

State of Charge Estimation of Lithium Sulfur Batteries using Sliding Mode Observer

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Abstract—The lithium-sulfur (Li-S) batteries are high energy storage systems that can be used for electric grid and solar power air vehicle applications. Such applications require an accurate state of charge (SOC) estimator to control and optimize battery performance. Modelling and estimation of discharging Li-S are highly challenging than other batteries as the discharge voltage of Li-S batteries has highly nonlinear and typical characteristics than the Lithium-Ion batteries. For Li-S battery SOC estimation, literature has proposed filters and machine learning techniques, but no literature on sliding mode observer (SMO). This paper presents the SMO for discharging Li-S SOC estimation and compares it to the extended Kalman filter (EKF). Both estimators use a first-order equivalent circuit network (ECN) model of Li-S cell parameters given in the literature. The performance of such ECN model based SOC estimators influenced by the Q-uncertainty, which is a perturbation in the form of process noise state-space model. Therefore, this work studies an optimal trade-off characteristic of SMO and EKF over the Q-uncertainty. With constant and mixed-amplitude pulse load current sequences, numerical simulation has performed. Simulation results illustrate that the SMO is optimal, converges to the true SOC than the EKF when the perturbation increased.

Keywords—Battery energy storage, nonlinear filters, Lithium-sulfur batteries, state of charge, sliding mode observer.

I. INTRODUCTION

Recently, the lithium-sulfur (Li-S) battery energy storage systems have been active research owing their own features and potential applications [1]. The features of Li-S include higher energy storage capabilities, more safe, lesser-weight and low cost. Therefore, the Li-S may be alternative to the Lithium-Ion (Li-Ion) batteries in applications such as electric grids and vehicles. For instance, Li-S used as energy storage systems in high altitude long-endurance solar-powered small aircraft [2]. These applications require an appropriate algorithm for the state of charge (SOC) estimation, as a critical component of battery management systems (BMSs). The SOC plays significant role to improve safety operation, control and the energy or power optimization of battery-operated systems.

The main goal of this paper is to design sliding mode observer for accurately estimate SOC of discharging Li-S batteries that can be used real-world battery applications. The SOC estimation of discharging Li-S batteries is challenging as its open circuit voltage (OCV) has deep-dive high-plateau and flatten, poorly observable low-plateau [1,3]. This is different than the OCV of traditional Li-Ion batteries. For SOC estimation of Li-S cells, literature [4-6] proposed EKF, unscented Kalman filter, and particle filter. nonlinear filters such as EKF, UKF and particle filter. Similarly, the adaptive neuro-fuzzy inference system (ANFIS) [7], and Support Vector Machine (SVM) [8] based Li-S SOC studied. But there

is not literature on SMO for Li-S batteries though SMO were widely used for SOC estimation of Li-Ion [9-11]. This paper fulfills such literature gap. This paper addresses the SOC estimation of Li-S battery cells under perturbation in the ECN parameters as battery parameters are time-varying, but the existing literature did not address such perturbation.

First, the SMO is designed for a deterministic first-order equivalent circuit network (ECN) model of discharging Li-S batteries. Next, on top of that, the perturbation is introduced into ECN model to study the robustness of the SMO. It is assumed that perturbation is a Gaussian noise, and it is bounded with known variance. The process noise is referred as Q-uncertainty. According to voltage and parameter characteristics, the gains of SMO SOC estimator are chosen. The stability of the SMO can be studied by the Lyapunov error function. Applying constant and dynamic load current, the performance of SMO is analyzed. The SMO's performance is compared to the traditional extended Kalman filter (EKF). While designing EKF, the discrete-time model is used, whereas a continuous-time model is used for SMO. However, both SMO and EKF use the ECN model and the prediction error minimization method of parameterization given in [1, 12]. Simulation results show that compared to the EKF the SMO is robust and much accurate, SOC estimation error is less than five percent, for considered Q-uncertainty.

The rest of the paper is organized as follow: Section II studies the modelling and SOC problem formulation. Section III and IV present the EKF and SMO, respectively. Simulation results and discussions are given in section V. Finally, section VI presents the conclusions and future works of this paper.

II. DISCHARGING LI-S BATTERY MODEL

A. Deterministic ECN Model

This section describes the deterministic state-space model of discharging Li-S battery model derived from ECN model given in Fig 1. Such model reported in the literature [12]. The modeling is repeated here because to contrast the discharge voltage of Li-S battery from the voltage of Li-Ion battery and to extend the deterministic model to uncertainty model.

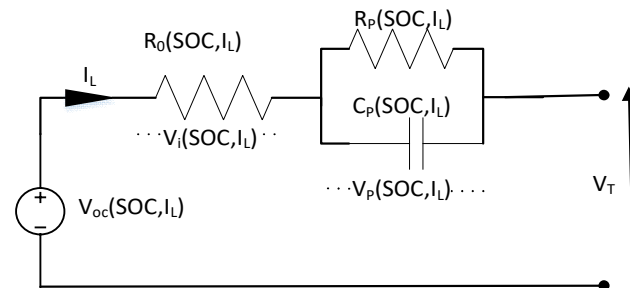


Fig.1 A first-order ECN model represents a Li-S battery cell.

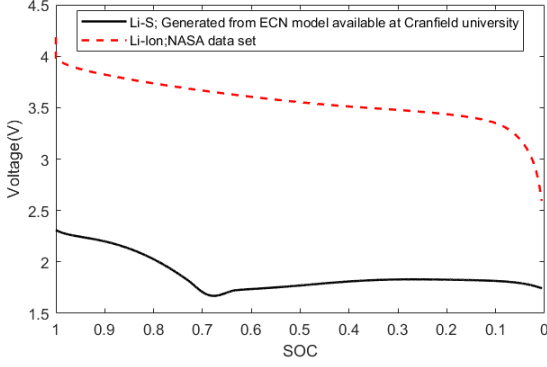


Fig.2 Li-S vs Li-Ion: voltage comparison of Li-S and Li-Ion batteries

Though the model is deterministic, the modeling complexity of Li-S batteries is higher than the transition Li-Ion because of the typical voltage characteristic. Fig 2 compares the voltage characteristics of typical Li-S and Li-Ion batteries. In Fig 2, the red-dashed line indicates the measured voltage of 2Ah Li-Ion battery given in [13], and the black-solid line represents the voltage of 2.72Ah Li-S battery that generated from the SIMULINK model [12]. These voltages are battery response to 2A constant load current and begin from maximum voltage and end at the cut-off voltage of respective battery cell. It is evident that the voltage of Li-S has a deep-dive high plateau during 0 to 1500s and a flat-in nature low plateau from 1500s to 5000s approximately. The high- and low-plateau have joined with a sharp curve, which is an additional complexity in the modeling of Li-S batteries. Therefore, the output or measurement model of state-space for discharging Li-S batteries is quite different from the traditional one. Such difference is explained with an example in rest of this section. The state-space model is written as

$$\dot{x}(t) = f_c(x(t), \theta_b(x_1(t)), I_L(t)); \quad x(t_0) \quad (1)$$

$$y(t) = V_T = h_c(x(t), \theta_b(x_1(t)), I_L(t)) \quad (2)$$

Where $x(t) = \begin{bmatrix} SOC \\ V_p \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ is a state vector, $\dot{x}(t)$ is the first derivative of $x(t)$, and $\theta_b(x_1(t))$ denotes parameters of ECN model, which are functions of SOC. The deterministic SOC is evaluated from the following first-order differential equation

$$\dot{x}_1(t) = \frac{-I_L}{3600 Q_{cap}}; \quad x_1(t_0) \quad (3)$$

where $\dot{x}_1(t)$ is the first derivative of SOC at time t , $x_1(t_0)$ is the initial SOC at time t_0 , the Q_{cap} is the capacity of the battery, and I_L is the load current or current taken out from the battery. The dynamic behavior of polarization voltage, V_p across the 1RC network is expressed as

$$\dot{x}_2(t) = \frac{-1}{C_p(x_1(t))R_p(x_1(t))}x_2(t) + \frac{I_L}{C_p(x_1(t))}; \quad x_2(0) \quad (4)$$

where $R_p(x_1(t))$ and $C_p(x_1(t))$ are resistor and capacitor of 1RC branch, respectively.

From Fig 1, the battery terminal voltage, $V_T(t)$, is expressed as

$$V_T(t) = V_{oc}(x_1) - x_2(t) - I_L R_o(x_1) \quad (5)$$

where $V_{oc}(x_1)$ is the open circuit voltage of the battery, $x_2(t)$ denotes polarization voltage, and $R_o(x_1)$ is the Ohmic resistance of the battery. Unlike the parameters of lithium-ion, the $V_{oc}(x_1)$ and $R_o(x_1)$ of Li-S battery have high and low-plateau are joined by a partial sinusoidal function (PSF) [4] as follow:

$$V_{oc}(x_1) = \gamma(x_1(t))V_{oc,high}(x_1(t)) + (1 - \gamma(x_1(t)))V_{oc,low}(x_1(t)) \quad (6)$$

$$R_o(x_1) = \gamma(x_1(t))R_{o,high}(x_1(t)) + (1 - \gamma(x_1(t)))R_{o,low}(x_1(t)) \quad (7)$$

where

$$V_{oc,high}(x_1(t)) = p_{1,vohi}x_1^n + p_{2,vohi}x_1^{n-1} + \dots + p_{n,vohi}x_1 + p_{n+1,vohi} \quad (8)$$

$$V_{oc,low}(x_1(t)) = p_{1,volo}x_1^n + p_{2,volo}x_1^{n-1} + \dots + p_{n,volo}x_1 + p_{n+1,volo} \quad (9)$$

$$R_{o,high}(x_1(t)) = p_{1,Rohi}x_1^n + p_{2,Rohi}x_1^{n-1} + \dots + p_{n,Rohi}x_1 + p_{n+1,Rohi} \quad (10)$$

$$R_{o,low}(x_1(t)) = p_{1,Rolo}x_1^n + p_{2,Rolo}x_1^{n-1} + \dots + p_{n,Rolo}x_1 + p_{n+1,Rolo} \quad (11)$$

and

$$\gamma(x_1(t)) = \begin{cases} 0, & \text{if } L_p \\ f_{PSF}(x_1(t)), & \text{if } J_{trans} \\ 1, & \text{if } H_p \end{cases} \quad (12)$$

In (12), the partial sinusoidal function $f_{PSF}(x_1(t))$ is expressed as

$$f_{PSF}(x_1(t)) = 0.5 + 0.5 \sin(2(5\pi)(x_1 - 0.68)) \quad (13)$$

and the logical conditions are expressed as

$$\begin{aligned} L_p: 0 \leq x_1 \leq 0.63 \\ J_{trans}: 0.63 < x_1 < 0.73 \\ H_p: 0.73 \leq x_1 \leq 1 \end{aligned} \quad (14)$$

In (8)-(11), the $p_{1,\dots}$, $p_{2,\dots}$ and p_{n+1} are the coefficients of n th order polynomial for each parameter are listed in Table I. These values are parameters of Li-S cell of 2.72 Ah at 20°C as same as Table 2 of [3].

TABLE I
POLYNOMIAL COEFFICIENTS OF LI-S CELL'S ECN PARAMETERS AT 20°C

Coeff	$V_{oc,high}$	$V_{oc,low}$	$R_{o,high}$	$R_{o,low}$	R_p	C_p
p_1	108.1	-752.62	-1300.2	12.96	140.636	89414.28
p_2	-361.1	2085.66	6470.07	-28.54	-613.186	-113090.7
p_3	444.73	-2392.87	-13363.0	25.46	1088.53	25401.3
p_4	-238.18	1466.98	14656.94	-11.65	-1005.91	15392.5
p_5	47.03	-517.42	-9000.23	3.09	512.386	-3017.3
p_6	1.88	105.21	2931.67	-0.42	-139.174	306.23
p_7		-11.69	-395.2	0.11	16.887	
p_8		0.62			-0.011	
p_9		2.1			-0.223	
p_{10}					0.074	

These parameters are used to characterize the discharging Li-S cell from the deterministic model (1)-(2) with two different types of load current. When applied mixed-amplitude pulse current sequence and assumed that the initial SOC was one, the Li-S cell have the typical high- and low-plateau voltage characteristic, as shown by Fig 3(a). Fig 3(b) shows the mixed-amplitude pulse current sequence. Fig 3(c) and Fig 3(d) denote the SOC and the dynamic polarization voltage of cell to the pulse current, respectively. Fig 4 shows a zoomed section of the voltage to current pulses with different amplitudes such as 0.5A(0.2C), 1A(0.4C) and 1.5A(0.55C) are considered. This exhibits the impact of polarization

voltage on the dynamic voltage behavior. Therefore, the SOC estimators need a first of ECN model and its parameters are shown by Fig 3(e)-(h) .

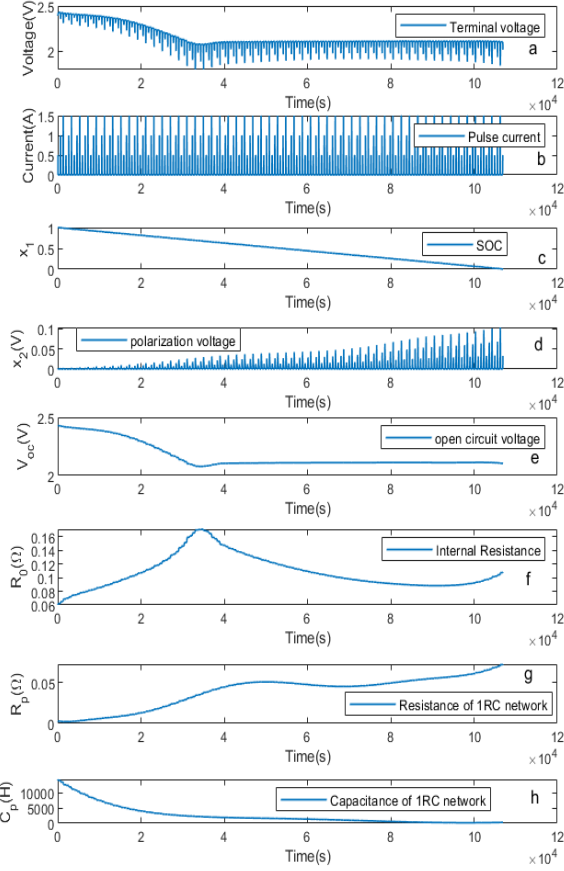


Fig.3 Characteristics of REVB's Li-S cell to mixed-pulse current: (a)voltage; (b) current; (c) True SOC, x_1 ; (d) True x_2 ; (e). $V_{oc}(x_1)$; (f). $R_o(x_1)$; (g). $R_p(x_1)$; (h). $C_p(x_1)$..

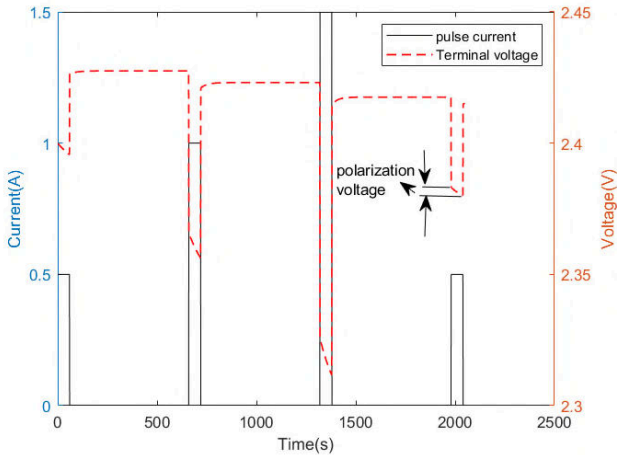


Fig. 4. Zoomed a section of Voltage of Li-S to the pulse current

B. Model with Perturbation

Due to unknown modeling error and sensor noise, the state and measurement models have perturbations. Such perturbations can be modelled as noise with known mean and variance. In this work, it is assumed the modelling error and sensor noise are Gaussian noise. The is assumed that the

actual noise variance of batteries varies in the form of additive noise of the state-space model as

$$\dot{x}(t) = f_c(x(t), \theta_b, u(t)) + w_a(t); x(t_0) \quad (15)$$

$$y(t) = h_c(t, \theta_b, u(t)) + v_a(t) \quad (16)$$

Where $w_a(t) \sim N(0, Q_a)$ and $v_a(t) \sim N(0, R_a)$ are they are the independent and identically distributed Gaussian noises, The noise $w_a(t)$ represents the unknown modelling error in the SOC model (3) and polarization voltage model (4). Furthermore the process noise assumes that the state variables are not correlated one to another. Therefore the process noise covariance Q_a can be expressed as

$$Q_a = \begin{bmatrix} Q_{soc} & 0 \\ 0 & Q_{vp} \end{bmatrix} \quad (17)$$

in which Q_{soc} and Q_{vp} (V^2) denote the variances of noise in the SOC and the polarization voltage, respectively. The R_a denotes the covariance of sensor noise. Both $w_a(t)$ and $v_a(t)$ are treated as perturbations that bounded with known variance, of process and measurement model respectively. These statistical parameters Q_a and R_a are uncertain, not known exactly when batteries are used for applications like EVs. Fig 5 shows a behavior of perturbed and unperturbed state (Fig 5c shows x_2) and output variable (Fig. 5b show terminal voltage), and parameter (Fig 5d shows V_{oc}) of Li-S cell that generated from the ECN model when pulse current is applied, Fig 5a.

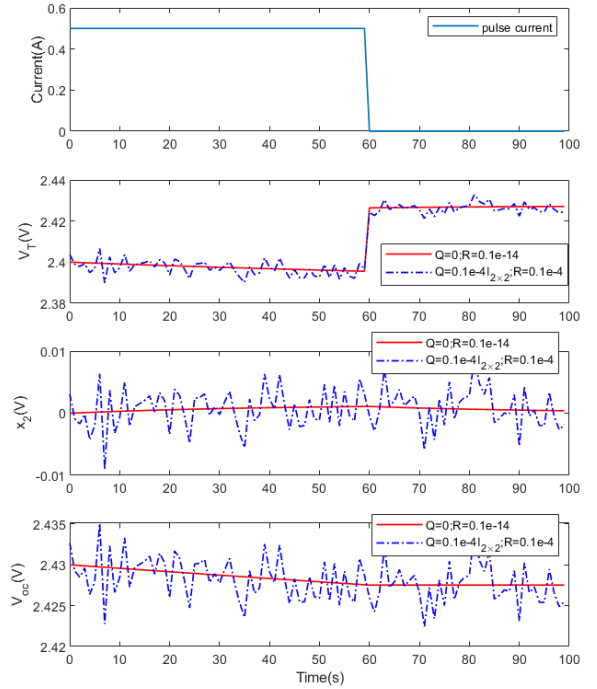


Fig.5 Perturbation on Li-S cell model when pulse current is applied (a) pulse current; (b) terminal voltage with noise; (c) x_2 ; (d). $V_{oc}(x_1)$

III. EKF BASED SOC ESTIMATOR

The EKF uses discrete state-space model. The discretized version of (15)-(16) is written as:

$$x_{k+1} = f_d(x_k, \theta_{b,k}, I_{L,k}) + w_k \quad (18)$$

$$y_k = h_d(x_k, \theta_{b,k}, I_{L,k}) + v_k \quad (19)$$

where k is the sampling time index,

$$f_d(x_k, \theta_k, I_{L,k}) = \begin{bmatrix} f_{d,1}(x_{1,k}, I_{L,k}) \\ f_{d,2}(x_{1,k}, \theta_k, I_{L,k}) \end{bmatrix} = \begin{bmatrix} x_{1,k} - \frac{\Delta t * I_{L,k}}{3600 * Q_{cap}} \\ x_{2,k} - \frac{\Delta t * x_{2,k}}{C_P(x_{1,k}) R_P(x_{1,k})} + \frac{\Delta t * I_{L,k}}{C_P(x_{1,k})} \end{bmatrix}, \quad (20)$$

$h_d(x_k, \theta_k, u_k) = V_{oc}(x_{1,k}) - x_{2,k} - R_p(x_{1,k}) I_{L,k}$ (21)
 w_k and v_k are the process and measurement noise.. Δt is sample time. $x_k = \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix}$ and $I_{L,k}$ are the discrete state and input same as the continuous model. The parameters of the discrete model are the same as the parameters of the continuous model. In the EKF, the following Jacobian matrices are used:

$$A_k = \frac{\partial f_d(x_k, I_{L,k})}{\partial x_k} = \begin{bmatrix} 1 & 0 \\ \frac{\partial f_{d,2}(x_k, I_{L,k})}{\partial x_{1,k}} & \frac{\partial f_{d,2}(x_k, I_{L,k})}{\partial x_{2,k}} \end{bmatrix} \quad (22)$$

$$B_k = \frac{\partial f_d(x_k, I_{L,k})}{\partial I_{L,k}} = \begin{bmatrix} -\frac{\Delta t}{3600 * Q_{cap}} \\ \frac{\Delta t}{C_P(x_{1,k})} \end{bmatrix} \quad (23)$$

$$C_k = \frac{\partial h_d(x_k, I_{L,k})}{\partial x_k} = \begin{bmatrix} \frac{\partial h_d(x_k, I_{L,k})}{\partial x_{1,k}} & -1 \end{bmatrix} \quad (24)$$

$$D_k = \frac{\partial h_d(x_k, I_{L,k})}{\partial I_{L,k}} = [-R_0(x_{1,k})] \quad (25)$$

The EKF algorithm includes five steps predicted state estimate, predicted covariance estimate, near-optimal Kalman gain calculation, state estimate update, and covariance of the state estimate. ‘-’ and ‘+’ at the top-right corner of each variable means before (prior) or after (posterior) the calculation [4]. Before the algorithm, the initial condition of the EKF algorithm is set as:

$$k = 0; \hat{x}_0^+ = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; P_0^+ = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \quad (26)$$

k in EKF is the step. $\hat{x}_k^-$ and $\hat{x}_k^+$ are the prior and posterior state estimates. P_k^- and P_k^+ are the prior and posterior error covariances. K_k is the Kalman gain. The EKF follows main steps:

The predicted state estimate:

$$\hat{x}_k^- = A_{k-1} \hat{x}_{k-1} + B_{k-1} I_{L,k-1} \quad (27)$$

The Predicted covariance estimate:

$$P_k^- = A_{k-1} P_{k-1}^+ A_{k-1}^T + Q \quad (28)$$

Q is the process noise. Q is the same as Δf_1 in SMO.

Near-optimal Kalman gain calculation:

$$K_k = P_k^- C_k^T (C_k P_k^- C_k^T + R)^{-1} \quad (29)$$

R is the measurement noise. R is the same as Δf_3 in SMO. In the result section, both Q and R will be changed to test the robustness.

State estimate update:

$$\hat{x}_k^+ = \hat{x}_k^- + K_k (y_k - C_k \hat{x}_k^- - D_k I_{L,k}) \quad (30)$$

The posterior states are estimated by Kalman gain, prior states and the measurement of terminal voltage y_k .

Covariance of the state estimate:

$$P_k^+ = (I - K_k C_k) P_k^- \quad (31)$$

After estimating covariance of the state, the five steps are finished, and a new loop will begin from (27) to (31). Finally, after all the data is processed, the loop finishes.

IV. SMO BASED SOC ESTIMATOR

This section presents design steps used for SMO based discharging Li-S SOC estimator. For designing the sliding-

mode observer and simulate the noise or perturbations, an equation of terminal voltage with uncertainty is given:

$$y(t) = V_{oc} - x_2(t) - V_L + \Delta y \quad (32)$$

Where $V_L = R_o I_L$, and Δy is the bounded uncertainty of measurement, which is equal to $v_a(t)$ with known covariance R_a . The time derivative for the SOC with an assuming uncertainty is:

$$\dot{x}_1 = -\frac{I_L}{3600 Q_{cap}} + \Delta x_1 \quad (33)$$

Where Δx_1 is a bounded perturbation of the SOC, which is characterized as a Gaussian noise with known covariance Q_{soc} . Having $V_L = R_o I_L$ and assuming no uncertainty in (32), the load current can be expressed as

$$I_L = \frac{V_{oc} - x_2(t) - y(t)}{R_o} \quad (34)$$

Substitution of (41) in (40) results the following first derivative of SOC :

$$\begin{aligned} \dot{x}_1(t) &= -\frac{V_{oc} - x_2(t) - y(t)}{3600 Q_{cap} R_o} + \Delta x_1 \\ &= a_1(y(t) + x_2(t) - V_{oc}) + \Delta x_1 \end{aligned} \quad (35)$$

where $a_1 = \frac{1}{3600 Q_{cap} R_o}$, and Δf_1 is the bounded uncertainty of the capacity term. The polarization voltage is written as:

$$\begin{aligned} \dot{x}_2(t) &= \frac{-1}{C_p R_p} x_2(t) + \frac{I_L}{C_p} + \Delta x_2 \\ &= -a_2 x_2(t) + b_1 I_L + \Delta x_2 \end{aligned} \quad (36)$$

which $a_2 = \frac{1}{C_p R_p}$, $b_1 = \frac{1}{C_p}$, and Δx_2 is the uncertainty of the polarization voltage, which is characterized by Gaussian noise with Q_{vp} .

Now, let express dynamic response of measured battery terminal voltage by taking derivative of (32). Such first derivative output voltage can be expressed with bounded perturbation $\Delta \dot{y}$ as:

$$\dot{y}(t) = \dot{V}_{oc} - \dot{x}_2(t) - \frac{d}{dt}(I_L R_o) + \Delta \dot{y} \quad (37)$$

Considering a fast-sampling time the derivative of the terminal voltage $y(t)$ to the load current is negligible ($\frac{\Delta V_L(t)}{\Delta I_L} \approx 0$). Therefore, the (14) can be expressed as

$$\dot{y}(t) = -\frac{I_L}{3600 Q_{cap}} + \frac{1}{C_p R_p} x_2(t) - \frac{I_L}{C_p} + \Delta \dot{y} \quad (38)$$

Substitution of $x_2(t) = V_{oc} - y(t) - I_L R_o$ in (45) results

$$\begin{aligned} \dot{y}(t) &= -\frac{I_L}{3600 Q_{cap}} + \frac{V_{oc} - y(t) - I_L R_o}{C_p R_p} - \frac{I_L}{C_p} + \Delta \dot{y} \\ &= -a_2 y(t) + a_2 V_{oc} - b_2 I_L + \Delta \dot{y} \end{aligned} \quad (39)$$

where $b_2 = \frac{1}{3600 Q_{cap}} + \frac{1}{C_p} + \frac{R_o}{C_p R_p}$. since the observability matrix is full rank, the state vector can be estimated.

To estimate the terminal voltage (46), the SMO uses the following model:

$$\dot{\hat{y}} = -a_2 \hat{y} + a_2 \hat{V}_{oc} - b_2 I_L + L_1 \text{sgn}(y(t) - \hat{y}) \quad (40)$$

where L_1 is a positive gain of the terminal voltage SMO, the $\hat{y}$ and $\hat{V}_{oc}$ are the estimations of $y(t)$ and V_{oc} . After defining the error as $e_L = (y(t) - \hat{y})$, the error dynamic equation of the terminal voltage can be expressed as:

$$\dot{e}_L(t) = -a_2 e_L(t) + a_2 (V_{oc} - \hat{V}_{oc}) + \Delta \dot{y} - L_1 \text{sgn}(e_L(t)) \quad (41)$$

where

$$\text{sgn}(e_L(t)) = \begin{cases} 1, & e_L(t) > 0 \\ -1, & e_L(t) < 0 \end{cases} \quad (42)$$

It is assumed that the gain L_1 satisfies an inequality

$$L_1 > |a_2 (V_{oc} - \hat{V}_{oc}) + \Delta \dot{y}| \quad (43)$$

The following Lyapunov function can be used to prove the error convergence:

$$V(e(t)) = \frac{1}{2} e_L^2(t). \quad (44)$$

In this work, the gain L_1 for considered Li-S cell's terminal voltage estimation is determined by

$$L_1 > |a_2(V_{oc} - \hat{V}_{oc}) + \Delta\dot{y}| \approx |\Delta\dot{y}| + 0.035 \quad (45)$$

This is according to the V_{oc} given in black solid-line of Fig. 2, perturbation in the measurement, and the maximum a_2 is 0.09842 by calculating the $\frac{1}{C_p R_p}$. When estimation of terminal voltage reaches the sliding surface, $\dot{e}_L$ and e_L reaches to 0. According to the equivalent control method, i.e., $\frac{L_1}{a_2} \text{sgn}(e_L)$ is replaced by $\left\{ \frac{L_1}{a_2} \text{sgn}(e_L) \right\}_{eq}$, the (45) can be written as an equivalent function as :

$$V_{oc} - \hat{V}_{oc} = \left\{ \frac{L_1}{a_2} \text{sgn}(e_L) \right\}_{eq} \quad (46)$$

The sliding mode observer model of $\hat{x}_1(t)$ is written as :

$$\dot{\hat{x}}_1 = a_1(\hat{y} + \hat{x}_2 - \hat{V}_{oc}) + L_2 \text{sgn}(e_1) \quad (47)$$

Where $e_1 = x_1 - \hat{x}_1$ is an error between actual SOC and estimated SOC $\hat{x}_1$. An error dynamic of SMO for $\hat{x}_1$ estimation is written as

$$\dot{e}_1 = a_1(e_L + e_2 - (V_{oc} - \hat{V}_{oc})) + \Delta x_1 - L_2 \text{sgn}(e_1) \quad (48)$$

Where $e_2 = (x_2 - \hat{x}_2)$. It is assume that the error between the actual V_{oc} and $\hat{V}_{oc}$ is an piecewise linear to e_1 , hence it takes

$$V_{oc} - \hat{V}_{oc} \approx m e_1 \quad (49)$$

where m is a positive gain that evolved the slope of V_{oc} over SOC. Having (48) the (49) results

$$\dot{e}_1 = a_1(e_L + e_2 - m e_1) + \Delta x_1 - L_2 \text{sgn}(e_1) \quad (50)$$

Similarly, the SMO model of

$$\dot{\hat{x}}_2 = -a_2 \hat{x}_2 + b_1 I_L + \Delta x_2 + L_3 \text{sgn}(e_2) \quad (51)$$

Having a relation between three error terms

$$e_2 = \left\{ \frac{L_2}{a_1} \text{sgn}(e_1) \right\} = \left[\frac{L_2}{a_1} \text{sgn} \left(\left\{ \frac{L_1}{m a_2} \text{sgn}(e_L) \right\}_{eq} \right) \right]_{eq} \quad (52)$$

the SMO model (57) results

$$\begin{aligned} \dot{\hat{x}}_2 &= -a_2 \hat{x}_2 + b_1 I_L \\ &+ L_3 \text{sgn} \left(\left[\frac{L_2}{a_1} \text{sgn} \left(\left\{ \frac{L_1}{m a_2} \text{sgn}(e_L) \right\}_{eq} \right) \right]_{eq} \right) \end{aligned} \quad (53)$$

For considered SOC problem, the SMO converges to the conditions $L_2 > |\Delta x_1|$ and $L_3 > |\Delta x_2|$

V. SIMULATION RESULTS AND DISCUSSIONS

This section presents simulation results of SMO based Li-S cell SOC estimation. Simulation has performed under two different ECN model conditions and two different load conditions. Having constant current, simulation demonstrates the performance of Li-S cell SOC estimators under conventional load condition. To demonstrate performance of estimators under dynamic load condition, the mixed-amplitude current is considered.

Firstly, performance of SMO is studied under deterministic model and two different load conditions: constant and mixed-amplitude load current. For both load condition the SMO performs well when model is deterministic, and it result is not presented here due to page limit. Results of SMO with perturbed state and measurement models, are studied. While comparing SMO performance under deterministic and uncertain models, the SOC estimation of deterministic model-based SMO is more than 98 percent

and the heavily perturbed model provides minimum 96.2 percent SOC accuracy.

Secondly, performance of SMO for model with Q-uncertainty is studied and compared it to the EKF under two load conditions.

A. Constant Load Current

The current is constant, but added with a random noisy as Fig 6a, which is actual measured current of NASA data set [2]. For this input load current, the Li-S cell's output voltage is generated from ECN model (16). Fig 6b shows measured voltage when measurement noise is set to $R_a = 0.00001V^2$. In this case, the perturbation in the model is set to be $Q_{soc}=0.0000003$ and $Q_{vp} = 0.1V^2$.

The SMO and EKF use the measured voltage and input current, and estimate the SOC of Li-S battery. The subplot of Fig 6c compares the estimated SOC and true SOC. In Fig 6c, the black dotted, red solid line and blue dashed lines denote true SOC, EKF and SMO, respectively. This comparison plot exhibits that the performance of SMO as close as the true SOC from the beginning (100 percent SOC) to the depletion state (zero percent SOC), whereas the EKF does not. After 25 percent of the SOC the EKF diverges from the true SOC and is unstable. Fig 6d shows the error in estimation by both EKF and SMO. The error of SMO is less than five percent through the discharge. On contrast to the SMO, the error of EKF is more than five percent in the discharging range 25 to zero percent though the EKF's error is less than five percent from 100 percent SOC to until 25 percent.

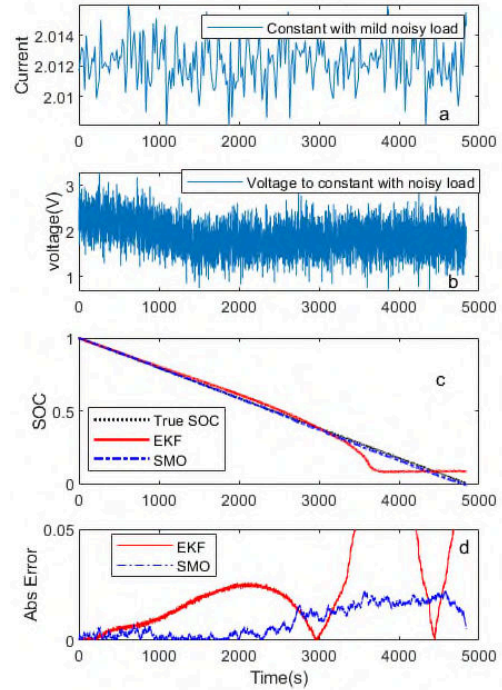


Fig.6 Results when load is constant: (a) voltage; (b) current; (c) SOC estimated Vs true SOC (d) absolute error SMO vs EKF

B. Mixed-Amplitude Pulse Load Current

This subsection illustrates the performance of both SMO and EKF based SOC estimation of Li-S batteries when mixed-amplitude current is applied. On top of that, the model perturbation is set to be $Q_{soc} = 0.00001$ and $Q_{vp} =$

$0.00001V^2$. Like previous case, the measurement noise is set to $R_a = 0.00001V^2$. Fig 7a and Fig 7b show the dynamic current and voltage of Li-S respectively.

Like constant current case, the SMO converges through the 100 percent SOC to zero percent, but the EKF starts diverges from beginning of the low-plateau around 73 percent SOC. Fig 7c illustrates the response of SMO and EKF when applied dynamic current. The red solid-line moves away from the true SOC, which is black dotted-line of Fig 7c. The red solid-line of Fig 7d shows that the absolute error by EKF is slowly increasing in low-plateau region because the poorly-observable low-plateau region. But the error by SMO is less than five percent in both high- and low-plateau.

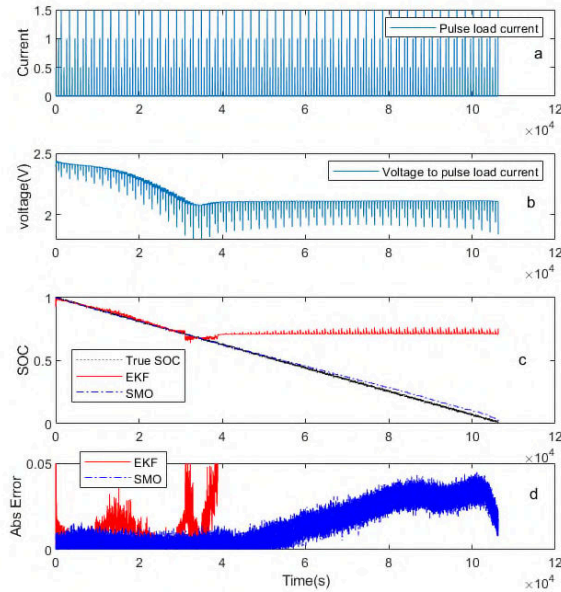


Fig.7 Results when load is mixed-amplitude pulses (a)pulse current; (b) voltage; (c) SOC estimated Vs true SOC, x_1 ; (d) absolute error SMO vs EKF.

VI. CONCLUSIONS AND FUTURE WORKS

A sliding mode observer has been designed for SOC estimation of discharging Lithium-Sulfur battery cell. The SOC estimation of discharging Li-S by sliding mode observer has been shown to be more robust and accurate than the traditional extended Kalman filter. The deterministic and perturbed models were considered while designing SMO based SOC estimator. When no perturbation, the SMO achieved more 98 percent accuracy, which is much higher than the accuracy of other methods studied in the literature. For sliding mode design the typical discharging voltage behaviour of Lithium-sulfur and potential uncertainty in process and measurement models are considered. The uncertainty in process, Q-uncertainty, is introduced by unknow covariance. As voltage behaviour of Li-S is different from the voltage Li-Ion, the design parameters of SMO based Li-S SoC estimator are different than that of SMO based Li-Ion SOC estimator available in the literature. The performance between SMO and conventional EKF has been compared with constant and mixed-pulse dynamic current. The SMO is much robust than the EKF when the model is heavily perturbed. The absolute error between estimated and true SOC is used to measure the accuracy of estimators. The absolute estimation error of SMO is less than five percent, whereas the error of

EKF is more than five percent for considered dynamic load and perturbed, that is Q-uncertain, Li-S battery model. The future works include a study on robustness against initial SOC uncertainty and a design of dual SMO for state of health estimation of Li-S batteries.

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