

CRANFIELD UNIVERSITY

BUDDHI ARACHCHIGE

Theoretical Approach to Predict Transverse Impact on Variable
Stiffness Curved Composite Plates

School of Aerospace, Transport and Manufacturing
Aerospace Engineering

PhD
Academic Year: 2014 - 2017

Supervisor: Dr. Hessam Ghasemnejad
September, 2017

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This thesis is submitted in partial fulfilment of the requirements for
the degree of Doctor of Philosophy
***(NB. This section can be removed if the award of the degree is
based solely on examination of the thesis)***

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Abstract

Composite materials are widely used in the industry nowadays due to its attractive advantages such as been lightweight, high strength to weight ratios and ability to design and manufacture complex shapes. However, one of the main disadvantages are that composites are susceptible to impact damage and this is an issue that needs to be addressed.

The main interests in this research lies in the analysis of curved variable stiffness composite plates under low velocity impact conditions. There is a need to develop a detailed analytical model to predict the low velocity transverse impact of curved variable stiffness panels. An analytical model is a useful tool in obtaining an initial understanding of the behaviour of such structures under impact loading due its efficient and cost effective nature. Analytical models do not usually require high computational processing power when compared to advanced numerical models and therefore is a time-saving technique. The proposed analytical model would be useful for the composite modelling and manufacturing industry since it is robust, efficient and cost effective when compared with experimental tests or numerical models.

The major research gap identified was the need to contribute to the research on analytical modelling of curved composite shell structures and extend the study to sandwich structures as well. The main novelty and contribution of the present research is the development of an analytical model to study impact response of variable stiffness curved composite plates and sandwich plates. The result of the work can be used as a tool to understand the low velocity impact response of curved composite shell structures, i.e. to study the effect of various parameters such as layups, velocities, impact masses, geometry, boundary conditions, etc. affect impact damage response.

Analytical methods provide an initial view on the behaviour of a composite structure and the need to build an analytical tool to predict the impact response of variable stiffness composite plates was identified. First order shear deformation theory coupled with two degree of freedom mass-spring system was used to derive analytical force functions for the variable stiffness plate. Contact force histories were plotted to study

the effect of geometry, layups and impact velocity on the impact response of variable stiffness curved composite plates.

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“The present is theirs; the future, for which I really worked, is mine.”- Nikola Tesla

Contents

1 Motivation	21
1.1 Introduction	21
1.2 Research Problem	22
1.3 Aim and Objectives	23
1.3.1 Aim	23
1.3.2 Objectives	23
1.3.3 Structure of thesis	23
1.3.4 Contributions to research	25
2 Literature Review	27
2.1 Composite materials	27
2.2 Classification of composites.....	28
2.3 Advantages and disadvantages of composite materials	29
2.4 Applications	32
2.5 Low velocity impact damage	33
2.6 Impact event flowchart.....	34
2.7 Failure modes in composites	35
2.7.1 Failure in longitudinal tension.....	35
2.7.2 Failure in longitudinal compression	37
2.7.3 Failure in transverse tension	37
2.7.4 Failure in transverse compression	38
2.7.5 In-plane shear	38
2.7.6 Delamination	39
2.8 Factors affecting impact damage of composite laminates	40
2.8.1 Projectile characteristics	40
2.8.2 Material properties	41
2.8.3 Laminate thickness, lay-up and stitching.....	42
2.8.4 Preload.....	44
2.8.5 Environmental conditions	44
2.8.6 Initiation of damage	44
2.8.7 Propagation of damage	45
2.9 Low Velocity impact testing methods.....	46
2.9.1 Charpy impact testing	46
2.9.2 Drop weight impact testing	47
2.10 Review of selected works and research gap.....	49
2.11 Review of low velocity impact damage models.....	53
2.11.1 Impact models developed in literature.....	53
2.12 Plate theories.....	66
2.12.1 First order shear deformation theory (FSDT)	66
2.12.2 Classical plate theory (CPT).....	73

2.13 Shell theory.....	74
2.14 Low Velocity impact modelling of curved sandwich plates.....	77
2.14.1 Introduction	77
2.15 Factors affecting low velocity impact on sandwich structures	79
2.15.1 Impact Mass.....	79
2.15.2 Impact velocity	79
2.15.3 Impactor Geometry	79
2.15.4 Face-sheet thickness	80
2.15.5 Boundary conditions.....	80
2.15.6 Environmental conditions	80
2.15.7 Impact angle.....	80
2.16 Damage mechanisms of composite sandwich structures	81
2.16.1 Energy absorption during an impact.....	81
2.16.2 Damage progression during impact of sandwich structures.....	81
2.17 Governing Equations of a sandwich plate.....	82
2.17.1 Strain Energy	84
2.17.2 Stiffness Matrices of Sandwich Plates	85
2.18 Studies on sandwich panels	87
2.19 Identified research gaps	91
Limitations of present analytical models	91
3 Analytical Modelling of Low velocity impact damage on variable stiffness curved composites	96
3.1 Introduction	96
3.2 Mathematical modelling	97
3.2.1 Modelling process	97
3.2.2 A, B, D stiffness matrices	98
3.2.3 Impact model.....	103
3.2.4 Variable stiffness model	107
3.2.5 Analytical modelling of variable stiffness curved plate	108
3.2.6 Natural frequency calculation	109
3.2.7 Analytical modelling flowchart of variable stiffness plate	114
3.2.8 Theoretical models developed	115
3.2.9 Validation (Curved plate).....	116
3.3 Results and discussions	119
3.3.1 Effect of contact stiffness on contact force history	120
3.3.2 Effect of stacking sequence (Equal widths).....	120
3.3.3 Effect of impact velocity (Equal width).....	123
3.4 Modified analytical model.....	124
3.4.1 Modifications	124
3.4.2 Effect of modifications	125
3.4.3 Effect of impactor mass.....	127
3.5 Structural Damping	127
3.5.1 Structural loss factor	128

3.6 Chapter Conclusions	131
4 Post Impact Damage Modelling of damaged variable stiffness curved composites	134
4.1 Introduction and review of previous works	134
4.2 Objective	136
4.3 Damage modelling-approach 1	136
4.4 Layups studied	139
4.5 Discussions (damage modelling approach 1)	140
4.6 Modified approach to damage modelling (Damage Modelling approach 2)	143
4.6.1 Modifications made	143
4.6.2 Layups studied	147
4.6.3 Results and discussions	149
4.6.4 Damage behaviour	152
4.6.5 Comparison of results	154
4.7 Chapter Conclusions	156
5 Analytical modelling Low Velocity Impact of Curved Sandwich Structures..	159
5.1 Impact modelling	159
5.1.1 Analytical modelling	159
5.1.2 Theoretical Formulations	159
5.1.3 Low velocity impact model	166
5.1.4 Validation of the model	169
5.1.5 Analytical modelling of impact on curved sandwich plate (constant stiffness face-sheets)	173
5.1.6 Analytical modelling of impact on curved sandwich plate (variable stiffness face-sheets)	175
5.2 Results and Discussions	177
5.2.1 Impactor velocity	178
5.2.2 Layups	178
5.2.3 Facesheet thickness (identical top and bottom facesheet with constant thickness)	179
5.2.4 Core thickness	180
5.2.5 Sandwich plate curvature	181
5.2.6 Effect of variable stiffness face-sheets	181
5.3 Chapter Conclusions	183
6 Numerical Modelling	186
6.1 Continuum damage modelling	186
6.2 Damage modes in composite laminates	186
6.3 Progressive damage analysis	187
6.3.1 Damage model	187
6.3.2 Damage evolution	188
6.4 Finite Element Modelling in LSDYNA	189
6.5 Modelling process	189

6.5.1 CAD modelling of the curved plate	190
6.5.2 LS-PrePost model	190
6.5.3 Element types and section properties	191
6.5.4 Definition of variable stiffness section	191
6.5.5 Integration points and ply orientations.....	192
6.5.6 Meshing	192
6.5.7 Material properties (Intra-laminar failure)	193
6.5.8 Velocity, contacts and boundary conditions	196
6.5.9 Delamination modelling (Inter-laminar failure).....	197
6.6 Results and Discussions.....	202
6.6.1 Mesh sensitivity analysis.....	202
6.6.2 LS-DYNA impact event	203
6.6.3 Verification	205
6.7 Post Impact Damage Model.....	208
6.7.1 Damage modelling approach 1	208
6.7.2 Damage modelling approach 2 (Localisation of damage region)	211
6.7.3 Damage model comparisons.....	212
6.8 Numerical modelling of foam core sandwich plate.....	213
6.9 Material models for foam-based sandwich models.....	214
6.10 Crushable foam	215
6.11 Modelling of sandwich panel.....	216
6.11.1 Validation of numerical model (with reference [125])	217
6.11.2 Validation of numerical model (with reference [136])	219
6.12 Modelling of curved finite element model (constant stiffness facesheets)	220
6.12.1 Impact energy of 10J.....	220
6.12.2 Impact energy of 25J.....	222
6.12.3 Force vs displacement plots.....	223
6.13 Modelling of curved sandwich plate with variable stiffness facesheets	224
6.14 Parametric study	225
6.14.1 Results and discussions (Numerical Model).....	227
6.15 Chapter Conclusions	232
7 Final Conclusions.....	233
7.1 Chapter 3.....	233
7.2 Chapter 4.....	233
7.3 Chapter 5.....	234
7.4 Chapter 6.....	235
8 Publications	236
9 References.....	237

List of Figures

Figure 1 Material development through history [2]	27
Figure 2 Fibrous composites [2].	28
Figure 3 Particulate composites [2].	28
Figure 4 Specific strength and modulus comparison [3].	30
Figure 5 Structural efficiency [3].	30
Figure 6 Fatigue properties of aerospace materials [3]	30
Figure 7 Impact damage in composites.....	31
Figure 8 Boeing 787 Dreamliner [3].	32
Figure 9 Automotive applications [3]	32
Figure 10 Marine applications [3]	33
Figure 11 Wind turbine blades [3]	33
Figure 12 Impact event flowchart (Composed by author).....	35
Figure 13 (a) Failure in Tension (b) Matrix based failure (c) Fibre based failure [146].	36
Figure 14 Failure modes in longitudinal compression (a) Elastic micro-buckling (b) Plastic micro-buckling (c) Fibre crushing (d) Matrix splitting (e) Buckle delamination (f) Shear band formation [4].	38
Figure 15 Fracture mechanics schematic of double cantilever beam (a) appearance of critical matrix cracks (b) delamination onset (c) transverse micro-cracking [147].	39
Figure 16 Effect of fibre on impact damage area [17].	41
Figure 17 Effect on resin on compression strength [17].	42
Figure 18 Effect of resin on impact damage area [17].	42
Figure 19 Charpy and Izod Tests [Composed by author]	47
Figure 20 Drop Weight Impact Tester [42]	48
Figure 21 Contact force history plot (Small and Large mass impacts [42]).....	48
Figure 22 Impact force vs impactor mass plot [42]	48
Figure 23 Schematic of Impact Event (a) Before impact $t > 0$, (b) During impact $t > 2$ [71]	55
Figure 24 Spring-mass model [71].	59
Figure 25 Gong's Modified Spring Mass Model [44].	61

Figure 26 Spring-Mass model	64
Figure 27 Deformation of a transverse normal [148]	67
Figure 28 Force Resultants	69
Figure 29 Cylindrical shell [83]	76
Figure 30 Sandwich Panel and Honeycomb core [90].....	78
Figure 31 Sandwich Panel Geometry [90].....	78
Figure 32 Deformation of sandwich plate in x-z plane [90].....	78
Figure 33 Impact event on a sandwich panel [108]	82
Figure 34 Failure modes of a sandwich panel [108]	82
Figure 35 Shear stress distribution (left) and approximate distribution (right) in a sandwich panel [84]......	87
Figure 36 Shear deformation of a sandwich panel [84]	87
Figure 37 Variable stiffness structures (a) application (b) Plate schematic	96
Figure 38 Mathematical modelling process	97
Figure 39 Modelling process	97
Figure 40 Coordinate System.....	99
Figure 41 Deformation of a plate in the x-z plane.....	99
Figure 42 Curvatures of reference plane [83]......	103
Figure 43 In plane forces at reference plane (left) and moments and transverse shear forces (right) [83].	103
Figure 44 Two degree of freedom mass=spring system	104
Figure 45 Variable stiffness curved plate	107
Figure 46 (a) Constant stiffness plate (b) Variable stiffness plate	107
Figure 47 Various designs of curved composite plates with a) uniform thickness, b) three variable thicknesses and c) five variable thicknesses and equal width (side view).	108
Figure 48 Stiffness variation across plate.....	113
Figure 49 Analytical Modelling Process (Variable Stiffness Plate)	114
Figure 50 Designs of curved composite plates with a) uniform thickness, b) three variable thicknesses and c) five variable thicknesses and equal width.....	115
Figure 51 Comparison of force-time history for a) Steel plate and b) composite plate between present results and available results in the literature.	117

Figure 52 Validation with curved composite experiment (Layup 1)	118
Figure 53 Validation with curved composite experiment (Layup 2)	118
Figure 54 Effect of contact stiffness on contact force history.	120
Figure 55 Effect of stacking sequence on the three sectioned plate with variable thicknesses a) 6m/s, b) 10m/s and c) 15m/s.	121
Figure 56 Effect of stacking sequence on five sectioned plate with variable thicknesses	122
Figure 57 Effect of impact velocity on three sectioned equal width plate (a) Layup 1 (b) Layup 2 (c) Layup 3.....	123
Figure 58 Modified analytical method.....	124
Figure 59 Effect of modification on Layups	125
Figure 60 Effect of modifications on impact velocity.....	126
Figure 61 Model comparisons (Impact velocity of 6m/s)	126
Figure 62 Effect of Impactor Mass	127
Figure 63 Two degree of freedom spring-mass-dashpot system	127
Figure 64 Impact on layered beam geometry [132]	128
Figure 65 Analytical Models (a) Undamaged plate, (b) Damaged Plate.....	137
Figure 66 Spring-Mass model (3 section damaged plate).....	138
Figure 67 Damage model (5 section plate)	138
Figure 68 Spring-Mass model (5 section damaged plate).....	138
Figure 69 Layup 1 (Contact force history)	141
Figure 70 Layup 2	142
Figure 71 Layup 3	142
Figure 72 Layup 4	142
Figure 73 Layup 5	143
Figure 74 Layup 6	143
Figure 75 Modified damage model (3 section plate)	146
Figure 76 Modified damage model spring-mass diagram.....	146
Figure 77 Modified damage model (5 section plate)	147
Figure 78 Modified spring-mass model (5 section plate).....	147
Figure 79 Modified damage model (Layup 1).....	150

Figure 80 Modified damage model (Layup 2)	151
Figure 81 Modified damage model (Layup 3)	151
Figure 82 Modified damage model (Layup 4)	151
Figure 83 Modified damage model (Layup 5)	152
Figure 84 Modified damage model (Layup 6)	152
Figure 85 Effect of curvature on damage behaviour for Layup 2 (a) Undamaged (b) one ply removed	154
Figure 86 Model comparisons (a) Undamaged (b) Modelling approach 1 (c) Modified damage modelling method	155
Figure 87 Model comparison (Layup 1)	155
Figure 88 Model comparison (Layup 2)	155
Figure 89 Sandwich Curved Plate	159
Figure 90 Local face-sheet indentation regimes [115]	160
Figure 91 Two degree of freedom spring-mass-dashpot system	167
Figure 92 Validation of present work with the literature. (a) honeycomb core (Simply-supported) & (b) foam core (Clamped).	172
Figure 93 Contact force history (Honeycomb sandwich plate)	172
Figure 94 Contact force history (Foam sandwich plate)	173
Figure 95 Impact force against time for clamped panel with different radius.	177
Figure 96 Effect of impactor velocity on impact response.	178
Figure 97 Effect of layups on impact response	179
Figure 98 Effect of face-sheet thickness	180
Figure 99 Effect of core thickness	180
Figure 100 Effect of plate curvature on impact response.	181
Figure 101 Effect of variable stiffness facesheets	182
Figure 102 Damage mechanics approaches (Composed by author)	188
Figure 103 Stress-strain graph of linear softening law [138]	188
Figure 104 Numerical modelling process	189
Figure 105 Finite element modelling process	189
Figure 106 Curved plate CAD model	190
Figure 107 LS-Prepost model	190

Figure 108 Definition of variable stiffness sections	191
Figure 109 Meshing (a) Meshed model (b) refined mesh.....	192
Figure 110 Velocity applied to solid impactor	197
Figure 111 Bi-linear traction separation law [140]	198
Figure 112 FE Models for impact on laminated plate [140]	198
Figure 113 Numerical Model with thin shell elements (Flat plate)	200
Figure 114 Shell element layers (a) 1 shell (b) 2 shells (c) 3 shells (d) 4 shells	200
Figure 115 Effect of shell layers on energy	201
Figure 116 Effect on contact force history	201
Figure 117 Mesh element sizes (a) 10mm (b) 5mm (c) 4mm (d) 3mm.....	202
Figure 118 Mesh convergence (Maximum contact force).....	203
Figure 119 Mesh convergence (Contact time)	203
Figure 120 Numerical impact energy.....	204
Figure 121 Impact event.....	204
Figure 122 von Mises stress distribution	204
Figure 123 Single shell (No tie-break).....	205
Figure 124 Double shell (Tie-break contact)	205
Figure 125 Model comparison (Layup 1).....	206
Figure 126 Model comparison (Layup 2).....	207
Figure 127 Model comparison (Layup 3).....	207
Figure 128 Validation (Small Mass Impact).....	208
Figure 129 Numerical model (Damage modelling approach 1)	209
Figure 130 Comparison between FE results and the developed theoretical model for layup 3.....	210
Figure 131 Finite element images of damaged curved composite panels with variable stiffness a) element deformation and b) stress distribution.....	210
Figure 132 Finite Element Model of damage modelling approach 2	211
Figure 133 Results comparison for Layup 3 (three sectioned plate)	211
Figure 134 Results comparison for Layup 4 (five sectioned plate).....	212
Figure 135 Foam core sandwich panel	213
Figure 136 Deformation zones in a foam material [142].....	215

Figure 137 Crushable foam behaviour [143].	216
Figure 138 Airex C70.55 foam core stress-strain curve [149]	218
Figure 139 Finite element model (Flat panel)	218
Figure 140 Facesheet damage [Composed by author]	218
Figure 141 Contact force history	218
Figure 142 Finite element model	219
Figure 143 Contact force history	219
Figure 144 Divinycell stress-strain curve [85]	220
Figure 145 Curved finite element model	220
Figure 146 Contact force history comparison for curved sandwich panel	221
Figure 147 (a) Curved sandwich panel (von Mises stress distribution) (b) damage in top facesheet (c) core crushing (d) no damage in bottom facesheet.	221
Figure 148 Contact force history (10J and 25J impact energy)	222
Figure 149 Curved sandwich panel (von Mises stress distribution) (b) damage in top facesheets (c) damage in bottom facesheets.	223
Figure 150 Comparison of force vs displacement at impact energy of 25J.	224
Figure 151 FE Model with variable stiffness facesheets	224
Figure 152 Contact force history (Variable stiffness facesheet model)	225
Figure 153 FE Models (a) Model 1 (b) Model 2 (c) Model 3 (d) Model 4 (e) Model 5	226
Figure 154 FE Models (a) Model 6 (b) Model 7 (c) Model 8 (d) Model 9 (e) Model 10	226
Figure 155 Force vs displacement (a) H=10mm (b) H=12.5mm (c) H=17.5mm (d) H=20mm	227
Figure 156 Combined plot for effect of core thickness	227
Figure 157 Combined plot for effect of facesheet thickness	229
Figure 158 Force vs displacement (a) h=0.3mm (b) h=0.4mm (c) h=0.5mm (d) h=0.6mm	229
Figure 159 Force vs displacement (a) R=100mm (b) R=120mm (c) R=140mm (d) R=160mm	230
Figure 160 Combined plot for effect of plate curvature	230

List of Tables

Table 1 Experimental parameters [71]	54
Table 2 Experimental parameters [44]	61
Table 3 Experimental parameters [82].	63
Table 4 Stiffness matrices of sandwich panel face-sheets [84]	85
Table 5 CFRP Properties [45]	116
Table 6 Properties of steel impactor [45]	116
Table 7 Laminate designs of three and five sectioned curved composite plates.	119
Table 8 Layups (Post impact damage model)	140
Table 9 Layups (Modified damage model)	149
Table 10 Contact Time duration for the different layups.	153
Table 11 Material properties of face-sheets and core	171
Table 12 Geometrical properties of face-sheets and core	172
Table 13 Dimensions and loading conditions of the impactor	172
Table 14 Comparison of stiffness matrices of sandwich panels	176
Table 15 Analytical and Experimental results comparison.	177
Table 16 Various layups for the parametric studies.	179
Table 17 Combinations of variable stiffness facesheets	182
Table 18 Material Model 54 inputs	195
Table 19 Numerical analysis layups	206
Table 20 Result Discussion	207
Table 21 Material model 63 input data [143].	216
Table 22 Models developed for parametric study	226
Table 23 Effect of core thickness on top and bottom facesheet maximum contact force	228
Table 24 Effect of curvature on energy absorption	231

Nomenclature

E	Young's modulus (GPa)
$F(t)$	Dynamic force
G_{12}	Shear modulus (GPa)
h	Thickness
t	Beam thickness (mm)
R	Radius
γ	Poisson's ratio
ρ	Density
M_1	Mass of plate
M_2	Mass of striker
K_1	Stiffness constant of plate
K_2	Stiffness constant of striker
K_2^*	Effective contact stiffness
V	Impact velocity
u, v, w	Displacements in x, y, z coordinate systems
x, θ, z	Axial, circumferential and radial coordinates for a curved shell
k	Mindlin shear correction factor
β_x, β_y	Bending slopes in the $x - z$ and $\theta - z$ planes
$N_x, N_\theta, N_{x\theta}$	In surface stress resultants in cylindrical polar system
$M_x, M_\theta, M_{x\theta}$	Bending and twisting moments per unit length
q_x, q_θ	Surface load components along the axial x-axis & circumferential θ axes
A_{ij}, D_{ij}	Extensional, bending stiffness of a laminated shell
Q_x, Q_θ	Transverse shearing force per unit length
D_{1d}	Dynamic bending of facesheet
h	Thickness of facesheet

h_k	Thickness of ply
H	Height of core
K_{gd}	Dynamic global stiffness of simply supported panel
K_{1d}	Dynamic local stiffness of top face sheet
m_f	Effective mass of top facesheet
Q_d	Total dynamic core crushing strength
[A]	Extensional stiffness matrix
[B]	Coupling stiffness matrix
[D]	Bending stiffness matrix
$[A_{sand}]$	Extensional stiffness matrix of sandwich panel
$[B_{sand}]$	Coupling stiffness matrix of sandwich panel
$[D_{sand}]$	Bending stiffness matrix of sandwich panel
X_t	Longitudinal tensile stress
X_c	Longitudinal compressive stress
Y_t	Transverse tensile stress
Y_c	Transverse compressive stress
S	Shear Strength
NFLS	normal tensile stress
SFLS	normal shear stress
U_s	Strain energy

Chapter 1:

Research Problem

1 Motivation

1.1 Introduction

Composites are materials built up of two or more different components. The use of composite materials in aerospace and automotive structures have highly increased in the last decades as a result of their attractive properties. Since most materials are stronger in fibre form than in bulk form, fibre reinforcement is ideal. There is an added advantage due to the reduction in the number of defects in fibre form compared to bulk form. The alignment of the fibre chains along the fibre contribute to the high strength of polymeric fibres when compared to the randomly arranged arrangement in the bulk fibre. The main advantages of the use of composites are weight reduction, resistance to corrosion and reduction in part count due to composites having reduced torsional stiffness. Other benefits that inspire some applications are low thermal expansion, high thermal conductivity, wear resistance and improved fatigue resistance. Weight reduction is the specific property of composites that have attracted applications in the transportation industry, generally in the aerospace sector. Low density of the fibres and the polymers used as matrices contributes to the lightweight nature of composites. More importantly, fibres have high strength to weight and stiffness to weight ratios when compared with most materials. Even though these fibres are very strong, these fibres cannot withstand compressive or transverse loads. Therefore, a matrix is required to hold the fibres together. The matrix also shields the fibres from environmental impacts and hence resistance to corrosion. Therefore, composites are increasingly used in the chemical industries due to its excellent resistance to corrosion. Polymer composites can be molded into complex shapes and consequently replace metallic parts that otherwise has to be assembled to obtain the desired shape. The part count reduction arising from reduced number of bearings and structural elements compensate for the higher cost of materials. [1]

Composites have lower mechanical properties compared to the properties of the fibres since fibres cannot be used alone and hence strength and stiffness matrix is insignificant compared to the fibres. However, strength and stiffness of composites are much higher than most traditional materials. Since transverse properties of composites

and strength of matrix are generally low, it is required to add laminae of different fibre orientations to strengthen the laminate and withstand the applied loads. [1].

1.2 Research Problem

Impact on composite structures significantly reduces the impact strength of such structures. The damage by low velocity impact is usually barely visible and therefore requires special attention. Analytical, numerical and experimental studies are widely found in literature in regards to low velocity impact of composite structures. The main interests in this research lies in the analysis of curved variable stiffness composite plates under low velocity impact conditions. There is a need to develop a detailed analytical model to predict the low velocity transverse impact of curved variable stiffness panels. An analytical model is a useful tool in obtaining an initial understanding of the behaviour of such structures under impact loading due its efficient and cost effective nature. Analytical models do not usually require high computational processing power when compared to advanced numerical models and therefore is a time-saving technique.

However, analytical modes need validating with either numerical or experimental work to make it more effective. There are several factors affecting impact response of composite structures and it is vital to analyse the effect of these parameters in the impact response of variable stiffness plates. This area of research requires more work to be performed and therefore it is the main motivation of the author. This piece of work aims to derive a novel analytical model to predict the transverse impact of curved variable stiffness plates by considering the different parameters involved in an impact event. This research also focuses on developing analytical models to predict impact response of curved sandwich plates with inclusion of variable stiffness face-sheets.

Research Problem: The need to develop an efficient analytical model capable of predicting low velocity impact response of curved composite plates with variable stiffness that would aid the composite designer in obtaining a first impression of how a composite shell structure would behave under impact loading, thus resulting in benefits such as been less computationally expensive and cost effective.

1.3 Aim and Objectives

1.3.1 Aim

The main aim of this research is to study the low velocity impact response of variable stiffness curved composite plates and curved sandwich plates through analytical and numerical modelling methods.

1.3.2 Objectives

Several objectives have been defined at the start of the research. These can be listed as:

- To study on the low velocity impact behaviour of curved composite plate
- Develop an analytical model capable of predicting the low velocity impact behaviour of variable stiffness curved composite plates
- Study how the various parameters such as contact stiffness, material properties, impact velocities, etc. affect the dynamic contact force history and the deformations.
- Develop analytical damage models capable of predicting post impact behaviour of damaged variable stiffness curved composite plates
- Develop analytical impact models to study impact response of curved sandwich plates with inclusion of variable stiffness curved composite plates. Then, analyse effects of various parameters governing impact response
- Advance numerical techniques (finite element analysis) to simulate impact damage in order to validate the proposed analytical models

1.3.3 Structure of thesis

This section presents a summary of the chapters present in this thesis.

Chapter 2: Literature Review

Low velocity impact on composite structures are studied in this chapter along with the different models adapted in literature to model impact behaviour. Factors affecting the

impact response of composite structures under low velocity impact are presented. The chapter also involves discussion of plate theories used to solve the impact problems.

Chapter 3: Analytical Modelling of Low Velocity Impact on Variable Stiffness Curved Composite Plates

This chapter involves a detailed analysis of variable stiffness curved composite plates subjected to low velocity impact. Theoretical formulations are presented with a thorough description of the mathematical modelling process. First order shear deformation theory has been used to predict the contact force histories of the plates and the response is depicted for a range of impact velocities, layups and different geometries.

Chapter 4: Post Impact Damage Modelling of Damaged Variable Stiffness Curved Composite Plates

This chapter is an extension of chapter 2 involving the analysis of variable stiffness plates with incorporated damage. Two new approaches to damage modelling are introduced where the stiffness in the damaged region is reduced through a reduction in thickness in that particular region. The effect of increasing the damage on variable stiffness plates are studied.

Chapter 5: Low velocity Impact Modelling of Curved Sandwich Plates

An analytical model to determine behaviour of curved sandwich plates under low velocity impact with foam core is presented. Initially, spring-mass-dashpot model was used to obtain the contact force function and through calculating natural frequencies of the plate and then the contact force histories were determined by a hemispherical-nose impactor. The analytical model was validated with experimental work performed on flat and curved sandwich plates and a good correlation was obtained. A parametric study was performed to understand the effects of several factors such as impactor velocity, face-sheet thickness, core thickness and layup orientation on the contact force histories of curved sandwich plates.

Chapter 6: Numerical Modelling

Numerical modelling acts as a validation technique for the previously created analytical models. Detailed numerical modelling is carried out within LSDYNA for curved variable stiffness plates and curved foam based sandwich plates.

1.3.4 Contributions to research

- Developed novel analytical methods to study impact response of variable stiffness curved composite plates under low velocity impact
- Developed novel analytical methods to study post impact behaviour of damaged variable stiffness curved composite plates.
- Developed analytical methods to predict impact response of curved foam based sandwich plates with inclusion of variable stiffness face-sheets
- Developed finite element models presented to determine impact response of variable stiffness curved plates and curved foam based sandwich panels for validation purposes.

Chapter 2:

Literature Review

2 Literature Review

2.1 Composite materials

Composite materials can be defined as a combination of two or more materials resulting in superior properties compared to the individual components. Composites offer many advantages such as high strength and stiffness, corrosion resistance, and greater weight reduction compared to traditional metal alloys [2]. Composite materials have been used for many centuries and importance of composites have experienced a steady growth over the years. There are two main constituents of a composite known as a reinforcement and a matrix. The reinforcement contributes to the high strength and stiffness, and mostly this reinforcement is stronger and stiffer than the matrix. This reinforcement is commonly either a fibre or a particulate [2]. The matrix can either be a polymer, metal or ceramic. The matrix holds the fibres in proper orientation and transmits loads to the fibres. In designing a composite material, many factors need to be considered such as type of reinforcement and matrix, volume fraction of constituents, loads experienced by the material and the operating conditions. A composite material is usually anisotropic (material properties different in all directions) or orthotropic (material properties different in three mutually perpendicular axes).

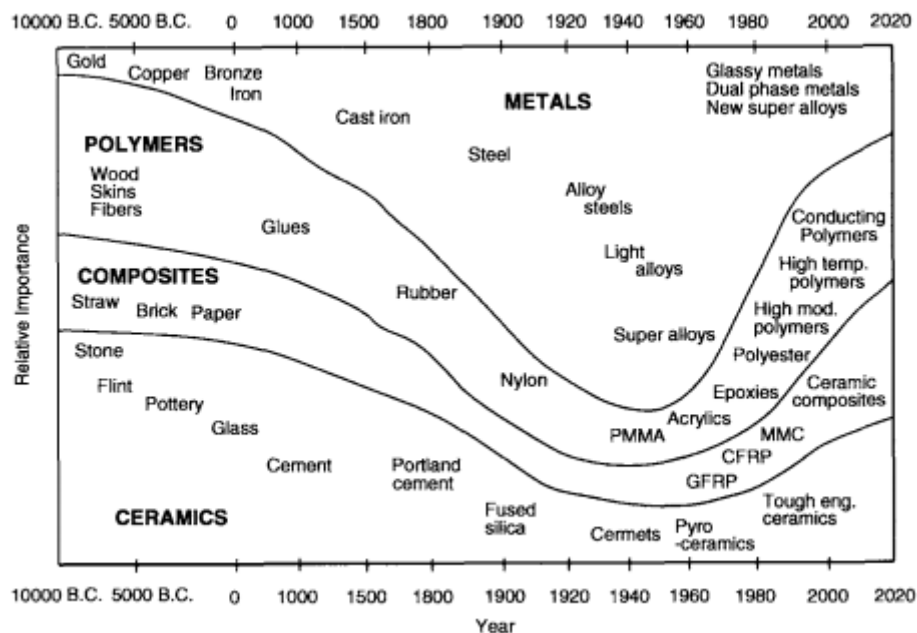


Figure 1 Material development through history [2]

2.2 Classification of composites

Composite materials are categorised according to the reinforcement type. The main two classes of composites are fibrous and particulate. Fibres have lengths that are much higher than their diameters and therefore a larger length to diameter ratio. They are generally stronger and stiffer than the bulk material. Continuous fibres have larger aspect ratios (length to diameter ratio), whereas discontinuous fibres have shorter aspect ratios. Common examples of continuous fibres are unidirectional, woven fabric and helical winding; whereas chopped fibres and random mat are examples of discontinuous fibres. Continuous fibre composites can be made into laminates through stacking layers of continuous fibres in different directions to achieve desired stiffness and strength characteristics. The high strength attributed with fibres are mainly due to their small diameter. Particulate composites comprises of particles suspended in a matrix. George [2] states that these particles can be of any shape, size or configuration. Strength and stiffness properties are much weaker compared to continuous fibre composites and therefore used in applications where strength is not a significant factor in the design.

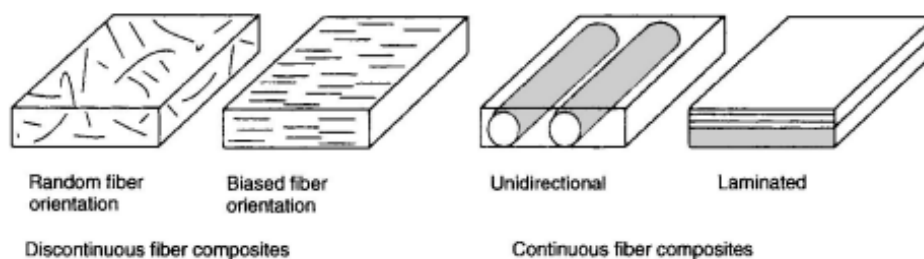


Figure 2 Fibrous composites [2].

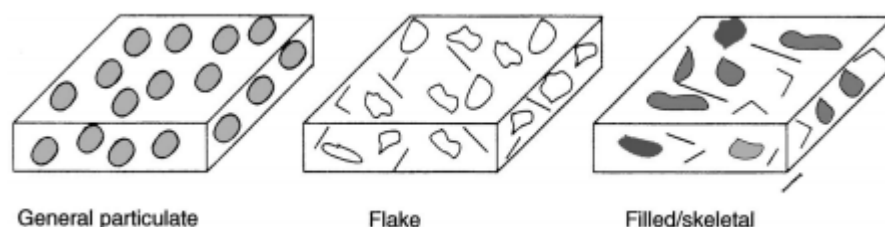


Figure 3 Particulate composites [2].

2.3 Advantages and disadvantages of composite materials

There are many advantages of composite materials including lighter weight, high stiffness and strength, greater corrosion and fatigue resistance and reduced assembly costs due to less parts and fasteners. The specific strength and specific modulus are greater when compared to other aerospace metallic structures. Since the direction of the fibres can be altered throughout the thickness of the laminate, the structural response of a component can be altered. As illustrated in Figure 4, composites offer superior strength to weight ratios and stiffness to weight ratios when compared with other aerospace metallic alloys. These greater weight savings results in efficient designs with improved performance and greater fuel savings. Figure 5 depicts the overall structural efficiency of composites when compared to Ti-6Al-4V and 7075-T6 aluminium. Since corrosion has been a major issue with aluminium alloys, composites are widely used in protecting the metal due to its high corrosion resistance properties. The fatigue resistance is also high compared to other high strength metals (Figure 6).

However, the main drawbacks of composites include high raw material costs, greater fabrication and assembly costs, adverse effects to moisture and temperature, low strength in out-of-plane direction and susceptibility to impact damage and delaminations (Figure 7), thus making the reparability process difficult [3]. The cost of conventional hand-layup in composite fabrication accounts for 40-60 percent of the total cost; and assembly costs are also expensive. Temperature has an adverse effect on the matrix dominated material properties and therefore decreases with increasing temperature. Composites are also susceptible to impact damage where delaminations can form and severely reduce the load carrying capabilities of the structure. This damage is mostly barely visible and can propagate through the structure in forms of matrix cracking and delaminations. Therefore, proper structural damage assessments are required to repair these structures after damage, thus increasing the costs.

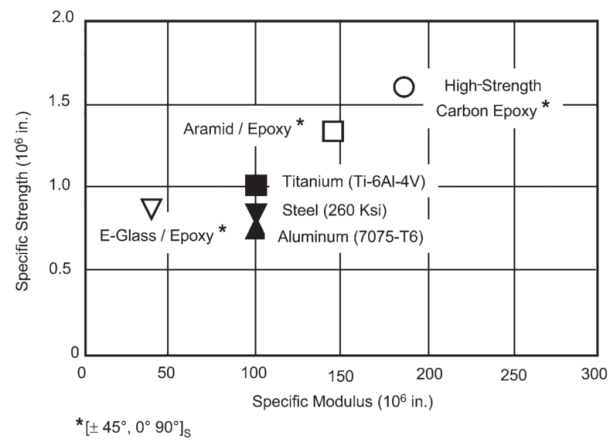


Figure 4 Specific strength and modulus comparison [3].

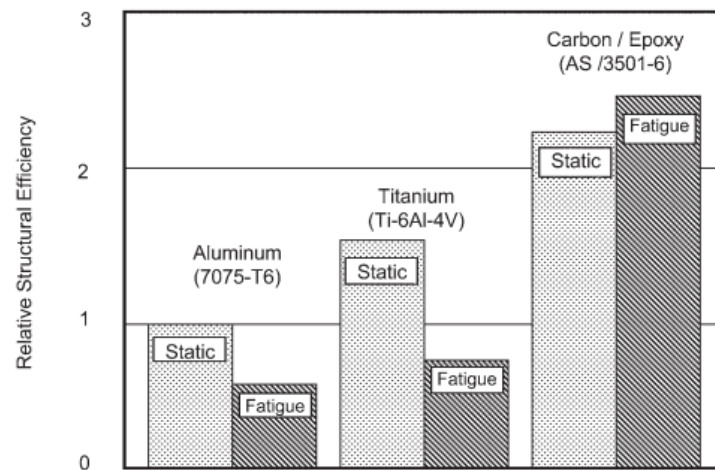


Figure 5 Structural efficiency [3].

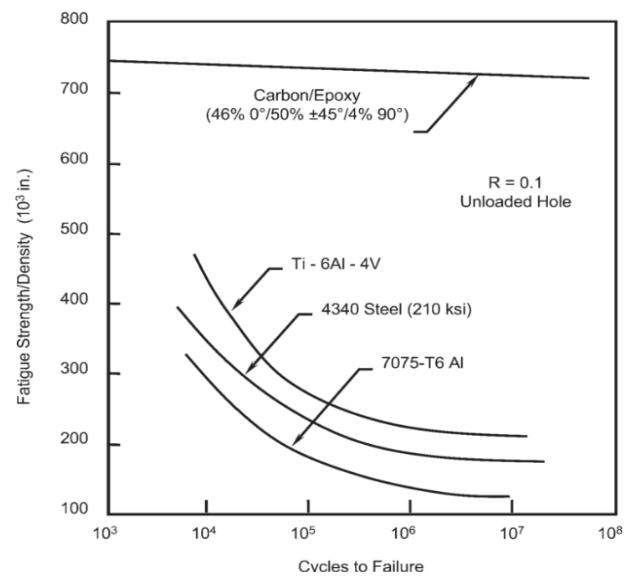


Figure 6 Fatigue properties of aerospace materials [3]

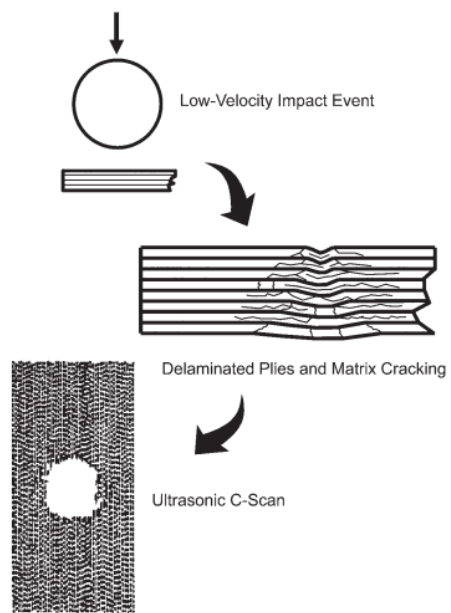


Figure 7 Impact damage in composites

2.4 Applications

The use of composites has widely increased and many industries such as aerospace, automotive, marine, construction utilise composites for applications where high strength and stiffness combined with greater weight savings is required. Aerospace industry use composites to decrease weight and thereby increase fuel consumptions. For example, the new Boeing 787 consists of a 50% composite airframe. Weight savings are around 20-25% compared to using aluminium as shown in Figure 8 [3]. Similarly, the automotive industry utilises composites for high performance Formula 1 cars to manufacture chassis, engine covers, etc (Figure 9). Marine industry uses composites to minimise the corrosion which is a major issue.

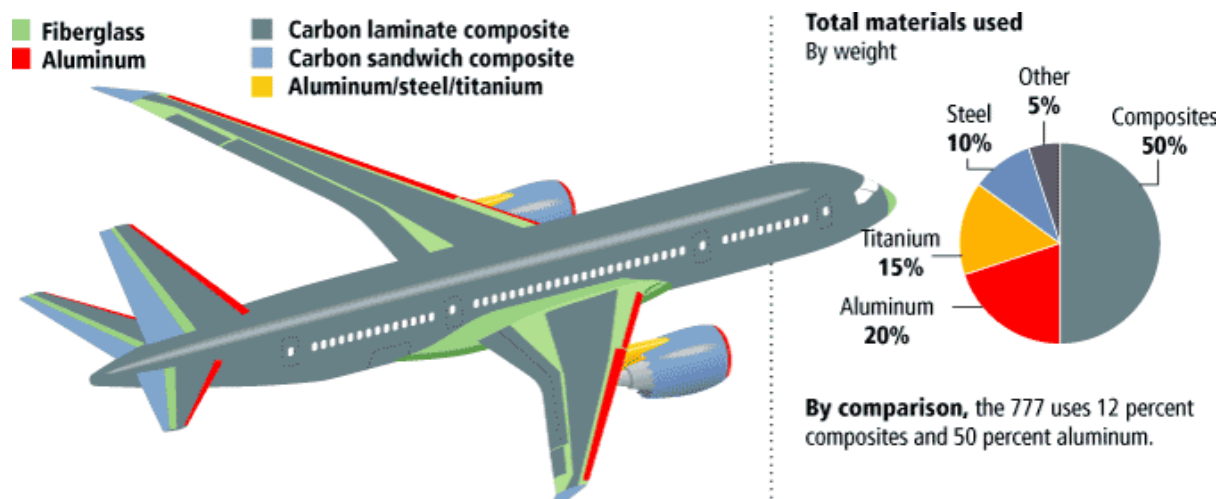


Figure 8 Boeing 787 Dreamliner [3].

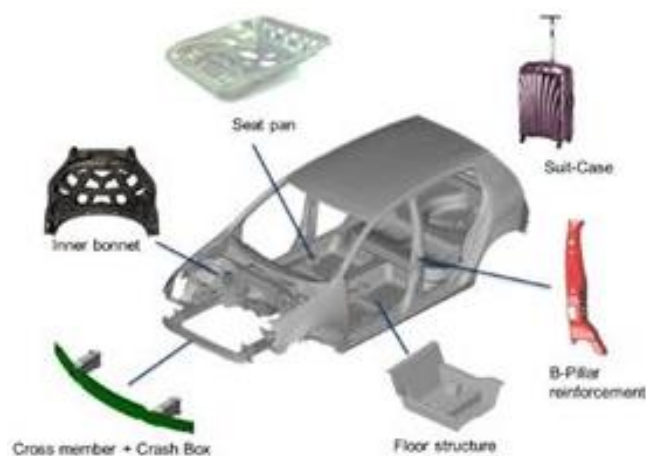


Figure 9 Automotive applications [3]



Figure 10 Marine applications [3]

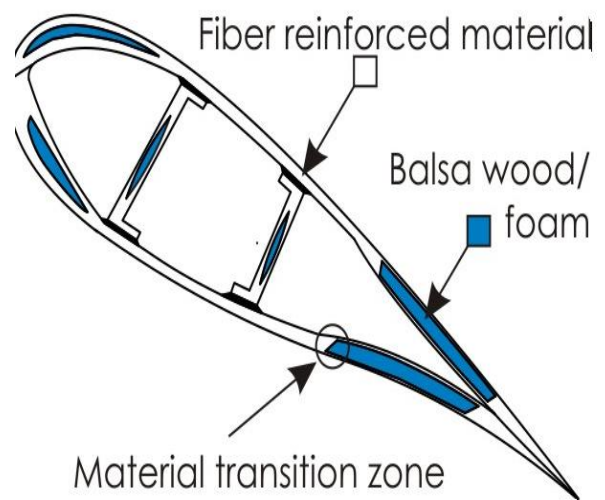


Figure 11 Wind turbine blades [3]

2.5 Low velocity impact damage

Generally, impact damage is not regarded as a threat in metallic structures due to the ductile nature of the material and the high level of energy it can absorb. However, composites can fail in a variety of failure modes and contain damages barely visible reducing the structural integrity of the component. The brittle nature of the composites attributes to its capability of absorbing energy only in elastic deformation and through damage mechanisms, and not through plastic deformation. Most impacts on a composite occurs in the transverse direction; however, the transverse damage resistance of a composite is generally poor due to the lack of through thickness reinforcement. Inter-laminar stresses in shear and tension cause primary failure due

to low inter-laminar strengths. Therefore, design failure strains are used in defending against impact failure, thus resulting in failure to take advantages of the excellent in-plane strength and stiffness properties of the composites. There are two main damage mechanisms associated with composites; (1) Intra-laminar damage mechanisms (fibre or matrix dominated failures due to tensile, compressive or out of plane stresses), and (2) inter-laminar damage commonly referred to as delamination. The internal damage from impact events can cause major reductions in strength and can increase under continuous loading.

2.6 Impact event flowchart

The impact event on a composite structure is a complex phenomenon governed by several factors. Figure 14 displays a typical description of an impact event. There are two classifications related to accompanying damage:

- Local damage or near far field damage at under point of impact. This form of damage is governed by the impact force and results in penetration of the target structure.
- Global damage or far field damage that occurs as a result of the impact, but is governed by bending and wave propagation effect.

The damage induced both locally and globally is described in terms of the damage mechanisms such as matrix cracking, interfacial de-bonding, crack coupling, delamination, fibre failure, indentation

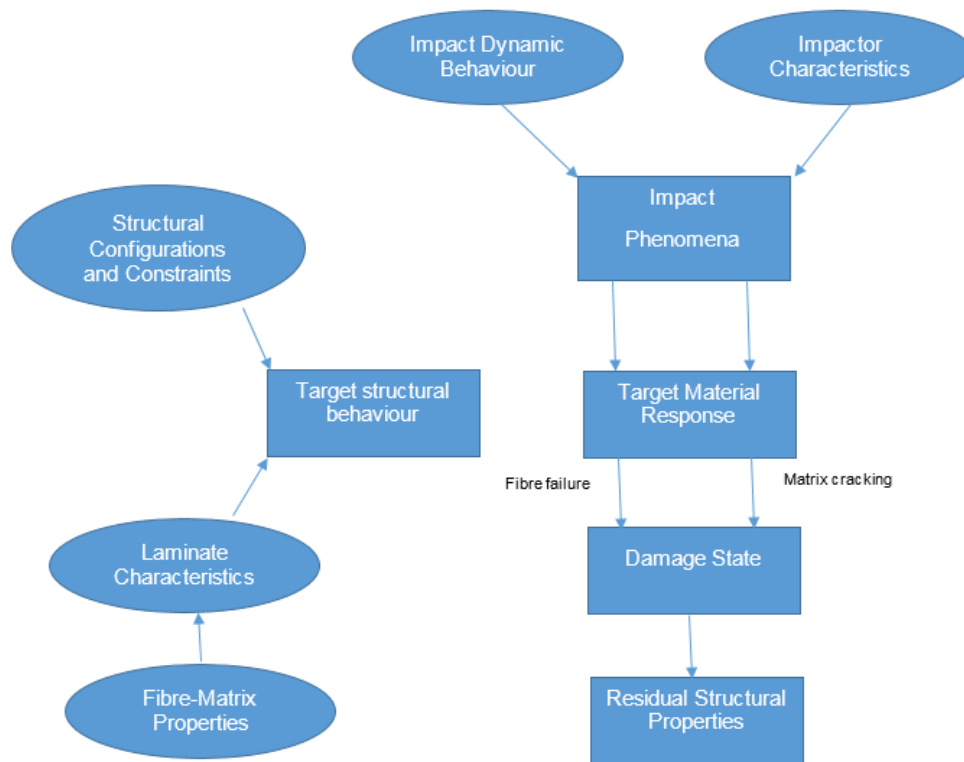


Figure 12 Impact event flowchart (Composed by author)

2.7 Failure modes in composites

2.7.1 Failure in longitudinal tension

The ultimate strain of the fibres governs the composite in this failure mode. The initial failure is usually a fibre break accompanied by loss of fibre length and high interfacial shear stresses near the break. A number of failure mechanisms in the matrix such as transverse cracking in the matrix extending to the fibres, fibre de-bonding or conical shear fracture of the matrix occur after this fibre break. The size of these failure sites remain relatively unchanged, however the number increases thereby causing the reducing the strength of the composite. Ultimate failure of the composite occurs as the number of localized fractures increases to provide a weaker path for complete fracture. Failure in longitudinal tension is largely associated with matrix based and fibre based failure mechanisms. If the failure strain of the matrix is lower than the fibre and when this strain is reached, matrix failure occurs and micro-cracking is initiated in the

material. These matrix cracks develop as the stress in the composite increases, thereby transferring the load to the fibres. In the scenario of fibre based failure, the fibres break before the matrix cracks transfers the entire load. The application of the tensile load causes the first fibre to break and the load it carried is transferred to the other surrounding fibres through the matrix. Since, these fibres are overloaded, they fail as the load is increased. This process repeats as more fibres fail in sequence and the load is transferred to un-failed fibres. [1]

The longitudinal tensile strength could be predicted through the expression:

$$S_{11T} = S_f v_f + \sigma_m (1 - v_f) \quad [2-1]$$

Where, S_f is the fibre tensile strength, σ_m is the stress in the matrix at a strain equal to the fibre failure strain.

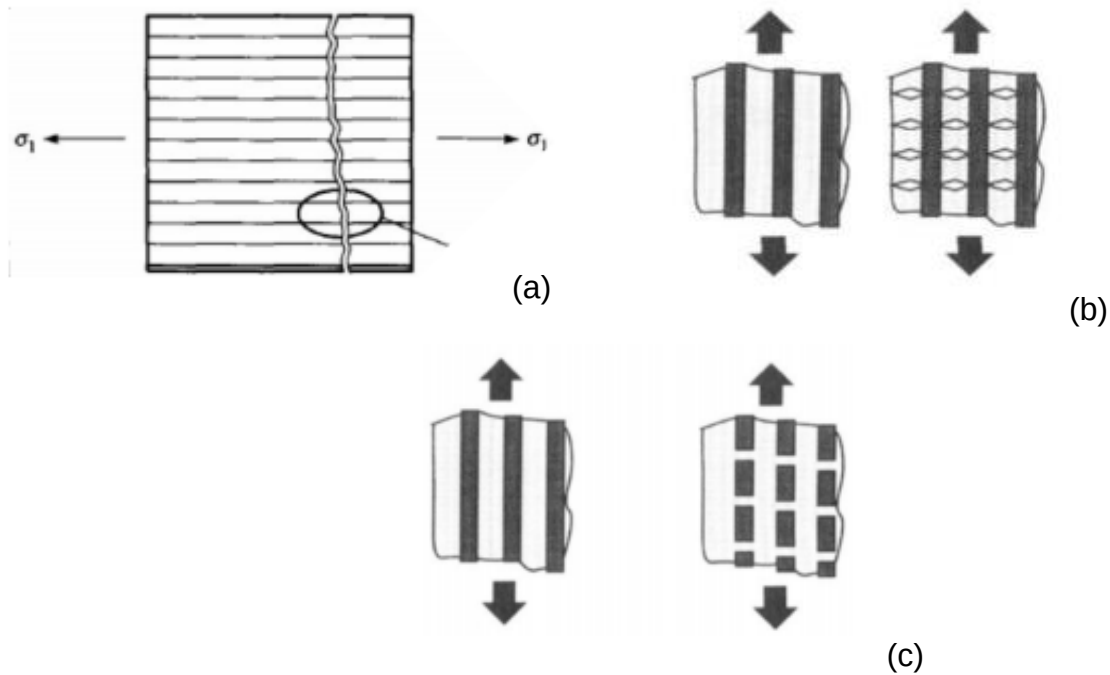


Figure 13 (a) Failure in Tension (b) Matrix based failure (c) Fibre based failure [146].

2.7.2 Failure in longitudinal compression

FRP composites are designed to retain a high-axial stiffness and strength. Therefore, the fibres are made from a strong and stiff material such as graphite; the matrix has a much higher toughness and lower strength than the fibres, thereby increasing the in-plane strength and ductility of the composite. Since damage mechanisms associated with compressive failure are governed by matrix properties, the longitudinal compressive strength of composites is generally low. The main damage mechanisms incorporated with compressive failure are:

- (a) Elastic micro-buckling: The matrix deforms in simple shear due to shear buckling instability.
- (b) Plastic micro-buckling: A shear buckling instability occurring at larger strains causing the matrix to deform in a non-linear pattern.
- (c) Fibre crushing: Failure occurs at fibre level due to an instability such as buckling within the fibres.
- (d) Splitting: The low toughness of the matrix causes matrix cracks parallel to the main axis fibre direction.
- (e) Buckle delamination: Buckle delamination occurs by the buckling of a surface layer from the de-bonding of a sub-surface. This failure mode is related to a low matrix toughness and the presence of a large flaw in the sub-surface.
- (f) Shear band formation: Matrix yields and fracture occurs in a band oriented at about 45° to the direction of the loading.

2.7.3 Failure in transverse tension

The failure of the composite is dominated by the high strain concentration in the matrix. Tensile matrix failure and fibre de-bonding are the primary modes of failure associated with transverse tensile loading. The transverse tensile strength is expressed as:

$$S_{22T} = \frac{\epsilon_{mT}^u}{k_\epsilon} E_{22} \quad [2-2]$$

Where, ϵ_{mT}^u is the ultimate tensile strength in the matrix.

2.7.4 Failure in transverse compression

Compressive and shear failure are the primary failure modes associated with transverse compressive loading. Compressive failure occurs in the matrix as the maximum compressive strain exceeds the ultimate compressive strain. The transverse compressive strength can be expressed as:

$$S_{22C} = \frac{\varepsilon_{mC}^u}{k_\varepsilon} E_{22} \quad [2-3]$$

Where, ε_{mC}^u is the ultimate compressive strength in the matrix.

2.7.5 In-plane shear

Failure mechanisms associated with this type of loading are matrix shearing, fibre debonding or a combination of the two. Uniaxial composite materials are weak in shear if the stress is applied in the direction of the fibres causing shearing of the weak matrix-fibre interface. This causes cracks to develop in areas with high percentages of matrix material. [4]

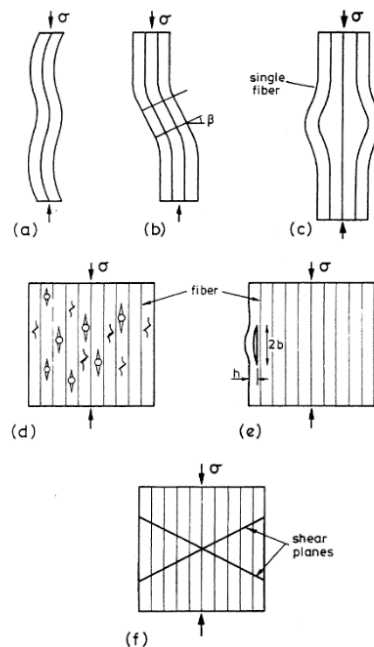


Figure 14 Failure modes in longitudinal compression (a) Elastic micro-buckling (b) Plastic micro-buckling (c) Fibre crushing (d) Matrix splitting (e) Buckle delamination (f) Shear band formation [4].

2.7.6 Delamination

Delamination in a composite is a form of inter-laminar damage resulting from inter-laminar stresses created by impacts, peculiarities in structural load paths or from discontinuities in the structure. These inter-laminar stresses usually occur due to the mismatch in engineering properties, i.e. mismatch in Poisson's ratio ν_{xy} and coefficient of mutual influence η_{xy} between layers. The mismatch in ν_{xy} gives rise to inter-laminar normal and shear stresses and mismatch in η_{xy} causes the rise of inter-laminar shear stresses near the free edge of the laminate. Layup sequence and ply thickness are also contributing factors for delamination. It was shown that high inter-laminar stresses occur in cases where angle plies are adjacent to each other. Composites with higher ply thickness encourage higher inter-laminar stresses, thus causing early delamination.

Low velocity impact can also cause high transverse shear and normal stresses, causing damage to propagate and reducing the strength of the laminate significantly. The internal stress waves and local out of plane displacements caused by impact initiate delamination at interfaces with major changes in ply orientations. The amount and type of damage in the laminate is governed by geometry of laminate, impact energy and the type of loading.

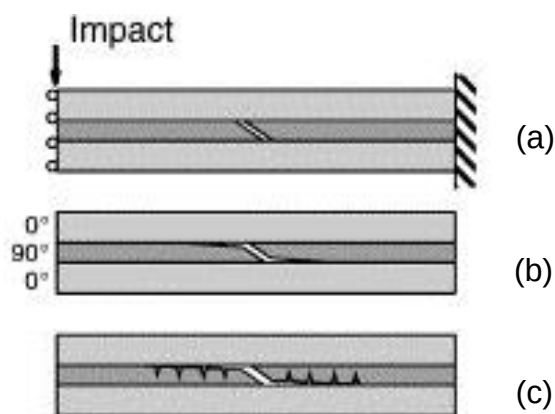


Figure 15 Fracture mechanics schematic of double cantilever beam (a) appearance of critical matrix cracks (b) delamination onset (c) transverse micro-cracking [147].

2.8 Factors affecting impact damage of composite laminates

The broad experimental work carried out on impact testing has created an understanding of the parameters affecting the initiation and growth of impact damage. The overall stiffness is governed by material properties and therefore having a substantial effect on the dynamic response of the structure. The properties of the matrix, fibres and the fibre-matrix interface controls the initiation and growth of impact. Parameters that control the stiffness of the target such as laminate thickness, size of panel and boundary conditions influences the impact dynamics of composites. Similarly, the features of the projectile such as weight, shape, properties and incident angle affect impact dynamics. Other factors affecting the impact response of composites are layup, stitching, preload and environment factors. [5]

2.8.1 Projectile characteristics

The shape and weight of the impactor plays a vital role in impact damage. The most commonly used impactor shape in research is hemispherical, however in reality other shapes impact composite structures. This has led to study of impact dynamics with conical and flat ended shapes. Mitrevski et al [6] studied the effects of impactor shapes on dynamic response of composite panels using hemispherical, ogival and conical impactors. They found out that energy absorption was higher in the specimens impacted by conical impactor and thus produced the largest penetration depth. The blunted hemispherical impactor produced the highest peak impact force and shortest contact duration. The hemispherical impactor predicted the highest damage threshold load. It was found that the residual tensile and compressive strengths of composite laminates are largely affected by damage area and damage mechanisms induced by impact. The studies [7–13] proved that different impactor shapes produced different damage mechanisms and damage areas in laminates, thereby changing the material properties. Lee et al. [14] found that flat and hemispherical impactors produced similar failure mechanisms and energy dissipation. The energy dissipated by flat and hemispherical indentors were higher than semi-cylindrical impactor due to the local indentation. Zhou et al. [15] carried out experimental studies on a carbon-epoxy laminate found that the change in the indenter nose shape resulted a change in the mode of failure. Matrix cracking and fibre failure was the dominant failure associated

with hemispherical indenter, whereas ply shear out was commonly observed with flat indentors, due to the spreading of the stress concentration under the impactor. The geometry of the indenter has an effect on the damage progression of a composite laminate and this effect was investigated by Siow and Shim [16] depicted that smaller radius of curvature of the impactor resulted in larger delamination areas and greater fibre breakage when compared to larger radius impactors.

2.8.2 Material properties

The overall properties of the FRP composite depend upon properties of constituent materials. The overall rigidities of the plate that affects the contact force history are defined by the material properties such as axial, transverse and shear stiffness of the composite. The laminate orientation also contributed to the impact response. The effect of fibre type was studied by Juska et al. [17] and they proved that glass fibre had the best strength and impact damage resistance, when compared with carbon, Kevlar and Spectra. Experiments on the effect of resin and hybrid systems were also investigated. The experiments proved that the compression and flexural modulus of the glass fibre increased with the resin modulus if the resin failure strain was above a critical value. The impact resistance of the glass fibre laminates were not dependent upon the resin failure strain if the resin failure strain was above a critical value. [17]

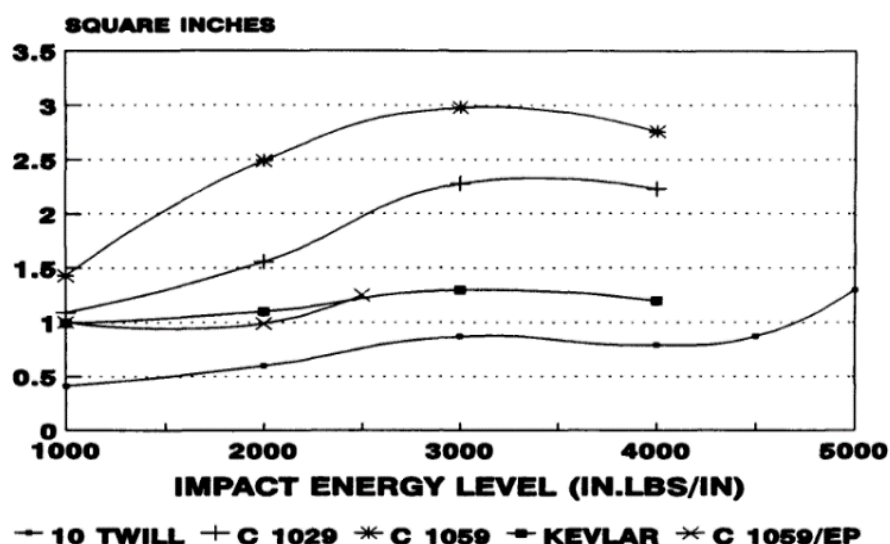


Figure 16 Effect of fibre on impact damage area [17].

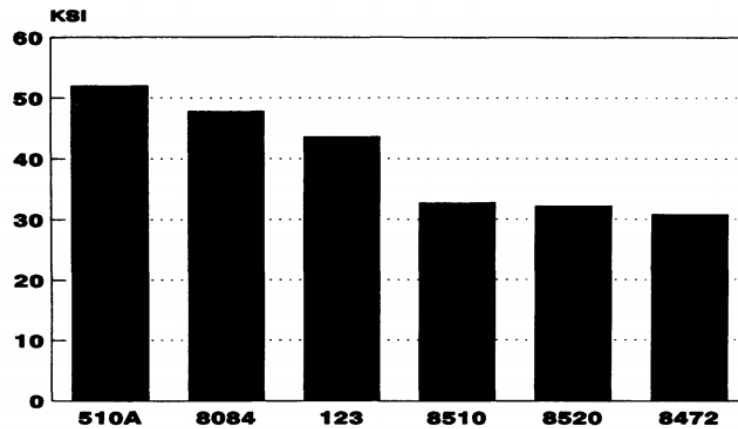


Figure 17 Effect on resin on compression strength [17].

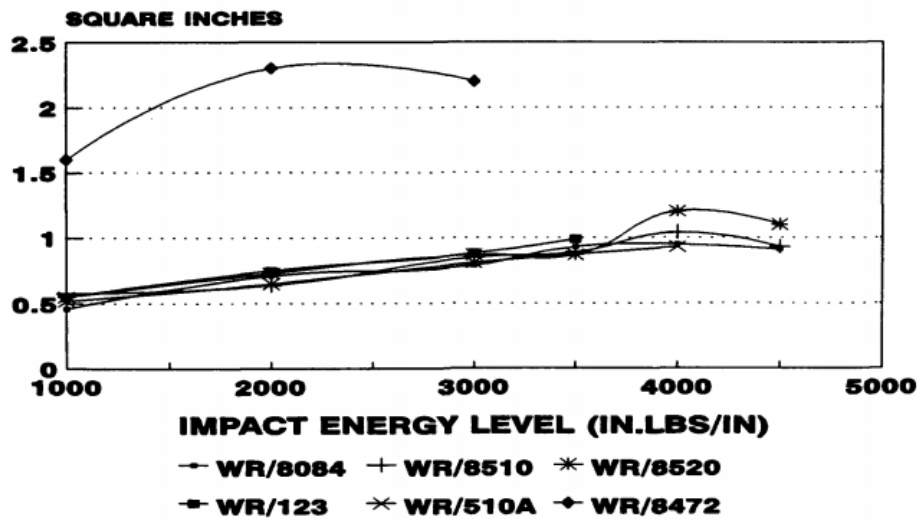


Figure 18 Effect of resin on impact damage area [17].

2.8.3 Laminate thickness, lay-up and stitching

Thickness of the laminate, layup, size and boundary conditions affect the target stiffness. The stiffness in the thickness direction have a major effect on the size of the maximum contact force which has a direct influence on the level of the damage. Similarly, a vital role is played by the stacking sequence on the impact damage resistance of composite laminates. In the scenario of a uni-directional laminate, since the fibres are aligned in the same direction, delamination does not occur. However, for two plates with similar thicknesses and different stacking sequences, the plate with the higher difference of angle between two adjacent plies will be subjected to higher

degree of delamination. Increased delamination also occur when the difference between longitudinal and transverse moduli of the material are higher, thus leading to higher mismatch in the bending stiffness. The stacking sequence has negligible effect on the damage initiation since it depends mostly on the matrix and fibre-matrix interface. The maximum contact force during an impact strongly depends upon the stacking sequence. [7]. Stitching is a method used in introducing through-the-thickness reinforcement. The stitching density, pattern and the properties of the thread affect the impact resistance of composite laminates. Caprino et al. [18] conducted low velocity impact tests on carbon/epoxy laminates of different thicknesses. They found that as the plate thickness increased, the force and energy at the onset of delamination increased approximately to the power of 1.5. Hosur et al [19] proved that hybrid laminates possessed a higher load carrying capability than carbon/epoxy laminates when subjected to low velocity impact. Baucom and Zikry [20] developed three composite systems: two dimensional (2D) plain woven laminate, 3D orthogonally woven monolith and a bi-axially reinforced warp-knit to study the effects of the geometry of reinforcement and progress of damage in woven composite panels. It was observed that 3D composites had a greater resistance to impact resistance, thus dissipating more total energy. Fuoss et al. [21,22] studied the effects of stacking sequence on the impact resistance of composite laminates and analysed the effect of the stress state in the laminate to predict delamination damage through the use of finite element modelling. They derived a parameter based on the bending strain of the laminate and used it to predict the impact response of composite laminates with variations in stacking sequences. Naik et al. [23,24] by analysing the effect of plate thickness and impact parameters on impact damage resistance of woven composites determined that the damage resistance is higher for low mass high velocity combinations compared to high mass low velocity combinations. Atas and Dahsin [25] and Liu [26] evaluated the effect of weaving angle on the impact response characteristics such as the peak force, contact duration and maximum deflection. Liu [27] introduced an energy profiling method and proved that penetration and perforation thresholds increase linearly with impactor size and nonlinearly with the thickness of the laminate.

2.8.4 Preload

Most researchers studied the low velocity impact behaviour of composite through analytical, numerical and experimental techniques. However, there is limited literature available on the impact response of composite structures with preload. A typical situation defining preload is in the case of an aircraft wing in flight where the upper skin is in compression and lower skin is in under tension and it is noted that most composite structures experiences impact loading in addition to the pre-existing stresses. Kelkar et al [28] analytically and experimentally studied the effect of tensile in-plane load on the impact response of composite laminate. They found that as tensile load increases, the impact duration decreased and impact force increases. Whittingham et al. [29] subjected carbon fibre reinforced composites to low velocity impact with tensile and compressive pre-stresses already applied to the plates. Their study proved that in the case of a low velocity impact, the pre-stresses had negligible effect on the depth of indentation and the maximum contact force.

2.8.5 Environmental conditions

Generally, composites are subjected to various environmental conditions such as moisture, low temperatures, UV rays, etc. Karasek et al. [30] studied the effect of moisture and temperature on the impact resistance of epoxy/graphite composite. They found that at higher temperatures, moisture had a major effect on the damage initiation energy and as the temperature decreased, the damage initiation energy decreased.

2.8.6 Initiation of damage

Impact damage in composites is a complex phenomenon, therefore it may be not possible to model the entire sequence of events leading to a given damage state. The first stage of low velocity impact damage is the development of a matrix crack. In some scenarios, the target is flexible and the crack initiates as a result of the tensile flexural stresses in the bottom ply of the laminate. This is a tensile crack and is perpendicular to the plane of the laminate. Shear cracks, created by contact stresses usually occur in thick laminates and appear near the top of the laminates. These cracks are inclined in relation to the laminate mid-plane. These matrix cracks cause delaminations at ply interfaces and initiate a damage evolution pattern. Therefore, it is possible to predict

the initiation of damage; however, predicting the final damage state is difficult to assess. [5]

There are two approaches for depicting the impact damage resistance of composites. The first approach involves the prediction of the overall damage area based on the theory that delaminations grow due to higher transverse shear stresses under the impactor. The distribution of the transverse shear stress is determined around the impact point in order to estimate the area of the damage zone, by using an appropriate failure criterion. The second approach involves the prediction of the threshold value of the contact force corresponding to the initiation of damage. There is a drastic increase in the damage area once the load reaches a critical value P_{crit} . The damage area is smaller below the critical value and it is known as a Hertzian surface. The contact force histories provide information about damage initiation and growth [9,31–34]. It is noted that the first load drop, in terms of Hertzian failure or substantial damage resembles the existence of initial damage in the form of matrix cracking, fibre failure and local indentation [15,35–38]. Davies and Zhang stressed that the first damage threshold was due to the initiation of delamination failure. Belingardi and Vadori [34] derived two damage thresholds from the contact force history. The first threshold corresponded to the first load drop for the first material damage and the second threshold corresponded to the maximum force value of the failure of first ply in the laminate.

2.8.7 Propagation of damage

A solid subjected to static or impact loading absorbs energy by two basic techniques and material deformation. The deformation of the material occurs first. As the impact energy increases and is large enough, a crack initiates and propagates, thus triggering the second energy absorbing mechanism. During the propagation of this crack, materials deformation advances. Brittle materials such as glass and ceramics associates a smaller amount of deformation; therefore, the accompanying energy absorbed is also small. Therefore, brittle materials display a low energy absorption capability. Two main parameters, impact energy (E_i) and absorbed energy (E_a) are used in assessing damage propagation in composite structures. The absorbed energy is calculated from calculating the area under a force-displacement curve. During the fracture process, the different mechanisms associated during crack propagation

relates to the total energy absorbed. Zaretsky et al. [39] measured the fracture toughness of glass-fibre reinforced epoxy composites and found that the three failure modes delamination, matrix cracking and de-bonding affected the fracture strength. Dandekar et al. [40] by developing a delamination model derived that the impact angle relative to the fibres affected the threshold and delamination tensile stresses. However, work on the low velocity impact response of different laminate combinations and impacts on complex geometries such as variable stiffness plates are less.

2.9 Low Velocity impact testing methods

2.9.1 Charpy impact testing

Charpy impact testing method involves a swinging pendulum mass as shown in Figure 19. Energy is transferred and work is done on the specimen as velocity and kinetic energy of the striking mass are changed. Energy absorption is in the form of elastic and plastic deformation, hysteresis effects, friction between the specimen and test fixture and specimen attaining kinetic energy. The outcome of an impact load in generating stress is dependent on the degree to which the energy is consumed in causing deformation. In the Charpy test, the specimen is supported as a simple beam, while in the Izod test is supported as a cantilever. The standards ASTM E 23 for metals and ASTM D 256 for plastics are used. In the case of homogenous isotropic materials, tests are usually performed with a notch on the side where the tensile load is applied. This notch acts as a stress concentration spot to reduce the energy necessary for the fracture initiation. Therefore, the total energy required to fracture is basically the energy essential for the fracture propagation. Since the fracture phenomenon of polymer composites is more complex, these impact tests might not be sufficient to accurately predict the impact dynamics of the system. [41]

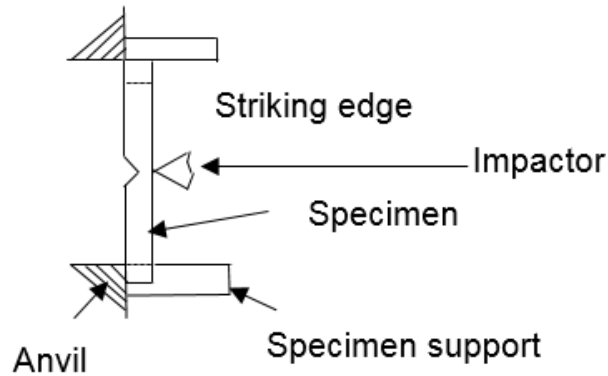


Figure 19 Charpy and Izod Tests [Composed by author]

2.9.2 Drop weight impact testing

This test is widely used in the transverse impact testing of laminated or sandwich composite. The instrumented impact tester shown in Figure 22 comprises of a drop tower fitted with an instrumented indenter (tup or dart). The tup consists of a force transducer and weights are added to the tup holder to deliver the required impact energy. Two optical flags parted by a distance of 75mm to 150mm are positioned in the path of the striker before it strikes the target specimen. The distance between the two flags and the time of flight of the striker is used in measuring the impact velocity. The contact force history is depicted from the point of initial contact with the specimen, and as the striker moves transversely through the specimen's thickness. Energy is obtained through integrating the force-time signal. Parameters such as force-displacement, force-time and energy-time are evaluated through the drop weight impact test. The contact force-time history plot provides a good representation of the damage progression in a composite laminate and is widely used in literature. Figure 23 shows a typical force-time history plot for large and small mass impacts. The large mass impact (4.6kg) consisted of many small oscillations superimposed on the impact response and are mainly due to vibrations of the plate against impactor during impact. Contact duration was significantly lower for the small-mass impacts (0.025kg) and the oscillations does not have much time to develop. Usually, the impact force curve is divided into three regions based on the impact mass as static (large mass), transitional and dynamic (small mass). [42]

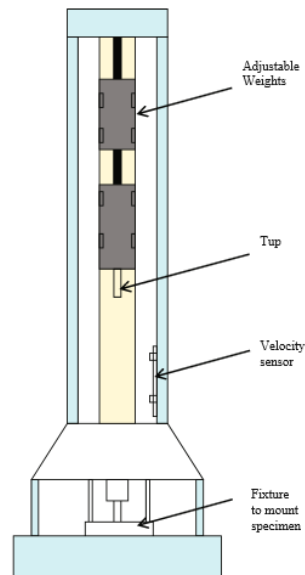


Figure 20 Drop Weight Impact Tester [42]

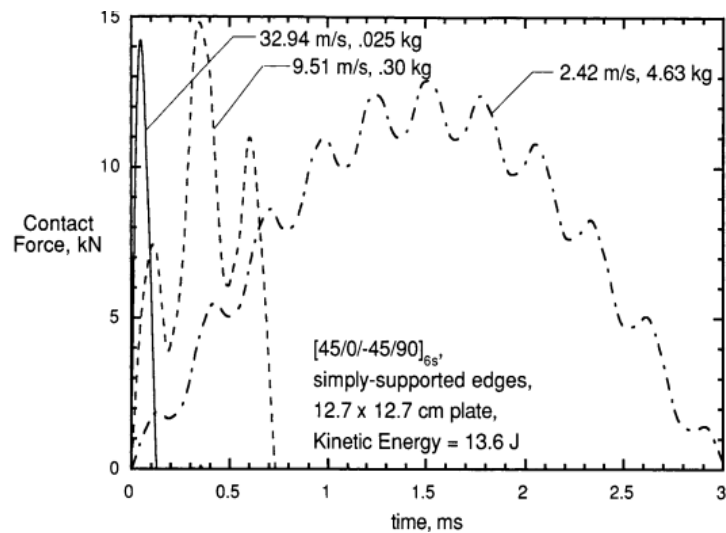


Figure 21 Contact force history plot (Small and Large mass impacts [42])

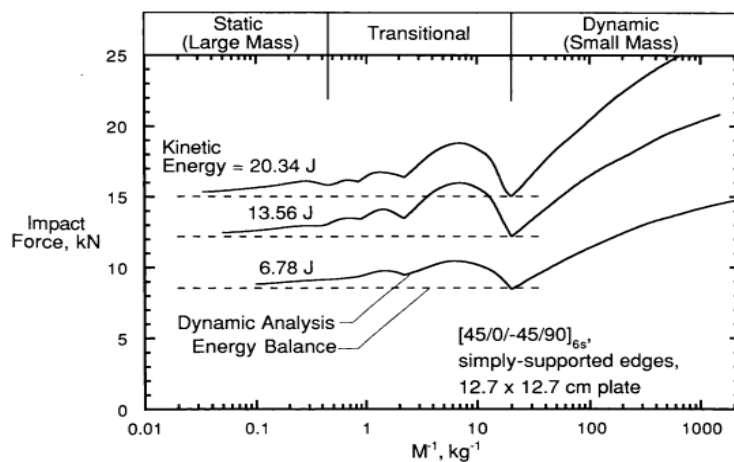


Figure 22 Impact force vs impactor mass plot [42]

2.10 Review of selected works and research gap

Various researchers have studied the impact behaviour of composite laminates. Several types of mathematical models have been built in order to study this impact behaviour. The initial literature reviews mostly focussed on the analytical models available in literature. The following research were used by the author [43].

Gong et al. [44] studied the elastic response of orthotropic laminated cylindrical shells to low-velocity impact. A spring-mass model was developed to determine the contact force between the shell and the impactor. An analytical function for the contact force was derived in terms of material properties and the mass of the shell and the impactor, as well as for the impact velocity. Khalili [45] studied the dynamic response of thin smart curved composite panel subjected to low-velocity transverse impact. First order shear deformation was used to obtain the structural field.

Martinez et. al. [46] performed impact tests over a carbon fibre reinforced epoxy using low energy in the striker. A non-conservative and nonlinear spring-clashpot series model was proposed to reproduce the material behaviour. The model considered simultaneously both flexural and indentation phenomena accounting for energy losses by means of the restitution coefficient. This model enabled the accurate prediction of the contact force duration.

Hassan et al. [47] addressed the response of Glass Fiber Reinforced Plastic laminates (GFRPs) under low-velocity impact. Experimental tests were performed according to ASTM: D5628 for different initial impact energy levels ranging from 9.8 J to 29.4 J and specimen thicknesses of 2, 3 and 4 mm. The impact damage process and contact stiffness were studied incrementally until a perforation phase of the layered compounds occurred, in line with a force–deflection diagram and imaging of impacted laminates. The effect that impact parameters such as velocity and initial energy had on deflection and damage of the test specimens was explored.

Ghasemnejad *et al* [48] studied the Charpy impact behaviour of single and multi-delaminated hybrid composite beam structures. The Charpy impact test was chosen to study the energy absorption capability of delaminated composite beam. It was shown that the composite beams with closer position of delamination to impacted

surface are able to absorb more energy in comparison with other delamination positions in hybrid and non-hybrid ones.

Krishnamurthy *et al.* [49] studied the impact response of a laminated composite cylindrical shell determined both by the classical Fourier series and the finite element methods. Impact response determined by the finite element method also includes a prediction of the impact-induced damage deploying the semi-empirical damage prediction model of Choi–Chang. The parametric study carried out by the finite element method investigated the effect of governing parameters such as impactor mass, its approach velocity, curvature of the shell, on both the impact response and on the impact-induced damage.

Caputo *et al* [50] proposed numerical techniques to describe the damage initiation and propagation of impact damages in composite structures. An explicit finite element analysis was developed through utilizing a global/local finite element model and the model predicted the propagation of both interlaminar and intralaminar damages.

Laminated theories have been widely used to study and predict the impact damage of composite structures. These theories can be classified as equivalent single layer (ESL), layer-wise and zig-zag theories. The ESL theories can be divided into three main categories: classical plate theory (CPT), first-order shear deformation theory (FSDT) and higher order shear deformation theories (HSDTs). Shear deformation effects are ignored by the CPT and it provides reasonable results for thin laminates. Nonetheless, it underestimates the deflection and overestimates the buckling load and frequency of thick laminates where shear deformation effects are more distinct. Shear deformation effects are accounted through the FSDT proposed by Reissner [51] and Mindlin [52] by the way of linear variation of in-plane displacements through the thickness. A shear correction factor is essential to compensate for the difference between actual stress state and presumed constant stress state since the FSDT violates the equilibrium conditions on the top and bottom surfaces of the plate. [53]

Impact on composite structures has attracted the attention of many researchers worldwide. Sun and Chattopadhyay [53] and Dobyns [54] manipulated the plate equations developed by Whitney and Pagano [55] in analysing a simply supported, orthotropic plate subjected to central impact. Ramkumar and Thakar [56] presented

an analysis for the low velocity impact analysis of a curved composite plate. Christoforou and Swanson [57] developed an analytical solution to study the impact loading of simply supported, orthotropic plates. Studies carried out by Singh and Mahajan [58] depicted that the force and deflection response of the laminate is largely dependent upon the extent of damage in composite laminates. Their model was able to predict the inter-laminar and intra-laminar damage effects on the stiffness of the structure.

Shahid et al [59] recognized the impact force response as a major parameter characterising impact damage resistance of laminated composites. They developed an analytical model to predict the damage resistance of composite laminates beyond the initial damage state through modifying the spring constant in Hertzian contact law which is a function of the extent of damage in composites.

Olsson [60–62] developed analytical models for the prediction of delamination growth and the resulting response during small and large mass impact on composite plates. The model was able to predict the delamination size, load and deflection history. Olsson [63] also developed an approximate analytical solution to the first phase of impact, or wave propagation dominated, response of composite plates. Robinson and Davies [64] studied the effects of impactor mass and specimen geometry on the low velocity impact behaviour of laminated composites. Thai et al.[65] investigated the contact law and impact response of a carbon-epoxy laminated plate. The effect on the laminate thickness and the ply orientation on the low velocity impact behaviour were studied experimentally and numerically using finite element methods. Wu and Shyu [66] experimentally studied the response of composite laminates to contact loads and determined that the laminate stacking sequence has an insignificant effect on the force-indentation relationship. Choi and Lim [67] introduced the linearized contact law to solve impact problems. A parametric study on the contact coefficient and the exponent of the contact law was studied. Comparisons were made between the modified Hertzian contact law and the experimental static indentation laws. Choi and Hong [68] used the first order shear deformation theory to predict impact force histories, deflections, dynamic strains in the laminate subjected to low velocity impact. Xiao et al. [69] developed an analytical model to predict the damage area in laminated composite plates subjected to low velocity impact. Riccio et al. [70] presented an

analytical model capable of predicting the impact induced damage onset and evolution in composites. Sivakumar [71] developed energy balanced and spring mass systems to determine the impact force and duration during low velocity impact of circular composite plates.

Low velocity impact damage of fiber-reinforced panels were considered an important factor in the research conducted by Sankar [72]. Many researchers have studied analytically, numerically and experimentally the low velocity impact response of composite plates. The low velocity impact response is strongly influenced by parameters such as fiber type, resin content, resin type, lay-up, thickness and projectile velocity. For low velocity impact events, testing methods such as Charpy and drop weight testers are commonly used [73]. Hufenbach et.al. [74] experimentally studied the effect of key parameters like fibre reinforcement type, hybrid materials, fibre type and architecture on the impact damage resistance of composite structures Mili [75] experimentally studied the impact of composites using a drop weight method for different stacking sequences. It was proved that as the stacking sequence of the laminate is varied; the plates present comparable behavior for the same projectile velocity. Mitrevski et al. [76] investigated the effect of impactor shapes on the impact response of thin woven carbon/epoxy laminates experimentally. They made use of hemispherical, ogival and conical impactors to conclude that the impactor shape made an important contribution to the damage and response of composite laminates. Shokrieh and Fakhar [77] studied the low velocity impact behaviour of rigidly supported and clamped composite sandwich panels using experimental, numerical and analytical techniques. They used a drop weight impactor to perform the experiments, the LS-DYNA software to do the numerical analysis and equivalent single and two degrees of freedom spring-mass models to predict the analytical results. Ghasemnejad et al. [78] explored technologies to improve damage response of single and multi-delaminated FRP composites using Flex yarn. They found that multi-delaminated beams pinned in the z-direction are able to arrest crack propagation and absorb more energy than simple ones in hybrid composite beams. Morioka and Tomita [79] investigated the effect of lay-up sequences on the mechanical properties of CFRP laminate composites and compared impact and bending fracture of uni-directional, cross-ply and quasi-isotropic laminates. David-West et al. [80] showed the effect of 45° plies in energy

absorption and bending stiffness in CFRP laminates. Caprino et al. [81] showed that for a given fiber type, the energy absorbed is mostly influenced by the total fiber volume and fiber diameter, whereas other factors such as resin type, fiber architecture, stacking sequences and orientations play a secondary role. Arachchige and Ghasemnejad [43] derived theoretical formulations based on a two degree of freedom system to predict the transverse response of variable stiffness curved composite plates. They studied the effects of various impact parameters such as layups, velocities, geometrical changes on the impact response of curved variable stiffness plates.

2.11 Review of low velocity impact damage models

2.11.1 Impact models developed in literature

This section of the report discusses about the different impact models developed by researchers in the past decade. These models have been the baseline for the improved models present today.

The study of the impact problems and associated analytical models has been divided into three main classifications namely deformation mechanics, damage mechanics and residual strength degradation

The models discussed in this section is related to deformation mechanics. The information gathered through these analyses yields in prediction of the damage region, damage growth and effects of damage

2.11.1.1 Analysis-Model 1

The first research in literature with regards to analytical modelling of low velocity impact on composite structures has been Shivakumar's and Elber's model [71]. This analytical model has paved way for the present advanced damage models developed in literature. Experimental parameters used in this model are classified in Table 1

Striker	Target
Mass-Large	Mass-small
Material-steel, aluminium	Material- Graphite/Epoxy
Geometry-spherical	Geometry-circular plate
Striker incidence-normal	Thickness-thick, intermediate, thin
Velocity-low	Ply layers-quasi-isotropic
	Boundary conditions-clamped, simply supported, movable, immovable

Table 1 Experimental parameters [71]

Two models were used in the analysis, the first an energy balance model to predict impact force magnitude, while the second, a spring-mass was used to obtain information about the impact force.

2.11.1.1.1 Energy-Balance Model

The impact force magnitude is predicted through this model with key features described below.

The kinetic energy of the striker is equal to the sum of the strain energy of the target, including that due to contact, bending, transverse shear, and membrane deformations. Major assumptions are:

- Impact duration is greater than the stress wave travel to the boundary
- Higher response models are neglected
- Energy losses from material damping, surface friction are neglected

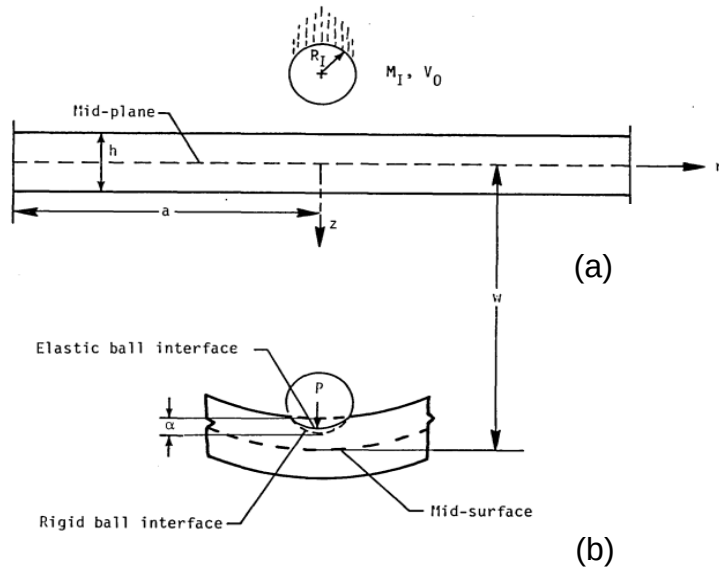


Figure 23 Schematic of Impact Event (a) Before impact $t > 0$, (b) During impact $t > 2$ [71]

2.11.1.1.1.1 Impact Force Deformations

For $t > 2$ these deformations consist of

- Contact deformations
- Transverse plate deflections

$t > 2$ is the condition where impactor strikes the target

The former relates to the striker and target interactions and is based upon the Hertz contact law, while the latter consists of

- Bending denoted by w_b
- Shear denoted by w_{sh}
- Membrane denoted by w_m

2.11.1.1.1.2 Target/striker interaction

Target/striker interaction consists of two main elements: contact area and plate load. The contact area is dependent on the force and moduli of the striker and the target, while the plate load is dependent on the plate bending (K_b), shear (K_s), and membrane stiffness (K_m). The plate bending and membrane stiffnesses as used are derived using vibrational methods for the case of simply supported and clamped boundary

conditions. The shear stiffness, K_s is derived by assuming a distributed impact over the contact area. Therefore, the governing equation for the energy balance model is

$$\frac{1}{2}m_s v_0^2 = E_c + E_{bs} + E_m \quad [2-4]$$

2.11.1.1.1.3 Contact Energy, E_c

The impact force, P and contact deformation α are related through the Hertz contact law

$$P = n\alpha^{3/2} \quad [2-5]$$

Where n , the contact parameter is dependent upon the material and geometrical properties of striker and target. The contact parameter for a spherical isotropic striker and transversely isotropic composite plate is given by:

$$n = \frac{4\sqrt{R_s}}{3\pi(K_1 + K_2)} \quad [2-6]$$

$$K_1 = \frac{(1-\nu_s^2)}{\pi E_s} \quad [2-7]$$

$$K_2 = \frac{\sqrt{A_{22}} \left[\sqrt{A_{11}A_{22} + (G_{Zr})^2} - A_{12} + G_{Zr} \right]^{1/2}}{2\pi \sqrt{G_{Zr}(A_{11}A_{22} - A_{12}^2)}} \quad [2-8]$$

$$A_{11} = E_z(1 - \nu_r)\beta \quad [2-9]$$

$$A_{22} = \frac{E\beta(1-\nu_{Zr}^2)\delta}{(1+\nu_r)} \quad [2-10]$$

$$A_{12} = E_r\nu_{Zr}\beta \quad [2-11]$$

$$\beta = \frac{1}{1-\nu_r-2\nu_{Zr}^2\delta} \quad [2-12]$$

$$\delta = E_r/E_z \quad [2-13]$$

Where,

E_s, ν_s = modulus and Poisson's ratio of the striker

E, G, ν = moduli and Poisson's ratio of the target

r, z = radial and thickness coordinates

Therefore, the contact energy is obtained using

$$E_c = \int_0^a P d\alpha \quad [2-14]$$

Using the Hertz contact law for $P(\alpha)$, the contact energy E_c can be expressed as:

$$E_c = \frac{2}{5} \frac{P^{5/3}}{n^{2/3}} \quad [2-15]$$

2.11.1.1.1.4 Bending, shear energy E_{bs}

In order to determine the target bending, shear energy E_{bs} , it is required to relate the target reactive force P_{bs} to the transverse plate deflection. This relation is expressed as,

$$P_{bs} = K_{bs} w \quad [2-16]$$

Where the quantity K_{bs} represents an effective, bending shear stiffness denoted by,

$$K_{bs} = \frac{K_b K_s}{K_b + K_s} \quad [2-17]$$

The value of bending stiffness varies accordingly to the various edge and boundary conditions. The following condition is considered as an example:

- Boundary condition-clamped
- Edge condition-immovable

The corresponding bending stiffness, K_{bs} for the above conditions are expressed as,

$$K_b = \frac{4\pi E_T h^3}{3(1-\nu_T^2)a^2} \quad [2-18]$$

The shear stiffness is given by

$$K_s = \frac{4\pi G_{Zr}}{3} \left(\frac{E_{rz}}{E_{rz} - 4\nu_{rz} G_{Zr}} \right) \left(\frac{1}{4/3 + \log a/a_c} \right) \quad [2-19]$$

Where, the contact radius, a_c , is expressed as

$$a_c = \left[\frac{3\pi}{4} P [K_1 + K_2] R_s \right]^{1/3} \quad [2-20]$$

The bending shear energy can therefore be expressed as

$$E_{bs} = \frac{1}{2} K_{bs} w^2 \quad [2-21]$$

2.11.1.1.1.5 Membrane Energy, E_m

The target reactive force for membrane effects can be expressed as

$$P_m = K_m w^3 \quad [2-22]$$

Similarly, the membrane energy is expressed as

$$E_m = \frac{1}{2} K_m w^4 \quad [2-23]$$

The value of membrane stiffness varies accordingly to the various edge and boundary conditions. The following condition is considered as an example:

- Boundary condition-clamped
- Edge condition-immovable

$$K_m = \frac{(353 - 191\nu_r)\pi E_r h}{648(1 - \nu_r)a^2} \quad [2-24]$$

The energy balance equation can then be expressed as,

$$m_s v_o^2 = K_{bs} w^2 + \frac{K_m w^4}{2} + \frac{4}{5} \left[\frac{(K_{bs} w + K_m w^3)^5}{n^2} \right]^{1/3} \quad [2-25]$$

2.11.1.1.1.6 Solutions

There are two methods to evaluate this problem. They are:

- Solving for w using numerical methods
- Solving for v_o for a given w

2.11.1.1.2 Spring-Mass Model

The energy balance method only provided the magnitude of the contact force. Therefore, a spring-mass model was developed to derive information about the force-time history of the impact event. In this model, the striker and target are considered rigid masses m_s and m_t with the target mass taken as an effective mass. The effective target mass is the target mass plus one fourth the target mass in order to account for inertial effects. The two masses are connected through springs as shown in Figure 24:

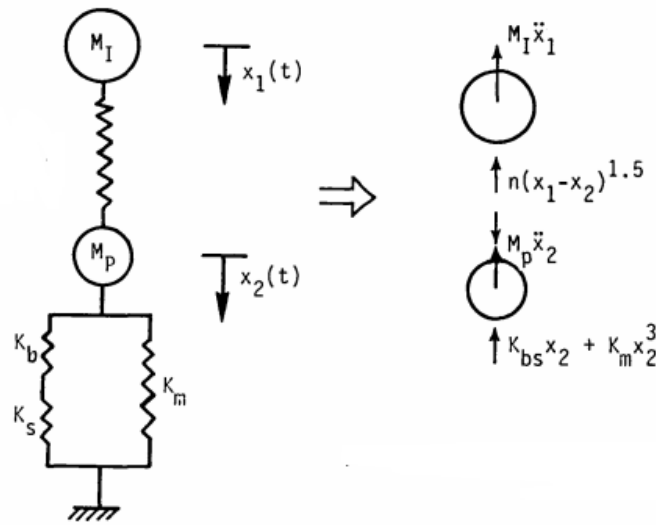


Figure 24 Spring-mass model [71].

The spring stiffness is represented by a contact spring between the striker and target, with bending, shear and membrane springs.

2.11.1.1.2.1 Model Development

By using Hertz contact law, that is,

$$P = n\alpha^{3/2} \quad [2-26]$$

The contact force between striker and target can be expressed as (the striker mass m_s is considered to be in contact with the target)

$$P = n(x_1 - x_2)^{1.5} \quad [2-27]$$

Where $w = x_2(t)$ is the transverse deflection of the target and

$$\alpha = x_1(t) - x_2(t) \quad [2-28]$$

is the contact deformation.

The governing equations of the spring-mass model can be expressed as

$$m_s \ddot{x}_1 + \lambda n(x_1 - x_2)^{1.5} = 0 \quad [2-28]$$

$$m_{ts} \ddot{x}_2 + K_{bs} x_2 + K_m x_2^3 - \lambda n(x_1 - x_2)^{1.5} = 0 \quad [2-29]$$

Initial conditions

$$x_1(0) = 0$$

$$\dot{x}_1(0) = v_o$$

2.11.1.1.2.2 Solutions to equations

The coupled nonlinear equations are solved using a numerical integration scheme with the solution terminated when the target displacement reaches either zero or a negative value. Similarly, to the energy balance model, the impact force is calculated by the expression,

$$P = K_{bs} w + K_m w^3 \quad [2-30]$$

2.11.1.2 Analysis-Model 2

Gong et al. [44] further developed the analytical modelling equations of low velocity impact behaviour of composite structures and introduced a relationship for the contact force history during the impact. The main outcomes of their study widened the understanding of the performance of composites under low velocity impact. They also studied the effects of various parameters affecting impact response. Experimental parameters in their work is summarised in Table 2.

Striker	Target
Mass-Large	Mass-small
Material-Steel	Material- Graphite/Epoxy
Geometry-spherical	Geometry-Curved Shell
Striker incidence-normal	Thickness-Thick (48 plies)
Velocity-low	Ply layers- $[(\pm 45/0_2)_2/\pm 45/0/90]_{2s}$
	Length=0.6096m, Width=0.2032, thickness= 0.0127m, Radius=0.25m
	Boundary conditions-simply supported

Table 2 Experimental parameters [44]

A spring-mass model was developed through modification of Shivakumar et al. [71] model.

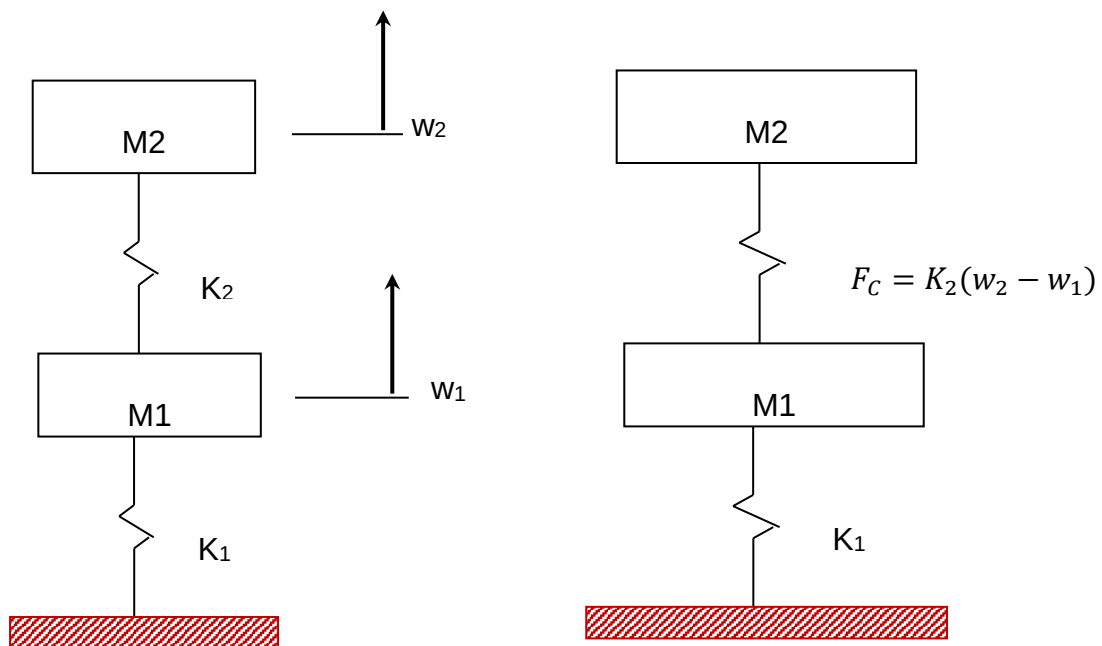


Figure 25 Gong's Modified Spring Mass Model [44]

In the present S-M model, M_1 and M_2 represent the mass of the laminated shell and the impactor. K_2 is the contact stiffness and K_1 is the equivalent stiffness of the simply supported laminated shell determined through;

$$K_1 = \omega_{11}^2 M_1 \quad [2-31]$$

Where ω_{11} is the fundamental frequency of the laminated shell

The radial displacements of the load point of the laminated shell and the impactor at any time t during impact is denoted by $w(t)$ and $w_2(t)$ respectively. Therefore, the contact deformation is expressed as:

$$\partial(t) = w_2(t) - w(t) \quad [2-32]$$

The contact force between the impactor and the laminated shell during impact is governed by the Hertzian theory and is expressed as:

$$F_c(t) = K_2 \partial^p \quad [2-33]$$

Where K_2 and p are material constants obtained through static indentation tests.

Since the above equation for the contact force was highly non-linear, it was a complex process to derive an analytical solution for the contact force. Therefore, the present approach by Gong made use of an effective contact stiffness K_2^* and the assumption of linear behaviour to relate an equivalent contact force to contact deformation.

$$F_c^*(t) = K_2^* \partial \quad [2-34]$$

or

$$F_c^*(t) = K_2^* [w_2(t) - w(t)] \quad [2-35]$$

The effective stiffness K_2^* was derived through dynamic experiments based upon the mechanical impedance

$$K_2^* = \sqrt{\pi} \Gamma\left(\frac{p+1}{2}\right) \frac{2\Gamma\left(\frac{p}{2}+1\right) + \sqrt{\pi} \Gamma\left(\frac{p+1}{2}\right)}{4\Gamma^2\left(\frac{p}{2}+1\right) + \pi \Gamma^2\left(\frac{p+1}{2}\right)} \delta_m^{p-1} K_2 \quad [2-36]$$

Where Γ is the gamma function which is considered to be equal to unity

The equations of motion of the two degree-of-freedom spring-mass model were expressed as;

$$M_1\ddot{w} + (K_1 + K_2^*)w - K_2^*w_2 = 0 \quad [2-37]$$

$$M_2\ddot{w}_2 - K_2^*w + K_2^*w_2 = 0 \quad [2-38]$$

2.11.1.3 Model 3

Another model based on deformation mechanics approach has been analysed. The following model was proposed by Madsen et al. [82]. They have used a two degree of freedom spring mass model to calculate natural frequencies of plate and impactor to derive a contact force function. Experimental parameters used in the model are shown in Table 3.

Striker	Target
Mass-small	Mass-large
Material-steel	Material- Graphite/epoxy
Geometry-hemispherical	Geometry-square plate
Striker incidence-normal	Thickness-thin
Velocity-low	Ply layers- $[\pm 18/90_4/\mp 18]$
	Boundary conditions- clamped-free

Table 3 Experimental parameters [82].

2.11.1.3.1 Analysis

A two degrees of freedom model consisting of a two-mass, two-spring system is considered using a mass m_1 to model the striker and m_2 to model the target. The analytical model is shown in Figure 26.

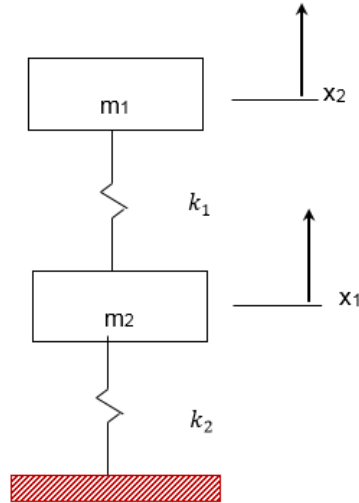


Figure 26 Spring-Mass model

The equations of motion for the two-mass-two-spring system is expressed as:

$$k_1(x_2 - x_1) = m_1\ddot{x}_1 \quad [2-39]$$

$$-k_1(x_2 - x_1) - k_2x_2 = m_2\ddot{x}_2 \quad [2-40]$$

In the above equations, the subscript 1 refers to the striker and subscript 2 refers to the target.

The initial conditions are defined by:

$$x_1(0) = x_2(0) = 0 \quad [2-41]$$

$$\dot{x}_1(0) = v_1\dot{x}_2(0) = 0 \quad [2-42]$$

A solution to this problem is derived as:

$$x_1(t) = v_1(A_I \sin \omega_I t + A_{II} \sin \omega_{II} t) \quad [2-43]$$

$$\dot{x}_1(t) = v_1(X_I A_I \sin \omega_I t + X_{II} A_{II} \sin \omega_{II} t) \quad [2-44]$$

The amplitudes are:

$$A_I = 1/(1 - X_I/X_{II})\omega_I \quad [2-45]$$

$$A_{II} = 1/(1 - X_{II}/X_I)\omega_{II} \quad [2-46]$$

The mode shapes are given by:

$$X_I = 1 - (m_1/k_1)\omega_I^2 \quad [2-47]$$

$$X_{II} = 1 - (m_1/k_1)\omega_{II}^2 \quad [2-48]$$

The frequencies ω_I and ω_{II} are defined by:

$$\omega^2 = \frac{(k_1+k_2)m_1+k_1m_2 \pm \sqrt{[(k_1+k_2)m_1+k_1m_2]^2 - 4k_1k_2m_1m_2}}{2m_1m_2} \quad [2-49]$$

The expression for the contact force function is:

$$F(t) = k_1(x_2 - x_1) = -m_1v_1(\omega_I^2 A_I \sin \omega_I t + \omega_{II}^2 A_{II} \sin \omega_{II} t) \quad [2-50]$$

It is observed that the mode shapes and frequencies are independent of the impact velocity. The impact velocity only affects the displacement and the contact force histories.

- Case 1: $\omega_1/\omega_2 \ll 1$

This scenario depicts a large striker mass impact. $\omega_1/\omega_2 \ll 1$ implies that

$$m_1 \gg (k_1/k_2) \quad [2-51]$$

The mass of the striker is large, therefore the equivalent velocity is low and therefore case 1 is classified as a low velocity impact. By expanding the solutions $x_1(t)$ and $x_2(t)$ in powers of ω_1/ω_2 and by retaining lower order terms, the impact response can be expressed as:

$$x_1(t) = \sqrt{\frac{m_1 v_1^2}{K^*}} \sin \sqrt{\frac{K^*}{m_1}} t \quad [2-52]$$

$$x_2(t) = \left(\frac{k_1}{k_1 + k_2} \right) \sqrt{\frac{m_1 v_1^2}{K^*}} \sin \sqrt{\frac{K^*}{m_1}} t \quad [2-53]$$

Similarly, the contact force $F(t)$ is defined as:

$$F(t) = -\sqrt{K^* m_1 v_1^2} \sin \sqrt{\frac{K^*}{m_1}} t \quad [2-54]$$

Where the equivalent contact stiffness K^* is defined by:

$$K^* = \frac{k_1 k_2}{k_1 + k_2} \quad [2-55]$$

2.12 Plate theories

This section of the literature review focusses on analysing the different plate theories that have been used in literature to solve low velocity impact problems of composite plates. Similarly with the dynamic behaviour of beams, shear deformation and rotary inertia are expected to have similar effects on the behaviour of plates, particularly for ones made of composite materials [83]. There are mainly three plate theories associated as first-order shear deformation theory (FSDT), classical plate theory (CPT) and higher order plate theory. All the equations listed in this sections 2.12.1-2.12.2 are obtained through derivations performed by Abrate [83].

2.12.1 First order shear deformation theory (FSDT)

The first order shear deformation theory proceeds similarly to the Timoshenko beam theory by assuming that the transverse normal strain ϵ_{zz} is negligible and the two transverse shear strains are constant through-the-thickness. The two assumptions are expressed as:

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} = 0 \quad [2-56]$$

$$\varepsilon_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \gamma_1 \quad [2-57]$$

$$\varepsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \gamma_2 \quad [2-58]$$

Suggest that the displacements are of the form

$$u(x, y, z) = u_0(x, y) + z(\gamma_1 - w_{0,x}) \quad [2-59]$$

$$v(x, y, z) = v_0(x, y) + z(\gamma_2 - w_{0,y}) \quad [2-60]$$

$$w(x, y, z) = w_0(x, y) \quad [2-61]$$

By changing the variables

$$\psi_x = \gamma_1 - w_{0,x}, \quad \psi_y = \gamma_2 - w_{0,y} \quad [2-62]$$

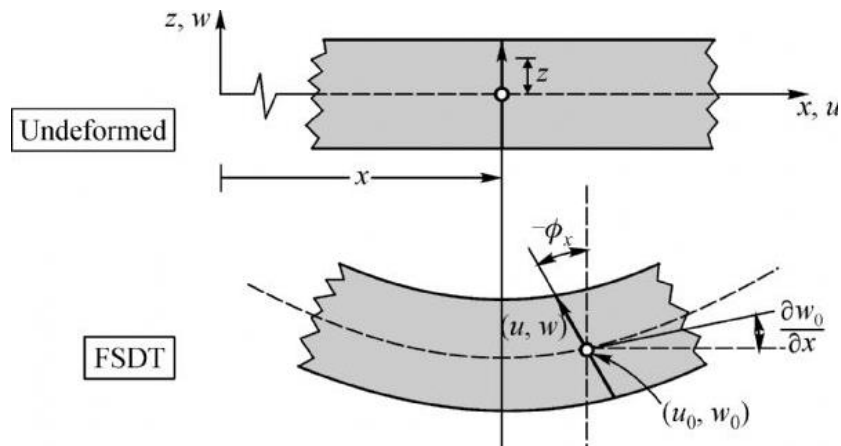


Figure 27 Deformation of a transverse normal [148]

The displacement field of the first order shear deformation theory can be obtained

$$u(x, y, z) = u_0(x, y) + z\psi_x \quad [2-63]$$

$$v(x, y, z) = v_0(x, y) + z\psi_y \quad [2-64]$$

$$w(x, y, z) = w_0(x, y) \quad [2-65]$$

where u_0 , v_0 , w_0 are the displacement components of a point on the midplane. ψ_x and ψ_y are the rotations of the normal to the midplane about the y and x-axis respectively.

The infinitesimal strain components are defined as:

$$\epsilon_{xx} = \frac{\partial u}{\partial x}, \quad \epsilon_{yy} = \frac{\partial v}{\partial y}, \quad \epsilon_{zz} = \frac{\partial w}{\partial z} \quad [2-66]$$

$$\epsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \epsilon_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \epsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad [2-67]$$

can be estimated from

$$\epsilon_{xx} = \frac{\partial u_0}{\partial x} + z \frac{\partial \psi_x}{\partial x}, \quad \epsilon_{yy} = \frac{\partial v_0}{\partial y} + z \frac{\partial \psi_y}{\partial y}, \quad [2-68]$$

$$\epsilon_{zz} = 0, \quad \epsilon_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z \left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right), \quad [2-69]$$

$$\epsilon_{xz} = \frac{\partial w_0}{\partial x} + \psi_x, \quad \epsilon_{yz} = \frac{\partial w_0}{\partial y} + \psi_y \quad [2-70]$$

A distributed load, q is applied over the surface of the plate, normal in-plane force resultants $\hat{N}_n$ over a part of the boundary Γ_1 , $\hat{N}_s$ over Γ_2 , moment resultants $\hat{M}_n$ and $\hat{M}_s$ over Γ_3 and Γ_4 respectively, and transverse shear forces, $\hat{Q}_n$ over Γ_5 . The in-plane force resultants are defined as

$$(N_x, N_y, N_{xy}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}) dz; \quad [2-71]$$

The bending moments expressed as

$$(M_x, M_y, M_{xy}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}) z dz; \quad [2-72]$$

And shear force resultants as

$$(Q_x, Q_{xy}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xz}, \sigma_{yz}) dz \quad [2-73]$$

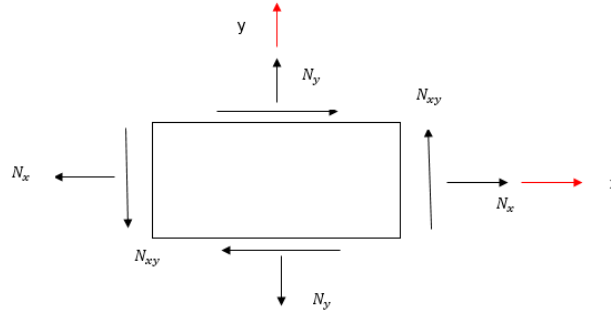


Figure 28 Force Resultants

The force resultants acting on a small plate element are shown in Figure 28. Along a line oriented by the unit normal vector $\bar{n}$, the in-plane force and moment resultants in relation to the x and y-components are expressed as:

$$N_n = N_x n_x^2 + N_y n_y^2 + 2n_x n_y N_{xy} \quad [2-74]$$

$$N_s = (N_y - N_x) n_x n_y + N_{xy} (n_x^2 - n_y^2) \quad [2-75]$$

$$M_n = M_x n_x^2 + M_y n_y^2 + 2n_x n_y M_{xy} \quad [2-76]$$

$$M_s = (M_y - M_x) n_x n_y + M_{xy} (n_x^2 - n_y^2) \quad [2-77]$$

The strain energy in a solid is defined as:

$$U = \frac{1}{2} \int_V (\sigma_{xx} \epsilon_{xx} + \sigma_{yy} \epsilon_{yy} + \sigma_{zz} \epsilon_{zz} + \sigma_{xy} \epsilon_{xy} + \sigma_{xz} \epsilon_{xz} + \sigma_{yz} \epsilon_{yz}) dV \quad [2-78]$$

Using the strain-displacement relationships derived earlier and substituting in above equation

$$U = \frac{1}{2} \int_{\Omega} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \sigma_{xx} \frac{\partial u_0}{\partial x} + \sigma_{yy} \frac{\partial v_0}{\partial y} + \sigma_{xy} \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right) + z \left[\sigma_{xx} \frac{\partial \psi_x}{\partial x} + \sigma_{yy} \frac{\partial \psi_y}{\partial y} + \sigma_{xy} \left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right) \right] + \sigma_{xz} \left(\frac{\partial w_0}{\partial y} + \psi_y \right) \right\} dz d\Omega \quad [2-79]$$

The strain energy expression is obtained by integrating over the thickness and using the force and moment resultants derived earlier in equations (2-75)-(2-78).

$$U = \frac{1}{2} \int_{\Omega} \left\{ N_x \frac{\partial u_0}{\partial x} + N_y \frac{\partial v_0}{\partial y} + N_{xy} \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right) + M_x \frac{\partial \psi_x}{\partial x} + M_y \frac{\partial \psi_y}{\partial y} + M_{x,y} \left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right) + Q_x \left(\frac{\partial w_0}{\partial x} + \psi_x \right) + Q_y \left(\frac{\partial w_0}{\partial y} + \psi_y \right) \right\} d\Omega \quad [2-80]$$

With the notation

$$\{N\} = [N_x, N_y, N_{xy}]^T \quad [2-81]$$

$$\{M\} = [M_x, M_y, M_{xy}]^T \quad [2-82]$$

$$\{\epsilon^0\} = [\epsilon_{xx}^0, \epsilon_{yy}^0, \epsilon_{xz}^0]^T = \left[\frac{\partial u_0}{\partial x}, \frac{\partial v_0}{\partial y}, \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right]^T \quad [2-83]$$

$$\{K\} = [K_x, K_y, K_{xy}]^T = \left[\frac{\partial \psi_x}{\partial x}, \frac{\partial \psi_y}{\partial y}, \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right]^T \quad [2-84]$$

Where by incorporating midplane strains by ϵ^0 and plate curvatures $\{K\}$, the strain energy can be rewritten as

$$U = \frac{1}{2} \int_{\Omega} \left[\{N\}^T \{\epsilon^0\} + \{M\}^T \{K\} + [Q_x, Q_y] \begin{Bmatrix} \epsilon_{xz} \\ \epsilon_{yz} \end{Bmatrix} \right] dV \quad [2-85]$$

The potential energy of the external forces are expressed as

$$V = - \int_{\Omega} q w_0 d\Omega - \int_{\Gamma} \bar{N}_n u_{n0} d\Gamma - \int_{\Gamma_2} \hat{N}_s u_{s0} d\Gamma - \int_{\Gamma_3} \hat{M}_n \psi_n d\Gamma - \int_{\Gamma_4} \hat{M}_s \psi_n d\Gamma - \int_{\Gamma_5} \hat{Q}_n w_0 d\Gamma \quad [2-86]$$

The kinetic energy of the plate is defined as:

$$T = \frac{1}{2} \int_V \rho [(\dot{u})^2 + (\dot{v})^2 + (\dot{w})^2] dV \quad [2-87]$$

By using displacement fields of FSDT, kinetic energy can be expressed as

$$T = \frac{1}{2} \int_{\Omega} \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho \left[(\dot{u}_0 + z\dot{\psi}_x)^2 + (\dot{v}_0 + z\dot{\psi}_y + (\dot{w}_0)^2) \right] dz d\Omega \quad [2-88]$$

Which can be written as

$$T = \frac{1}{2} \int_{\Omega} \{ I_1 (\dot{u}_0^2 + \dot{v}_0^2 + \dot{w}_0^2) + 2I_2 (\dot{u}_0 \dot{\psi}_x + \dot{v}_0 \dot{\psi}_y) + I_3 (\dot{\psi}_x^2 + \dot{\psi}_y^2) \} d\Omega \quad [2-89]$$

The inertia terms are expressed as:

$$(I_1, I_2, I_3) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho (1, z, z^2) dz \quad [2-90]$$

I_1 is the mass per unit length, I_3 is known as the rotary inertia, and I_2 is a coupling term that is zero when the plate is symmetric about the x-y plane.

For a single layer of orthotropic material, the stress-strain relations are expressed as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{Bmatrix} \quad [2-91]$$

The strain-displacement relations can be expressed as

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \psi_x}{\partial x} \\ \frac{\partial \psi_y}{\partial y} \\ \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \end{Bmatrix} \quad [2-92]$$

or

$$\{\epsilon\} = \{\epsilon_0\} + z\{\kappa\} \quad [2-93]$$

Where $\{\epsilon_0\}$ is the midplane strain vector and $\{\kappa\}$ denotes the plate curvatures. By using the definitions of in-plane forces, bending moments and shear force resultants, we find

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_y \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad [2-94]$$

Where

$$A_{ij} = \sum Q_{ij|k}(z_k - z_{k-1}) \quad [2-95]$$

$$B_{ij} = \frac{1}{2} \sum Q_{ij|k}(z_k^2 - z_{k-1}^2) \quad [2-96]$$

$$D_{ij} = \frac{1}{3} \sum Q_{ij|k}(z_k^3 - z_{k-1}^3) \quad [2-97]$$

The extensional-bending coupling is shown in above equation in the case of a general laminate. This means that both in-plane and out-of-plane deformations are produced only as a result of the application of in-plane forces. This coupling is zero when a symmetric laminate is considered. Therefore, the B matrix is zero.[45]

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \psi_y + \frac{\partial w_0}{\partial y} \\ \psi_x + \frac{\partial w_0}{\partial x} \end{Bmatrix} \quad [2-98]$$

$$A_{ij} = k_i k_j \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij} dz, \quad i, j = 4, 5. \quad [2-99]$$

k_i and k_j are two shear correction factors used in accounting for the non-uniformity of the distribution of shear stress through-the-thickness. k_i and k_j are usually considered to be 5/6.

The equations of motion in terms of force and moment resultants are derived by the Hamilton's principle:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1 \ddot{u}_0 + I_2 \ddot{\psi}_x \quad [2-100]$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_1 \ddot{v}_0 + I_2 \ddot{\psi}_y \quad [2-101]$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = I_1 \ddot{w}_0 \quad [2-102]$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_3 \ddot{\psi}_x + I_2 \ddot{u}_0 \quad [2-103]$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = I_3 \ddot{\psi}_y + I_2 \ddot{v}_0 \quad [2-104]$$

Equations of motion for a symmetric laminate ($[B] = 0$) is presented as:

$$\frac{\partial}{\partial x} \left[A_{45} \left(\psi_y + \frac{\partial w_0}{\partial y} \right) + A_{55} \left(\psi_x + \frac{\partial w_0}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[A_{44} \left(\psi_y + \frac{\partial w_0}{\partial y} \right) + A_{45} \left(\psi_x + \frac{\partial w_0}{\partial x} \right) \right] + q = I_1 \ddot{w}_0 \quad [2-105]$$

$$\frac{\partial}{\partial x} \left[D_{11} \frac{\partial \psi_x}{\partial x} + D_{12} \frac{\partial \psi_y}{\partial y} + D_{16} \left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[D_{16} \frac{\partial \psi_x}{\partial x} + D_{26} \frac{\partial \psi_y}{\partial y} + D_{66} \left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right) \right] - A_{45} \left(\psi_y + \frac{\partial w_0}{\partial y} \right) - A_{55} \left(\psi_x + \frac{\partial w_0}{\partial x} \right) = I_3 \ddot{\psi}_x \quad [2-106]$$

$$\frac{\partial}{\partial x} \left[D_{16} \frac{\partial \psi_x}{\partial x} + D_{26} \frac{\partial \psi_y}{\partial y} + D_{66} \left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[D_{12} \frac{\partial \psi_x}{\partial x} + D_{22} \frac{\partial \psi_y}{\partial y} + D_{26} \left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \right) \right] - A_{44} \left(\psi_y + \frac{\partial w_0}{\partial y} \right) - A_{45} \left(\psi_x + \frac{\partial w_0}{\partial x} \right) = I_3 \ddot{\psi}_y \quad [2-107]$$

2.12.2 Classical plate theory (CPT)

In the classical plate theory, the effect of shear deformation and rotary inertia are negligible along with the transverse normal strain. Therefore, γ_1 and γ_2 in equations are zero. According to equation [2-63]:

$$\frac{\partial w_0}{\partial x} + \psi_x = 0 \quad [2-108]$$

$$\frac{\partial w_0}{\partial y} + \psi_y = 0 \quad [2-109]$$

Since rotary inertias are zero, functions ψ_x and ψ_y are eliminated from the formulation and therefore the displacements are expressed as:

$$u(x, y, z) = u_0(x, y) - z \frac{\partial w_0}{\partial x}(x, y) \quad [2-110]$$

$$v(x, y, z) = v_0(x, y) - z \frac{\partial w_0}{\partial y}(x, y) \quad [2-111]$$

$$w(x, y, z) = w_0(x, y) \quad [2-112]$$

It is assumed that line segments normal to mid-plane before deformation remain straight and perpendicular to the mid-surface during deformation. Therefore, the mid-plane strains are unchanged and the plate curvatures take the form:

$$\{K\} = \left[-\frac{\partial^2 w}{\partial x^2}, -\frac{\partial^2 w}{\partial y^2}, -2\frac{\partial^2 w}{\partial x \partial y} \right]^T \quad [2-113]$$

Substituting into equation (2-104) and (2-105) and combination of these two equations results in the following equation in terms of moment resultants:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + q - I_1 \ddot{w}_0 = I_3 \left\{ -\frac{\partial^2 \ddot{w}_0}{\partial x^2} - \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right\} + I_2 \left\{ \frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right\} \quad [2-114]$$

The rotary inertia, I_3 is neglected since the effect of shear deformation has been neglected and it has been shown that the effect of rotary inertia for beams is negligible. $I_2 = 0, B_{ij} = 0$ for a symmetric laminate and the transverse motion is decoupled from the in-plane motion. Thus, the transverse displacements must satisfy a single equation of motion. The equation for symmetrically laminated plates according to the classical plate theory in terms of transverse displacement w is expressed as:

$$q = D_{11} \frac{\partial^4 w_0}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w_0}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w_0}{\partial x^2 \partial y^2} + 4D_{16} \frac{\partial^4 w_0}{\partial x^3 \partial y} + 4D_{26} \frac{\partial^4 w_0}{\partial x \partial y^3} + I_1 \ddot{w}_0 \quad [2-115]$$

2.13 Shell theory

Basic problems related with impact on composites can be studied on flat specimens, however in many applications curved panels are used and therefore mathematical models analysing effects on impacts on these structures must be evaluated. This section presents a brief review of the shell theory and a solution for the dynamic response of a simply supported plate is presented. Relatively few studies have analysed the impact modelling of composite shells. [83]

The Love's equations of motion for a cylindrical shell of length L , radius R and thickness h subjected to external loads are defined as [44]:

$$\frac{\partial N_x}{\partial x} + \frac{1}{R} \frac{\partial N_{x\theta}}{\partial \theta} + q_x(x, \theta, t) = \rho h \ddot{u} \quad [2-117]$$

$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R} \frac{\partial N_\theta}{\partial \theta} + \frac{Q_\theta}{R} + q_\theta(x, \theta, t) = \rho h \ddot{v} \quad [2-118]$$

$$\frac{\partial Q_x}{\partial x} + \frac{1}{R} \frac{\partial Q_\theta}{\partial \theta} - \frac{N_\theta}{R} + q_n(x, \theta, t) = \rho h \ddot{w} \quad [2-119]$$

$$\frac{\partial M_x}{\partial x} + \frac{1}{R} \frac{\partial M_{x\theta}}{\partial \theta} - Q_x = \frac{\rho h^3}{12} \ddot{\beta}_x \quad [2-120]$$

$$\frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R} \frac{\partial M_\theta}{\partial \theta} - Q_\theta = \frac{\rho h^3}{12} \ddot{\beta}_\theta \quad [2-121]$$

a dot in the above equations denotes differentiation with respect to time. For a specially orthotropic ply, the constitutive equations are defined by:

$$\begin{Bmatrix} N_x \\ N_\theta \\ N_{x\theta} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_\theta^0 \\ \gamma_{x\theta}^0 \end{Bmatrix} \quad [2-122]$$

$$\begin{Bmatrix} M_x \\ M_\theta \\ M_{x\theta} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_\theta \\ \kappa_{x\theta} \end{Bmatrix} \quad [2-123]$$

$$\begin{Bmatrix} Q_x \\ Q_\theta \end{Bmatrix} = \begin{bmatrix} kA_{55} & 0 \\ 0 & kA_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{\theta z}^0 \end{Bmatrix} \quad [2-124]$$

Where

$$(A_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \bar{Q}_{ij} (1, z^2) dz \quad (i, j = 1, 2, 6) \quad [2-125]$$

$$A_{ii} = \int_{-h/2}^{h/2} \bar{C}_{ii} dz \quad (i, i = 4, 5) \quad [2-126]$$

$\bar{Q}_{ij}$ denotes the reduced transformed stiffnesses in $x - \theta$ plane, $\bar{C}_{ii}$ are the transformed shear stiffnesses, and k is the Mindlin shear factor which is $\pi^2/12$. The relationship for the strain-displacement are defined by:

$$\varepsilon_x^0 = \frac{\partial u}{\partial x} \quad \varepsilon_\theta^0 = \frac{1}{R} \frac{\partial u}{\partial \theta} + \frac{w}{R} \quad \gamma_{x\theta}^0 = \frac{\partial u}{\partial x} + \frac{1}{R} \frac{\partial u}{\partial \theta} \quad [2-127]$$

$$\kappa_x = \frac{\partial \beta_x}{\partial x} \quad \kappa_\theta = \frac{1}{R} \frac{\partial \beta_\theta}{\partial \theta} \quad \kappa_{x\theta} = \frac{\partial \beta_\theta}{\partial x} + \frac{1}{R} \frac{\partial \beta_x}{\partial \theta} \quad [2-128]$$

$$\gamma_{xz}^0 = \beta_x + \frac{\partial w}{\partial x} \quad \gamma_{\theta z}^0 = \beta_\theta + \frac{1}{R} \frac{\partial w}{\partial \theta} - \frac{v}{R} \quad [2-129]$$

The boundary conditions are

$$N_x = v = w = M_x = \beta_\theta = 0 \quad \text{at} \quad x = \pm \frac{L}{2} \quad [2-130]$$

$$N_\theta = u = w = M_\theta = \beta_x = 0 \quad \text{at} \quad \theta = \pm \frac{\alpha}{2} \quad [2-131]$$

Double Fourier series expansion is used to represent the loading $q_n(x, \theta, t)$

$$q_n(x, \theta, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Q_{mn}(t) \cos \frac{m\pi x}{L} \cos \frac{n\pi \theta}{\alpha} \quad [2-132]$$

The Fourier coefficients are evaluated from:

$$Q_{mn}(t) = \frac{4}{\alpha L} \int_{-L/2}^{L/2} \int_{-\alpha/2}^{\alpha/2} q_n(x, \theta, t) \cos \frac{m\pi x}{L} \cos \frac{n\pi \theta}{\alpha} dx d\theta \quad m, n = 1, 3, 5, \dots \quad [2-133]$$

When a concentrated load $F(t)$ acts at a location (x_0, θ_0) , $Q_{mn}(t)$ can be expressed as:

$$Q_{mn}(t) = \frac{4F(t)}{L\alpha R} \cos \frac{m\pi x_0}{L} \cos \frac{n\pi \theta_0}{\alpha} \quad [2-134]$$

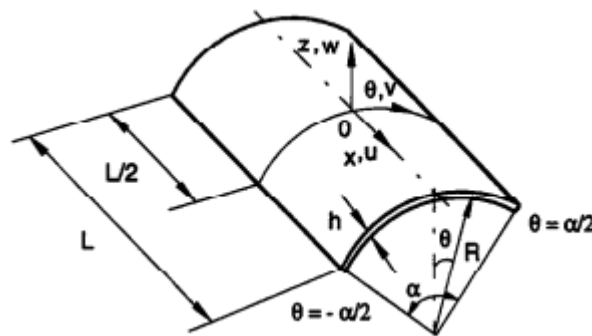


Figure 29 Cylindrical shell [83]

2.14 Low Velocity impact modelling of curved sandwich plates

2.14.1 Introduction

Sandwich plates consists of a core which is covered by facesheets and are more frequently used than solid plates due to their high bending stiffness-to-weight ratio. This high bending stiffness is mainly due to the distance between the facesheets, which carry the load, and the contributing factor for the light weight is due to the light weight of the core. [89]

Rectangular sandwich plates with facesheets on both sides of the core are considered. The facesheets may be an isotropic material or a fibre reinforced composite laminate, but must be thin when compared with the core. The core may be manufactured from foam or honeycomb and must have a material symmetry plane parallel to its midplane. (Figures 90, 91). The in-plane stiffness of the core must be small compared with the in-plane stiffness of the facesheets.

Kirchhoff hypothesis is used to analyse the behaviour of thin plates undergoing small deformations by assuming that normal remain straight and perpendicular to the deformed reference plane [84]. The first assumption of normals remaining straight is reasonable for a sandwich plate consisting of a core covered on both sides by facesheets. However, the second assumption may no longer be valid since normals do not necessarily remain perpendicular to the reference plane. Therefore, the x and y displacements of a point load located at a distance z from a randomly chosen reference plane are:

$$u = u^0 - z\chi_{xz} \quad v = v^0 - z\chi_{yz} \quad [2-135]$$

where u^0 and v^0 are the x and y displacements at the reference plane (where $z = 0$) and χ_{xz} , χ_{yz} are the rotations of the normal in the $x - z$ and $y - z$ planes as shown in Figure 32 . The first derivative of the deflection w^0 of the reference plane with respect to x is

$$\frac{\partial \omega^0}{\partial x} = \chi_{xz} + \gamma_{xz} \quad [2-136]$$

Similarly, the first derivative of the deflection w^0 of the reference plane with respect to y is:

$$\frac{\partial \omega^0}{\partial y} = \chi_{yz} + \gamma_{yz} \quad [2-137]$$

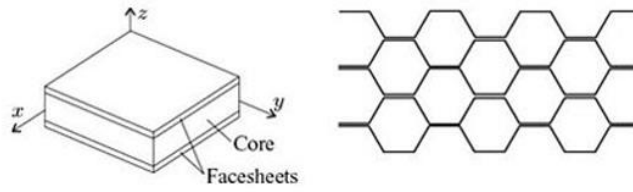


Figure 30 Sandwich Panel and Honeycomb core [90]

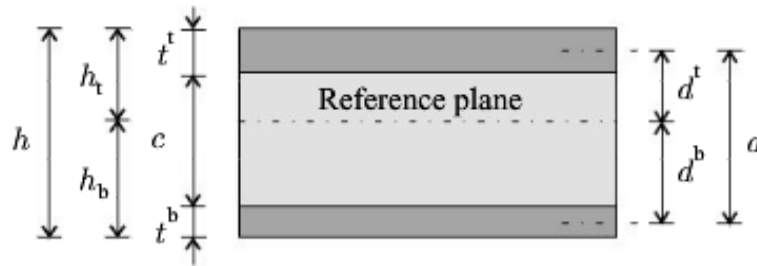


Figure 31 Sandwich Panel Geometry [90]

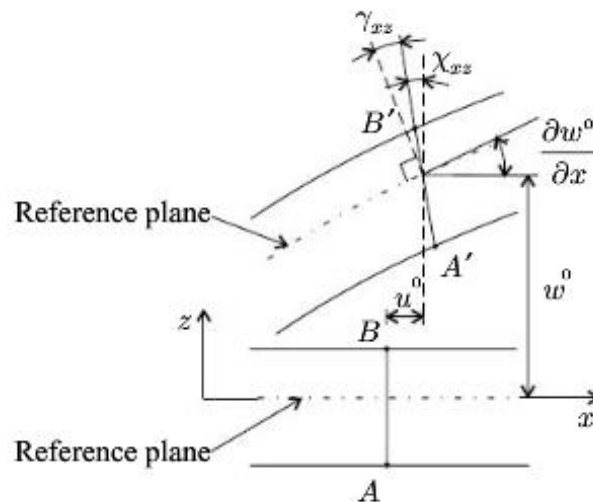


Figure 32 Deformation of sandwich plate in x-z plane [90]

2.15 Factors affecting low velocity impact on sandwich structures

This section involves the discussion of the various parameters affecting the impact response of sandwich structures. This analysis results in identifying the factors controlling the impact response which would be applied later in the actual modelling process of the sandwich structures [85].

2.15.1 Impact Mass

The mass of the impactor dominates the impact response. Larger deflections and longer impact durations are caused by larger impact masses. [86]. Researchers have identified that the mass ratio, $\bar{m}$ is a major parameter in identifying the type of impact response. i.e. large mass or small mass impact. [87]

2.15.2 Impact velocity

The extent of the impact velocity should be measured in conjunction with the impact mass [86,88]. In a low velocity impact, even though the impact velocity does not dominate the impact response, it would have an effect on the contact force and duration [88,89]. The maximum contact force is found to be proportional to the impact velocity [90]. Studies have proved that an increase of impact velocity on sandwich structures have a similar effect on the impact force and deflection similarly to composite laminate designs, but have negligible effect on the impact duration.

2.15.3 Impactor Geometry

The geometry of the impactor is a major parameter that governs the contact response between the impactor and the top facesheet of the sandwich structure. This effect on the impactor's shape on the impact response and failure mechanisms of sandwich panels have been studied extensively through experiments with both hemispherical and flat-ended indentors. [90–93] . It was found through the literature review that most research studied the effect of spherical impactors on the impact response [94–96]. Raju and Tomblin [96] and Raju and Smith [97] through experimental testing on $[(0/45)_n/core/(45/0)_n]$ sandwich plates with different radii discussed the effect of indenter size on the damage initiation, damage propagation and modes of damage. They proved that indentors with smaller radii mostly produces facesheet damage in

terms of matrix cracking and fibre fractures, whereas indentors with larger radii produced core crushing damage mechanism.

2.15.4 Face-sheet thickness

The thickness of the face-sheet plays a vital role in the impact response of a sandwich structure. The thicker plates result in a higher contact force and smaller central displacement when compared to thin plates subjected to a constant impact energy. However, the impact duration is less with thicker plates [98,99]. Experimental tests carried out on cross-ply skins with plies varying from 8 to 16 depicted that both the threshold load and initial slope are sensitive to skin thickness.

2.15.5 Boundary conditions

The impact response has been proved to be very subtle to boundary conditions for large mass impact, while not much for small mass impact. Published results from theoretical and numerical investigations shows that sandwich structures with larger structural stiffnesses result in larger contact forces and small contact durations. [98]

2.15.6 Environmental conditions

The effect of temperature and moisture on the impact response of sandwich structures has been evaluated experimentally [100,101]. The results proved that temperature had a major effect on the energy absorption while having a relatively lesser effect on the flexural stiffness. The maximum impact force has been found to decrease when the temperature is increased from 25°C to 75°C.

2.15.7 Impact angle

Most of the literature reviewed during this study focusses on the effect of direct impact. i.e. the impact velocity direction is perpendicular to the target surface. However, in practical scenarios this is not the case as the angle of impact could vary. Therefore, oblique impact on laminates and other composites have been evaluated theoretically and experimentally [102–104]. Zhou and Stronge [105] investigated the effect of ballistic speed limit on sandwich panels. They found out that the impactor must have a minimum ballistic speed limit at an oblique angle between 30° and 45° from the normal direction, in order for both spherical and flat ended impactors to be able to

penetrate thin metallic plates. A comparison between oblique and direct impacts depicted that laminated composite plates absorb more energy during oblique impacts.

2.16 Damage mechanisms of composite sandwich structures

This section focuses on the review done on the damage evolution, failure modes and energy absorption of composite sandwich structures.

2.16.1 Energy absorption during an impact

The total energy absorbed during an impact event by a sandwich structure consist of the following [106,107].

- The strain energy absorbed by the sandwich
- Kinetic energy of the impactor and the sandwich
- Absorbed strain energy of the impactor, but is usually ignored since the impactor is considered to be rigid.
- Fracture energy of the face-sheet.
- Energy for the buckling and crushing of the core
- Energy required to cause deboning between facesheet and the core
- Energy lost to friction as heat, sound, friction, etc. which is usually ignored since this is negligible.

2.16.2 Damage progression during impact of sandwich structures

Even though various experimental testing involving ultrasonic inspection, X-rays, thermography, etc. are available to distinguish the various forms of damage after impact [97] studies on damage progression on composite sandwich structures are somewhat limited. In the event of a low velocity impact damage on a sandwich panel, five different failure modes were identified [108]

- Buckling of the core and debonding
- Shearing of the core and cracking
- Top facesheet delamination
- Matrix cracking of the facesheet
- Fibre breakage in the facing

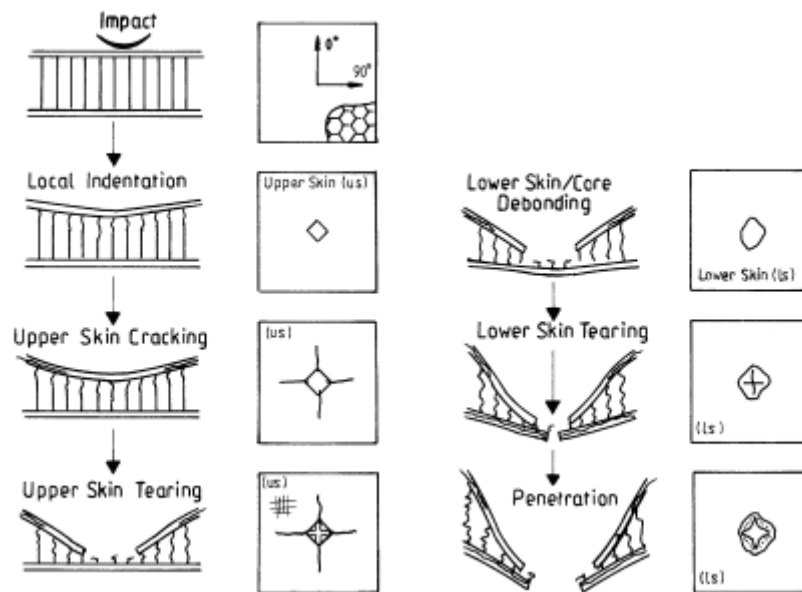


Figure 33 Impact event on a sandwich panel [108]

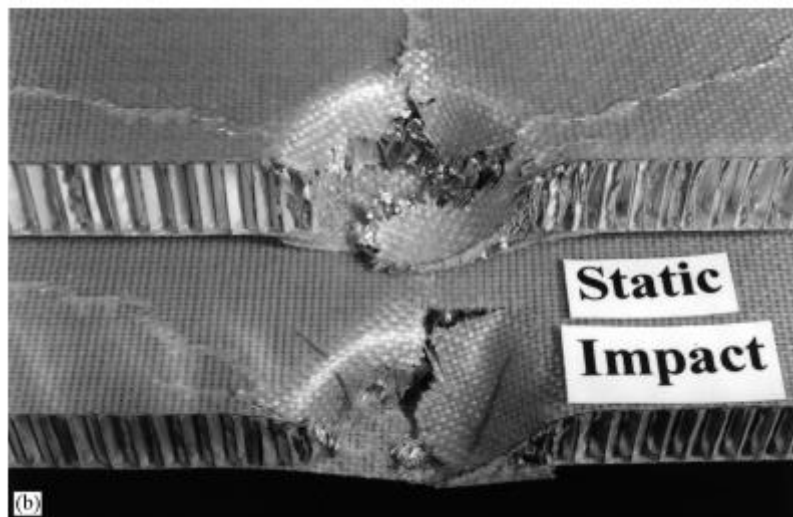


Figure 34 Failure modes of a sandwich panel [108]

2.17 Governing Equations of a sandwich plate

Governing equations of the sandwich panel are obtained from reference [84]

The strains at the reference plane are expressed as:

$$\epsilon_x^0 = \frac{\partial u^0}{\partial x} \quad \epsilon_y^0 = \frac{\partial v^0}{\partial y} \quad \gamma_{xy}^0 = \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} \quad [2-138]$$

The transverse shear strains are:

$$\gamma_{xz} = \frac{\partial \omega^0}{\partial x} - \chi_{xz} \quad \gamma_{yz} = \frac{\partial \omega^0}{\partial y} - \chi_{yz} \quad [2-139]$$

The reference plane curvatures are expressed as:

$$\kappa_x = -\frac{\partial \chi_{xz}}{\partial x} \quad \kappa_y = -\frac{\partial \chi_{yz}}{\partial y} \quad \kappa_{xy} = -\frac{\partial \chi_{xz}}{\partial y} - \frac{\partial \chi_{yz}}{\partial x} \quad [2-140]$$

The three equations above denotes the strain-displacement relationship for a sandwich plate.

The next step is to derive the force-strain relationships. The expressions for the forces and moments are given by

$$N_x = \int_{-h_b}^{h_t} \sigma_x dz \quad N_y = \int_{-h_b}^{h_t} \sigma_y dz \quad N_{xy} = \int_{-h_b}^{h_t} \tau_{xy} dz \quad [2-141]$$

$$M_x = \int_{-h_b}^{h_t} z \sigma_x dz \quad M_y = \int_{-h_b}^{h_t} z \sigma_y dz \quad M_{xy} = \int_{-h_b}^{h_t} z \tau_{xy} dz \quad [2-142]$$

Where N_i , M_i and V_i are the in-plane forces, moments, and the transverse shear forces per unit length. The stresses are:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad [2-143]$$

The strains at a distance z from the reference plane are:

$$\epsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u^0}{\partial x} - z \frac{\partial \chi_{xz}}{\partial x} \quad [2-144]$$

$$\epsilon_y = \frac{\partial v}{\partial y} = \frac{\partial v^0}{\partial y} - z \frac{\partial \chi_{yz}}{\partial y} \quad [2-145]$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} - z \left(\frac{\partial \chi_{xz}}{\partial y} + \frac{\partial \chi_{yz}}{\partial x} \right) \quad [2-146]$$

The force and moment matrices through combination of the strains and definitions of [A], [B], [D] matrices are:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = [A] \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [B] \begin{Bmatrix} -\frac{\partial \chi_{xz}}{\partial x} \\ -\frac{\partial \chi_{yz}}{\partial y} \\ -\frac{\partial \chi_{xz}}{\partial y} - \frac{\partial \chi_{yz}}{\partial x} \end{Bmatrix} \quad [2-147]$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = [B] \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [D] \begin{Bmatrix} -\frac{\partial \chi_{xz}}{\partial x} \\ -\frac{\partial \chi_{yz}}{\partial y} \\ -\frac{\partial \chi_{xz}}{\partial y} - \frac{\partial \chi_{yz}}{\partial x} \end{Bmatrix} \quad [2-148]$$

These expressions can be re-written as:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = [A] \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [B] \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad [2-149]$$

The relationship between the transverse shear forces and transverse shear strains are expressed as:

$$\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} = \begin{bmatrix} \tilde{S}_{11} & \tilde{S}_{12} \\ \tilde{S}_{12} & \tilde{S}_{22} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad [2-150]$$

where $[\tilde{S}]$ is the shear stiffness matrix of the sandwich plate.

2.17.1 Strain Energy

Solutions to plate problems may be obtained through the governing equations through energy methods. The expression for the strain energy of a sandwich plate when the thickness of the sandwich plate assumed to remain unchanged, $\epsilon_z = 0$ simplifies to

$$U = \frac{1}{2} \int_0^{L_x} \int_0^{L_y} \int_{-h_b}^{h_t} (\sigma_x \epsilon_x + \sigma_y \epsilon_y + \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz} + \tau_{yz} \gamma_{yz}) dz dy dx \quad [2-151]$$

Substituting earlier expressions into strain energy equation gives:

$$U = \frac{1}{2} \int_0^{L_x} \int_0^{L_y} \left\{ \begin{matrix} \begin{matrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \tau_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{matrix}^T \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \\ \{Y_{xz} \quad Y_{yz}\} \begin{bmatrix} \tilde{S}_{11} & \tilde{S}_{12} \\ \tilde{S}_{12} & \tilde{S}_{22} \end{bmatrix} \begin{Bmatrix} Y_{xz} \\ Y_{yz} \end{Bmatrix} \end{matrix} \right\} dydx \quad [2-152]$$

2.17.2 Stiffness Matrices of Sandwich Plates

The stiffness matrices are evaluated by assuming that the thickness of the core remains constant under loading and the in-plane stiffnesses of the core are negligible [84]. Under these assumptions the [A], [B], and [D] stiffness matrices of a sandwich panel are governed by the stiffnesses of the face-sheets are obtained through the parallel axes theorem. The resulting expressions are shown below in Table 10.

Layup of each face-sheet with respect to the face-sheet's mid-plane		
	Variable stiffness facesheets	Symmetrical (identical facesheets)
$[A_{sand}]$	$[A]^t + [A]^b$	$2 [A]^t$
$[B_{sand}]$	$d^t [A]^t - d^b [A]^b + [B]^t + [B]^b$	0
$[D_{sand}]$	$(d^t)^2 [A]^t + (d^b)^2 [A]^b + [D]^t + [D]^b + 2d^t [B]^t - 2d^b [B]^b$	$\frac{1}{2} d^2 [A]^t + 2[D]^t$

Table 4 Stiffness matrices of sandwich panel face-sheets [84]

As a consequence of the assumption of the in-plane stiffnesses to be negligible in the core, the transverse shear stress τ_{xz} acts uniformly. The shear stress distribution of the facesheets are shown in Figure 35. The transverse shear force V_x is:

$$V_x = \int_{-h_b}^{h_t} \tau_{xz} dz = \tau_{xz}^c c + \tau_{xz}^c \frac{t^t}{2} + \tau_{xz}^c \frac{t^b}{2} = \tau_{xz}^c d \quad [2-153]$$

Where superscripts c,t and b refer to the core, top and the bottom facesheets respectively. The distance $d = c + \frac{t^t}{2} + \frac{t^b}{2}$ is shown in Figure 35.

$$V_y = \tau_{yz}^c d \quad [2-154]$$

The stress-strain relationships for the core material is given by:

$$\begin{Bmatrix} \tau_{xz}^c \\ \tau_{yz}^c \end{Bmatrix} = \begin{bmatrix} \bar{C}_{55}^c & \bar{C}_{45}^c \\ \bar{C}_{45}^c & \bar{C}_{44}^c \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^c \\ \gamma_{yz}^c \end{Bmatrix} \quad [2-155]$$

where $\bar{C}_{ij}^c$ are elements of the core stiffness matrix.

Shear deformation effects are ignored in the thin face-sheets. The shear deformation γ_{xz}^c of the cross section is shown in Figure 36. This deformation is approximated by the average shear deformation γ_{xz} shown in Figure 36. The relationship between this average shear deformation and the core deformation is expressed in Figure 36.

$$\gamma_{xz}^c = \frac{d}{c} \gamma_{xz} \quad [2-156]$$

Similarly,

$$\gamma_{yz}^c = \frac{d}{c} \gamma_{yz} \quad [2-157]$$

The relationship between the transverse shear forces and the average shear deformation were derived in Equations [2-156] and [2-157]

$$\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} = \frac{d^2}{c} \begin{bmatrix} \bar{C}_{55}^c & \bar{C}_{45}^c \\ \bar{C}_{45}^c & \bar{C}_{44}^c \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad [2-158]$$

Comparing this Equation with Equation [2-155] yields:

$$\begin{bmatrix} \tilde{S}_{11} & \tilde{S}_{12} \\ \tilde{S}_{12} & \tilde{S}_{22} \end{bmatrix} = \frac{d^2}{c} \begin{bmatrix} \bar{C}_{55}^c & \bar{C}_{45}^c \\ \bar{C}_{45}^c & \bar{C}_{44}^c \end{bmatrix} \quad [2-159]$$

The four elements of the matrix $[\bar{C}^c]$ represents the core material, whereas $[\tilde{S}]$ is the shear stiffness matrix of the sandwich plate.

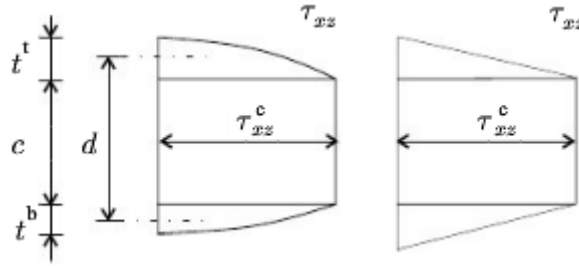


Figure 35 Shear stress distribution (left) and approximate distribution (right) in a sandwich panel [84].

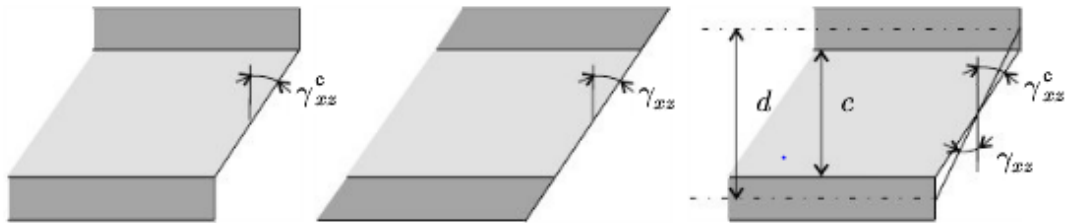


Figure 36 Shear deformation of a sandwich panel [84]

2.18 Studies on sandwich panels

Sandwich structures consisting of fibre reinforced composite face-sheets bonded to a lightweight core are being widely used for aerospace, marine, automotive applications mainly due to the advantages such as high strength and stiffness, high energy absorption capability and corrosion resistance. These structures experience low velocity impact events caused by tool drops, runway stones, hails, etc. which significantly reduce the load carrying capability of the structure. [33,109]. Therefore, it is vital to predict the impact damage resistance of composite sandwich structures.

Most research on the low velocity impact behaviour of sandwich panels provided numerical and experimental solutions. Few analytical models have been developed due to the complex interactions between the composite facesheet and core during deformation and failure [110]. Abrate [41] outlines present analytical modelling approaches to model impact on sandwich panels with laminated facesheets incorporating contact laws, plate theories and dynamic mass-spring systems.

Hoo Fatt and Park [110] used equivalent single and multi-degree of freedom systems to predict the low velocity impact response of composite sandwich panels subjected to various boundary conditions. Analytical solutions for the transient dynamic response of the sandwich panels were derived. The analytical predictions of the static load indentation and contact force between the projectile and facesheet were well matched with the experimental results for cases involving rigidly supported, clamped and simply supported panels.

Malekzadeh et al. [111] adopted the improved higher order sandwich plate theory (IHASAPT) and two models representing the contact behaviour of the panel and the impactor to predict the low velocity impact of sandwich panels comprising of a transversely flexible core and composite facesheets. A new three degree of freedom system consisting of spring-mass-damper-dashpot (SMDD) or spring-mass-damper (SMD) were used to derive analytical functions defining the contact force histories and deflections of the impactor and the panel.

Caprino et al. [112] used a single degree of freedom system to analyse drop weight impact tests on glass/polyester sandwich panels. The effect of different core densities and core thicknesses on the impact histories were examined. Anderson [113] investigated the use of single degree of freedom (SDOF) model for large mass impact on sandwich panels. Analytical contact force histories were compared with experimental force history results for six different facesheet and core configurations.

Olsson and McManus [114] derived an analytical model for the indentation of sandwich panels through incorporating core crushing and large facesheet deflections. Olsson [115] also suggested an engineering method for the prediction of impact response and damage in sandwich panels considering the effects of local core crushing, delamination and large facesheet deflections. Frosting et al. [116,117] used the higher order theory based on variational principles to predict the bending behaviour of a sandwich beam with a transversely flexible core. Hosseini et al. [118] derived an indentation law (force-indentation relationship) for a doubly curved composite sandwich panel impacted by a flat ended cylindrical impactor using the theory of minimum potential energy. It was found that as the radii of curvature of the panel increases, the indentation depth increases for the same indentation force.

Flores-Johnson and Li [119] conducted quasi-static indentation tests on a range of sandwich panels using different indenter nose shapes and their analysis proved that nose shape and foam core density significantly affected the low velocity impact behaviour of sandwich panels. Anderson and Madenci [120] analytically studied the indentation response of composite sandwich plates subjected to rigid spherical indentors through the use of the three dimensional elasticity theory. The contact pressure distribution between the sandwich plate and indenter and the contact area were derived. Hosseini et al. [121] proposed analytical methods to predict the indentation response of rigidly supported sandwich plates and derived relationships between the indentation load and deflections of the plates. Zhou and Stronge [122] derived analytical solutions for thin, lightweight circular sandwich panel impacted by a rigid spherical indenter. Their study assumed that the facesheets remained elastic, while the core has a rigid-plastic behaviour under normal compression. The theory of minimum potential energy was used in deriving a localized indentation law for the sandwich panel. Wierzbicki et al. [123] analytically studied the energy absorption capabilities of sandwich plates with honeycomb core. Baba [124] experimentally studied the impact resistance of curved sandwich composites with layer-wise graded cores. The perforation energy and failure modes of curved sandwich composites were studied. Results indicated that the perforation energies of sandwich panels are dependent upon geometrical and material properties. Impact tests on different core layer arrangement were performed to increase impact performance of curved sandwich panels. Yurddaskal and Baba [125] studied the effect of curvature on the impact response of foam-based sandwich composite panels. The energy absorbing capabilities of the sandwich panels were tested for a range of impact energies. The experimental tests proved that with the increase of panel curvature, the maximum contact force decreased while the displacement increased. Gunes and Arslan [126] experimentally and numerically studied the behaviour of aluminium honeycomb sandwich structures under low velocity impact. A drop weight impact test system was used to measure contact forces and impact energies. A finite element study validated the experimental work and the effect of cell size and core height on the impact response were studied. He et al.[127] performed experimental and numerical studies on investigating the low velocity impact behaviour of sandwich structures with a hybrid corrugated core. They carried out several impact tests to investigate the effect of

impact energy, core thickness and impact site on the impact resistance of such structures. A progressive damage model based on Hashin failure criteria was used in studying the effects of fibre and matrix damage. Abdolrahim et al.[128] analytically, numerically and experimentally investigated the impact behaviour of sandwich panels with honeycomb core. Studies on effect of parameters such as impact energy, thickness of composite laminates and shape of projectile on the impact response was evaluated. Their results proved that the accuracy of the analytical model decreased with the increase of impact energy.

The main motivation of this chapter is to study the impact response of curved sandwich plates analytically and the study has been extended to incorporate the effect of variable stiffness face-sheets which is the novelty. Most structural applications consist of curved sandwich structures and it is therefore required to evaluate the behaviour of such structures under impact loading. The present study develops an analytical model based on the theoretical formulations derived by Hoo Fatt and Park [110] for a curved sandwich panel with a Nomex honeycomb core. The present study considers the effect of core crushing and the equivalent spring-mass-dashpot models are used to predict the impact response of simply supported curved sandwich plates.

2.19 Identified research gaps

After performing a thorough literature study, it was identified that more work needs to be done on analytical modelling of low velocity impact modelling composite structures. Analytical models provide many advantages when compared with numerical and experimental methods since they are less computationally expensive than numerical methods and cost effective than experimental techniques. Therefore, this project focuses on analytical modelling of composite structures subjected to low velocity impact. The following research gaps were identified:

Limitations of present analytical models:

<i>Author</i>	<i>Research</i>	<i>Limitation</i>
Shivakumar et al [71]	Developed energy balanced and spring mass systems to determine impact force and duration during low velocity impact of circular composite plates	Not applicable for curved shells which require shell theory.
Madsen et al.[46]	Devised a method to determine natural frequencies of plate and impactor by solving differential equations for a two degree of freedom mass-spring system.	Not applicable for curved shells.
Gong et al. [44]	Studied elastic response of orthotropic laminated cylindrical shells subjected to low velocity impact	Need for a more detailed analytical study to examine effect of impact parameters such as layups, impact velocity. Only applicable for a constant stiffness curved composite shell. Analytical study was not validated with experimental or numerical analysis. Effect of lower impactor masses not studied

Khalili et al. [45]	Studied dynamic response of smart composite shell.	Only applicable for a constant stiffness curved composite shell. This study focussed more on effect of shape memory alloys on impact response of such curved shells
Shahid et al. [59]	An improved analytical model was developed for predicting damage resistance in composite panels beyond the initial damage state. This was achieved by modifying spring constant.	Limited to flat panels.
Olssen[61]	Suggested analytical models for prediction of impact damage initiation and growth during quasi-static response by large mass impactors. Structural spring mass models were modified to consider the reduced bending stiffness caused by delamination. The critical load for delamination growth is found independent to geometry and boundary conditions	Only applicable to flat composite plates. Research was more focussed on damage mechanics with analysing effect of delamination on force-displacement relationships and damage size of the specimens. Effects of factors such as layups, boundary conditions, impactor mass, impactor velocity, etc. were not analysed. This study was limited only to large mass impacts.
Singh and Mahajan[58]	Depicted that force and deflection response of the laminate is largely dependent upon the extent of damage in composite laminates. Inter-laminar and intra-laminar damage effects were simulated by progressively reducing the stiffness of the damaged region.	The need to analyse effects of layups on the impact response of damaged plates were identified. This was give an idea of the types of layups which are able to perform with greater superiority under impact loading. The present method was also only applicable to flat composite plates.

Akbarzadeh et al. [129]	Studied the vibration response of variable stiffness laminates. It was found that for simply-supported boundary conditions, a large proportion of fibers with orientation at the plate center close to 0° favors the in-plane stiffness, whereas a large proportion of fibers with angles close to 45° improves the out-of-plane stiffness and vibration re-sponses.	Even though this study focused on variable stiffness composites, analysis was limited to vibration response, and not low velocity impact response.
Ijsselmuiden et al.[130]	Performed numerical buckling analysis on variable stiffness laminated plates and found that there was a 100% improvement in buckling load when compared with constant stiffness plate	Study was limited to buckling of variable stiffness laminates.
Hoo Fatt and Park[110]	Derived analytical formulations to predict impact response of sandwich panels with various boundary conditions.	Models were only applicable for flat composite plates. The need for a more thorough analysis to study effect of impact parameters was identified.
Malekzadeh et al.[131]	Adopted improved higher order sandwich plate theory and a spring-mass-dashpot model to predict low velocity impact response of sandwich panels with a transversely flexible core.	Limited to flat sandwich plates. Validation with experimental or numerical analysis not performed.
Olsson and McManus[114]	Suggested a method for prediction of impact response and damage	Limited to flat sandwich panels.

of flat sandwich panels. This approach accounts for local core crushing, delamination and large face sheet deflections and does not rely on empirical indentation laws.

Limitations were identified from the present developed analytical models that predicted low velocity impact response of curved composite plates. Some of the major limitations identified were:

- Most studied focussed on flat composite plates. The need to perform thorough analysis on curved shells were identified.
- Analytical models developed in present literature were only applicable to study low velocity impact response of curved constant stiffness plates.
- None of the present models were capable of predicting impact response of damaged variable stiffness curved plates.
- Current analytical models related to sandwich plates were only applicable for flat specimens.
- Analytical models that studied impact response of sandwich plates were only limited to flat geometries.

Chapter 3:

Analytical Modelling of Low velocity
impact damage on variable stiffness
curved composites

3 Analytical Modelling of Low velocity impact damage on variable stiffness curved composites

3.1 Introduction

Analytical modelling of impact on composite structures plays a vital role in understanding the factors contributing to the impact response of such structures under impact loading conditions. An efficient analytical model would mean that less computational effort is required when compared to advanced finite element methods and less expensive compared to experimental techniques. However, any analytical model should be validated with either numerical or experimental data.

This chapter of the research focusses on developing an analytical model to predict the low velocity impact response of variable stiffness curved composite plates, which is the main novelty of the study. To present knowledge of the author, this is the first analytical model in literature to evaluate impact response of these structures. Variable stiffness/thickness sections are widely found in aerospace and automotive structures. An example would be a skin panel of an aircraft wing where the thickness changes from root to tip of the wing. Variable stiffness plates consists of varying thicknesses along the width of the plate as shown in Figure 37. These varying thicknesses results in varying stiffnesses of the plate. The stiffness of a plate is governed by extensional (A), coupling (B) and bending (D) matrices; and for a variable stiffness plate, the values of these matrices change across the plate width due to changes in the thickness. Therefore, the impact response would be different when compared to a plate with constant thickness. This effect of geometric change is analysed in this chapter.



Figure 37 Variable stiffness structures (a) application (b) Plate schematic

3.2 Mathematical modelling

Mathematical modelling aspect of this research aims to develop a contact force function able to predict impact response of variable stiffness curved composite plates. This force function is then adopted to perform a parametric analysis in studying effects of various parameters affecting impact response of such structures.

3.2.1 Modelling process

The following steps depicted in Figure 38 shows the general procedure adopted in this research to model low velocity impact response of composite laminates. Detailed flowchart for the variable stiffness plate is shown later in Figure 42.

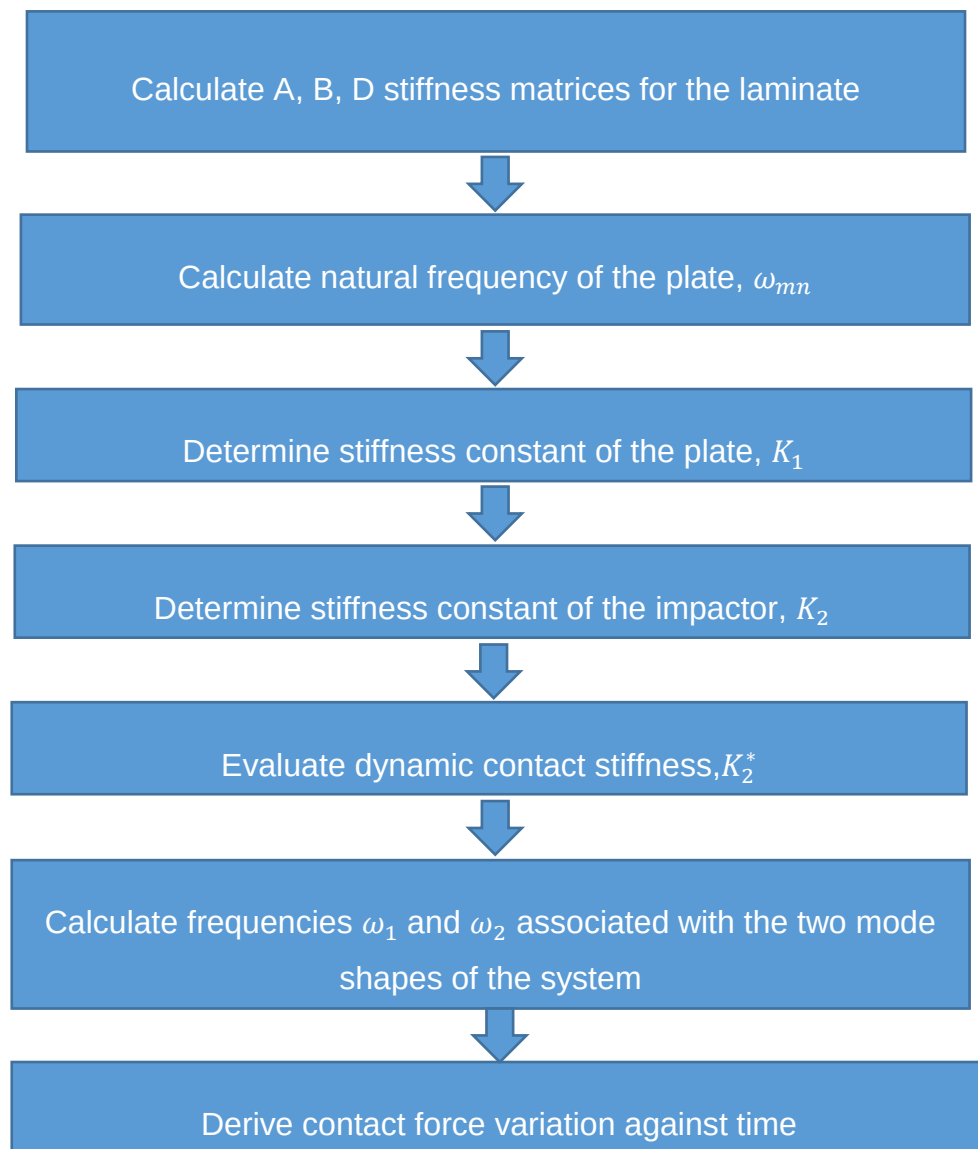


Figure 38 Mathematical modelling process

3.2.2 A, B, D stiffness matrices

Thin composite laminates are characterized by three stiffness matrices namely [A], [B], and [D]. This section explains the method in which these stiffness matrices are defined. Calculating these stiffness matrices are the starting point for the impact analysis problem. The analyses are based on the laminate plate theory and defined by approximating that the strains vary linearly across the laminate, out of plane shear deformations are negligible, and out-of-plane normal stress σ_z and the shear stresses τ_{xz} , τ_{yz} are small compared with the in-plane σ_x , σ_y , and τ_{xy} stresses. The stress-strain relationships under plane-stress conditions can be applied. The x and y coordinates in a suitably chosen reference plane, and z is perpendicular to this reference plane (Figure 33)

For convenience the reference plane is taken to be the mid-plane of the laminate. Unless the laminate is symmetrical with respect to the reference plane, the reference plane is not a neutral plane, and the strains in the reference plane are not zero under pure bending. The strains in the reference plane are:

$$\epsilon_x^0 = \frac{\partial u^0}{\partial x} \quad \epsilon_y^0 = \frac{\partial v^0}{\partial y} \quad \gamma_{xy}^0 = \frac{\partial u^0}{\partial x} + \frac{\partial v^0}{\partial y} \quad [3-1]$$

where u and v are the x, y components of the displacement and the superscript 0 refers to the reference plane.

Kirchhoff hypothesis is adopted considering that normal to the reference surface remain normal and straight (Figure 34). The angle of rotation of the normal of the reference plane for small deflections is:

$$\chi_{xz} = \frac{\partial \omega^0}{\partial x} \quad [3-2]$$

where ω^0 is the out of plane displacement of the reference plane. The total displacement in the x direction is:

$$u = u^0 - z\chi_{xz} = u^0 - z \frac{\partial \omega^0}{\partial x} \quad [3-3]$$

Total displacement in the y direction is:

$$\mathbf{v} = \mathbf{v}^0 - \mathbf{z} \frac{\partial \omega^0}{\partial y} \quad [3-4]$$

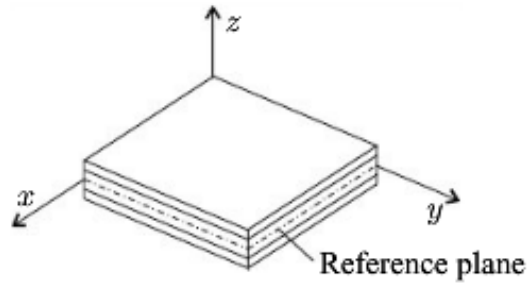


Figure 40 Coordinate System

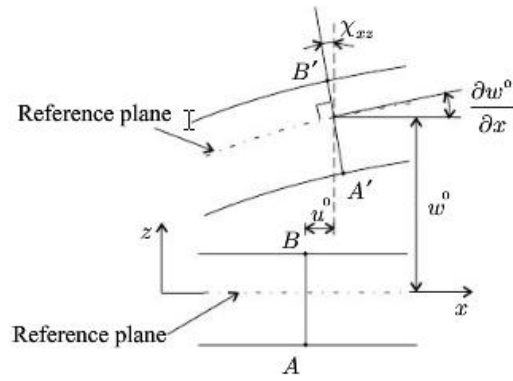


Figure 41 Deformation of a plate in the x-z plane

By definition, the strains are expressed as:

$$\epsilon_x = \frac{\partial u}{\partial x} \quad \epsilon_y = \frac{\partial v}{\partial y} \quad \tau_{xy} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad [3-5]$$

Substituting into above equations

$$\epsilon_x = \frac{\partial u^0}{\partial x} - \mathbf{z} \frac{\partial^2 \omega^0}{\partial x^2} \quad [3-6]$$

$$\epsilon_y = \frac{\partial v^0}{\partial y} - \mathbf{z} \frac{\partial^2 \omega^0}{\partial y^2} \quad [3-7]$$

$$\gamma_{xy} = \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} - \mathbf{z} \frac{\partial^2 \omega^0}{\partial x \partial y} \quad [3-8]$$

These expressions can be re-written in matrix form as:

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \tau_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad [3-9]$$

Where $\epsilon_x^0, \epsilon_y^0, \gamma_{xy}^0$ are strains in the reference plane and $\kappa_x, \kappa_y, \kappa_{xy}$ are the curvatures of the reference plane of the plate.

$$\kappa_x = -\frac{\partial^2 \omega^0}{\partial x^2} \quad \kappa_y = -\frac{\partial^2 \omega^0}{\partial y^2} \quad \kappa_{xy} = -\frac{2\partial^2 \omega^0}{\partial x \partial y} \quad [3-10]$$

The expressions for the in-plane forces and moments are given by

$$N_x = \int_{-h_b}^{h_t} \sigma_x dz \quad N_y = \int_{-h_b}^{h_t} \sigma_y dz \quad N_{xy} = \int_{-h_b}^{h_t} \tau_{xy} dz \quad [3-11]$$

$$M_x = \int_{-h_b}^{h_t} z \sigma_x dz \quad M_y = \int_{-h_b}^{h_t} z \sigma_y dz \quad M_{xy} = \int_{-h_b}^{h_t} z \tau_{xy} dz \quad [3-12]$$

Where N and M are the in-plane forces and moments, and h_t and h_b are the distances from the reference plane to the plate's surfaces. The transverse shear forces per unit length are expressed as:

$$V_x = \int_{-h_b}^{h_t} \tau_{xz} dz \quad V_y = \int_{-h_b}^{h_t} \tau_{yz} dz \quad [3-13]$$

The stress-strain relationship for each ply is defined by:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad [3-14]$$

$$[\bar{Q}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \quad [3-15]$$

Therefore,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\bar{Q}] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad [3-16]$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\bar{Q}] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad [3-17]$$

Where $[\bar{Q}]$ is the stiffness matrix of the ply in the $x - y$ coordinate system. Since composite laminates consists of different fibre orientations, the elements of the stiffness matrix $[\bar{Q}]$ are transformed from the elements of stiffness matrix $[Q]$.

$$\begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} = [T_\sigma]^{-1} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} [T_\epsilon] \quad [3-18]$$

Where $[T_\sigma]$ and $[T_\epsilon]$ are defined by:

$$[T_\sigma] = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{bmatrix} \quad [T_\epsilon] = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{bmatrix} \quad [3-19]$$

and $c = \cos\theta, s = \sin\theta$

The elements of the stiffness matrix in the global coordinate system are expressed in terms of the elements of the stiffness matrix in the local coordinate system.

$$\bar{Q}_{11} = c^4 Q_{11} + s^4 Q_{22} + 2c^2 s^2 (Q_{12} + 2Q_{66}) \quad [3-20]$$

$$\bar{Q}_{22} = s^4 Q_{11} + c^4 Q_{22} + 2c^2 s^2 (Q_{12} + 2Q_{66}) \quad [3-21]$$

$$\bar{Q}_{12} = c^2 s^2 (Q_{11} + Q_{22} - 4Q_{66}) + (c^4 + s^4) Q_{12} \quad [3-22]$$

$$\bar{Q}_{66} = c^2 s^2 (Q_{11} + Q_{22} - 2Q_{12}) + (c^2 - s^2)^2 Q_{66} \quad [3-23]$$

$$\bar{Q}_{16} = cs (c^2 Q_{11} - s^2 Q_{22} - (c^2 - s^2)(Q_{12} + 2Q_{66})) \quad [3-24]$$

$$\bar{Q}_{26} = cs (s^2 Q_{11} - c^2 Q_{22} - (c^2 - s^2)(Q_{12} + 2Q_{66})) \quad [3-25]$$

The force and moment matrices can be expressed in terms of the stiffness, strains and curvatures matrices:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-h_t}^{h_t} \left\{ [\bar{Q}] \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \tau_{xy}^0 \end{Bmatrix} + [\bar{Q}]z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right\} dz = \int_{-h_b}^{h_t} [\bar{Q}] dz \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \tau_{xy}^0 \end{Bmatrix} + \int_{-h_b}^{h_t} z [\bar{Q}] dz \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad [3-26]$$

$$\begin{Bmatrix} \mathbf{M}_x \\ \mathbf{M}_y \\ \mathbf{N}_{xy} \end{Bmatrix} = \int_{-h_b}^{h_t} \mathbf{z} \left\{ [\bar{\mathbf{Q}}] \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \tau_{xy}^0 \end{Bmatrix} + [\bar{\mathbf{Q}}] \mathbf{z} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right\} d\mathbf{z} = \int_{-h_b}^{h_t} \mathbf{z} [\bar{\mathbf{Q}}] d\mathbf{z} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \tau_{xy}^0 \end{Bmatrix} + \int_{-h_b}^{h_t} \mathbf{z}^2 [\bar{\mathbf{Q}}] d\mathbf{z} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

[3-27]

The stiffness matrices of the laminate are defined by:

$$[\mathbf{A}] = \int_{-h_b}^{h_t} [\bar{\mathbf{Q}}] d\mathbf{z} \quad [3-28]$$

$$[\mathbf{B}] = \int_{-h_b}^{h_t} \mathbf{z} [\bar{\mathbf{Q}}] d\mathbf{z} \quad [3-29]$$

$$[\mathbf{D}] = \int_{-h_b}^{h_t} \mathbf{z}^2 [\bar{\mathbf{Q}}] d\mathbf{z} \quad [3-30]$$

Since $[\bar{\mathbf{Q}}]$ is the stiffness matrix of the ply and is constant across each ply the integrals are replaced by summations so the whole laminate is considered.

$$\mathbf{A}_{ij} = \sum_{n=1}^N (\bar{\mathbf{Q}}_{ij})_n (\mathbf{z}_n - \mathbf{z}_{n-1}) \quad [3-31]$$

$$\mathbf{B}_{ij} = \frac{1}{2} \sum_{n=1}^N (\bar{\mathbf{Q}}_{ij})_n (\mathbf{z}_n^2 - \mathbf{z}_{n-1}^2) \quad [3-32]$$

$$\mathbf{D}_{ij} = \frac{1}{3} \sum_{n=1}^N (\bar{\mathbf{Q}}_{ij})_n (\mathbf{z}_n^3 - \mathbf{z}_{n-1}^3) \quad [3-33]$$

Where N is the total number of plies in the laminate; z_n and z_{n-1} are the distances from the reference plane to the two surfaces of the of the n th ply.

Therefore, the expressions for the in-plane forces and moments become:

$$\begin{Bmatrix} \mathbf{N}_x \\ \mathbf{N}_y \\ \mathbf{N}_{xy} \\ \mathbf{M}_x \\ \mathbf{M}_y \\ \mathbf{M}_{xy} \end{Bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{A}_{16} & \mathbf{B}_{11} & \mathbf{B}_{12} & \mathbf{B}_{16} \\ \mathbf{A}_{12} & \mathbf{A}_{22} & \mathbf{A}_{26} & \mathbf{B}_{12} & \mathbf{B}_{22} & \mathbf{B}_{26} \\ \mathbf{A}_{16} & \mathbf{A}_{26} & \mathbf{A}_{66} & \mathbf{B}_{16} & \mathbf{B}_{26} & \mathbf{B}_{66} \\ \mathbf{B}_{11} & \mathbf{B}_{12} & \mathbf{B}_{16} & \mathbf{D}_{11} & \mathbf{D}_{12} & \mathbf{D}_{16} \\ \mathbf{B}_{12} & \mathbf{B}_{22} & \mathbf{B}_{26} & \mathbf{D}_{12} & \mathbf{D}_{22} & \mathbf{D}_{26} \\ \mathbf{B}_{16} & \mathbf{B}_{26} & \mathbf{B}_{66} & \mathbf{D}_{16} & \mathbf{D}_{26} & \mathbf{D}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad [3-34]$$

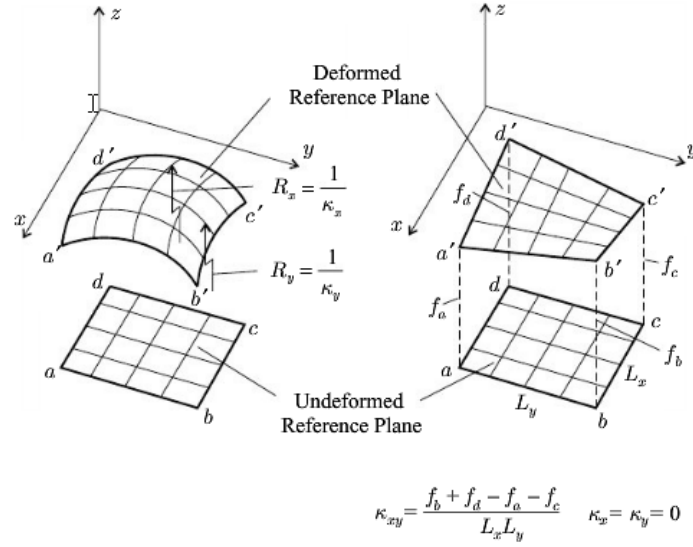


Figure 42 Curvatures of reference plane [83].

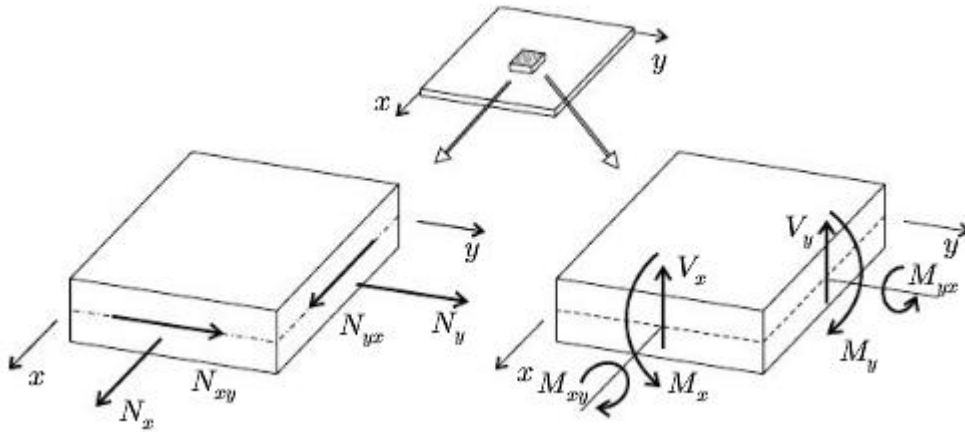


Figure 43 In plane forces at reference plane (left) and moments and transverse shear forces (right) [83].

3.2.3 Impact model

An impact model is required to derive a valid relationship between the dynamic contact force and time. In this work, a two degree of freedom mass-spring system is utilized. The initial model does not involve damping and the effect of damping on the contact force history is analyzed later.

The two degree of freedom mass-spring system (Figure 37) consists of the plate and the impactor, where M_1 and M_2 denotes mass of the plate and impactor respectively;

contact stiffness of the plate is represented as K_1 and K_2 is the stiffness of the impactor. The stiffness of the simply supported laminated plate is expressed as:

$$K_1 = \omega_{mn}^2 M_1 \quad [3-35]$$

where, ω_{mn} is the natural frequency of the laminated plate and obtained through equation (3-56) by assuming $i = j = 1$.

During an impact event, both the plate and the impactor undergoes displacement. In this model, the displacement of the plate is represented by $z_1(t)$ and $z_2(t)$ signifies the impactor movement at any time t during the impact. The contact deformation is expressed as:

$$\delta(t) = z_2(t) - z_1(t) \quad [3-36]$$

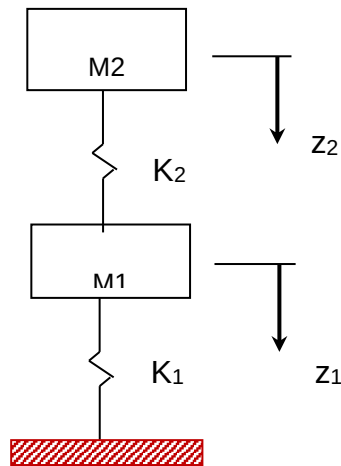


Figure 44 Two degree of freedom mass=spring system

The contact force between the plate and the impactor during an impact is estimated using the Hertzian theory expressed as:

$$F_c(t) = K_2 \delta^p \quad [3-37]$$

Where K_2 and p are material constants where p is 1.5

However, the highly non-linear nature of the equation in obtaining an analytical solution for the dynamic contact force signifies a risk. Therefore, this present study applies an effective contact stiffness denoted as K_2^* to relate the equivalent contact force to the contact deformation

$$\mathbf{F}_c^*(t) = \mathbf{K}_2^* \mathbf{\delta} \quad [3-38]$$

which is also be expressed as,

$$\mathbf{F}_c^*(t) = \mathbf{K}_2^* [\mathbf{z}_2(t) - \mathbf{z}_1(t)] \quad [3-39]$$

The effective contact stiffness $\mathbf{K}_2^*$ is calculated as [44]

$$\mathbf{K}_2^* = \sqrt{\pi} \Gamma \left(\frac{p+1}{2} \right) \frac{2\Gamma \left(\frac{p+1}{2} \right) + \sqrt{\pi} \Gamma \left(\frac{p+1}{2} \right)}{4\Gamma^2 \left(\frac{p+1}{2} \right) + \pi \Gamma^2 \left(\frac{p+1}{2} \right)} \delta_m^{p-1} \mathbf{K}_2 \quad [3-40]$$

where Γ is the Gamma function and the maximum contact deformation,

The stiffness constant of the impactor, K_2 is expressed as [44]:

$$\mathbf{K}_2 = \frac{2}{3} \sqrt{\Gamma} \frac{E}{(1-\nu^2)} \quad [3-41]$$

The plate geometry in this study is constrained by simply supported boundary conditions. The contact deformation δ_m for a simply supported target structure is expresses as [45]:

$$(\delta_m)_f < (\delta_m)_{ss} < (\delta_m)_c \quad [3-42]$$

where the subscripts f, ss, c refer to free, simply supported and clamped respectively.

The deformations for a free and clamped panels are expressed as:

$$(\delta_m)_f = \left(\frac{\mathbf{M}_1 \mathbf{M}_2}{\mathbf{M}_1 + \mathbf{M}_2} \right)^{0.4} \left[\frac{5V^2}{4k_2} \right]^{0.4} \quad [3-43]$$

$$(\delta_m)_c = (\mathbf{M}_2)^{0.4} \left[\frac{5V^2}{4k_2} \right]^{0.4} \quad [3-44]$$

The corresponding equations of motion for a two degree of freedom model are as follows:

$$\mathbf{M}_1 \ddot{\mathbf{z}}_1 = -\mathbf{K}_1 \mathbf{z}_1 - \mathbf{K}_2 (\mathbf{z}_1 - \mathbf{z}_2) \quad [3-45]$$

$$\mathbf{M}_2 \ddot{\mathbf{z}}_2 = -\mathbf{K}_2 (\mathbf{z}_2 - \mathbf{z}_1) \quad [3-46]$$

Employing the effective contact stiffness, $\mathbf{K}_2^*$ this equation becomes:

$$\mathbf{M}_1 \ddot{\mathbf{z}}_1 + (\mathbf{K}_1 + \mathbf{K}_2^*) \mathbf{z}_1 - \mathbf{K}_2^* \mathbf{z}_2 = \mathbf{0}$$

[3-47]

$$\mathbf{M}_2 \ddot{\mathbf{z}}_2 - \mathbf{K}_2 \mathbf{z}_1 + \mathbf{K}_2^* \mathbf{z}_2 = \mathbf{0}$$

[3-48]

The initial conditions are defined as:

$$\mathbf{z}_1 = \mathbf{z}_2 = \mathbf{0}, \quad \dot{\mathbf{z}}_2 = \mathbf{V} \quad \text{at} \quad t = 0$$

[3-49]

By using the initial conditions defined in Eq. [3-49] the analytical function for the force can be defined as: [44]

$$\mathbf{F}_c^*(t) = \mathbf{K}_2^* [\mathbf{A}_1 (\mathbf{C}_1 - 1) \sin \omega_1 t + \mathbf{A}_2 (\mathbf{C}_2 - 1) \sin \omega_2 t] \quad [3-50]$$

For a two degree of freedom mass-spring system, two mode shapes are associated with frequencies ω_1 and ω_2

$$\omega_{1,2}^2 = \frac{1}{2} \left(\frac{\mathbf{K}_1 + \mathbf{K}_2}{\mathbf{M}_1} + \frac{\mathbf{K}_2}{\mathbf{M}_2} \right) \mp \sqrt{\frac{1}{4} \left(\frac{\mathbf{K}_1 + \mathbf{K}_2}{\mathbf{M}_1} + \frac{\mathbf{K}_2}{\mathbf{M}_2} \right)^2 - \frac{\mathbf{K}_1 \mathbf{K}_2}{\mathbf{M}_1 \mathbf{M}_2}} \quad [3-51]$$

The coefficients of the force function are defined by:

$$\mathbf{C}_1 = \frac{\mathbf{K}_2}{\mathbf{K}_2 - \omega_1^2 \mathbf{M}_2} \quad [3-52]$$

$$\mathbf{C}_2 = \frac{\mathbf{K}_2}{\mathbf{K}_2 - \omega_2^2 \mathbf{M}_2} \quad [3-53]$$

$$\mathbf{A}_1 = \frac{\mathbf{V}}{\omega_1 (\mathbf{C}_1 - \mathbf{C}_2)} \quad [3-54]$$

$$\mathbf{A}_2 = \frac{\mathbf{V}}{\omega_2 (\mathbf{C}_2 - \mathbf{C}_1)} \quad [3-55]$$

3.2.4 Variable stiffness model

The main novelty of the present study is to derive an analytical force function to represent the contact force history of variable stiffness curved composite plates. The schematic of a variable stiffness curved composite plate is defined in Figure 38 below:

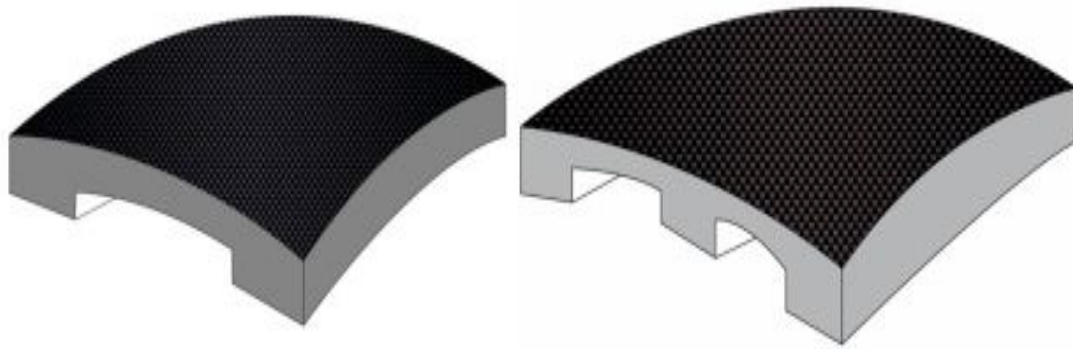


Figure 45 Variable stiffness curved plate

As the above schematic implies, the thickness of the plate varies across the width of the plate, thus signifying varying stiffnesses across the plate. This is governed by different A, B, D stiffness matrices across the plate. For instance, a plate with three variable stiffness sections would consist of three sets of stiffness matrices. The difference between a plate with constant stiffness and variable stiffness is depicted in Figure 39 (a) and 39 (b).

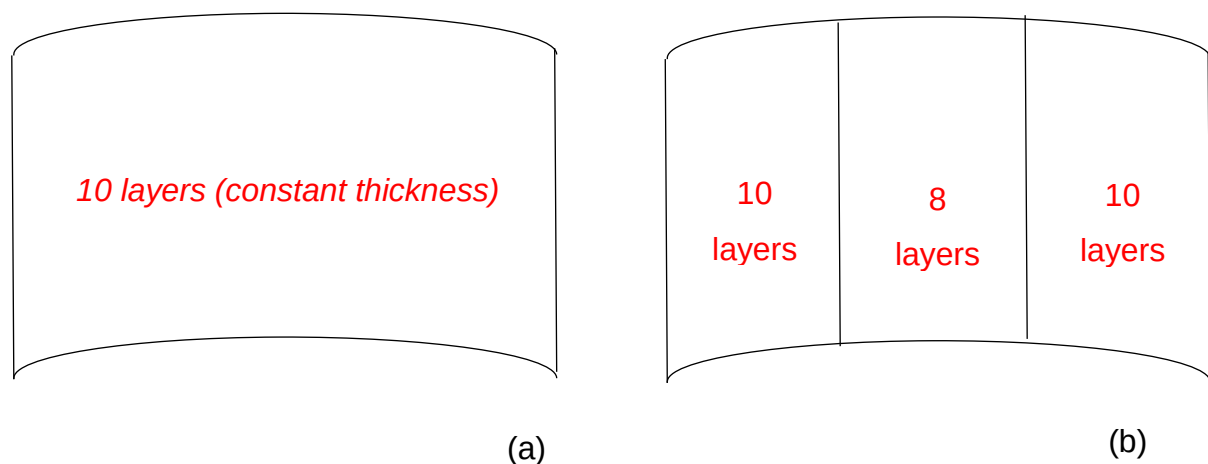


Figure 46 (a) Constant stiffness plate (b) Variable stiffness plate

3.2.5 Analytical modelling of variable stiffness curved plate

The stiffness variation is acquired through varying the thickness across the width of the plate. This variation in thickness is a result of using different number of composite layers throughout the plate as shown in Figure 39(b). Therefore, the stiffness matrices varies, thus resulting in variable stiffness sections.

The theoretical models developed in the present study are represented as [43]. The analysis is divided to two models, first the curved composite plate with three variable thicknesses (Figure 40b) and second one is with five variable thicknesses (Figure 40 c).

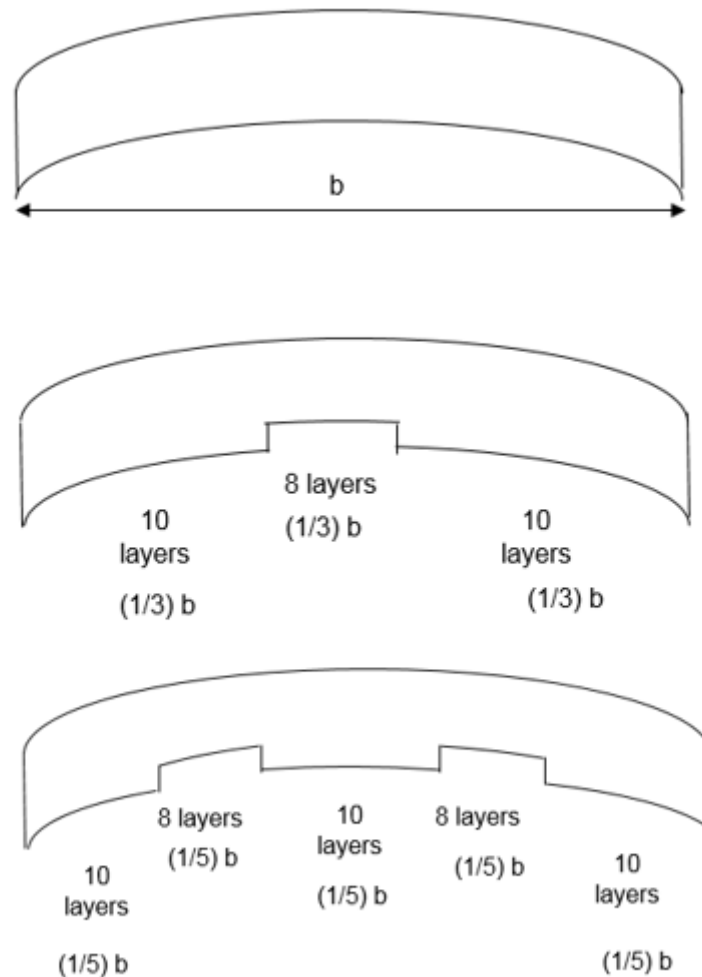


Figure 47 Various designs of curved composite plates with a) uniform thickness, b) three variable thicknesses and c) five variable thicknesses and equal width (side view).

Natural frequencies of the individual variable stiffness sections are calculated based on the approach suggested in [45]. For a constant thickness curved composite plate, natural frequency, ω_{mn} is expressed as:

$$\omega_{mn} = -\frac{(c_{13}K_U + c_{23}K_V + c_{33} + c_{34}K_X + c_{35}K_Y)}{\rho h} \quad [3-56]$$

h is the total thickness of the section

3.2.6 Natural frequency calculation

In this research, studying the impact response of composite plates involves relating natural frequency of the plate and impactor to derive the contact force function. First order shear deformation theory is used to express the equations of motion in a symmetric cross ply laminate. Then, double Fourier series is used to solve the equations to define Fourier coefficients for the impact problem.

Equations of motion of the symmetric cross ply composite is expresses as [45]:

$$A_{11} \left(\frac{\partial^2 u_0}{\partial x_1^2} + \frac{1}{R_1} \frac{\partial w_0}{\partial x_1} \right) + A_{12} \left(\frac{\partial^2 v_0}{\partial x_1 \partial x_2} + \frac{1}{R_2} \frac{\partial w_0}{\partial x_1} \right) + A_{66} \left(\frac{\partial^2 v_0}{\partial x_1 \partial x_2} + \frac{\partial^2 u_0}{\partial x_2^2} \right) + c_0 D_{66} \left(\frac{\partial^2 \varphi_1}{\partial x_2^2} + \frac{\partial^2 \varphi_2}{\partial x_1 \partial x_2} \right) + k_{sh} A_{55} \left(\frac{\varphi_1}{R_1} + \frac{1}{R_1} \frac{\partial w_0}{\partial x_1} - \frac{u_0}{R_1^2} \right) + \frac{\partial N_{11}^i}{\partial x_1} + \frac{N_{11}^i}{R_1} \left(\frac{\partial w_0}{\partial x_1} - \frac{u_0}{R_1} \right) - I_0 \frac{\partial^2 u_0}{\partial t^2} - I_1 \frac{\partial^2 \varphi_1}{\partial t^2} = 0 \quad [3-57]$$

$$A_{12} \left(\frac{\partial^2 u_0}{\partial x_1 \partial x_2} + \frac{1}{R_1} \frac{\partial w_0}{\partial x_2} \right) + A_{22} \left(\frac{\partial^2 v_0}{\partial x_2^2} + \frac{1}{R_2} \frac{\partial w_0}{\partial x_2} \right) + A_{66} \left(\frac{\partial^2 u_0}{\partial x_1^2} + \frac{\partial^2 u_0}{\partial x_1 \partial x_2} \right) + \left[k_{sh} A_{44} \left(\frac{\varphi_2}{R_2} + \frac{1}{R_2} \frac{\partial w_0}{\partial x_2} - \frac{v_0}{R_2^2} \right) \right] - c_0 D_{66} \left(\frac{\partial \varphi_1}{\partial x_1 \partial x_2} + \frac{\partial \varphi_2}{\partial x_1^2} \right) + \frac{\partial N_{22}^i}{\partial x_2} + \frac{N_{22}^i}{R_2} \left(\frac{\partial w_0}{\partial x_2} - \frac{v_0}{R_2} \right) = I_0 \frac{\partial^2 v_0}{\partial t^2} + \frac{\partial^2 \varphi_2}{\partial t^2} \quad [3-58]$$

$$k_{sh} A_{55} \left(\frac{\partial^2 w_0}{\partial x_1^2} + \frac{\partial \varphi_1}{\partial x_1} - \frac{1}{R_1} \frac{\partial u_0}{\partial x_1} \right) + k_{sh} A_{44} \left(\frac{\partial^2 w_0}{\partial x_2^2} + \frac{\partial \varphi_2}{\partial x_2} - \frac{1}{R_2} \frac{\partial v_0}{\partial x_2} \right) - \frac{1}{R_1} \left\{ A_{11} \left(\frac{\partial u_0}{\partial x_1} + \frac{w_0}{R_1} \right) + A_{12} \left(\frac{\partial v_0}{\partial x_2} + \frac{w_0}{R_2} \right) \right\} + \frac{1}{R_2} \left\{ A_{12} \left(\frac{\partial u_0}{\partial x_1} + \frac{w_0}{R_1} \right) + A_{22} \left(\frac{\partial v_0}{\partial x_2} + \frac{w_0}{R_2} \right) \right\} + \frac{\partial}{\partial x_1} \left[N_{11}^i \left(\frac{\partial w_0}{\partial x_1} - \frac{u_0}{R_1} \right) \right] + \frac{\partial}{\partial x_2} \left[N_{22}^i \left(\frac{\partial w_0}{\partial x_2} - \frac{v_0}{R_2} \right) \right] + q = I_0 \frac{\partial^2 w_0}{\partial t^2} \quad [3-59]$$

$$D_{11} \left(\frac{\partial^2 \varphi_1}{\partial x_1^2} \right) + D_{12} \left(\frac{\partial^2 \varphi_2}{\partial x_1 \partial x_2} \right) + D_{66} \left(\frac{\partial^2 \varphi_2}{\partial x_1 \partial x_2} + \frac{\partial^2 \varphi_1}{\partial x_2^2} \right) - \left[k_{sh} A_{55} \left(\varphi_1 + \frac{\partial w_0}{\partial x_1} - \frac{u_0}{R_1} \right) \right] = I_2 \frac{\partial^2 \varphi_1}{\partial t^2} + I_1 \frac{\partial^2 u_0}{\partial t^2} \quad [3-60]$$

$$D_{66} \left(\frac{\partial^2 \varphi_1}{\partial x_1 \partial x_2} + \frac{\partial^2 \varphi_2}{\partial x_1^2} \right) + D_{12} \left(\frac{\partial^2 \varphi_1}{\partial x_1 \partial x_2} \right) + D_{22} \left(\frac{\partial^2 \varphi_2}{\partial x_2^2} \right) - \left[k_{sh} A_{44} \left(\varphi_2 + \frac{\partial w_0}{\partial x_2} - \frac{v_0}{R_2} \right) \right] = I_2 \frac{\partial^2 \varphi_2}{\partial t^2} + I_1 \frac{\partial^2 v_0}{\partial t^2} \quad [3-61]$$

Deflections and rotations of a flat composite plate using Double Fourier series are expressed as [45]:

$$u_0(x_1, x_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{mn}(t) \cos \frac{m\pi x_1}{a} \sin \frac{n\pi x_2}{b} \quad [3-62]$$

$$v_0(x_1, x_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} V_{mn}(t) \sin \frac{m\pi x_1}{a} \sin \frac{n\pi x_2}{b} \quad [3-63]$$

$$w_0(x_1, x_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn}(t) \sin \frac{m\pi x_1}{a} \sin \frac{n\pi x_2}{b} \quad [3-64]$$

$$\varphi_1(x_1, x_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} X_{mn}(t) \cos \frac{m\pi x_1}{a} \sin \frac{n\pi x_2}{b} \quad [3-65]$$

$$\varphi_2(x_1, x_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Y_{mn}(t) \sin \frac{m\pi x_1}{a} \cos \frac{n\pi x_2}{b} \quad [3-66]$$

In equations [3-62] to [3-66], a and b represents the length and width of the plate respectively and $U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}$ are the time-dependant constants that needs to be evaluated. [45]

The terms of the Fourier series can be expressed as:

$$q_n(x_1, x_2, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Q_{mn}(t) \sin \frac{m\pi x_1}{a} \sin \frac{n\pi x_2}{b} \quad [3-67]$$

Q_{mn} represents terms of the Fourier series and for a concentrated load at a point (x_1^c, x_2^c) , it is expressed as:

$$Q_{mn}(t) = \frac{4F(t)}{ab} \sin \frac{m\pi x_1^c}{a} \sin \frac{n\pi x_2^c}{b}, m, n = 1, 3, 5 \dots \quad [3-68]$$

By substituting equations [3-62] to [3-66] in equations [3-57] to [3-61], the following relationship is obtained:

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} \end{bmatrix} \begin{Bmatrix} U_{mn}(t) \\ V_{mn}(t) \\ W_{mn}(t) \\ X_{mn}(t) \\ Y_{mn}(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \rho h \ddot{W}_{mn}(t) - Q_{mn}(t) \\ 0 \\ 0 \end{Bmatrix} \quad [3-69]$$

The operators of c_{ij} are defined by [45]:

$$c_{11} = -A_{11} \left(\frac{m^2 \pi^2}{a^2} \right) - A_{66} \left(\frac{n^2 \pi^2}{b^2} \right) - \left(\frac{k_{sh}}{R_1^2} \right) A_{55} - \left(\frac{1}{R_1^2} \right) N_{11}^i \quad [3-70]$$

$$c_{12} = - \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) (A_{12} + A_{66}) \quad [3-71]$$

$$c_{13} = \frac{1}{R_1} \left(\frac{m\pi}{a} \right) A_{11} + \frac{1}{R_2} \left(\frac{m\pi}{a} \right) A_{12} + \frac{k_{sh}}{R_1} \left(\frac{m\pi}{a} \right) A_{55} + \frac{1}{R_1} \left(\frac{m\pi}{a} \right) N_{11}^i \quad [3-72]$$

$$c_{14} = - \left(\frac{n^2 \pi^2}{b^2} \right) c_0 D_{66} + \frac{k_{sh}}{R_1} A_{55} \quad [3-73]$$

$$c_{15} = - \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) c_0 D_{66} \quad [3-74]$$

$$c_{22} = - \left(\frac{n^2 \pi^2}{b^2} \right) A_{22} - \frac{k_{sh}}{R_2^2} A_{44} - \left(\frac{1}{R_2^2} \right) N_{22}^i - \left(\frac{m^2 \pi^2}{a^2} \right) A_{66} \quad [3-75]$$

$$c_{23} = \frac{1}{R_1} \left(\frac{n\pi}{b} \right) A_{12} + \frac{1}{R_2} \left(\frac{n\pi}{b} \right) A_{22} + \frac{k_{sh}}{R_2} \left(\frac{n\pi}{b} \right) A_{44} + \frac{1}{R_2} \left(\frac{n\pi}{b} \right) N_{22}^i \quad [3-76]$$

$$c_{24} = \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) c_0 D_{66} \quad [3-77]$$

$$c_{25} = \frac{k_{sh}}{R_2} A_{44} + \left(\frac{m^2 \pi^2}{a^2} \right) c_0 D_{66} \quad [3-78]$$

$$c_{33} = -k_{sh} \left(\frac{m^2 \pi^2}{a^2} \right) A_{55} - k_{sh} \left(\frac{n^2 \pi^2}{b^2} \right) A_{44} - \left(\frac{1}{R_2^2} \right) A_{22} - \left(\frac{1}{R_1^2} \right) A_{11} - \left(\frac{m^2 \pi^2}{a^2} \right) N_{11}^i - \left(\frac{n^2 \pi^2}{b^2} \right) N_{22}^i \quad [3-79]$$

$$c_{34} = -k_{sh} \left(\frac{m\pi}{a} \right) A_{55} \quad [3-80]$$

$$c_{35} = -k_{sh} \left(\frac{n\pi}{b} \right) A_{44} \quad [3-81]$$

$$c_{44} = - \left(\frac{m^2 \pi^2}{a^2} \right) D_{11} - \left(\frac{n^2 \pi^2}{b^2} \right) D_{66} - k_{sh} A_{55} \quad [3-82]$$

$$c_{45} = -\left(\frac{m^2\pi^2}{a^2}\right)D_{12} - \left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)D_{66} \quad [3-83]$$

$$c_{55} = -\left(\frac{m^2\pi^2}{a^2}\right)D_{66} - \left(\frac{n^2\pi^2}{b^2}\right)D_{22} - k_{sh}A_{44} \quad [3-84]$$

Equation (3-72) can be changed into a second order differential equation in dynamic equilibrium:

$$U_{mn}(t) = K_U W_{mn}(t) \quad [3-85]$$

$$V_{mn}(t) = K_V W_{mn}(t) \quad [3-86]$$

$$X_{mn}(t) = K_X W_{mn}(t) \quad [3-87]$$

$$Y_{mn}(t) = K_Y W_{mn}(t) \quad [3-88]$$

Where

$$K_U = S_1 + S_2 K_X + S_3 K_Y \quad [3-89]$$

$$K_V = S_4 + S_5 K_X + S_6 K_Y \quad [3-90]$$

$$K_X = \Delta_X / \Delta \quad [3-91]$$

$$K_Y = \Delta_Y / \Delta \quad [3-92]$$

$$\Delta_X = (c_{35} + c_{15}S_1 + c_{25}S_4)(c_{45} + c_{14}S_3 + c_{24}S_6) - (c_{34} + c_{14}S_1 + c_{24}S_6)(c_{55} + c_{15}S_3 + c_{25}S_6) \quad [3-93]$$

$$\Delta_Y = (c_{45} + c_{15}S_2 + c_{25}S_5)(c_{34} + c_{14}S_1 + c_{24}S_4) - (c_{44} + c_{14}S_2 + c_{24}S_5)(c_{35} + c_{15}S_1 + c_{25}S_4) \quad [3-94]$$

$$\Delta = (c_{44} + c_{14}S_2 + c_{24}S_5)(c_{55} + c_{15}S_3 + c_{25}S_6) - (c_{45} + c_{14}S_3 + c_{24}S_6)(c_{45} + c_{15}S_2 + c_{25}S_5) \quad [3-95]$$

$$S_1 = \frac{c_{12}c_{23} - c_{22}c_{13}}{s} \quad [3-96]$$

$$S_2 = \frac{c_{12}c_{24} - c_{22}c_{14}}{s} \quad [3-97]$$

$$S_3 = \frac{c_{12}c_{25} - c_{22}c_{15}}{s} \quad [3-98]$$

$$S_4 = \frac{c_{12}c_{13} - c_{11}c_{23}}{s} \quad [3-99]$$

The natural frequency equation implies that for a variable stiffness plate, the variation in the stiffness gives rise to different natural frequencies across the plate. This is proven by the fact that the constants in equation (3-70 to 3-84) is governed by the stiffness matrices of the laminates.

For a variable stiffness curved plate, the natural frequency can be expressed as:

$$\omega_{eq} = \sqrt{\frac{\sum_{n=1}^m -(c_{13}K_U + c_{23}K_V + c_{33} + c_{34}K_X + c_{35}K_Y)_m}{\sum_{n=1}^m \rho h_m}} \quad [3-100]$$

where, m is the number of variable sections in a curved composite plate. Figure 41. portrays the stiffness variation for a plate with three variable stiffness sections implied by three different sets of A,B,D stiffness matrices. Therefore, natural frequencies are calculated for each section separately and the resultant equivalent natural frequency is the summation of the individual natural frequencies as shown in equation (3-100)

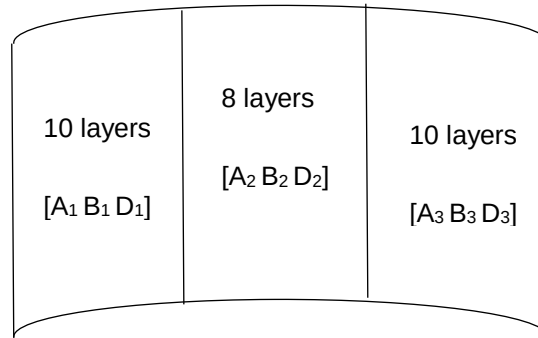


Figure 48 Stiffness variation across plate

For a variable stiffness plate, the stiffness constant, K_1 is defined by:

$$K_1 = \omega_{eq}^2 M_1 \quad [3-101]$$

Where M_1 is the mass of the variable stiffness sections. The final steps of the analytical modelling is to calculate the mode shapes frequencies ω_1 and ω_2 (equation) and the

constants of the analytical force function (equations 3-51). The analytical force function is defined as:

$$F_c^*(t) = K_2^*[A_1(C_1 - 1)\sin\omega_1 t + A_2(C_2 - 1)\sin\omega_2 t] \quad [3-102]$$

$$F_c^*(t) = \sqrt{\pi}\Gamma\left(\frac{p+1}{2}\right) \frac{2\Gamma\left(\frac{p}{2}+1\right) + \sqrt{\pi}\Gamma\left(\frac{p+1}{2}\right)}{4\Gamma^2\left(\frac{p}{2}+1\right) + \pi\Gamma^2\left(\frac{p+1}{2}\right)} \delta_m^{p-1} K_2 [A_1(C_1 - 1)\sin\omega_1 t + A_2(C_2 - 1)\sin\omega_2 t] \quad [3-103]$$

3.2.7 Analytical modelling flowchart of variable stiffness plate

A simple flowchart for the analytical modelling process of low velocity impact on curved variable stiffness plates is shown in Figure 49 below:

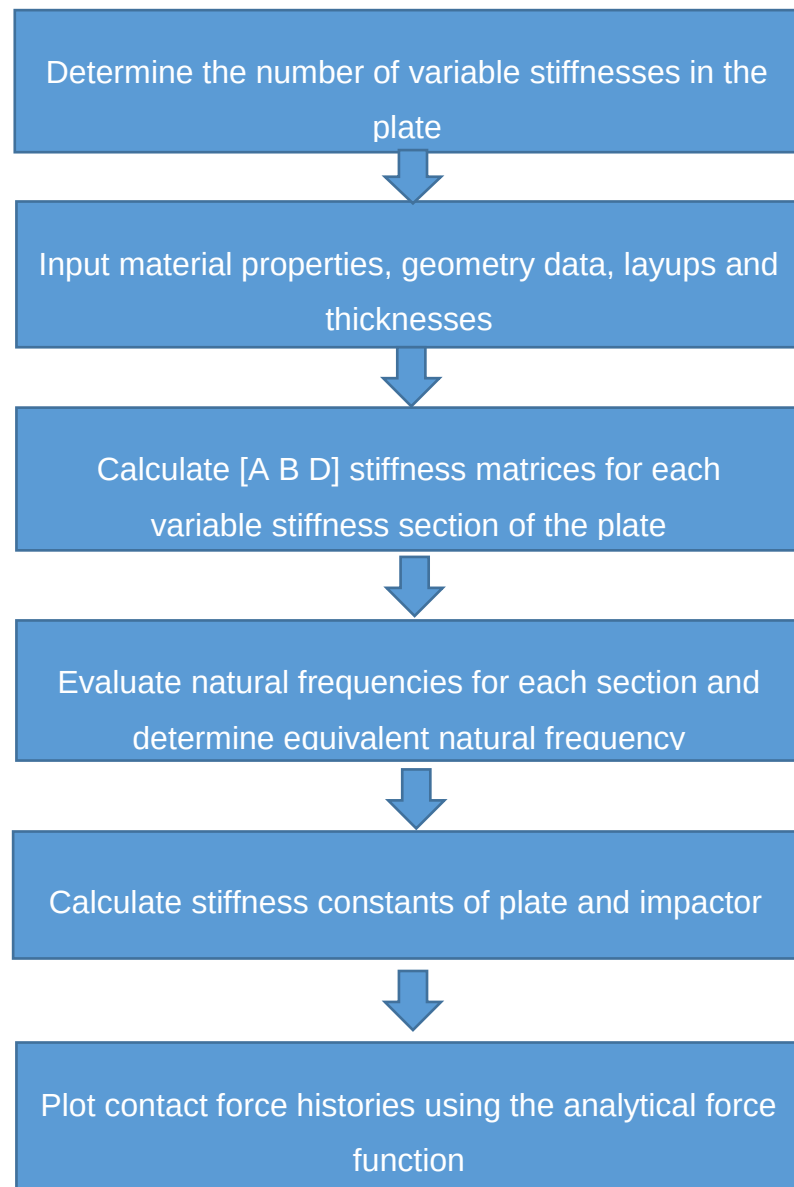


Figure 49 Analytical Modelling Process (Variable Stiffness Plate)

3.2.8 Theoretical models developed

Different geometries shown in Figures 50(a, b, c) are developed. In Figure 50, the length of the variable stiffness sections are of equal width. The main aim of developing these models is to analyze the effect of the number of variable stiffness sections in a curved plate and its location along the length of the plate.

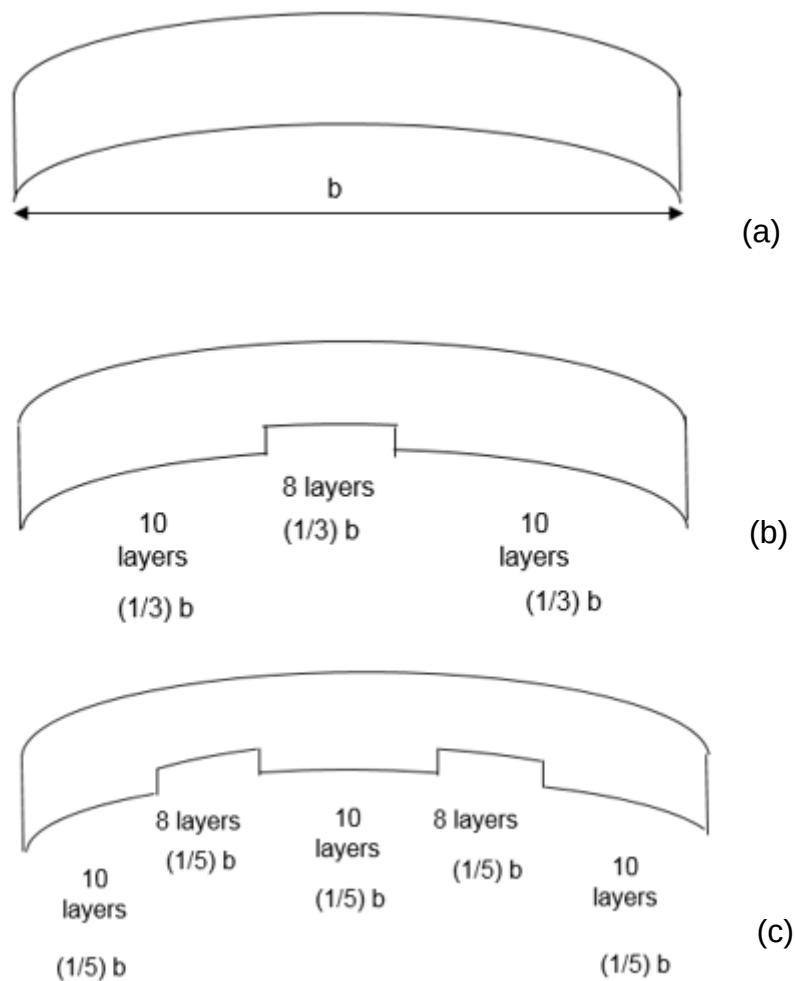


Figure 50 Designs of curved composite plates with a) uniform thickness, b) three variable thicknesses and c) five variable thicknesses and equal width

3.2.9 Validation (Curved plate)

3.2.9.1 Validation of present model with previous analytical studies

The initial step of analysing the theoretical models is validation with existing literature. The model was verified with low velocity impact studies on curved steel and composite plates. [44,45]. Gong [44] model consisted of a 48-ply laminate consisting a $[(\pm 45/0_2)_2/\pm 45/0/90]_{2s}$ lay-up fabricated from graphite/epoxy pre-preg. Mechanical properties and geometry dimensions can be found in Tables 4 and 5. The comparison of force – time history showed a good agreement between present results and previous published work (see Figure 51). Reference [44] is Khalili et al.'s work.

E_{11}	141.2GPa
$E_{22} = E_{33}$	9.72GPa
$G_{12} = G_{13}$	5.53GPa
G_{23}	3.74GPa
$\nu_{12} = \nu_{13} = \nu_{23}$	0.30
ρ	1536 kg/m ³

Table 5 CFRP Properties [45]

E(Young's Modulus)	207GPa
ν	0.30
ρ	7800kg/m ³
Tip diameter	20mm
Mass of the striker	3kg
Striker velocity	6m/s

Table 6 Properties of steel impactor [45]

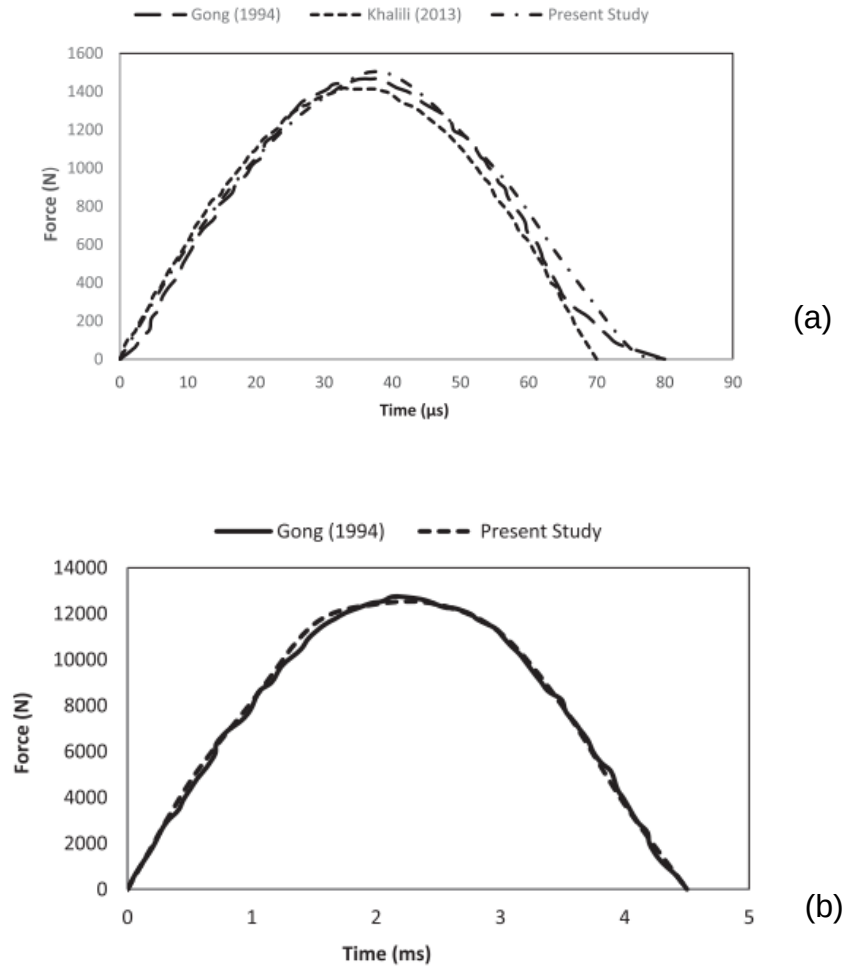


Figure 51 Comparison of force-time history for a) Steel plate and b) composite plate between present results and available results in the literature.

3.2.9.2 Validation of present analytical model with previous experimental research

The analytical model is then validated with a curved composite experiment. In this experiment, mass of the impactor is 30kg, with an impact energy of 6J. The layup 1 of the curved composite plate is $[0_2/45_2/-45_2/0_2/90_2]_s$ with a ply thickness of 0.125mm. Material properties of the composite are obtained from reference “Low-velocity heavy mass impact response of singly curved composites” [59]. The present analytical model matched well with the experimental research with an error less than 10%.

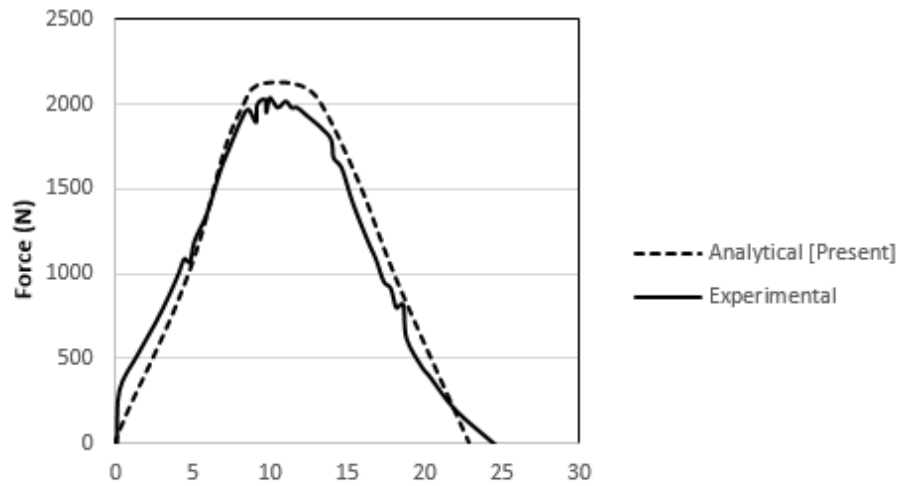


Figure 52 Validation with curved composite experiment (Layup 1)

Then the analytical model is validated successfully by using another layup 2 which was used in the experimental work. The stacking sequence of this layup is $[45_2/45_2/-45_2/0_2/90_2]_s$

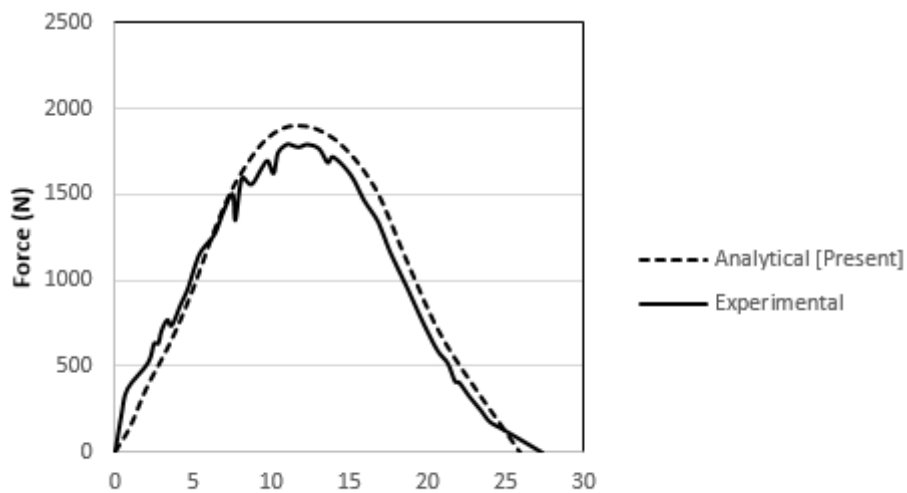


Figure 53 Validation with curved composite experiment (Layup 2)

3.3 Results and discussions

A parametric study is performed to analyze the effect of the governing factors affecting low velocity impact response of variable stiffness curved composite plates. Factors such as impactor velocity, layups, and geometries are studied. The layups studied are presented in Table 6 below.

Layup	Number of Sections	Layup
1	3	[-45 45 0 90 0] _s [-45 45 0 90] _s [-45 45 0 90 0] _s
2	3	[0 90 0 45 -45] _s [0 90 -45 45] _s [0 90 0 45 -45] _s
3	3	[0 90 0 90 0] _s [0 90 0 90] _s [0 90 0 90 0] _s
4	5	[-45 45 0 90 0] _s [-45 45 0 90] _s [-45 45 0 90 0] _s [-45 45 0 90] _s [-45 45 0 90 0] _s
5	5	[0 90 0 45 -45] _s [0 90 -45 45] _s [0 90 0 45 -45] _s [0 90 -45 45] _s [0 90 0 45 -45] _s
6	5	[0 90 0 90 0] _s [0 90 0 90] _s [0 90 0 90 0] _s [0 90 0 90] _s [0 90 0 90 0] _s

Table 7 Laminate designs of three and five sectioned curved composite plates.

3.3.1 Effect of contact stiffness on contact force history

The analysis on the effect of contact stiffness k_2 is shown in Figure 54. It can be seen from the developed models in section 3.2.8, the contact force increases with increase of the contact stiffness, k_2 , but the contact duration decreases. The higher k_2 yields to a higher effective contact stiffness, K_2^* . Therefore, higher contact stiffness between shell and striker indicates more spontaneous contact force. This means that a larger force is acting over a shorter time period. Likewise, higher modes of deformations are generated due to the shorter contact time as the impact by a stiffer striker is more spontaneous. (Figure 54)

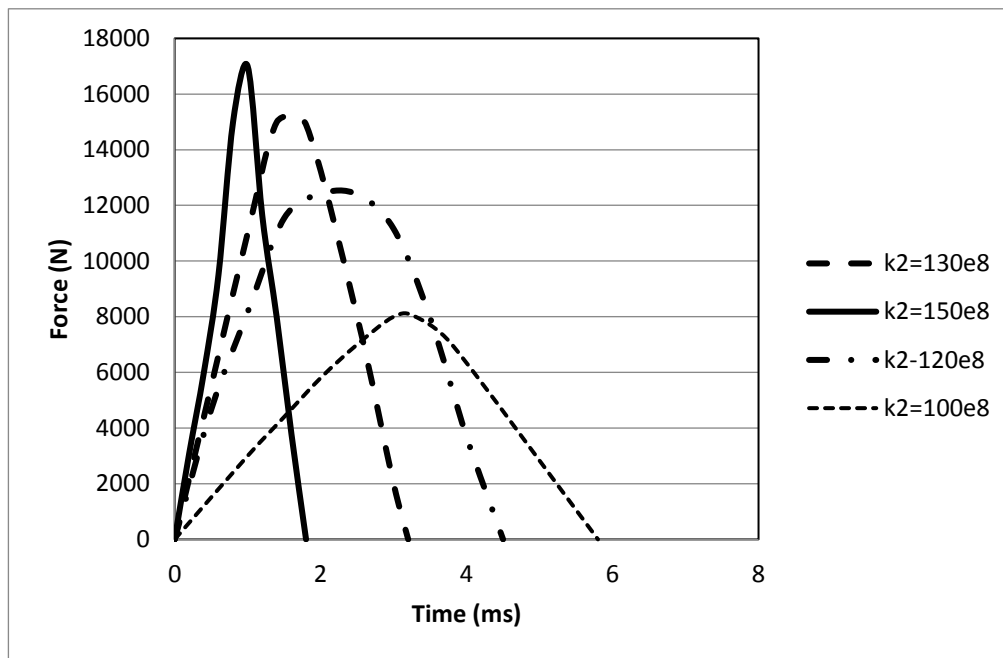


Figure 54 Effect of contact stiffness on contact force history.

3.3.2 Effect of stacking sequence (Equal widths)

The effect of layup and impact velocity was studied for two variable thickness composite plates with three and five sections respectively. Figure 55 shows the effect of fibre orientations and also impact velocity on the plate with three sections and equal widths. Layup 2 ([0/90/0/45/-45]_s // [0/90/-45/45]_s // [0/90/0/45/-45]_s) yielded the highest maximum contact force due to the reason that the plate stiffness and natural frequency was higher in this particular orientation. The contact duration for layup 2 is less than the similar time of other two cases. The maximum contact force for case 2

was 17% higher than the original sequence known as layup 1 ($[-45/45/0/90/0]_s // [-45/45/0/90]_s // [-45/45/0/90/0]_s$). The third sequence, layup 3 ($[0/90/0/90/0]_s // [0/90/0/90/0]_s // [0/90/0/90/0]_s$) yielded the smallest contact force and was 20% lesser than similar value of layup 1.

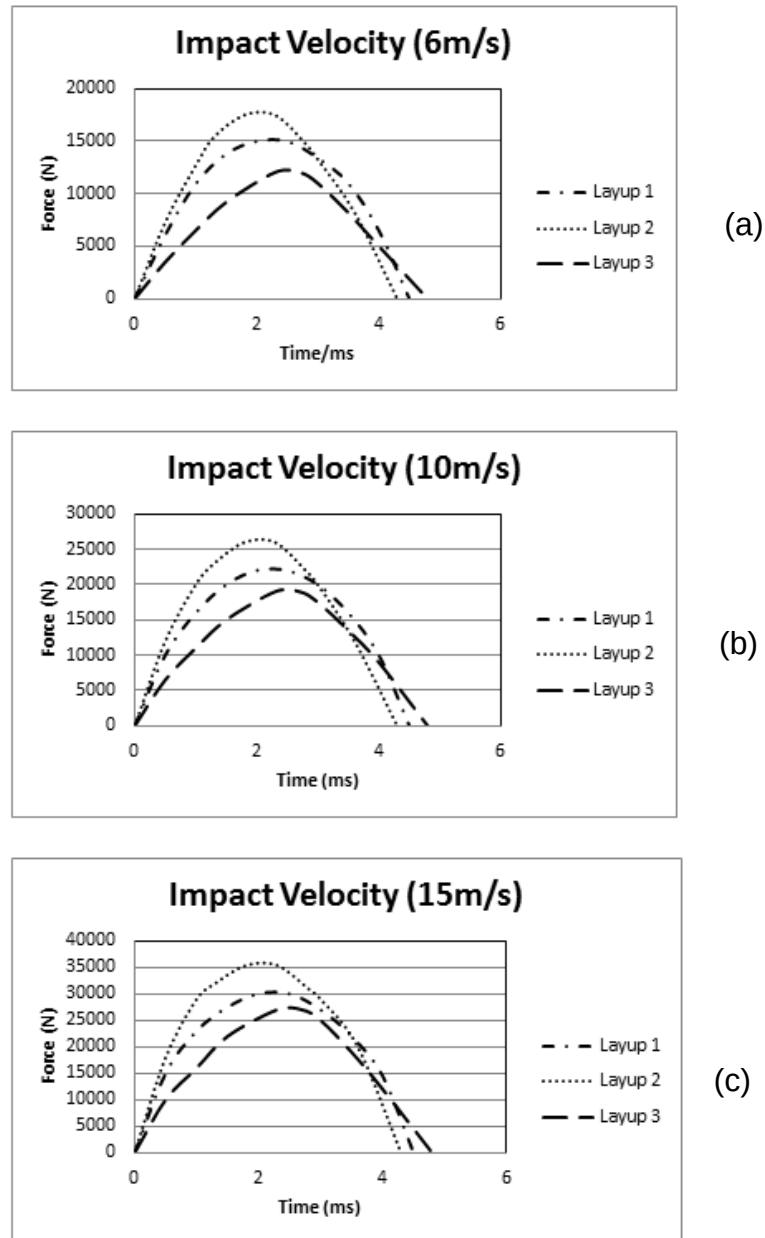
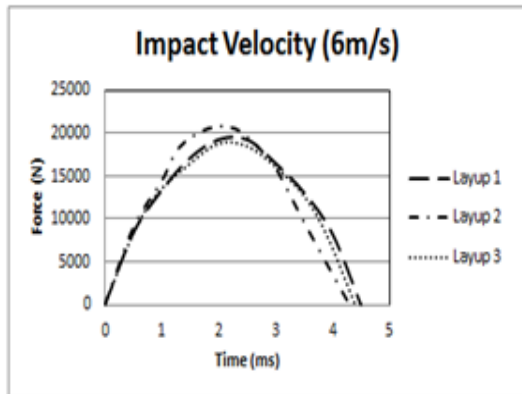


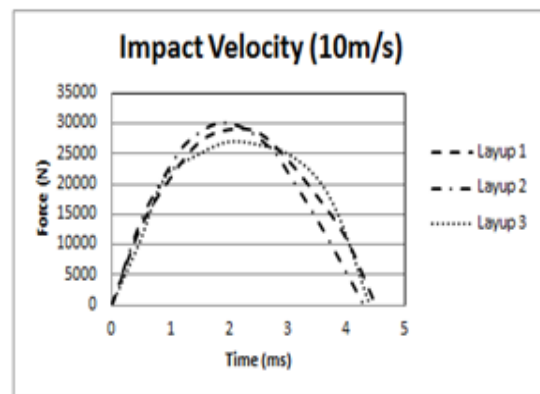
Figure 55 Effect of stacking sequence on the three sectioned plate with variable thicknesses a) 6m/s, b) 10m/s and c) 15m/s.

Even as the impact velocity increased to 10m/s in Figure 55(b), the contact force of layup 2 was higher than the other two layups. A change of impact velocity from 6-10m/s increased the maximum contact force from 17.5kN to 26.5kN. It can also be observed from Figure 55(a) and Figure 55(b) that the difference between the maximum contact forces decreases as the impact velocity rises. Increase of impact velocity to 15m/s increased the contact force of layup 2 to 35.4kN. It is noted that the contact duration remains unchanged.

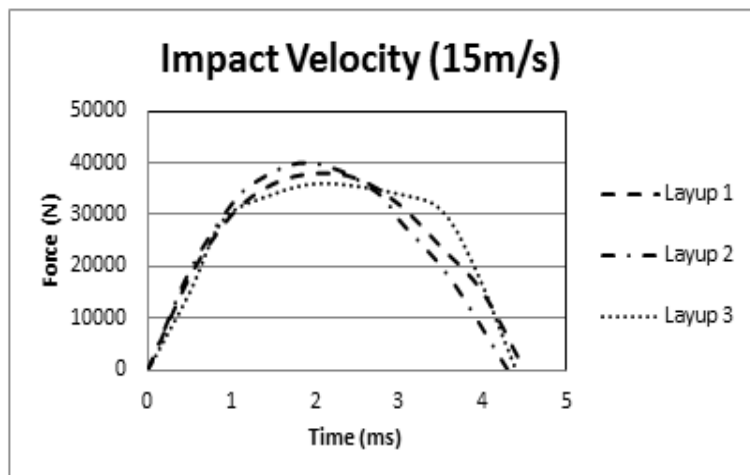
Figure 56(a, b, c) shows the effect of stacking sequence for an impact velocity of 6m/s for a five sectioned plate with variable thicknesses. It can be observed that as the number of sections increased, the effect of stacking sequence decreases.



(a)



(b)



(c)

Figure 56 Effect of stacking sequence on five sectioned plate with variable thicknesses

a) 6m/s b) 10m/s and c) 15m/s

3.3.3 Effect of impact velocity (Equal width)

The effect of impact velocity on the contact force is studied in this section. It is observed that the impact velocity has a direct effect on the contact force for all layups. Increasing of impact velocity from 10-15 m/s for layup 1 increased the maximum contact force by 36% as shown in Figure 57(a). The maximum contact force increased by 58% in layup 3 when the velocity increased from 6-10m/s. It is deduced that for all cases accompanied with a three sectioned plate, the increase of contact force was uniform, however, the contact duration remains unchanged. The reason for this is that the higher velocity gives rise to higher impact energy, thus requiring a larger deflection and contact force to dissipate it. The impact duration depends solely on the material properties of the striker and shell and therefore it remains constant.

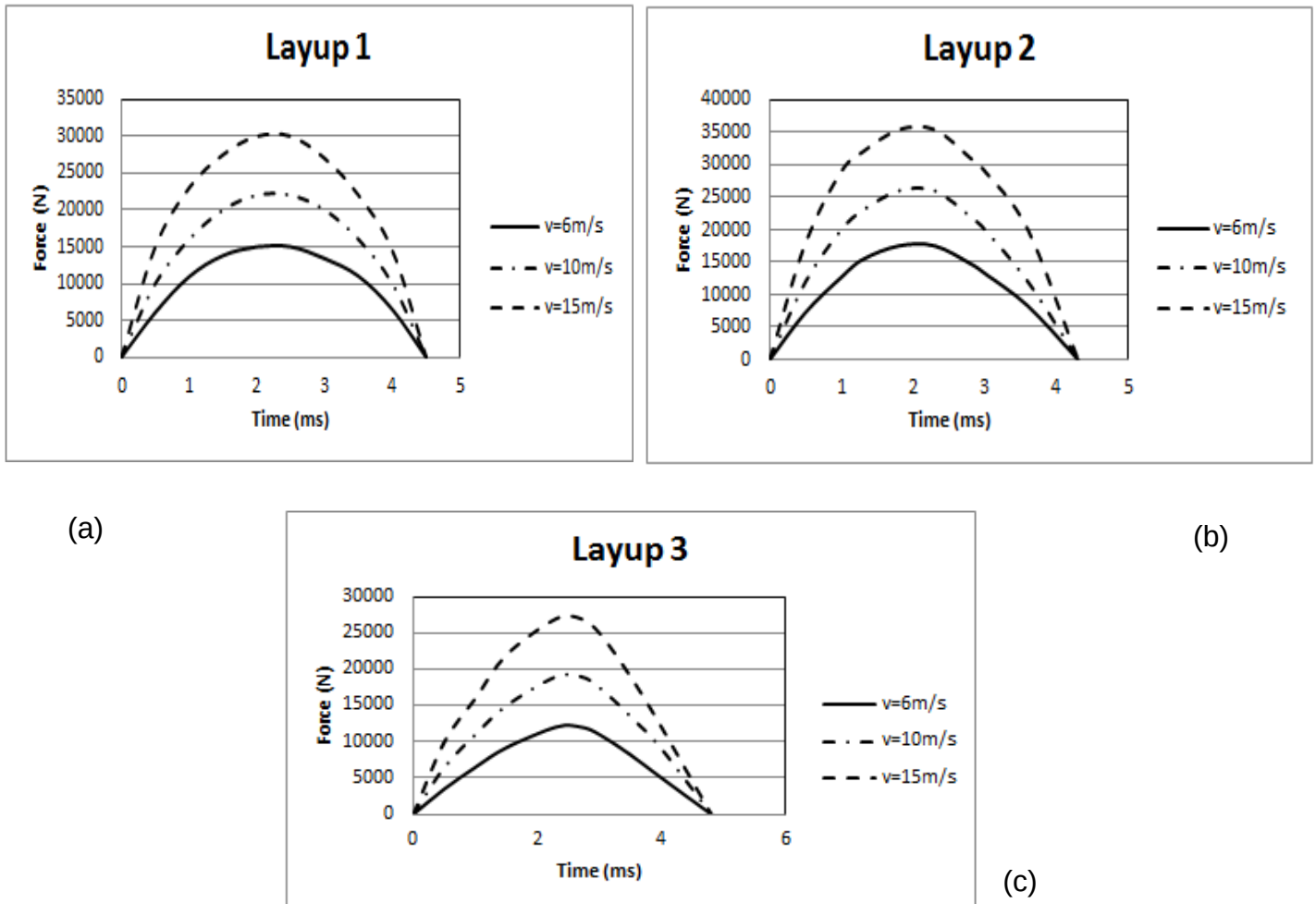


Figure 57 Effect of impact velocity on three sectioned equal width plate (a) Layup 1 (b) Layup 2 (c) Layup 3

3.4 Modified analytical model

3.4.1 Modifications

Modifications to the analytical model were made in order to incorporate the location of the variable stiffness section.

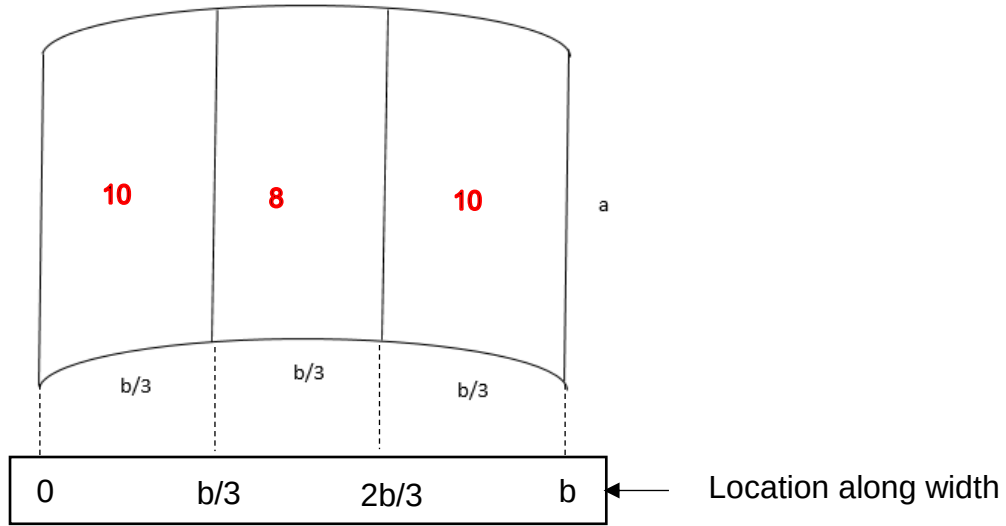


Figure 58 Modified analytical method

The proposed model is based on the natural frequency, ω_{eq} of the plate and $\omega_{1,2}$ is given as follows:

$$\omega_{eq} = \sqrt{\sum_{n=1}^m \frac{-(C_{13} K_U + C_{23} K_V + C_{33} + C_{34} K_X + C_{35} K_Y)_m}{\rho h_m}} \quad [3-104]$$

where,

$$\omega_{eq} = \omega_{eq(1)} + \omega_{eq(2)} + \omega_{eq(3)} \quad [3-105]$$

$$\omega_{eq(1)} = \int_0^{1/3b} \sqrt{\sum_{n=1}^m \frac{-(C_{13a} K_{Ua} + C_{23a} K_{Va} + C_{33a} + C_{34a} K_{Xa} + C_{35a} K_{Ya})_m}{\rho h_m}} \quad [3-106]$$

$$\omega_{eq(2)} = \int_{1/3b}^{2/3b} \sqrt{\sum_{n=1}^m \frac{-(C_{13b} K_{Ub} + C_{23b} K_{Vb} + C_{33b} + C_{34b} K_{Xb} + C_{35b} K_{Yb})_m}{\rho h_m}} \quad [3-107]$$

$$\omega_{eq(3)} = \int_{2/3b}^b \sqrt{\sum_{n=1}^m \frac{-(C_{13c} K_{Uc} + C_{23c} K_{Vc} + C_{33c} + C_{34c} K_{Xc} + C_{35c} K_{Yc}) m}{\rho h_m}} \quad [3-108]$$

$$\omega_{eq} = \omega_{eq(1)} + \omega_{eq(2)} + \omega_{eq(3)} \quad [3-109]$$

where, m is the number of variable sections in a curved composite plate.

3.4.2 Effect of modifications

It is observed that the modification of incorporating the location of the variable stiffness sections have reduced the maximum contact force. It is necessary to perform LS-DYNA numerical analysis to quantify the effect of these modifications. The accuracy of this modification is governed by the similarity with the numerical simulations performed in Chapter 6.

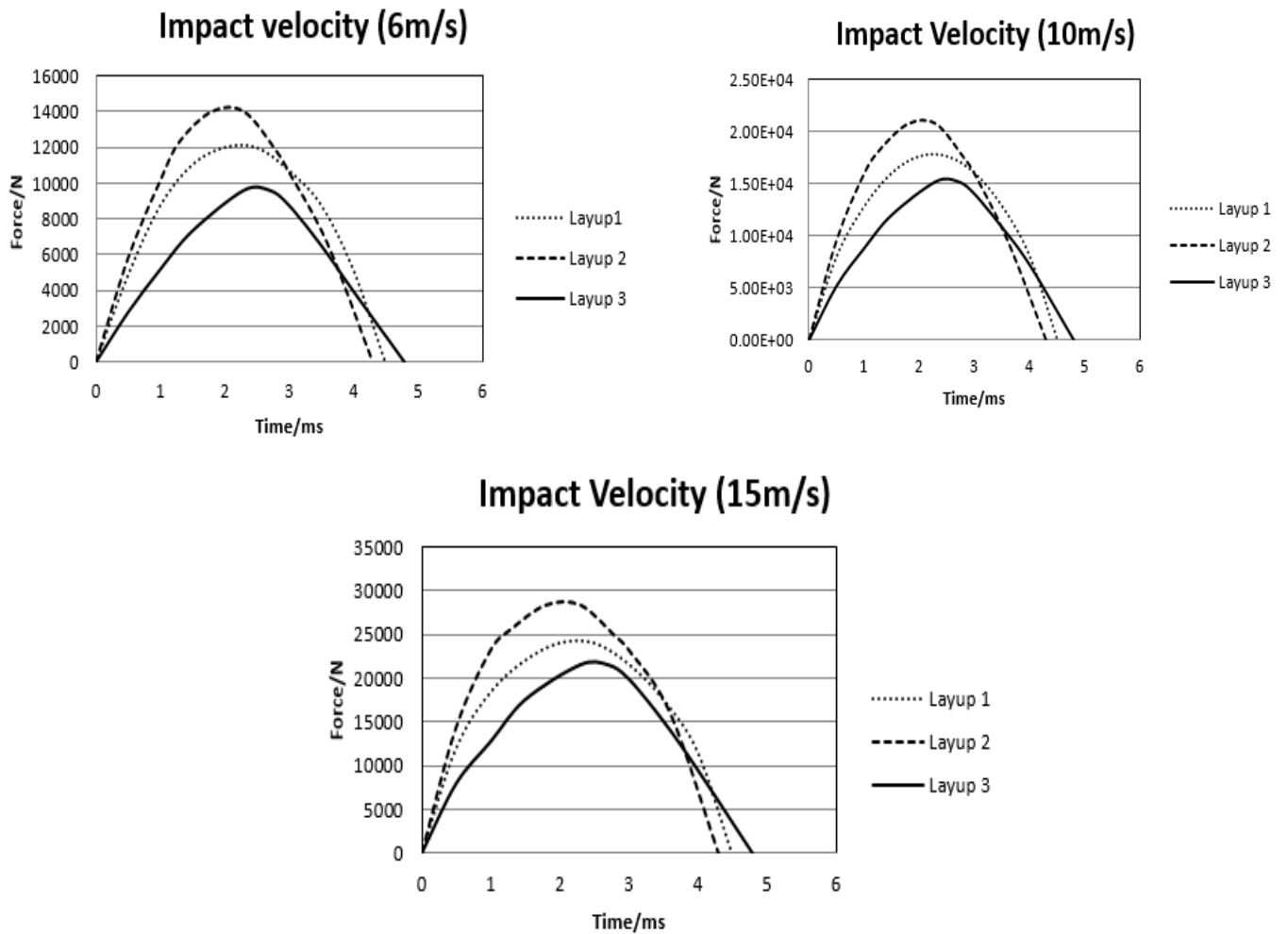


Figure 59 Effect of modification on Layups

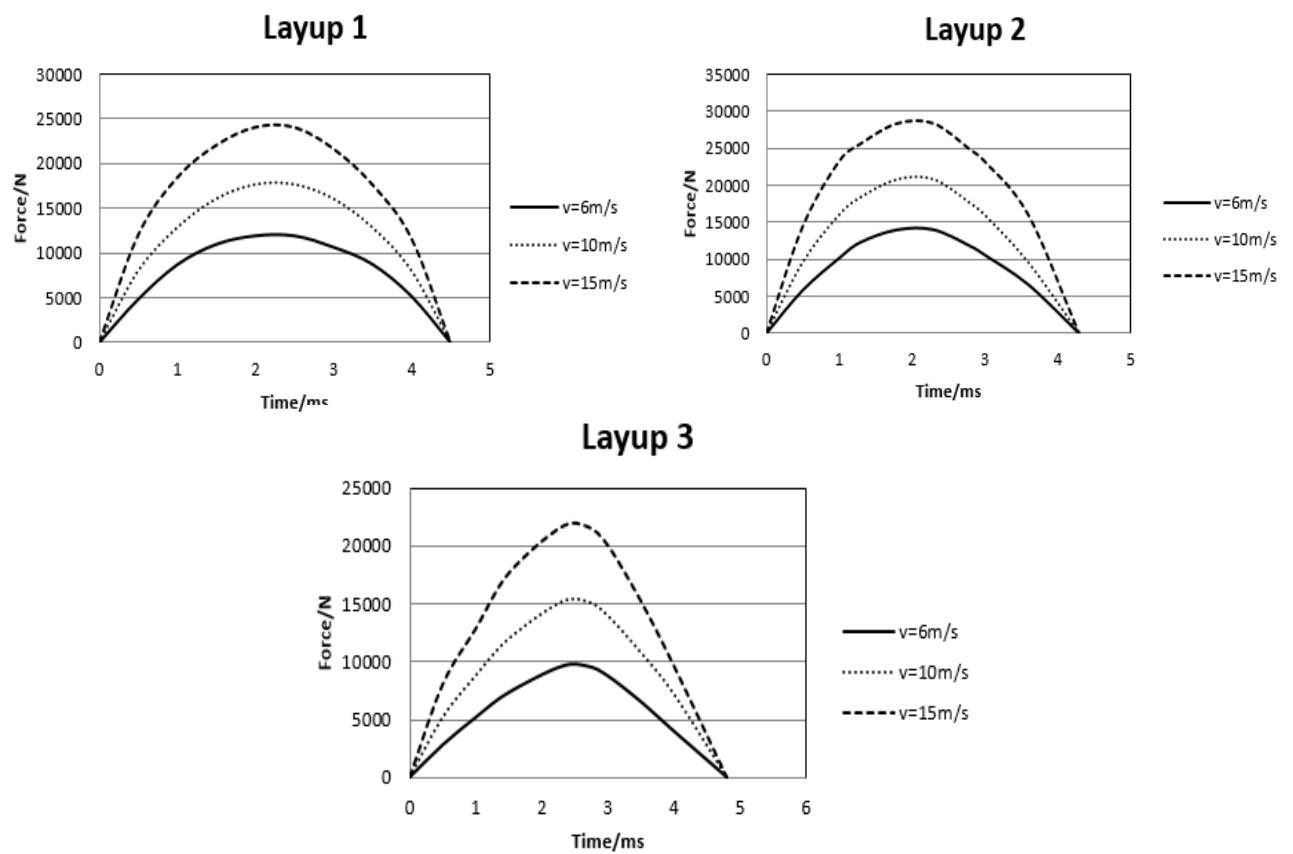


Figure 60 Effect of modifications on impact velocity

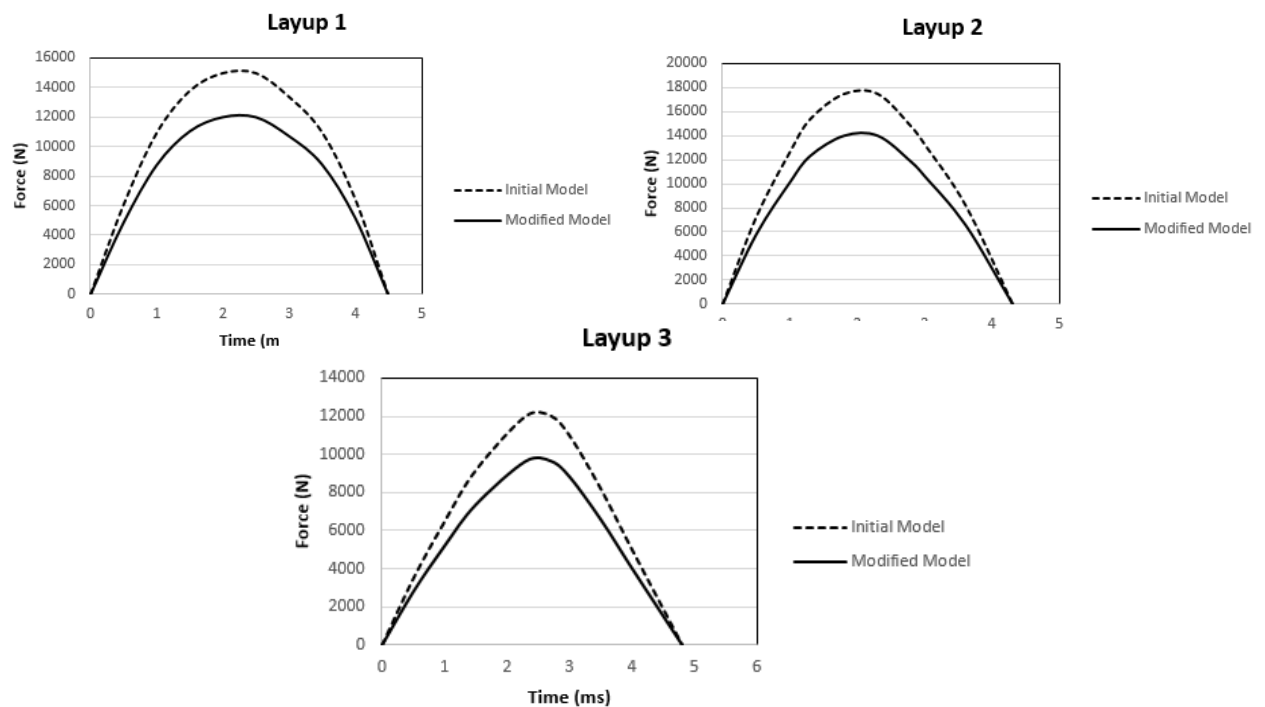


Figure 61 Model comparisons (Impact velocity of 6m/s)

3.4.3 Effect of impactor mass

The effect of impactor mass (including small mass impact) is also investigated. It is observed that as the impactor mass decreases, contact duration reduces.

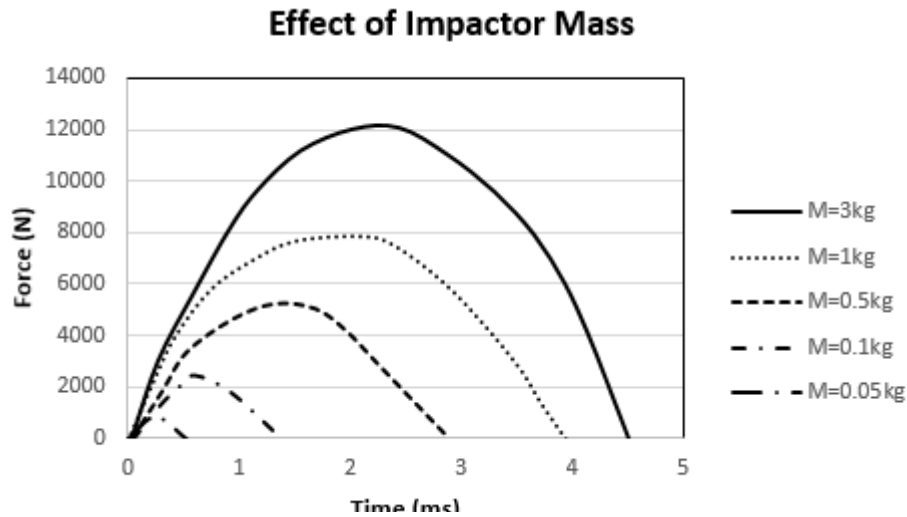


Figure 62 Effect of Impactor Mass

3.5 Structural Damping

The effects of incorporating structural damping into the system is also investigated.

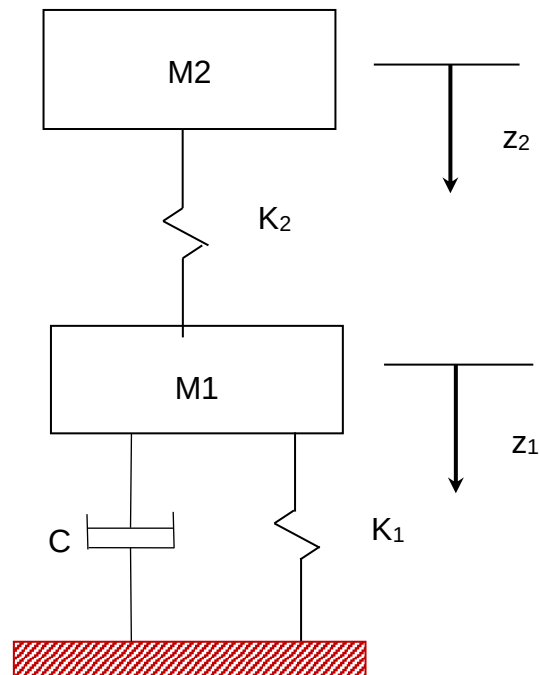


Figure 63 Two degree of freedom spring-mass-dashpot system

The frequencies for the two mode shapes and the force function are expressed in terms of material properties, stiffness and mass of striker/plate, initial velocity of the striker and also the structural damping of the system.

$$\omega_{1,2}^2 = \frac{1}{2} \left(\frac{k_1 + k_2}{M_1(1-\eta_{st})} + \frac{k_2}{M_2} \right) \mp \sqrt{\frac{1}{4} \left(\frac{k_1 + k_2}{M_1(1-\eta_{st})} + \frac{k_2}{M_2} \right)^2 - \frac{k_1 k_2}{M_1 M_2 (1-\eta_{st})}} \quad [3-110]$$

$$\mathbf{F}(t) = \mathbf{K}_2^* [\mathbf{A}_1 (\mathbf{C}_1 - \mathbf{1}) \sin \omega_1 t + \mathbf{A}_2 (\mathbf{C}_2 - \mathbf{1}) \sin \omega_2 t] \quad [3-111]$$

Incorporating structural damping into the system converts the equation of motion into:

$$[\mathbf{M}_s] \{\ddot{\chi}\} + [\mathbf{C}_s] \{\dot{\chi}\} + [\mathbf{K}_s] \{\chi\} = \{\mathbf{f}\} \quad [3-112]$$

Where $\{\chi\}$ is the element displacement vector; $[\mathbf{M}_s]$, $[\mathbf{C}_s]$ and $[\mathbf{K}_s]$ are the structural mass, damping and stiffness matrices respectively.

3.5.1 Structural loss factor

The method used to determine the structural loss factor, η_{st} is formulated by Gong and Lam [132] is adopted in this research. An impact event on a layered beam geometry is shown in Figure 64.

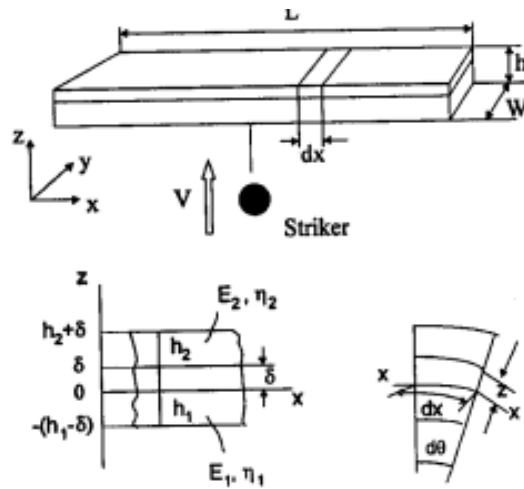


Figure 64 Impact on layered beam geometry [132]

The structural loss factor, η_{st} can be evaluated through the loss factor of the damping layer if the Young's Modulus and thickness ratios of the basic layers are known. [132]

$$\frac{\eta_{st}}{\eta_2} = \frac{\lambda_e \lambda_h}{(1 + \lambda_e \lambda_h)} \times \frac{3 + 6\lambda_h + 4\lambda_h^2 + 2\lambda_e \lambda_h^3 + \lambda_e^2 \lambda_h^4}{1 + 2\lambda_e(2\lambda_h + 3\lambda_h^2 + 2\lambda_h^3) + \lambda_e^2 \lambda_h^4} \quad [3-113]$$

where,

$$\lambda_h = \frac{h_2}{h_1} \quad \text{and} \quad \lambda_e = \frac{E_2}{E_1} \quad [3-114]$$

Results shown in Figure 65 depicts that inclusion of damping into the system has a low effect on the impact response for the impact mass of 3kg. Therefore, structural damping is not considered in the analytical models developed in latter sections.

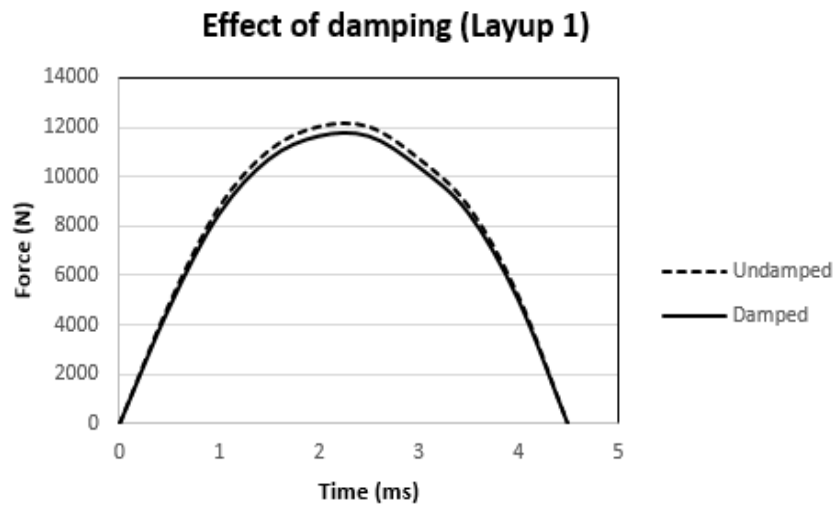
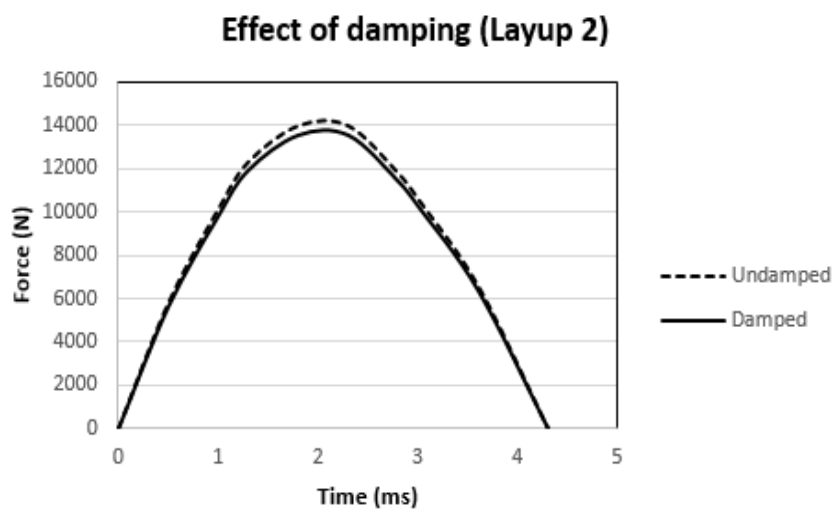
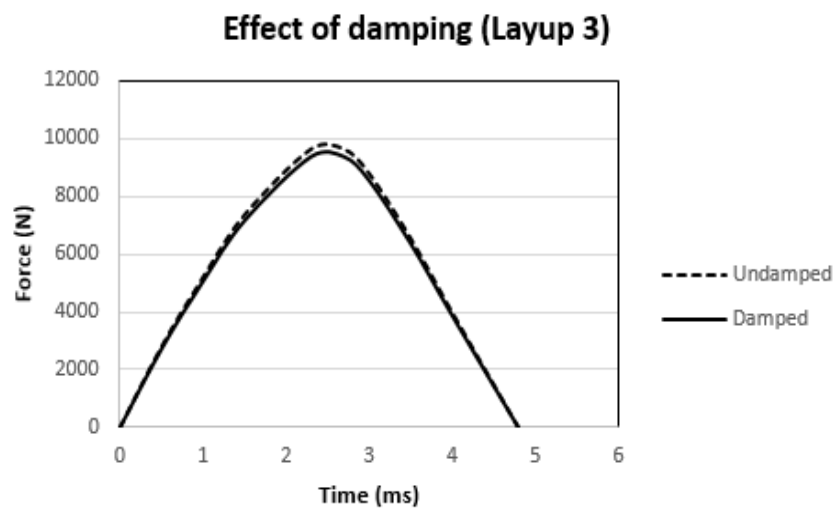


Figure 65 Damping effect on contact force history

3.6 Chapter Conclusions

In this chapter of the report, a mathematical model was developed to predict the impact response of curved fibre-reinforced polymer (FRP) composite plates with variable lateral stiffness which is a more realistic representation of composite structures compared to existing models. Variable stiffness composite sections are widely used in aerospace and automotive structures such as aircraft wing skins where usually the skin thickness varies from the root of the wing to the wing tip. Therefore, wing panels consist of composite sections with varying thicknesses. These varying thicknesses produces variable stiffness sections.

The analysis was performed first to validate the original model against available results in the literature and then it was used to study four geometry designs (3 sections equal/unequal widths, 5 sections equal/un-equal widths) with variable lateral stiffness. The outcomes were presented in the form of force-time history curves and maximum impact forces were calculated for each case. Then, a modified analytical model considering the effect the location of variable stiffness was introduced.

Main outcomes from the present chapter:

Natural frequency of the plate had an effect on the impact force. i.e. shells with higher natural frequencies at impact yields greater impact forces. This is mainly due to the fact that the plate stiffness increases with increase of natural frequency.

Higher contact stiffness between shell and striker gives yields in a higher maximum contact force.

Quasi-isotropic layup yielded a greater maximum contact force for both type of geometries of the variable stiffness curved plate. When compared with the cross-ply laminate sequence, quasi-isotropic layup produced an impact force which is 18% higher than that for the cross-ply layup.

Analytical predictions proved that the location has a significant effect on the impact response of variable stiffness plates. The incorporation of variable stiffness position reduced the maximum impact force by 20%.

When the impact velocity was increased from 6-10m/s, maximum impact force improved by 46%.

Inclusion of damping into the model had a lower effect on impact response for the specific mass of 3kg which was considered in the study, and therefore was neglected from the analytical model. It is anticipated that this knowledge may benefit the composite industry in providing a better understanding of structural health monitoring and protection mechanisms of advanced composite structures in aerospace and automotive sections. This may help to significantly reduce calculation time during design, analysis and manufacturing as well as improving the quality of final products.

Chapter 4:

Post Impact damage Modelling of
damaged variable stiffness curved
composites

4 Post Impact Damage Modelling of damaged variable stiffness curved composites

4.1 Introduction and review of previous works

Review of previous works proved that studies related to analytical modelling of low velocity impact response of damaged composite structures were limited and was identified as an area to be addressed specially focusing on variable stiffness curved composite plates. Almost all composite structures undergo damage under operations and therefore it is vital to understand how damaged structures perform under further impact loading. This section of the research aims to fill that gap through development of analytical methods.

Many researchers have investigated different mathematical models to predict the impact damage behaviour of various composite materials and structures. However, the need to develop a mathematical model to predict the damage of variable stiffness plates was identified. Some of these models which are used to study the impact behaviour of composite structures by external objects are briefly discussed below. There are three methods in which the impact load response of a laminate can be analysed: (1). Experimental Techniques (2). FE simulations (3). Analytical models. Experimental models are usually expensive in terms of manufacturing and time. However they are required to verify analytical and numerical models. FE simulations require large modelling efforts in regards to modelling geometry and materials to obtain accurate results. Analytical models are preferred sometimes to understand the first stage effect on load on a structure. The main aim of this study is to develop a mathematical model to accurately predict the response of a damaged curved composite plate with variable stiffnesses.

Studies carried out by Singh and Mahajan [58] depicted that the force and deflection response of the laminate is largely dependent upon the extent of damage in composite laminates. They developed an analytical model in order to predict the inter-laminar and intra-laminar damage effects on the stiffness of the structure. The damage caused a degradation of stiffness at the impact location and therefore a lesser contact force. The spring-mass model predicted by them matched well with the FE simulation results. Matrix cracking and delamination cause significant change in the characteristics of the

plate which are dependent on the extent of damage. Shahid et al. [59] recognized the impact force response as a major parameter characterising impact damage resistance of laminated composites. They developed an analytical model to predict the damage resistance of composite laminates beyond the initial damage state through modifying the spring constant in Hertzian contact law which is a function of the extent of damage in composites

Olsson [60] developed an analytical model to predict small mass impact on plates with delamination growth, however this model neglected stiffness degradation effects due to matrix cracking and delamination. Riccio et al. [70] used two methods in cohesive zone elements to simulate inter-laminar (delaminations) damage behaviour and Hashin failure criteria to simulate intra-laminar failure mechanisms. Comparison between these methods showed no appreciable difference for maximum contact force, maximum displacements and energy loss but were substantially different in damage evolution. The study proved that more refined models are required to predict the impact induced damage initiation and propagation, but efficient damage modelling methods such as the use of failure criteria are efficient at a preliminary design stage.

Akbarzadeh et al. [129] examined the vibration response of fiber-steered laminated plates manufactured by automated fibre placement. Third order shear deformation theory was used to predict the structural and vibration responses. They found that placing large proportion of 0° plies at the centre of the plate increased the in-plane stiffness and the placement of larger proportions of 45° plies increased the out-of-plane stiffness and vibration responses. Najafi et al. [133] analytically evaluated the impact response of functionally graded doubly curved panels using Winkler-Pasternak elastic foundation. This model also considered the thermal effects and time dependant material properties. The effect of key parameters such as materials property gradient, impactor velocity, curvatures, aspect ratio of panel on the impact response were studied. It was observed that aspect and curvature ratio had the greatest influence on the impact response. Park [134] numerically studied the behaviour of steel and composite plates under low-velocity impact. The composite plate demonstrated a displacement two time more compared to the steel plate for different impact loads.

Shi et al. [135] used the stress-based criteria for damage initiation and fracture mechanics approach for its evolution to predict the impact damage of composite laminates in the form of intra- and inter-laminar cracking.

This part of the research reports the development of new mathematical models and to employ this to predict the post impact damage response of curved composite panels with variable stiffness. The study also explores into the combinations of layups able to withstand higher impact damage, thus opening a new gateway to develop higher impact resistant composite structures.

4.2 Objective

The main objective of this chapter is to focus on the post impact damage modelling of damaged variable stiffness curved composite plates. The effect of subsequent impact on the contact force histories of such plates are studied. Two damage modelling approaches are proposed which involves localisation of the damaged region.

4.3 Damage modelling-approach 1

The post impact damage modelling is achieved through the reduction of stiffness in the damaged region. A new damage modelling approach is proposed through the reduction of thickness in the damaged region. The reduced stiffness is achieved through reducing the damaged ply thickness to zero. Since damage modelling is a complex process, it is assumed that the plies are damaged during the impacts. For instance, it is assumed that during the first impact none of the plies are damaged and during the second impact one ply is damaged from the damaged region. The damage model is split into five sections for the 3 sectioned plate (Figure 65) and seven sections for the 5 sectioned plate (Figure 67) with reduced stiffness. Ply removal is made from the bottom of the damaged region.

$$\omega_{eq(a)} = \int_0^{1/5b} \sqrt{\frac{\sum_{n=1}^m -(C_{13a} K_{Ua} + C_{23a} K_{Va} + C_{33a} + C_{34a} K_{Xa} + C_{35a} K_{Ya})m}{\rho h_m}} \quad [4-1]$$

$$\omega_{eq(b)} = \int_{1/5b}^{2/5b} \sqrt{\frac{\sum_{n=1}^m -(C_{13b} K_{Ub} + C_{23b} K_{Vb} + C_{33b} + C_{34b} K_{Xb} + C_{35b} K_{Yb})m}{\rho h_m}} \quad [4-2]$$

$$\omega_{eq(c)} = \int_{2/5b}^{3/5b} \sqrt{\frac{\sum_{n=1}^m -(C_{13c} K_{Uc} + C_{23c} K_{Vc} + C_{33c} + C_{34c} K_{Xc} + C_{35c} K_{Yc})m}{\rho h_m}} \quad [4-3]$$

$$\omega_{eq(d)} = \int_{3/5b}^{4/5b} \sqrt{\frac{\sum_{n=1}^m \frac{-(C_{13d} K_{Ud} + C_{23d} K_{Vd} + C_{33d} + C_{34d} K_{Xd} + C_{35d} K_{Yd})m}{\rho h_m}}{dx} \quad [4-4]$$

$$\omega_{eq(e)} = \int_{4/5b}^b \sqrt{\frac{\sum_{n=1}^m \frac{-(C_{13e} K_{Ue} + C_{23e} K_{Ve} + C_{33e} + C_{34e} K_{Xe} + C_{35e} K_{Ye})m}{\rho h_m}}{dx} \quad [4-5]$$

$$\omega_{eq(damaged)} = \omega_{eq(a)} + \omega_{eq(b)} + \omega_{eq(c)} + \omega_{eq(d)} + \omega_{eq(e)} \quad [4-6]$$

$\omega_{eq(c)}$ represents the natural frequency of the damaged region.

In this damage modelling approach, the stiffness constant of the plate is calculated by the sum of the individual stiffnesses of the five sections. Stiffness constant, K_1 is defined by:

$$K_1 = K_{1(a)} + K_{1(b)} + K_{1(c)} + K_{1(d)} + K_{1(e)} \quad [4-7]$$

Where

$K_{1(c)}$ is the stiffness constant of the damaged region.

The force function for the damaged model can be expressed as

$$F_{c(damaged)}^*(t) = K_2^*[A_1(C_1 - 1)\sin\omega_1 t + A_2(C_2 - 1)\sin\omega_2 t] \quad [4-8]$$

Constants of equation [4-8] are re-calculated for the plate with damage. In Figure 65b, a particular level of damage is already incorporated into the plate. This is the main concept in the study that analyses the effect of the level of damage on the impact response of variable stiffness plates.

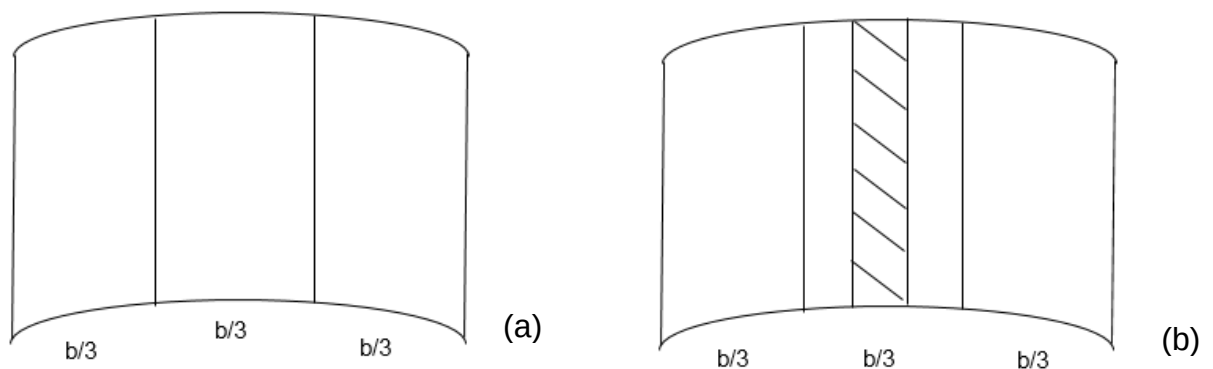


Figure 65 Analytical Models (a) Undamaged plate, (b) Damaged Plate

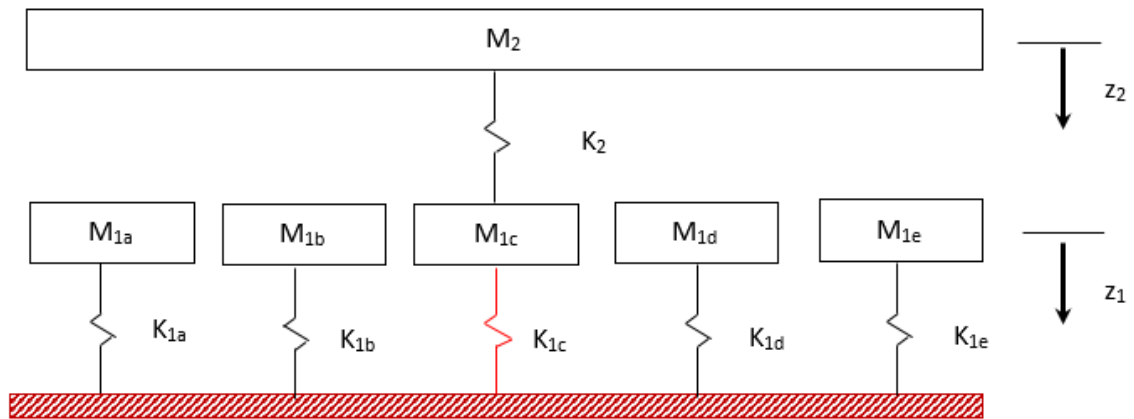


Figure 66 Spring-Mass model (3 section damaged plate)

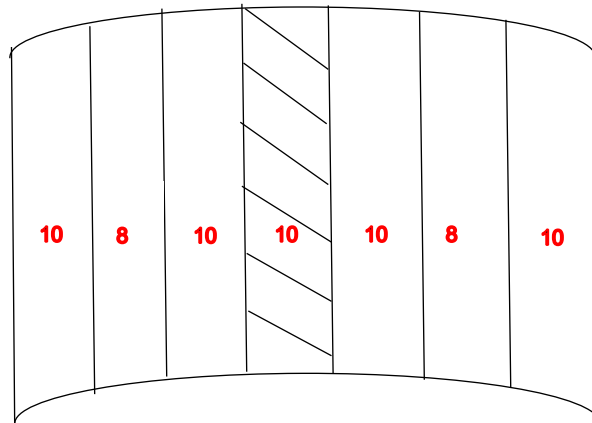


Figure 67 Damage model (5 section plate)

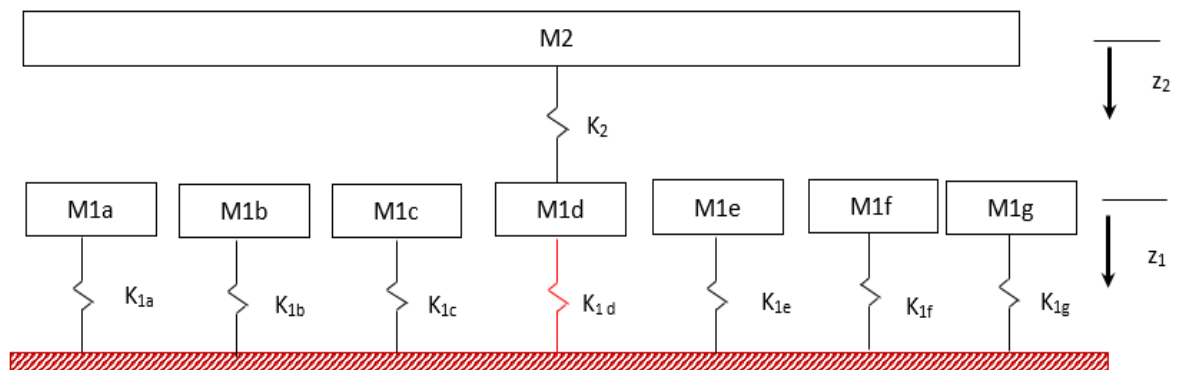


Figure 68 Spring-Mass model (5 section damaged plate)

4.4 Layups studied

The layups analysed in the study are listed in Table 7. The same layups are used as in Table 6 are used to maintain linearity in comparison. In each case the no. of sections and the layups of the undamaged and damaged model are displayed. The layup where the impact damage occurs is highlighted in red in the damage model. In all cases, the orthotropic nature of the laminate is maintained since the above derived equations are valid only for an orthotropic composite laminate (see Table 7).

<i>Layup</i>	<i>No. of sections (Undamaged Model)</i>	<i>No. of Sections (Damaged Model)</i>	<i>Layup (Damaged Model)</i>
1	3	5	$[-45/45/0/90/0]_s$ $[-45/45/0/90]_s$ $[-45/45/0/90]_s$ $[-45/45/0/90]_s$ $[-45/45/0/90/0]_s$
2	3	5	$[0/90/0/45/-45]_s$ $[0/90/-45/45]_s$ $[0/90/-45/-45]_s$ $[0/90/-45/45]_s$ $[0/90/0/45/-45]_s$
3	3	5	$[0/90/0/90/0]_s$ $[0/90/0/90]_s$ $[0/90/0/90]_s$ $[0/90/0/90]_s$ $[0/90/0/90/0]_s$
4	5	7	$[-45/45/0/90/0]_s$ $[-45/45/0/90]_s$ $[-45/45/0/90/0]_s$ $[-45/45/0/90/0]_s$ $[-45/45/0/90/0]_s$ $[-45/45/0/90/0]_s$ $[-45/45/0/90/0]_s$

5	5	7	$[0/90/0/45/45]_s$ $[0/90/-45/45]_s [0/90/0/45/-45]_s$ $[0/90/0/45/-45]_s$ $[0/90/0/45/-45]_s$ $[0/90/0/45/-45]_s$
6	5	7	$[0/90/0/90/0]_s$ $[0/90/0/90]_s$ $[0/90/0/90/0]_s$ $[0/90/0/90/0]_s$ $[0/90/0/90/0]_s$ $[0/90/0/90]_s$ $[0/90/0/90/0]_s$

Table 8 Layups (Post impact damage model)

4.5 Discussions (damage modelling approach 1)

This paper studies the effect of damage on the contact force histories of the plate with 3 sections and 5 sections respectively for the layups in Table 7. A ply removal technique is adopted which reduces the thickness of the damage region and thus the stiffness is degraded in that region. Removal of plies represent increase of damage level in the curved plate. Through this analysis, the best layups that can withstand impact damage for variable stiffness composites can be deduced. The present analysis proves that Layup 1 ($[-45/45/0/90\ 0]_s/[-45/45/0/90]_s/[-45/45/0/90/0]_s$) yielded the highest impact force without damage. The maximum contact force without damage was 2.5% higher than layup 2 and 24% higher than layup 3. The results showed that inclusion of more 0° plies resulted in a relatively lower maximum impact force. For the damaged model, the 3 sectioned plate was split into 5 sections to incorporate damage. The removal of a single ply from the damaged region resulted in an impact force drop of 7% of the undamaged plate for Layup1. For Layup 2, this drop was 12% and for Layup 3 was 9%. The highest drop in impact force for Layup 1 was noticed in between removing plies 6 and 7. This drop was 35% when compared to 36% of Layup 2's highest drop of force observed between removing plies 6 and 7. The highest difference between impact forces for Layup 3 which is 39% was observed in between ply removal 5 and 6. This analysis shows that significant weakening of the structure was observed between ply removals 5 and 7. The analysis also proved that Layup 2 was the best design for the plate with 3 sections, since the induced damage

affected the impact force history lesser and mainly since it had a higher contact force after removing all plies from the damaged region (see Figure 69-71).

The Layups 4, 5 and 6 are associated with the damage plate with 5 sections. The present analysis proved that Layup 5 $[0/90/0/45/-45]_s/[0/90/-45/45]_s/[0/90/0/45/-45]_s/[0/90/-45/45]_s/[0/90/0/45/-45]_s$ yielded the highest contact force of 14361N. This maximum force was 6.5% higher than that of Layup 4 and 9.1% higher than that of Layup 6. It is observed that shifting the position of the 45° and -45° plies in the laminate played an integral role in the change of the impact force. The 5 section plate was split into 7 sections to incorporate damage. The removal of a single ply from the damaged region resulted in an impact force drop of 6% of the undamaged plate for Layup 4. For Layup 5, this drop was 7% and 6% for Layup 6. Therefore, a higher impact force drop was observed at damage initiation in Layup 5. Similarly, in the case of the 3 section damage plate, the highest drop in impact force for Layup 1 was noticed in between removing plies 6 and 7. This force drop was 47% when compared to 41% of Layup 2's highest force drop observed between removing plies 6 and 7. The highest difference between impact forces for Layup 3 which is 60% was observed in between ply removal 6 and 7. Layup 4 was able to sustain the highest impact force after removal of all plies proving that it is the best layup design able to withstand damage even when Layup 5 yielded a higher impact force without damage induced (see Figure 72-74).

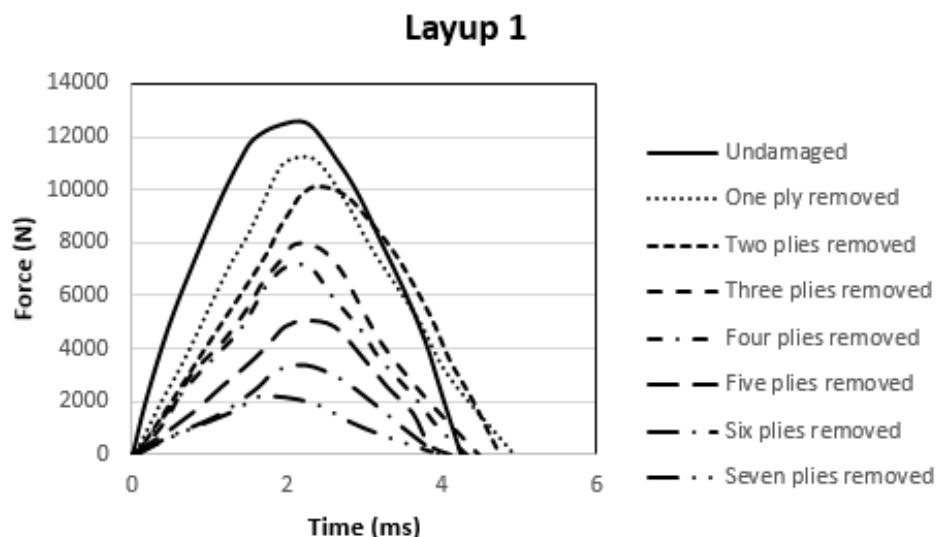


Figure 69 Layup 1 (Contact force history)

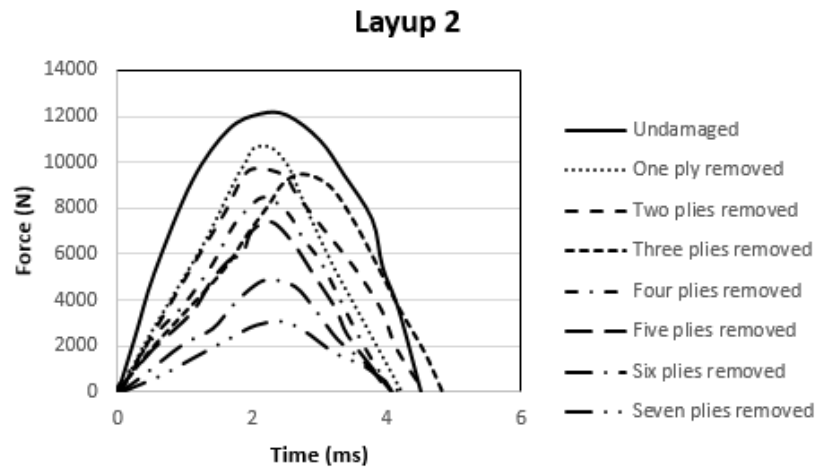


Figure 70 Layup 2

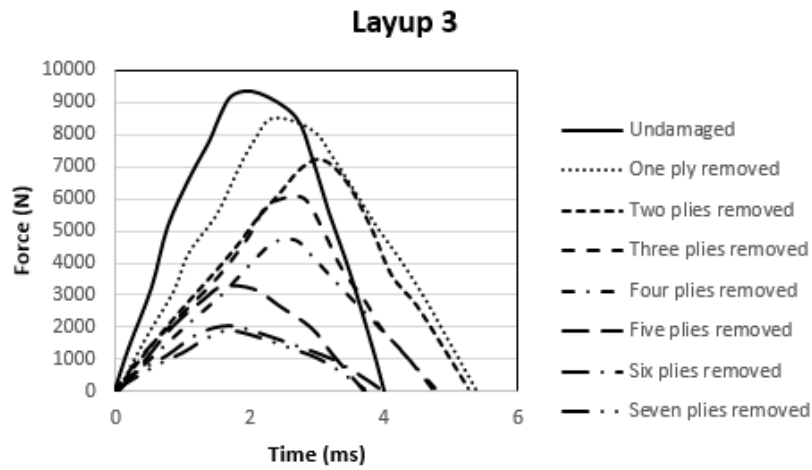


Figure 71 Layup 3

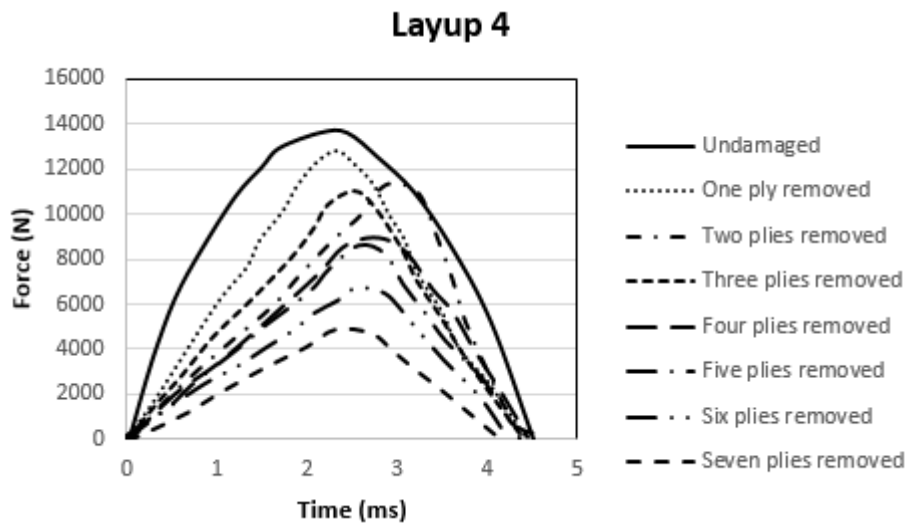


Figure 72 Layup 4

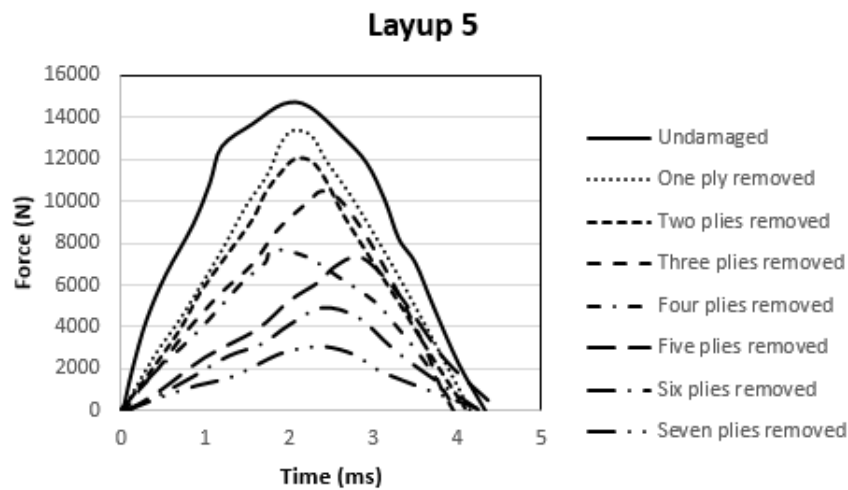


Figure 73 Layup 5

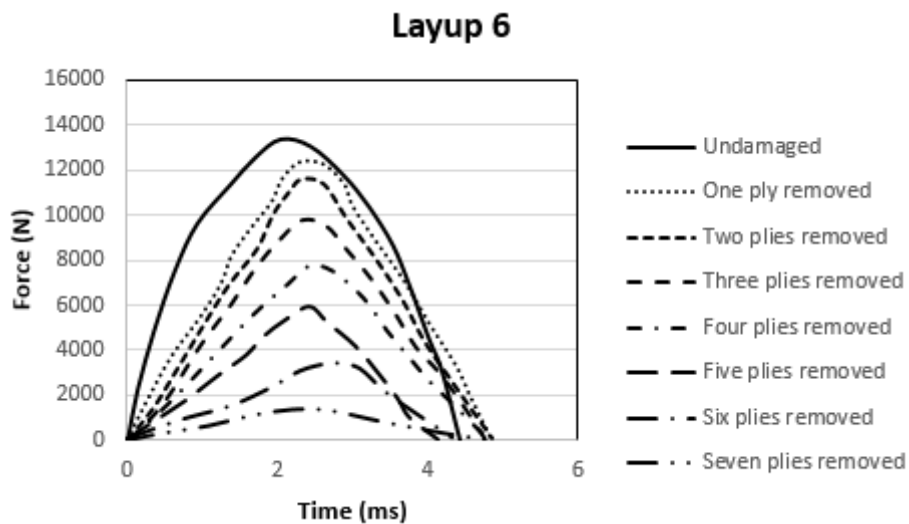


Figure 74 Layup 6

4.6 Modified approach to damage modelling (Damage Modelling approach 2)

4.6.1 Modifications made

A second damage modelling approach is proposed to further localise the damaged area of impact. The damage model is split into seven sections for the 3 sectioned plate (Figure 75) and nine sections for the 5 sectioned plate (Figure 77) with reduced stiffness. The thickness of the variable stiffness section subjected to impact is

decreased in order to simulate the damaged layers. Due to the introduction of the damaged region, there will be seven variable stiffness regions as shown in Eq. (4-9) – (4-17). The natural frequencies for the sections of the plate are calculated considering the location of the variable stiffness section. The damaged section is defined by the dimensions of the impactor and corresponds to an area $20mm \times 20mm$, thus refining the model. This damage model assumes that the damage caused by the spherical impactor is equivalent to a square region of $20mm \times 20mm$. In the following equations $\omega_{eq(a)}$ denotes the equivalent natural frequency of section 1 and similar for the other variable stiffness sections.

Section 1

$$\omega_{eq(a)} = \int_0^{1/5b} \sqrt{\frac{\sum_{n=1}^m -(C_{13(a)} K_{U(a)} + C_{23(a)} K_{V(a)} + C_{33(a)} + C_{34(a)} K_{X(a)} + C_{35(a)} K_{Y(a)})}{\rho t_m(a)}} m \quad [4-9]$$

Section 2

$$\omega_{eq(b)} = \int_{1/5b}^{2/5b} \sqrt{\frac{\sum_{n=1}^m -(C_{13(b)} K_{U(b)} + C_{23(b)} K_{V(b)} + C_{33(b)} + C_{34(b)} K_{X(b)} + C_{35(b)} K_{Y(b)})}{\rho t_m(b)}} m \quad [4-10]$$

Section 3

$$\omega_{eq(c)} = \int_{2/5b}^{\left(\frac{b}{5}\right)-20} \sqrt{\frac{\sum_{n=1}^m -(C_{13(c)} K_{U(c)} + C_{23(c)} K_{V(c)} + C_{33(c)} + C_{34(c)} K_{X(c)} + C_{35(c)} K_{Y(c)})}{\rho t_m(c)}} m \quad [4-11]$$

In the present model, Section 4 consists of 3 regions (two undamaged regions and one damaged region)

Section 4 (undamaged region)

$$\omega_{eq(da)} = \int_{\left(\frac{b}{5}\right)-20}^{20+\left(\frac{b}{5}\right)-20} \sqrt{\frac{\sum_{n=1}^m -(C_{13(da)} K_{U(da)} + C_{23(da)} K_{V(da)} + C_{33(da)} + C_{34(da)} K_{X(da)} + C_{35(da)} K_{Y(da)})}{\rho t_m(da)}} m \quad [4-12]$$

Section 4 (damaged region)

$$\omega_{eq(db)} = \int_{\left(\frac{b}{5}\right)-20}^{20+\left(\frac{b}{5}\right)-20} \sqrt{\frac{\sum_{n=1}^m -(C_{13}(db) K_U(db) + C_{23}(db) K_V(db) + C_{33}(db) + C_{34}(db) K_X(db) + C_{35}(db) K_Y(db))}{\rho t_m(db)}}} \quad [4-13]$$

Section 4 (undamaged region)

$$\omega_{eq(dc)} = \int_{\left(\frac{b}{5}\right)-20}^{20+\left(\frac{b}{5}\right)-20} \sqrt{\frac{\sum_{n=1}^m -(C_{13}(dc) K_U(dc) + C_{23}(dc) K_V(dc) + C_{33}(dc) + C_{34}(dc) K_X(dc) + C_{35}(dc) K_Y(dc))}{\rho t_m(dc)}}} \quad [4-14]$$

The values of the constants in these equations are calculated according to [45] considering the values for the length, a and widths, b of the respective damaged and undamaged regions.

Section 5

$$\omega_{eq(e)} = \int_{20+\left(\frac{b}{5}\right)-20}^{3/5b} \sqrt{\frac{\sum_{n=1}^m -(C_{13}(e) K_U(e) + C_{23}(e) K_V(e) + C_{33}(e) + C_{34}(e) K_X(e) + C_{35}(e) K_Y(e))}{\rho t_m(e)}}} \quad [4-15]$$

Section 6

$$\omega_{eq(f)} = \int_{3/5b}^{4/5b} \sqrt{\frac{\sum_{n=1}^m -(C_{13}(f) K_U(f) + C_{23}(f) K_V(f) + C_{33}(f) + C_{34}(f) K_X(f) + C_{35}(f) K_Y(f))}{\rho t_m(f)}}} \quad [4-16]$$

Section 7

$$\omega_{eq(g)} = \int_{4/5b}^b \sqrt{\frac{\sum_{n=1}^m -(C_{13}(g) K_U(g) + C_{23}(g) K_V(g) + C_{33}(g) + C_{34}(g) K_X(g) + C_{35}(g) K_Y(g))}{\rho t_m(g)}}} \quad [4-17]$$

The natural frequency of section 4 $\omega_{eq(4)}$ is defined by:

$$\omega_{eq(4)} = \frac{1}{\omega_{eq(4a)}} + \frac{1}{\omega_{eq(4b)}} + \frac{1}{\omega_{eq(4c)}} \quad [4-18]$$

Where $\omega_{eq(4b)}$ represents the natural frequency of the damaged section.

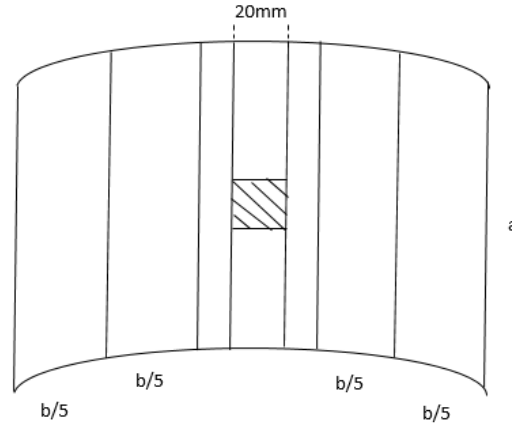


Figure 75 Modified damage model (3 section plate)

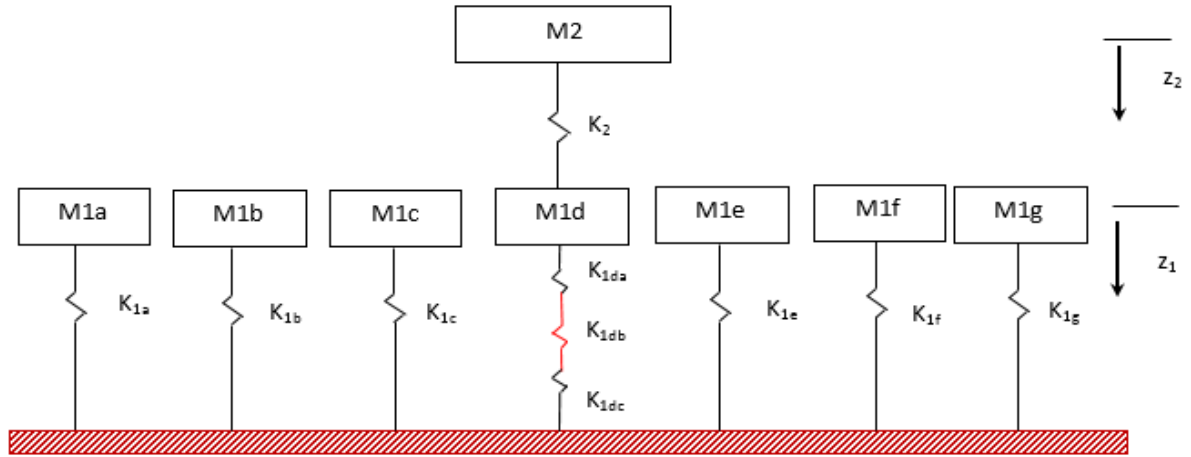


Figure 76 Modified damage model spring-mass diagram

In the above Figure 76, K_{1d} represents the equivalent stiffness of region 4 which consists of the damage region. The damage region is modelled as an area of $20mm \times 20mm$ where the diameter of the impactor is $20mm$. This damage model assumes that the damage caused by the spherical impactor is equivalent to a square region of $20mm \times 20mm$. The expression of the equivalent stiffness of region 4 is expressed as:

$$K_{1d} = \frac{1}{K_{1da}} + \frac{1}{K_{1db}} + \frac{1}{K_{1dc}} \quad [4-19]$$

Where,

$K_{1d}b$ represents the stiffness of the damaged region. In this model the stiffness constants $K_{1d}a$ and $K_{1d}c$ represents stiffnesses of undamaged regions. Therefore, in this model the damage region is refined.

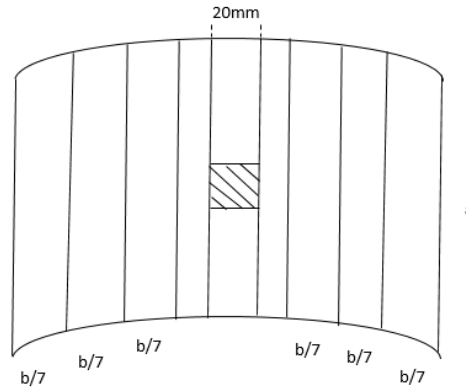


Figure 77 Modified damage model (5 section plate)

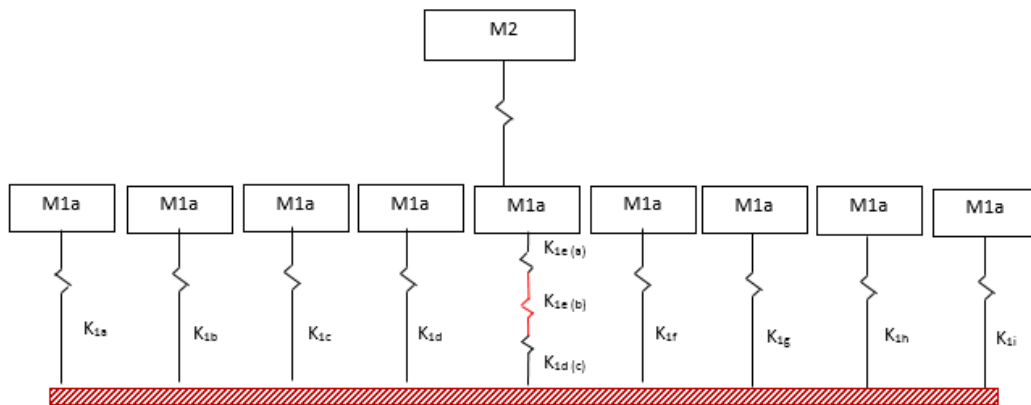


Figure 78 Modified spring-mass model (5 section plate)

4.6.2 Layups studied

Similarly, to the layups considered in the initial damage modelling approach, the following layups are analysed. Layups of the damaged section are highlighted in red.

Layup	No. of sections (Undamaged Model)	No. of Sections (Damaged Model)	Layup (Damaged Model)

1	3	7	$[-45/45/0/90/0]_s$ $[-45/45/0/90]_s$ $[-45/45/0/90]_s$ $[-45/45/0/90]_s$ $[-45/45/0/90]_s$ $[-45/45/0/90/0]_s$
2	3	7	$[0/90/0/45/-45]_s$ $[0/90/-45/45]_s$ $[0/90/-45/-45]_s$ $[0/90/-45/-45]_s$ $[0/90/-45/45]_s$ $[0/90/0/45/-45]_s$
3	3	7	$[0/90/0/90/0]_s$ $[0/90/0/90]_s$ $[0/90/0/90]_s$ $[0/90/0/90]_s$ $[0/90/0/90]_s$ $[0/90/0/90/0]_s$
4	5	9	$[-45/45/0/90/0]_s$ $[-45/45/0/90]_s$ $[-45/45/0/90/0]_s$ $[-45/45/0/90/0]_s$ $[-45/45/0/90/0]_s$ $[-45/45/0/90/0]_s$ $[-45/45/0/90]_s$ $[-45/45/0/90/0]_s$
5	5	9	$[0/90/0/45/45]_s$ $[0/90/-45/45]_s [0/90/0/45/-45]_s$ $[0/90/0/45/-45]_s$ $[0/90/0/45/-45]_s$ $[0/90/0/45/-45]_s$ $[0/90/0/45/-45]_s$ $[0/90/-45/45]_s$ $[0/90/0/45/-45]_s$
6	5	9	$[0/90/0/90/0]_s$ $[0/90/0/90]_s$ $[0/90/0/90/0]_s$ $[0/90/0/90/0]_s$ $[0/90/0/90/0]_s$ $[0/90/0/90/0]_s$ $[0/90/0/90/0]_s$

	[0 /90 /0 /90 /0] _s
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Table 9 Layups (Modified damage model)

4.6.3 Results and discussions

This section studies the effect of damage on the contact force histories of the plate with 3 sections and 5 sections respectively for the layups in Table 8 for the modified damage modelling approach. A ply removal technique is adopted which reduces the thickness of the damage region and thus the stiffness is degraded in that region. Through this analysis, the best layups that can withstand impact damage for variable stiffness composites can be deduced. The present analysis proves that Layup 1 ([-45/45/0/90 0]_s[/[-45/45/0/90]_s[/[-45/45/0/90/0]_s) yielded the highest impact force without damage. The maximum contact force without damage was 2% higher than layup 2 and 28% higher than layup 3. The results showed that inclusion of more 0° plies resulted in a relatively lower maximum impact force. For the damaged model, the 3 sectioned plate was split into 7 sections to incorporate damage where the damaged region was modelled as a 20mm × 20mm square. Removal of plies were done through this section. The removal of a single ply from the damaged region resulted in a maximum impact force drop of 6% of the undamaged plate for Layup1. For Layup 2, this drop was 16% and for Layup 3 was 17%. The highest drop in impact force for Layup 1 was noticed in between removing plies 5 and 6. This drop was 48% when compared to 41% of Layup 2's highest drop of force observed between removing plies 6 and 7. The highest difference between impact forces for Layup 3 which is 44% was observed in between ply removal 6 and 7. This analysis shows that significant weakening of the structure was observed between ply removals 5 and 7. The analysis also proved that Layup 2 was the best design for the plate with 3 sections, since the induced damage affected the impact force history lesser and mainly since it had a higher contact force after removing all plies from the damaged region (see Figures 79-81).

The Layups 4, 5 and 6 are associated with the damage plate with 5 sections. The present analysis proved that Layup 5 [0/90/0/45/-45]_s[/[0/90/-45/45]_s[/[0/90/0/45/-45]_s[/[0/90/-45/45]_s[/[0/90/0/45/-45]_s yielded the highest contact force of 14488N. This maximum force was 12% higher than that of Layup 4 and 17% higher than that of

Layup 6. It is observed that shifting the position of the 45° and -45° plies in the laminate played an integral role in the change of the impact force. The 5 section plate was split into 9 sections to incorporate damage as shown in Figure 76. The removal of a single ply from the damaged region resulted in an impact force drop of 18% of the undamaged plate for Layup 4. For Layup 5, this drop was 10% and 9% for Layup 6. Therefore, a higher impact force drop was observed at damage initiation in Layup 5. The highest drop in impact force for Layup 4 was noticed in between removing plies 6 and 7. This force drop was 46% when compared to 65% of Layup 5's highest force drop observed between removing plies 5 and 6. The highest difference between impact forces for Layup 6 which is 64% was observed in between ply removal 6 and 7. Layup 4 was able to sustain the highest impact force after removal of all plies proving that it is the best layup design able to withstand damage even when Layup 5 yielded a higher impact force without damage induced (see Figures 82-84).

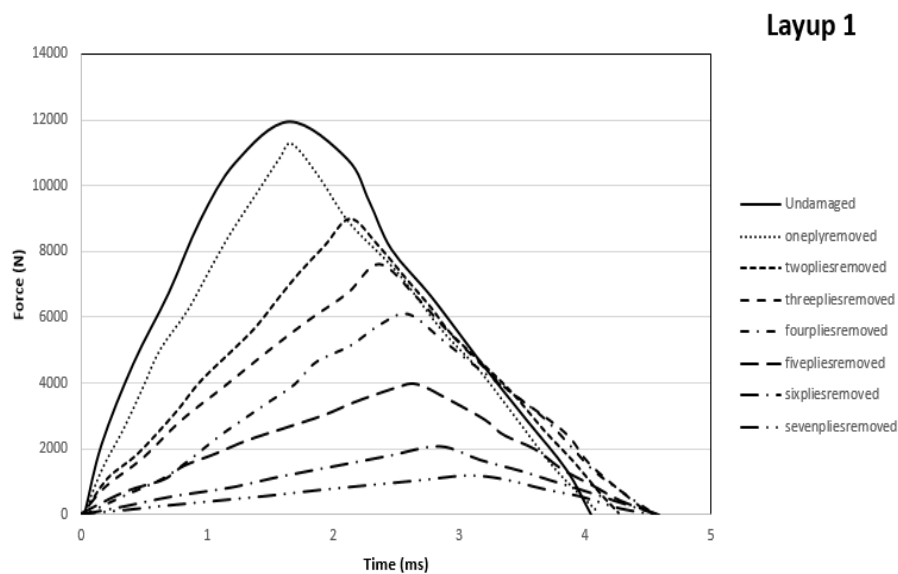


Figure 79 Modified damage model (Layup 1)

Layup 2

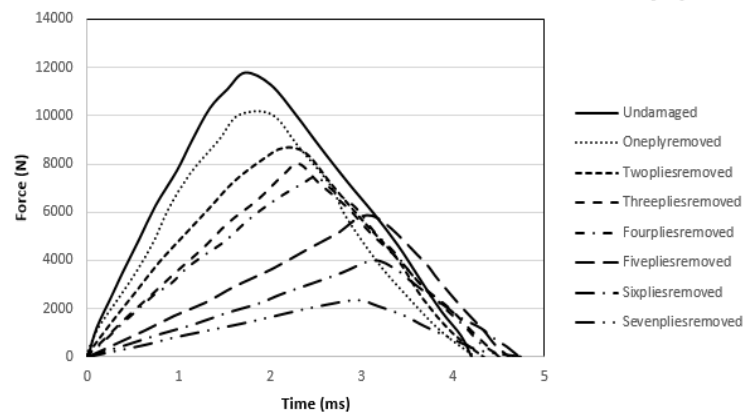


Figure 80 Modified damage model (Layup 2)

Layup 3

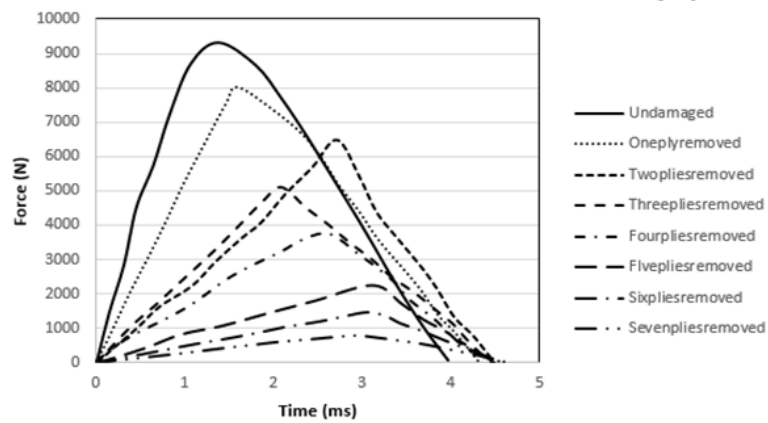


Figure 81 Modified damage model (Layup 3)

Layup 4

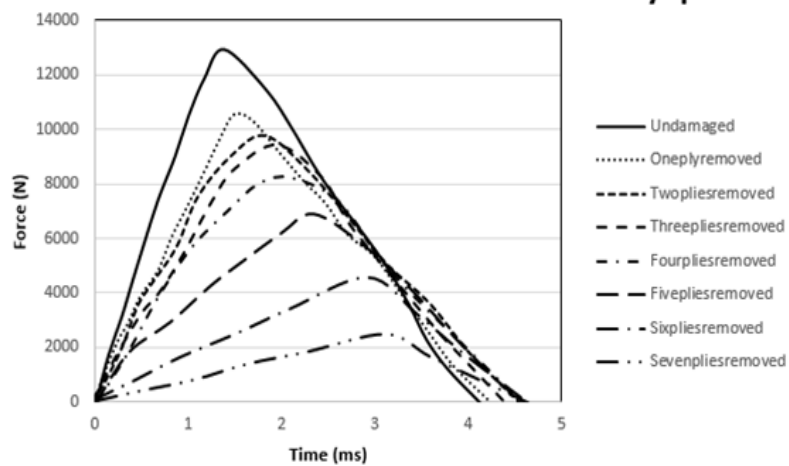


Figure 82 Modified damage model (Layup 4)

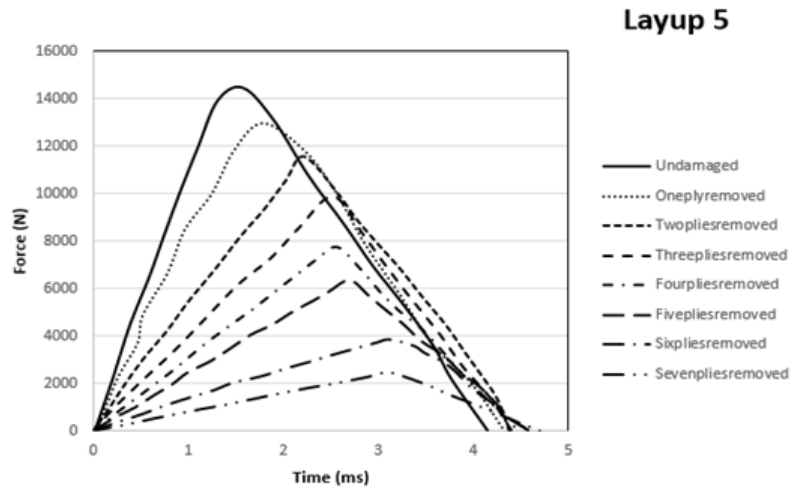


Figure 83 Modified damage model (Layup 5)

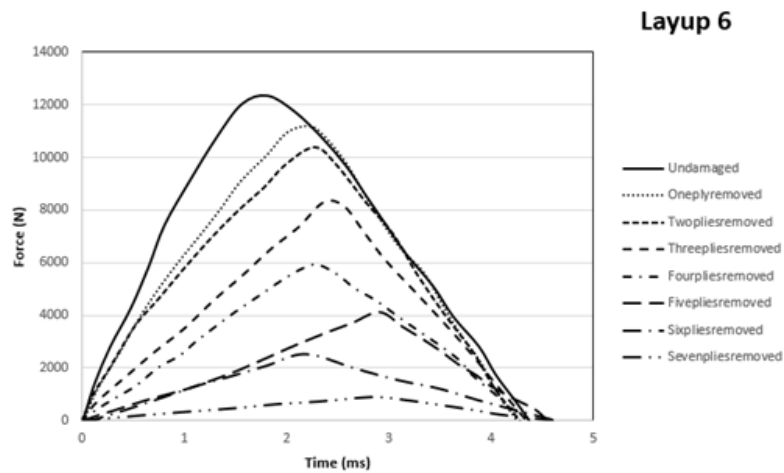


Figure 84 Modified damage model (Layup 6)

4.6.4 Damage behaviour

It can be observed from Figure 79-84 that damage causes a degradation in the overall stiffness of the plate. The damage was introduced into the model through the reduction in the thickness of the damaged section in the plate, thus degrading the overall natural frequencies of the plate. Therefore, the maximum contact force decreases as observed in Figures 79-84. The slope of the graphs in Figures 79-84 decreases as the thickness of the damaged region is reduced. This implies that the stiffness of the plate is decreasing, thereby weakening the laminate. However, contact duration is increasing as more damage is introduced. The first load drop, in terms of Hertzian failure or significant damage corresponds to occurrence of initial damage in the form

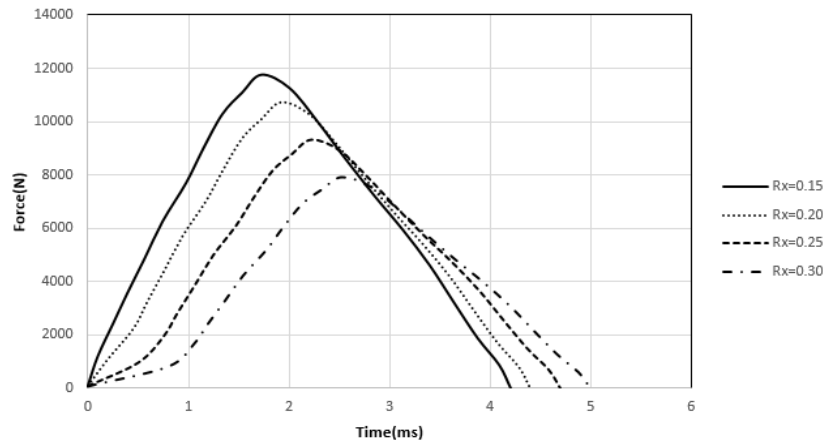
of matrix cracks or delamination as stated in literature. Damage initiation in the present model is based on removal of the first ply of the damaged region. In the present model, the total contact duration increases as the level of damage increases as shown in Table 9.

Layups	Contact time duration(ms)						
	1ply removed	2plies removed	3plies removed	4plies removed	5plies removed	6plies removed	7plies removed
1	4.1	4.3	4.5	4.6	4.6	4.6	4.6
2	4.2	4.3	4.5	4.5	4.6	4.7	4.7
3	4.3	4.5	4.5	4.5	4.6	4.6	4.6
4	4.2	4.4	4.5	4.6	4.6	4.7	4.7
5	4.3	4.4	4.4	4.5	4.6	4.6	4.7
6	4.3	4.3	4.4	4.4	4.6	4.6	4.7

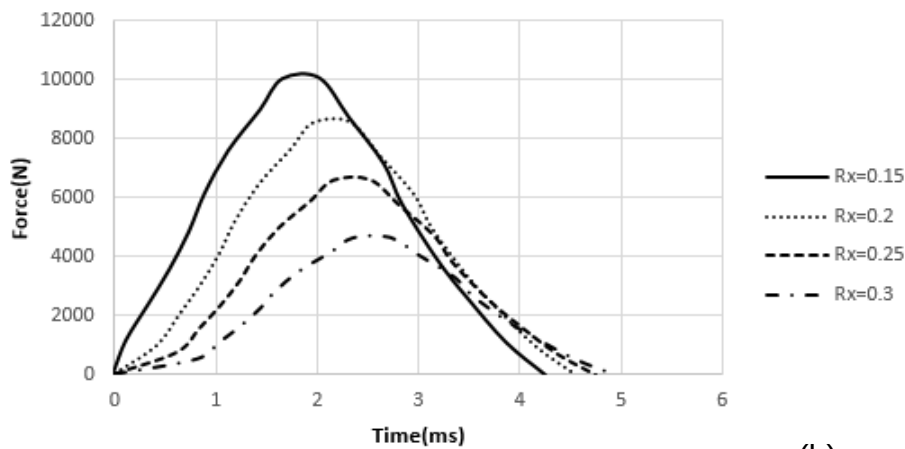
Table 10 Contact Time duration for the different layups.

4.6.4.1 Analysis of curvature effect on damage behaviour

The effect of curvature of the plate on the damage behaviour is studied analytically. It is observed that as the curvature of the plate increases, the contact force is reduced. In the case of the undamaged plate (plate without damage), a drop in the maximum contact force of 9% is observed when the radius of the plate increases from 0.15-0.20m. As damage is initiated through removal of the first ply, the force drop is 15% for the same increase in radius of the plate. This proves that the radius of the plate has an effect on the damage characteristics of the plate (see Figure 85).



(a)



(b)

Figure 85 Effect of curvature on damage behaviour for Layup 2 (a) Undamaged (b) one ply removed

4.6.5 Comparison of results

For comparison purposes, layups 1 and 2 from both damage models are analysed. A direct comparison of the schematic models are shown in Figure. Usually as damage progresses through the laminate, stiffness is reduced and impact response time increases. This reduction in stiffness is more distinct in the modified damage modelling approach as observed from Figures 87 and 88, where stiffness reduces progressively as damage increases. Contact duration also increases with damage in this model as opposed to the initial damage model. This proves that localisation of damage region improves the process of impact damage response evaluation.

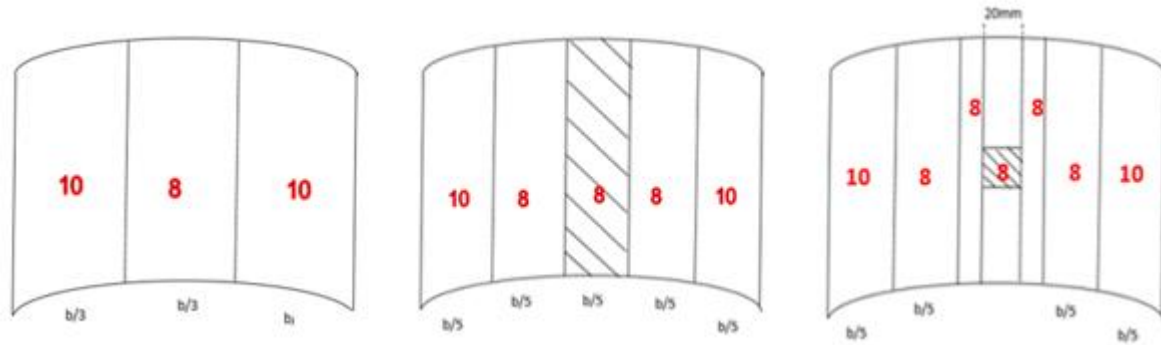


Figure 86 Model comparisons (a) Undamaged (b) Modelling approach 1 (c) Modified damage modelling method

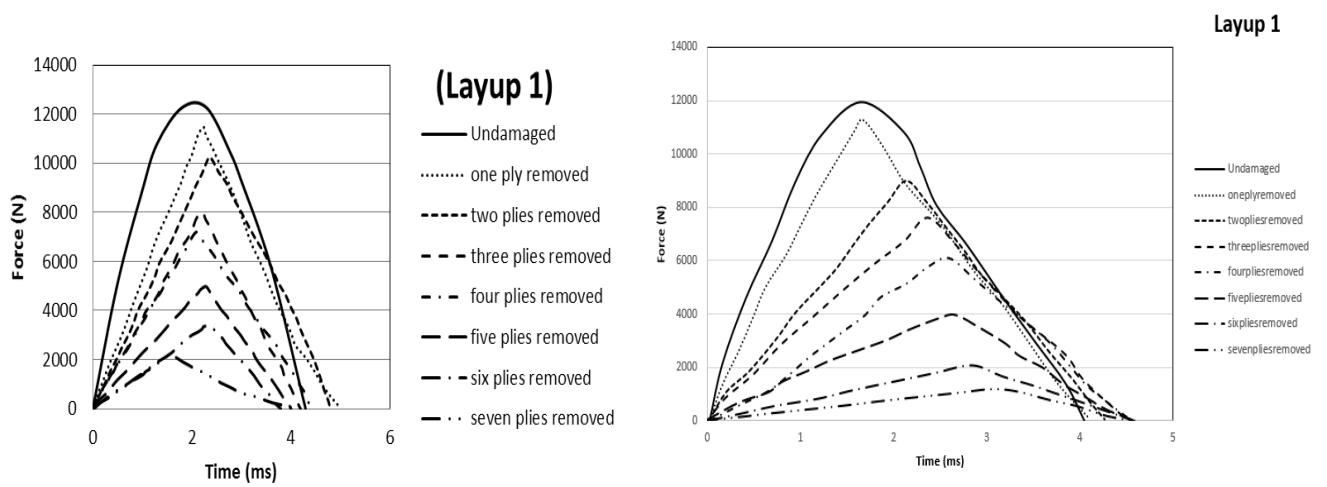


Figure 87 Model comparison (Layup 1)

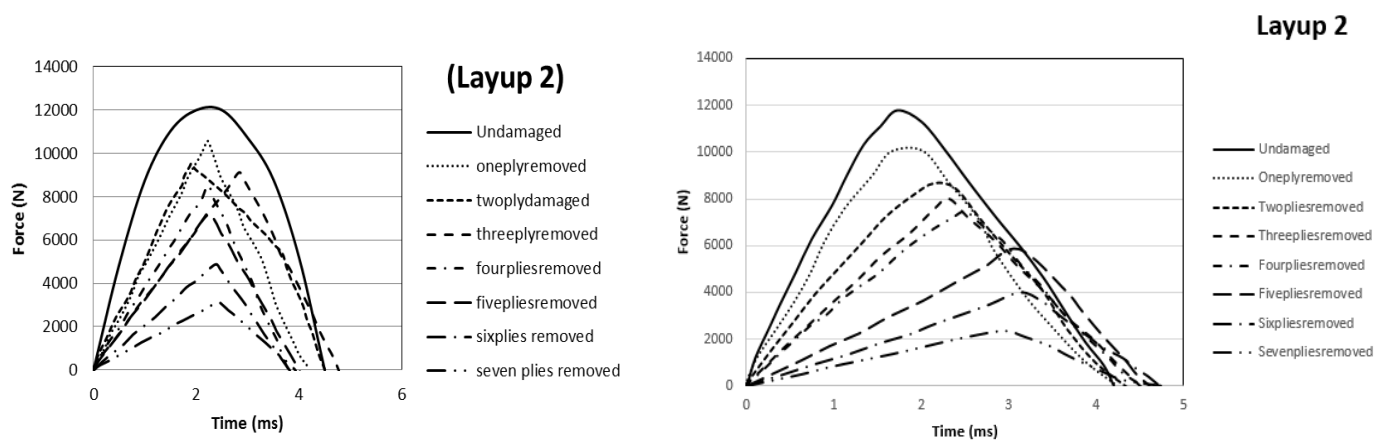


Figure 88 Model comparison (Layup 2)

4.7 Chapter Conclusions

This chapter of the report introduced methods to study the post impact behaviour of damaged variable stiffness curved composite plates. Variable stiffness sections are used widely and the need for studying the impact behaviour of these sections were identified. The existing damage models have not been developed to predict the impact damage behaviour of variable stiffness sections. In this regard, this paper adopts a new methodology to model the damage in the impacted region. The methodology of reducing the thickness of the damage region by ply removal was observed to be successful. The ply removal simulates weakening of the structure by degrading the stiffness in the impact region. Thus, the contact force is reduced when damage is introduced into the system.

The post impact damage analysis introduced two damage modelling methods. These damage modelling methods involves reduction of stiffness in the damaged region by reducing the thickness of the damaged ply to zero, thus reducing the natural frequency of the damaged region and the stiffness.

This chapter also investigated various types of layups able to withstand a greater impact damage mainly by focussing on the contact force at impact. Therefore, this research area gives an insight into the combinations of layups most suitable for designs manufactured from the technique of variable stiffnesses in composites.

Main outcomes from the present chapter:

The method of reducing the thickness of the damaged region, reduced natural frequency and stiffness of the plate.

Contact duration increased as the level of damage of the plate is higher

At higher levels of damage, the effect of the plate radius is much greater.

The second damage modelling approach with localisation of damage region depicted a better representation of contact force histories. The contact force was observed to decrease in a progressive manner and the contact duration was increasing.

It is observed that shifting the position of the 45° and -45° plies in the laminate played an integral role in the change of the impact force.

Curved plates with quasi-isotropic layups displayed higher contact forces with damage effects when compared with cross-ply layups.

Chapter 5:

Analytical Modelling of Low velocity
impact of Curved Sandwich Structures

5 Analytical modelling Low Velocity Impact of Curved Sandwich Structures

5.1 Impact modelling

5.1.1 Analytical modelling

The analytical model derived in the present study is for a curved sandwich plate. The plate is subjected to a central low velocity impact from a hemispherical nose indenter. A curved sandwich plate with dimensions, length a , width, b and radius, R_1 . In this study, the face-sheets are either of constant thickness of variable thicknesses. (Figure 89)

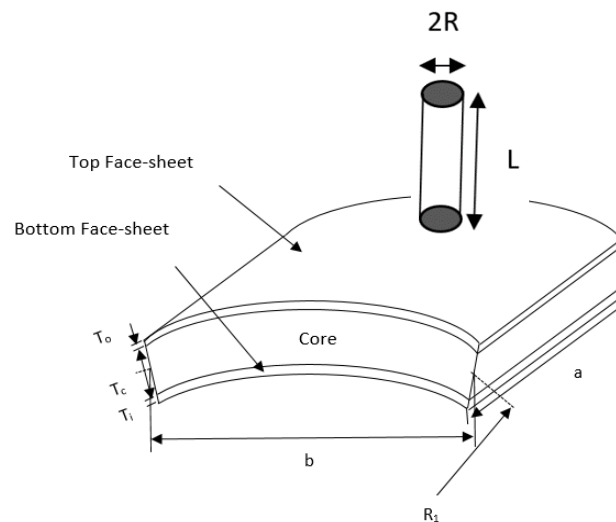


Figure 89 Sandwich Curved Plate

5.1.2 Theoretical Formulations

5.1.2.1 Local deformation

In a simply supported plate, two types of deformation occur: the local indentation of the top face-sheet into the core material, δ , and the global plate bending and shear deformation, Δ . Most models determined the local indentation through Hertzian contact methods [84]. However, the local deformation considered in this study causes transverse deflections of the entire top face-sheet and core crushing and therefore Hertzian contact laws are invalid in determining the local indentation response. Three

consecutive regimes are reflected for the local indentation of the plate. I- Plate on elastic foundation; II-Plate on a rigid-plastic foundation; III-Membrane on a rigid-plastic foundation. The plate on an elastic foundation can be used to determine the indentation response when the indentation is very small. The plate on rigid-plastic foundation can be used as the face sheet indentation becomes larger, but still less than half the thickness of the plate. The membrane on a rigid plastic foundation is used to find the indentation response when the face sheet indentation is greater than the face sheet thickness.[110] The local indentation response of a simply supported plate on an elastic foundation is presented by [110]. These three regimes are shown in Figure 90.

$$\delta_{mn} = \sum_{m=1,3,\dots}^{\infty} \times \sum_{n=1,3,\dots}^{\infty} \left\{ \left\{ 4P \sin\left(\frac{m\pi}{2}\right) \sin\left(\frac{n\pi}{2}\right) \right\} / \left\{ a^2 \left[\frac{\pi}{a^4} (D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2 + D_{22}n^4) + k^c \right] \right\} \right\}$$

[5-1]

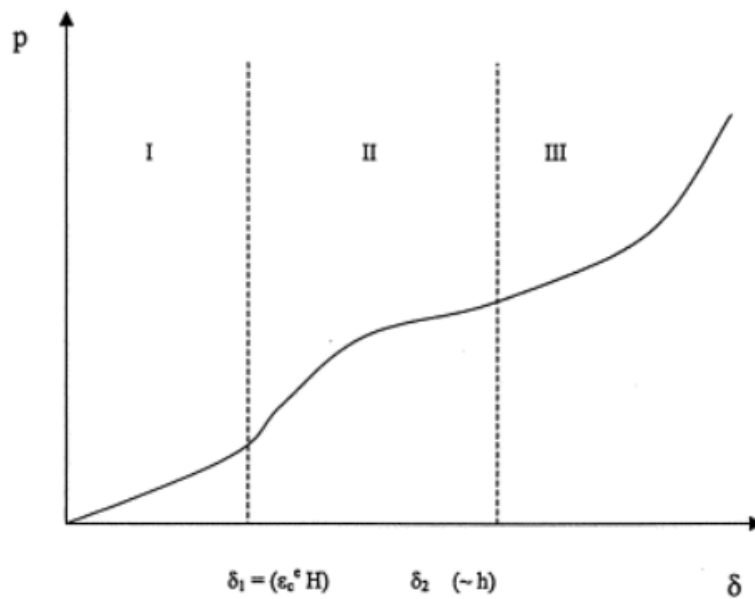


Figure 90 Local face-sheet indentation regimes [115].

5.1.2.1.1 Plate on rigid-plastic model

The load-indentation response is derived through the use of principle of minimum potential energy. The total potential energy Π is expressed as:

$$\Pi = U + D - V \quad [5-2]$$

Where U is the strain energy due to bending, D is the work due to core crushing, and V is the work done by the indentation force.

By presuming that the local indentation is due to bending only and therefore expressed as:

$$w = \begin{cases} \delta & \text{for } 0 \leq x^2 + y^2 \leq R^2 \\ \delta \left[1 - \frac{(x-R)}{(\zeta-R)}\right]^2 \left[1 - \frac{(y-R)}{(\zeta-R)}\right]^2 & \text{for } R^2 \leq x^2 + y^2 \leq \zeta^2, x \geq 0, y \geq 0 \end{cases} \quad [5-3]$$

where δ is the deflection under the impactor, ζ is the lateral extent of the deformation zone and R is the radius of the impactor. Bending of an orthotropic laminate face-sheet gives a strain energy expressed as:

$$U = \frac{1}{2} \int_S \left\{ D_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + D_{22} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 2D_{12} \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial y^2} \right) + 4D_{66} \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} dA \quad [5-4]$$

Where D_{ij} is the laminate bending stiffness matrix, $dA = dxdy$ and A is the surface area of the deformed facesheet. The integral can be approximated as:

$$U \approx 2 \int_R^\zeta \int_R^\zeta \left\{ D_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + D_{22} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 2D_{12} \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial y^2} \right) + 4D_{66} \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} dxdy \quad [5-5]$$

Then, by substituting derivatives of equation (5-3) into (5-5)

$$U \approx \frac{D_1 \delta^2}{(R-\zeta)^2} \quad [5-6]$$

Where

$$D_1 = \frac{16384}{11025} (7D_{11} + 7D_{22} + 4D_{12} + 8D_{66}) \quad [5-7]$$

D_1 is the bending stiffness of the orthotropic face-sheet. The energy absorbed due to core crushing is expressed as

$$D \approx \pi q R^2 \delta + 4 \int_R^\zeta \int_R^\zeta q w dxdy \quad [5-8]$$

Where q is the crushing strength of the core. The profile in equation (5-3) gives:

$$D \approx \pi q R^2 \delta + \frac{256}{255} q \delta (R - \zeta)^2 \quad [5-9]$$

The work done by the indentation force is given by:

$$V = P \delta \quad [5-10]$$

Therefore, the total potential energy is expressed as:

$$\Pi = \frac{D_1 \delta^2}{(R - \zeta)^2} + \pi q R^2 \delta + \frac{256}{255} q \delta (R - \zeta)^2 - P \delta \quad [5-11]$$

Differentiating Π with respect to δ gives

$$P = \frac{2D_1 \delta}{(R - \zeta)^2} + \pi q R^2 + \frac{256}{255} \delta (R - \zeta)^2 \quad [5-12]$$

Likewise, differentiation of P with respect to ζ gives

$$\frac{2D_1 \delta}{(R - \zeta)^2} = \frac{256}{255} \delta (R - \zeta)^2 \quad [5-13]$$

Then, the load-indentation response is established by removal of the length of the deformation zone, ζ from equations (5-12) and (5-13) and is expressed as:

$$P = \frac{32}{15} \sqrt{2D_1 q} \delta + \pi q R^2 \quad [5-14]$$

5.1.2.1.2 Membrane on rigid-plastic model

In this region, assumption is made that the transverse deformation profile $w(x, y)$ of the top face-sheet is expressed as:

$$w = \left\{ \begin{array}{l} \delta \text{ for } 0 \leq x^2 + y^2 \leq R^2 \\ \delta \left[1 - \frac{(x-R)}{(\zeta-R)} \right]^2 \left[1 - \frac{(y-R)}{(\zeta-R)} \right]^2 \text{ for } R^2 \leq x^2 + y^2 \leq \zeta^2, x \geq 0, y \geq 0 \end{array} \right\} \quad [5-15]$$

The load indentation response is defined by [110]:

$$P = \frac{8\sqrt{C_1 q}}{3} \delta^{3/2} + \pi q R^2 \quad [5-16]$$

Where

$$C_1 = 8 \left[\frac{(A_{11}+A_{22})}{45} + \frac{(2A_{12}+4A_{66})}{49} \right] \quad [5-17]$$

5.1.2.2 Global Deformation

The panel experiences two types of deformation when it is clamped around the edges: (1) local deformation of the top facesheet into the core material, δ , and (2) global panel bending and shear deformation, Δ . The local deformation is defined as the static indentation response of the top face-sheet as the core crushes. Global deformation is defined as the bending and shear deformation of a sandwich panel that has not experienced any local facesheet indentation and core crushing.

5.1.2.2.1 Simply supported panels

The principle of minimum potential energy is used in deriving approximate solutions for simply supported panels. The panel is considered to be square of dimensions $a \times a$ by setting $b = a$ to simplify the problem. The functions relating the transverse deformation, W , and the rotations with respect to the x - and y -axis, $\bar{\alpha}$ and $\bar{\beta}$, are expressed as:

$$W(x, y) = \Delta f(x)f(y) \text{ for } -a/2 \leq x \leq a/2, -a/2 \leq y \leq a/2 \quad [5-18]$$

and

$$\bar{\alpha}(x, y) = \alpha_0 g(x)h(y) \text{ for } -a/2 \leq x \leq a/2, -a/2 \leq y \leq a/2 \quad [5-19]$$

$$\bar{\beta}(x, y) = \beta_0 h(x)g(y) \text{ for } -a/2 \leq x \leq a/2, -a/2 \leq y \leq a/2 \quad [5-20]$$

where Δ, α_0 and β_0 are the amplitudes of shape functions for transverse deflections and rotations. Functions $f(x), f(y), g(x), g(y), h(x)$ and $h(y)$ satisfy displacement and force boundary conditions at the edges.

Since, in-plane deformations are considered to be negligible with respect to the transverse deformation, strain energy of the sandwich panel is mainly due to bending and shear:

$$\begin{aligned} U = & 4 \int_0^{a/2} \int_0^{a/2} \left\{ \frac{D_{11}^s}{2} \left(\frac{\partial \bar{\alpha}}{\partial x} \right)^2 + D_{12}^s \left(\frac{\partial \bar{\beta}}{\partial y} \right) \left(\frac{\partial \bar{\alpha}}{\partial y} \right) + \frac{D_{22}^s}{2} \left(\frac{\partial \bar{\beta}}{\partial y} \right)^2 + A_{55}^s \left[\frac{\bar{\alpha}^2}{2} + \bar{\alpha} \frac{\partial W}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 \right] + \right. \\ & A_{44}^s \left[\frac{\bar{\beta}^2}{2} + \bar{\beta} \frac{\partial W}{\partial y} + \frac{1}{2} \left(\frac{\partial W}{\partial y} \right)^2 \right] + D_{66}^s \left[\frac{1}{2} \left(\frac{\partial \bar{\alpha}}{\partial y} \right)^2 + \frac{\partial \bar{\alpha}}{\partial y} \frac{\partial \bar{\beta}}{\partial x} + \frac{1}{2} \left(\frac{\partial \bar{\beta}}{\partial x} \right)^2 \right] \left. \right\} dx dy \end{aligned} \quad [5-21]$$

The global load-indentation responses for simply supported panels are derived in the sections below. The resulting functions describing transverse deformation, W , and the rotations with respect to the x- and y-axis, $\bar{\alpha}$ and $\bar{\beta}$, are defined by:

$$W(x, y) = \Delta \left[1 - \left(\frac{2x}{a} \right)^2 \right] \left[1 - \left(\frac{2y}{a} \right)^2 \right] \text{ for } -a/2 \leq x \leq a/2, -a/2 \leq y \leq a/2 \quad [5-22]$$

$$\bar{\alpha}(x, y) = \alpha_0 \left[-\frac{3x}{a} + 4 \left(\frac{x}{a} \right)^3 \right] \left[1 - \left(\frac{2y}{a} \right)^2 \right] \text{ for } -a/2 \leq x \leq a/2, -a/2 \leq y \leq a/2 \quad [5-23]$$

$$\bar{\beta}(x, y) = \beta_0 \left[-\frac{3y}{a} + 4 \left(\frac{y}{a} \right)^3 \right] \left[1 - \left(\frac{2x}{a} \right)^2 \right] \text{ for } -a/2 \leq x \leq a/2, -a/2 \leq y \leq a/2 \quad [5-24]$$

Substituting derivatives of expressions in equations (5-22) -(5-24) into equation (5-21) gives the strain energy expression as:

$$U = F_1 \Delta^2 + F_2 \Delta \beta_0 + F_3 \beta_0^2 + F_4 \Delta \alpha_0 + F_5 \alpha_0^2 + F_6 \alpha_0 \beta_0 \quad [5-25]$$

Where

$$F_1 = \frac{2240}{1575} (A_{44}^s + A_{55}^s) \quad [5-26]$$

$$F_2 = \frac{1344}{1575} a A_{44}^s \quad [5-27]$$

$$F_3 = \frac{1}{1575} [204a^2 A_{44}^s + 2016 D_{22}^s + 2040 D_{66}^s] \quad [5-28]$$

$$F_4 = \frac{1344}{1575} a A_{55}^s \quad [5-29]$$

$$F_5 = \frac{1}{1575} [204a^2 A_{55}^s + 2016 D_{11}^s + 2040 D_{66}^s] \quad [5-30]$$

$$F_6 = \frac{4032}{1575} (D_{12}^s + D_{66}^s) \quad [5-31]$$

Substituting equation (5-25) and the relationship that external work done, $V = P\Delta$ into the potential energy equation gives:

$$\Pi = F_1\Delta^2 + F_2\Delta\beta_0 + F_3\beta_0^2 + F_4\Delta\alpha_0 + F_5\alpha_0^2 + F_6\alpha_0\beta_0 - P\Delta \quad [5-32]$$

By differentiating Π with respect to Δ and α_0 , the equilibrium of the system is restored and yields the load-indentation response. Differentiation of Π with respect to Δ gives

$$\frac{\partial \Pi}{\partial \Delta} = 2F_1\Delta + F_2\beta_0 + F_4\alpha_0 - P = 0 \quad [5-33]$$

Similarly, differentiating Π with respect to α_0 derives:

$$\frac{\partial \Pi}{\partial \alpha_0} = F_4\Delta + 2F_5\alpha_0 + F_6\beta_0 = 0 \quad [5-34]$$

And differentiation of Π with respect to β_0 expresses:

$$\frac{\partial \Pi}{\partial \beta_0} = F_2\Delta + 2F_3\beta_0 + F_6\alpha_0 = 0 \quad [5-35]$$

Then, by eliminating α_0 and β_0 in equations (5-33)-(5-35), the global force-deflection response is defined to be:

$$P = K_g\Delta \quad [5-36]$$

Where

$$K_g = \left\{ \left[(4F_1F_5 - F_4^2)(4F_3F_5 - F_6^2) + (2F_2F_3 - F_4F_6)(F_4F_6 - 2F_2F_5) \right] \right\} / \left\{ 2F_5(4F_3F_5 - F_6^2) \right\} \quad [5-37]$$

5.1.2.2.2 Clamped

Functions for the transverse deformation, W , and rotations respect to x - and y - axis are defined by $\bar{\alpha}$ and $\bar{\beta}$.

$$W(x,y) = \Delta \left(1 - \left(\frac{2x}{a} \right)^2 \right)^4 \left(1 - \left(\frac{2y}{a} \right)^2 \right)^4 \text{ for } -a/2 \leq x \leq a/2, -a/2 \leq y \leq a/2 \quad [5-38]$$

and

$$\bar{\alpha}(x, y) = \bar{\beta}(x, y) = \alpha_0 \left(1 - \left(\frac{2x}{a}\right)^2\right)^2 \left(1 - \left(\frac{2y}{a}\right)^2\right)^2 \text{ for } -a/2 \leq x \leq a/2, -a/2 \leq y \leq a/2$$

[5-39]

At the edges, boundary conditions of a four sided clamped plate $W = 0$ and $\bar{\alpha} = \bar{\beta} = 0$

Strain energy is expressed as:

$$U = F_1 \alpha_0^2 + F_2 \alpha_0 \Delta + F_3 \Delta^2$$

[5-40]

Where,

$$F_1 = \frac{1}{225} [320(D_{11}^s + D_{22}^s) + 225D_{12}^s + 865D_{66}^s + 64a^2 A_{55}^s]$$

[5-41]

$$F_2 = -\frac{2028a}{3465} (A_{44}^s + A_{55}^s)$$

[5-42]

$$F_3 = \frac{4294967296}{4927697775} (A_{44}^s + A_{55}^s)$$

[5-43]

The global load-displacement response for a clamped sandwich plate is defined as:

$$P = K_g \Delta$$

[5-44]

Where,

$$K_g = \frac{4F_1 F_3 - F_2^2}{2F_1}$$

[5-45]

5.1.3 Low velocity impact model

This section describes the dynamic models for the impact response of simply supported sandwich plates.

5.1.3.1 Simply supported plates

Both local and global deflections occur for a simply supported plate. The local deformation is the local indentation of the top face-sheet followed by core crushing. Global indentation is the bending and shear deformation of a sandwich plate that has not experienced any local face sheet indentation and core crushing. A two degree of freedom system is used to derive the function for the contact force. (Figure 91)

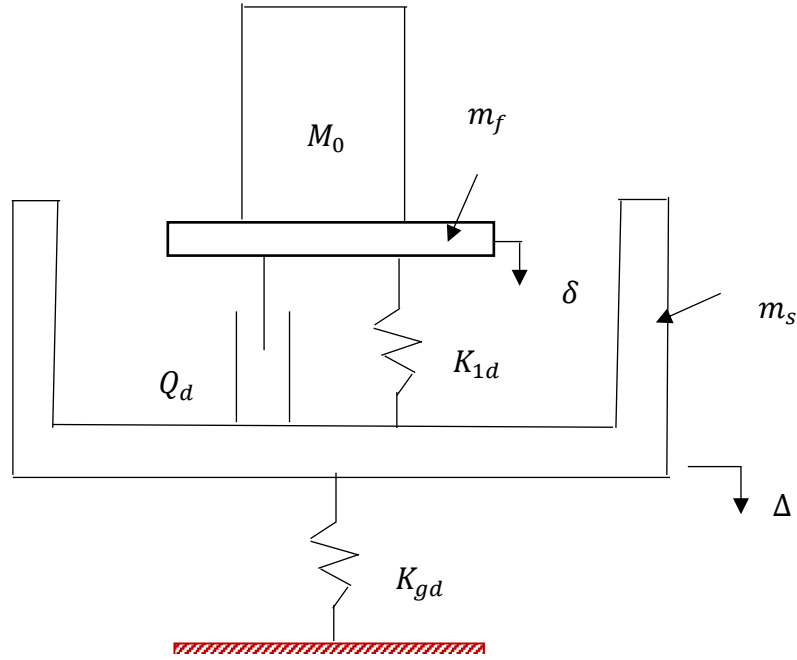


Figure 91 Two degree of freedom spring-mass-dashpot system

The equations of motion for the two degree-of-freedom system are:

$$(\mathbf{M}_0 + \mathbf{m}_f)(\ddot{\Delta} + \ddot{\delta}) + \mathbf{P}_1(\delta) + \mathbf{Q}_d = \mathbf{0} \quad [5-46]$$

and

$$\mathbf{Q}_d + \mathbf{P}_1(\delta) = \mathbf{m}_s \ddot{\Delta} + \mathbf{K}_{gd} \Delta \quad [5-47]$$

Where K_{gd} is the dynamic global stiffness of the sandwich. The above-coupled equations would be difficult to solve because of the nonlinear local spring response. Assume that the local spring response can be linearized.

$$\mathbf{P}_1(\delta) \approx \mathbf{K}_{1d} \delta \quad [5-48]$$

In this model, the mass of the sandwich panel is negligible compared to the mass of the impactor. Therefore, the above equations can be simplified to:

$$\mathbf{M}_0(\ddot{\Delta} + \ddot{\delta} + \mathbf{K}_{1d} \delta + \mathbf{Q}_d = \mathbf{0}) \quad [5-49]$$

and

$$Q_d + K_{1d}\delta = K_{gd}\Delta \quad [5-50]$$

Differentiating both sides twice with respect to time t gives:

$$\ddot{\Delta} = \frac{K_{1d}}{K_{gd}} \ddot{\delta} \quad [5-51]$$

Substituting $\ddot{\Delta}$ into equation (5-49) gives:

$$M_0 \left(1 + \frac{K_{1d}}{K_{gd}} \right) \ddot{\delta} + K_{1d}\delta + Q_d = 0 \quad [5-52]$$

The solution for δ is given by:

$$\delta = \frac{\delta_0}{\omega} \sin \omega t + \frac{Q_d}{K_{1d}} \cos \omega t - \frac{Q_d}{K_{1d}} \quad [5-53]$$

Where

$$\omega = \sqrt{\frac{K_{1d}K_{gd}}{(K_{1d}+K_{gd})M_0}} \quad [5-54]$$

The impact force is expressed as:

$$F(t) = -M_0(\Delta + \delta) = -M_0 \left(1 + \frac{K_{1d}}{K_{gd}} \right) \ddot{\delta} \quad [5-55]$$

$$\dot{\delta} = \dot{\delta}_0 \cos \omega t - \omega \frac{Q_d}{K_{1d}} \sin \omega t \quad [5-56]$$

$$\ddot{\delta} = -\omega \dot{\delta}_0 \sin \omega t - \omega^2 \frac{Q_d}{K_{1d}} \cos \omega t \quad [5-57]$$

Therefore, expression for the impact force can be re-arranged as:

$$F(t) = -M_0 \left(1 + \frac{K_{1d}}{K_{gd}} \right) (-\omega \dot{\delta}_0 \sin \omega t) - \frac{Q_d \omega^2}{K_{1d}} \cos \omega t \quad [5-58]$$

Where

$$\dot{\delta}_0 = \frac{V_0 K_g}{(K_g + K_1)} \quad [5-59]$$

5.1.3.2 Clamped panels

The expression for strain energy is defines by:

$$U = F_1 \alpha_0^2 + F_2 \alpha_0 \Delta + F_3 \Delta^2 \quad [5-60]$$

Where,

$$F_1 = \frac{1}{225} [320(D_{11}^s + D_{22}^s) + 225D_{12}^s + 865D_{66}^s + 64a^2 A_{55}^s] \quad [5-61]$$

$$F_2 = -\frac{2028a}{3465} (A_{44}^s + A_{55}^s) \quad [5-62]$$

$$F_3 = \frac{4294967296}{4927697775} (A_{44}^s + A_{55}^s) \quad [5-63]$$

The global load-displacement response for a clamped sandwich plate is defined as:

$$P = K_g \Delta \quad [5-64]$$

Where,

$$K_g = \frac{4F_1 F_3 - F_2^2}{2F_1} \quad [5-65]$$

5.1.4 Validation of the model

The present analytical model is validated with studies in literature for simply supported flat sandwich plate with honeycomb and foam cores respectively. The main aim of this comparison is to prove the validity of the analytical model with both type of core materials. For the honeycomb core, experimental study performed by Herup and Palzotto [84] on the low velocity impact testing of carbon/epoxy facesheets and Nomex honeycomb cores are considered for the validation along with the analytical study carried out by Malekzadeh et al [131]. For the foam core, experimental studies performed by Feng et al. [136] on a flat foam based panel are considered. The geometrical and material properties of the facesheet, core and impactor are presented in Tables 11 and 12. The effective masses of the facesheet is calculated from the

method proposed by Hoo [110]. The contact force history for the $[0/90]_{8s}$ composite honeycomb sandwich is compared with the analytical [131] and experimental [88] results for an impact velocity of 1.42m/s. The maximum contact force of 5577N is 5% less than the experimental result, whereas the analytical model derived in [135] is 6% less. This confirms that the present approach yields better results shown in Figure 93. A comparison of the analytical model with the foam based core proved that the maximum contact force of 4365N is 3% higher than the experimental results. (Figure 94.) These comparisons reveals that the present analytical model can be successfully integrated into a sandwich panel with either honeycomb or foam based cores.

References 90 and 135 in Tables 11-13 corresponds to the honeycomb plate, whereas reference 136 [Feng et al.] refers to the foam core sandwich plate.

<i>Examples</i>	<i>Ref (90,135)</i>		<i>Ref (136)</i>	
<i>Material properties</i>	<i>Facesheet</i>	<i>Core</i>	<i>Facesheet</i>	<i>Core</i>
E_{11} (GPa)	141.8	0.080	122	0.057
E_{22} (GPa)	9.72	0.080	6.2	0.060
E_{33} (GPa)	9.72	1.004	6.2	0.060
G_{12} (GPa)	7.0	0.032	4.3	0.020
G_{13} (GPa)	7.0	0.12	4.3	0.020
G_{23} (GPa)	3.8	0.076	2.8	0.020
ν_{12}	0.3	0.25	0.35	0.3
ν_{13}	0.3	0.02	0.35	0.3
ν_{23}	0.3	0.02	0.5	0.3
Density (kg/m ³)	1632	64	1600	65

Table 11 Material properties of face-sheets and core

<i>Properties</i>	<i>Ref (90,135)</i>	<i>Ref (136)</i>
Core thickness	12.7	10
Ply thickness	0.0635	0.032
Ply orientation	[0/90] _{8s}	[0 ₃ /90 ₃ /0]
Length of panel	178	250
Width of panel	178	250

Table 12 Geometrical properties of face-sheets and core

Examples	Ref (90,135)	Ref (136)
Mass (kg)	3.48	2.34
Diameter (mm)	25.4	12.7
Velocity (m/s)	1.4	2.3
Impact energy (J)	3.5	6.3

Table 13 Dimensions and loading conditions of the impactor

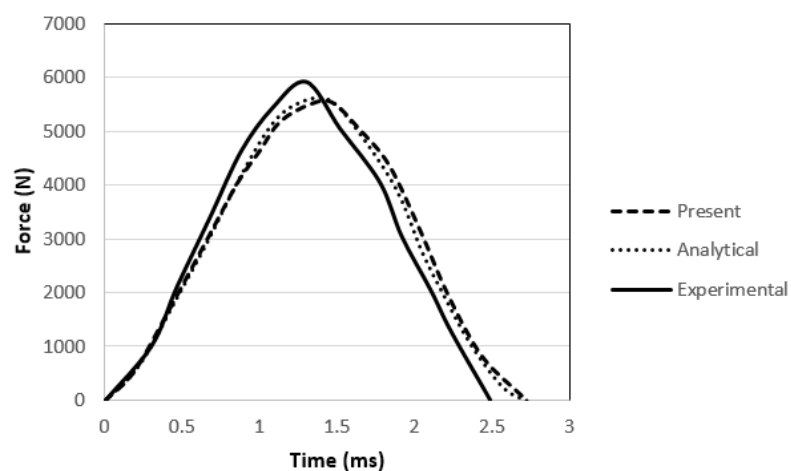


Figure 93 Contact force history (Honeycomb sandwich plate)

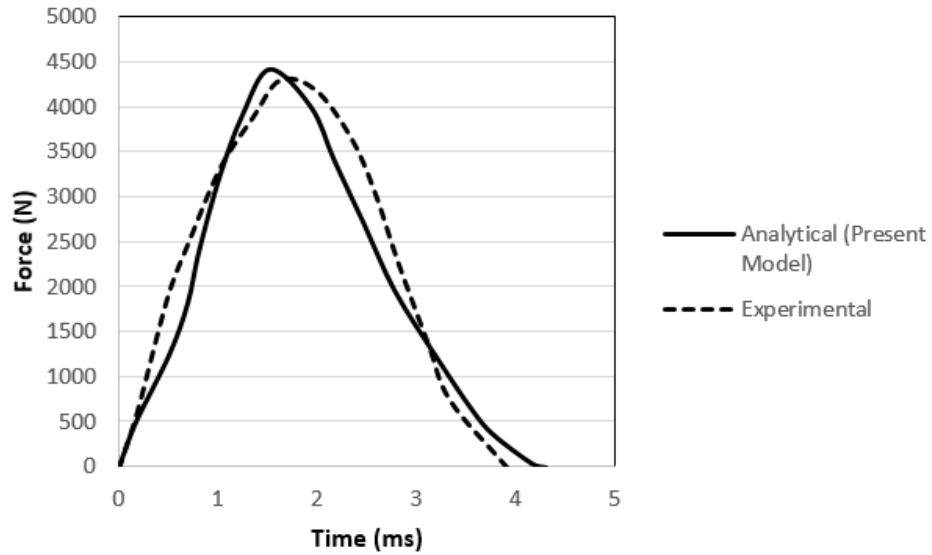


Figure 94 Contact force history (Foam sandwich plate)

5.1.5 Analytical modelling of impact on curved sandwich plate (constant stiffness face-sheets)

This study focusses on applying the two degree of freedom spring-mass-dashpot system to a curved sandwich plate. The natural frequency of the curved top facesheet is calculated by equation [5-66]. The constants in equation are defined in [45]. In calculating the natural frequency of the facesheet, the stiffness matrices, dimensions and curvature of the facesheets are considered.

$$\omega_{mn(top\ facesheet)} = \sqrt{\frac{-(C_{13} K_U + C_{23} K_V + C_{33} + C_{34} K_X + C_{35} K_Y)}{\rho h}} \quad [5-66]$$

The stiffness of the top facesheet is defined as:

$$K_{1d} = \omega_{mn(top\ facesheet)}^2 M_f \quad [5-67]$$

A method to derive the effective mass of facesheet is derived in [110] as:

$$M_f = \frac{4R^2}{25} \left[25 + \left(1 - \frac{a}{2R} \right)^2 \right] \rho_f h \quad [5-68]$$

The constants in equation [5-66] are defined as:

$$c_{11} = -A_{11} \left(\frac{m^2 \pi^2}{a^2} \right) - A_{66} \left(\frac{n^2 \pi^2}{b^2} \right) - \left(\frac{k_{sh}}{R_1^2} \right) A_{55} - \left(\frac{1}{R_1^2} \right) N_{11}^i \quad [5-69]$$

$$c_{12} = - \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) (A_{12} + A_{66}) \quad [5-70]$$

$$c_{13} = \frac{1}{R_1} \left(\frac{m\pi}{a} \right) A_{11} + \frac{1}{R_2} \left(\frac{m\pi}{a} \right) A_{12} + \frac{k_{sh}}{R_1} \left(\frac{m\pi}{a} \right) A_{55} + \frac{1}{R_1} \left(\frac{m\pi}{a} \right) N_{11}^i \quad [5-71]$$

$$c_{14} = - \left(\frac{n^2 \pi^2}{b^2} \right) c_0 D_{66} + \frac{k_{sh}}{R_1} A_{55} \quad [5-72]$$

$$c_{15} = - \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) c_0 D_{66} \quad [5-73]$$

$$c_{22} = - \left(\frac{n^2 \pi^2}{b^2} \right) A_{22} - \frac{k_{sh}}{R_2^2} A_{44} - \left(\frac{1}{R_2^2} \right) N_{22}^i - \left(\frac{m^2 \pi^2}{a^2} \right) A_{66} \quad [5-74]$$

$$c_{23} = \frac{1}{R_1} \left(\frac{n\pi}{b} \right) A_{12} + \frac{1}{R_2} \left(\frac{n\pi}{b} \right) A_{22} + \frac{k_{sh}}{R_2} \left(\frac{n\pi}{b} \right) A_{44} + \frac{1}{R_2} \left(\frac{n\pi}{b} \right) N_{22}^i \quad [5-75]$$

$$c_{24} = \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) c_0 D_{66} \quad [5-76]$$

$$c_{25} = \frac{k_{sh}}{R_2} A_{44} + \left(\frac{m^2 \pi^2}{a^2} \right) c_0 D_{66} \quad [5-77]$$

$$c_{33} = -k_{sh} \left(\frac{m^2 \pi^2}{a^2} \right) A_{55} - k_{sh} \left(\frac{n^2 \pi^2}{b^2} \right) A_{44} - \left(\frac{1}{R_2^2} \right) A_{22} - \left(\frac{1}{R_1^2} \right) A_{11} - \left(\frac{m^2 \pi^2}{a^2} \right) N_{11}^i - \left(\frac{n^2 \pi^2}{b^2} \right) N_{22}^i \quad [5-78]$$

$$c_{34} = -k_{sh} \left(\frac{m\pi}{a} \right) A_{55} \quad [5-79]$$

$$c_{35} = -k_{sh} \left(\frac{n\pi}{b} \right) A_{44} \quad [5-80]$$

$$c_{44} = - \left(\frac{m^2 \pi^2}{a^2} \right) D_{11} - \left(\frac{n^2 \pi^2}{b^2} \right) D_{66} - k_{sh} A_{55} \quad [5-81]$$

$$c_{45} = - \left(\frac{m^2 \pi^2}{a^2} \right) D_{12} - \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) D_{66} \quad [5-82]$$

$$c_{55} = - \left(\frac{m^2 \pi^2}{a^2} \right) D_{66} - \left(\frac{n^2 \pi^2}{b^2} \right) D_{22} - k_{sh} A_{44} \quad [5-83]$$

$$K_U = S_1 + S_2 K_X + S_3 K_Y \quad [5-84]$$

$$\mathbf{K}_V = \mathbf{S}_4 + \mathbf{S}_5 \mathbf{K}_X + \mathbf{S}_6 \mathbf{K}_Y \quad [5-85]$$

$$\mathbf{K}_X = \Delta_X / \Delta \quad [5-86]$$

$$\mathbf{K}_Y = \Delta_Y / \Delta \quad [5-87]$$

$$\Delta_X = (\mathbf{c}_{35} + \mathbf{c}_{15} \mathbf{s}_1 + \mathbf{c}_{25} \mathbf{s}_4)(\mathbf{c}_{45} + \mathbf{c}_{14} \mathbf{s}_3 + \mathbf{c}_{24} \mathbf{s}_6) - (\mathbf{c}_{34} + \mathbf{c}_{14} \mathbf{s}_1 + \mathbf{c}_{24} \mathbf{s}_6)(\mathbf{c}_{55} + \mathbf{c}_{15} \mathbf{s}_3 + \mathbf{c}_{25} \mathbf{s}_6) \quad [5-88]$$

$$\Delta_Y = (\mathbf{c}_{45} + \mathbf{c}_{15} \mathbf{s}_2 + \mathbf{c}_{25} \mathbf{s}_5)(\mathbf{c}_{34} + \mathbf{c}_{14} \mathbf{s}_1 + \mathbf{c}_{24} \mathbf{s}_4) - (\mathbf{c}_{44} + \mathbf{c}_{14} \mathbf{s}_2 + \mathbf{c}_{24} \mathbf{s}_5)(\mathbf{c}_{35} + \mathbf{c}_{15} \mathbf{s}_1 + \mathbf{c}_{25} \mathbf{s}_4) \quad [5-89]$$

$$\Delta = (\mathbf{c}_{44} + \mathbf{c}_{14} \mathbf{s}_2 + \mathbf{c}_{24} \mathbf{s}_5)(\mathbf{c}_{55} + \mathbf{c}_{15} \mathbf{s}_3 + \mathbf{c}_{25} \mathbf{s}_6) - (\mathbf{c}_{45} + \mathbf{c}_{14} \mathbf{s}_3 + \mathbf{c}_{24} \mathbf{s}_6)(\mathbf{c}_{45} + \mathbf{c}_{15} \mathbf{s}_2 + \mathbf{c}_{25} \mathbf{s}_5) \quad [5-90]$$

$$\mathbf{s}_1 = \frac{\mathbf{c}_{12} \mathbf{c}_{23} - \mathbf{c}_{22} \mathbf{c}_{13}}{\mathbf{s}} \quad [5-91]$$

$$\mathbf{s}_2 = \frac{\mathbf{c}_{12} \mathbf{c}_{24} - \mathbf{c}_{22} \mathbf{c}_{14}}{\mathbf{s}} \quad [5-92]$$

$$\mathbf{s}_3 = \frac{\mathbf{c}_{12} \mathbf{c}_{25} - \mathbf{c}_{22} \mathbf{c}_{15}}{\mathbf{s}} \quad [5-93]$$

$$\mathbf{s}_4 = \frac{\mathbf{c}_{12} \mathbf{c}_{13} - \mathbf{c}_{11} \mathbf{c}_{23}}{\mathbf{s}} \quad [5-94]$$

The coefficients of A_{ij}, B_{ij}, D_{ij} are obtained from extensional, coupling and bending stiffness matrices of the top facesheet.

5.1.6 Analytical modelling of impact on curved sandwich plate (variable stiffness face-sheets)

As to date, an analytical model to determine the impact response of curved sandwich composite plates with variable stiffness face-sheets and foam core has not been developed in the literature . Table 14 shows the equivalent A, B, D stiffness matrices. The global stiffness matrices are different for the sandwich panel with variable stiffness facesheets as observed from Table 14.

Stiffness Matrix	Variable stiffness face-sheets	Identical face-sheets (Symmetrical)
$[A_{sand}]$	$[A]^t + [A]^b$	$2 [A]^t$
$[B_{sand}]$	$d^t [A]^t - d^b [A]^b + [B]^t + [B]^b$	0
$[D_{sand}]$	$(d^t)^2 [A]^t + (d^b)^2 [A]^b + [D]^t + [D]^b + 2d^t [B]^t - 2d^b [B]^b$	$\frac{1}{2} d^2 [A]^t + 2[D]^t$

Table 14 Comparison of stiffness matrices of sandwich panels

It is also required to validate the present model with related experimental work. Yurddaskal and Baba [125] performed experimental studies on low velocity impact response of foam based curved sandwich panels. However, boundary conditions in their model were clamped. Therefore, in the calculation of the dynamic global stiffness, K_{gd} of the panel, the equations for the clamped boundary conditions were used. It is noted that in the devised analytical model, only top facesheet damage is considered. In reality, the bottom facesheet could also be damaged at higher impact energies, thus resulting in two force drops in contact force history plots. In the experimental study, facesheets were oriented at $[0^\circ/90^\circ/45^\circ/-45^\circ]$ configuration and the plate was impacted at 10J, 25J and 80J impact energies. At impact energy of 10J corresponding to impact velocity of 2.1m/s, only the top facesheet was damaged. The specimens were impacted by a 4.926kg hemispherical nose impactor having a 12.7mm diameter. The core of the sandwich panel was 15mm and radius of the panel were changed within the range of $R=100$ and 160mm.

However, graphical comparison of the contact force-time representation was not available in this experimental study for the curved sandwich plate. Since, data for the maximum contact force were available; it is considered as a method for comparison with the present developed analytical model. As presented in Table 15, the analytical model was proven to be accurate where the difference between the maximum contact force when compared with experimental results was less than 10% for two instances of plate curvatures.

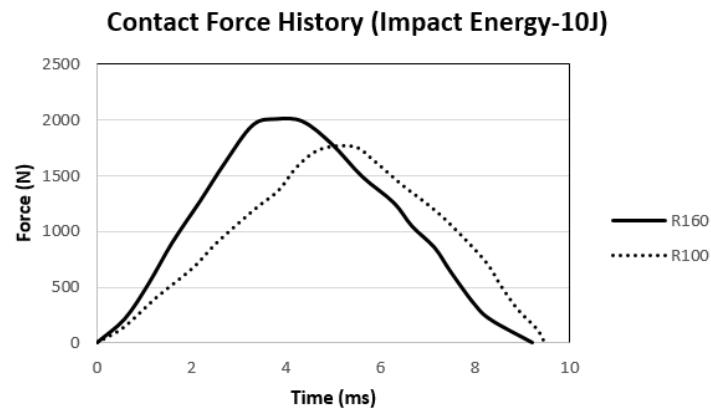


Figure 95 Impact force against time for clamped panel with different radius.

Radius (mm)	Maximum Impact Force (N)		% difference
	Experiment	Present Model	
160	2100	1990	5%
100	1900	1750	8%

Table 15 Analytical and Experimental results comparison.

5.2 Results and Discussions

After validating the present model with experimental studies for flat and curved sandwich panels, a parametric study is performed to analyse the effects of different factors affecting the low velocity impact response of curved sandwich panels with honeycomb core. For this study, a foam core panel with glass/epoxy facesheets are considered. Material properties for the glass/epoxy facesheets and impactor are presented in [125]. The radius of curvature of the panel 160mm. Boundary conditions of the plate are simply supported.

5.2.1 Impactor velocity

The effect of impactor velocity on the impact response of curved composite sandwich panels is investigated. The facesheets were oriented at $[0^\circ/90^\circ/45^\circ/-45^\circ]$ configuration for each velocities considered in the study. As the impact velocity increases, the maximum contact force increases. It is also observed that the contact duration increases slightly. An 20% increase in velocity gives rise to a 21% in the impact force and contact time by only 4%. The main reason been that the initial velocity of top facesheet, $\dot{\theta}_0$ is directly proportional to the impactor velocity.

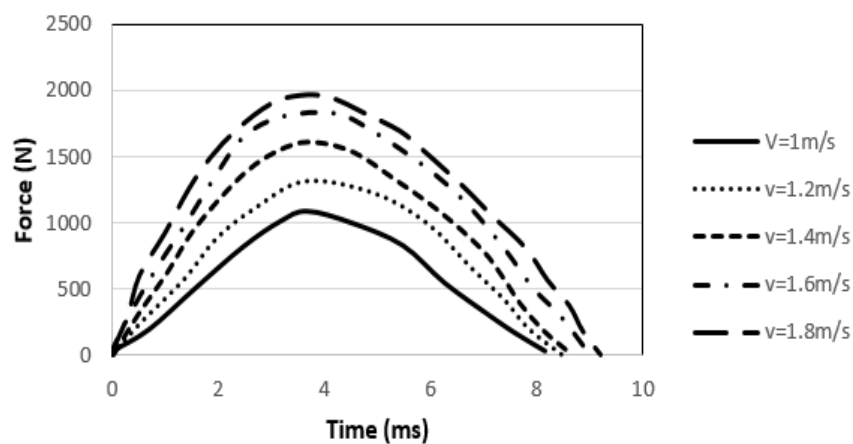


Figure 96 Effect of impactor velocity on impact response.

5.2.2 Layups

The effect of layups on the impact response of curved composite sandwich panels is investigated. Layups presented in Table 16 are studied. The results predicted that the stacking sequence of the facesheets played a minor role in the impact response. Negligible effect was observed in the contact duration. However, the maximum impact force of layup 1 was slightly higher than the other layups. Through the analysis, it is seen that 45° plies in the laminate increased the maximum contact force, even though interchanging the positions resulted in a negligible change.

Layup	Fibre Orientation
1	$[0^\circ/90^\circ/45^\circ/-45^\circ]$
2	$[0^\circ/90^\circ/-45^\circ/45^\circ]$
3	$[0^\circ/90^\circ/0^\circ/90^\circ]$
4	$[0^\circ/30^\circ/90^\circ/-30^\circ]$

Table 16 Various layups for the parametric studies.

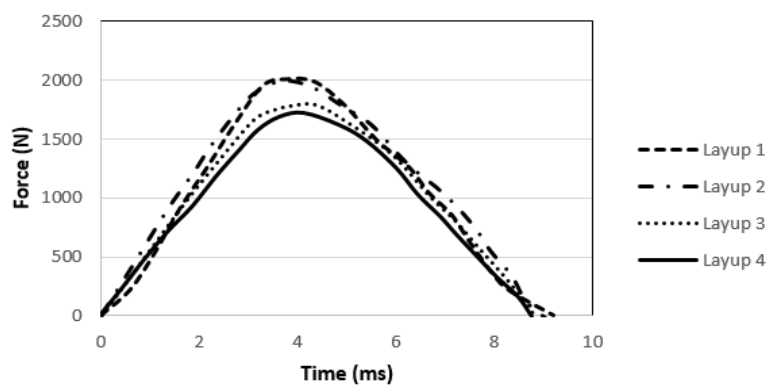


Figure 97 Effect of layups on impact response

5.2.3 Facesheet thickness (identical top and bottom facesheet with constant thickness)

Facesheet thickness also plays a key role in the impact response of sandwich panels. In all studies, four facesheets were used with the thickness of ply changed keeping the orientation constant. Higher facesheet thicknesses results in greater maximum contact force and lesser contact duration. For instance, a 33% increase in facesheet thickness results in a 17% rise in impact force and 5% decrease in contact duration. It is also noted that the drop in the maximum contact force occurs earlier when at higher

thicknesses. This force drop implies failure of the top facesheet and therefore confirms that thicker facesheets fail quickly since higher energy is absorbed during the impact.

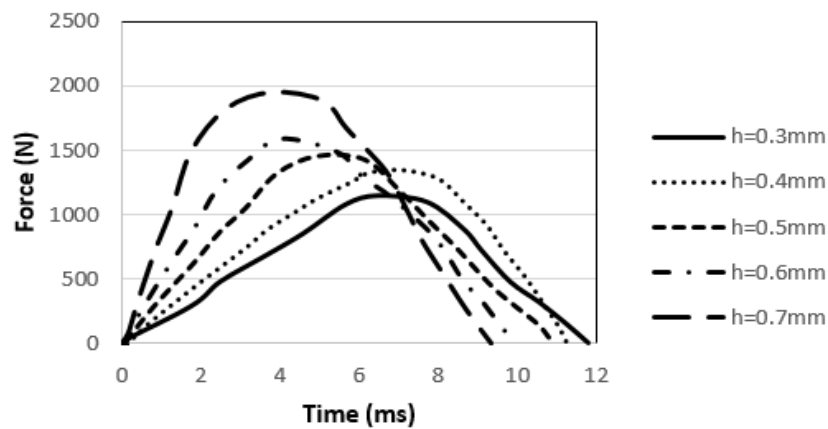


Figure 98 Effect of face-sheet thickness

5.2.4 Core thickness

The role played by core thickness in impact response is analysed here. As seen from Figure 99, as the core thickness increases, the maximum contact force increases. However, the contact duration decreases. It is also observed that the rise in the contact force decreases as the thickness increases. For instance, as the core thickness is raised from 10mm-12.5mm, the contact force increases by 22%, whereas when it is increased from 17.5mm-20mm, only 4% increase of contact force is observed. The pattern is similar for the contact duration. Therefore, it is proved that even though the maximum contact force increases with core thickness, it becomes insignificant at higher core thickness values.

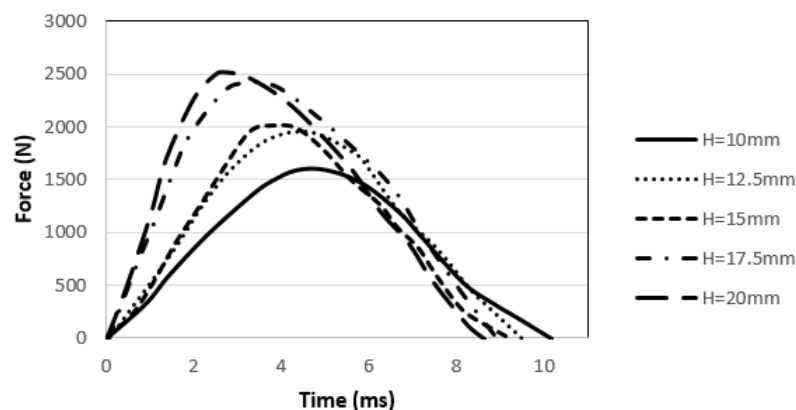


Figure 99 Effect of core thickness

5.2.5 Sandwich plate curvature

The final parametric study focusses on the effect of panel curvature on the impact response of the curved plate. Results depict that increase of curvature increases the maximum contact force of the plate. The drop in the impact force occurs earlier, thereby indicating that at higher curvatures, the top face-sheet fails earlier. A 7% increase in the plate curvature gives rise to a 5% rise in the maximum impact force and 2% decrease in the contact duration. (Figure 100)

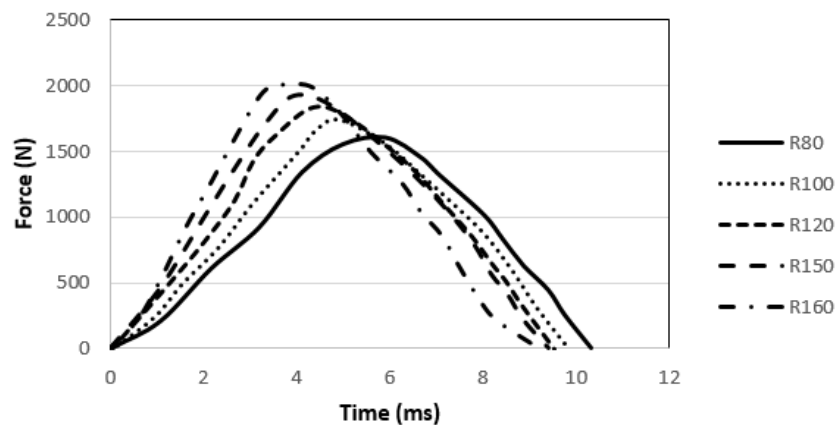


Figure 100 Effect of plate curvature on impact response.

5.2.6 Effect of variable stiffness face-sheets

The effect of variable stiffness face-sheets are analysed in this section. The stiffness matrices of the sandwich panel are derived through the formulation in Table 17. The stiffness variation is achieved through variation in thickness of top and bottom face-sheets. All other factors are kept constant. It is observed that both top and bottom facesheet contributes to the overall stiffness of the sandwich panel. Increase of thickness gives rise to an enhancement of stiffness. Combinations 1 and 2 corresponds to decrease of top facesheet thickness and combinations 3 and 4 corresponds to decrease of bottom facesheet thickness. The analysis proves that the top facesheet thickness has a greater effect on the maximum contact force than the bottom facesheet thickness. This is demonstrated by the higher drop in maximum contact force when compared with the sandwich panel with constant thickness facesheets. A 0.1mm reduction in thickness of only top facesheet reduces impact force

by 5.4% whereas 0.1mm reduction in thickness of only bottom facesheet gives a 4.1% reduction in maximum impact force

Combination	Top facesheet thickness (mm)	Bottom facesheet thickness (mm)	Maximum contact force/N	% decrease in maximum contact force
1 (Constant thickness facesheets)	0.5	0.5	1481	-
2 (Variable thickness)	0.4	0.5	1401	5.4%
3 (Variable thickness)	0.3	0.5	1221	17.6%
4 (Variable thickness)	0.5	0.4	1420	4.1%
5 (Variable thickness)	0.5	0.3	1284	13.3%

Table 17 Combinations of variable stiffness facesheets

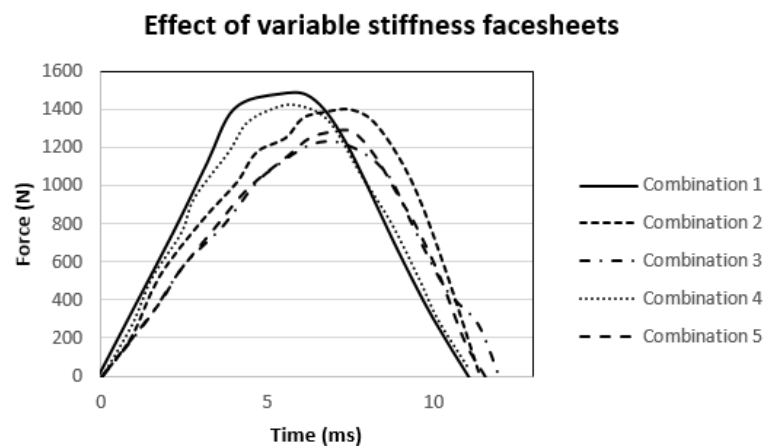


Figure 101 Effect of variable stiffness facesheets

5.3 Chapter Conclusions

This chapter focused on developing analytical models to study the low velocity of curved foam based sandwich plates. This was identified as the research gap in this chapter. Two degree of freedom spring-mass-dashpot system was used to derive analytical functions force functions. Effect of various parameters affecting impact response such as impactor velocity, layups, face-sheet thickness, core thickness and sandwich plate curvature are studied.

Main findings of this chapter are:

- 20% increase in impact velocity gives rise to a 21% increase in maximum impact force. Contact duration also increases by 4%.
- Stacking sequence plays a minor role in impact response of curved foam sandwich plates. However, it was observed that inclusion of 45 degree plies in the facesheets increases maximum contact force.
- When identical (constant) thickness facesheets were used for the top and bottom facesheets, a 33% increase in facesheet thickness results in a 17% rise in impact force and 5% decrease in contact duration. It is also noted that the drop in the maximum contact force occurs earlier when at higher thicknesses. This force drop implies failure of the top facesheet and therefore confirms that thicker facesheets fail quickly since higher energy is absorbed during the impact.
- Effect of core thickness was observed by increasing thickness from 10mm-12.5mm, the contact force increases by 22%, whereas when it is increased from 17.5mm-20mm, only 4% increase of contact force is observed.
- 7% increase in the plate curvature gives rise to a 5% rise in the maximum impact force and 2% decrease in the contact duration.
- Effect of variable stiffness facesheets were also analysed. For this study, non-identical facesheets with different thicknesses were used for the top and bottom facesheets. The analysis proves that the top facesheet thickness has a greater effect on the maximum contact force than the bottom facesheet thickness. This is demonstrated by the higher drop in maximum contact force when compared with the sandwich panel with constant thickness facesheets. 0.1mm reduction

in thickness of only top facesheet reduces impact force by 5.4% whereas 0.1mm reduction in thickness of only bottom facesheet gives a 4.1% reduction in maximum impact force.

Chapter 6:

Numerical modelling

6 Numerical Modelling

Numerical modelling aspect in this research aims at developing finite element models capable of validating the analytical models developed through this research in studying impact response of curved variable stiffness composite plates.

6.1 Continuum damage modelling

Fracture mechanics and progressive damage analysis are vital aspects of analysis tools utilised for damage tolerance designs capable of carrying the design load in the existence of damage. Recent advancements in computational damage mechanics in capturing complex details of damage and failure mechanisms have paved way for accurate representation and prediction of damage in composite structures. Continuum damage mechanics are widely used since they are efficiently implemented in finite element software. In this method, the effect of material degradation due to damage in the fracture process zone (FPZ) is modelled in a smeared technique and the stiffness is reduced by the use of an appropriate strain-softening method inside the FPZ [137]. The popularity of CDM has increased due to the inefficiencies of the linear elastic fracture mechanics (LEFM) approach such as LEFM's inability to predict the initiation of damage and it ignores the existence of the FPZ in front of the crack tip.

6.2 Damage modes in composite laminates

There are two main categories of damage modes in composites namely, intra-laminar and inter-laminar damage. Intra-laminar damage are formed by fibre dominated damage modes such as fibre breakage, fibre pull-out and fibre buckling as well as matrix dominated modes such as matrix cracking and fibre-matrix splitting. Inter-laminar damage is formed by the separation of layers of the laminate and is generally known as delamination. Initiation and progression of damage in laminated composites mainly depends on the material properties, laminate design and loading conditions. The damage mechanisms observed in composite laminates differ under in-plane tensile or compressive loading. Under tensile loading, the main form of damage is matrix cracking and fibre breakage. Delamination could also occur depending on the laminate layup. The main cause of delamination in a laminate is the difference in orientations between the laminates. Higher the angle between laminates, higher the

delamination area. Primary failure mode under compressive loading is the formation of kink-bands caused by matrix cracking and fibre rotation in plies oriented in the loading direction. Delamination and off-axis matrix cracking also are evident in composites under compression loading. Damage of composites is a complex phenomenon since several damage modes are present simultaneously and engagement of these damage modes increases the energy absorption capability of a laminate. Due to this complexity, majority of modelling methods simulate inter-laminar and intra-laminar damage modes separately [137]

Progression of damage in composites can be simulated through different approaches as shown in Figure 102. Delamination is usually modelled through the cohesive crack concept due to its discrete nature. In order to simulate intra-laminar damage, CDM technique is widely used. Intra-laminar damage commonly occurs in a wider and diffused pattern within composite laminates and therefore provides a better representation of damage patterns when compared with discrete methods.

6.3 Progressive damage analysis

6.3.1 Damage model

Progressive damage analysis based on CDM has been widely used in predicting the stiffness degradation and damage evolution by introducing damage tensors and damage evolution laws [138]. The damage constitutive model for the in-plane damage of composite based on the CDM is defined as [139]:

$$C_d = \frac{1}{D} \begin{bmatrix} (1 - d_f)E_1 & (1 - d_f)(1 - d_m)v_{21}E_1 & 0 \\ (1 - d_f)(1 - d_m)v_{12}E_2 & (1 - d_m)E_2 & 0 \\ 0 & 0 & (1 - d_s)GD \end{bmatrix} \quad [6-1]$$

$$D = 1 - (1 - d_f)(1 - d_m)v_{12}v_{21} \quad [6-2]$$

d_f , d_m and d_s are damage variables representing fibre breakage, matrix cracking and shear failure respectively.

Strength based failure criteria are then used to assess failure within the laminate. Commonly used failure criteria are maximum stress/strain criterion, Hashin and

Chang-Chang. These failure criteria are able to identify the detailed failure modes. Different finite element software incorporates different failure laws. For example, LS-DYNA which is used to model impact in this research uses Chang-Chang failure criterion to determine initiation of damage.

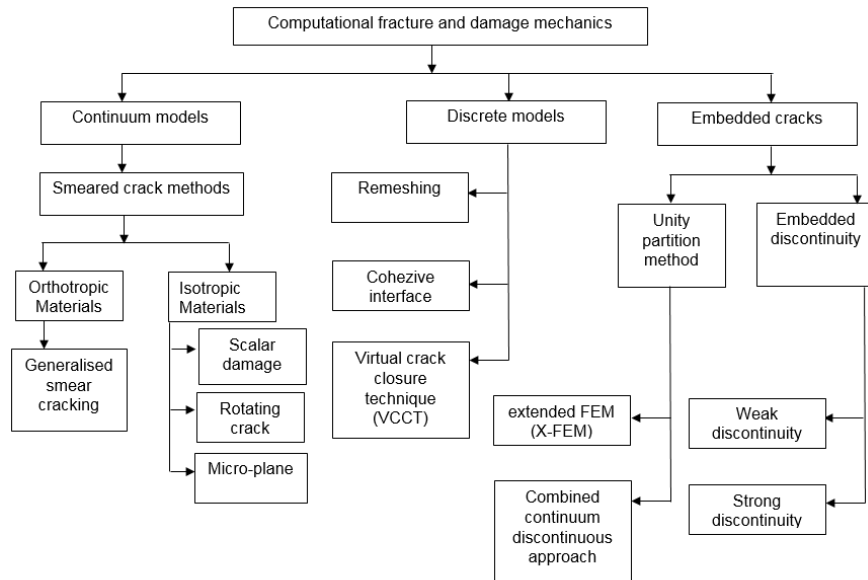


Figure 102 Damage mechanics approaches (Composed by author)

6.3.2 Damage evolution

The next major challenge of the CDM approach is the derivation of an evolution law for the multiple damage parameters. Evolution laws governing the conditions for initiation and growth of damage parameters are usually expressed in terms of the strain rate in the layer. A linear softening law is commonly used to gradually degrade the material stiffness of the damaged element once a failure criterion is met. The strength of the ply decreases with strain as shown in Figure 103.

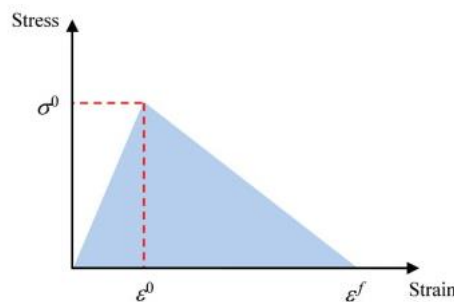


Figure 103 Stress-strain graph of linear softening law [138]

6.4 Finite Element Modelling in LSDYNA

The commercially available software LSDYNA was used for simulating the impact damage of the curved composite plate. The LS-DYNA module was used to simulate the impact behaviour.

6.5 Modelling process

The modelling process of the impact problem is defined by the following steps.

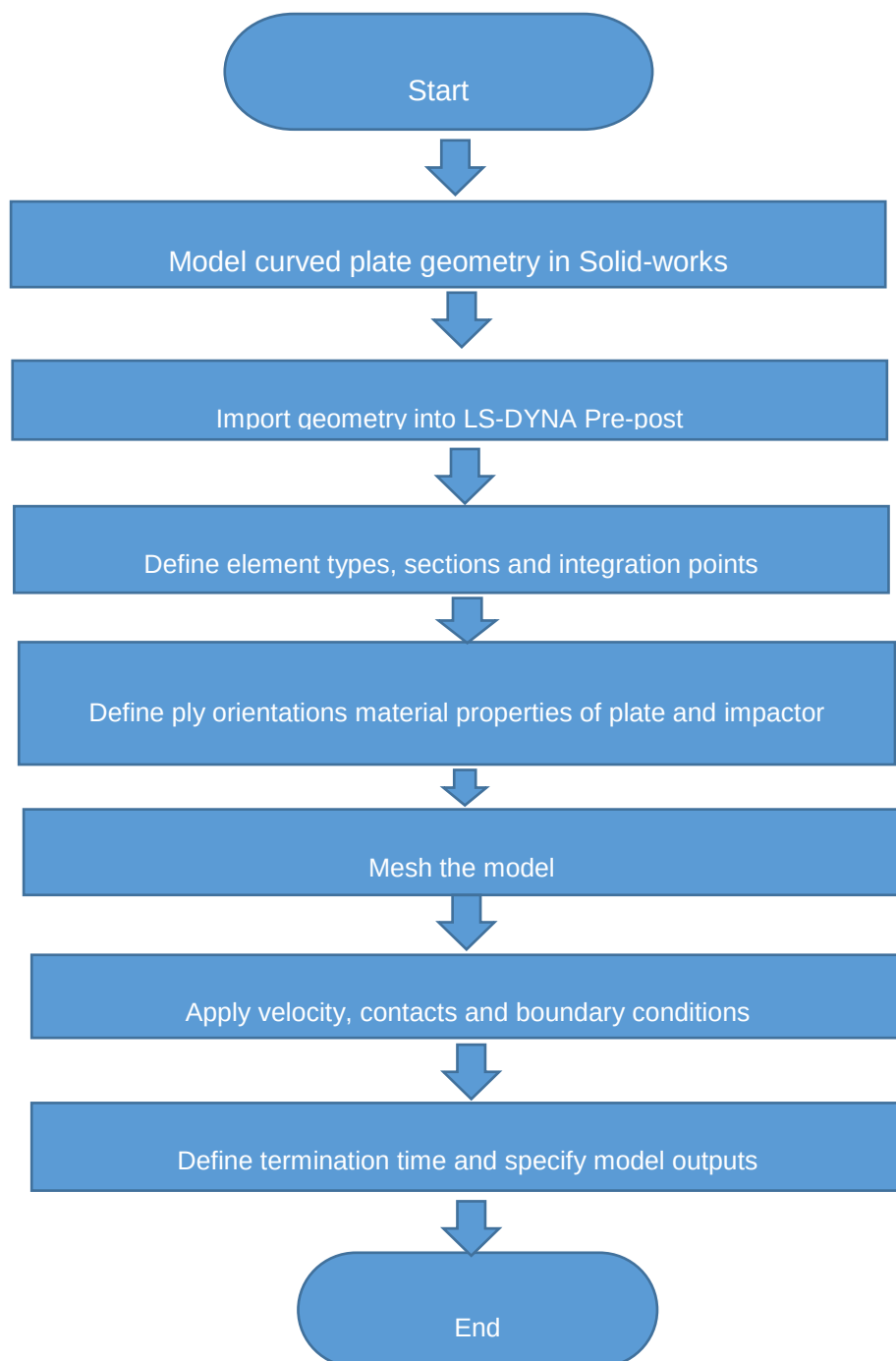


Figure 104 Numerical modelling process

6.5.1 CAD modelling of the curved plate

The first step of the finite element modelling process is to define the geometry. The geometry in the present study consists of a curved plate with three variable stiffness sections. This variation in the stiffness is achieved by varying the thicknesses of the sections across the width of the plate. Therefore, the surface of the plate is split into three sections as depicted in Figure 106.

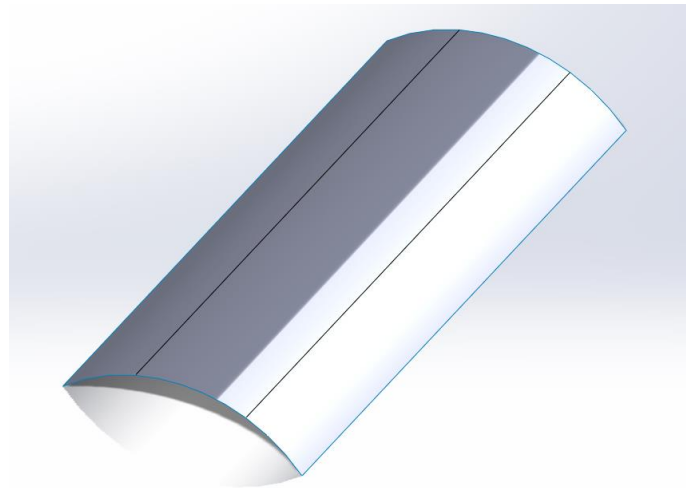


Figure 106 Curved plate CAD model

6.5.2 LS-PrePost model

The model is then imported into LS-Prepost. A spherical impactor was modelled with a diameter of 20mm

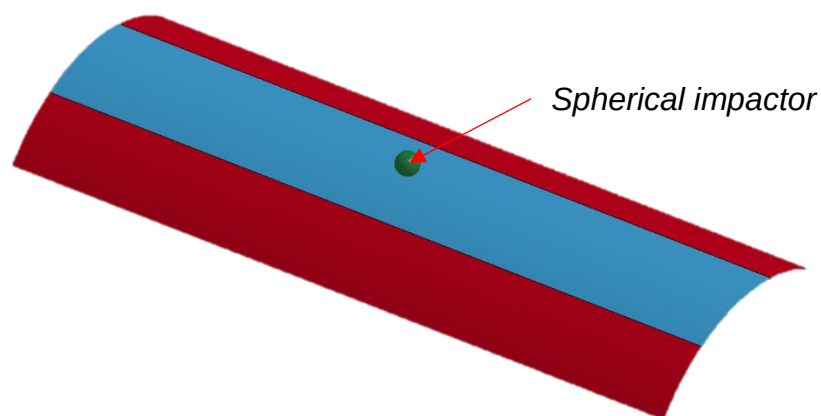


Figure 107 LS-Prepost model

6.5.3 Element types and section properties

An important aspect of the finite element model is to define the element types and section properties. The size of the composite plates were $600 \times 200 \text{ mm}^2$ and radius 150mm consisting of variable thicknesses through the length of the plate.

6.5.4 Definition of variable stiffness section

In the finite element model, the thicknesses across the width of the plate are varied to simulate the variable stiffness effect. This is achieved by using two shell element types defined by two different thicknesses as shown in Figure 108.

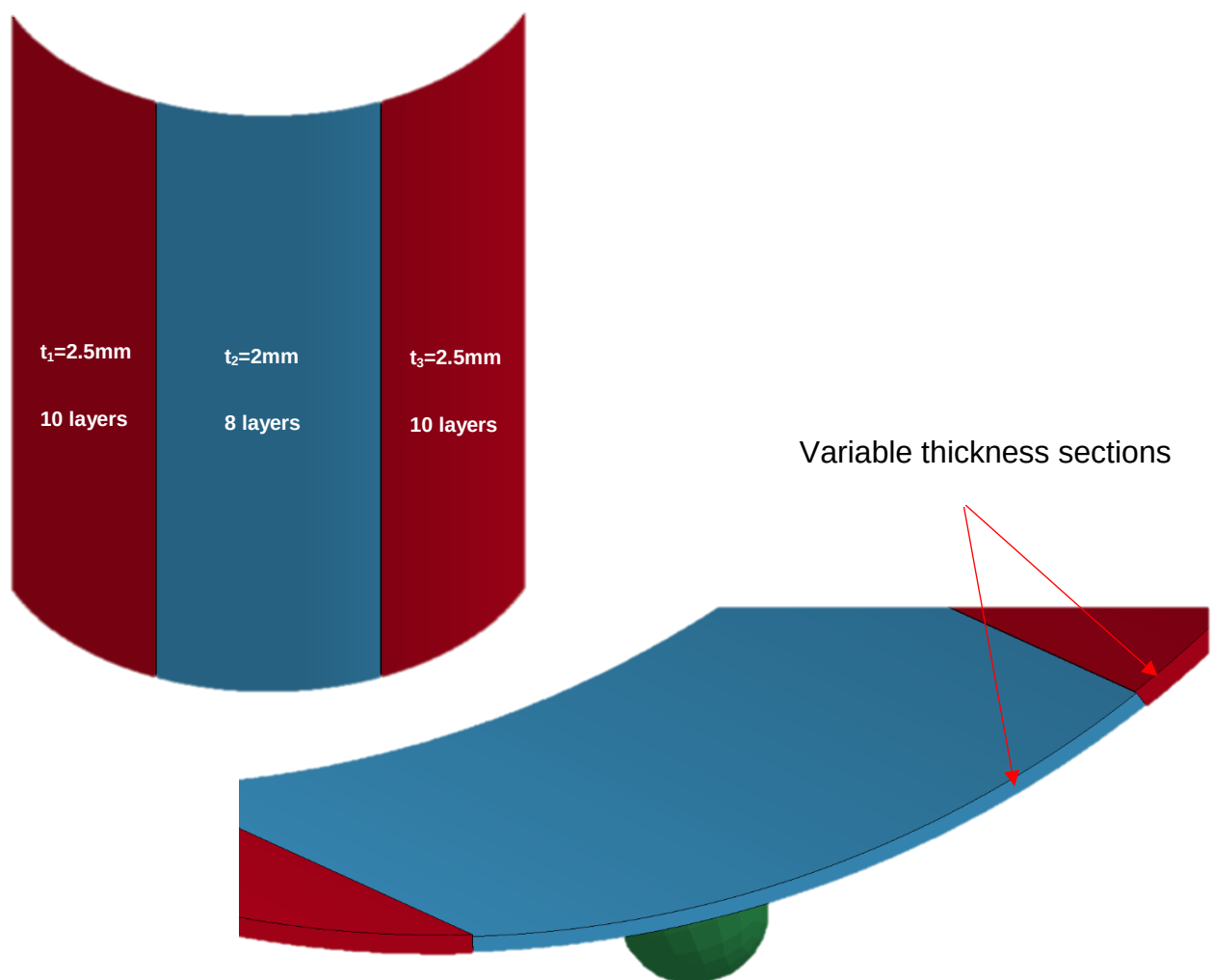


Figure 108 Definition of variable stiffness sections

6.5.5 Integration points and ply orientations

The no. of composite layers in a laminate are defined by integration points in LS-DYNA. Therefore, if 10 layers needs to be defined, 10 integration points need to be set up. The present model requires ten integration points for the outer sections of the plate and eight integration points for the mid-section. The ply orientation for the section with ten integration points is $[0/90/0/90/0]_s$ and $[0/90/0/90]_s$ for the section with eight integration points. This is achieved through defining the beta-angles in LS-Prepost.

6.5.6 Meshing

The plate was meshed based on Belytschko-Lin-Tsay quadrilateral shell elements. This shell element is based on a combined co-rotational and velocity strain. Since, the impact occurs at the centre, it was decided to use refined element size at the centre of the plate. Therefore, the outer surfaces of the plate were meshed using quadratic shell elements of size $4 \times 4 \text{ mm}^2$ and the mid-surface with eight integration points were meshed using shell elements sized $2 \times 2 \text{ mm}^2$. The striker was modelled as a rigid block using solid element. Results of mesh sensitivity analysis is shown in section 6.6.

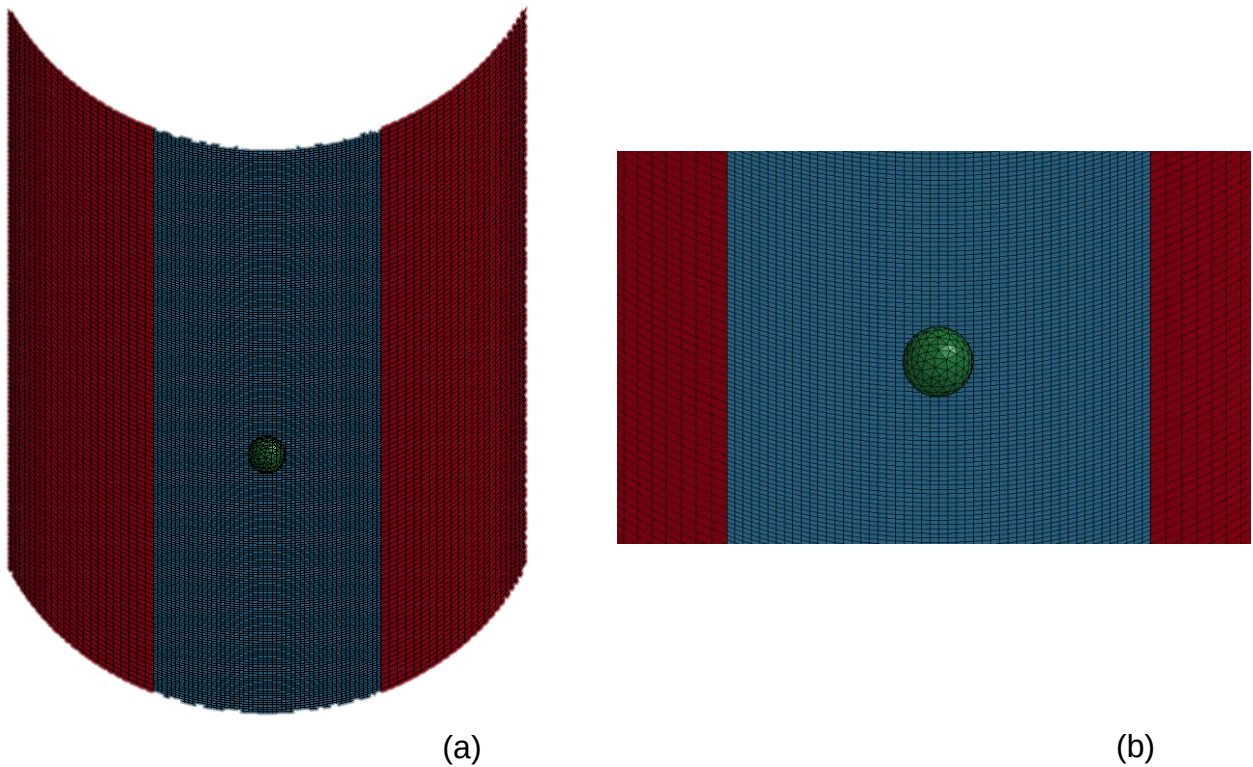


Figure 109 Meshing (a) Meshed model (b) refined mesh

6.5.7 Material properties (Intra-laminar failure)

A vital aspect of the finite element modelling process is the accurate definition of material properties. Material model 54 (ENHANCED_COMPOSITE_DAMAGE) of LS-DYNA was selected to model the intralaminar failure of curved composite plate. The Chang-Chang failure criterion which is the modification of the Hashin's failure criterion was chosen for assessing lamina failure. The post-failure conditions in the Material 54 model are somewhat different from the original Chang-Chang equations. In this model four failure modes are categorised. These failure indicators are appointed on total failure for the laminas, where both the strength and the stiffness are set equal to zero after failure is encountered. In this model as described below all material properties of lamina are checked using the following laws to determine the failure characteristic. Material model 54 assesses intra-laminar failure of the composite.

6.5.7.1 Tensile fibre failure mode: (fibre rupture)

If $\sigma_1 > 0$

$$\text{then } e_f^2 = \left(\frac{\sigma_1}{X_t}\right)^2 + \beta \left(\frac{\sigma_{12}}{\tau_s}\right)^2 - 1 \quad \begin{cases} e_f^2 \geq 0 & \text{failed} \\ e_f^2 < 0 & \text{elastic} \end{cases} \quad [6-3]$$

where β is a weighting factor for shear term in tensile fibre mode and its range is 0-1 and σ_1 is stress in the fibre direction, σ_{12} is transverse shearing stress, X_t is tensile strength in fibre direction and τ_s is shear strength. When lamina failure occurs, all material constants are set to zero.

6.5.7.2 Compressive fibre failure mode: (fibre buckling)

$$\text{If } \sigma_1 < 0 \quad \begin{cases} e_c^2 \geq 0 & \text{failed} \\ e_c^2 < 0 & \text{elastic} \end{cases}$$

$$\text{then } e_c^2 = \left(\frac{\sigma_1}{X_c} \right)^2 - 1 \quad [6-4]$$

where, X_c is compressive strength in fibre direction.

After lamina failure by fibre buckling E_1, ν_{12} and ν_{21} are set to zero.

6.5.7.3 Tensile matrix failure: (matrix cracking under transverse tension and in-plane shear)

If $\sigma_2 > 0$

$$\text{then } e_m^2 = \left(\frac{\sigma_2}{Y_t} \right)^2 + \left(\frac{\sigma_{12}}{\tau_s} \right)^2 - 1 \quad \begin{cases} e_m^2 \geq 0 \text{ failed} \\ e_m^2 < 0 \text{ elastic} \end{cases} \quad [6-5]$$

where σ_2 is stress in normal to the fibre direction, Y_t is tensile strength in normal to the fibre direction and Y_c is compressive strength in normal to the fibre direction. After lamina failure by matrix cracking, E_2, ν_{21} and G_{12} are set to zero.

6.5.7.4 Compressive matrix failure mode: (matrix cracking under transverse compression and in-plane shear)

$$\text{If } \sigma_2 < 0 \quad \begin{cases} e_d^2 \geq 0 \text{ failed} \\ e_d^2 < 0 \text{ elastic} \end{cases}$$

$$\text{then } e_d^2 = \left(\frac{\sigma_2}{2\tau_s} \right)^2 + \left[\left(\frac{Y_c}{2\tau_s} \right)^2 - 1 \right] \frac{\sigma_2}{Y_c} + \left(\frac{\sigma_{12}}{\tau_s} \right)^2 - 1 \quad [6-6]$$

6.5.7.5 Material Model 54

Composite damage in LSDYNA is commonly simulated by the material model 54. The inputs to this material model are defined in Table 18 below

Variable	Definition	Value
EA	Axial Young's modulus	141.9 GPa
EB	Transverse Young's modulus	9.72 GPa
GAB	Shear Modulus	5.53 GPa
PRBA	Minor Poisson's ratio	0.3
XT	Axial tensile strength	1500MPa
YT	Transverse tensile strength	50MPa
YC	Transverse-compressive strength	250MPa
SC	Shear strength	70MPa
DFAILT	Axial tensile failure strain	0.0106
DFAILC	Axial compressive failure strain	-0.008
DFAILM	Transverse failure strain	0.005
DFAILS	Shear failure strain	-
EFS	Effective failure strain	0
FBRT	Fibre tension damage factor after matrix failure	0.5
YCFAC	Fibre compression damage factor after matrix failure	1.2
SOFT	Crush front damage parameter	0
BETA	Shear factor in fibre tension	0.5

Table 18 Material Model 54 inputs

The values of failure strains are calculated as follows:

$$D_{FAILT} = \frac{X_T}{E_A} \quad [6-7]$$

$$D_{FAILC} = \frac{X_C}{E_A} \quad [6-8]$$

$$D_{FAILM} = \frac{Y_T}{E_B} \quad [6-9]$$

6.5.7.6 Material model 20

Material model RIGID-020 is used to model the solid impactor. The required inputs into this material model are the density, Young's modulus and Poisson's ratio. Since, the impactor is spherical, the volume is expressed as:

$$V = \frac{4}{3}\pi r^3 \quad [6-10]$$

where r is the radius of the impactor. Since the mass of the impactor is known to be 3kg, the density is calculated by the density relation between mass and volume. After all the material models are defined, the sections meshed with shell elements are assigned with material model 54 and the impactor which is meshed by solid elements is assigned with the material model 20.

6.5.8 Velocity, contacts and boundary conditions

The next step is the application of velocity and boundary conditions into the model. A velocity of 6m/s is applied to the impactor. The contact between the rigid plate and the specimens was modelled using a *nodes impacting surface* with a friction coefficient of 0.35 which was measured experimentally to avoid lateral movements. To prevent the penetration of the boundary by its own nodes, a *single surface* contact algorithm without friction was used. The plate was constrained by simply supported boundary conditions.

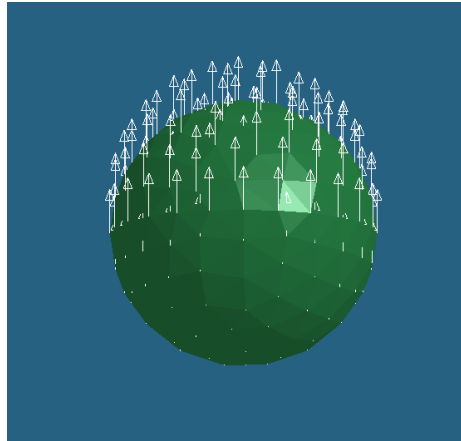


Figure 110 Velocity applied to solid impactor

6.5.9 Delamination modelling (Inter-laminar failure)

The failure modes observed in a composite laminate during a low velocity event are fibre breakage, matrix cracking, transverse matrix cracking, matrix crushing, fibre-matrix interface de-bonding and delamination. The Chang-Chang material failure model in Material 54 of LSDYNA models the failure modes except delamination. These failure modes incorporated in Material 54 can only be used with thin shell theory, since they are dependent upon in-plane stresses. However, delamination failure mode requires a three-dimensional representation of the constitutive equations and kinematics and the use of thin shell theory is incompatible to model delamination since it only deals with stresses at a macro-mechanical level. Therefore, when thin shell elements are used in composites failure modelling, delamination is usually ignored. Several methods exist to model the delamination growth in composites such as cohesive zone modelling (CZM), virtual crack extension technique, stress intensity factor, virtual crack closure technique.

Cohesive zone modelling (CZM) is used to analyse the failure behaviour of very thin layers such as composite delamination. In CZM, failure of the adhesive layer is modelled by a simple traction separation law instead of classical constitutive equations as shown in Figure 111.

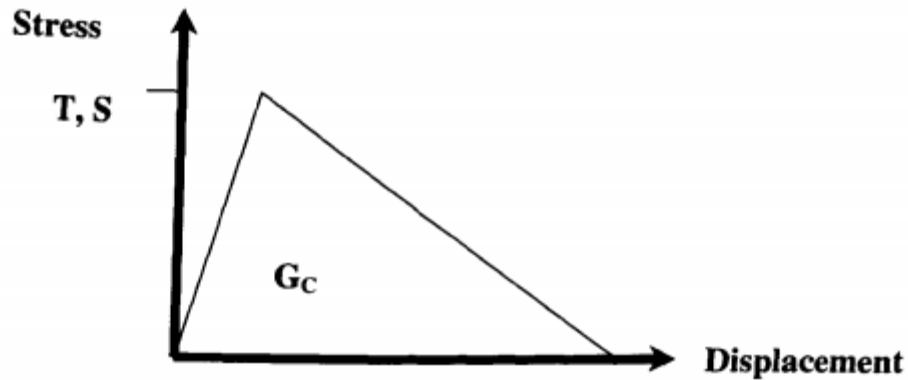


Figure 111 Bi-linear traction separation law [140]

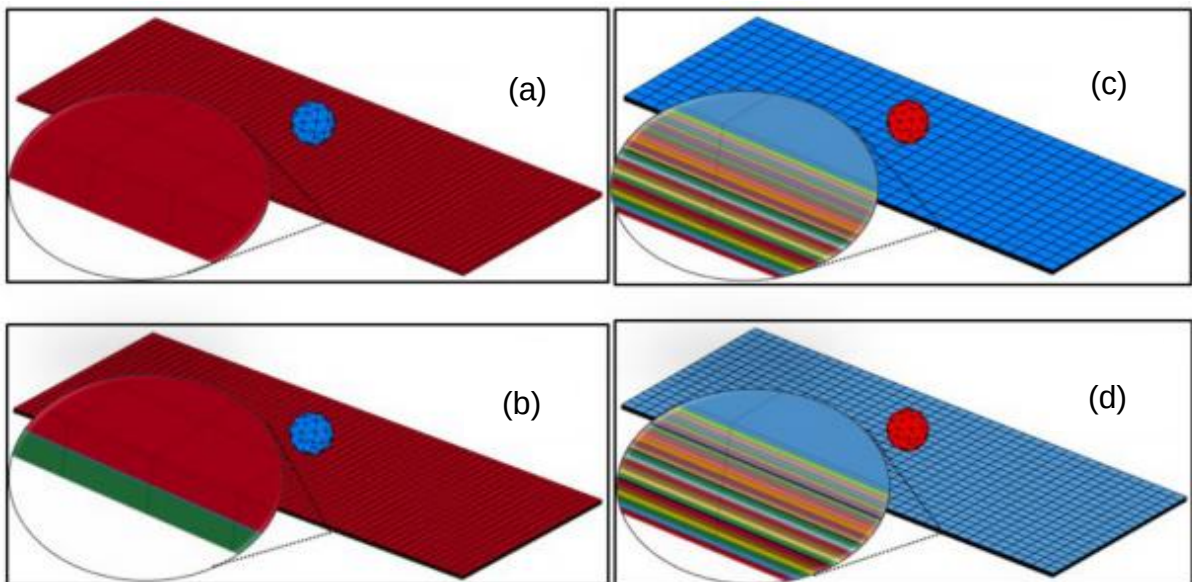


Figure 112 FE Models for impact on laminated plate [140]

Several methods have been adopted to model delamination behaviour during an impact event. The first method is to use thin shell elements with tiebreak contact which is explained later. The second method involves employing multilayer thick shell (TSHELL) element formulation by setting the number of integration points to the number of layers in the laminate. (Figure 112a and 112b). Delamination behaviour is modelled using cohesive zone elements [140]. Solid layers with their corresponding material angles and cohesive layers placed between those solid layers is also another delamination modelling technique. Most FEA analyses involves modelling thick composite structures by using shell elements, thus producing inaccurate results.

Therefore, 3D solid elements are required since they allow an accurate capture of the through thickness stress distribution. However, using solid elements for each layer significantly increases the computational time.

Delamination can also be modelled using tiebreak delamination contact using thin shell elements. This procedure is adopted for the present model since the laminate in the current model consists of a maximum of ten plies and can be considered as thin composite structures. Therefore, the present FEA model is considered accurate to validate the analytical model. However, the accuracy will decrease if more plies are used in the laminate meaning that other methods involving thick shells and cohesive zone modelling should be used. Tiebreak contacts are utilised as adhesive to bond the sub-laminates in the model. [140] Tiebreak contacts are active for nodes initially in contact. This type of contact requires normal and shear failure strengths to be defined to check for bond failure. Under compressive loading, tiebreak contact acts commonly to other types of contacts. During the loading process, damage of the material acts linearly to the distance between the two points initially in contact. This contact is broken and sub-laminates are converted into surface-to-surface contacts once the critical opening is reached. Under tensile loading, tiebreak allows separation of the surfaces and eventually the tied surfaces fail under the following failure criterion:

$$\left(\frac{|\sigma_n|}{NFLS}\right)^2 + \left(\frac{|\sigma_s|}{SFLS}\right)^2 \geq 1 \quad [6-11]$$

Where NFLS is tensile failure strength and SFLS is shear failure strength of the adhesive [140].

In the present model *surface-to-surface tiebreak* contact in LSDYNA was used to model the contact between the plies with properties of adhesive material, NFLS= 56MPa and SFLS= 44MPa for a typical epoxy adhesive [140].

6.5.9.1 Verification of the tie-brake contact

A verification is performed to analyse the validity of using the surface to surface tie-break contact to model delamination and impact damage of composites. For this purpose, impact simulations are performed on a flat composite plate similar to the

experimental study [141]. The flat plate has dimensions of $300\text{mm} \times 150\text{mm}$ with a total thickness of 2.7mm . The layup consists of 24 layers and oriented at $[-45/0/45/90]_{3s}$. The impactor was modelled as a rigid sphere of diameter 25.4mm , mass of 1.85kg and a velocity of 6.5m/s was applied. Influence of shell layers on force-time response is studied. Initially the plate was modelled as a single shell with all 24 layers (integration points) incorporated in the shell. In the subsequent models, the integration layers were split accordingly to the number of shells as shown in Figure 114. Surface to surface tie-break contacts were introduced in between shells for the model with two or more shells

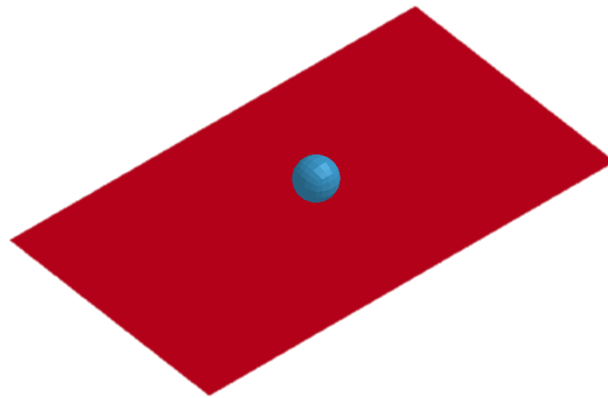


Figure 113 Numerical Model with thin shell elements (Flat plate)

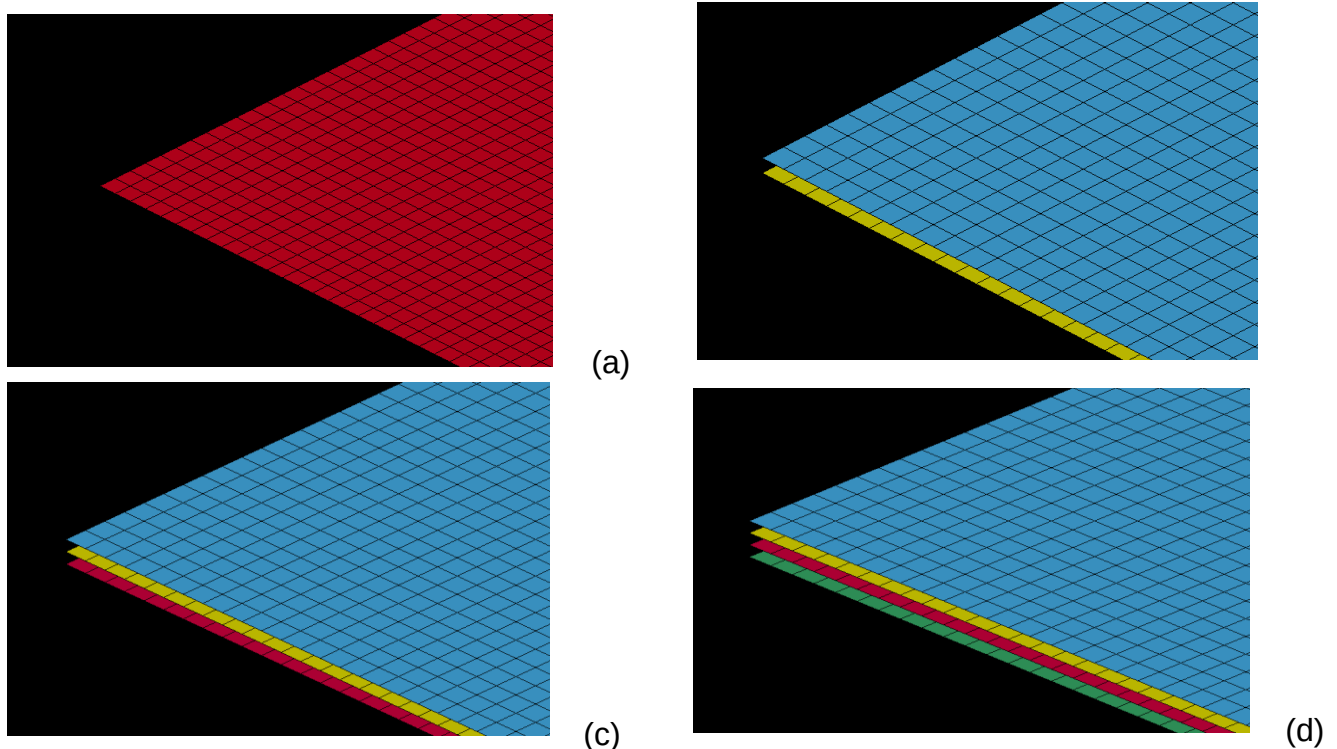


Figure 114 Shell element layers (a) 1 shell (b) 2 shells (c) 3 shells (d) 4 shells

Energy plots for the effect of shell layers on absorbed energy is shown in Figure 115. Absorbed energy is the lowest for the model with a single shell since it does not include any inter-laminar failure (delamination). However, failure in fibre tension occurs which is not observed with the other models with tie-break contacts. Another observation made was that the bending stiffness of a stacked shell model is higher than that of a single shell model. Therefore, the model with only one shell depicted the highest stiffness and the lowest deflection. Force-time response as in Figure 116 portrays that use of a tie-break contact with increased number of shells brings the numerical results closer to the experimental work. However, this effect is not significant when two or more shell layers are used. Use of two shells with the tie-break contact to model delamination improves contact force by 9% compared with experimental data.

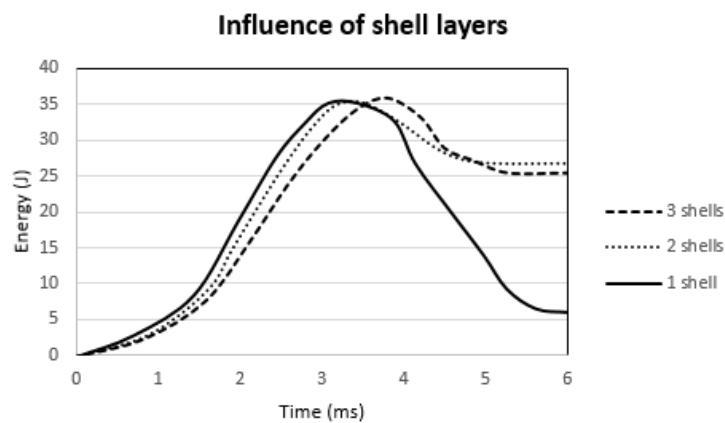


Figure 115 Effect of shell layers on energy

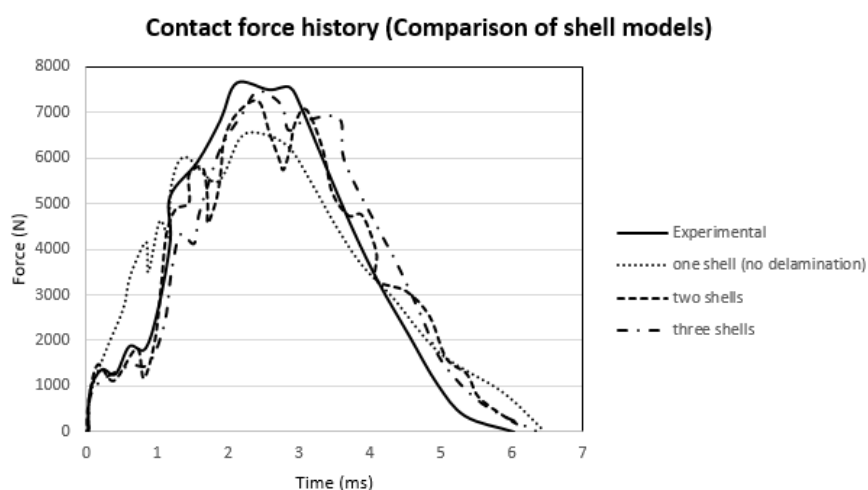


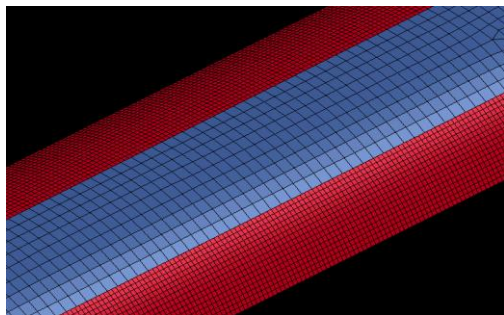
Figure 116 Effect on contact force history

6.6 Results and Discussions

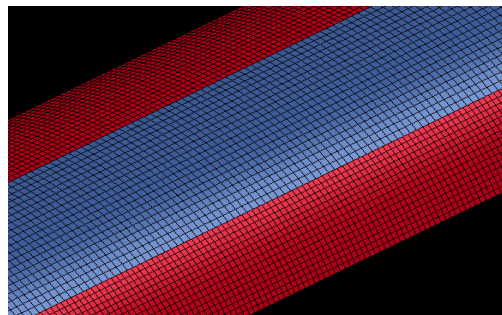
The main aim of this numerical model was to only serve as a validation tool for the analytical model. Validating the analytical model was a challenge since experimental work on low velocity impact on curved variable stiffness plates were not available in literature. In these models initial damage is not present. In Chapter 3, modifications were made to the impact model to incorporate the location of the variable stiffness section. The validity of this concept is verified through the numerical analysis. It proves that the position of the variable stiffness section is a vital factor in impact modelling of such structures. It is observed that neglecting this factor over-estimates the maximum contact force.

6.6.1 Mesh sensitivity analysis

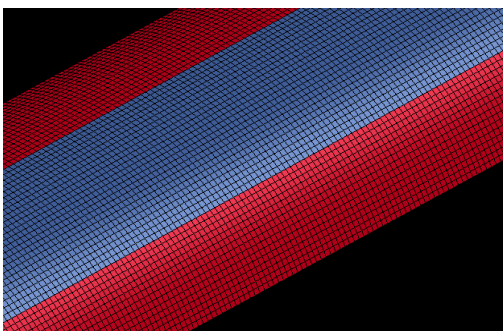
A mesh sensitivity analysis was performed to find the optimum mesh size for the study. In this analysis, the mesh size of the centre section was varied since the impact occurs at the centre of the plate. Element sizes of 10mm, 5mm, 4mm, 3mm and 2mm were used respectively as shown in Figure 117.



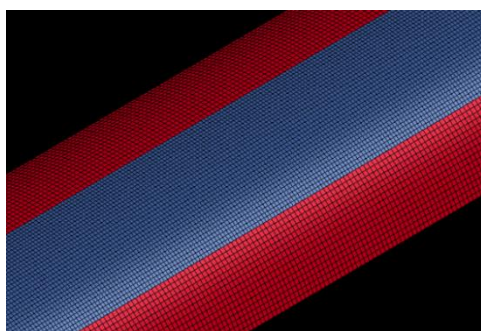
(a)



(b)



(c)



(d)

Figure 117 Mesh element sizes (a) 10mm (b) 5mm (c) 4mm (d) 3mm

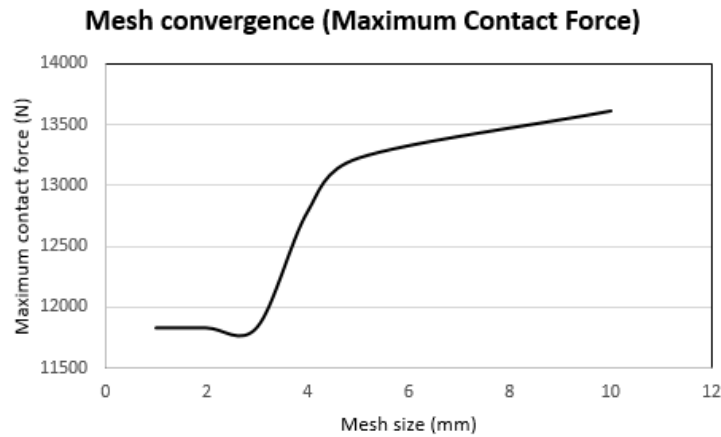


Figure 118 Mesh convergence (Maximum contact force)

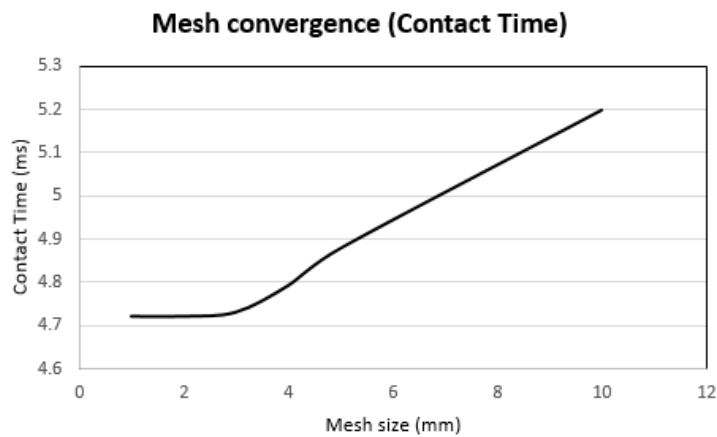


Figure 119 Mesh convergence (Contact time)

Results from the mesh convergence study proves that an element size of 2mm at the centre section is optimum for the numerical impact analysis. (Figures 118 and 119) 4mm mesh size is used in the outer variable stiffness sections to decrease computational time.

6.6.2 LS-DYNA impact event

The first step of verification in the simulation is the impact energy. In the analytical study, an impactor of 3kg mass and 6m/s impact velocity would yield 54J of kinetic energy. The kinetic energy of the LS-DYNA analysis is 52J as shown in Figure 120.

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Beginning of keyword reader
Memory required to process keyword      :      1220974
input of data is completed
initial kinetic energy = 0.52184818E+02

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Figure 120 Numerical impact energy

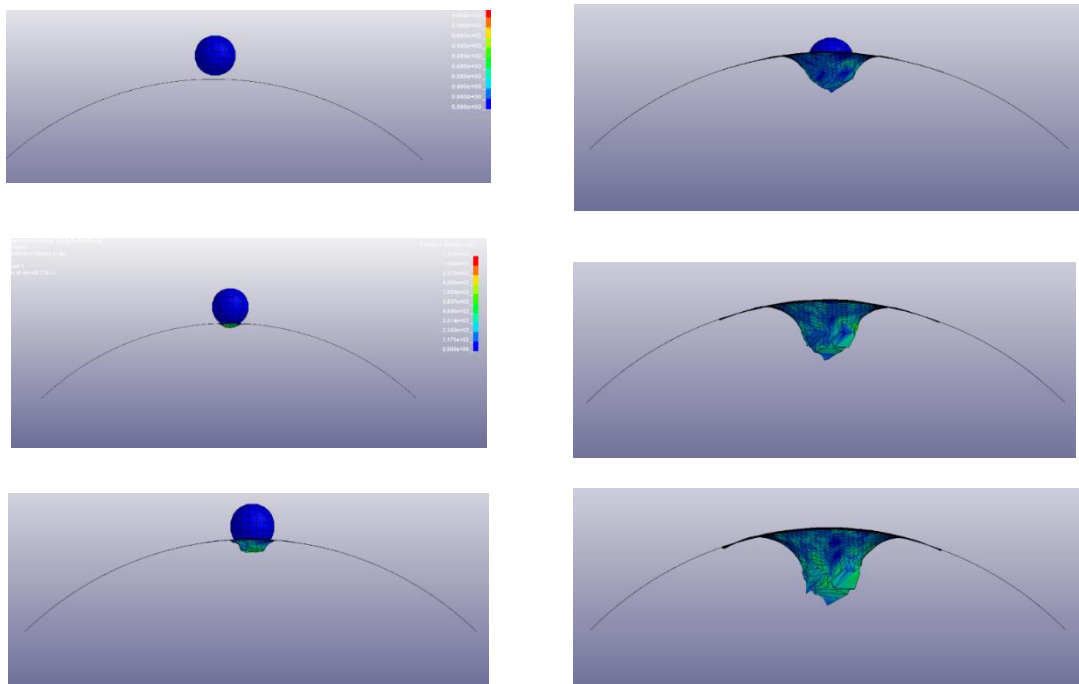


Figure 121 Impact event

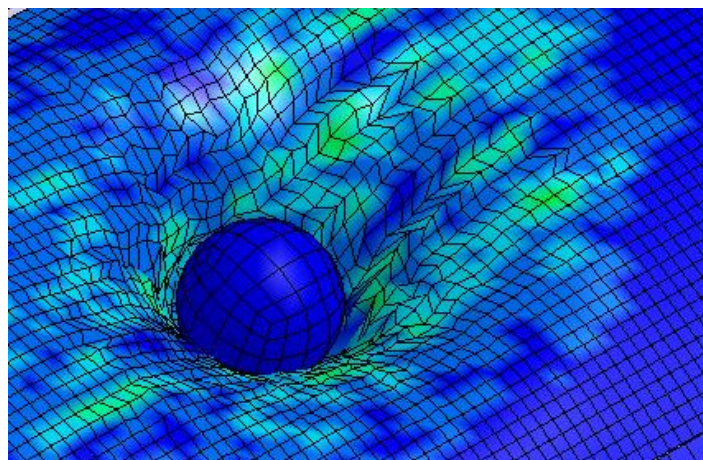


Figure 122 von Mises stress distribution

6.6.3 Verification

6.6.3.1 Tie-break contact modelling

The tie-break contact modelling approach was applied to the curved variable stiffness plate. Contact force histories are compared with analytical model results for using a single shell and two shells with a tie-break contact to model delamination. Delamination is omitted from the simulation in the scenario of using a single shell.

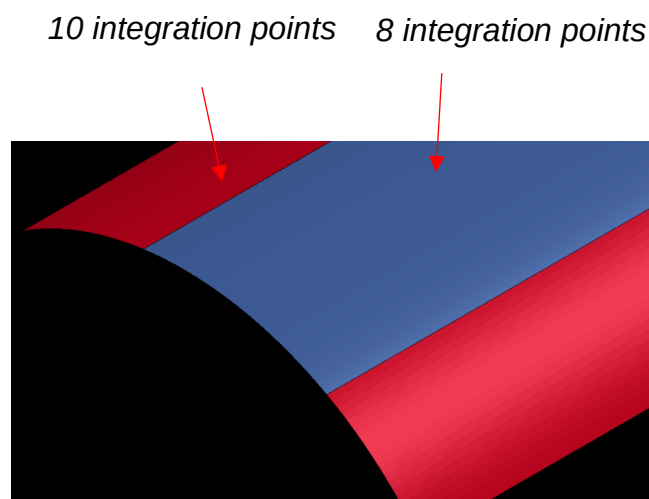


Figure 123 Single shell (No tie-break)

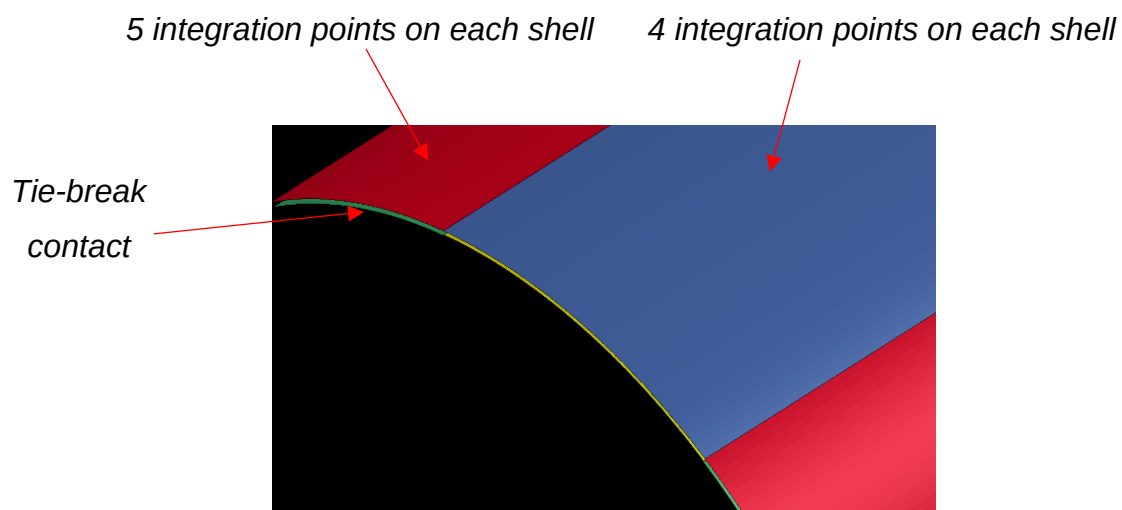


Figure 124 Double shell (Tie-break contact)

The main purpose of the numerical model was to validate the analytical model developed in the research. For comparison, curved plate with three variable stiffness sections are analysed with the layups shown in Table 19. Comparison of the analytical and numerical models are evaluated in Figures 125-127. The numerical analysis matches well with the modified analytical model with a discrepancy less than 5% for all layups studied. This study proves that the initially developed analytical model over-estimates the maximum contact force. This confirms the idea that the position of the variable stiffness section governs low velocity impact of curved variable stiffness sections. Layup 3 which includes 0 and 90° plies exhibit higher drop in loads in the initial region of the contact force history, thus suggesting the fact that delamination is higher.

Layup	Number of Sections	Layup orientation
1	3	Section 1- [-45 45 0 90 0] _s Section 2- [-45 45 0 90] _s Section 3- [-45 45 0 90 0] _s
2	3	Section 1- [0 90 0 45 -45] _s Section 2- [0 90 -45 45] _s Section 3- [0 90 0 45 -45] _s
3	3	Section 1- [0 90 0 90 0] _s Section 2- [0 90 0 90] _s Section 3- [0 90 0 90 0] _s

Table 19 Numerical analysis layups

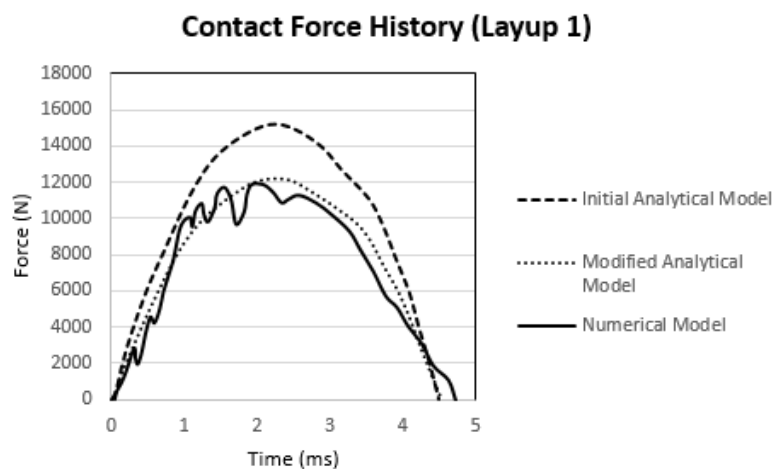


Figure 125 Model comparison (Layup 1)

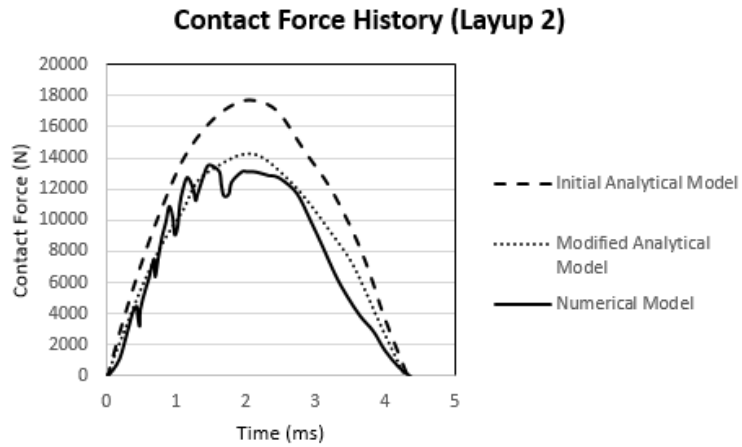


Figure 126 Model comparison (Layup 2)

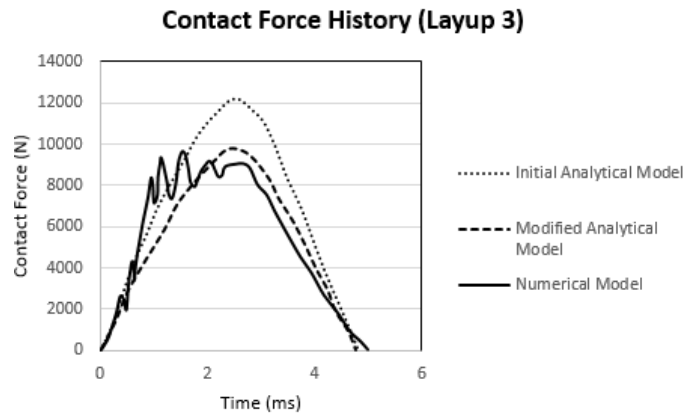


Figure 127 Model comparison (Layup 3)

Layup	Maximum impact force (N)			Discrepancy	
	Initial analytical model	Modified analytical model	Numerical model	Initial analytical model vs. Numerical model	Modified analytical model vs. Numerical model
1	15226	12184	11834	22%	3%
2	17720	14252	13561	23%	5%
3	12154	9880	9630	20%	2%

Table 20 Result Discussion

6.6.3.2 Validation with small mass impact

The developed numerical model is validated with the analytical study performed with a small mass impactor. The mass of the impactor is considered as 0.05kg. All other parameters such as layups, impactor velocity, laminate thickness are kept constant. The numerical model matched well with the analytical model for the small mass as well.

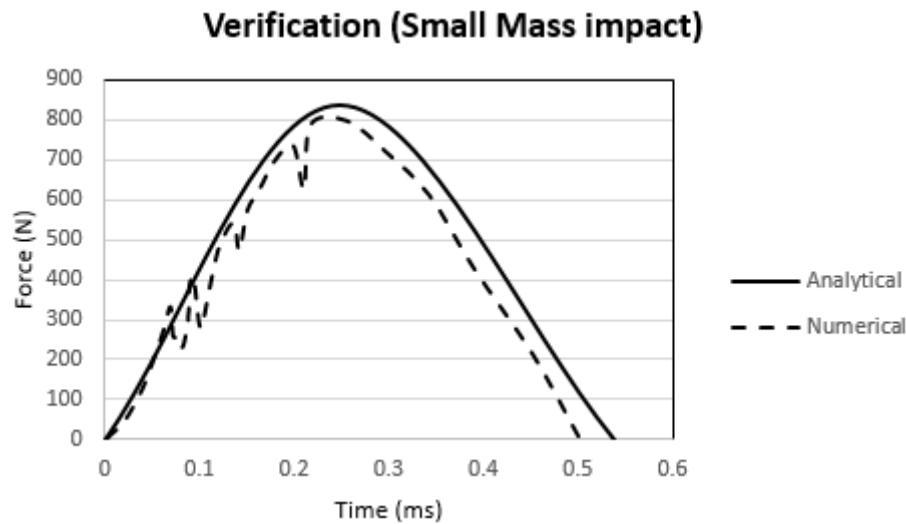


Figure 128 Validation (Small Mass Impact)

6.7 Post Impact Damage Model

The next aim of the numerical model is to validate the analytical results for the variable stiffness composite plate with damage inclusion.

6.7.1 Damage modelling approach 1

The damaged curved plates were modelled using finite element software LSDYNA. The size of the composite plates was $600 \times 200 \text{ mm}^2$ and radius 150mm with variable thickness through the length of plate. The plates were modelled based on Belytschko-Lin-Tsay quadrilateral shell elements. This shell element is based on a combined co-rotational and velocity strain. All surfaces of the model were meshed using quadratic shell element and the size of an element was $2 \times 2 \text{ mm}^2$ in the middle of beam (see Figure 136). The impactor was modelled as a rigid block using solid element. MAT_54 was used as the material model for the plate with Chang-Chang failure criterion.

The layers of each section was also defined using integration point (IP) through the thickness of the element and each integration point is used to represent each layer. In this case, the thickness of integration point layers at those places which are allocated for damaged zone was reduced to zero. This strategy introduces the damaged zone at the particular location.

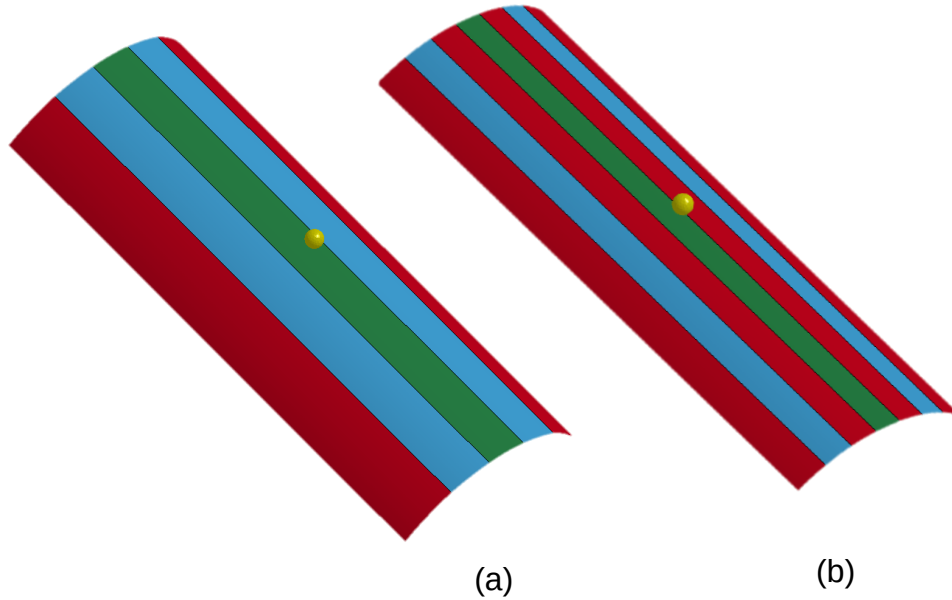


Figure 129 Numerical model (Damage modelling approach 1)

In order to validate the theoretical model, a numerical finite element was developed and the results are analysed and compared in this section. The analysis is validated using two different layups for the two plates with different geometries since the numerical modelling process is time consuming to be done for all the combinations discussed in the previous section. The 3 sectioned variable stiffness plate is validated through using Layup 3 for one and two plies removed. In Figures 130, the force-time history of damaged panels which were extracted from FEA model is presented. The main reason for difference between FEA and analytical results might come from deletion of elements after failure of all composite layers during the impact simulation. In this case there is no more resistance against the striker, therefore, some discrepancies are observed between analytical and FEA results (see Figure 130).

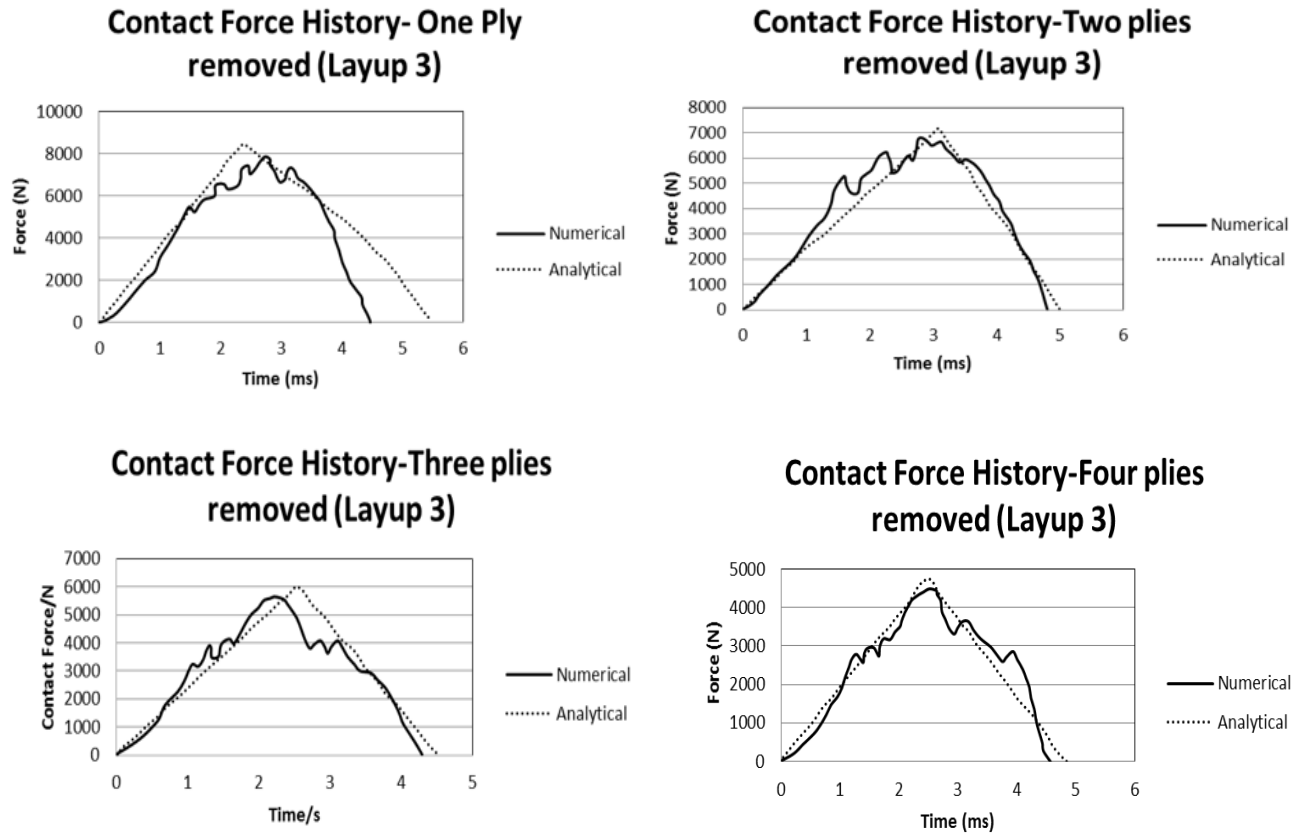


Figure 130 Comparison between FE results and the developed theoretical model for layup 3.

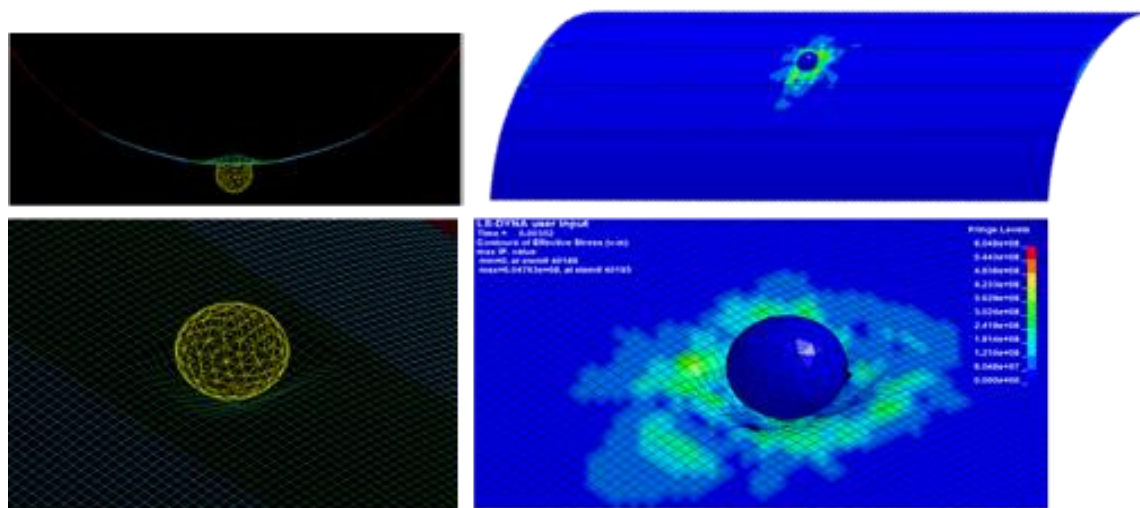


Figure 131 Finite element images of damaged curved composite panels with variable stiffness a) element deformation and b) stress distribution.

6.7.2 Damage modelling approach 2 (Localisation of damage region)

All surfaces of the model were meshed using quadratic shell element and the size of an element was $2 \times 2 \text{ mm}^2$ in the middle of beam similar to the analytical damage model. Results of the numerical simulations are shown in Figures 133-134.

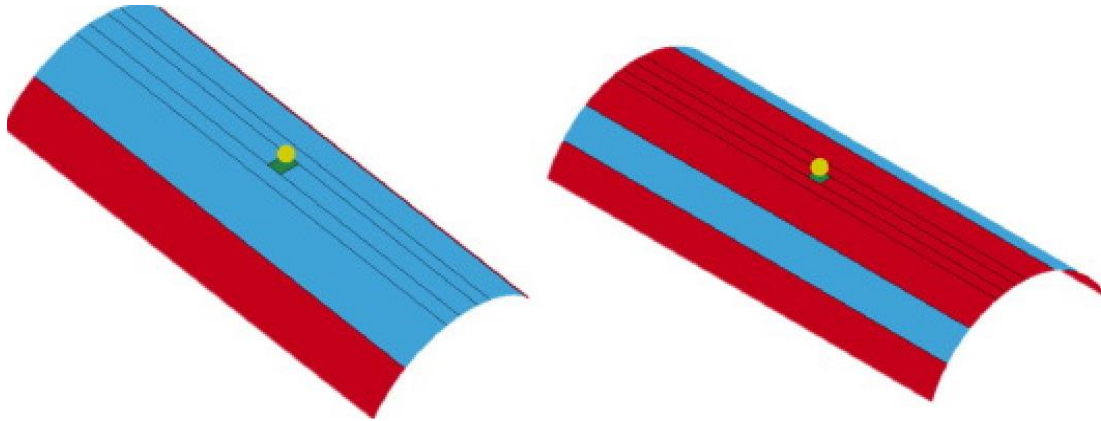


Figure 132 Finite Element Model of damage modelling approach 2

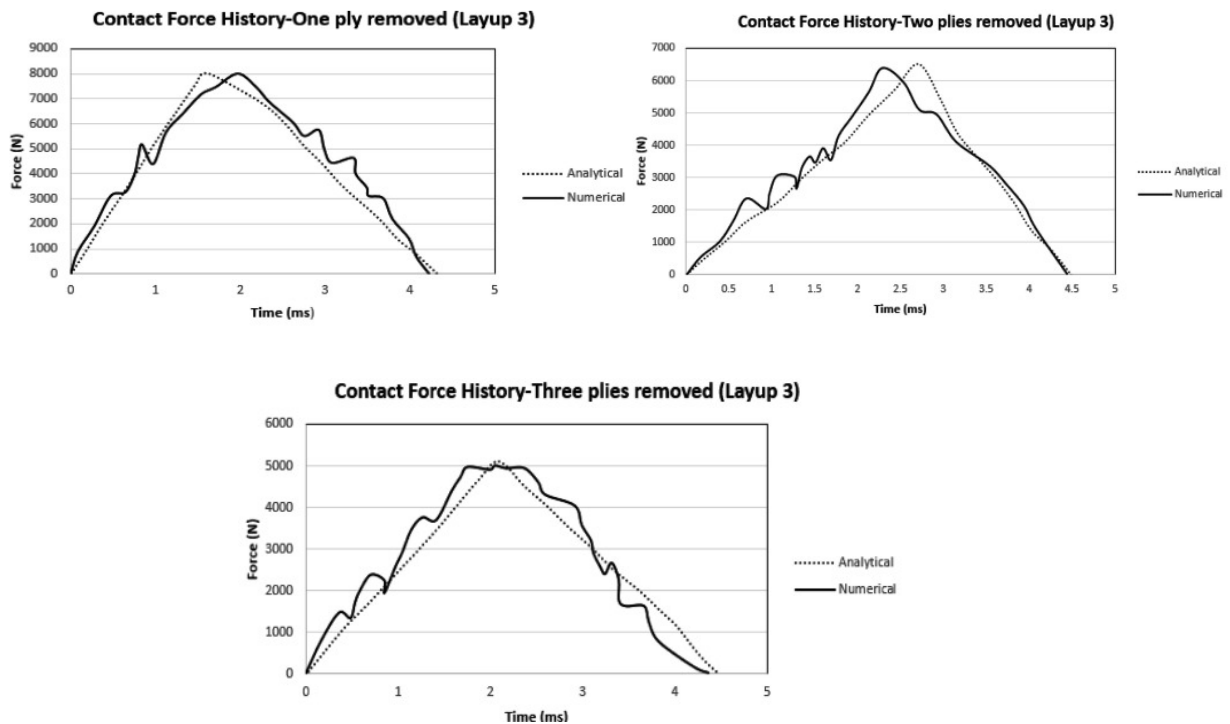


Figure 133 Results comparison for Layup 3 (three sectioned plate)

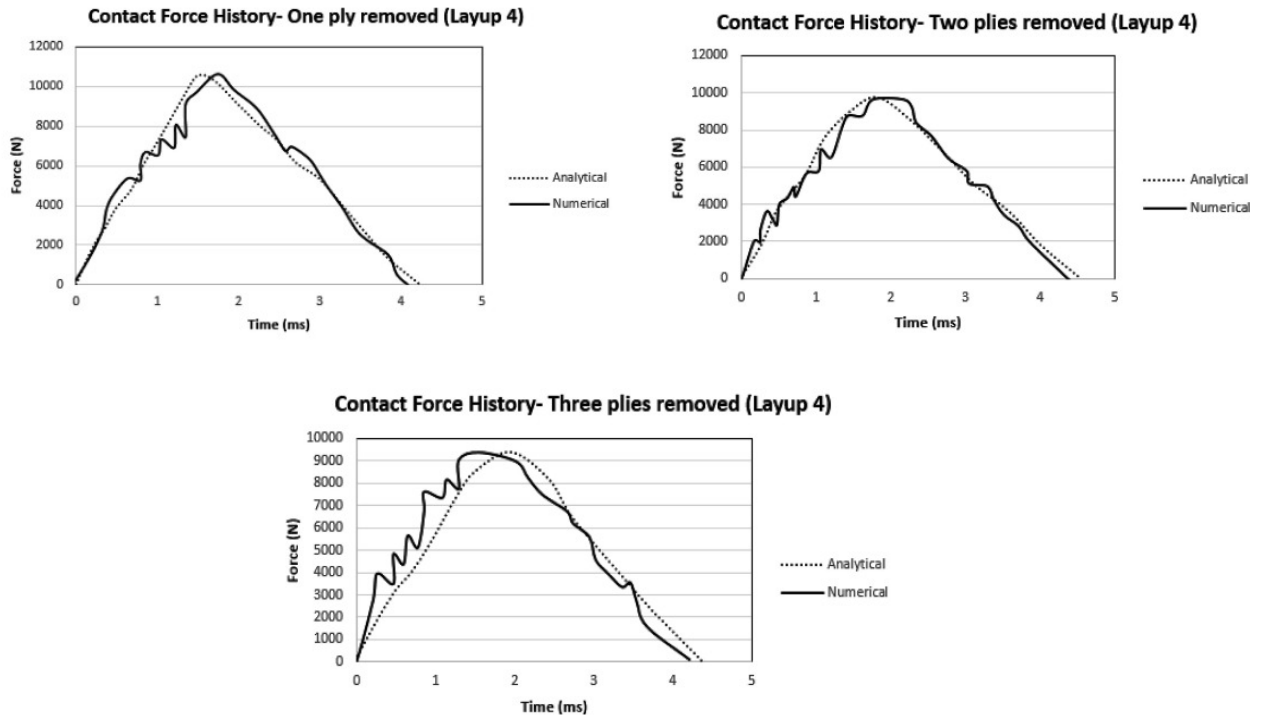


Figure 134 Results comparison for Layup 4 (five sectioned plate)

6.7.3 Damage model comparisons

6.7.3.1 Damage Modelling approach 1 (Two plies removed)

Layups studied	Maximum contact force		% difference
	Analytical	Numerical	
Layup 1	10100	9600	5%
Layup 2	9600	9210	4%
Layup 3	7200	6800	6%
Layup 4	11200	9700	13%
Layup 5	12000	10500	12%
Layup 6	11800	10250	13%

6.7.3.2 Modified damage modelling approach (Two plies removed)

Layups studied	Maximum contact force		% difference
	Analytical	Numerical	
Layup 1	9000	8600	4%
Layup 2	8800	8650	2%
Layup 3	6500	6450	1%
Layup 4	9800	9500	3%
Layup 5	11850	11600	2%
Layup 6	10200	10100	1%

Above analysis proves that the modified damage modelling provides better results when compared with the first model approach. This means that higher localisation of damage region increases the accuracy of the modelling approach.

6.8 Numerical modelling of foam core sandwich plate

The sandwich plate consists of top, bottom facesheets and a foam core as shown in Figure 135. This research mainly focusses on modelling foam based curved sandwich panels. The modelling process is similar to common practices observed in composite modelling in LS-DYNA; however, there are certain differences in modelling the core. The material models used are discussed in section 6.9.



Figure 135 Foam core sandwich panel

6.9 Material models for foam-based sandwich models

Foams are widely used for a variety of engineering applications such as energy absorption, increase of passenger safety in automobiles and also for protection of components. The extensively varying properties tailored for these high demand applications have contributed to the high complexities faced in modelling foam.

There are two vital variables in creating a foam material. They are the matrix material and the morphology of the gaseous phase. Two basic morphologies namely open cell and closed cell exist. The percentage of open or closed cell fraction are used to categorize foams. Matrix is made from a variety of materials, either rigid or flexible. The use of rigid materials for the matrix usually results in crushable foam. Crushable foams undergo deformation by ductile plasticity or brittle means [142]. The most evident mode of deformation in foams are through compression. Three zones exist in the compressive stress-strain relationship of foam: initial region (Zone 1) developing into a flat plateau compaction region (Zone 2) and finally densification in Zone 3. Initially in zone 1, foams have a certain amount of stiffness mainly due to the strength of the matrix. After yielding, gaseous component is affected. The gas escapes the foam through the open pores in an open cell foam and compressed in a closed cell foam. Therefore, the pressure of the gas is high enough to rupture the cell walls and the gas is released to the atmosphere. The result of this is permanent rupture of the cell and permanent damage to the foam. On the contrary, a stronger matrix would result in the cell remaining intact and collapsing soon immediately. Densification occurs when all of the cells are ruptured or collapsed and the foam behaves similarly to a typical matrix material in its stress-strain behaviour [142].

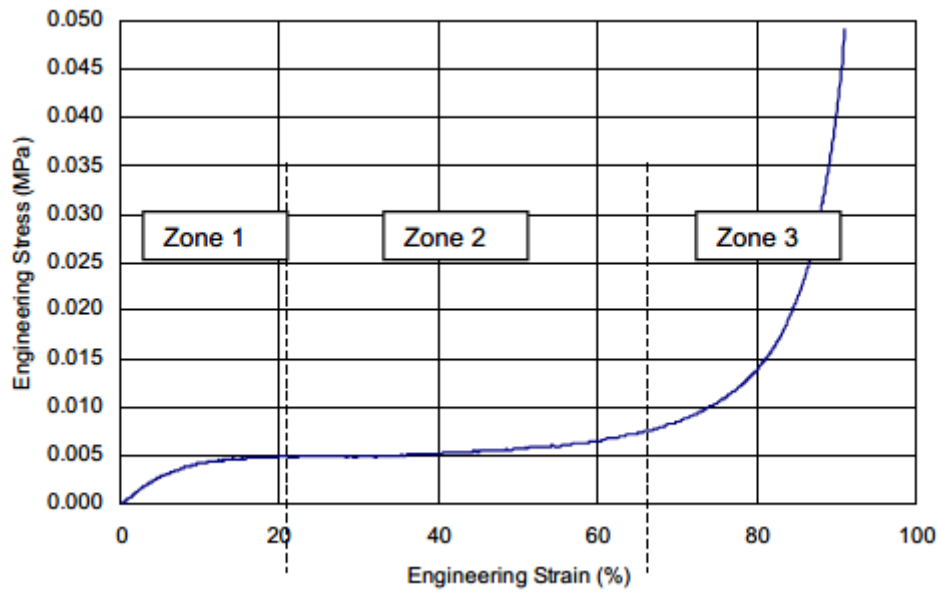


Figure 136 Deformation zones in a foam material [142]

6.10 Crushable foam

LS-DYNA offers several material models to model foam based materials with the ability to capture unique behaviours of different types of foam [142]. This material model is used to characterise the properties of isotropic crushable foam and also includes optional damping and a tensile cut-off stress [143]. Croop and Lobo [142] states that material model 63 in LSDYNA is a suitable choice to model EPS foams which undergo significant damage under impact. Slik et al. [144] used this material model in modelling a foam type with low elastic recovery. They predicted that material model 63 is suitable for foams with either no or low elastic recovery. Karthik et al. [145] also made use of the same material model to study impact behaviour of PMI foam. They assigned shell elements to the face-sheets and tetrahedral solid element to the core. Comparisons with relevant experimental results proved the validity of using material model 63 for foam modelling. The model allows the user to define the curve representing the yield stress against volumetric strain for a particular core type. The major parameters required to define the material model are listed in Table 21.

<i>Parameter</i>	<i>Description</i>
MID	Material identification ID
RO	Density of core
E	Young's Modulus
PR	Poisson's ratio
LCID	Load curve ID that defines yield stress against volumetric strain
TSC	Tensile stress cut-off value
DAMP	Damping coefficient

Table 21 Material model 63 input data [143].

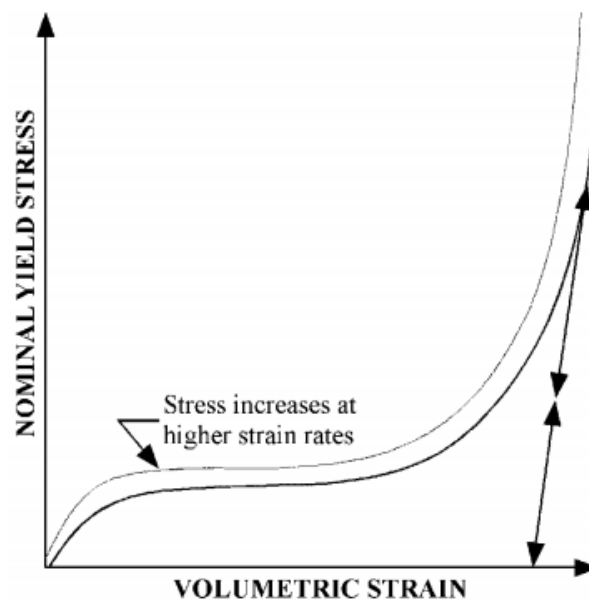


Figure 137 Crushable foam behaviour [143].

6.11 Modelling of sandwich panel

A numerical finite element model of the curved sandwich plate is developed in finite element program, LS-DYNA. The main aim of this numerical model is to validate the analytical model and compare with available experimental data from impact tests performed by Yurddaskal and Baba [125] for a curved sandwich plate. Initially, the

numerical model in LSDYNA is developed for the work by Feng [136] to validate the analytical model for a flat foam core sandwich panel. The facesheets were made using glass/epoxy and modelled using 4-node 1.5mm shell elements with Belytschko-Tsay element formulation with four integration points to define the four layers in the facesheet. The foam core is modelled using 2mm×2mm×2mm solid elements similarly to the impactor. Material model 54 in LS-DYNA with Chang Chang failure criterion is used to model the orthotropic facesheets with damage occurring by matrix cracking, compressive failure and fibre breakage. The foam core is modelled by the Crushable Foam material model 63 and material model 20 are used for the rigid impactor. An automatic surface to surface contact behaviour is defined for contact between the impactor and facesheets. Similarly, the same contact type was used to bond the facesheets to the foam core. Clamped boundary conditions were used for the numerical model for a valid comparison with experimental work. The numerical model requires the stress-strain curve for the foam core to be defined.

6.11.1 Validation of numerical model (with reference [125])

As the first step, the numerical model developed is validated with the literature [125] for a flat panel, and also with the analytical model proposed in the present study. Contact force history plot is identified as the main means of validating the present model. The curved model is developed after validation of the flat panel. Dimensions and material properties of the flat sandwich panel is obtained from literature [125]. It is observed that at an impact energy of 10J, only the top facesheet is damaged, thus validating the model with the experimental study. Material model 63 in LS-DYNA requires stress-strain behaviour of the foam to be defined. Therefore, in this work stress-strain response for Airex C70.55 foam core was obtained.

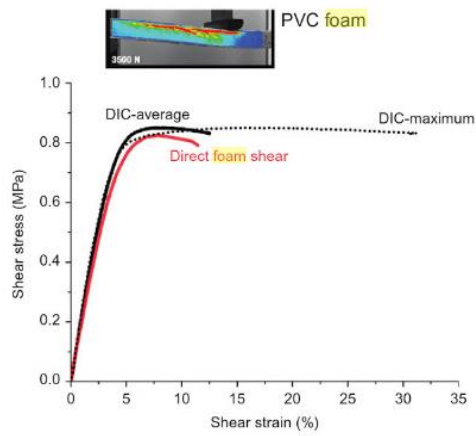


Figure 138 Airex C70.55 foam core stress-strain curve [149]

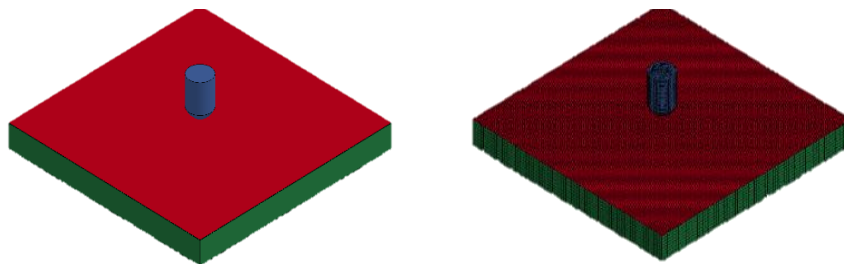


Figure 139 Finite element model (Flat panel)

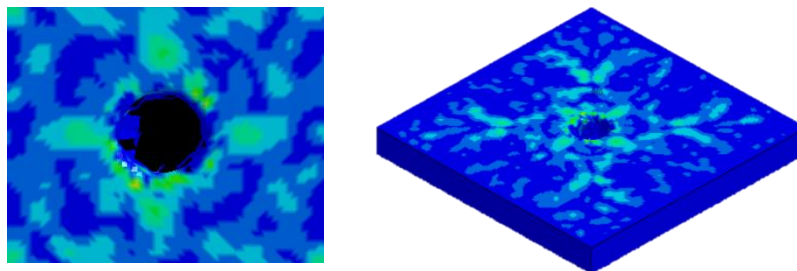


Figure 140 Facesheet damage [Composed by author]

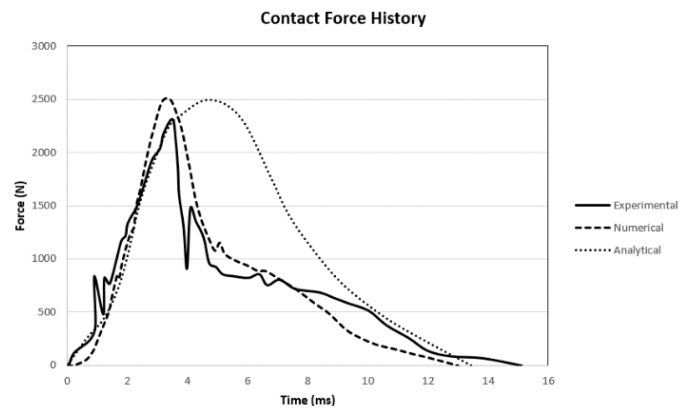


Figure 141 Contact force history

6.11.2 Validation of numerical model (with reference [136])

The numerical model is further validated with reference [136] for a flat sandwich plate with a different PVC foam core, DIAB Divinycell H100. Carbon/epoxy facesheets were used with 10 layers $[0_3/\pm 45]_s$ with a total thickness of 1.5mm. Dimensions of the sandwich plate were 250×250mm. Material properties of the facesheets and core are obtained from reference [134]. The diameter of the impactor was 12.5mm; however mass of the impactor was 2.34kg and impact energy was 6.3J. Similarly with the previous study, only top facesheet damage is observed at this energy level.

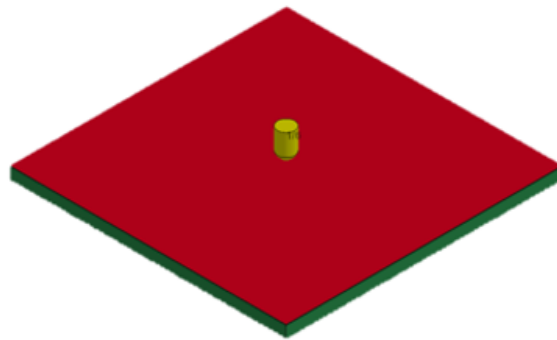


Figure 142 Finite element model

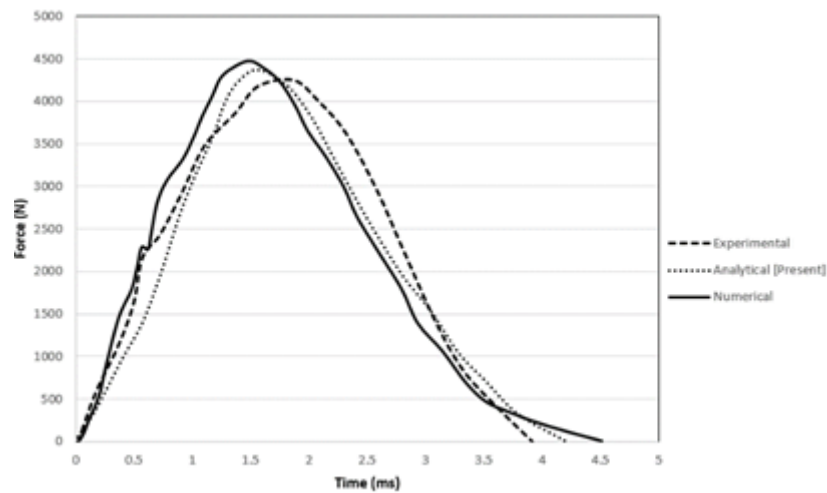


Figure 143 Contact force history

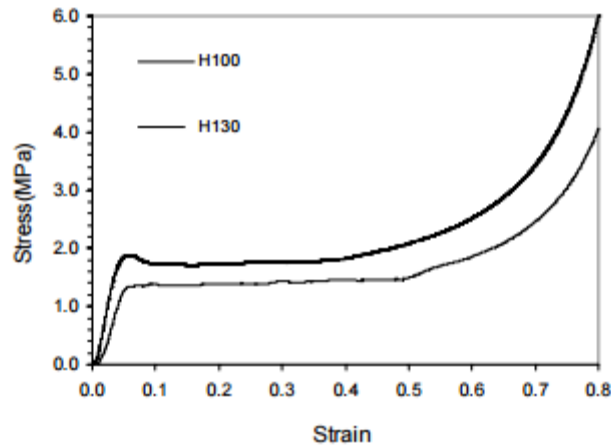


Figure 144 Divinycell stress-strain curve [85]

6.12 Modelling of curved finite element model (constant stiffness facesheets)

Figure 145 shows the numerical model for the curved sandwich plate with a foam based core. Shell elements were used for the plate and solid elements were used for the foam core and the impactor. Radius of the plate is 160mm. Material properties of facesheets, core and impactor are obtained from the experimental work [125].



Figure 145 Curved finite element model

6.12.1 Impact energy of 10J

The comparison between the analytical and numerical model is shown in Figure 146. It is concluded that at an impact energy of 10J, damage is observed only in the top facesheet. The fluctuations in the numerical study depicts the initiation of matrix cracking and the sudden load drop indicates failure of the top facesheet. The maximum contact force difference between the analytical and numerical studies is 7%.

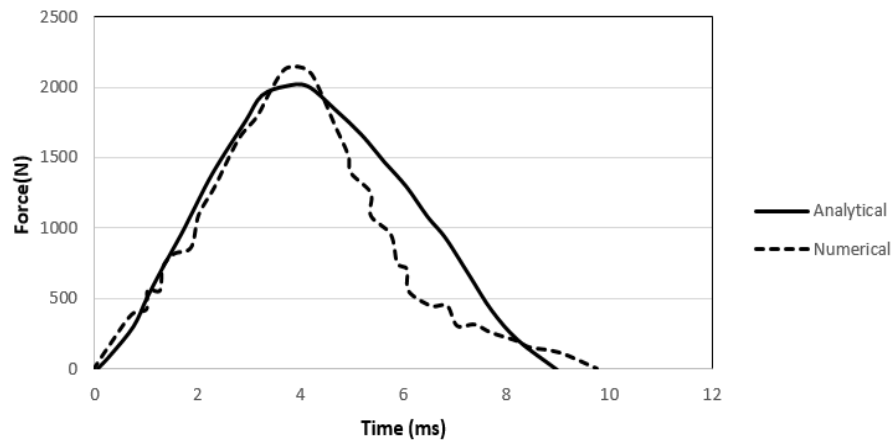


Figure 146 Contact force history comparison for curved sandwich panel

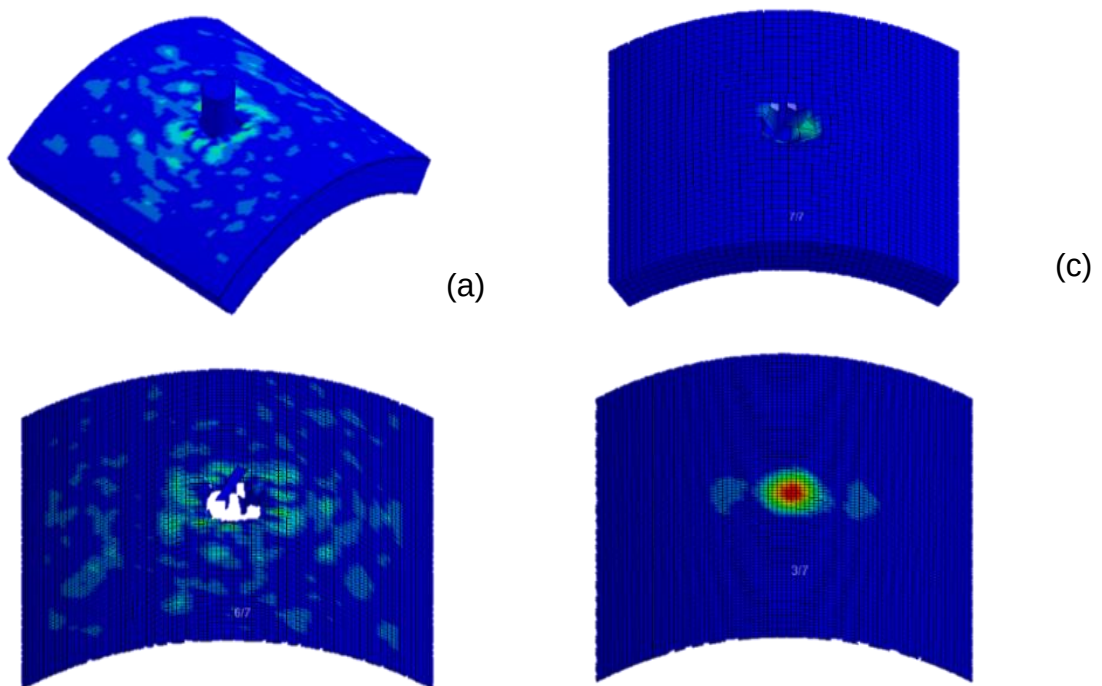


Figure 147 (a) Curved sandwich panel (von Mises stress distribution) (b) damage in top facesheet (c) core crushing (d) no damage in bottom facesheet.

6.12.2 Impact energy of 25J

As the impact energy is increased, increased amount of core crushing and perforation of the bottom facesheets is observed. This is indicated by two peaks in the contact force history depicted in Figure 148. The load drop caused by the failure of top facesheet is followed by deformation of the core where the solid elements are eroded due to distortion. The bottom facesheet then starts deflecting and this is indicated by the load rise. The final load drop is due to the failure of the bottom facesheet until final perforation. The comparison of the force vs displacement plot is shown in Figure 148; and it is observed that the numerical model matches well with the experimental result [125]. In the numerical model, top facesheet damage occurs at a higher maximum impact force at 2900N compared with 2700N of the experimental model. The maximum displacement of the numerical model is 27.7mm and this is a 3% increase when compared with the experimental results.

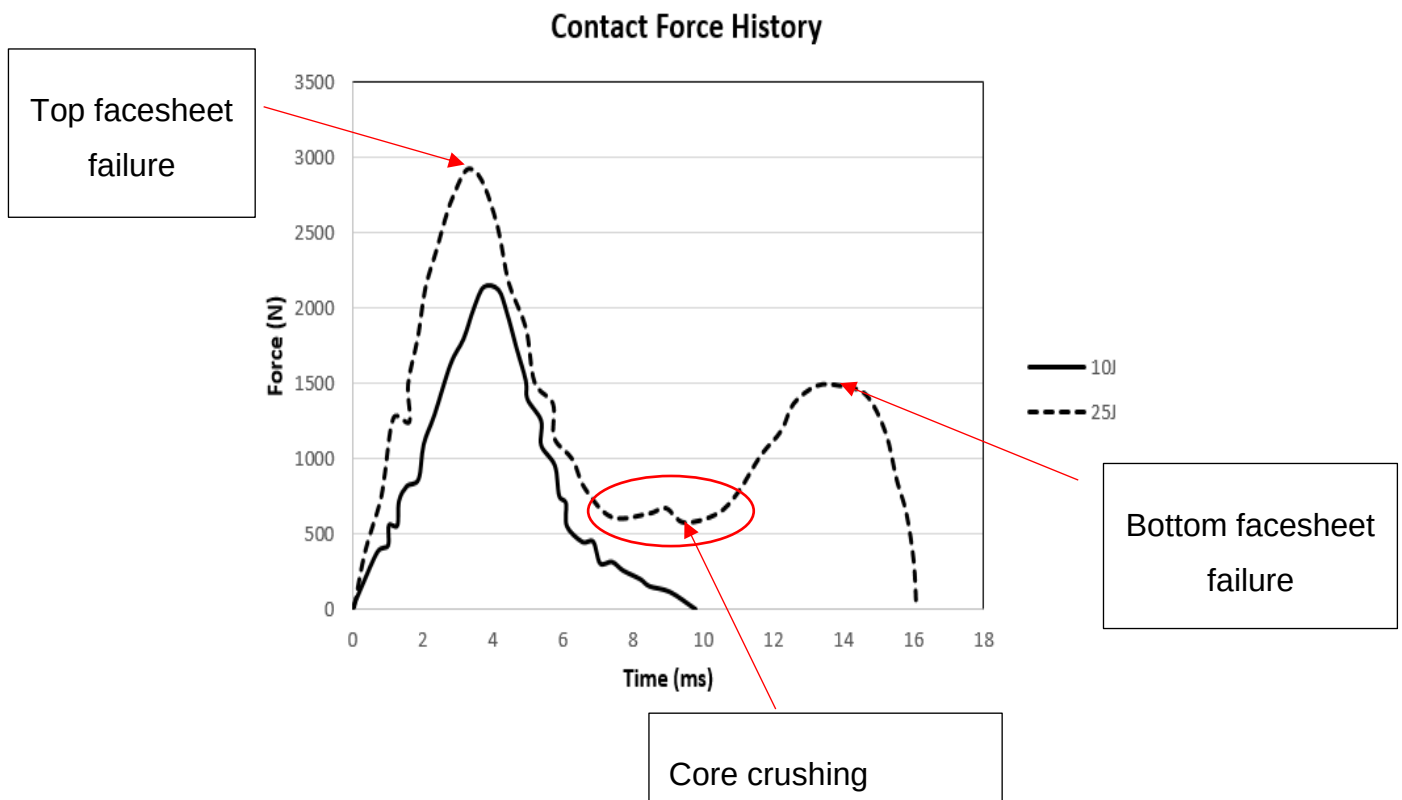


Figure 148 Contact force history (10J and 25J impact energy)

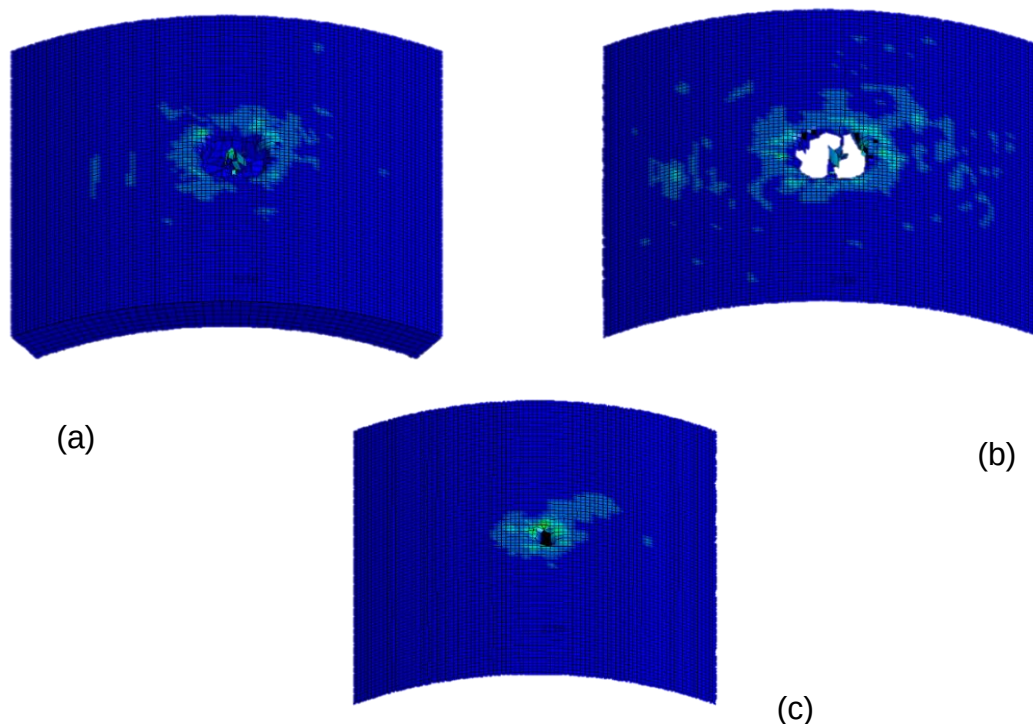


Figure 149 Curved sandwich panel (von Mises stress distribution) (b) damage in top facesheets (c) damage in bottom facesheets.

6.12.3 Force vs displacement plots

Figure 150 shows the force-displacement plot characteristics of the foam core sandwich plate. The first phase of the plate behaviour (A-B) consists of local indentation where the hemi-spherical indenter incrementally crushes the core and cause damage to the top facesheet. Figure 150 depicts that as the impactor loads the panel, shear deformation occurs in the core and upper skin debonds from the plate. The load drop at point B is due to tensile failure of the top facesheet. As the plate is continuously loaded, core crushing occurs and impactor penetrates the panel. This is shown by the section C-D in Figure 150. This phenomena is also known as core densification. This load is then transferred to the lower facesheet and it eventually fails at point E causing the second load drop.

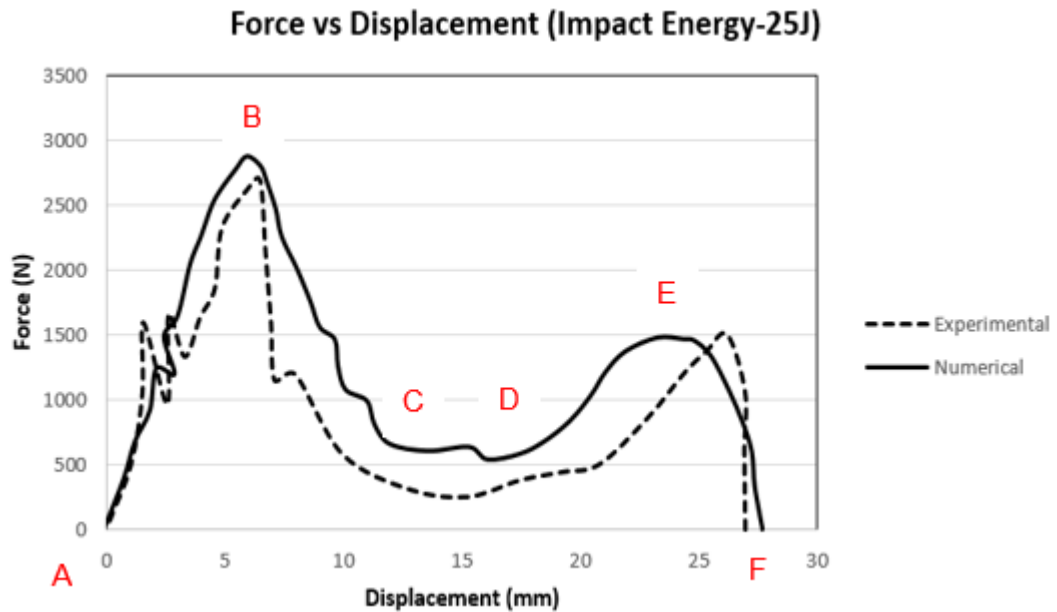


Figure 150 Comparison of force vs displacement at impact energy of 25J.

6.13 Modelling of curved sandwich plate with variable stiffness facesheets

An FE model (Figure 151) is developed to validate the analytical model for the curved sandwich panel with variable stiffness facesheets which was one of the main novelties in this study. For this purpose, combination 4 as in Table 18 with top facesheet thickness of 0.5mm and bottom facesheet thickness of 0.4mm is used. Numerical predictions are within 5% of the analytical result as depicted by Figure 152.

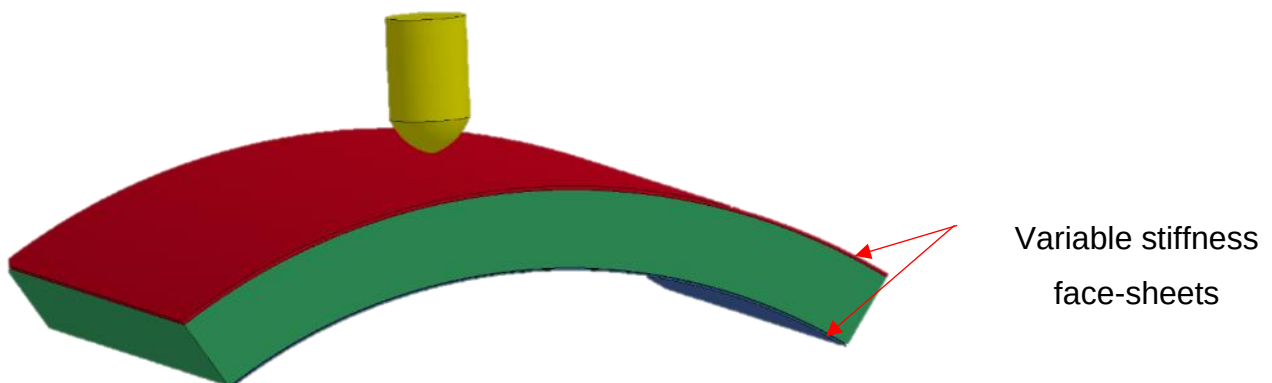


Figure 151 FE Model with variable stiffness facesheets

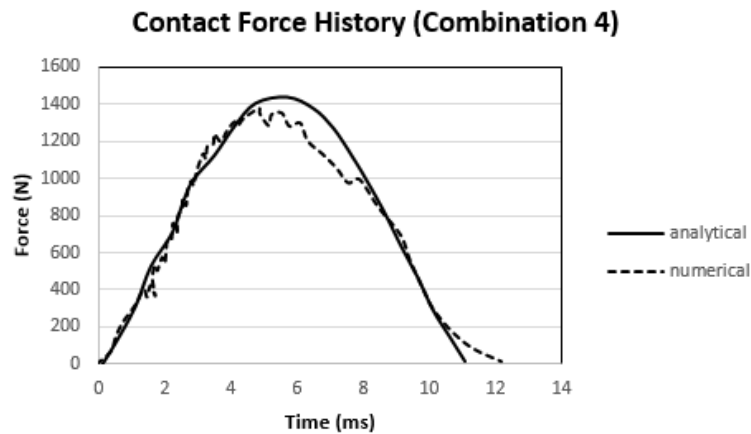


Figure 152 Contact force history (Variable stiffness facesheet model)

6.14 Parametric study

A finite element study is performed to analyse the effect of core thickness and facesheet thickness on the impact response of curved sandwich panels. This numerical study is similar to the analytical study carried out earlier, and therefore serves as a validation tool for the proposed analytical model. In Models 1-5, the thickness of the core is varied keeping the facesheet and all other geometrical properties constant. In Models 6-10, the facesheet thickness is varied. These FE models are shown in Figures 153 and 154.

Models	Core thickness (mm)	Facesheet thickness (mm)
1	10	0.75
2	12.5	0.75
3	15	0.75
4	17.5	0.75
5	20	0.75
6	15	0.3
7	15	0.4
8	15	0.5

9	15	0.6
10	15	0.7

Table 22 Models developed for parametric study

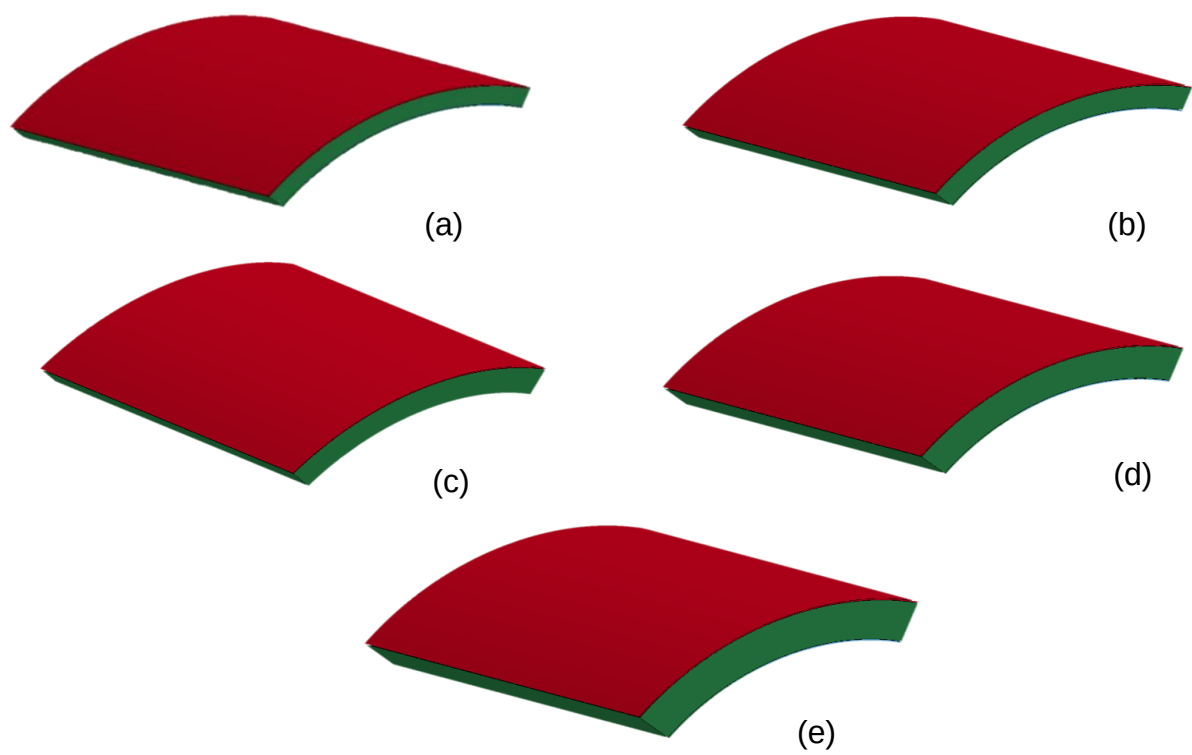


Figure 153 FE Models (a) Model 1 (b) Model 2 (c) Model 3 (d) Model 4 (e) Model 5

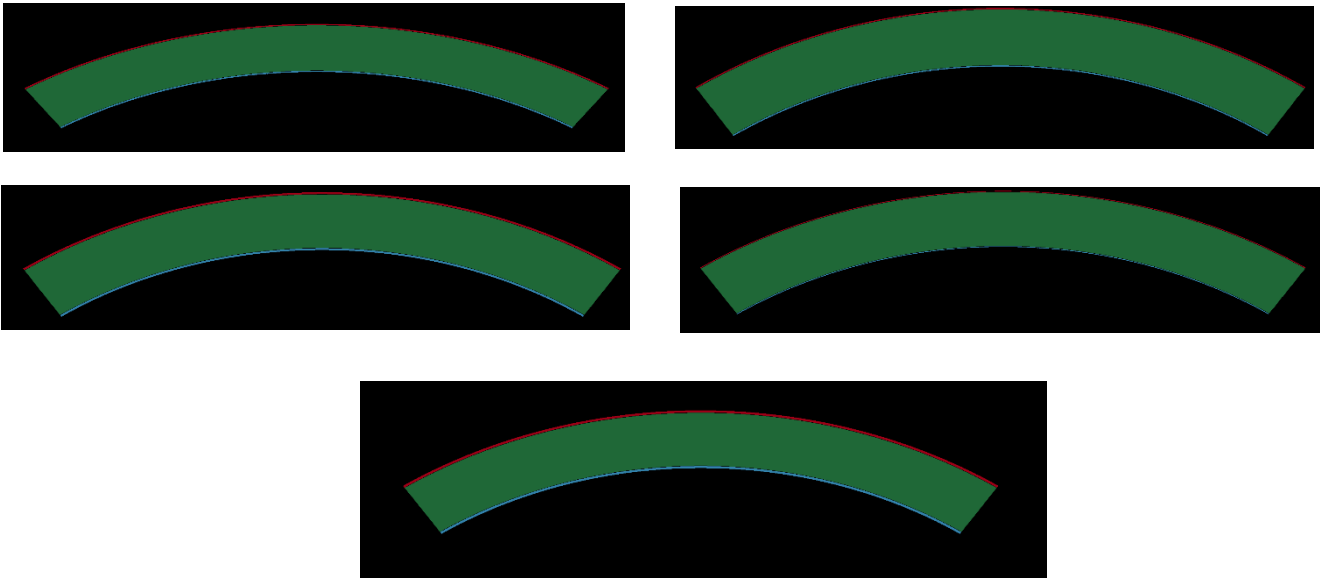


Figure 154 FE Models (a) Model 6 (b) Model 7 (c) Model 8 (d) Model 9 (e) Model 10

6.14.1 Results and discussions (Numerical Model)

6.14.1.1 Force against displacement plots (Effect of Core thickness)

These results are solely from the numerical analysis

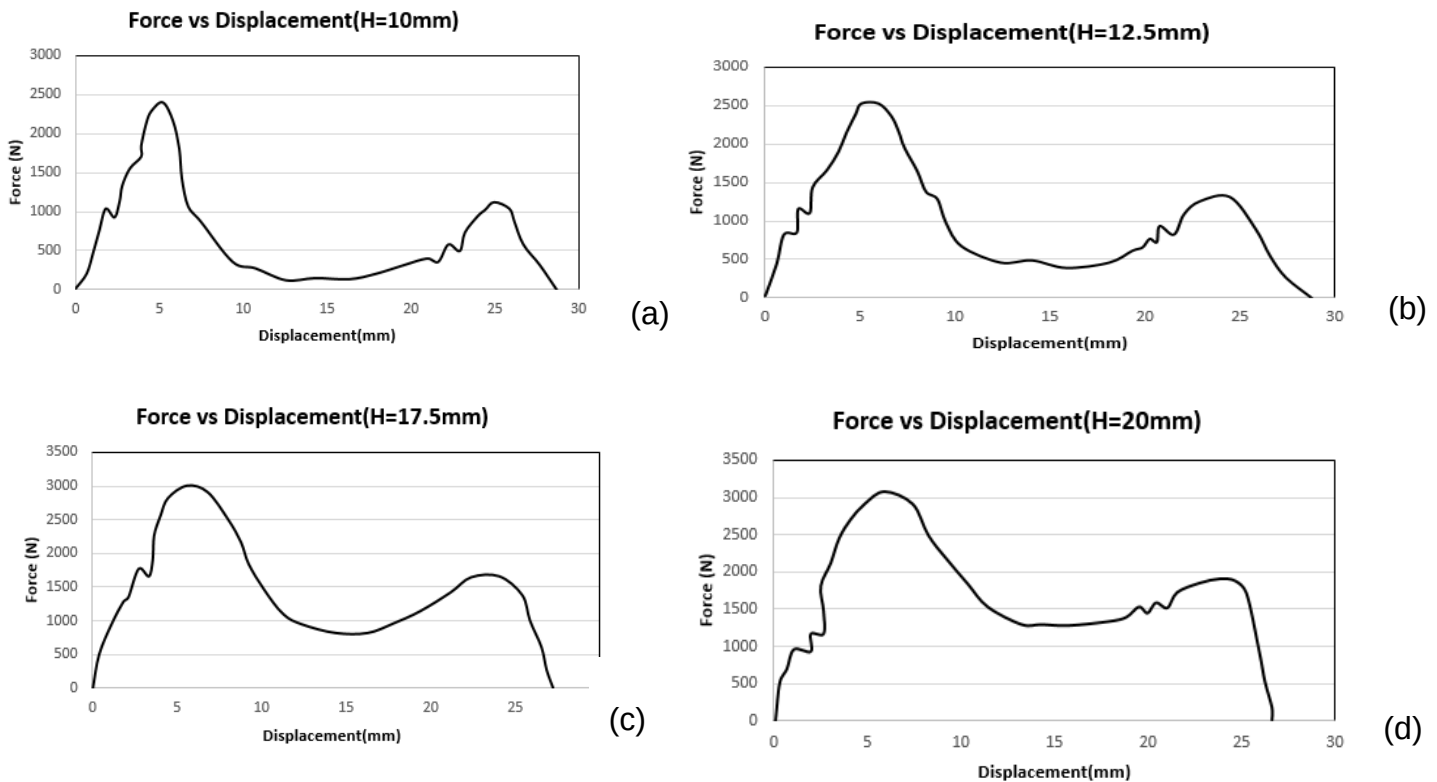


Figure 155 Force vs displacement (a) $H=10\text{mm}$ (b) $H=12.5\text{mm}$ (c) $H=17.5\text{mm}$ (d) $H=20\text{mm}$

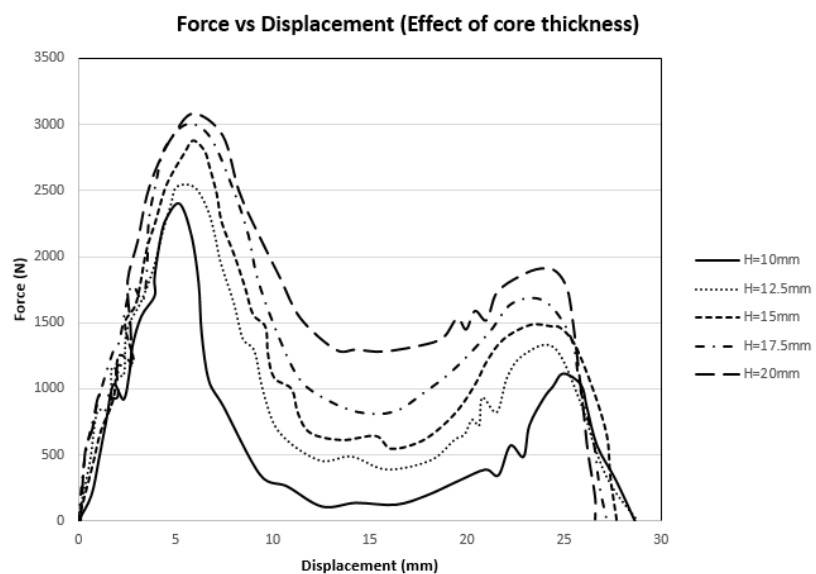


Figure 156 Combined plot for effect of core thickness

Core thickness (mm)	Maximum Contact Force at top facesheet failure (N)	Maximum Contact Force at bottom facesheet failure (N)	Maximum Displacement (mm)
10	2400	1200	28
12.5	2550	1350	27
15	2800	1500	27.5
17.5	3020	1730	27
20	3180	1900	26

Table 23 Effect of core thickness on top and bottom facesheet maximum contact force

Analysis of results proves that as the core thickness is increased, the energy absorption is increased. This is majorly due to the increase in the maximum contact force. Larger core thicknesses are able to resist larger impact loads before top and bottom facesheet occurs. For instance, a 20% increase in core thickness results in a 12% increase in maximum contact force at top face-sheet failure and 10% increase in maximum contact force at bottom face-sheet failure. The analysis also proved that with the increase of core thickness, the panel deflection decreases since the plate is able to withstand higher impact loads.

6.14.1.2 Force vs Displacement plots (Effect of face-sheet thickness)

It is observed that increase of face-sheet thickness enhances the maximum contact force at top and bottom face-sheet failure.

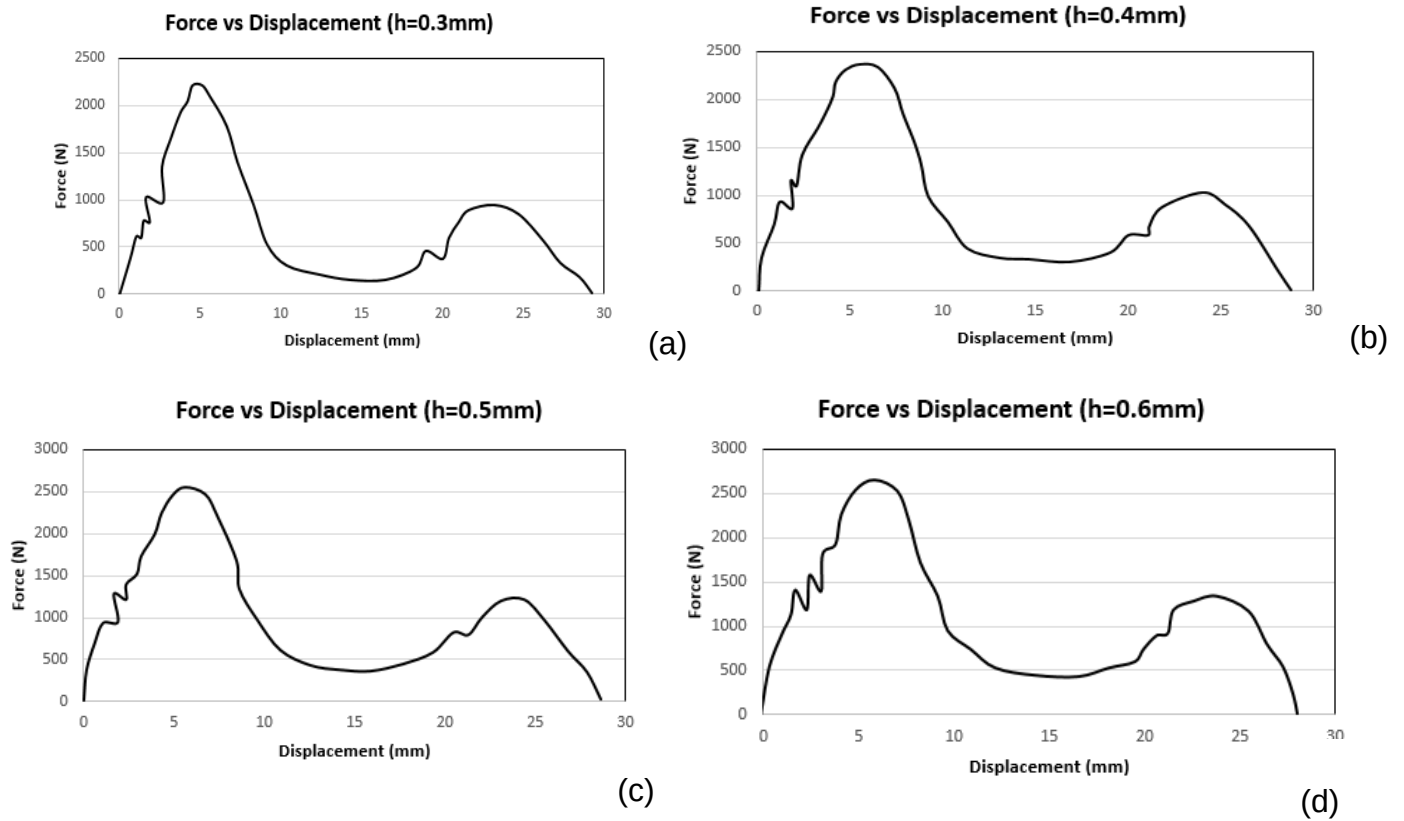


Figure 158 Force vs displacement (a) $h=0.3\text{mm}$ (b) $h=0.4\text{mm}$ (c) $h=0.5\text{mm}$ (d) $h=0.6\text{mm}$

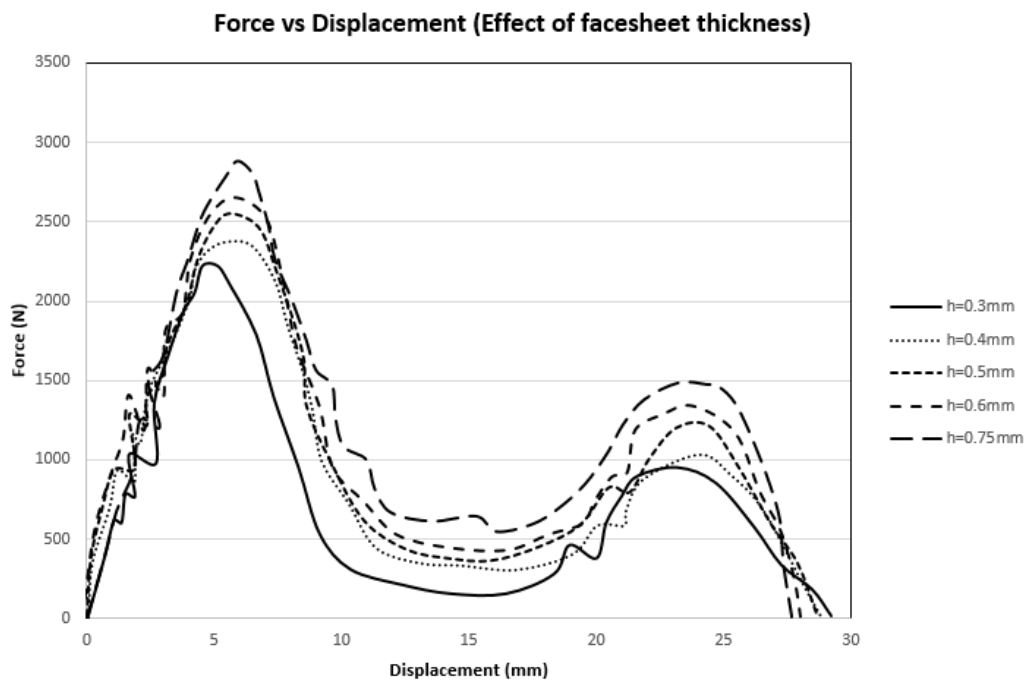


Figure 157 Combined plot for effect of facesheet thickness

6.14.1.3 Force vs Displacement (Effect of plate curvature)

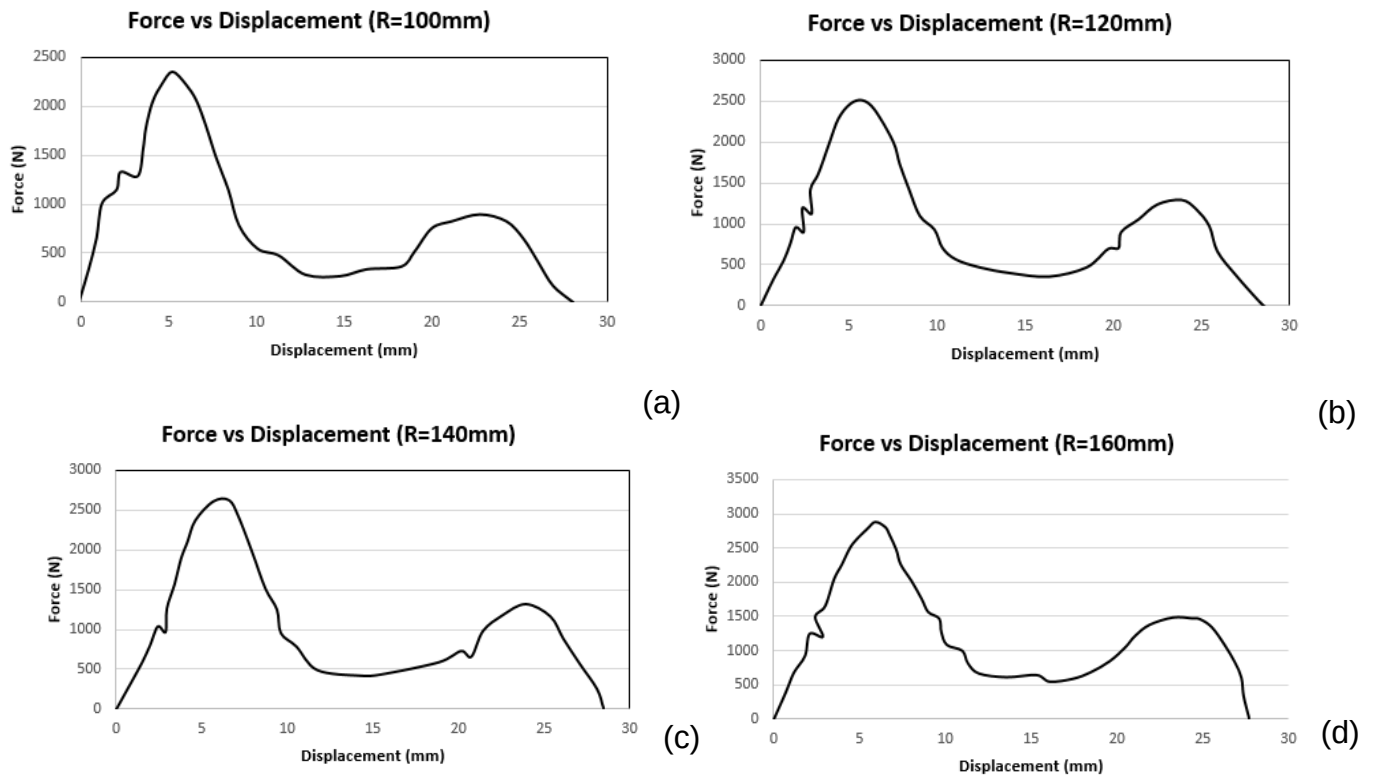


Figure 159 Force vs displacement (a) R=100mm (b) R=120mm (c) R=140mm (d) R=160mm

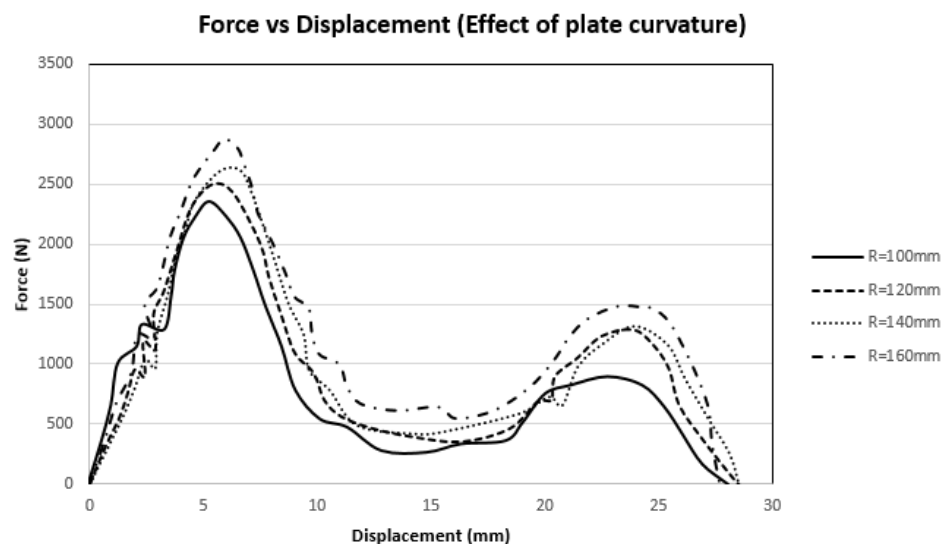


Figure 160 Combined plot for effect of plate curvature

Plate curvature (mm)	Maximum Contact Force at top facesheet failure (N)	Maximum Contact Force at bottom facesheet failure (N)	Displacement (mm)	Energy absorption (J)
100	2300	820	26	18.5
120	2510	1300	27	21.2
140	2700	1320	26.5	22.1
160	2850	2500	25	23.4

Table 24 Effect of curvature on energy absorption

Results from the above analysis suggest that increase of plate curvature increases the maximum contact force at top and bottom facesheet failures. The energy absorbed by the plate is in the form of bending energy and energy absorbed by damage modes such as delamination, fibre breakage, matrix cracking and core crushing. Energy absorption is critical in indicating a structure's impact resistance. The contact force-displacement graphs are used to obtain energy absorption which is given by the area under the curve. As observed from Figure 160, the energy absorption capability of the plate increases with plate curvature. This suggests that highly curved sandwich panels are able to withstand higher impact loads.

6.15 Chapter Conclusions

In this chapter, numerical models were developed and simulated in LS-DYNA finite element code to analyse low velocity impact on variable stiffness curved composite plates and sandwich plates. Main aim of the numerical study was to validate the developed analytical models which were the major highlight of the present research. The numerical model backed up the idea that the position of the variable stiffness location in the plate is a major aspect in impact dynamics of such structures. Inter-laminar and intra-laminar failure were also incorporated into the models through the relevant material models and tie-break contacts. Finite element models developed to study impact response of curved foam based sandwich plates validated the analytical models. It was observed that at higher impact energies, two peaks in the force-displacement plot were present due to failure of top and bottom face-sheets. Parametric analysis studied effects of core thickness, face-sheet thickness and plate curvature on impact response. This study proved that higher curvature of sandwich plates increases energy absorption capabilities similarly with enhanced core thickness and face-sheet thickness.

Main findings from this chapter are:

Use of double layer shells approach with tie-break contacts in modelling the variable stiffness plate improved the numerical model to match closely with analytical results.

Results of the contact force analysis depicted that delamination was highest in Layup 3 with 0° and 90° plies for the variable stiffness model.

The difference in analytical and numerical models were less than 10%.

Larger core thicknesses are able to resist larger impact loads before top and bottom facesheet occurs.

Results from the above analysis suggest that increase of plate curvature increases the maximum contact force at top and bottom facesheet failures

Energy absorption capability of the plate increases with plate curvature.

7 Final Conclusions

Overall conclusions of the thesis are summarised as:

7.1 Chapter 3

In this chapter of the report, a mathematical model was developed to predict the impact response of curved fibre-reinforced polymer (FRP) composite plates with variable lateral stiffness which is a more realistic representation of composite structures compared to existing models. First order shear deformation theory coupled with Fourier series was used to predict the contact force history of curved composite plates and the same approach was used for variable thickness composite plates, which provides the main novelty of this research. A formulated function for the natural frequency of the plate was used to derive the contact force function for the variable stiffness plate including effects of curvature. A parametric study was performed to study effects of layups, impact velocity and geometrical effects on contact force histories. Damping was also included into the two degree of freedom spring-mass system. However, as damping exhibited a negligible effect of the contact force, it was neglected. It is anticipated that this knowledge may benefit the composite industry in providing a better understanding of structural health monitoring and protection mechanisms of advanced composite structures in aerospace and automotive sections. This may help to significantly reduce calculation time during design, analysis and manufacturing as well as improving the quality of final products.

7.2 Chapter 4

This chapter of the report introduced methods to study the post impact behaviour of damaged variable stiffness curved composite plates. Variable stiffness sections are used widely and the need for studying the impact behaviour of these sections were identified. A method involving stiffness reduction in the damaged region was introduced. This stiffness reduction was achieved through reduction in the natural frequency in damaged region. The level of damage in the laminate was increased through reduction in thickness of the laminate of the damaged area. Studies carried out showcased that reduction in thickness caused a reduction in natural frequency and

thus a reduced maximum contact force. Two damage modelling approaches were introduced in this chapter that includes localisation of damaged region. Results suggested that increased localisation of damage region depicts a better representation of contact force histories for the damaged plate.

This chapter also investigated various types of layups able to withstand a greater impact damage mainly by focussing on the contact force at impact. Therefore, this research area gives an insight into the combinations of layups most suitable for designs manufactured from the technique of variable stiffnesses in composites.

7.3 Chapter 5

The research was extended in this chapter to study impact response of curved foam based sandwich plates. Two degree of freedom spring-mass system was used in deriving contact force function for this type of sandwich plate. Stiffness matrices were evaluated assuming that thickness of the core remains constant under loading and the in-plane stiffnesses of the core are negligible. Some important findings from the parametric study are summarised as:

- 20% increase in impact velocity gives rise to a 21% increase in maximum impact force. Contact duration also increases by 4%.
- Stacking sequence plays a minor role in impact response of curved foam sandwich plates. However, it was observed that inclusion of 45 degree plies in the facesheets increases maximum contact force.
- When identical (constant) thickness facesheets were used for the top and bottom facesheets, a 33% increase in facesheet thickness results in a 17% rise in impact force and 5% decrease in contact duration. It is also noted that the drop in the maximum contact force occurs earlier when at higher thicknesses. This force drop implies failure of the top facesheet and therefore confirms that thicker facesheets fail quickly since higher energy is absorbed during the impact.

- Effect of core thickness was observed by increasing thickness from 10mm-12.5mm, the contact force increases by 22%, whereas when it is increased from 17.5mm-20mm, only 4% increase of contact force is observed.
- 7% increase in the plate curvature gives rise to a 5% rise in the maximum impact force and 2% decrease in the contact duration.
- Effect of variable stiffness facesheets were also analysed. For this study, non-identical facesheets with different thicknesses were used for the top and bottom facesheets. The analysis proves that the top facesheet thickness has a greater effect on the maximum contact force than the bottom facesheet thickness. This is demonstrated by the higher drop in maximum contact force when compared with the sandwich panel with constant thickness facesheets. 0.1mm reduction in thickness of only top facesheet reduces impact force by 5.4% whereas 0.1mm reduction in thickness of only bottom facesheet gives a 4.1% reduction in maximum impact force.

7.4 Chapter 6

Chapter 6 focussed on developing numerical models to validate the developed analytical models in the previous chapters. LS-DYNA impact software was used to model the impact problem. The numerical model backed up the idea that the position of the variable stiffness location in the plate is a major aspect in impact dynamics of such structures. Inter-laminar and intra-laminar failure were also incorporated into the models through the relevant material models and tie-break contacts. Double shell method with tri-break contacts reduced the gap between analytical and experimental work. The difference in analytical and numerical models were less than 10% for both variable stiffness curved plates and the foam based sandwich plates.

8 Publications

8.1.1.1 Journals

Arachchige B, Ghasemnejad H. Theoretical approach to predict transverse impact response of variable-stiffness curved composite plates. *Composites Part B* 2016; 89, 34-43

Arachchige B, Ghasemnejad H. Post impact analysis of damaged variable-stiffness curved composite plates. *Composite Structures* 2017; 166, 12-21.

Arachchige B, Ghasemnejad H. Modelling of low velocity impact on curved sandwich plates with variable stiffness facesheets (Submitted). *Journal of Impact Engineering* 2017.

8.1.1.2 Conference proceedings

Arachchige B, Ghasemnejad H. An approach to predict low velocity impact damage modelling of variable stiffness curved composite plates. *World Academy of Science, Engineering and Technology: International journal of Materials and Metallurgical Engineering*; Vol 3, No:10, 2016.

Arachchige B, Ghasemnejad H. Investigating the effect of contact stiffness on the Low velocity impact response of Curved Composite Plates. *WSEAS Transactions on Applied and Theoretical Mechanics* 2015, vol 10, 227-234.

Arachchige B, Ghasemnejad H. Impact and Post Impact Analysis of variable stiffness curved composite plates. 21st International Conference on Composite Materials (ICCM 21), China.

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 143. Address C. LS-DYNA ® KEYWORD USER'S MANUAL VOLUME II Material

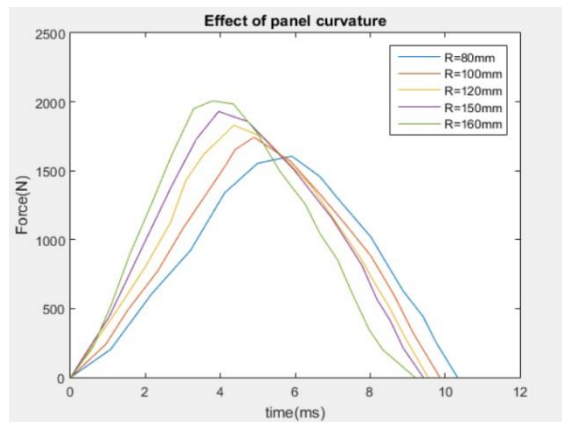
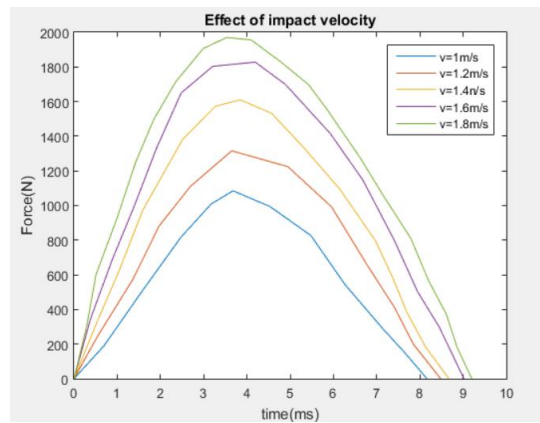
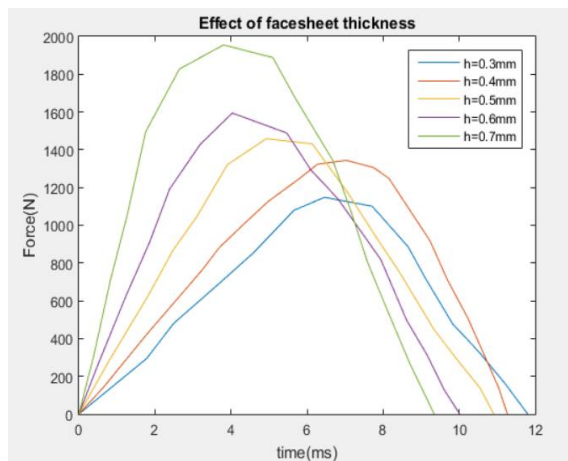
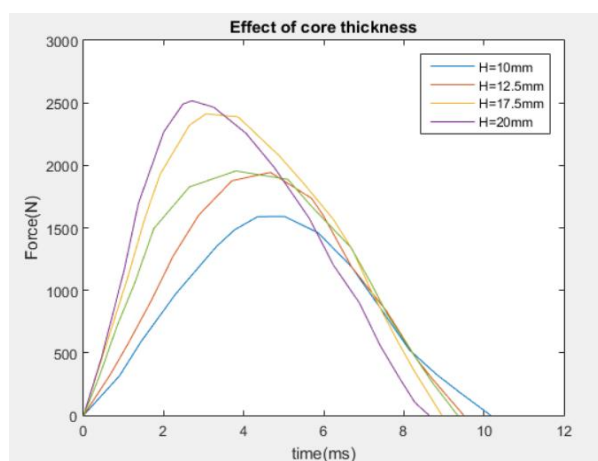
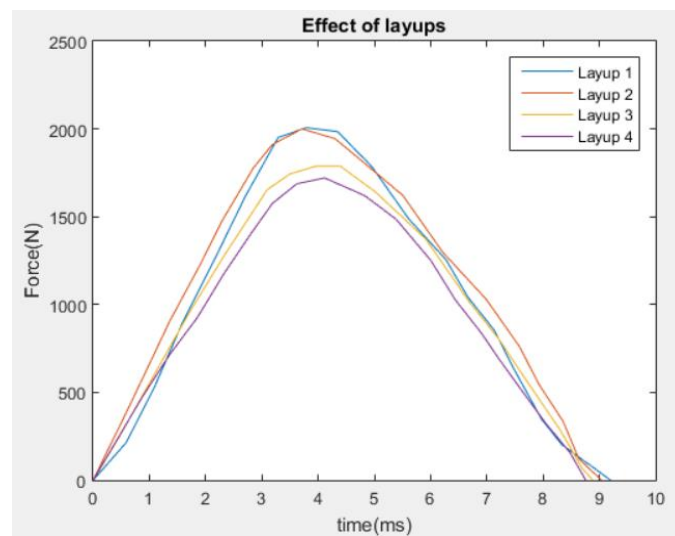
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Appendix A Sample MATLAB output

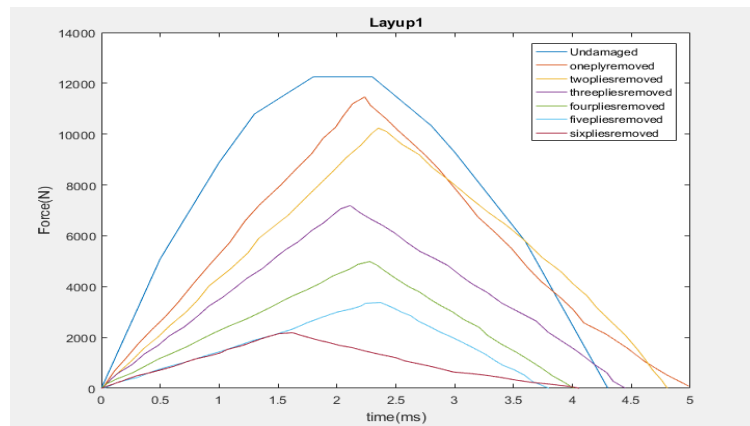
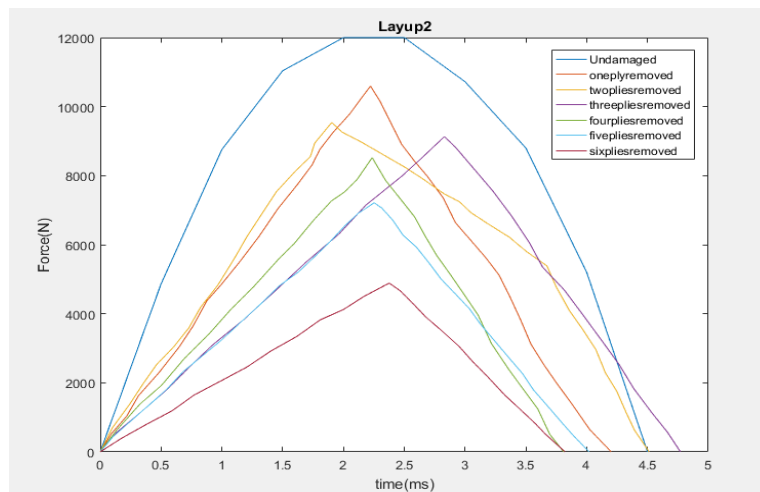
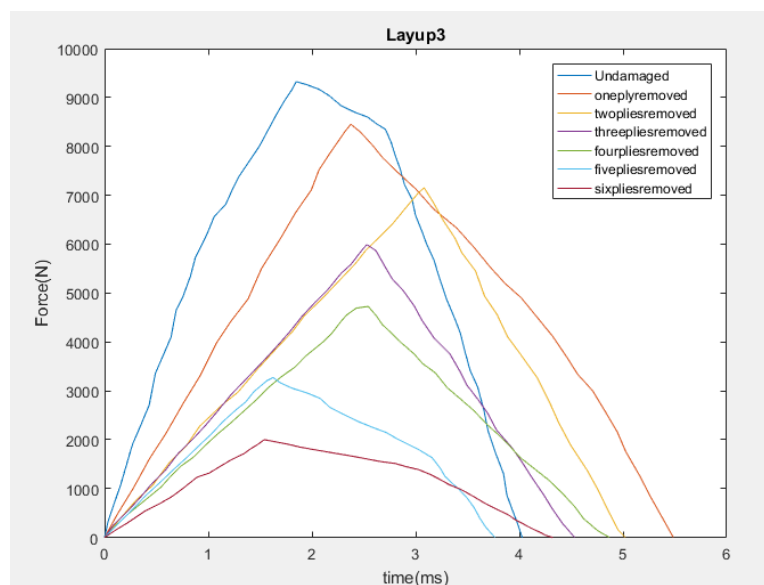
A.1 Curved sandwich plate

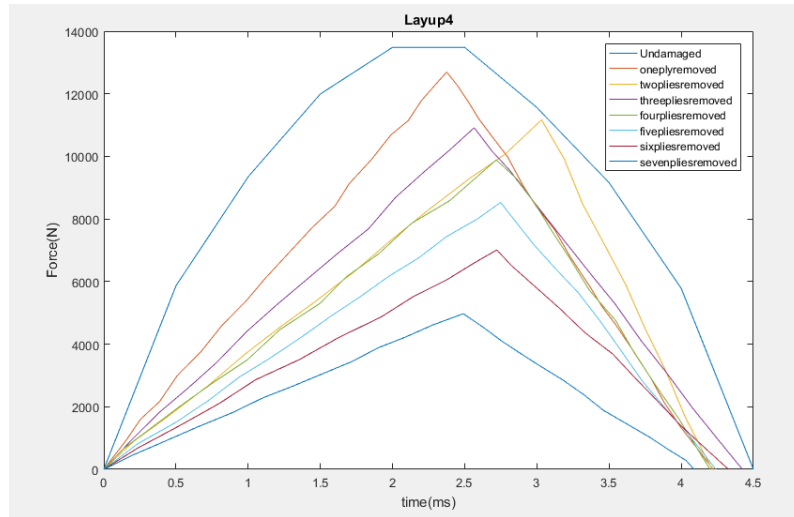
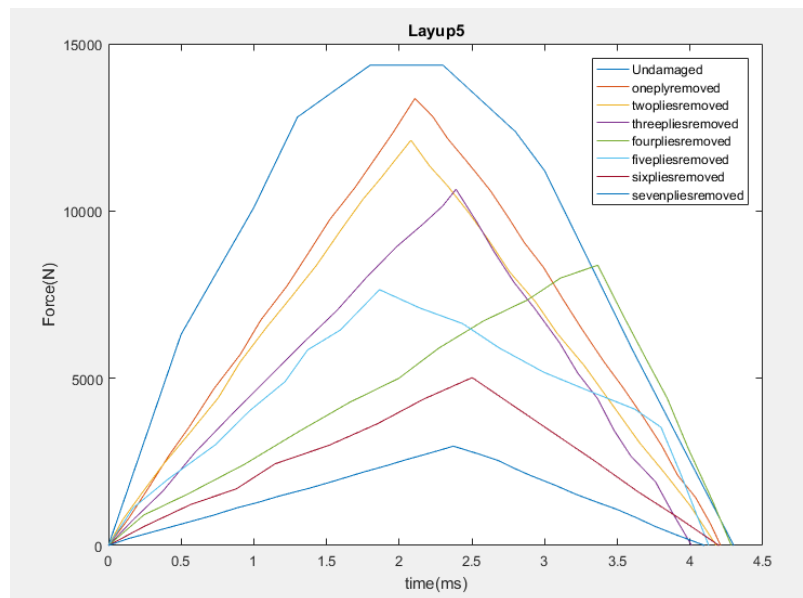
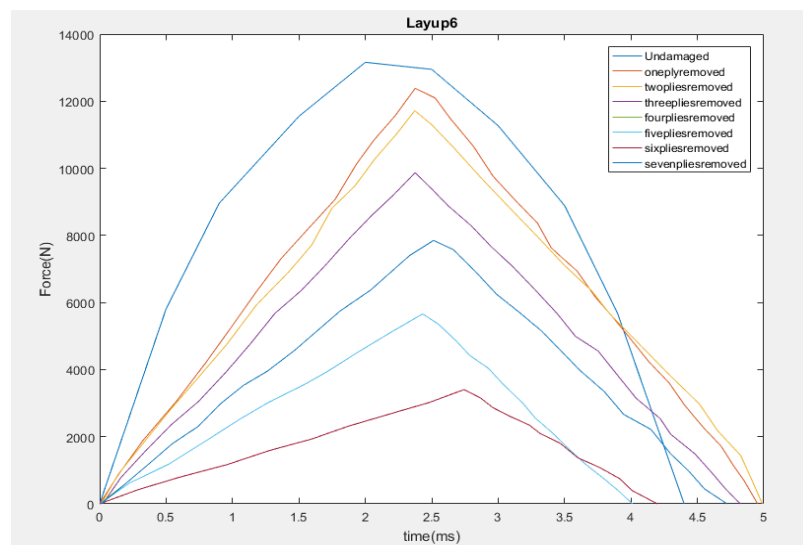
Damage Modelling approach 1



A.2 Damage modelling

A.2.1 Damage Modelling approach 1





A.2.2 Damage Modelling approach 2

