

Reduced-order model prediction of far-field mixing noise from internally-notched nozzles

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This work presents a numerical investigation of the effect of internal notches on the reduction of jet mixing noise from round nozzles. The baseline jet is produced by the University of Southampton's Doak Laboratory 40mm-diameter convergent, round nozzle. Numerical predictions of mixing noise for both round and internally-notched nozzles are conducted using a Generalized Acoustic Analogy that relies on Reynolds-Averaged Navier-Stokes (RANS) solutions of the nozzle flows, particularly the one proposed by Leib and Bridges. In this method, the RANS variables of interest, including mean axial velocity, Mach number, density, turbulence kinetic energy, and its dissipation rate, are interpolated onto a cylindrical structured grid suitable for aeroacoustic calculations. Subsequently, the respective Green's function and a hybrid spectral-time source model are computed, and power spectral densities at various polar and azimuthal angles are predicted. Comparison between predictions and experiments demonstrates good qualitative agreement for both nozzles, although the inversion in trends at certain Strouhal numbers is not captured by the numerical model. Additionally, the significance of the numerical scheme's order employed to solve the adjoint Green's function is evaluated. To elucidate the noise reduction attributed to internal notches, distributions of turbulent kinetic energy are analyzed at different azimuthal cross-sections.

I. Nomenclature

c	=	local speed of sound, m.s ⁻¹
CV	=	control volume
c_∞	=	ambient speed of sound, m.s ⁻¹
D	=	nozzle exit (averaged) diameter, mm
E	=	east CV center
f	=	frequency, s ⁻¹
$\hat{g}_0$	=	adjoint Green's function, m ^{-1.5}
I	=	acoustic spectrum, Pa.s.m ⁻³
$\vec{i}_r$	=	unit vector in the radial direction
M	=	acoustic Mach number
N	=	north CV center
NN	=	northernmost CV center in a 9-point stencil
P	=	central CV center
(r, ϕ, z)	=	cylindrical polar coordinate system, m ³
S	=	south CV center
$\vec{S}$	=	area vector perpendicular to CV face, m ²
St	=	Strouhal Number

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W	=	west CV center
U	=	jet local axial velocity, m s^{-1}
$\mathbf{x}$	=	observer location, (m,m,m)
$\mathbf{y}$	=	source location, (m,m,m)
Φ^*	=	source spectral tensor, $\text{m.Kg}^2.\text{s}^{-3}$
Δ	=	mesh spacing, m
γ	=	specific heat ratio
ρ	=	density, kg m^{-3}
ϕ	=	far-field azimuthal angle, $^\circ$
ω	=	radian frequency, rad.s^{-1}
∇_T	=	gradient in the transverse direction, m^{-1}
θ	=	far-field polar angle, $^\circ$

II. Introduction

JET engine noise reduction remains a significant concern in the aviation industry, particularly addressing the mixing noise generated from the exhausts of business jet aircraft. Among various strategies, passive exhaust lip treatments have gained considerable attention for their potential to reduce mixing noise. Serrations on exhaust nozzles, commonly referred to as chevrons, have proven to be an effective strategy for reducing jet mixing noise among various noise reduction techniques. Essentially, chevrons introduce streamwise vorticity, which in turn increases flow mixing near the nozzle and shifts acoustic energy from low to high frequencies. Chevrons exhibit the capability to specifically diminish low-frequency noise, particularly in downstream polar angles.[1],[2],[3]. Despite their effectiveness, chevrons are known to reduce the engine thrust because of the additional mixing. Therefore, there is a keen interest in developing treatments to reduce mixing noise while minimizing their negative impact on engine performance.

Passive exhaust lip treatments for noise reduction other than chevrons have been intensively investigated over the past few years, as indicated by the number of publications on them [2, 4–7]. Such techniques aim to mitigate mixing noise without significantly compromising engine performance, i.e., causing loss of thrust. Other challenges remain, e.g.:

- Design Complexity: Achieving optimal noise reduction without detrimental effects on engine efficiency requires intricate design considerations.
- Aerodynamic Effects: Changes to nozzle geometry may impact aerodynamics and fuel efficiency, necessitating careful optimization.

As discussed in a companion paper, incorporating triangular notches onto the internal wall of a circular nozzle lip seems to present a compelling compromise solution to achieve these objectives (Fig. 1). The thrust loss was also observed to be lower than for chevrons, for instance.

Of particular significance is the ability to predict the resultant impact on noise reduction stemming from what are often subtle geometric alterations. Although advanced techniques such as LES can effectively tackle this problem [3], they are still computationally too expensive for Multidisciplinary Optimization (MDO) and preliminary design, where the required prediction timescales are in the order of hours. Thus, there exists a demand for fast, yet reliable, predictive methods that can be used to estimate the impacts of nozzle lip modifications.

In response to the demand for faster predictive methods, numerous previous contributions have subsidized the development of so-called reduced order methods for predicting turbulent mixing noise. The main goal of this work is to assess the predictive capability of jet mixing noise by the Generalized Acoustic Analogy (GAA) model proposed by [8], which requires only RANS input data and has been found to yield satisfactory results for unheated, subsonic, non-axisymmetric jets. As a main conclusion of the current work, the method has been found to be sensitive to the presence of the internal notches, and in general captures the effects of such with reference to the baseline (round) nozzle across most frequencies.

III. Methodology

A. Experiments

Experiments have been conducted in the ISVR's Doak Laboratory, using the Flight Jet Rig (FJR), with two types of nozzle - a round baseline nozzle and an internally-notched nozzle. The Doak Laboratory comprises a chamber, fully

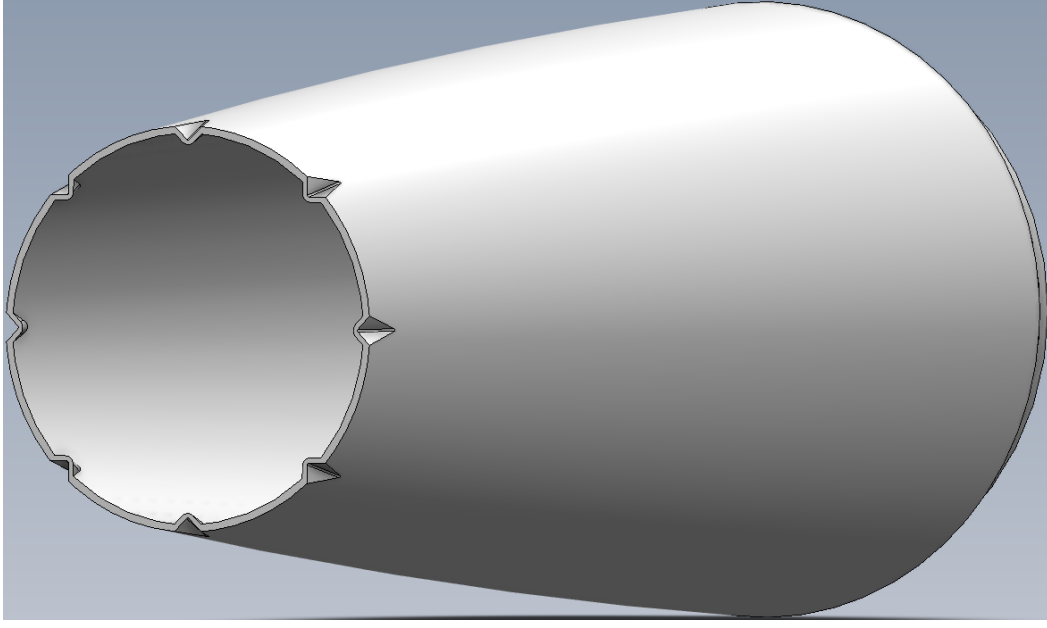


Fig. 1 Round 40mm-diameter nozzle with internal notches

anechoic above 400 Hz, with dimensions approximately equal to 15 m-long, 7 m-wide and 5 m-high. Two separate air supply systems allow in-flight simulations of single stream jet flows using the FJR. The primary ‘core’ jet flow is supplied by a high pressure compressor-reservoir system, capable of producing a maximum inlet pressure of 20 Bar. The present work deals only with static cases.

Total temperature and pressure sensors are mounted both in the plenum (located far upstream of the nozzle exit) and along the pipework of the core and flight jet ducts. Ambient chamber properties, such as pressure, temperature and relative humidity, are also recorded. These readings are required to set the nominal jet exit velocity condition and to apply corrections to the data (e.g. atmospheric attenuation and NPR variation during the long hot-wire traverse measurements). Far-field data are obtained using a linear “flyover” array. Ten microphones are used to cover polar observer angles ranging from 40° to 130°, at 10° intervals. The closest microphone, $\theta = 90^\circ$, is located at approximately 55 jet nozzle diameters from the nozzle centreline. An image of the anechoic chamber depicting the FJR and the far-field microphone array, from which the data presented in this report were recorded, is shown in Fig. 1. The internally notched nozzle used in the experiments is shown in Fig. 2.

All data were acquired using a 24-bit National Instruments dynamic signal acquisition system. Each test point was recorded for ten seconds at a sampling rate of 200k samples per second. The microphones were calibrated before and after the test campaign. The voltage data acquired by each microphone were converted to pressure using a frequency-dependent sensitivity transfer function obtained during the calibration procedure. The signals were also corrected for amplifier gain, microphone incidence, atmospheric attenuation and distance (assuming linear spherical wave propagation from the nozzle exit location) for each measurement. Power spectral density (PSD) information was obtained using Welch’s method and the final sound pressure levels (SPL) were calculated using a reference pressure of 20 μPa .

B. Mathematical model and numerical methods

As demonstrated in [8], the acoustic spectrum per unit volume in the absence of a co-flow reads:

$$I(x|y) = \frac{4\pi^2}{x^2} \frac{2\pi\omega \sin \theta}{c_\infty} [C^4 \Phi_{1111}^* + 2C(1-C) \Phi_{1122}^* + (1-C)^2 \Phi_{2222}^*] |\hat{g}_0|^2 + \frac{16\pi^2}{x^2} \frac{2\pi\omega \sin \theta}{c_\infty} \left| \frac{c \cos \theta}{c_\infty \omega (1-M \cos \theta)} \left[\nabla_T \left(\frac{c \hat{g}_0}{(1-M \cos \theta)} \right) - \frac{(\gamma-1)c_\infty \hat{g}_0}{2(1-M \cos \theta)} (\nabla_T M) \right] \right|^2 \Phi_{1212}^* \quad (1)$$

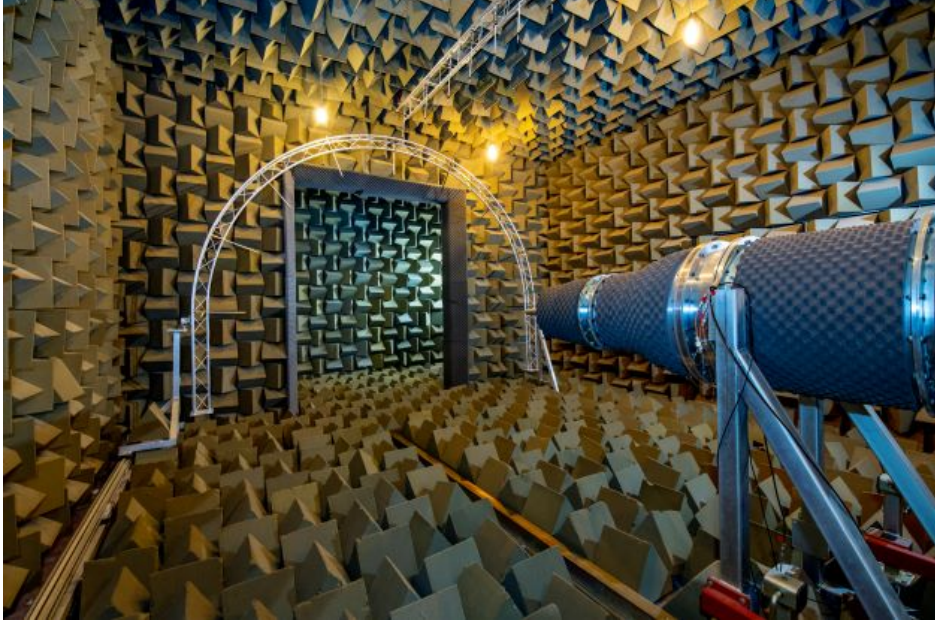


Fig. 2 Doak Laboratory (ISVR - University of Southampton, UK)

where

$$C^2 = \frac{c^2 \cos^2 \theta}{c_\infty^2 (1 - M \cos \theta)^2} \quad (2)$$

Equation 1 differs from Eq. (26) in [8] in that axisymmetric turbulence is assumed ($\Phi_{2222}^* = \Phi_{2323}^*$), therefore the last term on the RHS of Eq.(26) is zero. For a given frequency f , C , c , c_∞ , x , θ , M and γ can be obtained based on operating conditions and physical properties. Φ^* represent the spectrum tensor components, which rely on RANS results and are calculated based on the source model described in [9]. The source model constants were set as displayed in Table 1 and Table 2. Such values were adjusted by [9] considering unheated, axisymmetric jets.

Table 1 Coefficients in truncated series for turbulence spectrum

Component	$a_{1,0}/a_{0,0}$	$a_{2,0}/a_{0,0}$	$a_{1,0}/a_{0,0}$
1111	0.073	0.070	-0.0008
2222=3333	0.519	0.049	-0.0097
1122=1133	0.103	0.079	0.0
1212	0.388	-0.126	-0.031

Table 2 Constants used in length scales

Component	C_0	C_1	C_T
1111	0.7	1.2	0.4
2222=3333	0.7	0.8	0.89
1122=1133	0.7	1.05	1.0
1212	1.05	1.0	1.1

$\hat{g}_0$ is the adjoint Green's function, which acts as a propagator of the sources. Under some assumptions, [8] demonstrated that it can be calculated by Eq. 3

$$\frac{\partial}{\partial r} \frac{c^2}{(1 - M \cos \theta)^2} \frac{\partial \hat{g}_0}{\partial r} + \frac{\partial}{r \partial \phi} \frac{c^2}{(1 - M \cos \theta)^2} \frac{\partial \hat{g}_0}{r \partial \phi} + \omega^2 \left(1 - \frac{\frac{c^2}{c_\infty^2} \cos^2 \theta}{(1 - M \cos \theta)^2} \right) \hat{g}_0 = 0 \quad (3)$$

subject to the boundary conditions presented in [8]. The equation above is solved via the finite-volume method. Around each mesh vertex, a two-dimensional control volume (CV) is created.

Eq.3 is then integrated, and the Green-Gauss Theorem used to transform the volume integrals into surface integrals for the terms involving derivatives, so that fluxes across the control volume faces can be calculated. The process is illustrated below for the first term on the LHS of Eq. 3:

$$\int_{CV} \frac{\partial}{\partial r} \frac{c^2}{(1 - M \cos \theta)^2} \frac{\partial \hat{g}_0}{\partial r} dV = \oint_S \frac{c^2}{(1 - M \cos \theta)^2} \frac{\partial \hat{g}_0}{\partial r} \vec{i}_r \cdot d\vec{S} \quad (4)$$

The surface integral on the RHS of Eq. 4 runs over all CV faces, but since only faces s and n yield inner products which are not zero, it reduces to Riemann integrals across those faces:

$$\oint_S \frac{c^2}{(1 - M \cos \theta)^2} \frac{\partial \hat{g}_0}{\partial r} \vec{i}_r \cdot d\vec{S} = \int_n \frac{c^2}{(1 - M \cos \theta)^2} \frac{\partial \hat{g}_0}{\partial r} r d\phi - \int_s \frac{c^2}{(1 - M \cos \theta)^2} \frac{\partial \hat{g}_0}{\partial r} r d\phi \quad (5)$$

The integrals on the RHS of 5 are evaluated numerically. For face n for instance:

$$\int_n \frac{c^2}{(1 - M \cos \theta)^2} \frac{\partial \hat{g}_0}{\partial r} r d\phi \approx \left[\frac{c^2}{(1 - M \cos \theta)^2} \frac{\partial \hat{g}_0}{\partial r} r \right]_n \Delta\phi = \frac{c_n^2}{(1 - M_n \cos \theta)^2} \left[\frac{\partial \hat{g}_0}{\partial r} \right]_n r_n \Delta\phi \quad (6)$$

First derivatives must then be calculated across all CV faces. In the current work, two alternatives were tested: second-order centered scheme (geographic notation, [10]):

$$\left[\frac{\partial \hat{g}_0}{\partial r} \right]_n \approx \frac{\hat{g}_{0N} - \hat{g}_{0P}}{\Delta r} \quad (7)$$

and fourth-order centered schemes ([10]):

$$\left[\frac{\partial \hat{g}_0}{\partial r} \right]_n \approx \frac{1}{\Delta r} \left(-\frac{1}{24} \hat{g}_{0NN} + \frac{9}{8} \hat{g}_{0N} - \frac{9}{8} \hat{g}_{0P} + \frac{1}{24} \hat{g}_{0S} \right) \quad (8)$$

Applying the operations above to the other CV faces gives rise to a sparse linear complex system for $\hat{g}_0$, which is solved numerically by a direct banded solver. The solution of the adjoint Green's function was validated against analytical solutions.

The whole solution process can be summarized as below:

- Upon convergence of the RANS solution, the axial velocity, turbulent kinetic energy, dissipation ratio, density, and sound speed of the jet are extracted at axial planes ranging from 2 to 20 diameters downstream of the jet exit, and up to 6 diameters radially. These variables, which serve as input for the GAA code, are where the sources are located.
- The aforementioned variables are then interpolated onto a polar, uniform mesh.
- The linear system resulting from the Green's function (Eq. 3) is solved using a direct method.
- For a given frequency, the source tensor terms Φ^* are calculated based on the interpolated RANS results.
- Given the far-field observer distance x and the polar angle θ , the acoustic spectrum per unit volume is evaluated at each vertex via Eq. 1.
- Finally, the $PSD(Pa^2.s)$ is calculated by volume integrating $I(x|y)$ in cylindrical polar coordinates.

Regarding the RANS solutions, the domain was simulated assuming symmetrical cross planes, such that only a sector of 45 degrees was fully meshed with hexahedral cells. This was done to reduce computational costs. The final mesh count amounted to approximately 3.95 million cells, which produced mesh-independent results for the internally-notched nozzle. This mesh count is justified by the need to refine the grid at the notch locations to accurately capture the geometric features. Steady-state simulations, utilizing the standard k-epsilon turbulence model with enhanced wall treatment, were performed using the commercial CFD solver ANSYS-Fluent v23.2 [11].

IV. Results and Discussion

A. Effect of the numerical schemes used in the solution of the Green's function

To evaluate the influence of the numerical scheme's order on the solution of the adjoint Green's function (3), the far-field noise generated by the internally-notched nozzle was calculated using both 2nd and 4th order schemes and compared to experimental data. Results at three polar angles are depicted in Fig. 3. In these instances, a polar mesh consisting of 250 cells radially and 250 cells azimuthally yielded mesh-independent results.

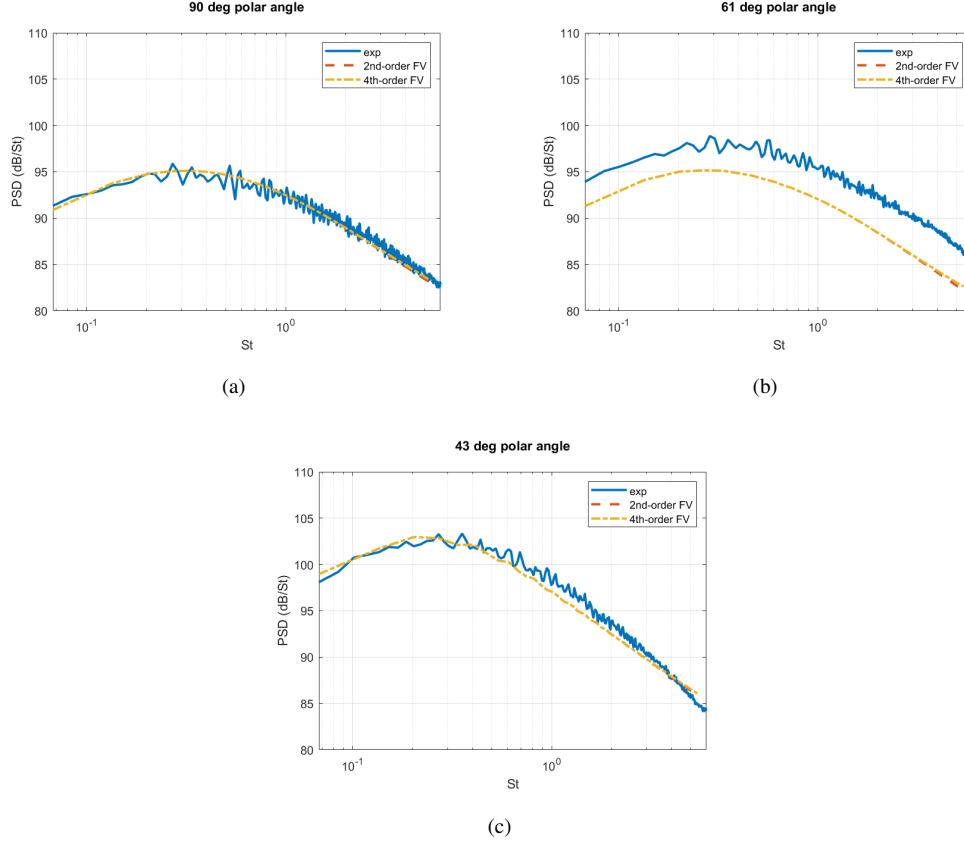


Fig. 3 Far-field spectra of the internally notched nozzle - simulation vs experiment at jet exit Mach number $M = 0.7$

As observed, there is negligible improvement when increasing the order of the numerical scheme, at least within the frequency range analyzed in this study. Given the nonlinear increase in computational cost, such a choice does not prove advantageous. Since the simulation involves two primary stages—sound generation and sound propagation—these findings suggest that enhancing the source modeling may be a more promising approach to improve overall accuracy. Subsequent results are obtained using the 2nd order scheme for the first derivatives in Eq. 3, due to its acceptable level of accuracy and lower computational cost.

In any event, the results depicted in Fig. 3 fulfill the purpose of validating the entire prediction procedure.

B. Internally-notched versus round nozzle

Numerical and experimental results are compared in Fig. 4 for both the baseline (R40) and internally notched (IN) nozzles. At first glance, it is evident that the simulations predict a variable, asymptotic noise reduction, reaching a maximum of 1.3 dB relative to the baseline nozzle at Strouhal numbers above 2. For clarification purposes, the noise reduction induced by the internal notches is presented in Fig. 5 for both experiments and simulations. Experimental data exhibits a reduction in PSD levels up to Strouhal numbers close to 3 or higher, followed by noise augmentation.

This shift occurs due to the redistribution of energy from lower to higher frequencies, akin to that caused by chevrons. However, the simulations fail to capture this behavior. Despite the under-prediction of the noise benefit from the internally notched nozzle by the model, it can be deemed reasonable considering the relatively subtle variation in geometry and the reliance of the noise model on steady RANS input data. Good agreement is observed at most polar angles, with deviations of up to 4 dB between experimental and predicted PSDs, particularly at lower Strouhal numbers. Results for two other jet Mach numbers, namely 0.5 and 0.8, are presented in Figures 6 and 7, respectively. Generally, trends are similar to those observed for a jet Mach number of 0.7, but a more pronounced under-prediction of the noise reduction between the notched and round nozzles can be noted, particularly at a polar angle of 43 degrees for a jet Mach number of 0.8. At this stage, identifying the cause of such behavior is challenging. Further investigation into the calibration of the source spectral components may shed light on how to enhance the accuracy of these predictions. Additionally, calibration of the source components using both experiments and LES of the internally notched nozzles is necessary to determine the extent of potential improvement.

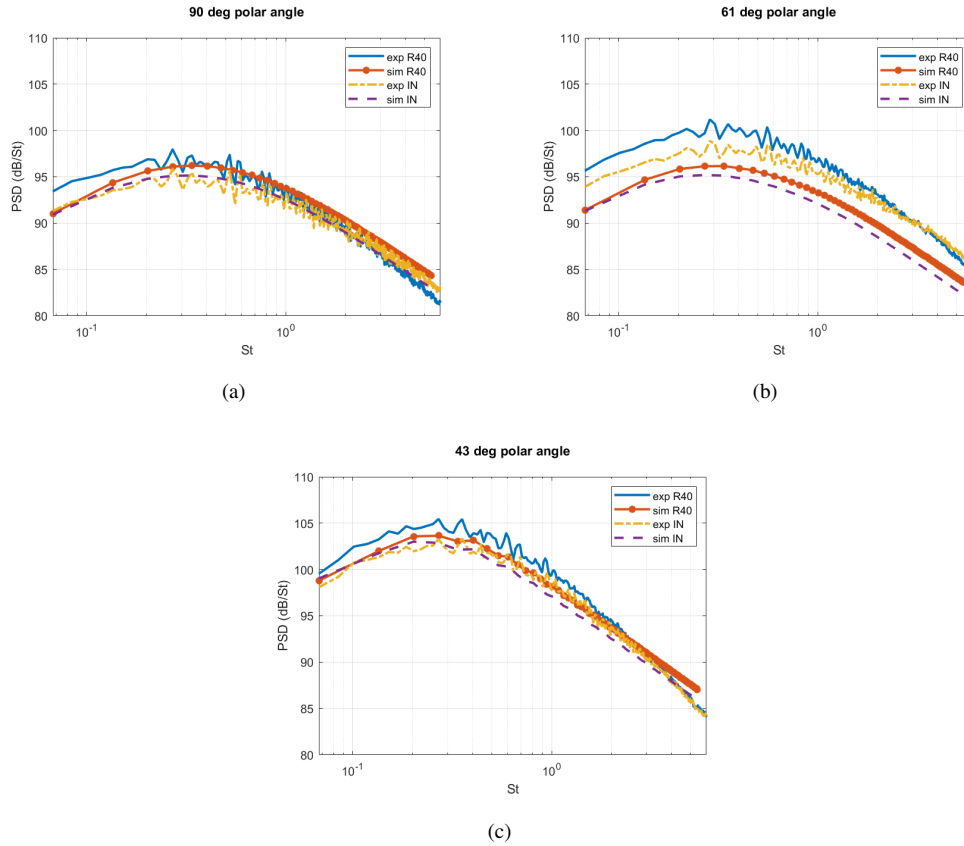


Fig. 4 Far-field spectra of both round and internally notched nozzle - simulation vs experiments at jet exit Mach number $M = 0.7$

V. Conclusions

The GAA methodology described in [8] provided reasonable predictions of far-field noise for both round and internally notched nozzles, particularly for Mach numbers of 0.5 and 0.7, and Strouhal numbers ranging from 0.3 to 1. The effects of internal notches are qualitatively captured by the proposed methodology across most frequencies. In general, the model under-predicts the absolute noise levels for a polar angle of 61 degrees at all Strouhal numbers, but shows excellent agreement with experiments for angles of 43 and 90 degrees. This discrepancy was also observed in [8]. Employing higher-order schemes (4^{th} order) for solving the Green's function did not significantly improve the predictions. This suggests that enhancements in accuracy would be better directed towards calibrating the source

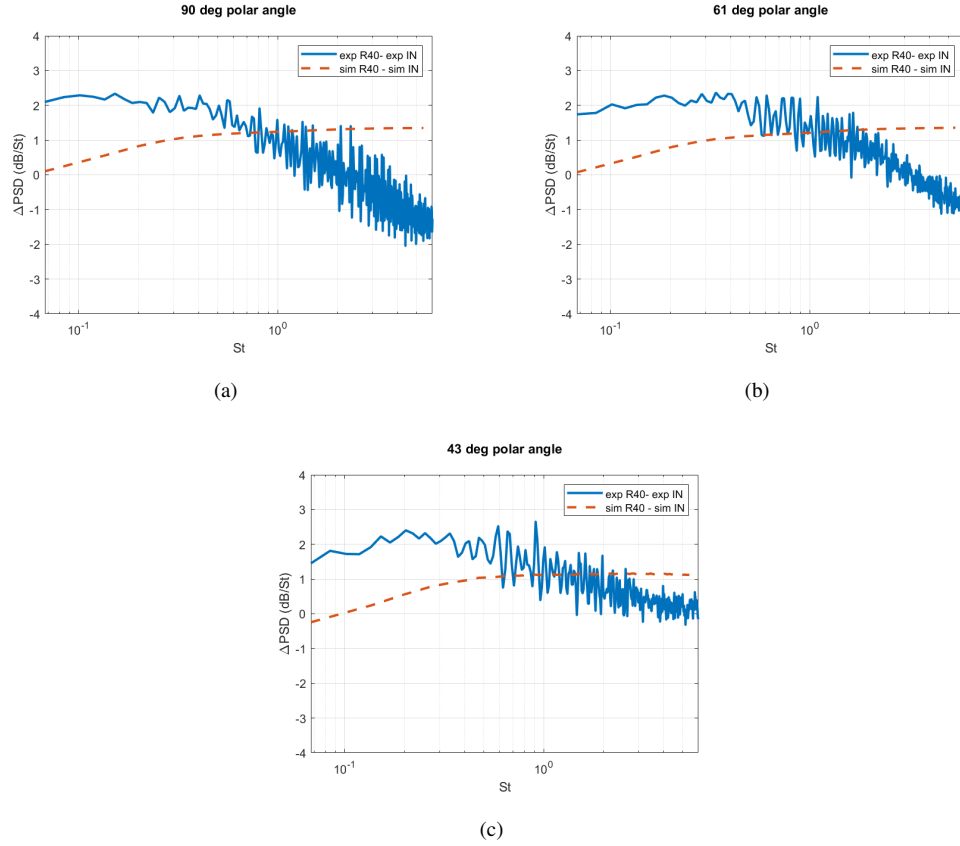


Fig. 5 Experimental and predicted noise reduction brought about the internal notches - jet exit Mach number $M = 0.7$

tensor. It is important to highlight that the constants used in this study were calibrated based on round, unheated jets, which likely explains why the PSD trend inversion between the internally notched nozzle and the round one at Strouhal numbers between 2 and 4 was not captured. Instead, similar Strouhal behavior was observed for both nozzles.

VI. Acknowledgements

This work is funded by the Innovate UK Research Programme FANTASIA - Future Aircraft Noise Technologies And Systems Integration Analytics (ref. 74217). The authors would like to acknowledge the support provided by the Rolls-Royce University Technology Centre for Propulsion Systems Noise, within the Institute of Sound and Vibration Research at the University of Southampton. Additionally, the first author would like to thank Stewart Leib (NASA Glenn Research Center) for valuable discussions on the GAA implementation, and the Federal University of Uberlandia.

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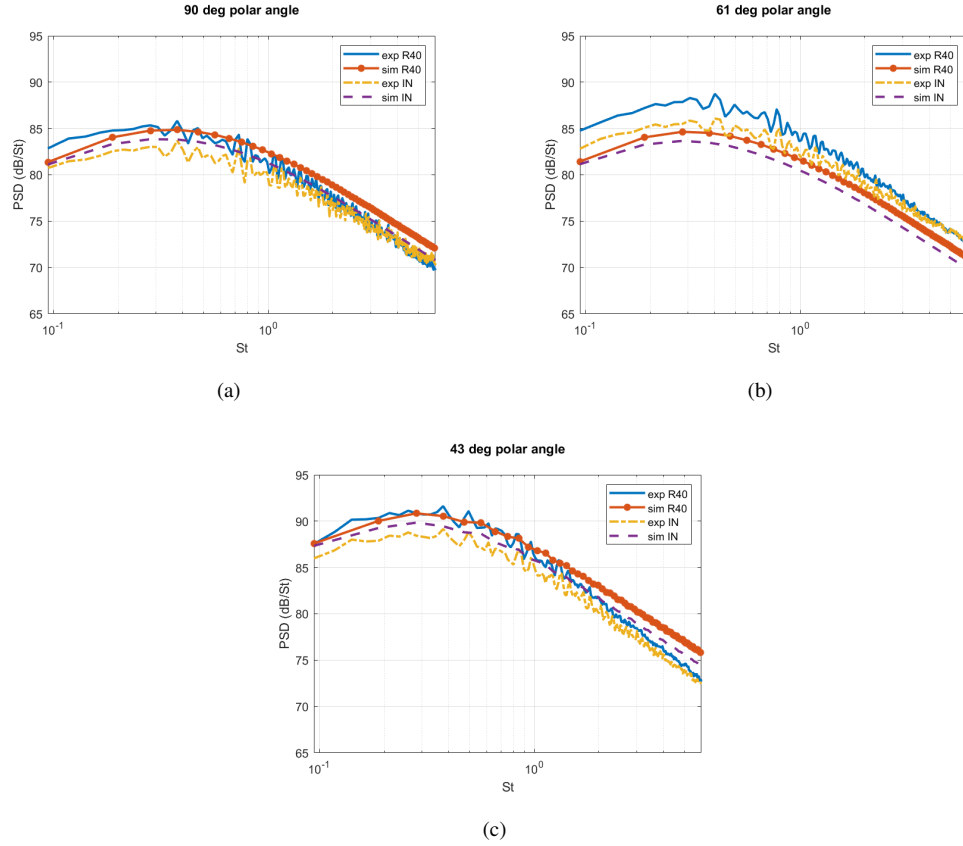


Fig. 6 Far-field spectra of both round and internally notched nozzle - simulation vs experiments at jet exit Mach number $M = 0.5$

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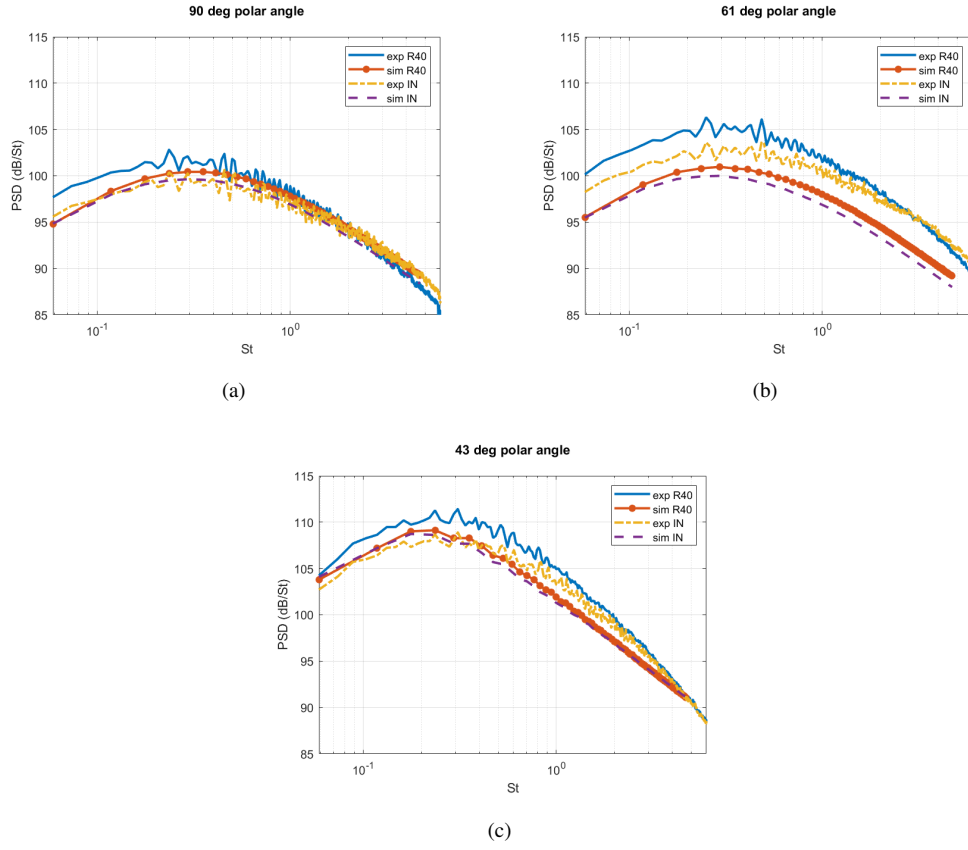


Fig. 7 Far-field spectra of both round and internally notched nozzle - simulation vs experiments at jet exit Mach number $M = 0.8$



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2024-05-30

American Institute of Aeronautics and Astronautics (AIAA)

<https://doi.org/10.2514/6.2024-3255>

de Souza FJ, Lawrence J, Proenca A. (2024) Reduced-order model prediction of far-field mixing noise from internally-notched nozzles. In: 30th AIAA/CEAS Aeroacoustics Conference (2024), 4-7 June 2024, Rome, Italy, Paper number AIAA 2024-3255