

Incorporating implicit condensation into data-driven reduced-order models for nonlinear structures

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ABSTRACT

The global climate effort is increasingly dependent on lightweight, flexible designs to provide engineering solutions capable of meeting ambitious emissions targets. Examples of these designs include high-aspect ratio wings, which are capable of achieving extended flight times using significantly less energy, but their complexity introduces geometric nonlinearity to the system, leading to a substantial increase in complexity. Although these nonlinear dynamics can be accurately modelled using finite element (FE) software, the required magnitude of such models are extremely computationally expensive, preventing their use in real-time applications or extensive modelling procedures. Non-intrusive reduced-order models (NIROMs) for the nonlinear behaviour are of great interest to the mechanical engineering community, as they are capable of capturing the full system dynamics using a significantly reduced coordinate system (typically a subset of the vibration modes). However, the generation of reliable NIROMs remains an active challenge. This paper combines the projection-based strategy adopted by the Implicit Condensation method with recent results from the field of machine learning to create a novel NIROM generation technique based on time series data. Specifically, a Variational Recurrent Autoencoder is applied to the system dynamics on the reduced modal basis. To complement the ability of VRAEs to reproduce time series and create statistically consistent synthetic data, a second decoder is added to recreate the true parameterisation of the nonlinear system of equations.

Keywords: Nonlinear Vibrations, Long Short-Term Memory, Physics-Informed Neural Networks, Reduced-Order Model

INTRODUCTION

Geometric nonlinearity occurs in structures experiencing vibration amplitudes that cause excessive strain of the structure, leading to a nonlinear force-displacement relationship, $F_{NL}(x)$. The equations of motion for nonlinear structures typically take the form:

$$M\ddot{x} + C_x\dot{x} + Kx + F_{NL,x}(x, \dot{x}) = F_x(t),$$

where M , C_x , and K are the mass, damping, and linear stiffness matrices, respectively, x is a vector of displacements, $F_{NL,x}$ is a nonlinear function in terms of these displacements (often assumed to be a cubic polynomial), F_x is an external forcing term, and the dot (·) notation denotes differentiation with respect to time, t .

While the majority of these terms can be calculated quickly and simply, either analytically or using FE software, the nonlinear restoring force, $F_{NL,x}$, provides more of a challenge. In FE software, this method finding the value of this force is not provided to the user, nor is the source code, which is the reasons a *non-intrusive* reduced-order model (NIROM) is sought. Essentially, this function must be approximated in some way without being able to directly access the calculations. There are numerous strategies for such an approximated—Mignolet *et al.* provide an excellent overview for these in [1]—but a common theme among several of these is to assume it takes a polynomial (typically cubic order) form, the coefficients of which are to be calculated based on the system behaviour. The Implicit Condensation (IC) method [2] (and subsequent Implicit Condensation and Expansion method [3]) applies a series of static modal forcing cases and utilises the resultant modal displacements in a regression model to calculate the most suitable coefficients. While this methodology has generated strong results [4], the magnitude of the forces applied are defined by the user, though it is difficult to correlate these to real-

world forces and there remains a lack of precise conditions that will guarantee the model is accurate. This has led to limited uptake of these methods (and, therefore, the application of nonlinear structures) in industrial applications.

The underlying aim of the present work is to build upon this condensation strategy by utilising recent results from the field of machine learning to condense the nonlinear components of the equations of motion from time series data. A key motivation in exploring this approach is the ability to include data that explicitly includes hysteretic time series, which is naturally not possible for static forcing cases. At IMAC-XLI, I presented my data-driven strategy for NIROM generation, which uses Long Short-Term Memory (LSTM), a recurrent neural network with in-built capabilities to “look back” through the previous n time steps. I demonstrated that hysteretic behaviour around natural frequencies can be accurately captured, even though these frequencies are not explicitly provided to the system. In the present work, the benefits of applying an LSTM neural network are compounded through the application of a Variational Recurrent Autoencoder (VRAE) [5].

VARIATIONAL RECURRENT AUTOENCODER (VRAE) NEURAL NETWORKS

Variational autoencoders (VAE) neural networks [6] are a probabilistic expansion of the autoencoder neural network, which comprises two key functions: the encoder and the decoder. The encoder learns a mapping from the input data to some latent space, which is assumed to be able to capture fundamental characteristics of the full system on some undefined, lower-order manifold. The decoder then learns to map the latent representation of the data back to the original data format. The *variational* component of VAEs arises from the introduction of a reparameterization of the latent space through the use of Gaussian distributions; through this adaptation, the decoder can be used to generate probabilistically consistent synthetic data, based solely on these Gaussian distributions [6].

In [5], the authors again expand the capabilities of the VAE method by introducing recurrent neural networks to the encoder and decoder functions. Full details on this are omitted here, but it should be noted that this update allows for LSTM layers to be included, with the important cell state of the method being the final layer prior to the latent space. As such, the ability to “look back” across time steps is inherently captured in the encoding procedure. The training of VRAE neural networks is undertaken through the typical backpropagation strategy, which is applied to both the encoder and decoder in parallel.

METHODOLOGY

A key aim of the proposed methodology is the ability to condense the nonlinear behaviour onto a low-order modal basis, producing an analytical NIROM. The projection of both the force and displacement time histories is very easily achieved through the application of a truncated mode shape matrix, Φ , which facilitates the projection $\mathbf{x} \approx \Phi \mathbf{q}$. The accuracy of this projection is dependent on the choice of the modal basis, $\mathbf{q}$, which is here selected using the Modal Assurance Criterion (MAC):

$$\text{MAC}(\phi_i, \phi_j) = \frac{(\phi_i^T \phi_j)^2}{(\phi_i^T \phi_i)(\phi_j^T \phi_j)}$$

Here, ϕ_i denotes the i^{th} mode shape, ϕ_j denotes the j^{th} Proper Orthogonal Mode [7], so that $\text{MAC}(\phi_i, \phi_j)$ determines the similarity between individual vectors of the two matrices. By setting a cutoff level for MAC scores, it is possible to optimally select a reduced modal basis based on both model error and efficiency.

The subsequent NIROM takes the following form:

$$\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{A}\mathbf{q} + \mathbf{F}_{NL}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{F}(t),$$

where $\mathbf{C}$, $\mathbf{A}$, $\mathbf{F}$ denote the modal damping, stiffness, and forcing matrices, respectively, all of which can be readily obtained through simple projection. As such, the modal nonlinear restoring force function, $\mathbf{F}_{NL}$, is the only term that cannot be directly and easily calculated.

The novel methodology proposed in this work aims to capitalize on the significantly lower order of the modal basis, which allows expansive time series data to be generated with minimal computational cost. Based on the rough assumption that the linear and nonlinear restoring force would be of a similar magnitude (i.e., $kx = \alpha x^3$ for a linear system), it is possible to randomly generate nonlinear functions of the form $\mathbf{F}_{NL,n}(\mathbf{q}, \dot{\mathbf{q}}) = \sum_{i,j,k=1}^N A_{ijk}^{(n)} q_i q_j q_k + \sum_{i,j=1}^N B_{ij}^{(n)} q_i q_j$, where $\{A, B\}$ are generated from a uniform distribution across the range $[0, \frac{\omega_{n,1}}{\max(x)}]$. For each of these functions, it is then possible to create time histories based on different forcing levels. A total of N_{TH} dynamical forcing cases are generated and combined to create a single time history, which aids VRAE training; note that “breaks” in the forcing time history are added to allow the damping effects to be observed. The corresponding displacement and velocity time series are then recorded for each parameterized nonlinear function. By consistently applying the same forcing time history, it is only necessary to use the state space time history as input for the VRAE. The VRAE framework is outlined in Fig. 1.

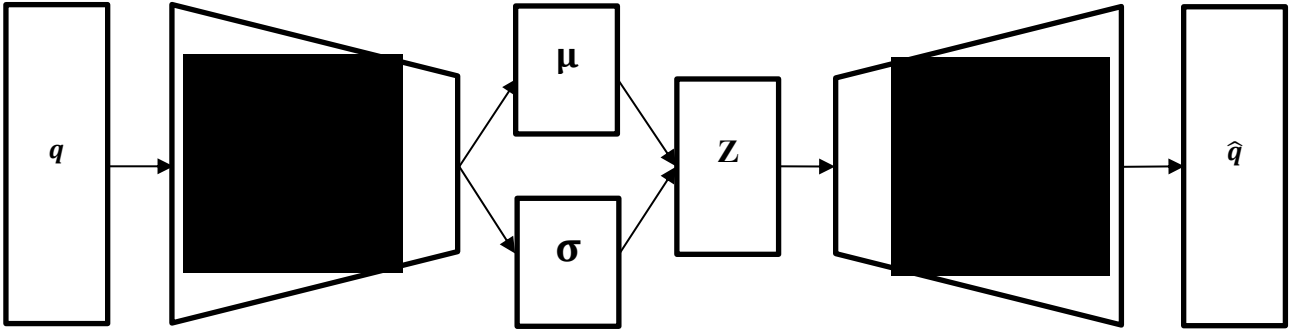


Figure 1: VRAE overview.

The structure and accuracy of LSTM neural networks was presented in [IMAC], so is not explored in detail here, though it can be observed from Fig. 2 that they are capable of producing very accurate surrogate models for the individual nonlinear parameterizations. As highlighted in Fig. 1, the LSTM encoder maps $\mathbf{q}$ to a latent space, $\mathbf{Z}$, that is characterized by the mean and standard deviation of each parameter. These Gaussian distributions are then sampled to generate synthetic data in the latent space, which is then decoded to reproduce the original time series. The backpropagation algorithm aims to minimize the mean square error of the true and reproduced time series data.

A second decoder will be added to map $\mathbf{Z}$ to $\{A, B\}$. Recent publications [8] have demonstrated that this approach can garner excellent results when the parameter vector is also provided as an input, which is useful in parameter variation studies. However, the aim of this study is to be able to characterize any system, regardless of whether such a parameterization is known. As such, the true values $\{A, B\}$ will not be included as an input to the VRAE, rather they will be implicitly present in the time series and, therefore, the latent space. Then, for reduced state space time history, the VRAE will be able to both produce synthetic time series data and parameterize the nonlinear restoring force function.

CONCLUSIONS AND FUTURE WORK

This study presents a novel VRAE approach for NIROM generation for geometrically nonlinear structures. The approach mirrors the condensation of higher-order strain effects developed by the IC approach, but replaces the static force and displacement cases with a neural network model that can handle (potentially hysteretic) time series data. The work builds upon the LSTM neural network results presented at IMAC-XLI by adding a latent space that captures the system dynamics in a lower-order manifold, characterized by Gaussian distributions. Currently, these distributions create new data that can be decoded to reproduce the original time series. In future work, a secondary decoder will be added, mapping this latent space—as well as the fundamental system characteristics it contains—to the true parameterization of the nonlinear restoring force function. Through this methodology, it will be possible to identify the nonlinear system equations solely from state space time history. Furthermore, the input data will be user-driven, so that the model can be tailored based on their knowledge of realistic operating conditions.

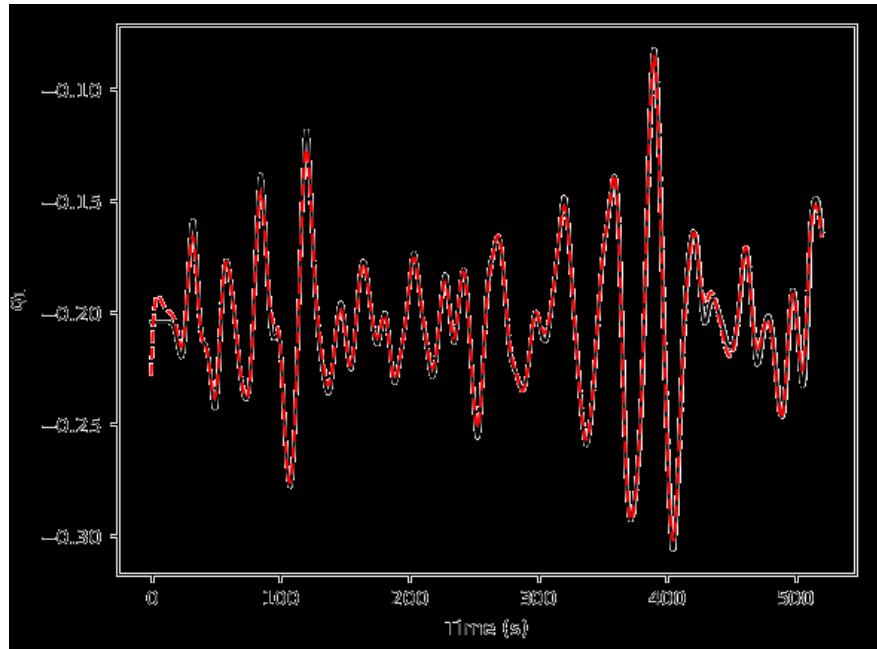


Figure 2: Time history for the true (black) and LSTM neural network predicted (red, dashed) for a randomly-parameterized, low-order nonlinear system.

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