

Network-entropy-based morphological polycentricity in 1851-1881 England and Wales

1 Abstract

A system of cities is morphologically polycentric when its cities are similarly sized and evenly spaced across its territory. In this paper, we adapt established spatial interaction models and entropy-based measures of heterogeneity of weighted networks to the problem of measuring morphological polycentricity. We study the evolution of the morphological polycentricity of the system of English and Welsh towns from 1851 to 1881, a period characterised by rapid urbanisation and expansion of railways. Our approach enables us to account for morphological aspects of the system that are often neglected by existing measures of morphological polycentricity, such as the evolution of transport infrastructure and its impact on travel distances. We show that the trend towards a greater concentration of the population in fewer urban centres in England and Wales was accompanied by a more diverse network of connections and potential intercity interactions.

2 Introduction

2.1 Polycentricity- evolving geographic phenomenon and policy objective

Polycentricity has gained considerable traction in planning but has remained for a long time an ambiguous concept [36]. This vagueness largely derives from its referring to two different aspects of a system of cities: the morphological one, relating to the size and spatial distribution of the cities [37], and the functional one, concerning the relations and exchanges between them [19, 22]. A system of cities is *morphologically polycentric* when its cities are similarly sized and evenly spaced across its territory [36, 26]. The distance between two cities may be computed as their geographical distance or, more aptly, as the travel time [19]. A system is *functionally polycentric* when the functional connections between the cities are more balanced and diverse [22]. Examples of functional connections include migration flows, commuting patterns, material and knowledge exchanges, among others. Real world observations suggest that a morphologically polycentric region is not necessarily a functionally polycentric one [43]. Yet, morphological and functional polycentricity are not independent, as the spatial distribution of cities and their sizes impacts the infrastructural linkages between them, which are indispensable for the creation of functional linkages [35, 13].

Another source of ambiguity is that polycentricity has been varyingly treated as an observable geographical phenomenon and as a policy objective to pursue [22]. Hall and Pain [23] described polycentricity as an emergent and ongoing process, discussing the advent of “Mega-city regions”: morphologically and functionally polycentric regions clustered around one or more major cities and developed through a process of decentralisation, whereby the cities take advantage of the division of labour and operate synergistically.

When polycentricity is interpreted as a policy objective, the primary aim is often that of analysing the connection between morphological and/or functional polycentricity and economic prosperity. In some cases, the theoretically neutral notion of polycentricity is inflected with the positive connotation of “desirable balance”. The European Spatial Development Perspective [16], for example, maintained that the disproportionate economic and demographic concentration in the core area of the European Union (which they defined as the pentagon with London, Paris, Milan, Munich, and Hamburg at its vertices) hinders the full realisation of the economic potential of its peripheral regions, and stressed the importance of multiplying hubs for accelerating growth and reducing regional imbalance. In other cases, though, the link between polycentricity and economic performance is called into question. For example, Seymour [43] discussed the different levels of economic success in similarly functionally polycentric regions such as the Dutch Randstad and German Ruhr, and the greater prosperity in the morphologically centralised South East of England compared to the much more morphologically polycentric North. Goetz et al. [21] observed that in the United

States counties that have high diversity in either their incoming or outgoing commuting flows tend to experience lower per capita income growth. However, counties that show high polycentricity of both their incoming and outgoing commuting flows are associated with higher income growth.

2.2 Aims

Whether polycentricity is treated as an emerging geographical phenomenon or as a policy objective, a fundamental problem is that of quantifying it. Most approaches to measuring morphological polycentricity consider cities in a region as existing “in isolation”, and focus on one or more aspects of interest, such as their population size, for which several mathematical measures of heterogeneity can then be used. The study of functional polycentricity presupposes instead a network approach [14], whereby cities are represented as nodes connected by links that correspond to a specific kind of functional linkages.

This paper examines the morphological structure of urban systems, focusing on population distribution and distances between towns, which can be seen as the underlying foundation for real interactions. While real interactions are influenced by the city system’s structure, they are also influenced by various other factors. We adopt an entropy-maximizing model that captures the *potential for interactions*, without making strong assumptions about their specific forms. We then use established measures of functional polycentricity to study the heterogeneity of these potential interactions. Our objective is to shed light on the intricate relationship between population concentration, improved transport connectivity, and their impact on the structure of the urban system and its balance. We do so by examining the system of English and Welsh towns between 1851 and 1881, a period marked by rapid urbanisation and expansion of the railways.

2.3 Structure

The rest of the paper is structured as follows. We begin by surveying several indicators of morphological polycentricity of systems of cities (Section 3.1) and explain why networks should play a role in measuring it (Section 3.2). We discuss the important role of Shannon entropy both as an established measure of the functional polycentricity and as the foundation of a statistically grounded spatial interaction model to predict intercity interactions (Section 3.3). In Section 4, we combine these two existing approaches, discussing how to connect cities into a network via the entropy-maximisation-based spatial interaction model and travel time distances, and then measure the morphological polycentricity of the system on the basis of the Shannon entropy of these predicted interactions. In Section 5, we illustrate the potential of this network-entropy-based approach by studying the evolution of the morphological polycentricity of the English and Welsh system of cities from 1851 to 1881, at the local level for each city, and for five main regions. We discuss advantages, limitations, and future opportunities offered by this approach (Section 6) and conclude the paper (Section 7).

3 Related work: polycentricity and network entropy

3.1 Simple and composite indices

Zipf’s law [51] is one of the most discussed [20, 11, 4] and statistically observed [40, 6] examples of regularity in the distribution of population sizes in system of cities around the world. It suggests a relation of inverse proportionality between a city’s population size and a power of its position in the national population ranking. The exponent in Zipf’s rank-size relation has been used to describe trends in the level of economic and demographic imbalance between one or few major centres in a national or transnational system of cities and the smaller centres in the system. Pumain and Moriconi-Ebrard [41], for example, remarked that, following the period of increasing centralisation during the 19th and early 20th century described by Bairoch et al. [3], a clear evolutionary pattern in national systems of cities did not emerge in more recent years from the analysis of the exponents. Hence, they advocated for more sophisticated methods that do not only take into account city sizes, but also their territorial contexts and spatial locations to quantify how centralised or polycentric a system of cities is. The exponent in the rank-size analysis has since been widely used as a simple measure of polycentricity at the regional scale [37, 13, 12]. This method is also mentioned in Hall and Pain [23], but the

authors stressed therein its crudeness. It is, indeed, sensitive to sample size (different exponents may result if a larger or smaller subset of cities in a region is selected), a system may not display a clear rank-size relation, and the measure's inability to capture spatial information about the cities' locations such as the distance between the cities.

Composite indices are adopted as more specialised and sophisticated measures of polycentricity, their main advantage being that of merging together information about various attributes of interest, such as employment or population, and the spatial location of the cities. Examples include the "polycentricity index" proposed by ESPON 1.1.1 – Potentials for Polycentric Development in Europe [19], which uses the slope in the rank-size relation for cities' population and Gross Domestic Product, the Gini coefficient of the sizes of the cities' service areas, and their multimodal accessibility, and the simpler "urban centrality index" [38], which combines inequity in the distribution of jobs with an index of the spatial dispersion of centres based on their geographical distance. Disadvantages of the polycentricity index are its reliance on the rank-size relation and its complexity, as it involves combining several pieces of information that may not be available for all systems of cities. The urban centrality index, on the other hand, uses geographical distance to compute heterogeneity in the spatial distribution, which may not reflect real travel time conditions.

3.2 Network theory and polycentricity

Network theory finds extensive application in the analysis of systems of cities, including modelling highway traffic flows between cities [27]; evaluating railway systems' robustness [10]; and reconstructing and tracking the evolution of interactions between settlements [29].

Studies of functional polycentricity invariably adopt a network approach, and explicitly define a network of cities in which the nodes represent cities and the links connecting them represent a functional relation of interest. The link (i, j) between cities i and j in a region R is associated with a non-negative weight w_{ij} , quantifying the intensity of the connection from i to j . For example, if the network represents commuting patterns, w_{ij} may be the number of people commuting from i to j .

For morphological polycentricity, the existence of a network is not always explicit. Yet, to account for the spatial distribution of cities, distances between the cities have to be computed, and this defines an implicit network structure where the weight w_{ij} of the link between cities i and j is some function of their distance.

The sum

$$s_i^{out} = \sum_{j:(i,j) \in L} w_{ij},$$

where L is the set of all the links in the network, is the sum of all the weights of the links issuing from the node i and is called the out-strength of i . Similarly,

$$s_i^{in} = \sum_{j:(j,i) \in L} w_{ji},$$

is called the in-strength of i . The set of link weights $\{w_{ij}\}_{i,j \in L}$ and the sets of nodes' strengths $\{s_i^{out}\}_{i \in R}$ and $\{s_i^{in}\}_{i \in R}$ are of particular interest in the study of polycentricity, as weights and strengths can be taken respectively as indices of the relative importance of a connection and of a node within the network. In fact, all network-based polycentricity measures are based on these sets.

First introduced in Wasserman et al. [45], the aptly named *Polycentricities* are a class of network-based measures that make use of the standard deviation of the distributions of link weights and nodes' strengths, and of the network density to give an indication of how polycentric the network is. Polycentricities have been adapted to the urban context by Green [22], who took the nodes in the network to be cities, and the weighted links to be either functional relations between them, or geographical distances.

Green developed Polycentricities with a functional approach in mind, but there have been attempts to apply Polycentricities to urban networks built using a morphological perspective. For instance, in [32, 31, 30] the networks were built by connecting the main centre and secondary centres and taking as weights the products $p_j \cdot d_j$ of the population p_j of the centre j and its distance d_j from the main centre. Issues of this adaptation include the fact that it assumes the existence of a single main centre, and it ignores the relations between smaller centres, which may be of great interest.

3.3 Entropy in Urban Science: potential interactions and heterogeneity

Alongside the exponent in Zipf's law, the Gini coefficient, and the standard deviation, Shannon entropy has been used to quantify heterogeneity and polycentricity in urban systems. In the context of what would become known as Information Theory, Shannon [44] defined the entropy H of a quantity q distributed among n objects as

$$H := - \sum_{j=1}^n \frac{q_j}{q} \log \frac{q_j}{q},$$

where q_j is the proportion of q associated with the object j , so that $\sum_{j=1}^n q_j = q$.

The Shannon entropy H is a natural measure of how balanced q is, with larger values indicating a more homogeneous distribution and smaller values a more concentrated and unequal one [17]. H attains its maximum value $H_{max} = \log n$, when q is uniformly distributed among all objects (i.e., $\frac{q_j}{q} = \frac{1}{n}$ for all j), and its minimum value $H_{min} = 0$ when q is maximally concentrated in one single object. The advantage of Shannon entropy is that it possesses other convenient mathematical properties that made it a versatile tool to analyse urban systems, well-established since its introduction in the pioneering works by Batty [5] and Wilson [47].

Batty's work (e.g., [5, 7]) clarified how to effectively adapt Shannon entropy to spatial data as a measure of its heterogeneity and complexity. A similar aim was later pursued by Heikkila and Hu [24]. Batty's approach is prevalent in the field of spatial statistics and has been applied to study the expansion and evolution of urban areas [1, 2], and the spatial association of employment data [28].

Wilson's original works [47, 48] applied entropy to study the configuration of spending flows $\{w_{ij}\}_{(i,j) \in L}$ in the presence of constraints such as the total money ($\sum_j w_{ij}$) spent by people from each location, the cost ($\sum_{ij} c_{ij} w_{ij}$) involved in spending in a different location and the total benefit ($\sum_{ij} b_j w_{ij}$). The spending flows are in all ways equivalent to the weights on an implicit network between the locations, and the cost c_{ij} can be equated to some measure of distance between locations i and j , for instance, the travel time from i to j . Maximising the Shannon entropy of the set of weights $\{w_{ij}\}_{(i,j) \in L}$ given these constraints is mathematically equivalent to finding the most likely configuration of the flows via maximum likelihood estimation [47]. This entropy-maximising spatial interaction model was adapted to the context of intercity migration in [49], where the population of cities is seen to evolve following the most likely flow configuration.

In their study of intercity functional connectivity, Yang et al. [50] linked cities in a network with weights $\{w_{ij}\}_{ij}$ representing railway passengers flows. They then used the Shannon entropy of the weights $\{w_{ij}\}_i$ of the links issuing from a city i as a measure of the polycentricity of i 's connections. A similar approach was followed by Marin et al. [33], who adopted Shannon entropy to assess the diversity of commuting flows $\{w_{ij}\}_{ij}$ between British cities, an example of functional polycentricity. Letting

$$H_i^{out} = - \sum_{j:(i,j) \in L} \frac{w_{ij}}{s_i^{out}} \log \left(\frac{w_{ij}}{s_i^{out}} \right), \quad (1)$$

and

$$H_i^{in} = - \sum_{j:(j,i) \in L} \frac{w_{ji}}{s_i^{in}} \log \left(\frac{w_{ji}}{s_i^{in}} \right), \quad (2)$$

Marin et al. [33] obtained measures of the heterogeneity of the outgoing and incoming connections between a city i and all the other cities.

4 Methodology

4.1 Network entropy as morphological polycentricity

We combine the entropy-maximisation-based spatial interaction model by Wilson [47] and the network-entropy-based approach of measuring functional polycentricity [50, 33] in the following method to study the morphological polycentricity of a system of cities.

Consider a region R . For each city i in R , let p_i be its population and associate i to a pair of coordinates $(lon_i, lat_i) \in R$, for example, the coordinates of its city hall or a main landmark. Compute for each pair of cities (i, j) a travel time c_{ij} from i to j , depending on the presence of transport infrastructure.

We assume that the total interactions from city i are proportional to its population, i.e., $\sum_j w_{ij} \propto p_i$. Following [48] and [47], if the population p_i is the only attribute known for each city, we assume the benefit b_j of interacting with city j to be $b_j = \log p_j$. With these assumptions, equation (2.8) in [48] for the interactions simplifies to

$$w_{ij} = \gamma \cdot p_i \frac{\exp(\alpha \log p_j - \beta c_{ij})}{\sum_k \exp(\alpha \log p_k - \beta c_{ik})}. \quad (3)$$

Here γ is a proportionality constant. Since we are interested in the network entropy and thus in w_{ij} normalised by their sums, we can safely take $\gamma = 1$. The parameters α and β represent the relative weight of cost and benefit, and they can be regulated according to the system. We thus obtain a network of cities whose connections represent the most likely interactions between the pairs of cities given their population, location, and the travel time between the pairs.

For each city i , we can now use eq. (1) and eq. (2) to compute the entropies H_i^{out} and H_i^{in} of the weights w_{ij} defined by eq. (3). Thus, we obtain measures of heterogeneity of $i \in N$ accounting for all aspects of interest in morphological polycentricity.

4.2 Entropy normalisation

To be able to compare cities with different number of connections and systems of cities with different number of cities and network topology, we adopt the following normalisation of H_i^{out} and H_i^{in} .

Remark that $H_i^{out} \geq 0$ and $H_i^{in} \geq 0$, with equality when all connections issuing from (resp. directed to) i end in (resp. issue from) a single city. This reflects the case when the set of connections is maximally centralised. The maximum value that H_i^{out} can attain is $H_{i,max}^{out} = \log d_i^{out}$, where d_i^{out} is the number of links issuing from i (also known as the out-degree of i). Similarly, H_i^{in} may attain as maximum $H_{i,max}^{in} = \log d_i^{in}$, where d_i^{in} is the number of links ending in i (also known as the in-degree of i). We normalise H_i^{out} and H_i^{in} by dividing them by $H_{i,max}^{out}$ and $H_{i,max}^{in}$, respectively. We thus obtain entropy measures that fall within the range $[0,1]$.

Eq. (3) defines a complete weighted network, connecting each city to every city in the system (including itself). In this case $d_i^{out} = N = d_i^{in}$, for all city i , where N is the number of cities in R . Note, however, that the equation can be adapted to different scales and purposes. Just as Green did with Polycentricities [22], we can impose a limit on the travel time c_{ij} and thus study the polycentricity of the connections within a certain travel time from i . In this case, d_i^{in} and d_i^{out} depend on the location of i and transport infrastructure. We can also average the normalised H_i^{out} and H_i^{in} over all nodes in a subregion R' of R to have overall measures of polycentricity of R' :

$$H_{R'}^{out} = \frac{1}{n} \sum_{i \in R'} H_i^{out} \text{ and } H_{R'}^{in} = \frac{1}{n} \sum_{i \in R'} H_i^{in},$$

where n is the number of cities in R' .

4.3 Dataset

We follow the above-described method to study the changes in the morphological polycentricity of the urban system in England and Wales from 1851 to 1881.

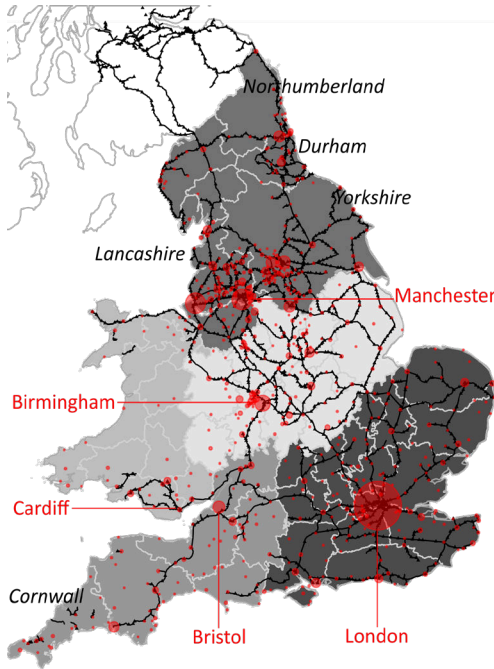
Bennett [8] collected the population of towns and cities in England and Wales from 1801 to 1911, at 10 years intervals. Not all centres have population data for 1851 and 1881. If the data for a town was not available in 1851 and/or 1881, but it was available at an earlier and at a later date, we estimate the population in the missing year(s) by linear interpolation.

To estimate travel time between localities at different times, we assume that people use the fastest land transport available. We use historic data on railway lines in England, Wales and Scotland [42, 34], and railway stations' locations [25, 34]. The data sets do not explicitly contain data on travel time or speed along the lines. We derive our estimate for average speed along the railway network from the online atlas that accompanies the data [9]. The atlas mentions that in 1830 (the year that saw the beginning of the railway era with the opening of the Liverpool to Manchester Railway) the average train speed was 12 miles per hour (mph), or equivalently 19.3 kilometres per hour (km/h). In 1900, the average speed had increased to 40 mph (64.4 km/h). Assuming a linear increase in the transport speed, we can estimate the average speed in 1851 and 1881 to be 20.4 mph (32.8 km/h) and 32.4 mph (52.1 km/h), respectively. This makes railway transport about 2.55 and 4.05 times faster in 1851 and 1881 respectively when compared to the 8 mph (12.9 km/h) average speed reached by the stagecoach, which was the fastest form of transport available to passengers before the advent of the railways.

Figure 1 shows the distribution of towns, railway lines and stations in England and Wales, as they evolved from 1851 to 1881. In these as in the following maps, we show the boundaries of the historical counties as provided by the Historic County Borders Project¹. For convenience, we grouped the historical counties into the following five regions: the Greater South East (GSE), i.e., the area including the capital London and its surroundings and corresponding to the modern-day regions of the South East and East of England; the Midlands, comprising the modern-day regions of the East and West Midlands, among the heartlands of the industrial revolution; Northern England, with its multitude of booming industrial centres, concentrated in the counties of Durham, Lancashire and around the mountain chain of the Pennines, in the south-western corner of the region; the South West (SW); and Wales. For each region, we singled out one main centre: London in the Greater South East; Birmingham in the Midlands, Manchester in the North, Bristol in the South West, and Cardiff in Wales.

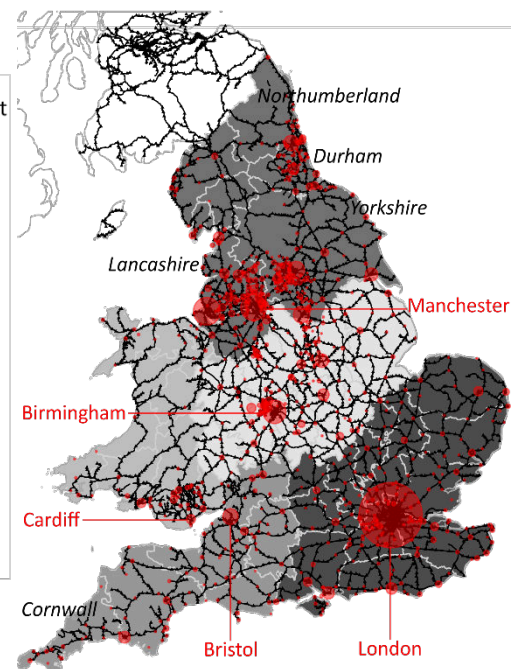
¹ The shapefile and relevant details are available at <https://www.county-borders.co.uk>.

English and Welsh towns and railways in 1851



a) Several towns remained disconnected from the railway network in 1851, especially in South West England and Central Wales.

English and Welsh towns and railways in 1881



b) Several towns remained disconnected from the railway network in 1851, especially in South West England and Central Wales.

Figure 1: Towns, railway lines and stations in England and Wales in 1851 and 1881, with historical county boundaries and regions of interest identified by different shades of gray. The number of towns in the data set [8] increases from 656 to 811. The area of the symbols representing a town is proportional to its population and shows the remarkable increase in urban population during this period. The railway network becomes denser and connects the vast majority of towns by 1881.

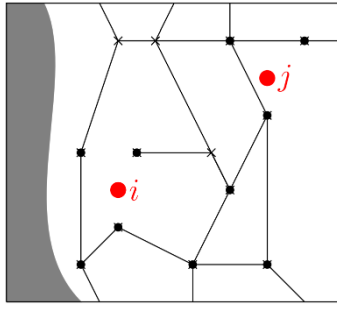
4.4 Network construction

4.4.1 Travel time computation

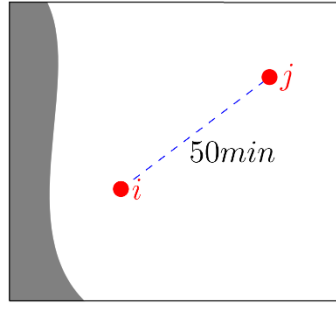
For each pair of cities i and j , first we found the geodesic distance separating them. We divided this distance by the average stagecoach speed of 12.9 km/h and obtained a baseline corresponding to the theoretical minimum time needed to reach j from i along the shortest line when faster transport means are not available. For each year and each city i , we found the closest three stations in the station data set and recorded the geodesic distances that separate them from i . The assumption here is that a passenger living in i may accept to travel to a station that is not the closest to their origin if this allows them to access a more direct railway line to their destination. Using the *igraph* package in R [18], for each pair of cities (i, j) , we computed the travel time distance along nine paths, each corresponding to a combination of travelling from i to one of the three stations closest to it, from this station to one of the closest three stations to j along the railway network, at the average speed for the specific year, and from there to j . We finally defined the travel time distance between cities i and j as the minimum between the travel time along the geodesic mentioned above and the fastest among these 9 paths. The “cost of travel” c_{ij} in the model was then taken to be equal to this shortest travel time. The process is schematically represented in Fig. 2.

4.4.2 Networks of potential interactions

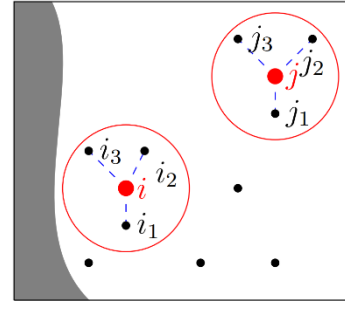
We combined p_i , p_j , and c_{ij} for each pair of cities i and j as detailed above to define two complete networks (one for each 1851 and 1881) with weights w_{ij} representing the most likely interactions given all constraints. We took $\alpha = \beta = 1$, as we noticed that this led to networks consistent with the migration networks described by two major historical studies discussing the evolution of the British urban system in the 19th century by Pooley and Turnbull [39] and Weber [46]. For a detailed discussion of this choice, see the Supplementary Information.



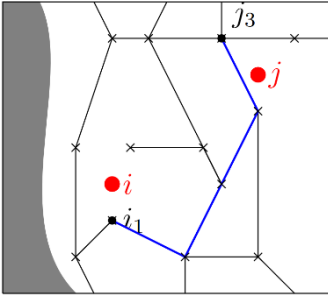
a) Region R with cities i and j , railway lines, stations, and crossings.



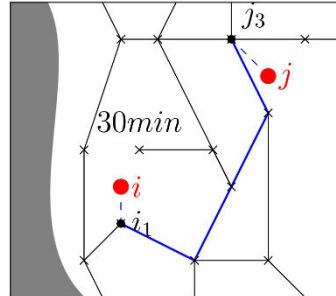
b) Step 1: Compute the travel time via stagecoach along the geodesic.



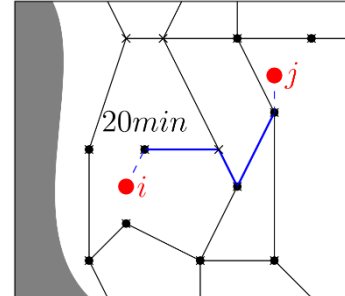
c) Step 2: Find the closest 3 stations to each city along geodesics.



d) Step 3: Find the shortest railway path(s) between each pair of stations $(i_m, j_n)_{1 \leq m, n \leq 3}$. In this example, i_1 to j_3 .



e) Step 4: For each shortest path, compute the total travel time (travel time along the railway plus time to reach station).



f) Step 5: Identify the fastest path overall, remarking that it could be the geodesic path via stagecoach from Step 1.

Figure 2: Computation of travel time between two cities i and j in region R . Red discs are cities, black dots are stations, crosses are railway crossings and endpoints.

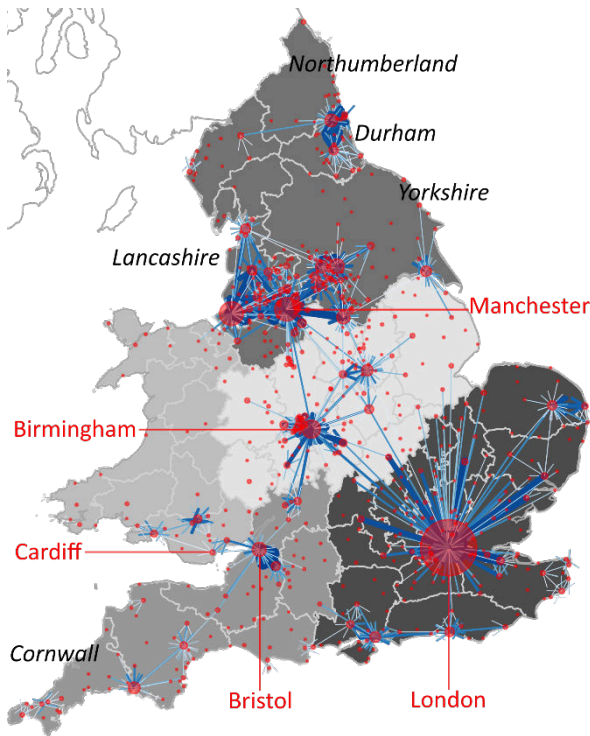
5 Results

5.1 Networks

Figure 3 shows the resulting networks in 1851 and 1881. For clarity, only the 600 links with the largest weights have been shown in each year.

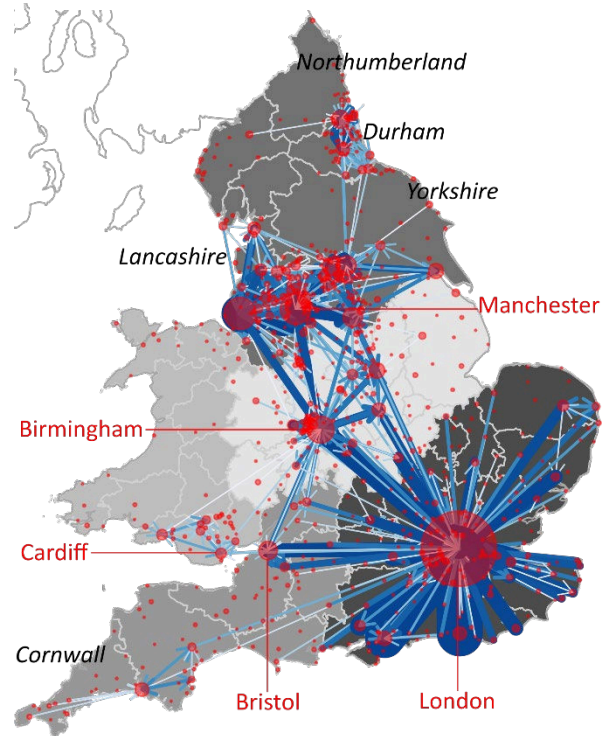
We remark an overall increase in the strength of the interactions, as a result of the combined effects of population growth, railway expansion and faster railway speeds. In 1851, a number of clusters of strong connections are apparent: a very centralised star pattern centred around London in the South-East, a strongly interconnected set of cities in Lancashire and South-Eastern Yorkshire, smaller star-shaped patterns in the Midlands that connect the aforementioned two large clusters, and a few small, isolated clusters in Durham, Yorkshire, southern Wales and in the South West. The situation in 1881 sees two major changes. First of all, the London-centred star-shaped pattern expands to the rest of the Greater South East, a sign of the consolidation of the preponderance of London on the region. Secondly, we observe a greater level of integration of the main clusters in the North and in the Midlands with the rest of the country, with the smaller Yorkshire and Durham clusters joining the former, and some centres in the latter being now strongly connected to London and to Southern Wales. This is likely an effect of both population concentration and better rail connectivity, that strengthen potential connections between larger centres even if these are geographically distant. Finally, we see that Bristol joins the main London-centred cluster and the towns in Cornwall reduce their isolation via stronger connections towards the East.

Strongest potential interaction in 1851



a) In 1851, most of the strongest links in the South-East are directed towards London, whilst the interactions in the other clusters are more polycentric.

Strongest potential interactions in 1881



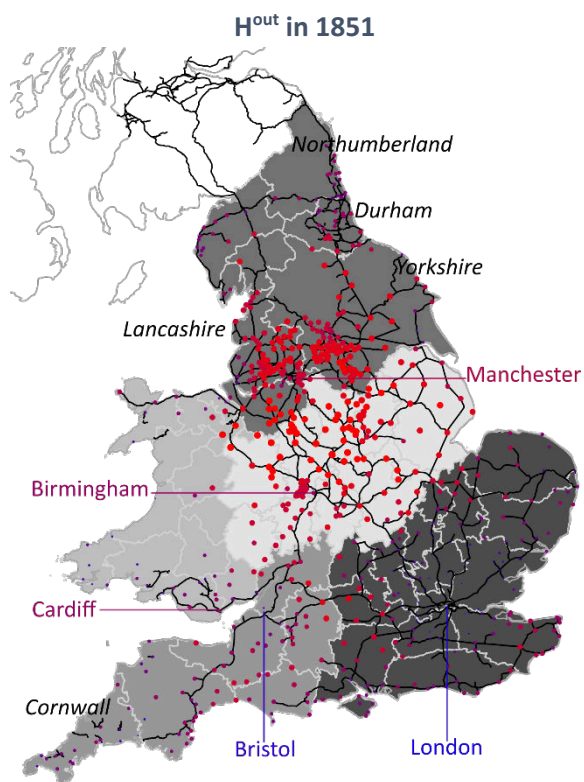
b) In 1881, the cominance of London in the South-East is reinforced and the urban clusterst in the North become more interconnected.

Figure 3: Potential interactions between English and Welsh towns in 1851 and 1881. The thickness of an arrow is proportional to the absolute magnitude of the interaction. Darker arrows represent relatively more intense interactions. For clarity, only the 600 largest connections are shown, but all pairs of towns are connected in the model, albeit some of them only very weakly.

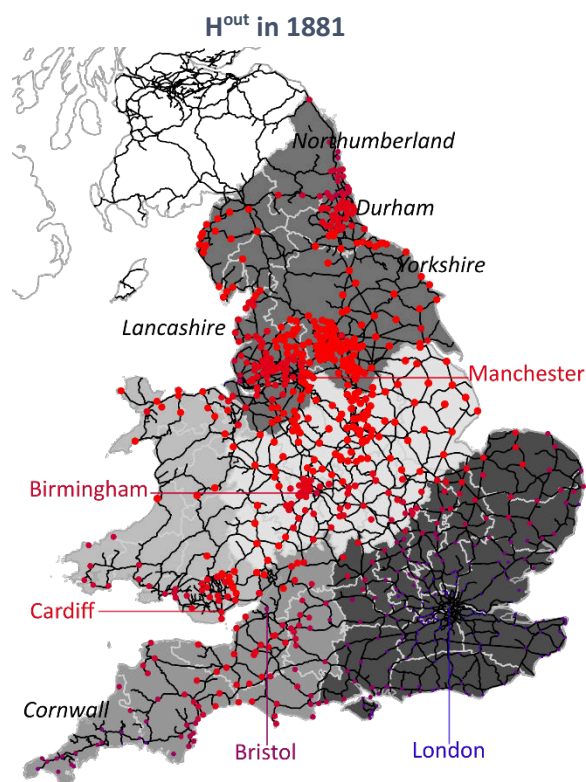
5.2 Morphological polycentricity

We computed the entropies H_i^{out} and H_i^{in} for each town i in 1851 and 1881. The maps in Fig. 4 show H_i^{out} in 1851 and 1881, when the whole network is considered and when a threshold of one hour on the travel time is imposed. In 1851 (Fig 4a), London and the surrounding towns in the Greater South East have substantially smaller entropy compared to towns in the main clusters in the Midlands and in the North. This result agrees with the centralisation of the connections in the Greater South East that we remarked in Fig. 3a. Indeed, a large part of the potential interactions issuing from cities in the South East are directed towards London and most of the potential interactions issuing from London are directed towards itself. Note, also, that in 1851, towns in South Western Wales, Northumberland and Cornwall, tend to have small entropy as the heterogeneity of their interactions is limited by their peripheral location compounded by poor infrastructure reach. In the case of Northumberland, this is due, at least in part, to artificial boundary effects caused by the fact that the population data refers to England and Wales alone, and including Scottish cities in the analysis might yield changes in the results. For the other regions the results are genuine.

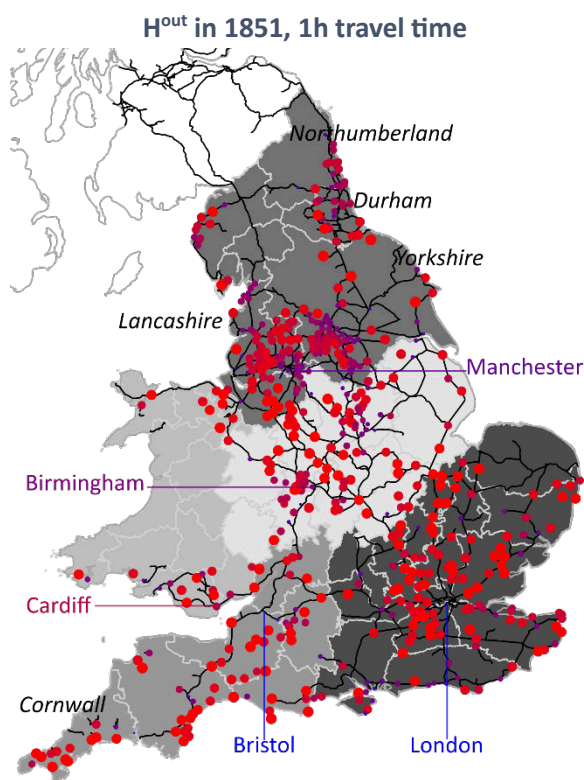
In 1881 (Fig 4b), better overall connectivity and faster transport speeds along the railway lines mitigate these effects: the potential interactions of geographically peripheral areas multiply and become more diverse, yielding larger entropy. The largest increase is observed for Aberystwyth, a coastal town in central Wales and one of the most geographically isolated ones in the area of study, whose potential interactions increase substantially after its connection to the railway network. The only centres that retain small entropy are London and those in its immediate surroundings, because of the dominance of the former, and, to a lesser extent, Cornish towns, that remain comparatively isolated.



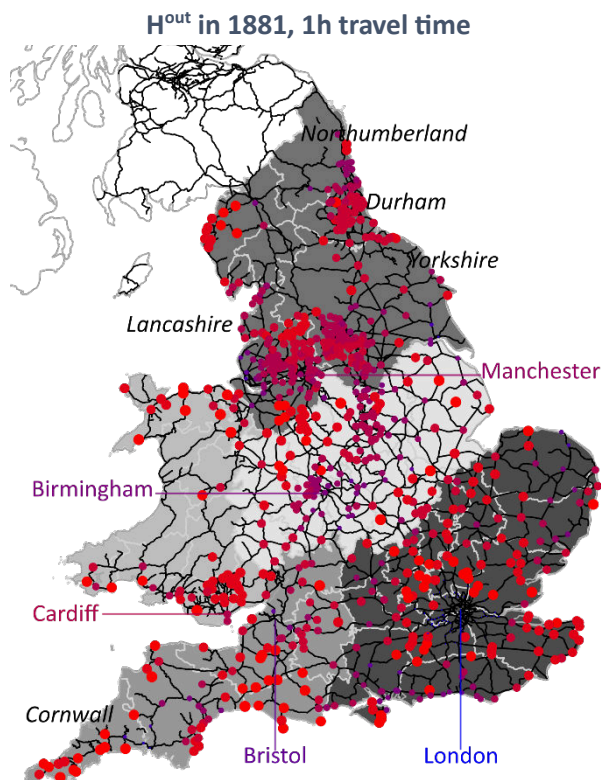
a) The entropy of potential outgoing interactions in 1851.



b) The entropy of potential outgoing interactions in 1881.



c) The entropy of potential outgoing interactions in 1851 within a 1h travel time radius.

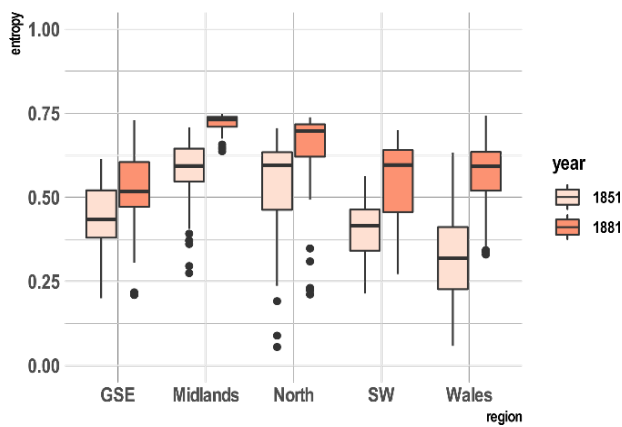


d) The entropy of potential outgoing interactions in 1881 within a 1h travel time radius.

Figure 4: Entropy of the outgoing potential interactions H^{out} in 1851 and 1881. The radius of a disc is proportional to the corresponding town's entropy, normalised so to fall within the range [0, 1]. The hue of the discs and city labels is redder when the town's entropy is larger and bluer when the town's entropy is smaller. For example, London's disc and label are bluer than Manchester's in all scenarios, so the former has consistently smaller entropy the latter; Bristol's disc and labels are bluer in 1851 than in 1881, whether the 1h travel time threshold is imposed or not, indicating that the city's entropy increases in time in both scenarios.

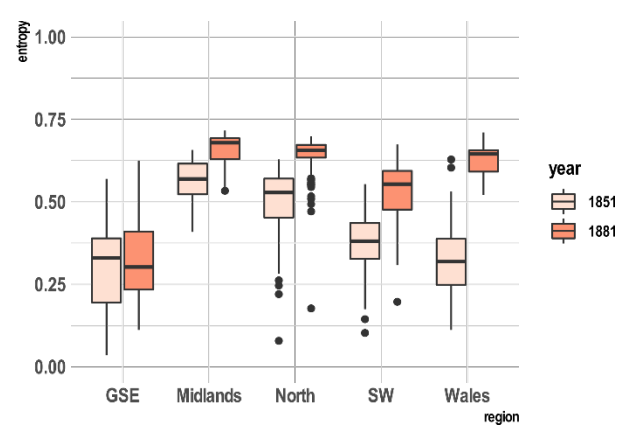
When a maximum travel time threshold is imposed, the effects of being located at the geographical extremes/centre of the area are mitigated. For a threshold of one hour in 1851 (Fig 4c), the connections in the South are substantially more polycentric, although still below the values reached in the North. This is because London remains out of reach from most areas of the South East when such a short travel time is allowed, so the towns are expected to have more balanced connections with one another. On the other hand, some towns register zero entropy, as they are further than one hour travel time from every other town, and thus all potential interactions are inevitably directed towards themselves. This is the case of several towns in Wales (including Aberystwyth), in Yorkshire, on the coast of Eastern England and, more generally, of most towns that were not reached by any railway line. In 1881 (Fig. 4d), the entropy values for the potential interactions in less than one hour of travel time resemble more closely those of the unconstrained case. In particular, the expected pattern of extreme centralisation re-emerges in parts of the “home counties”², from which one could by then reach London in less than one hour.

Entropy of incoming interactions, 1851 and 1881



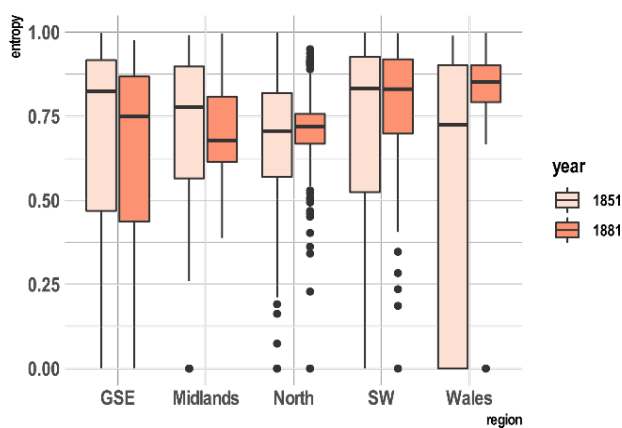
a) The median entropy of incoming connections increases from 1851 to 1881 in all regions.

Entropy of outgoing interactions, 1851 and 1881



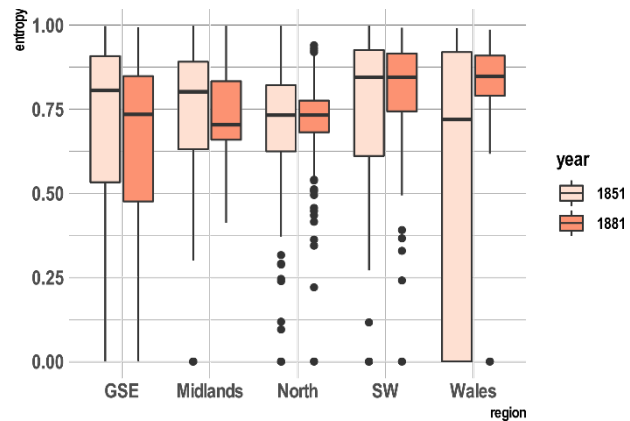
b) The median entropy of outgoing connections increases in all regions but the Greater South East.

Entropy of incoming interactions, 1851 and 1881 within 1h travel time radius



c) Imposing a 1-hour threshold in travel time, the Greater South East and the Midlands show increasing centralization in the incoming interactions.

Entropy of outgoing interactions, 1851 and 1881 within 1h travel time radius



d) Imposing a 1-hour threshold in travel time, the Greater South East and the Midlands show increasing centralization in the outgoing interactions.

Figure 5: Boxplots of the entropy of incoming and outgoing interactions, by region, with or without maximum travel time threshold. The top and bottom of each box correspond to the 1st and 3rd quartile respectively, and the horizontal line represents the median value. The lower (resp. upper) whisker extends by 1.5 times the distance from the median to the lower (resp. upper) quartile. Outliers are represented as points.

²The home counties include Berkshire, Buckinghamshire, Essex, Hertfordshire, Kent, Surrey. The name alludes to the fact that their residents could be based there but still reach London within a relatively short time.

The boxplots in Fig. 5 show the median and the upper and lower quartiles of H_R^{in} and H_R^{out} for the five regions under consideration. When no travel time threshold is imposed (Figs. 5a and 5b), the median entropies increase from 1851 to 1881 in all regions, with the exception of the entropy H_R^{out} of outgoing interactions in the Greater South East. Note, however, that both its 1st and 3rd quartiles (as well as the average) increase for this region. This suggests that, except hints towards centralisation in the South East, the increase in the number and size of cities and the expansion of the railway network led to more balanced regional systems from the perspective of potential interactions.

When a maximum travel time of one hour is assumed (Figs. 5c and 5d), the picture is more varied. The median, 1st and 3rd quartiles of the entropies of the interactions declined from 1851 to 1881 in the Greater South East, indicating growing local centralisation. In Wales, the median and 3rd quartile greatly increased. This results from the fact that several Welsh cities are geographically isolated and the expansion of the railway in 1881 increased the number of these that were reachable from at least another town in less than one hour. The same reason explains the reduction in the interquartile range. Such reduction is also observed for the remaining three regions, which is to be expected given on the one hand the decrease in isolated towns with zero entropy and on the other hand the strengthening of the influence of the main cities that are now reachable by a larger proportion of towns. Remarkable is the decrease in the median entropy values in the Midlands.

In Table 1, we show the entropy of interactions in the main city in each region. All cities become more polycentric with respect to their potential interactions from 1851 to 1881 when no threshold is imposed, indicating more balanced connectivity. Note that London attains generally smaller values for both H^{in} and H^{out} in 1881 compared to other cities, again emerging as a unique example of centralisation. When travel time is limited to one hour, the only city to become less polycentric as measured by both entropy values is Cardiff. This may be surprising given that the city saw the emergence of a growing cluster of towns in its surroundings. This result, however, is explained by fact that its population grew much faster than those of the towns in its immediate vicinity. This includes Merthyr Tydfil, whose population was still larger than Cardiff's in 1851 (whence Cardiff's very large entropy values in 1851 for this choice of travel time threshold), but only about 60% of Cardiff's by 1881.

Table 1: Entropy in the main city of each region.

City (Region)\Year	H^{in}		H^{out}		H^{in} in 1 hour		H^{out} in 1 hour	
	1851	1881	1851	1881	1851	1881	1851	1881
London (GSE)	0.035	0.134	0.200	0.341	0.051	0.137	0.025	0.084
Manchester (North)	0.554	0.667	0.430	0.595	0.447	0.602	0.375	0.574
Birmingham (Midlands)	0.519	0.649	0.409	0.580	0.532	0.578	0.480	0.530
Cardiff (Wales)	0.374	0.451	0.366	0.591	0.721	0.692	0.724	0.700
Bristol (South West)	0.308	0.419	0.174	0.442	0.000	0.366	0.000	0.406

Table 2 allows to compare these results with the Shannon entropy of the population sizes, a measure of polycentricity that, like the exponent in Zipf's law, only accounts for the populations and does not employ a network approach, but devoid of the limitations of the exponent mentioned in Section 3.1. The Shannon entropy, indeed, does not depend on any assumption on the distribution of the population sizes. According to this measure, four out of five regions in England and Wales (except the North) have seen their population become more concentrated from 1851 to 1881, following on a general trend in act since at least 1801.

Table 2: Shannon entropy of the population distribution for different regions in England and Wales between 1851 and 1881.

Region\Year	1801	1831	1851	1881	1911
North	0.841	0.793	0.764	0.774	0.780
GSE	0.467	0.447	0.436	0.418	0.453
South West	0.836	0.820	0.811	0.778	0.751
Wales	0.933	0.925	0.886	0.861	0.830
Midlands	0.877	0.851	0.833	0.795	0.783

6 Discussion

The years between 1851 and 1881 were characterised by rapid urbanisation and industrialisation in Britain, primarily in larger centres [46, 39]. They were also decades of major expansion and modernisation of the transport infrastructure. The results of our analysis suggest that while this period witnessed an increased concentration of the population into fewer urban centres, these became better integrated into a more heterogeneous network of connections. The years after 1881 brought on economic restructuring as older industries declined and newer sectors emerged and the growth shifted from large to smaller centres, especially suburban towns [39]. The economic and social factors driving these processes are complex, but certainly the conditions for this decentralisation process were made possible by the capillary expansion of the transport system in previous decades and by the strengthening of connections between larger and smaller centres. Entropy maximisation models, like the one employed in this analysis, are often used with a dynamical perspective in mind [48, 49]. Looking at the greater heterogeneity in population sizes after 1881 in the North and Greater South East (Table 2) suggests that the population growth of cities might eventually have followed the dynamics of their increasingly polycentric most-likely interactions. This observation suggests that a network-entropy-based approach to measuring morphological polycentricity might give a complementary description of the state of a region in addition to more traditional methods and shed further light on its subsequent evolution.

Several limitations and opportunities of future expansions emerge from our study. In our computation of travel time, we considered geographical distance divided by the stagecoach speed as a baseline and complemented it with distance along the railway network divided by an estimate of the average railway speed in the year of study. This approach tacitly assumes the presence of direct stagecoach roads between towns and ignores their real conditions (for instance, their steepness). Furthermore, it ignores variations in train travel speed, train frequency, cost, line capacity, line transfer times, and time needed by a typical person to reach a station along the actual street network of the towns. Lastly, it ignores the presence of alternative means of transport, such as ferries or waterway, and thus the effects of multi-modality. Our study aimed to provide a proof of concept, and any improvement in these directions would contribute to strengthen it. The Bradshaw's Monthly Railway and Steam Navigation Guides provide detailed train timetables for Britain from 1839 to 1961; transforming them into machine-readable data would greatly benefit our study, as well as any other study on historical transport networks and intercity interactions.

Another limitation is that, by drawing a boundary around a set of cities, we isolate them from the rest of the world, which is an unsatisfactory but necessary assumption. In the case of England and Wales, for example, we inevitably neglect the intense international interactions (e.g., emigration, commerce) in the 19th century and their impact on inter-county or inter-town interactions.

Finally, the entropy maximisation principle rests on the assumption that individuals' interactions at the basis of cities' interactions can be modelled as those of particles devoid of agency. More realistic models of interactions should attempt to include considerations of individuals' skills, socio-economic conditions, as well as the complexity of intentionality and decision making, as recently seen in work on cellular automata [15].

7 Conclusion

In this paper, we adapted network entropies as measures of the morphological polycentricity of systems of cities. We analysed in detail the case study of the system of towns in England and Wales and its evolution from 1851 to 1881 and remarked the contrasting pattern of increased population centralisation and growing heterogeneity of potential interactions. We showed how a network and entropy-based approach to morphological polycentricity can account for all the fundamental aspects beyond population size that such measure should capture but are frequently overlooked, including the heterogeneity of space and evolving technology- and human-centred notions of distances. It also provides a natural bridge between functional polycentricity and morphological polycentricity, unified through the lenses of network entropy. Future work will aim to improve the framework along the lines mentioned above and adapt the framework to contemporary systems of cities to construct morphological polycentricity measures better suited to a context of increased complexity in physical infrastructure and digital interactions.

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Mazzamurro, Matteo; Guo, Weisi

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