence the low pressure at the back of the compressor.

The event depicted in figs 3-6 and 3-7 shows four cell rotating stall changing directly to single cell rotating stall as the surge line is crossed. The graphical representation in figs 3-9 and 3-10 shows a three cell intermediate pattern. Some events show only three cell rotating stall on the stable side of the surge line. There is no obvious pattern as to whether three or four benign stall cells are present prior to the growth of the one cell leading to irreversible stall, as shown by the following table. All of these stall events were precipitated by overfuelling, and all were performed around the same point in the second experiment.

NAME	No. CELLS	SPEED AT STALL	BLEED
T050805	3 => 1	67%	ON
T180805	3 => 1	51%	OFF
T180806	4 => 1	67%	OFF
T180809	3 => 1	51%	ON
T180811	4 => 3 => 1	67%	ON

More analytical work could be done using the data from this test to determine the reasons why different numbers of stall cells were observed. It may be that the speed with which the stall line is approached is important, and/or the hysteresis effect discussed in section 3.4.2.2 with respect to stall drop-out may apply equally to the type of benign stall observed.

The adhesive Kulite transducers attached to two of the IGVs give some information as to the radial extent of the stall disturbances. They suggest that the initial benign stall cells are much more significant in the tip region, and that the axial growth of the cell leading to fully developed stall is paralleled by radial growth from the tip inwards (see section 3.4.3 for details).

3.4.2.2 *Boundary Between Low and Middle Speed Behaviour*

The point at which low speed stalling behaviour (three or four cells at the front of the machine) changes to middle speed behaviour (single cell, described in the next section) is dependent on whether or not bleed is taken from stage four. With bleed off, the compressor stalls at 73% speed in a 'low speed' manner with three cells at the front of the compressor present throughout the data record prior to stall (event T010816). Event T010812 on the other hand, also recorded with bleed off, shows the compressor stalling at 78% speed in a 'middle speed' manner (in fact, with more than one stall cell, but not regularly spaced). This is consistent with the observation that during a normal engine acceleration from idle benign stall 'drops out' (that is, rotating stall ceases and the flow becomes axisymmetric) at 75% speed (section 3.4.6).

With stage four bleed on, however, stall 'drop-out' can be expected to occur at a lower speed. Stall event T220804, at 67% speed, shows a period of 'three stall cells with one missing' prior to irreversible stall. This type of operation also occurs for a very brief period just before stall drop-out, and is described in section 3.4.6. Hence the change from low to middle speed behaviour with bleed on can be taken to be just above 67%.

It should be noted that there is a hysteresis effect with respect to stall drop-out. That is, there are some operating points that can exhibit either clean flow or benign rotating stall depending on the path taken to reach them. Stall drop-out for an accelerating engine, for instance, will be at a higher speed than stall drop-in for a decelerating engine, even on the same working line. Similar effects can be expected as a compressor is throttled at near constant speed. Thus in the boundary area between low and middle speeds it is not expected that the stalling behaviour can be determined from engine speed alone.

3.4.2.3 *Middle Speed*

At speeds between 69% (or 75% with stage 4 bleed switched off - see section 3.4.2.2) and 88% it will be seen that compressor stall is initiated by a single stall cell at the front of the compressor. This cell grows rapidly to the back of the compressor and then

spreads circumferentially. There is then a sharp difference in behaviour depending on engine speed. Below 80% speed the stall cell stops growing circumferentially, leaving the compressor in single cell rotating stall at very low flow, pressure ratio and efficiency, but with the compressor operating steadily in the mean. Above 80% speed, the stall cell grows until the entire compressor is operating in reversed flow, and the engine moves into violent multiple surge, where the flow direction changes at a frequency of about 8Hz. This dichotomy of post-stall behaviour agrees with the theory developed by Moore and Greitzer [1986] described in chapter 2. It is shown below that the stall *inception* pattern is identical above and below this critical engine speed, reflecting the fact that stall inception is an event local to the compressor, whereas post-stall behaviour is intrinsically related to the surrounding components. During the second series of tests the speed at which the post-stall behaviour changed was observed to be slightly higher at around 81% speed.

The change in post-stall behaviour is illustrated in fig 3-11, where pressure traces are shown from Kulites at compressor inlet and exit for two events recorded during the first experiment. Fig 3-11a shows a stall event from the second test series at 80.6% speed, that is just below the critical speed at which the behaviour changes. The engine has been decelerated slowly from 90% speed with air injected into the combustion chamber at a constant pressure of 500kPa. During the stall event the compressor exit pressure is seen to drop dramatically and remain at the new lower level. Fig 3-11b shows a similar event with a slightly higher level of air injection (700 kPa), causing the compressor operating point to cross the surge line at 81.2% speed. This time the overall pressure ratio drops nearly to unity, before climbing back to almost its original level. This process is repeated indefinitely, the gradual drop in peak pressure being related to a slow run-down in engine speed.

Figs 3-12 and 3-13 show time traces from the full set of Kulite pressure transducers during event T180804 in the second experiment, a stall event at 79% speed which results in stable rotating stall. For comparison, figs 3-14 and 3-15 show the same data for a stall event at 81% speed which results in multiple surge. These figures confirm that the stall inception pattern is very similar at the two speeds. Note that the high level of noise at compressor exit (figs 3-13, 3-15, and also fig 3-24) stems from the fact that these are measurements of total pressure as opposed to casing static pressure. A more detailed view of the traces from the compressor inlet transducers is shown in fig 3-16, and from the stage three transducers in fig 3-17.

The disturbance leading to stall can be recognised first from the compressor inlet time traces in fig 3-12 (first graph), indicating that the initial stalled region lies towards the

front of the machine. This is again supported by the fact that the stage 2 exit pressure perturbations are reversed sign with respect to the measurements at inlet. At around 21 rotor revolutions the measurements at stage 2 exit swap sign to be brought into alignment with those at compressor inlet. This point presumably marks a balance between the upstream and downstream blockage effects, as successive stages along the machine fall into stall. The nature of the growing stall cell can be seen more clearly from the static pressure maps presented below.

Close examination of the inlet Kulite time traces reveal disturbances of similar circumferential extent travelling at near stall cell speed between occurrences of the main cell. These additional disturbances travel in general slightly faster than the main cell and can sometimes be seen to be absorbed into the trailing edge of the main cell. The amplitude of these disturbances remains low, and they appear to play little part in the overall behaviour of the compressor.

Bands of rotating stall can be seen growing and decaying prior to the time period shown in the figures. This is discussed further in section 3.4.5.

Figs 3-18 and 3-19 show static pressure maps at various times during a stall event from the first experiment at 78% speed. The first sign of any disturbance (fig 3-18a) is an area of high static pressure (and hence low flow) at the front of the compressor, with a low static pressure 'tail'. This is consistent with the stage characteristics (section 3.4.2.6), which show the front stages of the machine to be furthest towards stall at this speed. The stall cell is rotating at roughly 56% of rotor speed. As in the low speed case, the cell grows almost axially to the back of the machine, only this time more quickly (figs 3-18b,c,d, at 5ms intervals, or roughly 1 rotor revolution). Having grown axially, it proceeds to grow circumferentially, as shown in fig 3-19. Fig 3-19a is the same as fig 3-18d, but with a new pressure scale. Eventually it occupies half the circumference and settles into stable rotating stall (figs 3-19c,d).

Figs 3-20 and 3-21 show static pressure maps for an event at 82% speed. The initial stall cell rotational speed is slightly higher than the 78% speed event, scaling with rotor speed. Figs 3-20 and 3-21 are taken at the same time intervals as figs 3-18 and 3-19, and are notable in that they show an almost identical pattern of stall cell development, even though the subsequent history will be seen to be entirely different. In this case the stall cell continues to grow circumferentially until the flow becomes axisymmetric once more (fig 3-21c). Having become axisymmetric, the flow returns to almost the same flow and pressure ratio as before the event, before repeating the cycle. Fig 3-21d

shows the map 85ms (16 rotor revolutions) into the event, where the pressure at the back is almost back to the original level.

3.4.2.4 Boundary Between Middle and High Speed Behaviour

The boundary between middle and high speed behaviour lies at approximately 88% speed, and is remarkable in that it is the only area where stall inception appears to occur on a long circumferential lengthscale. Fig 3-22 shows pressure traces from the inlet Kulites during stall event T290705 at 88.5% speed. There is a long period (at least 30ms) prior to stall where the pressures move up and down in a first order modal pattern (compare this figure with fig 3-16, which shows an event at 79.4% speed where stall inception occurs on a short circumferential lengthscale).

This long circumferential lengthscale behaviour only occurs over a narrow speed range (a few percentage points). The precise range cannot be deduced from the data available.

3.4.2.5 High Speed

Figs 3-23 and 3-24 show time traces from the Kulites in the front and rear stages respectively for a stall event at 100% speed. The first sign of a disturbance is the pressure rise on the first trace at stage 4. Unlike the behaviour at lower speeds, this disturbance does not immediately rotate, but stays fixed in the absolute frame of reference for four rotor revolutions. A more detailed view of the traces from the stage 4 Kulites is shown in fig 3-25. Smaller rises can be seen on the second and fifth traces at stage 4. This is consistent with a downstream blockage, presumably initiated by some geometrical non-uniformity, covering about 1/6th of the annulus near top dead centre. There is no known source of non-uniformity which would give this effect. Similar, but again smaller, rises can be seen on the second trace at stage 5, the second trace (especially) at stage 3, and the second trace at stage 2. These traces all relate to transducers around top dead centre (table 3-1). It is unclear, however, why the second trace on the second stage (K22) shows a larger effect than the first trace (K21), which is more in line with the other instruments downstream.

The movement of the point of stall inception from inlet at lower speeds to somewhere downstream of stage 4 at high speed is consistent with the way the compressor stage matching changes with speed (section 3.4.2.6).

After about 4 rotor revolutions the disturbance starts to rotate at around 47% of rotor speed. Assuming the fixed disturbance is the result of geometric non-uniformity, the stalling behaviour might be expected to be similar to the case of inlet distortion, where

a given level of distortion can give rise to instability in the form of rotating stall. The initial rotating disturbance can be seen first at stages 3 and 4, circumferentially coincident with the fixed disturbance. It can be seen from figs 3-23 and 3-24, however, that it spreads very quickly to the front and back of the compressor.

During high speed surge events the flame in the combustor was extinguished. Modelling by Freeman and Wilson [1993] suggests that this was due to rich extinction brought about by the reversed flow. Both the reversed flow and unburned fuel during the surge cycle could be seen dramatically during the test as flames appeared at the front and the back of the engine simultaneously. This feature may not mirror a 'natural' surge, however, as high pressure air was being pumped into the combustion chamber to raise the working line to the surge point.

Fig 3-26 shows static pressure maps at intervals of 2ms (0.45 rotor revolutions) during a surge event at 98% speed from the first experiment, starting at the point at which the disturbance starts to rotate. Fig 3-27 shows the subsequent development of the flow disturbance on a smaller pressure scale. The start of the rotating disturbance is indicated by the high pressure around stage 3 in fig 3-26a. The following graphs show the stall cell growing extremely rapidly, first in the axial direction and then circumferentially until the whole annulus is affected and the flow becomes more and more axisymmetric (fig 3-27d). The pressure ratio drops away rapidly to near unity.

The compressor exit pressure in the graphs in fig 3-26 shows a low level at the leading edge of the stall cell, and a high level at the trailing edge. This feature was also present, though less clearly, in the lower speed data. This is an unsteady effect, caused by the deceleration and subsequent reacceleration of the fluid downstream of the exit plane as the stall cell passes (an equivalent explanation can be obtained by considering the flow redistribution downstream of the exit plane in a frame of reference rotating with the stall cell, such that the stall disturbance is approximately steady). At the inlet the same effect is present in reverse; relative to the downstream measurements the disturbance appears to be shifted in the direction of rotation. At the leading edge, where the upstream fluid is being decelerated, the pressure is increased, and at the trailing edge, where the upstream fluid is being reaccelerated, the pressure is reduced. The same features can be observed in the measurements of stable rotating stall in a low speed compressor presented by Das and Jiang [1984].

3.4.2.6 Stage Characteristics

With casing static pressure transducers at inlet and exit to most stages it is possible to estimate the stage characteristics. Approximate stage characteristics are shown in figs

3-28 and 3-29. Because there were no measurements at stage 1 exit, stages 1 and 2 are shown combined (hence the large value of work input in that graph). The characteristics are based on pre-stall measurements during three transient events at low, middle, and high speeds. They are only approximate for three reasons;

- Transient changes affect both the measured flow and the measured pressure rise (in effect, the stage has to work to accelerate/decelerate the flow in addition to the measured static pressure rise). This effect has been minimised by using only pre-stall data, whilst the operating point is moving only slowly.
- The static pressures are measured using high response Kulite pressure transducers, which are not designed to give very accurate DC-coupled measurements.
- There are no angle or temperature measurements: the characteristics were derived from the static pressure measurements (or total pressure measurements at compressor exit) using a simple approximation of fixed deviation and efficiency.

In view of these limitations the stage characteristics shown in figs 3-28 and 3-29 should not be used quantitatively. Nonetheless they can be expected to represent qualitatively the effects of stage matching with engine speed.

At low speed the compressor operates in front end 'benign' rotating stall, which affects the mean static pressure as measured at the casing in the front stages. At the particular low speed operating condition used for the stage characteristics the main effect was at stage 0 exit, where the low static pressures recorded as the stall cells passed produced an artificially low stage work input at stage 0, and an artificially high one for stages 1 and 2. For this reason a correction was added to bring the stage 0 exit pressure into line with the median, as opposed to the mean, recorded pressure at that axial station (effectively discounting the data recorded in the stall cells). Even so, the presence of rotating stall means that the exact levels of work input in the front stages at low speed should be treated with some caution.

A second feature of low speed operation is the presence of handling bleed at stage 4. The main effect, that of changing the flow along the machine, is accounted for in the calculation of the stage characteristics. There is, however, a second effect, due to the fact that the stage 4 pressures were measured in the axial plane of the bleed off-take (in fact, each instrument head at that plane occupied one of the mid-stage bleed holes). At low speed, therefore, when handling bleed is taken, the stage four measurements are bound to be affected. No correction has been made for this effect. It is noted, however, that with bleed switched on a low reading might be expected at stage 4 exit,

and this would have the effect of depressing the measured work input of stage 4 and increasing that of stage 5, which is precisely what is seen on the stage characteristics in figs 3-28 and 3-29.

Taking into account the remarks above the change in stage matching with speed is entirely as expected. At low speeds the front stages are operating in a normally unstable region, stabilised by the effect of the rear stages. As the speed increases the stages become better matched, until at high speeds the rear stages are close to the peak of their characteristics, with the front stages operating some way down on the stable (high flow) side.

The change in stage matching described above explains the move in stall inception point from the front of the compressor at low speeds to the rear at high speeds, in that the disturbances start in the region of the compressor where the stages are operating at the most stalled position relative to their characteristics. It is also noteworthy that the long lengthscale behaviour seen at around 88% speed (section 3.4.2.4) is around the point where the front and back are operating close to the peak of their pressure rise characteristics. This is consistent with the theory put forward by Dr I J Day and T R Camp of the Whittle Laboratory that long circumferential lengthscale behaviour tends to occur when a compressor is well matched axially (Camp, 1995).

3.4.2.7 Repeatability

Stall/surge inception at any given speed is a deterministic event: the pattern of growth of the stall disturbance did not vary through the test. Figure 3-30 shows stage 3 pressure traces from three events recorded at roughly 82% engine speed some days apart. The stall cell growth is very similar between the three, even down to some quite minor variations in pressure. The best fit was obtained by overlaying Kulite 1 data from the second event on top of Kulite 4 data from the first and third events. This shows that the *circumferential* position of stall inception is not always the same, although other measurements such as the high speed case described above did show a preference for certain positions.

3.4.2.8 Summary of Stall/Surge in the VIPER Engine

Fig 3-31 summarises the different regions of stall behaviour in the VIPER engine compressor. At low speed (up to 69% or 75% with stage four bleed on) the compressor operates in 'benign' stall on the stable side of the surge line. This consists of three or four stall cells rotating at around 57% of rotor speed, limited axially to the front two stages, and considerably biased towards the outer part of the annulus. As the

surge line is crossed one of the cells grows axially to the back of the compressor, and then circumferentially. As it grows it slow down, swallowing the other cells. The final operating condition is fully developed rotating stall.

At middle speeds (up to around 88%) stall is initiated by a single cell at the front of the machine rotating at around 56% of rotor speed. This grows axially to the back of the compressor and then circumferentially during approximately 8 rotor revolutions before causing a significant drop in compressor exit pressure. Below 80% speed the final operating condition is fully developed rotating stall. Above this speed the final condition is multiple surge.

At high speeds, stall inception occurs extremely fast (about 10ms or 2.25 rotor revs from the initial rotating disturbance to a significant drop in combustor pressure). The point of inception moves towards the back of the machine, in line with the movement of the operating point towards stall in the rear stages (section 3.4.2.6). Unlike events at lower speeds, the initial disturbance grows for a period of time (around 4 rotor revolutions) at a fixed circumferential and axial location before starting to rotate. The speed of rotation of the stall cell is now slightly lower (47%) relative to the rotor speed. The cell is broader at the start than at lower speeds, though still only affecting a proportion of the annulus (about 120 degrees or 9.7 rotor 0 pitches). It grows rapidly, first in the axial direction, and then circumferentially, until the entire annulus is operating with axisymmetric reversed flow. The engine then flames out during the surge cycle, and the flow and pressure ratio never recover.

3.4.3 Radial Extent of Disturbances

The adhesive Kulites on the IGV vanes were used to measure the radial extent of stall disturbances, and in particular to distinguish between part span stall, which affects primarily the top few percent of span, and full span stall, where major velocity fluctuations are present across the whole span.

3.4.3.1 Three Cell Benign Rotating Stall at Low Speed

The benign rotating stall at low speed on the stable side of the surge line has already been shown to be part-length, in that it affects only the front of the compressor. It was previously assumed that it was also part-span, affecting only the tip section of the annulus. In this section the static pressure measurements from the adhesive Kulites on the Inlet Guide Vane (IGV) are combined with an incompressible irrotational duct flow solution of the inlet flow to show that this is not the case: the benign stall does affect the whole of the span, although biased toward the tip region.

The static pressure measurements from the adhesive Kulites are shown in figs 3-32 and 3-33. The latter graph shows the variation in pressure fluctuation with radius. The pressure disturbance stems both from blockage in the stalled region and from unsteady effects. The amplitude of the variation at the tip is almost twice as big as the dynamic head, and yet there is no source of loss upstream. Hence Bernoulli's equation is patently insufficient, and the shape of the axial velocity disturbance cannot be directly inferred from the pressure signals. In the inlet duct, however, the flow can be assumed irrotational and incompressible (at least at low speed as here). Under these circumstances it is theoretically possible to derive the entire flow from a knowledge of the static pressure field in the plane of the IGV. In practice, two further assumptions are required; that the inlet duct is approximately parallel annulus for a reasonable distance upstream, and that the measured static pressure profile (at mid-chord) approximates to that immediately upstream of the IGV.

Only the fluctuating component of the static pressures was recorded at IGV mid-chord, and not the DC level. Hence an alternative method was required to fix the absolute level (of pressure, or equivalently, of velocity) up the span. To this end the assumption was made that the maximum axial velocity around the annulus was the same at each height, with the overall level fixed by the annulus average mass flow. This approximation is justified below, but it is important first to note that the analysis as a whole is directed at the AC component of the disturbance: does the rotating stall extend to the hub? A small (or even large) error in the assumed radial profile of maximum axial velocity is unlikely to greatly affect the results.

The justification for the above approximation stems from two observations. Firstly, the mathematical analysis below shows that for a simple third circumferential harmonic disturbance the axial velocity reaches a maximum (or minimum) at the same circumferential positions at each radial height. The pressure measurements (fig 3-32), which are in phase radially, support this conclusion in practice. Secondly, the flow at any of the circumferential positions where the velocity has a turning point must satisfy (to a first approximation) the same conditions as axisymmetric flow, because at these positions the time and theta derivatives of the flow are zero, and therefore contribute nothing to the overall pressure balance. Thus the radial profile of axial velocity at these positions is equivalent to that in axisymmetric flow, which is fairly constant.

It is convenient to collect together all of the assumptions required for the calculation;

1. The pressure field as measured at part IGV-Chord mirrors that at IGV inlet.
2. The inlet flow is irrotational and incompressible.

3. The inlet duct is approximately parallel annulus.
4. The maximum axial velocity around the annulus is the same at each radial height.
5. The disturbances consist only of zeroth and third harmonics circumferentially. This is discussed later.

Dunham [1965] gives a general solution for incompressible ductflow including vorticity effects. What is required here is simpler, in that the inlet flow is also irrotational. Thus the flow can be calculated as a velocity potential which satisfies Laplace's equation. The solution takes the form of Fourier harmonics in the circumferential direction and Bessel harmonics in the radial direction;

$$\phi = \sum_{nk} (C_{nk}^J B_n^J(\mu_{nk} r) + C_{nk}^Y B_n^Y(\mu_{nk} r)) e^{in\theta - \mu_{nk} z},$$

where B_n^J and B_n^Y are Bessel-J and Bessel-Y functions respectively, of order n . n is also the order of the Fourier harmonic in the circumferential direction. μ_{nk} is a constant allowing the radial velocity (hence the radial derivative of the velocity potential) to be zero on the inner and outer walls; for each circumferential harmonic n , the set of μ_{nk} , with their respective Bessel functions, form an eigen-system for the radial profile. C_{nk}^J and C_{nk}^Y are constants which can be derived in the usual manner from any component of velocity using the orthogonality of the Bessel functions.

In order to apply the above equation directly, it is necessary to start with a known velocity component at the measurement plane (arbitrarily defined as $z=0$). In this case, however, only the static pressure was measured, and so the following procedure was adopted:

The variation in amplitude of static pressure fluctuations shown in fig 3-33 was approximated by a straight line to make the computations easier. The static pressure fluctuations (fig 3-32) are approximately third order circumferential harmonics. The velocity potential was thus assumed to consist only of third and zeroth circumferential harmonics ($n=3$ and $n=0$). A first guess at the profile of velocity perturbations was the constructed using Bernoulli's equation and ignoring circumferential velocities. As mentioned previously the pressure fluctuations are roughly double the dynamic head. For this reason they were scaled down (just to give an initial guess) such that the minimum axial velocity was exactly zero, on the grounds that a high negative velocity was unlikely.

The first guess of the radial profile of axial velocity described above was used in conjunction with the velocity potential equation (using 61 J and Y Bessel harmonics for each of $n=0$ and $n=3$) to generate the other velocity components at the

measurement plane and hence the static pressure profile. It had been intended to use this result to adjust the axial velocity profile, iterating until the calculated and measured pressure profiles coincided. In the event, no iteration was required (fig 3-34). Note that the unsteady effects do indeed account for around half of the amplitude of the static pressure fluctuations.

The axial velocity profile used to match the measured static pressure profile is shown in fig 3-35. It shows the velocity perturbations to be significant right down to the hub, although larger at the tip. Inasmuch as the static pressures line up, and given the assumptions stated earlier, it is the *only* solution for axial velocity, and so the benign rotating stall on the stable side of the surge line has been shown to be 'full span' in nature (though weighted toward the tip) rather than 'part span', as had earlier been assumed.

Although the calculated velocity fluctuations did generate the correct static pressure profile, it is important to see whether an equally good match might not have been obtained from a sharper velocity profile more reminiscent of part-span stall. To this end, a second 'sharp' velocity profile (also shown in fig 3-35) was introduced to the velocity potential equation. The resulting static pressure profile is shown in fig 3-34, compared to the measured profile. The fluctuations in static pressure are mainly confined to the tip; that is they do not spread significantly beyond the area where the velocity is fluctuating. The sharp peak in static pressure is a function of the sharpness of the discontinuity of the axial velocity profile (a slightly smoother profile would eradicate this peak), and therefore not relevant to the discussion. The mismatch between the calculated and measured static pressure profiles in this case, however, confirms the result that the VIPER engine exhibits full span part length rotating stall at this operating condition.

3.4.3.2 Other Stall Disturbances

The analysis in the previous section showed that in three cell benign stall at low speed, major velocity fluctuations occur across the whole span, though biased toward the outer wall. Fig 3-36 shows the amplitude of the static pressure disturbances as a function of radial height for this case, compared with operation at a number of other conditions. The profiles obtained for the four cell rotating stall pattern (further from stall) at low speed are qualitatively similar to those for the three cell pattern. The profile for the single cell fully developed rotating stall, on the unstable side of the surge line, is more nearly constant, indicating that the bias toward the outer radii has been considerably attenuated.

At middle and high speeds, where there is no benign stall prior to reaching the surge line, the picture is as the single cell low speed data. That is, no major spanwise effect is present, either during the transient or in stable rotating stall. By way of example, the solid circles in fig 3-36 represent the radial profile of the single cell disturbance leading to stall during event T121206 at 80% engine speed. In the case of the high speed events (above 88% speed), it should be remembered that the stall disturbance starts part way along the machine, and so the very start of the transient cannot be traced accurately using Kulites on the IGV.

3.4.4 Fast Data Acquisition Results

The last events in the second experiment were recorded with the high response Kulite pressure transducers coupled to the 48 channel data acquisition unit running at a sampling frequency of 25kHz/channel, with an anti-aliasing cut-off frequency of 12.5kHz. This compares with a blade passing frequency of 6670Hz at 100% speed. The sampling frequency was thus sufficient to resolve blade passing, but insufficient to record accurately the blade-to-blade variations in static pressure.

Within the limits described above, the blade passing signal was unchanged as stall was approached. Fig 3-37 shows the ensemble averaged once/rev static pressure signal away from stall and close to stall at 83% speed.

The most striking result of the tests with the high acquisition rate is the existence of high frequency signals which grow prior to stall. Fig 3-38 shows the average power in the 0-4kHz band as measured by the Kulite pressure transducers at inlet to the compressor during event T121210, an 'inbleed and decelerate' stall event (described in section 3.3.2.3) where the surge line was approached slowly. Time runs from top to bottom along the y axis, and the stall can be seen as the horizontal line just above 0.1s. Shaft orders have been removed using the process described in section 3.3.2.5. The spikes at the top of the graph represent shaft orders at the beginning of the event before the removal procedure has sufficient data to operate effectively. There is a clear band of activity in the 1000-2800Hz region that grows in amplitude right up to the stall point.

Much of the broadband noise shown in fig 3-38 is small amplitude, and incoherent between transducers. It has proved possible, however, to deduce that it consists of a series of disturbances that rotate around the annulus at approximately 126Hz. Fig 3-39 shows the cross-correlation between inlet Kulites 2 & 3 based on a 400 point window. The small peaks prior to stall correspond to multiples of the rotor 0 blade to blade passing time and are not significant. There is a significant peak however at a delay of

around 31 time units. Having taken account of the angle between the transducers this translates into a rotational speed of 126Hz (66% of shaft speed, which compares with the stall cell rotational frequency of 55% of shaft speed). Similar peaks are present away from stall and moving into stall, although the stronger correlation is that obtained moving into stall. Fig 3-40 shows the phase lag derived from a 1024 point cross-spectrum between Kulites 2 & 3 moving into stall during the same event. Points of strong amplitude (rather than coherence level, which could not be obtained because of the necessarily short time-frame) are marked with a star. A line has been overlaid corresponding to a constant delay of 198 time units (equivalent to a disturbance rotation frequency of 126Hz). This confirms that the 126Hz rotation speed applies right across the 1500-3000Hz activity range.

The 126Hz rotation speed should be treated with some caution, as it will be seen below that the disturbance is modulated by the presence of a stall cell. That is, the fundamental frequency of rotation may be different, but the rotation of the stall cell selectively amplifies the disturbance giving an apparent rotation rate close to that of the stall cell (105Hz).

Transducers 2&3 show the strongest correlation of any pair of transducers, although similar results can be obtained from other consecutive pairs. The coherence between non-consecutive pairs is lower, suggesting that the high frequency disturbances die away rapidly (or change form rapidly) as they move around the compressor. There was also negligible coherence between transducers from different stages, suggesting that the disturbances are very localised in nature.

Although the broadband high frequency noise is generally low amplitude with low coherence between transducers, something of its structure is revealed during stall inception, because these higher frequencies are amplified within the developing stall cell. Fig 3-41 shows the inlet Kulite time traces for event T121210, normalised and with shaft orders removed, skewed at the stall cell frequency of 105Hz (that is, each successive trace is moved toward the left by an amount commensurate with the angle between the transducers and the rotation rate of 105Hz). By repeating the traces with 360° and then 720° more skew the profile of the stall cell is traced over three revolutions. As well as showing the general evolution of the stall cell, figure 3-41 shows 'fingering', ie more than one peak of static pressure within the stall cell. This type of behaviour has been observed by the author during tests of other compressors not reported here. It is clear from fig 3-41 that the 'fingers' are not simply transported around the annulus with the stall cell. Fig 3-42 shows the same data, but this time skewed by 126Hz. Now the fingers line up between the traces. Thus the irregular

'fingering' of the stall cells is a direct result of the broadband noise described earlier being amplified in the stall cell region.

The 'fingering' described above can be seen to be limited to the tip region of the compressor. Fig 3-43 compares the time traces from the inlet (casing mounted) Kulites to those from the adhesive Kulites on the IGVs. The 'fingering' is seen at the casing, but only to a much reduced degree further inboard. Presumably, then, the broadband noise is also limited to the blade tips.

It is a general fact that many types of disturbances grow as a compressor, or stages within a compressor, move towards the top of their characteristics, simply because there is no change of pressure rise capability to stabilise fluctuations in flow through the compressor. The origin of this particular high frequency disturbance cannot be deduced from the measurements currently available. A modal signal at 1500-3000Hz, rotating at 126Hz around the compressor would have 12-24 lobes and, by way of comparison, there are only 29 blades in the first rotating bladerow, which is immediately downstream of the measurement plane. Thus the disturbance would cover only 1.2-2.4 of a blade pitch. More likely is some form of coherent vortex shedding (the Strouhal number based on the chord of the first rotor is 0.13).

Other disturbances are present in the compressor that grow prior to stall. Downstream stages show different frequency bands of such behaviour, although generally weaker and without any detectable coherence between the transducers. It is also true that low frequency (0-400Hz) noise grows prior to stall (section 3.4.5). The significance of the high frequency disturbances (if they prove sufficiently general) is that they can be detected faster than their low-frequency counterparts, and could thus prove more useful as a basis for stall detection.

3.4.5 Pre-Stall Behaviour

In the previous section the high frequency content of the pressure signals was shown to change as stall is approached. This section considers the pre-stall behaviour at lower frequency (0-400Hz).

3.4.5.1 Transient Rotating Stall Cells

Figs 3-12 to 3-15 showed time traces from the Kulite pressure transducers prior to and during stall at middle speeds taken with the lower sample rate of 4000 samples per second. Activity can be seen on the inlet traces, and at a lower level on the other traces as well, prior to the appearance of the stall cell that grows into the stall/surge event. A detailed examination of the time traces (such as shown in the next section)

shows the activity to be related to stall cells, limited to the front of the compressor, that last for up to 5 revolutions (10 rotor revolutions) before dying out.

The position of these 'transient' stall cells is tracked in fig 3-44 using the phase of the first circumferential harmonic of the static pressure at compressor inlet prior to stall. This type of analysis was first presented for stall results by McDougall [1988], using a program by Longley. Note that the input measurements have been lowpass filtered to remove other signals (shaft harmonics in particular) that would otherwise interfere with that of the transient stall cells. The raw time traces are shown underneath for reference. When transient stall cells are present the phase changes at a steady rate, showing up as straight parallel lines on the figure. Bands of stall can be seen that last for between two and ten rotor revolutions (1 to 5 stall cell revolutions) throughout the period. Such bands have been observed up to 100 rotor revolutions prior to stall, although the period of time is related to the speed at which stall is approached. It should be emphasised that although the phase of the first harmonic is plotted, the disturbances at inlet are much sharper than pure harmonic waves. Their precise nature is considered in the next section.

Pre-stall disturbances of this type only exist over the 'middle' speed range (defined in section 3.4.2) in which no benign rotating stall is present and yet stall initiates at the front of the compressor. No clear pre-stall disturbances were observed at high speed.

3.4.5.2 Possible Long Lengthscale Behaviour Prior to Stall/Surge

Flow disturbances leading to stall on the VIPER engine typically have a short circumferential lengthscale (ie "spikes"). Fig 3-16 shows time traces from the inlet during the 79% speed stall event T180804, where the disturbance that grows into stall has a short circumferential lengthscale. Fig 3-17 shows the time traces from the Kulites at stage 3. Although the disturbance is slightly broader in time (and therefore circumferentially) than at inlet, the area between occurrences is relatively unaffected by the presence of the stall cell.

Notwithstanding the above, there are bands of activity well before stall in the middle speed events which do appear to have a longer circumferential lengthscale. Figs 3-45 and 3-46 show one such band 0.3s prior to stall during the same event. These figures show the inlet and stage three Kulites respectively, normalised and with shaft orders removed according to the procedure described in section 3.3.2.5. In each case the measurements are plotted twice up the page to allow the path of the disturbance to be traced more easily. The inlet Kulites (fig 3-45) show a short circumferential lengthscale disturbance, but the stage 3 Kulites (fig 3-46) show more of a longer

lengthscale type of behaviour, where the disturbance affects the whole annulus. The disturbance at stage 3 is harder to distinguish than at inlet: successive troughs on the lowest trace occur at 38 and 47ms along the x-axis. By overlaying the two figures it can be seen that the short circumferential lengthscale disturbances at inlet coincide with the troughs at stage three, which is consistent with a blockage effect somewhere between the two axial planes.

It is possible that this pre-stall activity is fundamentally a long lengthscale disturbance (that is, one which can be modelled using low circumferential order or even linear approximations), with the peak pressures intensified at compressor inlet. Alternatively, it may be that the disturbance is fundamentally of short circumferential lengthscale at inlet, and that the stage 3 data is circumferentially mixed out in some manner.

It is significant that in no case has it proved possible to trace a single modal wave for more than a few revolutions before the stalling disturbance becomes clear to the eye from the time traces. Thus there is no substantial period of linear modal behaviour prior to the inception of short lengthscale disturbances that lead to stall. Also the short lengthscale disturbances start while the amplitude of any modal disturbances is of the same order of magnitude as the background noise. This implies that using active means to control the first harmonic (only) of the disturbance would be of little benefit in increasing the surge margin of the compressor, because at a slightly higher loading, random noise would be sufficient to initiate a short lengthscale disturbance.

3.4.5.3 Stall Detection Methods

A number of different algorithms were used on the pre-stall data in an attempt to derive an indication of impending stall. A very simple example is shown in fig 3-47, where the mean signal power in the low frequency range (up to first shaft harmonic) is plotted against time on a rolling average basis, averaged over all of the transducers at each axial plane. At this middle speed condition stall starts at the front of the compressor, and the inlet transducers show a general increase in the level of activity with a particularly marked jump at around 1400ms, which is roughly where the transient rotating stall cells (section 3.4.5.1) become visible. The stage 2 transducers show a smaller but still significant rise. Towards the back of the compressor little change is observed, which is again consistent with pattern of stall inception at this speed.

All of the algorithms tried responded to the transient stall cells at middle speeds described in section 3.4.5.1. None gave a robust stall indicator, however. There appear sometimes to be 'quiet' periods in the approach to stall where the flow behaves

as it does away from stall. At high speed it is difficult to find any distinctive features of the flow just prior to stall. One of the algorithms applied was used by Tryfonidis et al [1995] to detect pre-stall modal waves in a number of compressors prior to stall. This algorithm has become fairly well-known, and so its application is reported in detail.

The idea of the technique used by Tryfonidis et al is to use each of the measurements at an axial plane to derive a measure of the power of coherent signals travelling around the annulus. This is done using a time-windowed Fourier technique described below. This type of analysis was used on the data from the second experiment to see if any modal pre-stall waves could be detected.

The technique has three major limitations:

1. Because the signal has to be integrated over a reasonable time interval this technique cannot track rapidly changing signals. This is a particular drawback when the method is used prior to the stall disturbance becoming visible to the eye, as disturbances may last for only a short period of time.
2. Because there are only a small number of Kulites in each axial plane only the first two circumferential harmonics of any disturbance can be traced. Higher harmonics are aliased back onto these. This causes fewer problems than might be envisaged, however, because the frequency of a signal at a given circumferential velocity is proportional to the mode number. Since this technique gives a power spectrum over a range of frequencies the circumferential harmonics do not interfere.
3. In an engine environment external pipework and irregularly shaped casings limit the number of possible positions for instrumentation, and the transducers in the VIPER experiment could not be spaced evenly around the compressor (table 3-1). It can be shown that for the first harmonic this has little effect on the result. For higher orders of disturbance, however, the effect of circumferential displacement of the transducers is multiplied up, and the harmonic content of a wave can be incorrectly represented. Even the circumferential direction of the propagation was found to be frequently mis-represented for the fourth order benign stall (beyond the obvious reversal of direction for an aliased signal), although this was partly due to a genuine difference in response between the different transducers. An example of the effect of uneven spacing can be seen in the low speed results at stage 3 (described below) both in terms of the leakage of the third order signal into a first order signal and the reversal of direction of the aliased fourth order signal.

It should also be noted that in order to derive the rotating signal power estimate it is necessary to split the signal at each frequency into rotating and stationary waves. This split is somewhat arbitrary, as two waves travelling in opposite directions around the annulus are equivalent to a stationary wave. The best assumption, and the one used by Tryfonidis et al, is that the wave energy at a frequency is split between a stationary wave and a single rotating wave, either forwards or backwards. It has already been noted that for the higher circumferential harmonics, uneven transducer spacing and/or variable response of the transducers can affect the apparent direction of rotation of the disturbance. A better method for tracing these harmonics would thus be to trace the activity level regardless of the direction of motion (that is, a simple 2D FFT method).

The implementation of the technique was as follows:

1. Take all transducers in an axial plane.
2. Take successive time windows. A 1024 point (0.25s) window was found to be optimal for the VIPER measurements.
3. For each window:
 - 3.1. Remove any linear trend from each signal, to cancel the effect of the moving operating point.
 - 3.2. Normalise each signal by a standard deviation calculated well away from the stall event.
 - 3.3. Calculate the two-dimensional Discrete Fourier Transform.

The 'propagating wave energy' in the forward direction is then the difference between the squares of values corresponding to the positive and negative frequencies of the first circumferential harmonic (the second harmonic was also used, giving broadly similar results).

Figs 3-48 to 3-53 show typical results of the analysis at four engine speeds, in each case using measurements from the axial plane where the initial stalling disturbance was clearest. Stage 3 and stage 4 results are also shown for the 67% speed case, where the signals at inlet are dominated by the multiple cell benign front end rotating stall. In each case the three lines, moving up the page, represent time windows some distance apart. The three time windows chosen for each speed are shown in fig 3-54. The best representation of the data on the computer screen (but not for black and white reproduction) was found to be a contour plot of positive rotational energy against frequency and time, using all of the information available from successive time

windows. At low speed it proved necessary to scale the results from different stages due mainly to the changing amplitude of the four cell benign stall with axial distance.

Figs 3-48 to 3-50 show the results for the measurements at inlet, stage 3 and stage 4 at 67% speed. It will be remembered that at this speed the compressor operates in 'benign' stall even on the stable side of the surge line. With five transducers the four cell signal is aliased to a negatively rotating first mode signal, which is very evident at around 2.2 times shaft frequency at compressor inlet. Stages 3 and 4 also show activity at fourth order frequency. In the latter case this is not a directly aliased signal as there are six transducers at this stage. It is instead a leakage effect from the fourth order signal due to the irregularly spaced transducers. The apparent direction of rotation of the fourth order signal at stage 3 changes part way through the event. This cannot be accounted for by the uneven spacing of the transducers alone. It could stem either from a difference in response of the instruments to the passing cells due to local effects such as position with respect to blade wakes or corner separations, or less likely from a varying propagation speed as the cells rotate.

As stall is approached stages 3 and 4 show significantly increased first order activity at around 0.5 - 0.7 times shaft speed. Hence the propagating wave energy method does successfully track the change from four cell to single cell stall (as indeed does a windowed Fourier transform of a single channel). It is significant that spikes at a range of frequencies are produced, even though there is a clear single cell structure at the stall point (section 3.4.2.1). There are many possible explanations for this: the method picks out the first order difference between the four cells, and growth of this difference may well be very uneven.

In addition to the first order and aliased fourth order signals the plots at low speed (and higher speeds too) show some growth in activity at second and third order. This is a leakage effect of these higher harmonics into the first order signal and is due to the unevenly distributed transducers as considered above.

Fig 3-51 shows results from compressor inlet at 79% speed. Fig 3-52 shows similar data at 80% speed. The once per shaft revolution disturbance is now seen on all traces (at lower speeds it was not seen because the scaling of the graphs had to be reduced on account of the large disturbances related to the permanent benign rotating stall).

There is a significant change in the propagating wave signature between the top two traces on both graphs. The change is seen most clearly as an aliased signal at four times rotating stall frequency (as described above). The fact that this aliased signal is more significant than the first harmonic should not be taken as an indication of four

stall cells (the time traces clearly show a single cell structure). Rather, it is the effect of a circumferentially sharp stall disturbance. Note that there is little change in the signatures away from stall: the analysis reveals little before the stall cell becomes visible to the eye on a time trace.

The high speed results from stage 4 are shown in fig 3-53. There is relatively little change in the wave energy graphs as stall is approached. The only feasible indicator of approaching stall is the increased amplitude shown in the time window closest to stall (top trace). Unlike the results at lower speeds, however, the change in behaviour is indistinct and shows no obvious link to growing rotating stall activity.

In summary, the method used by Tryfonidis showed no clear indication of impending stall beyond what could be distinguished by eye from the pressure time traces. It does however accurately represent the low circumferential harmonic content of the growing stall cell, including the change from three or four cells to a single one at low speed. Care should be used in interpreting results from higher circumferential harmonics, particularly if the transducers are unevenly spaced around the circumference.

3.4.6 Stall Drop-Out

Fig 3-55 shows successive time traces from one of the inlet Kulites during event T081204 as the engine is accelerated along a normal working line from operation with the benign rotating stall described in section 3.4.2.1 to axisymmetric operation. The lowest trace represents operation with the usual four cell pattern of rotating stall. As the engine is accelerated the number of cells drops from four to three (second trace). One of these cells then decays away (third trace), and then the second (top trace) and then the third (axisymmetric operation). The significance of this is that the four and three cell structures represent forms of rotating stall modes for this particular compressor: there is no region of 5 cell stall, and likewise there is no region of two cell stall with the cells equidistant.

3.5 DISCUSSION

The results presented in this chapter represent the first detailed analysis of stall inception in an aeroengine based on a full coverage of high response instrumentation, and one of the first detailed analyses in a high speed compressor. Stall inception at all speeds in the VIPER engine has been shown to be deterministic and repeatable. For each stall event it has proved possible to trace the disturbance leading to stall/surge from its inception over a region limited in extent both circumferentially and axially,

through a period of axial and then circumferential growth, into fully developed rotating stall or axisymmetric surge.

The main thrust of the two experiments reported in this chapter was to provide analysed data on the nature of stall inception, that is the growth of a flow disturbance from nothing into fully developed stall or surge. This aspect of the results is discussed first. Pre- and post-stall behaviour are then considered, followed by comments on the importance of the results with respect to stall modelling and active control of stall/surge.

3.5.1 Stall Inception in an Aeroengine

The VIPER engine shows a remarkable variety of stall inception behaviour, dependent on engine speed. At low speed the compressor operates in three or four cell 'benign' rotating stall, limited to the front of the compressor, even on the stable side of the surge line. Stall inception occurs as one of the cells grows to the back of the compressor and then circumferentially. At middle speeds stall inception is related to the appearance and growth of a single stall cell at the front of the compressor, which again grows quickly to the back of the compressor and then circumferentially. At high speed the initial disturbance starts in the middle of the compressor, growing for four rotor revolutions at a fixed circumferential location before starting to rotate.

At nearly all speeds stall inception in the VIPER aeroengine starts with a stall cell of limited circumferential extent, consistent with the description of "finite stall cells" given by Day 1991a, rather than the 'modal' type of pre-stall disturbance observed by some experimenters. There is some evidence of long lengthscale behaviour, however, particularly in a very narrow speed range around 88% (section 3.4.2.4). This is consistent with the theory put forward by Dr I J Day and T R Camp that long circumferential lengthscale behaviour tends to occur when a compressor is well matched axially (Camp, 1995).

Many of the expectations of the stalling behaviour, based on previous mainly low speed experiments, have been borne out. The axial position of the initial disturbance leading to stall moves towards the rear at high speed, in line with expectation based on the stage characteristics. Also rotating stall is visible at the start of every stall/surge event, even those at high speed where the disturbance later develops into axisymmetric surge. This feature was observed at low speed by Day [1994a], and at high speed by Riess and Bloecker [1987].

The observation at high speed of the disturbance growing at a fixed circumferential point before starting to rotate appears to be new. It is presumed to be due to an unknown geometrical non-uniformity. Jackson [1987], testing a single stage compressor, reported a disturbance leading to stall that was initially fixed in a frame rotating with the rotor, although his results were later contested by McDougall et al [1990].

A multi-lobed disturbance has been discovered at middle speeds, limited to the tip region of the first stage, that rotates at a slightly faster rate than stall cell speed. This disturbance is only weakly coherent between measurement stations circumferentially, but can be seen to grow in amplitude prior to stall. When the stall cell appears it locally amplifies the multi-lobed disturbance giving rise to a 'fingered' appearance of the stall cell. Similar 'fingers' have been observed in a number of other stall inception tests.

The stall inception behaviour is generally similar to compressors tested on rigs. This is not surprising; the time period over which stall inception occurs is small compared to the filling and emptying times of upstream and downstream volumes, and the time integrated effect of heat addition on varying amounts of air in the combustion chamber. Thus stall *inception* can be considered as primarily a compressor event, whereas post-stall behaviour is heavily dependent on the effects of the surrounding components. This is confirmed in part by the fact that the stall inception behaviour at 78% and 82% engine speed is identical, even though the final operating condition is stable rotating stall / axisymmetric surge respectively.

3.5.2 Pre-Stall Behaviour

At low speed the VIPER compressor operates in permanent three or four cell rotating stall, even on the stable side of the surge line. This type of rotating stall is 'benign' in the sense that it does not prevent normal operation of the engine. The stall cells have been shown to be limited axially to the first stage, with a low static pressure 'tail' extending axially downstream part way along the compressor. The cells have also been shown to be full span type, although biased toward the outer annulus. This is contrary to expectation, but is consistent with the observation by Day et al [1978] that a large number of stages gives a propensity for full span rather than part span stall.

At middle speeds, bands of rotating stall are visible, sometimes for over 100 rotor revolutions prior to stall. These bands consist of small amplitude stall cells growing and dying out. It is unclear for some of these bands whether the disturbance is fundamentally 'spike' like (small circumferential lengthscale) or modal (long

circumferential lengthscale). No modal type of disturbance is seen however, that grows for a substantial period of time before the onset of spike-like stall inception. The type of analysis used by Tryfonidis et al [1995] captured the bands of rotating stall, but showed no sign of any disturbances not seen by eye from the time traces.

At high speed no pre-stall disturbances were found before the growth, initially at a fixed circumferential and axial position, of the disturbance leading to stall/surge.

3.5.3 Post-Stall Behaviour

There is a sharp dichotomy in the post stall behaviour at around 80% of engine speed. Below 80% speed the stall cell stops growing circumferentially, leaving the compressor in single cell rotating stall at very low flow, pressure ratio and efficiency, but with the compressor operating steadily in the mean. Above 80% speed, the stall cell grows until the entire compressor is operating in reversed flow, and the engine moves into violent multiple surge, where the flow direction changes at a frequency of about 8Hz. This dichotomy of behaviour agrees with the theory developed by Moore and Greitzer [1986].

At very high speed multiple surge is prevented by a flameout of the combustor, which modelling (Freeman and Wilson, 1992, also chapter 4) suggests is due to rich extinction as the air flow through the combustor drops off. This is of course a product of the engine environment of the compressor, and would not be reflected in a compressor rig test.

3.5.4 Implications for Stall Modelling

There are a number of significant aspects of the results with respect to stall modelling. Perhaps the most important is the short circumferential lengthscale of the stall inception process. This can cause some difficulty for pitchwise averaged stall inception models as discussed in chapter 2. It is also an indication of the importance of the non-linear terms, in that there is clear modal coupling; in the initial stages all of the circumferential harmonics travel together at the same speed.

The measurements of the benign stall at low speed carry implications for stall models designed to cover this speed range. It is unlikely that a model unable to simulate the front end benign rotating stall at low speed could accurately track the stalling process at these speeds in terms of the growth rate of the single cell disturbance, which must be different starting from an existing disturbance. This limits the performance of both lumped parameter (1D circumferential) models such as Moore and Greitzer's [1986]

and models with no circumferential dimension (such as developed by Greitzer before his paper with Moore).

The case for a radial dimension in a stall model is not proven by these results. On the one hand, the benign stall at low speed has been shown to have a strongly three-dimensional nature, being much stronger near the tip, although not so much as true part-span stall. On the other hand, stall cells appear to be broadly two dimensional for a large part of the period between inception and fully developed rotating stall or surge.

The high frequency disturbance observed prior to and during stall appears to play little part in the inception process, in that it is limited to the tip of the blades and rotates independently of any stall cells, apart from being of increased magnitude within the cells. Thus this high frequency disturbance need not be included in a stall model.

The observation that at high speed the disturbance leading to stall is fixed in a stationary frame must be of some concern to stall modellers. It is assumed to be related to a geometric non-axisymmetry, of which there may be many in an old engine. At the same time there is no known geometrical feature to cause this behaviour, and so there is no clear input to a stall model to allow it to be reproduced. It is a very significant feature of the stalling behaviour, however, in that it comprises the majority of the short time period between any flow disturbance becoming visible and a significant drop in combustor pressure.

3.5.5 Implications for Active Control of Stall

The results described in this chapter have four main implications with respect to active control of stall/surge;

1. The variety of stalling behaviour observed limits the generality of any stall control device. A system developed to control the stalling behaviour at middle speeds, where the disturbance starts at the front of the compressor, may not be effective either at low speeds, where the compressor operates in benign stall even on the stable side of the surge line (the system would have to be able to distinguish between this benign stall and stall inception proper), or at high speeds, where the disturbance starts towards the back of the compressor, and the initial disturbance does not even rotate. Similarly, a control system designed for one engine type cannot be expected to work on another engine type, where the stall patterns may be different again. The variety of behaviour also increases the risk of such a system; a control system that successfully increases stall margin by preventing the growth of

rotating stall may not add much stall margin if a non-rotating mode (such as is observed at high speed) quickly becomes unstable.

2. The short circumferential lengthscale of the disturbances leading to stall make active control more difficult; the modal type of stall inception controlled for example by Paduano et al [1993] allows for fairly simple linear control algorithms with relatively low-speed actuation devices. Nonetheless active control has been successfully applied to spike-like disturbances [Day, 1993b].
3. The speed of stall inception makes active control difficult at high speed. The time period between the first sign of a disturbance and a sizeable drop in compressor exit pressure is of the order of six rotor revolutions. Four of these relate to the growth of the disturbance at a fixed circumferential position, which may not prove to be the case in general. The task then becomes one of controlling a stall cell rotating at around 110Hz within one (stall cell) revolution with flightworthy actuators.
4. The position of detectors (and presumably actuators) is important. The initial disturbance at high speed, for instance, would not be picked up by detectors at the front of the compressor. Likewise the circumferential definition provided by a detection system would have to be carefully considered. This problem is particularly acute at high speed, where sufficient detectors would have to be deployed to ensure the initial stationary disturbance was resolved.

4. COMPRESSORS IN PARALLEL MODEL

4.1 OBJECTIVES

The primary objective of the work described in this section was to produce a simple model of stall/surge inception, growth, and post-stall transients of a compression system in an engine environment. The first use of the model was to test numerically various active control algorithms based on the type of control geometry used by Day [1993b, described in section 2.3]. The formulation of the model, however, also makes it suitable for investigation of other phenomena, such as those associated with combustor instabilities.

In order to test active control strategies using the model, it is necessary to represent accurately the short circumferential lengthscales of the disturbances leading to stall, as the control of such disturbances would be expected to be quite different from the control of low order circumferential harmonics. It is also necessary to capture the rate of decline of pressure in the downstream plenum, in order to gauge the reaction time required of a stall control device. It would be preferable to model the initial period of stall inception, when the disturbance leading to stall is very localised within the compressor. This last feature, however, is precluded by the simplicity of the model.

4.2 DESCRIPTION

4.2.1 Overview

For stall point prediction, concerned only with the growth of small amplitude flow disturbances, a linear model would be acceptable. In order to trace the disturbances to large amplitude a non-linear model is essential, particularly with regard to the compressor characteristic (that is, a constant slope characteristic is insufficient). Because of the difficulty posed by non-linear modelling the approach is numerical rather than analytic.

In the previous section it was noted that the model would have to represent accurately the short circumferential lengthscale over which the stall instability starts. Thus it is necessary that the model has a circumferential dimension. 32 circumferential nodes were selected as being the lowest number to adequately define the initial disturbances. The simplest such model was selected, namely one with no radial dimension, and with various assumptions being made (described in the next section) such that the whole compressor performance could be modelled as a form of semi-actuator disc (that is, the model accounts for the momentum of the fluid in the compressor, but the performance

of each individual bladerow is combined into a single characteristic). The two dimensional (circumferential and axial) flow in the inlet duct was solved analytically.

In order to capture the drop in pressure at the back of the compressor, it was necessary to model the combustion chamber. This is geometrically more complicated than the more usual case of a compressor rig with a simple plenum behind. For simplicity, the combustion chamber was split into a hot and a cold volume with flow through a single orifice from the cold to the hot (or vice versa in extreme conditions), and a choked exit nozzle from the hot volume. The two chambers were assumed to be totally mixed out (i.e. constant fluid properties) both circumferentially and axially. A very simple combustion efficiency characteristic was added, primarily to describe the flame-out which occurs during high speed surge events.

In the model, as in the experiment, some method is required to move the engine from its normal working line towards surge. To this end a simple choked nozzle calculation was performed to introduce high pressure air into the cold chamber of the combustor in the same way as the experiment described in chapter 3. Similar levels of inbleed pressure were used in the model to those in the experiment (at middle and high speeds; in the experiment at low speed the engine was moved to stall by overfuelling at a rate beyond that normally allowed). The effect of the inbleed on the stall behaviour is discussed in section 4.3.4.

A schematic view of the engine as modelled is shown in fig 4-1. The model is referred to here as a compressors in parallel model, but it should be noted that there is information exchange between circumferential nodes not only at inlet and exit, but also along the compressor in terms of axial momentum transfer. This is discussed further in section 4.2.3.

4.2.2 Assumptions/Approximations

The major assumptions inherent in the model, listed below, are those commonly used for lumped parameter pitchwise averaged models. A more general discussion of the various assumptions is given in section 2.2.

- 1. Two dimensional flow (hence high hub/tip ratio).**
- 2. Pseudo-incompressible flow in the compressor.** No mass storage is accounted for within the compressor, but the density is allowed to rise through the compressor as the duct area falls.

3. **Purely axial instabilities in the compressor.** That is, disturbances leading to stall/surge are arranged axially along the compressor. This has been generally observed, and is usually assumed in such models. For the particular case being modelled here, namely rotating stall in a VIPER engine, it has been experimentally verified (chapter 3).
4. **Negligible flow redistribution in the gaps between the bladerows.** This was the conclusion of Dunham [1965], although it is questionable for the small circumferential lengthscale disturbances being modelled here (see section 2.2 for a discussion). The more complex method described in chapter 5 models the gaps directly.
5. **Pitchwise averaged bladerow performance** so blade-to-blade effects are ignored.
6. **Quasi-steady bladerow performance.** Of the lag terms used by Moore and Greitzer [1986] (see section 2.2), the inertia of the fluid in the blade passages is included naturally in the formulation of the compressor model. The response of the blade forces, however, is assumed to be instantaneous (this was said to be a reasonable approximation in that paper). The blade forces are also assumed to be independent of the pressure ratio across the bladerow (in an unsteady model the two do not necessarily match).
7. **Bladerow performance (at a given shaft speed) solely determined by axial mass flow.** In fact, if the deviation from the lightly loaded IGV is assumed to be constant regardless of incidence, this can be seen to be a consequence of the earlier assumptions: if gap effects are ignored, the inlet flow angle to a bladerow is set purely by the axial flow and the deviation from the previous bladerow. Hence if the deviation of the IGV is purely a function of the axial flow, so is the deviation of all subsequent bladerows, and the velocity triangles for each stage are uniquely defined by the inlet flow.

Under the above assumptions, the entire compressor performance can be collapsed into a single 'axisymmetric pressure rise' characteristic (see section 2.2 for a definition of this term), which can be applied at each circumferential position independently.

In addition to the above there are a number of more specific assumptions, particularly with regard to the combustor, which are introduced as required below.

4.2.3 Compressor Model

The compressor model stems from a simple application of the axial momentum equation integrated axially from compressor inlet to compressor exit. For two dimensional flow, the axial momentum equation is

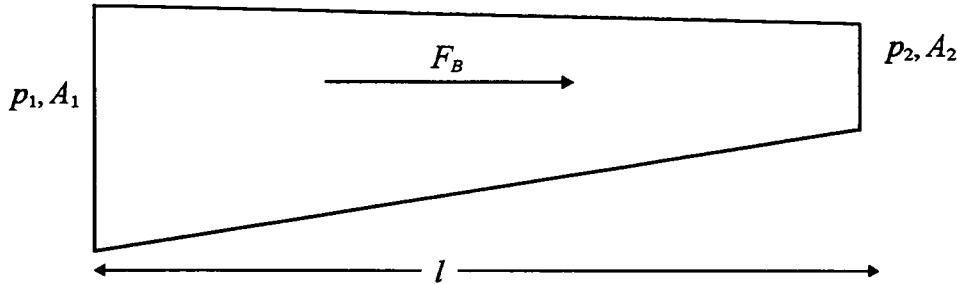
$$\rho \frac{\partial u_a}{\partial t} + \rho u_a \frac{\partial u_a}{\partial z} + \rho \frac{u_\theta}{r} \frac{\partial u_a}{\partial \theta} = - \cancel{\frac{1}{r} \frac{\partial p}{\partial \theta}} + F_B, \quad (\text{Equ 4.1})$$

$\swarrow \quad \searrow$
 $-\frac{\partial p}{\partial z}$

where F_B is the axial body force described later.

For a machine designed for near constant axial velocity, $\rho u_a \frac{\partial u_a}{\partial z} \approx 0$.

The above equation is applied in a form integrated over the length of the compressor:



$$M \frac{\partial u_a}{\partial t} + M \frac{\bar{u}_\theta}{r} \frac{\partial u_a}{\partial \theta} = p_1 A_1 - p_2 A_2 - p_{ave} (A_1 - A_2) + F_B, \quad (\text{Equ 4.2})$$

where $M = \bar{\rho} \bar{A} l$ is the total mass of fluid in the compressor. A better approximation is obtained using the 'effective' length of the compressor, $l_{eff} = l / \cos^2 \xi$, following Moore and Greitzer [1986], which accounts for the extra flow length caused by the stagger of the blades, and also for the fact that any change in axial velocity requires a proportionately greater change in flow velocity in the direction of the blade passage.

A more correct, but mathematically equivalent, approach is to perform the integration along the 'instantaneous gas path', that is the path (following the blade passages) that would be followed by an infinitely fast disturbance. This differs from the path taken by a fluid element in that the rotation of the rotor rows is discounted, and so the instantaneous gas path does not rotate around the annulus with axial distance. The incompressibility and gap flow assumptions above ensure that the throughflow level remains constant along this path, and hence axially along the machine. The transport of axial momentum is in reality transport of momentum in the direction of this

instantaneous gas path. The formulation above with an effective compressor length is retained as it is mathematically identical, but easier to conceptualise.

The whirl velocity $\bar{u}_\theta$ (averaged axially over the length of the compressor) is the combination of velocity due to blade shape (and deviation), and rotor speed. For a machine with around 50% reaction and similar chord in rotors and stators, the blade shape contributions roughly cancel, leaving the blade speed contribution in the rotors only. Thus the average whirl velocity is approximated to $\bar{u}_\theta = 0.5\Omega r$. The second term in the axially integrated equation above, which includes $\bar{u}_\theta$, represents the circumferential transport of axial momentum. It will be seen that in a long compressor such as this, where the compressor flow dominates the inlet effect, the stall disturbances are transported around the annulus at around half shaft speed. This is in agreement with the results reported in chapter 3, and also from a number of other compressors tested by the author but not reported here. The circumferential transport of axial momentum is thus seen to be the primary cause of the rotation of rotating stall in long compressors, over and above the incidence arguments forwarded by Emmons et al [1955], which modify the rotation rate only slightly from the half shaft speed level.

The axial body force, F_B , is derived according to the assumptions outlined above from the axial flow at each circumferential position together with an 'axisymmetric pressure rise' characteristic (fig 4-2). In steady flow this pressure rise is added to the static pressure at the compressor face to give the static pressure in the combustion chamber. As such it represents a static to total pressure rise less diffuser and combustor entry losses.¹

The shape of the stable (negatively sloped) part of the axisymmetric pressure rise characteristic at the high flow end was taken directly from measured data. The performance at low and negative flow is unknown. The unstable middle flow region was blended with the stable part and a value of $0.15 \Delta p / \rho u^2$ at zero flow, which is typical of known data. The shape of the reverse flow part of the characteristic follows Moore and Greitzer [1986]. It should be recognised that the gradient is very high on the negative flow side of the characteristic, reflecting the fact that a large pressure ratio is required to push air fast backwards through a rotating compressor. Given this fact, the precise form of the characteristic at this point is not very important: it only affects

¹ If the diffuser downstream of the compressor is long compared to the lengthscale on which the non-axisymmetric disturbances decay, then such a characteristic would give incorrect dynamic behaviour. Instead a static to static pressure rise characteristic should be used with a one dimensional diffuser model between the compressor and combustor. In the VIPER engine the diffuser length is of the same order as the decay lengthscale of the disturbances. Such a level of detail is beyond the scope of this simple model.

the compressor when stall is well underway, and if the flow were not exactly correct the only major effect would be to change the period of the surge cycle, increasing or decreasing slightly the time required for the combustor to empty. Even this latter change would not be that large, remembering that during this part of the surge cycle air is leaving the combustion chamber in both directions.

The characteristic was scaled in terms of flow and pressure ratio according to the shaft speed, which was assumed constant for the duration of each run. A mean line performance prediction programme was used to account for stage matching effects in the scaling process.

4.2.4 Inlet Duct

The flow in the inlet duct is assumed to be inviscid, incompressible and irrotational, with constant density right up to the compressor face. These are all reasonable assumptions for forward flow. Some errors have to be accepted, however, when reverse flow is present. Given these assumptions a velocity potential can be written down for the two dimensional case under consideration which satisfies Laplace's equation:

$$\phi = \bar{u}_a z + \sum_{n \geq 1} e^{nz/r} (a_n e^{in\theta} + a_n^* e^{-in\theta}) + C(t), \quad (\text{Equ 4.3})$$

where $\bar{u}_a$ is the circumferentially averaged axial velocity, and the coefficients a_n are calculated together with their complex conjugate values a_n^* from the axial velocity distribution at the compressor face using a discrete Fourier transform;

$$u_{a,z=0} = \frac{d\phi}{dz} \Big|_{z=0} = \bar{u}_a + \sum_{n \geq 1} n/r (a_n e^{in\theta} + a_n^* e^{-in\theta}) \quad (\text{Equ 4.4})$$

These equations do not represent the most general solution of Laplace's equation, which would also have terms growing exponentially in the downstream direction. These extra terms would normally be required to match the upstream boundary condition. In this case, however, the inlet duct length is several times the radius, and any circumferential variations decay with a lengthscale that is small compared to the length of the inlet duct. The upstream boundary condition is thus axisymmetry at the inlet plane, which for the long inlet duct case is correctly given by the equations above without the extra terms.

In order to couple the inlet duct model to the model of the compressor it is necessary to relate the velocity at the compressor face to the total pressure there. Using the incompressible approximation

$$P = p + 0.5\rho \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_i}, \quad (\text{Equ 4.5})$$

the axial momentum equation in the inlet duct can be written as follows;

$$P = P_{am} - \rho \frac{\partial \phi}{\partial t}. \quad (\text{Equ 4.6})$$

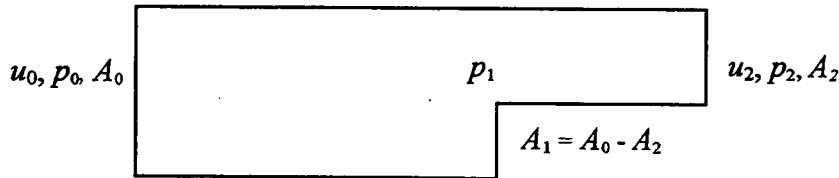
If the inertia of the fluid upstream of the inlet duct is ignored we can set $\phi = 0$ at the inlet plane, and $C(t) = \bar{u}_a l_{in}$. (Equ 4.7)

Then, at the compressor face, $z = 0$,

$$P = P_{am} - \rho l_{in} \frac{d\bar{u}_a}{dt} - \sum_{n \geq 1} \rho \frac{\partial}{\partial t} (a_n e^{in\theta} + a_n^* e^{-in\theta}). \quad (\text{Equ 4.8})$$

The static pressure p can then be deduced from the total pressure P using the known axial velocity and the circumferential velocity calculated from the velocity potential.

A correction was applied to the length of the inlet duct to account for differences in flow area from the compressor face (IGV inlet). The main difference is due to the absence of the central rotating body (or 'bullet'), which was modelled in a one-dimensional manner as follows;



The cross sectional area of the central rotating body (A_1) is assumed constant upstream of IGV inlet. Upstream of the bullet, the rate of change of momentum can be related to the static pressure differential using a control volume approach:

$$\begin{aligned} (p_0 - p_1)A_0 &= \frac{d}{dt}(\rho A_0 u_0 l_1) \\ &= \frac{d}{dt}(\rho A_2 u_2 l_1) \end{aligned} \quad (\text{Equ 4.9})$$

$$\text{or } p_0 - p_1 = \frac{d}{dt}(\rho u_2 l_{1,eff}), \quad (\text{Equ 4.10})$$

$$\text{where } l_{1,eff} = l_1 \frac{A_2}{A_0}. \quad (\text{Equ 4.11})$$

Thus the length of the duct upstream of the bullet was factored down by the ratio of the flow area to that at the compressor face. In reality the geometry of the ducting is more complex than that assumed and the correction is only approximate. Note that the correction only has an effect on the axisymmetric component of the flow disturbances; the lengthscale with which the non-axisymmetric disturbances decay is such that most of the momentum changes occur in the region where the central rotating body is present.

4.2.5 Combustion Chamber

The static pressure was assumed axisymmetric at the compressor exit plane (the diffuser was not modelled). Whilst this is only a rough approximation, any errors introduced will be much smaller for a long compressor (where most of the inertia is within the compressor itself), than for the short compressors described by previous modellers.

The Rolls-Royce VIPER engine has an annular combustor with the common arrangement of cooling air entering the hot part of the chamber through a series of cooling holes. Since there was the possibility of velocity fluctuations between the two sections (hot and cold) of the combustor, it was decided to model these two parts separately. The flow in both sections was considered fully mixed, that is, uniform. The conditions in each part were determined from conservation of mass and energy, ignoring losses within the combustor itself.

4.2.5.1 Mass

The mass flow into the cold section of the compressor was taken as the annulus average flow through the compressor. The flow between the cold and hot sections (through constant section holes) was modelled as

$$m = \rho A_h \sqrt{\frac{p_c - p_h}{\frac{1}{2} \rho_c}}, \quad (\text{Equ 4.12})$$

where A_h refers to the total area of the holes, and the subscripts c and h refer to the hot and cold section respectively. The flow out of the hot section of the combustor was approximated by a choked nozzle calculation, with a simple correction for unchoked conditions.

4.2.5.2 Energy

In order to calculate the energy input to the cold section of the combustor some level of efficiency has to be assumed for the compressor. For the purposes of the model the

following simple approach was adopted. At a flow corresponding to normal operation on the working line the effect of small changes in efficiency is negligible, and the isentropic temperature rise across the compressor was used (that is, the value assuming 100% efficiency). At zero flow a temperature rise was calculated assuming that the flow was accelerated to blade speed in each rotor row, and then decelerated again to zero velocity in each stator row;

$$T_{rise} = mu^2 / C_p \quad . \quad (\text{Equ 4.13})$$

This is only an approximation; the real flow pattern in this compressor at zero mean flow is unknown. At intermediate flows a linear combination of these two values was adopted, depending on the ratio of the flow to the normal working line flow. In reverse flow the temperature of the air leaving the combustor towards the inlet was naturally taken to be that of the air in the cold volume.

The energy equation for both sections can be written

$$\frac{dC_v MT}{dt} = FC_F \eta_{comb} + H_{in} - H_{out} \quad , \quad (\text{Equ 4.14})$$

where M refers to the mass in the section (hot or cold), T is the temperature, and FC_F is the calorific value of the fuel times the fuel flow, which is assumed constant for the stall transient (this term is clearly only relevant for the hot volume). η_{comb} is a combustion efficiency dependent on air/fuel ratio (fig 4-3). This model of combustor efficiency is due to Rowe [1992]. It is based partly on chemistry and partly on empirical data. The flame is assumed to be extinguished when the combustor efficiency drops below 40%. H_{in} and H_{out} are enthalpy fluxes calculated from the mass flux and temperature of the flows into and out of the volume.

In the case of the cold combustion chamber a second mass and energy contribution has to be added to represent the combustor inbleed that moves the engine towards stall. This was performed using a simple choked nozzle calculation, assuming that the incoming air is at ambient temperature.

4.2.6 Solution

The main variables for the calculations were the axial velocity at each circumferential node, and the temperature and pressure in each of the two combustor volumes. These variables were updated iteratively using a simple forward difference time-marching method, with a timestep equal to half the time taken for the rotor to move between two of the parallel compressors.

At each timestep the procedure was as follows. Firstly the rate of change of temperature in each of the combustor volumes was calculated from the enthalpy fluxes (equ 4.14). Then the equation of state was used to update the pressure in the combustor volumes based on the rate of change of temperature and the rate of change of mass, the latter being calculated from the mass flows into and out of each volume (section 4.2.5.1). Finally, the axial velocity at each circumferential node was calculated from a simultaneous solution of equations 4.2 and 4.8 using a discrete Fourier transform.

Previous modellers have tracked the growth of an initial disturbance of a given type. In this model a different approach was adopted, with the aim of producing circumferential disturbances that could grow into rotating stall without influencing overly the mode of the instability. At each timestep, a small random fluctuation was added to the velocity at each circumferential node. The magnitude of the random disturbance was broadly that of measured unsteadiness in multistage compressors, but due to the nature of the compressor model it was assumed to apply along the whole length of the compressor: no attempt was made to model the precise nature of the unsteadiness.

In order to make the code stable it proved necessary to upwind the discretisation of the term related to the circumferential transport of axial momentum. It is recognised that this does introduce some effective smoothing into the calculation, and this has been shown to have a significant effect on the type of stall behaviour exhibited. This is discussed further in section 4.3.3.

4.2.7 Input Parameter Values

Early runs of the model showed a bias towards rotating stall (as opposed to surge) as the final operating condition of the engine after a stall/surge event. The speed at which the change in behaviour occurred was found to be dependent not only on the Greitzer B parameter (defined under Greitzer [1976a,b] in section 2.2.3.1), but also on the level of smoothing used in the circumferential direction (section 4.3.3). It was found that the measured boundary at 80% of engine speed between stall and surge could be matched if a small amount of extra smoothing was applied beyond that introduced by the upwinding, and this was done for all of the results presented in the following section. The results in section 4.4 were produced using an earlier set of input values, with a lower level of smoothing, but with the compressor length artificially reduced.

4.3 RESULTS

4.3.1 Overall Behaviour

A comparison of the overall behaviour of the model with the experimental results from the VIPER compressor is shown in fig 4-4 to fig 4-7. The bottom half of each figure shows pressure measurements during stall transients from the experiments described in chapter 3. The top half gives the model results over the same timescale. Each graph shows the static pressure at the back of the compressor, along with two static pressures from the plane between the inlet duct and the compressor, positioned roughly 180° apart around the circumference. The four speeds chosen represent the four major types of stall behaviour exhibited by the VIPER engine during the experiments:

- 70% Multicell 'benign' front end stall => Single cell rotating stall => 'Stable' rotating stall
- 78% Single cell rotating stall at front => 'Stable' rotating stall
- 82% Single cell rotating stall at front => Multiple Surge
- 98% Single cell stall towards rear => Surge and Flameout

The two middle speeds were slightly different in the experiment (80.7% and 81.2%), but still represent either side of the boundary between stable rotating stall and surge as the final operating condition of the engine.

The very low measured pressures in the last three figures are probably due to temperature drift caused by the reversed flow from the combustion chamber: the compressor speed remains high throughout the time period shown on these graphs, and with high pressure air being pumped into the combustion chamber as well it is unlikely that the pressure there would fall to ambient.

It can be seen from the figures that the overall behaviour of the engine during a stall transient is well matched by the model:

- Each event starts with a period of rotating stall as measured by the 'transducers' at the front of the compressor (at compressor exit the flow in the model is assumed axisymmetric, so no sign of asymmetry can be expected to match the fluctuations shown in the measurements at that plane).

- The growth of the rotating stall coincides with a drop in the exit pressure; the rate at which this falls as calculated by the model is approximately correct, although slightly faster than in the experiment.
- Stable rotating stall is correctly predicted for the final operating condition at lower speeds, as is the return to axisymmetric reverse flow for the higher speeds. The multiple surges at middle to high speed are predicted, each with an initial period of non-axisymmetry as was usually observed in the measurements (the experimental data shown in fig 4-6 was slightly unusual in showing very little asymmetry at the start of the second surge cycle). The precise speed at which the behaviour changed from rotating stall to multiple surge was found to be a strong function of the amount of smoothing used in the circumferential direction (section 4.3.3).
- The 'hung' nature of the event at high speed, with axisymmetric near zero flow through the compressor, is captured by the model. As the pressure drops in the combustion chamber the dimensional mass flow through the exit nozzle falls to an extent where the nozzle is choked by the combustor inbleed alone. The compressor then operates axisymmetrically on the low-flow stable part of its characteristic. This effect is described further in section 4.3.4.
- The flameout at high speed is not predicted, although the calculated combustion efficiency does approach its cut-off value (and at lower levels of combustor inbleed a flameout *is* predicted - section 4.3.4). The low combustion efficiency levels predicted by the model suggest that the flameout observed experimentally at high speed is due to rich extinction: after the collapse of the pumping capability of the compressor, air starts to leave the combustion chamber in both directions, leaving a richer and richer fuel/air mixture. This result is likely to be correct, in view of the fact that at high speed flames could be seen coming out of the back of the engine, demonstrating the existence of unburned fuel leaving the combustion chamber.

The predicted inlet pressure in fig 4-6 drops to an unrealistically low level. This occurs also at high speed with low levels of combustor inbleed, and is discussed below in section 4.3.4.

4.3.2 Stall Inception Behaviour

A more detailed view of the stall inception process predicted by the model (that is, the initial stages of the stall transient) is shown for the 82% case in fig 4-8, again compared with data from the experiment. The time scale on this figure (and the two following) relates directly to that on the earlier figure showing the overall behaviour

(fig 4-6). The casing static pressure is plotted from five points roughly equispaced around the annulus between the inlet duct and the compressor. It can be seen that the stall inception behaviour of the model is reasonable, but does not match the experimental results as well as the overall behaviour described in the previous section. The form of the disturbance, a single short circumferential lengthscale rotating stall cell, is correctly predicted (though see the next section). Likewise the peak magnitude of the pressure increase is reasonable, although slightly lower than the measured level. The initial low amplitude disturbances are not captured at all. During this period the disturbance is limited axially to the front of the compressor (chapter 3), and so cannot be modelled effectively using a semi-actuator disk model for the whole compressor. Even discounting the initial stages, the period of non-axisymmetry is shorter in the model than in the experiment (this is balanced by a faster reduction in combustion chamber pressure, with no net effect on the critical speed at which the final operating condition changes from stable rotating stall to multiple surge). Finally, the shape of the predicted pressure disturbances does not match the experimental results, suggesting that the predicted stall cell shape is inaccurate.

The shape of the growing stall cell as predicted by the model can be defined more clearly by considering local flow rate rather than inlet pressure. In fig 4-9 the local flow rate (scaled to the area of the entire annulus) is plotted at five circumferential positions, roughly equispaced around the annulus. The flow reversal within the stall cell can be seen to be extremely fast, occurring over a fraction of a revolution of the cell. Once reversed flow is present the cell grows circumferentially to fill the annulus in 14ms (1.5 stall cell revolutions or 3 rotor revolutions).

The growth of the stall cell is tracked using a different format in fig 4-10. Time is plotted vertically up the page, and the shaded area represents the section of the annulus operating in reversed flow. The leading edge of the cell rotates at a fairly constant rate of 57% of rotor speed, slowing slightly as the cell grows. The trailing edge rotates initially at only 15% of rotor speed, but speeds up to almost the same speed as the leading edge by the time the clean flow region disappears. The speed changes can be explained qualitatively by the changing pressure ratio across the compressor. At the start of the stall cell development, when the pressure ratio is high, the clean flow region operates very close to the peak of the pressure rise characteristic (fig 4-2), and a small transfer of momentum from the stalled region will move its operation to the unstable positively sloped part of the characteristic. The stalled negative flow region, conversely, operates away from the minimum of the characteristic, and a larger transfer of momentum is required to move its operation to the unstable part of the

characteristic. Hence the trailing edge moves more slowly than the leading edge, and the stall cell grows. As the pressure in the combustor falls, the clean flow region operates further from the peak of the pressure rise characteristic, and the stalled flow closer to the minimum point. Thus the leading edge of the stall cell slows, and the trailing edge speeds up.

The same qualitative effects are visible in the 78% speed case, shown on the same figure. The trailing edge speed shows similar behaviour to the 82% speed case, but the leading edge slows down more as the cell grows, so that in this case the leading edge starts to travel more slowly than the trailing edge, and the cell starts to shrink again.

At other speeds the stall inception behaviour shows the same trends as at 82%, with a short initial transient, and without the low amplitude precursor which is visible in the experimental results, and is related to the axial growth of the stall disturbance (chapter 3). The model cannot be expected to show the 'benign' stall on the stable side of the surge line at low speed, as that affects only the front of the compressor.

4.3.3 Effect of Circumferential Smoothing

It was noted in the description of the model that the discretisation of the model in the circumferential direction had a significant effect on the type of stall behaviour exhibited. The model as developed uses a backward difference for the term connected with the circumferential transport of axial momentum. This is a type of upwinding which aids the stability of the model. Upwinding, however, carries with it implicitly a degree of second order smoothing: if f is an arbitrary function of circumferential position, then the upwinded derivative

$$\frac{df}{d\theta}_{upwind} = (f_n - f_{n-1})/\Delta\theta = (f_{n+1} - f_{n-1})/2\Delta\theta - (f_{n+1} - 2f_n + f_{n-1})/2\Delta\theta \quad (\text{Equ 4.15})$$

$$= \frac{df}{d\theta}_{central} - \frac{\Delta\theta}{2} \frac{d^2f}{d\theta^2}_{central} \quad (\text{Equ 4.16})$$

The up- as opposed to down- winding ensures that the second derivative is applied with the correct sign to smooth the result.

An investigation was carried out to determine the effect of smoothing in the circumferential direction. The model could not be run stably without the upwinded scheme. High levels of smoothing were tried, and as expected these caused the higher order circumferential modes to be damped out, allowing the model to exhibit a 'modal' (long circumferential lengthscale) stall inception pattern. By varying the amount of smoothing, the instability leading to surge could be changed from short circumferential

lengthscale rotating stall to a single circumferential harmonic modal pattern to pure axisymmetric surge.

With a small degree of extra smoothing in the circumferential direction (less than that introduced by the backward difference scheme) the stall inception pattern continued to be of the short circumferential lengthscale type. The critical speed at which the final operation changed from stable rotating stall to multiple surge, however, was found to be very sensitive to the amount of smoothing added. The higher the level of smoothing, the more prone was the model to multiple surge, and the higher the critical speed. The reason the critical stall/surge speed was sensitive to the amount of smoothing in the circumferential direction was not simply that the non-axisymmetric disturbances were damped out; if this were the case then the stall inception pattern would have been of the modal type or even axisymmetric.

The sensitivity of the critical stall/surge speed to the level of circumferential smoothing is related to the shape of the growing stall cell as predicted by the model. It was noted in the previous section that the flow in the stalled region reverses very quickly, such that for a long period whilst the combustion chamber empties some of the annulus is operating with positive flow, and some with negative flow, with a sharp boundary between the two. During this transient, the flow reversal is driven by the difference between the (local) pressure rise capability of the compressor and the actual pressure rise across it (modified by the transport of momentum as described in section 4.2.3). This difference is relatively small for both the forward flow (operating on the stable high flow end of the axisymmetric pressure rise characteristic) and the reverse flow (operating on the stable low/negative flow end of the characteristic). It is greatest for the flow between these two levels; that is, the flow in the boundary regions between the growing stall cell and the clean flow regions. The overall rate at which the flow reverses is therefore dependent on the size of the boundary regions, and this is in turn dependent on the smoothing in the circumferential direction. If the level of smoothing is high, the boundary regions are wider, and the difference between the actual pressure ratio and what the compressor is (locally) able to support acts over a greater area. This in turn leads to a faster rate of flow reversal; the leading edge of the stall cell travels faster, and the trailing edge slower. The fast rate of flow reversal makes the compression system more prone to axisymmetric surge than stable rotating stall as a final operating condition.

The level of smoothing most representative of a real compressor is unknown; the level used for the results described above was determined such that the change in behaviour from rotating stall to surge occurred at 80% speed, in line with experiment.

4.3.4 Effect of Combustor Inbleed

Having found the model to give good agreement with the experimental data, it was used to investigate the effect of the combustor inbleed on the post-stall behaviour of the engine. It was discovered that the level of inbleed could have a large effect on the post-stall behaviour. Fig 4-11 shows the different types of behaviour observed in the model, plotted against engine speed and combustor inbleed as a percentage of working line flow. The boundary lines were obtained by repeated use of the model (over 2000 runs in all). Because of the random element in the model the boundaries are ill-defined, with a range of typically $\pm 5\%$ inbleed within which more than one type of behaviour could be seen.

The most pronounced effect is shown at above 40% combustor inbleed, where after an initial surge event (initiated by a single rotating stall cell) the engine operates indefinitely in axisymmetric low flow ('ALF'). This effect has already been discussed briefly with respect to high speed surge (section 4.3.1). The underlying cause is that the combustor inbleed is sufficient to fill the combustor exit nozzle (at low pressure) without any contribution from the engine compression system. The compression system then operates stably on the negatively sloped very low or even reversed flow part of the axisymmetric pressure rise characteristic.

At slightly lower levels of combustor inbleed, between roughly 24% and 40%, a related event occurs which is an initial surge-like collapse to axisymmetric flow, followed by rotating stall on the return to forward flow which settles into stable rotating stall. At these conditions the inbleed is insufficient to fill the combustor exit nozzle (even at the minimum pressure point of the surge cycle), but blocks a large part of it, such that the effective nozzle size is smaller, and the pressure in the combustion chamber rises more rapidly than would be the case with no inbleed, leading to rotating stall rather than a multiple surge event. Between about 24% and 33% of combustor inbleed the inception of rotating stall occurs immediately after the initial surge event. At higher levels of inbleed a period of axisymmetric low flow ('ALF') operation occurs between the two, whilst the overall operating point is changing only very slowly. The period of 'ALF' operation grows in length as the level of inbleed is increased. Above 40% of inbleed, as already noted, this period becomes indefinite.

At high speed, no flameout is predicted with high levels of combustor inbleed as the inbleed itself is sufficient to keep the flame alight. A flameout is predicted, however, at low levels of combustor inbleed. For some of these cases the model failed to work. On investigation, the cause was identified as negative pressure at the compressor inlet

face. This was a result of the pseudo-incompressibility assumption (section 4.2.2). For the flow in the inlet duct and the compressor to move together the pressure drop from the inlet of the airmeter to the compressor face must be commensurate with the force supplied by the blading to the flow in the compressor less the pressure rise across it. Negative pressure is unphysical, however. In practice the pseudo-incompressibility assumption breaks down; the flow in the compressor would accelerate faster, leaving the flow in the inlet duct behind. One result is that the effective length of ducting over which the fluid has to be accelerated is less than the physical length, and a correction for this could be applied to the model.

The objective of the experiments reported in chapter 3 was to investigate the stall inception behaviour of the engine, rather than the post-stall transients. The stall inception behaviour is largely local to the compressor itself and should be independent of the level of combustor inbleed. In consequence only a small amount of data was collected with which to test these results. At low and middle speeds most of the experimental data was collected using as gentle a surging method as possible (chapter 3), and hence lies along the no-stall boundary line. Fig 4-12, however, shows the compressor exit static pressure level for two events conducted at a similar speed. The first, at 81% speed, used an inbleed and deceleration method of inducing surge (described in chapter 3) which places the inbleed level along the no surge boundary. The engine exhibits a multiple surge event as described in chapter 3. The second event was a more violent surge induced by introducing around 50% of combustor inbleed flow. The stalling speed of the engine for this second event was slightly higher at 85%, but still within the range where the gentler inbleed and deceleration stalling process gives rise to a multiple surge event (chapter 3). These two cases show good agreement with fig 4-11. The gentle stall event resulted in multiple surge behaviour, whilst the violent surge resulted in 'ALF' conditions such as those seen at high speed.

A final set of numerical calculations was performed to predict the post-stall behaviour of the engine if the stall were caused other than by combustor inbleed. For these calculations the initial flow was set on the stable side of the pressure rise characteristic, very close to the peak pressure rise point, with stall caused by the random element in the model. The results are compared to the stalling behaviour with combustion inbleed case in fig 4-13. For the purposes of this comparison just sufficient inbleed was used to cause stall/surge (that is, the results are taken from just above the no surge boundary in fig 4-11).

At low speed (up to 72% speed) the behaviour is essentially the same with and without combustor inbleed, with the compression system settling in stable rotating stall.

Between 73% and 76% the final operating condition of the engine is generally rotating stall, but can also be multiple surge. Above 76% speed without combustor inbleed the initial transient to axisymmetric reverse flow is similar to the multiple surge case with inbleed. Without the inbled air, however, the fuel/air ratio continues to worsen, and a flameout occurs. The quantitative prediction of the critical speed above which a flameout occurs should be treated with some caution in view of the simplicity of the combustor modelling (section 4.2.5).

From the above description it is clear that using combustor inbleed to raise the compressor working line towards stall has had a marked effect on the post-stall transient behaviour of the engine. The stall inception period, however, concerns flow disturbances that are localised within the compressor, and should be unaffected by the use of inbleed.

4.3.5 Discussion

It can be seen from the results in the previous sections that the post-stall behaviour of the VIPER engine is very well matched by the compressors in parallel model. The model put forward by Moore and Greitzer [1986], on which the present model is partly based, was developed primarily to examine the post-stall behaviour of compressors, and it is not surprising that the model works well here. The collapse of the pressure rise capability of the compressor causes huge swings in flow velocity as the combustion chamber empties. Likewise in rotating stall the flow in the clean flow areas is greater than that at the surge point, whereas in the low flow areas it is reversed. Hence the circumferential transport of momentum is also large compared to both the background noise and any detailed changes in the behaviour of any particular stage both in the real compressor and in the model. The change from rotating stall (at lower speeds) to multiple surge (at higher speeds), the 'Greitzer B' effect (section 2.2), is also a result of these gross flow changes. In addition, experimental results show that the flow in 'stable' rotating stall is generally more constant radially than during the stall inception process. Thus the post-stall behaviour is isolated from the major limitations of the model laid out below.

One interesting feature of the model is that the flow within a growing stall cell reverses very quickly. From that point on there is a division between the 'clean' flow and the 'stalled' flow, the two rotating around the annulus due to the circumferential transfer of momentum. The boundary regions, at intermediate levels of flow, occupy a very small sector of the annulus. The relative speed at which these boundaries move (and hence the rate of growth/decay of the stall cell) is dependent not only on the

pressure/inertia balance as represented in the Greitzer B parameter, but also on the level of smoothing in the circumferential direction. There are many possible sources of smoothing in a real compressor (mixing within blade passages, gap effects and bladerow response for instance). Many of these can be shown to be negligible for long circumferential lengthscale disturbances, but not necessarily for the sharp boundary regions shown by the model. A large number of comparisons with experimental data would be required to determine what parameters govern a representative level of mixing.

To some extent the difficulties outlined in the previous paragraph can be overcome with a more complex model, such as the one presented in chapter 5, where the gap effects in particular are treated explicitly. Even this model, however, is dependent on a pitchwise average description of bladerow performance, which is only accurate for disturbances long compared to a blade pitch.

The differences reported earlier between the time histories of inlet pressure during stall inception in the model and in the experiment suggests that the shape of the stall cell is not well captured by the model, and it might be thought that the above discussion is academic. Thin boundary regions between 'stalled' and 'clean' flow are regularly observed, however (Cumpsty and Greitzer [1982], Das and Jiang [1984], Riess and Walbaum [1996] among others). Cumpsty and Greitzer [1982] in particular relate the behaviour in the boundary region at the trailing edge of the stall cell to the stall cell speed. Critical to their argument, however, is the circumferential extent of the boundary region (which is in turn dependent on the amount of circumferential smoothing that exists), and little guidance is given by the authors in this respect. A model such as that of Cumpsty and Greitzer, considering only the boundary regions between 'clean' and 'stalled' flow, provides an alternative to the sort of model presented in this chapter and chapter 5. It is, however, subject to many of the same uncertainties (pitchwise average bladerow performance, gap effects, etc.).

Furthermore, it is only valid for periods when the flow can be characterised in this way, and hence cannot be used to model either the stall inception transient, where the flow in the stall cell is not fully reversed, or the return to axisymmetry during a surge event.

In addition to the above, three points in particular should be noted about the model results. Firstly, the rate of rotation of the disturbances in both the model and the experiment was roughly half of the shaft speed. This supports the hypothesis that the circumferential transport of axial momentum plays the dominant role in the rotation of stall cells in a long compressor. The same should not be expected to be true necessarily in a short compressor, because the unsteady effects at inlet and exit are

proportionately more significant. Secondly, the rate at which the pressure in the combustion chamber falls is reasonably represented in the model results. This is not surprising, in that it is a 'gross' effect which should be well modelled. It is important, however, in the field of active stall/surge control, as it gives a necessary condition for the response time of any active control system: if the pressure in the combustion chamber has dropped significantly before the effect of the control system is felt, then a great deal of energy is required to return the system to normal operation. The third point of interest is that, as expected (section 4.2.3), the poor modelling of the reversed flow regime has little effect on the results.

The way in which the overall stalling behaviour of the model matches the experimental results is impressive, and the model can be used to predict the behaviour of the VIPER and other engines. Application of the model to the VIPER has already shown (section 4.3.4) that the use of combustor inbleed to raise the compressor working line towards stall did have a significant effect on the post-stall behaviour of the engine, and hence that the *post-stall* behaviour is not necessarily representative of an engine in service (the stall inception behaviour is representative, however).

The most severe limitation of the model concerns the inability to accurately represent the initial stages of stall inception in compressors which exhibit short circumferential lengthscale, as opposed to 'modal' stall inception behaviour (see section 2.1 for a discussion of these types of behaviour). In these initial stages the flow disturbance is very localised within the compressor circumferentially, axially, and often radially. In the VIPER engine at middle to high speeds the duration of these initial stages is short (typically one to five rotor revolutions for the measurements reported in chapter 3), and in terms of the overall transient behaviour the detrimental effect is not great. With respect to active control of stall, however, this period is very important, because the initial local disturbance can act as a 'trigger' for the larger more axially uniform disturbance. That is, by the time the disturbance is of a shape such that it is accurately represented by this type of model it is already large. In order to achieve the ideal form of control, where disturbances are controlled at small amplitude with low energy actuators, it is necessary to control the disturbance while it is still localised within the compressor.

At low speeds the 'benign' front-end stall cannot be modelled due to the lack of axial definition in the model. In the engine one of the 'benign' stall cells grows to form the cell that leads to irreversible stall (chapter 3). Hence a model of this type, which cannot capture the benign stall, cannot be expected to accurately represent the dynamics of the initial stages of the stalling process even at low speed.

The limitation described above has been overcome using a more complex model (chapter 5) with axial and radial definition in addition to the circumferential dimension in the present model.

4.4 APPLICATION TO ACTIVE CONTROL OF STALL/SURGE

Ideally, active control would operate at negligible amplitude when the stall disturbance itself is small. The model described in this chapter is limited in its representation of this period, as discussed in the previous section. Nonetheless, useful guidelines can be obtained for active control of a real engine by devising a control scheme based on the model, as is reported in this section.

A number of actuator types have been suggested (section 2.3). The one chosen for investigation was the 'puffer jet' type used by Day [1993b]. The model was adjusted accordingly such that high pressure air could be introduced upstream of the compressor at each of the circumferential nodes. The injected air was assumed to be introduced at the local static pressure, but with high axial momentum, using a simple mixing sum. A number of control algorithms were tested. The most successful are described in the following two sections.

4.4.1 System Identification And Control: Axisymmetric Flow

As a simple initial case, active control of axisymmetric flow was considered. The non-axisymmetric behaviour of the engine model was excluded by removing the random unsteadiness added at each timestep. A system identification technique was used to characterise the flow in the model and the effects of the control system, rather than a direct linearisation of the fundamental equations of the model. The reason for doing this was that the technique is a general one that can be applied in future to other computer models of almost any description or in the course of a real engine control experiment.

4.4.1.1 Control with Axisymmetric Flow - Overview

It was attempted to characterise the model at a given unstable operating point by a linear differential equation of the flow and the puffer jet bleed input. This was done by feeding in a random time series as bleed input, and recording the resultant flow variations. A first order equation was found to be insufficient. It was found, however, that a good fit could be obtained from a second order equation with first and zeroth order terms in the control bleed. The coefficients of the differential equation could then be found at a particular operating point using a least squares method.

The characterisation process for the model is thus as follows:

1. Feed a series of random bleed inputs b into the system and measure the resulting flow q .
2. Calculate q'' , q' , q , b' , b , where a prime denotes a derivative with respect to time.
3. Use a least squares fit to calculate the coefficients a_i that best fit

$$q'' = a_1 q' + a_2 q + a_3 b' + a_4 b + a_5. \quad (\text{Equ 4.17})$$

The fit of this last equation can be checked graphically.

This characterisation process would be equally applicable to a real engine case, although the result may not be in the form of a second order differential equation.

Having characterised the system as a second order linear differential equation, the stability of the controlled system was analysed for control laws of a specific type, namely ones in which the control bleed is a linear combination of the compressor flow and its derivative. One complication here, which was found to be significant, is that the bleed input is based on the flow calculated one timestep previously. When this effect is accounted for, the equation derived in the characterisation process becomes a third order linear differential equation, which can be solved by finding the roots of the equivalent cubic equation (the 'eigenvalues' of the system). The real parts of the roots represent the stability of the controlled system; if any of these are positive the system is unstable. This fact can be used to show which combinations of the flow and its derivative will control the engine successfully. The controlling process is thus:

1. For any given values of control constants alpha and beta, set $b = \alpha \hat{q}' + \beta \hat{q}$, where $\hat{q} = q - dt \cdot q'$, and dt is the delay between the bleed input and the flow from which it was calculated.
2. Solve the resulting third order differential equation by solving the equivalent cubic equation.
3. Take the maximum real part of the three roots. This represents the fastest growing mode. If the maximum real part is negative, the system will be stable for the given control parameters alpha and beta.

The characterisation and controlling processes were checked by running the model with a range of control parameters and looking at the stability of the result.

4.4.1.2 Control with Axisymmetric Flow - Characterisation Process

The basic stall inception model described in the previous section used combustor inbleed to mimic the method by which stall was induced in the engine test. There was some difficulty in using this method for the current test, because the operating point of the compressor moves very fast as the bleed air is injected. This difficulty was avoided by setting up the model on the unstable side of the characteristic with no inbleed. It should be noted that the results then apply only to that operating point on the compressor characteristic. In order to obtain a general control law it would be necessary to repeat the calculations at various flow coefficients.

The random 'puffer jet' bleed input was two-sided. That is, positive and negative inputs were applied. Negative numbers correspond to 'outbleed', which is physically meaningless with the 'puffer jet' control system described above, but is allowed by the equations in the model and allows true linear control, as opposed to single-sided control. A more complicated control model would thus be required in the real engine case.

Typical input and output is shown in fig 4-14. Because the initial operating point is on the unsteady side of the compressor characteristic the flow falls away rapidly. Thus only a short period of between 40 and 150 timesteps (2.5 and 9.4ms) could be used for the characterisation work. The first 5 iterations were not used as there was a small starting transient due to a minor inconsistency between the calculations in the main loop and the part of the routine where the flow was initially set up. Despite the short time period the characterisation work was successful in that the fit of the linear equation to the data was exceptionally good, as described below. In a genuine engine case, however, where signals are noisy and the underlying physics more complex than the model equations, the fit would not be as good, and the problem of obtaining sufficient data on the unstable side of the characteristic would be harder to solve, perhaps necessitating some method of stabilising the compressor, such as an inlet gauze.

For the linearised flow equation (4.17) described above, the coefficients giving the least squared error are given by $\mathbf{a} = \mathbf{A}^{-1}\mathbf{v}$, where

$$\mathbf{v} = \sum_i \mathbf{w}_i \frac{d^2 q}{dt^2}_i, \text{ and} \quad (\text{Equ 4.18})$$

$$\mathbf{A} = \sum_i \langle \mathbf{w}_i, \mathbf{w}_i \rangle, \quad (\text{Equ 4.19})$$

where the angle brackets denote an outer product and the vector $\mathbf{w}_i$ is defined by

$$w_i = \left(\frac{dq}{dt}_i \quad q_i \quad \frac{db}{dt}_i \quad b_i \quad 1 \right)^T. \quad (\text{Equ 4.20})$$

All summations are over each timestep i in the sample.

Having calculated the coefficients a_i , $d^2 q/dt^2$ was calculated from the lower order terms (using equation 4.17) for comparison with the value calculated by differencing the 'measurements' from the model. An excellent fit was obtained (fig 4-15).

At 98% speed, the coefficients calculated for an initial flow of 22.68 kg/s are

$$a_1 = -0.006 \quad a_2 = 0.003 \quad a_3 = -0.884 \quad a_4 = 0.141 \quad a_5 = 0.0$$

The constant term a_5 has no contribution because the set-up flow is subtracted from the 'measured' flow in the least squares routine: in this axisymmetric case if there is no random input the compressor remains in unstable equilibrium, and hence the second derivative of the flow with respect to time is zero.

A slightly higher initial flow (24.04 kg/s) was run as a check on the results. The coefficients here were

$$a_1 = -0.025 \quad a_2 = 0.0016 \quad a_3 = -0.996 \quad a_4 = 0.118 \quad a_5 = 0.0$$

These are similar to those at the lower flow, perhaps because the slope of the compressor characteristic is similar. Note that the effect of the bleed flow and its derivative is considerably stronger than the effect of the compressor flow and its derivative.

4.4.1.3 Control with Axisymmetric Flow - Controlling Process

Linear control laws of the form $b_{act} = \alpha dq/dt + \beta q$ were investigated, where b_{act} is the controlling bleed flow through the puffer jets, and α and β are constants to be calculated from the linearised equation derived in the previous section in order to make the system stable. The bleed flow here is calculated from the compressor flow and its derivative, although other parameters such as inlet static pressure could equally have been chosen without changing the analysis. The above formula for b_{act} could be substituted directly into the linearised equation, but the result is not ideal, because the bleed flow at any timestep is calculated from the flow and its derivative one step earlier, whereas the differential equation was calculated using a current value for each of the parameters. This can be accounted for by noting that the value of q one timestep earlier was $q - \Delta t(dq/dt)$, where $\Delta t = 1$ (the timestep is normalised to unity throughout the characterisation and controlling processes). Thus if the control law is set up as above, the actual bleed input is defined by

$$b_{act} = \alpha \left(\frac{dq}{dt} - \Delta t \frac{d^2 q}{dt^2} \right) + \beta \left(q - \Delta t \frac{dq}{dt} \right). \quad (\text{Equ 4.21})$$

This formula can be used to predict stability including the effect of a time delay in the system by setting $\Delta t > 1$, provided the frequency of any disturbance is $\ll 1/\Delta t$.

When b_{act} is substituted into the earlier linearised differential equation (4.17) the result is a third order linear differential equation with constant coefficients. The stability of the system can then be analysed by looking at the real parts of the roots of the equivalent cubic equation. A routine was written to solve this equation for a range of values of α and β .

4.4.1.4 Control with Axisymmetric Flow - Results

The results of the stability analysis at an initial flow of 22.68 kg/s are shown in fig 4-16. The solid line encloses the region of α and β values over which the code should be stable to small disturbances at this initial flow. In order to check the result, the model was run repeatedly for up to 2000 timesteps with various values of alpha and beta, with stability gauged from the flow 'measurements'. These results are also shown in fig 4-16, where a tick represents a stable run, and a cross one in which the flow diverged. The agreement with the theoretical calculations is excellent. Similar calculations were done at an initial flow of 24.04 kg/s, with similar results.

The point of maximum stability can be found from a contour plot of the maximum real part of the roots of the cubic equation: in fact, the solid line on fig 4-16 is simply the zero contour level. The values at maximum stability turn out to be

$$\alpha = -0.73 \quad \beta = -0.05.$$

4.4.1.5 Control with Axisymmetric Flow - Discussion

Real compressors do not stall axisymmetrically (section 2.1, also chapter 3). The axisymmetric flow calculations described in this section are a 'trial run', both for the non-axisymmetric calculations described in the next section, and for future application to other stall models or a real engine experiment. As such, the results themselves are not as important as the close agreement they show with the stability of the model. Undoubtedly the close agreement stems from the good fit of the linearised equation; such a fit is unlikely to be obtained in a real engine case. Nonetheless, the method proved robust, required no prior knowledge of the mathematics of the model, and obtained an excellent result from a small sample of input data.

Although the first instability observed on throttling compressors to the surge point is non-axisymmetric, the axisymmetric mode is generally unstable at flows only slightly lower, and must be stabilised in these regions for active control to be effective. In this sense the results presented in this section are 'necessary but not sufficient' for successful active control at flows well below the surge point. Three features of interest stand out. Firstly, it is possible to stabilise the model with a control law of this type. Secondly, both elements (value and derivative) of the control law are necessary (the boundary of the stable region does not cross either of the zero axes). Thirdly, the boundary of the region is quite tight: extensive calculations with a series of speeds and operating points would be required to show whether fixed values of α and β could be found to stabilise operation over the entire range of interest. An overview of possible extensions to this work is given after the next section.

4.4.2 System Identification And Control: Non-Axisymmetric Flow

With the non-axisymmetric terms included, control of the compressors in parallel model described above becomes much more complex. The method used, however, is very similar. Again, a second order linear system was found as an approximation to the model. In this linear system the Fourier harmonics are independent, and the solution for each mode was found separately using the same cubic solver as for the axisymmetric case described above.

Having approximated the stall model to a linear system, the stability of the model was inferred from that of the linearised equations. One of the difficulties noted in the description of the axisymmetric case was that the details of the discretisation of the linearised equation proved significant, in that the controlling bleed flow was calculated based on the flow and its derivative one timestep earlier, whereas the linearised equations used current values for each parameter. This had to be taken into account before the stability of the linearised equations matched the stability of the stall model. In the non-axisymmetric case the discretisation in time and in the circumferential direction have to be matched, as described below.

4.4.2.1 Basic Equations

The basic linear equation used to describe the model is

$$q'' = \alpha q' + \beta q + \gamma b' + \delta b + \bar{\alpha} q' + \bar{\beta} q + \bar{\gamma} b' + \bar{\delta} b + \varepsilon \partial q / \partial n. \quad (\text{Equ 4.22})$$

In this equation, n refers to the node number in the circumferential direction (1-32). The basic flow variables q and b are now functions of circumferential position as well as time. The constants $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$, $\bar{\delta}$, are not themselves average values, but are

overscored to match the parameters they govern, which are annulus averaged quantities.

The 9 constants in the above equation were determined by applying random control signals to the stall model in exactly the same manner as the axisymmetric case. The values found for the lower flow (22.68 kg/s) case were

$$\begin{aligned}\alpha & -0.9855 \\ \beta & 0.0985 \\ \gamma & 3.6773 \\ \delta & 3.5777 \\ \bar{\alpha} & 0.8987 \\ \bar{\beta} & -0.0911 \\ \bar{\gamma} & -4.4867 \\ \bar{\delta} & -3.4086 \\ \bar{\varepsilon} & -0.2242\end{aligned}$$

Circumferential harmonics develop independently in the above linearised equation, and so it is useful to resolve the flow q into these harmonics:

$$q = \sum_{m=0}^{16} A_i(t) e^{2\pi i m n / 32} \quad (\text{Equ 4.23})$$

The zeroth order equation becomes simply

$$A_0'' = (\alpha + \bar{\alpha})A_0' + (\beta + \bar{\beta})A_0 + (\gamma + \bar{\gamma})\bar{b}' + (\delta + \bar{\delta})\bar{b}, \quad (\text{Equ 4.24})$$

which is identical in form to the axisymmetric equation described in the previous section. The calculated values of the constants are slightly different:

$$\alpha + \bar{\alpha} = -0.087, \quad \beta + \bar{\beta} = 0.007, \quad \gamma + \bar{\gamma} = -0.809, \quad \delta + \bar{\delta} = 0.169.$$

The form of the control law sought is again similar to the axisymmetric case:

$$b = \alpha_x q' + \beta_x q + \bar{\alpha}_x \bar{q}' + \bar{\beta}_x \bar{q} + \varepsilon_x \partial q / \partial n. \quad (\text{Equ 4.25})$$

This is an alternative to applying control to each harmonic individually. The reason for using it is that the dominant mode of stall failure appears from the experimental evidence (chapter 3) to be short circumferential lengthscale single cell rotating stall. In practice control of such disturbances would have to be performed by a smaller number of transducers and actuators than required to resolve the necessary harmonics. The data would then have to be combined with some sort of assumption, such as that of a slowly evolving rotating disturbance, in order to put this type of control into effect. In

these circumstances it was considered most convenient to have a purely local definition of the controlling bleed requirement, that is, one that depended only on transducers close to the disturbance itself.

Combining the control law equation with the harmonic description of the compressor flow q gives for mode $m > 1$,

$$A_m''(1 - \gamma\alpha_x) = A_m'(\alpha + \gamma\beta_x + \gamma\varepsilon_x i\omega + \delta\alpha_x) + A_m(\beta + \delta\beta_x + \delta\varepsilon_x i\omega + \varepsilon i\omega), \quad (\text{Equ 4.26})$$

where $\omega = 2\pi m/32$.

The constants $\bar{\alpha}_x$ and $\bar{\beta}_x$, related to the variation of annulus averaged flow and its derivative, do not occur in the equation for non-zero circumferential modes. This fact simplifies the work of finding coefficients which make the system stable. Instead of handling all five constants simultaneously, it is possible to define only the three constants occurring in this last equation, α_x , β_x and ε_x . The other two can then be chosen to bring $\alpha_x + \bar{\alpha}_x$ and $\beta_x + \bar{\beta}_x$ into the axisymmetric stability region, as defined by fig 4-16 (modified as necessary for the slightly different constants given above).

4.4.2.2 Discretisation Effects

In the description of the axisymmetric case (section 4.4.1.3) it was stated that a time delay was required to match the linearised stability analysis to the stall model itself. The same approach here again turns the linearised second order equation into a linear third order equation. For $m > 1$,

$$C_1 A_m''' + C_2 A_m'' + C_3 A_m' + C_4 A_m = 0, \quad \text{where} \quad (\text{Equ 4.27})$$

$$\begin{aligned} C_1 &= \gamma\alpha_x \Delta t \\ C_2 &= 1 - \gamma\alpha_x + \gamma\beta_x \Delta t + \gamma\varepsilon_x i\omega \Delta t + \delta\alpha_x \Delta t \\ C_3 &= -\alpha - \gamma\beta_x - \gamma\varepsilon_x i\omega - \delta\alpha_x + \delta\beta_x \Delta t + \delta\varepsilon_x i\omega \Delta t \\ C_4 &= -\beta - \delta\beta_x - \delta\varepsilon_x i\omega - \varepsilon i\omega \end{aligned} \quad (\text{Equ 4.28})$$

The best fit to the data was obtained with $\Delta t = 0.6$, as opposed to 1.0 in the axisymmetric case.

In the current non-axisymmetric case the discretisation of the linearised equations in the circumferential direction also becomes important. The coefficients of the linearised equations were calculated using a backwards difference. Now using a Taylor series expansion,

$$f(n) - f(n-1) = f'(n) - \frac{1}{2} f''(n) + \frac{1}{3} f'''(n) + \dots, \quad (\text{Equ 4.29})$$

and for a mode m , given any variable $\xi_m e^{im2\pi/32}$, the backwards difference is

$$\xi_m e^{i\omega n} - \xi_m e^{i\omega(n-1)} = \xi_m e^{i\omega n} (1 - e^{-i\omega}), \quad (\text{Equ 4.30})$$

where $\omega = 2\pi m/32$ as before.

In the analytical stability analysis, the analytic derivative $\xi_m i\omega e^{i\omega n}$ is of course assumed. Hence the effect of the discretisation was accounted for by replacing $i\omega$ with $(1 - e^{-i\omega})$ in the linearised equations given previously.

4.4.2.3 Non-Axisymmetric Flow Results

For simplicity, analysis was restricted at first to control laws where there is no dependency on the local time derivative of the flow coefficient (that is, $\alpha_x = 0$). The effect of the β_x and ε_x coefficients could then be analysed in isolation. In both the analytical work and the running of the model to check the results, the coefficients $\bar{\alpha}_x$ and $\bar{\beta}_x$, relating to the annulus averaged flow and its derivative, were calculated such that the axisymmetric equation (mode 0) lay on the maximum axisymmetric stability point as calculated in the previous section.

Having accounted for the discretisation of the linearised equations as described in the last section, the stability analysis shows very good agreement with the results obtained from running the model. Fig 4-17 shows lines of neutral stability for each of the harmonics in the circumferential direction for a range of beta and epsilon values. The parallelogram indicates the region in which the control law with the given coefficients should be expected to be stable. The same region is shown in fig 4-18, along with the results from a series of runs of the stall inception model with the control algorithm applied. The region of successful control can be seen to be mirrored effectively by these results. Furthermore, the model was observed to fail in a low mode where the high modes are within the neutral stability boundary and vice versa, in agreement with the analytical calculations.

There are two points at $\beta_x = -0.5$ outside the region of successful control at which the model runs stably. The reason for this is that the unstable modes of the linearised equations are at very high frequency in time (given by the imaginary part of the root), which cannot be matched in the model with the given timestep. If unstable modes with frequencies greater than $\pi/2$ are discounted, the lines of neutral stability move to those shown in fig 4-19, which match even more closely the results obtained by direct calculation. The 'steps' in this and previous figures are not genuine: they are a result of the coarseness of the mesh used for the calculations.

The influence of the α_x parameter on stability has not been studied in detail. It was noted, however, that a small positive value (0.1) had the effect of reducing the region

of successful control. At a value of -0.1 no region of successful control was present at all. This proved difficult to check directly with the stall model, however: the model seemed stable, but the disturbance predicted by the linearised analysis was low frequency and extremely slowly rotating, so it may have taken a very large number of timesteps to build up.

A brief analysis of the effect of the time delay was undertaken. Increasing the time delay had the effect of continuously reducing the size of the region of successful control. With time delays over one timestep the first order discretisation of the delay becomes insufficient. Instead a second order approximation was used, and attention limited to disturbances with frequencies sufficiently low to make the higher order terms negligible (with some judgement as to areas which were unstable at higher frequencies). With this proviso, the region of successful control disappeared completely at a delay of around 8 timesteps (0.5 ms).

4.4.2.4 Non-Axisymmetric Flow - Discussion

The most important result of the non-axisymmetric flow analysis is that a control law of the prescribed form could be found that stabilised the model in full non-axisymmetric mode. Further, the results in fig 4-17 show that the coefficients α_x and ε_x can be set to zero (given infinitely fast actuator mechanisms). This leads to a simpler control law with just one parameter related to the local flow, and two to the annulus averaged flow:

$$b = \beta_x q + \bar{\alpha}_x \bar{q}' + \bar{\beta}_x \bar{q} \quad (\text{Equ 4.31})$$

The values of the coefficients required for successful control can be taken from fig 4-16 and fig 4-17.

It should be stressed that the existence of such a control law for the stall model, which is based on relatively simple equations, is no guarantee of a similar law being successful for a real compressor, although it can be expected to give some indication as to its behaviour. Nonetheless the method used to derive the control law required no prior knowledge of the system being analysed, and could equally well be applied to a real compressor control experiment.

As with the axisymmetric results, a comprehensive set of results from a range of speeds and pressure ratios would be required to see if a constant set of coefficients could be found for the control law described above to successfully control stall at more than a single operating point. If not, the coefficients could fairly easily be made to follow a locus with changing steady state operating point.

The time delay (0.5ms) at which the region of successful control disappears represents a (tight) limit on the response time of the actuators involved. This delay is related to the time it takes for stall disturbances to grow, however, which is not very accurately represented by the stall inception model (section 4.3.2). A better estimate of the required response time in a real compressor can be obtained from the stall inception measurements in chapter 3.

4.4.2.5 Extensions to the Work

The most important extension to this work would be to apply the methods described above to a genuine compressor control experiment. Nonetheless there are other areas which could usefully be investigated:

1. The system could be applied to various operating points and speeds as described above to see if a generic control law is feasible.
2. The effect of time delays could be analysed more rigorously: the Δt approach used above is only applicable for disturbances with a cycle time large compared to the delay.
3. The effect of a 'single-sided' control process such as that used by Day [1993b] should be considered.
4. The analysis could be applied to the more complex stall model described in chapter 5.

5. 3D TIME-MARCHING MODEL

5.1 OBJECTIVES

The primary objective of the model is to provide a vehicle for numerical testing of possible strategies for active control of stall/surge in an aeroengine core compressor across the entire operating speed range. This objective gives rise to a number of requirements;

- The code must be time accurate. The results in chapter 3 show the initial stages of stall to occur on a timescale comparable to a practicable modern detection/reaction loop. Many of the important questions will therefore revolve around the detailed time response of the compressor to both the growing stall disturbance and the artificial controlling disturbance.
- The code must have axial, circumferential, and radial dimensions. This is discussed in detail in section 2.2. The requirement for a radial dimension is discussed further in chapter 3, where it is demonstrated that one of the differences between 'benign' stall and 'irreversible' stall at low speed in the VIPER compressor is the radial distribution of the flow disturbances.
- The code must be capable of multistage application. This precludes the use of a three dimensional code in each blade passage individually by reason of computing time/cost.
- The code must capture something of the initial breakdown of the flow field. Stall inception is often reckoned to start around the tips of rotors, as the tip leakage jet becomes progressively stronger. In an axial by circumferential model this could be accounted for by modifying the bladerow performance in some way, but then the differing effect of a control action at various spanwise positions would also have to be accounted for (this problem can be significant; 'puffer jets' in the casing, as used by Day [1993b] would be expected to give a quite different effect at hub, mean and tip). In a three dimensional model it is more attractive to model the initial flow breakdown by including the effect of the tip gap directly. Modelling the near wall flow in this way gives rise in turn to the requirement of some form of turbulence model.
- The code is not required to model post-stall transients, although this would be a valuable future addition. Thus operation in fully reversed flow is not required. It

also has a bearing on the real-time duration of the model (a relatively small number of rotor revolutions is sufficient).

- The code is not required to model low speed compressors. This allows the use of a time-marching scheme that would be unstable at very low Mach Numbers. Section 5.4 does describe an application of the model to a low speed compressor, but the numerical calculation was performed at a considerably higher rotational speed than the physical experiment in order to avoid stability problems. Attention is also confined to subsonic flow.
- The accuracy of the code in predicting the steady state flowfield prior to stall is of secondary importance (but see below).

Secondary objectives are to produce an accurate general-purpose throughflow model (that is, an axial by radial steady state compressor model) including spanwise mixing terms, and also to provide a method of stall point prediction. It should be noted that high accuracy (as required for a throughflow model) and a good time response (as required for a stall inception model) can be conflicting requirements.

The objectives for the code as outlined above are aggressive. Perhaps a more realistic objective for this first attempt is to demonstrate that such a model can effectively represent three-dimensional stall and surge inception events. Accordingly, when choices arose (such as the choice of turbulence model), it was decided to use the simplest technique which captured the essential physics with respect to the stall inception event (in the case of the turbulence model, a fairly crude mixing length model was adopted, as described below).

5.2 DESCRIPTION

5.2.1 Flow Equations

The equations to be solved are the Reynolds' averaged Navier-Stokes equations (presented here using convenient, if mixed, notation):

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\rho \mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \underline{F} + \frac{\partial}{\partial x_j} \left[\mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + \nabla (\lambda_t \nabla \cdot \mathbf{u}) \quad (\text{Equ 5.1})$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) \quad (\text{Equ 5.2})$$

$$\rho \frac{\partial e}{\partial t} = -\rho \mathbf{u} \cdot \nabla e - p \nabla \cdot \mathbf{u} + \frac{\partial Q}{\partial t} + \Phi + \nabla \cdot (\kappa_t \nabla t) - \nabla \cdot \mathbf{q}_r \quad (\text{Equ 5.3})$$

These equations can all be found in Hughes and Gaylord [1964]. Some work is required before discretisation to recast especially the last equation into a suitable form using the chosen basic flow variables (section 5.2.2.4).

Aside from the main flow variables there are a number of parameters in the above equations that need to be evaluated;

- $\underline{F}$ The body force supplied by the blading, calculated from the bladerow performance (loss and deviation) as described in section 5.2.6.
- μ_τ, λ_τ First and second turbulent viscosities, derived using a simple mixing length model as described in section 5.2.3.
- $\frac{\partial Q}{\partial t}$ Heat addition, assumed to be negligible. Note that the work supplied by the blading does not need to be included in the energy equation using this formulation.

$$\Phi = \mu_\tau \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} + \lambda_\tau \frac{\partial u_k}{\partial x_k} \frac{\partial u_l}{\partial x_l} \quad \text{[Hughes and Gaylord, 1964] is the dissipation.}$$

- κ_τ Turbulent heat transfer coefficient, considered further in section 5.2.3.
- $\mathbf{q}_r$ Radiation heat flux vector, assumed negligible.

One further parameter arising from the relationship between the energy and pressure terms is the ratio of specific heats, γ . This has been coded as a variable, but is fixed at a value of 1.4 for all of the results presented in the thesis.

5.2.2 Discretisation

5.2.2.1 Overview

There are of course many solution methods for the equations above, suited to different types of geometry and flow. One significant feature of a pitchwise averaged stall inception model as opposed to most three dimensional flow problems is that the calculation domain is a volume of rotation, and hence the same radial by axial grid can be used at each circumferential point around the annulus. The solution method presented here has been especially developed to take advantage of this simplifying feature of the geometry.

The central part of the model consists of a finite element solution to the flow at each timestep in each radial by axial plane. A description of the general form of finite

element modelling in fluid dynamics can be found in (among others) Chung [1978]. The primary reason for using a finite element technique is that it ensures a form of conservation (see section 5.2.2.4 below). All axisymmetric terms in the flow equations (that is, terms which can be non-zero for axisymmetric flow) are included in the finite element calculations, including those derived from theta-derivatives of the unit vectors $\hat{r}$ and $\hat{\theta}$. In this way the secondary objective of an accurate radial by axial steady state model can be met without discretisation errors in the theta direction.

In most three-dimensional flow solvers the time derivatives of the flow variables are calculated approximately at each timestep, with the 'residual' error calculated and added to the flow equations as a correction at the next timestep. The simplified geometry of the current task makes it possible to solve for the time derivatives with no residual. This involves a series of matrix inversions, described below in section 5.2.2.8. The matrices depend only on the geometry, and are calculated and inverted *once only*, before the first timestep. Even so, the storage space required would be prohibitive but for the fact that the same set of inverted matrices can be applied at each circumferential node.

The discretisation in the theta direction is handled differently. It has already been noted that accuracy is of prime importance for axisymmetric flow (the throughflow model), and that those terms appearing in axisymmetric flow are handled in the finite element part of the model. Hence the terms relating to the theta discretisation are related only to non-axisymmetric flow, and a simple finite difference calculation is sufficient.

It is to be emphasised that the discretisation is rigorous: in the limit $\{dr, d\theta, dz \rightarrow 0\}$ the discretised equations revert to those of section 5.2.1. In particular all of the viscous terms deriving from these equations are included in the model, although some very minor approximations are made to increase calculation speed (referred to in section 5.2.2.3). This is at variance with a number of other calculations (for instance Howard and Gallimore [1993]), where only the most significant spanwise mixing terms are addressed.

5.2.2.2 Finite Element Method

In this section the finite element method is described, but only as it is implemented in this particular model. A description of the general form of finite element modelling in fluid dynamics can be found in (among others) Chung [1978].

The discretisation of the annulus for the finite element calculation is based on hexagonal elements in the r - z plane as shown in fig 5-1. Flow conditions are assumed

to be constant over each circumferential section $[\theta, \theta + \delta\theta]$. Each element is divided into six triangular 'cells', over which the basic flow variables will be linearly interpolated; in finite element terminology, the 'basis' and 'test' functions S for each element are hexagonal based cones as shown at the right hand side of fig 5-1. Each timestep of the finite element method can then be conceptualised as a sequence of three steps:

1. The basic flow variables are interpolated linearly over the cells making up each element. In this way the flow is precisely defined not just at the nodes but between them as well.
2. The flow equations are used to calculate exactly what the time derivative of each flow variable should be everywhere in the calculation plane. This is then weighted by the function S and integrated over the hexagonal element.
3. An increment is calculated for the flow variables at each node such that the change over each element, when weighted by the function S and integrated, agrees with the calculated time derivative.

The interpolation in the above steps is what distinguishes the finite element method from the finite volume method. The first step above ensures *consistent* interpolation of variables. In a finite volume method difficulties can arise from conflicting interpolation methods across cells. For instance, the result of interpolating u^2 between nodes and integrating is different from interpolating u between nodes, squaring the result and then integrating. In the finite element method these difficulties do not arise, because the flow is uniquely defined everywhere in the calculation plane before the flow equations are applied.

The objective of the weighted integrals in the second and third steps is to reduce the time derivatives calculated everywhere in the flowfield back to the amount of data that can be stored at the nodes only. This is done in a way that ensures a form of conservation as described in section 5.2.2.4. An equivalent calculation is performed in the finite volume method. The calculation here is carried out exactly using a matrix inversion method described in section 5.2.2.8.

5.2.2.3 Example

A simple example of the discretisation procedure is afforded by the continuity equation;

$$\partial\rho/\partial t = -\nabla \cdot (\rho\mathbf{u}), \quad (\text{Equ 5.4})$$

which in 'local' finite element form becomes

$$\frac{d}{dt} \iiint_V \rho s dV = \iiint_V -s \nabla \cdot (\rho \mathbf{u}) dV . \quad (\text{Equ 5.5})$$

The integrals here are over the volume created by rotating the area of a hexagonal element in the r - z plane through an angle $\delta\theta$ around the z axis. Now

$$\begin{aligned} \iiint_V -s \nabla \cdot (\rho \mathbf{u}) dV &= \iiint_V \rho \mathbf{u} \cdot \nabla s dV - \iiint_V \nabla \cdot (s \rho \mathbf{u}) dV \\ &= \iiint_V \rho \mathbf{u} \cdot \nabla s dV - \iint_A s \rho \mathbf{u} \cdot d\mathbf{A} \end{aligned} \quad (\text{Equ 5.6})$$

by the divergence theorem, where A represents the total area surrounding the volume V . On the boundaries in the r - z plane, however, s is identically zero. Hence the last term reduces to

$$-\left[\iint_A \rho s u_\theta dA \right]_\theta^{\theta+\delta\theta} . \quad (\text{Equ 5.7})$$

As $\delta\theta \rightarrow 0$, $dV \sim r dA \delta\theta$ and the overall equation reduces to

$$\frac{\partial}{\partial t} \iint_A \rho s r dA = \iint_A \rho \mathbf{u} \cdot \nabla s dA - \frac{\partial}{\partial \theta} \iint_A \rho s u_\theta dA . \quad (\text{Equ 5.8})$$

The choice of $\omega = \frac{1}{\rho r}$ as the linearly interpolated density variable instead of ρ (as

described in the next section) poses some particular problems, which are discussed later. For simplicity it is supposed here that ρ as well as u is one of the linearly interpolated basic flow variables.

It is convenient at this point to see how integration is performed in the finite element method (as implemented here). Each of the terms in the above equation is an area integral to be performed over each hexagonal element. The integrand in each case is a product of either basic flow variables, or the weighting function s , or ∇s . The last of these, ∇s , is constant over the triangular 'cells' making up the hexagonal 'element'. The basic flow variables and the weighting function s are all linearly interpolated over each triangular cell. Hence the first step is to split the integration area into its constituent triangles¹. It is convenient to adopt the following notation for the linear interpolation of a variable x over a cell;

¹ in order to do this correctly, it is necessary first to manipulate the terms into a form such that they are bounded [in a mathematical sense] in the border regions between the cells. By way of example, it would not be

$$x = x_1 s_1 + x_2 s_2 + x_3 s_3, \quad (\text{Equ 5.9})$$

where x_1 (for instance) is the value of x at node 1, and s_1 is a linearly interpolated variable with value one at node 1 and zero at nodes 2 and 3. Using this notation, an integral of the form $\iint_{\text{Cell}} xyz dA$ can be written $\sum_{i,j,k=1}^3 x_i y_j z_k \iint_{\text{Cell}} s_1^m s_2^n s_3^p dA$, where $m = \delta_{i1} + \delta_{j1} + \delta_{k1}$, with n and p defined similarly. The advantage of this formulation is that this last integral is independent of the shape of the triangle, having the value $\frac{2Am!n!p!}{(m+n+p+2)!}$, where A is the area of the triangle [Chung 1978, page 77], and the

usual convention is followed of defining $0! = 1$. Terms with fewer or more factors in the integrand are handled in the same manner. It would of course be prohibitive to calculate this factor for each triangular cell for each hexagonal element for each calculation and for each iteration. Instead, the factors multiplying the cell areas are calculated before the first iteration and stored for subsequent use. Even so, the number of calculations is very large, particularly for terms with a large number of factors. For this reason some minor simplifications have been made (for instance interpolating r^2 instead of r) but only in terms that are expected to be small and that contain a viscous term (μ_τ or κ_τ), and hence are in any case subject to uncertainty due to the simplicity of the turbulence model.

Returning to equation 5.8, the three terms are representative of the three types occurring in each of the discretised equations. The first type, appearing on the left hand side of each of the discretised equations, is the time derivative of an area integral over each hexagonal element. The integral is simply related to the nodal values (as described above). The nodal values are known for the current timestep, but for the next timestep need to be deduced from the integral values using a matrix inversion described in section 5.2.2.8.

The second term in equation 5.8 is an area integral over a hexagonal element, calculated using the method described above.

The third term in equation 5.8 is the theta derivative of an area integral. The integration is identical in form to the previous ones and can be calculated from the

correct to split up the integral of a second derivative of one of the main flow variables, as with linear interpolation of these variables across cells the second derivative becomes infinite at the boundaries. Such terms can always be reduced by application of the divergence theorem to a form in which the terms remain finite at the boundaries. The principle outlined here can be formalised mathematically by assuming a type of interpolation over a small boundary width, and taking the limit as the boundary width tends to zero.

nodal values. The form of the finite difference discretisation in the theta direction is discussed in section 5.2.2.7.

5.2.2.4 Flow Variables and Conservation

The flow variables used in the calculation are velocity $\mathbf{u}$, internal energy e (for constant gamma related to static pressure by $e = p/(\gamma - 1)$), and $\omega = 1/r\rho$. This choice of variables provides for a simple finite element description, in which the number of independent factors in each term of the equations can be kept small. It is recognised however that these variables are not the natural choice with respect to conservation, which would be the conserved parameters of mass, momentum and energy.

Conservation in the finite element method is ensured in an integrated sense. A formalised proof is beyond the scope of this thesis, but an indication of the reasoning is given here using as an example a simple transport equation in two dimensions:

$$\partial\phi/\partial t = -\mathbf{u} \cdot \nabla\phi, \quad (\text{Equ 5.10})$$

where for simplicity it is assumed that $\nabla \cdot \mathbf{u} = 0$. In finite element form the transport equation reduces to the following for each hexagonal element:

$$\iint_A s \partial\phi/\partial t dA = -\iint_A s \mathbf{u} \cdot \nabla\phi dA \quad (\text{Equ 5.11})$$

$$= \iint_A -\nabla \cdot (s \mathbf{u} \phi) + \phi s \nabla \cdot \mathbf{u} + \phi \mathbf{u} \cdot \nabla s dA \quad (\text{Equ 5.12})$$

$$= -\oint_C s \phi \mathbf{u} \cdot \mathbf{n} dl + \iint_A \phi \mathbf{u} \cdot \nabla s dA, \quad (\text{Equ 5.13})$$

using the divergence theorem and noting that $\nabla \cdot \mathbf{u} = 0$. As $s = 0$ everywhere on the boundary C , the first term disappears, and the whole equation can be rewritten

$$\frac{\partial}{\partial t} \iint_A s \phi dA = \iint_A \phi \mathbf{u} \cdot \nabla s dA. \quad (\text{Equ 5.14})$$

The basis of the finite element method as laid out above is that if the integrations are performed using the pre-defined linear interpolation procedure, then the above equation will by definition hold exactly. But this equation can be shown to be a conservation equation; each triangular cell in the calculation region (except boundary cells which are affected by the boundary conditions) forms a part of three separate hexagonal elements, each having its central node at a different corner of the triangle (fig 5-2). Thus each triangle contributes to the weighted time derivative equation

(5.14) of three elements, weighted by s_1 , s_2 and s_3 respectively (defined as in the previous section). Noting that

$$s_1 + s_2 + s_3 = 1 \quad , \quad (\text{Equ 5.15})$$

$$\text{and hence } \iint_A \phi \mathbf{u} \cdot \nabla (s_1 + s_2 + s_3) dA = 0 \quad , \quad (\text{Equ 5.16})$$

the total contribution of the fluxes within the triangle can be seen to be zero. That is, any positive contribution to one of the adjoining elements is balanced by a negative contribution to the other two².

5.2.2.5 The Density Parameter w

Some particular difficulties derive from the use of the density parameter $\omega = \frac{1}{\rho r}$. The

use of this parameter ensures that the number of factors in most of the terms of the discretised equations is kept to four or less, but in some equations leads to a factor in the denominator. Equation 5.8 for instance becomes

$$\frac{\partial}{\partial t} \iint_A \frac{s}{\omega} dA = \iint_A \frac{\mathbf{u} \cdot \nabla s}{\omega} dA - \frac{\partial}{\partial \theta} \iint_A \frac{s u_\theta}{r \omega} dA \quad . \quad (\text{Equ 5.17})$$

Terms such as the final one can be satisfactorily calculated using the approximation $x^{-1} \approx \bar{x}^{-1} (1 - \hat{x} + \hat{x}^2)$, where $\bar{x}$ is the average of the nodal values, and $\hat{x} = (x - \bar{x})/\bar{x}$.

The first two terms are important for mass conservation, however, and are calculated exactly. The analytic solution is lengthy and not reproduced here. The integration of these terms is dependent on the shape of the triangular cell, which causes a problem with respect to the term on the left hand side of the equation as it prevents the use of the matrix inversion method described in section 5.2.2.8. For this reason it was decided to calculate the left hand side interpolating $1/\omega$ linearly and to use the matrix inversion technique to calculate the new nodal values, but then to use these new nodal values to calculate a small correction term to be applied at the next timestep. In practice this term was found to be small and was neglected for the calculations presented in the thesis.

² A formalised description would start with a large area formed by joining an arbitrary number of hexagonal elements, and use a similar procedure to the one described here to show that the time derivative of the area integral over that area was dependent only on the boundary cells, and then in such a way that fluxes out of the area were equal to fluxes entering adjoining areas.

5.2.2.6 Differentiation of Vectors

It was stated previously that 'axisymmetric terms', that is terms which appear in axisymmetric flow, are accounted for using the more carefully controlled finite element discretisation rather than the finite difference discretisation in the theta direction. This requires careful handling of vector differentiation. For example, the transport term in the momentum equation (in finite element form, using velocity as the main variable) is

$$\iint_A -sru \cdot \nabla u dA = \iint_A -sru \cdot \nabla^{\text{2D}} u - su_\theta \frac{\partial u}{\partial \theta} dA, \quad (\text{Equ 5.18})$$

where ∇^{2D} represents the gradient in the r - z plane. The first term on the right hand side can be integrated using the finite element discretisation methodology described above. Care has to be taken with the second term, however. A discrete vector differentiation $(\partial u / \partial \theta = [u]_\theta^{\theta+\delta\theta} / \delta\theta)$, while accounting for derivatives of the unit vectors $\hat{r}$ and $\hat{\theta}$, does introduce a discretisation error. Instead, the approach taken is to split the vector derivative into 'axisymmetric' and 'non-axisymmetric' parts:

$$\frac{\partial u}{\partial \theta} = \underbrace{u_r \hat{\theta} - u_\theta \hat{r}}_{\text{Axisymmetric}} + \underbrace{\frac{\partial u_r}{\partial \theta} \hat{r} + \frac{\partial u_\theta}{\partial \theta} \hat{\theta} + \frac{\partial u_z}{\partial \theta} \hat{z}}_{\text{Non-Axisymmetric}}. \quad (\text{Equ 5.19})$$

The axisymmetric terms are integrated using the finite element methodology described above. The non-axisymmetric terms are integrated in a similar way to the theta derivative in the third term of the example equation above (equation 5.8).

5.2.2.7 Discretisation in the Circumferential Direction

For reasons that have already been explained a finite difference method is used to calculate the effects of non-axisymmetry. In the absence of viscosity (which is neglected in the theta direction - section 5.2.3), the relevant terms are all derived from first derivatives with respect to angle θ . With limited circumferential definition there is a limit to the accuracy that can be obtained in the circumferential direction: in any case, some of the pitchwise averaging assumptions break down for high circumferential harmonics. It is most important, therefore, that a system of discretisation in the theta direction is chosen such that the growth rates and rotation rates of the low circumferential harmonics are accurately captured.

Early results (not reported in the thesis) were obtained using a simple central difference scheme with 15 nodes circumferentially. The results showed a growing 5th circumferential harmonic disturbance travelling around the annulus at around 20% of rotor speed, which is unexpectedly low. The reason can be elucidated using linear

analysis. For the m^{th} circumferential harmonic, the first derivative as calculated using a central difference scheme is

$$\begin{aligned}\frac{d(e^{im\theta})}{d\theta} &= (e^{im(\theta+\delta\theta)} - e^{-im(\theta-\delta\theta)})/2\delta\theta \\ &= e^{im\theta} (e^{im\delta\theta} - e^{-im\delta\theta})/2\delta\theta \\ &= e^{im\theta} (i \sin(m\delta\theta)/\delta\theta)\end{aligned}\quad (\text{Equ 5.20})$$

In the limit $\delta\theta \rightarrow 0$, this is of course the same as the analytic result $ime^{im\theta}$. With non-infinitesimal $\delta\theta$, however, the result is wrong by a factor of $\sin(m\delta\theta)/m\delta\theta$. In a simple circumferential transport equation, this factor represents the amount by which the propagation rate is miscalculated, and it is not necessarily small. For the fifth harmonic and fifteen nodes circumferentially, the factor is 0.41, which explains the unexpectedly low rotational speed of the observed disturbance.

Fig 5-3 shows propagation speed factors based on the above analysis for different numbers of nodes in the circumferential direction and different harmonic disturbances. It shows that a large number of nodes (at least 47) would be required to calculate a reasonable propagation rate for even the fifth circumferential harmonic. Under these circumstances, where the primary objective is to correctly model the low harmonics, and large numbers of nodes would give an unacceptable penalty in terms of calculation time, it is natural to consider the possibility of a higher order discretisation. The calculation time required for calculations associated with the higher order scheme would be negligible compared to the time required for the main flow calculations.

A very high order calculation would be inappropriate for this type of model, where the circumferential distribution of parameters can be highly non-uniform. Consideration was only given, therefore, to schemes involving at most two nodes either side of the one in question. A general scheme of this type for nodes $\delta\theta$ apart can be written

$$\frac{df}{d\theta_i} \approx \alpha f_{i+2} + \beta f_{i+1} + \gamma f_i + \delta f_{i-1} + \varepsilon f_{i-2} \quad , \quad (\text{Equ 5.21})$$

where the constants α , β , γ , δ and ε have yet to be specified. Using this scheme the theta derivative of the m^{th} circumferential harmonic is defined by

$$\frac{de^{im\theta}}{d\theta} = \alpha e^{im(\theta+2\delta\theta)} + \beta e^{im(\theta+\delta\theta)} + \gamma e^{im\theta} + \delta e^{im(\theta-\delta\theta)} + \varepsilon e^{im(\theta-2\delta\theta)} \quad , \quad (\text{Equ 5.22})$$

which can be written as $\zeta e^{im\theta}$, where ζ has a real part

$$(\alpha + \varepsilon) \cos 2m\delta\theta + (\beta + \delta) \cos m\delta\theta + \gamma , \quad (\text{Equ 5.23})$$

and an imaginary part

$$(\alpha - \varepsilon) \sin 2m\delta\theta + (\beta - \delta) \sin m\delta\theta . \quad (\text{Equ 5.24})$$

When applied in a simple wave propagation equation the real part corresponds to an artificial growth factor, and can be made identically zero for all mode numbers by setting

$$\gamma = 0, \quad \delta = -\beta, \quad \varepsilon = -\alpha . \quad (\text{Equ 5.25})$$

The imaginary part determines the propagation speed, and ideally should be close to the analytic result, which is the mode number m . Substituting equations 5.25 into equation 5.24 gives an imaginary part

$$2\alpha \sin 2m\delta\theta + 2\beta \sin m\delta\theta . \quad (\text{Equ 5.26})$$

A natural choice is to choose α and β in such a way as to force the propagation rate of the first two circumferential harmonics to be exactly correct. This is achieved by setting

$$\alpha = \frac{(\sin 2\delta\theta - 2 \sin \delta\theta)}{2(\sin^2 2\delta\theta - \sin 4\delta\theta \sin \delta\theta)} \quad (\text{Equ 5.27})$$

$$\beta = \frac{(1 - 2\alpha \sin 2\delta\theta)}{2 \sin \delta\theta}$$

With 15 nodes, the factor on the propagation rate of the fifth harmonic using this method is 0.66, compared with the 0.41 factor quoted above for the central difference scheme. With 31 nodes, however, the factor using this scheme is 0.97, compared to 0.84 for the central difference scheme.

The propagation speed factor for the five point theta discretisation described above with 31 nodes circumferentially is compared with that for the central difference scheme in fig 5-3, and shows a considerable improvement. The scheme was therefore adopted, with 31 nodes in the circumferential direction.

5.2.2.8 Matrix Inversion

The time derivative term on the left hand side of each of the flow equations reduces to the form $\frac{d}{dt} \iint_A \phi r s dA$, where ϕ represents the flow parameter in question. This integral

can be performed in the same way as in the terms on the right hand side of the

equations as described in section 5.2.2.3, and results in a linear combination of the values of $d\phi/dt$ at the 7 nodes in or at the edge of the hexagonal element. The constants in this linear combination depend only on the geometry in the r - z plane, and so do not change either in time from iteration to iteration, or circumferentially from node to node. For this reason the matrix inversions described below can be performed prior to the first iteration and so do not penalise the speed of the time-marching calculations.

The boundary conditions described in section 5.2.4 also give rise to equations involving the time derivatives of the flow variables at the nodes. Taken together with the main equations, the linear combinations of time derivatives can be recast into matrix form. A simultaneous solution of the entire set of linear equations is impracticable in terms of calculation time and memory requirements. The following procedure is efficient in terms of pre-calculation and memory storage requirements. It uses the fact that the equations relating to the elements at a given axial location i involve time derivatives from the nodes at that axial location, the previous one, and the following one (in a similar way to the Tri-Diagonal Matrix Algorithm (TDMA) as described, for instance, by Mitchell and Griffiths [1980, p32]). In matrix form;

$$\underline{\underline{M}}_i \underline{c}_{i-1} + \underline{\underline{E}}_i \underline{c}_i + \underline{\underline{P}}_i \underline{c}_{i+1} = \underline{f}_i, \quad (\text{Equ 5.28})$$

where $\underline{c}_i$ is a column vector of the time derivatives at axial position i , $\underline{f}_i$ is a column vector containing the integrated values on the right hand side of the flow equations, coupled with the top and bottom boundary conditions, and $\underline{\underline{M}}_i$, $\underline{\underline{E}}_i$ and $\underline{\underline{P}}_i$ are constant matrices (Minus, Equal and Plus, for convenience) relating the nodal time derivatives in the three relevant columns to the integrated time derivative over the element. Clearly $\underline{\underline{M}}_1 = \underline{0}$, $\underline{\underline{P}}_m = \underline{0}$, as there is no data outside of the calculation grid, and the boundary conditions are used to determine the time derivatives at inlet and exit.

The equation above relates the values at three adjacent axial positions. By marching towards compressor inlet from the exit boundary it is possible to derive an equation relating the values at just two axial positions; say we have already at column j

$$\underline{c}_j = \underline{\beta}_j \underline{c}_{j-1} + \underline{g}_j, \quad \text{for some } \underline{\beta}_j, \underline{g}_j \quad (\text{this is clearly true at the exit plane}).$$

Combining this with equ 5.28 applied at axial position $j-1$,

$$\underline{\underline{M}}_{j-1} \underline{c}_{j-2} + \underline{\underline{E}}_{j-1} \underline{c}_{j-1} + \underline{\underline{P}}_{j-1} \underline{c}_j = \underline{f}_{j-1}, \quad (\text{Equ 5.29})$$

$$\text{gives } \underbrace{\left[\underline{E}_{j-1} + \underline{P}_{j-1} \underline{\beta}_{j-1} \right]}_{\underline{\Gamma}_{j-1}} \underline{c}_{j-1} = \underline{f}_{j-1} - \underline{M}_{j-1} \underline{c}_{j-2} - \underline{P}_{j-1} \underline{g}_j \quad (\text{Equ 5.30})$$

$$\text{or } \underline{c}_{j-1} = \underline{\beta}_{j-1} \underline{c}_{j-2} + \underline{g}_{j-1} \quad (\text{Equ 5.31})$$

$$\text{where } \underline{\beta}_{j-1} = -\underline{\Gamma}_{j-1}^{-1} \underline{M}_{j-1} \quad (\text{Equ 5.32})$$

$$\text{and } \underline{g}_{j-1} = \underline{\Gamma}_{j-1}^{-1} \left[\underline{f}_{j-1} - \underline{P}_{j-1} \underline{g}_j \right]. \quad (\text{Equ 5.33})$$

Applying this procedure from both the exit and the inlet plane, together with equ 5.28 the following system of three equations is obtained at position i ;

$$\underline{M}_i \underline{c}_{i-1} + \underline{E}_i \underline{c}_i + \underline{P}_i \underline{c}_{i+1} = \underline{f}_i \quad (\text{Equ 5.28, repeated})$$

$$\underline{c}_{i+1} = \underline{\beta}_{i+1}^+ \underline{c}_i + \underline{g}_{i+1}^+ \quad (\text{Equ 5.34})$$

$$\underline{c}_{i-1} = \underline{\beta}_{i-1}^- \underline{c}_i + \underline{g}_{i-1}^-, \quad (\text{Equ 5.35})$$

from which can be deduced

$$\underbrace{\left[\underline{E}_i + \underline{M}_i \underline{\beta}_{i-1}^- + \underline{P}_i \underline{\beta}_{i+1}^+ \right]}_{\underline{\Omega}_i} \underline{c}_i = \underline{f}_i - \underline{M}_i \underline{g}_{i-1}^- - \underline{P}_i \underline{g}_{i+1}^+ \quad (\text{Equ 5.36})$$

$$\text{and hence } \underline{c}_i = \underline{\Omega}_i^{-1} \left(\underline{f}_i - \underline{M}_i \underline{g}_{i-1}^- - \underline{P}_i \underline{g}_{i+1}^+ \right). \quad (\text{Equ 5.37})$$

In the equations above the $\underline{\beta}$, $\underline{\Gamma}$ and $\underline{\Omega}$ matrices are entirely geometry dependent, and so can be calculated and inverted before the first iteration. The $\underline{g}$ vectors are dependent also on the right-hand-side integral values and boundary conditions ($\underline{f}_j$). Hence the nodal values can be determined from two passes in the axial direction, one downstream calculating the $\underline{g}^-$ vectors, and one upstream calculating the $\underline{g}^+$ vectors and deriving the nodal values $\underline{c}_i$.

In the integral of a parameter over a hexagonal element (particularly with the weighting function s), greatest weight is given to the central node. This ensures the dominance of the leading diagonal in the matrix $\underline{E}$, and also that the matrix $\underline{E}$ dominates $\underline{M}$ and $\underline{P}$. This in turn ensures that the matrices above are well behaved and invertible, and also that any errors in marching from the boundaries decay (as does the effect of the boundary conditions).

5.2.3 Turbulent Mixing

In line with the objectives of this first attempt at a 'viscous' stall inception model, a simple mixing length turbulence model was adopted, following the method of Howard and Gallimore [1993] for "spanwise mixing" throughflow calculations, first introduced by Gallimore and Cumpsty [1986a,b]. Two significant departures are made, however. Firstly, Howard and Gallimore calculated the flow near the annulus walls only to a position in the log law region. In the time marching code described here points may be placed in the laminar sublayer. Secondly, the previous authors applied the calculated viscosity only to flow variations in the radial direction. The current formulation uses the calculated level of turbulent mixing in both the radial and axial directions (the grid definition in the circumferential direction is too coarse for the viscous terms to be significant).

The basis for the turbulence model is the following equation for mixing length l after van Driest, which can be found in Schlichting [1979, page 604].

$$l = \chi y \left(1 - e^{-y^+/A} \right), \quad (\text{Equ 5.38})$$

where $\chi=0.4$ and $A=26$ are constants. $y^+ = \frac{y v_*}{\nu_{lam}}$ corresponds to the distance from the

wall normalised by the laminar viscosity ν_{lam} and the friction velocity, defined as

$v_* \approx l \left| \frac{du}{dy} \right|$, where u is the absolute value of the velocity and y is distance normal to the

wall. These formulae are to some extent misapplied, as they are intended for use in parallel flows with no pressure gradient. They have been shown, however, to give reasonable steady state boundary layer profiles in axial compressors [Howard and Gallimore, 1993]. Note that even three dimensional blade passage calculations can give doubtful results in the near wall regions, and so the degree of reasonableness is much looser than, say, would be required in a pure boundary layer calculation.

Combining the above formulae gives

$$l = \chi y \left(1 - e^{-ly \left| \frac{du}{dy} \right| / \nu_{lam}^A} \right). \quad (\text{Equ 5.39})$$

This equation is applied iteratively to determine the mixing length l .

Having found the mixing length l , the overall viscosity level is calculated as the sum of the laminar and turbulent components,

$$\mu_{\tau} = \mu_{lam} + \mu_{\tau} , \quad (\text{Equ 5.40})$$

$$\text{where } \mu_{\tau} = \rho l^2 \left| \frac{du}{dy} \right|. \quad (\text{Equ 5.41})$$

Two modifications are applied to ensure a reasonable profile of turbulent viscosity.

Firstly, the viscosity is prevented from exceeding a preset level $\mu_{\tau,main}$ for the

mainstream flow. Secondly, in all of the calculations the velocity gradient $\left| \frac{du}{dy} \right|$ is

subject to a minimum level to ensure that the viscosity does rise to this level.

The accuracy of the code in calculating steady flows using the van Driest expression was checked using a test case of flow through a duct with no blades, described in section 5.3.3.

In spanwise mixing calculations the level of turbulence has to account not only for true small lengthscale turbulence, but also larger lengthscale mixing effects due, for instance, to blade wakes or tip clearance flows. For this reason the value of turbulent viscosity used is higher than normal turbulence levels would suggest, especially in the endwall regions. In order to restrict the number of input variables in the code testing/demonstration phase, it was decided to use a fixed level of dynamic turbulent viscosity ($0.07 \text{ kg m}^{-1} \text{ s}^{-1}$) uniformly across the span for the stall test cases.

Measurements of mixing coefficients in the Deverson test rig by Goto [1992] show this level to be high by a factor of 3.5 for the high tip clearance test case and 6.5 for the low tip clearance case.

A second turbulent viscosity is defined by analogy with Stokes' hypothesis for laminar flow; $\lambda_{\tau} = -\frac{2}{3} \mu_{\tau}$. (Equ 5.42)

The turbulent heat transfer coefficient is defined by way of a turbulent Prandtl number

$$\text{Pr}_{\tau} = \frac{\mu_{\tau} c_p}{\kappa_{\tau}}. \quad (\text{Equ 5.43})$$

For the tests reported here the value of the turbulent Prandtl number has been set to 0.8.

5.2.4 Boundary Conditions

5.2.4.1 Endwalls

The boundary conditions on the inner and outer walls are

- No-slip, ($\mathbf{u} \wedge \mathbf{n} = 0$ in the wall relative frame)
- Zero normal static pressure gradient
- Mass conservation
- Energy conservation (no heat transfer)

The no-slip boundary condition is enforced by setting $\mathbf{u}_{rel} = 0$ on the endwalls.

It will be seen later that the radial (or more precisely 'normal') grid spacing close to the walls is too fine to resolve pressure waves in that direction, and so a correction to the radial velocity is devised to ensure that the second boundary condition holds (section 5.2.5). Strictly, this could give a non-zero normal velocity on the end-wall, but instead it is applied in such a way that the end-wall velocity is zero, with a subsequent negligible error to the 'freestream' radial velocity.

The final two boundary conditions are applied directly by integrating the continuity and energy equations over the 'half elements' at the top and bottom of the calculation region, using the conditions $\mathbf{n} \cdot \nabla t = 0$ and $\mathbf{u} = 0$ to define zero mass and heat addition on the boundary itself.

5.2.4.2 Inlet and Exit

In a time accurate code some care has to be taken with respect to the inlet and exit boundary conditions. Fixed values of total pressure at inlet, say, or static pressure at exit, can lead to reflection of pressure waves that with a long adjoining duct would be simply carried upstream or downstream. To prevent this, a one-dimensional non-reflecting boundary condition was implemented. As part of the work ensuring the stability of the code (as described in the next section), time derivatives of axial velocity and pressure were resolved into upstream and downstream (linear) waves. One boundary condition for axial velocity and static pressure was then set by ensuring that the amplitude of the wave entering the calculation zone is zero. The other condition was set by extrapolating the outgoing wave from within the calculation region (although as this wave is outgoing its significance is only in terms of numerical/discretisation error).

In reality (and in the model) some incoming wave information is required in order to set the operating point of the compressor, and so a slow return term was added towards a set value of inlet total pressure and exit static pressure. This 'required' exit static pressure was varied in time during the stall tests to mimic throttle transients.

The other flow properties were set at inlet to the calculation domain, and extrapolated at the exit plane. Inlet whirl velocity was fixed at a constant level (for the main test case described below the inlet flow was axial). Inlet radial velocity was set to zero. The inlet density was also set to a fixed level.

The boundary conditions at inlet and exit to the compressor could be improved. A one dimensional non-reflective boundary condition is essential. A two-dimensional non-reflective condition, however, would almost certainly be better. The radial and whirl velocity conditions are not unreasonable, provided the inlet plane is sufficiently far upstream of the compressor face. In the initial stall inception tests reported in section 5.4, the inlet plane is placed 0.2 radii upstream of rotor inlet. In this case some radial and circumferential flow does occur close to the inlet of the calculation domain, however, and the inlet duct would ideally be longer. The constant inlet density condition could be modified to account for the boundary layer at inlet.

5.2.5 Numerical Stability

For the test cases attempted so far numerical instability has been avoided. In particular the stall disturbance prediction in the case of the Deverson test compressor (section 5.4) is genuinely aerodynamic as opposed to numerical; stall inception follows the breakdown of the outer wall boundary layer, which is exactly the effect the model is intended to capture. Furthermore, the inception pattern is very close to that observed by McDougall. Finally, operation is shown to be stable further down the compressor characteristic.

The following five features have been included in the code to ensure numerical stability. This combination of features has evolved with the code and is not presented as being in any way optimal.

1. A standard four stage Runge Kutta algorithm is applied to the time derivative calculation.
2. The time derivatives of the axial velocity and static pressure parameters are first resolved into upstream and downstream wave components, and then upwinded slightly. This is equivalent to the Warming-Steger algorithm (described, for instance, in Hirsch [1990]), applied in one dimension only. The upwinding

coefficients were derived from a linearised eigenvalue analysis of the mid-height flow using the same discretisation as the full three-dimensional solution.

3. All other time derivatives are smoothed radially and axially using a second order implicit smoothing technique based on Lerat [1985]. The axial velocity and static pressure time derivatives are smoothed in the radial direction only.
4. A low level of second order axial and radial smoothing is applied to the flow parameters themselves. The smoothing terms are calculated as second order in time inviscid terms from the flow equations, scaled by a factor of 0.12 in the axial direction and 0.6 in the radial direction (hence they are small). The factors are variable; low values can be combined with a low time-step to give a more accurate steady state solution, whereas a higher timestep and higher smoothing levels can be used for modelling lengthy stall transients. A similar term is applied to smooth the flow parameters in the circumferential direction.
5. Stability is particularly difficult to maintain in the radial dimension, as the grid spacing is very small close to the walls. For this reason a correction is added to the radial velocity such that the time derivative of the static pressure is smooth in the radial direction. The correction is scaled by the local radial grid spacing, and so is small outside the boundary layer. Near the endwalls it is relatively large, and has the effect of ensuring nearly constant static pressure across the boundary layer. It would be possible to allow for wall curvature (circumferential and axial) in the 'required' pressure profile near the wall, but this was not done for the tests reported here.

With 31 nodes circumferentially the grid spacing in that direction is larger than the maximum spacing in the axial or radial direction by a factor of over six. The Courant-Friedrichs-Lewy number ($CFL = a\Delta t/\Delta x$, where a is the speed of sound, Δt is the timestep, and Δx is the local grid spacing) is therefore lower by the same factor, and no circumferential smoothing of the time derivatives is required to ensure stability, beyond the discretisation procedure described in section 5.2.2.7.

One immediate result of the importance of the CFL number in determining stability is that the maximum stable time-step is governed by the grid geometry. The grid definition, however, is largely set by the lengthscale of the disturbances being modelled, and hence the maximum time-step for a given application is pre-defined. Stall inception timescales, however, scale with wheel rotation speed, and hence more of the disturbance can be modelled with a given number of iterations if the wheel speed is set artificially high. This is done for the low speed test case described in section 5.4.

There are some implications in terms of compressibility effects which are discussed at that point.

The smoothing described above has greatest effect in the near wall areas where the grid spacing is finest. One of the effects is to slow the reaction of the boundary layers to unsteady disturbances in the freestream. An oscillating boundary layer test case was devised to measure this effect (section 5.3.4), and showed the boundary layer response rates to be acceptable, but only marginally so for low values of tip clearance. More work is required in the area of smoothing to improve the time response of the boundary layer.

5.2.6 Pitchwise Averaged Blade force Model

The blading in the compressor is modelled in a pitchwise averaged sense by introducing an axial and a tangential body force into the flow equations (no account is currently taken of radial forces). These forces are calculated in different ways depending on radial position. Across the majority of the span these forces are calculated from blade loss and deviation using the calculation procedure described below. The blade loss and deviation are in turn derived from the flow conditions at blade inlet. Towards the outer ends of the blades a 'constant blade force' model is implemented (described below in section 5.2.6.4). In the tip gap region, where there is no blade, the blade force is set to zero. The combination of a constant blade force model with zero force in the gap region has been shown to give reasonable agreement with measured steady state profiles [Gallimore, 1996].

5.2.6.1 Loss and Deviation Assumptions

For the initial test cases reported here the simplifying assumptions are made of zero loss and deviation calculated from the 'Miller rule' [Miller, 1987]. The absence of loss has some effect on the shape of the stage characteristic (section 5.4.3.1). Calculating loss and deviation from the blade inlet conditions is incorrect for reverse flow in the mainstream region, which limits the code to the stall inception period. A small amount of extra work would be required to define the blade row performance in reverse flow and hence to model the growth of the stall disturbance into fully developed rotating stall or axisymmetric surge.

5.2.6.2 Calculation of Blade Forces

The blade force terms arising from deviation are based on an assumed profile of 'ideal' relative flow angle with axial distance (the ideal profile is defined as the profile that would be achieved at that incidence level in steady state running). For simplicity the

ideal profile is taken to be a linear change from the inlet air angle at bladerow inlet to the exit metal angle plus deviation at bladerow exit. The force in the tangential direction per unit mass is then calculated from the simple formula

$$F_{\theta} = u_a \frac{\partial u_{\theta, ideal}}{\partial z}, \quad (\text{Equ 5.44})$$

where $u_{\theta, ideal}$ is the 'ideal' relative whirl velocity calculated from the ideal flow angle and the local value of axial velocity. Calculated in this way, variations in axial velocity along the blade are automatically accounted for. Furthermore, using the finite element discretisation, the calculation can be performed in such a way as to ensure negligible interpolation errors. Hence in steady state running the profile of relative angle with axial distance produced by the calculated forces will be exactly the 'ideal' one.

The axial force can be calculated from the tangential one using the fact that for zero loss the force must be everywhere perpendicular to the flow, hence

$$F_a = -u_{\theta} \frac{\partial u_{\theta, ideal}}{\partial z}. \quad (\text{Equ 5.45})$$

5.2.6.3 Bladerow Performance Time Lag

Mazzaway [1977], among others, has noted the existence of a time delay in bladerow loss and deviation beyond that caused by inertial effects. A time lag is included in this model using a simple first order technique;

$$F_{Current}^{Used} = \alpha F_{Current}^{Calculated} + (1 - \alpha) F_{Previous}^{Used}, \quad (\text{Equ 5.46})$$

where $\alpha = dt/dt_{resp}$ and dt_{resp} is the response time of the blade boundary layers, assumed to be of the order of the convection time through a blade passage. This procedure also has the stabilising effect of lowpass filtering the blade force in time.

5.2.6.4 Constant Bladeforce Model

In a compressor the force supplied to the air by the blading is derived from the static pressure distribution on the blade surfaces. Near the ends of blades the loss and deviation calculated in a two dimensional manner from the blade inlet conditions would imply an unsustainable static pressure gradient in the radial direction. Accordingly, measurements have shown that near the ends of blades the force supplied to the air by the blading is not solely determined by the blade inlet conditions, but is radially smoothed [Dring, 1993]. The region over which the smoothing is significant is dependent on whether the blade end is fixed or free, the level of tip clearance, the blading style, and presumably the inlet boundary layer thickness. Various models of

fixed and free ends have been adopted by previous authors. For the initial tests reported here the simple assumption was made that the blade force remains constant over the outer ten percent of the span. For the high tip clearance case the extent of the constant blade force was increased to twenty percent for rotor and stator at the outer wall only.

Note that the parameter that is kept constant is the body force per unit volume. The parameter used in the calculations (see above) is force per unit mass, and so has to be scaled by the local density in the constant blade force regions.

Early attempts at implementing the constant blade row model used a constant tangential blade force, but allowed the axial blade force to vary such that the force vector was always perpendicular to the flow and the flow thus remained loss-free. This approach naturally gave extremely high axial forces in regions of high swirl angle. Instead, it was decided to keep the force vector constant through the outer regions. This is in broad agreement with measurements [Dring, 1993]. Eventually the force angle could be derived from a near wall loss model.

Gallimore [1996] has implemented a constant blade force model similar to (but more complex than) the one described above, together with a near wall loss model, into a throughflow model incorporating the effects of spanwise mixing. Broad agreement is demonstrated between the predicted and measured steady state performance of three compressors over a range of operating conditions.

5.2.7 Random Input

A small random element (typically a small fraction of 1%) is added to the axial and circumferential velocities at each timestep, scaled by the local axial velocity. This procedure is analogous to that used for the multiple compressors in parallel model (described in section 4.2.6), with the same objective of introducing non-axisymmetric disturbances without overly influencing the mode of the instability being modelled.

5.3 CODE VALIDATION

Various checks have been performed on the code to ensure proper discretisation of the flow equations. Among these are four specific test cases described below. These tests were carried out during the development of the code and hence were not all performed with the latest version or the latest parameter settings. Nonetheless they demonstrate the capability of the discretisation scheme in a number of areas.

5.3.1 Hagen-Poiseuille Type Flow in an Annular Duct

This test case was used to check the velocity terms related to viscous flow in the axial direction. Schlichting [1979, page 85] gives an equation for Hagen-Poiseuille flow in a pipe, that is a fully developed axial flow profile at a set pressure gradient and with constant viscosity. The same method can be used with only minor modifications to determine a fully developed velocity profile in an annulus, with the result

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial z} \left[y^2 - \left(\frac{R_2^2 - R_1^2}{\ln R_2 - \ln R_1} \right) \ln y + \left(\frac{R_2^2 \ln R_1 - R_1^2 \ln R_2}{\ln R_2 - \ln R_1} \right) \right], \quad (\text{Equ 5.47})$$

where R_1 and R_2 are inner and outer radius respectively.

The ability of the code to match the above profile was tested in a blade-free parallel annulus. For this fully developed profile test case the inlet boundary condition was set to give zero normal derivative of axial velocity. The pressure gradient was calculated such that the peak velocity (according to the analytic solution) was 100m/s. A coarse mesh was used for the calculation, with 10 equispaced nodes radially, and a high hub/tip ratio (0.92).

The axial velocity profile calculate by the model at the inlet and exit of the duct is shown in fig 5-4, together with the analytic solution. The flow in the calculated solution is shown to accelerate along the duct. The cause is a combination of compressibility and adiabaticity. As the static pressure falls (by just over 4%), the density falls by the same ratio, and the velocity rises (again, by just over 4%). Thus the calculated solution is not fully developed (that is, axially constant). A fully developed flow is only possible if heat loss is allowed through the annulus walls (in a real situation, the heat generated by the viscous forces would increase the temperature of the fluid until the heat loss became significant). For the short duration stall transients at which the time marching model is directed any heat transfer to the walls is considered negligible, and adiabatic walls are assumed.

The acceleration of the fluid along the duct introduces an extra term into the axial momentum equation that reduces the effective pressure drop along the duct: the pressure drop now balances the material acceleration of the flow in addition to the viscous forces. The reduced level of the viscous forces is reflected in a lower peak axial velocity in the duct. A simple calculation at mid-height suggests that the effective pressure drop would be less than the actual pressure drop by 4.3%. The discrepancy between the actual and analytic peak velocity (fig 5-4) varies from 8% at inlet to 5% at exit. Better agreement could not be expected given the other assumptions implicit in

the model, primarily to do with the inlet and exit boundary conditions (constant density radially at inlet, possible heat transfer at inlet and exit).

Having compared the peak velocity as calculated by the model with the analytic solution, a comparison was made of the radial profiles by scaling the level of the analytic solution such that the peak value matched the peak value at duct inlet as calculated using the model. The scaled profile is also shown on fig 5-4, and is almost identical to the calculated profile.

5.3.2 Flow Between Rotating Cylinders

This test case is designed to test the viscous and inviscid terms in the flow equations relating to non-zero whirl velocity. The analytic solution for this test case with laminar viscosity, incompressible flow and zero axial velocity is given in Schlichting [1979, page 87].

The simplest way to model the test case without too much alteration to the inlet and exit boundary conditions was to apply a small gradient of static pressure axially, leading to a small axial throughflow in addition to the desired profile of whirl velocity. The circumferential velocity of the inner wall was set to 50m/s, and of the outer wall to 1m/s. One complication with respect to this test case is that the boundary conditions of the model require a profile of whirl velocity to be set at inlet. If this profile is too dissimilar to the steady state solution, then a large axial length is required for the flow to readjust. For this reason the analytic profile was applied at inlet, but with a small perturbation added (the profile used was the analytic solution with zero velocity on the outer wall). The same coarse calculation mesh was used as for the previous test case.

The perturbed profile of whirl velocity at inlet was observed to change with axial length, converging to a very similar exit profile to the analytic solution (fig 5-5). As the calculated and analytic profiles are almost linear a better idea of the accuracy is obtained by plotting the deviation from a straight line (fig 5-6). Again, good agreement is obtained. As an additional check, this time on the energy equation, the power input was calculated from the temperature profile at inlet and exit, and matched the (very small) power input of the rotating walls to within 1%, representing 0.006% of the total enthalpy flux through the duct.

5.3.3 Fully Developed Turbulent Boundary Layer

This test case is designed to check the near wall turbulence model. Again an axial pressure gradient is set up to drive flow down a blade-free duct. This time, however, the mixing length turbulence model is switched on (in the Hagen-Poiseuille test case

above laminar viscosity was assumed). A zero normal derivative inlet boundary condition was used for the axial velocity, such that the model converged slowly to something close to a fully developed profile with a relatively short duct length. The analytic result for flow in a pipe with the given turbulence model is given by Schlichting [1979, chapter 20].

In the calculated solution there is a small amount of variation axially due to an imposed inlet condition of constant density. The calculated axial velocity profile towards the end of the duct is shown in fig 5-7. The same profile (from the inner wall to mid-height) is shown again in fig 5-8, this time in normalised form and compared with the analytic result given by Schlichting [1979, page 601]. The x co-ordinate η represents the distance from the wall, normalised by the friction velocity and the laminar kinematic viscosity, on a logarithmic scale. The y co-ordinate ϕ is the axial velocity normalised by the friction velocity. The agreement is good except in the freestream regions (high η), where the turbulence modelling is slightly different (the code has a fixed level of freestream turbulence), and also the differences between a pipe wall and an annulus wall become significant. The convergence is such that the solution is improving very slowly with time.

5.3.4 Oscillating Boundary Layer

This test case was devised to test the unsteady response of the boundary layers in the model. The starting point is a converged steady flow down a blade-free duct. This steady flow is then perturbed by a small oscillation of the exit static pressure, at increasing frequency with time. An analytic solution is compared to results from the time marching model using the same calculation mesh and input parameters as in the stall inception test case reported in section 5.4.

The incompressible analytic solution (for oscillating flow over a flat plate with the turbulence model used in the code) is derived in analogy with the laminar case for pipe flow presented by Schlichting [1979, page 436]. The compressible solution should be similar provided the pressure perturbations are small and the wavelength of the freestream oscillation is large compared to the length of the duct (so that the change in freestream conditions is approximately constant along the duct). That is, $a/fl_{duct} \gg 1$, where f is the frequency in Hz. For the duct length used (0.50m) this limits the frequency to $f \ll 700$ Hz, which is well beyond the required range.

The starting point for the analytic solution is the axial momentum equation

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left(\nu_{\tau} \frac{\partial u}{\partial y} \right). \quad (\text{Equ 5.48})$$

The first term on the right hand side is ignored, as it is assumed that the velocity profile is uniform in the x direction. In the freestream, viscosity is negligible, and so the axial pressure gradient is proportional to the time derivative of the velocity;

$$\frac{\partial \hat{u}}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x}, \quad (\text{Equ 5.49})$$

where $\hat{u}$ is the deviation of the freestream velocity from its time average value.

Assuming that the static pressure remains constant through the boundary layer, the velocity fluctuations through the boundary layer are then related to those in the freestream by

$$\frac{\partial u}{\partial t} = \frac{\partial \hat{u}}{\partial t} + \frac{\partial}{\partial y} \left(\nu_{\tau} \frac{\partial u}{\partial y} \right). \quad (\text{Equ 5.50})$$

Taking a Fourier transform with respect to time yields

$$i\omega u = i\omega \hat{u} - \frac{d}{dy} \left(\nu_{\tau} \frac{du}{dy} \right), \quad (\text{Equ 5.51})$$

$$\text{or } u' = \frac{d}{dy} \left(\lambda^{-1} \frac{du'}{dy} \right), \quad (\text{Equ 5.52})$$

where $\lambda = -i\omega / \nu_{\tau}$, and the normalised velocity fluctuation $u' = \frac{u - \hat{u}}{\hat{u}}$.

The boundary conditions are then

$$u'(0) = -1, \quad (\text{Equ 5.53})$$

$$\text{and } u' \rightarrow 0 \text{ as } y \rightarrow \infty. \quad (\text{Equ 5.54})$$

For laminar (that is, constant) viscosity, the equation is easily solved with these boundary conditions to give

$$u' = -e^{-\sqrt{\lambda}y}. \quad (\text{Equ 5.55})$$

This result can be shown to be the limit of the pipe solution given by Schlichting as the radius increases to infinity.

$$\text{Now } -\sqrt{\lambda} = -\sqrt{-i\omega/\nu_{\tau}} = \left(\frac{i-1}{\sqrt{2}} \right) \sqrt{\omega/\nu_{\tau}}, \quad (\text{Equ 5.56})$$

and the real and imaginary parts of this function respectively correspond to an attenuation and a time lag of the boundary layer oscillations as compared to the freestream oscillations. The same features, a time lag as well as an attenuation, can be expected to occur in the turbulent case.

The basis of the turbulent calculation (loosely called analytic) is that the turbulence model used in the code is such that under a small perturbation to a mean flow the calculated turbulence level will be approximately constant. The calculated values of turbulent viscosity at the start of the test can then be used to integrate equation 5.52. This integration has to be done with some care. Firstly, note that in the code the turbulent viscosity is assumed constant over each cell. This can be mimicked in the analytic solution by integrating equ 5.52 directly between nodes in the radial direction to give

$$u' = Ae^{\sqrt{\lambda}y} + Be^{-\sqrt{\lambda}y} \quad (\text{Equ 5.57})$$

And the stress is (to a multiplicative constant)

$$\begin{aligned} \lambda^{-1} \frac{\partial u'}{\partial y} &= \lambda^{-1} (A\sqrt{\lambda}e^{\sqrt{\lambda}y} - B\sqrt{\lambda}e^{-\sqrt{\lambda}y}) \\ &= \lambda^{-1/2} (Ae^{\sqrt{\lambda}y} - Be^{-\sqrt{\lambda}y}) \end{aligned} \quad (\text{Equ 5.58})$$

In this analytic solution both the velocity and the stress must be continuous across each node. This gives two equations per internal node, with two variables (A and B) per inter-node gap. The remaining two equations come from the boundary conditions (eqs 5.53 to 5.54). The whole forms a linear set of equations that can be solved using standard matrix techniques.

The 'analytic' response of the boundary layer to a velocity fluctuation at 110Hz in the freestream is shown in fig 5-9, in terms of amplitude (normalised by the amplitude of the freestream fluctuation) and phase response. Compared to the laminar boundary layer result (equ 5.55) the turbulent viscosity profile produces a high response close to the wall, with little in the way of time lag. Also shown in fig 5-9 is the boundary layer response produced by the time-marching code. Again there is negligible time lag, but the response falls away further from the wall. Note, however, that the response is plotted against a log scale of radial distance from the wall, and so the size of the near wall regions is exaggerated.

As the time delay is negligible in both the model and the 'analytic' solution, a simple comparison can be made by normalising the response at each gridpoint by the 'analytic'

response at the gridpoint. The normalised response curves are shown for freestream fluctuations at a number of frequencies in fig 5-10, again plotted against a log scale of radial distance.

For the tip clearance calculations to be representative, it is necessary to have a fairly accurate response over at least half of the tip gap region at frequencies up to the passing frequency of the highest harmonic being modelled. The blade tip positions for the high and low tip clearance predictions of the Deverson rig (section 5.4) are marked on the plot. The code generally is intended to be accurate up to the fifth harmonic circumferentially, which (assuming a rotating stall frequency of half shaft speed) has a passing frequency of 125Hz. From this it can be seen that the boundary layer response is marginal for the high tip clearance case and poor for the low tip clearance test case.

The most significant factor in determining the boundary layer response was found to be the smoothing of the calculated time derivatives in the radial direction. Useful work could be done in the future either improving on the type of smoothing used or investigating the effects of a lower level of smoothing.

5.4 SINGLE STAGE COMPRESSOR STALL TEST CASE

The ability of the three dimensional time-marching code to capture stall inception dynamics was tested by comparing the predicted stalling behaviour of the Deverson low speed single stage compressor with the measurements made by McDougall [1988]. The predicted steady state behaviour prior to stall was compared also with the measurements of Goto [1992] on the same rig.

The rig used by McDougall is described in the following section. Most of his measurements were made with a large rotor tip clearance (3% of rotor tip chord, which is 2.2% of span) and stator sealed at both ends. McDougall detected a wave-like disturbance prior to stall, which rotated around the annulus at between 50% and 60% of shaft speed. The disturbance was strongest inside the rotor passages towards the trailing edge at around 85% of span and 18% of pitch from the suction surface. At the front and rear of the compressor the disturbance was seen to be fairly uniform radially, a feature interpreted by McDougall as being a 'modal' response of the compressor environment to the disturbance at rotor tip. McDougall made a smaller number of measurements at a lower level of tip clearance (1.2% of rotor tip chord, which is 0.9% of span). Here the results were interpreted as a hub stall. Day [1993a] however gives detailed measurements of stall in the same build, showing the same 'modal' stalling pattern at the high tip clearance, but a 'short circumferential

lengthscale' disturbance in the low tip clearance case (see section 2.1), noting that at both levels of clearance stall disturbances started at the tip of the rotor and grew towards the hub.

Day noted that the 'modal' behaviour observed by McDougall was not necessarily a precursor to irretrievable flow breakdown. The length of time over which such behaviour could be observed prior to stall varied considerably between events. Furthermore, if the throttle was opened while the modal behaviour was present axisymmetric operation could be regained. Both Day and McDougall, however, noted the smooth transition from the modal behaviour into irretrievable rotating stall, and so the initial stages of stall inception at high tip clearance as predicted by a successful stall model should reflect the type of behaviour reported by McDougall.

5.4.1 Rig

The Deverson rig is a low speed single stage compressor. The layout of the rig is shown in fig 5-11. The annulus is parallel, with a hub/tip ratio of 0.8. The blading is of free vortex design, based on C4 aerofoils. Details are given in table 5-1, which is reproduced from McDougall [1988, table 3.1]. Many of McDougall's measurements were made with the stators staggered closed by 10° (the 'unloaded configuration' in table 5-1) in an attempt to ensure that the stalling behaviour was dominated by the rotor.

A significant leakage flow into the annulus just upstream of the rotor hub has been reported by a subsequent experimenter using the same rig [Place, 1996], and was observed to give rise to a large separation region at rotor hub. This feature is considered briefly in section 5.4.3.2.

5.4.2 Application of Model

The calculations were performed using a 23000 point mesh, with 24 points in the axial direction, 31 radially, and 31 circumferentially. The grid used for the calculation is shown in fig 5-12. Note that the blade inlet and exit profiles were approximated by straight lines in the radial direction. This was done to simplify input and analysis of the results, and does not represent a limitation of the model.

As far as possible the input to the model was made to match the experimental conditions during McDougall's experiments. Many of the details have been discussed earlier in the chapter. Note in particular that the rotational speed of the rig was artificially increased by a factor of 6 to 49.8Hz, in order to allow the computation to be

performed in an acceptably small number of timesteps. The inlet boundary layer measured by McDougall was approximated by a seventh power law profile.

The stators were left at their nominal setting for the modelled transients. With the stator deviation model used the rate of deviation increase with inlet flow angle at the stall point was nearly identical with the stators at their nominal and unloaded settings, and so the 10° difference in stagger setting is not expected to affect the stall inception behaviour.

The stall inception model was applied with two levels of tip clearance, 1.4% and 2.7% of span. A full investigation was carried out at 2.7% clearance. Results from running the model in axisymmetric mode only are presented for the 1.4% clearance. The two values of clearance lie either side of the high tip clearance case (2.2% of span) reported by McDougall.

The procedure for identifying the stall inception pattern was as follows. First the model was run axisymmetrically, with the exit pressure gradually increasing. From this axisymmetric run the overall pressure ratio characteristic was derived (as described in the next section). The initial conditions for the non-axisymmetric runs were derived from the axisymmetric data at a point just below the peak of the total to static pressure rise characteristic (on the high flow, negatively sloped part). A slow throttle transient was then mimicked by gradually raising the exit static pressure until stall occurred. The random input (section 5.2.7) allowed a non-axisymmetric solution to develop.

5.4.3 Comparison with Experiment - Steady State Prior to Stall

Two fundamental comparisons were made with respect to the steady state behaviour prior to stall. Firstly the overall steady state operation of the code was checked by comparing the predicted and measured overall compressor characteristics. Secondly the predicted radial profile of axial velocity near the rotor exit plane is compared with measurement, as this profile (particularly in the tip region) is shown by the predictions to play a significant part in the stalling behaviour of the compressor.

5.4.3.1 Overall Characteristic

The measured [Goto, 1992] and predicted total-to-static stage characteristics are shown in fig 5-13. The leftmost point of each characteristic is the stall point (in the case of the model, the point at which the axisymmetric calculation exhibited surge behaviour). The calculations described here were performed neglecting blade profile loss (section 5.2.6.1). A simple correction, described below, has been applied to the model results to account for this. The flow parameter in the case of the rig is

calculated from the mass flow through the machine. In the case of the computational model, it is calculated on an average basis at mid-stage. The reason for using this axial plane is the fast rotation of the computational compared to the experimental rig, giving rise to a density ratio across the rotor and stator. Velocity triangles for the mid-height section with and without the density ratio are shown in fig 5-14. For a given level of rotor deviation the same level of work is achieved with the same axial velocity at rotor exit (rather than inlet), as a consequence of the axial inlet flow.

The agreement between the measured and calculated characteristics is reasonable, given the nature of the assumptions in the model (better agreement was obtained by accounting for the density ratio across the rotor in calculating rotor incidences, see below). The effect of the larger tip clearance in degrading the performance near the stall point is very similar to the measured effect (given the slightly different levels of tip clearance in rig and model) in terms of both pressure ratio and flow, which gives some confidence in the capability of the calculation to model the effect of tip clearance on stall inception.

The 'raw' pressure rise characteristic at low tip clearance is shown in comparison with the experimental results of Goto in fig 5-15. The computational predictions show a considerably higher pressure rise than the measurements, primarily as a result of the zero blade profile loss assumption. The predicted efficiency at low tip clearance is fairly constant at between 96 and 94% (adiabatic) along the whole characteristic. This loss derives from the viscous forces included in the model, particularly in the rotor tip region due to the recirculation there (see the next section). This efficiency is lower than might be expected (without any account taken of blade boundary layer loss), due to the high level and radial extent of mixing used for the calculations (section 5.2.3). Subsequent measurements to those of Goto, on a similar build with the same level of tip clearance (2.2% of span) showed an efficiency of 90% at the design point dropping to 80% just before stall [Place et al, 1995]. If a linear distribution with flow is assumed between these two points, the predicted characteristic (with the same work input) moves to the dotted line shown in fig 5-15, which is much closer to the measured characteristic. This corrected line is the one plotted in fig 5-13. If it is assumed that the difference between the measured and predicted efficiencies represents the blade profile loss, then the same efficiency correction (at a flow) is applicable at all clearances, and the high tip clearance characteristic shown in fig 5-13 was corrected accordingly.

It was noted above that the effect of the density rise across the rotor could be accounted for by plotting the characteristic against the flow coefficient at mid-stage.

The density ratio still causes a discrepancy between model and experiment, however, because the rotor deviation in the model is calculated from the conditions at rotor inlet. The second correction shown in fig 5-15 was obtained by repeating the calculations with the rotor deviation calculated from the rotor inlet flow, but with the axial velocity scaled by the density ratio across the rotor. With this second correction the predicted and measured characteristics agree well.

The third correction shown in fig 5-15 was obtained by repeating the calculations with the stators in their unloaded configuration (that is, closed 10°). The effect is negligible, partly because the zero blade profile loss assumption means that the exit total pressure is unaffected, and partly because the air exit angle is near axial, and so the angle change has little effect on the absolute velocity at stator exit (and hence little effect on the exit static pressure).

5.4.3.2 Axial Velocity Profile Near Rotor Exit

It will be seen in the next section that the behaviour of the flow in the rotor tip region is critical to stall inception in the Deverson rig. For this reason it is appropriate to compare the steady state behaviour of the model with experimental results prior to stall. Goto [1992] measured the radial profile of axial velocity behind the rotor on the same compressor build as studied by McDougall. A comparison of his measured profile (between the design flow and the stall point) with that predicted by the model at the same radially averaged flow coefficient is shown in fig 5-16.

The predicted velocity profile from 0 to 70% height is quite different to that measured by Goto: the predicted axial velocity falls with radial height, whereas the measured velocity rises. The reason is the large separation region at rotor hub. The primary cause of this separation was a leakage path into the annulus upstream of the rotor [Place, 1996], which is not represented in the model. In order to compare the flow in the tip region, calculated results are also shown at a higher radially averaged flow coefficient, such that the level of flow towards the tip away from the separated region is similar to the experimental results.

Between 70% height and the outer wall the agreement between the predicted profile and the measurements is better. The predicted velocity drops rather too rapidly towards the outer wall, but it should be noted that the predicted profile is changing rapidly in the axial direction at this point; within the rotor there is significant flow reversal at rotor tip. This feature appears to be reflected in McDougall's measurements, in that the on-rotor measurements near the tip show significantly lower axial velocities at a height than those in the stationary frame downstream of the rotor.

5.4.4 Axisymmetric Stalling Behaviour

The model presented in this chapter correctly predicts that stall inception occurs in a non-axisymmetric manner. When the model is run to stall axisymmetrically, however, a pattern of stall inception is observed that is qualitatively similar to the non-axisymmetric stall inception pattern (in a frame rotating with the disturbance).

The largest flow disturbances during the stall inception period as predicted by the model in axisymmetric mode were found at 75% of rotor chord. The radial profile of axial velocity over the outer half of the annulus at that plane is plotted at successive time intervals in fig 5-17. The compressor is being throttled with time, leading to a higher pressure ratio across the rotor, lower flow in the midstream regions, and a wider boundary layer region as shown by the solid lines, 1000 iterations (3.7 rotor revolutions) apart. The growth of the boundary layer is steady until at 5000 iterations it causes a significant drop in axial velocity at 80% height. This height represents the position from which constant blade force is assumed; once the flow has dropped significantly at that point, the whole outer annulus is affected. From then on the boundary layer grows much faster; the interval between the last solid line and the dotted line in fig 5-17 is a tenth of that between the solid lines. As the boundary layer grows the flow dips underneath the blockage, giving rise to a small velocity increase across much of the span. After a further 100 iterations the shape of the profile has changed entirely (dashed line). This pattern of stall inception behaviour is reflected in the non-axisymmetric results described below, and is discussed further in section 5.5.

5.4.5 Comparison with Experiment - Stall Inception Behaviour

The non-axisymmetric stall inception pattern predicted by the model for the high tip clearance case is demonstrated in fig 5-18, which shows for each iteration the static pressure around the circumference at rotor inlet relative to the mean level at the start of the event. The flow is initially axisymmetric (the left hand side of the graph). As the throttle is closed the disturbances in the compressor form themselves into two coherent bands representing a second circumferential harmonic pattern rotating at around 62% of shaft speed. As the disturbance grows in amplitude (the right hand side of the graph), the speed slows to around 50%.

The stall disturbance shown in fig 5-18 is made up primarily of the second circumferential harmonic. Figs 5-19 and 5-20 show the amplitude and modal rotation of the first five harmonics over the stalling period, where the modal rotation is defined as the 'unwrapped' phase (phase with multiples of 2π added as required to keep the resulting line smooth) divided by the mode number. The second and (later) the third

harmonic start to grow substantially before the others. The rotation rate of the third harmonic is lower than the second (52% of shaft speed).

The nature of the disturbance in the r - z plane can be elucidated by picking for a given iteration the circumferential node showing the largest disturbances from the mean level. Fig 5-21 shows velocity vectors for that node near the end of the time period shown in fig 5-18. Fig 5-22 shows the same plot for the flow at a point 90° around the circumference (that is, out of phase with the first position). The disturbance in the former case is seen to consist of a broadening of the recirculation region at rotor tip. This region acts as a local blockage; the upstream flow redistributes itself, dipping under the blocked region. Similar redistribution in the θ direction has been observed upstream of the blockage.

In accordance with this view of the disturbance, the largest variations in velocity are found at 80-90% height and 75% of rotor chord, which is exactly the region identified by McDougall as showing the largest fluctuations prior to stall. Fig 5-23 shows the radial profile of velocity disturbance relative to the mean level at 75% rotor chord and rotor inlet, which conforms to McDougall's observations of a disturbance at rotor tip giving rise to a response at inlet which is radially uniform.

5.5 DISCUSSION

The near wall regions of a compressor are known to play a large role in determining stall inception behaviour (chapter 2). The stall model presented in this chapter is the first to include directly time accurate calculations of the flow in these regions whilst at the same time being capable of multistage application. The results presented in this thesis, based on a single stage compressor, are preliminary, but demonstrate that the method can capture the stall dynamics of a real compressor. If similar results are obtained on multistage compressors, this type of model will provide a unique facility for investigation of stall and surge inception, whilst at the same time providing a vehicle for numerical simulation of active control of stall/surge.

The model was developed from scratch, and a considerable amount of work remains outstanding to do with the mechanics of a time accurate code, most notably the inlet and exit boundary conditions and the handling of the annulus boundary layers with respect to their response to unsteady disturbances. Likewise the bladerow performance model requires refinement, for instance to include losses and radial blade forces, before an accurate steady state representation of flow in a multistage compressor can be achieved. All of this work is beyond the scope of this thesis: the

main objective here is not to demonstrate a new three-dimensional time marching computational fluid dynamics code, nor to present a new set of calculations or correlations to do with blade/endwall interaction effects. The intention is rather to present a new type of stall inception model, where a viscous pitchwise average throughflow model is combined with a circumferential discretisation to give a time-accurate prediction of stall inception behaviour.

The stall inception results described in the previous section are very encouraging. Stall inception is found to be dominated by a low circumferential harmonic disturbance rotating at just above half shaft speed. The disturbance is correctly predicted to be strongest in the rotor tip region, whilst the corresponding disturbances upstream and downstream of the stage are almost uniform radially.

Investigation of the results from the model show that the stall inception pattern is determined by the interaction between the effect of the rotor tip gap and the constant blade force assumption in the rotor. The lack of blade force in the rotor tip gap causes a thin region of reverse flow at rotor tip. As the pressure ratio increases the size of the reversed flow region also increases, but has little effect on the overall flow because the blade force is determined by the flow conditions further away from the wall. As the region of reverse flow increases further, however, the effect reaches further inboard, until the height is reached at which the tip blade force is defined. Once the flow at this point has reduced significantly, the blade force in the tip region starts to reduce, and rotating stall is initiated.

The idea of a single point determining the blade force in the tip region is clearly unphysical, but it is a simple model of a genuine three-dimensional physical effect, namely that flow redistribution ensures a moderately uniform static pressure distribution radially, and therefore there cannot be large changes in blade force with radial position. Hence the conclusion is likely to be correct that the effect of the tip gap becomes important only as it grows radially to an extent where it influences the blade force in the rotor tip region. More complex endwall calculations could be added to the model in time, but the essential physics are captured by the model as presented.

The model showed the second and third circumferential harmonics to be less stable than the first. The reason for this is unclear; McDougall's measurements and linear theory [for instance Longley, 1994] suggest that the higher harmonics would be more stable. The reason may lie with the inlet and exit boundary conditions: the one-dimensional non-reflective conditions will naturally be more effective at removing the low circumferential harmonics. It should be noted, however, that linear theory

[Longley, 1994] suggests that the stability boundary for the different circumferential harmonics can be very similar.

The most severe limitation of the model as it currently stands is the response rate of the endwall boundary layers; the smoothing is currently such that the calculations of the flow in the tip gap region are only correct for high levels of tip clearance. If the inception of rotating stall is governed by the tip clearance region as here, then the flow within the tip gap must be capable of reacting to a disturbance of the size and rotational speed of a stall cell.

Axisymmetric runs of the model at high and low tip clearance show good agreement with the measured effect on the stall point, although it should be noted that a substantial correction has to be applied to correct for blade profile losses, which are not included in the model. This suggests that an axisymmetric model may be sufficient for predicting the effects of tip clearance on the stall point. This would not be surprising; linear modelling [Longley, 1994] (without a radial dimension) shows that the axisymmetric instability point is often close to the non-axisymmetric stall point (although the active control experiment of Paduano et al [1993] showed that this is not always the case). It is significant that the slope of the pressure rise characteristic at the stall point is quite different in the low and high tip clearance cases. This suggests that a surge-type model that depended on the slope of the pressure rise characteristic for stall point prediction would not give a correct indication of the effect of tip clearance changes.

5.6 FUTURE WORK

The most immediate requirement is to include the effect of losses in the calculations. A less significant improvement would be to include radial blade forces. Once these improvements are made the steady-state prediction capability of the code can be assessed in a multistage environment.

The unsteady flow prediction capability could be improved by addressing the following areas;

1. Improvement of derivative smoothing to improve the response of the annulus boundary layers.
2. Improvement of the speed at which the computations can be performed, by algorithm changes or simplifying assumptions, or improving the smoothing and hence increasing the timestep.

3. Implementation of two dimensional non-reflective boundary conditions at inlet and exit.
4. Differentiation between gap and bladerow elements with respect to circumferential communication.
5. Improvement of endwall model. This is likely to occur as a result of comparisons with measurements of steady state compressor performance.

6. DISCUSSION

The work presented in this thesis has contributed to the understanding of stall and surge in axial flow compressors in three main areas, corresponding to the three main chapters (3, 4 and 5). The experimental results presented in chapter 3 represent the first detailed analysis of stall inception in an aeroengine based on a full coverage of high response instrumentation, and one of the first detailed analyses in any high speed compressor (rig or engine). A wide variety of features, some of them new observations, are analysed and discussed. In chapter 4 a relatively simple stall model is developed based on previous modelling techniques but extended to include the effects of an aeroengine environment. The results of the model are compared to the measurements reported in chapter 3. The model is also used as the basis for an investigation into active control of stall. The more complex stall model presented in chapter 5 is a new idea, where the flow in the near wall regions of the annulus is calculated directly in a time accurate manner. The pitchwise average nature of the method means that (unlike a full multipassage unsteady CFD solution) it could be applied to a multistage compressor with current computing capability. The results are only preliminary, but comparison with test data from a single stage rig demonstrates that the method is capable of capturing the dynamics of the stall inception period.

The results presented in the thesis are discussed in sections 3.5, 4.3.5, 4.4.1.5, 4.4.2.4, and 5.5. This section therefore contains only a brief discussion of the most important aspects of the work.

6.1 STALL INCEPTION MEASUREMENTS

Many of the observations reported in chapter 3 confirm ideas on stall/surge based on previous, mainly low speed tests. Stall/surge is shown to be a deterministic, repeatable event starting in every case with a disturbance rotating around the annulus at around half shaft speed¹. The axial position of the initial disturbance leading to stall moves towards the rear at high speed, in line with expectation based on the effects of stage matching.

The post-stall behaviour of the VIPER engine compressor changes at 80% speed from stable rotating stall (at low speeds) to multiple surge cycles, and at 88% speed to a single surge followed by a combustor flameout. The change at 80% speed is

¹ At high speed the disturbance was initially fixed in a particular circumferential position before starting to rotate.

qualitatively in agreement with the theory of Greitzer [1976a,b, reviewed in chapter 2]. The modelling work presented in chapter 4 shows that the speeds at which the changes occur are affected by the combustor inbleed method used to move the compressor into stall/surge.

Perhaps the most significant observation in the experimental results from the VIPER engine is that the disturbances leading to stall are initially very localised within the compressor; that is, they show the short circumferential lengthscale behaviour reported by Day [1993a] as opposed to the 'modal' behaviour reported for instance by McDougall et al [1990]. This has important implications with respect to both stall modelling and active control of stall. The assumptions implicit in pitchwise average stall models (including the two models presented in chapters 4 and 5) are such that they are most accurate for low circumferential harmonic disturbances. Some of the assumptions frequently made about gap regions and bladerow performance are likewise less reliable for disturbances that are small circumferentially. Active control of stall is more difficult with short lengthscale disturbances. Higher response actuators are required to match the circumferential profile of the disturbance. Non-linear effects are also significant with disturbances of this type, which would make the simple linear feedback control systems (such as used by Paduano et al [1993], for example) less effective.

Other observations from the experiments indicate that active stall control would be difficult for this particular engine, especially at high speed. At design speed, the entire transient from initial rotating disturbance to a significant drop in combustor pressure takes only 10ms, which represents the available 'time-window' for detection, computation, actuation and any time delay as the flow responds to the actuator output. Furthermore, at these speeds, very little indication was found of impending stall prior to the event. At lower speeds the inception period was considerably longer, and evidence of impending stall was found, sometimes over 100 rotor revolutions before the event. Even within this period, however, the flow showed quiescent periods with no sign of impending stall.

The complexity of stall and surge as a subject is well demonstrated by the diversity of behaviour of the engine used in the experiments. Four different stall inception patterns are described in chapter 3; growth of a stall cell out of a 'benign' front-end stall pattern, growth of a single cell starting at the front, growth of a single cell starting towards the back and a 'modal' inception pattern (over a very small speed range). This variety of behaviour also indicates that active control of stall would be difficult; a successful control system designed for use across the whole speed range would have to

be capable of dealing with all of the stall inception patterns. In addition to the diversity of stall inception behaviour, three different patterns of post-stall behaviour are described in chapter 3; stable rotating stall, multiple surge, and surge followed by flameout. The modelling work described in chapter 4 suggests that with all combinations of speed and combustor inbleed level seven qualitatively different patterns of post-stall behaviour are possible.

The analysis provided in chapter 3 is detailed, with a number of new observations (for instance the initial period of stall inception at high speed, where the disturbance is fixed in circumferential position, and also the multilobed rotating pattern at the tip of the first rotor during stall inception at middle speeds). The significance of the new observations is difficult to assess in the absence of similar information from other tests. Nonetheless they represent a useful contribution to the available data on stall inception. The cumulative effect of such analyses is a greater understanding of stall and surge, with potential benefits in steady state compressor design and passive stall control as well as stall modelling and active control methodology.

6.2 COMPRESSORS IN PARALLEL MODEL

A multiple compressors in parallel model is presented in chapter 4 for stall in a single shaft aeroengine, based on existing compressor modelling techniques, but extended to include the engine environment. A thorough investigation of the capabilities and limitations of such a model is carried out, based on a comparison of the results with the experimental data presented in chapter 3. In general the model shows excellent agreement with respect to the overall behaviour and post-stall behaviour in particular. The detailed representation of the stall inception period is less accurate. This is as expected; the start of the stall inception period is characterised by highly non-uniform flow axially and radially as well as circumferentially, and the model has definition in the circumferential direction only.

The model showed considerable sensitivity to the level of smoothing in the circumferential direction. There are many possible causes of mixing in the circumferential direction (such as in-passage mixing and axial gap effects). Many of these can be shown to be negligible for low circumferential harmonic disturbances, but are not necessarily negligible for short circumferential lengthscale disturbances as demonstrated by both the VIPER engine and the model (at low levels of smoothing). Further work is required to determine a 'representative' level of smoothing in this direction.

The model was used to investigate the effect of using combustor inbleed to move the compressor toward stall as was done in the experiments reported in chapter 3, and showed that the post-stall behaviour was significantly altered by the presence of the inbleed. As expected the stall inception behaviour was similar with and without inbleed.

A system identification technique was used to develop a stall control system for the model. Simple control strategies were devised for both axisymmetric and non-axisymmetric instabilities. These were tested by repeated use of the model, and proved successful. In view of the limitations of the model (discussed above) with respect to the stall inception period, it is unlikely that the resulting control strategies could be applied directly to an engine. The objective of the control work is rather a demonstration of the system identification methodology, which is equally applicable to a real engine control experiment (although signal noise coupled with an underlying system that is inevitably more complex than that of the model would make application more difficult).

6.3 THREE-DIMENSIONAL TIME MARCHING MODEL

The major advantage of the model developed in chapter 5 over previous pitchwise average stall models is that the near wall flow (and the tip clearance region in particular) is calculated directly, including viscous effects. This is known to be a significant region with respect to stall inception; the position of the surge line is often critically dependent on the level of clearance. Stall inception results likewise (McDougall et al [1990], for instance) frequently show the end-wall regions to be significant in the initial development of stall disturbances. It would seem, therefore, that a good representation of these regions is essential if a model is to accurately represent the stall inception process. Once the disturbance has grown radially large, the endwall regions become proportionately less significant, and a less complex model would suffice.

An unsteady three dimensional blade passage calculation could in theory be used to provide more accurate stall inception results including near wall effects. In practice the computing time/cost is prohibitive for multistage applications. Although the model developed in chapter 5 is only tested against a single stage test case, it is not limited in this respect; the size of grid and number of calculations required for a multistage compressor is of the order of that required for a single conventional unsteady blade passage solution. The stall model presented in chapter 5 is thus the first to model the endwall regions directly that can be applied to multistage compressors.

Various tests are reported in chapter 5 which demonstrate the steady state and unsteady capability of the model. Some questions remain regarding the implementation of the model (as would be expected at this stage of development), particularly with respect to the unsteady response of the endwall boundary layers, the inlet and exit boundary conditions and the details of the constant blade force model. The objective of the work presented in chapter 5, however, was not to present a new type of three dimensional time marching CFD code, nor to develop a new set of assumptions regarding near wall flow. Rather, it was to present a new type of stall inception model, where a viscous pitchwise average throughflow model is combined with a circumferential discretisation to give a time-accurate prediction of stall inception behaviour.

The comparison of the model results with McDougall's [1988] test data presented in chapter 5 is very encouraging. McDougall observed an initial disturbance leading to stall that was centred at around 85% height towards the back of the rotor, with a radially uniform 'modal' response of the upstream and downstream flow. The same profile is observed in the model results, demonstrating that this new type of stall model is capable of capturing stall dynamics.

6.4 FUTURE WORK

One of the most significant observations made during the stall inception tests on the VIPER engine was that the initial disturbances leading to stall were not low circumferential harmonics, but were limited circumferentially to a small sector of the annulus. If this result proves general, then some of the assumptions underlying the most commonly used stall modelling techniques will have to be revisited. By way of example, Dunham [1965] demonstrated that the flow redistribution in axial gaps between bladerows is often negligible. This may not be the case, however, for the short circumferential lengthscale disturbances seen during stall inception. Likewise the lagged blade response investigated by Mazzawy [1977] may have to be modelled differently for short circumferential lengthscale disturbances. Perhaps the most significant area in this respect is that of the pitchwise average assumption (section 2.2.2.3) made in many stall models, including the two reported in this thesis.

The effects of the short circumferential lengthscale disturbances described above could all be accounted for in an unsteady blade passage CFD calculation. The computing power required for such a calculation could be considerably reduced for short circumferential lengthscale disturbances by following the disturbance around the annulus (that is, only performing the calculations over a limited portion of the annulus).

Even so, for the compressor in the VIPER engine, if three blade passages were modelled for each bladerow, then the computing power required would be around fifty times that required for the model developed in chapter 5, and (for a regularly used program) this is beyond currently available computing power.

In terms of active control there is a whole field of work to do with actuation schemes and control algorithms which has barely been touched in the course of the thesis. Many numerical studies have been carried out in this area (such as the one presented in section 4.4). Frequently the model on which the work is based is not sufficiently complex to represent the important early stages of stall inception. Provided that the initial encouraging results prove representative, the model developed in chapter 5 will offer a more accurate representation of stall inception, and hence will be an ideal vehicle for such studies.

In section 4.4 a system identification technique is used to develop an active stall control strategy based on the compressors in parallel model. Specific improvements and extensions to this work are listed in section 4.4.2.5. A framework for future development of the three dimensional time marching model is presented in section 5.6.

7. CONCLUSIONS

In this section the major conclusions of the work presented in the thesis are listed in numerical order.

7.1 ROLLS-ROYCE VIPER ENGINE EXPERIMENT

1. A thorough analysis has been presented of the stalling behaviour of a compressor in a jet engine environment.
2. A wide range of stall inception and post-stall transient behaviour has been observed.
3. The final operating condition of the engine after a stall event is stable rotating stall at low speed and multiple surge cycles at middle to high speeds. Near design speed the combustor flame is extinguished and compressor flow and pressure ratio remain at a low level.
4. All stall/surge events start with one or more disturbances rotating around the annulus near shaft speed. Near design speed this is preceded by a period with the disturbance fixed in a circumferential position.
5. The initial disturbance leading to stall is localised axially and circumferentially, except in a very narrow speed range. The localised disturbances conform to the short circumferential lengthscale disturbances described by Day [1993a].
6. The axial position of the initial disturbance moves rearwards as engine speed increases in line with stage matching effects.
7. At low speed a 'benign' form of rotating stall is present, limited to the front of the compressor, which does not greatly affect engine operation. This benign stall extends radially to the hub, but is strongest near the outer wall.
8. A secondary multi-lobed disturbance has been observed during the stall inception period. This disturbance is localised to the tip of the first rotor and rotates slightly faster than the stall cell. The disturbance is amplified in the stall cell region, giving the stall cell a 'fingered' appearance.
9. Active control of stall/surge in this engine would be difficult because of
 - a) the wide variety of stall inception behaviour observed.
 - b) the short circumferential lengthscale of the disturbances leading to stall (giving rise to a requirement for a high frequency response in the controlling system).

c) the growth rate of the initial disturbances; at high speed the 'time-window' for detection/actuation is only 10ms.

10. At middle speeds small amplitude rotating disturbances were observed up to and over 100 rotor revolutions prior to stall. Within this period quiet intervals were observed with little activity. No indication of impending stall was found at design speed.

11. The technique developed by Tryfonidis et al [1995] for tracking pre-stall disturbances was applied to the data, but showed no improvement over what could be seen by eye from the time traces.

7.2 COMPRESSORS IN PARALLEL MODEL

1. A compressors in parallel model has been developed using compressor modelling techniques similar to those of Moore and Greitzer [1986a,b], but extended to include the effects of an engine environment.
2. The results of the model have been compared to measurements from the VIPER engine experiment. The model gives a good representation of the overall stalling behaviour. In particular the effects of the engine environment (small exit volume, flameout) are captured.
3. The model has been used to demonstrate that the use of combustor inbleed in the VIPER experiment did have an effect on the post-stall behaviour of the engine.
4. The model is sensitive to the level of smoothing used in the circumferential direction. Further work is required to determine a representative level of smoothing.
5. The detailed behaviour during stall inception is not accurately predicted by the model.
6. The model has been used to demonstrate a system identification technique for devising a strategy to actively control both axisymmetric and non-axisymmetric flow disturbances. The control system devised has been shown to successfully control disturbances in the model.

7.3 3D TIME-MARCHING MODEL

1. A new method of stall modelling has been developed, where the flow in the annulus boundary layers is calculated directly, allowing the initial stages of the breakdown of the flowfield to be captured. At the same time the model is suitable for application

to multistage compressors, with a similar requirement in terms of computing power to a single blade passage solution using an unsteady CFD code.

2. A number of tests have been performed to validate the basic functionality of the model. Particular areas have been identified where further development is required.
3. Initial results from the model have been compared with test data from a low speed single stage rig. The good agreement demonstrates that this new type of model is capable of accurately predicting stall inception behaviour.

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TABLES AND FIGURES

AXIAL STATION	NAME	CIRCUMFERENTIAL POSITION					
		1	2	3	4	5	6
IGV EXIT	ki1-ki5	14	91	139	215	294	-
IGV EXIT	nki1-nki3	19	96	148	-	-	-
STAGE 0 EXIT	nk01-nk02	3	89	-	-	-	-
STAGE 2 EXIT	k21-k25	25	100	175	249	329	-
STAGE 3 EXIT	k31-k35	61	148	208	306	358	-
STAGE 4 EXIT	nk41-nk46	24	111	163	221	278	332
STAGE 5 EXIT	nk51-nk52	44	329	-	-	-	-
STAGE 6 EXIT	k61-k65	44	104	196	283	343	-
COMPR EXIT	kel	145	-	-	-	-	-
COMPR EXIT	nkel-nke6 [†]	30	70	134	226	290	332

[†] Transducers nkel-nke6 were connected to pitot heads measuring total as opposed to casing static pressure.

Table 3-1 Positions of High Response Kulite Pressure Transducers
Degrees from top dead centre viewed from the front, i.e. in the direction of rotor rotation.

Rotor Bladerow

Radius (m)	Camber (Degrees)	Stagger (Degrees)	Solidity
0.6096	40.3	38.8	1.48
0.6477	32.0	43.7	1.39
0.6858	26.5	47.9	1.31
0.7239	22.2	51.4	1.25
0.7620	18.5	54.4	1.18

51 Aerofoils, C4 profile based on a circular arc camberline.

Chord - 0.111m

Thickness to Chord Ratio - 0.10

Stator Bladerow, Unloaded Configuration

Radius (m)	Camber (Degrees)	Stagger (Degrees)	Solidity
0.6096	44.4	26.8	1.07
0.6477	44.2	25.6	1.01
0.6858	42.9	24.5	0.95
0.7239	42.3	23.6	0.90
0.7620	42.9	22.9	0.86

Stator Bladerow, Design Configuration

Radius (m)	Camber (Degrees)	Stagger (Degrees)	Solidity
0.6096	44.4	16.6	1.07
0.6477	44.2	15.4	1.01
0.6858	42.9	14.3	0.95
0.7239	42.3	13.4	0.90
0.7620	42.9	12.7	0.86

36 Aerofoils, C4 profile based on a circular arc camberline.

Chord - 0.114m

Thickness to Chord Ratio - 0.10

Table 5-1 Deverson Single Stage Compressor Geometry
Reproduced from table 3.1 of McDougall [1988]

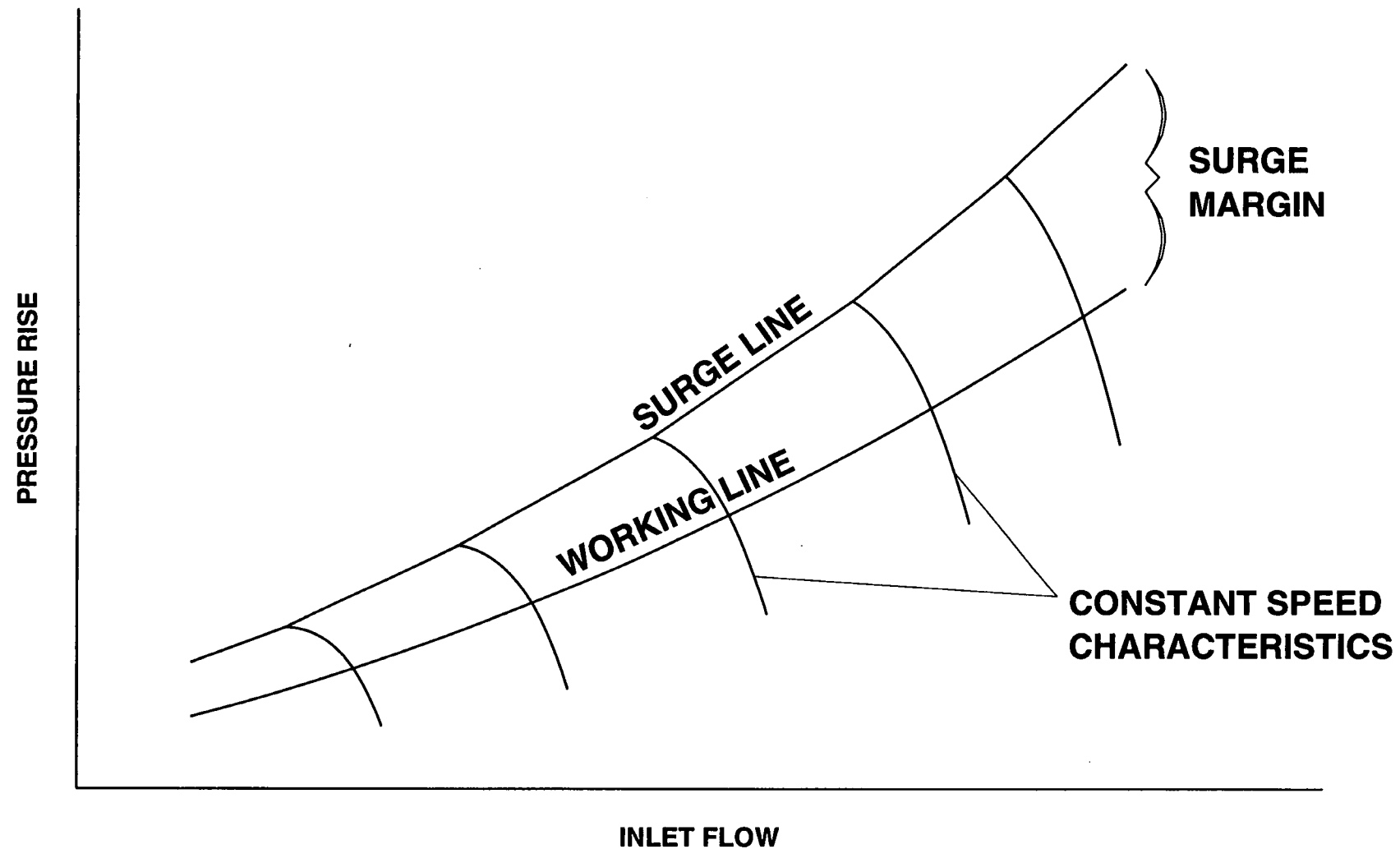


Fig. 1-1 Compressor Operating Map (Schematic).

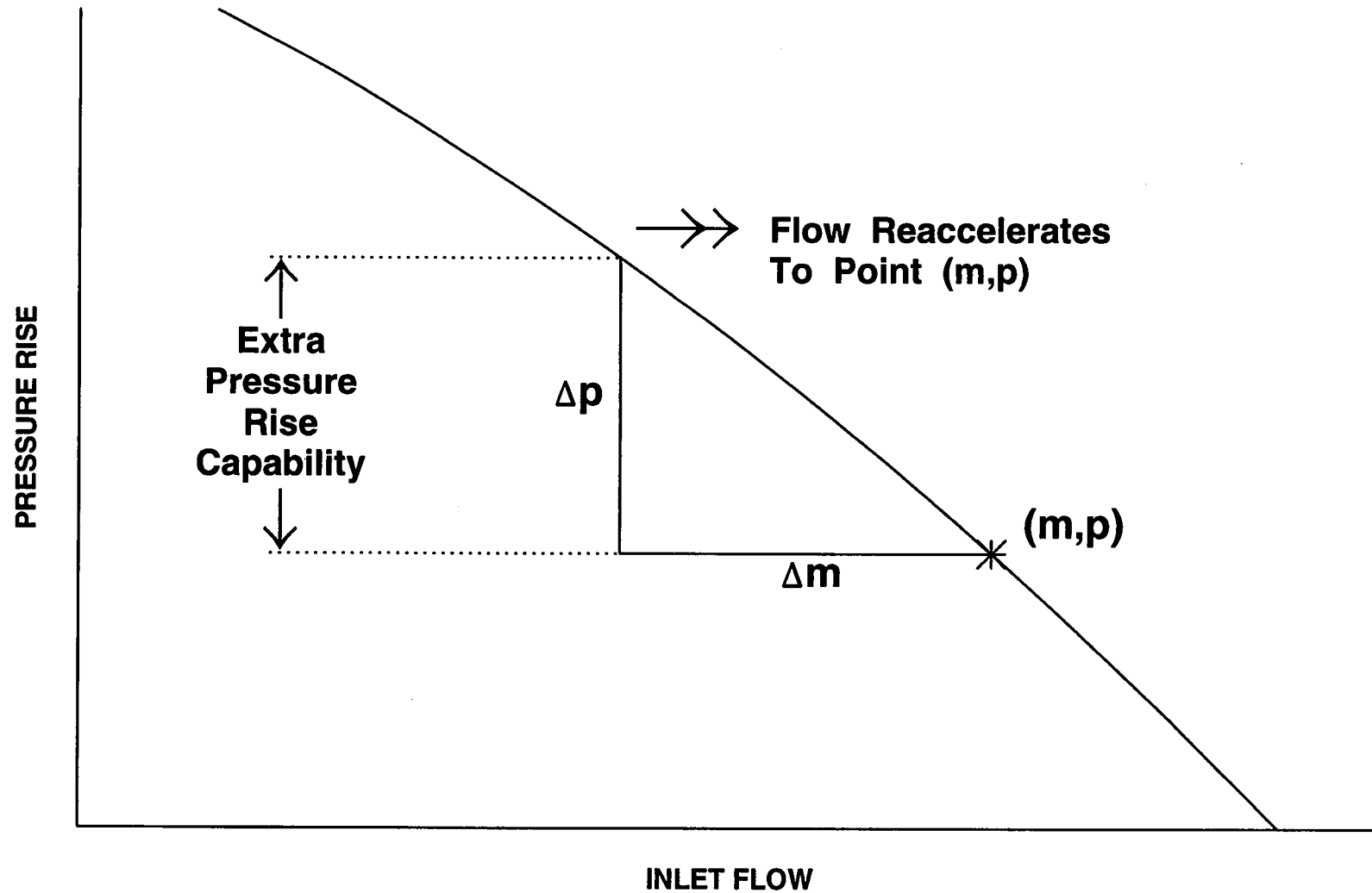


Fig. 1-2 Simple Explanation of Compressor Stability.

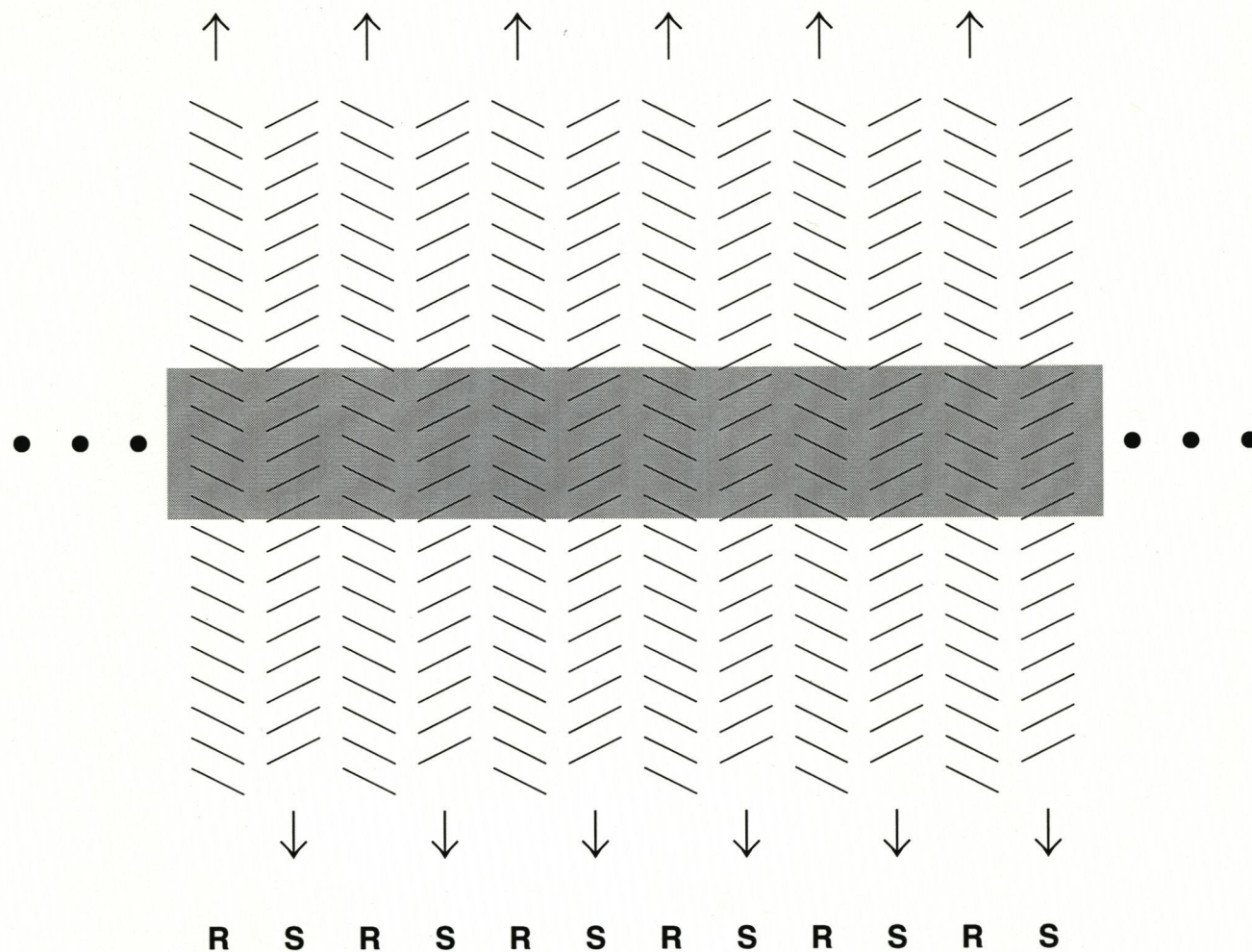


Fig. 1-3 Long Symmetrical Compressor (schematic).

Viewed in a Frame Rotating at Half Blade Speed. Stalled Region Shaded.

PRESSURE RISE (Inlet Total to Exit Static)

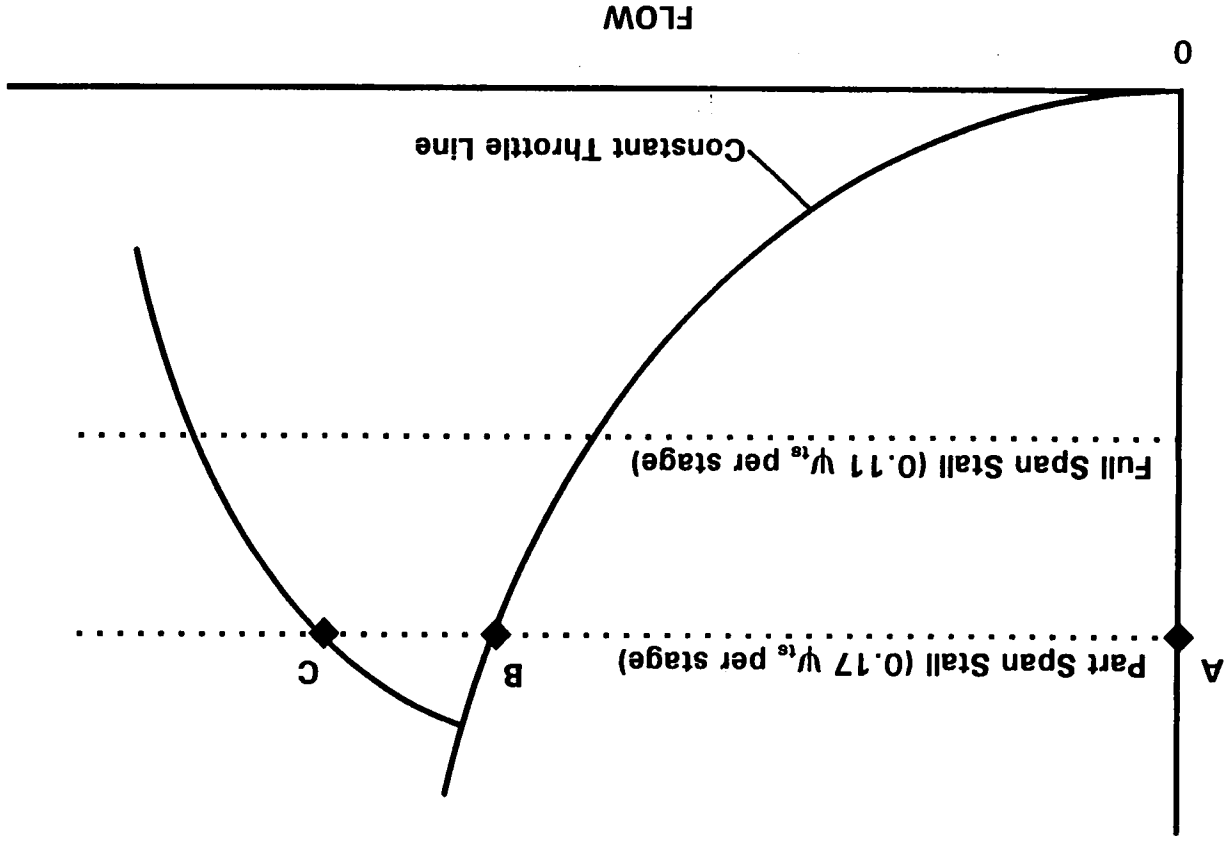


Fig. 2-1 Compressor Performance in Part Span and Full Span Stall.

Based on fig 6 of Day et al [1978].

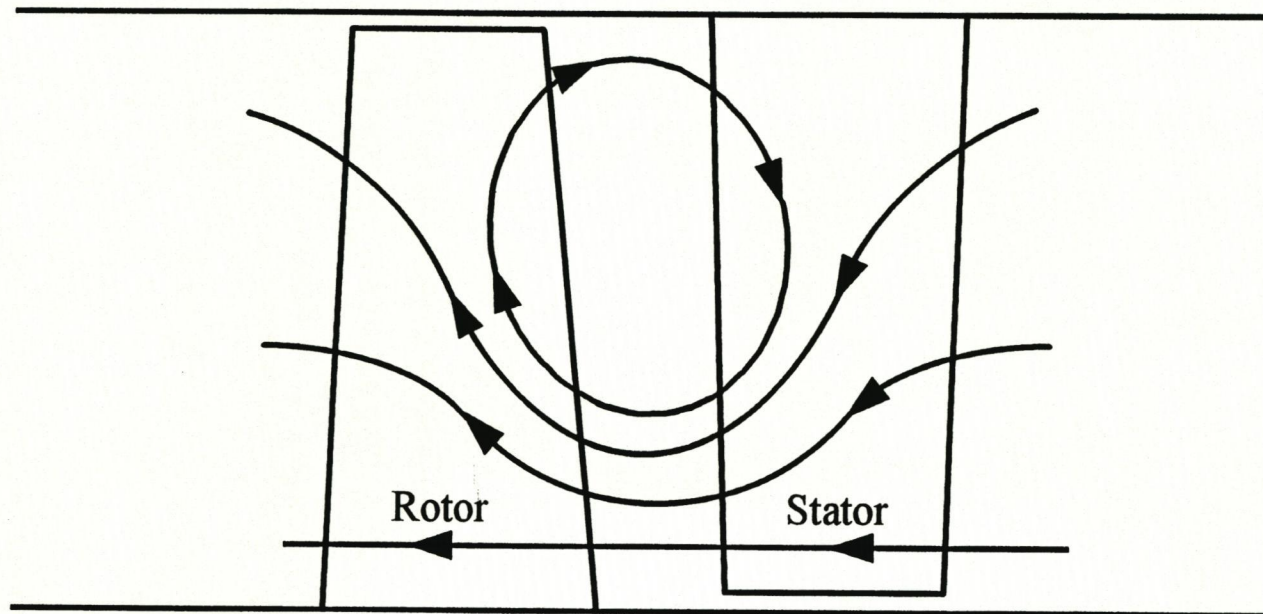


Fig. 2-2 Velocities in r - z Plane at Centre of Stall Cell.

After fig 13 of Das and Jiang [1984].

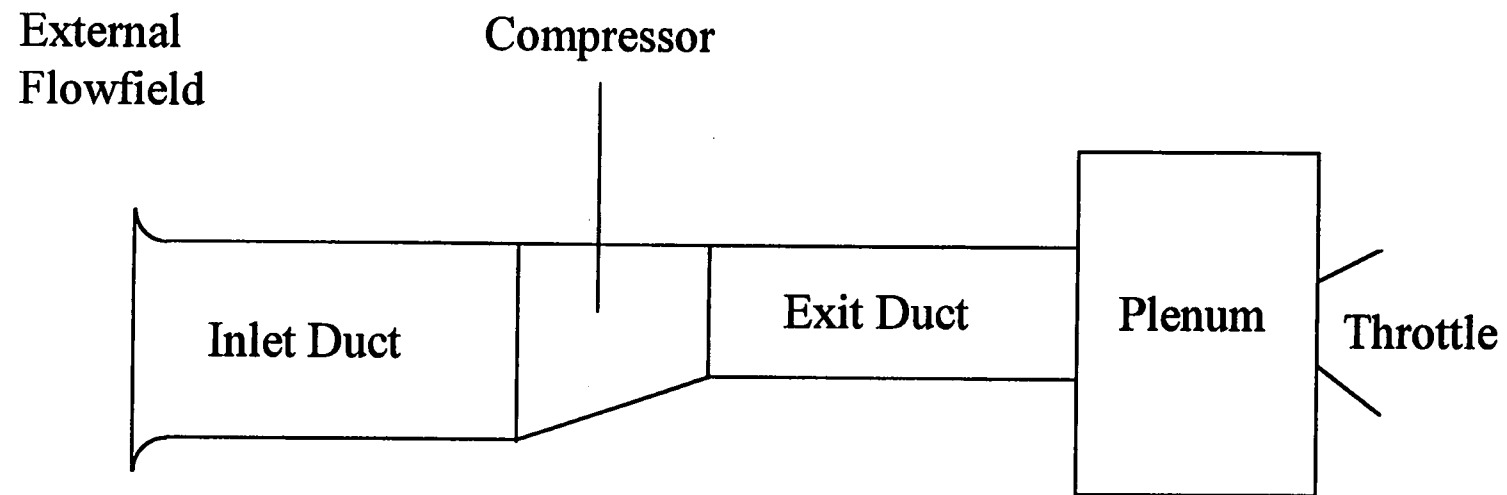


Fig. 2-3 Schematic View of a Compressor in a Rig Environment.

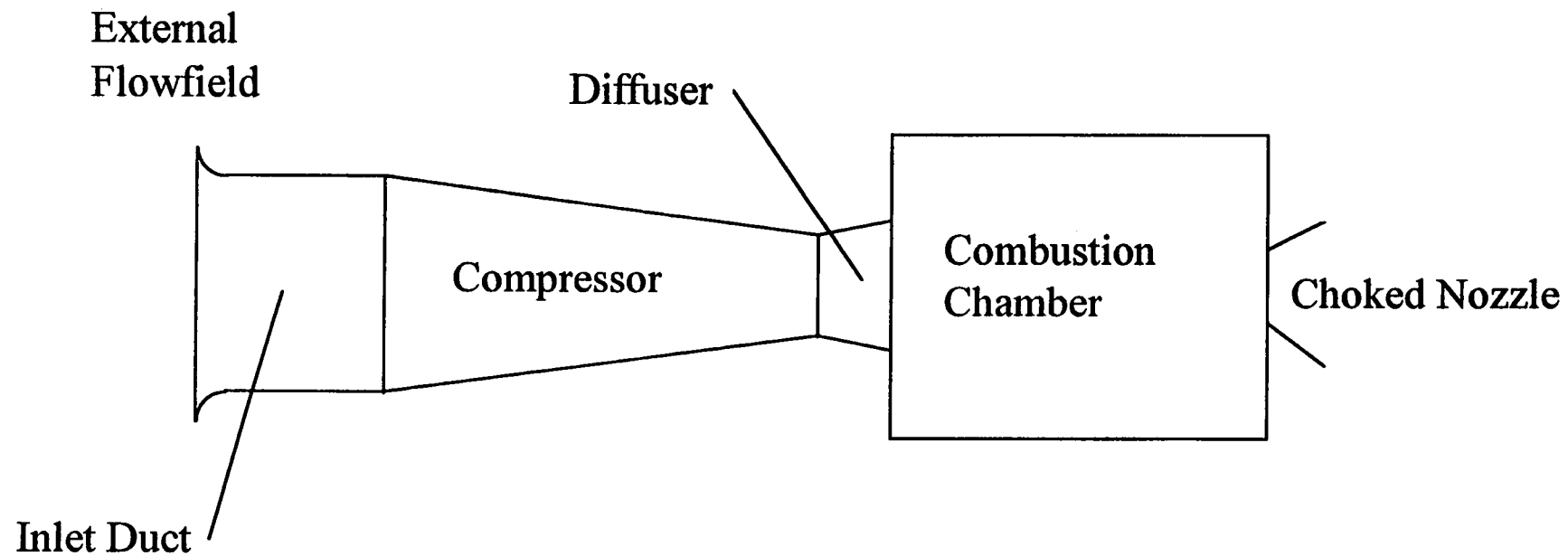


Fig. 2-4 Schematic View of an Engine Compression System.

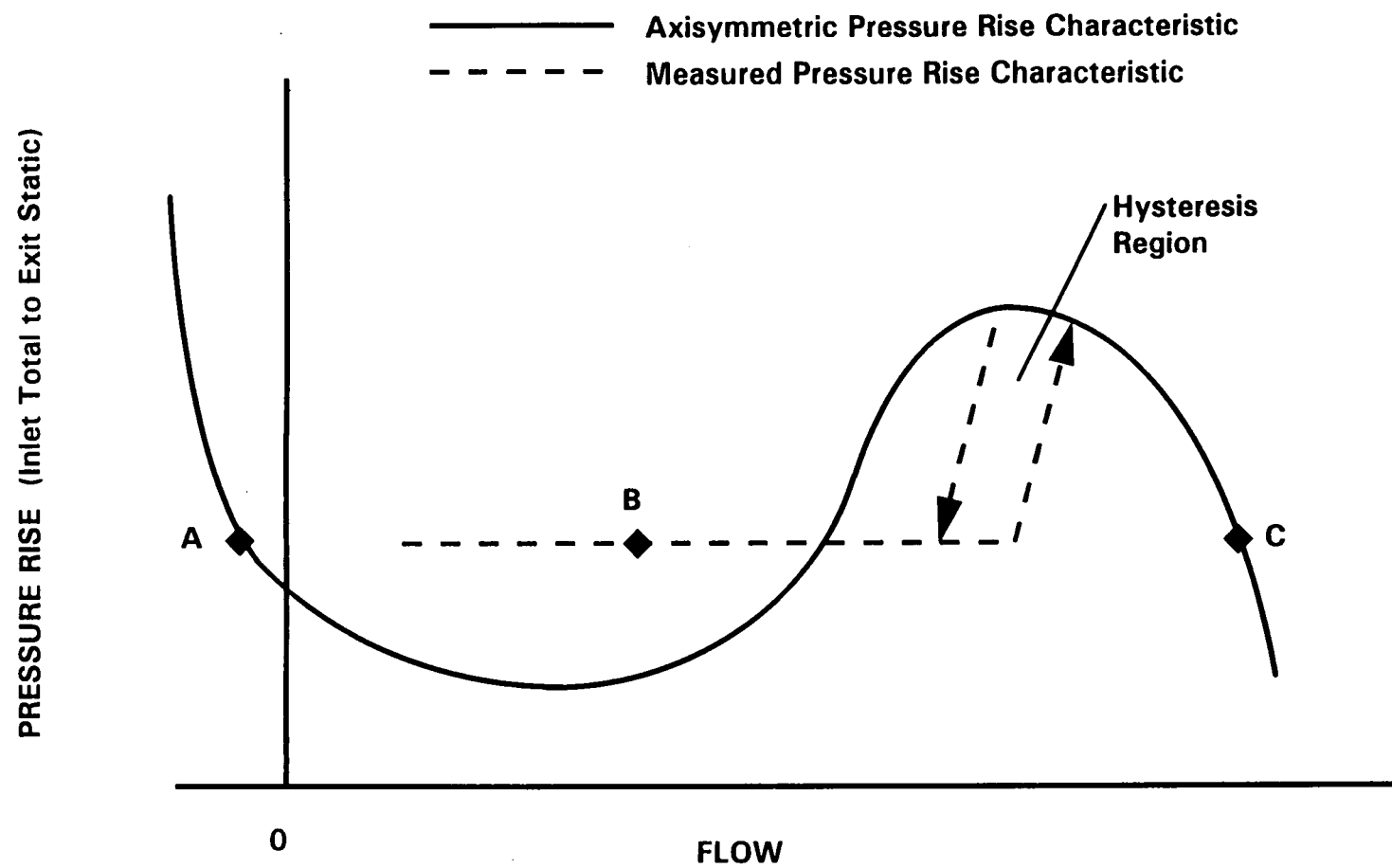


Fig. 2-5 Axisymmetric vs Measured Pressure Rise Characteristic (schematic).

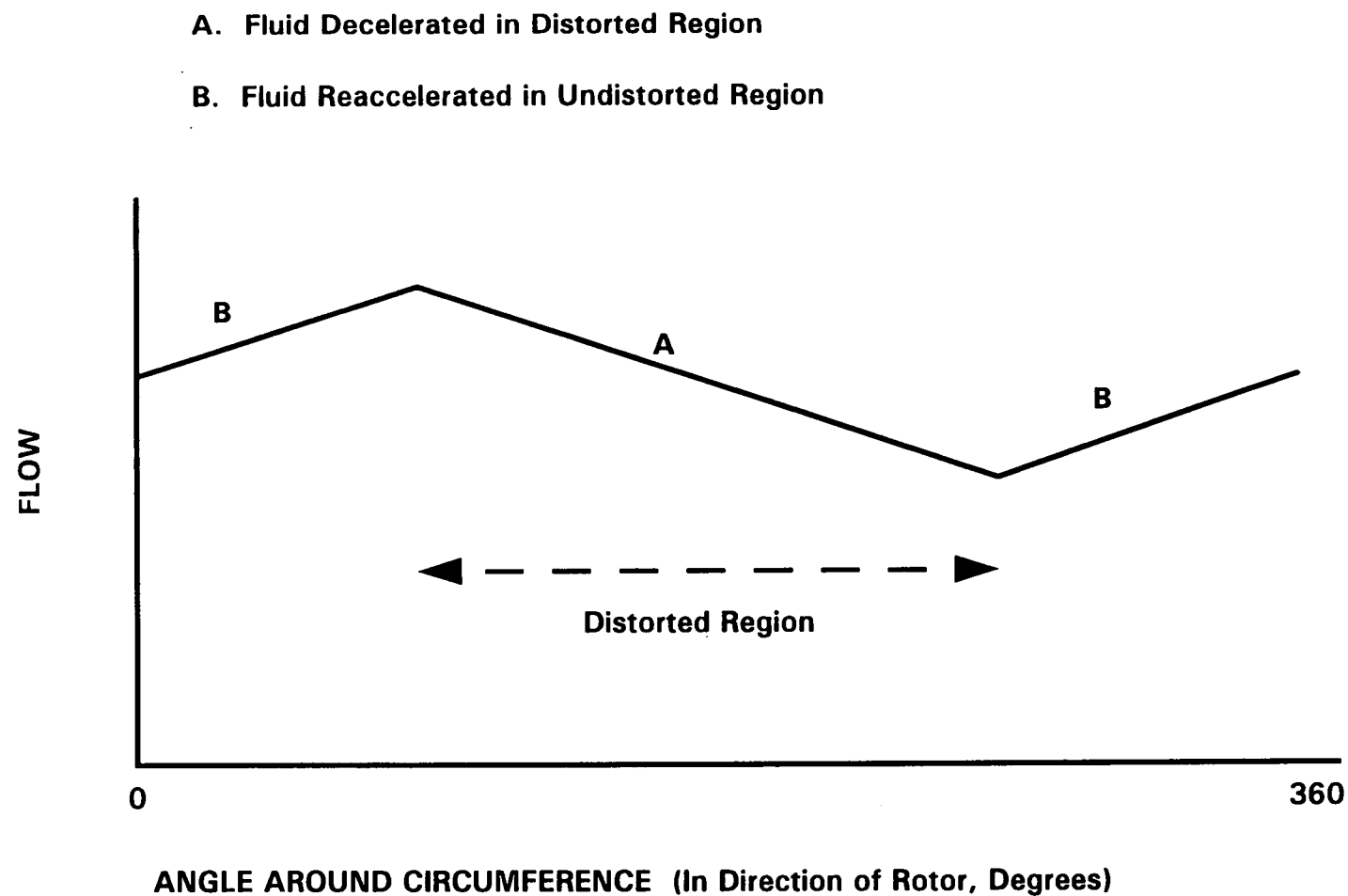


Fig. 2-6 Steady Background Flow With inlet Distortion (schematic).

After Hynes and Greitzer [1987].

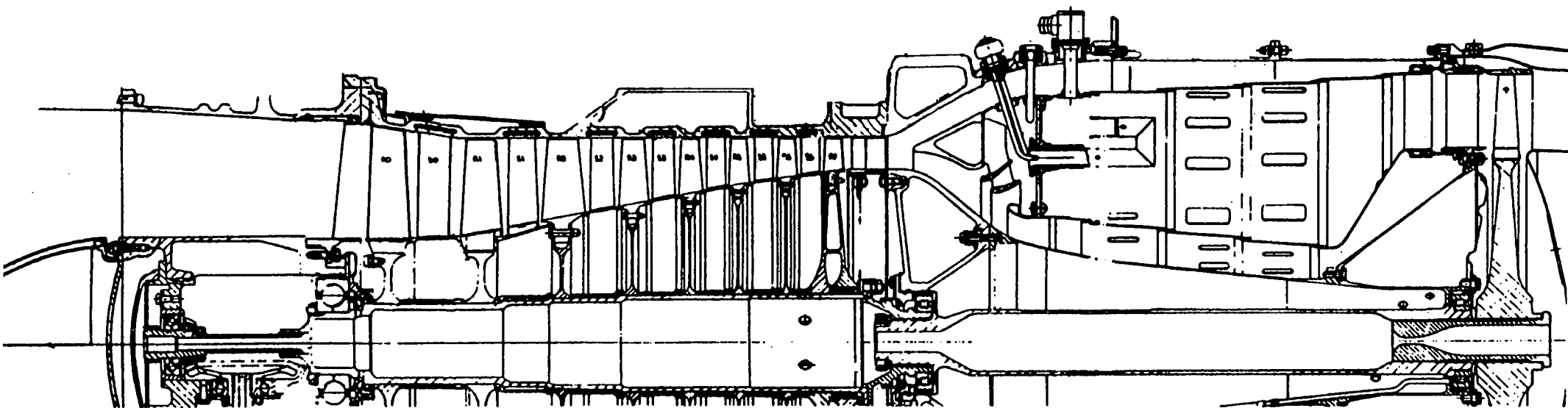


Fig. 3-1 VIPER Compressor and Combustor: General Arrangement.

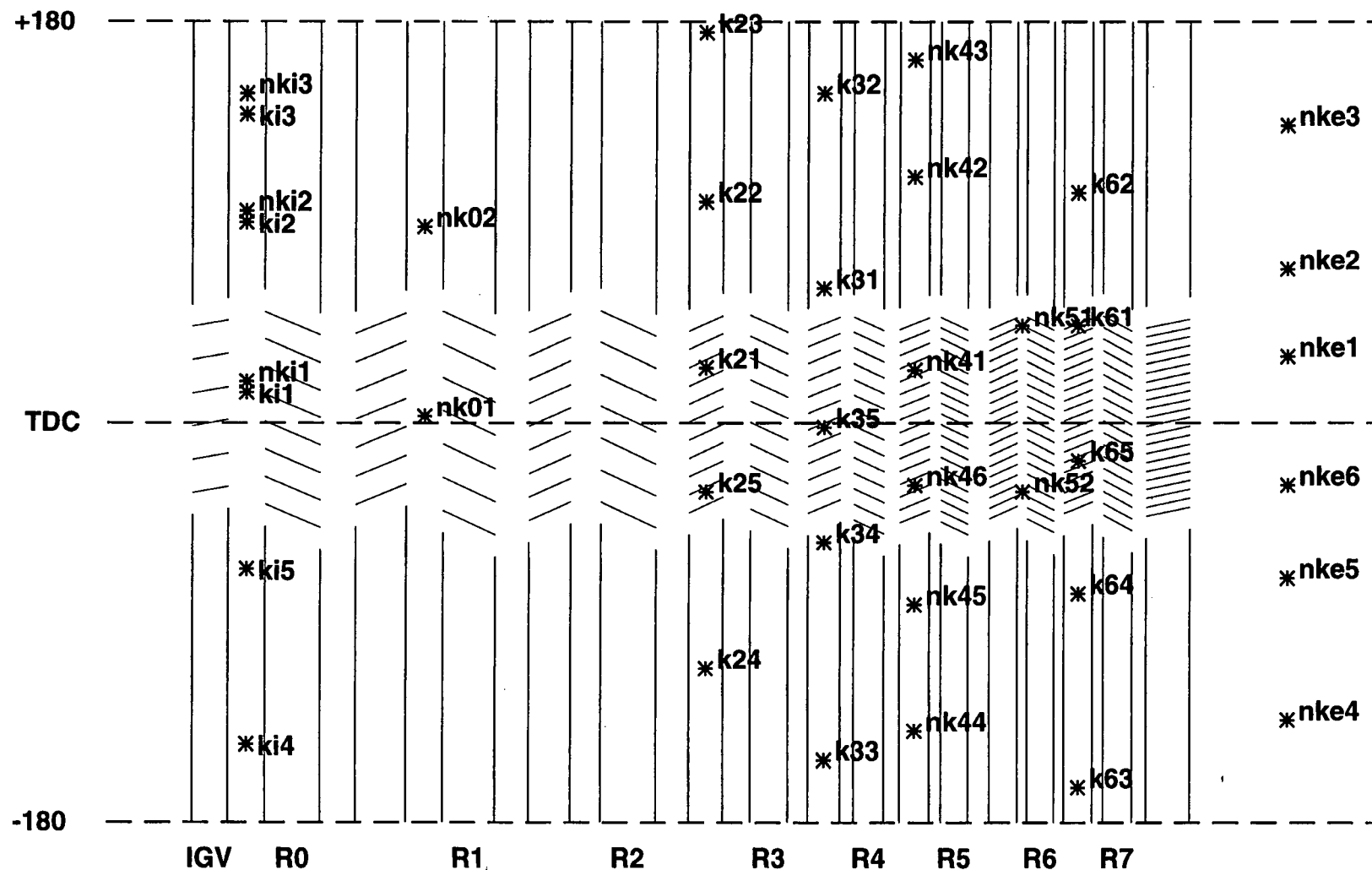


Fig. 3-2 VIPER Compressor: Unwrapped Layout.

Showing Positions of High Response Kulite Pressure Transducers

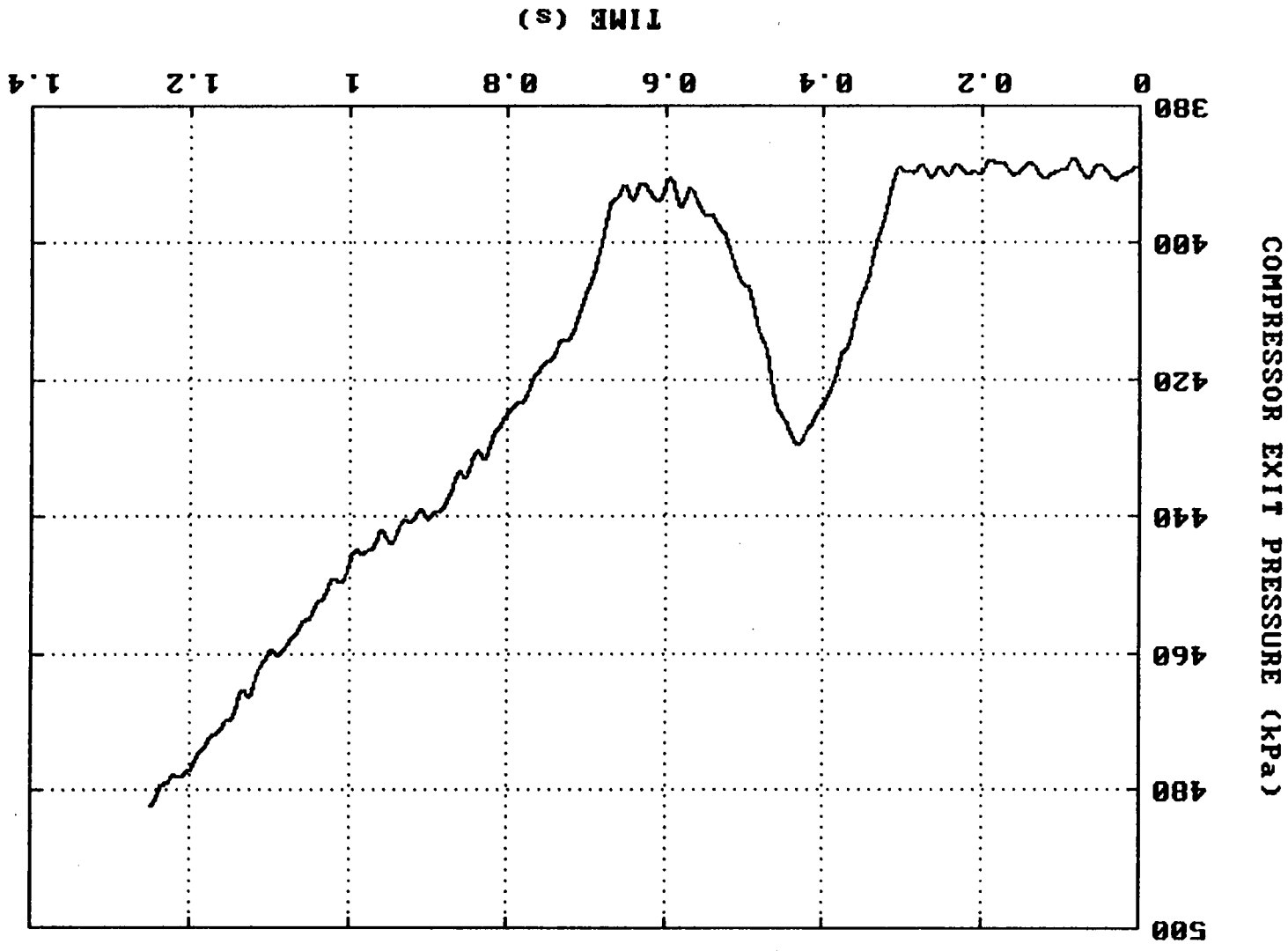


Fig. 3-3 Compressor Pressure (Gauge) During Inbleed at 100% Speed.

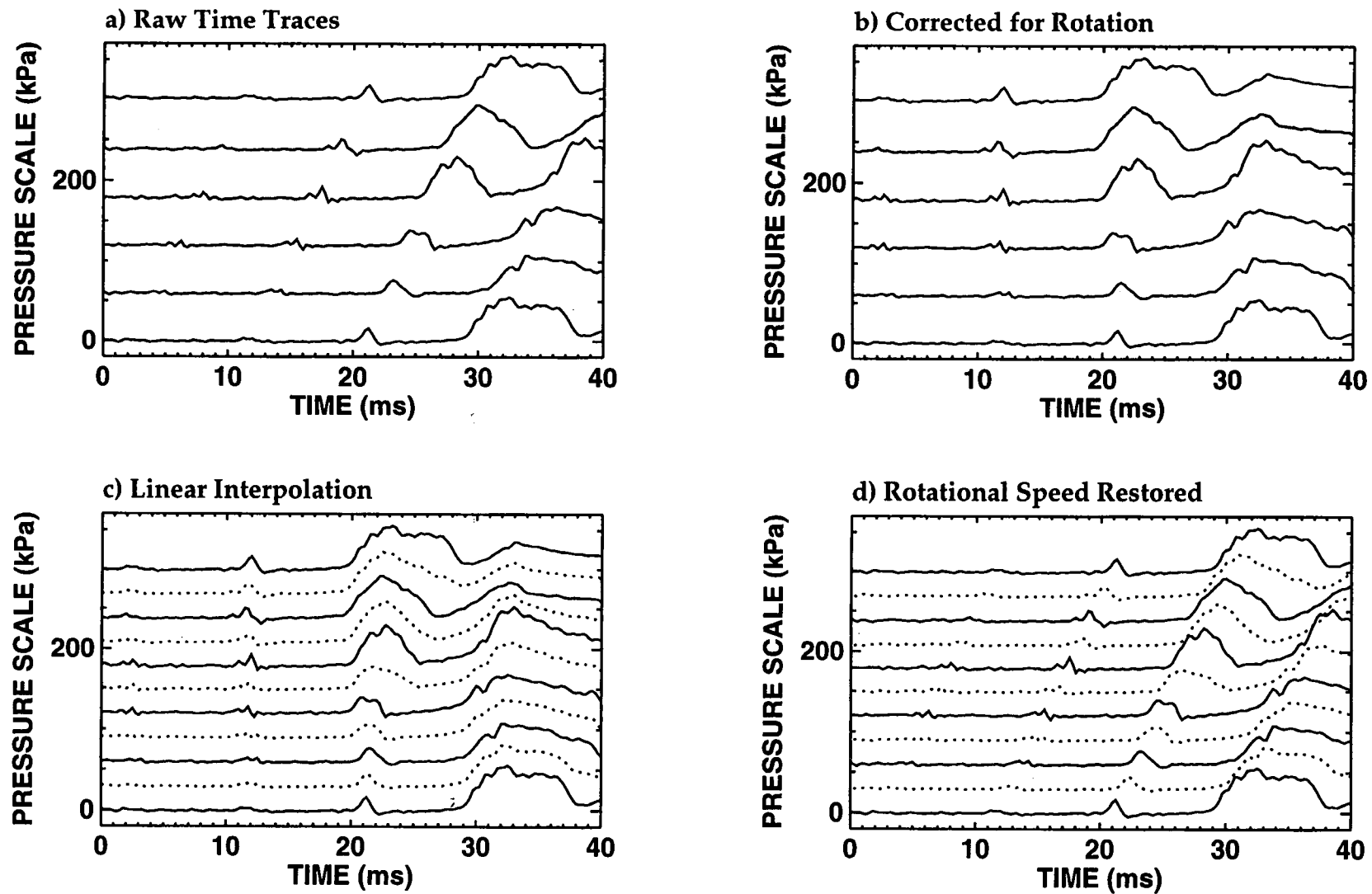


Fig. 3-4 Static Pressure Interpolation Procedure.

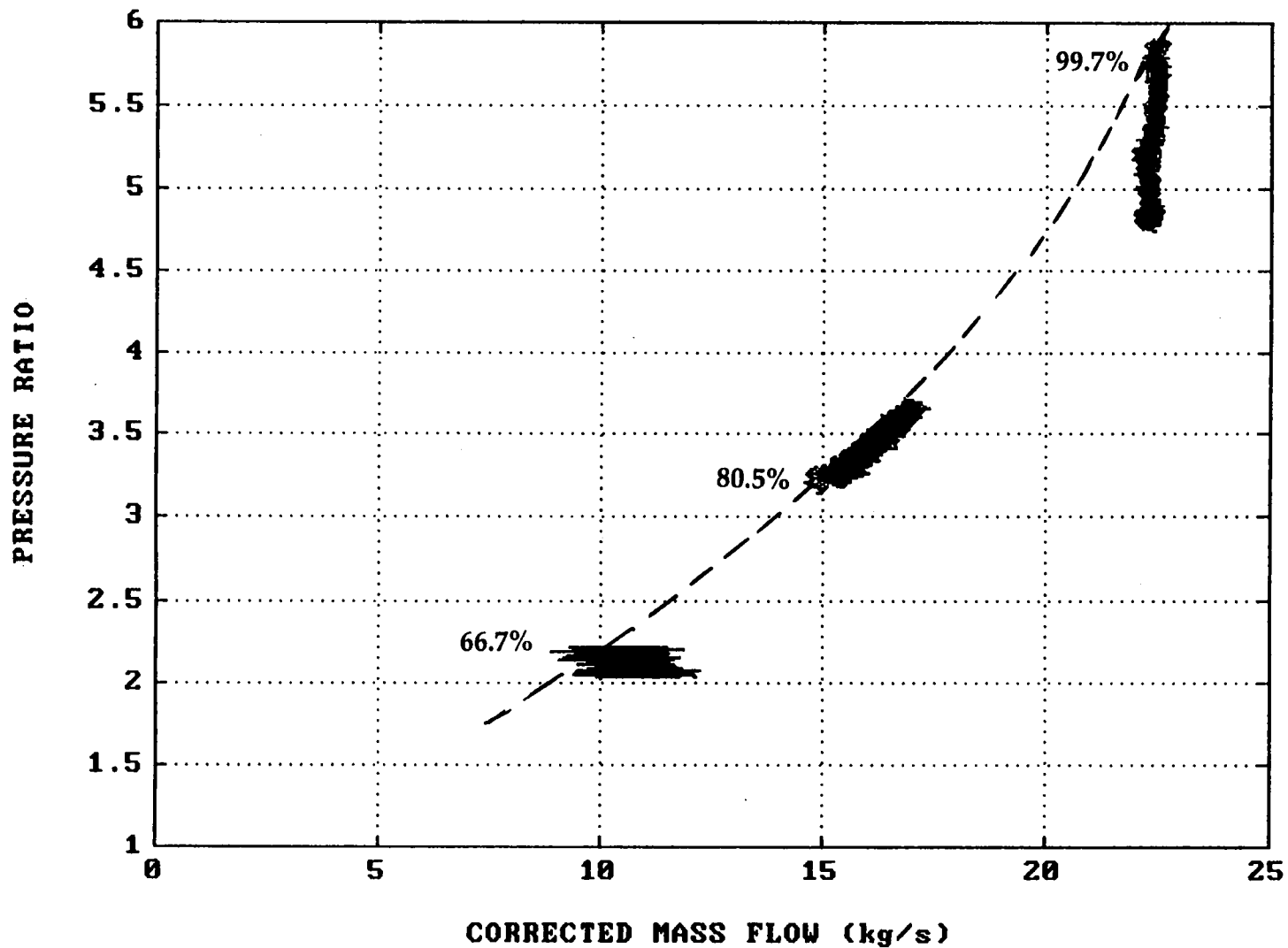


Fig. 3-5 VIPER Compressor Overall Performance.

Speeds marked are those at the stall point.

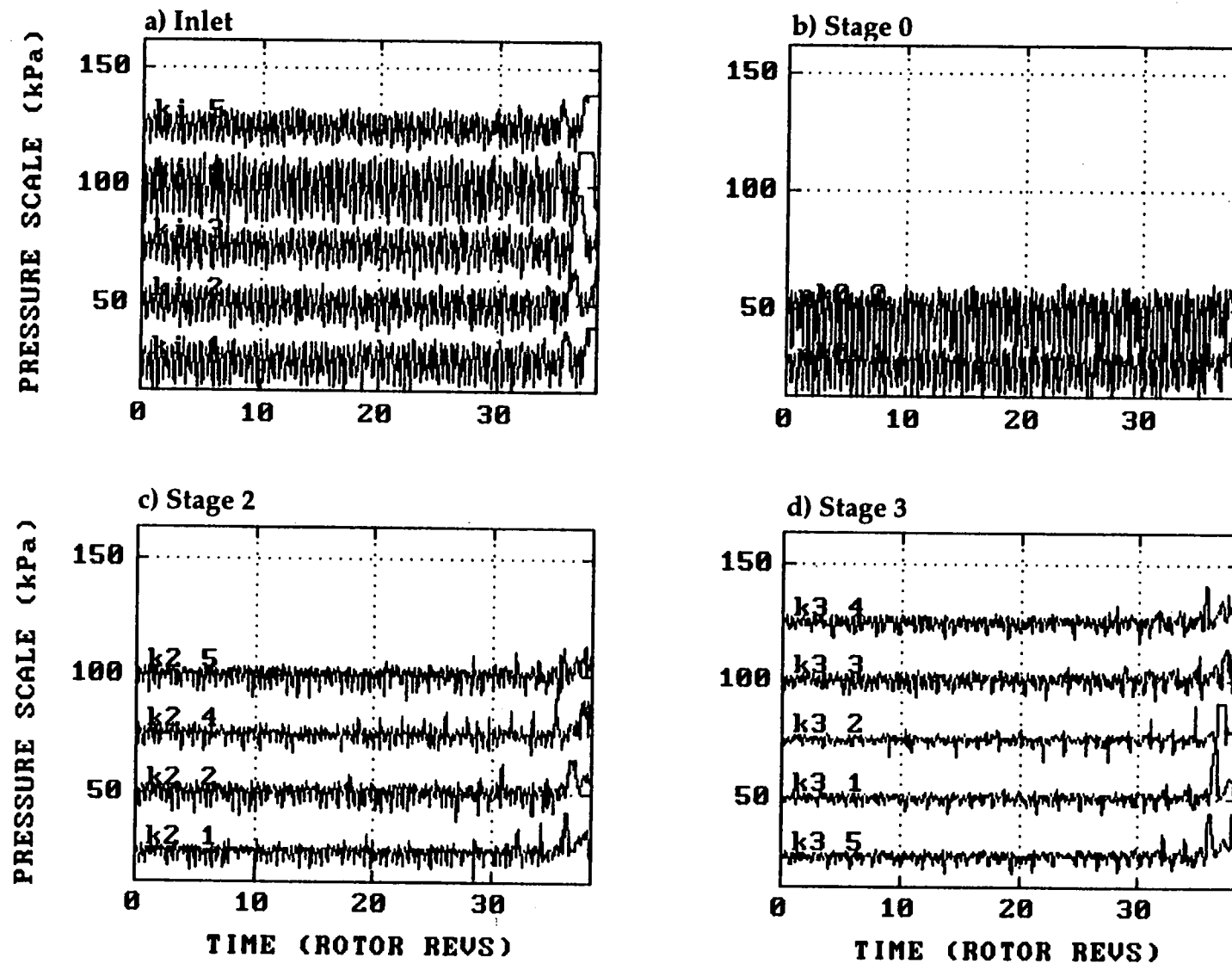


Fig. 3-6 T180806: 66.7% Speed Stall, Front Stages.

Casing static pressure traces from the Kulites at inlet, stage 0, stage 2 and stage 3.

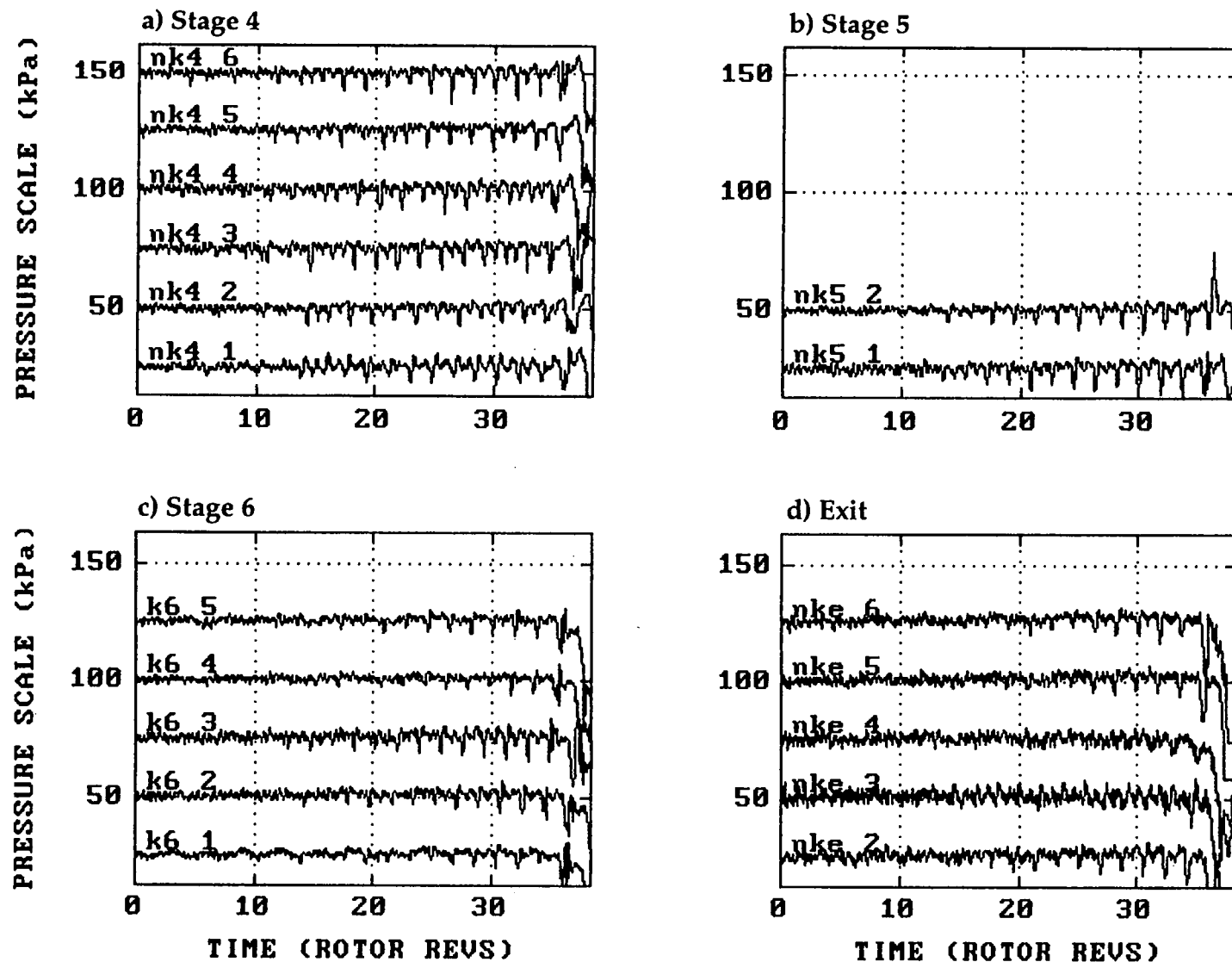


Fig. 3-7 T180806: 66.7% Speed Stall, Rear Stages.

Casing static pressure traces from the Kulites at stage 4, stage 5 and stage 6, and freestream total pressure traces from the Kulites at compressor exit.

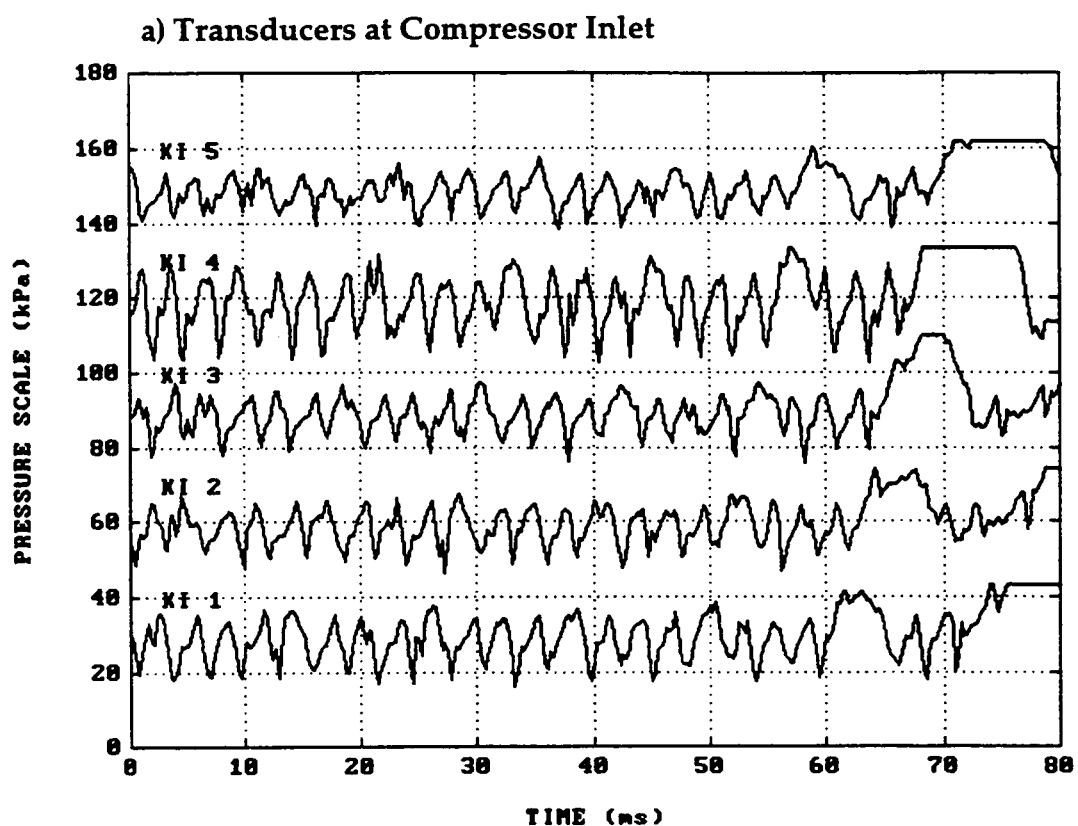
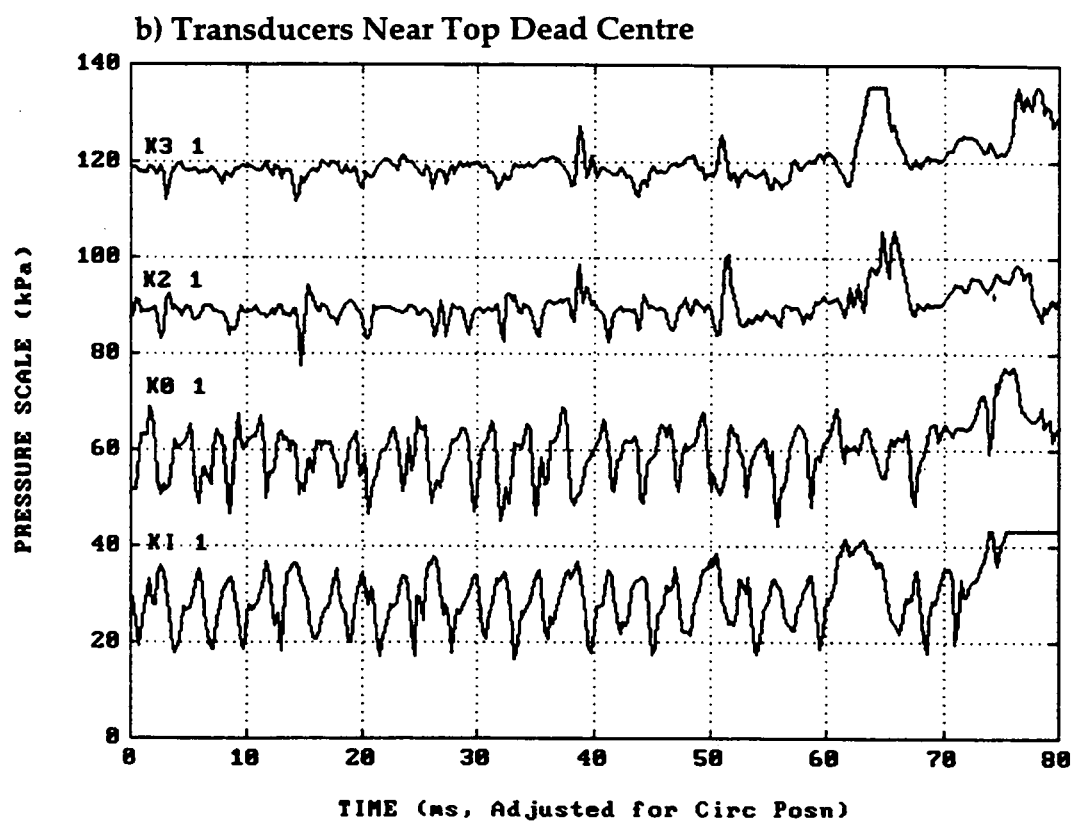


Fig. 3-8 T180806: 66.7% Speed, Transition from 'Benign' to 'Irreversible' Stall.

Casing static pressure traces.

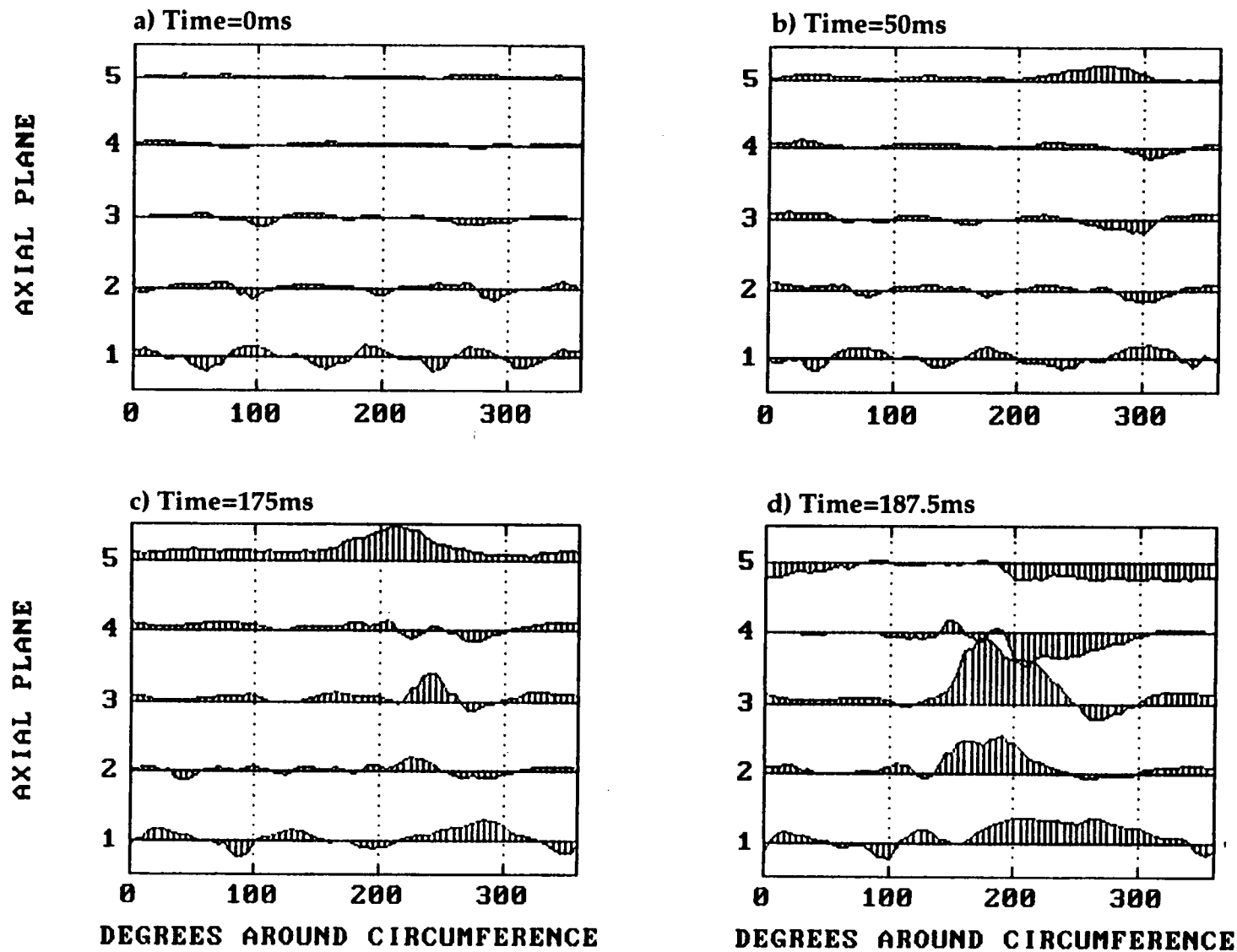


Fig. 3-9 Static Pressure Maps at 60% Speed: Development of Stall Cell.

One unit on the y axis represents 35kPa

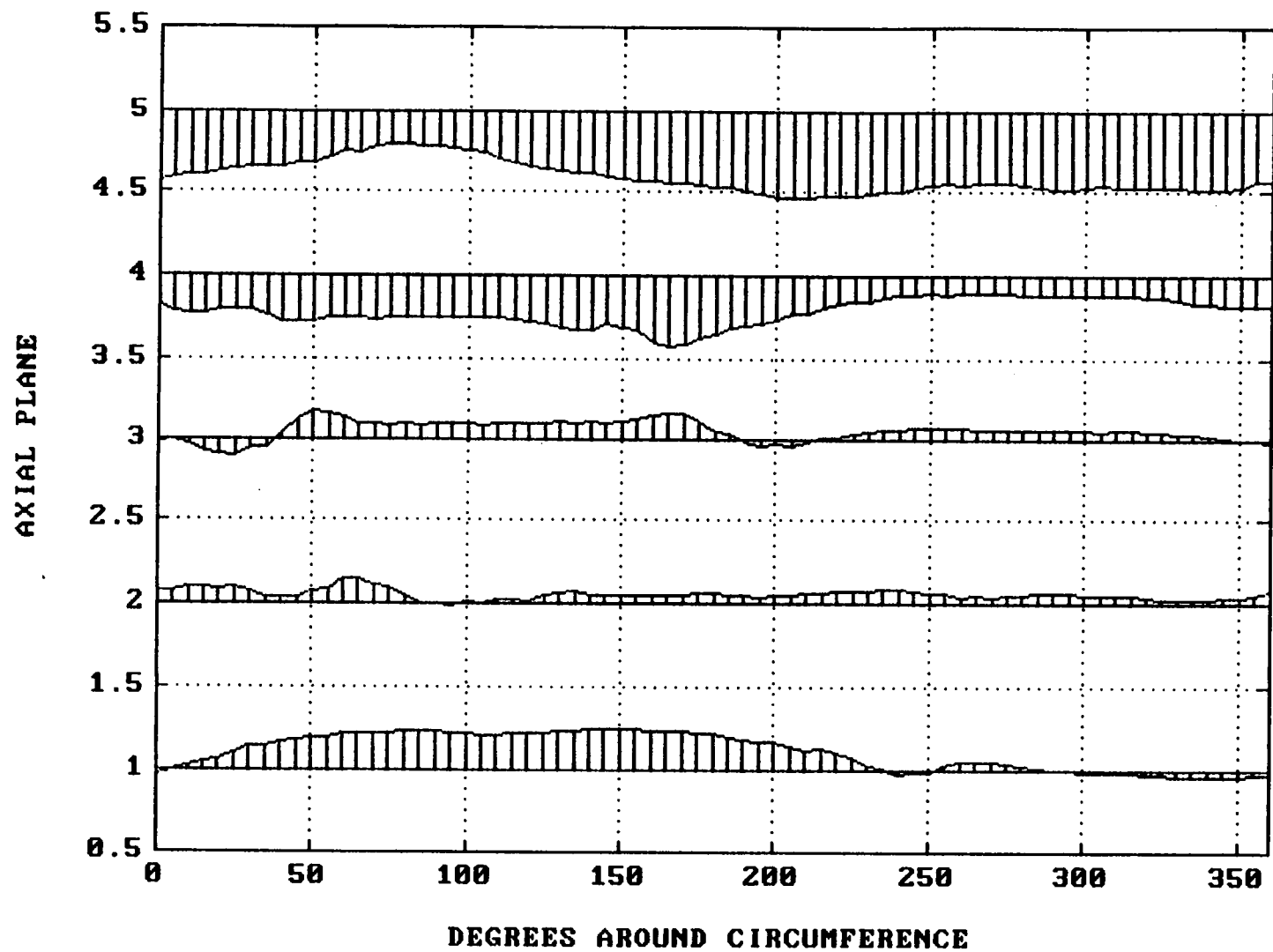


Fig. 3-10 Static Pressure Map at 60% Speed: Stable Rotating Stall (Time = 200ms).

One unit on the y axis represents 69kPa

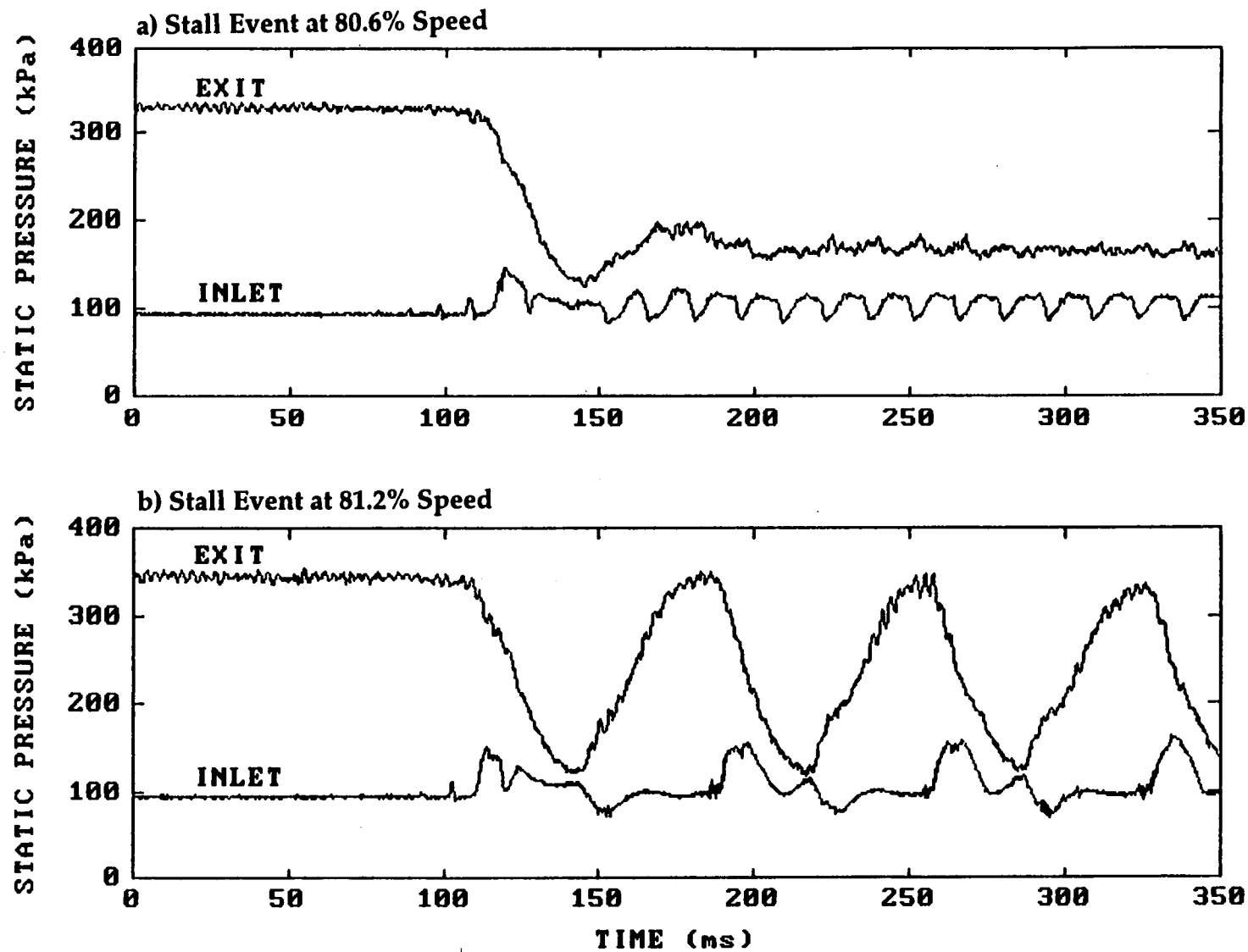


Fig. 3-11 Comparison of Two Mid-Speed Stall/Surge Events.

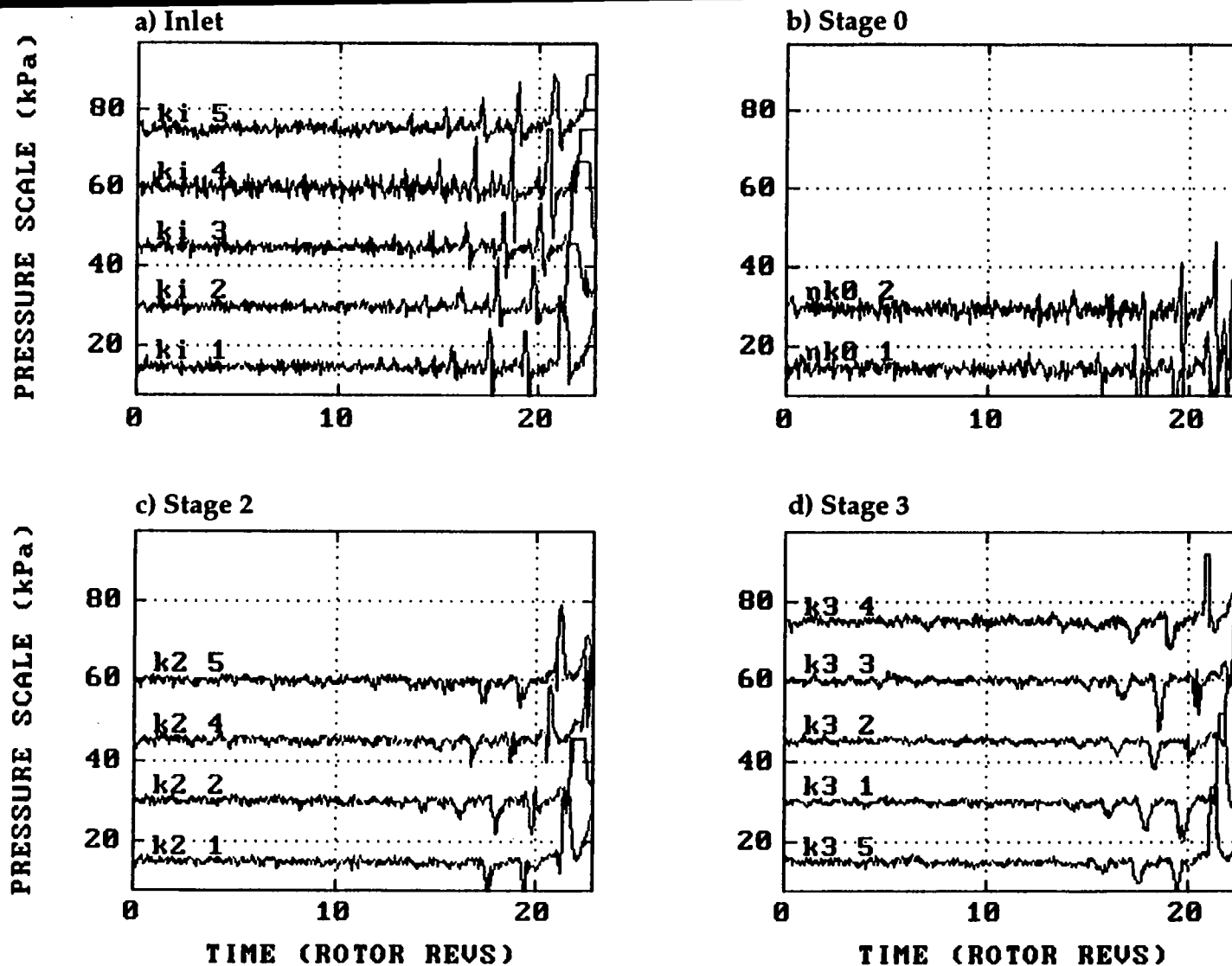


Fig. 3-12 T180804: 79.4% Speed Stall, Front Stages.

Casing static pressure traces from the Kulites at inlet, stage 0, stage 2 and stage 3.

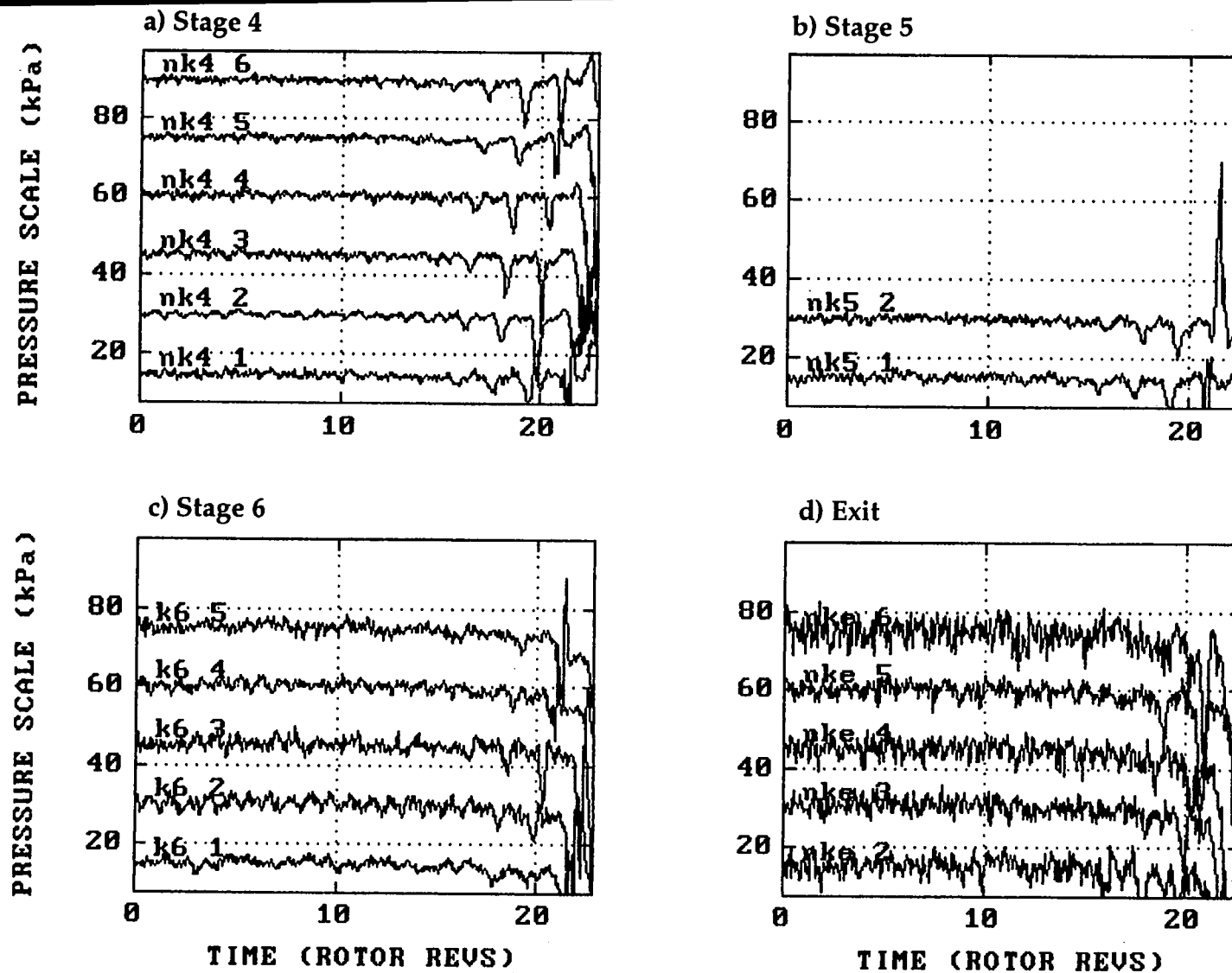


Fig. 3-13 T180804: 79.4% Speed Stall, Rear Stages.

Casing static pressure traces from the Kulites at stage 4, stage 5 and stage 6, and freestream total pressure traces from the Kulites at compressor exit.

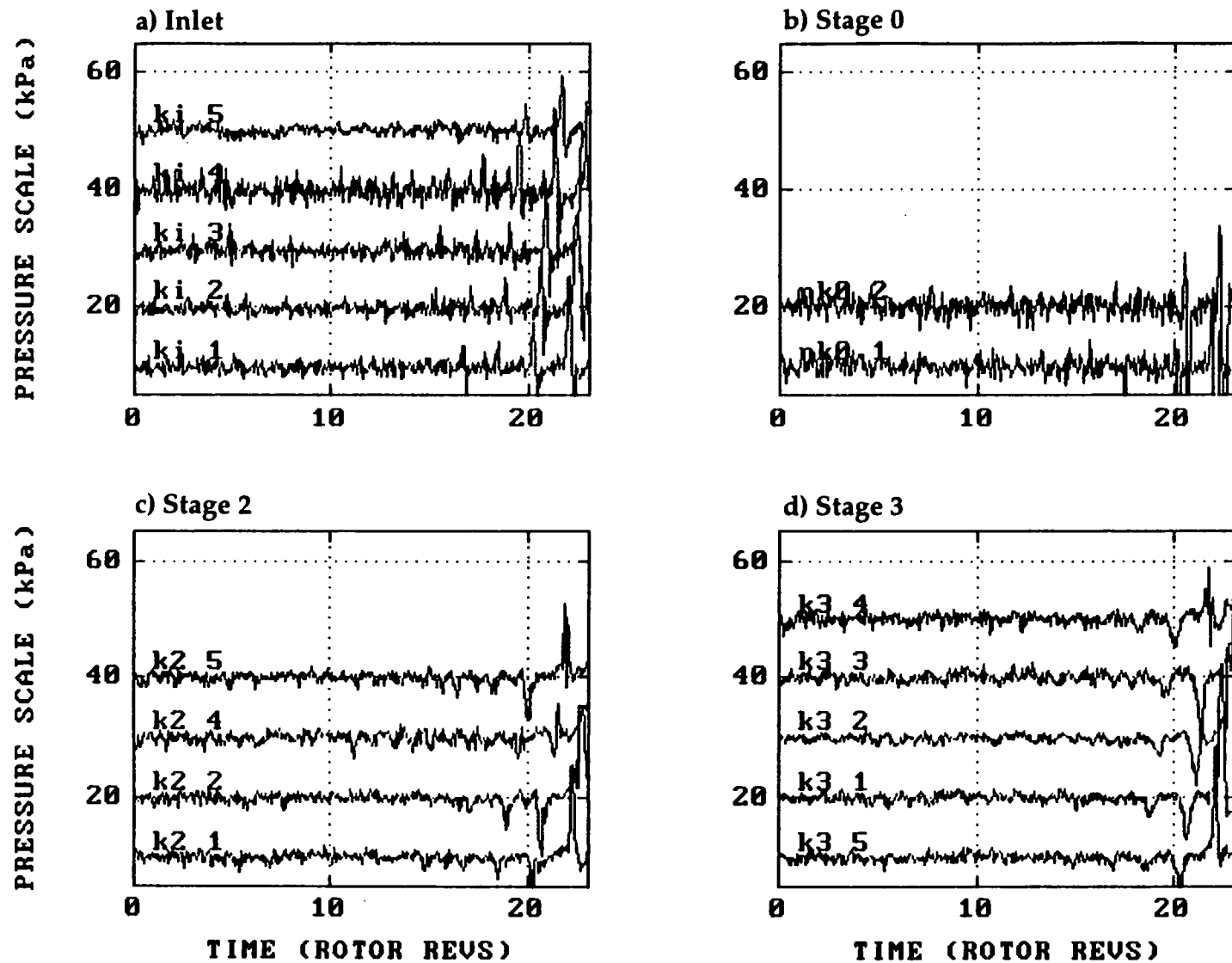


Fig. 3-14 T180802: 80.5% Speed Surge, Front Stages.

Casing static pressure traces from the Kulites at inlet, stage 0, stage 2 and stage 3.

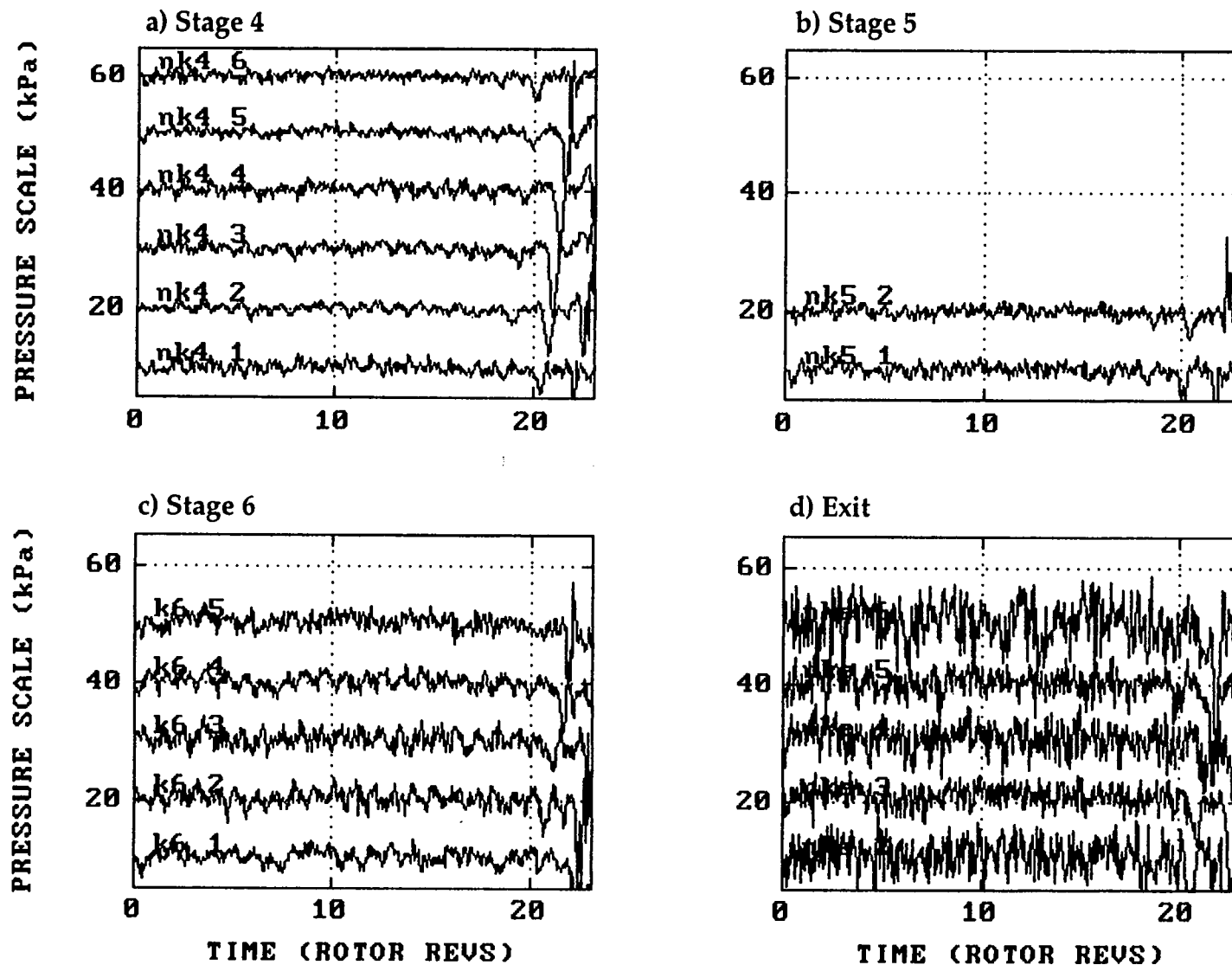


Fig. 3-15 T180802: 80.5% Speed Surge, Rear Stages.

Casing static pressure traces from the Kulites at stage 4, stage 5 and stage 6, and freestream total pressure traces from the Kulites at compressor exit.

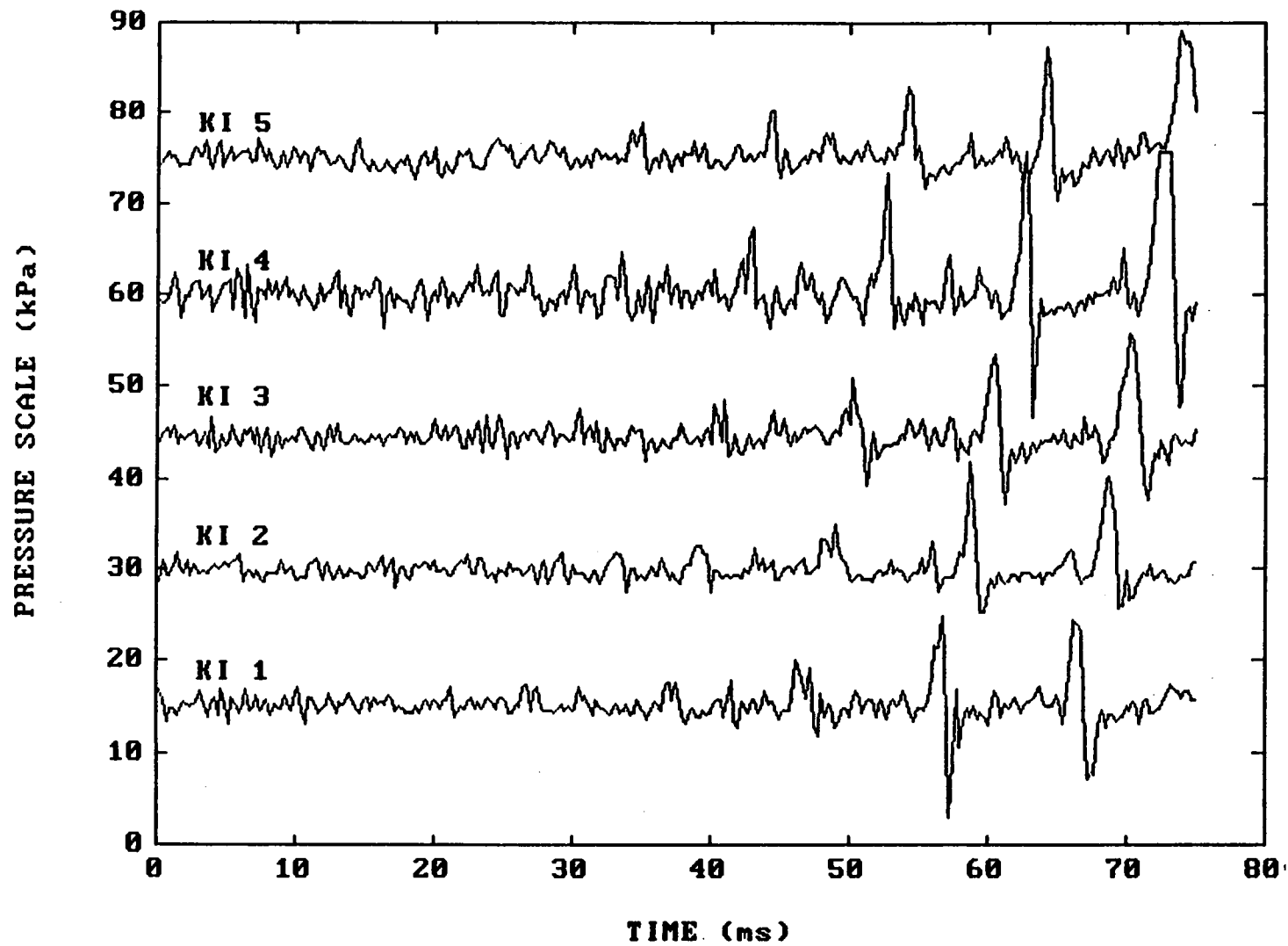


Fig. 3-16 T180804: 79.4% Speed Stall, Inlet.

Casing Static pressure traces.

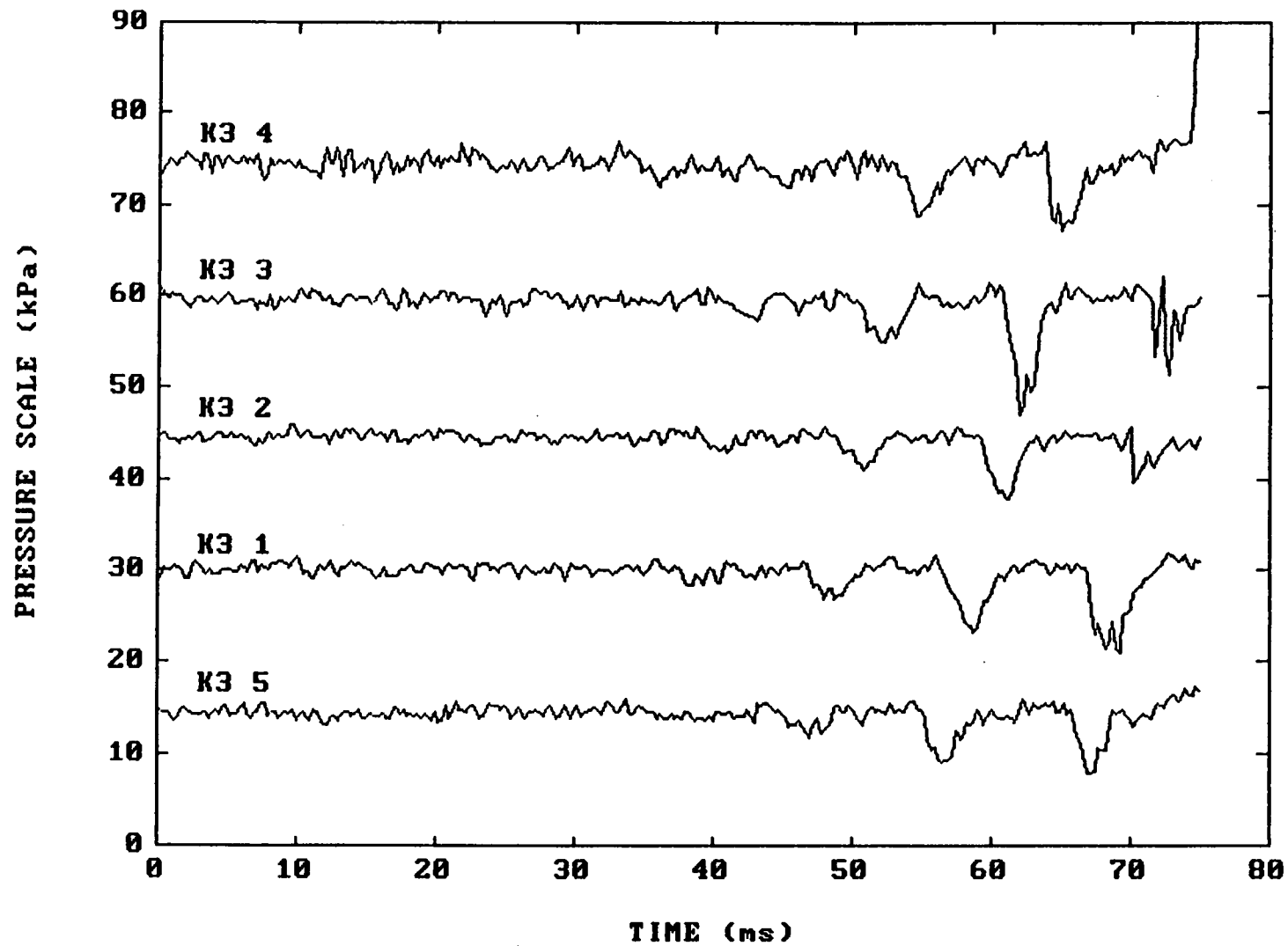


Fig. 3-17 T180804: 79.4% Speed Stall, Stage 3.

Casing static pressure traces.

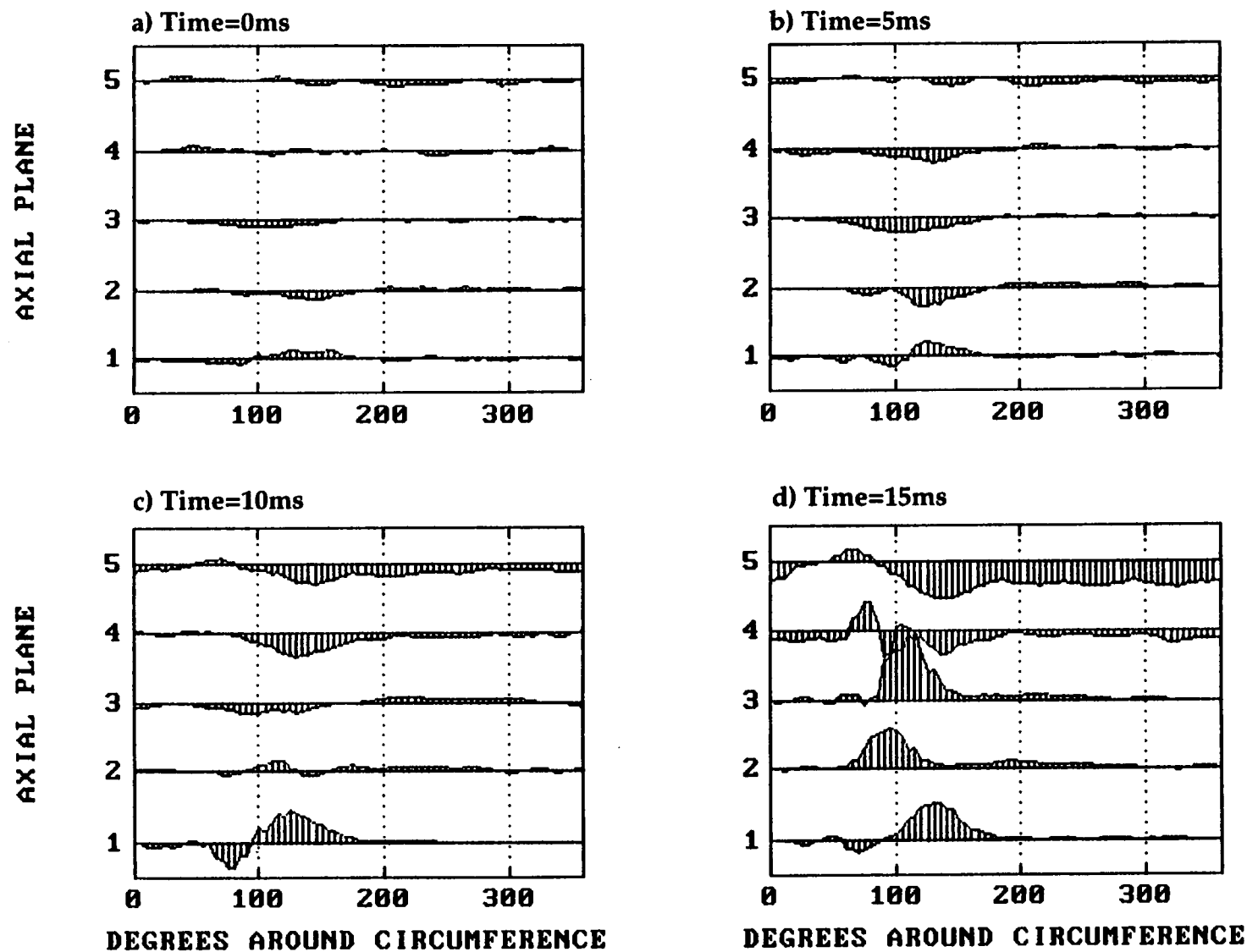


Fig. 3-18 Static Pressure Maps At 78% Speed: Development of Stall Cell.

One unit on the y axis represents 28kPa

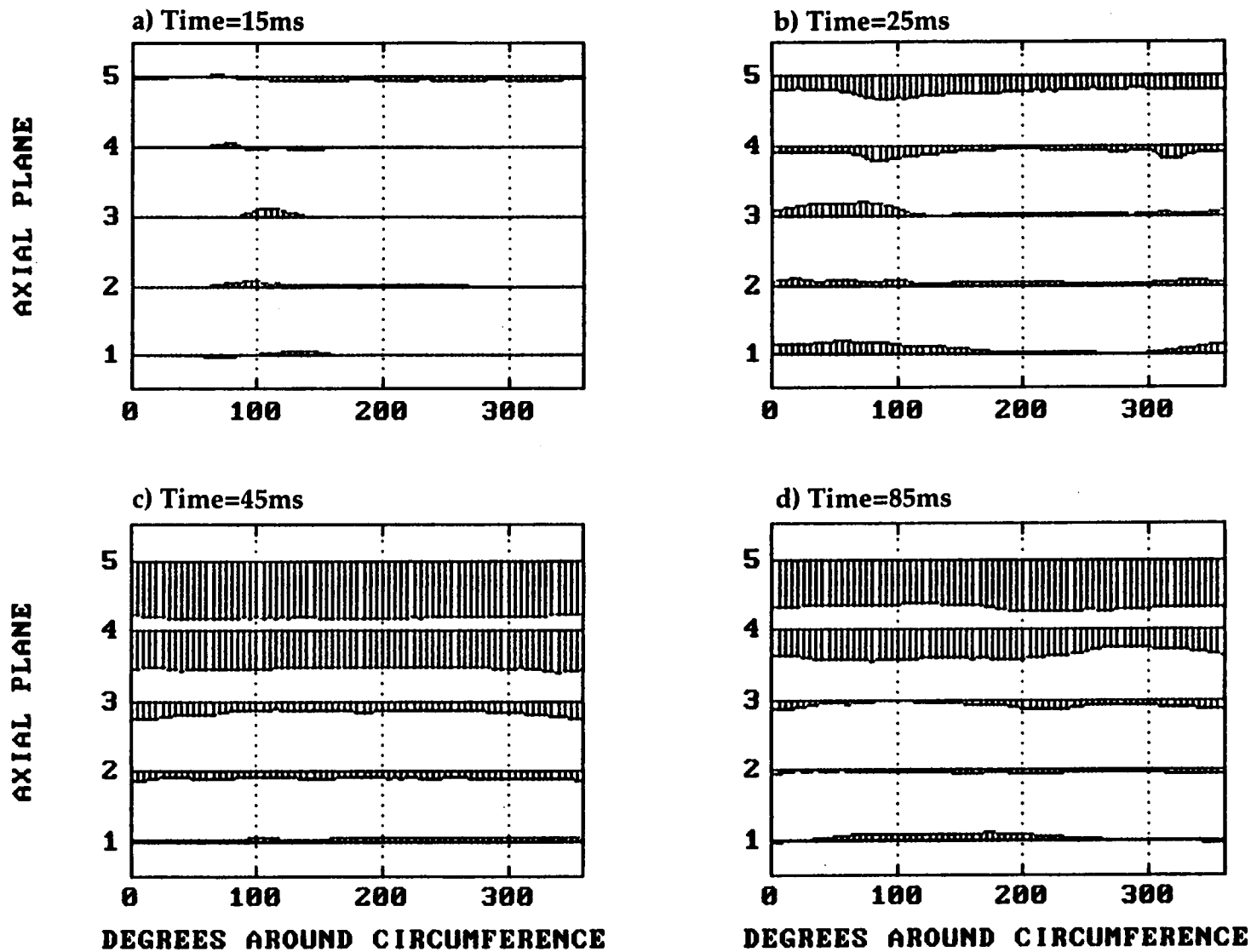


Fig. 3-19 Static Pressure Maps At 78% Speed (Ctd):
Growth of Stall Cell into Stable Rotating Stall.
 One unit on the y axis represents 230kPa

AXIAL PLANE

AXIAL PLANE

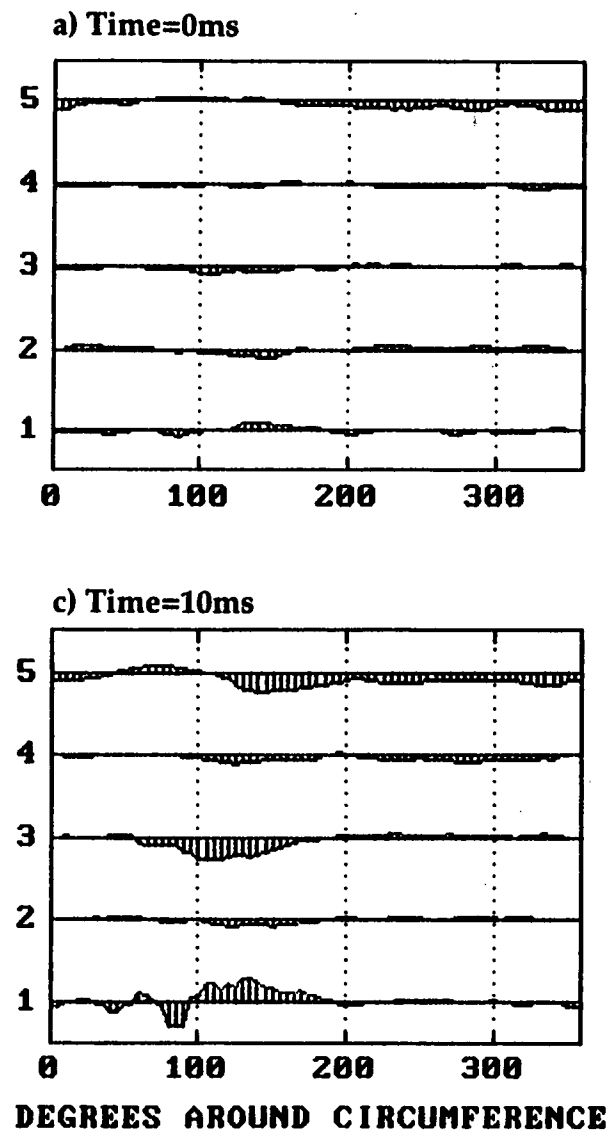
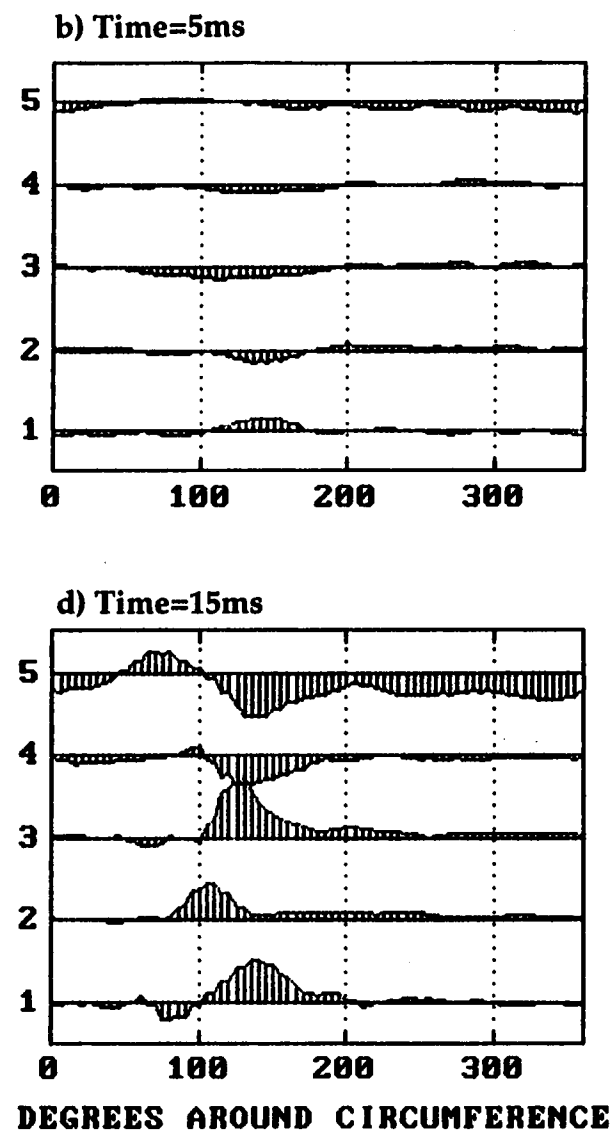
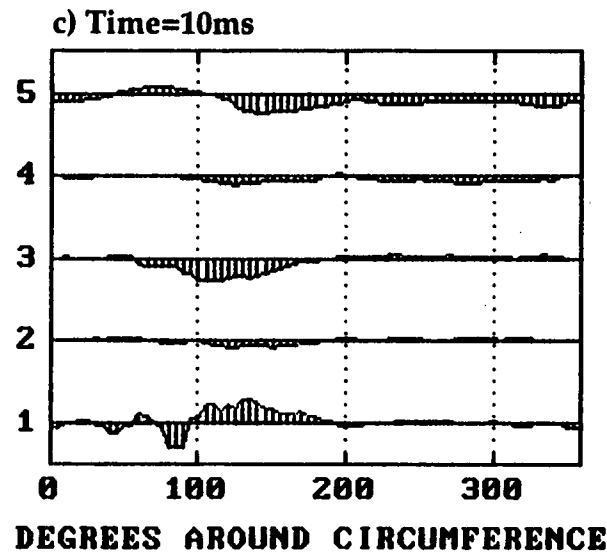
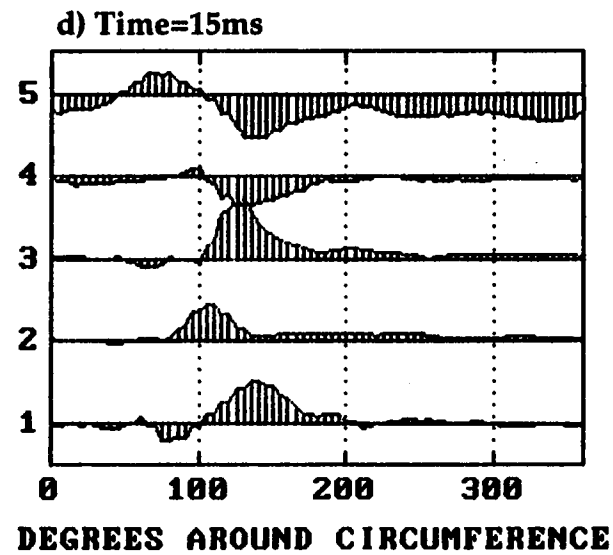


Fig. 3-20 Static Pressure Maps At 82% Speed: Development of Stall Cell.

One unit on the y axis represents 28kPa

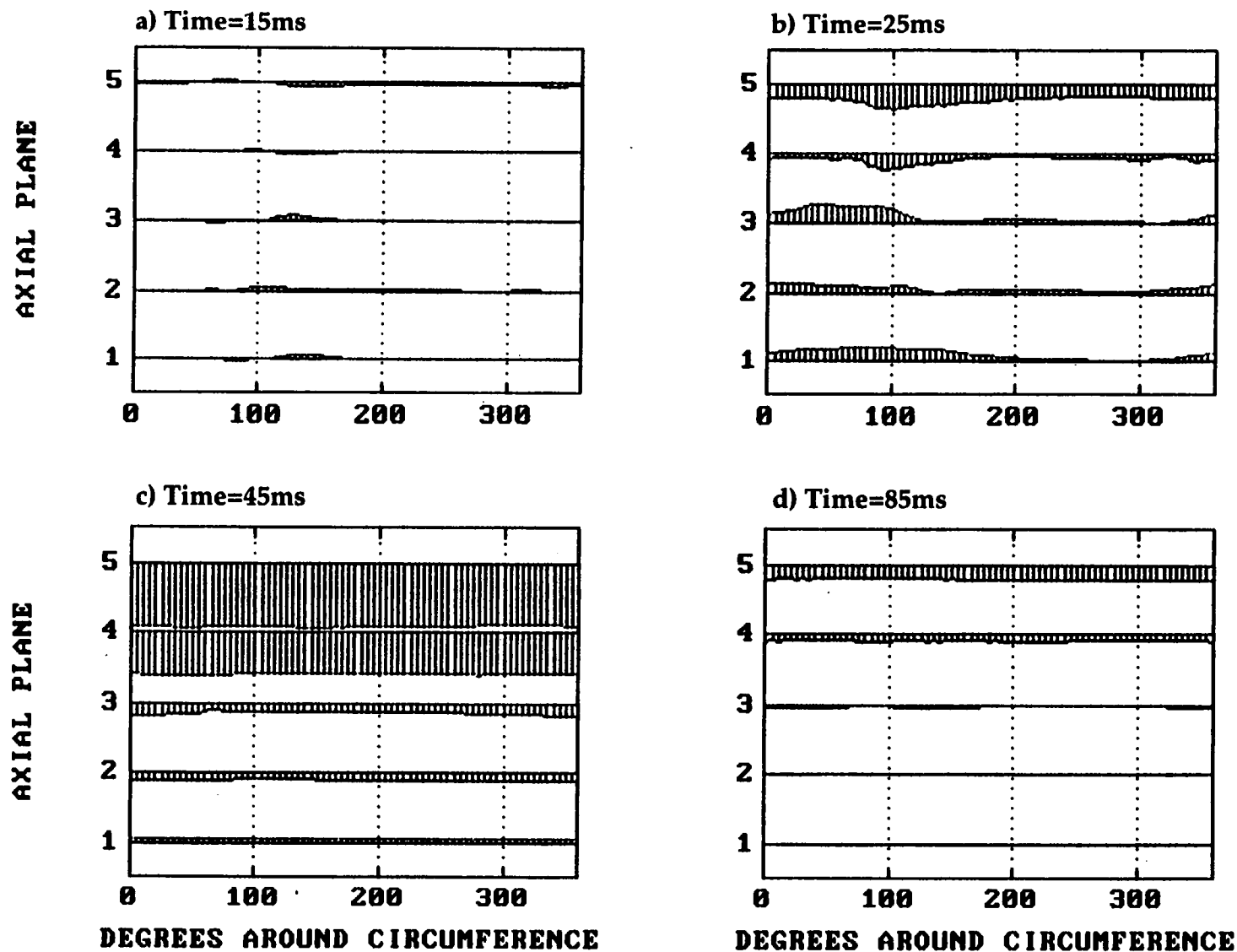


Fig. 3-21 Static Pressure Maps At 82% Speed (Ctd):
Growth of Stall Cell, Then Axisymmetric Flow.
 One unit on the y axis represents 230kPa

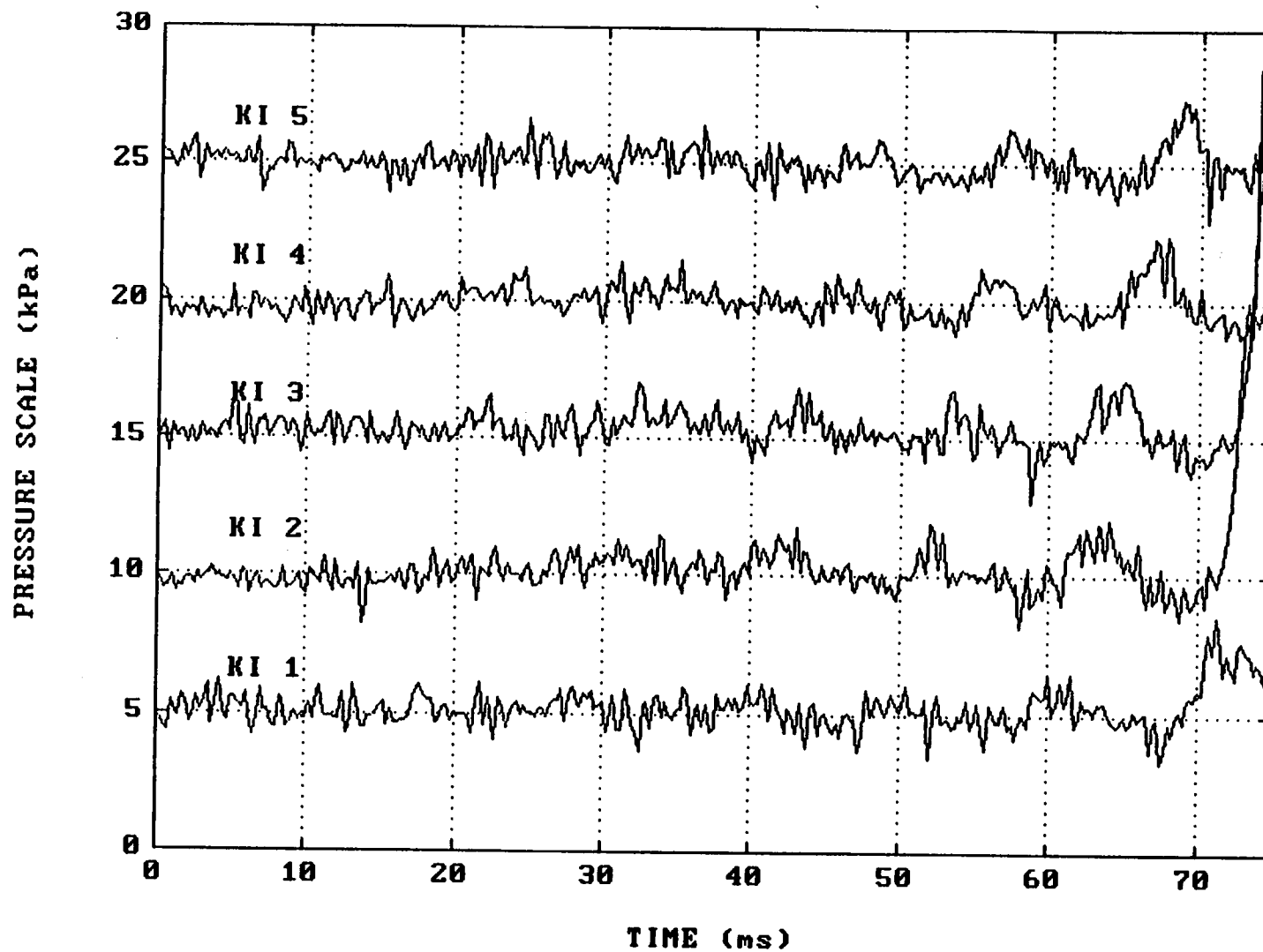


Fig. 3-22 T290705: 88.5% Speed Stall, Inlet.

Casing static pressure traces.

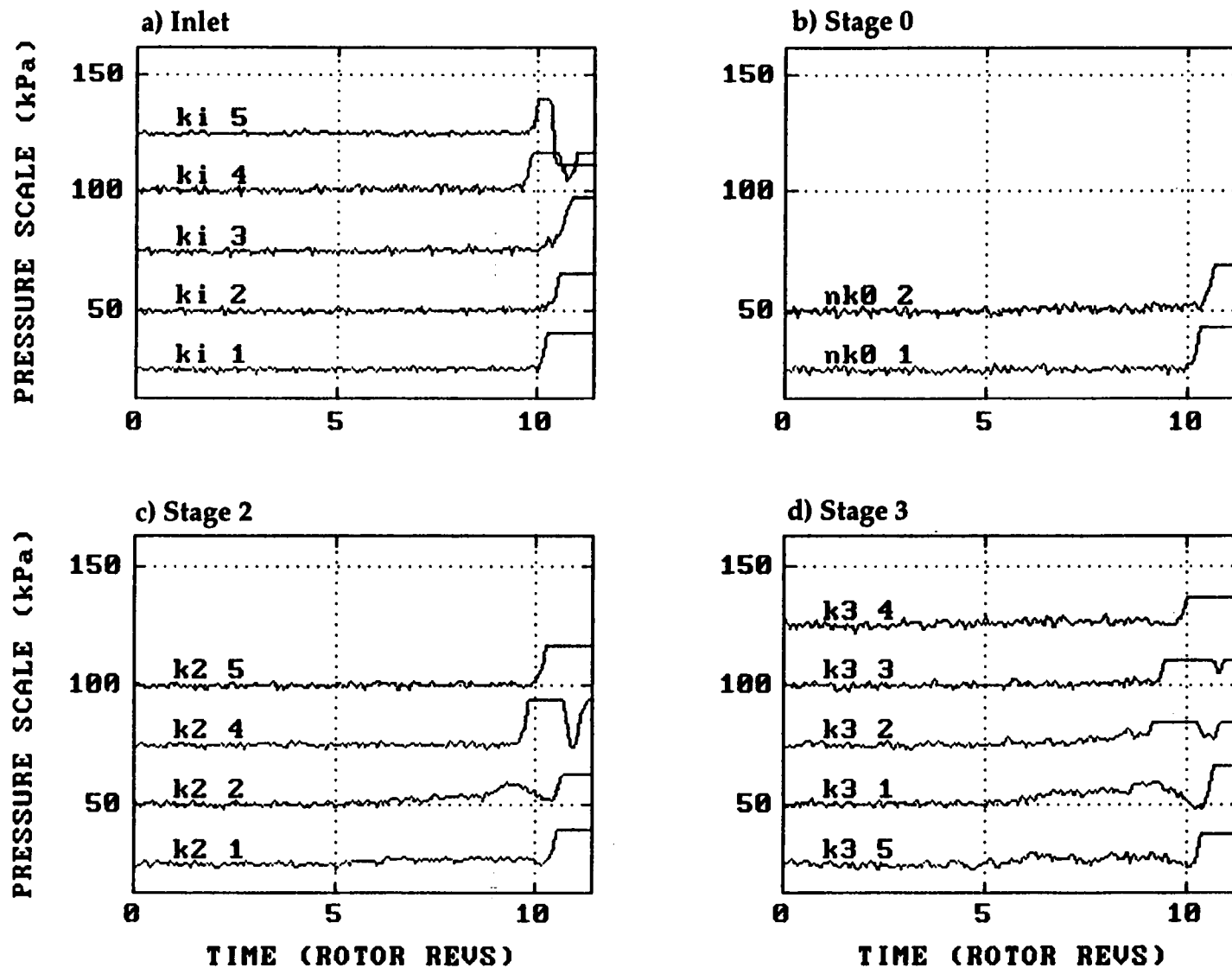


Fig. 3-23 T180803: 99.7% Speed Surge, Front Stages.

Casing static pressure traces from the Kulites at inlet, stage 0, stage 2 and stage 3.

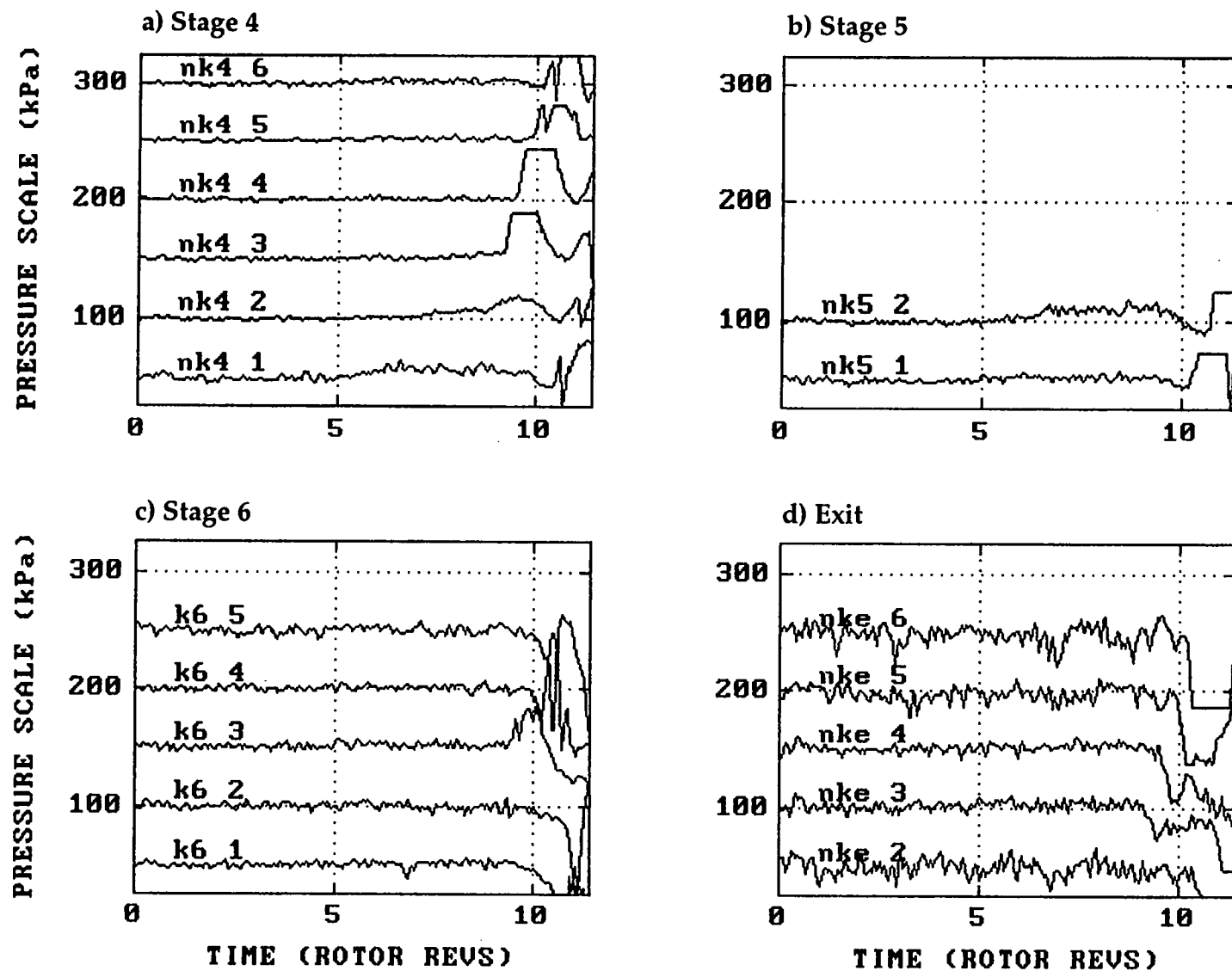


Fig. 3-24 T180803: 99.7% Speed Surge, Rear Stages.

Casing static pressure traces from the Kulites at stage 4, stage 5 and stage 6, and freestream total pressure traces from the Kulites at compressor exit.

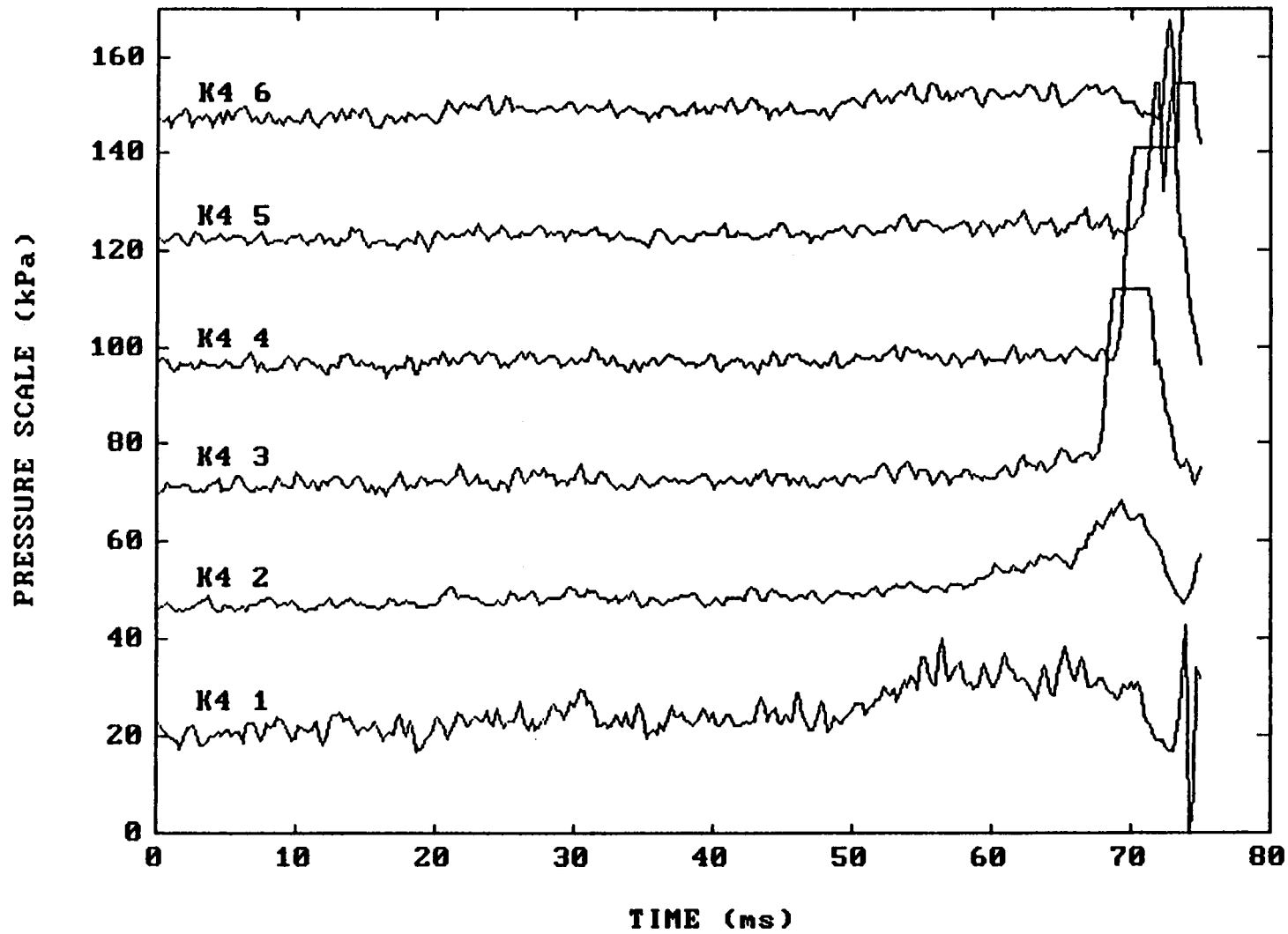


Fig. 3-25 T180803: 99.7% Speed Stall, Stage 4.

Casing static pressure traces.

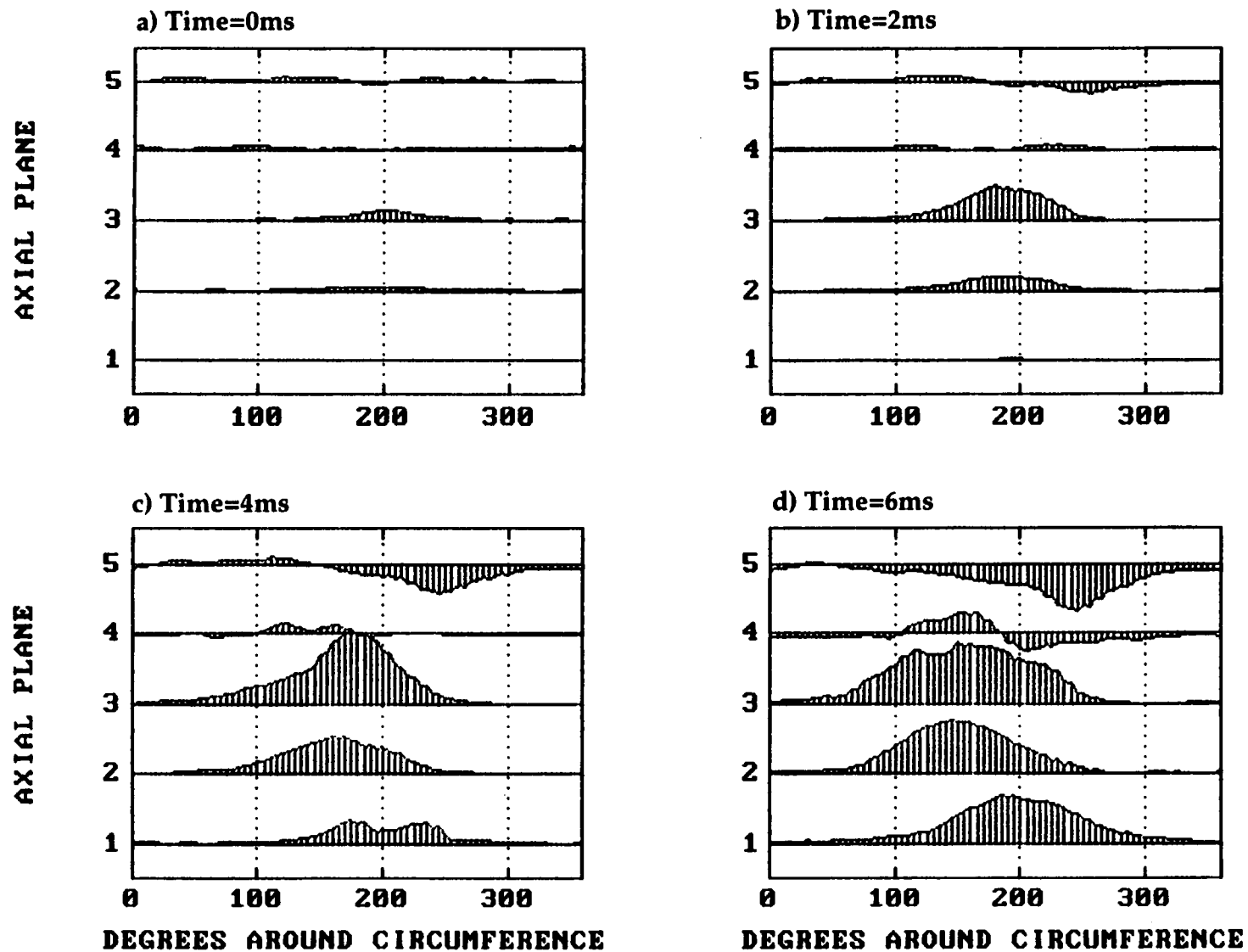


Fig. 3-26 Static Pressure Maps At 98% Speed: Development of Stall Cell.

One unit on the y axis represents 153kPa

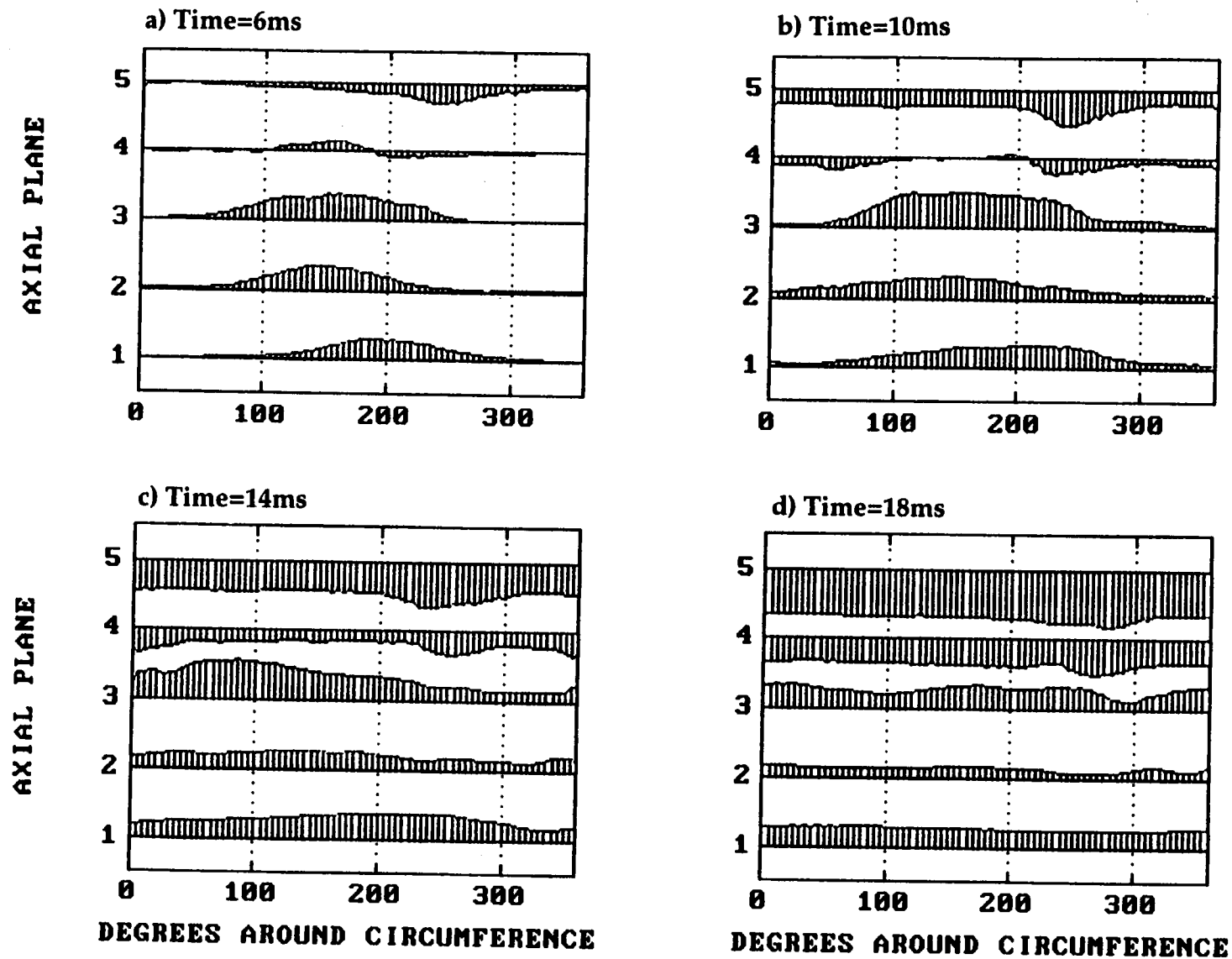


Fig. 3-27 Static Pressure Maps At 98% Speed (Ctd): Growth of Stall Cell.

One unit on the y axis represents 345kPa

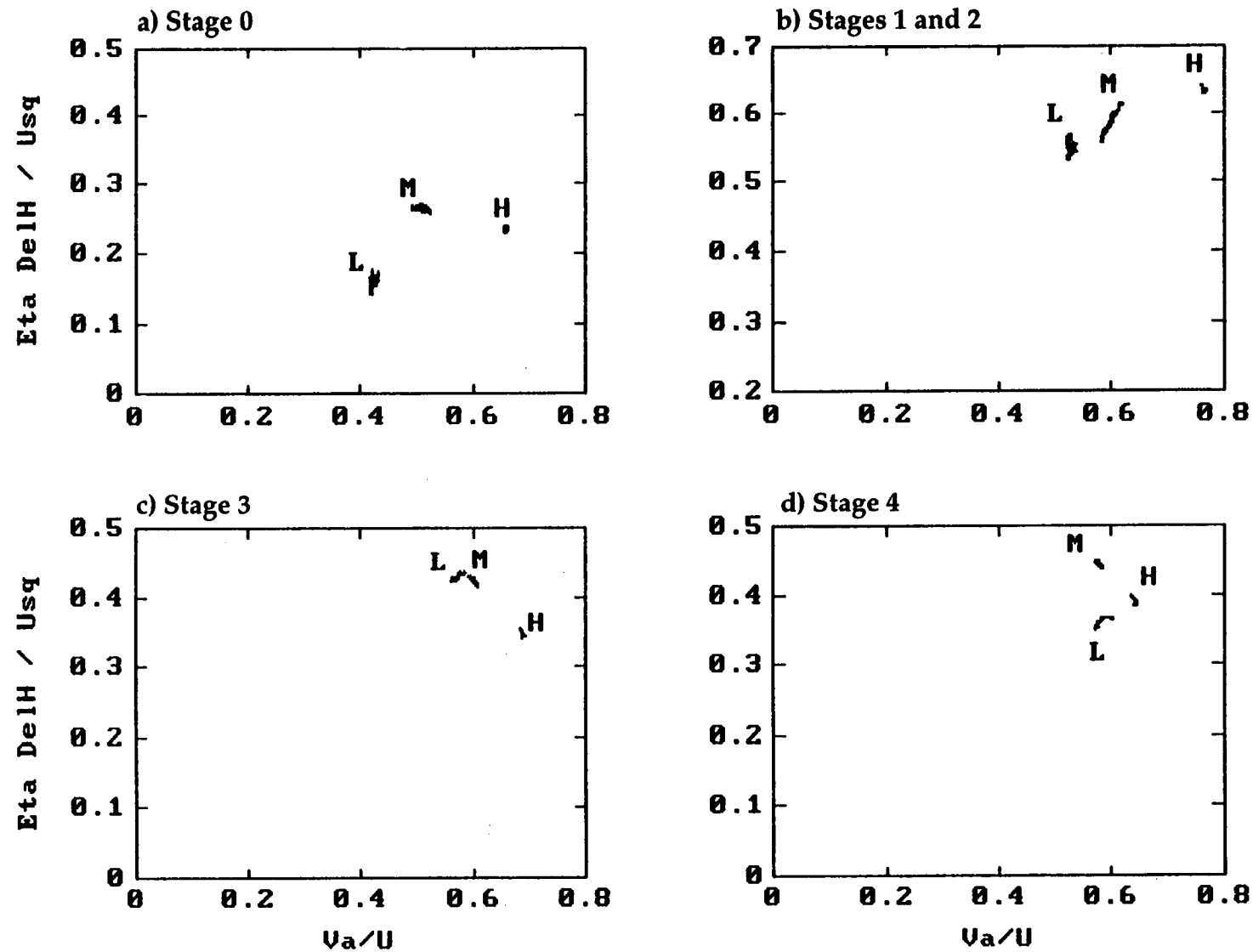


Fig. 3-28 Stage Characteristics: Front.

Approximate characteristics based on unsteady casing static pressure measurements. A combined characteristic is shown for stages 1 and 2.

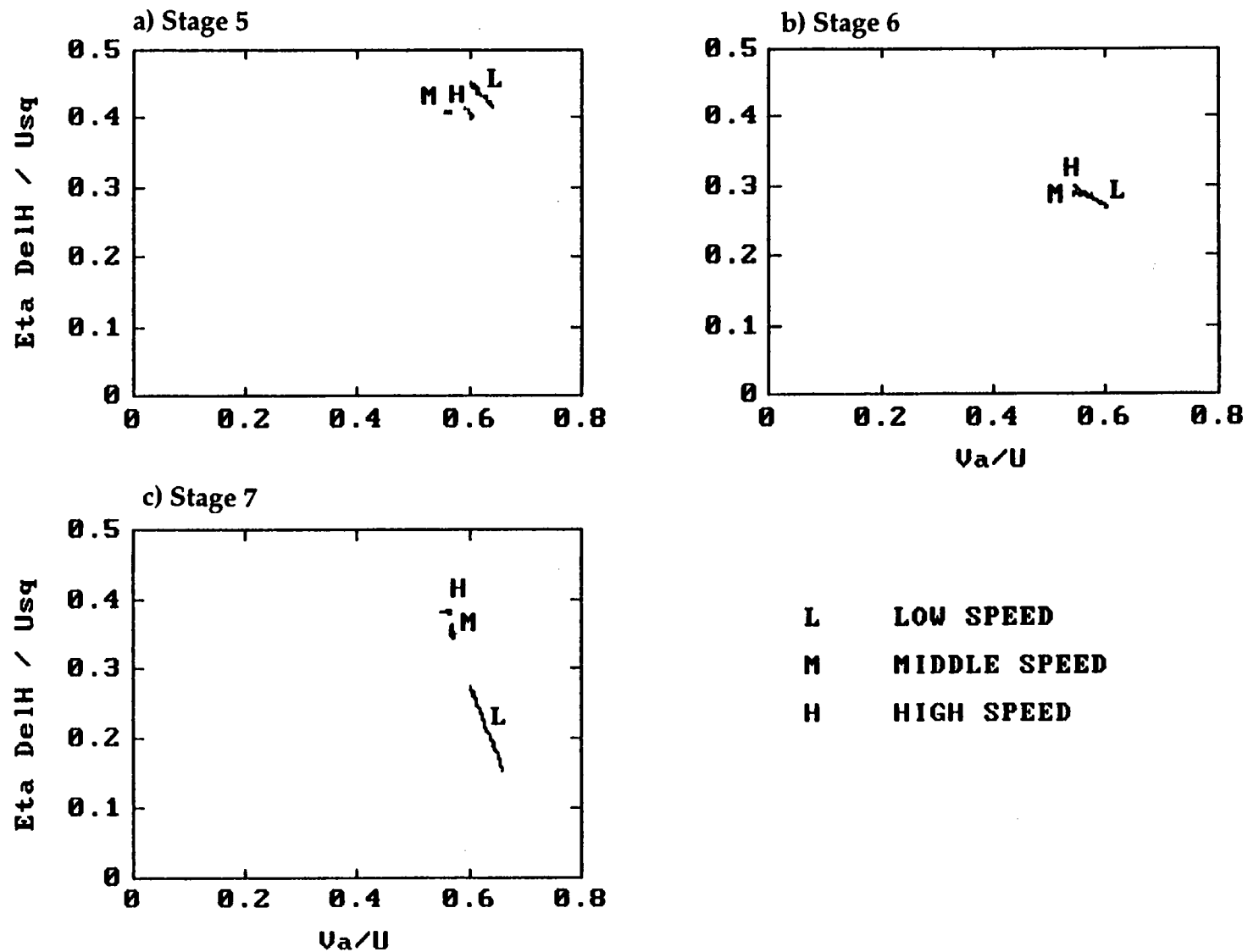


Fig. 3-29 Stage Characteristics: Rear.

Approximate characteristics based on unsteady casing static pressure measurements.

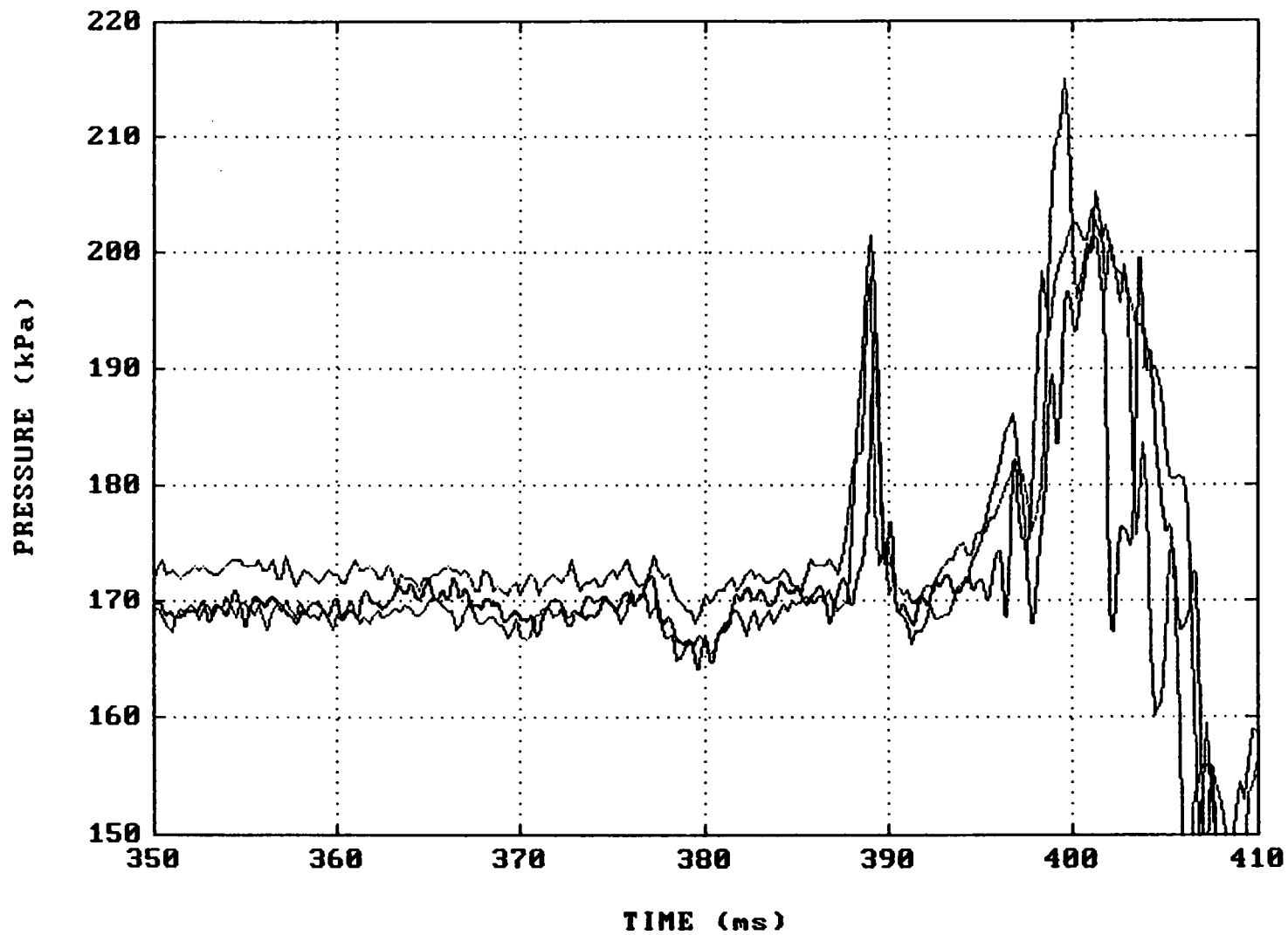
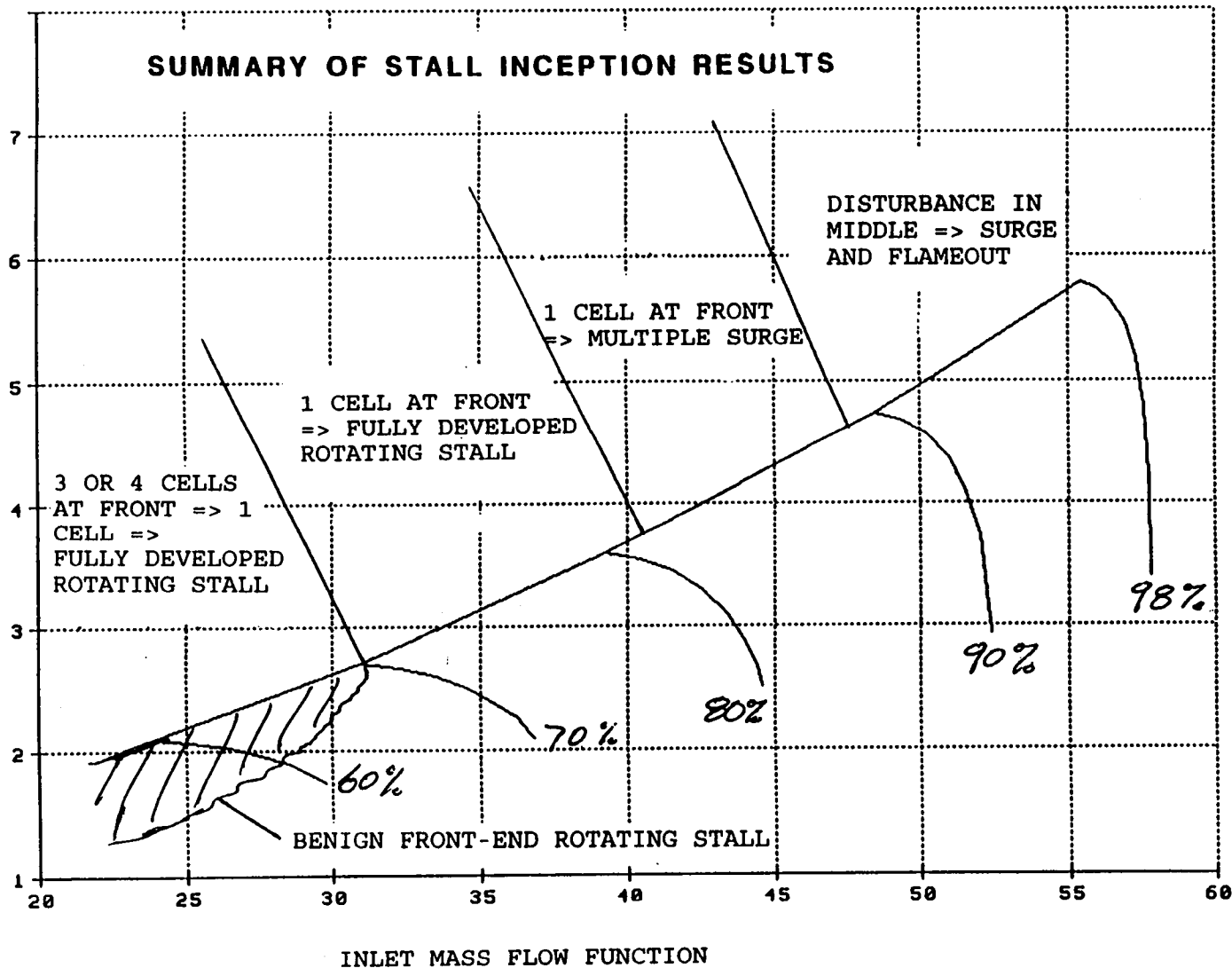


Fig. 3-30 Repeated Stall Events at Around 82% Speed.

Single trace of inlet casing static pressure shown for each event.

PRESSURE RATIO



(Imperial - Scale by $6.58 \cdot 10^{-5}$ for $\text{kg s}^{-1} \text{K}^{-1/2} \text{Pa}^{-1}$)

Fig. 3-31 Summary of Stall Inception Behaviour.

Background Map is Mean Line Performance Prediction

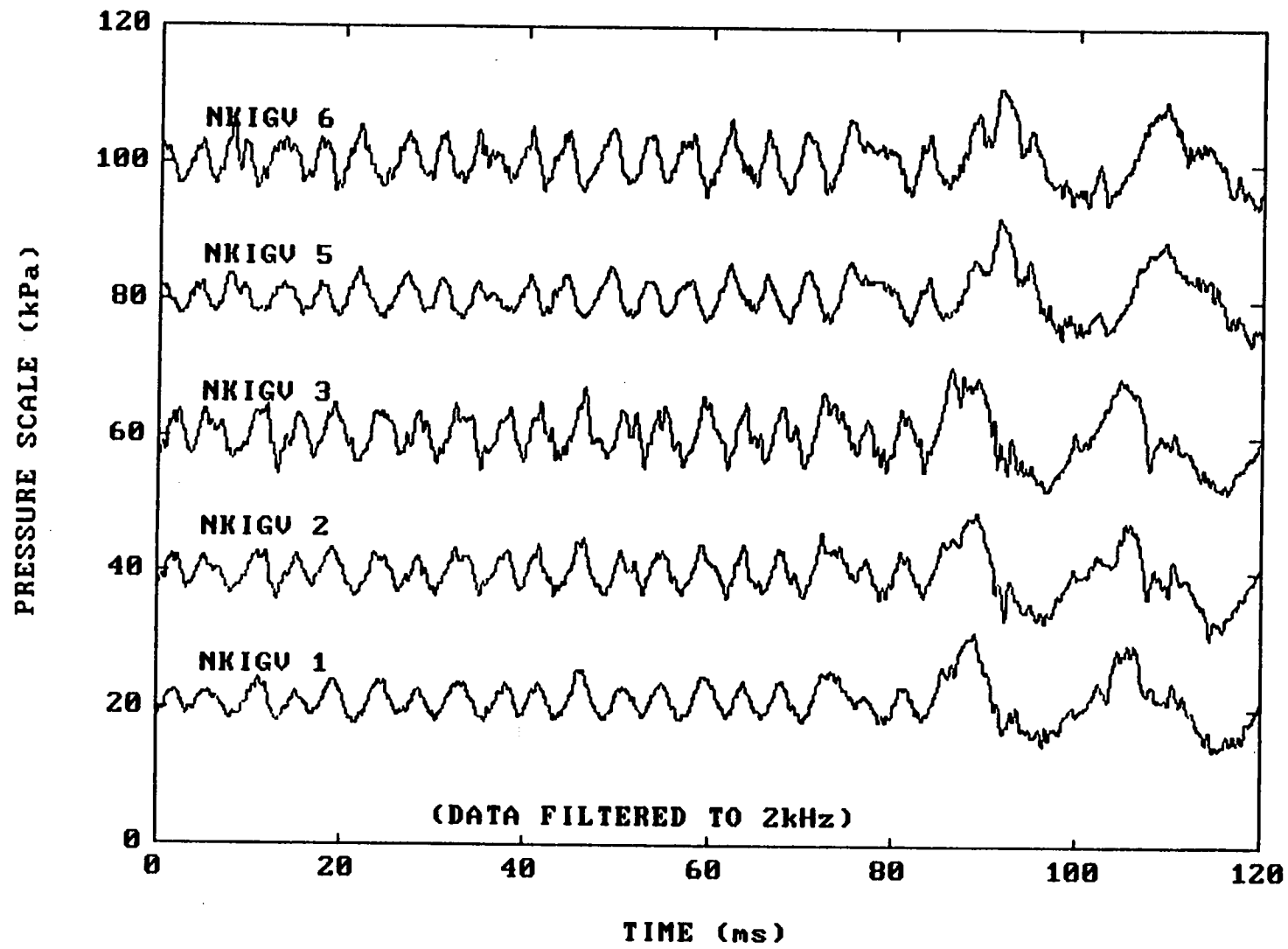


Fig. 3-32 T121204: 60% Speed Stall, IGV Adhesive Kulites.

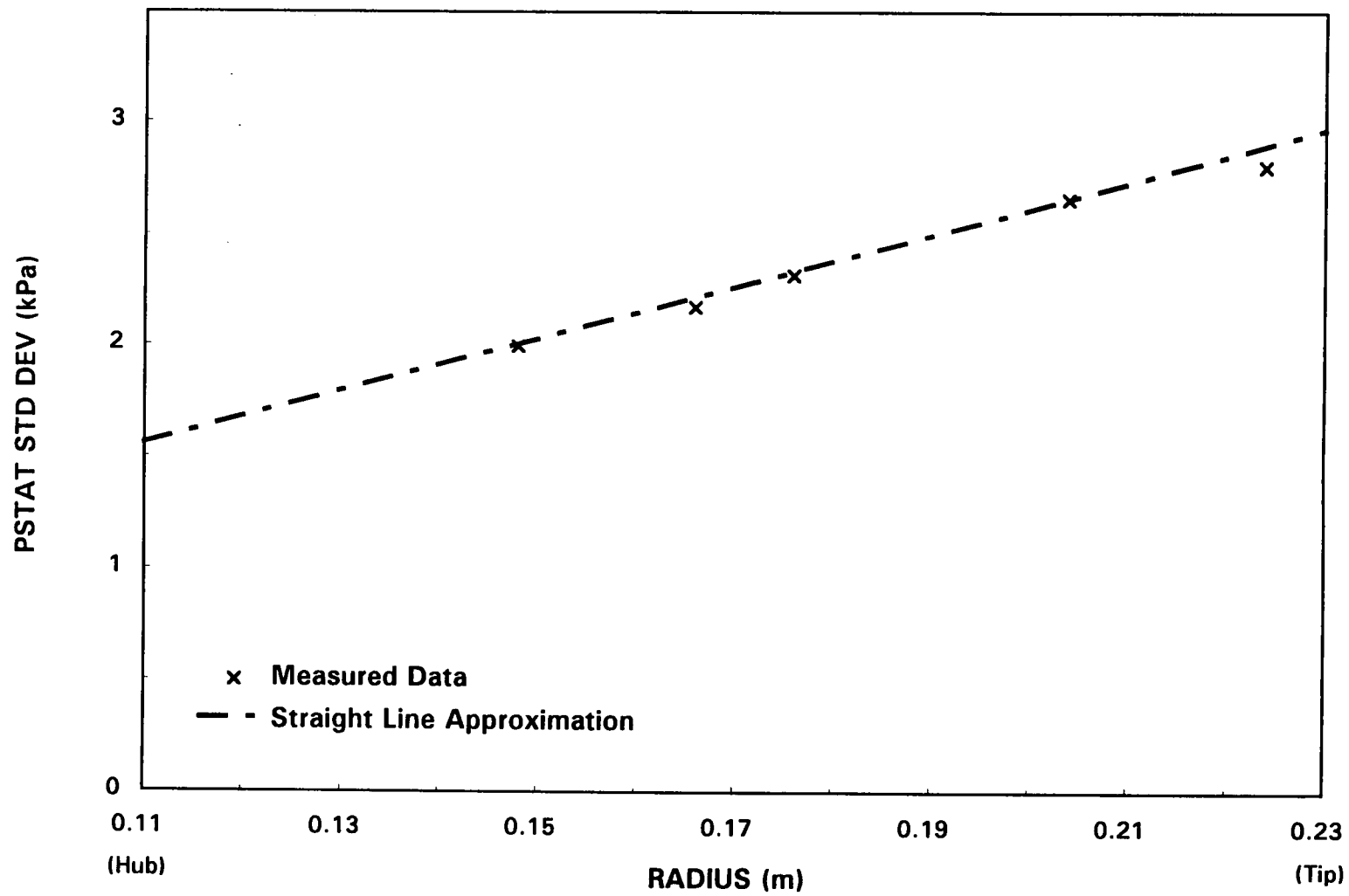


Fig. 3-33 T121204: 60% Speed, 3 Cell Disturbance Amplitude vs Radial Height.

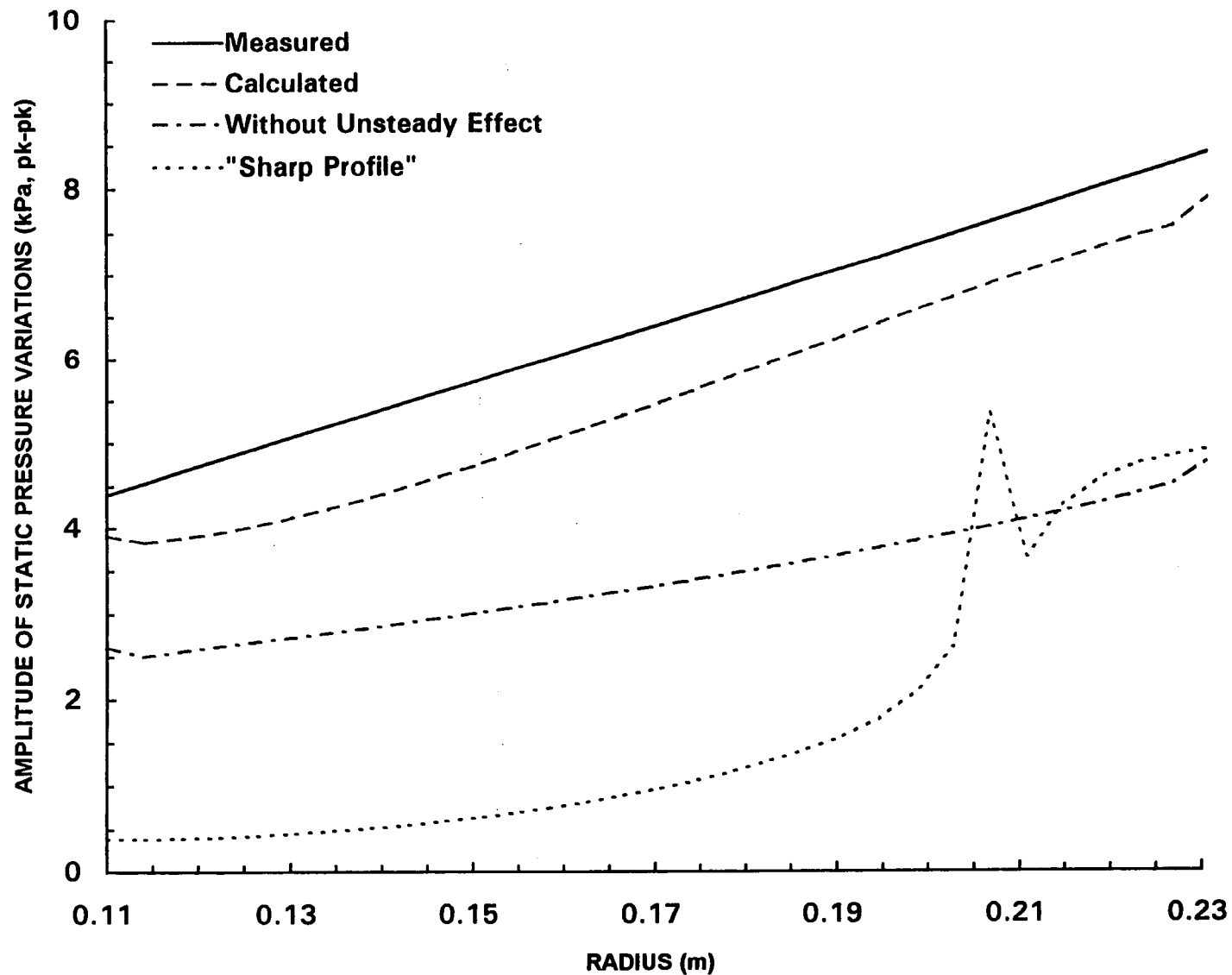


Fig. 3-34 Bessel-Fourier Fit to Rotating Static Pressure Disturbance at Inlet.

The "Sharp Profile", Plotted for Comparison, is Calculated from the Notional Part-Span Stall Velocity Profile Shown in Figure 35.

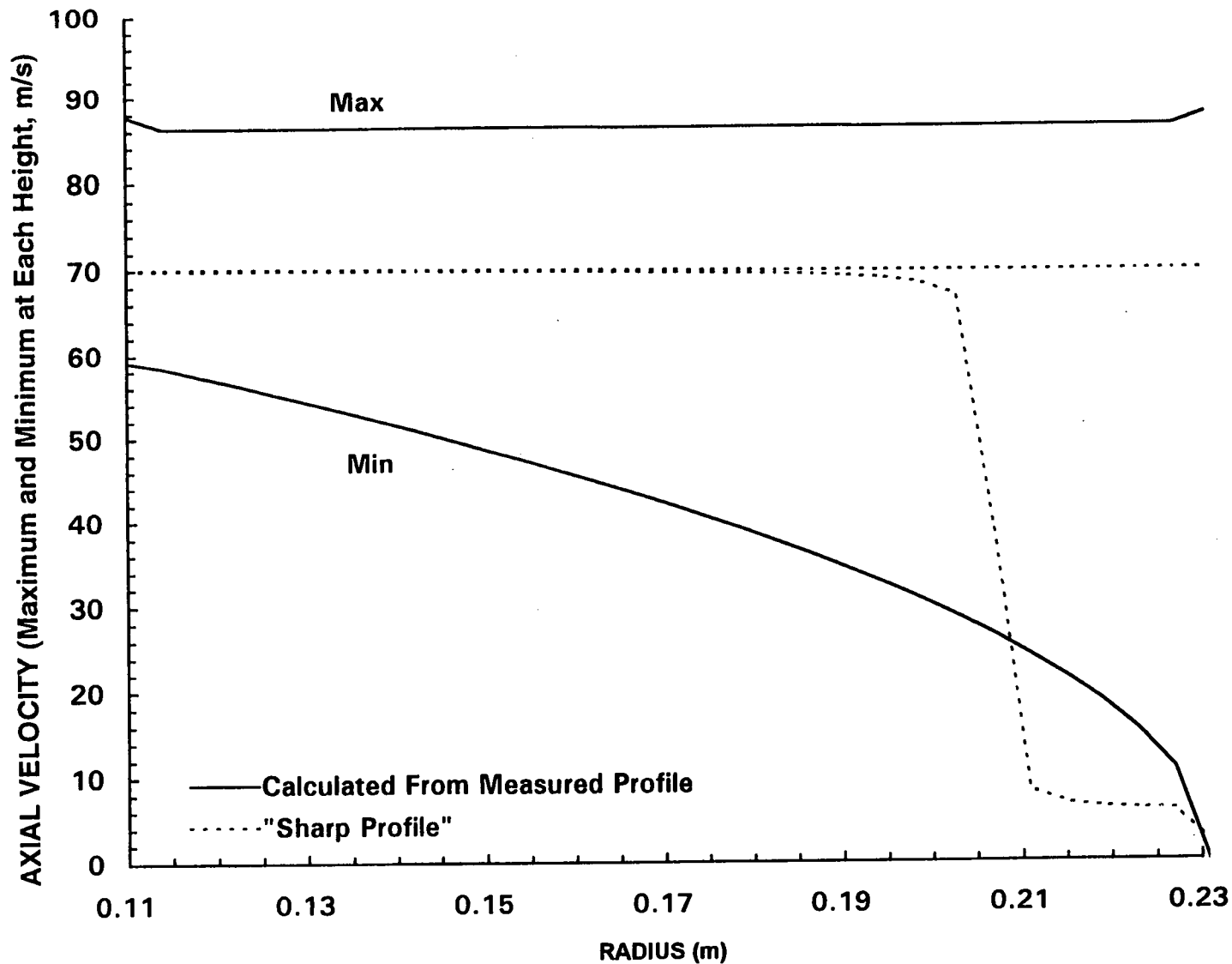


Fig. 3-35 Calculated Axial Velocity Profiles (Max and Min) at Inlet.

The "Sharp Profile", Plotted for Comparison, Represents a Notional Part-Span Stall Occupying Only the Outer 20% of Span.

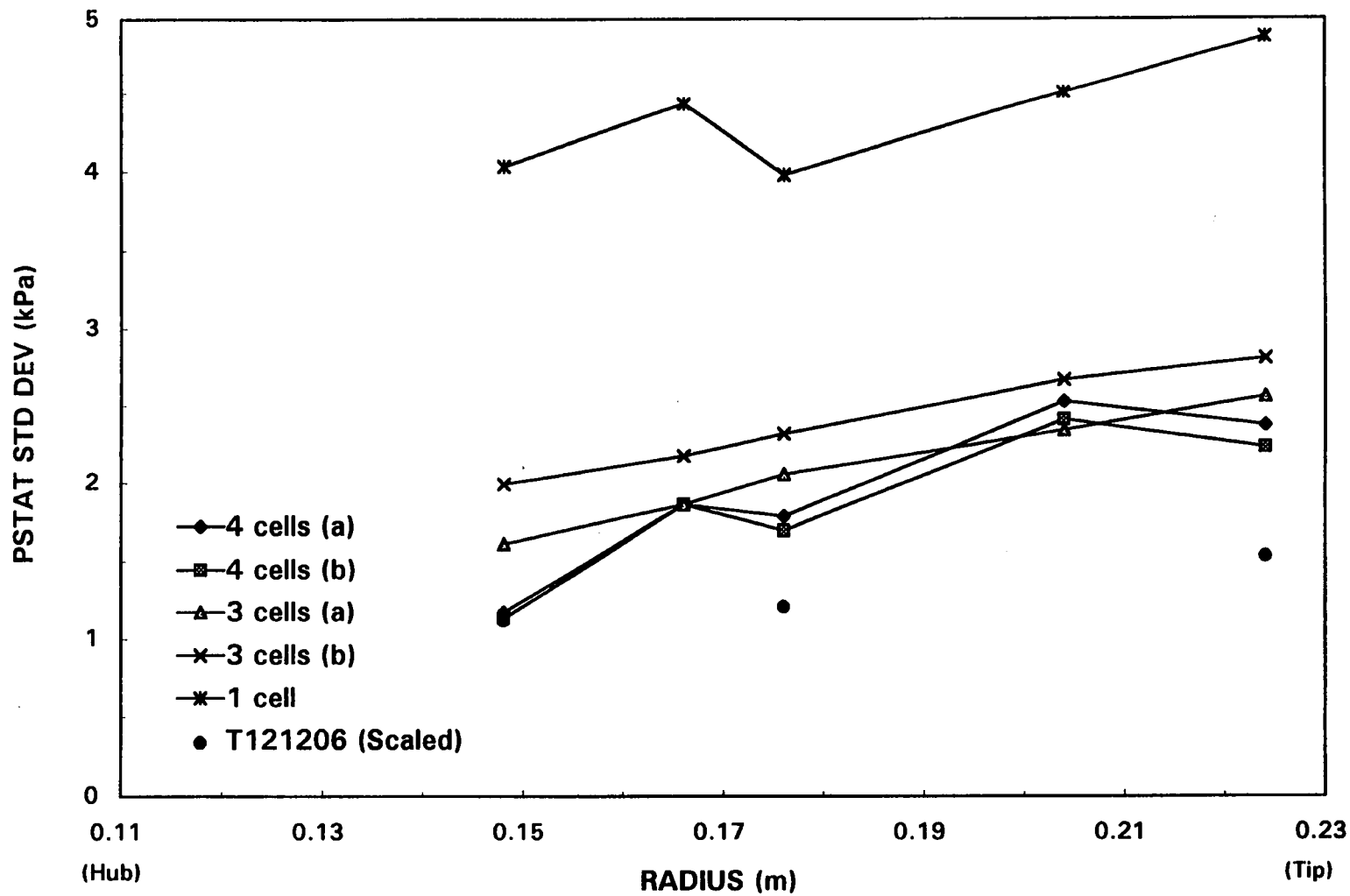


Fig. 3-36 Comparison of Radial Distribution of Various Stall Disturbances.

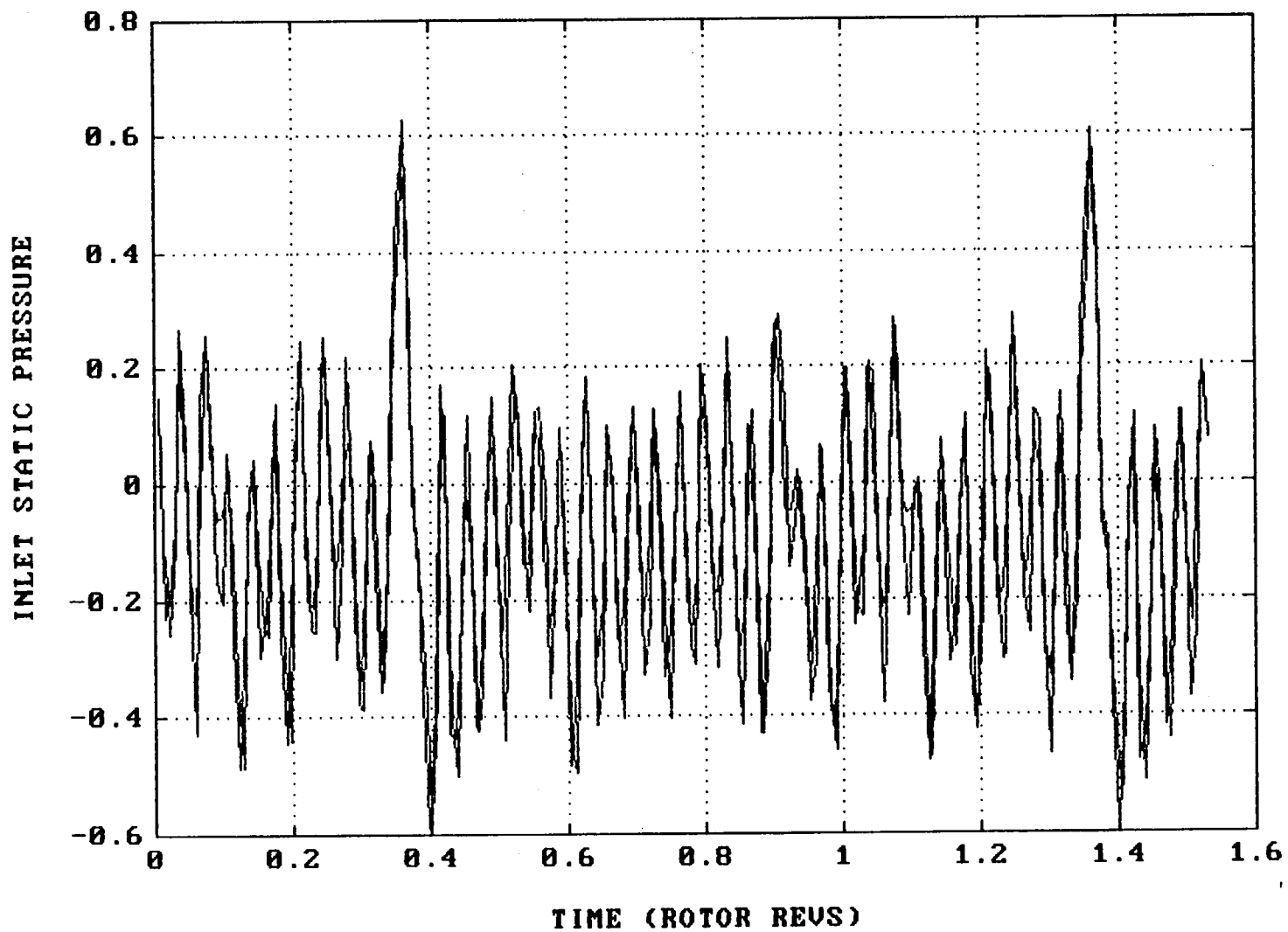


Fig. 3-37 Ensemble Averaged Once/Rev Signal.

Signals away from stall and near stall overlaid, showing almost identical behaviour.

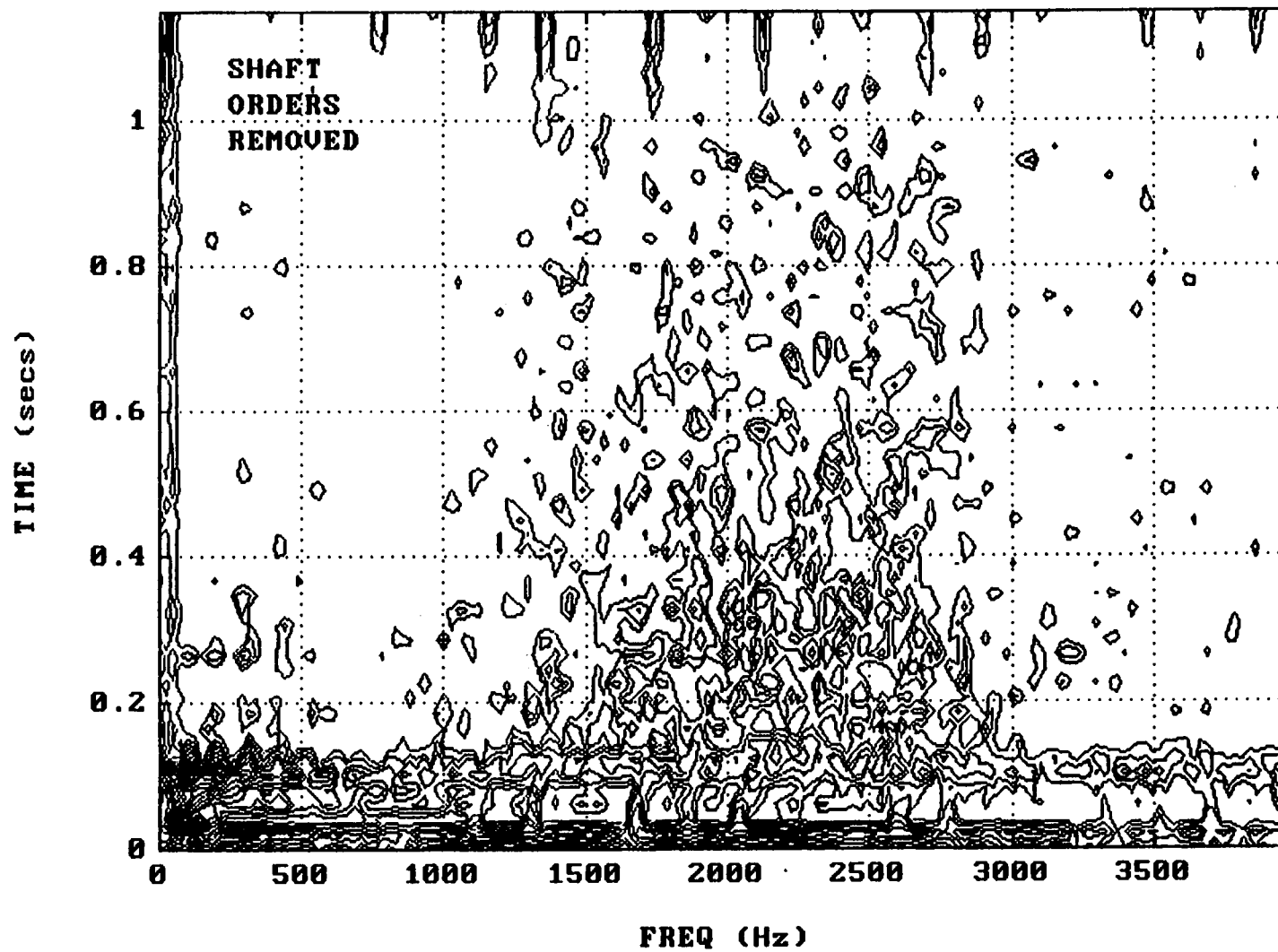


Fig. 3-38 T121210: Inlet Kulites High Frequency Signal Power.

Shows growing disturbances in 1.0-2.8kHz band prior to stall.

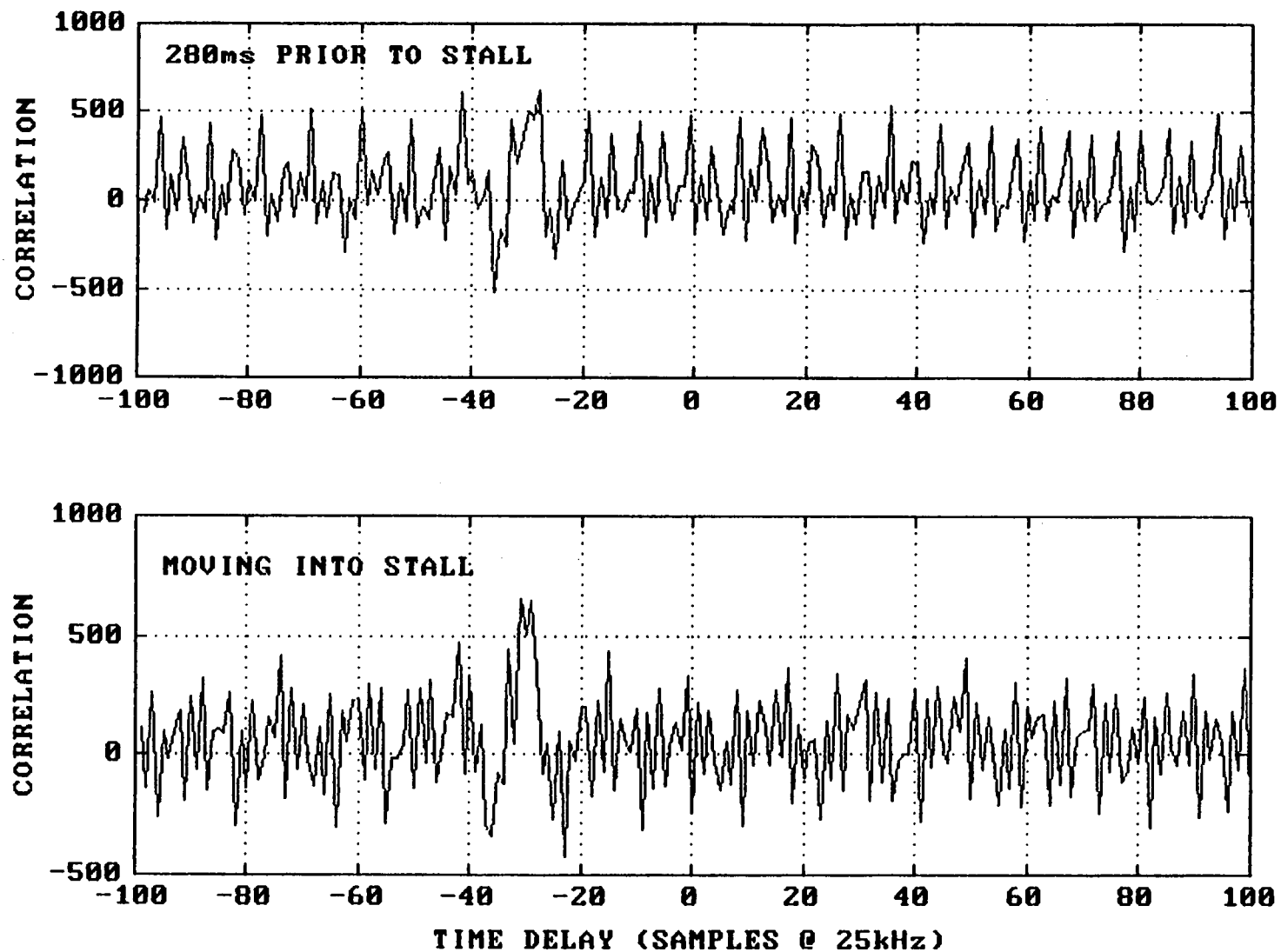


Fig. 3-39 T121210: Cross-Correlation of Measurements from Inlet Kulites 2 and 3.

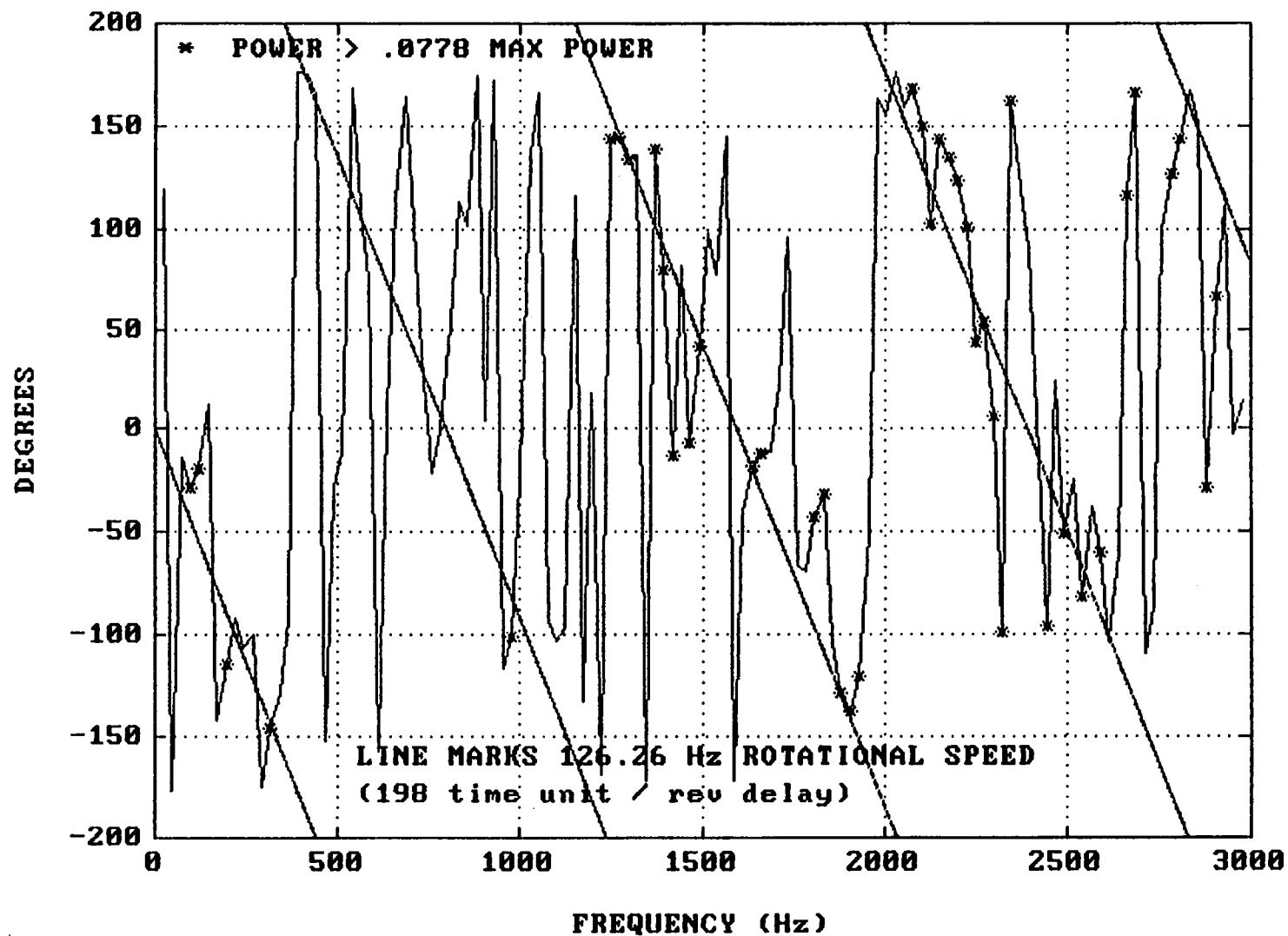


Fig. 3-40 T121210: Phase Change Between Measurements from Inlet Kulites 2 and 3.

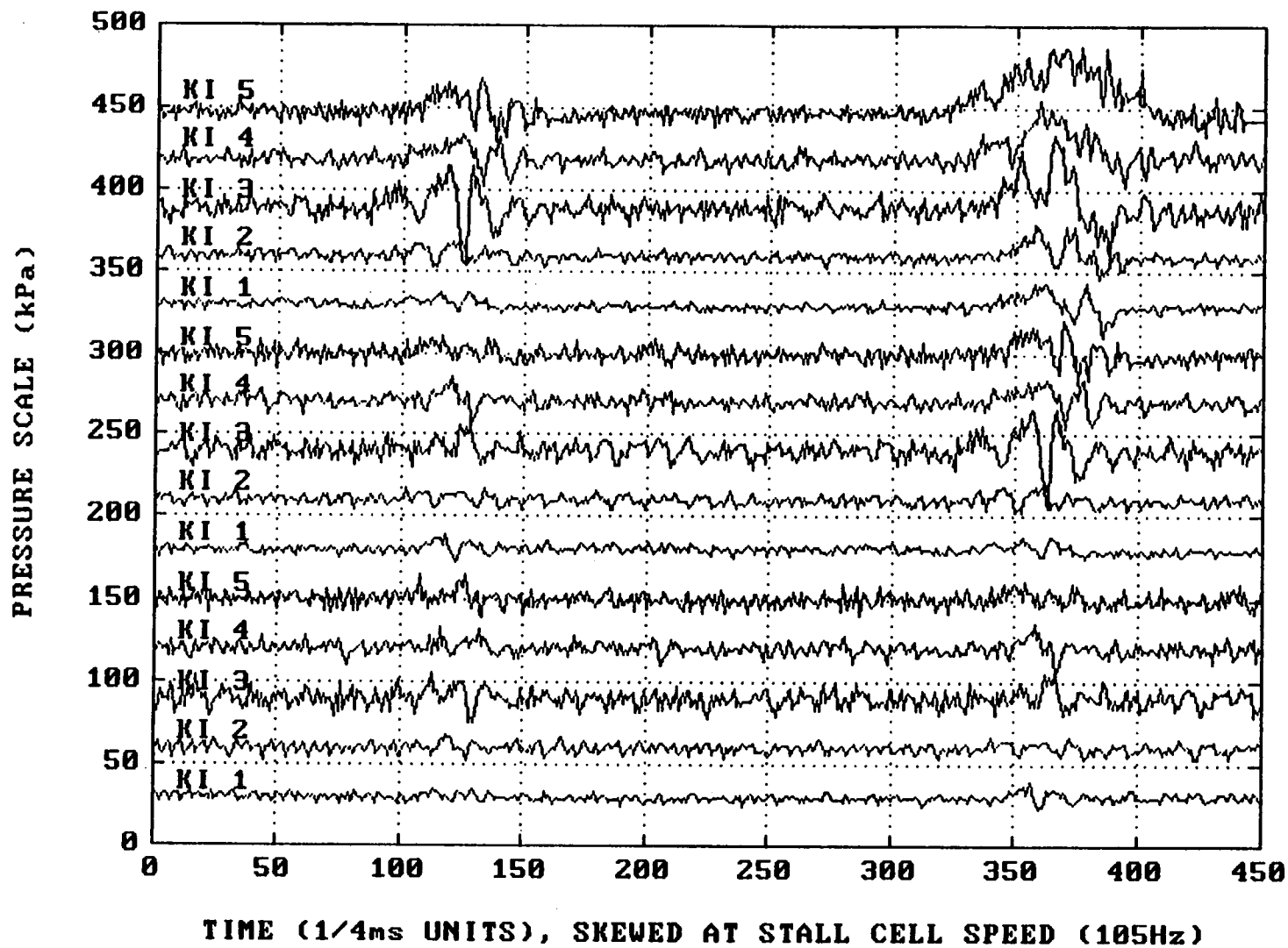


Fig. 3-41 T121210: Inlet Pressure Traces Skewed At Stall Cell Frequency.

PRESSURE SCALE (kPa)

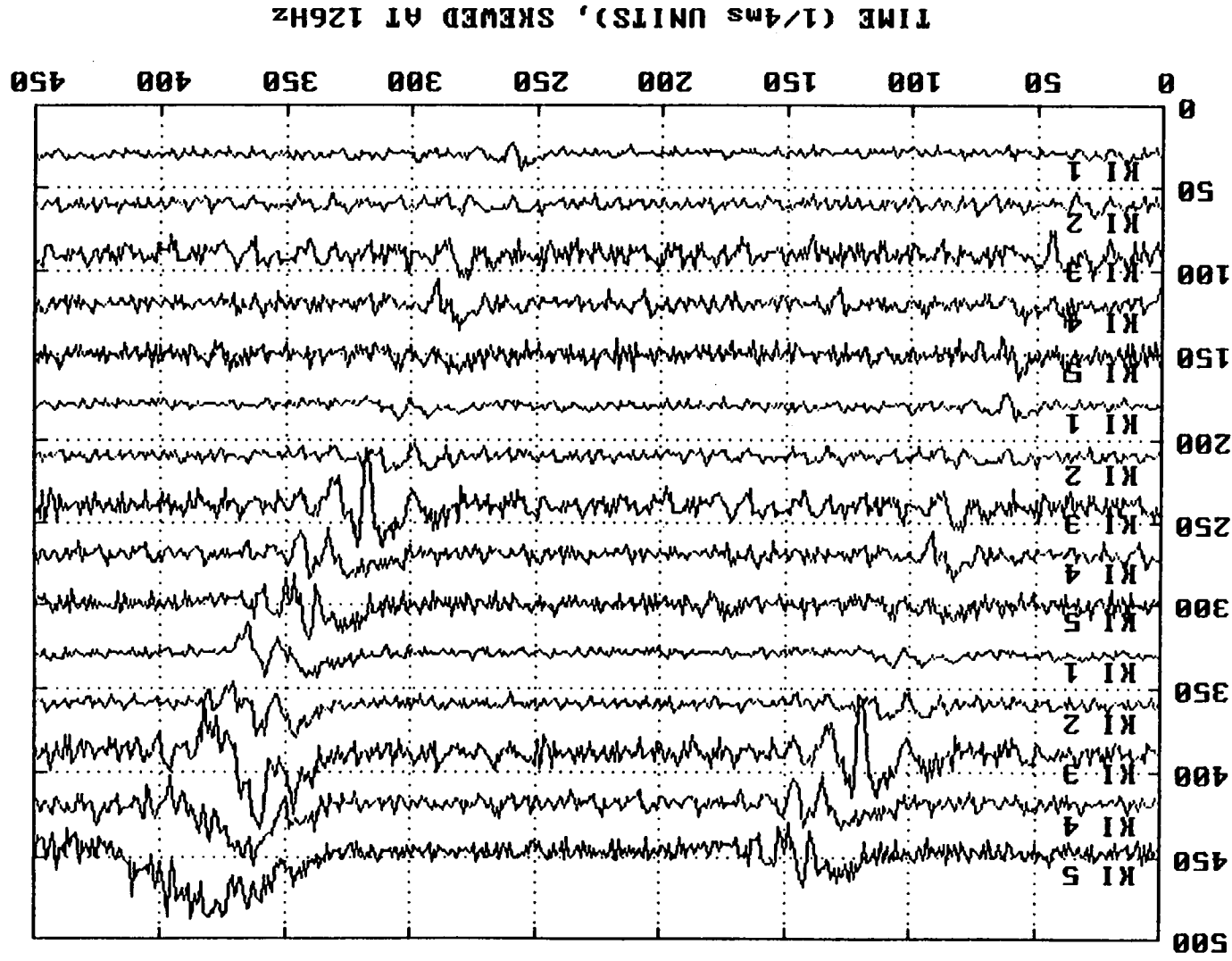


Fig. 3-42 T121210: Inlet Pressure Traces Skewed At 126Hz.

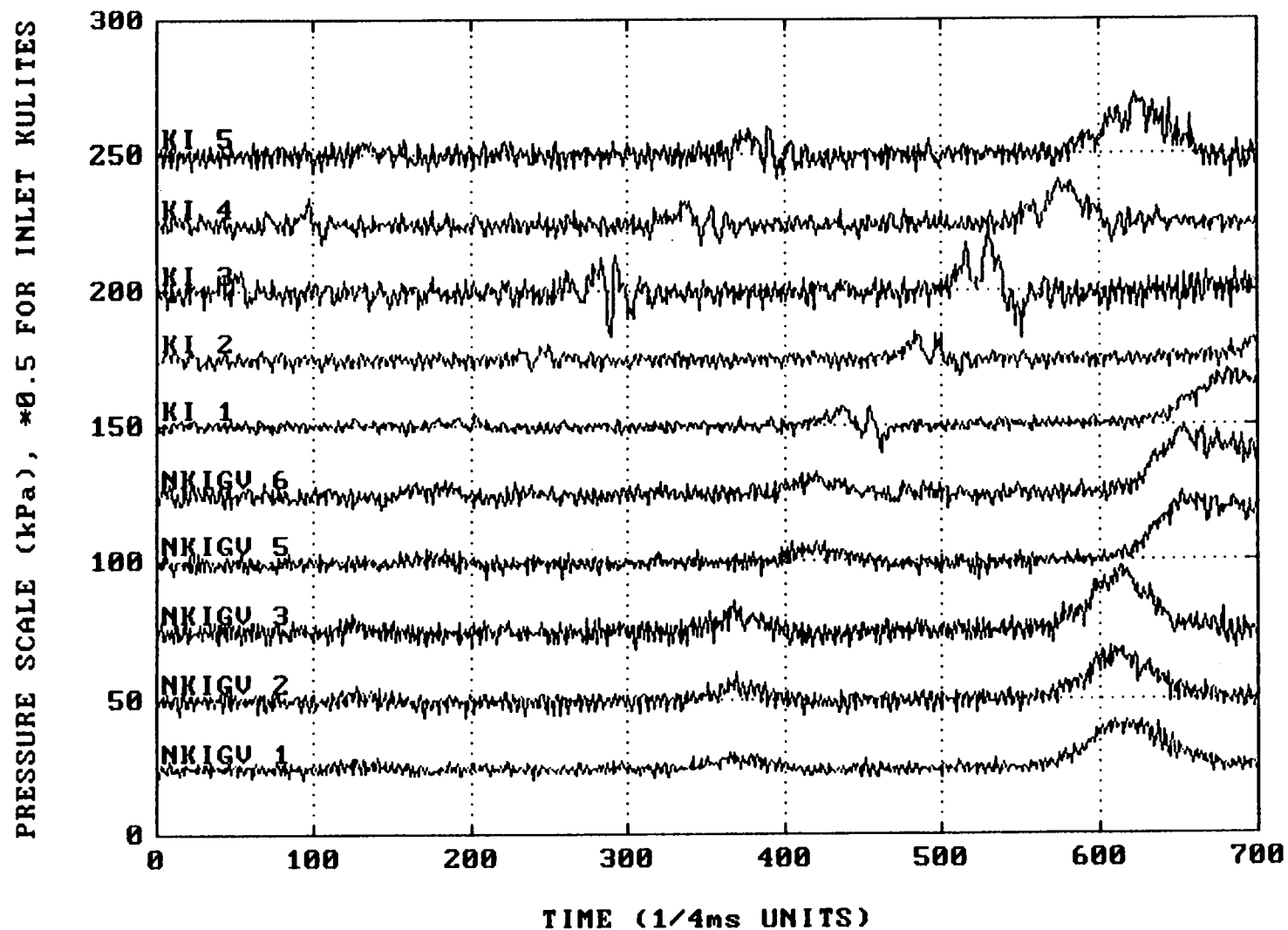


Fig. 3-43 T121210: 83% Speed Stall: Pressure Fluctuations on IGV Blade compared to IGV Casing Measurements.

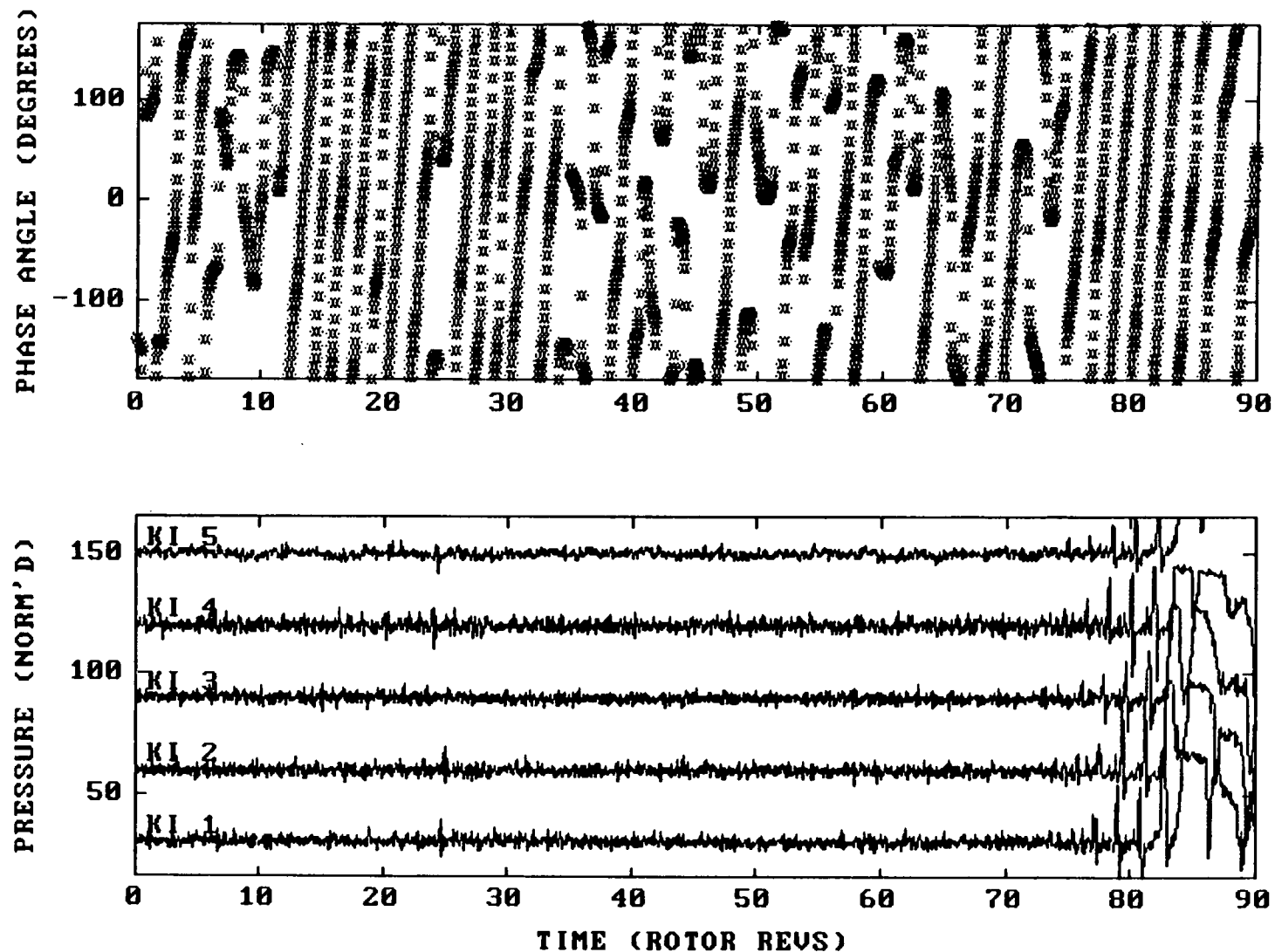


Fig. 3-44 T180804: 79% Speed Stall, Modal Phase Analysis of Casing Static Pressure at Inlet.

The lower graph shows the raw measurements from the inlet Kulites over the same time period.

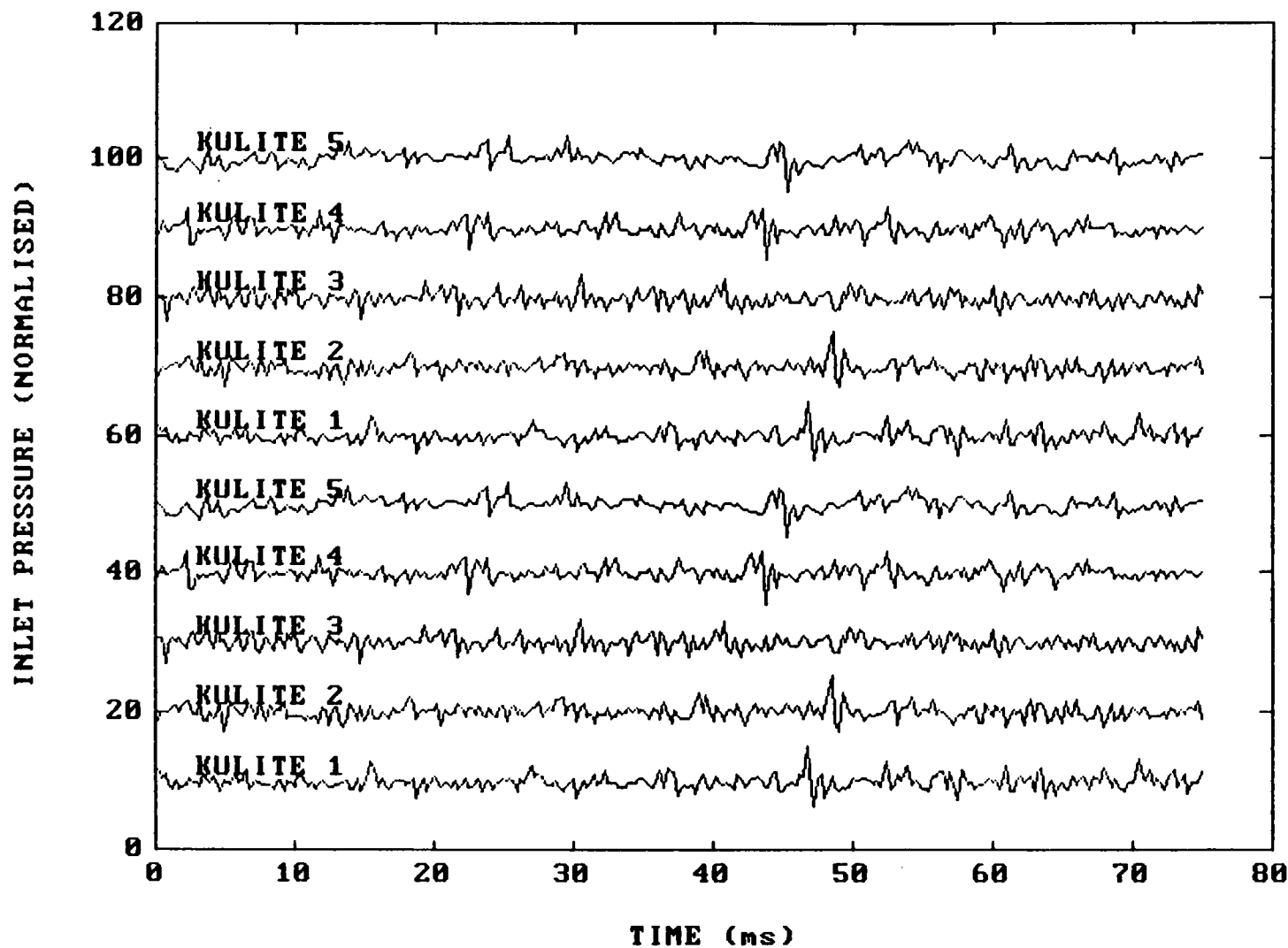


Fig. 3-45 T180804: Modal Behaviour Prior To Stall: Inlet.

Casing static pressure measurements, repeated to allow the disturbances to be traced more easily.

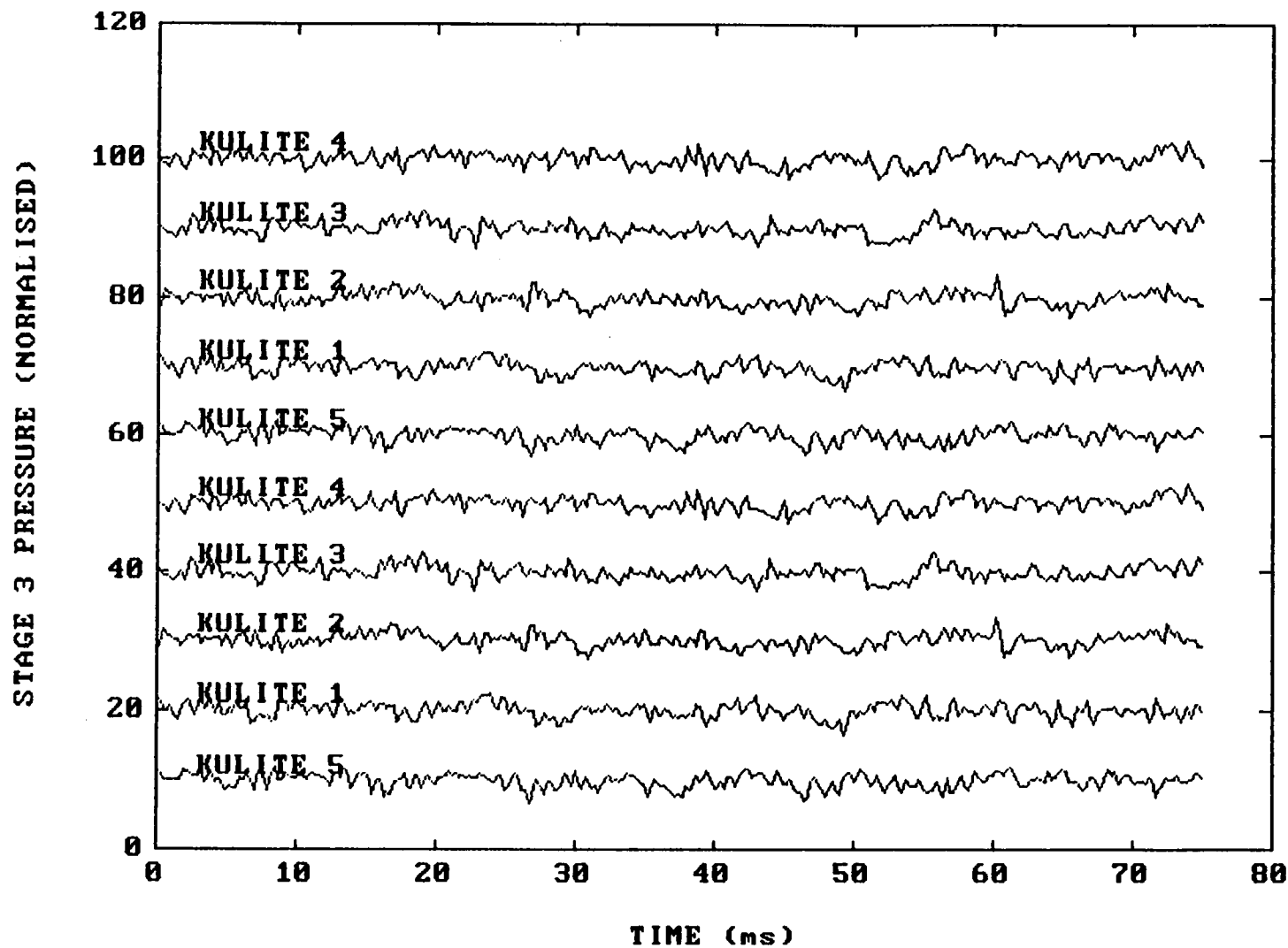


Fig. 3-46 T180804: Modal Behaviour Prior To Stall: Stage 3.

Casing static pressure measurements, repeated to allow the disturbances to be traced more easily.

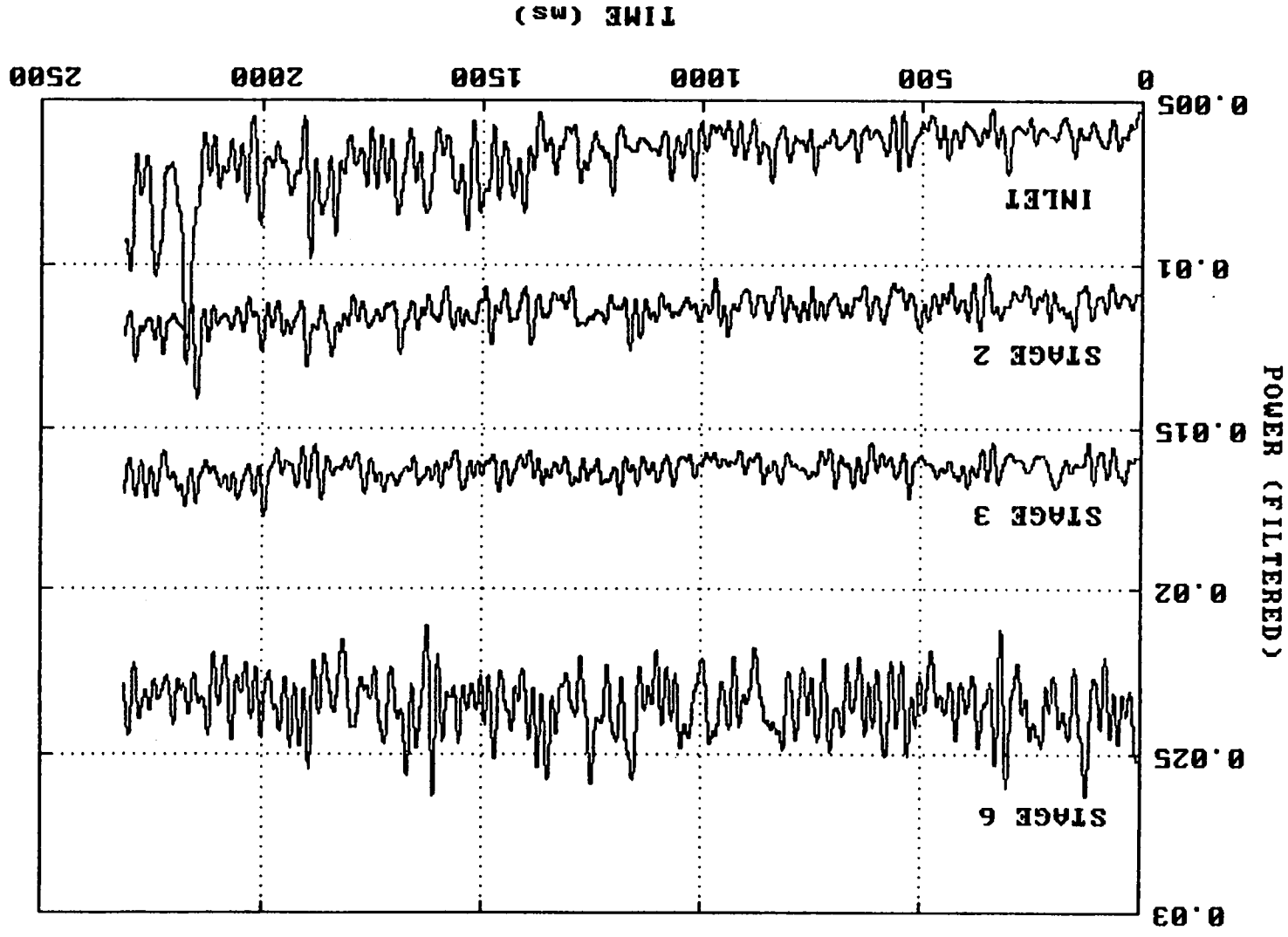


Fig. 3-47 T180813: 81% Speed Stall, Mean Signal Power.

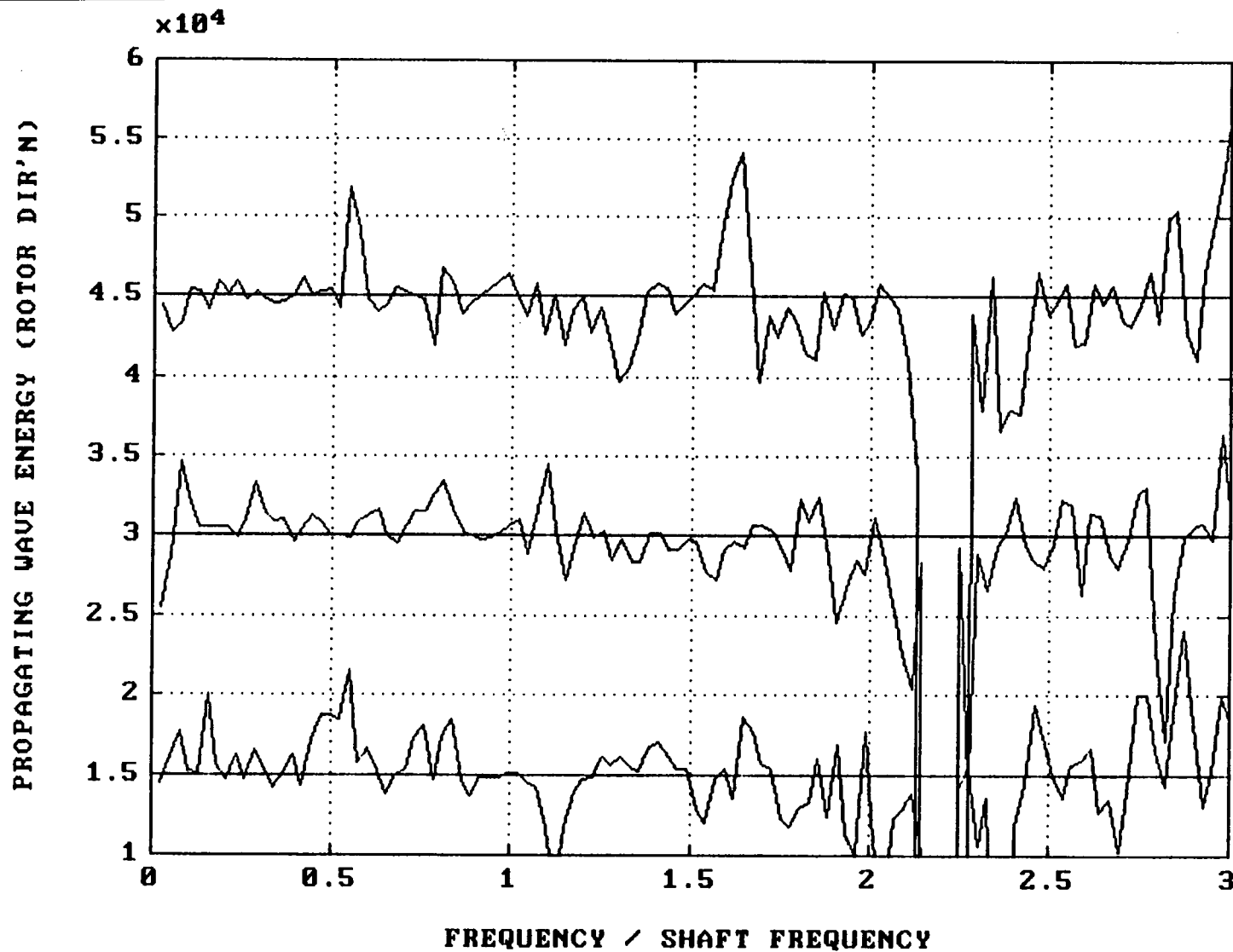


Fig. 3-48 T180806: 67% Speed Stall, Propagating Wave Energy Based on Inlet Casing Static Pressure Measurements.

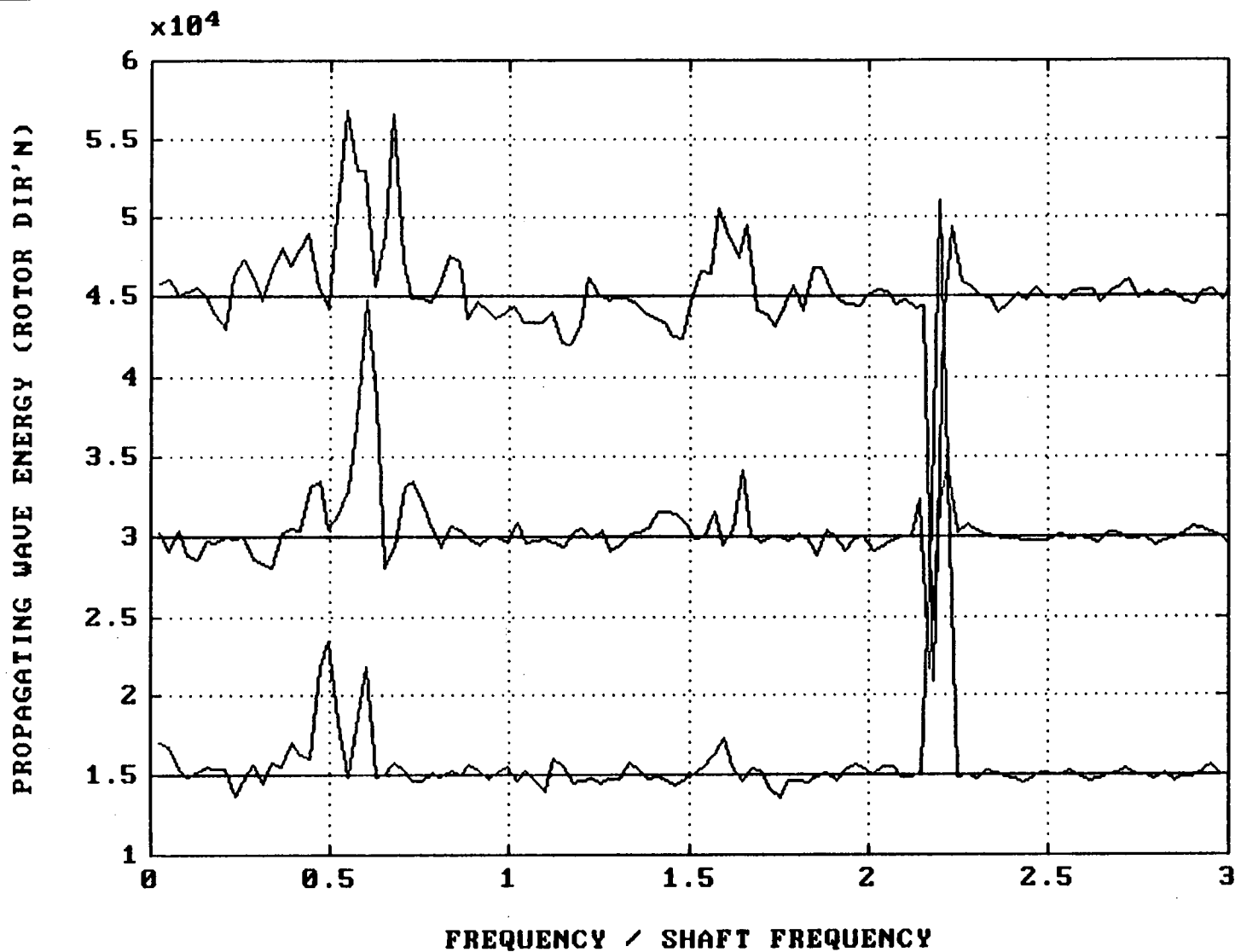


Fig. 3-49 T180806: 67% Speed Stall, Propagating Wave Energy Based on Stage 3 Casing Static Pressure Measurements.

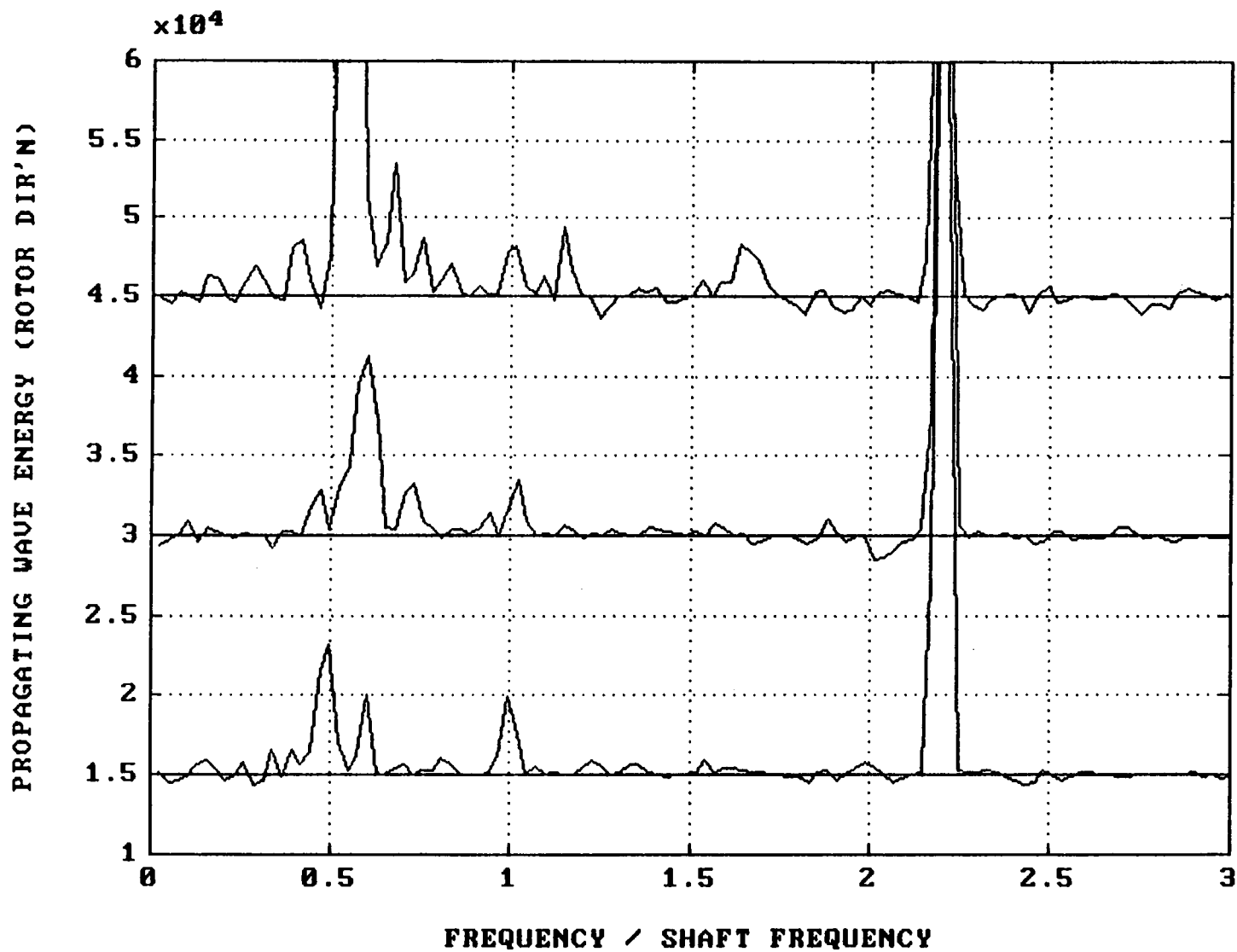


Fig. 3-50 T180806: 67% Speed Stall, Propagating Wave Energy Based on Stage 4 Casing Static Pressure Measurements.

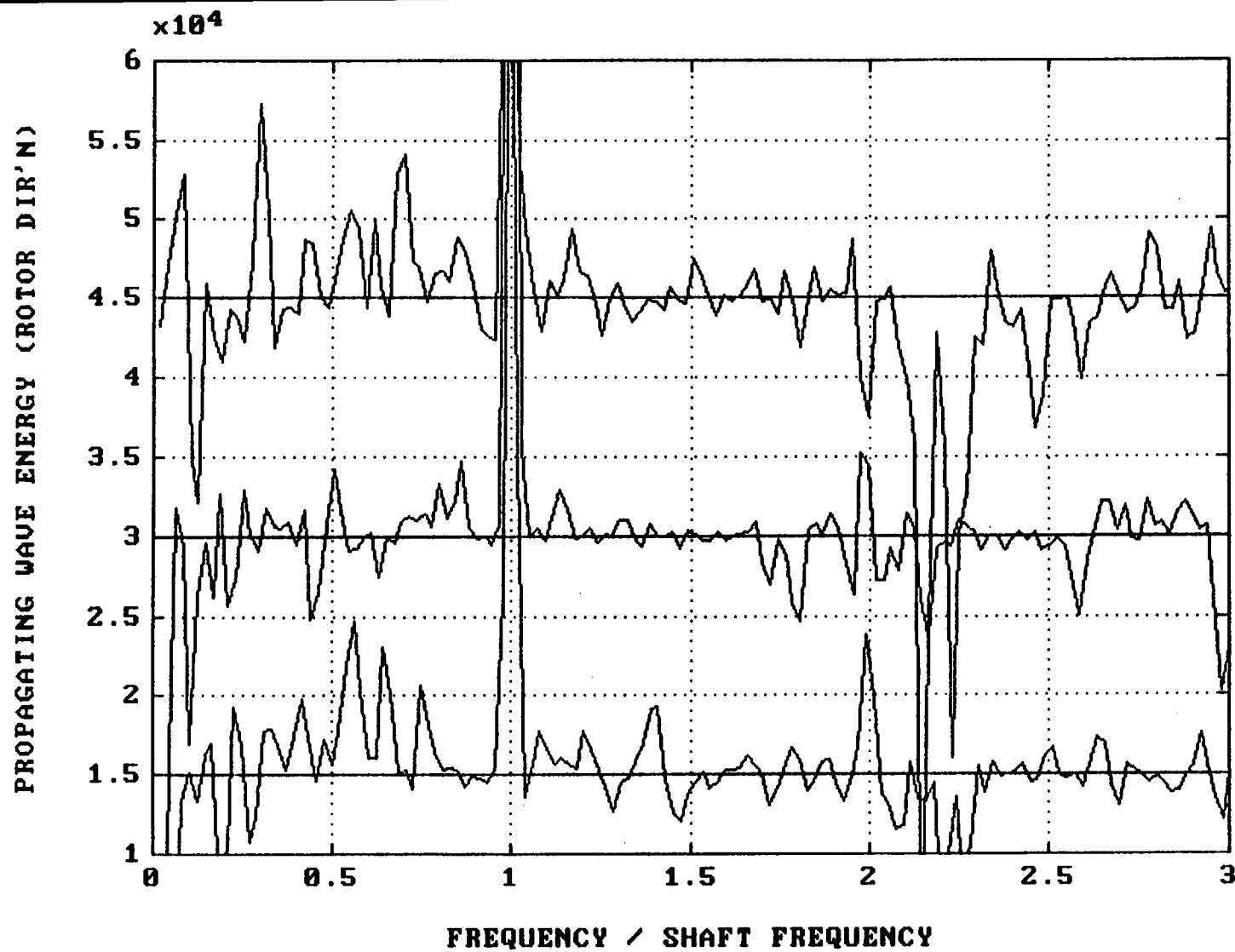


Fig. 3-51 T180804: 79% Speed Stall, Propagating Wave Energy Based on Inlet Casing Static Pressure Measurements.

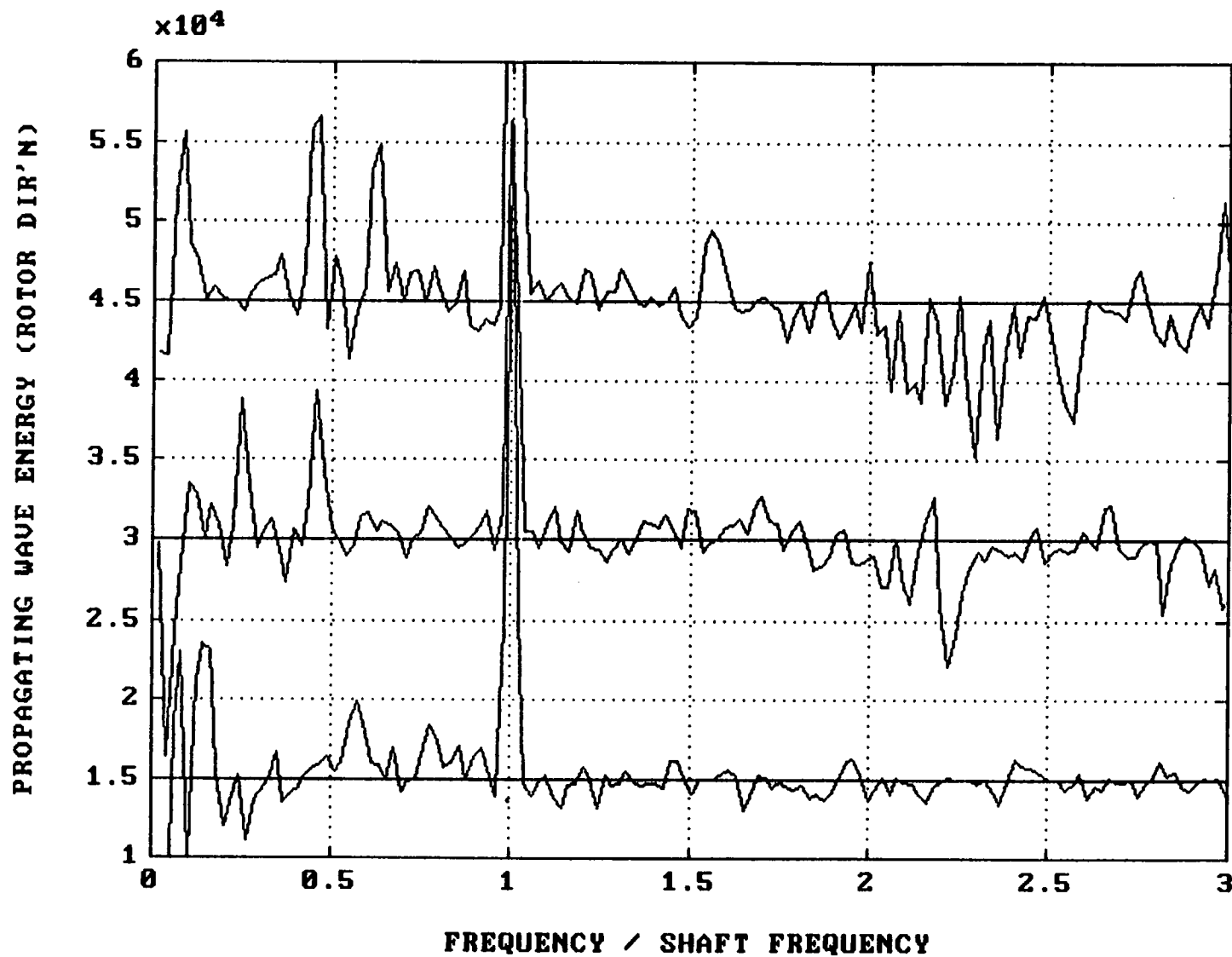


Fig. 3-52 T180802: 80% Speed Stall, Propagating Wave Energy Based on Inlet Casing Static Pressure Measurements.

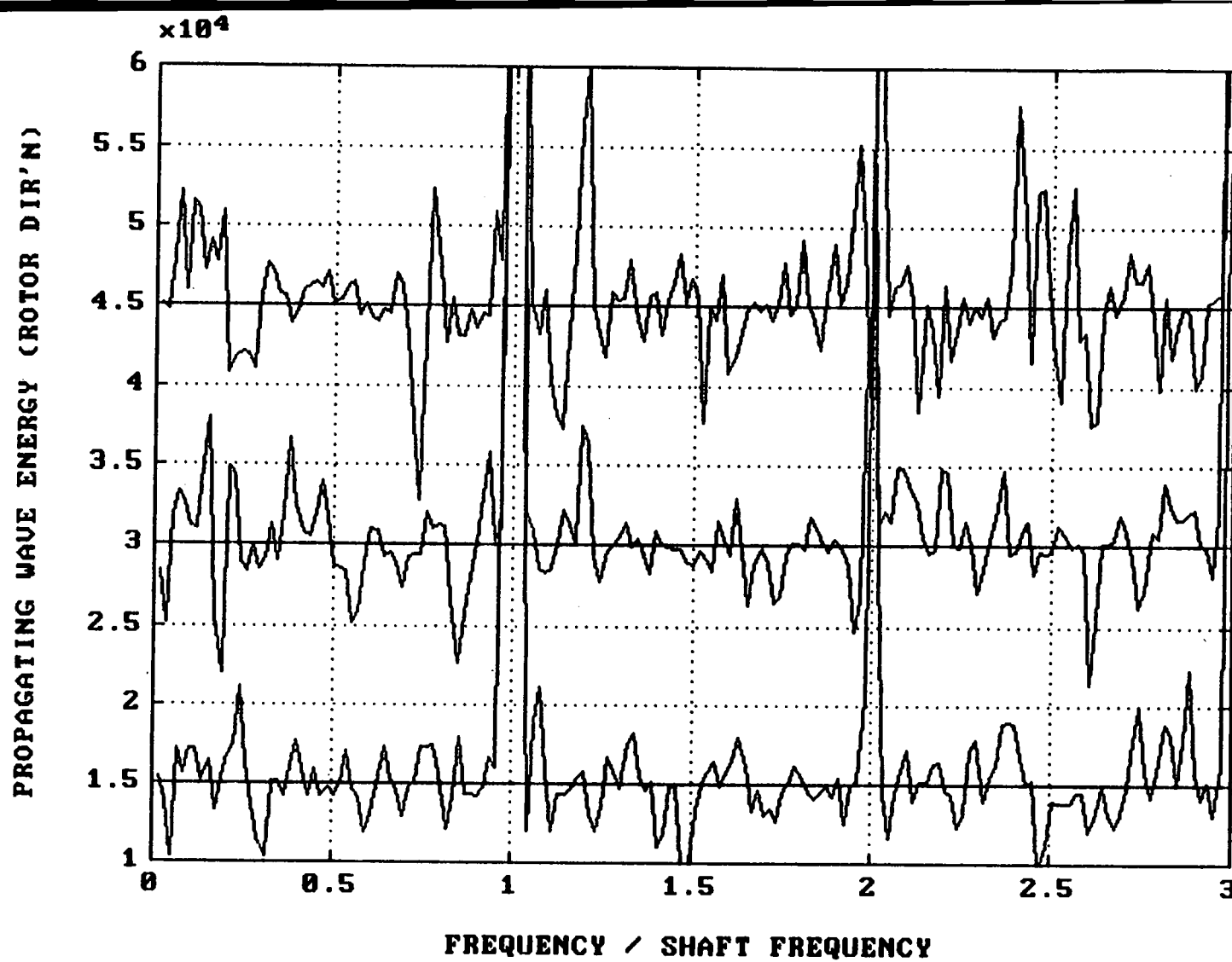


Fig. 3-53 T180803: 100% Speed Stall, Propagating Wave Energy Based on Stage 4 Casing Static Pressure Measurements.

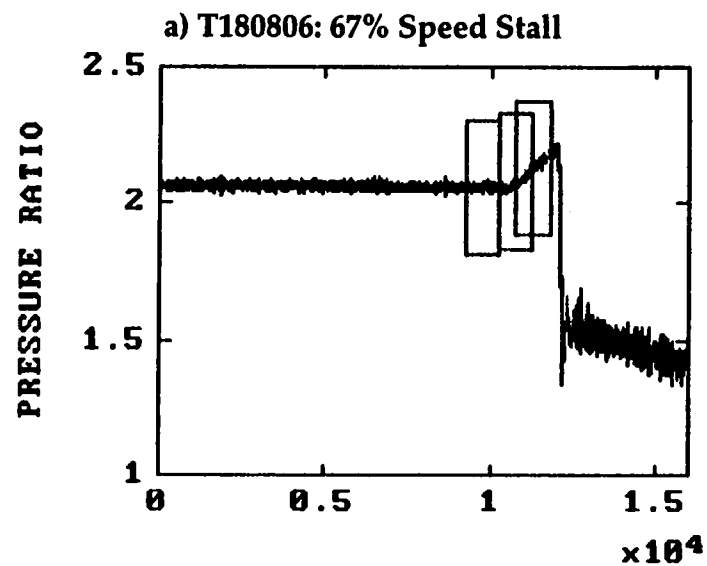
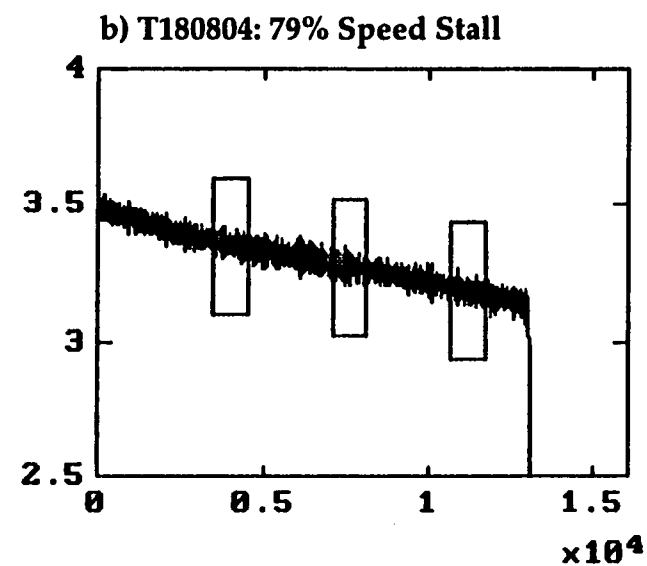
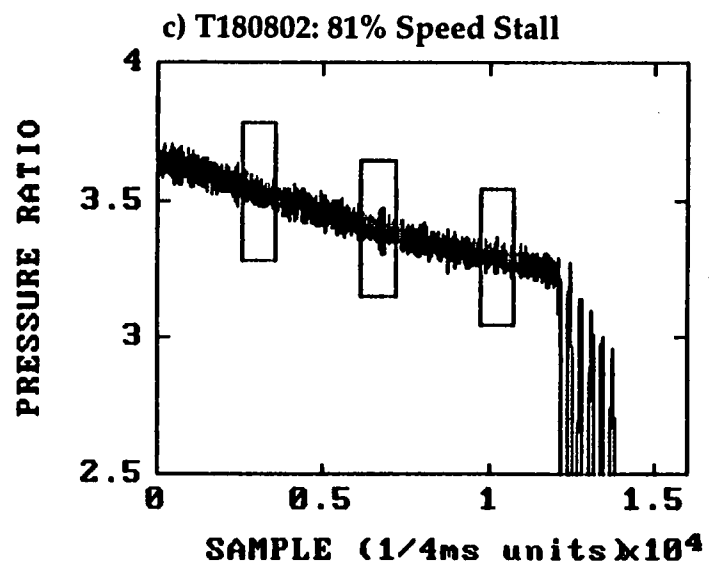
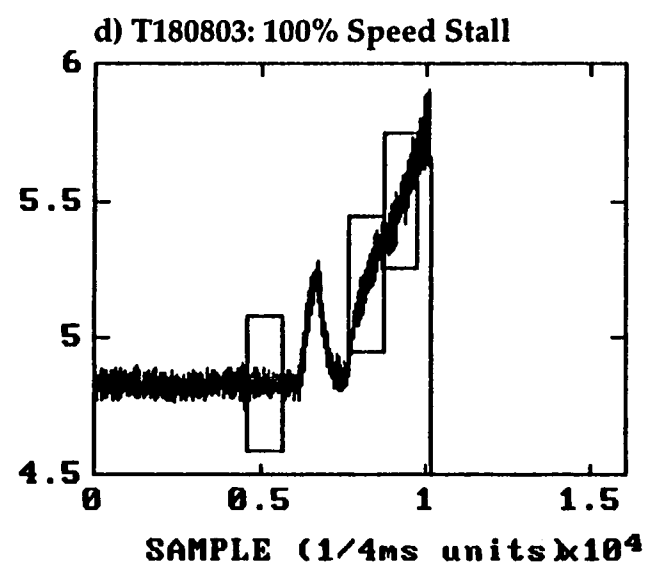


Fig. 3-54 Propagating Wave Energy: Positions of Time Windows.

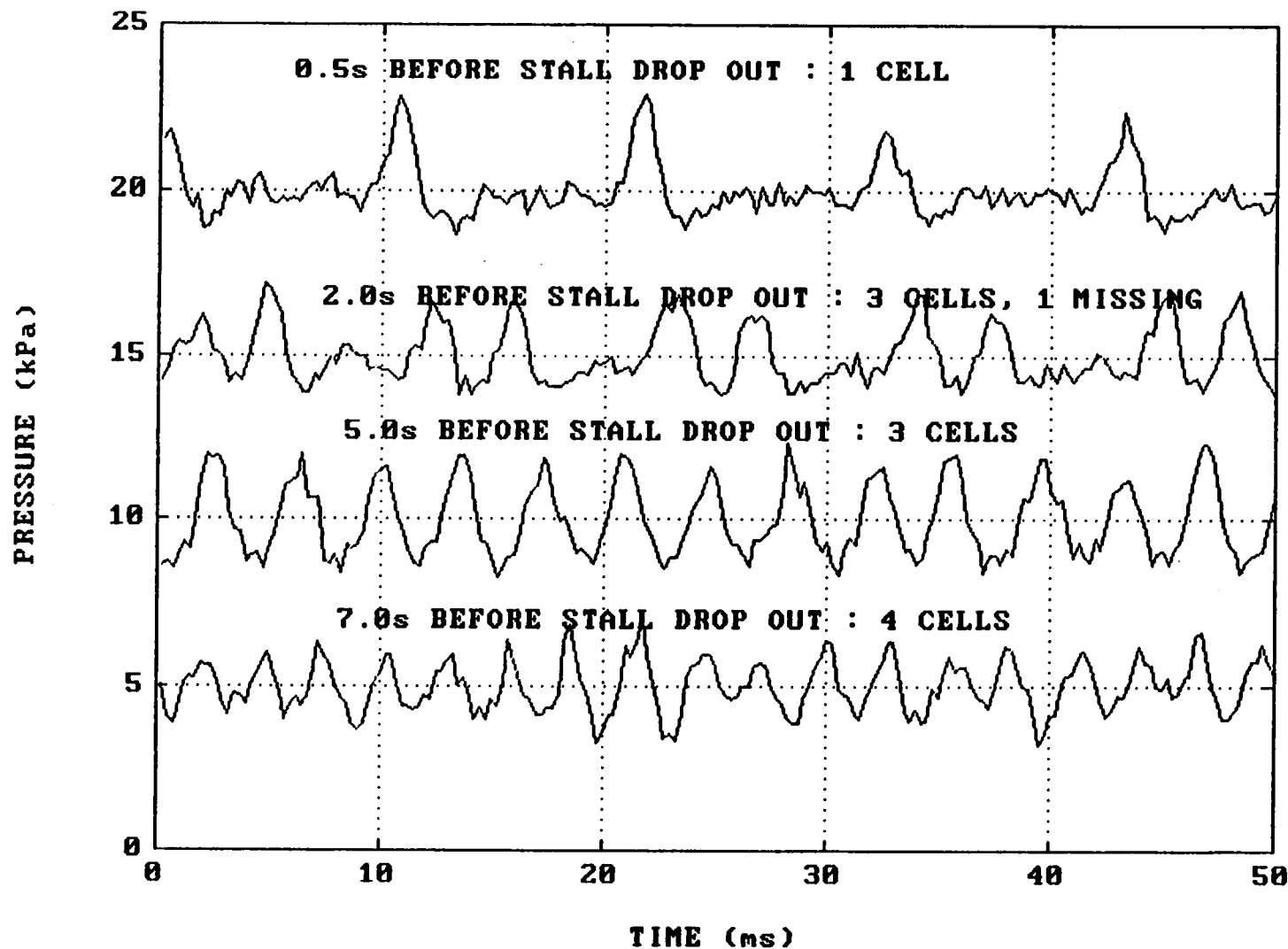
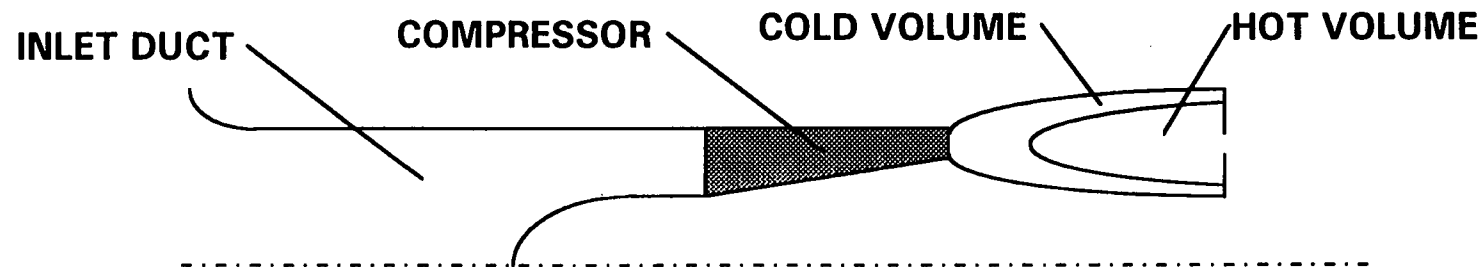


Fig. 3-55 T081204: IGV Exit Pressure During Stall Drop-Out.

Four sections of the casing static pressure trace from Kulite KI1.

a) Schematic View of Engine in Z-r Plane



b) Schematic View of Model in Z- θ Plane

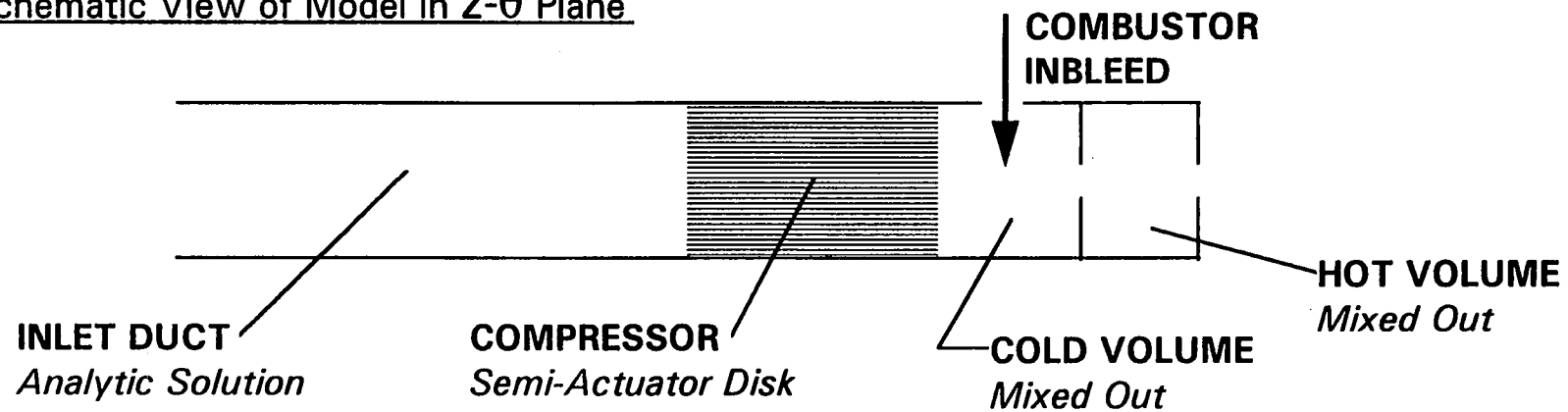


Fig. 4-1 Comparison of Engine Geometry and Compressors in Parallel Model (schematic).

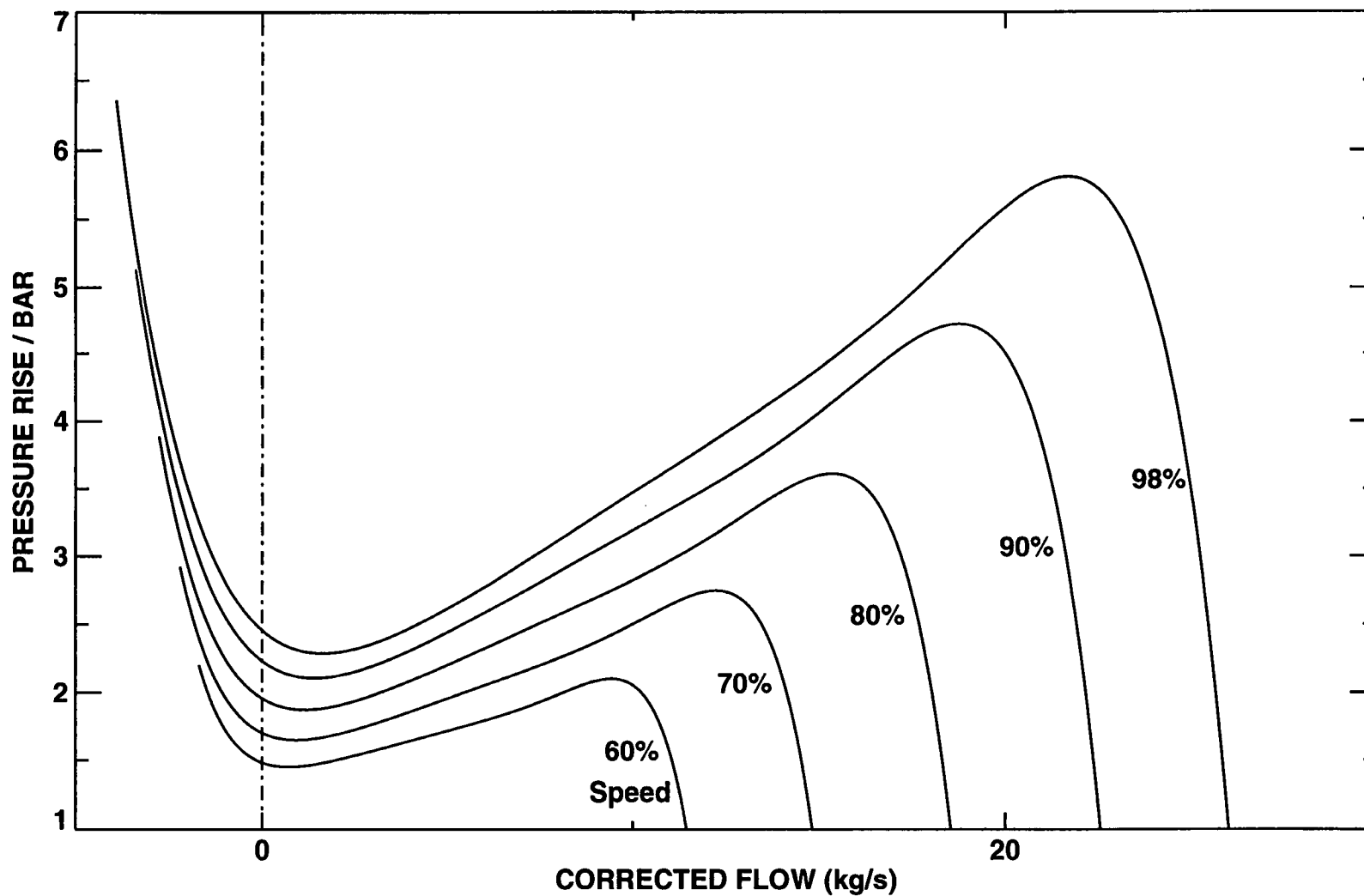


Fig. 4-2 Rolls-Royce VIPER Axisymmetric Pressure Rise Characteristic as Used in Compressors in Parallel Model.

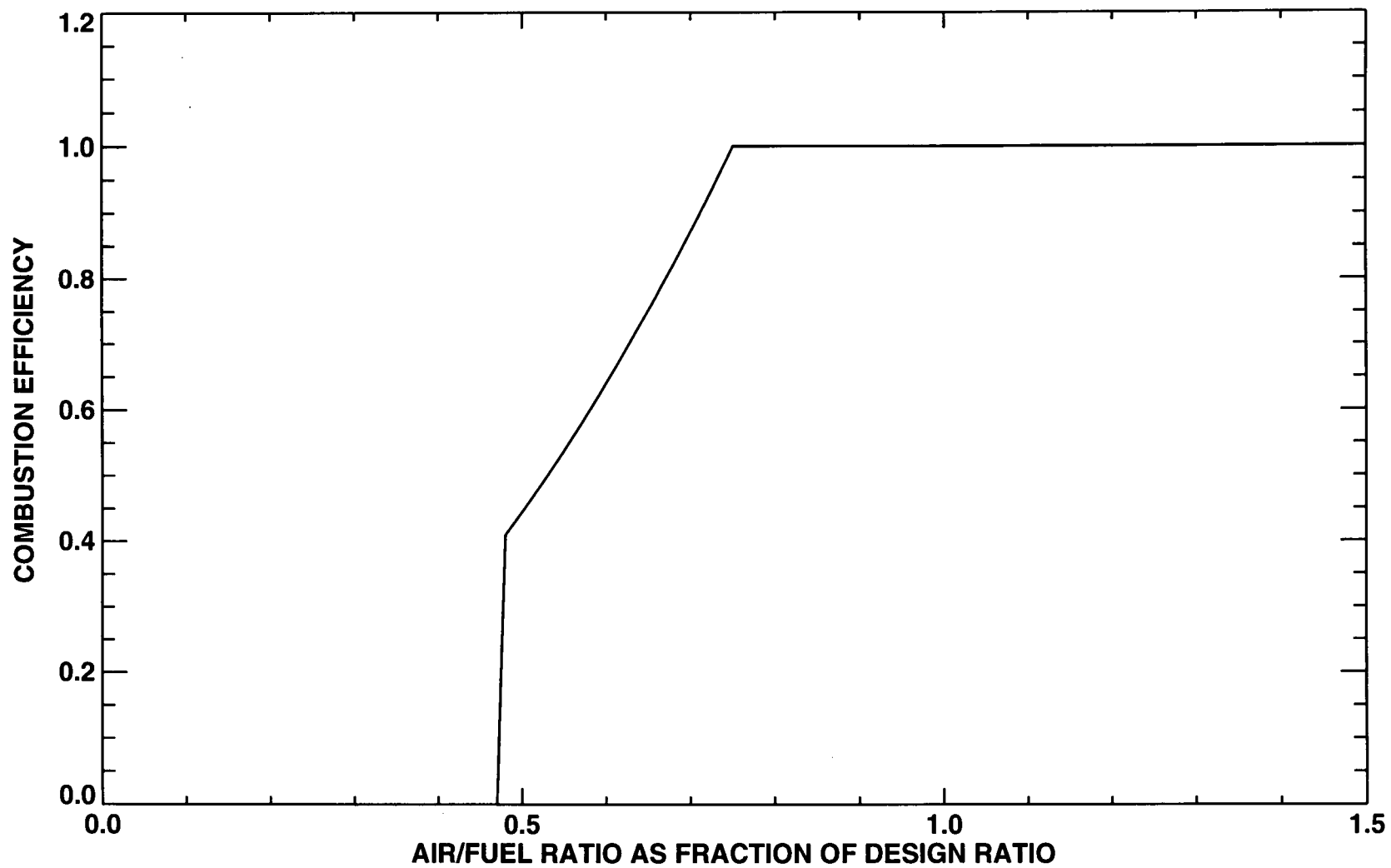


Fig. 4-3 Combustor Efficiency.

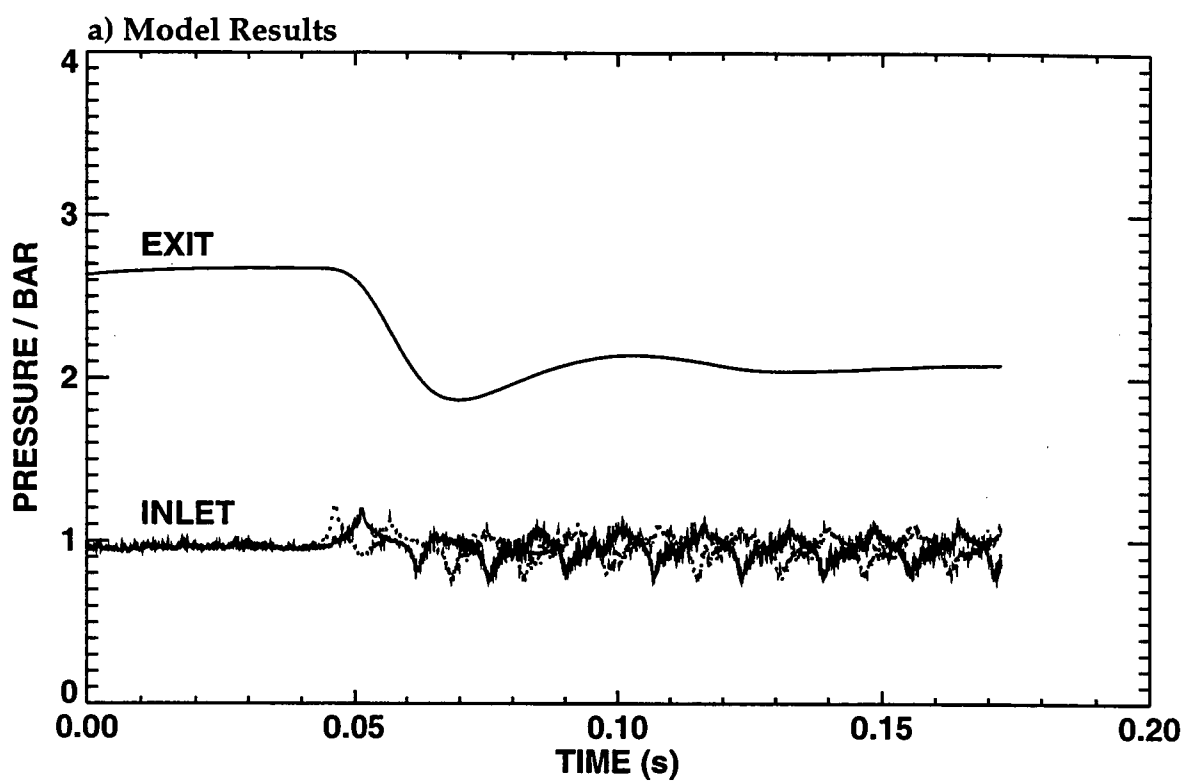
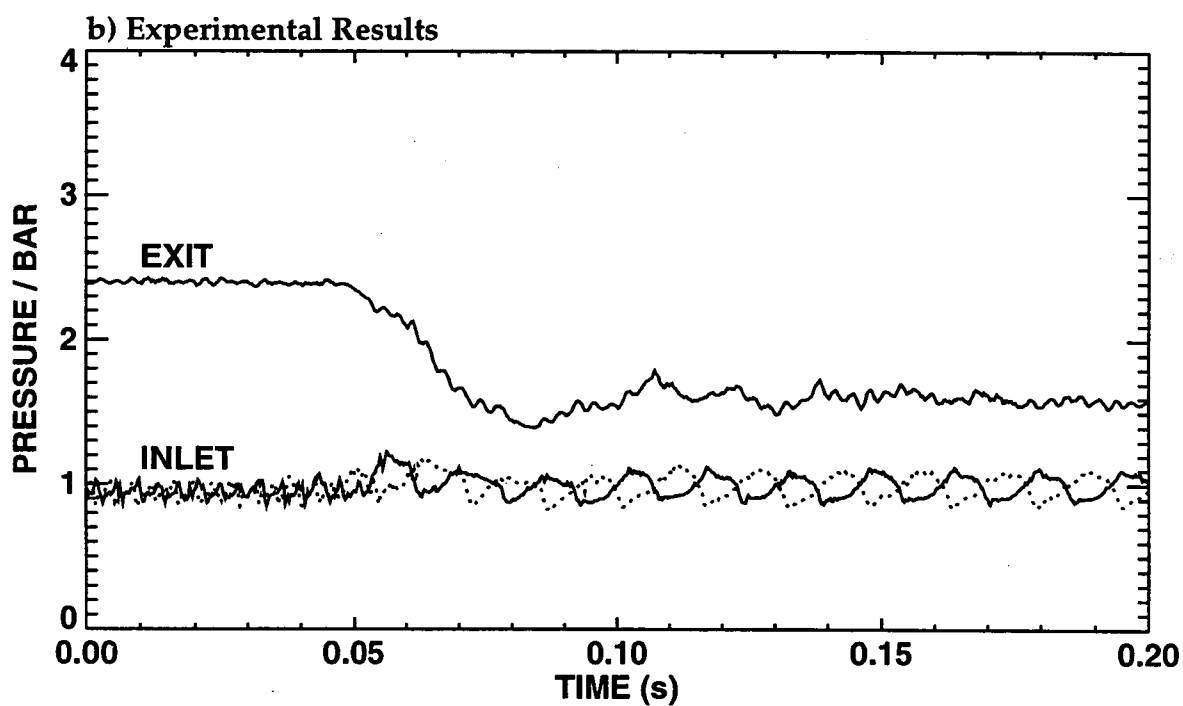


Fig. 4-4 Overall Behaviour at 70% Speed.

Comparison of Model Results with Measured Data.

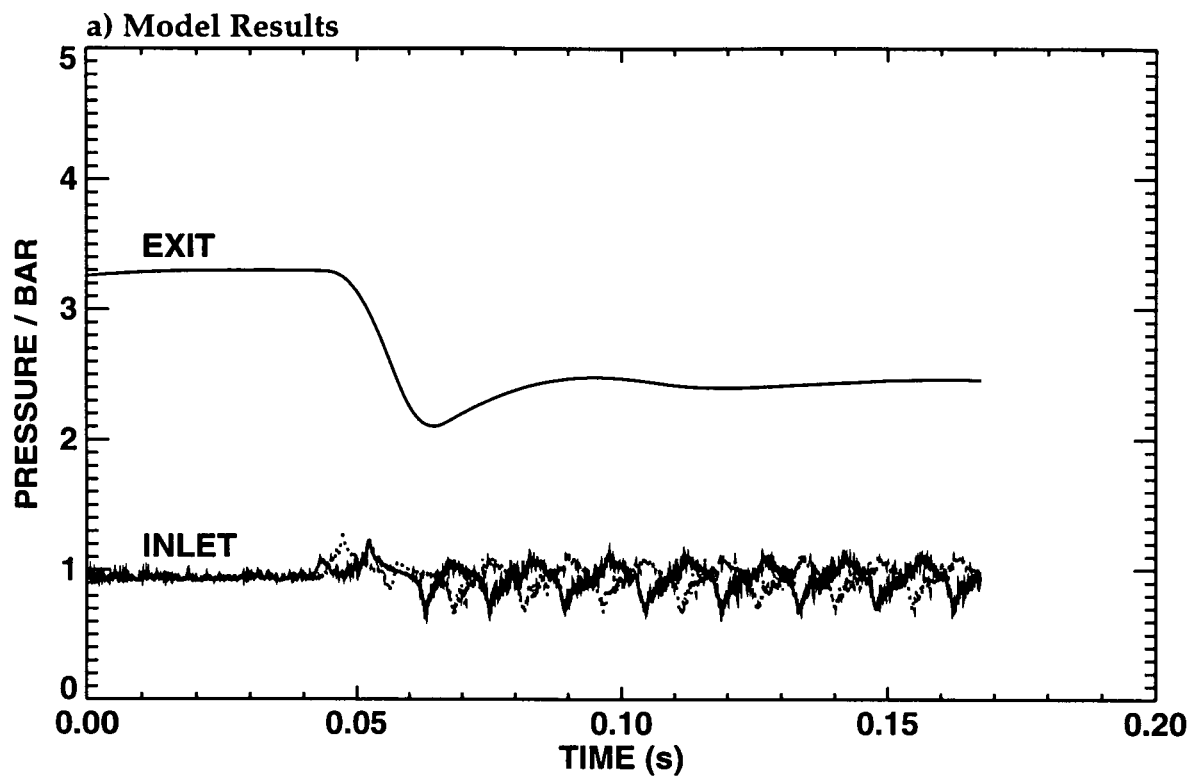
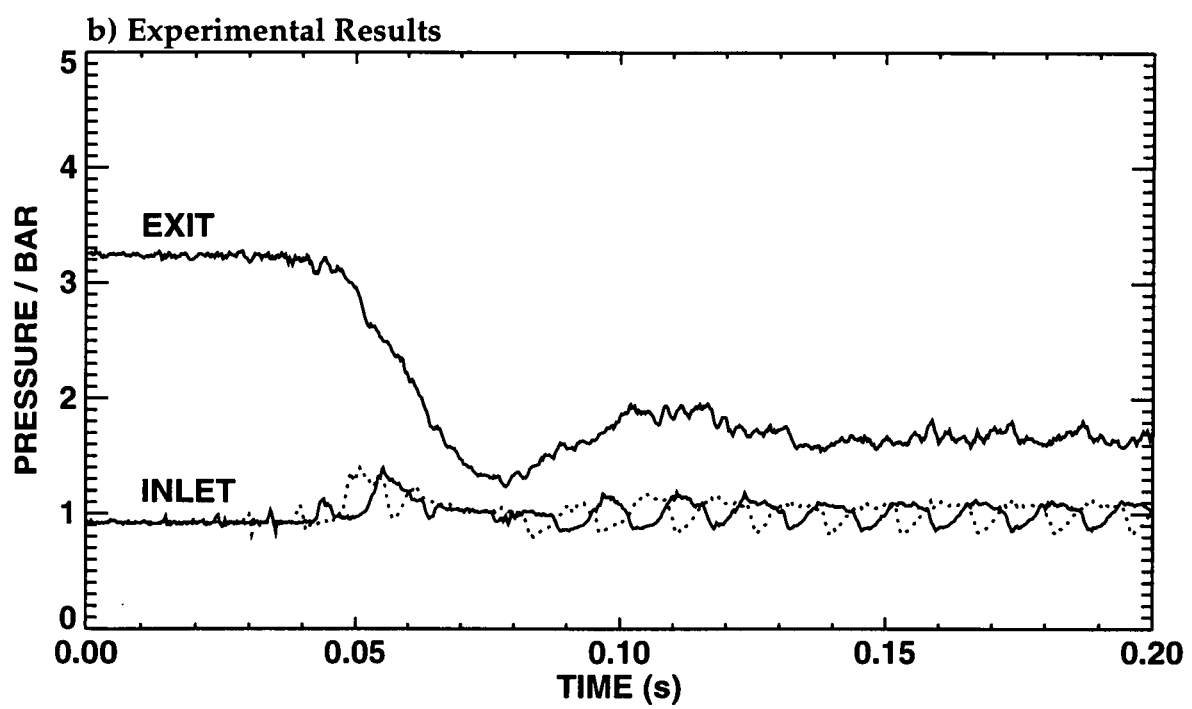


Fig. 4-5 Overall Behaviour at 78% Speed.

Comparison of Model Results with Measured Data.

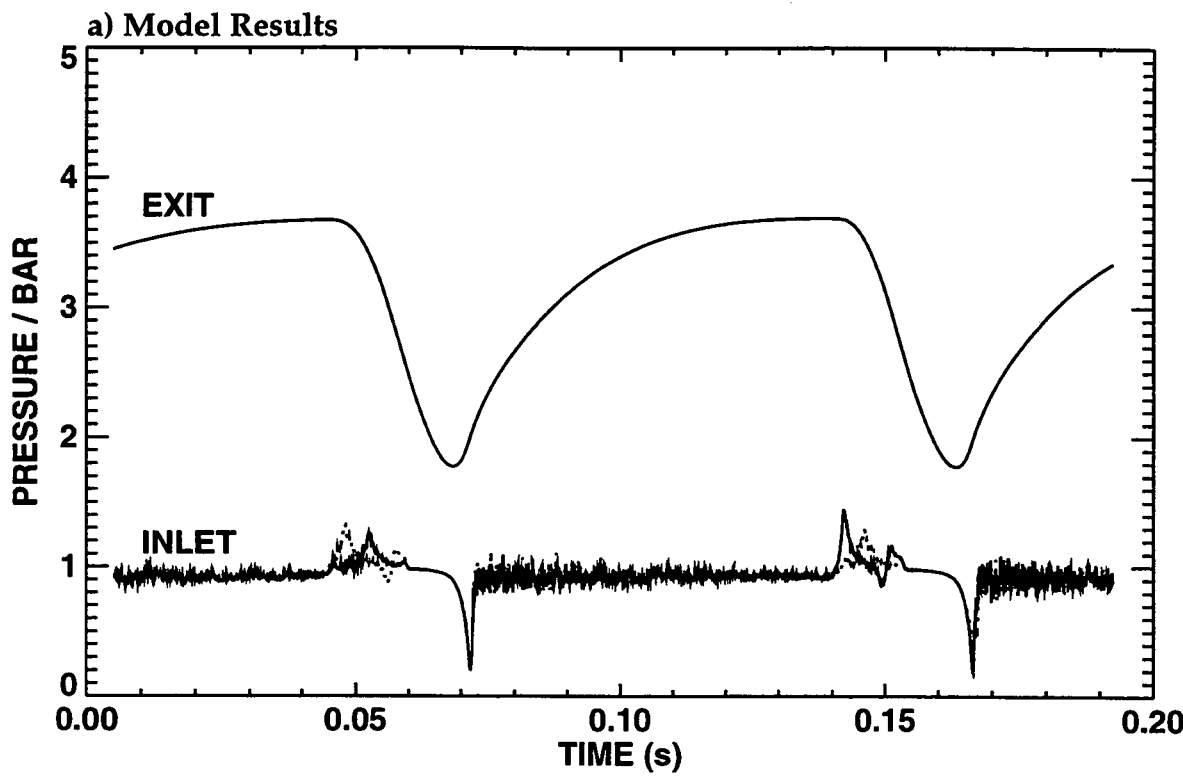
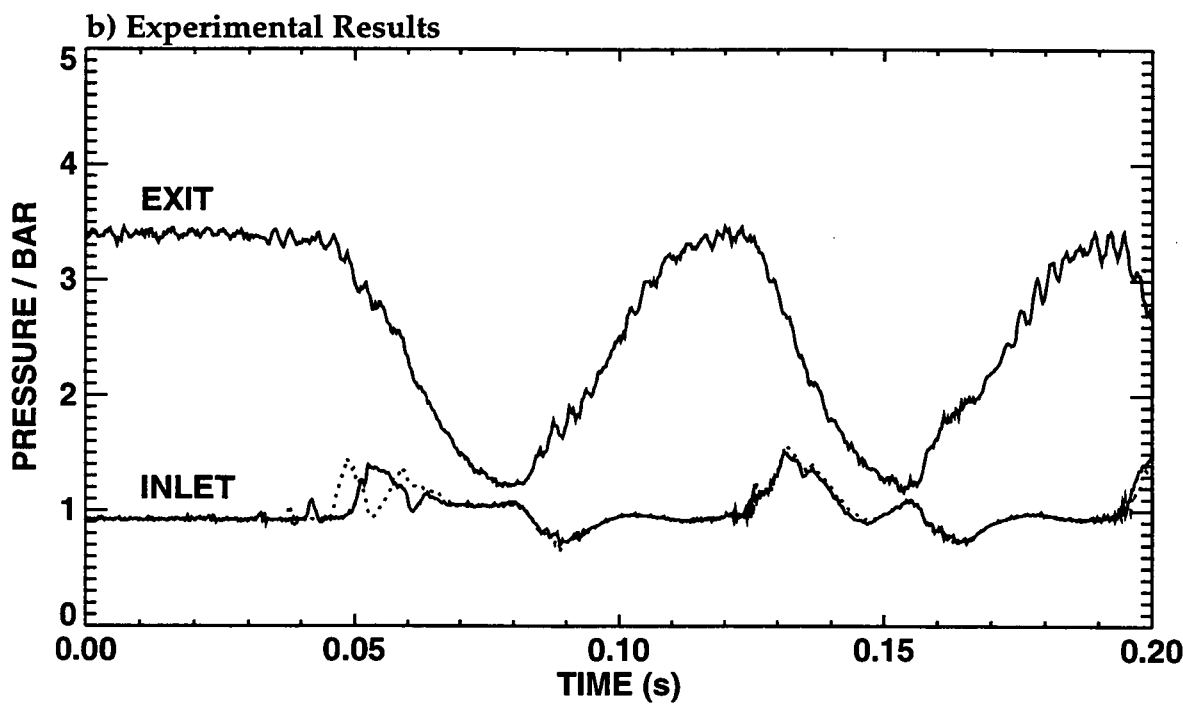


Fig. 4-6 Overall Behaviour at 82% Speed.
Comparison of Model Results with Measured Data.

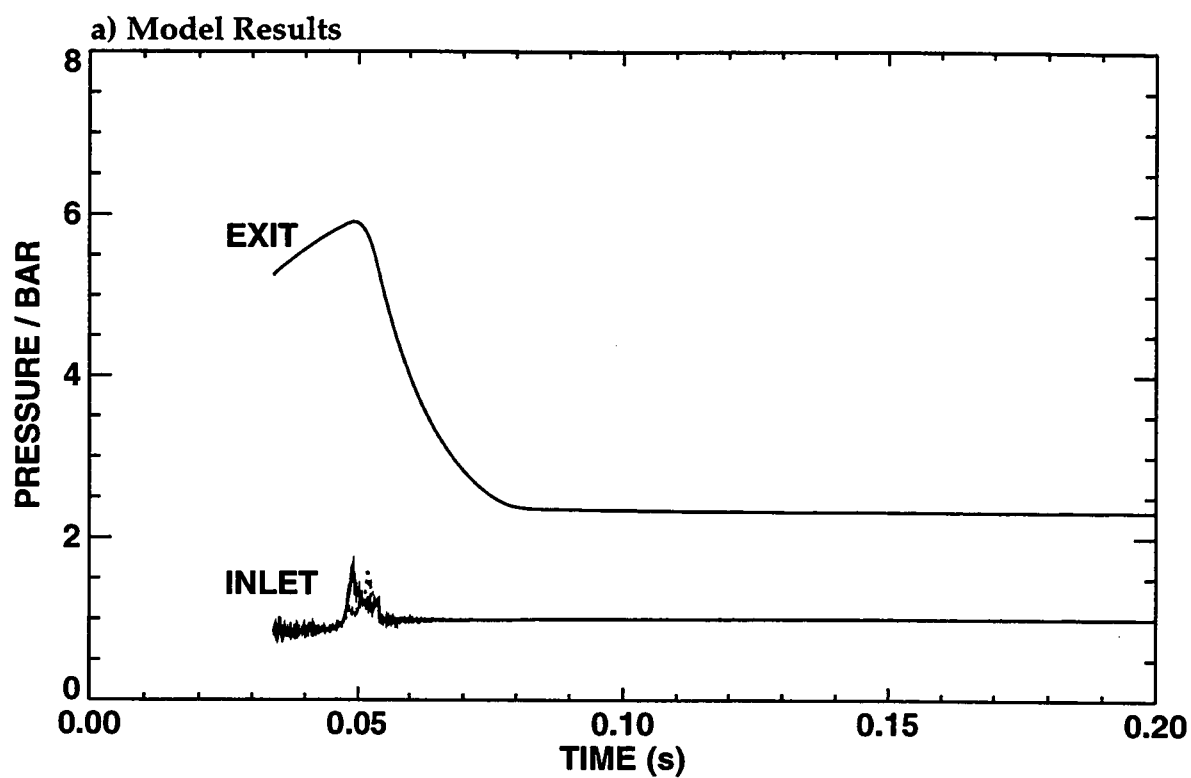
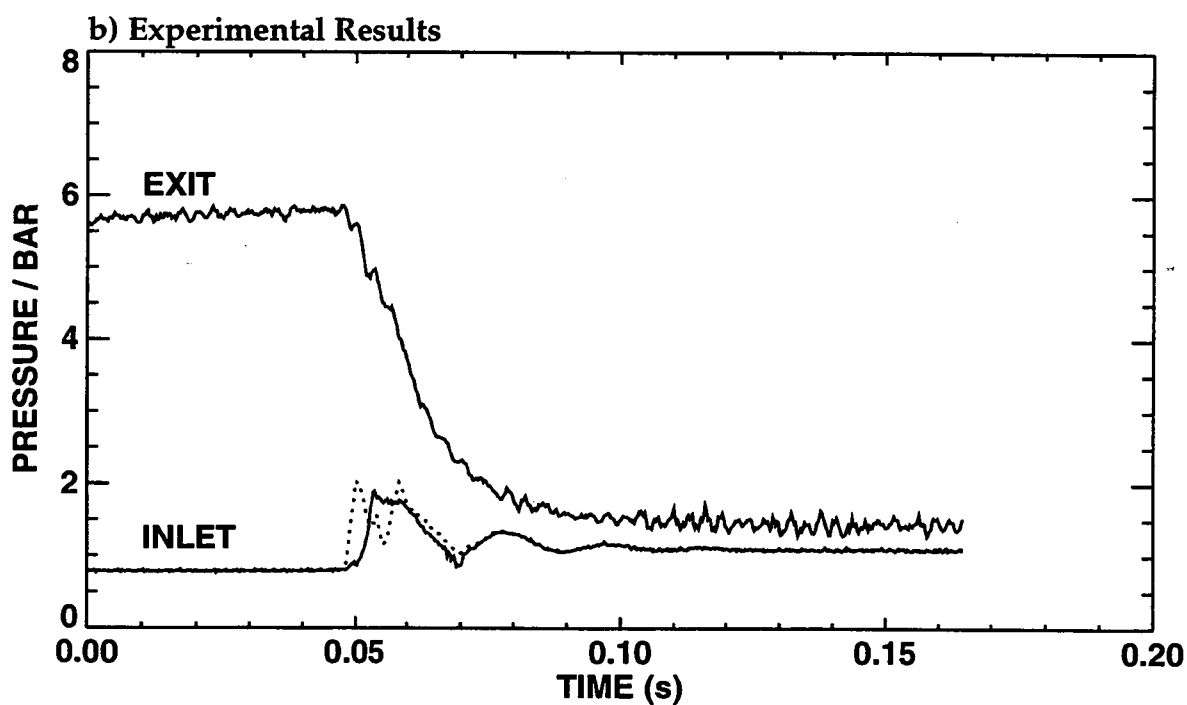


Fig. 4-7 Overall Behaviour at 98% Speed.

Comparison of Model Results with Measured Data.

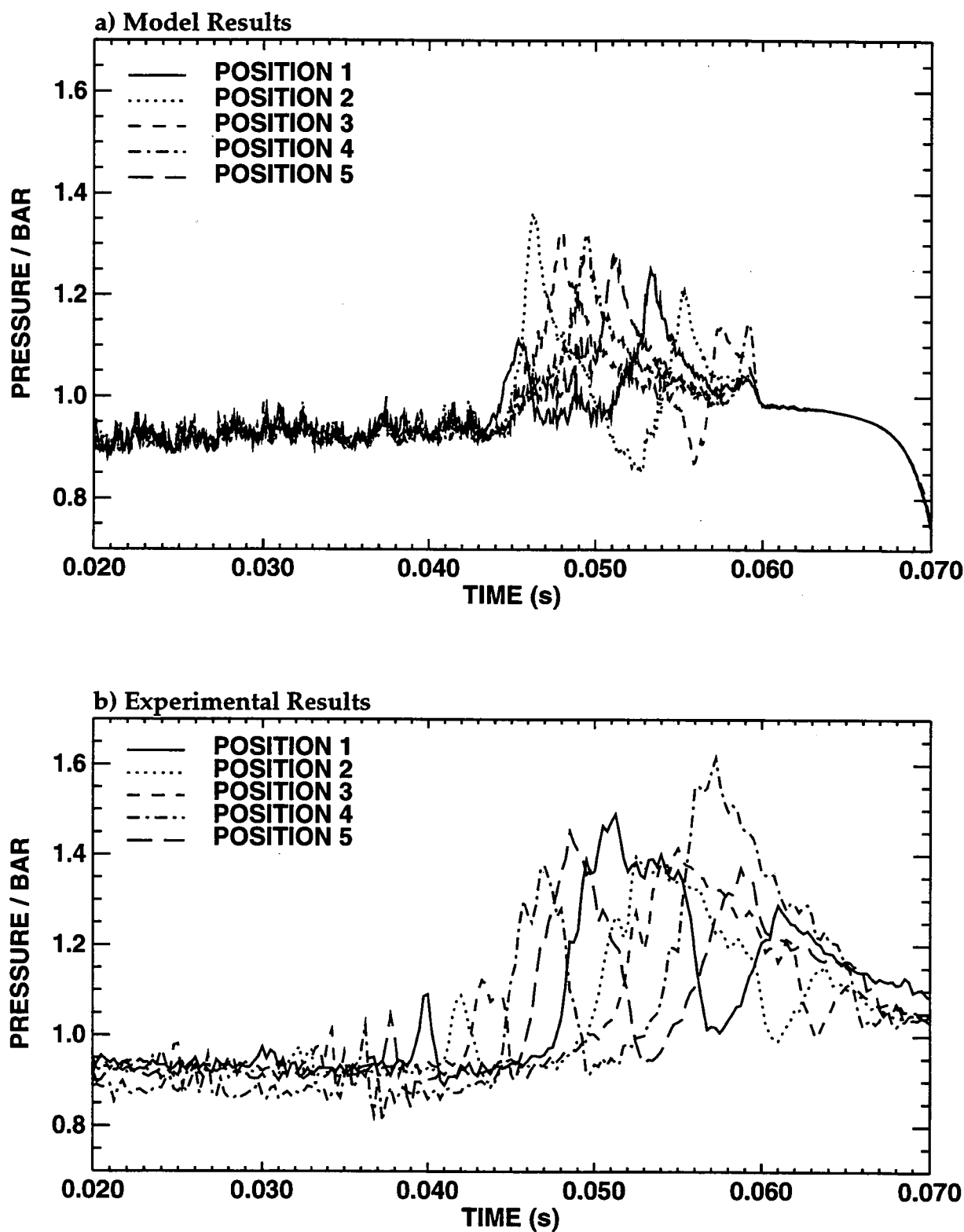


Fig. 4-8 Stall Inception Behaviour at 82% Speed.

Comparison of Model Results with Measured Data.

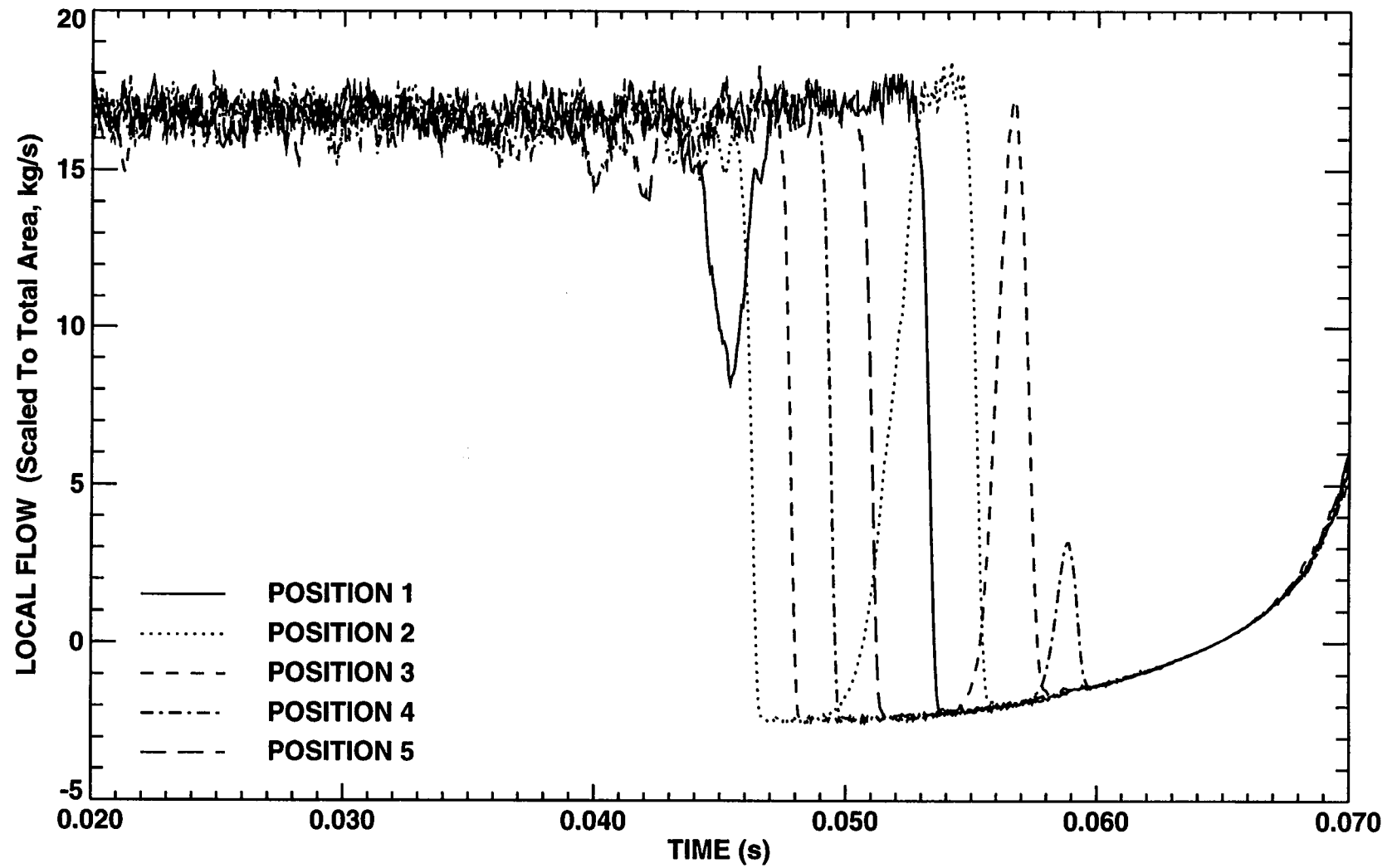


Fig. 4-9 Calculated Local Flow Level During Stall Inception at 82% Speed.

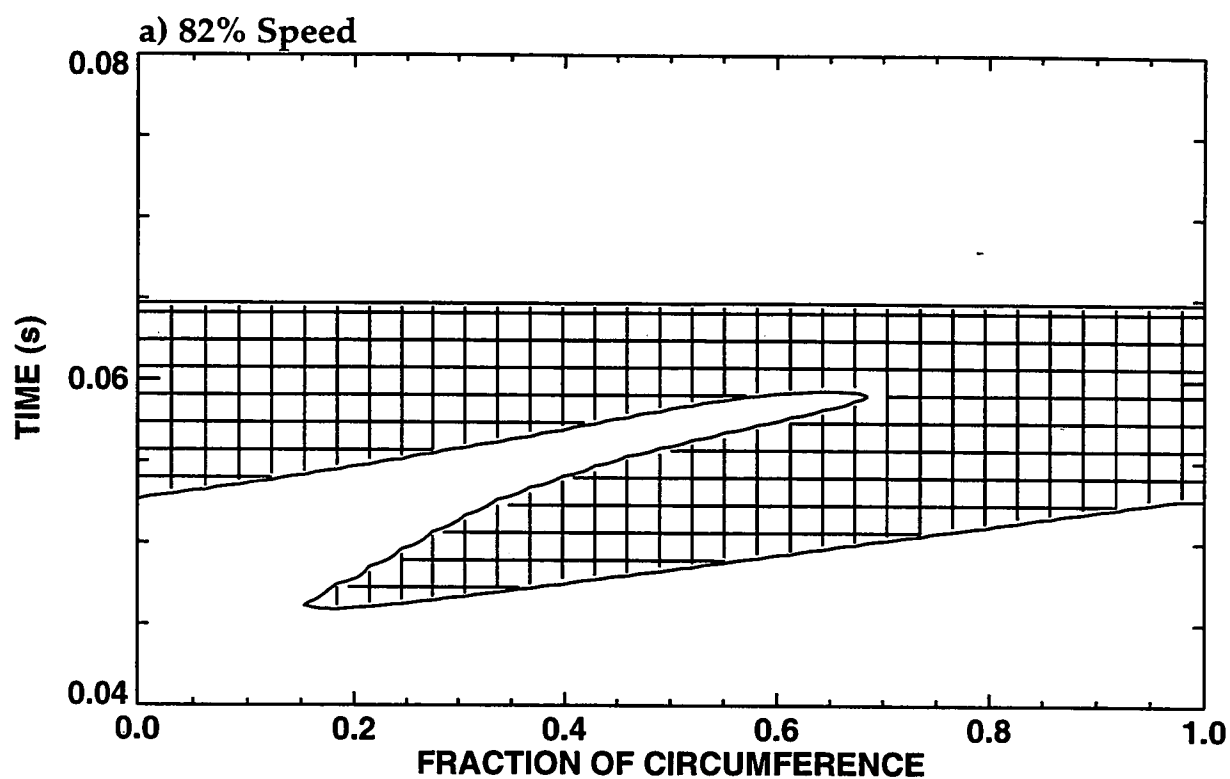
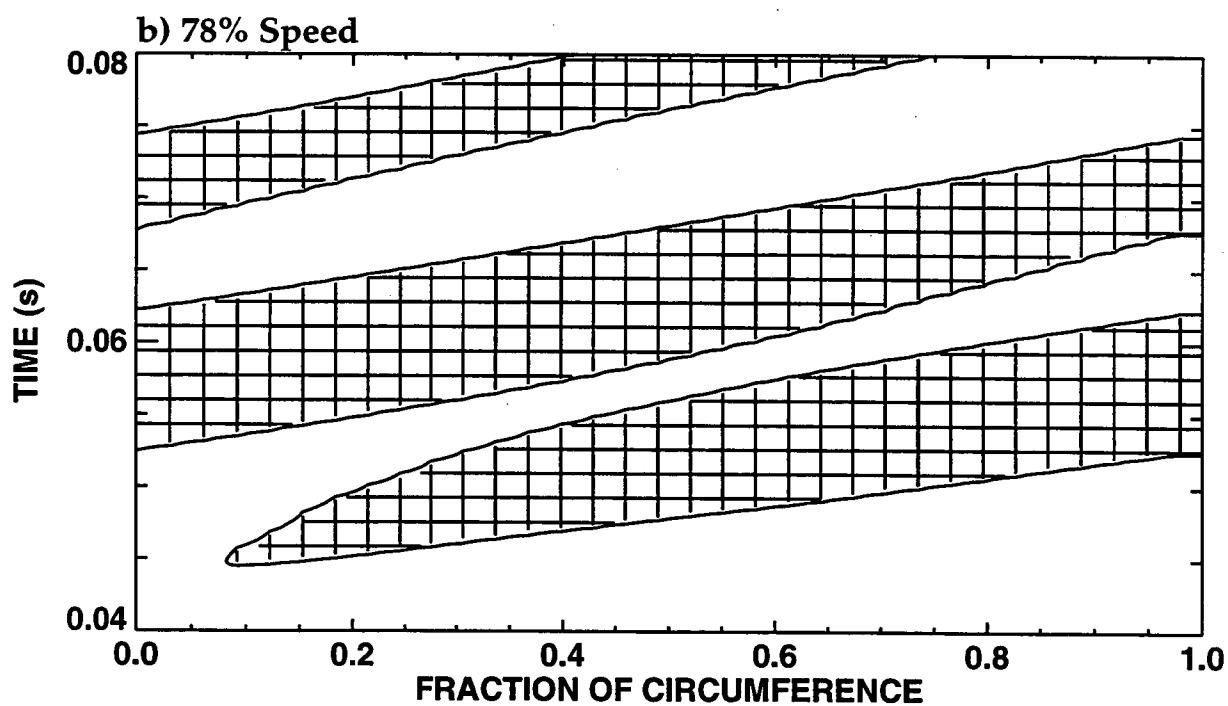


Fig. 4-10 Calculated Region of Reversed Flow
At 78% and 82% Speed.

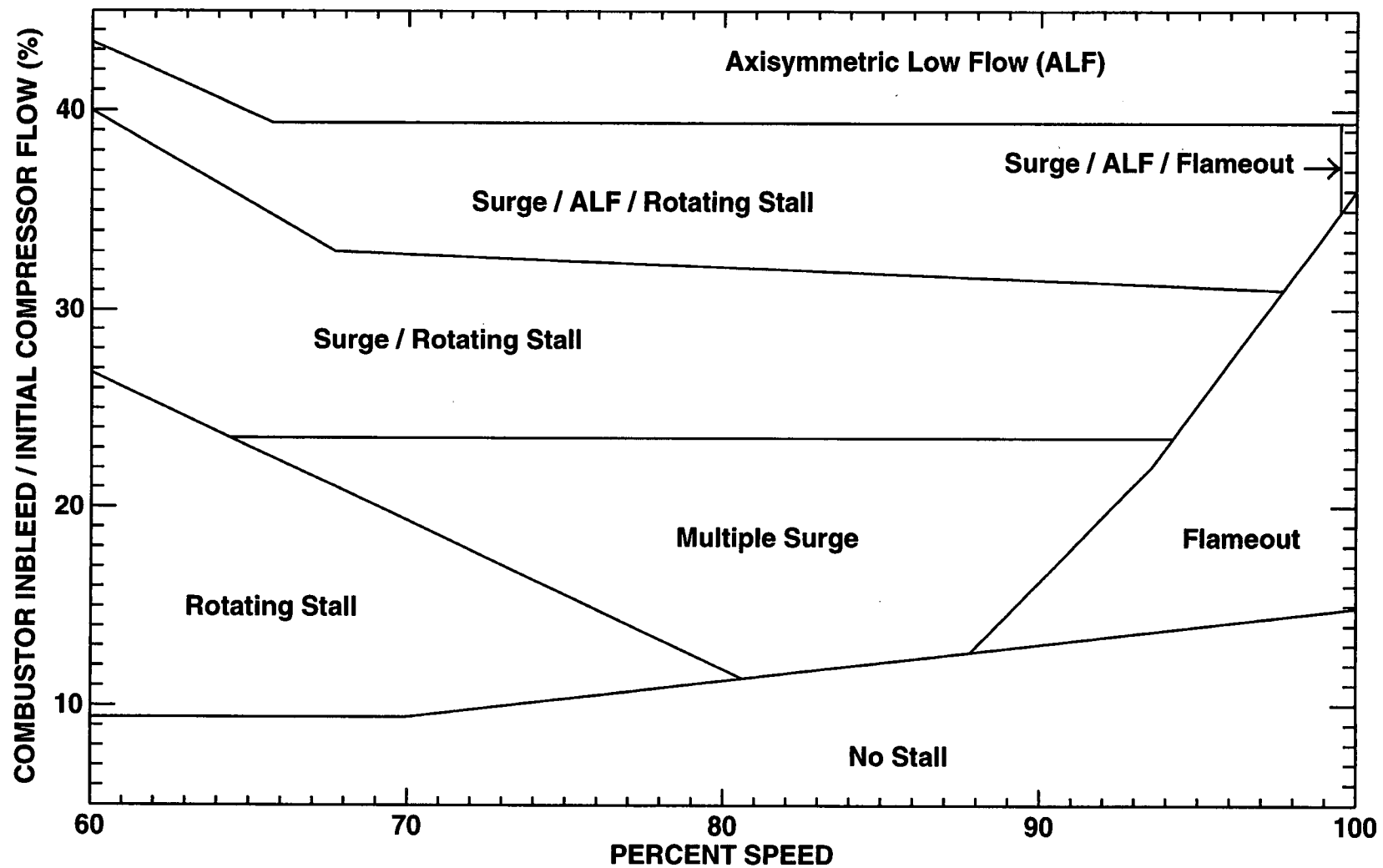


Fig. 4-11 Post Stall Behaviour vs Speed and Combustor inbleed.

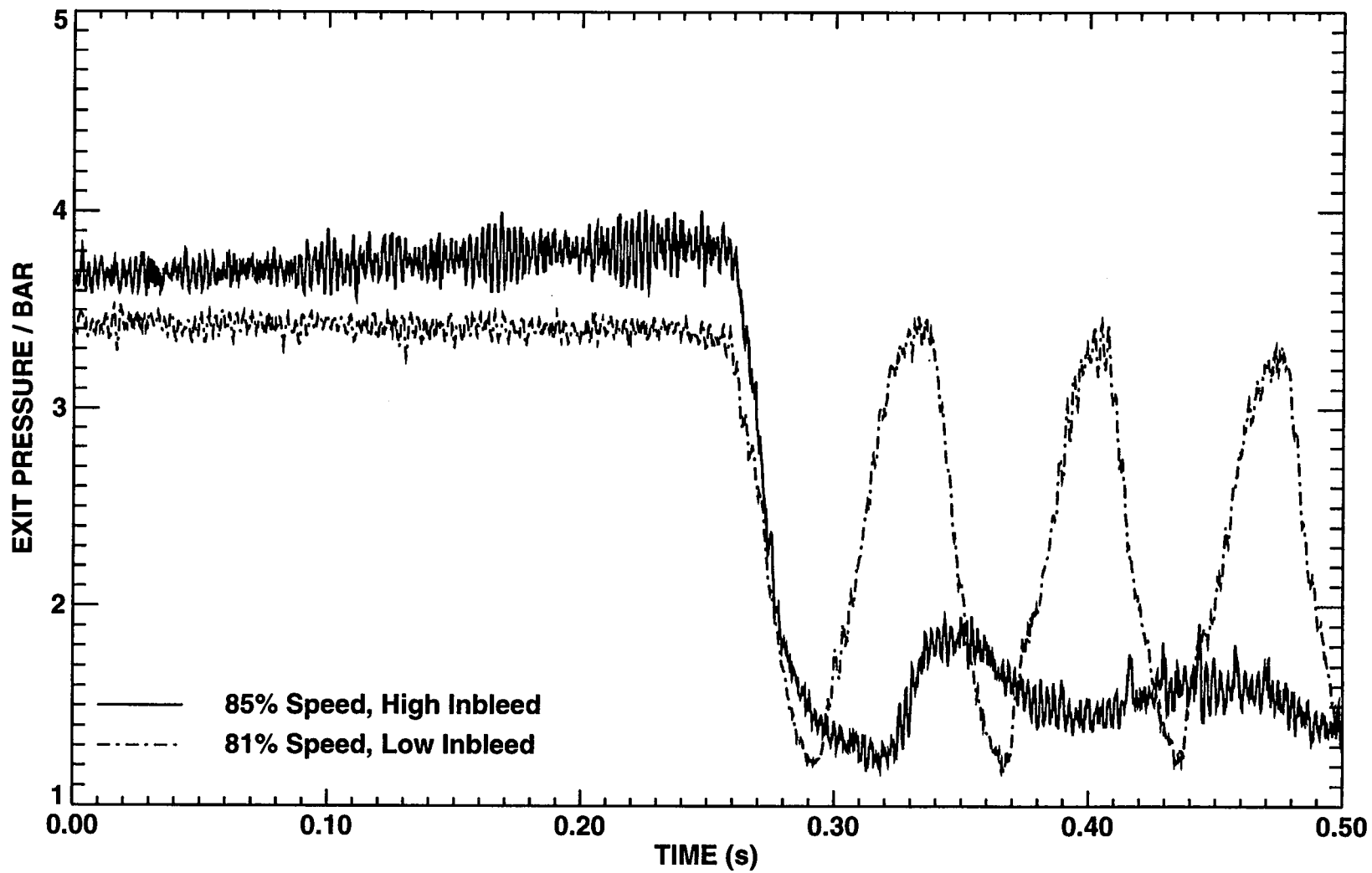
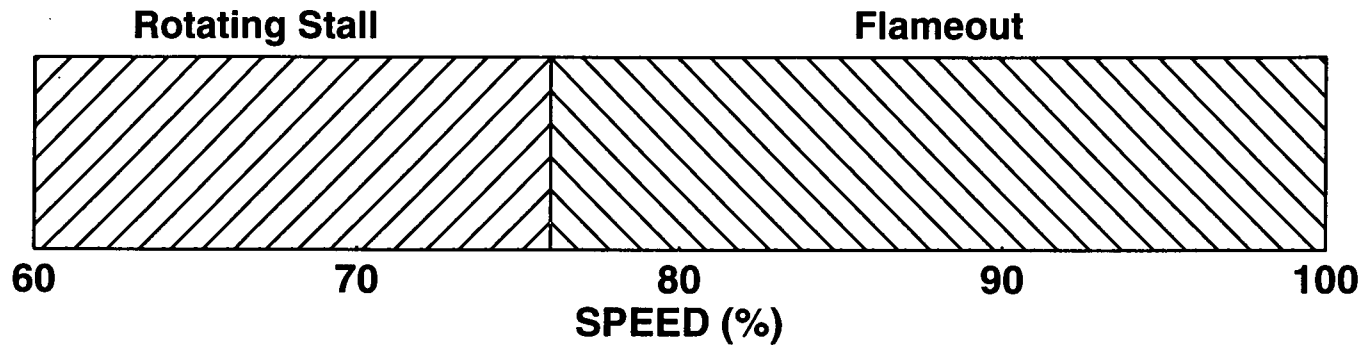


Fig. 4-12 Comparison of Two Measured Events With Different Levels of Combustor inbleed.

a) Without Combustor Inbleed



b) With Combustor Inbleed

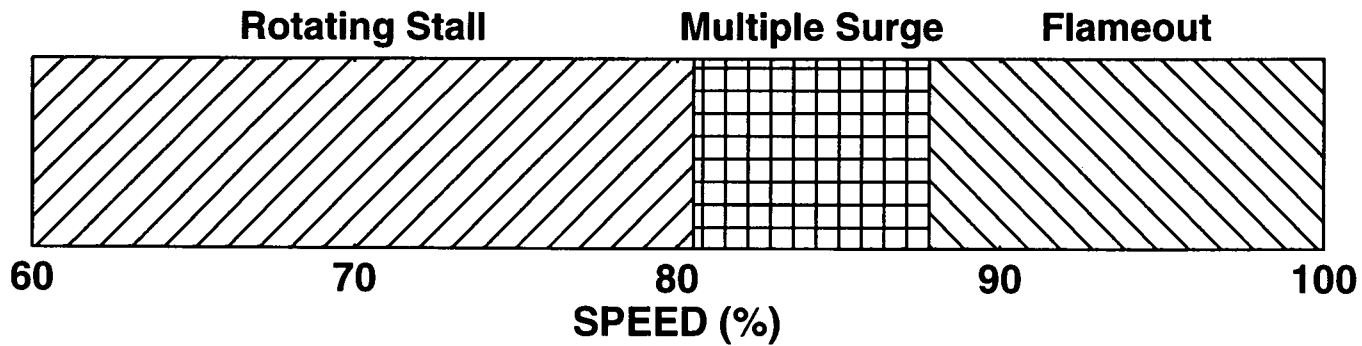


Fig. 4-13 Calculated Post-Stall Behaviour With and Without Combustor inbleed.

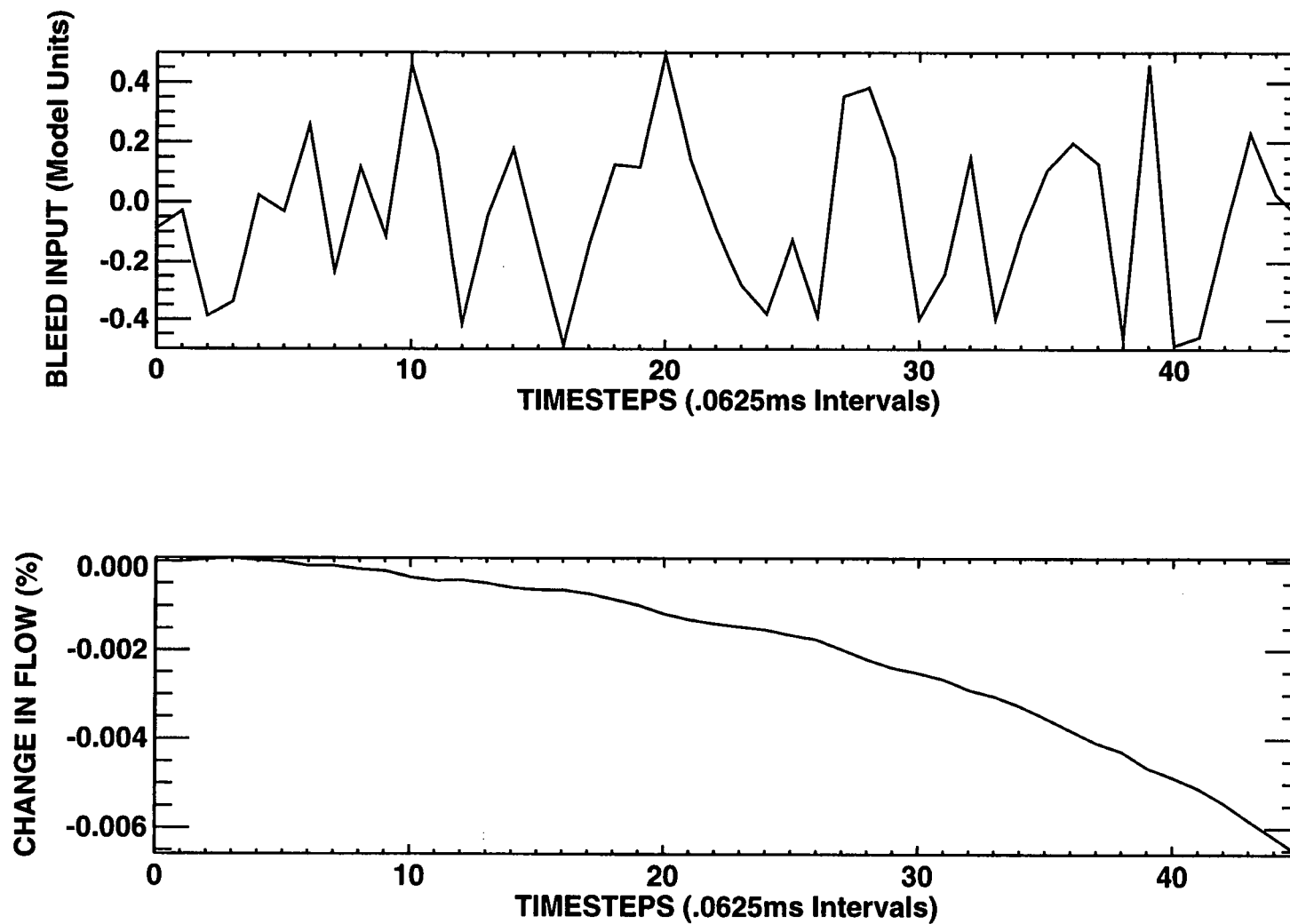


Fig. 4-14 Random Bleed Input and Resulting Flow Output from Model During Axisymmetric System Identification Work.

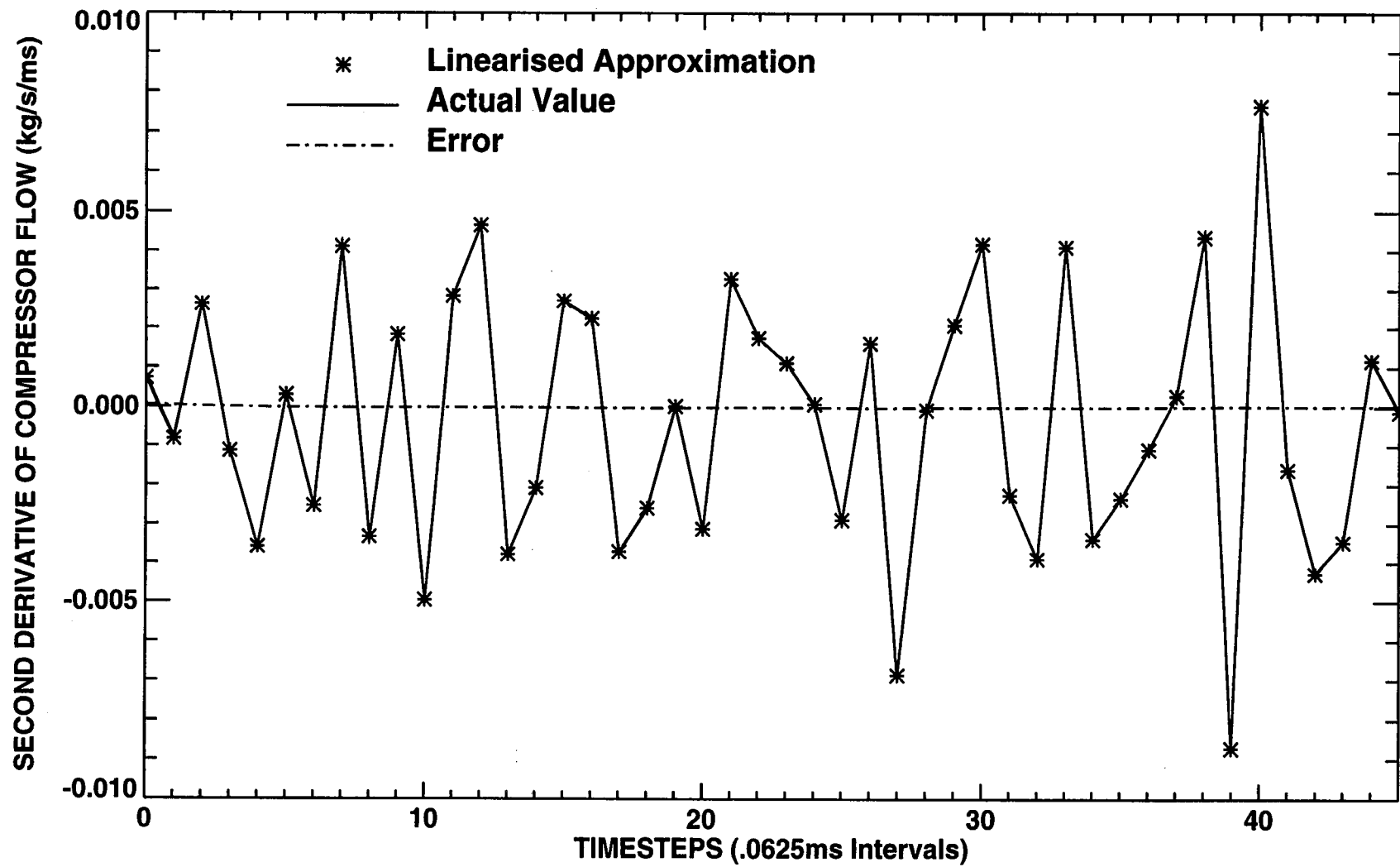


Fig. 4-15 Second Derivative of Compressor Flow vs Linearised Calculation.

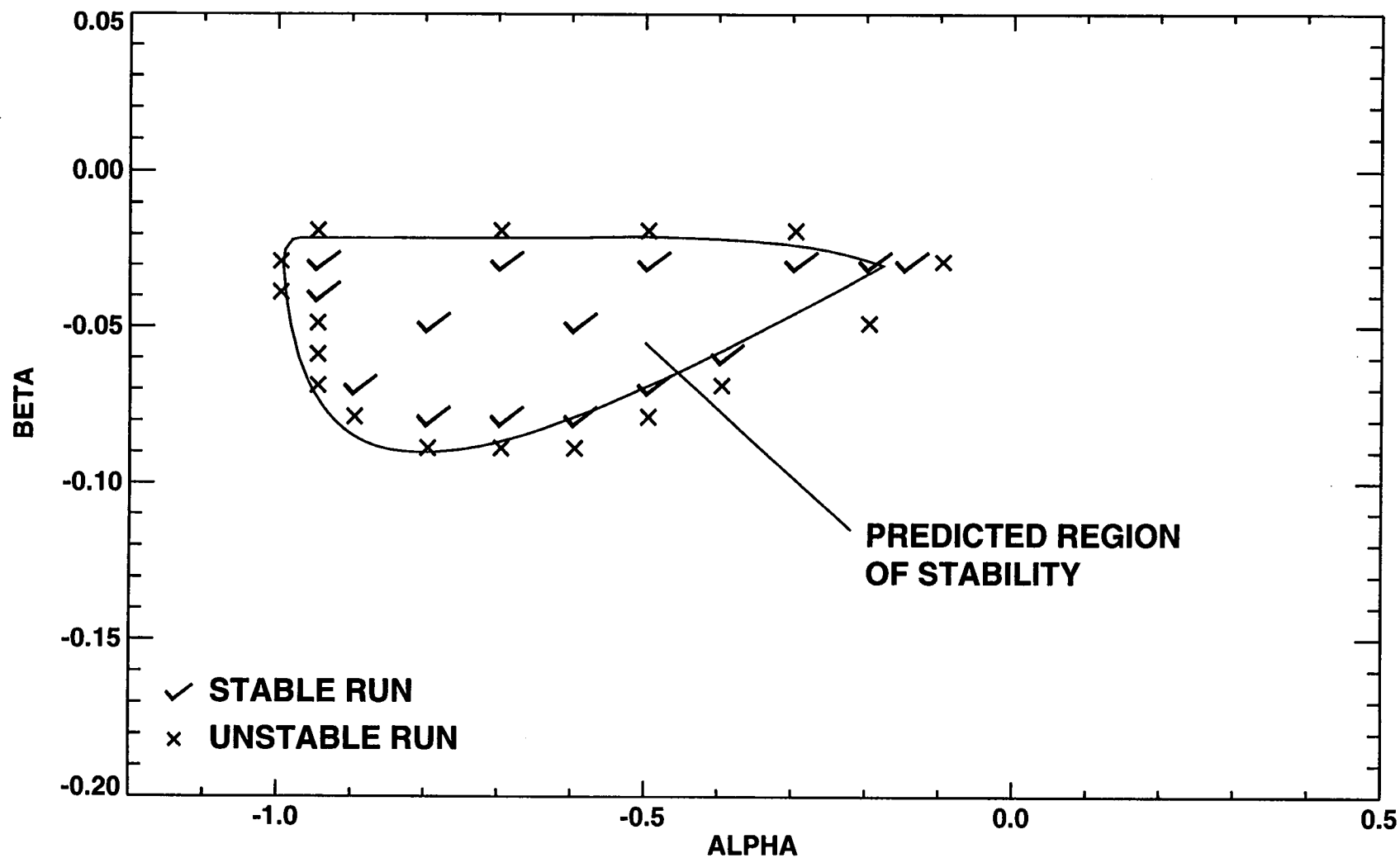


Fig. 4-16 Axisymmetric Flow, Region of Successful Control.

Region predicted using system identification procedure, checked by repeated runs of the model (ticks and crosses).

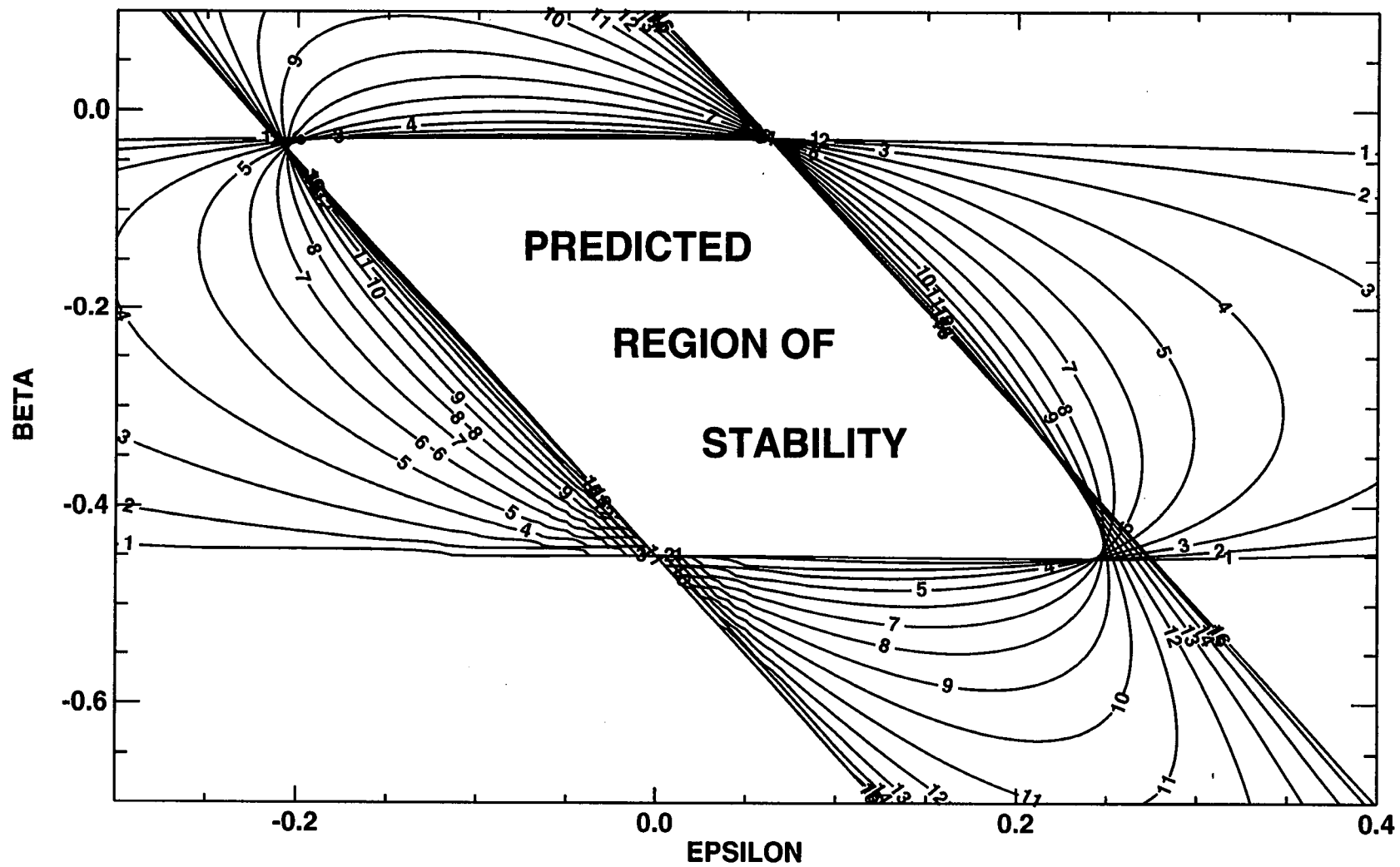


Fig. 4-17 Non-Axisymmetric Flow, Region of Successful Control.

Predicted stability boundaries are shown for circumferential harmonics 1 to 16.

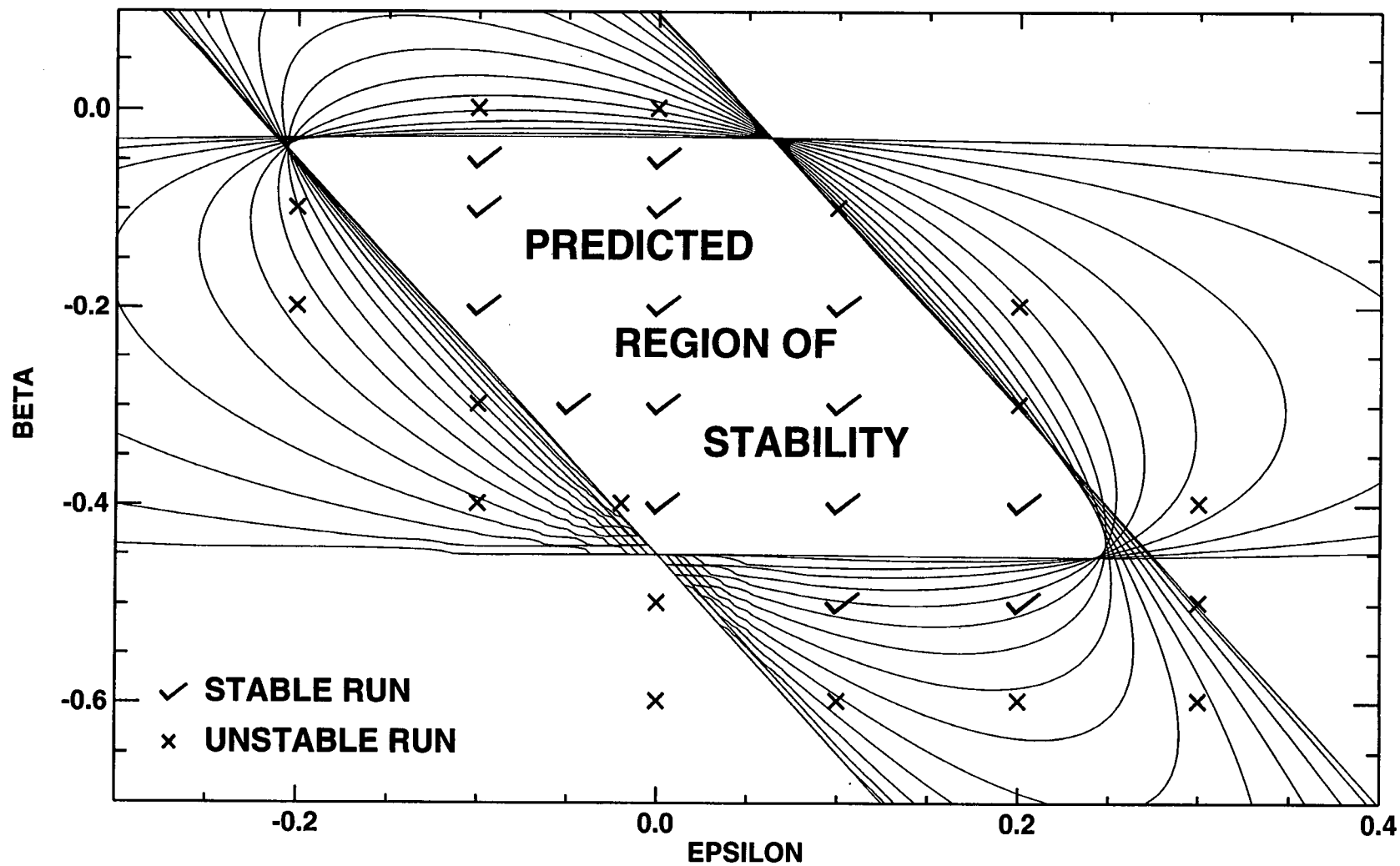


Fig. 4-18 Non-Axisymmetric Flow, Model Results as a Check on the Predicted Region of Successful Control.

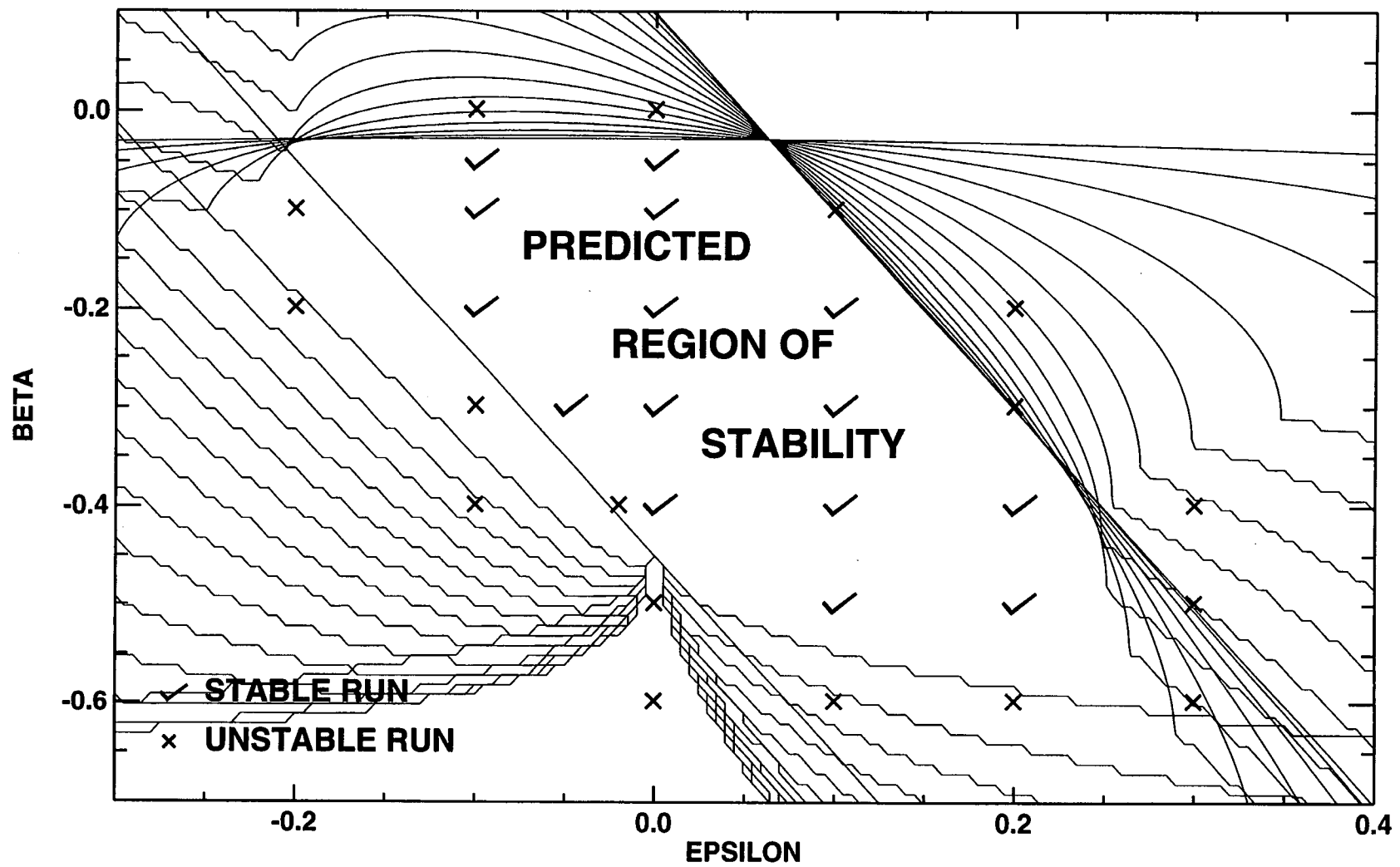
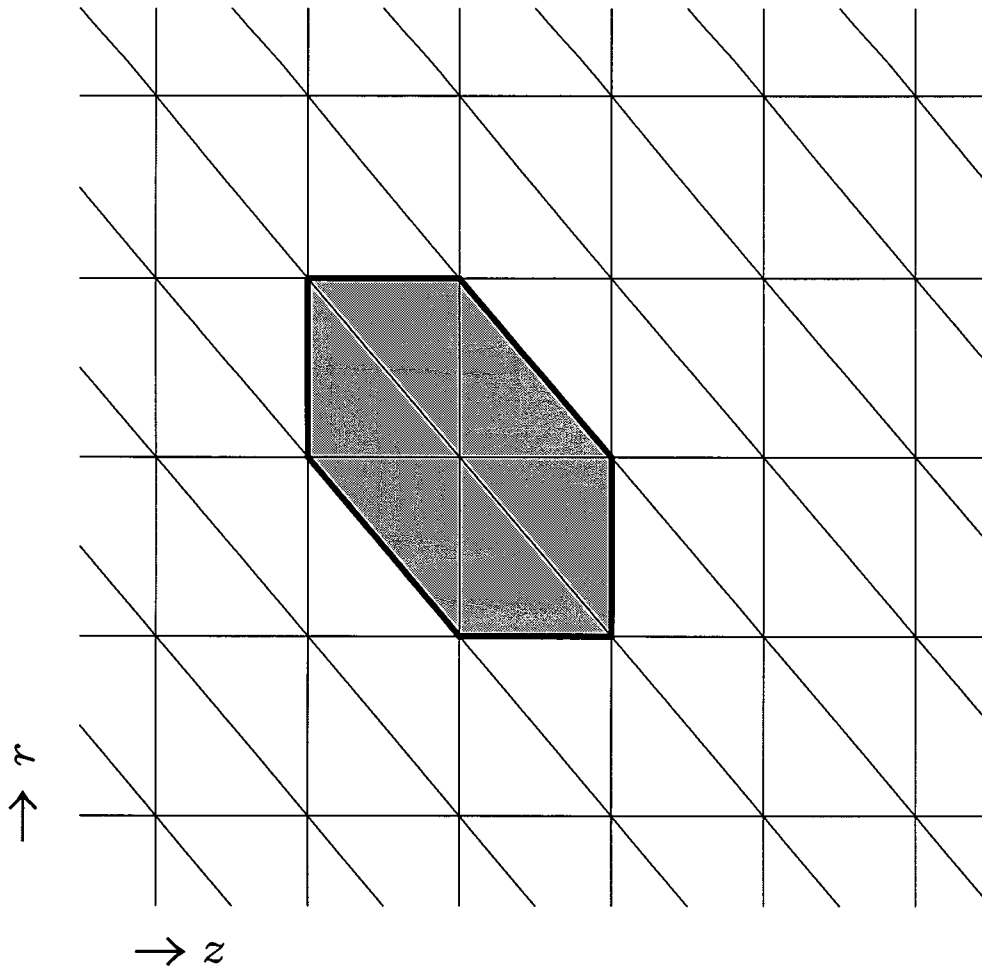
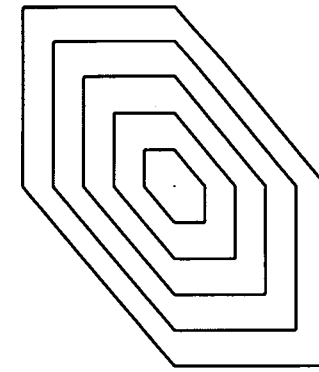


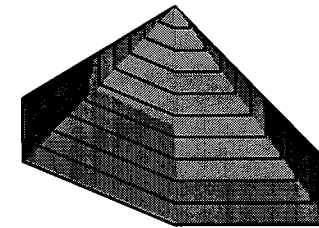
Fig. 4-19 Non-Axisymmetric Flow, Region of Successful Control with High Frequencies Discounted.



HEXAGONAL ELEMENT / TRIANGULAR CELLS



Contours (0.0 to 1.0 by 0.2)



Surface

WEIGHTING FUNCTION S

Fig. 5-1 Finite Element Grid: Hexagonal Elements, Triangular Cells and Weighting Function.

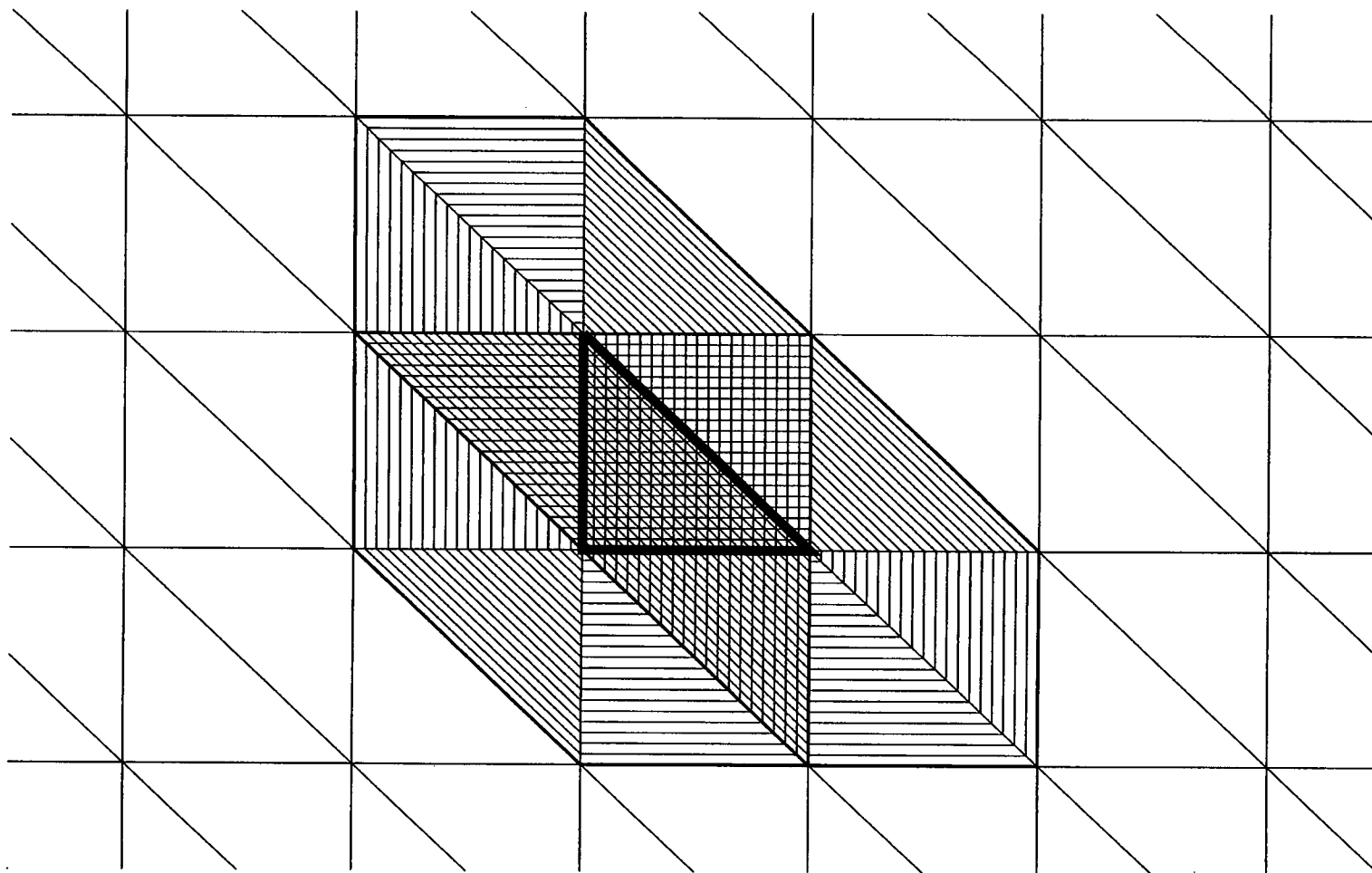


Fig. 5-2 Contribution of Triangular Cell to Three Hexagonal Elements.

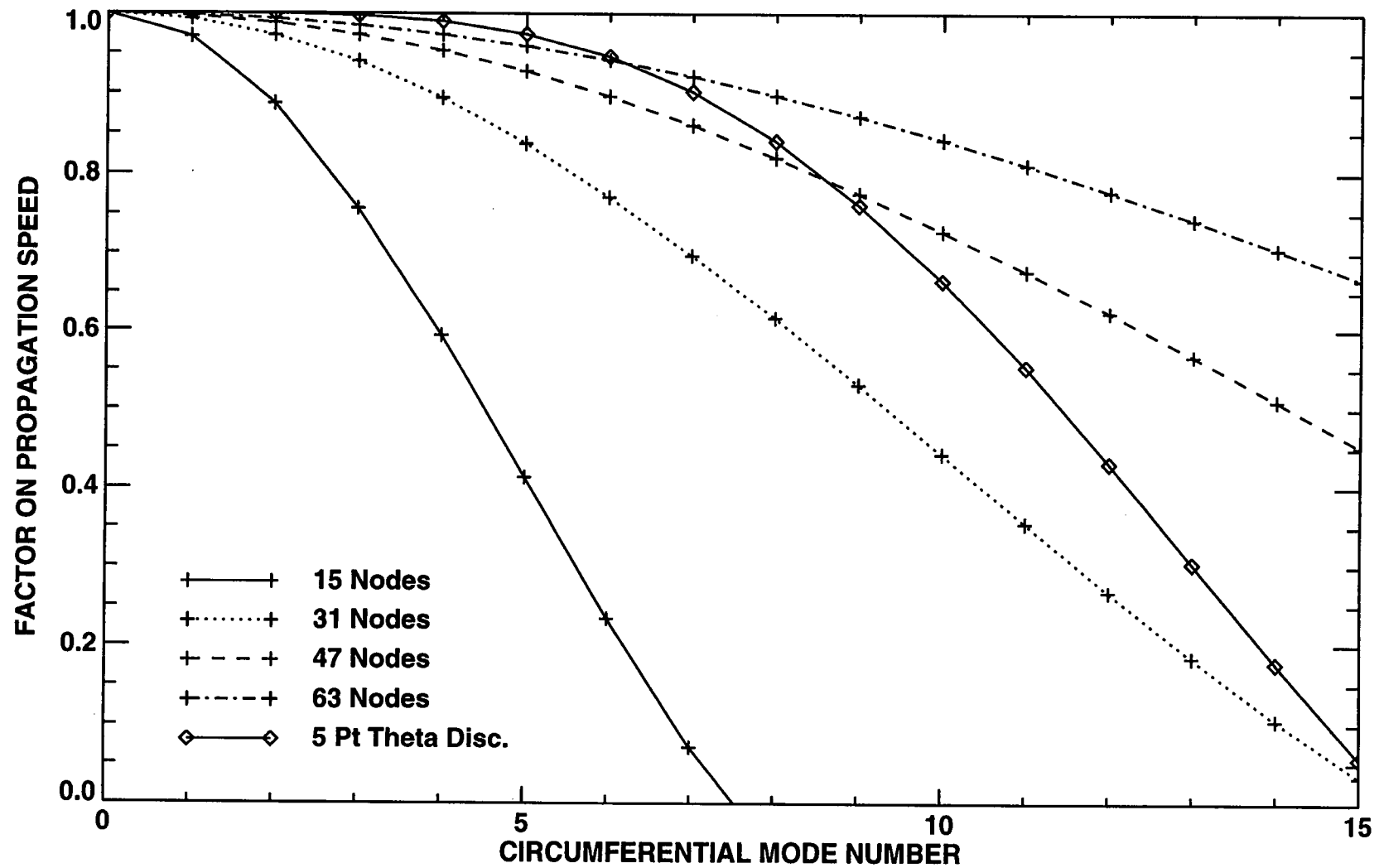


Fig. 5-3 Circumferential Propagation Speed Factors for Central Difference Scheme (various numbers of nodes) and Five Point Discretisation Method (31 nodes).

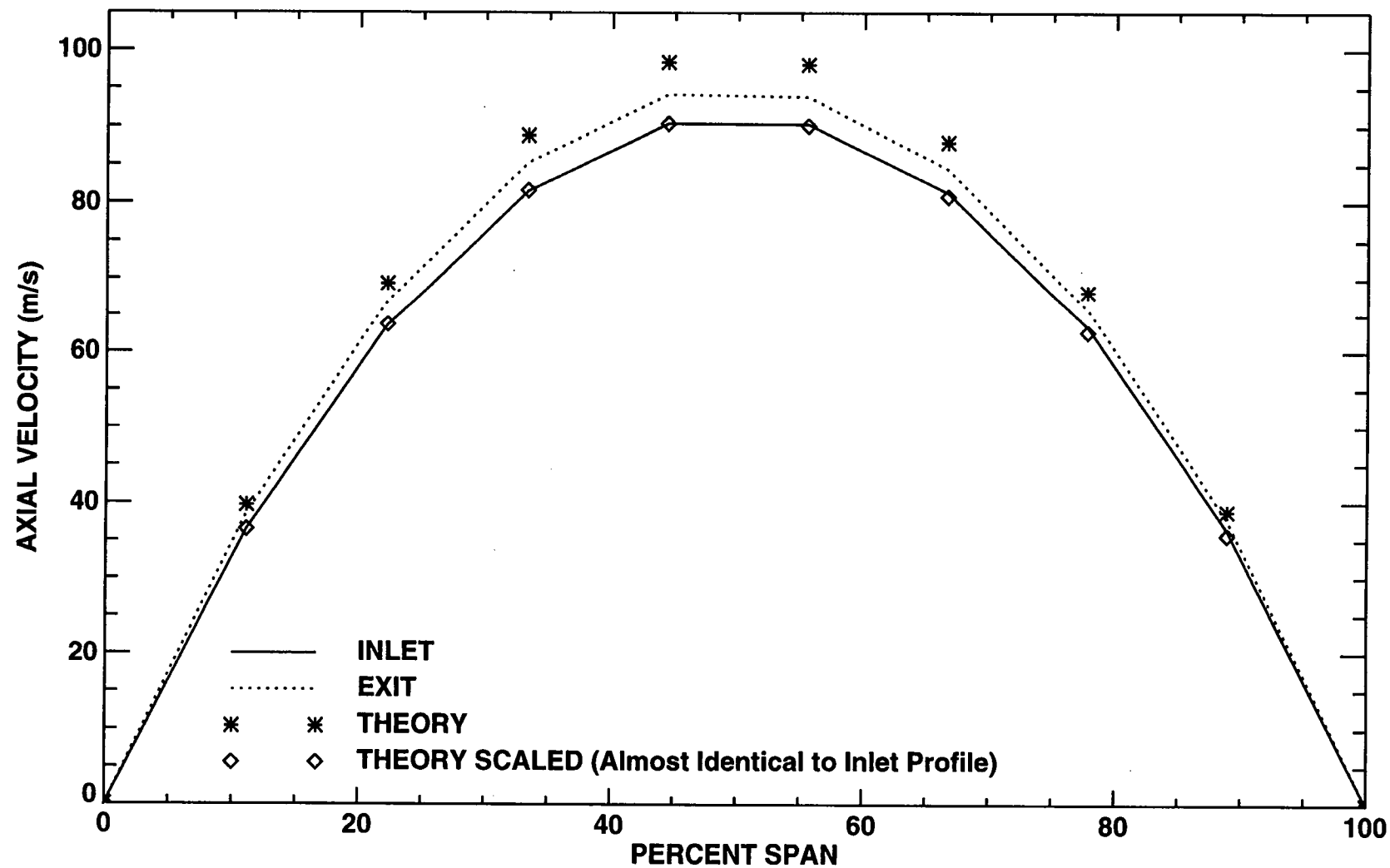


Fig. 5-4 Hagen-Poiseuille Type Flow in an Annular Duct: Radial Profile of Axial Velocity.

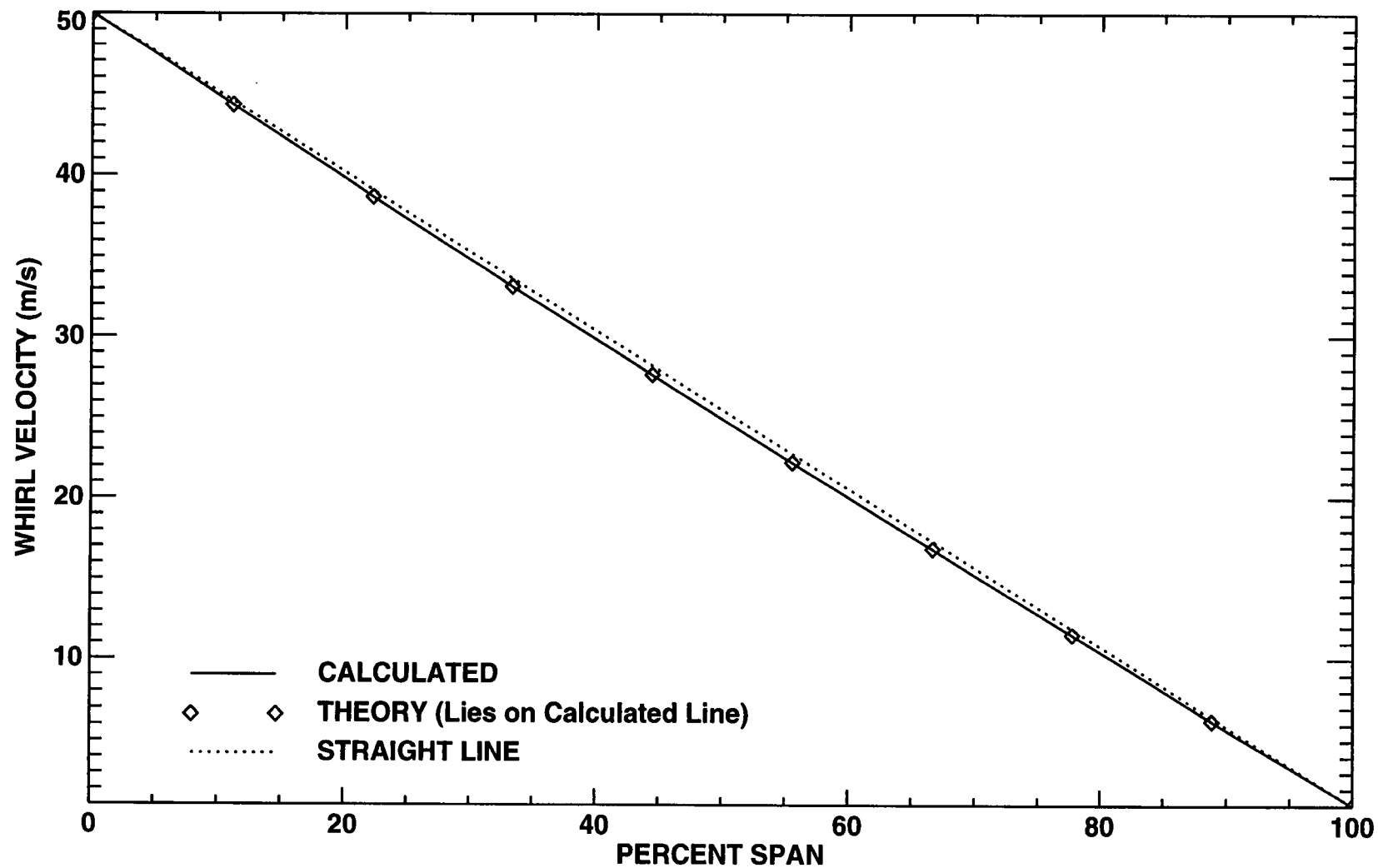


Fig. 5-5 Flow Between Rotating Cylinders: Radial Profile of Whirl Velocity.

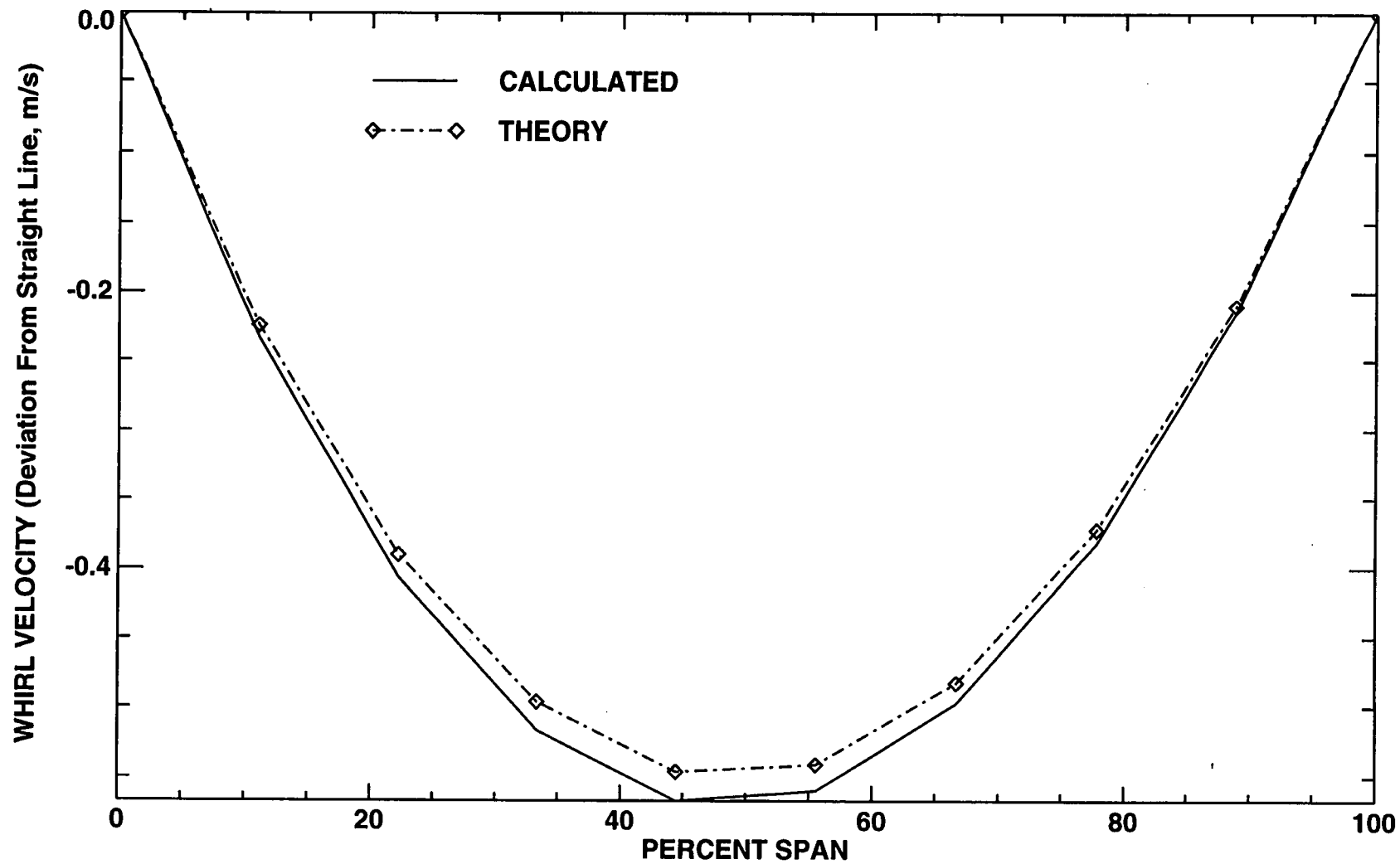


Fig. 5-6 Flow Between Rotating Cylinders: Deviation of Whirl Velocity From Straight Line.

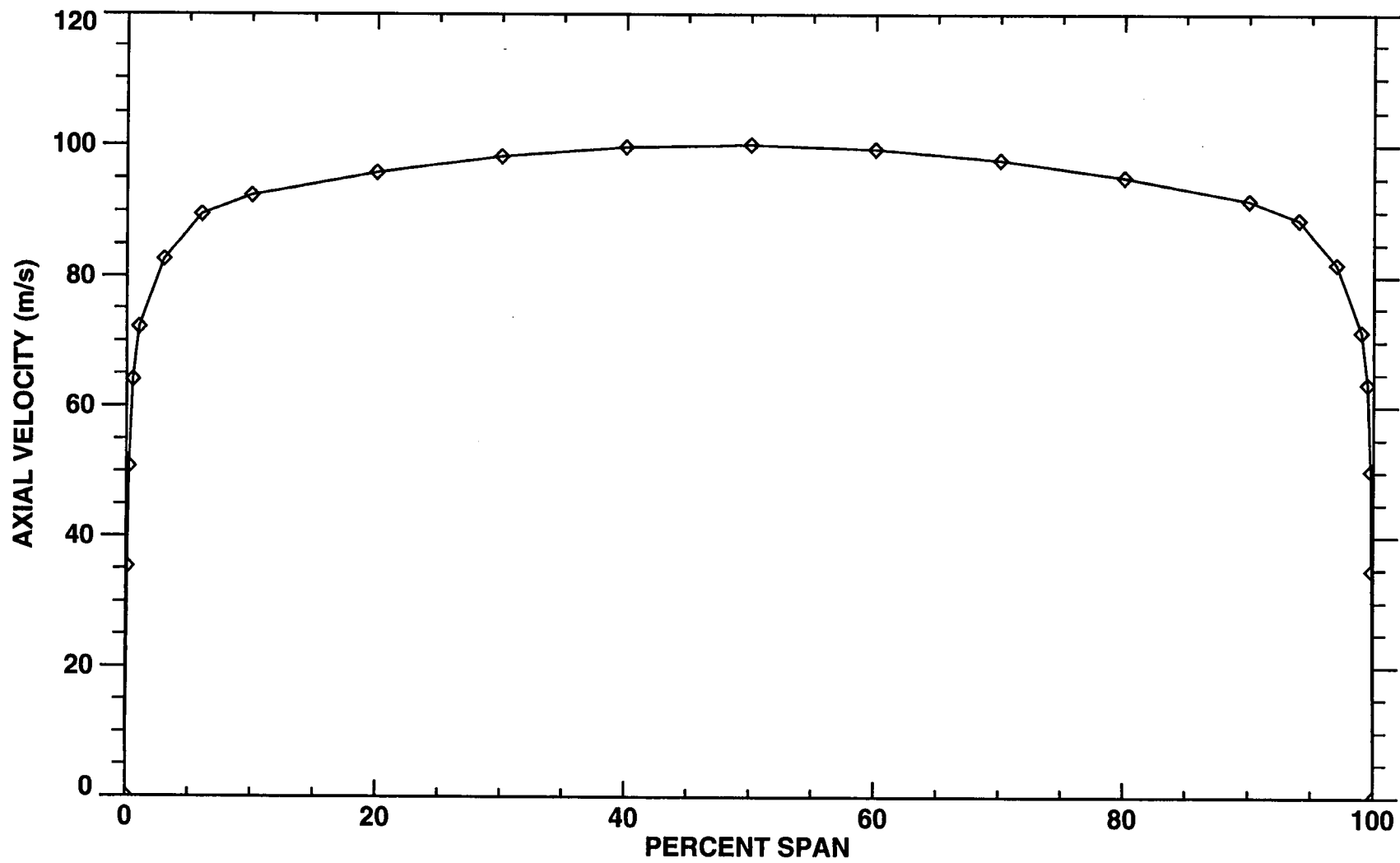


Fig. 5-7 Fully Developed Turbulent Boundary Layer: Radial Profile of Axial Velocity.

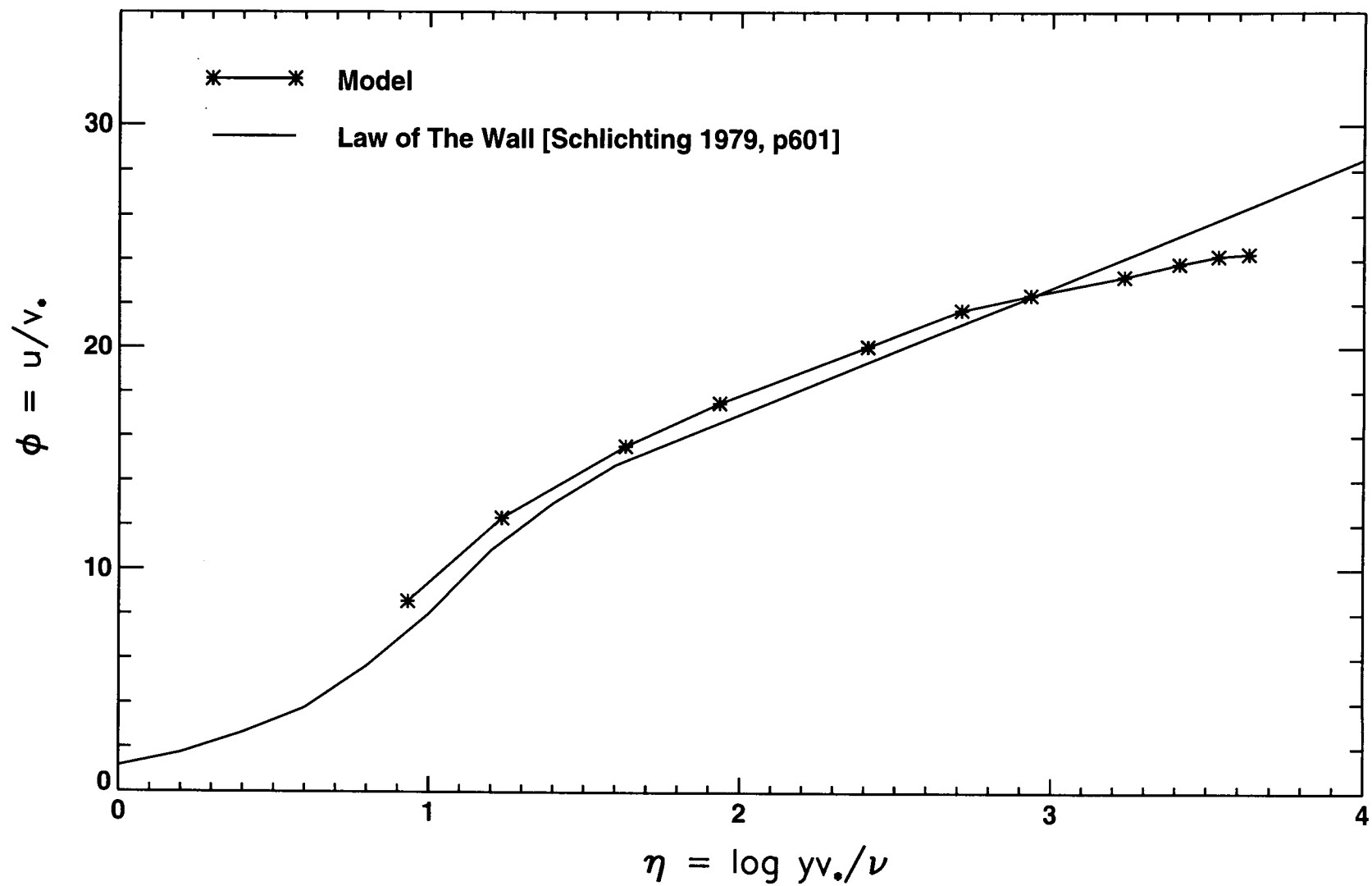


Fig. 5-8 Fully Developed Turbulent Boundary Layer: Radial Profile of Axial Velocity in Non-Dimensional Form.

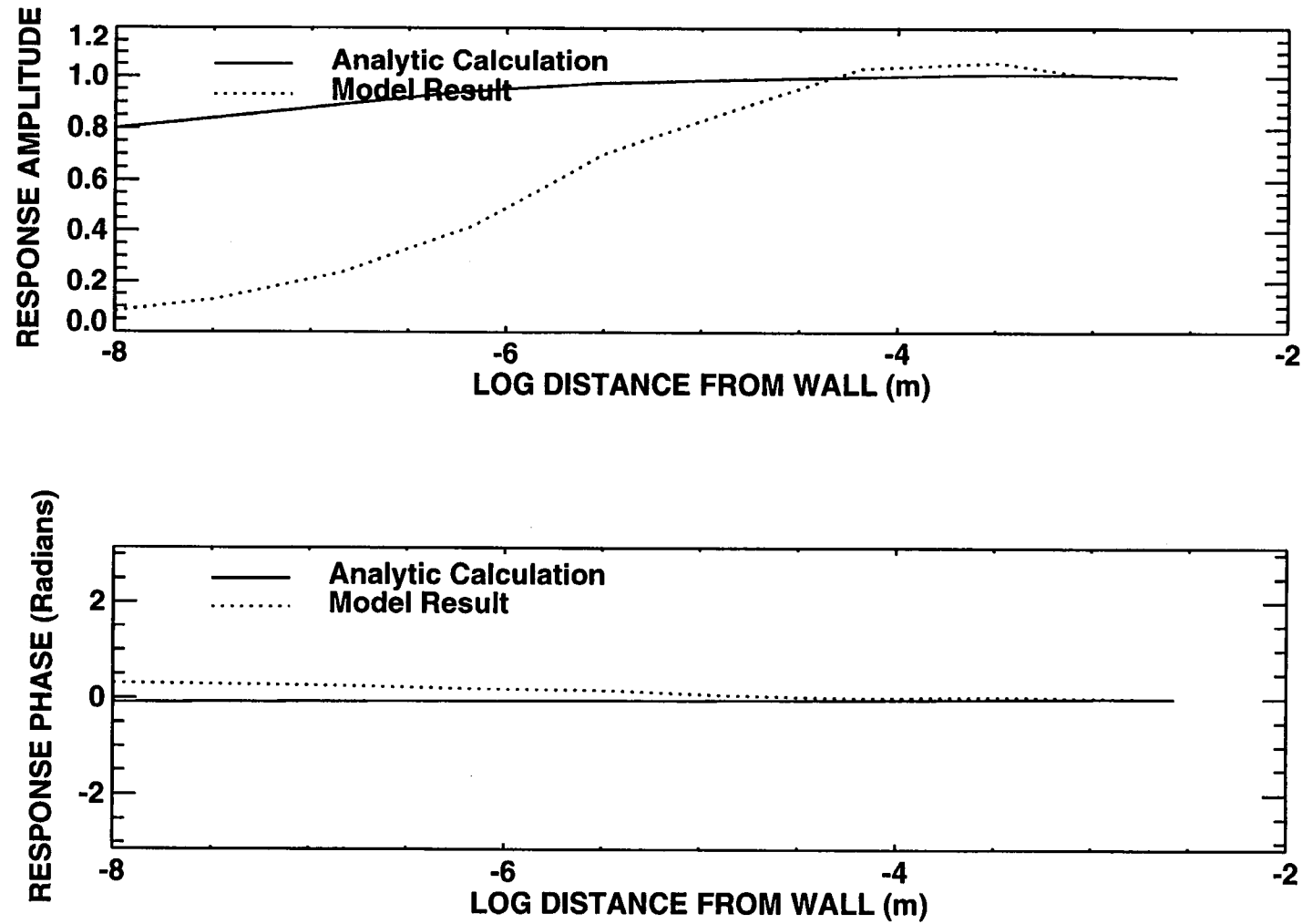


Fig. 5-9 Oscillating Boundary Layer: Near Wall Response to Unit Freestream Velocity Fluctuation.

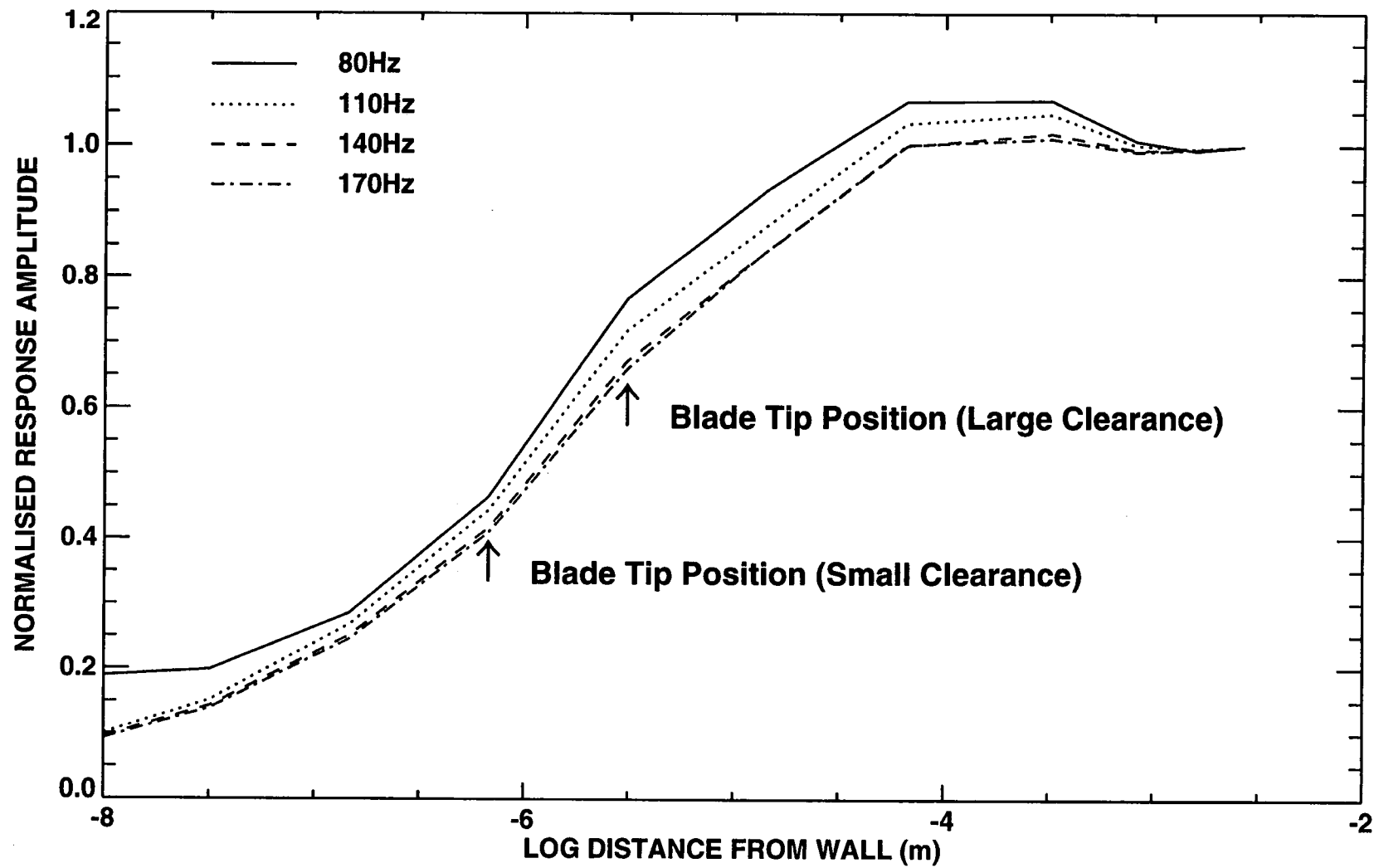


Fig. 5-10 Oscillating Boundary Layer: Model Response Normalised By Analytic Response.

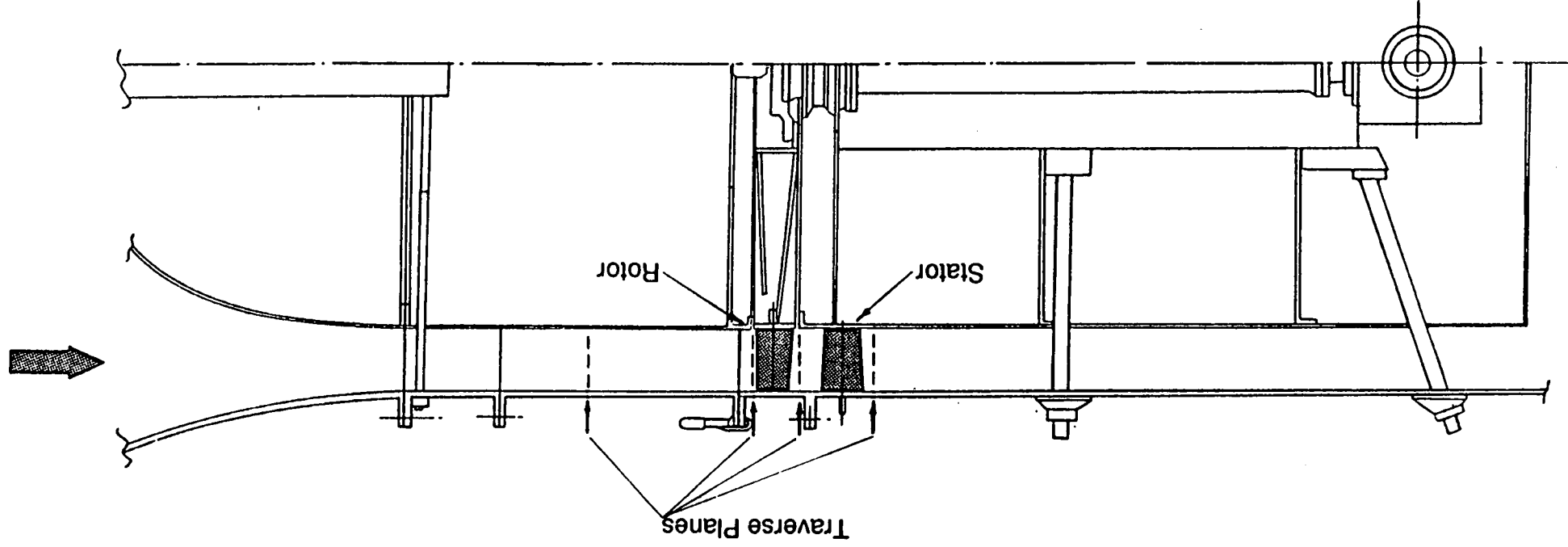


Fig. 5-11 Deverson Single Stage Low Speed Rig: Layout.

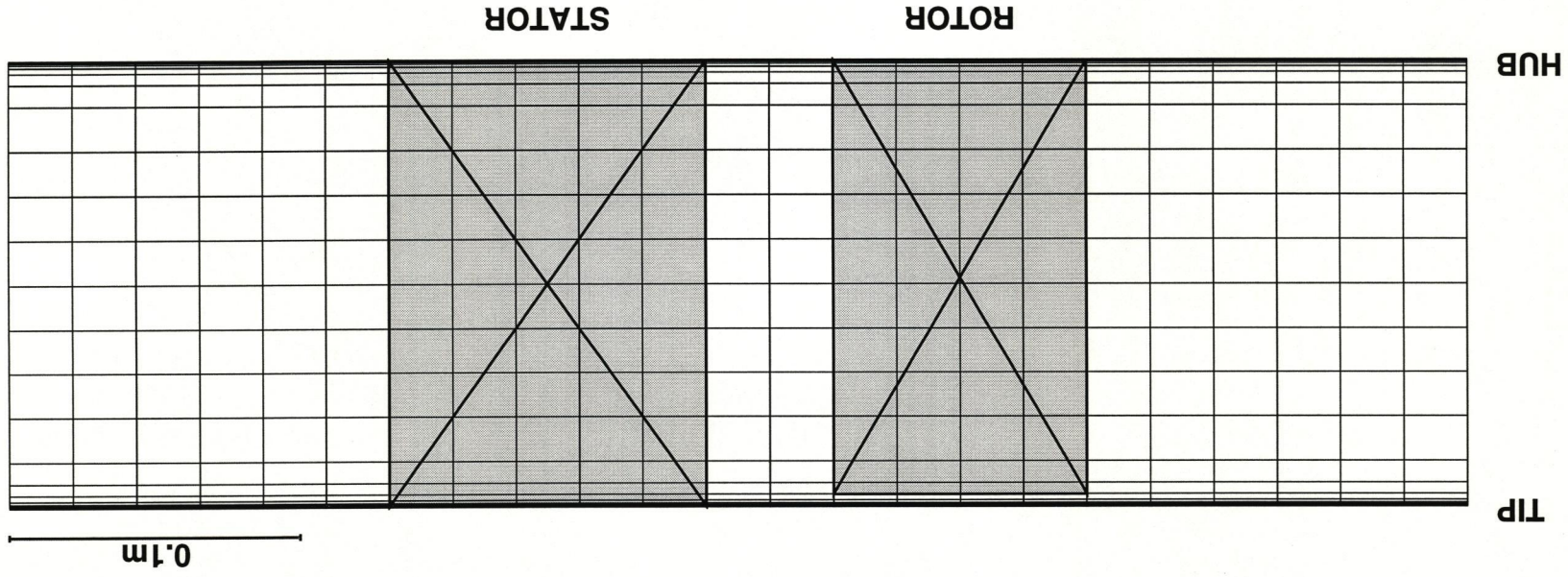


Fig. 5-12 Deverson Rig: Computational Mesh (approximately to scale).

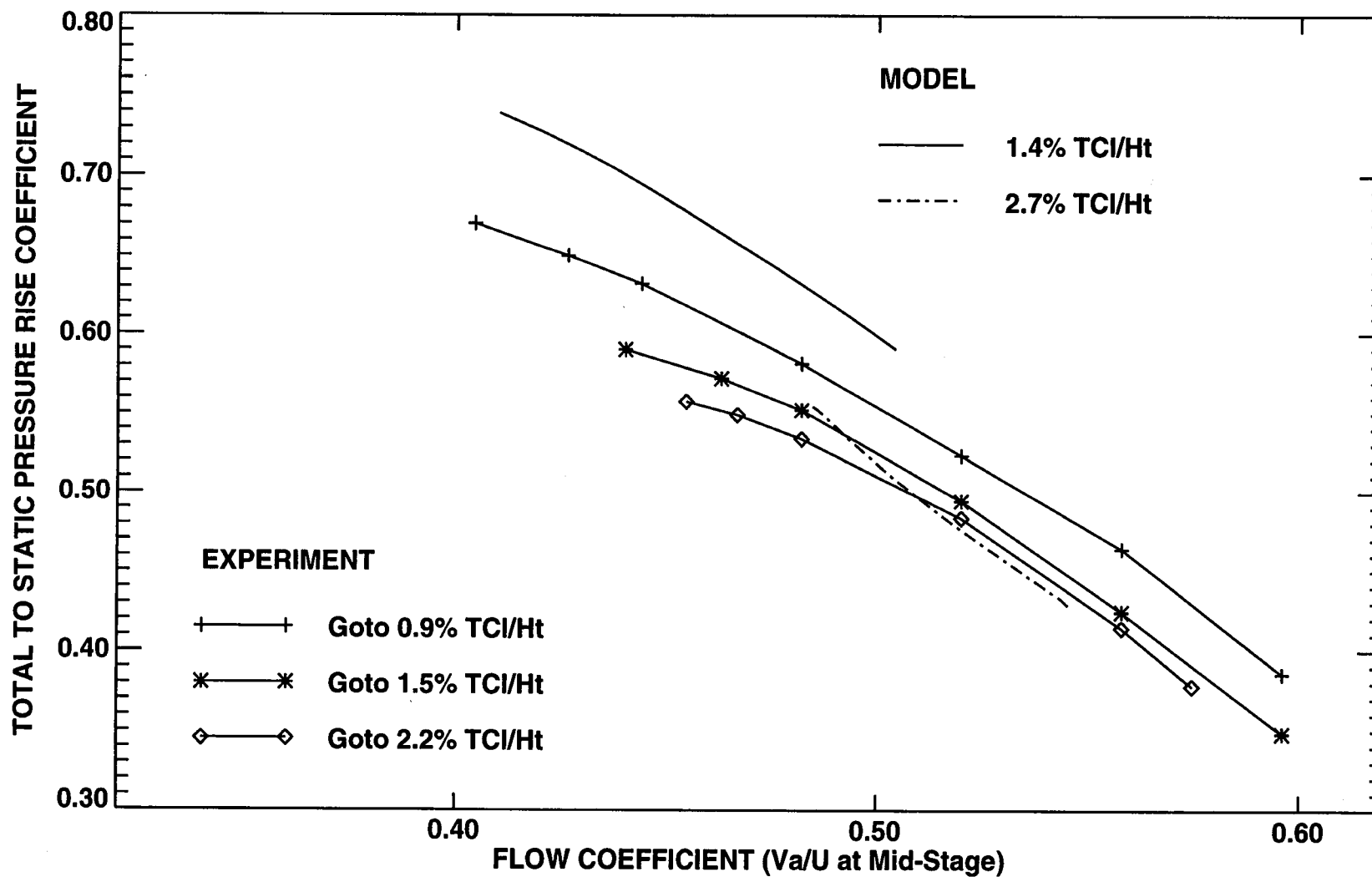


Fig. 5-13 Deverson Rig: Predicted vs Measured [Goto, 1992] Overall Pressure Rise Characteristics. 5-13

The lowest flow point on each line represents the stall point.

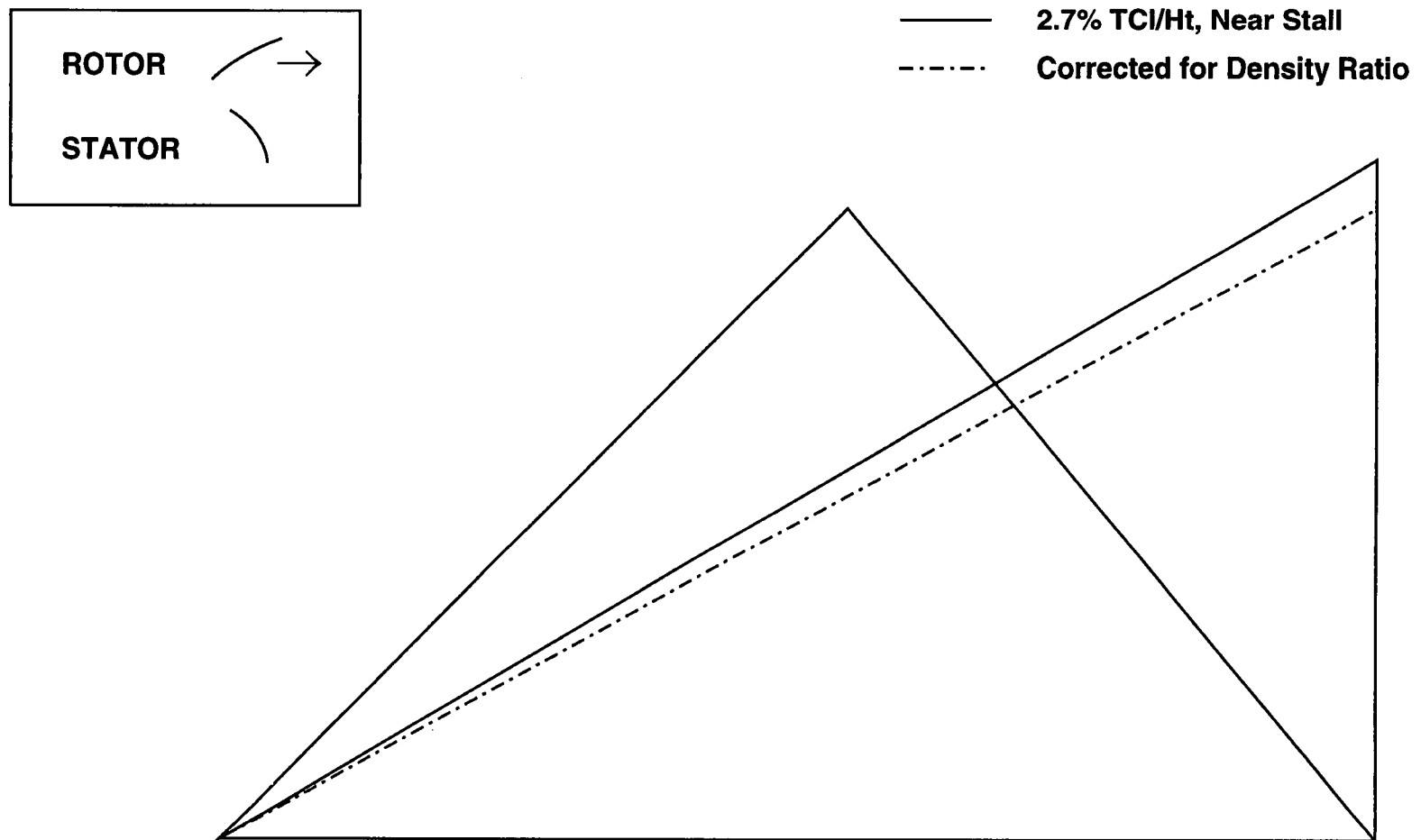


Fig. 5-14 Deverson Rig: Velocity Triangles With and Without Accounting for the Density Ratio.

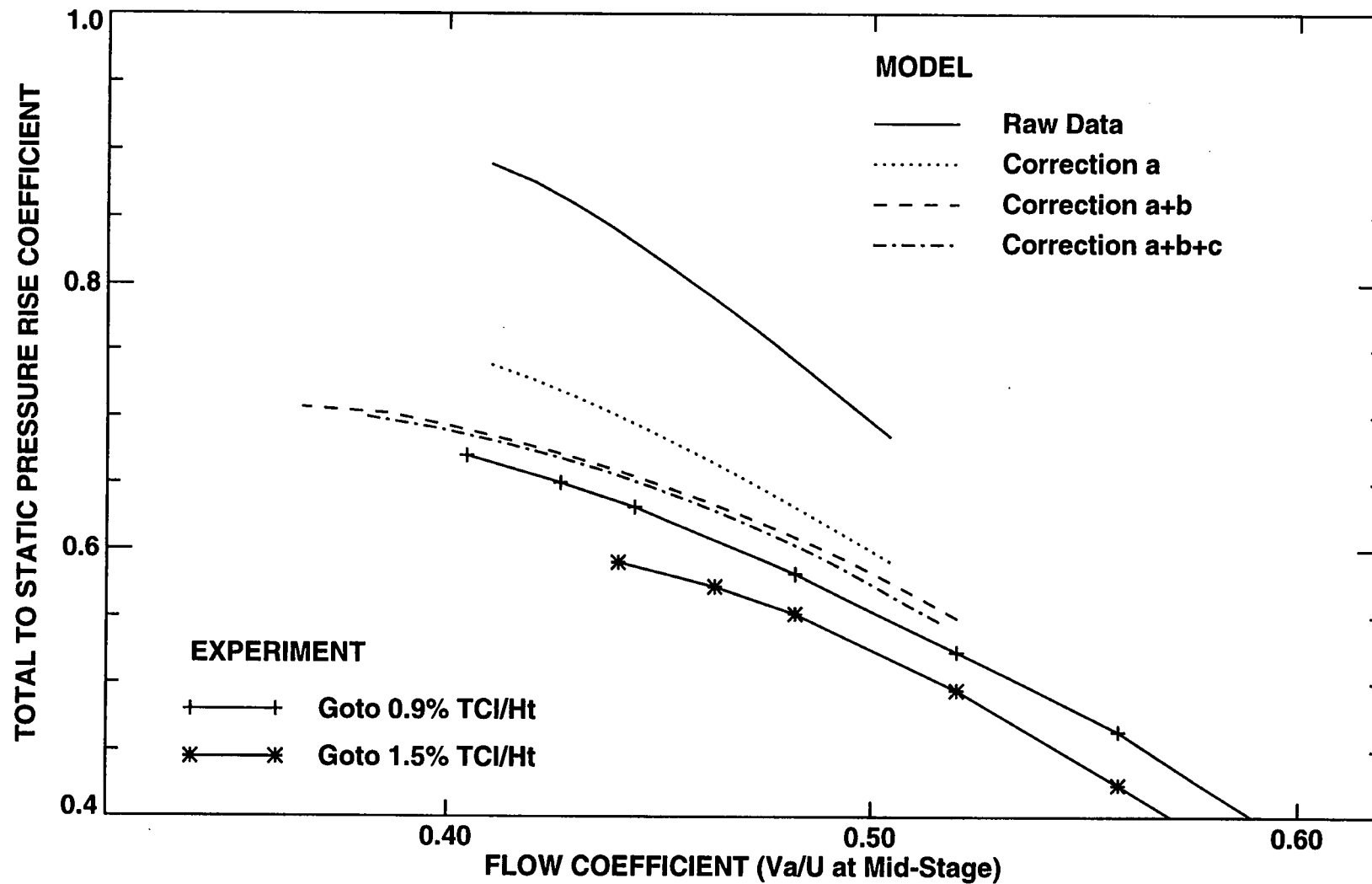


Fig. 5-15 Deverson Rig: Overall Pressure Rise Characteristics At Low Tip Clearance.

Corrected for a) Measured efficiency profile, b) Deviations calculated from inlet velocity accounting for density ratio across rotor, c) unloaded stator configuration.

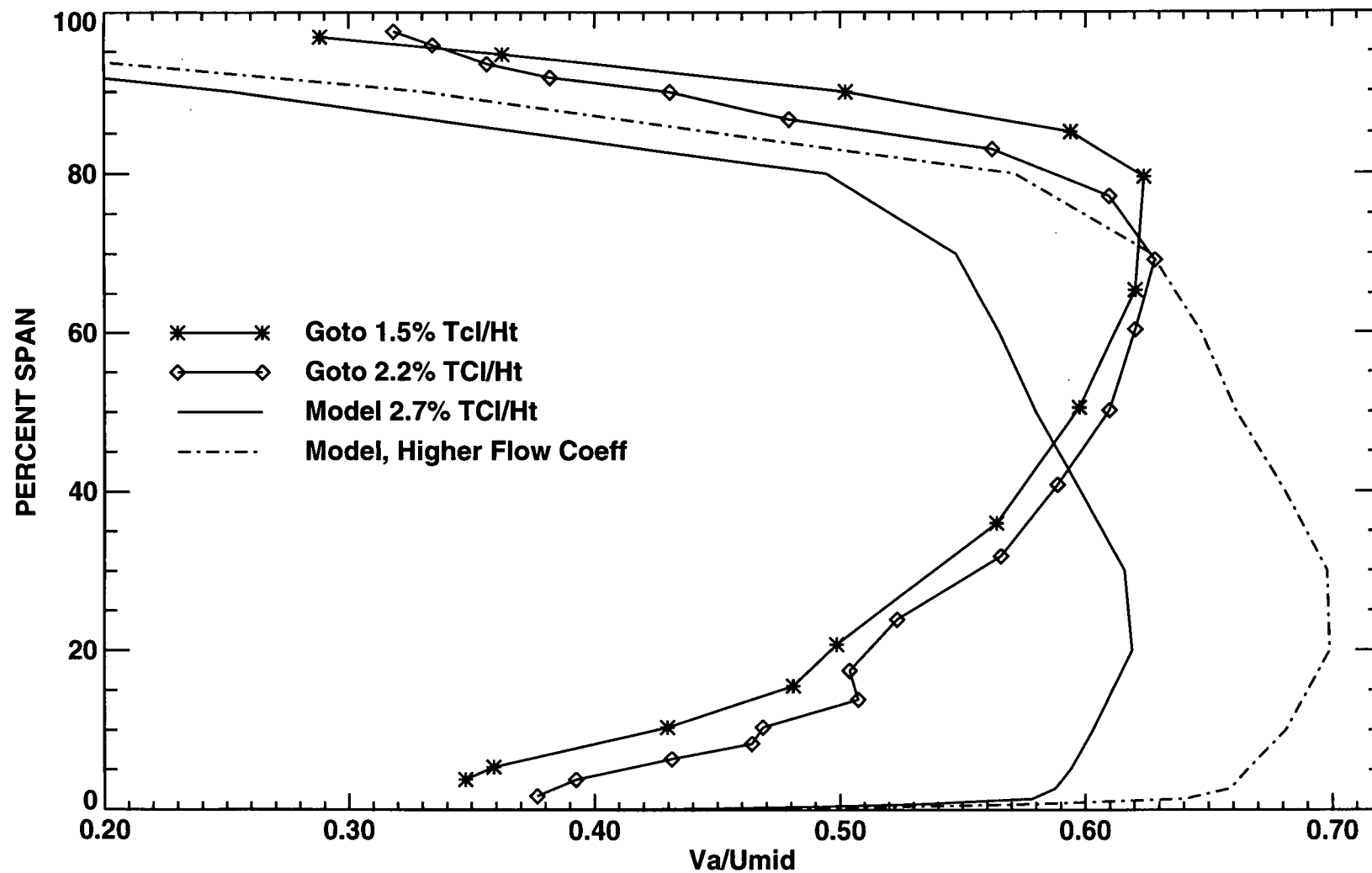


Fig. 5-16 Deverson Rig: Comparison of Predicted and Measured [Goto, 1992] Axial Velocity Profiles At Rotor Exit.

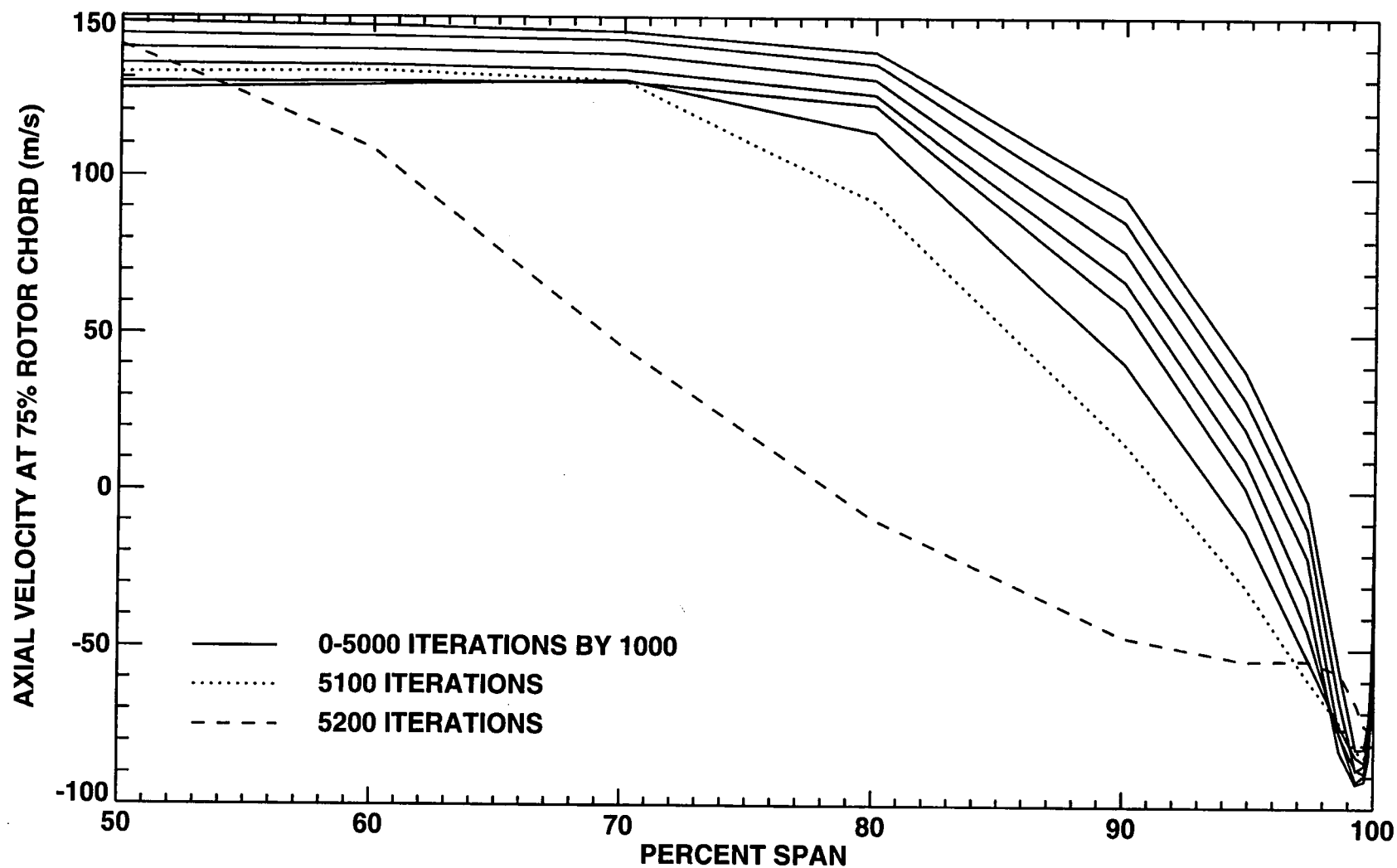


Fig. 5-17 Deverson Rig: Predicted Axisymmetric Stall inception Behaviour.

The first six (solid) lines represent times 1000 iterations apart, showing successively lower axial velocities. The dotted and dashed lines represent times 100 and 200 iterations respectively after the last solid line.

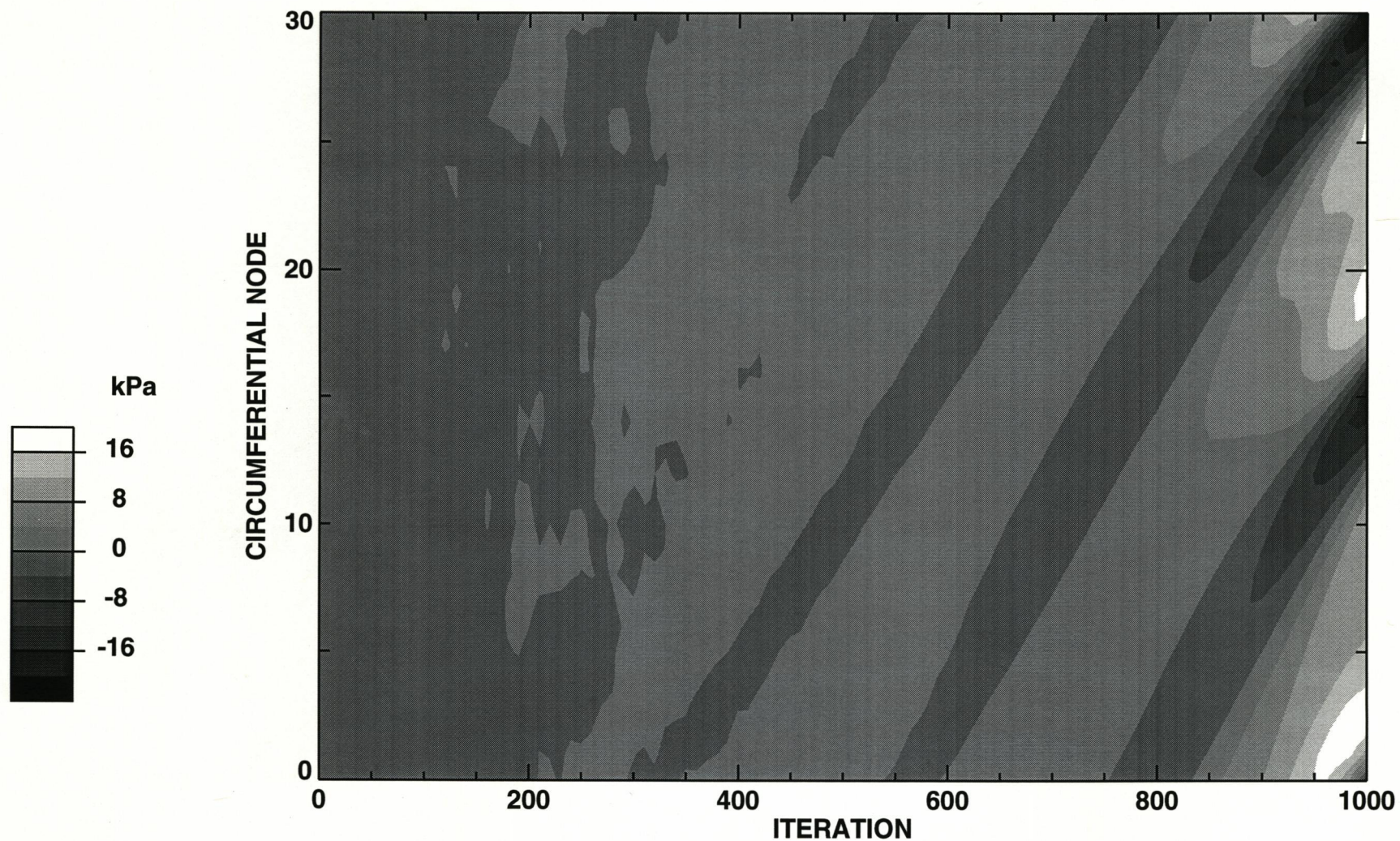


Fig. 5-18 Deverson Rig: Predicted Casing Static Pressures At Rotor inlet During Stall Inception.

268 iterations is equivalent to 1 rotor revolution or 20.1ms

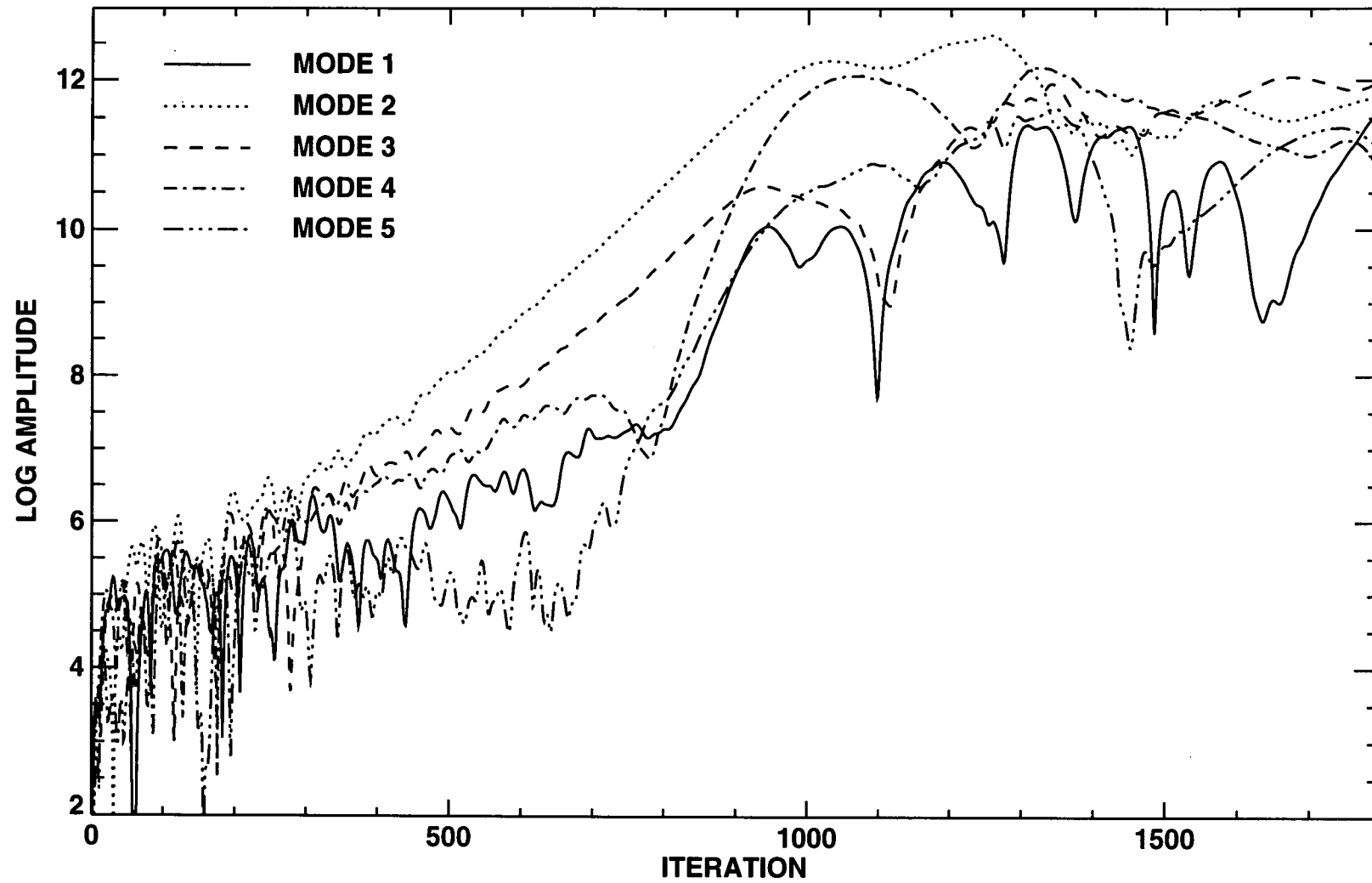


Fig. 5-19 Deverson Rig: Harmonic Content of Casing Static Pressures At Rotor inlet (Amplitude).

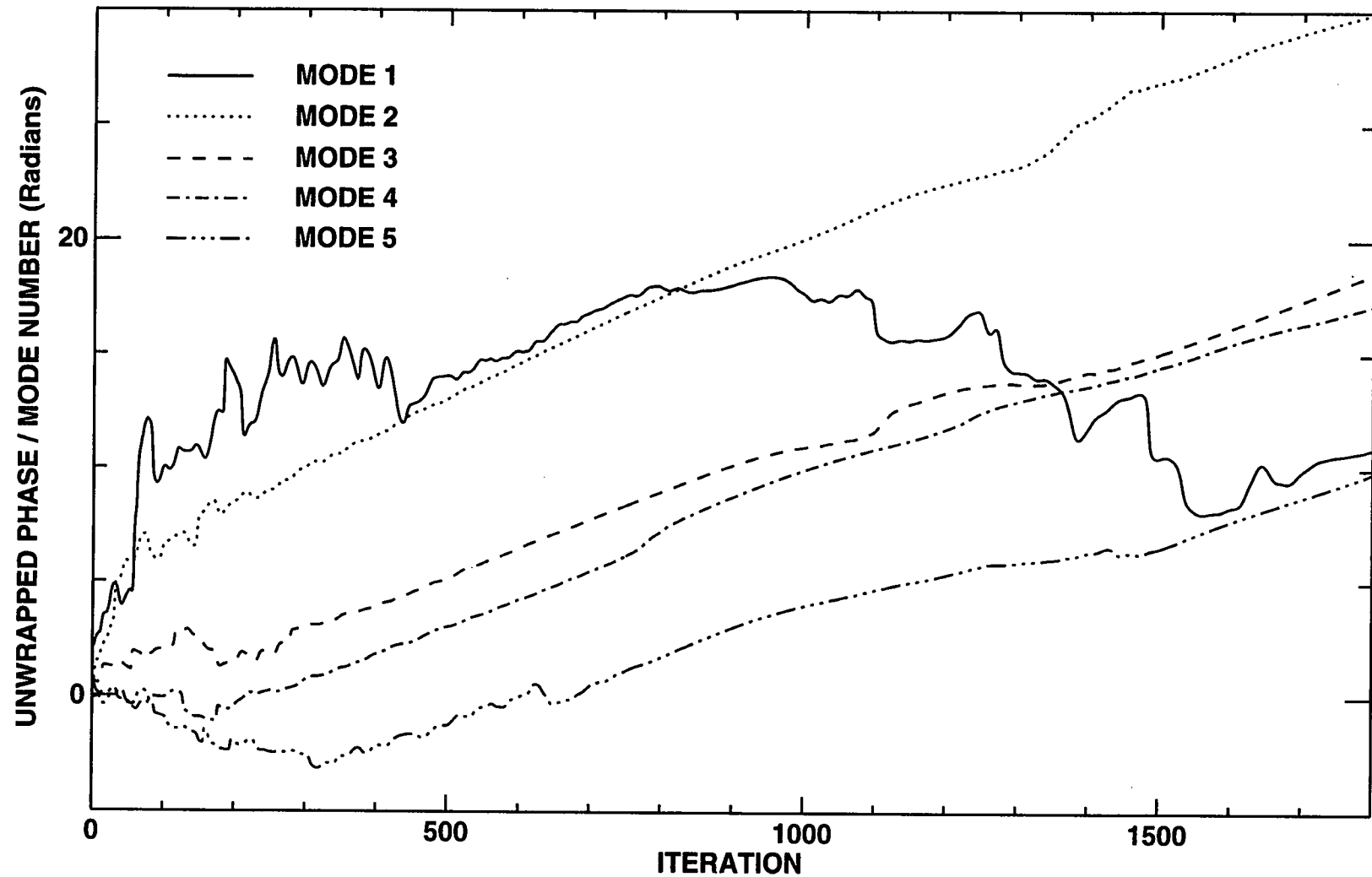


Fig. 5-20 Deverson Rig: Harmonic Content of Casing Static Pressures At Rotor inlet (Phase).

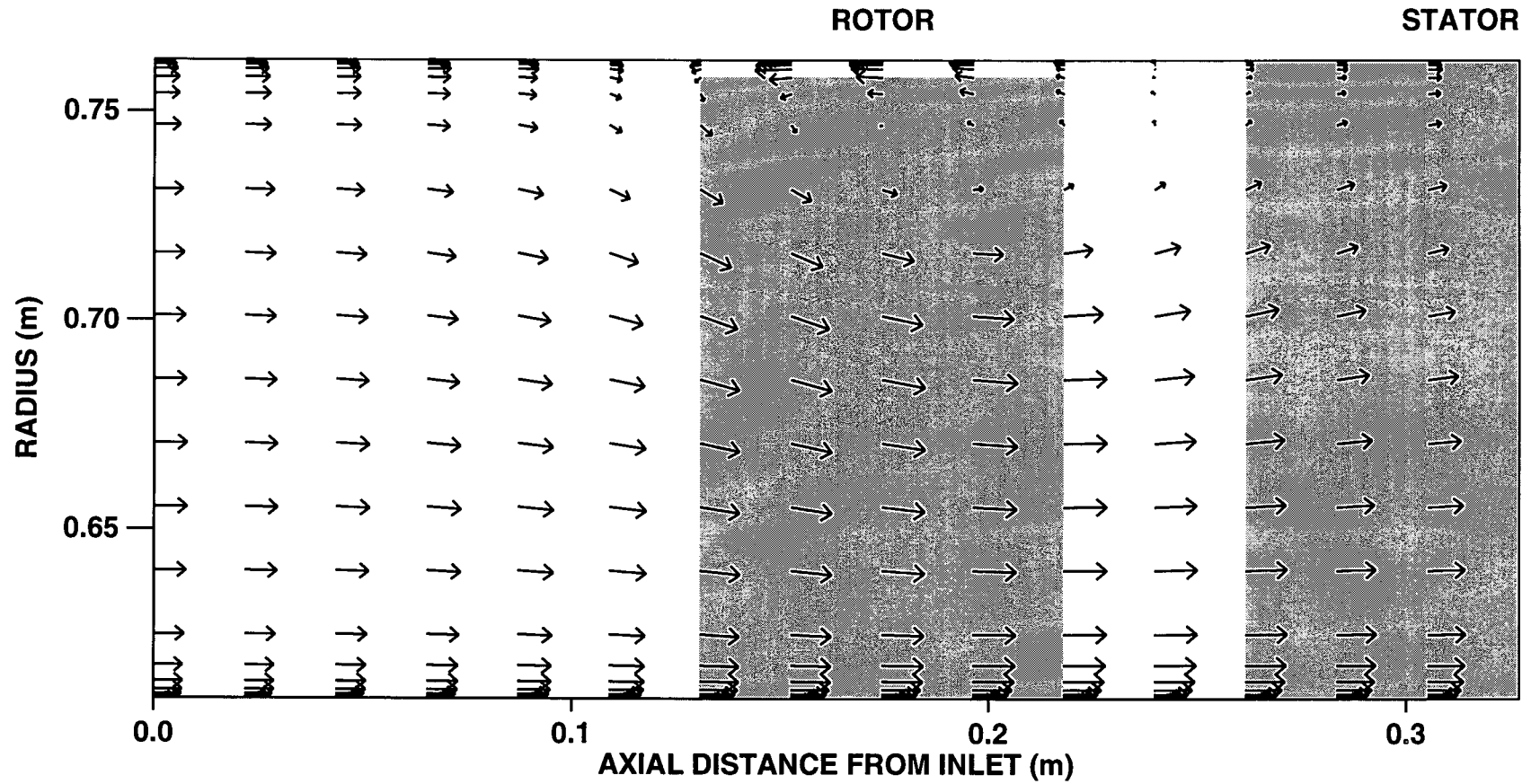


Fig. 5-21 Deverson Rig: Velocity Vectors in r-z Plane During Stall Inception at the Circumferential Node Showing the Greatest Disturbance.

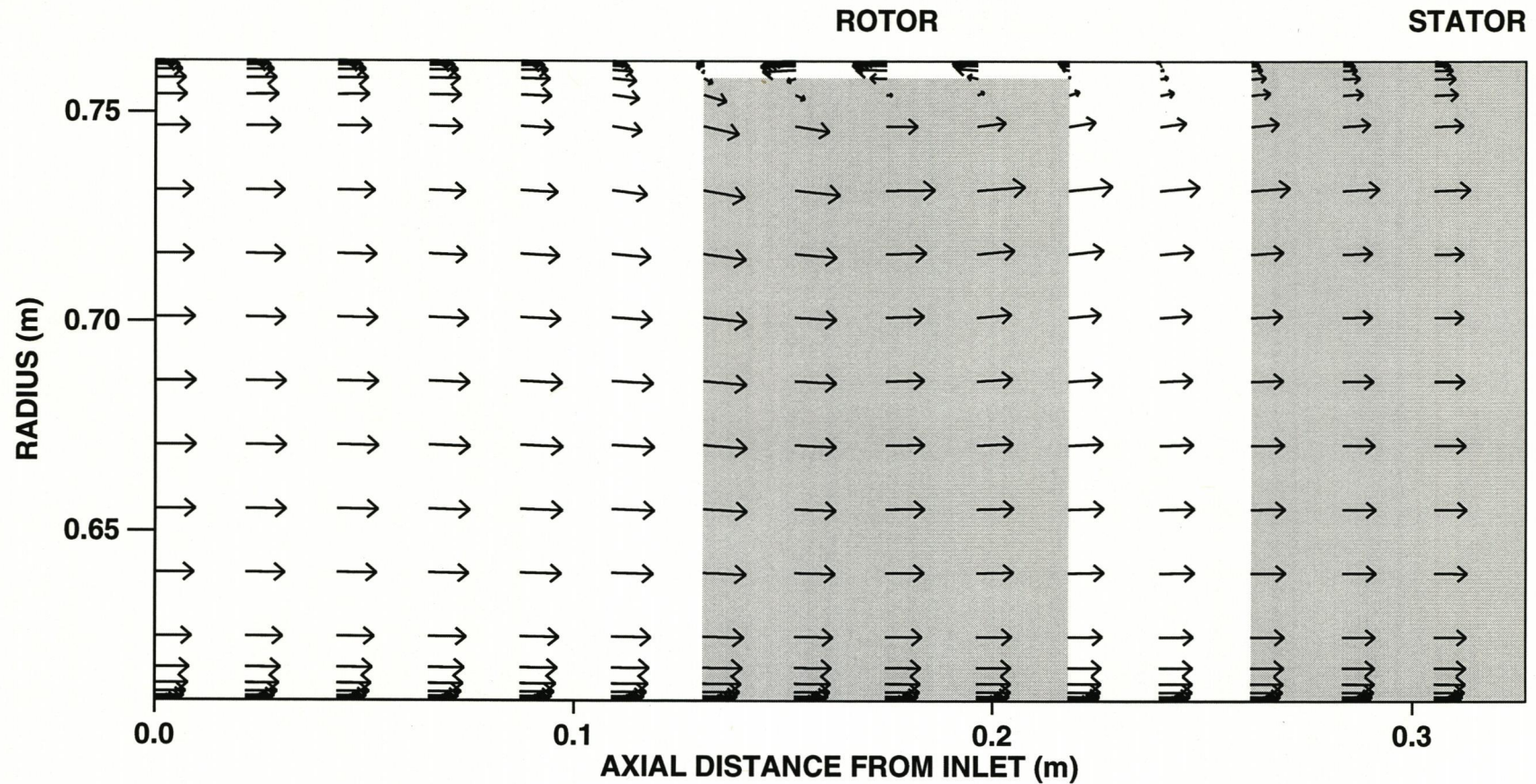


Fig. 5-22 Deverson Rig: Velocity Vectors In r-z Plane During Stall Inception, at a Node Circumferentially Out of Phase with the Position of Greatest Disturbance shown in Fig 5-21.

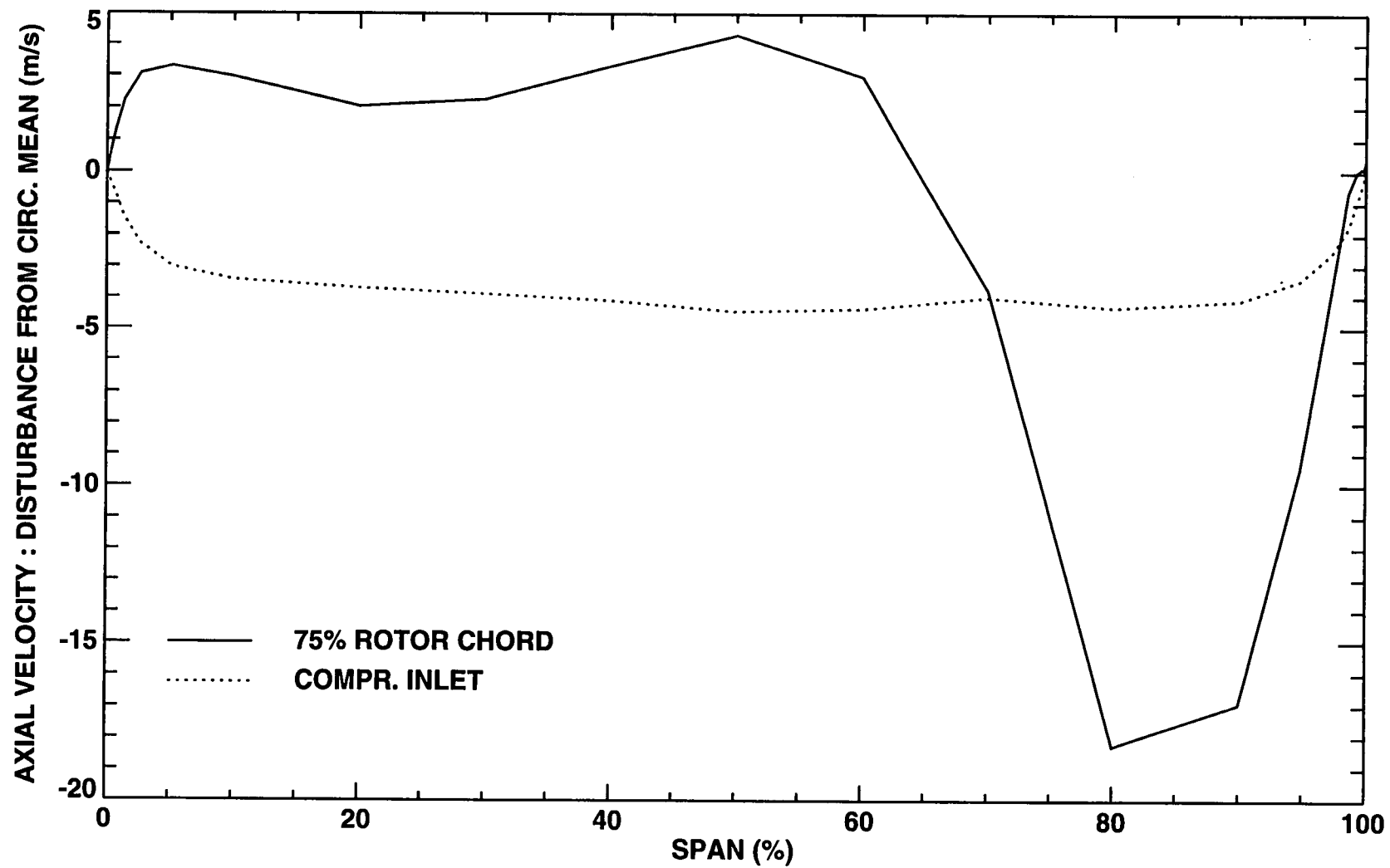


Fig. 5-23 Deverson Rig: Radial Profile of Axial Velocity Disturbance.