

Yibo Gao^{1,2}Linlin Geng^{1,3}Patrick G. Verdin²Ibra Fall¹Ruijie Zhang¹Zhongjie Tian¹Desheng Zhang^{1,*}

MODELING OF DUAL-FACTOR DRAG CORRECTION MODEL FOR BUBBLY FLOW UNDER ELEVATED PRESSURE

Abstract: A pressure correction method is proposed considering the influence of dual-factor. The applicability of a pressure correction method coupled with a drag model is discussed along with the accuracy of the simulation results obtained by such pressure correction method. It is found that the present pressure correction method combined with the DBS (Dual Bubble Size) drag model can accurately reflect the changing trend of gas holdup distribution with pressure. It is also established that results from this model applied to a bubble column match well experimental data. Finally, when comparing with other pressure correction models, the proposed model shows better robustness in 3D simulations, and can predict radial gas holdup distributions with better accuracy.

Keywords: Elevated pressure; Computational fluid dynamics; Bubble column; Drag; Gas holdup

Author affiliations

¹Research Center of Fluid Machinery Engineering and Technology, Jiangsu University, Zhenjiang, 212013, China.

²Energy & Sustainability, Cranfield University, Cranfield, Bedfordshire, MK43 0AL, UK.

³Wenling Fluid Machinery Technology Institute of Jiangsu University, Wenling, 317525, China.

Corresponding author: Prof. Desheng Zhang

Email corresponding author: zds@ujs.edu.cn

1 Introduction

Gas-liquid two-phase flow is present in numerous industrial processes and practical applications, particularly energy engineering. It is significantly used in oil-gas extraction and transportation, boiling systems, heat exchangers, and nuclear reactors [1-5]. Predicting the performance of these industrial processes requires knowledge of gas-liquid hydrodynamics and transport processes in the facilities. In most cases, there is significant interaction between the gas and liquid phases. Consequently, tackling the complexities of such specific gas-liquid flows presents significant challenges. The intricate nature of gas-liquid interactions has resulted in several unsolved interaction problems to date. These encompass various aspects such as phase distribution patterns, interphase interaction mechanisms, and the intricate structures of microscopic turbulence.

Computational Fluid Dynamics (CFD) simulations have emerged as a promising tool for investigating the local characteristics of gas-liquid multiphase flow, including gas holdup profiles, liquid velocity profiles,

mixing time, and shear rate profiles [6,7]. Currently, there are two main numerical methods for dealing with the gas-liquid two-phase flow: Eulerian-Lagrangian and Eulerian-Eulerian methods. The Eulerian-Lagrangian method considers the liquid phase as a continuous medium, and considers the gas phase as discrete bubbles which are tracked using equations of motion derived from the force balance principle. The primary superiority of this method lies in its ability to capture comprehensive motion information relating to individual bubbles within the flow field. However, it is essential to note that tracking each bubble demands significant computational resources, making this method more commonly applied in situations involving low volume fraction gas-liquid flow. As for the Eulerian-Eulerian approach, the dispersed and continuous phases are treated as interpenetrating continua. The governing equations based on the Eulerian-Eulerian method, solve for ensemble-averaged variables over time and space while determining the point phase fraction. The conservation equations for each phase, along with interphase exchange terms, are simultaneously solved. Interfacial force correlations are required as closures to solve the transport equations and model interactions between the phases. The Eulerian-Eulerian method offers a significant reduction in computational cost compared to the Eulerian-Lagrangian method, while maintaining a certain level of accuracy. Thus, this method is much more commonly used for modeling gas-liquid multiphase flows. The accuracy and applicability of Eulerian-Eulerian method depend on the Interfacial force correlations.

In order to better predict gas-liquid phenomena, several correlations have been developed [8-11]. They are widely used, especially in ambient pressure conditions as gas-liquid flow can be well predicted. However, many industrial facilities operate under extreme conditions characterized by high pressures [12-14]. In the literature, only a few studies have specifically addressed gas-liquid systems under high-pressure conditions. For a better design, the relationship between the flow characteristics and operating conditions must be established as per the expected objectives.

The effect of pressure can be attributed to multiple parameters. Firstly, an increase in pressure results in a slight and statistically insignificant rise in viscosity and liquid density [15]. However, the magnitude of this effect is marginal. Conversely, this effect diminishes as the bubble diameter decreases, implying that the impact of pressure is closely tied to an elevation in gas density. Similar reports have also appeared in the studies of other researchers, with the gas phase density increasing significantly with increasing pressure [12, 13, 14, 16, 17]. It is also worth noting that an increase in gas density due to pressure reduces the bubble rise velocity [16,17], resulting in higher gas holdup. In addition, the pressure-affected bubble drag force was studied; Mach et al. [18] conducted measurements on the bubble size distribution and drag coefficient of individual bubbles at four different pressures (0.14, 0.46, 2.00, and 4.00 MPa), aiming to establish the relationship between the two variables. Under constant gas holdup conditions, the drag coefficient of an individual bubble decreases as the pressure increases. Additionally, these smaller bubbles induce disturbances at the liquid edge, effectively suppressing turbulence and reducing drag forces.

Some researchers have incorporated the effect of pressure into the drag force model [19, 20]. Krishna et al. [19] added a simple gas phase density correction term (ρ_g/ρ_0) to the bubble drag force model to consider the influence of operating pressure. However, this correction factor overestimated the total gas holdup at higher apparent gas velocities and pressures. Then, Chen et al. [20] improved the density correction term to enable accurate prediction within the pressure range of 0.1-1.0 MPa. Based on Roghair's bubble swarm drag force model, Zhang et al. [21] employed a drag coefficient calculation formula with a pressure correction factor, which can be applied within the pressure range of 0.5-2.0 MPa. Tran et al. [22] used Eulerian-Eulerian method to simulate the air-water system at 0.1 MPa, 1.5 MPa and 3.5 MPa. They utilized a modified drag force model with pressure-dependent correction coefficients, which accurately captured the hydrodynamic characteristics of pressurized air-water bubble columns within a reasonable accuracy range. Tran et al. [23] tested the gas-liquid two-phase flow characteristics at a maximum of 16 MPa and measured the physical parameters such as density, viscosity and surface tension. The empirical correction factor is obtained by fitting the empirical

equation and is added to the Ishii-Zuber drag force model. Yang et al. [24] adopted 2-D simulation of a bubble column at high pressure to investigate the correction factor for such wake-accelerating and hindering effects, which rely on the bubble size distribution and local gas holdup. They postulated that local gas holdup should be considered in drag correction. Yan et al. [25] proposed three different resistance model schemes. They are optimized through density correction, the gas holdup of large and small bubbles and the energy minimization multiscale (EMMS) concept, respectively. The results obtained are of great significance for improving the gas-liquid two-phase model with high pressure and high gas holdup.

Although CFD simulation shows significant improvement in some cases through parameter-fitting, the crucial concern is whether these fitted correlations can be extrapolated to a broader range of operating conditions and physical properties. The literature shows that a parametric study is complex as the influence of a parameter itself depends on one or more other parameters. Wilkinson et al. [26] established that under a constant superficial gas velocity, the increase in gas holdup in a bubbling column with elevated pressure cannot be simply explained by the influence of gas density on bubble formation. The impact of pressure changes on fluid properties is complex, although gas density changes are the most obvious result. Therefore, introducing pressure (P) directly can more comprehensively demonstrate this effect. Besides, superficial velocity and pressure seem to have the same predominant effects [27-30]. Consequently, this work focuses on bubbles characterization under high-pressure conditions which can be used in gas-liquid flow working in these conditions. An improved pressure correction method is proposed, based on the DBS drag force model, enabling the improved model to predict a broader range of pressures.

In this paper, the necessity of pressure correction is discussed first. Such correction is embedded in different DBS drag models, and a bubble column is simulated under different pressure conditions to find the best drag model. Then, the pressure correction method is compared with different pressure correction models from the literature, and the average gas holdup distribution in the tower is analyzed under different operating conditions. The applicability and accuracy are further established, providing a valuable reference for further research on the numerical simulation of gas-liquid two-phase flow under elevated pressure.

2 Gas-liquid column experiment

The experiment of gas-liquid column under elevated pressure was described in detail by Yan et al. [25]. A sketch of the experimental setup is given in Fig. 1. The column has a circular cross-section with a diameter of 0.3m and a height of 6.6m. Air is injected into the system through a perforated plate with 128 holes of 5 mm diameter. The plate is positioned at the bottom of the column at a distance of 20 cm. In all experiments, the column is operated with an initial liquid level of 3.0 m and at a superficial gas velocity of 0.119-0.312 $\text{m}\cdot\text{s}^{-1}$. The experimental data are obtained in the column under elevated pressures (0.5-2.0 MPa) using a conductivity probe method. Electrical Resistance Tomography (ERT) is used to measure the hydrodynamic parameters of the bubble column on line, including the distribution of gas holdup. The ERT electrode matrices are installed 2.60 m far from the base of the column. Each ERT electrode matrix consists of 16 ERT electrodes, which are uniformly installed on the inner wall of the column. The cross-section is divided into ten concentric circles with radii as shown in Fig. 2. The conductivity distribution at the cross section is constructed according to the algorithm, so that the gas holdup contour of the cross-section can be interpreted through the image. The ERT data are collected after the flow reaches a steady state. 400 cross-sectional images are collected for each group. The ERT contour is obtained by superimposing these data and then averaging the corresponding positions. The experimental data used for model improvement in this paper are extracted from Yan et al. [25].

3 Numerical simulations

3.1 Eulerian-Eulerian method

The Eulerian–Eulerian method is used in this work to model the gas–liquid multi-phase flow. The derivation of the conservation equations is done by ensemble averaging the instantaneous local balance for each phase. The continuity and momentum equations are given as:

$$\frac{\partial}{\partial t}(\alpha_i \rho_i) + \nabla \cdot (\alpha_i \rho_i \bar{\mathbf{u}}_i) = 0 \quad (1)$$

$$\frac{\partial(\alpha_i \rho_i \bar{\mathbf{u}}_i)}{\partial t} + \nabla \cdot (\alpha_i \rho_i \bar{\mathbf{u}}_i \bar{\mathbf{u}}_i) = -\alpha_i \nabla P + \nabla(\alpha_i \mu_{eff,i}(\nabla \bar{\mathbf{u}} + \bar{\mathbf{u}}^T)) + \alpha_i \rho_i \mathbf{g} + \mathbf{F}_{i,j} \quad (2)$$

In the above equations, the subscript i denotes the different phases, with α , ρ and $\mathbf{u}_i$ the volume fraction, density and resolved velocity, respectively. P is the pressure and μ_{eff} is the effective viscosity which is the sum of turbulent viscosity and dynamic viscosity. $\mathbf{F}_{i,j}$ is the total interphase forces, as explained in Section 3.3.

3.2 Turbulence model

The turbulence model closes the conservation equations. It also determines the underlying turbulent structure in the two-phase flow. Turbulence becomes more complex due to bubbles in the liquid phase. No standard turbulence model is currently available for all types of gas-liquid flows. In this study, the standard k- ε model is considered.

Both phases are treated identically, in which the turbulent viscosity is calculated by:

$$\mu_{T,i} = C_\mu \rho_i \frac{k^2}{\varepsilon} \quad (3)$$

where C_μ is a constant, it was set to be 0.09, k represents the turbulence kinetic energy, and ε represents the turbulence dissipation rate.

3.3 Interphase force models

This study focuses on the gas-liquid interaction term in the momentum transfer equation. This term, known as the interphase force, represents the combined effect of multiple forces and is employed to account for momentum transfer at the gas-liquid two-phase interface [9]. The main axial forces include bubble drag force $\mathbf{F}_D$ and virtual mass force $\mathbf{F}_{VM}$. The primary radial forces consist of lift force $\mathbf{F}_L$, turbulent dispersion force $\mathbf{F}_{TD}$, and wall lubrication force $\mathbf{F}_{WL}$ [19]. A complete description of all interphase forces is shown in Fig. 3.

The term $\mathbf{F}_{i,j}$ in Eq. (2) represents the interfacial forces and is given by:

$$\mathbf{F}_{ij} = \mathbf{F}_D + \mathbf{F}_L + \mathbf{F}_{WL} + \mathbf{F}_{TD} + \mathbf{F}_{VM} \quad (4)$$

3.3.1 Drag force models

At present, the drag force is widely recognized as the predominant force between the gas and liquid phases, which arises from the relative motion between bubbles and the liquid, and serves as the primary force for momentum transfer between the two phases. The magnitude of the drag force is influenced by various factors including bubble shape, fluid properties, and the relative velocity between the two phases. The expression for the drag force experienced can be formulated as follows:

$$\mathbf{F}_D = -\frac{1}{2} C_D \rho_l |\mathbf{v}_r| \mathbf{v}_r A_b \quad (5)$$

In the equation, C_D , ρ_l , $\mathbf{v}_r$, and A_b represent the drag coefficient, the density of the liquid, the relative velocity between the two phases, and the projected area of the gas phase, respectively. Eq. (5) indicates that the definition of the drag coefficient C_D determines the magnitude of the drag force. However, the drag coefficient is uncertain for different flow states of the fluid. Thus, researchers mainly investigate the drag coefficient models to reduce this uncertainty. In this work, we evaluate different models of drag coefficient C_D under high-pressure conditions. These models include several DBS models as well as models proposed by Yan et al. [25], specifically designed to be applicable under elevated pressure conditions. And for comparison purposes, we also considered the Ishii-Zuber model. More details are provided in Tab. 1.

The DBS model has been developed by Yang et al., based on the EMMS (Energy-Minimization Multi-Scale) approach [31,32]. In the conceptual model, the dispersed phase of a gas-liquid system was resolved into a small-bubble phase and a large-bubble phase based on experimental observations from the literature. A stability condition was introduced to reflect the compromise in competition between two dominant mechanisms: the minimization of energy dissipation arising from bubble shape oscillation, representing a liquid-dominant mechanism in which smaller bubbles prevail, and the minimization of energy dissipation through the energy cascade process of liquid turbulence, representing a gas-dominant mechanism that favors the formation of larger bubbles. More details can be found in Yang et al. [31,32]. From the DBS model, the ratio of effective drag coefficient to bubble diameter C_D/d_b for the gas phase can be formulated as:

$$\frac{C_D}{d_b} = \left[\frac{\alpha_{g,small}}{d_{small}} C_{D,small} \left(\frac{U_{g,small}}{\alpha_{g,small}} \right)^2 + \frac{\alpha_{g,large}}{d_{large}} C_{D,large} \left(\frac{U_{g,large}}{\alpha_{g,large}} \right)^2 \right] \frac{\alpha_g}{U_g^2} \quad (6)$$

where α_g is the gas holdup, U_g is superficial gas velocity, and $C_{D,i}$ is the drag coefficient for the small- or large-bubble phase. The average drag force per unit volume is formulated as:

$$\mathbf{F}_D = -\frac{3}{4} \alpha_g \alpha_l \rho_l \frac{C_D}{d_b} |\mathbf{v}_r| \mathbf{v}_r \quad (7)$$

Xiao et al. [33] applied the mesoscale drag force model to the bubble column simulation, calculated the C_D/d_b of the bubble at different superficial gas velocities, and then established the mathematical correlation between C_D/d_b and superficial gas velocity U_g (Tab. 1). This model is also called "Global DBS model" because it is directly related to the macroscopic operating parameters. However, in a gas-liquid system, the flow has strong spatiotemporal heterogeneity, and the local flow effect creates the mesoscale structure which shows obvious spatiotemporal dynamic changes. Based on this, Jiang et al. [34] first tried to correlate C_D/d_b with the local superficial gas velocity U_g' and superficial liquid velocity U_l' . Then, they substituted the superficial velocity of each calculation cell into the C_D/d_b of the calculation cell. Similarly, Guan et al. [35] calculated the gas holdup and C_D/d_b of the mesoscale theoretical model at different superficial gas velocities. Then they achieved the correlation between C_D/d_b and gas holdup in each calculation cell. This is a drag closure without adjustable parameters. Therefore, the "Local DBS model" that considers the local flow characters effect is more suitable.

In addition, Yan et al. [25] recommended three drag force models for pressure correction, referred to as Model A, Model B and Model C, respectively (see Tab. 1). Model A is based on Roghair's small bubble swarm drag force model, and it is corrected by adding large bubble holdup, small bubble holdup and density correction items [36,37]. In model B, the threshold value of gas holdup is set, and the gas phase is divided into the sparse part and dense part with different processing methods, and then it is corrected based on Buffo's bubble swarm model [33,38,39]. Model C introduces a closed model of the relationship between the drag coefficient and the bubble diameter, and adjusts the model parameters by adding a density correction term accordingly [40,41].

3.3.2 Lift force model

The lift force acts on a moving bubble in the lateral direction whose direction is susceptible to liquid shear and bubble size. Eq. (8) approximates the shear-induced lift force of a bubble:

$$\mathbf{F}_L = -C_L \rho_l \alpha_g \mathbf{v}_r \times (\nabla \times \mathbf{v}_l) \quad (8)$$

where C_L represents the lift coefficient, $\mathbf{v}_r$ represents the velocity of the liquid phase. The lift force bears a major effect on the lateral distribution of the bubbles.

In this study, we use the lift model developed by Liu et al. [42]. It consider that the stable distribution of gas holdup is regarded as the dynamic equilibrium between the lift force and the turbulent dispersion force, as follows:

$$\frac{C_L}{C_{TD}} = -0.2(1 - \alpha_g) \frac{\alpha_l^2}{\alpha_l} \quad (9)$$

where α_l represents the liquid volume fraction.

3.3.3 Wall lubrication force model

The wall lubrication force is generated by changes in the velocity distribution around the air bubbles near the wall [43]. Its expression is as follows:

$$\mathbf{F}_{WL} = C_{WL} \frac{\rho_l \alpha_g}{d_b} \mathbf{v}_{rw}^2 \mathbf{n}_w \quad (10)$$

where C_{WL} represents the wall lubrication force coefficient, $\mathbf{v}_{rw}$ represents the tangential component of the relative velocity at the wall, and $\mathbf{n}_w$ represents the unit outer normal vector on the wall. The wall lubrication force adopts Tomiyama's model based on the bubble Eu (Eötvös) number [44], which is given by:

$$C_{WL} = \begin{cases} 0.47 & Eu < 1 \\ e^{-0.933Eu+0.179} & 1 \leq Eu < 5 \\ 0.00599Eu - 0.0187 & 5 \leq Eu < 33 \\ 0.179 & 33 \leq Eu \end{cases} \quad (11)$$

3.3.4 Turbulent dispersion force model

The turbulent dispersion force is due to the fluctuating component of the forces acting on bubbles, it can be expressed as:

$$\mathbf{F}_{TD} = -C_{TD} \rho_l k_l \nabla \alpha_g \quad (12)$$

The turbulent dispersion force coefficient C_{TD} is given by [25]:

$$C_{TD} = \frac{\alpha_{g,\max} - \alpha_g}{\alpha_{g,\max} - \alpha_{g,\min}} \quad (13)$$

where $\alpha_{g,\min}$ and $\alpha_{g,\max}$ represent constants of the limit function in the turbulent dispersion force. In this work, the values are taken as 0.3 and 0.7, respectively.

3.3.5 Virtual mass force model

The acceleration of bubbles relative to the liquid induces the acceleration drag, and is called virtual mass force. Eq. (14) represents this virtual mass force.

$$\mathbf{F}_{VM} = \alpha \rho_l C_{VM} \left(\frac{d_g \mathbf{v}_g}{dt} - \frac{d_l \mathbf{v}_l}{dt} \right) \quad (14)$$

The virtual mass force coefficient depends on the flow regime, the geometry of the discrete phase, and the volume fraction. Usually, a fixed virtual mass force coefficient is used. However, for some special conditions, such as pipe diameters greater than 0.15 m, the effect of virtual mass forces on simulation results can be ignored [45,46]. Although the study of interphase mechanisms should fully consider interphase forces [47], the virtual mass force is neglected in this study as we are primarily concerned with steady-state flow conditions.

3.4 Simulation settings

The CFD simulation was carried out through the commercial CFD code ANSYS Fluent 20.2, and the different drag models were integrated through user-defined functions (UDF). The transient and convective terms were discretized by a second-order implicit and second-order upwind schemes, respectively. The phase-coupled SIMPLE approach was selected to deal with the pressure-velocity coupling. When the simulation reaches a quasi-steady state (about 60s of physical time), the transient results for the next 30s were time-averaged to obtain steady-state flow fields.

The bubble diameter at each corresponding flow condition measured by Yan et al. [25] was used in the simulations (Tab. 2). The bottom column was modeled as a “velocity-inlet” boundary, similarly to Yan et al. An “outflow” type boundary was applied at the top outlet. The column wall was set as a no-slip boundary with a standard wall function. We assumed a constant pressure across the column and ignored the internal pressure drop.

It should be mentioned that a clear gas cap above the liquid-free surface was included in the simulations. In this gas cap, special measures must be taken to prevent numerical problems due to the high gas holdup [48]. This is accomplished as follows:

$$\text{if } \alpha_g > 0.9, \quad \begin{cases} C_D = 0.05 \\ C_L = 0 \\ C_{WL} = 0 \end{cases} \quad (15)$$

3.5 Development of density correction method

In many group simulations of this study, the gas-liquid flow presents significantly different distribution characteristics under different superficial gas velocities and different pressures. Therefore, the drag force model should consider the complex interaction of multiple factors, such as the fact that pressure has a significant effect on the density of the gas phase [16,19,20]. Based on Krishna et al.'s work [20] and considering the sensitivity of bubble hydrodynamics to various parameters, a novel correction method

coupling multiple operating parameters was further developed. By comparing the simulation data with the experimental values, we derived the correction parameter K_c used for the calculation, and the relationship of the correction coefficient K for the change of the double parameters was obtained by polynomial fitting, as shown in Fig. 4. The improved pressure correction formula is shown in Eq. 16, where the drag correction under different pressures is achieved through a density correction term.

To assess the goodness of fit of our model, we calculated the coefficient of determination (R^2). The R^2 value was found to be 0.9803, indicating that our model accounts for 98.03% of the variability in the data, shown in Tab. 2. This high R^2 value suggests a strong congruence between the model and the experimental data. To further validate the model's efficacy, we computed additional error metrics. The Root Mean Square Error (RMSE) was calculated to be 0.0604, while the Mean Absolute Error (MAE) was 0.0501. These relatively low error values provide additional support for the high precision of the model. It is noteworthy that despite the model's overall excellent performance, some discrepancies persist at specific data points. The maximum absolute error was observed to be 0.143, occurring at the negative edge of the data range ($K_c = -0.59$, $K = -0.447$). This observation suggests that the model's predictive accuracy may slightly decrease at extreme conditions.

Additionally, the correction coefficient K exhibited negative values under lower pressure conditions. This phenomenon can be attributed to the application of the density correction method proposed by Krishna et al. In this method, excessively large power exponents can lead to an overestimation of the gas holdup. Consequently, the negative values of K serve as a constraint to prevent such overestimation.

$$C_{D,correction} = \left(\frac{\rho_g}{\rho_{g,0}} \right)^K \times C_D \quad (16)$$

$$K = -2.3234 + 12.8126U_g + 1.6849P - 22.9952U_g^2 - 0.3029P^2 - 1.7152U_gP \quad (17)$$

where P and ρ_g are current operating pressure and gas density, $\rho_{g,0}$ is the atmospheric density of air. The applicability of this density correction method will be discussed in the following sections.

4 Results and discussion

In this section, the necessity of the drag force coefficient correction under elevated pressure is discussed first. By doing so, the present pressure correction method was coupled with different DBS models to find the most suitable combination model. Then, the present model was compared with other pressure correction models to further show its advantages.

4.1 The discussion of the applicability of the present method to DBS models

In this section, we discuss the applicability of this new pressure correction method to DBS models and to the Ishii-Zuber model. Fig. 5 compares the local average gas holdup distributions with or without corrections applied to different DBS models and to the Ishii-Zuber model. It can be seen from the figure that when the correction method is not applied, the distribution of gas holdup is the lowest. Besides, it cannot predict the tendency of gas holdup changing with pressure. On the contrary, when the correction method is applied, the results show an increment of gas holdup with the pressure elevation, which is consistent with the experimental results. Therefore, there is no doubt that the current correction method can not only help the drag model predict the trend of void fraction changing with pressure, but it can also be applied to different drag models, because the essence of the modified method is to reflect the effect of pressure on gas holdup.

In more detailed comparisons, it is also found from the comparison chart that the Ishii-Zuber model has the worst performance, and cannot accurately predict the gas holdup in any pressure stage. Among the three DBS models compared, Jiang et al.'s model shows poor applicability to the correction method,

which is caused by the model's sensitivity to local velocities, and the models of Guan et al. and Xiao et al. were well adapted to the correction method. However, after applying the present pressure corrections, the models show different levels of accuracy. Guan et al.'s model demonstrates improved results across all pressure ranges tested. Xiao et al.'s model performs well only at lower pressures ($P < 1.0$ MPa). At higher pressures ($P > 1.5$ MPa), this model tends to overestimate the gas holdup.

Fig. 6 shows the snapshots of gas holdup distribution predicted by different drag models at 60s. The contour plots reveal the gas holdup distribution with a core-peaking. The gas holdup distribution obtained from all the drag models is inhomogeneous in the axial direction. There is a wobbling motion, suggesting a heterogeneous flow state. The wobbling motion of all models first occurs at the bottom of the bubble column, and the inhomogeneity of these dynamic wobbling structures weakens as the height increases. Bubbles are mainly concentrated in the central region of the column, and the high gas holdup fluctuates as a spiral structure. Low gas holdup regions form in the near-wall region. This is consistent with the conclusion of Guan et al. [35] and Ojima et al. [49]. However, the axial variation of local gas holdup estimated by Jiang et al. and Ishii-Zuber's models with correction is still weak. In contrast, the variation from the other six drag models is much more evident, exhibiting meandering gas swirls. Moreover, Jiang et al. and Ishii-Zuber's models provide the highest local gas holdup, which may be due to inappropriate pressure correction. Furthermore, we also observed similar trends under other simulated conditions in this study.

Overall, the above facts prove that pressure correction methods are necessary when simulating bubble flow at different pressures. In addition, it is found that there is excellent applicability when applying the new pressure correction method to Guan et al.'s DBS model because the Ishii-Zuber model and Jiang et al.'s models exhibit poor predictive performance, indicating their limited applicability to the new pressure correction. Therefore, the pressure correction method and the DBS model of Guan et al. was coupled as a novel drag force model to conduct the following simulations and to compare with other pressure correction models.

4.2 Comparison of the present model with other pressure correction models

In this section, the performance of the present model was compared with three models from Yan et al., denoted Model A, Model B and Model C, by modelling the gas-liquid flow at higher and lower superficial gas velocities ($u_g = 0.121$ m/s and $u_g = 0.296$ m/s) and for four pressure conditions ($P = 0.5, 1.0, 1.5, 2.0$ MPa). It is noted that the radial gas holdup distributions predicted by Model A, B and C, are extracted from reference [25], in which the radial average gas holdup distributions in Fig. 7 and Fig. 9 is only obtained from 2D simulations, while the contours of the time-average gas holdup distribution in Fig. 8 and Fig. 10 is obtained from Yan's 3D simulation work.

Fig. 7 shows the radial gas holdup distributions at lower superficial gas velocity ($u_g = 0.121$ m/s). It can be seen that the gas holdup profiles predicted by these four pressure correction models are consistent with the experimental results, namely, the variation in gas holdup is associated with the magnitude of system pressure. At a pressure of 0.5 MPa, as shown in Fig.7(a), the gas holdup distribution of the four models is consistent in the central region of the bubble column. However, in the area close to the wall, the present model exhibits an advantage over other models proposed by Yan et al., but this advantage is limited to low-pressure conditions. At 1.0 MPa, Model A displays a poor match with the experimental data. This is different from the other three models, where the distribution is close to coincidence with the experimental data. Under a pressure of 1.5 MPa, the error between different drag force models is amplified, and Models A and C both underestimate the gas holdup, especially the former. The present model slightly underestimates the gas holdup in the central region, while it overestimates the gas holdup in the region close to the wall. In the highest-pressure environment tested ($P = 2.0$ MPa), the present model and Model A both overestimate the gas holdup near the wall. However, in general, the present model predicts reasonably well the gas holdup in the entire flow area. A comparison between these four models suggests that the present model predicts slightly higher local gas holdup and a bit

flatter distribution due to the higher ratio of effective drag coefficient to bubble diameter close to the wall.

Fig. 8 shows the average gas holdup distribution contours for different system pressures at a lower superficial gas velocity. The experimental contours are collected and averaged by the ERT after the flow reaches a steady state. The simulation contours are obtained by the average value of the simulation time from 60s to 90s. The simulation data used for comparison is from the 3D simulation results of Yan et al. At 0.5 MPa, experiments show that the gas holdup is higher at the center of the column, and lower near the wall. This distribution trend is well captured by the present model, while Yan et al.'s model presents a gas holdup distribution that is inconsistent with the experimental results, e.g. Model A underestimates the gas holdup on the right near the wall, Model B underestimates it on the left, and Model C shows a lower gas holdup distribution on the upper portion of the contour. At 1.0 MPa and 1.5 MPa, the superiority of the present model is not obvious, as the color differences corresponding to different gas holdup in the contours are small. But on closer observation, it is found that the results obtained by the present model at 1.5 MPa are more in agreement with the experimental results, which predict a relatively high gas holdup at the center of the column. At 2.0 MPa, it is found that the present model again predicts the gas holdup distribution well in agreement with the experiments, while Model A clearly underestimates it at the center of the column, and Models B and C predict a higher gas holdup on the left and upper left sides of the contours, respectively, which is completely inconsistent with the experimental results.

Based on the above information, the following conclusions can be drawn. Comparing to the Yan's model which shows inhomogeneous gas holdup distribution, the present model compares well with the experimental results under four pressure conditions, especially at 0.5 MPa and 2.0 MPa.

Fig. 9 shows the average gas holdup distribution predicted by different drag force models at higher superficial gas velocity ($u_g = 0.296$ m/s). The predictions given by these models are quite different. With an increased operating pressure, the gas holdup in each radial position increases and the profile becomes steeper.

At lower pressure conditions ($P = 0.5$ MPa), the present model shows good prediction. In contrast, all of Yan et al.'s models tend to underestimate the gas holdup near the wall to some extent while slightly overestimating it at the center of the column, with Model C showing the most significant deviation. At an operating condition $P = 1.0$ MPa, Model A exhibits an overall overestimation of the gas holdup. Model B predicts a lower gas holdup in the center region of the bubble column than the experimental value. The present model and model C perform well in estimating the local gas holdup. At $P = 1.5$ MPa, all models give good predictions except Model A. At the highest operating pressure tested ($P = 2.0$ MPa), all four models provide acceptable results except Model A, which overestimates the gas holdup in the area near the wall.

In summary, under higher superficial gas velocity conditions, Yan's three models encountered certain challenges at all four pressures considered. Specifically, the simulation results of Model C at 0.5MPa, Models A and B at 1.0MPa, and Model A at 1.5MPa and 2.0MPa were not ideal. This shows that it is difficult for these models to provide accurate simulation results under all four pressure conditions at the same time. In contrast, the new model proposed in this study showed good adaptability and accuracy under all four operating conditions.

Fig. 10 shows the time-average gas holdup distribution contours for different system pressures at higher superficial gas velocity ($u_g = 0.296$ m/s). The results obtained at higher superficial gas velocity are similar to those obtained at lower superficial gas velocity, with the column gas holdup decreasing as the distance from the wall decreases. At 0.5MPa, the present model captures the same distribution as the experiments, while Yan et al.'s 3D simulation underestimates the gas holdup at the column center to some extent. At 1.0MPa, the contour of the present model is consistent with the experiment result,

while Model A overestimates the gas holdup near the wall. Models B and C predict a higher gas holdup in the column, especially Model B. Under a pressure of 1.5 MPa, the present model and Model C both predict the gas holdup distribution well, but Models A and B overestimate the gas holdup near the wall, resulting in a too high overall gas holdup, a gradient of gas holdup distribution not obvious, and a poor similarity with the experiment. At 2.0 MPa, the present model predicts the gas holdup distribution well again. The gas holdup predicted by Models A and B deviate from the center of the column, especially Model B. Model C overestimates the overall gas holdup prediction, and there is an inconspicuous gas holdup gradient.

Based on the above comparison, it is found that the present model shows higher advantages under all pressure conditions at higher superficial gas velocity. The prediction accuracy is better than with other models from Yan et al, who explained the lack of accuracy by the complexity and variability of 3D simulations, which is reflected in the poor applicability of the model to 3D simulations. Therefore, the comparison results further establish the robustness of the present model. It should be noted that the same conclusions can be reached for all other working conditions, where it is shown that the present model is always more accurate.

The advantages of the present model are not so obvious in the radial gas holdup distribution, but all the present results are extracted from 3D simulations, while Yan's results are obtained by 2D simulations. Therefore, it is believed that the present model shows better robustness and applicability at different operating conditions, and it is more accurate for predicting the gas-liquid flow in actual industrial design applications.

4.3 Validation of the modified drag correction model

From the research of Reilly et al. [50], it is known that the influence of column diameter on gas-holdup can be neglected. We have therefore selected experimental data of gas-holdup versus superficial gas velocity for the water/air system to validate the modified drag correction model. Fig. 11 shows the comparison between simulation results from the modified drag force model and from Wilkinson et al., Tao et al. and Zhang et al.'s experimental data [21,26,51] under the pressures 0.5, 1.0, 1.5, and 2.0 MPa, at different superficial gas velocities. As can be seen, the simulation results of gas holdup are in agreement with the experimental data. This shows that the modified drag correction model, according to the experimental data of Yan et al., can be applied to the simulation under other experimental conditions.

5 Conclusions

The increase of system pressure can cause a dramatic increase of gas holdup in bubble columns. This cannot be captured by simply changing the liquid properties in CFD models. This work focused on simulating the effects of operating pressure on hydrodynamic behavior. A novel drag model is proposed by introducing a correction factor to reflect the effect of pressure and superficial velocity of bubble column. The pressure correction factor includes two parameters which means that it actually reflects more complex operating conditions.

The necessity of using the pressure correction method under the elevated pressure condition is discussed. The pressure correction method can make the bubble drag coefficient take different values under different pressures. This can help the drag force model to better predict the trend of gas holdup changing with pressure, and determine the most suitable drag model with this correction method. It is also found that the heterogeneous flow state predicted by the model is reasonable when comparing contours depicted in Fig. 6, and it is shown that the wobbling motion in the column and the reduction of inhomogeneity change with height.

In addition, the present model was compared with the pressure correction model from the literature by modelling the gas-liquid flow under a pressure range of 0.5-2.0 MPa and a superficial gas velocity of 0.121-0.296 m/s. It is found that the new drag model is able to take multiple factors into account, and can obtain more agreeable simulation results under various operating conditions. Overall, the present model shows good robustness and significant improvement for the CFD prediction of gas-liquid flow. It is believed that if computational models are carefully constructed, they can have great industrial application prospects in the field of multiphase flow.

Table 1. Drag force models.

Reference	Corrections of the drag force model
Xiao et al. (2013)	$\frac{C_D}{d_b} = \begin{cases} 431.14 - 6729.02U_g + 35092.2U_g^2 & U_g \leq 0.101 \text{ m/s} \\ 122.49 - 553.94U_g + 741.24U_g^2 & U_g > 0.101 \text{ m/s} \end{cases}$
Jiang et al. (2015)	$\frac{C_D}{d_b} = \begin{cases} 560.2 + 1108.9U_g' + 2812.6U_i' + 3681.5U_g'^2 + 316.8U_g'^3 & U_g' \geq 0 \text{ \& } U_i' \geq -0.5 \\ 5 & U_g' \geq 0 \text{ \& } U_i' < -0.5 \\ 500 & U_g' < 0 \end{cases}$
Guan et al. (2021)	$\frac{C_D}{d_b} = (1 - \theta) \times [723.26 \exp(-2.76\alpha_g) - 248.37] + \theta \times 488.54 \exp(-7.17\alpha_g)$ $\theta = 0.5 + 0.5 \times \tanh[300 \times (\alpha_g - 0.26)]$ <p>Model A</p> $\frac{C_D}{C_{D0}(1 - \alpha_g)} = 0.2 \left(\frac{\rho_g}{\rho_{g,0}} \right)^{0.85} + \frac{22}{Eo + 0.4} \alpha_{g,small} + \frac{Eo}{5} \alpha_{g,large}$ $C_{D0} = \begin{cases} 24(1 + 0.15Re_b)/Re_b & Re_b \leq 1000 \\ 0.44 & Re_b > 1000 \end{cases}$ <p>Model B</p> $\frac{C_D}{C_{D0}(1 - \alpha_{g,small})} = 1 + \frac{22}{Eo + 0.4} \alpha_{g,small} \quad \alpha_{g,large} \leq 0.08$ $\frac{C_D}{C_{D0}} = \left(\frac{\rho_g}{\rho_{g,0}} \right)^{0.87} \cdot (1 - \alpha_{g,large})^{8.23} \quad \alpha_{g,large} > 0.08$ <p>Model C</p> $\frac{C_D}{d_b} = \left(\frac{\rho_g}{\rho_{g,0}} \right)^{1.28} \cdot (4.7 + 8U_g - 69.47U_g^2)$ $C_D = \max \left(\min(C_{D_{ellipse}}, C_{D_{cap}}), C_{D_{sphere}} \right)$ $C_{D_{ellipse}} = \frac{24}{Re_b} (1 + 0.1Re_b^{0.75}), \quad 0 \leq Re_b < 1000$ $C_{D_{ellipse}} = \frac{2}{3} \sqrt{Eo}, \quad 1000 \leq Re_b \quad C_{D_{cap}} = \frac{8}{3}$
Ishii & Zuber (1979)	

Table 2. Coefficient of pressure correction method.

P (MPa)	ρ_g/ρ_0	U_g (m/s)	d_0 (mm)	K_c	K	R^2
0.5	5.265306	0.121	9.3	-0.59	-0.447	0.9803
		0.174	9.7	-0.20	-0.173	
		0.233	10.0	-0.01	-0.020	
		0.296	10.4	0.00	-0.033	
1.0	10.53061	0.121	8.8	0.23	0.165	
		0.174	9.4	0.33	0.293	
		0.233	9.9	0.38	0.396	
		0.296	10.2	0.40	0.329	
1.5	14.80816	0.121	8.4	0.50	0.425	
		0.174	9.2	0.65	0.608	
		0.233	9.7	0.58	0.660	
		0.296	10.1	0.50	0.539	
2.0	20.24490	0.121	7.7	0.61	0.633	
		0.174	8.4	0.70	0.771	
		0.233	9.3	0.75	0.772	
		0.296	9.6	0.55	0.597	

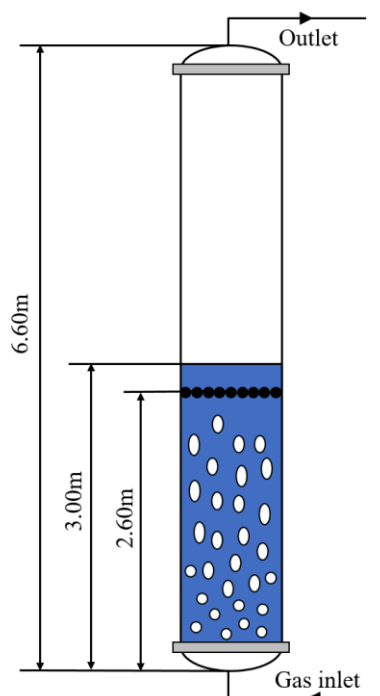


Figure 1. Schematic of the experimental column.

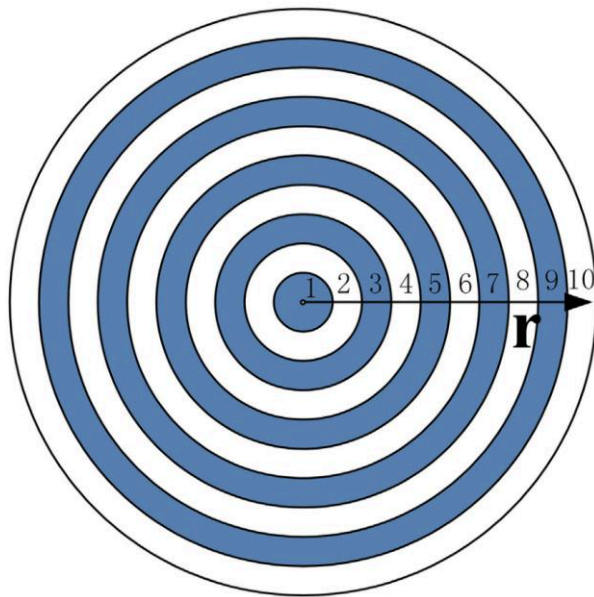


Figure 2. Schematic diagram of the radial distribution [25].

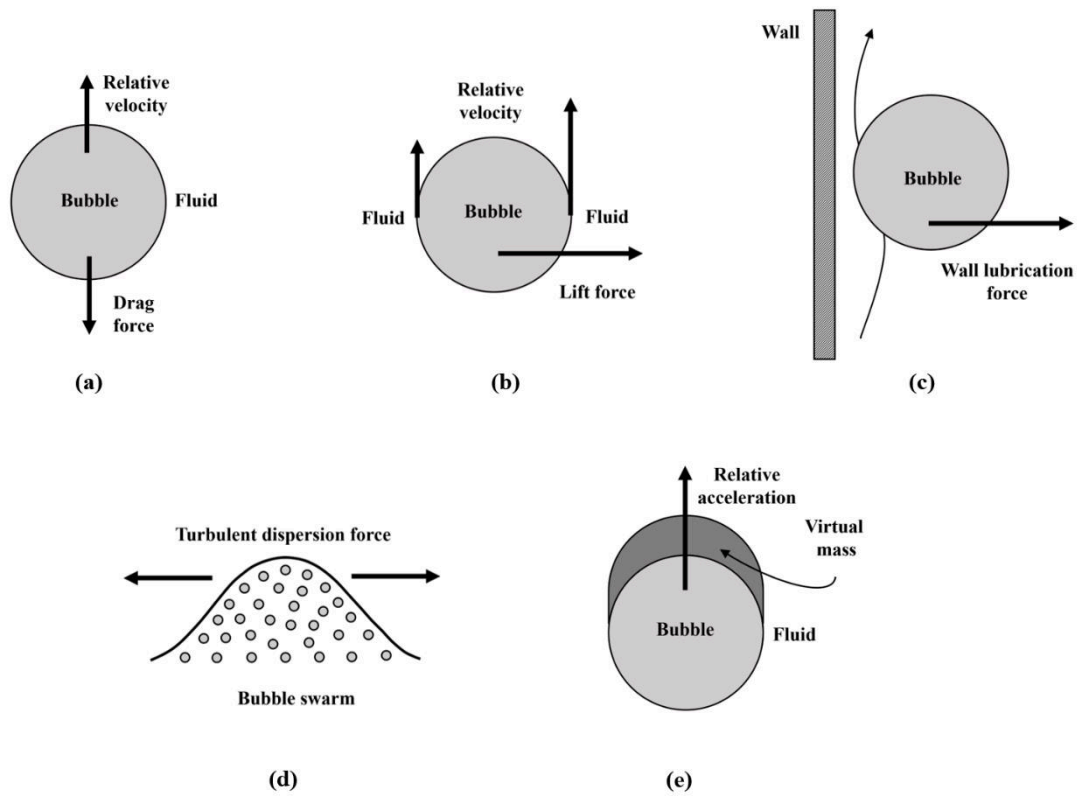


Figure 3. Schematic representation of various interfacial forces: (a) drag force, (b) lift force, (c) wall lubrication force, (d) turbulent dispersion force and (e) virtual mass force.

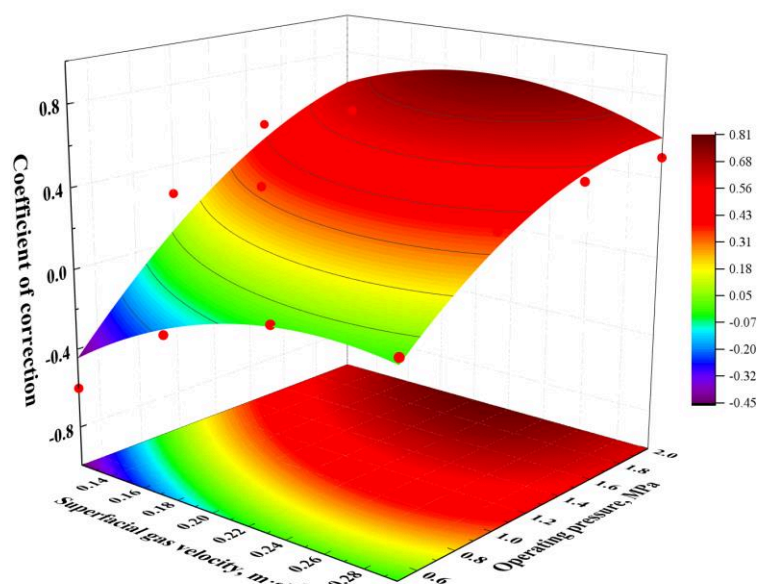


Figure 4. Correction of the relationship between parameters and operating conditions.

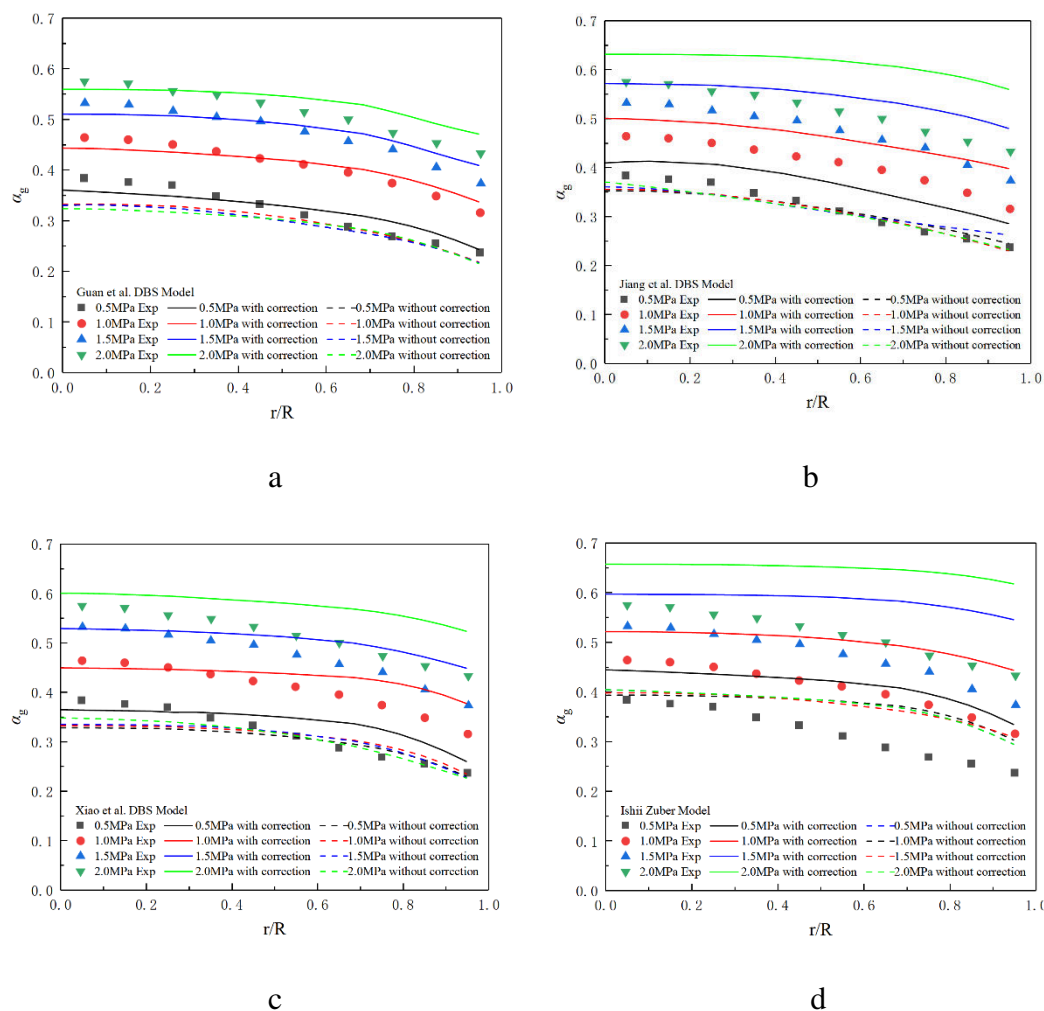


Figure 5. Radial gas holdup distributions at different system pressure ($z = 2.60$ m, $u_g = 0.174$ m/s).

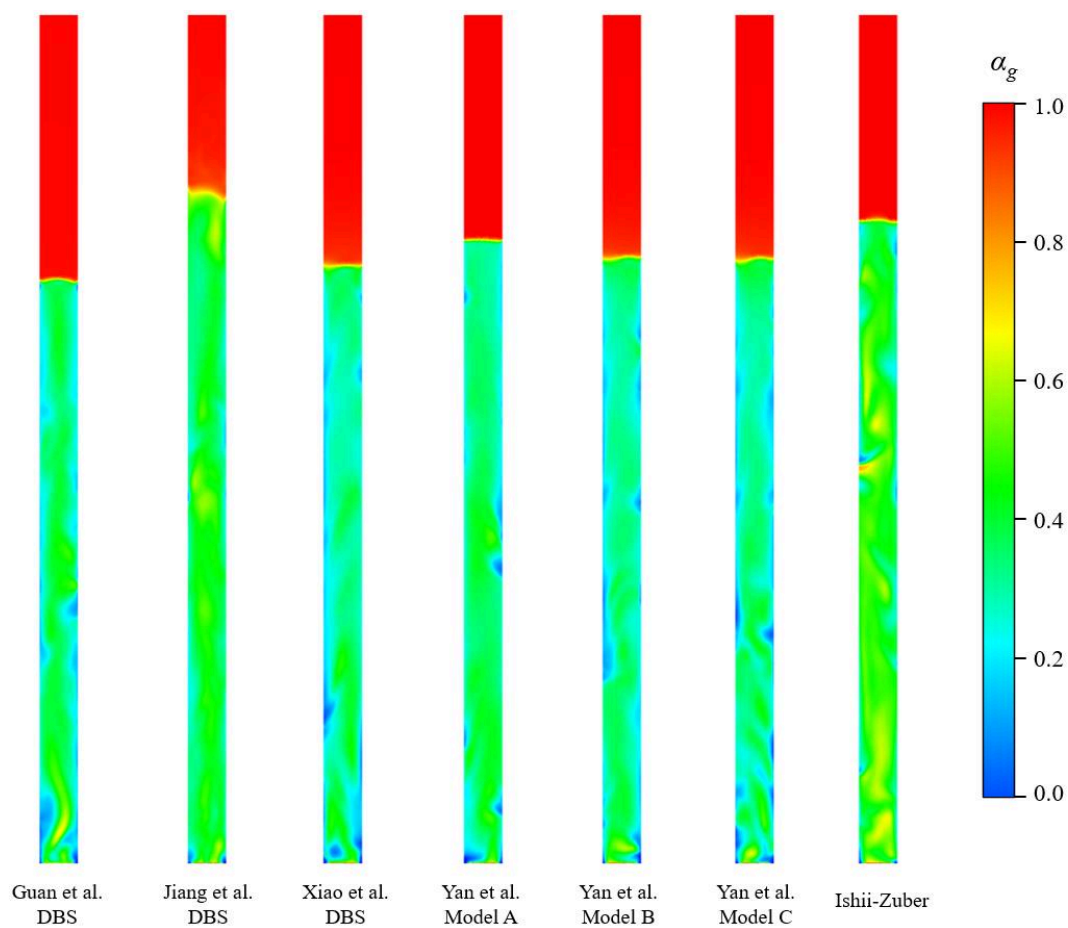


Figure 6. Contours of the gas holdup distribution with various drag force models for $t = 60$ s ($P = 0.5$ MPa , $u_g = 0.174$ m/s).

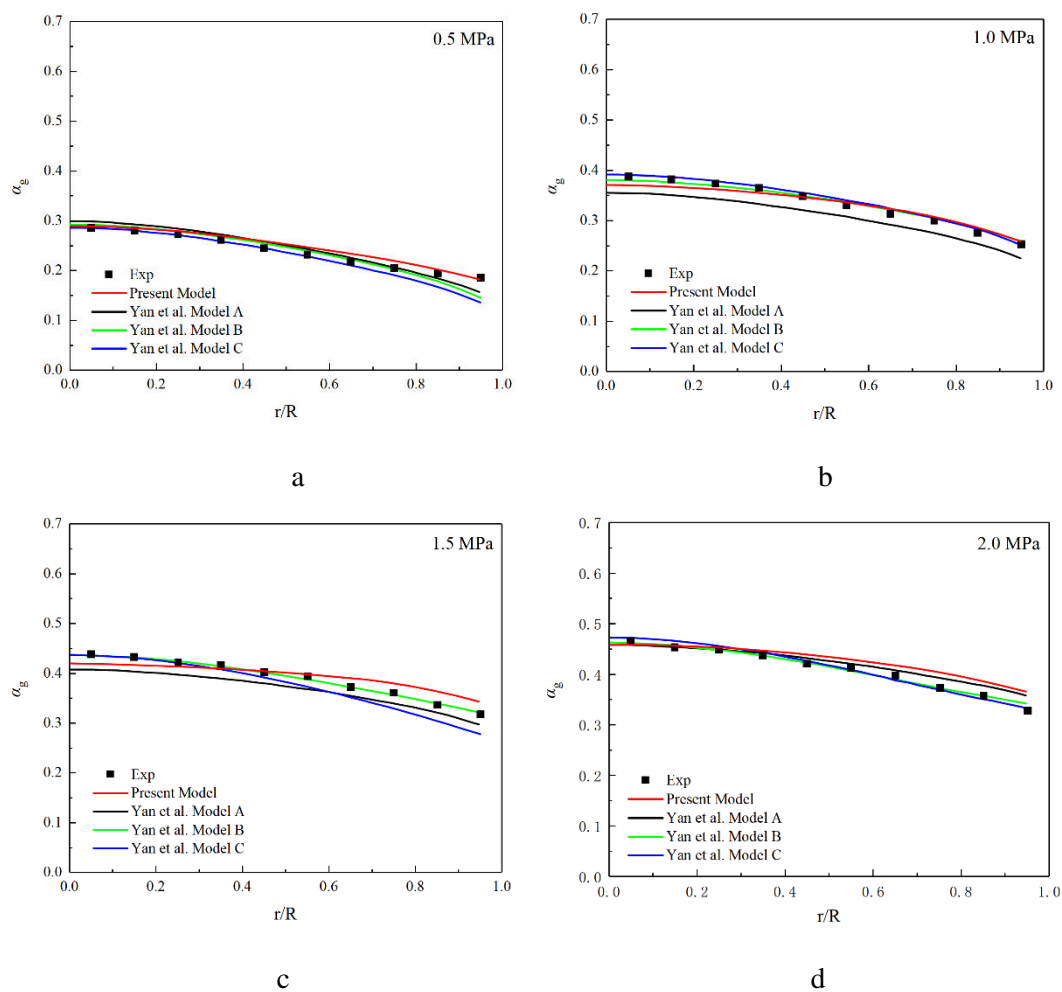


Figure 7. Radial gas holdup distributions at different system pressure ($z = 2.60$ m, $u_g = 0.121$ m/s).

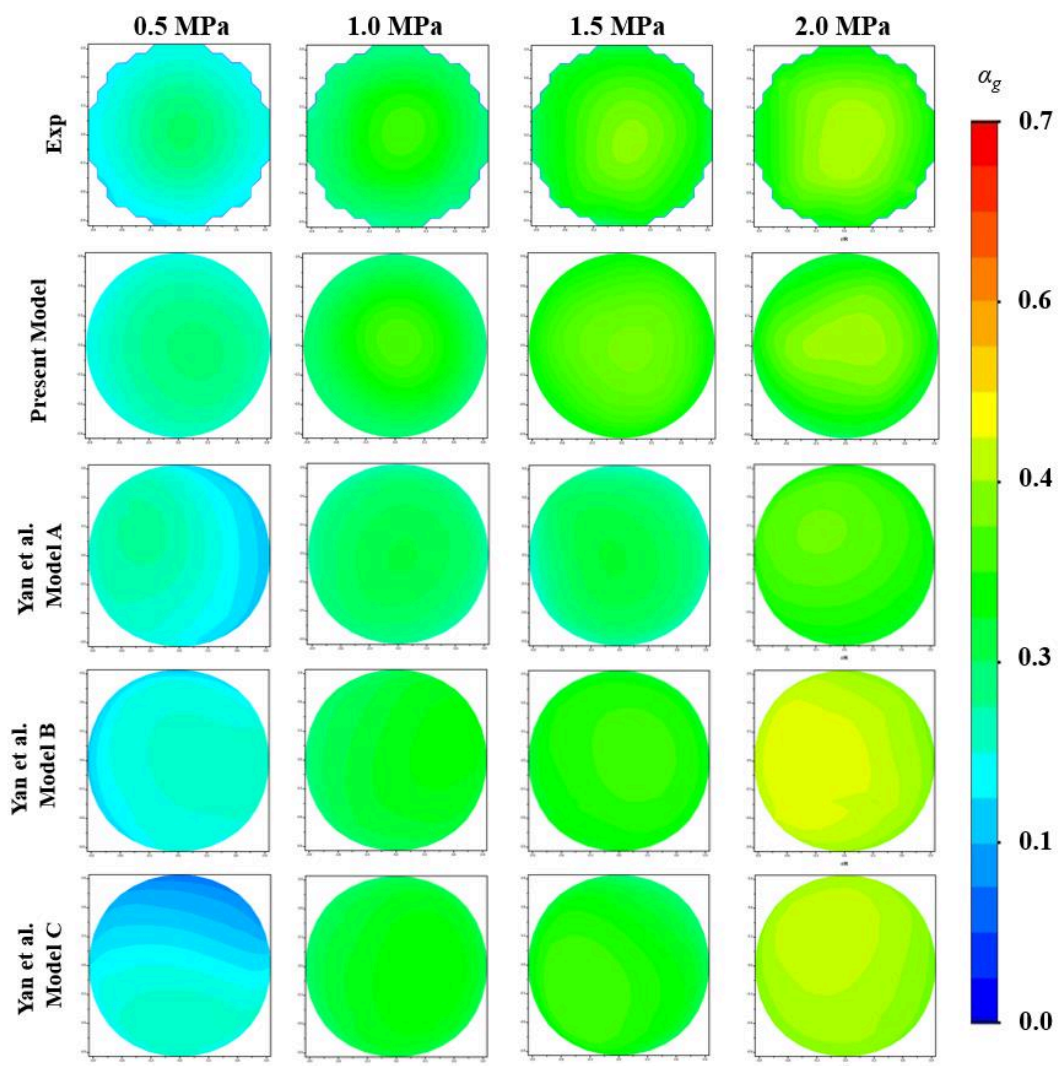


Figure 8. Contours of the mean gas holdup distribution at different system pressure ($z = 2.60$ m $u_g = 0.121$ m/s).

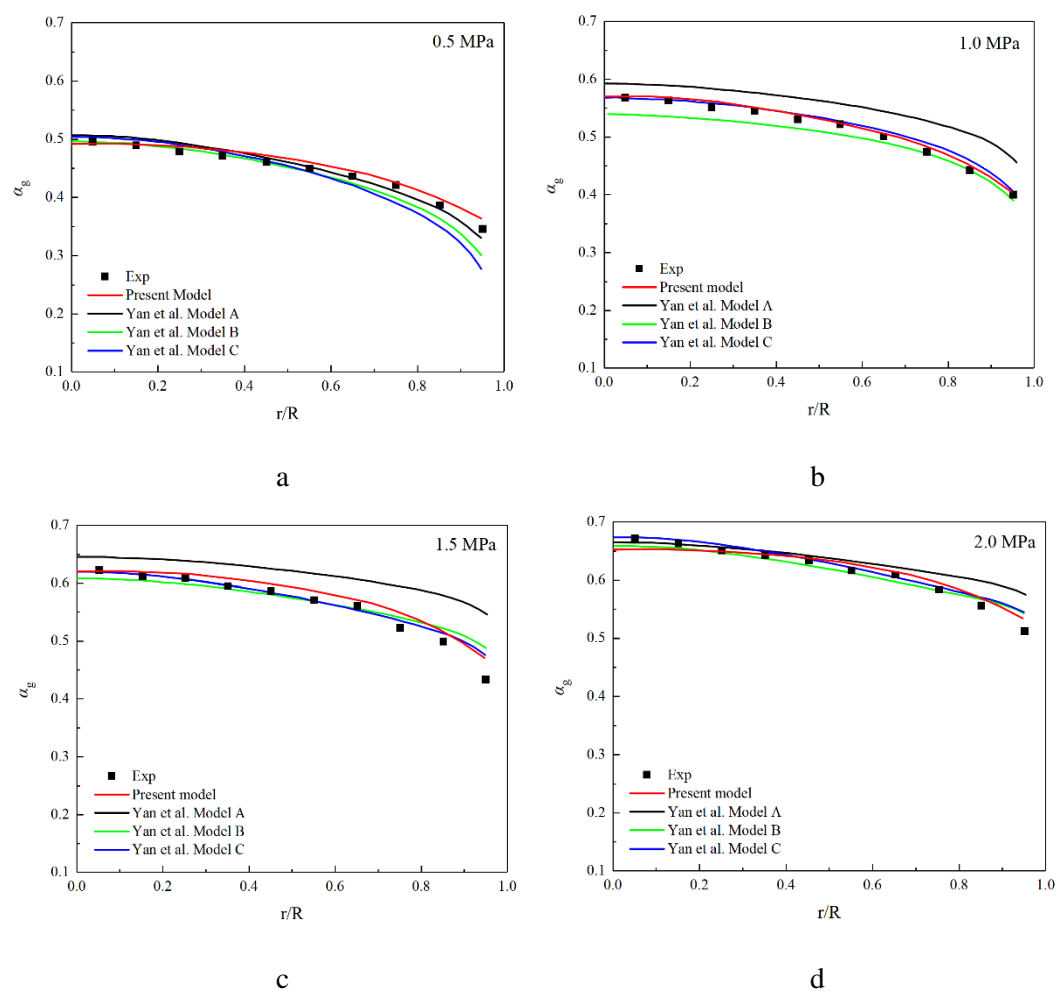


Figure 9. Radial gas holdup distributions at different system pressure ($z = 2.60$ m, $u_g = 0.296$ m/s).

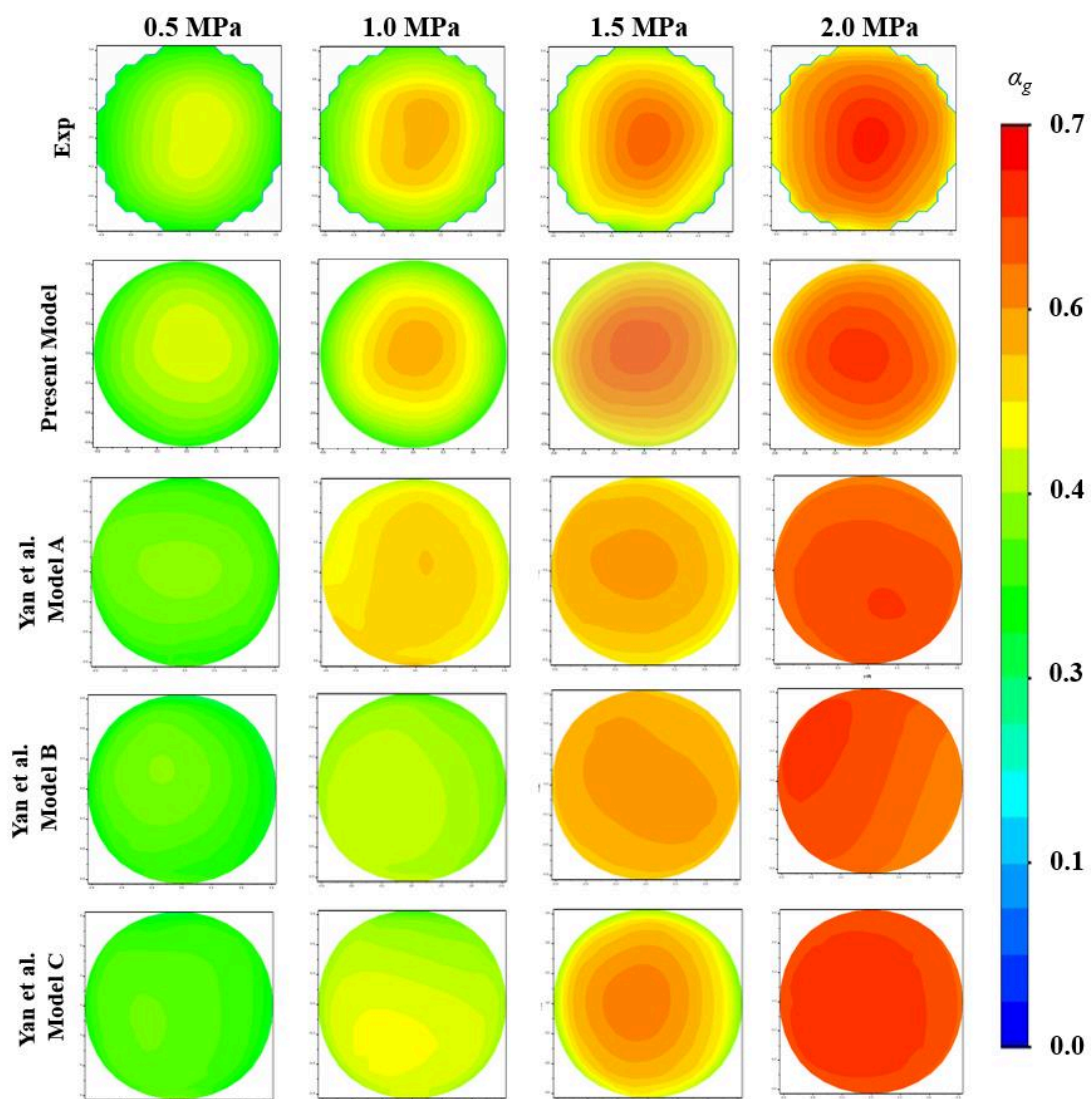


Figure 10. Contours of the mean gas holdup distribution at different system pressure ($z = 2.60$ m $u_g = 0.296$ m/s).

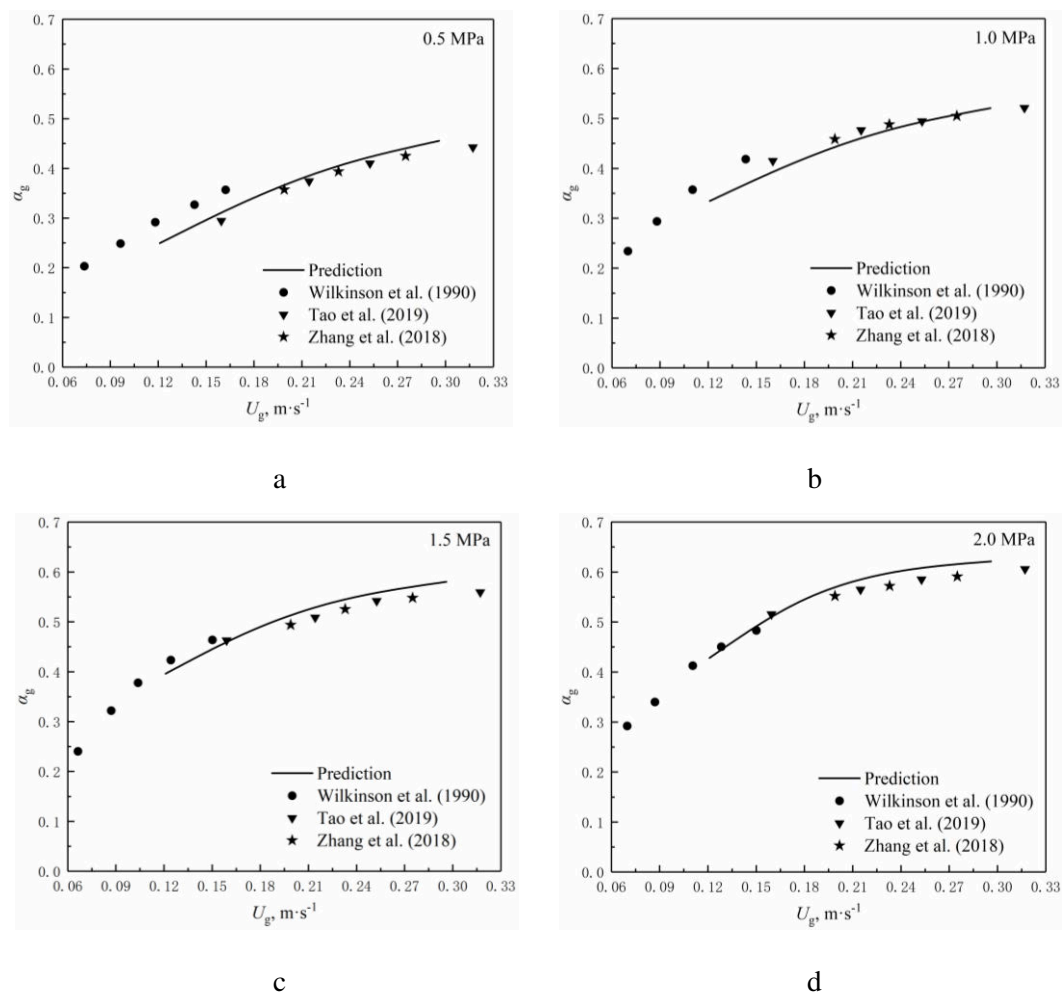


Figure 11. Gas holdup under different superficial gas velocities.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (No. U2106225), Outstanding Youth Foundation of Jiangsu Province of China (No. BK20211547), Jiangsu Province Science Foundation for Youths (No. BK20220538) and Program of China Scholarship Council (No. 202308690003).

Symbols used

A	[m ²]	area
C_{D0}	[-]	effective drag coefficient for a single bubble
C_D	[-]	drag coefficient
C_L	[-]	lift coefficient
C_{TD}	[-]	turbulent dispersion coefficient
C_{WL}	[-]	wall lubrication coefficient
C_{VM}	[-]	virtual mass coefficient
d	[m]	bubble diameter
D	[m]	bubble column diameter
Eo	[-]	Eötvös number
F_D	[N]	drag force
F_L	[N]	lift force
F_{TD}	[N]	turbulent dispersion force
F_{WL}	[N]	wall lubrication force
F_{VM}	[N]	virtual mass force
g	[m·s ⁻²]	gravitational acceleration
k	[m ² ·s ⁻²]	kinetic energy of turbulent flow
K	[-]	correction coefficient
K_c	[-]	correction coefficient for calculation
P	[MPa]	system pressure
r	[m]	radial distance
R	[m]	radius of bubble column
R^2	[-]	coefficient of determination
Re	[-]	Reynolds number
u	[m·s ⁻¹]	speed vector
u_g	[m·s ⁻¹]	gas velocity
u_l	[m·s ⁻¹]	liquid velocity
U_g	[m·s ⁻¹]	superficial gas velocity
U_l	[m·s ⁻¹]	superficial liquid velocity
v_r	[m s ⁻¹]	relative velocity between the two phases
Z	[m]	measuring height
Greek letters		
α_g	[-]	gas holdup
$\alpha_{g,small}$	[-]	gas holdup of small bubbles
$\alpha_{g,large}$	[-]	gas holdup of large bubbles
$\alpha_{g,max}, \alpha_{g,min}$	[-]	constant of the limit function in the turbulent dispersion force
α_l	[-]	liquid holdup
ε	[m ² ·s ⁻³]	turbulent dissipation rate
μ	[mPa·s]	viscosity

ρ	[kg·m ⁻³]	density
$\rho_{g,0}$	[kg·m ⁻³]	atmospheric density of air
σ	[N·m ⁻¹]	surface tension

Subscripts

<i>b</i>	bubble
<i>eff</i>	effective
<i>g</i>	gas phase
<i>l</i>	liquid phase
<i>large</i>	large bubble
<i>small</i>	small bubble

Abbreviations

CFD	computational fluid dynamics
DBS	dual bubble size
ERT	electrical resistance tomography
EMMS	energy minimization multi-scale
MAE	mean absolute error
RMSE	root mean square error

References

- [1] Y. D. Baba, A. M. Aliyu, A. E. Archibong, M. Abdulkadir, L. Lao, H. Yeung, *J. Pet. Sci. Eng.* **2018**, *165*, 397-411. DOI: <https://doi.org/10.1016/j.petrol.2018.02.003>
- [2] M. Gourma, P. G. Verdin, *J. Pet. Sci. Eng.* **2020**, *190*, 107062. DOI: <https://doi.org/10.1016/j.petrol.2020.107062>
- [3] S. Hamed, M. S. Amirhosein, D. Wei, L. Dirk, H. Uwe, *Appl. Therm. Eng.* **2021**, *204*, 117962. DOI: <https://doi.org/10.1016/j.applthermaleng.2021.117962>
- [4] P. Kapustenko, J. Kleme, O. Arsenyeva, L. Tovazhnyanskyy, V. Zorenko, *Energy*. **2021**.228 (C). DOI: <https://doi.org/10.1016/j.energy.2021.120583>
- [5] R. Chen, C. Dong, K. Guo, W. Tian, G. H. Su, *Prog. Nucl. Energy*. **2020**, *118*, 103057. DOI: <https://doi.org/10.1016/j.pnucene.2019.103057>
- [6] T. K. Gaurav, A. Prakash, C. Zhang, *Int. J. Multiphase Flow*. **2022**, *147*, 103902. DOI: <https://doi.org/10.1016/j.ijmultiphaseflow.2021.103902>
- [7] J. B. Joshi, K. Nandakumar, *Annu. Rev. Chem. Biomol. Eng.* **2015**, *6* (1), 347. DOI: <https://doi.org/10.1146/annurev-chembioeng-061114-123229>
- [8] R. Clift, J. R. Grace, M. E. Weber, *J. Fluid Mech.* **1978**, *94* (4), 795-796. DOI: <https://doi.org/10.1017/S0022112079221290>
- [9] M. Ishii, N. Zuber, *AIChE J.* **1979**, *25*, 843-855. DOI: <https://doi.org/10.1002/AIC.690250513>
- [10] L. Schiller, Z. Naumann, *Z. Ver. Deutsch. Ing.* **1935**, *77*, 318-320
- [11] A. Tomiyama, I. Kataoka, I. Zun, T. Sakaguchi, *JSME Int. J. Ser. B.* **2008**, *41* (2), 472-479. DOI: <https://doi.org/10.1299/jsmeb.41.472>

- [12] D. Feng, J. H. Ferrasse, A. Soric, O. Boutin, *Chem. Eng. Res. Des.* **2019**, *144*, 95-106. DOI: <https://doi.org/10.1016/j.cherd.2019.02.001>
- [13] C. Leonard, J. H. Ferrasse, O. Boutin, S. Lefevre, A. Viand, *Chem. Eng. Res. Des.* **2015**, *100*, 391-421. DOI: <https://doi.org/10.1016/j.cherd.2015.05.013>
- [14] R. Pohorecki, W. Moniuk, A. Zdrójkowski, P. Bielski, *Chem. Eng. Sci.* **2001**, *56* (3), 1167-1174, DOI: [https://doi.org/10.1016/S0009-2509\(00\)00336-5](https://doi.org/10.1016/S0009-2509(00)00336-5)
- [15] T. Lin, L. Fan, K. Tsuchiya, *AIChE J.* **1998**, *44* (3), 545-560. DOI: <https://doi.org/10.1002/aic.690440306>
- [16] H. Jin, Y. Qin, S. Yang, G. He, Z. Guo, *Chem. Eng. Technol.* **2013**, *36* (10), 1721-1728. DOI: <https://doi.org/10.1002/ceat.201200551>
- [17] C. Leonard, J. H. Ferrasse, S. Lefevre, A. Viand, O. Boutin, *Chem. Eng. Res. Des.* **2021**, *173*, 108-118. DOI: <https://doi.org/10.1016/j.cherd.2021.07.003>
- [18] J. Mach, J. Wiens, J. Adjaye, A. A. Donaldson, A. Macchi, *Chem. Eng. Sci.* **2020**, *223*, 115728. DOI: <https://doi.org/10.1016/j.ces.2020.115728>
- [19] P. Chen, J. Sanyal, M. P. Dudukovi, *Chem. Eng. Sci.* **2005**, *60* (4), 1085-1101. DOI: <https://doi.org/10.1016/j.ces.2004.09.070>
- [20] R. Krishna, J. Baten, *Chem. Eng. Sci.* **2001**, *56* (21-22), 6249-6258. DOI: [https://doi.org/10.1016/S0009-2509\(01\)00274-3](https://doi.org/10.1016/S0009-2509(01)00274-3)
- [21] B. Zhang, L. Kong, H. Jin, G. He, S. Yang, X. Guo, *Chin. J. Chem. Eng.* **2018**, *26* (6), 1350-1358. DOI: <https://doi.org/10.1016/j.cjche.2018.01.003>
- [22] B. V. Tran, S. I. Ngo, Y. I. Lim, K. Bae, N. S. Nho, *Chin. J. Chem. Eng.* **2020**, *33*, 190-202. DOI: <https://doi.org/10.1016/j.cjche.2020.08.031>
- [23] B. V. Tran, S. I. Ngo, Y. I. Lim, H. P. Hai, S. H. Lim, K. S. Go, N. S. Nho, *Chem. Eng. J.* **2021**, *418*, 129378. DOI: <https://doi.org/10.1016/j.cej.2021.129378>
- [24] G. Yang, H. Zhang, J. Luo, T. Wang, *Chem. Eng. Sci.* **2018**, *192*, 714-724. DOI: <https://doi.org/10.1016/j.ces.2017.01.013>
- [25] P. Yan, H. Jin, G. He, X. Guo, L. Ma, S. Yang, R. Zhang, *Chem. Eng. Sci.* **2019**, *199*, 137-155. DOI: <https://doi.org/10.1016/j.ces.2019.01.019>
- [26] P. M. Wilkinson, L. L. V. Dierendonck, *Chem. Eng. Sci.* **1990**, *45* (8), 2309-2315. DOI: [https://doi.org/10.1016/0009-2509\(90\)80110-Z](https://doi.org/10.1016/0009-2509(90)80110-Z)
- [27] G. Besagni, *Int. J. Multiphase Flow.* **2021**, *135*, 103510. DOI: <https://doi.org/10.1016/j.ijmultiphaseflow.2020.103510>
- [28] R. Demol, D. Vidal, S. Shu, F. Bertrand, J. Chaouki, *Chem. Eng. Process.* **2019**, *144*, 107647. DOI: <https://doi.org/10.1016/j.cep.2019.107647>
- [29] T. K. Gaurav, A. Prakash, C. Zhang, *Int. J. Multiphase Flow.* **2022**, *147*, 103902. DOI: <https://doi.org/10.1016/j.ijmultiphaseflow.2021.103902>
- [30] D. Lucas, T. Ziegenhein, *Int. J. Multiphase Flow.* **2019**, *120*, 103092. DOI: <https://doi.org/10.1016/j.ijmultiphaseflow.2019.103092>
- [31] N. Yang, J. Chen, H. Zhao, W. Ge, J. Li, *Chem. Eng. Sci.* **2007**, *62* (24), 6978-6991. DOI: <https://doi.org/10.1016/j.ces.2007.08.034>
- [32] N. Yang, J. Chen, W. Ge, J. Li, *Chem. Eng. Sci.* **2010**, *65* (1), 517-526. DOI: <https://doi.org/10.1016/j.ces.2009.06.014>

- [33] Q. Xiao, N. Yang, J. Li, *Chem. Eng. Sci.* **2013**, *100*, 279-292. DOI: <https://doi.org/10.1016/j.ces.2013.02.027>
- [34] X. Jiang, N. Yang, B. Yang, *Particuology*. **2016**, *27* (4), 95-101. DOI: <https://doi.org/10.1016/j.partic.2015.05.011>
- [35] X. Guan, N. Yang, *Chem. Eng. Sci.* **2021**, *243*, 116758. DOI: <https://doi.org/10.1016/j.ces.2021.116758>
- [36] I. Roghair, M. V. S. Annaland, H. J. A. M. Kuipers, *AIChE J.* **2013**, *59* (5), 1791-1800. DOI: <https://doi.org/10.1002/aic.13949>
- [37] G. Yang, K. Guo, T. Wang, *Chem. Eng. Sci.* **2017**, *170*, 251-262. DOI: <https://doi.org/10.1016/j.ces.2017.01.013>
- [38] T. Wang, J. Wang, Y. Jin, *AIChE J.* **2006**, *52* (1), 125-140. DOI: <https://doi.org/10.1002/aic.10611>
- [39] A. Buffo, M. Vanni, P. Renze, D.L. Marchisio, *Chem. Eng. Res. Des.* **2016**, *113*, 284-303. DOI: <https://doi.org/10.1016/j.cherd.2016.08.004>
- [40] J. Li, C. Cheng, Z. Zhang, J. Yuan, A. Nemet, F. Fett, *Chem. Eng. Sci.* **1999**, *54* (22), 5409-5425. DOI: [https://doi.org/10.1016/S0009-2509\(99\)00274-2](https://doi.org/10.1016/S0009-2509(99)00274-2)
- [41] W. Ge, J. Li, *Chem. Eng. Sci.* **2002**, *57* (18), 3993-4004. DOI: [https://doi.org/10.1016/S0009-2509\(02\)00234-8](https://doi.org/10.1016/S0009-2509(02)00234-8)
- [42] X. Liu, Y. Zhang, L. Zhang, H. Jin, *CIESC J.* **2017**, *68*, 87-96. DOI: <https://doi.org/10.11949/j.issn.0438-1157.20160762>
- [43] S. P. Antal, R. T. Lahey Jr, J. E. Flaherty, *Int. J. Multiphase Flow*. **1991**, *17* (5), 635-652. DOI: [https://doi.org/10.1016/0301-9322\(91\)90029-3](https://doi.org/10.1016/0301-9322(91)90029-3)
- [44] A. Tomiyama, *Multiphase Sci. Technol.* **1998**, *10* (4), 369-405. DOI: <https://doi.org/10.1615/MultScienTechn.v10.i4.40>
- [45] M. V. Tabib, S. A. Roy, J. B. Joshi, *Chem. Eng. J.* **2008**, *139* (3), 589-614. DOI: <https://doi.org/10.1016/j.ces.2007.09.015>
- [46] D. Zhang, N. G. Deen, J. Kuipers, *Chem. Eng. Sci.* **2006**, *61* (23), 7593-7608. DOI: <https://doi.org/10.1016/j.ces.2006.08.053>
- [47] C. Morel, *Mathematical Modeling of Disperse Two-Phase Flows. Springer Series in Fluid Mechanics and its Applications*, Vol. 114, Springer, Berlin **2015**.
- [48] D. Zhang, N. G. Deen, J. Kuipers, *Ind. Eng. Chem. Res.* **2009**, *48* (1), 47-57. DOI: <https://doi.org/10.1021/ie800233y>
- [49] S. Ojima, K. Hayashi, S. Hosokawa, A. Tomiyama, *Int. J. Multiphase Flow*. **2014**, *67*, 111-121. DOI: <https://doi.org/10.1016/j.ijmultiphaseflow.2014.05.008>
- [50] I. G. Reilly, D. S. Scott, T. D. B. J. Piskorz, *Can J Chem Eng.* **1986**, *64* (5), 705-717. DOI: <https://doi.org/10.1002/cjce.5450640501>
- [51] F. Tao, S. Ning, B. Zhang, H. Jin, G. He, *Processes*. **2019**, *7* (9), 594. DOI: <https://doi.org/10.3390/pr7090594>

Table and Figure captions

Table 1. Drag force models.

Table 2. Coefficient of pressure correction method.

Figure 1. Schematic of the experimental column.

Figure 2. Schematic diagram of the radial distribution.

Figure 3. Schematic representation of various interfacial forces: (a) drag force, (b) lift force, (c) wall lubrication force, (d) turbulent dispersion force and (e) virtual mass force.

Figure 4. Correction of the relationship between parameters and operating conditions.

Figure 5. Radial gas holdup distributions at different system pressure ($z = 2.60$ m, $u_g = 0.174$ m/s).

Figure 6. Contours of the gas holdup distribution with various drag force models for $t = 60$ s ($P = 0.5$ MPa, $u_g = 0.174$ m/s).

Figure 7. Radial gas holdup distributions at different system pressure ($z = 2.60$ m, $u_g = 0.121$ m/s).

Figure 8. Contours of the mean gas holdup distribution at different system pressure ($z = 2.60$ m $u_g = 0.121$ m/s).

Figure 9. Radial gas holdup distributions at different system pressure ($z = 2.60$ m, $u_g = 0.296$ m/s).

Figure 10. Contours of the mean gas holdup distribution at different system pressure ($z = 2.60$ m $u_g = 0.296$ m/s)

Figure 11. Gas holdup under different superficial gas velocities.



DSPACE

<https://dspace.org/>

Modeling of dualfactor drag correction model for bubbly flow under elevated pressure

Gao, Yibo; Geng, Linlin; Verdin, Patrick G.; Fall, Ibra; Zhang, Ruijie ...

2024-11

Wiley

<https://doi.org/10.1002/ceat.202300477>

Gao Y, Geng L, Verdin PG, et al., (2024) Modeling of dualfactor drag correction model for bubbly flow under elevated pressure. Chemical Engineering & Technology, Volume 47, Issue 11, November 2024, Article number e202300477