

CRANFIELD INSTITUTE OF TECHNOLOGY

SCHOOL OF AUTOMOTIVE STUDIES

Ph.D THESIS

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THE BEHAVIOUR OF POLYURETHANE FOAM BEAMS REINFORCED
WITH THIN WALLED STEEL SECTIONS FOR USE IN
BUS STRUCTURES

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SUMMARY

The thesis is directed to the theoretical determination of the behaviour of rigid skin polyurethane foam beams reinforced with thin walled steel sections. The aim is to enable the designer to predict the behaviour of bus body structure during accident situations in order to build safe buses.

A literature search has revealed that very little work has been done in this particular field of research. Thus a preliminary test programme was made to investigate the beam behaviour in bending particularly at high deflection. The test results shown in Chapter 1 gave promising results for continuing the research.

Chapter 2 is directed to elastic analysis of composite beams. One of the advanced methods to predict the elastic stiffness of sandwich beams is the interfacial shear method. This assumes an interfacial shear between the core and the faces of sandwich beams. So, by considering a similar assumption, i.e. an interfacial shear between the matrix and the reinforcement, an analysis was made to predict the stiffness of composite beams. Although this method highly overestimated the stiffness, it represents a good basis for the inelastic bending analysis which is more important for the present study.

Chapter 3 shows the inelastic bending analysis of composite beams. This analysis is based on TIMOSHENKO and JAMES curvature area method.

The experimental verification of the theory is shown in Chapter 4. The theory shows good agreement with the experimental results. The shape of the reinforcement was optimised for uniaxial bending. This optimum composite section showed a weight saving when it was compared with equivalent energy absorbing rectangular steel tube.

Impact tests were made using a pendulum designed specially to test cantilever beams at different speed and impact energies. The test results are presented in Chapter 5. These results showed that the composite beams behaviour is similar for both static and dynamic loading.

To compare the composite beam with the constituent beams, the large deflection behaviour of thin walled channel section beams made of thin sheet steel has been investigated. The experiments consisted of cantilever bending tests with the beam loaded through the shear centre and through the centroid. When loaded through the shear centre the beam buckling took place in the compression flange at the root of the cantilever. When loaded through the centroid however, it was noted that the compression flange buckled at a fixed distance from the fixed end. The general theory of thin walled beams developed by Vlasov was applied to the problem and indicated that the maximum compression stress at the free edge of the flange would be a maximum at some distance from the fixed end. The value of the maximum compression stress obtained by the general linear theory was small and its position did not coincide with the experimental position. The Vlasov analysis has been modified to include the increase in the twisting moment due to the lateral deformation of the beam along its length. Good agreement between the modified theory and experiment both for the position of the maximum compressive stress and for the twist of the cantilever at three points along its length.

Because of the very low torsional stiffness of thin walled channel sections, the small deflection theory is only applicable for small bending loads applied through the centroid and the modified theory should be used for practical loading cases.

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PRINCIPAL NOTATION

A, B, C, D	Integration constants
b	Width of the reinforcement
b_1	Width of the cross-section
C_1, C_2	Material constants
E, E_{ST}	Youngs modulus of steel
E_{FO}	Youngs modulus of foam
e	Distance from the neutral axis to the end of the elastic zone.
G	Shear modulus of foam material
h	Distance between the reinforcement flanges
h_1	Breadth of the cross-section
I	Second moment of area
I_C	Second moment of area of the core
I_m	Second moment of area of the matrix
K	Stiffness constant = $E_{FO} I_C$
K_1	Stiffness constant = $E_{FO} I_m$
L	Length of the beam
M_T	Moment due to interfacial shear
P	Load
R	Radius of curvature of the beam
t	Reinforcement flange thickness
t_1	Reinforcement web thickness
x, y, z	Cartesian co-ordinate axis
y_o	Cantilever tip deflection
α, β, λ	Constants
ϵ	Strain
ρ	Beam curvature
σ	Longitudinal stresses
τ	Shear stresses

INTRODUCTION

1. GENERAL

It is well known that when two or more materials are combined together they usually perform differently and often more efficiently than when used independently. There is the possibility of combining the advantages possessed by the individual components, e.g. strength and lightness, strength and corrosion resistance and so on.

Early examples of combining materials together include glued wood in the form of parallel-laminated members and plywood was used by ancient Egyptians. The Romans used plywood for fine furniture. The makers of arms and armour, in the Near and Far East as well as in Europe, understood the value of laminated metal. Pictures of these and other examples can be seen in Ref. (1).

Demands on materials imposed by today's advanced technologies have become so diverse and severe that they often cannot be met by simple single-component materials acting alone. It is frequently necessary to combine several materials into a composite to which each constituent not only contributes its share, but whose combined action transcends the sum of the individual properties and provides new performance unattainable by the constituents acting alone.

Rigid polyurethane structural foam has had many applications in recent years. Furniture, door and window frames, paddles, frames of many electrical appliances and many other applications can be found in industry. During the foaming process, the polyurethane can be united to nearly all materials without joints. This property is utilized when high demands are made on stiffness. By moulding in reinforcements, it is possible to strengthen the construction in line with particular demands. Reinforcements of metal or glass fibre for example can be used to improve mechanical properties. These advantages have made this composite material suitable for use in building vehicle structures.

2. A NEW METHOD FOR BUS BODY CONSTRUCTION

WARDILL Ref. (2) introduced a new method of bus body construction using modular plastic panels. In the given example, he mainly used two standard modular panels, one type for the sides of the vehicle and the other for the roof. The cut-outs for the wheel arches are provided by positioning loose pieces in the moulds. The moulding is made from rigid skin polyurethane foam reinforced with metal. The manufacturing process consists essentially of three sections: manufacture of the reinforcements, production of the moulded panels and assembly of the body with sub-assemblies of the accessories. The following were claimed for the new method:

- (a) low capital investment
- (b) low production costs
- (c) ease of labour training
- (d) suitability of the new method for the developing nations.

3. SAFETY REQUIREMENTS OF BUS STRUCTURAL MEMBERS

In recent years traffic safety has become a matter of major public concern. It includes various aspects ranging from adequate road construction, and the safety of vehicles, to complex human factors.

The safety of vehicles includes primary and secondary safety. The central objectives of secondary safety are the protection of human life, and the reduction of injuries in an accident situation. One of the most dangerous accidents is the roll over, particularly for buses. A bus roll over causes severe structural deformation and exposes a large number of people to great danger. This introduces a new aspect to the design problem, namely the study of the collapse behaviour of structural members.

KECMAN Ref. (3) studied the bending collapse of metal rectangular section tubes in relation to the bus roll over problem. He introduced a new theory to design optimum steel section tubes for building safe bus structures. He specified that the safety criteria may concern any one or all of the following:

- (a) Maximum strength which is the maximum load carrying capacity of the member.
- (b) Energy absorbed during the deformation of the member.
- (c) Remaining strength which is the load carrying capacity of the member after a certain deformation.

4. AIM OF THE THESIS

Initially two problems arise for the composite material used to build the new bus panels:

- (1) The optimum shape of the metal reinforcements to satisfy the requirements and the new safety criteria.
- (2) Comparison between the new members and the metal section tubes from the safety point of view.

Five chapters of this thesis are directed to the development of a theory which predicts the elastic and inelastic bending analysis of polyurethane foam beams reinforced by steel sections loaded in uniaxial bending. It also shows the behaviour of these beams under impact conditions.

A steel thin walled channel cantilever beam was tested in order to find its large deflection behaviour. This was to compare test results with those for a composite beam reinforced with an equivalent channel section. This experiment has thrown up an unexpected phenomenon. The channel cantilever buckled at a significant distance from the fixed end when loaded through the centroid, while a similar channel buckled at the fixed end when loaded through the shear centre.

Chapter 6 of this thesis is directed to an analysis of this phenomenon. An attempt to explain theoretically the buckling at this position using small deflection analysis, did not show the correct

trend. However, in this thesis, the author has developed a large deflection analysis which shows good agreement with experimental results.

The main aim of elastic analysis of beams is to predict their stiffness. This relates the increment of external load with the appropriate increment of displacement.

The theoretical boundary conditions of a cantilever beam are usually different from the real conditions.

In other words, it is very difficult practically to achieve fixed support conditions. This usually causes errors in the prediction of cantilever beams stiffness. Thus, a separate experiment was made (shown in the Appendix) to investigate the errors in the prediction of stiffness of a cantilever composite beam.

CHAPTER 1

PRELIMINARY TESTS

1.1. INTRODUCTION

A computer literature search to find publications on the inelastic deflection behaviour of reinforced plastic beams showed mainly three categories of paper:-

- (1) Elastic analysis of fibre reinforced plastics
e.g. Refs (4,5,6)
- (2) Elastic analysis of sandwich beams
e.g. Refs (7,8,9)
- (3) Elastic properties of foam material
e.g. Refs (10,11)

The first category does not relate to the subject of this thesis, since the composite material involved employs a different type of reinforcement. Although the second and third categories greatly helped the author of this thesis to solve the problem (as will be seen in later chapters), the references did not give any information on large deflection behaviour. In other words previous work considered only elastic analysis rather than inelastic behaviour which is more important in the case of large deformations. The author therefore decided to start with an experimental program to obtain some information about the inelastic behaviour of the composite beams in question.

1.2. TEST SAMPLES

The structural members we are dealing with in this thesis are beams. These beams should be as similar as possible to the beams used in the construction of the plastic bus. The beam cross section with dimensions shown in FIG.(1.1) was chosen for the first lot of test samples (following consultation with the designer of the plastic bus). It consisted of 40 x 60mm polyurethane rectangular section reinforced by a channel folded from sheet steel of 1.6mm thickness. The density of the foam used was 500kg/m³ (the same foam as used for the plastic bus). FIG.(1.1) shows the assembly drawing of the mould used to produce the composite test samples.

1.3. TEST METHOD

To determine the inelastic deflection behaviour, a cantilever test with a beam length of 700mm was chosen for the following reasons:-

- (a) For a cantilever, the point of load application (i.e. free end) is well separated from the collapsing part of the beam which is near to the fixed end. This will ensure the separation of the local effects at the loading point from the collapse hinge. A simply supported beam test would lead to mixed effects due to the interference of the loading point and the collapse hinge (they are almost in the same position).

- (b) For a long cantilever (i.e. high length to width ratio), the shear effect is negligible. This is very similar to the conditions which apply in most frameworks.
- (c) KECMAN Ref. (3) showed that in practice the most suitable test to determine the collapse behaviour is the cantilever test, since the other tests (e.g. simply supported beam and four point bending) become unstable at high deformations.

To provide restrained boundary conditions, the fixed end of the test beam were set in concrete blocks. To determine the inelastic bending behaviour, it is possible to use one of the test rigs shown in FIG.(1.2).

(1) Incremental increasing displacement rig.

This rig consists of a nut and screw as a loading equipment. The nut is fixed to a rigid frame. The screw has one end connected to a loading handle while the other end connected to the beam through a cable. The load through the cable is measured using an electric dynamometer and a digital indicator. The deflection of the beam is measured by a potentiometer using another digital indicator. This rig allows the load to decrease while the displacement increases.

(2) Incremental increasing load rig.

This rig has the same components used in the previous rig except that the loading equipment consists of simply a pulley and calibrated weights. This rig does not allow the load to decrease.

The loading condition of a structural member specifies which rig is suitable for determination of inelastic bending behaviour. The loading condition of a beam in a crane beam or a bridge structure is similar to the incremental increasing load rig. On the other hand, if a beam is hit with a pendulum, the load decreases when the beam collapses. This means that the load is allowed to decrease while the displacement increases and this is similar to the incremental increasing displacement rig.

1.4. TEST RESULTS

Six beams were tested using the incremental increasing displacement rig. These beams were 3 composites, two steel channels and one foam. FIG.(1.3) shows the load deflection curves for the beams tested. For the steel channel beam, the load deflection relation was linear (as expected) in the elastic zone. On further displacement the load increased non linearly till it reached a maximum value. It then started to drop continuously. This was due to buckling of the flange in compression.

For the unreinforced beam, the load deflection relation was almost linear till total collapse near to the fixed end occurred.

For the composite beam, the small load deflection relation was linear. But, by increasing the displacement the relation became non linear accompanied by a creep phenomenon. The values recorded are the readings after load and deflection has stabilized. Both the load and deflection before failure was considerably higher than for the pure steel or pure

foam beams. On further increase of displacement, the foam material on the tension side of the composite beam cracked. This caused a sudden drop-off of the load. Continued increase in the applied displacement caused the load to increase again until total collapse occurred.

The area under the load deflection curve represents the energy absorbed statically by the beam. For the samples tested this area was determined for first 180mm deflection (this is equivalent to 15° of beam rotation). The energy absorbed was (330J) for the composite beam, (53J) for the steel beam and (62J) for the foam beam.

Comparing the composite beam with constituents from the point of view of the safety criteria.

- (a) The composite beam absorbed 2.87 times the sum of the energy absorbed by the constituents.
- (b) The maximum strength of the composite (marked on FIG.(1.3)) is 2.5 times higher than the strength of the foam beam (the highest of the constituents).
- (c) The remaining strength of the composite (marked on FIG.(1.3)) after cracking is 2.5 times higher than that of the foam beam.

Another important comparison is that between the composite and rectangular steel tubes.

Kecman Ref. (3) developed a computer program to find the optimum dimensions of steel section rectangular tubes given the following:-

- (a) The energy absorbed after a certain angle of beam rotation.
- (b) Material properties of steel (i.e. σ_y , σ_U).
- (c) Width to depth ratio of the beam cross section.

The program will also give the maximum and remaining strength values. From the optimum dimensions of the beam we can find the mass per unit length. This is obviously a more relevant parameter for comparison with composite beams, because of its effect on the mass of the whole bus structure.

This program was used to find the optimum dimensions for a steel tube which could absorb the same amount of energy as the composite beams before cracking. The following table shows a comparison between the composite and optimum steel tubes with the same width to depth ratio as used for the composite.

Beam	mass/unit length (%)	maximum strength (N)	remaining strength (N)
Composite 2	100	2632	2211
Equivalent steel	80	2668	1240
Composite 3	100	2434	2106
Equivalent steel	70	2361	1170

1.5. CONCLUSIONS

The comparison with the constituents showed that the composite beam exhibited considerably improved performance in respect of safety criteria.

The comparison with optimum steel tubes was not totally representative, because the composite beams were not optimized. Although composite beams are slightly heavier, they have higher remaining strength. This is a considerable advantage, if we assume that the bus will probably stay on its roof after a roll-over accident.

There may be two reasons for the change of load deflection relation without any change in the external appearance of the beam:-

- (1) The foam changed from the elastic to the inelastic state.

This reason was rejected because:-

- (a) the tensile test of the foam material showed that the sample has fractured suddenly at the end of the elastic zone,
- (b) the behaviour of the foam beam before collapse was almost linear.

- (2) The steel changed from the elastic to the plastic state.

This reason was accepted because the steel channel beam buckled and the composite did not. This means that the foam prevented the steel section from buckling locally. In other words the steel was behaving plastically.

Finally, we can say that the preliminary tests gave useful results for continuing the research from two points of view:-

- (1) It showed the effectiveness of the new composite members.
- (2) It provided a starting point for a theoretical explanation of the inelastic behaviour.

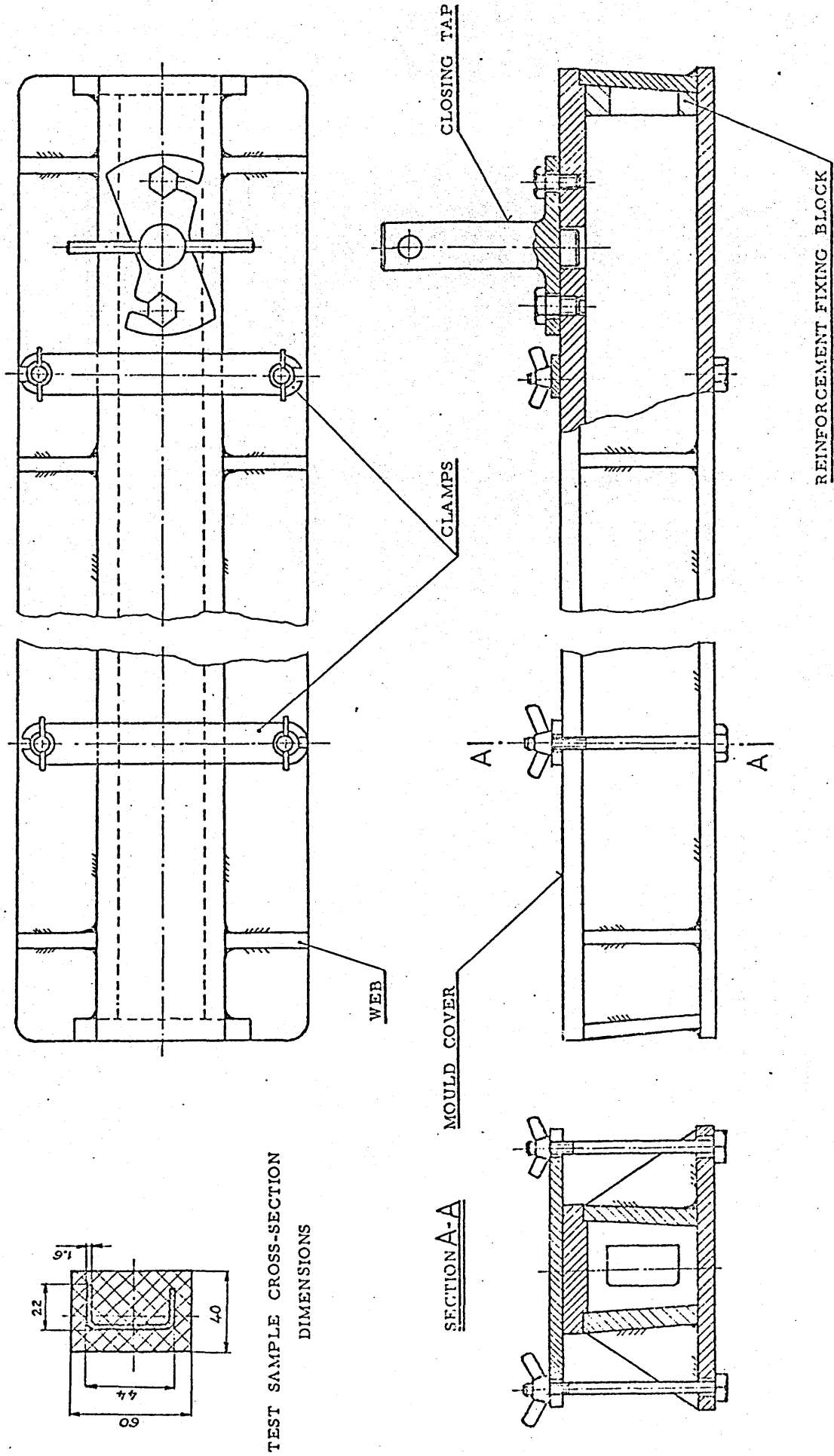
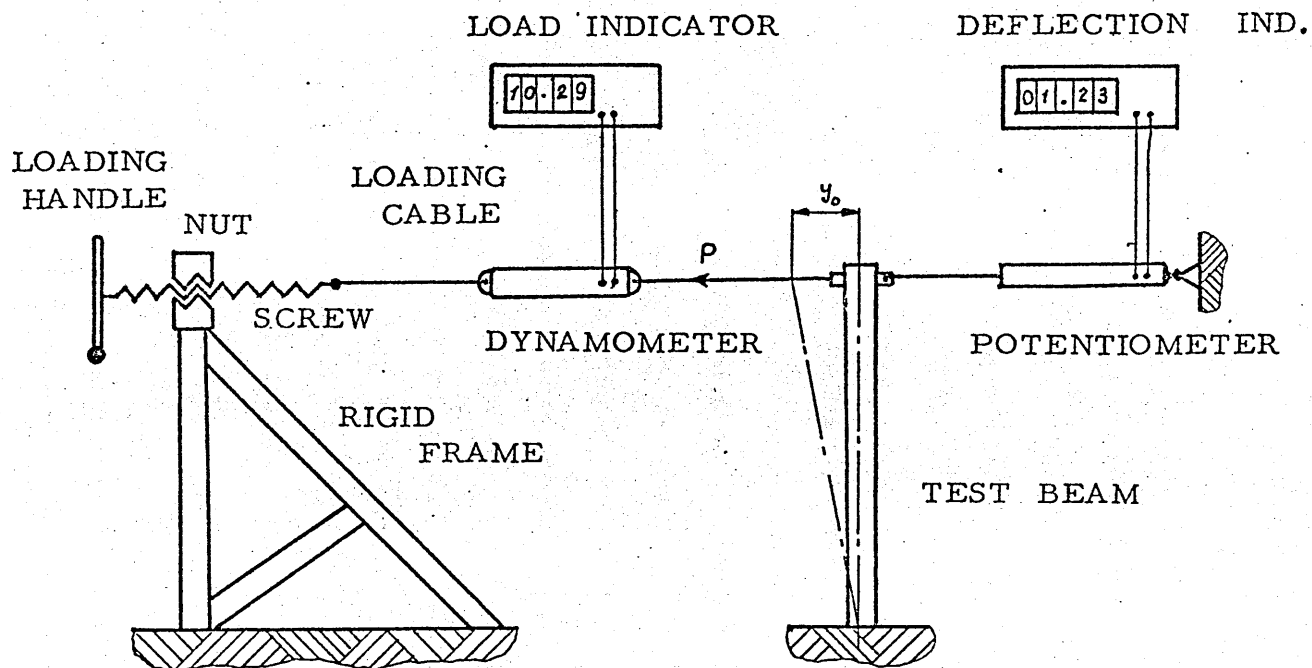
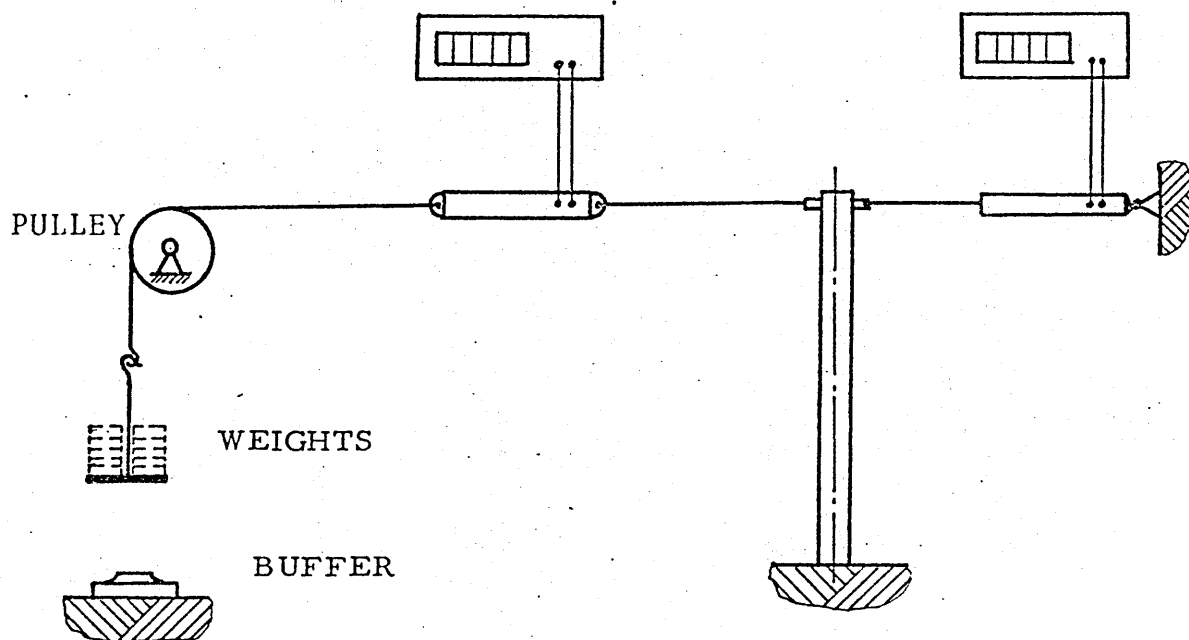


FIG. (1.1) ASSEMBLY DRAWING OF THE MOULD USED TO PRODUCE THE TEST SAMPLES

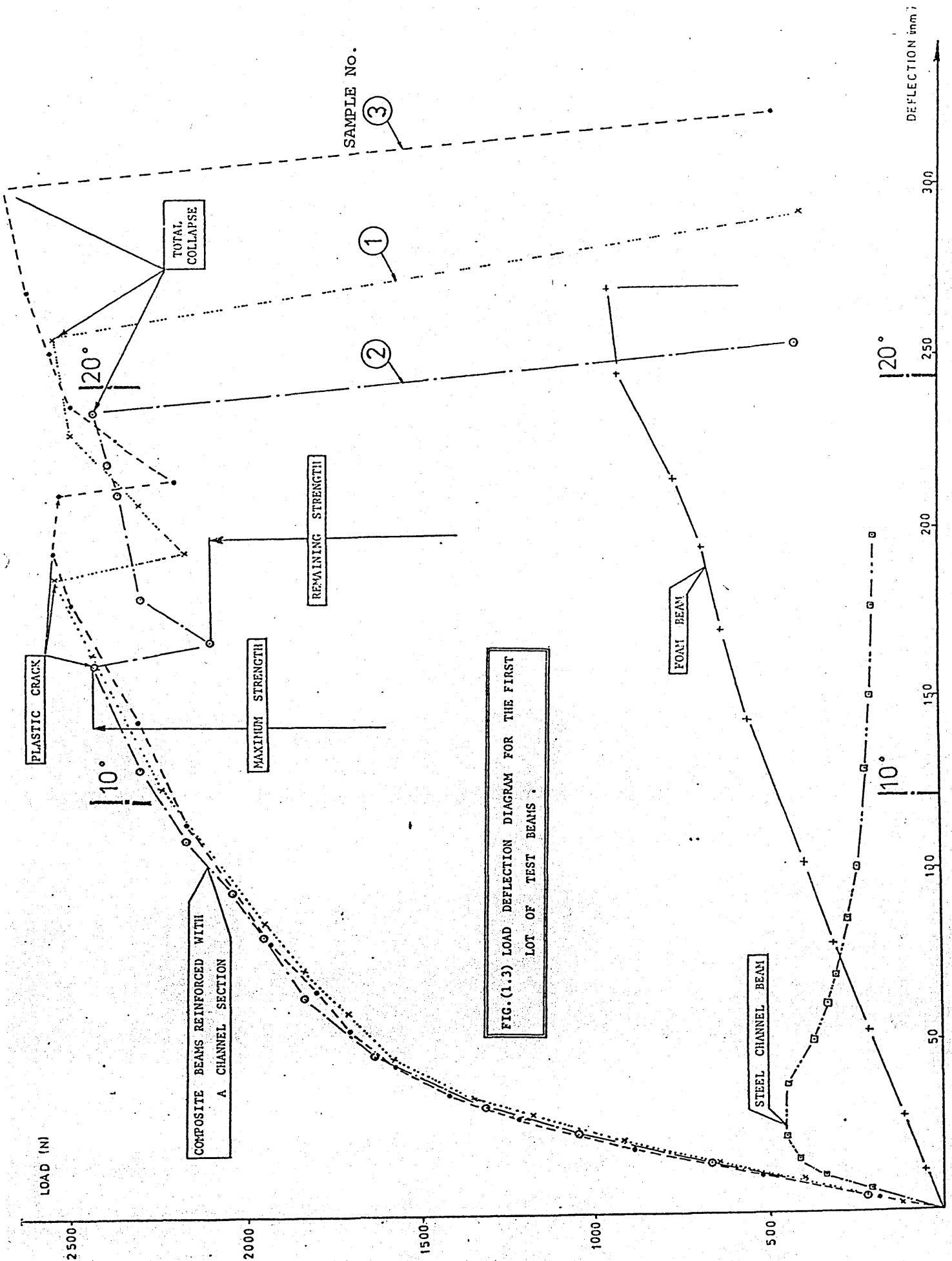


a. INCREMENTAL INCREASING DISPLACEMENT RIG



b. INCREMENTAL INCREASING LOAD RIG

FIG.(1.2) CANTILEVER BEAM TEST RIGS .



CHAPTER 2

ELASTIC ANALYSIS

2.1. INTRODUCTION

The composite beams which this thesis deals with, utilize foam material as matrix and thin walled steel sections for reinforcement. A sandwich beam consists of two thin layers of material on the outside, called the faces, with a thick core between them. The core is usually of light weight, low-strength material serving primarily as a filler or spacer, whereas the faces are of high-strength material. Sandwich beams are used in applications where light weight combined with high strength and high stiffness are needed.

It is clear that there is a considerable similarity between composite beams and sandwich beams. Therefore an attempt will be made in this chapter to derive an elastic analysis theory based on the theory of sandwich beams. Obviously, some modifications will be used to overcome the differences between composite and sandwich beams.

2.2. ELASTIC ANALYSIS OF A CANTILEVER SANDWICH BEAM

2.2.1. ANALYSIS USING THE ENGINEERS THEORY OF BENDING

An approximate theory based on the engineer's theory of bending was introduced in many references e.g. Réf.'s (12,13). This method is usually called the transformed section method. The procedure is to transform the cross section, consisting of more than one material, into an equivalent cross section composed of only one material. Then the later, called the transformed section, is analysed in the usual manner for a beam of one material. Therefore, a sandwich beam with steel faces and foam core shown in FIG.(2.1) can be transformed to an I section beam with equivalent web thickness t_1 ,

$$\text{where } t_1 = \frac{E_{FO} \cdot b}{E_{ST}}$$

E_{FO} Modulus of elasticity of foam,

E_{ST} Modulus of elasticity of steel,

and b width of the foam core

Therefore, for the cantilever sandwich beam shown in FIG.(2.1), the formula for tip deflection y_0 is

$$y_0 = \frac{P \cdot L^3}{3 \cdot E_{ST} \cdot I_{eq}}$$

where P applied load,
 L beam length

$$\text{and } I_{eq} = \frac{b t^3}{6} + \frac{b t}{2} (h+t)^2 + \frac{t_1 h^3}{12}$$

In fact, early investigations disclosed that the bending rigidity of sandwich beams is considerably smaller than may be anticipated on the basis of the engineer's theory of bending formulae mentioned in this section. The reason for this is the small shearing rigidity of the relatively thick core. Consequently, the theory of sandwich beams must take into account transverse shear deformations which are neglected in the previous case. Therefore, in the next section, more sophisticated theories derived by different investigators will be introduced.

2.2.2. STRAIN ENERGY METHOD

In this section only the formula of the tip deflection (y_0) of a cantilever sandwich beam as derived by HOFF Ref. (14) will be given. The differential equation of the problem was obtained using the principle of minimization of the essential portions of the strain energy stored in the beam during deformation, namely, the strain energy of extension and bending of the faces as well as the strain energy of shear of the core. Integrating this differential equation and considering the boundary conditions of the cantilever, gives

$$y_0 = \frac{PL^3}{3 E_{ST} I_b} \left(1 + \frac{I_1}{2 I_f} \frac{3}{(\alpha L)^3} (\alpha L - \tanh \alpha L) \right)$$

$$\text{where } I_1 = \frac{b t (h+t)^2}{2}, \quad I_f = \frac{b t^3}{12}, \quad I_b = I_1 + 2 I_f,$$

$$\alpha = \sqrt{\frac{G b h I_b}{2 E_{ST} I_1 I_f}}$$

and G is the shear modulus of the foam.

This method neglected the effect of bending stiffness of the core. This can be noticed from the absence of the elastic modulus of the foam in these formulae. HOFF showed that this method gave good agreement with experiment for elastic analysis.

2.2.3. STRESS FUNCTION SOLUTION

OGORKIEWICZ and SAYIGH Ref. (7) derived a formula for the central deflection of a simply supported carbon fibre/acrylic foam sandwich beam. The solution derived was based on stress functions (stress distribution in a rectangular strip of sandwich beam). This solution showed good agreement with experimental results. The formula was reduced and simplified by taking advantage of the disparity between the stiffness of the skin and the core materials, which makes terms involving the tensile modulus of the core negligibly small. The formula derived is not given here because it applied only to simply supported beams.

2.2.4. PARTIAL DEFLECTIONS METHOD

PLANTIMA Ref. (15) and HOWARD Ref. (16) introduced an analysis based on partial deflections due to shear and bending (see FIG.(2.2)). The differential equation of the problem is formed from the geometry of these deflections and the equilibrium conditions, considering the angles of rotations due to shear and bending as functions of the first and second derivatives of the partial deflections. The remaining analysis was classic i.e. solving the differential equation and applying the boundary conditions.

The previous investigators neglected the bending stiffness of the core, which does not cause a large error in elastic analysis.

The preliminary test of composite beams showed the load increases when the displacement increases, even at large deformations. It also showed that during large deflections the reinforcement behaves plastically. In contrast, the steel channel beam test showed that the load decreases at high deflections. This proves that the foam bending stiffness is important for high deflections.

A sandwich beam would be expected to have similar behaviour at large deflections i.e. the bending stiffness of the core is also important. Therefore, it is necessary to have an analysis which contains the bending stiffness of the core (matrix).

2.2.5. INTERFACIAL SHEAR METHOD

ADAMS and WEINSTEIN Ref. (9) developed a new theory for predicting the bending stiffness of sandwich beams. This theory analyses simply supported beams assuming an interfacial shear between the faces and the core of the sandwich beam. The solution considered all the properties of face and core. Therefore, this method is suitable as a basis for the inelastic analysis.

A modified theory to predict the elastic end deflections of a cantilever sandwich beam based on the interfacial shear method will be introduced.

For a cantilever sandwich beam defined earlier and shown in FIG.(2.1) the following assumptions were considered:

- (a) For simplicity, the foam core is homogeneously distributed and has the same properties in tension and compression.
- (b) The end load of a cantilever produced the following stresses in the parts of the beam:
 1. longitudinal stresses in the faces σ_f
 2. a shear stress τ induced by the bond between the facing and the core which is called the interfacial shear
 3. this shear will induce a longitudinal stress in the core, i.e. the beam will be equivalent to a foam beam with a variable shear stress acting on upper and lower surfaces and obviously loaded with a load P at the free end FIG.(2.3).
- (c) The stress distribution over the thickness of the faces is uniform (small thickness).

The equilibrium of moments on a general section at a distance z from the free end is

$$P.z = \frac{E_{FO} I_C}{R} + M_T \quad (2.1)$$

where R is the radius of curvature of the beam,

$$R = \left(1 / \frac{d^2 y}{dz^2} \right),$$

$$I_C = \frac{bh^3}{12} \text{ second moment of area of the core and}$$

M_T is the additional moment provided by the interfacial shear.

Defining $K = E_{FO} I_C$,

equation (2.1) becomes

$$P.z = \frac{K}{R} + M_T \quad (2.2)$$

The first part of the right hand side of this equation represents a bending moment due to stresses in the core and the second part represents the bending moment due to interfacial shear.

The interfacial shear moment results from the stresses σ_f in the faces

$$\text{i.e. } M_T = \sigma_f . b . t . (h+t)$$

Thus, equation (2.2) can be written in the form

$$P.z = \frac{K}{R} + \sigma_f \cdot \lambda \quad (2.3)$$

where $\lambda = b \cdot t(h+t)$

The equilibrium of forces on the faces for a general section of distance z (see FIG.(2.4)) gives,

$$\sigma_f \cdot b \cdot t = b \int_0^z \tau \, dz,$$

dividing by $b \cdot t$, gives

$$\sigma_f = \frac{1}{t} \int_0^z \tau \, dz,$$

differentiating w.r.t. z gives

$$\tau = t \frac{d\sigma_f}{dz} \quad (2.4)$$

, differentiating again w.r.t. z gives

$$\frac{d\tau}{dz} = t \frac{d^2\sigma_f}{dz^2} \quad (2.5)$$

Equation (2.3) is not sufficient to find σ_f , so that it is necessary to establish another relation which includes σ_f and R , then eliminate R between them. This requires strain compatibility at the interface between face and core.

From the engineer's bending theory, the longitudinal strain of a beam due to bending about its own axis is y/R , where y is the distance measured from the neutral axis.

Since the bending of faces around their own axis is negligible (σ_f is uniform over the thickness),

$$\text{then strain } \epsilon_{fb} = 0 \quad (2.6)$$

The tensile strain at the upper surface of the core due to bending ϵ_{cb} is given by:

$$\epsilon_{cb} = \frac{h}{2R} \quad (2.7)$$

The shear stress τ induces tensile stress σ_f in the upper face. This causes an additional tensile strain $\epsilon_{f\tau}$ which is given by:

$$\epsilon_{f\tau} = \frac{\sigma_f}{E_{ST}} \quad (2.8)$$

The remaining strain component is the strain induced by the shear stress acting on the core surface, whose value can be found as follows: FIG.(2.5) shows an element of the core with length Δz . The axial displacement ζ on the core surface due to a general value of τ is

$$\zeta = \frac{\tau}{G} \cdot \frac{h}{2} \quad \text{where } G \text{ is the shear modulus of the core}$$

Therefore, the values of axial displacements at distances z and $z + \Delta z$ are

$$\zeta(z) = \frac{\tau(z)}{G} \cdot \frac{h}{2}, \quad \zeta(z+\Delta z) = \frac{\tau(z+\Delta z)}{G} \cdot \frac{h}{2}$$

Then the strain $\epsilon_{c\tau}$ of an element Δz is

$$\epsilon_{c\tau} = \frac{\zeta(z+\Delta z) - \zeta(z)}{\Delta z} = \frac{h}{2G} \frac{\tau(z+\Delta z) - \tau(z)}{\Delta z}$$

and as $\Delta z \rightarrow 0$ $\epsilon_{c\tau} = \frac{h}{2G} \frac{d\tau}{dz}$

Substituting the value of $\frac{d\tau}{dz}$ from equation (2.5) in the last expression of $\epsilon_{c\tau}$ gives

$$\epsilon_{c\tau} = \frac{ht}{2G} \frac{d^2\sigma_f}{dz^2} \quad (2.9)$$

The compatibility condition requires that the total strain at the core surface must be equal to the total strain at the face surface, so that:-

$$\epsilon_{cb} + \epsilon_{c\tau} = \epsilon_{fb} + \epsilon_{f\tau}$$

and using equations (2.6), (2.7), (2.8), (2.9) gives

$$\frac{h}{2R} + \frac{ht}{2G} \frac{d^2\sigma_f}{dz^2} = 0 + \frac{\sigma_f}{E_{ST}}$$

rearranging gives $\frac{1}{R} = \frac{2\sigma_f}{E_{ST} \cdot h} - \frac{t}{G} \frac{d^2\sigma_f}{dz^2} \quad (2.10)$

substituting (2.10) in (2.3) to eliminate R gives

$$\frac{d^2 \sigma_f}{dz^2} - G \left(\frac{2K + \lambda E_{ST} h}{K h t E_{ST}} \right) \sigma_f = - \frac{P G}{K t} z$$

Defining $\alpha^2 = G \left(\frac{2K + \lambda E_{ST} h}{K h t E_{ST}} \right)$

and $\beta = \frac{E_{ST} h}{2K + E_{ST} \lambda h}$

so that $\frac{d^2 \sigma_f}{dz^2} - \alpha^2 \sigma_f = - P \alpha^2 \beta z$

The solution of this differential equation is

$$\sigma_f = A \sinh \alpha z + B \cosh \alpha z + P \beta z$$

Now, to determine the constants A and B, two boundary conditions of a cantilever sandwich beam must be used. These are:

(a) $\sigma_f = 0$ at the free end i.e. when $z = 0$
this implies that $B = 0$

(b) The interfacial shear (τ) vanishes at the fixed end and equation (2.4) gives

$$\tau = t \frac{d\sigma_f}{dz}, \text{ but } t \text{ is constant}$$

$$\frac{d\sigma_f}{dz} = 0 \quad \text{when } z = L$$

this implies $\frac{d\sigma_f}{dz} = A \alpha \cosh \alpha z + P \beta$

Thus $A = - \frac{P \beta}{\alpha \cosh \alpha L}$

Hence $\sigma_f = \frac{P \beta}{\alpha} \left(\alpha z - \frac{\sinh \alpha z}{\cosh \alpha L} \right) \quad (2.11)$

and $\tau = t \frac{d\sigma_f}{dz} = P \beta t \left(1 - \frac{\cosh \alpha z}{\cosh \alpha L} \right)$

To determine the beam deflection, substitute the values of σ_f and $\frac{1}{R} = \frac{d^2y}{dz^2}$ in the moment equation (2.3) to get

$$\frac{d^2y}{dz^2} = \frac{Pz}{K} - \frac{P\beta\lambda}{\alpha K} (\alpha z - \sinh \alpha z)$$

integrating twice w.r.t z gives

$$y = \frac{Pz^3}{6K} - \frac{P\beta\lambda}{\alpha^3 K} \left(\frac{\alpha^3 z^3}{6} - \frac{\sinh \alpha z}{\cosh \alpha L} \right) + Cz + D$$

where C and D are integration constants.

Now the boundary conditions for a cantilever beam are

a) $\frac{dy}{dz} = 0$ at $z = L$

i.e. $C = -\frac{PL^2}{2K} + \frac{P\beta\lambda}{\alpha^2 K} \left(\frac{\alpha^2 L^2}{2} - 1 \right)$

b) $y = 0$ at $z = L$

i.e. $D = \frac{PL^3}{3K} - \frac{P\beta\lambda}{\alpha^3 K} \left(\frac{\alpha^3 L^3}{3} + \tanh \alpha L - \alpha L \right)$

substituting the values of C and D in y - equation gives

$$y = \frac{P}{6K} (2L^3 + z^3 - 3L^2 z) + \frac{P\beta\lambda}{\alpha^3 K} \left[\frac{\alpha^3}{6} (3L^2 z - 2L^3 - z^3) + \alpha(L-z) - \tanh \alpha L + \frac{\sinh \alpha z}{\cosh \alpha L} \right]$$

And to determine tip deflection y_0 substitute $z = 0$ in the previous equation; this gives

$$y_0 = \frac{PL^3}{3K} \left[1 - \beta\lambda \right] + \frac{P\beta\lambda}{\alpha^3 K} \left[\alpha L - \tanh \alpha L \right] \quad (2.12)$$

This equation can also be determined using the curvature area theorem introduced by TIMOSHENKO Ref. (12) as follows:

$$y_0 = \int_0^L \rho \cdot z \, dz \quad \text{where } \rho \text{ is the beam curvature} = \frac{1}{R}$$

Then using equations (2.3), (2.11) gives

$$\rho = \frac{1}{R} = \frac{-Pz}{K} - \frac{P\beta\lambda}{\alpha K} \left(\alpha z - \frac{\sinh \alpha z}{\cosh \alpha L} \right)$$

$$\text{i.e. } y_0 = \frac{P}{K} \int_0^L z^2 dz - \frac{P\beta\lambda}{\alpha K} \int_0^L \left(\alpha z^2 - z \frac{\sinh \alpha z}{\cosh \alpha L} \right) dz$$

and finally

$$y_0 = \frac{PL^3}{3K} (1 - \beta\lambda) - \frac{P\beta\lambda}{\alpha^3 K} (\alpha L - \tanh \alpha L)$$

which is the same expression obtained before in equation (2.12).

The expression derived for the tip deflection of the cantilever sandwich beam contains the Young's modulus of the steel and the Young's and shear moduli of the foam. This makes this expression a suitable base for finding the inelastic deflection behaviour. An attempt to derive a similar expression for the tip deflection of a composite cantilever beam will be given.

2.3. ELASTIC ANALYSIS OF A CANTILEVER COMPOSITE BEAM

The composite beam in question is defined as a foam matrix with steel reinforcement. The general shape of the reinforcement to carry uniaxial bending is assumed to be an I section. Other shapes of reinforcement can be considered as particular cases of the I section. Considering the web thickness = 0, the reinforcement becomes two strips of metal. A channel section reinforcement in fact is similar to the I section for uniaxial bending.

Now consider a cantilever composite beam which has the cross section dimensions shown in FIG.(2.6). The assumptions considered for sandwich beams are also applied here. This means that the beam is considered as a cantilever foam beam with a cavity in the shape of the reinforcement and a variable shear stress acting on the inside surfaces of this cavity.

The equilibrium of moments on a general section at distance z from the free end gives

$$Pz = \frac{E_{FO} I_M}{R} + M_T$$

$$\text{where } I_M = \frac{b h^3}{12} \quad \text{second moment of area of the matrix.}$$

Assuming the stress distribution of the reinforcement shown in FIG.(2.7) gives

$$M_T = M_{T_{\text{flanges}}} + M_{T_{\text{web}}} \quad \text{where}$$

$$M_{\text{flanges}} = \sigma_r bt(h+t), \quad M_{\text{web}} = \sigma_r \frac{h^2 t_1}{6},$$

t_1 is the thickness of the web

and σ_r is the longitudinal stress in the flanges.

$$\text{Then } M_T = \sigma_r \left[bt(h+t) + \frac{h^2 t_1}{6} \right]$$

$$\text{or } M_T = \sigma_r \cdot \lambda_1 \quad \text{as} \quad \lambda_1 = bt(h+t) + \frac{h^2 t_1}{6}$$

Therefore, equation (2.13) can be written as

$$Pz = \frac{K_1}{R} + \lambda_1 \sigma_r \quad (2.14)$$

$$\text{where } K_1 = E_{FO} \cdot \frac{b_1 h_1^3}{12}$$

Considering the equilibrium of forces on the reinforcement:-

$$\text{a) For the flanges } \sigma_r bt = 2b \int_0^z \tau_f dz$$

$$\text{i.e. } \sigma_r = \frac{2}{t} \int_0^z \tau_f dz, \text{ by differentiating gives}$$

$$\tau_f = \left(\frac{t}{2} \right) \frac{d\sigma_r}{dz} \text{ and differentiating again gives}$$

$$\frac{d\tau_f}{dz} = \frac{t}{2} \frac{d^2\sigma_r}{dz^2}$$

$$\text{b) For the web } \sigma_r \cdot \frac{h}{2} \cdot \frac{t_1}{2} = 2 \int_0^{h/2} \int_0^z \tau_w dz dy$$

$$\text{i.e. } \sigma_r = \frac{8}{ht_1} \int_0^{h/2} \int_0^z \tau_w dz dy$$

Then as in the case of sandwich beam, the strains and compatibility condition at the lower surface of the upper flange can be written as:-

$$\epsilon_{rb} = 0, \quad \epsilon_{mb} = \frac{h}{2R}, \quad \epsilon_{r\tau} = \frac{\sigma_r}{E_{ST}}$$

$$\epsilon_{m\tau} = \frac{h}{2G} \frac{d\tau_f}{dz} = \frac{ht}{4G} \frac{d^2\sigma_r}{dz^2}$$

$$\text{and finally } \frac{1}{R} = \frac{2\sigma_r}{E_{ST}h} - \frac{d^2\sigma_r}{dz^2} \left(\frac{t}{2G} \right)$$

Eliminating R between the last equation and equation (2.14) to obtain the differential equation.

$$\frac{d^2\sigma_r}{dz^2} - \sigma_r \left(\frac{2G(2K_1 + E_{ST}\lambda_1 h)}{E_{ST}K_1 ht} \right) = - \frac{2PG}{K_1 t} z$$

$$\text{and defining } \alpha_1^2 = 2G \frac{(2K_1 + E_{ST}\lambda_1 h)}{E_{ST}K_1 ht}$$

$$\text{and } \beta_1 = \frac{E_{ST} h}{2K_1 + E_{ST}\lambda_1 h}$$

$$\text{so that } \frac{d^2\sigma_r}{dz^2} - \sigma_r \alpha_1^2 = - P \alpha_1^2 \beta_1 z$$

This is similar to equations obtained for the sandwich cantilever, so that:-

$$\sigma_r = \frac{P\beta_1}{\alpha_1} \left(\alpha_1 z - \frac{\sinh \alpha_1 z}{\cosh \alpha_1 L} \right)$$

$$\tau_f = \frac{P\beta_1 t}{2} \left(1 - \frac{\cosh \alpha_1 z}{\cosh \alpha_1 L} \right) \quad (2.15)$$

and finally

$$y_0 = \frac{PL^3}{3K_1} \left(1 - \beta_1 \lambda_1 \right) - \frac{P\beta_1 \lambda_1}{\alpha_1^3 K_1} \left(\alpha_1 L - \tanh \alpha_1 L \right)$$

The expression derived for the tip deflection of a cantilever composite beam is similar to that derived for the sandwich beam except for differences in the values of λ , K , α and β . The difference in the values of λ comes from including the longitudinal stresses in the web.

But when the web thickness is zero i.e. reinforcement is in form of strips the value of λ will remain the same as in a sandwich beam. The difference in the value of K comes from the difference in cross section breadth i.e. h and h_1 in FIG.(2.6). The difference in α comes from the interfacial shear. This shear is acting only on the inner surface of the sandwich beam faces but for the composite beam it acts on both surfaces of the reinforcement. The value of β is different because β is a function of K and λ .

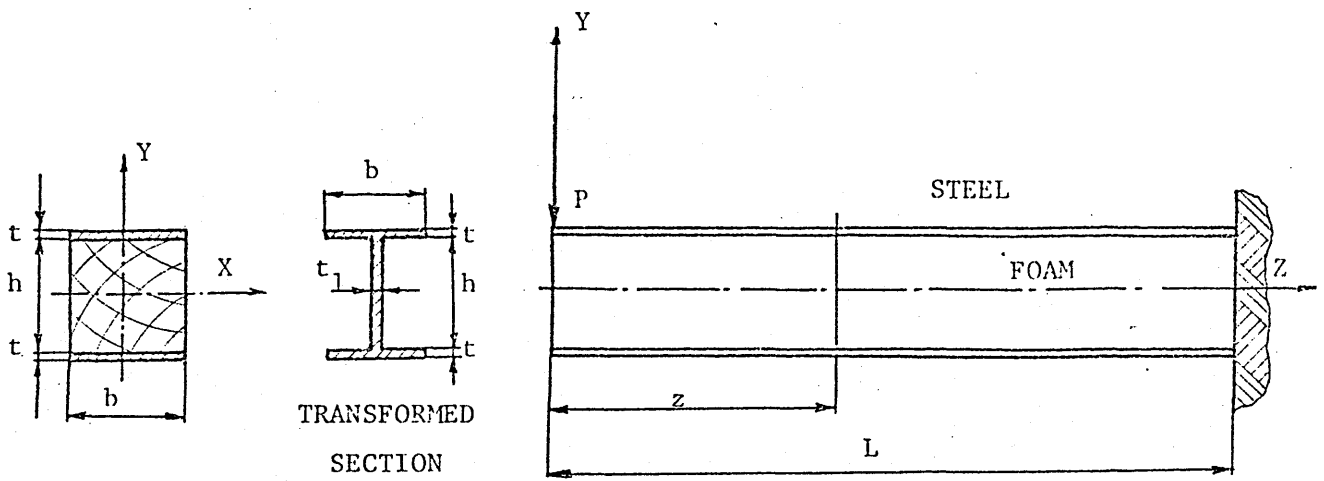
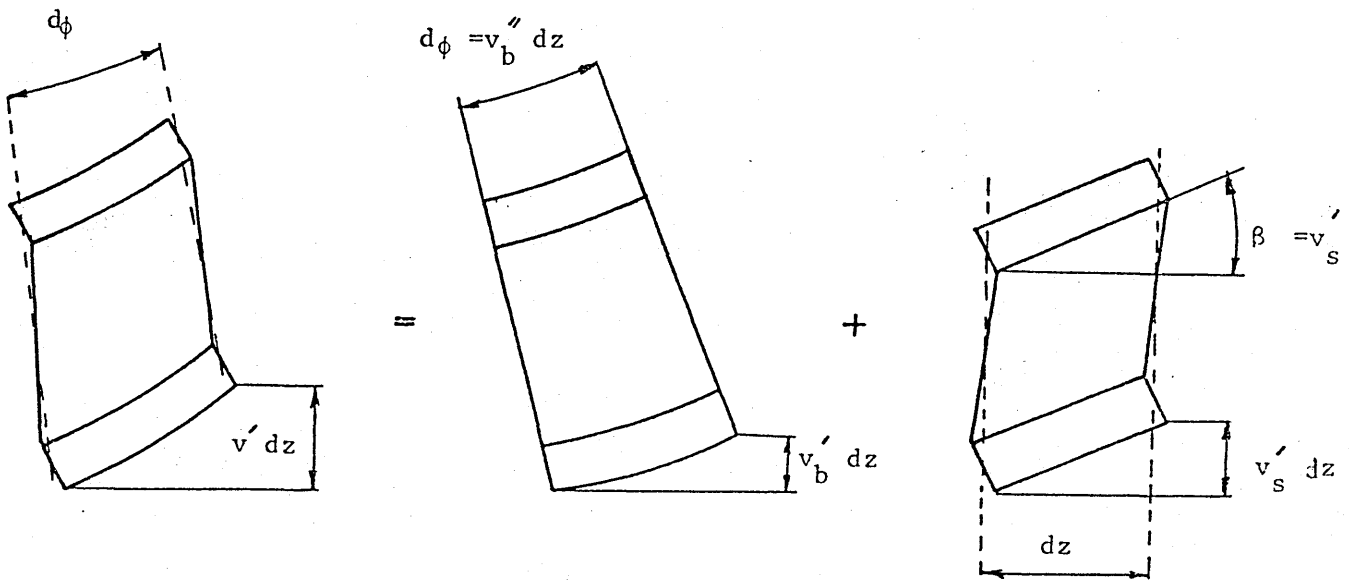


FIG.(2.1) CANTILEVER SANDWICH BEAM.



$$\begin{aligned} \text{TOTAL DEFORMATION} &= \text{BENDING DEFORMATIONS} + \text{SHEAR DEFORMATIONS} \\ v &= v_b + v_s \end{aligned}$$

FIG.(2.2) PARTIAL DEFLECTIONS OF SANDWICH BEAM ELEMENT DUE TO BENDING AND SHEAR.

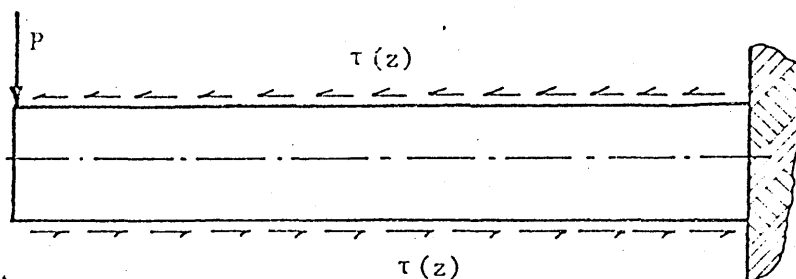


FIG.(2.3) EQUIVALENT BEAM ACCORDING TO THE ASSUMPTIONS.

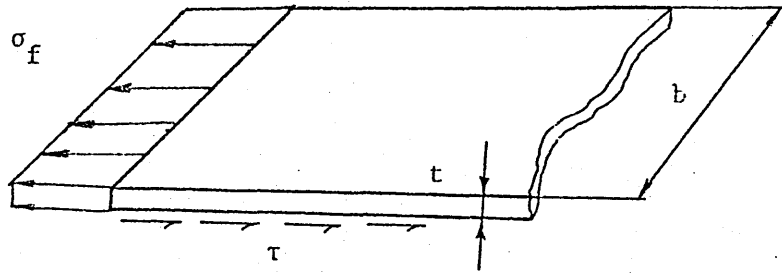


FIG.(2.4) STRESS DISTRIBUTION ON THE UPPER FACE AT A GENERAL DISTANCE z

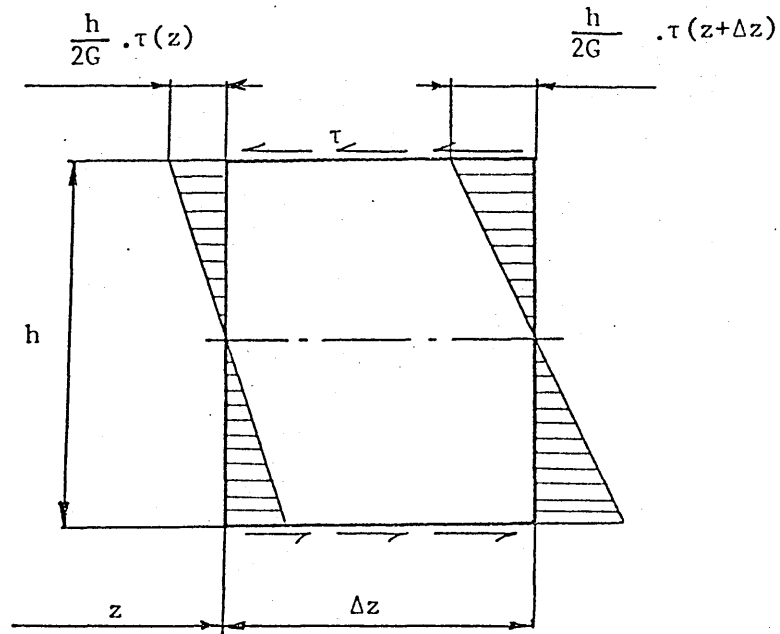


FIG. (2.5) CORE DEFORMATIONS DUE TO SHEAR

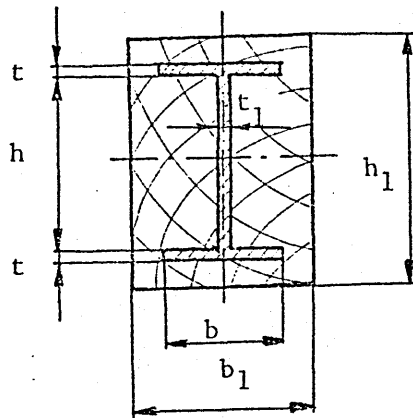


FIG.(2.6) CROSS SECTION DIMENSIONS OF COMPOSITE BEAM.

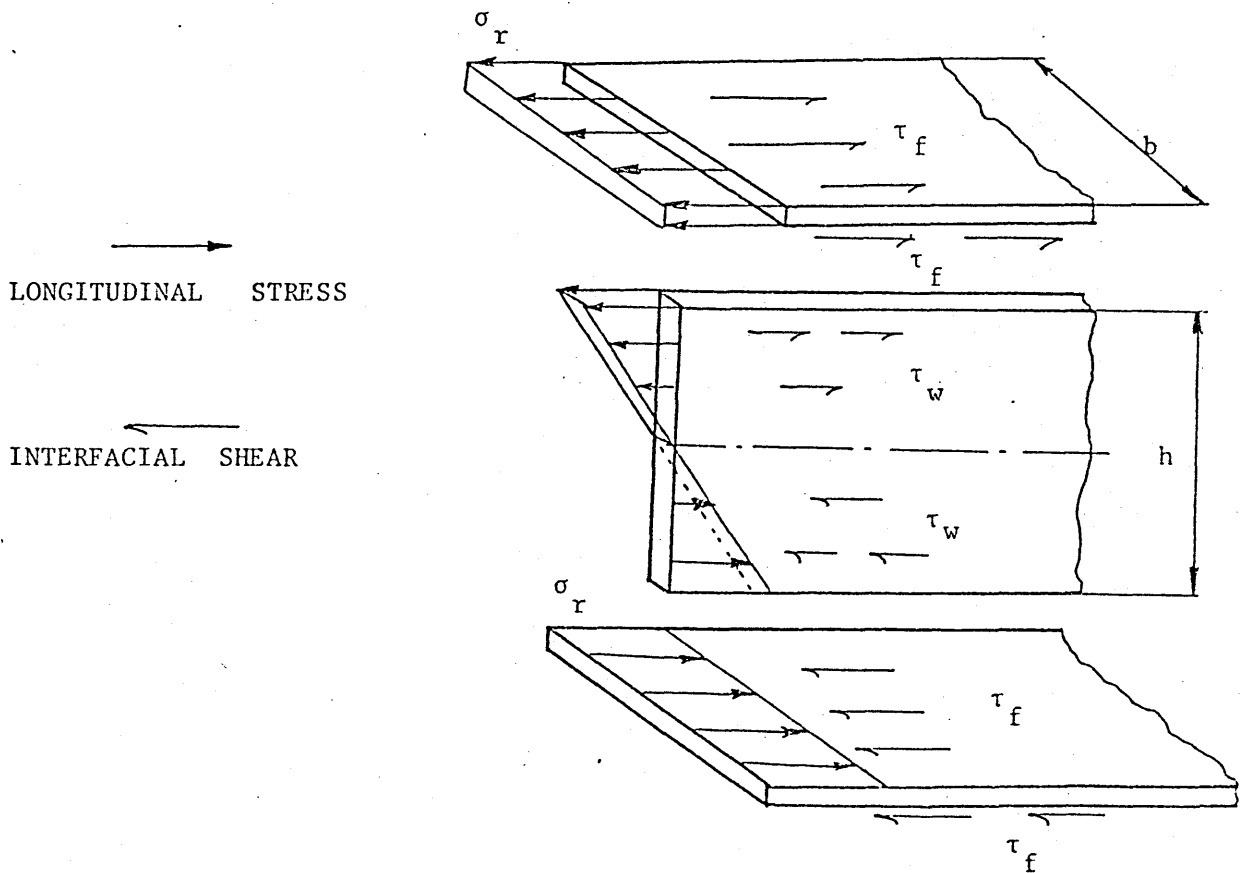


FIG.(2.7) STRESS DISTRIBUTION ON THE REINFORCEMENT.

CHAPTER 3

INELASTIC ANALYSIS

3.1. INTRODUCTION

Inelastic bending refers to bending of beams when the material does not follow Hooke's law. Inelastic behaviour of a beam depends upon the beam cross section. Solid and thick walled section beams behave inelastically when the stresses in the extreme external fibres attain the yield stress of the material. Otherwise, thin walled section beams behave inelastically due to local buckling.

3.1.1. INELASTIC BENDING OF SOLID AND THICK WALLED SECTION BEAMS

Many references have described the method of analysis of inelastic analysis of solid and thick walled section beams e.g. Ref. (12) and Ref. (13). A brief description of the method of analysis which represents a basis for composite beam analysis will be given.

- (a) The behaviour of a beam in inelastic bending is dependent upon the shape of the stress-strain diagram of the material.
- (b) The deflections of an inelastic beam may be calculated from the basic relationship between curvature and deflection.

$$\rho = \frac{1}{R} \approx \frac{d^2y}{dz^2}$$

This relation was derived from pure geometry, therefore it is valid for a beam of any material.

One of the methods for solving this equation is the curvature - area method introduced by TIMOSHENKO and JAMES Ref. (12). This method can be used to find the slopes and deflections for inelastic beams in the same manner that was used for elastic beams. This method can be used only for simple structures. For complicated structures numerical techniques are needed for finding the deflections.

This method will be demonstrated on a cantilever beam of an arbitrary solid cross section.

Assume the cantilever shown in FIG.(3.1a), loaded by a load P. FIG.(3.1b) shows the bending moment diagram as long as the value of maximum bending moment is less than the yield moment M_Y i.e. the beam is still fully elastic. The yield load which first produces yielding of the beam is given by the equation

$$P_Y = \frac{M_Y}{L} \quad \text{i.e.} \quad M_Y = P_Y \cdot L$$

The yield moment M_Y for an arbitrary solid cross section is given by

$$M_Y = Z \cdot \sigma_Y$$

where Z is the elastic modulus of the section.

When the maximum moment in the beam exceeds M_y , the beam will have two regions:

- (1) a region of fully elastic behaviour,
- (2) a region of elastic-plastic behaviour as shown in the curvature diagram of FIG.(3.1c).

In region (1) the elastic curvature ρ_e is given by

$$\rho_e = \frac{M}{EI} = \frac{P.z}{EI} \quad \text{for } z = 0 \quad \text{to } z = z_1$$

where z_1 is the length of the elastic region, and it can be found from the relation

$$z_1 = \frac{M_y}{P}$$

In region (2) the curvature ρ_p can be obtained by making use of the static equilibrium equation which states that the resultant of the stresses acting on the cross section is equal to the bending moment M :

$$\int \sigma.y.dA = M = P.z \quad \text{for } z = z_1 \quad \text{to } z = L.$$

Using this relation, it is possible to define e which is the distance from the neutral axis to the end of the elastic zone as a function of P and z i.e.

$$e = f(P, z)$$

The engineer's theory of bending states that the strain at distance y from the neutral surface is

$$\epsilon = \frac{y}{R} = \rho.y$$

Applying this relation to a point in the beam at the outer edge of the elastic zone i.e. at $y = e$ gives

$$\epsilon = \rho_p.e \quad \text{i.e.} \quad \rho_p = \frac{\epsilon}{e}$$

But strain ϵ at this point is $= \frac{\sigma_y}{E}$,

$$\text{then } \rho_p = \frac{\epsilon}{e} = \frac{\sigma_y}{E.e} = \frac{\sigma_y}{E.f(P, z)}$$

Applying the curvature area method gives

$$\begin{aligned}
 y_o &= \int_0^{z_1} \rho_e \cdot z \, dz + \int_{z_1}^L \rho_p \cdot z \, dz \\
 &= \int_0^{z_1} \frac{P \cdot z^2}{E \cdot I} \, dz + \int_{z_1}^L \frac{\sigma_Y \cdot z}{E \cdot f(P, z)} \, dz
 \end{aligned} \tag{3.1}$$

When the load increases further, the stress distribution over the cross section will be as in FIG.(3.2.b). At this stage a plastic hinge is formed and the beam will totally collapse if the load increases. FIG.(3.4) shows the load deflection diagram of a solid or thick walled beam.

3.1.2. INELASTIC BENDING OF THIN WALLED SECTION BEAMS

Both open and closed section thin walled beams are used in building vehicle structures. In bus bodies closed sections are most commonly used. A brief summary of the inelastic behaviour of rectangular steel tubes as studied by KECMAN Ref. (3) will be given.

A typical load-deflection diagram for a rectangular steel tube in bending is shown in FIG.(3.5). The linear part 0-1 represents the elastic behaviour of the beam. Next, due to local buckling near the fixed end, nonlinear behaviour takes place (segment 1-2 on the diagram). The load at point 2 represents the maximum strength of the beam. For standard metal tubes the length of segment 1-2 is almost negligible. The behaviour which follows point 2 depends upon the way in which the beam is loaded. If the loading allows decrease of the load while the displacement increases (e.g. the test rig shown in CHAPTER 1 FIG.(1.2.a), the relationship will follow curve 2-3. If the load does not decrease as the displacement increases (e.g. the test rig shown in FIG.(1.2.b), the beam will totally collapse, following line 2-5. Segment 3-4 of the diagram represents the beam unloading after a certain deflection.

3.2. INELASTIC BENDING OF A CANTILEVER SANDWICH BEAM

The ability of polyurethane foam to strongly unite to nearly all materials without joints provides an important advantage in sandwich beams. This ensures the stability of the faces against elastic buckling. This means that the faces behave plastically at the end of the elastic zone while the foam behaves elastically.

The yield load P_Y which induces yield stress in the faces, can be found using equation (2.11) derived in CHAPTER 2 which gives

$$P_Y = \frac{\sigma_Y \cdot \alpha}{\beta(\alpha L - \tanh \alpha L)} \tag{3.2}$$

The maximum elastic tip deflection of the beam can be found by substituting $P = P_Y$ in equation (2.12) derived in CHAPTER 2. This gives

$$y_O^Y = \frac{P_Y L^3}{3K} (1 - \beta\lambda) + \frac{P_Y \beta\lambda}{\alpha^3 K} (\alpha L - \tanh \alpha L) \quad (3.3)$$

To find the inelastic deflections, the curvature - area method demonstrated earlier in this chapter will be used.

In general, the stress strain diagram of mild steel can be approximated as two straight lines as shown in FIG.(3.6). The line O-1 represents the elastic behaviour and line 1-2 represents the plastic behaviour. The equation representing line 1-2 can be written in the form

$$\sigma_p = C_1 + C_2 \epsilon$$

where C_1 is the length of σ axis intersected by line 1-2 [see FIG.(3.6)]

$$\text{and} \quad C_2 = \tan \theta = \frac{\sigma_{\max} - \sigma_Y}{\epsilon_{\max} - \epsilon_Y}$$

The elastic curvature ρ_e can be written using equations (2.3) and (2.11) CHAPTER 2 as

$$\rho_e = \frac{1}{R} = \frac{P.z}{K} - \frac{P\beta\lambda}{\alpha K} \left(\alpha z - \frac{\sinh \alpha z}{\cosh \alpha L} \right) \quad (3.4)$$

The curvature in the inelastic zone ρ_p can be found by substituting $\sigma_f = \sigma_p$ in equation (2.3) CHAPTER 2. This gives

$$\rho_p = \frac{1}{R_p} = \frac{P.z}{K} - (C_1 + \epsilon C_2) \frac{\lambda}{K} \quad (3.5)$$

, but $\epsilon = \rho_p \frac{(h+t)}{2}$ at the edge of the elastic zone (i.e. at $z = z_1$)

Then substituting the value of ϵ in equation (3.5) gives:

$$\rho_p = 2 \frac{(P.z - C_1 \lambda)}{K + C_2 \lambda (h+t)} \quad (3.6)$$

The inelastic deflection can be found by substituting the values of ρ_e and ρ_p in equation (3.1) which gives

$$y_O^p = \int_0^{z_1} \frac{Pz^2}{K} (1 - \beta\lambda) dz + \int_0^{z_1} \frac{P\beta\lambda z \sinh \alpha z}{\alpha K \cosh \alpha L} dz +$$

$$+ \int_{z_1}^L \frac{2Pz^2}{K+C_2 \lambda (h+t)} dz - \int_{z_1}^L \frac{2 C_1 \lambda z}{K+C_2 \lambda (h+t)} dz$$

Evaluating these integrals gives

$$y_O^p = \frac{P(1-\beta\lambda)}{3K} z_1^3 + \frac{P\beta\lambda}{\alpha^3 K} \left[\frac{\alpha z_1 \cosh \alpha z_1 - \sinh \alpha z_1}{\cosh \alpha L} \right]$$

$$+ \frac{2P(L^3 - z_1^3)}{3[2K+C_2 \lambda (h+t)]} - \frac{C_1 \lambda (L^2 - z_1^2)}{[2K+C_2 \lambda (h+t)]} \quad (3.7)$$

where $z_1 = \frac{P_y}{P} \cdot L$

Experiments showed that failure occurred due to fracture of the foam. This means that, substituting the value of the fracture stress of the foam $\sigma_{FO}^{\max}$ into the moment equilibrium equation gives the maximum strength of the beam.

The moment equilibrium at fracture is given by

$$\sigma_{FO}^{\max} \cdot Z = P_{\max} \cdot L - \sigma_p \cdot \lambda.$$

where Z is the elastic modulus of the core $\approx \frac{bh^2}{6}$

$$\text{i.e. } P_{\max} = \sigma_{FO}^{\max} \frac{bh^2}{6L} + \frac{\sigma_p \lambda}{L} \quad (3.8)$$

The value of σ_p is a certain value of σ_p on line 1-2 in the stress-strain diagram shown in FIG.(3.6) which corresponds to the maximum strain of the foam. To be safe in finding the maximum strength of the beam the value of σ_p will be taken as σ_y because the maximum strain of the foam can change significantly due to strain rate change.

Thus, equation (3.8) becomes

$$P_{\max} = \sigma_{FO}^{\max} \frac{bh^2}{6} + \frac{\sigma_Y \lambda}{L} \quad (3.9)$$

3.3. INELASTIC BENDING OF A CANTILEVER COMPOSITE BEAM

The similarity between composite beam and sandwich beam make it possible to write directly the formulas which determines the inelastic behaviour of the composite beams. The differences in the formulas for these beams in fact, come from the differences in the constants, i.e. $h, b, \alpha, \beta, \lambda, K$ etc. Thus, there is no point in repeating the derivation of these equations again.

So, for a composite beam reinforced with steel thin walled section:-

the yield load P_Y is given by

$$P_Y = \frac{\sigma_Y \alpha_1}{\beta_1 (\alpha_1 L - \tanh \alpha_1 L)}$$

the maximum elastic tip deflection y_O^Y is given by

$$y_O^Y = \frac{P_Y L^3}{3K_1} (1 - \beta_1 \lambda_1) + \frac{P_Y \beta_1 \lambda_1}{\alpha_1^3 K_1} (\alpha_1 L - \tanh \alpha_1 L),$$

the inelastic deflection y_O^P is given by

$$y_O^P = \frac{P z_1^3}{3K_1} (1 - \beta_1 \lambda_1) + \frac{P \beta_1 \lambda_1}{\alpha_1^3 K_1} \left(\frac{\alpha_1 z_1 \cosh \alpha_1 z_1 - \sinh \alpha_1 z_1}{\cosh \alpha_1 L} \right) + \frac{2 P (L^3 - z_1^3)}{3 [2K_1 + C_2 \lambda_1 (h+t)]} - \frac{C_1 \lambda_1 (L^2 - z_1^2)}{[2 K_1 + C_2 \lambda_1 (h+t)]}$$

and finally the maximum strength $P_{\max}$ is given by

$$P_{\max} = \sigma_{FO}^{\max} \frac{b_1 h_1^2}{6L} + \frac{\sigma_Y \lambda_1}{L}$$

FIG.(3.7) shows the general theoretical load-deflection diagram of a cantilever composite (sandwich) beam using the formulae derived in this thesis. (2.12,3.2,3.3,3.7,3.8)

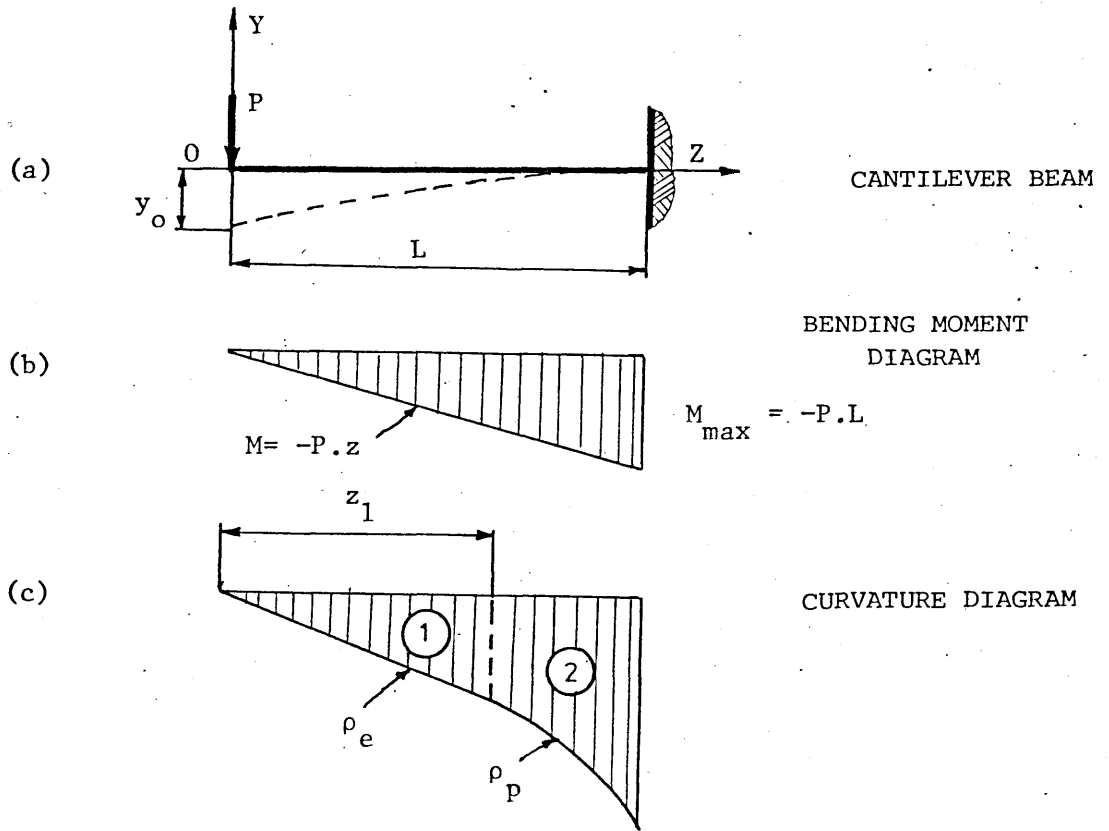


FIG. (3.1) INELASTIC BEHAVIOUR OF A BEAM OF ARBITRARY SOLID CROSS SECTION.

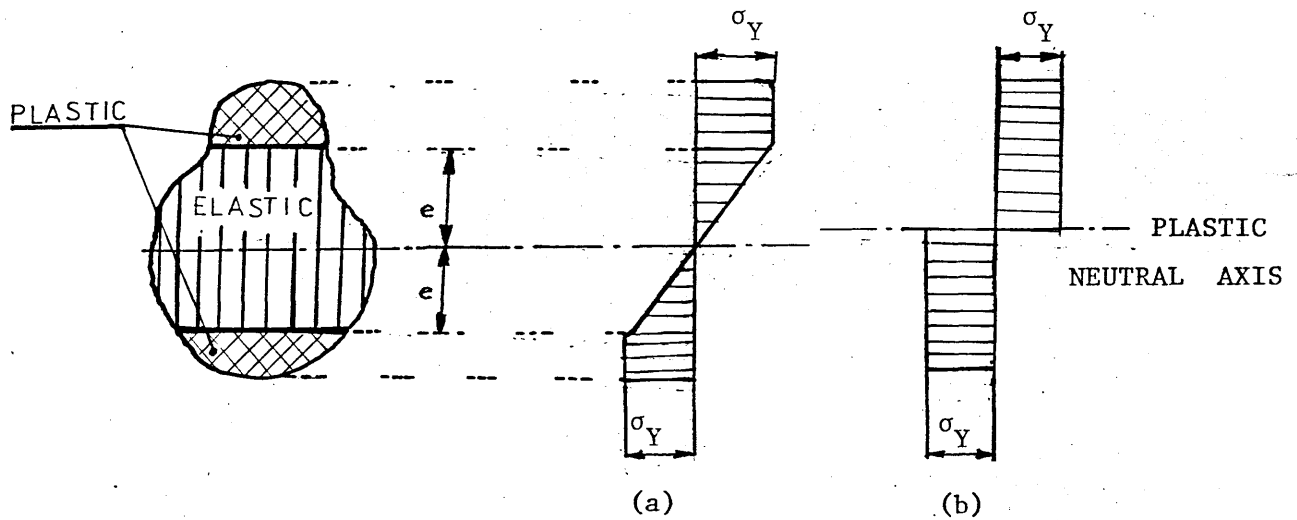


FIG. (3.2) STRESS DISTRIBUTION OF AN ARBITRARY SOLID SECTION

(a) IN ELASTIC-PLASTIC STATE

(b) IN FULLY PLASTIC STATE.

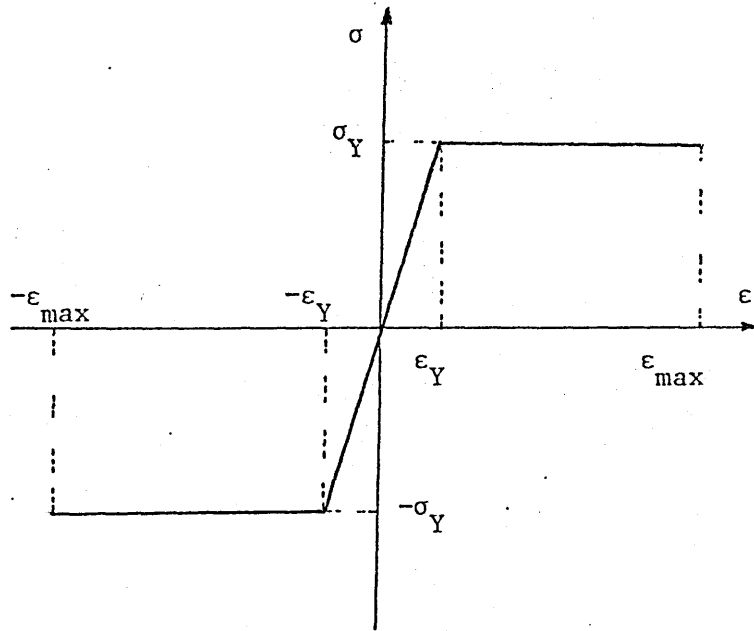


FIG. (3.3) STRESS STRAIN DIAGRAM FOR AN ELASTIC PLASTIC MATERIAL .

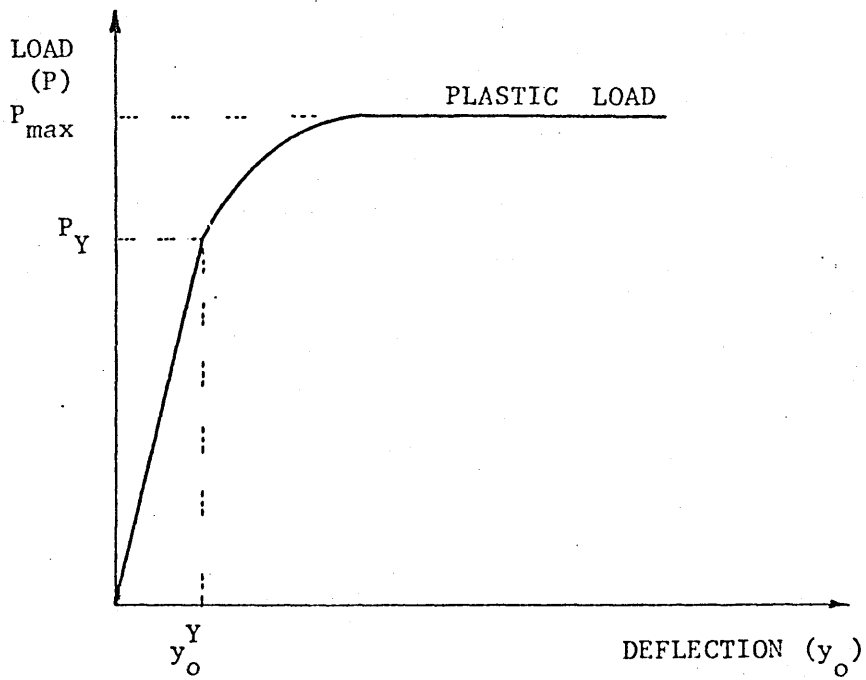


FIG. (3.4) LOAD DEFLECTION DIAGRAM OF THE CANTILEVER BEAM SHOWN IN FIG. (3.1).

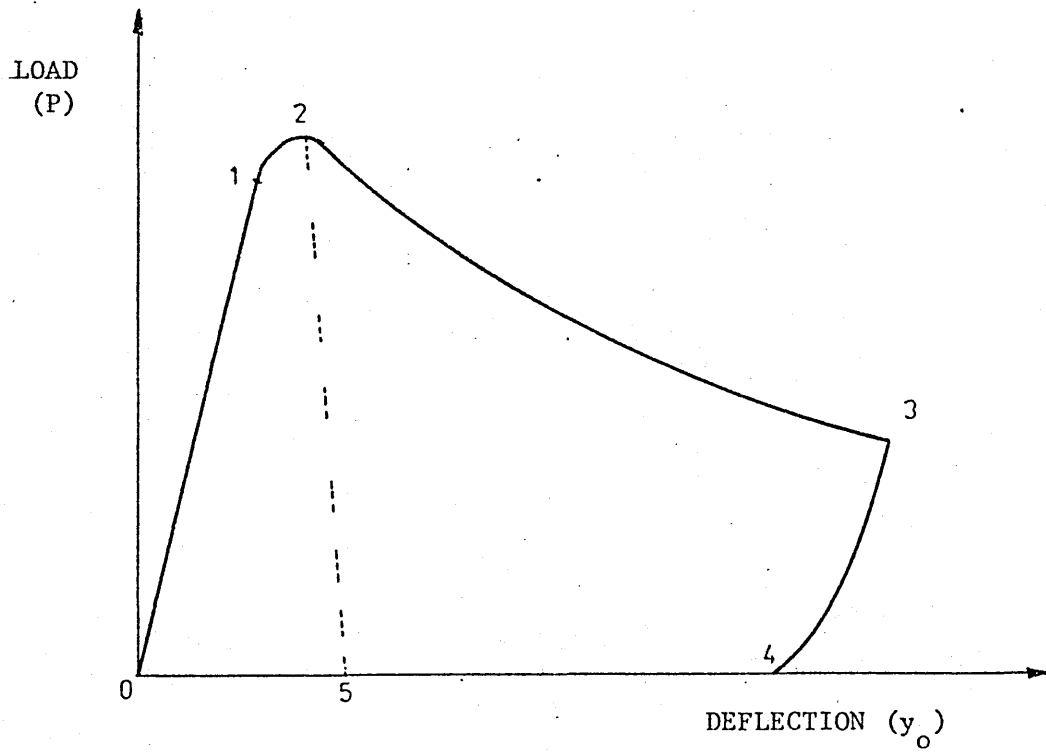


FIG. (3.5) LOAD DEFLECTION DIAGRAM OF RECTANGULAR STEEL TUBE .

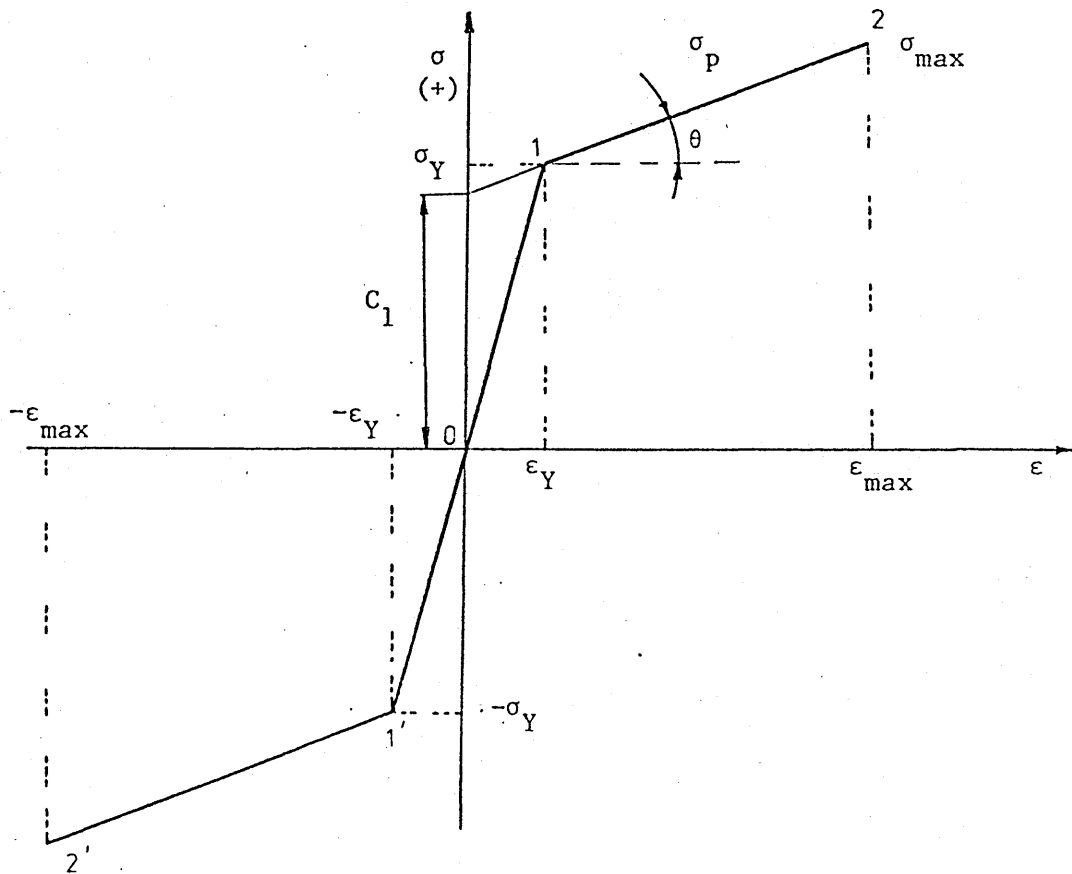


FIG. (3.6) GENERAL SHAPE OF STRESS STRAIN DIAGRAM FOR MILD STEEL .

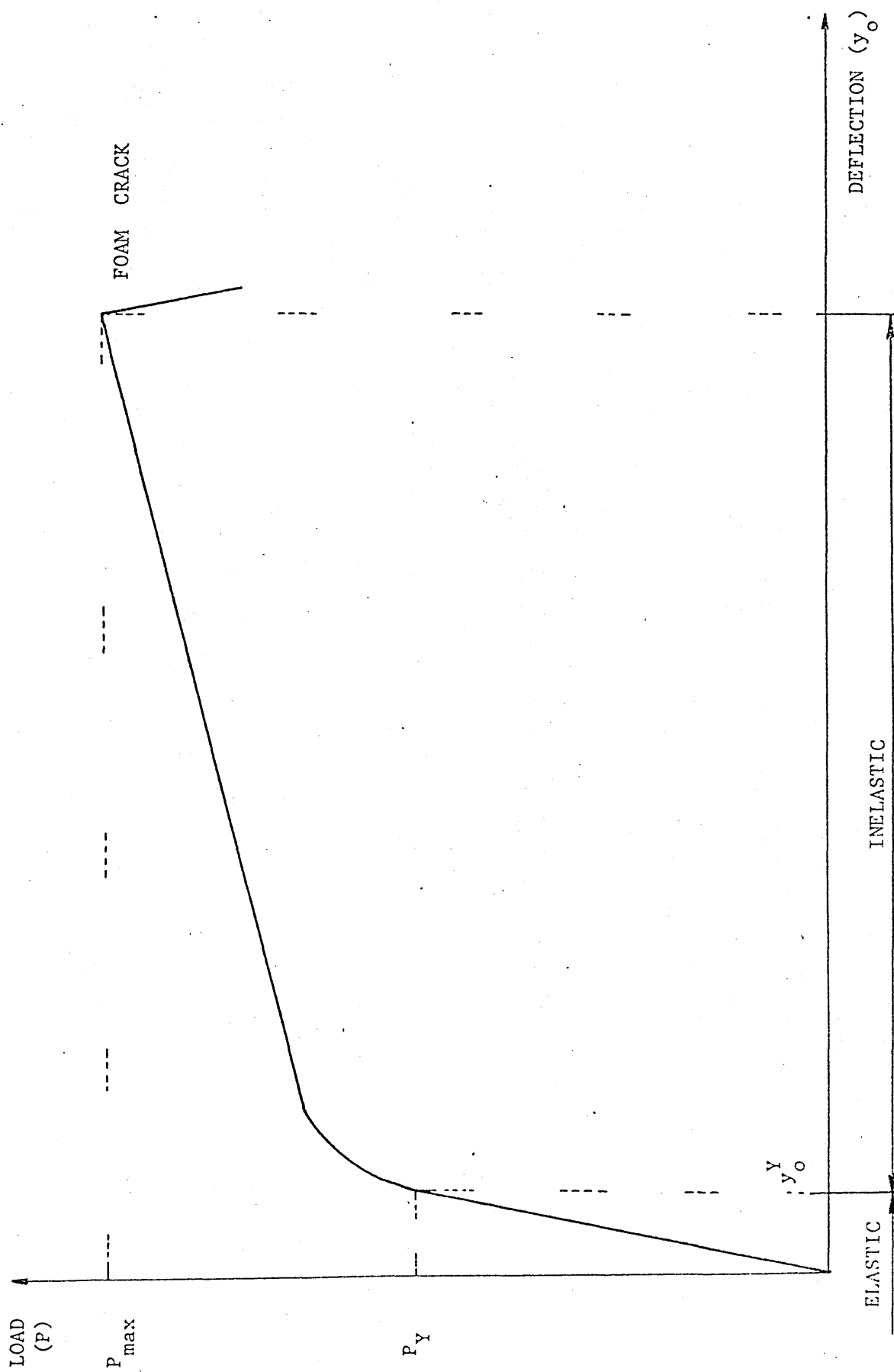


FIG. (3.7) THEORETICAL LOAD DEFLECTION DIAGRAM OF A CANTILEVER COMPOSITE BEAM.

CHAPTER 4

EXPERIMENTAL VERIFICATION OF THE THEORY

4.1. INTRODUCTION

In order to verify the theory developed in previous chapters of this thesis a program of testing beams was undertaken. The external dimensions of the composite beams used to construct the plastic bus are almost standard. Also, changing the external dimensions adds complications to the mould used for producing the test beams. Furthermore, changing these dimensions within a narrow range which would be the case if the beams are to function as bus ring components loaded in uniaxial bending, would not change significantly their load capacity. Thus, there is no reason to change the external dimensions. This means that the most suitable parameters to vary are shape and dimensions of the reinforcement.

The following categories of beams were chosen for testing to verify the theory:

- (1) Sandwich
- (2) Composite reinforced with two longitudinal steel strips (used as faces for the sandwich beam) positioned at distances of 4, 7 and 11mm from the beam surfaces.
- (3) Composite reinforced with I section steel.

The theory developed in this thesis requires exact information on the properties of the constituents used to construct the composite beams i.e. steel and foam. Therefore, the methods used for determination of these properties will be given.

4.2. STEEL PROPERTIES

The required properties of the steel are:

- (a) Young's modulus E_{ST}
- (b) Yield stress σ_y
- (c) Plastic behaviour i.e. value of the constants C_1 and C_2 introduced in CHAPTER 3.

This simply means that we need to know the stress strain diagram of the steel. Thus, a tensile test is required.

The reinforcement beam were cut from steel sheet so that metal grains were aligned with the beam axis. Thus, it was necessary to cut the tensile test samples with the same grain direction.

Two steel sheets of thickness 1.6 and 0.72mm were used to make all the reinforcements. The 1.6mm sheet was used for the sandwich beam faces and the strips of the composite beams. The 0.72mm was used to make the I section reinforcement for the other composite beams. Three standard tensile test samples from each sheet were tested.

FIG.(4.1) and FIG.(4.2) show the stress strain diagrams of the samples tested and the approximated diagram in the form of two straight lines. The test diagrams of the three samples tested were almost the same, thus, only one diagram for each material is given here. The values of C_1 and C_2 are also given in the diagram.

4.3. FOAM PROPERTIES

The trade name of the foam used to produce the test beams is BAYDUR. This foam has a density range within 400 and 700kg/m³. To make such moulding, a liquid charge has to be injected into the mould using a special injecting machine. This machine pumps the liquid at a constant rate, so that the density of the foam varies according to the duration of delivery of the charge. This liquid will then expand and fill the mould with foam (foaming process). The moulding liquid was prepared by:

- (1) Mixing of 100 parts by weight (P.B.W.) of a ready-formulated POLYOL (called Baydur 611OB) with 8 P.B.W. of ARCTON blowing agent (called R11), to form component A.
- (2) Mixing 1 P.B.W. of component A with 1.3 of component B which is modified 4,4 - diphenylmethane - diisocyanate (called Desmodur 44P90).

The component A was mixed outside the machine and then mixed with B inside the machine.

The polyurethane foam has a high density skin and a lower density cellular core. This property makes the mechanical properties change according to the product shape. The mechanical properties of the foam required by the theory are the Young's modulus, the tensile strength and the shear modulus.

PROGELHOF and EILERS Ref. (11) derived a formula to find the so called apparent modulus of a structural foam member. This formula is based on a knowledge of the densities and Young's moduli of both the skin and the core of the beam. Thus, rectangular section strips with 2mm thickness, 5mm width and 300mm length were cut from the skin and the core. These strips were tested using a standard tensile test machine (low range) to determine the Young's moduli. The densities were also obtained experimentally. The apparent modulus using the formula of Ref. (11) is 412N/mm².

The value of the Young's modulus of the foam obtained by this formula is based on tensile test data, which in fact is different from the loading condition in the composite beam which is in bending. A foam cantilever beam has the same loading condition as a composite cantilever. Therefore, the value of Young's modulus of the foam obtained by applying the engineer's theory of bending deflection to a foam cantilever gives more accurate data. Thus, the value obtained using PROGELHOF formula was rejected and the value obtained from cantilever test was used (490 N/mm²).

Beam failure is almost always due to cracking of the foam skin. This means that the tensile strength of the skin is the parameter required for obtaining the maximum strength of the beam. Thus, tensile test samples were cut from the skin of the beam according to ASTM

standard D638-72 (samples type IV) Ref. (26). These samples were tested using a standard tensile test machine. FIG.(4.3) shows a typical stress strain diagram from the samples tested. This diagram shows that $\sigma_{FO}^{max} = 30\text{N/mm}^2$ with 2.5% error. This test was carried out using a constant strain rate.

The core is the part of the foam which resists the shear stresses. Thus, rectangular torsional test samples were cut from the core according to ASTM standard No.1042-72 Ref. (26). These samples were tested on a special test rig shown in FIG.(4.4) designed according to the same standard. This test showed that the shear modulus of the foam is $= 97\text{N/mm}^2$. It must be borne in mind that the general relation $\{G=E/2(1+\nu)\}$ does not apply here because the E value is for the whole cross section while the G value is for the core only.

4.4. TEST RESULTS

Three samples from each kind of beam were tested using the incremental increasing displacement rig shown in FIG.(1.2.a) This was to ensure the repeatability of the test results. One beam from the composite reinforced with I section and one from the composite reinforced with two strips positioned at 7mm from beam surface were tested using the incremental increasing load rig shown in FIG.(1.2.b). This was to prove that the composite beam behaves adequately before cracking, regardless the method of loading.

FIG's. (4.5), (4.6), (4.7), (4.8) and (4.9) show the load deflection diagrams of the beams tested. The numbers between double brackets e.g. ((4)), ((7)) represent the distance of the reinforcement strips from the beam surface. The dashed lines connecting the points represent the experimental results. The continuous lines represent the theoretical curves. The dash and dot curves in FIG's. (4.7) and (4.9) represent the load deflection diagram of the incremental increasing load test rig, using a plotter. So, the loading steps and the creep phenomena were recorded on the diagram.

FIG's. (4.10), (4.11), (4.12), (4.13) and (4.14) show photographs of the test beams after test.

4.5. COMPARISON BETWEEN THE THEORETICAL AND EXPERIMENTAL RESULTS

The theory overestimates the elastic stiffness (P/y_0) of the beams, but it still gives better results than the engineer's theory of bending as shown in table (4.1).

The theory gives good agreement with experimental results for prediction of the yield load P_y as well as the inelastic bending behaviour. The theory underestimates the maximum strength P_{max} and the corresponding deflection. This is because the theory assumes the value of $\sigma_p = \sigma_y$ in the formula of the maximum strength P_{max} . However, it is an advantage from the safety point of view to underestimate the maximum strength of the beam. The theory overestimates the maximum strength of the composite beam reinforced with strips at a distance of 11mm from the surface. This particular beam had a failure mode different from the others. The beam collapsed due to slip of the reinforcement near to the free end. This means that the interfacial

shear exceeded the bonding strength of the steel and the foam. The formula of the interfacial shear (equation (2.15)) shows the logical agreement with this failure mode. This formula shows that the interfacial shear τ_f increases when the constant β_1 increases. But β_1 increases when h decreases, i.e. τ_f increases when the reinforcement is closer to the beam centroidal axis.

Table (4.2) shows the comparison between two of the composite beams and the optimum equivalent rectangular steel tube from the point of view of safety criteria. This comparison is made with the weakest beam tested from each type. The composite beam ((4)) showed that it has lower mass/unit length, higher maximum strength and lower remaining strength than the equivalent steel tube. The composite beam ((I)) is slightly heavier (only by 3%) but it has higher maximum and remaining strength.

4.6. OPTIMIZATION OF THE REINFORCEMENT FOR UNIAXIAL BENDING

The area under the load deflection curve represents the energy absorbed statically by the beam after a certain deformation. The main parameters limiting the extent of this area are:

- a) The elastic stiffness (P/y_0) of the beam.
- b) The yield load P_y
- c) The slope of the inelastic bending curve.
- d) The maximum strength P_{max} .

The elastic stiffness does not effect significantly the energy absorption of the beam. This is because the energy absorbed in the elastic range is small compared with the total energy absorbed.

The yield load P_y has a large effect on the energy absorption of the beam. Two main factors affect the magnitude of this load:

- (1) The yield stress σ_y of steel reinforcement.
- (2) The distance of the flanges of the reinforcement from the beam centroid axis.

The higher the material yield stress the higher is the value of P_y . This is clear from the formula of P_y (equation (3.2)) as P_y is directly proportional to σ_y . The experimental results also support this argument. In both composite ((I)) and composite ((7)), the reinforcement is at 7mm from the surface. However, the yield stress of the reinforcement in composite ((I)) is 273N/mm^2 whereas that in composite beam ((7)) is 180N/mm^2 . This makes the yield load for the composite beam ((I)) higher than that for the composite beam ((7)) (experimentally $\approx 1300\text{N}$ and 670N respectively).

Also the greater the distance between the flanges the higher is the yield load. This fact is very clear from the comparison of the load deflection diagrams of the sandwich beam and the composite beam ((11)) FIG's. (4.5) and (4.8). Both these beams have the same reinforcement, but the distance between the reinforcement is different. The sandwich beam has a higher yield load than the composite ((11)) because it has larger distance between the flanges.

Two main factors affect the slope of the inelastic behaviour curve:

- (1) The stiffness of the foam material.
- (2) The strain hardening of the steel (constant C_2 introduced in CHAPTER 3)

The more stiff the foam the greater is the slope of the inelastic curve of the composite beam. Strain hardening of the steel also has a similar effect. The higher the value of constant C_2 the greater is the slope of the inelastic bending curve. The experimental as well as the theoretical load deflection curves show that the sandwich and composite beams ((4)), ((7)) and ((11)) have the same slope in the inelastic bending zone because they have the same material properties. The composite beam ((I)) has lower slope than the other beams because the value of C_2 is lower for this material (see FIG's. (4.1) and (4.2)).

The maximum strength formula (3.8) has two terms. The first term deals with the maximum load of the foam which is almost the same for all the beams tested. The value of the second term is directly proportional to the yield strength of the reinforcement σ_y and the reinforcement constant λ . The latter is also mainly dependent on the distance between the flanges of the reinforcement. Thus, the higher the yield stress σ_y and the greater the distance between the reinforcement the higher is the maximum strength of the beam.

The energy absorption of the beam is not the only important parameter to be considered. The second important parameter is the remaining strength. The theory developed in this thesis does not deal with this parameter. This is because the beams with webless reinforcement behave in an unstable manner after foam cracking due to separation of steel and foam. Beams containing web reinforcement ((I)), ((C)) behave in a stable manner. This is because the web restrains the flanges from movement after the foam cracking. The value of the remaining strength in this case can be considered approximately equal to the yield load P_y . In fact, the actual value of the remaining strength is higher than P_y because the foam cracks on the tension side of the beam while the compression side is still stable. But to be safe, the remaining strength will be considered = P_y . This explains the importance of a web in the reinforcement for stable behaviour after cracking and consequently for improvement of the maximum strength.

It is clear now that an optimum shape of reinforcement for uniaxial bending is an I section with the flanges positioned on the beam outer surface. Positioning the flanges on the outer surface is a disadvantage due to the poor corrosion resistance of mild steel. But, positioning the flanges inside the section at a small distance (2:3mm) from the beam surface does not significantly decrease the beam loading capacity and improves the corrosion resistance. The thickness of the web should be minimal i.e. just sufficient to restrain the flanges movement after foam cracking. The most convenient thickness of the web from the point of view of reinforcement manufacture for the present work is half the thickness of the flanges. This is because the reinforcement is made by folding steel sheet into I shape. Thus, the section shown in FIG.(4.15) is the optimum composite beam section for uniaxial bending.

FIG. (4.16) shows the theoretical and experimental load deflection diagram of this optimum composite section. The reinforcement were folded from a steel sheet with 0.72mm thickness. The properties of the steel and the foam of this composite section were obtained by the methods explained previously in this chapter. These are:

$$\frac{\text{Foam}}{E_{FO}} = 640 \text{ N/mm}^2$$

$$G = 139 \text{ N/mm}^2$$

$$\sigma_{FO}^{\max} = 30 \text{ N/mm}^2$$

$$\frac{\text{Steel}}{\sigma_y} = 230 \text{ N/mm}^2 \approx C_1$$

$$\sigma_u = 320 \text{ N/mm}^2$$

$$C_2 = 724$$

The theory again showed good agreement with the experimental results except for the maximum load capacity $P_{\max}$. The reason for this is that the foam properties change significantly from beam to beam particularly $\sigma_{FO}^{\max}$ for the optimized beams (these were made by a different factory).

The comparison between this optimum composite beam and the optimum equivalent rectangular steel tube is given in Table 4.3. This comparison was made using the weakest of the two composite beams tested. The steel tube has 6% higher strength than the optimum composite section. However, the composite section has higher remaining strength and lower weight per unit length. The optimum steel tube has dimensions of 45 mm width, 76.2 mm breadth and 1.49 mm thickness.

	Type of Beam				
	SANDWICH	COMPOSITE ((4))	COMPOSITE ((7))	COMPOSITE ((11))	COMPOSITE ((I))
EXPERIMENTAL	55	45	42	33	63
THESIS THEORY	112	106	85	60	84
ENGINEERS TH.	176	132	103	70	108

TABLE (4.1) ELASTIC STIFFNESS OF BEAMS (P/y_o) (N/mm)

Beam	MASS/UNIT LENGTH (%)	MAX. STRENGTH (N)	REMAINING STRENGTH (N)
COMPOSITE ((4))	100	2000	530
EQUIVALENT STEEL	116	1815	890
COMPOSITE ((I))	100	2000	1340
EQUIVALENT STEEL	97	1888	1097

TABLE (4.2) COMPARISON BETWEEN COMPOSITE BEAMS AND EQUIVALENT STEEL TUBES FROM THE SAFETY POINT OF VIEW.

Beam	MAX. STRENGTH (%)	REMAINING STRENGTH (%)	MASS/UNIT LENGTH (%)
OPTIMUM COMPOSITE	100	100	100
EQUIVALENT OPTIMUM STEEL TUBE	106	63	112

TABLE (4.3) COMPARISON BETWEEN OPTIMUM COMPOSITE BEAM AND THE EQUIVALENT ENERGY ABSORBING OPTIMUM STEEL RECTANGULAR TUBE.

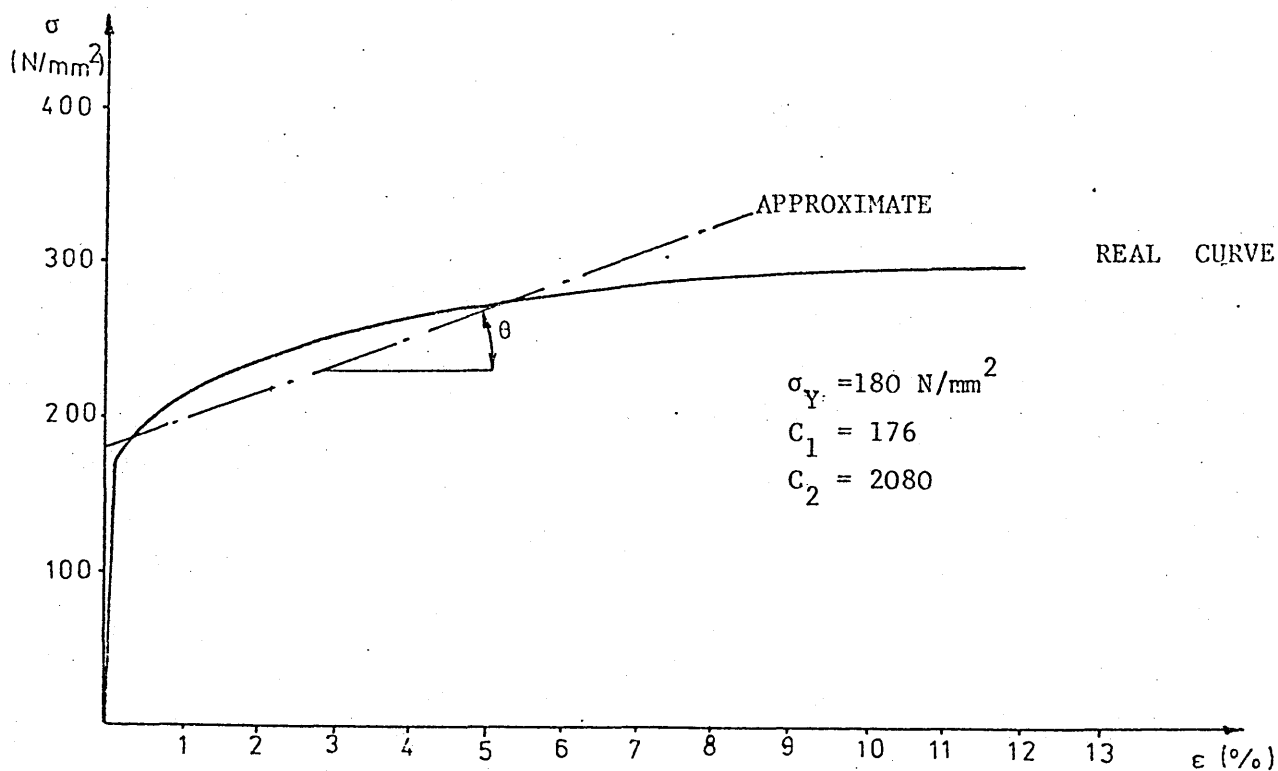


FIG.(4.1) STRESS STRAIN DIAGRAM OF THE SHEET OF 1.6 mm THICKNESS.

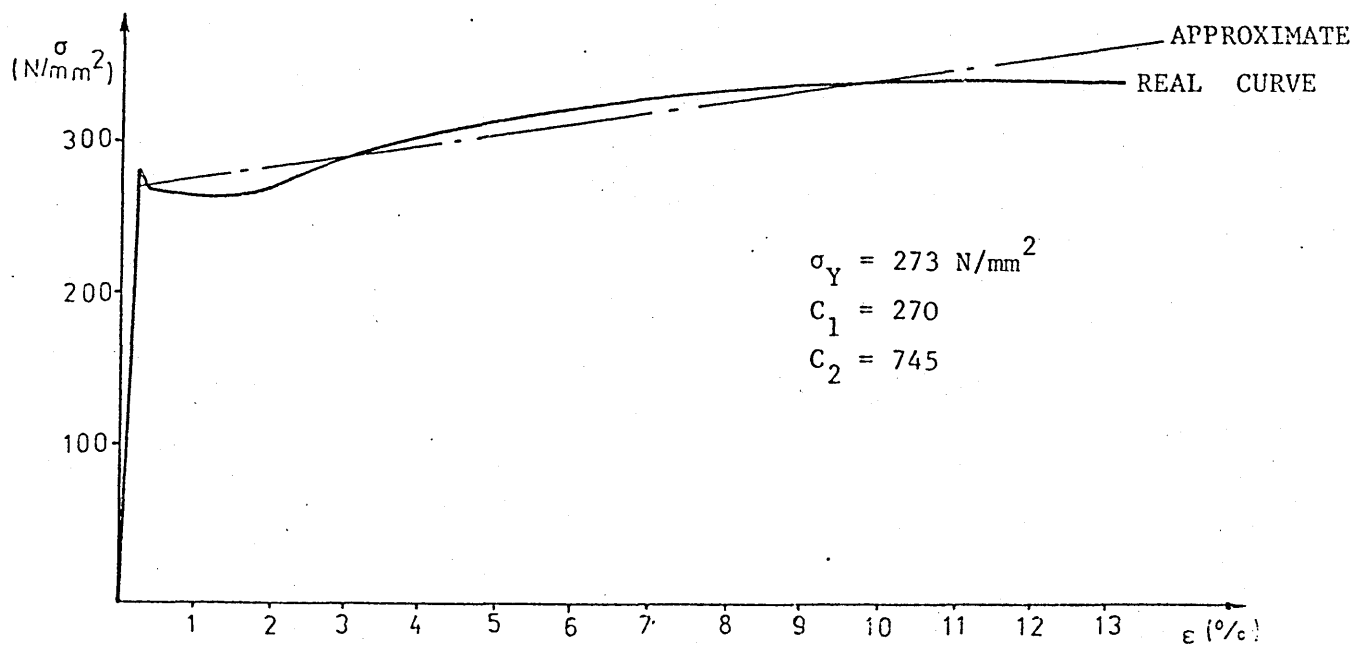


FIG.(4.2) STRESS STRAIN DIAGRAM OF THE SHEET OF 0.72 mm THICKNESS.

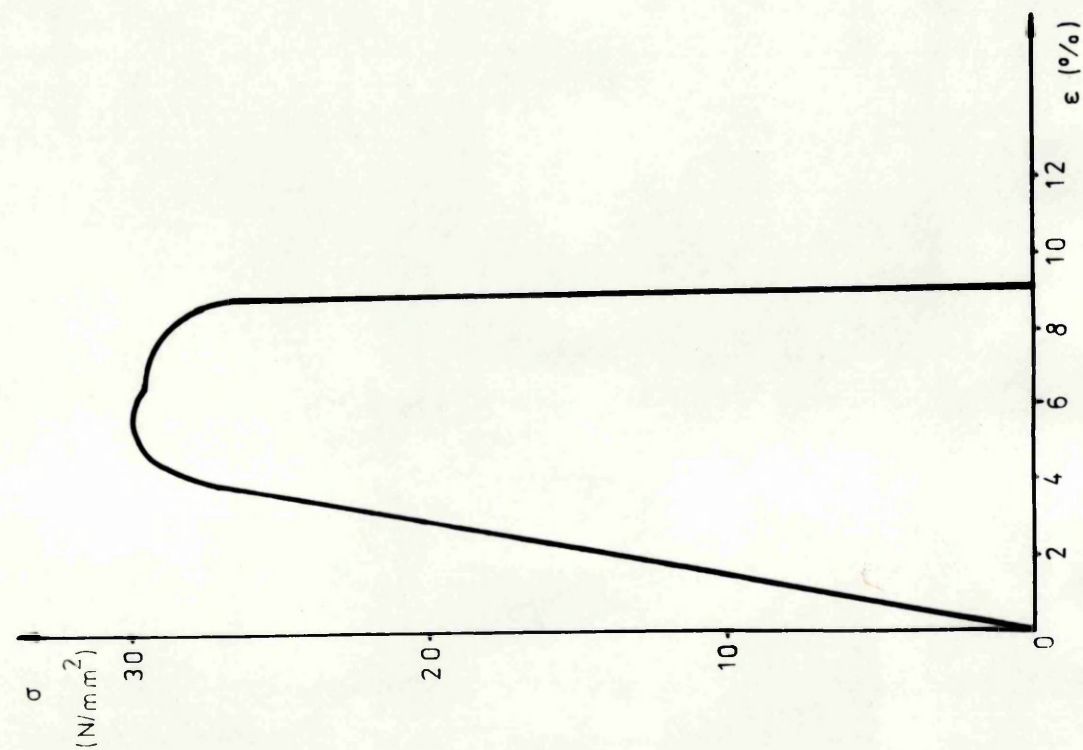


FIG.(4.3) STRESS STRAIN DIAGRAM OF
FOAM SKIN MATERIAL.

CONSTANT STRAIN RATE

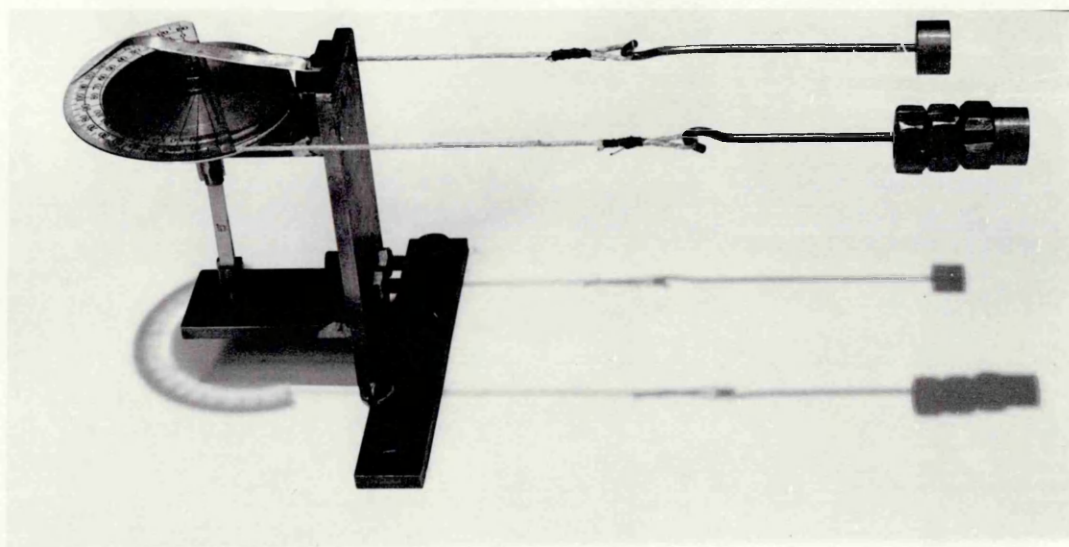
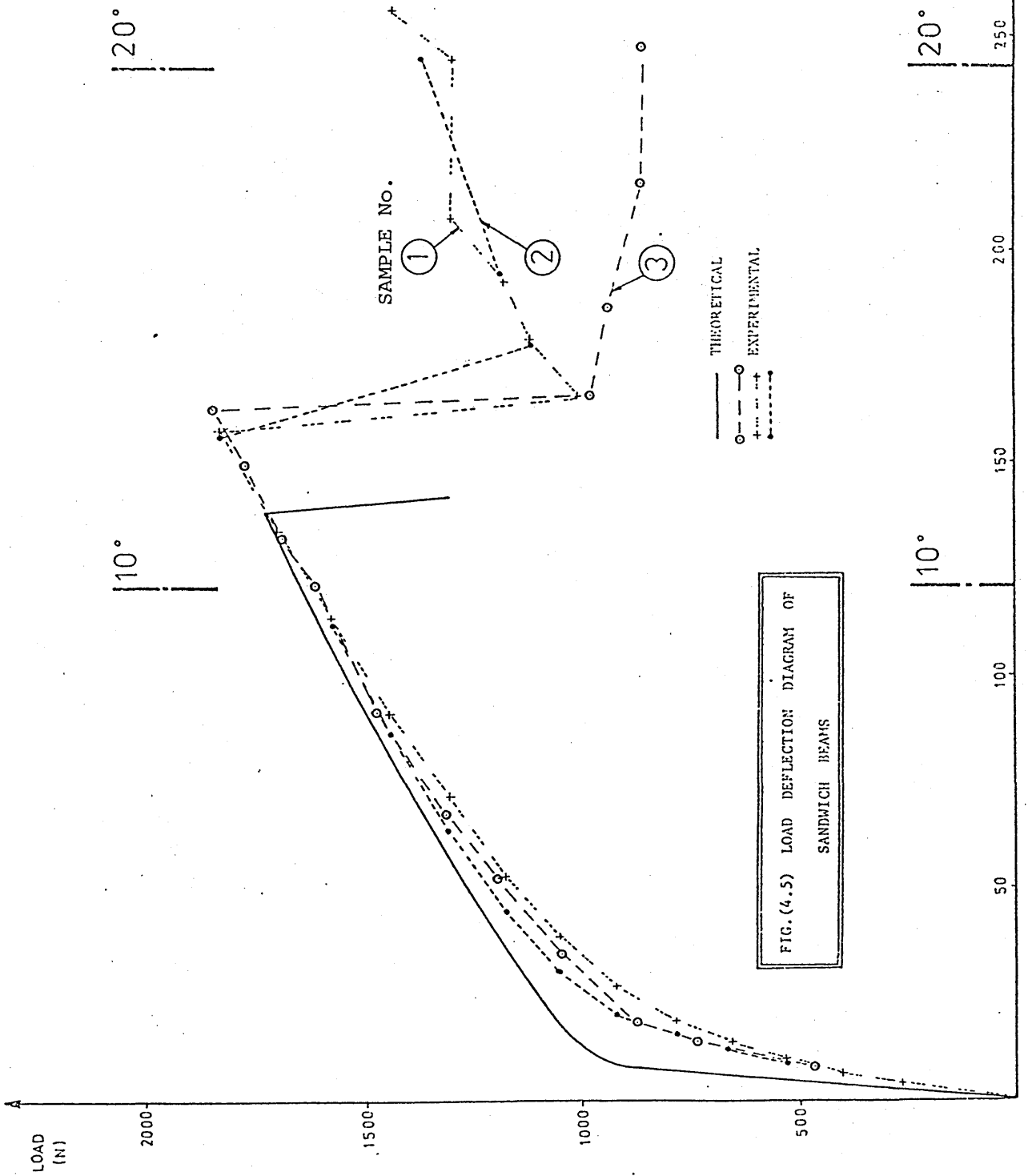
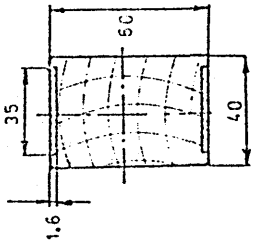
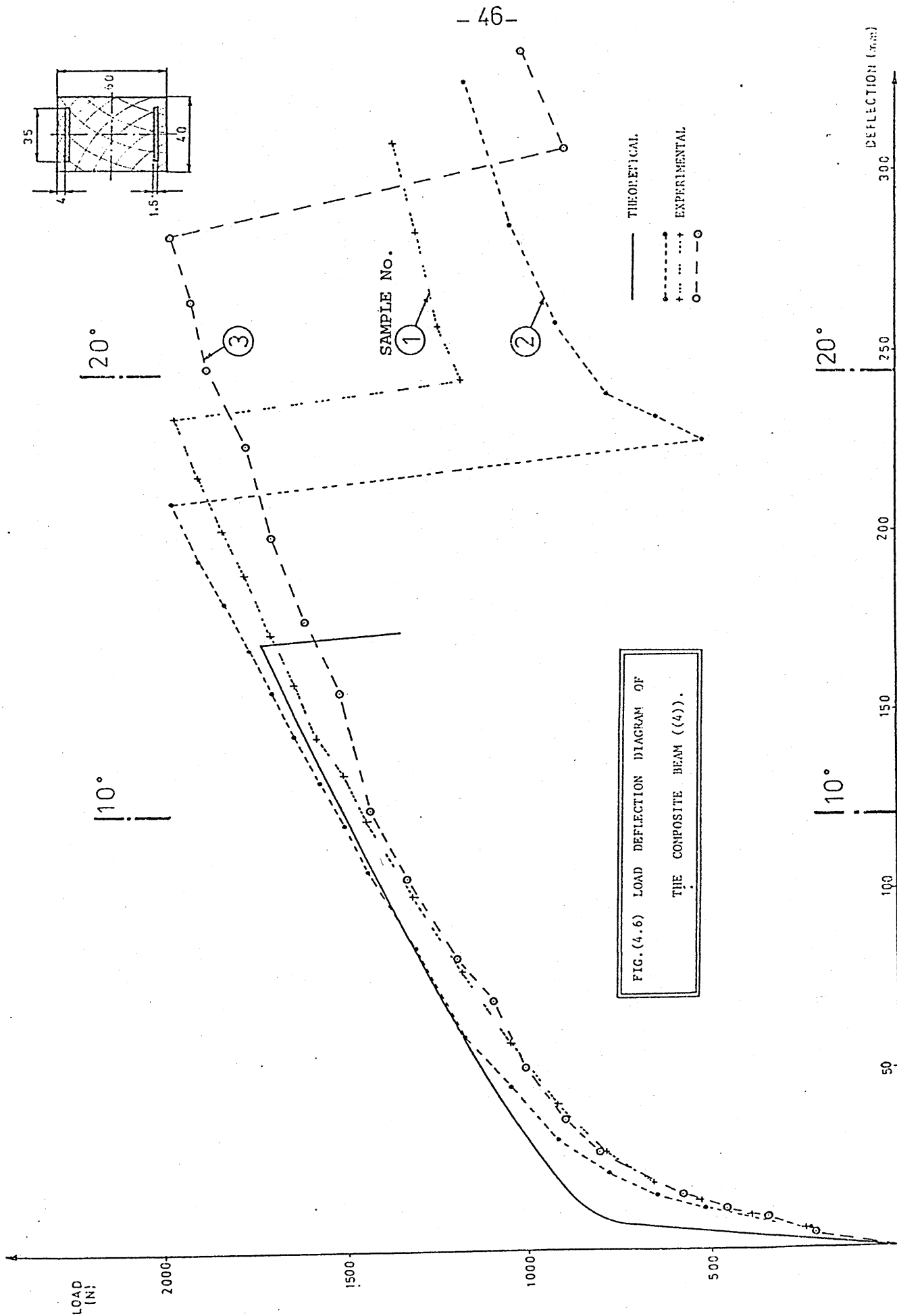
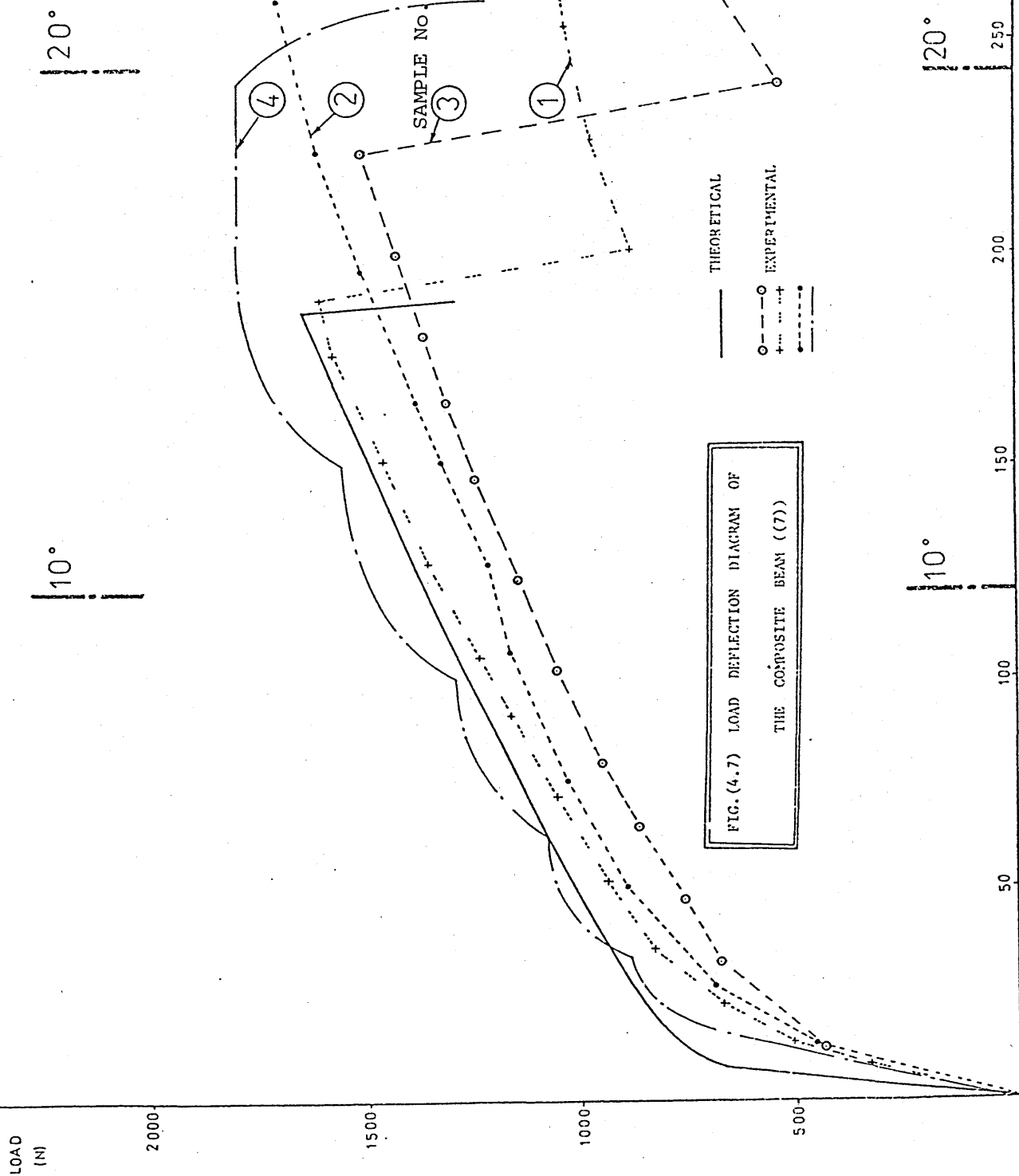
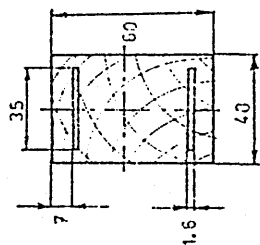
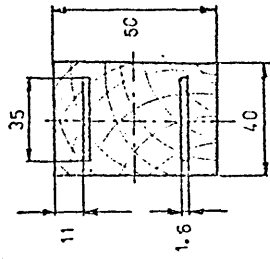


FIG. (4.4) TEST RIG TO DETERMINE THE SHEAR
MODULUS OF THE FOAM.





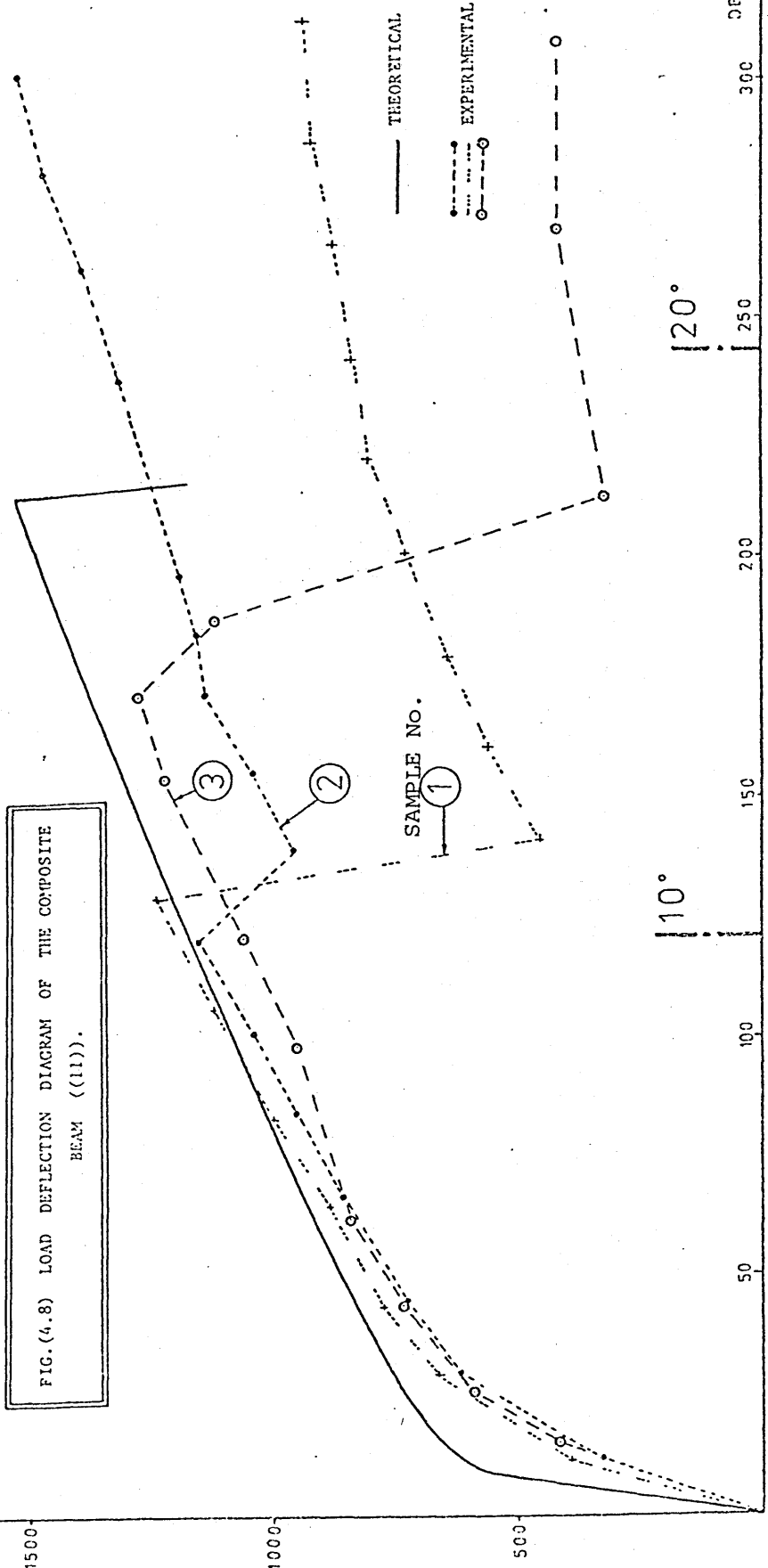


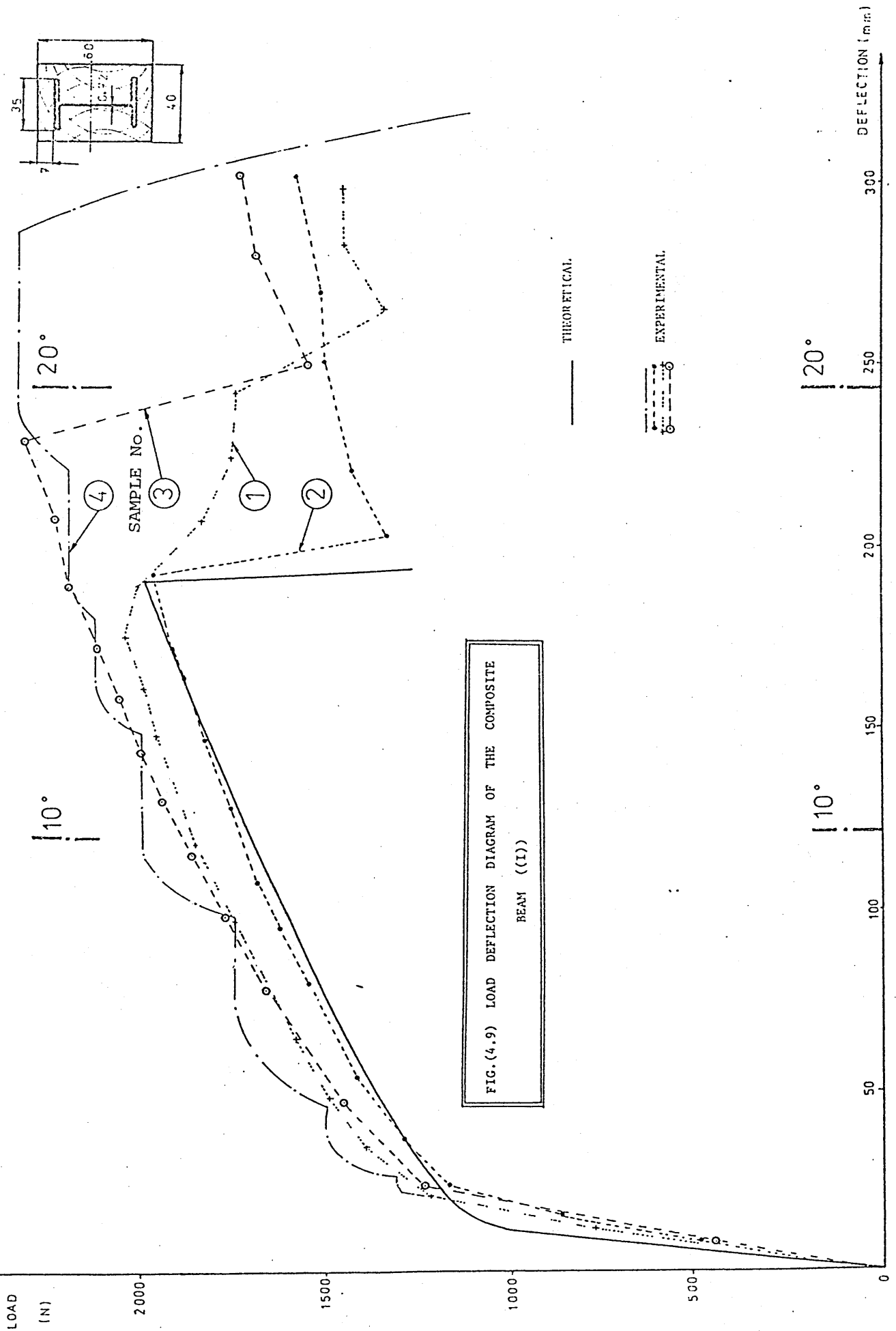
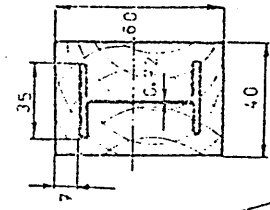


10°

20°

FIG. (4.8) LOAD DEFLECTION DIAGRAM OF THE COMPOSITE BEAM ((11)).





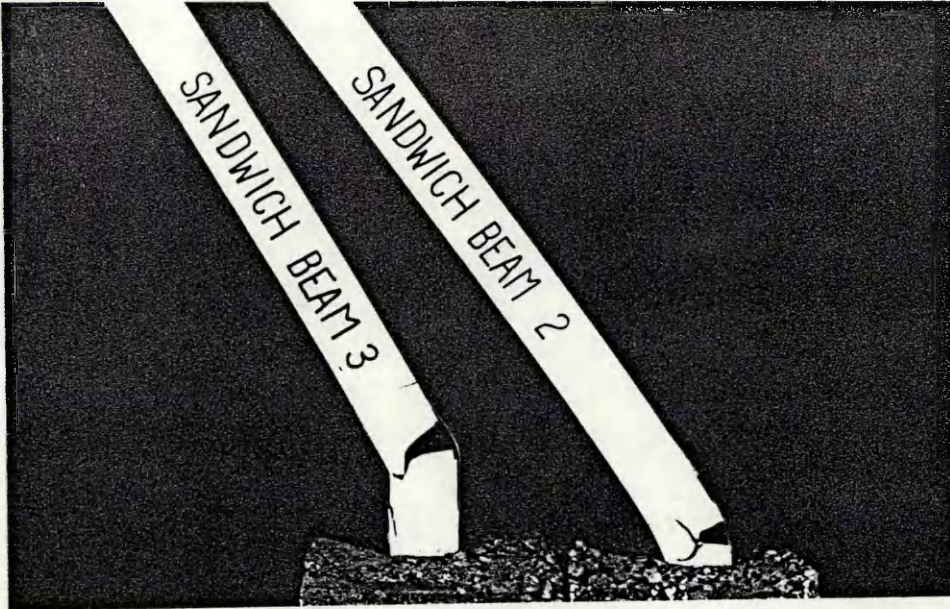


FIG. (4.10) SANDWICH BEAMS AFTER TEST.

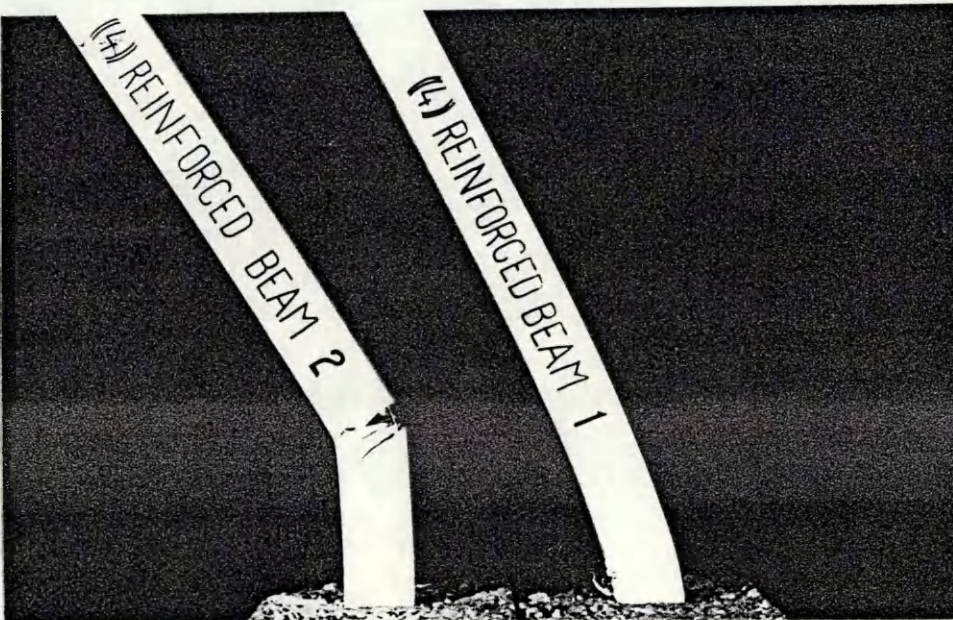


FIG. (4.11) COMPOSITE BEAM ((4)) AFTER TEST.

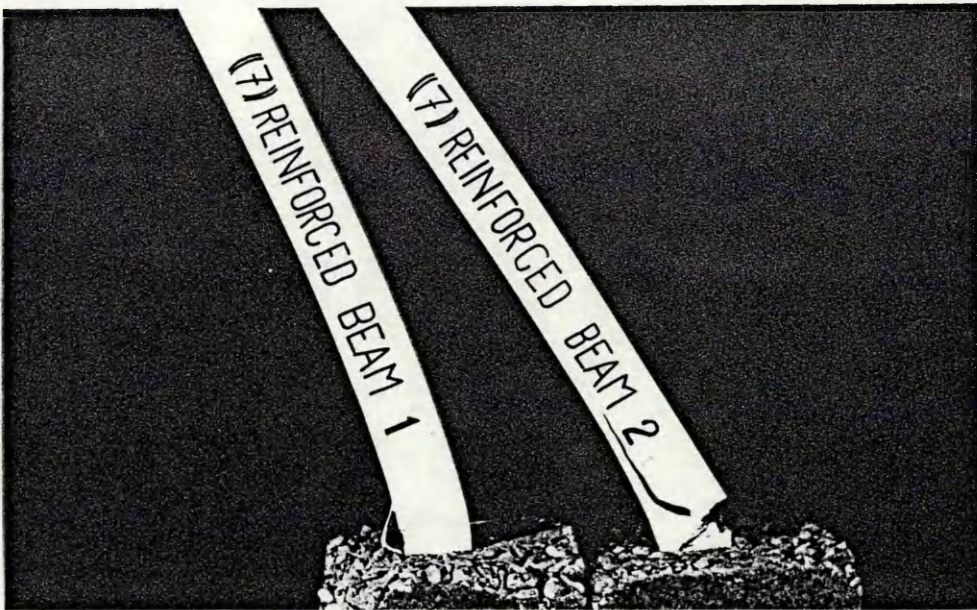


FIG.(4.12) COMPOSITE BEAM ((7)) AFTER TEST .



FIG. (4.13) COMPOSITE BEAM ((11)) AFTER TEST.

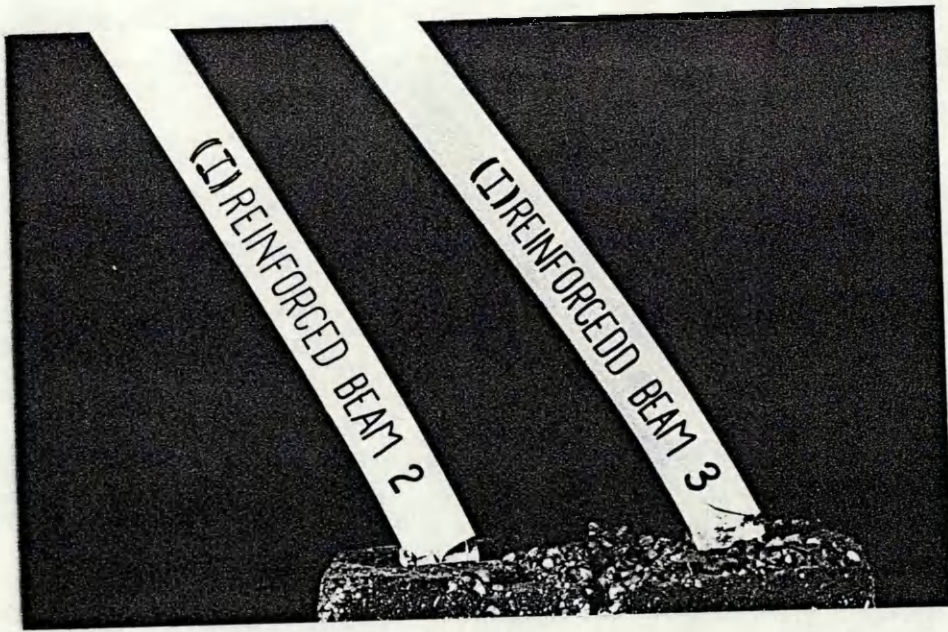


FIG.(4.14) COMPOSITE BEAM ((I)) AFTER TEST.

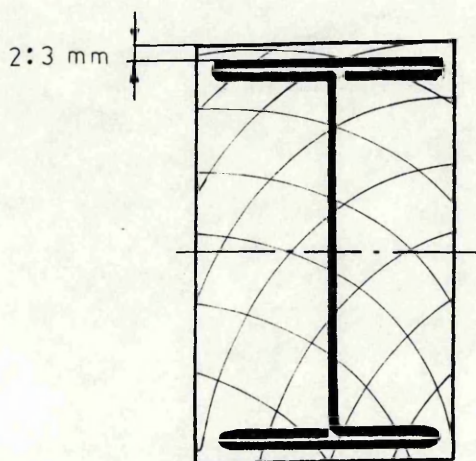
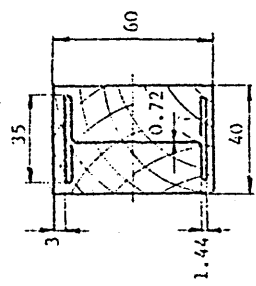
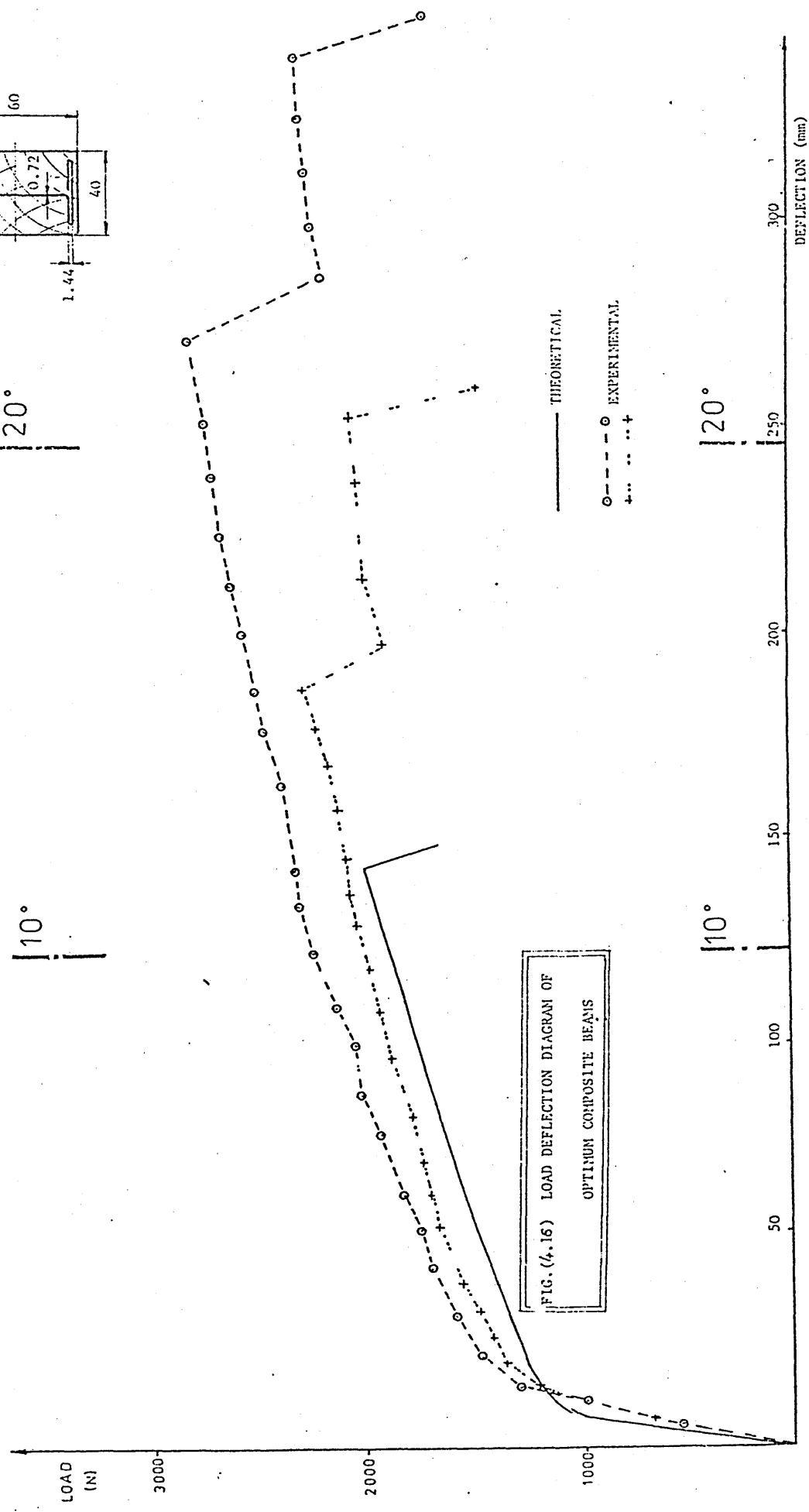


FIG.(4.15) OPTIMUM COMPOSITE BEAM SECTION FOR
UNIAXIAL BENDING.



10°

20°



CHAPTER 5

IMPACT PROPERTIES

5.1. INTRODUCTION

In an accident situation structural members of vehicle bodies are subjected to dynamic loads which make them very difficult to analyse. This is due to the random character of accident conditions. Some classification is still possible by selecting certain characteristic types of accident such as frontal, rear or side impact and roll over. Different events may still exist within each type, but narrowing the scope may identify an appropriate method of analysis which will probably give satisfactory results.

Destructive testing of the whole vehicle structure is still the most effective method of studying structure behaviour in a certain type of accident. However, this is a very expensive and time consuming method particularly in the early stages of design as the whole vehicle structure may not be available for testing. Thus, it is more economical in costs and time to predict theoretically the performance of vehicle structural components subject to impact loads.

In previous chapters of this thesis analysis was made of the large deflection behaviour of composite beams subjected to static loading. But to simulate an accident situation it is necessary to study their behaviour under impact conditions. This is usually different from the static behaviour because of the effect of inertia and material performance under high strain rate conditions. Therefore, in this chapter the results of tests of composite beams under impact conditions are presented.

5.2. TEST RIG

Two main factors affect the behaviour of a structural member during impact, energy and speed of impact. So, the special pendulum shown in FIG.(5.1) is able to test cantilever beams at different speeds and impact energies. This pendulum was built in the School of Automotive Studies as an MSc project. Although the pendulum was still under development, it was sufficiently advanced to enable the testing of composite beams.

The pendulum is powered by an air spring which accelerates it up to the required speed of impact. To achieve a certain speed of impact the pressure inside the air spring has to be raised to a definite value using a pneumatic control system. By adding masses to the basic weight of the pendulum it is possible to increase the impact energy.

The axis of the pendulum arm pivots was made to coincide with the fixed end of the test beam. This ensures that the pendulum and the test beam will move together to allow large deformation with negligible relative motion in the axial direction of the beam during the impact. Some other possible test rig arrangements would cause high relative motion and the beam may lose contact with the pendulum at large deflections which is undesirable (see FIG.(5.2)).

The velocity of the pendulum immediately before impact is measured using a digital time counter. This reads the time interval during which a pointer attached to the pendulum interrupts a light beam. This light beam is directed on to two photocells 60mm apart placed in the path of the pointer just before the point of impact. It could be argued that measuring the speed between two points only is not accurate enough because the pendulum may be still accelerating. This was proved to be incorrect since the displacement time diagram obtained during preliminary tests of the rig showed a constant slope in the range between the speed measuring points and the impact position (FIG.(5.4)).

The force during the impact is measured using a strain gauge force transducer. FIG.(5.5) shows the force transducer calibration diagram. This transducer has a solid steel semicylindrical nose to transfer the force of impact to the transducer. The weight of this nose is large ($\approx 1.5\text{kg}$) and caused an error in the value of the force measured due to its inertia. This caused a decrease of the force readings during the impact interval proportional to the pendulum deceleration. An attempt to quantify this error showed for a high speed of impact (10 m/sec), the nose subtracts $\approx 250\text{ N}$ from the real impact force at the beginning of the impact interval. But, this value decreases through the impact interval and vanishes at the end. So, the force values in the diagrams, shown later in this chapter, are lower ($\approx 10\%$) than the real force diagram particularly at the early stage of the impact interval. Using a stiff light metal alloy nose in future tests would significantly decrease this error.

To measure the displacement the test rig originally was equipped with an angular potentiometer which measures the angle of rotation of the pendulum. An attempt was made to measure the beam displacement using a parallel link mechanism attached to the free end of the test beam, as shown in FIG.(5.3). This mechanism indicated the angle of rotation of the beam. A high speed film (2000 frames/sec) was made during the preliminary tests of the pendulum. This proved that the link of the mechanism vibrated in bending with large amplitudes and caused a fluctuation of the displacement trace during the impact interval as shown in FIG.(5.6). Therefore, the link mechanism idea was rejected and the original design was used, i.e. the rotary potentiometer indicating the pendulum rotation.

The preliminary tests confirmed that the second and following impacts of the pendulum after rebound showed no further plastic deformations. The rig did not incorporate a device to prevent the pendulum bouncing at the time the tests were carried out. This means that the beams tested absorbed the energy of the first impact only and the following impacts caused only elastic bending of the test beam as shown in FIG.(5.7). This is clear from the displacement diagram as second and third impacts did not cause further plastic deformations.

The high speed film also showed that the beam loses contact with the pendulum just after the first peak of impact, but this did not cause any noticeable effect. From the above discussion it is clear that measuring the displacement of the pendulum is accurate enough for the present purpose.

Since the impact lasts for tens of milliseconds, the only equipment which was available, to record the force and displacement signals at the time the tests were carried out, was a high speed recorder. Before the test the recorder was connected to the force transducer and the rotary potentiometer. During the test the recording speed was set on its maximum (1.587m/sec). After the test the recorder was reconnected to a plotter which plots the force deflection diagram when replaying the tape on lower speed (1/64 of the recording speed).

The preliminary tests of the pendulum showed that high peaks of force occurred particularly at the beginning of the impact interval. This caused some difficulties for the plotter to follow the recorded signal. Also these transients do not contribute any significant information for the present test. Hence, low pass filters, which cut all frequencies higher than 150 Hz, were connected to both force and displacement channels before recording the signal. Usually filters cause a phase shift between the filtered signals, but FIG. 5.4 shows that displacement time diagram changes its slope at the point of impact. This implied that the force and displacement are reasonably in phase. For the tests made at high speeds a thin sheet of reinforced rubber (3mm thickness) was glued at the beam contact point with the pendulum nose in order to damp the impact vibrations. This is clear from the comparison of the peaks of force at the beginning of impact in FIG.(5.6) and (5.13). The force peak in FIG.(5.13) is lower because the rubber sheet was used during the test while it was not used in the other test.

FIG.'s (5.14) and (5.15) show the detail drawings of the original design of the pendulum.

5.3. TEST RESULTS

The nature of failure of material of vehicle body structure is very important from point of view of safety particularly during accident situations. The breaking of glass screens during accidents is a representative example of dangerous material failure. Due to impact, glass breaks into small pieces with sharp edges and scatter in random directions. These often hit the occupants of the vehicle and cause serious injuries. Likewise, it was interesting to study the behaviour of the polyurethane foam failure due to impact. Two unreinforced foam beams were tested for this purpose using the pendulum. These beams were set in concrete blocks to retain the cantilever boundary conditions.

The first beam was tested using the basic weight of the pendulum. The pendulum was released to accelerate under the effect of gravity only i.e. with atmospheric pressure in the air spring. This made the pendulum speed ≈ 3.8 m/sec. and the energy ≈ 108 J i.e. slightly higher than the energy necessary to break a similar beam by static loading (90 J). When this test was made the beam did not break and no plastic deformation occurred as what happened when a similar beam was loaded statically to the same deflection.

The second foam beam was tested also using the basic weight of the pendulum. This time a certain pressure was set in the air spring. The speed of impact reached ≈ 8 m/sec. and a pendulum energy of 480 J. The basic weight of the pendulum was heavy so that lower energies could not be achieved. The beam was broken near the fixed end and propelled a distance of 15 metres. This test showed that the beam fracture has sharp edges which can be very dangerous and may cause injuries. So, it can be concluded from this test that the existence of reinforcement is very important to restrain the beam separation. This emphasises the importance of reinforcing the panels. This can be done by wire net reinforcement or use of sandwich panels with thin aluminium faces.

The next tests were made on six optimum composite beams: a pair at different speed to ensure repeatability. The first two beams (1,2) were tested at the bus roll over speed which is 4.3 m/sec Ref. (3). Two masses each of 6.5 kg were added to the original pendulum to make the energy of the pendulum at this speed ≈ 300 J. FIG's. (5.8) and (5.9) show the load deflection diagrams of beams 1 and 2 (shown by the continuous curve). The dashed line shows the theoretical load deflection curve of the optimum beam. The dashed and dot line shows the experimental diagram of a similar beam (the weakest) when loaded statically. The energies absorbed by these beams were 235 J and 240 J respectively. Beam 2 has a foam crack on the tension side near the fixed end similar to the crack which occurred for a similar beam when loaded statically (shown in FIG.(4.14)). However, no crack was noticed for beam 1.

The next beams to be tested were 3 and 4. This time the test was performed using only the basic weight of the pendulum, but at a higher speed. The speed of impact was 6.9 m/sec which made the pendulum energy ≈ 350 J i.e. slightly higher than the previous test. FIG'S. (5.10) and (5.11) show the load deflection of these beams. Beam 3 absorbed 255 J and no crack occurred, while beam 4 absorbed 295 J and the foam cracked.

The load deflection diagram shown in FIG.(5.11) shows that the crack caused a drop in the force before the end of the impact, similar to the drop occurred for the beams tested statically when the crack occurred. The crack shape and position again was similar to the crack occurred for the beams loaded statically.

The last two beams tested also used the basic weight of the pendulum but at a higher speed of 8 m/sec, which made the pendulum energy ≈ 480 J. FIG's.(5.12) and (5.13) show the test results. Beam 5 absorbed an energy of 375 J and cracked while beam 6 absorbed 428 J, an almost total collapse of the beam occurred. The collapse of beam 6 again was similar to the collapse which occurred statically. The tension side half of the beam had material separation for both the foam and the reinforcement. The thickness of the reinforcement at the fractured section was thin relative to the original thickness i.e. similar to the fracture of a tensile test specimen after testing.

A high speed film was made to indicate qualitatively the deflection of the beam when the crack started during the testing of beam 5. This showed that the beam cracked near the end of impact interval and proved that the force drop in the diagram shown in FIG.(5.12) was due to cracking of the foam as expected. Table 5.1 shows the test results.

Finally, the impact tests of the optimum composite beams have shown some important results which can be summarised as follows:

- 1) The beam crack took place near the fixed end and has the same form as for the beams tested statically.
- 2) The beams absorbed higher energy when loaded dynamically at different speeds than when loaded statically to the same deflection.
- 3) The unreinforced foam beams or panels cannot be used as parts from the vehicle body structure due to their dangerous fracture i.e. the reinforcement is essential from point of view of safety.

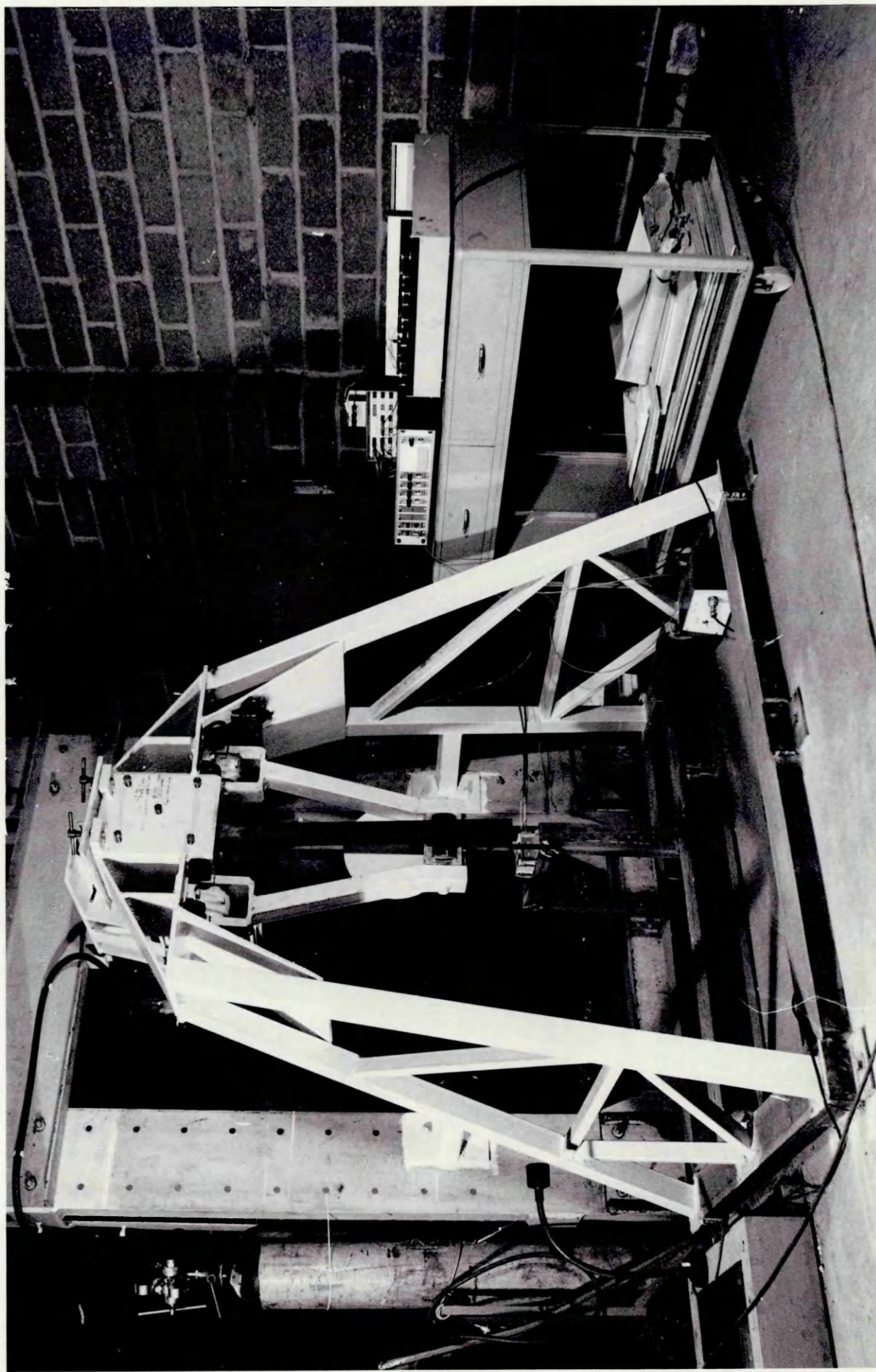


FIG. (5.1) HIGH SPEED PENDULUM TEST RIG.

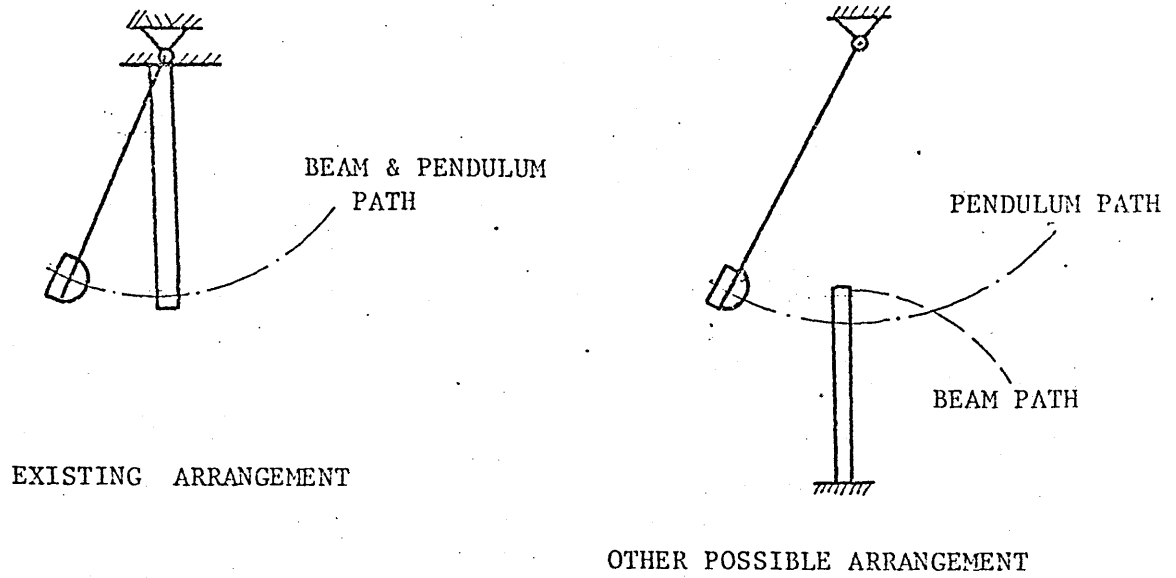


FIG.(5.2) DIFFERENT PENDULUM ARRANGEMENTS

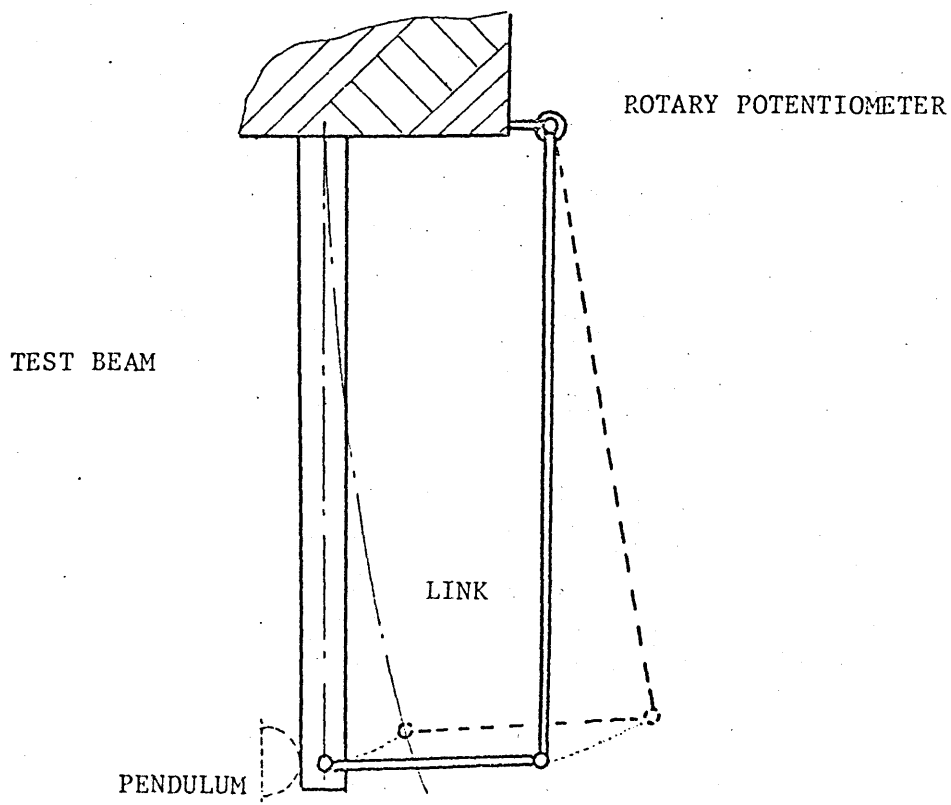


FIG.(5.3) PARALLEL LINK MECHANISM TO MEASURE BEAM DEFLECTIONS:

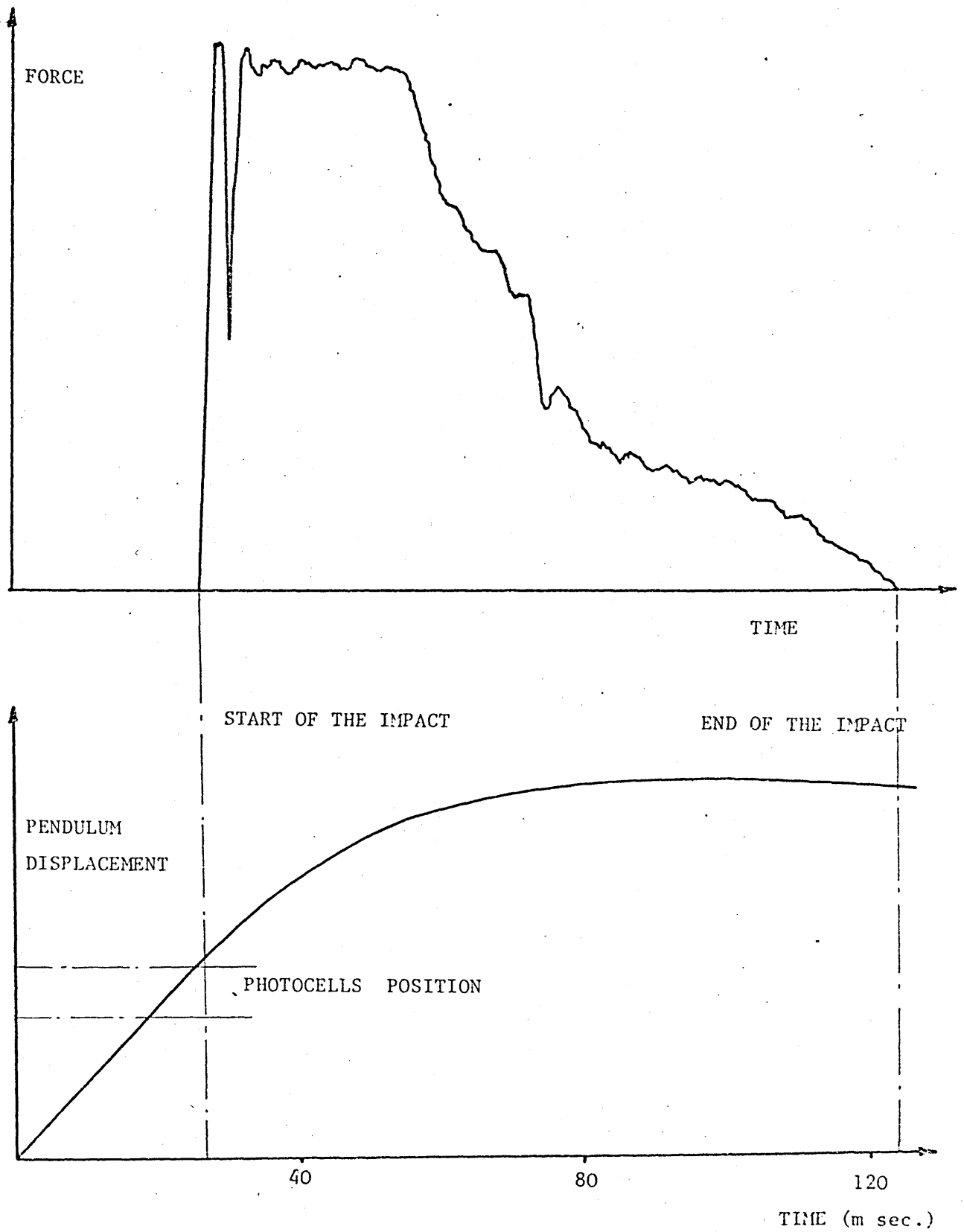


FIG. (5.4) FORCE AND DISPLACEMENT-TIME DIAGRAMS OF
PRELIMINARY TESTS OF THE PENDULUM.

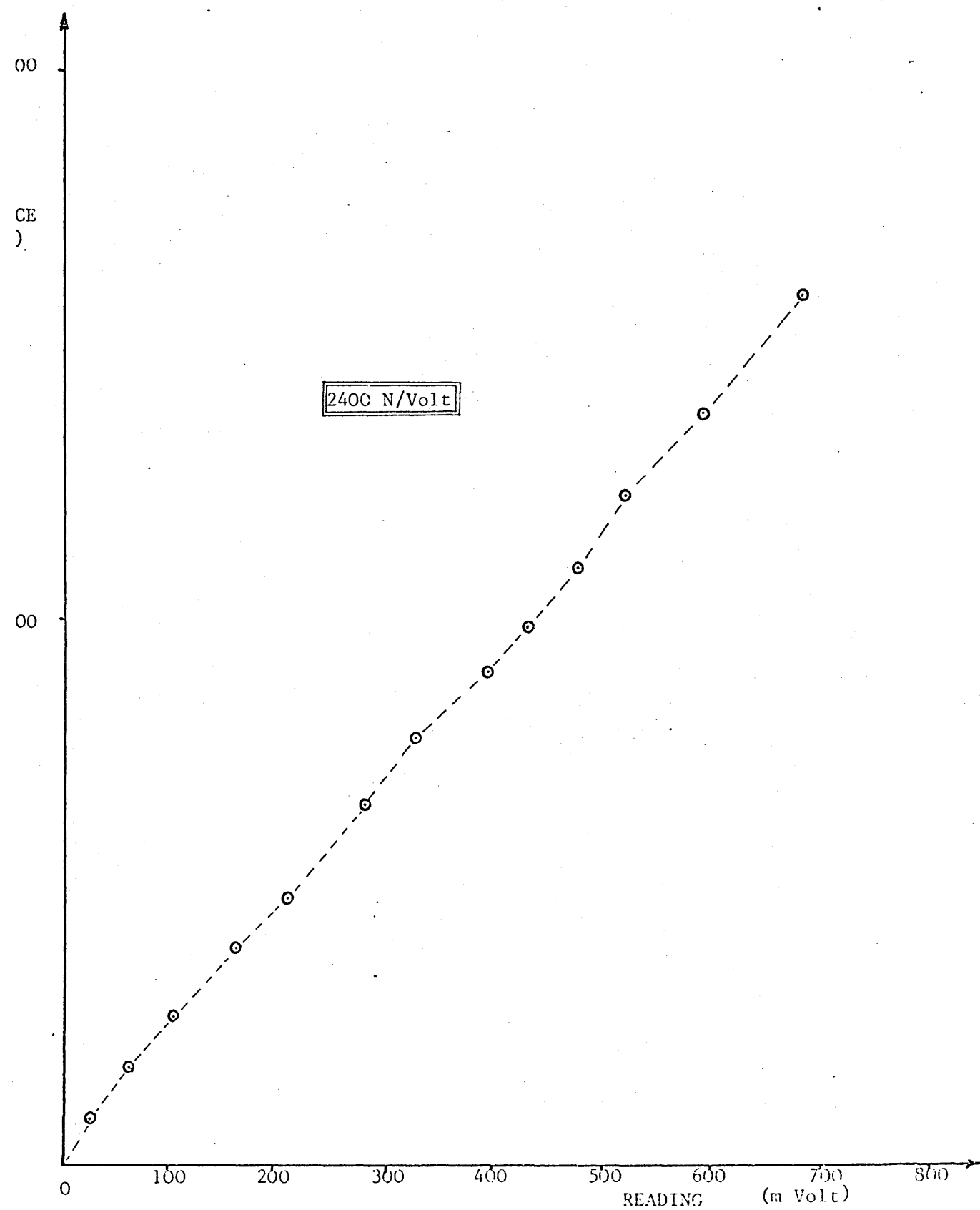


FIG.(5.5) FORCE TRANSDUCER CALIBRATION

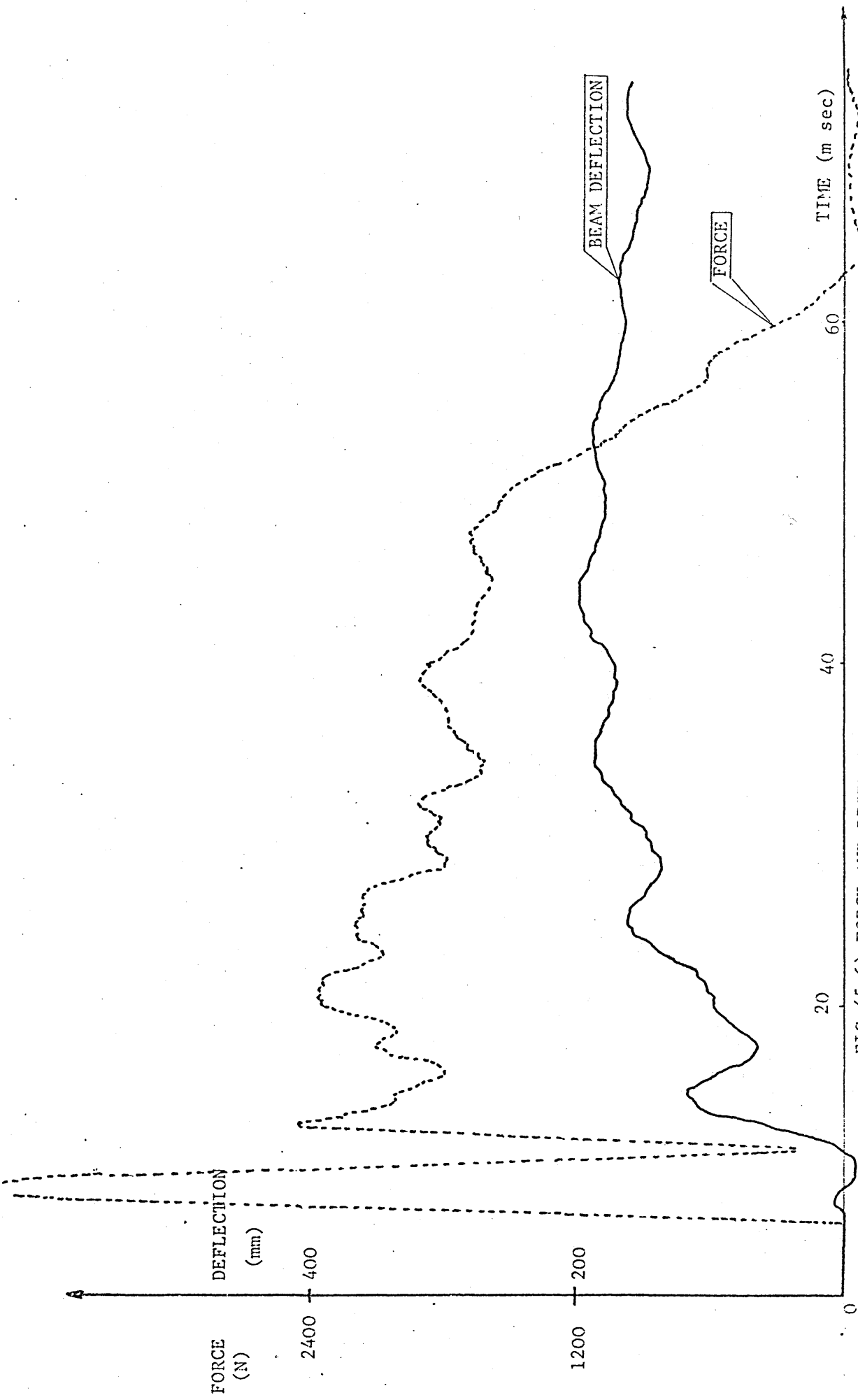


FIG. (5.6) FORCE AND DISPLACEMENT DIAGRAM OF FIRST IMPACT USING THE

PARALLEL LINK MECHANISM.

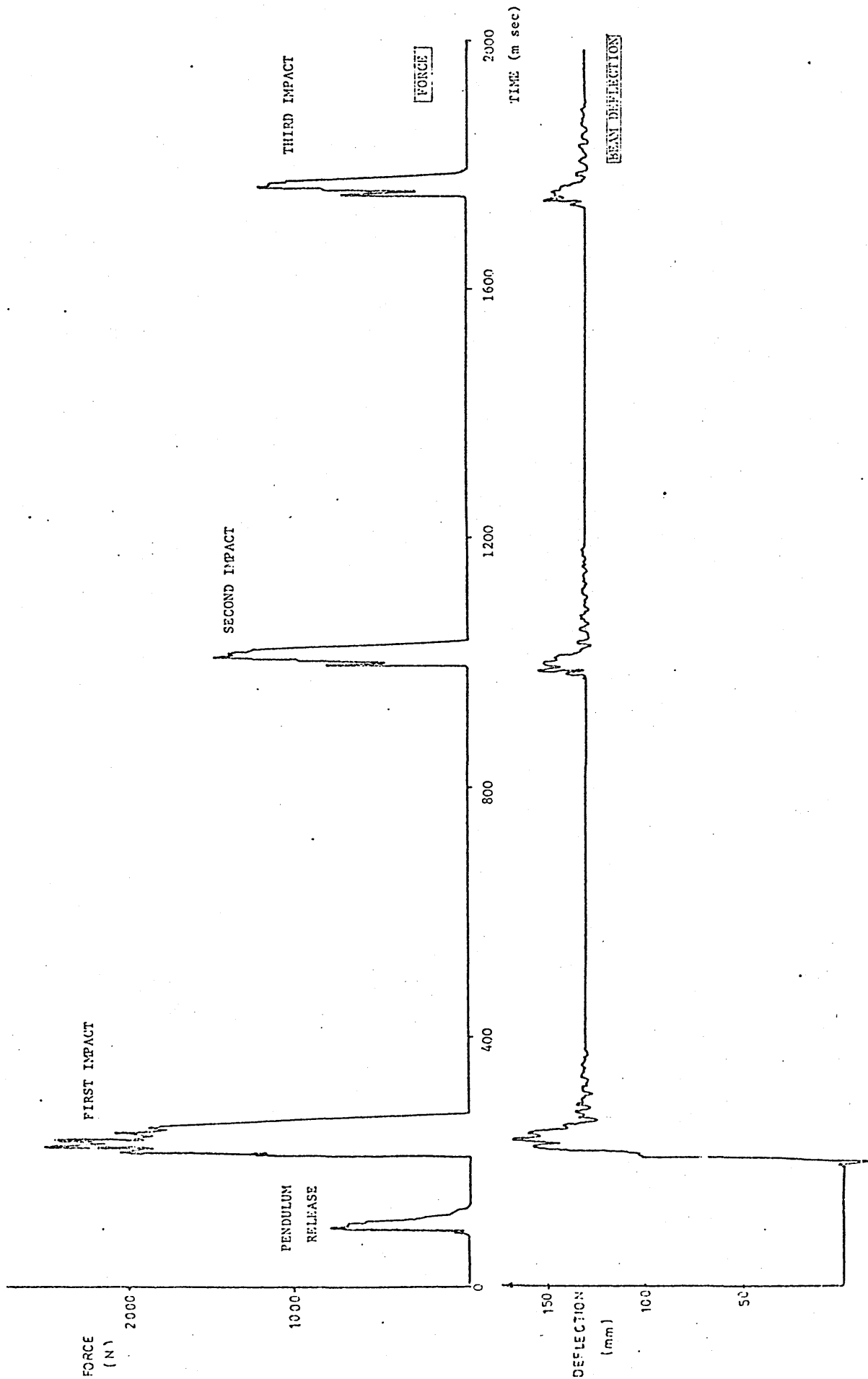


FIG. (5.7) FORCE AND DISPLACEMENT TIME DIAGRAM OF PRELIMINARY TESTS USING THE PARALLEL LINK MECHANISM.

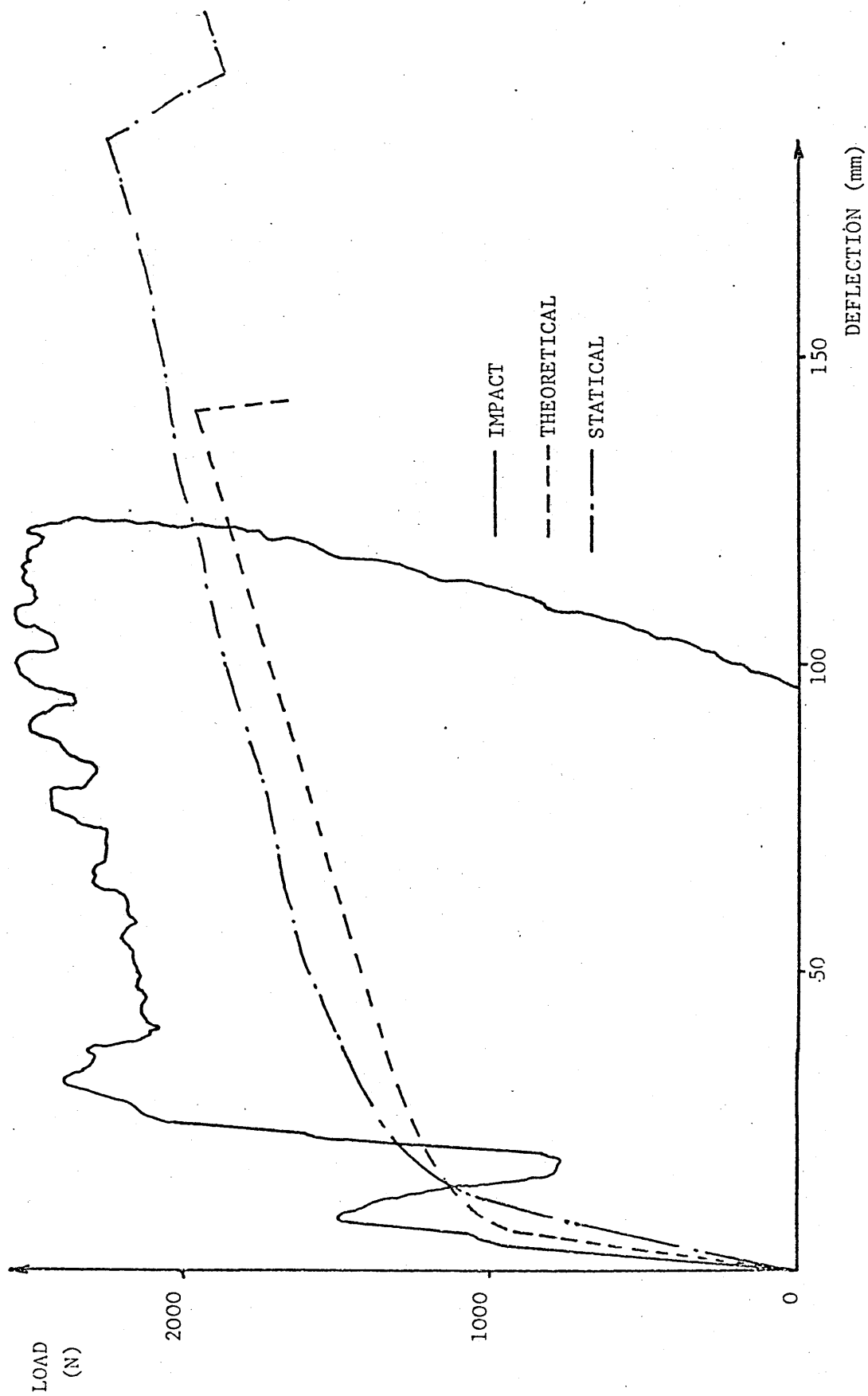


FIG. (5.8) LOAD DEFLECTION DIAGRAM OF BEAM 1 (SPEED =4.3 m/sec , PENDULUM ENERGY =300 J)

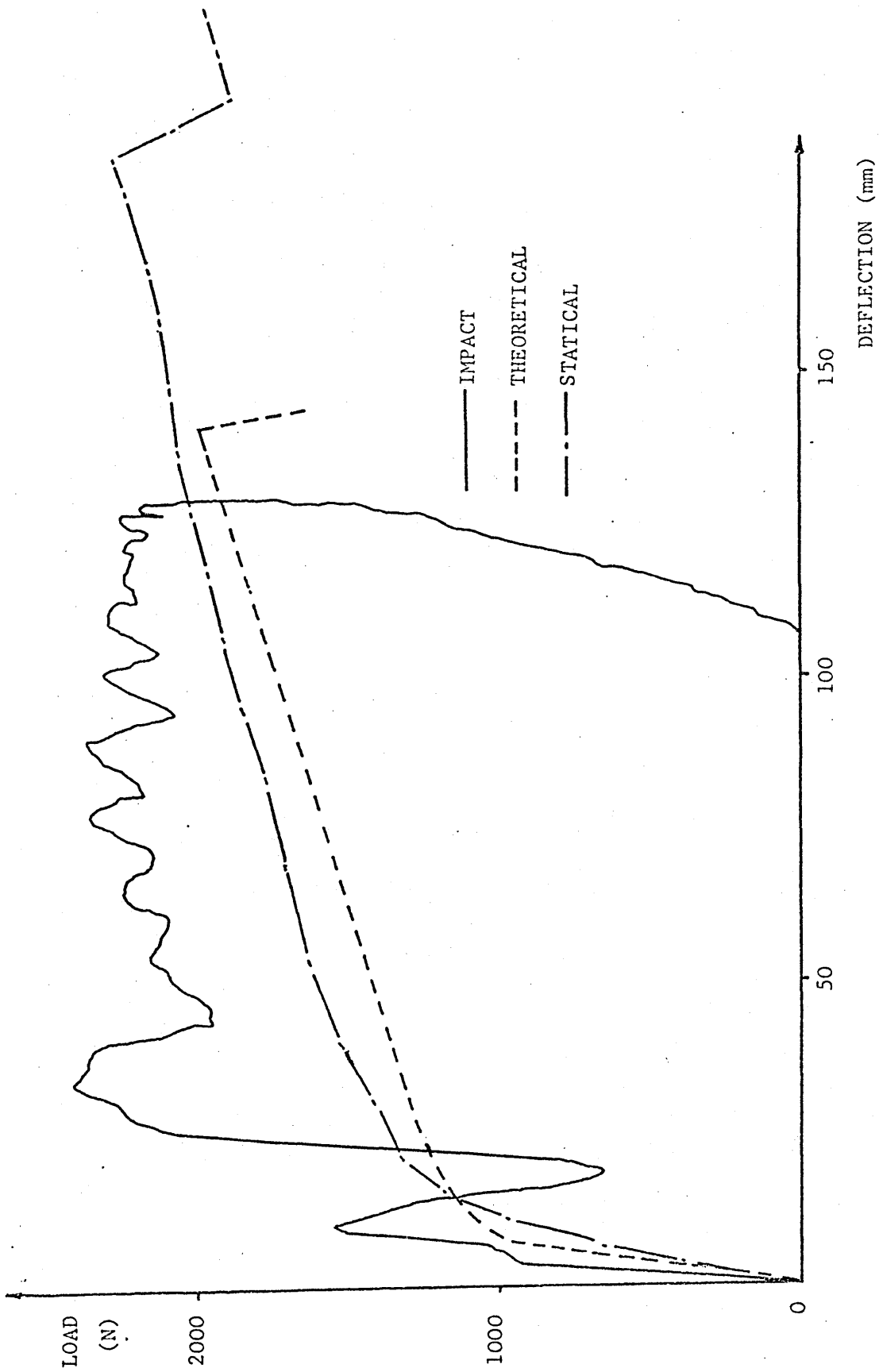


FIG.(5.9) LOAD DEFLECTION DIAGRAM OF BEAM 2 (SPEED =4.3 m/sec , PENDULUM ENERGY = 300 J)

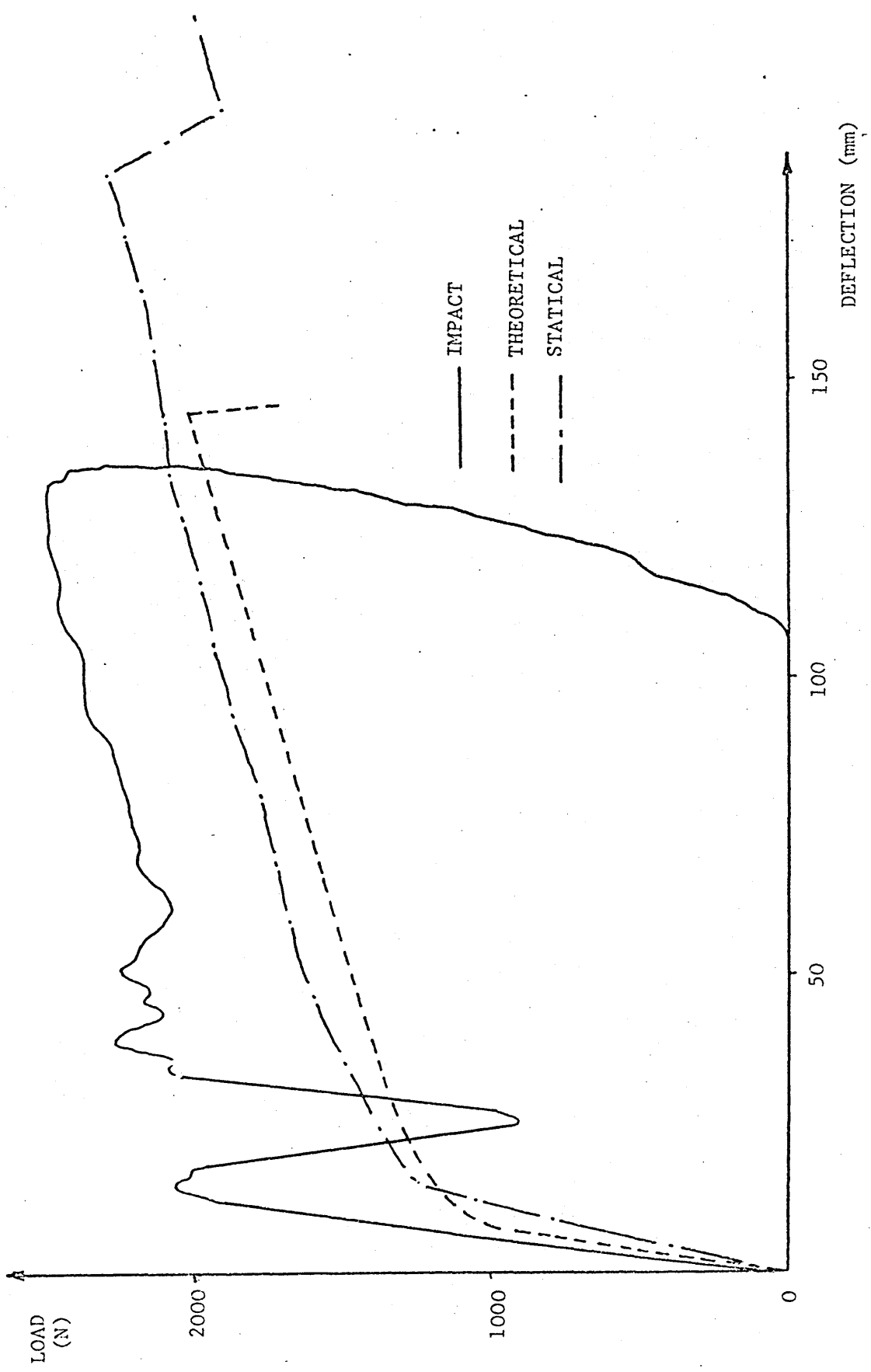


FIG. (5.10) LOAD DEFLECTION DIAGRAM OF BEAM 3 (SPEED = 6.8 m/sec , PENDULUM ENERGY = 350 J)

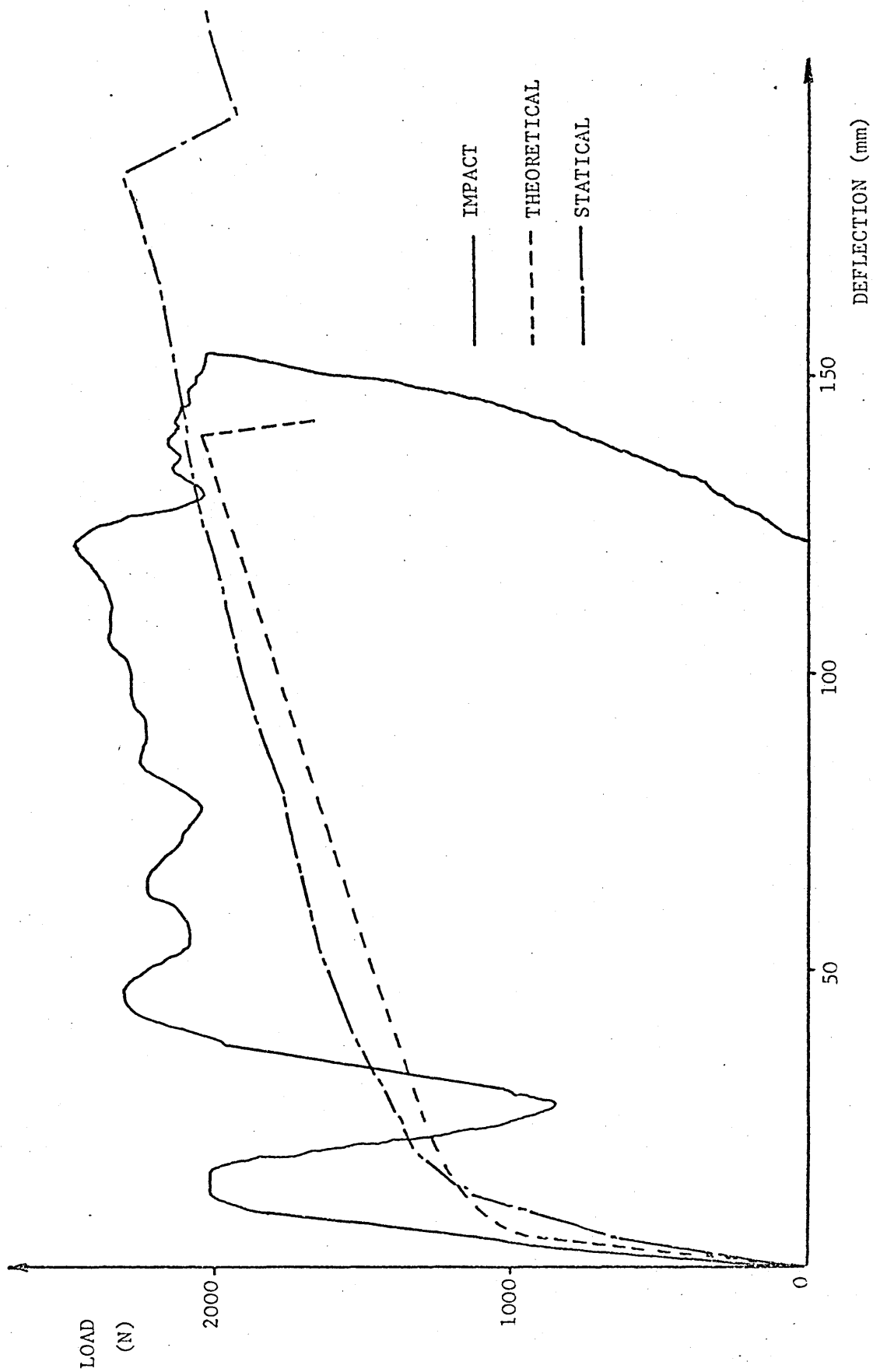


FIG.(5.11) LOAD DEFLECTION DIAGRAM OF BEAM 4 (SPEED = 6.8 m/sec , PENDULUM ENERGY = 350 J)

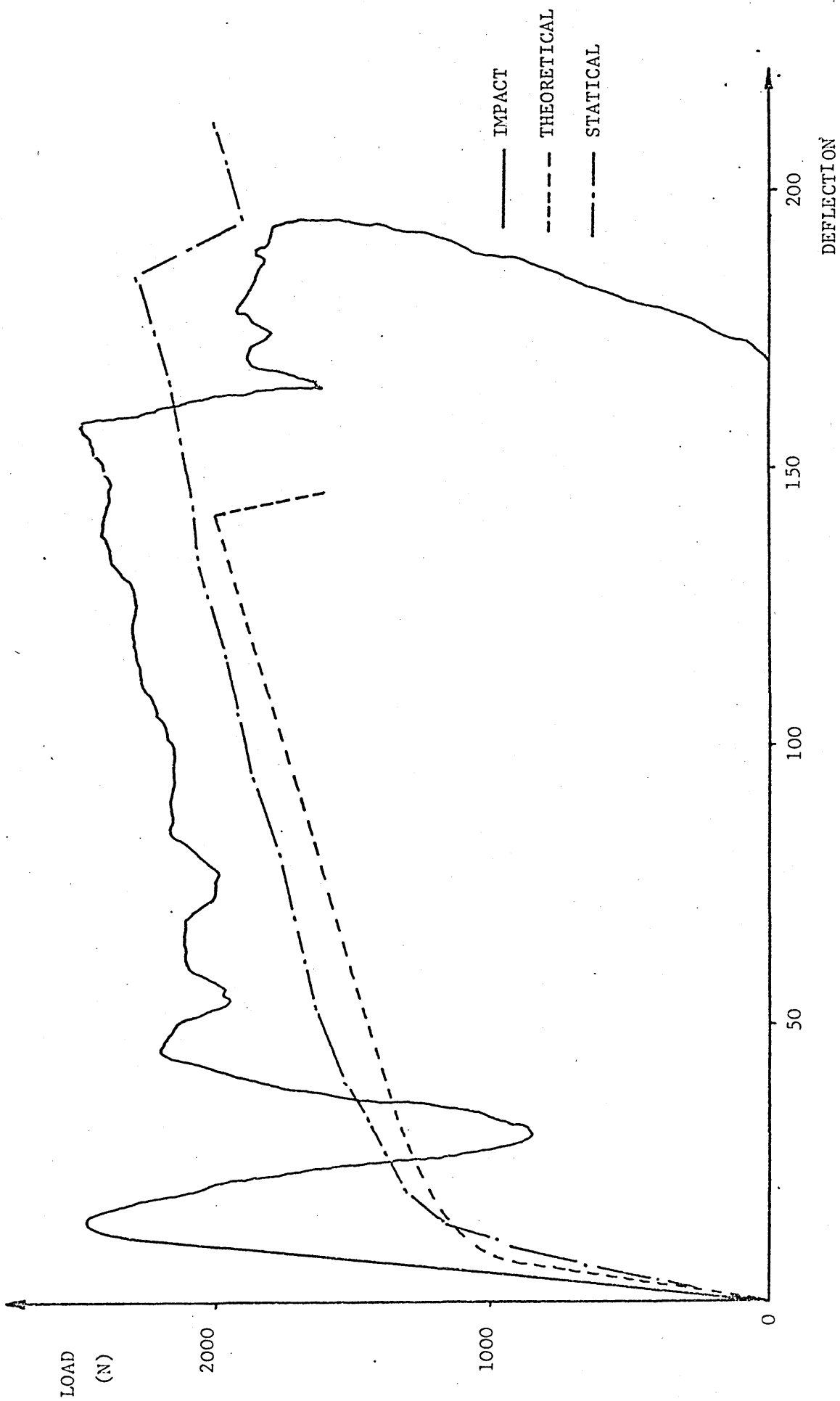


FIG. (5.12) LOAD DEFLECTION DIAGRAM OF BEAM 5 (SPEED=8 m/sec , PENDULUM ENERGY = 480 J)

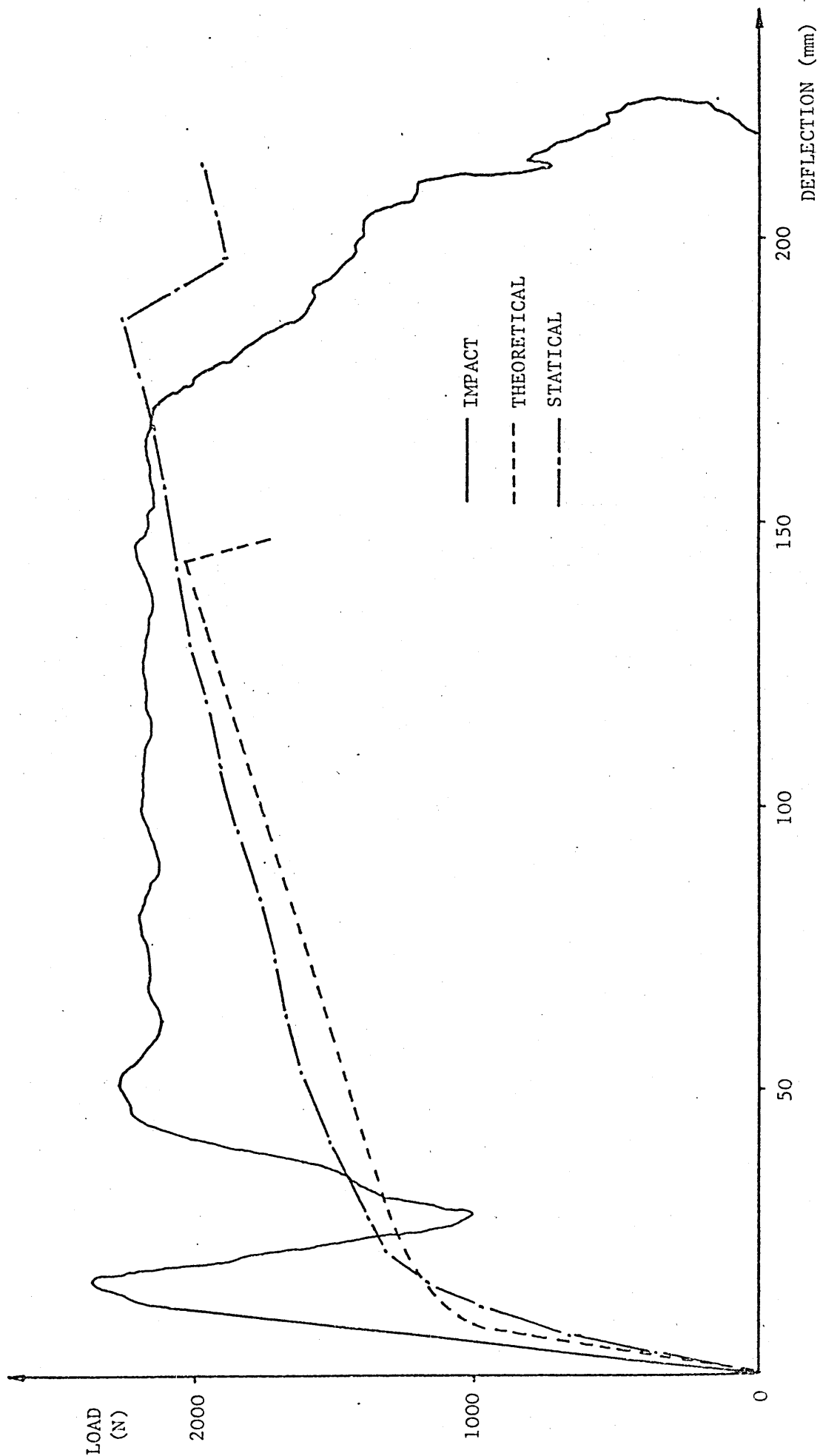


FIG. (5.13) LOAD DEFLECTION DIAGRAM OF BEAM 6 (SPEED =8 m/sec , PENDULUM ENERGY =480 J)

Beam No	Speed of Impact m/sec	Pendulum arm energy (J)	Beam Dynamic Energy (J)	Beam static energy to the same deflection (J)	Beam failure
1	4.3	300	235	170	NONE
2	4.3	300	240	190	FOAM CRACK
3	6.9	350	255	200	NONE
4	6.9	350	295	230	FOAM CRACK
5	8.0	480	375	320	FOAM CRACK
6	8.0	480	482	408	TOTAL COLLAPSE

TABLE 5.1. TEST RESULTS OF THE COMPOSITE BEAMS

CHAPTER 6

LARGE DEFLECTION ANALYSIS OF A THIN
WALLED CHANNEL SECTION CANTILEVER BEAM.

PRINCIPAL NOTATION OF CHAPTER 6

A	Cross section area of the beam
a	Beam width
B(z)	Internal Bimoment at distance z
B ₀	External Bimoment at the support z = 0
E	Youngs Modulus of beam material
E_1	$= \frac{E}{1-\nu^2}$
e	Distance between shear centre axis and centroidal axis
F	Axial load
G	Modulus of rigidity of beam material $= \frac{E}{2(1-\nu)}$
I_x, I_y	Second moments of area about axes x,y respectively
I_ω	Principal sectorial moment of inertia $I_\omega = \int_A \omega^2 dA$
J	Torsion constant of the section (St. Venant)
μ	Constant $= \sqrt{\frac{1-\nu}{2} \frac{J}{I_\omega}}$
L	Length of the beam
m(z)	External torque per unit length at z
M _x	Bending moment acting in a plane normal to the x axis
M _y	Bending moment acting in a plane normal to the y axis
t	An arbitrary parameter
T(z)	Internal torque at point z
T ₀	External torque at the support z = 0
x,y,z	Cartesian co-ordinate axes
ν	Poisson's ratio of beam material
$\sigma(s)$	Longitudinal stress in the beam at point (s)
ϕ	Angle of twist
ϕ_z	Angle of twist of the beam at a distance z from the origin
ϕ_L	Angle of twist at the beam free end
ϕ_0	Angle of twist at the support z = 0
$\omega(s)$	Principal sectorial co-ordinates of the section

6.1. INTRODUCTION

Early investigators disclosed that thin walled metal sections behave differently compared with thick walled sections. "The distinctive feature of thin walled beams is that they can undergo longitudinal extension as a result of torsion. Consequently, longitudinal normal stresses proportional to these strains are created which lead to an internal equilibrium of the longitudinal forces in each cross section. These complementary longitudinal normal stresses, which arise as a result of the relative warping of the section and which are not examined in the theory of pure torsion, can attain very large value in thin walled beams with open (rigid or flexible) cross sections and also in beams with closed flexible cross sections". These few sentences from the introduction of VLASOV's book Ref. (17) well explain the special behaviour of thin walled sections.

It is mentioned in the introduction of this thesis that, the experimental investigation of the large deflection behaviour of thin walled channel section in bending has thrown up an unexpected phenomenon. This thin walled channel cantilever buckled at a significant distance from the fixed end, when loaded through the centroid of the free end FIG.1. Meanwhile, a similar channel buckled at the fixed end as expected when loaded through the shear centre FIG.2. The theoretical explanation of this phenomenon according to VLASOV's theory (small deflection analysis) as well as a modified large deflection analysis will be given in this part of the thesis.

6.2. SMALL DEFLECTION ANALYSIS

The general formula for the longitudinal stresses $\sigma(s)$ in a thin walled beam according to VLASOV's theory Ref.'s (17,13) is:

$$\sigma(s) = \frac{F}{A} + \frac{M_x}{I_x} \cdot y(s) + \frac{M_y}{I_y} \cdot x(s) + \frac{B(z)}{I_\omega} \cdot \omega(s) \quad (1)$$

The general equation formulating the relation between the angle of twist of the beam and the external load is:

$$\frac{d^4 \phi}{dz^4} - \mu^2 \frac{d^2 \phi}{dz^2} = - \frac{m(z)}{E_1 I_\omega} = - \frac{dT(z)}{dz} \frac{1}{E_1 I_\omega} \quad (2)$$

The solution for a straight beam having a constant cross section is:

$$\begin{aligned} \phi_z = & \phi_0 + \frac{d\phi_0}{dz} \frac{\sinh \mu L}{\mu} + \frac{B_0}{GJ} (1 - \cosh \mu z) \\ & + \frac{T_0}{\mu GJ} (\mu z - \sinh \mu z) + \frac{1}{\mu GJ} \int_0^z \left(\mu(z-t) - \sinh \mu(z-t) \right) m(t) dt \quad (3) \end{aligned}$$

where $m(t) = \frac{dT(t)}{dt}$

The internal bimoment at any section z is:

$$\begin{aligned}
 B(z) &= -E_1 I_\omega \frac{d^2 \phi_z}{dz^2} \\
 &= -\frac{d\phi_0}{dz} \frac{GJ}{\mu} \sinh \mu z + B_0 \cosh \mu z + \frac{T_0}{\mu} \sinh \mu z \\
 &\quad + \frac{1}{\mu} \int_0^z \sinh \left[\mu(z-t) \right] m(t) dt \quad (4)
 \end{aligned}$$

Assume a cantilever thin walled channel section is loaded through the centroid of the free end by a load P . The fixed end is fully fixed and the other end is free to warp FIG.3. The effect of this load is equal to an equal load P acting through the shear centre axis and a torque $T = P.e$, constant with z . Therefore, the values of different loads for this cantilever are:

Axial force $F = 0$

Bending moments $M_x = P(L-z)$
 $M_y = 0$

Torque $T_z = T_0 = -T = -P.e$

The boundary conditions are:

$$\phi_0 = 0, \quad \frac{d\phi_0}{dz} = 0, \quad B(L) = 0$$

For $z = L$ equation (4) becomes:

$$B(L) = 0 = B_0 \cosh \mu L - \frac{T}{\mu} \sinh \mu L, \text{ so that}$$

$$B_0 = \frac{T}{\mu} \frac{\sinh \mu L}{\cosh \mu L} = \frac{T}{\mu} \tanh \mu L$$

$$\text{i.e. } B(z) = \frac{T}{\mu} (\tanh \mu L \cosh \mu z - \sinh \mu z)$$

$$= \frac{T}{\mu} \frac{\sinh \mu(L-z)}{\cosh \mu L}$$

The angle of twist ϕ_z is given by:

$$\phi_z = \frac{T}{\mu GJ} (\tanh \mu L - \tanh \mu L \cosh \mu z - \mu z + \sinh \mu z) \quad (5)$$

Therefore, the longitudinal stress for a point (s) a distance z along the beam will be

$$\sigma(s) = \frac{P(L-z)}{I_x} y(s) + \frac{P.e}{\mu} \frac{\sinh \mu(L-z)}{\cosh \mu L} w(s) \quad (6)$$

A further experiment using strain gauges was made to check the stress distribution through the different beam sections. FIG.4 shows the stress distribution along the beam at the position of the strain gauges. The continuous line shows the theoretical stresses using equation (6) and the dashed lines show the measured stresses. Although the theoretical value of the stress on the buckled edge σ_4 has its maximum value far from the fixed end, it shows very low stress compared with the measured stresses. On the same experiment for measuring the stresses the angle of twist of the beam were measured at three cross sections of the beam at different loads. The measured angles of twist showed good agreement with the theoretical value calculated from equation (5) but only at small loads. At higher loads the small deflection theory did not show the correct trend. Therefore, a large deflection analysis was developed to explain both the high compression stresses on the buckled edge and the error in the estimation of the angle of twist at higher loads.

6.3. LARGE DEFLECTION ANALYSIS

Many papers have been published using matrix and finite element methods for the large deflection analysis of thin walled beams e.g. Ref.'s (19,20,21,22,23). A similar approach to that published by MARTIN M. BLACK Ref. (24) is used and is essentially a continuous function analysis rather than a finite element approximation.

In the small deflection analysis the torque was considered to be constant along the length of the beam. But in fact for large deflections this torque will vary according to the lateral displacement of the beam FIG.6.

The following assumptions are made

- (1) The angle of twist $\phi_z = \phi_L \cdot \frac{z}{L}$

This assumption satisfies cantilever boundary conditions. For the general case of a beam with arbitrary cross-section a non-linear expression would probably be necessary, but for the particular cross-section tested the experimental results shown in FIG.8 confirms that a linear approximation is reasonable.

- (2) Although it is a large deflection analysis, the angle of twist is still a small angle (experiment showed that ϕ_L is less than 15°).

$$\text{i.e. } \cos \phi_z = 1, \quad \sin \phi_z = \phi_z = \phi_L \cdot \frac{z}{L}$$

Thus, the loads at an arbitrary cross-section will be:

Axial force $F = 0$

Bending moments $M_x = P(L-z) \cos \phi_z = P(L-z)$

$M_y = P(L-z) \sin \phi_z$

Torque from FIG.4 is given by

$$T(z) = -P (U_L - U_z - R \sin \phi_L + e \cos \phi_z) \quad (7)$$

The displacements U_L and U_z in the plane of the cross-section can be found using the engineers theory of bending Ref.'s (17,24,25). So that,

$$\begin{aligned} EI_y \frac{d^2 U_z}{dz^2} &= -M_y = -P(L-z) \sin \phi_z \\ &= -P(L-z) \frac{\phi_L}{L} \cdot z = -P \frac{\phi_L}{L} (Lz - z^2) \end{aligned}$$

Integrating w.r.t. z gives

$$EI_y \frac{dU_z}{dz} = -\frac{P \phi_L}{L} \left(\frac{Lz^2}{2} - \frac{z^3}{3} \right) + C_1$$

But for a cantilever $\frac{dU_z}{dz} = 0$ at $z = 0$

which implies $C_1 = 0$

Integrating again w.r.t. z gives

$$EI_y U_z = -P \frac{\phi_L}{L} \left(\frac{Lz^3}{6} - \frac{z^4}{12} \right) + C_2$$

But also for a cantilever $U_z = 0$ at $z = 0$

which implies $C_2 = 0$

$$\text{Thus, } U_z = -\frac{P \phi_L}{12EI_y L} (2Lz^3 - z^4) \quad (8)$$

$$\text{and } U_L = \frac{P \phi_L L^3}{12EI_y} \quad (9)$$

Considering the assumptions and substituting equations (8) and (9) in equation (7) gives:

$$T(z) = \frac{P^2 \phi_L}{12 EI_Y} \left(L^3 - 2 z^3 + \frac{z^4}{L} \right) - P (e + R \phi_L) ,$$

$$m(z) = \frac{dT(z)}{dz} = \frac{P^2 \phi_L}{6 EI_Y L} (2 z^3 - 3 z^2 L) \quad \text{and}$$

$$T_0 = T(z) \Big|_{z=0} = \frac{P^2 \phi_L L^3}{12 EI_Y} - P(e + R \phi_L)$$

Substituting the value of T_0 and $m(z)$ in equation (4) and since $B(L) = 0$ gives:

$$\begin{aligned} B(L) = 0 = B_0 \cosh \mu L + \left(\frac{P^2 \phi_L L^3}{12 EI_Y} - P(e + R \phi_L) \right) \frac{\sinh \mu L}{\mu L} \\ + \frac{P^2 \phi_L}{6 EI_Y \mu L} \left(2 \int_0^L z^3 \sinh \mu (L-z) dz - 3L \int_0^L z^2 \sinh \mu (L-z) dz \right) \quad (10) \end{aligned}$$

The values of the integrals are

$$\int_0^L z^3 \sinh \mu (L-z) dz = - \left(\frac{L^3}{\mu} + \frac{6L}{\mu^3} - \frac{6}{\mu^4} \sinh \mu L \right)$$

$$\text{and} \quad \int_0^L z^2 \sinh \mu (L-z) dz = - \left(\frac{L^2}{\mu} + \frac{2}{\mu^3} - \frac{2}{\mu^3} \cosh \mu L \right)$$

Substituting the values of these integrals in equation (10) gives the value of B_0 which is:

$$\begin{aligned} B_0 = - \frac{P^2 \phi_L}{6 EI_Y \mu^4} \left(\frac{\mu^2 L^2 - 6}{\cosh \mu L} - 6 - 12 \frac{\tanh \mu L}{\mu L} \right) \\ - \left(\frac{P^2 \phi_L L^4}{12 EI_Y} - PL (e + R \phi_L) \right) \frac{\tanh \mu L}{\mu L} \end{aligned}$$

Using the values of T_o , B_o and $z = L$ for equation (3) to find ϕ_L gives

$$\begin{aligned}\phi_L &= \frac{B_o}{GJ} (1 - \cosh \mu L) + \frac{T_o}{\mu GJ} (\mu L - \sinh \mu L) \\ &+ \frac{1}{\mu GJ} \int_0^L \left(\mu(L-z) - \sinh \mu(L-z) \right) m(z) dz \\ &= \frac{B_o}{GJ} (1 - \cosh \mu L) + \frac{T_o}{\mu GJ} (\mu L - \sinh \mu L) \\ &+ \frac{P^2 \phi_L}{6EI_Y \mu L GJ} \left[\int_0^L \mu(L-z) (2z^3 - 3z^2 L) dz + 2 \int_0^L z^3 \sinh \mu(L-z) dz \right. \\ &\quad \left. - 3L \int_0^L z^3 \sinh \mu(L-z) dz \right]\end{aligned}$$

Substituting B_o and T_o and the values of the integrals gives:

$$\begin{aligned}\phi_L &= \frac{Pe}{\mu GJ} (\tanh \mu L - \mu L) \\ &\left[1 + \frac{P^2}{6\mu^4 EI_Y GJ} \left(\frac{\mu^2 L^2 - 6}{\cosh \mu L} + \frac{12 \tanh \mu L}{\mu L} - 6 \right) \right. \\ &\quad \left. + \frac{P^2 L^4}{12EI_Y GJ} \left(\frac{\tanh \mu L}{\mu L} - 0.7 \right) - \frac{PR}{\mu GJ} (\tanh \mu L - \mu L) \right]\end{aligned}$$

It is possible now to find the value of bimoment at any point z from equation (3) as follows

$$B(z) = B_o \cosh \mu z + \frac{T_o}{\mu} \sinh \mu z + \frac{P^2 \phi_L}{6EI_Y \mu L} \int_0^z (2t^3 - 3t^2 L) \sinh \mu(z-t) dt$$

Evaluating the integral and substituting in the last equation gives:

$$\begin{aligned}B(z) &= \left(B_o - \frac{P^2 \phi_L}{EI_Y \mu^4} \right) \cosh \mu z + \left(\frac{T_o}{\mu} + \frac{2P^2 \phi_L}{EI_Y L \mu^5} \right) \sinh \mu z \\ &+ \frac{P^2 \phi_L}{6EI_Y \mu^4 L} (3\mu^2 L z^2 + 6L - 2z^3 \mu^2 - 12z)\end{aligned}$$

And finally the equation for the stress will be:

$$\sigma(s) = \frac{P(L-z)}{I_x} y(s) + \frac{P(L-z)}{I_y} \phi_z x(s) + B(z) \frac{\omega(s)}{I_\omega} \quad (11)$$

The stress distribution along the beam at the same four points in the cross-section according to equation (11) is shown in FIG.7. The experimental results from the strain gauged beam are in reasonable agreement with the theoretical values developed by the large deflection analysis particularly the slopes of the stress distribution curves.

FIG.9 shows the comparison between experimental values for the angle of twist at different loads and the theoretical values using both small and large deflection theories. The results show good agreement with the small deflection theory for angles up to 4 degrees and with large deflection theory at larger angles. This proves that assumption (1) is sufficient for this particular case.

6.4. TEST METHOD

The cantilever channel sections consisted of commercial grade mild steel channel section with nominal dimensions and 700mm length (L). The channels were folded from sheet and consequently an indeterminate corner radius was present at the corners between the flanges and the web of the section (points 2 and 3 in FIG.3). The channel was set in a concrete block to provide a fully restrained boundary condition at the fixed end. The load was applied by a cable attached to the bracket shown in FIG.5 at the free end. The point of loading was positioned on the bracket at a distance 40mm (R) from the axis of symmetry of the section. The bracket only applied loads to the cross-section at the three points where the principal sectorial area is zero so that free warping could be assumed.

The original tests were made with uninstrumented channels and only the load deflection curves at the free end were obtained. For the three specimens tested in this conditions the buckle occurred at 230mm $\pm$ 5mm from the fixed end FIG.1. Another three specimens were loaded through the shear centre and the buckle occurred at the fixed end FIG.2. FIG.5 shows the instrumented specimen in the test rig with potentiometers arranged to measure the bending and torsional displacement at three cross sections along the beam. Strain gauges were also fitted. Although this test confirmed that the twist angle was linear with distance along the beam the results were suspect due to friction in the linear potentiometers. The tests were therefore repeated with stress measurements by strain gauges only. Since the strain gauges were 7mm wide and therefore covered a substantial proportion of the flange width, the adhesive would be expected to stiffen the cross-section. No attempt has been made to quantify this error, but it may be a contributory factor in the lack of quantitative agreement between theory and practice, particularly at the flange edge position 1. The error for the other flange edge position 4, where the stress is compressive, is not so large as on the tension side. Although both the theoretical and measured curves are flat in the region of maximum compression, this occurs near the measured buckle distance of 230mm from the fixed end.

A separate experiment to measure the twist angle only at the same three cross-sections was carried out using plumb bobs to project the position of the arms originally used to carry the potentiometers on a horizontal plane. The results of this test which were not contaminated by potentiometer friction or by strain gauge local stiffening are shown in FIG.8 and FIG.9.

A further experiment was carried out to investigate the effect of the beam cross-section dimensions on the buckling behaviour. Three different sections were chosen for this experiment, all made from sheet of 1.6mm thickness with flange widths 15, 22 and 30mm. The breadth of the webs were made double the flange width for the three chosen sections, i.e. 30, 44 and 60mm respectively. The load this time were applied at the centroid point i.e. $R = 0$ FIG.6. Two specimens were tested from each kind, one with three arms carrying the plumb bobs to measure the angle of twist at the same sections along the beam chosen for the previous tests and the second to determine the buckling position without the effect of interference from the stiffening of the plumb bobs arm carrier.

FIG.'s 10, 11 and 12 show the beams tested. The buckle occurred at distances of 210, 320 and 340mm from the fixed end for the beams with the arm carriers. The other beams buckled at distances of 240, 350 and 440mm from the fixed end.

FIG.'s 13, 14, 15, 16, 17, 18 show the stress distribution along the beam at the section corners for the sections tested using small and large deflection analysis. The large deflection analysis generally showed good agreement between the theoretical maximum stress on the buckled edge 4 and the experimental buckling position. The small deflection analysis showed that stress on the beam edge 4 for the section with 30mm flange width is a tensile stress, while buckling occurred due to compression. However, the large deflection analysis showed that it is a compressive stress.

FIG.'s 19, 20 and 21 show the experimental and theoretical values for the variation of the angle of twist. The large deflection analysis showed again better agreement with the experimental results particularly at high loads. Larger errors appeared using the large deflection analysis for the new sections tested. This is due to two main reasons:

- (1) The theory assumes a linear relation between angle of twist and the distance along the beam which is not the case for the new sections tested. This can be noticed from the ϕ -z diagrams shown in FIG.'s 19, 20, 21.
- (2) The theory considers small angles of twist which is not the case for the new sections tested (angles exceeded 30°).

Equation (5) shows the relation between the angle of twist of a cantilever and the distance along the beam as a hyperbolic function. It is noticed that for the sections tested $\sinh \mu z$ cancels $\tanh \mu L \cos \mu z$ and $\tanh \mu L$ tends to unity so that the angles of twist ϕ_z and ϕ_L can be written as:

$$\phi_z = \frac{T}{\mu GJ} (1 - \mu z) \quad \text{and}$$

$$\phi_L = \frac{T}{\mu GJ} (1 - \mu L)$$

Therefore a new relation between ϕ_Z and ϕ_L can be written in the form

$$\phi_Z = \frac{1 - \mu_Z}{1 - \mu_L} \phi_L$$

An attempt has been made to use this relation instead of assumption (1) in the large deflection analysis. It did not give good agreement with experiment and it gave almost the same results as those obtained by the small deflection analysis.

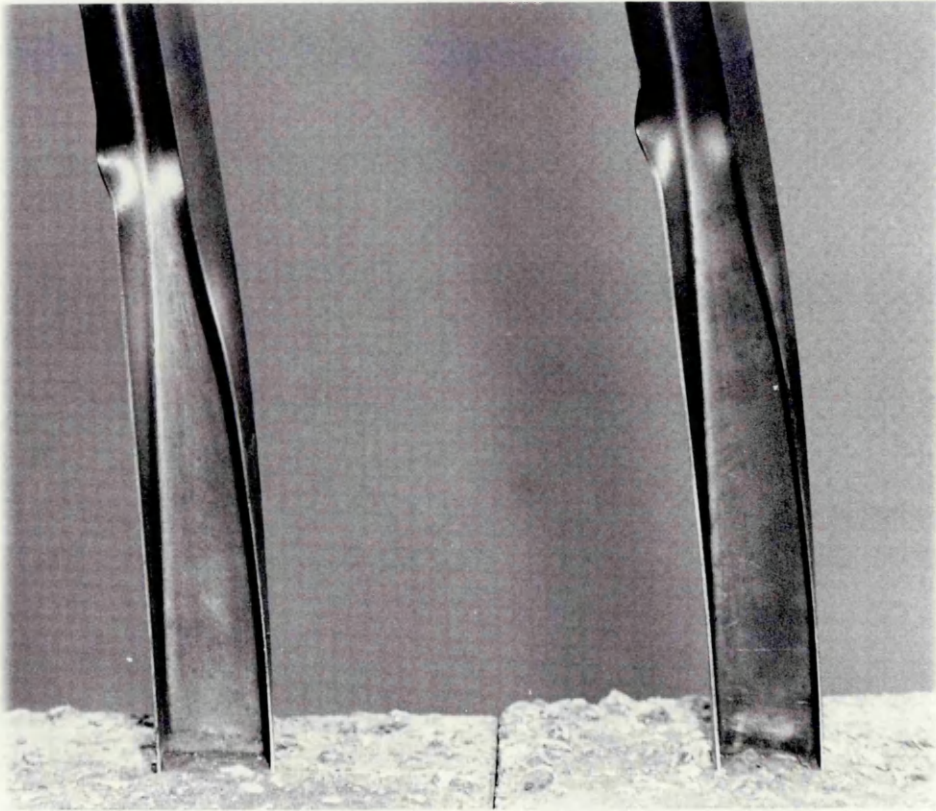


FIG.1. POSITION OF BUCKLING FOR THE BEAMS
LOADED THROUGH THE CENTROID.

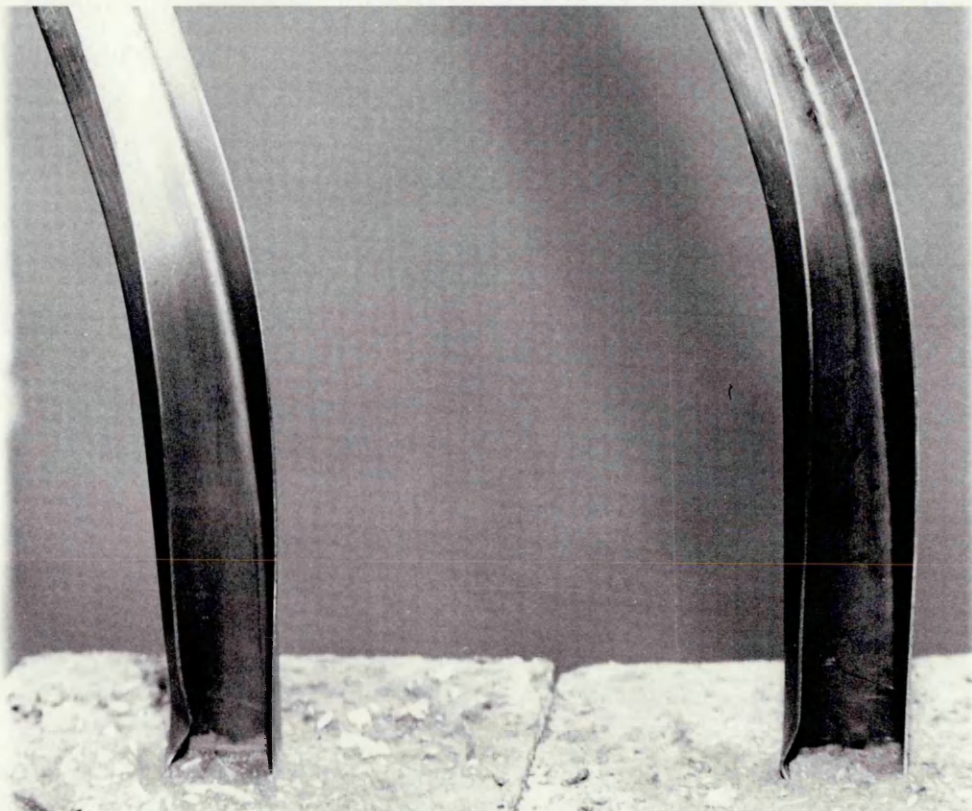


FIG.2. POSITION OF BUCKLING FOR THE BEAMS
LOADED THROUGH THE SHEAR CENTRE.

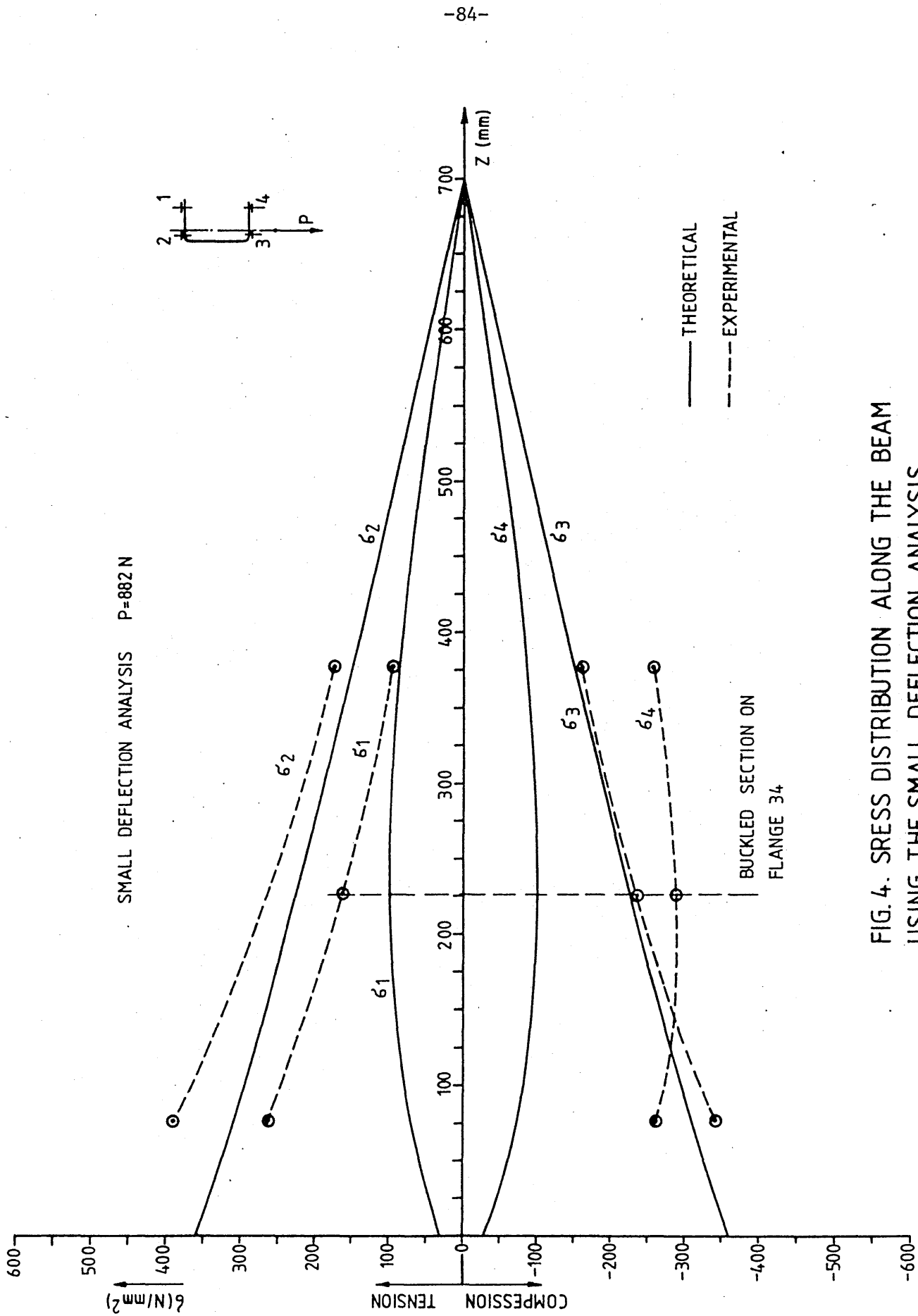


FIG. 4. STRESS DISTRIBUTION ALONG THE BEAM
USING THE SMALL DEFLECTION ANALYSIS

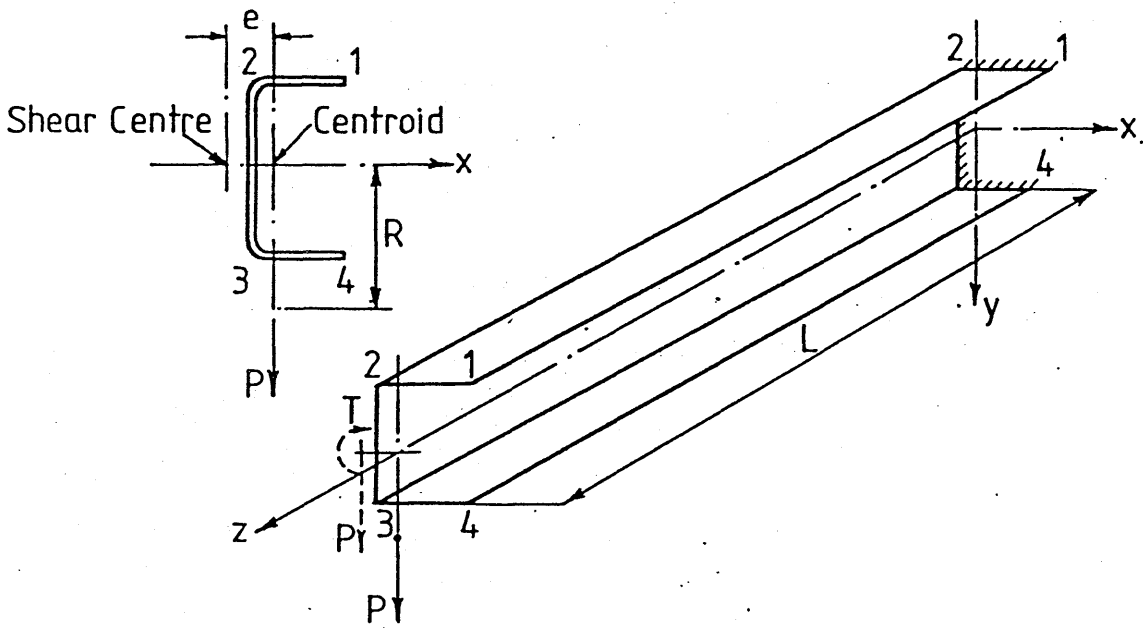
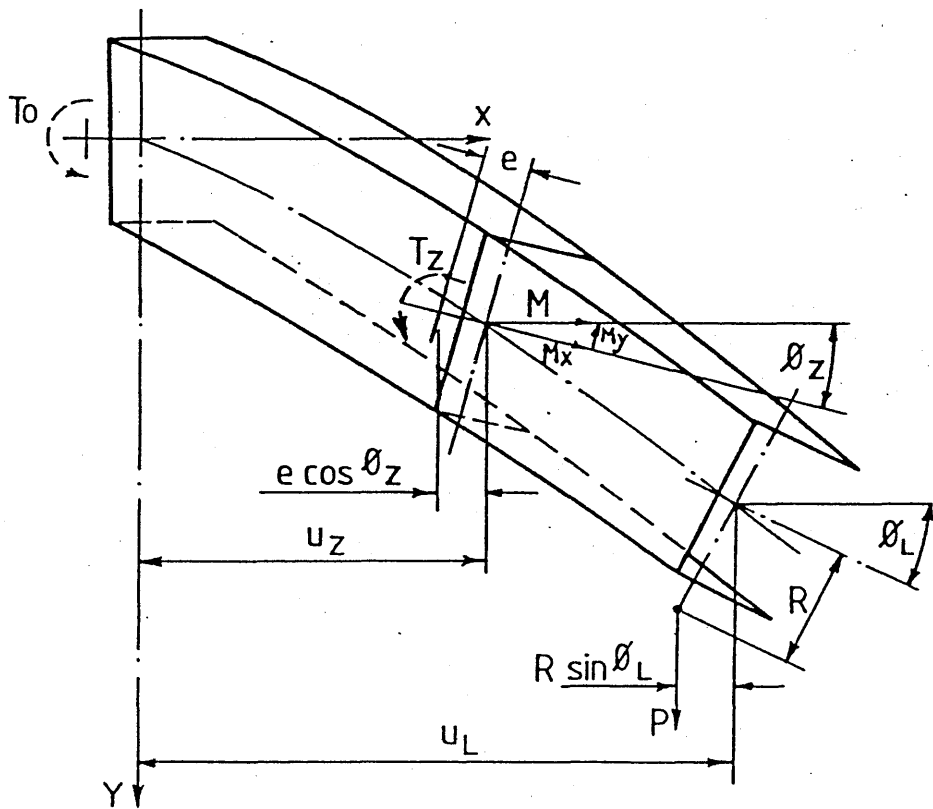


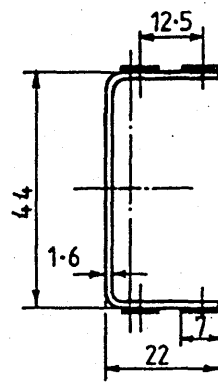
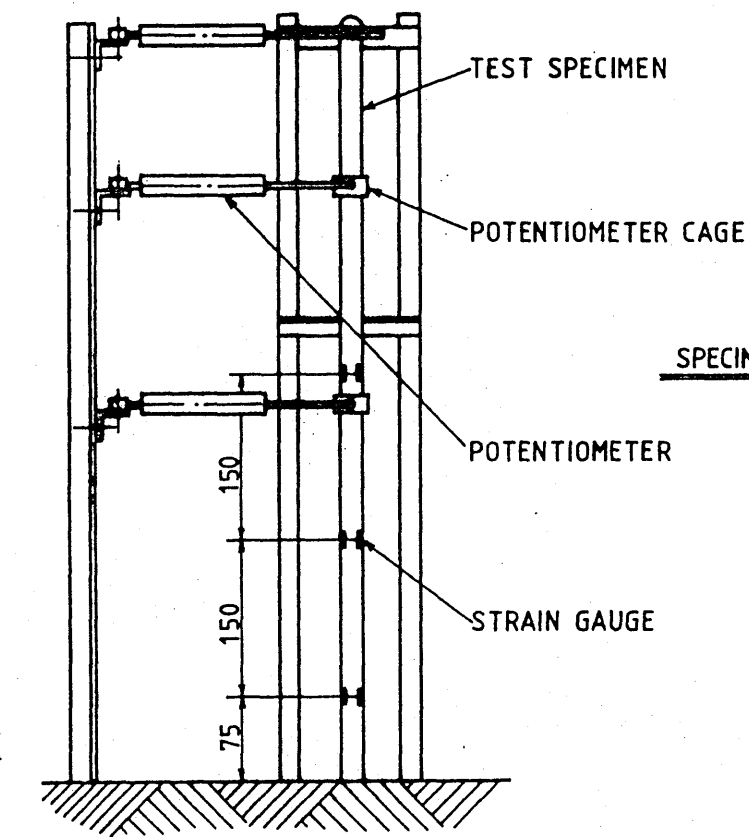
FIG. 3. CANTILEVER THIN WALLED CHANNEL SECTION BEAM LOADED THROUGH THE CENTROID AT THE FREE END



$$T(z) = -P(u_L - u_Z - R \sin \theta_L + e \cos \theta_Z)$$

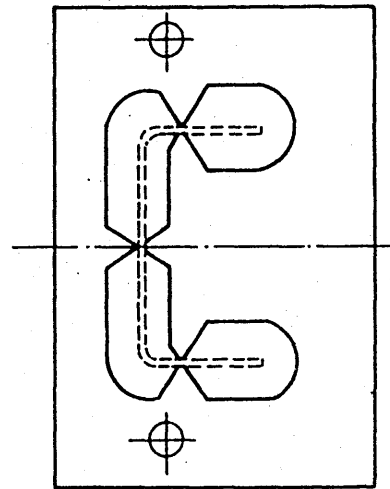
θ_L is a negative angle

FIG. 6. CHANNEL SECTION CANTILEVER WITH LARGE DEFLECTION



BEAM LENGTH $L=700\text{mm}$

SPECIMEN SECTION WITH STRAIN GAUGES



LOADING BRACKET

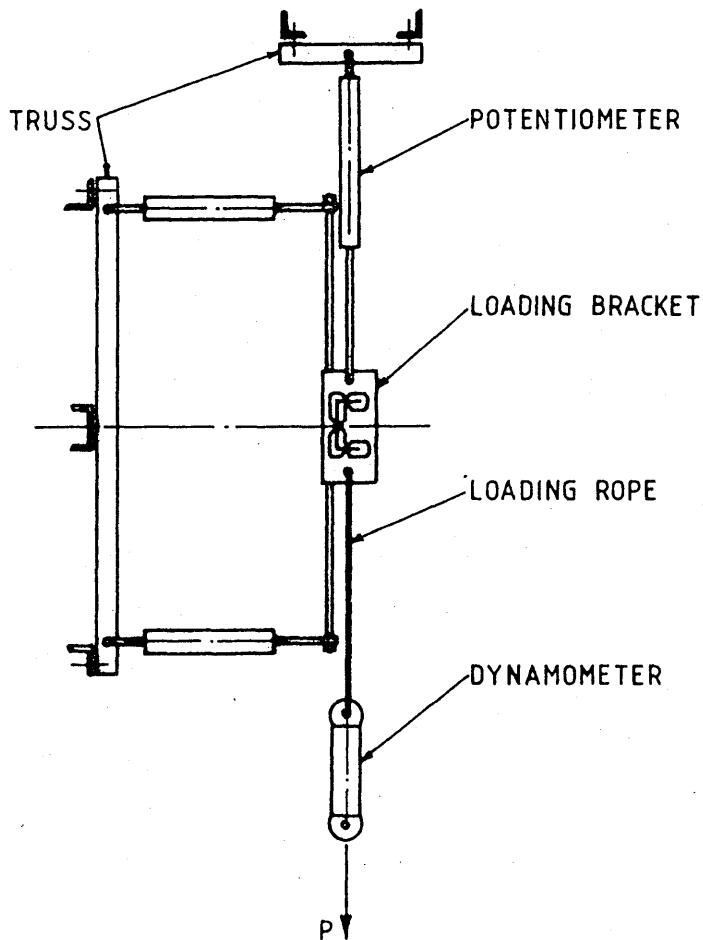


FIG. 5. TEST RIG FOR MEASURING THE STRESS DISTRIBUTION ALONG BEAM EDGE AND TO VERIFY THE ASSUMPTION

$$\phi_Z = \phi_L \cdot \frac{Z}{L}$$

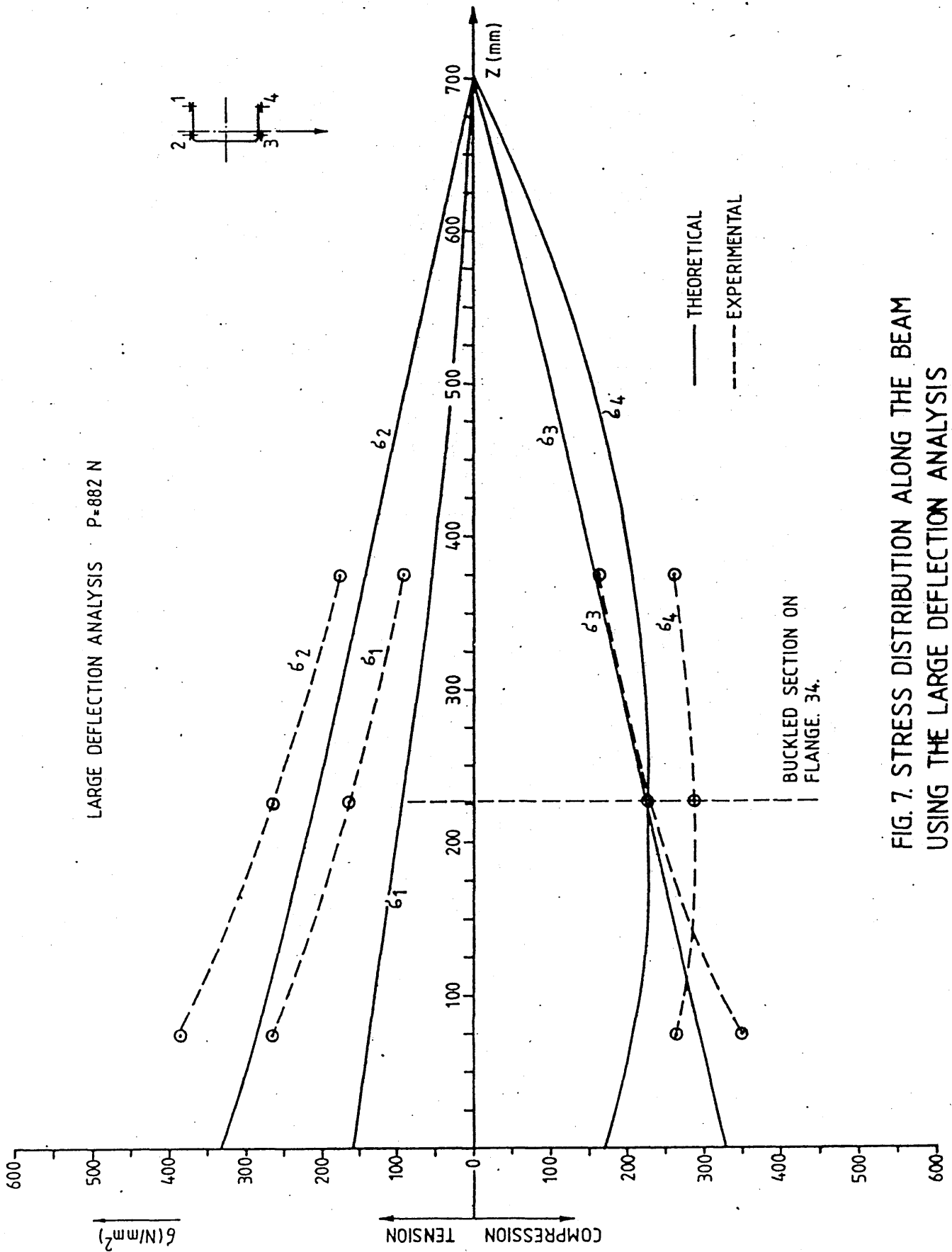


FIG. 7. STRESS DISTRIBUTION ALONG THE BEAM USING THE LARGE DEFLECTION ANALYSIS

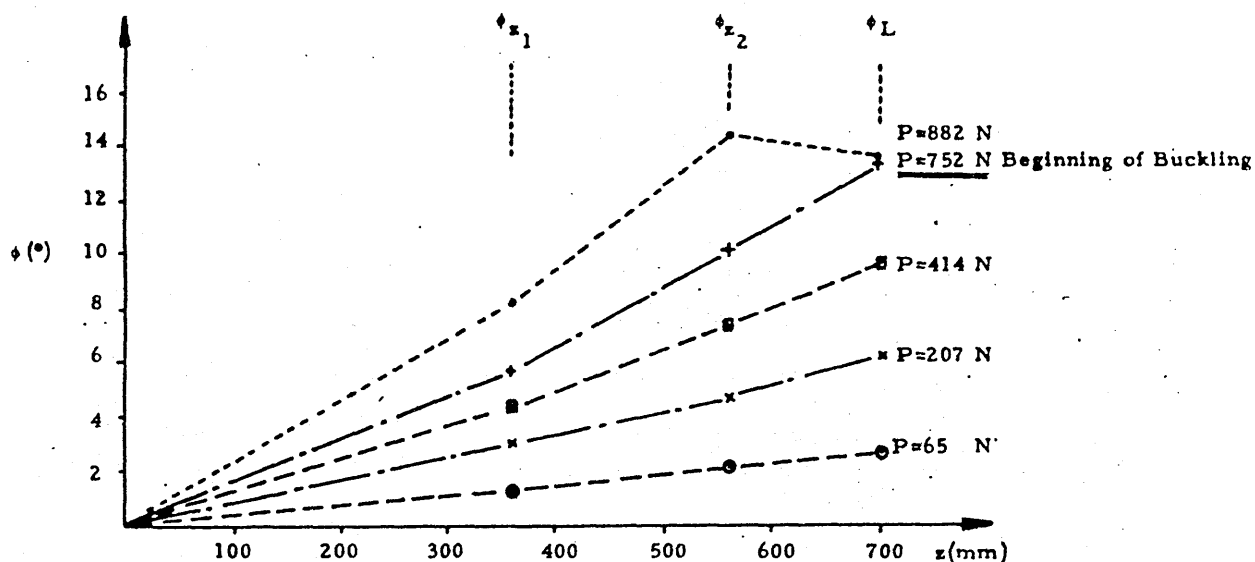


FIG. 8. EXPERIMENTAL VALUES FOR ANGLE OF TWIST ALONG THE BEAM AT DIFFERENT LOADS

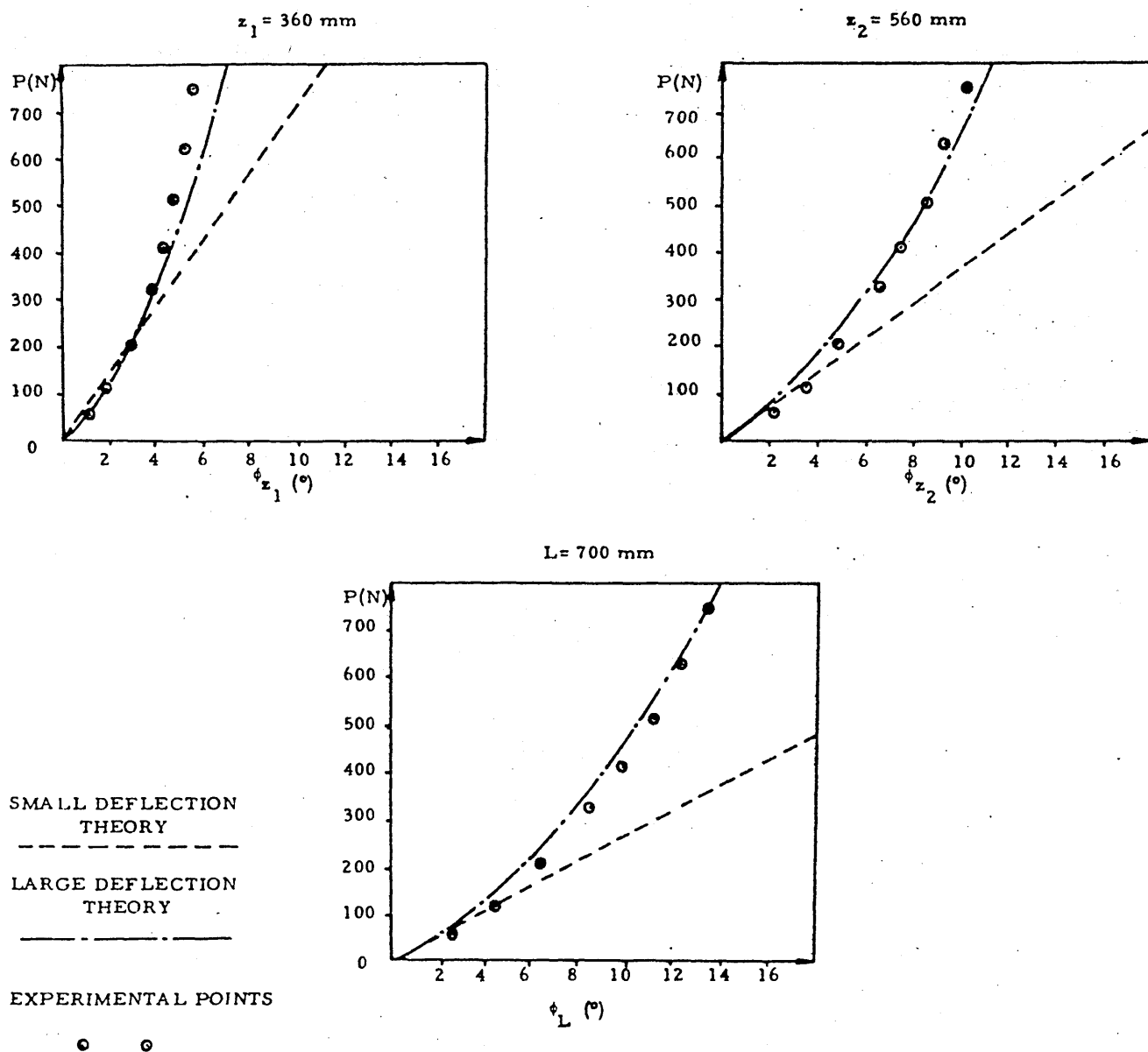


FIG. 9. THEORETICAL AND EXPERIMENTAL RELATION BETWEEN ANGLE OF TWIST AND BEAM LOAD.

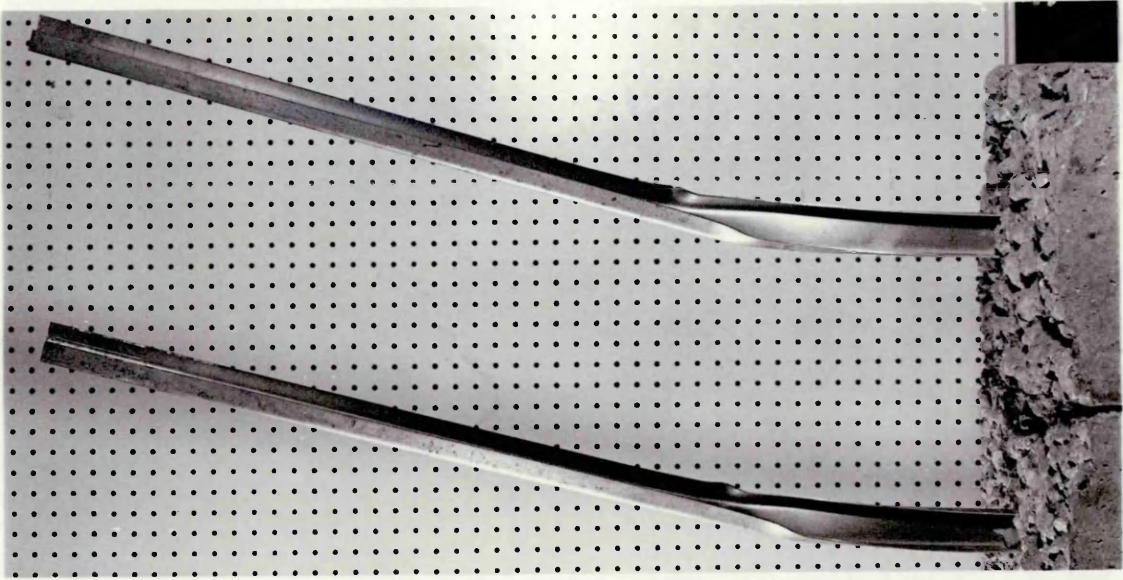


FIG. 10

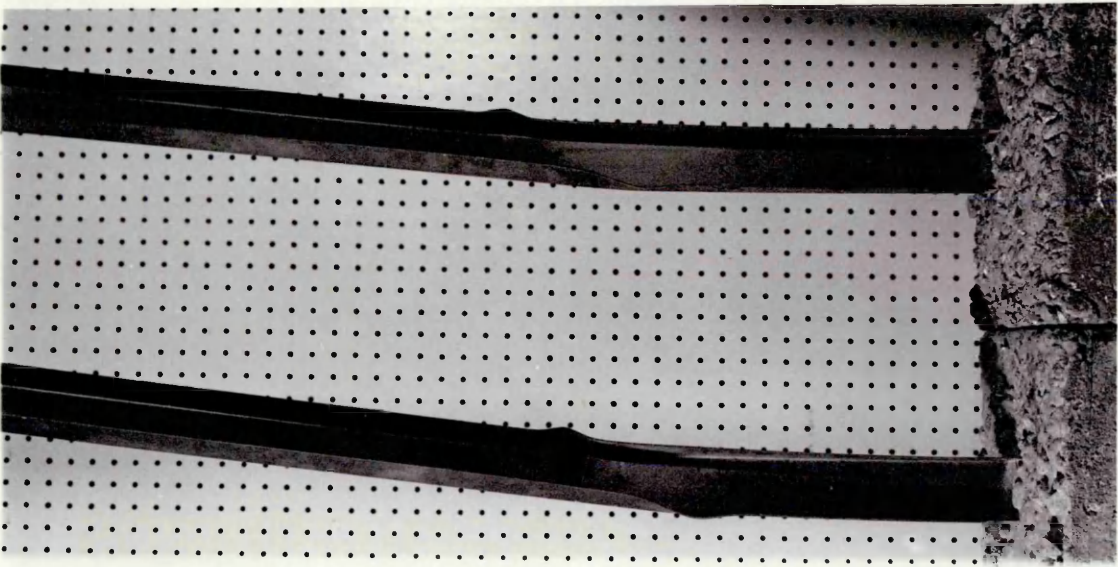


FIG. 11

POSITION OF BUCKLING FOR THE BEAMS TESTED.

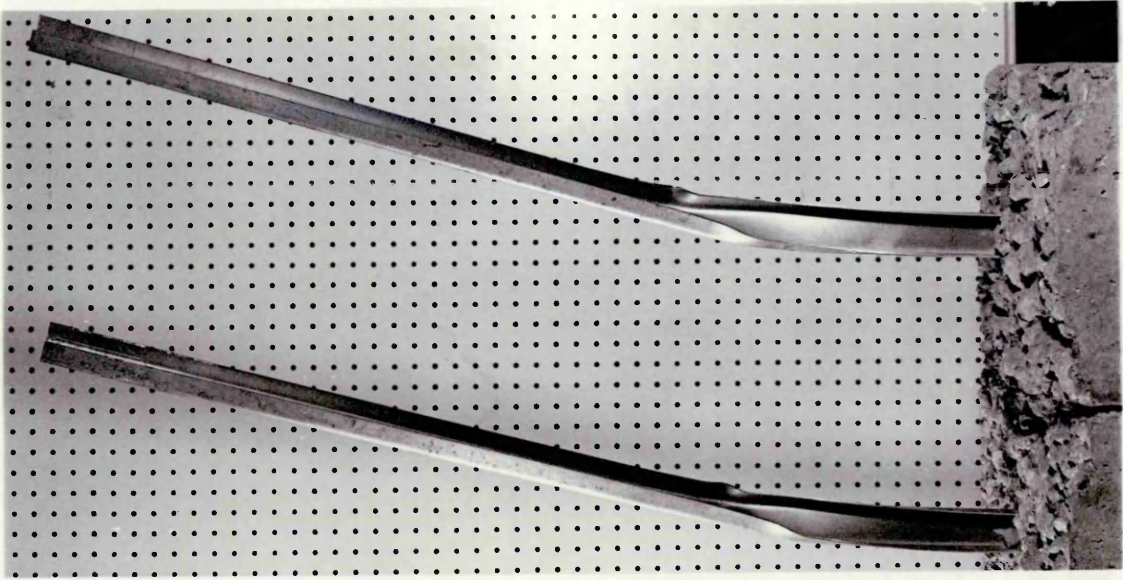


FIG. 12

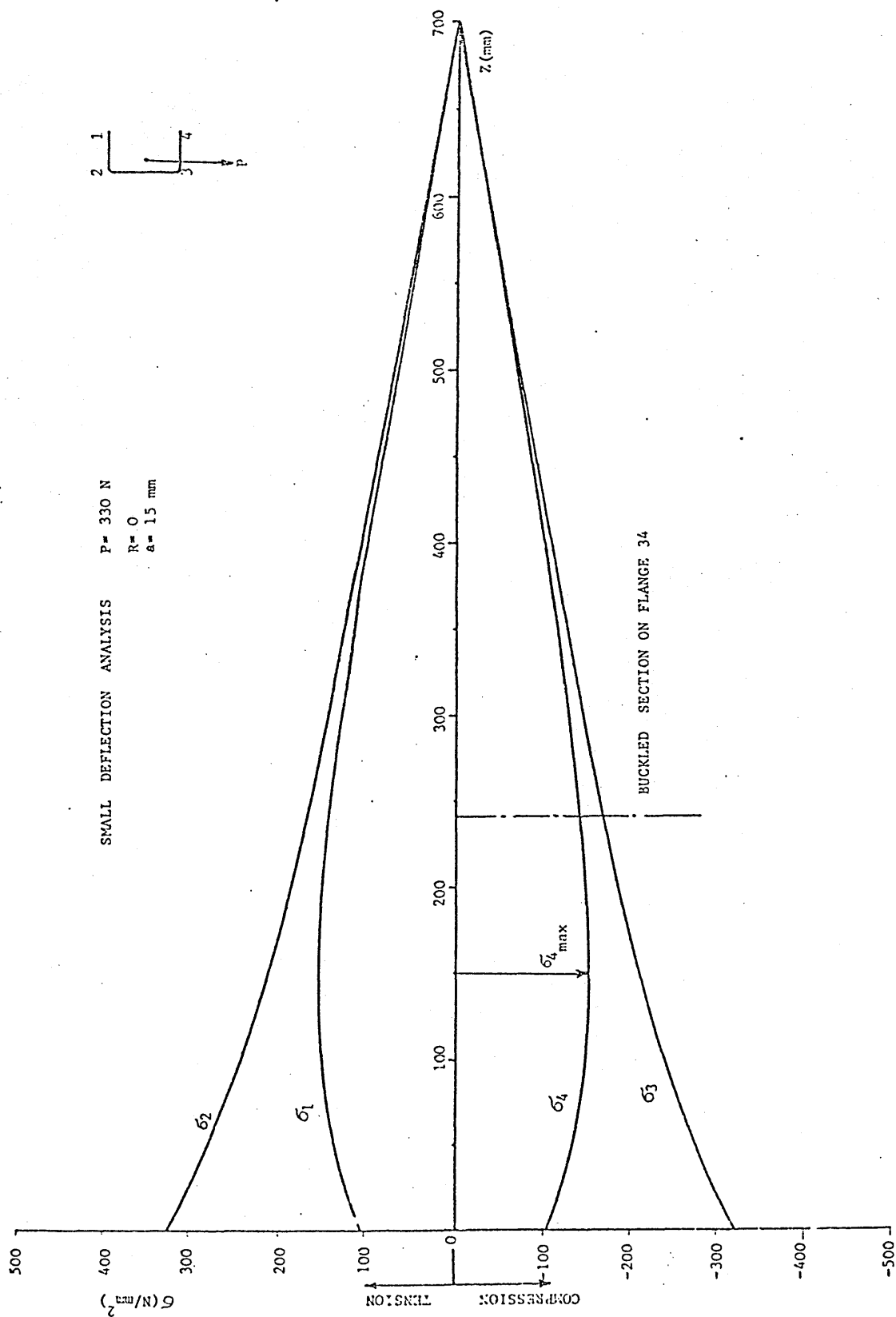


FIG. 13. STRESS DISTRIBUTION ALONG THE BEAM WITH FLANGE WIDTH = 15mm
USING THE SMALL DEFLECTION ANALYSIS .

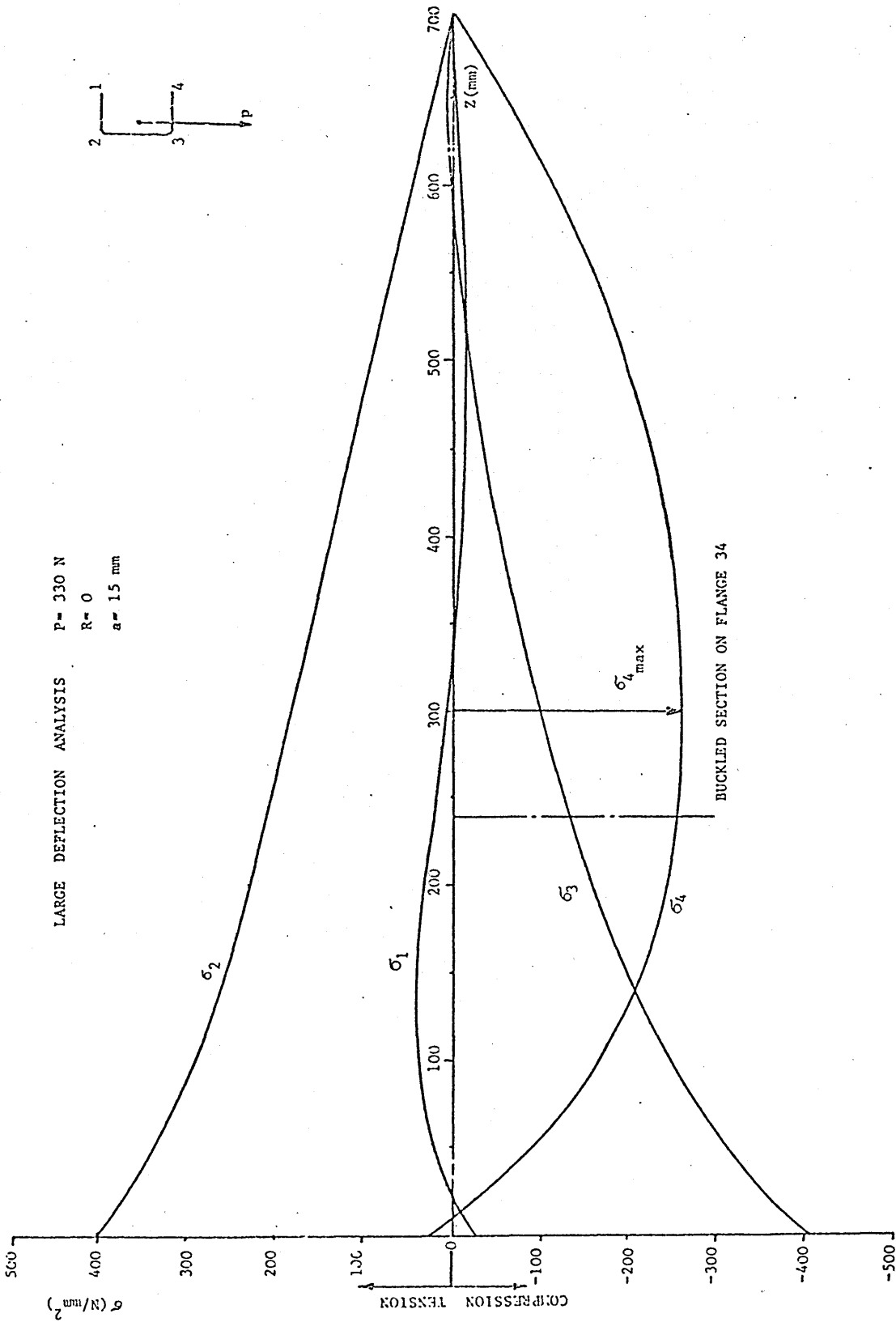


FIG. 14. STRESS DISTRIBUTION ALONG THE BEAM WITH FLANGE WIDTH -15 mm
 USING THE LARGE DEFLECTION ANALYSIS.

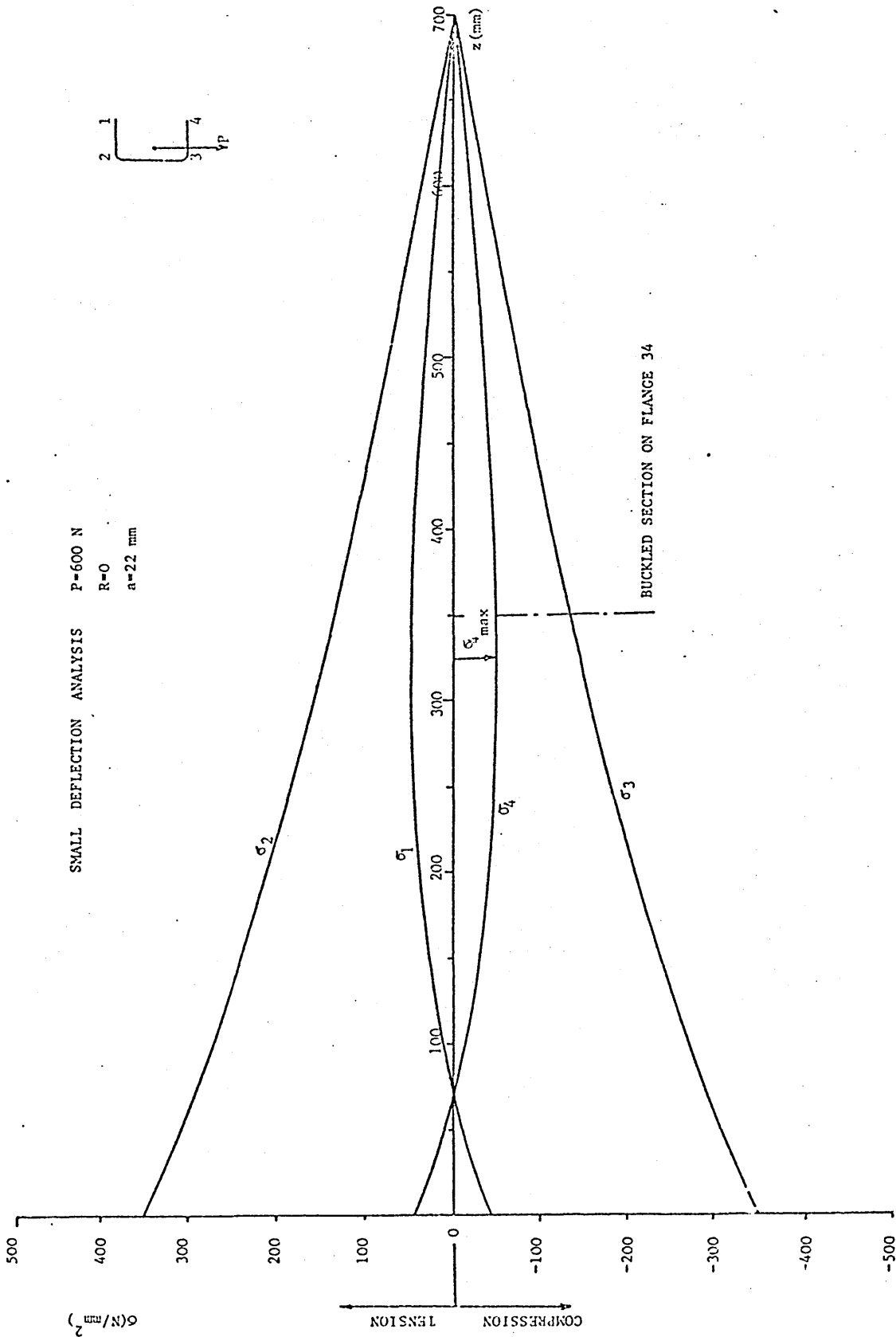


FIG. 15. STRESS DISTRIBUTION ALONG THE BEAM WITH FLANGE WIDTH $=22 \text{ mm}$
 USING THE SMALL DEFLECTION ANALYSIS.

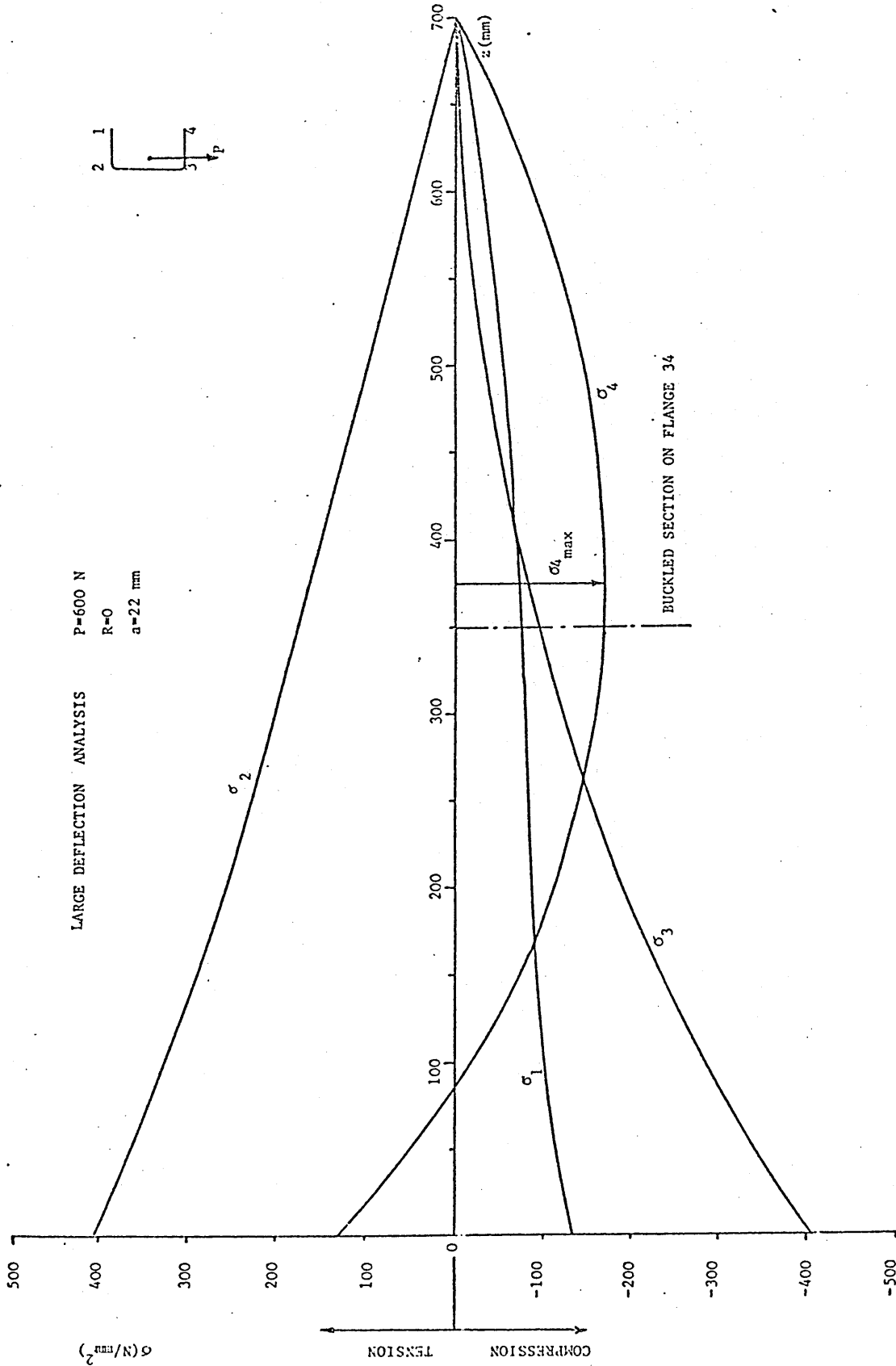


FIG.16. STRESS DISTRIBUTION ALONG THE BEAM WITH FLANGE WIDTH $=22\text{mm}$

USING THE LARGE DEFLECTION ANALYSIS.

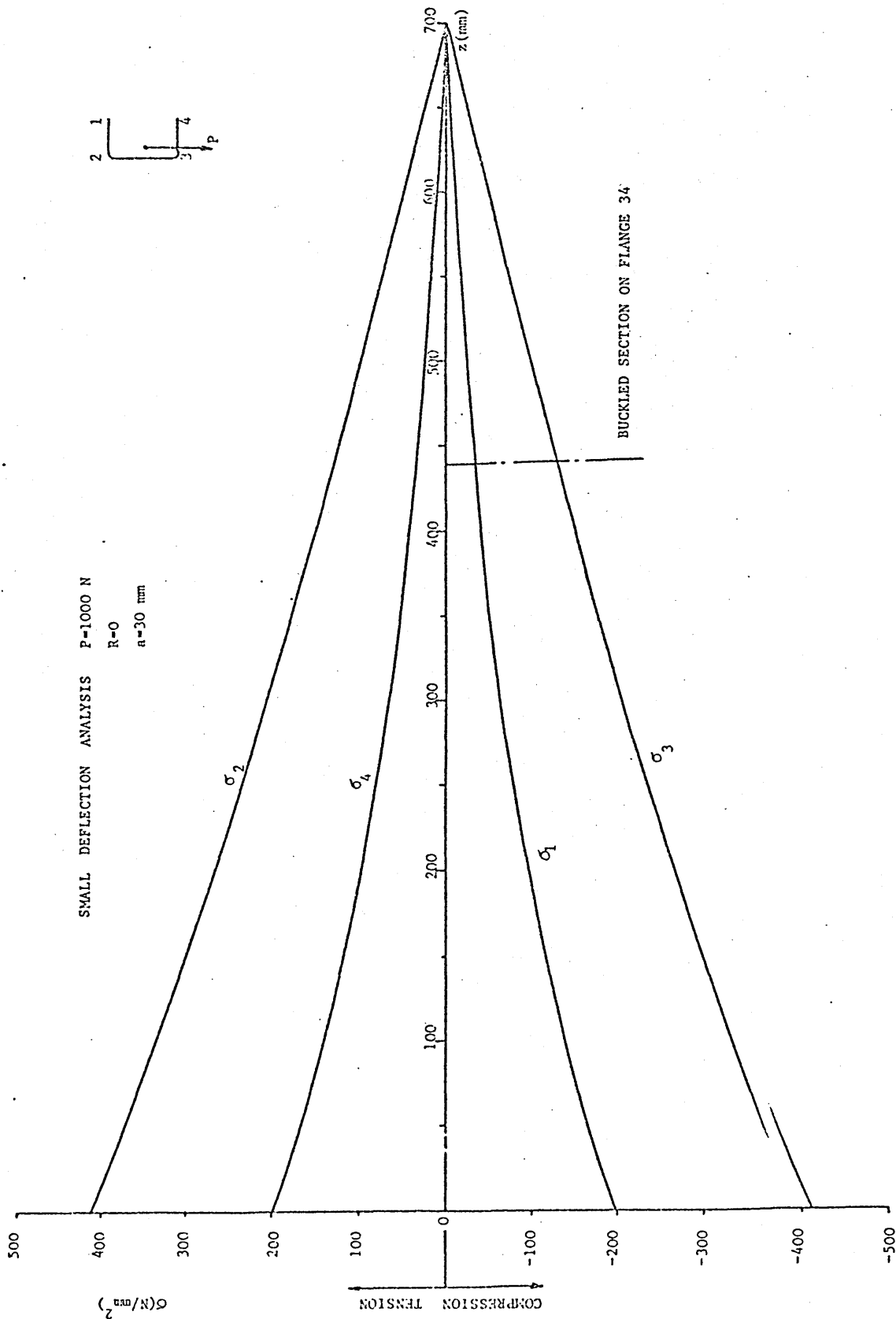


FIG.17. STRESS DISTRIBUTION ALONG THE BEAM WITH FLANGE WIDTH = 30 mm
 USING THE SMALL DEFLECTION ANALYSIS.

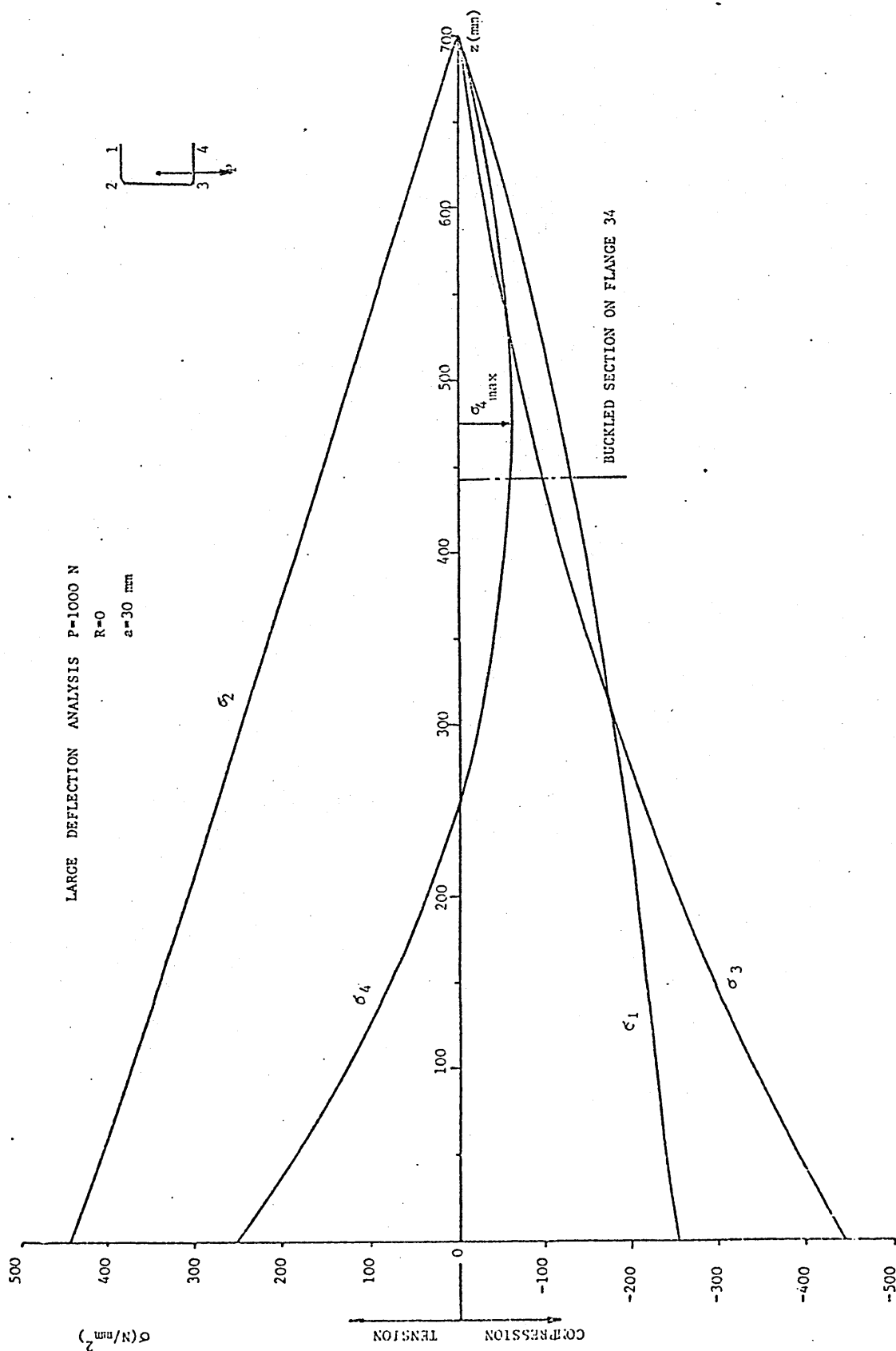
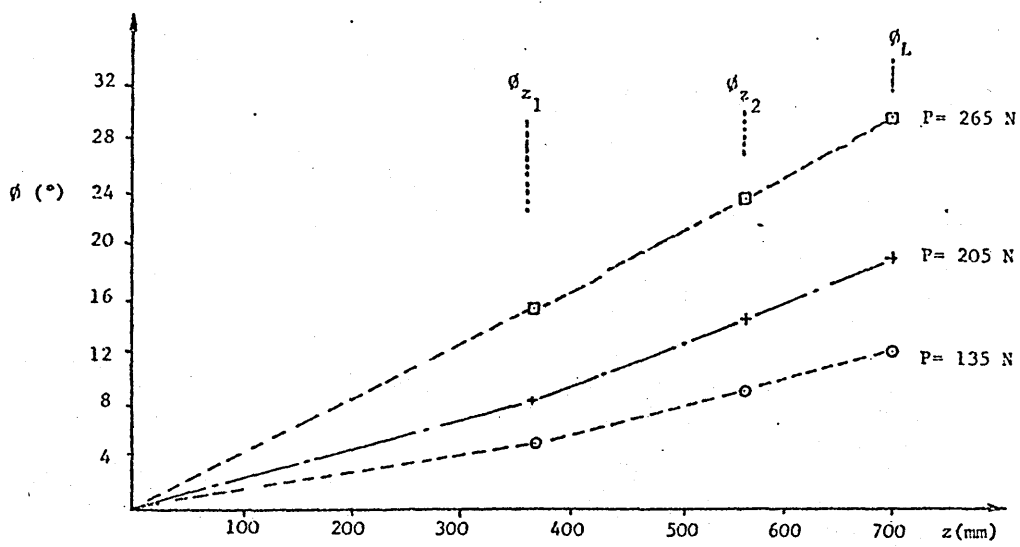


FIG. 18. STRESS DISTRIBUTION ALONG THE BEAM WITH FLANGE WIDTH $a=30\text{ mm}$ USING THE LARGE DEFLECTION ANALYSIS.



EXPERIMENTAL VALUES FOR ANGLE OF TWIST ALONG THE BEAM AT DIFFERENT LOADS.

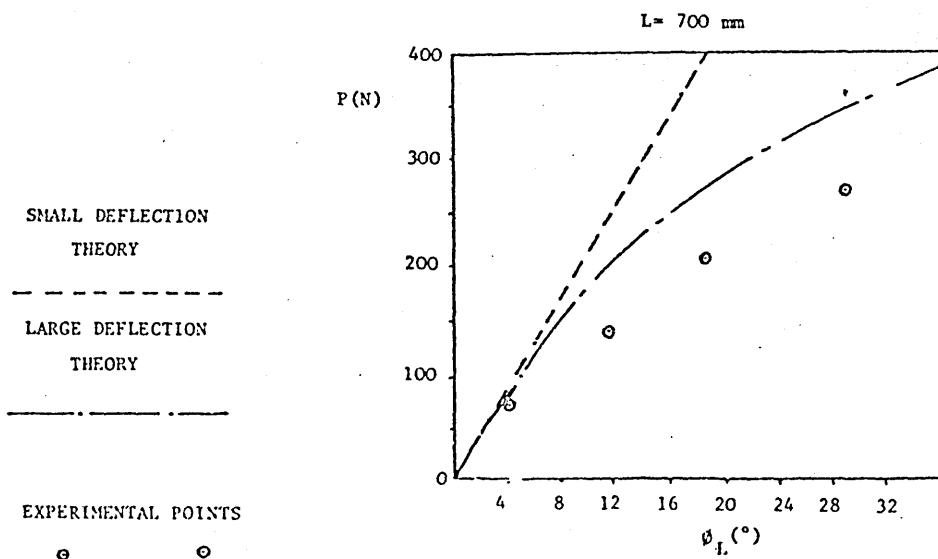
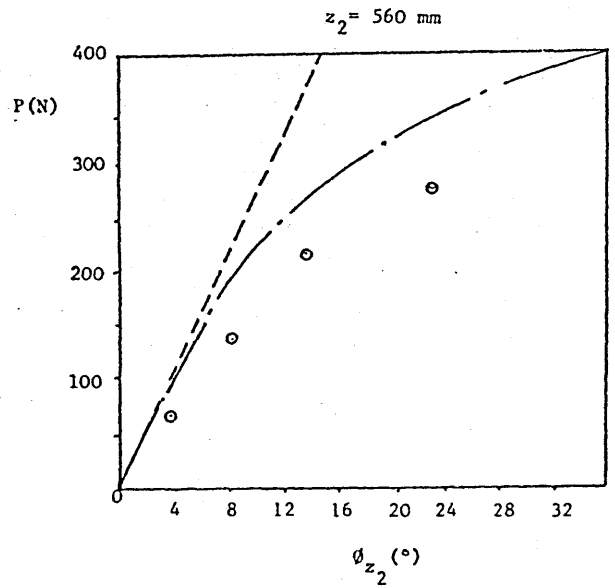
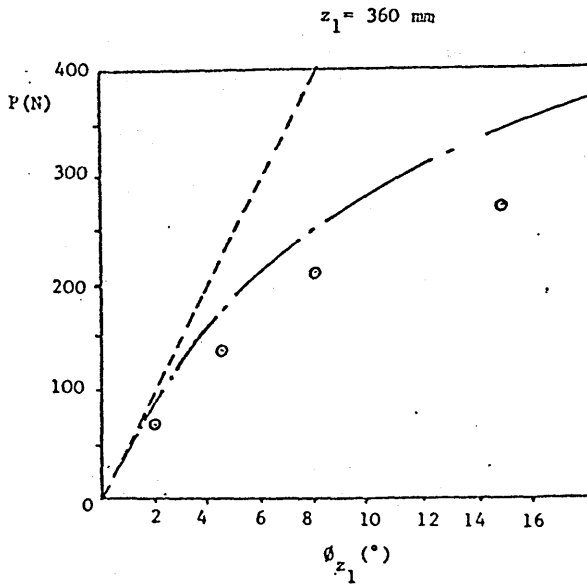
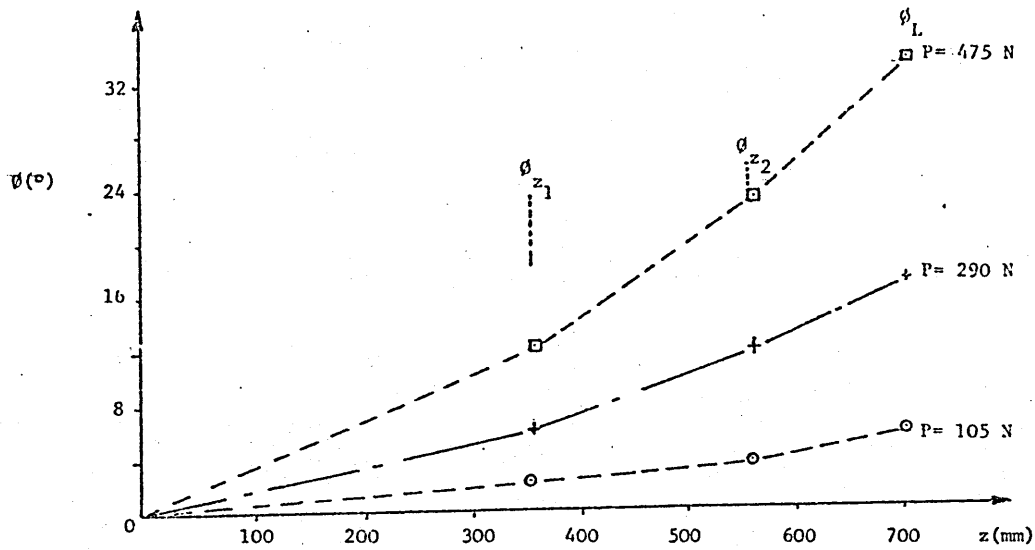


FIG. 19. THEORETICAL AND EXPERIMENTAL RELATION BETWEEN ANGLE OF TWIST AND LOAD FOR THE BEAM WITH FLANGE WIDTH ≈ 15 mm.



EXPERIMENTAL VALUES FOR ANGLE OF TWIST ALONG THE BEAM AT DIFFERENT LOADS.

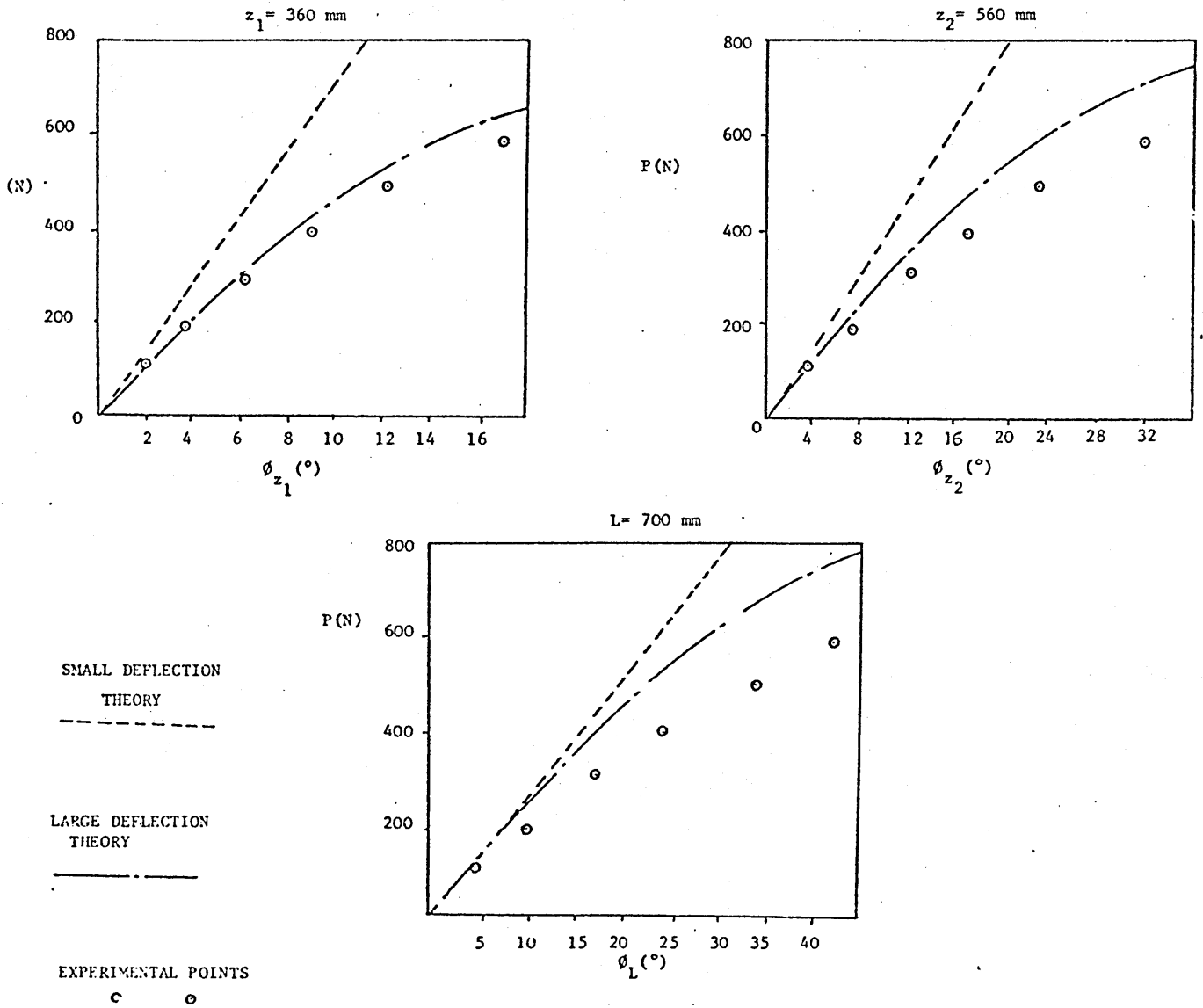
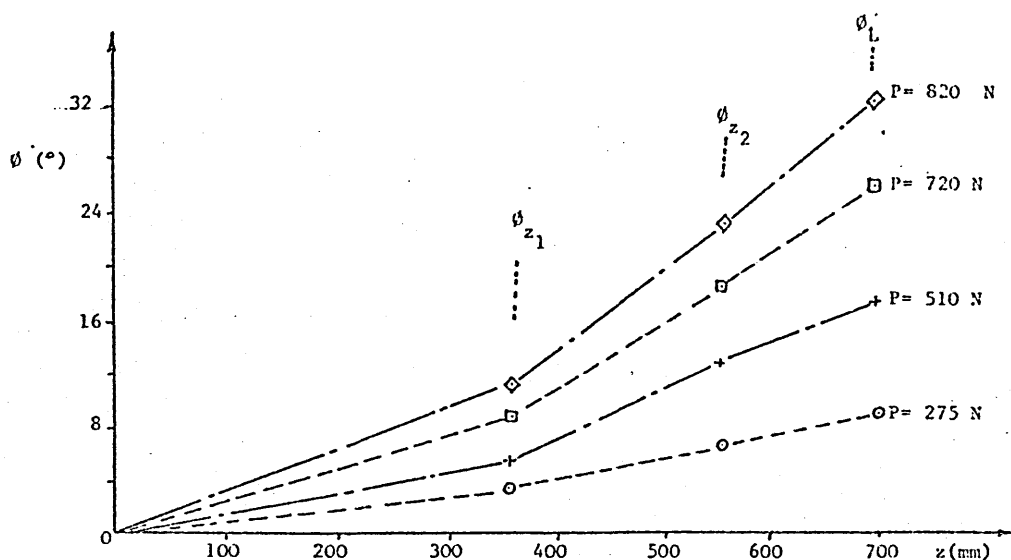
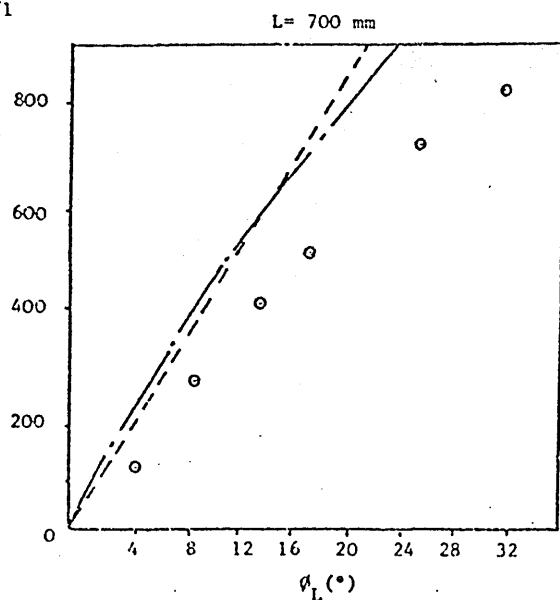
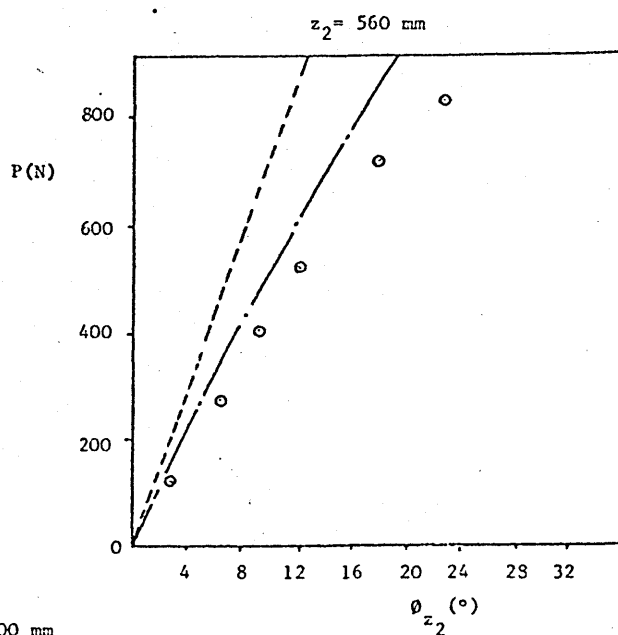
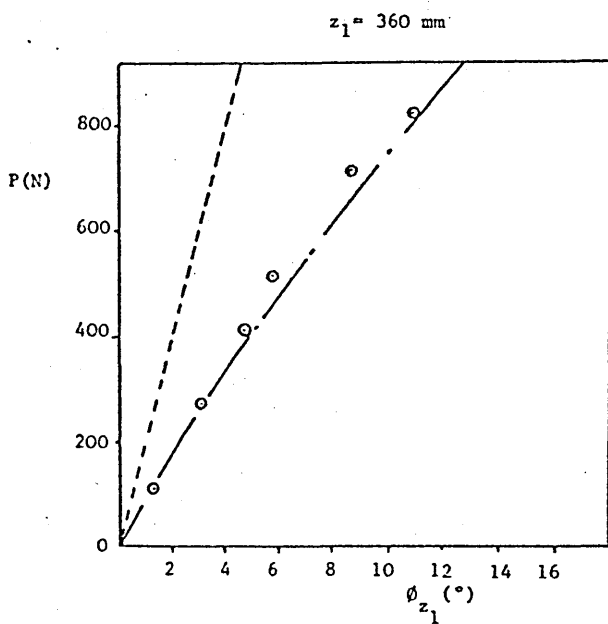


FIG. 20. THEORETICAL AND EXPERIMENTAL RELATION BETWEEN ANGLE OF TWIST AND LOAD FOR THE BEAM WITH FLANGE WIDTH = 22 mm.



EXPERIMENTAL VALUES FOR ANGLE OF TWIST ALONG THE BEAM AT DIFFERENT LOADS.



SMALL DEFLECTION
THEORY

LARGE DEFLECTION
THEORY

EXPERIMENTAL POINTS

○ ○

FIG. 21. THEORETICAL AND EXPERIMENTAL RELATION BETWEEN ANGLE OF TWIST AND

LOAD FOR THE BEAM FROM FIGURE 20.

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

A new method for bus body construction was introduced recently. This method claimed to be more economical than the existing methods. It utilizes a composite material for building two standard modular panels, one type for the sides of the vehicle and the other for the roof. The composite material used consists of rigid skin polyurethane foam as a matrix and thin walled metal sections as a reinforcement. The original design contained aluminium and steel sections for reinforcement. However, the research in this thesis was made on those reinforced by steel sections only. A further investigation can be made for beams reinforced with aluminium or other types of reinforcement such as fibre glass and carbon fibres.

A bus roll over accident causes high structural deformations. These deformations must not exceed certain limits and intrude into the passengers survival space. This introduces a new aspect to the design problem, namely the study of large deflection behaviour of structural members. This mainly identifies three safety criteria for structural members (defined early in Chapter 1 of this thesis).

- 1) Maximum strength
- 2) Energy absorbing ability
- 3) Remaining strength

The preliminary tests of cantilever beams made from this composite material in bending to high deformations showed a good performance from the point of view of safety criteria. A comparison was made between the composite beam with the constituent beams and also with an equivalent energy absorbing optimum rectangular steel tube. The comparison with the constituents showed that the composite beam has performed more effectively in respect of safety criteria. This was because the foam prevented the thin walled steel reinforcement from buckling locally. In other words, the steel reinforcement performed plastically and did not collapse. The comparison with the equivalent optimum steel tube showed that the composite beam has higher remaining strength and the ability to absorb higher energy if higher deformations are applied. It also showed that by optimizing the shape of the reinforcement a weight saving can be achieved.

Elastic analysis of beams is a classical design aspect. The main aim of elastic analysis is to predict beam stiffness. This relates the increment of external load with the appropriate increment of deflection. Elastic deflections are usually small in comparison with inelastic deflections. So stiffness is significant only for small deflection behaviour. The aim of the elastic analysis of composite beams introduced in this thesis was to form a suitable base for the inelastic analysis.

Investigators usually choose simply supported beams to predict elastic stiffness. This is because the theoretical boundary conditions of a simply supported beam are more close to reality compared with the boundary conditions of a cantilever. Many investigators published several methods to predict elastic stiffness of composite beams using different approaches. Most of these investigations were made for simply supported sandwich beams. Only one approach from these was suitable to

be a basis for the inelastic analysis. This was using the interfacial shear method. It assumes an interfacial shear between the faces and the core of the sandwich beam. It also considers the whole mechanical properties of the beam constituents particularly the core (matrix) bending stiffness which is neglected by other methods. This is a very important parameter in the inelastic analysis. Thus, the interfacial shear method was modified to predict the stiffness of a cantilever sandwich beam. Also a new theory using the same approach was introduced to predict the stiffness of composite beams with a general shape of reinforcement for uniaxial bending.

A comparison was made in Chapter 2 between the new elastic analysis and the analysis using the engineer's theory of bending. This showed that the thesis theory gave better agreement with the experimental values of beam stiffness. While some errors remain, the theory is sufficient for the present study. For further study if more exact elastic analysis is needed, research should be directed to simply supported beams. This may probably give better results using the same assumptions considered in this thesis.

Inelastic bending analysis of beams explains the behaviour when the relation of the increment of external load with the appropriate increment of deflection becomes nonlinear. This is either due to the change of the material at the extreme external fibres to the yield state or to local buckling, particularly in thin walled sections. In thick walled and solid sections the load-deflection curve slope after yield transformations soon becomes zero or positive. This depends upon the strain hardening behaviour of the beam material. Otherwise, in thin walled sections the slope becomes negative due to the beam collapse.

The preliminary tests made to investigate the foam material behaviour showed that it behaves almost elastically until failure occurs. It was also concluded from the preliminary tests of the composite beams that the load deflection relation changed from linearity to nonlinearity due to yielding of the reinforcement. Thus, the load which caused the beam to yield (yield load) was found by substituting the value of the yield stress in the equation expressing the stress in the reinforcement. This gave good agreement with the experimental results. A small error in the prediction of the yield load was noticed for the beams reinforced with folded I section. This is mostly due to the strain hardening effect gained during folding of the reinforcement.

TIMOSHENKO and JAMES curvature - area method was used to derive the inelastic bending analysis of composite beams introduced in this thesis. This analysis has shown very good agreement with the experimental results. In order to establish inelastic bending analysis for more complicated structures, more sophisticated numerical techniques are needed. The mechanical properties of the composite beam constituent must be determined using the methods described in Chapter 4 to obtain satisfactory results from the analysis.

The maximum strength of composite beams was predicted by finding the sum of the load capacities of both the steel and the foam. The value of ultimate stress of the foam σ_{FO}^{max} varies significantly from one

beam to another. This makes the maximum strength of the composite beams unpredictable. But certain improvements in the design of the mould (such as providing hot water jackets to maintain constant temperature during the foaming) will improve and stabilize the character of the foam. Another possible method of improving the stability of the foam is by decreasing the foam density. This will decrease the number of air bubbles in the beam skin material since a longer creaming time will be available.

The composite beams were optimised for uniaxial bending. A comparison was made between this optimum composite and an equivalent energy absorbing optimum steel tube. The comparison showed that weight saving has been achieved. This comparison is not absolutely fair because rectangular steel tubes have better performance in the perpendicular direction. So, a further study is needed to optimise the composite beams for biaxial bending to justify the comparison.

In accident situations structural members are subjected to impact conditions. So, it is important to know the behaviour of composite beams due to impact loading. Several beams were tested using a special pendulum at different speeds, and impact energies. In these tests the composite beams performed adequately. The behaviour of the beams was similar for both static and impact loading.

Hence the theory developed for predicting beam behaviour can be used safely to estimate the performance due to impact loads.

The energy absorbing capacity of the beams tested using the pendulum was limited. This makes it impossible to raise the speed of impact above a certain value. Thus, to investigate the behaviour of composite beams at higher speeds ($> 8\text{m/sec} \approx 18\text{ mile/hr}$), two alternatives are available: either test stronger beams or use lighter pendulum arms than the existing design.

The large deflection theory developed here which is based on VLASOV's General theory of thin walled beams satisfactorily explains the buckling behaviour noticed during testing of light channel sections. While some quantitative errors remain, the theory is sufficient to indicate the position along the beam where the maximum stress will occur to cause buckling.

In order to predict a general large deflection theory which can be applied on different section dimensions a non-linear relation between the angle of twist and the distance along the beam must be considered. This relation also must include the section characteristics. The non-linear coefficients can probably be obtained using limit analysis theories such as minimization of the strain energy. Also a wider testing program is needed in this case to confirm the theoretical analysis.

Thus, VLASOV's general theory for thin walled beams can be applied satisfactorily for small loads and deformations only. For high loads and deformations some modifications are needed to give a satisfactory result.

Finally, the thesis demonstrates a method which enables the designer to predict composite beam behaviour during accident situations and to incorporate this into the building of safe bus structures. It must be borne in mind that the whole study was made at room temperature and further research is needed to investigate the effect of temperature variations on the beam behaviour.

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APPENDIX

ERRORS IN PREDICTION OF THE STIFFNESS OF A CANTILEVER COMPOSITE BEAM

A.1. INTRODUCTION

The aim of the thesis was to investigate the large deflection behaviour of composite beams. So, the theoretical and experimental investigations were directed to cantilever beams (the reasons are given in Chapter 1). Elastic stiffness is significant for small deflection behaviour which represents the normal loading conditions of structural members. Usually, the experimental values of the stiffness of cantilever beams are lower than the theoretical values. This is due to difficulties in construction of a fully fixed support. Thus, in this appendix the results of a separate experiment to investigate the errors which may occur due to cantilever support movements will be given.

A.2. TEST METHOD

The experiment consisted of a cantilever beam test. One composite beam, with optimum cross section dimensions [shown in Fig (4.16) page 59] was chosen for the test. One end of the beam was set in a concrete block which has a square base with dimensions of 220 x 220 mm. The height of the block was 270 mm. The beam length measured from the concrete block to the point of loading was 650 mm.

The concrete block was fixed in a relatively rigid frame of four bolts as shown in Fig(A.1). The test rig shown in Fig (1.2.a) (page 9) was used to carry out the test with an addition of three capacitance probes (≈ 19 mm diameter) to measure the beam deflections near to the fixed end. These probes were attached to an independent frame to avoid the error which may occur due to the movement of the original frame used to fix the beam. The axis of the probes were positioned at distances of 20, 46, 72 mm from the fixed end of the beam. The probes were connected to a digital counter which reads the beam displacement with an accuracy of ± 0.01 mm. The accuracy of the electric dynamometer (indicating the load) was ± 4 N. The beam tip deflection readings, using a potentiometer had an accuracy of ± 0.1 mm.

A.3 TEST RESULTS

Fig (A.3) shows the beam deflections at the positions of the probes for different loads. These are shown by the dotted lines connecting the experimental points. The extrapolation of these curves (graphically using a french curve) to the fixed end of the beam showed that the concrete block has a translational motion. The theoretical deflection curves of the beam at the same loads were found according to beam deflection equation shown in line 13, page 18. Then, the origin of these curves were shifted to the point of intersection of the extrapolated curves and the y-axis.

The purpose of this was to compare the slopes of the theoretical and experimental curves. The dash and dotted lines in Fig (A.3) show the theoretical beam deflections shifted to the new origins at different loads. The comparison of theoretical and experimental curves shows that

the concrete support has also a rotational motion. This means that the support has both translation and rotation motions.

Fig (A.2) shows the theoretical and the real deformed central axes of the beam. For simplicity these axes will be considered straight. The error in the value of tip deflection, due to support movement, can be defined by the relation $y - y_0 = y_\delta + y_\theta$

where y is the experimental deflection
 y_0 is the theoretical deflection
 y_δ is the deflection due to support translation
and y_θ is the deflection due to support rotation.

Fig (A.4) shows the load deflection diagrams of the beam in the elastic state. The continuous line represents the theoretical relation according to Equation (2.12), page 18, which shows the beam stiffness (P/y_0 is 152 N/mm. The dashed line passing through the points shows the experimental results. The experimental average value of the beam stiffness (P/y) is 119 N/mm which gives an error of 21.7% with respect to the theoretical value. The dashed and dot line represents the corrected experimental diagram. This was obtained by subtracting the values of the deflections of the beam due to support movement ($y_\delta + y_\theta$) from the measured value of y , using the relation $y_\theta = L\theta$. The angle θ was determined graphically as shown in Fig (A.3). This made the stiffness value = 142 N/mm with an error of 6.6%

For a more exact investigation of this error, numerical methods can be used to extrapolate the experimental results to find the support translation and the angle of rotation. This can be made by assuming a third order polynomial (the same as theoretical beam deflections) passing through the experimental points. In this case, the deflections should be measured at four points near to the fixed end to determine the polynomial coefficients. However, for the first approximation the graphical technique is sufficient.

A.4 CONCLUSIONS

The theoretical analysis of composite cantilever beams introduced in this thesis gives a good estimation of beam stiffness for fully fixed beams. For other boundary conditions it is necessary to quantify the error which may occur due to support movements.

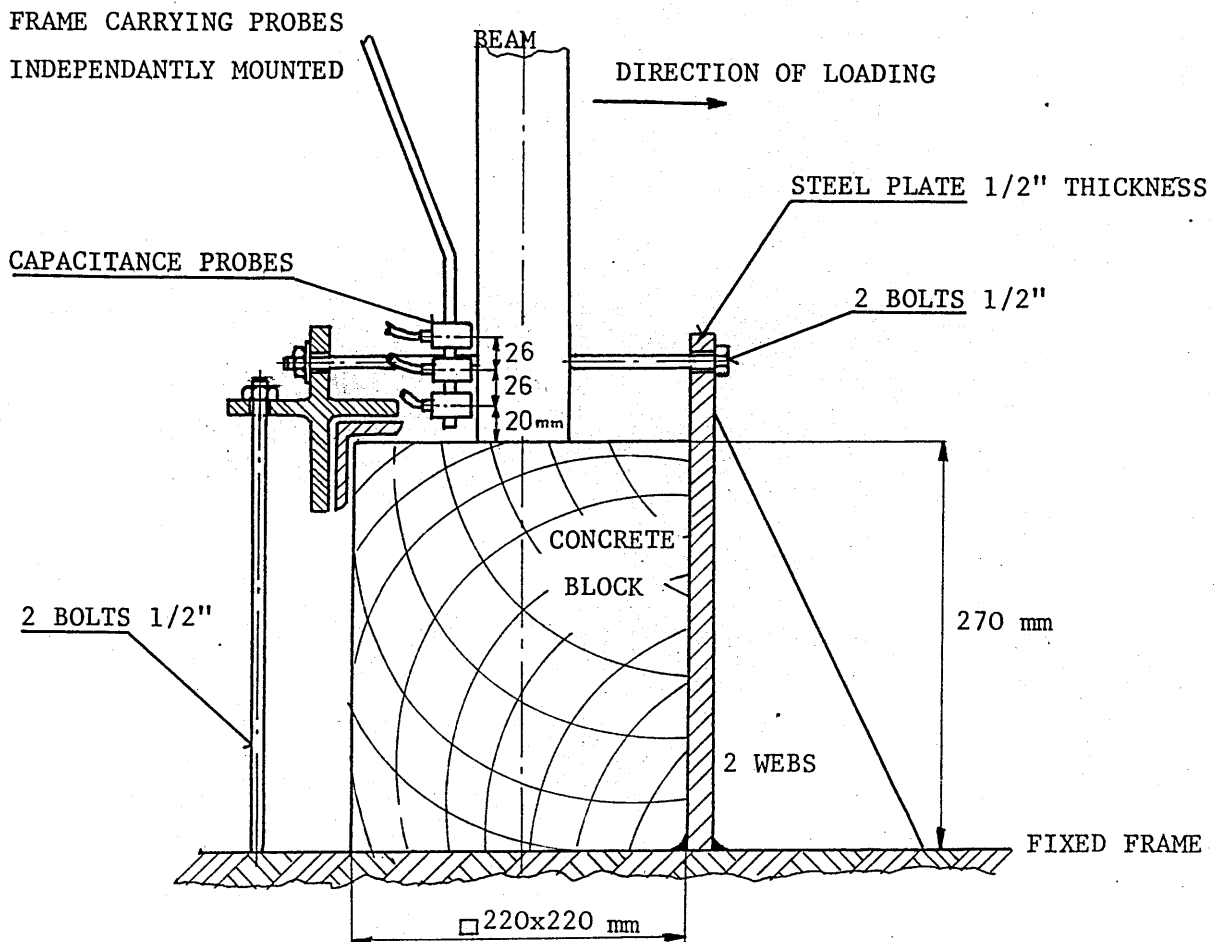


FIG.(A.1) BEAM FIXED END

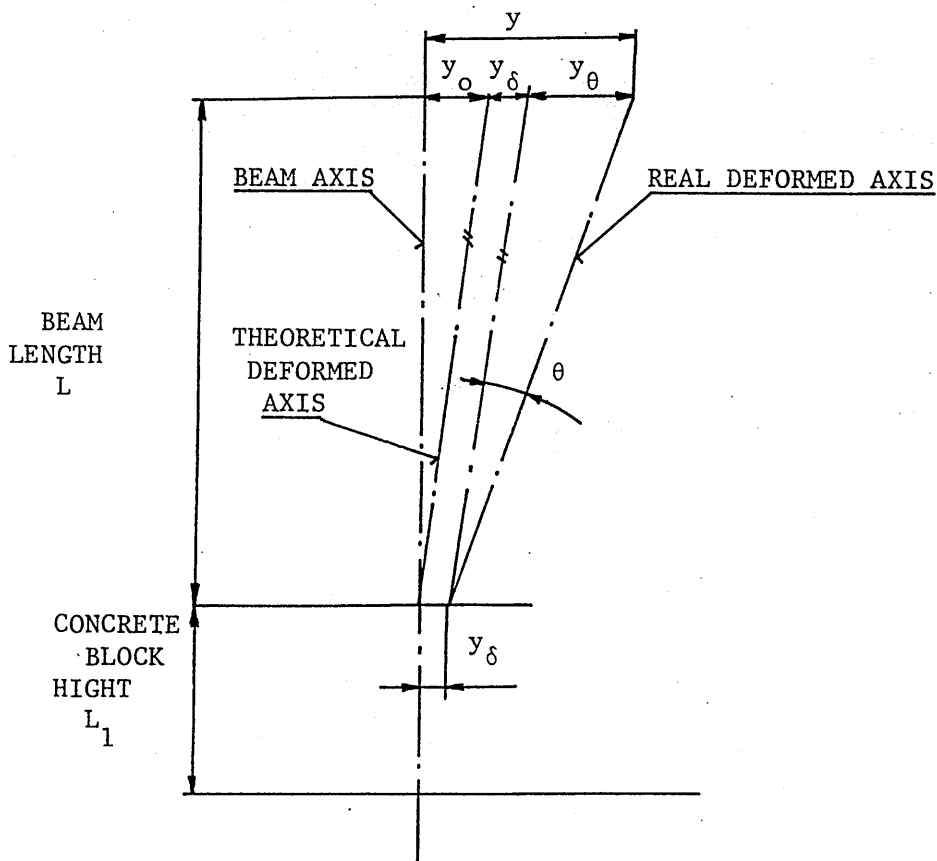


FIG.(A.2) THEORETICAL AND REAL DEFORMED BEAM AXIS

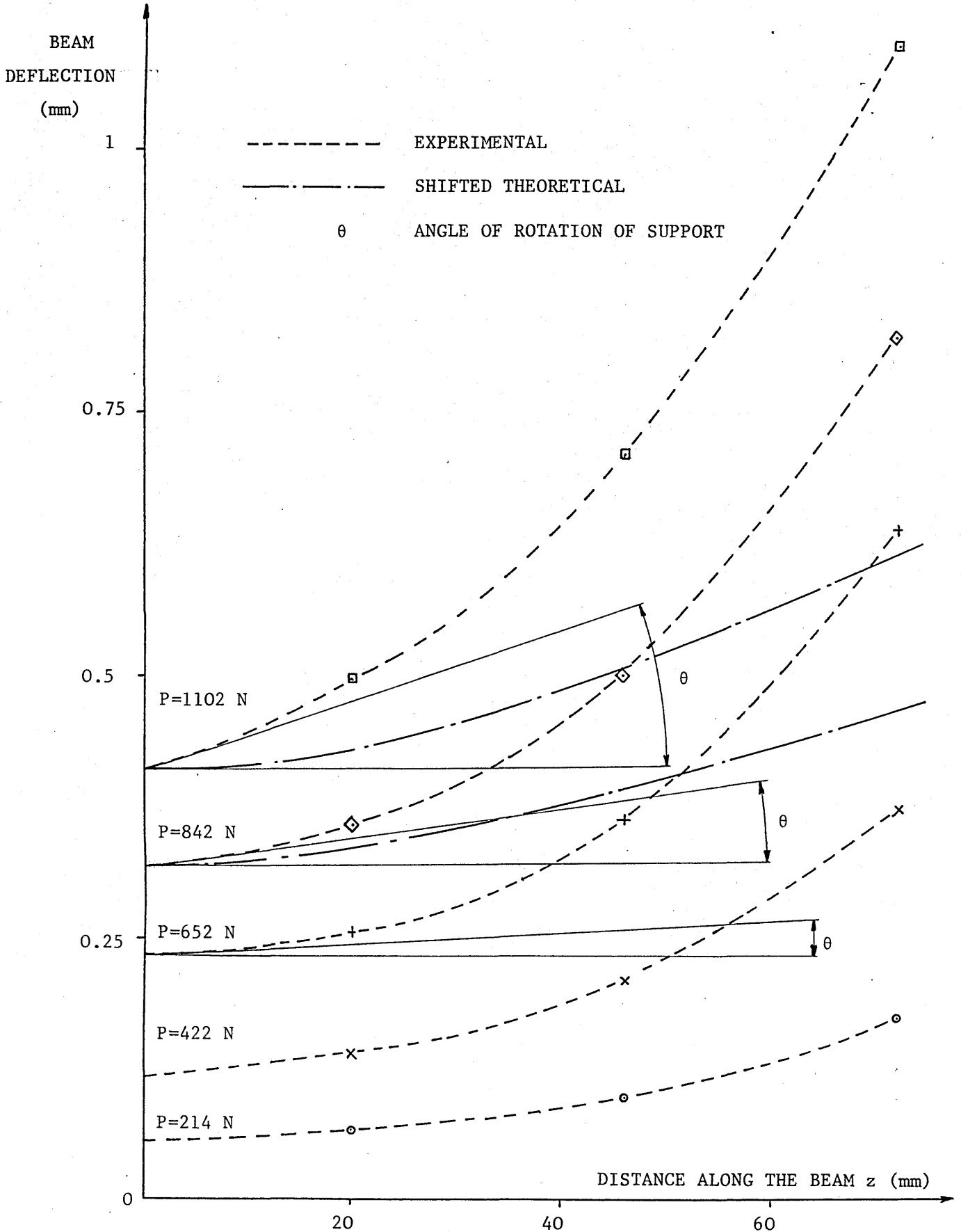


FIG.(A.3) BEAM DEFLECTIONS NEAR TO THE FIXED END.

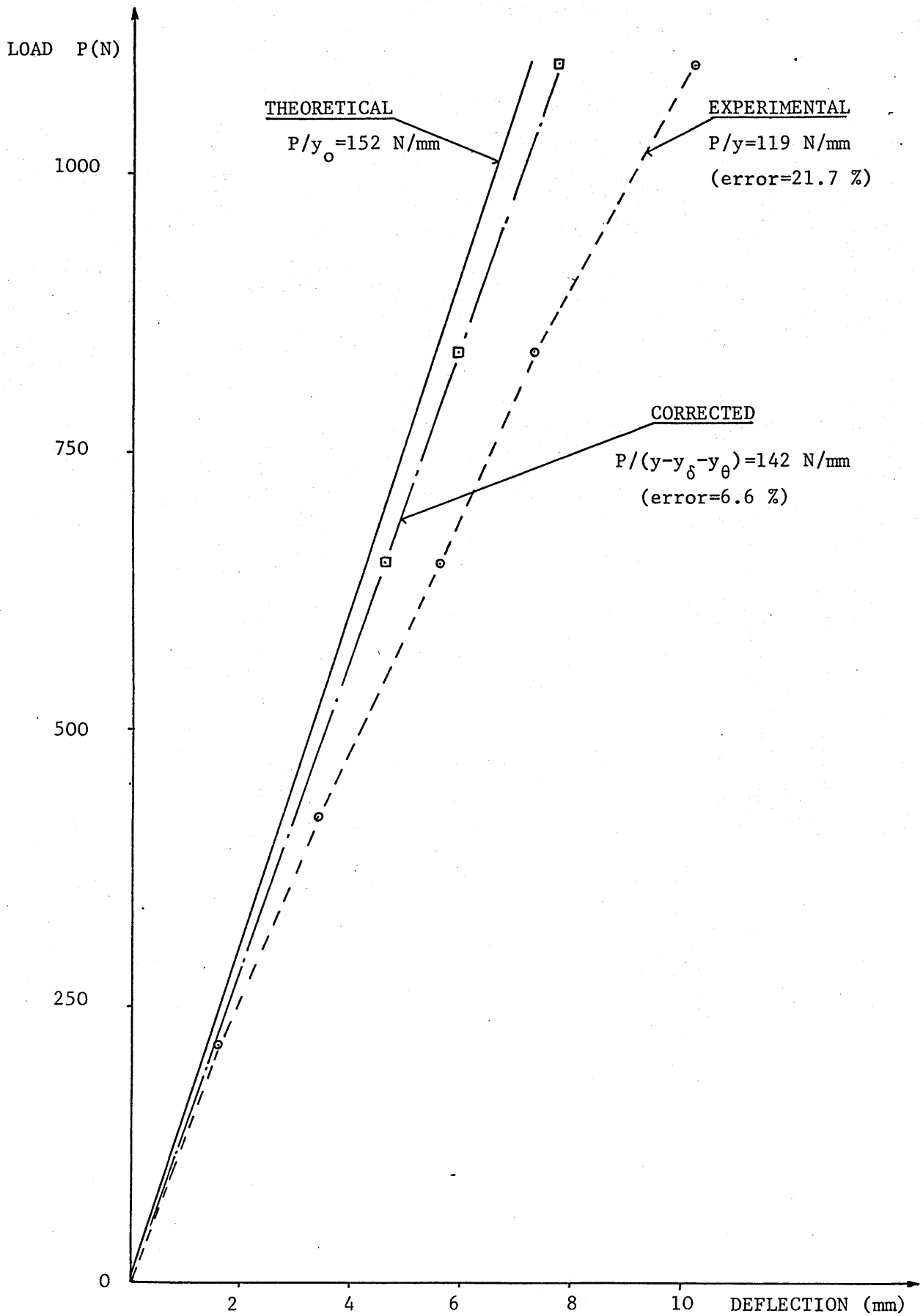


FIG.(A.4) LOAD DEFLECTION DIAGRAMS.