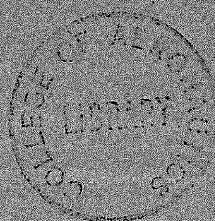
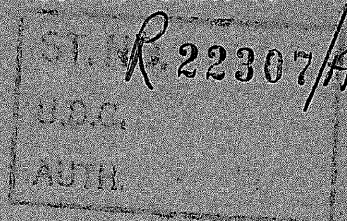


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THE COLLEGE OF AERONAUTICS
CRANFIELD



REPORT OF A VISIT TO U. S. S. R.

25TH JUNE 1960

by

R. J. A. PAUL



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THE COLLEGE OF AERONAUTICS

CRANFIELD

Report of a visit to U.S.S.R. for the International
Federation of Automatic Control Conference in Moscow

25th June - 9th July, 1960

- by -

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1. PLACES VISITED

U.S.S.R. (Moscow, Leningrad and Kiev).

2. PURPOSE OF VISIT

2.1. To attend the I.F.A.C. (International Federation of Automatic Control) Conference in Moscow. (See Appendix 1).

2.2. To visit educational and industrial establishments.

3. DATE OF VISIT

25th June, 1960 - 9th July, 1960 (inclusive).

4. GENERAL COMMENTS ON THE I.F.A.C. CONFERENCE

4.1. Technical

The background of the International Federation of Automatic Control is given in Appendix 1. The Conference held at Moscow State University (June 27th, 1960 to 1st July, 1960) was the first I.F.A.C. Conference.

As stated in Appendix 1, I.F.A.C. is composed of 29 national organisations and the United Kingdom is represented by the British Conference on Automation and Computation (B.C.A.C.).

There were approximately 80 members in the British delegation, led by J.F. Coales (Cambridge University). The total number of delegates was approximately 1,100. The technical content of the Conference, as presented in 274 papers, embraced theory, components and industrial applications of automatic control. With eleven simultaneous sessions, the overall impression of any one individual are restricted to attendance at a small percentage of the number of sessions and informal discussions with other delegates. A feature which impressed most Western delegates was the concentration of Russian mathematicians and scientists on the theory of non-linear control systems, and the excellent quality of Russian papers on this topic. Russian contributions to non-linear mathematics, of course, are well known, but it was interesting to learn of the concentrated effort to apply these theoretical aspects to typical physical systems. On the other hand, practical realisation of these techniques, in the form of working equipment, was not impressive and was reflected in the small number of Russian papers devoted to this topic. This point was also confirmed during visits to industrial establishments.

Detailed reviews of the technical content of the Conference have now been published (Ref. 1), and my personal contribution to this paper is given in Appendix 2.

In addition a Symposium was held at the Institution of Mechanical Engineers (27th to 28th September, 1960) at which the British papers and a review of the most significant papers from other countries were presented by chief reporters. These reports and discussions will be published in the Journal. I acted as chief reporter at two of the sessions and copies of these reports are given in Appendix 3.

4.2. Organisation and Official Russian Viewpoint

One aspect of the Conference, worthy of note, was the support given to the Russian Organising Committee by the Soviet Government, the Academy of Sciences and the Institute of Automatics and Telemechanics. The importance attached to automatic control by the Soviet Government was illustrated by the fact that the opening meeting was attended by several members of the Government (equivalent to cabinet rank). The Government also acted as hosts at a formal reception at the Kremlin on the last day of the Conference. This proved to be a most spectacular and elaborate affair, with a sumptuous buffet and an excellent cabaret, in which dancing groups from various republics, members of the Bolshoi Ballet Company and others performed. This offset, to some extent, one's impression of Russian hospitality which was gained at an earlier reception given by the Organising Committee. The solid and liquid refreshments here were also excellent but delegates were somewhat surprised when they were presented with an equally impressive bill. The official Russian attitude to the importance of industrial applications of automatic control may be illustrated by the opening remarks by V.A. Trapeznikov, President of the Institute of Automatics and Telemechanics - "Not only will automation raise productivity but it will also radically change the very nature of labour. The higher labour productivity and expanded material production stimulated by automation will sharply improve the standard of living and reduce the working day to but a few hours in the near future".

Another illustration was the publicity given to the Conference in the press, on radio and television. We were also told that 7.5% of the U.S.S.R. Academicians were directly working in the automatic control field. This figure did not include those working in associated fields of interest such as computing techniques. This emphasis on automatic control was also illustrated by the impressive array of Russian text books paper on the subject, which was estimated at about 1,000. Equally impressive was the fact that most of the available Western text

books and reports were translated into Russian and available to students at a remarkably low cost, e.g. the Russian translation of a book costing about 40 shillings in this country, was available at the rouble equivalent of about five shillings. Sales of books we were told, were very high, and this fact pained some Western authors who, of course, received no royalties.

4.3. Summary

In retrospect, the papers and discussions at the Conference, form a significant contribution to available knowledge.

The Russians are attaching considerable importance to the application of automatic control to industrial processes. Although there is little evidence, as yet, of significant practical applications, it would appear likely that they will make significant advances in the next ten years.

At present, it would appear that the Russians are at least as advanced on theoretical topics as the Western world but in general they are behind in terms of development and practical applications.

5. TECHNICAL VISITS

Various official visits to educational, research and industrial establishments were organised. I personally concentrated on educational and research institutes, and took the opportunity of visiting establishments in Leningrad and Kiev in addition to those in Moscow. A review of technical visits is given in Ref. 2 and I include some brief comments on the more important visits which I made.

5.1. Educational Establishments

5.1.1. Moscow State University

The conference was held in this University but unfortunately there was no official tour included in our list of visits. It was difficult, therefore, to get a clear picture of the method of teaching, lecture hours, etc.

The building itself is vast and was built about 1954. Architecturally it reminded me of a gigantic wedding cake, comprising a functional core with adornments added in the form of spires, turrets, etc. I managed to talk to several under graduates who told me that the total student body was about 24,000*. Languages, Literature, Science and Mathematics form the

* It was reported by other delegates that the University caters for more than 10,000 students in Natural Science.

main feature, but every student is compelled to read Soviet Philosophy throughout the 5 - 6 years course. Engineering is not taught at the University, as this topic is dealt with in the various education institutes. The age at entrance appears to be in the range of 18 to 22.

I noticed that all under graduates spoke extremely good English and I formed the impression that this was also a compulsory subject.

Attendance at lectures varied from 1,000 for a general topic such as Soviet Philosophy to about 30 for a very specialised course.

Judging by the text books on scientific topics used on the course it would appear that the standard of the degree awarded, compares favourably with British University degrees. The standard of mathematics seemed to be exceptionally high.

The major lecture halls were very well equipped (individual head phones, good projection facilities, etc.). Several of these halls were extremely large and could comfortably house about 1,200 people.

5.1.2. The Leningrad Institute of Electrical Engineering

Director - Professor N.P. Bogoroditsky.

This Institute, established about 75 years, is an educational establishment, which specialises in preparing engineers in all modern branches of Electrical Engineering. The specialisations vary according to demand and there are at present five departments.

- (i) Radio Engineering
- (ii) Electronic Engineering and devices
- (iii) Automatic and Computing Devices
- (iv) Industrial Automatic Control
- (v) Electro-physics - link between University and Technical studies - especially in the field of electro-acoustics.

Day and evening courses.

Duration - day course - 5 years, 10 months.

evening course - 3 times a week, 6 hours of lectures, giving a total of 18 hours per week.

Vacations: 2 fortnightly periods in Winter, 2 months in Summer.

During the six year courses, some eighteen months are spent in industrial concerns. Throughout the course an examination is held every six months which the student must pass to proceed to the next stage. The total failure rate was stated to be about 10% from start to finish. The first two years are common to all students with 150 hours devoted to engineering drawing. In the third year the student may specialise in any one of the five departments but Physics (Solid State and Quantum Mechanics) is compulsory. With successive years more time is spent in the laboratories at the expense of formal lectures. The total hours per week decrease from 34 in the second year to 24 in the final year which is devoted to a research topic, the quality of which varies from Ph.D. standard to a project study. The theses are examined by a state engineering board composed mainly of industrialists, academicians and research workers. A Diploma of the Institute is awarded to successful students provided that they also prove themselves to be proficient in swimming, skiing and politics. There are 6,000 day students (20% female) and about 1,000 part-time day and evening students. The numbers attending each lecture course may exceed 200 but the main teaching is tutorial based on groups of 25. The total staff exceeds 500, thirty of whom are doctors, and 250 of whom are candidates actively engaged on research for their doctoral degree, (awarded by Leningrad University) - Doctor of Engineering Science. Further research work after the award of Doctor paves the way for entry into the Academy of Science.

Entrance to the Institute course is by examination and only 25% of candidates can be accepted. The age limits are 17 and 35 but the majority of students are in the range 18 - 22 years old.

The laboratories which we saw were well equipped and there was particular emphasis on experimental investigations of two-dimensional field theory in the teaching laboratories.

Most of the laboratories are associated with research activities and this results in an active and vigorous laboratory staff.

5.2. Research Institutes

5.2.1. Institute of Automation and Telemechanics (I.A.T.) - Moscow

This Institute was visited by most delegates because of its world wide reputation as judged by the excellence of the papers published in the Institute's Journal, translated reviews of which are available in most Western Countries.

My first impression was one of disappointment in respect of the rather dingy and small buildings in which the Institute is housed. However, we were informed that they are shortly moving into a new building with properly equipped laboratories.

I. A. T. employs some 400 scientists engaged on ten main themes, with each group under the direction of a well known name. It was interesting to learn that each particular group was confined to a very narrow field of interest. The Institute has no direct connection with Moscow University, (where very little research is carried out), but it employs about seventy post-graduate students. Members of the Institute, however, lecture at the University on mathematical topics.

The Institute has some 20 specialists who translate documents in any language and who claim that they see all available literature pertaining to their subjects of interest.

The staff appear to be very proud that they are members of a pure research institute and in general prefer to have little interest in the possible applications of their studies. However, it was apparent that some members were engaged on development topics and an interesting example was Professor Aiserman's work on the use of pneumatic and hydraulic devices as computing units (digital and analogue).

Computing facilities at the Institute included two analogue computers the performance of which were not impressive by Western standards. We did not find any digital computers but it would appear that they had access to such a facility.

The general standard of laboratory equipment was not impressive and most delegates were disappointed in this respect. Due to the rather lengthy introductory speech in the Director's office the tour of the Institute, because of the time limitation, was confined to visits to relatively few laboratories, so impressions gained should be viewed with reserve.

Finally it may be of interest to note that the salary of a scientist having a doctorate is about £2,000 per annum (based on the tourist's exchange rate of 28 roubles per pound sterling) and about £1,300 per annum for the less qualified. These salaries are not subject to income tax. However, if a more realistic exchange rate (60 roubles to the pound sterling) is taken, these salaries do not appear to be impressive.

5.2.2. Computing Centre (Kiev)

Created in 1957.

This is a new research institute engaged on the investigation of digital and analogue techniques. They also appear to act as consultants to industrial organisations regarding the use of digital and analogue computers. A specific task on which they are engaged is the application of computers to the control of industrial processes and in particular a Bessemer steel making plant. The Institute has designed the KIEV digital computer, which by Western standards is not very impressive. It was estimated by delegates that we were about 5 years in advance regarding computing techniques and the institute staff agreed with this point of view. They stressed, however, that they were a very young institute and they hoped to be on level terms in the next few years.

The Institute also uses a URAL digital computer which is an earlier design and is comparable in performance to the earliest American commercial form of a small digital machine.

It was interesting to note that some 300 URAL computers are now installed throughout the Soviet Union.

During the visit we did not see any general purpose analogue computers which was rather surprising. They demonstrated a small conventional resistance network analyzer which again appeared to have no novel features.

The staff were extremely friendly and seemed to be very well acquainted with the work of Western authors.

6. GENERAL EDUCATION IN THE U.S.S.R.

It may be interesting to note some aspects of education which emerged during conversations with students and others during the visits. It appears that a Russian child starts school at the age of 7 and continues his studies until he is 17 years old. If the child at this stage is considered to be in the super grade, a further 1 year study course follows.

Marks are given out of a total of 5 and the students with whom I spoke felt that marks out of 10 would have certain advantages - they seemed to consider this point worthy of mention.

I understood also that some special schools give all tuition in English for the age group 11 - 17. The standard on leaving school at the age of 17 was difficult to assess, particularly as I only talked to the super-grade students, but I formed the impression that general standards were inferior to the average British student at this age. However, if a statement attributed to Krushchev comes into effect, all children in the future will leave school at the age of 15, and those in the super-grade class will take a sandwich course at the University or Technical Institute.

7. GENERAL IMPRESSIONS OF THE U.S.S.R.

It will be obvious that impressions formed on such a short visit and based on conversations restricted to English speaking Russians, must be regarded with some reserve. However, for what they are worth, I give below a few general comments.

1. Without exception, the people whom I met, were convinced that all Western scientists were subjected to the tyranny and exploitation of capitalists. They would simply not believe that conditions here were not so bad as they were led to believe.

2. British people were regarded with more favour than Americans. This is probably due to propaganda but the Russians seemed to dislike so-called American bad manners. I was told on many occasions that Americans never said "please" or "thank you" and gave the impression that they were a superior race.

3. In general the people in Moscow were reserved and rarely smiled. We were told that only first class citizens are allowed to live in Moscow and presumably this applies to all walks of life from lawyer to brick-layer.

Despite the enormous building programme now in full swing, living conditions were poor by our standards. However, despite the Russian protestations that there was no discrimination in the type of accommodation offered to different groups of the community, I found that in G. Orwell's words, some people were more equal than others. For example, the tenancy of the most central apartment building was restricted to a group of people of approximately the same social standing as judged by Western standards, e.g. lawyers, professors, ballerinas, doctors, but no labourers or bus drivers.

A colleague had the good fortune to be invited to the house of a Russian

Professor who had a small daughter. The accommodation consisted of three small rooms, (kitchen, living room and bedroom), with shared toilet facilities. This example may not be typical but it would appear that accommodation generally is not of a high standard.

4. The people in Leningrad seemed to be more relaxed and to be enjoying life. However, they seemed to suffer from an inferiority complex in respect of the people living in Moscow.

5. In Kiev I found that the people had an even more carefree manner and were willing to alter the rigid instructions from Moscow regarding our visits programme. The Ukrainians indeed seemed to be most likeable, but were probably not good communists.

6. There appears to be certain levels in Russian Society. Status in descending order of merit seem to be based on the following groups - politicians, ballerinas of the Bolshoi Ballet, top clowns of the Moscow State Circus, Academicians, musicians and finally a general group comprising doctors, scientists, lawyers, factory managers, professional engineers and those who have made some contribution to art and literature.

7. Travelling arrangements throughout the Soviet Union were in our experience chaotic and innumerable delays were encountered due to sheer inefficiency of the planning staff.

8. Russian foods are rather stodgy but this is a personal opinion. It is possible, however, to vary the diet by visiting some of the more expensive restaurants.

The beer is not good but drinkable. Georgian wine, (red) is very bad but the dry white wines and the champagne are palatable. Vodka tastes like London vodka.

9. Table manners are somewhat crude. In particular one must get accustomed to retain the one and only knife and fork offered between courses.

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APPENDIX 1

The International Federation of Automatic Control (I.F.A.C.)

In the Autumn of 1956 a conference on automatic control was held in Germany, and it was there that an international federation was first proposed. The proposal was not received with acclamation. One objection was that there were already enough international bodies to call conferences together. The representatives were finally persuaded by the possibility that a new federation might encourage collaboration between east and west, and a first constitutive assembly consequently gathered in Paris in September 1957. There, after some discussion of details, the International Federation of Automatic Control ('Ifac') was formally brought into existence, and its first congress was planned to meet in Moscow in 1960. The arrangements then went steadily and the congress met on June 27th to July 7th.

By its constitution Ifac is composed of national organisations. There are 29 of these at the present time*. The United Kingdom is represented by the British Conference on Automation and Computation (B.C.A.C.), which is a combination of established institutions gathered into three groups: Group A for engineering applications, Group B for computation and automatic control, and Group C for sociological and economic aspects. Further details may be obtained on each of these three Groups from their Honorary Secretaries.

There were about eighty members in the British delegation to the Moscow Congress, led by Mr. John, F. Coales (Cambridge University) with Professor G. D. S. Maclellan (Glasgow University) as his deputy. The British papers were discussed at a symposium in London on September 27th to 28th.

* Argentine, Australia, Austria, Belgium, Bulgaria, Canada, China, Czechoslovakia, Denmark, Finland, France, German Democratic Republic, German Federal Republic, Hungary, India, Israel, Italy, Japan, Netherlands, Norway, Poland, Roumania, Sweden, Switzerland, Turkey, U.K., U.S.A., U.S.S.R., and Yugoslavia.

APPENDIX 2

I.F.A.C. Conference in Moscow

General Comments

The coverage of the conference was extremely broad as may be illustrated by the fact that 285 pre-prints of papers were available out of which 274 reports were presented and discussed.

It is important to emphasize the fact that any remarks given about the conference are personal and are based on the attendance at a relatively few number of papers and formal discussions, together with informal discussions with other delegates.

There seemed to be a preponderance of purely theoretical papers (a total of 140) at the expense of more erudite engineering applications. I felt that existing mathematical techniques had not been exploited to their best advantage in the field of control engineering and there seemed to be a wide gulf between theoretical considerations and design techniques pertaining to overall systems and components. Nevertheless many of these papers were of fundamental importance and I was particularly impressed by the papers dealing with the Theory of Invariance and Multivariable Control Theory. Discussions on the whole were rather disappointing and the requirement for a formal written contribution to be submitted in advance, restricted to some degree the amount of spontaneous discussion. Moreover in some sessions, discussers used the opportunity to present another paper, rather than discuss in a critical manner the paper presented. One fact was obvious, in that the Russian delegates had found time to peruse the relevant papers in some detail.

It is difficult to spotlight, without further reflection, the major trends in the theoretical aspects, with which this review is mainly concerned. The following might serve to indicate major topics, in addition to those already noted.

Further development of linear statistical theory and synthesis.
Application of statistical theory to non-linear systems.
The Maximum Principle, i.e. the optimum design of a permissible control system to minimise the expenditure of energy in performing a required task.

Dynamic Programming, i.e. the study of control systems as multi-stage decision processes with emphasis on methods suitable for high speed digital computation.

Development of the theory of discrete systems (sampled data and relay systems).

Theory of self adaptive systems.

Over 50 papers were presented in the Components Section which covered computing techniques, transducers, data handling and components of systems. The reviewer was particularly interested in the computing techniques described and a selection of these are described below.

In the final section dealing with applications over 60 papers were presented covering the automatic control of machine tools, power systems, electrical machines, steel industry, chemical and oil refining processes, nuclear power plants. In his capacity as Chairman of the Application Committee, Mr. J. Mozley stated that there was an urgent requirement for the definition of a criterion of merit for a process taking into account all aspects including cost.

Apart from the formal lecture content of the Conference an interesting feature was a book exhibition together with a classified list of text books related to automatic control. It was interesting to note that several standard American and British books had been translated into Russian.

One disappointment to the reviewer was the fact that one could find few occasions on which to discuss topics in an informal manner with the Russian delegates. The main contact, in this respect, was obtained during the technical visits which will be the subject of a later review.

There is no doubt that the Russians are putting a large amount of effort into the field of automatic control. Although there is not much evidence as yet of practical and economic realization of optimum systems for process plant it would be foolish to suppose that this deficit will not be rectified to some extent in the near future.

Finally, in summing up the Conference, I would make the following points. The significance of the Russian contribution was reaffirmed. There was a lack of "down to earth" engineering applications of optimum control theory and the gulf between theory and practical realization seems to be widening. The mathematical content of some papers may be such as to appal the engineer, but it is comforting to reflect that the end result of a control system or process plant can be specified in simple terms. On economic grounds, the practical realization of a theoretical technique is the important thing, and this aspect deserves more attention by engineers in the future. In the field of process control, the application of sophisticated

control theory must be delayed in many cases until a deeper understanding is obtained of the process itself.

As stated before it is difficult to assess at this stage the relative significance of the main theoretical themes but it would appear that the use of digital computers to control process plant to give an economic optimum result is more firmly established on a theoretical basis.

The main benefits which I derived were not obtained by formal attendance at papers and discussions, but rather from informal talks with other colleagues. The opportunity to see a small part of the Russian way of life also made the journey worth while.

A brief discussion of a selection of papers included in the main headings listed, is given below. The coverage of these main topics was the specific task of the reviewer. Other topics are reviewed by Dr. J. H. Westcott.

THEORY OF CONTINUOUS LINEAR SYSTEMS

Marsik, J. (Czechoslovakia) - "Some Control Possibilities of Systems with Variable Parameters"

Interesting paper in which the control of a linear system with time varying parameters, is effected by a controller comprising conventional integral term or proportional plus integral term followed by a transfer function which is the inverse of the control system transfer function. Unfortunately experimental results are not included as an indication of the success of this approach. There would appear to be some restrictions or realizability conditions and it will be interesting to read of further work on this topic.

Buckley, P. S. (U.S.A.) - "Automatic Control of Processes with Dead Time"

This paper discusses various techniques for the compensation of processes with dead time. Comparisons are made between conventional integral controller, feed-forward techniques and the use of a Smith linear predictor. Experimental verification of theoretical results are included by considering the control of a liquid mixing process. The Smith type linear predictor is shown to give the best results.



Kulebakin, V. S. (U.S.S.R.) - "The Theory of Invariance of Regulating and Control Systems"

The invariance principle, i.e. independence of the controlled magnitude in respect of disturbances acting on the controlled member, is discussed. The analysis is simplified by linear approximation with particular reference to a plant characterised by two co-ordinates. One practical realization of the theory is to measure the external disturbances acting on the controlled object and supply the information regarding these disturbances in a certain form to components which react on the regulator system.

Another paper by Petrov, B. N. (U.S.S.R.) discusses the application of the Invariance Principle to Linear and Non-Linear Systems. The realization conditions are discussed in some detail. Active work is being carried out in the U.S.S.R. on this topic which appears to be of primary importance.

THEORY OF DISCRETE SYSTEMS (SAMPLED DATA, AND RELAY SYSTEMS)

Jury, E. I. (U.S.A.) - "Recent Advances in the Field of Sampled Data and Digital Control Systems"

As the title implies, this important paper reviews the present state of the art, covering linear theory, statistical theory, non-linear theory, time-varying theory and digital control systems. This paper did not evoke much discussion but written by such an expert it gives a very good appraisal of present-day techniques.

Tsyarkin, Ya. Z. (U.S.S.R.) - "Elements of Theory of Computing Automatic Systems"

This paper discusses the automatic digital computer as a system which quantises both in time and in amplitude level. The quantisation of the latter is a non-linear operation. The author discusses a law of optimum control by the use of Bellman's Dynamic Programming method. This appears to be a most important paper and no doubt will form the basis for further work.

Other interesting papers included the application of operational methods (Brown, - U.K.) and Time Series and Z Transform methods

(Cruickshank - U.K.), Systems with non-linear elements (Andeen - U.S.A.) and Multi-variable systems (Nishidia - Japan). A paper by Hopkin and Wang - U.S.A. discusses the optimum (minimum settling time for predicted error and derivatives to reach zero) design of relay type systems designed for certain classes of random inputs. The switching equations for some categories of second and third order systems are given.

I. Flugge - Lotz - U.S.A., in her paper entitled the Synthesis of Third Order Contactor Systems, discusses the behaviour of systems with linear switching functions of the form $F = e + K_1 \dot{e} + K_2 \ddot{e}$.

During the discussion (Kalman - U.S.A.) presented a short note dealing with the problem of transfer functions containing zeros.

As stated in last month's issue a paper by Fuller (U.K.) investigates optimisation of relay systems for performance criteria other than the usual minimum settling time. He considers first order and second order systems and points out that Newton's power restraint method gives a poor assessment of the minimum error squared for systems subjected to a random square wave.

It would appear that further work is necessary to formulate, if possible, a technique for dealing with high order systems with an indication of the flatness of the optimum switching function.

THEORY OF CONTINUOUS NON LINEAR SYSTEMS

Macura, A. - "Static Analysis of Non-Linear Systems with two-Input Elements".

This paper describes a method of static analysis of non-linear systems comprising elements with two inputs (e.g. electronic or pneumatic valves). Techniques are presented for splitting complex block diagrams into simpler ones. A two input element may be transformed into two - one input elements, and this technique may be used to simplify and synthesize the static performance.

It would appear that the method could be extended to the transient analysis of certain non-linear systems and this would increase considerably its effective use.

Gille, J. C. and Decaulne, P. - "Note Concerning Forced Oscillations of Non-Linear Servo Systems".

An extension of the describing function method is presented for the study of systems with one non-linear element. Three types of non-linearities are discussed. The conditions for instability of the forced oscillation and for jump effect are determined by the relationship of the transfer locus of the linear elements to the envelope of a family of circles characterising the non-linear element.

West, J. C. (U.K.) - "Gain Modulated Control Systems".

West considers the parameter changes necessary to achieve an error power as a percentage of power input of a stationary random signal. A second order linear system and one with a torque saturation are considered to illustrate the method of determining necessary parameter changes which the author calls Gain Modulation Control.

The gain modulation element output signal is in general a function of the input signal and another modulation signal. Separation of variables is suggested by using equivalent gain techniques as an approximation. The design of the element can then be achieved by the use of a multiplier.

This is an interesting paper as it spotlights a basic concept in the form of adaptive system considered, and paves the way for further work. The discussion on this paper did not do justice to it.

Other interesting papers included one by Popov (U.S.S.R.) who considers the design of non-linear systems by using the method of harmonic and statistical linearization. Systems with two non-linearities are considered and several criteria are presented.

Syrk (West Germany) describes in a practical manner the control of the effect of saturation in a process control by an extension of the describing function.

Oldenburger and Nakada (U.S.A.) extend the work published by Oldenburger and Liv in 1959, by considering the application of a stabilizing signal to a generalised control system with one non-linearity, to remove self oscillation. The determination of the required periodic signal is given and the method is illustrated by considering a system which includes a limiter form of non-linearity.

COMPUTING TECHNIQUES

Solodovnikov, V. V. et. al. (U.S.S.R.) - "Analysis and Synthesis of Automatic Control Systems by means of Computers".

A method is described for dealing with statistical analysis of non-linear control systems. A specialised analogue computer (called a synthesiser) capable of solving Fredholm equations of the first kind, is also described. The solution is obtained by solving a set of approximate algebraic equations in an iterative manner. An experimental model has been constructed and is intended to be used as part of a process control system. The synthesiser is fed with the random waveforms of input and output of the system and produces the impulse response function of the system. The time required for the solution is 10-40 secs. and accuracy is in the range 5-10%. During the discussions several speakers were unsuccessful in their attempts to get more detailed information about the apparatus. The use of a digital computer to solve correlation functions was also described.

Goodman, T. P. (U.S.A.) - "Theory and Application of a Tapered Electronic Delay Line Synthesizer".

In the delay line synthesizer (D.L.S.) the unit impulse response or weighting function of a linear system is synthesized from a sequence of pulses $h_1(t)$; $h_2(t)$ $h_n(t)$ delayed with respect to the input of times t_1 ; t_2 , t_n respectively when the input to the delay line is unit impulse function $u_1(t)$. The output of the synthesizer is of the form $h(t) = a_0 u_1(t) + \sum_{n=1}^N a_n h_n(t)$. The coefficient a_n can be adjusted between limits ± 1 and so the device acts as a versatile analogue computer.

The paper describes the use of the D.L.S. to determine a dynamic model of a linear system subject to random inputs. The equipment described is an extension of Reswick's earlier work. The design procedure for determining the R.C. delay line with optimum overlap between successive impulses (tapered delay) is given.

During the discussion several applications were outlined.

Morgan, M. L. (U.S.A.) - "A New Computer for Algebraic Functions of a Complex Variable"

This computer uses tele-deltos paper as a representation of the complex plane. Pole and zero locations are represented by feeding voltages to the paper by means of probes. Two sheets of paper are used, one for plotting the logarithmic amplitude contours and one for angle contours. The machine can be used for root plotting, root locus plotting and frequency response analysis. The motion of the probe assembly is directly coupled to a stylus on a plotting table which plots the required contours. This computer is to be marketed under the name E.S.I.A.C. at a cost of about 10,000 dollars. The reviewer, during discussion, questioned the possible accuracy particularly with regard to the consistency of the paper. Precise figures however were not available.

Ginsburg, S. A. (U.S.S.R.) - "Logical Method of Synthesizing Function Generators".

A method of synthesizing functions of one or more variables is described based on a piece-wise non-linear approximation. It is claimed that the method can be extended to functions of three or more variables. The determination of the logical networks required would obviously be a lengthy process.

During the discussion, Paynter - U.S.A., stated that similar work had been carried out in the U.S.A. by Philbrick in 1950.

I found the paper rather difficult to follow and the discussion was not very informative. The results obtained from this application of mathematical logical synthesis are extremely important and the paper deserves a more detailed study.

Karplus, W. J. (U.S.A.) - "A Discrete-Space-Discrete Time Computer".

This is a network analyser containing precision resistors and d.c. amplifiers, designed for simulation of field problems including the diffusion equation and the wave equation. The computer is a hybrid analogue digital machine. Electrical voltages are proportional to field potentials as in the case of the conventional analogue network analyser, but the time variable as well as the space variables, is approximated by finite difference expressions as in a digital computer. The design is based on an extension of a method proposed by Liebmann in 1956, but differs in that some of the resistors at each node have negative magnitudes.

The latter are simulated by means of active two-port networks. The solution is obtained in discrete steps in time.

This is a useful paper and the computer should prove its worth in field investigations.

Stakhovskiy, et. al. - "Automatic Optimizers and their use for solving Variational Problems and for Automatic Synthesis".

This paper together with an experimental model of the system exhibited at the Institute of Automation and Telemechanics, Moscow, created wide interest. The main component of the system is a device which automatically searches for the minimum value of a controlled quantity Q , with limits on the range of possible search. The output quantity Q may be a function of n input variables and the paper describes a model with $n = 12$. The output is the function $Q(x_1, x_2, \dots, x_n)$.

The apparatus selects automatically a combination of values of $x_1, \dots, x_n$ in an iterative manner to yield the minimum value of Q . It is important that over the range of variation of the input variables, there is only one minimum value of Q , as otherwise the logic of piece-wise representation is very complicated. Details of the method of search and the logic of switching are described in a paper by Fieldbaum published in Autom. Telemekh. Moscow 19, No.8. (1959). The model was demonstrated acting as an automatic function generator of one variable and the detailed results are given in the paper. During the discussion it was difficult to ascertain whether the apparatus had yet been put to practical use.

APPENDIX 3

Copies of the author's reports presented
at the Institution of Mechanical Engineers
Symposium held on 27th - 28th Sept. 1960.

Session 4 - Mathematical Methods

This report is a brief survey of the papers presented in Section 1.8 (Special Problems of Mathematics) of the I.F.A.C. Conference. Out of a total of thirteen papers, seven were contributed from the U.S.S.R., three from U.S.A. and one each from W. Germany, Japan and Poland. The subject matter of the papers may be specified under two main groups. The first group is concerned with the derivation and application of Liapunov functions in the determination of stability of non-linear systems, whilst the main theme of the second group is the study of steady-state oscillations in non-linear systems. This second group may be divided into two sub-sections, one dealing with discontinuous systems and the other with the stability of periodic movements of continuous systems. Some details of the papers are given in the following paragraphs.

Four papers were concerned with Liapunov's second method of investigating stability and the construction of Liapunov functions. It may be relevant, therefore to describe briefly the theorems of Liapunov.

Consider a dynamical system which can be described by the equations :

$$\begin{aligned}\frac{dx_1}{dt} &= x_1(x_1, x_2 \dots x_n) \\ \frac{dx_2}{dt} &= x_2(x_1, x_2 \dots x_n) \\ \frac{dx_n}{dt} &= x_n(x_1, x_2 \dots x_n)\end{aligned}\tag{1}$$

where the functions $x_1 \dots x_n$ are functions of the dependent variables $x_1, x_2 \dots x_n$.

Let the solution of (1) with zero initial conditions ($x_1 = x_2 = x_n = 0$) at ($t = 0$) be

$$x_1 = x_1(t), x_2 = x_2(t) \dots \dots \dots x_n = x_n(t)\tag{2}$$

Consider a function $V = V(x_1, x_2 \dots x_n)$.

Since the dependent variables $(x_1, x_2 \dots x_n)$ are functions of time t , then V itself is a function of t

$$\text{i.e. } \frac{dV}{dt} = \sum_{r=1}^n \frac{\partial V}{\partial x_r} \dot{x}_r \quad (3)$$

Theorems

(1) if it is possible to find a function $V(x_1, x_2 \dots x_n)$ which is positive for all values of $x_1, x_2 \dots x_n$ such that $|x_1| \leq h \dots |x_n| \leq h$ (4) (except for origin $x_1 = x_2 = \dots x_n = 0$ where $V > 0$) and also such that

$\frac{dV}{dt} \leq 0$ in the same region (4) then the undisturbed motion defined by (2) is stable.

(2) If $\frac{dV}{dt} = 0$ only when $x_1 \dots x_n = 0$ then the undisturbed motion is asymptotically stable. There are two further theorems concerned with the instability of disturbed motion.

(3) If $V(x_1 \dots x_n)$ can be found which takes on both positive and negative values in region (4) and whose total derivative $\frac{dV}{dt}$ is always either positive or negative in the same region (and only zero at origin) then the undisturbed motion is unstable.

(4) If $V(x_1 \dots x_n)$ can be found such that

$$\frac{dV}{dt} = \lambda V + W(x_1 \dots x_n) \quad (5)$$

where $\lambda > 0$ and $W = 0$ or is a function always positive or always negative (except possibly zero at origin) and if V is not constant in sign, opposite to that of W , then the undisturbed motion is unstable.

There is no general rule for the construction of Liapunov functions to meet all cases.

References

- (a) "Theory of Stability of Motion", by I. G. Malkin
AEC-tr-3352 translated by U.S.A.E.C., Oak Ridge,
Tennessee. Available from Office of Technical
Services, Dept. of Commerce, Washington 25 D.C. \$4.75.
- (b) "Some Non-Linear Problems in the Theory of Automatic
Control" by A.I. Lur'e 1957. H.M.S.O. 17/6.
- (c) "Probleme General de la Stabilite du Mouvement"
by A. Liapunov, Princeton 1947.

A paper by Barbasain (U.S.S.R.) is worthy of note in that a list is included of 51 references (46 of them to Russian publications) to relevant papers and books on the use and construction of Liapunov functions.

The author referred to several of these papers to indicate that the importance of the Liapunov function is not restricted to the problem of stability. Estimates of variation of controlled value, control time and quality of control may be made based on the knowledge of the Liapunov function for the particular system under consideration. In addition it may also be used to establish the presence of forced oscillations. However, it is stressed that these estimates are not always satisfactory from the point of view of application.

Several methods of constructing Liapunov functions are given in the paper including quadratic forms, total power method and the method of dividing variables.

Lehnigk (W. Germany) described a quadratic form of Liapunov function, based on a linear comparison system. Criteria are developed for the stability of the linear comparison system, and limit boundaries obtained. The non-linear system will also be stable if the non-linearity satisfies the same boundary circuit as its linear comparison circuit. The controlled motion of a missile under the influence of a non-linear law is included as an example, and certain facts regarding stability deduced. This example is a good feature, as it indicates the practical application of Liapunov Functions when most of the literature is concerned solely with theory.

Quadratic forms of the Liapunov function are formulated by Bedelbayev

(U.S.S.R.) in his paper to reduce stability criteria for particular regulator systems to the simple verification of a few straightforward inequalities. These define, in finite terms, the permissible region in which the parameters of the regulators being analysed may vary. Examples are given for systems with one regulator and extended to control systems with several regulators.

Lur'e and Rozen Wasser (U.S.S.R.) discussed the stability of direct and indirect non-linear control systems. Reference is made to previous work and Liapunov functions are used, which reduce condition for stability to the statement that a system of numbers $a_1, \dots, a_2$ exists which constitutes a solution to a system of quadratic equations. Examples have been investigated up to the fifth order.

Three papers dealt with the theory of non-linear systems with discontinuous characteristics. Andre and Siebert (U.S.A.) discussed the after end point motions (chattering) of contactor type systems. Relay systems with and without a constant time delay in switching are described and equations derived for stability, which are based on previous work by authors and other workers. A basic stability theorem is postulated which states in effect that a system with constant switch delay is stable at origin, if the corresponding system with infinitesimal switch delay has this property. A one degree of freedom problem is included and it is shown that small limit cycles exist near the state of equilibrium.

Filippov (U.S.S.R.) also considered contactor type systems and a generalised definition of the solution of the set of differential equations is put forward, which in the case of a contactor system is equivalent to supposing that the contactor operates with a time delay and then allowing the time delay to tend to zero. Reference is made to original work in which the theory is derived and results obtained with reference to oscillation conditions.

A paper by Ayzerman and Gantmakher (U.S.S.R.) discussed the equations describing the dynamic performance of systems with one or more discontinuities. An exact method is given for the presence of hunting for systems with one discontinuous characteristic (e.g. relay system or system with coulomb friction). The stability of hunting for the more general case is studied and a linear approximation (based on an extension of Liapunov's Theorem), (which is valid for certain cases), is given. It is shown that only the discontinuities can be responsible for positions of equilibriums on the surfaces of discontinuity.

The existence of steady state regions and the stability of periodic movements of non-linear control systems were the main themes of four papers. Neymark (U.S.S.R.) described the stability of periodic movement, as a function of several parameters which may be continuously changed in a random manner. Reference to previous work shows that if all the roots of the characteristic equation of the periodic motion lie within a given circle, the motion will be stable. Based on this fact several theorems are postulated and for a change in parameters it is shown that the system will either pass into a new stable periodic motion or become unstable. The conditions which determine these modes are discussed.

Nemickii (U.S.S.R.) in his paper, pointed out the vague description of steady state regimes in non-linear systems with more than one degree of freedom. The author introduced "almost periodic Bohr Functions" (which are similar to periodic functions) as a means of determining steady state regimes. The almost periodic Bohr Function may be developed in a trigonometrical series of the type
$$\sum_{k=1}^{\infty} A_k e^{j\lambda_k x},$$
 where in

contrast to periodic functions, λ_k (Fourier indices) are not whole numbers.

If the number of the Fourier indices is a finite number, the function is described as quasi-periodic functions. These define the movement coordinates of a stable trajectory in phase space and the author suggested that this might lead to a method of determining all steady state regimes.

A further development of linear differential equations with periodic coefficients to that given in his text book, was presented by Cosoriff U.S.A. (Non-Linear Control Systems - McGraw Hill). The author investigates the transient behaviour of nonlinear differential equations in the vicinity of the steady state periodic solution and the resulting class of linear differential equations, is shown to have periodic coefficients. Methods of solution of this type are given. The techniques are applied to the determination of limit cycle parameters, stability and subharmonic instability. A solution for the limit cycle condition is assumed and small signal differential equations determined. Parameters of the assumed limit cycle solutions are then adjusted until the exponential term with a real argument is zero.

Quasi-periodic oscillations were described by Hayashi (Japan) with particular reference to the analysis of a resonant circuit with direct current imposed on two non-linear inductors. Conditions for limit cycles were given and the results obtained were checked by an analogue computer study.

In the two remaining papers, Bass (U.S.A.) discussed in a rigorous manner the mathematical legitimacy of the describing function technique. It is shown that the implied heuristic assumptions in its application are valid provided that the system does not exhibit resonance effects. The work is a simplified and generalised development of the author's earlier work (Proc. Symposium on Non-Linear Circuit Analysis M.R.I. Series VI. (1956) 163.). A good feature of the paper is the extensive bibliography.

Finally, Skowron'ski and Ziemba (Poland) described the motion of a structural system represented by lumped parameters. The existence and stability of limit regimes for non-linear, vibrating systems is discussed.

Session 9 - Telecommunications, Information
Processing and Analogue Computing

This report covers the papers given in Section 1.9 (Simulation and Experimental Methods) and Section 2.2 (Electric Computing and Analogue Devices, Programming Elements and Controlling Machines), of the I.F.A.C. Conference.

There were no British contributions in the total of 21 papers presented in the two sections. Eight papers were contributed from the U.S.A., four from the U.S.S.R., two from Poland, two from W. Germany and one each from Austria, China, France, Japan and Roumania.

The subject matter included descriptions of two new types of analogue computer, the formulation of mathematical models of control systems, the application of analogue, digital and hybrid analogue digital techniques, to the simulation of control systems, and the logical synthesis of function generation.

A discrete space-discrete time computer was described by Karplus (U.S.A.). This is essentially a resistance network analyzer containing precision resistors and directly coupled amplifiers designed for the simulation of partial differential equations. Hybrid analogue digital techniques are used with the field potentials represented by electrical voltages as in the conventional network analyser, but the time variable and the space variable are represented by finite difference expressions in a similar manner to that used in a digital computer approach. The method used is an extension of Liebmann's work on heat transfer but differs in that some of the resistors at each node have negative values, and one set of node voltages is adjusted automatically. These tasks are achieved by employing feedback computer amplifier techniques in each node module. The solution is obtained in discrete steps in time and it is therefore possible to adjust the network resistor magnitudes after each step to simulate parameter variations.

Morgan (U.S.A.) described a computer using teledeltos paper as a representation of the complex plane. Pole and zero locations are represented by voltages of opposite signs fed to the paper by means of probes. Two sheets are used, one for plotting logarithmic amplitude contours and one for angle contours. The motion of the probe assembly is directly coupled to a stylus on a plotting table which plots the required contours. The main purpose of the machine is for root-locus plotting

and frequency response analysis. No figures were given regarding accuracy or consistency of the teledeltos paper.

Mathematical models for control systems were discussed by Stout (U.S.A.) and Eckman and Lefkowitz (U.S.A.). Stout outlined the two basic types of computer control systems, the first being that in which the computer experiments systematically with the process itself and the second where the computer operates on a mathematical model of the system. Three kinds of mathematical models were discussed, comprising batch, continuous and semi-continuous techniques. Several examples of dynamic models were described including mixing, chemical reaction, material transport and separation. Eckman and Lefkowitz presented self checking or model adaptation concepts as a means of extending the model approach in which parameters are varied to give a best fit to observed system behaviour.

Delay line synthesis techniques were described by Goodman (U.S.A.) and Schneider (W. Germany). Goodman described the application of the technique for the determination of the dynamic model of a linear system subjected to random inputs. The design procedure for determining the R.C. delay line with optimum overlap between successive impulses (tapered delay) was included. The development of linear difference equation units was the main theme of Schneider's paper who discussed the design of a polynomial generator to reduce the number of discrete delay units.

A method was described by Solodounikov et. al. (U.S.S.R.) for statistical analysis of models of non linear control systems by analogue techniques. An analogue auto-correlator employing magnetic tape was discussed. Other apparatus included a specialised hybrid analogue digital computer called a synthesiser capable of solving one dimensional Fredholm and Volterra integral equations based on algebraic equation approximations. The solution is carried out in an iterative manner and the apparatus essentially comprises a store and arithmetic unit. Details of the unit were not given but each cycle of operation was reported to be 1.33 secs. The solution time for one equation is in the range 10-40 seconds with an accuracy of about 10%.

Leonhard (W. Germany) illustrated in a practical manner the use of an analogue computer to investigate non-linear control loops. Use of the describing function technique is made for the improvement of systems with simple types of non-linear elements. The parameters are first adjusted in the equivalent case to give a damping factor $\zeta = 0.414$

of the dominant complex roots. The parameter for the non-linear case are then selected so that for a given magnitude of the input signal to the non-linear element, the describing function attains the same values of parameters derived previously for the linear system. Qualitative consideration of amplitude dependence of the describing function indicates corrections which are usually necessary. Several applications of the technique are included in the paper.

The synthesis of transcendental functions was the subject of a paper by Perret (France). The functions considered were

$\cosh \lambda_s$ and $\frac{1}{\cosh \lambda_s}$ where s is the complex variable and λ is a real constant. The synthesis of $\frac{1}{\cosh \lambda_s}$ is achieved from the expression

$$\frac{1}{\cosh \lambda_s} = \sum_{k=0}^{k=\infty} \frac{(-1)^k}{\lambda} \left(\frac{2\omega_k}{s^2 + \omega_k^2} \right)$$

The series with $k = 2$ is used as an approximation of the transfer function of an open ended transmission line in frequency spectrum 0 to ω_0 (the first resonant frequency). The series is realised with three L.C. circuits associated with computing amplifiers. $\cosh \lambda_s$ synthesis is based on a Mac Laurin's Series.

An interesting paper by Paynter (U.S.A.) outlined the formulation of an electronic model of polyphase alternating current systems, which may be realised by high speed analogue and digital computing techniques. Ginsburg (U.S.S.R.) discussed the logical synthesis of functions of one or more continuous variables by the application of the calculus of propositions and Boolean algebra to continuous quantities. The laws of mathematical logic for the latter are the same as those for two valued logic. The logical structure of the approximation depends on the sign of the second derivative in different segments of the given function. The simulation of the function based on the logical diagram has the advantage that for any value of the independent variable, the resulting function is equal to one of the factors. Diodes resistors and operational amplifiers are used to realise the approximation procedure.

Another paper on function generation by Chin Lan (China) described the use of non-linear silicon carbide resistors associated with linear resistors and operational amplifiers. The volt ampere characteristic of the non-linear resistor is approximated by the formula $I_T = aV_T + bV_T^3 + cV_T^5 + \dots$

A coefficient of non-linearity is defined as
$$= \frac{\frac{V_T}{I_T}}{\frac{dV_T}{dI_T}}$$

These equations are normalised with respect to the maximum values of V_T and I_T respectively and four terms only of the series are used as an approximation.

A generalised non-linear coefficient is calculated for various combinations of linear and non-linear resistors. With the aid of these basic configurations, the design procedure for synthesizing a cube law and a sine function is given. Results quoted showed that the maximum deviation between experimental and calculated figures did not exceed 1.5% of the original value. This figure could be reduced by slight adjustment of the linear resistor values.

A hybrid digital analogue model of a nuclear reactor was the subject of a paper by Rechberger and Sequenz (Austria). To overcome the disadvantages of the conventional analogue computer with regard to accuracy and time scaling the authors proposed a digital form of multiplier consisting essentially of two resistors connected by an electronic gate, and a pulse doubler. With the use of this device the solution of the equation

$$T \frac{dQ}{dt} = (k - 1)Q + S$$

was discussed.

T = life time of a neutron, k = number of new neutrons (function of time):
 Q = neutron density, S = strength S of external source of neutrons. The value of k is obtained by pulse width modulation in a statistical manner based on a noise generator. The solution of problems with delayed neutrons is also discussed with the delays being represented by R. C. networks.

Voronov et. al. (U.S.S.A.) described a method of digital function generation by using combinations of digital integrators. The chief advantage of this method compared with the analogue method is the relative ease of performing the various mathematical functions since the independent variable is not restricted to time. Arrangements of integrators are shown for the solution of polynomials, and second order curves including ellipses, paraboles and hyperboles. Reference is made to an original paper by Voronov. During the discussion Voronov commented that fifth order problems could be solved in about three hours. He also stated, in answer to a question, that the methods proposed

were general techniques and not restricted to particular functions.

A paper by Findeisen and Mardal (Poland) dealt with the simulation of a digital type velocity servo-mechanism, on a conventional analogue computer. Two conventional function generators were used to represent the in-phase and the reversed phase controlling voltage characteristic of a two-phase motor. On/Off elements were represented by relays and hold circuits by switched integrators.

A device called an automatic optimizer was the subject of a paper by Stakhovskiy, et. al. (U.S.S.R.) and an experimental model, which created much interest, was demonstrated at the Moscow Institute of Automatics and Telemechanics. The main component of the system is a device which automatically searches for the minimum value of a controlled quantity Q , with limits on the range of possible search. The principles involved are given in six references at the end of the paper with the original work based on Feldbaum's paper. (Autom. Telemech. Moscow 19, No.8 (1958)). Q is a function of n variables i.e. $Q = Q(x_1, x_2, \dots, x_n)$ and the automatic search is performed by carrying out

trial variations of the input variables x_i , with the minimization of Q taking the form of stored values of the first partial derivatives $\frac{\partial Q}{\partial x_i}$.

The gradient vector $\sum_{i=1}^n \frac{\partial Q}{\partial x_i}$ has the direction of the most rapid

variation of the function. The operation of trial variations are alternated with working steps along the line of gradient. When

$\sum_{i=1}^n \frac{\partial Q}{\partial x_i} = \Delta Q$ is far removed from the extremum, the process

changes over to alternation between determination of the gradient and of a single working step (gradient method). The procedure for dealing with constraints is outlined. A model with $n = 12$ was demonstrated acting as an automatic function generator of one variable and detailed results are given in the paper.

The remaining five papers presented, dealt with the simulation of partial differential equations, a computer compensated process control, the automatic control of zero level of directly coupled amplifiers, a description of the Polish "SKRZAT" digital computer and a design basis for positional servo-mechanisms.

In conclusion, three main trends may be stated,

- (i) the use of model techniques associated with computers,
- (ii) development of hybrid analogue digital techniques for function generation; and simulation of control systems,
- (iii) iterative automatic function generation.