

Interactive effects of deformable wave energy converters operating in close proximity

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ABSTRACT

Flexible wave energy converters (FlexWECs) have been gaining increasing research and industrial interest as their deformable nature can potentially remedy the structural issues that limit the development of rigid WECs. To maximise the usage of space and infrastructure and improve energy efficiency, FlexWECs are normally deployed in close proximity, where the wave interaction with one device can influence others, signifying the opportunity to obtain energy efficiency enhancement from the interactions. To investigate the power capture performance of a FlexWEC array, this study employed a validated three-dimensional high-fidelity computational method to simulate the wave interaction with three FlexWECs in various array arrangements including power-take off. Based on systematic simulation cases, the present work analysed the relation between the geometrical characteristics of an isolated FlexWEC's perturbed wave field and the array's overall energy capture efficiency. The constructive interaction of the array was found the strongest when the longitudinal and lateral spacings of the array were 0.6 and 1 times of incident wavelength respectively, with a 15 % enhancement of overall captured energy compared to three devices operating in isolation. Overall, this study provides insights into the fluid-structure interaction of waves with multiple deformable structures, facilitating the modelling and planning of FlexWECs.

Nomenclature

Symbol	Description	Unit
$\mathbf{u}$	Flow field velocity	m/s
t	Time	s
ρ_f	Fluid density	kg/m ³
p	Pressure	Pa
μ	Fluid dynamic viscosity	kg/(m·s)
$\mathbf{g}$	Gravitational acceleration	m/s ²
$\mathbf{S}$	Damping source	m/s ²
$\mathbf{F}$	Immersed boundary forcing	Pa/m
α	Volume fraction of the fluid	Dimensionless
$\mathbf{d}$	Displacement of structure	m
ρ_s	Structure density	kg/m ³
σ	Cauchy stress tensor	Pa
σ^{pto}	Stress tensor of the PTO	Pa
ν^{pto}	PTO damping coefficient	MPa·s
P_a	Power absorbed of FlexWEC	W
CWR	Capture width ratio	Dimensionless

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Symbol	Description	Unit
q	Impact factor	Dimensionless
H	Wave height	m
T	Wave period	s
L	Wave length	m
h_s	Water depth	m
h	Length of FlexWEC	m
D	Width of FlexWEC	m
b	Thickness of FlexWEC	m
E	Young modulus	Mpa
η	Free water surface elevation	m
X_k	Longitudinal separation distance	m
Y_k	Lateral separation distance	m

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1. Introduction

There has been a range of conceptual designs for rigid wave energy converters (WECs), such as oscillating water columns [1–5], wave-activated bodies [6–10], and overtopping devices [11]. A rigid WEC typically consists of components including generators, hydraulic pistons, bearings, and gears, which are susceptible to mechanical failures in complex marine environments. Moreover, the rigid structure of a WEC can endure significant environmental loads and stress concentrations under extreme sea conditions [12]. This limits the economic feasibility and commercial development of rigid WEC technologies [13]. However, recent advances in dielectric elastomer generators (DEGs) may overcome these challenges. DEGs are made up of deformable materials and electrodes. When embedded within a structure, the DEGs form a power take-off (PTO) system that generates electricity from structural deformations. This new trend of flexible-material-based WEC, known as FlexWEC, not only avoids complex mechanical systems but also mitigates extreme wave loads, thereby enhancing feasibility [13, 14].

Therefore, a growing number of research has focused on FlexWECs. Reviews by Collins et al. [13] and Renzi et al. [14] outlined various design options, including deformable membranes [15], bulge waves [12, 16], floating flexible disks [17–19], and flapping cantilevered beams [20–22]. So far, such FlexWECs have been studied using several reduced-order models, which neglect the nonlinearities of the fluid, the structure, and their coupling. Babarit et al. [12] proposed a frequency-domain model to study an elastic tube WEC using potential flow theory and a one-dimensional model for the inner flow of the tube. The vibration characteristics of the elastic tube were predicted, taking into account the effects of surrounding fluid pressure loads and PTO impedance. McDonald et al. [23] combined a quasi-static finite element model with a frequency-domain dynamic model to analyze the interaction between waves and a liquid-filled membrane WEC. Shabara and Abdelkhalik [24] proposed a dynamic model using a Lagrangian approach for a deformable spherical heaving WEC. The model is coupled with the open-source boundary element method software NEMOH to explore the effect of flexible shape buoys on WEC trajectories and power generation. Zheng et al. [17] and Michele et al. [18] developed a theoretical model based on linear potential flow theory and eigenfunction matching methods for studying the hydroelastic properties and capture width ratio (CWR) of disk-shaped elastic plate WECs. These reduced-order models are computationally efficient and thus widely used in the early design stages of FlexWECs. However, for the strong nonlinearity and unsteady flow phenomena inherent in fluid-solid coupled systems under large interfacial displacements, the accuracy of reduced-order models is greatly limited. With increased computational capabilities, high-fidelity computational approaches for fluid-structure interactions in waves have been demonstrated [25–27]. Recently, different types of isolated FlexWECs have been modeled using a two-way coupling approach [28,29].

To apply WECs at a commercial level, multiple WECs are expected to be deployed in close proximity, forming a wave farm. Studying the array effect of a wave farm is significant because waves interacting with one device will influence its neighboring ones, impacting energy efficiency. Relevant research began with rigid WECs, showing that the effect of hydrodynamic interactions in a wave energy array on captured power is closely related to the spatial position of the WECs [30,31]. An optimal array layout can make such interactions constructive. Researchers have attempted to determine the optimal arrangement of WEC arrays using various metaheuristic algorithms [32–34], finding that the optimal layouts are commonly staggered and parallel to the wave crest. Based on symmetry, studying sub-unit arrays of several WECs as part of a staggered array can help determine the layout and spacing of large arrays. Child and Venugopal [35] deduced the optimal layout of a heaving cylinder type of WEC array by analysing the perturbed wave field. They found that the region of in-phase superposition of scattered or radiated

waves from an isolated heaving cylinder with the incident wave forms parabolic curves. Therefore, a high-performance layout can be obtained by placing the WECs sequentially at the parabolic curves' intersection points. Renzi and Dias [36] investigated inline, three-unit flap-type WEC arrays and found that at specific longitudinal intervals, these arrays could utilise system transverse mode resonance to achieve a high level of CWR. Wu et al. [37] investigated arrays consisting of two and three pitching non-symmetric WECs, and the results indicated that the constructive and destructive interactions were correlated with the diffracted wave patterns of each device. Han et al. [38] analysed the energy distribution characteristics of arrays of multiple two-body heaving WECs and the correlation between the array layout and the interference wave field of the devices to determine the optimal separation distance. Meanwhile, Benites-Munoz et al. [39] investigated the performance of three oscillating WECs in staggered and in-line layouts, finding that CWR can be increased by 30 % through constructive wave interactions.

However, there has been little study of the hydroelastic interactions of FlexWEC arrays. Nové-Josserand et al. [20] proposed a 1D empirical model for predicting upstream and downstream FlexWEC interference, showing that the global reflection coefficient is large, but the amplitude of the structural oscillations decreases when the lateral separation distance is half of the wavelength, while the opposite is true when the separation distance is 1/4 of the wavelength. Wei et al. [40] performed computational simulations for arrays of 2D FlexWECs and showed that the array performs best when the lateral separation distance between the devices is in the range of 0.28–0.41 of the incident wavelength. However, it is important to note that the above works have not considered 3D wave-structural interactions that contain important features of staggered arrays. The review by Huang et al. [41] suggested that the study of multiple dynamic devices in waves is lacking in existing literature, especially deformable devices.

To enhance research in the array development of FlexWECs, this study firstly establishes a fully coupled fluid-structure interaction (FSI) simulation framework. The FSI framework contains a fluid solver governed by the Navier-Stokes equations with multiphase modelling, and a solid solver for modelling nonlinear deformations of Euler-Bernoulli beams with PTO, coupled using an immersed boundary method. Subsequently, the effects of different arrangements on the three-unit array of bottom-mounted FlexWEC with DEG-PTO are investigated (illustrated in Fig. 1). The study explores the relationship between the perturbed wave field distribution, the array layout, and how the optimal array layout is determined.

The paper is organised as follows. Section 2 is dedicated to the computational approach and the establishment of the simulation model. Section 3 presents the verifications of the model's setups as well as the validation of the model against experiments. Section 4 first evaluates the basic characteristics of an isolated FlexWEC, then investigates the energy capture characteristics of three FlexWECs arranged in 109 different array configurations, and finally examines the performance of four array layouts under different wave periods. Section 5 summarises the main findings of this work.

2. Computational approach

2.1. Computational domain and boundary conditions

The sketch of the three-unit FlexWEC array is shown in Fig. 2, where the origin o is the set at the midpoint of the wetted area of the middle FlexWEC. The x , y , z is the global Cartesian coordinate system, where x represents the direction of wave propagation, z is the opposite direction of gravitational acceleration, and $z = 0$ represents the still water level.

The computational domain ranges in the xyz direction are $[-3L, 4L] \times [-4L, 4L] \times [-1.0h_s, 0.6h_s]$, where L is the wavelength and h_s is the water depth. Three bottom-mounted FlexWECs are arranged in the computational domain. FlexWECs are numbered from 1[#] to 3[#] along the y -axis. FlexWEC 3[#] is located at a mirror position of FlexWEC 1[#] relative

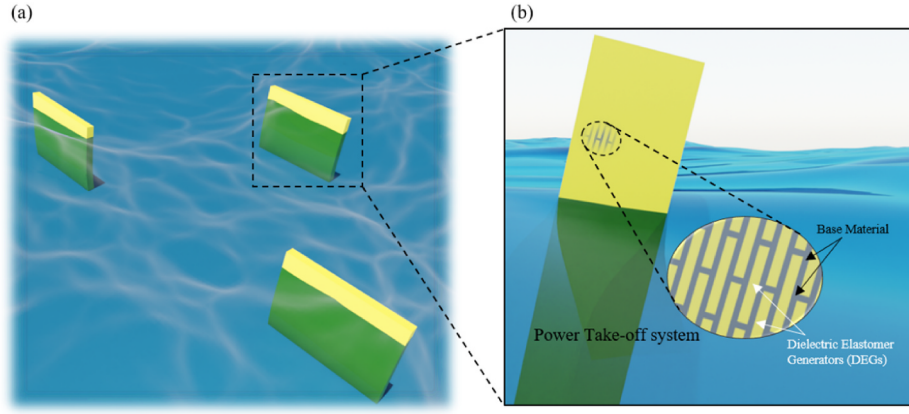


Fig. 1. Schematic of the three-unit FlexWEC array studied in this work; (a) overall view; (b) inner DEG-PTO systems.

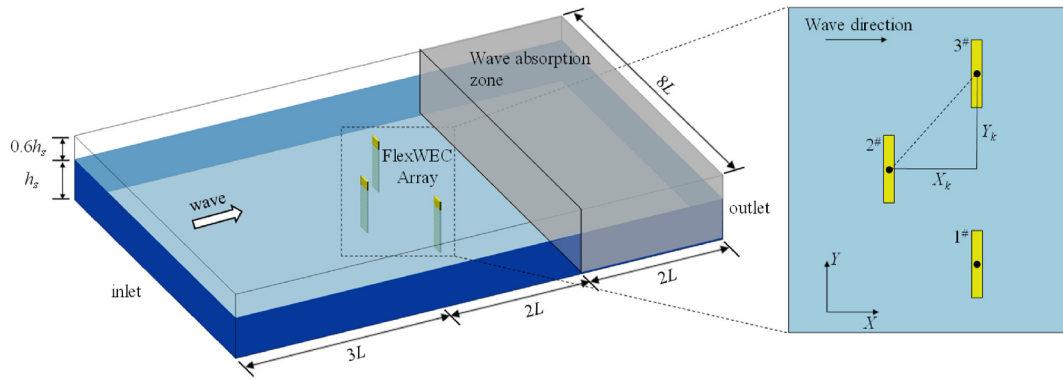


Fig. 2. Schematic of the computational domain and partial enlargement of the FlexWEC array.

to FlexWEC 2[#]. As shown in Fig. 2, the separation distance between the centers of FlexWEC 2[#] and FlexWEC 3[#] is denoted by X_k and Y_k in the coordinate system. The FlexWEC surface is an impermeable boundary, the bottom is fixed to the seabed, and the free end can oscillate along the x -axis under wave action. Each FlexWEC shares the same density, elasticity, and geometric properties. The dimensions of the FlexWEC in the x , y , and z -axis directions are denoted as b , D , and h , respectively.

The boundary conditions are established as follows: At the upper boundary, the pressure and fluid velocity are set to the open boundary, meaning the gradient is set to zero. The lower boundary is set as a no-slip condition. The boundaries on both sides perpendicular to the y -axis are set as free-slip boundary conditions. The boundaries perpendicular to the x -axis are set as the wave inlet and outlet boundaries, respectively. The water level and fluid velocities at the inlet boundary are determined according to second-order Stokes wave theory. Sommerfeld radiation boundary conditions are used for the outlet boundary [42]. To absorb the reflection caused by the outlet boundary, an absorption region of $2L$ is placed at the right end of the domain, as shown in the gray area in Fig. 2. The fluid-structure boundary is subject to a no-slip boundary condition. Both the displacement and angle at the fixed end of the FlexWEC are set to zero.

2.2. Fluid solutions

In this study, the numerical wave tank is developed based on the Navier-Stokes solver. The incompressible viscous fluid is governed by the continuity equation and the modified momentum equation:

$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

$$\frac{\partial(\rho_f \mathbf{u})}{\partial t} + \nabla \cdot (\rho_f \mathbf{u} \mathbf{u}^T) = -\nabla p + \nabla \cdot (\mu(\nabla \mathbf{u} + \nabla \mathbf{u}^T)) + \rho_f \mathbf{g} + \rho_f \mathbf{S} + \mathbf{F} \quad (2)$$

where $\mathbf{u}$ is the fluid velocity vector, p is the pressure, ρ_f is the fluid density, t is the time, $\mathbf{g}$ is the gravitational acceleration, μ is the fluid dynamic viscosity, $\mathbf{F}$ is the momentum forcing imposed by the moving fluid-structure boundary, which is further detailed in Eq. (14), and $\mathbf{S}$ is the damping term that is equal to zero except for the wave absorption zone. The equation for $\mathbf{S}$ is given by Ref. [43] as:

$$\mathbf{S} = -\beta \left(\frac{\mathbf{x} - \mathbf{x}_s}{x_e - x_s} \right)^2 \frac{z_b - z}{z_b - z_{fs}} \mathbf{u}_z \quad (3)$$

where β is a numerical coefficient that damps the undesirable wave reflections at boundaries (mimicking an open-sea computational domain), which is set to 4 in this study; x_s and x_e are the coordinates of the starting point and the ending point of the dissipation zone in the x -direction, respectively; z_b and z_{fs} are the coordinates of the bottom of the dissipation zone and the free surface in the z -direction.

Additionally, the free water surface is captured by the piecewise linear interface calculation Volume of Fluid (PLIC-VOF) method in this study [44]. To reconstruct the air-water interface, it is necessary to solve the convection equation for the volume fraction of the fluid α :

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) = 0, \quad (4)$$

Thus, the density and viscosity of the two-phase fluid can be weighted by the fluid volume fraction:

$$\rho_f = \rho_w \alpha + \rho_a (1 - \alpha), \mu = \mu_w \alpha + \mu_a (1 - \alpha). \quad (5)$$

In the present study, $\rho_w = 1000 \text{ kg/m}^3$, $\rho_a = 1.29 \text{ kg/m}^3$, $\mu_w = 0.89 \times 10^{-3} \text{ kg/(m}\cdot\text{s)}$, $\mu_a = 1.86 \times 10^{-5} \text{ kg/(m}\cdot\text{s)}$ are generally taken.

The four-step projection method proposed by Choi and Moin [45] is used to numerically solve the governing equations. The computational domain of fluid is discretised using a Cartesian grid, which is refined near the free surface and structures. Initial velocities are explicitly advanced to intermediate velocities as in Eq. (2). The convective term of Eq. (2) is discretised using a consistent mass-momentum scheme, ensuring computational stability for high-density ratio flow as outlined in Ref. [25]. Subsequently, the pressure is obtained by solving the Poisson pressure equation, and finally, the velocity is updated for the next time step. The fluid volume fraction is updated for the next time step based on the fluid velocity, as described in Eq. (4).

2.3. Structural solutions with PTO

The FlexWEC is assumed to be elastic, with its dynamics governed by:

$$\rho_s \frac{\partial^2 \mathbf{d}}{\partial t^2} = \nabla \cdot \boldsymbol{\sigma} + \nabla \cdot \boldsymbol{\sigma}^{pto} + \rho_s \mathbf{g}, \quad (6)$$

where $\mathbf{d}$ is the displacement, ρ_s is the density of the FlexWEC, $\mathbf{g}$ is the gravitational acceleration, $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}^{pto}$ are the Cauchy stress tensor and the additional stress tensor of the Power Take-Off (PTO) system, respectively. $\boldsymbol{\sigma}^{pto}$ is proportional to the normal strain rate, and the component can be written as:

$$\sigma_{ij}^{pto} = \begin{cases} -\nu^{pto} \frac{\partial^2 d_i}{\partial x_j \partial t}, & i = j, \\ 0, & \text{else} \end{cases} \quad (7)$$

where i and j are the stress tensor indices, and ν^{pto} is the PTO damping coefficient. In this study, the structure is simplified using the Euler-Bernoulli beam assumption, meaning that each cross-section remains rigid and perpendicular to the central axis during deformation. This study neglects the shear deformation of FlexWEC in the cross-section, which could introduce errors when simulating structures with a

length-to-thickness ratio of 10. In Appendix A, the error of the beam model was evaluated to be approximately 2.1 %, which is not significant. Therefore, the displacement $\mathbf{d}_M = (d_{xM}, d_{yM}, d_{zM})$ of any material point named M in the structure can be represented by translations (d_{xC}, d_{yC}, d_{zC}) and rotations ($d_{rxC}, d_{ryC}, d_{rzC}$) of point C on the central axis (as shown in Fig. 3(a)). The formulations can be written as:

$$d_{xM} = d_{xC} - y_M d_{rzC} + z_M d_{ryC}, d_{yM} = d_{yC} - z_M d_{rxC}, d_{zM} = d_{zC} + y_M d_{rxC}, \quad (8)$$

$$\frac{\partial d_{yC}}{\partial x} = d_{rzC}, \frac{\partial d_{zC}}{\partial x} = -d_{ryC}, \quad (9)$$

where y_M and z_M are the coordinates of M .

As shown in Fig. 3(b), the flexible body is discretised using beam elements. Each element, arranged on both sides of the central axis, has two nodes, denoted as A and B , as shown in Fig. 3(c). A local coordinate system can be established with the x -axis aligned positively along the central axis of the structure, and the y -axis and z -axis on the cross-section. Each node has three translational and three rotational degrees of freedom, including one axial force, two shear forces, two bending moments, and one torque that can be transmitted. The horizontal displacements d_{xC} and torsion d_{rxC} are obtained by linear interpolation of the corresponding displacements d_x and d_{rx} at points A and B . The lateral displacements d_{yC} and d_{zC} are obtained by cubic interpolation, and the rotational displacements are obtained by Eq. (9). Therefore, the dynamic equations of the FlexWEC can be discretised as:

$$[m]\{\ddot{\mathbf{d}}\} + ([k_e] + [k_g])\{\mathbf{d}\} + [C^{pto}]\{\dot{\mathbf{d}}\} = \{\mathbf{f}_h\} - \{\mathbf{f}_{internal}\}, \quad (10)$$

where $\{\ddot{\mathbf{d}}\}$ and $\{\dot{\mathbf{d}}\}$ are the acceleration and velocity of the node, respectively, $[m]$ is the mass matrix, and $[k_e]$ and $[k_g]$ represent the linear and geometrically nonlinear stiffness matrices, respectively. $\{\mathbf{f}_h\}$ is the hydrodynamic load vector and $\{\mathbf{f}_{internal}\}$ is the internal force equivalent load. More detailed information on the mass and stiffness matrix and vector formulas can be found in Wang et al. [46]. $[C^{pto}]$ is the damping matrix due to the PTO system, which can be written as:

$$[C^{pto}] = \begin{bmatrix} [C^1] & [C^1] \\ [C^1]^T & [C^1] \end{bmatrix}, \quad (11)$$

$$[C^1] = \begin{bmatrix} a & 0 & 0 & 0 & 0 & 0 \\ & b & 0 & 0 & 0 & c \\ & & e & 0 & -f & 0 \\ & & & 0 & 0 & 0 \\ & & & & g & 0 \\ \text{Symm.} & & & & & h \end{bmatrix}, [C^2] = \begin{bmatrix} -a & 0 & 0 & 0 & 0 & 0 \\ 0 & -b & 0 & 0 & 0 & c \\ 0 & 0 & -e & 0 & -f & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & f & 0 & l & 0 \\ 0 & -c & 0 & 0 & 0 & m \end{bmatrix}, [C^3] = \begin{bmatrix} a & 0 & 0 & 0 & 0 & 0 \\ & b & 0 & 0 & 0 & -c \\ & & e & 0 & f & 0 \\ & & & 0 & 0 & 0 \\ & & & & g & 0 \\ \text{Symm.} & & & & & h \end{bmatrix}. \quad (12)$$

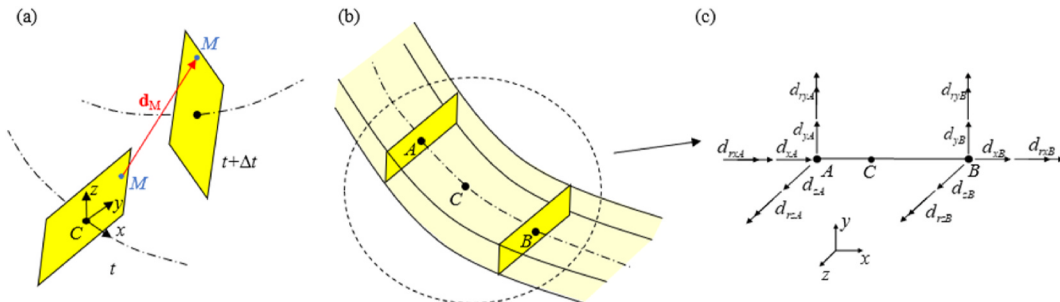


Fig. 3. Illustration of the structural model; (a) The displacement of a point on the cross-section from t to $t + \Delta t$ with the central axis; (b) Schematic diagram of a discretised slender body; (c) A beam element with 12 degrees of freedom.

The coefficients of the matrix are as follows: $a = \nu^{pto} A/L$, $b = 12\nu^{pto} I_z/L^3$, $c = 6\nu^{pto} I_z/L^2$, $e = 12\nu^{pto} I_y/L^3$, $f = 6\nu^{pto} I_y/L^2$, $g = 4\nu^{pto} I_y/L$, $h = 4\nu^{pto} I_z/L$, $l = 2\nu^{pto} I_y/L$, $m = 2\nu^{pto} I_z/L$. A represents the cross-sectional area of the beam, I_y and I_z represent the moments of inertia of the cross-section along the y -axis and z -axis, respectively, and L represents the length of the element.

2.4. Fluid-structure interaction

The solid-fluid interface boundary condition is obtained by velocity and stress continuum conditions, which include no-slip, no-permeability, and traction conditions. The velocity boundary condition for no slip and no infiltration can be written as:

$$\mathbf{u}_s = \left(\frac{\partial d_x}{\partial t} - Y_s \frac{\partial^2 d_y}{\partial x \partial t} + Z_s \frac{\partial^2 d_z}{\partial x \partial t} \right) \mathbf{i} + \left(\frac{\partial d_y}{\partial t} - Z_s \frac{\partial^2 d_x}{\partial t} \right) \mathbf{j} + \left(\frac{\partial d_z}{\partial t} + Y_s \frac{\partial^2 d_x}{\partial t} \right) \mathbf{k}, \quad (13)$$

where the left side of Eq. (13) is the fluid velocity and the right side is the structural velocity. The structural velocity is obtained from the velocity $\partial d_i / \partial t$ of the cross-section. $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the basis vectors along the x -, y -, and z -axes of the local coordinate system of the cross-section, respectively.

The fluid-structure interaction is modeled as a momentum forcing term $\mathbf{F}$ on the right sides of Eq. (2):

$$\mathbf{F} = \frac{\rho^{n+1} \tilde{\mathbf{u}} - \rho^n \mathbf{u}^n}{\Delta t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u})^n - \nabla \cdot (\mu^n (\nabla \mathbf{u} + \nabla \mathbf{u}^T)^n) + \nabla p^n - \rho^{n+1} \mathbf{g} - \rho^{n+1} \mathbf{S}^n, \quad (14)$$

where the superscript n denotes the time step. As shown in Fig. 4(a), $\mathbf{F}$ is not zero only in the interior of the structure and in the outermost fluid grid of the structure. $\tilde{\mathbf{u}}$ represents the weighted average between the Navier-Stokes solution and the velocity reconstruction:

$$\tilde{\mathbf{u}} = (1 - \gamma) \mathbf{u}^{**} + \gamma \mathbf{U}, \quad (15)$$

$$\rho^{n+1} \mathbf{u}^{**} = \rho^n \mathbf{u}^n + \Delta t [\nabla \cdot (\rho \mathbf{u} \mathbf{u})^n + \nabla \cdot (\mu^n (\nabla \mathbf{u} + \nabla \mathbf{u}^T)^n) - \nabla p^n], \quad (16)$$

where $\mathbf{U}$ represents the structural surface velocity reconstruction (see more details in Ref. [46]). In this study, a quadratic least-squares reconstruction near the immersed boundary is used. $\mathbf{u}^{**}$ is the intermediate velocity without the effect of momentum forcing and is also

used to determine the velocity reconstruction stencil. As the boundary gradually intrudes into the fluid domain, in order to achieve a smooth transition of the boundary velocity, γ is gradually changed from 1 to 0. In this study, γ is calculated with the following expression:

$$\gamma = V_{\min} / \Delta x \Delta y \Delta z, \quad (17)$$

where $V_{\min}$ is the minimum volume occupied by the fluid in the neighboring grid. For example, in Fig. 4(b), $V_{\min}$ is the smallest value of V_1 , V_2 , V_3 , and V_4 . Δx , Δy , Δz are the grid sizes in the x , y , and z directions. The traction condition can be written as:

$$-\mathbf{p} \mathbf{n} + \mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T) \cdot \mathbf{n} = \mathbf{t}, \quad (18)$$

where $\mathbf{t}$ is the surface traction. Integration of surface traction results in the hydrodynamic forces of the structure:

$$\{\mathbf{f}_h\} = \mathbf{f}_{hl} = \int_{S^n} \left[-p \mathbf{n}_j + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) n_i \right] N_{jl} dS. \quad (19)$$

where N is the shape function of a beam element. The following table gives the fluid-solid coupling process over a complete time step.

- (i) Evaluate Eq. (19) to obtain the hydrodynamic forces, solve the structural dynamics equations by Eq. (10), and update the new positions of the structural nodes $\mathbf{d}^{n+1}$.
- (ii) Obtain $\mathbf{u}^{**}$ by solving the preliminary explicit momentum step, using Eq. (16).
- (iii) The volume occupied by the structure in the fluid grid is calculated, and the weighting factor γ is computed according to Eq. (17).
- (iv) The surface velocity reconstruction is applied to obtain $\tilde{\mathbf{u}}$. Then the momentum forcing $\mathbf{F}$ in Eq. (14) can be calculated.
- (v) A four-step projection method is used to calculate the pressure p^{n+1} and flow velocity $\mathbf{u}^{n+1}$.
- (vi) Eq. (4) is used to update the fluid volume fraction α^{n+1} and calculate the mass fluxes for the velocity control volume and the pressure control volume.

2.5. Data analysis

The power absorbed (P_a) by each FlexWEC can be calculated by integrating the PTO stress and strain as:

$$P_{a,i} = \int_V \sigma_i^{pto} \dot{\epsilon}_i dV = \{\dot{\mathbf{d}}\}^T [C_{pto}] \{\dot{\mathbf{d}}\} \quad (20)$$

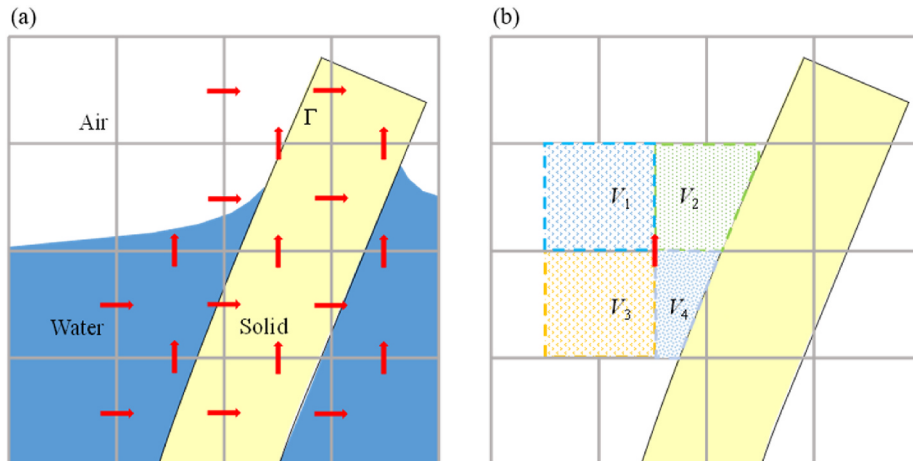


Fig. 4. Schematic diagram of the discretised problem; (a) Momentum forcing applied on the inner and outermost fluid grids of the structure; (b) V_1 , V_2 , V_3 , and V_4 are the volumes occupied by the fluid in the neighboring grid used to calculate γ .

The capture width ratio (*CWR*) is defined as the ratio of average absorbed power $P_{a,avg}$ to the incident wave power $P_w D$ [47,48]:

$$CWR_i = P_{a,avg,i} / P_w D \quad (21)$$

$$P_{a,avg,i} = \frac{1}{T} \int_t^{t+T} P_{a,i} dt = \frac{1}{T} \int_t^{t+T} \{\dot{d}\}^T [C_{pto}] \{\dot{d}\} dt \quad (22)$$

where $P_w = \rho g c_g H^2 / 8$ denotes the wave energy flux per unit width, c_g represents the wave group velocity, H denotes the wave height, T is the wave period, and D is the width of the FlexWEC. To further analyze the overall performance of the array, an impact factor q for each FlexWEC and the entire array is used to measure the change in efficiency:

$$q_i = \frac{CWR_i}{CWR_{isolated}}, q_{array} = \frac{\sum_{i=1}^3 CWR_i}{NCWR_{isolated}}, N = 3, \quad (23)$$

where $CWR_{isolated}$ is the result obtained from simulating one FlexWEC in isolation. When q is greater than 1, it means that the interaction between FlexWECs enhances energy absorption, indicating the array is in a state of constructive interaction. When q_{array} is equal to 1, the effect between the devices is nullified; and when q_{array} is less than 1, the array is in a state of destructive interaction.

3. Verification and validation

To verify the accuracy of the computational model, a comparison is conducted between the computational results with a series of experimental results by Nové-Josserand [49]. In the experiment, two flexible plates are arranged one in front of the other, and the displacement and shear force of the front plate are measured. Fig. 5(a) illustrates the computational domain setup. The geometric and physical parameters used in the computational model are summarized in Table 1.

To demonstrate the validity of the adopted mesh accuracy, a mesh convergence test was first performed. As shown in Fig. 5(b), the minimum grid in the x - and y -directions is set in the FlexWEC refinement area, and the minimum grid in the z -direction is set in the wave surface refinement area. Based on the minimum grid size, three grid resolutions are designed, namely coarse, medium, and fine grid. Table 2 shows the minimum grid size of the three grids. Table 2 shows that the relative deviation of the computational results between the medium and fine grids is about 1.83 %, indicating that the medium grid has reached convergence. Considering both computational efficiency and the necessity for numerical accuracy, the medium grid is considered adequate for subsequent studies.

Fig. 6 illustrates the experimental and computational results for the relative displacement and fixed end shear force of the front side plate with varying X_k/L . The results are normalised by the case of a single isolated plate. These results show that both plate response and shear amplitude oscillate as X_k/L increases, peaking at $X_k/L = 0.23$ and troughing at $X_k/L = 0.45$. This pattern aligns with the results of Nové-

Table 1
Parameter setting for the validation.

Parameter	Value
Wave height, H (m)	6.0×10^{-3}
Wave period, T (s)	0.2
Wavelength, L (m)	6.24×10^{-2}
Water depth, h_s (m)	8.0×10^{-2}
Plate length, h (m)	9.0×10^{-2}
Plate widths, D (m)	1.4×10^{-3}
Plate thickness, b (m)	3.5×10^{-4}
Solid density, ρ_s (kg/m ³)	1380
Young modulus, E (Mpa)	5000
Longitudinal separation distance, X_k/L	0.23, 0.34, 0.45, 0.57, 0.67, 0.79, 0.90

Table 2
Mesh details and mesh convergence results in terms of maximum displacement and maximum shear force for $X_k/L = 0.23$.

	Coarse grid	Medium grid	Fine grid	Nové-Josserand (exp.) [49]
Minimum Δx	$b/2$	$b/4$	$b/8$	/
Minimum Δy	$D/14$	$D/28$	$D/56$	/
Minimum Δz	$H/10$	$H/20$	$H/40$	/
Maximum displacement (mm)	2.817 (+7.78 %)	2.698 (+3.22 %)	2.650 (+1.39 %)	2.614
Maximum shear force (mN)	5.152 (11.2 %)	4.809 (4.54 %)	4.791 (4.16 %)	4.600

Josserand [49], where $X_k/L = 0.45$ corresponds to the Bragg resonance case, leading to lower energy absorption efficiency. To further evaluate the accuracy of the computational model, the Mean Absolute Percentage Error (MAPE) is used to analyze the difference between the experiment and the simulation:

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{\hat{y}_i - y_i}{y_i} \right| \times 100\% \quad (24)$$

where $\hat{y}_i$ and y_i represent the computational and experimental results, respectively. n is the number of data points. The MAPE values of the displacement and shear force are 1.79 % and 9.19 %. There is a good agreement between the computational and experimental results of displacement, but there is some difference in the shear force. This difference is primarily attributed to the selection of the standard value for non-dimensionalization. In Nové-Josserand [49], the maximum shear force of a single plate was used as the standard value to non-dimensionalize the maximum shear force of two plates, but the exact standard value was not specified. Therefore, in this study, the maximum shear force of the isolated plate is averaged based on the peak values of the shear force over 10 periods of the experimental results. Using this standard value for non-dimensionalization resulted in higher non-dimensional shear forces in our results. Overall, the computational

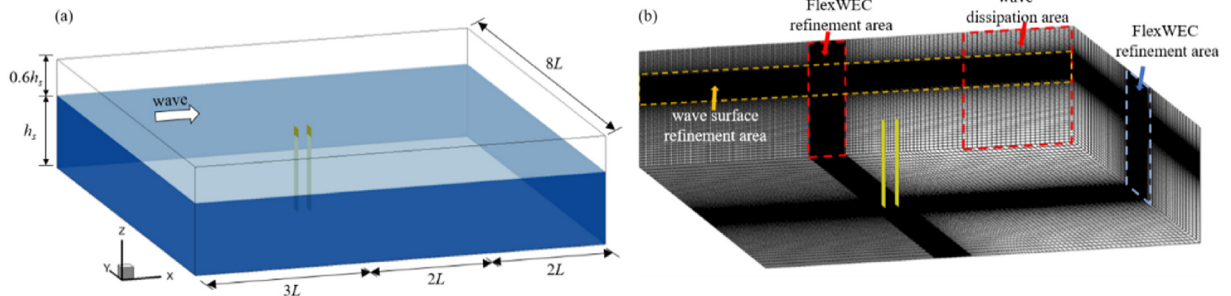


Fig. 5. Schematic diagram of the validation model; (a) The computational domain; (b) Grid of the domain with refinements implemented for the free surface region and the region around the FlexWECs.

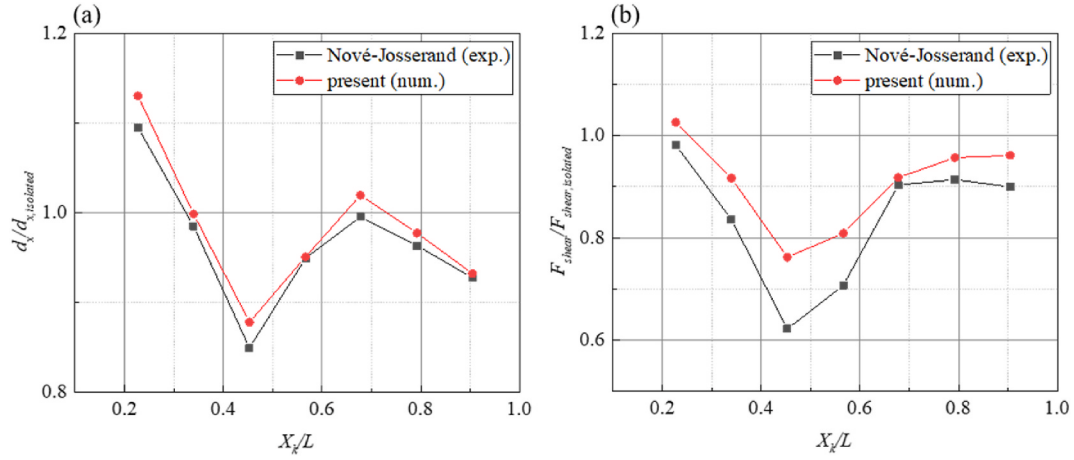


Fig. 6. Comparison between the experimental and computational results of different separating distances X_k/L ; (a) Maximum displacement; (b) Maximum shear force.

Table 3

Parameters setting for the wave condition and FlexWEC.

Parameter	Value
Wave steepness, kH	0.1
Water depth, h_s (m)	5.0
Wave period, T (s)	2.95, 3.74, 4.54, 6.26, 8.04
FlexWEC length, h (m)	6.0
FlexWEC width, D (m)	8.0
FlexWEC thickness, b (m)	0.6
Relative wave length, L/D	1.67, 2.5, 3.33, 5, 6.67
Solid density, ρ_s (kg/m^3)	1200
Young modulus, E (Mpa)	12.0
PTO damping coefficient, ν_{PTO} (MPa·s)	20

simulation results have consistent patterns with the experiments, thus demonstrating the validity and reliability of the present code as a tool for predicting the performance of FlexWEC in regular wave conditions.

4. Results and discussion

In this section, full-scale FlexWECs are tested. The wave condition and FlexWEC parameters are presented in Table 3. To obtain the $CWR_{isolated}$, an isolated FlexWEC is first studied. Subsequent methodical simulations are conducted to analyze the CWR_i and q for three-unit arrays with different separation distances, with the aim of identifying the optimal array arrangement.

4.1. Performance of an isolated FlexWEC

Fig. 7(a) shows the variation of $CWR_{isolated}$ for an isolated FlexWEC

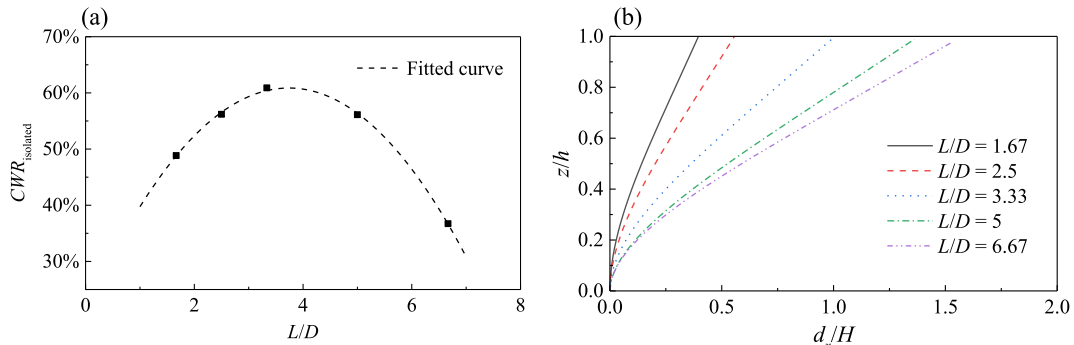


Fig. 7. Energy capture and motion characteristics of FlexWEC under different L/D ; (a) CWR , (b) Dimensionless deformation amplitude along the length.

with relative wave length L/D . The $CWR_{isolated}$ increases with increasing L/D and then decreases after reaching its maximum value. It can be seen that the $CWR_{isolated}$ reaches the peak value at 61 % when $L/D = 3.33$. Fig. 7(b) illustrates the dimensionless deformation amplitude along the length of FlexWEC. The results indicate that FlexWEC vibrates in its first mode of deformation. As L/D increases, the displacement gradually increases. When $L/D > 3.33$, the FlexWEC exhibits significant bending deformation (corresponding to a relative deflection $d_{x,max}/h > 0.1$). However, after $L/D > 3.33$, the rate of increase in deformation amplitude slows down as L/D continues to rise, at which point the geometric nonlinearity limits the structural deformation. Consequently, the energy capture due to the deformation becomes insufficient to match the increase in incident wave energy, corresponding to the decrease in $CWR_{isolated}$ observed in Fig. 7(a).

Fig. 8 illustrates the variation of free end motion (d_x , d_z) in x - and z -directions for an isolated FlexWEC at two wave periods, accompanied by the corresponding power generation profile. t_1 and t_2 correspond to the maximum and minimum horizontal displacement d_x of the FlexWEC in the x -axis, respectively, during which the instantaneous power generation is zero. The absolute value of the minimum value of d_x is smaller than the maximum value, indicating that the FlexWEC oscillates more toward the shore side than the seaward side, and thus there are also two different troughs of d_z . t_3 and t_4 represent the timeframe during which the beam returns to its equilibrium position, the FlexWEC reaches its maximum velocity, and the instantaneous power reaches its peak value. The peaks corresponding to the t_3 and t_4 are significantly different at $L/D = 6.67$. This discrepancy highlights the nonlinear behavior, where the velocity from the maximum seaward displacement to the equilibrium position is slower than from the maximum landward displacement to the equilibrium position. These nonlinearities in the motion and power

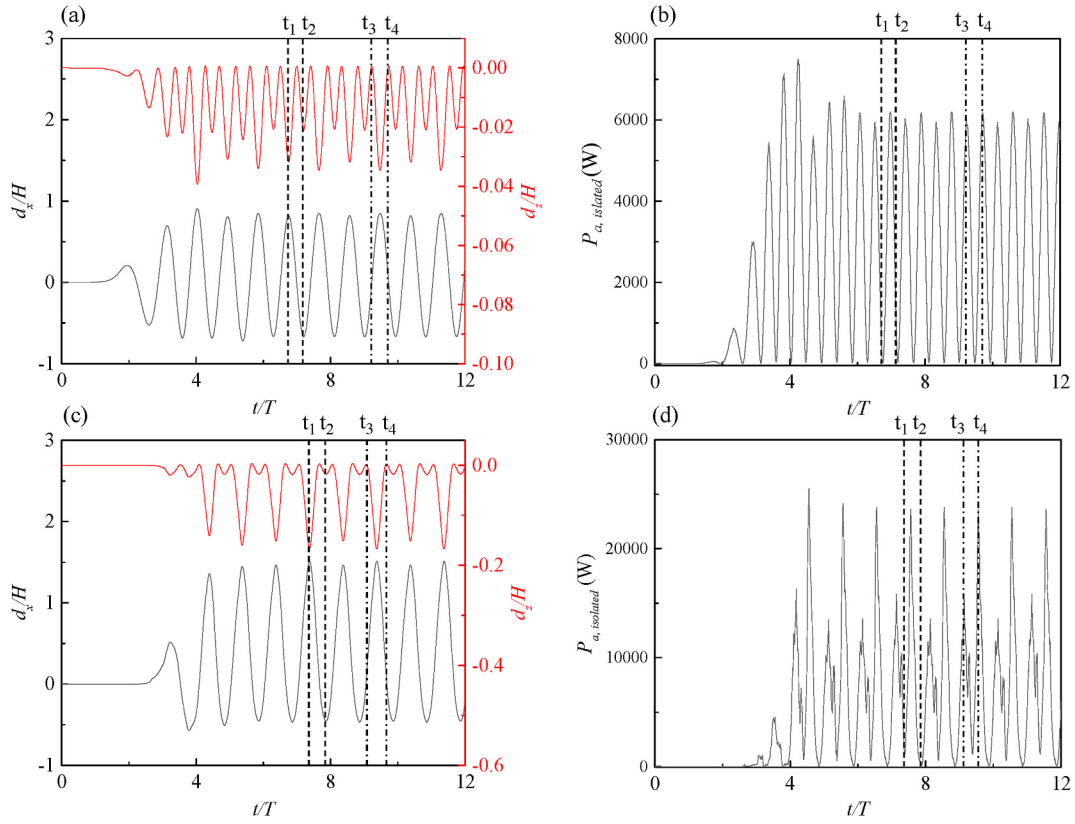


Fig. 8. Motion and power generation of an isolated FlexWEC; (a) Free end motion for $L/D = 3.33$ (Black line: horizontal-direction displacement, red line: vertical-direction displacement); (b) Power extracted for $L/D = 3.33$. (c) Free end motion for $L/D = 6.67$ (Black line: horizontal-direction displacement, red line: vertical-direction displacement); (d) Power extracted for $L/D = 6.67$.

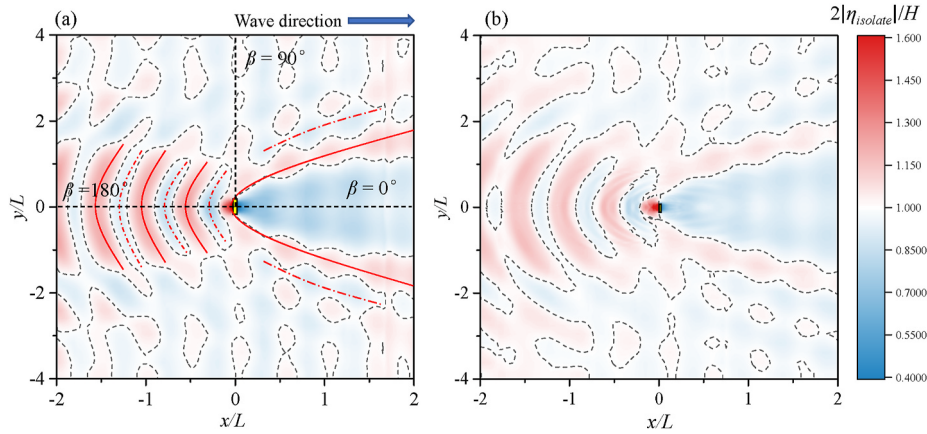


Fig. 9. Spatial distribution of wave amplitudes around an isolated FlexWEC; (a) $L/D = 3.33$, (b) $L/D = 6.67$.

generation profiles underscore the complex dynamics of the FlexWEC system.

Fig. 9 illustrates the spatial distribution of wave amplitude surrounding an isolated FlexWEC with $L/D = 3.33$ and 6.67 . The wave amplitude is obtained by calculating the average wave amplitude over five wave periods using the zero crossing method. The red signifies the region of wave amplification, while the blue area indicates the region with weakened wave amplitude. In both wave conditions, the wave height shows a similar distribution pattern. The water surrounding the FlexWEC with $y > 0$ can be divided into three zones based on the distribution form of the red and blue regions, which is associated with the azimuthal angle β . In the frontal zone (i.e., β near 180°), the wave amplitude undergoes periodic changes with spatial variation, and the

spatial interval for this periodic variation is $0.5L$. In the rear zone (i.e., β near 0°), the wave height is noticeably attenuated due to the shielding effect, resulting in minimal variation in wave amplitude with spatial changes. Wave height fluctuations are more pronounced, and the wave height shielding effect is more significant at $L/D = 3.33$, indicating a greater impact of the FlexWEC on the wave field. In the side zone (i.e., β close to 90°), the wave amplitude is distributed in a checkerboard grid-like pattern, and the spatial interval for the same wave amplitude is approximately L . The distribution of wave amplitude is a result of the superposition of the incident wave, scattered wave, and radiated wave from the FlexWEC. The wave elevation resulting from superposition can be written as:

$$\eta_{\text{isolate}}(x, y, t) = H/2 \cos(-\omega t + kx) + A_d H/2 \cos(-\omega t + k \cos \beta x + k \sin \beta y + \phi), \quad (25)$$

where A_d is the relative amplitude of the composite wave from the scattered and radiated wave, ϕ is the phase of the composite wave, and H is the wave height of the incident wave. The trigonometric function of the above equation can be further simplified as:

$$\eta_{\text{isolate}}(x, y, t) = |\eta_{\text{isolate}}(x, y)| \cos(-\omega t + \psi), \quad (26)$$

$$|\eta_{\text{isolate}}(x, y)| = H / \sqrt{2 \sqrt{(1 + A_d^2 + 2A_d \cos(kx - ky \cos \beta - ky \sin \beta - \phi))}}, \quad (27)$$

where ψ is the phase of the superposed wave elevation. It is evident that the cosine function on the right side of Eq. (27) results in a periodic variation of the superimposed wave amplitude. The cosine function changes from $\cos(-\phi)$ to $\cos(kx - ky - \phi)$ as the angle β varies from 0° to 90° . At the angle of 180° , the cosine function transforms into $\cos(2kx - \phi)$. This explains the different spatial interval of the wave amplitude as the azimuthal angle β varies. Solid and dashed red characteristic lines, based on the simulated wave height distribution, are plotted in Fig. 9. The solid lines indicate where $|\eta_{\text{isolate}}|$ takes an extremely large value, i.e., the waves from the FlexWEC are in-phase with the ambient incident wave. The dashed lines indicate where an extremely small value occurs, representing the out-of-phase condition. In addition, it is worth noting that the existence of the standing wave in front of the FlexWEC is confined to a certain range of angles, i.e., the presence of dipole-pattern waves. This is caused by the fact that the width of FlexWEC is comparable to L ($D = 0.3L$) and that the vibration mode of the FlexWEC is predominantly surge and pitch [50].

4.2. Performance of three-unit array

In the case of the three-unit array, given the symmetry of FlexWECs $1^\#$ and $3^\#$ with respect to $2^\#$, the array arrangement is determined by the variables X_k and Y_k . The values of X_k and Y_k used in our study are presented in Table 4. X_k varies from one wavelength upstream of $2^\#$ to one wavelength downstream, with intervals of $0.2L$. Y_k initiates at $0.25L$ and gradually increases to $2.5L$ in intervals of $0.25L$. The wave condition is set to $L/D = 3.33$. The joint effect of X_k and Y_k is analysed in this study, with a total of 109 sets of simulations conducted. To ensure that FlexWECs avoid collision, the case where $X_k = 0$ and $Y_k = 0.25L$ is not included. Fig. 10 illustrates the spatial range of $1^\#$ and $3^\#$ encompassed by different array arrangements. In this study, a typical rubber material with $E = 12$ MPa is selected.

4.2.1. Wave energy conversion performance

The variations in the power contribution of $1^\#$ are first studied. Fig. 11(a) shows a color block chart of q_1 with the dimensionless distance. q_1 exhibits a range of variation from 0.641 to 1.21, gradually converging to 1.0 as Y_k/L increases. This trend indicates a weakening interaction between the FlexWECs as Y_k/L increases. The variation of q_1 is mainly due to the actual incident wave elevation, η_1 , at the location of $1^\#$. Since $3^\#$ is farther away from $1^\#$, η_1 is assumed to be a superposition of the incident wave and the scattered and radiated wave from $2^\#$:

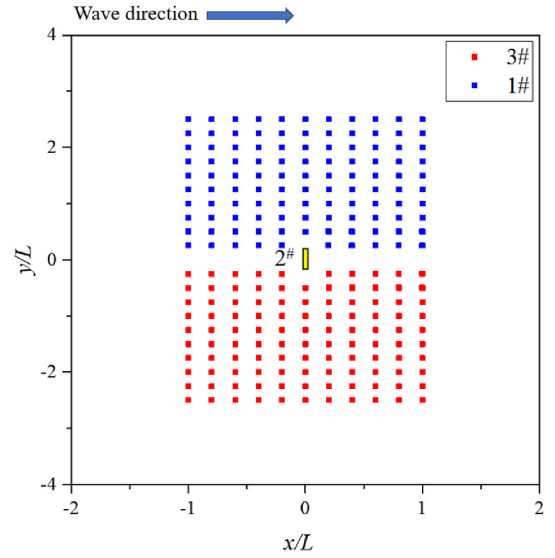


Fig. 10. Different arrangements of the array, where $2^\#$ is fixed at the origin of the coordinates.

$$2\eta_1(X_k, Y_k, t) / H = \cos(-\omega t + kX_k) + A_d \cos(-\omega t + k \cos \beta X_k + k \sin \beta Y_k + \phi), \quad (28)$$

Comparing Eq. (22) and Eq. (25), it is evident that η_1 under various separation distances is equivalent to the perturbed wave of an isolated device. Thus, the amplitude of η_1 can be expressed as:

$$|\eta_1(X_k, Y_k)| = |\eta_{\text{isolate}}(X_k, Y_k)|, \quad (29)$$

Fig. 11 (b) shows the contour plot of the $|\eta_1|$, i.e., the wave amplitude corresponding to $x/L = [-1, 1]$, $y/L = [0.25, 2.5]$ in Fig. 9. In Fig. 11(a), it can be observed that the variation of q_1 is spatially sensitive, evident from different enhancement and reduction stripes. In a manner similar to the periodic variation of $|\eta_1|$, the spatial period of q_1 changes from $0.5L$ to L as β decreases from 180° to 90° . A low q_1 is noticeable when the angle β is 0° , indicating that the device is placed in the wave attenuation region from $1^\#$. Between the three angles, the spatial period exhibits transitional features.

It is noteworthy that the maximum value of q_1 is obtained at $(X_k/L = -0.2, Y_k/L = 0.5)$, corresponding to the location where the amplitude of the superposition wave is at its minimum. However, another type of WEC, such as a heaving WEC, has been observed to demonstrate a higher CWR in the region of wave amplification [35], which is different from the region with weakened wave amplitude as investigated in this study. The difference arises from the different wave excitation forces acting on the two types of WECs. The motion of the heaving WEC is driven by the vertical wave force, and the magnitude of the superposition wave excitation force is associated with the superimposed wave amplitude. Consequently, higher wave amplitudes result in an increased vertical excitation force, leading to greater WEC motion and enhanced wave energy capture. In contrast, the FlexWEC's motion is primarily induced by the horizontal wave force, and the magnitude of the superposition wave excitation force is influenced by both the amplitude and propagation direction of each component wave.

As shown in Fig. 12, at the position $(X_k/L = -0.2, Y_k/L = 0.5)$, $1^\#$ is subjected to both the forward-propagating incident wave and the backward-propagating reflected wave from $2^\#$. This leads to $1^\#$ experiencing the maximum horizontal combined wave force when the two waves are out-of-phase. Consequently, if $1^\#$ is positioned before $2^\#$, the q_1 is maximised in the region with weakened wave amplitude. Conversely, at the position $(X_k/L = 0.8, Y_k/L = 1.0)$, $1^\#$ is subjected to the forward-propagating incident wave and the scattered and radiated wave from $2^\#$. This results in $1^\#$ experiencing the maximum horizontal

Table 4
Setting for array arrangements of three FlexWECs.

Parameter	Value
Longitudinal separation distance, X_k/L	-1, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 1
Lateral separation distance, Y_k/L	0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, 2, 2.25, 2.5

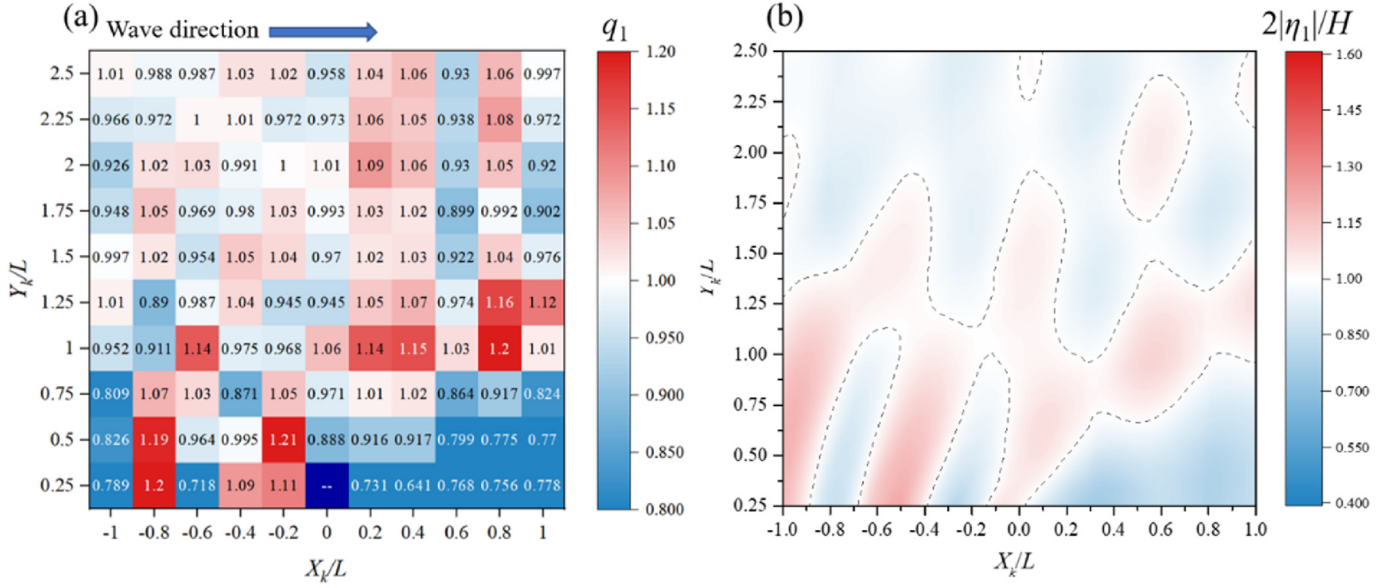


Fig. 11. Variations in q_1 and $|\eta_1|$ as a function of the non-dimensional separation distance when $L/D = 3.33$; (a) Color block chart of q_1 (b) Contour plot of the superposed wave amplitude of the incident wave and the scattered and radiated wave from $2^\#$ when $1^\#$ and $3^\#$ are removed from the wave field.

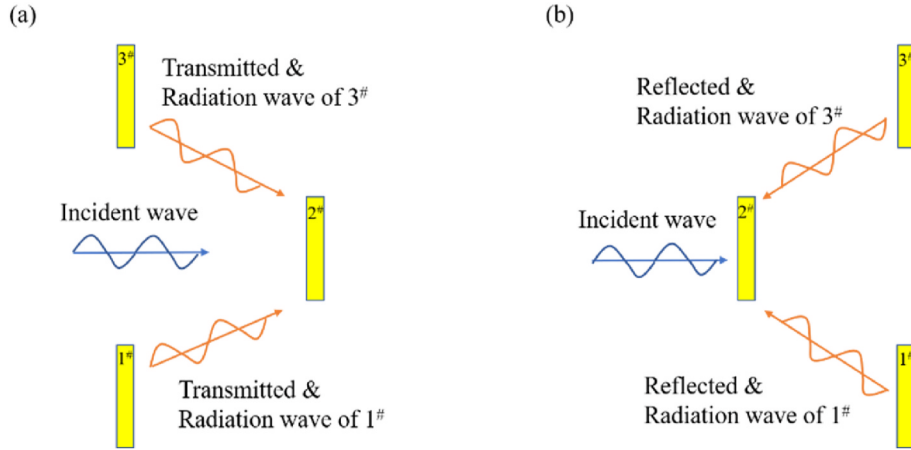


Fig. 12. The propagation directions of radiated, diffracted and incident waves in different configurations: (a) $X_k/L = -0.2$, $Y_k/L = 0.5$, (b) $X_k/L = 0.8$, $Y_k/L = 1.0$.

combined wave force when the two waves are in-phase. Therefore, if $1^\#$ is placed after $2^\#$, the q_1 is maximised in the region of wave amplification. The wave energy capture performance of $3^\#$ is identical to that of $1^\#$ due to symmetry and is not repeated in this paper.

$$2\eta_2(X_k, Y_k, t)/H = \cos(-\omega t - kX_k) + 2A_d \cos(-\omega t - k\cos\beta X_k + k\sin\beta Y_k + \varphi), \quad (30)$$

$$2\eta_2(X_k, Y_k, t)/H = \sqrt{(1 + 4A_d^2 + 4A_d \cos(-kX_k + kX_k \cos\beta - kY_k \sin\beta - \varphi))} \cos(-\omega t + \psi), \quad (31)$$

Secondly, the variations of the power contribution of $2^\#$ are investigated. Fig. 13(a) shows the color block chart of q_2 with the dimensionless distance. q_2 varies from 0.548 to 1.56, which is twice as much as the variation of q_1 because of the simultaneous influence of waves from $1^\#$ and $3^\#$. The superposition of the incident wave and the scattered and radiated waves from $1^\#$ and $3^\#$ is the actual incident wave elevation, η_2 , at the location of $2^\#$. According to Eq. (25), η_2 can be written as:

Comparing Eq. (27) and Eq. (31), the amplitude of η_2 can be expressed as:

$$|\eta_2(X_k, Y_k)| = \sqrt{2|\eta_{isolated}(-X_k, Y_k)|^2 - 1} \quad (32)$$

Fig. 13 (b) shows the contour plot of the $|\eta_2|$, which is obtained by transforming $|\eta_{isolated}|$ according to Eq. (32). A similar manner of periodic variation is observed in $|\eta_2|$ and q_2 . This suggests that the radiated and diffracted waves from $1^\#$ and $3^\#$ cause spatially periodic variations in

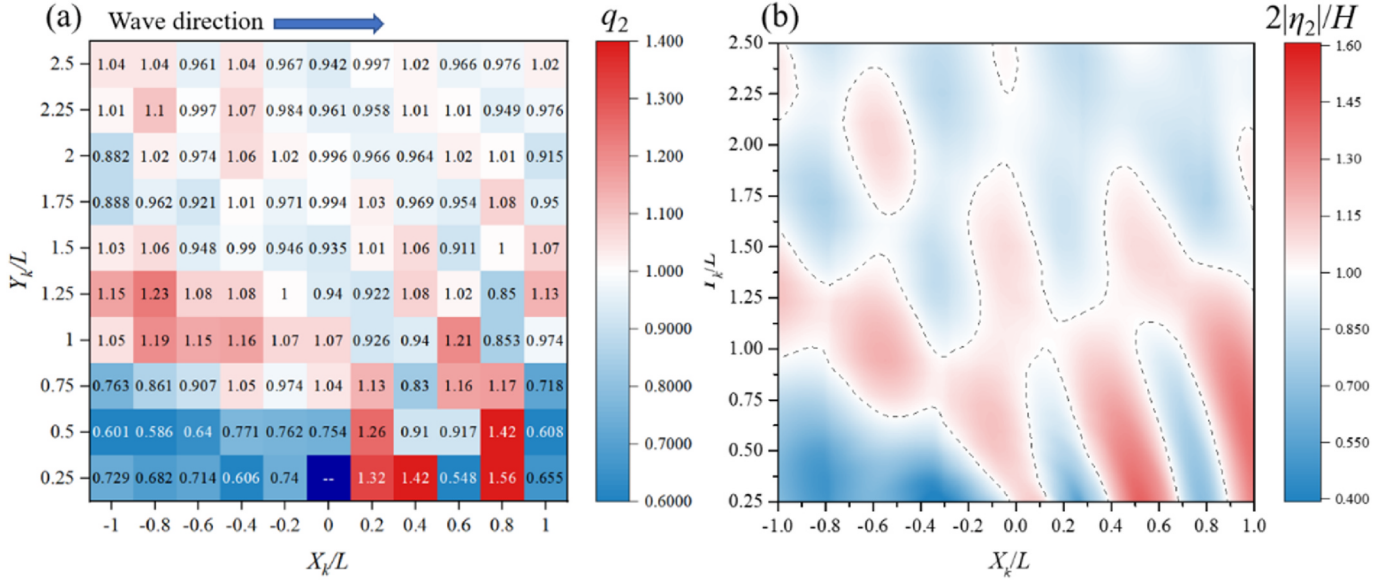


Fig. 13. Variations in q_2 and $|\eta_2|$ as a function of the non-dimensional separation distance when $L/D = 3.33$; (a) Color block chart of q_2 . (b) Contour plot of the superposed wave amplitude of the incident wave and the scattered and radiated wave from 1[#] and 3[#] when 2[#] is removed from the wave field.

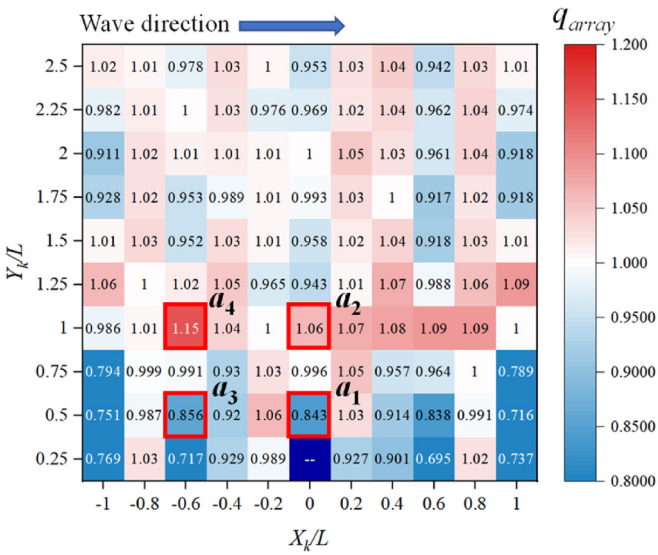


Fig. 14. Color block chart of the q_{array} as a function of the non-dimensional separation distance when $L/D = 3.33$, where a_1 , a_2 , a_3 and a_4 correspond to the four array arrangements used for the analysis in Section 4.2.3.

the $|\eta_2|$, which results in alternating gain and loss of q_2 variation with different dimensionless distances. The spatial overlap of the gain and loss stripe of $|\eta_2|$ and q_2 occurs when the azimuth angle $\beta < 0$, i.e., $X_k < 0$. While for azimuth angle $\beta > 0$, $|\eta_2|$ and q_2 enhancement and weakening stripes are staggered. This is similar to the observation in q_1 , where the propagating angle of the radiated and diffracted waves from 1[#] and 3[#] varies with azimuth angle β . The maximum value of $q_2 = 1.56$ is obtained at $(X_k/L = 0.8, Y_k/L = 0.25)$ when 2[#] is located in the reflected wave of 1[#] and 3[#]. Similarly, as shown in Fig. 11, the maximum value of q_1 also corresponds to 1[#] being in the region of the reflected wave of 2[#]. In summary, the optimal design of the FlexWEC array should prioritize the effective utilization of reflected wave energy.

Fig. 14 shows the color block chart of the q_{array} with respect to the dimensionless separation distance. A clear finding is that constructive and destructive interactions are staggered similarly to a checkerboard pattern. In the regions where $Y_k < 0.75L$, strong destructive interactions

($q < 0.9$) are observed. Despite a maximum improvement of 56 % in energy capture efficiency for 2[#] in this region, the remaining FlexWECs (i.e., 1[#] and 3[#]) experience a maximum decrease of 25 % in energy capture due to their location in the sheltered region, resulting in $q_{array} = 1.02$. Conversely, constructive interactions ($q > 1.05$) prevail over a wide range of X_k for $Y_k = L$, with $X_k = -0.6L$ yielding the maximum value of 1.15 for q_{array} . At this specific point, 1[#] and 3[#] are positioned before 2[#], and 1[#] and 3[#] are both in the region with weakened wave amplitude, while 2[#] is in the region of wave amplification. As previously discussed concerning Figs. 11 and 13, all three devices, 1[#], 2[#], and 3[#], experience a high q , resulting in the maximum q_{array} . When $X_k = 0$, the three-unit array is in an in-line arrangement. The overall efficiency of the array is maximised when the lateral spacing is L . Similar phenomena have been observed in the previous study of rigid flap-type WEC arrays [36]. This suggests the occurrence of resonance of the system transverse mode. However, in comparison to the $q_{array} = 1.6$ when resonance occurs in Renzi and Dias [36], the present resonance phenomenon is weak ($q_{array} = 1.06$). The reasons for this difference will be analysed in detail in the next section. Furthermore, q_{array} gradually approaches 1.0 as the separation distance increases, due to the decrease in amplitude of the scattered and radiated wave.

4.2.2. Spatial distribution of wave heights

The impact of the array on the surrounding wave field is investigated, a necessary step before the array could be constructed. Fig. 15 shows the free surface amplitude $|\eta|$ and power extraction impact factor q_i for six configurations of the three-unit array. For a more in-depth analysis of the relationship of the array's perturbed wave field, q_b and the interference wave field of each FlexWEC, the characteristic lines of each FlexWEC, as previously illustrated in Fig. 9, are specifically marked in Fig. 15.

When $Y_k/L = 1$ (as seen in Fig. 15(a)–(c), and (e)), wave reflection is stronger. Conversely, when $Y_k/L = 1.6$ (as shown in Fig. 15(b)–(d), and (f)), wave reflection is weaker. This phenomenon can be explained with the assistance of characteristic lines. At $Y_k/L = 1$, the lines of different FlexWECs intersect, implying that the radiated waves generated by the two FlexWECs are in-phase at the point of intersection, thus producing a significant reflection wave height. Conversely, at $Y_k/L = 1.6$, the characteristic lines do not coincide, indicating weaker interaction of the radiated waves by the different FlexWECs, resulting in a smaller reflection wave height. In addition, it can be found that there is a larger

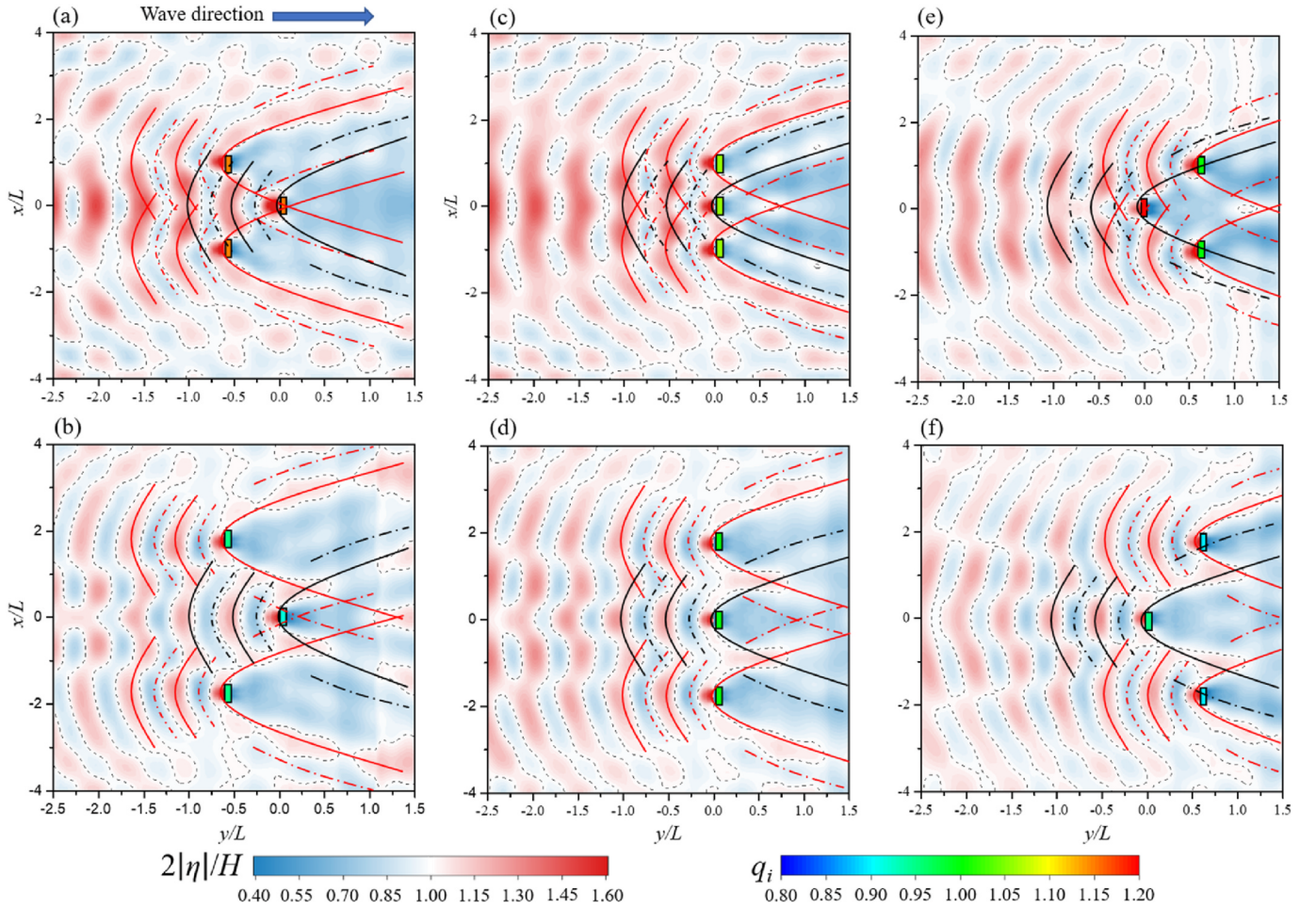


Fig. 15. Free-surface amplitude $|\eta|$ contours for six configurations of three-unit FlexWEC array when $L/D = 3.33$. (a) Configuration P_1 , $X_k/L = -0.6$, $Y_k/L = 1.0$. (b) Configuration P_2 , $X_k/L = -0.6$, $Y_k/L = 1.75$. (c) Configuration P_3 , $X_k/L = 0.0$, $Y_k/L = 1.0$. (d) Configuration P_4 , $X_k/L = 0.0$, $Y_k/L = 1.75$. (e) Configuration P_5 , $X_k/L = 0.6$, $Y_k/L = 1.0$. (f) Configuration P_6 , $X_k/L = 0.6$, $Y_k/L = 1.75$. The body colour denotes the power extraction impact factor q_i of each body (according to the right colour bar).

shaded area on the shore side for $Y_k/L = 1.6$, which can reduce the efficiency of utilization within the wave farm.

The q_i also varies significantly under different arrangement schemes. Fig. 15 (a) corresponds to the most efficient array arrangement scheme among the 109 cases. Here, $1^\#$ and $3^\#$ are positioned along the out-of-phase characteristic line of $2^\#$. As previously discussed regarding Figs. 11 and 13, since the radiated and incident waves propagate in opposite directions, the wave from $2^\#$ increases the wave excitation of $1^\#$ and $3^\#$, leading to high energy conversion efficiency for both $1^\#$ and $3^\#$. Additionally, $2^\#$ is located at the intersection of the in-phase characteristic lines of $1^\#$ and $3^\#$, resulting in higher wave excitation forces and increased energy conversion efficiency.

As shown in Figs. 15 (e), $2^\#$ is located at the intersection of the out-of-phase characteristic lines of $1^\#$ and $3^\#$, resulting in significantly higher energy capture efficiency. However, the overall energy conversion efficiency improvement of the array is limited due to the insignificant energy enhancement effect on $1^\#$ and $3^\#$.

For Fig. 15(c) and (d), the three-unit array is in an in-line arrangement. The impact of FlexWEC on neighboring devices is mainly

determined by the diffracted and radiated wave pattern. As previously discussed, the wave pattern of an isolated FlexWEC is a dipole-pattern, so the characteristic line exists only within a finite width on both sides of the FlexWEC. When the array is in an in-line arrangement, each FlexWEC is situated outside the range of characteristic lines of the other FlexWECs, leading to weak interfering effects between FlexWECs and $q_i \approx 1$. In the study of Renzi and Dias [36], the flap-type WEC has a width of $0.05L$, which is much smaller than the wavelength, so that the wave pattern of the diffracted and radiated waves is closer to a circular-pattern [50]. This explains the smaller q_{array} when transverse resonance occurs in the present study compared to Renzi and Dias [36].

For the cases in Fig. 15 (b) and (f), the conversion efficiency of the array is generally low. This is due to the fact that the front-side FlexWEC is situated outside the characteristic lines of the other FlexWECs, while the back-side FlexWEC is on the out-of-phase characteristic lines of the front-side FlexWECs. Based on the above analysis, two basic conditions can be inferred for the optimal array arrangement of any number of FlexWECs: 1) the upstream FlexWEC should be positioned at the intersection of the out-of-phase characteristic lines of the downstream FlexWECs; 2) the downstream FlexWEC should be situated at the intersection of the in-phase characteristic lines of the upstream FlexWECs.

4.2.3. Array performance at different wave periods

In this section, the performance of the array at different wave periods is analysed. Two inline cases (a_1 , a_2), and two staggered cases (a_3 , a_4) as

Table 5

Physical separation distance for different arrangements.

Parameter	a_1	a_2	a_3	a_4
Longitudinal separation distance, X_k (m)	0	0	-16	-16
Lateral separation distance, Y_k (m)	13.33	26.67	13.33	26.67

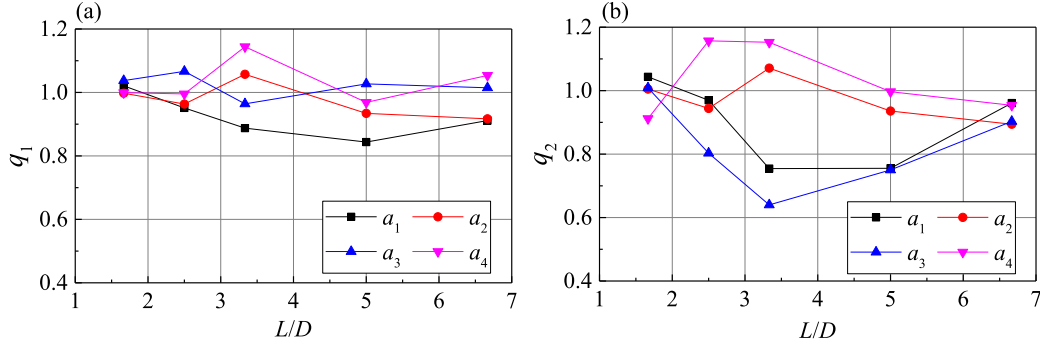


Fig. 16. Value of impact factor q as a function of different L/D . (a) q_1 ; (b) q_2 .

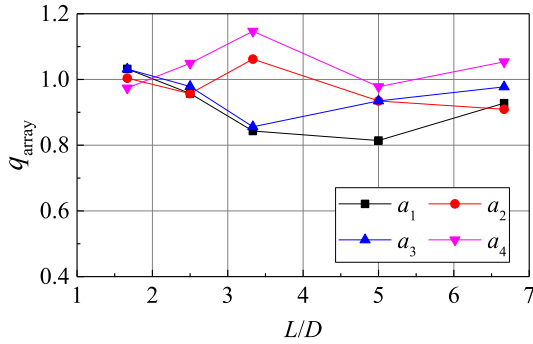


Fig. 17. Value of impact factor q_{array} as a function of different L/D .

shown in Fig. 14 are chosen. When $D/L = 3.33$, strong constructive interactions occur at a_4 , while strong destructive interactions happen at a_1 and a_3 , and only weak interactions are observed at a_2 . The physical separation distance for different arrangements is listed in Table 5.

Fig. 16 shows the value of the impact factor q of $1^\#$ and $2^\#$ as a function of different L/D . In all cases, the q factor exhibits slight oscillations with L/D under short-wave conditions ($L/D = 1.67$ – 2.5) and long-wave conditions ($L/D = 6.67$), while showing significant oscillations under medium-wave conditions ($L/D = 3.33$ – 5). This is consistent with previous conclusions. Under long-wave conditions, the interference of FlexWEC on the wave field weakens, leading to gradually diminished interaction effects. The oscillation of q is induced by the variation of dimensionless separation distance ($X_k/L, Y_k/L$). For example, in case a_2 ,

at $L/D = 3.33$, the dimensionless separation distance ($X_k/L, Y_k/L$) = (0.0, 1.0), and they become (0.0, 0.5) at $L/D = 6.67$. The color block chart in Figs. 11(a) and 13(a) can approximately describe the variation of q_1 and q_2 with X_k/L and Y_k/L under different wave periods. It can be seen that as ($X_k/L, Y_k/L$) moves from (0.0, 1.0) to (0.0, 0.5), q_1 and q_2 transition from constructive to destructive interactions in Figs. 11(a) and 13(a), which is confirmed by the variation of q_1 and q_2 in Fig. 16. When L is relatively short, the dimensionless separation distance between arrays is relatively large, resulting in weaker interactions. Fig. 16 also confirms this, showing that the oscillation amplitude of q is smaller under short-wave conditions.

When comparing the changes in the q factor across four array arrangements, it is observed that for the same D/L variation, the number of oscillations of q increases as the physical separation distance increases. This is because a larger physical separation distance results in a greater variation in the dimensionless separation distance for the same wave period change, spanning more regions of constructive and destructive interference. Within the range of $D/L = 3.33$ – 5 , the values of q_2 for a_1 and a_3 are lower than those for a_2 and a_4 . This is because, at smaller separation distances, $2^\#$ is shadowed by the upstream device. Fig. 17 shows the variation of q_{array} for the four array configurations, indicating that the a_4 configuration ensures higher energy conversion efficiency across different wave periods ($D/L = 2.5$ – 6.67).

Fig. 18 illustrates the instantaneous distribution of wave elevations and z -vorticity at $L/D = 6.67$ for the FlexWEC array. The FlexWECs have a relatively weak influence on the waves, corresponding to the reduced fluctuations in q for longer waves. In Fig. 18(a), the two upstream FlexWECs are positioned $L/4$ from the wave crest and are accelerating in the positive x -direction, while the downstream FlexWEC is situated at

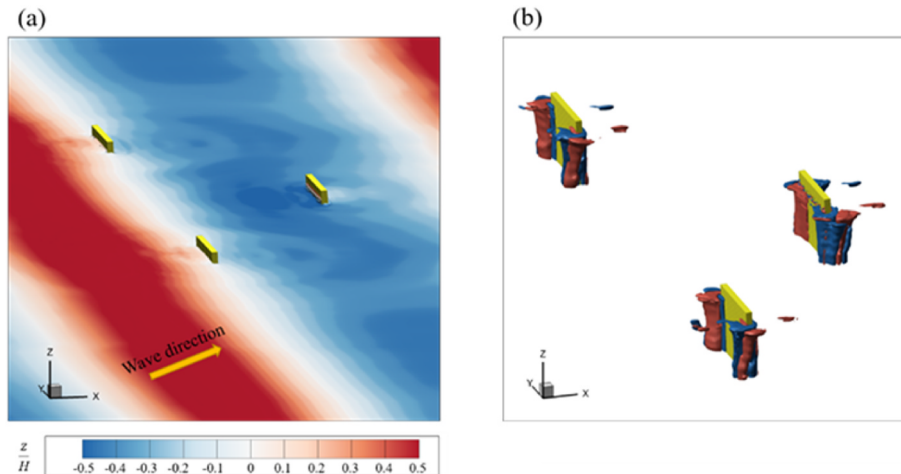


Fig. 18. The spatial distribution of wave elevations and iso-surfaces of z -vorticity at $L/D = 6.67$. (a) Wave elevations. (b) Z -vorticity ($4\pi\omega_z D/TgkH = \pm 5$).

the wave trough and decelerating in the negative x -direction. Vortices shed at the straight corner on the leeward side in the direction of FlexWEC motion. The negative pressure generated by these vortices impedes the motion of the FlexWEC and further reduces the power absorbed.

5. Conclusions

To support FlexWECs deployed in wave farm arrays, this work developed a fully coupled dynamic FSI model and studied the optimal longitudinal and lateral separations of three bottom-mounted FlexWECs operating in close proximity.

Key findings include:

- For an isolated FlexWEC, as the relative wavelength L/D increases, the $CWR_{isolated}$ first increases and then decreases, reaching a peak value of 61 % at $L/D = 3.33$.
- For the three-unit array, 109 different cases were systematically tested at $L/D = 3.33$. The middle device has a greater variation in captured power (q_2 : 0.548–1.56), compared to the side ones (q_1 : 0.641–1.21). The optimal configuration ($X_k/L = -0.6$, $Y_k/L = 1.0$) enhanced overall power extraction by up to 15 %.
- Since each FlexWEC is situated outside the range of characteristic lines of the other FlexWECs, the interaction effect between devices in the in-line arrangement is weak. With a staggered arrangement, a higher energy capture efficiency is achieved when the upstream device is positioned along the out-of-phase characteristic line of the downstream device and the downstream device is arranged on the in-phase characteristic line of the upstream device.
- The impact factor q oscillations are smaller for both shorter and longer waves. This is because the non-dimensional separation distance is larger for shorter waves, and FlexWEC's disturbance to the waves is minimal for longer waves.

These insights offer practical implications for optimizing the deployment and performance of FlexWEC arrays, aiding in design and commercial decisions. While the current study focuses on the width D for CWR calculations, the complexity of three-dimensional wave interactions is acknowledged. Future research could explore the influence

of device length and depth to further refine the understanding of wave energy capture efficiency.

CRediT authorship contribution statement

Chao Wang: Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Formal analysis, Data curation, Investigation. **Yujia Wei:** Writing – original draft, Investigation. **Wenchuang Chen:** Writing – review & editing, Supervision, Resources, Methodology, Investigation, Funding acquisition, Conceptualization. **Luofeng Huang:** Writing – review & editing, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

All data underlying the results are available as part of the article and no additional source data are required.

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Appendix A

In this study, the FlexWEC is modeled as a rubber material with a Poisson's ratio of 0.5. The Euler-Bernoulli beam model neglects the shear deformation, effectively assuming infinite shear stiffness, equivalent to a Poisson's ratio of -1 . To verify the impact of shear deformation on the accuracy, the deformation of the FlexWEC was compared between the present Euler-Bernoulli beam and Timoshenko beam in ANSYS with three Poisson's ratios (0.5, 0, and -0.99).

A pressure of 250 Pa was applied on one side of the FlexWEC (as shown in Fig A1(a)) to approximate the load it might encounter in water. The FlexWEC undergoes oscillation under this pressure, and Fig A1(b) presents the predictions of free-end motion for both models.

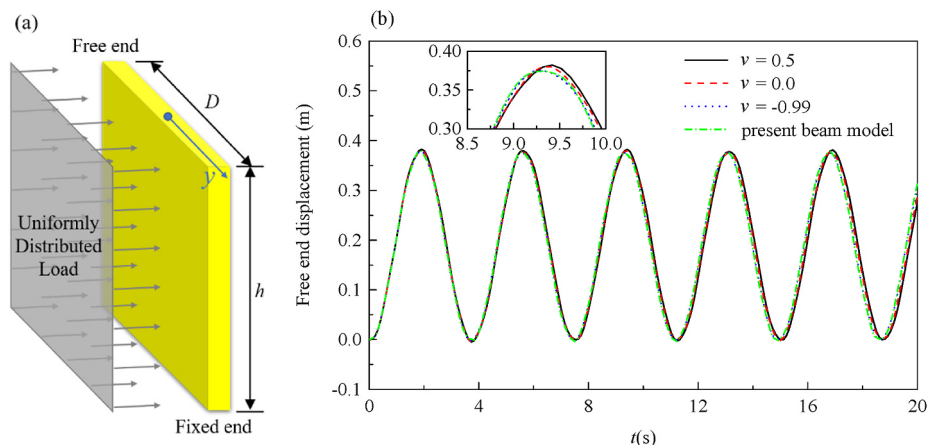


Fig. A1. Plate structure test: (a) Computational setup, (b) Comparison between ANSYS Timoshenko beam elements and present Euler-Bernoulli beam elements.

The Euler-Bernoulli beam model used in this study predicts structural displacements with an average difference of 2.1 % and a frequency difference of −1% compared to the ANSYS results with a Poisson's ratio of 0.5, which is within an acceptable range. Additionally, when the Poisson's ratio is −0.99, the ANSYS results closely match the present predictions.

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