

CRANFIELD UNIVERSITY

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SYNERGISTIC AERODYNAMIC FORCE ASSESSMENT THROUGH  
AN EXTENDED EXERGY APPROACH

SCHOOL OF AEROSPACE, TRANSPORT AND MANUFACTURING  
Centre For Propulsion And Thermal Power Engineering

PhD

Academic Year: 2020 - 2023

Supervisor: Prof. P. Laskaridis  
Associate Supervisor: Dr. D. Sanders  
June 2023

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This thesis is submitted in partial fulfilment of the requirements for the  
degree of Doctor of Philosophy

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## ABSTRACT

Drag decomposition using energy and exergy-based methods has shown large utility for aerodynamic performance assessment through their flow-field decompositions into different physical mechanisms. A particularly significant advantage of these methods is their ability to identify recoverable energy, which describes the available energy imparted to the flow by the aircraft as it traverses through the fluid. This type of assessment is not possible with traditional momentum analysis. Thus, energy/exergy analysis uniquely evaluates the potential benefits of wake energy utilisation for thrust production through novel architectures such as boundary layer ingestion. The velocity decomposition approach has introduced notable improvements to this analysis framework. This allows for a phenomenological drag decomposition into reversible and irreversible components by splitting the velocity field into its isentropic and non-isentropic contributions within the flow. From this, the reversible drag originating from the bulk flow can be obtained through the isentropic field, whilst the non-isentropic field provides the irreversible dissipative drag arising from the boundary layer and wake zones. The work conducted in this thesis aims to improve the velocity decomposition approach by combining it with partial pressure field analysis, enabling the decomposition of pressure into Euler and dissipative parts, previously not achievable with velocity decomposition alone. Assessment in this manner improves the evaluation of recoverable energy by identifying the additional pressure work potential within the dissipative field. Additionally, the unification extends energy/exergy-based analysis principles to the near-field, providing a unique decomposition capable of evaluating the local accumulation of viscous drag through dissipative pressure and skin friction, whilst the induced drag is assessed from the non-dissipative pressure.

Keywords:

near-field, far-field, momentum, mechanical energy analysis, velocity decomposition, partial pressure fields, drag decomposition

## ACKNOWLEDGEMENTS

This journey has been quite long and filled with many jumps and turns, but finally, it is nearing its end. None of this would have been possible without the fantastic support from everyone who helped me along the way; it would not have been possible without you all. This all started with a heartfelt conversation with Dr Helen Inglis; thank you for being such an instrumental piece in starting my jump into academia; your wise words of encouragement helped me take the leap and see where I fit in. I would also like to thank Dr Lelanie Smith for her amazing networking, without which I wouldn't have been surrounded by such great talent and all-around wonderful people. Thank you for seeing my potential and continually mentoring me through my academic journey. I also would like to thank my supervisors, Prof. Panagiotis Laskaridis and Dr Drewan Sanders, for their continual guidance throughout my PhD. Through your excellent management and professionalism, I have matured to the academic level I have today. Thank you for the constant advice and support you showed me personally through the good and the bad.

Thank you also to the great minds of Prof. Sven Schmitz and Pierce Hart. Working with you has been an absolute pleasure, and I appreciate the continual nuggets of knowledge you both shared with me along the way. Also, a huge thanks to the beautiful PhDs in the UTC and our small BLI team; even if I was only around briefly, you all made Cranfield feel like home and brightened my day in more ways than one.

My deepest gratitude to Barrie Hopewood for giving me the strength and courage to march this far, even when things seemed dire and I was close to giving up. You helped me pick myself back up and move forward with a hopeful smile and a positive outlook.

To Charisa, we've finally managed to reach the end, and we can breathe a sigh of relief. I will always hold dear; thank you for carrying me through the most challenging times and for your patience, selflessness and loving heart that compassionately helped me through my journey.

To Nadine, my partner in crime, I hope seeing me soldier through this inspires you for years to come through your journey. I will be there for you the same way you have for me. Thank you so much.

Finally to my mother, thank you for being my number one fan; it has been a roller coaster ride of emotions, and you've managed to see me through all of them. Thank you for instilling the principles within me that make me who I am today. I am because you are. I love you dearly, thank you.

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## NOMENCLATURE

|                          |   |                                                      |
|--------------------------|---|------------------------------------------------------|
| $AR$                     | = | aspect ratio [°]                                     |
| $\alpha$                 | = | angle of attack [°]                                  |
| $\dot{A}$                | = | total anergy deposition rate [W] = $\dot{A}_{total}$ |
| $\dot{A}_{\nabla T}$     | = | thermal anergy deposition rate [W]                   |
| $\dot{A}_{\Phi}$         | = | viscous anergy deposition rate [W]                   |
| $\dot{A}_w$              | = | wave anergy deposition rate [W]                      |
| $\dot{A}_{\mu_g g}$      | = | dilational anergy dissipation rate [W]               |
| $\dot{A}_{\mu \omega^2}$ | = | enstrophy-related anergy dissipation rate [W]        |
| $c$                      | = | chord length [m]                                     |
| $C( )$                   | = | coefficient quantity                                 |
| $C_d$                    | = | airfoil drag coefficient                             |
| $C_D$                    | = | wing drag coefficient                                |
| $C_{D,0}$                | = | zero-lift drag coefficient                           |
| $C_l$                    | = | airfoil lift coefficient                             |
| $C_L$                    | = | wing lift coefficient                                |

|                      |   |                                     |
|----------------------|---|-------------------------------------|
| $C_p$                | = | static pressure coefficient         |
| $C_p$                | = | power coefficient                   |
| $C_L$                | = | lift coefficient                    |
| $\delta$             |   | velocity boundary layer thickness   |
| $\delta_t$           |   | thermal boundary layer thickness    |
| $\mathbf{D}$         | = | density defect                      |
| $D$                  | = | drag force [N]                      |
| $D_{nf}$             | = | near-field drag [N]                 |
| $D_{far}$            | = | far-field drag [N] = $D_{ff}$       |
| $D_{\dot{E}}$        | = | energy-based drag [N]               |
| $D_{\dot{\epsilon}}$ | = | exergy-based drag [N]               |
| $D_{pr}$             | = | profile drag [N]                    |
| $D_{\tau}$           | = | skin friction drag [N]              |
| $D_p$                | = | pressure drag [N]                   |
| $D_v$                | = | viscous drag [N] = $D_{\tau} + D_p$ |
| $D_w$                | = | wave drag [N]                       |
| $D_i$                | = | induced drag [N]                    |

|                |   |                                                                         |
|----------------|---|-------------------------------------------------------------------------|
| $D_{\delta s}$ | = | entropy drag [N]                                                        |
| $D_{sp}$       | = | spurious drag [N]                                                       |
| $D_{RVF}$      | = | real viscous flow drag [N] = $D$                                        |
| $D_E$          | = | Euler drag [N]                                                          |
| $D_\mu$        | = | dissipative drag [N]                                                    |
| $dS$           | = | surface element of the control volume                                   |
| $dV$           | = | volume element of the control volume                                    |
| $\dot{E}_a$    | = | axial kinetic energy deposition rate [W]                                |
| $\dot{E}_v$    | = | transverse kinetic energy deposition rate [W]                           |
| $\dot{E}_p$    | = | pressure-work deposition rate [W]                                       |
| $\dot{E}_k$    | = | kinetic energy deposition rate [W] = $\dot{E}_a + \dot{E}_v$            |
| $\dot{E}_m$    | = | mechanical energy deposition rate [W] = $\dot{E}_k + \dot{E}_p$         |
| $\mathbf{e}$   | = | x, y, and z axis directions = $e_x \hat{x} + e_y \hat{y} + e_z \hat{z}$ |
| $e_i$          | = | inviscid spanwise efficiency                                            |
| $e_v$          | = | viscous spanwise (Oswald) efficiency                                    |
| $e$            | = | mass-specific internal energy [J kg <sup>-1</sup> ]                     |
| $\mathcal{E}$  | = | exergy                                                                  |

|                            |   |                                                                                          |
|----------------------------|---|------------------------------------------------------------------------------------------|
| $\dot{\mathcal{E}}_{\Phi}$ | = | rate of thermal exergy generation by viscous dissipation [W]                             |
| $\dot{\mathcal{E}}_{th}$   | = | thermal exergy deposition rate [W]                                                       |
| $\dot{\mathcal{E}}_{phy}$  | = | physical exergy deposition rate [W] = $\dot{\mathcal{E}}_{th} + \dot{\mathcal{E}}_{phy}$ |
| $\mathbf{F}$               | = | integrated force quantity [N]                                                            |
| $\mathbf{F}_B$             | = | force acting on body surface quantity [N]                                                |
| $\mathbf{F}_p$             | = | propulsive force [N]                                                                     |
| $F_x$                      | = | streamwise force quantity [N]                                                            |
| $\mathbf{F}_{rf}$          | = | integrated force quantity [N]                                                            |
| $\mathbf{F}_{far}$         | = | far-field integrated force quantity [N]                                                  |
| $\mathbf{F}_f$             | = | aerodynamic force expression based on vortex force theory [N]                            |
| $\mathbf{F}_{\ell}$        | = | vortex force [N]                                                                         |
| $\mathbf{F}_{\rho}$        | = | compressible generation force term [N]                                                   |
| $\mathbf{F}_{\nabla\rho}$  | = | compressible correction force term [N]                                                   |
| $\mathbf{F}_{\Sigma}$      | = | lamb vector surface integral force [N]                                                   |
| $\mathbf{F}_{\mu\theta}$   | = | viscous compression force term [N]                                                       |
| $\mathbf{F}_{\mu\omega}$   | = | viscous shearing force term [N]                                                          |

|                            |   |                                                                      |
|----------------------------|---|----------------------------------------------------------------------|
| $\mathbf{F}_{\dot{E}}$     | = | integrated energy-based force quantity [N]                           |
| $\mathbf{F}_{\dot{e}}$     | = | integrated exergy-based force quantity [N]                           |
| $\mathbf{F}_{irr}$         | = | irreversible aerodynamic force [N]                                   |
| $\mathbf{F}_{rev}$         | = | reversible aerodynamic force [N]                                     |
| $h$                        |   | mass specific enthalpy [J.kg <sup>-1</sup> ]                         |
| $k$                        | = | space dimension minus one                                            |
| $\mathbf{K}$               | = | kinetic energy defect                                                |
| $k_{eff}$                  | = | effective thermal conductivity (W.m <sup>-1</sup> .K <sup>-1</sup> ) |
| $\ell$                     | = | lamb vector = $\ell = \boldsymbol{\omega} \times \mathbf{V}$         |
| $\mathcal{G}$              | = | divergence of velocity = $\nabla \cdot \mathbf{V}$                   |
| $L$                        | = | lift [N]                                                             |
| $L/D$                      | = | lift-to-drag ratio = aerodynamic efficiency = $\eta_{aero}$          |
| $M$                        | = | Mach number                                                          |
| $\mathbf{M}$               | = | mass defect                                                          |
| $\mathbf{n}$               | = | unit normal vector                                                   |
| $\hat{\mathbf{n}}$         | = | unit normal vector = $-\mathbf{n}$                                   |
| $\hat{\mathbf{n}}_{shock}$ | = | unit normal vector for the shock wave                                |
| $\varphi$                  | = | potential velocity function [m <sup>2</sup> .s <sup>-1</sup> ]       |

|                            |   |                                                                          |
|----------------------------|---|--------------------------------------------------------------------------|
| $\psi$                     | = | stream function [ $\text{m}^2.\text{s}^{-1}$ ]                           |
| $\sigma$                   | = | in-plane velocity gradient [ $\text{s}^{-1}$ ]                           |
| $\sigma_z$                 | = | compressibility corrected in-plane velocity gradient [ $\text{s}^{-1}$ ] |
| $\xi$                      | = | axial vorticity [ $\text{s}^{-1}$ ]                                      |
| $\mathcal{G}$              | = | divergence of velocity = $\nabla \cdot \mathbf{V}$                       |
| $\Theta$                   | = | volumetric power [W]                                                     |
| $\Phi$                     | = | viscous dissipation rate [W]                                             |
| $\Phi_{\mu_g \mathcal{G}}$ | = | dilational viscous dissipation rate [W]                                  |
| $\Phi_{\mu \omega^2}$      | = | enstrophy-related viscous dissipation rate [W]                           |
| $\Phi_w$                   | = | shockwave dissipation rate [W]                                           |
| $\Gamma$                   | = | circulation [ $\text{m}^2.\text{s}^{-1}$ ]                               |
| $n, t$                     |   | normal and tangential axes                                               |
| $\rho$                     | = | fluid density [ $\text{kg}/\text{m}^3$ ]                                 |
| $p$                        | = | static pressure [Pa]                                                     |
| $p_G$                      | = | gauge pressure [Pa]                                                      |
| $P$                        | = | total pressure [Pa]                                                      |
| $\mathbf{P}$               | = | momentum defect                                                          |
| $\Pi$                      | = | pressure defect                                                          |

|                                 |   |                                                                                   |
|---------------------------------|---|-----------------------------------------------------------------------------------|
| $P_S$                           | = | shaft power [W]                                                                   |
| $Pr$                            |   | Prandtl number                                                                    |
| $Pr_{lam}$                      |   | laminar Prandtl number                                                            |
| $Pr_{turb}$                     |   | turbulent Prandtl number                                                          |
| $\mathbf{q}$                    | = | effective heat flux by conduction [ $\text{J.s}^{-1}$ ] = $-k_{eff} \nabla T$     |
| $\mathbf{Q}$                    | = | velocity defect                                                                   |
| $Re$                            | = | Reynolds number                                                                   |
| $R$                             |   | gas constant [ $\text{J. kg}^{-1} \cdot \text{K}^{-1}$ ]                          |
| $S$                             |   | control surface                                                                   |
| $S_\phi$                        |   | source/sink term                                                                  |
| $s$                             | = | mass-specific entropy [ $\text{J. kg}^{-1} \cdot \text{K}^{-1}$ ]                 |
| $\bar{\mathbf{S}}$              | = | mean strain rate [ $\text{s}^{-1}$ ]                                              |
| $T$                             | = | static temperature [K]                                                            |
| $\bar{\boldsymbol{\tau}}$       | = | viscous stress tensor [N]                                                         |
| $\bar{\boldsymbol{\sigma}}$     | = | Cauchy stress tensor [N]                                                          |
| $\bar{\boldsymbol{\tau}}_{eff}$ | = | effective viscous stress tensor [N] = $(\mu_{lam} + \mu_{turb}) \bar{\mathbf{S}}$ |
| $\mu$                           | = | dynamic viscosity [Pa.s]                                                          |
| $\mu_{lam}$                     | = | laminar viscosity [Pa.s]                                                          |

|                       |   |                                                                                                   |
|-----------------------|---|---------------------------------------------------------------------------------------------------|
| $\mu_{turb}$          | = | turbulent viscosity [Pa.s]                                                                        |
| $\mu_{artificial}$    | = | artificial viscosity [Pa.s]                                                                       |
| $\mu_{eff}$           | = | artificial viscosity [Pa.s] = $(\mu_{lam} + \mu_{turb})$                                          |
| $\mu_g$               | = | bulk viscosity [Pa.s]                                                                             |
| $u, v, w$             | = | perturbation velocities [m/s]                                                                     |
| $x, y, z$             | = | cartésien axes                                                                                    |
| $V_\infty$            | = | freestream velocity [m/s]                                                                         |
| $\mathbf{V}$          | = | fluid velocity $(V_\infty + u)\hat{x} + v\hat{y} + w\hat{z}$ [m/s]                                |
| $\mathbf{V}_A$        | = | absolute velocity vector [m/s]                                                                    |
| $\mathbf{V}_B$        | = | aerodynamic body flight velocity [m/s]                                                            |
| $\mathbf{V}_{CS}$     | = | control surface velocity [m/s]                                                                    |
| $V^2$                 | = | fluid speed squared = $\mathbf{V} \cdot \mathbf{V}$ [m/s]                                         |
| $V_n$                 | = | side cylinder normal velocity $vn_y + wn_z$ [m/s]                                                 |
| $\boldsymbol{\omega}$ | = | local vorticity vector [s <sup>-1</sup> ] = $\omega_x\hat{x} + \omega_y\hat{y} + \omega_z\hat{z}$ |
| $W$                   | = | weight of aircraft [N]                                                                            |
| $W\Gamma$             | = | Potential energy rate [W]                                                                         |
| $\mathbf{x}$          | = | position vector = $\hat{x} + \hat{y} + \hat{z}$                                                   |

$$\gamma = \text{specific heat ratio} = c_p / c_v$$

## SUBSCRIPTS & SUPERSSCRIPTS

$$( )_{\infty} = \text{freestream quantity}$$

$$( )_{\mu} = \text{dissipative flow quantity}$$

$$( )_E = \text{Euler flow quantity}$$

$$( )_B = \text{quantity on the body surface}$$

$$( )_O = \text{quantity on the outer-boundary surface}$$

$$( )_{shock} = \text{quantity on shockwave interface}$$

$$( )_O^{FP} = \text{quantity on the front plane surface}$$

$$( )_O^{TP} = \text{quantity on the transverse plane surface} = ( )_{TP}$$

$$( )_O^{SC} = \text{quantity on the side cylinder surface} = ( )_{SC}$$

$$( )_{CS} = \text{control surface associated quantity}$$

$$( )_{CV} = \text{control volume associated quantity}$$

$$( )_{ref} = \text{reference quantity}$$

$$( )_{RVF} = \text{Real viscous flow quantity}$$

|                 |   |                                              |
|-----------------|---|----------------------------------------------|
| $( )_{EIF}$     | = | equivalent viscous flow quantity             |
| $( )_{DF}$      | = | defect flow quantity                         |
| $( )_{nf}$      | = | near-field quantity                          |
| $( )_{far}$     | = | far-field quantity = $( )_{ff}$              |
| $( )_{\dot{E}}$ | = | mechanical energy quantity                   |
| $( )_{\dot{e}}$ | = | exergy quantity                              |
| $( )^{lam}$     | = | laminar flow quantity                        |
| $( )^{turb}$    | = | turbulent flow quantity                      |
| $( )^*$         | = | numerically adjusted/corrected flow quantity |
| $( )^*$         | = | numerically adjusted/corrected flow quantity |
| $( )_c$         | = | compressible flow quantity                   |
| $( )_{inc}$     | = | incompressible flow quantity                 |
| $( )_{wall}$    | = | wall quantity                                |
| $( )_{VII}$     | = | viscous inviscid interaction quantity        |
| $( )_{sp}$      | = | spurious flow quantity                       |

|             |   |                                           |
|-------------|---|-------------------------------------------|
| $( )_p$     | = | pressure drag related quantity            |
| $( )_{pr}$  | = | profile drag related quantity             |
| $( )_i$     | = | induced drag related quantity             |
| $( )_w$     | = | wave drag related quantity = $[ ]_{wave}$ |
| $( )_v$     | = | viscous drag related quantity             |
| $( )_{rec}$ | = | recoverable flow quantity                 |
| $c( )$      | = | coefficient quantity                      |

## OPERATORS

|                         |   |                                                      |
|-------------------------|---|------------------------------------------------------|
| $\llbracket \rrbracket$ | = | discontinuous jump of a quantity                     |
| $\approx$               | = | approximately equal                                  |
| $\delta( )$             | = | quantity relative to freestream = $( ) - ( )_\infty$ |
| $\Delta( )$             | = | measured change of flow quantity                     |
| $\nabla( )$             | = | gradients of a vector quantity                       |
| $\nabla \cdot ( )$      | = | divergence of a vector quantity                      |
| $\nabla^2( )$           | = | Laplacian operator                                   |

$\otimes$  = tensor product

## LIST OF ABBREVIATIONS

|      |                                                     |
|------|-----------------------------------------------------|
| 2D   | two-dimensional                                     |
| 3D   | three-dimensional                                   |
| CV   | control volume                                      |
| VII  | viscous inviscid interaction                        |
| PPF  | partial pressure field                              |
| VD   | velocity decomposition                              |
| UPVD | unified partial pressure and velocity decomposition |
| RVF  | real viscous flow                                   |
| EIF  | equivalent inviscid flow/Euler inviscid flow        |
| DF   | defect flow/dissipative flow                        |
| CFD  | computational fluid dynamics                        |
| KJ   | Kutta-Joukowski                                     |
| KJMB | Kutta-Joukowski-Maskell-Betz                        |
| MSK  | Maskell                                             |
| RANS | Reynolds Averaged Navier-Stokes                     |
| BL   | boundary layer                                      |
| BLI  | boundary layer ingestion                            |

|      |                                             |
|------|---------------------------------------------|
| BLIP | boundary layer ingesting propulsor          |
| FIP  | freestream ingesting propulsor              |
| EWG  | exergy waste coefficient                    |
| PER  | potential for energy recovery               |
| PSC  | power saving coefficient                    |
| IBLT | integral boundary layer theory              |
| IBL  | integral boundary layer                     |
| ARF  | absolute reference frame                    |
| RRF  | relative reference frame                    |
| RHS  | right-hand side                             |
| LHS  | left-hand side                              |
| NACA | National Advisory Committee for Aeronautics |

# 1 Research Background and Introduction

## 1.1 Context

For most aerodynamicists, aerodynamic force assessment begins with control volume analysis through the Navier-Stokes Momentum Equation. This is commonly introduced through near-field drag assessment, where drag is considered as the component of stress imposed by the fluid on the body integrated over its physical surfaces. In this instance, the drag is split into binary *mechanical* components: skin friction and pressure drag.

Skin friction is commonly understood to result from viscous, shearing effects. On the other hand, pressure drag is more complex and involves contributions from viscous effects, shockwaves, and induced effects caused by the overall impact of lift, also known as induced drag [1]. This diverse nature of pressure drag highlights a primary difficulty in obtaining contributions tied to precise physical mechanisms from near-field mechanical decompositions.

Furthermore, this type of drag decomposition is affected by what Drela [2] describes as the “force cancellation problem”, which prevents any meaningful evaluation of local pressure drag contributions, as can be done with the skin friction. This is because local pressure drag evaluation considers the regional flow around an unclosed body. The drag evaluated would be intensely dependent on the unclosed body’s local pressure field, which may not have a net contribution toward drag when considering the flow field around the entire body.

Although a significant fraction of the body’s pressure variation is inviscid, a small contribution exists due to the action of viscosity, referred to as form resistance [3]. This represents the viscous drag contribution from the pressure field, owing to the boundary layer’s displacement effect on the surrounding flow. Partial pressure field (PPF) analysis [4–6] can be used to quantify this contribution via the dissipative pressure drag.

PPF analysis considers the static pressure as a linear split consisting of an Euler (inviscid) part denoted  $P_E$  and a dissipative (viscous) part  $P_\mu$ . The viscous drag obtained from PPF analysis is described as the combined contribution of skin friction and dissipative pressure. At the same time,

the Euler drag is associated with contributions originating from bulk flow phenomena such as shockwaves and lift-induced drag.

Considering aerodynamic bodies with *no active propulsion*, drag contributes toward the accumulation of dissipation within the flow, representing an irreversible loss of energy to the flow as heat. Per the work-energy relationship from the first law of thermodynamics, the manifestation of this heat requires the body to do work on the flow. This work exerts a net force on the flow, which following Newton's third law, would be the retarding force experienced by the body in the direction opposing its motion, i.e., drag. This intelligible use of Newton's third law, called far-field analysis, allows drag to be assessed from the perspective of the fluid instead of a near-field body-focused study.

The illustration in Figure 1-1 aims to show a fascinating element of these two opposing vantage points. The total drag from either view is mathematically equivalent; however, no immediate summation of far-field components can intuitively be obtained to return a combination that resembles the near-field pressure or skin friction components. This highlights the difficulty in obtaining a coherent understanding of drag from individual component contributions, even if assessed from the same momentum-based perspective. As a result, most studies only use near- and far-field total drag in tandem to verify/validate numerical or experimental accuracy for drag bookkeeping, as these two approaches should return the same overall evaluation of the aerodynamic force experienced by the body.

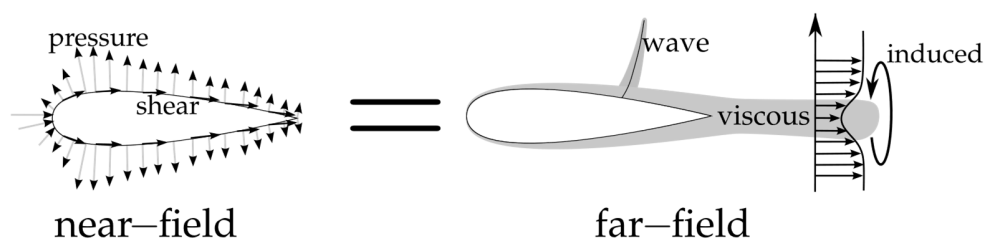


**Figure 1-1 Comparison of near and farfield drag contribution interpretations**

A practical advantage of far-field analysis is observed through its well-defined phenomenological decomposition, shown in Figure 1-1, which allows for the physical drag contributing mechanisms to be evaluated through wake measurements, notably applicable for wind tunnel testing.

A substantial body of work conducted by Kusunose [7–13] discusses the implications of drag extraction from wind tunnel wake measurements for subsonic and transonic flows through thermodynamic methods. These methods are simply an extension of far-field analysis offering a broadened understanding of the physical processes contributing toward aerodynamic drag, delineating these into viscous, wave and induced drag contributions. The viscous drag is associated with entropy-generating mechanisms localised to the vortical boundary layer and wake regions. In contrast, the wave drag is associated with entropy generation outside the vortical zones owing to shockwave formation. Lastly, the induced drag is associated mainly with the shed vorticity from the wingtips accompanying lift production.

The foundational basis of the formulations presented by Kusunose [7–13] was derived through the work of Betz [14] and Maskell [15], who showed the extensive wind tunnel applicability of wake-based formulations to extract profile and induced drag, although limited to analysis of incompressible flows. To address this, Kusunose [7–13] began by first rederiving the classical expressions from the work of Betz [14] and Maskell [15], now considering the effects of compressibility.

The resulting profile drag formulation (which included wave drag) was observed to be a modified form of Oswatitsch’s entropy drag integral [16], with the entropy terms replaced by an equivalent total pressure loss expression to present the formulation in terms of flow variables extractable within the wind tunnel.

The induced drag formulation resembled that of Maskell [15], modified to represent the scalar functions of the cross-sectional velocities using an analytical Green’s function approach [7,17,18]. This modification offered the benefit of reducing grid dependency previously experienced with Maskell’s formulation while also providing the ability to evaluate the effect of wind tunnel walls on experimentally obtained induced drag values. Although immensely beneficial from the

perspective of far-field analysis, no active effort was made to explicitly relate the phenomenological drag contributions to the near-field mechanical breakdown, thus highlighting a potential area of investigation. The benefits of such a relation would allow for a fuller understanding of the near-field manifestation of phenomenological drag-producing mechanisms and their associated downstream evolution in the far-field, as depicted in Figure 1-2.

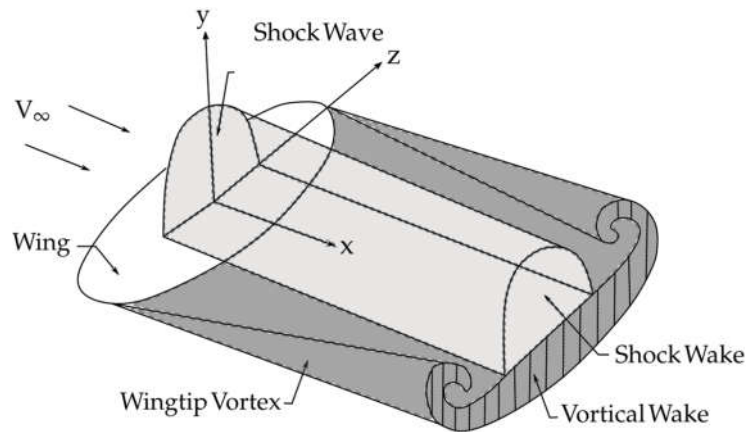


**Figure 1-2 Schematic of drag contributing mechanisms, wake formation and downstream evolution adapted form [10]**

Considering bodies with *active propulsion*, the non-local nature of pressure drag presents a significant thrust-drag bookkeeping challenge, particularly when differentiation between thrust and drag is required for sub-components of a whole aircraft configuration. As mentioned, meaningful pressure-drag assessment relies on its evaluation over complete closed bodies; analysis of subdivisions of body segments returns partially unbalanced pressure force contributions strongly dependent on the local pressure field of the isolated part [3]. When integrated over the closed body surface, these pressure forces are counterbalanced by mutually opposing force contributions. Furthermore, for closely coupled bodies, the continuum nature of fluids is strongly emphasised through mutual buoyancy force interactions where no action/reaction occurs in isolation, as they are aerodynamically connected. As a result, the idea of a “thrust” or “drag” force solely appearing on a single element of this interacting body system becomes difficult to define.

Instead of relying on this momentum-based performance assessment approach where the thrust and drag definition requirement is unavoidable, Drela [2] analyses the balance between mechanical energy production from the aircraft and its consumption through dissipative mechanisms within the flow. The resulting analysis, referred to as the power balance method, returned a formulation where neither definition of “thrust” nor “drag” was required, thus avoiding the force decomposition ambiguity. Furthermore, this method could decompose the various wake energy loss mechanisms into independent contributions from viscous dissipation, mechanical energy, and thermo-compressibility [2,19], as shown in Figure 1-3.

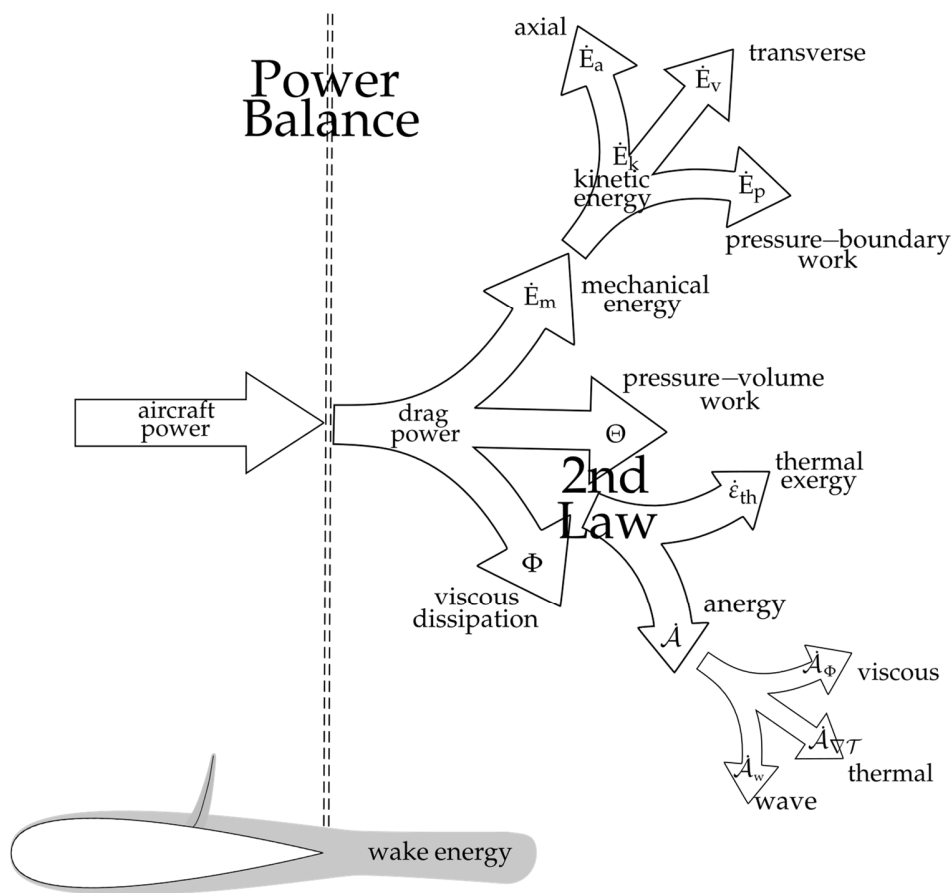


**Figure 1-3 Schematic of energy transfer pathways observed through energy and exergy-based parameters**

The mechanical energy is defined as the sum of the kinetic energy (directionally split into axial and transverse components) and the pressure boundary-work rate representing the pressure-related flow power. The irreversible loss of kinetic energy to the surroundings as heat is tallied through the viscous dissipation, while pressure-related losses are evaluated through the pressure-volume work rate.

Arntz [20,21] effectively expanded this approach by coupling it with the second law of thermodynamics to obtain an exergy-based analysis approach. This allowed for an expanded interpretation of energy, where its properties are split into a two-part superposition consisting of a portion theoretically convertible into useful mechanical work and another irreversibly lost to the surroundings, referred to colloquially as exergy and anergy, respectively, per the work of Rant [22].

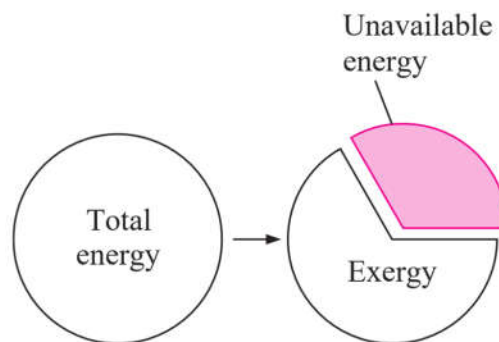


**Figure 1-4 Total energy partition into useful work potential (exergy) and unavailable energy (anergy), adapted from [23]**

Figure 1-4 provides a simple illustration of the *total energy* partition of an arbitrary isolated system, which is always conserved through the 1<sup>st</sup> Law of Thermodynamics, regardless of the nature of the process, i.e., reversible (ideal) or irreversible (real). The concept of exergy conservation is distinct from energy conservation. While energy conservation holds for all processes, exergy conservation only applies to ideal processes. In real processes, exergy is destroyed through entropy-generating processes, as per the 2<sup>nd</sup> Law of Thermodynamics. Therefore, the amount of exergy decreases

continuously, following the *decrease of exergy principle* [23], which states that exergy destruction always accompanies entropy generation.

Exergy is usually measured relative to the environment as depicted in the mass specific formulation in equation (1.1), with the environment being denoted by the subscript  $( )_{\infty}$ , representing an *exergetic dead state* [23]. This state occurs when the system is in thermodynamic equilibrium with its surroundings.

$$\underbrace{\left( h + \frac{1}{2}V^2 + gz \right) - \left( h_{\infty} + \frac{1}{2}V_{\infty}^2 + gz_{\infty} \right)}_{\text{Total Energy}} = \underbrace{\mathcal{E}}_{\text{Exergy}} + \underbrace{T_{\infty}(s - s_{\infty})}_{\text{Unavailable energy (Anergy)}} \quad (1.1)$$

The exergy is then identified by measuring the system's departure from this *dead state*, indicating its useful work potential if returned to thermodynamic equilibrium with its surroundings through some reversible process.

Arntz [20,21] successfully showed that coupling mechanical energy methods with the 2<sup>nd</sup> law aids in identifying the thermal energy emanating from the pressure-volume work and viscous dissipation contributions. The combined contribution of the domain *exergy* from thermal and mechanical mechanisms, referred to as *physical exergy* [24], then defines the total amount of available energy within the flow which can be utilised to produce mechanical work. This metric is useful for novel technologies, such as boundary layer ingestion, as physical exergy identifies the work potential within the flow, which can hypothetically be used for thrust production. In contrast to exergy, which can be used to do work, *anergy* refers to energy unavailable for work and results from irreversible loss-producing mechanisms. These losses include viscous, thermal, and wave anergy, as shown in Figure 1-3. Viscous anergy represents the loss of kinetic energy in the form of heat due to viscous dissipation and turbulence. Thermal anergy results from dissipative losses from thermal mixing, while wave anergy is generated from the entropy produced through shockwaves.

Recent advancements to exergy-based analysis have been made through the work of Aguirre [25,26], who extended the applicability of exergy analysis for wind tunnel testing. This was

achieved by first decomposing the velocity field into isentropic and non-isentropic components, similarly for other wake-reduced [14] and axial velocity deficit formulations [27,28]. Aguirre’s approach differs from these, focusing on the decomposition of axial and transverse velocity components. At the same time, the density and temperature contributions are obtained through the isentropic and the ideal gas equations. Combining these results allows for a complete reconstruction of the isentropic and non-isentropic flow fields for irreversible and reversible drag evaluation.

Implementation of this approach involves the assumption that the isentropic and actual viscous flow static pressures are equivalent. The applicability of this assumption, however, has not been formally examined. Furthermore, Aguirre [25] identifies regions of underprediction obtained through this approach for irreversible viscous drag evaluation, referring to this as the “missing drag” attributed to viscous-inviscid interactions within the flow field. These highlighted areas present an opportunity for further investigation to understand the nature of these inaccuracies and assess whether they are a consequential result of the initially applied static pressure assumption.

## **1.2 Problem Statement**

A significant challenge for holistic drag decomposition obtaining contributions associated with profile and induced drag phenomena is hindered mainly by the variability in formulaic definitions presented in literature. An excellent example is the profile drag definitions proposed by Betz [14] and Jones [29], whose enforced simplifying assumptions affect the coherence of the respective drag quantities, mainly when assessed in the near wake. When surveyed far downstream, however, these equations produce similar results, as noted by Takahashi [30]. This variability may confuse or lead to debate based on the subjective nature of the drag quantities returned due to their positional sensitivity and formulaic representation.

This research aims to develop a Multiphysics based approach for aerodynamic force decomposition, allowing for a coherent link of phenomenological contributions obtained through momentum, energy, and exergy-based methods. This will be done by exploring various force decomposition approaches for steady attached flows to investigate the possibility of defining an

entirely consistent phenomenological drag decomposition that can be correlated between different approaches. The point of view taken in this work considers force decomposition as the quantitative separation of forces into components due to viscous and inviscid/Euler effects [30]. Considering this, the findings presented by Aguirre [26] already suggest that conventional far-field momentum and energy-based approaches offer further separability to expose their irreversible (viscous) and reversible (Euler) drag contributions through a component-based decomposition of flow quantities into isentropic and non-isentropic parts.

To this end, the main objectives of the research will be to:

- Establish a means of relating *mechanical* near-field decomposition components with respective far-field *phenomenological* constituents.
- Provide a means of assessing the local cumulative drag from various analysis perspectives, i.e., near-field, far-field, energy and exergy. Mainly this will focus on problems associated with local pressure drag evaluation discussed earlier.
- Extend the application of this approach to complex flow scenarios involving shockwave formation, thermo-compressibility, and lift by considering the incurred effects from the additional flow physics on the drag decomposition.
- Implement the approach on verifiable test cases through data obtained from literature to validate and further develop the methodology by identifying its possible limitations in practice.

### 1.3 Thesis Outline

To address the set-out objectives, this thesis documents the development of a synergistic aerodynamic performance assessment approach through the unification of momentum, energy, and exergy-based force decomposition approaches. The main body of work is organised into three main chapters summarised as follows:

- Chapter 2 develops a unified approach for aerodynamic force assessment by combining velocity and partial pressure decomposition approaches. Additionally, a method for quantifying viscous-

inviscid interaction contributions is introduced. This allows for a corrected decomposition to be performed where all drag contributions are correctly attributed to their phenomenological origin in the near-field.

- Chapter 3 extends the unified approach to transonic flows to assess its performance on more complex flows involving shockwave formation, thermo-compressibility and lift production. Through this, an alternative energy-based wave drag quantification method is introduced, obtaining its contribution by assessing the shockwave's dissipation via the entropy equation.
- Chapter 4 presents a holistic drag decomposition for lifting bodies by evaluating the contributions of reversible and irreversible flow mechanisms to viscous and induced drag. Particular interest is taken in the induced drag and its relation to Prandtl's classical formulation for finite wings.

## **1.4 Communications**

The work in this dissertation has been presented in part at two international conferences, while the content provided in Chapters 2 – 4 is in preparation for submission for publication in the AIAA Journal.

### **1.4.1 Conference Proceedings**

- N. Mutangara, D. Sanders, P. Laskaridis, "Examining the Benefits of Boundary Layer Ingestion on Powered Bodies," presented at the 3AF Aero Conference 2023, Paris, France, Mar. 2023.
- N. Mutangara, D. Sanders, P. Laskaridis, P. Hart and S. Schmitz, "A unified partial pressure field and velocity decomposition approach toward improved energetic aerodynamic force decompositions," presented at the 3AF Aero Conference 2023, Paris, France, Mar. 2023.
- N. Mutangara, D. Sanders, P. Laskaridis, "Aerodynamic energy recovery mapping and force decomposition for future ultra-efficient aircraft applied to a finite wing," presented at the 12th EASN International Conference, Barcelona, Spain, Oct. 2022.

### 1.4.2 Peer-Reviewed Journals

- N. Mutangara, D. Sanders, P. Laskaridis, P. Hart and S. Schmitz, “Unified Exergy & Partial Pressure Field Aerodynamic Decomposition Approach,” *AIAA Journal*. **(in preparation for submission)**
- N. Mutangara, D. Sanders, P. Laskaridis, P. Hart and S. Schmitz, “Extension of Unified Exergy & Partial Pressure Field Aerodynamic Decomposition Approach to Transonic Flows,” *AIAA Journal* **(in preparation for submission)**
- N. Mutangara, D. Sanders, P. Laskaridis, P. Hart and S. Schmitz, “Flow-Field Reversibility and irreversibility relations toward induced drag decomposition,” *AIAA Journal* **(in preparation for submission)**

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## **2 Unified Exergy & Partial Pressure Field Aerodynamic Decomposition Approach**

### **2.1 Introduction**

Traditionally, aircraft performance assessment primarily focuses on evaluating and quantifying aerodynamic forces such as lift and drag. This has been done using momentum conservation, commonly conducted from two essential perspectives. The first is the fluid action on the body, referred to as the near-field approach, where drag is obtained through force integration over all the physical surfaces and decomposed into contributions from pressure and skin friction. From this perspective, the aerodynamic force is understood physically as stress (normal or shear) acting over a specific area. The normal stress is attributed to the on-body pressure distribution and shear stress due to the no-slip condition over the body surface.

On the other hand, far-field methods evaluate drag via analysis of the flow perturbations resulting from the body being immersed within the fluid. This allows for a broader, more descriptive decomposition of drag into its specific phenomenological contributions. This is categorised into the viscous drag, associated with losses within the boundary layer and wake; induced drag, tied to the shed vorticity and downwash from lifting bodies; and wave drag, related to shockwave formation.

These methods are limited when analysing closely coupled bodies, particularly when studying highly integrated configurations encompassing technologies such as boundary layer ingestion (BLI), wingtip propulsion and novel airframe designs. For such arrangements, the performance of single components is not aerodynamically decoupled from the entire system. Potential field effects originating from the generated pressure distributions mutually influence the flow around each segment [1,2], thus affecting the aerodynamic performance. Forces arising specifically from the potential field, such as buoyancy forces, do not contribute directly to the overall system drag. Instead, they tend to self-balance, cancelling when the entire configuration is considered. However, when force contributions of individual components are desired, values obtained using either near- or far-field analysis are considerably misleading as ‘fictitious’ buoyancy force

contributions toward the overall drag are present as their effects have not yet self-balanced. Drela [1] proposes a mechanical energy-based drag build-up approach, i.e., the power balance method, using viscous dissipation and kinetic energy defect to circumvent this. These parameters are less sensitive to changes in local pressure distribution [2] and the accumulation of drag over closely coupled bodies.

This approach can only be exploited through a mechanical energy viewpoint, which separates a real viscous flow (RVF) into an equivalent inviscid (EIF) and boundary layer defect flow (DF). This flow decomposition relies on the fact that viscous and turbulence effects are confined to the boundary layer (BL) and wakes whilst flow outside these regions can be described through inviscid Euler equations. The procedure for obtaining the equivalent inviscid and defect flow fields is achieved by solving the inviscid and boundary layer flow equations separately and matching their solutions through an iterative process. Implementation of such an approach within CFD has successfully been shown through the work conducted by Sato [2] using MSES<sup>1</sup>. In addition, Sato [2] shows that for power balance analysis of isolated bodies, a *wake-reduced* formulation for profile mechanical energy loss evaluation can be obtained using integral mechanical energy defect equations directly applied to the boundary DF (also referred to as the dissipative field in this work).

Mele et al. [3–6] show that integral momentum equations describe aerodynamic drag through volume integrals of non-negligible quantities within the fluid domain. From this, an exact aerodynamic drag expression based on vortex force theory is possible through derivative moment transformation vector identities. The resulting formulation decomposes the aerodynamic drag into a reversible contribution given by the vortex force and an irreversible part provided by the surface integral flux of the Lamb vector ( $\ell = \boldsymbol{\omega} \times \mathbf{V}$ ) moment within the wake. The reversible drag is directly associated with bulk flow phenomena, such as induced drag effects resulting from lift generation. Additionally, this formulation intuitively highlights that irreversible drag contributions are confined to the wake. Kang et al. [7] show that the viscous drag surface integral of the vortex

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<sup>1</sup> MSES is a 2D interacting boundary layer theory code developed by Mark Drela [34]

force formulation can be wake-reduced, as non-zero vorticity contributions ( $\omega \neq 0$ ) are found mainly within this region. The vortex force drag breakdown does not attempt to decompose the flow field into an inviscid and defect flow. Instead, it provides an intelligible grouping of forces that associates particular drag-producing mechanisms with the EIF and boundary layer defect flow, such as induced and viscous drag.

Aguirre [8] acknowledges the Lamb-vector force breakdown of reversible and irreversible components but refers to these as isentropic and non-isentropic, respectively. The proposed decomposition takes advantage of the artificial/isentropic velocity concept introduced by Betz [9], which describes the flow's velocity if no energy losses were present. This concept is used to produce an approximate reconstruction of the EIF, referred to as the isentropic field, with the associated velocity, temperature, and density fields obtained through the isentropic flow equations and ideal gas law.

Finally, this decomposition is applied to exergy analysis, which Arntz et al. [10–13] introduce for aerodynamic performance assessment. Exergy analysis combines the 1<sup>st</sup> and 2<sup>nd</sup> laws of thermodynamics to identify the available energy within the flow domain, which can produce further useful work, i.e., thrust. The 1<sup>st</sup> law helps highlight the conservation and transfer of energy into different forms when an aerodynamic body moves through a fluid. The 2<sup>nd</sup> law, on the other hand, defines energy *quality*, explicitly assessing loss mechanisms and the degradation of useful-work potential through an evaluation of entropy generation. As a result, the exergy-based analysis provides an intuitive decomposition of energy, defining one portion as the useful work potential, termed *exergy*, which theoretically can be used to produce thrust.

In comparison, the remaining energy is referred to as “useless energy”, i.e., *anergy*, which refers to the portion that cannot be converted into mechanical work and is associated with irreversible losses to the surroundings, which leads to an increase in entropy. Combining this with the velocity decomposition (VD) approach allows for a clear separability of isentropic and non-isentropic exergy. Aguirre et al. [14] show that for an isentropic flow, no work potential is available as the exergy associated with this field self-recovers because of the reversible process followed by the

fluid. As a result, this implies that the isentropic exergy does not contribute toward the useful work potential, which is instead fully represented by the non-isentropic exergy. Therefore, an essential advantage of this combination of the exergy and VD approach is that an improved description of the useful work potential is obtained, associated solely with the non-isentropic exergy.

Schmitz and Coder [15] also achieve a form of flow field separability through a similar approach but consider a linear component decomposition of the pressure field instead. It is assumed that the RVF static pressure is given by the sum of the Euler (inviscid) and viscous components, each governed by a separate Poisson equation. The solution of these equations provides a pressure field split into non-dissipative and dissipative contributions. This terminology is analogous to reversible and irreversible component breakdown proposed by Mele et al. [3–6], Kang et al. [7] and Aguirre's [8] isentropic and non-isentropic decomposition mentioned before. It should be noted that partial pressure field (PPF) decomposition is seamless and allows for complete separability of form and induced drag in the limit of high Reynolds numbers. However, more importantly, PPF analysis offers a unique and comprehensive understanding of near-field pressure drag characterisation for a finite-volume CFD solution.

It can be noted that Betz [9] shows that the velocity decomposition approach can be applied to farfield momentum analysis for viscous drag evaluation. However, the validity of its use was limited to non-lifting incompressible flows. Aguirre [8] suggests improvements that extend this method's applicability to compressible and lifting flows and applies this new method successfully on exergy-based farfield drag decomposition. However, a limitation of this method is that the pressure field is omitted from the flow field decomposition. This occurs due to the assumption that the isentropic and RVF static pressure fields are equivalent, which implies that all pressure-related phenomena are isentropic, which is strictly not the case.

This paper aims to improve the abovementioned approach by addressing this assumption and obtaining the isentropic pressure field using partial pressure field analysis. Section 2.2 provides a derivation of this new unified approach, which is easily extended to momentum-based analysis in the near- and far-field as well as mechanical energy and exergy analysis to obtain force decomposition representations for non-dissipative and dissipative flowfields. The validity of the

assumptions and formulaic manipulations proposed through the derivations are then tested via a series of case studies. The first consists of a zero pressure gradient flat plate, which is used to assess the numerical implementation and accuracy of the PPF Poisson equation-solving procedure. As pressure-related contributions to the overall flat plate drag are known to be null for this test case, any significant deviations in force decomposition obtained via the unified approach can be associated with numerically related errors from an inadequate Poisson equation-solving procedure. The second consists of a NACA 0012 airfoil studied under various operating conditions, including variations in Reynolds number, Mach number and angle of attack. This is used to understand and observe the effects of changes in operating conditions regarding the overall aerodynamic drag and the constitutive force components obtained through the various force decomposition methods.

## 2.2 Mathematical Models and Numerical Methodology

### 2.2.1 Partial Pressure and Velocity Decomposition Approaches

This decomposition begins with the compressible Navier-Stokes equations assuming steady conditions with no external body forces, volumetric heat generation or conduction. The static pressure is split into a linear superposition consisting of an Euler and dissipative contribution, i.e.,  $p = p_E + p_\mu$ . Since pressure is a linear term, a splitting of this nature is plausible [15]. The partial pressures are grouped by their associated physical terms, as shown in equation (2.1). The Euler pressure is grouped with the inertial force terms, satisfying an inviscid Euler-type momentum equation when equated to zero. The same can be done for the RHS dissipative (viscous) pressure, which equivalently relates to viscous stress transport.

$$\nabla p_E + \rho(\mathbf{V} \cdot \nabla)\mathbf{V} = -\nabla p_\mu + \nabla \cdot \bar{\boldsymbol{\tau}} = 0 \quad (2.1)$$

The velocity decomposition (VD) proposed by Aguirre [8] takes a similar stance. It decomposes the velocity as follows  $\mathbf{V} = \mathbf{V}_E + \mathbf{V}_\mu$ , allowing the inertial forces to be identified as either dissipative or non-dissipative.

Implementing the VD approach assumes the static and isentropic pressure (referred to here for the subsonic analyses as the non-dissipative Euler pressure) are equivalent, i.e.,  $p_E \approx p$ . The suitability of this assumption is now testable through partial pressure fields (PPFs) as this approach can effectively provide the non-dissipative (isentropic) pressure field throughout the domain.

It is proposed that this decomposition of pressure and velocity can be extended to the density and temperature fields through the isentropic and ideal gas equations similarly used in [8]. Therefore, through a combination of PPF and VD approaches coupled with isentropic relations and ideal gas law, it is possible to expand equation (2.1) into the unified form given in equation (2.2).

$$\nabla p_E + \rho_E (\mathbf{V}_E \cdot \nabla) \mathbf{V}_E = -\nabla p_\mu - [\rho (\mathbf{V} \cdot \nabla) \mathbf{V} - \rho_E (\mathbf{V}_E \cdot \nabla) \mathbf{V}_E] + \nabla \cdot \bar{\boldsymbol{\tau}} = 0 \quad (2.2)$$

### 2.2.2 Momentum and force relations for the Euler field

The LHS of equation (2.2) provides the divergence form of the Euler flow field, which can be written in integral form for an arbitrary control volume  $V$  enclosed by the surface,  $S$  as shown in equation (2.3). The surface  $S$  is split into an inner boundary  $S_B$  that lies over the aerodynamic body and  $S_o$  represents the outer boundary within the flow field.

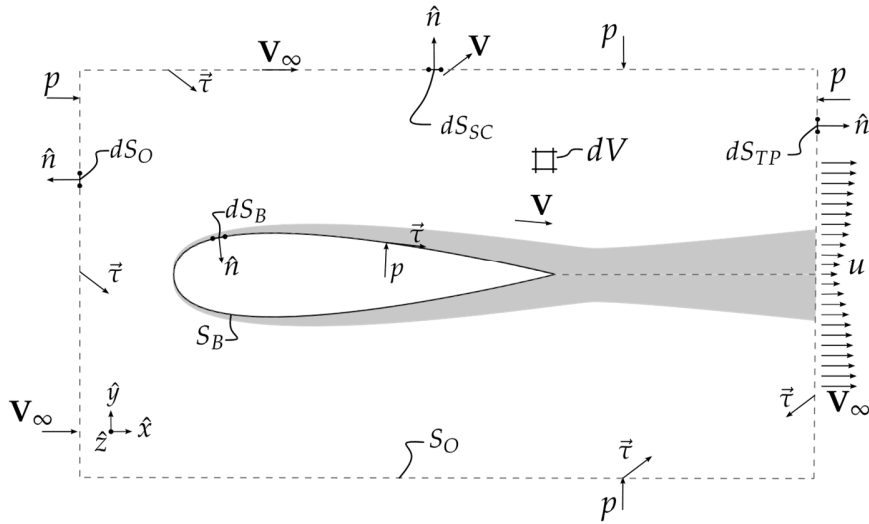


Figure 2-1 Control volume encapsulating an unpowered aerodynamic body

These formulations differ from the conventional as they represent the fluid's Euler, density, pressure, and velocity contributions  $\mathbf{V}_E$  is replaced with  $\mathbf{V}_E - \mathbf{V}_\infty$  permissible based on the assumption of continuity for the Euler field, i.e.,  $\nabla \cdot (\rho_E \mathbf{V}_E) = 0$ . The Euler pressure  $p_E$ , on the other hand, is replaced with the Euler gauge pressure  $p_{G_E} = p_E - p_\infty$  in equation (2.3), permissible due to the general relation  $\oint_S \mathbf{n} dS = 0$ .

$$\begin{aligned}
\oint_S (p_E \mathbf{n} + \mathbf{V}_E \rho_E \mathbf{V}_E \cdot \mathbf{n}) dS &= 0 \\
\underbrace{\iint_{S_B} (p_E \mathbf{n} + \mathbf{V}_E \rho_E \mathbf{V}_E \cdot \mathbf{n}) dS}_{\mathbf{F}_E} &= - \iint_{S_O} (p_E \mathbf{n} + \mathbf{V}_E \rho_E \mathbf{V}_E \cdot \mathbf{n}) dS \\
&= - \iint_{S_O} [(p_E - p_\infty) \mathbf{n} + (\mathbf{V}_E - \mathbf{V}_\infty) \rho_E \mathbf{V}_E \cdot \mathbf{n}] dS \\
\mathbf{F}_E &= - \iint_{S_O} [p_{G_E} \mathbf{n} + (\mathbf{V}_E - \mathbf{V}_\infty) \rho_E \mathbf{V}_E \cdot \mathbf{n}] dS
\end{aligned} \tag{2.3}$$

In general, for conventional 2D Euler flows where no lift is being produced, it is expected that no drag contribution is observed in the absence of shock waves. However, through this decomposition, the Euler drag  $F_{x_E} = D_E$  returns a non-zero contribution for a finite Reynolds number due to the body having a finite wake thickness. This results in a non-closed body with a finite displacement and momentum thickness, contributing to the overall drag referred to as a viscous inviscid interaction effect [15].

### 2.2.3 The Euler Mechanical Energy formulation

Taking the dot product of LHS of equation (2.2), i.e., the Euler pressure momentum equation, with the local non-dissipative velocity, it is possible to obtain the Euler mechanical energy equation shown in equation (2.4).

$$\nabla p_E \cdot \mathbf{V}_E = -\nabla \cdot \left( \rho_E \mathbf{V}_E \frac{1}{2} V_E^2 \right) \quad (2.4)$$

Expanding and simplifying the above mechanical energy formulation using the vector identity  $\nabla p_E \cdot \mathbf{V}_E = \nabla \cdot (p_E \mathbf{V}_E) - p_E \nabla \cdot \mathbf{V}_E$  is possible, as shown in equation (2.5).

$$\begin{aligned} \nabla \cdot (p_E \mathbf{V}_E) - p_E \nabla \cdot \mathbf{V}_E &= -\nabla \cdot \left( \rho_E \mathbf{V}_E \frac{1}{2} V_E^2 \right) \\ \oint_S p_E \mathbf{V}_E \cdot \mathbf{n} \, dS - \iiint_V p_E \nabla \cdot \mathbf{V}_E \, dV &= -\oint_S \rho_E \frac{1}{2} V_E^2 \mathbf{V}_E \cdot \mathbf{n} \, dS \end{aligned} \quad (2.5)$$

The substitutions  $V_E^2 \rightarrow V_E^2 - V_\infty^2$  and  $p_E \rightarrow p_E - p_\infty = p_{G_E}$  have been made for this simplification and are permissible due to the relations provided below:

$$\begin{aligned} \frac{V_\infty^2}{2} \oint_S \rho_E \mathbf{V}_E \cdot \mathbf{n} \, dS &= 0 \\ p_\infty \iiint_V (\nabla \cdot \mathbf{V}_E) \, dV &= p_\infty \oint_S \mathbf{V}_E \cdot \mathbf{n} \, dS \\ \oint_S (p_E - p_\infty) \mathbf{V}_E \cdot \mathbf{n} \, dS - \iiint_V (p_E - p_\infty) \nabla \cdot \mathbf{V}_E \, dV &= -\oint_S \rho_E \frac{1}{2} V_E^2 \mathbf{V}_E \cdot \mathbf{n} \, dS \\ \oint_S p_{G_E} \mathbf{V}_E \cdot \mathbf{n} \, dS - \iiint_V p_{G_E} \nabla \cdot \mathbf{V}_E \, dV &= -\oint_S \rho_E \frac{1}{2} (V_E^2 - V_\infty^2) \mathbf{V}_E \cdot \mathbf{n} \, dS \\ 0 &= -\iint_{S_O} \rho_E \frac{1}{2} (V_E^2 - V_\infty^2) \mathbf{V}_E \cdot \mathbf{n} \, dS - \iint_{S_O} p_{G_E} \mathbf{V}_E \cdot \mathbf{n} \, dS + \iiint_V p_{G_E} \nabla \cdot \mathbf{V}_E \, dV \end{aligned} \quad (2.6)$$

Taking the dot product of the Euler drag formulation in equation (2.3) and combining this with the vector identity  $[(\mathbf{V}_E - \mathbf{V}_\infty) \rho_E \mathbf{V}_E \cdot \mathbf{n}] \cdot \mathbf{V}_\infty = \rho_E \frac{1}{2} [(\mathbf{V}_E^2 - \mathbf{V}_\infty^2) - (\mathbf{V}_E - \mathbf{V}_\infty)^2] \mathbf{V}_E \cdot \mathbf{n}$  provides the aerodynamic drag power shown in equation (2.7).

$$\begin{aligned}
\mathbf{F}_E \cdot \mathbf{V}_\infty &= \iint_{S_O} -p_{G_E} (\mathbf{V}_\infty \cdot \mathbf{n}) dS - \iint_{S_O} (\mathbf{V} - \mathbf{V}_\infty) \rho (\mathbf{V} \cdot \mathbf{n}) dS \cdot \mathbf{V}_\infty \\
\mathbf{F}_E \cdot \mathbf{V}_\infty &= \iint_{S_O} -p_{G_E} (\mathbf{V}_\infty \cdot \mathbf{n}) dS - \iint_{S_O} \rho \frac{1}{2} \left[ (V^2 - V_\infty^2) - (\mathbf{V} - \mathbf{V}_\infty)^2 \right] \mathbf{V} \cdot \mathbf{n} dS \quad (2.7) \\
&= \iint_{S_O} p_{G_E} (\mathbf{V} - \mathbf{V}_\infty) \cdot \mathbf{n} dS - \iiint_V p_{G_E} \nabla \cdot \mathbf{V} dV + \iint_{S_O} \rho \frac{1}{2} (\mathbf{V} - \mathbf{V}_\infty)^2 \mathbf{V} \cdot \mathbf{n} dS
\end{aligned}$$

The summation of the drag power and simplified mechanical energy formulations in equations (2.6) and (2.7) provides a power balance formulation for the non-dissipative Euler field. This consists of the non-dissipative pressure-boundary work  $\dot{E}_{p_E}$ , kinetic energy, which can be split into streamwise  $\dot{E}_{a_E}$  and transverse  $\dot{E}_{v_E}$  components, and pressure-volume work, shown in equation (2.8). A unique property of this formulation is that it provides a volumetric formulation for non-dissipative pressure-volume work. It should be noted that this separability of the pressure-volume work and identification of its non-dissipative contribution is not possible with conventional power balance and mechanical energy approaches. Furthermore, the prior observation of the non-zero contribution of the Euler drag force extends to the Euler drag power as these terms only differ due to a constant multiplier, i.e.,  $\mathbf{V}_\infty$ . This implies that it is also possible to quantify the viscous inviscid interaction through the mechanical energy formulation, as suggested in equation (2.3).

$$\begin{aligned}
\mathbf{F}_E \cdot \mathbf{V}_\infty &= \iint_{S_O} p_{G_E} (\mathbf{V}_E - \mathbf{V}_\infty) \cdot \mathbf{n} dS - \iiint_V p_{G_E} \nabla \cdot \mathbf{V}_E dV + \iint_{S_O} \rho \frac{1}{2} (\mathbf{V}_E - \mathbf{V}_\infty)^2 \mathbf{V} \cdot \mathbf{n} dS \\
\therefore \mathbf{F}_E \cdot \mathbf{V}_\infty &= \dot{E}_{p_E} + \Theta_{p_E} + \underbrace{\dot{E}_{k_E}}_{\dot{E}_{a_E} + \dot{E}_{v_E}} \quad (2.8)
\end{aligned}$$

## 2.2.4 Euler field Exergy-Based Analysis

Following the work of Arntz [10,12,13], the formulations are extended to the exergy domain via the Gibbs relation, introducing the second law of thermodynamics through its associated link with entropy and internal energy. However, as a fully isentropic flow is considered, a constant entropy

field is obtained throughout the entire domain, resulting in the entropy-related term of the Gibbs relation in equation (2.9) falling away.

$$\begin{aligned}
T \nabla \cdot (\rho_E s \mathbf{V}_E) &= \nabla \cdot (\rho_E e_E \mathbf{V}_E) + p_E \nabla \cdot \mathbf{V}_E \\
\text{invoking continuity the following is permissible:} \\
\underbrace{T \nabla \cdot (\rho_E \delta s \mathbf{V}_E)}_{= 0 \text{ isentropic}} &= \nabla \cdot (\rho_E \delta e_E \mathbf{V}_E) + p_E \nabla \cdot \mathbf{V}_E
\end{aligned} \tag{2.9}$$

Where:

$$\begin{aligned}
\delta(\cdot) &= (\cdot) - (\cdot)_\infty \\
\therefore 0 &= \nabla \cdot (\rho_E \delta e_E \mathbf{V}_E) + p_\infty \nabla \cdot \mathbf{V}_E + p_{G_E} \nabla \cdot \mathbf{V}_E
\end{aligned}$$

Substituting the final expression in equation (2.9) into the mechanical energy Euler formulation in equation (2.8) yields the exergy-based Euler formulation in equation (2.10). From this, it can quickly be realised that the energy and exergy-based Euler formulations bear significant similarities, highlighting the equivalence of the Euler thermal exergy  $\dot{\mathcal{E}}_{th_E}$  (a surface integral through Gauss theorem) and the Euler pressure-volume work  $\Theta_{p_E}$  (a pure volume integral evaluation). This becomes particularly advantageous for the numerical assessment as it provides multiple means of obtaining this quantity by taking either a surface or volume integral approach, depending on whichever is more straightforward for numerical implementation.

$$\begin{aligned}
\underbrace{-\iiint_V p_{G_E} \nabla \cdot \mathbf{V}_E dV}_{\Theta_{p_E}} &= \underbrace{\iiint_V [\nabla \cdot (\rho_E \delta e_E \mathbf{V}_E) + p_\infty \nabla \cdot \mathbf{V}_E] dV}_{\dot{\mathcal{E}}_{th_E}} \\
\therefore \mathbf{F}_E \cdot \mathbf{V}_\infty &= \dot{E}_{p_E} + \dot{\mathcal{E}}_{th_E} + \underbrace{\dot{E}_{k_E}}_{\dot{E}_{a_E} + \dot{E}_{v_E}}
\end{aligned} \tag{2.10}$$

### 2.2.5 The Dissipative flow field and its Link to Boundary Layer Defect Methods

The principle of RVF momentum and mechanical energy formulations represented as superpositions of an EIF (also referred to as the Euler inviscid field in this work), and DF is primarily considered in the derivation of integral boundary layer theory (IBLT) formulation shown

in work in [1,2,16,17]. This section highlights the connection between the unified partial pressure and velocity decomposition approach adopted here and boundary layer defect analysis.

The approach shown in equation (11) makes it possible to identify the specific link between the decomposition methods carried out in this study and by Aguirre et al.[8] with boundary layer defect formulations. Although fundamentally equivalent, the decomposition approaches centre around evaluating the LHS of equation (2.11), mainly focussing on the accuracy through which the EIF is constructed to retain accuracy when assessing the DF quantities.

$$\begin{aligned} & [\text{equation}]_{RVF} - [\text{equation}]_{EIF} = [\text{equation}]_{DF} \\ \Rightarrow \iint_{CV} [\text{equation}]_{RVF} dS - \iint_{CV} [\text{equation}]_{EIF} dS &= \iint_{CV} [\text{equation}]_{DF} dS \end{aligned} \quad (2.11)$$

Taking the difference between RVF and EIF formulations, as shown in equation (2.11) and integrating over a control volume containing the DF will return an integral confined to the viscous boundary layer and wake, as depicted in Figure 2-2. Therefore, similar to the EIF formulations, the DF momentum and mechanical energy formulations can be obtained, as shown in equations (2.12) and (2.13).

$$[\nabla p + \rho(\mathbf{V} \cdot \nabla) \mathbf{V} - \nabla \cdot \bar{\boldsymbol{\tau}}] - \nabla p_E + \rho_E (\mathbf{V}_E \cdot \nabla) \mathbf{V}_E = 0 \quad (2.12)$$

$$\nabla p \cdot \mathbf{V} + \rho \mathbf{V} \cdot \frac{1}{2} V^2 - (\nabla \cdot \bar{\boldsymbol{\tau}}) \cdot \mathbf{V} - \left[ \nabla p_E \cdot \mathbf{V}_E + \nabla \cdot \left( \rho_E \mathbf{V}_E \frac{1}{2} V_E^2 \right) \right] = 0 \quad (2.13)$$

As a result, the decomposition approaches provide an *indirect* method of obtaining the DF quantities via subtraction while avoiding *direct* evaluation of the DF integrals shown on the RHS of equation (2.11) as considered in IBLT methods.

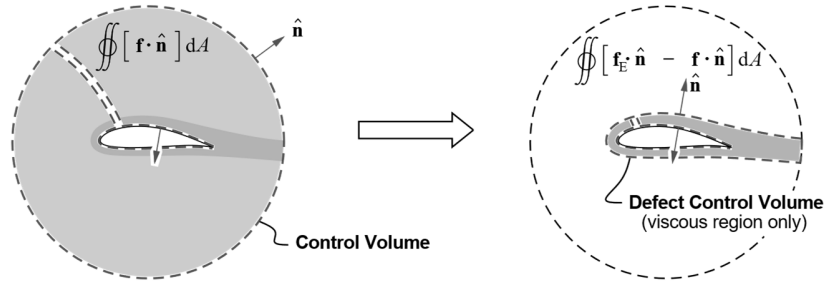


**Figure 2-2 Real viscous and defect flow control volume decompositions adapted from [18]**

Although there is a simple distinction between an indirect and direct evaluation method, a characteristic difference shown by IBLT and a significant advantage would be that the mathematical derivation process preserves the physical meaning of the various terms to boundary layer parameters, for instance, the momentum expression provided in equation (2.2) in the relative reference frame (RRF), as well as its associated mechanical energy formulation, can be expressed in defect form through the formulations derived in the work of Sato [2] and Zhang [19] provided in equations (2.14) and (2.15)<sup>2</sup>. In these formulations, the variables  $\mathbf{P}$ ,  $\mathbf{M}$ ,  $\mathbf{K}$  and  $\mathbf{D}$  represent the momentum, mass, kinetic energy and density flux defects, respectively.

$$\nabla \cdot \mathbf{P} + (\nabla \mathbf{V}_E) \cdot \mathbf{M} + \nabla \Pi + \nabla \cdot \bar{\boldsymbol{\tau}} = 0 \quad (2.14)$$

$$\nabla \Pi \cdot \mathbf{V} + \nabla \cdot (\bar{\boldsymbol{\tau}} \cdot \mathbf{V}) - (\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V} = \nabla \cdot \mathbf{K} + \mathbf{D} \cdot \left( \frac{1}{2} \nabla V_E^2 \right) \quad (2.15)$$

Notably, these defect formulations have been obtained in the absolute reference frame (ARF) through the work of Lamprakis et al. [16], which significantly aids in highlighting the Galilean invariance of forces and, more importantly, the Galilean covariance of work energy relationships [20].

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<sup>2</sup> For the readers convenience the derivatio of the momentum and mechancal energy defect equations has been provided in A.1 and A.2, respectively.

Also noted for the implementation of IBLT methods is the inclusion of the thin shear layer approximation, which considers the EIF pressure as being approximately equivalent to the local static pressure, i.e.,  $p_E \simeq p$ . In actuality, this is the same assumption applied by Aguirre [8] in the VD approach, thus implying that this thin shear layer approximation is inadvertently imposed when the DF is considered for the VD approach.

This point aids in highlighting the primary difference of the unified partial pressure and velocity decomposition (UPVD) proposed here. Instead of taking the thin shear layer approximation for the EIF pressure, it is evaluated directly through the Euler PPF Poisson equation using the procedure outlined in [15,21].

As seen before with EIF, the DF derivations can be extended for exergy-based analysis through the internal energy equation (2.16).

$$\left[ \nabla \cdot (\rho e \mathbf{V}) + p \nabla \cdot \mathbf{V} - (\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V} + \nabla \cdot \mathbf{q} \right] - \left[ \nabla \cdot (\rho_E e_E \mathbf{V}_E) + p_E \nabla \cdot \mathbf{V}_E \right] = 0 \quad (2.16)$$

For the EIF, this extension was priorly obtained using the Gibbs equation discussed in Section 2.2.4. Furthermore, the entropy formulation in equation (2.17) is also included in the DF formulations to introduce various anergy-related terms, as shown in prior work by Arntz [10,12,13]. For the analyses conducted in this work, the EIF is considered fully isentropic, thus yielding an overall contribution of zero for the entropy formulation, as shown in equation (2.17).

$$\left[ \underbrace{T_\infty \nabla \cdot (\rho s \mathbf{V})}_{\dot{A}_{total}} - \underbrace{\frac{T_\infty}{T} (\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V}}_{\dot{A}_\Phi} - \underbrace{\frac{T_\infty}{T^2} k (\nabla \mathbf{T})^2}_{\dot{A}_{VT}} + \nabla \cdot \frac{T_\infty}{T} \mathbf{q} \right] - 0 = 0 \quad (2.17)$$

### 2.2.5.1 Assessment of flow field continuity

For classical IBLT methods, the decomposition of an RVF into an EIF and DF is strictly governed by two constraints provided in equation (2.18). The first is that the EIF is an irrotational continuation of the outer inviscid flow into the shear layer [18,22]. The second maintains that the

EIF velocity must be equal to the RVF velocity outside the boundary layer region, naturally satisfied as the outer bulk flow is commonly viewed as inviscid.

$$\begin{aligned}\nabla \times \mathbf{V}_E &\equiv \omega_E = 0 \quad (\text{everywhere}) \\ \mathbf{V}_E &= \mathbf{V} \quad (\text{outside boundary layer})\end{aligned}\tag{2.18}$$

The RVF's normal and tangential velocities at the wall are zero due to the no-slip condition, which implies a zero-flux boundary. The EIF, on the other hand, consists of a non-zero *wall transpiration velocity*  $v_{E_{wall}}$  shown in Figure 2-4, which results in the flow not being tangent to the body. In addition, wall transpiration is included to account for the displacement effect due to the boundary layer in RVFs, ensuring this effect is also observed in the EIF.

$$\begin{aligned}\nabla \cdot (\rho \mathbf{V} - \rho_E \mathbf{V}_E) &= 0 \quad (\text{a}) \\ \nabla \cdot \mathbf{M} - \underbrace{(\rho \mathbf{V}_E)_{wall} \cdot \mathbf{n}_{wall}}_{\text{wall transpiration}} &= 0 \quad (\text{b})\end{aligned}\tag{2.19}$$

For a simple inviscid flow model approach, also shown in Figure 2-3, the wall transpiration is negligible, therefore assuming flow tangency to the body surface and  $\nabla \cdot \mathbf{M} \simeq 0$  from the continuity formulation in equation (2.19). This assumption does not incur significant errors for thin boundary layers and can thus be omitted without extensive penalties. However, the displacement effect significantly affects the bulk flow tangency for thicker boundary layers, resulting in significant differences in velocity, pressure, and lift obtained from the EIF using this *simple inviscid flow* model.

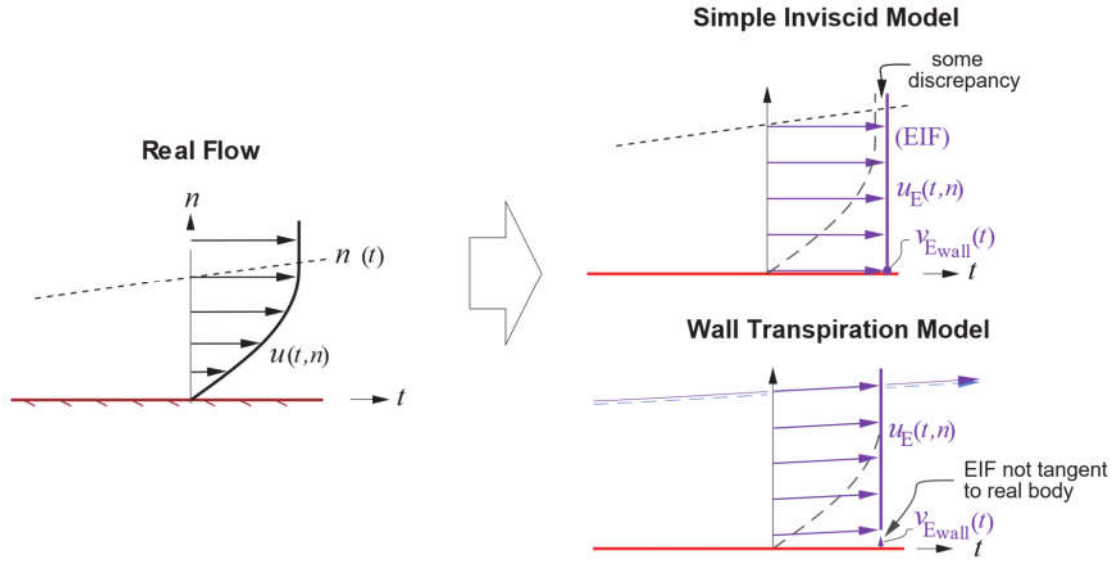


**Figure 2-3 Real viscous flow EIF reconstruction using a simple inviscid (top) and wall transpiration model (bottom) [22]**

Similar to the VD approach [8], the unified approach assumes that loss convection in the DF is aligned with the RVF, and consequently, the EIF vectors will similarly be aligned. By taking this assumption for the reconstruction of the velocity field and its associated vectorial direction, a simple inviscid flow model is implied as no wall transpiration velocity is included to modify the EIF. As a result, assessing the applicability of the simple inviscid flow model for the various test cases presented in this work becomes paramount. Particularly of interest is satisfying the continuity condition stated in equation (2.19)(a), as this becomes heavily reliant on the applicability of the simple inviscid model chosen for the reconstruction of the EIF. Control volumes of varying sizes are selected to assess this, as shown in Figure 2-4.

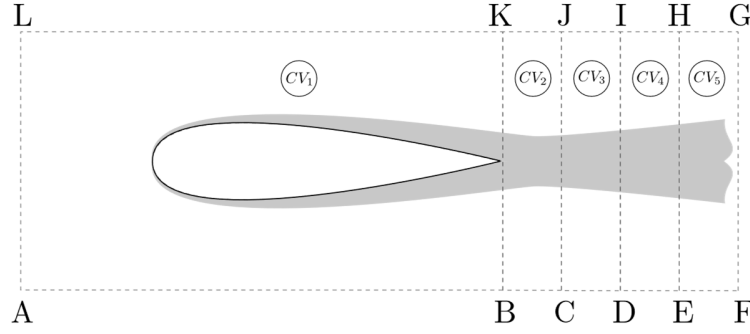


**Figure 2-4 Various contours for continuity condition assessment**

### 2.2.5.2 Dissipative Force and Power Evaluations

The DF force contribution can be obtained using equation (2.11) by taking the difference between the RVF and EIF solutions. Starting with the *near-field* forces, conventionally, this can be split into its binary pressure and skin friction contributions. However, equation (2.20) shows that the EIF force comprises only the Euler pressure contribution, while the DF combines the dissipative pressure and skin friction contributions [21,23].

$$\begin{aligned} \underbrace{\mathbf{F}_{nf}}_{\mathbf{F}_{RVF}} &= \mathbf{F}_p + \mathbf{F}_\tau \\ \therefore \underbrace{\mathbf{F}_p + \mathbf{F}_\tau}_{\mathbf{F}_{RVF}} - \underbrace{\mathbf{F}_{p_E}}_{\mathbf{F}_{EIF}} &= \underbrace{(\mathbf{F}_{p_\mu} + \mathbf{F}_\tau)}_{\mathbf{F}_{DF}} \end{aligned} \quad (2.20)$$

$$\underbrace{\mathbf{F}_{far}}_{\mathbf{F}_{RVF}} - \underbrace{\mathbf{F}_i}_{\mathbf{F}_{EIF}} = \underbrace{\mathbf{F}_v}_{\mathbf{F}_{DF}} \quad (2.21)$$

*Farfield* drag decomposition into viscous and induced drag, from equation (2.21), highlights that for an ideal decomposition, the EIF would provide an accurate measure of the induced drag ( $\mathbf{F}_i \cdot \mathbf{e}_x$ ) whilst the DF would provide the viscous drag contribution ( $\mathbf{F}_v \cdot \mathbf{e}_x$ ). However, as mentioned prior, due to the EIF having a BL and wake of finite thickness, a small portion of the viscous drag would be contained within the EIF due to viscous inviscid interaction.

$$\underbrace{\mathbf{F}_{\dot{E}} \cdot \mathbf{V}_{\infty}}_{\mathbf{F}_{RVF} \cdot \mathbf{V}_{\infty}} - \underbrace{\mathbf{F}_E \cdot \mathbf{V}_{\infty}}_{\mathbf{F}_{EIF} \cdot \mathbf{V}_{\infty}} = \underbrace{\mathbf{F}_{\mu} \cdot \mathbf{V}_{\infty}}_{\mathbf{F}_{DF} \cdot \mathbf{V}_{\infty}} \quad (2.22)$$

$$\left( \dot{E}_m + \Phi + \Theta \right) - \left( \dot{E}_{m_E} + \Theta_E \right) = \left( \dot{E}_{m_{\mu}} + \Phi + \Theta_{\mu} \right)$$

$$\underbrace{\mathbf{F}_{\dot{E}} \cdot \mathbf{V}_{\infty}}_{\mathbf{F}_{RVF} \cdot \mathbf{V}_{\infty}} - \underbrace{\mathbf{F}_E \cdot \mathbf{V}_{\infty}}_{\mathbf{F}_{EIF} \cdot \mathbf{V}_{\infty}} = \underbrace{\mathbf{F}_{\mu} \cdot \mathbf{V}_{\infty}}_{\mathbf{F}_{DF} \cdot \mathbf{V}_{\infty}} \quad (2.23)$$

$$\left( \dot{E}_m + \dot{\mathcal{E}}_{th} + A_{total} \right) - \left( \dot{E}_{m_E} + \dot{\mathcal{E}}_{th_E} \right) = \left( \dot{E}_{m_{\mu}} + \dot{\mathcal{E}}_{th_{\mu}} + A_{total} \right)$$

$$\left( \dot{\mathcal{E}}_{phy} + A_{total} \right) - \left( \dot{\mathcal{E}}_{phy_E} \right) = \left( \dot{\mathcal{E}}_{phy_{\mu}} + A_{total} \right)$$

All dissipative field components can be obtained for mechanical energy and exergy analysis by taking the difference between the RVF and EIF, including the mechanical energy  $\dot{E}_m$ , thermal exergy  $\dot{\mathcal{E}}_{th}$  and pressure-volume work  $\Theta$  as shown in equations (2.22) and (2.23).

The resulting DF formulation can capture viscous loss mechanisms within the boundary layers (BLs) and wakes, contributing toward the overall drag. Aside from the influence of viscous inviscid effects, the positive monotonically increasing trend of the dissipative force is similar to the characteristics of viscous dissipation. This can evaluate the accumulation of drag over aerodynamic bodies while filtering out the non-dissipative/ isentropic bulk flow force contributions.

The benefit of this flow decomposition is its region-specific identification of the phenomenological drag contributions. Mechanisms associated with the wake, such as the viscous drag, can now be quantified through the dissipative field, and the remaining bulk flow phenomena, such as lift and lift-induced drag, can be assessed through the EIF.

The formulations used in the numerical case studies in this work aim to highlight the interconnected nature of near- and farfield methods as well as energy & exergy approaches. For example, as summarised in equation (2.20) - (2.23), the dissipative force and its associated drag power can be expressed in various forms, following a momentum-based approach through near- and farfield methods or extending to take advantage of power balance and exergy-based

decompositions. This becomes useful to verify the posed formulaic manipulations and provide alternative means for performance evaluation.

### 2.2.5.3 Numerical implementation of the partial pressure field and velocity decomposition approaches

An advantage of the UPVD approach is its ability to *approximate* the EIF pressure field for a real viscous flow CFD solution through the Euler Poisson expression expressed in equation (2.24). As a result, the static pressure assumption, i.e.,  $p_E \approx p$  priorly taken in the VD approach [8], can be avoided while extending the flow field decomposition approach to the near-field.

$$\nabla^2 p_E \approx -\rho \nabla \cdot (\mathbf{V} \cdot \nabla) \mathbf{V} \quad (2.24)$$

Reynolds-averaged Navier–Stokes simulations were performed on all analyses conducted in this work using the pressure-based solver in ANSYS Fluent ©, using the Green-Gauss cell-based method with the coupled pseudo-transient algorithm. Turbulence was modelled using the one-equation Spalart–Allmaras eddy-viscosity model [24] with second-order up-winding for all numerical fluxes. Flow residuals convergence was set to below  $1 \times 10^{-6}$  while maintaining a  $y^+ \leq 0.5$  everywhere around the body for compatibility with the Spalart–Allmaras model and PPF analysis.

Numerically, the Euler Poisson expression in equation (2.24) was implemented through a user-defined scalar transport formulation shown in equation (2.25)(a) as done in the work of Hart and Schmitz [21]. The transient, convective and diffusion coefficient terms were set such that the transport equation matched equation (2.24), i.e., no transient term (as the flow analyses are steady), no convective term and a diffusion coefficient  $\Xi$  of 1. The RHS source term in the reduced transport equation (2.25)(b) was then set to the RHS of equation (2.24) to obtain the Euler pressure scalar field whose equation was converged to a residual below  $1 \times 10^{-12}$  as suggested in [15].

$$\begin{aligned} \frac{\partial(\rho\phi)}{\partial t} + \rho(\mathbf{V} \cdot \nabla)\phi &= \nabla \cdot (\Xi \nabla \phi) + S_\phi & (a) \\ \nabla^2 \phi &= -S_\phi & (b) \end{aligned} \quad (2.25)$$

The solved Euler pressure field can then be substituted into the isentropic relations in equation (2.26) to obtain the reconstructed Euler velocity, temperature, and density fields.

$$\begin{aligned} V_E &= V_\infty \sqrt{1 - \frac{2}{(\gamma-1)M_\infty^2} \left[ \left( \frac{p_E}{p_\infty} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]} & (a) \\ T_E &= T_\infty \left[ 1 + \frac{(\gamma-1)}{2} M_\infty^2 \left( 1 - \frac{V_E^2}{V_\infty^2} \right) \right] & (b) \\ \rho_E &= \rho_\infty \left[ 1 + \frac{(\gamma-1)}{2} M_\infty^2 \left( 1 - \frac{V_E^2}{V_\infty^2} \right) \right]^{\frac{1}{\gamma-1}} & (c) \end{aligned} \quad (2.26)$$

The computational grids used were of the C-H type, as shown in Figure 2-5, with far-field and outlet grid boundaries located 30 reference chord lengths away from the respective body. Appropriate Dirichlet and Neuman boundary conditions were set to solve the Euler-Poisson equation. In the far field, a Dirichlet boundary condition was set assigning the Euler pressure to the boundary's static pressure, i.e.,  $p_E = p$  at the outlets and walls where a no-slip boundary is enforced, a natural Neumann boundary condition exists where  $\nabla p_E = 0$ .

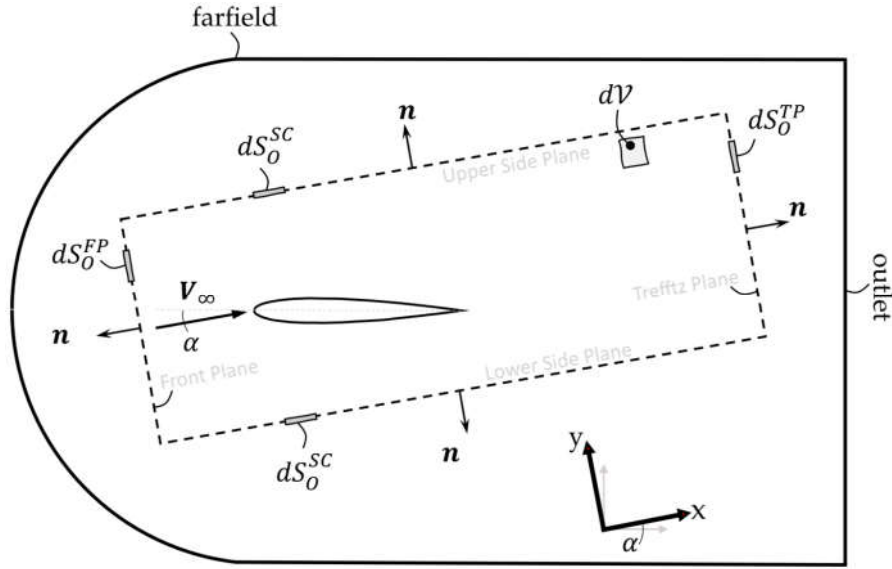


Figure 2-5 Schematic of C-H domain, illustrating fluid domain and control volume boundaries

### 2.2.6 The potential for energy recovery and the absolute reference frame perspective

Integral analysis through the Reynolds transport theorem allows the governing fluid equations to be transferred from a Lagrangian (system) approach following a single particle to an Eulerian (control volume) approach observing fluid particles travelling through a fixed point in space [25].

$$\left[ \frac{D}{Dt} ( \quad ) \right]_{\text{system}} = \frac{\partial}{\partial t} \iiint_{CV} ( \quad ) dV + \oint_{CS} ( \quad ) \mathbf{V} \cdot \mathbf{n} dS \quad (2.27)$$

For a *fixed control volume* in an *arbitrary reference frame*, the velocity field is measured relative to this frame of reference, with the fluid velocities being expressed relative to the control volume. Therefore, an observer fixed to the control volume will see a relative velocity  $\mathbf{V} = \mathbf{V}_A - \mathbf{V}_{CS}$  where  $\mathbf{V}_A$  is the fluid velocity relative to the same coordinate system in which the control volume motion  $\mathbf{V}_{CS}$  is observed [26].

$$\left[ \frac{D}{Dt}(\cdot) \right]_{\text{system}} = \frac{\partial}{\partial t} \iiint_{CV} (\cdot) dV + \oint_{CS} (\cdot) (\mathbf{V}_A - \mathbf{V}_{CS}) \cdot \mathbf{n} dS \quad (2.28)$$

Considering a control volume moving at a constant flight velocity equal to that of the aerodynamic body, i.e.,  $\mathbf{V}_{CS} = \mathbf{V}_B = -\mathbf{V}_\infty$  it is possible to realise that  $\mathbf{V} = \mathbf{V}_A + \mathbf{V}_\infty$ , which is the same expression provided by Sanders and Laskaridis [27] and Lamprakis et al.[20] for relative to absolute reference frame (ARF) velocity transformation. Although not explicitly considered as a reference frame transformation, momentum, energy, and exergy derivations initially in the relative reference frame (RRF) perform a Galilean conversion into the ARF through the *perturbation velocity vector* [1] substitution, which is analogous to the absolute velocity vector  $\mathbf{V}_A$  in the ARF.

In principle, as viewed from the RRF, exergy-based analyses consider perturbations of the system as having an inherent energetic potential that can be converted into useful work [8]. As a result, it becomes theoretically possible to extract work from these perturbations through mechanisms such as BLI. Mainly focusing on the velocity, these perturbations can be of either sense, i.e., positive ( $>0$ ) or negative ( $<0$ ), with positive perturbations being associated with propulsive jets. In contrast, negative perturbations are associated with the slower-moving boundary layer and wake flow as viewed from the RRF.

The ARF aids in identifying the sense of the perturbations (observed from RRF) directly associated with BLI benefits. In the ARF, all energy required for flight is supplied by the aircraft and is transferred to the atmosphere via the no-slip condition. This reveals the energy transformation pathway providing available energy to BLI systems, which is present within the local flow ARF kinetic energy, shown in Figure 2-6. This is also linked to the negative perturbations within the BL and wake as perceived from the RRF, which provides available kinetic energy as observed from this frame of reference, also shown in Figure 2-6.

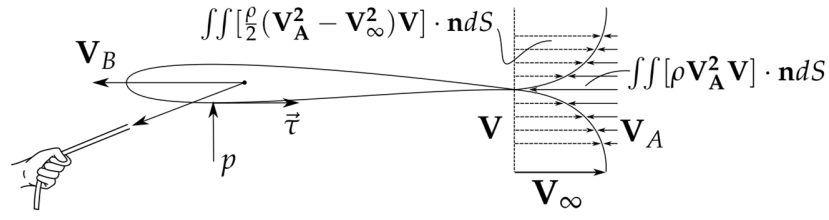


**Figure 2-6 Kinetic energy comparison in the relative vs absolute reference frame adapted from [27]**

Various performance metrics from power balance and exergy analysis have been introduced to track and estimate this work potential. For example, the exergy waste coefficient (EWC) introduced by Arntz [10,12], in equation (2.29)(a), tallies the amount of physical exergy (i.e., combined mechanical and thermal exergy) relative to the total drag power of the configuration under test. On the other hand, the potential for energy recovery (PER) from the work of Sanders and Laskaridis [27] is obtained via mechanical energy analysis and was later improved by Lamprakis et al. [20] for compressible flows. The formulation proposed in [27] tallies the accumulation of viscous dissipation compared to the total drag power, as shown in equation (2.29)(b). In contrast, the formulation by Lamprakis et al. [20], given in equation (2.29)(c), additionally includes the thermo-compressibility losses, which provides a more conservative estimate of the PER as the resulting loss fraction is higher relative to the total drag power. The work herein introduces a further modification to PER as well as EWC shown in equations (2.29)(d) and (e), respectively. This approach takes advantage of the flow field decomposition brought about through partial pressure field and velocity decomposition and considers recoverable energy as only being associated with the dissipative mechanical and thermal energy components.

$$EWC = \frac{\dot{\mathcal{E}}_{phy}}{DV_{\infty}} \quad (a)$$

$$PER = 1 - \frac{\Phi}{DV_{\infty}} \quad (b)$$

$$PER_c = 1 - \left( \frac{\Phi + \Theta_{\infty}}{DV_{\infty}} \right) \quad (c) \quad (2.29)$$

$$PER_{\mu} = \frac{\dot{E}_{m_{\mu}}}{DV_{\infty}} \quad (d)$$

$$EWC_{\mu} = \frac{\dot{\mathcal{E}}_{phy_{\mu}}}{DV_{\infty}} \quad (e)$$

## 2.3 Case Study Numerical Implementation in CFD

### 2.3.1 Numerical Case Study I: Flat Plate

#### 2.3.1.1 Analytical Assessment

This section assesses the accumulation of energy imparted to the flow for a zero-pressure gradient flat plate. As there is no pressure gradient, the non-dissipative and static pressure fields are identical, i.e.  $p_E = p = p_{\infty}$ , a natural assumption already taken by the VD approach. Therefore, as this is physically true, the VD and UPVD formulations become equivalent.

$$\mathbf{V}_E = \begin{Bmatrix} u_E + V_{\infty} \\ v_E \\ w_E \end{Bmatrix} \text{ and } \mathbf{V}_{\infty} = \begin{Bmatrix} V_{\infty} \\ 0 \\ 0 \end{Bmatrix}$$

Substituting  $p_E = p_{\infty}$  into isentropic relations : (2.30)

$$\mathbf{V}_E = \mathbf{V}_{\infty} \Rightarrow \mathbf{V}_E - \mathbf{V}_{\infty} = \begin{Bmatrix} u_E \\ v_E \\ w_E \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

By substituting the aforementioned pressure field relation into equation (2.26)(a), the non-dissipative velocity is equal to the freestream as given in equation (2.30). After simplifying these

relations, the non-dissipative mechanical exergy quantities become zero, as shown in equation (2.31).

$$\begin{aligned}
\dot{E}_{a_E} &= \iint_{S_o} \frac{1}{2} \rho_E \underbrace{u_E^2}_{=0, u_E=0} (V_\infty + u_E) dS = 0 \\
\dot{E}_{v_E} &= \iint_{S_o} \frac{1}{2} \rho_E \underbrace{(v_E^2 + w_E^2)}_{=0, v_E+w_E=0} (V_\infty + u_E) dS = 0 \\
\dot{E}_{p_E} &= \iint_{S_o} \underbrace{(p_E - p_\infty)}_{=0, p_E=p_\infty} (V_\infty + u_E) dS = 0
\end{aligned} \tag{2.31}$$

A similar approach can be taken for the thermal exergy as shown in equation (2.32), wherein the non-dissipative contributions are also null as their respective temperature and density quantities are equal to the freestream.

$$\begin{aligned}
&\text{Substituting } p_E = p_\infty \Rightarrow T_E = T_\infty; \rho_E = \rho_\infty \\
\therefore \dot{\mathcal{E}}_{th_E} &= \iint_{S_o} \rho_E c_v T_E \underbrace{\left[ 1 - \frac{T_\infty}{T_E} \ln \left( \frac{T_E}{T_\infty} \right) \right]}_{=0, T_E=T_\infty} (V_\infty + u_E) dS = 0 \\
\dot{\mathcal{E}}_{th_{p_E}} &= \iint_{S_o} p_\infty \underbrace{\left[ 1 - \frac{\rho_\infty}{\rho_E} \ln \left( \frac{\rho_E}{\rho_\infty} \right) \right]}_{=0, \rho_E=\rho_\infty} (V_\infty + u_E) dS = 0
\end{aligned} \tag{2.32}$$

As shown through the derivations in Section 2.2 and summarised in equation (2.33), the mechanical energy, thermal exergy and pressure-volume work terms can be split into dissipative and non-dissipative contributions.

$$\begin{aligned}
\dot{E}_m &= \dot{E}_{m_\mu} + \underbrace{\dot{E}_{m_E}}_{=0} \\
\Theta &= \Theta_\mu + \underbrace{\Theta_E}_{=0} \\
\dot{\mathcal{E}}_{th} &= \dot{\mathcal{E}}_{th_\mu} + \underbrace{\dot{\mathcal{E}}_{th_E}}_{=0}
\end{aligned} \tag{2.33}$$

Furthermore, as all non-dissipative contributions are equal to zero, the total drag power can be evaluated to the same level of accuracy for conventional power balance and exergy analysis. Similarly, velocity decomposition and the newly formulated unified approach would also provide the same results, as all these methods become equivalent for this particular test case.

### 2.3.1.2 Operating Condition and Modelling Considerations

The case study operating conditions and geometric lengths were chosen following the work of Diskin et al. [28] for validation. The flat plate length was taken as  $c = 2\text{m}$  with a unit length Reynolds number of  $5 \times 10^6$  at a freestream Mach number of  $M_\infty = 0.20$ .

The grid convergence study summarised in Table 2-1 was conducted at several mesh refinement levels, with L1 representing the finest grid. Grid convergence was assessed based on the numerical agreement of the drag coefficients obtained from near- and far-field momentum methods and the accompanying energy and exergy-based quantities. For example, at the finest grid refinement level, all drag coefficients showed at most a  $\pm 1\%$  difference between their respective values while showing a percentage error of  $< 4\%$  in comparison to the near-field validation data obtained from Diskin et al. [28] provided in brackets in Table 2-1.

**Table 2-1 Flat plate drag coefficient grid convergence study reported in drag counts ( $C_d \times 10^{-4}$ )**

| M = 0.20, Re/c = $5 \times 10^6$ , $\alpha = 0^\circ$ [28] |             |          |        |        |
|------------------------------------------------------------|-------------|----------|--------|--------|
| Grid (# cells)                                             | Near-field  | Farfield | Energy | Exergy |
| L4 (23496)                                                 | 29.4        | 29.6     | 30.5   | 30.4   |
| L3 (66696)                                                 | 29.4        | 29.5     | 29.8   | 29.6   |
| L2 (257208)                                                | 29.4        | 29.5     | 29.9   | 29.7   |
| L1 (315772)                                                | 29.4 (28.7) | 29.4     | 29.7   | 29.6   |

As mentioned in Sections 2.2.5.1, the UPVD and VD approaches neglect the effects of wall transpiration in the reconstruction of the EIF and assume it to be negligible. As a result, this causes

metric evaluation error using EIFs reconstructed using the simple inviscid model. These errors manifest themselves as viscous inviscid interaction effects, also referred to as “missing drag” in the work of Aguirre [29]. This occurs due to the imposed effects of omitting wall transpiration on *continuity*, as shown in equation (2.19). The case studies conducted here evaluate continuity on various control volumes described in Figure 2-4 to assess the viscous inviscid impact extent. All the control volumes have a common front plane located at ten chord lengths upstream of the leading edge and top and bottom planes ten chord lengths above and below the airfoil trailing edge. Apart from continuity, the effects of the omission of wall transpiration are assessed for the ideal gas formulation. This is tested by obtaining the average gas constant (R) around the control volumes, which is expected to remain constant for the RVF, EIF and DF.

**Table 2-2 Flat plate continuity and ideal gas constant assessment for various control volumes**

| Contour | x/c | RVF                              |        | EIF (using velocity decomposition)   |        | EIF (using unified approach)         |        |
|---------|-----|----------------------------------|--------|--------------------------------------|--------|--------------------------------------|--------|
|         |     | $\nabla \cdot (\rho \mathbf{V})$ | R      | $\nabla \cdot (\rho^* \mathbf{V}^*)$ | $R^*$  | $\nabla \cdot (\rho_E \mathbf{V}_E)$ | $R_E$  |
| ABKL    | 1   | 0.05                             |        | 0.20                                 |        | 0.17                                 |        |
| ACJL    | 3   | 0.04                             |        | 0.09                                 |        | 0.05                                 |        |
| ADIL    | 5   | 0.04                             | 287.04 | 0.07                                 | 287.06 | 0.04                                 | 287.06 |
| AEHL    | 7   | 0.03                             |        | 0.06                                 |        | 0.03                                 |        |
| AFGL    | 9   | 0.03                             |        | 0.05                                 |        | 0.03                                 |        |

Following the procedure described in Section 2.2.5.1, a continuity assessment is performed along the various control volumes for the finest grid (L1). The RVF continuity quantities shown in Table 2-2 are obtained to provide benchmark values for comparison against the reconstructed EIFs. The VD reconstructed EIF is then assessed to indicate the degree of relative change observed by switching from the static pressure assumption to the partial pressure field evaluation of the Euler pressure field. It can be observed from Table 2-2 that near the trailing edge (i.e., ABKL), the VD and UPVD approach both provide values much higher than those shown by the RVF. This is due

to the strong viscous inviscid interaction (VII) effects observed at the body trailing as a result of the high pressure gradients experienced in this region.

Further from the trailing edge, where the magnitude of the VII effects has diminished, the values reported by the UPVD and VD approaches tend to show good agreement with those observed for the RVF. This indicates that flow quantities obtained from EIF and DF decompositions will primarily have a high uncertainty near the trailing edge, becoming more accurate further downstream as VII effects diminish. Finally, for the gas constant comparison, both VD ( $R^*$ ) and UPVD ( $R_E$ ) approaches provide the same value, which deviates slightly from the RVF by approximately  $< 0.01\%$ .

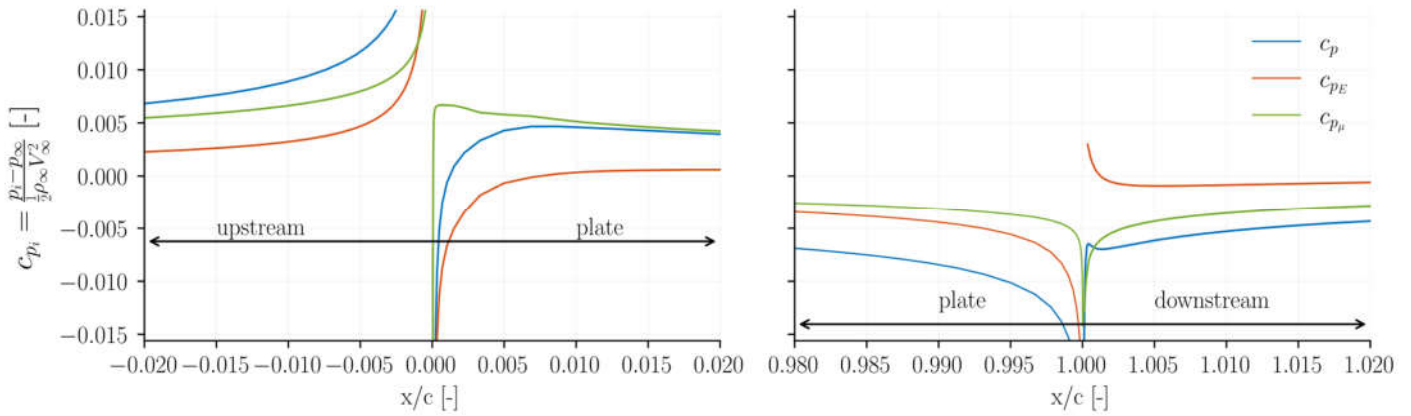


**Figure 2-7 Surface pressure coefficients near the leading (left) and trailing edge (right)**

Figure 2-7 shows the pressure distribution over the flat plate, additionally decomposed into its constituent Euler and dissipative pressure components. It can be observed that in the upstream region, there is a pressure gradient which develops at the leading edge. Considering the ARF, the body travels through a quiescent fluid, forming a stagnation point at the leading edge. High pressures are experienced in this region due to the flow being displaced by the body during its motion. The non-penetration/zero-flux condition at the body surface results in the flow streamlines being displaced outwards to conform to its shape; this streamline deflection propagates outward into the bulk flow in all directions, forming a pressure and corresponding velocity field.

The pressure gradient formed at the leading edge shows a monotonic decay ahead of the body, as shown in Figure 2-7. An important aspect to note is the rate of decay of the pressure ahead of the body, which can be seen for the Euler pressure tends to decay more rapidly than the RVF pressure ( $c_p$ ). This occurs due to viscosity, dampening its decay rate and further accelerating its propagation.

The streamlines are deflected by an effective body shape consisting not only of the body contour, as with the Euler flow, but now include the viscous boundary layer formed over its surface—the combined displacement results in the overall change in pressure observed for  $c_p$ . The dissipative pressure then describes the sole effect of the boundary layer on the pressure field developed around the body.

The leading and trailing edge pressures over the flat plate are discontinuous, as observed in [28] which tends to affect all the pressure distributions near the leading and trailing edge. However, this effect diminishes at a fixed distance from the edges, primarily observed due to discretisation errors resulting from the singularities observed at the leading and trailing edges.

Pressure recovery is observed in the downstream region behind the flat plate, where the Euler pressure is observed to rapidly asymptote back toward the ambient pressure leaving the dissipative pressure as the primary contributor toward the RVF static pressure. Therefore, there may be significant Euler pressure contributions at the trailing edge that rapidly asymptotes toward the ambient pressure within a few chord lengths. In this instance, for this particular region, the assumption  $p_E = p$  proposed by Aguirre [8] would not be valid, and instead, the contrary would be true, i.e.,  $p_\mu = p$ . This observation essentially highlights the difference between the VD and unified approaches. Furthermore, the decomposition of the pressure field allows for the dissipative pressure contribution within the wake to be quantified, as shown in Figure 2-7, which impacts the method through which energy and exergy formulations evaluate dissipative pressure work components, as mentioned in Section 2.2.5.2.

### 2.3.1.3 Generalised Flat Plate Dissipative Drag Analysis

Conventionally near-field drag over an aerodynamic body consists of pressure and skin friction; as the classic example of a zero-pressure gradient flat plate with no angle of incidence is considered, only the skin friction component contributes toward the aerodynamic force, as shown in Figure 2-8. This becomes a prime example for aerodynamic assessment in the absence of pressure gradients, particularly concerning exploring the relationship between the potential for energy recovery (PER) and fluid momentum diffusivity ( $\nu$ ).

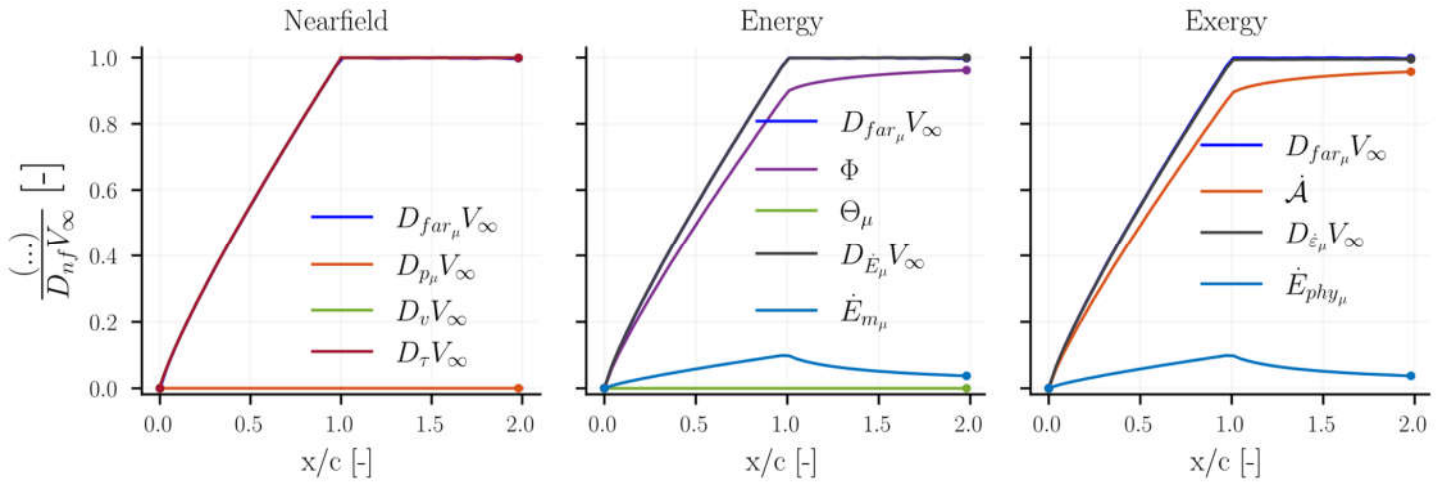


**Figure 2-8 Flat plate near-field, farfield, energy and exergy dissipative drag accumulation plots**

Figure 2-8 shows that energy recovery only becomes intuitively apparent after introducing energy and exergy-based methods, particularly in the ARF, through the dissipative mechanical energy  $\dot{E}_{m_\mu}$ . This term is strongly related to the potential for energy recovery, i.e., PER, through the expressions detailed in equation (2.29). Additionally, PER can be split into two main contributions shown in (2.34), the first being the momentum diffusivity due to shear stress, which is a strong function of fluid properties such as the kinematic viscosity ( $\nu$ ). The second is momentum diffusivity due to pressure, related to characteristics affecting the pressure gradients over the body.

$$PER = PER_\tau + PER_{\nabla P} \quad (2.34)$$

Therefore, momentum diffusion is strictly due to the applied shear stress for a zero-pressure gradient flat plate. In this instance,  $PER_\tau$  can be loosely described as a macroscopic non-dimensional factor highlighting the *viscous lag rate*<sup>3</sup> between the accumulation of drag (i.e., applied stress per unit area) and viscous dissipation (associated with the strain-rate magnitude) whose behaviour is depicted in the *energy-based* dissipative drag accumulation plots in Figure 2-8. From an *exergy-based* perspective, this lag represents an accumulation of physical exergy over the body that can be utilised to produce mechanical and thermodynamic work. The exergy accumulation peaks at the trailing edge and is deposited into the downstream wake, gradually decaying as it is irreversibly lost to the surroundings as heat.

At the fluid element level, the applied stress ( $\bar{\tau}$ ) is related to the strain rate ( $\bar{S}$ ) via the fluid viscosity ( $\mu$ ) using the general expression  $\bar{\tau} = \mu \bar{S}$ . Through this, it is possible to realise that for a given applied stress, a higher  $\mu$  improves the *momentum diffusion* between the body and fluid by reducing the strain rate, which lowers viscous dissipation. Therefore, regarding  $PER_\tau$ , higher viscosities are beneficial as they increase the *viscous lag rate*, indicating a decrease in viscous dissipation accumulation.

## 2.3.2 Numerical Case Study II: NACA 0012 Airfoil

### 2.3.2.1 Operating Condition and Modelling Considerations

The NACA 0012 analyses assessed in this section extend the application of the UPVD approach to *non-lifting* bodies with geometric curvature as well as *lifting* bodies in compressible flow. The *non-lifting* test case considers a NACA 0012 airfoil at zero degrees angle of attack, whose operating conditions have been chosen based on the work of Jespersen et al. [30]. The airfoil chord is taken as  $c = 0.435\text{m}$  operating at a chord-based Reynolds number of  $6 \times 10^6$  and a subsonic incompressible freestream Mach  $M_\infty = 0.15$ .

---

<sup>3</sup> The use of the term *viscous* is chosen intentionally, to highlight that without viscous effects no *PER* can exist, a deduction similarly observed by Aguirre [14]

The *lifting* test case operating conditions are taken from the work of Aguirre et al. [8] and consider a NACA 0012 airfoil at a subsonic compressible Mach number of  $M_\infty = 0.30$ , a chord-based Reynolds number of  $3 \times 10^6$  and an angle of attack of  $10^\circ$ .

Table 2-3 shows the grid convergence study for both the lifting and non-lifting test cases. Similar to the procedure before for the flat plate, several mesh refinement levels were assessed, with L1 representing the finest grid level. Grid convergence was evaluated based on the numerical agreement of the drag coefficients for the various force assessment methods as well as their relative agreement with numerical data from literature. Results at the finest grid refinement level showed good agreement for the different force assessment methods. Their values agreed to within  $\pm 2\%$  with a relative error of  $< 5\%$  compared with the validation data from literature indicated in brackets.

**Table 2-3 NACA 0012 drag coefficient grid convergence study reported in drag counts ( $C_d \times 10^{-4}$ )**

| M = 0.15, Re = $6 \times 10^6$ , $\alpha = 0^\circ$ [30] |               |               |        |               |
|----------------------------------------------------------|---------------|---------------|--------|---------------|
| Grid (#cells)                                            | Near-field    | Farfield      | Energy | Exergy        |
| L5 (46107)                                               | 82.2          | 81.7          | 83.7   | 83.4          |
| L4 (184428)                                              | 83.7          | 82.2          | 83.4   | 83.2          |
| L3 (414963)                                              | 83.6          | 83.0          | 83.5   | 83.3          |
| L2 (737712)                                              | 83.6          | 82.5          | 83.3   | 83.2          |
| L1 (1070925)                                             | 83.6 (83.0)   | 82.5          | 83.3   | 83.2          |
| M = 0.30, Re = $3 \times 10^6$ , $\alpha = 10^\circ$ [8] |               |               |        |               |
| Grid (#cells)                                            | Near-field    | Farfield      | Energy | Exergy        |
| L5 (46107)                                               | 169.2         | 165.0         | 162.5  | 161.5         |
| L4 (184428)                                              | 163.9         | 160.4         | 161.6  | 161.0         |
| L3 (414963)                                              | 163.1         | 159.6         | 161.5  | 160.8         |
| L2 (737712)                                              | 162.8         | 159.3         | 161.4  | 160.7         |
| L1 (1070925)                                             | 162.7 (154.1) | 159.2 (151.5) | 161.3  | 160.6 (154.0) |

The ideal gas and continuity assessments shown in Table 2-4 are conducted on the finest grid (L1) and show similar behaviour as observed priorly with the flat plate. Notably, near the trailing edge (i.e., control volume ABKL), the UPVD and VD approaches return values with a high offset from those obtained from the RVF. As identified earlier, this occurs due to the strong VII in this region, with the performance assessment improving further downstream as these effects diminish.

**Table 2-4 NACA 0012 lifting and non-lifting continuity and ideal gas constant assessment for various control volumes**

| $M = 0.15, Re = 6 \times 10^6, \alpha = 0^\circ$  |                 |                                  |        |                                      |        |                                      |        |
|---------------------------------------------------|-----------------|----------------------------------|--------|--------------------------------------|--------|--------------------------------------|--------|
| Contour                                           | $\Delta x/c$ of | $\nabla \cdot (\rho \mathbf{V})$ | R      | $\nabla \cdot (\rho^* \mathbf{V}^*)$ | $R_E$  | $\nabla \cdot (\rho_E \mathbf{V}_E)$ | $R_E$  |
| ABKL                                              | 1               | 0.005                            |        | -0.35                                |        | -0.26                                |        |
| ACJL                                              | 3               | 0.006                            |        | 0.04                                 |        | 0.08                                 |        |
| ADIL                                              | 5               | 0.005                            | 287.04 | 0.06                                 | 287.06 | 0.03                                 | 287.06 |
| AEHL                                              | 7               | 0.002                            |        | 0.06                                 |        | -0.03                                |        |
| AFGL                                              | 9               | 0.002                            |        | 0.05                                 |        | -0.09                                |        |
| $M = 0.30, Re = 3 \times 10^6, \alpha = 10^\circ$ |                 |                                  |        |                                      |        |                                      |        |
| ABKL                                              | 1               | 0.004                            |        | 0.48                                 |        | 0.63                                 |        |
| ACJL                                              | 3               | 0.005                            |        | 0.04                                 |        | 0.05                                 |        |
| ADIL                                              | 5               | 0.005                            | 287.04 | 0.05                                 | 287.06 | 0.01                                 | 287.06 |
| AEHL                                              | 7               | 0.004                            |        | 0.05                                 |        | 0.05                                 |        |
| AFGL                                              | 9               | 0.004                            |        | 0.05                                 |        | 0.09                                 |        |

Figure 2-9 shows the airfoil pressure distributions for the non-lifting and lifting scenarios in (a) and (b). Upstream of the airfoil, the Euler pressure shows a more rapid decay to ambient pressure than the RVF static pressure. Over the surface, the non-lifting test case shows an RVF pressure distribution that compares well with the CFL3D SA validation data from [30], as shown in Figure 2-9(a). The difference between the RVF and EIF distributions represents the dissipative pressure contribution, which indicates the boundary layer displacement effect on the airfoil pressure field. Additionally, the dissipative pressure provides a non-zero pressure distribution over the surface, contributing toward the viscous drag.

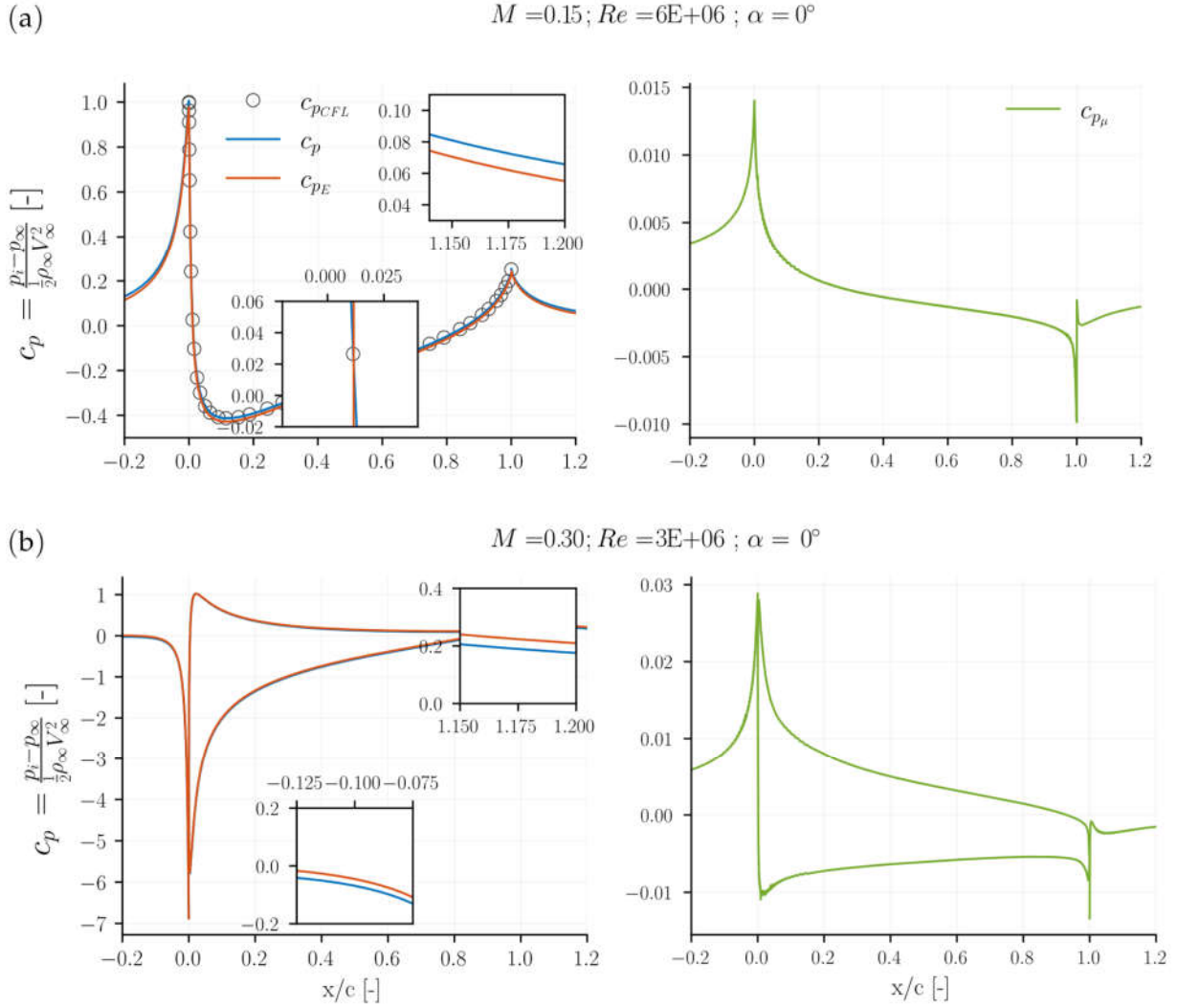


Figure 2-9 Surface pressure coefficients for the (a) non-lifting and (b) lifting NACA 0012 airfoil

### 2.3.2.2 Generalised Planar Body Dissipative Drag Analysis

The prior section showed that the mechanism through which momentum is transferred from the body to the fluid for a zero-pressure gradient flat plate is through shear stress. Furthermore, it was shown that through the relationship between shear stress and the fluid viscosity, the amount of energy imparted to the flow is governed mainly by the Reynolds number.

When considering aerodynamic bodies with a pressure gradient, an additional mechanism for momentum and energy transfer exists, i.e., momentum diffusivity due to pressure. The pressure gradient created around an aerodynamic body facilitates the momentum transfer between the body and fluid. A significant example assessed in this section involves the pressure gradient around a lifting airfoil, which transfers momentum through the downward turning of the flow, i.e., downwash, resulting in lift via Newton's third law. In this instance, PPF analysis identifies the momentum diffusivity due to the dissipative pressure. This helps quantify the recoverable exergy explicitly associated with the pressure field, which can be utilised to produce useful work. The remaining energy imparted to the flow associated with the non-dissipative pressure field is predominantly associated with lift generation. Therefore, for non-lifting scenarios, the lift-dependent PER given in equation (2.35)(a) would be zero reducing the expression to the form shown in equation (2.35)(b).

$$\begin{aligned}
 PER &= \underbrace{PER_\tau + PER_{\nabla p_\mu}}_{\text{recoverable exergy}} + \underbrace{PER_{\nabla p_E}}_{\text{lift-dependent}} \quad (\text{a}) \\
 \therefore PER_{\text{non-lifting}} &= \underbrace{PER_\tau + PER_{\nabla p_\mu}}_{\text{recoverable exergy}} \quad (\text{b})
 \end{aligned} \tag{2.35}$$

For the non-lifting test case, Figure 2-10 shows the dissipative drag accumulation plots similarly done for the flat plate in Figure 2-8. However, the trends presented do not exhibit the same behaviour as priorly observed due to the introduction of viscous inviscid interaction (VII). This effect uniquely affects both the near and farfield (energy and exergy) dissipative plots. For the near-field analyses, it is commonly known that an actual Euler flow returns a net drag contribution of zero for a non-lifting scenario. However, in this instance, the Euler pressure drag provides a non-zero contribution due to VII shown in the *near-field* section of Figure 2-10 through  $D_{p_E}$ , which implies that a portion of the dissipative drag is contained within the Euler pressure field as mentioned in [15,23]. For far-field assessment, the VII affects the evaluation of the cumulative drag over the airfoil due to constraints necessary to reconstruct a true EIF being relaxed. The first is the irrotationality condition throughout the flow field described in equation (2.18), while the second is the continuity relation in equation (2.19) for the RVF and EIF. This relation results in

the introduction of a *wall transpiration velocity* mentioned in Section 2.2.5.1, which becomes necessary to mimic the boundary layer displacement effect experienced by the RVF. The omission of these two conditions results in viscous inviscid interaction effects, particularly noticeable in regions with high pressure gradients, such as the trailing edge, as mentioned by Lock et al. [31,32]. This explains the noticeable discontinuous behaviour at the trailing edge for the dissipative drag, mechanical energy and physical energy in Figure 2-10.

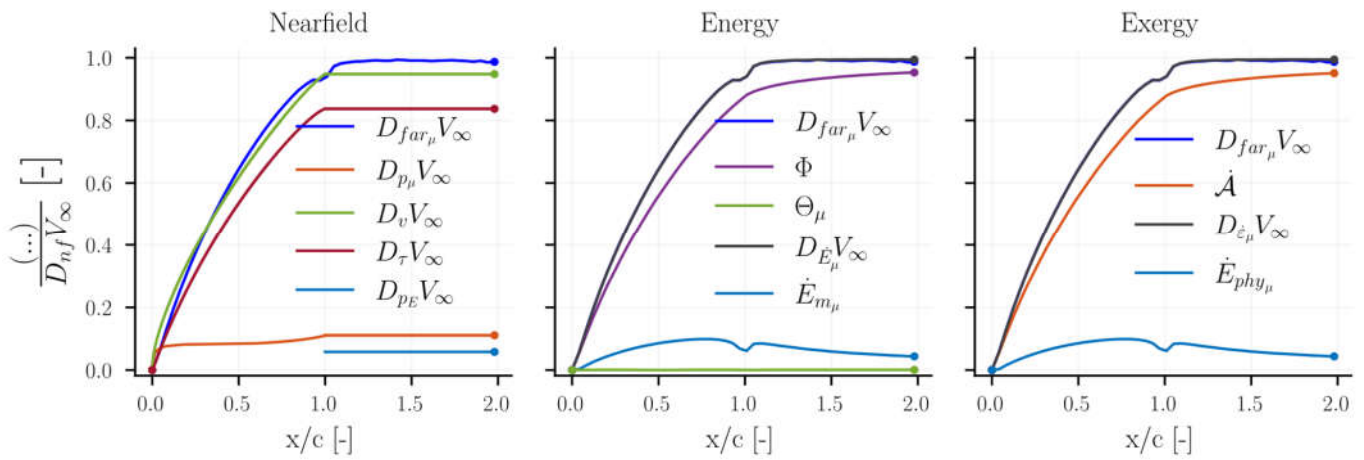


**Figure 2-10 Uncorrected NACA0012 airfoil near-field, farfield, energy and exergy dissipative drag accumulation plots**

### 2.3.2.3 Dissipative Drag Viscous Inviscid Interaction Correction

For a complete decomposition of drag components, it becomes essential that the viscous inviscid interaction drag is filtered out. Whether assessed from the near-field through the local force accumulation over the body surface or the farfield via the fluid's perturbative responses, the cumulative drag observed from either perspective should be equivalent. However, Figure 2-10 shows that the near- and far-field dissipative drag accumulation plots provide different values. On the other hand, all methods provide good agreement within the wake, highlighting that VII interaction effects are concentrated around the body and become less significant downstream.

The process of obtaining the VII drag distribution over the body begins with the difference between the dissipative farfield drag and near-field viscous drag obtained through PPF analysis, as shown in equation (2.36).

$$\begin{aligned}
D_{sp_\mu} V_\infty &= D_{\dot{\epsilon}_\mu} V_\infty - D_v V_\infty \\
&\text{where:} \\
D_{\dot{\epsilon}_\mu} V_\infty &= \dot{E}_m + \dot{\epsilon}_{th} + \underbrace{\Phi - \dot{\epsilon}_\Phi}_{\dot{A}_\Phi} + \dot{A}_{\nabla T} \\
D_v V_\infty &= D_\tau V_\infty + D_{p_\mu} V_\infty
\end{aligned} \tag{2.36}$$

Taking the definition from work in [11], the difference between a near and farfield drag solution provides a spurious drag quantity. Since the drag values are from the dissipative field, the result can be referred to as a spurious dissipative drag  $D_{sp_\mu} V_\infty$  containing the viscous inviscid interaction contribution and a spurious recoverable energy portion, as shown in equation (2.37).

$$D_{sp_\mu} V_\infty = D_{p_{VII}} + D_{sp_{rec}} \tag{2.37}$$

In the near-field, the dissipative pressure is related to the shear stress through the *dissipative PPF Poisson equation* [15,21,33] shown in equation (2.38).

$$\nabla^2 p_\mu = \nabla \cdot (\nabla \cdot \bar{\boldsymbol{\tau}}) \tag{2.38}$$

As shown for the flat plate, the shear stress contributes toward the momentum diffusivity from the body to the fluid. For bodies with geometric curvature, the momentum diffusivity due to pressure results from the relationship between the dissipative pressure and shear stress through the dissipative Poisson equation. Therefore, a near-field recoverable energy assessment can be done directly through dissipative pressure. Additionally, the thermal exergy gained through viscous dissipation is added to the near-field recoverable energy for compressible flows to account for thermo-compressibility effects in the near-field, as shown in equation (2.39).

$$D_{sp_{rec}} V_{\infty} = \underbrace{\dot{E}_{m_{\mu}}}_{D_{far_{rec}} V_{\infty}} - \underbrace{\left( D_{p_{\mu}} V_{\infty} + \dot{\varepsilon}_{\Phi} \right)}_{D_{nf_{rec}} V_{\infty}} \quad (2.39)$$

After this, the viscous inviscid drag power can be evaluated through equation (2.40)(a) and used to obtain the corrected dissipative pressure and viscous drag shown in equation (2.40)(b) and (c), respectively.

$$\begin{aligned} D_{p_{VII}} V_{\infty} &= D_{sp_{\mu}} V_{\infty} - \underbrace{\left[ \left( \dot{E}_{m_{\mu}} - \dot{\varepsilon}_{\Phi} \right) - D_{p_{\mu}} V_{\infty} \right]}_{D_{sp_{rec}}} = \dot{\varepsilon}_{th_{\mu}} + \Phi + \dot{A}_{\nabla T} - D_{\tau} V_{\infty} \quad (a) \\ D_{p_{\mu}}^* V_{\infty} &= D_{p_{\mu}} V_{\infty} + D_{p_{VII}} V_{\infty} \quad (b) \\ D_v^* V_{\infty} &= D_v V_{\infty} + D_{p_{\mu}}^* V_{\infty} \quad (c) \end{aligned} \quad (2.40)$$

The corrected near-field viscous drag thus provides the cumulative drag behaviour over the body without viscous inviscid interaction. As a result, its value is used as a reference for correcting the farfield cumulative drag distribution via a spurious residual drag obtained through the formulation shown in equation (2.41).

$$D_{sp_{res}} V_{\infty} = D_v^* V_{\infty} - D_{far_{\mu}} V_{\infty} \quad (2.41)$$

The resulting corrected near-field distributions in Figure 2-11 show that the cumulative viscous drag trend is coherent with the flat plate in Figure 2-8, with no abrupt dip observed at the trailing edge, priorly seen for the *uncorrected* trends in Figure 2-10. The VII drag contribution  $D_{p_{VII}} V_{\infty}$  shown in the *corrected* near-field plot allows for its contribution to be correctly appreciated as a dissipative drag contribution along the body and filtered out from the Euler drag, which returns  $D_{p_E} V_{\infty} \simeq 0$  as expected for an actual Euler flow.

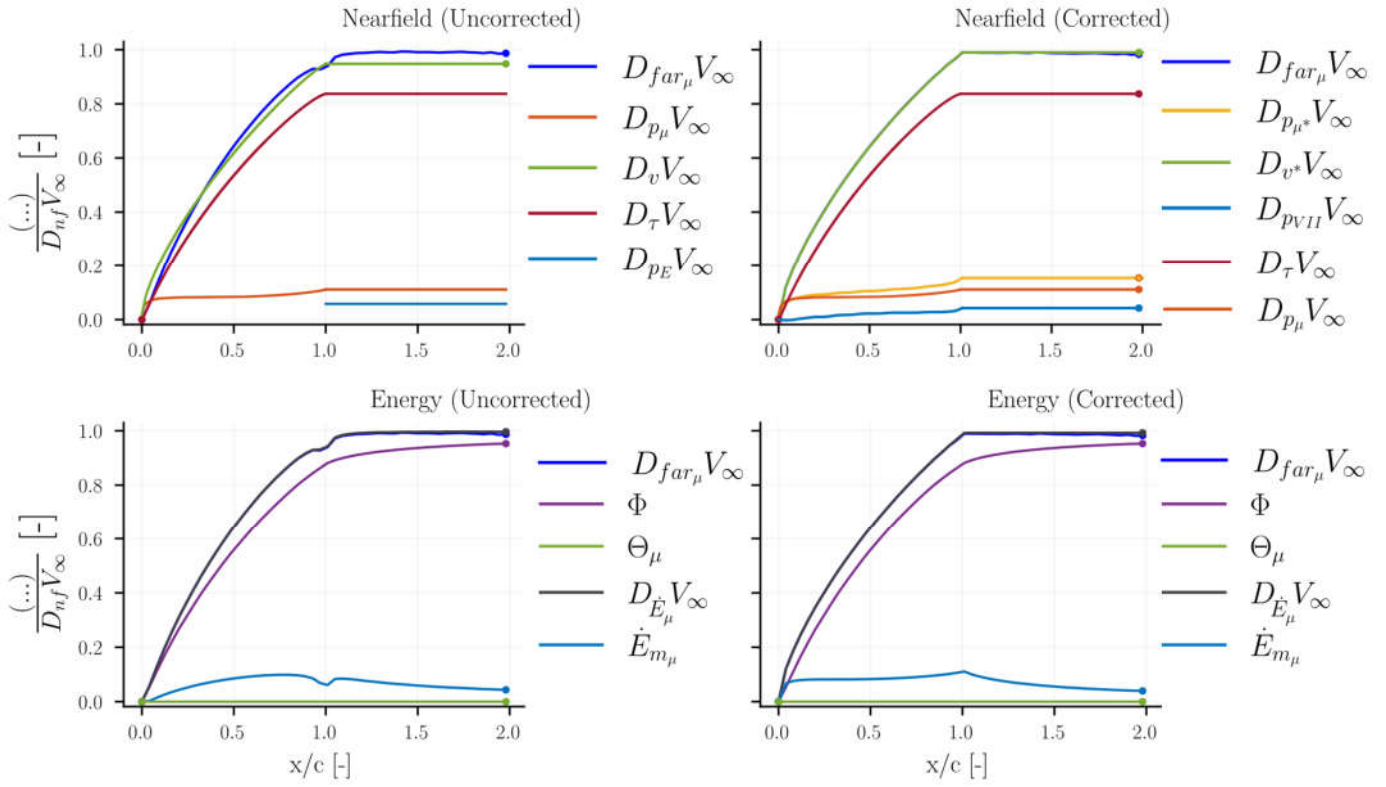


**Figure 2-11 Non-lifting NACA 0012 comparison of uncorrected and corrected dissipative drag accumulation plots for near-field and energy-based analysis**

The corrected near-field viscous drag accumulation will then be explicitly represented as a summation of the cumulative shear stress and corrected (total) dissipative pressure drag, including the VII drag contribution. When assessed over the body, cumulative shear stress drag has a monotonically increasing trend with a gradually decaying gradient from LE to TE due to BL growth. The BL can be viewed as a viscous interface between the body and fluid, facilitating momentum transfer. Therefore, a thicker boundary layer tends to affect the momentum diffusivity (i.e., momentum diffusivity due to shear stress) between the body and the fluid as the viscous interface dampens momentum transfer. Particularly for the near-wall velocity gradients, regions where the BL is thin, usually at/near the LE, will exhibit much higher local shear stresses. This

results in higher local contributions toward the cumulative drag than in regions further downstream near the TE due to the viscous damping effect of the larger BL on the velocity profiles.

On the other hand, the corrected cumulative dissipative pressure curve  $D_{p_\mu} V_\infty$  represents near-wall loss accumulation due to momentum diffusivity through pressure within the boundary layer. The dissipative pressure curve adopts a cumulative behaviour highlighting the increase in viscous drag due to the steep pressure gradients at the LE and TE regions, as shown in Figure 2-11.

The spurious residual drag can then be extended to *energy* and *exergy*-based analysis to correct terms affected by viscous inviscid interaction. In particular, this pertains to dissipative mechanical energy and physical exergy. The cumulative *energy-based* dissipative drag plots in Figure 2-11 show that by applying this correction procedure and including the spurious residual drag, the localised trailing edge dip due to VII for the uncorrected energy analyses is removed, and all dissipative quantities can be evaluated with less uncertainty in this region.

The trends observed in Figure 2-12 become consistent with those obtained for the flat plate, particularly regarding the cumulative total drag over the body surface being equivalent for all evaluation methods, whether assessed from a near- or farfield perspective through momentum and energy/exergy-based approaches. Secondly, the behaviour of the dissipative mechanical energy and physical exergy is shown to have a cumulative behaviour, similarly displaying a maximum at the trailing edge, decaying downstream as the energy imparted to the flow is irreversibly lost to the surroundings as heat due to viscous losses.

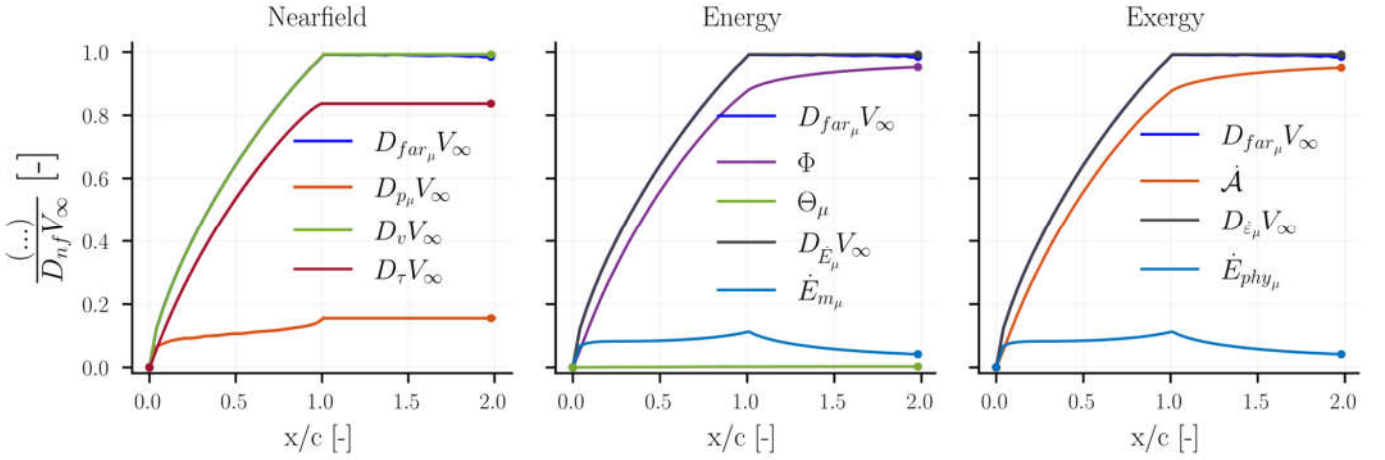


**Figure 2-12 Corrected non-lifting NACA0012 airfoil near-field, farfield, energy and exergy dissipative drag accumulation plots**

As a note, the flow analyses priorly considered in this section are in the low-subsonic Mach regime. As a result, the flow can effectively be regarded as incompressible. This results in the mechanical aspects of the flow being the major contributors toward the aerodynamic drag leaving a negligible contribution from thermal effects. This causes the mechanical energy and exergy-based analyses to exhibit similar trends, as shown prior in Figure 2-8, as the exergy formulation reduces to mechanical energy in the limit where  $M_\infty \rightarrow 0$ .

#### 2.3.2.4 Extension of Corrected Generalised Planar Body Dissipative Drag Analysis to Moderately Compressible Lifting Flows

The VII correction method applied in the previous section is extended to a moderately compressible lifting flow and tested on a NACA 0012 airfoil at  $M_\infty = 0.30$  and  $\alpha = 10^\circ$ . Firstly, the uncorrected near-field cumulative drag is evaluated in Figure 2-13, which shows that the near- and far-field cumulative viscous drag curves, i.e.  $D_v V_\infty$  and  $D_{far_\mu} V_\infty$ , tend to show poor agreement between their respective solutions. However, the summation of the viscous and Euler pressure drag  $D_{p_E} V_\infty$ , specifically at the trailing edge, recovers the total drag over the body. As observed prior, this occurs due to VII interaction effects strongly concentrated around the airfoil surface.

Compared to the non-lifting scenario, the intensity of the VII interaction is seen to rise, evidenced by the increased deviation in the cumulative viscous drag for the near- and far-field solutions.

Following the work of Lock et al. [31,32], it is known that VII is most significant in regions of high-pressure gradients. Therefore, apart from the pressure gradient development due to the airfoil's geometric curvature for the lifting test case, an additional pressure gradient is experienced due to lift production. Therefore, due to this increase in pressure gradients, there is an increased intensity of the VII effects.

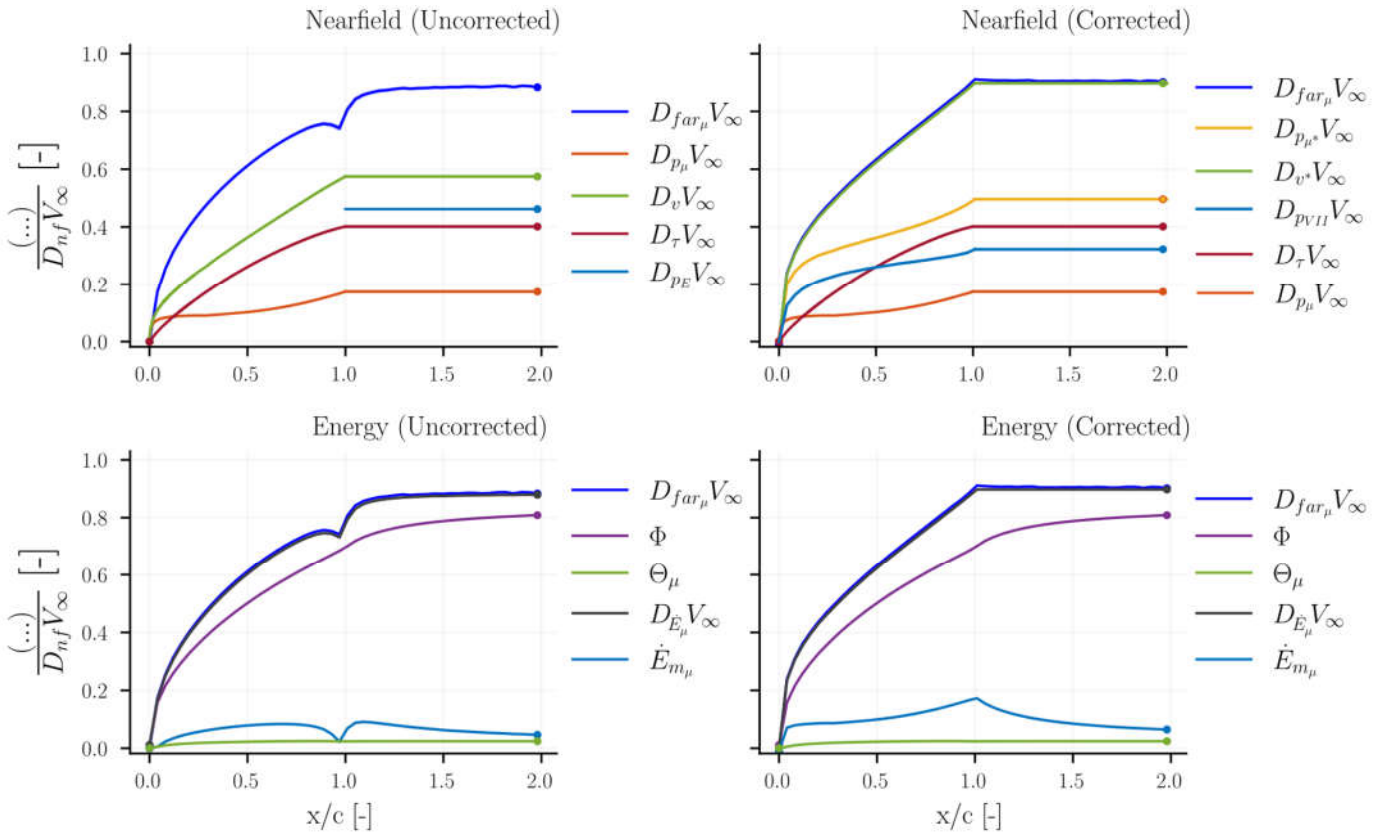


**Figure 2-13 Lifting NACA 0012 airfoil comparison of uncorrected and corrected dissipative drag accumulation plots for near-field and energy-based analysis**

For an actual Euler flow without VII, the drag returned by a lifting body provides the *lift-induced* drag. This represents the minimum drag of the body, occurring due to lift production. Therefore,

after employing VII corrections, the portion of viscous drag present within the Euler pressure drag is filtered out, returning the lift-induced drag contribution. At this point, the *lift-induced* drag can be obtained either by taking the difference between the uncorrected Euler and VII drag in Figure 2-13 or by taking the difference between the total drag and the corrected cumulative viscous drag as shown in equation (2.42)(a) and (b), respectively.

$$\begin{aligned} D_{p_E}^* V_\infty &= D_l V_\infty = D_{p_E} V_\infty - D_{p_{VII}} V_\infty & (a) \\ &= D_{\eta_f} V_\infty - D_v^* V_\infty & (b) \end{aligned} \quad (2.42)$$

As the VII drag contribution represents a portion of the body's viscous drag, it is a dissipative quantity. As a result, it would have a locally cumulative behaviour when assessed over the body, confirmed in the corrected near-field analysis shown in Figure 2-13. Additionally, it can be observed that the VII contribution toward the overall drag is significant, with an almost doubled contribution in comparison with the uncorrected dissipative pressure quantity  $D_{p_\mu} V_\infty$ . This shows that for adequate local cumulative drag assessment over a body, it becomes paramount to consider the effects of VII, especially for lifting scenarios, as the severity of their influence on the local drag increases under these circumstances.

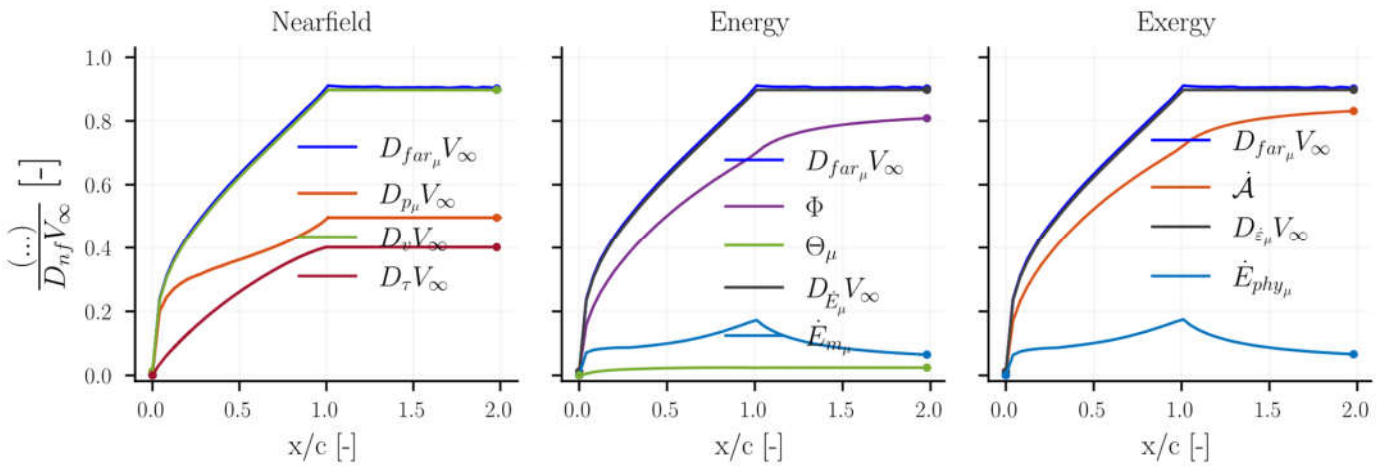


**Figure 2-14 Corrected lifting NACA0012 airfoil near-field, farfield, energy and exergy dissipative drag accumulation plots**

Extension of this correction to mechanical energy and exergy-based analysis shows the local decrease in dissipative mechanical energy and physical exergy is no longer observed. This indicates the VII correction procedure adequately identifies and redistributes the cumulative VII contributions even for lifting scenarios in moderately compressible flow. Removal of the trailing edge discontinuity serves as a significant advantage for recoverable energy evaluation through the formulations provided in equations (2.29)(d) and (e), as these can now be assessed with better confidence in this region.

Furthermore, regarding recoverable energy using mechanical energy analysis, Lamprakis et al. [20] introduce a compressibility-corrected formulation of PER stated in equation (2.29)(c) which builds from the work of Sanders and Laskaridis [27]. Apart from mechanical energy losses, the compressibility corrected formulation additionally accounts for thermo-compressibility losses (evaluated through  $\Theta_\infty$ ), which improves both the evaluation and interpretation of recoverable energy, which can be utilised to produce useful work through mechanisms such as boundary layer ingestion. At high Mach numbers, the thermo-compressibility losses originate from the dissipative portion of the pressure-volume work, which is associated with the baroclinic power imparted to the flow due to non-isentropic flow compression and expansion [20]. Numerically,  $\Theta_\infty$  is evaluated by taking a volume integral over the entire domain as the formulation provided in the mechanical energy balance in [20] cannot obtain the local non-isentropic pressure-volume work contributions. However, the UPVD approach introduces a formulation capable of evaluating the local contributions toward the non-isentropic/dissipative pressure-volume work  $\Theta_\mu$ , shown in Figure 2-14, while additionally allowing for VII corrections when assessed over a body.

## 2.4 Discussion of Results

### 2.4.1 Velocity and Pressure Decomposition

The work conducted by Aguirre [8] shows that exergy-based formulation components can be decomposed into dissipative and non-dissipative contributions. This serves as an advantage as it allows for wake-reduced exergy expressions to be obtained, which can be applied in wind-tunnel

testing for drag evaluation. Furthermore, considering the dissipative contributions allows for the cumulative viscous drag to be traced as it develops over the body.

However, two assumptions are made in its implementation, and this work addresses one of these, specifically focusing on the decomposition of pressure into its dissipative and non-dissipative portions without assuming all contributions associated with the pressure field are non-dissipative, as done prior in [8]. This allows for dissipative pressure drag to be evaluated and considered in new viscous drag formulations.

Furthermore, in the limit where the Reynolds number approaches infinity, the non-dissipative (Euler) pressure contribution readily approaches the static pressure. Therefore, as Reynolds increases, the applicability of Aguirre's [8] initial assumption is further supported. However, when the Reynolds number is low, the contribution of dissipative pressure toward overall drag increases and should thus be adequately considered through PPF analysis and its coupling with farfield momentum, power balance and exergy-based methods.

PPF analysis intuitively highlights VII influence, as Schmitz et al. [15] explained, due to the wake having a finite momentum and displacement thickness, resulting in a non-zero drag contribution from the non-dissipative flow field. This is further supported by evidence provided by Aguirre [29] through analysis of isolated and closely coupled aerodynamic bodies. The numerical discrepancy near the trailing edge, referred to in [29] as the "*missing drag*", is caused by the VII experienced in high-pressure gradient zones, as mentioned in [31,32].

The zero-pressure gradient flat plate study was used for numerical verification of PPF analysis for a test case where the pressure-related behaviour was already known. The derivations provided in Section 2.3.1.1 show that Power balance [1,27], VD [8], and UPVD approaches give the same formulation for this test case. PPF implementation is assessed through the cumulative viscous drag plots in Figure 2-8. As expected, the UPVD approach returns no dissipative pressure-related contribution toward the flat plate drag.

Furthermore, this analysis aids in confirming the conditions through which VII occurs, mentioned in [31,32], as no observable VII effects are realised for the zero-pressure gradient scenario. As a

result, the flat plate analyses offer a benchmark for understanding cumulative drag analysis in the absence of VII. Notably, all evaluation methods from a near- or far-field perspective provide the same cumulative viscous drag behaviour over the body and within the downstream wake shown in Figure 2-8.

Significant pressure gradients are experienced in the airfoil studies due to the geometric curvature and lift production. As a result, the VII intensifies as the omission of the wall transpiration velocity by the UPVD approach becomes more evident. The Euler pressure drag can quantify VII effects through near-field PPF analysis. For example, a non-lifting airfoil for an actual Euler flow returns a drag contribution of zero; therefore, the non-zero Euler pressure drag observed in Figure 2-10 indicates the VII contributions. For the farfield momentum, energy and exergy-based methods, VIIs are most observable through the local TE discontinuous dip in cumulative mechanical energy, physical exergy and viscous drag shown in Figure 2-10. This effect is further highlighted by the difference in cumulative viscous drag over the body assessed through near- and far-field analyses shown in Figure 2-11 and Figure 2-13. Additionally, the good agreement observed for the cumulative viscous drag for these same plots but slightly downstream of the trailing edge highlights that VII effects are locally concentrated around the body with their intensity decaying downstream.

A notable advantage of the proposed unified approach is the ability to filter out the VII drag contributions from the Euler field. Taking the non-lifting scenario as an example, PPF analysis provides a near-field method for evaluating the total VII contribution through the near-field Euler pressure drag. From the uncorrected cumulative viscous drag distributions in Figure 2-11, it can be observed that the LE and TE values from near- and far-field analysis are similar. This implies that the local deviation in their viscous drag distributions primarily occurs due to VII experienced from two different perspectives: near-field VII over the body surface for PPF analysis and VII within the BL for the dissipative far-field analyses. Realising this, obtaining the cumulative VII drag contribution through the correction procedure described in Section 2.3.2.2 becomes possible.

**Table 2-5 Euler and dissipative drag decomposition for near-field, far-field, energy, and exergy analysis reported in drag counts ( $C_d \times 10^{-4}$ )**

| Case Study       | Near-field |             | Farfield  |             | Energy    |             | Exergy    |             |
|------------------|------------|-------------|-----------|-------------|-----------|-------------|-----------|-------------|
|                  | $C_{d_E}$  | $C_{d_\mu}$ | $C_{d_E}$ | $C_{d_\mu}$ | $C_{d_E}$ | $C_{d_\mu}$ | $C_{d_E}$ | $C_{d_\mu}$ |
| I                | 29.4       |             | 29.4      |             | 29.7      |             | 29.6      |             |
|                  | 0          | 29.4        | 0         | 29.4        | 0         | 29.7        | 0         | 29.6        |
| II (non-lifting) | 83.6       |             | 82.5      |             | 83.3      |             | 83.2      |             |
|                  | 4.4(0.9)   | 79.2        | 0         | 82.5        | 0         | 83.3        | 0         | 83.2        |
| II (lifting)     | 162.7      |             | 159.2     |             | 161.3     |             | 160.6     |             |
|                  | 69.0(16.   | 93.7        | 15.       | 143.        | 18.3      | 143.0       | 17.6      | 143.0       |

Table 2-5 shows the total drag obtained using various evaluation methods and a region-specific drag decomposition into Euler and dissipative components. The viscous drag confined mainly to the BL and wake can now be assessed through the dissipative drag ( $D_\mu$ ), while the remaining bulk flow contributions, such as the lift-induced drag, can be examined through the Euler drag ( $D_E$ ). Additionally, provided in brackets is the VII corrected drag, which is used to assess the effectiveness of the correction method in comparison to values obtained using far-field momentum and energy/exergy approaches. For the *non-lifting* test case, it can be observed that the VII correction procedure provides appreciable correction, with the near-field PPF drag decomposition now showing good agreement within <1% difference for viscous drag in comparison with the other methods employed. Similarly, for the *lifting* case study, good agreement is observed for the dissipative ( $D_\mu/D$ ) and Euler ( $D_E/D$ ) drag shares, showing respective contributions of  $\approx 0.9$  (90%) and  $\approx 0.1$  (10%) toward the overall drag share with a difference of <1% for all solution evaluations.

### 2.4.2 Potential for Energy Recovery

The flat plate analyses provided a simple means of assessing the accumulation of drag over an aerodynamic configuration with no pressure gradient. Incorporating energy and exergy-based methods allowed for the various physical mechanisms associated with the accumulation of drag to be studied. From observation, it was noted that the drag over the flat plate could be categorised into two distinct forms. The first is energy being irreversibly lost to the surroundings, quantified through the viscous dissipation and anergy, while the second represents mechanical energy imparted to the flow. The mechanical energy imparted to the flow indicates energy that has not yet dissipated and can thus be considered as available energy, i.e., exergy, which a propulsor can use to produce useful work, i.e., thrust. Equation (2.29) has introduced various formulations to evaluate this recoverable energy. From an exergy-based perspective, Arntz et al. [10–12] introduce the exergy waste coefficient (EWC), which assesses the total available energy present within the flow, considering both mechanical and thermal energy aspects. However, as highlighted by Aguirre [8,14,29], exergy contributions can be split into two distinct forms, the first being the non-recoverable isentropic contributions which effectively self-recover as they follow an isentropic path. The second is non-isentropic contributions which represent the actual recoverable energy. Taking this into account, Sanders and Laskaridis [27] introduce the mechanical energy-based potential for energy recover factor (PER), [27] introduce the mechanical energy-based potential for energy recover factor (PER), which evaluates the recoverable en[21] via an assessment of the viscous dissipation, later modified by Laskaridis et al. [20] ( $PER_c$ ) to include thermo-compressibility. The work presented here revisits the EWC formulation, revising it as shown in equation (2.29)(e) to consider only the dissipative mechanical and thermal energy contributions. Additionally, this was extended to mechanical energy analysis through equation (2.29)(d), allowing for an alternative mechanical energy-based formulation for recoverable energy evaluation.

Table 2-6 compares these various recoverable energy formulations evaluated at the trailing edge on the different case studies conducted in this work. Good agreement is observed for the incompressible non-lifting flat plate and airfoil analyses for all the evaluation methods. This occurs

because the flows considered are in the low-subsonic Mach range; therefore, thermal energy aspects are negligible, resulting in the exergy-based formulations being reduced simply to mechanical energy analyses.

**Table 2-6 Case Study Recoverable Energy Evaluation Comparison**

| Case                | (%) EWC [11,12] | (%) PER [27] | (%) PER <sub>c</sub> [20] | (%) EWC <sub>μ</sub> | (%) PER <sub>μ</sub> |
|---------------------|-----------------|--------------|---------------------------|----------------------|----------------------|
| I                   | 9.92            | 9.92         | 9.92                      | 9.92                 | 9.92                 |
| II<br>(non-lifting) | 11.6            | 11.5         | 11.5                      | 11.3                 | 11.3                 |
| II<br>(lifting)     | 27.7            | 29.9         | 17.7                      | 17.5                 | 17.3                 |

A significant deviation between the results is observed for the lifting test cases. For the EWC, this is due to the inclusion of isentropic contributions at the trailing edge, which self-recover further downstream, resulting in an overestimation of the recoverable energy. For the PER, the overestimation in recoverable energy occurs since compressibility-related losses are omitted from the formulation. The inclusion of compressibility losses is done through PER<sub>c</sub> which results in a reduction of the evaluated recoverable energy. The resulting value shows good agreement with the newly proposed dissipative formulations of EWC<sub>μ</sub> and PER<sub>μ</sub>, increasing confidence in these evaluations' validity.

## 2.5 Conclusions

This work considers aerodynamic performance assessment in the absolute reference frame, which allows for the underlying principles behind the ideology of recoverable energy within the downstream wake of an aerodynamic body to be understood. Mechanical energy devices such as boundary layer propulsors can theoretically utilise this energy content to produce thrust more efficiently by reducing the downstream mechanical energy losses. This work shows that exergy-based analysis offers a natural decomposition of phenomenological drag constituent terms into exergetic and anergetic contributions through the 2<sup>nd</sup> law of thermodynamics and Gibbs equations. This is essential in understanding that recoverable energy is available within the flow or, more aptly, referred to as exergy. The VD approach is employed to understand that there is a theoretical limit to the amount of exergy that can be utilised to produce useful work, i.e., thrust. Additionally, the VD approach partitions exergy into the portion which will be irreversibly lost to the surrounds and the remainder, which self-recovers through isentropic energy exchange between pressure-work and kinetic energy.

This work aimed to extend the applicability of the VD approach to the near-field by replacing Aguirre's isentropic static pressure assumption through PPF analysis. This enabled a more robust decomposition of the phenomenological drag components using momentum and energy/exergy-based methods. The prior VD approach could only decompose the velocity field into dissipative and non-dissipative contributions. The unification proposed herein allows for the pressure to be decomposed similarly, and as a result, the dissipative pressure drag contribution can be obtained.

This unification with PPF analysis offers a two-fold benefit. Firstly, it allows for a corrected decomposition of the pressure field and dissipation during energy and work transfer. Secondly, PPF analysis enables the dissipative field to be quantified and included in recoverable energy evaluations. Lastly, PPF analysis extends this decomposition to the near-field, where forces can now be split into their respective dissipative and non-dissipative contributions. The dissipative forces consist of the dissipative pressure and skin friction drag, collectively making up what is referred to in this work as the viscous drag. The remaining contribution is the non-dissipative drag which consists of the Euler drag contribution. In classical aerodynamic theory, Euler flows without

shocks would have no wake development downstream of the body and yield a zero-drag contribution. For the analyses conducted herein, the non-zero Euler drag contribution occurs due to the finite wake thickness and thus would have a small contribution toward form drag. This contribution is formally referred to as viscous inviscid interaction as it indicates the influence of the non-dissipative flow on the overall drag, which would otherwise not be present for an actual Euler flow.

The flat plate and airfoil analysis highlight that the potential for energy recovery is a function of the momentum diffusivity. Therefore, any mechanism that aids in enhancing the diffusivity would improve PER. It was postulated that the PER could be characterised into two distinct portions, dependent on the momentum diffusivity. These were the shear stress and pressure gradient. The main contributor toward PER for the flat plate was the momentum diffusivity due to shear stress. As fluid properties solely govern this, augmenting PER can be achieved by manipulating the local momentum diffusivity or, more simply, the Reynolds number. An additional contribution which enhances PER for bodies with geometric curvatures, such as airfoils, is the momentum diffusivity from the pressure gradients developed around the body. The pressure gradient enhances the momentum diffusivity resulting in more energy being imparted to the flow.

As a result, particular care must be taken when dealing with lifting bodies as there will be a significant increase in pressure gradients and downstream energy deposition due to the downward momentum imparted to the flow, i.e., downwash, due to lift. In this instance, the recoverable energy should be considered as all energy imparted to the flow that is not directly associated with lift production. The flow decomposition allows the lift-related phenomena to be identified and isolated through the non-dissipative field, which can then be omitted from recoverable energy evaluations. This helps highlight the advantage of the separability introduced by the flow decomposition as it provides a clearer view of the energy available for producing useful work through mechanisms such as boundary layer ingestion, even for lifting bodies.

Lastly, a significant advantage of the unified approach is VII correction, allowing for the viscous drag contribution within the Euler field to be filtered out and correctly attributed to the dissipative field. The resulting cumulative drag contribution helps highlight that the VII effects are naturally

strongly local, concentrated around the body and decaying downstream. This implies that local drag evaluation over aerodynamic bodies in the absence of VII corrections must be performed tentatively, as this effect offers a cumulative distribution contributing toward viscous drag.

## Appendix A

The momentum and mechanical energy boundary layer defect equations are derived in this section following the procedures outlined in the work of Sato [2] and Zhang [19]. For these derivations, various integral defects are defined as follows:

$$\begin{aligned}
\Pi &\equiv p_E - p && : \text{pressure defect} \\
\mathbf{M} &\equiv (\rho_E \mathbf{V}_E - \rho \mathbf{V}) && : \text{mass defect} \\
\mathbf{P} &\equiv \rho (\mathbf{V}_E - \mathbf{V}) \mathbf{V} && : \text{momentum defect} \\
\mathbf{K} &\equiv \frac{1}{2} \rho (V_E^2 - V^2) \mathbf{V} && : \text{kinetic energy defect} \\
\mathbf{D} &\equiv (\rho_E - \rho) \mathbf{V} && : \text{density defect} \\
\mathbf{Q} &\equiv \mathbf{V}_E - \mathbf{V} && : \text{velocity defect}
\end{aligned} \tag{2.43}$$

### A.1 Boundary Layer Defect Momentum Formulation

To obtain the momentum defect equations, the difference between the Euler and real viscous flow momentum equations is taken as shown in Section 2.2.5 and rearranged to get the form provided in equation (2.44).

$$\begin{aligned}
&\nabla \cdot (\rho_E \mathbf{V}_E \mathbf{V}_E - \rho \mathbf{V} \mathbf{V}) + \nabla (p_E - p) + \nabla \cdot \bar{\boldsymbol{\tau}} = 0 \\
&\nabla \cdot [\rho (\mathbf{V}_E - \mathbf{V}) \mathbf{V} + \mathbf{V}_E (\rho_E \mathbf{V}_E - \rho \mathbf{V})] + \nabla (p_E - p) + \nabla \cdot \bar{\boldsymbol{\tau}} = 0
\end{aligned} \tag{2.44}$$

Using the momentum, mass and pressure defect identities in equation (2.43):

$$\nabla \cdot \mathbf{P} + \nabla \cdot (\mathbf{V}_E \mathbf{M}) + \nabla \Pi + \nabla \cdot \bar{\boldsymbol{\tau}} = 0 \tag{2.45}$$

The vector identity  $\nabla \cdot (\mathbf{V}_E \mathbf{M}) = (\nabla \mathbf{V}_E) \cdot \mathbf{M} + (\nabla \cdot \mathbf{M}) \mathbf{V}_E$  can expand the mass defect term in equation (2.45) to obtain the expression given in equation (2.46).

$$\nabla \cdot \mathbf{P} + (\nabla \mathbf{V}_E) \cdot \mathbf{M} + \underbrace{(\nabla \cdot \mathbf{M})}_{= 0, \text{ continuity}} \mathbf{V}_E + \nabla \Pi + \nabla \cdot \bar{\boldsymbol{\tau}} = 0 \quad (2.46)$$

The continuity assumption  $(\nabla \cdot \mathbf{M}) = 0$  is applied to obtain the reduced form of the defect momentum formulation in equation (2.47).

$$\nabla \cdot \mathbf{P} + (\nabla \mathbf{V}_E) \cdot \mathbf{M} + \nabla \Pi + \nabla \cdot \bar{\boldsymbol{\tau}} = 0 \quad (2.47)$$

## A.2 Boundary Layer Defect Mechanical Energy Formulation

Similarly, for the momentum equation, the difference between the Euler and real viscous flow mechanical energy formulation is taken after applying the vector identity.

$\nabla \cdot (\bar{\boldsymbol{\tau}} \cdot \mathbf{V}) = (\nabla \cdot \bar{\boldsymbol{\tau}}) \cdot \mathbf{V} + (\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V}$ , to obtain the form provided in equation (2.48).

$$\begin{aligned} \nabla p_E \cdot (\mathbf{V}_E - \mathbf{V}) - \nabla (p - p_E) \cdot \mathbf{V} + \nabla \cdot (\bar{\boldsymbol{\tau}} \cdot \mathbf{V}) - (\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V} = \\ -\frac{1}{2} \nabla \cdot \left[ \rho (V_E^2 - V^2) \mathbf{V} + V_E^2 (\rho_E \mathbf{V}_E - \rho \mathbf{V}) \right] \end{aligned} \quad (2.48)$$

Using the velocity, pressure, kinetic energy and mass defect identities in equation (2.43):

$$\nabla p_E \cdot \mathbf{Q} + \nabla \Pi \cdot \mathbf{V} + \nabla \cdot (\bar{\boldsymbol{\tau}} \cdot \mathbf{V}) - (\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V} = - \left[ \nabla \cdot \mathbf{K} + \mathbf{M} \cdot \left( \frac{1}{2} \nabla V_E^2 \right) \right] \quad (2.49)$$

Substituting the expressions  $\mathbf{M} - \rho_E \mathbf{Q} = \mathbf{D}$  and  $\nabla p_E = -\rho_E \nabla \frac{1}{2} V_E^2$  into equation (2.49) yields the defect mechanical energy formulation shown in equation (2.50), similarly obtained in the work of Sato [2] and Zhang [19].

$$\nabla \Pi \cdot \mathbf{V} + \nabla \cdot (\bar{\boldsymbol{\tau}} \cdot \mathbf{V}) - (\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V} = \nabla \cdot \mathbf{K} + \mathbf{D} \cdot \left( \frac{1}{2} \nabla V_E^2 \right) \quad (2.50)$$

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## **3 Extension of Unified Exergy & Partial Pressure Field Aerodynamic Decomposition Approach to Transonic Flows**

### **3.1 Introduction**

External aerodynamic analysis through computational fluid dynamics (CFD) provides flow field solutions by solving the Navier-Stokes equations within a flow domain. Classical aerodynamic research shows that particular flow regions can be modelled accurately using comparatively less complicated formulations governed by flow physics [1,2]. For example, shear and boundary layer zones can be modelled through the boundary-layer defect equations and the remaining bulk flow through Euler formulations. Furthermore, these equations can be solved simultaneously using a technique known as viscous inviscid interaction (VII) [2], which allows for the solutions of these equations to be combined through an iterative process until a converged solution is obtained for the entire flow field. The composite solution describes the real viscous flow (RVF) as a superposition of an Euler equivalent inviscid flow (EIF) and defect flow (DF). Notably, for exergy-based analysis, this flow decomposition was acknowledged by Aguirre [3], who introduced the velocity decomposition approach, which assists in splitting a composite CFD flow field into its dissipative and non-dissipative parts, thus allowing for zonal drag decomposition to be established, where the dissipative field characterises drag originating from the boundary layer (BL) and wake. In contrast, the non-dissipative field is primarily associated with contributions arising from the bulk flow.

The advantages of zonal drag decomposition have been acknowledged in the work of Paparone and Tognaccini [4], who highlight that a significant limitation of the conventional near-field analysis is its binary definition of drag into pressure and skin friction components. Instead, they suggest that far-field assessment would provide a more informative phenomenological drag decomposition through an identification of physical drag-producing mechanisms assigned to viscous drag (associated with the BL), wave drag (associated with shockwaves) and lift-induced or vortex drag (associated with the shed vorticity for three-dimensional lifting bodies).

The work of Steger and Baldwin [5] introduced the use of far-field methods for wave-drag extraction for fully potential transonic flows through an assessment of the difference in momentum experienced across the shockwave interface. This was later modified by van der Vooren [6], who proposed a more general approach by taking the volume integral of the artificial viscosity in and around the shockwave's region of influence. Extension of far-field wave drag analysis to RANS flows was proposed by van der Vooren and Destarac [7] and later improved by Paparone and Tognaccini [4] by including a second-order correction to Oswatitsch formulation [8].

Oswatitsch [8] provided a far-field entropy-drag formulation considering only first-order effects, which proved immensely valuable for zonal drag identification. From this, Chao and van Dam [9] showed that viscous and wave drag could be obtained by applying the entropy-drag formulation to the BL and shockwave zones separately. This approach has also been adopted for exergy-based wave drag analysis through the wave anergy, as shown in the work of Arntz [10]. Schmitt and Destarac [11] show that using the Gauss theorem to convert the priorly defined surface integral entropy-drag formulation to a volume integral is possible, which carries the advantage of filtering out the spurious drag contributions arising from numerical discretisation error and artificial dissipation. Similarly, this Gauss theorem conversion has been employed in the work of Aguirre [12], proposing the transformation of the surface integral wave anergy given by Arntz [10] to a volume integral, allowing for a relatively more robust method of wave drag evaluation for transonic exergy analysis.

Zonal exergy-based drag decomposition takes advantage of the artificial velocity principle introduced by Betz [13], whose direction vector is parallel to the freestream flow. In so doing, these conditions inadvertently enforce a simple inviscid flow model for the reconstruction of the equivalent inviscid flow (EIF), which leads to VII effects in regions of high-pressure gradients, particularly noticeable at the trailing edge and shockwave interfaces, as mentioned by Lock and Williams [1,2]. An additional assumption in [3] is the equivalence of the EIF and RVF static pressure fields, i.e.,  $P_E \approx P_\infty$  imposing a thin shear layer approximation commonly used in integral boundary layer (IBL) analysis. IBL analyses such as those in [14–16] present a zonal drag decomposition primarily focusing on evaluating drag originating from the BL, commonly called

the DF zone<sup>4</sup>. In this approach, the DF is obtained by taking the difference between the RVF and reconstructed EIF, with the resulting flow components being related to boundary layer parameters such as the momentum and mass defect for momentum IBL drag assessment, as well as the kinetic energy and density flux defect used in mechanical energy IBL drag assessment.

Following from prior work in [17], Lamprakis et al. [18] suggest that a more intuitive means of expressing IBL formulations, usually described in the relative reference frame (RRF), would be to adopt an absolute reference frame (ARF) perspective. The ARF considers an aerodynamic body moving through a quiescent fluid, where the BL facilitates the associated momentum and energy transfer between the body and fluid. This perspective intuitively highlights energy conversion pathways and is particularly well suited for identifying local flow mechanisms containing available energy that can be utilised to produce useful work through energy recovery systems such as boundary layer ingestion (BLI) and heat exchangers. Secondly, through this perspective, the Galilean invariance of force relations can be realised, but more importantly, the Galilean covariant nature of the work energy relationships in flight. This is important in recognising the independence of the reference frame of force-based relations, which guarantees their consistency irrespective of the reference frame. However, energy and exergy-based analysis rely on the dot product of the momentum equation with the local flow velocity, which is reference frame dependent, thus immediately inferring its variation with the reference frame. The ARF becomes particularly useful when extending past momentum-based analysis as it becomes essential for interpretative consistency to use an approach that provides a clear and intuitive trace of the various energy transformation pathways.

The present work's main objective is to demonstrate the applicability of the unified velocity decomposition and partial pressure field approach presented in Chapter 2 for complex flow studies, including shockwave formation and lift production. The proposed unification is a modification of Aguirre's velocity decomposition approach [3], which adopts an ARF perspective. As the case

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<sup>4</sup> In the relative reference frame (RRF) the BL represents a region of mass, momentum and energy deficit and thus is intuitively referred to as the defect zone to highlight this feature

studies chosen for this work are carried out in transonic flow, farfield momentum and exergy wave drag assessment is conducted through entropy-drag formulations applied individually to the shockwave zone as done in [9] with the shock and viscous zones identified through flow feature identification methods suggested in [4] and [19]. In addition, energy-based wave drag assessment is explored first by testing the efficacy of the proposed wave drag power  $\dot{E}_w$  introduced in Drela's work [20]. Secondly, an improved energy-based wave drag formulation is presented that considers the entropy rise across the shockwave interface, similar to the exergy-based wave anergy. Finally, near-field wave drag assessment is conducted through PPF decomposition via flow projection [21] and VII correction techniques shown in Chapter 2. The collective of drag decompositions then provides a means of comparative quantitative contrasting of the various evaluation methods to assess their relative consistency and agreement for lifting and non-lifting transonic test cases.

## 3.2 Mathematical Methodology

### 3.2.1 Generalised Velocity Decomposition and Partial Pressure Field Unification

The velocity decomposition approach Aguirre [3] proposed anchors around Betz's [13] artificial velocity principle, which describes the behaviour of a hypothetical velocity field where no losses in the total energy are experienced. Before the modifications proposed by Aguirre [3], Méheut [22] similarly introduced a velocity decomposition approach, which enabled the evaluation of profile drag using far-field momentum analysis. A significant advantage of this was that it allows for a considerably accurate profile drag analysis applicable for experimental wind tunnel testing by evaluating downstream wake data. However, it is later identified in [3] that this approach would not be valid when coupled with exergy-based formulations as it assumes that the transverse velocities are isentropic throughout the entire flow field. However, this is only accurate for flow outside the BL and wake zones, i.e., bulk flow as flow within the shear layers consists of both isentropic and non-isentropic components<sup>5</sup>.

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<sup>5</sup> Isentropic and non-isentropic are analogous to non-dissipative and dissipative as mentioned in the work of Schmitz et al. [23,24]

Aguirre [3] introduces a new procedure for velocity decomposition, which obtains the isentropic flow components of the velocity, density and temperature through the isentropic flow equations and ideal gas law for the entire flow field. Implementation of this methodology relies on two primary assumptions, the first being that the static pressure of the RVF is equivalent to the isentropic static pressure, which then allows for the isentropic velocity to be obtained from the isentropic flow equations. The second assumes the direction of the isentropic velocity vector is the same as that of the RVF at all points in the domain.

PPF analysis [23–25] introduces an equally functional decomposition of the pressure field for steady flows. The static pressure is represented as a superposition of an Euler and dissipative pressure, each governed by a separate Poisson equation, as shown in equation (3.1).

$$\begin{aligned}\nabla^2 p_E &= \nabla \cdot (-\rho(\mathbf{V} \cdot \nabla) \mathbf{V}) & (a) \\ \nabla^2 p_\mu &= \nabla \cdot (\nabla \cdot \bar{\boldsymbol{\tau}}) & (b)\end{aligned}\tag{3.1}$$

The Euler pressure provides an improved approximation of the isentropic pressure field, which can then be coupled with Aguirre’s velocity decomposition approach, avoiding the previously mentioned static pressure assumption. Under transonic conditions, it is essential to highlight that the isentropic field can no longer be considered as isentropic due to shockwave formation and is thus more aptly referred to as an Euler inviscid flow. This unification with PPF decomposition highlights the duality of shockwave drag interpretation; in the near-field, this contribution is realised through a surface integration of the Euler pressure, whereas the dissipative pressure integration returns the shockwave boundary layer interaction drag. Meanwhile, in the far field, the wave drag is realised through the dissipative field primarily due to its entropic nature.

This flow field separability into Euler and dissipative parts can naturally be extended to force contributions, as shown in equation (3.2). Generally, the RVF drag can be represented as a superposition of the dissipative drag originating from the shear layers and the Euler drag associated with the bulk flow, which is extended for transonic flows to differentiate between the profile, wave and lift-induced, physical drag mechanisms.

$$\begin{aligned}
D_{RVF} &= \underbrace{D_\mu}_{\text{shear-layer}} + \underbrace{D_E}_{\text{bulk flow}} \\
&= \underbrace{D_\mu}_{\text{viscous drag}} + \underbrace{D_w}_{\text{wave drag}} + \underbrace{D_i}_{\text{lift induced drag}}
\end{aligned} \tag{3.2}$$

The Euler momentum, energy, and exergy-based formulations in equation (3.3) become particularly useful for lifting flows as they provide a unique means of quantifying the drag contribution linked to downwash, i.e., lift-induced drag, as shown in Chapter 2.

$$\begin{aligned}
\mathbf{F}_E &= -\iint_S \left[ p_{G_E} \mathbf{n} + (\mathbf{V}_E - \mathbf{V}_\infty) \rho_E \mathbf{V}_E \cdot \mathbf{n} \right] dS \quad (\text{a}) \\
\mathbf{F}_E \cdot \mathbf{V}_\infty &= \dot{E}_{phy_E} = \dot{E}_{m_E} + \Theta_E = \dot{E}_{m_E} + \dot{\varepsilon}_{th_E} \quad (\text{b})
\end{aligned} \tag{3.3}$$

### 3.2.2 Unification Effects on Recoverable Energy Quantification

An advantage supporting the adoption of energy and exergy-based analyses is their ability to quantify the recoverable energy present within the flow. For example, exergy-based performance metrics such as exergy waste coefficient (EWC) [10,26,27] can quantify the total recoverable energy (mechanical + thermal) present within the flow. This differs slightly from the energy-based potential for energy recovery factor (PER) [17,28], which only considers the recoverable mechanical energy and regards the thermal energy as a thermo-compressibility loss contributing toward the total domain energy losses.

$$\begin{aligned}
EWC_\mu &= \frac{\dot{\varepsilon}_{phy_\mu}}{DV_\infty} \quad (\text{a}) \\
PER_\mu &= 1 - \frac{\Phi + \Theta_\mu}{DV_\infty} = \frac{\dot{E}_{m_\mu}}{DV_\infty} \quad (\text{b})
\end{aligned} \tag{3.4}$$

The proposed unification modifies the conventional recoverable energy formulations, i.e., EWC and PER, presented in the literature. As shown in equations (3.4)(a) and (b), this modification considers that only dissipative terms be used while evaluating the recoverable energy [17]. Particularly for energy-based analysis, this entails including the dissipative pressure-volume work

as part of the total mechanical energy loss, which is analogous to an evaluation of the dissipative mechanical energy, as shown in equation (3.4)(b).

### 3.2.3 Transonic Flow Analysis and Shockwave Treatment

#### 3.2.3.1 Partial Pressure Field Analysis

A particular limitation of transonic drag decomposition using entropy-drag formulations coupled with flow feature identifiers, such as in the work of Paparone and Tognaccini [4], is the potential mixing of the shockwave and viscous wake phenomena [21]. This highlights that the viscous and wave drag quantities obtained partially depend on how effectively the chosen identifiers delineate the various phenomenological contributions. Hart et al. [21] introduce a PPF decomposition which aims to avoid the aforementioned potential mixing issue and provides a unique wave drag evaluation technique through near-field analysis.

This method evaluates the wave drag by isolating compressibility effects through the Euler pressure and velocity fields by splitting their contributions into compressible and incompressible parts, as shown in equation (3.5).

$$\begin{aligned} p_E &= p_c + p_{inc} & (a) \\ \mathbf{V} &= \mathbf{V}_c + \mathbf{V}_{inc} & (b) \end{aligned} \tag{3.5}$$

The velocity field  $\mathbf{V}_{inc}$  is obtained from an incompressible flow solution using the same flow conditions as the RVF. Taking the difference between the RVF and incompressible fields in equation (3.5)(b) returns the isolated compressible velocity field  $\mathbf{V}_c$ , which is used in the definition of the compressible and incompressible pressure Poisson equations shown in equation (3.6)(a) and (b), respectively.

$$\begin{aligned} \nabla^2 p_c &= \nabla \cdot (-\rho(\mathbf{V} \cdot \nabla) \mathbf{V}_c) & (a) \\ \nabla^2 p_{inc} &= \nabla \cdot (-\rho(\mathbf{V} \cdot \nabla) \mathbf{V}_{inc}) & (b) \end{aligned} \tag{3.6}$$

The benefits of this PPF decomposition for transonic flows are twofold; firstly, the wave drag can be quantified through the compressible pressure drag from  $P_c$  obtained via a near-field assessment. Secondly, this decomposition introduces the concept of compressible and incompressible lift, which is similarly realised in compressible vortex force lamb vector derivations as shown in [29–34]. The decomposition of lift in this manner aids in highlighting the physical importance of vorticity in the generation of lift. Firstly, incompressible lift generation is directly related to the circulation developed around the airfoil through the commonly known Kutta Joukowski theorem. For 2D viscous flows, the circulation is associated with bound vorticity, which manifests as a result of the physical vorticity distribution due to the velocity and shear gradients within the boundary layer, as shown in Figure 3-1(a). Compressible lift generation is associated with baroclinic torque [35], which influences vorticity within the BL through its density variations. Like the baroclinic power [16,18], its influence is a strong function of the Mach number becoming negligible for incompressible flows. Physically, the baroclinic torque describes an inertial moment that occurs due to forces acting on fluid particles of different densities, with non-zero contributions existing in areas where the density and pressure gradients are perpendicular, depicted in Figure 3-1(b).

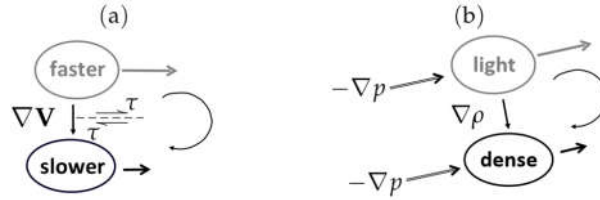


**Figure 3-1 Schematics illustrating vorticity generation mechanisms adapted from [35]**

### 3.2.3.2 Far-field and Exergy-based Shockwave Treatment

Similar to the research in [10], this work considers shockwaves as adiabatic, inviscid flow discontinuities characterised by an infinitesimal width labelled  $S_{\text{shock}}$  in Figure 3-2. Numerical assessment of shockwave effects involves isolating its volumetric region of influence  $V_{\text{shock}}$ , identified through flow feature extraction methods.

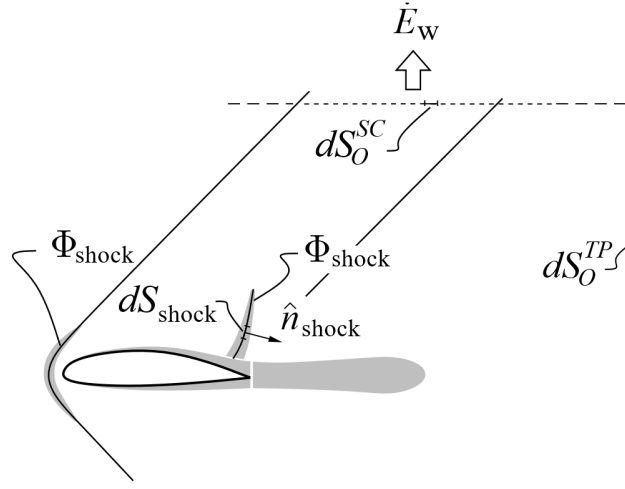


**Figure 3-2 Surface and normal vector definition for theoretically discontinuous shockwaves  
adapted from [20]**

The case studies in this research identify the shockwave's region of influence through a total pressure identifier ( $P_t / P_{t\infty}$ ) suggested by Aguirre and Duplaa [19] and delineate its contribution from the BL and wake zones using a viscous identifier ( $\mu_{eff} / \mu$ ) used in the work of Paparone and Tognaccini [4]. After identifying the shockwave region, its drag contribution can be evaluated through the wave anergy formulation introduced by Arntz's [10,26,27,36] and given in equation (3.7).

$$A_w = T_\infty \iint_{S_{shock}} [\rho \delta s \mathbf{V}] \cdot \hat{\mathbf{n}}_{shock} dS = D_w V_\infty \quad (3.7)$$

This method exploits the idea behind Kusonose's profile drag decomposition [37,38], which considers that aerodynamic flows consist of two distinct sources of entropy. The first is the boundary and wake, while the second is the shockwave region. As a result, the wave drag can be estimated by evaluating the entropy drag [8], i.e., equation (3.8), originating from the shockwave region, an approach similarly adopted in the work of Chao and van Dam [9].

$$D_{\delta s} = \frac{T_\infty}{V_\infty} \iint_{S_o} \rho \delta s (\mathbf{V} \cdot \mathbf{n}) dS \quad (3.8)$$

Aguirre [12] modifies the surface integral in equation (3.7) through a Gauss theorem conversion to obtain a volume integral formulation given in equation (3.9), similarly proposed in the work of Schmitt and Destarac [11].

$$D_w V_\infty = A_w = T_\infty \underbrace{\iint_{S_{\text{shock}}} [\rho \delta s \mathbf{V}] \cdot \tilde{\mathbf{n}} dS}_{\text{Arntz method}} = \underbrace{\iiint_{V_{\text{shock}}} \nabla \cdot (T_\infty \rho \delta s \mathbf{V}) dV}_{\text{Aguirre method}} \quad (3.9)$$

### 3.2.3.3 Power Balance Shockwave Treatment

Drela [20] introduces wave drag evaluation techniques using energy-based methods and proposes its evaluation by measuring the pressure work and kinetic energy deposition rate crossing the side cylinder using the formulation provided in equation (3.10). This method, however, is highly dependent on the amount of mechanical energy interacting with the side cylinder<sup>6</sup>, as shown in Figure 3-2. As a result, large control volumes with no pressure work or kinetic energy crossing the side cylinder would return a wave drag contribution of zero. Therefore, evaluation in this manner becomes less accurate as the control volume size increases, prompting the need for a more robust energy-based wave drag evaluation technique.

$$\dot{E}_w = \iint_{S_{SC}} \left[ p - p_\infty + \frac{1}{2} \rho (u^2 + v^2 + w^2) \right] V_n dS_O^{SC} \quad (3.10)$$

Arntz [10] highlights the dependency of dissipation analysis on Boussinesq's approximation for turbulent flow Reynolds averaged Navier Stokes (RANS) analyses within CFD. This occurs due to the additional shear stress and associated heat flux flow phenomena encountered for turbulent flows. The Boussinesq approximation considers that local shear stresses share a linear relationship with the mean strain rate  $\bar{\mathbf{S}}$ , having a proportionality constant equal to the eddy viscosity. This results in an *effective* stress tensor that combines viscous and turbulent effects, thus defining a similar *effective* dissipation rate due to the combined viscous and turbulent dissipation shown in equation (3.11).

---

<sup>6</sup> For dimensionality consideration, when assessing two-dimensional flows the side cylinder reduces to a side plane.

$$\begin{aligned}
(\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V} &= \underbrace{(\mu_{lam} + \mu_{turb})}_{\mu_{eff}} \phi \\
\text{where:} & \\
\phi &= 2 \left[ \bar{\mathbf{S}} : \bar{\mathbf{S}} - \frac{1}{3} (\nabla \cdot \mathbf{V})^2 \right]
\end{aligned} \tag{3.11}$$

This notion of an *effective* shear stress tensor highlights that the local shear stress behaviour is strongly related to the phenomenological characteristics of the flow region in question. For example, within the boundary layer and wake, the non-zero viscosity results in cumulative loss mechanisms over the aerodynamic body associated with *viscous* dissipation. For turbulent flows, however, this conversion of kinetic energy to heat, i.e., dissipation, is intensified through *turbulent mixing* losses.

It then becomes plausible to assume that for highly compressible transonic flows, an additional dissipation mechanism arises from the compressibility of the flow and, more importantly, the formation of shockwaves. It is commonly known from literature through work such as that of Turkel [39] that central differencing schemes introduce a numerically stabilising *artificial viscosity*. This introduction also creates an “entropy-like” condition, enabling the capturing of shockwave phenomena. Therefore, in the instance of transonic flows, it is possible to introduce an additional hypothetical proportionality factor equal to the *artificial* viscosity, which aids in capturing the shockwave dissipation and the wave drag.

$$(\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V} = \underbrace{(\mu_{lam} + \mu_{turb} + \mu_{artificial})}_{\mu_{eff}^*} \phi \tag{3.12}$$

### 3.3 Numerical RANS Analysis Results and Discussion

#### 3.3.1 Case Study I: NACA 0012 Airfoil Inviscid Transonic Flow Analysis

The modified effective viscosity concept introduced in equation (3.12) was tested by comparing conventional power balance and exergy-based drag assessment for an inviscid NACA 0012 airfoil.

Two case studies from references [10,21] were chosen; the first consisted of a non-lifting NACA 0012 airfoil at zero degrees angle of attack operating at a Mach number of 0.80 and a chord-based Reynolds number  $3 \times 10^6$ . The second case study consisted of a lifting NACA 0012 airfoil at a moderate angle of attack of  $3.04^\circ$ , operating at a Mach number of 0.70 and a chord-based Reynolds number of  $9 \times 10^6$ .

As mentioned, Drela [20] proposes evaluating the wave drag through equation (3.10). This approach is suitable to an extent but is strongly dependent firstly on the strength of the shock since the method relies on the propagation of shockwave perturbations into the downstream flow, which crosses the side cylinder. Therefore, weak shocks not directly interacting with the side cylinder will tend to under-approximate the wave drag. Furthermore, this approach depends on the control volume size with the side cylinder mechanical energy contribution going to zero when sufficiently far, as shown in Figure 3-3.

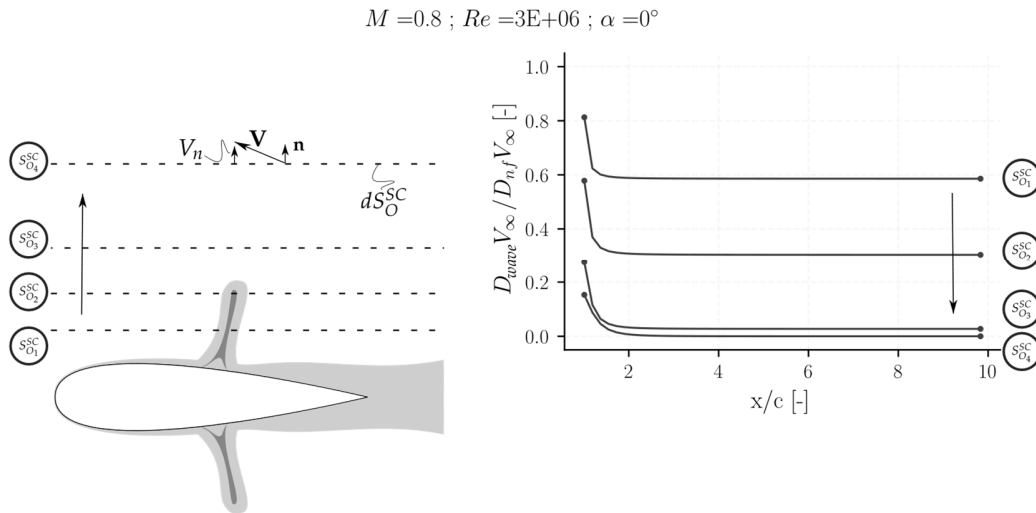


**Figure 3-3 Assessment of Wave Drag Variation with Side Cylinder Placement**

A power balance analysis shows that the mechanical energy and thermo-compressibility contributions mutually counterbalance, returning no overall drag contribution. Secondly, as the flow is inviscid, there is no boundary layer or wake development, and the associated viscosity in this instance is zero by definition. Therefore, no dissipative mechanisms are captured by equation

(3.11) as this formulation's applicability is limited to turbulent flows without flow discontinuities such as shockwaves. This would imply that a conventional power balance analysis would return a total drag contribution of zero, which is incorrect, as the airfoil returns a non-zero near-field pressure drag equal to the wave drag.

As expected, the exergy analyses in Figure 3-4 highlight the significant contribution of the total anergy toward drag, as its contribution in this instance for inviscid flow captures the wave drag as described in the formulations introduced by Arntz [10] and Aguirre [12] in equation (3.7). Furthermore, a term-by-term comparison of the energy and exergy-based Euler formulations provided in equation (3.13) shows that from an exergy perspective, the pressure-volume work is analogous to the thermal exergy, also confirmed through the Euler internal energy equation shown in Chapter 2.

$$\begin{aligned} DV_\infty &= \dot{E}_{m_E} + \Theta_E + \Phi_w \\ DV_\infty &= \dot{E}_{m_E} + \dot{\mathcal{E}}_{th_E} + \dot{A}_w \end{aligned} \quad (3.13)$$

More importantly, for transonic flow analysis, any dissipation for this flow scenario solely occurs due to the shockwave, as this term becomes analogous to the wave anergy from an exergy-based perspective.

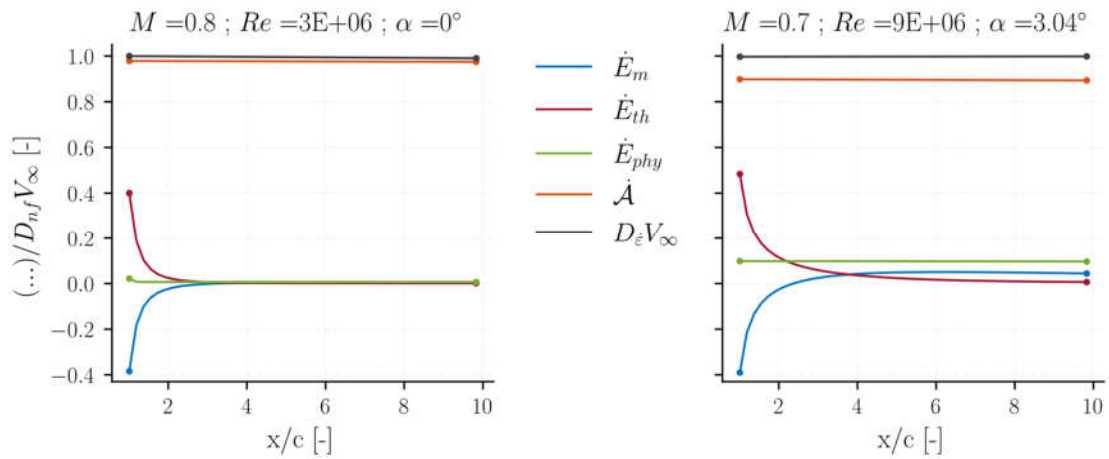


**Figure 3-4 NACA 0012 airfoil wake energy assessment in transonic flow**

As a result, a plausible research question arises, i.e., *from a power balance perspective, can entropy explicitly be introduced to evaluate shockwave dissipation and its associated drag similarly to exergy analyses?*

The entropy formulation provided in equation (3.15) addresses this and is obtained by combining the Gibbs and internal energy expressions in equation (3.14)(a) and (b), respectively.

$$\begin{aligned} T\nabla \cdot (\rho s \mathbf{V}) &= \nabla \cdot (\rho e \mathbf{V}) + p \nabla \cdot \mathbf{V} & (a) \\ \nabla \cdot (\rho e \mathbf{V}) &= -\nabla \cdot \mathbf{q} - p \nabla \cdot \mathbf{V} + (\bar{\tau} \cdot \nabla) \cdot \mathbf{V} & (b) \end{aligned} \quad (3.14)$$

The Gibbs relation serves the purpose of providing a connection between entropy and internal energy which when coupled with the internal energy formulation by combining equation (3.14)(a) + equation (3.14)(b) produces the dissipation expression in equation (3.15) capable of capturing the shockwave-induced entropy rise for transonic flows. This expression would now represent the hypothesised modified effective dissipation formulation presented in equation (3.12).

$$(\bar{\tau} \cdot \nabla) \cdot \mathbf{V} = T\nabla \cdot (\rho s \mathbf{V}) + \nabla \cdot \mathbf{q} \quad (3.15)$$

As shockwaves are considered adiabatic phenomena, this implies that the heat flux term yields no contribution by definition, simplifying the dissipation expression to the form given in equation (3.16) when applied to transonic shockwave analyses.

$$[(\bar{\tau} \cdot \nabla) \cdot \mathbf{V}]_{\text{wave}} = [T\nabla \cdot (\rho s \mathbf{V})]_{\text{wave}} + \underbrace{[\nabla \cdot \mathbf{q}]_{\text{wave}}}_{= 0, \text{ adiabatic}} \quad (3.16)$$

Equation (3.16) can be extended to exergy analysis through the procedure detailed by Arntz [10], which introduces the wave energy  $\dot{A}_w$  shown in equation (3.17) and is responsible for capturing the shockwave-induced entropy rise through the priorly defined expressions in equations (3.7) - (3.9).

$$\begin{aligned}
\lim_{T \rightarrow T_\infty} [T \nabla \cdot (\rho s \mathbf{V})]_{wave} &= \underbrace{[T_\infty \nabla \cdot (\rho s \mathbf{V})]_{wave}}_{\dot{A}_w} \\
&= \underbrace{[(\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V}]_{wave}}_{\Phi_w}
\end{aligned} \tag{3.17}$$

By considering that thermal and velocity gradients predominantly exist in the viscous BL region, as shown in Figure 3-5, the shockwave wake, which lies mainly in the bulk flow region, will have local static temperatures very near freestream (i.e.,  $T \simeq T_\infty$ ) due to the lower thermal gradients observed in the bulk flow.

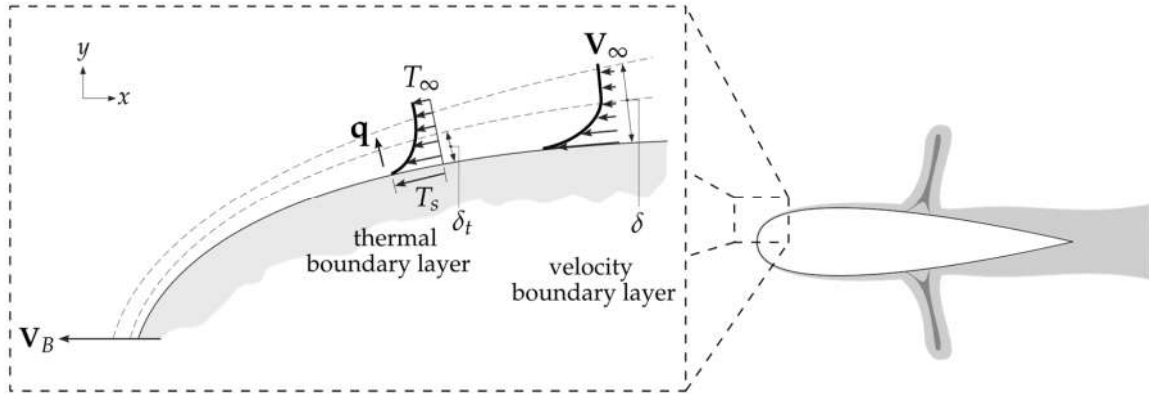


**Figure 3-5 Development of Thermal and Velocity Boundary Layers over the airfoil surface adapted from [40]**

As a result, the shockwave dissipation  $\Phi_w$  can be formulaically represented through the wave anergy, as shown in equation (3.17). The validity of this formulation for wave drag assessment via shockwave dissipation using mechanical energy methods will be assessed for more complex viscous flow scenarios in the following sections.

### 3.3.2 Case Study II & III: NACA 0012 Airfoil RANS Transonic Flow Analysis

For viscous flow analysis, an important aspect necessary to obtain an accurate depiction of the drag contributions from shear and bulk flow effects as described in equation (3.2), comes through the consideration of the separability between the shear layer over the aerodynamic body and bulk

flow. The shear layer is usually defined through the *velocity* boundary layer thickness  $\delta$  arbitrarily defined at a point where the velocity has reached a certain percentage of the freestream, usually 99% [41]. The remaining region above  $\delta$  would then define the bulk flow. This condition automatically creates the requirement that the local temperature be near freestream i.e.  $T \approx T_\infty$  within the bulk flow for effective use of the shockwave dissipation formulation proposed in equation (3.17). For this requirement to be satisfied it becomes necessary that the velocity boundary layer thickness be either equal or greater than the thermal boundary layer i.e.,  $\delta \geq \delta_t$  to ensure that the local flow velocity and temperature are near freestream within the bulk flow.

Conveniently, this can be assessed effectively through the Prandtl number, as depicted in Figure 3-6, which describes the ratio of momentum and thermal diffusivities i.e.,  $Pr = \nu/\alpha$

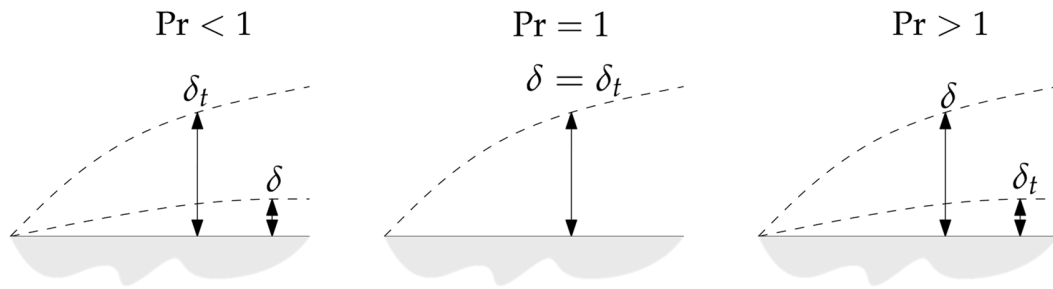


**Figure 3-6 Thermal and Velocity Boundary Layer thickness characteristics at various Prandtl numbers adapted from [42]**

When  $Pr > 1$ , the momentum diffusivity is greater than thermal diffusivity, resulting in a thicker *velocity* boundary layer due to its higher growth rate over the body. When  $Pr < 1$  the opposite occurs where the thermal diffusivity is larger resulting in a thicker thermal boundary layer instead.

For the numerical analyses in this work, the freestream conditions considered a pressure of 1 atm at a temperature of 300K. At these conditions, the laminar and assumed turbulent Prandtl number of air would be  $Pr_{lam} = 0.73$  and  $Pr_{turb} = 0.9$ , respectively. This results in an effective Prandtl number i.e.,  $Pr = (Pr_{lam} + Pr_{turb}) > 1$  ensuring that  $\delta \geq \delta_t$  as required for transonic shockwave analysis using equation (3.17).

### 3.3.2.1 Boundary and Operating Conditions

The prior inviscid test cases were extended to turbulent RANS flow in ANSYS Fluent © using the Spalart–Allmaras eddy-viscosity turbulence model [43]. Simulations were run using a coupled pseudo-transient pressure-based solver with the Green-Gauss cell-based method and second-order up-winding. The analyses were conducted in a C–H flow domain with pressure farfield and outlet boundaries at the peripheries located 30 chord lengths ( $c = 0.435\text{m}$ ) from the airfoil no-slip wall surface. The near-wall mesh refinement ensured the  $y^+$  was kept below 0.5 for turbulence model compatibility and PPF analysis.

As a requirement for solving the dissipative and Euler pressure Poisson equations, the domain boundary conditions were set following the work of Schmitz et al. [23–25]. First, the farfield was set as a Dirichlet boundary condition, wherein the Euler pressure was set to equal the freestream static pressure. Next, the outlet and airfoil walls were both taken as Neumann boundaries. The gradient of the Euler pressure normal to the outlet was set to zero, and at the airfoil surface where the no-slip condition was enforced, naturally resulted in a state where  $\nabla p_E = 0$ . Lastly, the dissipative pressure gradient was set to equal the shear stress tensor divergence at the walls and zero normal to the outlet, respectively.

The additional boundary conditions necessary for the compressible and incompressible pressure Poisson equations were set following the work of Hart and Schmitz [21]. In the far-field, the incompressible pressure was set to equal the freestream, which is plausible as viscous effects tend to zero in the far-field, resulting in incompressible flow behaviour. In contrast, this implies that the complementary compressible pressure becomes zero in the farfield. Lastly, the incompressible pressure requirements at the outlet and walls consist of a Neumann ( $\nabla p_{inc} = \nabla \cdot \bar{\tau}$ ) and Dirichlet ( $p_{inc} = 0$ ) condition, whilst the compressible pressure consists of a Neumann boundary, i.e.  $\nabla p_c = 0$ , similar to the Euler pressure.

Table 3-1 provides a mesh convergence study monitoring the total drag of the lifting and non-lifting airfoil test cases, assessed using near-field, far-field, energy, and exergy-based methods.

For each, several mesh refinement levels were used, each with increasing mesh density, with L1 representing the finest grid.

**Table 3-1 NACA 0012 drag coefficient grid convergence study reported in drag counts ( $C_d \times 10^{-4}$ )**

| $M = 0.80, Re = 3 \times 10^6, \alpha = 0^\circ$ [10]    |               |          |        |        |
|----------------------------------------------------------|---------------|----------|--------|--------|
| Grid (# cells)                                           | Near-field    | Farfield | Energy | Exergy |
| L5 (42837)                                               | 175.4         | 176.2    | 175.9  | 175.9  |
| L4 (171348)                                              | 173.9         | 174.3    | 174.3  | 174.3  |
| L3 (385533)                                              | 173.8         | 174.1    | 174.0  | 174.0  |
| L2 (685392)                                              | 173.8         | 173.9    | 173.9  | 173.9  |
| L1 (1070925)                                             | 173.8 (168.1) | 173.9    | 173.8  | 173.8  |
| $M = 0.70, Re = 9 \times 10^6, \alpha = 3.04^\circ$ [21] |               |          |        |        |
| Grid (# cells)                                           | Near-field    | Farfield | Energy | Exergy |
| L5 (46107)                                               | 145.6         | 145.7    | 144.7  | 144.7  |
| L4 (184428)                                              | 144.8         | 144.6    | 143.3  | 143.3  |
| L3 (414963)                                              | 144.8         | 145.0    | 143.7  | 143.7  |
| L2 (737712)                                              | 145.1         | 144.7    | 143.3  | 143.3  |
| L1 (1152675)                                             | 145.2 (147)   | 144.9    | 143.5  | 143.5  |

Mesh convergence was assessed by comparing the numerical agreement of the drag coefficients obtained through the various methods as well as their relative coherence with validation data obtained from literature. On observation, at the finest grid refinement level, all drag coefficients showed a  $< 1\%$  difference between their respective solutions while also providing a percentage error of  $< 4\%$  in comparison to the validation data obtained from [10] and [21], shown in brackets in Table 3-1.

### **3.3.2.2 Partial Pressure Field Decomposition**

The distributions in Figure 3-7 show the non-lifting and lifting pressure trends for the NACA 0012 airfoil; the EIF and RVF static pressure show good agreement between the respective trends, with their difference giving the dissipative pressure representing the pressure-related viscous drag. A non-zero dissipative pressure contribution exists ahead of the airfoil due to the boundary layer's displacement effect on the pressure field as the airfoil traverses through the flow. Characteristic of transonic flows, shockwave effects are observed through the rapid change in EIF and RVF static pressures over the airfoil, similarly realised through the discontinuous change in dissipative pressure at the shockwave region. Downstream of the airfoil trailing edge, the non-zero dissipative pressure indicates a pressure work potential via the energy imparted to the flow. This pressure work then contributes toward the recoverable energy present within the downstream wake, gradually decaying downstream due to irreversible viscous losses.

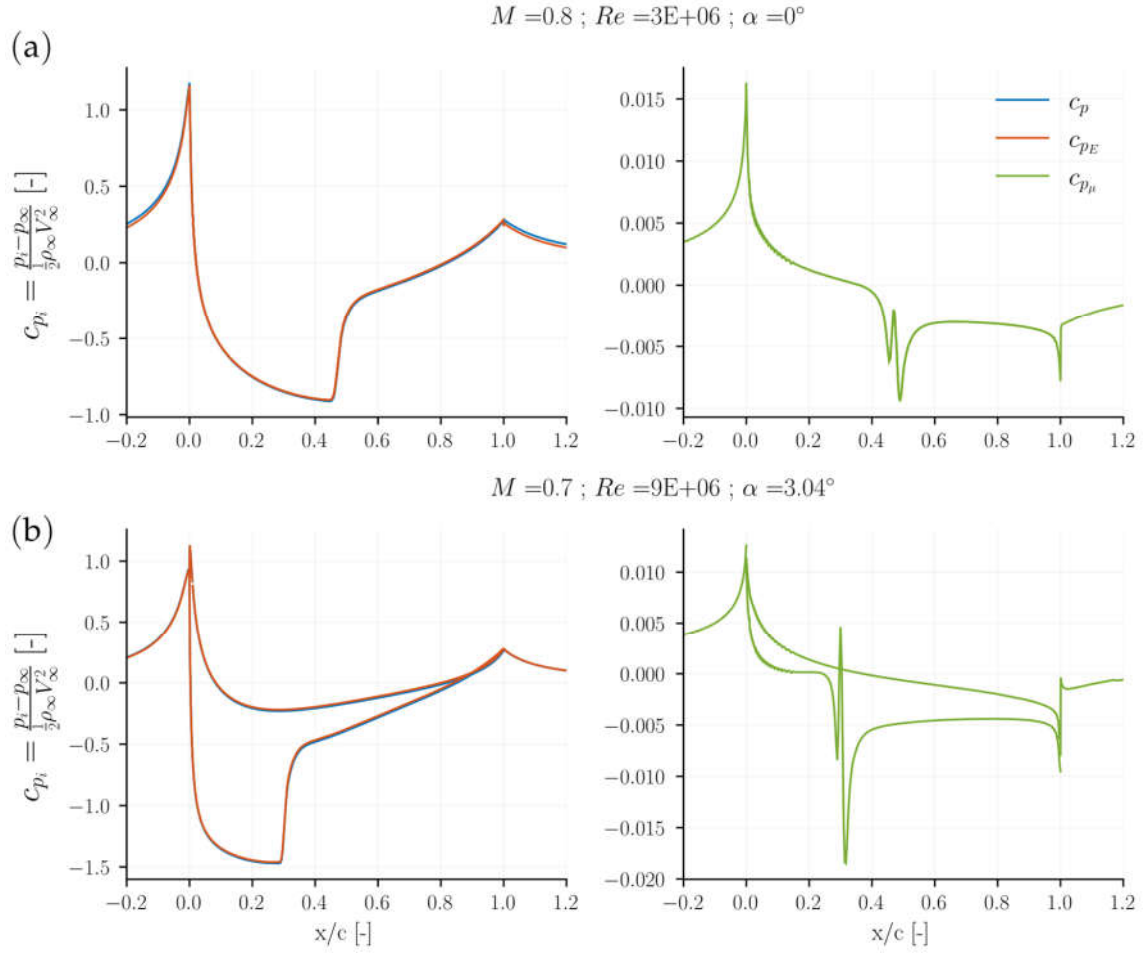


**Figure 3-7 Transonic airfoil isolated flow regions for profile, viscous and wave drag evaluation**

It is well known that an additional phenomenological drag contribution exists under transonic conditions due to shockwave formation. This section looks into the mechanisms responsible for the increase in drag caused by shockwaves, i.e., wave drag, from a near-field perspective through PPF decomposition utilising the pressure field separability shown in Figure 3-7.

The wall shear stress of a Newtonian fluid can be described via equation (3.18), where  $\mu$  describes the dynamic viscosity and  $(\partial u / \partial y)$  the fluid velocity gradients. The effects of shockwave boundary layer interaction can change these two terms through several mechanisms.

$$\bar{\tau} \simeq \mu \frac{\partial u}{\partial y} \quad (3.18)$$

Firstly, through boundary layer (BL) growth, the thicker BL after the shockwave increases the area of action of the BL's viscous damping effect, which causes a decrease in the local velocity gradients. As the airfoil traverses through the fluid, the BL acts as a medium of momentum and energy transfer between the body and the fluid, with thin BLs representing rapid momentum and energy transfer through a small area, resulting in steeper gradients near the wall. Thicker BLs offer a much larger area for this work transfer; therefore, the diffusion of momentum and energy is more gradual, resulting in less steep velocity gradients.

Secondly, through a decrease in fluid velocity, the shock wave results in a localised abrupt reduction in velocity, which reduces the BL velocity gradients and decreases the shear stress. Relating this to the skin friction would broadly imply a decrease in this quantity downstream of the shockwave, confirmed in Figure 3-8, where an apparent reduction in the gradient of the cumulative shear stress drag contribution  $(D_{\tau} V_{\infty})$  is observed.

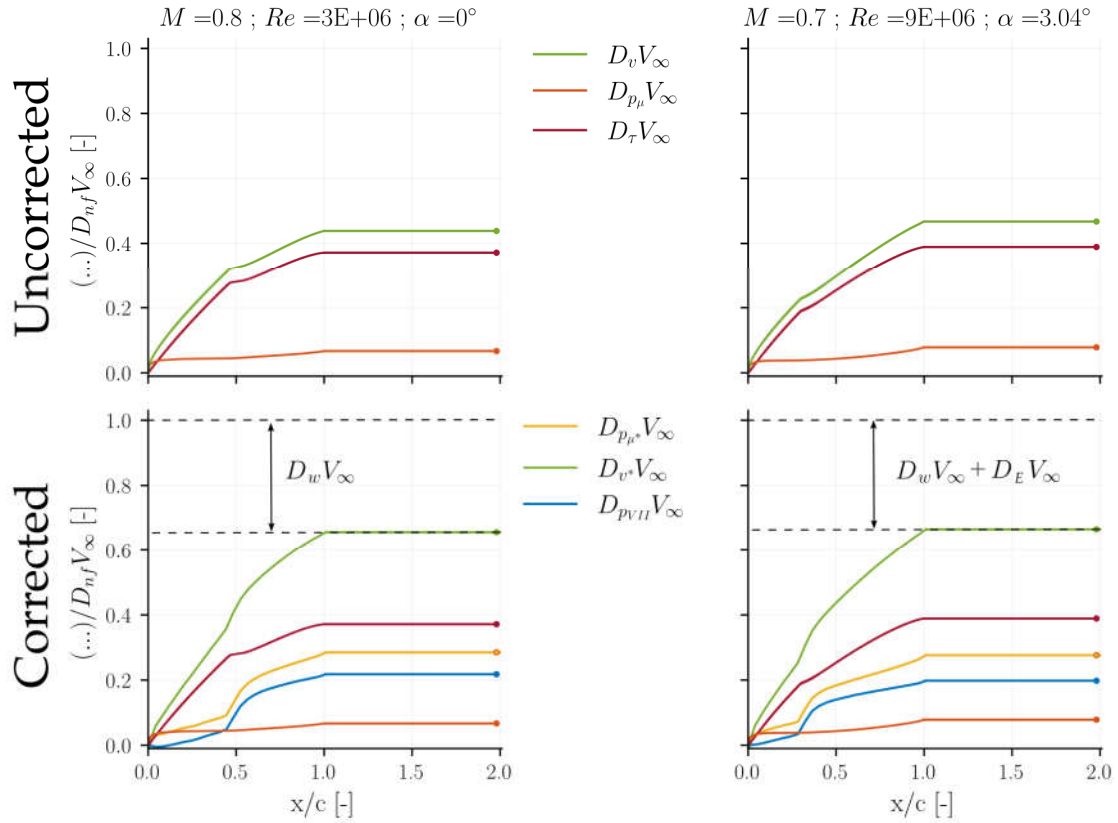


**Figure 3-8 Near-field comparison of uncorrected (top) and corrected (bottom) dissipative drag behaviour for lifting and non-lifting transonic NACA 0012 airfoil**

Shockwave theory suggests that increased static pressure and temperature accompany shockwave formation. Particularly of interest from a near-field perspective is the change in pressure following the shock. The changes in pressure across a shockwave are usually abrupt, as seen in Figure 3-7. However, for the uncorrected data in Figure 3-8, no significant discontinuous behaviour is observed for the dissipative pressure drag  $(D_{p\mu} V_\infty)$ , showing a reasonably continuous cumulative distribution.

Physically, the dissipative pressure drag power is a near-field representation of the recoverable mechanical energy, which was shown to be relatively insensitive to changes in Mach number through the work of Lamprakis et al. [17]. More aptly, it should be noted that the recoverable

energy is weakly linked to compressibility through thermal and mechanical energy cross-coupling as expressed by Arntz [10] and later improved by Aguirre [44] through a mathematical link between the thermal energy due to pressure and pressure boundary work.

The abrupt change in pressure drag expected over the airfoil is only realised after employing the VII correction procedure outlined in Chapter 2. This procedure aids in filtering out the viscous drag contributions present within the Euler field and correctly attributes their effects toward the dissipative pressure drag. Through this, the discontinuous rise in pressure drag is captured, as well as its contribution toward viscous drag through the VII pressure drag term  $(D_{p_{VII}} V_\infty)$  shown through the corrected trends in Figure 3-8.

The VII pressure represents the pressure field induced by the EIF wall transpiration within the BL. As the unified partial pressure and velocity decomposition approach employs a simple inviscid model that omits wall transpiration, the effects of this cause VII in regions with significant pressure interactions, such as areas of high pressure gradients, as mentioned by Lock and Williams [1,2]. The VII effects become most noticeable at the trailing edge and shockwave boundary layer interfaces due to the significant pressure gradients experienced in these regions. As a result, the corrected viscous drag shown  $(D_{p_\mu^*} V_\infty)$  in Figure 3-8 now accurately represents the expected cumulative drag behaviour without VII effects.

### 3.3.2.3 Power Balance and Exergy-based Drag Decomposition

Through a systematic analysis of Potential, Euler, and RANS subsonic test cases, Aguirre [45] shows that the Euler field<sup>7</sup> has no contribution toward the recoverable energy as flow mechanisms associated with this field all mutually self-recover as they follow an isentropic path. For transonic flows, it becomes imperative to note that the Euler field is no longer isentropic due to the shockwave, which is a direct source of entropy production within the fluid. However, the notion

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<sup>7</sup> Aguirre [45] refers to the Euler field as the isentropic, which is plausible only for *subsonic* flows, as shockwave formation results in entropic phenomena within the bulk flow in transonic conditions

of the Euler field having no contribution toward recoverable energy remains true as the shockwave's influence is directly related to anergy which has no direct work potential and thus does not facilitate any potential for useful work production. As a result, it is solely through the dissipative field that the recoverable energy can be quantified. As such, energy recovery can be viewed as entropy generation prevention by utilising available energy within the flow before it is irreversibly lost to the surroundings.

Power balance and exergy-based analysis offer the advantage of identifying the various physical mechanisms contributing toward aerodynamic drag. Compared to force-based decomposition, this approach allows for the energy conversion pathways of the various phenomenological drag contributors to be traced as they develop along the body and within the wake. This provides more physical insight into the flow physics, with exergy-based analysis, in particular, introducing concepts such as exergy, which describes energy imparted to the flow by the body, which can theoretically still be used to produce thrust through mechanisms such as BLI. However, for conventional configurations, this available energy represents a lost work potential as it is irreversibly lost to the surroundings as heat. An additional aspect introduced through exergy analysis is the concept of anergy, which describes the various loss mechanisms within the flow originating either from viscous losses and turbulence, i.e., viscous anergy, or through thermal mixing dissipation, i.e., thermal anergy, or lastly, entropy generation via shockwave formation referred to as wave anergy.

Figure 3-9 shows the normalised drag density ( $C_d$ ) of a zoomed-in section of the wake of the transonic NACA 0012 airfoil studies. This provides a qualitative overview of how exergy-based analysis can be used to identify the various physical flow mechanisms contributing toward drag while also gaining more insight into the flow physics. The total profile drag density  $DV_\infty$  in Figure 3-9 shows that two wake regions exist, the first being attributed to the shockwave region, which extends into the bulk flow and the second to the viscous wake and BL regions. As mentioned, the exergy analysis in Figure 3-9 shows the shockwave's drag contribution associated with the anergy field through the wave anergy. This is important because it highlights that the energy within the

shockwave's wake does not present a potential for energy recovery to produce useful work as it is associated with energy which (theoretically) has no work potential.

On the other hand, analysis of the viscous wake region in Figure 3-9 shows that both mechanical and thermal exergy is present. This immediately pinpoints that for energy recovery, a plausible area for installing a BLI propulsor for energy utilisation would be within the airfoil's viscous wake. More notably, propulsor installation at the trailing edge would be advantageous for maximal energy recovery due to the high work potential observed in this area.

From the qualitative wake energy analysis in Figure 3-9, it can be observed that for the lifting scenario, there is an additional energy contribution  $\dot{E}_{m_E}$  not seen for the non-lifting test case, which imparts energy, mainly to the bulk flow, due to the downward flow turning, i.e., downwash as it produces lift. However, from an energy recovery perspective, this energy would not be considered a useful work potential to maintain a constant lift as its recovery would counteract the downwash, causing an overall detriment in the lift.

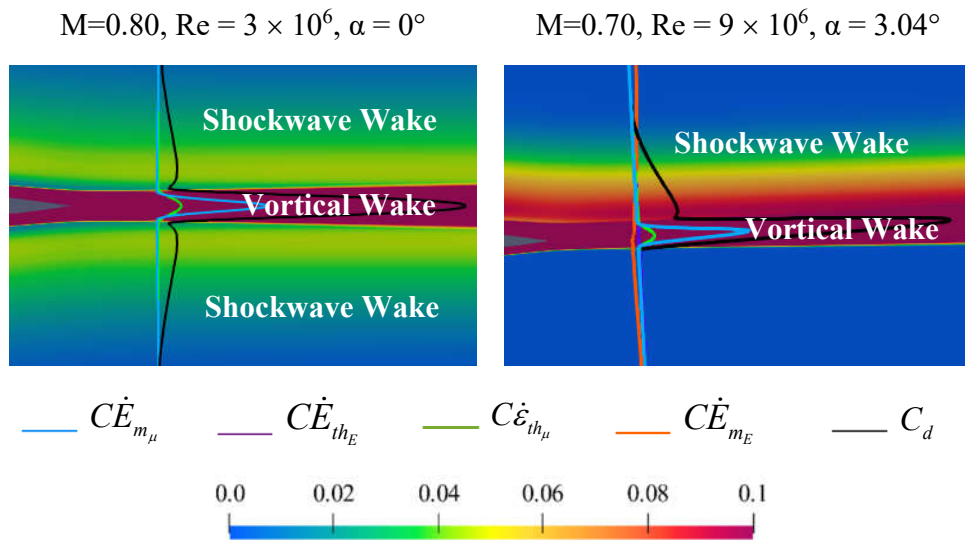


**Figure 3-9 Non-lifting transonic airfoil qualitative wake energy and drag density visualisation**

The top row of Figure 3-10 provides the uncorrected dissipative drag trends for the lifting and non-lifting test cases. The wave drag contribution is also quantified through the wave energy. Similar

to the subsonic analyses conducted in Chapter 2, the effects of VII are primarily noticeable through the discontinuous dip in the cumulative behaviour at the trailing edge for the dissipative physical exergy and total dissipative drag, which extends to the combined total dissipative and wave drag for the transonic test cases. Furthermore, this erratic behaviour is also observed near the region where the shockwave initiates, shown through the sharp rise in dissipative physical exergy. The behaviour observed at the shockwave region as well as at the airfoil trailing edge is in direct agreement with the sentiments made by Lock and Williams [1,2] who point out that the shock wave and trailing edge exhibit the most significant VII influence due to the substantial pressure gradients in these regions.

Therefore, it should be noted that VII effects are locally concentrated around the airfoil, having significantly observable behaviour at shockwave and trailing edge zones, as mentioned prior. This localised behaviour has been demonstrated from a near-field perspective in the context of dissipative pressure drag in the corrected near-field analyses in Figure 3-8. Furthermore, the VII contribution shows a cumulative trend when analysed over the airfoil surface, contributing toward the overall dissipative pressure drag. For exergy-based analysis, the dissipative physical exergy shows an abrupt rise at the shockwave zone, followed by a monotonic decay.

From an ARF perspective, the physical exergy is viewed as the mechanical and thermal exergy imparted to the quiescent atmosphere by the airfoil. As a greater surface area interacts with the surrounding flow, there is a consistent increase in the momentum and energy transfer between the body and the fluid, which explains the cumulative trends observed for the subsonic analyses in Chapter 2. As a result, the decreasing trend observed after the shockwave for the uncorrected physical exergy curves in actuality signifies momentum and energy transfer in the opposite direction, i.e., from the fluid to the body, where the body is seen as extracting available energy from the flow. In the absence of energy recovery mechanisms such as boundary layer ingestion, as is the case here, this observation is counterintuitive as the airfoil is the sole source of energy necessary for flight.

This behaviour is due to VII directly tied to the omission of wall transpiration. Within the context of exergy analysis, a portion of the dissipative physical exergy is not correctly captured and

accounted for within the DF and is tallied as a contribution within the EIF. The VII correction procedure is employed to identify this contribution, with the corrected trends shown in the bottom row of Figure 3-10. In keeping with the work conducted in Chapter 2, the discontinuous dip priorly observed at the trailing edge is removed. The corrected curves provide a representation of the evolution of physical exergy over the body in the absence of VII providing a maximal energy recovery potential at the trailing edge of  $\approx 10\%$  for both test cases, which downstream of the trailing edge begins to decay due to viscous losses, contributing toward the increase in overall dissipation accumulation.

Additionally, the abrupt rise in physical exergy at the shockwave interface is no longer observed. Instead, a change in the gradient of the dissipative physical exergy now represents the interaction in this region. This behaviour is in keeping with shockwave treatment for finite-volume computations. For these, shockwaves are primarily characterised by an abrupt rise in entropy (captured through the wave anergy), resulting in a change in flow properties across the shockwave interface. The change in flow properties in response to the entropy addition is then captured through the change in the cumulative gradient of physical exergy.

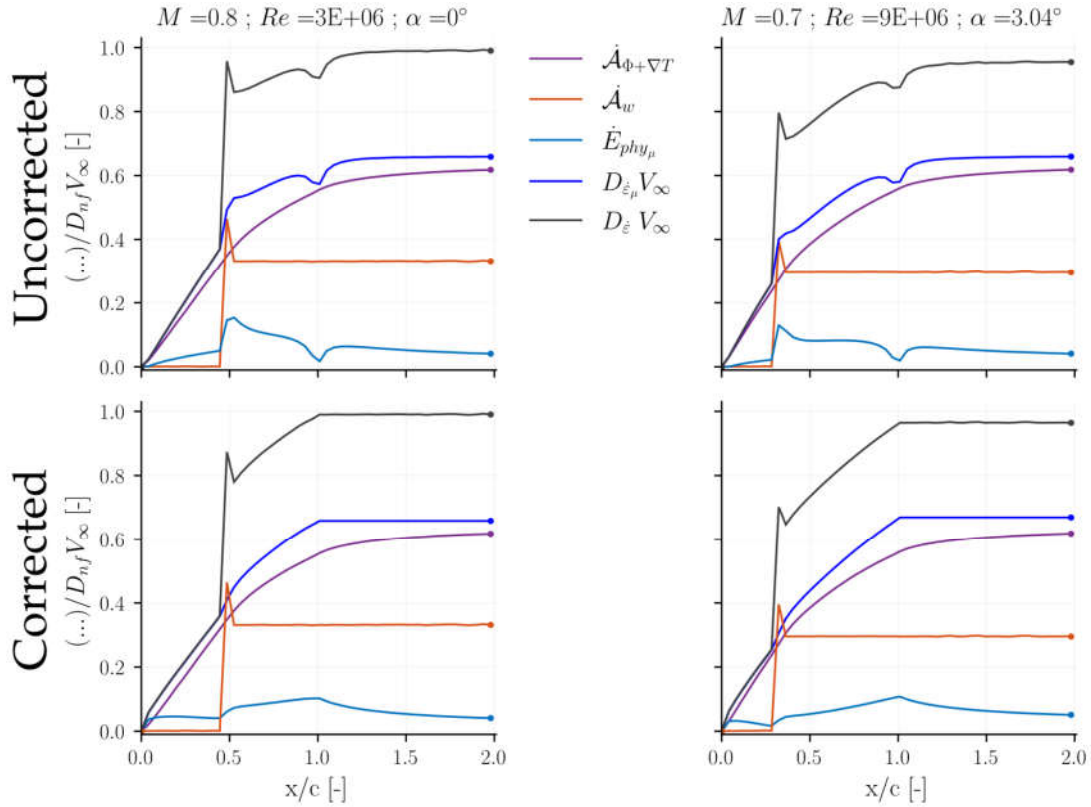


**Figure 3-10 Exergy-based comparison of uncorrected (top) and corrected (bottom) drag evolution over lifting and non-lifting transonic NACA 0012 airfoil**

Similar to the exergy-based analysis in Figure 3-10, Figure 3-11 provides an aerodynamic force decomposition from a power balance-based perspective. A major differentiating factor between energy and exergy analysis is how thermo-compressibility phenomena are treated. For exergy-based analysis, these are split into thermal exergy, representing the theoretical work output which can be extracted from the thermal field, and thermal anergy associated with the thermal losses due to mixing dissipation. In the context of doing work, as mentioned by Arntz [10], thermal energy is considered lower quality as only a portion of its contribution can be used to produce useful work through a Carnot cycle with an efficiency of less than unity. As such, Power balance analysis, which mainly focuses on evaluating the higher quality mechanical energy, considers thermo-compressibility-related phenomena (exergetic and anergetic contributions) as losses quantifying

their contributions through the pressure-volume work  $\Theta_\mu$  illustrated in Figure 3-11 through the rise in this quantity initiating at the shockwave interface. This represents the change in thermo-compressibility-related fluid properties across the shockwave, priorly observed via the gradient change in cumulative dissipative physical exergy in Figure 3-10, from the exergy-based perspective.

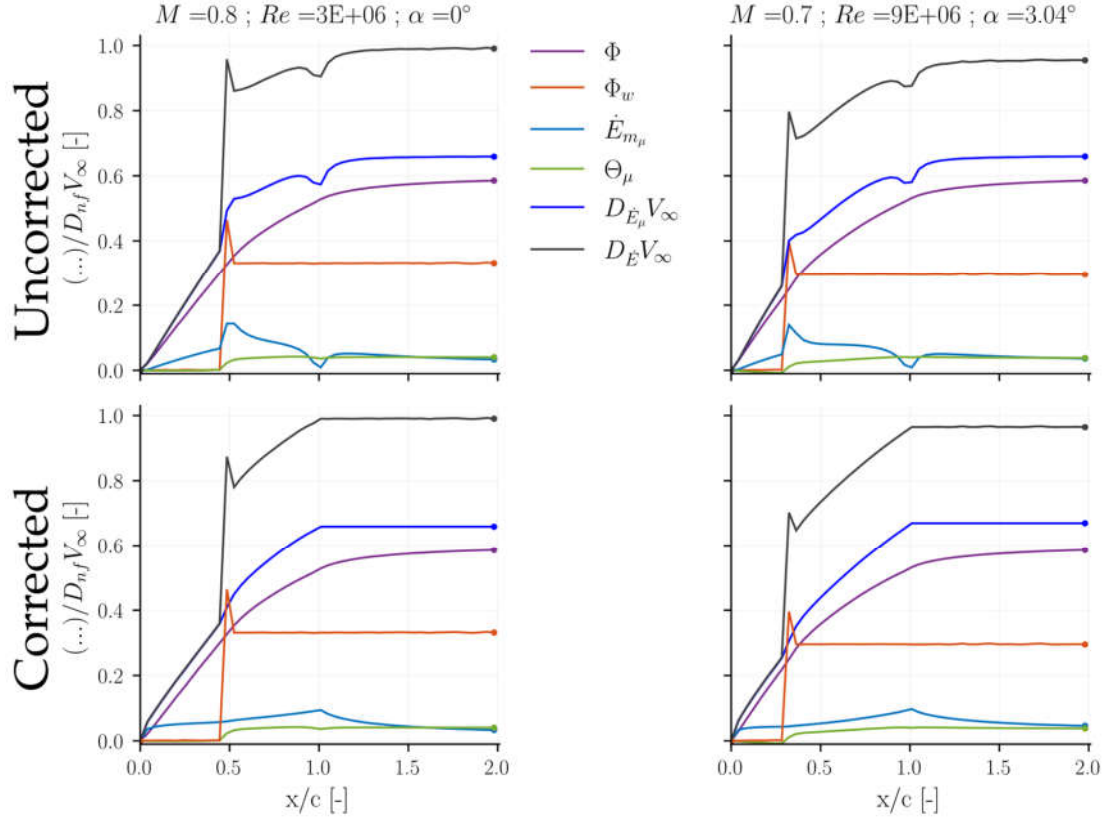


**Figure 3-11 Energy-based comparison of uncorrected (top) and corrected (bottom) drag evolution over lifting and non-lifting transonic NACA 0012 airfoil**

The Power balance analyses in Figure 3-11 were also conducted to assess the performance of the alternative dissipation formulation provided in equation (3.16) for quantifying the shockwave drag and its impact on the overall accuracy of the drag decomposition considering its modification. It can be observed that the shock dissipation  $\Phi_w$  in Figure 3-11 and the wave anergy  $\dot{A}_w$  in Figure

3-10 have similar trends showing a rapid increase at the shockwave interface due to entropy production.

The relative strength of the shockwave can be tallied through the changes in the dissipative mechanical energy trends. For the uncorrected dissipative mechanical energy, the strength of the shockwave can be measured through the gradient of decay observed after the abrupt rise at the shockwave interface. Figure 3-11 shows that the non-lifting shockwave is much stronger than the lifting test case due to the steepness of the decay rate. Quite notably, this interpretation is preserved for the corrected dissipative mechanical energy; however, a more intuitive behavioural trend is observed in this instance, which ties more closely to the various physical flow phenomena.

Firstly, the shockwave changes the rate of energy imparted to the flow. This occurs due to the change in boundary layer thickness experienced after the shockwave, as discussed in Section 3.3.2.2, resulting in a decrease in near-wall shear stress gradients depicted in Figure 3-8 through  $D_\tau V_\infty$ . This decrease in the near-wall shear stresses results in a reduction of viscous dissipation  $\Phi^{lam}$  which increases the rate of energy transfer between the body and fluid, evidenced by the increase in cumulative dissipative mechanical energy gradient downstream of the shockwave. Considering this, for comparison of various test cases, such as the lifting and non-lifting scenarios assessed here, the gradient steepness of the cumulative trends reflects the shockwave's strength. A low slope/rate of change would represent a more substantial shock as less energy is imparted to the flow, instead being lost via the various loss mechanisms incurred due to the shockwave, particularly those associated with boundary layer turbulence effects.

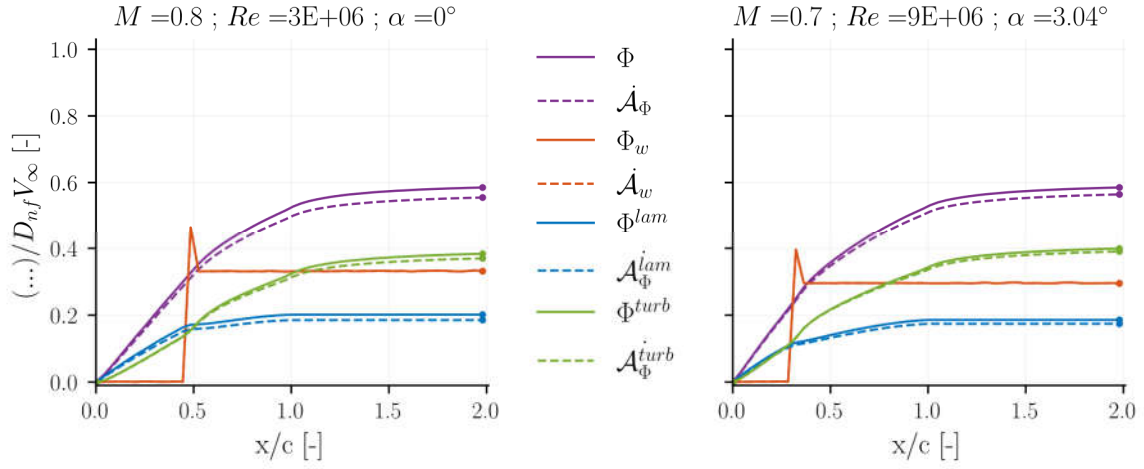


**Figure 3-12 Comparison of uncorrected (top) and corrected (bottom) energy-based drag evolution over a lifting and non-lifting transonic NACA 0012 airfoil**

To elaborate, the shockwave significantly amplifies the turbulence within the BL, and the increased turbulence causes an increased momentum exchange between the lower and upper layers of the BL. This exchange changes the BL velocity profile, with higher velocities and velocity gradients experienced within the lower layers of the BL, increasing turbulent dissipation  $\Phi^{turb}$ , as shown in Figure 3-12. Therefore, the viscous dissipation tallies the amount of lost work potential during the transfer of energy from the body to the fluid, whilst the turbulent dissipation tallies the additionally lost work potential incurred due to turbulent mixing within the BL and wake.

The various dissipation components are plotted alongside their corresponding anergy terms in Figure 3-12. The relative difference between their distributions is captured via exergy-based analysis through the gain in thermal exergy  $\dot{\mathcal{E}}_\phi$ , which occurs during the conversion of kinetic to thermal energy (i.e., heat), as shown in equation (3.19). The irreversible loss of this thermal exergy then contributes toward thermal anergy losses via thermal mixing, i.e.  $\dot{\mathcal{A}}_{\nabla T}$ .

$$\underbrace{\iiint_V \Phi \, dV}_{\Phi} = \underbrace{\iiint_V \frac{T_\infty}{T} \Phi \, dV}_{\dot{A}_\Phi} + \underbrace{\iiint_V \left(1 - \frac{T_\infty}{T}\right) \Phi \, dV}_{\dot{E}_\Phi} \quad (3.19)$$

Figure 3-12 also compares the shockwave evaluation formulations from exergy and power balance analysis. Comparing the two formulations shows good agreement with overlaying trends, reinforcing that significant thermal gradients are concentrated within the viscous wake and BL. As a result, the shockwave wake, which spans into the bulk flow, experiences minimal temperature perturbations. Therefore, there is no useful thermal exergy within the shockwave's wake.

#### 3.3.2.4 Near-field, Far-field, Energy and Exergy-based Drag Bookkeeping Comparison

This section provides a drag decomposition comparison between near-field, far-field, energy- and exergy-based analysis methods. Generally, for transonic flows, the drag can be split into three distinct portions, i.e., profile drag, wave drag and lift-induced drag, which can be identified based on their zone of influence through flow field decomposition. For instance, profile drag contributions can be recognised by isolating the BL and evaluating the dissipative drag contributions within this region. The wave drag can be identified by assessing the Euler drag force component from near-field analysis or, more conveniently, by isolating the shockwave wake and evaluating its entropy drag contribution commonly reverted to for farfield methods.

For lifting body analysis from a near-field perspective, the lift-induced drag shares its Euler drag contribution with the wave drag. Therefore, separability between the wave and lift-induced drag is obtained by taking the difference between the total near-field drag and the sum of the viscous and wave drag, leaving the remaining lift-induced contribution following equation (3.2). However, from a farfield perspective, this separability is more apparent as the lift-induced drag can be obtained directly from the total non-dissipative drag formulation in equation (3.3) and the wave drag from the entropy drag formulation in equation (3.7).

**Table 3-2 Aerodynamic force assessment method comparison reported in drag counts ( $C_d \times 10^{-4}$ )**

| Case | Near-field  |           |           | Far-field   |           |           | Energy      |           |           | Exergy      |           |           |
|------|-------------|-----------|-----------|-------------|-----------|-----------|-------------|-----------|-----------|-------------|-----------|-----------|
|      | 173.8       |           |           | 173.9       |           |           | 173.8       |           |           | 173.8       |           |           |
| II   | $C_{d_\mu}$ | $C_{d_E}$ | $C_{d_w}$ | $C_{d_\mu}$ | $C_{d_E}$ | $C_{d_w}$ | $C_{d_\mu}$ | $C_{d_E}$ | $C_{d_w}$ | $C_{d_\mu}$ | $C_{d_E}$ | $C_{d_w}$ |
|      | 114.5       | 0         | 59.3      | 115.7       | 0         | 58.2      | 115.6       | 0         | 58.2      | 115.6       | 0         | 58.2      |
|      | 145.2       |           |           | 144.9       |           |           | 143.5       |           |           | 143.5       |           |           |
| III  | $C_{d_\mu}$ | $C_{d_E}$ | $C_{d_w}$ | $C_{d_\mu}$ | $C_{d_E}$ | $C_{d_w}$ | $C_{d_\mu}$ | $C_{d_E}$ | $C_{d_w}$ | $C_{d_\mu}$ | $C_{d_E}$ | $C_{d_w}$ |
|      | 95.8        | 4.4       | 45.0      | 97.2        | 4.4       | 43.3      | 95.8        | 4.4       | 43.3      | 95.8        | 4.4       | 43.3      |

Table 3-2 shows this decomposition between the dissipative, non-dissipative and wave drag contributions obtained through momentum, energy, and exergy-based analyses. Comparison, particularly of the wave drag using the newly proposed power balance shock wave dissipation  $C_{d_w}$ , shows good agreement with the other evaluation methods, with a maximum difference of  $< 1\%$  observed between their various wave drag solutions. This good agreement between the multiple methods also advertises the performance of the VII correction procedure, which allows for an improved near-field force decomposition, priorly affected by VII.

In general, the force decomposition provides a profile, wave and induced drag share of approximately 65%, 35% and 0% for non-lifting airfoil and 65%, 30%, and 5% for the lifting airfoil relative to the total airfoil drag, with a  $< 1\%$  difference between the near-field, far-field, energy, and exergy-based solutions. A noteworthy observation from the dissipative drag trends observed here is their resemblance to those provided in the work of Schmitt and Destarac [11], with the results showing a clear cumulative trend for viscous drag contributions and an abrupt, discontinuous rise in drag due to shockwave formation as depicted in Figure 3-13.

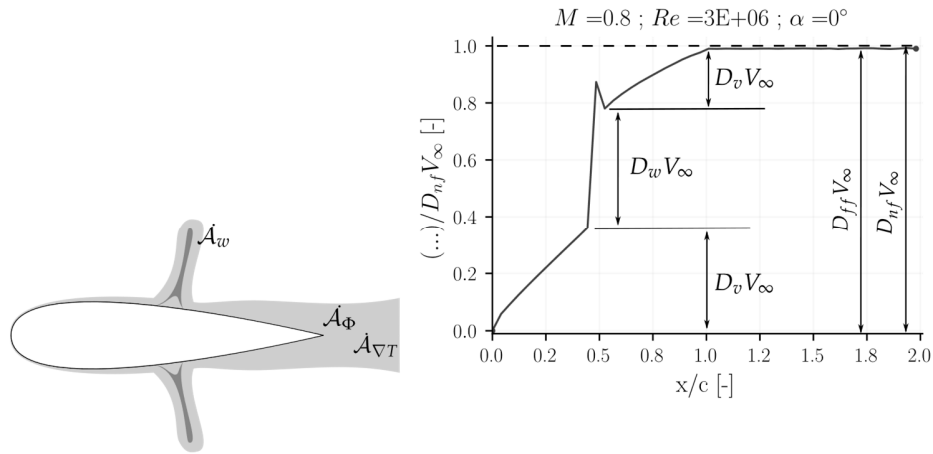


**Figure 3-13 Generalised viscous and wave drag analysis in RANS computations for NACA 0012 airfoil in transonic flow**

An additional noteworthy observation comes about by introducing the flow projection approach proposed by Hart et al. [21], which allows for the Euler pressure to be represented as a decomposition consisting of an incompressible and compressible pressure contribution, as shown in Figure 3-14. Hart et al. [21] show that this pressure decomposition is particularly useful in obtaining the wave drag from the compressible component for high Reynolds number flows.

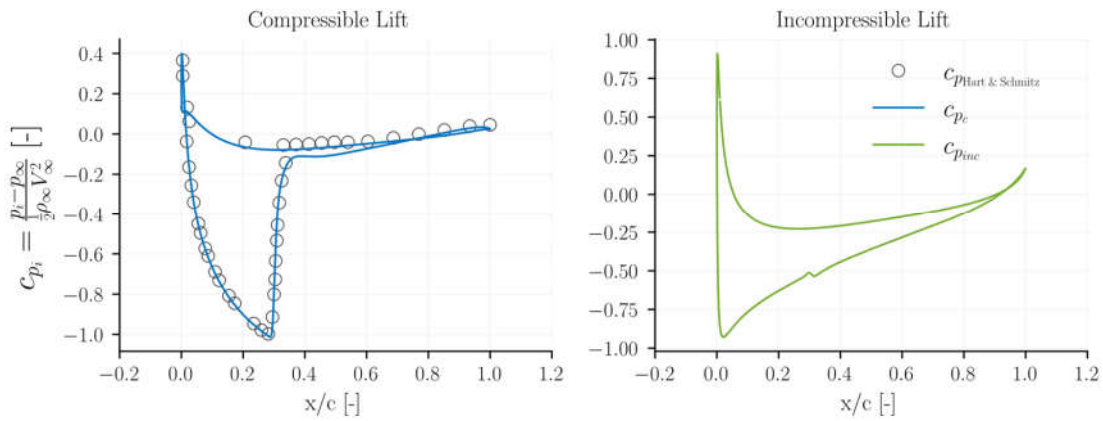


**Figure 3-14 Transonic airfoil isolated flow regions for profile, viscous and wave drag evaluation**

However, in this work, this approach is appreciated more through its ability to introduce the concept of compressible and incompressible lift, where the incompressible lift describes lift generation from the vorticity accumulation within the BL, whilst the compressible lift is a form of lift augmentation occurring due to compressibility. Compressibility effects introduce a baroclinic torque which contributes toward an increase in lift due to its accumulation, particularly within the BL, due to flow compressibility.

**Table 3-3 NACA 0012 Total Lift Coefficient Decomposition into Compressible and Incompressible Components**

| Case | Total | Incompressible | Compressible |
|------|-------|----------------|--------------|
| III  | 0.49  | 0.27           | 0.22         |

Table 3-3 shows the decomposition of lift into these components showing a slightly more significant contribution to lift due to the “incompressible” or, more aptly, the viscous BL vorticity-related lift production. In addition, the baroclinic torque-related lift provides a beneficial augmenting influence which increases lift (shown by its positive contribution) at higher Mach and, in this instance, is observed in the transonic regime with shock formation.

### 3.4 Conclusions

This work explored the extension of the unified partial pressure and velocity decomposition approach to transonic flows. Under these conditions, a new phenomenological drag mechanism exists not exhibited for subsonic flows, i.e., shockwave drag, resulting in a significant increase in the overall drag. Farfield analysis utilising thermodynamic methods allows the wave drag contribution to be quantified through the entropy drag evaluated within the shockwave’s region of influence. For power balance analysis, prior suggestions propose the wave pressure-work and kinetic energy outflow term  $\dot{E}_w$  for shockwave’s drag analysis. However, initial investigations showed that the quantity returned through this term largely depends on the control volume size. As a result, it was concluded that although it measures the pressure work rate and kinetic energy passing through the side cylinder of the control volume, it cannot provide a consistent and robust

means of wave drag evaluation due to its sensitivity to the control volume size. Instead, a shockwave dissipation formulation was introduced, which relates the dissipation accumulation to entropy generation. Applying this formulation to the shockwave's region of influence provided a wave drag quantity which showed good agreement against the entropy drag and exergy-based wave anergy formulations.

Adopting exergy-based analysis allowed for the shockwave's drag contribution to be evaluated through the wave anergy. This phenomenon is associated with the anergy field, and as a result, the energy associated with this mechanism cannot be utilised to produce useful work. Therefore, flow field decomposition methods become of particular importance to identify the portion of energy that can be utilised. The work here uniquely shows that these decomposition methods can be coupled with partial pressure field analysis, which allows for a linear decomposition of the pressure field into its non-dissipative and dissipative contributions. By connecting this with the velocity decomposition approach, fewer assumptions are made in separating the flow field into its EIF and DF regions. This unified decomposition approach allows for more precise identification of the energy imparted to the flow as well as its contribution toward aerodynamic drag. For instance, the energy imparted to the bulk flow can be evaluated through EIF primarily associated with lift-induced drag contributions.

Additionally, through near-field PPF analysis coupled with VII corrections, the wave drag can similarly be obtained from the EIF for transonic flows. The introduction of VII correction becomes necessary due to the simple inviscid flow model used in the reconstruction of the EIF. This model omits the use of wall transpiration, which neglects the effects of boundary layer displacement for the EIF. This causes VII effects, where a portion of the viscous drag is contained within the Euler field. This effect is concentrated mainly around the body and shows visibly significant contributions in regions of high pressure gradients, such as the trailing edge and shockwave zones.

Near-field PPF drag decomposition showed that this correction allows for the VII dissipative drag to be filtered out of the Euler field, leaving a more accurate depiction of the true Euler drag contributions, namely, the lift-induced and wave drag. Furthermore, the energy and exergy-based analyses show that the VII correction procedure can correctly account for the cumulative VII

contribution toward the dissipative field. As a result, a more accurate evaluation of the useful work potential can be obtained, providing a better overview of the amount of recoverable energy that can be utilised by devices such as BLI propulsors and heat exchangers.

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## 4 Flow-Field Reversibility and irreversibility relations toward induced drag decomposition

### 4.1 Introduction

Significant efforts have been undertaken to reduce aviation's environmental impact by designing novel fuel-efficient aircraft. As a result, accurately quantifying the aerodynamic forces acting over these configurations becomes crucial for reporting performance benefits. Conventionally, aerodynamic forces are evaluated through near-field integration of the pressure and friction forces acting over the body surface. This, however, only provides a binary mechanical decomposition of drag into pressure and skin friction. A particular caveat of this decomposition is the lack of knowledge regarding the apportionment of these drag contributions to physical mechanisms, such as viscous effects within the boundary layer and wake, shockwave formation and wingtip shed vorticity. As a result, significant interest has been taken in developing thermodynamic methods [1–5] capable of identifying the phenomenological sources of drag and quantifying their contributions. Notably from these is Oswatitsch formulation [3], which, although limited to the analysis of entropy-generating phenomena, allows for the evaluation of viscous and wave drag. However, the assessment of induced drag through this approach can only be obtained indirectly via subtraction from the total near-field force, leading to a relatively ambiguous representation of its physical meaning.

Direct evaluation of induced drag is made possible through methods such as Maskell's induced drag formulation [6], popularised primarily through modifications made by Kusunose [7–12]. Kusunose highlights the extensive applicability of this approach to numerical and experimental analyses by expressing Maskell's formulation analytically through a Green's function approach [8,13,14]. This modification improved evaluation accuracy by circumventing commonly encountered grid dependency issues observed from the conventional solving procedure.

Another state-of-the-art method is the vortex force/lamb vector approach [15–18]. This method allows for a zonal identification of the various drag contributions by splitting them into *irreversible* and *reversible* parts. The irreversible drag contributions are associated with drag emanating from

the viscous wake/boundary layer, including shockwave regions for transonic flows. Conversely, the reversible part conveniently accounts for bulk flow mechanical energy transfer mainly associated with lift-induced drag.

Following the work of Arntz [19–22] on drag decomposition from an exergy-based point of view, Aguirre[23,24] introduced a more thorough approach by categorising the phenomenological contributions into *irreversible* and *reversible* parts through a flow field split of the velocity into isentropic and non-isentropic components. The findings showed that each element (namely velocity, density, pressure and temperature) has individual contributions toward both reversible and irreversible drag, particularly when assessed in the viscous boundary layer (BL) and wake, as shown in Figure 4-1.

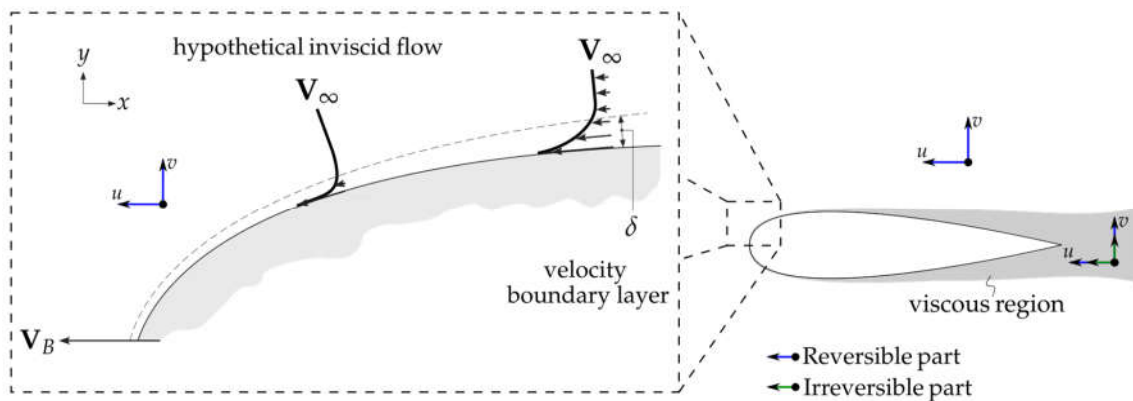


**Figure 4-1 Reversible and Irreversible component decomposition adapted from [25]**

This finding questions the full efficacy of the irreversible and reversible drag decomposition presented through vortex force analysis, as no component-based reversible/irreversible decomposition is performed. This omission can partly be tied to the near-wake variability in drag observed in [15,16,26,27] owing to the significant interaction between the irreversible and reversible flow phenomena. However, the performance of these methods is seen to improve in the far wake where weaker interaction between these phenomena is experienced.

By approaching performance assessment through this component-based reversible/irreversible decomposition, Aguirre [23,24] was able to show, via low subsonic finite wing analyses, that all

irreversible drag contributions were associated with the wing's viscous drag in agreement with the propositions from vortex force methods. Similarly, the reversible contributions were associated with bulk flow energy transfer, thus providing an alternative means of induced drag evaluation through exergy analysis.

The unified drag decomposition approach presented in Chapter 2 allows for the ideology of irreversible and reversible force contribution decomposition can be extended to the near-field through partial pressure field analysis [28–30]. In this instance, the dissipative pressure and skin friction drag contributions form the irreversible drag portion, whilst the reversible contributions are associated with the Euler pressure drag. This approach provides a novel near-field method of assessing induced drag, successfully demonstrated on a classical ONERA M6 wing under subsonic and transonic conditions in the work of Hart and Schmitz [28].

This work will focus on the numerical implementation of these state-of-the-art methods in commercial CFD code Ansys Fluent © to assess the insights obtained through their application in tandem on simple airfoil and rectangular finite wing cases. Interest is taken into decomposing the aerodynamic force into reversible and irreversible contributions, assessing their relative sensitivities to flow conditions. The studies focus on compressibility through a variation in Mach number, turbulence extent by variation in Reynolds number, and lift production through a variation in the angle of attack.

The benefits of this assessment are two-fold, firstly by contributing toward knowledge concerning the computation and decomposition of aerodynamic forces, which popularly present challenges in the differentiation of drag, lift, and thrust-related contributions for highly integrated configurations. Secondly, it comprehensively overviews the various work and energy relationships observed in flight. Lastly, it allows for an enriched representation of the phenomenological origins of drag through the different physical mechanisms involved and their evolution in the flow.

## 4.2 Mathematical Models and Numerical Methodology

### 4.2.1 Momentum Relations

#### 4.2.1.1 Near- and Far-field Momentum Formulations

This section covers the derivation of near- and far-field aerodynamic force evaluation techniques from the same form of the Navier-Stokes momentum equation presented by [17]. This provides a consistent derivation path for all formulations, beginning from the same primary starting point. The evaluation starts with the general compressible form of the Cauchy momentum equation in (4.1)(a). In this form, the Cauchy stress tensor  $\bar{\sigma}$  can conveniently be separated into its *isotropic* (*hydrostatic pressure*) and *deviatoric* components. By coupling this with the constitutive model of Newtonian viscous fluids, the deviatoric stress can be related to the shear strain rate and viscosity.

$$\begin{aligned} \frac{\partial(\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) &= \nabla \cdot \bar{\sigma} & (a) \\ &= -\nabla p + \nabla \cdot \bar{\tau} & (b) \\ &= -\nabla p + (\mu_g + \mu) \nabla (\nabla \cdot \mathbf{V}) + \mu \nabla^2 \mathbf{V} & (c) \end{aligned} \quad (4.1)$$

Many far-field force decomposition approaches focus on obtaining their constituent terms from the Navier Stokes momentum equation in the form provided in equation (4.1)(b). However, further expanding the deviatoric stress, as shown in equation (4.1)(c), allows for the convenient inclusion of vorticity via the last term, containing the Laplacian through Lamb's vector identity given in equation (4.2).

$$\nabla \times (\nabla \times \mathbf{V}) = \underbrace{\nabla (\nabla \cdot \mathbf{V})}_g - \nabla^2 \mathbf{V} \text{ where } \nabla \times \mathbf{V} = \boldsymbol{\omega} \quad (4.2)$$

The Navier Stokes equation, in this instance, simplifies to the form provided in equation (4.3), where the pressure and dilation terms are grouped, as their effects are commonly associated with macroscopic and molecular expansion/compression mechanisms.

$$\frac{\partial(\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) = -\nabla \cdot \underbrace{(p - \mu_\theta \theta)}_{\Pi} + \nabla \times \mu \omega \quad (4.3)$$

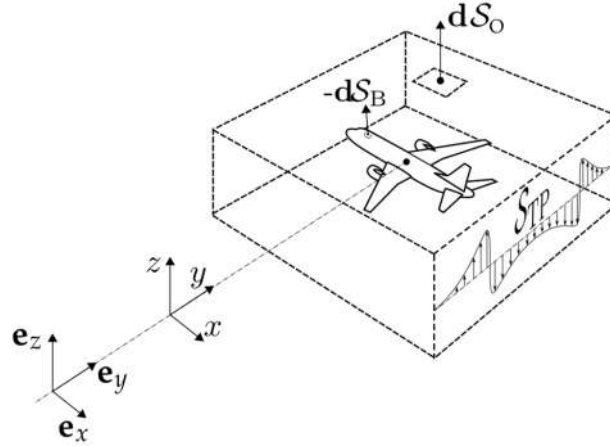


**Figure 4-2 3D control volume surrounding the aerodynamic body bounded by the body surface  $S_B$  and far-field surface  $S_O$**

The near- and far-field force expressions are obtained from the steady form of equation (4.3) converted to its equivalent integral form and assessed through a control volume defined within the flow, as depicted in Figure 4-2. The solid aerodynamic body is immersed in a viscous and compressible fluid, bounded by surface  $S = S_B + S_O$ , where  $S_B$  is the inner boundary on the body surface. At the same time,  $S_O$  is the outer periphery far from the body within the flow field.

Applying Gauss theorem to the integral form of equation (4.3), it is possible to obtain the expression given in equation (4.4)(a). As shown in equation (1.4)(b), this can be modified to introduce perturbation quantities through the permissible substitutions  $\mathbf{V} \rightarrow \mathbf{V} - \mathbf{V}_\infty$  and

$p \rightarrow p - p_\infty = p_G$  as  $\mathbf{V}_\infty \oint_S (\rho \mathbf{V}) \cdot \mathbf{n} dS = 0$  and  $p_\infty \mathbf{V}_\infty \oint_S \mathbf{n} dS = 0$ , respectively.

$$\begin{aligned}
\oint_S \mathbf{V} \rho \mathbf{V} \cdot \mathbf{n} \, dS &= \oint_S -\Pi \mathbf{n} + \underbrace{(\mu \boldsymbol{\omega} \times \mathbf{n})}_{\boldsymbol{\tau}} \, dS & (a) \\
\oint_S (\mathbf{V} - \mathbf{V}_\infty) \rho \mathbf{V} \cdot \mathbf{n} \, dS &= \oint_S -(p_G - \mu_g \mathcal{G}) \mathbf{n} + \boldsymbol{\tau} \, dS & (b)
\end{aligned} \tag{4.4}$$

For the near-field aerodynamic force  $\mathbf{F}_B$  in equation (4.5), the inertial term becomes zero due to the no-slip condition, leaving contributions only from the pressure and viscous interactions. The far-field aerodynamic force is then defined by  $\mathbf{F}_O$ , with the difference of this formulation in comparison to traditional forms [19,31,32] being the decomposition of the shear stress tensor to expose the dilation term  $\mu_g \mathcal{G}$  in both near and far-field formulations.

$$\mathbf{F} = \underbrace{\iint_{S_B} (p_G - \mu_g \mathcal{G}) \mathbf{n} - \boldsymbol{\tau} \, dS}_{\mathbf{F}_B} = - \underbrace{\iint_{S_O} [(\mathbf{V} - \mathbf{V}_\infty) \rho \mathbf{V} \cdot \mathbf{n} + (p_G - \mu_g \mathcal{G}) \mathbf{n} - \boldsymbol{\tau}] \, dS}_{\mathbf{F}_O} \tag{4.5}$$

#### 4.2.1.2 Vortex Force Formulation

The momentum equation provided in equation (4.1) can be modified by applying the chain rule to the local acceleration and the vector identity  $\nabla \cdot (s\mathbf{A}) = \nabla s \cdot \mathbf{A} + s(\nabla \cdot \mathbf{A})$  to the convective acceleration. In this instance,  $s$  and  $\mathbf{A}$  represent scalar and vector quantities, respectively.

This reduces the LHS of the momentum formulation in equation (4.6) to a material derivative as the second RHS bracketed term is equal to zero through a continuity assumption.

$$\frac{\partial(\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) = \underbrace{\rho \left[ \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right]}_{\rho \frac{D\mathbf{V}}{Dt}} + \underbrace{\mathbf{V} \left[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) \right]}_{0, \text{continuity}} \tag{4.6}$$

Similar to equation (4.2), vorticity can be introduced into the momentum equation via the vector identity in equation (4.7)(a). This is particularly advantageous as it naturally introduces the Lamb vector  $\boldsymbol{\ell} = \boldsymbol{\omega} \times \mathbf{V}$ , defined as the cross product of the vorticity with velocity [33].

$$\begin{aligned}\mathbf{V} \cdot \nabla \mathbf{V} &= \nabla \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) - \mathbf{V} \times \boldsymbol{\omega} \quad (\text{a}) \\ \nabla \left( \rho \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) &= \rho \nabla \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) + \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) \nabla \rho \quad (\text{b})\end{aligned} \quad (4.7)$$

The second term in equation (4.7)(a) can then be expanded via the identity in equation (4.7)(b) to produce two terms, one representing the gradient of the kinetic energy and the other the compressibility generation through density gradients.

Considering the right-hand side of the momentum formulation in equation (4.1), the first term signifies the static pressure gradient, which can be combined with the kinetic energy to describe the total pressure gradient permissible since  $P = p + \rho V^2/2$  when considering subsonic flows.

By substituting this into equation (4.3) and combining the result from equation (4.7) yields the Navier-Stokes momentum formulation given in [17], stated here in equation (4.8).

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho \ell - \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) \nabla \rho = -\nabla P + \nabla (\mu_g \mathcal{G}) - \nabla \times (\mu \boldsymbol{\omega}) \quad (4.8)$$

This form of the momentum equation can provide an alternative means of defining the aerodynamic force relying on volume integrals of non-negligible flow quantities in the flow domain [34] through equation (4.9).

$$\mathbf{F}_{vf} = -\iiint_V \left( \rho \ell - \frac{\mathbf{V} \cdot \mathbf{V}}{2} \nabla \rho \right) dV - \iint_S P \mathbf{n} dS + \iint_S (\mu_g \mathcal{G} \mathbf{n} + \mu \boldsymbol{\omega} \times \mathbf{n}) dS \quad (4.9)$$

Within equation (4.9), the total pressure integral is expanded through a *surface integral* derivative moment transformation (DMT) [17,35]. Combined with the steady form of equation (4.8) and the volume integral DMT<sup>8</sup>, this exposes the compressibility generation expression via a grouping of

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<sup>8</sup> For the reader's convenience more detail regarding the derivative momentum transformations and their implementation can be found in the Appendix A.1 under the title Derivative Moment Transformation

the density gradient terms, resulting in the decomposition of the vortex force  $\mathbf{F}_{vf}$  shown in equation (4.10).

$$\begin{aligned}
\mathbf{F}_{vf} = & \underbrace{-\iiint_V \rho \ell \, dV}_{\mathbf{F}_\ell} + \underbrace{\frac{1}{k} \iiint_V \mathbf{x} \times \left[ \nabla \times \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \nabla \rho \right) \right] dV}_{\mathbf{F}_\rho} - \underbrace{\frac{1}{k} \iint_S \mathbf{x} \times [\mathbf{n} \times \rho \ell] \, dS}_{\mathbf{F}_\Sigma} \\
& + \underbrace{\iint_S \left\{ \mu_\theta \mathcal{G} \mathbf{n} - \frac{\mathbf{x}}{k} \times [\mathbf{n} \times \{-\nabla(\mu_\theta \mathcal{G})\}] \right\} dS}_{\mathbf{F}_{\mu\theta}} + \underbrace{\iint_S \left\{ \mu \boldsymbol{\omega} \times \mathbf{n} - \frac{\mathbf{x}}{k} \times [\mathbf{n} \times (\nabla \times \mu \boldsymbol{\omega})] \right\} dS}_{\mathbf{F}_{\mu\omega}}
\end{aligned} \tag{4.10}$$

Mele et al. [34] proposed the force breakdown presented equation (4.11) for subsonic viscous flows. The main advantage of this was the explicit depiction of phenomenological contributions toward lift and lift-induced drag originating from the *reversible* part of the aerodynamic force and the remaining viscous drag attributed to the *irreversible* part.

$$\begin{aligned}
L^{MELE} &= \mathbf{e}_z \cdot (\mathbf{F}_\ell + \mathbf{F}_\rho) \quad (a) \\
D_i^{MELE} &= \mathbf{e}_x \cdot (\mathbf{F}_\ell + \mathbf{F}_\rho) \quad (b) \\
D_v^{MELE} &= \mathbf{e}_x \cdot \mathbf{F}_\Sigma \quad (c)
\end{aligned} \tag{4.11}$$

Fournis et al. [16,26] point out that no preliminary analyses had been conducted to explicitly prove that the compressibility generation term was directly related to the lift and lift-induced drag for higher Mach number flows. To assess their relative coherence, they proposed a direct term-by-term comparison of the vortex force and classical far-field momentum formulations. Through this, they observed that the breakdown in equation (4.11) did not show good agreement with the more conventional thermodynamic methods of Destarac and Van der Vooren [4] in the near wake. This was attributed to the non-negligible nature of the compressibility correction term  $\mathbf{F}_{\nabla\rho}$  in the near-wake.

For the formulations presented by Mele et al. [34], the contribution of this compressibility term is assumed negligible and thus omitted from the overall formulation, as shown in equation (4.11). However, this is only valid in two scenarios, first for incompressible flows where the

compressibility generation, by definition, would not be significant. The second is in the far-wake where the behaviour of the compressibility correction term shown in equation (4.12) vanishes, i.e.,

$\lim_{S_0 \rightarrow \infty} \mathbf{F}_{\nabla \rho} = 0$  as the local velocity approaches freestream, i.e.  $\lim_{S_0 \rightarrow \infty} V = V_\infty$ .

$$\mathbf{F}_{\nabla \rho} = \frac{1}{k} \oint_{S_0} \mathbf{x} \times \left( \mathbf{n} \times \frac{V^2 - V_\infty^2}{2} \nabla \rho \right) dS \quad (4.12)$$

As a result, Fournis et al. [16,26] propose that the lift, lift-induced drag, and viscous drag formulations in the absence of the far-wake assumption can be expressed as shown in equation (4.13). These formulations are strictly equivalent to the classical analyses of Kutta, Joukowski, Maskell and Betz, as illustrated in [16,26].

$$\begin{aligned} L^{KJMB} &= \mathbf{e}_z \cdot (\mathbf{F}_\ell + \mathbf{F}_\rho - \mathbf{F}_{\nabla \rho}) = \mathbf{e}_z \cdot \mathbf{F}_{\text{KJ}} & (a) \\ D_i^{KJMB} &= \mathbf{e}_x \cdot (\mathbf{F}_\ell + \mathbf{F}_\rho - \mathbf{F}_{\nabla \rho}) = \mathbf{e}_x \cdot \mathbf{F}_{\text{MSK}} & (b) \\ D_v^{KJMB} &= \mathbf{e}_x \cdot (\mathbf{F}_\Sigma + \mathbf{F}_{\nabla \rho}) = \mathbf{e}_x \cdot \mathbf{F}_{\text{BETZ}} & (c) \end{aligned} \quad (4.13)$$

The implications of the findings from Fournis et al. [16,26] work is exceptionally noteworthy, as they represent a formal attempt toward force decomposition definition consistency through a coherence assessment between classical and state-of-the-art methods (in this case, vortex force analysis).

Primarily, the viscous drag contributions from the lamb vector and compressibility forces are directly linked to Betz's classical formulation and categorised through vortex force analysis as an *irreversible* drag. Similarly, the induced drag obtained through Maskell's formulation characterises the *reversible* drag contribution.

#### 4.2.2 Mechanical Energy and Exergy Formulations

This section uses the momentum equation to obtain an expression for the power developed by the net force acting on the aircraft. Firstly, the LHS of the momentum equation is reverted to the form given in equation (4.6), which in this context is easier to convert into its integral format via Gauss Theorem.

$$\rho \frac{D\mathbf{V}}{Dt} = \rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla p + \nabla(\mu_g \mathcal{G}) - \nabla \times (\mu \boldsymbol{\omega}) \quad (4.14)$$

From this, the mechanical energy equation can be formulated by taking the scalar dot product of the momentum equation (4.14) with the local velocity vector, resulting in equation (4.15).

$$\rho \mathbf{V} \cdot \frac{D\mathbf{V}}{Dt} = \rho \left[ \mathbf{V} \cdot \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot (\mathbf{V} \cdot \nabla \mathbf{V}) \right] = -\nabla p \cdot \mathbf{V} + \nabla(\mu_g \mathcal{G}) \cdot \mathbf{V} - \nabla \times (\mu \boldsymbol{\omega}) \cdot \mathbf{V} \quad (4.15)$$

The local acceleration in equation (4.15) can be expressed alternatively using the dot product rule in equation (4.16), which is helpful for formulaic simplification through the continuity assumption.

$$\begin{aligned} \frac{\partial}{\partial t}(\mathbf{V} \cdot \mathbf{V}) &= \mathbf{V} \cdot \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \frac{\partial \mathbf{V}}{\partial t} \\ \Rightarrow \frac{\partial}{\partial t} \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) &= \mathbf{V} \cdot \frac{\partial \mathbf{V}}{\partial t} \end{aligned} \quad (4.16)$$

Similarly, the convective acceleration can be expanded through the vector identity in equation (4.7)(a) while also noting that  $\mathbf{V} \cdot [\mathbf{V} \times (\nabla \times \mathbf{V})] = 0$  as  $\mathbf{V} \perp \mathbf{V} \times (\nabla \times \mathbf{V})$ . These modifications lead to a new definition of the material derivative given in equation (4.15), resulting in the form provided in equation (4.17).

$$\rho \frac{D}{Dt} \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) = \rho \left[ \frac{\partial}{\partial t} \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) + \mathbf{V} \cdot \nabla \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) \right] \quad (4.17)$$

The first RHS term in equation (4.17) can be decomposed through the chain rule, as shown in equation (4.18). This aids in developing the final form of the mechanical energy equation by simplification through continuity.

$$\frac{\partial}{\partial t} \left( \rho \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) = \rho \frac{\partial}{\partial t} \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) + \frac{\mathbf{V} \cdot \mathbf{V}}{2} \frac{\partial \rho}{\partial t} \quad (4.18)$$

The second RHS term in equation (4.17) can also be decomposed so that when in its integral form, it is described by a volume and surface integral via Gauss Theorem.

$$\nabla \cdot \left( \rho \mathbf{V} \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) = \rho \mathbf{V} \cdot \nabla \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) + \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) \underbrace{\nabla \cdot \rho \mathbf{V}}_{=0, \text{continuity}} \quad (4.19)$$

Combining these modifications provides a simplified form of the LHS of the mechanical energy formulation shown in equation (4.20), where the third term can be omitted as its contribution is null due to continuity.

$$\rho \frac{D}{Dt} \left( \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) = \frac{\partial}{\partial t} \left( \rho \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) + \nabla \cdot \left( \rho \mathbf{V} \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) - \frac{\mathbf{V} \cdot \mathbf{V}}{2} \underbrace{\left[ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{V} \right]}_{0, \text{continuity}} \quad (4.20)$$

Conversely, the RHS of the mechanical energy equation can be expanded via the identity  $\nabla \cdot [\mathbf{V} \times (\mu \boldsymbol{\omega})] = \mu \boldsymbol{\omega} \cdot (\nabla \times \mathbf{V}) - \mathbf{V} \cdot [\nabla \times (\mu \boldsymbol{\omega})]$ , introducing the enstrophy term  $\mu \omega^2$  into the differential mechanical energy equation (4.21).

$$\frac{\partial}{\partial t} \left( \rho \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right) = \nabla \cdot \left\{ \mathbf{V} \mu_g \mathcal{G} - \underbrace{[(\mu \boldsymbol{\omega}) \times \mathbf{V}]}_{\mu \ell} - p \mathbf{V} - \rho \mathbf{V} \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right\} + \Pi \mathcal{G} - \mu \omega^2 \quad (4.21)$$

Taking the integral of the mechanical energy equation (4.21) and following the procedure detailed in Appendix A.3, it is possible to obtain a drag power decomposition in terms of both energy and exergy-based components, as shown in equation (4.22).

The main advantage of an energy/exergy type decomposition is that it readily identifies the various work and energy transfer pathways of the phenomenological drag contributions. This enables the physical mechanisms to be understood more deeply and from an alternate perspective than the conventional momentum-based approach.

A primary difference between the mechanical energy decomposition presented in this work and that of Drela [31] or Sanders and Laskaridis [32] is the identification of the individual component

contributions toward viscous dissipation. This is done mainly through vorticity (via the enstrophy term) and expansion/contraction (via the dilatational dissipation).

Lastly, this mechanical energy formulation can be extended for thermal energy assessment by including the second law of thermodynamics, following the work of Arntz [19,22] and Aguirre [23,24]. As a result, the enstrophy and dilatational dissipation in equation (4.22)(a) extends to the exergy domain, producing the enstrophy and dilatational anergy depicted in equation (4.22)(b).

$$\begin{aligned} \mathbf{F} \cdot \mathbf{V}_\infty &= \underbrace{\dot{E}_a + \dot{E}_v + \dot{E}_p}_{\dot{E}_m} + \Theta + \Phi_{\mu\omega^2} \quad (\text{a}) \\ &= \underbrace{\dot{E}_m + \dot{\epsilon}_{th}}_{\dot{\epsilon}_{phy}} + \dot{A}_{\mu\omega^2} + \dot{A}_{\nabla T} \quad (\text{b}) \end{aligned} \quad (4.22)$$

### 4.3 Reversible and Irreversible Aerodynamic Force, Energy & Exergy Decomposition

The decomposition presented here begins by defining a generic categorisation of the total drag into reversible (dissipative) and irreversible (non-dissipative) parts, shown in equation (4.23). This allows for a consistent definition of component contributions traceable from their near-field origin to their phenomenological evolution in the far-field.

$$\underbrace{\mathbf{F}}_{RVF} = \underbrace{\mathbf{F}_{rev}}_{EIF} + \underbrace{\mathbf{F}_{irr}}_{DF} \quad (4.23)$$

This is done by decomposing the real viscous flow (RVF) using the unified partial pressure, velocity decomposition procedure introduced in [36] to obtain the Euler (EIF) and dissipative flow fields (DF).

In the *near-field*, the reversible force contribution is quantified via the Euler pressure, whilst the irreversible force contributions are evaluated through a summation of the skin friction and dissipative pressure obtained through partial pressure field (PPF) analysis. The pressure terms are obtained by solving the governing partial pressure Poisson equations in equation (4.24), subject to the boundary conditions suggested by Schmitz et al. [28,29,37].

$$\begin{aligned}\nabla^2 p_E &= -\rho \nabla \cdot (\mathbf{V} \cdot \nabla) \mathbf{V} & (a) \\ \nabla^2 p_\mu &= \nabla \cdot \boldsymbol{\tau} & (b)\end{aligned}\tag{4.24}$$

In the *far-field*, momentum-based force decomposition is firstly represented, as shown in equation (4.25), through the combined KJMB (Kutta, Joukowski, Maskell, Betz) formulation presented by Fournis et al. [16,26].

$$\begin{aligned}\mathbf{F}_{\mathcal{V}} &= \mathbf{F}_{BETZ} + \mathbf{F}_{MSK} = \mathbf{F}_{KJMB} \\ \text{where:} \\ \mathbf{F}_{BETZ} &= \iint_{S_o} \left[ (P_\infty - P) \mathbf{n} - \frac{V_\infty^2}{2} (\rho_\infty - \rho) \mathbf{n} + \bar{\boldsymbol{\tau}} \cdot \mathbf{n} \right] dS \\ \mathbf{F}_{MSK} &= \iint_{S_o} \frac{\rho}{2} (v^2 + w^2 - u^2) \cdot \mathbf{n} dS\end{aligned}\tag{4.25}$$

The reversible and irreversible force contributions are then obtained through the unified approach [36], representing the formulations via their respective dissipative and non-dissipative components, as shown in equation (4.26).

$$\begin{aligned}\mathbf{F}_{KJMB} &= \mathbf{F}_{KJMB_E} + \mathbf{F}_{KJMB_\mu} \\ &= \underbrace{(\mathbf{F}_{BETZ_E} + \mathbf{F}_{MSK_E})}_{\text{reversible}} + \underbrace{(\mathbf{F}_{BETZ_\mu} + \mathbf{F}_{MSK_\mu})}_{\text{irreversible}}\end{aligned}\tag{4.26}$$

Lastly, for *energy* and *exergy*-based analysis, the power, given in equation (4.27), is similarly decomposed using the unified approach, with the resulting terms grouped according to their reversible and irreversible contributions, as done prior.

$$\begin{aligned}
\underbrace{\mathbf{F}_{\dot{E}} \cdot \mathbf{V}_{\infty}}_{\mathbf{F}_{RVF} \cdot \mathbf{V}_{\infty}} &= \underbrace{\mathbf{F}_E \cdot \mathbf{V}_{\infty}}_{\mathbf{F}_{EIF} \cdot \mathbf{V}_{\infty}} + \underbrace{\mathbf{F}_{\mu} \cdot \mathbf{V}_{\infty}}_{\mathbf{F}_{DF} \cdot \mathbf{V}_{\infty}} \\
&= \left( \dot{E}_{m_E} + \Theta_E \right) + \left( \dot{E}_{m_{\mu}} + \Phi_{\mu\omega^2} + \Theta_{\mu} \right) \\
&= \underbrace{\left( \dot{E}_{m_E} + \dot{\mathcal{E}}_{th_E} \right)}_{\text{reversible}} + \underbrace{\left( \dot{E}_{m_{\mu}} + \dot{\mathcal{E}}_{th_{\mu}} + \dot{A}_{\mu\omega^2} \right)}_{\text{irreversible}}
\end{aligned} \tag{4.27}$$

#### 4.3.1 Two-dimensional airfoil reversible drag evaluation through inviscid flow analysis

Betz [2] introduced the concept of an *isentropic velocity* for use in wind tunnel testing (WTT) to reduce the amount of data necessary for profile drag analyses from an infinite/entire survey plane, which is not practical for WTT and rather more inclined for CFD analysis, to only requiring wake plane measurements. This concept introduces a unique means of understanding the phenomenological origin of aerodynamic drag by decomposing its contributions into non-dissipative reversible (i.e., isentropic) and dissipative irreversible (i.e., non-isentropic) parts.

The *isentropic velocity* field, in this context, provides only an *estimation* of the velocity field that would be observed in an equivalent inviscid Euler flow (EIF). The discrepancy exists as a result of a few criteria necessary for the modelling of an EIF not being considered for the reconstruction of the isentropic field, namely:

1. Irrotational flow in the BL zone of the EIF
2. Wall transpiration to account for the boundary layer displacement effect in the EIF

For energy and exergy-based analysis, Drela [31] and Aguirre [38] provide compelling test cases to provide insight into the expected behaviour and physical mechanisms from *low-speed subsonic* Euler flows for *non-lifting* scenarios. Without dissipation, any fluid perturbations return to equilibrium fully reversibly. From an energy-based perspective, the fluid pressure-boundary work  $\dot{E}_p$  fully compensates for the kinetic energy  $\dot{E}_a + \dot{E}_v$  transferred to the fluid, as illustrated in Figure 4-3.

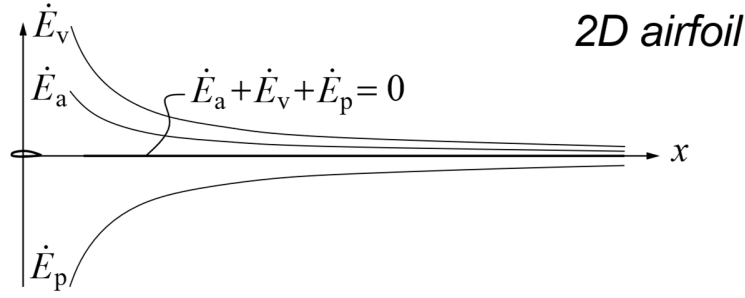


**Figure 4-3 Energy loss rate components for incompressible inviscid 2D airfoil adapted from [31]**

To further explain this behaviour, the transverse kinetic energy  $\dot{E}_v$  is combined with Maskell's induced drag expression [6], presented as a drag power, as shown in equation (4.28). As no wingtip vortices are shed for two-dimensional airfoils, the first term in equation (4.28) representing the drag power associated with the streamwise wake vorticity would be zero. Furthermore, as there is no shockwave formation, the total airfoil drag for this non-lifting inviscid scenario would then be zero, thus agreeing with the observations shown in Figure 4-3 from the work of Drela [31].

$$\begin{aligned}
 \dot{E}_v &= \iint_{S_O^{TP}} \frac{\rho}{2} (v^2 + w^2) (V_\infty + u) dS \\
 &= D_i^{MSK} V_\infty + \iint_{S_O^{TP}} \frac{\rho}{2} (v^2 + w^2) u dS \\
 &= \underbrace{D_{i_{w\xi}} V_\infty}_{0, \text{ no shed vorticity}} + D_{i_{\varphi\sigma}} V_\infty + \underbrace{\iint_{S_O^{TP}} \frac{\rho}{2} (v^2 + w^2) u dS}_{\dot{E}_{v_u}}
 \end{aligned} \tag{4.28}$$

This knowledge of the total drag behaviour makes it possible to express the drag power in terms of its physical exergy through exergy analysis, as shown in equation (4.29). This exposes the transverse kinetic energy through the mechanical energy term, allowing for the transverse kinetic energy decomposition provided in equation (4.28) to be substituted.

This substitution notably shows the cancellation between the cross-flow induced drag power and energy/exergy terms, which, in effect, represents Hackett's cross-flow cancellation principle [39]

from an energy/exergy perspective. Commonly expressed through momentum analysis, this cancellation describes the mutual counter-balancing of the streamwise and cross-flow velocity contributions toward the inviscid airfoil drag.

$$\begin{aligned}
 DV_{\infty} &= \underbrace{\dot{\mathcal{E}}_m + \dot{\mathcal{E}}_{th}}_{\dot{\mathcal{E}}_{phy}} \\
 &= (\dot{E}_a + \dot{E}_v + \dot{E}_p) + \dot{\mathcal{E}}_{th}
 \end{aligned}
 \tag{4.29}$$

Substituting equation (4.28)

$$\begin{aligned}
 &= \dot{E}_a + (D_{i_{\varphi\sigma}} V_{\infty} + \dot{E}_{v_u}) + \dot{E}_p + \dot{\mathcal{E}}_{th} \\
 &= 0
 \end{aligned}$$

#### 4.3.1.1 Compressibility Effects on Airfoil Drag

The analyses performed by Drela [31] and Aguirre [38] focused on assessing flows with negligible thermo-compressibility effects. Considering this, the numerical studies in this section extend to cases with more appreciable compressibility to observe the impact on the various energy/exergy-based terms.

The study examines a NACA 0012 airfoil in a classical C-H domain with farfield and pressure outlet boundaries located 30 chord lengths ( $c = 0.435\text{m}$ ) from the body. The operating conditions were set to zero degrees angle of attack and a chord-based Reynolds number of  $3 \times 10^6$ .

The analyses in Figure 4-4 highlight that as the Mach number increases, i.e.,  $0.15 < M_{\infty} < 0.60$ , an additional drag contribution is observed due to compressibility, represented through the thermal exergy  $\dot{\mathcal{E}}_{th}$ . This thermal exergy is shown in all instances to share a mutual counter-balancing behaviour with the mechanical exergy, returning a total drag result of zero, as expected.

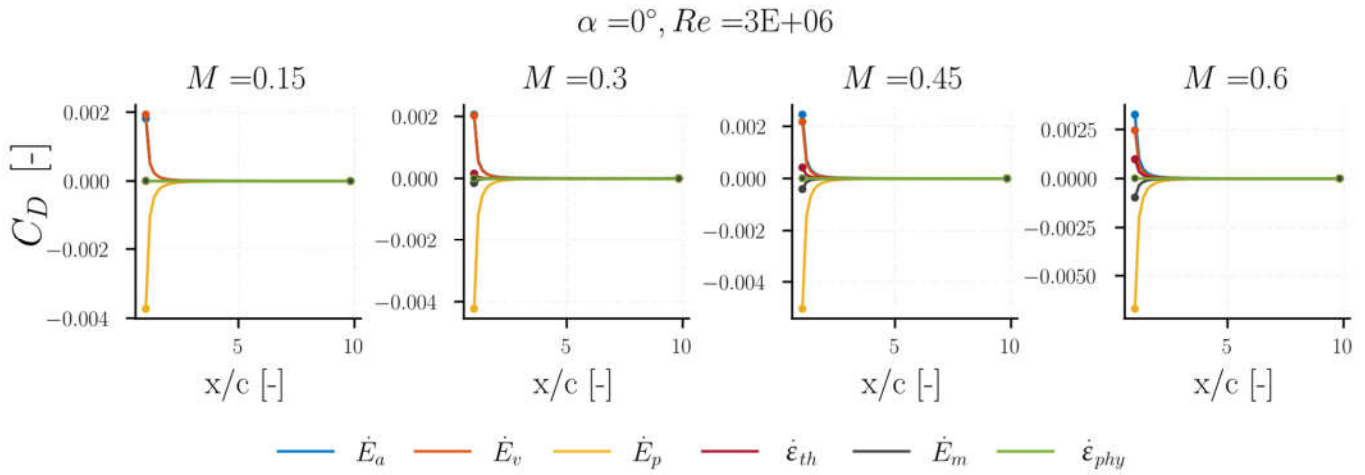


**Figure 4-4 Wake energy loss rate components for NACA 0012 airfoil at various Mach numbers and zero degrees angle of attack.**

### 4.3.2 Three-dimensional finite wing reversible drag evaluation through inviscid flow analysis

Drela [31] extensively analyses three-dimensional inviscid lifting flows in low-speed subsonic conditions. The analyses showed that the mechanical energy flow rate, formed by the summation of energy rates (i.e., streamwise, transverse and pressure boundary work), would represent the wing's induced drag power, as shown in Figure 4-5.

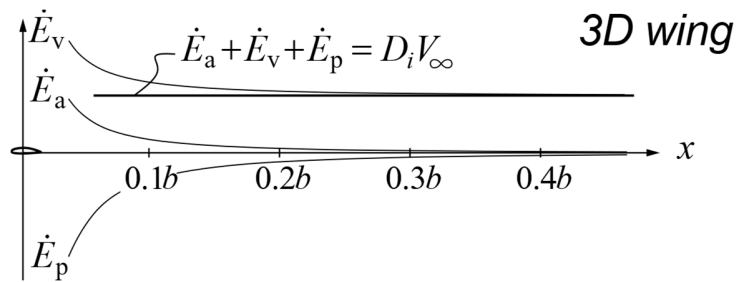


**Figure 4-5 Energy loss rate components for incompressible inviscid 3D wing adapted from [31]**

Unlike the airfoil, the total drag of the finite lifting wing is non-zero; this immediately highlights that aerodynamic bodies producing lift inevitably lose energy to the surroundings in the

downstream wake, corresponding to their lift-induced drag. This observation is essential in establishing the definition of induced drag used in this work. *The induced drag was viewed as the reversible energy loss to the surroundings that inevitably accompanies lift production.*

The reversible nature of this drag contribution in the context of subsonic flows comes through its association with the Euler field, where no dissipative phenomena exist. Therefore, although examined in the context of low-speed energy-based analysis in the work of Drela [31], the Euler drag generally describes the induced drag regardless of the force evaluation method applied. For example, the Euler pressure drag would quantify the induced drag in the near-field, providing a matching value to the energy-based assessment depicted in Figure 4-5.

Another state-of-the-art method commonly used for induced drag evaluation popularly within the wind tunnel was developed by Maskell [6]. Like the work of Drela [31], the simplifying assumption of incompressible flow was used in the formulation's derivation. As a result, when applied to high Mach number flows with appreciable thermo-compressibility, the formulation's accuracy diminishes, as demonstrated in the work of Aguirre [40].

Therefore, in keeping with the objective of the airfoil analyses, the formulation applicability is extended to compressible flows. This is done through a re-derivation following the procedure outlined by Diebold [41] with *no* incompressible flow assumption enforced. The derivation procedure has been provided in Appendix A.2 for the reader's convenience.

Summarising the main modifications, firstly, the compressible form of continuity is used, as shown in equation (4.30). This leads to the natural transformation of the cross-flow divergence to account for compressibility effects, represented as  $\sigma_z$

$$\frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = \rho \sigma_z \quad (4.30)$$

Secondly, the scalar function expressions of the transverse velocities in equations (4.31) and (4.32) are modified by multiplying the second RHS term by a compressibility correction factor ( $\rho_{ref}/\rho$ ).

For CFD analyses, an appropriate reference density would be the freestream density<sup>9</sup>. It should be noted that under the conditions of incompressible flow and a reference density set to the freestream density ( $\rho_{ref} = \rho_{\infty}$ ) would result in a compressibility correction factor of unity, thus returning the original formulation presented by Maskell [6].

$$v = \frac{\partial \psi}{\partial z} + \frac{\rho_{ref}}{\rho} \left( \frac{\partial \varphi}{\partial y} \right) \quad (4.31)$$

$$w = -\frac{\partial \psi}{\partial y} + \frac{\rho_{ref}}{\rho} \left( \frac{\partial \varphi}{\partial z} \right) \quad (4.32)$$

Expansion of the induced drag in equation (4.33)(a) using modified transverse velocity expressions leads to the new scalar function form presented in equation (4.33)(b). Essentially this expression is similar in structure to those found in literature [6–12,40], with the distinct difference in the source strength's ability to account for compressibility effects.

$$\begin{aligned} D_i &= \iint_{S_O^{TP}} \frac{\rho}{2} (v^2 + w^2) dS \quad (a) \\ &= \iint_{S_O^{TP}} \frac{\rho}{2} (\psi_{\xi}^2 - \varphi \sigma_z) dS \quad (b) \end{aligned} \quad (4.33)$$

The streamwise vorticity  $\psi_{\xi}$  and transverse cross-flow  $\varphi \sigma_z$  are calculated using an analytical Green's function approach [8,13,14] assessed through a Para View© plugin called Epsilon [42,43] which is an open-source code developed by the National Higher French Institute of Aeronautics and Space (ISAE- SUPAERO).

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<sup>9</sup> In the event this is applied to wind tunnel analysis, the reference density may be prescribed by a known tunnel reference density instead.

#### 4.3.2.1 Compressibility Effects on finite wing drag

Drag decomposition is considered in a similar manner as the work of Takahashi [44], where the component contributions are grouped into viscous (irreversible) and inviscid (reversible) parts.

As the analyses focused on inviscid flows with no shockwave formation, no irreversible drag contributions were observed. Thus the main interest is directed toward understanding the reversible induced drag through the various momentum and energy/exergy-based approaches shown in equation (4.34).

$$\text{induced drag} = \underbrace{\underbrace{D_i}_{\text{far-field}} V_\infty}_{\text{momentum}} = \underbrace{D_{i_{\psi\zeta}}}_{\text{Maskell}} V_\infty = \underbrace{\dot{\mathcal{E}}_{phy_E}}_{\text{energy/exergy}} \quad (4.34)$$

The level of agreement of these expressions was examined on an inviscid NACA 0012 rectangular wing planform with an aspect ratio of 8 and a chord length of 0.435m. The wing was set to operate at a slight angle of attack of  $2^\circ$  and chord-based Reynolds number of  $3 \times 10^6$  for a Mach number range  $0.15 < M_\infty < 0.60$ . The analyses were carried out in a classical C-H domain for consistency, with farfield and pressure outlet boundaries located 30 chord lengths from the wing planform.

Figure 4-6 shows the finite wing's wake energy and force contributions downstream of the trailing edge. Maskell's induced drag formulation indicates no induced drag contribution from the cross-flow term  $D_{i_{\phi\sigma}}$ , representing the incurred drag from the downstream propagation of bound vorticity flow effects through the wake. As no wake formation occurs for the modelled inviscid flows, this term intuitively returns no contribution toward the wing's overall drag.

On the other hand, as the shed wingtip vorticity *is* propagated downstream, the asymptotic non-zero drag contributions thus represent the wing's induced drag. This is confirmed by the good agreement shown for the various solutions in Figure 4-6 obtained through the induced drag formulations provided in equation (4.34). The consistency of the results for the range of Mach numbers assessed highlights the formulation applicability for subsonic incompressible and compressible flow analysis through momentum and energy/exergy-based approaches.

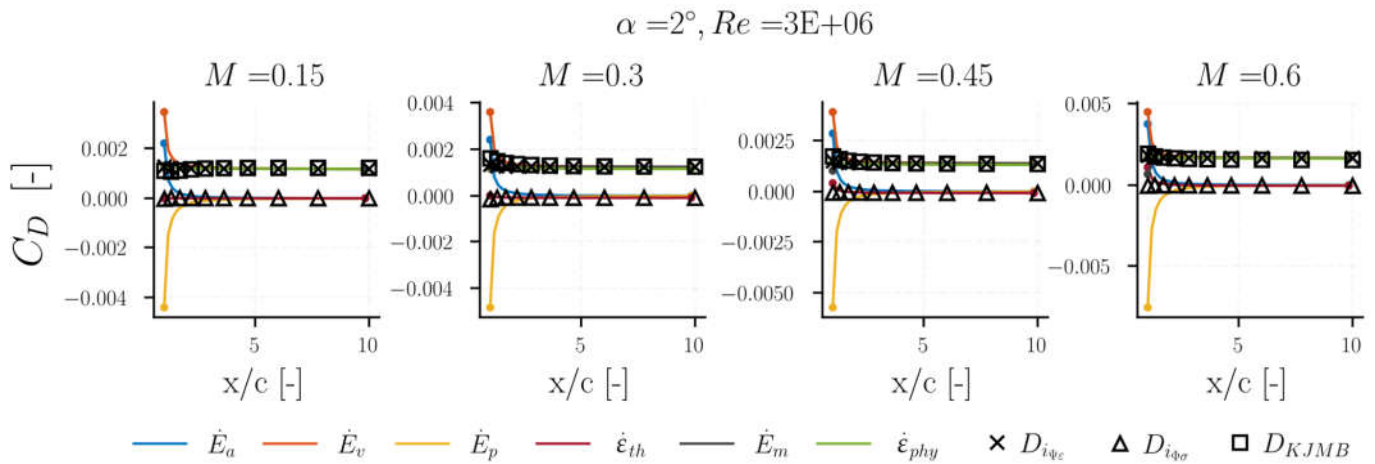


**Figure 4-6 Wake energy and force assessment for NACA 0012 rectangular wing at various Mach numbers**

For clarity, the wing's induced drag is provided in Table 4-1 for the various wake assessment approaches examined in Figure 4-6. The results are all reported through the trend averages assessed from the trailing edge to the far-wake approximately ten chord lengths downstream, i.e.,  $0 < x/c < 10$ . The results show that all solutions provide good agreement, with a percentage difference of no more than 5% between the various assessment methods for the Mach number range assessed.

**Table 4-1 Comparative analysis of induced drag quantification methods at various Mach numbers (in drag counts  $C_d \times 10^{-4}$ )**

| Mach No. | KJMB           | Maskell       | Energy/Exergy            |
|----------|----------------|---------------|--------------------------|
|          | $C_{D_{KJMB}}$ | $C_{D_{MSK}}$ | $C_{D_{\dot{\epsilon}}}$ |
| 3D Wing  |                |               |                          |
| 0.15     | 118.1          | 118.4         | 118.5                    |
| 0.30     | 130.2          | 125.2         | 124.7                    |
| 0.45     | 140.1          | 139.1         | 137.3                    |
| 0.60     | 159.2          | 167.4         | 162.9                    |

#### 4.4 Benchmarking CFD models

This section extends the prior Euler analyses to viscous flows for the two-dimensional NACA0012 airfoil and three-dimensional finite rectangular wing. The studies were carried out at various combinations of Reynolds, Mach and angles of attack, summarised in Table 4-2. Reynolds-averaged Navier–Stokes (RANS) simulations were performed on all analyses using a pressure-based solver in ANSYS Fluent ©, with the Green-Gauss cell-based method and a coupled pseudo-transient algorithm. Turbulence was modelled using the one-equation Spalart–Allmaras eddy-viscosity model [45] with second-order up-winding for all numerical fluxes. Flow residual convergence was below  $1 \times 10^{-6}$  while a  $y^+ \leq 0.5$  was maintained around the bodies for compatibility with turbulence modelling and PPF analysis.

**Table 4-2 Two- and three-dimensional NACA 0012 wing operating conditions**

| Reynolds No. | $1 \times 10^6$     | $3 \times 10^6$ | $6 \times 10^6$ | $1 \times 10^7$ |
|--------------|---------------------|-----------------|-----------------|-----------------|
| Mach No.     | Angle of attack [°] |                 |                 |                 |
| 0.15         | 0                   | 2               | 4               | 6               |
| 0.30         | 0                   | 2               | 4               | 6               |
| 0.45         | 0                   | 2               | 4               | 6               |

Mesh and solution validation was done through a comparative analysis of the total drag obtained through near-field, far-field KJMB, energy, and exergy-based analysis for both the airfoil and finite wing. This was conducted at several angles of attack shown in Table 4-3 at constant Reynolds number  $3 \times 10^6$ . For validation, the near-field results were compared against data from literature [23], with the percentage error provided in brackets in Table 4-3. Solution validity was considered when this near-field percentage error was below 5%. These results' reliability was reinforced by comparing the level of coherence with the other methods. It was observed that the respective quantities all showed good agreement, with no more than a 5% difference between their solutions for the airfoil and finite wing.

**Table 4-3 Aerodynamic lift and drag characteristic prediction comparison of analysis methods (in drag counts  $C_d \times 10^{-4}$ )**

| $\alpha$<br>[°] | Near-field   |              | KJMB           |                | Energy            | Exergy            |
|-----------------|--------------|--------------|----------------|----------------|-------------------|-------------------|
|                 | $C_{d_{nf}}$ | $C_{l_{nf}}$ | $C_{d_{KJMB}}$ | $C_{l_{KJMB}}$ | $C_{d_{\dot{E}}}$ | $C_{d_{\dot{e}}}$ |
| 2D Airfoil      |              |              |                |                |                   |                   |
| 0               | 92.6 (1.1%)  | 0            | 92.6           | 0              | 92.3              | 92.2              |
| 2               | 94.9 (1.6%)  | 0.23         | 94.5           | 0.23           | 94.4              | 94.3              |
| 4               | 102.2 (3.1%) | 0.46         | 100.1          | 0.45           | 101.1             | 101.1             |
| 6               | 114.7 (4.8%) | 0.68         | 111.0          | 0.67           | 113.2             | 113.3             |
| 3D wing         |              |              |                |                |                   |                   |
| $\alpha$<br>[°] | $C_{D_{nf}}$ | $C_{L_{nf}}$ | $C_{D_{KJMB}}$ | $C_{L_{KJMB}}$ | $C_{D_{\dot{E}}}$ | $C_{D_{\dot{e}}}$ |
| 0               | 94.7 (0.1%)  | 0            | 94.3           | 0              | 93.4              | 91.7              |
| 2               | 107.9 (2.4%) | 0.17         | 107.4          | 0.17           | 106.0             | 104.0             |
| 4               | 146.7 (1.8%) | 0.33         | 146.2          | 0.33           | 143.4             | 141.4             |
| 6               | 211.9 (0.8%) | 0.51         | 211.1          | 0.50           | 205.2             | 202.9             |

#### 4.4.1 Aerodynamic flow examples for two-dimensional flows - Airfoil

Considering an aerodynamic flow from an energy/exergy-based perspective allows for the following inferences from the introduced flow terms. Firstly, the kinetic energy deposition rates  $\dot{E}_a + \dot{E}_v$  explain a phenomenon where the body does work on the fluid with the energy transfer at the fluid-solid interface being facilitated by viscosity. As the airfoil moves through the quiescent fluid, there is a consistent transfer of work and energy between the two media. The pressure boundary work term  $\dot{E}_p$  conveniently identifies the work contribution associated with the pressure field.

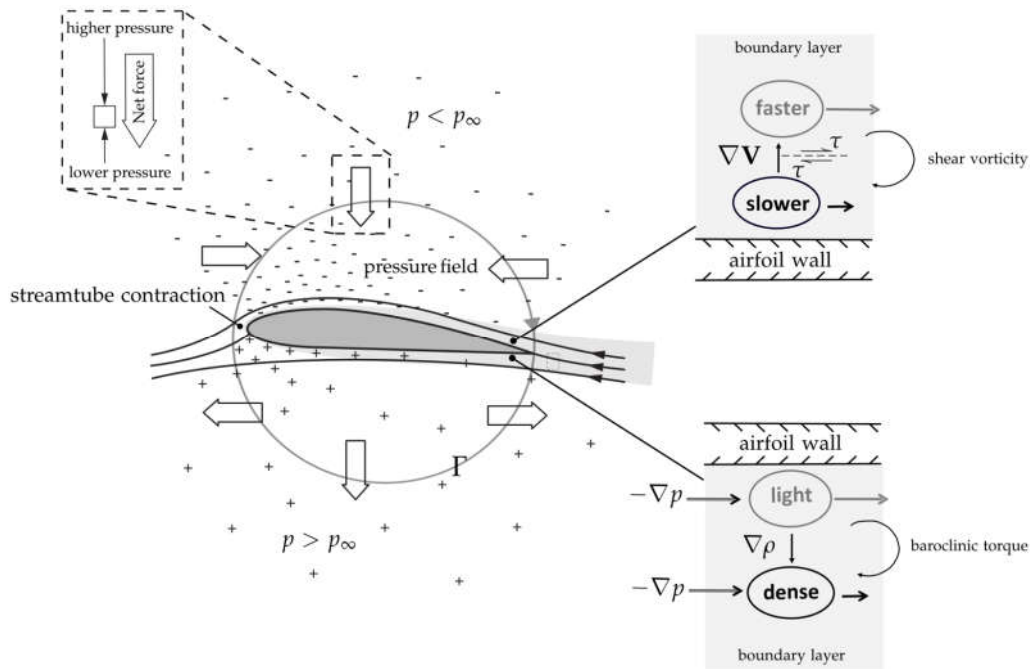


**Figure 4-7 Lifting airfoil Pressure and velocity field illustration, with indicated circulation development adapted from [46]**

The pressure field development around the airfoil, as depicted in Figure 4-7, relies on a circular cause-and-effect relationship between pressure and velocity. To maintain this pressure field, the fluid behaves as a continuous medium, deforming as the airfoil moves through it. This results in a contraction of the stream tubes near the leading edge as the flow follows the direction of the airfoil's curvature [47,48]. To preserve mass through continuity, this contraction causes an increase in flow velocity within the stream tube, which according to Newton's second law, is associated with a pressure gradient that imposes a net force on the fluid parcels, thus causing their acceleration (i.e., increased velocity). However, as mentioned by McLean [46], the existence of this pressure gradient is reliant on the parcel's motion; it is the resistance of the fluid parcels to a change in speed or direction (i.e., their inertia, due to their mass) that sustains the observed pressure gradient.

The fluid parcels located directly above the airfoil see a higher pressure above them than below, resulting in a net force generally in the downward direction, as shown in Figure 4-7. As stated

earlier, this causes the parcel's path to be deflected downward, with the pressure gradient sustained by its inertia, resisting its path from being deflected from a straight line.

Babinsky [8] explains that for a streamline to be curved, a pressure gradient across it must exist with pressure increasing in the direction away from the centre of curvature. This means that far away from the airfoil, the air is undisturbed as the pressure approaches the freestream, and the streamlines become straight.

For viscous flows, the work transfer between the body and fluid is a non-conservative process (due to friction) arising again due to the action of viscosity. The non-conservative work manifests as dissipation at fluid-body interfaces (i.e., boundary layer) and fluid-fluid interfaces within the wake. For the generation of lift, any effects inherently affecting the vorticity budget within the thin viscous boundary layer tend to be propagated into the bulk flow via viscous-inviscid interactions. Notably, the vorticity budget of compressible flows describes lift as being generated through a combined contribution from the vorticity originating from the velocity and shear gradients within the BL and through *baroclinic torque* due to the flow's compressibility. The baroclinic torque, represented in Figure 4-7, is an additional flow inertial, moment-generating effect due to forces acting on fluid parcels of different densities.

The principle of this physical mechanism and its contribution toward the aerodynamic force is conveniently introduced in vortex force theory via the Lamb vector and compressibility terms described in equations (4.11) and (4.13) from the work of Mele et al. [34] and Fournis et al. [16,26]. These effects and their quantification are further supported by their emergence in compressible partial pressure field analysis presented in [30].

Energetic assessment of lifting bodies ties nicely with the theoretical ideologies of lift, as explored by McLean [46]. The transverse kinetic energy  $\dot{E}_v$ , for instance, is associated with the starting vortex, which facilitates the development of the bound vorticity (as well as the circulation) around the airfoil. The starting vortex is formed through a largely transient process wherein the airfoil begins by moving through a quiescent fluid, causing large velocity gradients at the trailing edge, consequently leading to high vorticity levels.

The pressure difference of the lower and upper surfaces moves the trailing edge flow upwards, rolling up the thin sheet of vorticity to give a point vortex of counterclockwise sense, which induces a clockwise circulation around the airfoil in agreement with *Kelvin's Circulation Theorem* [47,48]. This system of vortices builds up during this transient process until the vortex around the airfoil gains the correct strength to satisfy the Kutta condition. After this point, the shed vorticity ceases, and steady conditions prevail around the airfoil.

#### 4.4.1.1 Mach, Reynolds, and Angle of Attack Effects on Two-dimensional Aerodynamic force characteristics

Figure 4-8 shows drag polar plots for the two-dimensional airfoil, decomposing the resulting drag into reversible and irreversible contributions. At the same time, the lift is split into Lamb vector  $\mathbf{e}_z \cdot \mathbf{F}_\ell$  and compressibility  $\mathbf{e}_z \cdot (\mathbf{F}_\rho - \mathbf{F}_{\nabla\rho})$  components following vortex force theory formulations as provided in equation (4.13).

The dissipative irreversible drag coefficient trends highlight that increasing the *Reynolds number* decreases viscous drag. The rise in Reynolds entails an increase in inertial forces relative to viscous forces, which implies that the fluid becomes less viscous, reducing the skin friction-related drag.

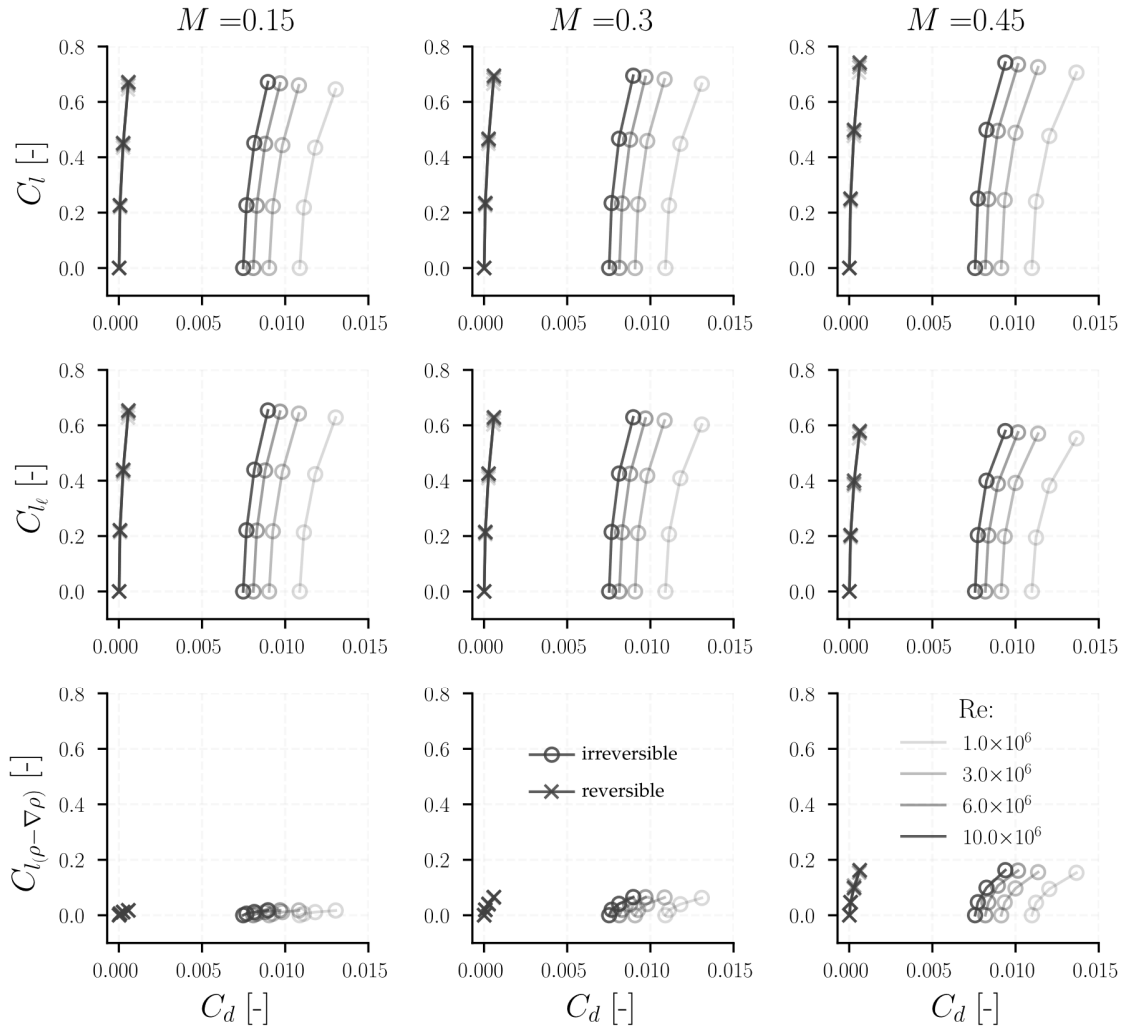


**Figure 4-8 NACA0012 airfoil Reynolds, Mach, and angle of attack influence study on reversible and irreversible drag**

Additionally, a higher Reynolds number indicates lower momentum diffusivity between the body and fluid, this results in slow propagation of viscous effects within the flow, such as boundary layer growth. The thinner boundary layer reduces dissipative pressure drag through the lowered intensity of the boundary layer displacement effect on the pressure field. These Reynolds number effects decrease skin friction and pressure drag, reducing the overall viscous drag, as observed in Figure 4-8.

Secondly, it can be observed that an increase in the *angle of attack* leads to a rise in the irreversible drag, which is expected due to the increase in BL and wake thickness with angle of attack. Considering the pressure coefficient plot shown in Figure 4-9, it can be observed that there is a favourable pressure gradient at the leading edge, which is accompanied by a rapid acceleration of the flow in this region. This results in a localised increase in the near-wall shear stresses resulting in an accompanying localised rise in skin friction drag. After the pressure coefficient inflexion point, an adverse pressure gradient exists, which persists to the trailing edge. The adverse pressure gradient facilitates the transfer of momentum and energy into the surrounding flow through the action of pressure leading to an increase in the boundary layer thickness. This results in a larger area for viscous damping effects, resulting in less steep velocity and shear gradients being observed, thus lowering the local skin friction drag.

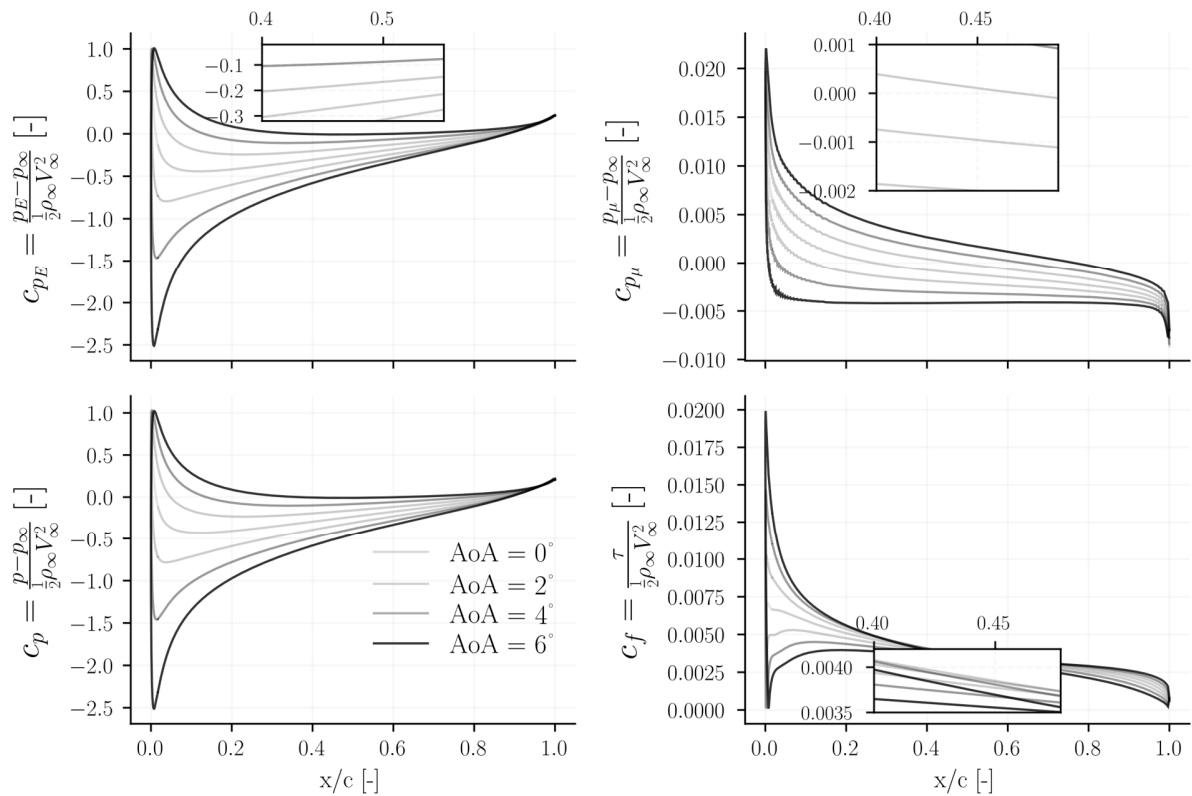


**Figure 4-9 Pressure and skin friction coefficient comparison at various angles of attack for NACA 0012 airfoil at Reynolds number  $3 \times 10^6$**

Conversely, the thicker BL increases the BL displacement effect, intensifying its induced impact on the pressure field and increasing *dissipative* pressure drag. The collective influence of the increased localised skin friction drag in the regions of high acceleration and the increased dissipative pressure drag due to the intensified BL displacement effect result in an overall increase in the irreversible drag contributions with angle of attack, as depicted in Figure 4-8.

Lastly, the effects of the compressibility on drag, observed through the *Mach number*, are negligible, agreeing with the sentiments made by Anderson [48]. In the absence of shockwave formation, the drag coefficient is a weak function of the Mach number remaining relatively constant until the drag divergence Mach number. Although the overall drag coefficient remains constant for the Mach number range assessed, an interesting observation lies in the behaviour of the pressure and skin friction components as compressibility effects become more significant.

This is assessed through the NACA 0012 test case at zero degrees angle of attack shown in Figure 4-11. As the Mach number increases, the skin friction drag is reduced in favour of increased pressure drag.

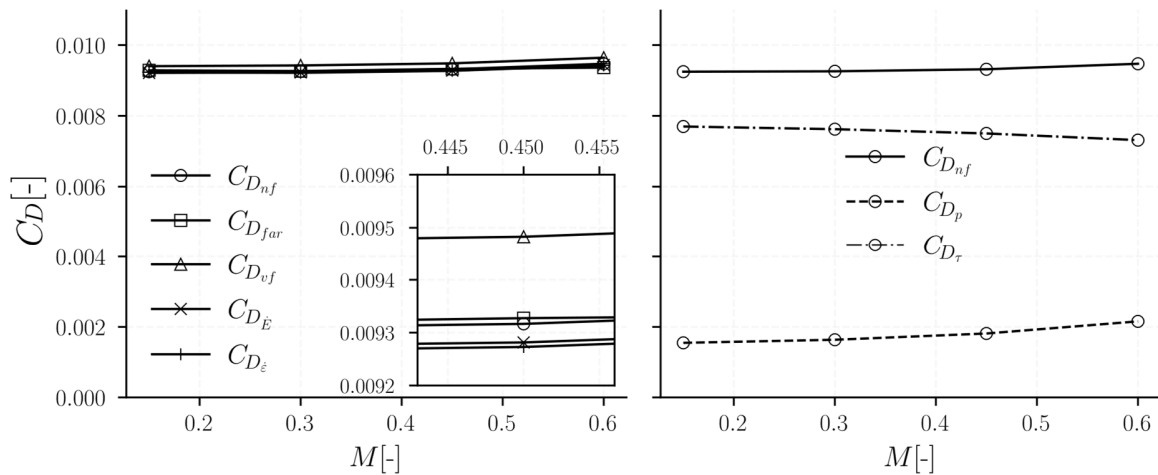


**Figure 4-10 Mach number variation effects on NACA 0012 airfoil drag coefficient for several methods (left), and near-field method breakdown (right) at  $\alpha = 0^\circ$  and Reynolds number  $3 \times 10^6$**

This can be explained as follows, an increase in compressibility entails an increase in fluid element's sensitivity to volumetric changes due to applied pressure. Therefore, comparing an

incompressible and compressible test case for the same applied pressure, the compressible flow would exhibit a more intense response. This is evidenced through the changes in pressure and density coefficients with Mach number attack shown in Figure 4-11, where the favourable gradients are seen to become steeper with increasing Mach, indicating both an increase in lift and drag-related characteristics. Instead of a thickening of the BL, as observed with the angle of attack and Reynolds number cases, the streamline deformation becomes more pronounced for compressible flows due to the fluid's increased responsiveness to pressure-induced phenomena.

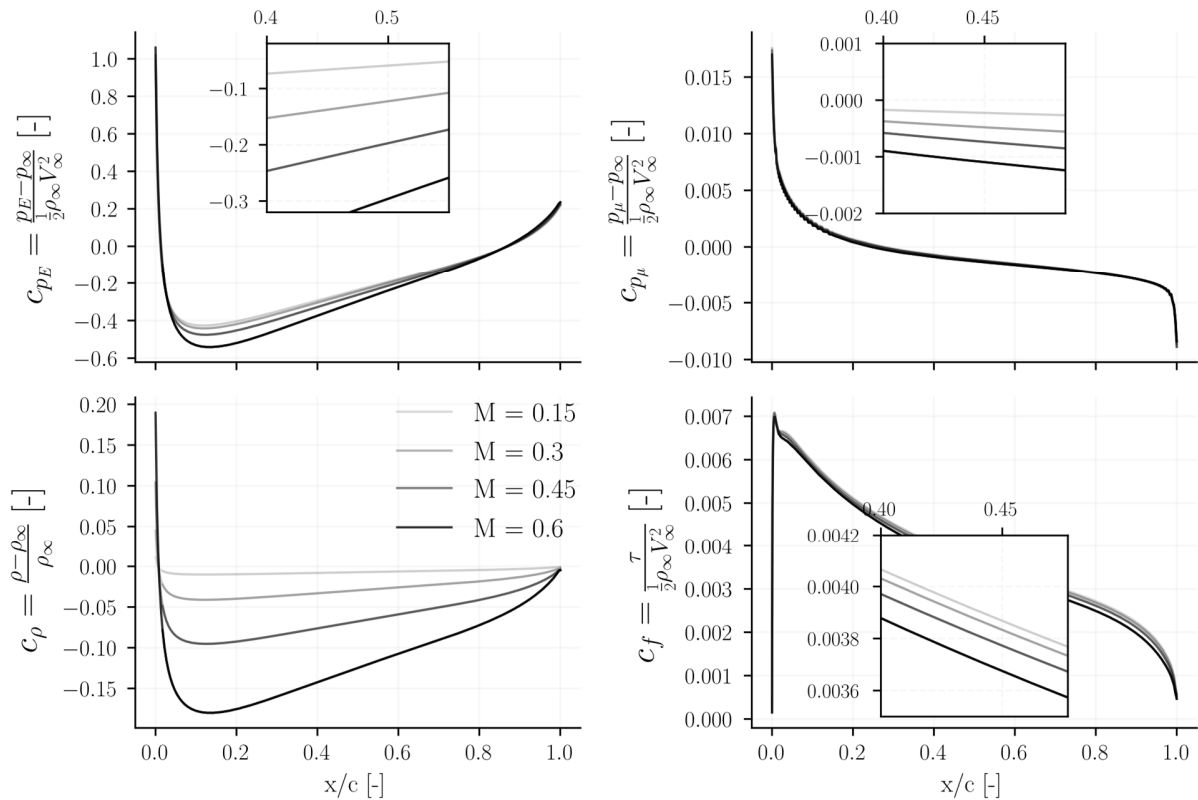


**Figure 4-11 Mach number variation effects on Euler pressure, dissipative pressure, density, and skin friction coefficients for a NACA 0012 airfoil at  $\alpha = 0^\circ$  and Reynolds number  $3 \times 10^6$**

The decomposition of lift into its shear-vorticity and baroclinic torque contributions aids in showing how compressibility augments lift at higher Mach numbers. An increase in lift with compressibility is observed in Figure 4-8, attributed to the effects of the baroclinic torque. For

incompressible flows, the central contributing mechanism toward lift is the shear-vorticity as the fluid compressibility increases the inertial moment caused by the baroclinic torque, resulting in a lift augmenting effect.

Furthermore, Figure 4-8 shows that the lift is a weak function of Reynolds number; the slight differences obtained with increasing Reynolds number occur due to the change in BL thickness. As mentioned, the BL facilitates the diffusion of momentum and energy from the body to the fluid, making the fluid a natural sink of momentum and energy. As a result, thicker BLs result in more work transfer between the body and fluid. This lost work potential reduces the intensity of the favourable pressure gradients developed around the airfoil necessary for lift production.

Lastly, a notable advantage of the drag polar decomposition into irreversible and reversible contributions is its ability to identify and separate Reynolds-dependent and independent contributions. The dissipative irreversible drag coefficient is directly related to the BL and wake. As a result, the BL change in thickness due to Reynolds number results in the irreversible drag being a strong function of the Reynolds number, as shown in Figure 4-8.

The non-dissipative reversible drag coefficient's contribution is primarily associated with bulk flow phenomena, such as lift-induced drag. As the reconstruction of the EIF utilises a simple inviscid flow model, the boundary layer displacement effect is omitted in its reconstruction. As a result, changes in the BL thickness and wake with Reynolds number have little to no influence on the overall trends. The reversible drag is instead more strongly tied to the angle of attack, as it is directly associated with the lift-induced effects at higher angles of attack.

#### **4.4.2 Aerodynamic flow examples for three-dimensional flows – Rectangular finite wing**

This section extends the two-dimensional analyses to finite wings in three-dimensional flow. The two-dimensional flow decompositions allowed for identifying the lift and lift-induced drag through the non-dissipative flow field, observed due to the airfoil's bound vorticity. Like the two-dimensional flows, the circulation around the finite wing is conserved through Kelvin's theorem. However, the main difference between them is the flow's spanwise motion. The pressure

differential between the lower and upper wing surfaces results in the formation of tip vortices shed downstream, creating a closed vortex loop, as shown in Figure 4-12(a). The coupled contribution from the bound and tip vorticities raises the flow's downward momentum, i.e., downwash, compared to the two-dimensional airfoil scenario, as depicted in Figure 4-12(b).

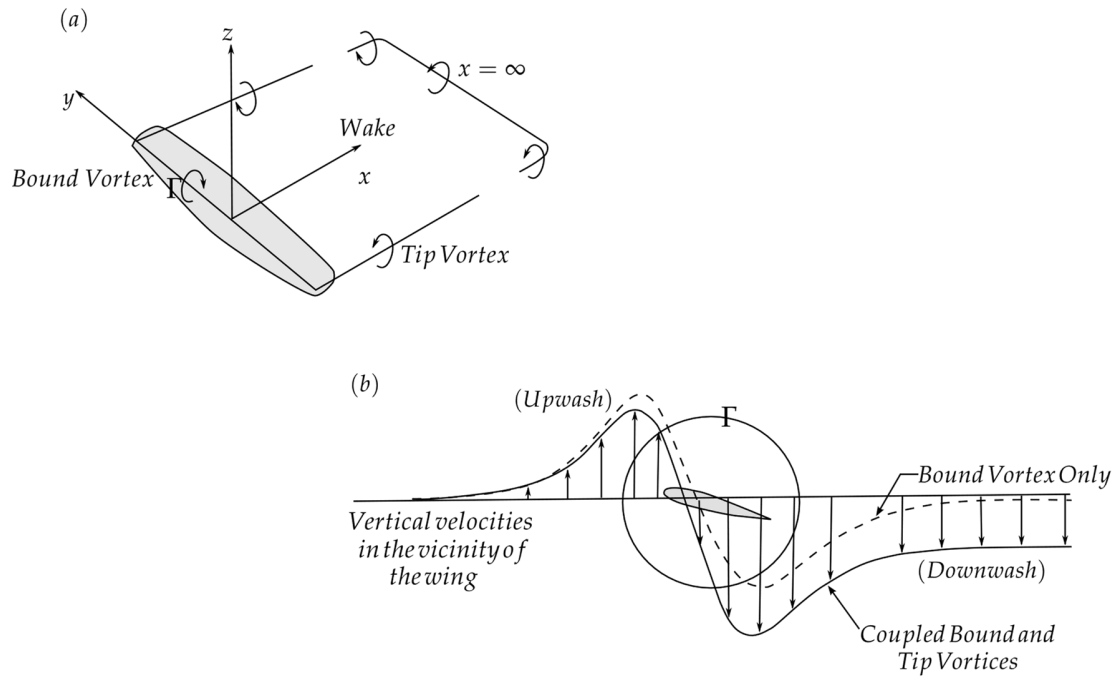


**Figure 4-12 Downwash behaviour of wing vortex system adapted from [50]**

From an energy-based perspective, these effects are represented through the transverse kinetic energy imparted to the flow. By comparing the two- and three-dimensional scenarios in Figure 4-13, the transverse kinetic energy increases for the three-dimensional analysis. This occurs due to the coupled vortical effects that raise the drag penalty, i.e., lift-induced drag, during lift production compared to the two-dimensional test cases, where no shed wingtip vorticity is present.



advantage of the fact that for subsonic flows, the predominant contribution toward induced drag originates from the Euler field, confirmed through the inviscid analyses in Section 4.3.2.

$$D_i^{PPF} = \iint_{S_B} p_E \mathbf{n} dS \quad (4.35)$$

Induced drag quantification through *far-field* momentum methods takes advantage of two variations of Maskell's induced drag formulation, the *first* of which corresponds to the lamb vector KJMB formulation proposed by Fournis et al. [16,26]. This formulation is advantageous in unifying aerodynamic force assessment methods as it directly links vortex force and far-field techniques.

$$\begin{aligned} D_i^{KJMB} &= \mathbf{e}_x \cdot (\mathbf{F}_\ell + \mathbf{F}_\rho - \mathbf{F}_{\nabla\rho}) \\ &= \mathbf{e}_x \cdot \iint_{S_O^{TP}} \frac{1}{2} \rho (v^2 + w^2 - u^2) dS \end{aligned} \quad (4.36)$$

As can be seen in equation (4.36), the induced drag formulation is not wake-reduced and applies to the entire transverse survey plane downstream of the aerodynamic body. Maskell [6] proposed a wake reduction procedure to assess induced drag in the wind tunnel through wake measurements. This involved introducing the *artificial velocity* concept [2] through the axial isentropic perturbation velocity  $u_E$ . This allowed for the isentropic perturbation velocity to be expressed as a wake-reduced blockage correction term when coupled with the wake blockage velocity  $u_b$  considering the presence of tunnel walls.

The blockage term allowed for the induced drag formulation to be simplified to the form shown in equation (4.37)(b) with the wake-reduced form in (4.37)(c) for clean wing configurations obtained by assuming the contribution of the second term in equation (4.37)(b) is small.

$$\begin{aligned}
D_i^{MSK} &= \iint_{S_O^{TP}} \frac{1}{2} \rho (v^2 + w^2) dS & (a) \\
&= \iint_{S_O^{TP}} \frac{\rho}{2} (\psi \xi - \varphi \sigma_z) dS & (b) \\
D_i^{MSK} &\approx \iint_{Wake} \frac{\rho}{2} \psi \xi dS & (c)
\end{aligned} \tag{4.37}$$

Following the studies in Section 4.3.2, extending induced drag analysis to energy/exergy-based methods is possible by evaluating the non-dissipative physical exergy  $\dot{\mathcal{E}}_{phyE}$  as shown in equation (4.38).

$$\begin{aligned}
D_{\dot{\mathcal{E}}_E} V_\infty &= \underbrace{\dot{E}_{m_E} + \dot{\mathcal{E}}_{th_E}}_{\dot{\mathcal{E}}_{phyE}} \\
&= \iint_{S_O^{TP}} \left[ p_E - p_\infty + \frac{1}{2} \rho (V_E^2 - V_\infty^2) \right] (V_\infty + u_E) dS \\
&\quad + \iint_{S_O^{TP}} \left\{ \rho_E [\delta e_E + T_\infty \delta s] + p_\infty \right\} (V_\infty + u_E) dS
\end{aligned} \tag{4.38}$$

#### 4.4.2.1 Mach, Reynolds, and Angle of Attack Effects on Three-dimensional Aerodynamic force characteristics

An impressive aspect of *vortex force* methods is their natural introduction of reversible and irreversible aerodynamic force contributions without explicitly referencing entropy formulations for identifying the irreversible drag, commonly done for thermodynamic methods [4,5]. This method makes it possible to uniquely identify the various phenomenological drag contributions, namely the induced drag, given by the vortex force, and the irreversible viscous drag provided by the surface integral of the Lamb vector moment in the body wake. Additionally, the vortex force formulations presented by Mele et al. [34] can be fully related to classical aerodynamic methods through the procedure outlined by Fournis et al. [16,26], as shown in given in equation (4.13).

Kang et al. [17] show that even though no velocity [23] or partial pressure decomposition [28–30] is used for reversible and irreversible component identification, the *total drag* can still be correctly predicted at various downstream locations. However, it was seen that when the individual viscous and induced drag components were assessed, significant variation in the near-wake was observed, with their contributions only reaching asymptotic values at downstream locations of approximately two chord lengths behind the trailing edge of an aerodynamic body. This near-wake behaviour reinforces that conventional force decomposition utilising only RVF components does not provide complete separability between reversible and irreversible force contributions. As a result, the identification of phenomenological contributions can be improved by employing flow-field decomposition approaches. This is confirmed by the improved near-wake behaviour shown for the reversible and irreversible force decompositions provided in Figure 4-13.

#### **4.4.2.2 Total drag decomposition relation with traditional approaches**

The decomposition of forces into reversible and irreversible components modifies the definition of viscous and induced drag by considering the phenomenological *origins* of the force contributions. For instance, for *two-dimensional lifting flows* due to the lack of wingtip shed vorticity, the standard inference taken by conventional methods in literature [48,51–53] is to attribute the total drag force to profile drag, implying no drag due to lift. In this instance, the differentiating characteristic between lifting and non-lifting bodies would be the rise in profile drag with angle of attack associated with the increase in BL and wake thickness. Agreeably, an increase in the BL and wake thickness does lead to an increase in drag; however, the nature of this rise is not immediately realisable when assessed in this manner.

As the angle of attack increases, the pressure gradients over the aerodynamic body increase momentum diffusivity to the flow due to pressure. This results in a more considerable amount of momentum being diffused through the action of pressure and less through the shear stress. The increase in angle of attack also results in higher flow circulation around the airfoil, intensifying the downwash experienced by the flow.

To produce lift, an airfoil must develop circulation; this process is a power sink and transfers energy from the body to the flow observable in the downstream wake as the flow downwash. The associated energy imparted to the flow then acts as a source of drag (i.e., *lift-induced drag*), becoming the trace of lift, differentiating lifting and non-lifting bodies through their difference in downstream wake-energy behaviour.

Hypothetically, the minimum drag of a lifting body would be this lift-induced drag, being present as long as lift is being produced, even for two-dimensional airfoils. This 2D lift-induced drag can be confirmed through the non-zero near-field pressure drag returned from Euler inviscid *lifting* airfoil analyses.

This can be extended to three-dimensional flows; in subsonic flow, the total drag can be split into a profile and drag due to lift contribution. Flow-field decomposition approaches allow these to be further segmented into respective dissipative and non-dissipative components. These can be grouped to represent the irreversible (viscous) and reversible (lift-induced) contributions toward the aerodynamic force following the same ideology presented for vortex force analyses [17,54]. The primary difference between the reversible and irreversible definitions provided in [17,54] and the ones used in this work is the slight re-definition of components toward these respective contributions. This re-definition carries the same significance as the Oswald and spanwise efficiency transformation discussed by Spedding and McArthur [55].

Although commonly referred to interchangeably, there are fundamental differences between these efficiencies. Equation (4.39)(a) consists of the *Oswald efficiency*  $e_v$ , which is a viscous correction factor accounting for the departure of the lift distribution from an ideal elliptical distribution, as well as the drag penalty incurred due to the wing planform shape mainly linked to the wing's aspect ratio (AR). On the other hand, the *spanwise efficiency*  $e_i$  in equation (4.39)(b) is an inviscid correction factor accounting solely for the wing lift distribution departure from elliptical.

The second term in equation (4.39)(a) is commonly called the *drag due to lift* [55], as it includes viscous drag variation effects with lift. In comparison, the second term in (4.39)(b) is referred to

as *lift-induced drag*, as the first term  $C_d$  fully accounts for all viscous drag contributions, including those incurred through lift variations.

$$\begin{aligned} C_D &= C_{D,0} + \frac{C_L^2}{\pi(AR)e_v} \quad (a) \\ C_D &= C_d + \frac{C_L^2}{\pi(AR)e_i} \quad (b) \end{aligned} \tag{4.39}$$

Similarly, the drag book-keeping approach taken here, particularly in the far-field, considers the viscous drag as solely being associated with dissipative contributions regardless of their downstream region of influence, i.e., viscous BL/wake or the vortex. This is plausible because all observable downstream dissipative phenomena originate due to on-body near-field boundary layer development, later convected downstream and partitioned into contributions from the viscous wake and vortex.

The remaining non-dissipative contributions are associated with the now purely reversible induced drag directly related to lift production. The resulting expression from this modification is provided in equation (4.40), which now represents the viscous and induced drag in a similar manner as in equation (4.39)(b).

$$\begin{aligned} C_D &= C_{D_{pr}} + C_{D_i} \\ &= \underbrace{C_{D_{pr\mu}} + C_{D_{i\mu}}}_{C_{D_{irrev}}} + \underbrace{C_{D_{prE}} + C_{D_{iE}}}_{C_{D_{rev}}} \\ &= \underbrace{C_d}_{C_{D_{irrev}}} + \frac{C_L^2}{\underbrace{\pi(AR)e_i}_{C_{D_{rev}}}} \end{aligned} \tag{4.40}$$

Figure 4-14 shows the decomposed drag polar plots at various Reynolds, Mach, and angles of attack for the finite wing test cases. The dissipative viscous drag trends closely resemble the two-dimensional test cases in Figure 4-8, showing similar responses to angle of attack, Reynolds, and Mach number changes. Observable differences between the two- and three-dimensional analyses

lie mainly in the reversible (induced) drag behaviour, which shows a substantially higher contribution for the finite wings. This is because the finite wings are not horizontally constrained (as are the two-dimensional airfoil), and as a result, the flow wraps around the wingtips, forming wingtip vortices.

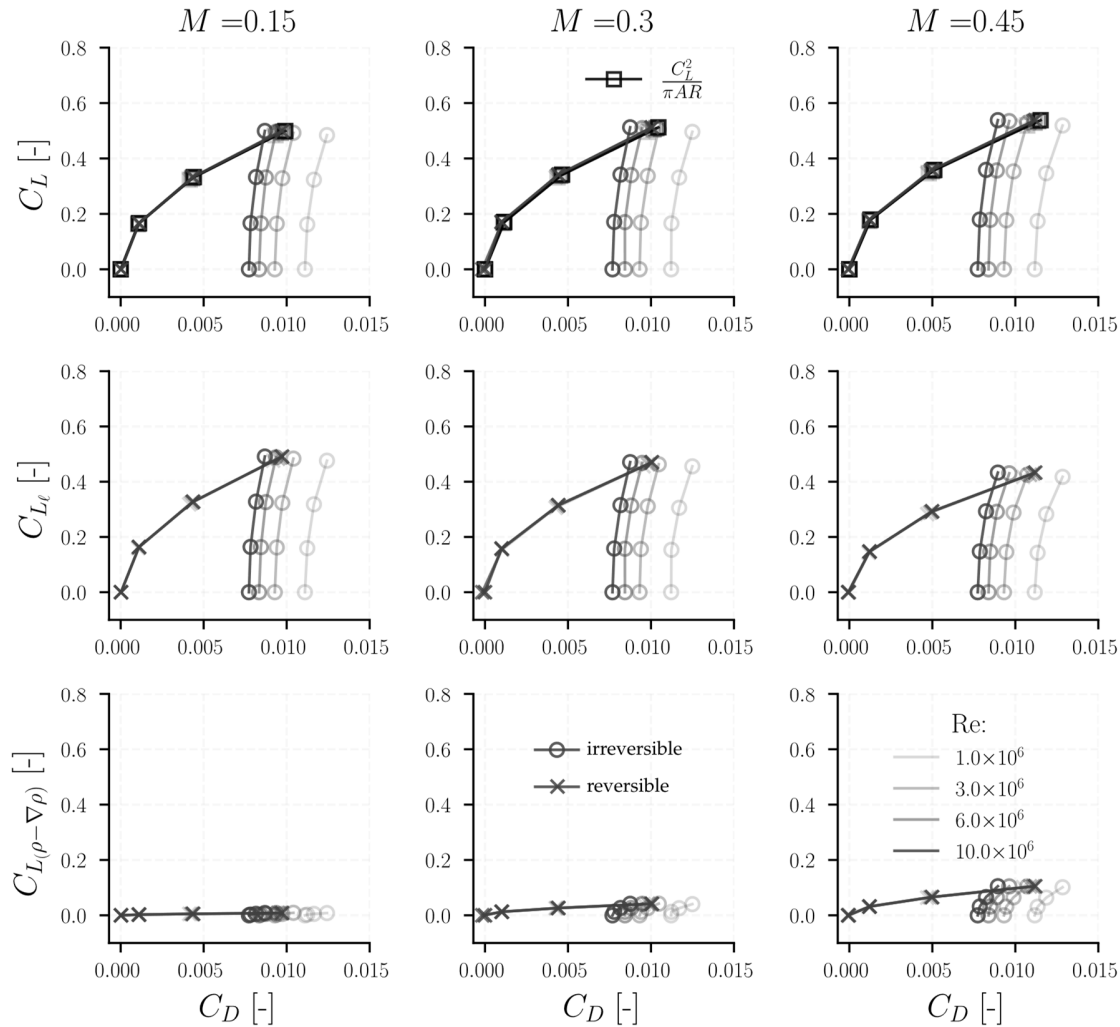


**Figure 4-14 Finite rectangular NACA 0012 wing Reynolds, Mach, and angle of attack influence study on reversible and irreversible drag**

The reversible drag shows the same Reynolds number invariance previously observed for the two-dimensional airfoil. More importantly, the induced drag relationship with the total lift shows good

agreement with Prandtl's [56] classical induced drag formulation, returning an apparent *spanwise efficiency* factor of approximately one  $e_i \approx 1$ .

This outcome agrees with the statements made by Spedding and McArthur [55] regarding spanwise efficiency as being a number slightly below unity. Furthermore, the work of Glauert [57] highlighted that for low aspect ratio planar wings ( $AR = 2$ ) for taper ratios (tip chord/root chord) between 1 and 0.25, the spanwise efficiency values varied within a range between  $0.95 \leq e_i \leq 0.98$ .

As a result, the spanwise efficiency obtained in this work highlights that the downwash distribution approaches ideality with an increase in aspect ratio, necessary to achieve a spanwise efficiency of unity. However, to present a more conclusive argument and provide more insight into spanwise efficiency sensitivity, further analyses would have to be conducted at various sweep angles, aspect, and taper ratios over a more extensive operating condition range, particularly those including significant thermo-compressibility effects, which will be addressed in future work.

## 4.5 Conclusions

This work presents a synergistic assessment of a two-dimensional airfoil and finite wing through the simultaneous application of momentum, energy, and exergy-based methods to identify the phenomenological origins of physical mechanisms leading toward lift and drag. The first section begins with deriving force decomposition approaches through the Navier Stokes Momentum equation with the stress tensor decomposed into its pure shear and expansion/compression components.

The derivative moment transformation allows for the derivation of vortex force formulation decompositions. These are significantly advantageous as they immediately introduce the conceptual principle of reversible and irreversible aerodynamic force contributions. This allows a direct association of phenomenological mechanisms within the flow to be linked to respective force contributions such as lift, lift-induced and viscous drag.

Although the conceptual argument of reversible/irreversible force decomposition is valid, observations show that vortex force formulations, particularly for the aerodynamic drag, show large variability when analysed in the near-wake of an aerodynamic body. This occurs due to viscous inviscid interaction because the formulations are cast using real viscous flow components, which neglect separability into reversible and irreversible parts.

To address this, partial pressure field and velocity decomposition approaches are used, allowing for the real viscous flow to be represented as a superposition of an Euler inviscid and dissipative flow, representing the reversible and irreversible flow phenomena, respectively. The reversible force contributions (i.e., lift and lift-induced drag) represent the isentropic part of the aerodynamic force originating from the bulk flow, whilst the irreversible contributions (i.e., viscous drag) describe forces of non-isentropic origin associated with entropy-generating mechanisms within the viscous boundary layer and wake.

Approaching drag decomposition in this manner carries considerable advantages. For near-field momentum analysis, this enables the pressure drag to be decomposed into its dissipative and non-dissipative parts. This allows the viscous drag to be expressed as a summation of the skin friction and dissipative pressure drag while, more importantly, providing a means of evaluating the lift-induced drag via the non-dissipative pressure drag. Similarly, for far-field momentum analysis, the surface-integral momentum flux can also be separated into reversible and irreversible components representing the viscous and lift-induced drag, respectively.

Extending this aerodynamic assessment to introduce the first and second laws of thermodynamics provides a unique performance evaluation approach through energy/exergy analysis. From this perspective, drag is interpreted as mechanical energy being lost from the aerodynamic body and transferred to the surrounding flow. This energy is categorised through the second law of thermodynamics into two distinct forms. The first can theoretically be converted into mechanical work, referred to as exergy, and the remainder is irreversibly lost to the surroundings, referred to as anergy.

The viscous stress tensor decomposition mentioned earlier aids in identifying the most significant mechanical energy loss contribution within the flow attributed to the enstrophy-related energy associated with the accumulation of vorticity within the boundary layer and wake.

A notable aspect of the reversible/irreversible drag decomposition is its ability to identify lift-induced drag. As an airfoil produces lift, the generated downwash imparts energy to the downstream flow contributing as an inevitable drag source. From this perspective, the lift-induced drag represents the minimum amount of power transferred from the aerodynamic body to the flow necessary to produce lift. This is more evident when extended to the three-dimensional finite wings where the lift-induced drag can directly be related to lift through Prandtl's classical induced drag formulation.

For three-dimensional flows, the utility of reversible and irreversible force decomposition is realised through its ability to differentiate between formulations utilising Oswald and spanwise efficiency factors. Formulations using Oswald efficiency evaluate the drag due to lift and contain corrections that account for lift distribution departure from ideal elliptical and viscous losses due to the wing planform. As a result, the drag due to lift is not a purely reversible contribution and contains irreversible losses incurred due to the wing planform.

On the other hand, formulations utilising the span efficiency contain corrections only accounting for the departure from an elliptic lift distribution. The associated induced drag evaluated in this instance coincides with classical induced drag formulations. The planform-related irreversible losses are then accounted for via the dissipative viscous drag, thus providing complete separability between reversible and irreversible drag contributions.

## **Appendix A**

### **A.1 Derivative Moment Transformation**

Kang et al. [17] define two vectorial integral identities referred to as derivative-moment-transformations (DMT), based on the work of Wu et al. [35], which are necessary to produce the unique form of the vortex for expressions. These formulations, shown in equation (4.41), are

applicable in  $n$ -dimensional space (where  $n = 2,3$ ) for any piecewise differentiable vector field  $\mathbf{g}$  and scalar field  $\zeta$ .

$$\begin{aligned} \iiint_V \mathbf{g} dV &= \frac{1}{n-1} \iiint_V \mathbf{x} \times (\nabla \times \mathbf{g}) dV - \frac{1}{n-1} \oint_S \mathbf{x} \times (\mathbf{n} \times \mathbf{g}) dS \\ \oint_S \zeta \mathbf{n} dS &= -\frac{1}{n-1} \oint_S \mathbf{x} \times (\mathbf{n} \times \nabla \zeta) dS + \frac{1}{n-1} \oint_C \zeta \mathbf{x} \times d\mathbf{x} \end{aligned} \quad (4.41)$$

## A.2 Compressibility Correction Derivation for Maskell's induced drag formulation

Maskell's formulation [6] allows for a wind tunnel applicable induced drag evaluation approach using wake plane measurements. The simplifying assumptions for the derivation of this formulation enforce an incompressible flow condition, affecting the accuracy of results when analysis of compressible flows is required. This section addresses this by modifying the cross-flow divergence source strength  $\sigma_z$  in equation (4.42) to consider thermo-compressibility effects following a derivation procedure like Diebold [41].

$$\begin{aligned} \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} &= \rho \sigma_z \\ \Rightarrow \left( \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + \frac{1}{\rho} \left( v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} \right) &= \sigma_z \\ \text{using Leibniz or product rule} & \\ \Rightarrow \left( \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) - \rho \left( v \frac{\partial \left( \frac{1}{\rho} \right)}{\partial y} + w \frac{\partial \left( \frac{1}{\rho} \right)}{\partial z} \right) &= \sigma_z \end{aligned} \quad (4.42)$$

The derivation begins with the streamwise vorticity and compressible continuity formulations in equations (4.43) and (4.42), noting that these expressions are both satisfied by the transverse velocities  $v$  and  $w$ .

Both  $\xi$  and  $\sigma_z$  are functions of  $y$  and  $z$  in the transverse plane (see Figure 4-2), where  $\xi$  denotes the x-component of the vorticity vector while  $\rho\sigma_z$  represents  $-\partial(\rho u)/\partial x$  in equations (4.42), per the three-dimensional continuity equation.

$$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 2\omega_x = \xi \quad (4.43)$$

Two scalar functions of  $y$  and  $z$  are introduced through  $\psi$  and  $\phi$  in equations (4.44) and (4.45), both satisfying the vorticity and source term equations (4.43) and (4.42), respectively.

$$\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = -\xi \quad (4.44)$$

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \rho\sigma \quad (4.45)$$

Equations (4.43) and (4.42) are satisfied if

$$v = \frac{\partial \psi}{\partial z} + \frac{\rho_{ref}}{\rho} \left( \frac{\partial \phi}{\partial y} \right) \quad (4.46)$$

$$w = -\frac{\partial \psi}{\partial y} + \frac{\rho_{ref}}{\rho} \left( \frac{\partial \phi}{\partial z} \right) \quad (4.47)$$

The transverse velocity formulations provided in equations (4.46) and (4.47) differ from the conventional expressions in literature [6–11], as they contain a compressibility correction factor  $\rho_{ref}/\rho$  acting on the scalar function  $\phi$ .

Maskell [6] shows that induced drag for a lifting body can be evaluated through:

$$D_i^{MSK} = \iint_{S_o^{TP}} \frac{\rho}{2} (v^2 + w^2) dS \quad (4.48)$$

Substituting the transverse velocity expressions in equations (4.46) and (4.47) for  $v$  and  $w$  results in the following

$$\begin{aligned} \iint_{S_O^{TP}} (v^2 + w^2) dS &= \iint_{S_O^{TP}} \left[ v \left( \frac{\partial \psi}{\partial z} + \frac{\rho_{ref}}{\rho} \left( \frac{\partial \varphi}{\partial y} \right) \right) + w \left( -\frac{\partial \psi}{\partial y} + \frac{\rho_{ref}}{\rho} \left( \frac{\partial \varphi}{\partial z} \right) \right) \right] dS \\ &= \iint_{S_O^{TP}} \left[ \left( v \frac{\partial \psi}{\partial z} - w \frac{\partial \psi}{\partial y} \right) + \frac{\rho_{ref}}{\rho} \left( v \frac{\partial \varphi}{\partial y} + w \frac{\partial \varphi}{\partial z} \right) \right] dS \end{aligned} \quad (4.49)$$

Adding and subtracting  $\iint_{S_O} \psi \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) dS$  and  $\rho_{ref} \iint_{S_O} \varphi \left( \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dS$  to the RHS of equation (4.49) while keeping in mind that,

$$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = \xi$$

and

$$\left( \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = \underbrace{\sigma - \frac{1}{\rho} \left( v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} \right)}_{\sigma_z},$$

allows for the *quotient rule* to be used, yielding the expression

$$\begin{aligned} \iint_{S_O^{TP}} (v^2 + w^2) dS &= I_a + I_b + I_c \\ I_a &= \iint_{S_O^{TP}} \left[ \left( v \frac{\partial \psi}{\partial z} - w \frac{\partial \psi}{\partial y} \right) - \psi \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \right] dS \quad (a) \\ I_b &= \rho_{ref} \iint_{S_O^{TP}} \left[ \frac{1}{\rho} \left( v \frac{\partial \varphi}{\partial y} + w \frac{\partial \varphi}{\partial z} \right) + \varphi \left( \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right] dS \quad (b) \\ I_c &= \iint_{S_O^{TP}} (\psi \xi - \varphi \sigma_z) dS \quad (c) \end{aligned} \quad (4.50)$$

Using the *product rule*  $I_a$  and  $I_b$  can be rewritten as

$$\begin{aligned} I_a &= \iint_{S_O^{TP}} \left( \frac{\partial \psi v}{\partial z} - \frac{\partial \psi w}{\partial y} \right) dS \quad (a) \\ I_b &= \iint_{S_O^{TP}} \left( \frac{\partial \phi v}{\partial y} + \frac{\partial \phi w}{\partial z} \right) dS \quad (b) \end{aligned} \quad (4.51)$$

Through Stokes Theorem,  $I_a$  and  $I_b$  can be modified and represented as line integrals along the boundary surface  $S_O^{TP}$  (see Figure 4-2).

$$I_a + I_b = \oint_C (-\psi v_t + Z \phi v_n) dl = 0 \quad (4.52)$$

Where  $v_n$  and  $v_t$  represent the normal and tangential velocities to the transverse plane boundaries.

For the expression in equation (4.52), the enforced boundary conditions  $\psi = 0$  and  $v_n = 0$  lead to the mutual cancellation of  $I_a$  and  $I_b$ , leaving only  $I_c$  as a non-zero contributor toward induced drag.

$$D_{i,MSK} = \iint_{S_O^{TP}} \frac{\rho}{2} (v^2 + w^2) dS = \iint_{S_O^{TP}} \frac{\rho}{2} (\psi \xi - \phi \sigma_z) dS \quad (4.53)$$

### A.3 Mechanical Energy and Exergy Formulations

Energy-based analysis allows aerodynamic forces acting on an aircraft to be obtained by assessing its mechanical equilibrium. The approach represents the aircraft's resultant force through its associated rate of work done through the total flight power defined in equation (4.54). This relates the associated resultant force power with the aircraft's potential and kinetic energies.

$$-\mathbf{F} \cdot \mathbf{V}_\infty = \iint_{S_O} [(\mathbf{V} - \mathbf{V}_\infty) \rho \mathbf{V} \cdot \mathbf{n} + (p_G - \mu_g g) \mathbf{n} - \boldsymbol{\tau}] dS \cdot \mathbf{V}_\infty \quad (4.54)$$

Although expressed in an energy-based form, equation (4.54) is simply a multiplication of the momentum equation with the aircraft flight velocity, i.e., equation (4.5)  $\cdot \mathbf{V}_\infty$ . To present the flight power in a familiar energy-based form per the work of Drela [31] and Arntz [19,22], the following modifications are applied.

Firstly, the identity  $[\rho(\mathbf{V} - \mathbf{V}_\infty)\mathbf{V} \cdot \mathbf{n}] \cdot \mathbf{V}_\infty = \frac{\rho}{2}[(\mathbf{V}^2 - \mathbf{V}_\infty^2) - (\mathbf{V} - \mathbf{V}_\infty)^2]\mathbf{V} \cdot \mathbf{n}$  is used, allowing the flight power to be expressed in the form given in equation (4.55).

$$\begin{aligned} \mathbf{F} \cdot \mathbf{V}_\infty = & - \iint_{S_o} \frac{\rho}{2} [(\mathbf{V}^2 - \mathbf{V}_\infty^2) - (\mathbf{V} - \mathbf{V}_\infty)^2] \mathbf{V} \cdot \mathbf{n} dS \\ & - \iint_{S_o} p_G (\mathbf{V}_\infty \cdot \mathbf{n}) dS \\ & + \iint_{S_o} (\mu_g \mathcal{G}) (\mathbf{V}_\infty \cdot \mathbf{n}) dS \\ & + \iint_{S_o} \boldsymbol{\tau} \cdot \mathbf{V}_\infty dS \end{aligned} \quad (4.55)$$

Considering steady viscous flow and applying Gauss Theorem alongside the substitution  $\frac{\mathbf{V}^2}{2} \rightarrow \frac{\mathbf{V}^2}{2} - \frac{\mathbf{V}_\infty^2}{2}$  to equation (4.21) yields the integral form of the mechanical energy formulation shown in equation (4.56).

$$\begin{aligned} 0 = & - \oint_S \frac{\rho}{2} [\mathbf{V}^2 - \mathbf{V}_\infty^2] \mathbf{V} \cdot \mathbf{n} dS - \oint_S p (\mathbf{V} \cdot \mathbf{n}) dS \\ & + \oint_S (\mu_g \mathcal{G}) (\mathbf{V} \cdot \mathbf{n}) dS - \oint_S [(\boldsymbol{\mu} \boldsymbol{\omega}) \times \mathbf{V}] \cdot \mathbf{n} dS + \underbrace{p_\infty \oint_S \mathbf{V} \cdot \mathbf{n} dS - \iiint_V p_\infty \mathcal{G} dV}_{0, \text{ equal Gauss forms}} \\ & + \iiint_V p \mathcal{G} dV - \iiint_V \mu_g \mathcal{G}^2 dV - \iiint_V \mu \omega^2 dV \end{aligned} \quad (4.56)$$

Combining equations (4.55) + (4.56) applied to the control volume outer boundary surface  $S_o$  (see Figure 4-2) expands the flight power expression into the form presented in equation (4.57).

$$\begin{aligned}
\mathbf{F} \cdot \mathbf{V}_\infty = & \\
& + \iint_{S_o} \frac{\rho}{2} [(\mathbf{V} - \mathbf{V}_\infty)^2] \mathbf{V} \cdot \mathbf{n} dS + \iint_{S_o} p_G (\mathbf{V} - \mathbf{V}_\infty) \cdot \mathbf{n} dS - \iint_{S_o} \boldsymbol{\tau} \cdot (\mathbf{V} - \mathbf{V}_\infty) dS \\
& - \iiint_V p_G \vartheta dV - \left[ \iint_{S_o} (\mu_g \vartheta) (\mathbf{V} - \mathbf{V}_\infty) \cdot \mathbf{n} dS - \iiint_V \mu_g \vartheta^2 dV - \iiint_V \mu \omega^2 dV \right]
\end{aligned} \tag{4.57}$$

From equation (4.57), as per the definitions provided by Drela [31], the first term describes the flow kinetic energy, which can be directionally split into streamwise  $\dot{E}_a$  and transverse  $\dot{E}_v$  components. The second term is the pressure-boundary work representing the flow power in regions where the pressure differs from the ambient. The third term is the viscous boundary work shown by Sanders and Laskaridis [32] to have a negligible contribution toward the aircraft flight power. The fourth term describes the pressure-volume work, which is the flow power provided by the fluid during expansion. Lastly, the remaining terms represent viscous aerodynamic losses through dissipation from fluid element deformation due to dilation  $\mu_g \vartheta^2$  and enstrophy  $\mu \omega^2$  shown in equation (4.58).

$$\mathbf{F} \cdot \mathbf{V}_\infty = \underbrace{\dot{E}_k}_{\dot{E}_a + \dot{E}_v} + \dot{E}_p + \dot{E}_\tau + \Theta + \underbrace{\dot{\mathcal{E}}_{\mu_g \vartheta} + \Phi_{\mu_g \vartheta} + \Phi_{\mu \omega^2}}_{\Phi} \tag{4.58}$$

As per the work of Arntz [19,22], the dilational dissipation is considered negligible, as the most significant contribution to viscous dissipation is through the flow enstrophy via boundary layer and wake-related vorticity.

Applying the dilational dissipation and viscous boundary work simplifications leads to the reduced form of the mechanical energy formulation provided in equation (4.22)(a). On the other hand, obtaining the exergy-based formulation in equation (4.22)(b) requires the introduction of the second law of thermodynamics facilitated through the substitution of the internal energy equation (4.59) into equation (4.58).

$$\begin{aligned}
& -p_G \mathcal{G} + (\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V} = \nabla \cdot (\rho \mathbf{V} e) + p_\infty \mathcal{G} + \nabla \cdot \mathbf{q} \quad (\text{b}) \\
& \text{adding and subtracting } T_\infty \nabla \cdot (\rho \mathbf{V} s) \\
& -p_G \mathcal{G} + (\bar{\boldsymbol{\tau}} \cdot \nabla) \cdot \mathbf{V} - T_\infty \nabla \cdot (\rho \mathbf{V} s) = \nabla \cdot (\rho \mathbf{V} e) - T_\infty \nabla \cdot (\rho \mathbf{V} s) + p_\infty \mathcal{G} + \nabla \cdot \mathbf{q} \quad (\text{c})
\end{aligned} \tag{4.59}$$

The second law of thermodynamics allows for the useful work potential provided through compressibility from the pressure-volume work and viscous dissipation to be identified via the thermal exergy  $\dot{\mathcal{E}}_{th}$  shown in equation (4.60).

$$\begin{aligned}
\Theta + \Phi - \dot{A}_{total} &= \iiint_V \left[ \nabla \cdot (\rho \mathbf{V} e) - T_\infty \nabla \cdot (\rho \mathbf{V} s) + p_\infty \mathcal{G} + \nabla \cdot \mathbf{q} \right] dS \\
&= \oint_S \left[ \rho (\delta e - T_\infty \delta s) + p_\infty \right] \mathbf{V} \cdot d\mathbf{S} + \oint_S \mathbf{q} \cdot \mathbf{n} dS \\
&= \underbrace{\iint_{S_o} \left[ \rho (\delta e - T_\infty \delta s) + p_\infty \right] \mathbf{V} \cdot \mathbf{n} dS}_{\dot{\mathcal{E}}_{th}}
\end{aligned} \tag{4.60}$$

The remaining contributions evaluated through the total anergy  $\dot{A}_{total} = \iiint_V T_\infty \nabla \cdot (\rho \mathbf{V} s) dS$  represent the collective irreversible energy losses through viscous and thermal phenomena.

The enstrophy and dilational losses can be identified from the total anergy and an additional contribution from thermal mixing losses evaluated through the thermal anergy  $\dot{A}_{\nabla T}$  shown in equation (4.61).

$$\mathbf{F} \cdot \mathbf{V}_\infty = \dot{E}_k + \dot{E}_p + \dot{\mathcal{E}}_{th} + \underbrace{\dot{A}_{\mu_g \mathcal{G}} + \dot{A}_{\mu \omega^2} + \dot{E}_{\mu_g \mathcal{G}} + \dot{A}_{\nabla T}}_{\dot{A}_{total}} \tag{4.61}$$

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## 5 Summary, Conclusions and Perspectives

### 5.1 Summary

Force decomposition is an undeniably crucial aspect of aerodynamic performance assessment. Classical methods primarily focus on evaluating these forces from a momentum-based perspective, which carries many benefits; one significant advantage is the Galilean invariance of force. This implies that assessment from either the absolute reference frame, where the body is moving through a quiescent fluid, or from the relative reference frame considering a stationary body with fluid moving past it, returns a consistent aerodynamic force unaffected by the frame transformation. This highlights the significance of momentum-based analysis for wind tunnel testing, naturally conducted in the relative reference frame, as it allows for simple scalability of force measurements for direct comparison with in-flight tests.

Introducing the *artificial velocity* concept to momentum-based analysis allows for zonal profile drag analysis of non-lifting bodies evaluated through wake-based measurements. This can be extended to more complex flow scenarios by combining farfield momentum and thermodynamic methods to introduce an *axial velocity deficit*. This allows for phenomenological drag decomposition identifying physical mechanisms contributing toward aerodynamic drag corresponding to particular spatial regions within the flow. For instance, viscous and wave drag are associated with entropy-generating agents localised to the boundary layer, wake and shockwave regions. In contrast, induced drag is related mainly to mechanical energy losses from large flow vortex structures, usually those shed from wingtips during lift production.

The axial velocity deficit approach assumes the negligible contribution of transverse velocities, which can be adopted for non-lifting scenarios without significant error. However, the extension of this approach to lifting flows (without separation) where the transverse velocities become substantial must be investigated through a complete decomposition of the velocity's axial *and* transverse components. The work conducted in this research identifies that this extension is not limited to exergy-based analysis, as studied in prior literary work, and shares its applicability with far-field momentum and energy-based analysis. Its application improves the distinction between

viscous and induced drag for force decomposition by identifying the reversible isentropic and irreversible non-isentropic flow contributions.

Additionally, it was observed that identifying reversible and irreversible force contributions in this manner was analogous to integral boundary layer analysis. The flow field was considered a superposition of a simple inviscid Euler and dissipative viscous flow. The distinction of force contributions obtained allowed for a consistent definition of viscous, induced, and wave drag contributions from *near-field*, *far-field*, *energy*, and *exergy*-based methods allowing for coherent descriptions of the observed phenomenological contributions.

A significant element of this research comes from introducing partial pressure field analysis into the abovementioned decompositions. This inclusion proposes splitting the pressure field into dissipative and non-dissipative components obtained by solving a system of governing Poisson equations. Directly evaluating these pressure fields allows for the non-dissipative isentropic pressure assumption taken by the velocity decomposition approach to be avoided and replaced by the explicitly solved pressure fields. The resulting unified method provides an improved interpretation of loss mechanisms associated with the pressure field, allowing for evaluating local dissipative pressure drag contributions. This is particularly advantageous for energy and exergy-based analyses as it allows for a better assessment of recoverable energy as contributions associated with the dissipative pressure are now included. These would have otherwise been omitted for velocity decomposition analysis as all pressure field contributions are assumed non-dissipative and thus do not provide any potential for energy recovery.

For near-field analysis, partial pressure decomposition modifies the binary pressure and skin friction drag decomposition representing this instead as viscous drag, which combines the dissipative pressure and skin friction drag, and Euler drag which describes the lift-induced drag for subsonic flows, and the combined wave and lift-induced drag for transonic flows with shockwave formation. Separability of the Euler drag for transonic flows is achieved by applying the entropy drag formulation, which isolates the wave drag contribution.

From this, a conclusive argument can be made that the combined use of pressure and velocity decomposition allows for improved identification of flow-field components associated with reversible and irreversible contributions. This component decomposition allows for distinct zonal drag definitions applicable to all evaluation methods, i.e., near-field momentum, far-field momentum, energy, and exergy-based approaches. It intuitively associates viscous drag with the boundary layer and wakes, lift-induced drag with the bulk flow phenomena accompanying lift production, and wave drag with bulk flow entropy generation.

## 5.2 Research Overview and Knowledge Contributions

Chapter 2 - “Unified Partial Pressure and Velocity Decomposition Approach”: This chapter begins by introducing a sequential subsonic Euler flow derivation procedure to obtain expressions for near- and far-field momentum drag and extends these to energy and exergy-based analysis to provide subsequent drag power expressions. This allows Euler drag contributions to be traced through the various force accounting perspectives. This provides an improved understanding of the origins of Euler drag-producing mechanisms as they manifest in the near-field and evolve downstream.

Non-zero Euler drag contributions primarily exist for two-dimensional subsonic airfoil flows due to lift production. The pressure differential above and below the airfoil surface facilitates the development of flow circulation around the airfoil, inducing a small downward velocity component, i.e., downwash. The flow downwash essentially signifies the downward turning of the surrounding flow in the vicinity of the airfoil, which alters the near-field surface pressure distributions in the direction of increased drag. In the far-field, the transfer of momentum from the body to the fluid via the flow’s downwash is considered an inevitable loss in mechanical energy accompanying lift production. The body supplies the associated “lost” power through the lift-induced drag during flight to produce a specific amount of lift.

Extending this to three-dimensional finite wings, as the flow is not horizontally confined (as is the case with the two-dimensional airfoil), the pressure differential of the upper and lower wing surfaces results in additional downwash being generated via shed vorticity emanating from the

wingtips. This additional downwash intensifies the priorly mentioned effects, i.e., more significant alterations in near-field surface pressure and increased downstream mechanical energy losses, implying an increase in the lift-induced drag.

The Euler flow derivations were used to obtain the dissipative flow-field by taking the difference between the Euler and real viscous fields, as done with velocity decomposition analysis. This approach considers the real viscous flow as a superposition of a dissipative flow, whose drag contributions are linked to entropy-generating mechanisms confined to the boundary layer and wake, and an equivalent inviscid Euler flow whose drag contributions originate outside the viscous region within the bulk flow. For this approach, the Euler flow is reconstructed by assuming the equivalence of the Euler and static pressure fields, i.e.  $p_E = p$ , followed by the use of isentropic flow equations as well as the ideal gas law to obtain the Euler velocity, density, and temperature fields. However, the work conducted in this research shows that coupling the velocity decomposition approach with partial pressure field analysis makes the priorly mentioned static pressure assumption unnecessary, as the Euler pressure field can be explicitly defined through the governing partial pressure Poisson equations.

Following this approach, the resulting dissipative field becomes analogous to a defect flow obtained through integral boundary layer analysis, subject to the assumption of a simple inviscid Euler flow neglecting wall transpiration. The importance of the wall transpiration boundary condition is to mimic the boundary layer displacement effect in the Euler field. Its omission results in viscous-inviscid interaction, where a portion of the viscous drag is incorrectly attributed to the Euler field. To address this, a proposed correction procedure identifies the associated interaction pressure field and correctly attributes its contribution to dissipative pressure. For *near-field* analysis, this provides an improved separability of the partial pressure field drag into its viscous and Euler components.

Extending the correction procedure to far-field momentum, energy, and exergy analysis, provides improved separability between dissipative and non-dissipative drag contributions. This was observed through a dissipative drag analysis tracing the accumulation of viscous drag over

aerodynamic bodies. In the absence of viscous-inviscid correction, a discontinuous dip in the accumulation trend was observed at the trailing edge. This occurred due to the high-pressure gradients in this region, which signify the strong pressure-related interactions between the viscous boundary layer/wake and surrounding bulk flow. However, with the correction applied, the discontinuous behaviour was removed, providing a continuous cumulative viscous drag trend over the body.

Chapter 3 – “Extension of Unified Partial Pressure and Velocity Decomposition on Transonic Flows”: The main aim of this chapter was to extend the unified partial pressure and velocity decomposition approach introduced in *Chapter 2* to transonic two-dimensional flows, particularly those involving lift production. To achieve this, the chapter begins by identifying the various methods used for wave drag quantification, which were used to compare the level of agreement between the multiple solutions. Particular focus centred on energy-based wave drag assessment, where the proposed formulation in literature was observed to be sensitive to the analysis control volume size and did not provide consistent results when this was varied. An alternative approach was proposed by calculating the shockwave dissipation through the entropy equation. The resulting formulation was later shown to reduce to the total anergy formulation proposed in exergy-based analysis. This provided a coherent wave drag assessment which did not directly rely on the control volume size.

Like the subsonic cases in *Chapter 2*, the flow field was decomposed into its Euler and viscous contributions. In this instance, a distinct difference is that for near-field transonic flow analysis, the Euler pressure drag now contains contributions from both the lift-induced and wave drag. For further separability of the Euler pressure drag, two approaches were suggested; the first was a purely near-field assessment involving the decomposition of the Euler pressure and velocity into constituent compressible and incompressible parts, with the wave drag being represented by the compressible pressure drag.

An additional advantage of the Euler pressure decomposition into compressible and incompressible portions is its implications on lift production. Introducing this separability proposes that the lift force can also be decomposed into contributions of compressible and

incompressible origin. This ideology is not foreign to aerodynamic performance assessment, as this principle is naturally introduced in vortex force analysis. This expansion of the Euler field opens the possibility of unification between near-field Euler pressure drag and vortex force compressibility assessment.

The second wave drag evaluation approach relies on the classical entropy drag formulation, which evaluates the entropy variation over the shockwave's region of influence. This method is commonly used in thermodynamic and exergy-based approaches. Therefore, its adoption for near-field decomposition of the Euler pressure drag provides a consistent wave drag value for the various methods.

Chapter 4 – “A Synergistic Viewpoint on Aerodynamic Force Assessment”: This chapter also extends the unified partial pressure and velocity decomposition approach, focusing on three-dimensional finite wing flows in subsonic conditions. The first section begins with an exploratory derivation study to assess whether there is any utility in deriving the various aerodynamic force formulations starting from an expanded form of the Navier-Stokes momentum equations, a common starting point for vortex force derivations. This form expressed the stress tensor as a decomposition consisting of pure shear and pure compression/expansion terms related to linear proportionality constants  $\mu$  and  $\mu_g$  representing the shear and expansion viscosities, respectively.

Vortex force decompositions show that the associated terms related to these viscosities are higher order terms and thus become negligible for high Reynolds number flows and are usually omitted. However, particularly for energy and exergy analysis, this decomposition of the stress tensor naturally introduces the concept of flow *enstrophy and expansion dissipation*. The enstrophy describes the local dissipation rate into heat by viscous stresses, primarily through vorticity. Expansion dissipation is associated with local viscous losses through the volumetric deformation of fluid elements. The subsonic analyses showed that flow enstrophy was the most significant contributor to the overall dissipation, with the expansion dissipation showing negligible contributions throughout the assessed Mach number range.

The exploratory study was followed by component decompositions through partial pressure and velocity decomposition, as in *Chapters 2 and 3*, to identify the irreversible and reversible force contributions. In the finite wing drag assessment context, the irreversible drag is linked to viscous drag contributions whose local origin occurs through near-field boundary layer development. This would involve all dissipative contributions, including propagated downstream phenomena such as the wingtip vortex core and viscous wake. Conversely, the reversible drag is linked to the reversible mechanical energy loss accompanying lift production. This interpretation allows the lift-induced drag to share the same meaning as for Euler flows, providing a consistent definition of induced drag applicable to both Euler and viscous Reynolds Averaged Navier-Stokes flows.

Additional interest was taken into the definition of induced drag for compressible flows as standard reports primarily focus on its evaluation under incompressible conditions, thus leading to formulations which only provide approximate estimates of induced drag under compressible conditions. This was firstly addressed from an exergy-based perspective, where the induced drag was defined as the reversible contribution of the physical exergy. The unique difference of the proposed formulation is its combination of mechanical and thermal exergy contributions to account for thermo-compressibility. From a momentum-based perspective, Maskell's induced drag formulation was similarly modified through a rederivation considering compressibility. Under incompressible conditions, these formulations reduce to the commonly cited definitions in literature, thus providing consistent descriptions of induced drag applicable to compressible and incompressible flows.

Notably, reversible and irreversible component-based decomposition becomes significantly impactful, providing a unique insight into the phenomenological contributions toward the total drag. In particular, it highlights the main difference between the *Oswald* and *span* efficiency factors that relate the induced drag with lift through Prandtl's classical induced drag formulation. This difference naturally affects the interpretation of the component contributions toward the total drag. For instance, the reservable contributions describe the induced drag related to the lift through the span efficiency factor. This factor acts as a correction for departures of the wing's lift

distribution from elliptical, and any incurred irreversible losses are collectively considered through the viscous drag.

“Induced drag”, obtained through classical methods that do not employ the reversible/irreversible component decomposition, should more aptly be referred to as the *drag due to lift* related directly to the lift through Oswald’s efficiency factor. In this instance, the Oswald efficiency acts as a correction factor for departures of the wing’s lift distribution from elliptical and any incurred viscous losses due to lift production. This results in both the profile and drag due to lift sharing contributions of irreversible nature.

Lastly, although the central premise of the chapter focused on drag decomposition, the proposed synergistic approach offered the additional possibility of lift decomposition through vortex force analysis into incompressible and compressible contributions. This principle was priorly introduced in *Chapter 3* via partial pressure analysis. Its natural re-emergence within vortex force derivations presents a clear pathway for future work progression to investigate the relative coherence between the partial pressure field and vortex force lift decompositions. Furthermore, as these components can directly be related to the lift-induced drag through Prandtl’s induced drag formulation, an analysis of the component sensitivity, particularly for compressible flows, would provide further insight into the development and evolution of lift-induced drag.

### **5.3 Conclusions**

The research conducted in this thesis presents a synergistic viewpoint toward aerodynamic performance assessment. Particular interest was taken into the interconnected nature of the various force decomposition perspectives, utilising near-field, far-field, energy, and exergy-based formulations. The central premise of this work focused on highlighting that all aerodynamic force assessment methods are derived from the same foundational basis, i.e., the Navier-Stokes Momentum equations. As a result, this presented the opportunity to directly relate the near-field mechanical and far-field phenomenological drag decomposition components through their equivalence using Newton’s third law. To explore this, the research delved into various drag decomposition definitions, realising that categorising force components into Euler inviscid and

viscous phenomena would provide a consistent description of the observed physical mechanisms contributing toward specific phenomenological drag contributions. An initial drawback in this regard was obtaining a means of delineating between inviscid and viscous force contributions. This led to investigating flow-field decomposition approaches applicable to RANS CFD solutions. This introduced the concepts of partial pressure field and velocity decomposition, which had only broadly been applied independently within their fields of focus before this work. Partial pressure field analysis would focus more on a near-field understanding of force contribution origins, while velocity decomposition focused primarily on far-field exergy-based applications. As a result, through their unification, the first contributions to knowledge were established, addressing the main aim described at the outset of this work, namely:

- ✓ Develop a Multiphysics based approach for aerodynamic force decomposition from RANS CFD solutions allowing for a coherent link of phenomenological contributions obtained through momentum, energy, and exergy-based methods

Flow assessments showed partial pressure field analysis could obtain local near-field viscous pressure drag contribution. This is useful as it allows for the ambiguities highlighted in the Introduction in *Chapter 1* to be addressed, namely those associated with the inability to obtain local pressure drag contributions due to inviscid flow interaction. The cumulative local pressure drag was obtained by identifying the viscous portion of the pressure field through partial pressure field analysis.

Without the unified approach, partial pressure analysis could not filter out the viscous-inviscid interaction force contributions from the Euler pressure drag. As a result, this would lead to slight Euler pressure drag inaccuracies, particularly in conditions involving lift and thermo-compressibility effects. The work covered in this thesis addresses this and proposes a viscous-inviscid interaction correction procedure which stems from the force component equivalence through Newton's third law. Through this balance, the sensitivities associated with the interaction pressure field could be understood and quantified within the context of RANS CFD solutions.

The drag component decomposition obtained through the unified approach allowed for a near-field identification of the origins of drag contributions from inviscid (non-dissipative) and viscous (dissipative) phenomena, achieving the first step in irreversible/reversible force contribution identification starting at the near-field.

This was then extended to far-field momentum, energy and exergy-based methods, where it was observed that the component decomposition into inviscid and viscous contributions was analogous to integral boundary layer analysis. As a result, the extension of this unified approach to energy and exergy-based formulations was able to provide the following:

- Improved wake-reduced energy and exergy-based formulation

The improvements toward wake-reduced formulations were obtained through a series of investigations, which independently enhanced the physical understanding of drag-contributing mechanisms from an energy/exergy-based perspective. These mechanisms could now be related to near-field phenomenological origins, which enabled traceability from their near-field source to their associated downstream evolution. The investigations in question included:

- Investigation into the suitability of the static pressure assumption taken for velocity decomposition implementation

Partial pressure field analysis unification allowed for the static pressure assumption priorly taken by velocity decomposition to be omitted entirely. Additionally, this allowed for the assumption's applicability to be further understood.

In the advent of high Reynolds number flows, the assumption would not lead to significant error as the dissipative pressure contribution toward the total static pressure diminishes with increasing Reynolds number, vanishing in the limit of an infinite Reynolds number.

- Investigation into a means of accounting for “missing drag” for far-field assessment

The “missing drag”, leading to the local underprediction of drag using exergy-based methods, was addressed through the viscous-inviscid interaction correction procedure. The implications of this allowed for tracking the cumulative development of drag over the aerodynamic body. Notably, this also allowed for improved tracking of recoverable energy over the body surface, which was not previously possible due to the significant inviscid interactions observed in the near-field.

- *Investigation into wave and induced drag evaluation techniques*

The utility of the unified approach was further realised through its application to transonic and lifting flows. For instance, transonic flow analysis allowed for an improved wave drag assessment approach for energy analysis through an entropy-based shock dissipation formulation directly linked to classical formulations presented in literature.

For lifting flows, the reversible and irreversible drag decomposition allowed for a clear differentiation between the Oswald and spanwise efficiency factors. This allowed for the inherent differences between lift-induced drag and drag due to lift, usually referred to interchangeably, to be appreciated from the perspective of purely reversible mechanisms and classical phenomenological decomposition.

## **5.4 Future work Perspectives**

The research covered in this work forms part of the natural progression in the maturity of exergy-based aerodynamic performance assessment geared toward obtaining a fully energy-based aircraft design and analysis framework. This work shows the various strengths of the myriad of force and energy/exergy-based aerodynamic assessment methods, highlighting that through a synergistic application of these methods, an enriched view of the phenomenological mechanisms contributing toward aerodynamic forces such as lift and drag can be obtained. This approach leans on the benefits of principles from classical fluid flow solving techniques such as viscous-inviscid interaction, where solutions from the inviscid Euler flow external to the viscous boundary layer/wake and the viscous flow within the boundary layer/wake are combined through an iterative solving procedure to form a composite solution over the entire flow field. This flow separability

into Euler and viscous flow regions forms the central premise toward consistent definitions of aerodynamic force contributions from physical mechanisms originating from various spatial areas of the flow. The direct relation of this classical approach with integral boundary layer analysis allows for simplified general flow separability of a real viscous CFD solution into its equivalent inviscid and viscous flow contributions through the velocity decomposition approach. The associated simplifying assumptions of this approach can be improved through partial pressure field analysis, allowing for the concept of viscous and Euler force decomposition to be extended to near-field analysis, which is not possible with the velocity decomposition approach alone. This extension is crucial in consistently tracing force contributions from viscous and Euler-related drag phenomena from both momentum and energy/exergy perspectives.

An immediate area of impact of this methodology is its implementation for performance analysis of highly integrated aerodynamic configurations such as those employing boundary layer ingestion (BLI). Current work efforts in progress to this end have been presented at the 57th Edition of the 3AF International Conference on Applied Aerodynamics under the paper titles:

- N. Mutangara, D. Sanders, P. Laskaridis, “Examining the Benefits of Boundary Layer Ingestion on Powered Bodies,” presented at the 3AF Aero Conference 2023, Paris, France, Mar. 2023.
- P. Hart, S. Schmitz, N. Mutangara, D. Sanders and P. Laskaridis, “Thrust/Drag Decomposition using Partial-Pressure Fields,” presented at the 3AF Aero Conference 2023, Paris, France, Mar. 2023.

This work’s benefits are described in the following section to highlight the immediate utility of this synergistic assessment approach for highly integrated configurations involving BLI.

#### **5.4.1 Examining the Benefits of Boundary Layer Ingestion on Powered Bodies**

The studies in this section assess the effects of propulsor integration on a highly integrated boundary layer ingesting configuration. This begins with a *qualitative* flow analysis to observe the changes within the flow field, with the primary objective of highlighting the independent response

of the viscous and Euler fields to propulsor integration through their respective partial pressure fields.

The configurations assessed, shown in Figure 5-1, consisted of a NACA 0012 airfoil operating at a low subsonic compressible Mach number of 0.3 and a chord-based Reynolds number of  $3 \times 10^6$  with a chord length of 0.435m. Three angles of attack were assessed, namely 0, 4 and 8 degrees, with the propulsor being sized based on the trailing edge boundary layer thickness of the unpowered airfoil at 0 degrees angle of attack. A wake-attenuating propulsor is adopted to ensure maximal energy recovery providing a thrust force equal to the total configuration drag, similarly done in the work of Lamprakis et al. [1].

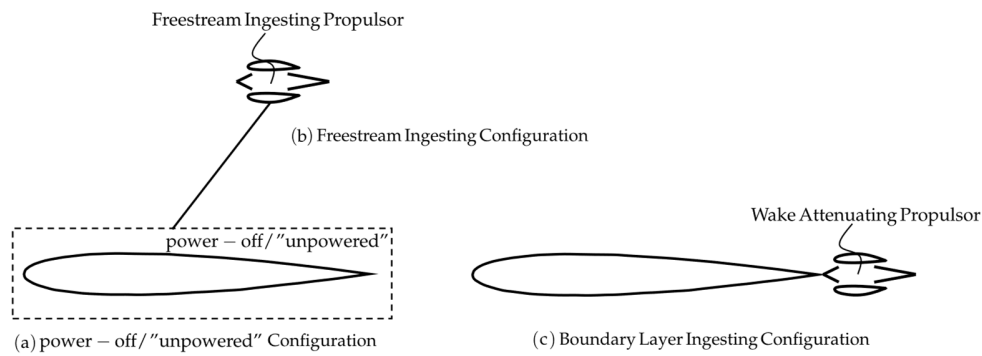


**Figure 5-1 Configuration classification into (a) power-off/unpowered, (b) freestream ingesting, and (c) boundary layer ingesting configurations**

The configurations displayed in Figure 5-1 each play an essential role in the qualitative and quantitative performance assessment studies. Firstly, the “unpowered” design quantifies the amount of recoverable energy imparted to the flow by the body via the potential for energy recovery factor ( $PER_\mu$ ), discussed in *Chapter 2*. This factor indicates the upper limit of the expected propulsor power reduction through wake energy utilisation for thrust production. The freestream ingesting configuration provides a point of comparison with the boundary layer ingesting design to evaluate a power saving coefficient (PSC) through the formulation provided in equation (5.1) following the work of Smith [2].

$$PSC = \frac{P_{FIP} - P_{BLIP}}{P_{FIP}} \quad \text{where:} \quad (5.1)$$

FIP → freestream ingesting propulsor configuration

BLIP → boundary layer ingesting propulsor configuration

Additionally assessed were the changes in configuration forces, i.e., lift and drag, split into viscous and Euler contributions using the approaches discussed in *Chapters 2– 4*. This is done to obtain a *quantitative* view of the effects of integration on the propulsor power requirements and airframe aerodynamic forces.

#### 5.4.1.1 Qualitative Flow Assessment of BLI Effect on Powered Airfoil

Figure 5-2 shows the pressure coefficient contour behaviour for power-on and off cases at angles of attack 0, 4, and 8 degrees. As the angle of attack increases, the suction pressure region on the airfoil's upper forebody section becomes more pronounced, signifying an increased flow velocity and favourable pressure gradient in this region. At first glance, the only difference between the power-on and off cases appears to be the slight increase in pressure at the airfoil trailing edge due to the boundary layer ingesting propulsor. These effects cause flow perturbations that propagate upstream and downstream of the propulsor location modifying the flow field and resulting in changes in the configuration's lift and drag.

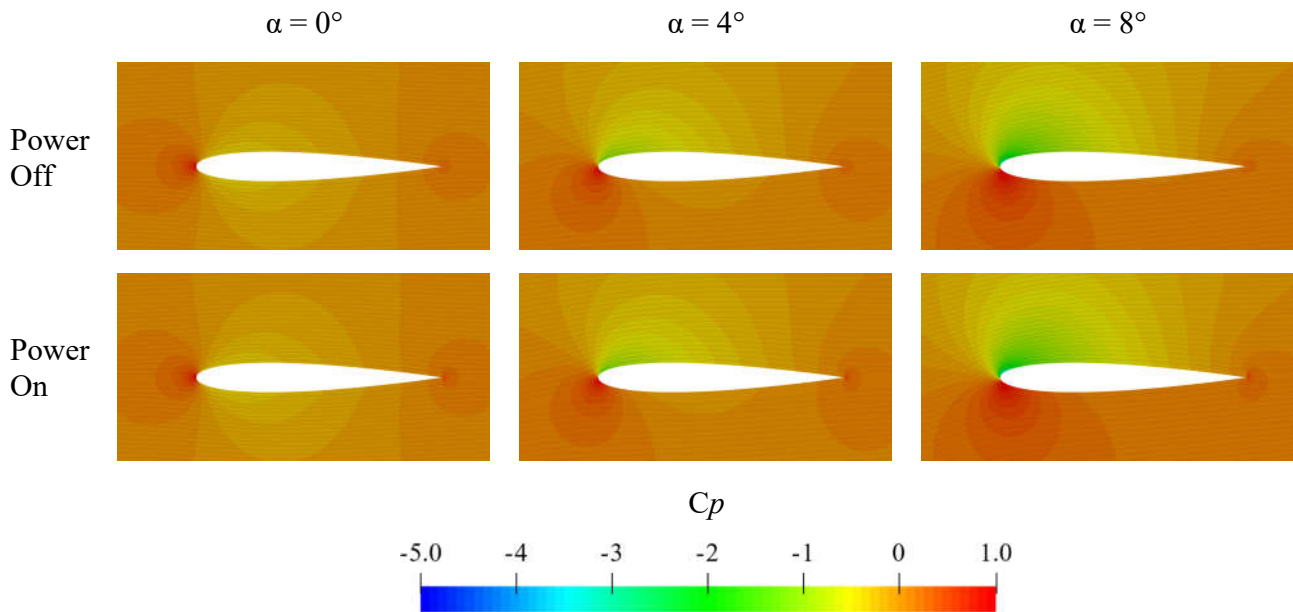


**Figure 5-2 Pressure coefficient comparison of boundary layer ingesting airfoil in power-on and off conditions at multiple angles of attack**

Generally speaking, as the angle of attack of an airfoil increases, the pressure differential between the airfoil's lower and upper surface allows the fluid to exert a lift force directly on the airfoil's physical surfaces [3–5], with the lower surface experiencing a higher pressure than the upper surface.

The plots in Figure 5-3 show the change in pressure components between the power-on and off conditions. Specifically, a visibly higher Euler pressure differential can be seen, with the lower surface experiencing a higher pressure and the upper surface experiencing a lower pressure for the power-on condition compared to power-off. Collectively this highlights an increase in the configuration's lift for power-on conditions.

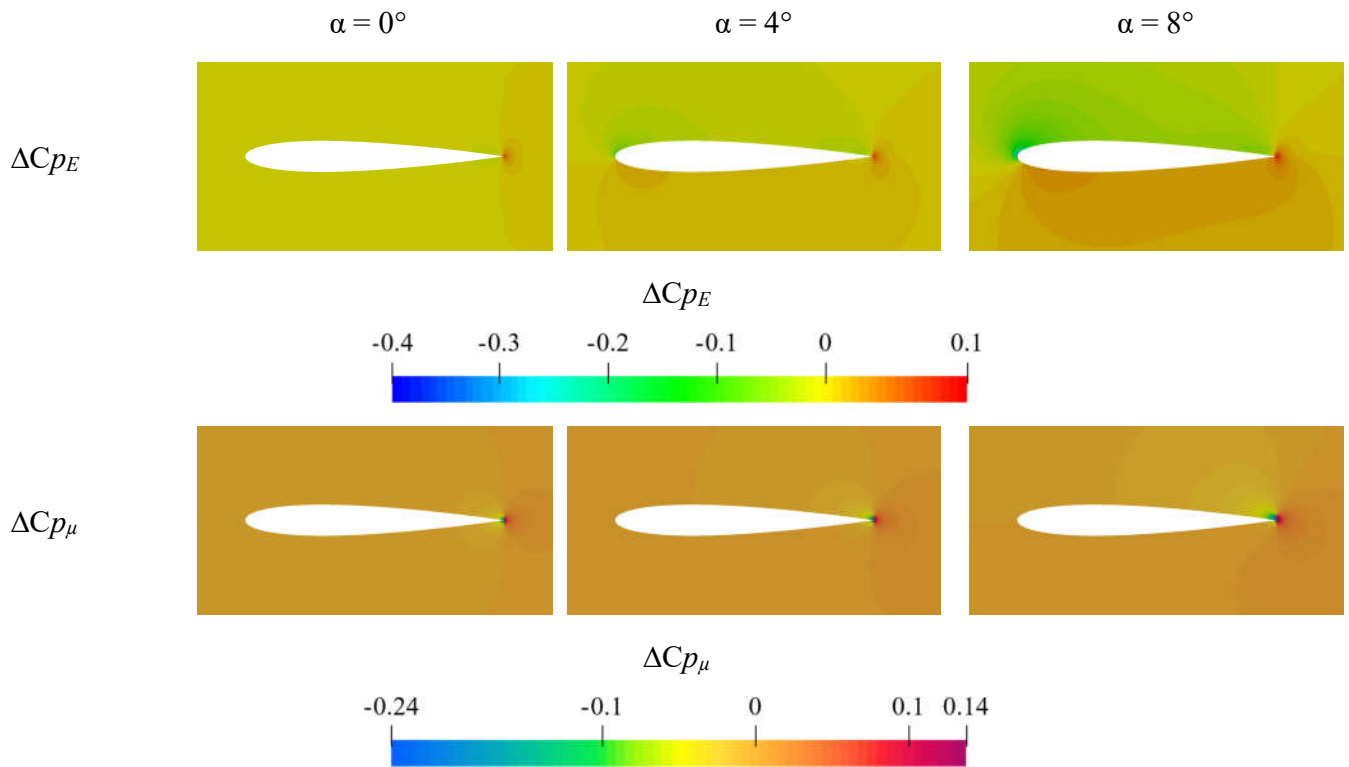


**Figure 5-3 Euler and dissipative static pressure contour plot difference for power-on and off conditions at multiple angles of attack**

Figure 5-4 shows the velocity field change between power-on and off conditions. For 0 degrees AoA, it can be observed that there is a slight increase in velocity near the trailing edge of the upper and lower airfoil surfaces due to the action of the propulsor., which leads to a localised increase in viscous drag, also shown in the dissipative pressure contours in Figure 5-3. The additional drag fraction experienced near the airfoil trailing edge occurs due to the rapid flow acceleration experienced through the propulsor's suction effect.

Furthermore, as the angle of attack increases in Figure 5-4, the amount of momentum imparted by the propulsor also rises, highlighted by the increased intensity in the velocity magnitude. This occurs due to the larger wake size experienced at higher angles of attack, which raises the power necessary to attenuate the wake through the propulsor. However, the increased wake size is not the only reason for the rise in propulsor power. As the angle of attack increases, the lift-induced drag

contribution rises proportionally with the configuration's lift; thus, more energy is required from the propulsor to compensate for this additional drag source.

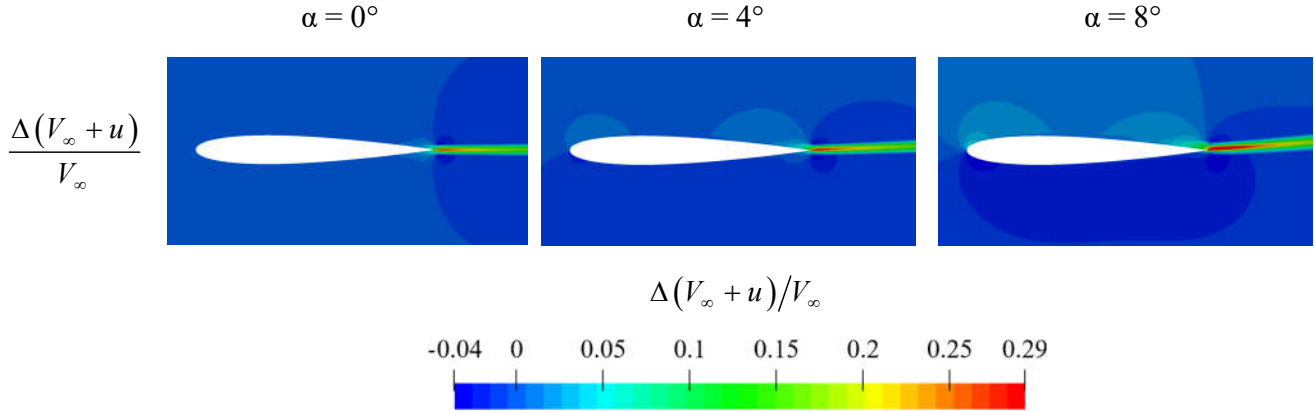


**Figure 5-4 Normalised streamwise velocity contour plot difference for power-on and off conditions at multiple angles of attack**

#### 5.4.1.2 Quantitative flow assessment of powered effects on the dissipative and non-dissipative flow field components

This section quantitatively assesses the change in performance characteristics between the power on and off test cases conducted. Firstly, for the *power-off* condition in Figure 5-5, it can be observed that there is an increase in lift-induced drag  $D_E V_\infty$  with the angle of attack.

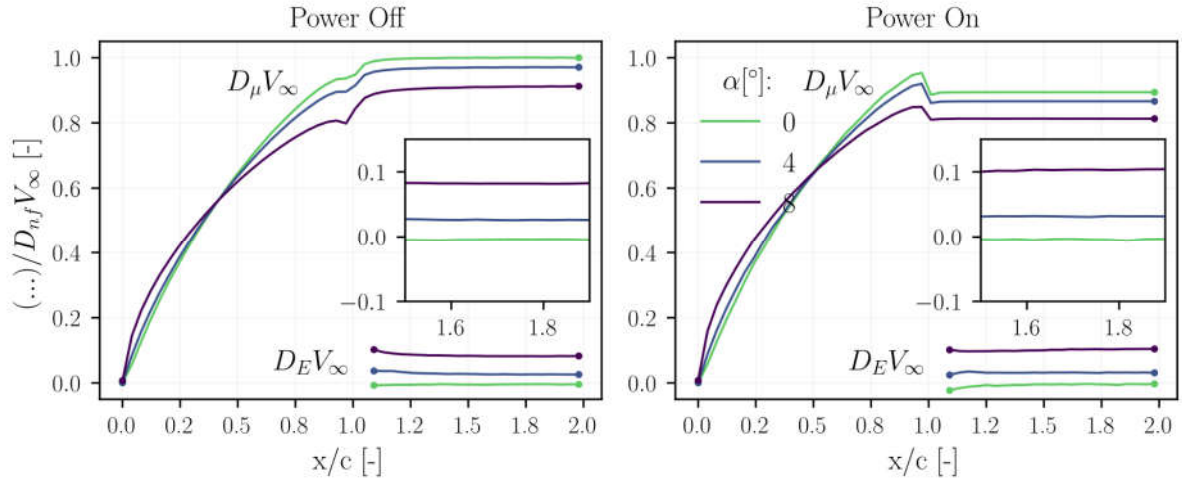


**Figure 5-5 Comparison of power-on and off behaviour of dissipative and Euler drag contributions for angles of attack 0, 4 and 8 degrees**

This is expected as there is an increase in momentum diffusivity to the flow due to pressure via the strong pressure gradients developed around the lifting body. This leads to a rise in pressure-related contributions toward the overall drag at the expense of viscous contributions. This is confirmed in Figure 5-6 via the dissipation accumulation rate, which is shown to increase at the airfoil forebody region near the leading edge due to the rapid increase in flow velocity through the favourable pressure gradient observed in this region. This behaviour is shared with the dissipative drag trends in Figure 5-5, which confirm that the overall contribution of viscous drag toward the total drag tends to decrease with the angle of attack due to incurred lift-induced losses associated with the Euler field.

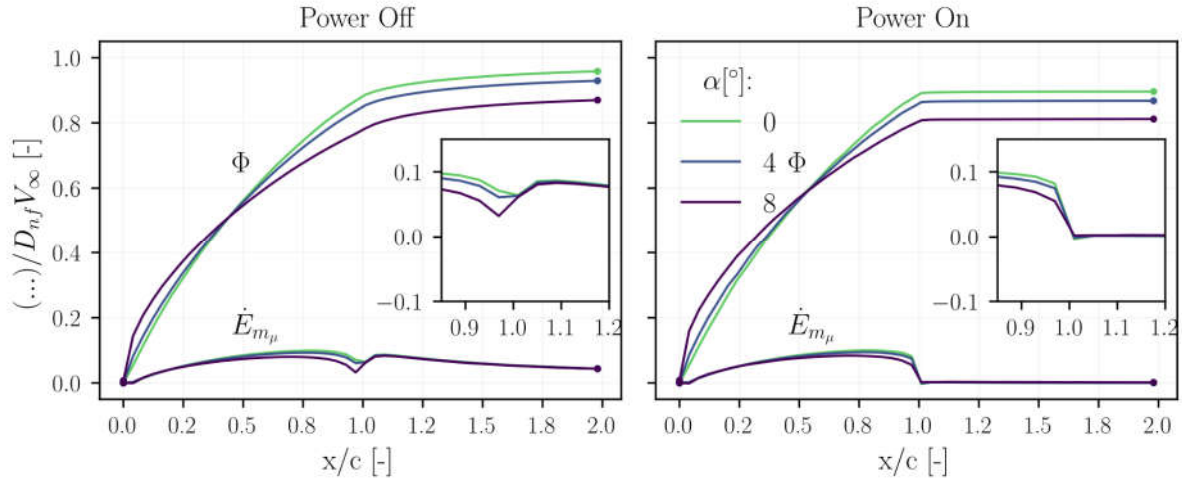


**Figure 5-6 Comparison of power-on and off behaviour of viscous dissipation and dissipative mechanical energy for angles of attack 0, 4 and 8 degrees**

The power-on analyses in Figure 5-5 highlight the effects of the boundary layer ingesting propulsor on the dissipative and non-dissipative flow fields. The BLI propulsor's wake-attenuating characteristics can be observed by reducing the downstream mechanical energy deposition. This is more clearly illustrated through the dissipative mechanical energy assessment in Figure 5-6, where the energy priorly imparted to the flow in the power-off case is effectively utilised to thrust, reducing the downstream dissipative mechanical energy losses.

The reduction in wake energy loss has the knock-on effect of decreasing the accumulation of viscous dissipation in the wake. Figure 5-6 shows that the resulting viscous dissipation accumulation levels off at the trailing edge, indicating no further build-up after this point. Through this, it becomes possible to rank BLI performance by assessing the overall downstream mechanical energy losses with well-performing configurations returning considerably lower magnitudes of mechanical energy and viscous dissipation within the wake.

Figure 5-5 highlights the increase in lift-induced drag at higher angles of attack, an indirect by-product of the increase in lift observed due to propulsor integration. When compared against the power-off configurations, drag rises from 1 to 5%, whilst lift increases from 5 to 8%. The range

of increase in lift relative to the power-off case is much greater than the indirect increase in drag due to propulsor integration. As a result, BLI analysis of this nature becomes a trade-off study between the reduction in wake energy losses and lift augmented compared to the incurred induced and viscous penalties.

Additionally, Figure 5-5 highlights that propulsor effects propagate upstream, affecting the accumulation of viscous drag over the body surface. Coupling this with the qualitative partial pressure field analysis in Figure 5-4 allows these effects' origin to be further understood. For instance, the dissipative pressure field in Figure 5-4 shows how the trailing edge suction drag intensifies with the angle of attack, which from an energy-based perspective, is illustrated through the increase in dissipative drag in Figure 5-5.

The Euler pressure, on the other hand, aids in highlighting the modification of the lift-related effects. For power-on conditions, there is a visible decrease in the pressure over the airfoil's upper surface and an increase on the lower surface, collectively resulting in an increased lift. Through the circular relationship between pressure and velocity, the decrease in pressure over the upper surface intensifies the pressure gradients, resulting in faster-moving flow in this region. As the flow is accelerated, this leads to an increase in the accumulation of viscous losses, observed as an increase in the *local* rate of viscous dissipation accumulation over the airfoil forebody surface, as shown in Figure 5-6.

#### **5.4.1.3 Benefit assessment of propulsor integration effects on configuration performance**

The analyses shown in Figure 5-7 compare the different configurations through their respective aerodynamic efficiencies. The formulation proposed in equation (5.2) modifies the classical  $C_L/C_D$  through energy-based metrics. This is done to present the classical formulation in the context of energy-based analysis while also allowing for the configuration performance to be assessed at different net streamwise forces through the potential energy rate term  $W\Gamma$ .

$$P_S - \underbrace{(\mathbf{F}_B + \mathbf{F}_P) \cdot \mathbf{V}_\infty}_{W_\Gamma} = \dot{E}_m + \Phi + \Theta$$

$$\eta_{aero} = \frac{C_L}{C_P} = \frac{LV_\infty}{P_S - \underbrace{(\mathbf{F}_B + \mathbf{F}_P) \cdot \mathbf{V}_\infty}_{W_\Gamma}} = \frac{LV_\infty}{\dot{E}_m + \Phi + \Theta} \quad (5.2)$$

It is possible to observe that the modified formulation in equation (5.2) reduces to the classical  $C_L/C_D$  formulation under various conditions.

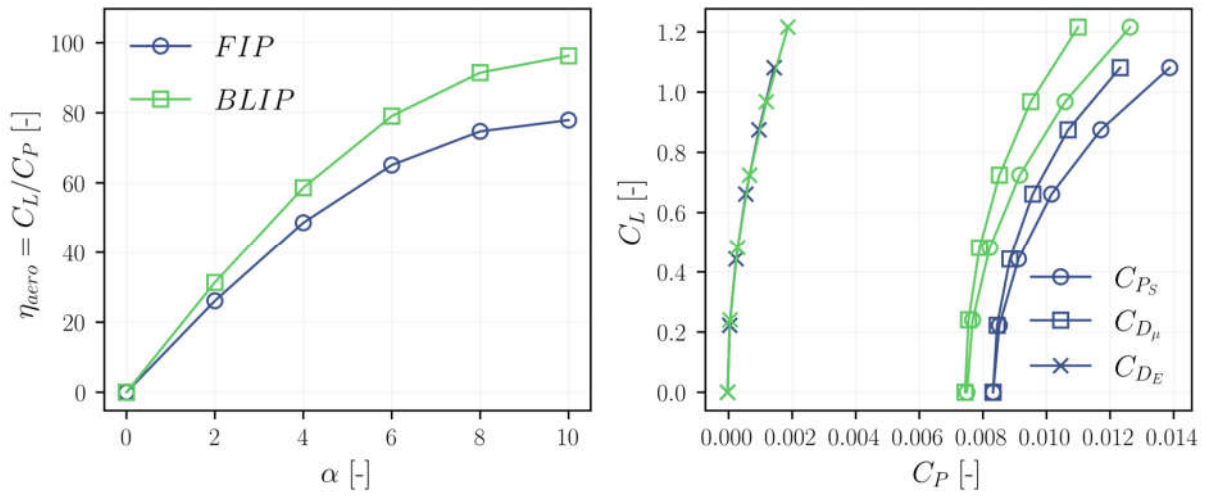


**Figure 5-7 Aerodynamic efficiency and drag polar comparison of freestream and boundary layer ingesting configurations**

The most popular condition is the so-called “unpowered” configuration which is the simplest to explain. This design has no active propulsion devices, as illustrated in Figure 5-1. In this instance, the shaft power  $P_s$  and propulsive total flight power  $\mathbf{F}_P \cdot \mathbf{V}_\infty$  are zero, i.e.,  $P_s = \mathbf{F}_P \cdot \mathbf{V}_\infty = 0$ . This leaves a non-zero contribution only from the configuration drag power  $\mathbf{F}_B \cdot \mathbf{V}_\infty$ , equal to the mechanical energy imparted to the flow and viscous losses through dissipation and pressure-volume work shown on the RHS of equation (5.3). As a result, the ratio of the lift and drag power ( $LV_\infty/\mathbf{F}_B \cdot \mathbf{V}_\infty$ ) would give the classical aerodynamic efficiency formulation.

$$P_S - W\Gamma = \underbrace{\dot{E}_{m_\mu} + \Phi + \Theta_\mu}_{D_\mu V_\infty} + \underbrace{\dot{E}_{m_E} + \Theta_E}_{D_E V_\infty} \quad (5.3)$$

The second scenario is a configuration in cruise flight with an aerodynamically decoupled ideal propulsor, i.e., propulsive efficiency of 100%. For this case, the net streamwise force would be balanced, i.e., the shaft power would be equivalent to the total flight power, i.e.  $P_s = \mathbf{F}_p \mathbf{V}_\infty \neq 0$ . This differs from the first instance as the two quantities are now non-zero but balanced. As a result, the aerodynamic efficiency formulation would also reduce to the standard  $C_L/C_D$ , similar to the “unpowered” scenario.

Analysis of the test cases in Figure 5-7 shows that the BLI configuration provides higher aerodynamic efficiency, implying that these configurations require less input power to achieve a particular lift target flight condition. Additionally, the lift-induced drag polar  $C_{D_E}$  shows an increase in drag with the angle of attack for both power-on and off states. This is expected due to the increased mechanical energy deposition due to downwash as more lift is produced. Furthermore, it can be observed that for the same amount of input propulsor power, the lift of the BLI configuration is higher, incurring a minute lift-induced drag penalty.

The viscous drag polar  $C_{D_\mu}$  agrees with prior observations of the decrease in dissipative mechanical energy and viscous dissipation accumulation, which collectively lead to lower viscous drag. This tends to offset the viscous drag polar of the BLI configuration, showing lower viscous drag values compared to the freestream ingesting case.

Through these analyses, it can be observed that for power-on conditions, ingestion of the boundary layer leads to a decrease in the dissipative drag, which would have otherwise contributed toward the overall viscous drag for a freestream ingesting configuration scenario. This decrease in the dissipative drag power is thus a benefit as less propulsor power is required to counteract this force contribution.

On the other hand, the non-dissipative drag is seen to increase for all power-on conditions caused by the increase in lift occurring due to the modified pressure field around the configuration. The increase in lift leads to a rise in the lift-induced drag, considered a detriment as the propulsor would have to provide additional power to counteract this added force contribution.

Therefore, to assess the propulsor integration's overall benefit, a trade-off study must be conducted where the collective influence of the dissipative and non-dissipative flow field characteristics are considered. For lifting flows, ranking the configuration performance would require an additional comparison metric. The introduced metric, shown in equation (5.4), evaluates the change in aerodynamic efficiency between the BLI and ideal freestream ingesting designs to assess the effects of integration on the configuration's lift.

$$\Xi = \frac{\eta'_{aero} - \eta_{aero}}{\eta'_{aero}} \quad \text{where:} \quad (5.4)$$

$\eta_{aero} \rightarrow$  freestream ingesting configuration aerodynamic efficiency  
 $\eta'_{aero} \rightarrow$  boundary layer ingesting configuration aerodynamic efficiency

Including lift in the analysis of boundary layer ingesting configurations becomes particularly interesting as the propulsor modifies the velocity and pressure fields around the design. As the lift is strongly tied to the mutual interaction between the pressure and velocity fields, the power-on scenarios provide significant lift augmentation benefits, thus improving the configuration's aerodynamic efficiency, as shown in Figure 5-7. This is further supported in Figure 5-8, which indicates that the BLI configuration's aerodynamic efficiency can improve by as much as  $\Xi \approx 20\%$  compared to the freestream ingesting configuration.

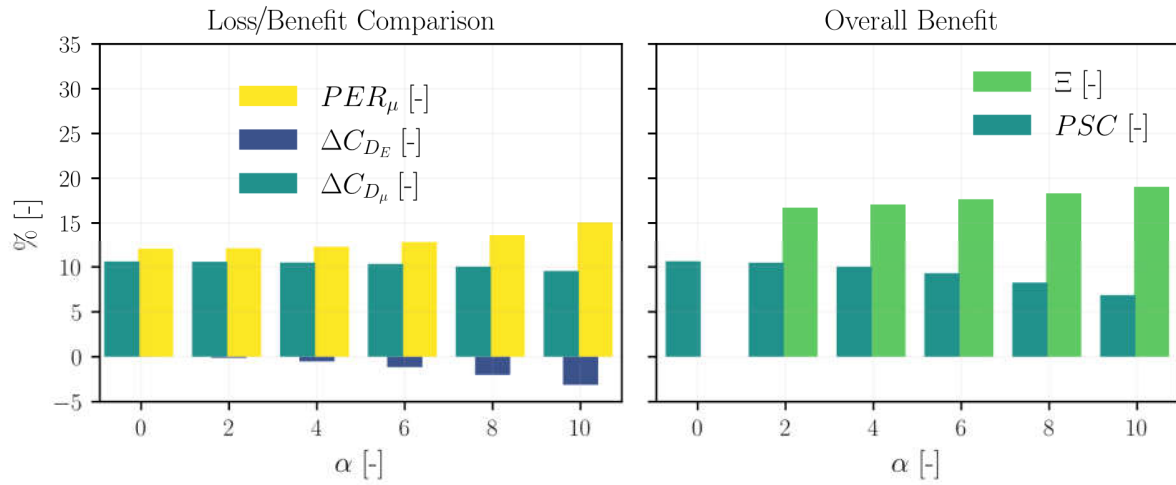


**Figure 5-8 Comparative loss/benefit assessment tallying potential for energy recovery, power saving coefficient, aerodynamic efficiency, as well as Euler and dissipative drag**

Additionally shown in Figure 5-8 is the BLI configuration power consumption reduction through wake energy utilisation observed through the power saving coefficient (PSC), which compares the required propulsor power between the BLI and freestream ingesting configurations to attain a similar flight condition. As the angle of attack increases, the amount of power saved decreases, owing to the increased production of lift-induced drag, although it must be noted that even at these higher angles of attack, a power benefit is still observed ( $PSC \approx 6\%$ ).

It is evident from this analysis that the introduction of synergistic aerodynamic assessment through partial pressure field and flow decomposition allows for the various phenomenological characteristics of the flow to be further examined, understood and assessed within the context of powered BLI configurations. This is to say, approaching aerodynamic performance assessment in this manner allows for broad application for more informative aerodynamic analysis of the flow physics spanning through various operating conditions for unpowered and powered aerodynamic flows.

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