

General fault factor method for health monitoring of a liquid rocket engine

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Nomenclature

A	=	cross-sectional area of pipe
A_o, B_o, C_o	=	coefficients of oxidizer pump
A_f, B_f, C_f	=	coefficients of fuel pump
C	=	characteristic velocity
C_d	=	discharge coefficient
I	=	moment of inertia of turbopump rotor
K	=	resistance coefficient of pipe or valve
L	=	length of pipe

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$\dot{m}$	=	mass flow rate
P	=	pressure
R	=	gas constant
r	=	pressure or mass flow rate ratio
T	=	temperature
V	=	volume
γ	=	heat specific ratio
η	=	efficiency
ρ	=	density
ω	=	rotational velocity
$\mathcal{N}(\boldsymbol{\mu}, \mathbf{C})$	=	multivariate normal distribution with mean $\boldsymbol{\mu}$ and covariance $\mathbf{C}$

Subscripts

a	=	oxidizer pipe branch point
b	=	fuel pipe branch point
c	=	combustion chamber
f	=	fuel
fic	=	combustion chamber fuel injector
fig	=	gas generator fuel injector
fp	=	fuel pump
fs	=	fuel tank
$f1$	=	fuel pump outlet
$f2$	=	gas generator fuel
$f3$	=	combustion chamber fuel
g	=	gas generator
o	=	oxidizer
oic	=	combustion chamber oxidizer injector
oig	=	gas generator oxidizer injector
op	=	oxidizer pump
os	=	oxidizer tank
$o1$	=	oxidizer pump outlet
$o2$	=	gas generator oxidizer
$o3$	=	combustion chamber oxidizer

tbe	=	turbine exit
tbi	=	turbine inlet
tp	=	turbopump

I. Introduction

LIQUID rocket engines (LREs) are complex and comprehensive systems, and they can break down easily because of the minimal safety margins used to reduce weight, so if they fail, they can incur severe accidents [1]. Hence, studies focused on fault detection and diagnosis (FDD) in LREs commenced with the development of space launch vehicles (SLVs), categorized into the following: i) model-free methods, ii) analytical model-based approaches, and iii) soft model-based techniques [2–7]. Furthermore, the recent development of reusable launch vehicles (RLVs) emphasizes the need for enhanced safety and reliability [8–10]. Therefore, given the potential for inherent faults to accumulate or arise in engines due to repeated usage or testing under extreme conditions, implementing FDD systems has become paramount in guaranteeing the safety and reliability of SLVs or RLVs. Consequently, there has been an increase in global research efforts in this area [11, 12].

Although there have been various previous studies on FDD algorithms for LREs, they mainly focused on the steady state of the engine. Even when applied to a transient state, most FDD algorithms have been based on soft model-based approaches, such as neural network algorithms, after obtaining and analyzing many samples of data from system-level simulations or experiments due to the complexity of the transient state [11–17]. Only two analytical model-based FDD approaches for transient LREs have been reported, as cited in Refs. [18, 19]. These approaches are based on nonlinear Kalman filters and particle filters, but implementing them in real-time is challenging due to the numerous steps involved in the process and the heavy computational requirements.

To solve this problem, we focus on the real-time diagnosis capabilities of the fault factor method, described in Refs. [20, 21], which was only for a steady state because a linearization process with an already-known constant state (generally design point or full thrust) is required. Unlike in steady-state conditions, during transient states, the time derivative of state variables in a nonlinear dynamic equation is non-zero, leading to continuous changes in the healthy state until reaching a steady state. Because of this, it is hard to determine whether each changing state is within the range of a healthy condition and to apply the previous fault factor method. Therefore, it is difficult to use the fault factor method considering the non-zero time derivative state and to obtain accurate state information for linearization in a transient state.

In the case of obtaining state variables, although it is possible to use a pre-constructed dataset, it is hard to prepare for transient states that consider all circumstances or conditions beforehand, and errors can occur if there is even a slight timing difference between the dataset and the actual state. While health monitoring or fault diagnosis of a system can be

achieved using a nonlinear model-based state estimation algorithm to generate residuals, relying solely on residuals for FDD has limitations in accurately determining faults. Hence, additional processes are required for diagnosing faults, which may pose limitations for real-time analysis or increase processing complexity, as in Refs. [18, 19]. Therefore, this study proposes a general fault factor method for real-time health monitoring during a transient state of LRE. This method integrates the previously introduced fault factor method (Refs. [20, 21]) with a nonlinear model-based state estimation technique to acquire changing unfaulty state information. To address the non-zero time-derivative state variables in estimating fault factors, we employ two different approaches: the Euler-based derivative and the time-derivative-ignoring approaches. We then validate the efficiency of the proposed algorithms through numerical simulations, demonstrating that both approaches yield similar results.

In this context, we first review and summarize the previous steady-state fault factor method presented in Refs. [20, 21] in Sect. II. Then, in Sect. III, we propose an extended fault factor method by incorporating nonlinear state estimation for transient states. In Sect. IV, we apply the developed methods to a numerical simulation of a transient-state open-cycle LRE, considering two types of faults to verify its effectiveness. Finally, Sect. V concludes the study.

II. Steady-State Fault Factor Method

The mathematical model of a nonlinear system can be represented as differential and algebraic equations (DAEs). The state vector ($\mathbf{x}$) can be categorized into the differential-equation state vector and algebraic equation state vector, as $\mathbf{x} = [\mathbf{x}_d^T, \mathbf{x}_a^T]^T$. Therefore, the state equations (DAE), which incorporate a multiplicative fault factor in each equation ($a_i = 1$, for healthy condition and takes other values for fault, $i = 1, 2, \dots, n$ and $\mathbf{a} = [a_1, \dots, a_i]^T$) and discrete output (measurement) can be expressed using a vector equation in semi-explicit form:

$$\dot{\mathbf{x}}_d(t) = f(\mathbf{x}(t), \mathbf{a}(t)) + \mathbf{w}(t) \quad (1)$$

$$\mathbf{0} = g(\mathbf{x}(t), \mathbf{a}(t)) \quad \text{or} \quad \mathbf{x}_a(t) = g_a(\mathbf{x}_d(t), \mathbf{a}(t)) \quad (2)$$

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k \quad (3)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ ($\mathbf{x}_d(t) \in \mathbb{R}^d$, $\mathbf{x}_a(t) \in \mathbb{R}^a$ and $d + a = n$), $\mathbf{a} \in \mathbb{R}^n$ and $\mathbf{y}_d^k \in \mathbb{R}^c$ are state, fault factor and output vectors, respectively, and $\mathbf{w}(t) \in \mathbb{R}^d$ and $\mathbf{v}_k \in \mathbb{R}^c$ are process and measurement noise vectors, respectively.

The state equations (Eqs. 1, 2), we can formulate a vector equation as follows.

$$\begin{bmatrix} \dot{\mathbf{x}}_d \\ \mathbf{0} \end{bmatrix} = F(\mathbf{x}, \mathbf{a}) + \begin{bmatrix} \mathbf{w} \\ \mathbf{0} \end{bmatrix} \quad (4)$$

with

$$F(\mathbf{x}, \mathbf{a}) = \begin{bmatrix} f(\mathbf{x}, \mathbf{a}) \\ g(\mathbf{x}, \mathbf{a}) \end{bmatrix} \quad (5)$$

In a steady state, the time derivatives of the state variables $\dot{\mathbf{x}}$ in Eq. 4 are zero. Then, the dynamic equations for a steady state, also known as the energy balance equations, are represented as:

$$\mathbf{0} = F(\mathbf{x}, \mathbf{a}) \quad (6)$$

Since the state variables $\mathbf{x}$ can also be classified into measured variables $\mathbf{x}_m$ and unmeasured variables $\mathbf{x}_u$, the fault factor is estimated using $\mathbf{x}_m$ under the assumption that $\delta\mathbf{x}_u$ is small and can be ignored. To do this, we linearize the dynamic equations at the equilibrium state $(\mathbf{x}^*, \mathbf{a}^*)$ and then estimate the deviation $\delta\hat{\mathbf{a}}$ of $\mathbf{a}$ from the normal equilibrium value $\mathbf{a}^*$.

$$\hat{\mathbf{a}} = \mathbf{a}^* + \delta\hat{\mathbf{a}} = \mathbf{1} + \mathbf{A}^\dagger \mathbf{B} \cdot \delta\mathbf{x}_m \quad (7)$$

Here $\mathbf{A} \triangleq \frac{\partial F}{\partial \mathbf{a}} \Big|_{(\mathbf{x}^*, \mathbf{a}^*)}$, $\mathbf{B} \triangleq - \frac{\partial F}{\partial \mathbf{x}_m} \Big|_{(\mathbf{x}^*, \mathbf{a}^*)}$ and $\mathbf{A}^\dagger \triangleq (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T$ is the Moore-Penrose pseudoinverse of matrix $\mathbf{A}$.

We also obtain the confidence intervals for $\hat{\mathbf{a}}$, assuming that the difference between the estimated $\hat{\mathbf{a}}$ and true $\mathbf{a}$ fault factors follows the distribution $\hat{\mathbf{a}} - \mathbf{a} \triangleq \mathbf{e} \in \mathcal{N}(\mathbf{0}, \mathbf{C})$, where the error covariance matrix (ECM) is given by $\mathbf{C} = (\mathbf{A}^T \mathbf{\Sigma}^{-1} \mathbf{A})^{-1}$. Then, the i -th element of the distribution is given as

$$\hat{a}_i - a_i \in \mathcal{N}(0, \mathbf{C}_{i,i}) \quad \text{or} \quad \frac{\hat{a}_i - a_i}{\sqrt{\mathbf{C}_{i,i}}} \in \mathcal{N}(0, 1) \quad (8)$$

where $\mathbf{C}_{i,i}$ indicates the (i, i) entry of the ECM $\mathbf{C}$.

Through this process, we estimate the fault factors and assess their robustness using the confidence interval, which quantifies the uncertainty introduced by noise. More detailed procedures can be found in Refs. [20, 21].

III. General Fault Factor Method

In the case of a steady-state system, the time derivatives of state variables are zero, so the approach described in Sect. II can be applied to estimate the fault factors with the already-known constant state vector $\mathbf{x}_{ref}$, as shown in Fig. 1a. However, due to the inapplicability of the energy balance equations (Eq. 6) in a transient state, we cannot determine

whether the changing state variable falls within the range of the healthy condition. To address this issue within the framework of the fault factor method, we may take the following two remedial approaches: First, we approximate the time-derivative term using the Euler approximation and then construct algebraic-type equations. Afterward, we can apply the steady-state fault factor method. It is essential to note that, to implement the algorithm in this case, we need to specify the values at two consecutive instants. The other approach is to ignore the time-derivative term based on the assumption that the time-derivative state variables are small and can be neglected. This case can naturally be handled directly by the steady-state fault factor method. In this context, we refer to the first method as the 'Euler-based derivative approach' and to the second as the 'time-derivative-ignoring approach.' Furthermore, we must estimate the state vector to calculate the energy balance-like equations during the transient process. For this, we use a nonlinear model-based state estimation to obtain accurate changing healthy-state information $\hat{\mathbf{x}}_{ref}$ at each sampling time, as illustrated in Fig. 1b. Then, the fault factors can be estimated similarly to previous research [20, 21], allowing real-time system condition monitoring in a transient state.

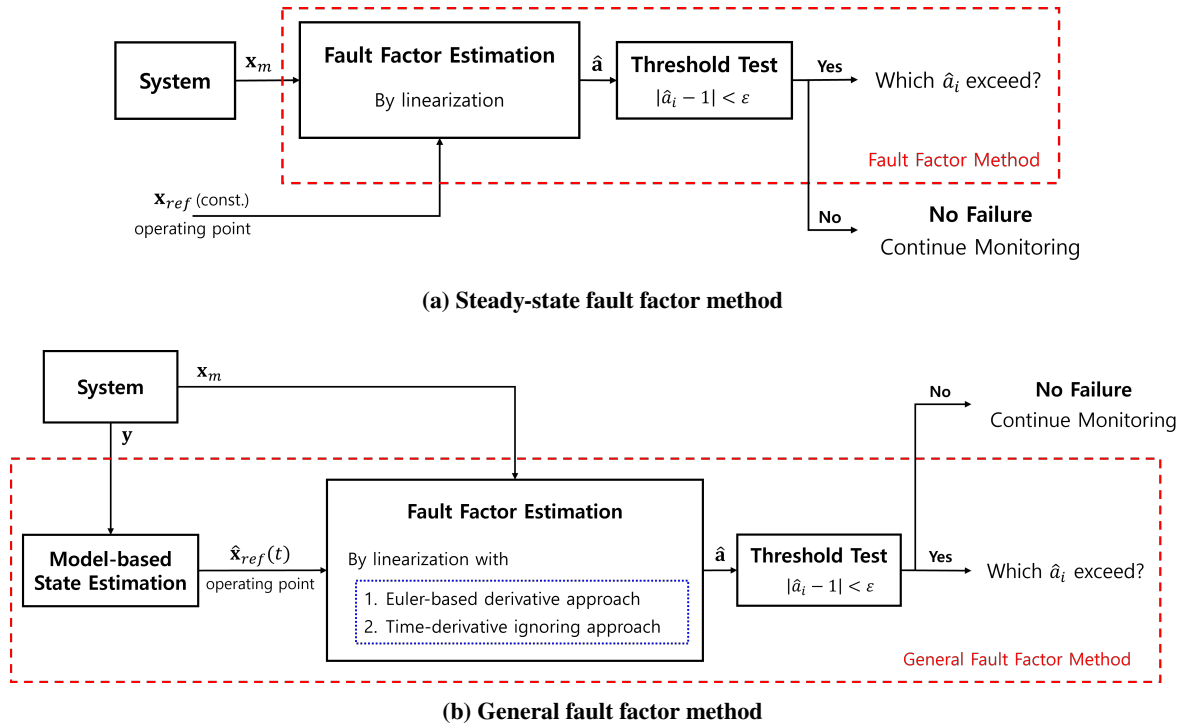


Fig. 1 Block diagram of the health monitoring system

A. Implementation of general fault factor method

1. Euler-based derivative approach

For a sufficiently small sampling time Δt , the time-derivative of the state vector can be approximated as

$$\dot{\mathbf{x}}_{d,k} = f(\mathbf{x}_{k-1}, \mathbf{a}_{k-1}) \simeq \frac{\mathbf{x}_{d,k} - \mathbf{x}_{d,k-1}}{\Delta t} \quad (9)$$

where $\mathbf{x}_{d,k} \triangleq \mathbf{x}_d(t_k)$, $\mathbf{x}_{d,k-1} \triangleq \mathbf{x}_d(t_{k-1})$ and $\dot{\mathbf{x}}_{d,k} \triangleq \dot{\mathbf{x}}_d(t_k)$.

Similar to the process in the steady-state fault factor method, we linearize the dynamic equation (Eq. 4) using this method described in Eq. 9 with $\mathbf{x}_{d,k}$ and $\mathbf{x}_{d,k-1}$. This linearization is based on the $(k-1)$ -th time step measured variables $\mathbf{x}_{m,k-1}$ and unmeasured variables $\mathbf{x}_{u,k-1}$ for the $(k-1)$ -th time step operating point $\mathbf{x}_{m,k-1}^*$ and $\mathbf{x}_{u,k-1}^*$.

$$\begin{aligned} \begin{bmatrix} \frac{\mathbf{x}_{d,k} - \mathbf{x}_{d,k-1}}{\Delta t} \\ \mathbf{0} \end{bmatrix} &\simeq F(\mathbf{x}_{m,k-1}, \mathbf{x}_{u,k-1}, \mathbf{a}_{k-1}) + \begin{bmatrix} \mathbf{w} \\ \mathbf{0} \end{bmatrix} \\ &= F(\mathbf{x}_{m,k-1}^* + \delta \mathbf{x}_{m,k-1}, \mathbf{x}_{u,k-1}^* + \delta \mathbf{x}_{u,k-1}, \mathbf{a}_{k-1}^* + \delta \mathbf{a}_{k-1}) + \begin{bmatrix} \mathbf{w}_k \\ \mathbf{0} \end{bmatrix} \\ &\simeq F(\mathbf{x}_{m,k-1}^*, \mathbf{x}_{u,k-1}^*, \mathbf{a}_{k-1}^*) + \left. \frac{\partial F}{\partial \mathbf{x}_m} \right|_{(\mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{x}_{m,k-1} + \left. \frac{\partial F}{\partial \mathbf{x}_u} \right|_{(\mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{x}_{u,k-1} + \left. \frac{\partial F}{\partial \mathbf{a}} \right|_{(\mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{a}_{k-1} + \begin{bmatrix} \mathbf{w}_k \\ \mathbf{0} \end{bmatrix} \end{aligned} \quad (10)$$

where $\mathbf{x}_k^* \triangleq \hat{\mathbf{x}}_{k|k}$, $\delta \mathbf{x}_k = \mathbf{x}_k - \mathbf{x}_k^*$, $\delta \mathbf{x}_{m,k-1} \triangleq \mathbf{x}_{m,k-1} - \mathbf{x}_{m,k-1}^*$, $\delta \mathbf{x}_{u,k-1} \triangleq \mathbf{x}_{u,k-1} - \mathbf{x}_{u,k-1}^*$ and $\delta \mathbf{a}_{k-1} \triangleq \mathbf{a}_{k-1} - \mathbf{a}_{k-1}^*$, and $\mathbf{w}_k \triangleq \mathbf{w}(t_k)$.

Similar to Eq. 9, we can approximate the dynamic equation at the operating point with $\mathbf{x}_{d,k}^*$ and $\mathbf{x}_{d,k-1}^*$ as:

$$\dot{\mathbf{x}}_{d,k}^* = F(\mathbf{x}_{m,k-1}^*, \mathbf{x}_{u,k-1}^*, \mathbf{a}_{k-1}^*) \simeq \frac{\mathbf{x}_{d,k}^* - \mathbf{x}_{d,k-1}^*}{\Delta t} \quad (11)$$

Therefore, by combining Eqs. 10 and 11, we obtain the linearized form of the dynamic equation as follows:

$$\begin{aligned} \begin{bmatrix} \frac{\mathbf{x}_{d,k} - \mathbf{x}_{d,k-1}}{\Delta t} \\ \mathbf{0} \end{bmatrix} - F(\mathbf{x}_{m,k-1}^*, \mathbf{x}_{u,k-1}^*, \mathbf{a}_{k-1}^*) &\simeq \begin{bmatrix} \frac{\mathbf{x}_{d,k} - \mathbf{x}_{d,k-1}}{\Delta t} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \frac{\mathbf{x}_{d,k}^* - \mathbf{x}_{d,k-1}^*}{\Delta t} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \frac{\delta \mathbf{x}_{d,k} - \delta \mathbf{x}_{d,k-1}}{\Delta t} \\ \mathbf{0} \end{bmatrix} \\ &\simeq \left. \frac{\partial F}{\partial \mathbf{x}_m} \right|_{(\mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{x}_{m,k-1} + \left. \frac{\partial F}{\partial \mathbf{x}_u} \right|_{(\mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{x}_{u,k-1} + \left. \frac{\partial F}{\partial \mathbf{a}} \right|_{(\mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{a}_{k-1} + \begin{bmatrix} \mathbf{w}_k \\ \mathbf{0} \end{bmatrix} \end{aligned} \quad (12)$$

Since linearization is performed at each sampling time and the difference in the $(k-1)$ -th unmeasured variable

$\delta \mathbf{x}_{u,k-1}$ can be ignored through estimation using a nonlinear filter when measuring the significant state variables [22], Eq. 12 can be expressed as

$$\begin{bmatrix} \frac{\delta \mathbf{x}_{d,k} - \delta \mathbf{x}_{d,k-1}}{\Delta t} \\ \mathbf{0} \end{bmatrix} - \frac{\partial F}{\partial \mathbf{x}_m} \Big|_{(\mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{x}_{m,k-1} = \frac{\partial F}{\partial \mathbf{a}} \Big|_{(\mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{a}_{k-1} + \begin{bmatrix} \mathbf{w}_k \\ \mathbf{0} \end{bmatrix} \quad (13)$$

By denoting $\hat{\mathbf{A}}_{E,k-1} \triangleq \frac{\partial F}{\partial \mathbf{a}} \Big|_{(\mathbf{x}^*, \mathbf{a}^*)}$ and $\hat{\mathbf{B}}_{E,k-1} \triangleq -\frac{\partial F}{\partial \mathbf{x}_m} \Big|_{(\mathbf{x}^*, \mathbf{a}^*)}$, the fault factor in a transient state can be obtained using the least square estimate.

$$\hat{\mathbf{a}}_{k-1} = \mathbf{a}^* + \delta \hat{\mathbf{a}}_{k-1} = \mathbf{1} + \left(\hat{\mathbf{A}}_{E,k-1} \right)^\dagger \left(\hat{\mathbf{B}}_{E,k-1} \cdot \mathbf{x}_{m,k-1} + \begin{bmatrix} \frac{\delta \mathbf{x}_{d,k} - \delta \mathbf{x}_{d,k-1}}{\Delta t} \\ \mathbf{0} \end{bmatrix} \right) \quad (14)$$

In addition, if the state becomes a steady state ($\hat{\mathbf{x}}_{m,k|k} = \mathbf{x}_m^*$, $(\frac{\delta \mathbf{x}_{d,k} - \delta \mathbf{x}_{d,k-1}}{\Delta t} \simeq \delta \dot{\mathbf{x}}_{d,k} = \mathbf{0})$, the matrices in Eq. 14 become the same as those in Eq. 7 ($\hat{\mathbf{A}}_{E,k-1} = \mathbf{A}$, $\hat{\mathbf{B}}_{E,k-1} = \mathbf{B}$). Furthermore, since the Euler method has a first-order error, we can estimate more accurately if the central difference method is applied. However, for this, we delay the two sampling times.

2. Time-derivative-ignoring approach

The dynamic equations (Eqs 1-3) can be expressed in the form of a single vector equation as

$$G(\dot{\mathbf{x}}, \mathbf{x}, \mathbf{a}) \triangleq \begin{bmatrix} f(\mathbf{x}, \mathbf{a}) - \dot{\mathbf{x}}_d \\ g(\mathbf{x}, \mathbf{a}) \end{bmatrix} = \begin{bmatrix} \mathbf{w}_k \\ \mathbf{0} \end{bmatrix} \quad (15)$$

Then, Eq. 15 with fault factor can be linearized directly as

$$\begin{aligned} G(\dot{\mathbf{x}}, \mathbf{x}, \mathbf{a}) &= G(\dot{\mathbf{x}}_k^* + \delta \dot{\mathbf{x}}_k, \mathbf{x}_{m,k}^* + \delta \mathbf{x}_{m,k}, \mathbf{x}_{u,k}^* + \delta \mathbf{x}_{u,k}, \mathbf{a}_k^* + \delta \mathbf{a}_k) \\ &\simeq G(\dot{\mathbf{x}}_k^*, \mathbf{x}_{m,k}^*, \mathbf{x}_{u,k}^*, \mathbf{a}_k^*) + \frac{\partial G}{\partial \dot{\mathbf{x}}} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \dot{\mathbf{x}}_k + \frac{\partial G}{\partial \mathbf{x}_m} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{x}_{m,k} + \frac{\partial G}{\partial \mathbf{x}_u} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{x}_{u,k} + \frac{\partial G}{\partial \mathbf{a}} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{a}_k \\ &= \begin{bmatrix} \mathbf{w}_k \\ \mathbf{0} \end{bmatrix} \end{aligned} \quad (16)$$

where $\mathbf{x}_k^* \triangleq \hat{\mathbf{x}}_{k|k}$, $\delta \dot{\mathbf{x}}_k = \dot{\mathbf{x}}_k - \dot{\mathbf{x}}_k^*$, $\delta \mathbf{x}_{m,k} = \mathbf{x}_{m,k} - \mathbf{x}_{m,k}^*$, $\delta \mathbf{x}_{u,k} = \mathbf{x}_{u,k} - \mathbf{x}_{u,k}^*$ and $\delta \mathbf{a}_k = \mathbf{a}_k - \mathbf{a}_k^*$.

Given that the single vector equation form of DAEs always satisfies $G(\dot{\mathbf{x}}, \mathbf{x}, \mathbf{a}) = \mathbf{0}$, and that linearization is performed

at each sampling time, we assume that the effect of the k -th differential variable $\delta \dot{\mathbf{x}}_k$ is negligibly small. This assumption is based on the fact that the elements in the coefficient matrix for $\delta \dot{\mathbf{x}}_k$ are 0 and 1, whereas those for $\delta \mathbf{x}_{m,k}$ are either 0 or greater than 100 as

$$\left\{ x \mid x \in \frac{\partial G}{\partial \mathbf{x}_m} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)} \wedge x \neq 0 \right\} \gg \left\{ x \mid x \in \frac{\partial G}{\partial \dot{\mathbf{x}}} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)} \wedge x \neq 0 \right\} \quad (17)$$

In the case of the differential variable, simulation results indicate that the initial proportion of the ignored differential variable constitutes up to 20% of the total measured variable but decreases to less than 2% shortly thereafter. This behavior, combined with the relatively small and negligible coefficient matrix, supports the validity of the assumption as

$$\left\| \frac{\partial G}{\partial \mathbf{x}_m} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{x}_{m,k} \right\| \gg \left\| \frac{\partial G}{\partial \dot{\mathbf{x}}} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \dot{\mathbf{x}}_k \right\| \quad (18)$$

Furthermore, since the variation in the k -th unmeasured variable $\delta \mathbf{x}_{u,k}$ can be ignored, as described in Sect. III.A.1, we can express Eq. 16 as

$$-\frac{\partial G}{\partial \mathbf{x}_m} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{x}_{m,k} = \frac{\partial G}{\partial \mathbf{a}} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)} \cdot \delta \mathbf{a}_k \quad (19)$$

Let $\hat{\mathbf{A}}_{T,k} \triangleq \frac{\partial G}{\partial \mathbf{a}} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)}$ and $\hat{\mathbf{B}}_{T,k} \triangleq -\frac{\partial G}{\partial \mathbf{x}_m} \Big|_{(\dot{\mathbf{x}}^*, \mathbf{x}^*, \mathbf{a}^*)}$. Then, we can estimate the fault factor in each step of the transient state using the least square estimation as

$$\hat{\mathbf{a}}_k = \mathbf{a}^* + \delta \hat{\mathbf{a}}_k = \mathbf{1} + \hat{\mathbf{A}}_{T,k}^\dagger \hat{\mathbf{B}}_{T,k} \cdot \mathbf{x}_{m,k} \quad (20)$$

Additionally, if the state converges to a steady state ($\mathbf{x}_{m,k}^* = \mathbf{x}_m^*$), then the matrixes in Eq. 20 become the same as those in Eq. 7 ($\hat{\mathbf{A}}_{T,k} = \mathbf{A}$, $\hat{\mathbf{B}}_{T,k} = \mathbf{B}$).

B. Nonlinear state estimation

Linearization requires known state information for an operating point in the fault factor estimate process. Therefore, we obtain the healthy changing state variable by estimating it through a nonlinear model-based state estimation algorithm [23–25] and use it as an operating point to estimate the fault factor at each sampling time. In various state estimation methods developed for an open-cycle LRE [18, 19], we employ the extended Kalman filter (EKF), one of the most commonly used model-based estimation algorithms in this study.

To design the nonlinear model-based state estimation, we use Eqs. 1-3 with healthy condition ($\mathbf{a} = \mathbf{1}$) to derive the mathematical framework necessary for accurate estimation as

$$\dot{\hat{\mathbf{x}}}_d(t) = f(\hat{\mathbf{x}}(t)) + \mathbf{w}(t) \quad (21)$$

$$\dot{\mathbf{P}}(t) = \mathbf{F}(t)\mathbf{P}(t) + \mathbf{P}(t)\mathbf{F}^T(t) + \mathbf{Q}(t) \quad (22)$$

$$\mathbf{V}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k \quad (23)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{V}_k^{-1} \quad (24)$$

$$\hat{\mathbf{y}}_{d,k} = h(\hat{\mathbf{x}}_{d,k|k-1}) + \mathbf{v}_k \quad (25)$$

$$\hat{\mathbf{x}}_{d,k|k} = \hat{\mathbf{x}}_{d,k|k-1} + \mathbf{K}_k (\mathbf{y}_{d,k} - \hat{\mathbf{y}}_{d,k}) \quad (26)$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \quad (27)$$

$$\hat{\mathbf{x}}_{a,k|k}(t) = g_a(\hat{\mathbf{x}}_{d,k|k}(t)) \quad (28)$$

where $\mathbf{F} \triangleq \left. \frac{\partial f}{\partial \mathbf{x}} \right|_{\mathbf{x}^*}$, $\mathbf{H} \triangleq \left. \frac{\partial h}{\partial \mathbf{x}} \right|_{\mathbf{x}^*}$, and $\mathbf{P}(t)$, $\mathbf{V}_k$, $\mathbf{K}_k$, $\mathbf{P}_{k|k}$, $\hat{\mathbf{x}}_{d,k|k}$, $\hat{\mathbf{x}}_{a,k|k}$ are predicted covariance estimate, residual covariance, Kalman gain, updated covariance estimate, updated differential state estimate, updated algebraic state estimate, respectively, and $\mathbf{w}(t) \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}(t))$ and $\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k)$ are the process and measurement noises, respectively.

Through state estimation, the estimated healthy state variable at k -th time step $\hat{\mathbf{x}}_{k|k}$ can be used as the operating point, i.e., $\mathbf{x}_k^* \triangleq \hat{\mathbf{x}}_{k|k}$. Additionally, we can utilize the estimated state information for the k -th unmeasured state variables as $\mathbf{x}_{u,k} \triangleq \hat{\mathbf{x}}_{u,k|k}$, such that the variation in the k -th unmeasured variable $\delta \mathbf{x}_{u,k}$ becomes zero. This occurs because the operating point of the unmeasured variable is also $\mathbf{x}_{u,k}^* \triangleq \hat{\mathbf{x}}_{u,k|k}$.

IV. Application of General Fault Factor Method to LRE

In this section, we conduct simulations using Matlab/Simulink to evaluate the performance of the general fault factor methods for detecting faults in an open-cycle LRE during a transient state. We focus on the startup process, which is validated by comparing normalized simulation data (red-dashed line) with experimental data (blue-solid line) under healthy conditions, as shown in Fig. 2. To simulate fault conditions, we inject fault signals following the established approach described in Refs. [26–28], which has been validated using actual datasets. Given the scope of this study, accounting for all possible faults is impractical. Therefore, we focus on turbopump faults and pipeline blockages as

representative fault types [27–30]. Additionally, faults can be categorized into two types: internal faults and valve faults. In this study, we simulate turbine efficiency fault and fuel pipeline blockage as internal faults, while a valve malfunction represents a valve fault. To illustrate these categories, we introduce three representative faults into the healthy-condition simulation: i) a first internal component fault (turbine efficiency fault, black-dotted line), ii) another internal component fault (fuel pipeline blockage, green dashed line), and iii) a valve fault (main oxidizer valve (MOV) malfunction, purple dash-single dotted line), as illustrated in Fig. 2. We then apply the proposed algorithms under these fault conditions, including noise, to validate its performance. The health monitoring process is illustrated in Fig. 3.

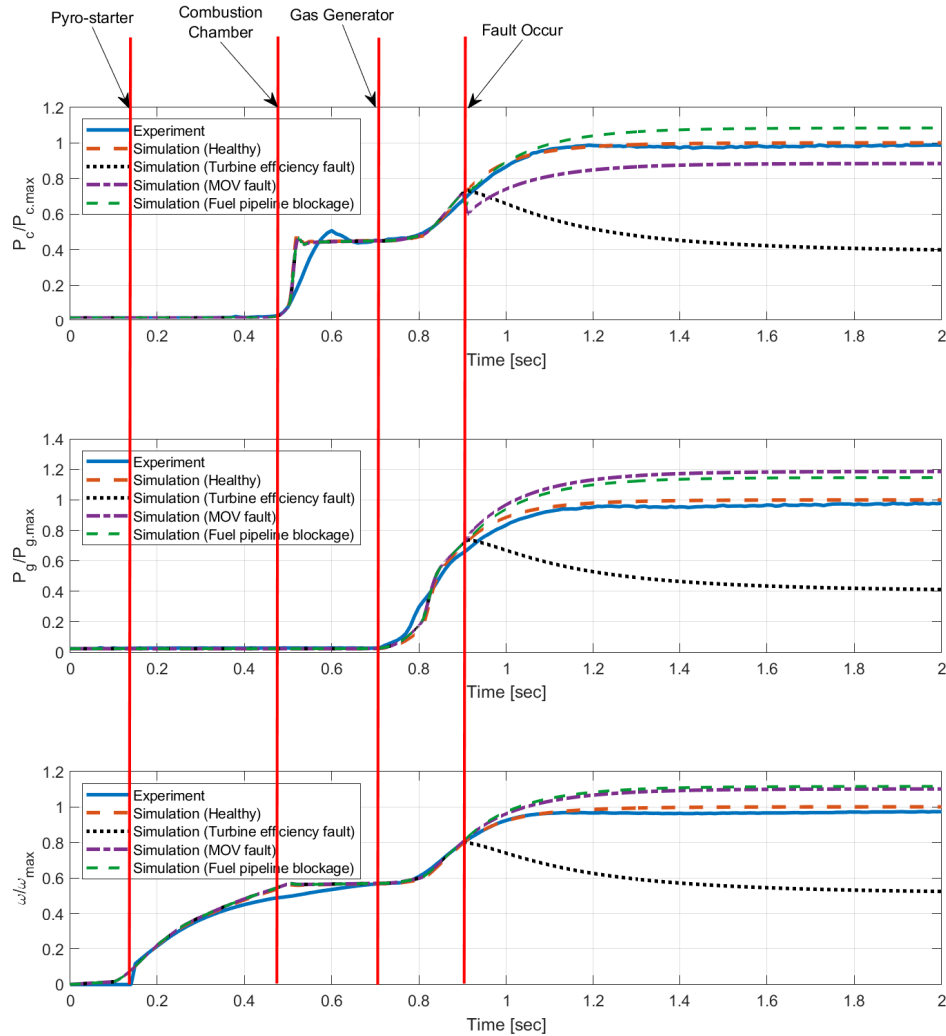


Fig. 2 Normalized empirical and simulated startup results of a liquid rocket engine: (top) chamber pressure; (middle) gas generator pressure; (bottom) turbopump rotational velocity [30]

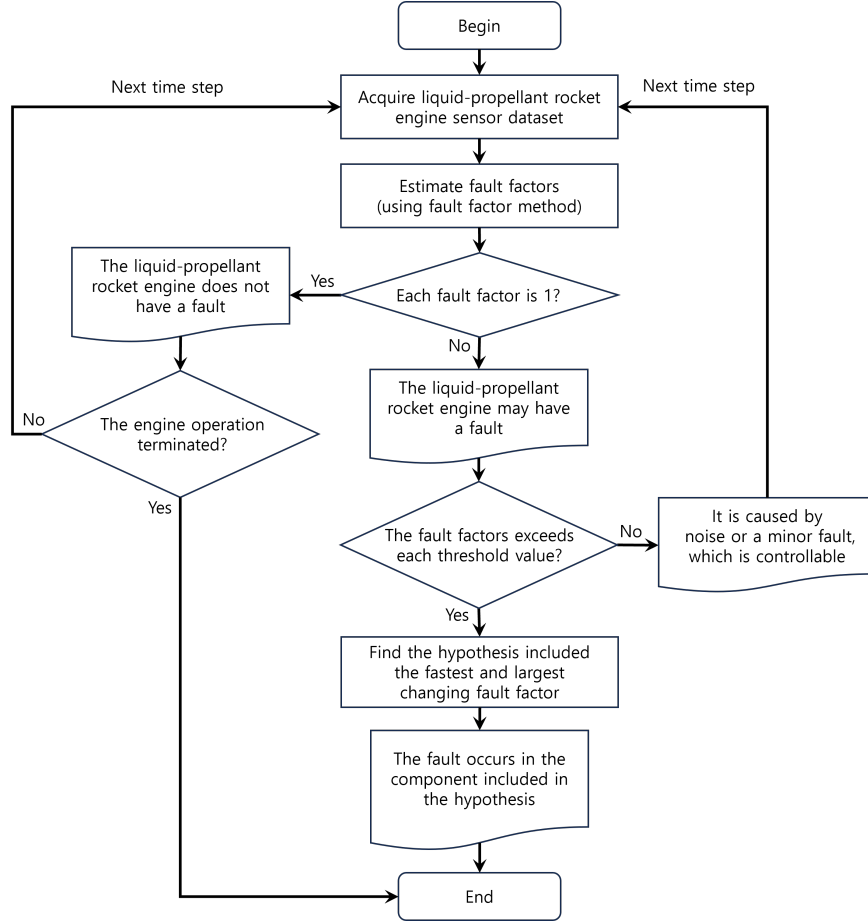


Fig. 3 Flowchart of the health monitoring process

A. Dynamic equations of LRE with fault factor

The constructed dynamic equations with fault factors are shown in Table 1 [21].

Using the 26 equations in Table 1, the described FDD algorithms can be applied by estimating the corresponding 26 fault factors. However, in real systems, limitations in sensor placement often make it impossible to measure all necessary states. Consequently, a comprehensive estimation of all fault factors may lead to inaccuracies due to insufficient information. To address this issue, we employ a hypothesis-based approach that groups relevant specific components. This method effectively reduces the number of unknown states by leveraging overlapping measured states and decreases computation time by analyzing multiple smaller groups rather than a single large group. In this study, we focus on the subsystems of the LRE, optimizing the grouping of each component system as illustrated in Fig. 4: i) oxidizer pipeline, ii) fuel pipeline, iii) main combustion chamber, iv) gas generator, and v) turbopump [21]. The abbreviations used in the figure are defined in the Subscripts section of the Nomenclature.

Table 1 Dynamic equations [21]

Pressure equations			
No	Dynamic equation	No	Dynamic equation
1	$P_{o1} - a_1 (P_a + [K_{o1}] \dot{m}_{o1}^2) = 0$	7	$P_b - a_7 \left(P_{fic} + \left[\frac{1}{K_{MEV}} + K_{f3} \right] \dot{m}_{f3}^2 \right) = \left(\frac{L_{f3}}{A_{f3}} \right) \frac{d}{dt} \dot{m}_{f3}$
2	$P_a - a_2 \left(P_{oic} + \left[\frac{1}{K_{MOV}} + K_{o3} \right] \dot{m}_{o3}^2 \right) = \left(\frac{L_{o3}}{A_{o3}} \right) \frac{d}{dt} \dot{m}_{o3}$	8	$P_{fic} - a_8 (P_c + [K_{fic}] \dot{m}_{fic}^2) = 0$
3	$P_{oic} - a_3 (P_c + [K_{oic}] \dot{m}_{oic}^2) = 0$	9	$P_b - a_9 \left(P_{fig} + \left[\frac{1}{K_{GFV}} + K_{f2} \right] \dot{m}_{f2}^2 \right) = \left(\frac{L_{f2}}{A_{f2}} \right) \frac{d}{dt} \dot{m}_{f2}$
4	$P_a - a_4 \left(P_{oig} + \left[\frac{1}{K_{GOV}} + K_{o2} \right] \dot{m}_{o2}^2 \right) = \left(\frac{L_{o2}}{A_{o2}} \right) \frac{d}{dt} \dot{m}_{o2}$	10	$P_{fig} - a_{10} (P_g + [K_{fig}] \dot{m}_{fig}^2) = 0$
5	$P_{oig} - a_5 (P_g + [K_{oig}] \dot{m}_{oig}^2) = 0$	11	$P_{tbe} - a_{11} \left(\frac{\dot{m}_{tbi}}{A_{tbe} C_d \sqrt{\frac{\gamma}{RT} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}}} \right) = 0$
6	$P_{f1} - a_6 (P_b + [K_{f1}] \dot{m}_{f1}^2) = 0$		
Mass flow rate equations			
No	Dynamic equation	No	Dynamic equation
12	$\dot{m}_{o1} - a_{12} (\dot{m}_{o2} + \dot{m}_{o3}) = 0$	15	$\dot{m}_{f1} - a_{15} (\dot{m}_{f2} + \dot{m}_{f3}) = 0$
13	$-\dot{m}_c + a_{13} (\dot{m}_{oic} + \dot{m}_{fic}) = \left(\frac{V_c}{R_c T_c} \right) \frac{d}{dt} P_c$	16	$-\dot{m}_{tbi} + a_{16} (\dot{m}_{oig} + \dot{m}_{fig}) = \left(\frac{V_g}{R_g T_g} \right) \frac{d}{dt} P_g$
14	$\dot{m}_{cn} - a_{14} \left(\frac{P_c}{\sqrt{T_c}} C_c \right) = 0$	17	$\dot{m}_{tbi} - a_{17} \left(\frac{P_g}{\sqrt{T_g}} C_g \right) = 0$
Turbopump pressure increment equations			
No	Dynamic equation	No	Dynamic equation
18	$P_{o1} - a_{18} (P_{os} + (A_o \omega^2 - B_o \omega \dot{m}_{o1} - C_o \dot{m}_{o1}^2)) = 0$	19	$P_{f1} - a_{19} (P_{fs} + (A_f \omega^2 - B_f \omega \dot{m}_{f1} - C_f \dot{m}_{f1}^2)) = 0$
Turbopump power equations			
No	Dynamic equation	No	Dynamic equation
20	$\tau_{op} - a_{20} \left(\frac{\dot{m}_{o1} (P_{o1} - P_{os})}{\omega \eta_{op} \rho_o} \right) = 0$	22	$\tau_{tb} - a_{22} \left(\frac{\dot{m}_{tbe} T_{tbi}}{\omega} \left(1 - \left(\frac{P_{tbe}}{P_{tbi}} \right)^{\frac{\gamma-1}{\gamma}} \right) \frac{\gamma R_g}{\gamma-1} \right) = 0$
21	$\tau_{fp} - a_{21} \left(\frac{\dot{m}_{f1} (P_{f1} - P_{fs})}{\omega \eta_{fp} \rho_f} \right) = 0$	23	$\dot{m}_{tbe} T_{tbi} \left(1 - \left(\frac{P_{tbe}}{P_{tbi}} \right)^{\frac{\gamma_g-1}{\gamma_g}} \right) \frac{\gamma_g R_g}{\gamma_g-1} - a_{23} \left(\frac{\dot{m}_{o1} (P_{o1} - P_{os})}{\eta_{op} \rho_o} + \frac{\dot{m}_{f1} (P_{f1} - P_{fs})}{\eta_{fp} \rho_f} \right) = I_{tp} \frac{d}{dt} \omega$
Pressure or mass flow rate ratio equations			
No	Dynamic equation	No	Dynamic equation
24	$r_{tb} - a_{24} \left(\frac{P_{tbi}}{P_{tbe}} \right) = 0$	26	$r_g - a_{26} \left(\frac{\dot{m}_{oig}}{\dot{m}_{fig}} \right) = 0$
25	$r_c - a_{25} \left(\frac{\dot{m}_{oic}}{\dot{m}_{fic}} \right) = 0$		

B. Case study

We demonstrate the proposed algorithms through three fault scenarios: reduced turbine efficiency, fuel pipeline blockage and a 'not fully open' malfunction in the MOV. The process follows a similar approach to that described in Ref. [21], except for the two methods employed to address derivative states and nonlinear model-based state estimation in Sect. III.A and III.B. However, accounting for the variability in torque values and ratios—such as pressure or mass flow rate ratios—during transient states using direct sensor measurements within the algorithms is difficult to implement in a real system. Therefore, the six equations (nos. 20, 21, 22, 24, 25, and 26 in Table 1), which pertain to torque and these ratios, are excluded during transient states. As a result, this study utilizes 20 equations to estimate 20 fault factors, and the arranged fault factors within each hypothesis are as follows.

- Hypothesis 1 (oxidizer pipeline): $a_1, a_2, a_3, a_4, a_5, a_{12}, a_{18}$
- Hypothesis 2 (fuel pipeline): $a_6, a_7, a_8, a_9, a_{10}, a_{15}, a_{19}$

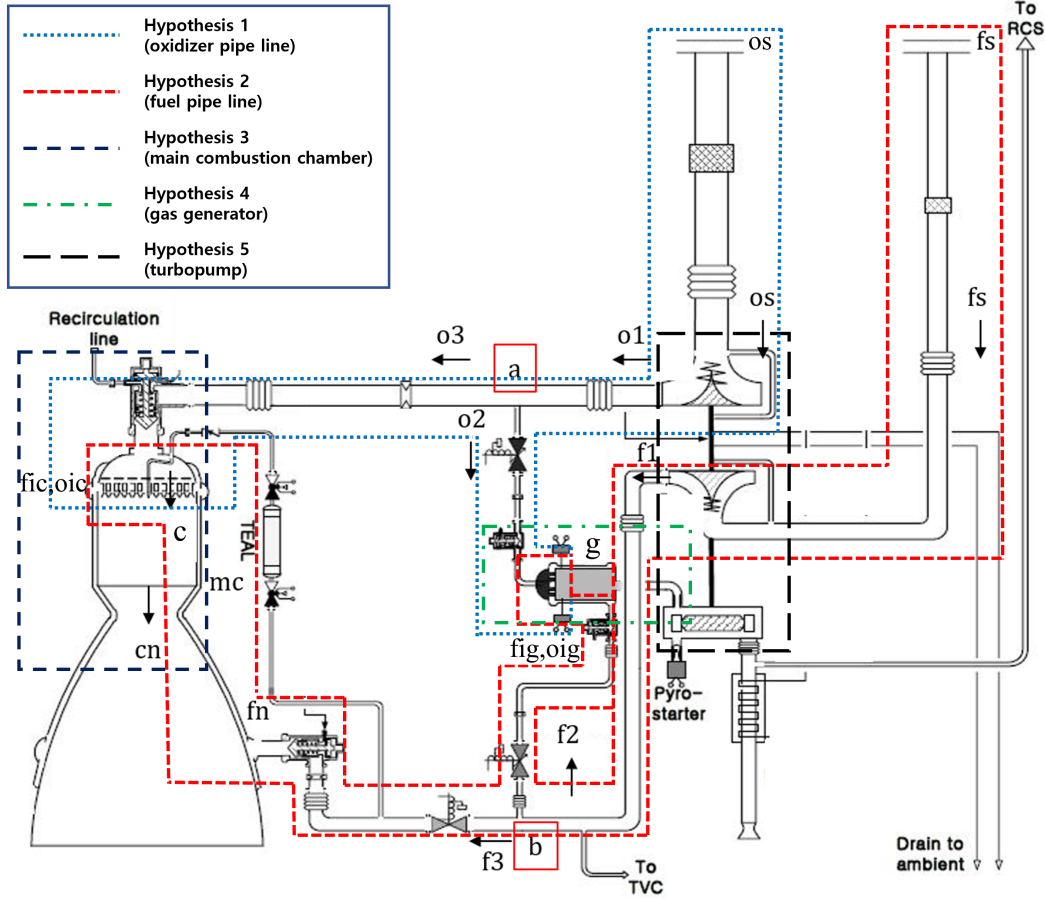


Fig. 4 Hypothesis groups for a liquid rocket engine [21]

- Hypothesis 3 (combustion chamber): a_3, a_8, a_{13}, a_{14}
- Hypothesis 4 (gas generator): $a_5, a_{10}, a_{16}, a_{17}$
- Hypothesis 5 (turbopump): $a_{11}, a_{18}, a_{19}, a_{23}$

To determine whether a fault occurs using the developed methods, we establish a threshold value of $\pm 10\%$ for fault detection based on the minimum margin of safety (MS) or factor of safety (FoS) values. These values, which typically range from 1.1 to 2.0, are considered in the design of LREs, as detailed in Ref. [31].

1. Internal component fault (turbine efficiency fault)

For an internal fault, we artificially inject a turbine efficiency-reducing fault at 0.9 seconds into the startup process, as illustrated by the black dotted line in Fig. 2. Next, we apply the proposed method using two approaches—the Euler-based derivative approach and the time-derivative-ignoring approach—to detect the fault by determining whether at least one fault factor exceeds the threshold, as shown in Fig. 3. To evaluate the performance of the approaches, we employ two methods: i) analyzing the changes in fault factors over time to assess detection speed, and ii) examining the

Table 2 Final value of the estimated fault factors of the turbine efficiency fault in the startup process

Hypothesis 1: Oxidizer pipeline		Hypothesis 2: Fuel pipeline	
$\hat{a}_i$	Mean Value	$\hat{a}_i$	Mean Value
$\hat{a}_1$	1.0000 ± 0.0152	$\hat{a}_6$	0.8304 ± 0.0135
$\hat{a}_2$	1.0516 ± 0.0152	$\hat{a}_7$	1.2772 ± 0.0135
$\hat{a}_3$	1.0574 ± 0.0037	$\hat{a}_8$	1.0569 ± 0.0036
$\hat{a}_4$	1.0865 ± 0.0152	$\hat{a}_9$	1.2955 ± 0.0135
$\hat{a}_5$	1.0362 ± 0.0066	$\hat{a}_{10}$	1.0497 ± 0.0063
$\hat{a}_{12}$	1.0000 ± 0.0039	$\hat{a}_{15}$	1.0000 ± 0.0025
$\hat{a}_{18}$	0.9982 ± 0.0001	$\hat{a}_{19}$	0.9998 ± 0.0001
Hypothesis 3: Main combustion chamber		Hypothesis 4: Gas generator	
$\hat{a}_i$	Mean Value	$\hat{a}_i$	Mean Value
$\hat{a}_3$	1.0574 ± 0.0037	$\hat{a}_5$	1.0362 ± 0.0066
$\hat{a}_8$	1.0569 ± 0.0036	$\hat{a}_{10}$	1.0497 ± 0.0063
$\hat{a}_{13}$	1.0001 ± 0.0028	$\hat{a}_{16}$	1.0004 ± 0.0136
$\hat{a}_{14}$	0.9821 ± 0.0009	$\hat{a}_{17}$	0.9993 ± 0.0136
Hypothesis 5: Turbopump			
$\hat{a}_i$	Mean Value	$\hat{a}_i$	Mean Value
$\hat{a}_{11}$	1.0175 ± 0.0000	$\hat{a}_{19}$	0.9998 ± 0.0001
$\hat{a}_{18}$	0.9982 ± 0.0001	$\hat{a}_{23}$	1.7033 ± 0.0005

final converged mean values of the fault factors after fault detection to estimate severity. Figs. 5 and 6 illustrate the changes in fault factors over time for the Euler-based derivative and time-derivative-ignored approaches, respectively, with the thresholds indicated by green dashed lines (at 0.9 and 1.1). Table 2 presents the final converged mean values of the fault factors along with 95% confidence intervals for both approaches. The results indicate that the largest and fastest-changing factor is $\hat{a}_{23}$, related to the turbopump equation (no. 23 in Table 1) in hypothesis 5 (turbopump). In Figs. 5 and 6, the magnitude and speed of changes in the other factors are all less than those of $\hat{a}_{23}$, although some factors ($\hat{a}_6$, $\hat{a}_7$ and $\hat{a}_9$) exceed the $\pm 10\%$ limit (green-dashed line). Therefore, we can conclude that a fault occurred in the turbopump. Additionally, since all fault factor results in Figs. 5 and 6 remain within the confidence intervals presented in Table 2, the results are statistically reliable, confirming the validity of both approaches.

2. Internal component fault (fuel pipeline blockage)

The second internal fault, a fuel pipeline blockage, is introduced following the same procedure as the turbine efficiency fault. The results for this case are presented in Table 3, which shows the final converged mean values, and in Figs. 7 and 8, which illustrate the changes in fault factors for each approach. The results indicate that the most prominent factor is $\hat{a}_7$, associated with the main combustion chamber fuel pipeline equation (no. 7 in Table 1) under hypothesis 2 (fuel pipeline). In contrast, although some fault factors ($\hat{a}_6$, $\hat{a}_9$, and $\hat{a}_{14}$) exceed the $\pm 10\%$ limit (green-dashed line),

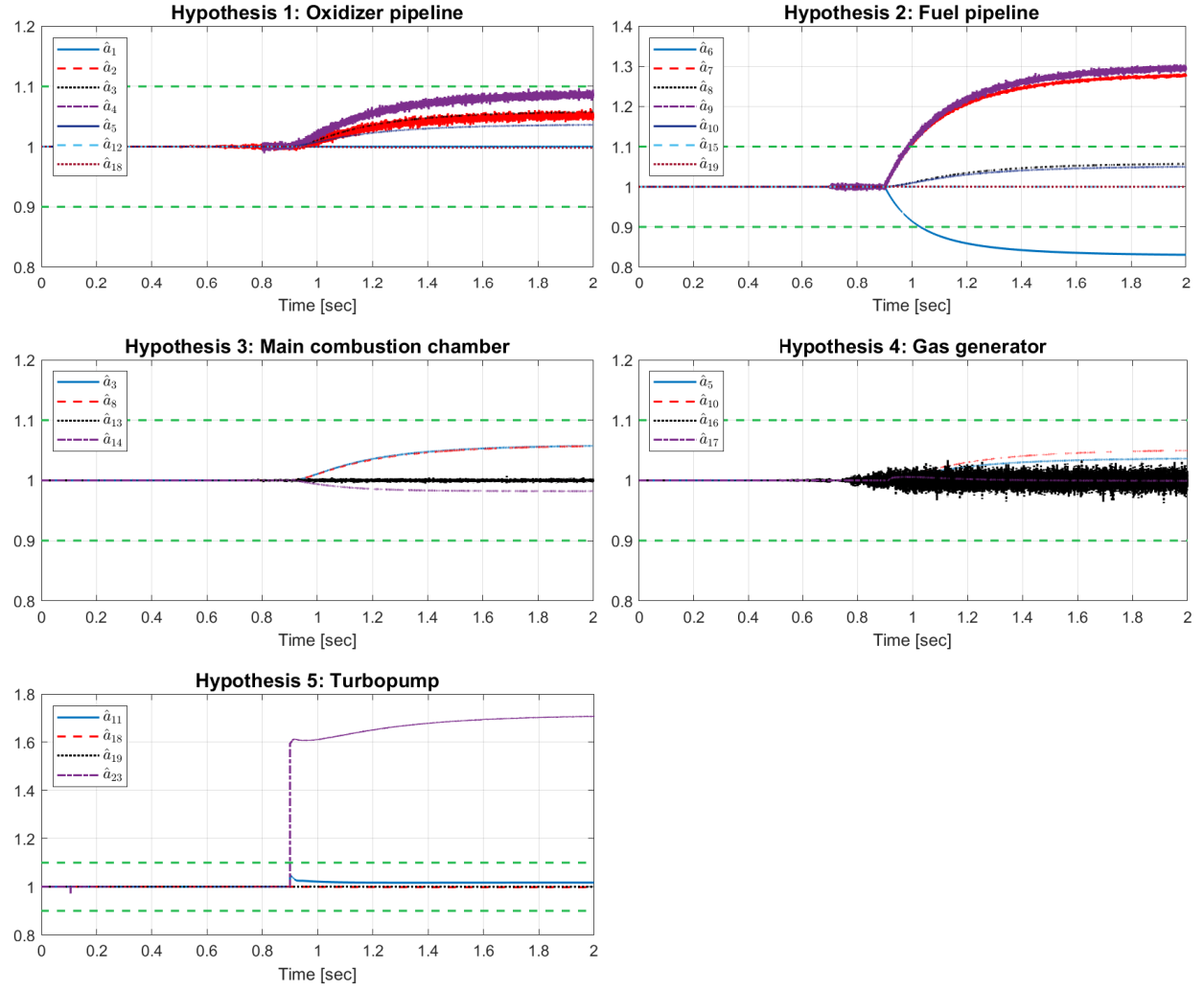


Fig. 5 Euler-based derivative approach results of the estimated fault factors of the turbine efficiency fault in the startup process

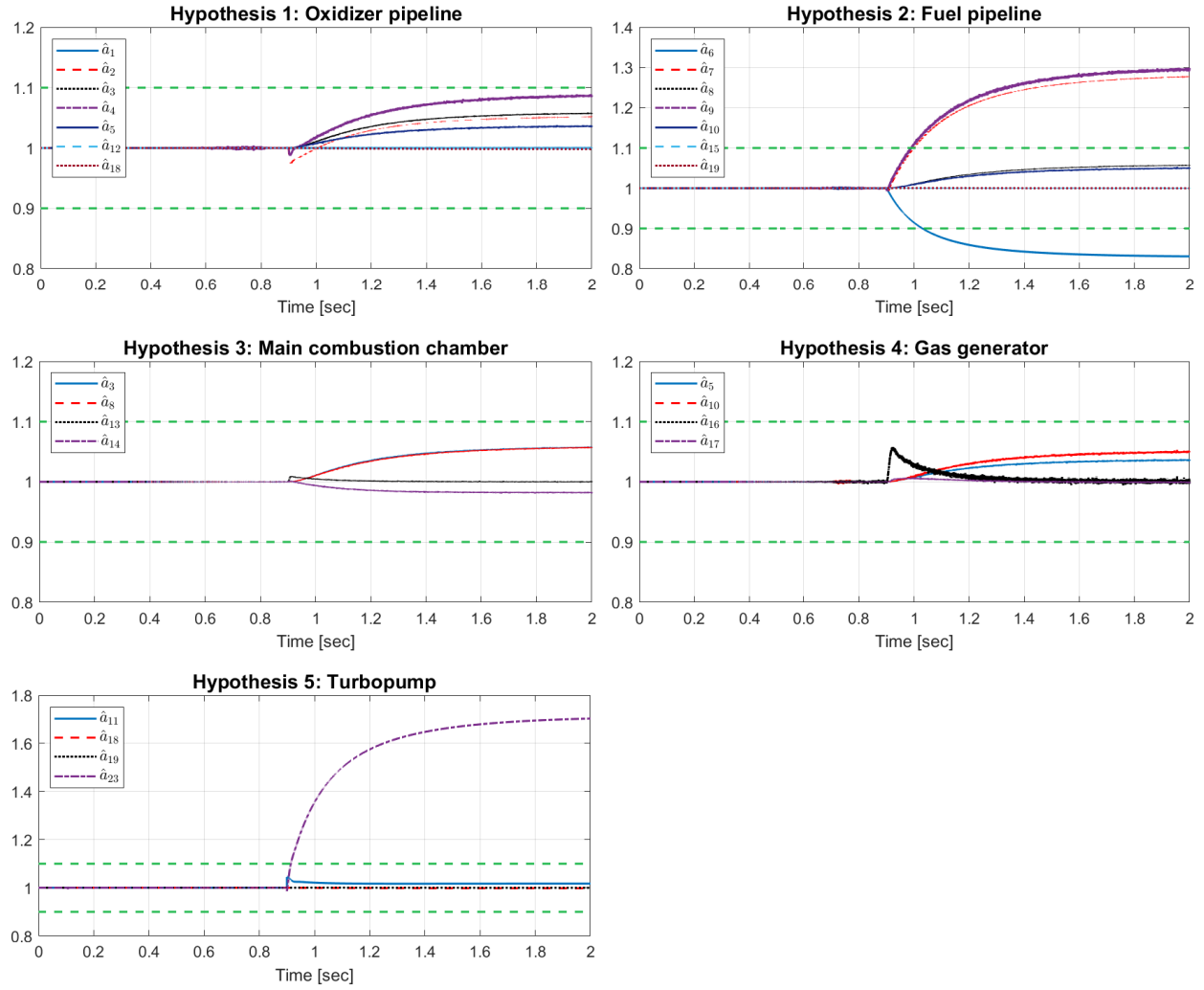


Fig. 6 Time-derivative-ignoring approach results of the estimated fault factors of the turbine efficiency fault in the startup process

Table 3 Final value of the estimated fault factors of the fuel pipeline blockage in the startup process

Hypothesis 1: Oxidizer pipeline		Hypothesis 2: Fuel pipeline	
$\hat{a}_i$	Mean Value	$\hat{a}_i$	Mean Value
$\hat{a}_1$	1.0000 ± 0.0152	$\hat{a}_6$	1.1377 ± 0.0135
$\hat{a}_2$	1.0072 ± 0.0152	$\hat{a}_7$	1.4383 ± 0.0135
$\hat{a}_3$	1.0075 ± 0.0037	$\hat{a}_8$	1.0276 ± 0.0036
$\hat{a}_4$	1.0055 ± 0.0152	$\hat{a}_9$	0.8783 ± 0.0135
$\hat{a}_5$	1.0017 ± 0.0066	$\hat{a}_{10}$	1.0065 ± 0.0063
$\hat{a}_{12}$	1.0006 ± 0.0039	$\hat{a}_{15}$	0.9997 ± 0.0025
$\hat{a}_{18}$	0.9999 ± 0.0001	$\hat{a}_{19}$	1.0005 ± 0.0001
Hypothesis 3: Main combustion chamber		Hypothesis 4: Gas generator	
$\hat{a}_i$	Mean Value	$\hat{a}_i$	Mean Value
$\hat{a}_3$	1.0075 ± 0.0037	$\hat{a}_5$	1.0017 ± 0.0066
$\hat{a}_8$	1.0276 ± 0.0036	$\hat{a}_{10}$	1.0065 ± 0.0063
$\hat{a}_{13}$	1.0002 ± 0.0028	$\hat{a}_{16}$	0.9996 ± 0.0136
$\hat{a}_{14}$	1.2467 ± 0.0009	$\hat{a}_{17}$	1.0294 ± 0.0136
Hypothesis 5: Turbopump			
$\hat{a}_i$	Mean Value	$\hat{a}_i$	Mean Value
$\hat{a}_{11}$	0.9442 ± 0.0000	$\hat{a}_{19}$	1.0005 ± 0.0001
$\hat{a}_{18}$	0.9999 ± 0.0001	$\hat{a}_{23}$	0.9563 ± 0.0005

their changes are less significant than $\hat{a}_7$. Thus, we can confirm a fault in the fuel pipeline.

Furthermore, these results show the difference between the Euler-based approach and the time-derivative-ignoring approach. Because differential variables are ignored, the fault factors in dynamic equations that include differential terms (differential equations) change sharply immediately after the fault occurs. This effect is particularly noticeable in $\hat{a}_{23}$. Although $\hat{a}_{23}$ sharply exceeds the limit immediately after the fault occurs and later converges within the limit in the time-derivative-ignoring approach (Fig. 8), it remains consistently within the limit in the Euler-based approach (Fig. 7).

3. Valve fault

To simulate valve faults, we implement the proposed algorithms for the fault scenario in which the MOV is not fully open, as was previously done for the turbine efficiency fault. The results for the valve fault case are presented in Table 4, showing the final converged mean values, and in Figs. 9 and 10, which illustrate the changes in fault factors for each approach. In the results, the largest and fastest-changing factor is $\hat{a}_2$ related to the main combustion chamber oxidizer pipeline equation (no.2 in Table 1) in hypothesis 1 (oxidizer pipeline), while the other fault factors exhibit smaller changes than $\hat{a}_2$ and remain within the $\pm 10\%$ limit (green-dashed lines). Thus, we can confirm that a fault occurs in the oxidizer pipeline.

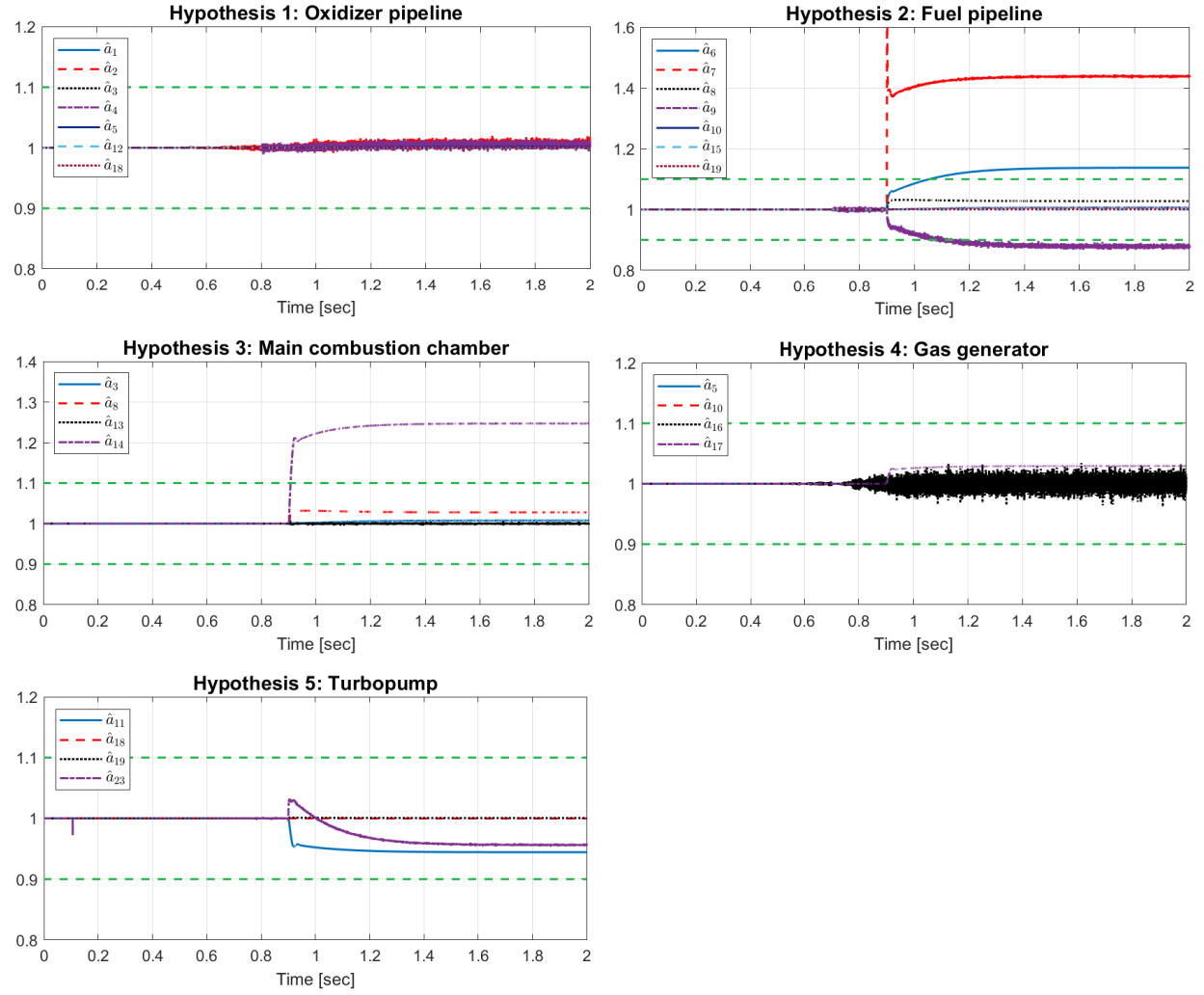


Fig. 7 Euler-based derivative approach results of the estimated fault factors of the fuel pipeline blockage in the startup process

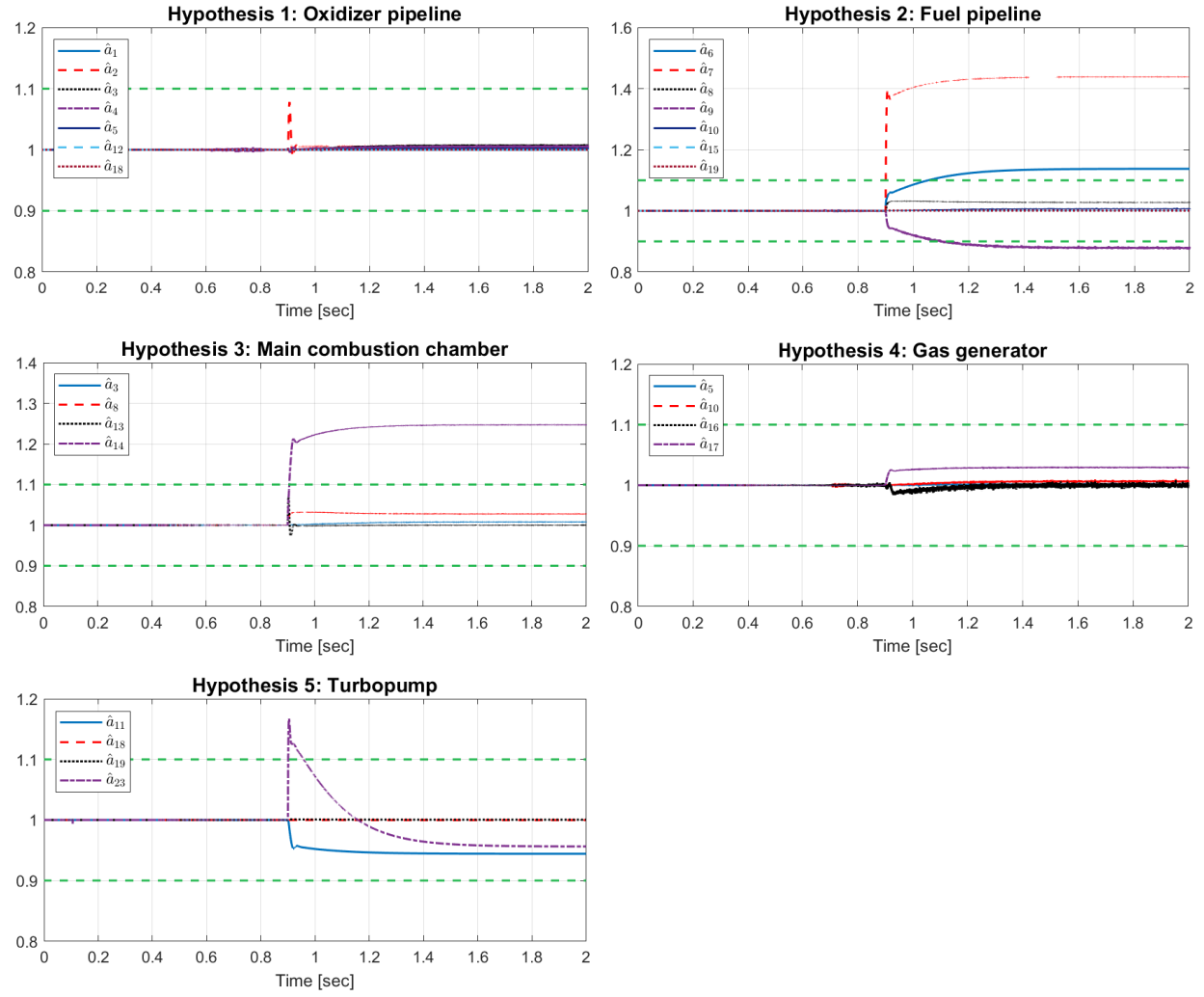


Fig. 8 Time-derivative-ignoring approach results of the estimated fault factors of the fuel pipeline blockage in the startup process

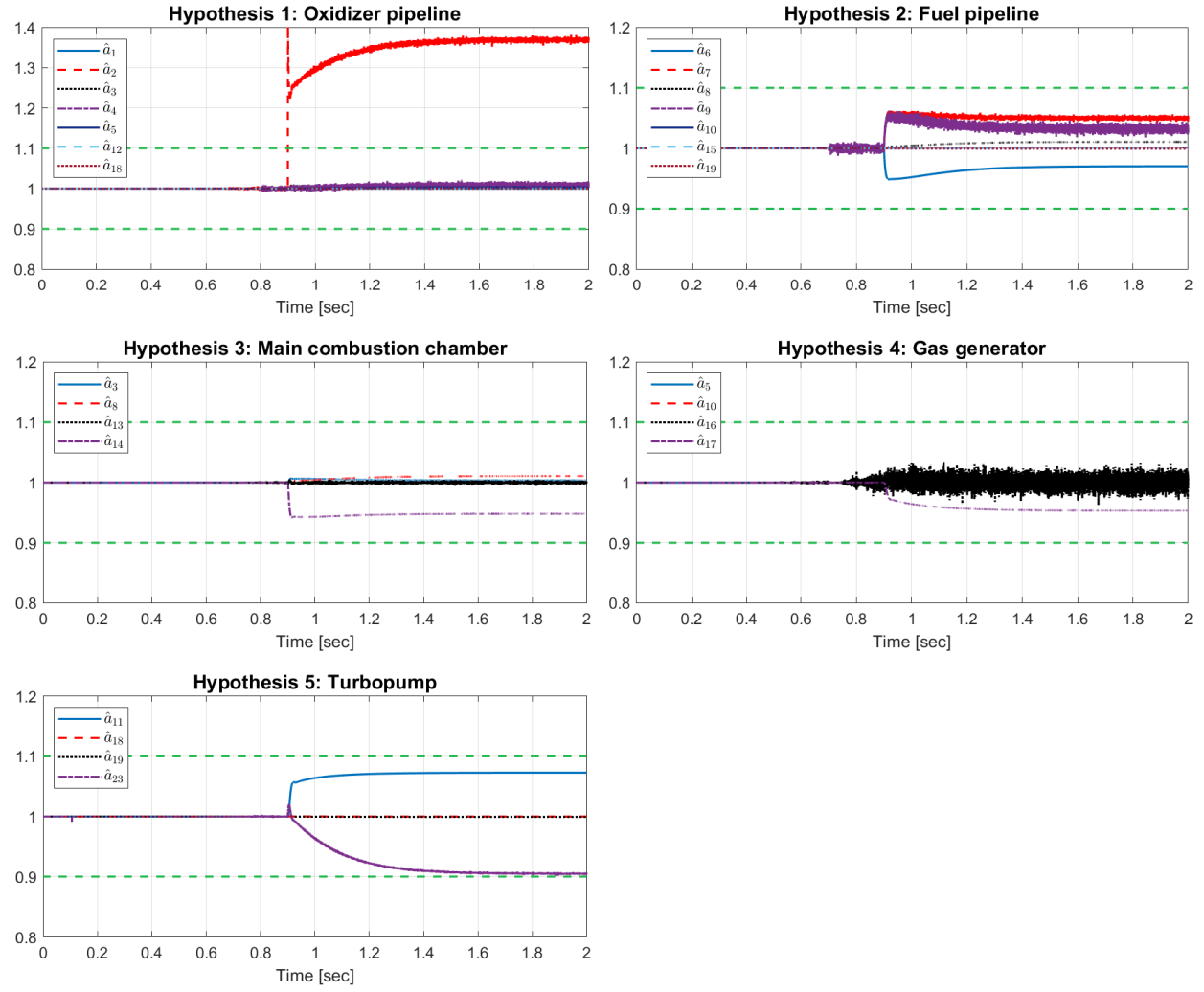


Fig. 9 Euler-based derivative approach results of the estimated fault factors of the MOV fault in the startup process

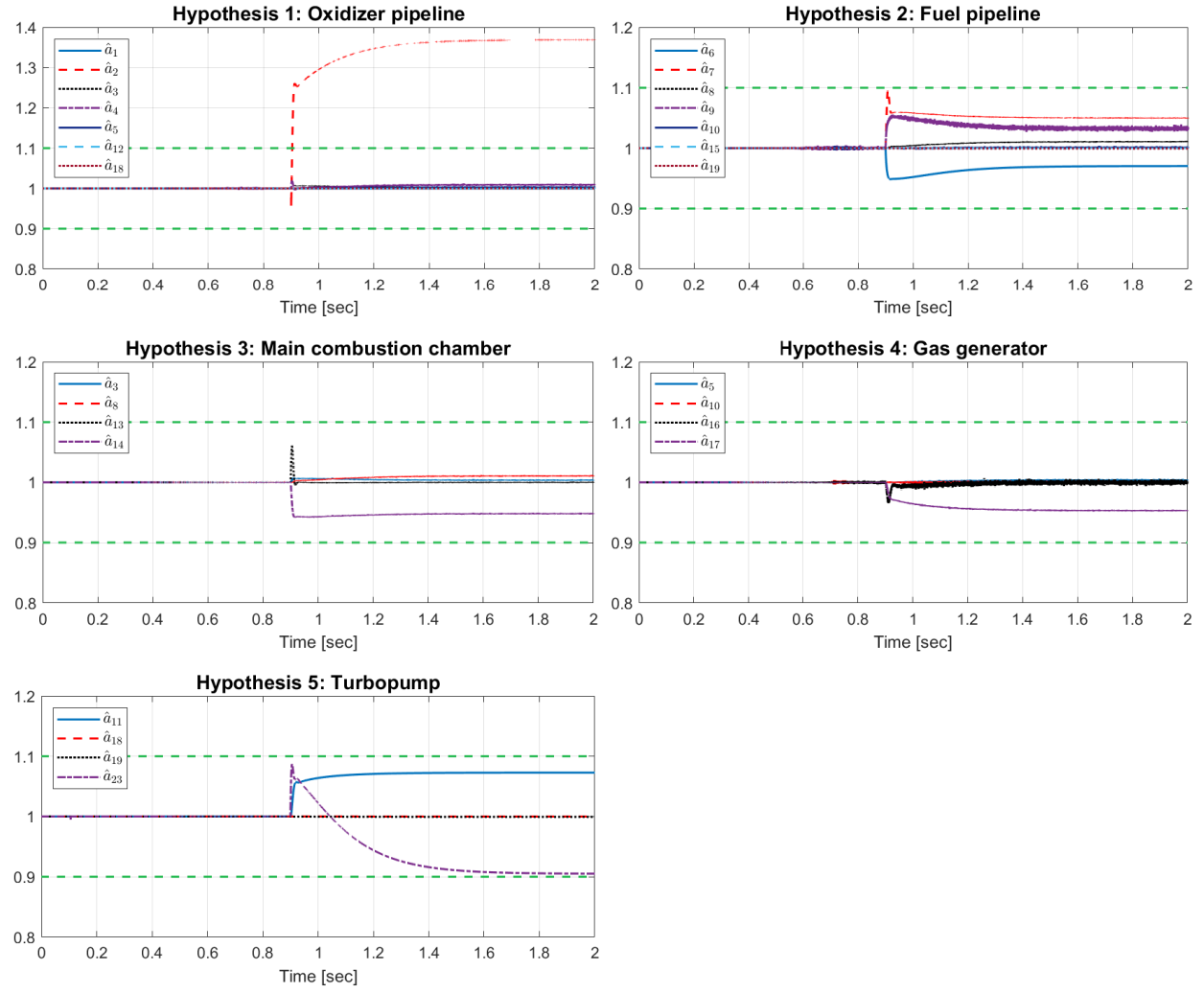


Fig. 10 Time-derivative-ignoring approach results of the estimated fault factors of the MOV fault in the startup process

Table 4 Final value of the estimated fault factors of the MOV fault in the startup process

Hypothesis 1: Oxidizer pipeline		Hypothesis 2: Fuel pipeline	
$\hat{a}_i$	Mean Value	$\hat{a}_i$	Mean Value
$\hat{a}_1$	1.0000 ± 0.0152	$\hat{a}_6$	0.9703 ± 0.0135
$\hat{a}_2$	1.3690 ± 0.0152	$\hat{a}_7$	1.0500 ± 0.0135
$\hat{a}_3$	1.0036 ± 0.0037	$\hat{a}_8$	1.0107 ± 0.0036
$\hat{a}_4$	1.0092 ± 0.0152	$\hat{a}_9$	1.0322 ± 0.0135
$\hat{a}_5$	1.0038 ± 0.0066	$\hat{a}_{10}$	1.0010 ± 0.0063
$\hat{a}_{12}$	1.0000 ± 0.0039	$\hat{a}_{15}$	1.0000 ± 0.0025
$\hat{a}_{18}$	0.9999 ± 0.0001	$\hat{a}_{19}$	0.9993 ± 0.0001
Hypothesis 3: Main combustion chamber		Hypothesis 4: Gas generator	
$\hat{a}_i$	Mean Value	$\hat{a}_i$	Mean Value
$\hat{a}_3$	1.0036 ± 0.0037	$\hat{a}_5$	1.0038 ± 0.0066
$\hat{a}_8$	1.0107 ± 0.0036	$\hat{a}_{10}$	1.0010 ± 0.0063
$\hat{a}_{13}$	1.0000 ± 0.0028	$\hat{a}_{16}$	1.0000 ± 0.0136
$\hat{a}_{14}$	0.9480 ± 0.0009	$\hat{a}_{17}$	0.9531 ± 0.0136
Hypothesis 5: Turbopump			
$\hat{a}_i$	Mean Value	$\hat{a}_i$	Mean Value
$\hat{a}_{11}$	1.0728 ± 0.0000	$\hat{a}_{19}$	0.9993 ± 0.0001
$\hat{a}_{18}$	0.9999 ± 0.0001	$\hat{a}_{23}$	0.9050 ± 0.0005

4. Discussion

Through simulations, we verify the performance of the developed method for nonlinear systems in a transient state. The method detects faults in real-time: the Euler-based derivative approach detects faults within one sampling step (0.0001 seconds), while the time-derivative-ignoring approach detects faults within four sampling steps (0.0004 seconds). The results indicate that the Euler-based derivative approach exhibits more significant fluctuation due to noise from local truncation errors. However, as shown in Figs. 5–10, these fluctuations remain within the confidence intervals presented in Tables 2, 3 and 4, respectively, confirming the validity of both approaches.

In the time-derivative-ignoring approach, the omission of differential variables can initially lead to errors in estimating fault factors of differential equations following an abrupt fault, even though the most prominent change is observed in the same fault factor directly associated with the fault in both approaches. However, as discussed in Sect. III.A.2, these errors gradually diminish over time. As a result, both approaches detect faults based on the most prominent fault factor change, and initial discrepancies caused by differential variables gradually converge, ultimately yielding identical mean values for the fault factors, except noise, which remains within the confidence intervals. Therefore, while the Euler-based approach facilitates faster fault detection by incorporating differential state variables, it is more susceptible to truncation errors. In contrast, the time-derivative-ignoring approach demonstrates greater robustness to noise but operates more slowly and may introduce errors in the fault factors of differential equations. To mitigate

the limitations of each approach, the noise effects in the Euler-based derivative approach can be reduced through higher-order methods, such as the central difference method. Additionally, special attention should be given to fault factors in differential equations when employing the time-derivative-ignoring method.

The proposed method also infers fault locations within component groups. In an LRE, urgent faults—such as fuel leakage into the oxidizer pump or turbine faults—can result in catastrophic failures, including explosions. Therefore, in real-time FDD for LREs, detecting faults before propagation and shutting down the engine is more critical than determining the exact fault cause. However, for postmortem analysis, precise fault diagnosis is required, necessitating additional processing steps. These steps can be achieved through residual signal analysis, as in steady-state analysis [21], fault propagation modeling and physical cause investigation [26], or machine learning-based approaches that learn complex fault patterns from historical data [7].

Moreover, the proposed method can be implemented in real time by integrating nonlinear model-based estimation techniques, such as the EKF, which has been widely adopted in various real-time applications due to its ability to provide accurate state estimation with manageable computational demand, with the fault factor method, which requires only matrix inversion calculations. In addition, nonlinear model-based estimation allows for the use of unmeasured states, enhancing the method's applicability even in scenarios with limited sensor placement. Performance can be further improved by optimizing sensor placement, as suggested in Refs. [32, 33].

V. Conclusion

This paper presents a real-time health monitoring method for LREs operating in transient states, using a general fault factor approach. Unlike previous methods [20, 21], which are limited to steady-state conditions, this method addresses non-zero time derivative state variables in transient states by integrating Euler-based derivative and time-derivative-ignoring approaches with a nonlinear model-based state estimation. Then, numerical simulations are conducted to evaluate the efficacy of the proposed method in addressing two types of faults in a transient-state open-cycle LRE. As part of our ongoing research, we plan to extend the method to address various combined or cascading fault scenarios and incorporate an additional process for accurate fault-cause diagnosis.

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