



An optimization-driven design framework for inertance-integrated hydraulic shock absorbers: incorporating nonlinear and parasitic effects

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Abstract

Integrated spring-damper-inerter systems have been shown to outperform traditional damper-only absorbers in suppressing mechanical vibrations, driving interest in incorporating hydraulic stiffness, damping, and inertance components into automotive shock absorbers to enhance ride comfort. Extensive research has been conducted to identify the optimal absorber configuration from a vast design space. However, existing design approaches often neglect the nonlinear and parasitic effects (NPEs) inherent in hydraulic components and consider only limited topological layouts, thereby limiting the comprehensiveness and accuracy of the design space exploration. This oversight can result in discrepancies between simulated and real-world performance, potentially leading to suboptimal designs. To address this, a novel computer-aided engineering (CAE) framework is proposed for optimizing the configuration of inertance-integrated hydraulic shock absorbers. The framework follows a three-step process: (i) a graph-based method for enumerating all feasible hydraulic network layouts from a predefined catalog of components, (ii) an automated MATLAB subroutine for modeling these networks in Simscape, incorporating component models that explicitly account for NPEs, and (iii) a MATLAB-CarMaker co-simulation workflow for optimizing the hydraulic networks within a high-fidelity vehicle model. A case study involving a saloon car subjected to an ISO 8608 rough road input demonstrates the effectiveness of the framework. The optimized absorber design improves ride comfort by 17.6% when the NPE associated with hydraulic inertance realization is neglected and by 8.3% when it is considered, both while meeting dynamic tire load and suspension travel constraints. These results emphasize the importance of incorporating NPEs in the design process and validate the framework as an effective CAE tool for developing high-performance hydraulic absorbers for practical applications.

Keywords Inertance · Absorber · Optimal · Ride comfort · Graph theory · Parasitic effects

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1 Introduction

Automotive suspension systems are pivotal for ensuring ride comfort and maintaining vehicle handling safety, particularly during extreme maneuvers. In most mass-produced vehicles, the suspension typically consists of a coil spring and a hydraulic damper [1]: the spring supports the chassis load, while the damper suppresses vibrations caused by road irregularities and steering inputs. In addition to these components, the inerter—a passive mechanical element first formalized in [2], which generates a resistive force proportional to the relative acceleration between its terminals—offers new design possibilities for suspension systems. Owing to this unique dynamic characteristic, incorporating inertance and stiffness to create a hydraulic stiffness-damping-inertance

absorber holds significant potential for enhancing vibration isolation performance.

Extensive research efforts have been devoted to realizing stiffness and inertance within the hydraulic domain. For hydraulic stiffness, one approach is to use the bulk modulus of the hydraulic oil, though this method provides only limited tunability. Alternatively, a gas accumulator can be used, where hydraulic stiffness is achieved via the expansion and compression of air inside the accumulator. This approach, however, only allows for a one-terminal stiffness element, as the accumulator must connect in series with the hydraulic network, as outlined in [3]. To achieve embedded stiffness within the network, two feasible methods have been proposed. The first method incorporates a rubber membrane between the two chambers of a cylinder [3, 4], while the second method employs a spring connecting the cylinder's piston and casing [5]. In both cases, changes in fluid volume within either chamber lead to deformation of the rubber membrane or the spring, thereby producing the desired stiffness effect. Hydraulic inertance can be achieved through a helical tube [6, 7], where inertial effects arise from the acceleration and deceleration of fluid within the coiled structure. This design leverages the fluid's mass and the tube's geometry to generate the desired inertance. Notably, Tran et al. [8] further developed a semi-active fluid inerter by incorporating a controllable bypass valve into the helical tube configuration. Another approach employs a hydraulic motor, where inertance is realized by converting hydraulic flow into motor rotation and can be tuned by adjusting the flywheel attached to the motor [9]. Compared to their idealized linear properties, the physical realizations of hydraulic components typically exhibit nonlinear and parasitic effects (NPEs). Specifically, Liu et al. [10, 11] investigated the impact of fluid leakage between the piston and the cylinder in the implementation of a helical tube inerter. Chillemi et al. [12, 13] examined the stiffness variation caused by residual air within the inerter chamber. Zhang et al. [14] identified a parasitic damping effect during the implementation of a displacement-dependent fluid inerter and subsequently utilized this effect in the design of a tunable inerter-damper suspension system [15].

Building on the aforementioned methods for achieving hydraulic stiffness and inertance, several innovative suspension designs that integrate both effects have been proposed. For instance, Yang et al. [16] employed a helical tube connecting the gas accumulator to the absorber cylinder to realize hydraulic inertance, resulting in improved damping performance across both low and high frequency ranges. Zhang et al. [17, 18] introduced a configuration with dual gas accumulators linked by a helical tube, effectively reducing vehicle body vibration without increasing the required suspension working space. Zhu et al. [19] incorporated an electromagnetic proportional valve to enable semi-active

control of the inertance coefficient. Nie et al. [20] applied this layout to interconnected suspension systems and demonstrated simultaneous improvements in ride comfort and handling safety. While these studies underscore the benefits of incorporating hydraulic inertance and stiffness into suspension systems, they have primarily focused on a limited set of topological configurations, leaving a broad design space yet to be explored.

Several studies have attempted to identify the optimal configuration of integrated hydraulic shock absorbers from the vast design space. These approaches typically begin by enumerating all possible network topologies constructed from a predefined catalog of elements (e.g., stiffness, damping, and inertance), which define how these elements are interconnected within the hydraulic network. The parameters of each potential layout are then optimized to identify the best-performing configuration for a given vibration suppression problem. For instance, the structural-immittance approach proposed by Zhang et al. [21] focuses on analyzing series-parallel networks composed of pre-defined element types (e.g., springs, dampers, and inerters). While effective for low-complexity vibration absorbers composed of few elements (see e.g., those used in engines [22], buildings [23], and vehicles [24]), this method excludes non-series-parallel configurations, potentially leading to suboptimal results. Herber et al. [25, 26] proposed an enumeration technique capable of covering all possible absorber layouts. However, this approach requires predefining not only the number of elements but also the number of ports (e.g., 3-port and 4-port joints representing T-shaped and cruciform interconnections, respectively). To address these limitations, He et al. [27] introduced a novel approach that represents absorber networks as weighted and colored multigraphs, allowing the use of graph theory techniques to enumerate both series-parallel and non-series-parallel configurations without using the port elements. Building on the structural-immittance approach, Li et al. [5] developed a multi-domain synthesis method to realize absorber layouts across mechanical, hydraulic, pneumatic, and electrical domains.

Despite progress in optimal network design through enumeration-based approaches, existing methods typically model vibration absorbers as idealized networks composed of linear elements. These methods fail to integrate the NPEs inherent in hydraulic components into the entire design space exploration. As a result, the performance of networks optimized under linear assumptions may deviate significantly from their real-world behavior, potentially leading to suboptimal designs. Another critical challenge associated with linear element assumptions is the difficulty in formulating practically meaningful side constraints on their linearized properties. These properties (e.g., hydraulic inertance) depend not only on the physical parameters of the components (e.g., pipe radius and length) but also on fluid

properties (e.g., density, viscosity) and flow conditions (e.g., Reynolds number). Improperly defined side constraints can result in impractical physical designs. Therefore, it is essential to employ physical component models that explicitly account for NPEs during the design stage and consider the physical component parameters as design variables (DVs). This enables designers to more accurately predict real-world performance and develop more robust suspension solutions suited for practical applications.

To address these challenges, this paper introduces a novel computer-aided engineering (CAE) framework for optimizing the configuration (i.e., both layout and parameters) of inertance-integrated hydraulic shock absorbers to meet car body dynamics objectives while satisfying relevant performance constraints. The framework comprises three key components: (i) a graph-based method for enumerating all feasible hydraulic network layouts from a predefined catalog of elements (e.g., stiffness, damping, inertance); (ii) an automated MATLAB subroutine that models these layouts in Simscape using component models that explicitly capture NPEs; and (iii) a MATLAB-CarMaker co-simulation workflow for optimizing absorber networks within a high-fidelity vehicle model under realistic test scenarios. By adopting hydraulic component models that directly treat physical parameters as DVs and explicitly incorporate NPEs, the framework ensures that the resulting absorber configurations are both physically realizable and closely aligned with actual performance. The primary novelty of this work lies in its unified and automated solution to a bi-level constrained design optimization problem within a high-fidelity simulation environment, where the upper level addresses absorber layout generation and the lower level focuses on parameter tuning. Compared to prior studies, which often assume linear or idealized component behavior and explore only limited topological layouts, the proposed framework combines graph-based enumeration and model-based co-simulation in the context of nonlinear hydraulic systems, enabling a level of automation, design-space coverage, and real-world applicability that has not been previously demonstrated in the literature.

The structure of this paper is as follows: Sect. 2 introduces a full-car model of a BMW 5 Series saloon subjected to an ISO 8608 rough road profile, which serves as the testbed for the numerical demonstration of the proposed framework. Building on the conventional hydraulic damper design, this section then proposes a new design by replacing the orifice connecting the two chambers of the traditional hydraulic damper with a generalized hydraulic network that integrates stiffness, damping, and inertance effects. Additionally, it formulates a corresponding optimization problem for the new shock absorber designs. Section 3 outlines the proposed CAE design framework, which includes sequentially graph-based layout enumeration, automated modeling

of absorber networks with NPEs, and parameter optimization of these networks within the full-car model. Using this framework, Sect. 4 determines the optimal configuration for the hydraulic shock absorber in the testbed model and examines the impact of inertance's parasitic damping on vehicle performance improvements. Finally, Sect. 5 concludes the paper with a summary of key findings and discusses potential directions for future research.

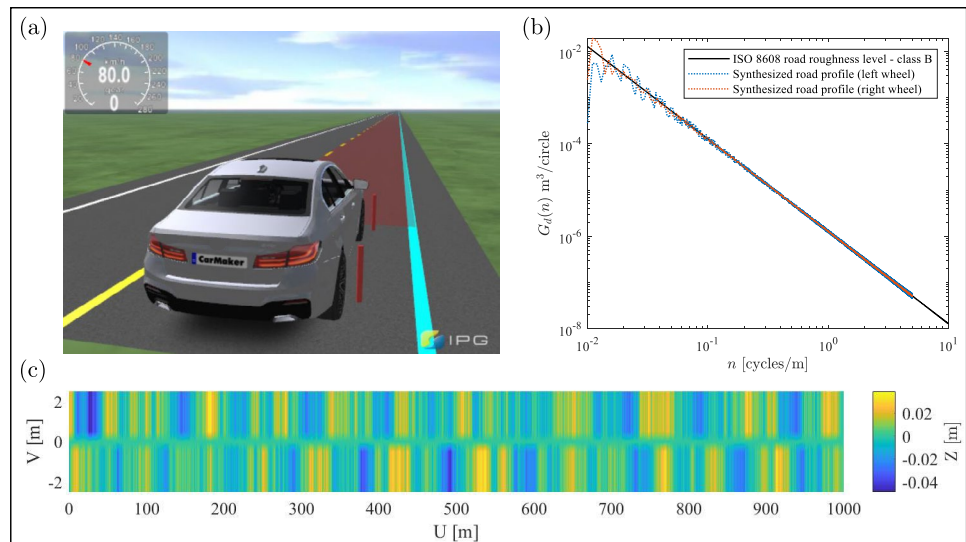
2 Design problem formulation

This section presents the formulation of the design problem, using a high-fidelity CarMaker vehicle model subjected to a codified rough road profile as the testbed for absorber design optimization. While this case study focuses on vehicle dynamics under rough road excitations, the proposed framework is readily extensible to other scenarios, including cornering, acceleration, braking, and discrete disturbances such as potholes and speed bumps. This section begins with a description of the vehicle model, road input, and conventional hydraulic damper to be improved. The key performance metrics for absorbers in vehicle dynamics are then introduced. Finally, the design optimization problem for a stiffness-damping-inertance integrated absorber is formulated, with these performance metrics as design objectives and constraints.

2.1 Full-car model, road input, and traditional damper

The IPG CarMaker[®] is utilized to simulate vehicle dynamics, with a demo BMW 5 Series saloon model serving as an example to illustrate the design problem, as shown in Fig. 1(a). The road input is assumed to follow the ISO 8608 [28], where the power spectral density (PSD) of the road displacement is given by $G_d(n) = G_d(n_0) \cdot (n_0/n)^2$. In this equation, n is the spatial frequency of the road in the unit of cycles/m, and n_0 is the reference spatial frequency ($= 0.1$ cycles/m). For this study, $G_d(n_0)$ is set to $128 \times 10^{-6} \text{ m}^3$ to simulate a B-class road, which corresponds to the upper limit of this road classification. The left and right wheels are assumed to experience two independent random roads, ensuring that both the roll and pitch motions of the vehicle are excited. Figure 1(b) illustrates the PSDs of this ISO 8608 road profile, along with the PSDs of the synthesized road for the left and right wheels. The 3D road profile, generated using OpenCRG [29], has a length of 1000 m ($U = 1000$ m) and a width of 4 m ($V = 4$ m), with the elevation variations along the road's length and width shown in Fig. 1(c). In this profile, the upper segment, $0 \leq V \leq 2$ m, is assumed to have the same profile as the one corresponding to the PSD indicated by the blue curve in Fig. 1(b), while the lower

Fig. 1 Full car model and road profile. **a** A demo BMW 5 model in CarMaker, with a speed set at 80 km/h. **b** Power spectrum densities (PSDs) of the ISO 8608 Class B rough road profile, alongside the synthesized road profiles for left and right wheels. **c** Road elevation over a length of 1000 m and a width of 4 m



segment, $-2 \text{ m} \leq V < 0$, is assumed to share the same profile as the one corresponding to the PSD indicated by the red curve in Fig. 1(b). The vehicle speed is set to 80 km/h.

The suspension model in CarMaker can be simplified to the schematic shown in Fig. 2(a), consisting of a spring and a damper connected between the wheel and the car body. The conventional/default hydraulic damper, widely adopted in the automotive market and discussed in [1, 5], is illustrated in Fig. 2(b). It generates damping by allowing fluid to pass through the orifice in the piston, thereby resisting relative movement between its two ends. A gas accumulator is connected to the cylinder's lower chamber to balance the fluid volume changes in the cylinder as the piston moves. Additionally, since fluid flowing in and out of the gas accumulator compresses and extends the internal gas, the accumulator also provides a stiffness effect to this damper. Figure 2(c) presents an equivalent layout to that shown in Fig. 2(b), with the orifice positioned in an external channel connecting the upper and lower chambers of the cylinder. This layout will be generalized for the design of a hydraulic shock absorber

that integrates stiffness, damping, and inertance effects. The key parameters for the default hydraulic shock absorber used in the case study are listed in Table 1, consistent with the data reported in [30]. It is important to note that although the front and rear shock absorbers share the same fluid properties, they differ in mechanical dimensions.

2.2 Performance metrics for absorber evaluation

Three key metrics are used to evaluate the dynamic performance of the conventional and new shock absorber designs. The first metric is the root-mean-square (RMS) value of car body's vertical acceleration, $\ddot{x}_s$, over the simulation period [31]. This metric, denoted as J_r , is given by

$$J_r = \text{r.m.s.}(\ddot{x}_s) \quad (1)$$

which is commonly used to assess passenger riding comfort by quantifying the level of vibrations transmitted to the car body.

Fig. 2 Default suspension design. **a** The suspension model in CarMaker, assumed to consist of a damper and a spring. **b** The default hydraulic damper, which dissipates vibrations through fluid flow via an orifice in the piston. **c** An equivalent layout of (b), which is employed later to determine the optimal stiffness-inertance absorber

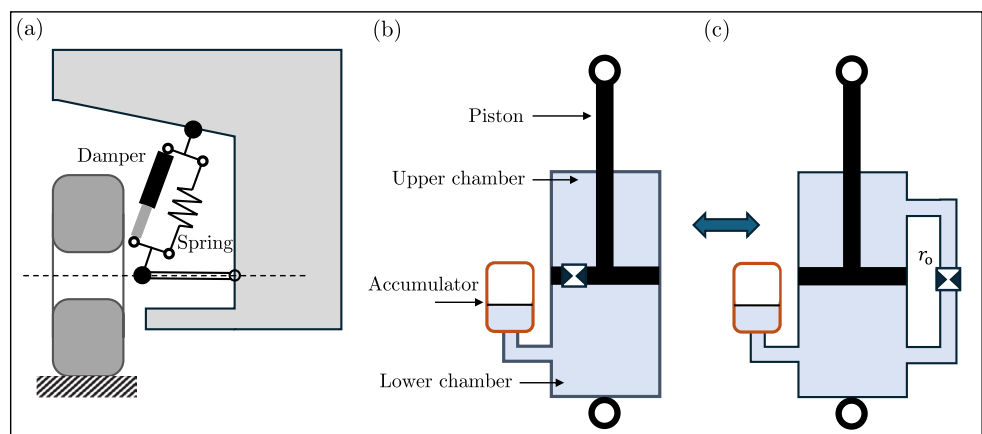


Table 1 Key parameters of the default hydraulic shock absorber used in the case study [30]

Parameters	Front	Rear
Piston diameter	27 mm	25.4 mm
Rod diameter	14 mm	12.4 mm
Accumulator volume	0.09 L	0.09 L
Pre-charge pressure	7 bar	7.5 bar
Equilibrium pressure	12 bar	11.5 bar
Fluid density ρ	827 kg/m ³	
Fluid viscosity ν	22 cSt	
Friction factor f_L	2000	
Friction factor f_T	4000	
Discharge coefficient C_D	0.7	
Roughness height r	1.5×10^{-5} m	
Spring device area A_s	314.2 mm ²	
Laminar flow pressure ratio B_{lam}	0.999	

The pre-charge pressure is gauge pressure

The second metric is the dynamic tire load, used to evaluate the vehicle's road-handling ability [31]. It is defined as

$$J_t^i = \max[\text{r.m.s.}(F_t^{iL}), \text{r.m.s.}(F_t^{iR})], \quad (2)$$

for front tires if $i = F$, and rear tires if $i = R$. In these equations, F_t^{ij} ($j = L$ or R) represents the tire dynamic forces (subtracting the vehicle weight) for each wheel, specifically the front left (FL), front right (FR), rear left (RL), and rear right (RR) wheels. Since the left and right wheels experience two random road profiles with the same PSD and given the vehicle's left-right symmetry, it is theoretically expected that $\text{r.m.s.}(F_t^{iL}) = \text{r.m.s.}(F_t^{iR})$ for $i = F$ and R . However, considering that only a 1000 m length of the road is evaluated, we can only approximate this relationship, leading to $\text{r.m.s.}(F_t^{iL}) \approx \text{r.m.s.}(F_t^{iR})$ for $i = F$ and R . Therefore, the dynamic tire load metric for the front (rear) wheels is

assumed to be the maximum value of the loads from both the left and right wheels.

The third metric is peak suspension travel, used to prevent the suspension mechanism from reaching saturation [32]. It is defined as

$$J_s^i = \max[\max(x_{su}^{iL}), \max(x_{su}^{iR})] \quad (3)$$

for front tires if $i = F$, and rear tires if $i = R$. In these equations, x_{su}^{ij} represents the travel for each suspension of the vehicle, specifically the front left (FL), front right (FR), rear left (RL), and rear right (RR). Given that $\max(x_{su}^{iL}) \approx \max(x_{su}^{iR})$ for $i = F$ and R , similar to the dynamic tire load metric, the peak suspension travel metric for the front (rear) wheels is defined as the maximum travel between the left and right suspensions.

2.3 Design optimization problem formulation

To further enhance performance of the a hydraulic shock absorber, this paper proposes replacing the orifice in Fig. 2(c) with a general hydraulic network, as shown in Fig. 3(a). This general hydraulic network can be configured to include any pre-determined number of physical components. In this study, three types of passive components are considered: hydraulic damping, hydraulic stiffness, and hydraulic inertance. Figure 3(b) illustrates typical physical realizations of these properties. For instance, hydraulic damping can be achieved using an orifice or a valve, hydraulic inertance can be implemented with a helical tube (as suggested by [10, 11]) or a hydraulic motor (as suggested by [9]), and hydraulic stiffness can be realized with a cylinder featuring a membrane fixed at the center (as suggested by [3, 4]) or with a spring placed between the piston and the casing (as suggested by [5]).

To determine the optimal configuration for this general hydraulic network, it is first necessary to establish the benchmark performance for this study. Here, the benchmark shock

Fig. 3 Proposed hydraulic shock absorber design. **a** A general hydraulic network, which can consist of any pre-determined number of hydraulic damping, stiffness, and inertance components. **b** Typical physical realizations of hydraulic damping, stiffness, and inertance components

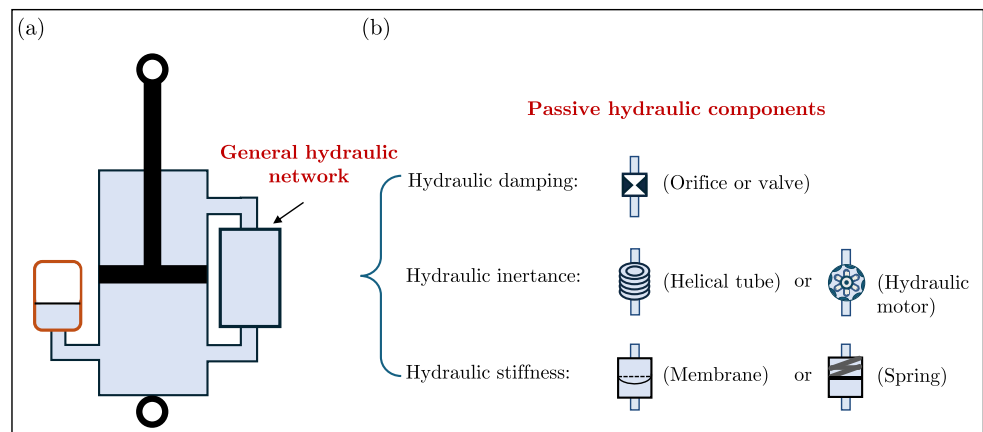
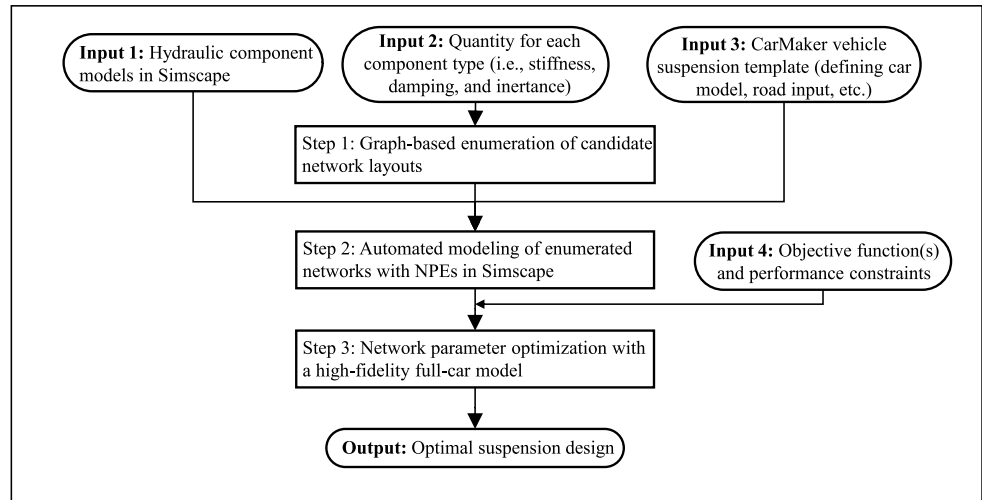


Fig. 4 Flowchart summarizing the main steps of the CAE design framework



absorber is defined as a default hydraulic damper with orifice diameters optimized to achieve minimal RMS car body acceleration. The goal of the new design is to further reduce the RMS car body acceleration for ride comfort, while ensuring that suspension travel and dynamic tire load metrics remain no greater than those of the benchmark suspension. Based on these considerations, the constrained design optimization problem is reformulated as an unconstrained problem by incorporating two penalty terms as follows:

$$\begin{aligned} \text{Min } J_r(g, \theta) + \frac{1}{2} \left\{ p_t \sum_{i=\{F,R\}} \max \left(\frac{J_t^i(g, \theta) - J_t^{i,\text{Ben}}}{J_t^{i,\text{Ben}}}, 0 \right)^2 \right. \\ \left. + p_s \sum_{i=\{F,R\}} \max \left(\frac{J_s^i(g, \theta) - J_s^{i,\text{Ben}}}{J_s^{i,\text{Ben}}}, 0 \right)^2 \right\} \quad (4) \\ \text{s.t. } \begin{cases} g \in G \\ \theta_{\text{low}} \leq \theta \leq \theta_{\text{up}} \end{cases} \end{aligned}$$

where g denotes the layout of a hydraulic network, and G is the full set of all possible network layouts with a predetermined number of components. θ denotes the DVs subjected to the side constraint $\theta_{\text{low}} \leq \theta \leq \theta_{\text{up}}$. The specific parameters included in θ will be further explained in Sections 4.2 and 4.3. The superscript ‘Ben’ in the equation denotes benchmark performance. The penalized objective function is defined as the sum of the RMS car body acceleration metric (the objective function of the original constrained optimization problem) and two penalty terms, which incorporate the dynamic tire load constraint and the suspension travel constraint through penalty coefficients p_t and p_s , respectively. When the constraints are satisfied, the second and third terms become zero, contributing nothing to the penalized objective function. However, if the constraints are violated, i.e., $J_t^i(g, \theta) \geq J_t^{i,\text{Ben}}$ and/or $J_s^i(g, \theta) \geq J_s^{i,\text{Ben}}$, the

penalty coefficients p_t and p_s significantly amplify the penalized objective function. In this study, p_t and p_s are set to 10,000, determined through trial and error to strike a balance between minimizing the RMS car body acceleration value and satisfying the two constraints. As a result, the minimal value of the penalized objective function is achieved when the RMS car body acceleration is optimized while both constraints are satisfied.

3 Proposed design optimization framework

Based on the design problem outlined in Eq. (4), this section presents a novel CAE framework for optimizing the configuration of inertance-integrated hydraulic shock absorbers. The proposed framework follows a systematic three-step process, as shown in Fig. 4. First, a graph-based enumeration technique [27] is employed to generate all possible layouts for the hydraulic network from a predefined catalog of stiffness, damping, and inertance elements. Second, a scripted MATLAB subroutine is employed to automate the modeling of the enumerated layouts, utilizing nonlinear hydraulic component models in Simscape that explicitly account for NPEs. Finally, the nonlinear models of the hydraulic shock absorber are integrated into a high-fidelity full-car model through a MATLAB-CarMaker co-simulation workflow for parameter optimization, guided by specified design objectives and performance constraints. By incorporating NPEs into the optimization process, this framework ensures that the identified shock absorber configuration is practically realizable. It is worth noting that although the current implementation uses CarMaker and Simscape, the framework is fully compatible with other vehicle simulation platforms, including CarSim, the Simscape Vehicle Template, and multi-domain modeling environments such as Amesim. This flexibility enables seamless integration of the proposed

framework into standard industrial workflows for hydraulic absorber design. The details of these three steps are elaborated in the following subsections.

3.1 Graph-based enumeration of candidate network layouts

This step aims to generate all possible layouts for a hydraulic network with given numbers for each component type (i.e., stiffness, damping, and inertance). Fundamental to this step is the representation of the hydraulic network as a weighted and colored graph. Here, components in the hydraulic network are mapped to graph edges, while the connection points between components are represented by graph nodes. The component type is distinguished by the numerical weight assigned to its corresponding edge: hydraulic stiffness, damping, and inertance components are represented by edge weights of 1, 2, and 3, respectively, as shown in Fig. 5(a). To differentiate the two terminal points of the hydraulic network from other internal points, the nodes corresponding to the terminal points are colored red, while the internal nodes remain black. With these definitions, a one-to-one correspondence between the graph and the hydraulic network can be established. For instance, Fig. 5(b) illustrates two example graphs alongside their corresponding hydraulic networks. It is important to note that in the second example graph in Fig. 5(b), there are three red nodes; however, two of them are connected and will ultimately merge into a single node. This facilitates the enumeration of all possible layouts for a hydraulic network with pre-determined component quantities by transforming the task into an enumeration of graph representations with a specified number of edges.

The authors have previously developed a method [27] for enumerating all possible graphs with a given number

of mechanical stiffness, damping, and inertance components. This method can be directly applied here by replacing mechanical components with hydraulic components, without further detailed discussion. For illustration, Fig. 5(c) shows all possible graphs containing one hydraulic stiffness, one hydraulic damping, and one hydraulic inertance, resulting in a total of 8 different configurations.

3.2 Automated modeling of enumerated networks with NPEs in Simscape

Based on the hydraulic network layouts generated in the previous step, corresponding simulation models can be readily constructed in a model-based multiphysics simulation environment, such as Simscape, which is used in this study for subsequent parameter optimization. As a remark, Simscape is a high-fidelity toolbox widely recognized for its accuracy in simulating physical systems. Its component models are based on well-established physical principles that have been validated in previous studies [33, 34]. These built-in models ensure that the simulations conducted in this study are grounded in experimentally verified physics. However, as the number of components increases, the number of possible absorber layouts grows exponentially, making the manual generation of all simulation models extremely time-consuming, if not impractical. To address this challenge, this section introduces a scripted MATLAB subroutine that automatically constructs simulation models for all feasible absorber layouts, based on a given catalog of hydraulic components. The models are built using nonlinear hydraulic elements in Simscape, explicitly accounting for NPEs.

One input for this step is the CarMaker vehicle-suspension template, as shown in Fig. 6(a). This template is derived from ‘CarMaker for Simulink®’ (a Simulink

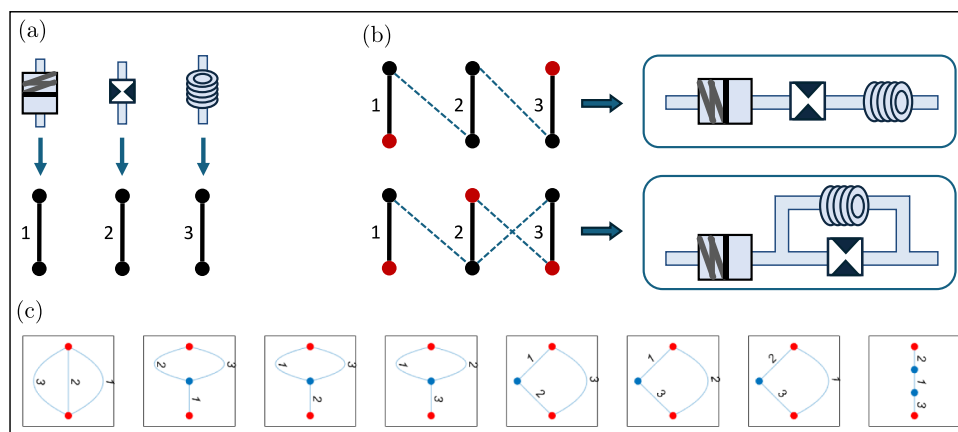
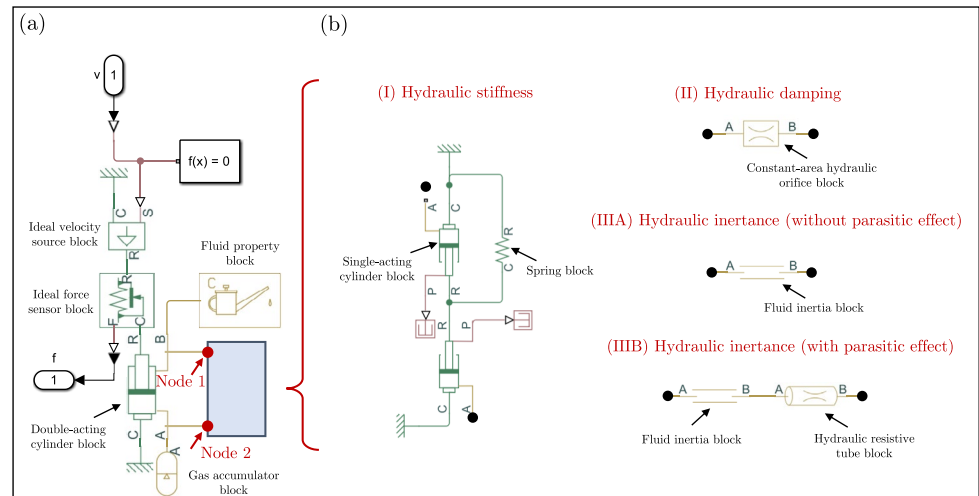


Fig. 5 Enumeration of all possible hydraulic networks using graph theory. **a** Representation of a stiffness component, a damping component, and an inertance component as an edge with weights of 1, 2, and 3, respectively. **b** Two possible configurations for connecting a

stiffness component, a damping component, and an inertance component, along with their corresponding hydraulic networks. **c** All possible graphs with one stiffness component, one damping component, and one inertance component

Fig. 6 The Simscape modeling of the hydraulic shock absorber. **a** The model of the hydraulic shock absorber, where the blue block represents a generic hydraulic network to be designed. **b** The physical components modeled in Simscape to represent hydraulic stiffness, damping, and inductance. Note that two types of hydraulic inductance are considered: an ideal realization without parasitic tube damping and a practical realization with parasitic tube damping effect



template integrated with CarMaker for co-simulating vehicle dynamics) with its suspension section replaced by a generic hydraulic suspension. The second input is a catalog of typical hydraulic models constructed in Simscape, representing stiffness, damping, and inductance properties, as shown in Fig. 6(b). A detailed explanation of these models is given below.

For hydraulic compliance, a Simscape model corresponding to the spring design depicted in Fig. 3(b) is established, which consists of two Single-Acting Hydraulic Cylinder blocks grounded at their connection point by a Spring block. Letting p_A and p_B represent the pressures at the two terminals of this component, and q denote the flow rate across it, the governing equation can be derived as:

$$q = C(\dot{p}_A - \dot{p}_B) \quad (5)$$

Here, C is the hydraulic stiffness, expressed as

$$C = \frac{A_s^2}{k_e} \quad (6)$$

where A_s is the area of the spring piston, and k_e is the embedded spring stiffness. In this context, A_s is considered a constant, while k_e is treated as a DV. The value of A_s for the case study presented in Sect. 4 is listed in Table 1. Once k_e is determined, the hydraulic stiffness, as defined by Eq. (6), becomes fixed. Consequently, this Simscape implementation can be regarded as exhibiting a linear stiffness characteristic. It is important to note that the friction between the piston and the cylinder in this realization is neglected, as it can vary significantly and needs to be identified through experiments.

Hydraulic damping is modeled using the Constant Area Hydraulic Orifice block with a circular cross-section provided in Simscape [35]. Using the same definitions for p_A , p_B , and q , its governing equation is expressed as:

$$q = R(p_A - p_B) \quad (7)$$

Here, R is the hydraulic damping, which depends on pressures at the two terminals, and is expressed as:

$$R = C_D \pi r_o^2 \sqrt{\frac{2}{\rho}} \frac{1}{(p_A - p_B)^2 + p_{cr}^2}^{1/4} \quad (8)$$

where C_D is the flow discharge coefficient, r_o is the radius of the orifice, ρ is the fluid density, and $p_{cr} = ((p_A + p_B)/2 + p_{atm})(1 - B_{lam})$. Here, p_{atm} is the atmospheric pressure, and B_{lam} is the pressure ratio at the transition between laminar and turbulent regimes. In this context, r_o is the DV, while C_D , ρ , B_{lam} , and p_{atm} are considered as constants. The values of these constants for the case study are listed in Table 1. Since the hydraulic damping, as described by Eq. (8), depends on pressure, this Simscape model inherently exhibits a nonlinear damping characteristic.

To realize hydraulic inductance, the helical tube design illustrated in Fig. 3(b) is selected. Two Simscape models are provided: one represents the ideal realization without considering the parasitic damping effect, while the other represents the practical realization with this parasitic effect. The ideal realization model uses a Fluid Inertia block [36], as shown in Fig. 6(iiiA). Assuming a circular cross-sectional shape, the governing equation for this block can be derived as:

$$\dot{q} = \frac{1}{I}(p_A - p_B) \quad (9)$$

where I is the hydraulic inductance, given by

$$I = \rho \frac{l_p}{\pi r_p^2} \quad (10)$$

In this equation, l_p and r_p denote the tube's length and radius, respectively, and are treated as DVs. Once l_p and r_p are specified, the hydraulic inertance, as defined by Eq. (10), becomes constant. Consequently, this Simscape implementation can be regarded as exhibiting a linear inertance characteristic. The second inertance model includes a Resistive Tube block [37] in series with the Fluid Inertia block to capture both the inertance effect and the substantial parasitic damping observed in practical tube implementations, as shown in Fig. 6(iiiB). This modeling approach is consistent with that adopted in [10, 11]. The governing equation of this resistive tube is given as

$$p_A - p_B = f \frac{\rho l_p}{4\pi^2 r_p^5} q |q| \quad (11)$$

where f is a friction factor that depends on the Reynolds number, Re , of the fluid, i.e.,

$$f = \begin{cases} 64/Re, & Re \leq Re_L \\ f_L + \frac{f_T - f_L}{Re_T - Re_L} (Re - Re_L), & Re_L < Re < Re_T \\ \frac{1}{\left(-1.8 \log_{10} \left(6.9/Re + (r/(7.4r_p))^{1.11} \right) \right)^2}, & Re \geq Re_T \end{cases} \quad (12)$$

Here, f_L and f_T represent the friction factors at the laminar and turbulent boundaries, respectively. r denotes the height of roughness on the pipe internal surface. Re_T and Re_L denote the minimum and maximum Reynolds number at turbulent flow, respectively. The Reynolds number is defined as $Re = \frac{2q}{\pi r_p v}$, where v is the fluid's kinematic viscosity. In this model, l_p and r_p are DVs, while f_L , f_T , Re_T , Re_L , v , and r are considered constants. The values of these constants for the case study are listed in Table 1. Since the friction factor f depends on the flow rate q , this model exhibits nonlinear behavior. This paper adopts these two models for hydraulic inertance to investigate the impact of its parasitic effect on the performance improvements, which will be discussed in Section 4.2.

It is worth noting that the modeling approach of these three hydraulic components is grounded in well-established physical principles, supporting the reliability of the results within the scope of this study. Nevertheless, certain NPEs – such as piston-cylinder friction in the hydraulic stiffness component, which requires experimental characterization – are not included in the current model due to a lack of available experimental data and will be addressed in future work.

Algorithm 1 Automatic generation of Simscape models for the combined vehicle-suspension system

Input:

Simscape models for the hydraulic components
The graph representation, g , of a hydraulic network
CarMaker vehicle-suspension template

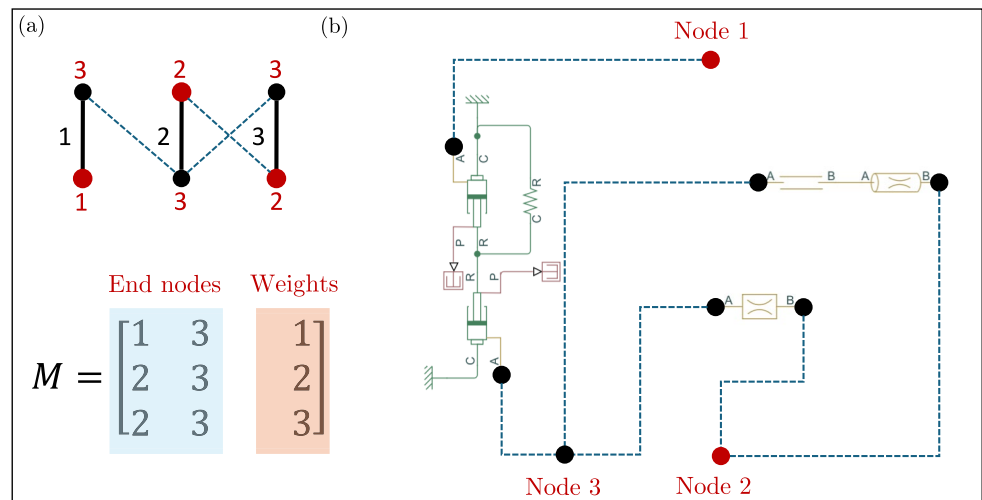
Output:

Simscape vehicle model equipped with the hydraulic shock absorber represented by g

```

1 Load the CarMaker vehicle-suspension template to Simulink
  Obtain the matrix representation,  $M$ , of the graph,  $g$ 
  for  $i = 1 : \text{number of rows of } M$  do
2    switch  $M(i, 3)$  do
3      case 1 do
4        Add stiffness component to the Simulink
5      case 2 do
6        Add damping component to the Simulink
7      case 3 do
8        Add inertance component to the Simulink
9    Add lines connecting the two terminals of the component and the nodes
      specified in  $M(i, [1, 2])$ 
      // Nodes 1 and 2 represent the upper and lower chambers of the cylinder. For any
      additional nodes, ensure they are created first if they are not already present in
      the Simulink model.
```

Fig. 7 Automated generation of the Simscape model for an example hydraulic stiffness-damping-inertance shock absorber. **a** Matrix representation of the example graph, where the first two columns represent the end nodes of each edge, and the third column represents the weight/type of each edge (i.e., 1 for stiffness, 2 for damping, and 3 for inertance). **b** The topological connection of a hydraulic shock absorber corresponding to the graph representation in (a)



Algorithm 1 outlines the developed technique for this step, and a worked example is illustrated in Fig. 7. After loading the CarMaker vehicle-suspension template into Simulink, this technique begins by deriving the matrix representation for any graph obtained in the previous step. Figure 7(a) illustrates an example graph and its corresponding matrix representation. Specifically, this graph contains three edges, represented by black lines, with weights labeled in black numerals. Node indices are marked in red numerals, and the connection relationships between these nodes are indicated by blue dashed lines. For simplicity, the nodes that correspond to the terminals of the hydraulic networks are indexed as 1 and 2, with the other internal node indices starting from 3. The resulting matrix representation, M , for this graph consists of three columns: the first two columns specify the end nodes of each edge, and the third column represents the weight of each edge. The number of rows in the matrix corresponds to the number of edges in the graph. With the end-node and weight data from this matrix, the Simscape model for the complete vehicle-suspension system can be constructed systematically. For example, when the developed algorithm reads the first row of the matrix in Fig. 7(a) and identifies '1' and '3' in the first two columns with a '1' in the third column, it instructs the loaded CarMaker vehicle-suspension template model to insert the stiffness component and connect it between Node 1 and Node 3. Since Nodes 1 and 2 are the terminals of the hydraulic network, any component connected to these two nodes can be directly linked to the upper and lower chambers of the shock absorber cylinder. However, since any node with an index greater than 2 is actually an internal node in the hydraulic network, it must be created in the model upon its first occurrence. Figure 7(b) presents the connected hydraulic components based on the graph in Fig. 7(a).

3.3 Network parameter optimization with a high-fidelity full-car model

In this step, the Simscape models of the enumerated networks with NPEs, generated in Section 3.2, are integrated into a high-fidelity CarMaker vehicle model. Parameter optimization and performance evaluation are then conducted for all network configurations, guided by the specified objective function and performance constraints outlined in the design problem. This process is carried out using a MATLAB-CarMaker co-simulation workflow, where MATLAB handles parameter optimization and coordinates with CarMaker to assess the performance of each candidate network. Although MATLAB's optimization toolbox offers various algorithms, this study selects the gradient-based local optimization solver, `fmincon`, for its efficiency compared to population-based stochastic methods like genetic algorithms and particle swarm optimization (PSO). To increase the chance of finding the global optimum, the Latin Hypercube Sampling (LHS) technique [38] is used to generate ten near-random initial parameter points from a multi-dimensional distribution within the bounded search region. This stratified sampling technique ensures comprehensive exploration of the parameter space. It is worth noting that using fewer initial points increases the risk of converging to a local optimum. The gradient-based optimization process seeks to identify a stationary point of the penalized objective function described in Eq. (4), where the gradient with respect to the DVs approaches zero. Convergence is deemed achieved when any of the following criteria are met: (1) The maximum number of function evaluations reaches 200 times the number of DVs; (2) The percentage change in the objective function value between consecutive iterations is less than 10^{-4} ; (3) The percentage change in all DVs between consecutive iterations is less than 10^{-4} .

4 Identification of optimal absorber configuration

In this section, the proposed CAE design framework is applied to identify the optimal configuration of a hydraulic shock absorber that incorporates integrated stiffness, damping, and inertance effects. For the sake of simplicity in demonstrating the benefits of the proposed framework, we assume that the hydraulic network consists of one stiffness, one damping, and one inertance. In this context, there are a total of eight different network layouts, with their corresponding graph representations illustrated in Fig. 5(c). The section begins by establishing the benchmark performance through the optimization of the default hydraulic shock absorber. It then proceeds by determining the optimal stiffness-damping-inertance configuration, assuming an ideal inertance realization. Finally, the section incorporates this parasitic damping effect and re-determines the optimal suspension configuration based on a practical inertance realization.

4.1 Benchmark performance of default shock absorber

This subsection focuses on evaluating the three performance metrics—RMS car body acceleration, dynamic tire load, and suspension travel—for the default shock absorber and establishes the benchmark performance for the new suspension design. It is assumed that the piston orifices in the left and right shock absorbers have identical diameters, while variations are allowed between the front and rear absorbers. Thus, the DVs for optimizing the default shock absorber are the orifice diameters: r_o^F for the front shock absorbers and r_o^R for the rear absorbers. Figure 8(a) is a contour plot illustrating the variation in RMS car body acceleration as a function of different r_o^F and r_o^R . It is evident that both r_o^F and r_o^R significantly influence RMS car body acceleration. Notably, the optimal RMS car body acceleration, $J_r^{\text{Ben}} = 0.333 \text{ m/s}^2$, for the default hydraulic shock absorber is achieved when $r_o^F = 0.67 \text{ mm}$ and $r_o^R = 0.88 \text{ mm}$. Figure 8(b) further investigates the effect of orifice diameters on the frequency components of the car body's vertical acceleration, using three example orifice sizes: the optimal size, 'a smaller size', and 'a larger size', which correspond to the red points marked in Fig. 8(a). In this paper, 'the smaller size' is defined as both the front and rear piston orifices having a diameter that is 0.8 times the identified optimal diameters, while 'the larger size' indicates that both the front and rear piston orifices have a diameter that is 1.2 times the identified optimal diameters. It can be observed that all three curves in the figure exhibit two resonance peaks

within the frequency range of 0 to 11 Hz. The first peak corresponds to the vertical vibration of the sprung mass (generally includes vehicle body, chassis, engine, and passengers), and the second peak corresponds to the vertical vibration of the unsprung mass (generally includes the wheels, tires, brake assemblies, and portions of the suspension system itself). Notably, the second resonance peak is not as sharp as the first peak due to the presence of four wheels, each having similar but not identical resonance frequencies, which collectively contribute to the formation of the second peak. By comparing the three curves in the figure, it is evident that changes in orifice diameters have opposing effects on the two resonance peaks. Specifically, an increase in orifice diameters deteriorates the first peak while suppressing the second peak. This is because, as commonly observed in vehicle dynamics (e.g., [39]), the sprung mass experiences significantly greater movement than the unsprung mass in the first mode. In this scenario, higher damping effectively absorbs and dissipates the vibration energy of the sprung mass, functioning as a vibration absorption mechanism. Conversely, in the second mode, the unsprung mass exhibits significantly greater movement than the sprung mass. When damping is high, the suspension between the sprung and unsprung masses behaves as if it is 'stiff,' facilitating the transmission of vibration energy from the unsprung mass to the sprung mass and thereby amplifying the latter's vibration. In contrast, low damping effectively isolates the vibration, preventing its transmission to the sprung mass and functioning as a vibration isolation mechanism. Therefore, it is essential to carefully tune the orifice diameters to achieve an optimal balance between the first and second modes, ultimately enhancing the overall ride comfort.

Figures 8(c) and (d) show the dynamic tire load and peak suspension travel with varying orifice diameters, respectively. Unlike RMS car body acceleration, as examined in Fig. 8(a), the dynamic tire load and peak suspension travel for the vehicle's front (rear) primarily depend on the orifices within the corresponding front (rear) shock absorbers, i.e., the same side. Although minimal, changes in the orifice diameters on the opposite side also have some impact due to the coupling between the front and rear suspensions, which arises from the vehicle's pitch motion. For further details on coupling characteristics and methods to reduce this coupling, readers can refer to [40, 41].

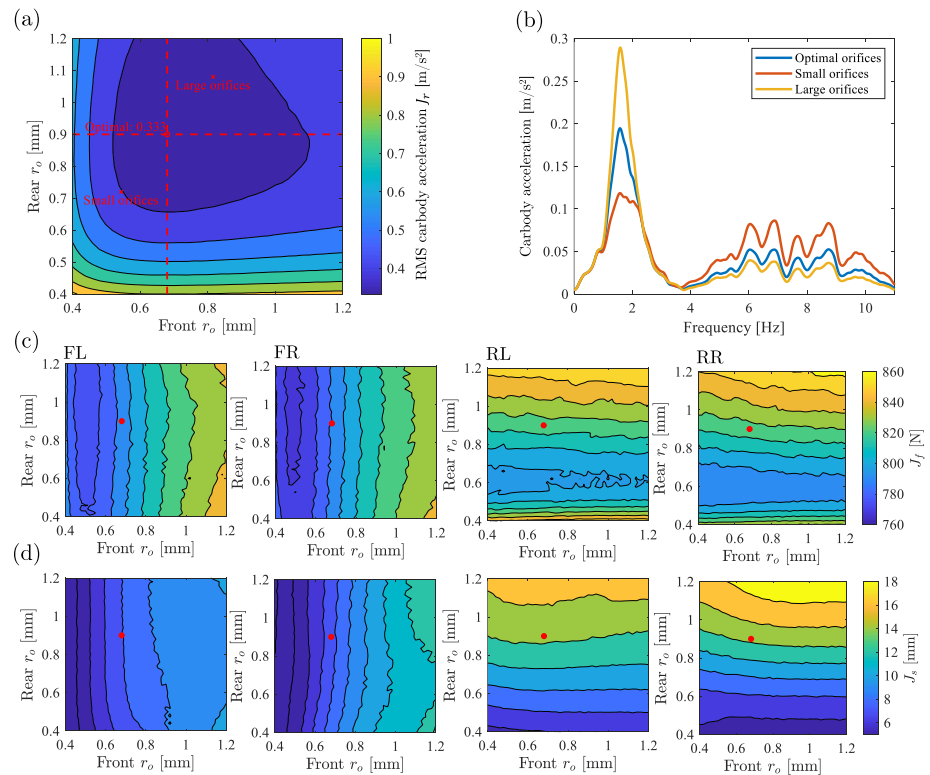
As outlined in Section 2.3, the benchmark performance is defined by the default shock absorber optimized to minimize the RMS car body acceleration. The dynamic tire load and suspension travel corresponding to the optimal RMS car body acceleration are plotted as red points in Fig. 8(c) and (d), which are used to form the constraints for the new absorber design. Based on the definitions of the dynamic

Table 2 Component parameters and RMS car body acceleration in different hydraulic absorber configurations

Configurations	Component parameters	RMS car body acceleration
Benchmark	$r_o^{F,Ben} = 0.67$ mm $r_o^{R,Ben} = 0.88$ mm	$J_r^{Ben} = 0.333$ m/s ²
Optimal-1	$r_o^{F,Opt1} = 0.69$ mm, $k_e^{F,Opt1} = 248.5$ kN/m $r_p^{F,Opt1} = 0.58$ mm, $l_p^{F,Opt1} = 1$ m $r_o^{R,Opt1} = 0.90$ mm, $k_e^{R,Opt1} = 66.1$ kN/m $r_p^{R,Opt1} = 1.05$ mm, $l_p^{R,Opt1} = 1$ m	$J_r^{Opt1} = 0.274$ m/s ² (17.6%↓)
Optimal-2	$r_o^{F,Opt2} = 0.65$ mm, $k_e^{F,Opt2} = 143.1$ kN/m $r_p^{F,Opt2} = 1.02$ mm, $l_p^{F,Opt2} = 0.78$ m $r_o^{R,Opt2} = 1.02$ mm, $k_e^{R,Opt2} = 44.92$ kN/m $r_p^{R,Opt2} = 1.82$ mm, $l_p^{R,Opt2} = 2.20$ m	$J_r^{Opt2} = 0.306$ m/s ² (8.3%↑)

The configurations ‘Optimal-1’ and ‘Optimal-2’ represent the optimal configurations without and with parasitic damping, respectively. The superscripts ‘F’ and ‘R’ denote the front and rear suspensions, respectively

Fig. 8 Performance evaluation of the default hydraulic shock absorber. **a** Variation in RMS car body acceleration J_r with different orifice diameters. **b** Car body acceleration for various orifice sizes in the frequency domain. **c** Dynamic tire load J_f of the four wheels (Front Left: FL; Front Right: FR; Rear Left: RL; Rear Right: RR) with varying orifice diameters. **d** Peak suspension travel J_s of the four suspensions with varying orifice diameters

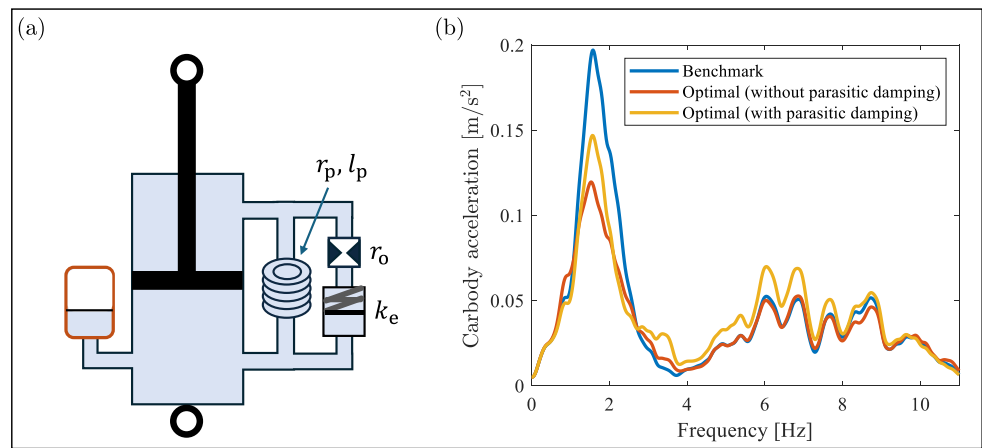


tire load and suspension travel metrics provided in Section 2.2, the corresponding values for the benchmark shock absorber are as follows: $J_t^{F,Ben} = 792.4$ N, $J_t^{R,Ben} = 822.7$ N, $J_s^{F,Ben} = 7.9$ mm, and $J_s^{R,Ben} = 14.1$ mm, respectively. Table 2 provides a summary of the component parameters and RMS values of car body acceleration for the benchmark suspension system.

4.2 Optimal absorber configuration without parasitic damping

To examine the influence of the parasitic damping in the realization of hydraulic inertance on performance enhancement, this subsection excludes this specific effect first, while still accounting for other NPEs, such as the nonlinear

Fig. 9 **a** The optimal shock absorber configuration identified using the proposed framework. The configuration is identical for both front and rear suspensions, regardless of the presence of parasitic effects. **b** Amplitude spectra of car body acceleration for the benchmark absorber, the optimal absorber without parasitic effects, and the optimal absorber with parasitic effects



damping of the orifice. Since there is one component for each type, the DVs in this optimization include: the radius r_o of the orifice block for achieving hydraulic damping, the stiffness k_e of the spring block for achieving hydraulic stiffness, and the radius r_p of the fluid inertia block for achieving ideal hydraulic inductance, i.e., $\theta = [r_o, k_e, r_p]$. The parameter constraints are: $r_o \in [0.3, 5]$ mm, $k_e \in [1, 500]$ kN/m, and $r_p \in [0.3, 5]$ mm. The length of the fluid inertia block, l_p , is fixed at 1 m, and is not treated as a DV. This is because, according to the governing equation of this block (Eq. (10)), l_p and $1/r_p^2$ have equivalent effects on the resulting inductance. By fixing l_p , r_p can be directly adjusted to achieve the desired inductance.

As this paper allows for differences in the network layouts of the front and rear shock absorbers, the eight possible layouts for each absorber results in a total of 64 distinct unique combinations. Among these possibilities, the optimization results indicate that the optimal front and rear absorbers share the same layout, as depicted in Fig. 9(a), despite differing component parameters. The component parameters and the corresponding optimal RMS car body acceleration are detailed in Table 2, where the superscript 'opt1' denotes this ideal inductance case. The reason the optimal front and rear absorbers share the same network layout may be that the pitch motion does not significantly influence vehicle's dynamics in a constant-velocity straight-running scenario. Consequently, both absorbers tend to adopt the same optimal topology to effectively suppress vibrations. However, if the vehicle's acceleration and braking performance were to be optimized—where the pitch motion has a more pronounced effect—different configurations for the front and rear absorbers might emerge, though this aspect is not addressed in the current study.

The optimal layout identified above can achieve approximately a 17.6% improvement in RMS car body acceleration compared to the benchmark damper, without deteriorating dynamic tire load and suspension travel constraints. To

understand the positive impact of this new design on enhancing RMS car body acceleration across different frequency compositions, Fig. 9(b) presents the frequency-domain sprung mass acceleration from 0 to 11 Hz. It can be observed that the new shock absorber primarily focuses on reducing vibrations in the first mode, while showing limited improvement in the second mode. This is because the hydraulic network in the identified absorber has a natural frequency close to the first mode of the vehicle. To determine the natural frequency of the hydraulic network, the damping component can be removed. This allows the relationship between the pressure difference across the hydraulic network, Δp , and the flow rate, q , passing through it to be expressed in the frequency domain as

$$q = \left(Cs + \frac{1}{Is} \right) \Delta p \quad (13)$$

where s is the Laplace variable. C and I represent the stiffness and inductance values, which can be calculated from the component parameters listed in Table 2 using Eqs. (6) and (10). The natural frequency, f_n , of this network is then given by

$$f_n = \frac{1}{2\pi} \sqrt{\frac{1}{CI}} \quad (14)$$

Using this equation, the natural frequencies for the front and rear absorbers are 1.4 Hz and 1.3 Hz, respectively, both of which are close to the natural frequency of the vehicle's first mode.

4.3 Effects of parasitic damping on absorber performance

The results obtained in the previous subsection are promising; however, they may not fully reflect practical scenarios. This is because the helical tube implementation of hydraulic inductance inevitably introduces some damping to the fluid

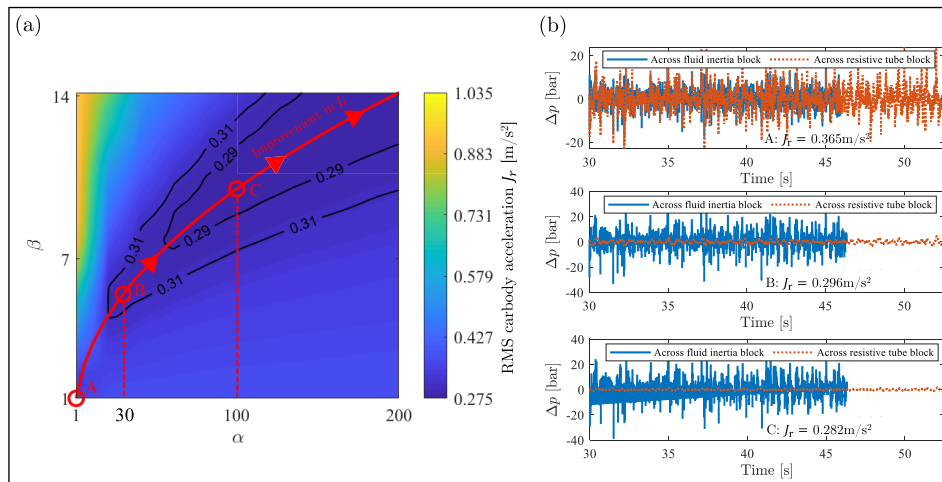


Fig. 10 The influence of parasitic tube damping on RMS car body acceleration. **a** Variation of RMS car body acceleration with length amplification factor α and diameter amplification factor β when tube damping is considered in the inertance modeling. The red curve shows the constant inertance curve, where the fluid inertia remains

flow. Therefore, it is essential to investigate the impact of this parasitic damping on the performance improvements discussed in the previous subsection. To achieve this, the hydraulic inertance is realized using a fluid inertia block plus a hydraulic resistive tube block, as shown in Fig. 6(iiiB), to replace the ideal hydraulic inertance realization without such a resistive tube. Figure 10(a) presents a contour plot depicting RMS car body acceleration across various lengths and radii of the fluid inertia and resistive tube blocks, with parameters of other components set to those specified for the optimal absorber identified in Section 4.1. Here, α represents the length amplification factor, $\alpha = l_p^i / l_p^{i, \text{Opt1}}$ for $i = F$ and R , while β represents the radius amplification factor, i.e., $\beta = r_p^i / r_p^{i, \text{Opt1}}$ for $i = F$ and R . For the ideal inertance realization without parasitic effect, as expressed by Eq. (10), changes in α and β will not affect absorber performance as long as they satisfy $\alpha/\beta^2 = 1$. This relationship is depicted by the red curve in Fig. 10(a). However, when the parasitic damping is considered, performance is no longer consistent along this curve. For instance, with the parasitic effect considered and tube dimensions unchanged from the optimal suspension identified in Section 4.1 (corresponding to Point A in Fig. 10(a)), the RMS car body acceleration J_r is significantly degraded from 0.274 m/s² to 0.365 m/s². As both α and β increase, however, the RMS car body acceleration steadily improves, eventually converging back to the optimal performance previously identified, as indicated by the red arrow along the curve. To understand this behavior, Fig. 10(b) shows the differential pressures across the fluid inertia block and the resistive tube block under the road

constant as long as $\alpha/\beta^2 = 1$. **b** Differential pressures, Δp , across the fluid inertia block and the resistive tube block for points A, B, and C in (a). It shows that as the tube's length and diameter increase, the influence of the resistive tube block on differential pressure becomes increasingly smaller compared to that of the fluid inertia block

excitation, as defined in Section 2.1, for Points A, B, and C along the red curve. The results indicate that, with larger tube dimensions while keeping inertance value unchanged, the differential pressure across the resistive tube block diminishes relative to that across the fluid inertia block, suggesting that the parasitic damping becomes progressively smaller. While exceedingly large tubes may be impractical in automotive applications due to space constraints, this analysis offers theoretical guidance for mitigating parasitic damping in inertance realization: by increasing the tube dimensions.

Given the significant impact of parasitic damping on absorber performance, as demonstrated above, the optimal hydraulic network configuration that accounts for this effect may differ from the one obtained without considering it. Therefore, the proposed CAE design framework must be re-applied to maximize improvements in RMS car body acceleration while maintaining dynamic tire load and suspension travel constraints. Unlike the optimization in the previous subsection, this optimization employs the nonlinear inertance model shown in Fig. 6(iiiB) to replace the linear inertance model depicted in Fig. 6(iiiA). In this optimization, in addition to the DVs and their value ranges specified in Section 4.1, the length l_p of the fluid inertia (with the resistive tube having the same length) is also considered as a DV, with a range of [0.5, 5] m. Therefore, the DVs for this optimization become $\theta = [r_o, k_e, r_p, l_p]$. It is important to note that all 64 combinations of the front and rear absorber configurations have been re-optimized. Interestingly, the optimal configuration remains unchanged and corresponds to the one depicted in Fig. 9(a) for both the front and rear

Fig. 11 Comparison of car body accelerations for three shock absorber designs: the benchmark absorber, the optimal absorber without parasitic damping, and the optimal absorber with parasitic damping. **a** Raw acceleration time histories. **b** Low-pass filtered accelerations with a cutoff frequency of 4 Hz. **c** Band-pass filtered accelerations in the frequency range of 4–10 Hz

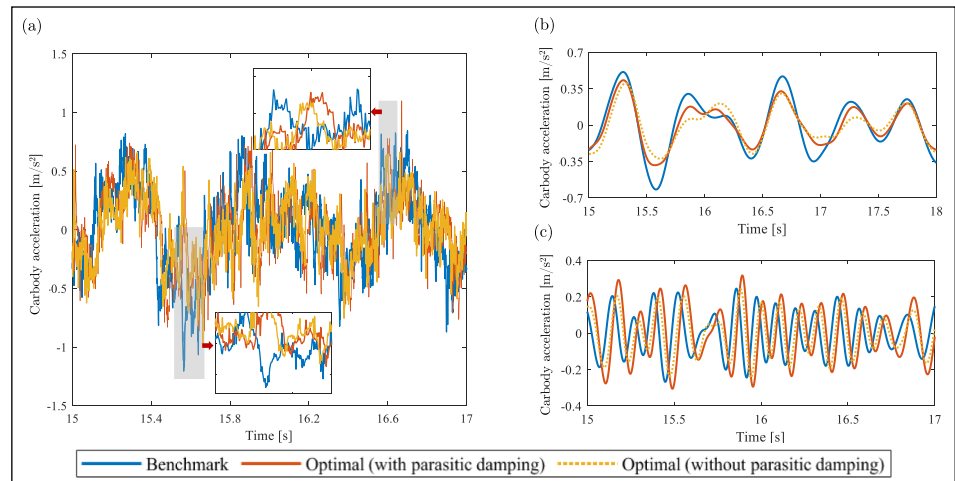
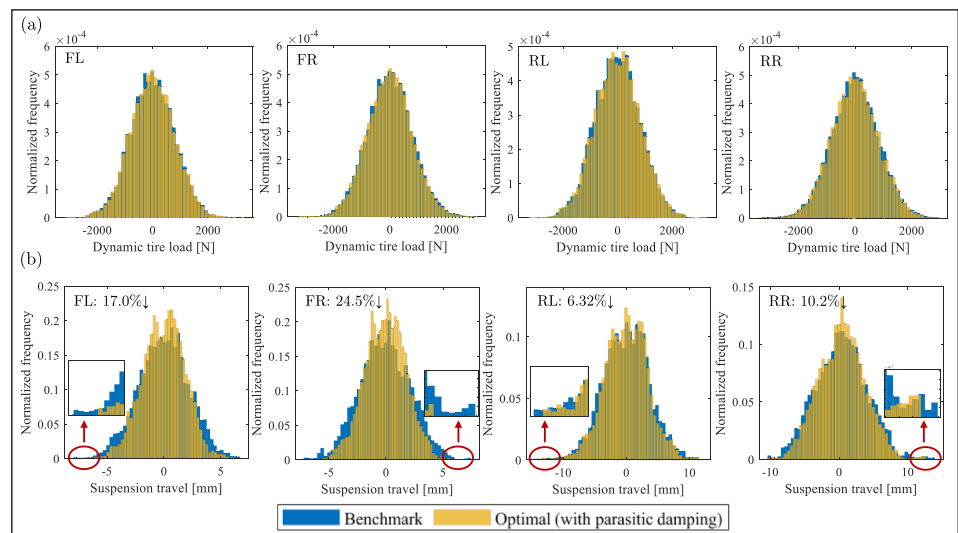


Fig. 12 Empirical histograms of **(a)** dynamic tire load and **(b)** suspension travel. From left to right, the plots correspond to the FL, FR, RL, and RR corners of the vehicle. The percentages in **(b)** represent the reduction in peak suspension travel achieved with the optimal configuration considering parasitic damping. The optimal configuration with parasitic damping achieves a dynamic tire load nearly identical to that of the benchmark while significantly reducing the peak travel across all four suspensions



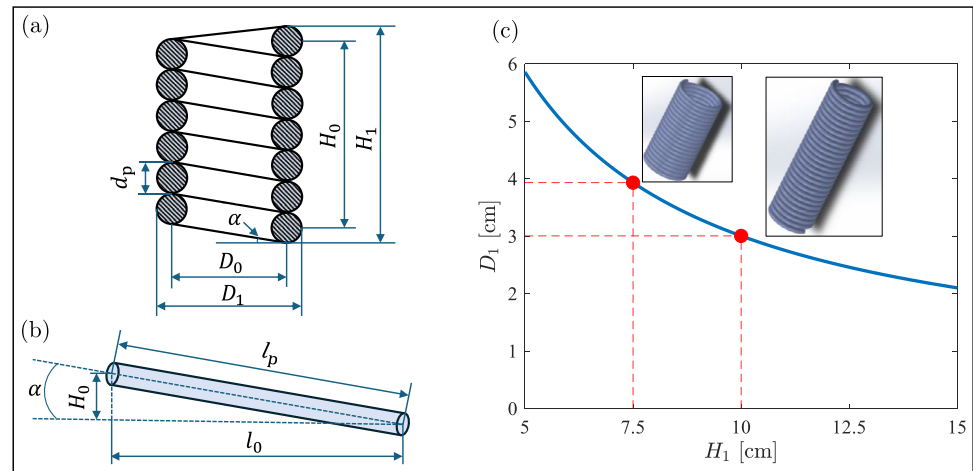
absorbers, despite differences in their component parameters. The component parameters and the resulting optimal RMS car body acceleration are summarized in Table 2, revealing that the improvement in RMS car body acceleration decreases from 17.6% to 8.3%. To distinguish this case from the ideal inertance scenario, it is labeled as ‘Opt2’ in the table. The car body acceleration response in the [0, 11] Hz range is plotted in Fig. 9(b). Compared to the response obtained with ideal hydraulic inertance, the introduction of parasitic damping leads to a degradation in both the first and second modes. However, for the first mode, performance still shows significant improvement relative to that of the benchmark absorber. Therefore, to accurately predict absorber performance in practical applications, it is crucial to consider parasitic damping when implementing inertance using helical tubes.

Figure 11(a) illustrates the time history of the car body acceleration response. It is evident that the optimal configuration without parasitic damping effectively suppresses

vibration of the car body, whereas the inclusion of this effect partially compromises this improvement. Figures 11(b) and (c) provide a closer examination of the band-pass filtered results from Fig. 11(a). Specifically, Fig. 11(b) depicts the response in the frequency range [0, 4] Hz, corresponding to the first mode in Fig. 9(b), while Fig. 11(c) shows the response in the range [4, 10] Hz, corresponding to the second mode in Fig. 9(b). The results indicate that both configurations, with and without parasitic damping, effectively suppress vibrations associated with the first mode. However, the configuration without parasitic damping demonstrates superior suppression performance. It is also worth noting that while this configuration reduces vibrations in the second mode, the configuration including the parasitic damping slightly amplifies them. These observations align with the frequency-domain trends in Fig. 9(b).

Figure 12(a) illustrates the empirical histograms of the dynamic tire load for all four wheels. The histogram for the optimal configuration with parasitic damping nearly overlaps

Fig. 13 The helical tube dimensions. **a** The helical tube is wound with no gaps between successive turns. **b** The dimensions of the helical tube when unrolled into a straight line. **c** The relationship between the overall helix height, H_1 , and the overall helix diameter, D_1



with that of the benchmark shock absorber. This indicates that the dynamic tire load metric for the optimal configuration achieves almost the same level as the benchmark, without violating the constraints. Figure 12(b) presents the empirical histogram of the suspension travel for all four corners. The optimal configuration with parasitic damping demonstrates a narrower distribution, suggesting a tendency for smaller suspension travel compared to the benchmark shock absorber. Additionally, the peak suspension travels have been significantly reduced with the optimal configuration, by up to 24.5% for the front suspension and 10.2% for the rear suspension. This reduction can help minimize the suspension mechanism's dimensions, resulting in a more compact overall design.

It is important to note that while the rear suspension requires a helical tube with a length of 2.20 m to achieve the necessary hydraulic inertance, the dimensions of this helix can still be made compact, allowing for practical implementation in the vehicle suspension system. An example realization involves a helix with an overall diameter of 3.93 cm and an overall height of 7.5 cm, as detailed in the Appendix A.

5 Conclusions

This paper presents a novel computer-aided engineering framework aimed at enhancing the vibration suppression performance of conventional hydraulic shock absorbers by integrating stiffness and inertance elements into the absorber's hydraulic network. The framework employs a graph-based method to enumerate all feasible network layouts using a predefined set of hydraulic stiffness, damping, and inertance elements. A scripted MATLAB subroutine automates the modeling of these layouts using Simscape hydraulic blocks, explicitly accounting for the nonlinear and parasitic effects (NPEs) inherent in hydraulic components. These models are integrated into a high-fidelity full-vehicle

model through a MATLAB-CarMaker co-simulation workflow for parameter optimization.

A numerical case study is conducted to demonstrate the effectiveness of the framework, wherein the orifice on the absorber piston is replaced with an optimized hydraulic network to minimize the root-mean-square (RMS) car body acceleration while satisfying suspension travel and dynamic tire load constraints. Without loss of generality, the network in the case study consists of one stiffness, one damping, and one inertance element. The optimization results show that:

(1) Under ideal inertance conditions (neglecting parasitic damping), the optimized absorber reduces the RMS car body acceleration by approximately 17.6% compared to a conventional absorber;

(2) When the parasitic damping associated with inertance is included, the reduction in RMS car body acceleration is moderated to 8.3%. However, the design achieves 24.5% and 10.2% reductions in peak suspension travel at the front and rear axles, respectively;

(3) For the physical realization of hydraulic inertance, increasing the helical tube's length and radius – within spatial constraints – is found to mitigate parasitic damping effects and is therefore recommended.

These findings underscore the importance of explicitly accounting for NPEs and demonstrate the framework's capability in designing high-performance, inertance-integrated hydraulic shock absorbers. Future work will focus on the experimental validation of the optimized designs in this study to further demonstrate the framework's effectiveness and industrial applicability.

Appendix A Helical tube dimensions

To minimize the space occupied by the helical tube, it is essential to ensure that the tubes are tightly wound with no gaps between successive turns, as shown in Fig. 13(a). In

this figure, d_p is the tube diameter (i.e., $d_p = 2r_p$), D_0 is the effective helix diameter, D_1 denotes the overall helix diameter (i.e., $D_1 = D_0 + d_p$), H_0 is the effective helix height, and H_1 is the overall helix height (i.e., $H_1 = H_0 + d_p$). Given the tube length, l_p , and its diameter d_p , the key dimensions of the helix, specifically D_0 and H_0 , can be determined by the following process.

By unrolling the helical tube into a straight line with the same helix angle α , a right triangle can be formed, as illustrated in Fig. 13(b). In this triangle, the hypotenuse corresponds to the tube length l_p , while the two legs have lengths H_0 and l_0 , respectively, where

$$l_0 = \pi D_0 \frac{H_0}{d_p} \quad (\text{A1})$$

Therefore, we can have

$$l_p^2 = H_0^2 + l_0^2 \quad (\text{A2})$$

Substituting Eq. (A1) into Eq. (A2), the relationship between H_0 and D_0 can be derived as

$$D_0 = \frac{1}{\pi} \sqrt{\frac{l_p^2 d_p^2}{H_0^2} - d_p^2} \quad (\text{A3})$$

The possible outline dimensions (D_1, H_1) for the helical tube identified in Section 4.3 for the rear suspensions, with $l_p = 2.20$ m and $r_p = 1.82$ mm, can be obtained, as shown in Fig. 13(c). To realize this identified helical tube, two possible dimensions can be considered: $H_1 = 7.5$ cm, $D_1 = 3.93$ cm and $H_1 = 10$ cm, $D_1 = 3.00$ cm. The 3D models for these two cases are also provided in Fig. 13(c). While other dimensions can be considered according to specific application requirements, these two example compact dimensions demonstrate the practicality of implementing the identified helical tube in vehicle suspension.

Author Contributions Haonan He: Conceptualization, Methodology, Formal analysis, Validation, Writing—Original Draft. Zixiao Wang: Conceptualization, Methodology, Investigation, Validation, Writing—Review and Editing. Yiyuan Li: Methodology, Software, Investigation, Writing—Review and Editing. Zhiguang Song: Investigation, Writing—Review and Editing.

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Data availability No datasets were generated or analysed during the current study. Models available upon reasonable request.

Declarations

Conflict of interest The authors declare no Conflict of interest.

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