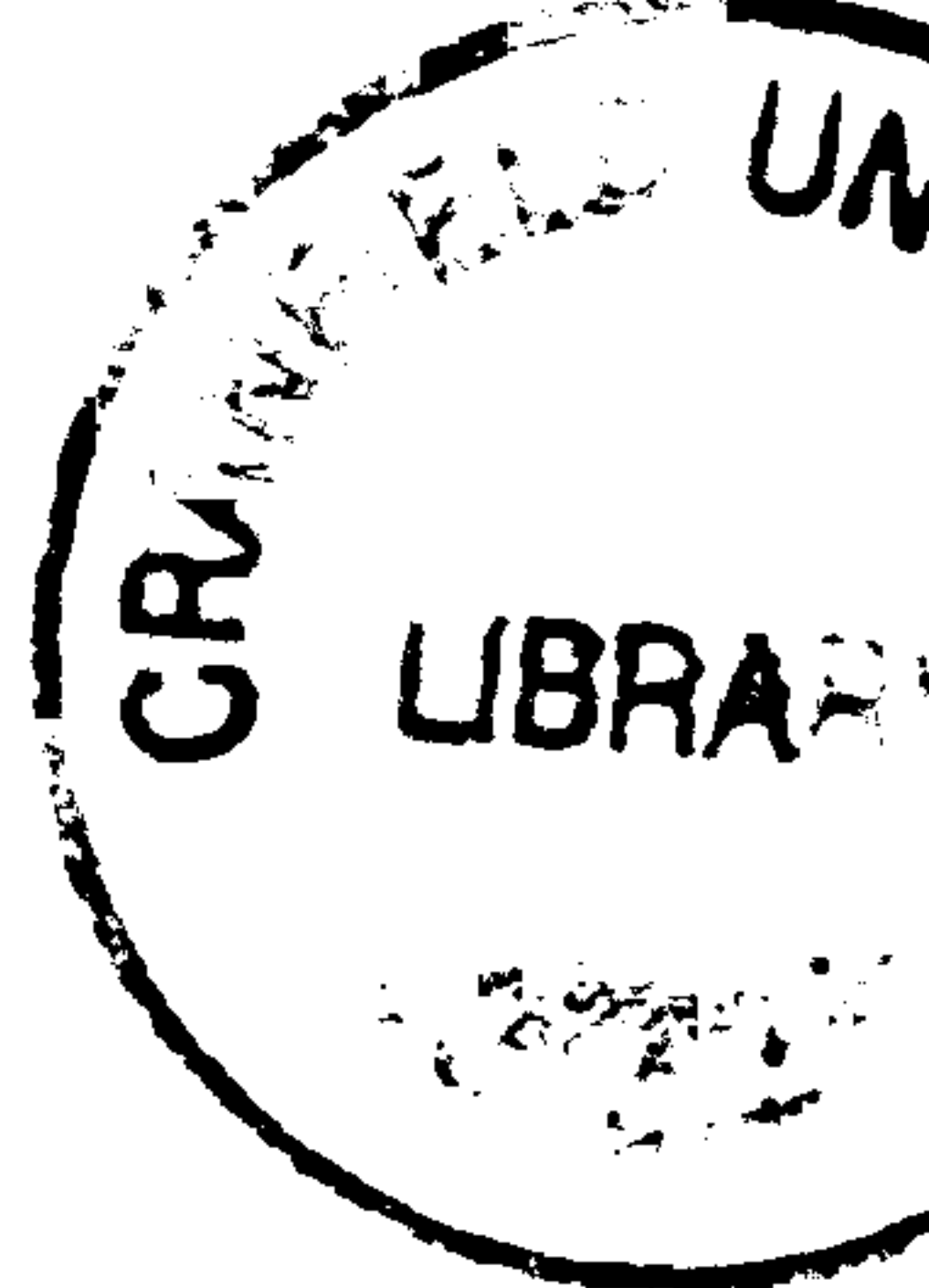




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Modelling of Transient Gas-liquid Flow and Pigging in Pipes

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**MODELLING OF TRANSIENT GAS-LIQUID FLOW
AND PIGGING IN PIPES**

Supervisor: Dr H Yeung

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ABSTRACT

More and more transient gas-liquid operations in pipes are being successfully applied in the oil and gas industry. Pigging in two-phase pipelines, to remove liquid accumulation or for cleaning purposes, is an important transient operation. Another important operation is the injection of gas to transport the accumulated liquid in the pipeline to process facilities. Analysis of such transient two-phase flow in a pipeline is necessary not only for designing the liquid and gas handling facilities, but also for safe operating procedure. In pipeline-riser system such operations cause even more severe changes in flow conditions.

A two-fluid model has been developed to determine the transient behaviour of fluids during these operations. The derived one-dimensional set of equations for each flow pattern describe the flow of fluids in all regions. Semi-implicit finite difference schemes were used to solve the initial and boundary value problem for each phase of the process: gas/pig injection, gas shut-in, slug production and gas flow out of the system.

An extensive experimental program has been carried out to acquire two-phase transient flow and pigging data on a 67 m long, 0.0525 m diameter, 9.9 m high pipeline-riser system. A computer based data acquisition system has been utilised to obtain rapidly changing and detailed information of the flow behaviour during the transient tests. The model results compare well with the experimental data for characteristics such as inlet pressure, hold-up and pig velocity.

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This dissertation is dedicated to the my wife, Luiza, whose love, understanding and constant encouragement made this work possible, to my parents and to Frederico and Henrique, my sons.

NOMENCLATURE

<u>Variables</u>	<u>Description</u>
A	pipe cross sectional area
AF	area of flow
C_o	distribution coefficient
c	shear or friction coefficient
D	diameter
E, F	velocity coefficients
G	body force
f	Fanning friction factor
g	acceleration caused by gravity
h	liquid height in stratified flow
k	shear force multiplying factor
m	mass
p	pressure
P	perimeter
t	time
v	velocity
z	position from pipeline inlet
α	volume fraction, hold-up
ε	absolute pipe roughness

δ	liquid film thickness in annular flow
Δ	increment
Θ	pipe inclination angle
μ	viscosity
ρ	density
τ	shear stress

Subscripts

Description

b	bubble
d	drift
f	film
g	gas
i	gas-liquid interface
h	hydraulic
j	one-dimensional cell index
l	liquid
m	mixture
s	slug
w	pipe wall

Superscripts

Description

n	time index
$\diamond$	value in the basic step of SETS method

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CHAPTER 1. INTRODUCTION

The use of long gas-liquid pipelines is becoming an attractive alternative to conventional monophasic systems. Utilisation of two-phase pipeline-riser systems will continue to grow since the vast majority of recent discoveries have been located offshore and because of the growing number of horizontal wells. Conventional gas-liquid pipeline-riser pigging, gas pump and pig lift are severe cases of transient operations.

1.1 Conventional Pigging

A pig is a tool used for various maintenance tasks in pipelines. It is defined as 'a projectile, forced through the inside of a pipe using either hydraulic or pneumatic pressure, while maintaining a positive seal with the pipe wall'. Maintenance tasks include functions such as cleaning the line, removal of liquid hold-up or condensate from gas pipelines and inspection. In gas-liquid flow the pig acts as a moving boundary, scooping the liquid ahead of it into an expanding liquid slug region, and leaving behind a region with almost no liquid as shown in Fig. 1.1, causing a severe case of transient flow. Knowledge of the dynamic behaviour of the flow is very important to properly design and operate gas-liquid pipelines and their upstream and downstream facilities. A frequently pigged wet-gas pipeline is capable of transporting up to 70% more gas as compared to operation without pigging, according to McDonald and Baker (1964). Gas-liquid flowlines connecting subsea wellheads or manifolds to surface facilities in deep water are being regularly pigged to avoid accumulation of organic deposits causing great operational and safety problems due to the severity of the phenomena according to Lima and Neto (1995) and Lima and Yeung (1998).

Standard design procedure for gas-liquid pipeline-riser systems subject to pigging still rely on steady-state empirical correlations and mechanistic models, or sometimes, on simplified quasi-steady-state pigging models. The use of steady-state empirical correlations and mechanistic models for predicting pressure, flow rates and liquid hold-up in the pipeline can result in oversized or undersized facilities.

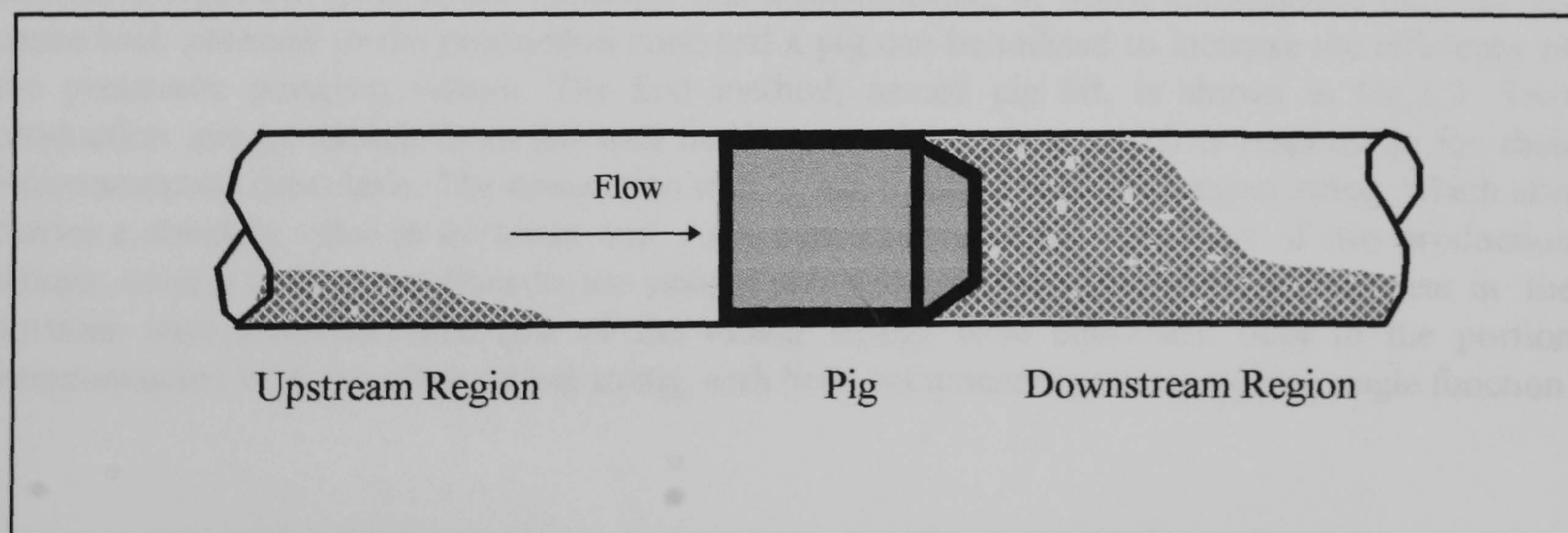


Figure 1.1 - Conventional gas-liquid pipeline pigging

1.2 Pig Lift and Gas Pump

Petroleum is found below the ground filling porous medium regions called reservoirs. One of the great challenge of the oil industry is to transport the oil from the reservoir to surface. After an oil well has been completed the oil flow to the surface will only occur if the reservoir pressure is sufficient to overcome the back pressure exerted by the head of the production piping system. In cases where the reservoir pressure is not high enough it then becomes necessary to utilise some artificial method in order to lift the oil to the surface. Some of these methods are mechanical pumping systems. A common feature shared by all these systems is that they require the supply of some kind of energy to drive the pumping equipment by some physical means. This may be an electrical cable to feed the motor of a submersed centrifugal pump, or a string of mechanical rods for driving a rod pump or a progressive cavity pump. All of these systems share the common feature that they consist of a great number of components which are prone to failure.

A widely employed method is pneumatic pumping, known by 'gas lift' (Brown - 1980), which consists basically of injecting gas into the annular space that exists between the production strings and the well casing as shown in Fig. 1.2. Gas is injected into the production string by means of special valves with the purpose of gasifying the oil. This gasification reduces the fluid density and facilitates the flow off to the surface. There are two 'gas lift' systems namely 'continuous gas lift' and 'intermittent gas lift'. As the very name suggests, gas is injected continually into the annular space until it reaches a valve at the bottom of the well which allows the gas to be injected into the interior of the production string. An efficient continuous gas lift requires a flow rate of gas 5 times greater than the flow rate of liquid. In the 'intermittent gas lift', contrary to the previous one, the well is allowed to produce for some time without injecting gas. Gas is then injected into the annular space at quite high pressures. Special valves installed in the 'gas lift mandrel' allow the gas to be injected into the production string thus causing the effect of pumping the oil to the surface.

In spite of some advantages of the conventional gas lift methods, it has some inconveniences, such as, for instance, the high the back pressure exerted by the gas and oil accumulated in vertical or inclined pipes, which is a limiting factor in oil production. Lima (1996) proposes two methods of intermittent gas lift systems that eliminate this inconvenience, in which accumulated oil does not cause back pressure to the production zone and a pig can be utilised to increase the efficiency of the pneumatic pumping system. The first method, named pig lift, is shown in Fig.1.3. Two production strings stretch from the well head to one joint piece which is responsible for their interconnection downhole. The connection tool is run together with the longest string, which also carries a standing valve in its lower end. As a consequence of the utilisation of two production strings, several surface components are utilised pair wise. Certain components that appear in the portions interconnected with one of the tubing strings have equivalent ones in the portion interconnected with the other tubing string, with both components performing one single function.

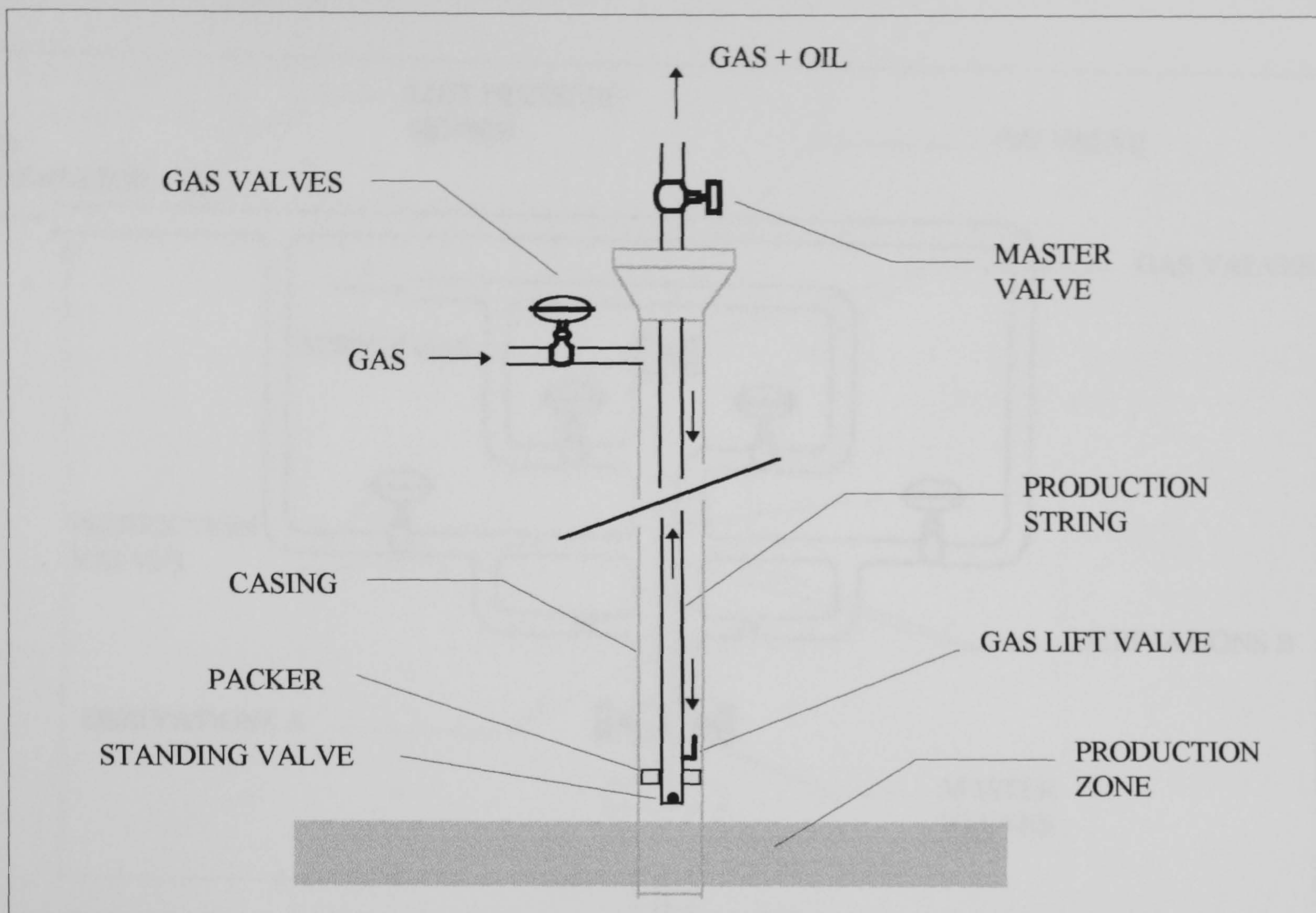


Figure 1.2 - Gas Lift

The pig lift operation begins with the opening of the pig valve of the launching device, allowing the introduction of at least one pig. Production valves are open while gas valves are closed. This procedure assures the accumulation of oil in both production strings. After a certain period of time, the pig is launched. For this, either one of the production valves is closed and either one of the gas valves is opened; they must be the ones installed in the same portion where the pig was introduced before.

Inasmuch as the pressure of the supply gas is higher than the pressure of the existing production strings, the pig is pushed by the gas. The pig descends through the whole production string until it reaches the special coupling; it then enters through the other production string and rises, causing the displacement to the surface of the liquid accumulated in both production strings. The upward movement of the pig is naturally interrupted right after its passage through first derivation, because the pressures upstream and downstream of the pig become equal; the pig then stops between derivations. A pressure sensor installed in the production line detects the moment at which oil starts flowing out of the well. At this moment, this sensor commands the closure of the gas feeding valve. After certain adjusted time, that assures the pig arrival, the production valve that was closed is opened, thus enabling the beginning of a new filling cycle of the two production strings. After the amount of oil has reached an adequate accumulated value, the pig is launched again, now in the opposite direction and the same operations has to be done in the opposite side.

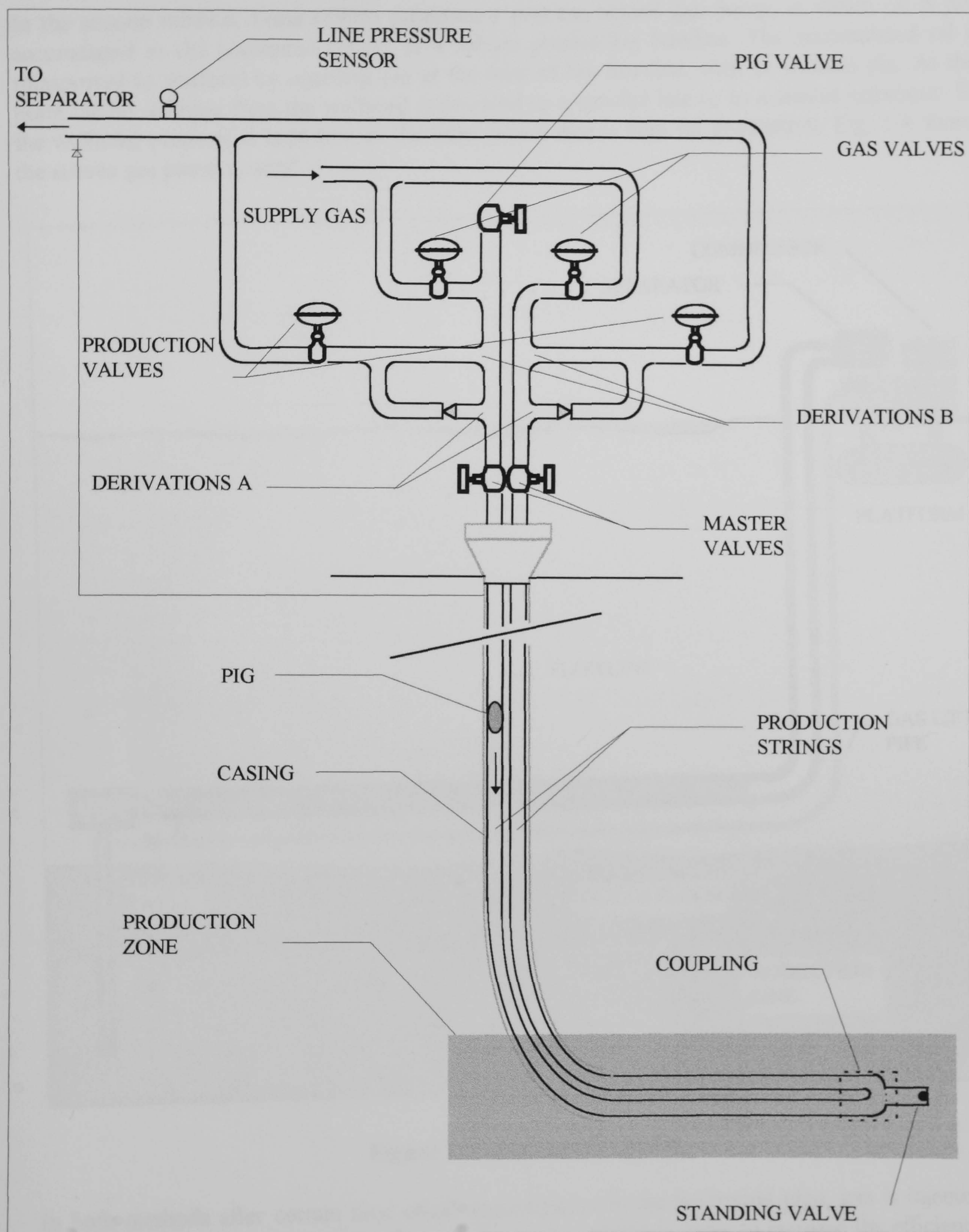


Figure 1.3 - Pig lift system in horizontal well

In the second method, Lima (1996) proposes a process named gas pump, in which oil is just accumulated in the horizontal region of a subsea production flowline. The accumulated oil is transported to platform by injecting gas at the inlet of the flowline, with or without pig. At this point the oil coming from the wellhead is deviated to a parallel line or to a subsea separator. So the wellhead pressure is kept low all the time, which means high oil production. Fig. 1.4 shows the subsea gas pump system.

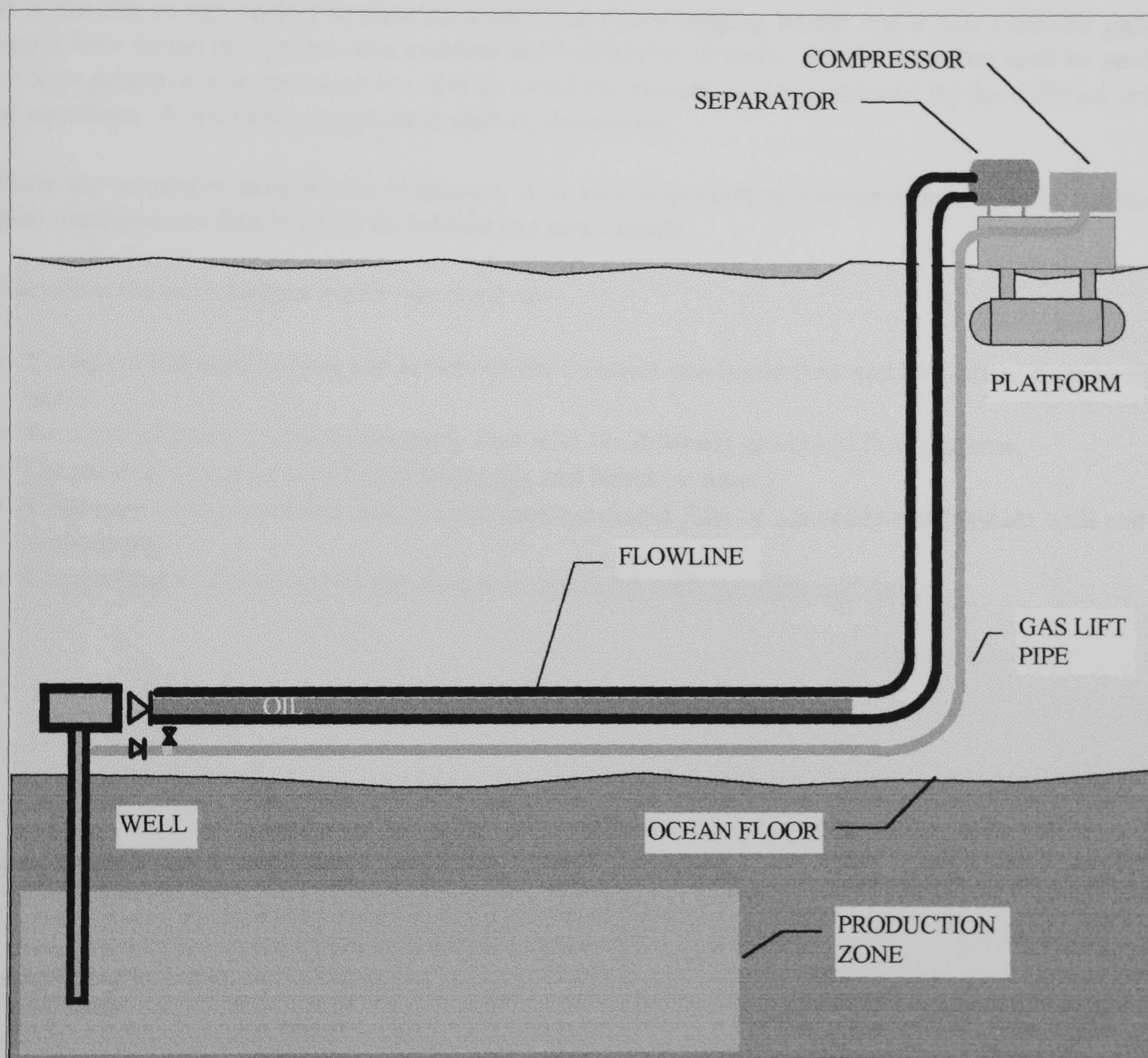


Figure 1.4 - Gas pump system

In both methods after certain time of oil accumulation in the horizontal pipe, gas is injected in order to transport the oil to the surface. A pig can be utilised in order to increase the efficiency of the oil transportation. A complex transient gas-liquid modelling is required to simulate these two methods and also for conventional gas-liquid pipeline pigging so common in the oil industry.

1.3 Objective of this Work

There are no previous works on pipeline-riser multiphase pigging and gas pump. The evaluation of existing tools is difficult because there are no experimental data sets made available to the public and due to the proprietary nature of these tools. The necessity of a theoretical and experimental study on gas-liquid pipeline-riser transient flow and pigging is obvious.

It is the aim of this project to develop a new two-phase pigging model and a new transient gas-liquid flow model in pipeline-riser systems and horizontal oil wells. Special attention shall be paid to flow pattern characterisation in order to avoid the discontinuities generated by the different set of equations. A computer programme shall be developed.

Once the computer programme is created, it is very important to compare the obtained results with experimental data in order to validate the new models.

Therefore the main targets within this work are:

- Complete literature survey and review of the transient gas-liquid flow and two-phase pigging in pipes.
- Creation of a new model to smoothly deal with the different gas-liquid flow patterns.
- Creation of a pigging model simulating gas and liquid by-passes.
- Collection of experimental data on transient gas-liquid flow in a pipeline-riser system with and without pig.
- Comparison of the results of the computer simulator with experimental data.

CHAPTER 2. REVIEW OF RELEVANT LITERATURE

2.1 Flow Regimes and Two-phase Flow Models

During concurrent gas-liquid flow in pipes, a variety of flow patterns can exist. Each pattern results from the particular manner by which the liquid and gas distribute in the pipe. Authors differ somewhat in the name they assign to each of the flow patterns. However, the differences are small and most agree on four flow regimes: stratified, slug, annular and dispersed bubble as shown in Fig. 2.1.

- Stratified: in this case the liquid flows at the bottom of the pipe with gas at the top.
- Slug: liquid slugs separated by gas pockets move violently downstream.
- Annular: the gas flows in the centre of the pipe while liquid flows as an annular.
- Dispersed bubble: the gas is dispersed in the form of small bubbles within a continuous liquid phase.

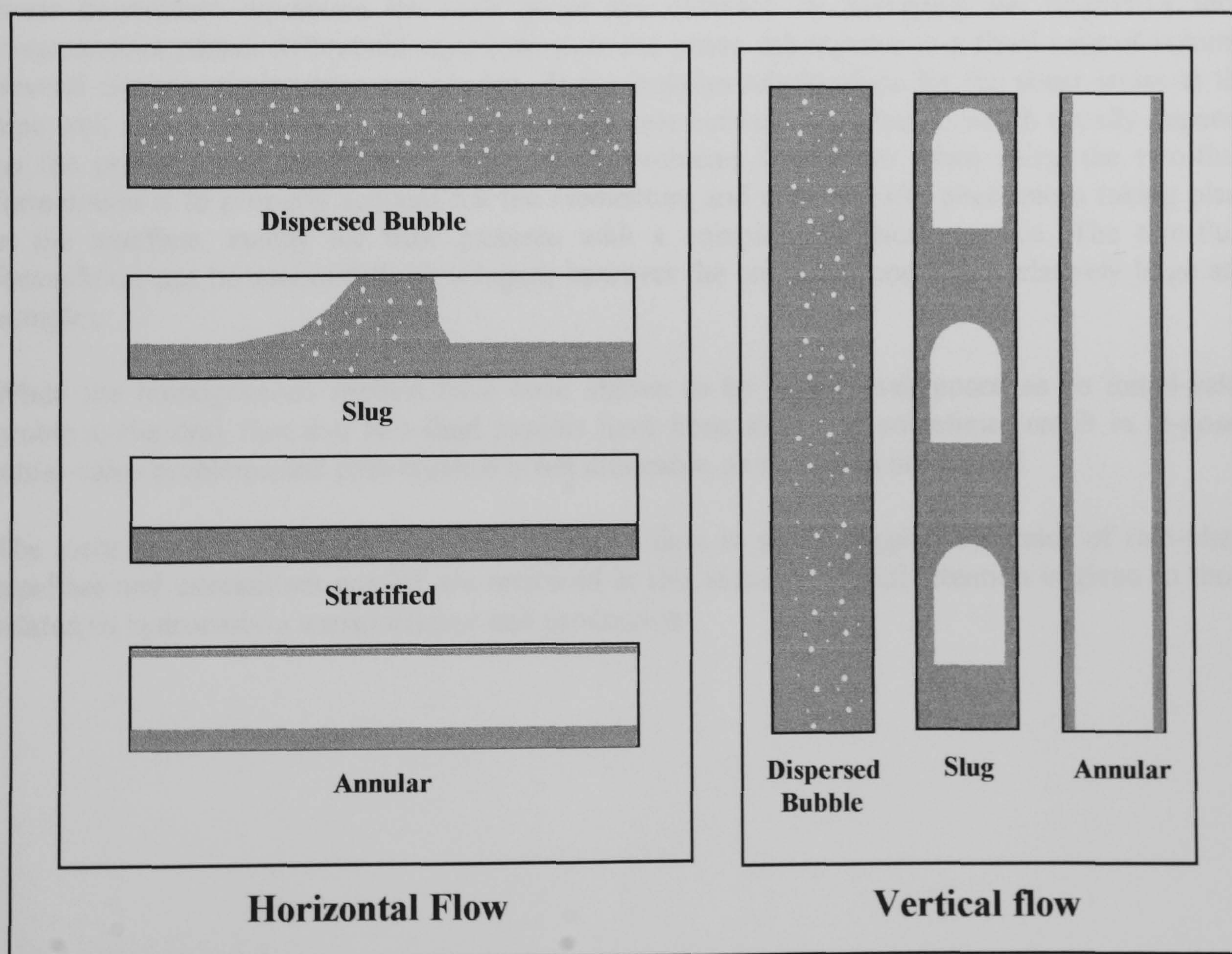


Figure 2.1 - Flow patterns in gas-liquid flow in pipes

Theoretical developments for transient two-phase gas-liquid flow in pipes can be classified into three categories: no-slip flow or homogeneous models, slip mixture flow or drift-flux models, and separated flow or two-fluid models.

Homogeneous mixture models are too simplified and in general do not perform well in comparison with experimental data.

The drift flux models are sometimes called diffusion models. The basic concept of this formulation is to consider the mixture as a whole, rather than two phases separately. This is obtained by using a mixture momentum equation that results from the combination of the gas and liquid linear momentum equations. The mixture momentum equation does not contain the interfacial transfer terms, because they cancelled out in the summation process. Some additional manipulations to convert phase velocities into mixture and drift velocities are also done to express the mixture velocity, the pressure, and the liquid hold-up as dependent variables. This formulation is simpler than two-fluid models.

Two-fluid formulations are very complex. The equations describing the conservation of mass and linear momentum equations for each phase are obtained by averaging the respective local instantaneous partial differential equations over the phase sub-volume in a fixed control volume. Several closure relationships are needed. These include relationships for the shear stress at the pipe wall and at the interface, the mass transfer rate between the phases, which usually depends on the pressure and temperature. One of the problems that arises when using the two-fluid formulation is to properly account for the momentum and mass transfer phenomena taking place at the interface, mainly for flow patterns with a complex interfacial surface. The two-fluid formulation can be successfully developed, however the computer codes are relatively large and complex.

While the homogeneous models have been shown to be always well-posed as an initial-value problem, the drift flux and two-fluid models have been shown to sometimes result in ill-posed initial-value problems and convergence is not attainable, as described in item 2.2.

The most relevant works on transient gas-liquid flow in pipes, pigging dynamics of two-phase pipelines and intermittent gas lift are reviewed in this section. Special attention is given to those related to hydrocarbon transportation and production.

2.2 Transient Gas-liquid Flow

A broad description of gas-liquid flow categories was given by Scoggins (1977), who was one of the first investigators on this subject in the oil industry. Scoggins presented a relatively comprehensive literature survey, after which he decided to use a drift flux formulation in his model for horizontal two-phase transient flow. Homogeneous mixture models were discarded as being too simplified and for not performing well in comparison with experimental data. Two-fluid models were discarded for the ill-posedness consideration of the equation sets (Lyczkowski et al. - 1975), and for the lack of practical and reliable means to account for the flow regime dependent interfacial friction and transient flow forces.

Scoggins determined the slippage between liquid and gas phases through commonly accepted steady-state empirical liquid hold-up correlations. The concepts of slip velocity and slip ratio, both used in earlier works, were not considered. The Eaton (1967) and Dukler et al. (1964) correlations were used as closure relationships. Fluid physical properties and mass transfer between phases were calculated by the black-oil model approach.

The polynomial characteristic equation of the Scoggins model yielded all real-valued roots for a broad range of operating conditions common to gas-oil two-phase flow pipeline operations. This non-linear equation was solved using a finite difference method, and an implicit sequential solution algorithm, which was based upon a Newton-Raphson iterative procedure. Finally, a comparison between measured transient flow data and the prediction of the model tended to validate the proposed formulation.

Taitel et al. (1978) developed a theory to predict flow pattern transition under transient conditions using two-fluid flow model equations. Comparison with experimental data was also presented. It was found that under transient conditions, flow pattern transitions can take place at flow rates substantially different from those occurring under steady-state conditions. It was also concluded that certain unexpected 'spurious' flow with slugging would temporarily occur when the gas and liquid flow rates were suddenly increased after the establishment of a steady-state flow. This occurred even though the initial and the final steady-state flow patterns were stratified for both conditions. Taitel et al. also showed that if the flow rates were increased gradually, slugging would not have been observed.

A theoretical and experimental work on two-phase transient flow in pipes was carried out by Dutta-Roy (1982). He compared the formulations used by Scoggins (1977) and Taitel et al. (1978), and concluded that the Scoggins formulation did not include all the interfacial terms in the mixture momentum equation. The two-fluid model formulation used by Taitel et al. (1978) was coded and compared with the experimental results. Transients were created by increasing the flow rates after steady-state condition was reached. The comparison showed that the Taitel et al. transient flow pattern prediction method did not accurately predict the time period for the slug formation, but gave the same trends as the experimental data. The Scoggins transient model and the two-fluid model for stratified flow were compared with the field data of Cunliffe (1978). The results of the comparison show that the Scoggins formulation performed better than the two-fluid model.

The use of a two-fluid model with the inclusion of the pressure differential term was attempted by Sharma (1983). The inclusion of such a small scale flow property has been shown by several researchers to improve the stability of the equation set (Banerjee and Chan - 1981, Roy and Ho - 1980). The analysis of the characteristic polynomial equation for the two-fluid equation set showed that all characteristics were real in the range of parameters investigated, indicating that the inclusion of the phase pressure difference indeed yielded a hyperbolic and well-posed set of equations. The numerical results were consistent with other stratified transient flow formulations, but the predictions were poorer than the ones obtained from the Scoggins drift flux model. No significant difference in results between unequal and equal-phase pressure formulations was found, although the last is known to yield an ill-posed set of equations.

Sharma (1985) proposed a transient slug flow model based on the coupling of an unequal phase velocity and unequal phase pressure two-fluid model with the hydrodynamic slug flow model developed by Dukler and Hubbard (1975). His method included averaging techniques for the slug flow parameters in order to allow the use of the separated flow model. The proposed formulation, however, was not evaluated against any experimental transient slug flow data.

The well known and one of the few commercially available two-phase transient computational codes, OLGA, resulted from a joint research program conducted by the Institute for Energy Technology (IFE) and SINTEF in Norway (Bendiksen et al. - 1986, 1991). This code has been continuously updated since 1983 and is now comprised of tens of thousands of code lines. It is based on an 'extended two-fluid model', which assumes the existence of three separate phases, namely, gas, liquid film, and liquid droplets. Separate continuity equations are applied to each of these phases, and two momentum equations are used: a combined equation for the gas and the liquid droplets, and a separate equation for the liquid film. A mixture energy conservation is also used. The possible flow patterns are grouped into two major categories, separated (stratified and annular) and distributed (dispersed bubble and slug). Equations for the interfacial terms and slippage between the phase were given for each of these two categories. Switching between the two sets of equations is done using the minimum slip concept, that is, the roots yielding the minimum liquid hold-up were picked as the correct ones. Although transition criteria for determining the flow pattern at a specific location and time were presented, this information was used as an indication only and was not utilised in the transient calculations.

Other commercially available two-phase transient codes are PLAC and TACITE. PLAC (AEA Technology - 1996) was developed from the nuclear reactor code TRAC. The PLAC code solves mass, momentum and energy equations for each phase using a one-dimensional finite difference scheme. The SETS (Stability-Enhancing Two-Step) method, used in PLAC, is a semi-implicit method which treats the convective terms implicitly. SETS is a two step method, consisting of a basic step and a stabilising step. The basic step is a semi-implicit equation set which provides information about pressure wave propagation. The second step is thus added as a stabilising step, and it provides information about the propagation of density, energy and momentum. PLAC has flow regime maps for vertical and horizontal pipes. The flow regime boundaries in vertical flow are mainly based on void fraction: bubbly, plug, churn and annular. In horizontal flow the transition from stratified flow to other flow regimes is determined using the method devised by Taitel & Dukler (1976), based on gas velocity and the transition between slug flow and annular flow are based simply on void fraction. Studies by Mahaffy (1982) showed that in some circumstances numerical instabilities can arise and so the method is stability enhancing rather than totally stable.

TACITE (Pauchon et al. - 1993) has been developed under a joint research program between IFP, TOTAL and ELF AQUITANE. The TACITE code is based on numerical resolution of a drift flux model. The time advancing scheme is explicit.

Due to the proprietary nature of OLGA, PLAC and TACITE it is difficult to know the details of these codes.

Taitel et al. (1989) presented a new simplified approach for modelling two-phase transient flow in pipes. This model assumes that the gas phase can be considered in quasi-steady condition. Thus, the time dependent term in the gas continuity equation can be neglected. Local momentum equilibrium between the gas and liquid phases is also assumed. In order to compensate for some inaccuracies incurred in the simplification process, Minami (1991) used mechanistic models for predicting flow pattern, the slippage between phase and the pressure drop. Minami performed an extensive experimental program showing this simplified approach is physically sound for some flow conditions. However the quasi-steady state gas flow assumption is considered a serious restriction in situations where there is a considerable gas accumulation as proposed in this work.

Vigneron et. al. (1995) carried out an experimental programme to acquire multiphase transient data. Comparisons were presented between the data and predictions with TUFFP simplified model (Minami - 1991), PLAC and OLGA. The results show that further work should be done in order to have a better prediction. Even in a simple 420 m horizontal loop, the models predictions were not so good.

2.3 Pigging

McDonald and Baker (1964) were probably the first investigators to present a study on pigging. They assumed a successive steady-state approach to model the phenomena. The pipeline under the pigging operations was divided into four flowing zones. The front of each zone was moved at every time-step based on a volumetric material balance. Using steady-state correlations average pressure drop and average liquid hold-up were calculated. The pig velocity was determined through a gas volumetric balance, assuming no gas leakage through the pig. A pressure drop correlation through the pig was also provided. The successive steady-state assumption is the main weakness of McDonald and Baker pigging model. It fails to predict the hydrodynamic flow behaviour after the delivery of the liquid slug and the pig into the downstream liquid handling facility.

Barua (1982) pursued an attempt to improve the McDonald and Baker pigging model. He proposed a procedure to model the liquid slug acceleration during its delivery into the separator. He considered that the pig was moving at the gas phase velocity immediately behind it, and used his own empirical correlation for predicting pressure drop across the pig. However, Barua did not remove the main weakness of successive steady-state conditions.

Kohda et. al. (1988a and 1988b) proposed the first pigging model based on full two-phase transient flow formulation. Their model includes both the Kohda et al. (1987) drift flux transient code, which is based on the Scoggin's study, and a pigging model. The pigging model composed of a correlation for the pressure drop across the pig, a correlation for liquid hold-up in the slug zone, a correlation for the pigging efficiency as a function of the pig to pipe diameter ratio, a pig velocity model, and a gas and liquid mass flow boundary condition applied to the slug front. The resulting set of equations was solved numerically by finite difference method, using two co-ordinate systems, one fixed and the other adaptive. No detail was given on how the difference equations were coupled and solved simultaneously. In the experimental part of the study, two pigging test results were reported that were obtained from a 1436.5 m long, 105.3 mm diameter, low pressure horizontal pipeline, using compressed air and water as the two-phase flow mixture. The experimental data compared relatively well with the predicted values from the numerical simulator. Other than the fact that the Kohda et al. pigging model is still based on a drift flux model, and that it uses flow pattern independent steady-state liquid hold-up and pressure drop correlations, no other deficiencies are apparent.

Minami (1991) developed a pigging model and coupled it with the Taitel simplified transient model. An Eulerian-Lagrangian approach using a fixed and moving co-ordinate system is used. He used mechanistic models for predicting flow pattern, the slippage between phases and the pressure drop. Minami performed an extensive experimental program showing this simplified approach is physically sound. However the quasi-steady state approach is not suitable for pipeline-riser system gas pump and pigging due to the high accumulation of gas upstream the pig.

2.4 Intermittent Gas Lift

Although intermittent gas lift has been around for almost seventy years, it was not until the work of Brown and Jessen (1962), that the basis for modern design was established. Increasingly sophisticated modelling efforts have been realised in the last years (Schmidt et al - 1984, Zimmerman -1980, and Liao -1991). The work of Liao deserves special attention, since it represents a complete modelling effort of the whole intermittent cycle. The results of this model were compared with the experimental data of Brown and Jessen (1962), Brill et al (1967) and Neely et al (1974), and good matches were obtained. The results of White (1963) regarding the gas bubble penetration of the liquid slug as being a natural constant of lift system under consideration are highly encouraging.

Results imply that sophisticated modelling can successfully predict complicated flow behaviour, provided that the appropriate closure relationships are used.

However all of these studies consider only vertical flow. So there is a need to investigate a similar phenomena in a pipeline-riser system or horizontal well.

2.5 Summary

The following conclusions can be made based on the Literature Review:

- The complexity of the gas-liquid transient flow has raised difficulties in the development of easy-to-use and proven codes for the oil industry to design and operate pipelines under transient conditions.
- The drift-flux model requires the use of empirical correlations to account for the slippage between the phases limiting the degree of confidence of this formulation. It also suffers from ill-posedness problems.
- The quasi-steady state gas flow assumed in the Taitel et al. simplified model is a serious restriction for gas pump and pipeline-riser pigging.
- The existing pigging models are not suitable for pipeline-riser systems.

The above considerations show that further studies should be done for predicting gas-liquid transient flow and pigging in pipeline-riser systems. The objectives of the present work are: to collect experimental data on gas-liquid pigging, gas pump with and without pig, transient gas-liquid flow in pipeline-flow and to develop a model to predict these complex phenomena. The model should be a transient two-fluid model avoiding the complexity of highly non-linear momentum equations of OLGA or PLAC and the simplicity of Taitel et al., drift flux or homogenous flow models.

CHAPTER 3. TRANSIENT MODEL

The two-fluid formulation was assumed to be the more suitable in this work. Two-fluid formulation equations taken from Sharma (1985) are shown below.

The area averaged mass conservation equations, neglecting the mass fluxes across the gas-liquid interface, are given by:

- Gas continuity equation

$$\frac{\partial(\alpha_g \rho_g)}{\partial t} + \frac{\partial(\alpha_g \rho_g v_g)}{\partial z} = 0 \quad (1)$$

- Liquid continuity equation

$$\frac{\partial(\alpha_l \rho_l)}{\partial t} + \frac{\partial(\alpha_l \rho_l v_l)}{\partial z} = 0 \quad (2)$$

where α_g and α_l are the volumetric fractions, v_g and v_l are velocities, ρ_g and ρ_l are densities and t and z are time and space, respectively.

The area averaged momentum equations neglecting the momentum transfer across the gas-liquid interface are given by:

- Gas momentum equation

$$\frac{\partial(\alpha_g \rho_g v_g)}{\partial t} + \frac{\partial(\alpha_g \rho_g v_g^2)}{\partial z} = -\alpha_g \frac{\partial p}{\partial z} + \alpha_g \rho_g G_z + \tau_{ig} + \tau_{wg} \quad (3)$$

- Liquid momentum equation

$$\frac{\partial(\alpha_l \rho_l v_l)}{\partial t} + \frac{\partial(\alpha_l \rho_l v_l^2)}{\partial z} = -\alpha_l \frac{\partial p}{\partial z} + \alpha_l \rho_l G_z + \tau_{il} + \tau_{wl} \quad (4)$$

where G_z is body force in z direction and τ_{ig} , τ_{il} , τ_{wg} , τ_{wl} are shear forces per unit volume rather than shear stresses.

3.1 Proposed Transient Model

Taitel et al. (1989) suggested the use of standard flow pattern dependent steady state model, neglecting the two terms on the left hand side of momentum equations 3 and 4. However, for rapid transient events, the inertia terms could not be ignored. As the liquid density is order of magnitude higher than gas, it is proposed that only the inertia of liquid is included in the model. Rather than solving the complete equations, the effect of inertia of liquid is taken into account as a pseudo shear force.

The equation (4) can be written as:

$$\frac{\partial(m_l v_l)}{\partial t} + \frac{\partial(m_l v_l^2)}{\partial z} = -A_l \partial p - A_l \rho_l g \sin \Theta \hat{c}z + \tau_i P_i \hat{c}z - \tau_{wl} P_{wl} \hat{c}z \quad (5)$$

where τ_i and τ_{wl} are shear stresses, P_i and P_{wl} are the perimeters.

The equation (5) above can be rewritten as:

$$A_l \partial p + A_l \rho_l g \sin \Theta \hat{c}z - \tau_i P_i \hat{c}z + (1 + K) \tau_{wl} P_{wl} \hat{c}z = 0 \quad (6)$$

where

$$K = \left(\frac{\partial(m v)}{\partial t} + \frac{\partial(m v^2)}{\partial z} \right) / (\tau_{wl} P_{wl} \hat{c}z) \quad (7)$$

For gas, K is taken as zero.

The horizontal ($-10^\circ < \Theta < 10^\circ$) flow pattern is determined by Beggs and Brill (1973) criteria. If $\Theta \geq 10^\circ$ slug flow is tried. If liquid hold-up is smaller than 0.1 annular flow is assumed and for liquid hold-up greater than 0.9 bubble flow is assumed. Otherwise slug flow is assumed. The slug flow is treated as a combination of other flow patterns as described in 3.1.4. This new slug flow approach assures a smooth two-phase flow structure transition.

3.1.1 Shear Coefficients

The shear coefficient is given by:

$$c = 0.5 f \frac{P}{A} \quad (8)$$

where f is the Fanning friction factor and P is the perimeter. The shear stresses are given by $\tau_w = f_w(\rho v/v_l)^2$ and $\tau_i = f_i[\rho_g(v_g - v_l)/v_g - v_l]/2$.

The wall shear coefficients for the gas and liquid phase are calculated from friction factors given by the Swamee et al. (1976) explicit equation for turbulent flow (Reynolds number greater than 4000) as follows:

$$f_w = \frac{1.325}{\left[\ln \left(\frac{\varepsilon}{3.7 D_h} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \quad (9)$$

where ε is the absolute pipe roughness, $D_h = 4AF/P$ and $\text{Re} = \rho v D_h / \mu$. P and AF are the perimeter and area of flow, respectively. For annular flow the hydraulic diameter D_h is the liquid film thickness δ .

If Reynolds number is less than 2000, the friction factor is $64/\text{Re}$. Between 2000 and 4000 the friction factor is determined by linear interpolation.

The gas-liquid interface friction factor is calculated by the Andritsos and Hanratty (1987) correlation for stratified flow given by

$$f_i = f_{wg} \quad (\text{for } v_{sg} \leq v_{sg}^{\bullet}) \quad (10)$$

and

$$f_i = f_{wg} \left[1 + 15 \sqrt{\frac{h}{D}} \left[\frac{v_{sg}}{v_{sg}^{\bullet}} - 1 \right] \right] \quad (\text{for } v_{sg} > v_{sg}^{\bullet}) \quad (11)$$

where, $v_{sg}^{\bullet} = 1 + 5 \sqrt{\frac{p_{atm}}{p}}$, in the SI unit system (m/s).

The correlation recommended by Wallis (1969) is used for the calculation of the interface friction factor for annular flow as follows:

$$f_i = 0.005 \left[1 + 300 \frac{\delta}{D} \right] \quad (12)$$

where δ is liquid film thickness.

3.1.2 Annular Flow

- Gas momentum equation

$$\frac{1}{\rho_g} \frac{dp}{dz} + \frac{c_i}{\alpha_g} (v_g - v_l) |v_g - v_l| + g \sin \Theta = 0 \quad (13)$$

- Liquid momentum equation

$$\frac{1}{\rho_l} \frac{dp}{dz} + (1 + K) \frac{c_{wl}}{(1 - \alpha_g)} v_l |v_l| - c_i \frac{\rho_g}{(1 - \alpha_g) \rho_l} (v_g - v_l) |v_g - v_l| + g \sin \Theta = 0 \quad (14)$$

- Gas continuity equation

$$A \frac{\partial(\alpha_g \rho_g)}{\partial t} + \frac{\partial[(\alpha_g \rho_g A) v_g]}{\partial z} = 0 \quad (15)$$

- Liquid continuity equation

$$A \frac{\partial[(1 - \alpha_g) \rho_l]}{\partial t} + \frac{\partial[(1 - \alpha_g) \rho_l A v_l]}{\partial z} = 0 \quad (16)$$

where p is the pressure, c is the shear coefficient, g is the acceleration caused by gravity, Θ is the pipe inclination angle and A is the pipe cross sectional area.

3.1.3 Stratified Flow

In the case of stratified flow the term $\frac{c_{wg}}{\alpha_g} v_g |v_g|$ should be added to equation (13).

3.1.4 Bubble or Mist Flow

For bubble or mist flow it is assumed that there is no slippage between gas and liquid phases. So the homogeneous model equations can be used.

- Mixture momentum equation

$$\frac{1}{\rho_m} \frac{dp}{dz} + (1 + K) c_m v_m |v| + g \sin \Theta = 0 \quad (17)$$

where p_m is the mixture density, c_m is the mixture shear coefficient and v_m the mixture velocity.

- Gas continuity equation

$$A \frac{\partial(\alpha_g \rho_g)}{\partial t} + \frac{\partial[(\alpha_g \rho_g A) v_m]}{\partial z} = 0 \quad (18)$$

- Liquid continuity equation

$$A \frac{\partial[(1 - \alpha_g) \rho_l]}{\partial t} + \frac{\partial[(1 - \alpha_g) \rho_l A v_m]}{\partial z} = 0 \quad (19)$$

3.1.5 Slug Flow

The slug flow is divided into two regions. Dispersed bubble and stratified flow for horizontal pipe flow. Otherwise a dispersed bubble and annular combination is assumed. Fig. 3.1 shows the slug flow structure. This approach smoothly deals with flow pattern transitions avoiding the common solution difficulties of the previous works on transient gas-liquid flow and pigging.

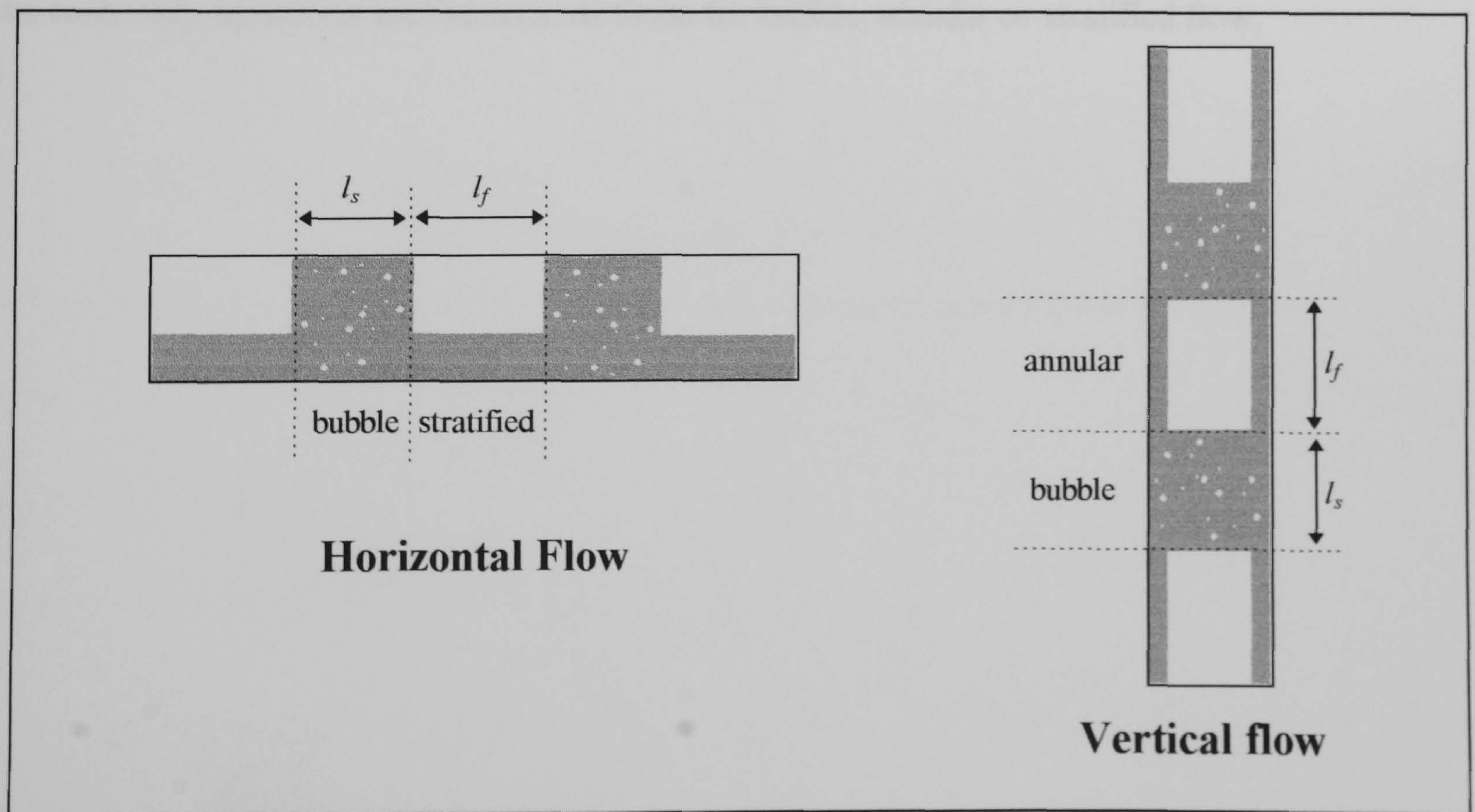


Figure 3.1 - Slug flow structure

- Gas momentum equation in the film zone

$$\frac{1}{\rho_g} \frac{dp}{dz} \Big|_f + \left[\left[\frac{c_{wgf}}{\alpha_{gf}} v_{gf} |v_{gf}| \right] + \left[\frac{c_{if}}{\alpha_{gf}} (v_{gf} - v_{lf}) |v_{gf} - v_{lf}| \right] + [g \sin \Theta] \right] \frac{l_f}{l} = 0 \quad (20)$$

- Liquid momentum equation in the film zone

$$\frac{1}{\rho_l} \frac{dp}{dz} \Big|_f + \left[\left[\frac{(1+k)c_{wlf}}{(1-\alpha_{gf})} v_{lf} |v_{lf}| \right] - \left[c_{if} \frac{\rho_g}{(1-\alpha_{gf})\rho_l} (v_{gf} - v_{lf}) |v_{gf} - v_{lf}| \right] + [g \sin \Theta] \right] \frac{l_f}{l} = 0 \quad (21)$$

- Momentum equation in the bubble zone

$$\frac{1}{\rho_s} \frac{dp}{dz} \Big|_s + \left[\frac{(1+K)c_{ws}}{\alpha_{ls}} v_s |v_s| + g \sin \Theta \right] \frac{l_s}{l} = 0 \quad (22)$$

where $v_s = \alpha_{gf} v_{gf} + (1-\alpha_{gf})v_{lf}$ and c_{wgf} is zero for annular flow.

The variables α_{ls} , v_l , v_g , l_s and l_f are determined based on the steady-state slug model based on Taitel et. al. (1989) as shown in Appendix I.

The continuity equations are the same as those for bubble, annular or stratified flow.

3.2 Model Solution

The simplified continuity and momentum equations are solved by a two step semi-implicit method (Mahaffy - 1982) using the finite difference equations given below. The basic step is a simply semi-implicit equation set. A stabilising step is added to provide information about density and momentum being transported across cell boundaries. The spatial mesh used is staggered, with thermodynamic properties evaluated at the cell centres and the velocity evaluated at the cell edges as shown in Fig. 3.2.

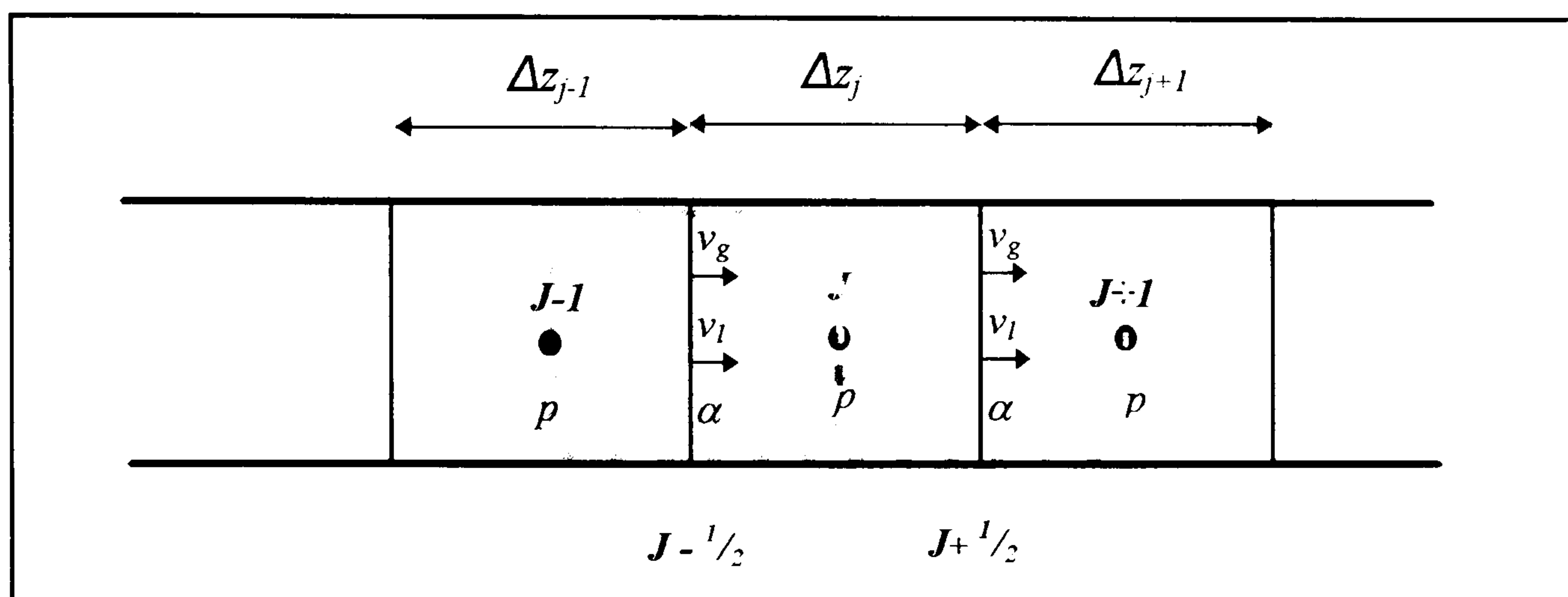


Figure 3.2 - Spatial mesh

The finite difference divergence operator is

$$\nabla_j \cdot (xv) = \left(\langle xv \rangle_{j+1/2} - \langle xv \rangle_{j-1/2} \right) / \Delta z_j \quad (23)$$

To improve stability, the flux terms at cell edges use donor cell averages of the form

$$\begin{aligned} \langle xv \rangle_{j+1/2} &= x_j v_{j+1/2} & \text{if } v_{j+1/2} \geq 0 \\ &= x_{j+1} v_{j+1/2} & \text{if } v_{j+1/2} < 0, \end{aligned} \quad (24)$$

where x is any group of state variables.

The finite difference equations for momentum and continuity equations are given below.

3.2.1 Annular Flow

Basic Equations

- Gas momentum equation

$$\frac{1}{\langle \rho_g \rangle_{j+1/2}^n \Delta z_{j+1/2}} (p_{j+1}^{\delta n+1} - p_j^{\delta n+1}) + \frac{c_i^n}{\langle \alpha_g \rangle_{j+1/2}^n} \left[v_g^n - v_l^n \right] \left[2(v_g^{n+1} - v_l^{n+1}) - (v_g^n - v_l^n) \right] + g \sin \Theta = 0 \quad (25)$$

- Liquid momentum equation

$$\begin{aligned} & \frac{1}{\left\langle \rho_l \right\rangle_{j+\frac{1}{2}}^n \Delta z_{j+\frac{1}{2}}} (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + \frac{(1+K)c_{wl}^n}{\left\langle 1-\alpha_g \right\rangle_{j+\frac{1}{2}}^n} \left| v_l^n \right| \left[2v_l^{n+1} - v_l^n \right] \\ & - c_i^n \left\langle \frac{\rho_g}{(1-\alpha_g)\rho_l} \right\rangle_{j+\frac{1}{2}}^n \left| v_g^n - v_l^n \right| \left[2(v_g^{n+1} - v_l^{n+1}) - (v_g^n - v_l^n) \right] + g \sin \Theta = 0 \end{aligned} \quad (26)$$

- Gas continuity equation

$$A \left[\frac{(\alpha_g \rho_g)^{\diamond n+1} - (\alpha_g \rho_g)^n}{\Delta t} \right]_j + \nabla_j \cdot \left[A \rho_g^n \alpha_g^{\diamond n+1} v_g^{n+1} \right] = 0 \quad (27)$$

- Liquid continuity equation

$$A \left[\frac{\left[(1-\alpha_g)\rho_l \right]_j^{\diamond n+1} - \left[(1-\alpha_g)\rho_l \right]_j^n}{\Delta t} \right]_j + \nabla_j \cdot \left[A \rho_l^n (1-\alpha_g)^{\diamond n+1} v_l^{n+1} \right] = 0 \quad (28)$$

where the superscript $\diamond$ is the value in the basic step of SETS method, n is the time index and j is the cell index.

Stabilising Equations

- Gas continuity equation

$$A \left[\frac{(\alpha_g \rho_g)^{n+1} - (\alpha_g \rho_g)^n}{\Delta t} \right]_j + \nabla_j \cdot \left[(\alpha_g \rho_g A)^{n+1} v_g^{n+1} \right] = 0 \quad (29)$$

- Liquid continuity equation

$$A \left[\frac{\left[(1-\alpha_g)\rho_l \right]_j^{n+1} - \left[(1-\alpha_g)\rho_l \right]_j^n}{\Delta t} \right]_j + \nabla_j \cdot \left[\left[(1-\alpha_g)\rho_l A \right]^{n+1} v_l^{n+1} \right] = 0 \quad (30)$$

3.2.2 Stratified Flow

In the case of stratified flow the term $\frac{c_{wg}^n}{\langle \alpha_g \rangle_{j+\frac{1}{2}}^n} |v_g^n| (2v_g^{n+1} - v_g^n)$ should be added to equation (25).

3.2.3 Bubble or Mist Flow

For bubble flow it is assumed that there is no slippage between gas and liquid phases. So the homogeneous model equations can be used.

Basic Equations

- Mixture momentum equation

$$\frac{1}{\langle \rho_m \rangle_{j+\frac{1}{2}}^n \Delta z_{j+\frac{1}{2}}^1} (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + (1+K) c_m^n |v_m^n| [2v_m^{n+1} - v_m^n] + g \sin \Theta = 0 \quad (31)$$

- Gas continuity equation

$$A \left[\frac{(\alpha_g \rho_g)^{\diamond n+1} - (\alpha_g \rho_g)^n}{\Delta t} \right]_j + \nabla_j \cdot [A \rho_g^n \alpha_g^{\diamond n+1} v_m^{n+1}] = 0 \quad (32)$$

- Liquid continuity equation

$$A \left[\frac{[(1 - \alpha_g) \rho_l]_j^{\diamond n+1} - [(1 - \alpha_g) \rho_l]_j^n}{\Delta t} \right]_j + \nabla_j \cdot [A \rho_l^n (1 - \alpha_g)^{\diamond n+1} v_m^{n+1}] = 0 \quad (33)$$

Stabilising Equations

- Gas continuity equation

$$A \left[\frac{(\alpha_g \rho_g)^{n+1} - (\alpha_g \rho_g)^n}{\Delta t} \right]_j + \nabla_j \cdot [(\alpha_g \rho_g A)^{n+1} v_m^{n+1}] = 0 \quad (34)$$

- Liquid continuity equation

$$A \left[\frac{\left[(1 - \alpha_g) \rho_l \right]_j^{n+1} - \left[(1 - \alpha_g) \rho_l \right]_j^n}{\Delta t} \right] + \nabla_j \cdot \left[\left[(1 - \alpha_g) \rho_l A \right]^{n+1} v_m^{n+1} \right] = 0 \quad (35)$$

3.2.4 Slug Flow

- Momentum equations

$$\begin{aligned} & \frac{1}{\left\langle \rho_g \right\rangle_{j+\frac{1}{2}}^n \Delta z_{j+\frac{1}{2}}} (p_{j+1}^{n+1} - p_j^{n+1}) \\ & + \left[\left[\frac{c_{wgf}^n}{\left\langle \alpha_{gf} \right\rangle_{j+\frac{1}{2}}^n} \left| v_{gf}^n \right| \left[2v_{gf}^{n+1} - v_{gf}^n \right] \right] + \left[\frac{c_{lf}^n}{\left\langle \alpha_{gf} \right\rangle_{j+\frac{1}{2}}^n} \left| v_{gf}^n - v_{lf}^n \right| \left[2(v_{gf}^{n+1} - v_{lf}^{n+1}) - (v_{gf}^n - v_{lf}^n) \right] \right] + [g \sin \Theta] \right] \frac{l_f}{l} \\ & + \left\langle \frac{\rho_s}{\rho_g} \right\rangle_{j+\frac{1}{2}}^n \left[\frac{c_{ws}^n}{\left\langle \alpha_{ls} \right\rangle_{j+\frac{1}{2}}^n} \left| v_s^n \right| \left[2v_s^{n+1} - v_s^n \right] + g \sin \Theta \right] \frac{l_s}{l} = 0 \end{aligned} \quad (36)$$

$$\begin{aligned} & \frac{1}{\left\langle \rho_l \right\rangle_{j+\frac{1}{2}}^n \Delta z_{j+\frac{1}{2}}} (p_{j+1}^{n+1} - p_j^{n+1}) \\ & + \left[\left[\frac{(1+K)c_{wlf}^n}{\left\langle 1 - \alpha_{gf} \right\rangle_{j+\frac{1}{2}}^n} \left| v_{lf}^n \right| \left[2v_{lf}^{n+1} - v_{lf}^n \right] \right] - \left[\left\langle \rho_g \right\rangle_{j+\frac{1}{2}}^n \frac{c_{lf}^n}{\left\langle \rho_l \right\rangle_{j+\frac{1}{2}}^n \left\langle 1 - \alpha_{gf} \right\rangle_{j+\frac{1}{2}}^n} \left| v_{gf}^n - v_{lf}^n \right| \left[2(v_{gf}^{n+1} - v_{lf}^{n+1}) - (v_{gf}^n - v_{lf}^n) \right] \right] + [g \sin \Theta] \right] \frac{l_f}{l} \\ & + \left\langle \frac{\rho_s}{\rho_l} \right\rangle_{j+\frac{1}{2}}^n \left[\frac{(1+K)c_{ws}^n}{\left\langle \alpha_{ls} \right\rangle_{j+\frac{1}{2}}^n} \left| v_s^n \right| \left[2v_s^{n+1} - v_s^n \right] + g \sin \Theta \right] \frac{l_s}{l} = 0 \end{aligned} \quad (37)$$

By means of the above momentum equations and steady-state slug flow model of Taitel et al. (1989) described in Appendix I it is possible to determine the phase velocities as a function of pressure. The continuity equations of basic and stabilising steps are treated in the same way as for annular or stratified flow. The term c_{wgf} is zero for annular flow.

3.2.5 Linearization of the Finite Difference Equations

The set of finite difference equations is non-linear and hence to solve them at each time step it is necessary to use an iterative method, based on Newton's Method. Starting with some estimated value for the independent variables at the new time step, the derivatives of the equations with respect to those variables are used to give the next best estimates-based on linear extrapolation from the last value - continuing until the latest estimates are equal (within prescribed tolerance) to the previous ones.

At a time step $n+1$, given an initial guess of the independent variables (in this case p and α), the values on the next iteration (variables without primes) are assumed to be related to those at the last (with primes) by the relations:

$$p^{n+1} = p'^{n+1} + \delta p \quad (38)$$

$$\alpha_g^{n+1} = \alpha_g'^{n+1} + \delta \alpha_g \quad (39)$$

Since the finite difference equations are functions of p and α_g , a Taylor expansion about the last iteration's value, retaining only the terms linear in δp and $\delta \alpha_g$ gives:

$$\begin{aligned} f(p, \alpha_g) &= f(p' + \delta p, \alpha_g' + \delta \alpha_g) \\ &= f(p', \alpha_g') + \left. \frac{\partial f}{\partial p} \right|_{p=p'} \delta p + \left. \frac{\partial f}{\partial \alpha_g} \right|_{\alpha_g=\alpha_g'} \delta \alpha_g + \text{higher order terms} \end{aligned} \quad (40)$$

In practice, an expansion is performed only on the gas and liquid continuity equations. In the basic equations v_g^{n+1} and v_l^{n+1} are calculated from momentum equation as a function of the differential pressure of the last iteration. In the stabilising step the v_g^{n+1} and v_l^{n+1} are kept constant and final value of p and α_g for the new time step are obtained.

Basic Step

From the momentum equation phase velocities, as shown in Appendix J, can be written as:

$$v_g^{n+1} = E_g(p_{j+1}^{n+1} - p_j^{n+1}) + F_g \quad (41)$$

$$v_l^{n+1} = E_l(p_{i+1}^{n+1} - p_i^{n+1}) + F_l \quad (42)$$

- Gas Phase

$$f_g^\diamond(p, \alpha_g) = \left[\frac{(A\alpha_g\rho_g)^{\diamond n+1} - (A\alpha_g\rho_g)^n}{\Delta t} \right]_j + \frac{\left(\left\langle \alpha_g \rho_g A v_g \right\rangle_{j+1/2}^{\diamond n+1} - \left\langle \alpha_g \rho_g A v_g \right\rangle_{j-1/2}^{\diamond n+1} \right)}{\Delta z} \quad (43)$$

$$\frac{\partial f_g^\diamond}{\partial \alpha_g} = \frac{[\rho_g A]_j^{\diamond n+1}}{\Delta t} + \frac{\left(\left\langle \rho_g A v_g \right\rangle_{j+1/2}^{\diamond n+1} - \left\langle \rho_g A v_g \right\rangle_{j-1/2}^{\diamond n+1} \right)}{\Delta z} \quad (44)$$

$$\frac{\partial f_g^\diamond}{\partial p} = \frac{\left[\alpha_g \frac{\Delta \rho_g}{\Delta p} A \right]_j^{\diamond n+1}}{\Delta t} + \frac{\left(\left\langle \alpha_g \rho_g A E_g (p_{j+1} - p_j) \right\rangle_{j+1/2}^{\diamond n+1} - \left\langle \alpha_g \rho_g A E_g (p_j - p_{j-1}) \right\rangle_{j-1/2}^{\diamond n+1} \right)}{\Delta z} \quad (45)$$

- Liquid Phase

$$f_l^\diamond(p, \alpha_g) = \left[\frac{(A(1-\alpha_g)\rho_l)^{\diamond n+1} - (A(1-\alpha_g)\rho_l)^n}{\Delta t} \right]_j + \frac{\left(\left\langle (1-\alpha_g)\rho_l A v_l \right\rangle_{j+1/2}^{\diamond n+1} - \left\langle (1-\alpha_g)\rho_l A v_l \right\rangle_{j-1/2}^{\diamond n+1} \right)}{\Delta z} \quad (46)$$

$$\frac{\partial f_l^\diamond}{\partial \alpha_g} = \frac{[-\rho_l A]_j^{\diamond n+1}}{\Delta t} + \frac{\left(\left\langle -\rho_l A v_l \right\rangle_{j+1/2}^{\diamond n+1} - \left\langle -\rho_l A v_l \right\rangle_{j-1/2}^{\diamond n+1} \right)}{\Delta z} \quad (47)$$

$$\frac{\partial f_l^\diamond}{\partial p} = \frac{\left[(1-\alpha_g) \frac{\Delta \rho_l}{\Delta p} A \right]_j^{\diamond n+1}}{\Delta t} + \frac{\left(\left\langle (1-\alpha_g)\rho_l A E_l (p_{j+1} - p_j) \right\rangle_{j+1/2}^{\diamond n+1} - \left\langle (1-\alpha_g)\rho_l A E_l (p_j - p_{j-1}) \right\rangle_{j-1/2}^{\diamond n+1} \right)}{\Delta z} \quad (48)$$

Stabiliser Step

- Gas Phase

$$f_g(p, \alpha_g) = \left[\frac{(A\alpha_g\rho_g)^{n+1} - (A\alpha_g\rho_g)^n}{\Delta t} \right]_j + \frac{\left(\left\langle \alpha_g \rho_g A v_g \right\rangle_{j+1/2}^{n+1} - \left\langle \alpha_g \rho_g A v_g \right\rangle_{j-1/2}^{n+1} \right)}{\Delta z} \quad (49)$$

$$\frac{\partial f_g}{\partial \alpha_g} = \frac{\left[A\rho_g \right]_j^{n+1}}{\Delta t} + \frac{\left(\left\langle \rho_g A v_g \right\rangle_{j+1/2}^{n+1} - \left\langle \rho_g A v_g \right\rangle_{j-1/2}^{n+1} \right)}{\Delta z} \quad (50)$$

$$\frac{\partial f_g}{\partial p} = \frac{\left[\alpha_g \frac{\Delta \rho_g}{\Delta p} A \right]_j^{n+1}}{\Delta t} + \frac{\left(\left\langle \alpha_g \frac{\Delta \rho_g}{\Delta p} A v_g \right\rangle_{j+1/2}^{n+1} - \left\langle \alpha_g \frac{\Delta \rho_g}{\Delta p} A v_g \right\rangle_{j-1/2}^{n+1} \right)}{\Delta z} \quad (51)$$

- Liquid Phase

$$f_l(p, \alpha_g) = \left[\frac{(A(1-\alpha_g)\rho_l)^{n+1} - (A(1-\alpha_g)\rho_l)^n}{\Delta t} \right]_j + \frac{\left(\left\langle (1-\alpha_g)\rho_l A v_l \right\rangle_{j+1/2}^{n+1} - \left\langle (1-\alpha_g)\rho_l A v_l \right\rangle_{j-1/2}^{n+1} \right)}{\Delta z} \quad (52)$$

$$\frac{\partial f_l}{\partial \alpha_g} = \frac{\left[-A\rho_l \right]_j^{n+1}}{\Delta t} + \frac{\left(\left\langle -\rho_l A v_l \right\rangle_{j+1/2}^{n+1} - \left\langle -\rho_l A v_l \right\rangle_{j-1/2}^{n+1} \right)}{\Delta z} \quad (53)$$

$$\frac{\partial f_l}{\partial p} = \frac{\left[(1-\alpha_g) \frac{\Delta \rho_l}{\Delta p} A \right]_j^{n+1}}{\Delta t} + \frac{\left(\left\langle (1-\alpha_g) \frac{\Delta \rho_l}{\Delta p} A v_l \right\rangle_{j+1/2}^{n+1} - \left\langle (1-\alpha_g) \frac{\Delta \rho_l}{\Delta p} A v_l \right\rangle_{j-1/2}^{n+1} \right)}{\Delta z} \quad (54)$$

or

$$\begin{vmatrix}
 \text{D} & \text{U} & & & & \\
 \text{L} & \text{D} & \text{U} & & & \\
 & \text{L} & \text{D} & \text{U} & & \\
 & & \cdot & \cdot & \cdot & \\
 & & & \cdot & \cdot & \cdot \\
 & & & & \cdot & \cdot \\
 & & & & & \text{L} & \text{D} & \text{U} \\
 & & & & & \text{L} & \text{D} & \text{U} \\
 & & & & & & \text{L} & \text{D}
 \end{vmatrix} \times \begin{vmatrix} \text{Y} \\ \text{Y} \\ \text{Y} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \text{Y} \\ \text{Y} \end{vmatrix} = \begin{vmatrix} \text{R} \\ \text{R} \\ \text{R} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \text{R} \\ \text{R} \end{vmatrix}$$

where,

L, D and U are block sub-matrices on the diagonals of the Jacobian tridiagonal matrix, Y is the solution vector containing the increment values (δp and $\delta \alpha_g$) for each cell and R is the vector containing the independent terms.

3.2.6 Inertial Force Adjustment

The liquid momentum equations contain the inertial force adjustment term, K , which is lumped together with the shear forces. The value of K for the present time step is estimated from the previous time step by the following equation:

$$K = \left(\frac{\left(m_l^n v_l^n \right) - \left(m_l^{n-1} v_l^{n-1} \right)}{\Delta t} + \frac{\left(m_l^n v_l^n \right)_{j+\frac{1}{2}} - \left(m_l^n v_l^n \right)_{j-\frac{1}{2}}}{\Delta z_{j+\frac{1}{2}}} \right) / \left(c_{wl} A \rho_l v_l |v_l| \right). \quad (55)$$

3.2.7 Two-phase Transient Model Flow Chart

The solution strategy used in the code is shown in the Fig. 3.3.

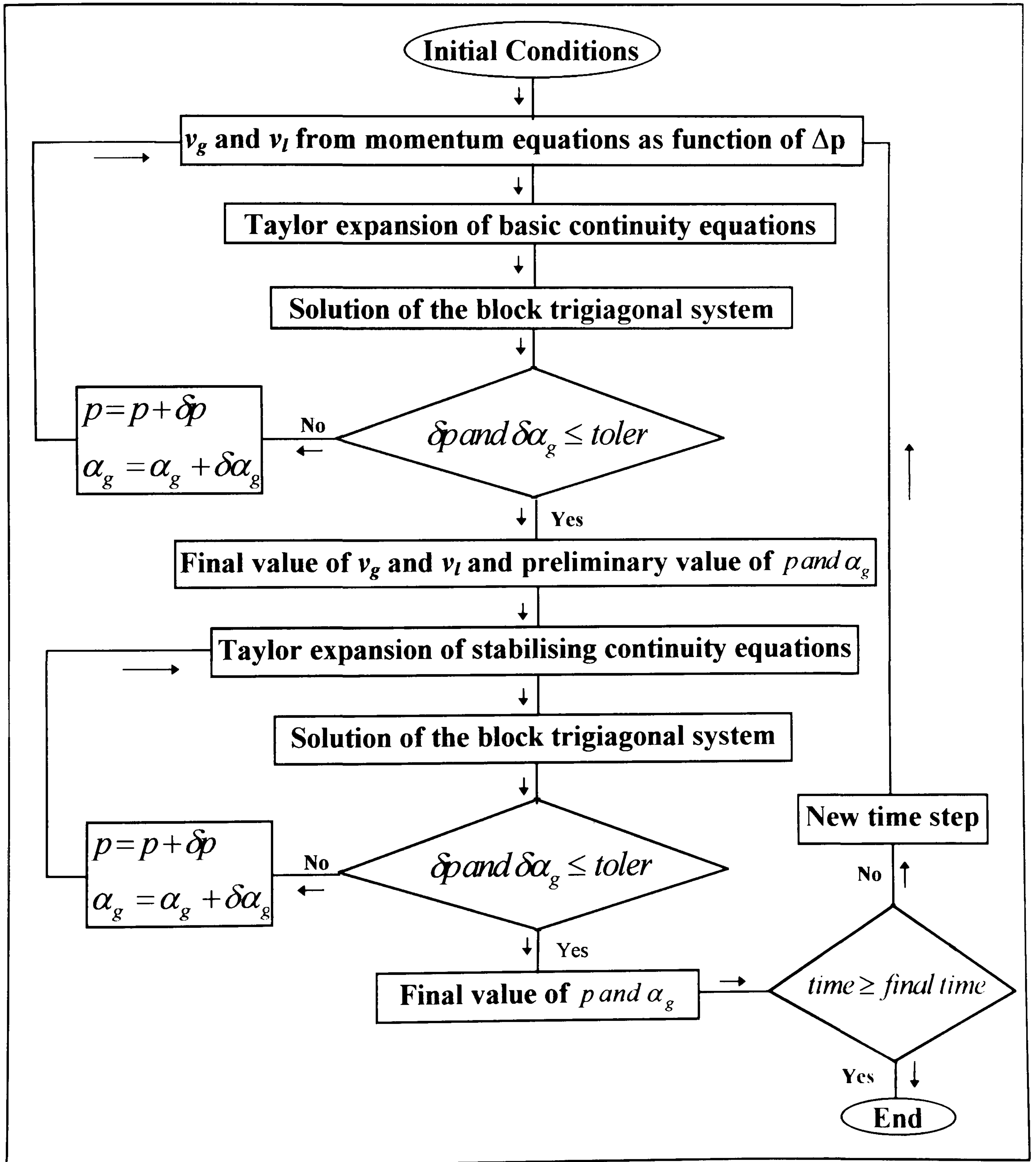


Figure 3.3 - Code flow chart

CHAPTER 4. EXPERIMENTAL FACILITIES

An extensive experimental program has been carried out to acquire two-phase transient flow, gas pump and pigging data on a 67 m long, 0.0525 m diameter, 9.9 m high pipeline-riser system. Fig. 4.1 shows a schematic diagram of the experimental facility. Description of the various components of the system follows.

4.1 Air and Water System

Air was supplied by two compressors C1 and C2. Compressor C1 is a screw compressor rated at $0.0717 \text{ m}^3/\text{s}$ at 8.5 Bara. Compressor C2 is a reciprocating compressor rated at 0.1887 at 19 Bara. The compressors C1 and C2 were connected to the buffer vessel V2. Upstream from the mixing section the compressed air can pass through two turbine meters: FG1 and/or FG2. Beside these flowmeters a thermocouple was wired into the data acquisition system to provide the temperature signal.

Water was pumped from a storage tank by pump P1. Pump P1 is a progressive cavity pump with $0.00972 \text{ m}^3/\text{s}$ capacity and its maximum discharge pressure is 7 Bara. The water phase can be measured by two magnetic flowmeters: FL1 and/or FL2. The electrical signal from the turbine meter was sent to the analogue-to-digital (A/D) converter of the data acquisition system.

Before the test section air and water are mixed through a mixing section.

After the test section, the mixture flows into a separator S1 at the top of the riser. At the upper part of the top separator there is a gas line and at the lower part there is a liquid line. These two lines merge into a two-phase line that connects the separator S1 to the big horizontal separator S2 on the floor. The differential pressure signal from the upper to the lower part of the separator S1 was sent to the level control valve LCV1 installed in the liquid line. A turbine flowmeter FG3, a pressure transducer and a thermocouple was installed in the gas line. At the top of the floor separator S2 there was a line to permit venting of air to the atmosphere through a back pressure valve and there was also a liquid level control valve LCV2. The separated liquid was sent from S2 to water coalescer vessel WC1 and then to its storage tank WT1. The main specifications of the equipment and vessels are shown in Table 4.1 and 4.2.

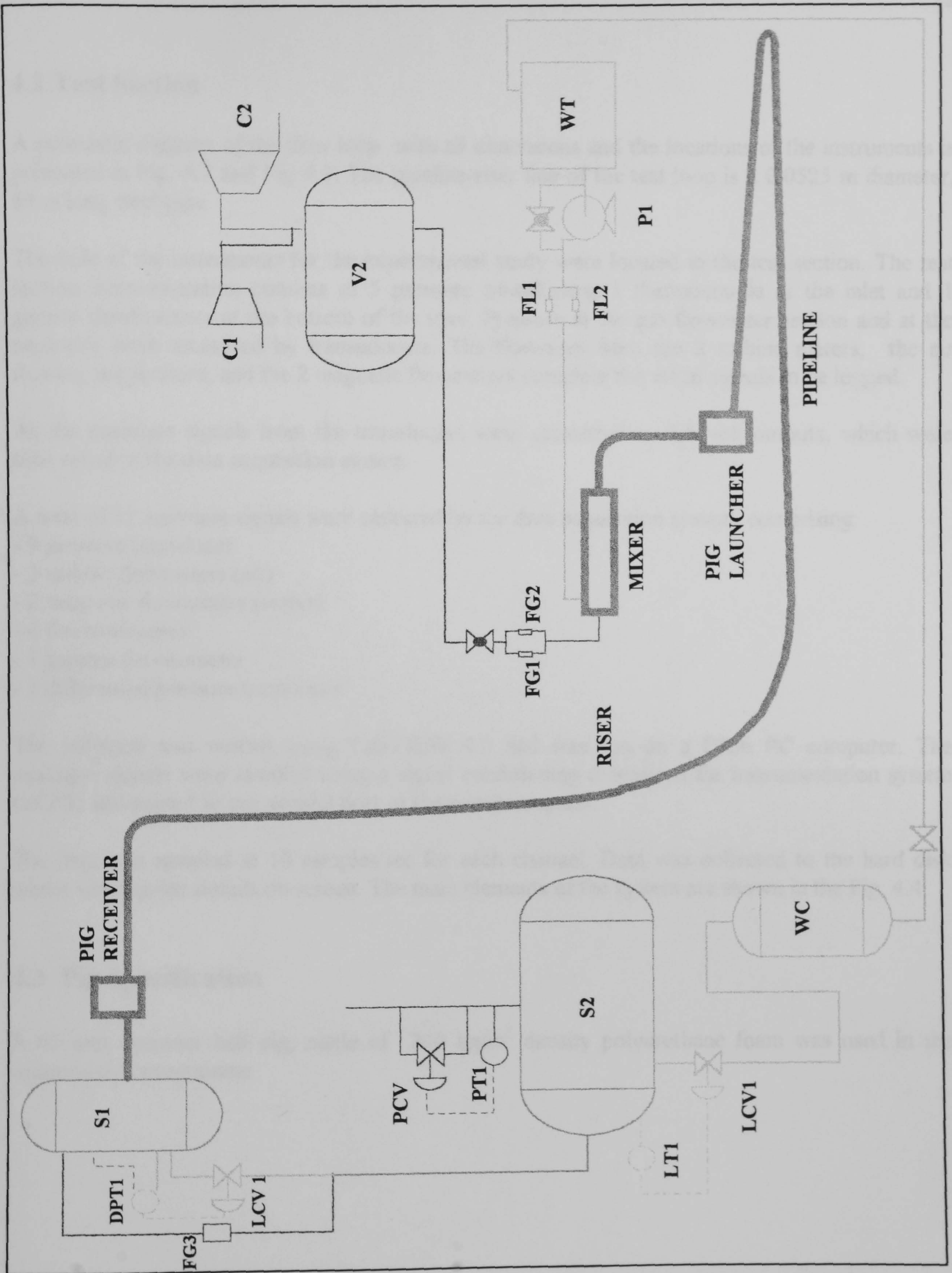


Figure 4.1 - Schematic diagram of the experimental facility

4.2 Test Section

A schematic diagram of the flow loop with all dimensions and the locations of the instruments is presented in Fig. 4.2 and Fig 4.3. The pipeline-riser line of the test loop is a 0.0525 m diameter, 69 m long steel pipe.

The bulk of the instruments for the experimental study were located in the test section. The test section instrumentation consists of 5 pressure transducers, 1 thermocouple at the inlet and 1 gamma densitometer at the bottom of the riser. Pressure at the gas flowmeter section and at the separator were measured by transducers. The flowrates from the 2 turbine meters, the air flowing temperature, and the 2 magnetic flowmeters complete the set of signals to be logged.

All the analogue signals from the transducers were converted to 0-5 volt outputs, which were then wired to the data acquisition system.

A total of 21 analogue signals were captured by the data acquisition system, comprising:

- 9 pressure transducer
- 3 turbine flowmeters (air)
- 2 magnetic flowmeters (water)
- 4 thermocouples
- 1 gamma densitometer
- 1 differential pressure transducer

The software was written using LabVIEW 4.0 and was run on a P166 PC computer. The analogue signals were sampled using a signal conditioning extensions for instrumentation system (SCXI) and passed to the parallel port of the microcomputer.

The data was sampled at 10 samples/sec for each channel. Data was collected to the hard disk whilst viewing the signals on screen. The main elements of the system are shown in the Fig. 4.4.

4.3 Pig Specification

A 60 mm diameter ball pig, made of 240 kg/m³ density polyurethane foam was used in the experimental programme.

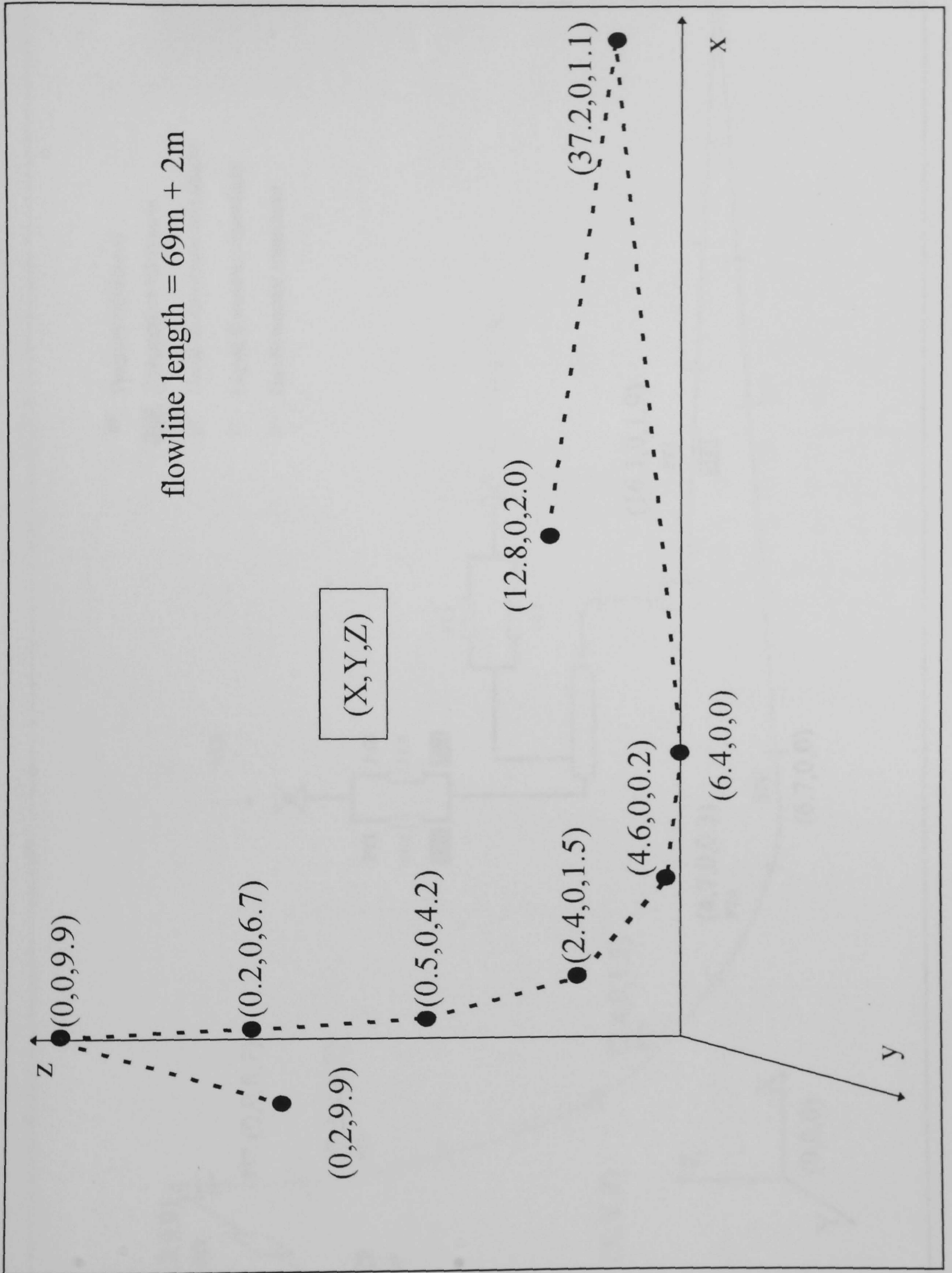


Figure 4.2 - Test section dimensions

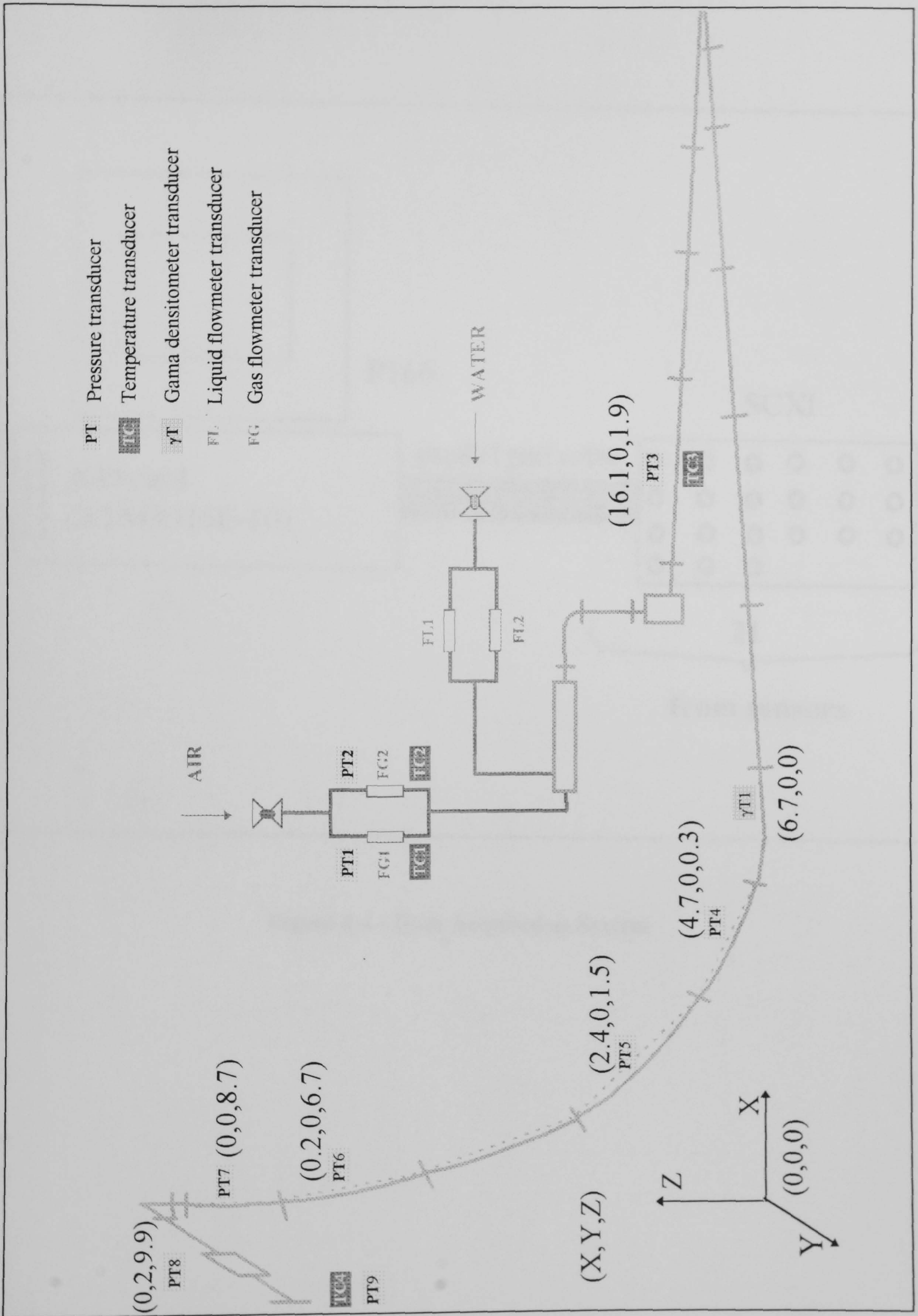


Figure 4.3 - Test section position of the instruments

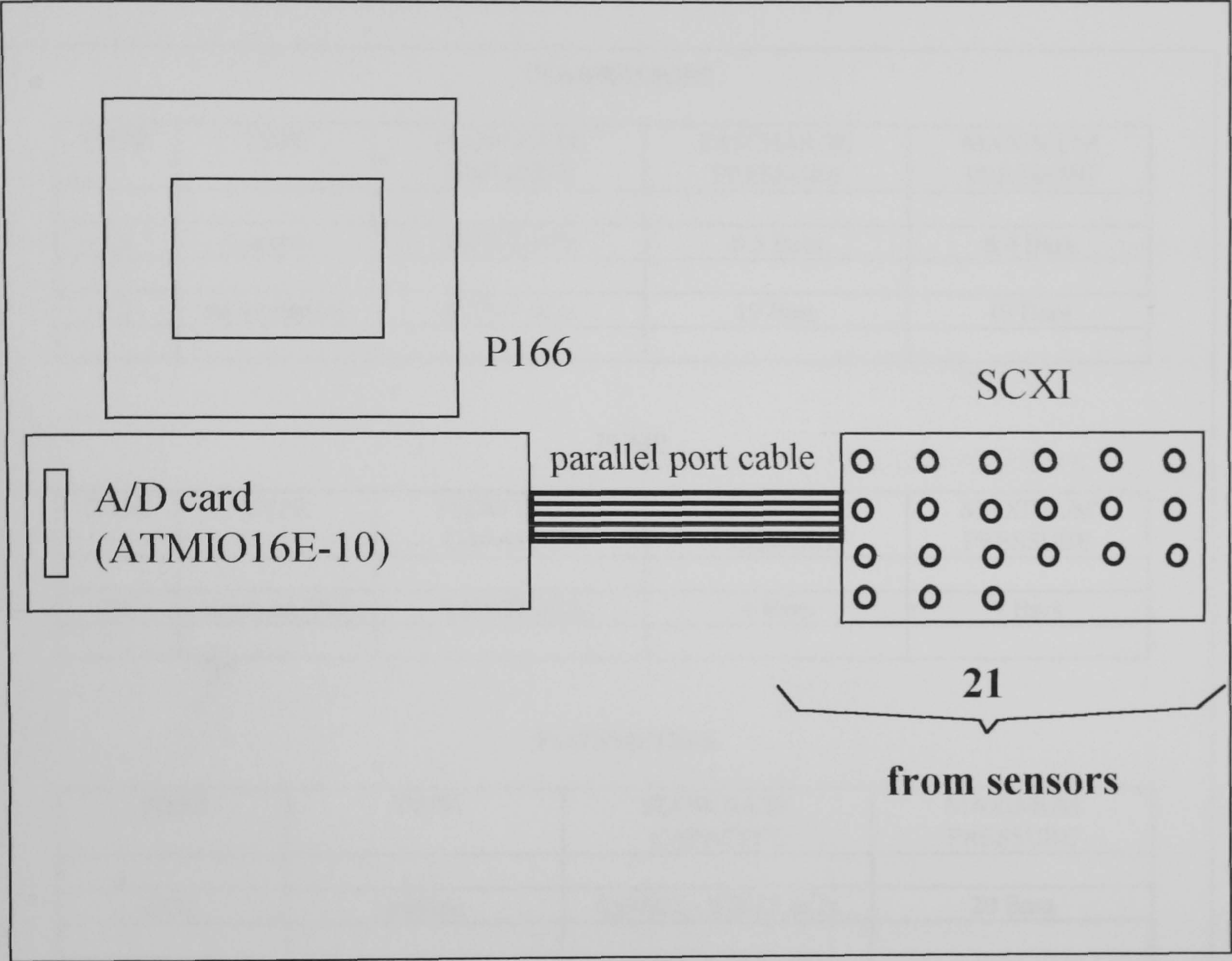


Figure 4.4 - Data Acquisition System

Table 4.1 - Equipment

COMPRESSORS

ITEM	TYPE	FLOW RATE CAPACITY	DISCHARGE PRESSURE	MAXIMUM PRESSURE
C1	screw	0.0717 m ³ /s	8.5 Bara	8.5 Bara
C2	reciprocating	0.1887 m ³ /s	19 Bara	19 Bara

PUMP

ITEM	TYPE	FLOW RATE CAPACITY	DISCHARGE PRESSURE	MAXIMUM PRESSURE
P1	moyneau (PD)	0.00972 m ³ /s	4 Bara	7 Bara

FLOWMETERS

ITEM	TYPE	FLOW RATE CAPACITY	MAXIMUM PRESSURE
FG1	turbine	0.00022 - 0.0022 m ³ /s	20 Bara
FG2	turbine	0.0025 - 0.025 m ³ /s	25 Bara
FG3	turbine	0.00139 - 0.0167 m ³ /s	25 Bara
FL1	magnetic	0.00011 - 0.00167 m ³ /s	12 Bara
FL2	magnetic	0.00083 - 0.0125 m ³ /s	12 Bara

Table 4.2 - Vessels

TWO-PHASE SEPARATORS				
ITEM	POSITION	DIAMETER (m)	VOLUME (m ³)	MAXIMUM PRESSURE (Bara)
S1	UPSTAIRS/ VERTICAL	0.5	0.33	25
S2	FLOOR/ HORIZONTAL	1.5	12	25

BUFFER VESSELS				
ITEM	POSITION	DIAMETER (m)	VOLUME (m ³)	MAXIMUM PRESSURE (Bara)
V2	FLOOR/ HORIZONTAL	1.7	2.76	26.5

WATER SYSTEM					
ITEM	TYPE	MAXIMUM PRESSURE (Bar)	DIMENSIONS		
			DIAMETER	HEIGHT	
WC1	COELESCE VESSEL	25	0.915	3.277	
			HEIGHT	WIDTH	LENGTH
WT1	STORAGE TANK (10 m ³)	atm	1.68	2.2	2.75

CHAPTER 5. PIGGING MODEL

To simulate the pig motion in the pipe, the pipeline is divided into two sections as shown is Fig. 5.1. The first section is from the inlet to the pig, and the second from the pig to the outlet of the pipeline. The velocity of the pig is given by the velocity of the mixture pushing the pig in the previous time step. The pig is assumed to run one cell in each pig time step. As the mass of the cell in front of the pig is known and also the time to run it, we know the mass flow rate entering the section in front of the pig. This mass flow rate will be the boundary condition for the transient calculation from the pig to the outlet. This transient calculation gives the pressure in the pig which is the boundary condition for the transient calculation from the inlet to the pig. Thus the transient calculation throughout the pipeline in this pig time step is finished. This procedure is repeated until the pig reaches the end of the pipeline. After that the normal transient calculation can be done. The pigging model described above makes it unnecessary to track the position of the slug front. The pressure drop across the pig is neglected.

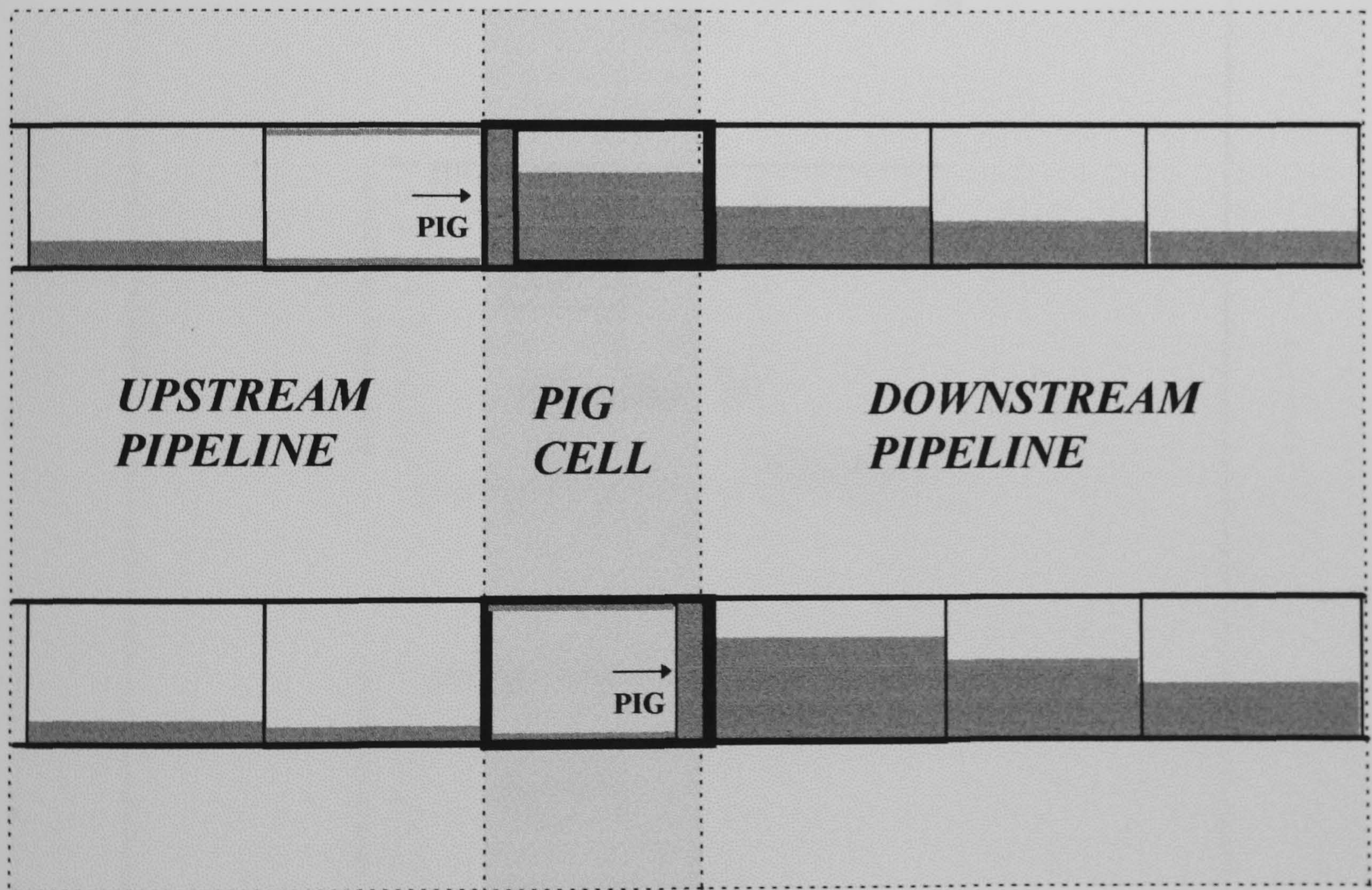


Figure 5.1 - Pigging model

5.1 Pig By-pass

The pig by-pass introduced in the pigging model comes from the runs of gas pump with pig. In these runs the test section is filled with water and followed by pig and gas injection in the inlet. Experimental results, shown in Appendix C, indicate that there is gas by-pass in the same direction of pig motion and that liquid is left behind the pig. The gas by-pass was evaluated from the gamma densitometer signals shown in Fig. 5.2.

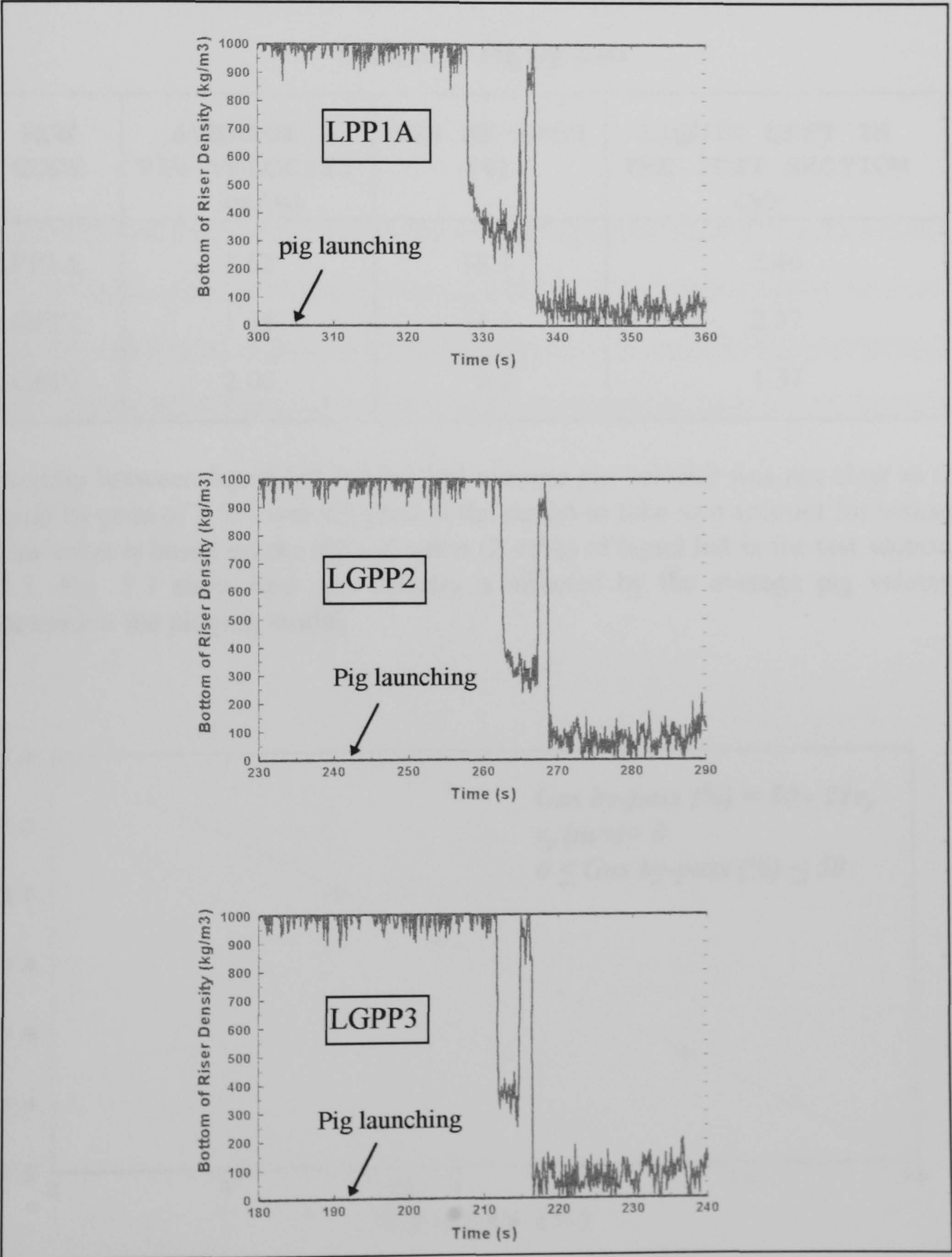


Figure 5.2 - Density at the bottom of the riser

The gas by-pass is evaluated by comparing the gas mass in front of the pig to the total mass of gas injected up to the time when the pig reach the gamma densitometer.

The liquid by-pass (liquid left behind the pig) is evaluated by draining the system after the pig arrival at the receiver and comparing the measured liquid to the total volume of the test section.

Table 5.1 presents the gas by-pass and the liquid in the test section for the gas pump runs with pig as a function of the average pig velocity from the inlet to the densitometer.

Table 5.1 - Pig By-pass

RUN CODE	AVERAGE PIG VELOCITY (m/s)	GAS BY-PASS (%)	LIQUID LEFT IN THE TEST SECTION (%)
LPP1A	1.52	18.3	2.46
LGPP2	1.85	11.1	2.37
LGPP3	2.00	8.2	1.37

The relationship between liquid left behind and average pig velocity was not clear as the gas by-pass. A liquid by-pass of 2.5% was adopted in the model to take into account for leakage through the pig. This value is based on the highest value (2.46%) of liquid left in the test section as shown in table 5.1. Fig. 5.3 show how gas by-pass is affected by the average pig velocity and the correlation used in the pigging model.

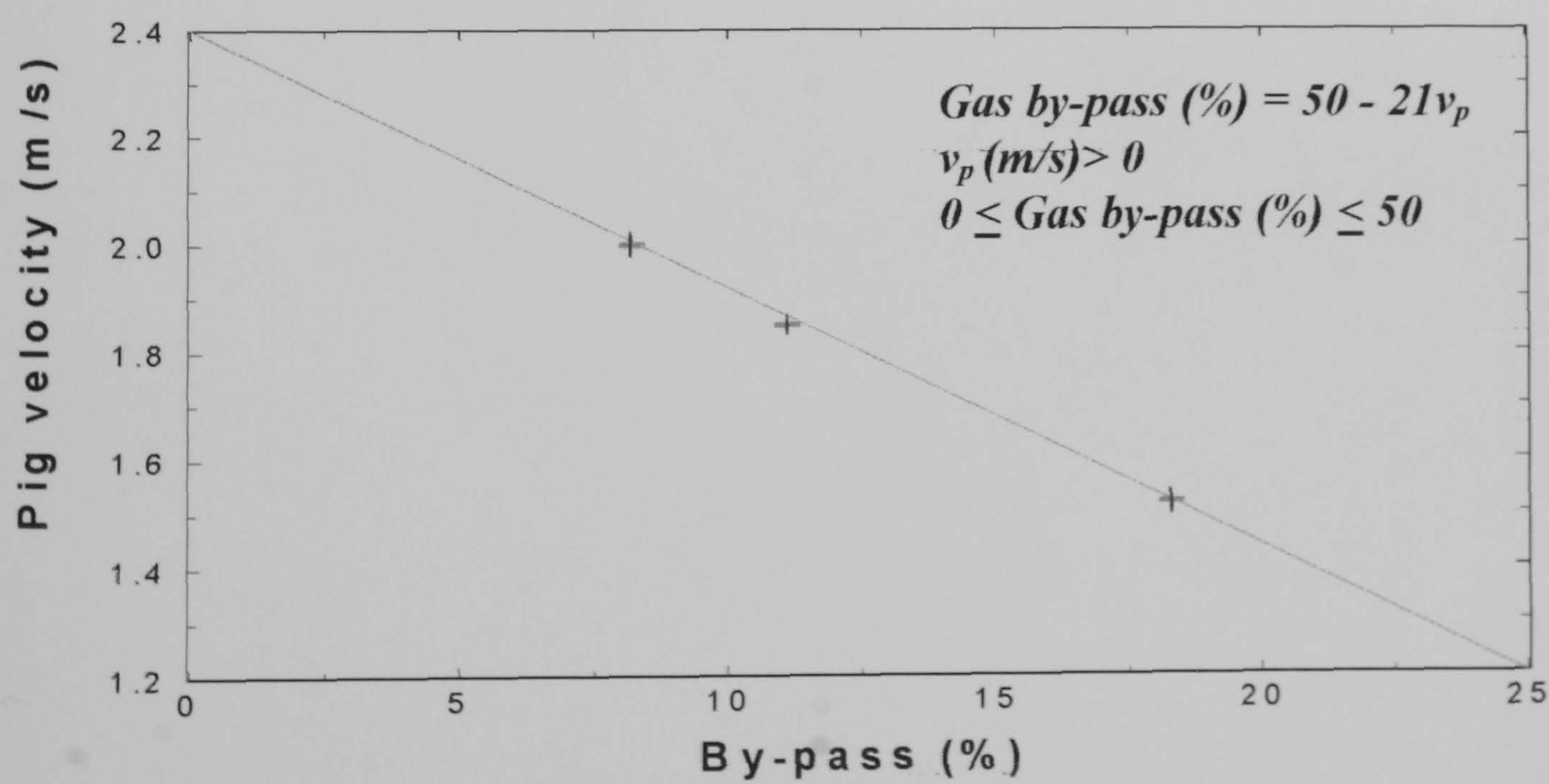


Figure 5.3 - Pig By-pass

5.2 Pigging Model with By-pass

The velocity of the pig is given by the corrected velocity of the mixture pushing the pig. This velocity is calculated considering the flow rate reduction due to the gas by-pass (Fig. 5.3) and the reduction of the area (2.5%) due to liquid left behind the pig as a film. The pig is assumed to run one cell in each pig time step. The mass of gas in the cell in front of pig should be added by the by-pass mass of gas. The mass of liquid should be reduced by the mass of liquid left behind the pig. As the mass of the cell in front of the pig is known and also the time to run it, we know the mass flow rate entering the section in front of the pig. This mass flow rate will be the boundary condition for the transient calculation from the pig to the outlet. This transient calculation gives the pressure at the outlet of the pig cell. The initial condition of the pig cell is assumed to be annular steady flow due to the film left behind the pig. This steady state calculation gives the pressure at the inlet of the pig cell which will be the boundary condition for the transient calculation from the inlet of the pipeline to the inlet of the pig cell. The transient model calculation in the pig cell gives a new inlet pressure which will be the new boundary condition for the transient calculation from the inlet of the pipeline to the inlet of the pig cell. This process is repeated until pressure convergence is obtained. Thus the transient calculation throughout the entire pipeline in this pig time step is finished. This procedure is repeated until the pig reaches the end of the pipeline. Fig.5.4 shows the pigging model flow chart.

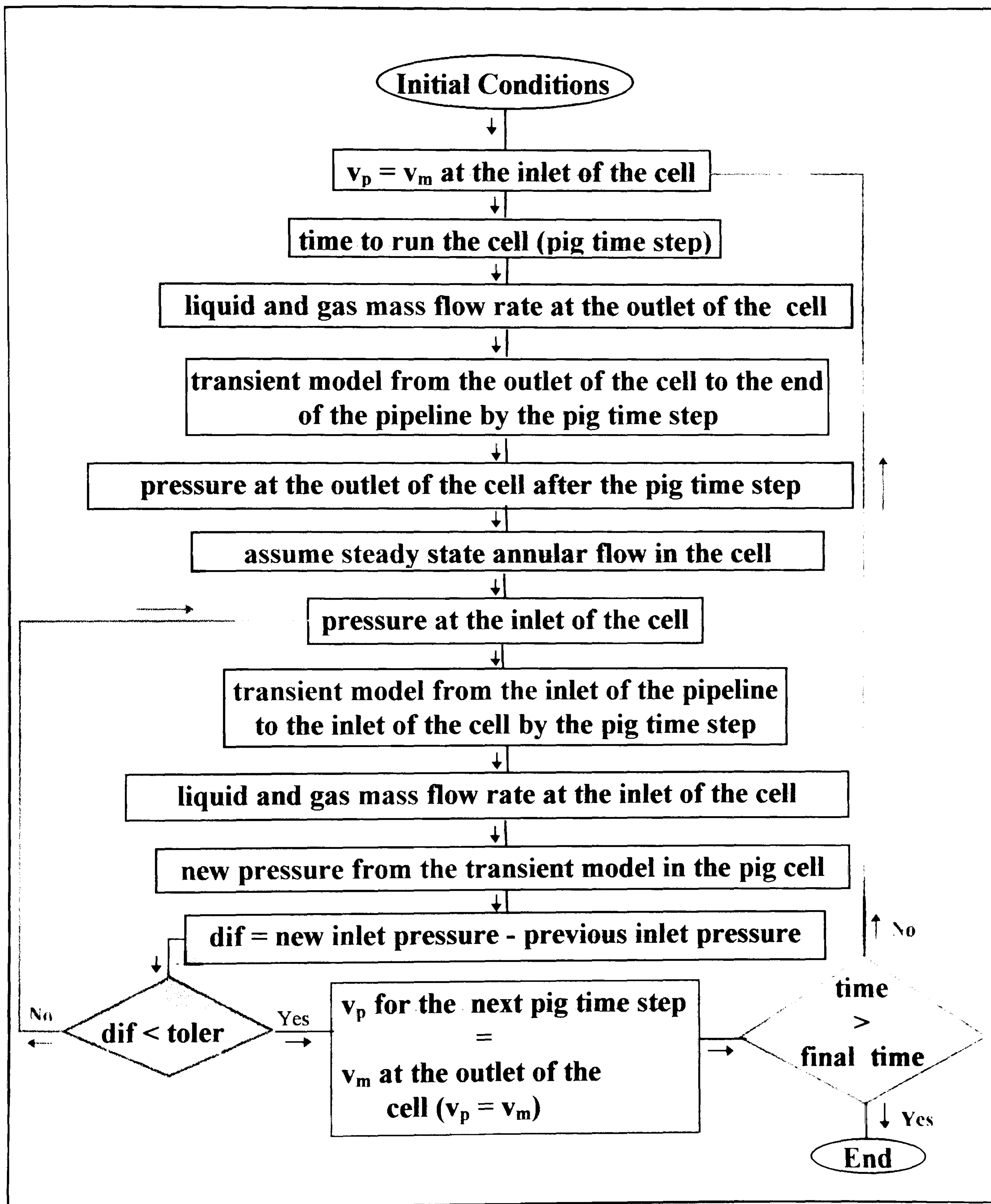


Figure 5.4 - Pigging model flow chart

CHAPTER 6. RESULTS AND DISCUSSIONS

In this chapter the transient data collected in the experimental programme are compared with the model predictions. Several types of transient flow behaviour are observed for the different runs:

1. Conventional Pigging: a pig is launched from the inlet of the test section after the establishment of a steady-state gas-liquid flow.
2. Gas Pump without pig: the test section is filled with water and followed by injection of gas from the inlet.
3. Gas Pump with pig: the test section is filled with water and followed by pig and gas injection from the inlet.
4. Transient Flow: change of boundary conditions such as inlet gas flow rate, inlet liquid flow rate and/or outlet pressure.

A total of 32 experimental runs were carried out. A summary of the experimental results of each run can be found in Appendixes A, B, C and D. The Appendixes E, F, G and H show the model results of these runs.

The transient flow parameters of interest to the pipeline and oil industry, such as pressure, liquid hold-up, and flow rates are analysed and discussed based on the experimental data and the prediction of the model. In some tests the gas flow rate at the outlet of the separator is greater than the maximum value measured by the flowmeter ($0.0167 \text{ m}^3/\text{s}$). A change of 1m in the water level of the separator corresponds to 5 mV.

The use of a fast computer data acquisition system produced experimental transient data sets of unique characteristics. Very detailed information of the flow structure can be determined from these data sets. This includes, for instance, flow pattern, slug frequency, slug hold-up, film height and pig velocity. The sampling rate of the data sets are 10 samples per second per each channel.

6.1 Conventional Pigging

Table 6.1 presents the experimental conditions and comparison between the experimental data and the model prediction.

A total of 13 experimental conventional pigging runs are carried out. A summary of the experimental results of each run can be found in Appendix A. The Appendix E shows the model results of these runs. Once the pig is launched the separator pressure tends to decrease because the pig starts to block the gas and to generate the liquid slug in its front. The inlet pressure does not change too much while the liquid slug front is in the near horizontal pipe. As soon the liquid slug front reaches the riser the inlet pressure starts to increase due to the head exerted by the liquid in the riser. This period of gas accumulation means low pig velocity. When the back of the liquid slug reaches the riser, the head reduces dramatically due to gas phase in the riser. At this point the pig accelerates and high flow rate of liquid is observed in the separator followed by high flow rate of gas, causing a sharp increase and decrease of the separator pressure. After some time the system reaches a stable condition again.

Table 6.1 - Conventional pigging experimental data and model results

RUN CODE	SUP. GAS VELOCITY (m/s)	SUP. LIQ. VELOCITY (m/s)	INITIAL INLET PRESSURE		PIG TRAVEL TIME	
			Exp. (Bara)	Model (Bara)	Exp. (s)	Model (s)
LPG3W1	2.6	0.16	2.3	2.3	48	40.6
LPG4W1	6.2	0.16	2.4	2.6	25	26.6
LPG3W2	2.2	0.48	2.7	2.5	42	34.0
LPG4W2	5.9	0.45	3.2	2.9	18	17.0
LPG3W3	1.7	0.81	2.8	2.8	39	33.0
LPG4W3	4.0	0.77	3.1	3.2	23	21.0
HPG2W1	0.8	0.27	4.7	4.6	79	78.0
HGP3W1	2.8	0.16	5.6	5.7	34	32.6
HPG4W1	5.9	0.24	5.8	6.0	16	18.0
HPG4W2	5.4	0.34	6.2	6.2	18	20.4
HPG2W3	0.5	0.99	6.2	6.1	40	44.0
HPG3W3	2.5	0.94	6.3	6.3	27	25.3
HPG4W3	6.4	0.67	6.9	6.8	14	13.6

Fig. 6.1 and Fig. 6.2 show the predicted pig travel time and the initial inlet pressure respectively. In these diagrams no special trends of the error can be detected. Good agreement is observed.

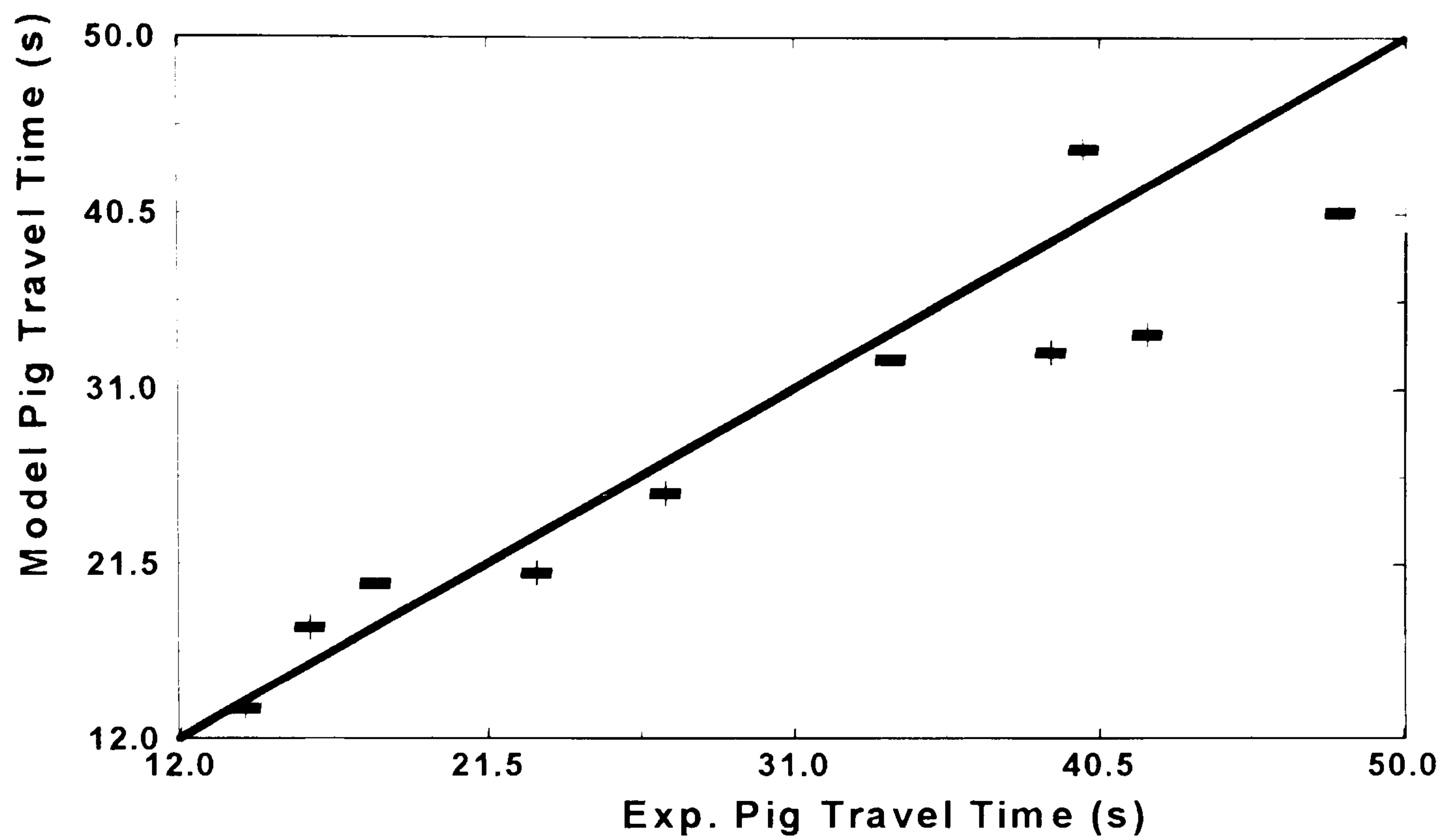


Figure 6.1 - Measured and predicted time between pig launching and receiving

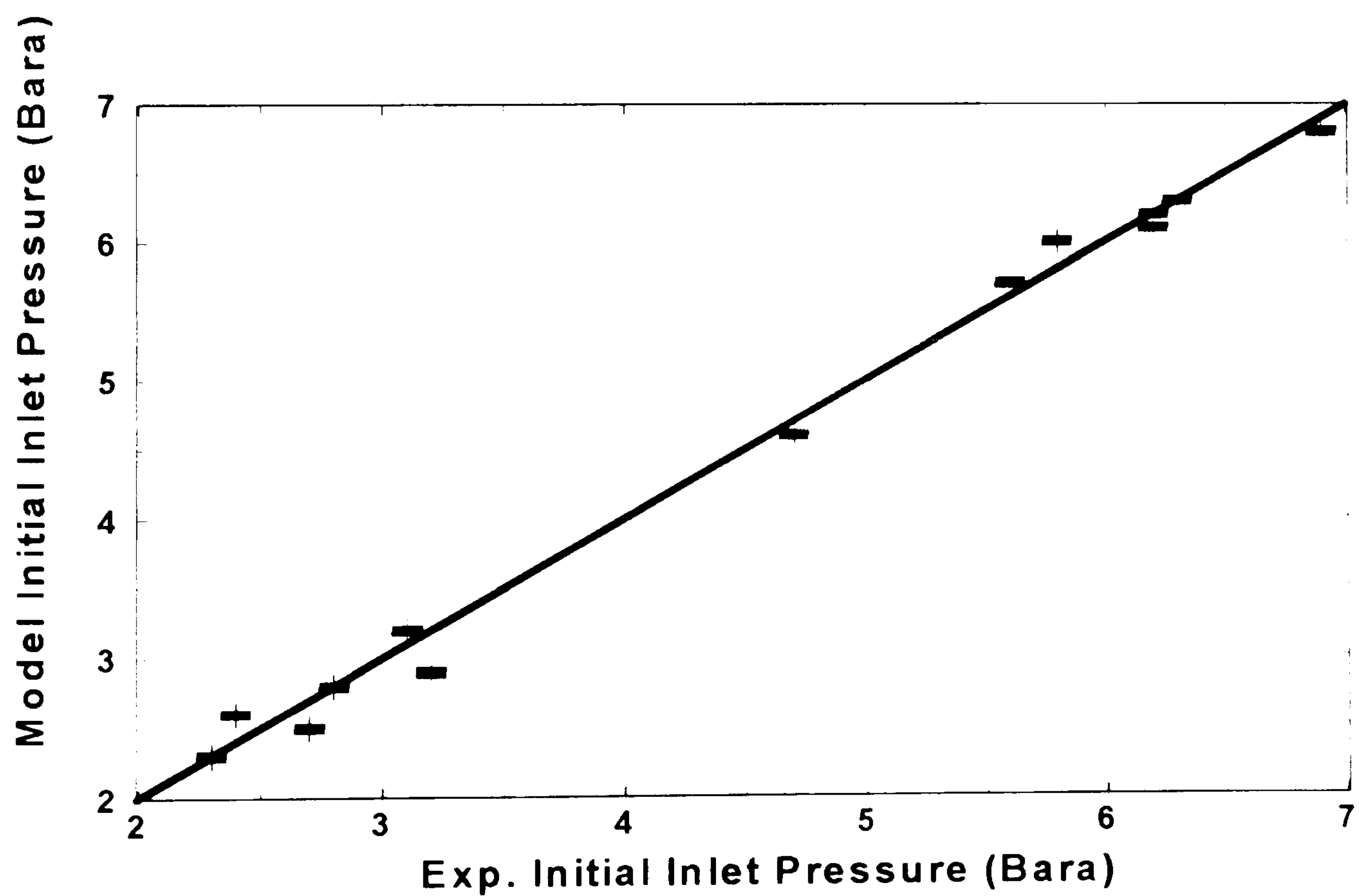


Figure 6.2 - Measured and predicted initial inlet pressure

Experimental runs LPG4W1, LPG3W3, HPG4W1, and HPG3W3 are used to analyse conventional pigging case.

Figures 6.3, 6.5, 6.7 and 6.9 show the results of the model simulation of bottom riser density along with the experimental measurements. All these figures show a high hold-up for a certain time. This time is proportional to the slug length in front of the pig. After the pig, a very low liquid hold-up is observed. This means that the pig is mainly pushed by gas. After certain time, liquid start flowing again at the bottom of the riser.

Figures 6.4, 6.6, 6.8 and 6.10 show the results of the model simulation of inlet pressure along with the experimental measurements. All these figures show an increase of the inlet pressure followed by a sharp decrease. After some time the pressure stabilises.

6.1.1 Run LPG4W1

The initial and final flow conditions are in stratified flow pattern. The initial inlet gas and liquid superficial velocities are 6.2 m/s and 0.16 m/s, respectively. The inlet pressure is 2.4 Bara. The pig is launched at 458 s. Fig. 6.3 and Fig. 6.4 show the transient behaviour of the bottom riser density and the inlet pressure during and after the pigging run.

The predicted steady state inlet pressure is higher than observed. The measured density at the riser bottom is higher than predicted. The time and the maximum value of inlet pressure are very well predicted. The model predicts a faster recovery of pressure, but the time interval of the low pressure after the arrival of pig is well predicted. The time interval of the liquid front to reach the gamma densitometer after the passage of the pig is also well predicted. The measured slug length is smaller than predicted, however the total volume of liquid in the slug shows good agreement. The predictions of the model can be considered good.

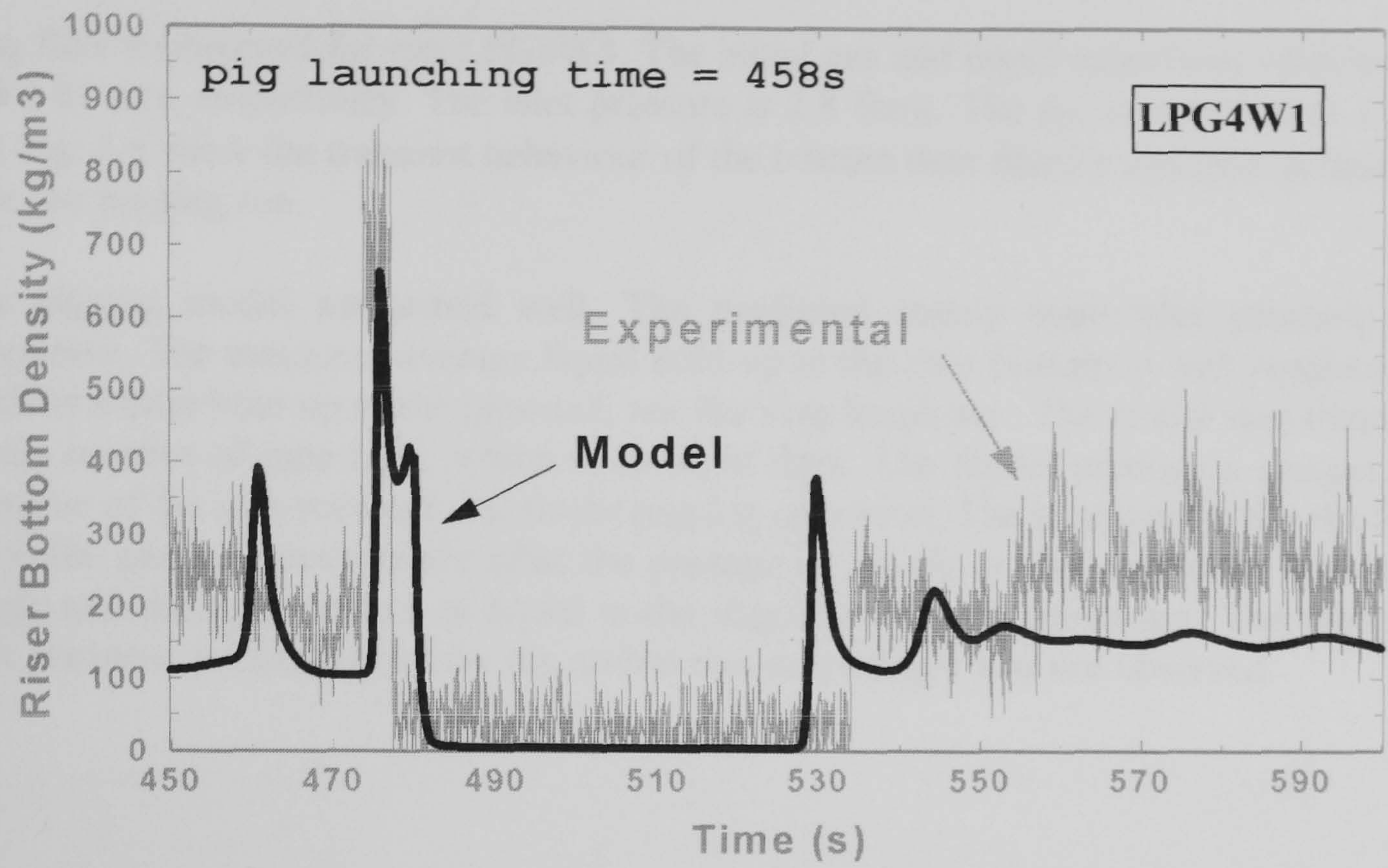


Figure 6.3 - Measured values and model prediction for the riser bottom density with time

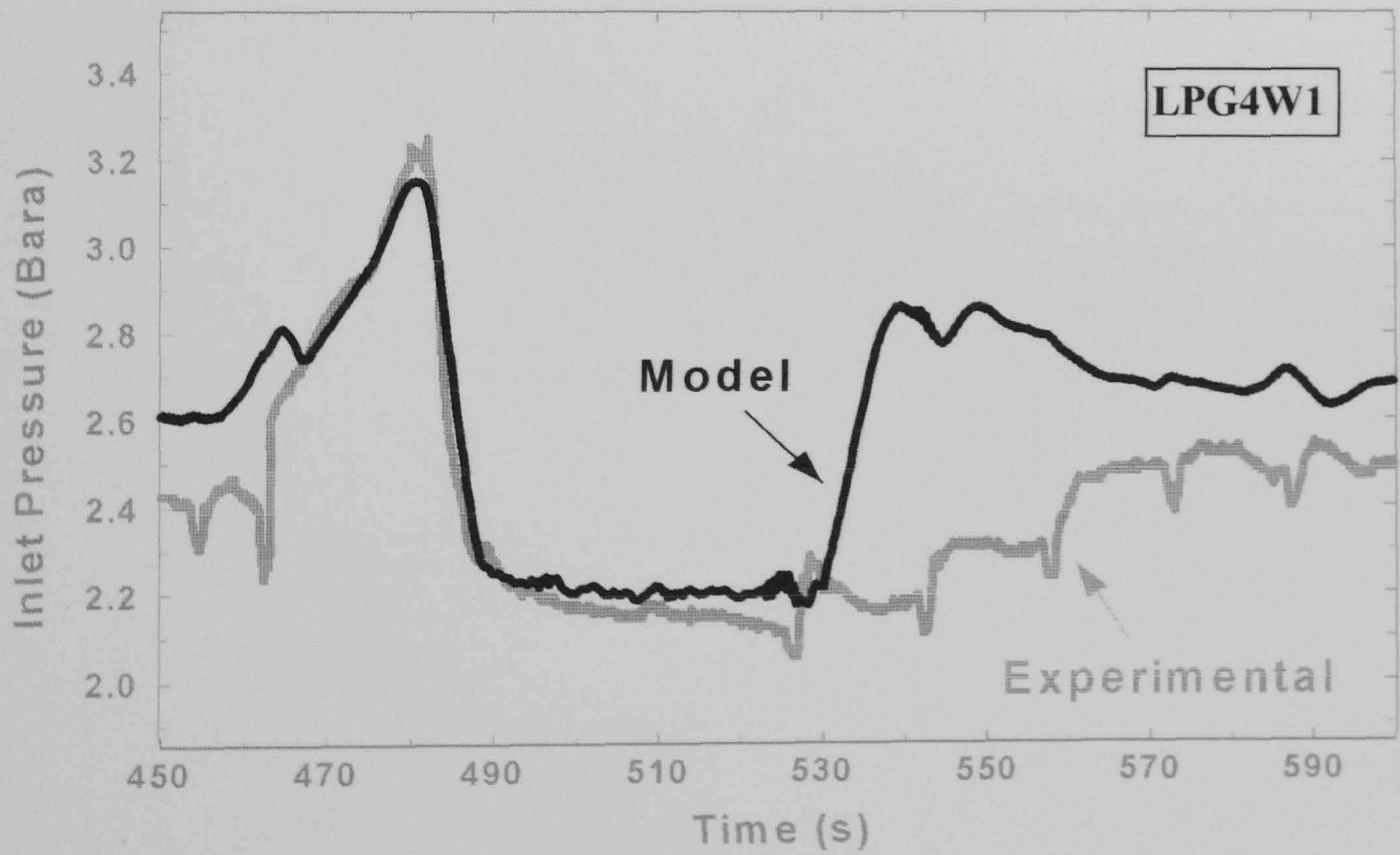


Figure 6.4 - Measured values and model prediction for the inlet pressure with time

6.1.2 Run LPG3W3

Slug flow is observed for run LPG3W3. The initial gas and liquid superficial velocities are 1.7 m/s and 0.81 m/s, respectively. The inlet pressure is 2.8 Bara. The pig is launched at 193s. Fig. 6.5 and Fig. 6.6 show the transient behaviour of the bottom riser density and inlet pressure during and after the pigging run.

The pigging model performed well. The predicted steady state inlet pressure shows good agreement. The measured average liquid hold-up at the riser bottom is well predicted. The model predicts a total hold-up of the pipe cell, not the slug frequency. The model slug frequency is given by the number of pipe cells, which is an input data. The model predicts a sharper increase and decrease of the inlet pressure due to the pigging operation. The time interval for the liquid front to reach the gamma densitometer after the passage of the pig is well predicted. The measured slug length and the total volume of liquid in the slug shows good agreement. The model predicts an inlet pressure reduction after the pig arrival that surprisingly was not observed.

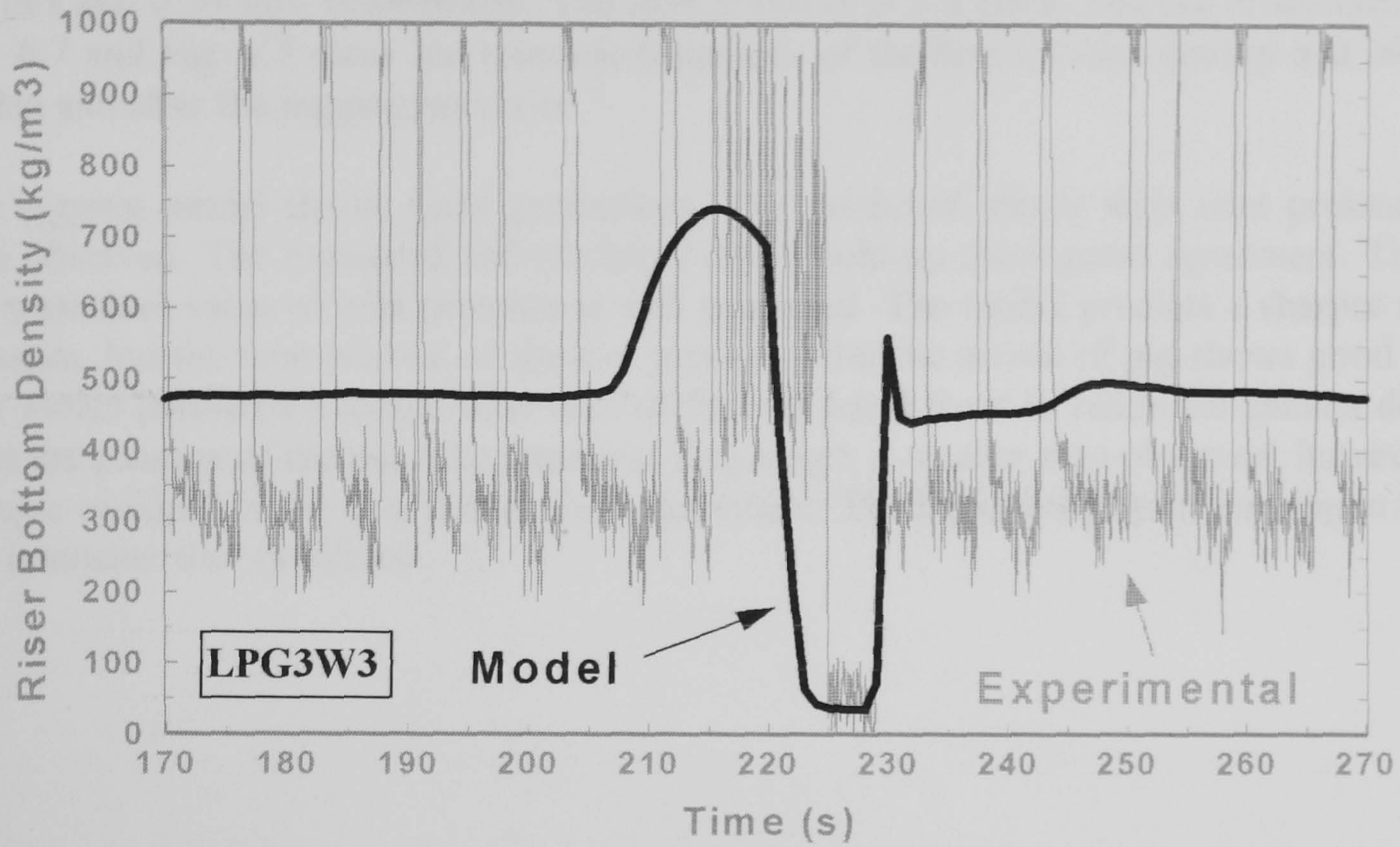


Figure 6.5 - Measured values and model prediction for the riser bottom density with time

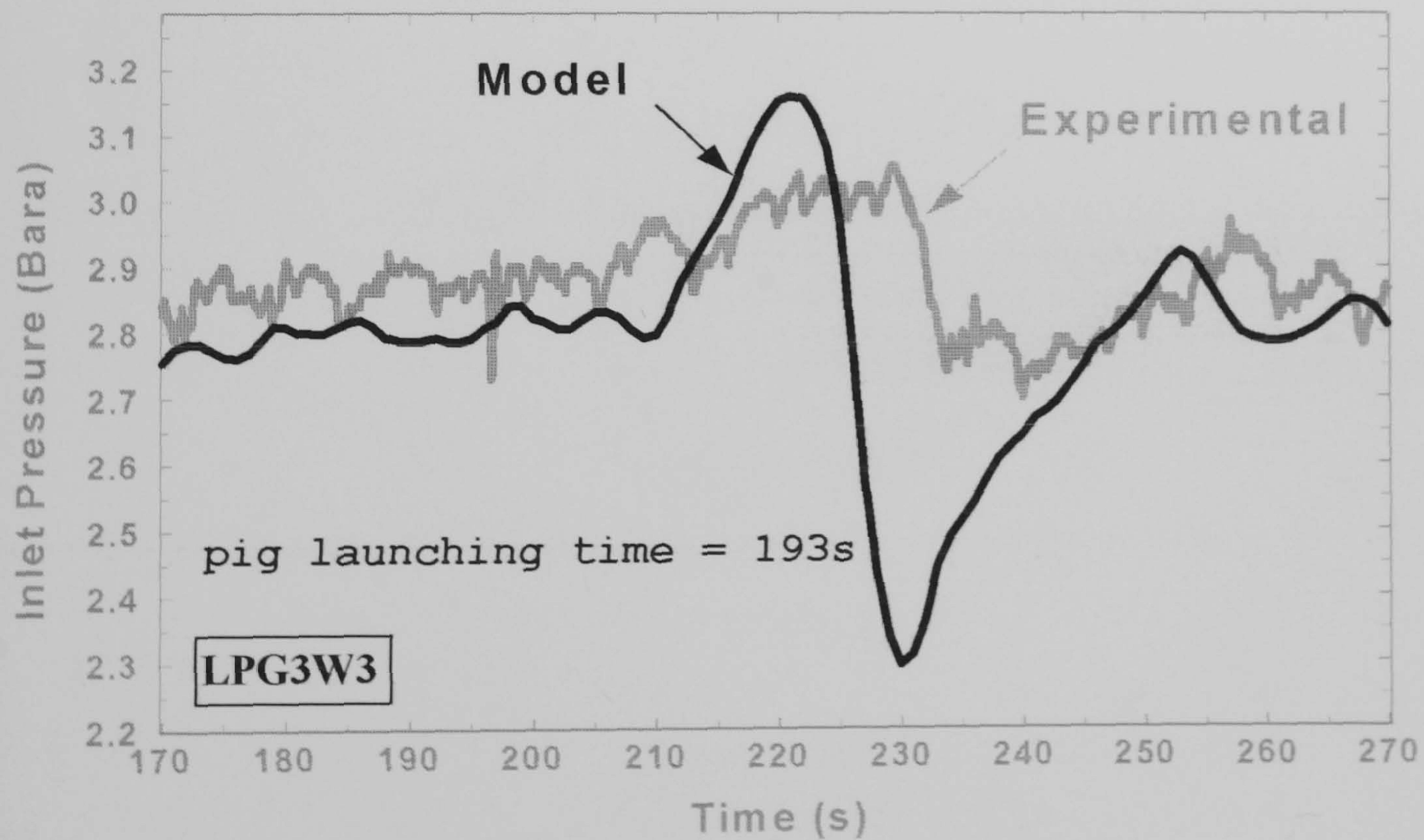


Figure 6.6 - Measured values and model prediction for the inlet pressure with time

6.1.3 Run HPG4W1

Stratified flow is observed for run HPG4W1. The initial gas and liquid superficial velocities are 5.9 m/s and 0.24 m/s, respectively. The inlet pressure is 5.8 Bara. The pig is launched at 118 s. Fig. 6.7 and Fig. 6.7 show the transient behaviour of the bottom riser density and inlet pressure during and after the pigging operation.

The pigging model shows good predictions. The predicted steady state inlet pressure is higher than observed. The measured and predicted liquid hold-up show good agreement. The time and the maximum value of inlet pressure is well predicted. The model predicts a sharper recovery of pressure, but the time interval of the low pressure after the arrival of pig shows good agreement. The model predicted a longer time interval for the liquid front to reach the gamma densitometer after the passage of the pig. The measured slug length is smaller than observed, however the total volume of liquid in the slug shows good agreement. The liquid hold-up of the slug in front of the pig is smaller than predicted.

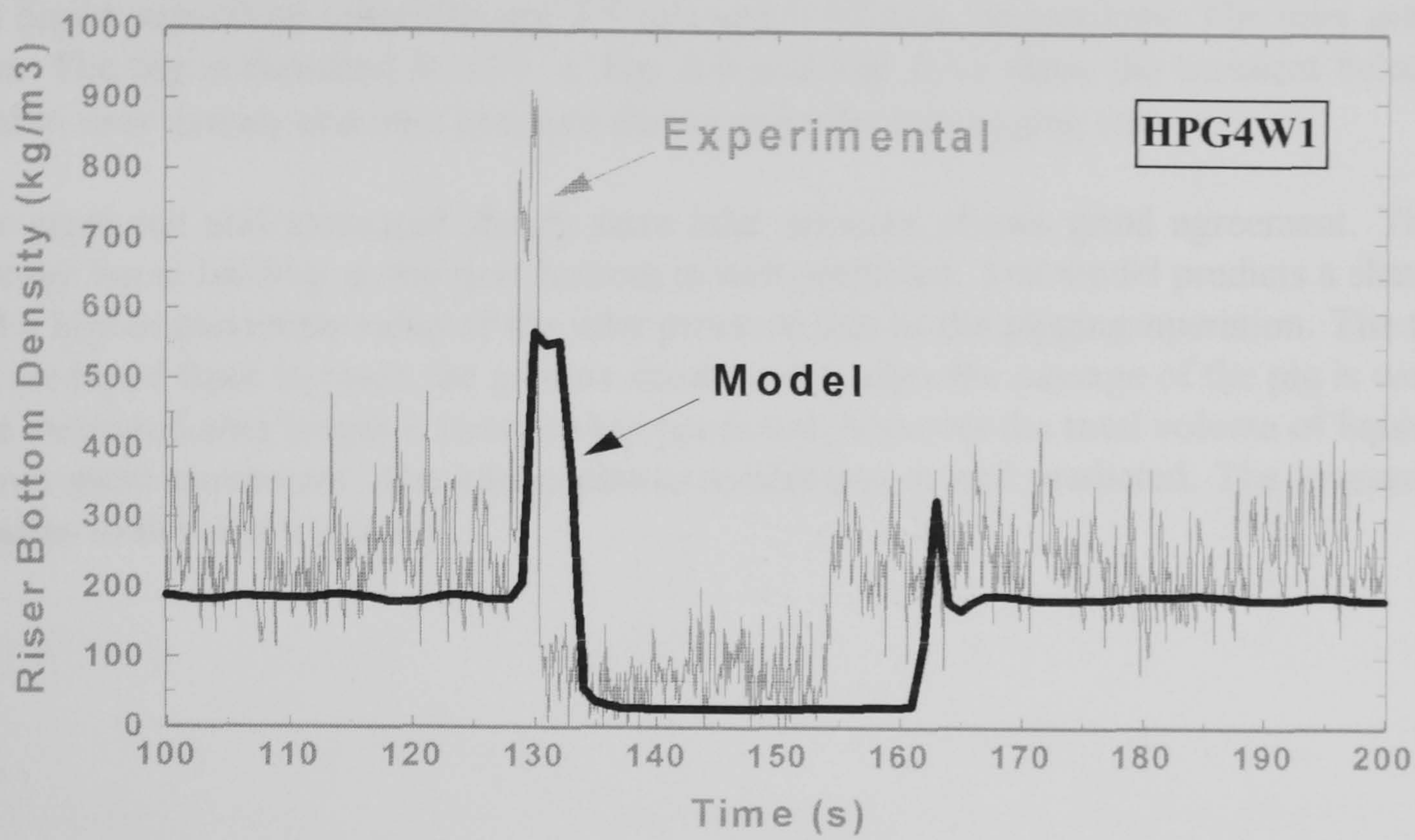


Figure 6.7 - Measured values and model prediction for the riser bottom density with time

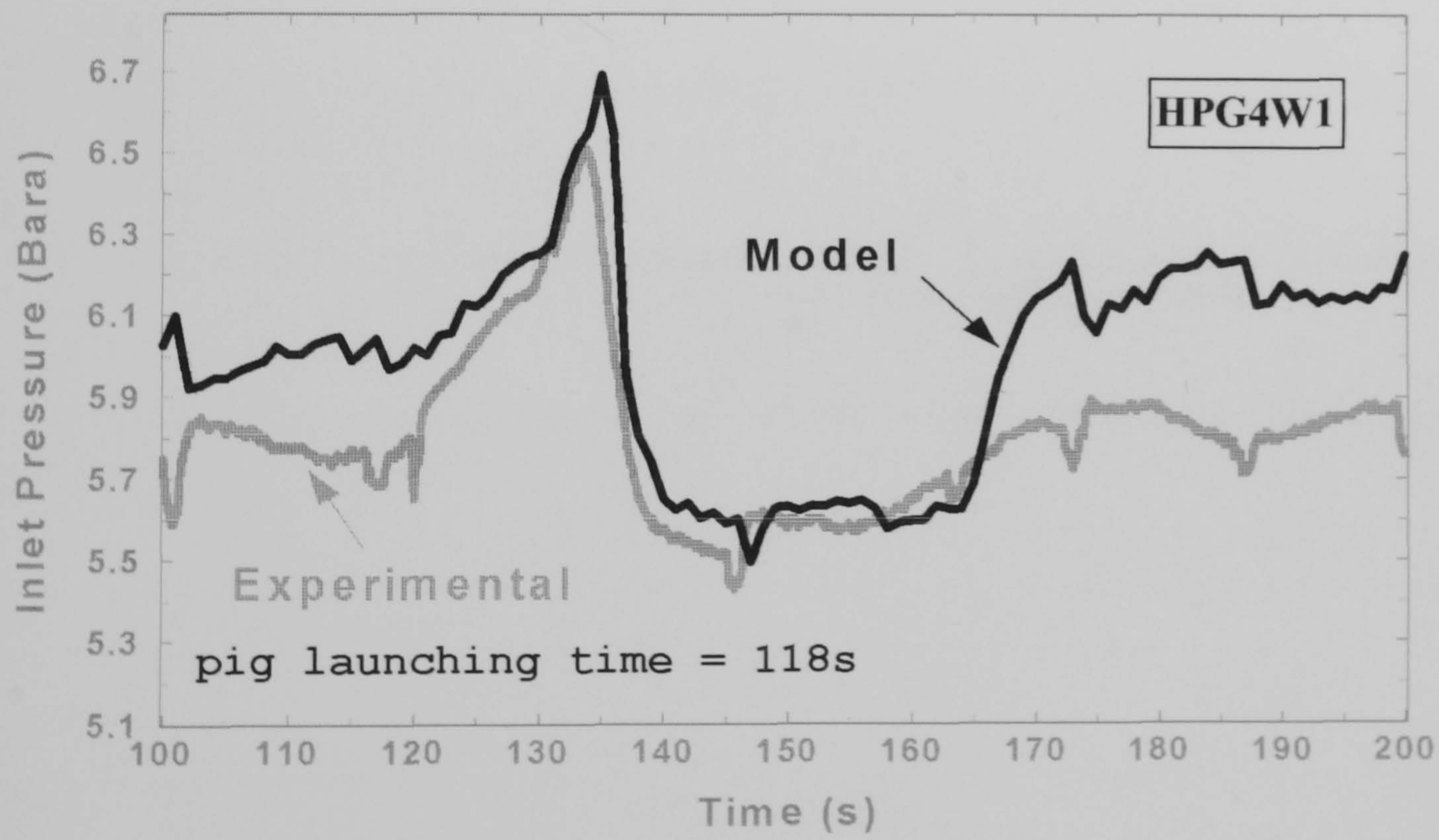


Figure 6.8 - Measured values and model prediction for the inlet pressure with time

6.1.4 Run LPG3W3

The initial and final flow conditions of run HPG3W3 are in the slug flow pattern. The initial gas and liquid superficial velocities are 2.5 m/s and 0.94 m/s, respectively. The inlet pressure is 6.9 Bara. The pig is launched at 408 s. Fig. 6.9 and Fig. 6.10 show the transient behaviour of the bottom riser density and inlet pressure during and after the pigging test.

The predicted and measured steady state inlet pressure shows good agreement. The measured average liquid hold-up at the riser bottom is well predicted. The model predicts a sharper increase and a higher maximum value of the inlet pressure due to the pigging operation. The time interval for the liquid front to reach the gamma densitometer after the passage of the pig is well predicted. The measured slug length is shorter than predicted, however the total volume of liquid in the slug shows good agreement. The inlet pressure oscillations is well predicted. The pressure oscillation equates to the slug frequency.

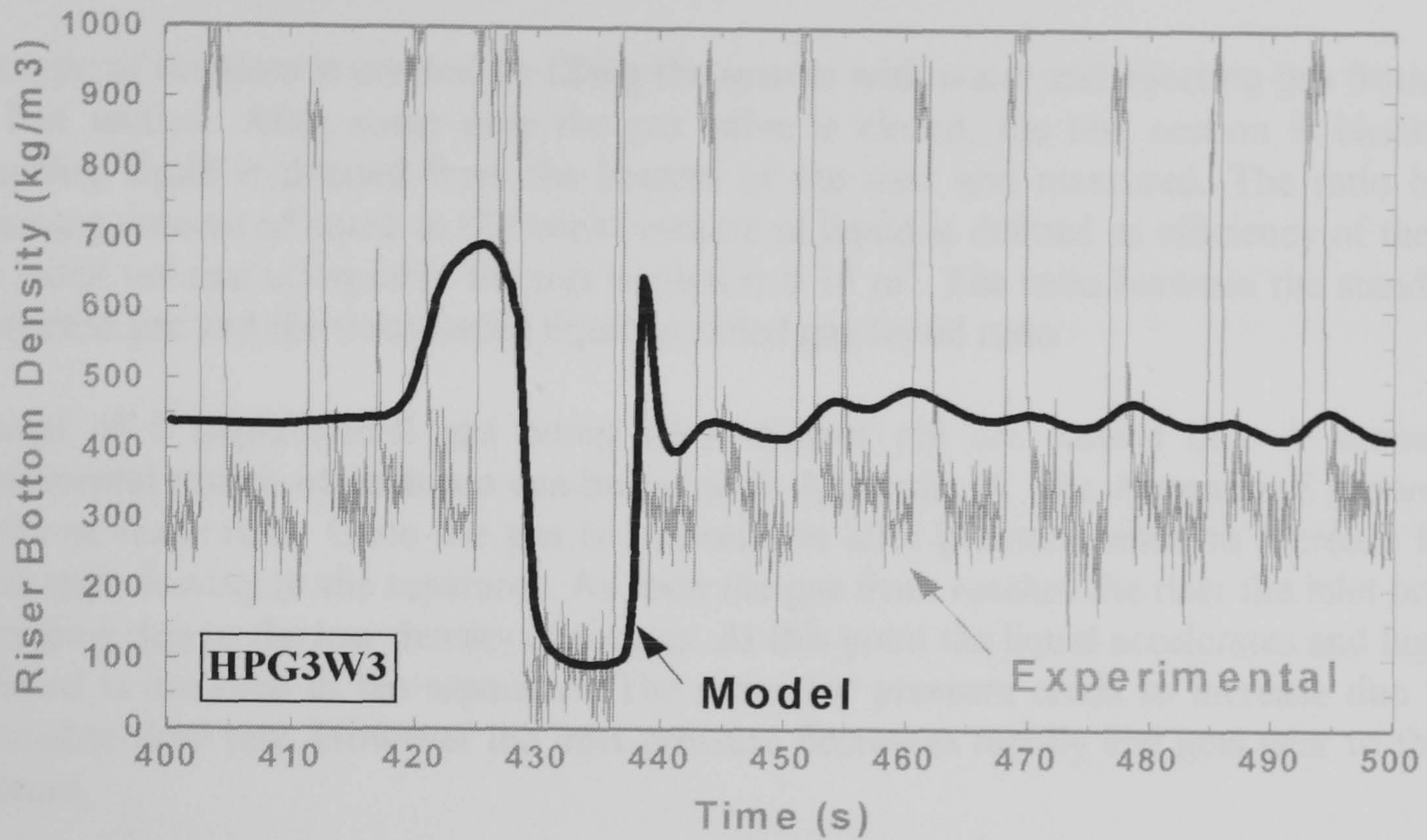


Figure 6.9 - Measured values and model prediction for the riser bottom density with time

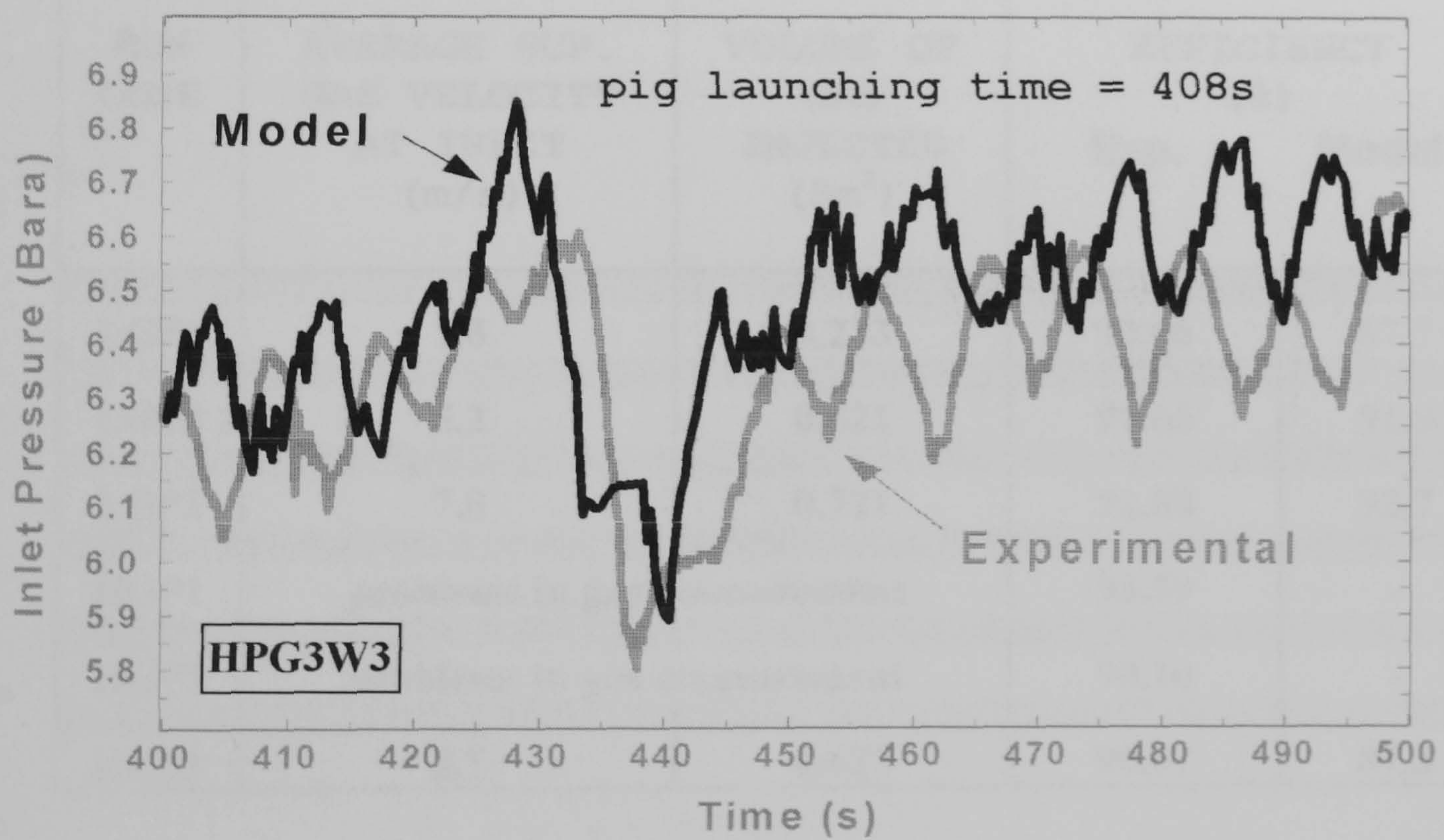


Figure 6.10 - Measured values and model prediction for the inlet pressure with time

6.2 Gas Pump without Pig

This type of transient is created by filling the system with water and injecting gas from the inlet of the test section. After some time the gas valve is closed, the test section is blocked and the remaining liquid is drained from the bottom of the riser and measured. The ratio between the remaining volume of liquid to the initial volume of liquid is defined as efficiency of the gas pump. The initial volume of liquid in the test section is 0.14 m³. The ratio between the standard volume of injected gas and the transported liquid is called gas-liquid ratio.

A total of 6 experimental gas pump runs without pig are carried out. A summary of the experimental results of each run can be found in Appendix B. The Appendix F shows the model results of these runs. Once the gas is injected the inlet pressure tends to increase because the liquid start flowing to the separator. As soon the gas front reaches the riser the inlet pressure start decreasing due to the low density of the gas. At this point the liquid accelerates and high flow rate of liquid is detected in the separator. The separator pressure tends to increase due to the high volumetric flow rate. However the inlet pressure decreases rapidly and gets near to the separator pressure.

Table 6.2 presents data to allow comparison between the experimental data and the model prediction.

Table 6.2 - Gas pump without pig experimental and model results

RUN CODE	AVERAGE SUP. GAS VELOCITY AT INLET (m/s)	VOLUME OF GAS INJECTED (Sm ³)	EFFICIENCY (%)	
			Exp.	Model
LGP1	1.8	0.233	90.66	87.7
LGP2	5.2	0.521	91.65	91.9
LGP3	7.0	0.721	92.88	92.7
HGP1	problems in gas measurement		93.59	-
HGP2	problems in gas measurement		90.10	-
HGP3	3.7	0.627	90.07	87.3

In Fig. 6.11 the efficiency of the gas pump without pig is reported. Good agreement is observed. However this diagram shows that the predictions are lower than measured values. This indicates that more liquid is transported than predicted, possibly liquid droplets not accounted for by the model.

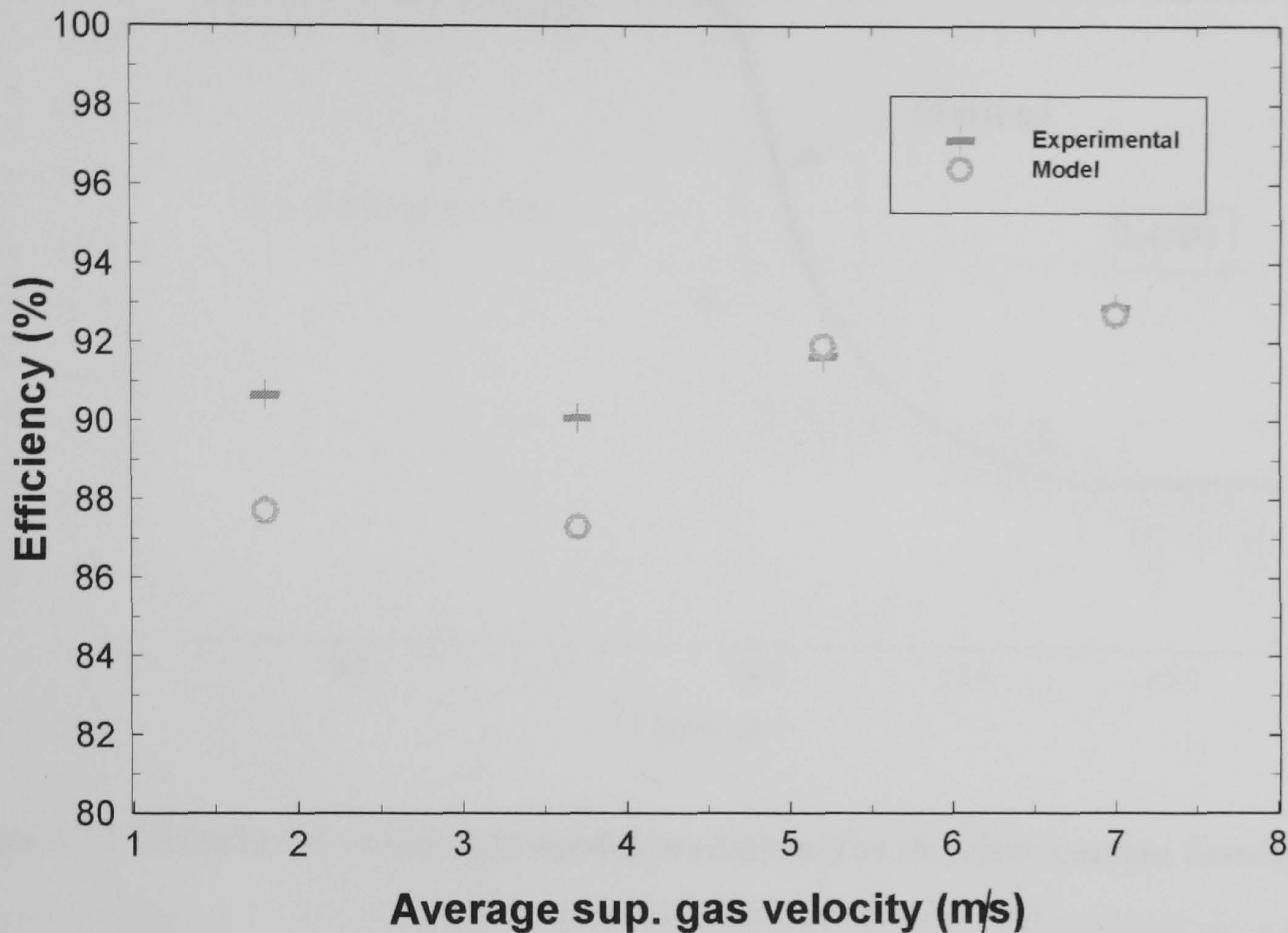


Figure 6.11 - Measured values and model predictions for the efficiency

6.2.1 Run LGP1

At 162s gas is injected at an average flow rate of $0.00803 \text{ Sm}^3/\text{s}$. At 191s the gas valve is closed. At 180s the hold-up at the bottom of the riser decreases sharply as shown in Fig. 6.12. The model prediction is good, however a slower decrease was predicted. The final hold-up is well predicted as shown in Fig. 6.12. Fig. 6.13 shows the inlet pressure increase due to the liquid flow. When the gas reaches the riser the pressure starts dropping. The model prediction of the inlet pressure is good, although the peak pressure was higher than predicted. At 210s the test section is blocked and the remaining liquid is drained and measured to evaluate the efficiency. The predicted efficiency is 87.7% and the measured efficiency is 90.66%. The volume of injected gas is 0.233 m^3 which gives a gas-liquid ratio of $1.84 \text{ Sm}^3/\text{m}^3$. This value is very small compared to the conventional gas lift system used in the oil industry. A gas-liquid ratio of $9.2 \text{ Sm}^3/\text{m}^3$ is required for an efficient continuous gas lift. Even though the bottom riser pressure will be 0.3 Bar greater than the separator pressure according to Hagedorn-Brown correlation (1980).

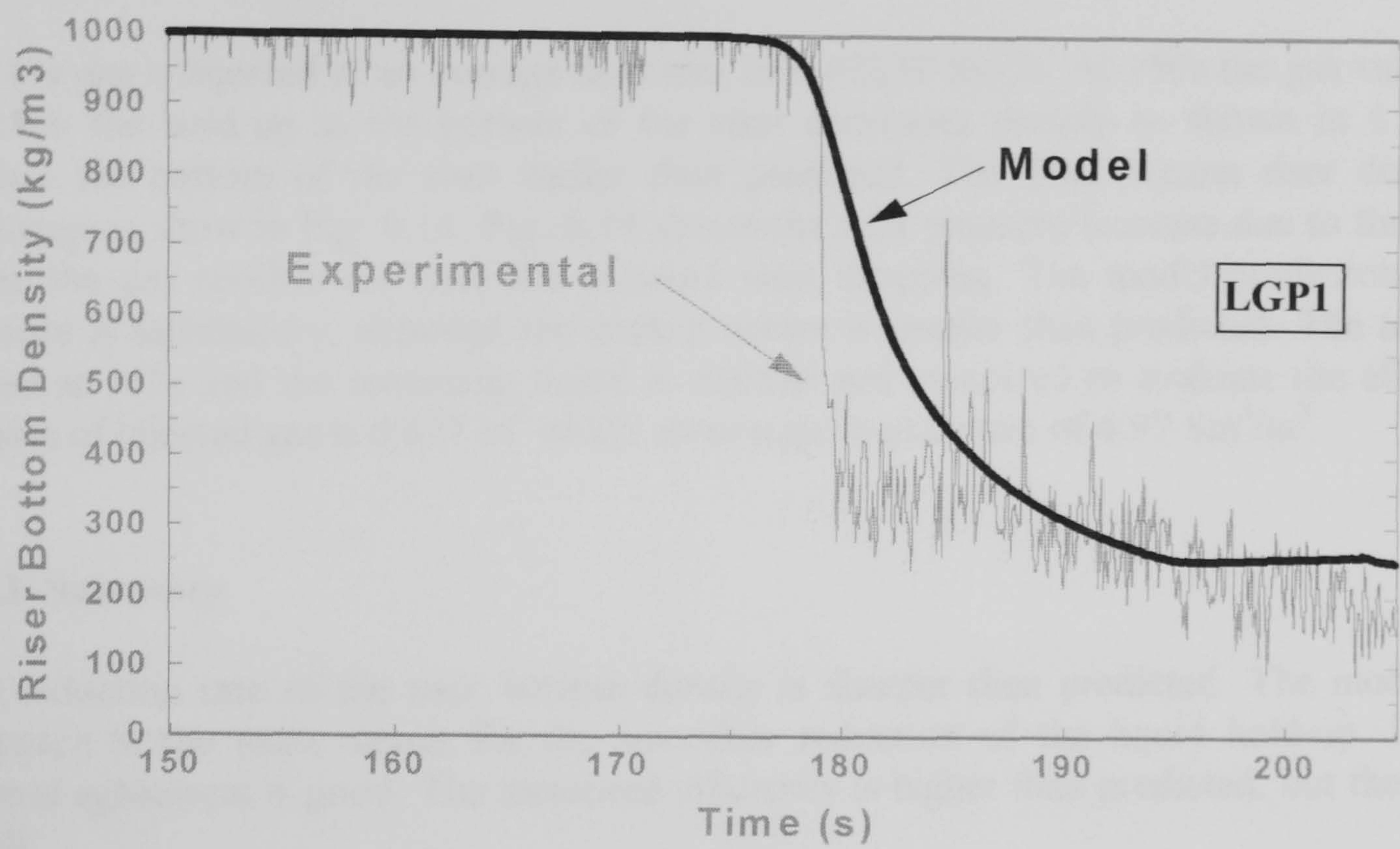


Figure 6.12 - Measured values and model prediction for the riser bottom density with time

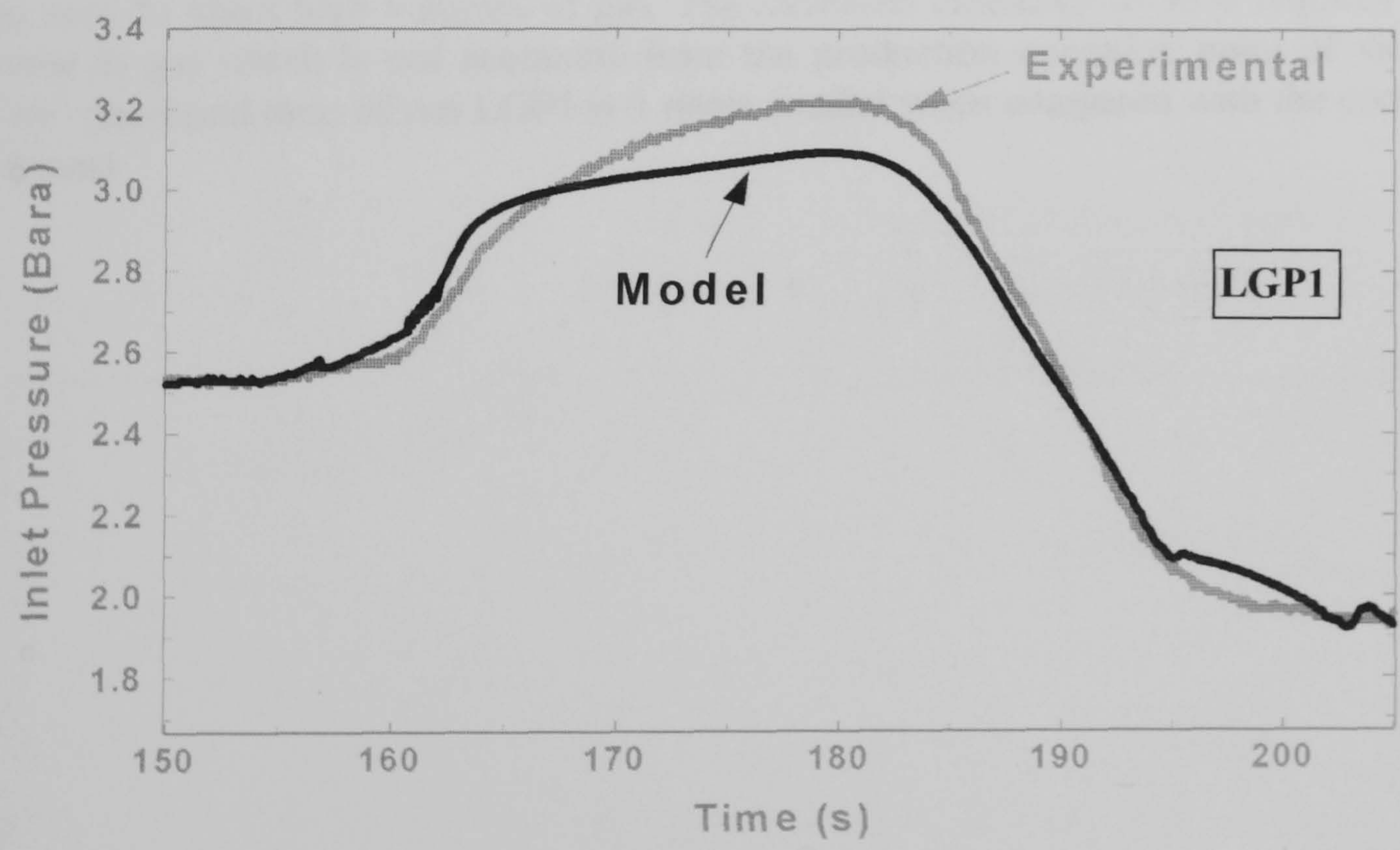


Figure 6.13 - Measured values and model prediction for the inlet pressure with time

6.2.2 Run HGP3

At 131s gas is injected at an average flow rate of $0.02239 \text{ Sm}^3/\text{s}$. At 159s the gas valve is closed. At 140s the hold-up at the bottom of the riser decreases sharply as shown in 6.12. The gas reaches the bottom of the riser earlier than predicted. The final bottom riser density is well predicted as show in Fig. 6.14. Fig. 6.15 shows the inlet pressure increase due to the liquid flow. When the gas reaches the riser the pressure start dropping. The model prediction of the inlet pressure is satisfactory, although the peak pressure is smaller than predicted. The test section is closed at 165s and the remaining liquid is drained and measured to evaluate the efficiency. The volume of injected gas is 0.627 m^3 which gives a gas-liquid ratio of $4.97 \text{ Sm}^3/\text{m}^3$.

6.2.3 Summary

The reduction rate of the riser bottom density is sharper than predicted. The model averaging approach is the main reason for the smoother reduction of the liquid hold-up. However the general agreement is good. The measured efficiency is higher than predicted, but the difference is small.

The inlet pressure prediction is good. In these diagrams no special error trends can be detected.

In run LGP1 is injected just 0.233 Sm^3 of gas and the efficiency is 90.66%. Increasing the gas volume to 0.721 Sm^3 according to LGP3 there is a small increase of the efficiency to 92.88%. This means that levels of efficiency of around 90% can be achieved by small volume of gas, there is no need to inject high volumes of gas. The increased efficiency to 93% requires a three fold increase in gas which is not economic from the production operation point of view. The $1.84 \text{ Sm}^3/\text{m}^3$ gas-liquid ratio of run LGP1 is 5 times smaller when compared with the conventional gas lift system.

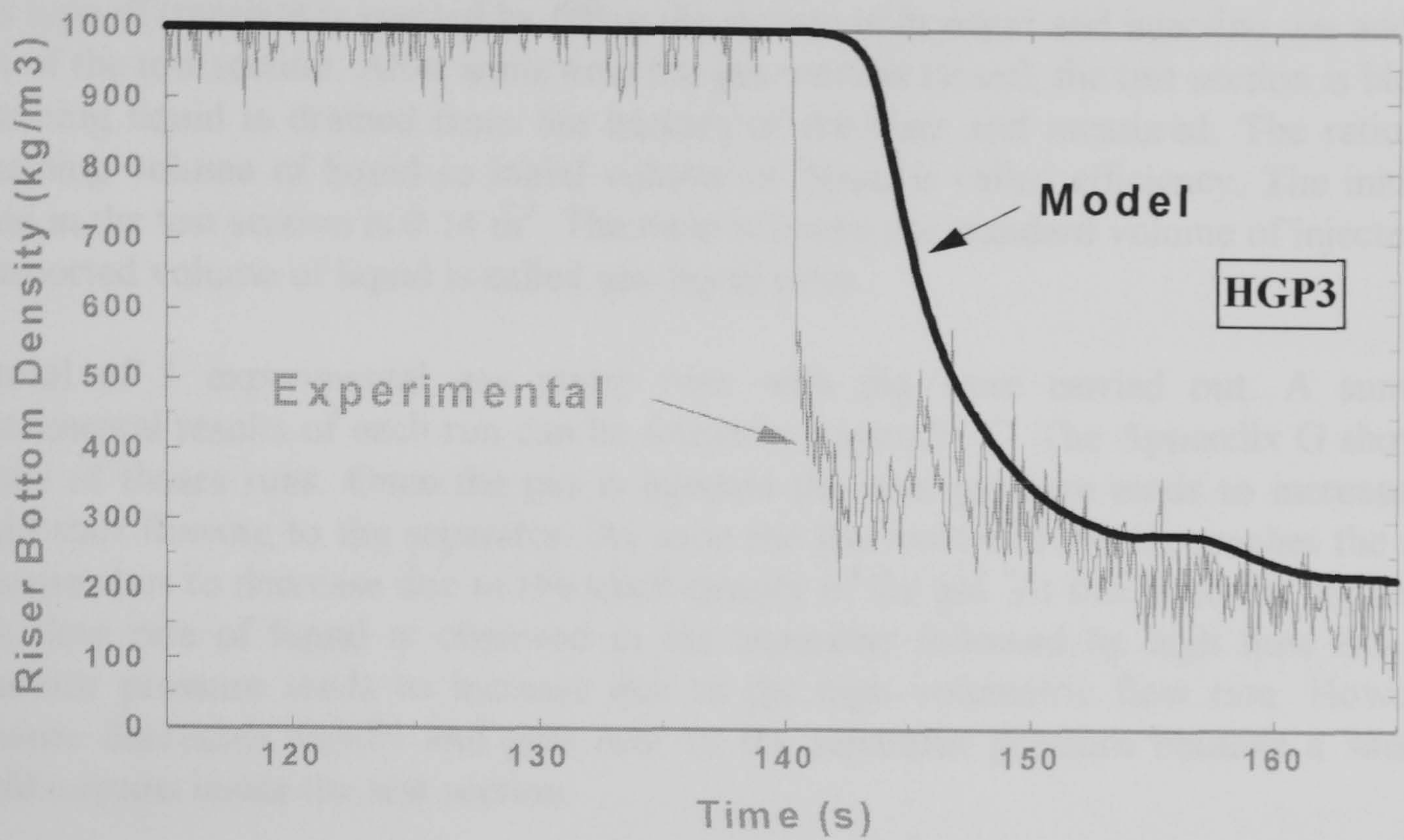


Figure 6.14 - Measured values and model prediction for the riser bottom density with time

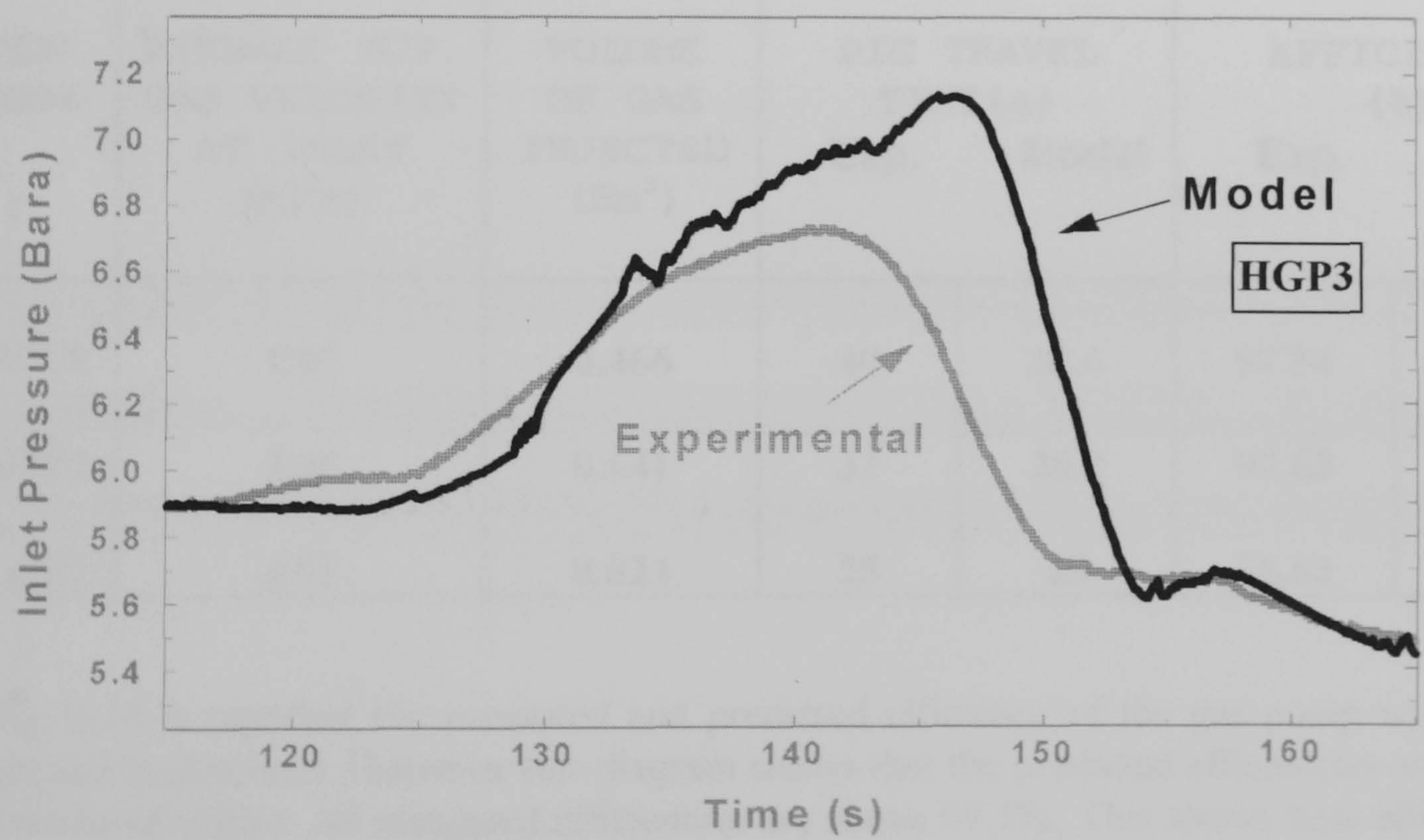


Figure 6.15 - Measured values and model prediction for the inlet pressure with time

6.3 Gas Pump with Pig

This type of transient is created by filling the system with water and injecting gas and pig from the inlet of the test section. After some time the gas valve is closed, the test section is blocked and the remaining liquid is drained from the bottom of the riser and measured. The ratio between the remaining volume of liquid to initial volume of liquid is called efficiency. The initial volume of liquid in the test section is 0.14 m³. The ratio between the standard volume of injected gas and the transported volume of liquid is called gas-liquid ratio.

A total of 3 experimental gas pump runs with pig were carried out. A summary of the experimental results of each run can be found in Appendix C. The Appendix G shows the model results of theses runs. Once the gas is injected the inlet pressure tends to increase because the liquid start flowing to the separator. As soon the gas front and the pig reaches the riser the inlet pressure start to decrease due to the small density of the gas. At this point the pig accelerates and high flow rate of liquid is observed in the separator followed by high flow rate of gas. The separator pressure tends to increase due to the high volumetric flow rate. However the inlet pressure decreases rapidly and gets near to the separator pressure because a small volume of liquid remains inside the test section.

Table 6.3 presents data to allow comparison between the experimental data and the model prediction.

Table 6.3 - Gas pump with pig experimental and model results

RUN CODE	AVERAGE SUP. GAS VELOCITY AT INLET (m/s)	VOLUME OF GAS INJECTED (Sm ³)	PIG TRAVEL TIME (s)		EFFICIENCY (%)	
			Exp.	Model	Exp.	Model
LPP1A	1.97	0.466	40	38.6	97.54	99.18
LGPP2	3.45	0.641	32	28.2	97.63	98.62
LGPP3	3.95	0.824	25	25	98.63	99.7

In Fig. 6.16 is reported the measured and predicted efficiency of the gas pump with pig. Good agreement is observed. However this diagram shows that the predicted efficiencies are higher than the measured values. All measured efficiencies are above 97.5%. This shows how efficient is foam pig to transport liquid. Based on this value it is assumed in the model calculation that the liquid hold-up behind the pig is 0.025 (2.5%). The measured efficiencies are greater than 97.5% because some liquid left behind the pig is transported by the gas phase. The model predicts that a higher than observed volume of liquid is transported by the gas. The reason for this is the model assumption of mist flow in case of extremely low liquid hold-up.

In Fig. 6.17 is reported the measured and predicted pig travel time. Good agreement is observed. However this diagram shows that the predicted times are smaller when compared with the measured values.

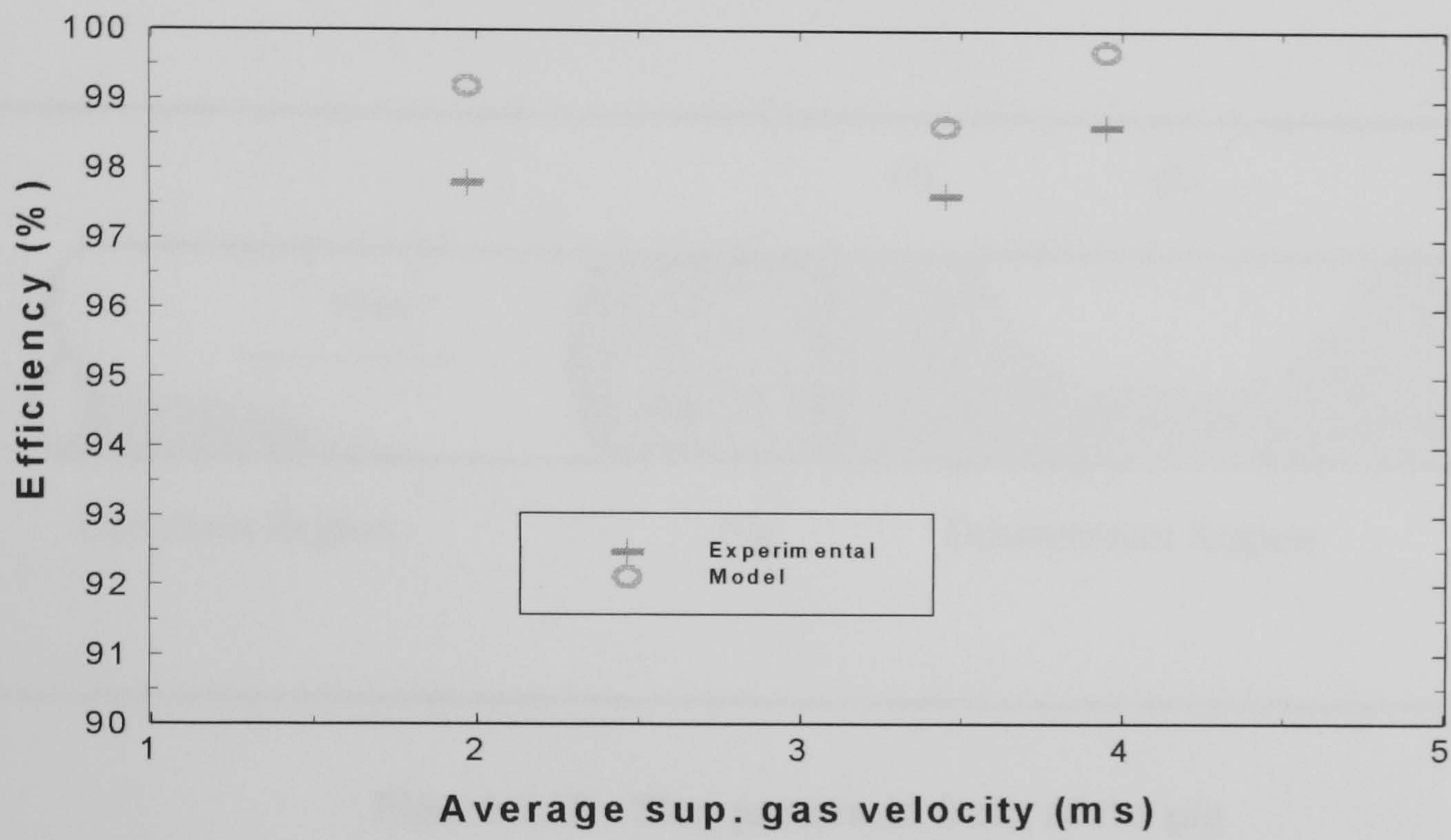


Figure 6.16 - Measured values and model predictions for the efficiency

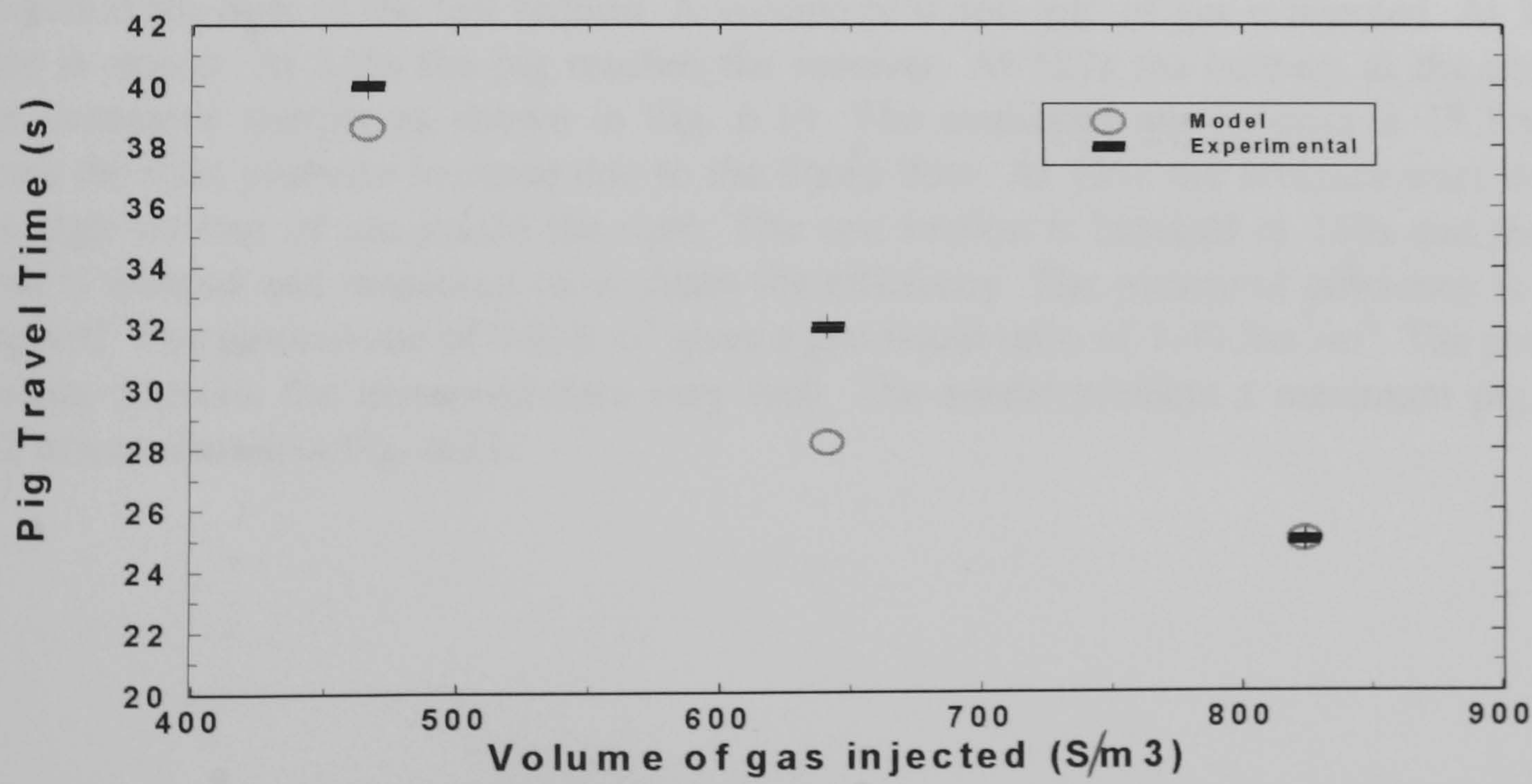


Figure 6.17 - Measured values and model predictions for pig travel time with gas injected

Figures 6.19, 6.22 and 6.25 show the results of the model simulation of the bottom riser density along with the experimental measurements. The measured liquid hold-up shows a non predicted behaviour as shown in Fig. 6.18. The experimental results show a low density region (2) between two high densities regions (1) and (3). It is postulated that the low density region (2) is due to high leakage through the pig during and after the launching when the pig velocity is low. The leakage rate reduces as the pig speeds up generating a liquid slug in front of the pig, region (1).

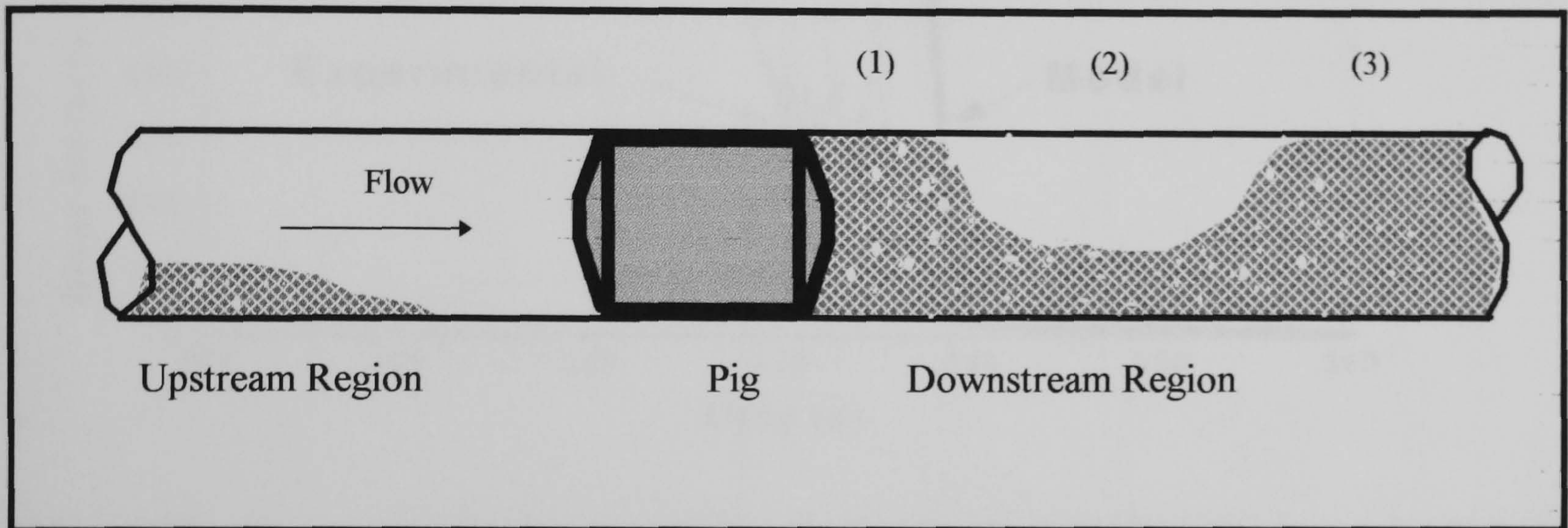


Figure 6.18 - Slug pattern in front of the pig

6.3.1 Run LPP1A

The results of run LPP1A are shown in Figures 6.19, 6.20 and 6.21. At 307s gas and pig are injected at the inlet of the test section. A volume of 0.466 Sm^3 of gas is injected. At 339s the gas valve is closed. At 345s the pig reaches the receiver. At 327s the hold-up at the bottom of the riser decreases sharply as shown in Fig. 6.19. The evaluated gas by-pass is 18.3%. Fig. 6.20 shows the inlet pressure increase due to the liquid flow. At 331s the pressure start dropping due to a high volume of gas inside the riser. The test section is blocked at 360s and the remaining liquid is drained and measured to evaluate the efficiency. The measured efficiency is higher than observed. The gas volume of 0.466 m^3 gives a gas-liquid ratio of $3.40 \text{ Sm}^3/\text{m}^3$. The predicted inlet pressure matches the measured data very well. The model predicts a maximum pig velocity of 18.2 m/s as shown in Fig. 6.21.

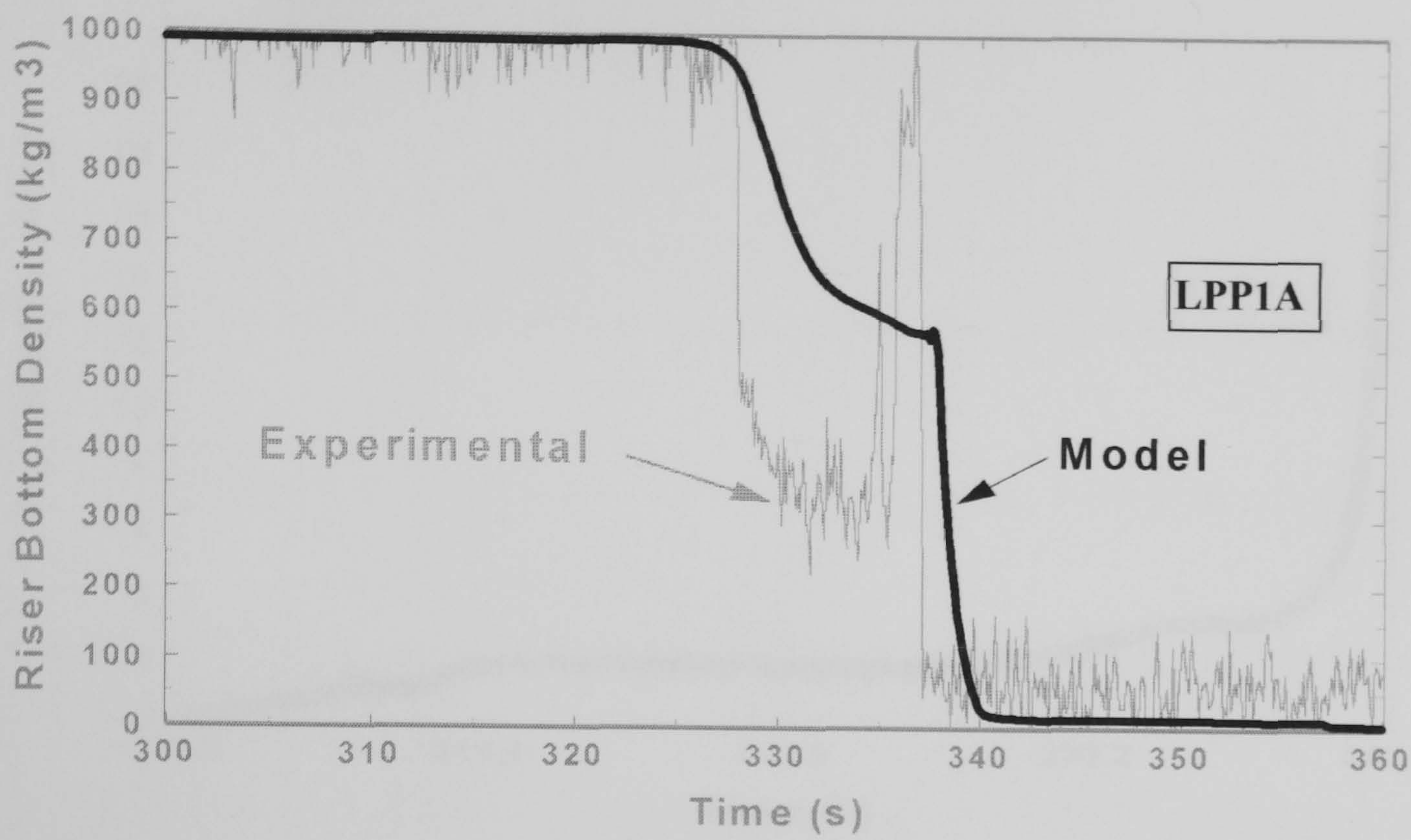


Figure 6.19 - Measured values and model prediction for the riser bottom density with time

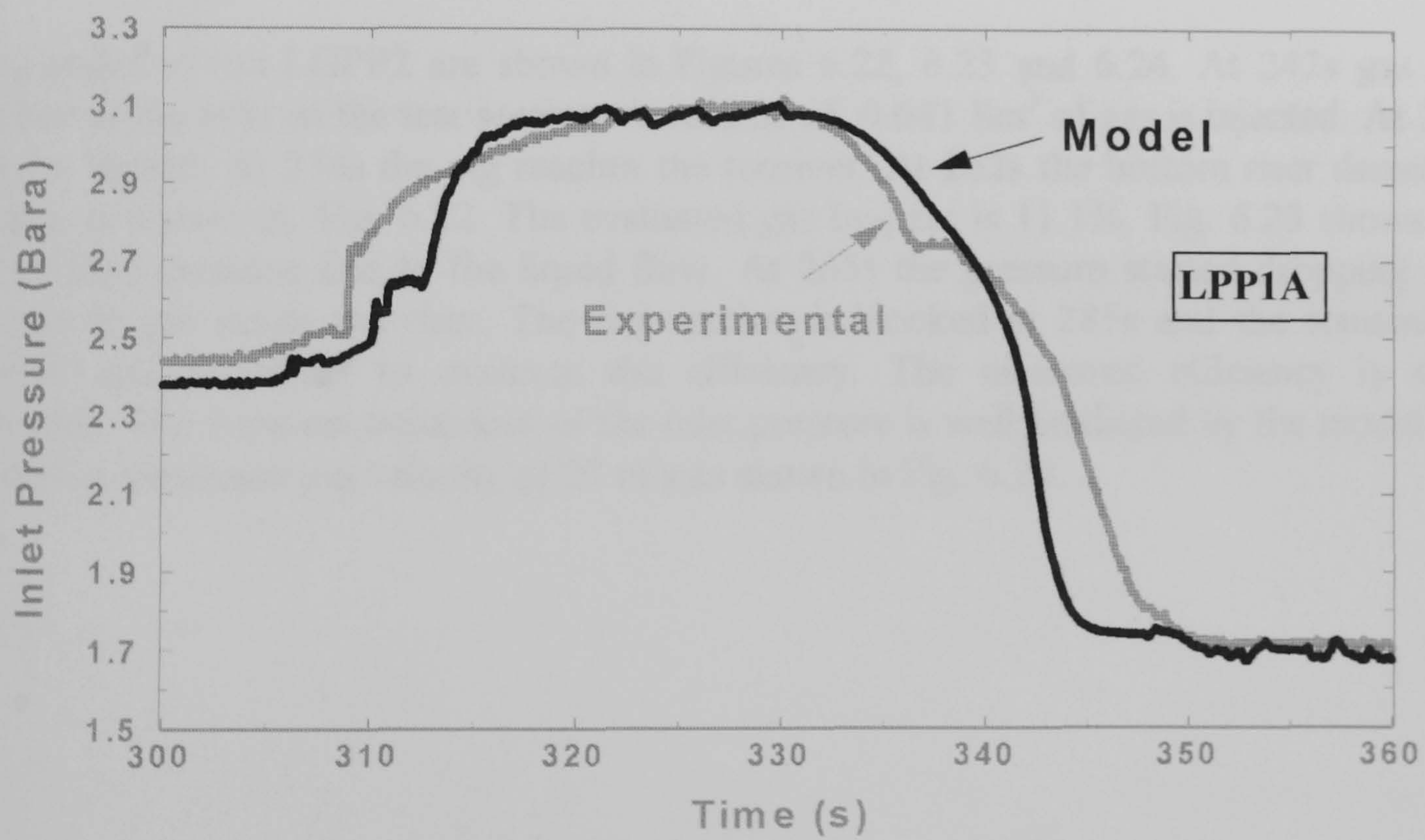


Figure 6.20 - Measured values and model prediction for the inlet pressure with time

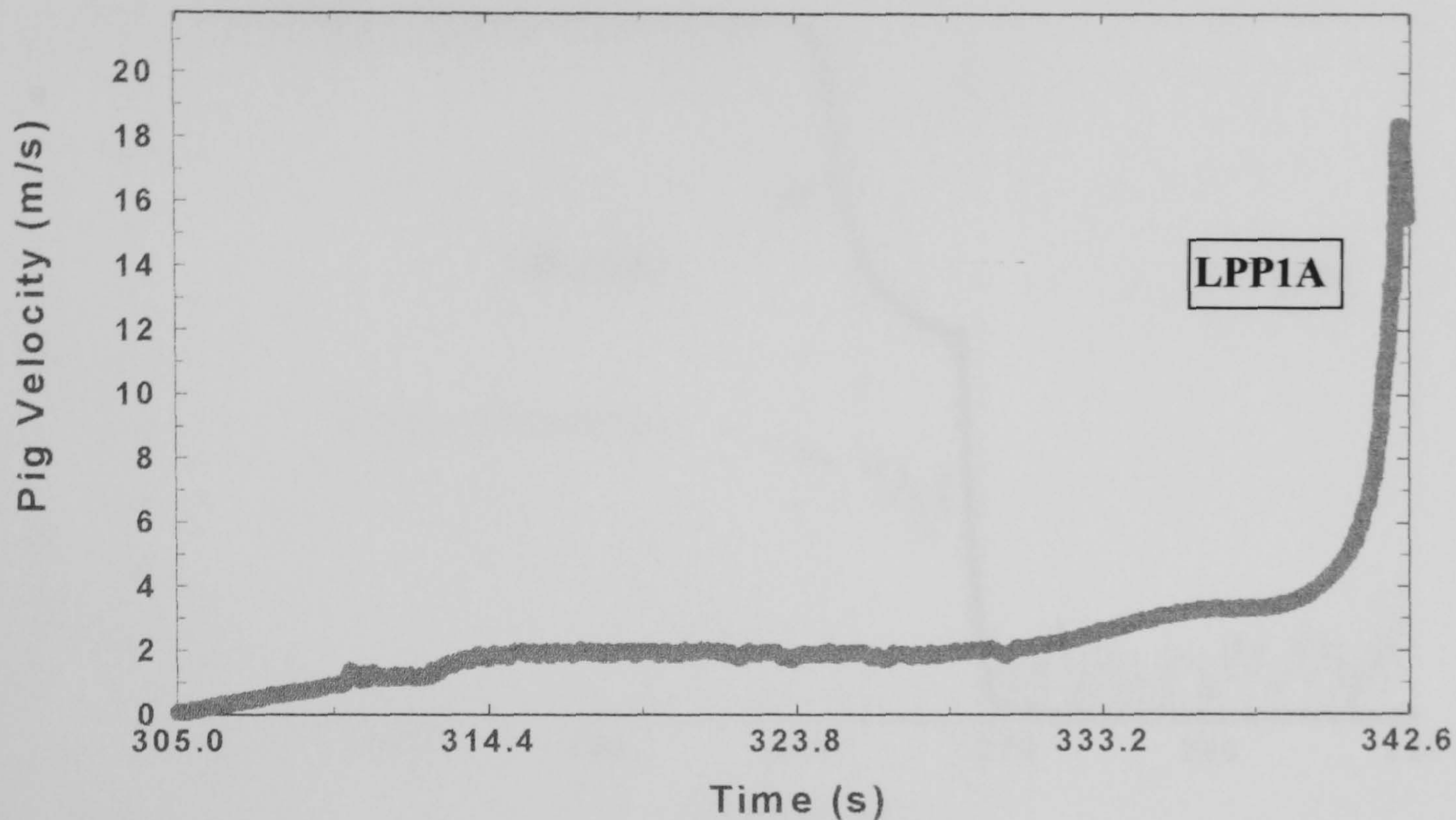


Figure 6.21 - Model prediction for pig velocity with time

6.3.2 Run LGPP2

The results of run LGPP2 are shown in Figures 6.22, 6.23 and 6.24. At 242s gas and pig are injected at the inlet of the test section. A volume of 0.641 Sm^3 of gas is injected. At 271s the gas valve is closed. At 274s the pig reaches the receiver. At 262s the bottom riser density decreases sharply as shown in Fig. 6.22. The evaluated gas by-pass is 11.1%. Fig. 6.23 shows an increase of the inlet pressure due to the liquid flow. At 265s the pressure started dropping due to high volume of gas inside the riser. The test section is blocked at 285s and the remaining liquid is drained and measured to evaluate the efficiency. The measured efficiency is smaller than predicted. The transient behaviour of the inlet pressure is well predicted by the model. The model predicts a maximum pig velocity of 27 m/s as shown in Fig. 6.24.

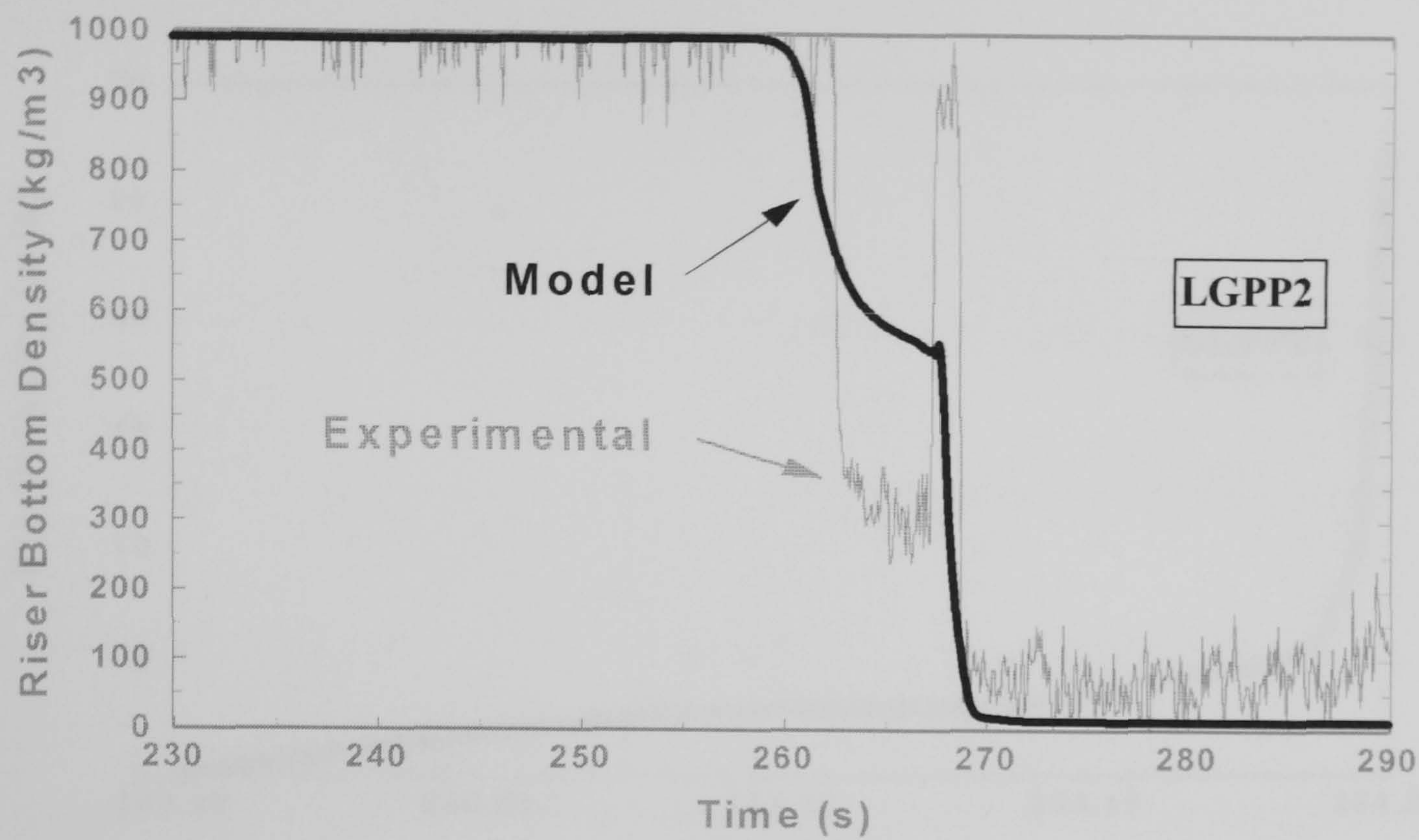


Figure 6.22 - Measured values and model prediction for the riser bottom density with time

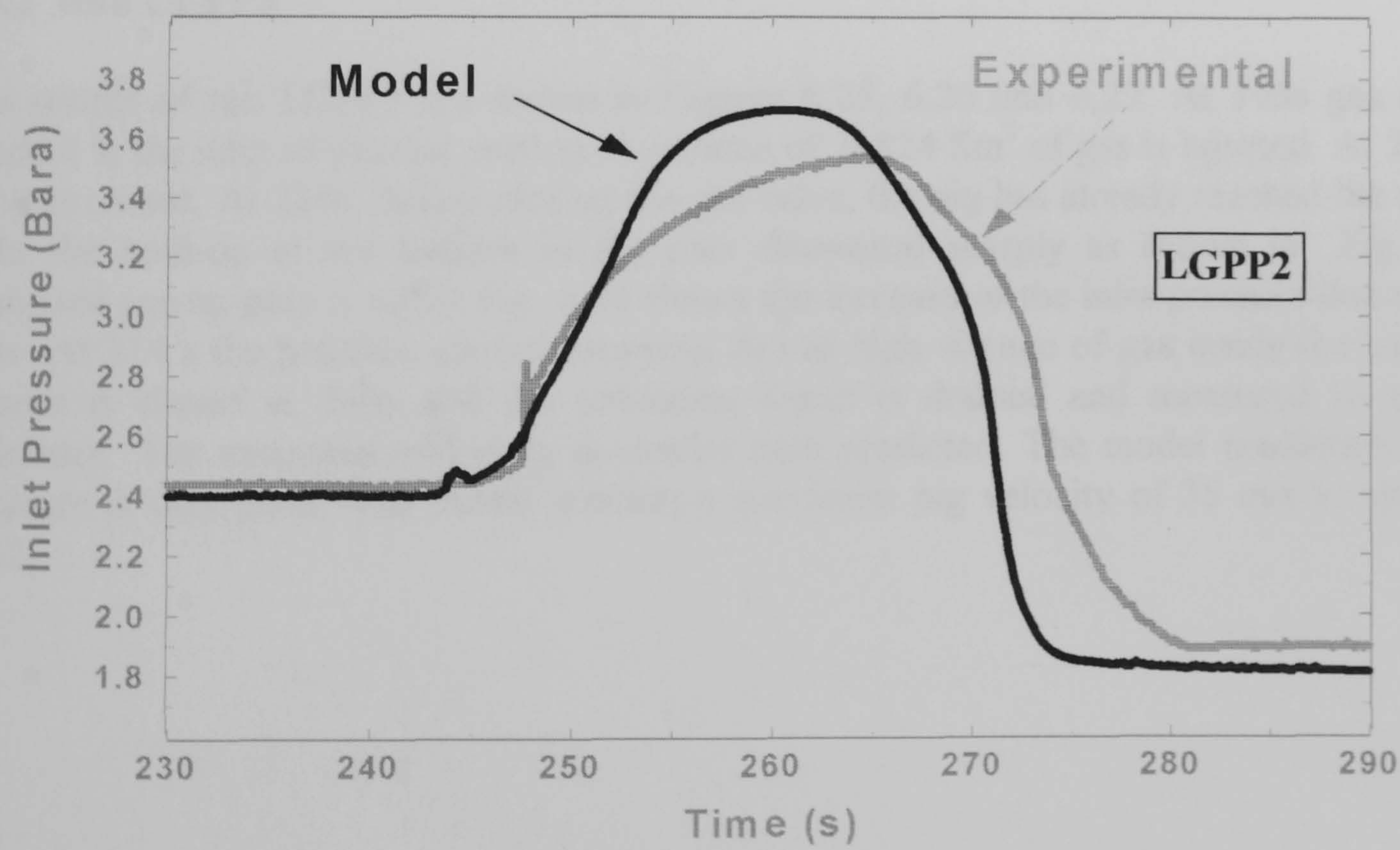


Figure 6.23 - Measured values and model prediction for the inlet pressure with time

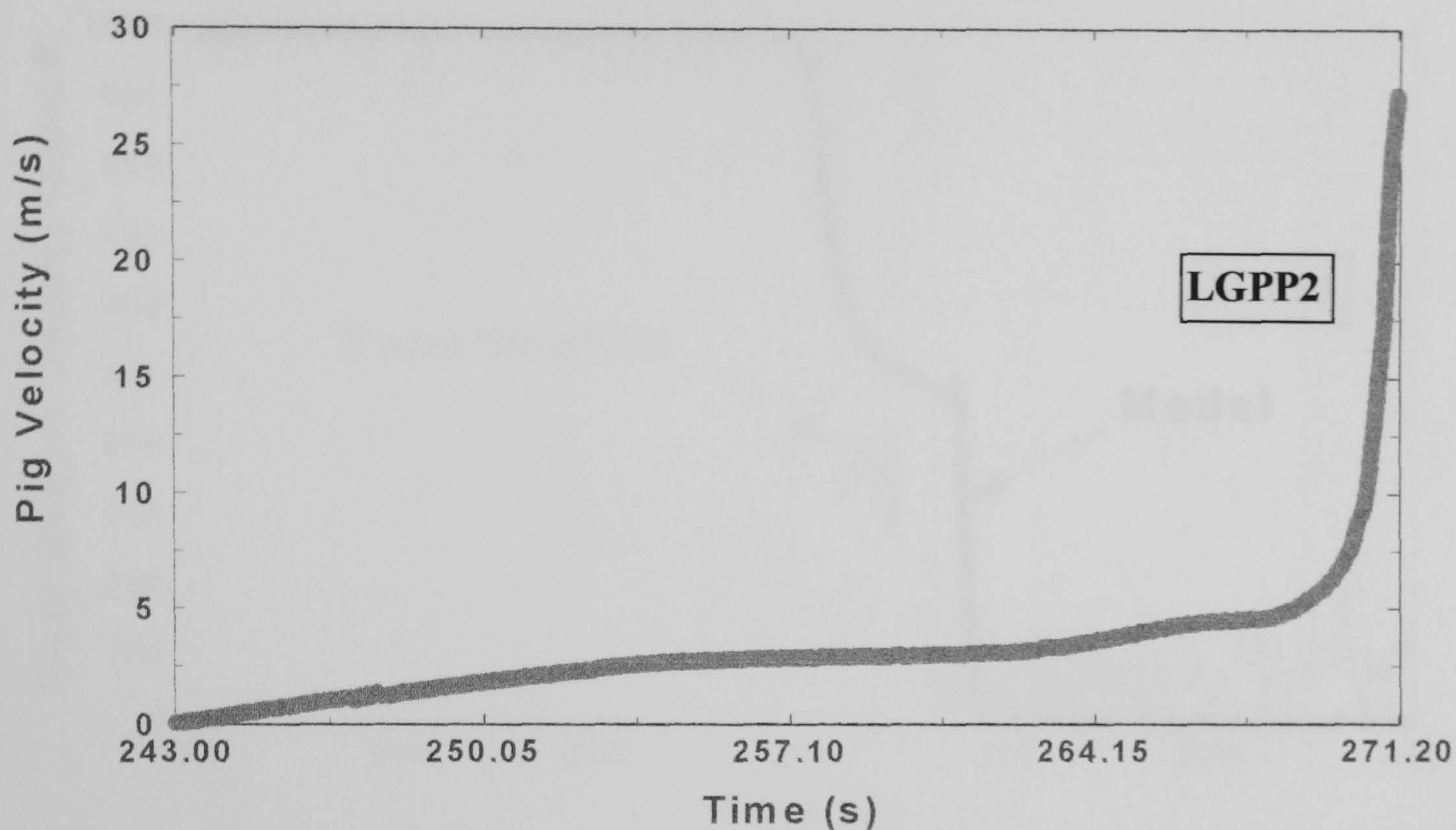


Figure 6.24 - Model prediction for pig velocity with time

6.3.3 Run LGPP3

The results of run LGPP3 are shown in Figures 6.25, 6.26 and 6.27. At 195s gas and pig are injected at the inlet of the test section. A volume of 0.824 Sm^3 of gas is injected. At 223s the gas valve is closed. At 220s, before closing the gas valve, the pig has already reached the receiver. At 211s the hold-up at the bottom of the riser decreased sharply as shown in Fig. 6.25. The evaluated gas by-pass is 8.2%. Fig. 6.26 shows the increase of the inlet pressure due to the liquid flow. At 214 s the pressure started dropping due to high volume of gas inside the riser. The test section is closed at 240s and the remaining liquid is drained and measured to evaluate the efficiency. The measured efficiency is smaller than predicted. The model prediction of the inlet pressure is very good. The model predicts a maximum pig velocity of 35 m/s as shown in Fig. 6.27.

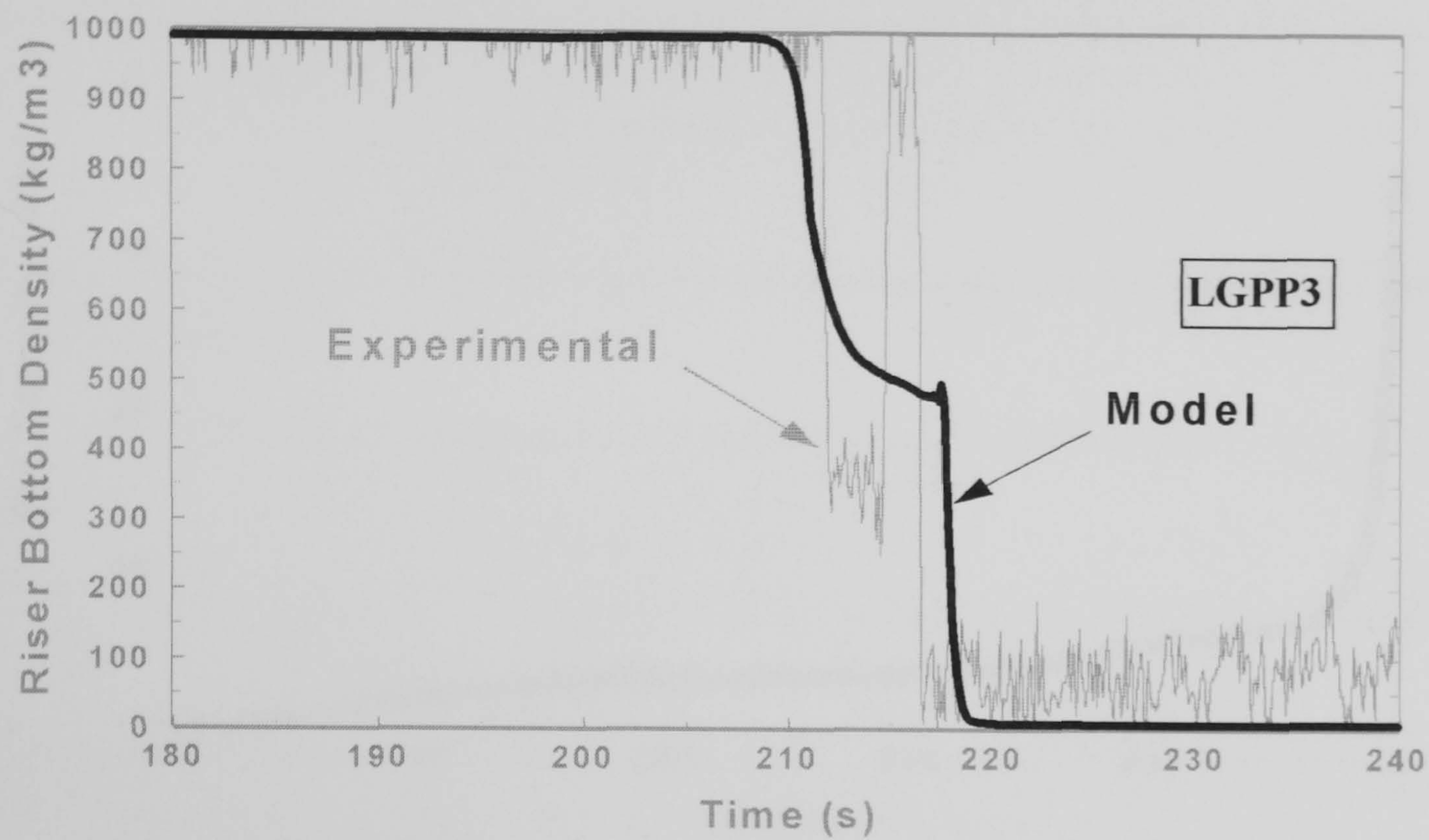


Figure 6.25 - Measured values and model prediction for the riser bottom density with time

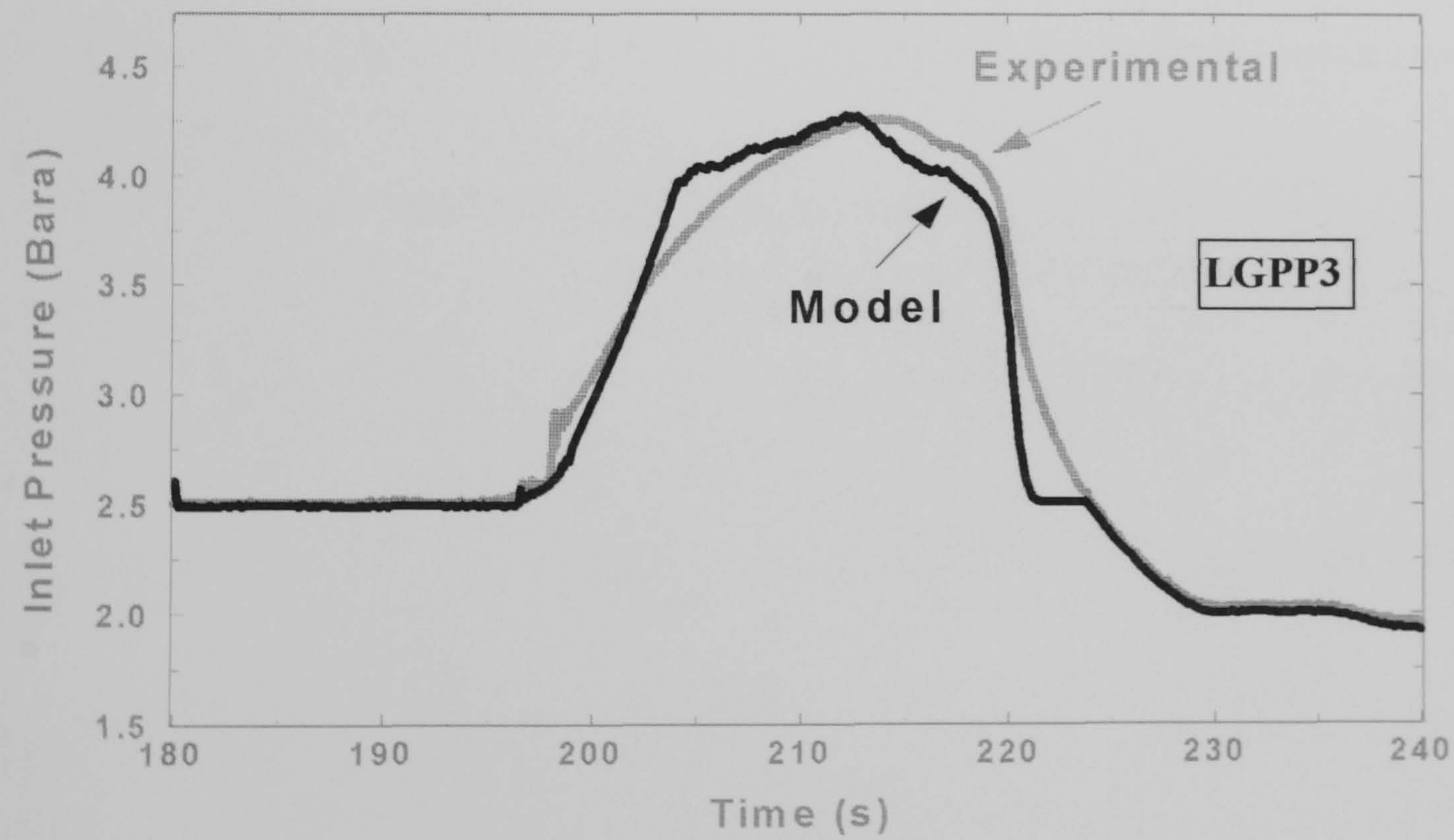


Figure 6.26 - Measured values and model prediction for the inlet pressure with time

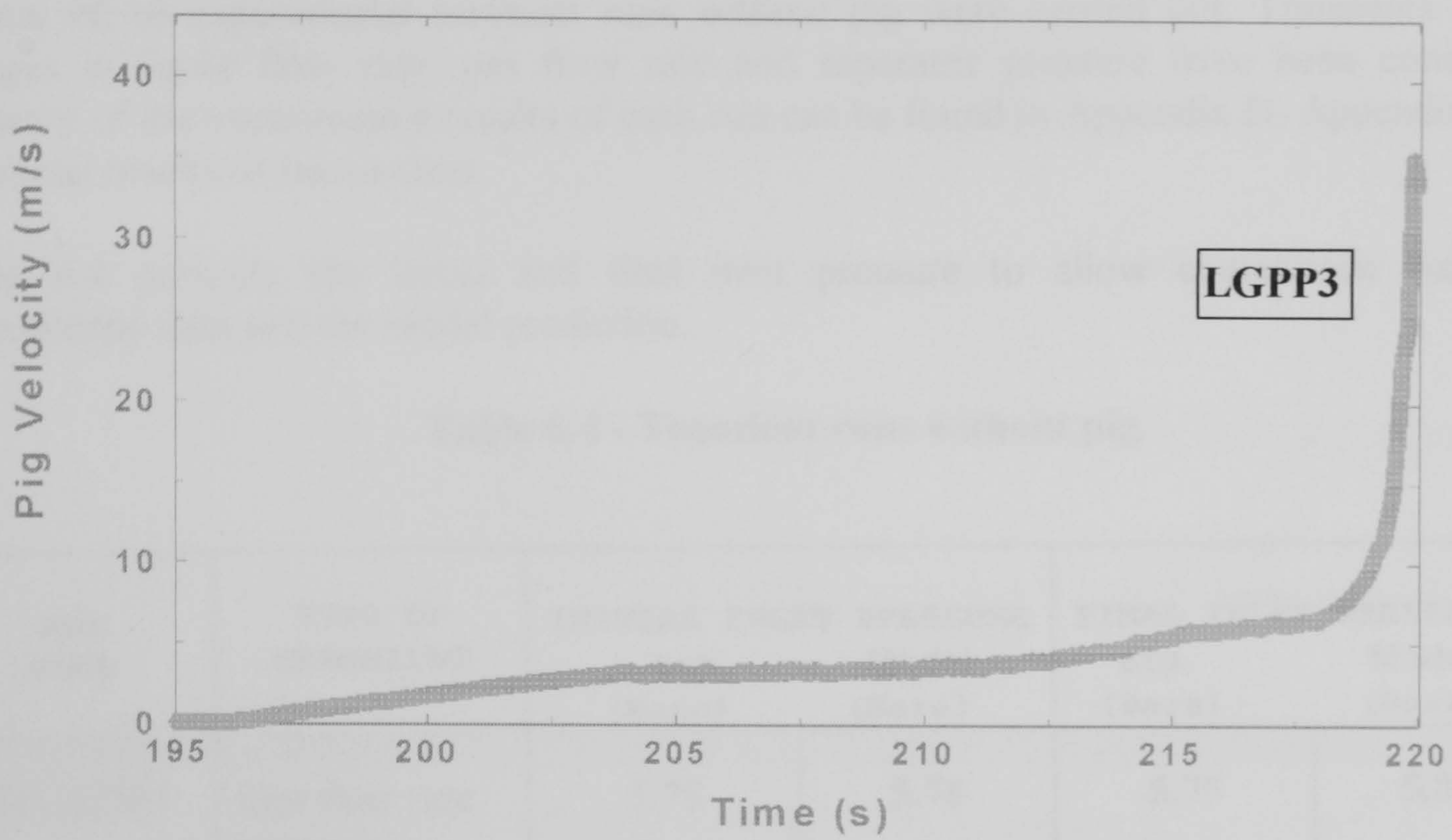


Figure 6.27 - Model prediction for pig velocity with time

6.4 Transient runs

A total of 10 experimental transient runs without pig were carried out. Transients caused by changes in liquid flow rate, gas flow rate and separator pressure have been considered. A summary of the experimental results of each run can be found in Appendix D. Appendix H shows the model results of theses runs.

Table 6.4 presents the initial and final inlet pressure to allow comparison between the experimental data and the model prediction.

Table 6.4 - Transient runs without pig

RUN CODE	TYPE OF TRANSIENT	INITIAL INLET PRESSURE		FINAL INLET PRESSURE	
		Exp. (Bara)	Model (Bara)	Exp. (Bara)	Model (Bara)
HTG12W3	Gas flow rate increase	5.75	5.78	5.75	5.80
HTG34W1	Gas flow rate increase	4.76	4.70	5.93	6.30
HTG34W2	Gas flow rate decrease	6.55	7.00	5.60	5.60
HTG34W3	Liquid flow rate decrease	6.13	6.20	6.85	7.30
LTG12W1	Gas flow rate increase	2.60	2.80	2.60	2.48
LTG23W2	Gas flow rate increase	2.80	2.63	2.80	2.75
LTG34W2	Gas flow rate increase	2.70	2.70	3.20	3.20
LTG34W3	Liquid flow rate decrease	2.85	2.75	2.90	3.00
LTG43W1	Gas flow rate decrease	2.35	2.60	2.30	2.40
LTG32W1	Gas flow rate decrease	2.35	2.40	2.25	2.27

In Figures 6.28 and 6.29 are reported the prediction of the initial and final inlet pressure. Good agreement is observed. There is no special error trend for these runs.

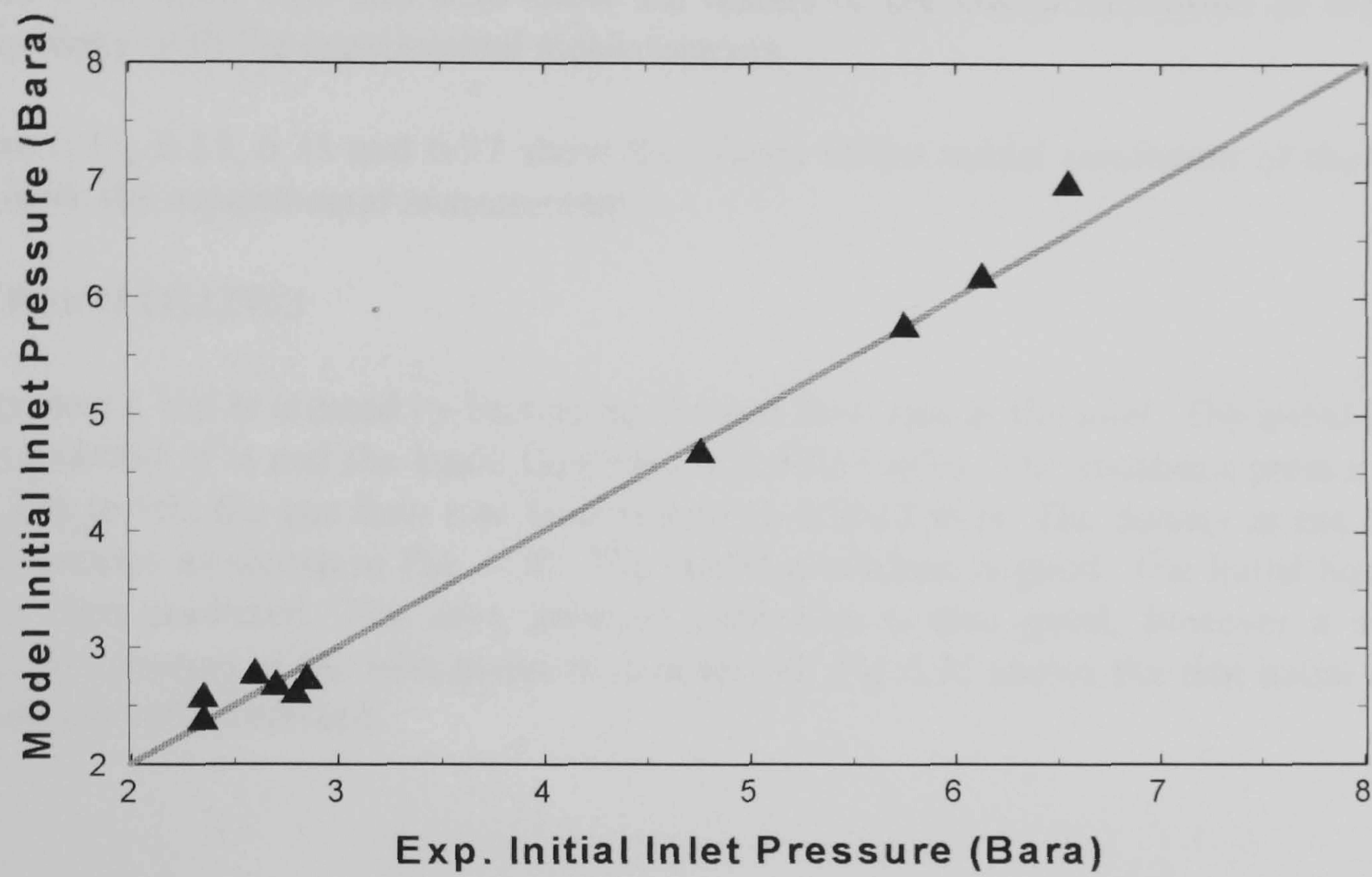


Figure 6.28 - Measured and predicted initial inlet pressure

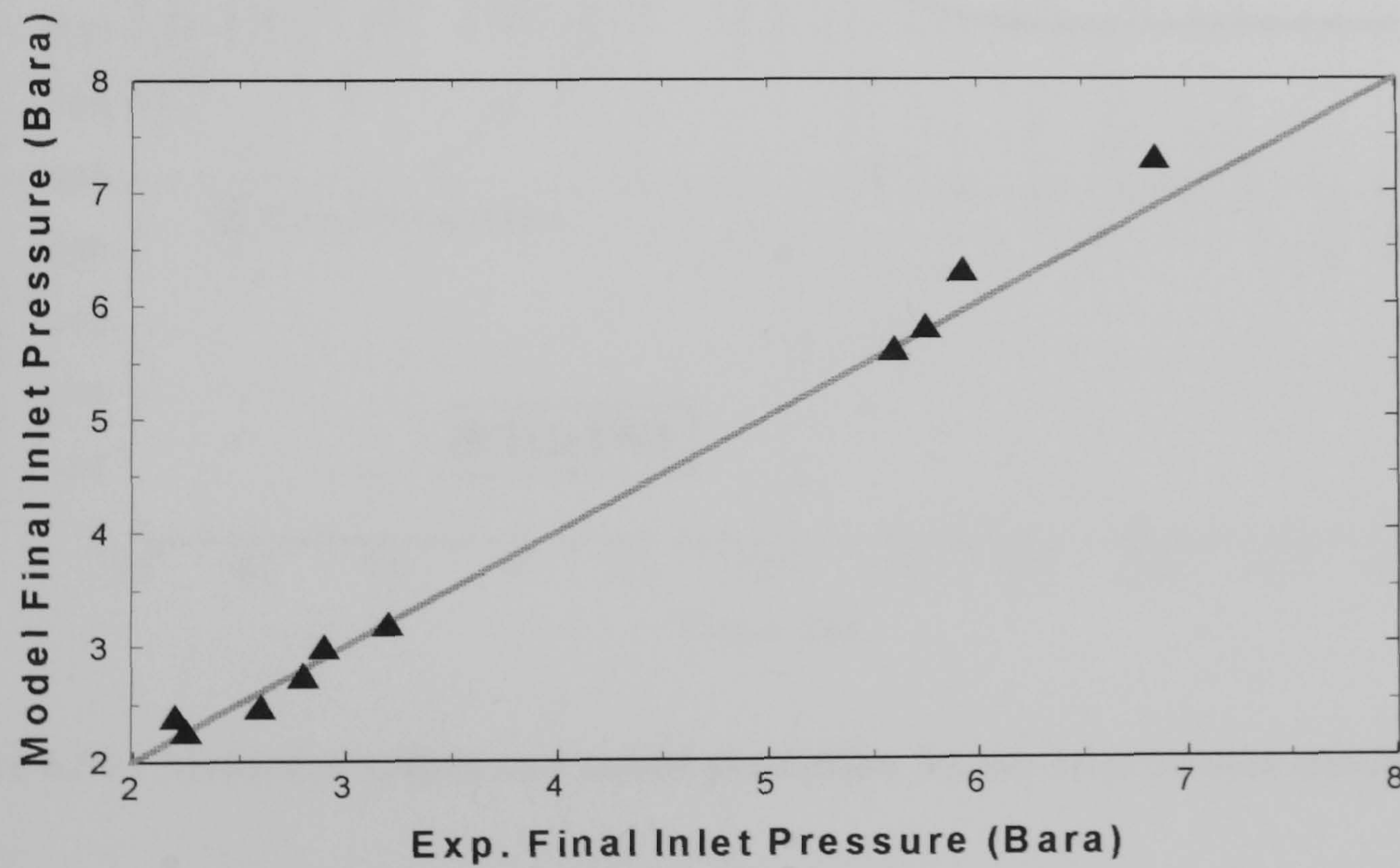


Figure 6.29 - Measured and predicted final inlet pressure

Transient runs HTG12W3, LTG12W1, LTG34W3 and HTG34W2 are used to analyse the transient tests.

Figures 6.30, 6.32, 6.34 and 6.36 show the results of the model simulation of the bottom riser density along with the experimental measurements.

Figures 6.31, 6.33, 6.35 and 6.37 show the results of the model simulation of the inlet pressure along with the experimental measurements.

6.4.1 Run HTG12W3

This transient run is created by increasing the gas flow rate at the inlet. The initial inlet gas flow rate is $0.00034 \text{ m}^3/\text{s}$ and the liquid flow rate is $0.00235 \text{ m}^3/\text{s}$. The separator pressure is 4.4 Bara. From 68s to 95s the gas flow rate is increased to $0.0012 \text{ m}^3/\text{s}$. The density at the bottom of the riser decreases as shown in Fig. 6.30. The model prediction is good. The initial liquid hold-up is smaller than predicted. The inlet pressure prediction is also good, however a slightly higher temporary increase of the inlet pressure is observed. Fig 6.31 shows the that initial and final inlet pressure are well predicted.

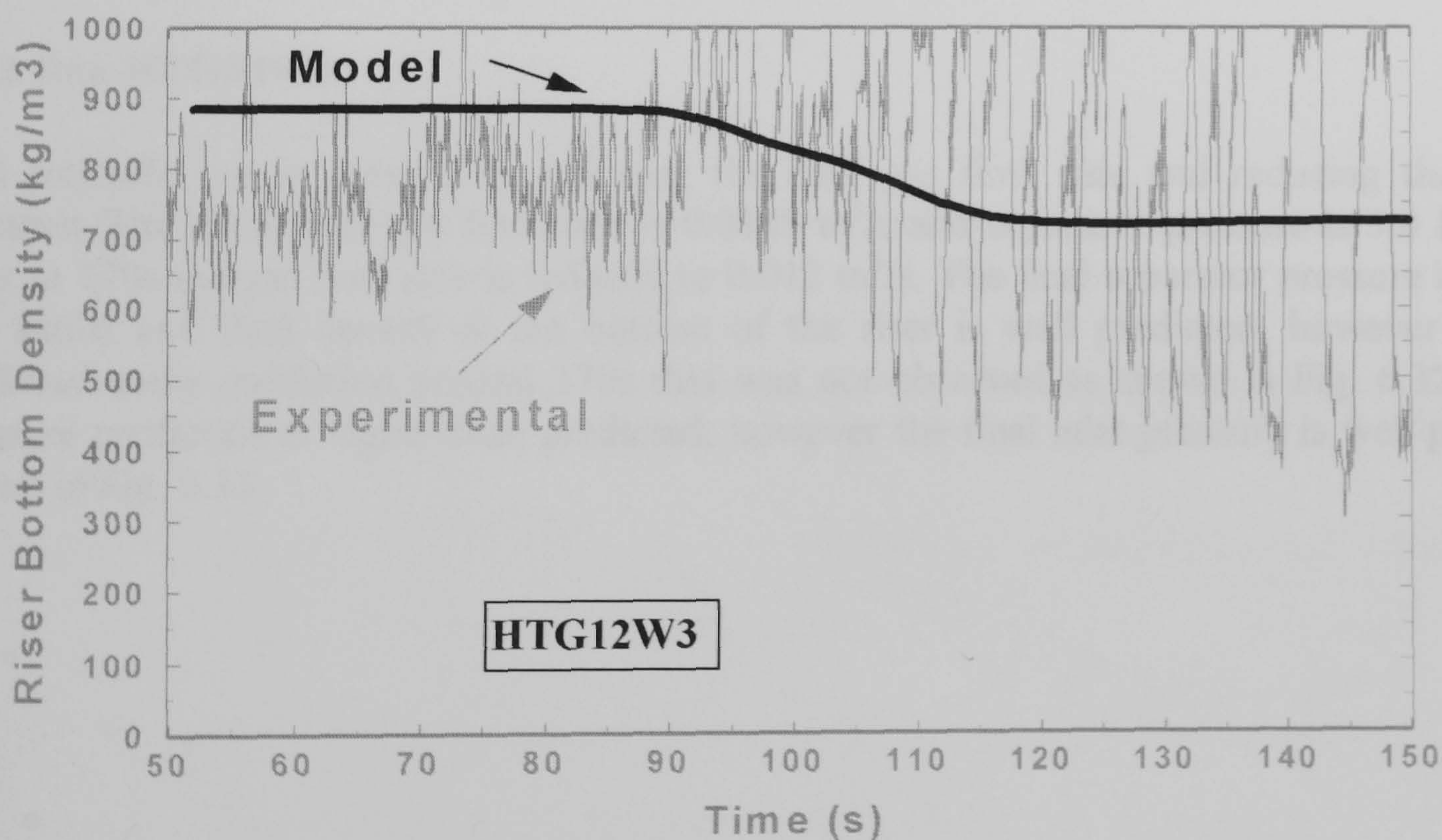


Figure 6.30 - Measured values and model prediction for the riser bottom density with time

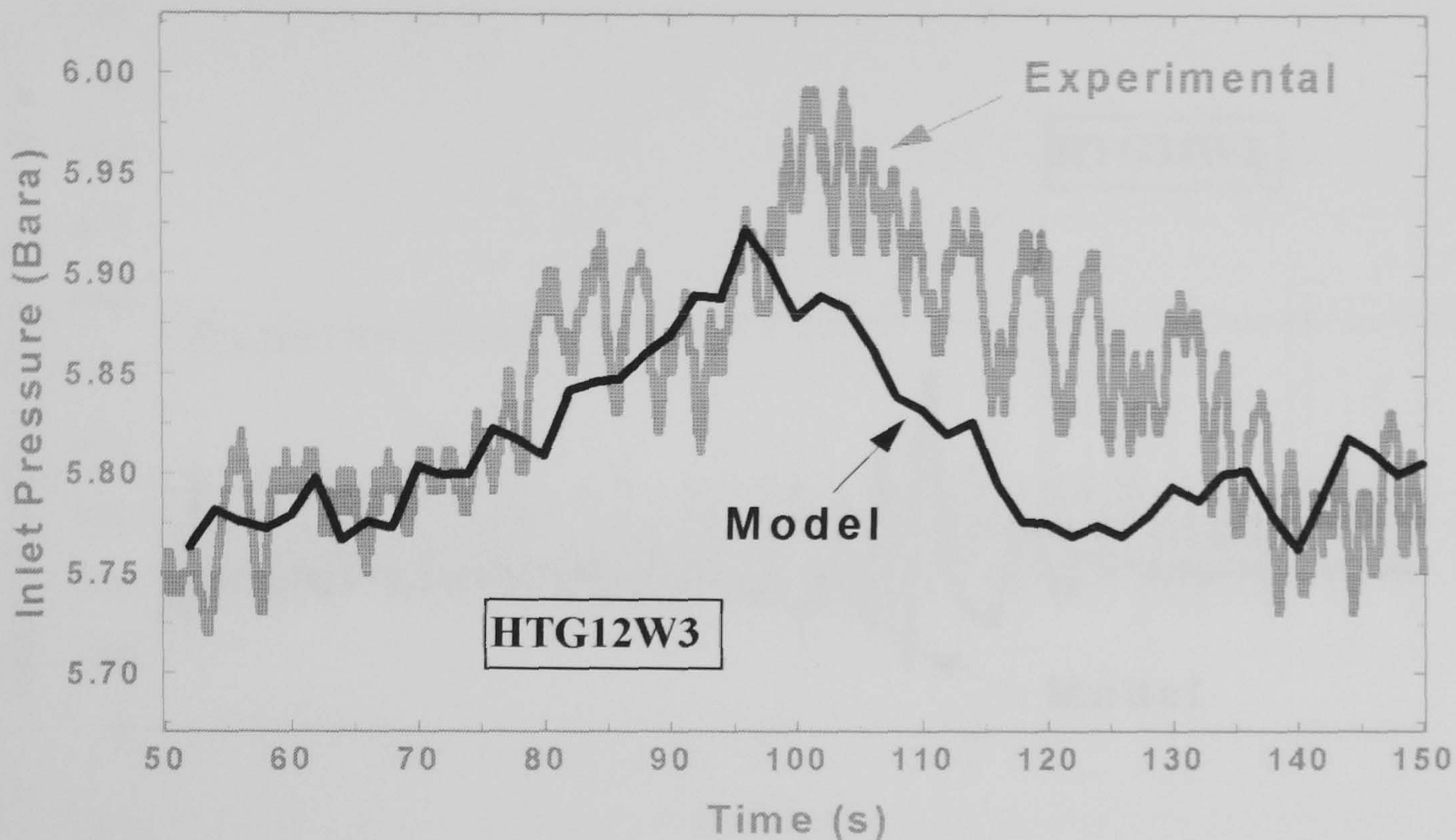


Figure 6.31 - Measured values and model prediction for the inlet pressure with time

6.4.2 Run HTG34W2

This transient run is created by reducing the inlet gas flow rate and reducing the separator pressure. The initial inlet gas flow rate is $0.0145 \text{ m}^3/\text{s}$ and separator pressure is 5.8 Bara. From 130s to 170s the gas flow rate is reduced to $0.012 \text{ m}^3/\text{s}$. The final separator pressure is 4.9 Bara. The initial and final density at the bottom of the riser is well predicted, however the model predicted some oscillation around 170s that was not observed as shown in Fig. 6.32. The inlet pressure prediction is higher than predicted, however the final inlet pressure is well predicted as shown in Fig. 6.33.

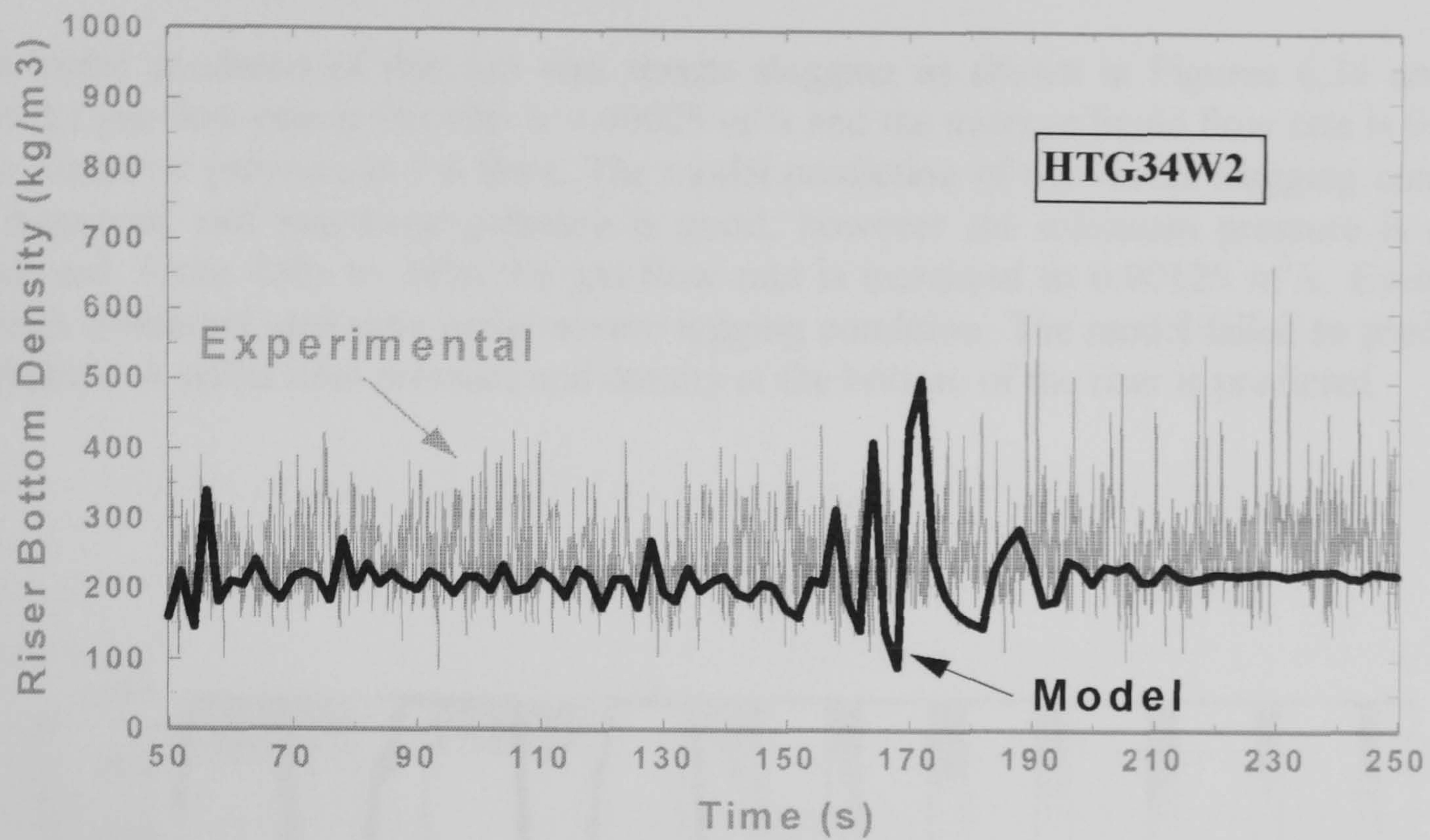


Figure 6.32 - Measured values and model prediction for the riser bottom density with time

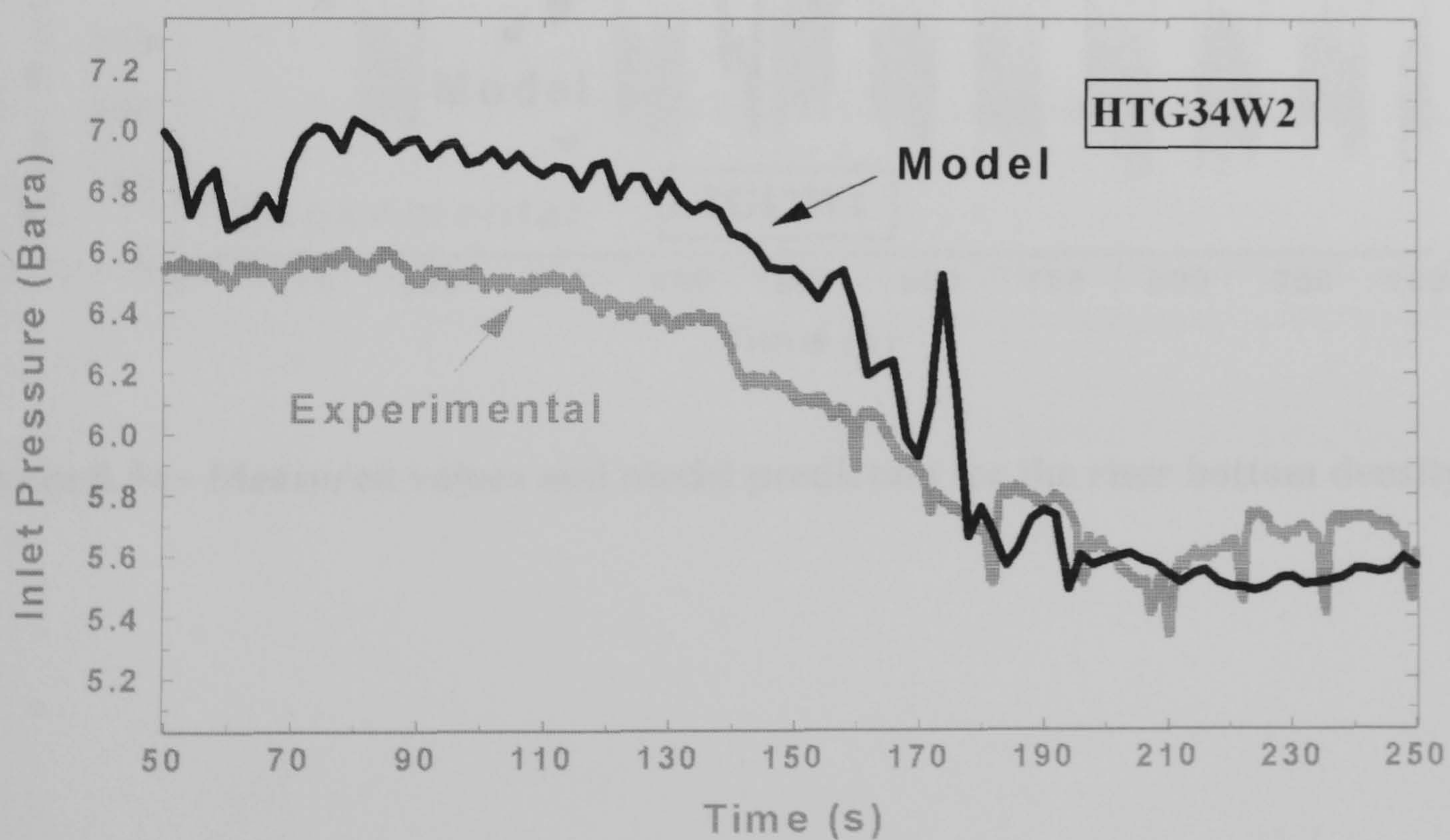


Figure 6.33 - Measured values and model prediction for the inlet pressure with time

6.4.3 Run LTG12W1

The initial condition of this run was severe slugging as shown in Figures 6.34 and 6.35. The average gas flow rate at the inlet is $0.00025 \text{ m}^3/\text{s}$ and the average liquid flow rate is $0.00033 \text{ m}^3/\text{s}$. The separator pressure is 1.9 Bara. The model prediction of the severe slugging conditions such as frequency and maximum pressure is good, however the minimum pressure is smaller than predicted. From 430s to 490s the gas flow rate is increased to $0.00125 \text{ m}^3/\text{s}$. Even though the system continued operating under severe lugging condition. The model failed to predict this final condition. A stable inlet pressure and density at the bottom of the riser is predicted.

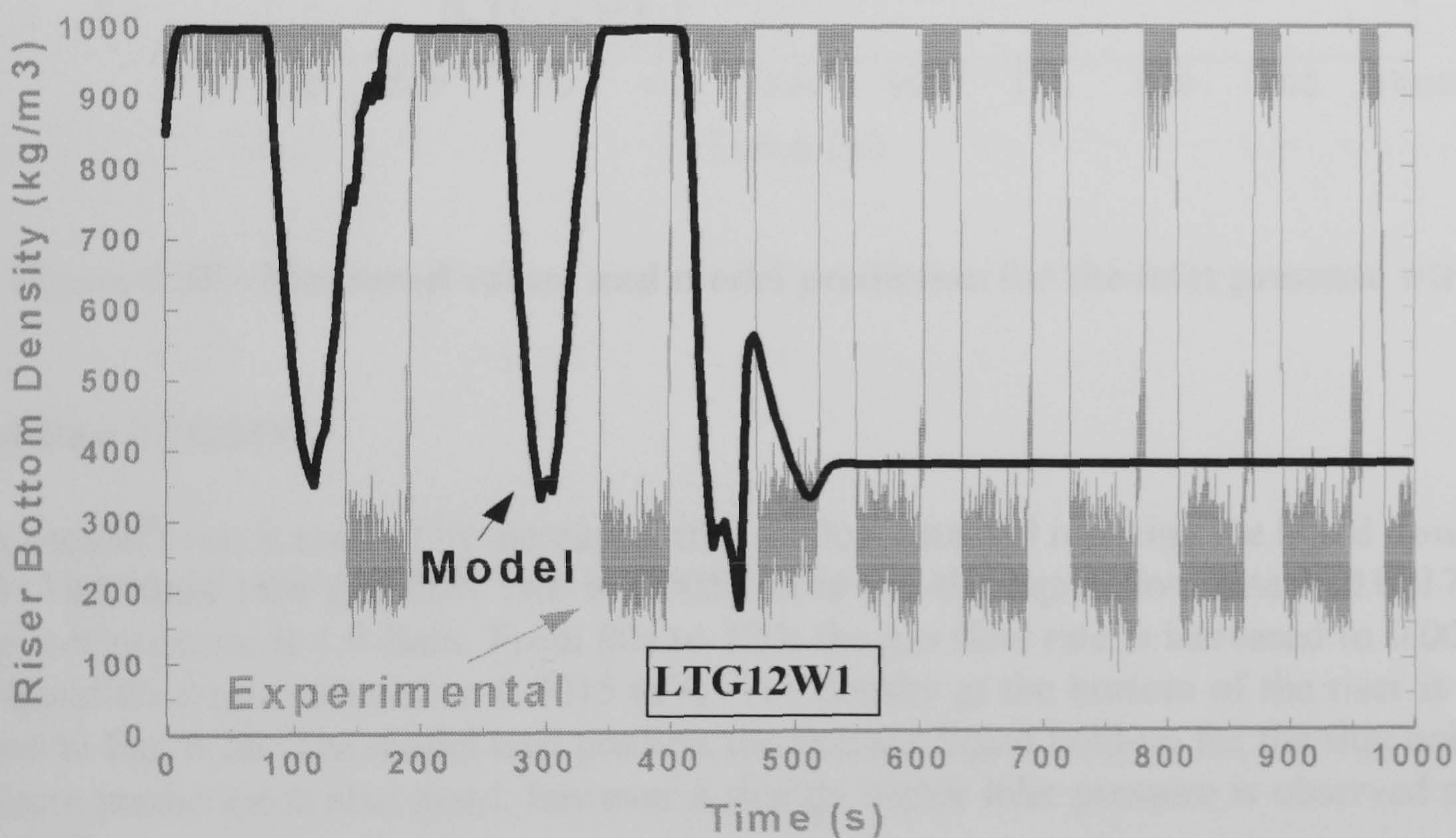


Figure 6.34 - Measured values and model prediction for the riser bottom density with time

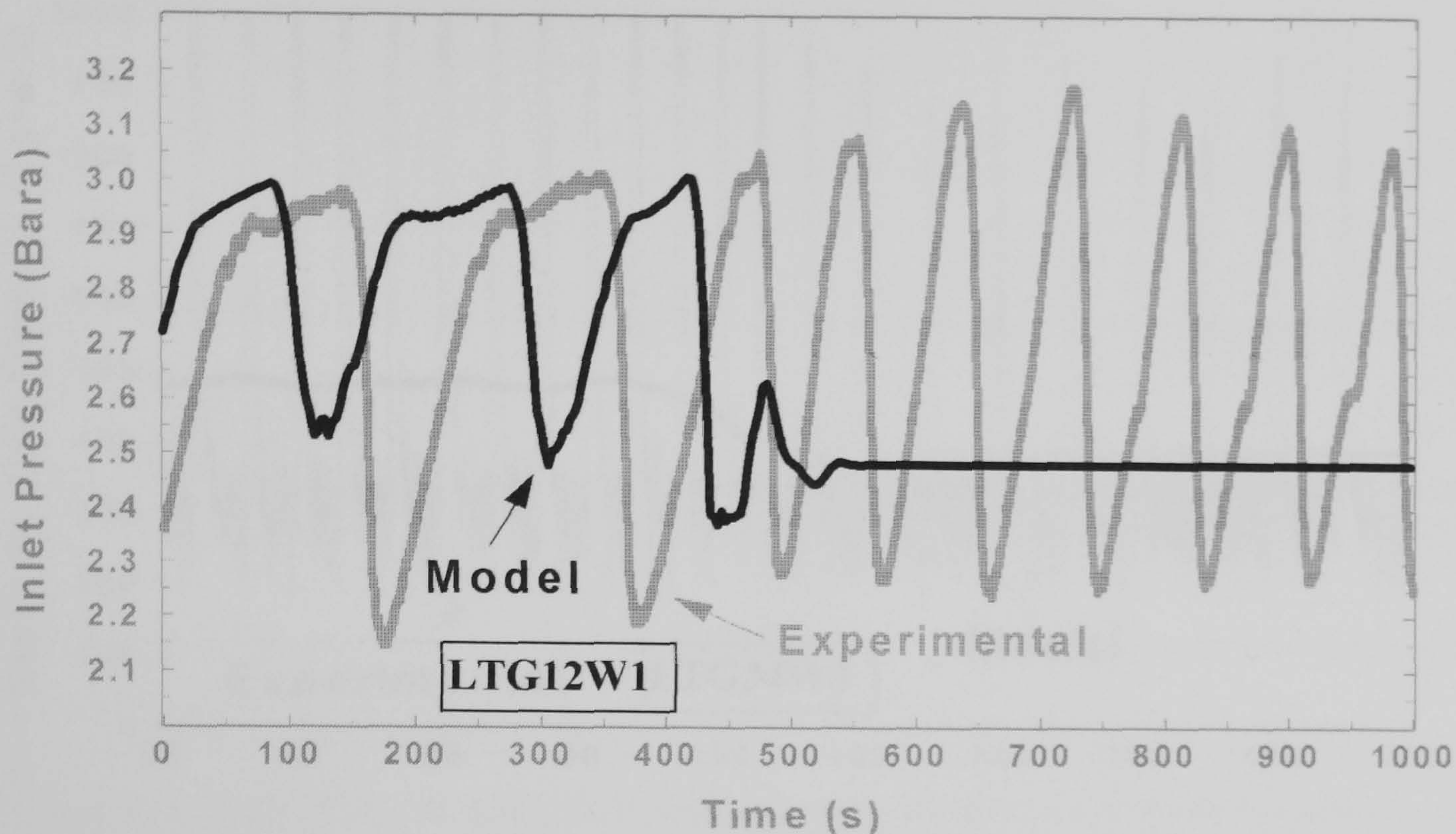


Figure 6.35 - Measured values and model prediction for the inlet pressure with time

6.4.4 Run LTG34W3

This transient run is created by increasing the gas flow rate and reducing the liquid flow rate at the inlet. The initial inlet gas flow rate is $0.0037 \text{ m}^3/\text{s}$ and the liquid flow rate is $0.0017 \text{ m}^3/\text{s}$. The separator pressure is 1.9 Bara. From 80s to 120s the gas flow rate is increased to $0.006 \text{ m}^3/\text{s}$ and the liquid flow rate reduced to $0.0015 \text{ m}^3/\text{s}$. The density at the bottom of the riser is reduced as shown in Fig. 6.36. The model well predicts the average liquid hold-up for the slug unit. The inlet pressure prediction is also good, however a slightly higher inlet pressure is observed as shown in Fig. 6.37.

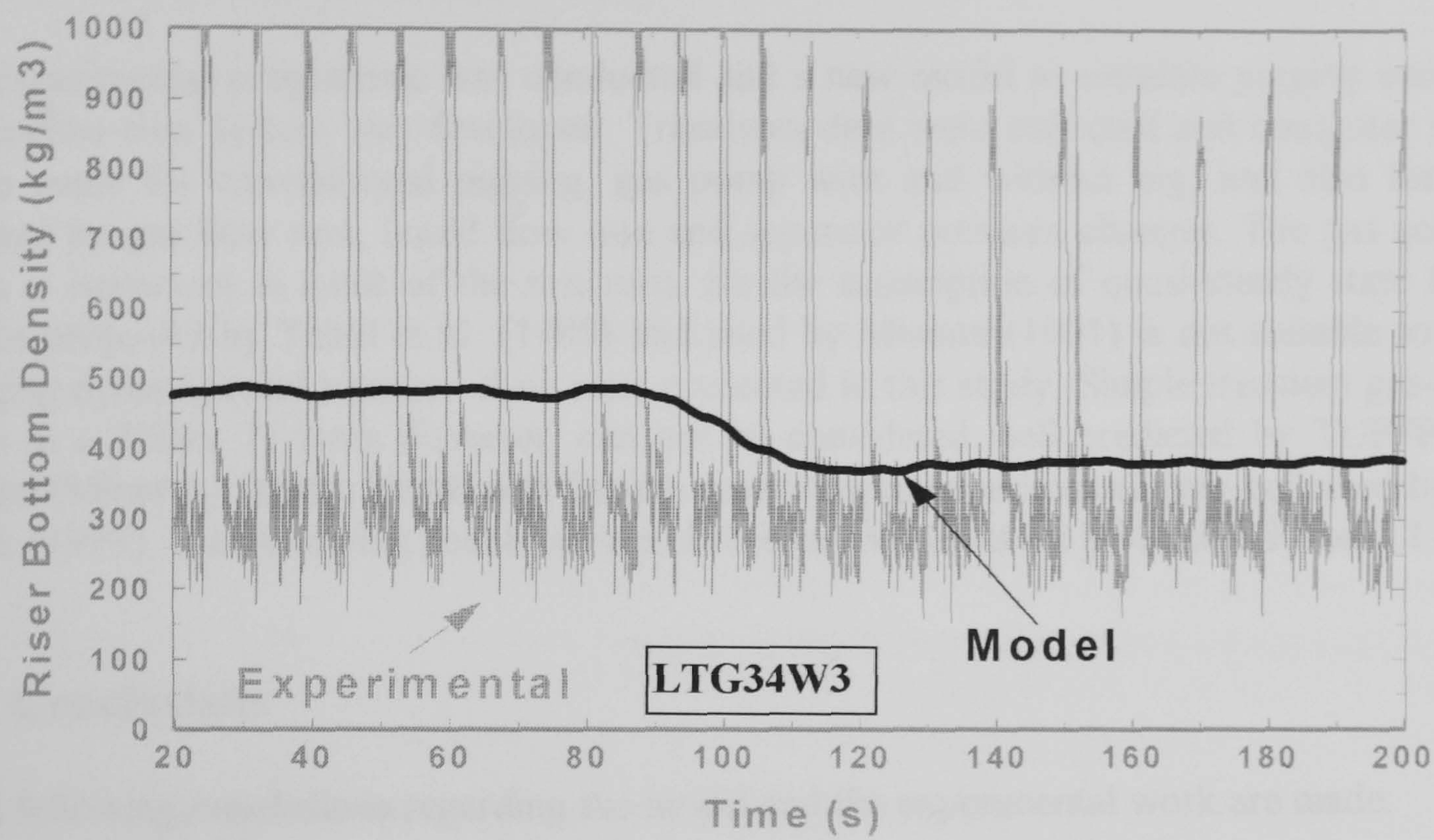


Figure 6.36 - Measured values and model prediction for the riser bottom density with time

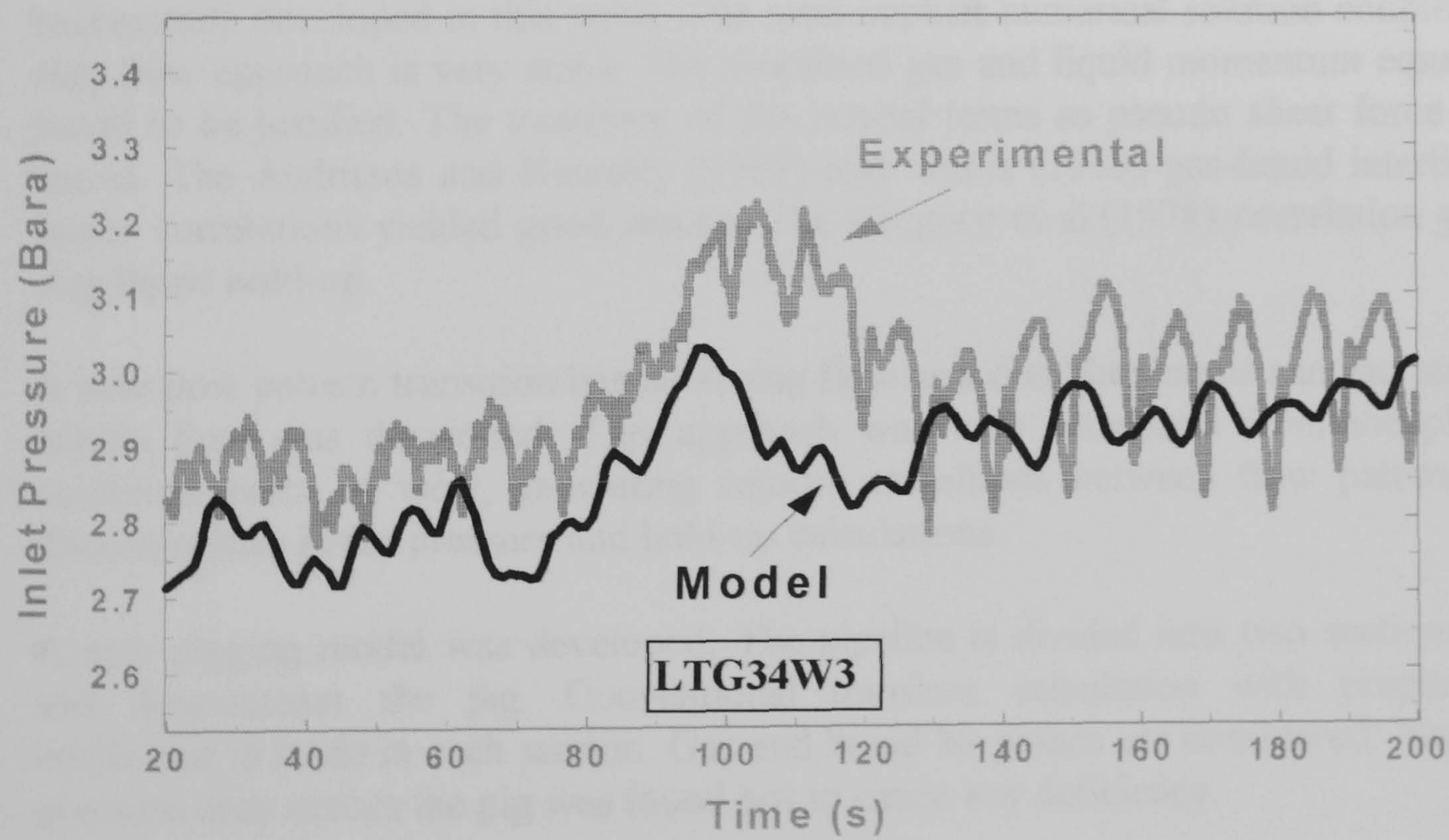


Figure 6.37 - Measured values and model prediction for the inlet pressure with time

7. CONCLUSIONS AND RECOMMENDATIONS

An experimental programme was conducted and a new model to simulate pigging and gas pump in pipeline-riser system was developed. Transients data were collected and computer simulations were made for conventional pigging, gas pump with and without pig, and also for transients created by gas flow rate, liquid flow rate and separator pressure changes. The gas accumulation term is important in most of the test runs. So the assumption of quasi-steady state for the gas phase proposed by Taitel et al. (1989) and used by Minami (1991) is not suitable to model the pigging dynamics and transient flow tests presented in this study. Simple transient gas-liquid flow tests in a 420m, 77.9mm diameter, can not be considered well predicted by TUFFP simplified model (Minami - 1991), PLAC and OLGA according to experimental data collected by Vigneron et al. (1995). The following conclusions and recommendations are given in 7.1 and 7.2.

7.1 Conclusions

The following conclusions regarding the model and the experimental work are made:

- (1) Transient gas-liquid flow and pigging data in a 0.0525 m diameter, 67 m long, 9.9 m high pipeline-riser system has been collected using a high speed computer based data acquisition system. Unprecedented detailed information on pipeline-riser transient and pigging dynamics was obtained.
- (2) A new transient two-fluid model based on flow pattern dependent set of equations was successfully developed in this work. The semi-implicit numerical solution coupled to a new slug flow approach is very stable. The simplified gas and liquid momentum equations were found to be justified. The treatment of the inertial terms as pseudo shear force was found sound. The Andritsos and Hanratty (1987) and Wallis (1969) gas-liquid interface friction factor correlations yielded good results. The Gregory et al.(1978) correlation yielded low slug liquid hold-up.
- (3) A new flow pattern transition based on slug flow as a combination of annular, stratified and bubble flow was developed. This approach was very successful from the physical and numerical point of view, generating smooth transitions between flow patterns avoiding discontinuities in the pressure and hold-up calculations.
- (4) A new pigging model was developed. The pipeline is divided into two sections: upstream and downstream the pig. Conventional transient calculation with proper boundary conditions is made in each section. Gas and liquid by-passes are considered. Neglecting the pressure drop across the pig was found not to cause any deficiency.

- (5) For conventional pigging the model predictions of the pig travel time and the inlet pressure are good. A smaller hold-up in front of the pig is predicted. The use of Gregory et al. correlation for the calculation of the liquid hold-up in the slug region is the main reason for this. It should be expected a high increase of the inlet pressure and high liquid and gas flow rates at the separator. Operational problems can be caused.
- (6) In case of gas pump without pig the simulated results matches the experimental data very well. A slightly greater than predicted efficiency was observed. This means that the gas phase transports more liquid than expected for liquid hold-up near 0.10. The reason for this is the fact that liquid droplets in the gas phase is neglected by the model. The gas-liquid ratio are smaller when compared with the conventional gas lift system.
- (7) For gas pump with pig runs the model predictions of inlet pressure are very good. However the model fails to predict a low density region between two high densities regions in front of the pig. However the model predicts well the average density and length of the gas-liquid mixture region in front of the pig. The predicted inlet pressure values matched the measured data very well.
- (8) Gas pump can be a efficient process of liquid transportation. However the observed and theoretical flow behaviours indicate that high liquid and gas flow rates at the separator should be expected, specially when pig is used. Efficiencies of 90% was obtained without pig, even for low gas-liquid ratio.

7.2 Recommendations

The following recommendations regarding the model and the experimental work are made:

- (1) The pigging model was found to be inaccurate in the prediction of the two regions of different liquid hold-up in front of the pig. The development of these regions can be better investigated by using gamma-densitometer in other points of the pipeline section.
- (2) Pig by-pass should be further investigated specially during the pig launching. A transparent pig launcher can provide a better understanding of the phenomena.
- (3) The model should be improved by using a more accurate pig by-pass model. A correlation based on a wider range of experimental data can be developed. There is a need to develop a liquid by-pass correlation based on pig velocity, fluid properties and pipe wall conditions.
- (4) In this work it was used just one type of pig. In the oil industry a huge variety of pigs is used. The evaluation of the effect induced in the gas-liquid flow by different types of pig is essential. The gas and liquid by-passes models for different pigs are important issues for pipeline-riser systems. High gas by-pass through the pig can reduce the severity of the problems detected in this work, allowing a safer pigging operation.
- (5) Different riser shapes, other inclination angles of the pipeline and different fluids should be tested. Due to the high level of instability of pipeline-riser systems, the flow regimes could be tremendously affected by different combinations of geometrical configurations and fluid properties.
- (6) The transient gas-liquid flow and pigging models developed in this work should be further tested against experimental data collected from larger and higher pressure pipeline-riser systems. The increase of pressure tends to eliminate the pressure and hold-up fluctuation. So it is important to perform a detailed investigation on the pressure effects.
- (7) Additional experimental verification of the proposed models is needed for transient flow conditions not covered in this work such as severe slugging. The simultaneous combination of severe slugging and pigging should also be investigated.

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APPENDIX A

EXPERIMENTAL DATA OF THE CONVENTIONAL PIGGING RUNS

A. EXPERIMENTAL DATA OF THE CONVENTIONAL PIGGING RUNS

A total of 13 conventional pigging runs are summarised in this appendix. Table A.1 presents the initial conditions of the steady state gas-liquid flow, pig launching schedules and other transient characteristics related to the runs. Figs A.1 to A.13 show the transient flow behaviour caused by a conventional pigging operation.

Table A.1 - Summary of the conventional pigging

RUN CODE	IN SITU GAS FLOW RATE (m ³ /s)	IN SITU LIQUID FLOW RATE (m ³ /s)	PRESSURE		PIG SCHEDULE	
			Inlet (Bara)	Separator (Bara)	Launching (s)	Arrival (s)
LPG3W1	0.0053	0.00033	2.3	1.9	510	558
LPG4W1	0.0125	0.00032	2.4	2.0	458	483
LPG3W2	0.0045	0.00098	2.7	2.0	288	330
LPG4W2	0.0120	0.00091	3.2	2.2	520	538
LPG3W3	0.0035	0.00170	2.8	1.9	193	232
LPG4W3	0.0080	0.00160	3.1	2.0	265	288
HPG2W1	0.0016	0.00054	4.7	4.2	271	350
HGP3W1	0.0057	0.00033	5.6	5.6	322	356
HPG4W1	0.0120	0.00049	5.8	5.8	118	134
HPG4W2	0.0110	0.00069	6.2	6.1	123	141
HPG2W3	0.0011	0.00200	6.2	6.2	440	480
HPG3W3	0.0050	0.00190	6.3	6.3	408	435
HPG4W3	0.0130	0.00135	6.9	6.7	94	108

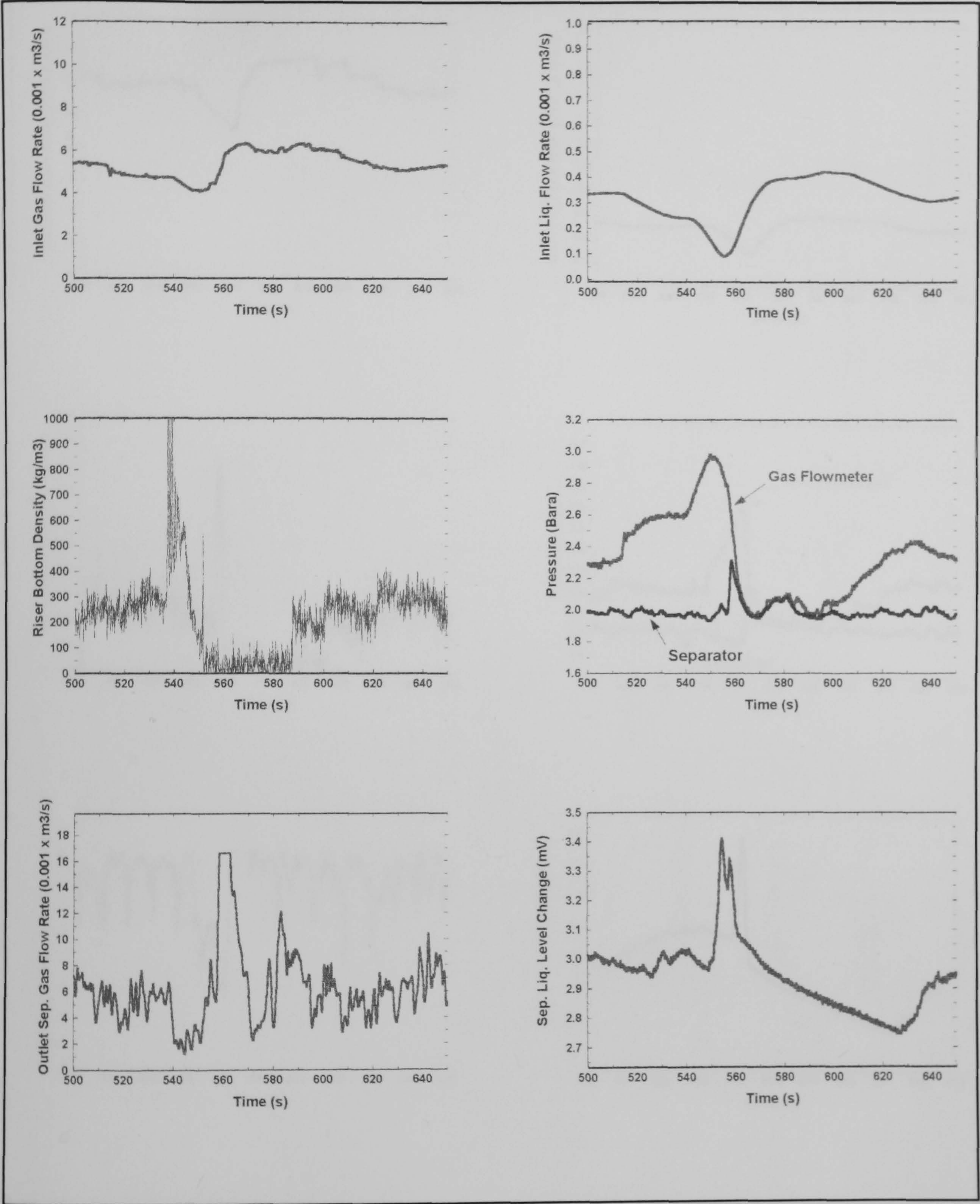


Figure A-1 - Summary of run LPG3W1

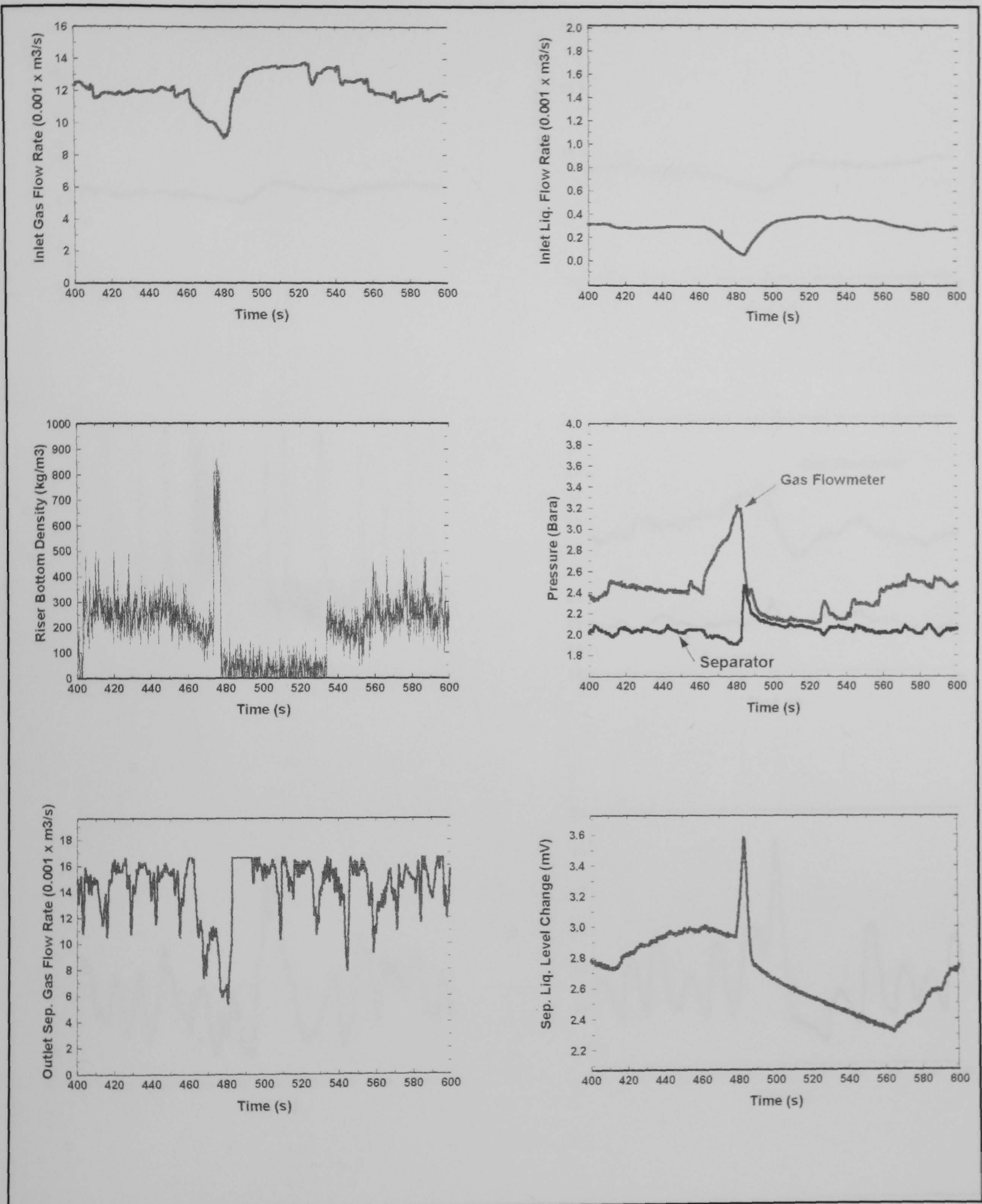


Figure A-2 - Summary of run LPG4W1

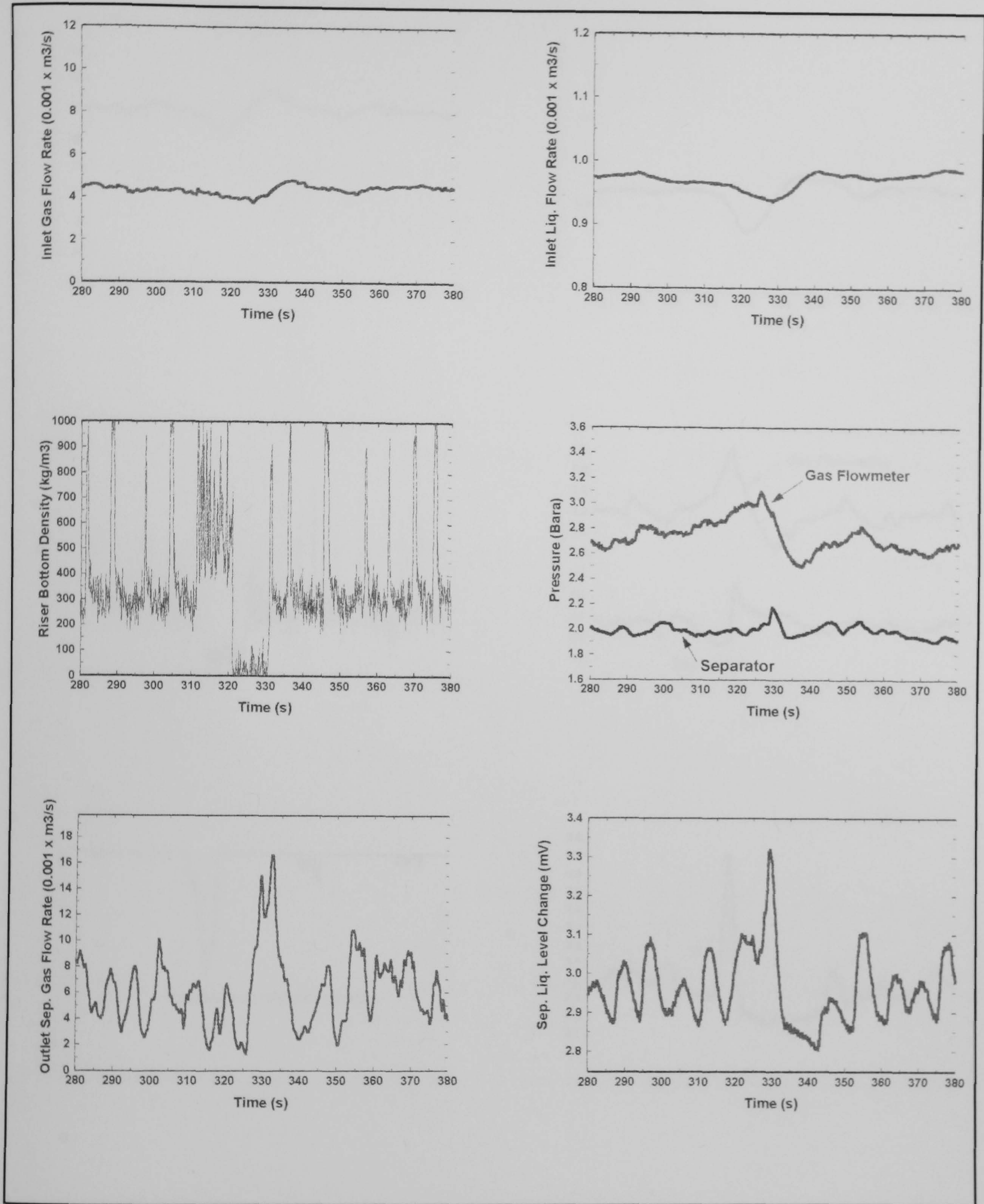


Figure A-3 - Summary of run LPG3W2

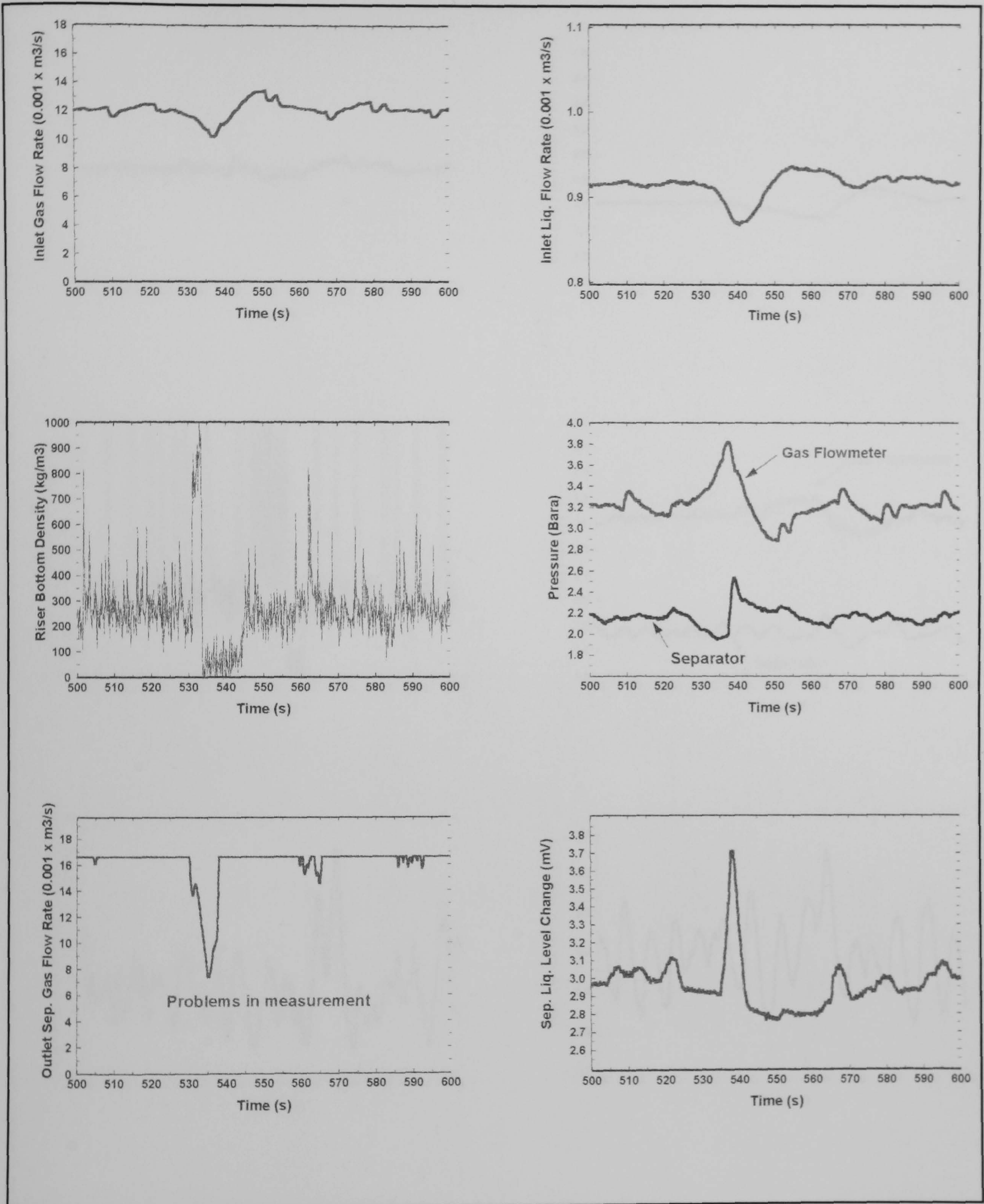


Figure A-4 - Summary of run LPG4W2

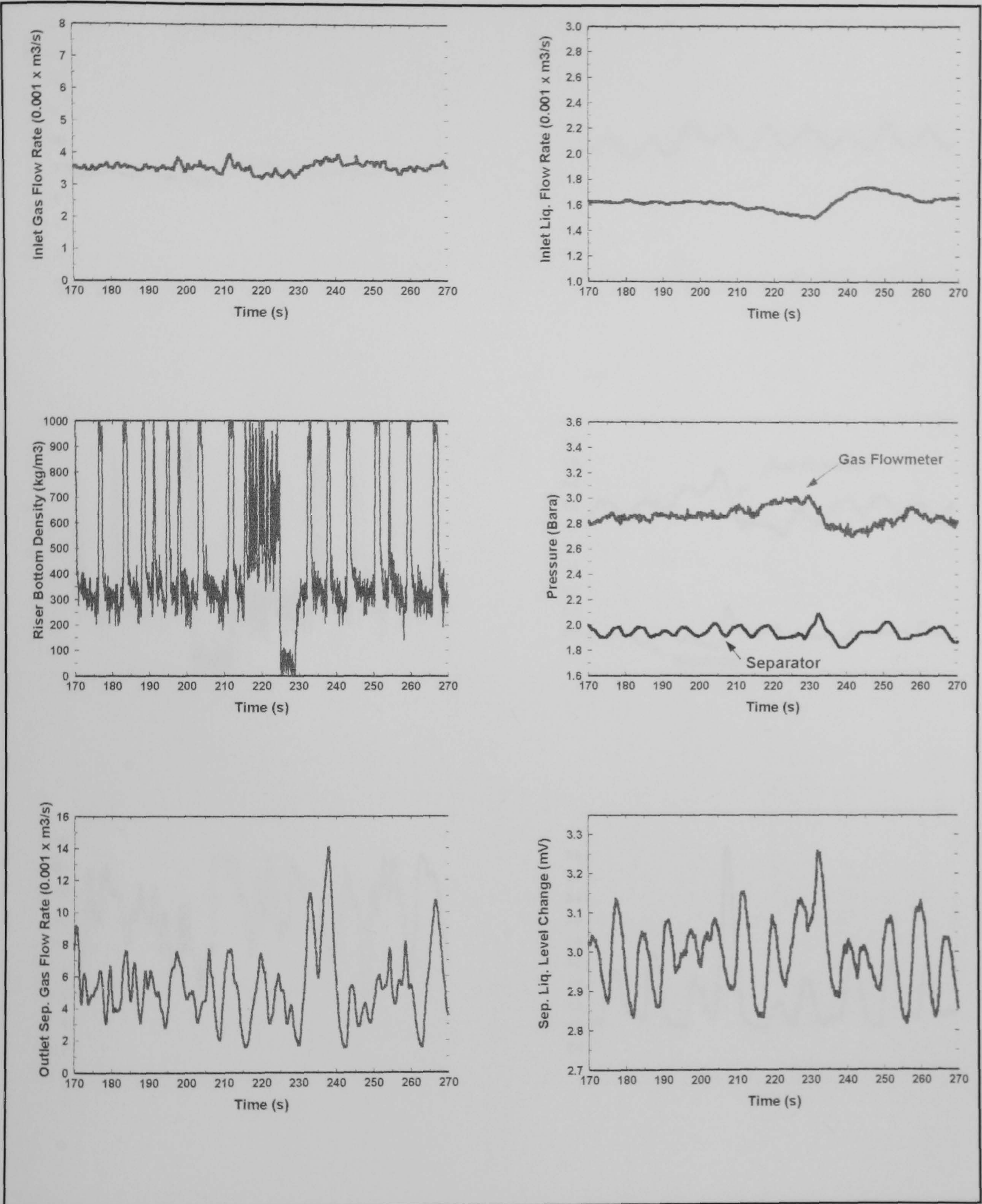


Figure A-5 - Summary of run LPG3W3

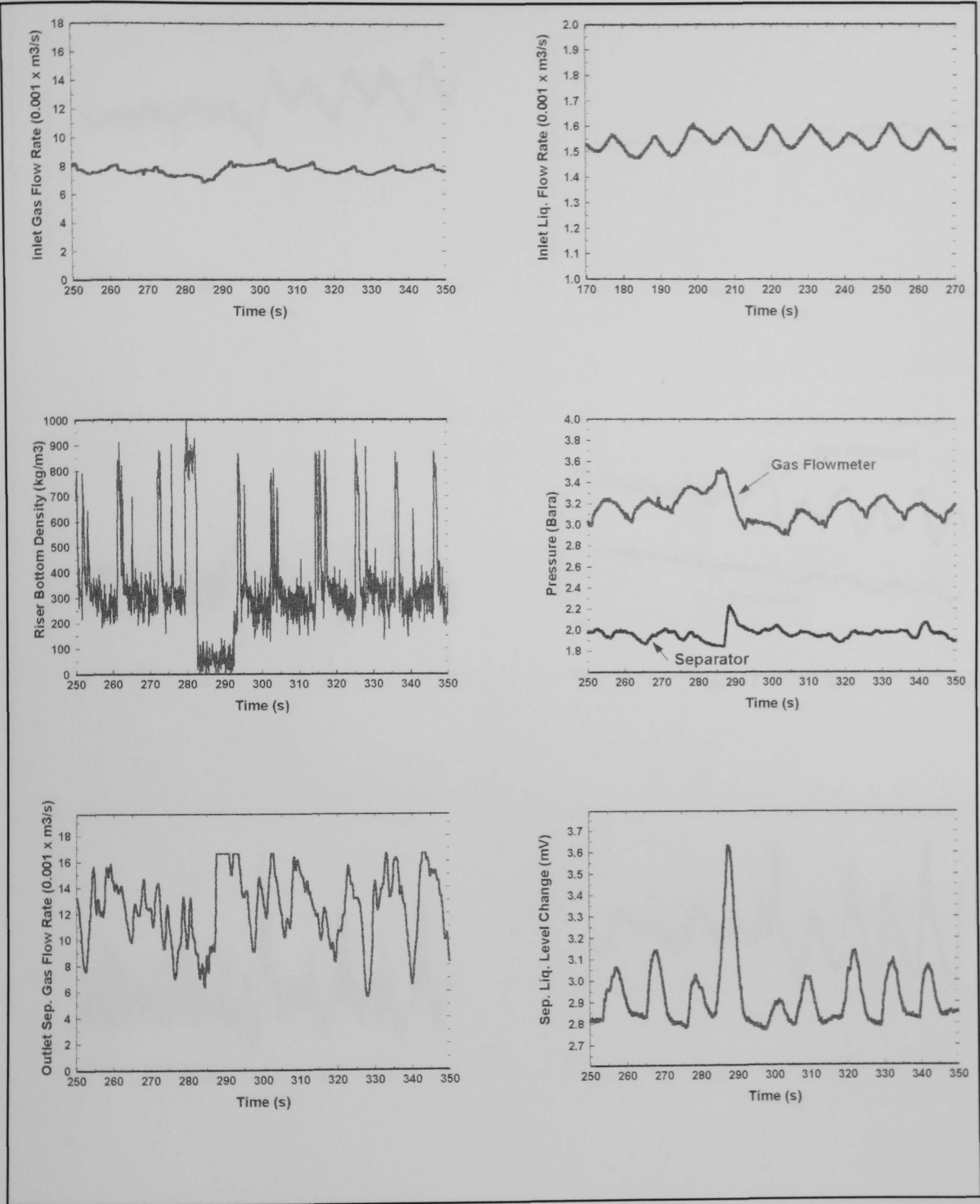


Figure A-6 - Summary of run LPG4W3

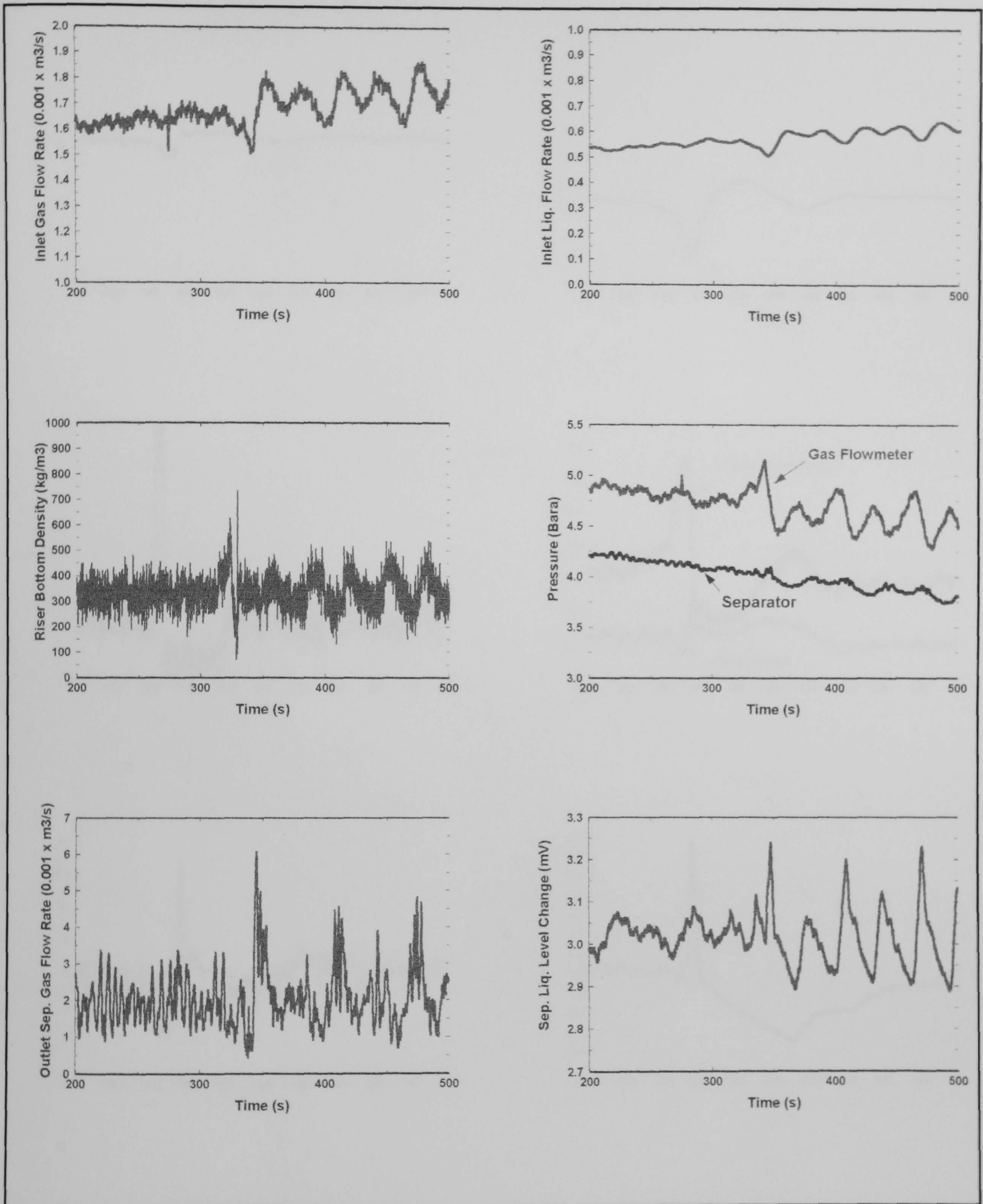


Figure A-7 - Summary of run HPG2W1

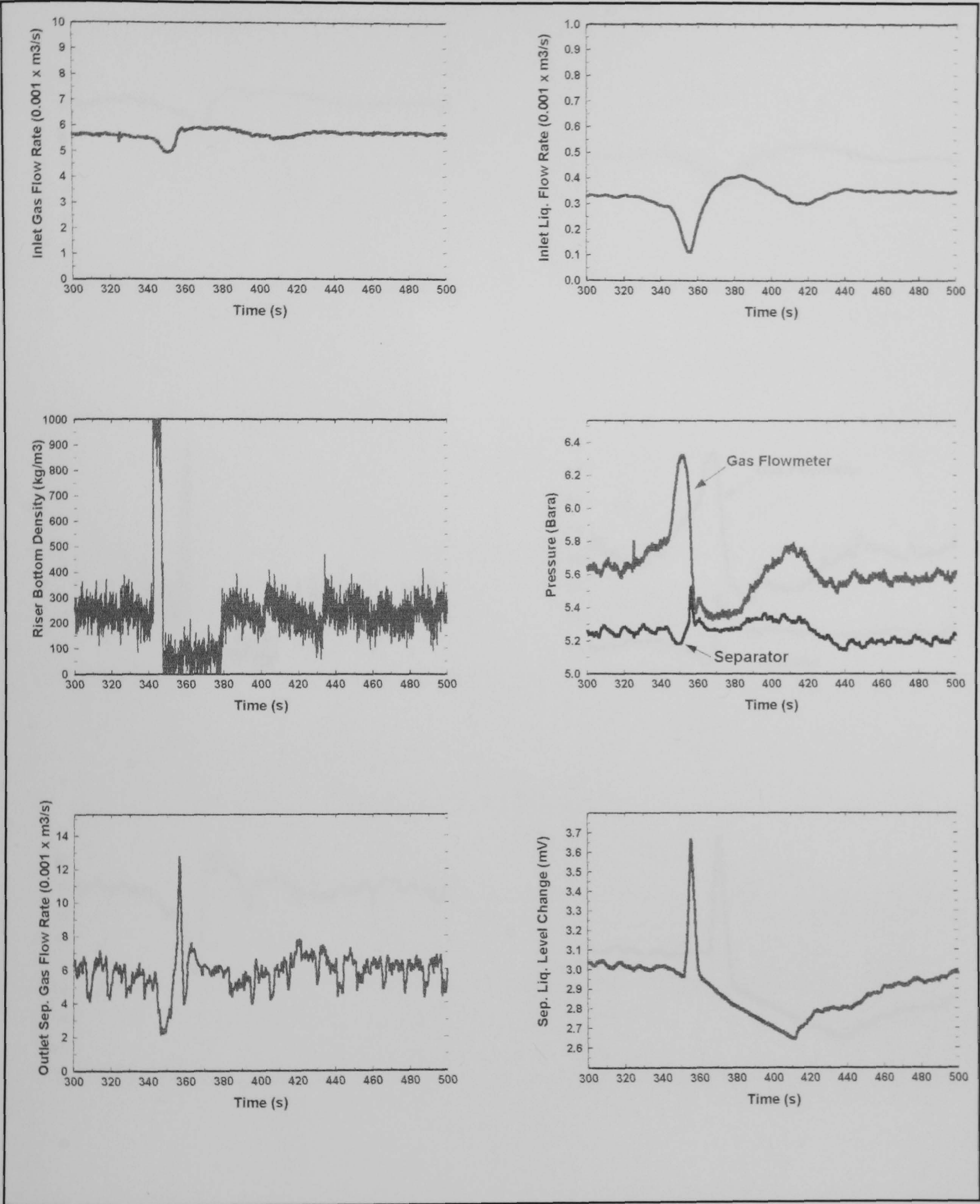


Figure A-8 - Summary of run HPG3W1

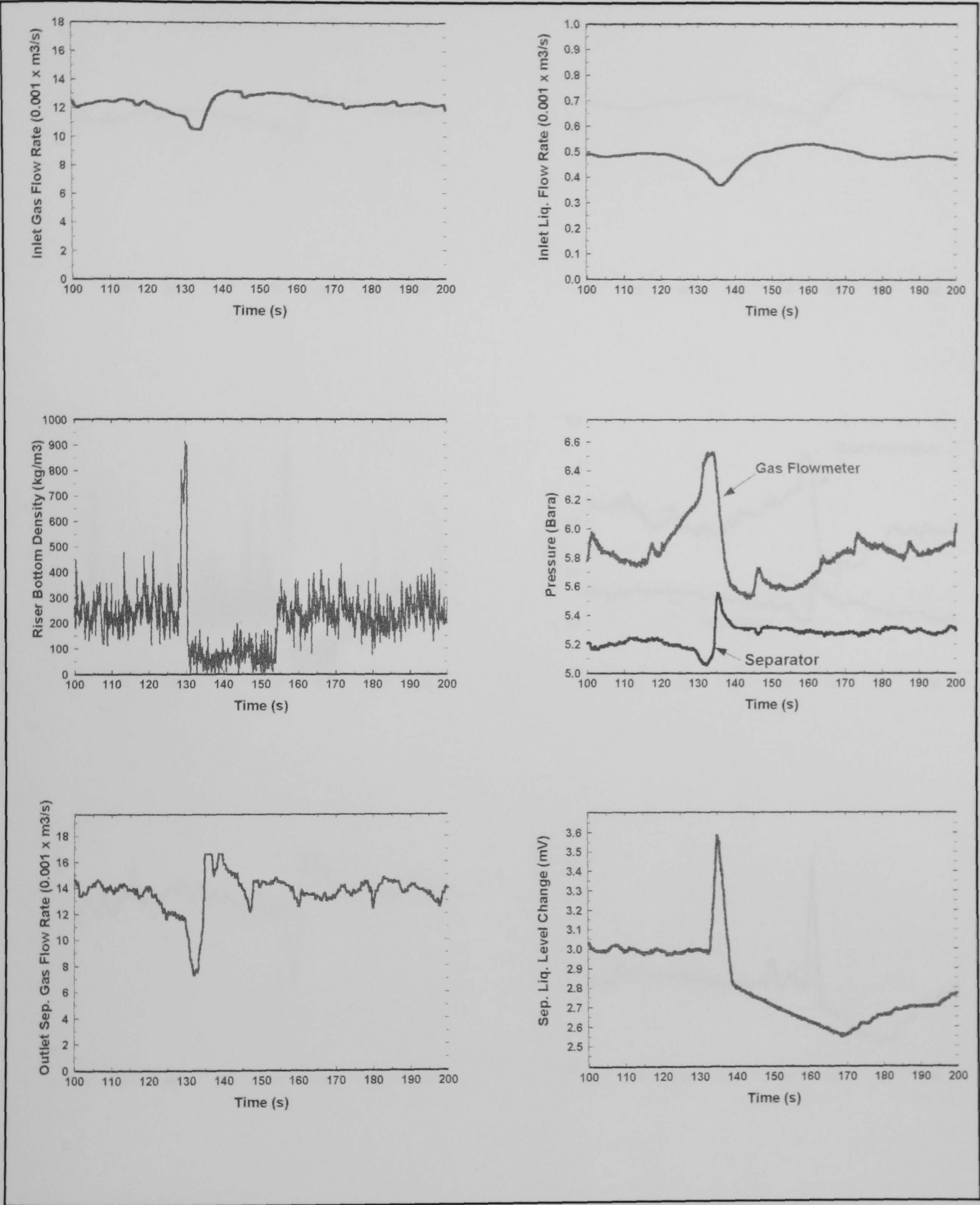


Figure A-9 - Summary of run HPG4W1

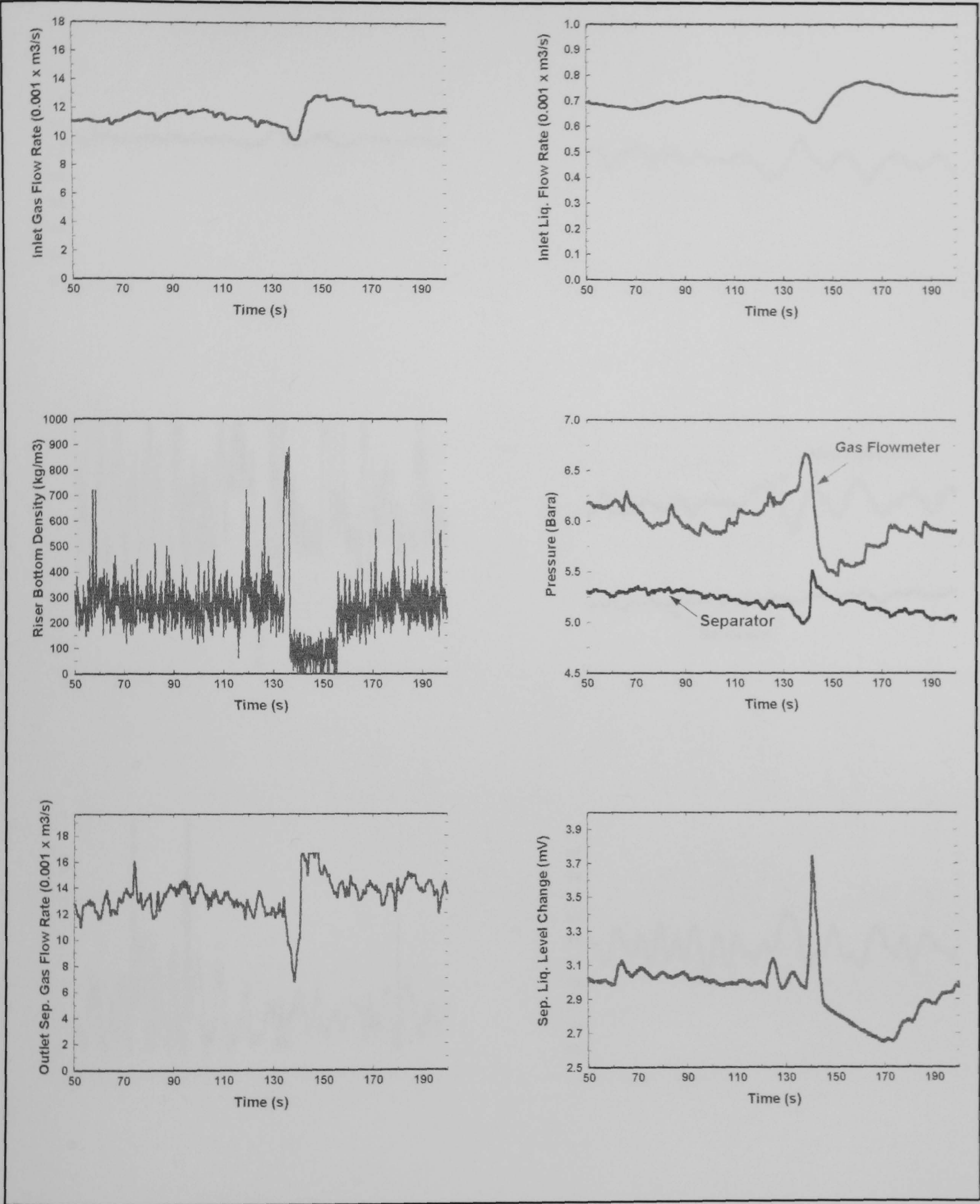


Figure A-10- Summary of run HPG4W2

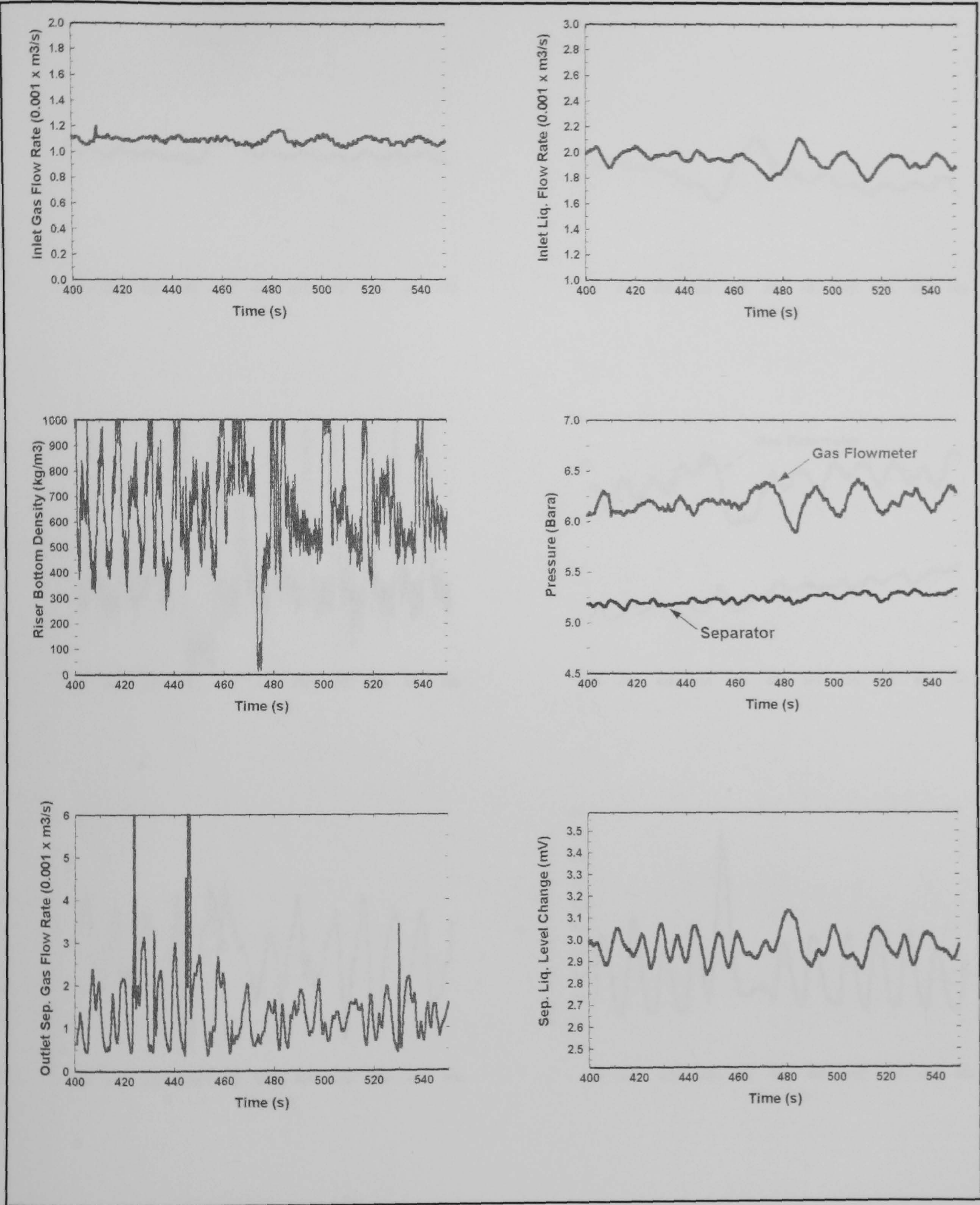


Figure A-11- Summary of run HPG2W3

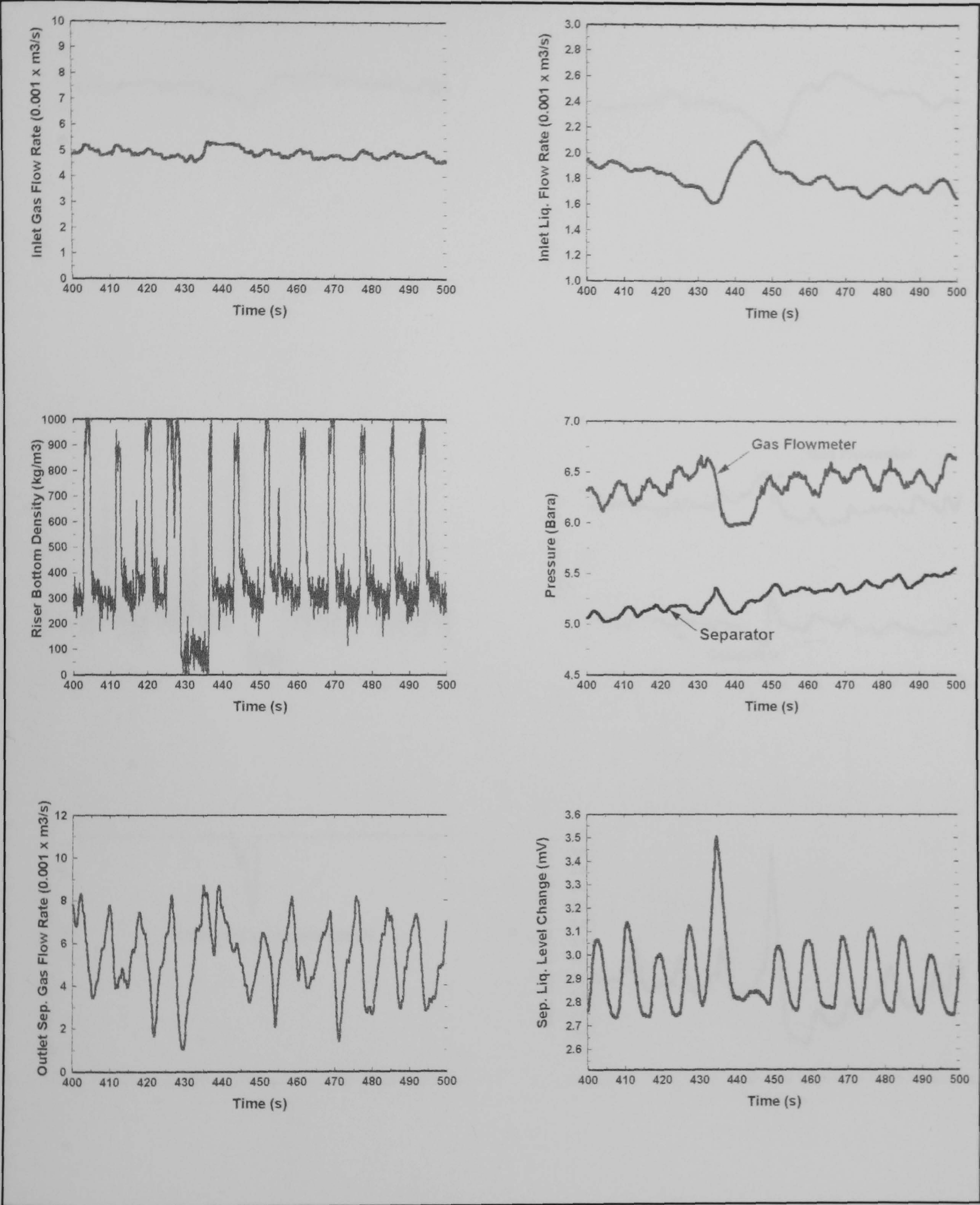


Figure A-12- Summary of run HPG3W3

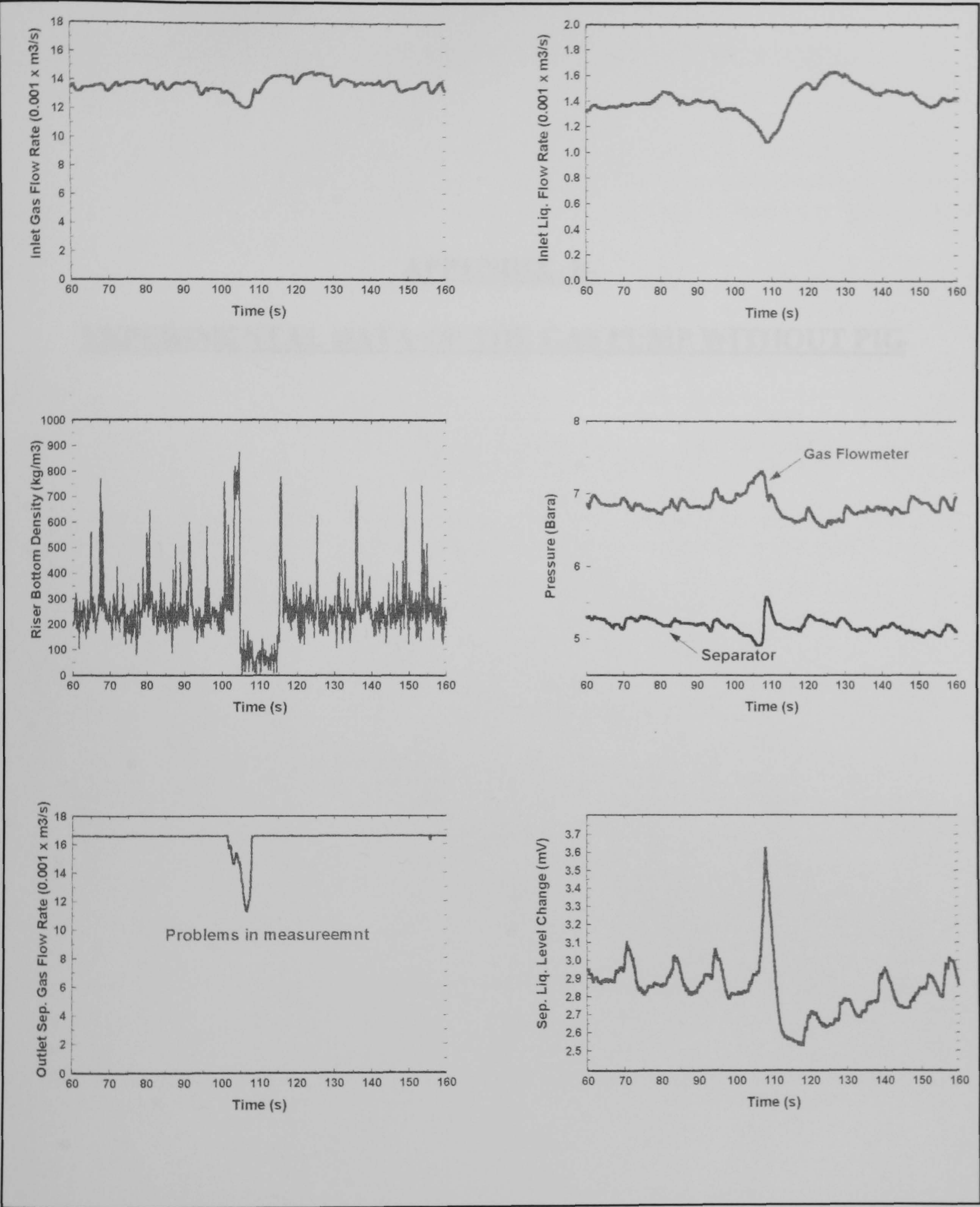


Figure A-13- Summary of run HPG4W3

B. EXPERIMENTAL DATA OF THE GAS PUMP WITHOUT PIG

A total of 6 gas pump runs without pig are summarised in this appendix. Table B.1 presents the initial conditions of the test section, the gas injection conditions and the efficiency of liquid transportation. Figs B.1 to B.6 show the transient flow behaviour caused by a gas injection, without pig, in a pipeline-riser system full of liquid.

Table B.1 - Summary of the gas pump without pig

RUN CODE	INITIAL INLET PRESSURE (Bara)	INJECTED GAS		AVERAGE GAS INJECTION	EFFICIENCY (%)	GAS- LIQUID RATIO (Sm ³ /m ³)
		Volume (Sm ³)	Time* (s)	(Sm ³ /s)		
LGP1	2.52	0.233	29	0.00803	90.66	1.84
LGP2	2.51	0.521	29	0.01797	91.65	4.06
LGP3	2.60	0.721	28	0.02575	92.88	5.54
HGP1	6.03	-	55	-	93.59	-
HGP2	5.91	-	44	-	90.10	-
HGP3	5.90	0.627	28	0.02239	90.07	4.97

*Duration of gas injection

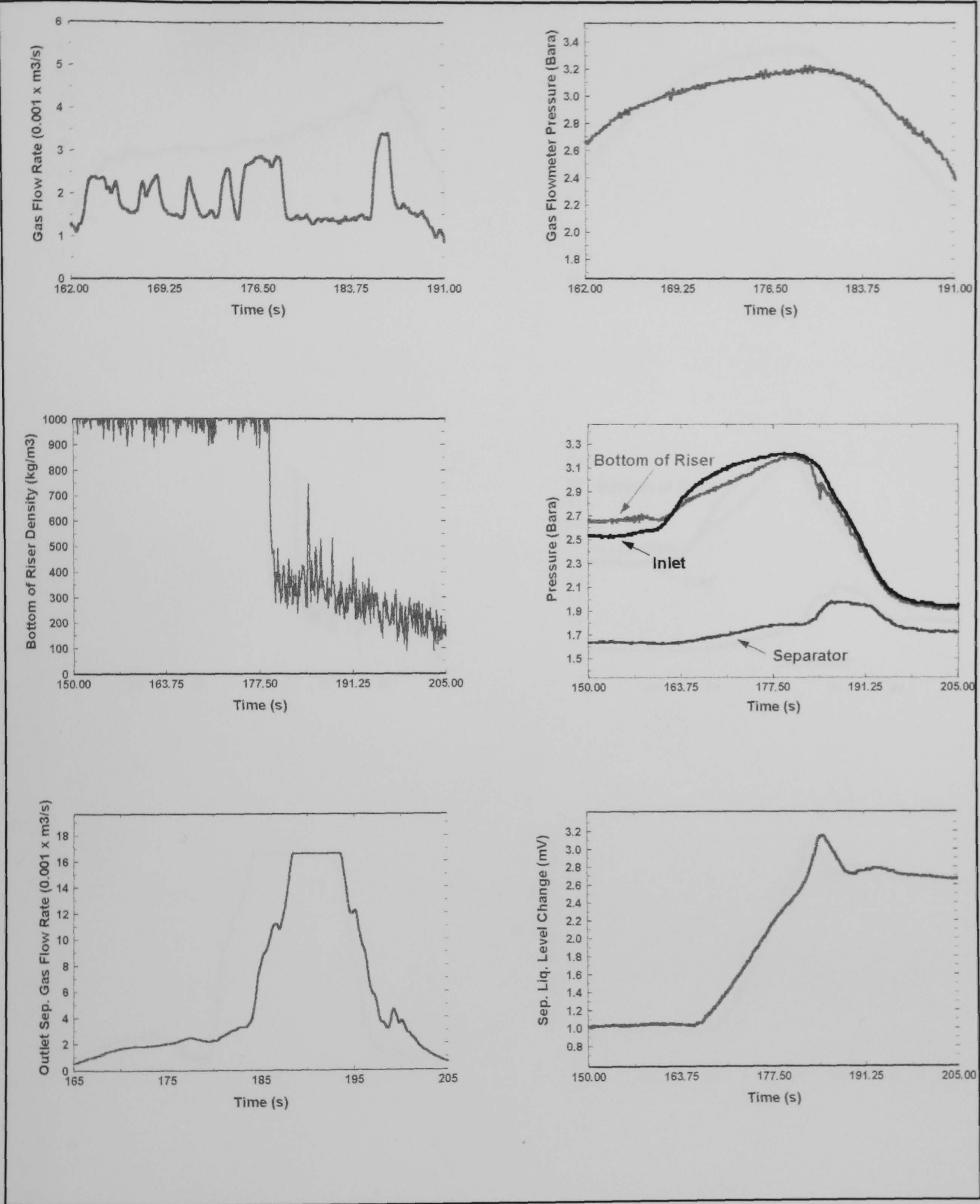


Figure B-1 - Summary of run LGP1

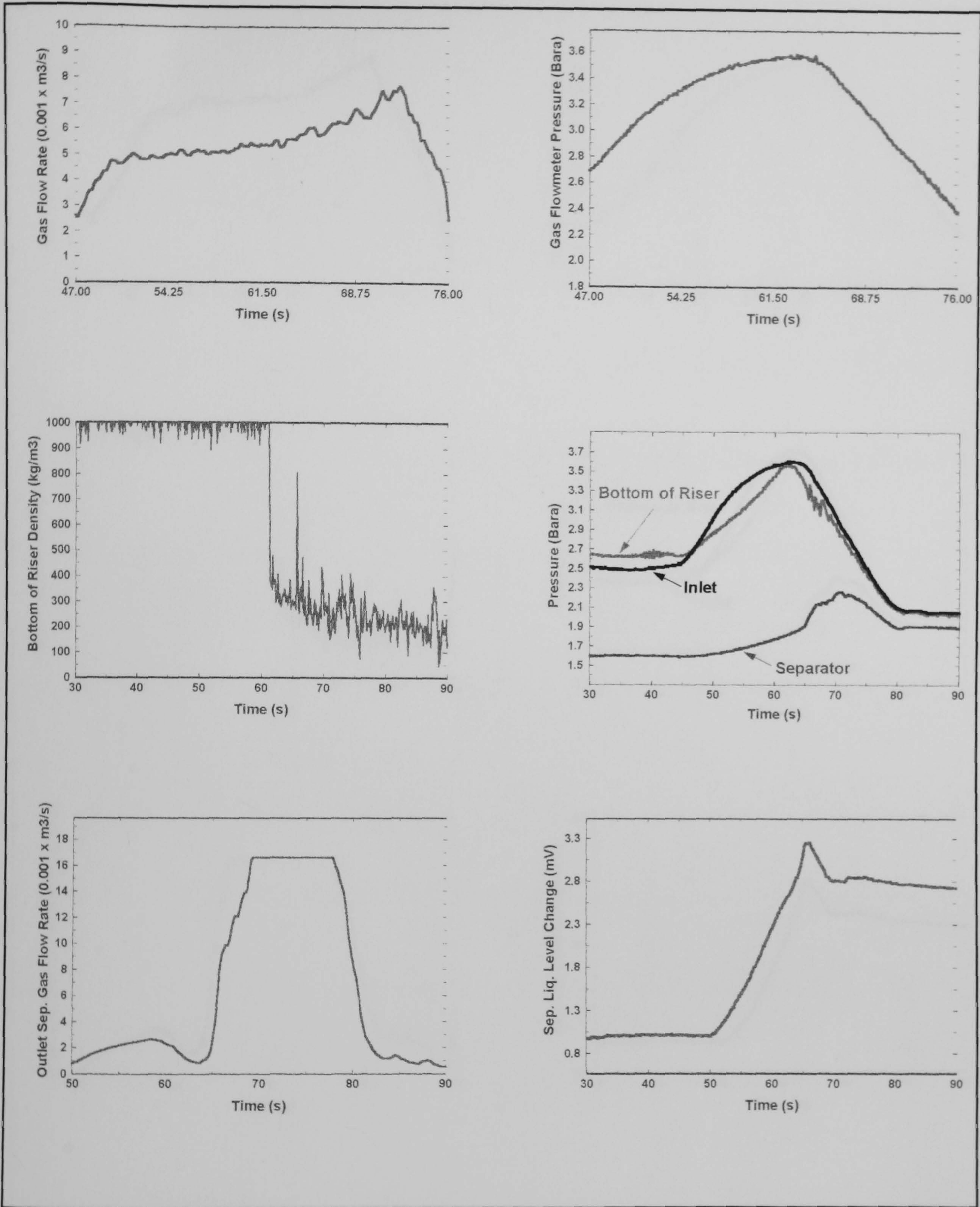


Figure B-2 - Summary of run LGP2

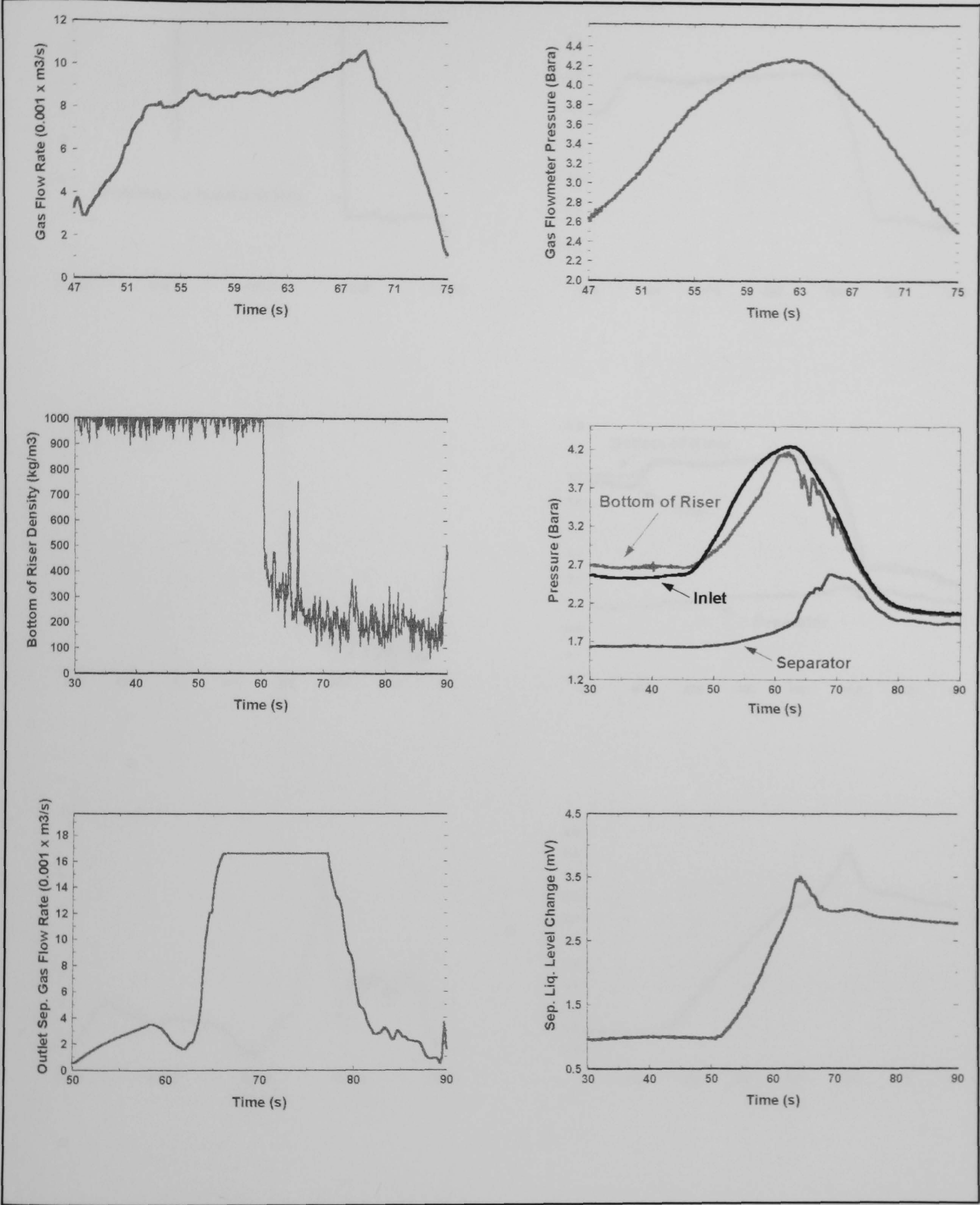


Figure B-3 - Summary of run LGP3

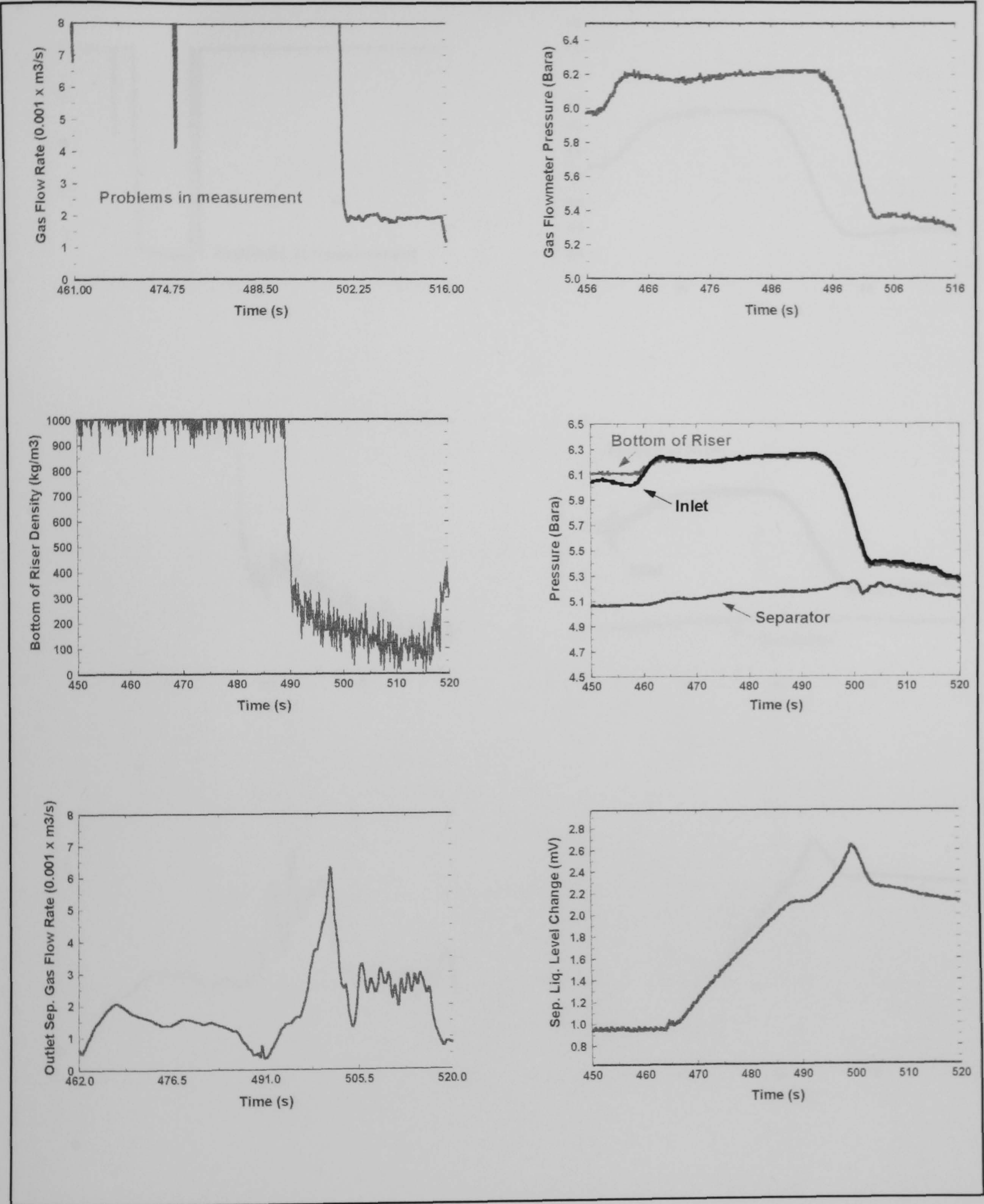


Figure B-4 - Summary of run HGP1

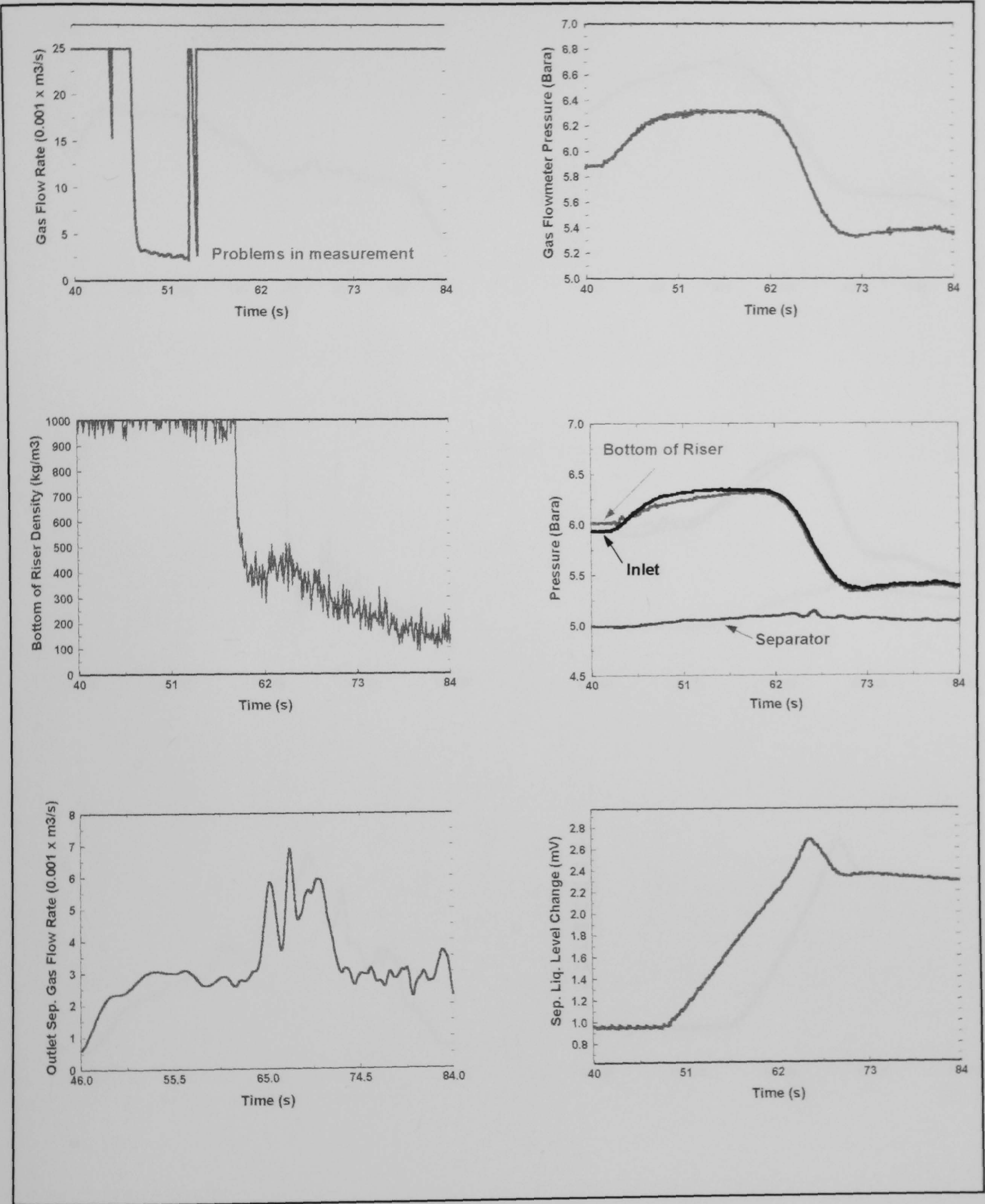


Figure B-5 - Summary of run HGP2

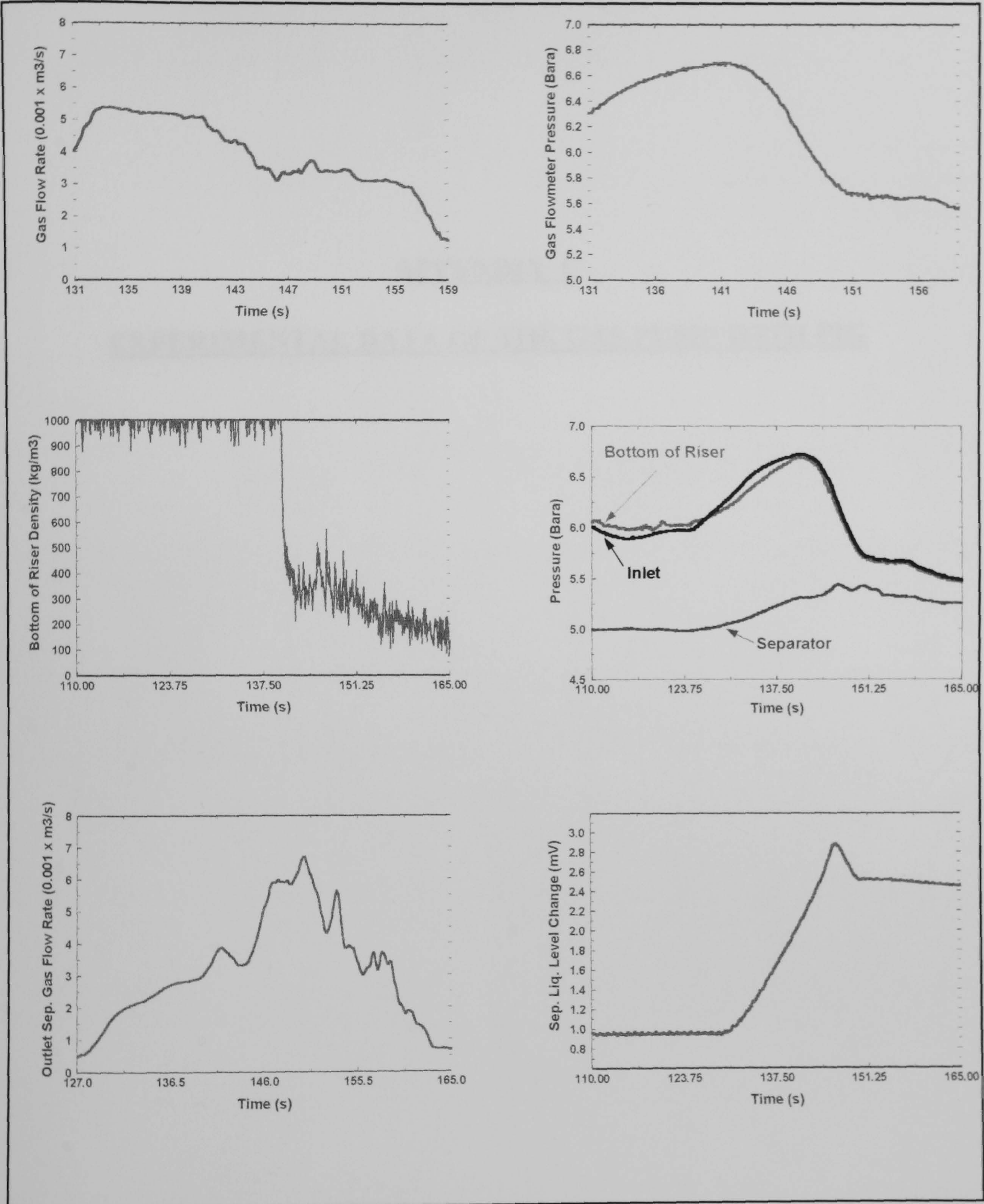


Figure B-6 - Summary of run HGP3

EXPERIMENTAL DATA OF THE GAS PUMP WITH PIG

Experimental data of the gas pump with pig is presented in Table 1. The data shows the variation of the gas flow rate, the gas pressure, the gas temperature and the gas density. The data is presented in the following table.

APPENDIX C

EXPERIMENTAL DATA OF THE GAS PUMP WITH PIG

TIME (min)	INITIAL TEMPERATURE (°C)		PRESSURE OF GAS (kPa)		PIG SPEED (m/s)		REFERENCE (%)
	Before (°C)	After (°C)	Before (kPa)	After (kPa)	Before (m/s)	After (m/s)	
15754	24	25	2.45	2.45	2.45	2.45	97.54
15771	24	25	2.45	2.45	2.45	2.45	97.54
15771	24	25	2.45	2.45	2.45	2.45	97.54

C. EXPERIMENTAL DATA OF THE GAS PUMP WITH PIG

A total of 3 gas pump runs with pig are summarised in this appendix. Table C.1 presents the initial conditions of the test section, the gas and pig injection conditions and the efficiency of liquid transportation. Figs C.1 to C.3 show the transient flow behaviour caused by a gas and pig injection in a pipeline-riser system full of liquid.

Table C.1 - Summary of the gas pump with pig

RUN CODE	INITIAL PRESSURE		VOLUME OF GAS INJECTED (Sm ³)	PIG SCHEDULE		EFFICIENCY (%)
	Inlet (Bara)	Separator (Bara)		Launching (s)	Arrival (s)	
LPP1A	2.4	1.5	0.466	305	345	97.54
LGPP2	2.4	1.5	0.641	242	274	97.63
LGPP3	2.5	1.6	0.824	195	220	98.63

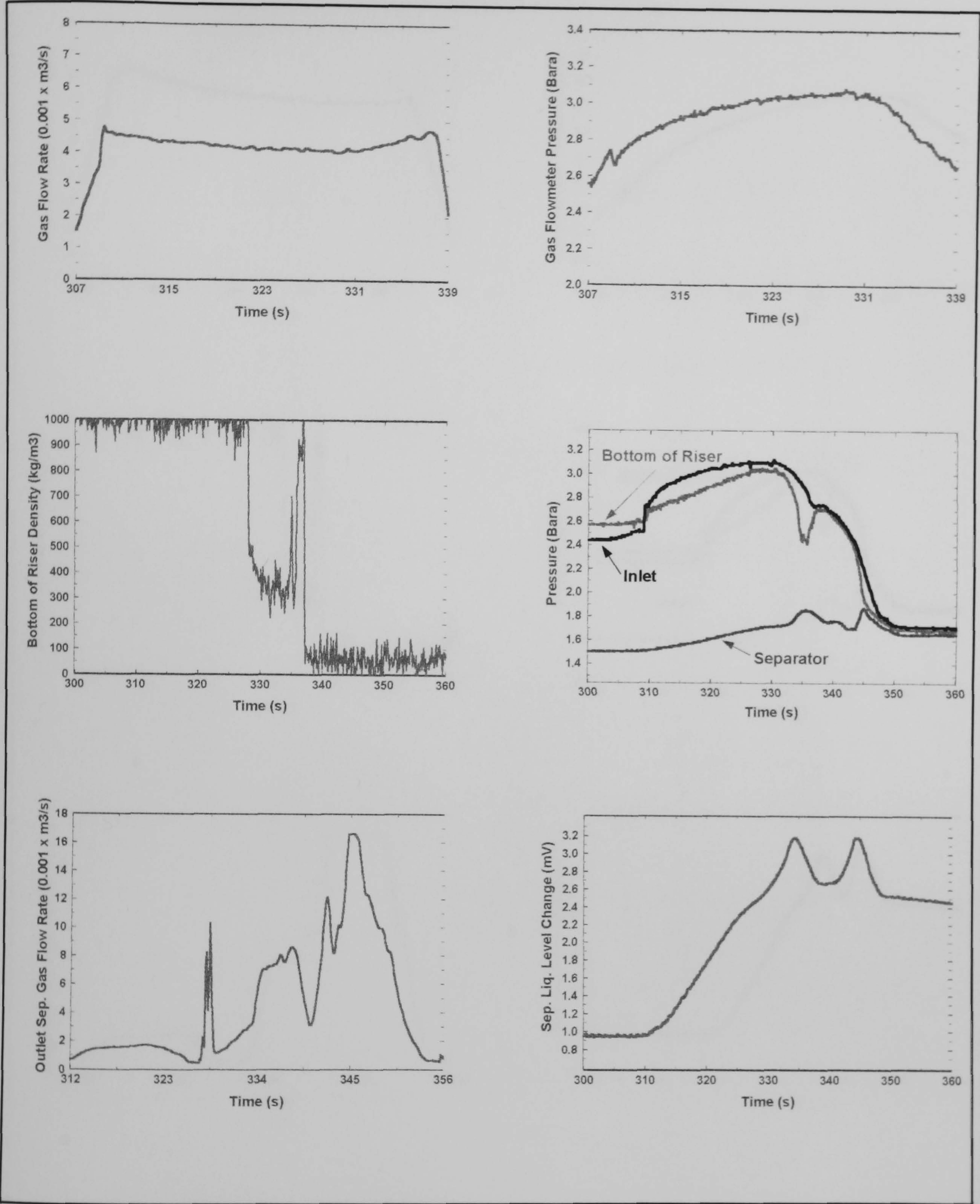


Figure C-1 - Summary of run LPP1A

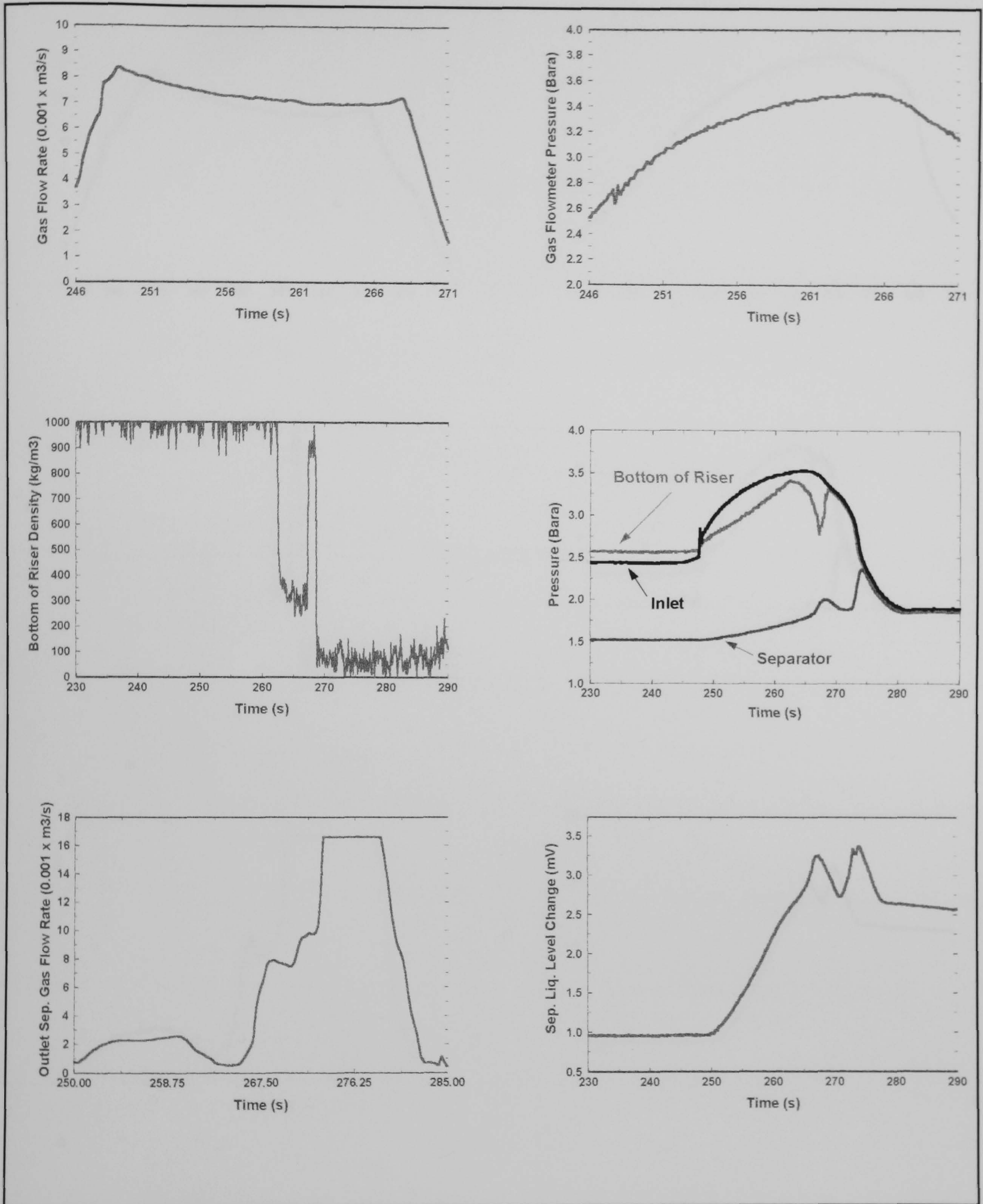


Figure C-2 - Summary of run LGPP2

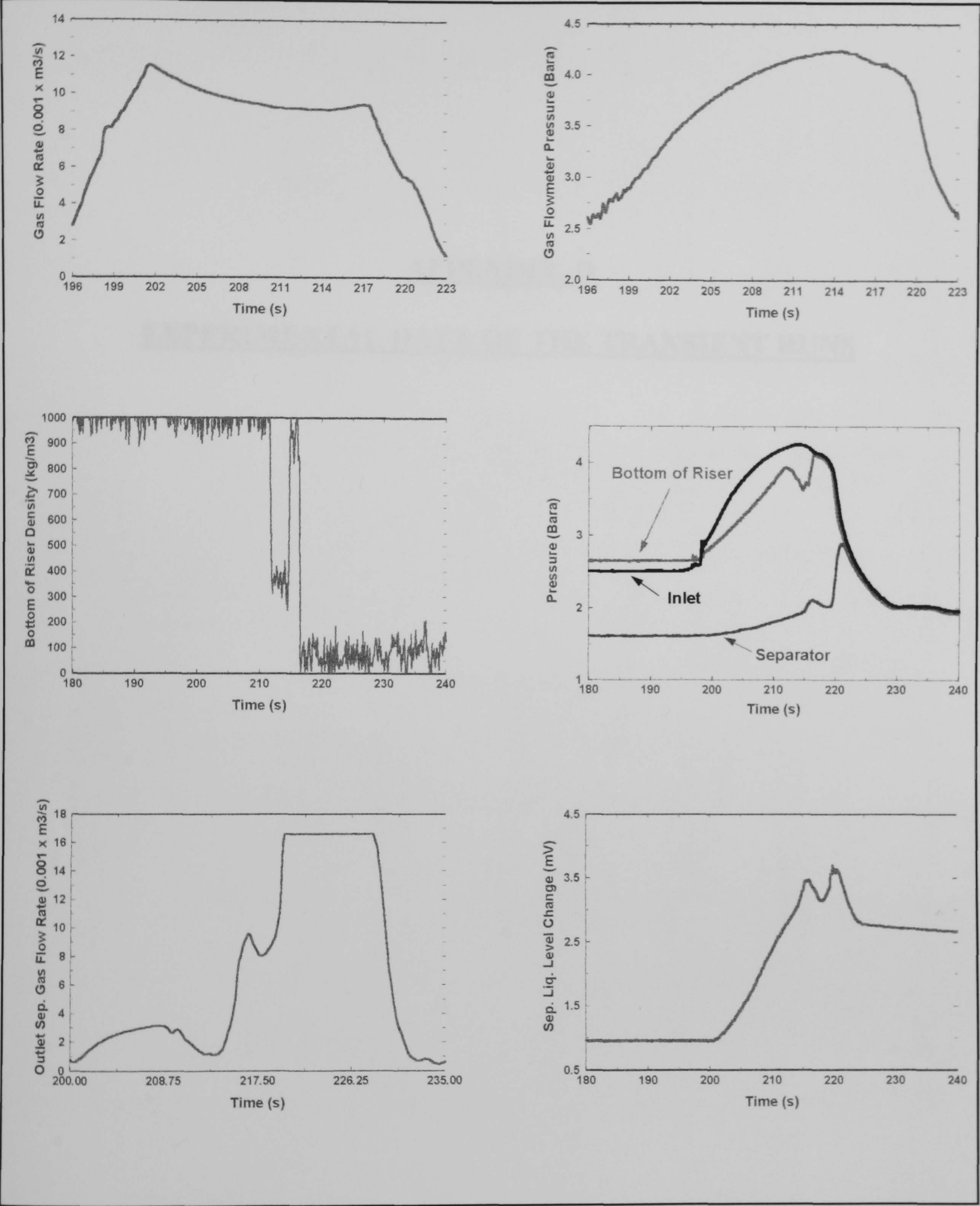


Figure C-3 - Summary of run LGPP3

APPENDIX D

EXPERIMENTAL DATA OF THE TRANSIENT RUNS

D. EXPERIMENTAL DATA OF TRANSIENT RUNS

A total of 10 transient runs with pig are summarised in this appendix. Table D.1 presents the initial and final conditions of the test section. Figs D.1 to D.10 show the transient flow behaviour caused by a change of boundary conditions.

Table D.1 - Summary of the transient runs

RUN CODE	INITIAL PRESSURE		FINAL PRESSURE	
	Inlet (Bara)	Separator (Bara)	Inlet (Bara)	Separator (Bara)
HTG12W3	5.75	4.65	5.75	4.75
HTG34W1	4.76	4.31	5.93	5.37
HTG34W2	6.55	5.88	5.60	4.90
HTG34W3	6.13	4.94	6.85	5.95
LTG12W1	2.60	1.80	2.60	2.00
LTG23W2	2.80	2.00	2.80	2.10
LTG34W2	2.70	2.05	3.20	2.20
LTG34W3	2.85	1.90	2.90	2.05
LTG43W1	2.35	2.05	2.30	2.00
LTG32W1	2.35	2.00	2.25	1.95

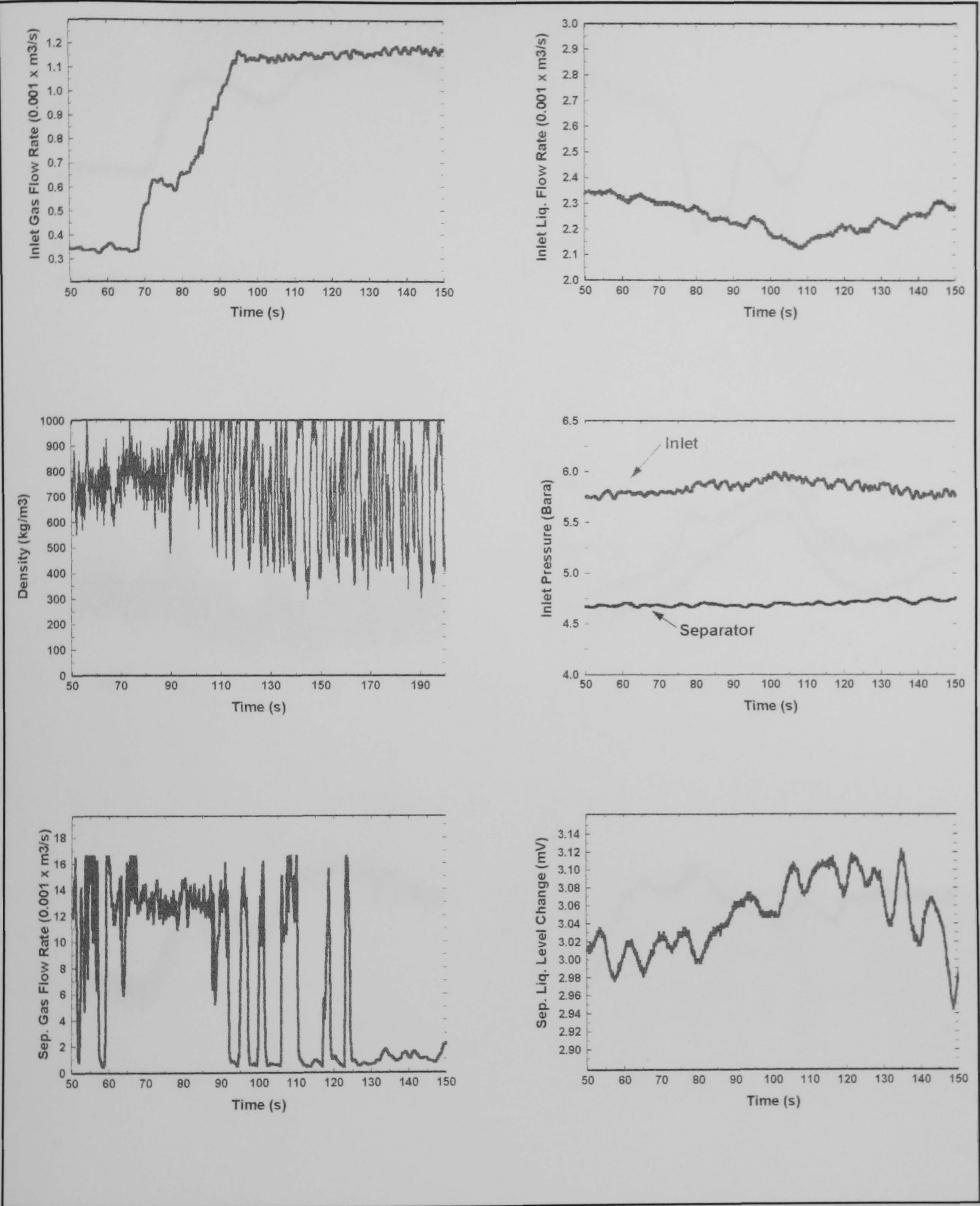


Figure D-1 - Summary of run HTG12W3

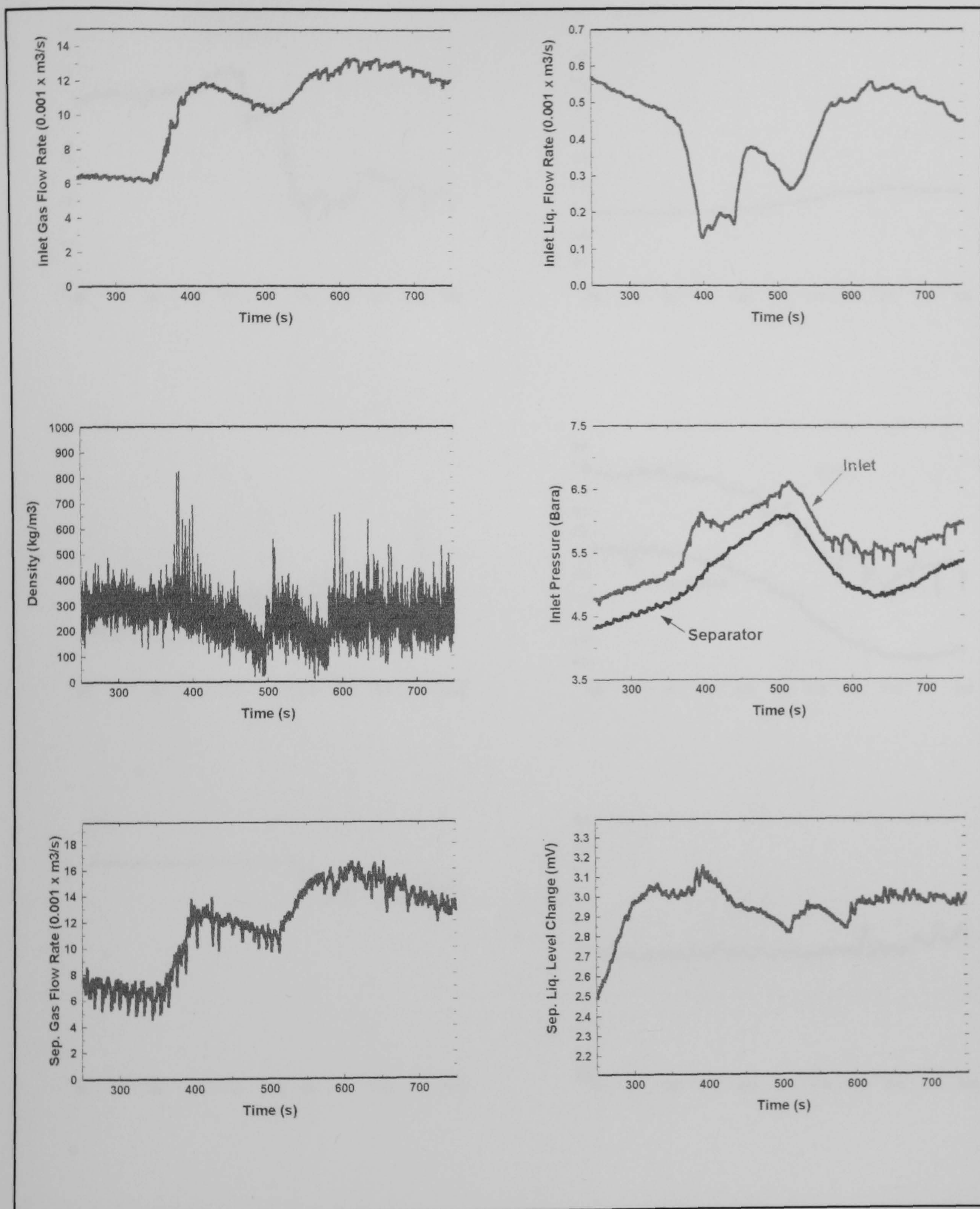


Figure D-2 - Summary of run HTG34W1

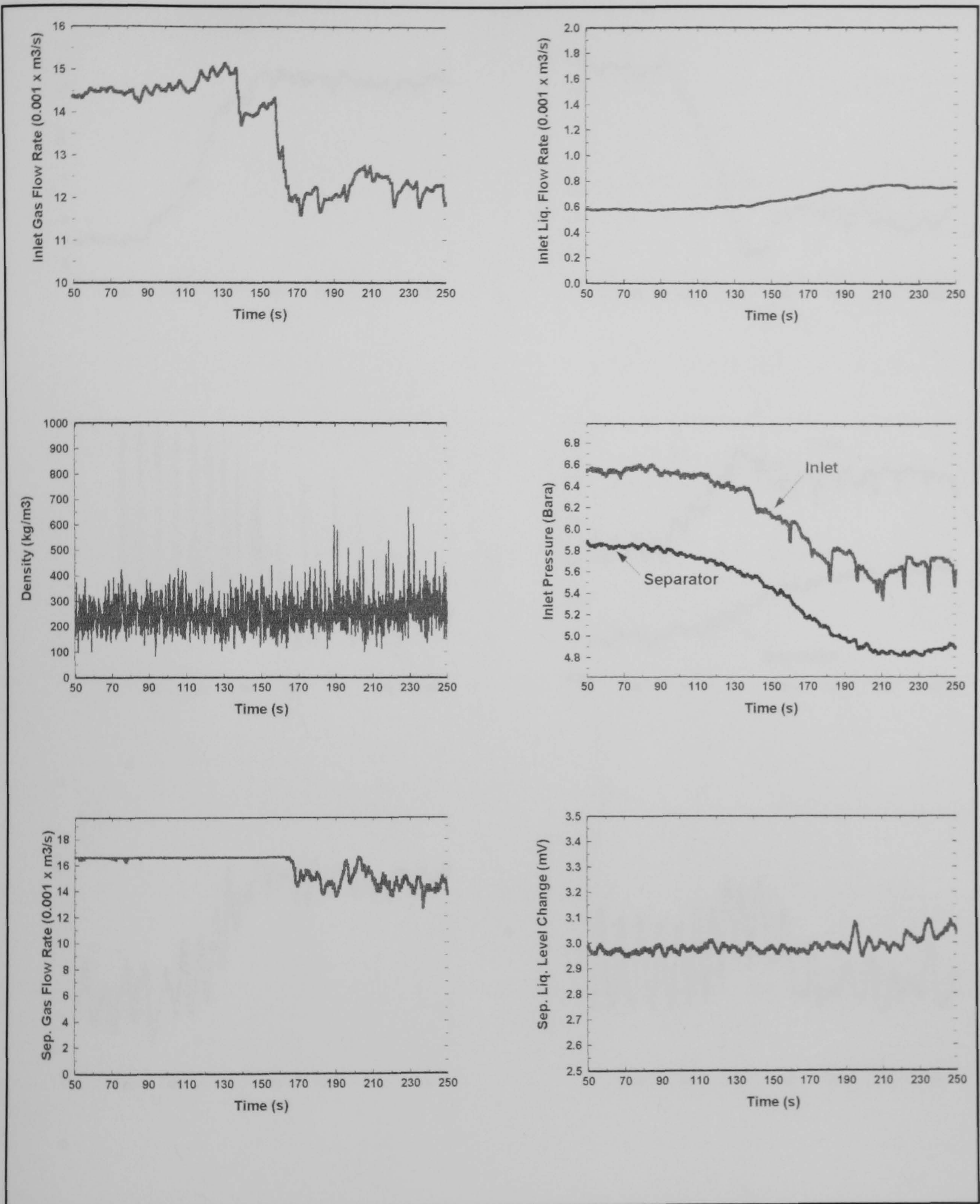


Figure D-3 - Summary of run HTG34W2

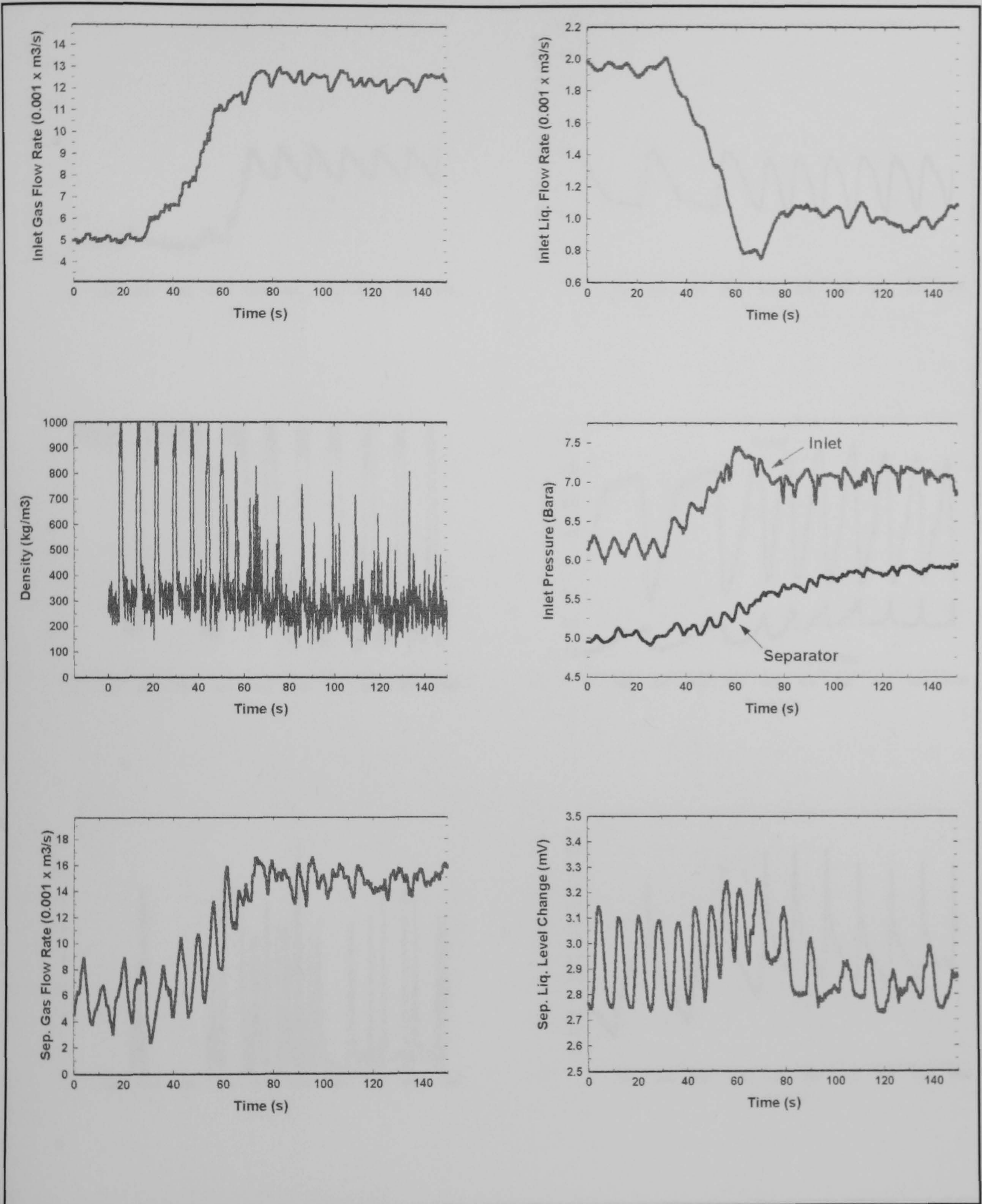


Figure D-4 - Summary of run HTG34W3

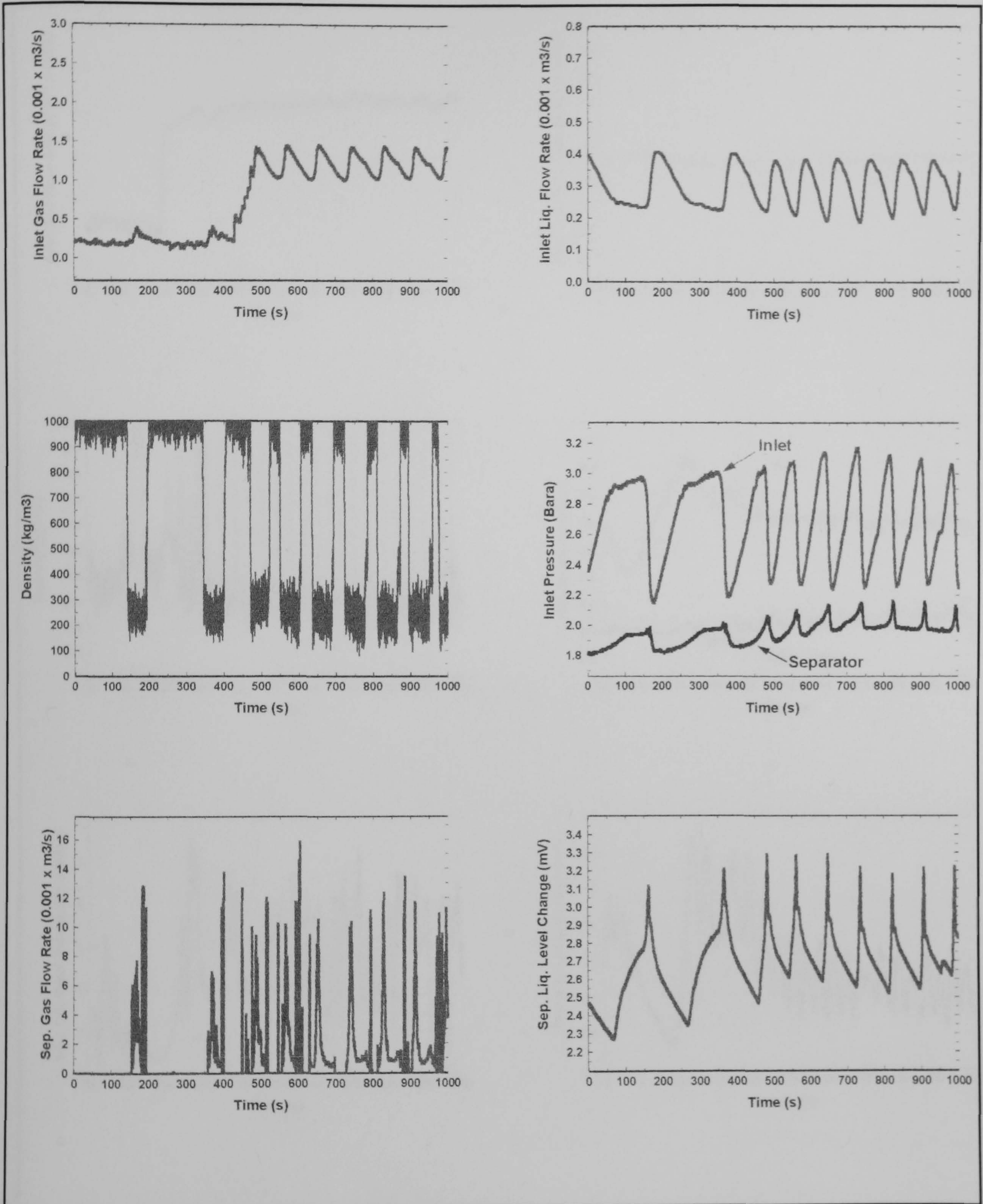


Figure D-5 - Summary of run LTG12W1

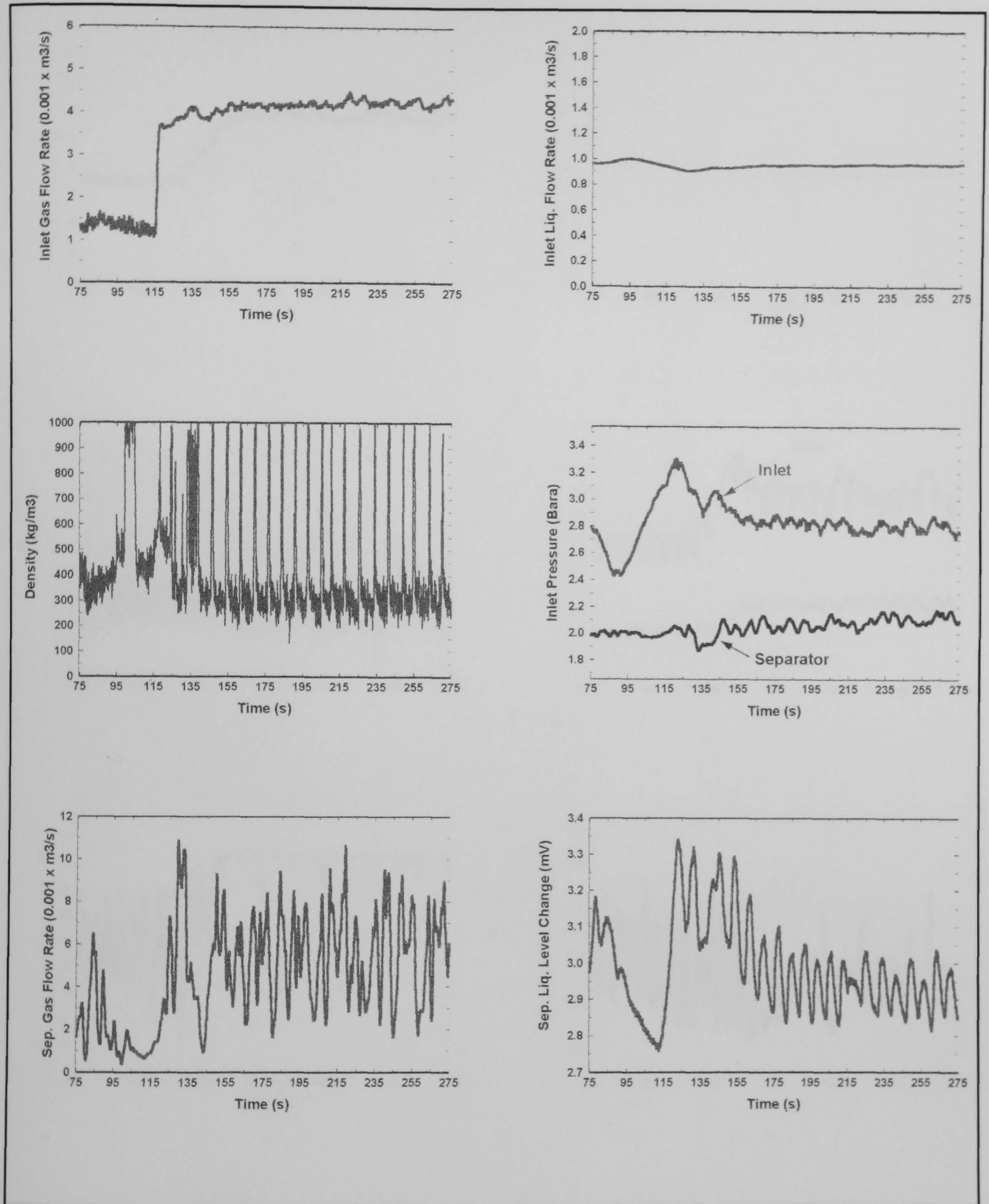


Figure D-6 - Summary of run LTG23W2

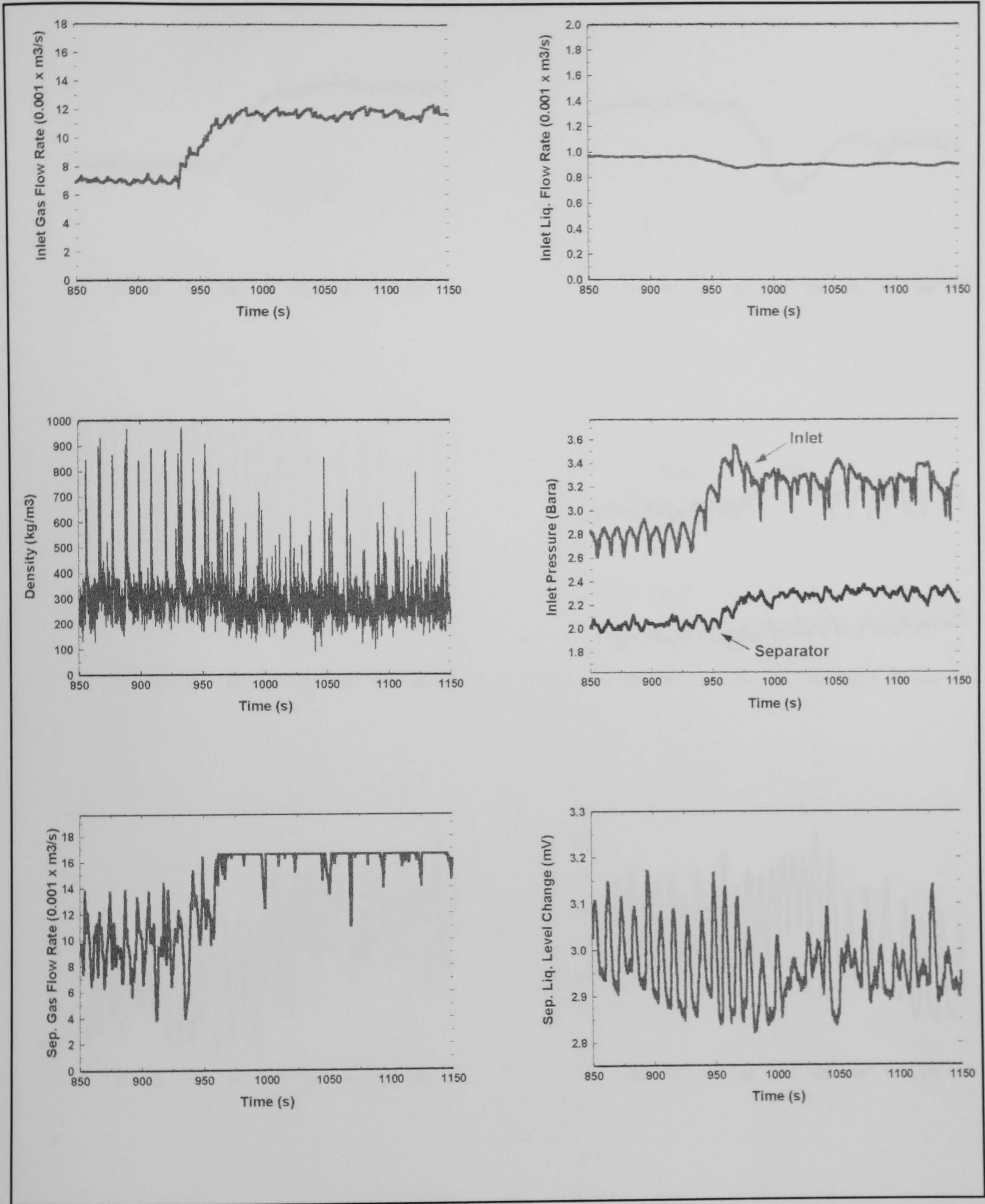


Figure D-7 - Summary of run LTG34W2

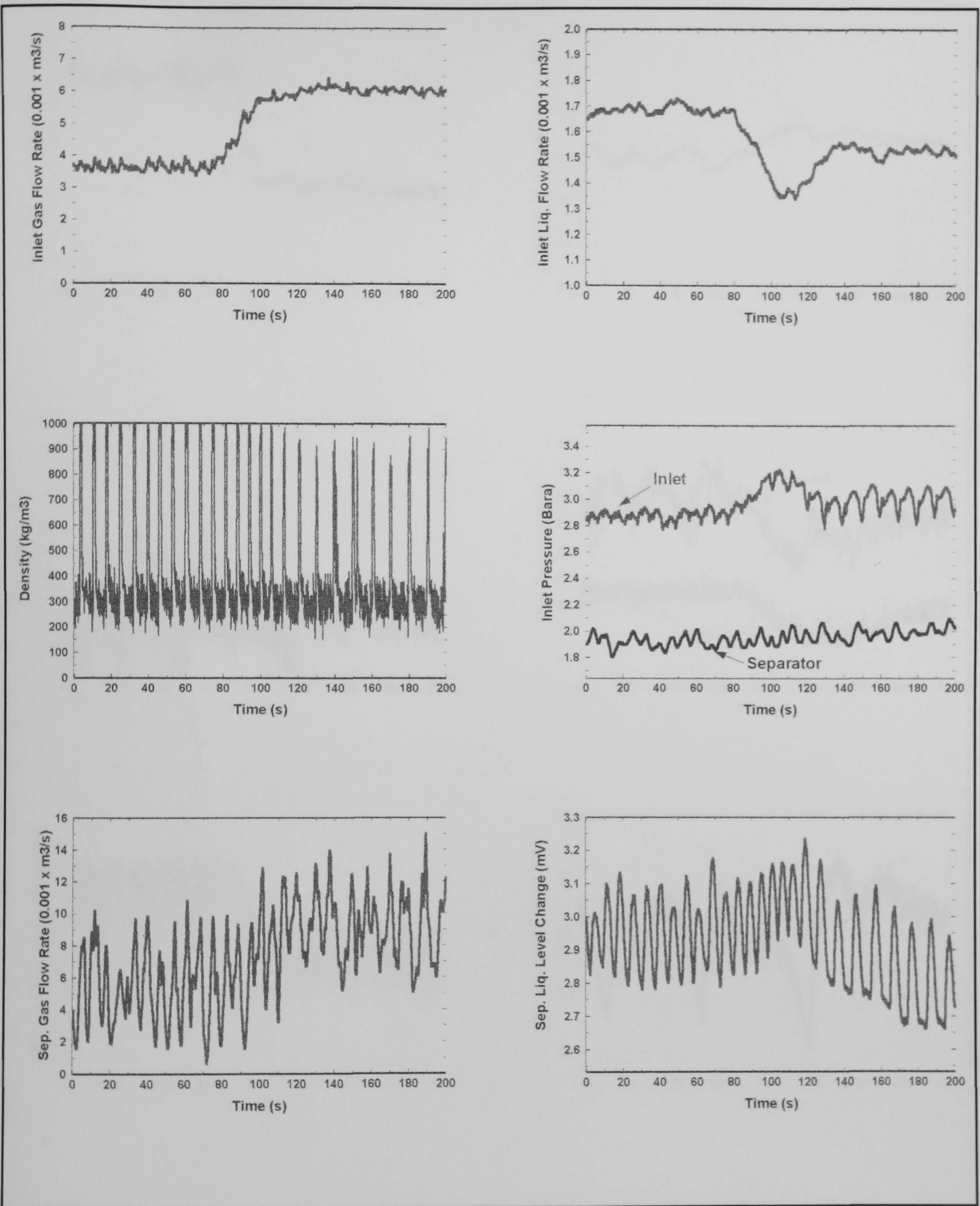


Figure D-8 - Summary of run LTG34W3

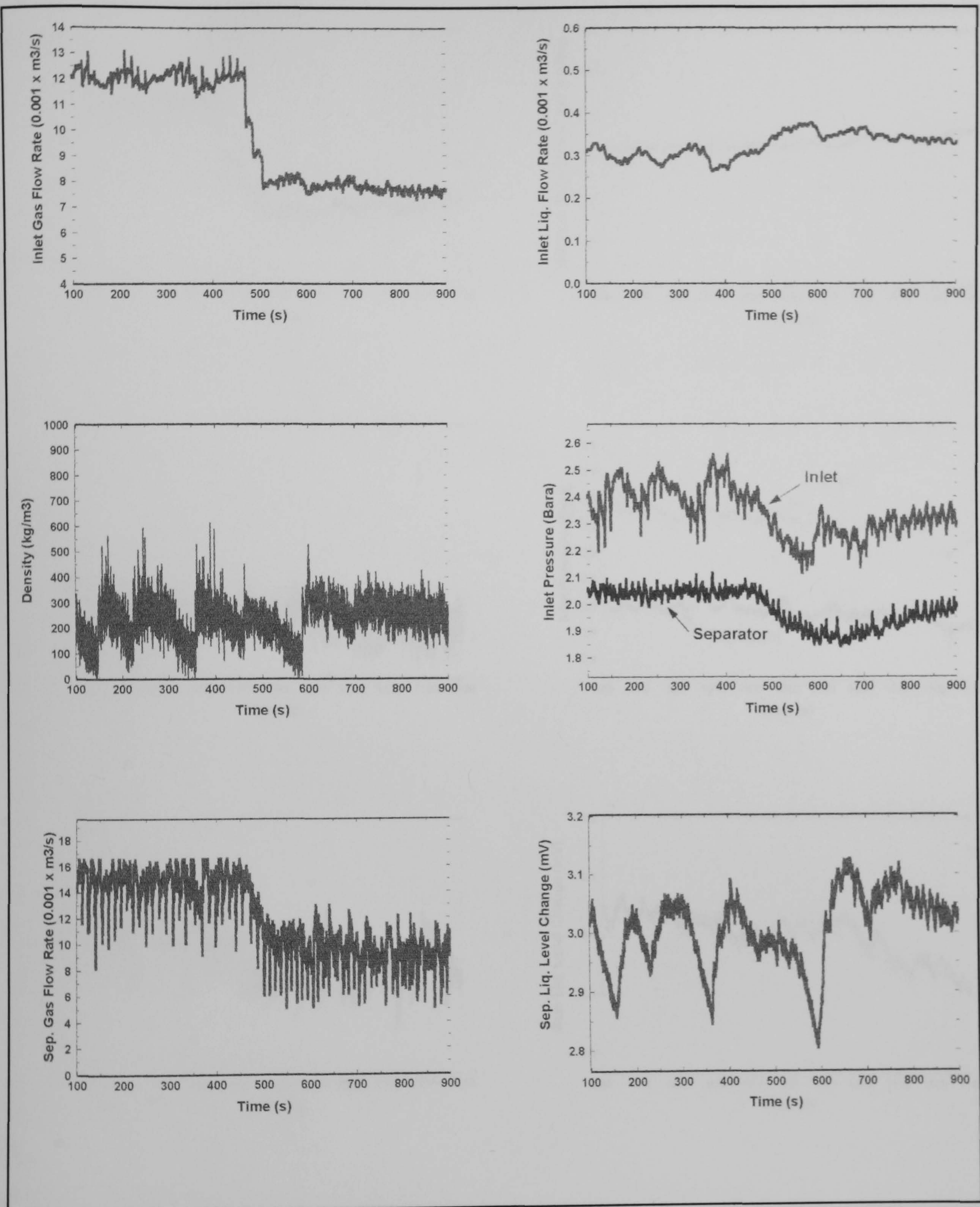


Figure D-9 - Summary of run LTG43W1

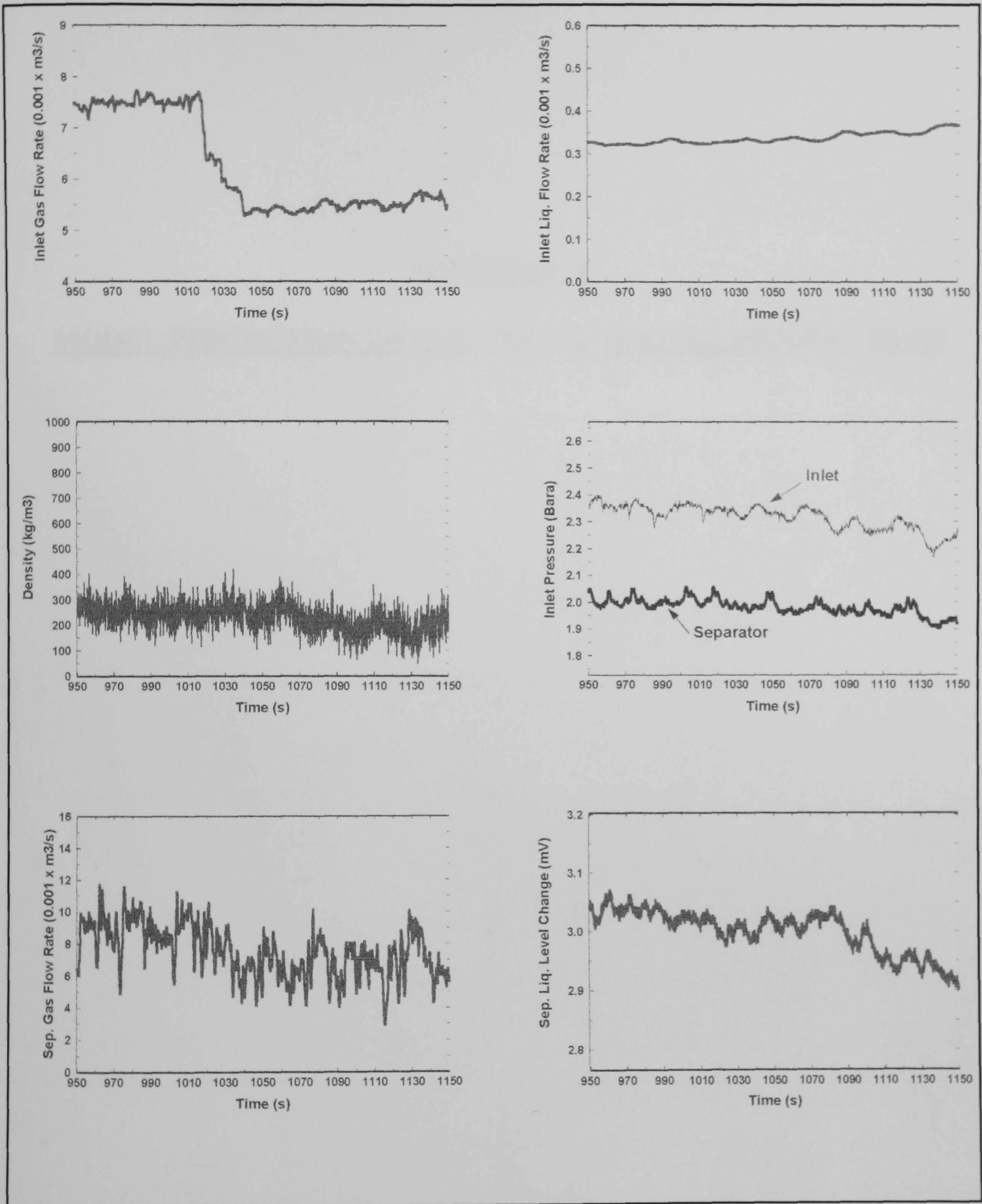


Figure D-10 - Summary of run LTG32W1

E. MODEL PREDICTION OF THE CONVENTIONAL PIGGING RUNS

The model prediction of the 13 conventional pigging runs are shown in this appendix. Table E.1 presents the prediction of initial conditions of the steady state gas-liquid flow, superficial velocities and pig launching schedules of the runs. Figs E.1 to E.13 show the transient flow prediction due to conventional pigging operations.

Table E.1 - Conventional pigging results

RUN CODE	SUP. GAS VELOCITY (m/s) Exp.	SUP. LIQ. VELOCITY (m ³ /s) Exp.	PRESSURE		PIG SCHEDULE	
			Inlet (Bara)	Separator (Bara) Exp.	Launching (s) Exp.	Arrival (s)
LPG3W1	2.6	0.16	2.3	1.9	510	550.6
LPG4W1	6.2	0.16	2.6	2.0	458	484.6
LPG3W2	2.2	0.48	2.5	2.0	288	322.0
LPG4W2	5.9	0.45	2.9	2.2	520	537.0
LPG3W3	1.7	0.81	2.8	1.9	193	226.0
LPG4W3	4.0	0.77	3.2	2.0	265	286.0
HPG2W1	0.8	0.27	4.6	4.8	271	349.0
HGP3W1	2.8	0.16	5.7	5.6	322	354.6
HPG4W1	5.9	0.24	6.0	5.8	118	136.0
HPG4W2	5.4	0.34	6.2	6.1	123	143.4
HPG2W3	0.5	0.99	6.1	6.2	440	484.0
HPG3W3	2.5	0.94	6.3	6.3	508	533.3
HPG4W3	6.4	0.67	6.8	6.7	94	107.6

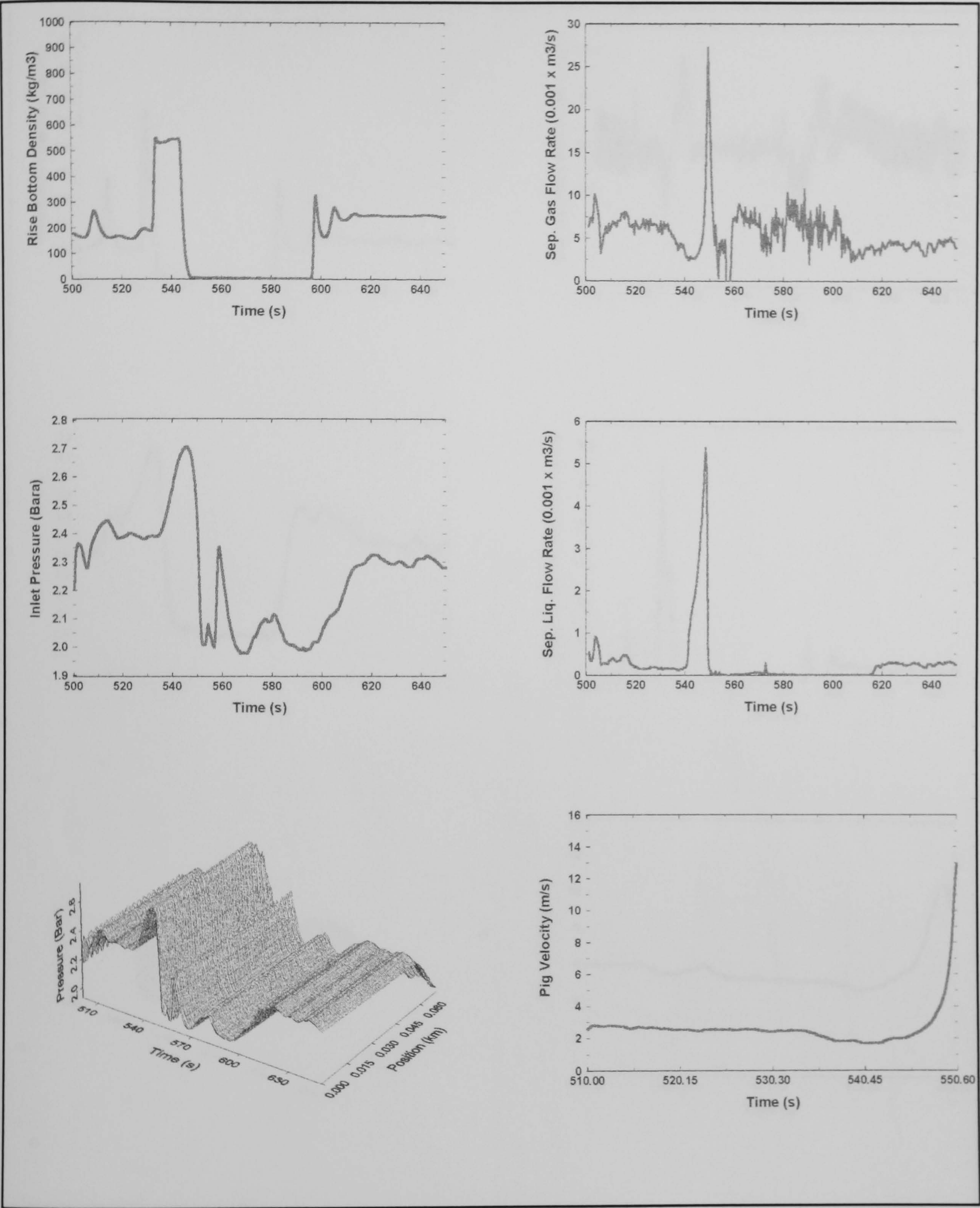


Figure E-1 - Model results of run LPG3W1

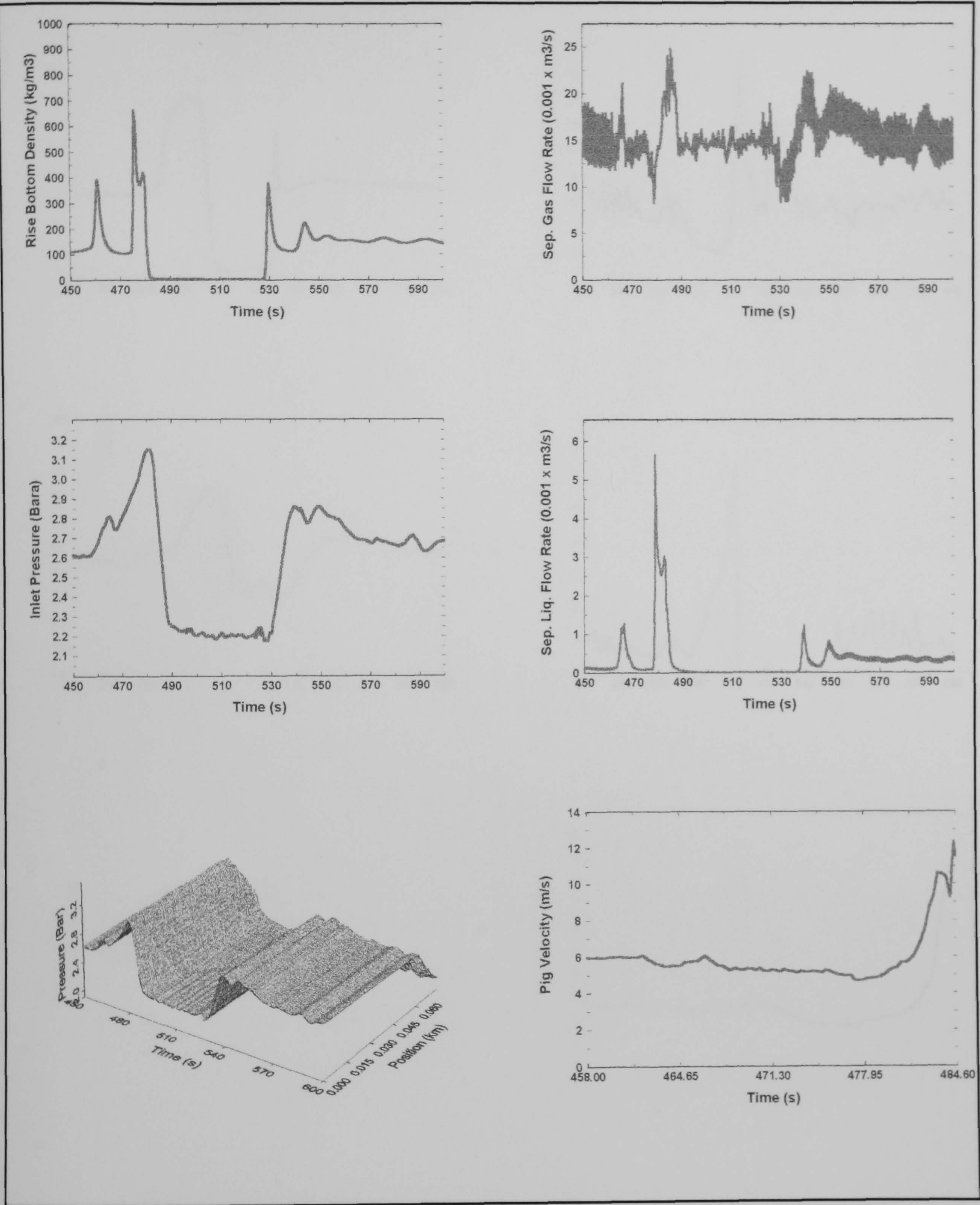


Figure E-2 - Model results of run LPG4W1

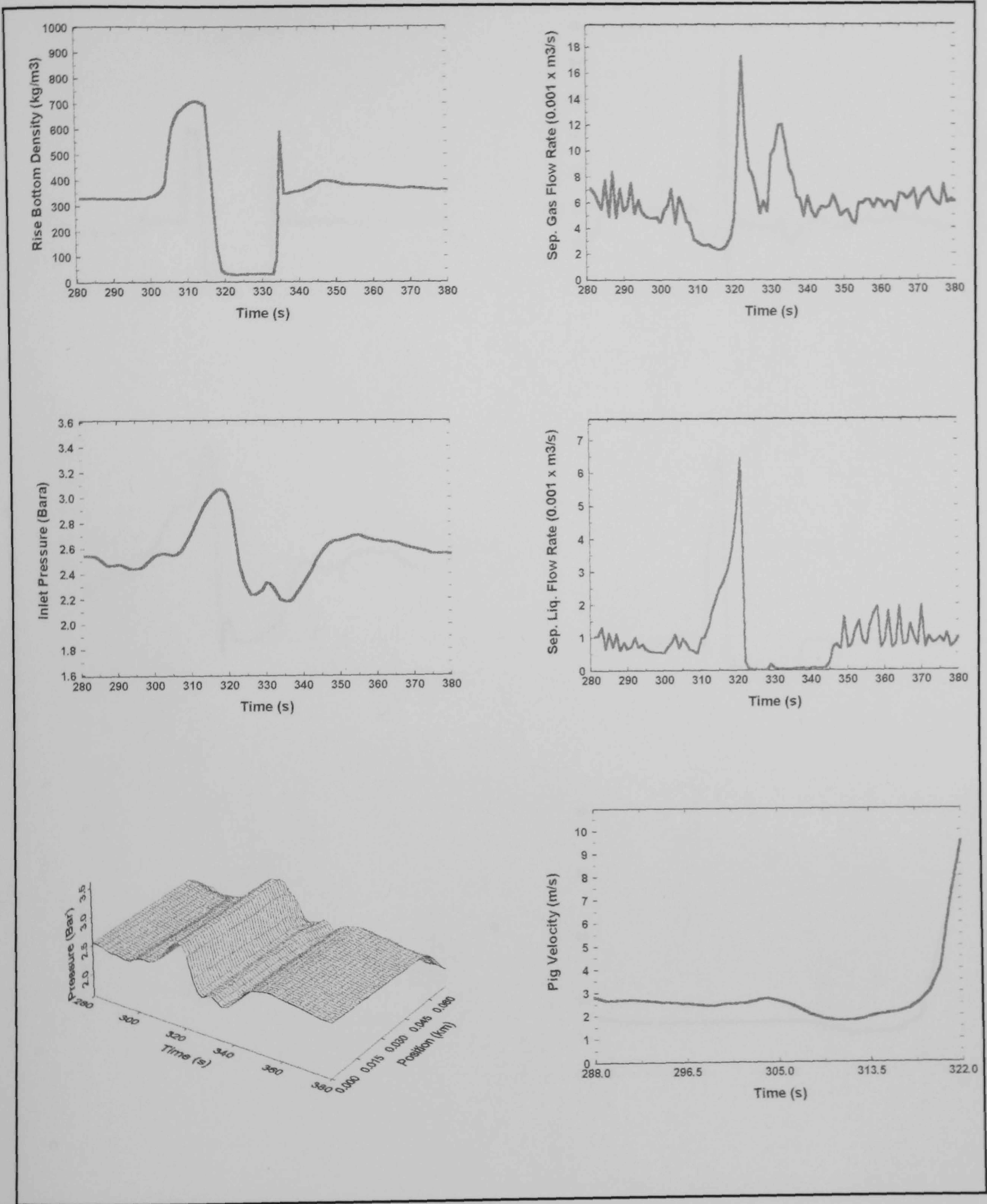


Figure E-3 - Model results of run LPG3W2

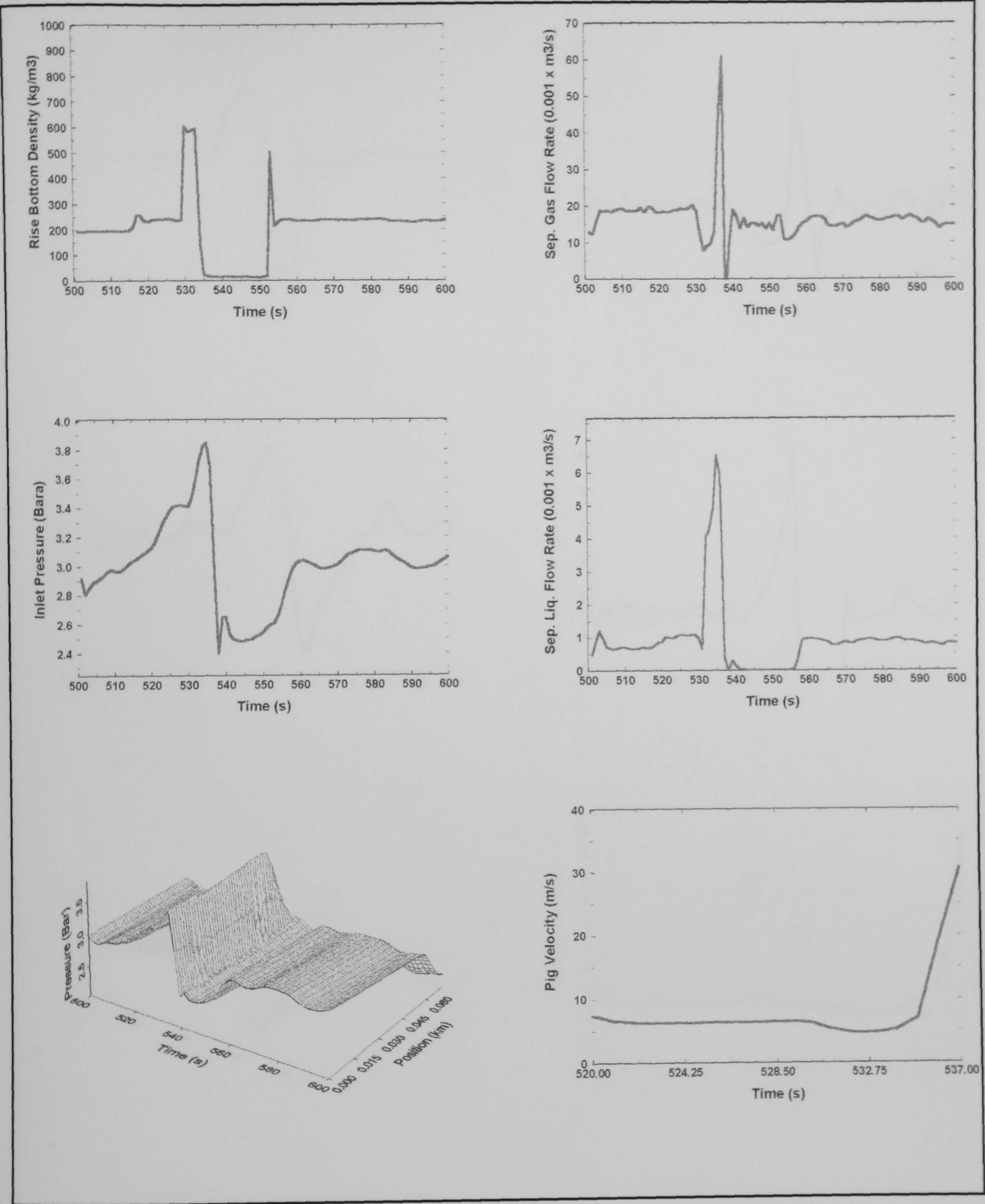


Figure E-4 - Model results of run LPG4W2

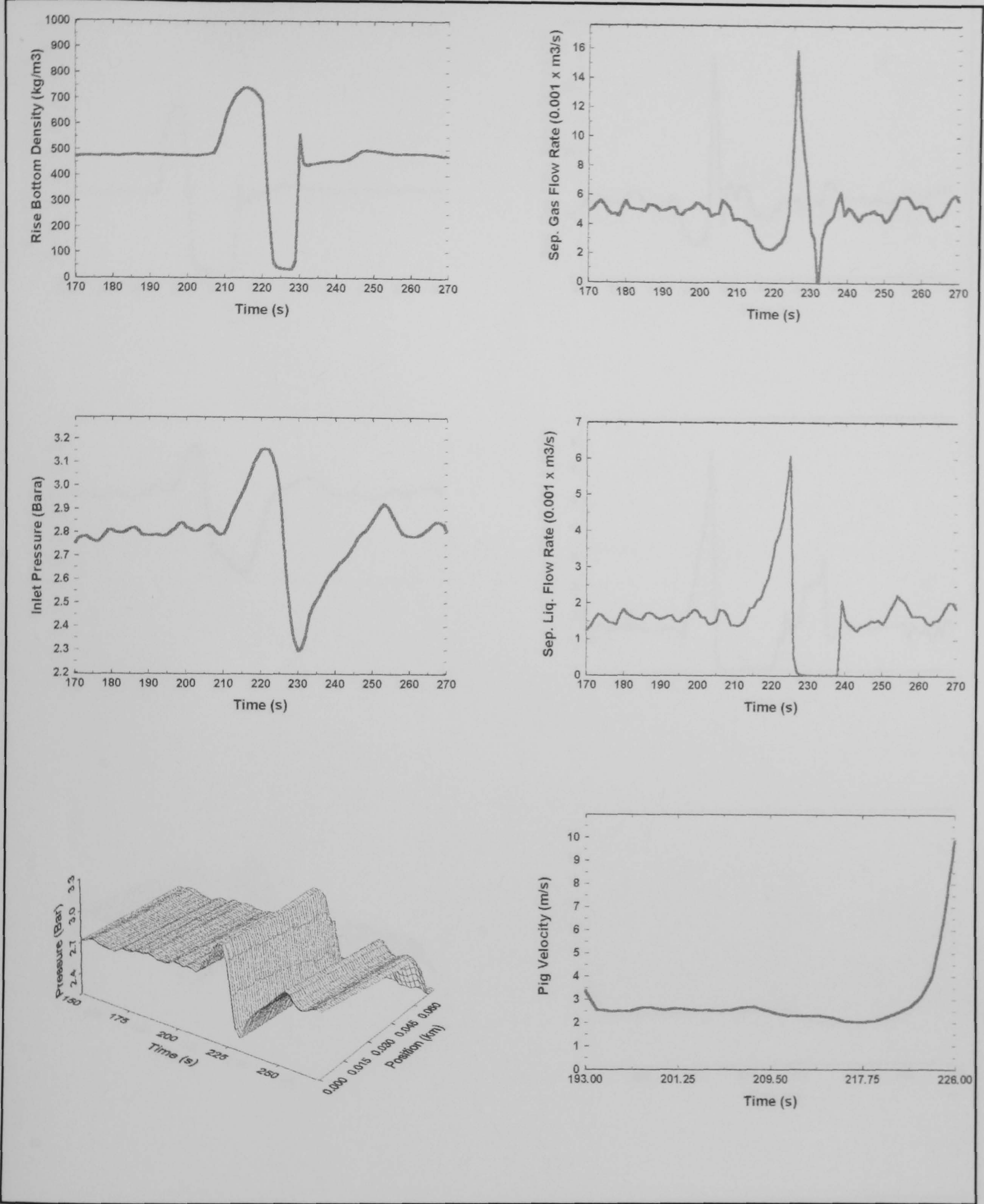


Figure E-5 - Model results of run LPG3W3

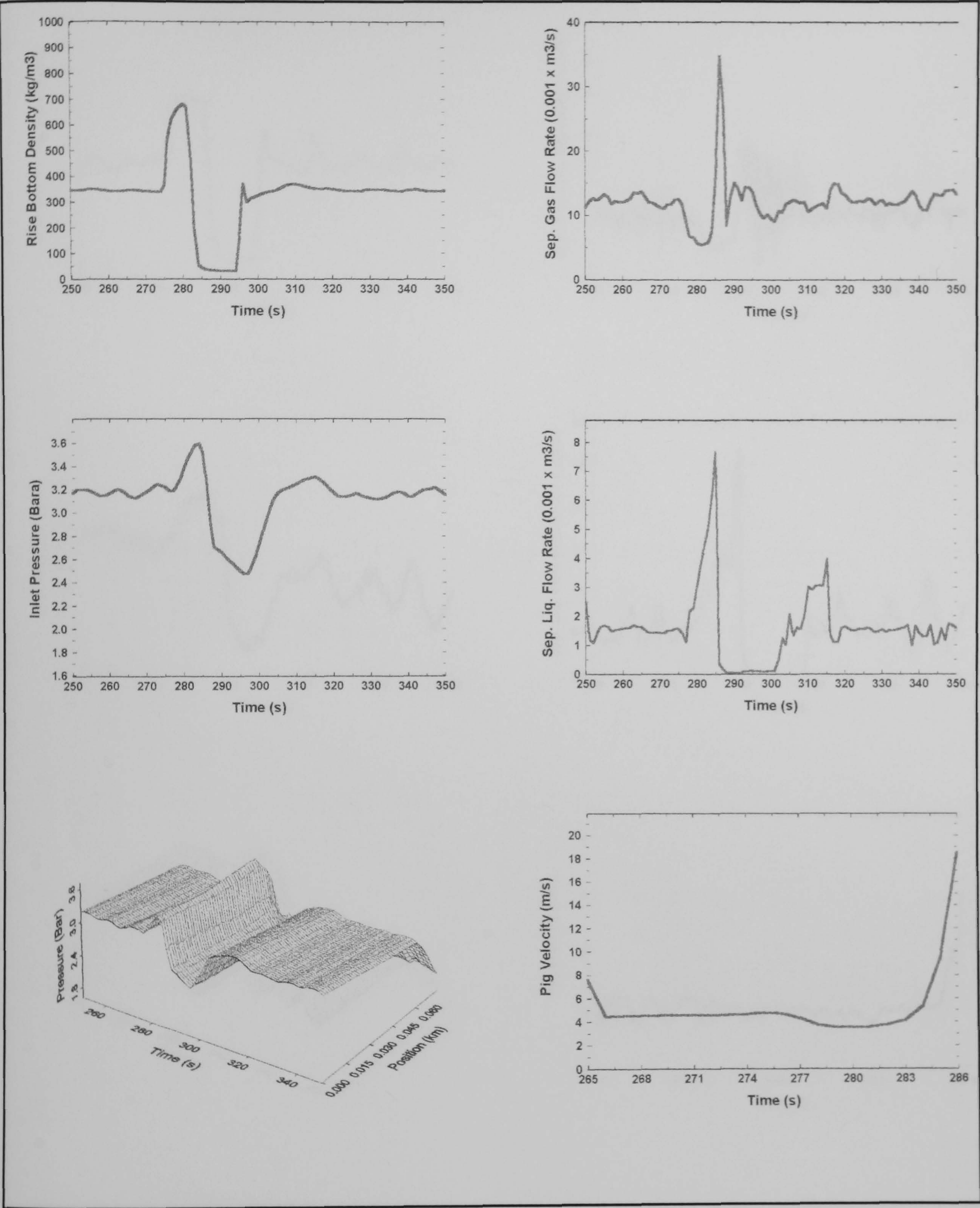


Figure E-6 - Model results of run LPG4W3

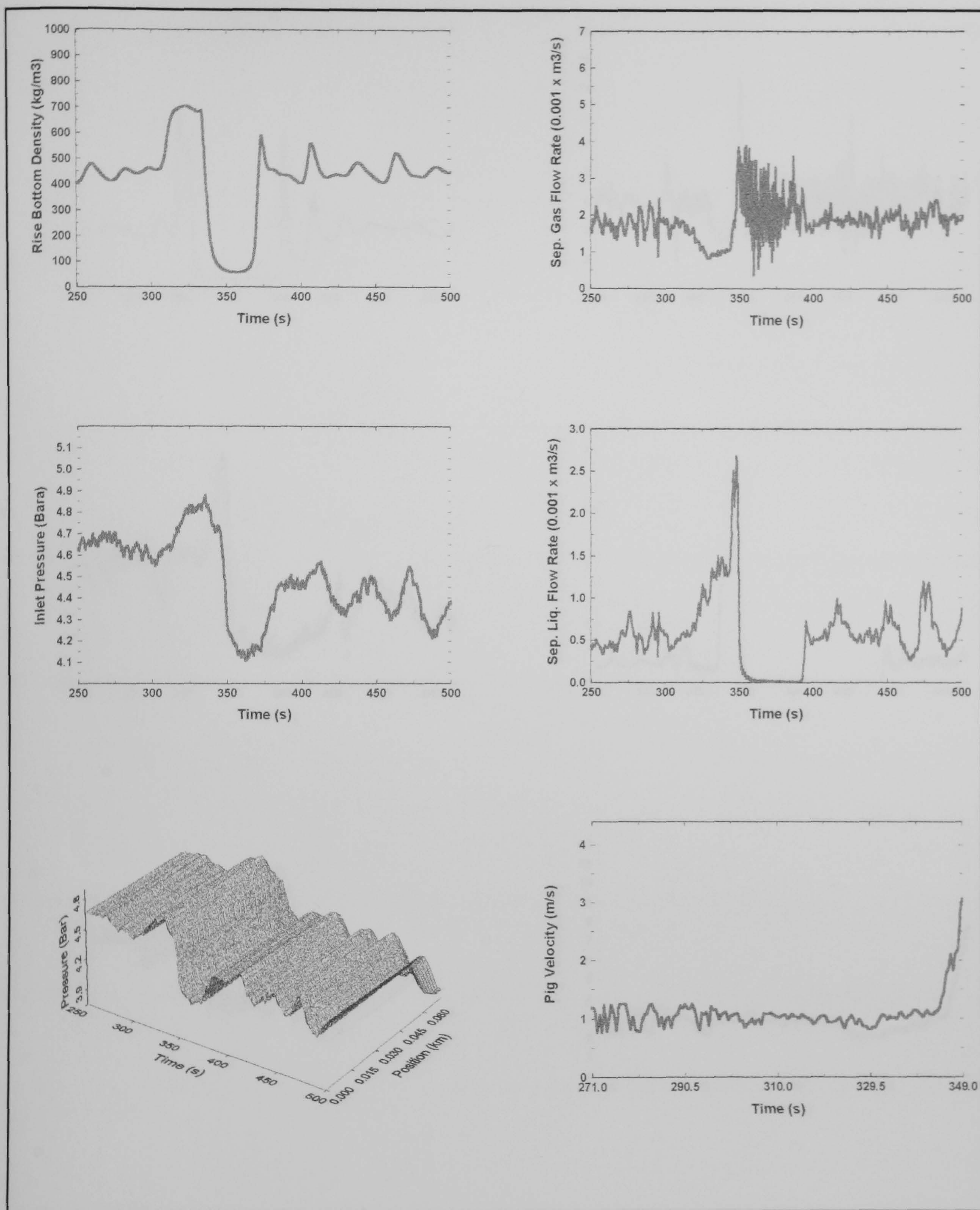


Figure E-7 - Model results of run HPG2W1

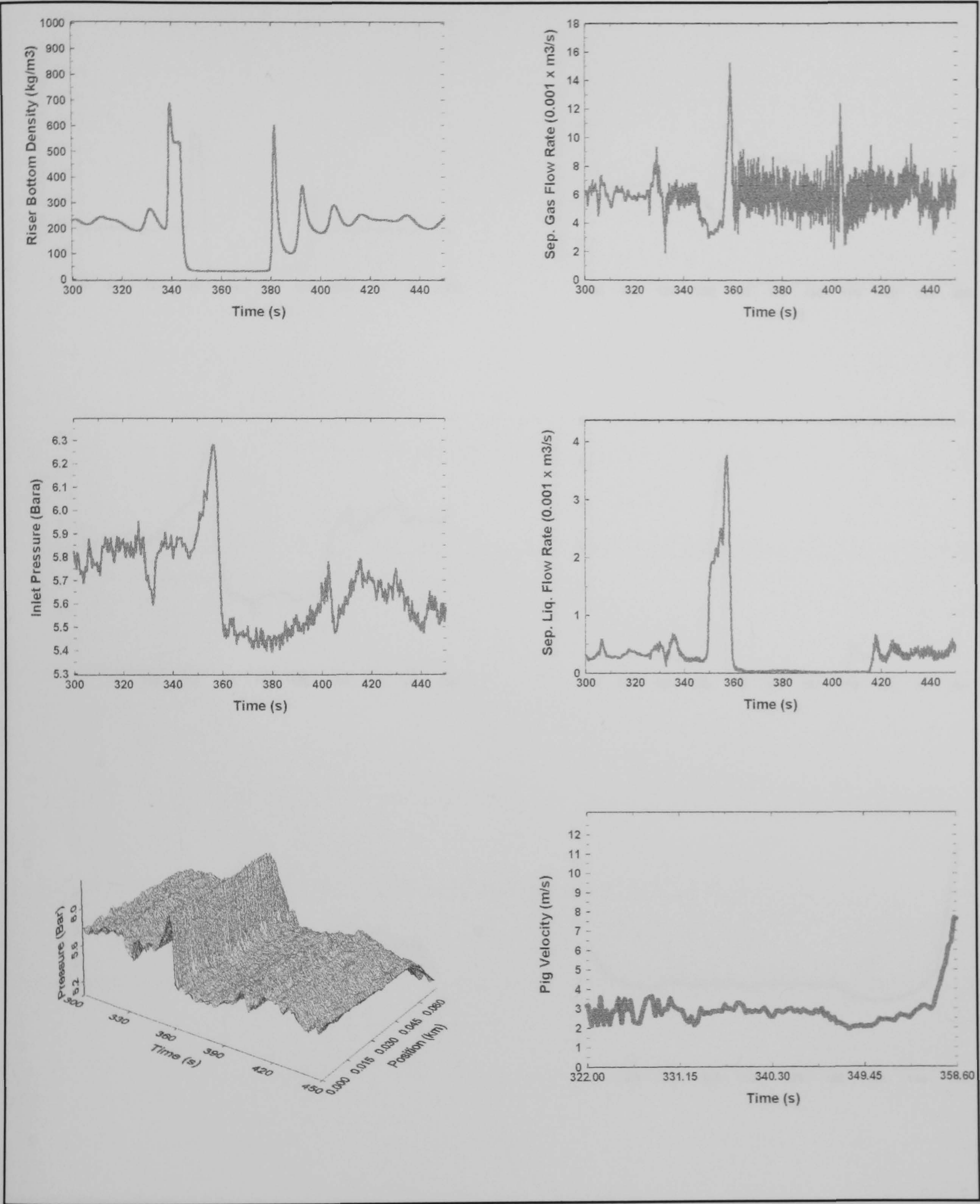


Figure E-8 - Model results of run HPG3W1

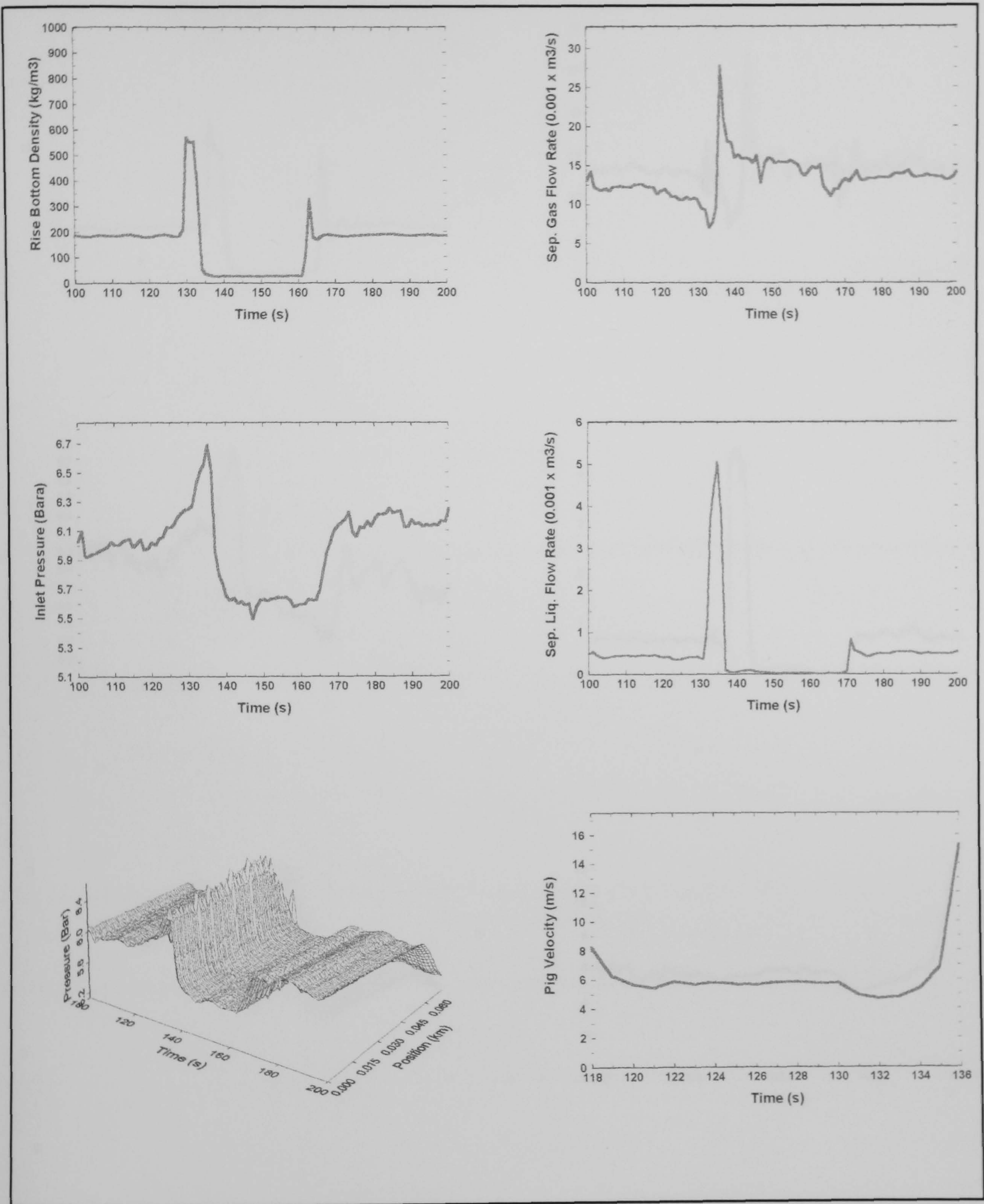


Figure E-9 - Model results of run HPG4W1

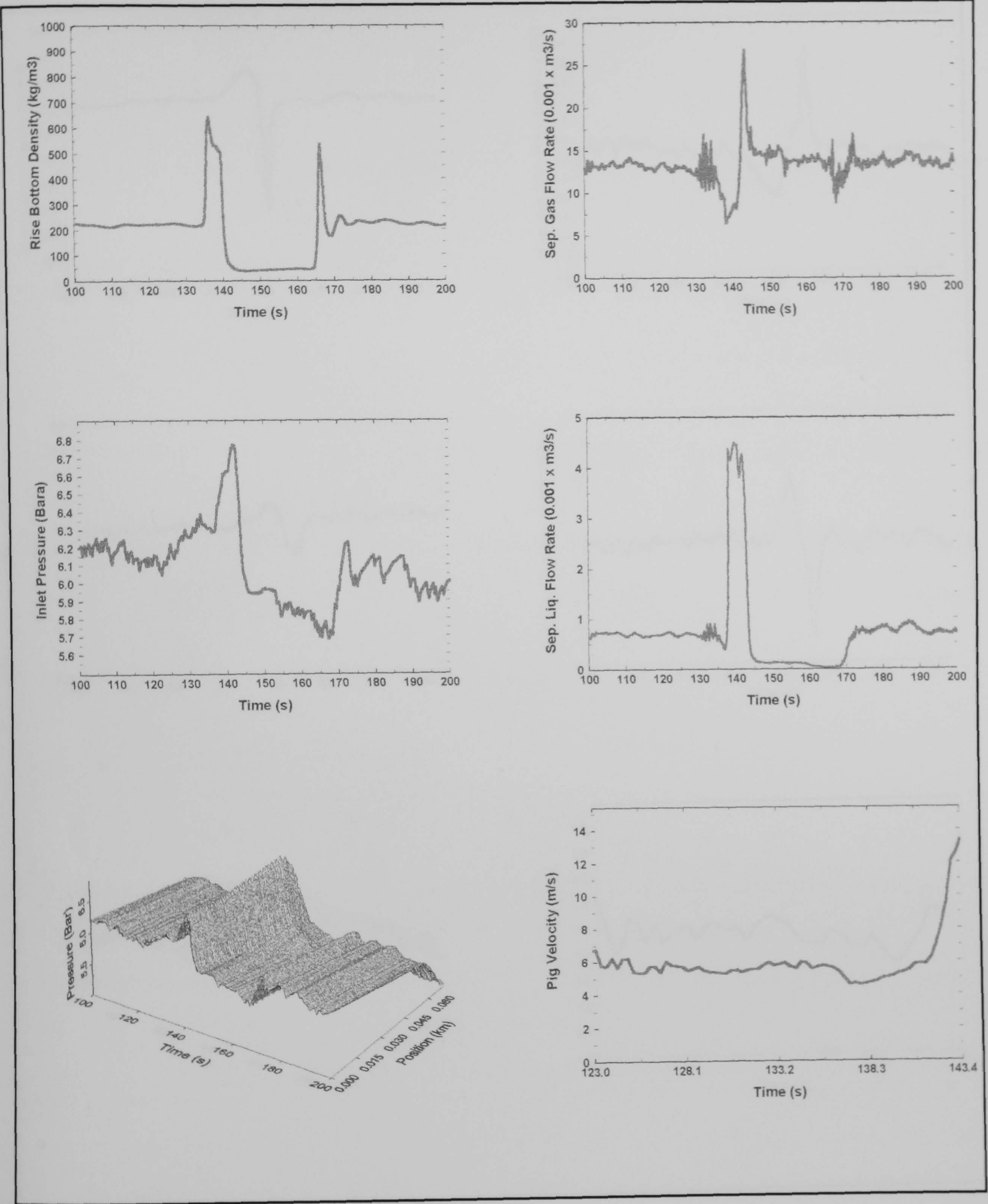


Figure E-10 - Model results of run HPG4W2

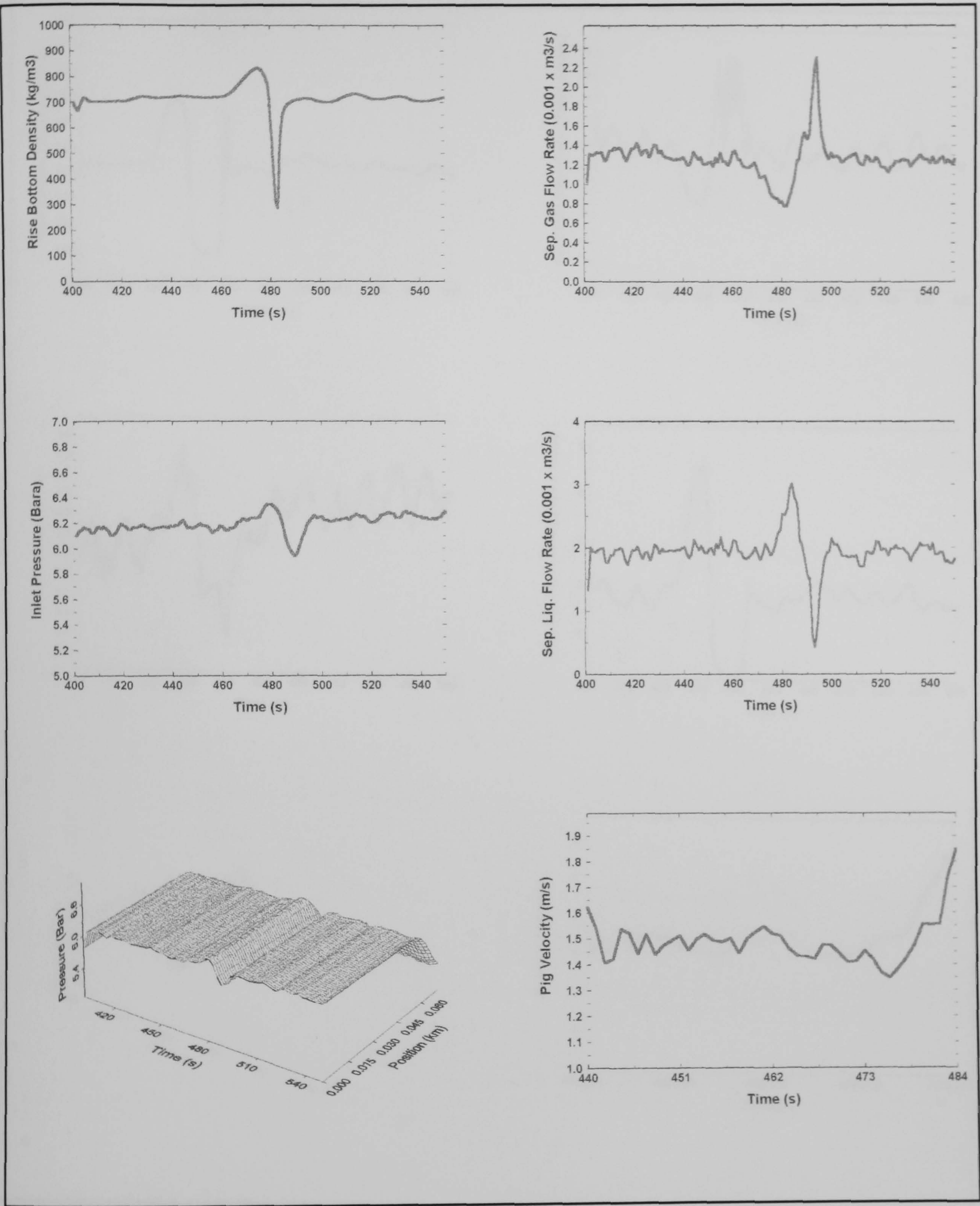


Figure E-11 - Model results of run HPG2W3

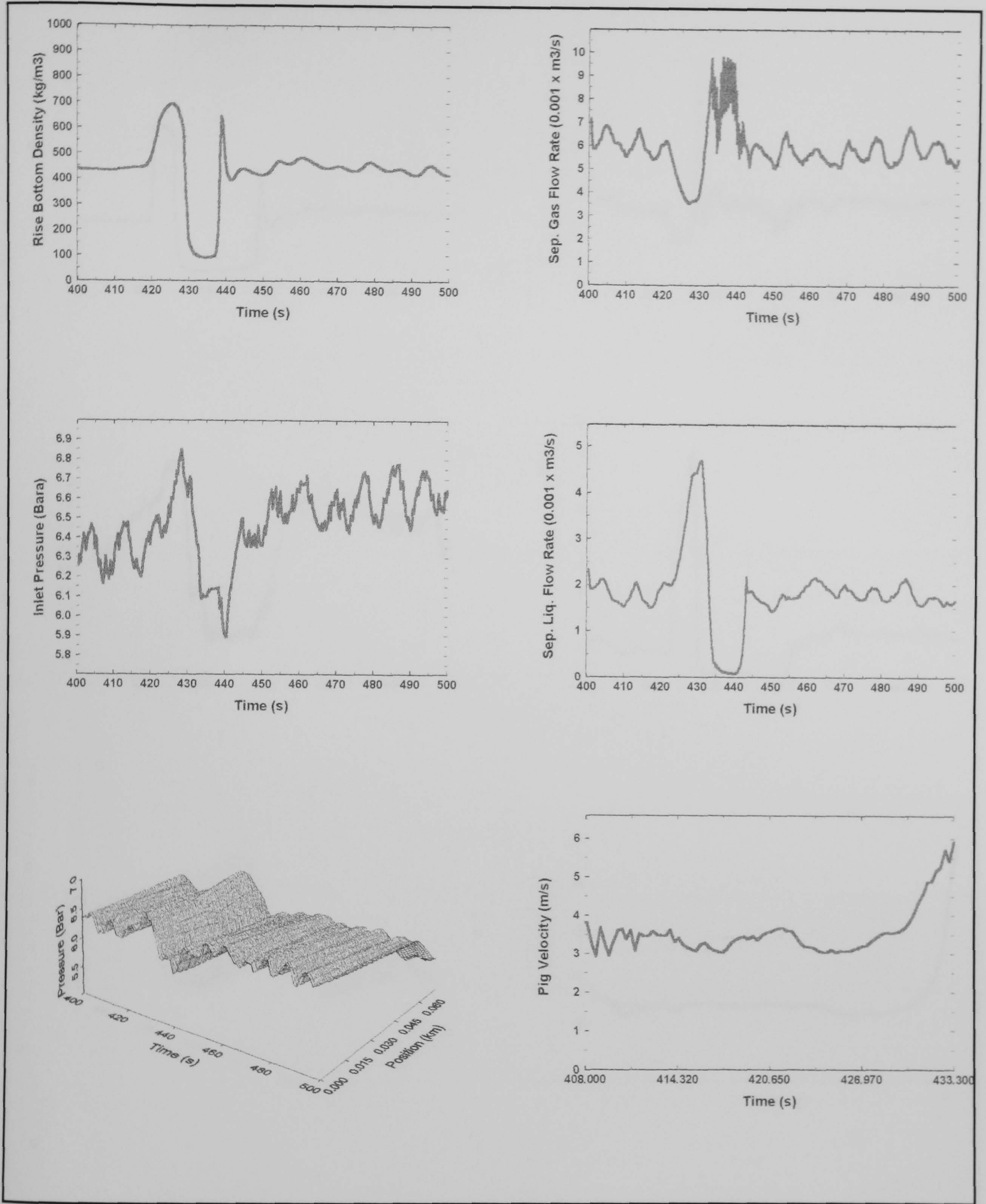


Figure E-12 - Model results of run HPG3W3

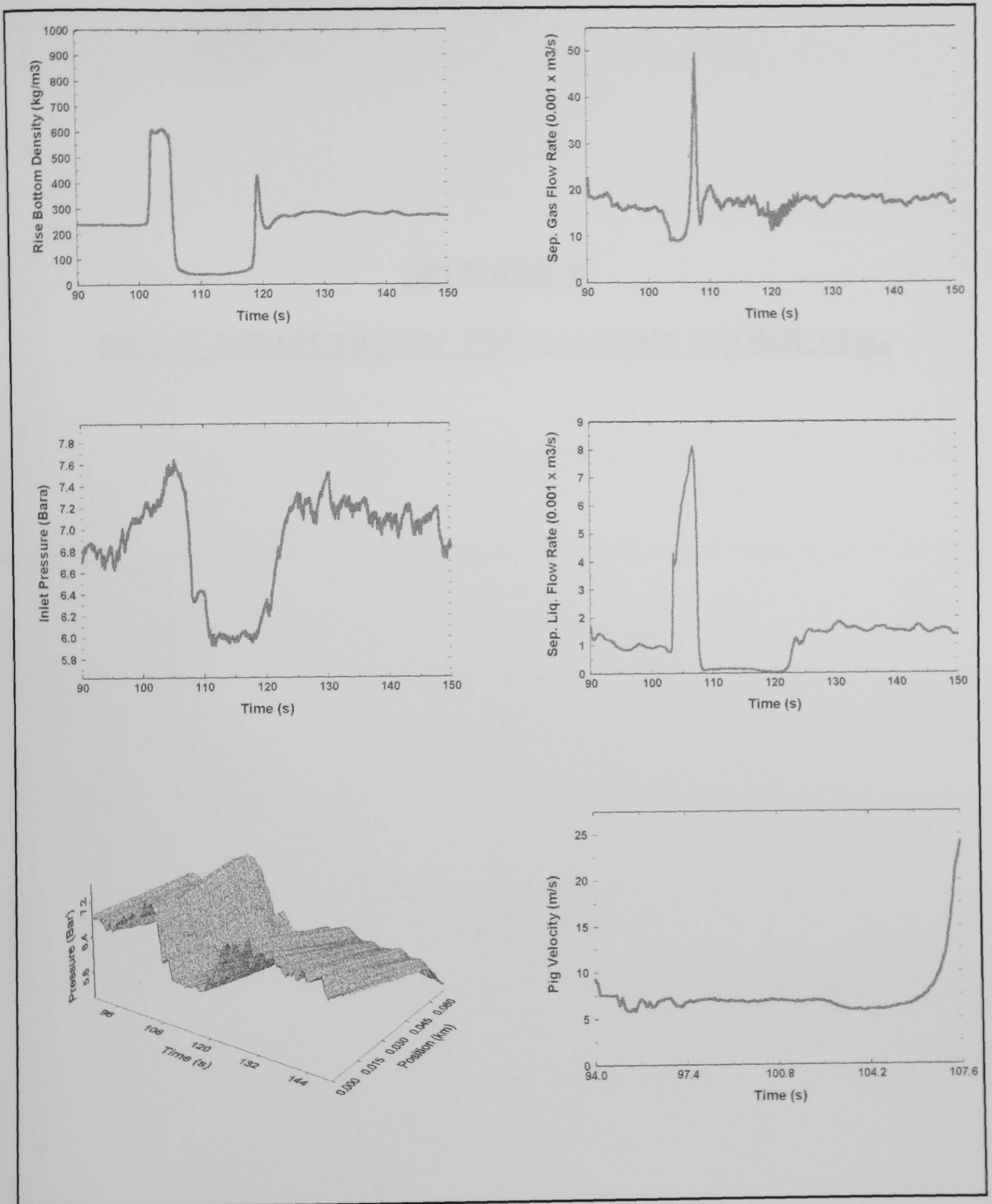


Figure E-13 - Model results of run HPG4W3

APPENDIX F

MODEL PREDICTION OF THE GAS PUMP WITHOUT PIG

F. MODEL PREDICTION OF THE GAS PUMP WITHOUT PIG

The model predictions of the six gas pump runs without pig are shown in this appendix. Table F.1 presents the initial conditions of the test section, the gas injection conditions and the efficiency of liquid transportation. Figs F.1 to F.6 show the transient flow behaviour predicted by the model when gas is injected, without pig, in a pipeline-riser system full of liquid.

Table F.1 - Model prediction of the gas pump without pig

RUN CODE	INITIAL INLET PRESSURE (Bara)	GAS INJECTED		EFFICIENCY (%)
		Volume (Sm ³)	Time (s)	
LGP1	2.52	0.233	29	87.7
LGP2	2.51	0.521	29	91.9
LGP3	2.60	0.721	28	92.7
HGP1	6.03	0.777*	55	83.6
HGP2	5.91	0.750*	44	84.1
HGP3	5.90	0.627	28	87.3

* assumed value (problems in gas measurement)

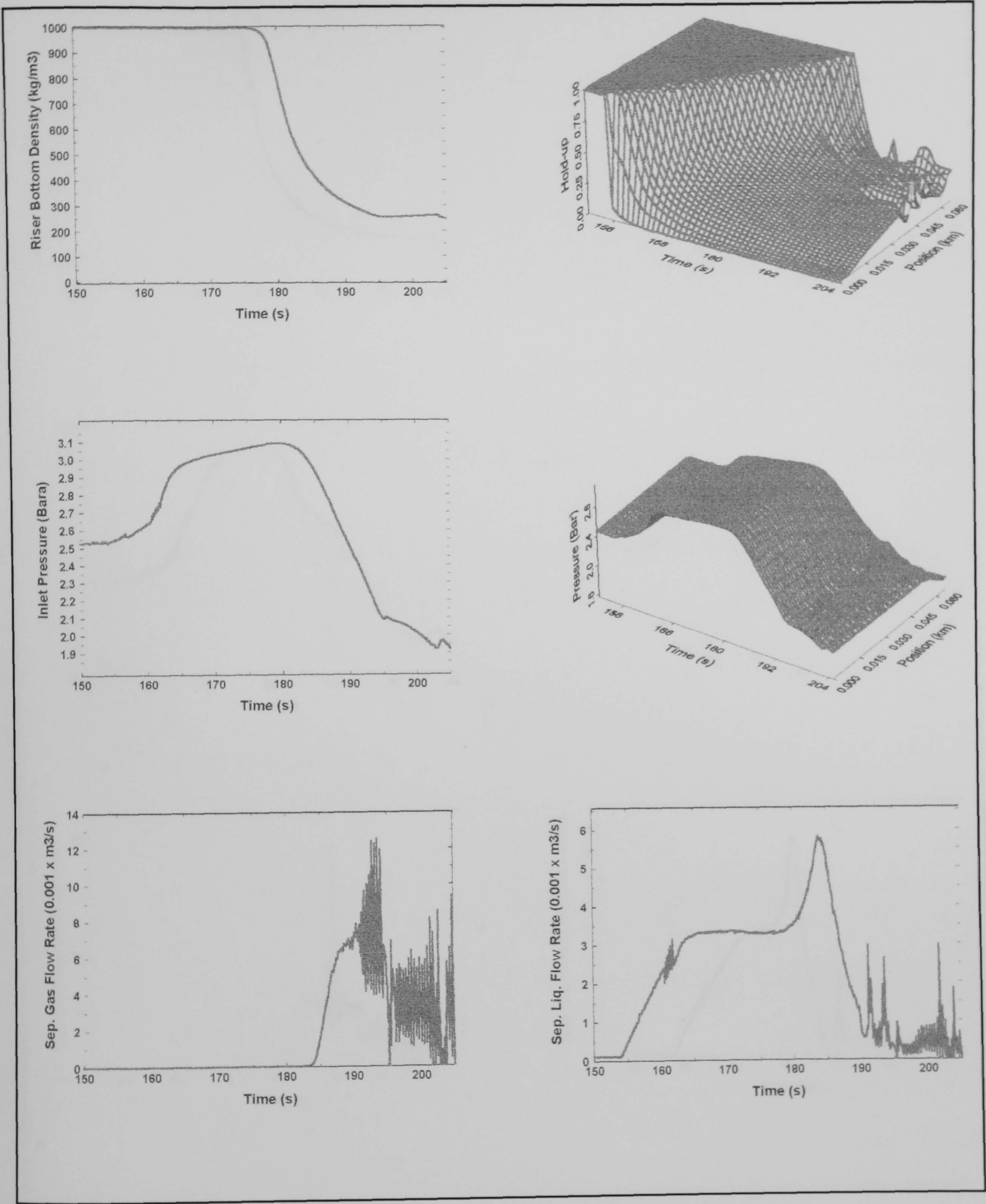


Figure F-1 - Model results of run LGP1

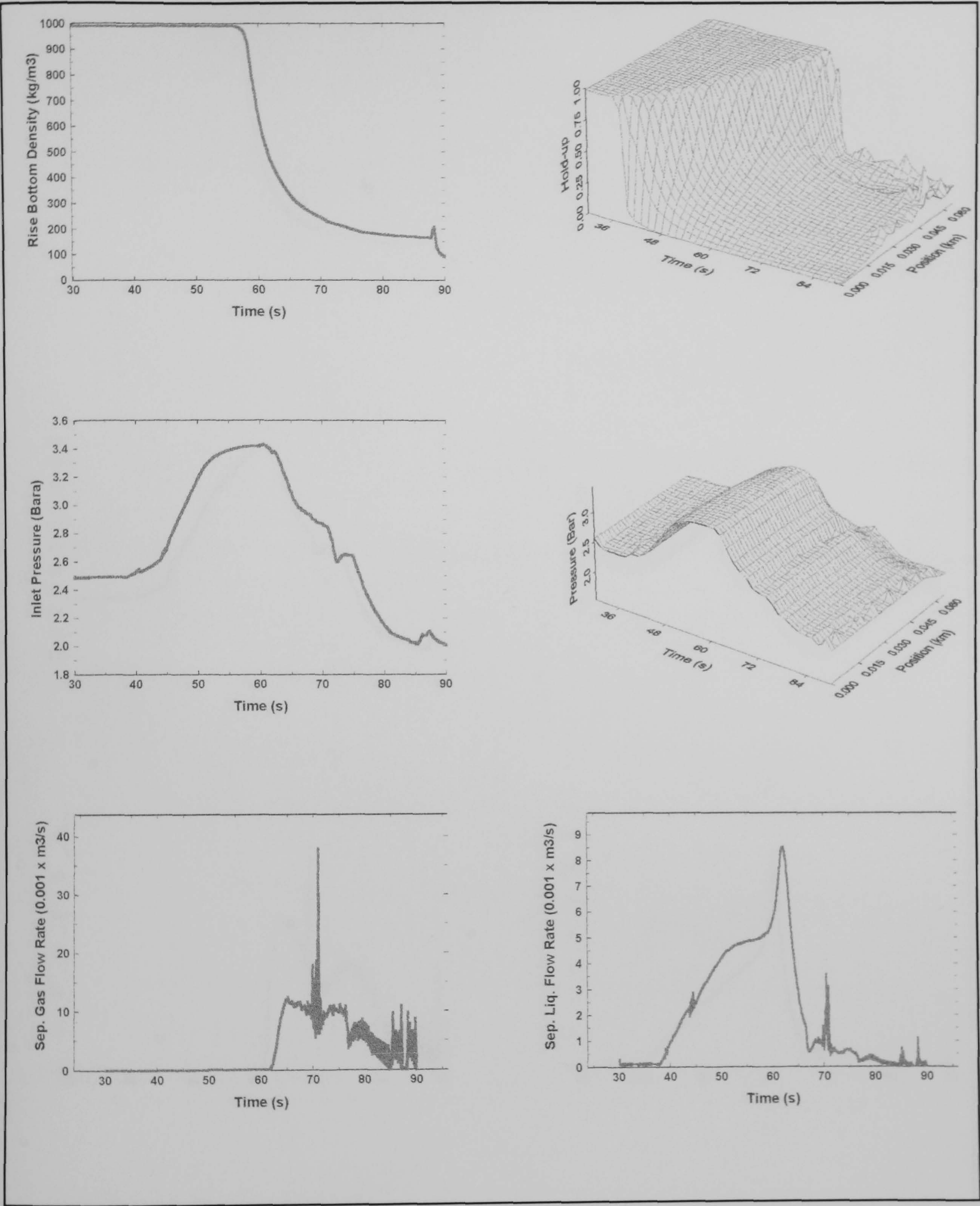


Figure F-2 - Model results of run LGP2

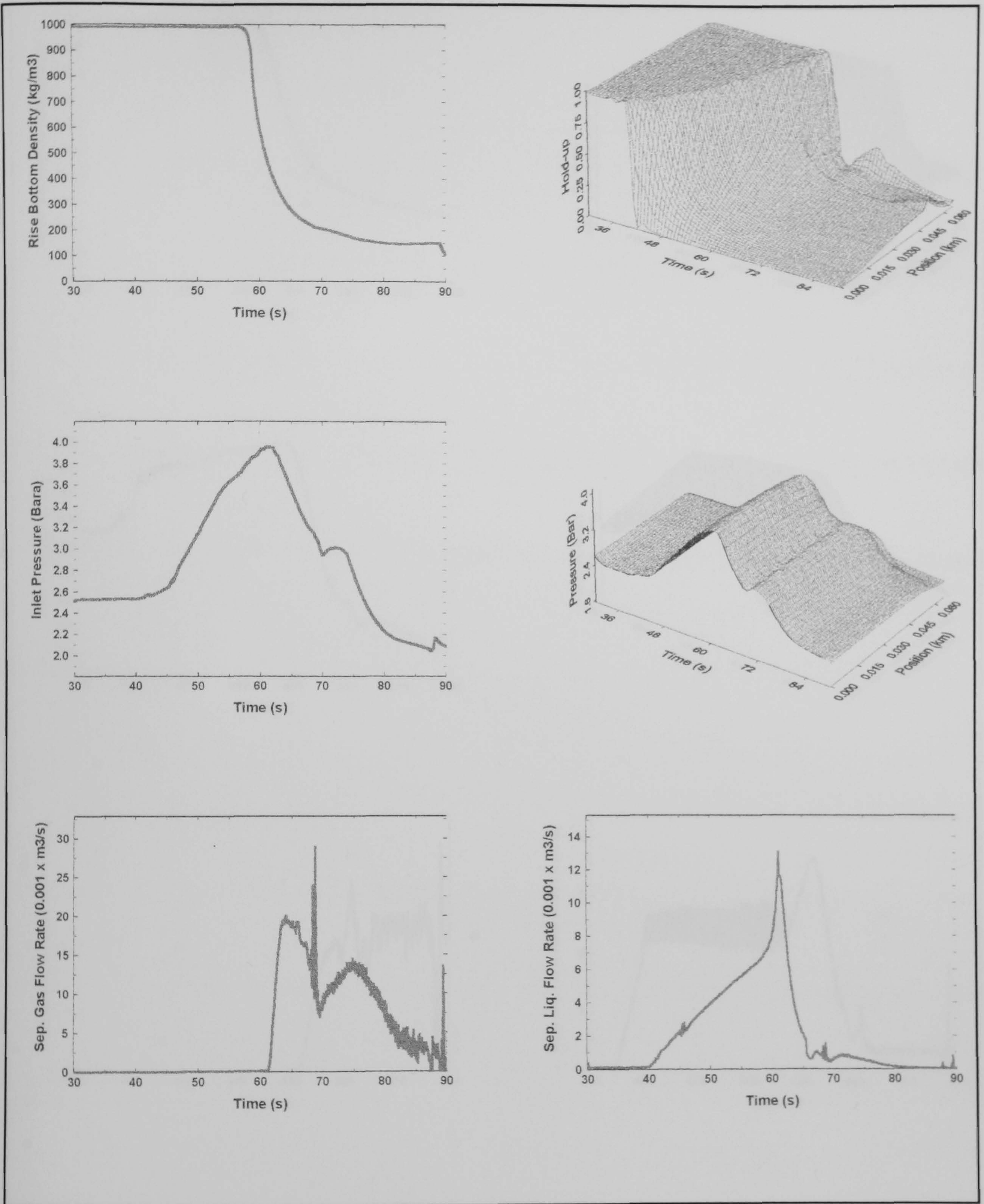


Figure F-3 - Model results of run LGP3

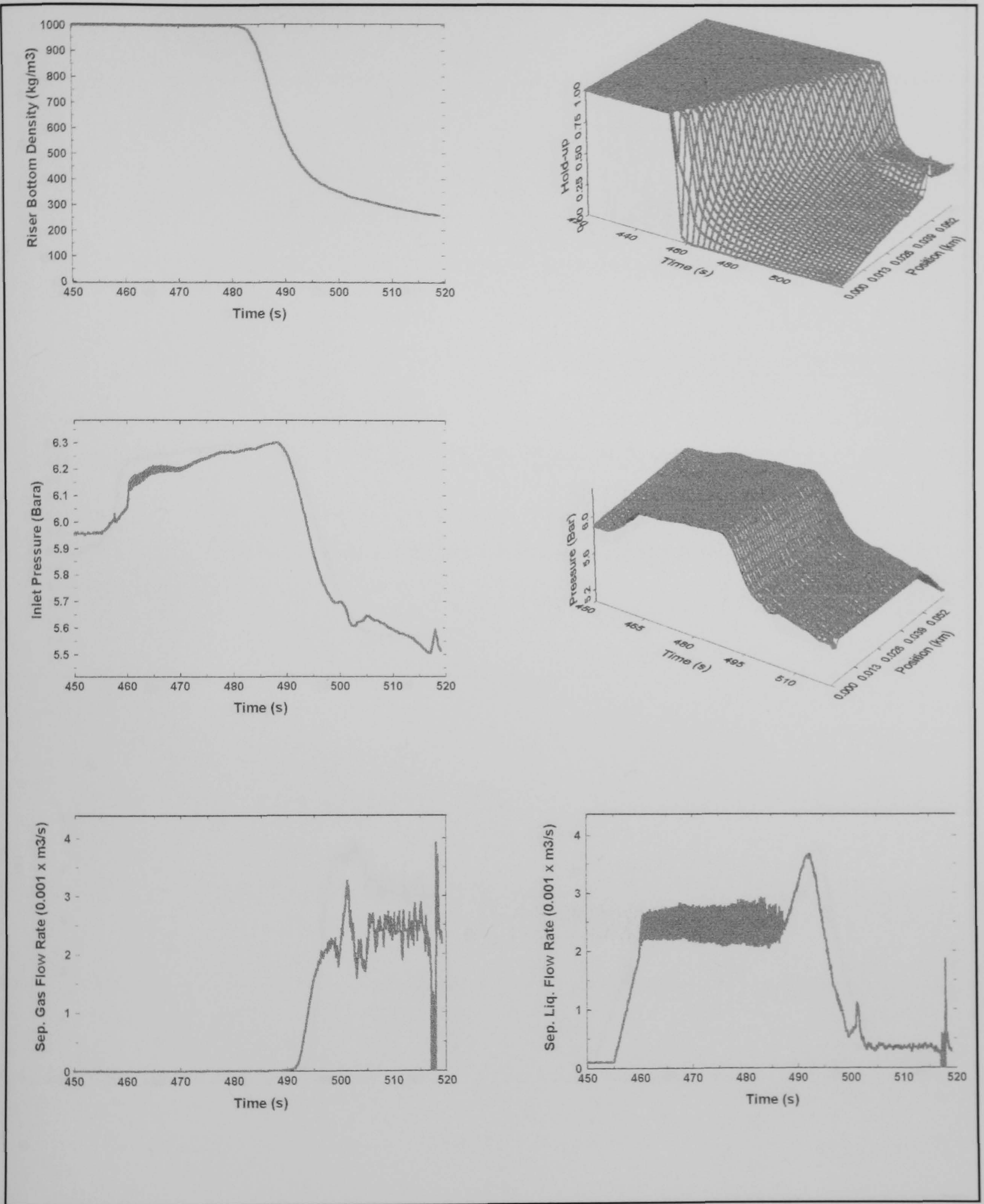


Figure F-4 - Model results of run HGP1

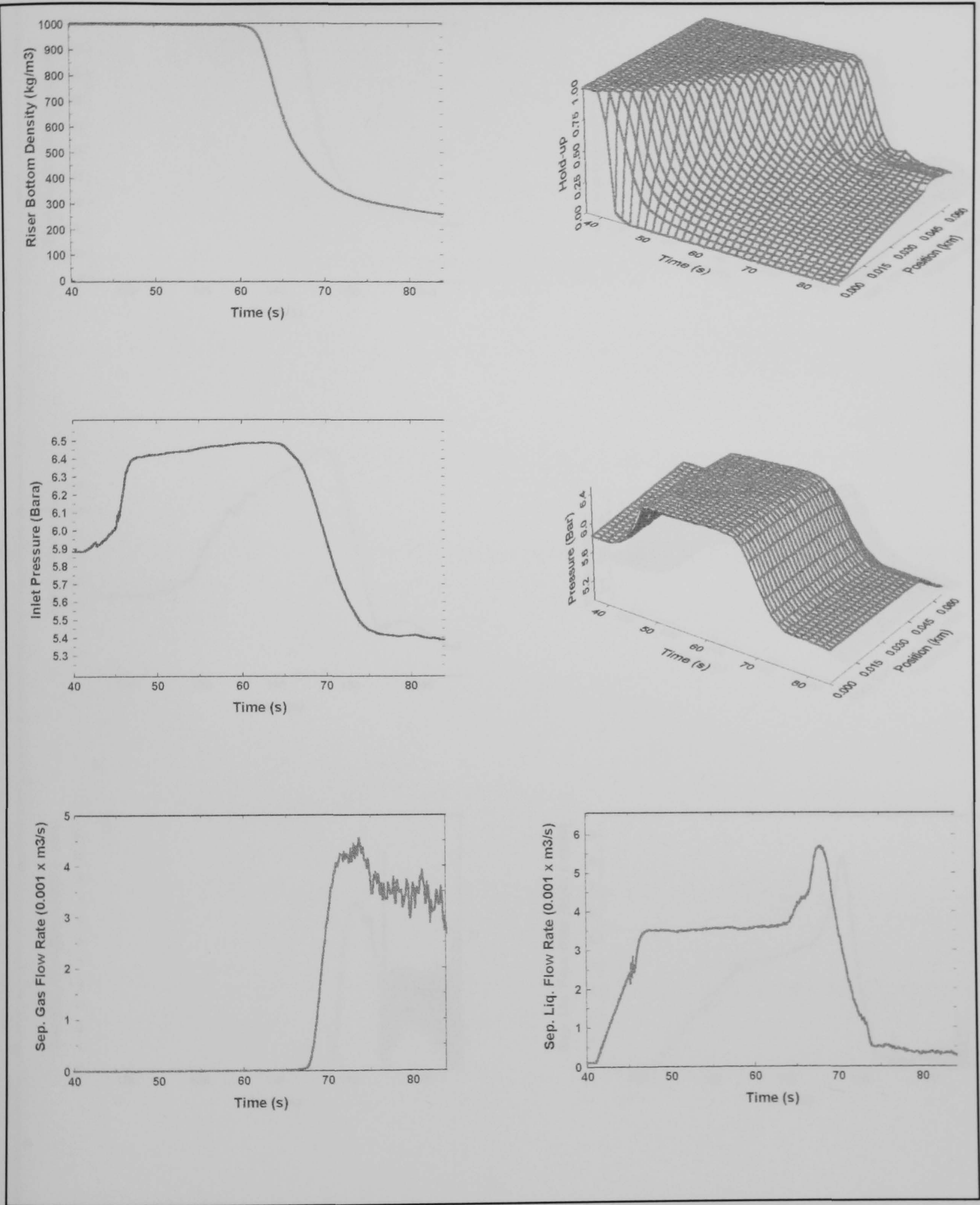


Figure F-5 - Model results of run HGP2

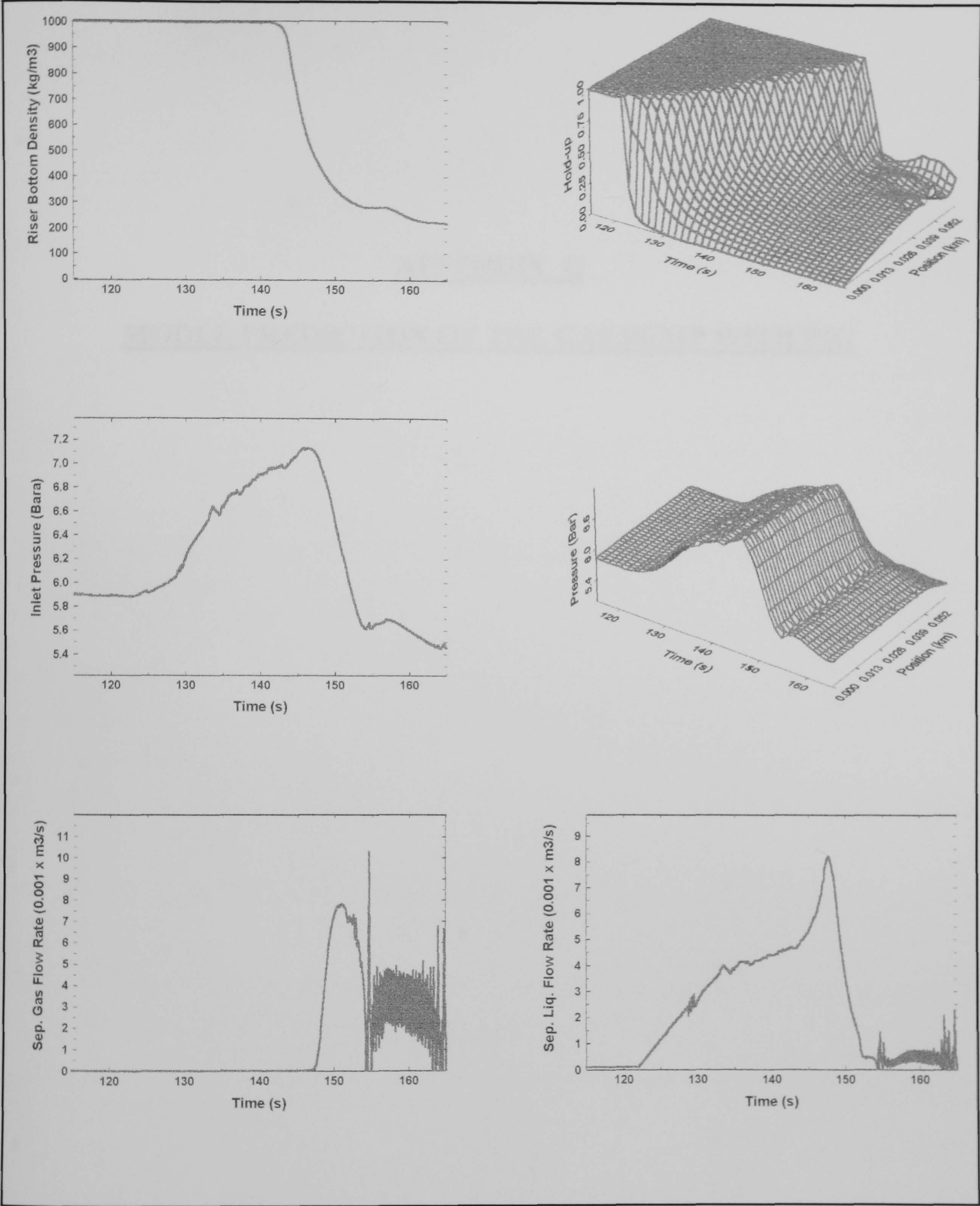


Figure F-6 - Model results of run HGP3

APPENDIX G

MODEL PREDICTION OF THE GAS PUMP WITH PIG

G. MODEL PREDICTION OF THE GAS PUMP WITH PIG

The model prediction of the three gas pump runs with pig is shown in this appendix. Table G.1 presents the initial conditions of the test section, the gas and pig injection schedules and the predicted efficiency of liquid transportation. Figs G.1 to G.3 show the predicted transient flow behaviour caused by a gas and pig injection in a pipeline-riser system full of liquid.

Table G.1 - Model results of the gas pump with pig

RUN CODE	INITIAL PRESSURE		VOLUME OF GAS INJECTED (Sm ³)	PIG SCHEDULE		EFFICIENCY (%)
	Inlet (Bara)	Separator (Bara)		Launching (s)	Arrival (s)	
LPP1A	2.4	1.5	0.466	305	343.6	99.18
LGPP2	2.4	1.5	0.641	242	270.2	98.62
LGPP3	2.5	1.6	0.824	195	217.0	99.7

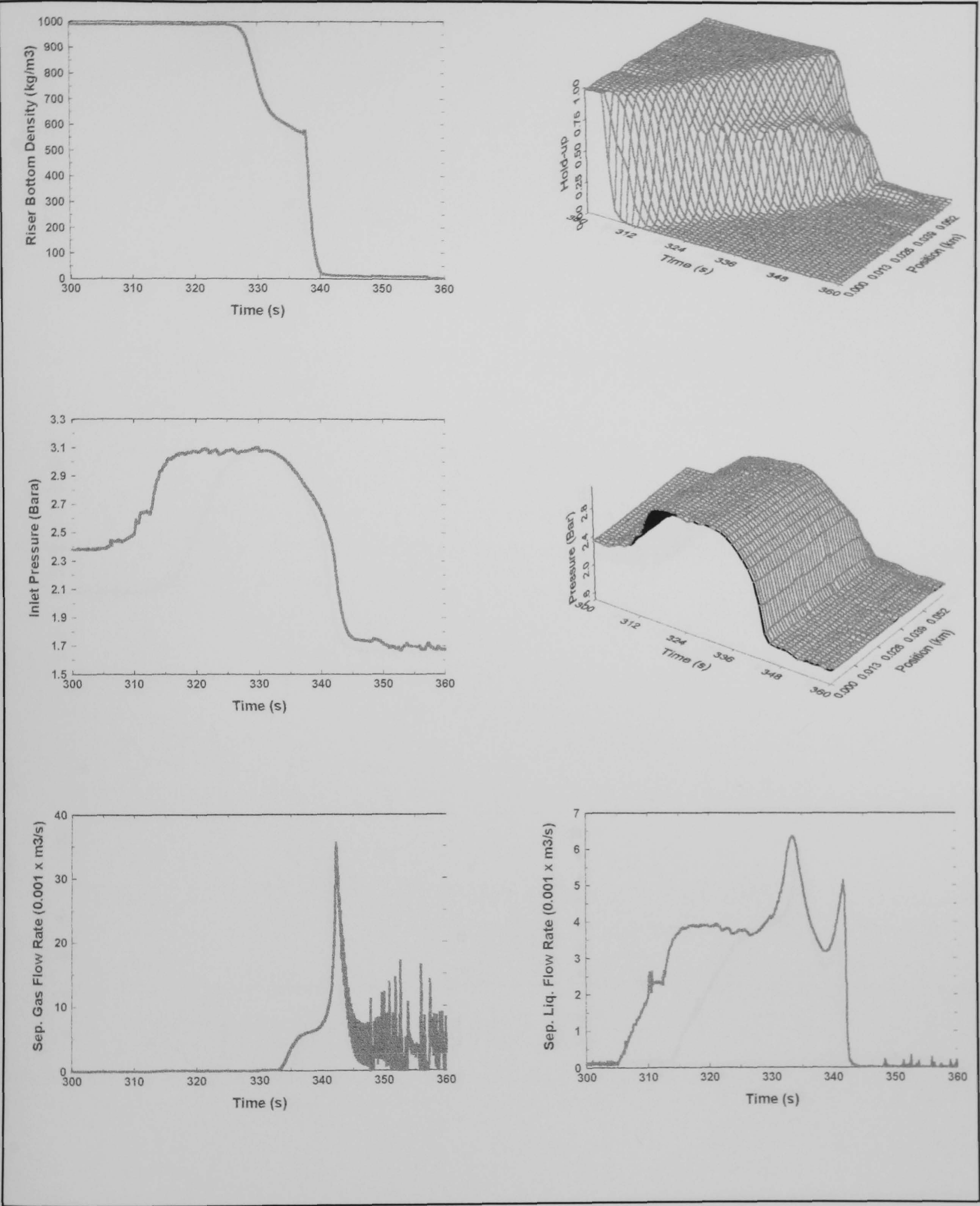


Figure G-1 - Model results of run LPP1A

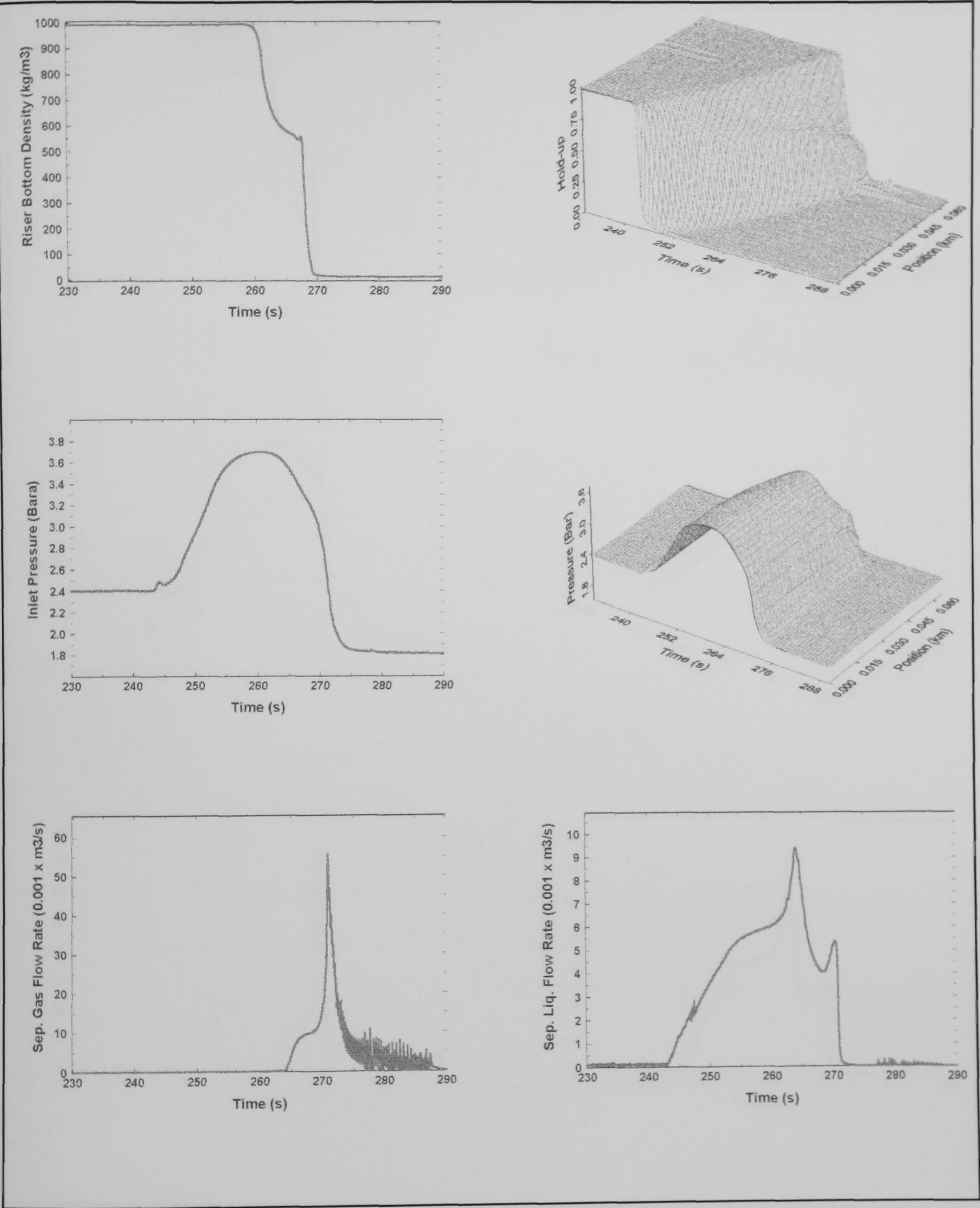


Figure G-2 - Model results of run LGPP2

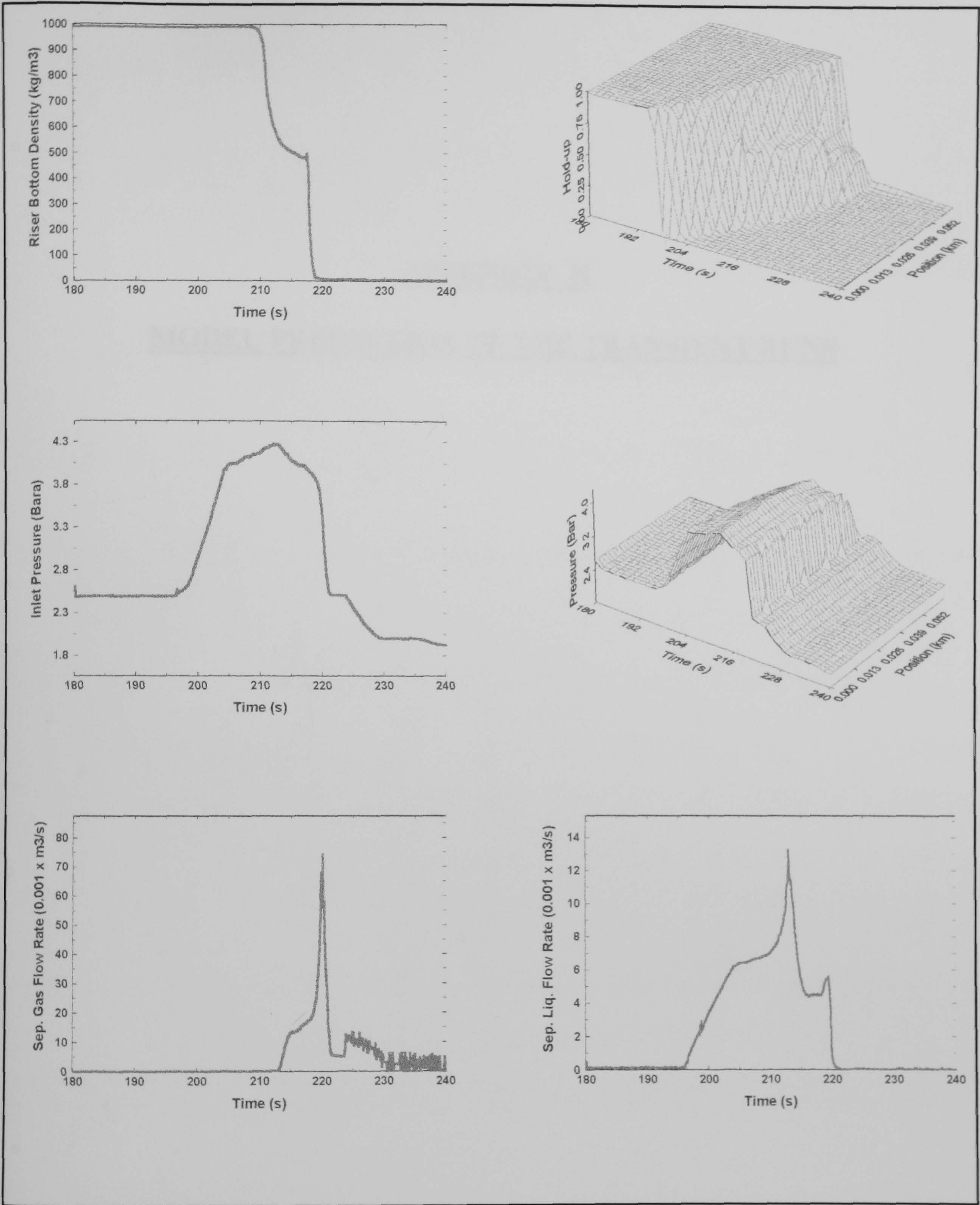


Figure G-3 - Model results of run LGPP3

APPENDIX H

MODEL PREDICTION OF THE TRANSIENT RUNS

H. MODEL PREDICTION OF TRANSIENT RUNS

The model prediction results of the ten transient runs is shown in this appendix. Table H.1 presents the predicted results for initial and final conditions of the test section. Figs H.1 to H.10 show the predicted transient flow behaviour caused by a change of boundary conditions.

Table H.1 - Model results of the transient runs

RUN CODE	INITIAL PRESSURE		FINAL PRESSURE	
	Inlet (Bara)	Separator (Bara)	Inlet (Bara)	Separator (Bara)
HTG12W3	5.78	4.65	5.80	4.75
HTG34W1	4.70	4.31	6.30	5.37
HTG34W2	7.00	5.88	5.60	4.90
HTG34W3	6.20	4.94	7.30	5.95
LTG12W1	2.80	1.80	2.48	2.00
LTG23W2	2.63	2.00	2.75	2.10
LTG34W2	2.70	2.05	3.20	2.20
LTG34W3	2.75	1.90	3.00	2.05
LTG43W1	2.60	2.05	2.40	2.00
LTG32W1	2.40	2.00	2.27	1.95

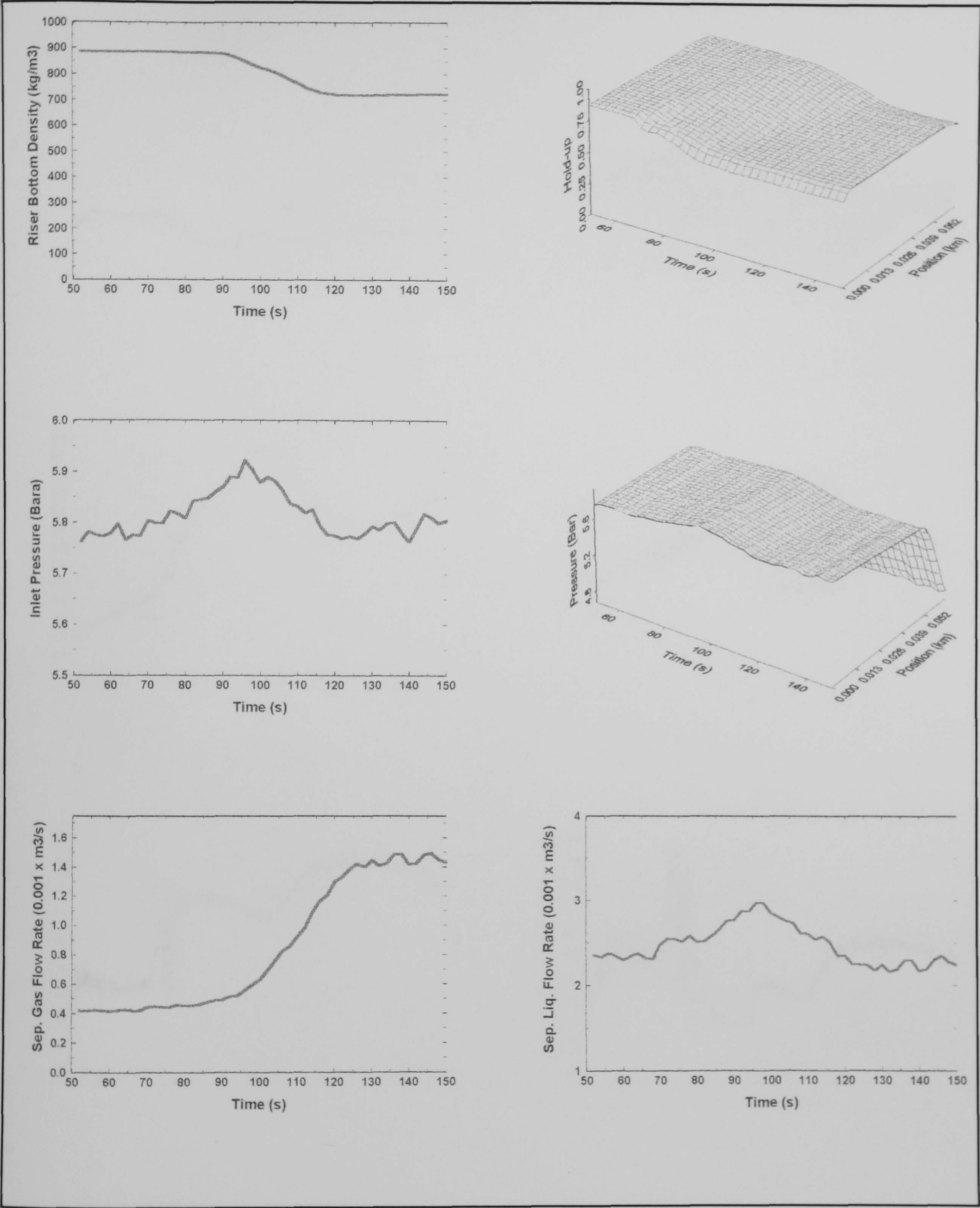


Figure H-1 - Model results of run HTG12W3

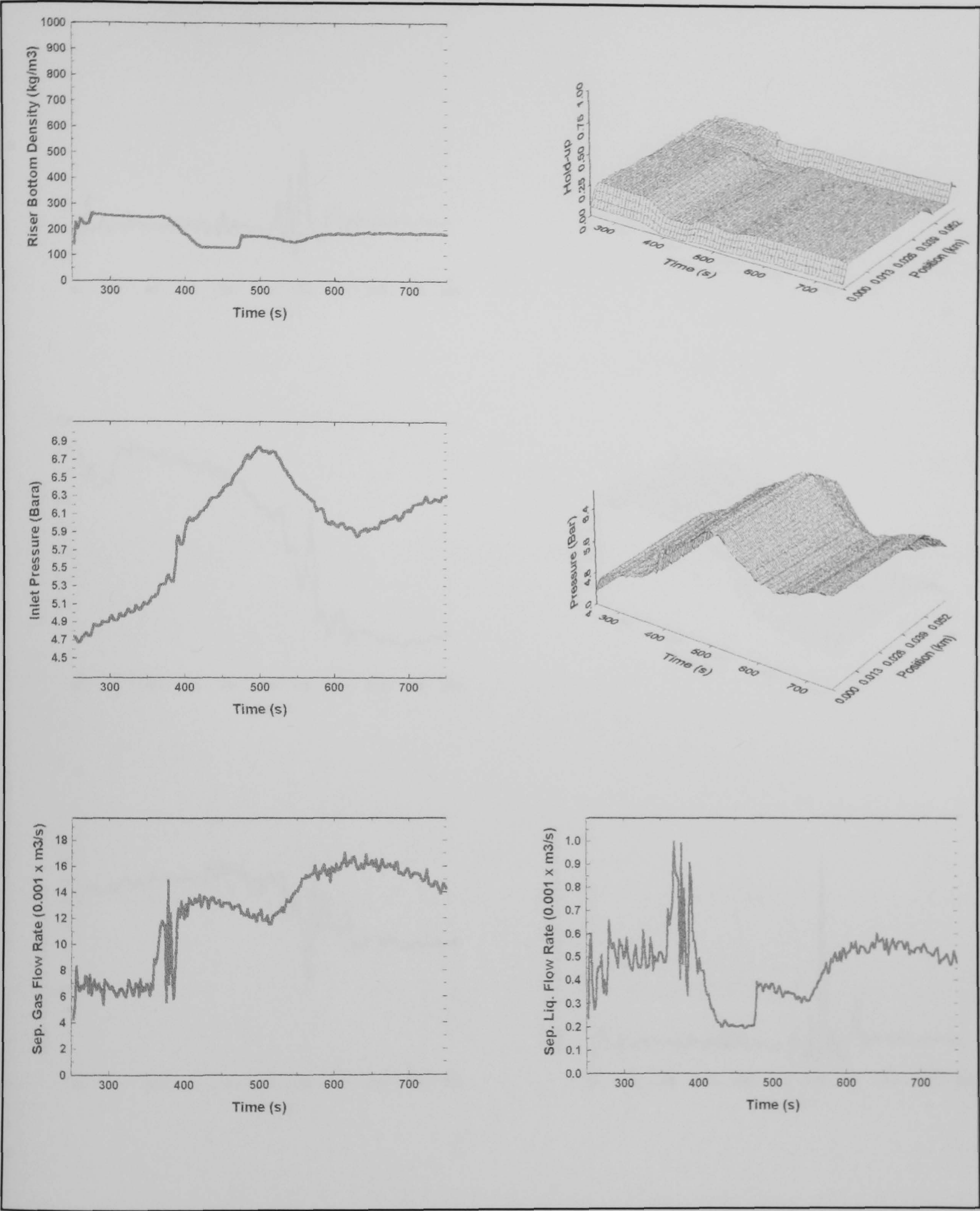


Figure H-2 - Model results of run HTG34W1

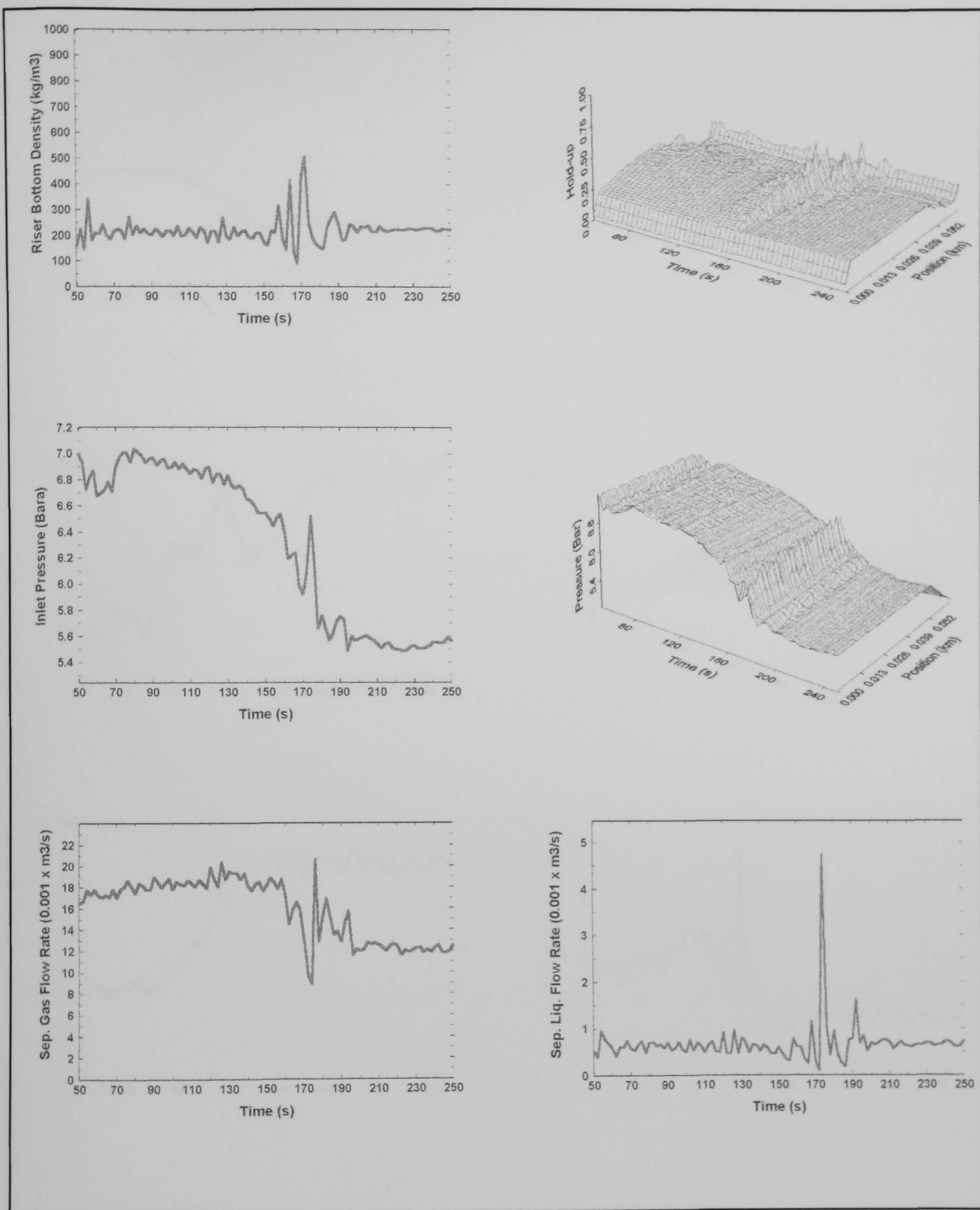


Figure H-3 - Model results of run HTG34W2

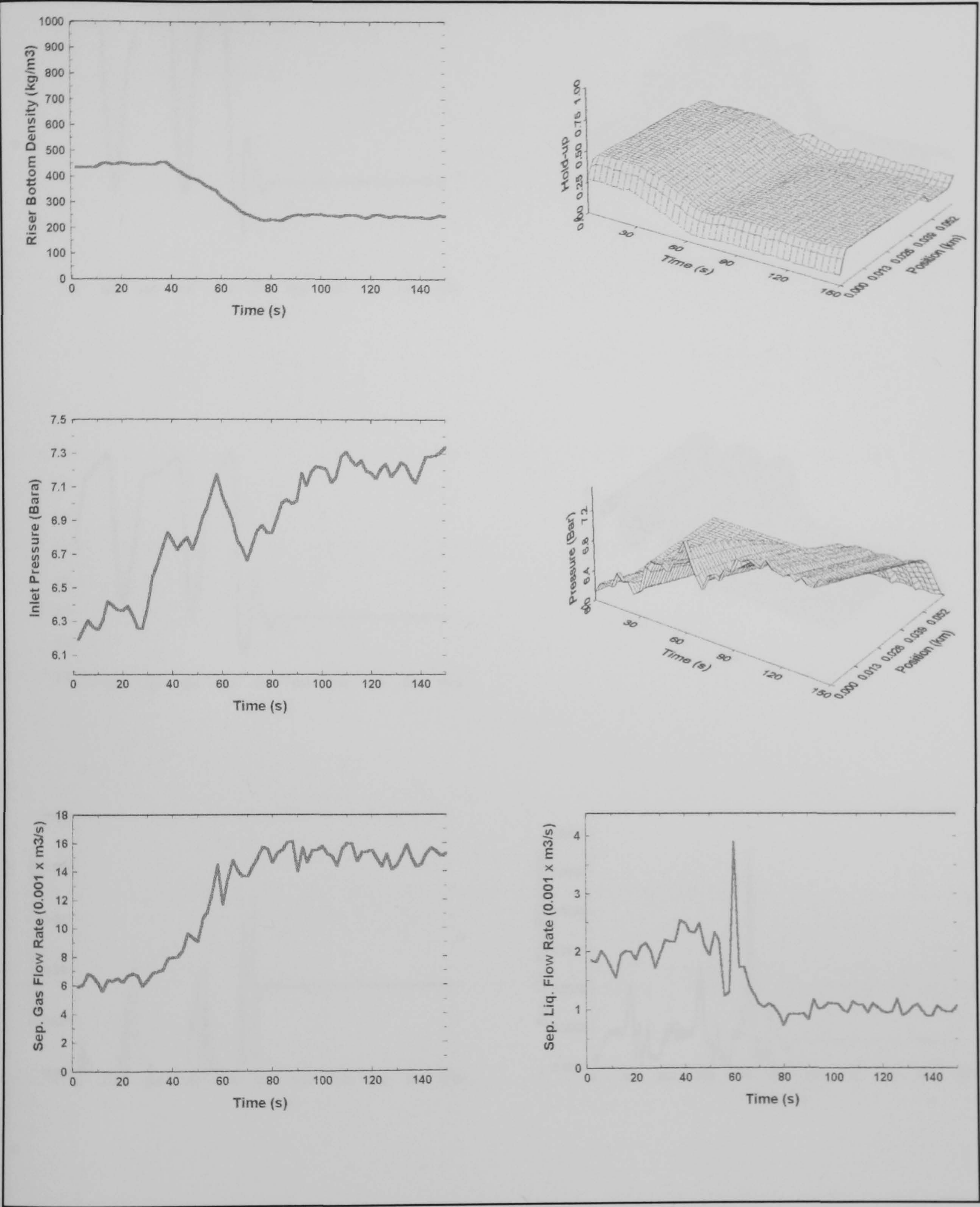


Figure H-4 - Model results of run HTG34W3

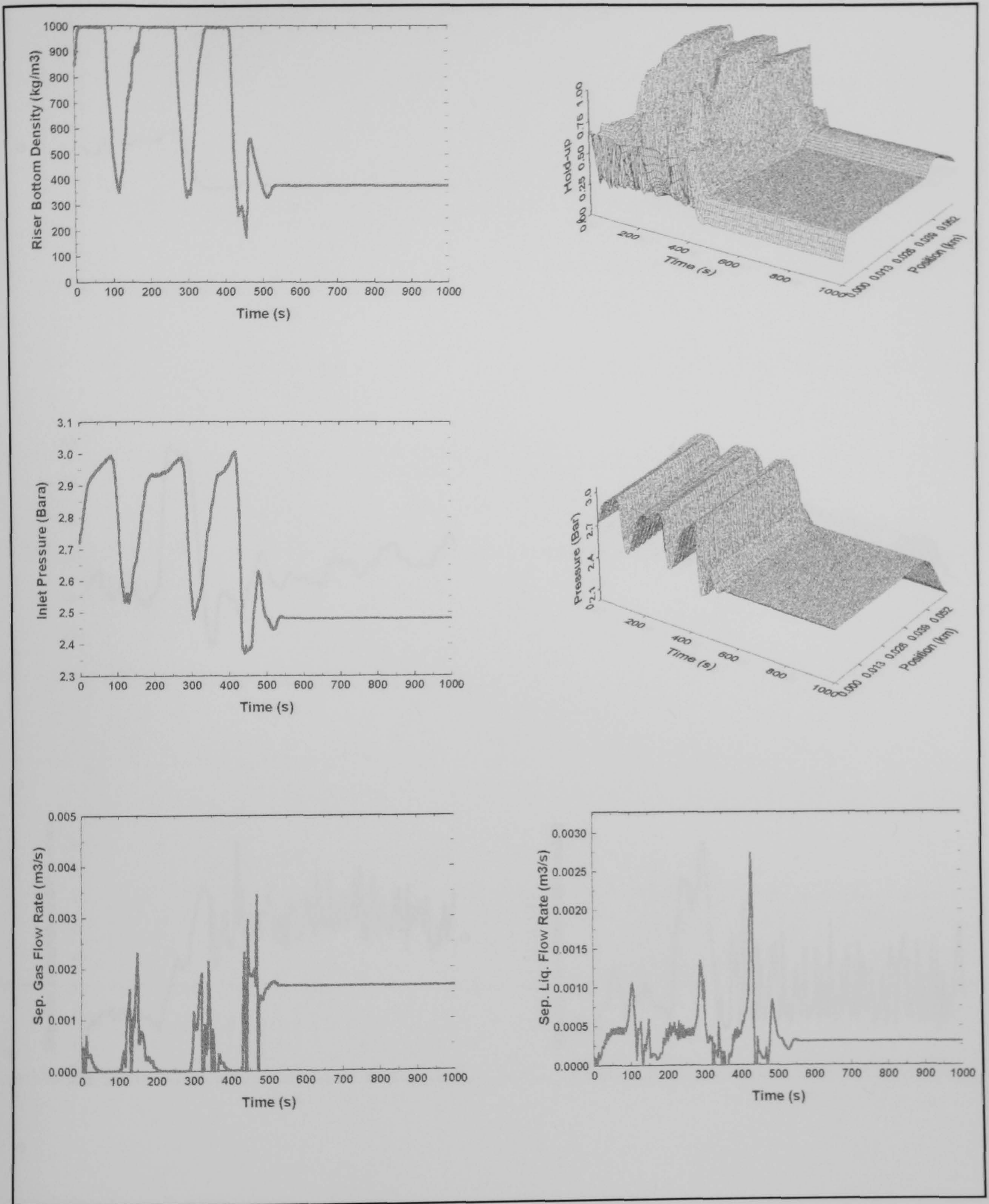


Figure H-5 - Model results of run LTG12W1

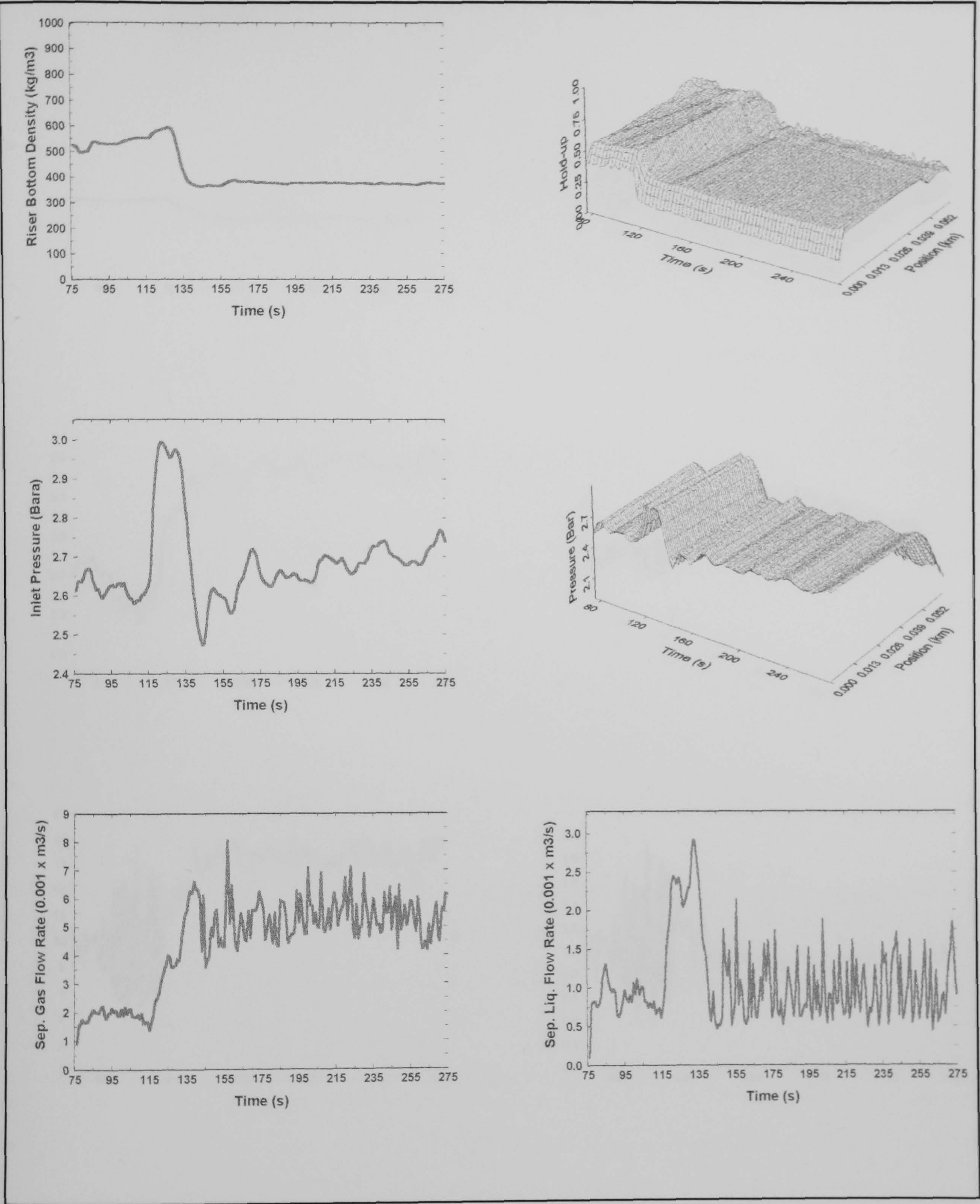


Figure H-6 - Model results of run LTG23W2

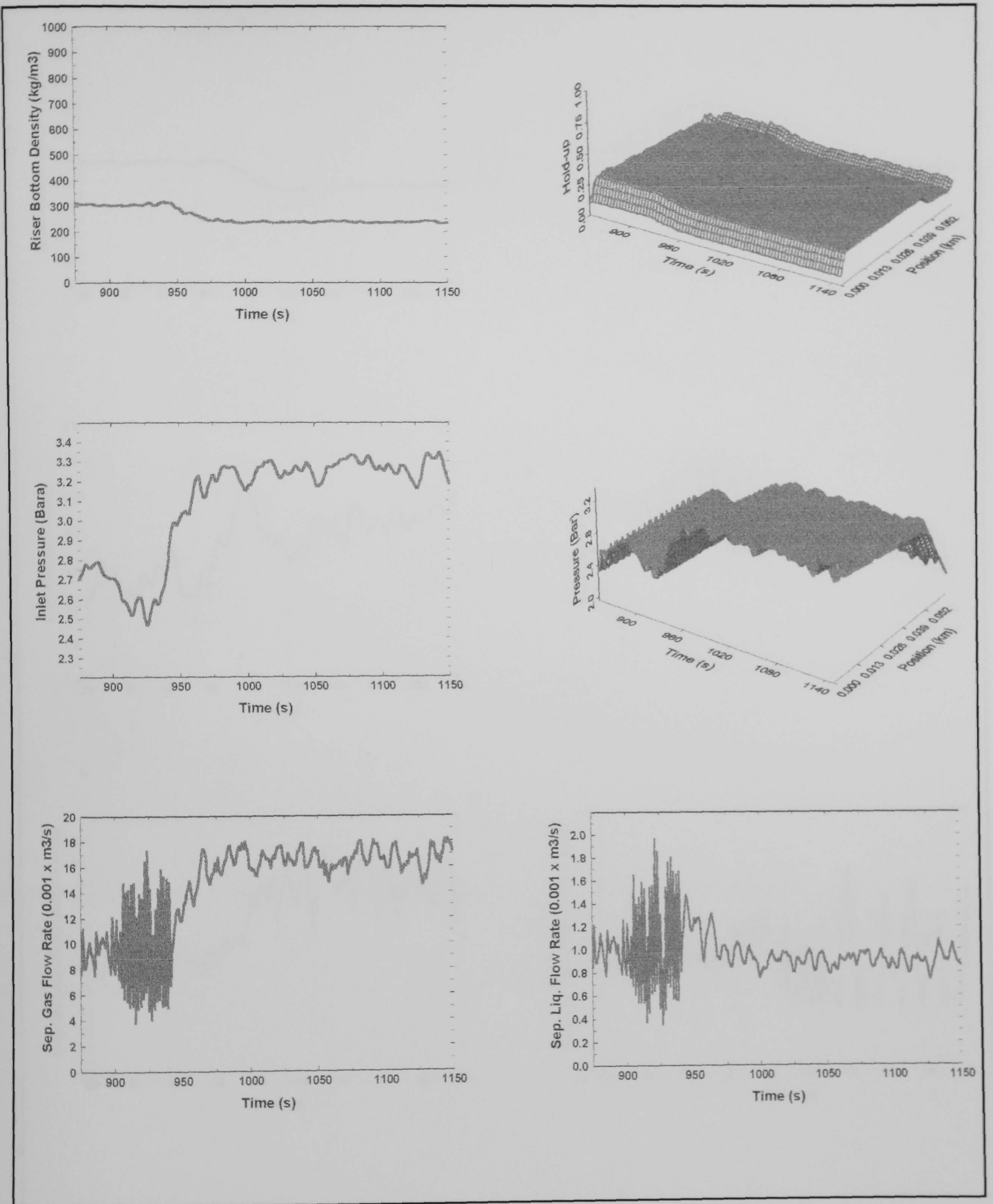


Figure H-7 - Model results of run LTG34W2

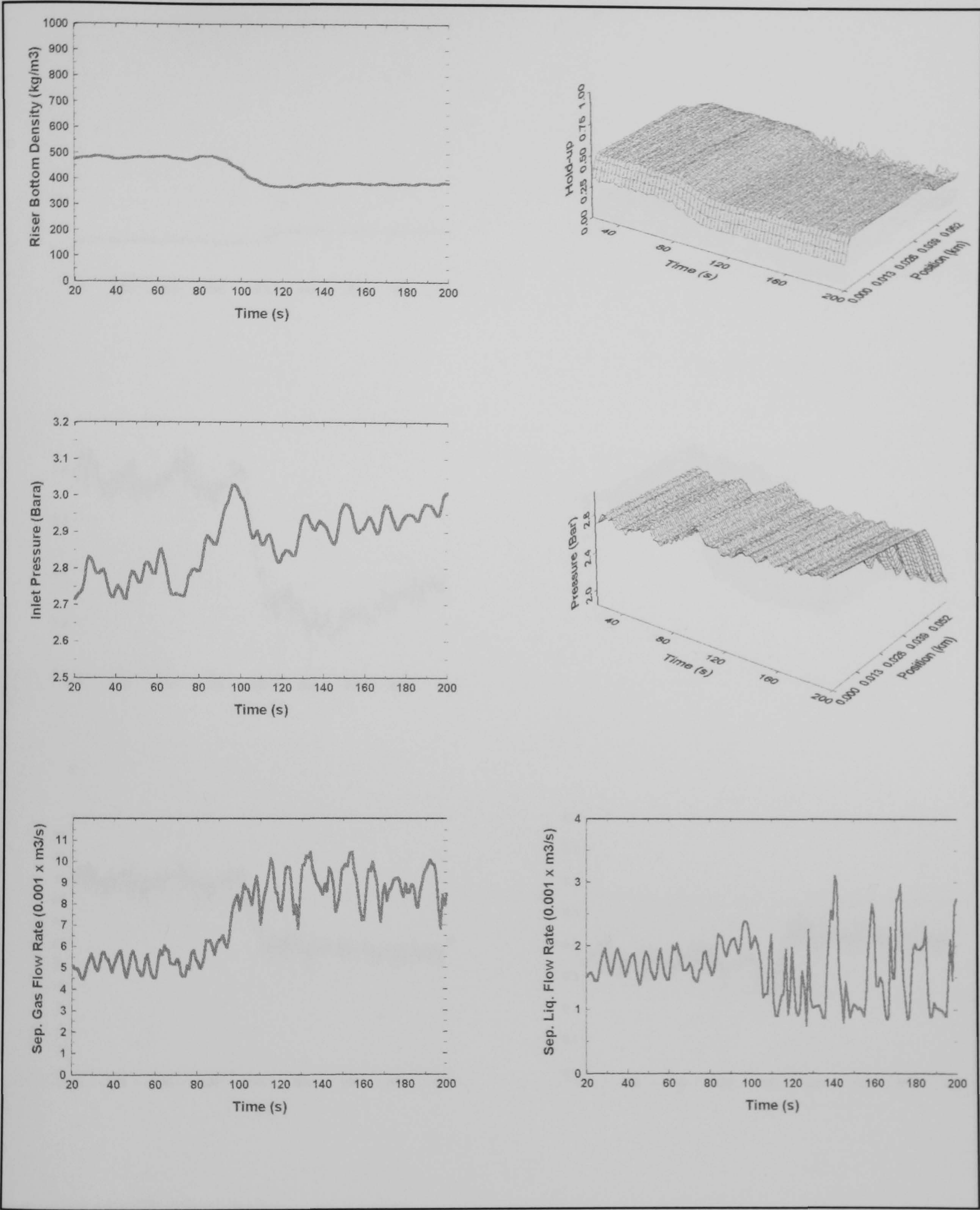


Figure H-8 - Model results of run LTG34W3

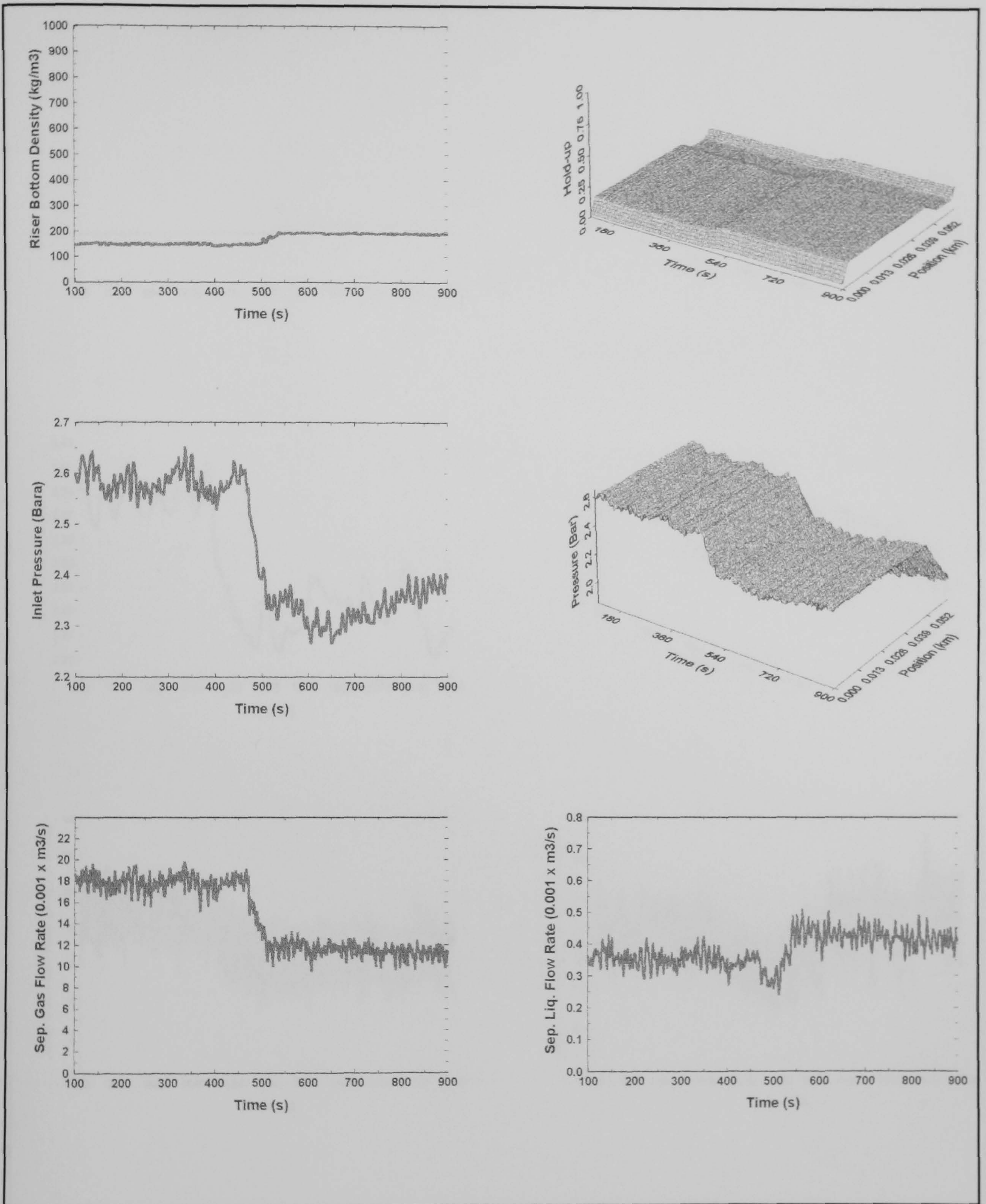


Figure H-9 - Model results of run LTG43W1

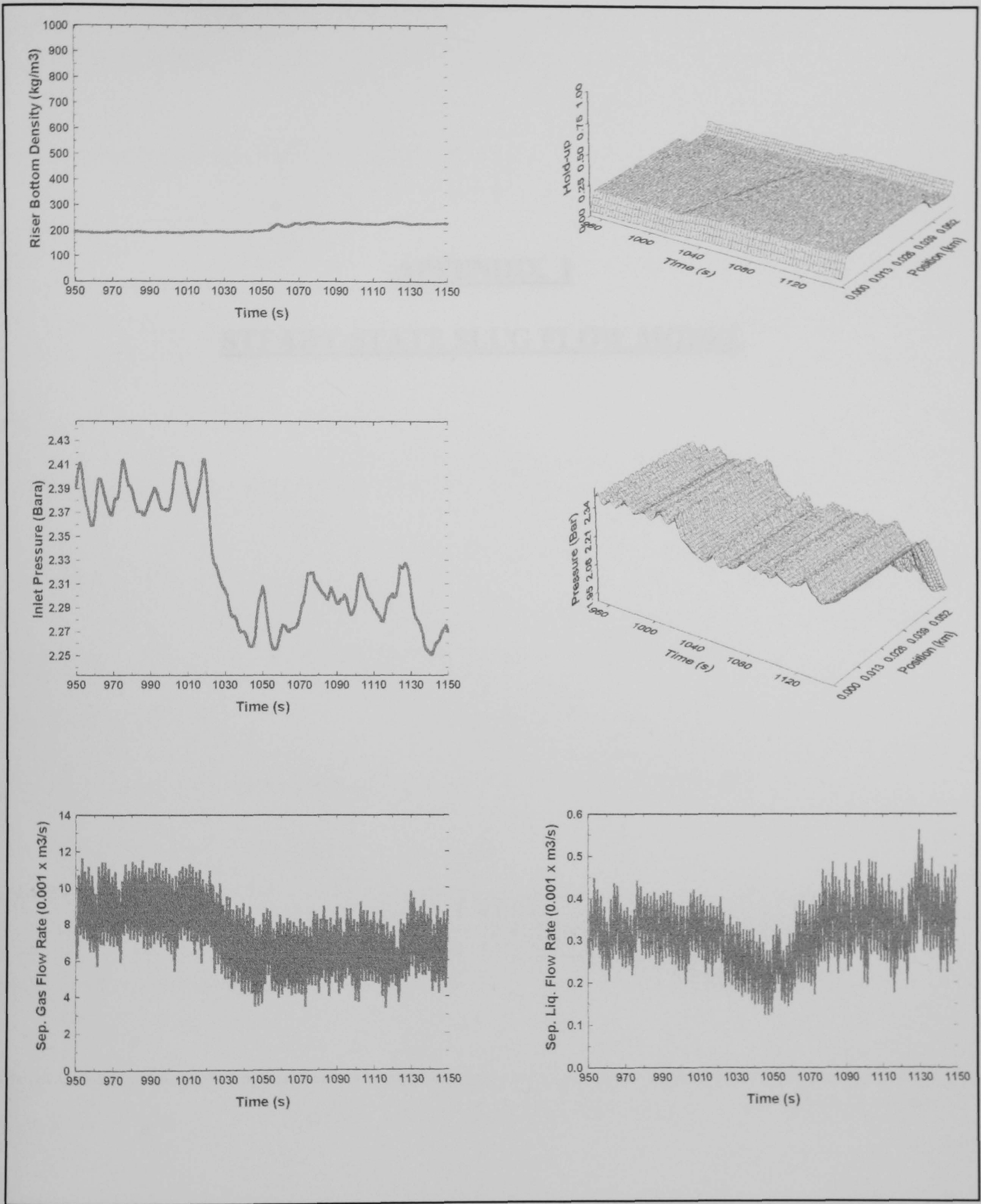


Figure H-10 - Model results of run LTG32W1

APPENDIX I

STEADY-STATE SLUG FLOW MODEL

I. STEADY-STATE SLUG FLOW MODEL

For slug flow the average liquid flow rate is given by:

$$v_l \alpha_l = \alpha_{ls} v_s \frac{l_s}{l} + \alpha_{lf} v_{lf} \frac{l_f}{l} \quad (I-1)$$

The v_{lf} in eq. (I-1) can be eliminated using a liquid continuity balance relative to a moving co-ordinate system that travels with the slug translational velocity v_t :

$$\alpha_{lf} (v_t + v_{lf}) = \alpha_{ls} (v_t - v_s) \quad (I-2)$$

The liquid hold-up for a slug unit can be defined as

$$\alpha_l = \alpha_{ls} \frac{l_s}{l} + \alpha_{lf} \frac{l_f}{l} \quad (I-3)$$

Solving eq. I-3 for α_{lf} and combining the result with eq. I-1 and I-2 yields

$$v_l \alpha_l = v_t \alpha_l - \alpha_{ls} (v_t - v_s) \quad (I-4)$$

The translational velocity is correlated in terms of the slug velocity v_s as follows:

$$v_t = C_o v_s + v_d \quad (I-5)$$

Substituting v_t and $v_l \alpha_l = v_s - v_g \alpha_g$ into eq. I-4 yields:

$$v_s = \frac{v_g \alpha_g - v_d (\alpha_{ls} - \alpha_l)}{1 - \alpha_l C_o + (C_o - 1) \alpha_{ls}} \quad (I-6)$$

Eq. I-6 allows the calculation of the average slug velocity for given liquid hold-up for a slug unit and gas flow rate. α_{ls} is correlated in terms of slug velocity by Gregory et al. (1978) as follows:

$$\alpha_{ls} = \frac{1}{1 + (v_s / 8.66)^{1.38}} \quad (I-7)$$

for $\alpha_{ls} > 0.48$; otherwise, a minimum value of 0.48 was used (Barnea and Brauner - 1985).

The drift velocity for vertical flow was determined from Nicklin et al. (1962)

$$v_d = 0.35\sqrt{gD} \quad (1-8)$$

For inclined flow, the value was multiplied by $\sin \Theta$.

A value of 1.2 was taken for the constant C_o (Nicholson et al. 1978).

Once v_s is known, v_l can readily be obtained from $v_l = [v_s - v_g (1 - \alpha_l)] / \alpha_l$.

Applying the momentum balance on the film region yields:

$$\begin{aligned} & \left[\left[\rho_g \frac{c_{wgf}}{\alpha_{gf}} v_{gf}^2 \right] + \left[\rho_g \frac{c_{if}}{\alpha_{gf}} (v_{gf} - v_{lf})^2 \right] + [\rho_g g \sin \Theta] \right] \\ & - \left[\left[\rho_l \frac{c_{wlf}}{(1 - \alpha_{gf})} v_{lf}^2 \right] - \left[c_{if} \frac{\rho_g}{(1 - \alpha_{gf})} (v_{gf} - v_{lf})^2 \right] + [\rho_l g \sin \Theta] \right] = 0 \end{aligned} \quad (1-9)$$

Eq. (1-9) is solved by a trial-and-error procedure for the gas volume fraction in the film zone α_{gf} . For a given α_{gf} , the liquid film velocity v_{lf} is calculated using eq. 1-2. The gas velocity in the film zone, v_{gf} , is calculated using the following mass balance:

$$v_{gf} \alpha_{gf} - v_{lf} (1 - \alpha_{gf}) = v_s \quad (1-10)$$

The shear coefficients used here are also the same as for stratified or annular flow.

The ratios l_s/l and l_f/l can be calculated from eq. 1-3.

APPENDIX J
VELOCITY COEFFICIENTS

J. VELOCITY COEFFICIENTS

The momentum equations in the Basic Step are:

- Gas momentum equation

$$\frac{1}{\left\langle \rho_g \right\rangle_{j+\frac{1}{2}}^n \Delta z_{j+\frac{1}{2}}} (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + \frac{c_i^n}{\left\langle \alpha_g \right\rangle_{j+\frac{1}{2}}^n} \left| v_g^n - v_l^n \right| \left[2(v_g^{n+1} - v_l^{n+1}) - (v_g^n - v_l^n) \right] + g \sin \Theta = 0 \quad (\text{J-1})$$

- Liquid momentum equation

$$\frac{1}{\left\langle \rho_l \right\rangle_{j+\frac{1}{2}}^n \Delta z_{j+\frac{1}{2}}} (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + \frac{(1+K)c_{wl}^n}{\left\langle 1-\alpha_g \right\rangle_{j+\frac{1}{2}}^n} \left| v_l^n \right| \left[2v_l^{n+1} - v_l^n \right] - c_i^n \left\langle \frac{\rho_g}{(1-\alpha_g)\rho_l} \right\rangle_{j+\frac{1}{2}}^n \left| v_g^n - v_l^n \right| \left[2(v_g^{n+1} - v_l^{n+1}) - (v_g^n - v_l^n) \right] + g \sin \Theta = 0 \quad (\text{J-2})$$

Equations (J-1) and (J-2) can be rewritten as:

$$\bullet \quad G_1 (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + G_2 \left[2(v_g^{n+1} - v_l^{n+1}) - G_3 \right] + G_4 = 0 \quad (\text{J-3})$$

$$\bullet \quad L_1 (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + L_2 (2v_l^{n+1} - L_3) - L_4 \left[2(v_g^{n+1} - v_l^{n+1}) - L_5 \right] + G_4 = 0 \quad (\text{J-4})$$

$$\text{where } G_1 = \frac{1}{\left\langle \rho_g \right\rangle_{j+\frac{1}{2}}^n \Delta z_{j+\frac{1}{2}}}, G_2 = \frac{c_i^n}{\left\langle \alpha_g \right\rangle_{j+\frac{1}{2}}^n} \left| v_g^n - v_l^n \right|, G_3 = (v_g^n - v_l^n), G_4 = g \sin \Theta, L_1 = \frac{1}{\left\langle \rho_l \right\rangle_{j+\frac{1}{2}}^n \Delta z_{j+\frac{1}{2}}},$$

$$L_2 = \frac{(1+K)c_{wl}^n}{\left\langle 1-\alpha_g \right\rangle_{j+\frac{1}{2}}^n} \left| v_l^n \right|, L_3 = v_l^n, L_4 = c_i^n \left\langle \frac{\rho_g}{(1-\alpha_g)\rho_l} \right\rangle_{j+\frac{1}{2}}^n \left| v_g^n - v_l^n \right|, L_5 = (v_g^n - v_l^n).$$

Equations (J-3) and (J-4) can be rewritten as:

$$\bullet \quad G_1 (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + 2G_2 v_g^{n+1} - G_2 v_l^{n+1} - G_2 G_3 + G_4 = 0 \quad (\text{J-5})$$

$$\bullet \quad L_1 (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) - 2L_4 v_g^{n+1} + (L_2 + L_4) v_l^{n+1} - L_2 L_3 + L_4 L_5 + G_4 = 0 \quad (\text{J-6})$$

or

$$\bullet \quad G_1 (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + 2G_2 v_g^{n+1} - G_2 v_l^{n+1} - G_5 = 0 \quad (\text{J-7})$$

$$\bullet \quad L_1 (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) - 2L_4 v_g^{n+1} + L_6 v_l^{n+1} - L_7 = 0 \quad (\text{J-8})$$

where $G_5 = G_2 G_3 - G_4$, $L_6 = L_2 + L_4$ and $L_7 = L_2 L_3 - L_4 L_5 - G_4$.

Multiplying eq. (J-7) by L_4 and eq. (J-8) by G_2 yields:

$$\bullet \quad G_1 L_4 (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + 2L_4 G_2 v_g^{n+1} - L_4 G_2 v_l^{n+1} - L_4 G_5 = 0 \quad (\text{J-9})$$

$$\bullet \quad L_1 G_2 (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) - 2L_4 G_2 v_g^{n+1} + L_6 G_2 v_l^{n+1} - G_2 L_7 = 0 \quad (\text{J-10})$$

Adding eq. (J-9) to eq. (J-10) yields:

$$(G_1 L_4 + L_1 G_2) (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + (L_6 G_2 - L_4 G_2) v_l^{n+1} - (L_4 G_5 + G_2 L_7) = 0 \quad (\text{J-11})$$

Eq. (J-11) can be rewritten as:

$$L_8 (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + L_9 v_l^{n+1} - L_{10} = 0 \quad (\text{J-12})$$

where $L_8 = G_1 L_4 + L_1 G_2$, $L_9 = L_6 G_2 - L_4 G_2$, $L_{10} = L_4 G_5 + G_2 L_7$.

Multiplying eq. (J-7) by L_6 yields:

$$G_1 L_6 (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + 2L_6 G_2 v_g^{n+1} - L_6 G_2 v_l^{n+1} - L_6 G_5 = 0 \quad (\text{J-13})$$

Adding eq. (J-13) to eq. (J-10) yields:

$$(G_1 L_6 + L_1 G_2) (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + 2(L_6 G_2 - L_4 G_2) v_g^{n+1} - (L_6 G_5 + G_2 L_7) = 0 \quad (\text{J-14})$$

Eq. (J-14) can be rewritten as:

$$G_6(p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + G_7 v_g^{n+1} - G_8 = 0 \quad (\text{J-15})$$

where $G_6 = G_1 L_6 + L_1 G_2$, $G_7 = 2(L_6 G_2 - L_4 G_2)$, $G_8 = L_6 G_5 + G_2 L_7$.

Equations (J-12) and (J-15) can be rewritten respectively as:

$$v_l^{n+1} = E_l (p_{i+1}^{\diamond n+1} - p_i^{\diamond n+1}) + F_l \quad (\text{J-16})$$

$$v_g^{n+1} = E_g (p_{j+1}^{\diamond n+1} - p_j^{\diamond n+1}) + F_g \quad (\text{J-17})$$

where $E_l = -\frac{L_8}{L_9}$, $F_l = \frac{L_{10}}{L_9}$, $E_g = -\frac{G_6}{G_7}$ and $F_g = \frac{G_8}{G_7}$.