



A bio-inspired hybrid flapping wing rotor for high-efficiency micro rotorcraft

Xun Huang¹ · Linghai Lu¹ · James Whidborne¹ · Marilena Pavel²

Received: 24 December 2025 / Revised: 12 April 2026 / Accepted: 27 April 2026
© The Author(s) 2026

Abstract

Enhancing propulsive efficiency at micro aerial vehicle (MAV) scale remains challenging because low Reynolds number aerodynamics, structural flexibility, and severe power constraints limit the effectiveness of conventional rotor design strategies. This paper investigates a new hybrid flapping-rotary propulsion concept, termed the Hybrid Flapping Wing Rotor (Hybrid FWR), which superposes controlled flapping on a rotating blade to exploit stroke-wise asymmetry while retaining a compact rotorcraft architecture. A unified analytical framework is developed, comprising (i) a kinematic model that captures mechanically constrained flapping and inertia-driven passive pitching with experimentally informed transition coefficients, (ii) a blade-element-based aerodynamic model to estimate stroke-resolved forces, and (iii) an experimentally fitted motor-power model to enforce constant input power while varying the hybridisation ratio. The resulting lift-coefficient evaluation accounts explicitly for unequal upstroke and downstroke durations. Model predictions indicate a consistent optimum hybridisation ratio near 0.7–0.8, where aerodynamic loading in the upstroke is minimised, and lift production is concentrated in the downstroke, maximising the cycle-averaged lift coefficient for a given power. More than 200 bench-top trials using a two-motor prototype corroborate the existence of an optimum near a hybrid ratio of 0.7, demonstrating up to a 2.148-fold improvement in power efficiency relative to pure rotation under comparable lift conditions. The findings clarify the physical mechanism governing the optimum and provide a practical basis for efficiency-oriented design and further high-fidelity refinement.

Keywords Hybrid flapping-rotary propulsion · Micro air vehicles (MAVs) · Low Reynolds number aerodynamics · Bio-inspired rotorcraft · Power-efficient lift generation

List of symbols

A, B, C	Coefficients for the quadratic power consumption equation	$C_{L,(down,up)}$	Average lift coefficient during the downstroke
C_D	Aerodynamic drag coefficient (airfoil section)	C_x, C_z	Instantaneous drag coefficient projected in the blade-element coordinate system (x_{bl} and z_{bl} direction, respectively)
$C_{D,(down,up)}$	Average drag coefficient during the downstroke or upstroke	C_Z	Cycle-averaged lift coefficient
$C_{f,up}$	Sum of average lift and drag coefficients during the upstroke	$\bar{C}_Z$	Alternative notation for cycle-averaged lift coefficient
$C_{f,up,(max,min)}$	Maximum or minimum value of the upstroke aerodynamic force coefficient	$C_{X,(down,up)}$	Average drag coefficient during the downstroke or upstroke phase
C_L	Aerodynamic lift coefficient (airfoil section)	f	Flapping frequency (Hz)
		$F_{x_{bl}}$	Total integrated drag force along the blade x_{bl} axis
		$F_{z_{bl}}$	Total integrated lift force along the blade z_{bl} axis
		h_{bl}	Vertical height offset of the blade

✉ Linghai Lu
l.lu@cranfield.ac.uk

¹ Cranfield University, Bedfordshire, UK

² Delft University of Technology, Delft, The Netherlands

I	relative to the body coordinate system Motor current (A)	β_{INIT}	Initial flapping angle
k	Proportion of time spent at the stepping phenomenon relative to stroke duration	$\beta_{(max,min)}$	Maximum or minimum mechanical flapping angle limit
L	Average lift force	$\dot{\beta}$	Flapping velocity
M	Motor torque (N·m)	$\Delta T_{(down,up)}$	Transition time for the stepping phenomenon during downstroke or upstroke
n	Number of sampling points used for averaging within a stroke phase	ε	Power efficiency (g/W)
P	Input power of a single motor (W)	ϑ	Hybrid ratio parameter governing the balance between flapping and rotation
P_{flap}, P_{rot}	Power consumption of the flapping and rotation motors, respectively.	μ	Proportion of upstroke time within a total cycle
P_{total}	Total power consumption of the Hybrid FWR system	ρ	Air density
r	Radial position of the blade element along the wingspan	φ	Passive blade pitch angle
t	Current time variable within the flapping cycle	ϕ	Alternative notation for blade pitch angle
T	Duration of one full flapping cycle (1/f)	$\varphi_{(max,min)}$	Maximum or minimum passive pitch angle limit
T_{down}	Duration of the downstroke phase	Ψ	Azimuth angle/active in-plane rotation angle
T_p	Period of the pitching-angle oscillation	Ω	Rotor rotational speed (rad/s)
T_{up}	Duration of the upstroke phase	—	Average
Thr	Pulse width modulation (PWM) duty-cycle percentage	Abbreviations	
Thr_{flap}	PWM duty-cycle input for the flapping motor	AoA	Angle of attack
Thr_{Input}	Total system PWM duty-cycle input	BET	Blade element theory
Thr_{rot}	PWM duty-cycle input for the rotation motor	C.G	Centre of gravity
U	Motor voltage (V)	DoF	Degrees of freedom
$v_{\beta,r,t}$	Linear flapping velocity at a specific blade-element radius and time	FWR	Flapping wing rotor
v_i	Induced velocity	MAV	Micro aerial vehicle
V	Magnitude of the average inflow velocity vector	MTOW	Maximum takeoff weight
$V_{\infty,hub,(x,y,z)}$	Inflow velocity (x, y, z)-component in the hub coordinate system	NED	North–east–down coordinate system
W	Resultant inflow velocity vector	PWM	Pulse width modulation
$W_{(x,y,z)}$	Inflow velocity component along the horizontal blade axis ($x_{bl} - y_{bl} - z_{bl}$)	RPM	Revolutions per minute
$(x, y, z)_{bl}$	Axes of the blade-element coordinate system	UAV	Unmanned aerial vehicle
$(x, y, z)_{body}$	Axes of the body coordinate system	1 Introduction	
$(x, y, z)_{earth}$	Axes of the Earth (NED) coordinate system	Micro Air Vehicles (MAVs), commonly defined as aerial platforms with wingspans or rotor diameters in the 15–30 cm range, are increasingly used in missions where compact size, agility, and hovering capability are critical [1–5]. Among MAV configurations, rotorcraft architectures remain practical due to their hover and manoeuvrability [6, 7]; however, extending hover/endurance at MAV scale remains constrained by low-Reynolds-number aerodynamics, limited onboard energy, and strict mass/volume envelopes.	
$(x, y, z)_{hub}$	Axes of the hub coordinate system	At MAV scale, classical rotorcraft efficiency-improvement strategies, such as increasing rotor disk area, reducing hub and swashplate complexity, and using lightweight structures, have been demonstrated in representative	
α^*	Effective angle of attack of the blade element (alternative notation)		
β	Active flapping angle		
β^*	Incoming flow angle relative to the blade		
β_{AMP}	Amplitude of the flapping motion		

systems (Table 1). Yet further gains become difficult without growing rotor diameter beyond the MAV envelope, and the resulting structural and control penalties can offset benefits [8, 9]. Consequently, a growing body of work has explored non-conventional kinematics and bio-inspired mechanisms to improve force production and lift-per-power at low Reynolds numbers.

Hybridising rotary and flapping/pitching kinematics at low Reynolds numbers has been explored from multiple perspectives. Fitchett (2007) [10] developed and tested a MAV-scale rotor testbed (“Flotor”) with independently controlled blade rotation and powered flapping, demonstrating that adding powered flapping can increase thrust and reduce torque at moderate thrust levels, while highlighting mechanical and energetic trade-offs. More recent studies have examined composed unsteady kinematics and fluid–structure interactions (FSI) in low-Re rotors, indicating that blade flexibility and non-steady motion can significantly affect loads and performance. Alguacil et al. investigated FSI and unsteady kinematics of a low-Re rotor in AIAA Journal, providing evidence that coupled unsteady kinematics and structural response can influence rotor performance [11]. In parallel, the “flapping rotary wing” (FRW) literature has systematised concepts that merge flapping-wing features into micro rotors. Chen et al. studied pitching–flapping perturbations on revolving wings and reported that such perturbations can alter lift, torque and power loading at low Re, providing mechanistic motivation for stroke-wise asymmetry [12]. A 2024 Progress in Aerospace Sciences review by Chen et al. provides a comprehensive overview of FRW concepts, aerodynamic mechanisms and prototypes, and frames the debate on when and how hybrid kinematics may outperform conventional micro rotors [13].

Building on the above research context, this study introduces a semi-biomimetic Hybrid Flapping Wing Rotor (Hybrid FWR) [14, 15], which combines conventional rotor rotation with insect-inspired flapping to realise an optimised motion hybridisation. A central feature of this architecture

is the integration of a flapping mechanism within a standard rotorcraft layout. By leveraging existing drive shafts and rotor hubs, the flapping motion operates in parallel with rotation, generating additional lift through unsteady aerodynamic effects while preserving a compact and rotorcraft-compatible configuration.

The present work does not aim to reduce mechanical complexity relative to the simplest micro-rotor layouts. Instead, it addresses a different MAV bottleneck: improving hover-relevant lift-per-power under strict size, mass, and energy constraints, without increasing rotor diameter or abandoning a compact hub–shaft architecture. The Hybrid FWR therefore trades a modest increase in mechanism complexity for a measurable gain in aerodynamic efficiency, and this paper focuses on quantifying and explaining this trade-off within a bench-top, hover-like operating envelope.

It should be further clarified that the introduction of the flapping mechanism does not result in a disproportionate increase in system mass. Compared to a conventional rotor configuration, the additional flapping-related components contribute approximately 15 g to the total system weight, remaining within the typical mass range of MAV platforms. Therefore, the efficiency improvements reported in this study are not achieved at the expense of substantial weight increase, but rather through a more effective utilisation of the available power via motion hybridisation.

In the context of existing flapping rotary wing (FRW) and hybrid-kinematics studies, the novelty of this work lies in three main aspects. First, a power-consistent hybridisation framework is proposed, in which electrical input is allocated between independently actuated rotation and flapping through a single hybrid ratio η , enabling direct and fair comparisons across operating modes under fixed power. Second, a kinematic model grounded in high-speed experimental observations is developed, explicitly capturing mechanically constrained passive pitching, including a stepping behaviour at pitch limits and the resulting stroke-time asymmetry. Third, an intermediate efficiency optimum is identified and experimentally validated, and a physical mechanism is established linking this optimum to upstroke aerodynamic unloading and downstroke lift concentration.

The specific objectives of this study are to: (i) evaluate the feasibility of the Hybrid FWR concept for MAV propulsion; (ii) investigate whether an intermediate operating mode between pure rotation and pure flapping yields a performance optimum under constant power; (iii) explain the aerodynamic and energetic mechanisms governing such an optimum; and (iv) validate the proposed models and trends experimentally using a bench-top prototype across the available operating envelope.

The paper is structured as follows. Section 2 introduces the design features and mechanical structure of the

Table 1 Comparison between representative rotorcraft MAV and UAV configurations

Name	Architecture	Weight	Rotor diameter	Endurance
PD-100 black hrnet	MAV Single rotor	~16–18 g	~123 mm	~20–25 min
MuFly	MAV Coaxial rotor	~30 g	~120 mm	~6 min
ARS micro coax	MAV Coaxial rotor	~80 g	~140 mm	~10 min
ETH CoaX	UAV Coaxial rotor	~400–460 g	~480 mm	~7–10 min
Yamaha	UAV Single rotor	~60–65 kg	~3.1 m	~60–90 min

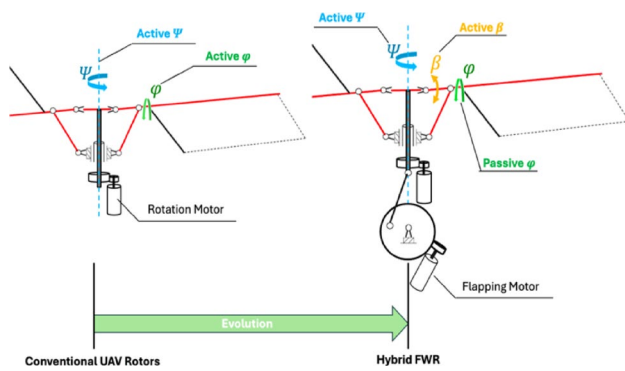


Fig. 1 Illustration of design transition from conventional rotors to bio-inspired rotors

Hybrid FWR, illustrating how conventional rotor systems can evolve into bio-inspired hybrid configurations [14–16]. Section 3 establishes the Hybrid FWR mathematical model, providing a unified description of its kinematics, aerodynamics, and power consumption, and forming the analytical basis for the above procedure. Section 4 explains the high-power efficiency observed in the Hybrid FWR, derived from the results of that analytical procedure. Section 5 presents the experimental campaign conducted to validate the optimal condition identified in Sect. 4. Finally, Sect. 6 summarises the findings and contributions of the work.

2 Design of the hybrid FWR

2.1 Conceptual design

Figure 1 illustrates how traditional UAV rotors ‘evolve’ into the Hybrid FWR concept, proposed in the current study, and highlights their differences in active and passive degrees of freedom (DoF). In a conventional rotorcraft UAV, the rotor is directly driven by a rotational motor, thereby providing an active in-plane rotation (Ψ) that generates lift. This

design typically employs an active blade pitch angle (φ) for cyclic pitch control while relying on passively induced out-of-plane flap motion governed by aerodynamic forces. By contrast, the Hybrid model, derived from the conventional rotorcraft UAV concept, introduces an additional flapping motor shown in Fig. 1 (right). Through a system of gears and linkages, the flapping motor actively drives the rotor’s flapping motion, resulting in an active flap angle (β) reminiscent of insect or avian flight. Due to spatial limitations in the Hybrid FWR’s structure, the new design does not include an active pitch control mechanism. Instead, a pitch (feathering) hinge is incorporated to accommodate dynamically coupled pitch variations arising from flapping, torsional, inertial, and other effects. Hence, φ becomes passive in this new design.

2.2 Mechanical structure and core components

The Hybrid FWR MAV prototype fabricated using 3D-printed nylon material in Fig. 2 has a height of 100 mm and a base diameter of 25 mm × 25 mm. Beneath the base where the prototype is mounted, a lightweight flight controller weighing only 3 g, along with the battery, is installed on the frame. The flight controller runs the flight control software, while the battery provides power. Furthermore, as illustrated in Fig. 3, the maximum mechanical range of β motion extends from $\beta_{min} = -10^\circ$ during the downstroke to $\beta_{max} = 35^\circ$ during the upstroke.

A limit-stop mechanism in Fig. 4 is employed to ensure φ remains within its permissible physical bounds. By adjusting the position of the limit-stop rod, the motion range of the free-sliding block is constrained, thereby maintaining φ between 10° and 25° . Moreover, repositioning the limit-stop rod enables further refinement of these upper and lower angle limits.

Mechanical angle limits and parameter selection. The flapping angle range (β) and passive pitching limits (φ)

Fig. 2 Illustration of the Hybrid FWR test prototype designed in the current study

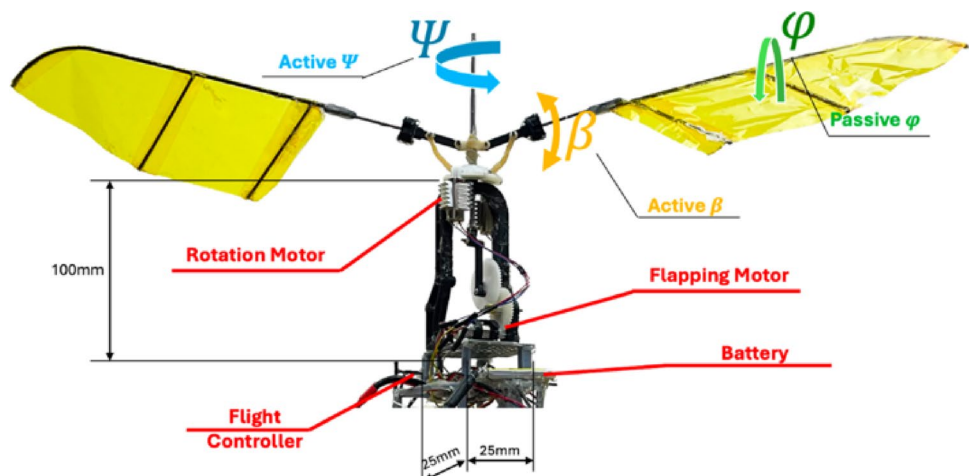


Fig. 3 Limitation of flapping angle range (β) (Left – upstroke, Right – downstroke)

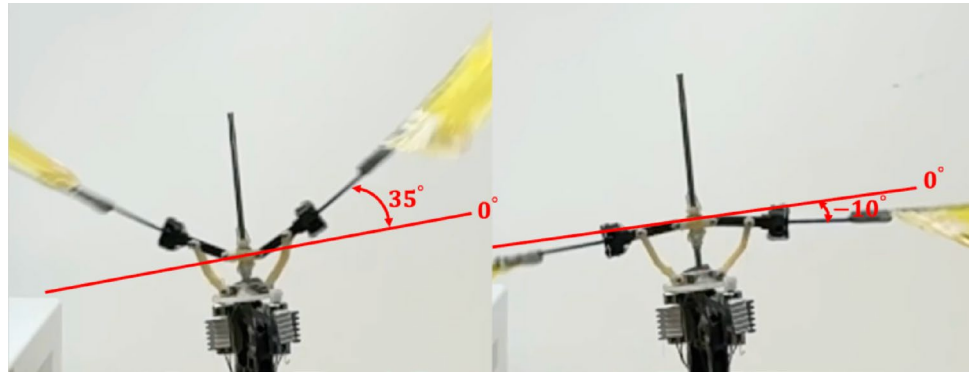
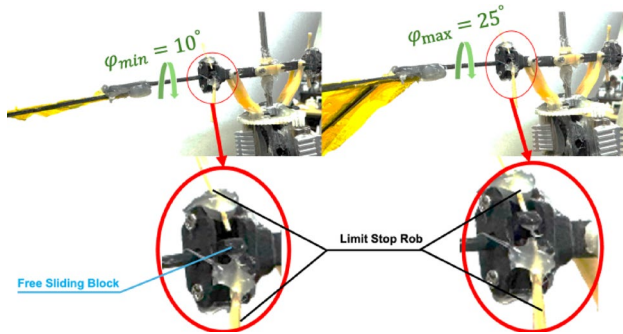


Fig. 4 Illustration of pitch angle φ limitation

are primarily dictated by mechanical constraints of the prototype, including linkage travel, geometric interference, and structural clearance. In particular, the flapping range of approximately -10° to $+35^\circ$ is defined by the allowable motion of the linkage mechanism without collision between the wing and the supporting structure. Similarly, the passive pitching limits (10° to 25°) are imposed by mechanical limit stoppers to ensure repeatable and stable pitching behaviour.

These limits are not arbitrarily chosen but are designed to promote favourable aerodynamic asymmetry. Specifically, a higher pitch angle during the upstroke reduces the effective angle of attack and associated aerodynamic loading, thereby facilitating upstroke unloading, while a lower pitch angle during the downstroke enhances lift generation.

It should be noted that the theoretical model does not depend on a single fixed set of angle limits. Instead, β and φ are treated as parametric inputs within the modelling framework. While the precise optimal hybrid ratio (ϑ) may shift with different angle bounds, the existence of an intermediate optimum remains robust. The sensitivity of the

predicted performance to variations in these parameters will be investigated in future work. The present study focuses on the efficiency characteristics associated with the proposed hybridisation mechanism.

A unique feature that warrants further examination is the rotor's passively governed pitch angle φ . During rotor lift generation, both the rotation motor and the flapping motor are powered simultaneously, enabling active rotation as well as active flapping. Meanwhile, as mentioned above, the passive φ in this new design is determined by the combined influence of aerodynamic and inertial forces. High-speed camera observations in Fig. 5 show that φ is primarily driven by the inertial forces from the flapping motion and the aerodynamic forces from the incoming flow, resulting in an asymmetric periodic pattern akin to the asymmetrical wing-pitching motion of dragonflies [17]. Typically, at the onset of the upstroke, φ increases quickly to $\varphi_{\max} = 25^\circ$ and remains at this value throughout the stroke, whereas during the downstroke, it decreases rapidly to $\varphi_{\min} = 10^\circ$ and stays there. The next section provides a more detailed kinematic model of this passively induced pitching motion stemming from the flapping dynamics.

To improve reproducibility and enable system-level interpretation of the proposed concept, the key prototype specifications and mass budget are summarised in Table 2. The Hybrid FWR prototype is constructed as a compact dual-actuation system integrating a flapping motor and a rotation motor within a shared rotor hub. The overall system mass, including motors, structural components, onboard electronics, and battery, is approximately 80 g (maximum take-off weight, MTOW). The wing structure is fabricated using a polyimide membrane supported by a lightweight

Fig. 5 Hybrid FWR wing motion during one flapping cycle

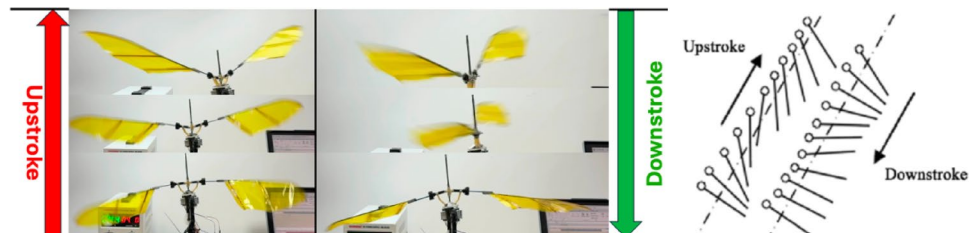


Table 2 Prototype specification and mass budget

Item	Value	Notes
Rotor architecture	Hybrid flapping–rotary (dual-actuator)	One rotation motor + one flapping motor
Number of blades	2	Symmetrical flapping wings
Rotor diameter	~34 cm	From FWR prototype description
Wingspan	34 cm	Same as above
Blade material	Polyimide film + carbon fibre skeleton	Ultra-light flexible wing
Wing thickness	0.015 mm	From digital twin modelling section
Prototype height	100 mm	Explicitly stated
Base footprint	25 mm × 25 mm	Explicitly stated
Flapping angle range	−10° to +35°	Mechanical constraint
Pitch angle range	10° to 25°	Passive pitch limiter
Flapping frequency range	6–15 Hz (tested), up to 20 Hz	From experiments
Rotation speed	3.3–9.1 rev/s	From Tables 3 and 4
Motors	8520 coreless motors ×2	One for flapping, one for rotation
Motor mass (each)	5.9 g	Motor spec
Flight controller mass	3 g	Explicitly stated
Battery mass	15 g	
Total system mass (MTOW)	110 g	
Reynolds number range	~2000–7000	

carbon-fibre skeleton, allowing controlled elastic deformation under flapping motion.

The table further reports the geometric parameters, flapping and pitching angle limits, and operating conditions used in the present study. These include a flapping range of approximately −10° to +35° and a passive pitching range constrained between 10° and 25° through mechanical limiters. The propulsion system employs two 8520 coreless motors, one dedicated to flapping actuation and the other to rotor rotation, enabling controlled variation of the hybridisation ratio.

It should be emphasised that all efficiency comparisons reported in this study are conducted under an identical hardware configuration, with only the hybridisation ratio varied. Therefore, the observed improvement in power efficiency does not arise from changes in system mass or scale, but from the redistribution of input power between flapping and rotational motion.

It is important to clarify that the present experimental campaign focuses on propulsion-level validation using a bench-top rotor test platform rather than full free-flight testing. The objective of this setup is to isolate the aerodynamic and energetic effects of the hybrid flapping–rotary mechanism under controlled conditions. As such, the reported lift

and efficiency metrics correspond to the rotor subsystem and do not represent the complete vehicle flight performance. This approach allows systematic evaluation of the hybridisation mechanism while avoiding the additional uncertainties introduced by flight dynamics and control coupling.

3 Hybrid FWR model development

The optimal motion-hybridisation strategy requires the development of a kinematic model, an aerodynamic model, a power-consumption model, and a lift-coefficient evaluation model for the Hybrid FWR. This section provides a detailed formulation of each of these components.

3.1 Kinematics modelling

The kinematic model serves as the most critical component among all models. Its accuracy directly influences the predictive reliability of the subsequent aerodynamic model. Moreover, its formulation is the most complex aspect of the overall modelling process, as it requires a mathematical description of the passive characteristics of the Hybrid FWR's φ angle and its induced kinematic coupling effects.

As stated in the previous section, the flapping motion of β and the asymmetric periodic kinematics of φ in the Hybrid FWR closely resemble those observed in dragonfly flight [18]. Within the dragonfly's body, each wing is actuated by two muscles arranged in a configuration analogous to a parallel mechanical structure. When both muscle attachment points exert force in the same direction, a flapping motion occurs, actively inducing β changes. Conversely, if the forces applied at the two muscle attachment points differ, a twisting motion is generated, actively modifying φ . Although the mechanism governing φ in the Hybrid FWR closely resembles that observed in biological dragonflies, significant differences remain between the flapping kinematics of the Hybrid FWR and those of natural dragonfly flight. These differences primarily arise from two factors: mechanical structural limitations and the passive pitching kinematics of φ . The mechanical structural constraints, as discussed earlier, are defined by the limitations on the flapping angles, $\beta_{\max} = 35^\circ$ and $\beta_{\min} = -10^\circ$, as well as the limitations on the passive pitching angle, $\varphi_{\max} = 25^\circ$ and $\varphi_{\min} = 10^\circ$. In contrast, the passive pitching kinematics of φ reveal a distinct divergence from dragonfly flight mechanics, which can primarily be attributed to the two key aspects.

Firstly, the pitch angle φ of biological dragonflies is actively regulated by specialised musculature [18], enabling precise control of wing flapping. In contrast, in the Hybrid FWR, φ is governed passively by the flapping velocity,

primarily driven by the inertial forces of the wings. As a result, the timing for reaching φ_{max} and φ_{min} , as well as the duration of the wing's residence at these extreme positions, vary with changes in flapping velocity. This fundamental difference, active control in dragonflies versus passive, inertia-driven kinematics in the Hybrid FWR, underscores the necessity of considering flapping velocity and acceleration effects when modelling the wing pitching motion of the Hybrid FWR.

Figure 6 summarises high-speed camera observations, illustrating the relationship between flapping velocity $\dot{\beta}$ and the passive pitching angle φ . The blue curve represents the variation in flapping velocity over a single flapping cycle. It demonstrates the changes in wing passive pitching angle, the timing of reaching φ_{max} and φ_{min} , and the duration of the wing's residence at these limitation positions is closely related to the acceleration of the flapping velocity.

Taking the upstroke as an example, when the flapping velocity is in its acceleration phase, shown in the green oval in Fig. 6, the inertial forces acting on the wing produce a lag, causing it to rotate about the pitch hinge (the red shaft) and pitch upward until it reaches the mechanical stop at $\phi_{max} = 25^\circ$ (green square). During the subsequent deceleration phase of the upstroke, highlighted by the red oval, the inertial forces instead drive a downward pitching motion. As a result, the wing moves away from the $\phi_{max} = 25^\circ$ limit and gradually decreases its pitch angle, as indicated by the red rectangular frame.

During the downstroke, the combination of acceleration and inertial forces quickly drives the wing to the mechanical stop at $\phi_{min} = 10^\circ$, where it remains until the downstroke velocity begins to decelerate. At that point, the wing gradually pitches upward again (red square in the downstroke sequence). These passive responses demonstrate a strong coupling between flapping velocity, acceleration, and pitching angle, as well as the timing and duration of the wing's contact with ϕ_{max} and ϕ_{min} . This behaviour underscores the need to incorporate these dynamics into the kinematic

model to capture the observed motion accurately. In this study, the intervals during which the wing becomes temporarily locked at ϕ_{max} or ϕ_{min} are referred to as the stepping phenomenon.

Secondly, the passive pitching characteristics of the Hybrid FWR significantly influence the duration distribution between upstroke and downstroke within a flapping cycle. Experiments on the mechanical prototype show that, under constant motor input power, the time spent in each half-stroke is strongly shaped by the aerodynamic forces acting on the wing. This behaviour is consistent with engineering principles: when aerodynamic loading is high, the motor experiences greater resistance, leading to lower acceleration for the same power input and therefore longer stroke durations. Conversely, lower aerodynamic forces reduce motor loading, allowing higher acceleration and shorter motion times. In the Hybrid FWR, this asymmetry is particularly evident. During the upstroke, the passive pitch-up motion induced by inertial effects reduces the effective angle of attack and thus the aerodynamic load on the motor. The opposite occurs during the downstroke, where the larger angle of attack increases aerodynamic loading. As a result, the upstroke is completed more quickly than the downstroke.

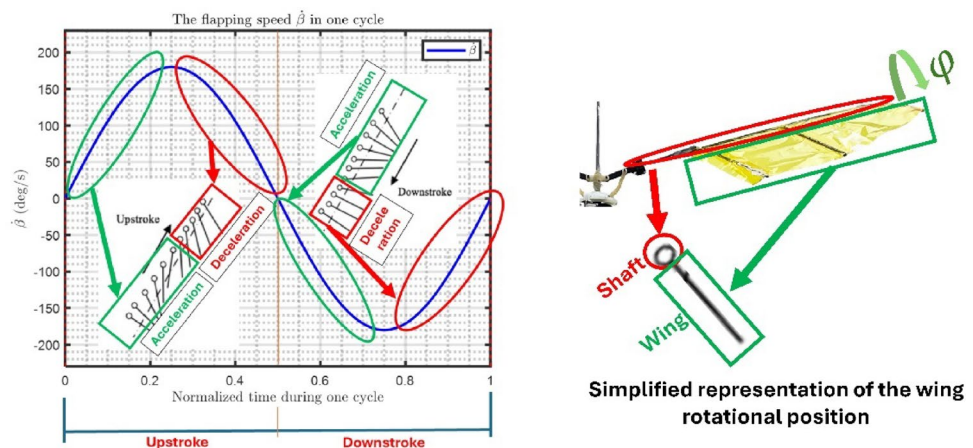
The incoming flow angle β_* is defined,

$$\beta_* = \arctan \frac{W_z}{W_x} \quad (1)$$

and illustrated in Fig. 7.

in which W_z represents the inflow velocity along the vertical blade (wing) element axis (Z_{bl}), which is primarily influenced by the flapping velocity. W_x denotes the inflow velocity along the horizontal blade axis (X_{bl}), which is mainly affected by the blade elements' tangential velocity. The small circular marker on the wing in Fig. 7 represents the passive rotational shaft of the wing (corresponding to the structure highlighted by the red frame in Fig. 6).

Fig. 6 Relationship between flapping speed and wing rotation position



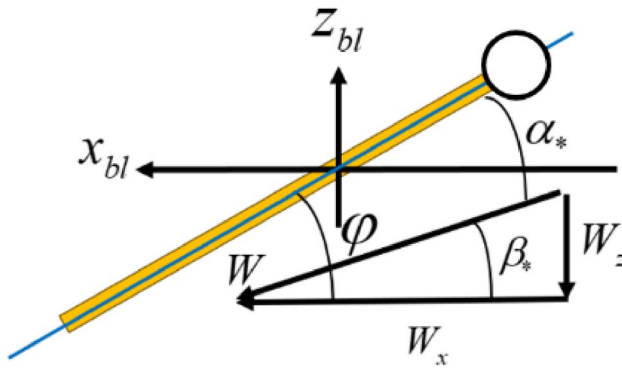


Fig. 7 Definition of Hybrid FWR blade element's angles β_*

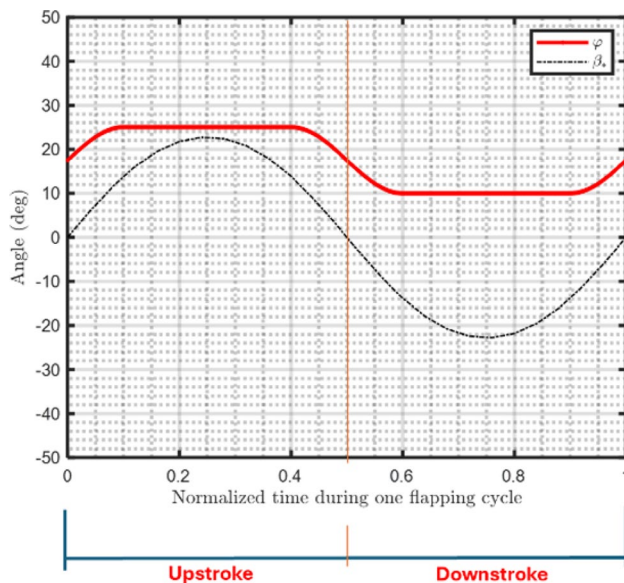


Fig. 8 Approximate relationship between β_* and φ derived from high-speed camera observations

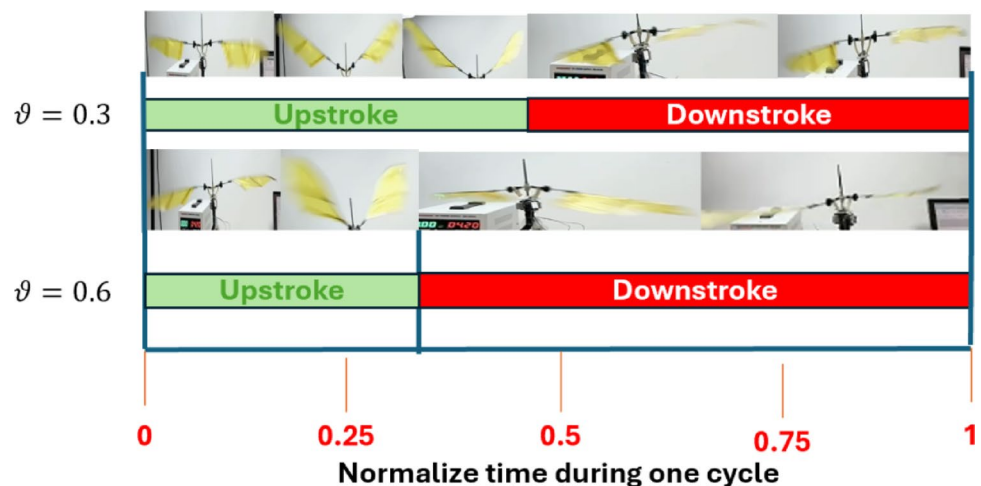
Furthermore, the difference between φ and β_* defines the effective angle of attack a_* for a blade element, expressed below:

$$a_* = \varphi(t) - \beta_* \quad (2)$$

Figure 8 shows the relationship between β_* and φ of the wing, derived from high-speed camera observations. To obtain a consistent and analytically tractable representation, the raw kinematic data were processed using multi-cycle averaging combined with piecewise profile fitting. In particular, the flapping motion is approximated using sinusoidal segments, while the pitching motion is represented using a combination of transition and plateau phases to capture the observed stepping behaviour at the pitch limits. It should be noted that the raw measurements do exhibit small cycle-to-cycle variations and high-frequency fluctuations, arising from structural vibration, measurement noise, and aeroelastic effects. However, these fluctuations are relatively small compared to the dominant stroke-scale motion and do not materially affect the cycle-averaged kinematic trends that are relevant for the present quasi-steady modelling framework. The fitted profiles are therefore used to provide a simplified but physically representative description of the wing kinematics.

During the upstroke, a_* reduces, resulting in lower aerodynamic loads during the upstroke. Conversely, during the downstroke, the negative β_* produces a much larger a_* , significantly increasing the aerodynamic loads. This discrepancy in aerodynamic loads between the upstroke and downstroke has been observed through high-speed camera analysis of the Hybrid FWR. It was found to have a significant impact on the time distribution of the upstroke within the overall flapping cycle. Figure 9 illustrates the proportion of time spent in the upstroke and downstroke under varying ϑ , which is defined as a combined measure of flapping frequency and rotational speed and will be further detailed

Fig. 9 Illustration of upstroke time under different flapping conditions



in the next Subsection. The results in the figure reveal that under different flapping conditions ($\vartheta = 0.3$ or $\vartheta = 0.6$), the proportion of time experienced during the upstroke and downstroke differs, leading to variations in aerodynamic loads applied to the upstroke within the total flapping cycle. Moreover, the aerodynamic loads on the wing of both scenarios during the upstroke are consistently lower than those during the downstroke. Consequently, under constant motor power, the upstroke always occupies a shorter duration of the overall flapping cycle compared to the downstroke.

Due to the significant kinematic differences between the Hybrid FWR and dragonflies as described above, the wing kinematic model of the Hybrid FWR cannot be directly adapted from that of dragonflies. Instead, the model should be refined to capture two unique kinematic phenomena observed in the Hybrid FWR:

- Point 1: The duration of the stepping phenomenon at φ_{max} and φ_{min} are strongly influenced by the correlation between β and φ . Hence, it must be accurately modelled to reflect this dependency.
- Point 2: Aerodynamic load asymmetry causes discrepancies in the durations of upstroke and downstroke. This asymmetry can be quantified through the upstroke time proportion within the overall flapping cycle.

Therefore, unlike the flapping kinematics theories for dragonflies presented in [19, 20], the Hybrid FWR model introduces two transition time coefficients ΔT_{up} and ΔT_{down} to represent the stepping phenomenon durations at φ_{max} and φ_{min} , addressing Point 1. Additionally, the coefficient μ is introduced to represent the proportion of upstroke time within the total cycle T , addressing Point 2.

The next step moves on to build the kinematic model, starting with the flapping kinematics of the rotor. The equation governing the flapping velocity is expressed as below,

$$\dot{\beta}(t) = \begin{cases} \pi \cdot \beta_{AMP} \cdot f \cdot \sin\left(\frac{\pi t}{\mu \cdot T}\right) & t < \mu \cdot T \\ -\pi \cdot \beta_{AMP} \cdot f \cdot \sin\left[\frac{\pi(t-\mu \cdot T)}{(1-\mu) \cdot T}\right] & t \geq \mu \cdot T \end{cases} \quad (3)$$

in which μ characterises the kinematic asymmetry of the Hybrid FWR between upstroke and downstroke. $T = \frac{1}{f}$ denotes the one whole cycle duration, where f is the flapping frequency, and β_{AMP} is the flapping amplitude.

By integrating Eq. (3), the flapping dynamics can be derived in Eq. (4):

$$\beta(t) = \begin{cases} \beta_{INIT} - \beta_{AMP} \cdot f \cdot \mu \cdot T \cdot \cos\left(\frac{\pi t}{\mu \cdot T}\right) & t < \mu \cdot T \\ \beta_{INIT} + \beta_{AMP} \cdot f \cdot [(1-\mu) \cdot T] \cos\left[\frac{\pi(t-\mu \cdot T)}{(1-\mu) \cdot T}\right] & t \geq \mu \cdot T \end{cases} \quad (4)$$

in which β_{INIT} represents the initial flapping angle, according to the mechanical constraints defined in Fig. 3,

which is set to $10^\circ + \frac{35^\circ}{2} = 27.5^\circ$. β_{AMP} is constrained in Fig. 6 and limited to $\frac{35^\circ}{2} = 17.5^\circ$.

The pitching kinematics of the Hybrid FWR are significantly more complex. Unlike the flapping kinematics, which primarily follows trigonometric characteristics, the pitching motion exhibits the stepping phenomenon caused by mechanical constraints at two limiting angles φ_{max} and φ_{min} , as shown in Fig. 4 and further illustrated by the red curve in Fig. 8. Therefore, in addition to the parameter μ , two new transition time coefficients, ΔT_{up} and ΔT_{down} are introduced to characterise the duration of the stepping phenomenon at the mechanical limits. The pitching angle kinematics are formulated as follows:

if t belongs $\left[-\frac{\Delta T}{2}, \frac{\Delta T_{up}}{2}\right]$, and then:

$$\varphi(t) = \frac{\varphi_{max} + \varphi_{min}}{2} + \frac{\varphi_{max} - \varphi_{min}}{2} \sin\left(\pi \frac{t}{\Delta T_{up}}\right) \quad (5)$$

if t belongs $\left(\frac{\Delta T_{up}}{2}, T_{up} - \frac{\Delta T_{up}}{2}\right)$, and then:

$$\varphi(t) = \varphi_{max} \quad (6)$$

if t belongs $\left[T_{up} - \frac{\Delta T_{up}}{2}, T_{up}\right]$, and then:

$$\varphi(t) = \frac{\varphi_{max} + \varphi_{min}}{2} + \frac{\varphi_{max} - \varphi_{min}}{2} \sin\left(\pi \frac{T_{up} - t}{\Delta T_{up}}\right) \quad (7)$$

if t belongs $\left(T_{up}, T_{up} + \frac{\Delta T_{down}}{2}\right]$, and then:

$$\varphi(t) = \frac{\varphi_{max} + \varphi_{min}}{2} + \frac{\varphi_{min} - \varphi_{max}}{2} \sin\left(\pi \frac{t - T_{up}}{\Delta T_{down}}\right) \quad (8)$$

if t belongs $\left(T_{up} + \frac{\Delta T_{down}}{2}, T_p - \frac{\Delta T_{down}}{2}\right)$, and then:

$$\varphi(t) = \varphi_{min} \quad (9)$$

if t belongs $\left[T - \frac{\Delta T_{down}}{2}, T_p\right]$, and then:

$$\varphi(t) = \frac{\varphi_{max} + \varphi_{min}}{2} + \frac{\varphi_{min} - \varphi_{max}}{2} \sin\left(\pi \frac{T_p - t}{T_{down}}\right) \quad (10)$$

in which t is the current time during one flapping cycle. $T_p = \frac{1}{f}$ is the period of pitching angle, T_{up} denotes the duration of the upstroke as a proportion of the total cycle, while T_{down} represents the corresponding proportion for the downstroke. Their expressions are defined as follows:

$$\begin{cases} T_{up} = \mu T_p \\ T_{down} = (1 - \mu) T_p \end{cases} \quad (11)$$

The transition time coefficients ΔT_{up} and ΔT_{down} are given:

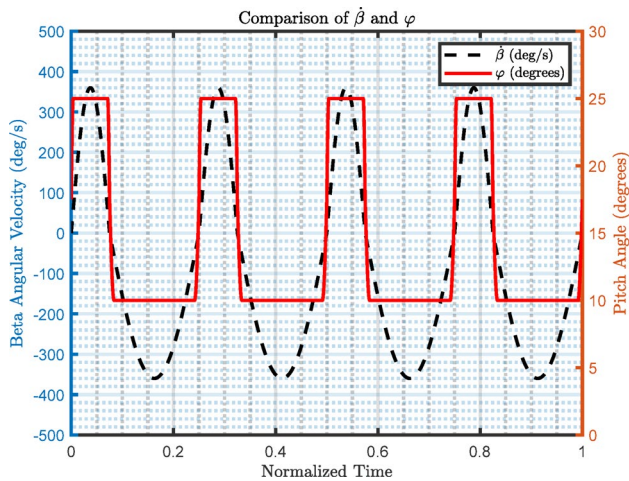


Fig. 10 A wing kinematics case of Hybrid FWR (where $f = 4\text{ Hz}$, $\mu = 0.3$, and $k = 0.9$)

$$\begin{cases} \Delta T_{up} = (1 - k)T_{up} \\ \Delta T_{down} = (1 - k)T_{down} \end{cases} \quad (12)$$

in which k represents the proportion of time spent at the stepping phenomenon, relative to the total upstroke or downstroke duration. It is influenced by factors such as flapping velocity, acceleration, mechanical smoothness, and rotational axis damping. In practice, k is highly sensitive to flapping conditions and manufacturing quality, such as the fit tolerance of the bearing, which can significantly affect the motor power. Moreover, given its dependency on these empirical variables, k is almost exclusively determined through experimental observation. Based on repeated high-speed camera analyses of the mechanical structure under various operating conditions, k is set to an average value of 0.9 for the operating range 1–20 Hz of our Hybrid FWR. The parameter k is sensitive to mechanical factors such as manufacturing tolerances, friction at the pitch hinge, and structural damping. Similarly, the μ parameter, which represents the upstroke time proportion, is influenced by aerodynamic loading and system dynamics.

In the present study, these parameters are treated as representative values corresponding to the tested prototype. A systematic parametric investigation of their variation is beyond the scope of this work, and the reported results should therefore be interpreted within this modelling assumption.

The kinematic model described above provides a comprehensive representation of the wing kinematics observed during high-speed camera experiments with the Hybrid FWR. Using the equations outlined, the typical kinematic patterns are illustrated in Fig. 10.

The results shown in Fig. 10 align well with the passive wing motion principles depicted in Fig. 6. However, it should be noted that this example represents only one of

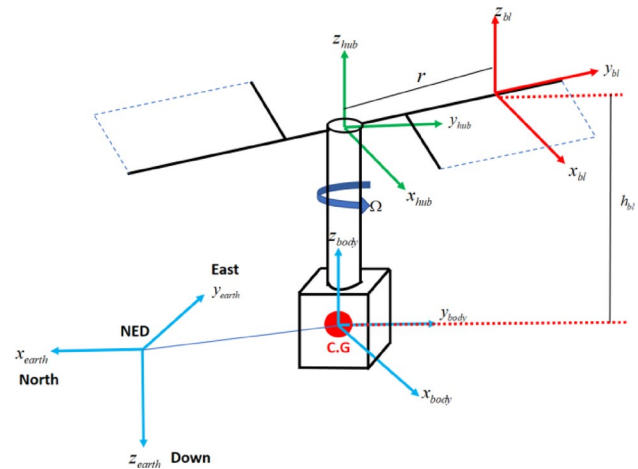


Fig. 11 Hybrid FWR coordinate system definition

the many complex motion patterns exhibited by the Hybrid FWR. In practice, due to the varying aerodynamic loads on the wing, the parameter μ , dynamically changes in response to these aerodynamic forces. In addition, the governing equations from Eqs. (3)–(12) adequately describe the kinematics of the Hybrid FWR, but in practical use, they require periodic recalibration to reflect mechanical condition, with particular attention to the parameter k , which must be refined using high-speed camera observations.

3.2 Aerodynamics modelling

Based on the wing kinematics established in the previous section, the aerodynamic forces acting on the Hybrid FWR can be readily evaluated. This is achieved by applying Blade Element Theory (BET) [21–23] to compute the aerodynamic loads on individual blade elements and subsequently integrating these contributions over the full wingspan. The blade-element coordinate is defined in Fig. 11.

The inflow velocity components in the wing element coordinate system $x_{bl} - y_{bl} - z_{bl}$ in Fig. 11 can be calculated,

$$\begin{cases} W_x(r, t) = -\Omega r \cos[r \cdot \beta(t)] \\ \quad -V_{\infty hub_x} \sin \Psi + V_{\infty hub_y} \sin\left(\frac{\pi}{2} - \Psi\right) \\ W_y(r, t) = [V_{\infty hub_x} \cos \Psi \\ \quad + V_{\infty hub_y} \cos\left(\frac{\pi}{2} - \Psi\right)] \cos[r \cdot \beta(t)] \\ W_z(r, t) = (v_i - V_{\infty hub_z}) \cos[r \cdot \beta(t)] \\ \quad + v_\beta(r, t) + V_{\infty hub_x} \cos \Psi \sin[r \cdot \beta(t)] \end{cases} \quad (13)$$

in which W_y denotes the lateral velocity of the blade element. The symbol Ω (in rad/s) indicates the rotor rotational speed, and r corresponds to the blade element position along the wingspan. Ψ defines the azimuth angle of the blade element. The parameters $V_{\infty hub_x}$ and $V_{\infty hub_y}$ define the

inflow velocities along x_{bl} and y_{bl} directions of the blade element, respectively, while v_i denotes the induced velocity. The flapping linear velocity at the current blade element position, denoted as $v_\beta(r)$, is expressed by the following equation, which incorporates $V_{\infty hub_z}$, the inflow velocity in the z_{bl} direction.

$$v_\beta(r, t) = -[r \cdot \dot{\beta}(t)](v_i - V_{\infty hub_z}) \cos[r\beta(t)] \quad (14)$$

Next, β^* and α^* can be determined from Eqs. (1) and (2), and then, the aerodynamic forces can be calculated. The first step is to transform C_L and C_D in Fig. 7, into the blade coordinate system in Fig. 11, as described below:

$$\begin{cases} C_z(a_*, \beta_*) = C_L(a_*) \cdot \cos\beta_* - C_D(a_*) \cdot \sin\beta_* \\ C_x(a_*, \beta_*) = C_D(a_*) \cdot \cos\beta_* + C_L(a_*) \cdot \sin\beta_* \end{cases} \quad (15)$$

Furthermore, by performing integration across all blade elements along the span, the total lift force $F_{zbl}(a_*, \beta_*, r, t)$ and total drag force $F_{xbl}(a_*, \beta_*, r, t)$ in the $x_{bl} - z_{bl}$ coordinate system can be obtained:

$$\begin{cases} F_{xbl}(a_*, \beta_*, r, t) = \frac{1}{2} \sum C_x(a_*, \beta_*) \rho [W_x^2(r, t) + W_z^2(r, t)] \\ F_{zbl}(a_*, \beta_*, r, t) = \frac{1}{2} \sum C_z(a_*, \beta_*) \rho [W_x^2(r, t) + W_z^2(r, t)] \end{cases} \quad (16)$$

It is important to note that the above equations represent only the most fundamental lift and drag models for the Hybrid FWR. In practice, the key to constructing accurate lift and drag models lies in developing precise $C_L(a_*)$ and $C_D(a_*)$ specifically tailored for micro, highly flexible rotors like the Hybrid FWR. This is because its wings exhibit high flexibility, operate in a low Reynolds number regime [24], and experience large a_* due to flapping. These characteristics render traditional steady-state $C_L(a_*)$ and $C_D(a_*)$ inadequate [25–27]. Previous research by the authors [16] has demonstrated that employing a physically driven digital twin is essential to obtaining reliable $C_L(a_*)$ and $C_D(a_*)$ for such micro flexible rotors. Since high-fidelity aerodynamics model development is not the primary focus of this study, the equations provided above are sufficient to address the research questions raised in the paper.

The present aerodynamic model is based on a quasi-steady blade element formulation, which provides a simplified yet computationally efficient framework for capturing the dominant aerodynamic trends of the Hybrid FWR. However, at MAV-scale Reynolds numbers, additional

unsteady aerodynamic effects such as leading-edge vortex (LEV) formation, delayed stall, wake interaction, and aero-elastic deformation may influence the instantaneous force generation.

In the current model, induced flow variations and vortex-wake interactions are treated in a simplified manner, and their coupling with the flapping-rotational kinematics is not fully resolved. These effects may introduce deviations in the predicted aerodynamic loading and shift the location of the optimal hybridisation ratio.

As such, the present model is intended to capture first-order aerodynamic trends rather than provide a fully resolved description of unsteady flow physics. The conclusions drawn in this work should therefore be interpreted within this modelling scope.

3.3 Power consumption modelling based on hybrid ratio variation

To evaluate the efficiency of the Hybrid FWR, a model describing both power consumption and the balance between flapping and rotation is required, as shown in Fig. 1. The central aim, as discussed earlier, is to improve flight efficiency, which in practical terms means producing as much lift as possible for a given power input. This principle applies equally to fixed-wing aircraft and rotorcraft and is commonly framed in terms of maximising lift, or lift coefficient, under constant power conditions. Efficiency optimisation for the Hybrid FWR therefore hinges on the relationship between power and lift, making the development of a representative power model a necessary first step in assessing performance across different flapping-rotation combinations.

As shown in Fig. 1, the power system of the Hybrid FWR consists of two identical 8520 coreless motors, with specifications listed in Table 3.

If the Pulse Width Modulation (PWM) duty cycle percentage (Thr (%), ranging from 0% to 100%) of the electronic speed controller on the Hybrid FWR's flight control system is used as the input parameter for motor system power control, the relationships between Thr , motor voltage, current, and power have been experimentally fitted as follows:

$$\begin{aligned} I &= 0.08811 \times Thr + 0.5217 \\ U &= 0.02817 \times Thr + 0.1742 \\ P &= IU \end{aligned} \quad (17)$$

Here, I represents the motor current (in amperes), U denotes the motor voltage (in volts), and P represents the input power of a single motor (in watts). The power of the flapping motor is designated as P_{flap} , while the power of the rotational motor in Fig. 3 is denoted as P_{rot} . The total

Table 3 Specifications of 8520 coreless motors

	Diameter	8.6 mm
	Length	19.5 mm
	Weight	5.9 g
	Maximum voltage	4.2 V
	Maximum speed	60,000 RPM

power consumption of the Hybrid FWR system, P_{total} , is defined as the sum of these two components: $P_{total} = P_{rot} + P_{flap}$, which represents the overall power required by the Hybrid FWR system.

Further experimental tests were conducted to establish the relationship between the PWM duty cycle input Thr and the torque output of the rotational motor, as well as the relationship between Thr and the flapping frequency of the flapping motor, as expressed in Eq. (18). Here, f is the flapping frequency (in Hz) and M is the torque (in Nm). With these parameters, the motor power can be mathematically linked to the wing kinematics, enabling a comprehensive integration of the power model in Eq. (17) and the kinematics characteristics from Eqs. (3)–(12) of the Hybrid FWR.

$$\begin{cases} f = 0.2 \times Thr + 3.386 \times 10^{-16} \\ M = 0.00143 \times Thr + 0.000108 \end{cases} \quad (18)$$

In the Hybrid FWR motor control flight program, the power distribution between the flapping motor and the rotational motor is determined by a parameter referred to as ϑ . For clarity, ϑ , along with the PWM duty cycle inputs for these two motors, are denoted as Thr_{rot} and Thr_{flap} , respectively. Their relationship with Thr_{Input} is expressed as follows:

$$\begin{cases} Thr_{flap} = Thr_{Input} \times \vartheta \\ Thr_{rot} = Thr_{Input} \times (1 - \vartheta) \end{cases} \quad (19)$$

in which Thr_{Input} represents the total PWM duty cycle percentage of the electronic speed controller. The variable ϑ , ranging from 0 to 1, allows the system to transition between different operational modes: pure active rotation (no active flapping, $\vartheta = 0$), a hybrid of active rotation and active flapping ($\vartheta \in [0, 1]$), and pure flapping (with passive rotation, $\vartheta = 1$). This ϑ parameter forms the foundation for studying the optimal combination of rotation and flapping to maximise efficiency.

The hybrid ratio effectively quantifies the degree of blending between rotational and flapping motions. By further combining Eqs. (17) and (19), the relationship between Thr_{Input} and P_{total} can be derived as follows:

$$A \cdot Thr_{Input}^2 + B \cdot Thr_{Input} + C = 0 \quad (20)$$

in which

$$\begin{cases} A = 0.002481 \times (1 + 2\vartheta^2 - 2\vartheta) \\ B = 0.03006 \\ C = 0.18188 - P_{total} \end{cases} \quad (21)$$

Further, solving the quadratic equation yields:

$$Thr_{Input} = \max \left(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \right) \quad (22)$$

Given a specified P_{total} and ϑ , Thr_{Input} can be solved using Eq. (22). Once Thr_{Input} is obtained, it can be distributed between the two motors according to Eq. (19). The result from Eq. (22) requires selecting the maximum value because the throttle input Thr_{Input} must be non-negative. Hence, the process in Fig. 1 can be implemented to explore the optimal ϑ yielding the highest lift coefficient output under a fixed input power P_{total} . The derived ϑ corresponds to the maximum C_z defines the optimal efficiency point. When $\vartheta = 0$, the rotor operates in pure rotorcraft mode, allowing a direct comparison between the efficiency of pure rotorcraft mode and the optimal efficiency of the Hybrid FWR.

3.4 Lift coefficient evaluation model

This subsection will build a lift coefficient evaluation model that accounts for the asymmetry between the upstroke and downstroke phases. This asymmetry significantly influences the calculation of the cycle-averaged lift coefficient $\overline{C_z}$, which directly impacts the actual aerodynamic performance in flight. However, the lift coefficient C_z varies continuously during the upstroke and downstroke due to persistent changes in the angle of attack (AoA), as shown in Fig. 8. Moreover, Fig. 9 highlights that the durations of the upstroke and downstroke cycles vary under different flapping conditions, which further affects the $\overline{C_z}$ computation. Therefore, $\overline{C_z}$ is defined in Eq. (23), after accounting for these factors.

$$\overline{C_z} = \mu \frac{\sum_{i=0}^n C_{z,up}}{n} + (1 - \mu) \frac{\sum_{i=0}^n C_{z,down}}{n} \quad (23)$$

in which μ is used as a weighting factor to determine the contribution to $\overline{C_z}$ during the upstroke and downstroke, and n represents the number of sampling points used within each stroke phase. To calculate μ , the total aerodynamic force during the upstroke must first be evaluated. In this study, it is simplified using the parameter $\overline{C_{f,up}}$, which is defined as the sum of the average lift coefficient $\left| \frac{\sum_{i=0}^n C_{z,up}}{n} \right|$ and the average drag coefficient $\left| \frac{\sum_{i=0}^n C_{x,up}}{n} \right|$ during the upstroke, expressed as:

$$\overline{C_{f,up}} = \left| \frac{\sum_{i=0}^n C_{z,up}}{n} \right| + \left| \frac{\sum_{i=0}^n C_{x,up}}{n} \right| \quad (24)$$

Afterwards, μ is derived as below,

$$\mu = 0.65 \times \frac{\overline{C_{f,up}} - \overline{C_{f,up,min}}}{\overline{C_{f,up,max}} - \overline{C_{f,up,min}}} \quad (25)$$

in which a normalising factor is introduced with regard to its maximum and minimum values $\overline{C_{f,up,max}}$ and $\overline{C_{f,up,min}}$, respectively. Moreover, an empirical factor of 0.65 is included based on high-speed camera experimental results. This μ value varies according to the aerodynamic forces acting on the wing during the upstroke. When $C_{f,up}$ is small, the upstroke time proportion μ is reduced. Conversely, when $C_{f,up}$ is large, μ increases, accurately reflecting the relationship between aerodynamic forces and the upstroke duration.

After completing the formulation of Eqs. (23)–(25), we can proceed to analyse the effect of varying ϑ values ranging from 0 to 1 on $\overline{C_Z}$ under a given constant power P_{total} . This allows for identifying a typical value ϑ at which the fixed P_{total} produces the maximum lift coefficient, representing the potential optimal efficiency point for the Hybrid FWR system. The details are elaborated in the next section.

4 Optimal hybrid ratio analysis

4.1 Optimal condition predicted by the developed model

Figure 12 shows the results generated from the developed model in the previous section, between ϑ and the cycle-averaged lift coefficient $\overline{C_Z}$ under a steady rotor with constant flapping frequency and rotational speed for power inputs P_{total} ranging from 10 W to 30 W, with 5 W increments. It is evident that the rotor consistently achieves the highest efficiency when ϑ lies between 0.7 and 0.8.

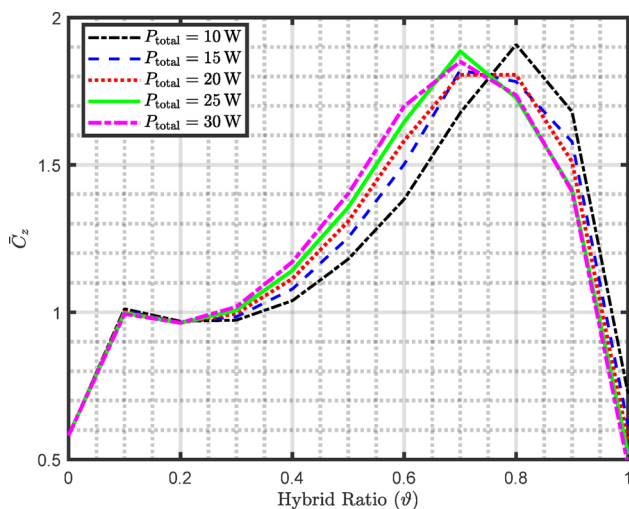


Fig. 12 Relationship between ϑ and $\overline{C_Z}$ w.r.t various defined power settings

Notably, this efficiency is significantly higher than that observed at $\vartheta = 0$, which corresponds to the conventional rotorcraft mode.

The case of $P_{total} = 15W$ in Fig. 12 was selected to illustrate the reason behind the existence of the optimal efficiency point. Figure 13 shows the variations of φ , the average lift coefficient, and the average drag coefficient as ϑ ranging from 0 to 1.

Figure 13d indicates that the highest efficiency points of $\overline{C_Z}$ locating at $\vartheta = 0.7$ for the case of $P_{total} = 15W$. This coincides with the point where the upstroke time proportion μ reaches its minimum in Fig. 13a. The same conclusion applies to other P_{total} values as well, but they are omitted here for simplicity.

4.2 Physical mechanism governing the observed optimal lift coefficient

While Fig. 13 highlights the efficiency trend via averaged aerodynamic coefficients, a detailed analysis was carried out in this Subsection to investigate the physical mechanism underlying the relationship between μ and the optimal lift coefficient, focusing on variations in the average lift and drag coefficients during the upstroke ($\overline{C_{Z,up}}$ and $\overline{C_{x,up}}$) and downstroke ($\overline{C_{Z,down}}$ and $\overline{C_{x,down}}$) over a single flapping cycle. The relationship between φ and β_* at different ϑ values for $P_{total} = 15W$ is plotted in Fig. 14.

As observed in Fig. 14a, the β_* curves at $\vartheta = 0.6$ and $\vartheta = 0.7$ appear significantly closer to the pitch angle curve during the upstroke compared to other values of ϑ . Figure 14(b) show the difference between the pitch angle and β_* during the upstroke was computed using Eq. (2), and the resulting effective angle of attack was integrated over time. As shown there, the integrated value is smallest at $\vartheta = 0.7$, suggesting the closest alignment between β_* and the pitch angle curve. This implies that, under $\vartheta = 0.7$, the aerodynamic forces acting on the wing during the upstroke are minimal, resulting in a reduced $\dot{C}_{f,up}$ in Eq. (24). Consequently, the upstroke time proportion μ becomes very small according to Eq. (25), indicating that the upstroke phase terminates earlier at $\vartheta = 0.7$ compared to other hybrid ratios. The negative region at $\vartheta = 0.9$ corresponds to a negative effective angle of attack, which induces detrimental negative lift during the upstroke and reduces overall aerodynamic efficiency.

During the downstroke, the increased W_z in Eq. (1), driven by the higher downstroke velocity, results in a larger $\dot{\beta}$. According to Eq. (2), this indicates a significantly greater effective angle of attack during the downstroke. Consequently, a substantially increased aerodynamic force

Fig. 13 Aerodynamic parameters ($\vartheta = 0$ to 1) when $P_{total} = 15W$

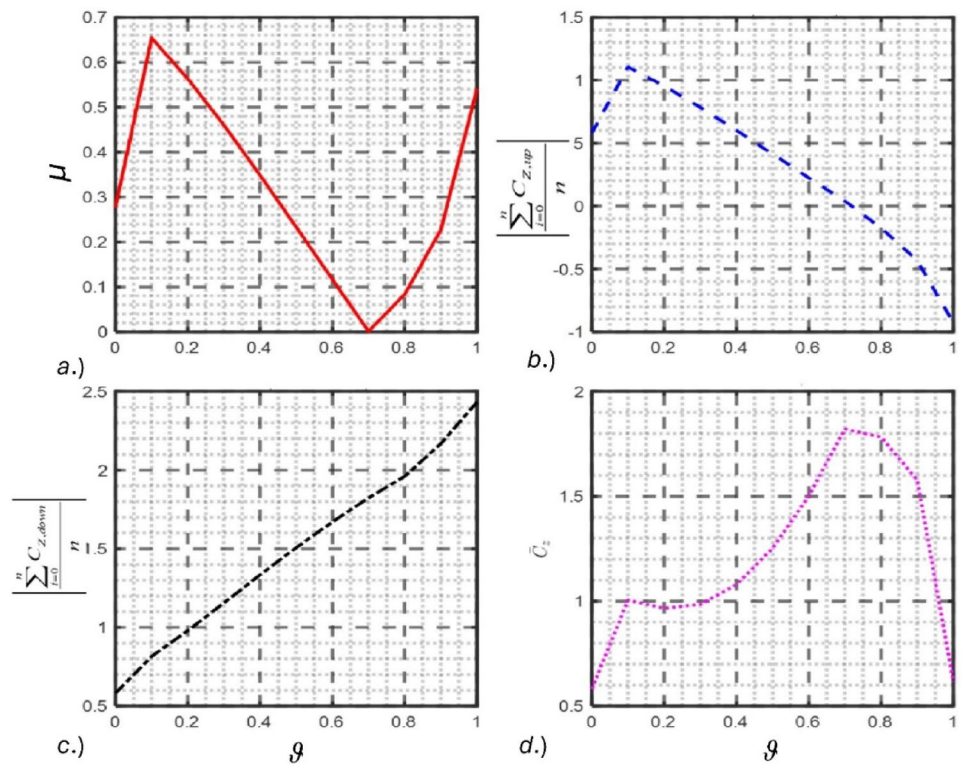
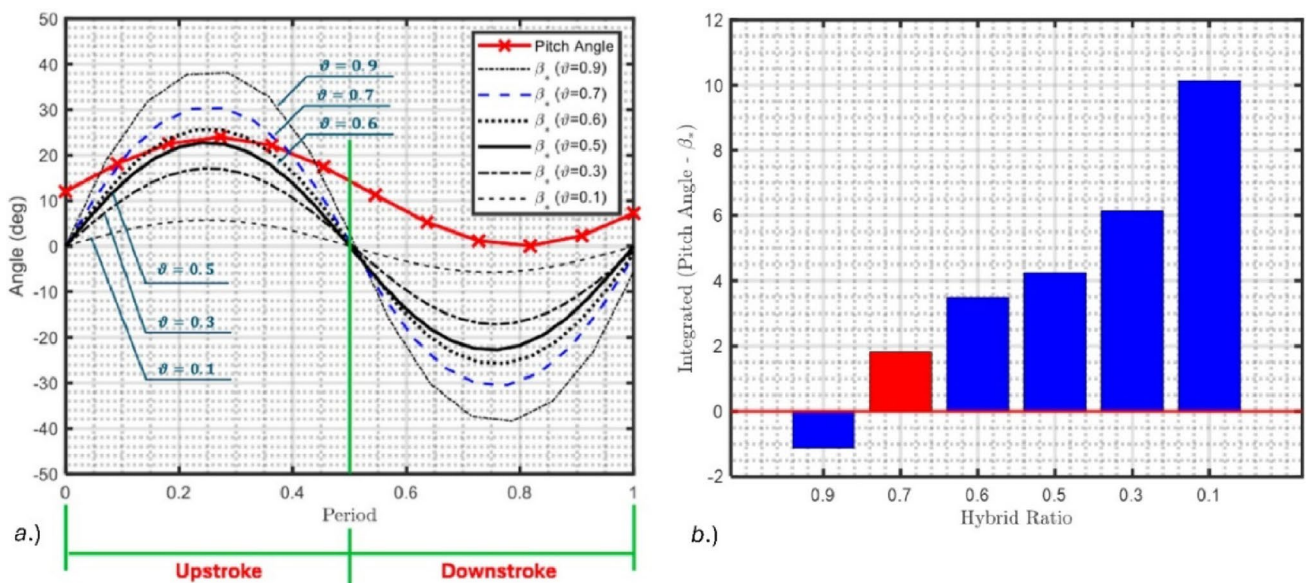


Fig. 14 The relationship between φ and β_* under different ϑ values

acts on the wing during this phase, causing the downstroke to occupy a larger fraction of the overall flapping cycle.

Figure 15 shows the average lift and drag coefficients, the magnitude of the average inflow velocity vectors $\bar{V}$, as well as their position and direction during the upstroke and downstroke phases under different g values at $P_{total} = 15W$.

Figure 15 (a₁, b₁) correspond to g 0.1–0.6, Fig. 15(a₂, b₂) to $\vartheta = 0.7$, and Fig. 15 a₃, b₃) to g 0.8–1. During the

upstroke (left column), the case of $\vartheta = 0.7$ exhibits the smallest lift and drag coefficients, consistent with Fig. 14, where α_* reaches its minimum. As a result, aerodynamic forces during the upstroke are minimised. At lower g (< 0.7), the wing still generates positive lift during the upstroke, but this is accompanied by substantial drag. In contrast, at higher g (0.8–1), the resultant flow impinges on the upper surface of the wing at an unfavourable angle, producing

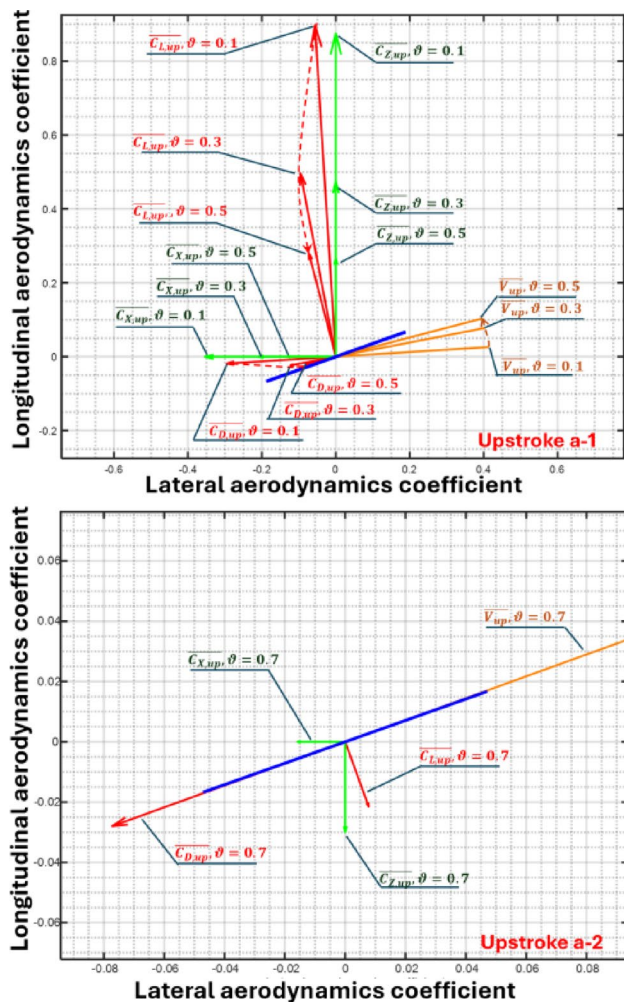


Fig. 15 Variation of average aerodynamic coefficients and incoming flow $\bar{V}$, under different ϑ values

negative lift and significantly reducing aerodynamic efficiency. Except for the case $\vartheta = 0.7$, the relatively large aerodynamic loads during the upstroke lead to increased values of μ , as also observed in the left column.

During the downstroke (right column), the large induced vertical velocity component W_Z generates a significant β^* , which substantially increases α^* according to Eqs. (1) and (2). This results in markedly higher mean lift and drag coefficients, $\bar{C}_{L,down}$ and $\bar{C}_{D,down}$. When transformed into the $x_{bl}-z_{bl}$ coordinate system using Eq. (15), these forces yield a very large $\bar{C}_{Z,down}$, which increases with ϑ , while the corresponding drag coefficient $\bar{C}_{X,down}$ remains relatively low. The combination of minimal aerodynamic loading during the upstroke and strong lift generation during the downstroke identifies $\vartheta = 0.7$ as the optimal efficiency point for $P_{total} = 15$ W. At this ϑ , the reduced upstroke forces shorten its duration, allocating more of the cycle to the downstroke and thereby maximising the cycle-averaged lift coefficient $\bar{C}_Z$. Although higher ϑ values generate larger

downstroke lift, the accompanying negative lift during the prolonged upstroke offsets this benefit, while lower ϑ values produce insufficient downstroke lift. Consequently, the Hybrid FWR achieves superior lift efficiency at $\vartheta = 0.7$, outperforming a conventional micro rotorcraft rotor ($\vartheta = 0$) in terms of overall lift performance.

5 Experimental validation

The experimental setup in Fig. 16 was employed to validate the high-lift mechanism of the Hybrid FWR discussed in Sect. 4.

The setup includes a weight scale, equipped with the Hybrid FWR hardware introduced in Sect. 2, securely mounted to the table to measure the average lift ($\bar{L}$). A power meter records voltage and current (power) consumption during the rotor's operation. A data acquisition computer transmits precise control commands for ϑ and input throttle (Thr_{input}) to the Hybrid FWR flight controller, enabling motor operations that correspond to the desired ϑ . A laser tachometer measures the rotor speed by detecting a reflective marker affixed to the rotor blade. These measurements, together with the flapping frequency captured by a high-speed camera, enable validation of the rotor's coupled flapping and rotational dynamics. It should be noted that the orange auxiliary propeller shown at the bottom of the figure was not used in the present experiments; it was prepared for subsequent control experiments. The cross-shaped black arms visible beneath the prototype are mounting struts employed to stabilise the fixture and accommodate reaction loads; they do not generate lift and do not interact aerodynamically with the rotor. Once the experimental setup was complete, tests were conducted. More than 200 experimental trials were conducted to assess its aerodynamic efficiency. To capture the performance data across the entire operating envelope of the Hybrid FWR, experiments were performed for various combinations of $P_{total} = 0 - 20$ W and $\vartheta = 0 - 1$, measuring the resulting lift. Following data collection, rather than directly plotting lift against different ϑ values at a constant power level as shown previously in Fig. 12, a more practical approach, reflective of typical UAV efficiency assessments, was implemented. Specifically, for predefined constant lift values (30 g, 35 g, and 40 g), the total power consumption was recorded across varying ϑ . The rotor's power efficiency (P_{total}) was then calculated using the following equation:

$$\epsilon \text{ (g/W)} = \frac{\bar{L}}{P_{total}} \quad (26)$$

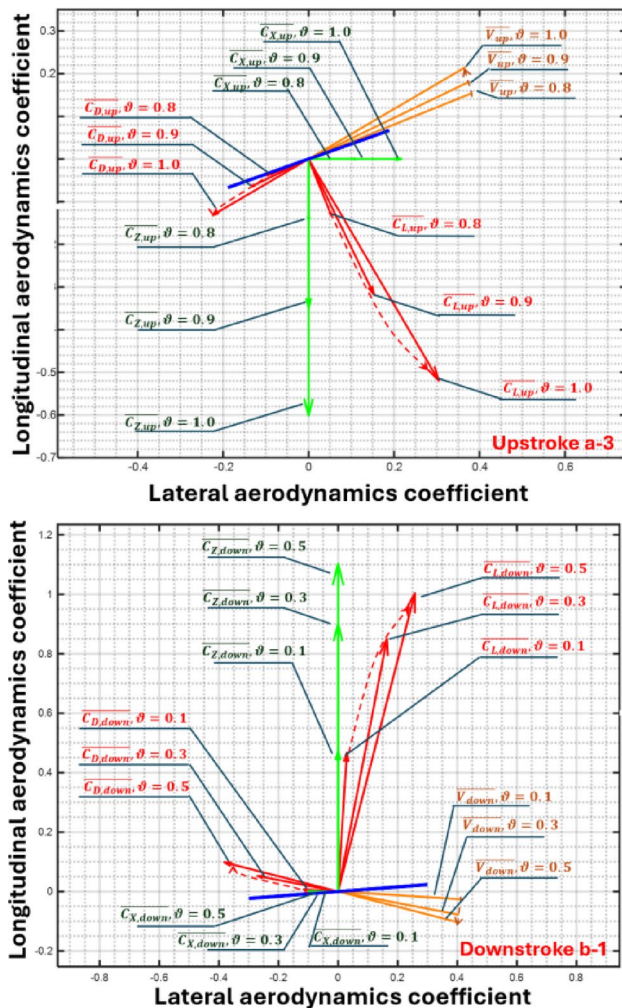


Fig. 15 (continued)

If the theoretical framework presented in Sect. 4 is valid, comparing the power efficiency (ϵ) across various ϑ at a given constant lift ($\bar{L}$) should produce results consistent with those depicted in Fig. 12. A summary of the experimental findings is provided in the figure below:

Each data point represents the average of approximately 200 repeated measurements. Error bars indicate the standard deviation, reflecting experimental uncertainties including motor fluctuations, power supply variations, and unsteady aerodynamic effects. It can be observed from Fig. 17 that at $\vartheta = 0.5$ – 0.7 , the power efficiency across various lift conditions reaches its peak. Furthermore, the shape and trend of the efficiency curves closely resemble the theoretical predictions in Fig. 16, confirming the validity of the high-lift mechanism proposed in Sect. 4. Moreover, these experimental results show that the optimal efficiency point occurs at $\vartheta = 0.5$ – 0.7 , which is slightly smaller than the theoretical prediction of $\vartheta = 0.7$ – 0.8 in Fig. 15. Additionally, the curves there exhibit a trend of continuing to shift with increasing ϑ , which was less pronounced in the theoretical predictions.

This discrepancy is likely due to the simplifications in the aerodynamics model, which did not account for certain nonlinear aerodynamic phenomena associated with flapping wings [28], such as the Weis-Fogh effect [29], delayed stall [30, 31], and the Kármán vortex street [32, 33]. Moreover, the model also did not consider chordwise deformations, aeroelastic effects [34], and resonance phenomena caused by the flexible wing and its structural design, which could influence aerodynamic forces and the upstroke-to-downstroke time proportion μ .

Despite these limitations, the experimental results robustly support the existence of the optimal high-lift coefficient point described in Sect. 4, along with the mechanism and trends leading to its occurrence. The results further demonstrate that when the Hybrid FWR operates at its optimal efficiency point ($\vartheta = 0.5$ – 0.7), its power efficiency surpasses that of traditional micro rotorcraft rotors ($\vartheta = 0$) by a factor of up to 2.148 (in the 40 g case) and at least 1.743 (in the 30 g case), showcasing exceptional performance far beyond that of conventional rotors.

6 Conclusions

This paper has examined a bio-inspired Hybrid Flapping Wing Rotor (Hybrid FWR) for MAV-scale lift generation, using a combined modelling and experimental approach to determine whether blending rotary and flapping motion can improve efficiency relative to pure rotation. A kinematic

description of the mechanically constrained flapping and inertia-driven passive pitching was developed and used to estimate cycle-averaged aerodynamic performance under fixed electrical power. The resulting trends with hybridisation ratio were interpreted in terms of stroke-wise aerodynamic asymmetry and then assessed experimentally using an instrumented bench-top rig and a two-motor prototype.

- The combined modelling-and-test approach shows that motion hybridisation produces a clear efficiency optimum, rather than a simple monotonic trade-off between flapping and rotation.
- Under the conditions studied, the model predicts that the cycle-averaged lift coefficient is maximised at an intermediate hybridisation ratio (typically ≈ 0.7 – 0.8) for constant power, exceeding the pure-rotation baseline.
- More than 200 experiments confirm an optimum close to $\vartheta \approx 0.7$, with power-efficiency gains up to $2.148\times$ relative to pure rotation at comparable lift.

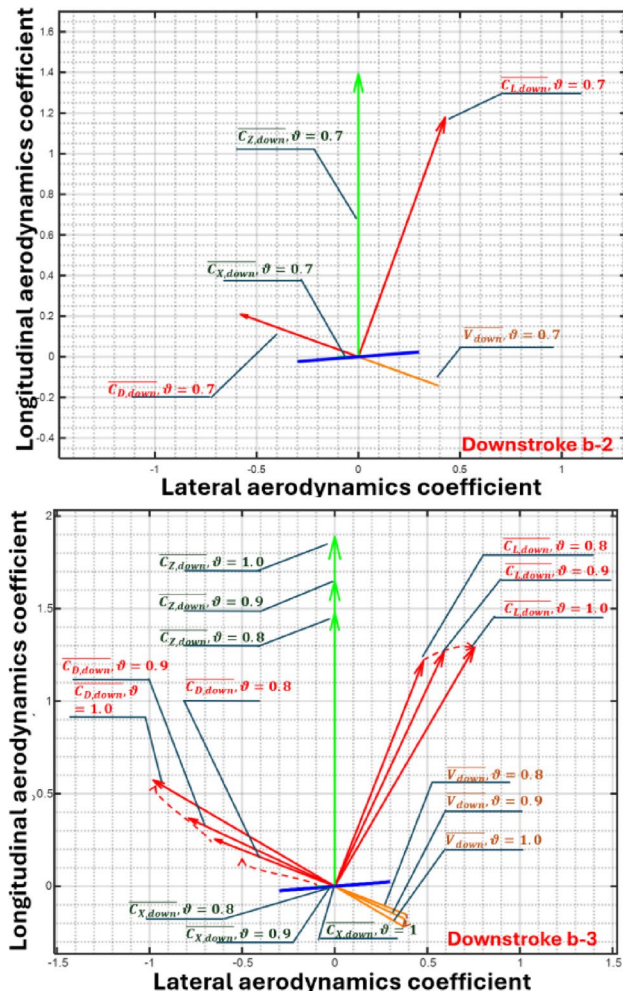


Fig. 15 (continued)

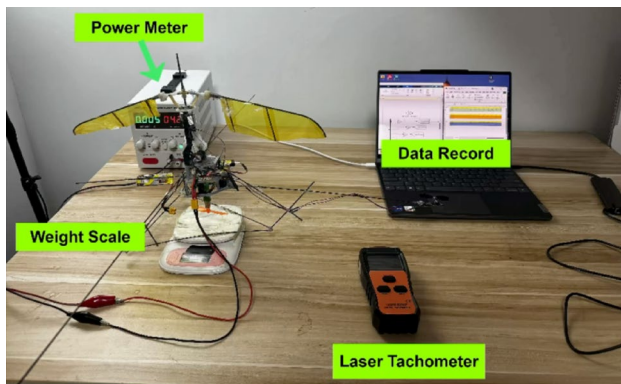


Fig. 16 Equipment testbed built to validate Hybrid FWR efficiency

- The optimum is explained by stroke-wise asymmetry: near $\vartheta \approx 0.7$, upstroke aerodynamic loading is reduced (shortening the upstroke fraction), while downstroke lift remains strong, increasing cycle-averaged lift for the same power.

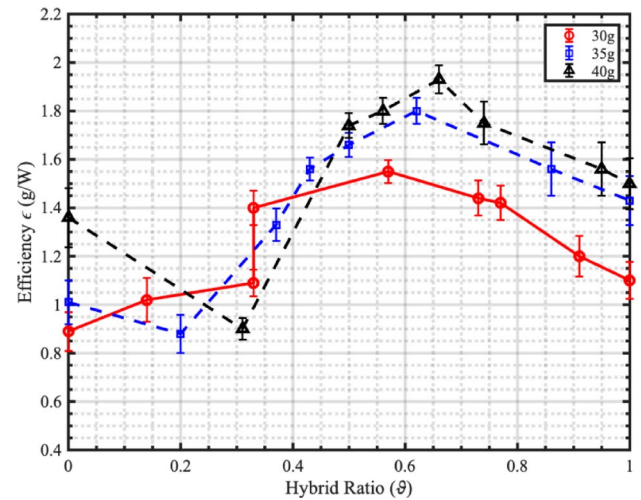


Fig. 17 Relationship between power efficiency ϵ (g/W) and hybrid ratio ϑ

While the present study demonstrates the aerodynamic and energetic advantages of the Hybrid FWR concept under controlled experimental conditions, further work is required to translate these findings into practical MAV systems. Future efforts will focus on integrating the hybrid propulsion system into a complete airframe, including attitude stabilisation, control allocation, and validation through free-flight testing.

In parallel, the modelling framework should be extended to incorporate unsteady aerodynamic and aeroelastic effects, including improved representations of induced flow

and wake interaction, supported by dedicated flow-field and structural deformation measurements, in order to reduce reliance on empirical tuning. A more systematic investigation of the sensitivity of the predicted performance to variations in aerodynamic coefficient models and key kinematic parameters (including k and μ) will also be conducted. Furthermore, validation across a broader operating envelope, together with assessment of sensitivity to manufacturing tolerances, mechanical wear, structural durability and fatigue, as well as acoustic characteristics, will be essential to establish robustness. The implications of the proposed hybridisation mechanism across different scales will also be investigated to assess its applicability beyond the current prototype configuration.

These developments will enable a more comprehensive system-level evaluation of the Hybrid FWR concept and clarify its feasibility for real-world MAV applications.

Author contributions Dr Xun Huang generated the key results during his PhD research and contributed to the underlying analysis and interpretation. Dr Linghai Lu (PhD supervisor) drafted the manuscript

based on Dr Huang's results and led the overall paper development and revision process. Professor James Whidborne and Professor Marilena Pavel provided supervisory guidance throughout the PhD programme and contributed to technical direction, critical review, and refinement of the manuscript. All authors reviewed and approved the final version of the manuscript and accept accountability for their contributions.

Data availability No datasets were generated or analysed during the current study.

Declarations

Conflict of interest Marilena Pavel, co-author of this paper, is also an editor of this journal, but was not involved in the review process. All other authors have no conflict of interests to declare that are relevant to the content of this article.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

- Berkil, E.: The diverse applications of aerospace drones: from defense to humanitarian and commercial uses. *Next Generation J. Young Researchers*. **8**(1), 69 (2024). <https://doi.org/10.62802/6hwvpp26>
- Derricott, J.C., Willis, J.B., Peterson, C.K., Franke, K.W., Hedengren, J.D.: Disaster reconnaissance using multiple small unmanned aerial vehicles. *Mech. Eng.* **141**(6), S7–S11 (2019). <https://doi.org/10.1115/1.2019-JUN5>
- Gad-el-Hak, M.: Micro-air-vehicles: can they be controlled better? *J. Aircr.* **38**(3), 419–429 (2001). <https://doi.org/10.2514/2.2807>
- Li, H., Guo, S., Zhang, Y.L., Zhou, C., Wu, J.H.: Unsteady aerodynamic and optimal kinematic analysis of a micro flapping wing rotor. *Aerosp. Sci. Technol.* **63**, 167–178 (2017). <https://doi.org/10.1016/j.ast.2016.12.025>
- Liu, Y., Zhang, Y., Wu, J.: Design and experimental study on a flapping wing micro air vehicle. In: *Proc. Asia-Pacific Int. Symp. Aerosp. TechnolAPISAT*. pp. 1330–1342. Springer (2019). (2018). https://doi.org/10.1007/978-981-13-3305-7_106
- Haider, N., Shahzad, A., Mumtaz Qadri, M.N., Shah, A., S.I.: Recent progress in flapping wings for micro aerial vehicle applications. *Proc. Inst. Mech. Eng. Part. C J. Mech. Eng. Sci.* **235**(2), 245–264 (2021). <https://doi.org/10.1177/0954406220917426>
- Chen, C., Zhang, T.: A review of design and fabrication of the bionic flapping wing micro air vehicles. *Micromachines*. **10**(2), 144 (2019). <https://doi.org/10.3390/mi10020144>
- McMichael, J.M., Francis, M.S.: Micro air vehicles – toward a new dimension in flight. *Unmanned Syst.* **15**(3), 10–17 (1997)
- Hwang, I.S., Min, S.Y., Lee, C.H., Kim, S.J.: Development of a four-rotor cyclocopter. *J. Aircr.* **45**(6), 2151–2157 (2008). <https://doi.org/10.2514/1.35575>
- Fitchett, B.K.: Development and investigation of a flapping rotor for micro air vehicles. M.S. Thesis, University of Maryland (2007)
- Alguacil, A., Jardin, T., Gourdain, N.: Fluid–structure interactions and unsteady kinematics of a low-reynolds-number rotor. *AIAA J.* **58**(2), 955–967 (2020). <https://doi.org/10.2514/1.J058230>
- Chen, L., Wu, J., Zhou, C., Hsu, S.-J., Cheng, B.: Unsteady aerodynamics of a pitching-flapping-perturbed revolving wing at low reynolds number. *Phys. Fluids*. **30**(5), 051901 (2018). <https://doi.org/10.1063/1.5024925>
- Chen, L., Cheng, C., Zhou, C., Zhang, Y., Wu, J.: Flapping rotary wing: a novel low-reynolds number layout merging bionic features into micro rotors. *Prog. Aerosp. Sci.* **146**, 100984 (2024). <https://doi.org/10.1016/j.paerosci.2024.100984>
- Huang, X., Lu, L., Pavel, M., Whidborne, J.: A bio-inspired hybrid mode for enhancing rotary MAV performance. In: *Vert. Flight Soc. 81st Annu. Forum Technol. Disp.* (2025). <https://doi.org/10.4050/F-0081-2025-120>
- Huang, X., Lu, L., Whidborne, J., Guo, S.: Modelling and simulation of a novel bioinspired flapping-wing rotary MAV. In: *49th Eur. Rotorcraft Forum (ERF 2023)*, Bückeburg. DGLR-Bericht 2023-01, pp. 1–19. DGLR, Bonn (2023)
- Guo, S., Li, D., Wu, J.: Theoretical and experimental study of a piezoelectric flapping wing rotor for micro aerial vehicle. *Aerosp. Sci. Technol.* **23**(1), 429–438 (2012). <https://doi.org/10.1016/j.ast.2011.10.002>
- Wang, Z.J.: Dissecting insect flight. *Annu. Rev. Fluid Mech.* **37**(1), 183–210 (2005). <https://doi.org/10.1146/annurev.fluid.36.050802.121940>
- Nakata, T., Liu, H., Tanaka, Y., Nishihashi, N., Wang, X., Sato, A.: Aerodynamics of a bio-inspired flexible flapping-wing micro air vehicle. *Bioinspir. Biomim.* **6**(4), 045002 (2011). <https://doi.org/10.1088/1748-3182/6/4/045002>
- Wang, H., Zeng, L., Liu, H., Yin, C.: Measuring wing kinematics, flight trajectory and body attitude during forward flight and turning maneuvers in dragonflies. *J. Exp. Biol.* **206**(4), 745–757 (2003). <https://doi.org/10.1242/jeb.00183>
- Truong, Q.T., Nguyen, Q.V., Truong, V.T., Park, H.C., Byun, D.-Y., Goo, N.-S.: A modified blade element theory for estimation of forces generated by a beetle-mimicking flapping wing system. *Bioinspir. Biomim.* **6**(3), 036008 (2011). <https://doi.org/10.1088/1748-3182/6/3/036008>
- Li, H., Guo, S.: Aerodynamic efficiency of a bioinspired flapping wing rotor at low Reynolds number. *R Soc. Open. Sci.* **5**(3), 171307 (2018). <https://doi.org/10.1098/rsos.171307>
- Wan, J.: M.D. P. Pavel Designing the ornicopter, a tailless helicopter with active flapping blades: a case study. *Proc. Inst. Mech. Eng. Part. G J. Aerosp. Eng.* **230** 12 2195–2219 <https://doi.org/10.1177/0954410015622228> (2016)
- Deng, S., Percin, M., van Oudheusden, B., Remes, B., Bijl, H.: Experimental investigation on the aerodynamics of a bio-inspired flexible flapping wing micro air vehicle. *Int. J. Micro Air Veh.* **6**(2), 105–115 (2014). <https://doi.org/10.1260/1756-8293.6.2.105>
- Lee, Y.J., Lua, K.B., Lim, T.T., Yeo, K.S.: Corrigendum: a quasi-steady aerodynamic model for flapping flight with improved adaptability (2016 *Bioinspir. Biomim.* 11 036005). *Bioinspir. Biomim.* **11**(4), 049501 (2016). <https://doi.org/10.1088/1748-3190/11/4/049501>
- Rylko, A., van 't Hoff, S., Lu, L., Padfield, G., Podzus, P., White, M., Quaranta, G.: Rotorcraft flight simulation to support aircraft certification: a review of the state of the art with an eye to future applications. In: *49th Eur. Rotorcraft Forum (ERF 2023)*, Bückeburg. DGLR-Bericht 2023-01, pp. 1–12

26. Lu, L., Agarwal, D., Padfield, G.D., White, M.D., Cameron, N.: A new heuristic approach to rotorcraft system identification. *J. Am. Helicopter Soc.* **68**(2), 022005 (2023). <https://doi.org/10.4050/JAHS.68.022005>
27. Padfield, G., Casolaro, D., Hamers, M., Pavel, M., Roth, G., Taghizad, A.: Fidelity of helicopter real-time simulation models. In: AHS 61st Annu. Forum, Grapevine, TX (2005)
28. Kumar, G.C.V., Juliyana, M.R.: Design and analysis of flapping wing. *Appl. Mech. Mater.* **110–116**, 3495–3499 (2011). <https://doi.org/10.4028/www.scientific.net/AMM.110-116.3495>
29. Kolomenskiy, D., Moffatt, H.K., Farge, M., Schneider, K.: The Lighthill–Weis-Fogh clap–fling–sweep mechanism revisited. *J. Fluid Mech.* **676**, 572–606 (2011). <https://doi.org/10.1017/jfm.2011.83>
30. Usab, W.J., Hardin, J., Bilanin, A.J.: Bioinspired delayed stall propulsor. *IEEE J. Ocean. Eng.* **29**(3), 756–765 (2004). <https://doi.org/10.1109/JOE.2004.833112>
31. Sudhakar, Y., Vengadesan, S.: The functional significance of delayed stall in insect flight. *Numer. Heat. Transf. Part. Appl.* **58**(1), 65–83 (2010). <https://doi.org/10.1080/10407782.2010.490453>
32. Thoraval, M.-J., Takehara, K., Etoh, T.G., Popinet, S., Ray, P., Josserand, C., Zaleski, S., Thoroddsen, S.T.: Von Kármán vortex street within an impacting drop. *Phys. Rev. Lett.* **108**(26), 264506 (2012). <https://doi.org/10.1103/PhysRevLett.108.264506>
33. Chen, S., Li, H., Guo, S., Tong, M., Ji, B.: Unsteady aerodynamic model of flexible flapping wing. *Aerosp. Sci. Technol.* **80**, 354–367 (2018). <https://doi.org/10.1016/j.ast.2018.07.017>
34. Shyy, W., Aono, H., Chimakurthi, S.K., Trizila, P., Kang, C.-K., Cesnik, C.E.S., Liu, H.: Recent progress in flapping wing aerodynamics and aeroelasticity. *Prog. Aerosp. Sci.* **46**(7), 284–327 (2010). <https://doi.org/10.1016/j.paerosci.2010.01.001>

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.



DSPACE

<https://dspace.org/>

A bio-inspired hybrid flapping wing rotor for high-efficiency micro rotorcraft

Huang, Xun; Lu, Linghai; Whidborne, James F.; Pavel, Marilena

2026-12-31

Springer

<https://doi.org/10.1007/s13272-026-00973-z>

Huang X, Lu L, Whidborne J, Pavel M. (2025) A bio-inspired hybrid flapping wing rotor for high-efficiency micro rotorcraft. CEAS Aeronautical Journal, Available online 18 June 2026