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ZHUOJUN LI

AIRCRAFT ENGINE TRANSIENT PERFORMANCE MODELLING
WITH HEAT SOKAGE EFFECTS

SCHOOL OF AEROSPACE, TRANSPORT AND
MANUFACTURING
PROPULSION ENGINEERING

PhD in Aerospace
Academic Year: 2016 - 2019

Supervisor: Dr Yiguang Li
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CENTRE FOR PROPULSION ENGINEERING
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ABSTRACT

Transient performance design and assessment is a very crucial step of aircraft engine development, especially for acceleration and deceleration process. Normally, the assessment of transient performance stability would be done during the detained design stage while component design parameters are available. As a result, design iterations might be necessary and costly if the transient performance assessment is not satisfactory. To make engine design more cost and time efficiently, it has become more and more important to assess the transient performance stability at conceptual and preliminary design stage with the inclusion of key impact factors such as fuel control schedule, rotor dynamics, volume dynamics and heat soakage.

However, due to the lack of detailed engine structural and geometrical information at the initial design stage, such transient performance simulation and assessment may have to ignore heat soakage effects. Therefore, in this paper, a novel generically simplified heat soakage and tip clearance model for three major gas path components of gas turbine engines including compressors, turbines and combustors and has been developed to support more realistic transient performance simulation of gas turbine engines at conceptual and preliminary design stage. Such heat soakage model including heat transfer and tip clearance only requires thermodynamic design parameters as input, which is normally available during such design stages. This generic heat soakage method has been applied to two engine models to test its effectiveness through an in-house developed performance code. The case study of heat-soakage effects could demonstrate that results are promising and the simplified heat soakage model is satisfactory.

Keywords:

Transient performance, Heat Soakage, Heat Transfer, Tip Clearance, Modelling.

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TABLE OF CONTENTS

ABSTRACT	iii
ACKNOWLEDGEMENTS.....	iv
LIST OF FIGURES.....	i
LIST OF TABLES	vi
LIST OF EQUATIONS.....	vii
LIST OF ABBREVIATIONS	xvii
1 Introduction.....	1
1.1 Background.....	1
1.2 Aim and objectives.....	2
1.3 Contributions.....	2
1.4 Thesis Structures	2
2 Literature Review	4
2.1 Chapter Introduction	4
2.2 Gas Turbine Performance.....	4
2.2.1 Steady State Performance	4
2.2.2 Transient Performance.....	5
2.3 Development of Gas Turbine Performance Simulation	6
2.3.1 Overview	6
2.3.2 Typical Gas Turbine Performance Programs	7
2.4 Heat Soakage Phenomena.....	14
2.4.1 Transient Manoeuvre	14
2.4.2 Impacts of Heat Soakage Phenomena on Engine Performance	15
2.4.3 Thermodynamics Effects.....	15
2.4.4 Geometry Effects	16
2.4.5 Aerodynamics Effects.....	19
2.5 Existing Heat Soak Models	23
2.5.1 Heat Transfer	24
2.5.2 Tip Clearances	31
2.5.3 Compressor Transient Characteristics Map	35
2.6 Research Gap.....	37
3 Methodology.....	37
3.1 Introduction	37
3.2 Performance Model.....	41
3.2.1 Model Framework.....	41
3.2.2 Simulation Methods.....	42
3.2.3 Engine Components.....	44
3.2.4 Control System.....	65
3.2.5 Numerical approaches	66
3.3 Heat Soakage Model	69
3.3.1 Geometry	69

3.3.2 Heat Transfer	83
3.3.3 Tip Clearance	99
3.4 Models Integration	103
4 Results and Discussion	106
4.1 Engine Model	106
4.1.1 Single Spool Turbojet	107
4.1.2 Twin Spool Turbojet	108
4.2 Verification of Engine Performance Model	110
4.2.1 Verification of Steady State Performance	110
4.2.2 Verification of Transient Performance	119
4.2.3 Summary of Verification on Performance Model	128
4.3 Verification of Heat Soakage Model	129
4.3.1 Results of Geometry Model	129
4.3.2 Verification of Heat Transfer Model	129
4.3.3 Results of Tip Clearance Model	133
4.3.4 Summary of Verification on Heat Soakage Model	136
4.4 Case Study of Heat Soakage Effects	137
5 Conclusion	156
6 Future Work	159
6.1 Geometry Model Improvement	159
6.2 Turbine Heat Transfer Model Improvement	159
6.3 Radiative Heat Transfer Model Improvement	159
6.4 Choice of Heat Transfer Coefficients	160
6.5 Thermal Properties	160
6.6 Engine Performance Model Improvement	160
6.7 Control System Improvement	161
REFERENCES	162
APPENDICES	i

LIST OF FIGURES

Figure 2-1 GasTurb Structure (NATO, 2007)	8
Figure 2-2 GasTurb Transient Simulation (GasTurb 12)	8
Figure 2-3 Types of Engines in GasTurb (GasTurb 12)	9
Figure 2-4 GSP Window with an Turbofan Model (NATO, 2007)	9
Figure 2-5 GSP Standard Component Architecture (NATO, 2002)	10
Figure 2-6 GSP Engine System Model Architecture (NATO, 2002)	11
Figure 2-7 NLR TERTS Engine-Model with Sub-Levels (NATO, 2007).....	12
Figure 2-8 Overview of NPSS	13
Figure 2-9 Zooming principle (Reed & Afjeh 2000)	13
Figure 2-10 Typical Turbofan Temperature Distribution at Idle and Full Power	14
Figure 2-11 Typical Singe Stage Turbomachinery (Naylor, 2004).....	17
Figure 2-12 HPT tip clearance over a given mission profile	18
Figure 2-13 Tip clearance changes with acceleration slam (Naylor 2004)	18
Figure 2-14 Flow profile losses in blades (Zachos, 2016)	20
Figure 2-15 Flow profile losses contributors and weightings (Zachos, 2016) ...	21
Figure 2-16 Thermal effect on the boundary layers (Maccallum, 1977)	21
Figure 2-17 Thermal effects on compressor maps (Maccallum, 1977).....	22
Figure 2-18 Effects of tip clearance on compressor map (Kurzke, 1992).....	23
Figure 2-19 Representation of gas turbine component hardware (Naylor, 2004)	24
Figure 2-20 Stream one dimensional method (Naylor, 2004).....	26
Figure 2-21 Heat estimation using fixed temperature (Naylor, 2004)	27
Figure 2-22 Lumped capacitance method (Naylor, 2004)	27
Figure 2-23 Transient convective heat transfer (Naylor, 2004).....	28
Figure 2-24 Finite volumes conductive heat transfer (Naylor, 2004)	29
Figure 2-25 Tip clearance model integrated with engine model (Chapman, 2016)	31
Figure 2-26 Diagram of HPT model (Chapman, 2016).....	32
Figure 2-27 1D Heat transfer representation for shroud.....	33

Figure 3-1 Heat Soakage Model Flow Chart	40
Figure 3-2 TurboCycle Architecture.....	41
Figure 3-3 TurboCycle Interface.....	42
Figure 3-4 A Typical Compressor Map.....	43
Figure 3-5 Engine Components.....	45
Figure 3-6 Burner Efficiency (Kurzke, 2004).....	54
Figure 3-7 Engine PID Control System Work Flow Chart.....	65
Figure 3-8 A Single Spool Turbojet Engine (Kurzke, 2004).....	67
Figure 3-9 Engine compressor cutaway	70
Figure 3-10 Equivalent Engine Compressor Model	71
Figure 3-11 Compressor Annulus Diagram	72
Figure 3-12 Compressor Equivalent Annulus Diagram	75
Figure 3-13 Equivalent Engine Turbine Model	76
Figure 3-14 Annulus Combustion Chamber (Rolls-Royce, 2015).....	78
Figure 3-15 Annulus Combustor Equivalent Configuration.....	78
Figure 3-16 Annulus Combustor Simplified Configuration.....	79
Figure 3-17 Annulus Combustor Inlet Configuration (Priyant Mark & Selwyn, 2016)	79
Figure 3-18 Turbomachinery single averaged stage	84
Figure 3-19 Turbomachinery 0-D heat transfer model.....	84
Figure 3-20 Turbomachinery 1-D heat transfer model.....	85
Figure 3-21 Temperature and Pressure Profile of Compressor (Rolls-Royce, 2015)	86
Figure 3-22 Numerical Integration (intmath 2019)	89
Figure 3-23 Blade and Disc Geometry	90
Figure 3-24 Gas Temperature in Combustor (Priyant Mark & Selwyn, 2016)... 94	
Figure 3-25 Table of coefficients	103
Figure 3-26 Transient Performance with Heat Soakage Methodology Diagram	105
Figure 4-1 Single Spool Turbojet Engine Configuration	107
Figure 4-2 Twin Spool Turbojet Engine Configuration.....	108

Figure 4-3 Vertical and horizontal Part of Speed Line in GasTurb (Kurzke, 2004)	110
Figure 4-4 Comparison between TurboCycle and GasTurb in Terms of Steady State Work Lines of the Single Spool Turbojet	112
Figure 4-5 Comparison between TurboCycle and GasTurb Based on a Single Spool Turbojet in Terms of Air Flow and Fuel Flow	112
Figure 4-6 Comparison between TurboCycle and GasTurb Based on a Single Spool Turbojet in Terms of Compressor and Turbine Pressure Ratio Respectively	113
Figure 4-7 Comparison between TurboCycle and GasTurb Based on a Single Spool Turbojet in Terms of Thrust and Turbine Entry Temperature Respectively	113
Figure 4-8 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on Air Flow and Fuel Flow	115
Figure 4-9 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on LPC Exit Pressure and Temperature	115
Figure 4-10 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on HPC Exit Pressure and Temperature	115
Figure 4-11 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on Burner Exit Pressure and Temperature	116
Figure 4-12 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on HPT Inlet and Outlet Temperature and Exit Temperature	117
Figure 4-13 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on LPT Exit Temperature and Exit Pressure	118
Figure 4-14 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on Net Thrust	118
Figure 4-15 Fuel Schedule for the Single Spool Turbojet	119
Figure 4-16 Transient Working Lines of the Twin Spool Turbojet	120
Figure 4-17 Transient Fuel Flow Comparison between TurboCycle and GasTurb	121
Figure 4-18 Transient Results of Air Flow and Compressor Pressure Ratio	121
Figure 4-19 Transient Results of Turbine Entry Temperature and Pressure Ratio	122
Figure 4-20 Transient Results of Relative Corrected Speed and Net Thrust	122
Figure 4-21 Transient Results Comparison between IMF and ICV Method Based on the Single Spool Turbojet	123

Figure 4-22 Combustor Mass Flow Storage Comparison between IMF and ICV Method.....	123
Figure 4-23 Fuel Flow Schedule for the Twin Spool Turbojet.....	124
Figure 4-24 Comparison of Fuel Flow and Comparison of Air Flow	125
Figure 4-25 Comparison of Low Spool Relative Corrected Speed and Comparison of High Spool Relative Corrected Speed.....	125
Figure 4-26 Comparison of LPC Pressure Ratio and Comparison of HPC Pressure Ratio.....	126
Figure 4-27 Comparison of Combustor Exit Temperature and Comparison of Turbine Entry Temperature.....	126
Figure 4-28 Comparison of HPT Expansion Ratio and Comparison of LPT Expansion Ratio	127
Figure 4-29 Comparison of EGT and Comparison of Net Thrust.....	127
Figure 4-30 Variation of LPC Heat Transfer Coefficients	130
Figure 4-31 Comparison of LPC Exit Temperature Deviation Due to Heat Soakage between 0D Method and Numerical 1D Method	131
Figure 4-32 Variation of LPC Heat Transfer Coefficients	131
Figure 4-33 Variation of LPC Heat Transfer Coefficients	132
Figure 4-34 Variation of LPC Heat Transfer Coefficients	132
Figure 4-35 Heat Soak Effect of all Components	133
Figure 4-36 Compressor heat transient heat transfer.....	134
Figure 4-37 Deflection of HP Compressor Parts from another Acceleration Case	135
Figure 4-38 Compressor tip clearance	136
Figure 4-39 Individual Heat Soakage Effect on Thrust.....	137
Figure 4-40 Individual Heat Soak Impact on Thrust	139
Figure 4-41 Individual Heat Soak Effect on Air Flow	139
Figure 4-42 Individual Heat Soak Impact on Air Flow.....	140
Figure 4-43 Individual Heat Soak Effect on Compressor Pressure Ratio	141
Figure 4-44 Individual Heat Soak Impact on Compressor Pressure Ratio	142
Figure 4-45 Individual Heat Soak Effect on TET	143
Figure 4-46 Individual Heat Soak Impact on TET.....	144

Figure 4-47 Individual Heat Soak Effect on HPT Expansion Ratio	145
Figure 4-48 Individual Heat Soak Impact on HPT Expansion Ratio	145
Figure 4-49 Individual Heat Soak Effect on SFC	146
Figure 4-50 Individual Heat Soak Impact on SFC	147
Figure 4-51 Individual Heat Soak Impact on XNR	147
Figure 4-52 Individual Component Heat Soak Effect on XN1R	148
Figure 4-53 Individual Component Heat Soak Effect on Thrust	149
Figure 4-54 Individual Component Heat Soak Effect on Thrust	149
Figure 4-55 Individual Component Heat Soak Impact on Thrust.....	150
Figure 4-56 Individual Component Heat Soak Effect on Air Flow.....	150
Figure 4-57 Individual Component Heat Soak Impact on Air Flow	151
Figure 4-58 Individual Component Heat Soak Effect on TET	151
Figure 4-59 Individual Component Heat Soak Impact on TET	152
Figure 4-60 Individual Component Heat Soak Effect on EGT	153
Figure 4-61 Individual Component Heat Soak Impact on EGT.....	153
Figure 4-62 Individual Component Heat Soak Impact on LPC Pressure Ratio	154
Figure 4-63 Individual Component Heat Soak Impact on HPC Pressure Ratio	154
Figure 4-64 Individual Component Heat Soak Impact on HPT Pressure Ratio	155
Figure 4-65 Individual Component Heat Soak Impact on LPT Pressure Ratio	155

LIST OF TABLES

Table 3-1 Intake Brick Information.....	46
Table 3-2 Compressor Brick Information	49
Table 3-3 Premas Brick Information	51
Table 3-4 Burner Calculation Information	53
Table 3-5 Turbine Calculation Information	57
Table 3-6 Mixer Calculation Information.....	59
Table 3-7 Volume Calculation Information	61
Table 3-8 Nozzle Calculation Information.....	63
Table 3-9 Compressor Parameters from Design Point.....	71
Table 3-10 Turbomachinery Design Criterion of Compressor	71
Table 3-11 Turbine Parameters from Design Point	76
Table 3-12 Turbomachinery Design Criterion of Turbine.....	76
Table 3-13 Combustor Parameters from Design Point.....	79
Table 3-14 Pressure Loss in Combustor (Lefebvre & Ballal, 2010).....	80
Table 4-1 Primary Parameters of Single Spool Turbojet at Design Point Condition	108
Table 4-2 Key Parameters of Twin Spool Turbojet at Design Point Condition	109
Table 4-3 Map Scaling of Single Spool Turbojet at Design Point Condition ...	111
Table 4-4 Map Scaling of Twin Spool Turbojet at Design Point Condition	114
Table 4-5 Summary of Verification of TurboCycle	128
Table 4-6 Component Mass and Area estimated by Geometry Model.....	129

LIST OF EQUATIONS

(2-1)..... 16

(2-2)..... 25

(2-3)..... 25

(2-4)..... 25

(2-5)..... 25

(2-6)..... 25

(2-7)..... 28

(2-8)..... 28

(2-9)..... 28

(2-10)..... 28

(2-11)..... 28

(2-12)..... 29

(2-13)..... 29

(2-14)..... 29

(2-15)..... 29

(2-16)..... 30

(2-17)..... 30

(2-18)..... 30

(2-19)..... 30

(2-20)..... 30

(2-21)..... 30

(2-22)..... 32

(2-23)..... 32

(2-24)..... 33

(2-25)..... 33

(2-26)..... 33

(2-27)..... 33

(2-28)..... 33

(2-29).....	34
(2-30).....	34
(2-31).....	34
(2-32).....	34
(2-33).....	34
(2-34).....	34
(2-35).....	34
(2-36).....	34
(2-37).....	35
(2-38).....	35
(2-39).....	35
(2-40).....	35
(2-41).....	35
(2-42).....	35
(2-43).....	36
(2-44).....	36
(2-45).....	36
(2-46).....	36
(2-47).....	36
(3-1).....	43
(3-2).....	46
(3-3).....	46
(3-4).....	46
(3-5).....	46
(3-6).....	46
(3-7).....	47
(3-8).....	47
(3-9).....	47
(3-10).....	47

(3-11).....	47
(3-12).....	47
(3-13).....	47
(3-14).....	47
(3-15).....	49
(3-16).....	49
(3-17).....	49
(3-18).....	50
(3-19).....	50
(3-20).....	50
(3-21).....	51
(3-22).....	51
(3-23).....	52
(3-24).....	52
(3-25).....	52
(3-26).....	52
(3-27).....	52
(3-28).....	53
(3-29).....	54
(3-30).....	54
(3-31).....	54
(3-32).....	54
(3-33).....	55
(3-34).....	55
(3-35).....	55
(3-36).....	55
(3-37).....	55
(3-38).....	57
(3-39).....	57

(3-40).....	57
(3-41).....	57
(3-42).....	58
(3-43).....	58
(3-44).....	59
(3-45).....	59
(3-46).....	59
(3-47).....	59
(3-48).....	61
(3-49).....	61
(3-50).....	62
(3-51).....	62
(3-52).....	62
(3-53).....	63
(3-54).....	64
(3-55).....	64
(3-56).....	64
(3-57).....	64
(3-58).....	64
(3-59).....	64
(3-60).....	64
(3-61).....	65
(3-62).....	65
(3-63).....	65
(3-64).....	65
(3-65).....	66
(3-66).....	66
(3-67).....	66
(3-68).....	66

(3-69).....	66
(3-70).....	67
(3-71).....	67
(3-72).....	67
(3-73).....	68
(3-74).....	68
(3-75).....	68
(3-76).....	68
(3-77).....	68
(3-78).....	68
(3-79).....	69
(3-80).....	69
(3-81).....	69
(3-82).....	69
(3-83).....	72
(3-84).....	72
(3-85).....	72
(3-86).....	72
(3-87).....	72
(3-88).....	72
(3-89).....	72
(3-90).....	73
(3-91).....	73
(3-92).....	73
(3-93).....	73
(3-94).....	73
(3-95).....	73
(3-96).....	73
(3-97).....	73

(3-98).....	74
(3-99).....	74
(3-100).....	74
(3-101).....	74
(3-102).....	74
(3-103).....	74
(3-104).....	74
(3-105).....	74
(3-106).....	75
(3-107).....	75
(3-108).....	75
(3-109).....	75
(3-110).....	77
(3-111).....	77
(3-112).....	80
(3-113).....	80
(3-114).....	80
(3-115).....	80
(3-116).....	81
(3-117).....	81
(3-118).....	81
(3-119).....	81
(3-120).....	81
(3-121).....	81
(3-122).....	81
(3-123).....	81
(3-124).....	81
(3-125).....	81
(3-126).....	82

(3-127).....	82
(3-128).....	82
(3-129).....	82
(3-130).....	82
(3-131).....	82
(3-132).....	82
(3-133).....	86
(3-134).....	86
(3-135).....	87
(3-136).....	87
(3-137).....	87
(3-138).....	87
(3-139).....	87
(3-140).....	87
(3-141).....	87
(3-142).....	87
(3-143).....	87
(3-144).....	87
(3-145).....	87
(3-146).....	87
(3-147).....	88
(3-148).....	88
(3-149).....	88
(3-150).....	88
(3-151).....	88
(3-152).....	88
(3-153).....	89
(3-154).....	89
(3-155).....	89

(3-156).....	89
(3-157).....	89
(3-158).....	90
(3-159).....	90
(3-160).....	90
(3-161).....	90
(3-162).....	90
(3-163).....	91
(3-164).....	91
(3-165).....	91
(3-166).....	91
(3-167).....	91
(3-168).....	91
(3-169).....	91
(3-170).....	91
(3-171).....	92
(3-172).....	92
(3-173).....	92
(3-174).....	92
(3-175).....	92
(3-176).....	92
(3-177).....	92
(3-178).....	92
(3-179).....	92
(3-180).....	93
(3-181).....	93
(3-182).....	93
(3-183).....	93
(3-184).....	93

(3-185).....	93
(3-186).....	93
(3-187).....	94
(3-188).....	94
(3-189).....	94
(3-190).....	95
(3-191).....	95
(3-192).....	95
(3-193).....	95
(3-194).....	95
(3-195).....	95
(3-196).....	95
(3-197).....	95
(3-198).....	95
(3-199).....	96
(3-200).....	96
(3-201).....	96
(3-202).....	96
(3-203).....	96
(3-204).....	96
(3-205).....	96
(3-206).....	96
(3-207).....	96
(3-208).....	97
(3-209).....	97
(3-210).....	97
(3-211).....	97
(3-212).....	98
(3-213).....	98

(3-214).....	98
(3-215).....	98
(3-216).....	98
(3-217).....	98
(3-218).....	98
(3-219).....	98
(3-220).....	99
(3-221).....	99
(3-222).....	100
(3-223).....	100
(3-224).....	101
(3-225).....	101
(3-226).....	101
(3-227).....	101
(3-228).....	101
(3-229).....	101
(3-230).....	102
(2-231).....	102
(3-232).....	103
(3-233).....	103
(3-234).....	103
(3-235).....	103

LIST OF ABBREVIATIONS

Symbols

A	Heat Transfer Wetted Area(m^2)
AR	Aspect ratio of blade
Cp	Specific heat (kJ/kg. K)
CMF	Continuity mass flow
CW	Compressor power (W)
D	Diameter of interesting part (m)
DH	Enthalpy drop of turbine(J)
deltaH	Enthalpy drop of turbine per stage (J)
dt	Length of time interval (s)
$\bar{E}$	Error function Vectors
F	Ratio of heat into fluid to work out of fluid
GSP	Gas Turbine Simulation Programme
Gap	Baseline tip clearance
h	Heat transfer coefficient
HCF	Heat capacity of fluid
HCS	Heat capacity of solid
HPC	High pressure compressor
HPT	High pressure turbine
I	Moment of inertial ($kg.m^2$)
ICV	Inter-component volume
IMF	Imbalanced-mass flow
k	Heat conductivity ($Wm^{-1}K^{-1}$)
L	Length of component (m)
LHV	Fuel low heat value
LPC	Low pressure compressor
LPT	Low pressure turbine
Ma	Mach Number
M	Mass of component or part (kg)
MOPS	Modular performance synthesis program
N	Shaft speed (RPM)
n	Number of blades or stages

Nu	Nusselt Number
P	Total pressure (Pa)
PLA	Power level angle
PLDM	Piecewise linear dynamic modelling
Ploy	Ploy-tropic index of compression or expansion
PR	Pressure Ratio
PRC	Pressure Recovery Coefficient
Pr	Prandtl Number
Q	Heat flux to fluid (W)
Qa	Mass flow function
q	Quantity of heat absorbed/released by the metal
qc	Convective heat energy (J)
qr	Radiative heat energy (J)
R	Radius of interesting part (m)
r	Radius of interesting part (m)
Re	Reynolds number
RG	Universal gas constant ($\text{Jmol}^{-1}\text{K}^{-1}$)
Rho	Density of component(kg/ m^3)
SM	Surge Margin
SR	Space chord ratio of blade
T	Total Temperature(K)
Tau	Thermal time constants of the metal part
TC	Tip Clearance
TERTS	Turbine engine real time simulator
TW	Turbine Power (W)
U	Total conductance (W/K)
u	Deformation of component part (m)
V	Volume of Component (m^3)
Va	Flight Speed (m/s)
W	Mass flow rate of flow (kg/s)

Greek letters

α	Thermal expansion coefficient
σ	Stefan-Boltzmann constant
ε	Radiative emissivity
γ	Specific Heat
τ	Thermal time constant
δ	Total deformation of component part
η	Film cooling effectiveness

Subscripts

a	Air
ad	Adiabatic
avg	Average
B	Burner
b	Blade
bs	Blade surface area
bd	Blade and disc interference part
bds	Blade and disc interference area
c	Casing
cl	Clearance
cs	Casing surface area
D	Diffusor
d	Disc
ds	Disc area
f	Fluid
fb	Fore-body
g	gas
h	Hub
ht	Heat transfer
in	Inlet
m	Mean
n	Neighbor
Noz	Nozzle
out	Outlet
p	Primary zone

rb	Rear Body
s	Solid
ss	Steady state
t	Tip
tr	Transient
w	Wall
WF	Fuel Flow
0	Beginning of time interval
1	Station of inlet
2	Station of Outlet

1 Introduction

1.1 Background

Nowadays, engine development becomes more and more expansive and also takes a very long period, even decades. As the requirement for engine performance is always increasing, the design of such a high-performance engine becomes more complicated and difficult. Consequently, if the engine performance is not satisfactory, which is likely to be at the beginning, the design iterations may be required and would also be costly. If the engine performance could be assessed at the preliminary design and conceptual stage, it probably would help the engineer to design the engine more precisely with less number of design and test iterations.

On the other hand, transient performance is the critical part of engine performance, because it would probably lead an engine performance assessment unsatisfactory even if its steady-state performance is satisfactory.

To make engine design more cost and time effective, it has become more and more essential to assess the transient performance stability at the conceptual and preliminary design stage. The assessment should also include key impact factors such as fuel control schedule, rotor dynamics, volume dynamics and heat soakage.

Volume packing and rotor dynamics are relatively more comfortable to be solved by employing the volume dynamics and shaft inertial momentum. However, the simulation of heat soakage effects becomes more difficult as the calculation of heat transfer and tip clearance needs a lot of trivia component design details, which are not available when it is at the preliminary design stage.

Therefore, in this paper, the research area would be focused on the development of the novel heat soakage model. The first step is to rebuild such a performance model which could predict both steady-state and transient performance by consideration of volume dynamics, rotor dynamics and corresponding control system.

The second step is to develop a generically simplified heat soakage model for three major gas path components of gas turbine engines including compressors, turbines and combustors to support transient performance simulation of gas turbine engines. Such heat soakage model only requires thermodynamic design parameters as input, which are commonly available during conceptual and preliminary design stages of the engine.

1.2 Aim and objectives

The aim of this paper is to develop and rebuild two models to support the transient performance simulation with heat soakage effects, which are the performance model and the simplified heat soakage model. To effectively achieve this goal, several objectives are stated as below.

1. Rebuilding of the performance model (TurboCycle)
2. Development of the simplified component geometry model
3. Development of the numerical 1D heat transfer model considering heat convection, heat conduction and heat radiation
4. Development of the simplified tip clearance model
5. Verification of a performance model
6. Integration of the heat soakage model with performance model
7. Verification of heat soakage model

1.3 Contributions

The potential contributions to this paper is generically simplified heat soakage model. It could help to assess the heat soakage effects by only requiring the design point performance parameters as input when the component design details are not available, particular at the conceptual and preliminary design stage of an engine.

1.4 Thesis Structures

The thesis is structured into six different chapters. The first chapter presents the general background, aim and objectives, potential contributions to the dissertation.

Chapter 2: Literature Review

In this chapter, the previous works on engine performance simulation method and software are reviewed. The earlier researches on heat soakage simulation are briefly introduced. In addition, existing methods of heat soakage, including heat transfer and tip clearance, are presented and studied. At the same time, the different ways of heat transfer were compared and discussed.

Chapter 3: Methodology

This chapter describes the construction of the entire simulation platform. The development of performance model and heat soakage model are explained step by step in details. In the end, the integration of all models has been demonstrated.

Chapter 4: Results and discussion

In this chapter, the verification results of the performance model and heat soakage are presented and discussed. Besides, the case study of heat soakage effects has also been carried out based on two engine models.

Chapter 5: Conclusion

The chapter summarises the current crucial finding, outcomes and also gives the evaluation of accomplishments.

Chapter 6: Future Work

The last chapter gives some recommendations on the improvement of the current model, which might be subjected to the prospective works.

2 Literature Review

2.1 Chapter Introduction

The objective of this literature review is to collect and present the gas turbine performance and heat soakage information available in open data resource related to this paper.

Firstly, it briefly introduces traditional widely used methods on gas turbine performance simulation. The second section generally presents the typical examples of gas turbine performance code in public resource. The third part of this chapter focuses on typical phenomena which are related to gas turbine transient performance and heat soakage. Finally, it describes that the existing methods for heat soakage simulation, including heat transfer and tip clearance.

2.2 Gas Turbine Performance

2.2.1 Steady State Performance

The fundamental principles of the gas turbine theory and cycle calculation method could be easily found through the open resource. Steady state performance comprises the design point performance and off design performance.

Design point performance is the first step to define a new engine including engine configuration, cycle parameters, component performance levels, and component sizes selected to meet the requirements from the customers or a given specification (Walsh & Fletcher 2004).

Off design performance is varied steady state performance of a fixed engine at different operation conditions. It only could be carried out when design point performance has been defined.

The calculations of off design performance are determined by satisfying the fundamental condition of the equilibrium of mass flow, shaft power and physical rotational speed among different components, which are usually referred to as matching procedures (Saravanamutto & Cohen 2008).

2.2.2 Transient Performance

Transient performance is often referred to as handling a gas turbine engine from one steady state to another, generally above idle. It deals with the working regime where engine performance is varying with time (Walsh & Fletcher 2004).

The transient behaviour usually would be critical, especially for the application of aircraft engine. For example, one of the key performance indicators of civil aircraft is for a fast thrust response to provide the required thrust in case of emergency, when an aircraft has to go round even if it is very close to a touchdown. According to the FAA regulations of airworthiness, the civil engine must have the capability to meet the requirement of certification, which is to accelerate from fight idle to 95% of maximum thrust within 5 seconds (Saravanamuttoo & Cohen 2008).

In terms of calculations for transient performance, there are two typical widely used approaches. The first one is the continuity of mass flow (CMF) method, and it assumes the mass flow entering component is always equal to that leaving the component. Therefore, it is more suitable for slow transient, which is quite similar to the off design performance iterative matching pattern. Nevertheless, only the mass flow and rotational speed are matched in this case. The unmatched powers are used to calculate the shaft accelerations and decelerations (Pilidis & Palmer, 2006). It was first applied with the analogue computer by Larrowe et al. (1957). The limitation for this method is that it could simulate the behaviour of the volume packing when an engine is handled with a rapid transient.

The other one is called inter-component volumes (ICV) method, which could address the limitation of CMF method mentioned previously. It was firstly introduced by Fawke and Saravanamuttoo (1971).

The ICV method is direct approach instead of an iterative way, but it requires a smaller time interval to keep the computational instability. The difference in mass flow between inlet and outlet at each time interval is used to calculate the temperatures and pressures at inlet and outlet of components. As the ICV method considers the volume packing, it is more suitable for fast transient (Naylor, 2004).

2.3 Development of Gas Turbine Performance Simulation

2.3.1 Overview

In this section, the past gas turbine performance programs and codes would be briefly reviewed according to the survey concluded by Sanghi et al. (2000) and Naylor (2004). It is mentioned that recent advancements for engine performance simulation are substantially dealing with computational platforms, implementation techniques and programming to speed up the execution time with friendly graphic interfaces. Up to the present, there is no radical change in mathematical modelling and representation of components (Sanghi et al. 2000).

The initial analytical and experimental researches on the transient behaviour of gas turbine are recorded at the early 1950s. Otto and Taylor (1950) were likely to be the first to demonstrate that the speed response speed of a single spool turbojet could be approximated by a first-order system (Sanghi et al. 2000).

Exploration is always driven by the pioneer's inspiration and passion. The linear model was replaced by a more flexible component based on analogue and digital simulation techniques by Fawke and Saravanamuttoo (1971). The digital simulation has become worldwide and adopted to develop substantially all of the performance programs today (Naylor 2004).

TURBOCODE was developed by Palmer (1968) at Cranfield University, which was further developed to be TURBOMATCH later on. Then a transient performance code, TURBOTRANS was developed by Palmer and Yan (1982).

A transient performance code named GTTrans was developed by Pilidis (1983) at the University of Glasgow, which could estimate tip clearance variation and seal changes through a thermomechanical module inside. It was developed to integrate heat soakage research works achieved by Maccullum (1970-1979).

In the early years, NASA has developed a series of performance codes such as DYNGEN (Seller & Daniele, 1975), GETRAN (Schobeiri et al., 1994). GETRAN is capable of simulating arbitrary engine transient performance even with variable

geometry and air cooling system. Beyond that, the heat transfer is also included in the code.

In recent years, NASA also has further developed a brand new numerical propulsion system simulation (NPSS). The breakthrough is that it is capable of carrying out multidisciplinary analysis on system and subsystem to at the various level of details.

2.3.2 Typical Gas Turbine Performance Programs

In this chapter, several commercial gas turbine performance programs would be introduced, which are now widely used by industry and academia.

2.3.2.1 GasTurb

GasTurb is an extensive widely used commercial gas turbine performance program developed by Kurzke (1996). Kurzke worked for MTU Aero Engine Company as a performance expert and until his retirement in 2004. He began to create this program since 1991 as his parallel job. Before the development of GasTurb, the performance programs used by industry could not provide such as a friendly interface to deal with complicated component matching issues, and only the experienced engineers could handle those programs. The friendly oriented task interface and the high quality of the visual contour output make it much easier to apply.

Figure 2-1 shows the structure of GasTurb. It is capable of simulation steady state and transient performance. Before simulating transient performance, the off design performance simulation must be done. Moreover, the control schedules also have to be specified in advance, which could be defined as fuel flow, power lever angle (PLA), rotational speed and others customised by users. The transient performance simulation window is shown in figure 2-2.

The 15 different types of engine are presented in figure 2-3. The engine library almost covers mainstream engines today even including the variable geometry engine. It is really convenient for the user to call them directly without any modification. Nevertheless, this kind engine library would also bring a specific

limitation, which means the engine configuration could not be customised if it is necessary to do so.

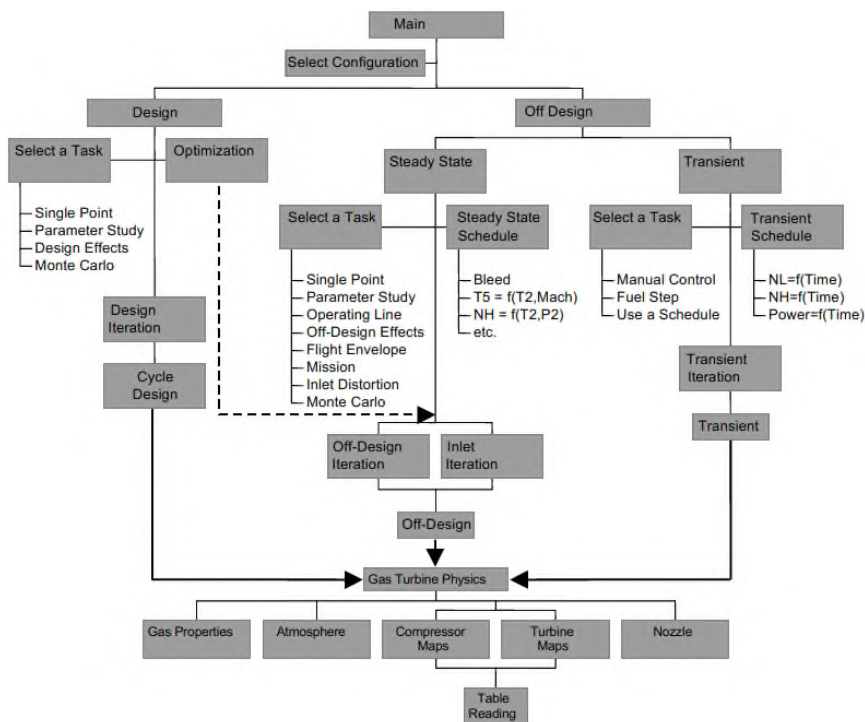


Figure 2-1 GasTurb Structure (NATO, 2007)

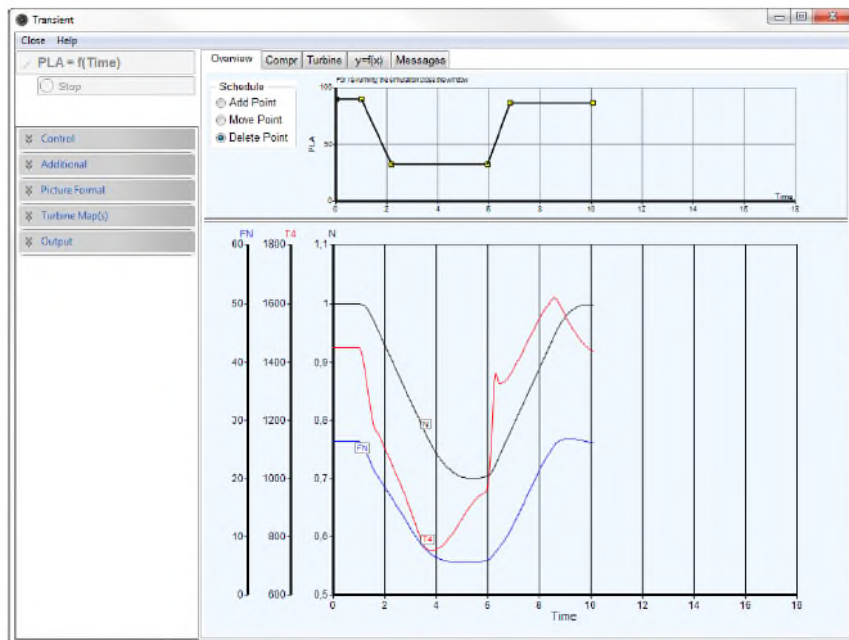


Figure 2-2 GasTurb Transient Simulation (GasTurb 12)

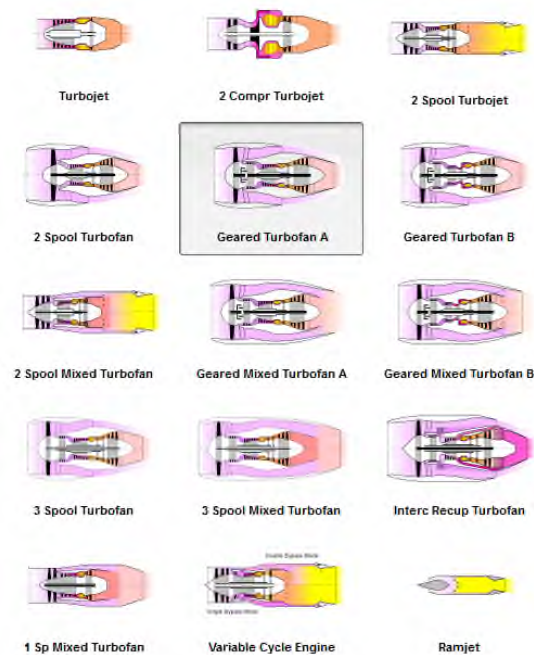


Figure 2-3 Types of Engines in GasTurb (GasTurb 12)

2.3.2.2 Gas Turbine Simulation Program (GSP)

The Gas Turbine Simulation Program (GSP) is an object-orientated component-based modelling environment for performing both steady state and transient performance, which was developed by Visser & Broomhead (2000) at NLR. The object-orientation with user-friendly interface offers many advantages which would help engineers to deal with many difficulties in reality. Figure 2-4 shows GSP window with an example of a turbofan model.

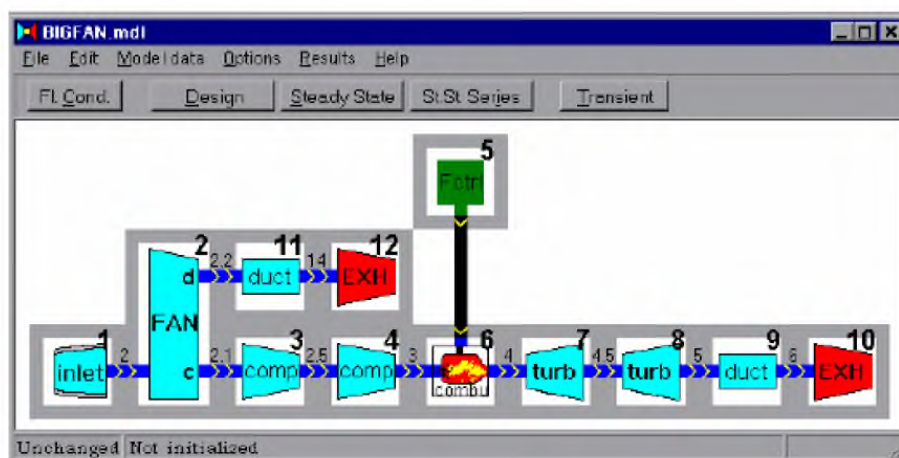


Figure 2-4 GSP Window with an Turbofan Model (NATO, 2007)

Unlike GasTurb, the GSP consists of a lot of component modules or subroutines which allow the user to call the individual component and make some changes in case of need. That gives much more flexibility, and the programme architecture is more advanced. The figure 2-5 presents the GSP standard component architecture.

Additionally, a separate mechanism is required to link all components selected into a whole engine model. It employs the installation and connection of the arbitrary dynamic of elements to define any gas turbine engine configuration. The engine system architecture of GSP is given in figure 2-6.

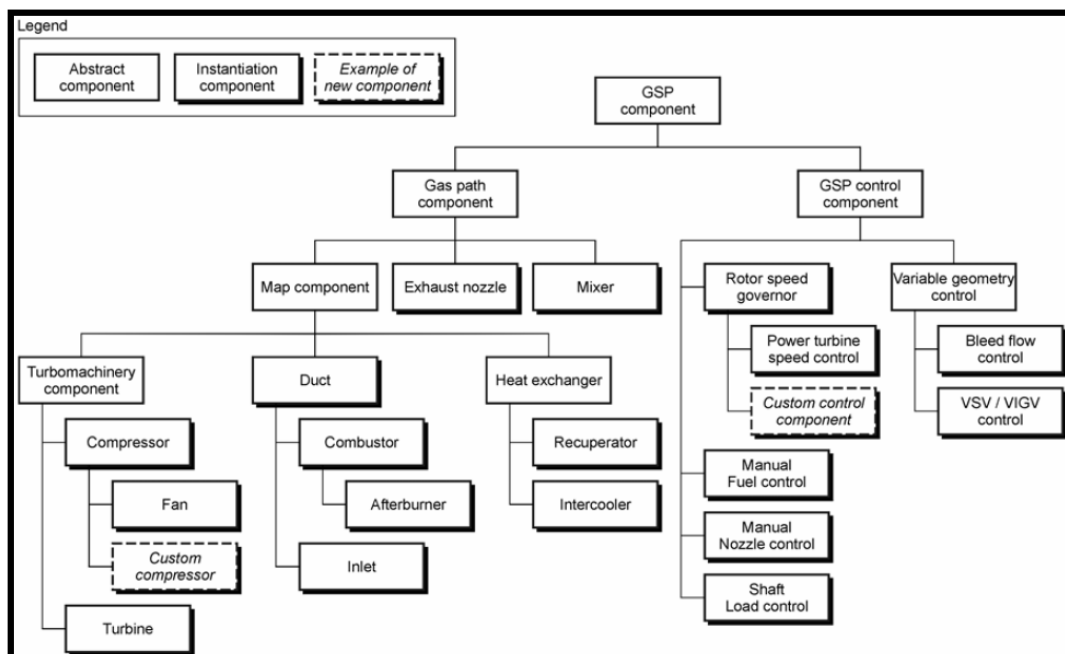


Figure 2-5 GSP Standard Component Architecture (NATO, 2002)

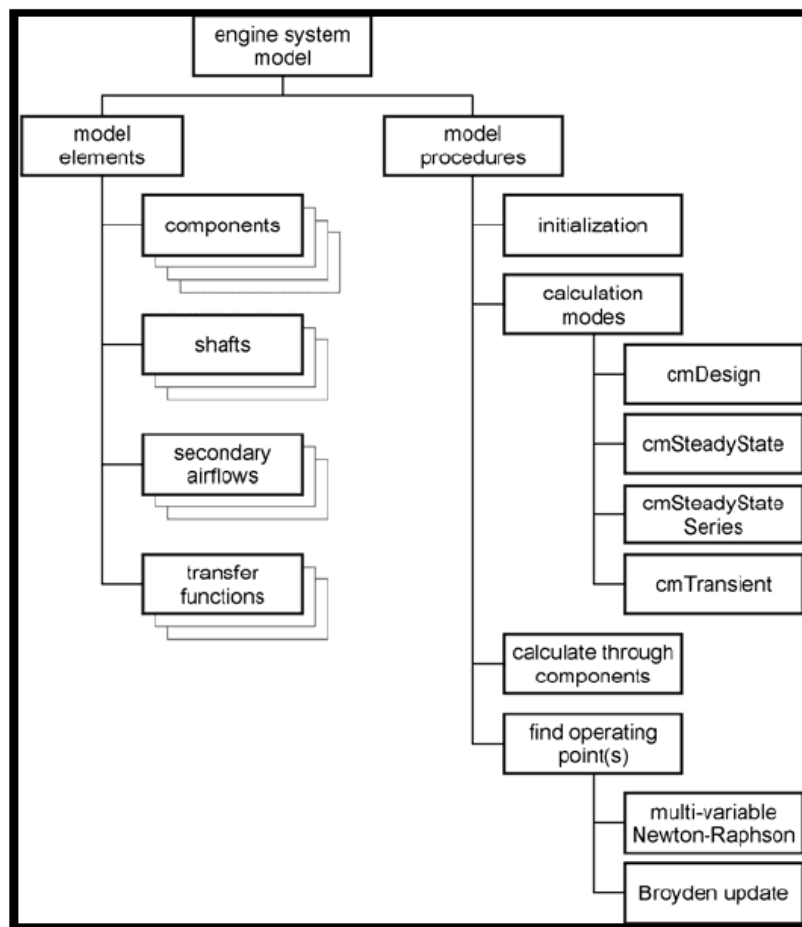


Figure 2-6 GSP Engine System Model Architecture (NATO, 2002)

2.3.2.3 Turbine Engine Real Time Simulator (TERTS)

The TERTS is a real-time gas turbine performance program developed based on MATLAB/Simulink environment by Visser et al. (2002). The Majority of performance programs like GasTurb and GSP can only be applied in the case of offline or ground simulation. They probably might face a significant challenge if real-time performance of the aircraft engine on-wing needs to be simulated immediately.

Compared with two programs introduced previously, it applies the one iteration per time step in order to fulfil the requirement of limited computation time but offering good accuracy. Therefore, it can compute the engine performance instantaneously, which have great advantages on applications for engine monitoring, diagnostics and airworthiness certification.

As TERTS uses Simulink, it can involve subsystems in its libraries. Therefore, it could allow the user to add different components in case of a need. The figure 2-7 shows the NLR TERTS Thermodynamic Engine-Model with Sub-Levels.

The ordinary used real-time computational approach is named Piecewise Linear Dynamic Modelling (PLDM) method (Wang, 2013). Visser et al. (2000) mentioned that the pre-calculated Inverse Jacobian Matrix is employed to reduce running time for new real-time simulation.

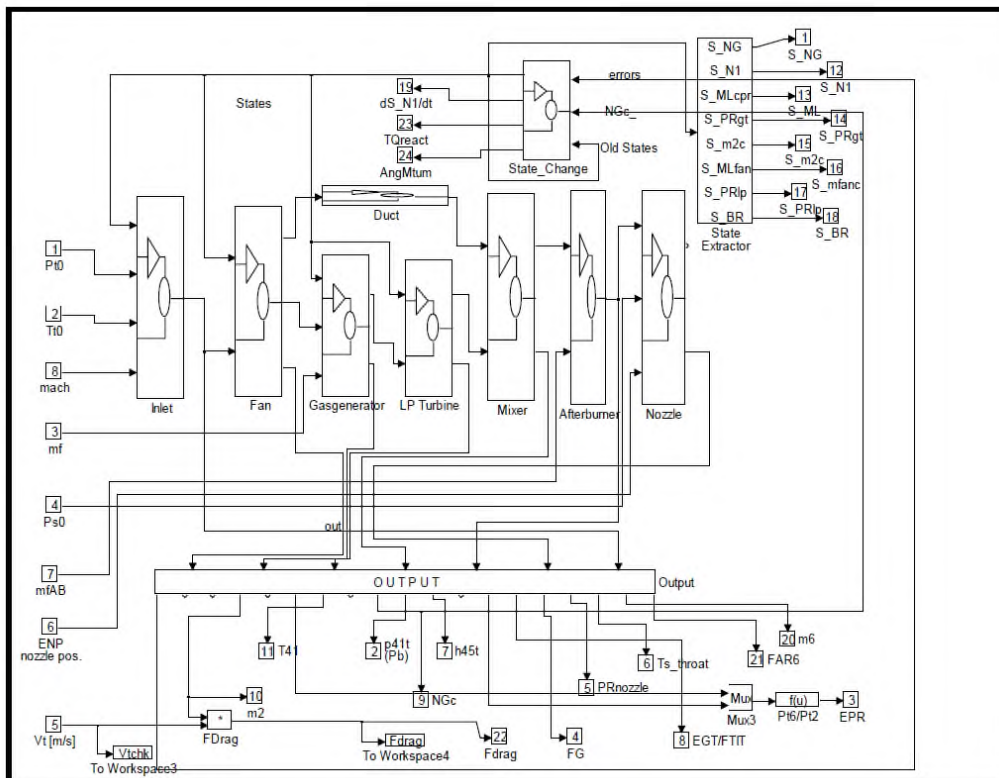


Figure 2-7 NLR TERTS Engine-Model with Sub-Levels (NATO, 2007)

2.3.2.4 Numerical Propulsion System Simulation (NPSS)

Numerical Propulsion System Simulation is developed by NASA Glen Research Centre involving aerospace industry and academia. The great progress of NPSS is by integrating multiple disciplines such as aerodynamics, thermodynamics, heat transfer and structures along with expanding 0-D to 1-D, 2-D and 3-D component models. For 0-D simulation, it is like another performance program by using 0-D component maps to meet the matching patterns. The 1-D simulation considers the gas properties along with engine axial direction. 2-D and 3-D

analysing using unique zooming concept by employment of Computational Fluid Dynamics (CFD) and Finite Elements Analysis (FEA) approaches. The overview of NPSS and its zooming principle are presented in figure 2-8 and figure 2-9, respectively.

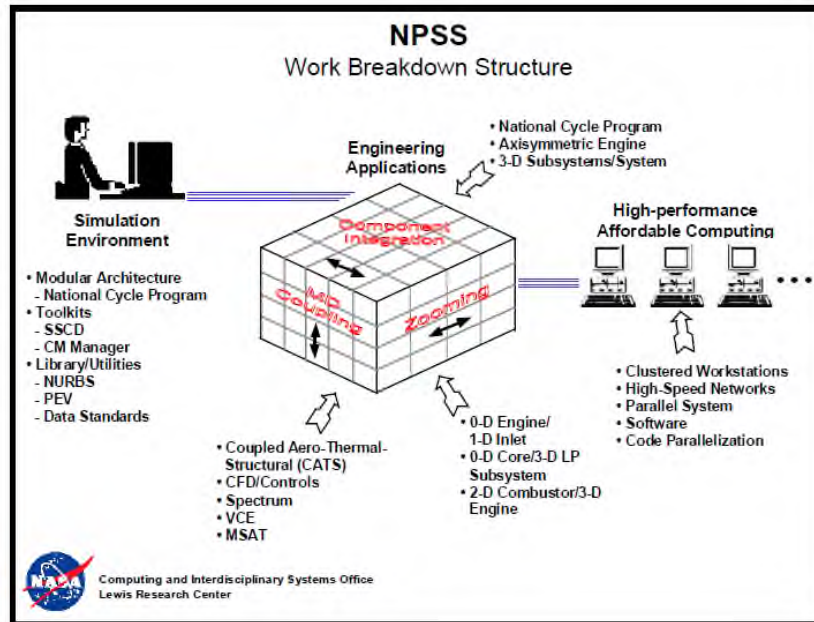


Figure 2-8 Overview of NPSS

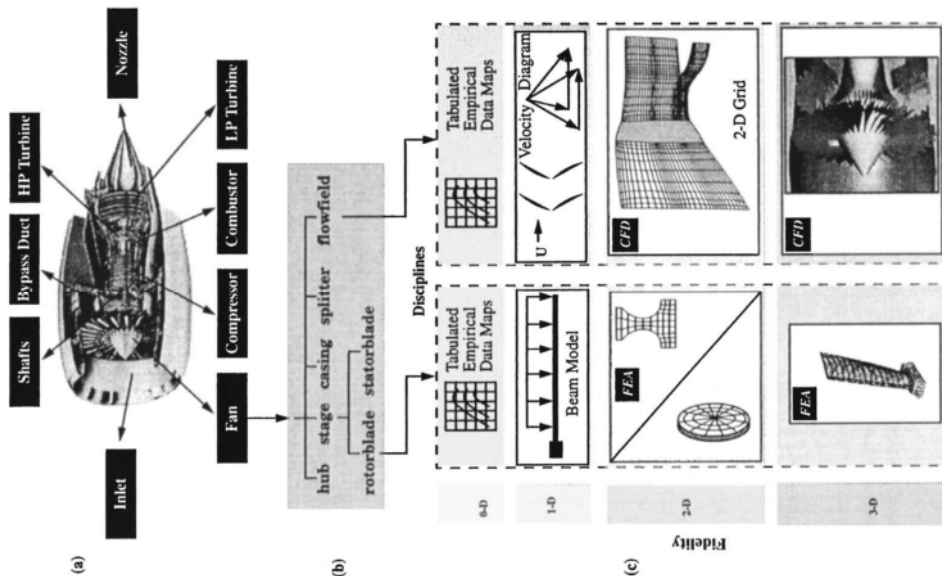


Figure 2-9 Zooming principle (Reed & Afjeh 2000)

2.4 Heat Soakage Phenomena

In this section, it begins with the cause of heat soakage and follows an explanation of heat soakage impact on engine transient performance.

Finally, the heat soakage effects would be discussed in terms of aerodynamics, thermodynamics and engine geometry.

2.4.1 Transient Manoeuvre

The figure 2-10 illustrates the difference of temperature profiles in engine stations at conditions of idle and full power for a twin-spool high bypass turbofan engine.

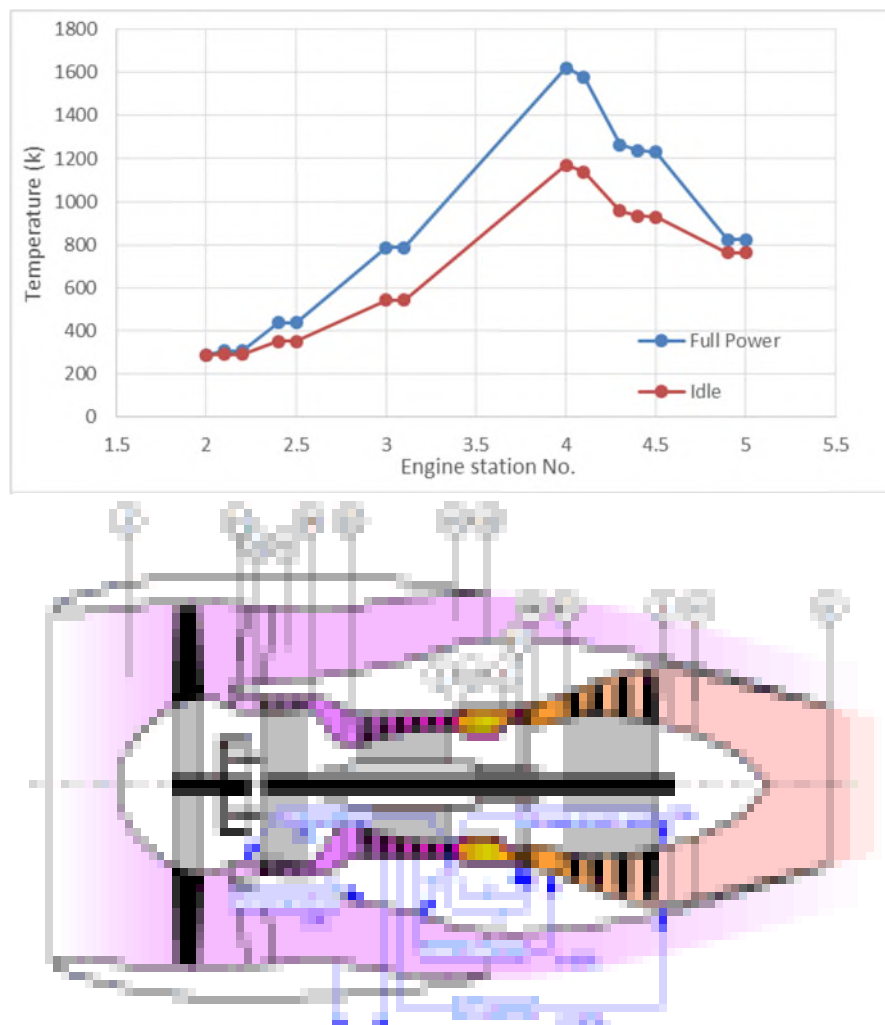


Figure 2-10 Typical Turbofan Temperature Distribution at Idle and Full Power

The substantial temperature differences between the primary gas path and engine metals would lead to heat soakage phenomena which have notable impacts on engine performance during and after a transient manoeuvre.

2.4.2 Impacts of Heat Soakage Phenomena on Engine Performance

In the past, heat soakage phenomena impacts on engine performance have already been studied by a lot of researches. According to the public resource, it is reported that the time taken for an engine to operate transient manoeuvring without considering heat soakage may be 20 to 30 percent shorter than that in real condition (Bauerfeind 1968 & Thomson 1975). Maccallum (1970) reported the engine thrust was lost 1.7 percent when completing engine acceleration, and it took more than 60s to reach its equilibrium thrust. Maccallum (2000) described that compressor surge is more likely to occur when slam deceleration to idle is followed immediately by re-slam to high power which is so-called hot re-slam or Bodie. Those effects are the results of heat soakage phenomena and their consequences.

Additionally, the impacts of heat soakage on engine performance also is up to size and type of engine as it becomes lighter and more powerful by applying many new technologies like air cooling system, coatings and ACC controls. Moreover, heat soakage will be less observable if the engine operates a slow transient and vice versa (Naylor 2004). Pilidis (1983) reported the heat soakage is not so evident at altitude because it would take longer when acceleration is completed. As a consequence, the heat is transferred is more equally over the period of time.

2.4.3 Thermodynamics Effects

Changing the temperature profile of the primary gas path flow will alter the engine thermodynamics. During the steady state performance calculations, the processes of compression and expansion are assumed as adiabatically. That is reasonable because the heat energy lost is quite tiny compared with the power generated by the engine component. However, during the transient, this effect cannot be ignored any longer because the heat would be transferred between engine metals and working fluid, which is relatively much bigger. This is because

the temperature of gas would be increasing greatly due to the over fuelling, which can produce extremely high level of heat transfer flux. Therefore, the F parameter is defined as the heat transferred between fluid and component divided by the work done by fluid or component. According to Pilidis and Palmer (2006), it shows the maximum value of F can be up to 0.4 during the transient manoeuvre. The effect of the F parameter is given in the equation 2-1.

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{(1-F)/(P \log \left(\frac{\gamma-1}{\gamma}\right))} \quad (2-1)$$

From this equation, the F will alter the component pressure ratio for a specific temperature ratio. Then engine cycles will be changed consequently.

2.4.4 Geometry Effects

The engine component geometry changes due to the mechanical and thermal effects during the transient manoeuvres. The heated metals will expand and cooled parts will contract. In addition, the centrifugal stress or the different pressures acting on metals also makes them deflection. Therefore, the primary flow path area changes, as a consequence, the tip clearance and seal clearance alter as well.

The rate of heat transfer depends on the heat capacity of metals, the temperature difference between the metals and fluids, and heat transfer coefficients.

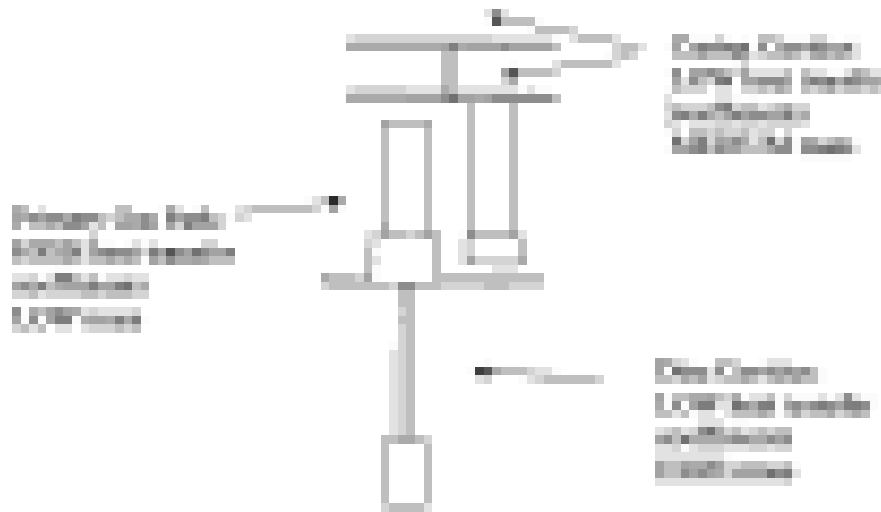


Figure 2-11 Typical Single Stage Turbomachinery (Naylor, 2004)

Figure 2-11 shows a schematic diagram of a single stage of turbomachinery, which could represent compressor and turbine. In the primary flow path, as results of mass flow, the smaller contacting surface area and lower metal mass, the thermal response for rotor blades and vanes are relatively short. In addition, it is rotating part and the heat transfer is forced, therefore, its heat transfer coefficient is high. For the casing part, as the thin thickness, the heat transfer coefficients are quite small. Therefore, its thermal response is generally longer than that of blades.

In some cases, when thermal barrier coatings are used, the heat conduction decides the rate of heat changing rather heat convection. The mass flow in disc cavities is the roughly same amount of that in casing cavities, but its rotation speed leads to a higher heat transfer coefficient. As the disc mass is relative high, its thermal response is longer than that of casing generally.

Figure 2-12 shows conventional engine HPT tip clearance profiles during a flight envelope. During the cruise phase, the active tip clearance control (ACC) function is presented. This is a tip clearance manage system which helps to avoid the tip clearance too close or too large for engine safety and economic operating.

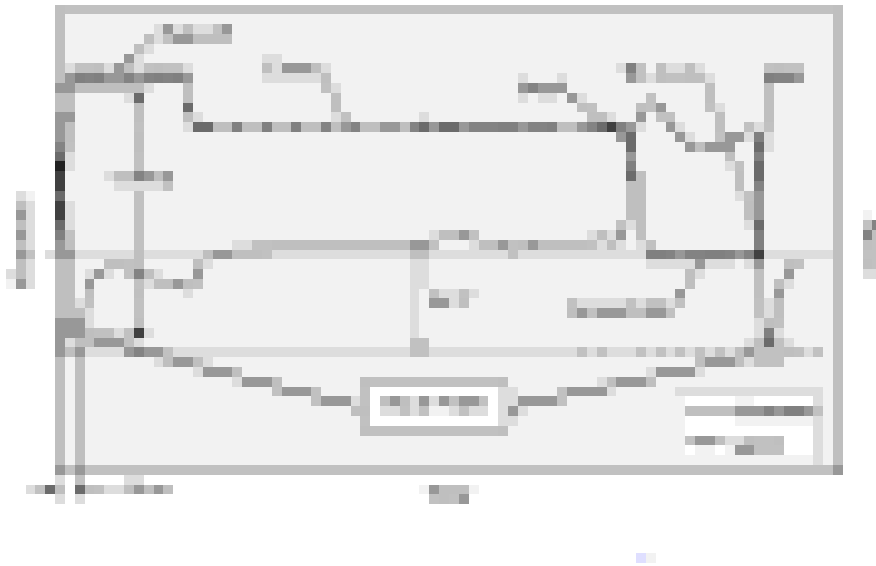


Figure 2-12 HPT tip clearance over a given mission profile

The figure 2-13 describes how the tip clearance may be expected to vary following a slam acceleration.

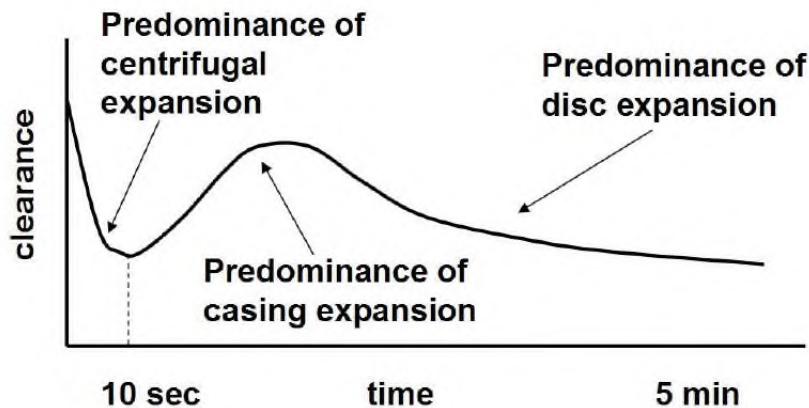


Figure 2-13 Tip clearance changes with acceleration slam (Naylor 2004)

In the early phase, due to the shaft speed accelerates rapidly, the centrifugal force makes the blade expansion so that rapid reduction in clearance. After completing the transient manoeuvre, the rotor speed keeps stable, and the casing expansion rate is faster than that of the blade, therefore the tip clearance increases. As casing mass is relatively smaller than disc, the time taken for the casing to complete thermal response is shorter than that of the disc. As a result, the disc continues expansion make tip clearance down again at a later stage.

A lot of studies and researches have shown that the engine loses the performance (thrust or SFC) during the transient manoeuvres due to clearance changes.

2.4.5 Aerodynamics Effects

As the airflow is sucked through the engine intake and then squashed between component stages with different velocity profiles and temperature profiles, therefore, the flow profiles inside the component are very complex, which are laminar flow, flow separation, turbulence, boundary layer, and shockwaves. Those factors have a significant impact on component performance.

According to the public resource, there are descriptions of following flow profiles in a compressor are given below by Foley (1995):

Passage vortices: the slower moving flow around the blade hub turns with bigger angle than that of bulk of flow forming the exit section pressure profile, which results in passage vortices.

Rotor blade boundary layers: the blade boundary layers grow along with the chord length from leading edge to trailing edge. The thinner boundary layer, the less profile loss. Turbulences form from the transition area, which will destroy the boundary layers and make more losses.

Annulus wall boundary layers: huge pressure gradient, tip leakage flows, and other rapid variations in end wall flows make the convectional boundary layer not valid any longer in end wall region. New kind of boundary layer needs to be represented.

Tip leakage flow: believed to appear due to the pressure difference pressure along with blade tip and unturned flow in the end wall boundary layer. This is a recognised most significant contributor to lowering efficiency and surge margin.

Shock waves: shock waves appear at high-speed zone, and these effects become more and more crucial for modern turbomachinery designs as they are more likely to be designed with higher rotational speed. It is also one of the largest contribution to component performance.

Unsteady effects: engine vibration causes the wake chopping.

The flow profile in turbine is roughly the same with that in compressor but with slightly more complicated due to the blade filming cooling and thermal barrier coating effects.

Figure 2-14 shows the visualised flow profiles losses in blades, including the main contributors mentioned before.

Figure 2-15 presents the main weightings of contributors on the impact of losses of flow profile in turbomachinery.

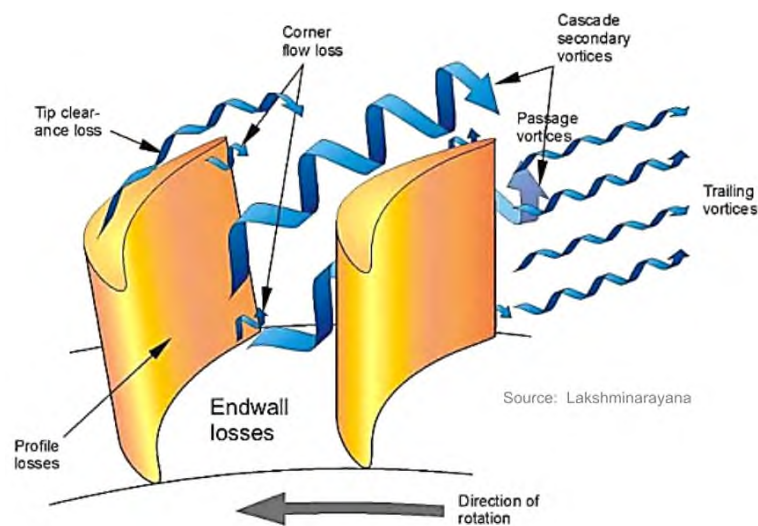
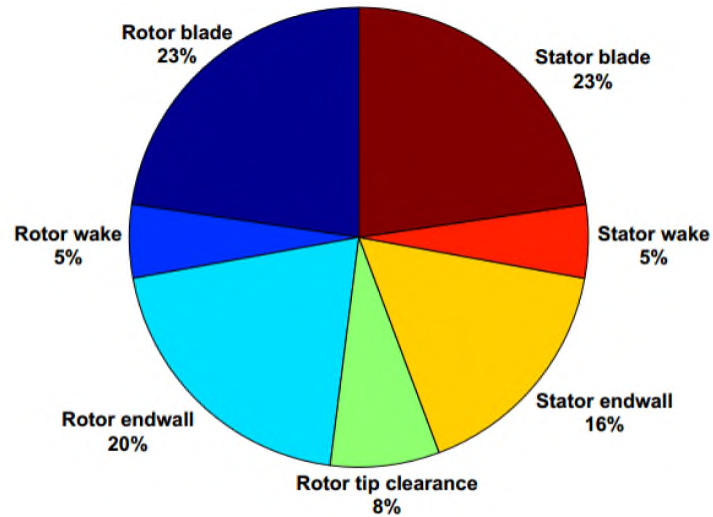


Figure 2-14 Flow profile losses in blades (Zachos, 2016)



Source: David K. Hall, MIT

Figure 2-15 Flow profile losses contributors and weightings (Zachos, 2016)

Maccallum (1979) developed an approach to predict changes to compressor map during the transients as a result of heat soak phenomena by heating and cooling the blade cascade boundary layers, which affects the density and viscosity of fluid in terms of the boundary layer. It is reported that the boundary layer grows thicker on the suction side than on the pressure side for heated compressor blade. Figure 2-16 shows the heat transfer effect on boundary development.

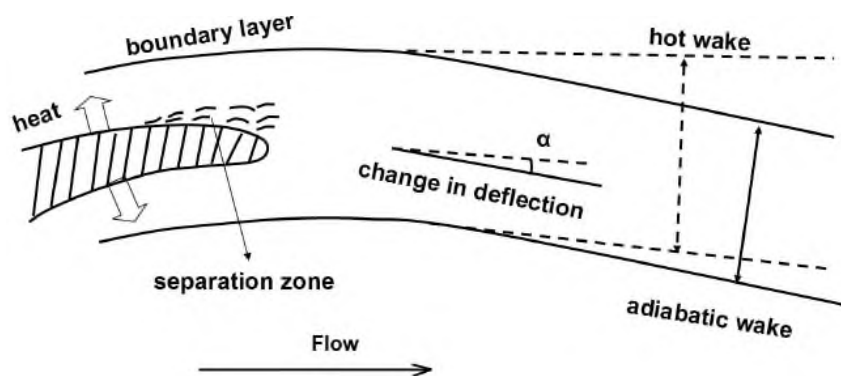


Figure 2-16 Thermal effect on the boundary layers (Maccallum, 1977)

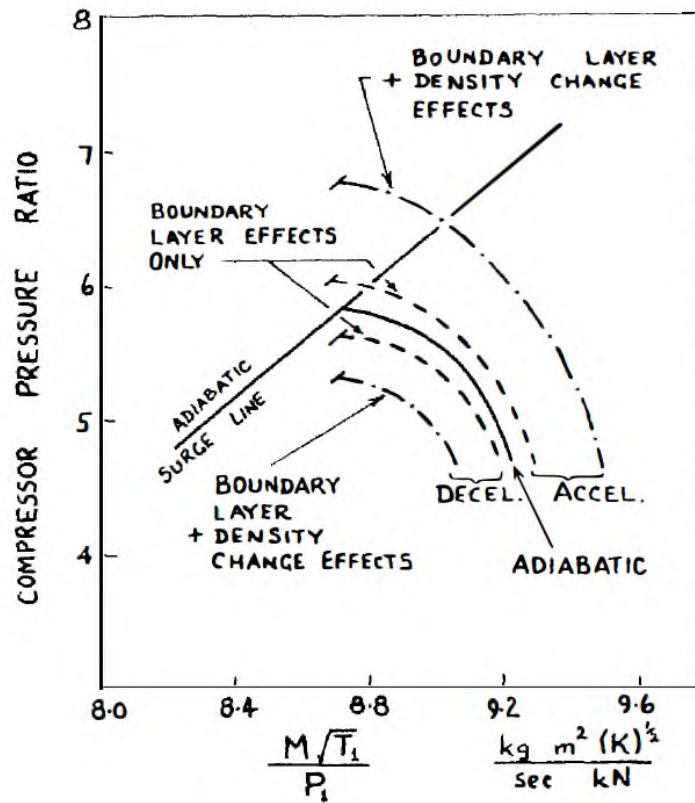


Figure 2-17 Thermal effects on compressor maps (Maccallum, 1977)

According to figure 2-17, the transient changes to the compressor characteristics map is shown. Comparing to the adiabatic condition, accelerating engine moves the engine speed lines upwards and decelerating moves the speed line downwards. In addition, the surge line also shifts.

Surge margin is very sensitive to tip clearance variations according to publications of Maccallum & Pilidis (1986). Kurzke (1992) demonstrated the effects of the tip clearance variation to the compressor maps shown in figure 2-18.

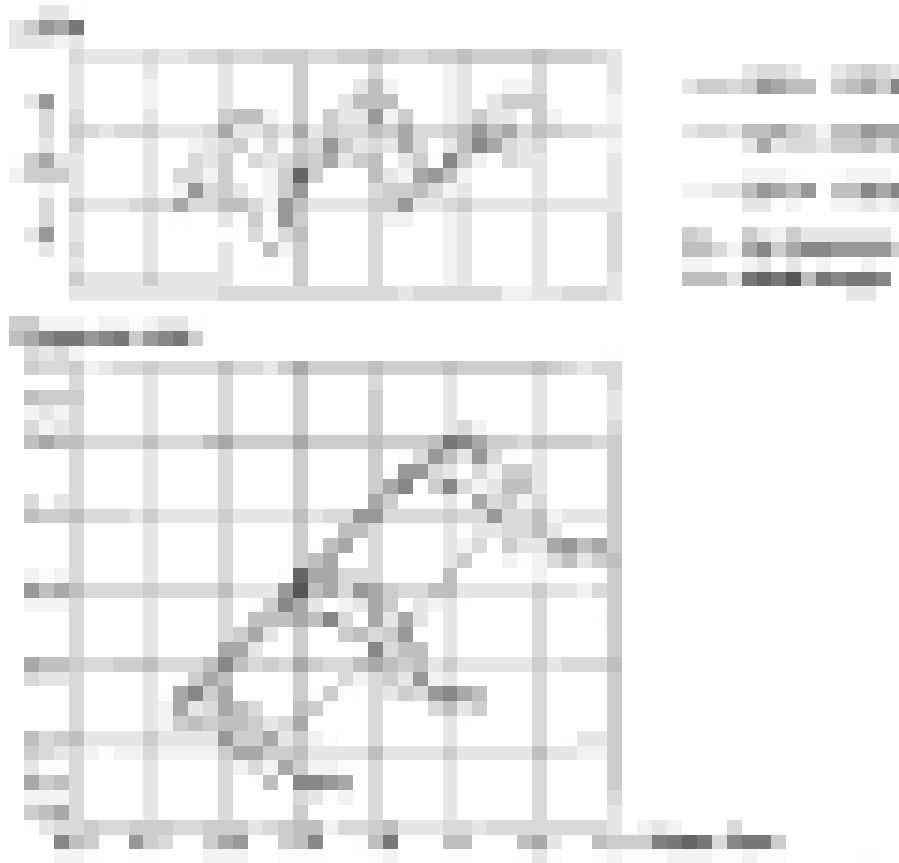


Figure 2-18 Effects of tip clearance on compressor map (Kurzke, 1992)

2.5 Existing Heat Soak Models

Heat soakage model is a part of gas turbine performance simulation program, its calculation procedure like the performance model can vary from very simple to very complicated upon on requirement of the accuracy and time consuming. For general estimation, the 0-D or 1-D model is good enough. If the very high accuracy is required, the CFD or FEA 2-D or 3-D approaches should be employed. However, those tools take plenty of time to get the converged results due to a large number of computations by manipulating the matrixes. The methods introduced are 0-D or 1-D models, which are less complicated than 2-D and 3-D models like NPSS. However, those models are also good enough for heat soak phenomena estimation. In order to describe more convenient and logically, they are classified into three categories which are heat transfer, tip clearances and compressor map.

2.5.1 Heat Transfer

Elder (1974) Method

Elder (1974) introduced a method to calculate the heat exchange between the fluid in the primary gas path of an axial flow compressor and the metal of its blades and annulus casings.

Naylor (2003) described the RR transient heat exchange method to calculate the primary gas path heat soakage in all of the major engine components. Rolls Royce Aero Engine Performance program (RRAP) uses a subroutine named BR081 to calculate the heat soakage, which is the same approach with Elder (1974). It represents the physical component in a straightforward way. The annulus part of the engine component is treated as a tube of equivalent hydraulic diameter, and for component with a blade; the blades are treated as a flat plate.

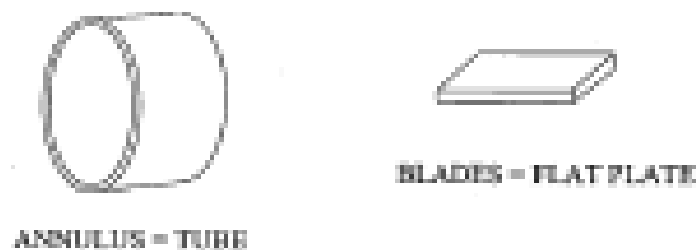


Figure 2-19 Representation of gas turbine component hardware (Naylor, 2004)

The thermal mass, wetted area and metal specific heat are employed as a representative of the component. Meanwhile, it is also assumed that the component has infinite thermal conductivity, and no thermal transfer takes place between blade and annulus. The heat transfer coefficients for annulus and blade are calculated by using standard Nusselt number correlations for turbulent flow in a tube and over a flat plate in a pipe, respectively. The transient heat transfer calculations use the lumped capacitance approach and are carried out by calculating the thermal time constants for both the annulus and blades. In BR081, annulus means the component without blade such as combustor etc. The Blades represent the turbomachinery components. Initial fluid and metal temperatures

are assumed equal to the mean temperatures between inlet and outlet at steady state condition.

$$\tau = M * Cp_s / (h * A) \quad (2-2)$$

$$T_{m1} = T_{f0} - (T_{f0} - T_{m0})^{(-\frac{dt}{\tau})} \quad (2-3)$$

$$q = (T_{s0} - T_{s1}) * M * Cp_s / dt \quad (2-4)$$

$$Nu_L = \frac{hL}{k} = 0.023 * Re^{0.8} Pr^{0.4} \quad (2-5)$$

$$F = \frac{q}{CW/TW} \quad (2-6)$$

Calculation procedures:

1. Calculate the heat convective transfer coefficient through (2-5)
2. Calculate the thermal time constants
3. Set time interval
4. Calculate the new metal temperature as setting a new transient gas temperature
5. Then calculate the amount energy absorbed or released by the metal during the time interval
6. Assuming the amount of energy absorbed or released by metal is the same amount of heat absorbed or released by the fluid (Energy Conservation)
7. The use equation (2-3) to calculate the updated gas temperature as resulting heat transfer (Iterative method due to the specific heat of fluids varies when its temperature changes)
8. Read the adiabatic component pressure and temperature ratio from the maps at the start of the time interval, and then calculate the instantaneous polytropic index.
9. Calculate the instantaneous value of F parameter from the heat transferred and the instant component power by using Equation(2-6)
10. Use equation (2-1) to calculate the non-adiabatic component exit pressure.

Pilidis (1983) Method

Pilidis (1983) developed a kind of gas turbine transient performance code named GTTrans, which takes a step further than Elder (1974). The main difference is Pilidis (1983) method calculates not only heat transfer but also the blade tip clearances and seal clearance variation. Lumped capacitance method was also applied based on the same assumption of infinite thermal conductivity of metals.

Naylor (2004) Stream Method

Naylor (2004) developed a stream method, which applied the lumped capacitance principle divided the component into a number of discrete solids like figure 2-20 below. This method is one-step ahead of the original lumped capacitance by allowing both fluids and body temperatures to change due to exponential decay. This method involves both the transient heat convection and conduction with the finite volume method.



Figure 2-20 Stream one dimensional method (Naylor, 2004)

The figure 2-21 shows the heat exchange estimation by using constant fluid temperature, comparing to the real situation this can lead an overestimation of heat quantity of heat transferred during a time interval. For a short time intervals, this method is ok, but it makes a big error when for a long time interval. As a consequence of that, for the next time interval, the heat exchange would be an underestimation due to the lower temperature difference between metals and fluid.

Figure 2-22 demonstrates the conventional lumped capacitance method. This method assumes the solid temperate follows the exponential decay, which is much more close to reality.



Figure 2-21 Heat estimation using fixed temperature (Naylor, 2004)

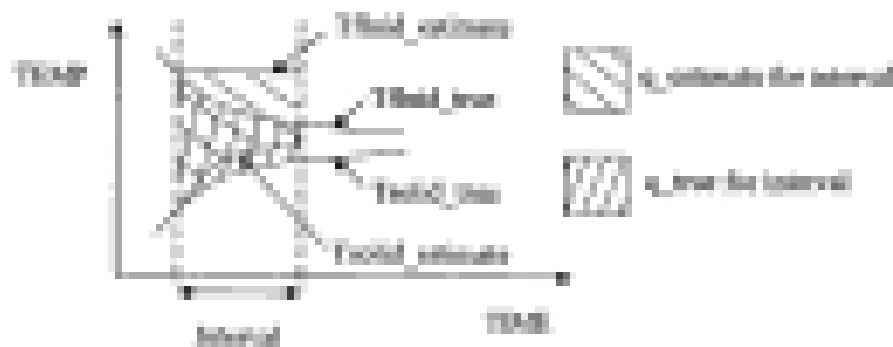


Figure 2-22 Lumped capacitance method (Naylor, 2004)

Naylor (2004) improved the original lumped capacitance method further by allowing both solid and fluid temperature changes following exponential decay. The finite thermal conductivity is used to calculate convective heat transfer as well. Those two improvements give much more accurate results for convective heat calculations.

Heat Convection

For Naylor's Method, the solid and fluid are regarded as thermal mass points. During the time interval, the fluid mass is mass flow rate divided by time interval. As transferred heat depends not only by the boundary layer at the interface but also by the thermal resistance of solid, those two heat transfer effects are combined.

The figure 2-23 shows the heat transfer is from the centre of fluid to the centre of solid. X is the distance solid from its centre to the surface. A is the normal surface wetted area of solid.



Figure 2-23 Transient convective heat transfer (Naylor, 2004)

The total thermal transfer factor can be expressed as equation below

$$\frac{1}{U_{tot}} = \frac{1}{U_1} + \frac{1}{U_2} + \dots \quad (2-7)$$

$$U_{tot} = \frac{1}{\left[\left(\frac{1}{hA} \right) + \frac{1}{\frac{KA}{X}} \right]} \quad (2-8)$$

The heat capacity of fluid and solids can be given as equation 2-8 and 2-9.

$$HCF = W_f * C_{pf} * dt \quad (2-9)$$

$$HCS = M_s * C_{ps} \quad (2-10)$$

The total thermal characteristic of the system is defined as below:

$$Z = -\left(\frac{U}{HCF} + \frac{U}{HCS} \right) \quad (2-11)$$

The heat quantity transferred during the time interval is shown below

$$q = U * \left(\frac{Tf0 - Ts0}{Z} \right) * (e^{Z*dt} - 1) \quad (2-12)$$

Heat Conduction

For heat conduction calculation, Naylor (2004) developed a 1-D finite volume method to sort it out illustrated in figure 2-24. The method is to calculate the heat transfers between adjacent surfaces of solids.

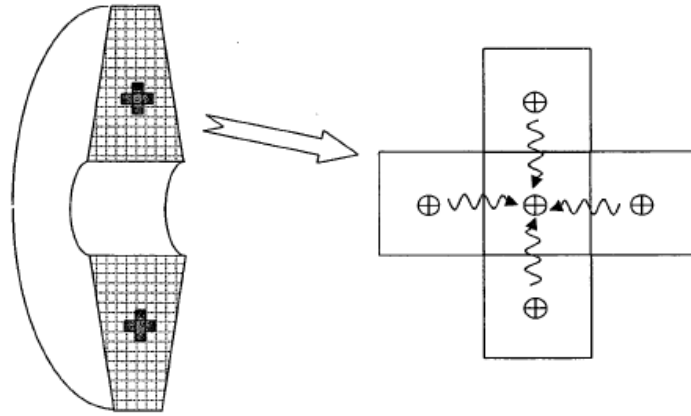


Figure 2-24 Finite volumes conductive heat transfer (Naylor, 2004)

For the adjacent of volumes, the total thermal conductance is derived as Eq.2-13

$$U_{tot} = \frac{1}{\frac{1}{\frac{K1A}{X1}} + \frac{1}{\frac{K2A}{X2}}} \quad (2-13)$$

The pseudo steady state temperature of the volume is given as

$$T_{ss} = \frac{\text{Sum}(\text{VolumeNeighboursConductance} * \text{NeighboursTemp})}{\text{Sum}(\text{NeighboursConductance})} \quad (2-14)$$

The time constant is given by;

$$\text{Tau} = \text{VolumeMass} * \text{Cp} / \text{Sum}(\text{NeighboursTConductance}) \quad (2-15)$$

The conductive heat transferred to volume from one of its neighbours is described as below

$$q = -U_{tot} * [(T_{ss} - T_n) * t + (T_{ss} - T_v) * Tau) * (e^{-\frac{t}{Tau}} - 1)] \quad (2-16)$$

Heat Radiation

Mellor (1990) developed a simple method to estimate the radiative heat transfer from a gas to the black wall surrounded,

$$q_{rad} = \sigma(\varepsilon_g T_g^4 - \alpha_g T_w^4) \quad (2-17)$$

Where

σ = Stefan-Boltzmann constant $5.67 \times 10^{-8} \text{ W/ (m}^2\text{K}^4\text{)}$

ε_g = gas emissivity at temperature T_g

α_g = gas absorptivity at temperature T_w

In reality, there is no such black body wall. Therefore, the empirical value for wall emissivity ε_w could be given as 0.8. Eq.2-17 then is derived as (Lelaj, 2018).

$$q_{rad} = \sigma\left(\frac{1 + \varepsilon_w}{2}\right)(\varepsilon_g T_g^4 - \alpha_g T_w^4) \quad (2-18)$$

The radiative heat transfer for a non-luminous flame is given by (Jagtenberg, 2018)

$$q_{rad} = \sigma\left(\frac{1 + \varepsilon_w}{2}\right)\varepsilon_g T_g^{1.5}(T_g^{2.5} - T_w^{2.5}) \quad (2-19)$$

The gas emissivity could be calculated by (Lefebvre, 1983)

$$\varepsilon_g = 1 - e^{-290P(q l_b)^{0.5} T_g^{-1.5}} \quad (2-20)$$

$$l_b = 3.4 * \frac{Vol}{Area} \quad (2-21)$$

Where,

q = fuel to air ratio locally

l_b = characteristic length

2.5.2 Tip Clearances

Chapman et al (2016) Method

Chapman et al. (2016) developed a tip clearance model by using a finite difference method for a single-stage turbine. This model is integrated with C-MAPSS40K engine model shown in figure 2-25. This method only considers the changes in turbine temperature, not including the flow temperature's variations. The cooling temperature is assumed to be equal to the compressor outlet temperature.

Figure 2-26 describes the typical turbine stage and the equivalent model by Chapman (2016). In this model, the blade part and rotor part are insulated, which mean no heat communications.

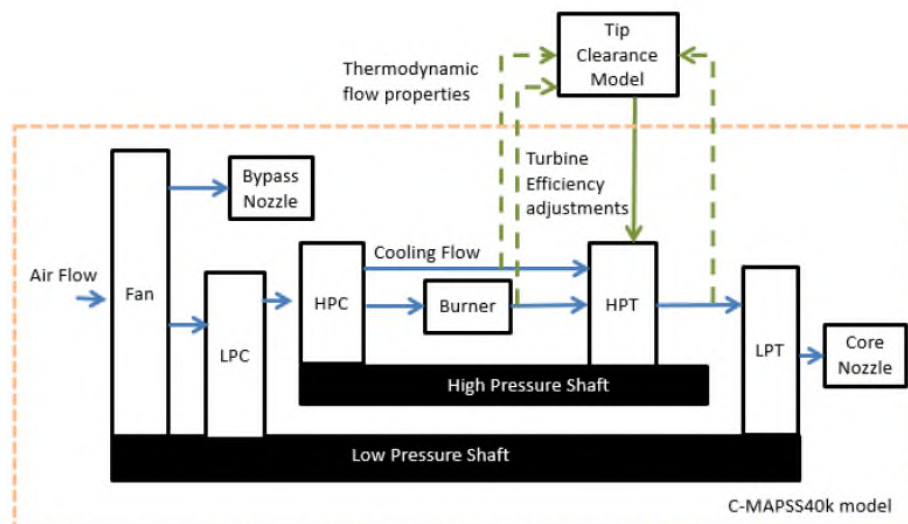


Figure 2-25 Tip clearance model integrated with engine model (Chapman, 2016)



Figure 2-26 Diagram of HPT model (Chapman, 2016)

The tip clearance modelling is defined by three strategies still using the way of Kypuros and Melcher (2002), which are shroud, rotor and blades. The shroud is divided into an inner and outer layer to consider the thermal barrier coatings and super-alloy structure, respectively. The tip clearance gap (TC) is the distance between of blade tip and shroud inner surface and is a function of some parameters like shaft speed, primary gas flow, cooling airflow, and turbine inlet temperature and component geometries. Instant tip clearance is determined by nominal tip clearance and mechanical deformation of the global system.

$$TC(t) = TC_{nom} + (\delta_{shroud} - \delta_{rotor} - \delta_{blade}) \quad (2-22)$$

$$TC_{nom} = R_{shroud} - (R_{rotor} + L_{blade}) \quad (2-23)$$

A. Shroud

There is a finite number of nodes selected along the shroud Y direction, and energy transferred between adjacent components determine variations in node temperatures.

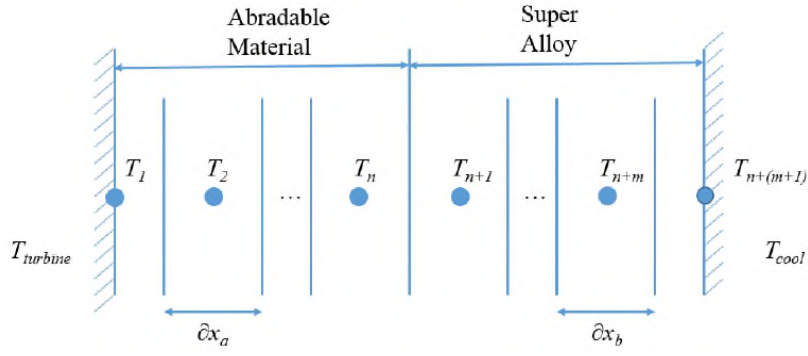


Figure 2-27 1D Heat transfer representation for shroud

The interaction between nodes is derived from the heat equation by using Crank & Nicolson (1996) method, which utilizes the central forward finite difference method to solve partial differential equations.

The calculation procedure of this model is to calculate the current (t) temperature by using a guessed future temperature (t+1), then actual current temperature is compared with the calculated current temperature to generate error function, then employ with the Jacobian matrix and Newton-Raphson method to tune the guessed future temperature to the actual future temperature(t+1).

The node heat energy equations are following the energy conservation shown below

$$Q_{node} = \sum Q_{conduction} + \sum Q_{convection} \quad (2-24)$$

$$Q_{node} = \frac{\rho * vol * Cp}{\partial t} (T_n^{t+1} - T_n^t) \quad (2-25)$$

$$Q_{conduction} = \frac{k * A}{2 * \partial x} [(T_{n+1}^t - T_n^t) + (T_{n+1}^{t+1} - T_n^{t+1})] \quad (2-26)$$

$$Q_{conduction} = \frac{h * A}{2} [(T_{n-1}^t - T_n^t) + (T_{n-1}^{t+1} - T_n^{t+1})] \quad (2-27)$$

For convenience, the Fourier number and Bi number are given as below,

$$Fo = \frac{\partial t * k}{\rho * Cp * \partial x^2} \quad (2-28)$$

$$Bi = \frac{\partial x * h}{k} \quad (2-29)$$

Exterior or edge (Convective) node, T_1 and $T_{n+(m+1)}$:

$$T_{edge}^t = Fo(1 - Bi) * (T_{edge}^{t+1} + T_{edge}^t) - Fo[T_{interior}^{t+1} + T_{interior}^t + Bi(T_{\infty}^{t+1} + T_{\infty}^t)] + T_{edge}^{t+1} \quad (2-30)$$

Interior (Conductive) nodes with heat insulation material or super-alloy:

$$T_n^t = Fo * (T_n^t + T_n^{t+1}) - \frac{Fo}{2} [T_{n-1}^t + T_{n-1}^{t+1} + T_{n+1}^t + T_{n+1}^{t+1}] + T_n^{t+1} \quad (2-31)$$

Interface (Conductive) nodes T_n and T_{n+1} :

$$T_n^t = Fo * (T_n^t + T_n^{t+1}) - \frac{Fo}{2} [T_{n-1}^t + T_{n-1}^{t+1} + T_{n+1}^t + T_{n+1}^{t+1}] + T_n^{t+1} \quad (2-32)$$

$$Fo_{inter} = \frac{\partial t * (\frac{\partial x_a * k_a + \partial x_b * k_b}{\partial x_a + \partial x_b})}{\rho_a * Cp_a * \partial x_a (\frac{\partial x_a + \partial x_b}{2})} \quad (2-33)$$

The temperatures from node 1 to node $n+m+1$ could be expressed by using the equation (2-24), (2-25) and (2-26). With the help of Newton Raphson method, it can calculate each node temperature, thus the average temperature T_{avg} .

The shroud deformation then is defined as

$$\delta_{shroud} = \alpha_{shroud} * r_{shroud} * T_{avg} \quad (2-34)$$

B. Rotor

The same 1-D finite difference method is used to calculate the average temperature of rotor. The rotor deformation comes from centrifugal stress and thermal expansion, the deformation equations are summarized below:

$$\delta_{rotor} = u_{rotor1} + u_{rotor2} \quad (2-35)$$

$$u_{rotor1} = \alpha_{rotor1} * r_{rotor1} * T_{avg} \quad (2-36)$$

$$u_{rotor2} = \frac{1}{4E_{rotor}} (1 - V_{rotor}) \rho_{rotor} * r_{rotor}^3 * \left(\frac{2\pi}{60} N\right)^2 \quad (2-37)$$

E_{rotor} is the modulus of elasticity for the rotor, V_{rotor} is Poisson's ratio for the rotor. N is shaft speed.

C. Blades

HPT blade is regarded as a single mass for heat transfer modelling due to the low ratio of volume to surface area. Deflection of blade is also affected by thermal expansion and centrifugal force.

$$\delta_{blade} = u_{blade1} + u_{blade2} \quad (2-38)$$

$$U_{blade1} = \alpha_{blade} * L_{blade} * T_{blade} \quad (2-39)$$

$$T_{blade}^{t+1} = (T_{blade}^t - T_{ref}) * e^{-\frac{h*A}{\rho*Vol*Cp}*dt} + T_{ref} \quad (2-40)$$

$$T_{ref} = \eta(T_{cool} - T_{turb}) + T_{turb} \quad (2-41)$$

$$U_{blade2} = \frac{\rho_{blade} * L_{blade}^2 * (L_{blade} + r_{rotor})}{E_{blade}} * \left(\frac{2\pi}{60} N\right)^2 \quad (2-42)$$

2.5.3 Compressor Transient Characteristics Map

Maccallum (1985) Method

Modification in compressor characteristics is from the publications of Maccallum & Pilidis (1986). The component characteristic would change during produces that heat exchange occurring between engine materials and working fluid. These changes are as results of changing in fluid density, affecting the axial velocity, which changes the blade performance.

Maccallum (1979 & 1985) summarized the three main effects on changes in compressor characteristics, which are following as:

1. Fluids (air) density changes due to the heat transfer
2. Boundary layer development of blade aerofoil and end wall

3. Tip clearance variation

Then consequences mentioned above will change the both the speed line and surge margin of steady state compressor map during the transients. The speed line after moving during the transient may treat as similar to the steady state characteristics when the engine is running adiabatically at a different speed, $(\Delta N + N)$, where ΔN is the difference from the actual compressor speed N .

The corrected speed of compressor could express as

$$\left(\frac{\Delta N}{N}\right) = C1 * \frac{(T_b - T_{air})}{T_{air}} + C2 * \frac{\dot{Q}}{(mC_p)T_m} + C3 * \frac{\Delta(\delta)}{gap} \quad (2-43)$$

This the first term, second term and third term are boundary layer, density changes and tip clearance effects, respectively. C1, C2 and C3 are got from the experimental correction. For LP compressor are -0.07, -0.07 and 0.3, respectively and for HP compressor are -0.1, -0.1 and 0.3.

Therefore, the instantaneous speed then is

$$N_{new} = \Delta N + N \quad (2-44)$$

The changed surge margin as results of heat transfer effect could express as following:

$$\frac{SM_{ht} - 1}{SM_{ad} - 1} = 1 - 0.36 * F \quad (2-45)$$

Similarly, the change in surge margin caused by tip clearance is given as

$$\frac{SM_{cl} - 1}{RSM_{ref} - 1} = 1 - 1.1 * \frac{\Delta(\delta)}{gap} \quad (2-46)$$

Finally, the magnitude of changes in surge margin may express as

$$\frac{SM_{cl} - 1}{SM_{ht} - 1} = 1 - 1.1 * \frac{\Delta(\delta)}{gap} \quad (2-47)$$

2.6 Research Gap

The methods mentioned above are either limited to the heat soakage effects on particular gas turbine components or rely on accurate geometrical information.

Therefore In this paper, a generically simplified heat soakage model for three main gas path components including compressors, combustor and turbines has been developed for the purpose of transient performance simulations during gas turbine engine conceptual and preliminary design stage when the accurate geometrical information of the engines are not available.

3 Methodology

3.1 Introduction

The purpose of this chapter is to describe the construction of the performance model (TurboCycle) and the heat soakage model (Heat Soak).

The performance model consists of three parts, which are design point performance, steady-state performance and transient performance, respectively.

The heat soakage model is divided into three modules, which are geometry model, heat transfer model, and tip clearance model shown in figure 3-1.

Firstly, the geometry model calculates the engine component mass and wetted area, including components of turbomachinery and combustor.

Then the heat transfer model estimates the heat quantity exchange between the fluid and metal through heat convection, heat conduction and heat radiation by using the results of the geometry model. Consequently, that heat exchange would lead to the variation of engine thermal dynamic performance.

After that, the Tip clearance model would estimate the tip clearance variations during the transient manoeuvres through the thermal expansion, centrifugal stress expansion and pressure stress expansion. As a result, the turbomachinery characteristic map could also be corrected by the tip clearance model due to the variation of tip clearance and heat transfer.

The updated engine thermodynamics and component maps would lead to the transient performance results with heat soakage phenomena.

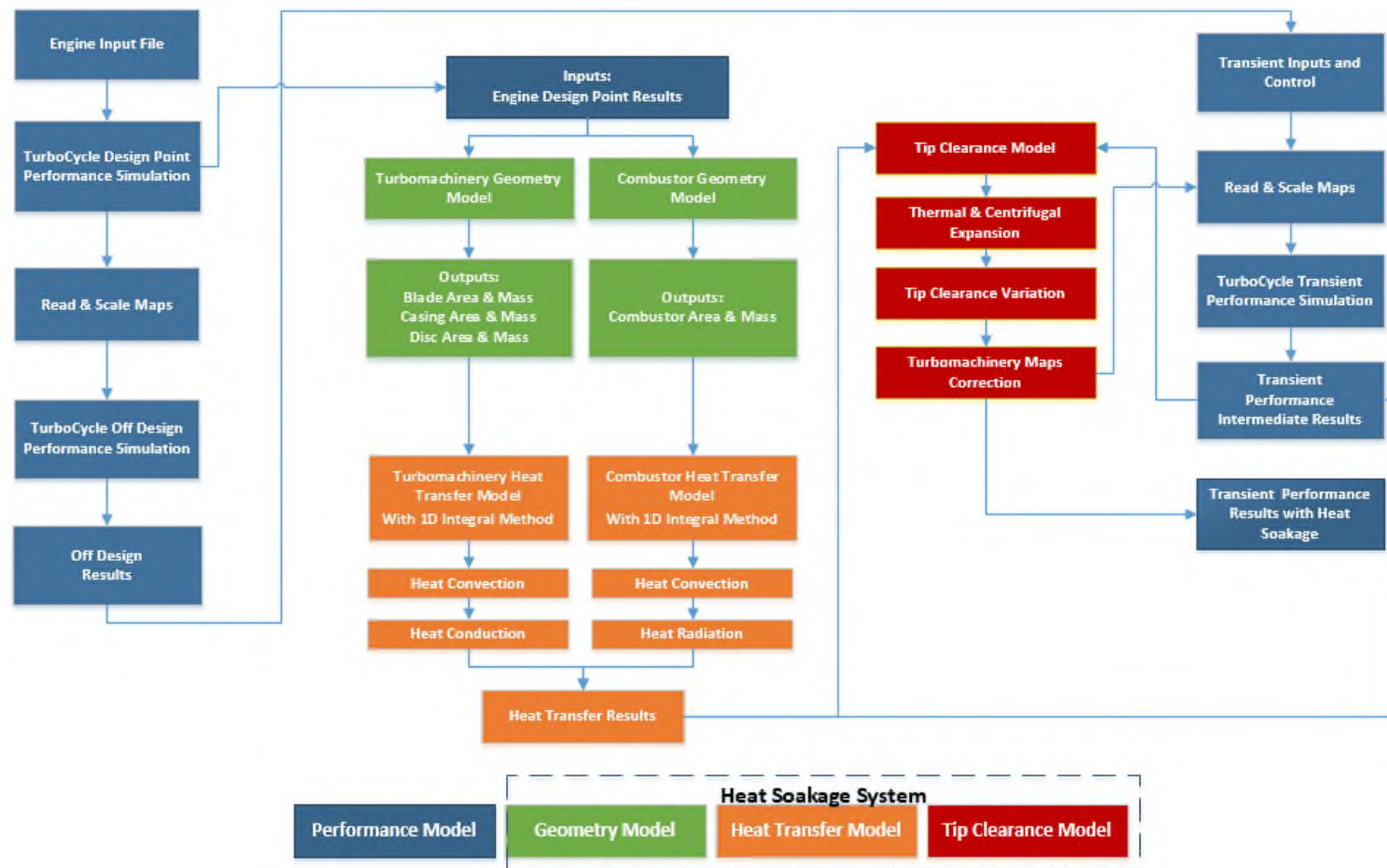


Figure 3-1 Heat Soakage Model Flow Chart

3.2 Performance Model

3.2.1 Model Framework

The performance model (TurboCycle) is designed by under modular architecture shown in Figure 3-3. It has the capability to implement the tasks including design point, steady state and transient performance simulations with an interface shown in figure 3-4.

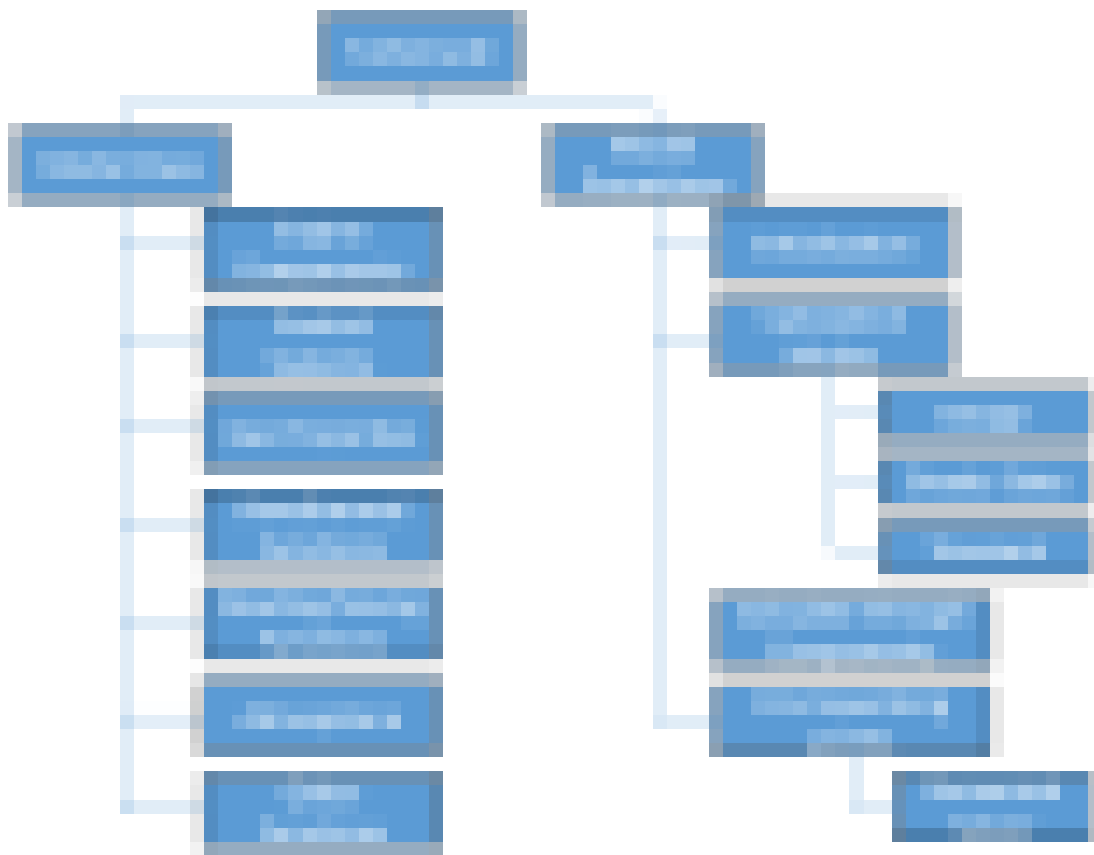


Figure 3-2 TurboCycle Architecture

TurboCycle contains seven different classes shown in figure 3-3. The following chapters would introduce the engine components class and control system class. The gas properties class is very similar to NASA CEA standard, which calculates chemical equilibrium product concentrations from any set of reactants and determines thermodynamic and transport properties for the product mixture (NASA GLENN RESEARCH CENTRE, 2019). The aerothermal functions class contains aerodynamic and thermodynamic functions, such as the isentropic flows,

entropy and enthalpy function, and so on. The reading and saving class is built for reading the input files and saving the outputs.

The atmosphere class is the international standard atmosphere model, which could calculate static and total temperature and pressure by given a flight and ground condition. The final class (other functions) contains some mathematic functions such as linear and non-linear interpolation functions, matrix operating functions, algorithm, and so on.

TurboCycle graphical interface makes it more convenient to use and also gives real-time results on display, which is shown in figure 3-4.

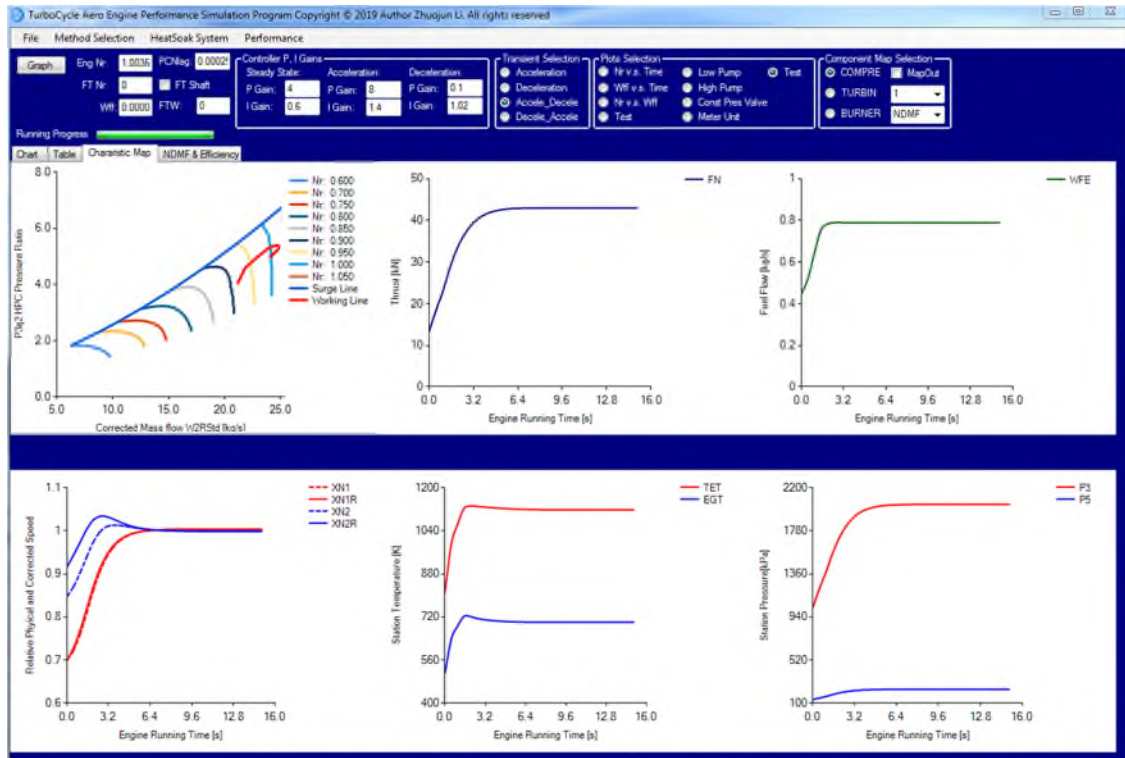


Figure 3-3 TurboCycle Interface

3.2.2 Simulation Methods

Tradition methods for transient performance simulation are continuity of mass flow (CMF) and Inter-component volume (ICV) as mentioned before. However, both of these two methods have certain limitations. For example, the CMF method could not reflect the phenomenon of mass storage during the rapid transient manoeuvre. ICV method could not read the points on such a speed line

which is likely to be vertical and horizontal one. For example, in the figure below, the speed line around the red circle is nearly flat. As the ICV method is to read the map by using the calculated pressure ratio, it would be more than two points for such a pressure ratio at that case. As a result, ICV could predict engine performance in this case.

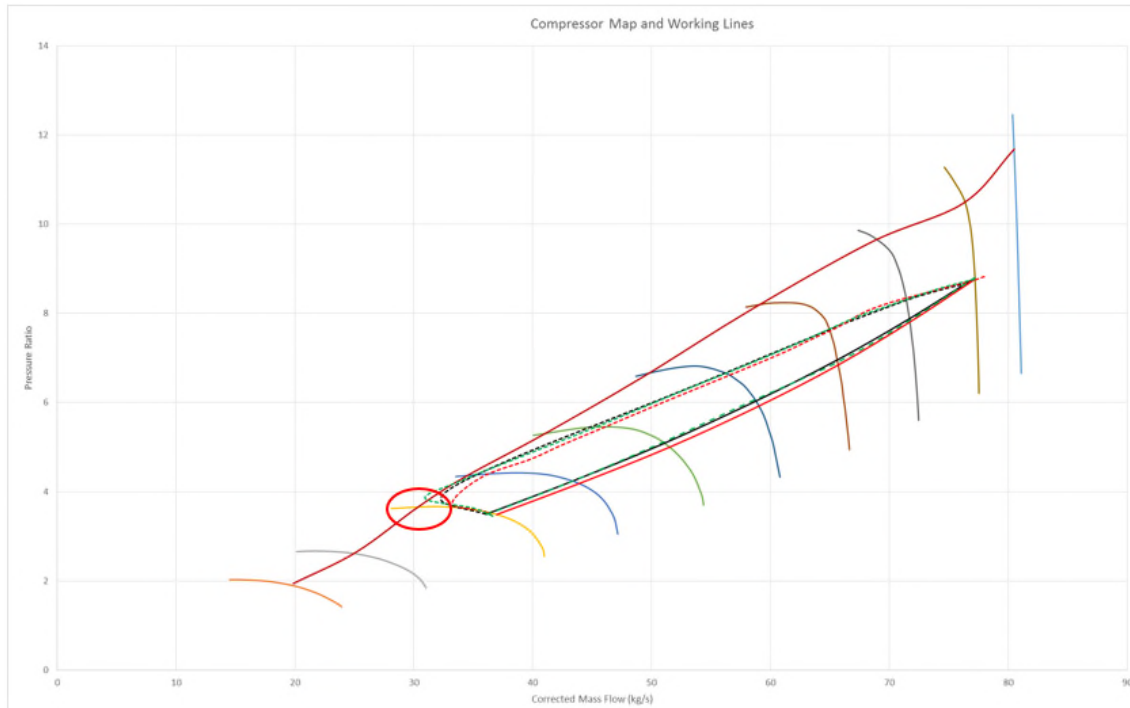


Figure 3-4 A Typical Compressor Map

Therefore, the imbalanced mass flow (IMF) method, an improved method CMF , is used for transient performance simulation(Palmer & Yan, 1982). The basic idea of the IMF method is based on the CMF method by introducing an imbalanced mass flow IMF term, which is produced by a volume by using volume dynamics.

The mass flow between the inlet and outlet of a volume could be determined through Eq. 3-68

$$W_{out} = W_{in} - IMF \quad (3-1)$$

The advantage of the IMF method could calculate any point performance on a given map even if the speed line is flat. Meanwhile, the mass storage effect could also be simulated all the time.

For steady state calculation, the inlet and outlet of mass flow is continuity, and the power of components on the same shaft is equal to each other.

For transient calculation, the power of component on the same shaft is not equal any more. The mass flow difference between inlet and outlet of the volume is equal to IMF term, which is calculated through iterations. However, the mass flow is still continuity for all the other components. The detailed calculation of IMF would be explained by Eq.(3-49) to (3-51) in the following chapter.

3.2.3 Engine Components

There are eight different components in Component Class shown in figure 3-5, which are intake, compressor, turbine, burner, convergent nozzle, volume, splitter and mixer.

In this chapter, the functionality of each component would be introduced in details in terms of input, output and calculation equations.



Figure 3-5 Engine Components

Intake

There are two functions for intake. The first one is to calculate free stream conditions at a given altitude and Mach number by using an International Standard Atmosphere (ISA), with allowance for deviation from standard static temperature and pressure. The second one is to calculate the exit condition of intake by using the term of pressure recovery.

For intake brick, there are six inputs and seven outputs without variables and errors, which are listed in table 3-1.

Intake		
Inputs		
1	H	Altitude
2	DT	Deviation from ISA Temperature
3	DP	Deviation from ISA Pressure
4	Ma	Flight Mach Number
5	dP	Pressure Recovery
6	Wa	Inlet Mass Flow
Outputs		
1	P1	Inlet Total Pressure
2	T2	Inlet Total Temperature
3	P2	Outlet Total Pressure
4	T2	Outlet Total Temperature
5	W2	Outlet Mass Flow

6	Va	Flight Speed
7	MD	Momentum Drag
Variables		Nil
Errors		Nil

Table 3-1 Intake Brick Information

International Standard Atmosphere (ISA) model are described by the following equations (Kurzke, 2004). Given the flight altitude *alt*, the static temperature and pressure are calculated by ISA.

Below 11000m, the ambient static temperature is

$$T_{amb, ISA} = 288.15 - 6.5 * \frac{alt}{1000} \quad (3-2)$$

Ambient static pressure is

$$P_{amb, ISA} = 101.325 * \left(1 - \frac{0.0225577 * alt}{1000}\right)^{5.25588} \quad (3-3)$$

From 11000m to 25000m, the static temperature remains as a constant and is equal to 216.65K. The ambient pressure is

$$P_{amb, ISA} = 22.632 * e^{\frac{11000 - alt}{6341.62}} \quad (3-4)$$

Above 25000m, the ambient temperature is

$$T_{amb, ISA} = 216.65 + 3 * (alt - 25000)/1000 \quad (3-5)$$

And ambient pressure is

$$P_{amb, ISA} = 2.4886 * \left(\frac{216.65}{T_{amb, ISA}}\right)^{11.8} \quad (3-6)$$

Then the total temperature and pressure at inlet could be calculated from equation 3-6 and 3-7 by given an ambient condition and flight Mach number *Ma*.

$$\frac{T_a}{T_{amb}} = \left[1 + \frac{\gamma - 1}{2} * Ma^2 \right] \quad (3-7)$$

$$\frac{P_a}{P_{amb}} = \left[1 + \frac{\gamma - 1}{2} * Ma^2 \right]^{\frac{\gamma}{\gamma - 1}} \quad (3-8)$$

As intake does not do any work, the total energy at the inlet is always equal to the outlet. Therefore, the total temperature of the inlet is equal to the outlet, thus

$$T_2 = T_1 \quad (3-9)$$

The total outlet pressure is given from the equation 3-9 by given a pressure recovery coefficient PRC,

$$P_2 = P_1 * PR \quad (3-10)$$

PRC is between 0 to 1.0, which could be calculated from the USAF standard (Nikolaidis, 2017) shown in equation 3-10

$$PRC = \begin{cases} 1.0 & Ma < 1.0 \\ 1 - 0.075 * (Ma - 1)^{1.35} & 1.0 < Ma < 5.0 \\ \frac{800}{Ma^4 + 935} & Ma > 5.0 \end{cases} \quad (3-11)$$

The outlet mass flow is equal to the inlet, thus

$$W_2 = W_1 \quad (3-12)$$

The flight speed is then calculated from the equation 3-12 by knowing flight Mach number and ambient temperature.

$$V_a = Ma * \sqrt{\gamma * R * T_{amb}} \quad (3-13)$$

Finally, the momentum drag could be calculated from equation 3-13 by given mass flow and flight speed

$$MD = W_1 * V_a \quad (3-14)$$

Compressor (Fan)

Compressor (Fan) is one of the turbomachinery components of gas turbine engines, which squashes air stream to increase the total pressure to which required by engine cycles. The outlet conditions of compressor and compressor work could be calculated by given the inlet conditions, design point of pressure ratio and isentropic efficiency, and compressor map.

The basic parameters for compressor calculation are shown in the table below.

Compressor		
Inputs		
1	Beta	Beta value of compressor map (0-1) as an initial guess parameter
2	XNr	Relative corrected speed
3	PRdp	Design Point Pressure Ratio
4	Etadp	Design Isentropic Efficiency (0-1)
5	ES	Error selection (1:Error required 0: No Error)
6	Mno	Compressor Map Number
7	Sno	Shaft number
Data receiving from upstream component		
1	P1	Inlet Total Pressure
2	T1	Inlet Total Temperature
3	W1	Inlet Mass flow
4	FAR1	Inlet Fuel-Air Ratio
Outputs		

1	CW	Compressor Work
2	P2	Outlet Total Pressure
3	T2	Outlet Total Temperature
4	W2	Outlet Mass Flow
5	FAR2	Outlet Fuel-Air Ratio
Variables		
1	Beta	Beta value of compressor
2	XNr	Relative corrected speed
Errors		
1	(W1-W2)/W1	Mass Flow Error

Table 3-2 Compressor Brick Information

The calculation here for compressor is based on the entropy and enthalpy function and variable specific heat.

Firstly, the the compressor map is scaled based on the design point and reference point of the original map. Then it gives pressure ratio, efficiency and mass flow by reading the scaled map shown as

$$(PR, W2, ETA) = \text{ReadMap}(\text{Beta}, \text{XNr}) \quad (3-15)$$

Thus, the compressor outlet of pressure is given as

$$P2 = P1 * PR \quad (3-16)$$

According to the definition of the isentropic process, the entropy of inlet is equal to the outlet if it is a.n ideal compression process. Then, the entropy equation applied is shown

$$S(T2is, P2) = S(T1, P1) \quad (3-17)$$

It could give the isentropic exit temperature T_{2is} by finding the inverse value of the entropy function.

$$T_{2is} = S(P_2)^{-1} \quad (3-18)$$

Then the real T_2 is calculated according to the definition of isentropic efficiency η ,

$$\eta = \frac{T_{2is} - T_1}{T_2 - T_1} \quad (3-19)$$

Applying the enthalpy equation, finally, the work of compressor could be given as

$$CW = W_2 * [h(T_2) - h(T_1)] \quad (3-20)$$

Here the variables and errors are used for off design and transient calculation by solving the non-linear equations.

Premas

The function of Premas is to calculate the outlet conditions from a component such as a splitter, compressor bleed valve, bypass duct and jet pipe by separating mainstream into primary stream and bypass stream and bleeding and turbine cooling. For example, for the bypass, it is used as a splitter, and for the air system, it is used as turbine NGV cooling.

The table below contains the essential information for Premas calculation,

Premas		
Inputs		
1	λ_w	Mass flow coefficient (%)
2	ΔW	Mass flow change due to bleed (absolute)
3	λ_p	Pressure ratio coefficient (%)
4	ΔP	Pressure change (absolute)
Data receiving from upstream component		

1	P1	Inlet Total Pressure
2	T1	Inlet Total Temperature
3	W1	Inlet Mass flow
4	FAR1	Inlet Fuel-Air Ratio
Outputs		
1	P21	Primary flow total pressure
2	P22	Bypass flow/cooling air pressure
3	T21	Primary flow total temperature
4	T22	Bypass flow/cooling air temperature
5	W21	Primary mass flow
6	W22	Bypass flow/cooling air flow
7	FAR2	Outlet Fuel-Air Ratio
Variables		
1	λ_w	For bypass splitter of turbofan use only
Errors		Nil

Table 3-3 Premas Brick Information

The coefficient of mass flow and pressure ration are defined respectively from the equations below,

$$\lambda_w = \frac{W_{21}}{W_1} = \frac{1}{BPR + 1} \quad (3-21)$$

$$\lambda_p = \frac{P_{21}}{P_1} \quad (3-22)$$

The outlet mass flow of two streams are given from

$$W_{21} = \lambda_w * W_1 - \Delta W \quad (3-23)$$

$$W_{22} = W_1 - W_{21} \quad (3-24)$$

Thus the outlet pressure of two streams are calculated from

$$P_{21} = \lambda_p * P_1 - \Delta P \quad (3-25)$$

$$P_{22} = P_1 \quad (3-26)$$

It is assumed as the adiabatic process during flow bleeding or separation. Therefore, the outlet temperature of the two streams are given as

$$T_{22} = T_{21} = T_1 \quad (3-27)$$

For off design and transient calculation, the flow coefficient is a variable, and there are no errors generated by Premas.

Burner

The burner is a component of an engine where the mixing of air and fuel and combustion take place. The chemical energy would transfer to thermal energy during the combustion process. As a result, the outlet total gas temperature would be increased rapidly according to the law of energy conservation. The function of the burner is to calculate the outlet conditions from the combustion chamber either total outlet temperature or fuel flow by given inlet conditions, pressure loss and combustion efficiency.

The primary parameters of burner brick are listed in the table below,

Burner		
Inputs		
1	$\Delta P/P_1$	Pressure Loss (%)
2	η_{dp}	Design Combustion Efficiency (0-1)
3	WF	Fuel Flow

Data receiving from upstream component		
1	P1	Inlet Total Pressure
2	T1	Inlet Total Temperature
3	W1	Inlet Mass flow
4	FAR1	Inlet Fuel-Air Ratio
Outputs		
1	P2	Outlet total pressure
2	T2	Outlet total temperature
3	W2	Outlet Mass flow
4	FAR2	Outlet Fuel-Air Ratio
Variables		
1	COT	Outlet Total Temperature
Errors		
1	(WF-WFest)/WF	Mass Flow Error Only for Transient

Table 3-4 Burner Calculation Information

At the design point, combustion efficiency is given as an input. However, at off design point, the combustion efficiency is varied along with the burner loading, which is defined by (Kurzke, 2004)

$$\Omega = \frac{W1}{P1^{1.8} * e^{\frac{T1}{300}} * Vol} \quad (3-28)$$

Where vol is the volume of burner.

Burner loading at design point is equal to 100% by its definition. According to (Kurzke, 2004), the burner efficiency at off design could be given from

$$\log(1 - \eta_{od}) = a + b * \log\left(\frac{\Omega_{od}}{\Omega_{des}}\right) \quad (3-29)$$

The constant a is correlated with design point efficiency given as

$$a = \log(1 - \eta_{des}) \quad (3-30)$$

The constant b is defined as part- load constant shown in the figure below as an example with cycle design point efficiency of 0.995:

Therefore, the value of 1.6 is selected here for the constant b.

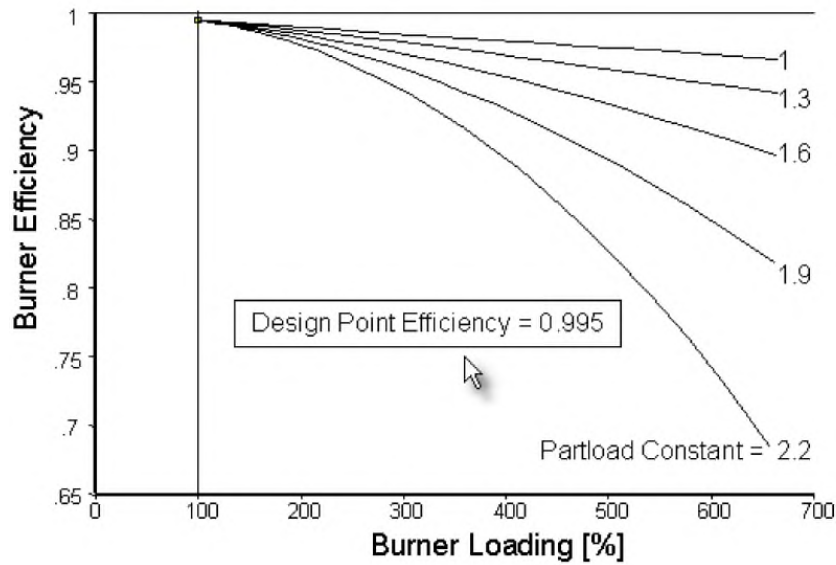


Figure 3-6 Burner Efficiency (Kurzke, 2004)

According to the continuity of mass flow, the outlet mass flow is given from Eq.3-31

$$W_2 = W_1 + W_F \quad (3-31)$$

Then, the outlet of fuel to air ratio (FAR2) is calculated from Eq.3-32

$$FAR_2 = \frac{W_F}{W_2} \quad (3-32)$$

Thus, the outlet enthalpy $h(T2)$ is given from Eq.3-33 by knowing the fuel low heat value LHV and temperature of fuel flow T_{WF} , which are constants

$$h(T2) = \frac{h(T1) + FAR2 * (LHV * \eta_{od} + h(T_{WF}))}{1 + FAR2} \quad (3-33)$$

The outlet total temperature $T2$ is calculated through Eq.3-34

$$T2 = \frac{h(T2)}{Cp} \quad (3-34)$$

For transient simulation, the outlet temperature $T2$ is initially guessed at beginning. Then the estimated fuel flow WF_{est} is calculated based on the guessed $T2$ from the Eq.3-35

$$WF = \frac{(W1 + WF_{est.}) * h(T2) - W1 * h(T1)}{LHV * \eta_{od} + T_{WF} * Cp} \quad (3-35)$$

Then the fuel flow WF from the control system is against the WF_{est} mentioned above to generate a mass flow error term. As long as the error is close to 0, $T2$ would be guessed accurately.

$$\text{Error} = (WF - WF_{est.})/WF \quad (3-36)$$

Finally, the outlet pressure is calculated directly from Eq.3-36

$$P2 = P1 * (1 - \Delta P/P1) \quad (3-37)$$

Turbine

The turbine is another turbomachinery component in the hot section. It drives the compressor on the same shaft by converting thermal energy to kinetic energy from the hot gas with high pressure. It also provides the power to aircraft electrical equipment such as air conditioning.

The inputs and outputs and other important parameters are listed in the table below,

Turbine		
Inputs		
1	Beta	Turbine Beta Value
2	XNr	Design Relative Corrected Speed
3	AW	Auxiliary Work or Power Off Take
4	ETA	Design Isentropic Efficiency
5	SNo	Shaft No.
6	MNo	No. of Map
7	I	Inertial of Rotor
8	N	Design Physical Speed (RPM)
Data receiving from upstream component		
1	P1	Inlet Total Pressure
2	T1	Inlet Total Temperature
3	W1	Inlet Mass flow
4	FAR1	Inlet Fuel-Air Ratio
Outputs		
1	P2	Outlet Total Pressure
2	T2	Outlet Total Temperature
3	W2	Outlet Mass Flow
4	FAR2	Outlet Fuel-Air Ratio
Variables		
1	Beta	Turbine Beta Value

Errors		
1	$(W1-W2)/W1$	Mass Flow Error
2	$(CW-TW)/CW$	Power Error Only for Steady State

Table 3-5 Turbine Calculation Information

The cooling flow for the turbine here is assumed for NVG only not for the rotor. Therefore, the cooling flow would be accounted for turbine power.

The calculation process of the turbine is similar to that of the compressor. In the same way, turbine map scaling is based on the design point and reference point in an original map. Thus, it gives expansion ratio, efficiency and mass flow by reading the scaled map.

$$(EPR, W2, ETA) = \text{ReadMap}(\text{Beta}, XNr) \quad (3-38)$$

Then, the outlet pressure P2 is calculated by Eq.3-39

$$P2 = P1/EPR \quad (3-39)$$

Based on the definition of the isentropic process, the entropy of inlet is equal to the outlet. Thus, the entropy function is given as Eq.3-40

$$S1(T1, P1) = S2(T2is, P2) \quad (3-40)$$

Here, an iteration method has been applied to estimate the T2is by knowing T1, P1, and P2. As long as S1-S2 is close to 0, then T2is would be iterated accurately.

Afterwards, the real outlet temperature T2 could be calculated through the definition of turbine isentropic efficiency which is slightly different with that of compressor..

Finally, the turbine work TW could be determined by applying the enthalpy function shown in Eq.3-41

$$TW = W1 * h(T1) - W2 * h(T2) \quad (3-41)$$

For the steady state simulation, there are two errors shown below,

$$\text{Error1} = (W1 - W2)/W1 \quad (3-42)$$

$$\text{Error2} = (TW - AW - CW)/TW \quad (3-43)$$

The first error is the mass flow difference between a reading value from the map and a calculated value from a component ahead. The second error is the difference between turbine work and compressor.

During the transient simulation, as the power is not balanced; therefore, there is only mass flow error term.

Mixer

The mixer is an imaginary component which is employed to mix the cooling flow with the primary stream without total pressure loss. This mixing process follows the mass and energy conversation laws, and this mixing process is only valid when the cooling flow is much smaller than primary flow. Otherwise, the total pressure loss cannot be ignored.

The inputs and outputs and other important parameters for Mixer calculation are listed in the table below,

Mixer		
Inputs		
1	P12	Inlet Total Pressure of Cooling Flow
2	T12	Inlet Total Temperature of Cooling Flow
3	W12	Inlet Mass flow of Cooling Flow
4	FAR12	Inlet Fuel-Air Ratio of Cooling Flow
Data receiving from upstream component		
1	P11	Inlet Total Pressure of Primary Flow
2	T11	Inlet Total Temperature of Primary Flow

3	W11	Inlet Mass flow of Primary Flow
4	FAR11	Inlet Fuel-Air Ratio of Primary Flow
Outputs		
1	P2	Outlet Total Pressure
2	T2	Outlet Total Temperature
3	W2	Outlet Mass Flow
4	FAR2	Outlet Fuel-Air Ratio
Variables		Nil
Errors		Nil

Table 3-6 Mixer Calculation Information

According to the law of mass continuity, the outlet mass flow of mixer is given from Eq.3-44.

$$W2 = W11 + W12 \quad (3-44)$$

As this mixing process is assumed no total pressure loss, there the total outlet pressure is equal to that of the inlet, thus

$$P2 = P11 = P12 \quad (3-45)$$

Finally, the total outlet temperature of the mixer is calculated by applying enthalpy function and law of conservation of energy, then.

$$W2 * h(T2) = W11 * h(T11) + W22 * h(T12) \quad (3-46)$$

$$T2 = h(T2)^{-1} \quad (3-47)$$

Volume

The volume is also an imaginary component employed for transient simulation by using the volume dynamic method. The reason for introducing the volume is because the mass flow equilibrium is broken when during the quick transient

manoeuvre. Therefore, the volume applied here is to determine the mass storage by using the previous and current step of temperature, pressure and mass flow. Then the initial mass flow storage variable is introduced here, finally the error term of the mass storage difference between guessed one and calculated one is produced. As long as the initial variables are matched, then this error term would be close to zero. Then the real mass flow storage could be simulated accurately by this matching procedure. This matching procedure is only active for transient simulation, and the mass flow of inlet and outlet is always equal at the case of steady state performance simulation.

The basic information of volume is summarised in the table below,

Volume		
Inputs		
1	V	Volume Value m^3
Data receiving from upstream component		
1	P1	Mean Total Pressure of Primary Flow at Current Step
2	T1	Mean Total Temperature of Primary Flow at Current Step
3	W1	Inlet Mass Flow of Primary Flow at Current Step
4	P0	Mean Total Pressure of Primary Flow at Previous Step
5	T0	Mean Temperature of Primary Flow at Previous Step
6	W0	Inlet Mass Flow of Primary Flow at Previous Step

Outputs		
1	IMF	Imbalance Mass Flow (Storage Mass Flow)
2	W2	Outlet Mass Flow of Primary Flow at Current Step
Variables		
1	Wa	Guessed Mass Flow Storage
Errors		
1	(Wa-IMF)/IMF	Imbalanced Mass Flow Error

Table 3-7 Volume Calculation Information

It should be noticed that the previous mean total temperature, pressure and outlet mass flow would be stored in advance before calculation.

The ideal gas law is given as

$$m = \frac{PV}{RT} \quad (3-48)$$

Where P is total pressure of a control volume

V is volume value

R is universal gas constant

T is the total temperature of a control volume

By differentiating this equation in term of the time interval dt , then Eq.3-48 is derived as

$$\frac{dm}{dt} = \frac{dP}{dt} * \frac{V}{RT} - \frac{dT}{dt} * \frac{PV}{RT^2} \quad (3-49)$$

The term on the left-hand side of Eq.3-49 is the mass flow storage in a control volume during a time interval. It is caused by changes in pressure and temperature. The first term of the right-hand side is the changes in pressure, and

the second term is the changes in temperature. It is found that the second term would have a relatively smaller impact on mass flow storage compared with the pressure term as it contains the second-order of the inverse value of temperature. Therefore, it is reasonable to assume that this term could be ignored for simplification.

Finally, this equation could be re-arranged as

$$IMF = \frac{dm}{dt} = \frac{dP}{dt} * \frac{V}{RT} \quad (3-50)$$

Then the imbalanced mass flow IMF is given as

$$IMF = \frac{dm}{dt} = \frac{(P2 - P1)}{dt} * \frac{V}{RT1} \quad (3-51)$$

Finally, the outlet mass flow of volume W2 is given from Eq.3-52

$$W2 = W1 - IMF \quad (3-52)$$

Nozzle

The nozzle described here is a convergent nozzle, which is like a duct. It has a fixed throat area (exit area), and subsonic flow accelerates to the throat. There are two kinds of conditions for the nozzle. The first one is un-choked, which mean the total pressure divided by the ambient pressure is less than the critical value of choking derived from Q-lambda functions. Then the flow is subsonic at the throat. If the pressure ratio of the nozzle is larger than the critical value, then the Mach number flow at the throat is equal to one.

The primary information for nozzle calculation is shown in the table below,

Nozzle		
Inputs		
1	A	Nozzle Throat Area m ²
Data receiving from upstream component		

1	P1	Inlet Total Pressure
2	T1	Inlet Total Temperature
3	MD	Flight Momentum Drag
4	P_{amb}	Ambient Static Pressure
5	W1	Inlet Mass Flow
Outputs		
1	FN	Net Thrust
2	W2	Outlet Mass Flow of Primary Flow at Current Step
3	SFC	Specific Fuel Consumption
4	V_e	Exhausted Gas Velocity
5	p2	Static Pressure at Outlet
6	t2	Static Temperature at Outlet
Variables		Nil
Errors		
1	$(W1-W2)/W1$	Mass Flow Error

Table 3-8 Nozzle Calculation Information

The critical value of pressure ratio and temperature ratio could be calculated from the Eq.3-53

$$\left(\frac{P2}{p2}\right)_{critical} = \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma}{\gamma - 1}} \quad (3-53)$$

If $\left(\frac{P_2}{P_{amb}}\right) > \left(\frac{P_2}{p_2}\right)_{critical}$, then the nozzle is determined under choked condition, then, the static outlet pressure p_2 is calculated by Eq.53 and static temperature t_2 is given from Eq.3-54

$$\left(\frac{T_2}{t_2}\right)_{critical} = \left(1 + \frac{\gamma - 1}{2}\right) \quad (3-54)$$

If the term $\left(\frac{P_2}{P_{amb}}\right) < \left(\frac{P_2}{p_2}\right)_{critical}$, then the nozzle is at the un-choked condition. Thus, the static outlet pressure p_2 is equal to ambient pressure P_{amb} shown as

$$p_2 = P_{amb} \quad (3-55)$$

Then the static temperature is calculated from

$$\frac{T_2}{t_2} = \left(\frac{P_2}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \quad (3-56)$$

If the nozzle is choked, then the exit velocity of exhausted gas Ve is given from

$$Ve = \sqrt{\gamma * R * t_2} \quad (3-57)$$

Otherwise, then the exit velocity of exhausted gas Ve is calculated from

$$Ve = \sqrt{2 * cp * (T_1 - t_2)} \quad (3-58)$$

As the area of nozzle throat is fixed, the area at design point is always equal to off design. According to the definition of mass flow rate, the relation between throat area and mass flow rate is given as

$$W = \rho * A * Ve \quad (3-59)$$

Re-arrange the ideal gas law, and it could be given as

$$\rho = \frac{P}{RT} \quad (3-60)$$

Substitute Eq.3-60 into Eq.59 and re-arrange again, then the area of the throat is calculated based on design point condition from the Eq.3-61

$$A = \frac{W2_{DP} * R * t2_{DP}}{p2_{DP} * Ve_{DP}} \quad (3-61)$$

For off design and transient calculation, the mass flow rate $W2$ at exit could be estimated through the Eq.3-62

$$W2 = \frac{A * p2 * Ve}{R * t2} \quad (3-62)$$

Then net thrust FN is given by taking into account the momentum drag MD from the intake,

$$FN = W2 * Ve - MD + A * (p2 - Pamb) \quad (3-63)$$

Finally, the specific fuel consumption SFC is calculated from

$$SFC = \frac{Wf}{FN} \quad (3-64)$$

3.2.4 Control System

There are two kinds of fuel schedules in TurboCycle. The first one is the direct transient fuel schedule without any control. The second one is the PID control with steady state fuel schedules. In this paper, all simulations were carried out based on a direct transient fuel schedules by switching off the control system.

The engine control system in TurboCycle is a simple control loop mechanism PID Control. The PID system here is to determine the fuel flow for engine transient manoeuvre by applying the Proportional constant CP , integral constant CI and differential constant CD and speed error. The control system logic chart is shown in figure 3-7.

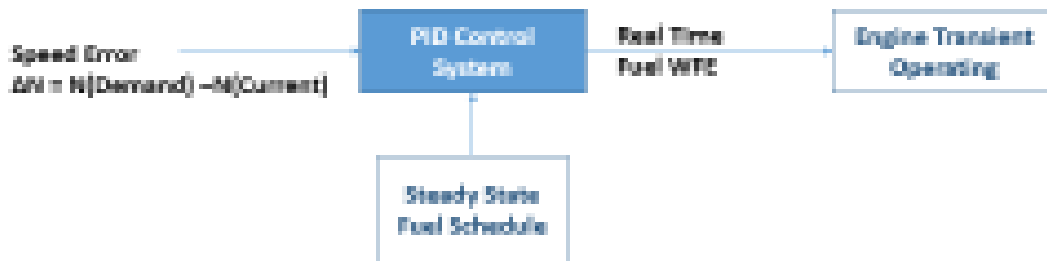


Figure 3-7 Engine PID Control System Work Flow Chart

The modulated fuel flow caused by proportional term is calculated through the Eq.3-65

$$\Delta Wf_P = C_P * (N_{Demand} - N_{Current}) \quad (3-65)$$

While the integral fuel is given as

$$\Delta Wf_I = C_I * \int (N_{Demand} - N_{Current}) dt \quad (3-66)$$

Then the differential fuel is calculated from

$$\Delta Wf_D = C_D * \frac{d(N_{Demand} - N_{Current})}{dt} \quad (3-67)$$

Finally, the real time fuel flow is given through

$$Wf_{transient} = Wf_{steady} + \Delta Wf_P + \Delta Wf_I + \Delta Wf_D \quad (3-68)$$

The PID control system could also be inactive in case of a need.

3.2.5 Numerical approaches

In this chapter, the numerical approaches employed for the engine performance simulation are explained.

Multi-Variable Newton Raphson Method

For engine steady state and transient performance simulation, a multi-variable Newton Raphson method has been applied.

The engine operating point is represented by engine model state vector, which is defined by the state variables. For a single spool turbojet engine shown in figure 3-8, there are three variables to determine the engine steady state performance when a target spool speed N_r is given, which is shown in Eq.3-69,

$$\bar{V} = \begin{bmatrix} V1 \\ V2 \\ V3 \end{bmatrix} = \begin{bmatrix} \beta_c \\ \beta_t \\ T_4 \end{bmatrix} \quad (3-69)$$

Where they are beta values of compressor and turbine, and combustor outlet temperature respectively.

$$\bar{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \quad (3-73)$$

It is worth mentioning that the number of error functions should always be equal to the number of variables. Then errors functions could also be defined in the same way,

$$\bar{E} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} \quad (3-74)$$

The corresponding Jacobian matrix could be calculated as

$$J = \begin{bmatrix} \frac{\partial E_1}{\partial V_1} & \dots & \frac{\partial E_1}{\partial V_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial E_n}{\partial V_1} & \dots & \frac{\partial E_n}{\partial V_n} \end{bmatrix} \quad (3-75)$$

Thus, the Eq.3-72 could be linearized as

$$J = \frac{\Delta \bar{E}}{\Delta \bar{V}} \quad (3-76)$$

Re-arrange this equation by taking the inverse of Jacobian matrix J, and new running point could be found from the equations below,

$$\bar{V}_{i+1} - \bar{V}_i = \Delta \bar{V} \quad (3-77)$$

$$\bar{V}_{i+1} = \bar{V}_i + J^{-1} \Delta \bar{E} \quad (3-78)$$

Applying the newton iteration process, finally, the state variables for that specific operating point could be iterated as long as the error is small enough.

For the transient simulation, it is very similar to steady state. The difference is the error function terms contains only mass flow error and does not include the power errors any more.

Finally, the performance results could be calculated by substituting the iterated variables into the performance function,

$$PerfReults = PerfFun(V_1, V_2, \dots V_n) \quad (3-79)$$

Euler Explicit method

Because transient simulation is dependent on time, then the Euler explicit method has been employed to solve the power difference during current time interval.

As mentioned before, the mass flow balance with IMF terms could be solved by the Newton Raphson method. When the mass flow error terms are close to zero, the power difference between compressor and turbine on the same shaft SP is given from

$$SP(t_i, N_i) = TurbWork(t_i, N_i) - CompWork(t_i, N_i) \quad (3-80)$$

Thus, the speed increment is derived as

$$\frac{\partial N}{\partial t} = \frac{SP * 3600}{4\pi^2 * I * N_i} \quad (3-81)$$

Finally, the shaft speed for next time interval is given from

$$N_{i+1} = N_i + \frac{\partial N}{\partial t} * dt \quad (3-82)$$

3.3 Heat Soakage Model

In this chapter, a generic heat soakage method would be presented in details, which could be applied to compressor, turbine and combustor. The heat soakage method contains three different modules which are the component geometry, heat transfer and tip clearance, respectively.

3.3.1 Geometry

As the compressor and turbine are turbomachinery components, their geometry is a very similarity to each other. Therefore, the geometry method for compressor and turbine are based on the essential turbomachinery design criterion. However,

this no general criterion for combustor design, and combustor in this paper is regarded as a simple circular duct with double casings.

3.3.1.1 Compressor

The figure 3-9 shows a compressor cutaway diagram, and it contains shafts, discs, blades (rotors and stators) and casing generally.

Therefore, the equivalent compressor geometry model can be made shown in figure 3-10.

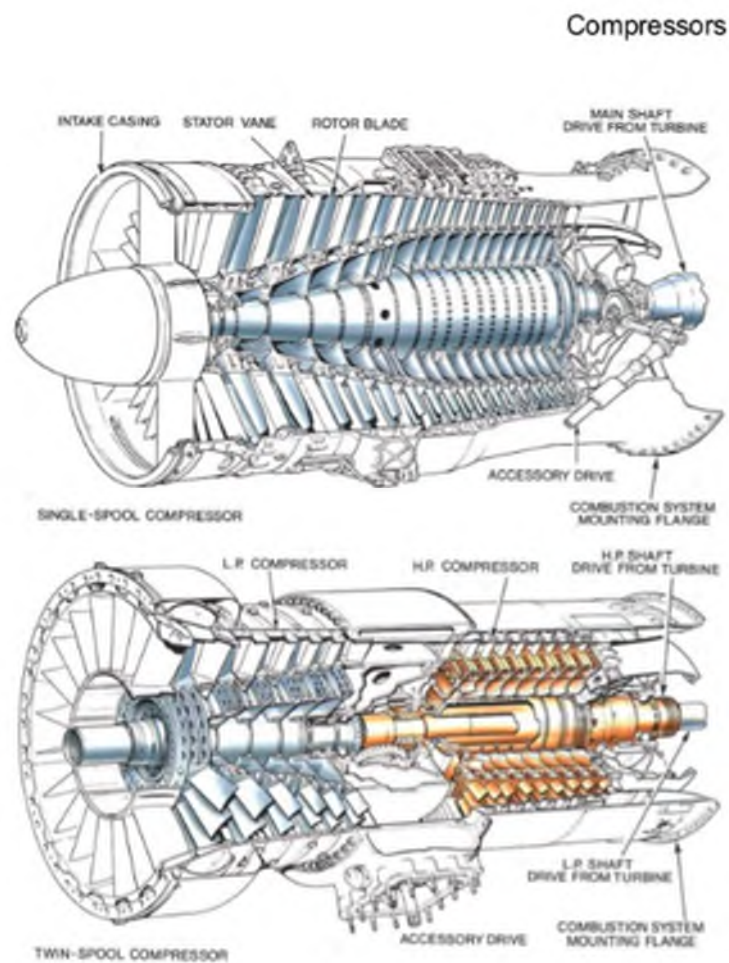


Figure 3-9 Engine compressor cutaway

The compressor configuration keeps the constant tip diameter with large inlet annulus area and small outlet annulus area.

The parameters from engine design point and turbomachinery design criterion are shown in table 3-9 and table 3-10, respectively.

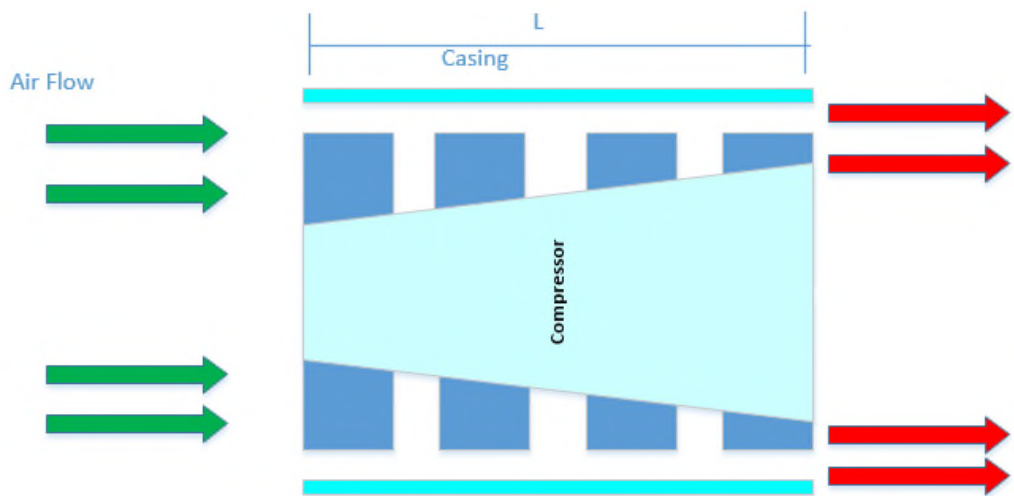


Figure 3-10 Equivalent Engine Compressor Model

Design Point	Shaft speed (100%)	Mass flow rate	Pressure ratio	Fuel to air ratio	Total pressure and temperature
	RPM	W_a	PR	FAR	P_{in} T_{in} P_{out} T_{out}

Table 3-9 Compressor Parameters from Design Point

Maximum tip speed (m/s)	Inlet/Outlet Mach No.	Length per stage (m)	Pressure ratio per stage
400-450	0.3-0.45	around 0.07	1.365

Table 3-10 Turbomachinery Design Criterion of Compressor

According to the design point (DP) performance parameters, such as the RPM, T_{in} , P_{in} , T_{out} , P_{out} , and Turbomachinery essential design criteria such as flow function Q_a , max tip speed, then the simplified compressor geometry can be defined as followings:

Flow function:

$$Q_a = Ma * \sqrt{\frac{\gamma}{R}} * \left(1 + \frac{\gamma - 1}{2} * Ma^2\right)^{\left(-\frac{\gamma + 1}{\gamma - 1}\right)} \quad (3-83)$$

$$\text{Area} = \frac{W\sqrt{T}}{PQ_a} \quad (3-84)$$

Then the inlet flow area and outlet flow area can be estimated from equation (3-84) by using corresponding T, P and Q, respectively.

$$R_{tip} = U_{tmax} / \left(2 * \text{RPM} * \frac{\text{PI}}{60}\right) \quad (3-85)$$

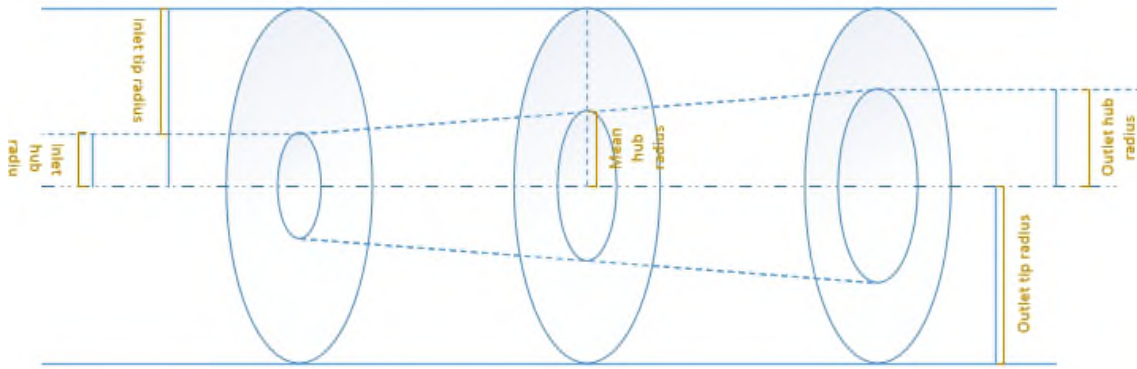


Figure 3-11 Compressor Annulus Diagram

Then tip radius can be calculated from equation (3-85) by using max tip speed from the design point. In this case, the tip diameter is constant. Therefore inlet tip radius R_{tin} is equal to outlet tip radius R_{tout} , and the front area A_{tin} is equal to the rear area A_{tout} .

$$A_{tin} = \text{PI} * R_{tin}^2 \quad (3-86)$$

$$A_{tout} = \text{PI} * R_{tout}^2 \quad (3-87)$$

$$A_{in} = A_{tin} - A_{hin} \quad (3-88)$$

$$A_{out} = A_{tout} - A_{hout} \quad (3-89)$$

According to Eq.3-86 to Eq.-89, then A_{hin} and A_{hout} can be determined.

$$A_{hin} = \pi * R_{hin}^2 \quad (3-90)$$

$$A_{hout} = \pi * R_{hout}^2 \quad (3-91)$$

Thus inlet hub radius R_{hin} and outlet hub radius R_{hout} can be determined.

It is assumed that the area between the inlet and outlet is linear distributions.

Then the mean annulus area tip diameter R_{tm} can be defined as below

$$R_{tm} = 0.5 * (R_{tin} + R_{tin}) = R_{tin} \quad (3-92)$$

Thus mean annulus area hub diameter R_{hm} is also defined by the same way

$$R_{hm} = 0.5 * (R_{hin} + R_{hout}) \quad (3-93)$$

It is assumed that state of the art compressor stage pressure ratio is 1.365 (GE Aviation, 2019), then the number of stage of compressor can be estimated by

$$N_{stage} = \text{Ceiling} [\log_{1.365} PR] \quad (3-94)$$

It is also assumed that the average stage length of compressor is 0.0657m (GE Aviation, 2019), then the total length is estimated as

$$L = N_{stage} * 0.0657 \quad (3-95)$$

The mean annulus area of disc A_m can be calculated as

$$A_m = \pi * R_{hm}^2 \quad (3-96)$$

As the blade bottom and disc surface are connected, there the coefficient (around 0.5 to 0.8) is employed to describe the ratio of interference area to total area.

Therefore, the disc effective wetted area could be expressed as below

$$A_{ds} = \pi * (R_{hin} + R_{hout}) * L * \text{Coefficient} \quad (3-97)$$

Disc and blade effective inter-connected surface area can be written as below

$$A_{dbs} = \pi * (R_{hin} + R_{hout}) * L * (1 - \text{Coefficient}) \quad (3-98)$$

Casing surface area is given by

$$A_{cs} = \pi * 2 * R_{tin} * L \quad (3-99)$$

Mean annulus blade chord is estimated by

$$\text{Chord} = (R_{tm} - R_{hm}) / AR \quad (3-100)$$

Where AR is aspect ratio (input or given)

Blade row space is calculated from

$$\text{Space} = SR * \text{Chord} \quad (3-101)$$

Where SR is Space chord ratio (input or given)

Number of blades per stage can be estimated by

$$n_b = \pi * (R_{tm} + R_{hm}) / \text{Space} \quad (3-102)$$

Total effective blades area through the primary flow path is given by

$$A_{bs} = N_{stage} * N_b * 2 * 2 * \text{Chord} * (R_{tm} - R_{hm}) \quad (3-103)$$

Where the first term 'two' means the blade sheet double side surface, the second term 'two' means the sum area of rotor and stator blades.

Volume and Mass

Total volume of blades is given as

$$V_b = A_{bs} * \text{Thick}_b \quad (3-104)$$

Total mass of blades is calculated by

$$M_b = \rho_{hob} * V_b \quad (3-105)$$

Where the density of blade ρ_{hob} and thickness are inputs.

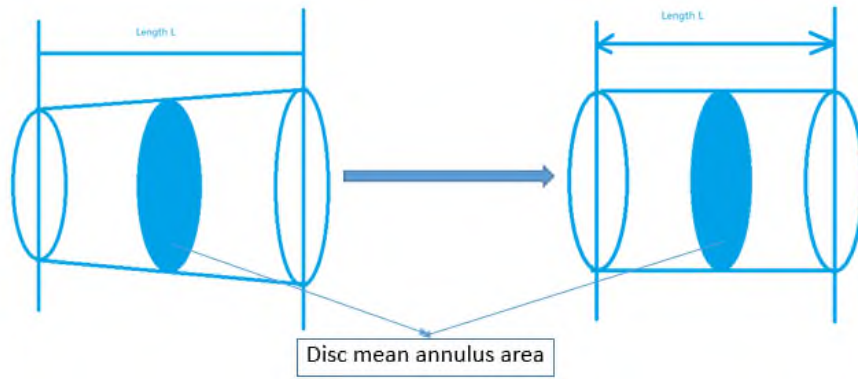


Figure 3-12 Compressor Equivalent Annulus Diagram

As the volume of the disc is not regular, the best way is to calculate by integrals. However, using the equivalent uniform volume is the simplest way with roughly the same results.

Total volume of disc can be described as

$$V_d = A_m * L \quad (3-106)$$

Total mass of disc is determined by

$$M_d = \rho_{hod} * V_d \quad (3-107)$$

Total volume of casing is calculated from

$$V_c = A_{cs} * Thick_c \quad (3-108)$$

Total mass of casing is estimated by

$$M_c = \rho_{hoc} * V_c \quad (3-109)$$

3.3.1.2 Turbine

Like a compressor, turbine contains similar casing, blades, shaft and disc, and the only difference is the inlet area is much smaller than the outlet area. Therefore, the equivalent turbine geometry model could be simplified, as shown in figure 3-13. The turbine configuration keeps a constant tip diameter with decreasing hub diameter.

The turbine parameters from the design point and turbomachinery design criterion for the turbine are shown in table 3-11 and table 3-12, respectively.

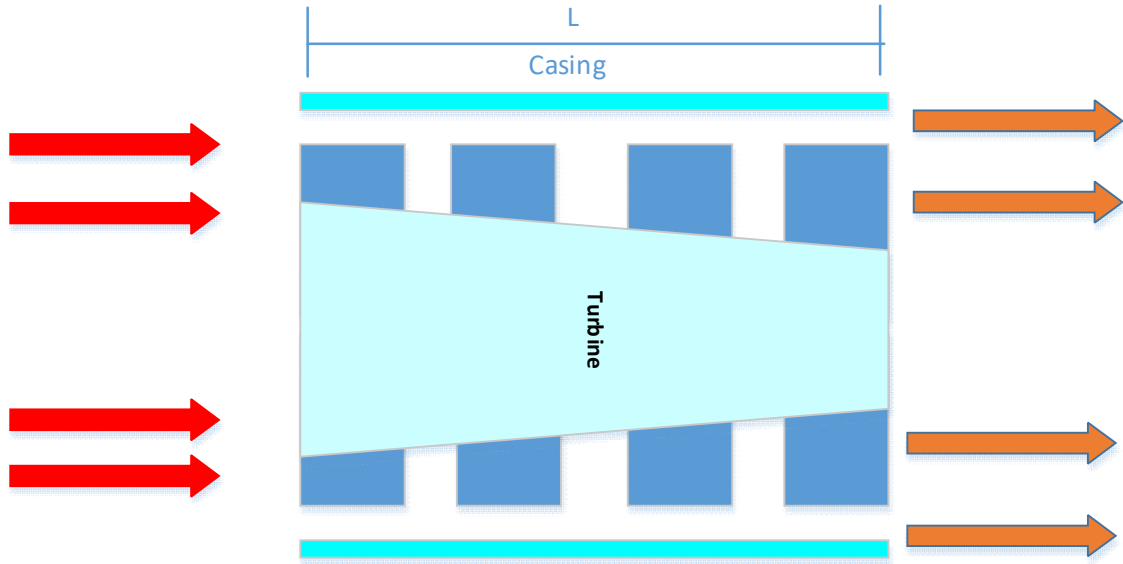


Figure 3-13 Equivalent Engine Turbine Model

Design Point	Shaft speed (100%)	Mass flow rate	Enthalpy Drop	Fuel ratio	air	Total pressure and temperature
	RPM	Win	DH	FAR		Pin Tin Pout Tout

Table 3-11 Turbine Parameters from Design Point

Turbine parameters are roughly the same as the compressor, but the only different one here is the enthalpy drop DH across inlet to outlet.

Maximum tip speed (m/s)	Inlet/Outlet Mach No.	Length per stage (m)	Stage Loading
400-430	0.3-0.5	Around 0.07	Around 2.2

Table 3-12 Turbomachinery Design Criterion of Turbine

The turbomachinery design criteria for the turbine is also slightly different compared with the compressor. The number of stage of the turbine is determined by using the stage loading instead of stage pressure ratio. Therefore, each single stage enthalpy drop ΔH is given from Eq.3-110,

$$\Delta H = \frac{U_{tip}^2}{StageLoading} \quad (3-110)$$

Then the number of stage is calculated by the ceiling function

$$n_{stage} = \text{Ceiling}\left(\frac{\Delta H}{\Delta H_{stage}}\right) \quad (3-111)$$

Where the Ceiling function is to estimate the least number of the stages required to achieve the required to achieve the required total enthalpy drop for turbines.

Except for the number of stages, the rest of the procedure of turbine geometry calculation is exactly the same as the compressor. Therefore, the method of compressor could be applied here for turbine geometry.

Applying the same procedure of compressor and replacing Eq.3-94 by Eq.3-111, then the turbine geometry of area and mass could be estimated as well.

3.3.1.3 Combustor

The modern combustor of the aircraft engine is evaluated from multiple combustion chambers to the annulus combustion chamber. It consists of a single flame tube formed in a ring, which is located between inner and outer casing shown in figure 3-14.

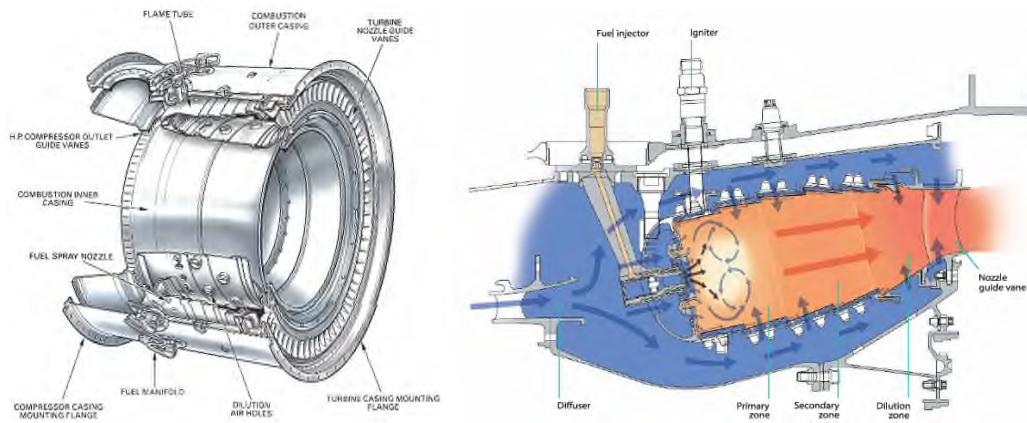


Figure 3-14 Annulus Combustion Chamber (Rolls-Royce, 2015)

Hence, the equivalent combustor configuration could be defined in figure 3-15. The combustor contains diffuser part at the head and primary zone in the rear with the inner and outer casing. The combustion liners are at the inlet of the primary zone.

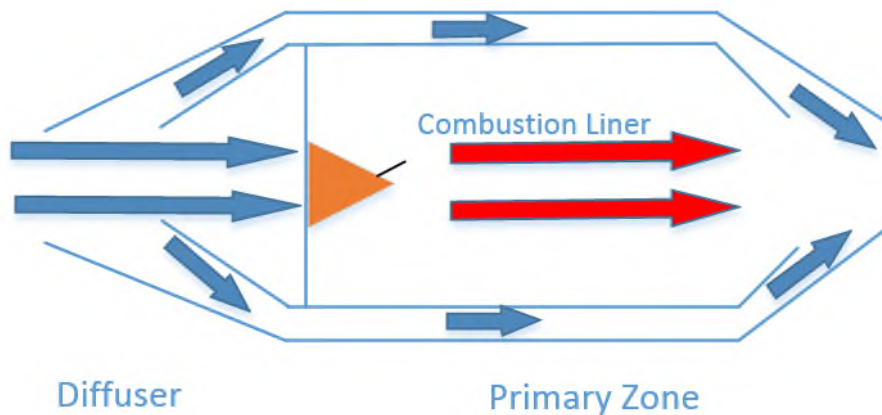


Figure 3-15 Annulus Combustor Equivalent Configuration

As the shape of the combustor equivalent configuration is still a little complicated for weight and area calculation, the simplified combustor configuration has been come up with, which is shown in figure 3-16.

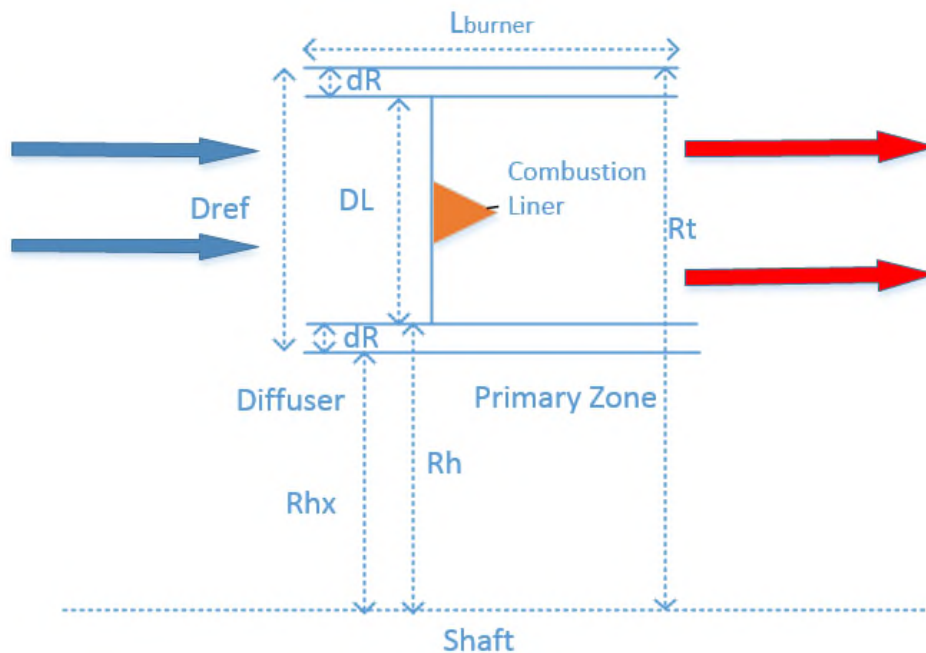


Figure 3-16 Annulus Combustor Simplified Configuration

The primary dimensions of simplified combustor have also been marked in figure 3-17. The hub diameter R_h is defined as the distance between the lower inner casings and the shaft and the tip radius R_t is the distance between the upper outer casing and the shaft. It is assumed that the distance dR between the outer casing and the inner casing is twenty percent of hub radius R_h , which is same as the hub radius of the upstream compressor.

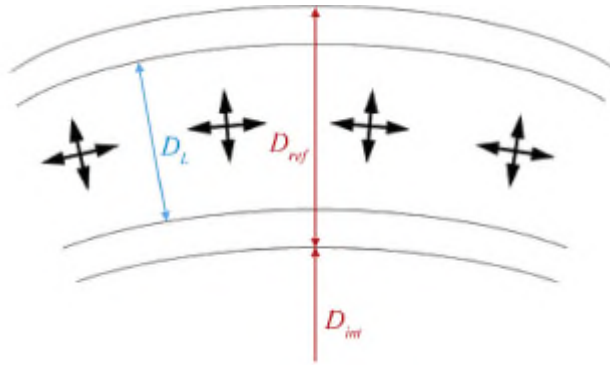


Figure 3-17 Annulus Combustor Inlet Configuration (Priyant Mark & Selwyn, 2016)

According to the combustor inlet configuration shown in figure 3-18, the combustion liner length is defined as D_L , and inlet radius of the combustor is D_{ref} .

Combustor parameters from the design point for geometry calculation are shown in the table below,

Design Point	Mass flow rate	Total pressure and temperature	Fuel air ratio
	Win	Pin Tin	FAR

Table 3-13 Combustor Parameters from Design Point

The inlet of combustor reference area A_{ref} could be calculated by Eq.3-112

$$A_{ref} = \left[\frac{RG}{2} \left(\frac{\dot{m}_3 T_3^{0.5}}{P_3} \right)^2 \frac{\Delta P_{3-4}}{q_{ref}} \left(\frac{\Delta P_{3-4}}{P_3} \right)^{-1} \right]^{0.5} \quad (3-112)$$

Where RG is universal gas constant, $\dot{m}_3$ is the inlet mass flow rate, P_3 is total inlet pressure and T_3 is the total inlet temperature.

According to the table shown in 3-14, the pressure drop factor $\frac{\Delta P_{3-4}}{q_{ref}}$ for annular combustor is 20 and the corresponding pressure loss factor is 0.06. By substituting those values into the equation above, then reference area could be determined.

Type of Chamber	$\frac{\Delta P_{3-4}}{P_3}$	$\frac{\Delta P_{3-4}}{q_{ref}}$	$\frac{\dot{m}_3 T_3^{0.5}}{A_{ref} P_3}$
Tubular	0.07	37	0.0036
Tuboannular	0.06	28	0.0039
Annular	0.06	20	0.0046

Table 3-14 Pressure Loss in Combustor (Lefebvre & Ballal, 2010)

The distance R_{hx} between the shaft and lower casing of combustor is given from

$$R_{hx} = R_h - d_R = 0.8R_h \quad (3-113)$$

Then the length of combustion DL liner could be estimated by

$$D_L = \sqrt{\frac{A_{ref} + \pi * R_{hx}^2}{\pi}} - R_{hx} \quad (3-114)$$

Thus the tip diameter of combustor R_t is calculated as

$$R_t = D_L + R_{hx} \quad (3-115)$$

As the modern annular combustor is designed shorter than cannular combustor, the length of annular combustor L_B is roughly two times of the length of its liner. Hence, the length of combustor is given from

$$L_B = D_L * 2 \quad (3-116)$$

It is assumed that the length of combustor diffuser L_D is half of the liner length, then L_D is calculated from

$$L_D = D_L * 0.5 \quad (3-117)$$

Then the length of primary zone is given by

$$L_P = L_B - L_D \quad (3-118)$$

Due to the inner and outer casing configuration, the effective wetted area of the annular combustor is the sum of the area of the inner surface of outer casing (two sides) and the area of the inner and outer surface of inner casing (four sides).

Therefore, the corresponding inner and outer perimeter of the inner and outer casing could be determined respectively from Eq.3-119 to Eq.3-122.

$$\text{InnerPerimeterInnerCasing} = 2 * \pi * Rh \quad (3-119)$$

$$\text{OuterPerimeterInnerCasing} = 2 * \pi * (Rh + D_L) \quad (3-120)$$

$$\text{InnerPerimeterOuterCasing} = 2 * \pi * Rh \quad (3-121)$$

$$\text{OuterPerimeterOuterCasing} = 2 * \pi * Rt \quad (3-122)$$

Then the effective wetted area of inner casing $A_{\text{InnerWetted}}$ and outer casing $A_{\text{OuterWetted}}$ can be estimated through Eq.3-123 and Eq.3-124.

$$A_{\text{InnerWetted}} = (\text{InnerPerimeterInnerCasing} + \text{OuterPerimeterInnerCasing}) * 2 * L_P \quad (3-123)$$

$$A_{\text{OuterWetted}} = (\text{InnerPerimeterOuterCasing} + \text{OuterPerimeterOuterCasing}) * L_B \quad (3-124)$$

It is assumed that the thickness of combustor metal casing $Burner_{th}$ is uniform, then volume of inner casing V_{Inner} and outer casing V_{Outer} could be determined through Eq.3-125 and Eq.3-126, respectively.

$$V_{\text{Inner}} = 0.5 * A_{\text{InnerWetted}} * Burner_{th} \quad (3-125)$$

$$V_{Outer} = A_{OuterWetted} * Burner_{th} \quad (3-126)$$

The reason for multiplying by 0.5 for the inner casing is the effective wetted area is double of the geometric area. Therefore, it should be corrected back to the geometric area for volume calculation.

Finally, the total mass of combustor M_{Burner} is obtained by the volume and density

$$M_{Burner} = (V_{Outer} + V_{Inner}) * Rho \quad (3-127)$$

The total wetted area of combustor A_{Burner} is calculated as the sum of inner outer casing wetted area

$$A_{Burner} = A_{InnerWetted} + A_{OuterWetted} \quad (3-128)$$

As the temperature profiles of diffuser and primary zone are quite different, it is necessary to determine the area of mass of them separately.

Thus, the diffuser mass M_D and primary zone M_P mass are given respectively by the Eq.3-129 and Eq.3-130

$$M_D = (L_D/L_B) * M_{Burner} \quad (3-129)$$

$$M_P = (L_P/L_B) * M_{Burner} \quad (3-130)$$

Similarly, the diffuser wetted area A_D and primary zone wetted area A_P mass are determined respectively by Eq.3-131 and Eq.3-132

$$A_D = (L_D/L_B) * A_{Burner} \quad (3-131)$$

$$A_P = (L_P/L_B) * A_{Burner} \quad (3-132)$$

3.3.2 Heat Transfer

In this chapter, a 1D integral heat transfer method has been developed, including the convection, conduction and radiation applied for three main engine components, which are compressor, turbine and combustor.

3.3.2.1 1D Heat transfer Model Vs 0D

Unlike a traditional 0D method, the 1D integral method would estimate energy transferred by integrating temperature changes from inlet to outlet of the component rather than using the mean average temperature changes. It is more realistic because this method accounts for heat transfer from the surface to surface instead of point to point. The main differences between 1D model and 0D model are summarised as below

1. 0D model does not consider the length of component whereas 1D model does.
2. 1D model would linearize the component wetted area, mass and heat transfer coefficients over the length.
3. 1D model would calculate the energy transferred using integral method from inlet to outlet by considering the length whereas the 0D model calculate energy transfer between the points of averaged value.

3.3.2.2 Turbomachinery

It is assumed that the heat transfer follows the same process in turbomachinery components like compressor and turbine. There are two reasons to support this assumption. The first one is that the compressor and turbine would force the heat transfer due to the high rotational speed, and the second one is that the geometry structures of compressor and turbine are similar. The only difference is that the turbine is in the hot section while the compressor is in the cold one. Besides, it is assumed that no radiation in the turbine section as the turbine entry temperature of the models is relatively lower

The heat transfer model of turbomachinery component is regarded as a single averaged stage component in a control volume, which contains the flow, casing, blade and discs shown in the figure 3-18.

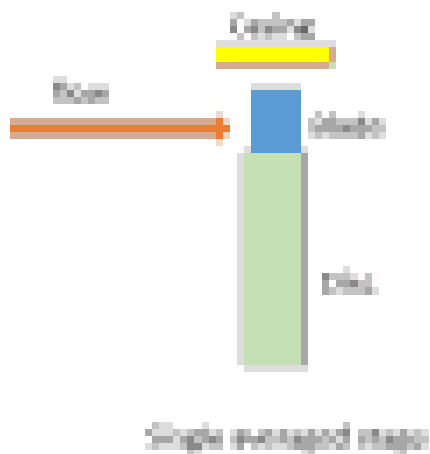


Figure 3-18 Turbomachinery single averaged stage

The traditional 0-D method shown in Figure 3-19 usually would treat each solid component as one point located at the centre of mass. Thus, the temperature at the centre is assumed to be equal to the average temperature of the inlet and outlet. Then the heat energy transferred is calculated through the temperature difference at the centre of the solid component.

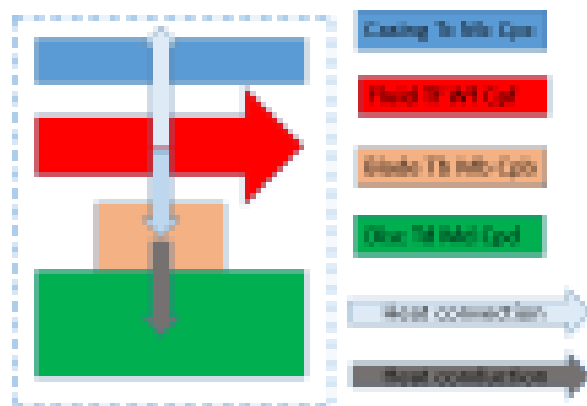


Figure 3-19 Turbomachinery 0-D heat transfer model

However, there are two apparent disadvantages for 0D method. Firstly, mean temperature centre does not coincide with the mass centre of component, and secondly, it is the point to point pattern of heat transfer. Heat transfer is based on the model of the surface to surface.

Hence, the innovative 1D method has been developed to overcome the defects mentioned above, of which the schematic diagram is shown the figure 3-20.

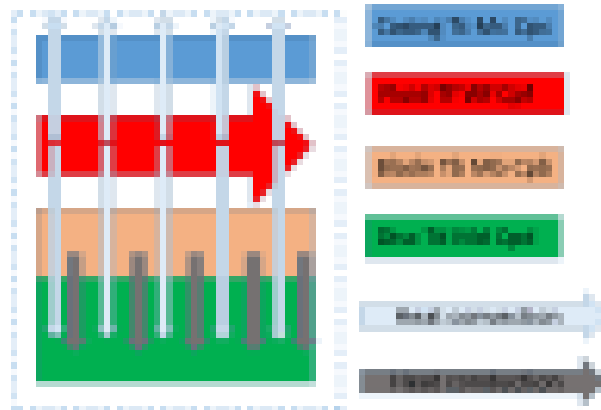


Figure 3-20 Turbomachinery 1-D heat transfer model

The key of the 1D method is to get the profile of temperature, mass and area along the length of the turbomachinery component. The red line shows the temperature distribution along the length of the compressor according to Figure 3-21 shown below. It could be clearly found that the temperature remains the same when crossing the stator of the compressor and rises linearly through the rotor. Therefore, it is assumed that the temperature profile is linear distribution along the length of turbomachinery. Similarly, the area and mass are also considered to be linear, which is not entirely accurate but a reasonable assumption.

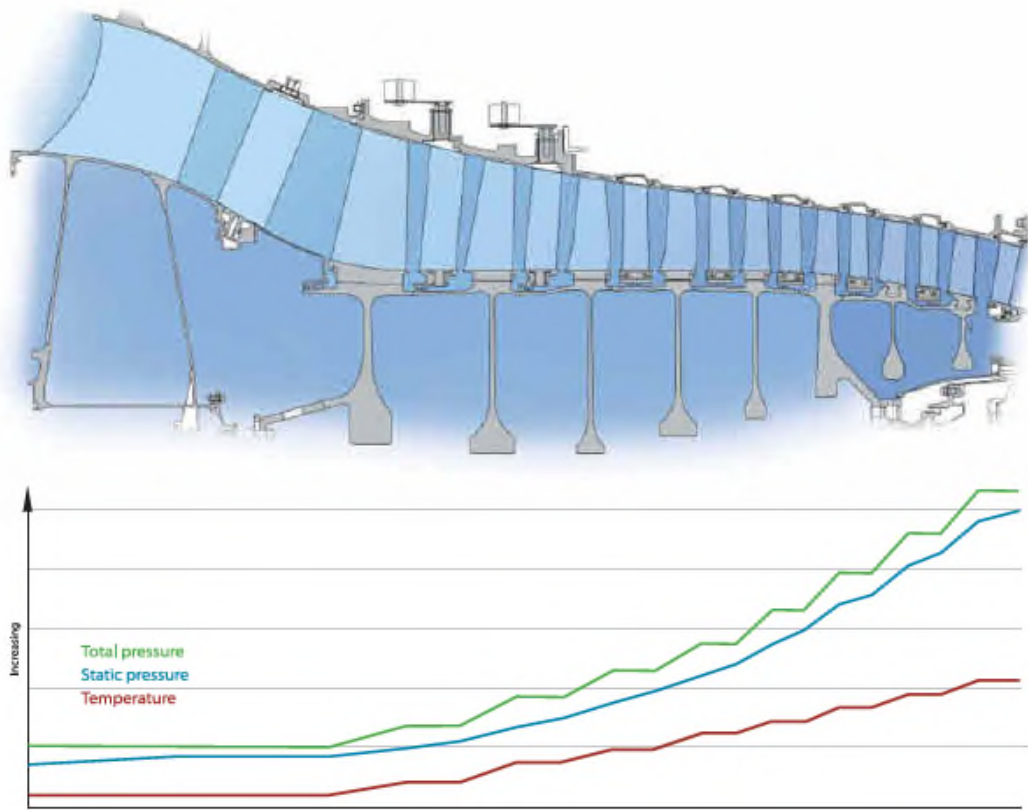


Figure 3-21 Temperature and Pressure Profile of Compressor (Rolls-Royce, 2015)

Heat Convection of Turbomachinery

As 1D heat transfer model would use the integral method, all related parameters should be linearized or non-linearized along the length of the component, which is temperature, mass, area and heat transfer coefficient.

Inlet temperatures of flow and metal are assumed to be equal at the beginning of the transient process (initial steady-state condition). The temperature profile $T_f(x)$ of gas is defined as Eq.3-133

$$T_f(x) = A * x + T_{in} \quad (3-133)$$

$$A = \frac{T_{fout} - T_{in}}{L} \quad (3-134)$$

Where T_{fout} is gas (air) flow outlet temperature, T_{fin} is gas (air) flow inlet temperature and L is the length of the component.

In the same way, the temperature profile of blade $T_b(x)$, casing $T_c(x)$ and disc $T_d(x)$ could be got through the Eq.3-135 to Eq.3-140

$$T_b(x) = B * x + T_{in} \quad (3-135)$$

$$B = \frac{T_{bout} - T_{in}}{L} \quad (3-136)$$

$$T_c(x) = C * x + T_{in} \quad (3-137)$$

$$C = \frac{T_{cout} - T_{in}}{L} \quad (3-138)$$

$$T_d(x) = \frac{T_{dout} - T_{in}}{L} * x + T_{in} \quad (3-139)$$

$$D = \frac{T_{dout} - T_{in}}{L} \quad (3-140)$$

As the mass and area results could obtain from the geometry model in the previous chapter, the mass profile of blade, casing and disc are defined as Eq.3-141 to Eq.3-143

$$M_b(x) = \frac{M_b}{L} * x \quad (3-141)$$

$$M_c(x) = \frac{M_c}{L} * x \quad (3-142)$$

$$M_d(x) = \frac{M_d}{L} * x \quad (3-143)$$

Similarly, the area profile of blade, casing, disc and interference surface between blade and disc are defined as Eq.3-144 to Eq.3-147

$$A_b(x) = \frac{A_b}{L} * x \quad (3-144)$$

$$A_c(x) = \frac{A_c}{L} * x \quad (3-145)$$

$$A_d(x) = \frac{A_d}{L} * x \quad (3-146)$$

$$A_{db}(x) = \frac{A_{dbs}}{L} * x \quad (3-147)$$

Recalling the definition of heat convection, it is defined as Eq3-148

$$\frac{dq}{dt} = h * A * (T_f - T_s) \quad (3-148)$$

Applying the lumped method and conservation of energy, the heat convection could be derived as Eq.3-149

$$q = M_s * C_p * (T_f - T_s) \left(e^{-\frac{\Delta t}{\tau}} - 1 \right) \quad (3-149)$$

Time constant τ is defined as Eq.3-150

$$\tau = M_s * \frac{C_p}{h * A} \quad (3-150)$$

Where M_s is mass of component; C_p is specific heat of component.

According to (Naylor 2004), the heat transfer coefficient h could be determined by the Rolls Royce heat transfer correlations, which would be provided in the Appendix. Here the heat transfer coefficient h is simply described as function of Reynold Number Re , Prandtl Number Pr and Thermal conductivity k .

$$h = f(Re, Pr, k) \quad (3-151)$$

As Reynolds Number is related to flow density and flow viscosity, the heat transfer coefficient should be varied along with the length of the component. Therefore, it is assumed that the heat transfer coefficient is a linear relation along the length of the component. Hence, the heat transfer coefficient function is defined as

$$h(x) = h_{in} + \frac{h_{out} - h_{in}}{L} * x \quad (3-152)$$

Where h_{in} and h_{out} are the heat transfer coefficients of inlet and outlet, respectively.

By substituting temperature profile, mass profile and area profile into Eq.3-153, the heat convection between flow and blade could be written as

$$q_b = \frac{Mb}{L} * x * Cp * \left(\frac{T_{fout} - T_{in}}{L} - \frac{T_{bout} - T_{in}}{L} \right) * x * (e^{-\frac{\Delta t}{\tau}} - 1) \quad (3-153)$$

By rearranging this equation, the local point heat energy via convection q_b is got as

$$q_b = \frac{Mb}{L} * Cp * \left(\frac{T_{fout} - T_{in}}{L} - \frac{T_{bout} - T_{in}}{L} \right) * x^2 * (e^{-\frac{\Delta t}{\tau}} - 1) \quad (3-154)$$

Then the total energy transferred during the time interval Δt via heat convection is estimated by integral from the inlet to the outlet.

$$\int_0^L q_b = \int_0^L \frac{Mb}{L} * Cp * \left(\frac{T_{fout} - T_{in}}{L} - \frac{T_{bout} - T_{in}}{L} \right) * x^2 * (e^{-\frac{\Delta t}{\tau}} - 1) dx \quad (3-155)$$

To solve this integral equation, the numerical integration of rectangular rule was employed, of which the illustration is shown in figure 3-22.

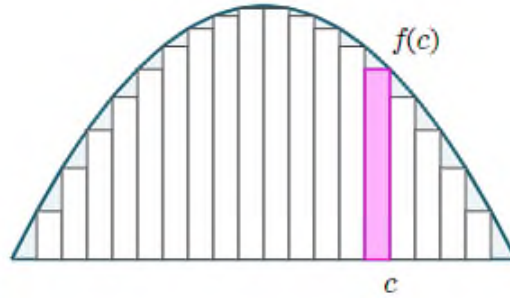


Figure 3-22 Numerical Integration (intmath 2019)

Similarly, the heat energy transferred from or to casing and discs are given from

$$\int_0^L q_c = \int_0^L \frac{Mc}{L} * Cp * \left(\frac{T_{fout} - T_{in}}{L} - \frac{T_{cout} - T_{in}}{L} \right) * x^2 * (e^{-\frac{\Delta t}{\tau}} - 1) dx \quad (3-156)$$

$$\int_0^L q_d = \int_0^L \frac{Md}{L} * Cp * \left(\frac{T_{fout} - T_{in}}{L} - \frac{T_{dout} - T_{in}}{L} \right) * x^2 * (e^{-\frac{\Delta t}{\tau}} - 1) dx \quad (3-157)$$

Heat Conduction of Turbomachinery

As the materials of the turbomachinery component are metals, the heat energy would also transfer through the metal conductivity. The turbomachinery component contains the blade and discs which are physically contacted with each

other. Therefore, the heat conduction between the blade and discs over the touched surfaces should be taken into account.

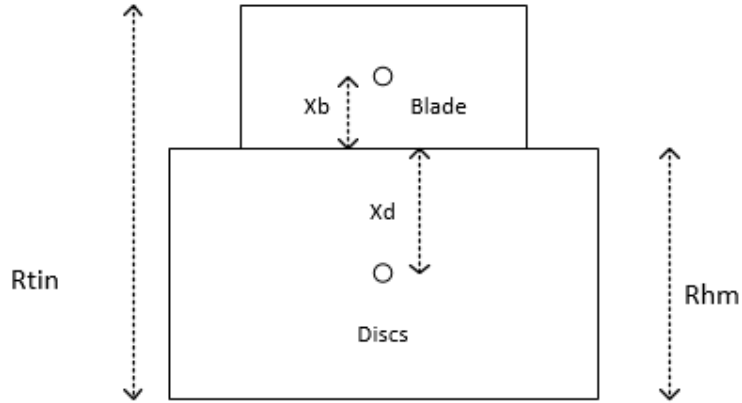


Figure 3-23 Blade and Disc Geometry

According to the Figure 3-23, the centre of blade and disc to the inter-surface distances are determined from Eq.3-158 and Eq.3-159

$$X_b = 0.5 * (R_{tin} - R_{hm}) \quad (3-158)$$

$$X_d = 0.5 * R_{hm} \quad (3-159)$$

From the (Naylor 2004), the inverse of thermal resistance of blade U_b and disc U_d are given as, respectively

$$U_b = k_b * A_{ref} / X_b \quad (3-160)$$

$$U_d = k_d * A_{ref} / X_d \quad (3-161)$$

Where A_{ref} is the contacted surface area of blade and disc; k_b is blade conductivity and k_d is disc conductivity.

The total thermal conductance is determined by Eq.3-162

$$U_{tot} = 1 / ((1 / U_b) + (1 / U_d)) \quad (3-162)$$

It could be rearranged by substituting the Eq.3-160, Eq.3-161 and Eq.3-147 into Eq.3-162

$$U_{tot} = Z * x \quad (3-163)$$

$$Z = \frac{kb * kd / (Xb * Xd)}{\left(\frac{kb}{Xb} + \frac{kd}{Xd}\right)} * Adbs \quad (3-164)$$

The hypothetical steady state temperature T_{ss} of a whole system including blade and disc is defined as

$$T_{ss} = (U_b * T_b + U_d * T_d) / (U_b + U_d) \quad (3-165)$$

By substituting temperature profile function of blade and disc into Eq.3-165, hence the hypothetical temperature could be re-written as

$$T_{ss} = Y * x + T_{in} \quad (3-166)$$

$$Y = \frac{\left(\frac{kb}{Xb} * \frac{T_{bout} - T_{in}}{L} + \frac{kd}{Xd} * \frac{T_{dout} - T_{in}}{L}\right)}{\left(\frac{kb}{Xb} + \frac{kd}{Xd}\right)} \quad (3-167)$$

The conductive thermal constant τ of the whole system is given from

$$\tau = (M_b * C_{pb} + M_d * C_{pd}) / (U_b + U_d) \quad (3-168)$$

In the same way, by substituting the mass and area profile function of blade and disc, hence then conductive thermal constant τ could be derived as

$$\tau = \frac{(M_b * C_{pb} + M_d * C_{pd})}{\left(\frac{kb}{Xb} + \frac{kd}{Xd}\right) * Adbs} \quad (3-169)$$

Then, the quantity of conductive heat exchange along with time is calculated by Eq.3-170

$$q_{bd} = -U_{tot} * \{(T_{ss} - T_d) * \text{time} + [(T_{ss} - T_b) * \tau] * [\text{Math. Exp}(-\text{time} / \tau) - 1]\} \quad (3-170)$$

In order to get integral heat function, the temperature profile function of blade and disc are substituted into the equation above. Then it could be written as

$$q_{bd} = -Zx^2 * \{(Y - D) * \text{time} + [(Y - B) * \text{Tau}] * [\text{Math. Exp}(-\text{time} / \text{Tau}) - 1]\} \quad (3-171)$$

$$B = \frac{T_{bout} - T_{in}}{L} \quad (3-172)$$

$$D = \frac{T_{dout} - T_{in}}{L} \quad (3-173)$$

Finally, the total energy exchange q_{bd} through heat conduction is got by integrating from inlet to outlet shown below

$$\int_0^L q_{bd} = \int_0^L -Zx^2 * \{(Y - D) * \text{time} + [(Y - B) * \text{Tau}] * [\text{Math. Exp}(-\frac{\text{time}}{\text{Tau}}) - 1]\} dx \quad (3-174)$$

Heat Transfer Calculation of Turbomachinery

According to the law of conservation of energy and the law of mass continuity, four energy equations for fluid, blade, casing and disc are given respectively.

$$\int_0^L q_b + \int_0^L q_c + \int_0^L q_d = \int_0^L Wf * dt * Cp_f(t) * [T_f(t + 1) - T_f(t)] \quad (3-175)$$

$$\int_0^L q_b + \int_0^L q_{bd} = \int_0^L Mb(x) * Cp_b * [T_b(t + 1) - T_b(t)] \quad (3-176)$$

$$\int_0^L q_d = \int_0^L Mc(x) * Cp_c * [T_c(t + 1) - T_c(t)] \quad (3-177)$$

$$\int_0^L q_d - \int_0^L q_{bd} = \int_0^L Md(x) * Cp_d * [T_d(t + 1) - T_d(t)] \quad (3-178)$$

Then by substituting the mass functions and temperature functions into those four equations, and they could be re-written as

$$\int_0^L q_b + \int_0^L q_c + \int_0^L q_d = \int_0^L Wf * dt * Cp_f(t) * [A(t + \Delta t) - A(t)] * x dx \quad (3-179)$$

$$\int_0^L q_b + \int_0^L q_{bd} = \int_0^L \frac{M_b}{L} * C_{pb} * [B(t + \Delta t) - B(t)] * x^2 dx \quad (3-180)$$

$$\int_0^L q_c = \int_0^L \frac{M_c}{L} * C_{pc} * [C(t + \Delta t) - C(t)] * x^2 dx \quad (3-181)$$

$$\int_0^L q_d - \int_0^L q_{bd} = \int_0^L \frac{M_d}{L} * C_{pd} * [D(t + \Delta t) - D(t)] * x^2 dx \quad (3-182)$$

There are two methods available to solve those equations efficiently, which are direct algebraic approach and numerical approach. Here, the numerical approach is applied as a preference.

Firstly, the temperature coefficients for the next time interval are initially guessed then iterated by the Newton Raphson method.

Then, the temperatures at the outlet station of the component could be estimated by iterated temperature coefficients shown below

$$T_{fout}(t + \Delta t) = A(t + \Delta t) * L + T_{in} \quad (3-183)$$

$$T_{bout}(t + \Delta t) = B(t + \Delta t) * L + T_{in} \quad (3-184)$$

$$T_{cout}(t + \Delta t) = C(t + \Delta t) * L + T_{in} \quad (3-185)$$

$$T_{dout}(t + \Delta t) = D(t + \Delta t) * L + T_{in} \quad (3-186)$$

As long as time is long enough, the temperature of fluid, blade, casing and disc should be infinity close to each other.

3.3.2.3 Combustor

The fuel is burning at the combustion chamber. As a consequence, the flame which has the highest temperature would produce huge heat radiation flux to the combustor metal surrounding of it.

According to (Priyant Mark & Selwyn, 2016), a typical example of a gas temperature profile in a combustor is given as figure 3-25. It is found that the gas temperature rises to the pick and drops to a certain level linearly. Therefore, it is

assumed that this combustor temperature distribution is linear as well in this paper.

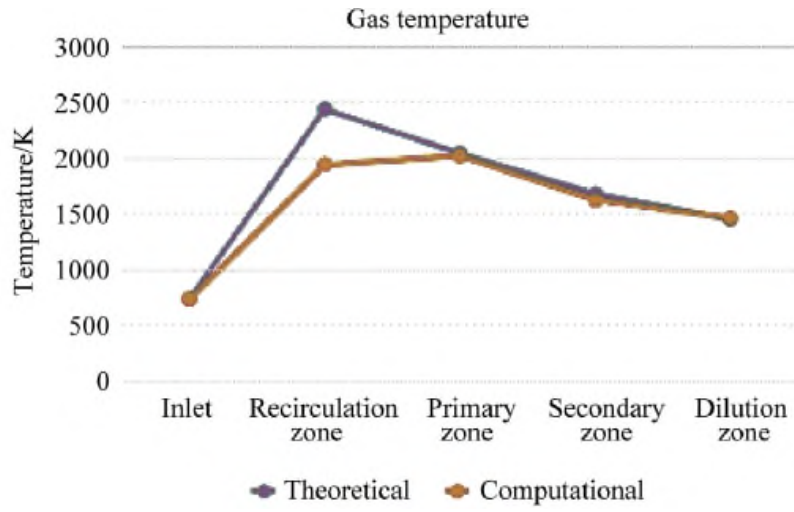


Figure 3-24 Gas Temperature in Combustor (Priyant Mark & Selwyn, 2016)

Due to this temperature distribution, it is reasonable to split combustor into two parts, which are front-body and rear-body, respectively. Therefore, the heat convection and radiation would be calculated separately in those two parts. At the same time, the heat conductivity between those two parts is assumed to be infinite. Therefore, heat conduction could be eliminated in this case.

Heat Convection of Combustor

The heat convection of combustor is very similar to that of the turbomachinery component.

Firstly, the metal temperature functions of front-body T_{Bfb} and rear body T_{Brb} are defined as

$$T_{Bfb}(x) = E * x + T_{Bin} \quad (3-187)$$

$$E = \frac{T_{flame} - T_{Bin}}{L_{fb}} \quad (3-188)$$

$$T_{Brb}(x) = F * x + T_{Bm} \quad (3-189)$$

$$F = \frac{T_{Bm} - T_{Bout}}{L_{rb}} \quad (3-190)$$

Where T_{Bin} and T_{Bout} are inlet and outlet temperature of combustor; L_{fb} and L_{rb} are the length of front body and rear body; T_{Bm} is the local combustor metal temperature close to the flame.

The flame temperature is determined by the function according to (Goodger, 2015) through knowing the fuel-air ratio FAR , inlet air temperature T_{in} , inlet pressure P_{in} and fuel temperature T_{fuel} .

$$T_{flame} = FlameFunction(FAR, P_{in}, T_{in}, T_{fuel}) \quad (3-191)$$

The gas temperature functions for front body T_{ff} , rear body T_{fr} and whole combustor T_f are given as below

$$T_{ff}(x) = G * x + T_{fin} \quad (3-192)$$

$$G = \frac{T_{flame} - T_{fin}}{L_{fb}} \quad (3-193)$$

$$T_{fr}(x) = H * x + T_{flame} \quad (3-194)$$

$$H = \frac{T_{flame} - T_{fout}}{L_{rb}} \quad (3-195)$$

$$T_f(x) = I * x + T_{fin} \quad (3-196)$$

$$I = \frac{T_{fout} - T_{fin}}{L_{fb} + L_{rb}} \quad (3-197)$$

Here the temperature function for the whole combustor is also assumed as linear. The reason why making this assumption is to estimate the overall heat transfer more realistic as gas flow is continuous rather than segmented.

Then, time constants of the front body and rear body are written, respectively

$$\tau_{fb} = M_{fb} * \frac{Cp}{h_{fb} A_{fb}} \quad (3-198)$$

$$\tau_{rb} = M_{rb} * \frac{Cp}{h_{rb}A_{rb}} \quad (3-199)$$

The heat transfer coefficient functions Eq.3-151 and Eq.3-152 for combustor are applied again, then the heat coefficient functions of combustor front body h_{fb} and rear body h_{rb} are given as

$$h_{fb}(x) = h_{in} + \frac{h_m - h_{in}}{L_{fb}} * x \quad (3-200)$$

$$h_{rb}(x) = h_m + \frac{h_{out} - h_m}{L_{rb}} * x \quad (3-201)$$

Where h_m is the local heat transfer coefficient of the critical point between front body and rear body.

similarly, the mass and area functions of fore body and rear body could be described as

$$M_{fb}(x) = \frac{M_{fb}}{L_{fb}} * x \quad (3-202)$$

$$M_{rb}(x) = \frac{M_{rb}}{L_{rb}} * x \quad (3-203)$$

$$A_{fb}(x) = \frac{A_{fb}}{L_{fb}} * x \quad (3-204)$$

$$A_{rb}(x) = \frac{A_{rb}}{L_{rb}} * x \quad (3-205)$$

Finally, applying of lumped capacitance method again, and the convective heat functions of fore body and rear body could be derived as respectively,

$$\int_0^{L_{fb}} qc_{fb} = \int_0^{L_{fb}} \frac{M_{fb}}{L_{fb}} * Cp * (G - E) * x^2 * (e^{-\frac{\Delta t}{\tau_{fb}}} - 1) dx \quad (3-206)$$

$$\int_0^{L_{rb}} qc_{rb} = \int_0^{L_{rb}} \frac{M_{rb}}{L_{rb}} * Cp * (H - F) * x^2 * (e^{-\frac{\Delta t}{\tau_{rb}}} - 1) dx \quad (3-207)$$

Heat Radiation of Combustor

In reality, the heat transfer of radiation takes places in the form of an electromagnetic wave, which is invisible. It is usually related to the black body which could absorb all radiation that falls on its surface. However, in the real world, there is no such a black body, a concept of a grey body is accepted in this paper, which would partly absorb and reflect the radiation.

According to the Stefan-Boltzmann Law, the radiative heat transfer between the flow and grey metal of combustor is defined by

$$qr = \sigma * \varepsilon * A * (T_f^4 - T_B^4) \quad (3-208)$$

Where, σ is Stefan-Boltzmann constant $= 5.56 * 10^{-8} \text{W}/(\text{m}^2 \text{K}^4)$;

ε is emissivity coefficient, typical value around 0.5 ($0 < \varepsilon < 1$) for metal wall;

A is the area of combustor.

By applying the lumped capacitance method for radiative heat transfer, the transient radiative heat equation could be derived as

$$qr = A * \sigma * \varepsilon * (T_f^4 - T_B^4) (e^{-\frac{\Delta t}{\tau}} - 1) \quad (3-209)$$

Therefore, the radiative heat transfer functions of combustor fore body and rear body are given as respectively

$$\int_0^{L_{fb}} qr_{fb} = \int_0^{L_{fb}} A_{fb}(x) * \sigma * \varepsilon * [T_{ff}(x)^4 - T_{Bfb}(x)^4] (e^{-\frac{\Delta t}{\tau_{fb}}} - 1) dx \quad (3-210)$$

$$\int_0^{L_{rb}} qr_{rb} = \int_0^{L_{rb}} A_{rb}(x) * \sigma * \varepsilon * [T_{rf}(x)^4 - T_{Brb}(x)^4] (e^{-\frac{\Delta t}{\tau_{rb}}} - 1) dx \quad (3-211)$$

Heat Transfer Calculation of Combustor

Based on the law of energy conservation, the total heat energy absorbed or released by metal is always equal to heat energy released or absorbed by air or gas flow.

Therefore, the conservation of energy equation of gas flow during each time interval Δt is given by

$$\begin{aligned} \int_0^{L_{fb}} qc_{fb} + \int_0^{L_{rb}} qc_{rb} + \int_0^{L_{fb}} qr_{fb} + \int_0^{L_{rb}} qr_{rb} \\ = \int_0^L W_g * dt * Cp_f * [T_f(x, t + \Delta t) - T_f(x, t)] dx \end{aligned} \quad (3-212)$$

In the same way, the conservation of energy equations of fore body and rear body during each time interval could be written as

$$\int_0^{L_{fb}} qc_{fb} + \int_0^{L_{fb}} qr_{fb} = \int_0^{L_{fb}} M_{fb}(x) * Cp_B * [T_{ff}(x, t + \Delta t) - T_{ff}(x, t)] dx \quad (3-213)$$

$$\int_0^{L_{rb}} qc_{rb} + \int_0^{L_{rb}} qr_{rb} = \int_0^{L_{rb}} M_{rb}(x) * Cp_B * [T_{rf}(x, t + \Delta t) - T_{rf}(x, t)] dx \quad (3-214)$$

Substituting the temperature functions into expressions above, those equations with temperature coefficients could be re-written as

$$\begin{aligned} \int_0^{L_{fb}} qc_{fb} + \int_0^{L_{rb}} qc_{rb} + \int_0^{L_{fb}} qr_{fb} + \int_0^{L_{rb}} qr_{rb} \\ = \int_0^L W_g * dt * Cp_f * [I(t + \Delta t) - I(t)] dx \end{aligned} \quad (3-215)$$

$$\int_0^{L_{fb}} qc_{fb} + \int_0^{L_{fb}} qr_{fb} = \int_0^{L_{fb}} M_{fb}(x) * Cp_B * [G(t + \Delta t) - G(t)] dx \quad (3-216)$$

$$\int_0^{L_{rb}} qc_{rb} + \int_0^{L_{rb}} qr_{rb} = \int_0^{L_{rb}} M_{rb}(x) * Cp_B * [H(t + \Delta t) - H(t)] dx \quad (3-217)$$

The temperature coefficients I , G and H for next time could be iterated by using the Newton Raphson method.

Finally, the corresponding temperatures could be determined by using the iterated temperature coefficients, which are shown below

$$T_{fout}(t + \Delta t) = I(t + \Delta t) * (L_{fb} + L_{rb}) + T_{in} \quad (3-218)$$

$$T_{bm}(t + \Delta t) = G(t + \Delta t) * L_{fb} + T_{in} \quad (3-219)$$

$$T_{bout}(t + \Delta t) = H(t + \Delta t) * L_{rb} + T_{flame} \quad (3-220)$$

3.3.3 Tip Clearance

Compressor and turbine tip clearance could vary on account of thermal expansion or contraction. Beyond that, tip clearance could also change due to the centrifugal force and pressure imbalance. As a result, the changed tip clearance would change the aerothermal characteristics of turbomachinery.

Therefore, the tip clearance model in this paper is divided into two parts which are tip clearance variation and map variation, respectively.

Tip Clearance variation part includes thermal expansion, centrifugal stress expansion and pressure stress expansion, whereas, performance variation is to use the outcome of tip Clearance variation to reflect the deviation on the component map. In addition, the tip clearance model is based on a single average stage configuration.

3.3.3.1 Tip Clearance Variation

A. Thermal Expansion or Contraction (blade, casing and disc)

In this paper, it is assumed that thermal expansion or contraction of blade, casing and disc only would reflect the changes in vertical direction excluding horizontal direction.

As it is a simplified tip clearance model, the temperature used here is the mean temperatures of the blade, casing and disc. The temperature gradients across the width and length are ignored consequently.

According to the definition of the thermal expansion, and deflection δ due to the thermal expansion or contraction is given by

$$\delta(t + \Delta t) = \alpha * L * [T(t + \Delta t) - T(t)] \quad (3-221)$$

α Expansion rate of corresponding material

L The original length of component part

$T(t + \Delta t)$ The mean temperature of component part at the end of time interval

$T(t)$ The mean temperature of component part at the start of time interval

B. Centrifugal Stress Expansion (blade and disc)

According to (Kypyuros & J Melcher, 2003), the radial deflection of discs Δd due to the centrifugal stress is given by

$$\Delta d_{t+\Delta t} = \frac{1}{4 * E_d} * (1 - \nu_d) * \rho_d * R_d^3 * (2 * PI * \frac{N_t}{60})^2 \quad (3-222)$$

E_d Young's modulus of disc

ν_d Poisson's ratio of disc

ρ_d Density of disc

N_t Rotational speed at start of time interval

R_d Unstressed radius of disc

The stress expansion of blade Δb is estimated by the equation according to (Timoshenko & Goodier, 1970)

$$\Delta b_{t+\Delta t} = \rho_b * L_b^2 * (L_b + R_b) / E_b * (2 * PI * \frac{N_t}{60})^2 \quad (3-223)$$

E_b Young's modulus of blade

L_b Length of blade

ρ_b Density of blade

N_t Rotational speed at start of time interval

R_b Unstressed tip radius of blade

C. Pressure Stress Expansion (casing)

The casing expansion due to the pressure could be estimated by obtaining the exterior side and interior side pressures of the casing. The interior pressure acting on the casing could be treated as the pressure of the primary flow, whereas, the

exterior pressure could be assumed to be equal to ambient pressure or bleeding pressure if there is.

Then mechanical grow of casing Δc could be determined by

$$\Delta c_{t+\Delta t} = \Delta P * \frac{R_c^2}{E_c} / Th \quad (3-224)$$

ΔP Pressure difference between interior and exterior side

E_c Young's modulus of disc

N_t Rotational speed at start of time interval

R_c Radius of casing

Th Thickness of casing

The overall growth Δu of blade u_b , casing u_c and disc u_d could be estimated by summing of their thermal growth and mechanical growth

$$u_b(t + \Delta t) = \delta b_{t+\Delta t} + \Delta b_{t+\Delta t} \quad (3-225)$$

$$u_c(t + \Delta t) = \delta c_{t+\Delta t} + \Delta c_{t+\Delta t} \quad (3-226)$$

$$u_d(t + \Delta t) = \delta d_{t+\Delta t} + \Delta d_{t+\Delta t} \quad (3-227)$$

$$\Delta u = u_c(t + \Delta t) - u_b(t + \Delta t) - u_d(t + \Delta t) \quad (3-228)$$

Finally, the real-time tip clearance TC is determined based on the original tip clearance TC_{Nom} and its variation Δu

$$TC(t + \Delta t) = TC_{Nom} + \Delta u \quad (3-229)$$

3.3.3.2 Map Variation

From the (Pilidis & Maccullum, 1986), it is known that the compressor characteristic map could be changed due to the effects of heat transfer and tip clearance variation. As a result, both of speed line and surge margin of compressor map would be altered by the effects mentioned above.

According to their previous research work, it is found that the changes in speed and surge margin are directly determined. Therefore, the outputs of the map are obtained by reading a new speed. If it is accelerating, the new speed is the original speed plus the change in speed, while original speed minus the change in speed for deaccelerating. Its new surge margin is determined based on the original speed. Firstly, the adiabatic surge margin is obtained by reading the map of the original speed line. Then the new surge margin is estimated by a correlation related to the original surge margin.

Theoretically, the map of turbomachinery should always be varied as long as the state of tip clearance variation and heat transfer have not reached in a new equilibrium. Therefore, the map correction method for turbomachinery component has been developed. In this paper, the map correction method for the turbine is assumed to be the same as that of the compressor.

The basic idea is also to obtain a new speed by estimating the change in speed. Then two different groups of mass flow, pressure ratio and efficiency are obtained by reading the map by using new speed and original speed respectively. Finally, the scaling factors of mass flow, pressure ratio and efficiency could be calculated by those two groups. At each time interval, the scaling factors would be updated. Then the real-time map is corrected by those updated scaling factors.

The changes in corrected speed ΔN could be determined by

$$\frac{\Delta N}{N} = C1 * \frac{(T_b - T_f)}{T_f} + C2 * \frac{\dot{Q}}{W_a * C_p * T_f} + C3 * \frac{\Delta u}{TC} \quad (3-230)$$

$$\dot{Q} = \int_0^L q_b + \int_0^L q_c + \int_0^L q_d \quad (2-231)$$

T_b Mean temperature of blade

T_f Mean temperature of flow

$\dot{Q}$ Heat energy transferred to or from flow

W_a Mass flow rate of primary stream

The new speed N_{new} is obtained by

$$N_{new} = N + \Delta N \quad (3-232)$$

The coefficients for low and high spool were obtained from the experimental for a particular engine according to (Maccullum, 1979), which is shown in figure 3-26.

Compressor	C1	C2	C3
LP	-0.07	-0.07	0.3
HP	-0.1	-0.1	0.3

Figure 3-25 Table of coefficients

The scaling factors of mass flow MF_{Scl} , pressure ratio PR_{Scl} , efficiency EFF_{Scl} are given from

$$MF_{Scl} = \frac{MF(N_{new}, Beta)}{MF(N, Beta)} \quad (3-233)$$

$$PR_{Scl} = \frac{PR(N_{new}, Beta) - 1}{PR(N, Beta) - 1} \quad (3-234)$$

$$EFF_{Scl} = \frac{EFF(N_{new}, Beta)}{EFF(N, Beta)} \quad (3-235)$$

3.4 Models Integration

Since all components of heat soakage model have been developed, it is possible to demonstrate the integration of all individual module into a whole system.

Therefore, the transient performance with heat soakage methodology diagram is shown in Figure 3-26.

This architecture of the whole system could clearly illustrate the work procedure between the performance model and the heat soakage model. The red block represents the heat soakage system, including the geometry model marked in green, and heat transfer model in orange and tip clearance in red.

The transient performance simulation is started from steady state performance. Geometry model receives the result of the design point and calculates area and mass of components.

Then heat transfer model would calculate heat exchange between metal and gas flow and vary the temperatures of component and gas flow by using the result from the geometry model and results from the transient performance without heat soakage.

Hence, the tip clearance model could correct the components' map by receiving results of heat transfer model and results of speed change due to the transient process.

Finally, the temperature differences of component and gas flow and corrected maps would be returned to the whole system to reflect the effects of heat soakage phenomena.

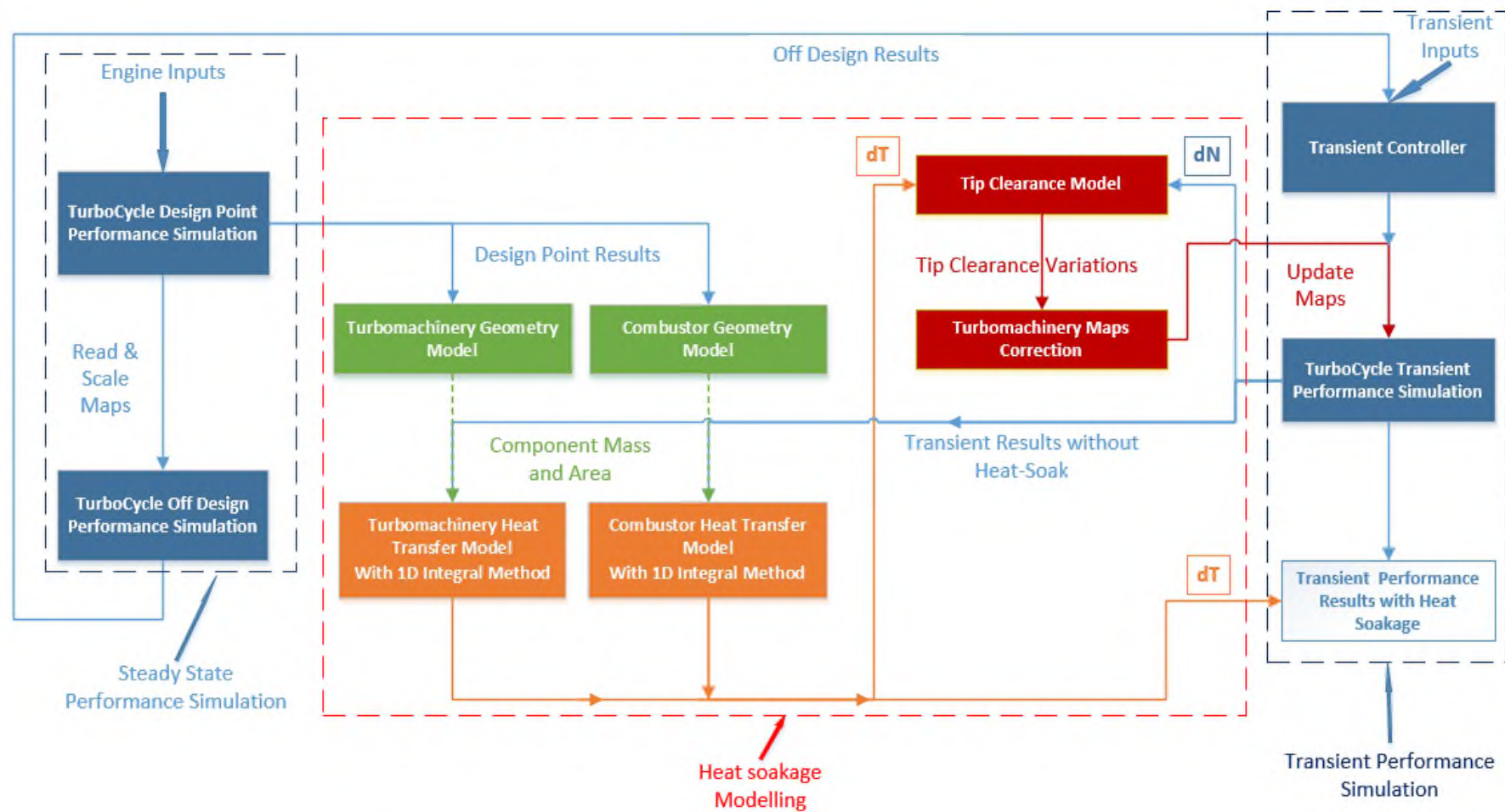


Figure 3-26 Transient Performance with Heat Soakage Methodology Diagram

4 Results and Discussion

In this chapter, the verification of the performance model and heat soakage model have been carried out, respectively. After verification, the case study of heat soakage has been conducted and analysed, and all related results have been discussed in details.

The verification and case study are based on two different engines which are single spool turbojet and twin-spool turbojet, sequentially.

Chapter 4.1 describes the two different engine configurations and their design parameters, severally.

Chapter 4.2 focuses on the verification of the performance model through on those two different engines. The verification has been carried out by the comparison with commercial software. Both steady state and transient performance of the two engine models have been verified.

Chapter 4.3 describes the verification of heat soakage model based on the twin-spool turbojet.

Chapter 4.4 represents the case study of heat soakage effects based on the two different engines.

4.1 Engine Model

To test and validate the in house developed engine performance model and heat soakage model, the integrated whole system has been applied to model single spool and twin-spool turbojet engines.

There are two reasons for selecting those two engines. Firstly, two models could comprehensively check modularity of the system that could potentially be applied to any engine.

The second reason is to avoid any contingency factors may be caused by only one model.

4.1.1 Single Spool Turbojet

The model single spool turbojet engine configuration is shown in figure 4-1. It includes seven real components of intake, compressor, splitter, burner, mixer, turbine, nozzle and two imaginary volumes A and B. The volume A and B would work only at the transient process. They are employed to reflect the effects of mass flow storage during rapid transient manoeuvring since the chamber of combustor and nozzle are relatively large, which could not be ignored at this moment. In addition, cooling air is bleeding from the inlet of the splitter to the outlet of the mixer. It is assumed that the primary flow and cooling air would be mixed finely before entering the turbine rotor blade.

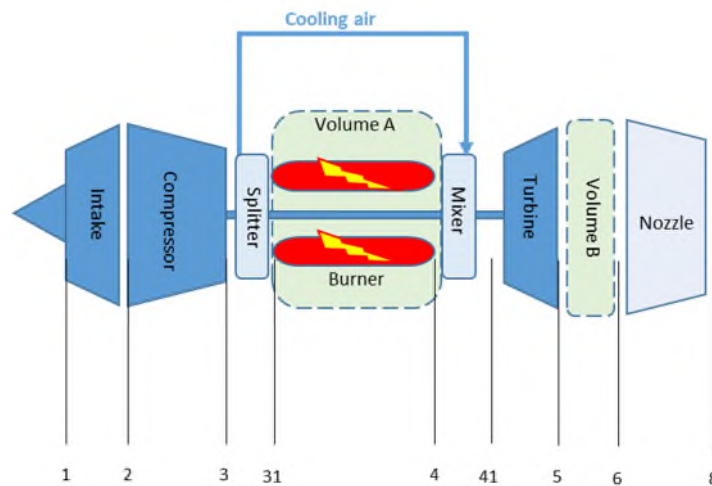


Figure 4-1 Single Spool Turbojet Engine Configuration

The primary design point parameters of the single turbojet engine are shown in table 4-1 at ISA sea level taking off condition.

Parameters	Value	Unit
Air Flow	77.2	kg/s
Compressor Pressure Ratio	8.8	/
Compressor Isentropic efficiency	0.84	
Premas mass flow efficient	0.9	/

Burner Pressure loss	0.95	/
Burner Efficiency	0.99	/
Burner Outlet Temperature	1141	K
Turbine Efficiency	0.87	/
Engine Physical Speed	9000	RPM
Volume A	0.2	m ³
Volume B	0.2	m ³
Net Thrust	47.48	kN
SFC	22.26	g/kN·s

Table 4-1 Primary Parameters of Single Spool Turbojet at Design Point Condition

4.1.2 Twin Spool Turbojet

The schematic configuration of the model twin-spool turbojet engine is shown in figure 4-2. It contains nine actual components and two imaginary components, including volume A and Volume B. Volume A and Volume B are assumed to represent the combustion chamber and nozzle chamber, respectively. The cooling air is also bleeding from inlet of the splitter to outlet of the mixer.

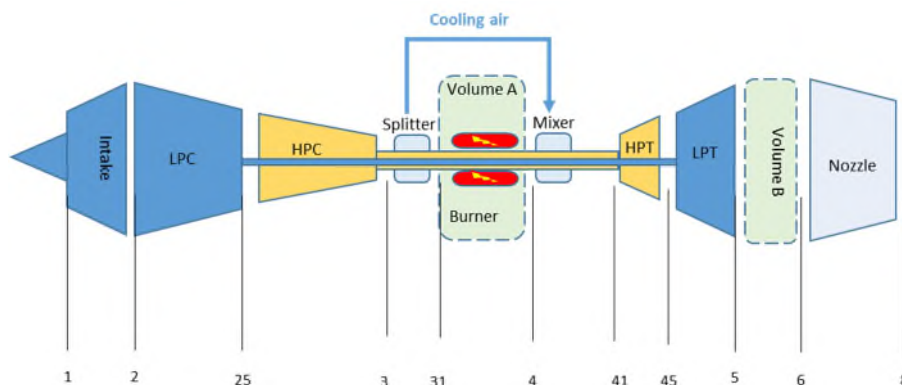


Figure 4-2 Twin Spool Turbojet Engine Configuration

In addition, the main parameters of the twin spool turbojet engine are summarised as in the table 4-2.

Parameters	Value	Unit
Air Flow	77.2	kg/s
LP Compressor Pressure Ratio	4	/
LP Compressor Isentropic Efficiency	0.86	/
HP Compressor Pressure Ratio	5.0	/
HP Compressor Isentropic Efficiency	0.84	/
Premas Mass Flow Coefficient	0.90	%
Burner Pressure Loss	5	%
Burner Efficiency	0.99	/
Burner Outlet Temperature	1150	K
HP Turbine Efficiency	0.89	/
LP Turbine Efficiency	0.90	
Engine Low Spool Physical Speed	9000	RPM
Engine high Spool Physical Speed	14000	RPM
Volume A	0.2	m ³
Volume B	0.2	m ³
Net Thrust	44.30	kN
SFC	18.36	g/kN·s

Table 4-2 Key Parameters of Twin Spool Turbojet at Design Point Condition

4.2 Verification of Engine Performance Model

As the performance model (TurboCycle) is an in house developed code, it needs to be verified before integrating with heat soakage model. One of the efficient ways for verification of TurboCycle is compared to a widely used commercial software GasTurb (Kurzke, 2004) directly. Both steady state performance and transient performance of TurboCycle have been verified.

4.2.1 Verification of Steady State Performance

Model Single Spool Turbojet Engine

In this section, all comparisons are referred to as the model single turbojet jet engine.

Besides using same engine models, it is also vital to keep component maps as same. Beyond that, the location of the design point in the reference map should also be selected as equal.

It does not work correctly in some conditions with the standard component map because of the speed line in a part of the reference map, which may be flat and vertical. At this case, it is impossible to read corresponding values by a given pressure ratio or mass flow shown as the blue area in figure 4-3.

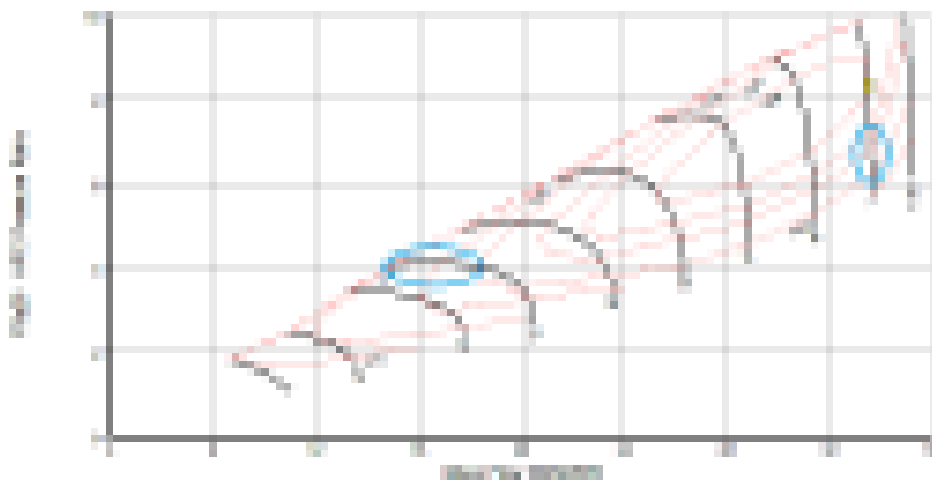


Figure 4-3 Vertical and horizontal Part of Speed Line in GasTurb (Kurzke, 2004)

Therefore, the component map with beta line format has accepted in TurboCycle, which is default format maps of GasTurb.

Consequently, the reference speed lines and beta values are claimed for the model single spool turbojet in the table below.

Reference Map Scaling Parameters	XNR	Beta
Compressor	1.00	0.5
Turbine	1.00	0.5

Table 4-3 Map Scaling of Single Spool Turbojet at Design Point Condition

As simple as possible, all relative corrected speed and beta value are chosen as 1.0 and 0.5 correspondingly.

After those settings, it could make sure all inputs as same as possible, including engine cycle selection, component maps and scaling factors.

Hence, the steady-state performance of two model engines has been simulated and compared comprehensively at standard ISA ground condition.

The engine throttling characteristic is governed by varying the first compressor relative corrected speed XNR1. The XNR1 range is from 70% to 100%.

Firstly, the steady state work lines of the single turbojet are projected on the same compressor map, which is simulated by GasTurb and TurboCycle respectively.

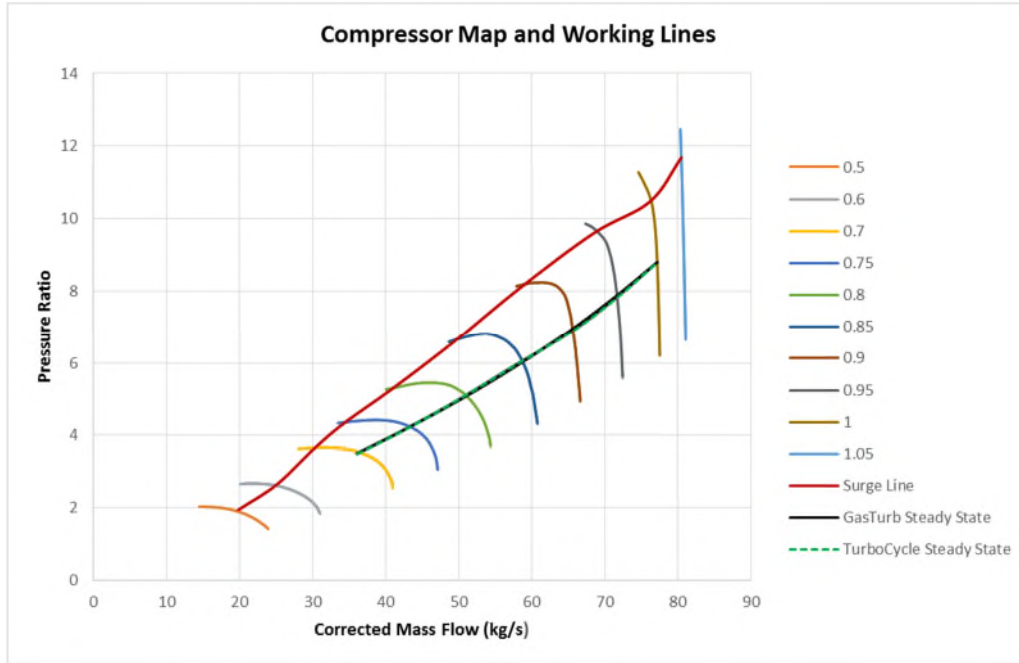


Figure 4-4 Comparison between TurboCycle and GasTurb in Terms of Steady State Work Lines of the Single Spool Turbojet

From figure 4-4, it is clear to find that two lines are almost coincident from 70% to 100% of corrected speed.

That means that firstly, the compressor maps and maps scaling for those two different platforms are kept same and secondly, compressor behaviors are roughly in those two platforms.

Then the numerical results are represented and compared to further validate the TurboCycle in terms of accuracy based on the single spool turbojet.

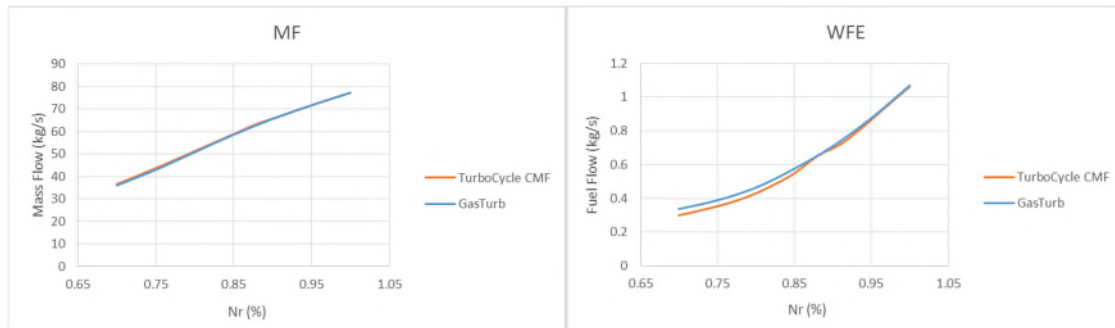


Figure 4-5 Comparison between TurboCycle and GasTurb Based on a Single Spool Turbojet in Terms of Air Flow and Fuel Flow

Figure 4-5 shows the comparison between TurboCycle and GasTurb based on the single spool turbojet in terms of airflow and fuel flow. From the air flow picture, there is no difference to observe, and it could identify the difference in fuel flow. However, the peak difference occurred at the lowest speed is roughly around 5% compared to GasTurb. Therefore, it is acceptable.

Next left picture in figure 4-6 shows a comparison of TurboCycle and GasTurb in compressor pressure ratio. It is easy to see that the two-line almost coincident, which means there is no much difference. The right picture tells that there is a relatively small difference compared with GasTurb at the low-speed range between 70%-80%. Its maximum error is less than 2%.

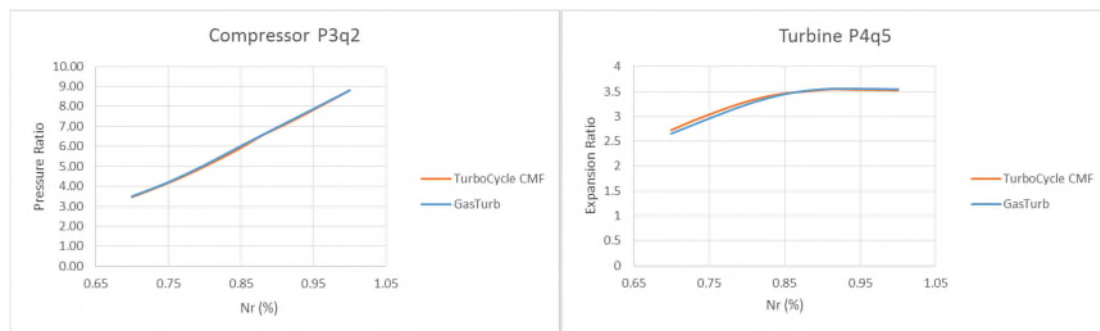


Figure 4-6 Comparison between TurboCycle and GasTurb Based on a Single Spool Turbojet in Terms of Compressor and Turbine Pressure Ratio Respectively

The last group of pictures are illustrations of comparison of thrust and comparison of turbine entry temperature shown in figure 4-7.

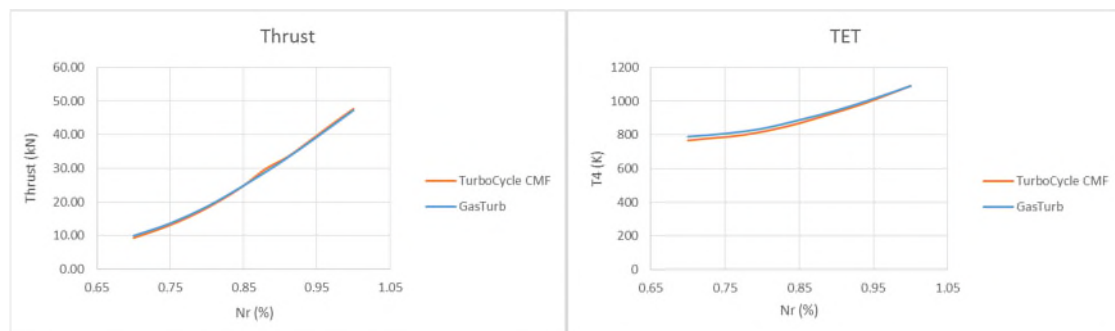


Figure 4-7 Comparison between TurboCycle and GasTurb Based on a Single Spool Turbojet in Terms of Thrust and Turbine Entry Temperature Respectively

According to the left picture of figure 4-7, the thrust calculated by TurboCycle is very close to that by GasTurb. In terms of turbine entry temperature, it is found that there is a small difference at low speed. When the speed goes up, the difference will disappear. However, the maximum error is still very tiny round, 2.5% even at the speed of 70%.

Model Twin Spool Turbojet Engine

In this section, all comparisons between TurboCycle and GasTurb are based on the twin-spool turbojet engine.

The map scaling information for the model twin-spool turbojet are collected in table 4-4 shown below

Reference Map Scaling Parameters	XNR	Beta
LP Compressor	1.00	0.5
HP Compressor	1.00	0.5
HP Turbine	1.00	0.5
LP Turbine	1.00	0.5

Table 4-4 Map Scaling of Twin Spool Turbojet at Design Point Condition

The design point (take off at ISA) of all compressors and turbines are scaled based on the reference point of which relative corrected speed and beta value are 1.0 and 0.5 respectively.

Similarly, the engine throttling characteristic is controlled by changing low spool relative corrected speed XNR1 from 70% to 100%. Hence steady state performance of the model twin-spool turbojet has been run by TurboCycle and GasTurb respectively. Then numerical results from that two software have been gathering and compared again.

First of all, the results of airflow and fuel flow are plotted against low spool relative corrected speed XNR1, which are shown in figure 4-8. According to the two images, there is nearly no difference between TurboCycle and GasTurb.

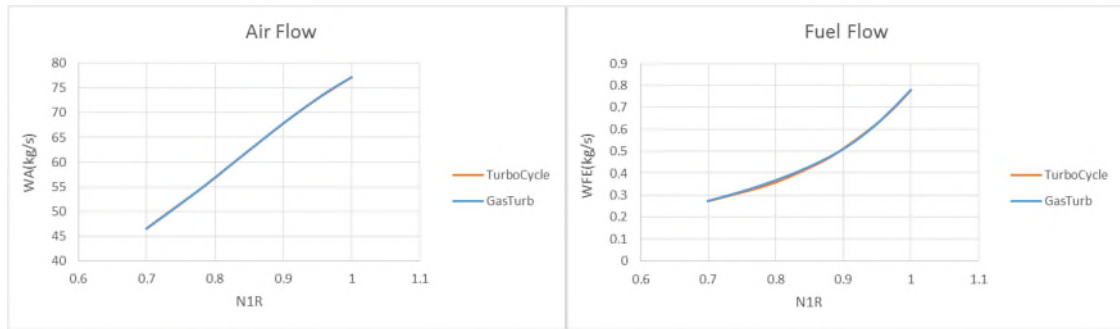


Figure 4-8 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on Air Flow and Fuel Flow

The next a pair of graphs reflects the low-pressure compressor outlet parameters' comparison between TurboCycle and GasTurb. According to the results, it is found that there always is a tiny difference in exit pressure. For exit temperature, a small gap at low speed occurs and disappears when speed is over 80%.

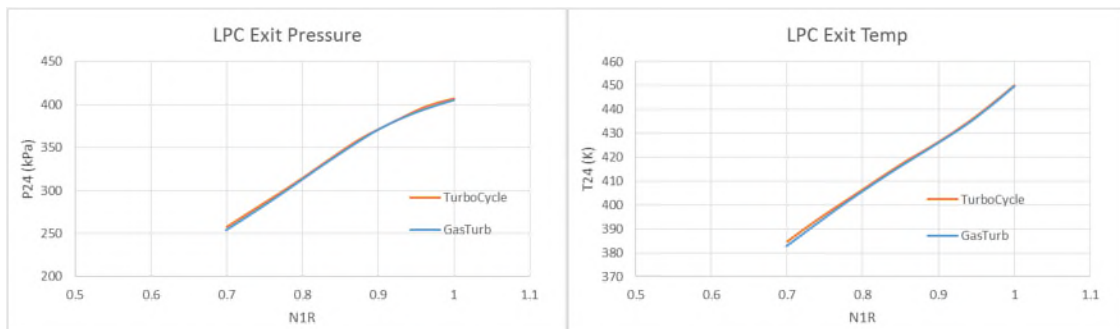


Figure 4-9 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on LPC Exit Pressure and Temperature

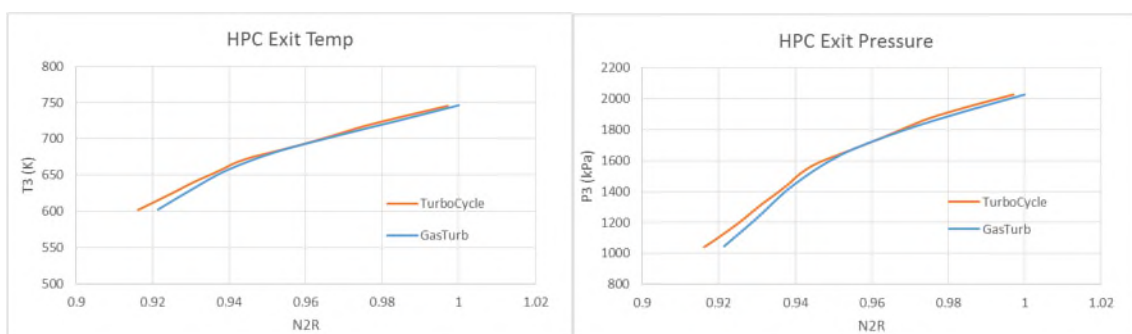


Figure 4-10 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on HPC Exit Pressure and Temperature

The figure 4-10 gives the results of high-pressure compressor exit temperature and pressure against the high spool relative corrected speed XNR2. According to the results, the difference in exit temperature and pressure could be observed. The main difference is caused by the difference between high spool relative corrected speed. It looks like be shifting backwards.

The main reason for that is an error produced in the correction of high spool speed. Because high spool is corrected by its inlet temperature, which would bring in its errors, consequently, the error of corrected speed is produced by the errors in temperature, which comes from the enthalpy equation. However, the maximum error in NXR2 is less than 0.5%, which could be ignored.

Two graphs in figure 4-11 illustrate the comparison of the results of the combustion chamber. The results are plotted against the low spool relative corrected speed XNR1.

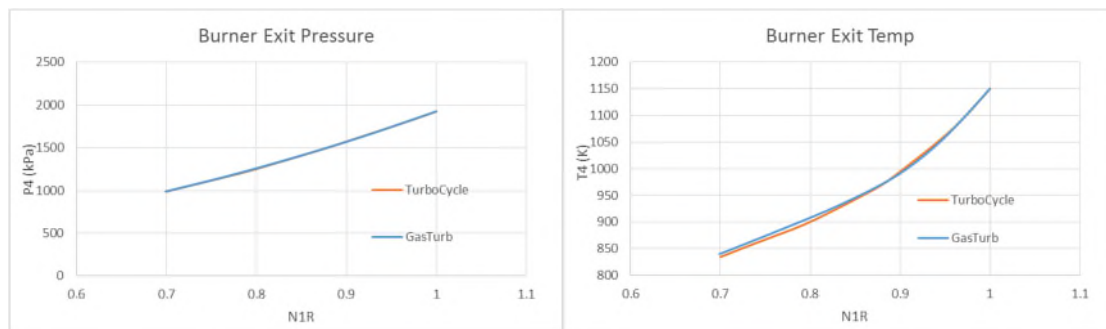


Figure 4-11 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on Burner Exit Pressure and Temperature

From the left picture, it is found that exit pressure got from TurboCycle is very pretty much close to that from GasTurb.

According to the right picture, the exit temperature calculated by TurboCycle is a slight different from that determined by GasTurb from 70% to 80% of corrected speed. After 80% of corrected speed, the difference becomes smaller gradually.

The figure 4-12, including three images, shows the results on high-pressure turbine inlet temperature and outlet temperature and pressure, respectively.

By observing and analysing those three images, it is found that all results got from TurboCycle looks be shifted to left a little bit comparing that from GasTurb. This case is very similar to the high-pressure compressor.

This is because of the same reason about error producing during correcting of high-pressure spool speed. That error is caused by temperature calculation. Both of TurboCycle and GasTurb use entropy function to determine the temperature. However, the iteration process is employed to determine the temperature in the entropy equation at TurboCycle, which might different comparing to GasTurb. That different calculation way in entropy leads the temperature difference, which is the main contributor to the error generated in high speed corrected speed XNR2.

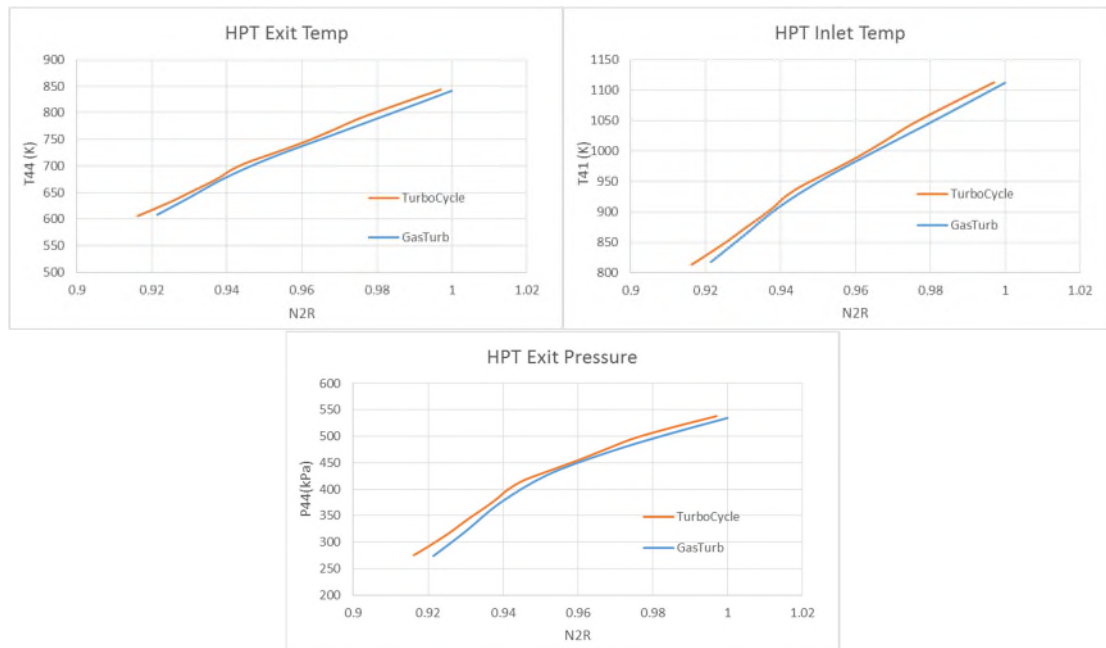


Figure 4-12 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on HPT Inlet and Outlet Temperature and Exit Temperature

Figure 4-13 illustrates the results of the low-pressure turbine on exit temperature and pressure. As the low-pressure turbine is on the low-pressure spool, all results are drawn against low spool relative corrected speed XNR1.

On the view of two pictures, it is easy to see that there is no much difference in low-pressure turbine exit temperature and pressure.

The only difference is majorly at part of the low-speed range, and the error between TurboCycle and GasTurb is less than 0.5%.

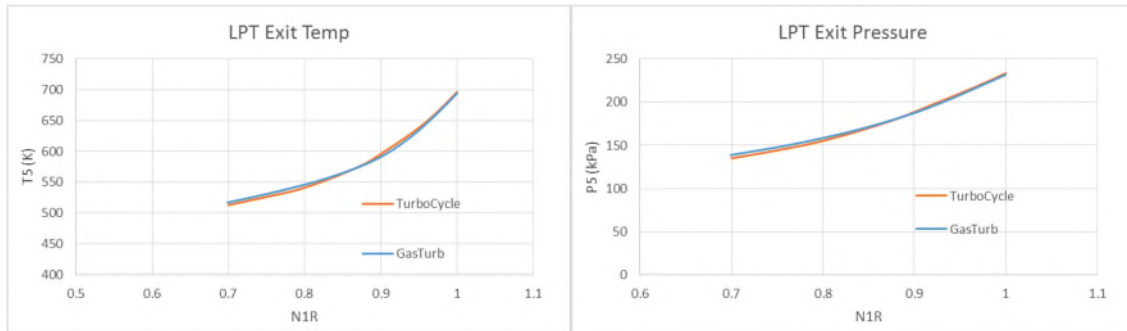


Figure 4-13 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on LPT Exit Temperature and Exit Pressure

The last figure 4-14 shows the result of the comparison of net thrust between TurboCycle and GasTurb.

According to the results, it is clear to see that thrust from TurboCycle is roughly the same as that from GasTurb when speed is over 80% of corrected speed. The thrust difference is less than 1% when speed is below 80% of corrected speed.

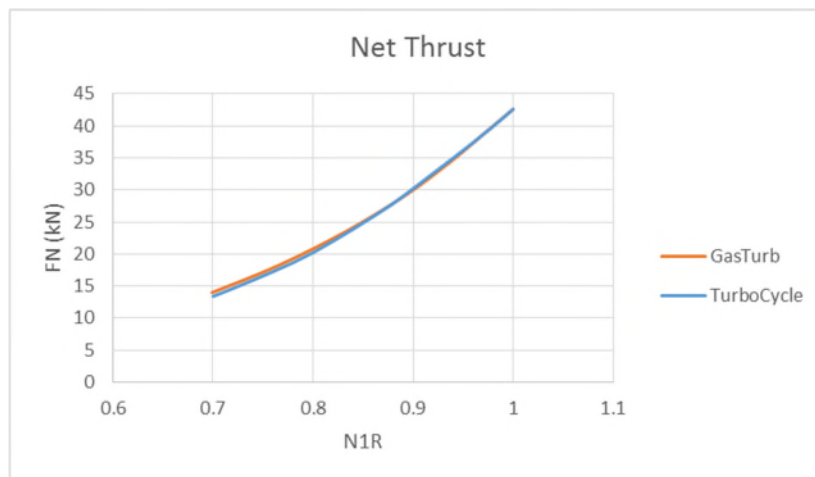


Figure 4-14 Comparison between TurboCycle and GasTurb Based on Twin Spool Turbojet on Net Thrust

Through the performance comparison with GasTurb, it is concluded that the accuracy of TurboCycle is acceptable in terms of steady state performance calculation.

4.2.2 Verification of Transient Performance

As heat soakage effects are simulated based on transient performance, it is essential to validate the transient performance function of TurboCycle. This verification would be implemented based on the single turbojet and the twin-spool turbojet models and compared with GasTurb as well.

As control law or fuel schedule could have a big impact on the engine transient performance for the same engine, it is very necessary to make the control law or fuel schedule as same as possible. Only based on that same control or fuel schedule, it is possible to identify the difference between TurboCycle and GasTurb and whether the TurboCycle could reach the requirement or not.

In transient performance verification, the PI control is not active. Instead, the fuel schedule is directly used, whose format is low spool relative corrected speed with fuel flow.

In addition, an acceleration process is selected for this transient performance comparison, which is from 70% to 100% of corrected at ISA static ground condition. The reason for choosing this process is because this acceleration process is very close to the take-off condition of modern aircraft.

Model Single Spool Turbojet Engine

The fuel flow schedule for the model single spool turbojet is given in figure 4-15.

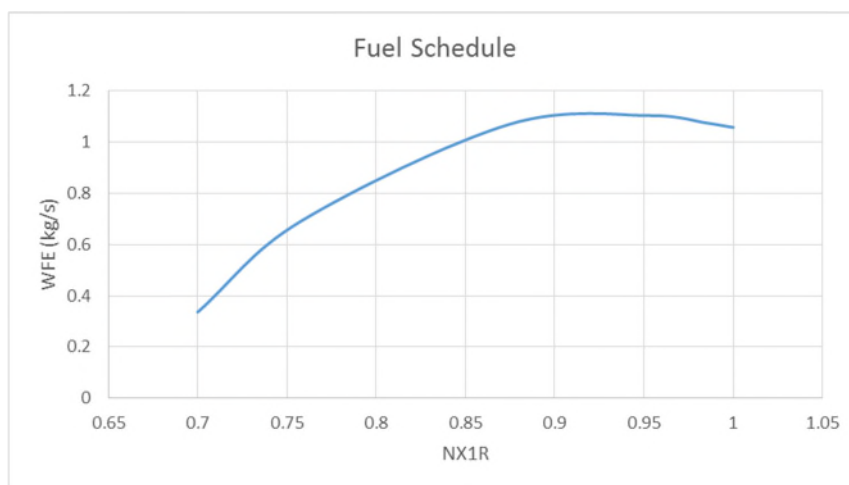


Figure 4-15 Fuel Schedule for the Single Spool Turbojet

Then this take-off acceleration process has been simulated at TurboCycle and GasTurb respectively. Hence, the transient working lines of take-off are plotted together with steady state lines in figure 4-16.

The blue lines are from TurboCycle, while the red lines are got from GasTurb. The red and blue dash line represents the transient work line of GasTurb and TurboCycle, respectively.

According to the comparison of transient working lines, it is found that behaviors of the compressor in two platforms are very close to each other, which shows a good signal for this comparison.

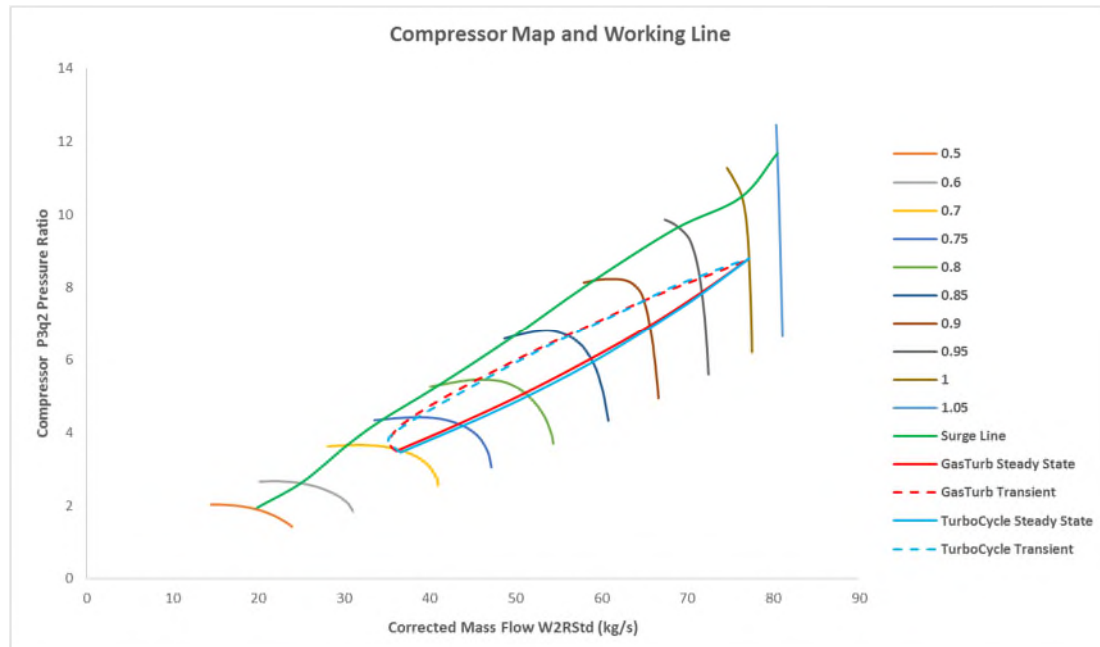


Figure 4-16 Transient Working Lines of the Twin Spool Turbojet

First of all, the transient fuel flow from the two platforms has been shown in figure 4-17. It is obvious to see that there is minimal difference in fuel flow supplying for the twin-spool turbojet in both codes. As fuel flow format in GasTurb is time with the fuel, it is very difficult to keep the fuel flow exactly the same in those two codes. The maximum difference is less than 0.5%. Therefore, it is within a reasonable tolerance.

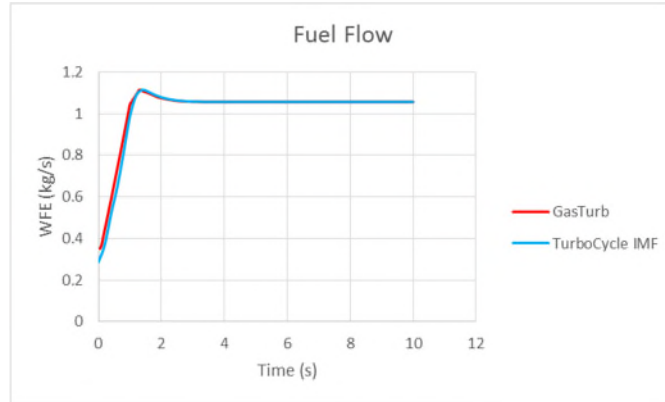


Figure 4-17 Transient Fuel Flow Comparison between TurboCycle and GasTurb

The results of the airflow and compressor pressure ratio are shown in figure 4-18. Based on the left picture, it is known that the mass flow difference appears from 1 to 4 seconds. During this period, the maximum difference reached is less than 3.9%.

The picture on the right-hand side shows the results of the compressor pressure ratio. According to the comparison of the results, it is recognised that the maximum difference on the compressor pressure ratio is less than 3.7%.

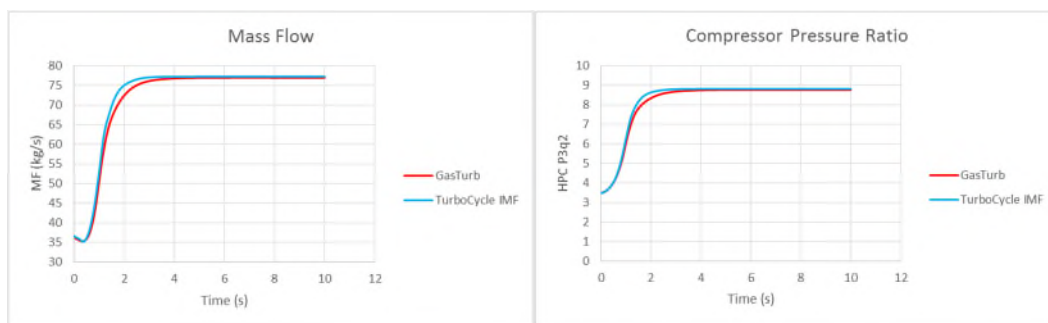


Figure 4-18 Transient Results of Air Flow and Compressor Pressure Ratio

Results of turbine entry temperature and turbine pressure ratio are demonstrated in the next pair of images shown in figure 4-19.

From the left picture, it is known that temperature reaches its peak value around 1 second then decreases gradually. The maximum difference in TET is with 3.1%.

According to the results of the right picture, it is seen that there is always a difference in turbine pressure ratio, and its maximum difference is within 1.7%.

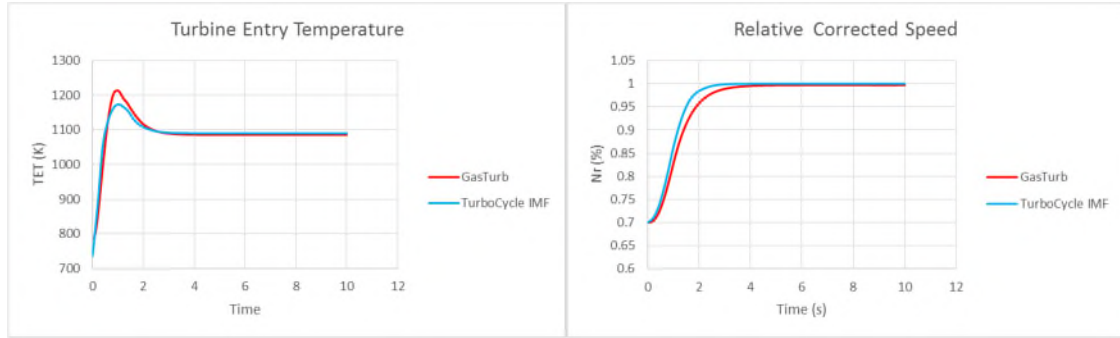


Figure 4-19 Transient Results of Turbine Entry Temperature and Pressure Ratio

The last two pictures in Figure 4-20 illustrate the results of relative corrected speed and net thrust, respectively.

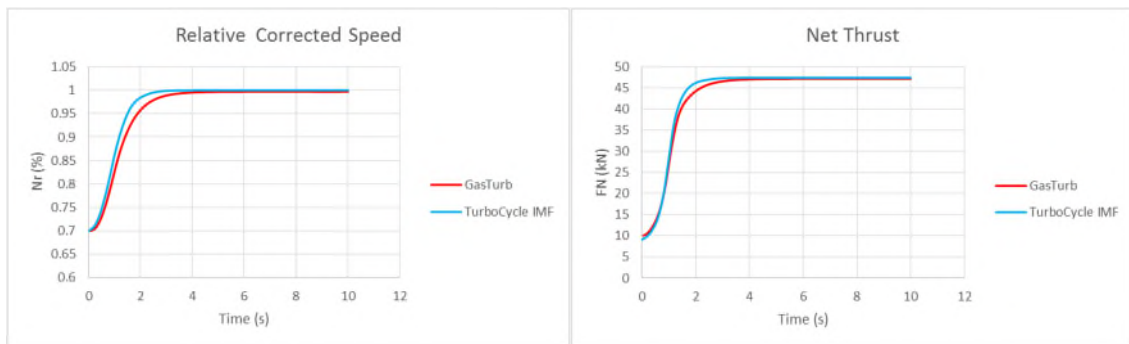


Figure 4-20 Transient Results of Relative Corrected Speed and Net Thrust

According to the results, it could be known that the differences in relative corrected speed and net thrust are around 2.8% and 4.7% respectively.

In addition, the volume effect simulated by TurboCycle Imbalanced Mass Flow (IMF) method is also verified through compared with Inter-Component Volume (ICV) method developed by (Wang, 2016) which has been integrated into TurboCycle.

Hence, the results estimated by IMF and ICV method are against in figure 4-21 in terms of airflow, relative corrected speed, compressor pressure ratio, turbine pressure ratio, turbine entry temperature and thrust.

According to the comparison, it is found that IMF results are quite close to ICV results, and the maximum difference comes from thrust comparison, which is less than 1.5%.

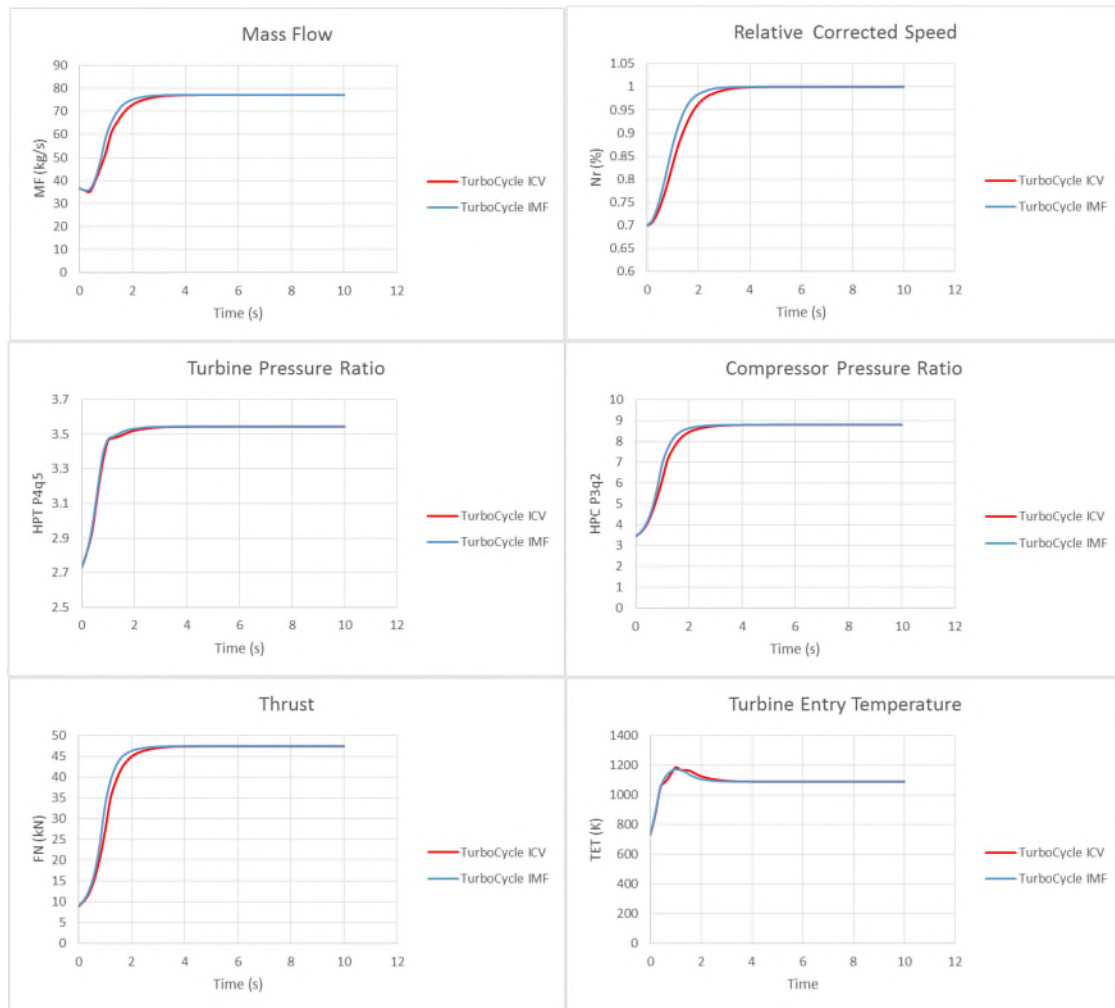


Figure 4-21 Transient Results Comparison between IMF and ICV Method Based on the Single Spool Turbojet

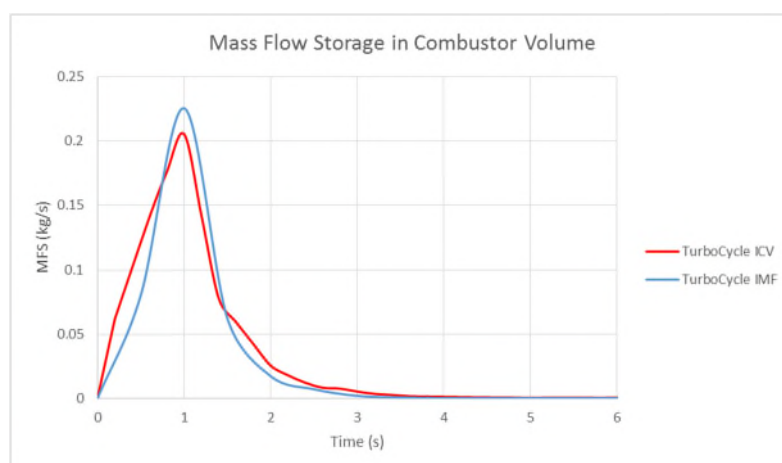


Figure 4-22 Combustor Mass Flow Storage Comparison between IMF and ICV Method

Figure 4-22 shows the mass flow storage comparison in the combustor chamber between the IMF and ICV method.

From the results, it is known that the maximum difference in mass flow storage is around 9.8%. As the quantity mass flow storage is very small compared with airflow, that value of the difference is acceptable as a result.

Model Twin Spool Turbojet Engine

First of all, the fuel flow schedule for the mode twin-spool turbojet is given in figure 4-23, and its format is low spool relative corrected speed against with fuel flow rate.

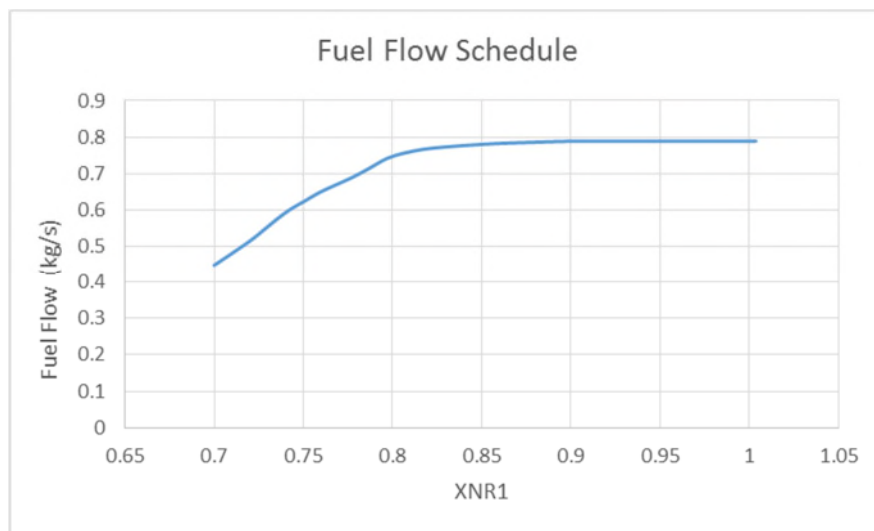


Figure 4-23 Fuel Flow Schedule for the Twin Spool Turbojet

The transient process for the twin-spool turbojet is selected as same as the single spool turbojet, which is the take-off condition (from 70% to 100% of XNR1) at ISA.

Then the take-off acceleration for the twin-spool turbojet has been simulated at TurboCycle and GasTurb respectively. Hence, the numerical results are compared between TurboCycle and GasTurb, which are plotted along with acceleration time.

The first group of two pictures in figure 4-24 shows the airflow results comparison and fuel flow results comparison, respectively.

According to those results, it is hard to tell the difference between TurboCycle and GasTurb regarding airflow and fuel flow.

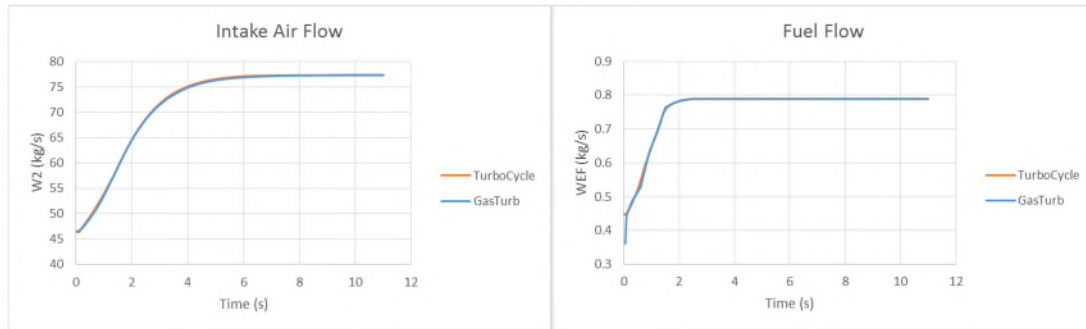


Figure 4-24 Comparison of Fuel Flow and Comparison of Air Flow

Two groups of comparisons are carried out upon low spool, and high spool relative corrected speed shown in figure 4-25.

In the light of comparisons, the difference could be ignored on the low spool relative corrected. It is also found the visible difference in high spool relative corrected speed is lower than 1.7% compared with GasTurb. The main contributor to this error is also the different temperature calculation method.

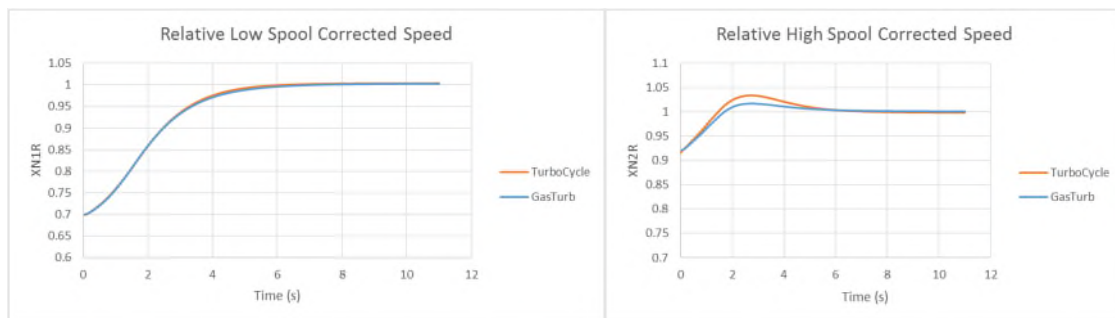


Figure 4-25 Comparison of Low Spool Relative Corrected Speed and Comparison of High Spool Relative Corrected Speed

The following two images in figure 4-26 demonstrate the comparison of low compressor pressure ratio and comparison of high compressor pressure ration separately.

According to the figure, the results of two comparisons have proved a good accuracy of TurboCycle in the aspect of performance prediction of the compressor.

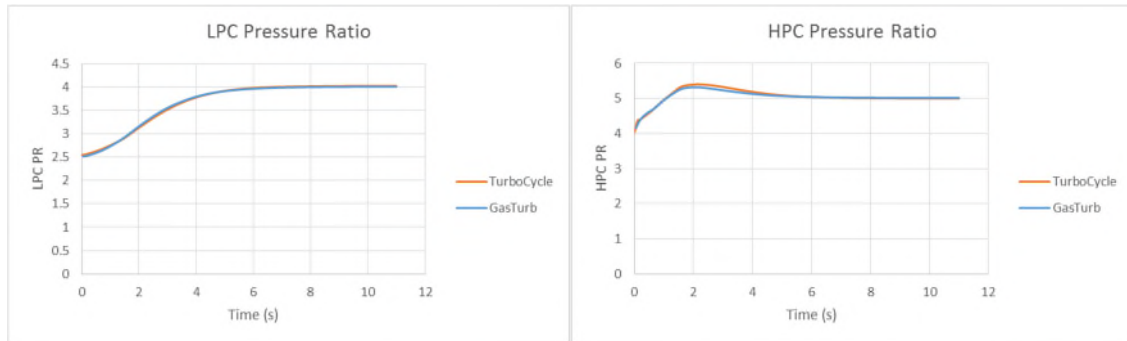


Figure 4-26 Comparison of LPC Pressure Ratio and Comparison of HPC Pressure Ratio

The figure 4-27 shows the comparison of combustor exit temperature and comparison of turbine entry temperature, respectively.

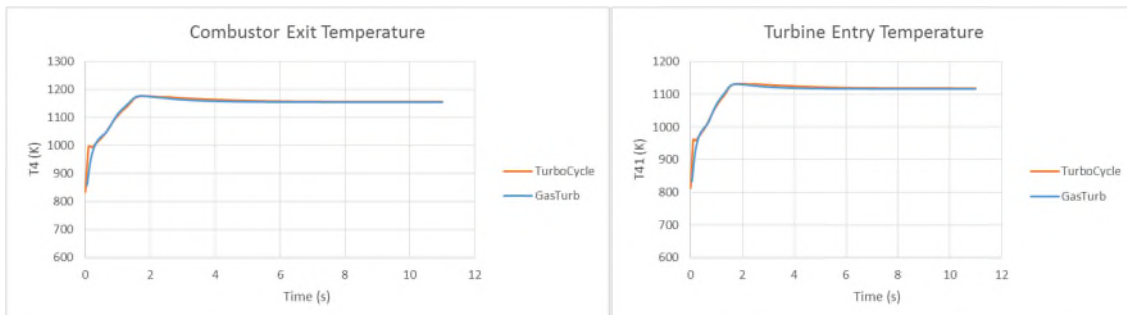


Figure 4-27 Comparison of Combustor Exit Temperature and Comparison of Turbine Entry Temperature

Based on the results of the comparison, it is known that the function of combustor temperature estimation is reliable and cooling system of TurboCycle is trusted excellently.

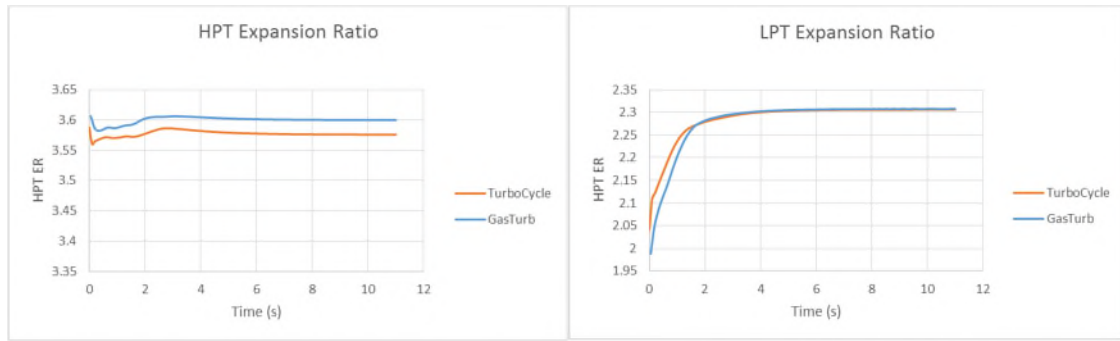


Figure 4-28 Comparison of HPT Expansion Ratio and Comparison of LPT Expansion Ratio

The two pictures above in Figure 4-28 give the comparison of high turbine expansion ratio and low turbine expansion ratio at the same time.

According to the left picture, the difference in the HPT expansion ratio could be observed clearly. It is seen that the shape or the trend of two lines are quite similar, which looks like a pair of parallel lines. That means there is a small error on this map reading accused by an error on the corrected speed of the high spool. Be that as I may, it is measured that the maximum error is less than 0.84%.

From the results of the right picture, it could be estimated that the maximum difference is less than 2.5% in terms of low-pressure turbine expansion ratio compared with GasTurb.

The last pair of two pictures in figure 4-29 illustrates the comparison of exhaust gas temperature and the comparison of the net thrust separately.

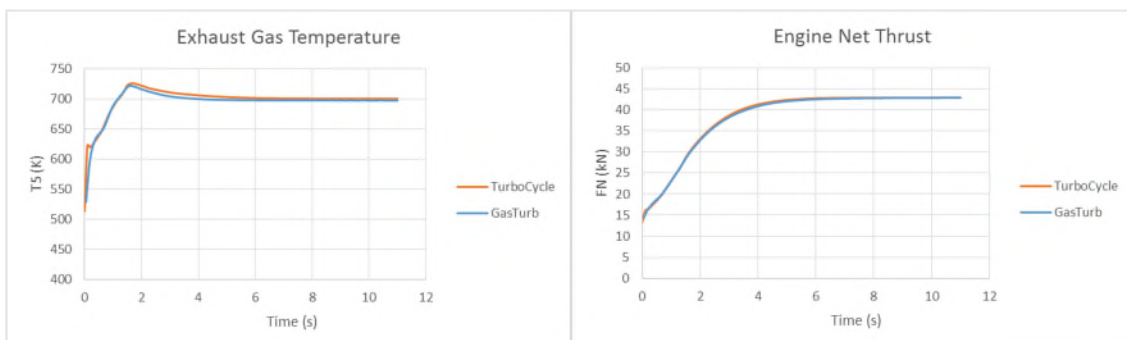


Figure 4-29 Comparison of EGT and Comparison of Net Thrust

Based on comparisons from figure 4-29, it is got that the temperature estimation of and thrust calculation of TurboCycle is very close to that of GasTurb with an error no more than 0.5%.

4.2.3 Summary of Verification on Performance Model

The overall comparison results between TurboCycle and GasTurb have been listed as in table 4-5.

According to the results in the table, it is known that maximum errors of steady state performance of Turbojet A and Turbojet B simulated in TurboCycle are 2.5% and 1.0% respectively, which is less than the target error of 5%.

Beyond that, the maximum errors of transient performance of Turbojet A and Turbojet B simulated in TurboCycle are 3.9% and 2.5% respectively, which is also less than the target of 10%.

In addition, the maximum error of volume effects of Turbojet A simulated by TurboCycle IMF method is less 10% compared with traditional ICV method.

Verification of TurboCycle	Single Spool Turbojet A	Twin Spool Turbojet B	Criterion	Results
Verification Classification	Maximum Error (%)	Maximum Error (%)	Target Error (%)	✓ ✗ □
Steady State Performance	2.50%	1.00%	5%	✓
Transient Performance	3.90%	2.50%	10%	✓
Volume Effects	9.80%	\	10%	✓

Table 4-5 Summary of Verification of TurboCycle

On the strength of results of the comparison above, it could be concluded that functionality and accuracy of TurboCycle are verified successfully.

4.3 Verification of Heat Soakage Model

Since the Twin Spool Turbojet B has more accurate results on transient performance simulation, the verification of heat soakage model is based on the turbojet B.

4.3.1 Results of Geometry Model

Firstly, the component area and mass of turbojet B are estimated by the geometry model of heat soakage system shown in table 4-6.

According to the result of the geometry model, it is known that the mass of the high-pressure turbine is the lightest component as it has only two stages. The weight of low pressure of the compressor is the heaviest component as its volume is largest with five stages. The second heaviest component is the high-pressure compressor because of its second-largest of volume. The weight of low pressure is high than the burner and lower than the HP compressor. Therefore, it is believed that the results of geometry mode are reasonable more or less.

Geometry Model	Compressor LP	Compressor HP	Burner	Turbine HP	Turbine LP
Area (m^2)	9.611	3.950	1.450	0.705	1.465
Mass (kg)	301.8	187.7	59.0	55.0	126.3

Table 4-6 Component Mass and Area estimated by Geometry Model

The heat transfer model would use the estimated component mass and area for heat exchange calculation.

4.3.2 Verification of Heat Transfer Model

The 1D heat transfer methods of convection and conduction are verified through the comparison with the traditional 0D method based on the compressor because the 1D heat transfer method for the compressor is the same as that of the turbine and combustor. This verification has been implemented in TurboCycle.

First of all, the heat transfer coefficients are significant for the calculation of heat exchange.

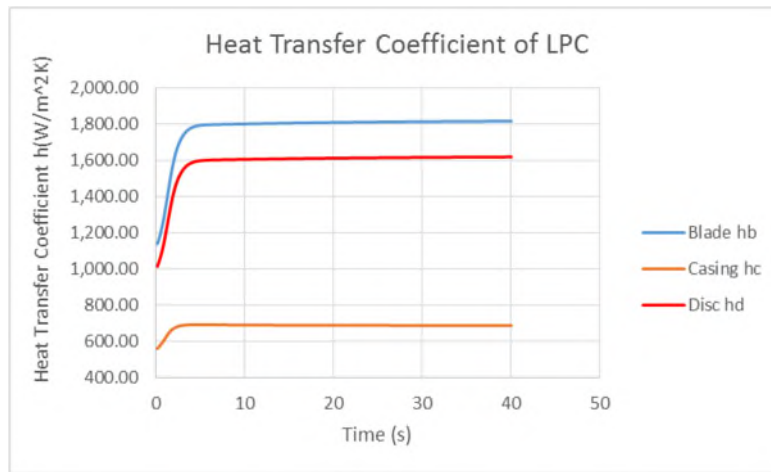


Figure 4-30 Variation of LPC Heat Transfer Coefficients

The figure 4-30 shows variations of the heat transfer coefficients determined by heat correlation functions attached in the appendix.

According to the results, it is known that the value of heat transfer coefficient of the blade is the highest. There are two main reasons, and the first one is due to its high rotational speed. As a consequence, it would lead to a higher Reynolds number. The second one is due to its relatively small conductive thermal resistance, which would lead to a larger Prandtl number.

It is also observed that the value of heat transfer coefficient of the casing is the lowest since it is a stationary component. Those results are likely to be very reasonable.

The first verification result in figure 4-31 shows the comparison of the low-pressure compressor exit temperature deviation due to heat soak effects.

According to the result, it is found that the peak temperature deviation estimated by the numerical 1D method is only roughly 0.6 degree less than that of the old OD method, which means the result of the new method is pretty reliable.

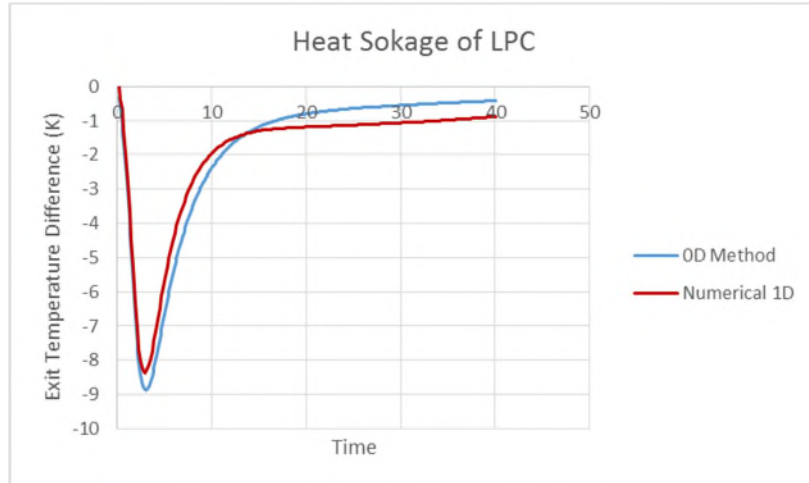


Figure 4-31 Comparison of LPC Exit Temperature Deviation Due to Heat Soakage between 0D Method and Numerical 1D Method

The next graph in figure 4-32 shows the comparison of the power of PLC heat exchange caused by heat soakage.

Based on the comparison, it is found that there is no much difference at the beginning of the acceleration process, and the difference becomes a little bit large after 10 seconds and remains for a long period. This is because the 0D method would estimate less quantity of heat at each time interval. Consequently, it would lead to a faster process for heat transfer compared with the 1D method. The root reason for that is heat transfer of point to point is slower than that of surface to surface due to the different mass and temperature distribution.

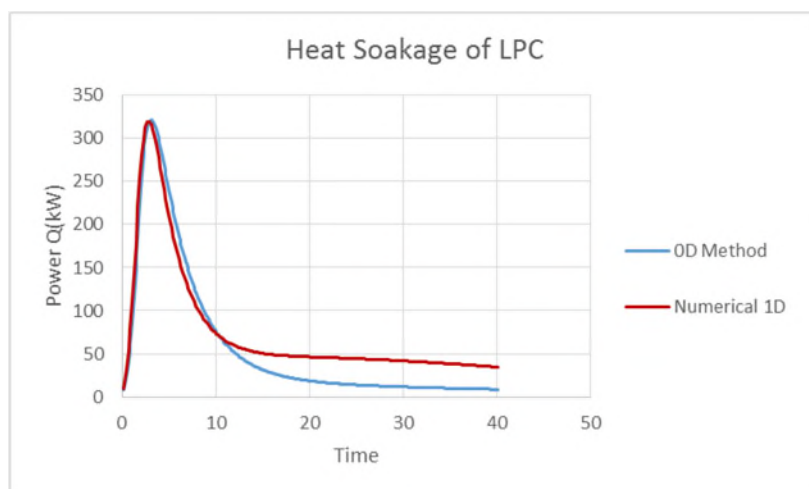


Figure 4-32 Variation of LPC Heat Transfer Coefficients

The following picture in figure 4-33 illustrates the comparison of the high-pressure compressor exit temperature deviation.

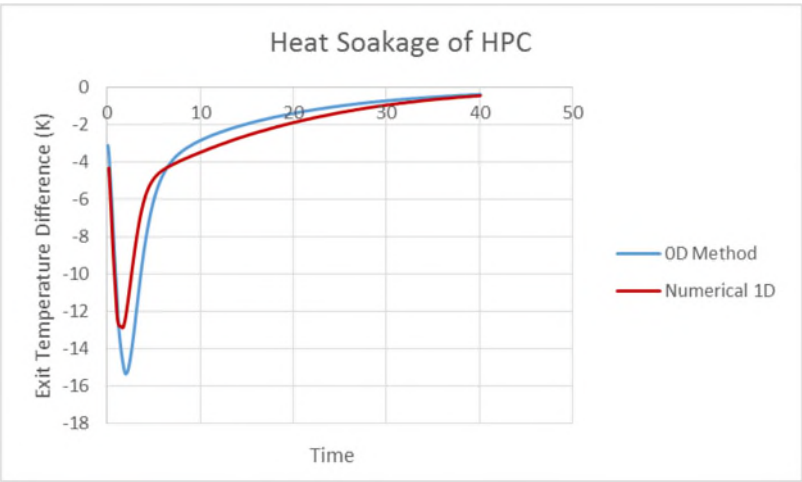


Figure 4-33 Variation of LPC Heat Transfer Coefficients

Depending on the result, it is found to be around 3 degrees that maximum exit temperature difference of HPC between OD method and the 1D method.

The figures 4-34 shows the comparison of power on HP compressor heat exchange during this take off acceleration.

From the comparison result, it is found that the peak power of the OD method is higher than the 1D method and power of 1D method becomes higher later from the time at six-second.

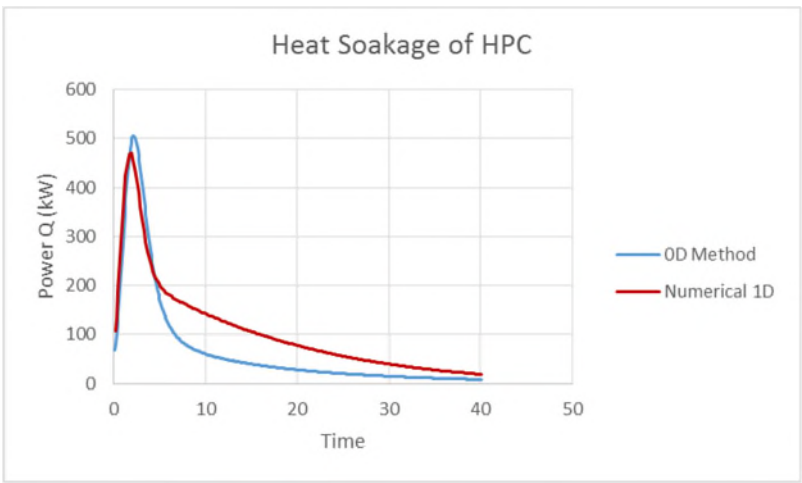


Figure 4-34 Variation of LPC Heat Transfer Coefficients

Finally, figure 4-35 shows the heat soakage effects of all components of including LPC, HPC, Burner, LPT and HPT, respectively.

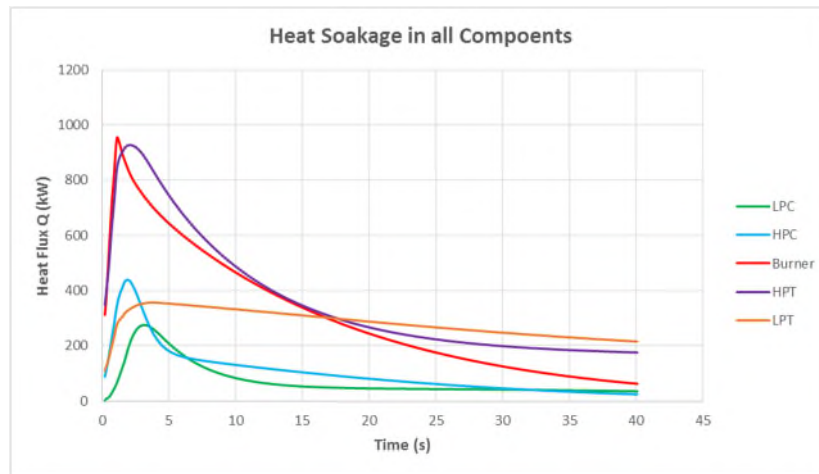


Figure 4-35 Heat Soak Effect of all Components

By observing the figure, it is found that the heat power of the combustor and HP turbine would dominate the whole engine heat soakage effects. The reason for that is the initial over fuelling would lead the temperature increased dramatically in the combustor, which would produce a high insensitive field with radiative and convective heat flux. The HP turbine followed with the combustor would face the hot gas instantaneously. As a result, the heat flux produced by heat convection and conduction is also very stronger than the rest of the components.

As LP compressor and HP compressor are located in the cold section, their heat exchange is relatively smaller than the components in the hot section.

So far, the 1D heat transfer method is verified effectively in terms of convection, conduction and radiation.

4.3.3 Results of Tip Clearance Model

In this section, the results of tip clearance are presented based on the case of Turbojet B take-off condition.

Figure 4-36 shows the temperature variation among the airflow, blade, casing and disc of HPC. At the beginning of acceleration, the fluid temperature raised rapidly, and instantaneous temperatures of metals remain roughly the same.

Then, the heat energy is transferred from flow to metals. Finally, their temperatures reach to the same level as long as time is long enough.

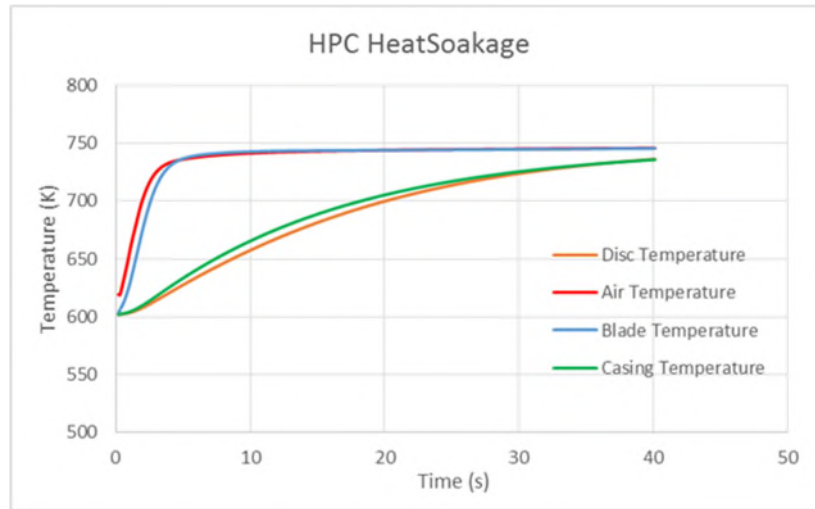


Figure 4-36 Compressor heat transient heat transfer

As the thermal time constant of the blade is relatively small, the temperature rise of the blade is as fast as airflow. The casing temperature increases slower than the blade, and the reason for that is the heat coefficient of the casing is lowest with a higher conductive thermal resistance. With regards to disc, speed change of its temperature is slowest owing to the largest the mass, and of course a big thermal time constant.

Figure 4-37 shows the deflection variation of HPC components. The peak deflection of blade emerges at the earliest while that of the disc appears at latest from the results.

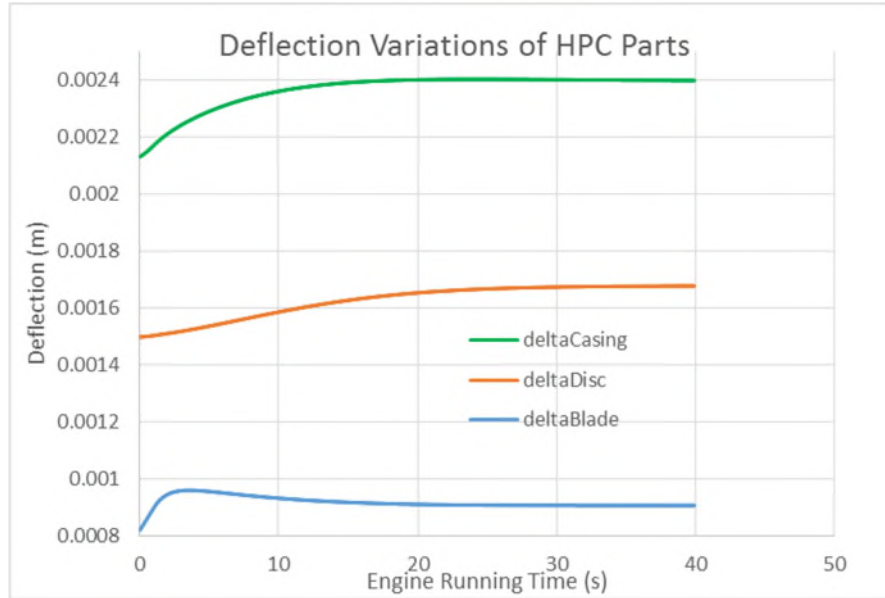


Figure 4-37 Deflection of HP Compressor Parts from another Acceleration Case

Figure 4-38 describes the compressor tip clearance variation along with engine running time from another acceleration case.

Based on the graph, the blade expands faster than others due to the high centrifugal stress at the start of acceleration. Therefore, the main factor on tip clearance is the centrifugal expansion. During the middle phase, casing deflection continues to grow with faster speed than the disc. As a result, the main contributor for tip clearance variation becomes the casing expansion. For the last phase of acceleration, the only deflection of the disc is still increasing because of a longer thermal response.

Despite lacking comparison data, it is still believed that the trend of the tip clearance variation for acceleration is in line with the actual situation according to public reference.

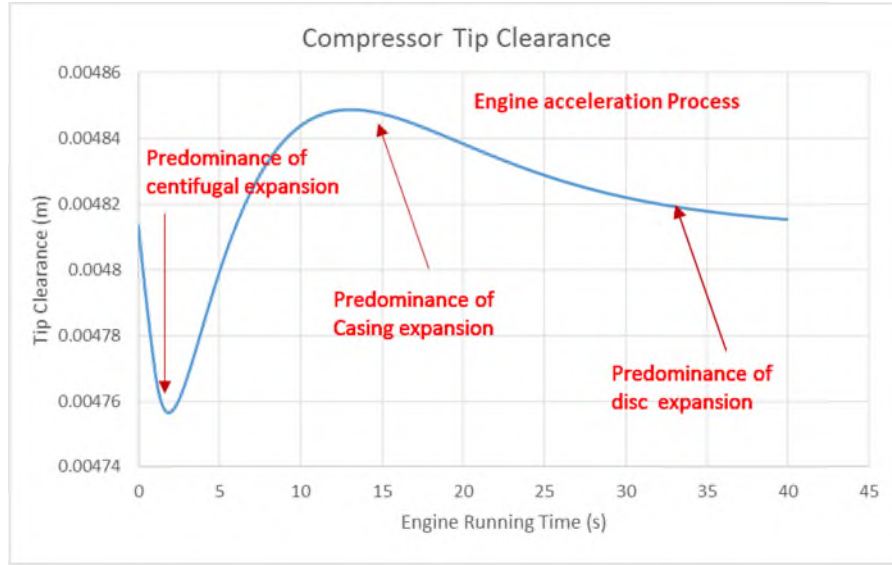


Figure 4-38 Compressor tip clearance

4.3.4 Summary of Verification on Heat Soakage Model

The 1D numerical heat transfer model has been verified effectively through the comparison with the traditional 0D method.

The results of the geometry model and tip clearance model are analysed to be feasible as they are consistent with empirical knowledge.

Therefore, it is believed that heat soakage model could predict the heat soak effects effectively.

4.4 Case Study of Heat Soakage Effects

In this chapter, the case study of heat soakage effects has been carried out based on the two engine models mentioned in the previous chapter. The heat soakage effects of each component would be presented and discussed. Furthermore, the tip clearance impacts of compressor and turbine also illustrated and discussed.

Model Single Spool Turbojet

There are main three different components associated with heat soakage effects in this paper for Turbojet A, which are including compressor, turbine and combustor. Therefore, their heat transfer impacts on engine transient performance have been simulated and analysed individually. Nevertheless, heat soakage of compressor and turbine contain the effects of not only the heat transfer but also the tip clearance. As the thermal expansion effects of tip clearance are coupled with heat transfer, it is complicated to split them. Therefore, the effect of tip clearance is determined by comparison through heat transfer with tip clearance and heat transfer with no tip clearance.

Then heat soakage impacts are determined by comparing between transient performances at the adiabatic condition and heat soakage transient performance caused by each individual aspect and the combination of all.

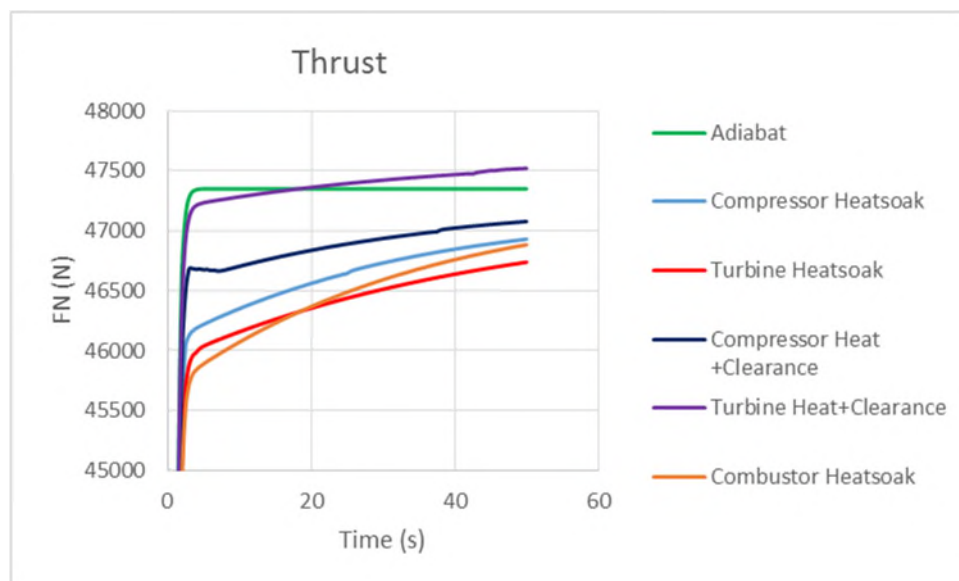


Figure 4-39 Individual Heat Soakage Effect on Thrust

Figure 4-39 shows the heat soakage effect on thrust response comparing to the adiabatic case in terms of those five different individual cases, which are combustor heat transfer, compressor heat transfer, turbine heat transfer, compressor heat transfer with tip clearance and turbine heat transfer with clearance respectively.

From the picture, it is clear to see that the combustor heat transfer would decrease the thrust most significantly at the beginning of acceleration. At the time of 18 seconds, the biggest impactor becomes the turbine heat transfer. The compressor heat transfer has the smallest influent on the thrust reduction.

The dark blue line represents the compressor heat transfer with tip clearance. It is discovered that the dark blue line has a less thrust reduction than compressor heat transfer, which means that compressor tip clearance would lead an increment on thrust.

The purple line indicates the turbine heat transfer with tip clearance. Through the comparison with turbine heat transfer, it is known that the turbine tip clearance could help to build up the thrust during this acceleration.

In order to compare more conveniently, all relevant performance parameters are compared at 99.12% of corrected speed for all cases.

The results of individual heat soakage impact on thrust variation are shown in figure 4-40.

According to the result, it is found that the turbine heat transfer dominates the thrust deviation around -2.0%. This is because of its largest heat transfer coefficient and facing the second-highest temperature. Although combustor is facing the highest temperature with heat radiation, the impact of it around -1.2% takes the second place after the turbine. The reason is probably that the heat transfer coefficient is smaller as it is a stationary component without forced convection. The flame temperature inside combustor would generate heat radiation flux. Therefore, its impact is bigger than that of the compressor, which is around -0.25%.

From the result, it is also known that the tip clearance would help to build up thrust for an acceleration. In this case, it is observed that compressor clearance could increase the thrust around 0.5% while turbine tip clearance could add 0.7% on thrust increment.

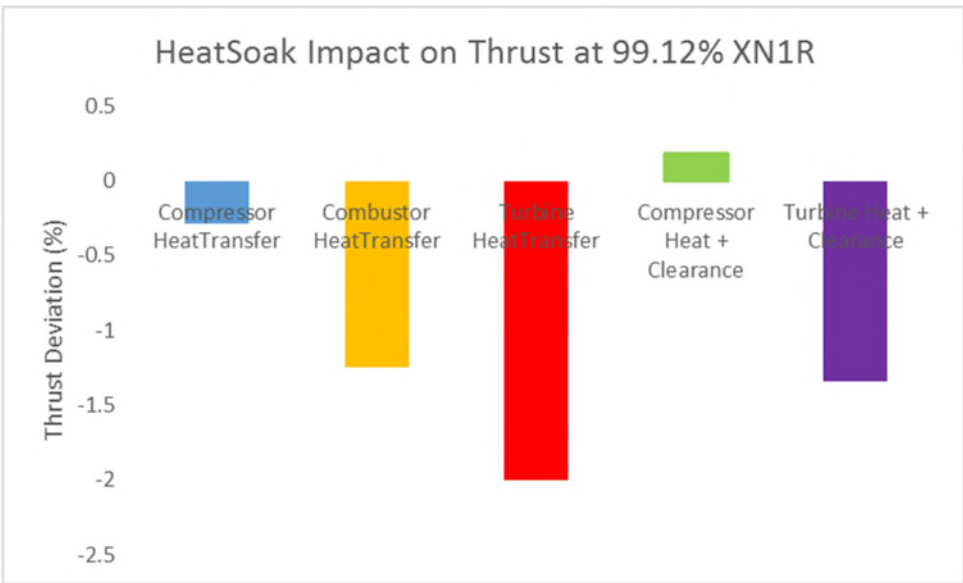


Figure 4-40 Individual Heat Soak Impact on Thrust

The effects of heat soakage on air mass flow are presented in figure 4-41.

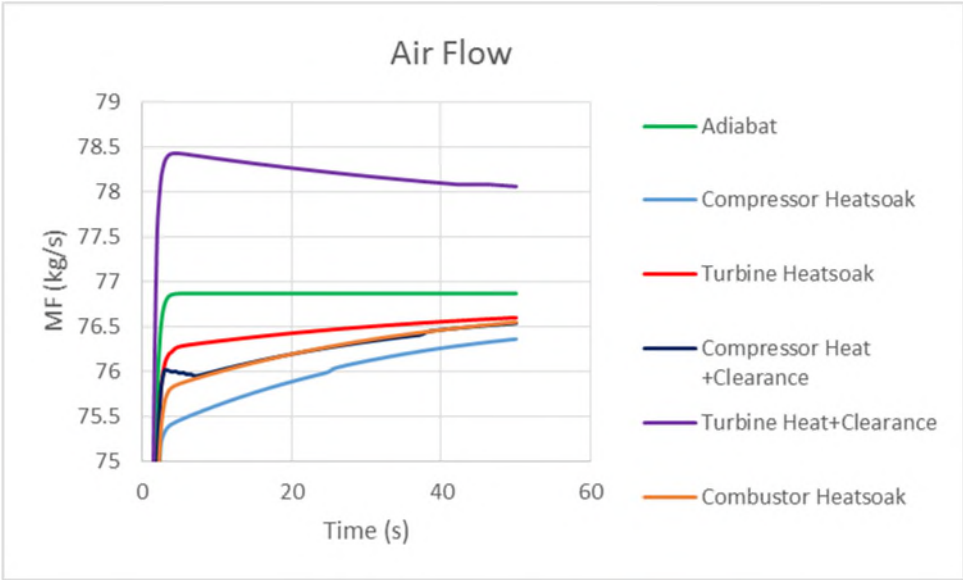


Figure 4-41 Individual Heat Soak Effect on Air Flow

During this transient process, it is known that compressor heat transfer could lead the biggest airflow reduction and compressor tip clearance would give an effect of airflow increasing compared with the comparison between compressor heat transfer and compressor heat transfer with tip clearance.

In addition, combustor heat transfer and turbine heat transfer could also reduce the airflow. Compared with turbine heat transfer, turbine heat transfer would lead to a smaller airflow reduction.

Through the comparison between turbine heat transfer and turbine transfer with tip clearance, it could be found that the turbine tip clearance would help to increase the airflow even over the adiabatic case.

The results of heat soakage impact on airflow are presented in figure 4-42.

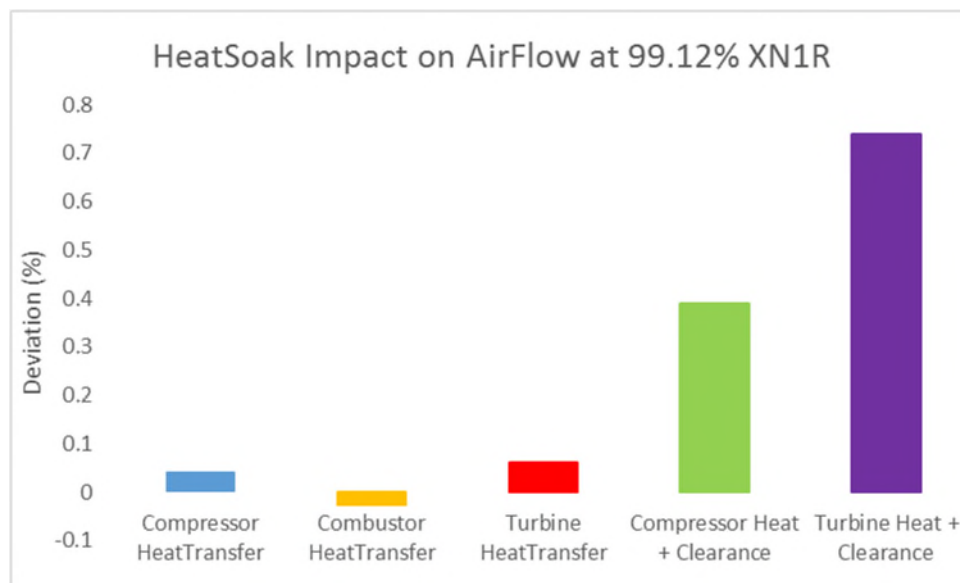


Figure 4-42 Individual Heat Soak Impact on Air Flow

Based on the bar chart, it is known that combustor heat transfer would contribute to airflow variation around by -0.02%. Whereas, the heat transfer of the compressor and turbine would increase the airflow round by 0.04% and 0.06%, respectively.

By comparing compressor heat transfer and compressor heat transfer with tip clearance, it is seen that compressor tip clearance would increase the airflow by 0.35% roughly.

Similarly, the turbine tip clearance would contribute a 0.67% increase in airflow.

The heat soakage effects and impact on compressor pressure ratio are illustrated in figure 4-43 and figure 4-44, respectively.

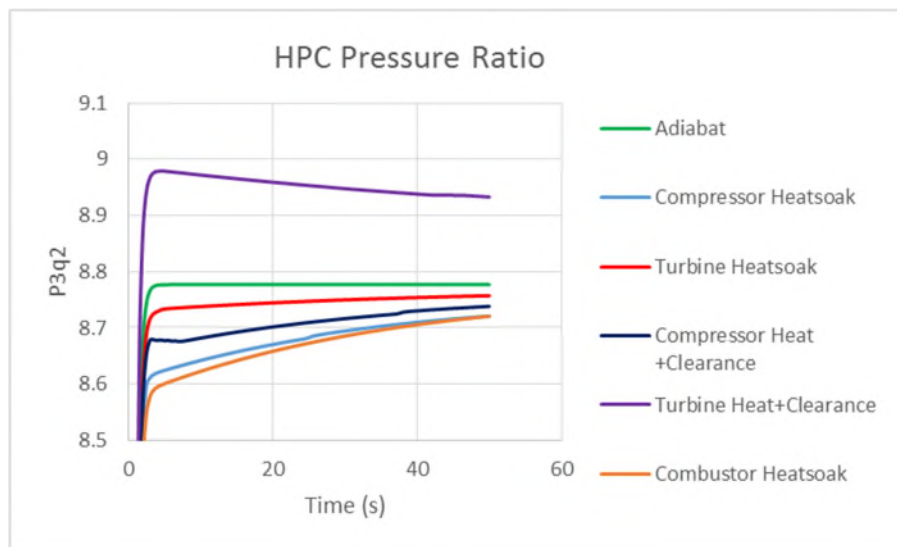


Figure 4-43 Individual Heat Soak Effect on Compressor Pressure Ratio

According to the figure 4-43, it is to see that heat transfer of compressor, turbine and combustor would lead a result of compressor pressure ratio reduction while the tip clearance of compressor would lead an increasing of compressor pressure ratio.

For the whole transient process, it is also observed that the combustor heat transfer has the biggest impact on the compressor pressure ratio.

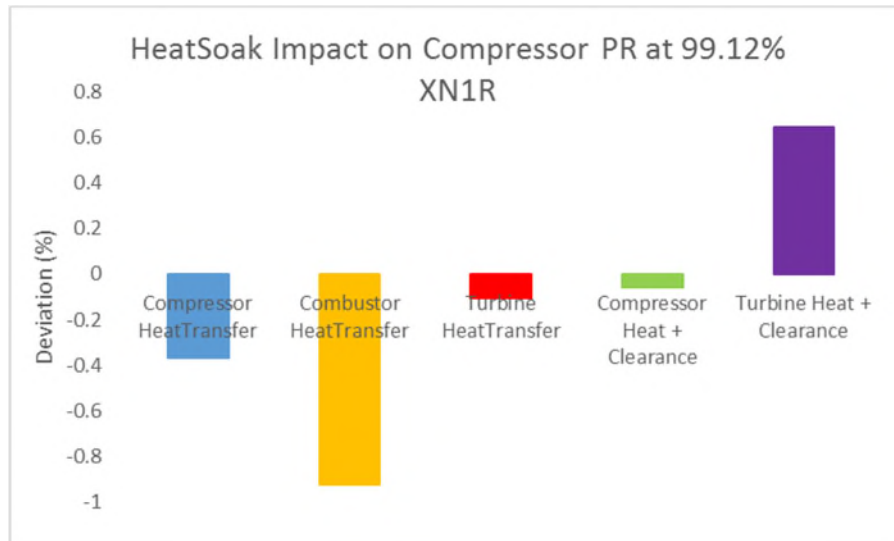


Figure 4-44 Individual Heat Soak Impact on Compressor Pressure Ratio

The compressor pressure ratios of all five cases are compared at the correct speed of 99.12%.

Based on the results, combustor heat transfer would decrease the compressor pressure ratio of around 0.9%. At the same time, the heat transfer of the compressor and turbine would decrease the compressor pressure ratio by 0.35% and 0.1% respectively.

In terms of compressor tip clearance and turbine tip clearance, they are 0.3% and 0.5% that the corresponding increments on compressor pressure ratio, respectively.

The heat soakage effects on turbine entry temperature are shown in figure 4-45.

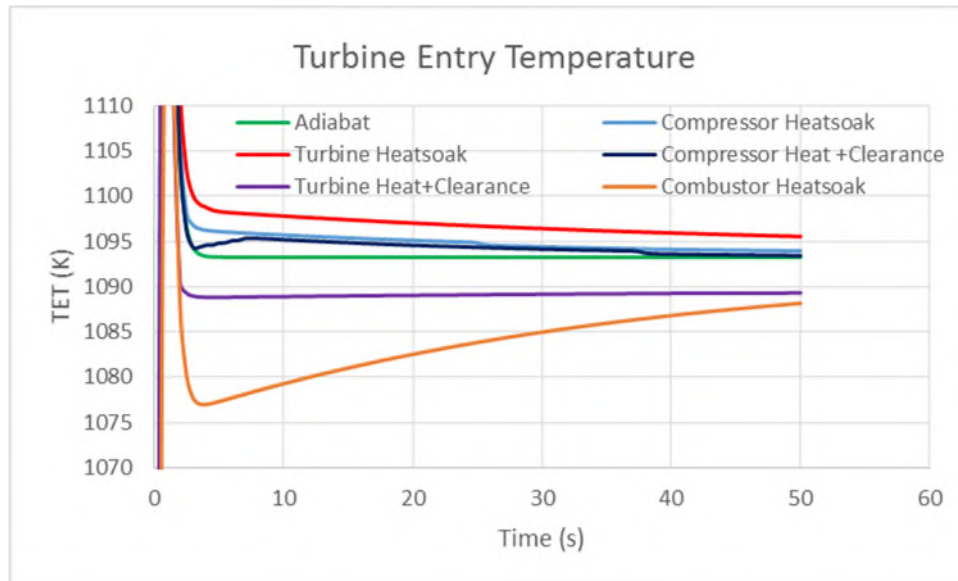


Figure 4-45 Individual Heat Soak Effect on TET

From the figure, it is observed that the combustor heat transfer has the most significant influence on reducing the TET for this acceleration. However, the heat transfer of turbine and compressor would lead the results on increasing TET. The reason for that is they are compared at a different speed.

The result of heat soakage impact on TET at a speed of 99.12% is given in figure 4-46.

In light of results, it proves that all heat transfer would lead to a lower turbine entry temperature. Moreover, the combustor heat transfer is dominant, which could decrease the TET by around 14%. Since the turbine is at downstream, its heat transfer would not affect its inlet temperatures theoretically. The turbine is connected to the compressor. Therefore, the turbine heat transfer would reduce the turbine work to drive the compressor. Accordingly, the temperature delivered by the compressor should be slightly reduced. That is why the turbine heat transfer here could still reduce the TET by around 0.26%. While the compressor is in the cold section, its heat transfer impact is relatively small comparing combustor. The compressor heat transfer would decrease the TET by around 3.7%.

Beyond that, it is found that the tip clearance of the compressor and turbine would further decrease TET by 0.6% and 3.0%.

The reason might be that the tip clearance could improve the efficiency of the compressor and turbine. As a result, the lower TET is required to reach the same speed.

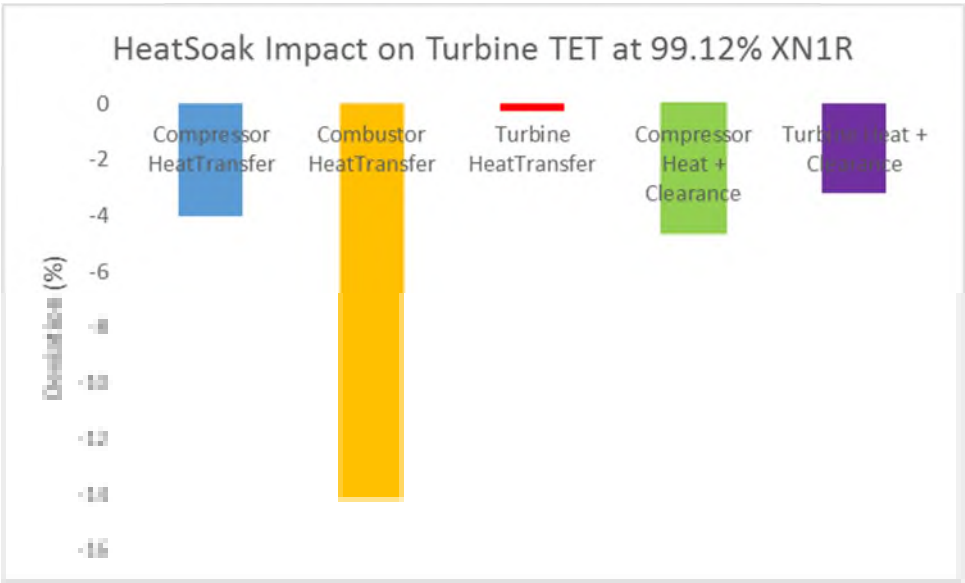


Figure 4-46 Individual Heat Soak Impact on TET

The following picture in figure 4-47 shows the heat soakage effect on turbine expansion ratio. Based on the figure, it is seen that heat transfer and tip clearance of the turbine have the biggest influences on the turbine pressure ratio. Moreover, the rest of the other factors are relatively much smaller.

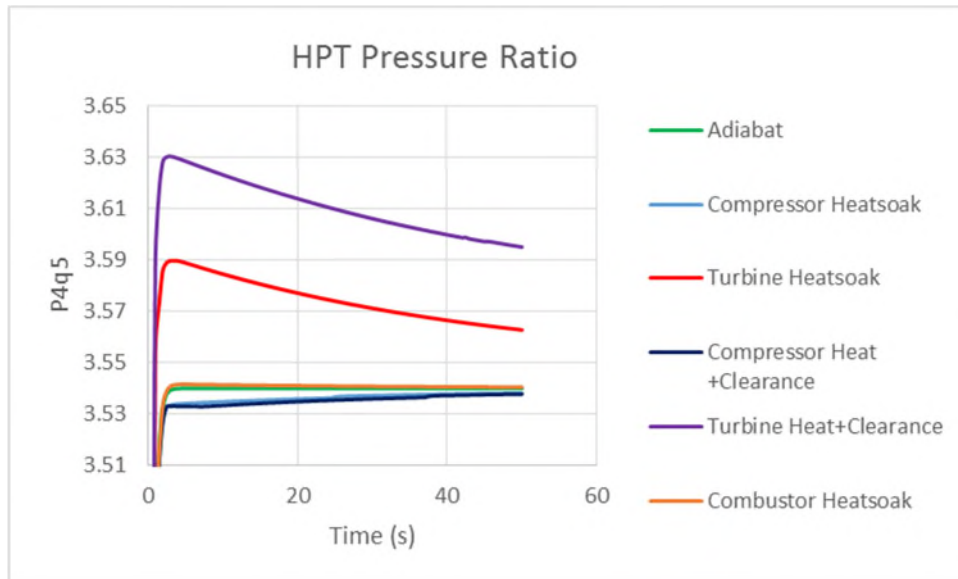


Figure 4-47 Individual Heat Soak Effect on HPT Expansion Ratio

The figure 4-48 presents the heat soakage impacts on turbine expansion ratio at a corrected speed of 99.2%.

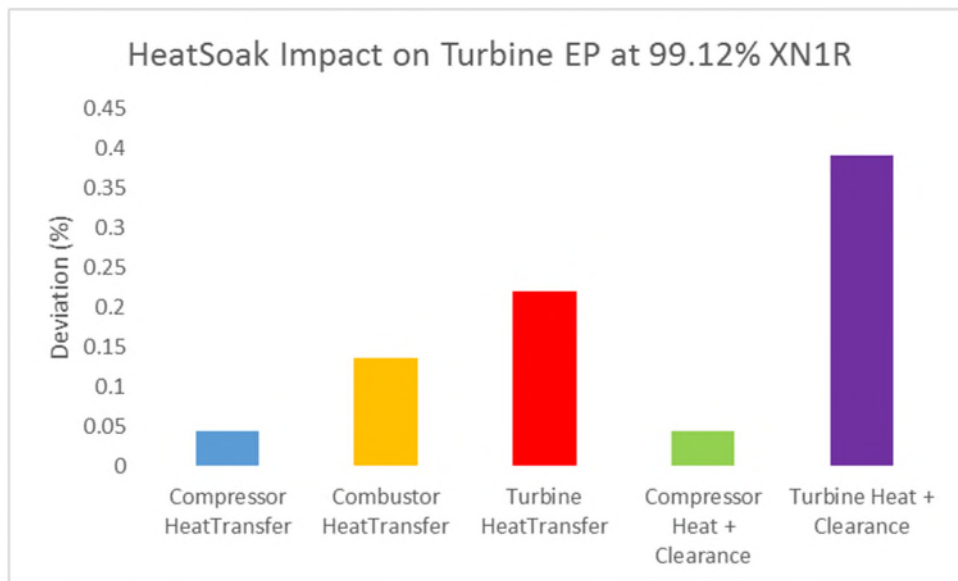


Figure 4-48 Individual Heat Soak Impact on HPT Expansion Ratio

According to the figure, it is found that heat transfer and tip clearance would increase the turbine expansion ratio by 0.22% and 0.17%, respectively. As turbine heat transfer would bring a part power, which is to deliver to the compressor, in order to maintain the compressor work, the work line point of the turbine has to move up at same speed line to produce total work. Consequently,

the expansion ratio would be increased slightly. In addition, the transfer of combustor and compressor would also increase the turbine expansion ratio by 0.14% and 0.4%, respectively.

The next figure 4-49 shows the result of heat soakage effect on specific fuel consumption.

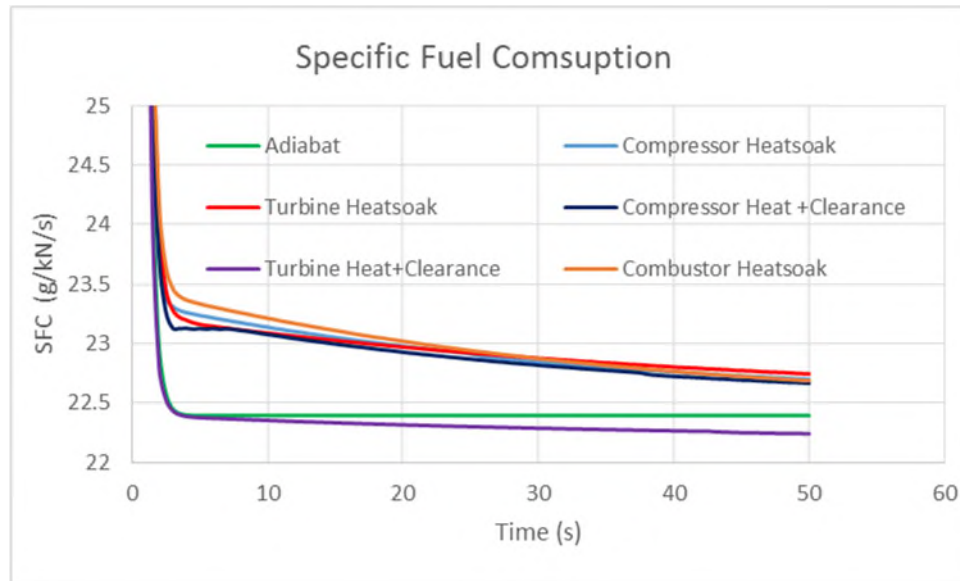


Figure 4-49 Individual Heat Soak Effect on SFC

From the results, it could be found that the heat transfer of all component would result in increasing the specific fuel consumption.

Beyond that, it is discovered that the tip clearance of the turbine and compressor is the other way around, which could help to save the fuel economy.

Regarding figure 4-50, it could be known that the turbine heat transfer would increase the SFC by around 1.4%. The heat transfer of combustor could increase the SFC by 1.0%. Besides, the compressor heat transfer would contribute an incensement of 0.3% on SFC.

Moreover, the tip clearance of the compressor and turbine would decrease the SFC by 0.39% and 0.42% compared with heat transfer with tip clearance of the compressor and turbine, respectively.

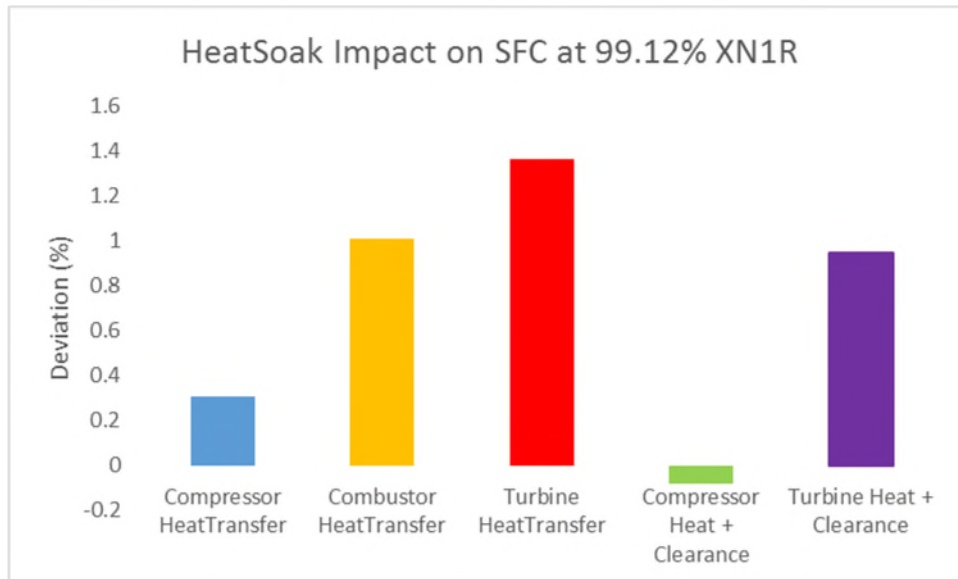


Figure 4-50 Individual Heat Soak Impact on SFC

The last figure in 4-51 shows the heat soakage effects on speed response. According to the figure, it could be known that turbine tip clearance has a significant impact, which could let the engine to reach speed with shorter time.

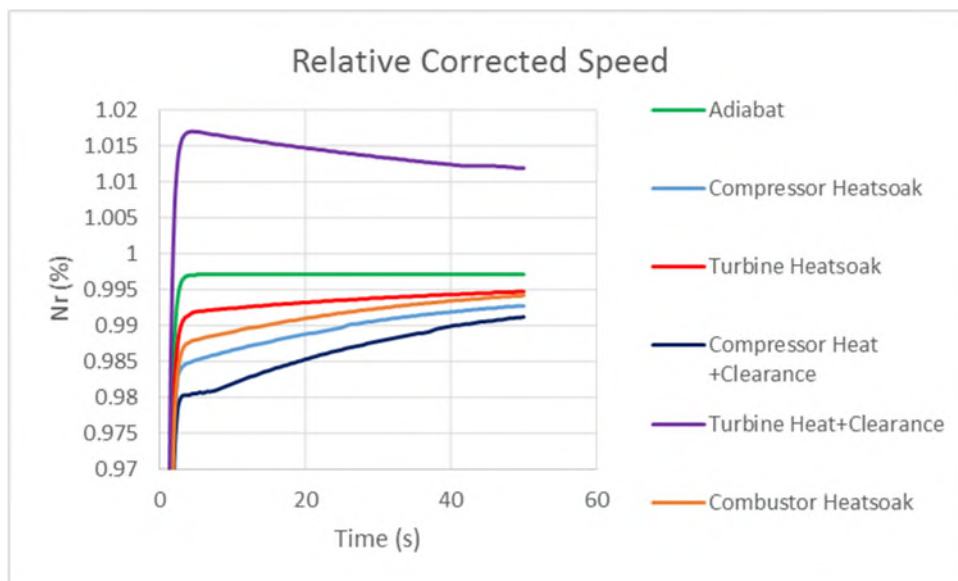


Figure 4-51 Individual Heat Soak Impact on XNR

Model Twin Spool Turbojet

There are main five different components associated with heat soakage effects for twin-spool turbojet, which are including LP compressor, HP compressor, HP turbine, LP turbine and combustor. As a result, the analysis of single-factor has been carried out for each component. Finally, the results of all component are compared with the adiabatic condition.

The heat soakage effects on low spool corrected speed are presented in figure 4-52.

According to the result, it is known that combustor heat soak would delay this acceleration tremendously.

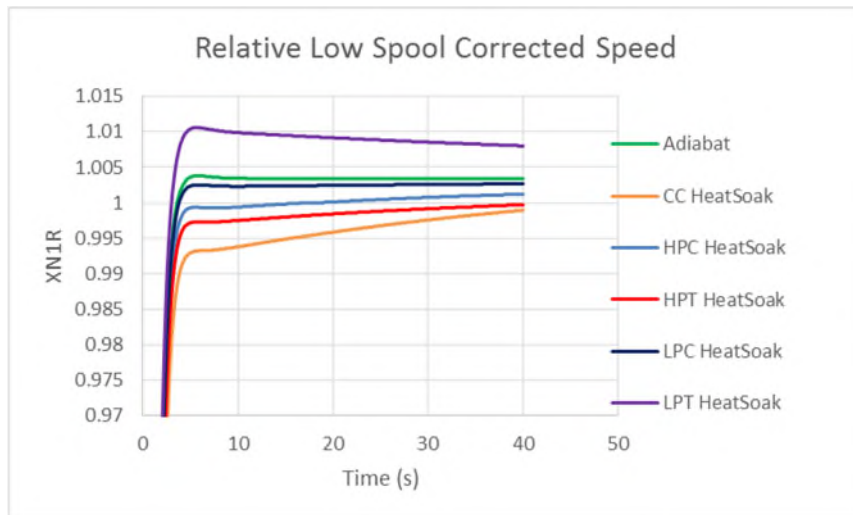


Figure 4-52 Individual Component Heat Soak Effect on XN1R

In order to compare more reasonably, it is selected that 99.7% of low spool corrected speed.

The heat soak impacts on acceleration time are illustrated in the 4-53. From the results, it is known that the acceleration time to 99.7% speed of combustor heat soak is around 26 seconds, which is more than six times of adiabatic case. Other components could also delay this acceleration more or less, but not very significantly.

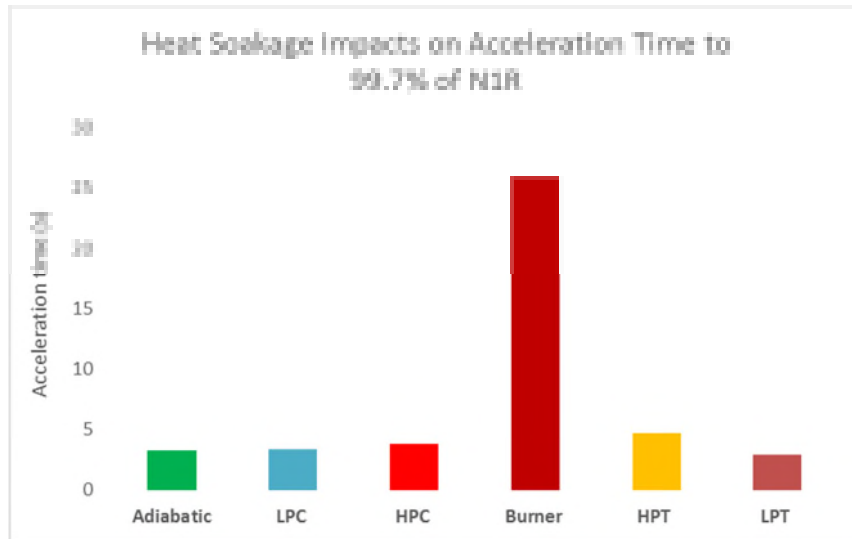


Figure 4-53 Individual Component Heat Soak Effect on Thrust

Figure 4-54 shows the heat soakage effects on thrust. It is easy to see that the combustor has the most significant impact on thrust and LP compressor has the smallest influence on thrust.

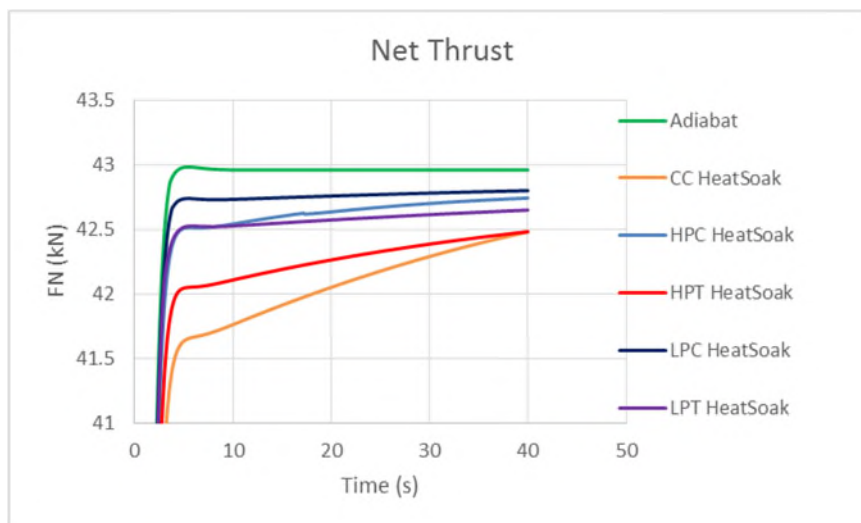


Figure 4-54 Individual Component Heat Soak Effect on Thrust

According to figure 4-55, it is known that combustor heat soak would reduce the thrust by around 1.8% at 99.7% of corrected speed. Turbine heat soak would contributor the second-largest reduction on thrust around 1.6%. The heat soak of the HP compressor takes third place by 0.76%. The last two are the LP compressor and LP turbine.

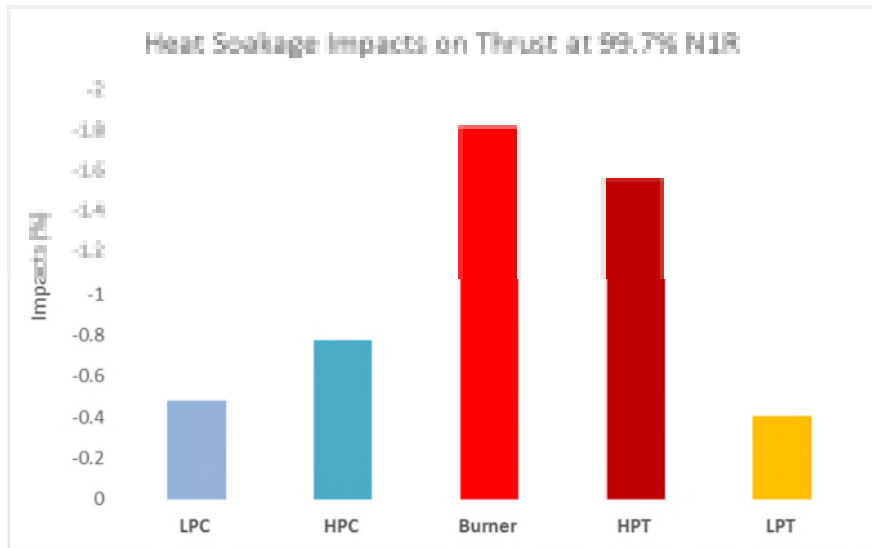


Figure 4-55 Individual Component Heat Soak Impact on Thrust

The figure 4-56 shows the heat soakage effects on airflow sucked by the engine. It could be observed that the combustor heat soakage has the greatest reduction in airflow. In addition, only the LP turbine heat soak would lead to an increase of airflow.

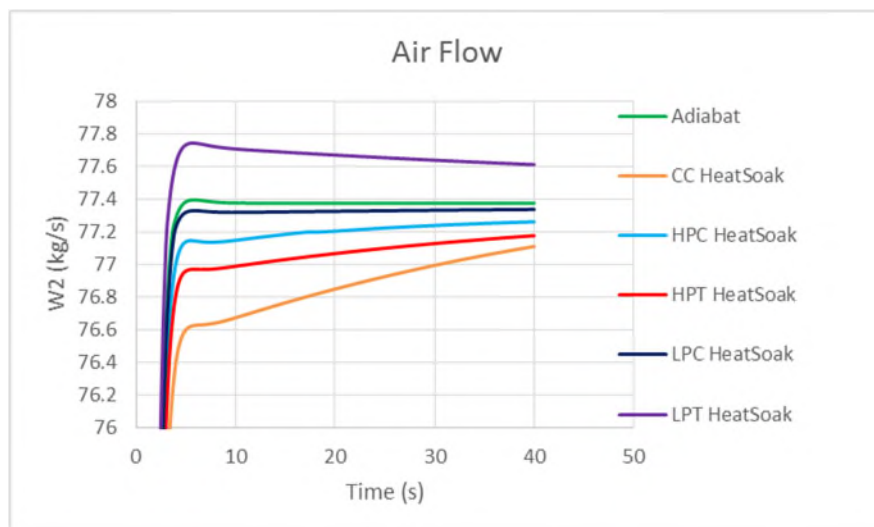


Figure 4-56 Individual Component Heat Soak Effect on Air Flow

The next picture shows the heat soakage impact on airflow at 99.7% of corrected speed.

According to the figure, the combustor heat soak would decrease airflow by around 0.24%. The airflow reductions caused by the HP compressor and HP

turbine are 0.024% and 0.019%, respectively. Beyond that, it is to see that the LP turbine heat soak would increase the airflow by around 0.018%.

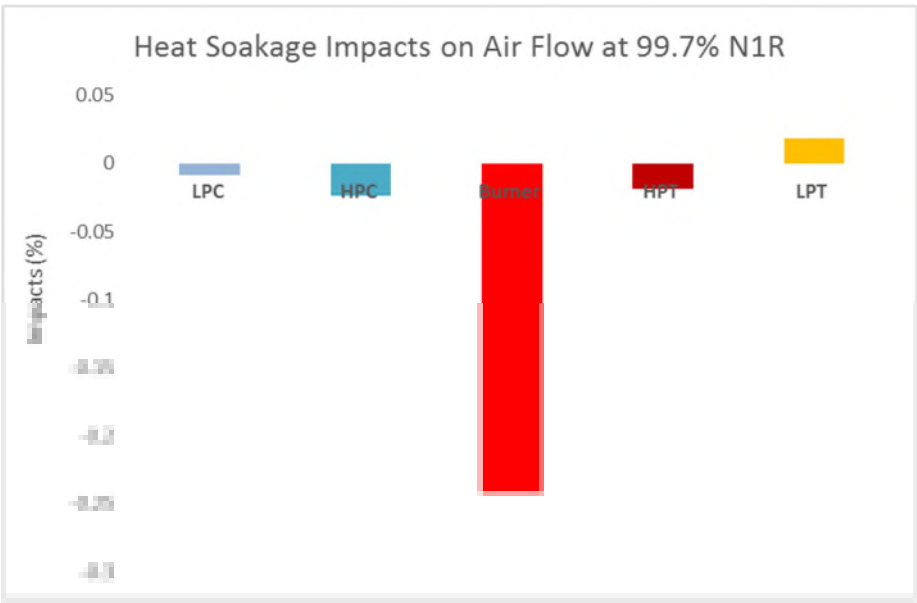


Figure 4-57 Individual Component Heat Soak Impact on Air Flow

The results of heat soakage effects on TET are presented in figure 4-58. From this figure, it could be found that the combustor heat soak would be dominant on TET reduction as it would bring a large amount of energy from the hot gas in the chamber. Consequently, the exit temperature of combustor and inlet temperature would be reduced dramatically.

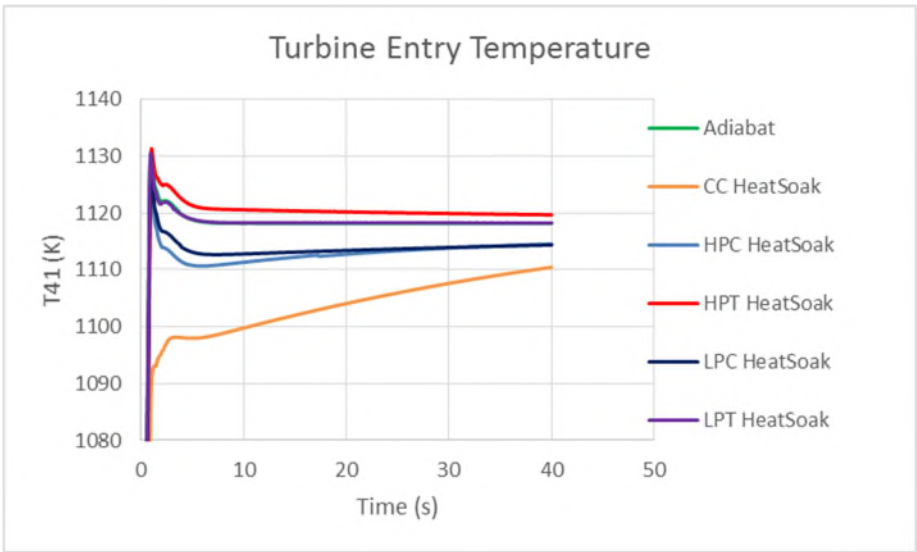


Figure 4-58 Individual Component Heat Soak Effect on TET

Additionally, the heat soakage of LPC and HPC also could reduce the TET. However, the behaviours of HPT and LPT are the other way around. Their heat soak would result in TET increasing slightly. The reason for that is the efficiency of LPT and HPT are reduced. As a result, the higher TET is required for acceleration to the given speed.

The heat soak impacts on TET at a corrected speed of 99.7% are compared and shown in figure 4-59.

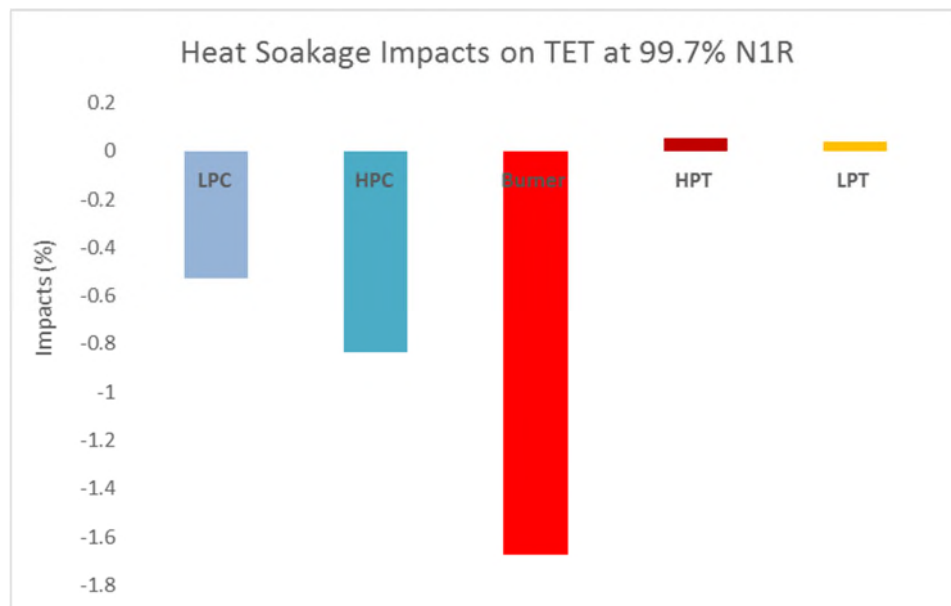


Figure 4-59 Individual Component Heat Soak Impact on TET

Based on the results shown, it is understood that heat soak impact on TET reductions of LPC, HPC and combustor is 0.6%, 0.95% and 1.67%, respectively.

Moreover, the heat soak impact on TET increments of HPT and LPT are 0.05% and 0.04%, respectively.

The figure 4-60 illustrates results of the heat soak effect on EGT. According to the figure, it is known that all component heat soak could reduce the EGT. Furthermore, the LPT would have the biggest influence on the decreasing of EGT.

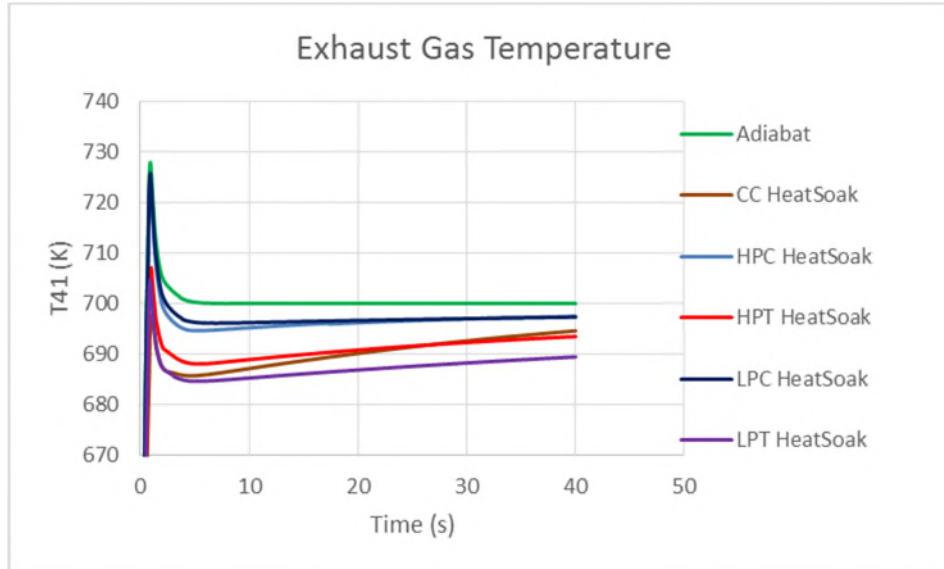


Figure 4-60 Individual Component Heat Soak Effect on EGT

The figure 4-61 presents the heat soakage impact on EGT at 99.7% of corrected speed. According to the results, the impact on EGT reduction of LPT is around by 2.27%. The impact on EGT reduction of HPT and combustor are very close, which are 1.95% and 1.85%, respectively. Additionally, the impact on EGT reduction of LPC and HPC are around 0.6% and 0.95%, respectively.

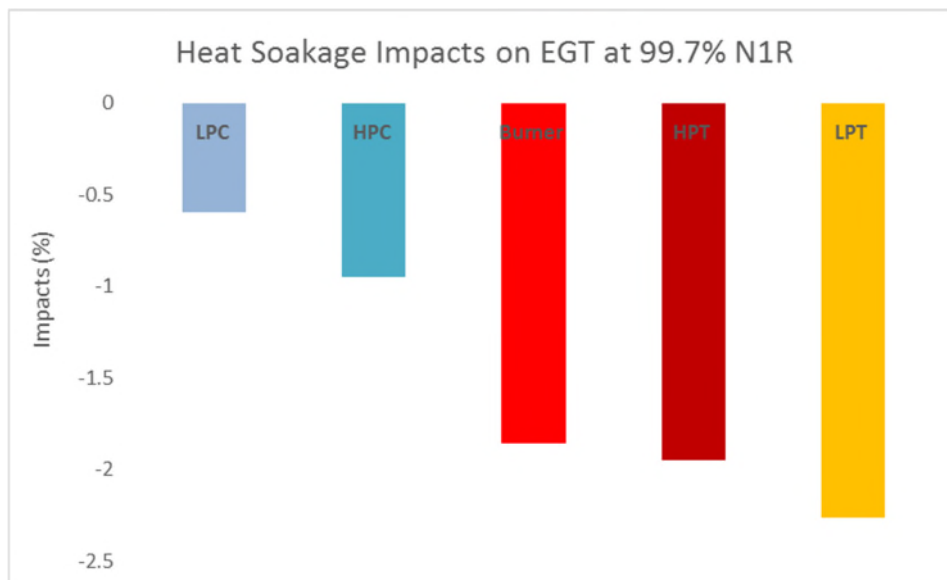


Figure 4-61 Individual Component Heat Soak Impact on EGT

The figure 4-62 shows the heat soak impact on LPC pressure ratio. According to the figure, it is known that the heat soak of combustor would have the biggest

impact on LPC pressure ratio increment, which is around 1.37%. Besides combustor, the HPC heat soak would also increase the LPC pressure ratio but only with 0.5%. Heat soak of LPC, HPT and LPT would lead reductions on the LPC pressure ratio at 99.7 of speed, which are 0.76%, 0.21% and 0.1%, respectively.

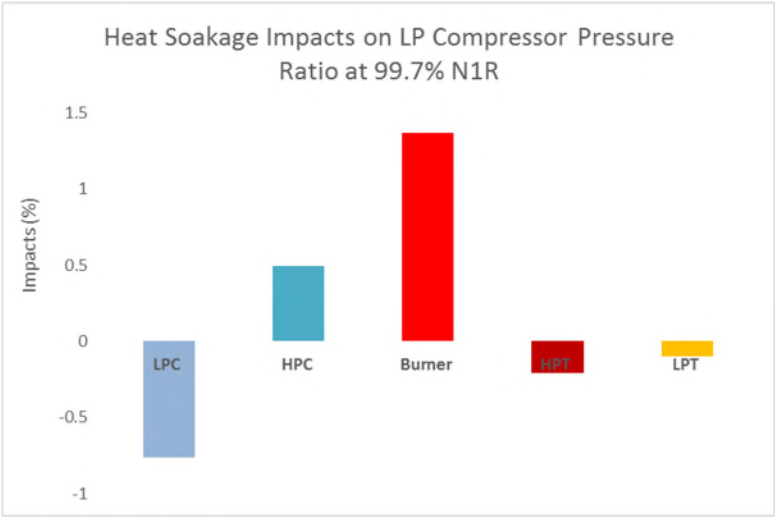


Figure 4-62 Individual Component Heat Soak Impact on LPC Pressure Ratio

From figure 4-63, it is found that heat soak of HPC and combustor would lead decrements of HPC pressure ratio, which are 0.93% and 2.43%, respectively. Heat soak of LPC, HPT and LPT would lead increments on HPC pressure ratio, which are 0.49%, 0.22% and 0.14%, respectively.

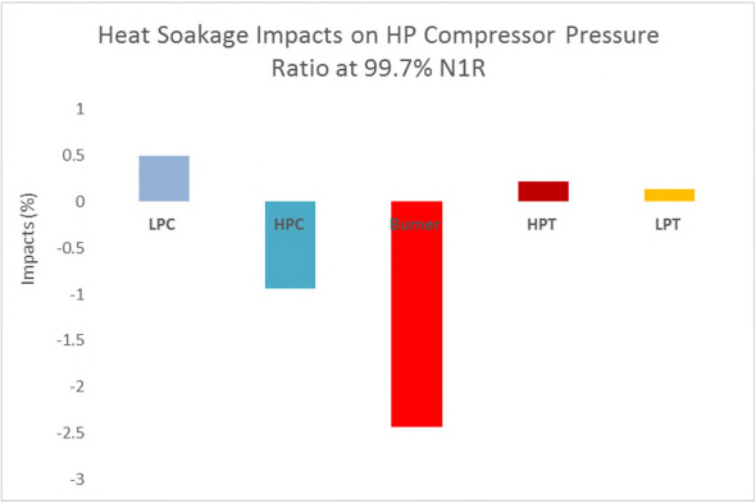


Figure 4-63 Individual Component Heat Soak Impact on HPC Pressure Ratio

The heat soak impact on HPT pressure ratio could be obtained from the figure 4-64. Based on the result, it is discovered that HPT heat soak has the biggest impact on its pressure ratio increment around by 0.95%, whereas the second largest impactor, LPT has only an increase of 0.08%. Impact of others are too small, which could be ignored in this case.

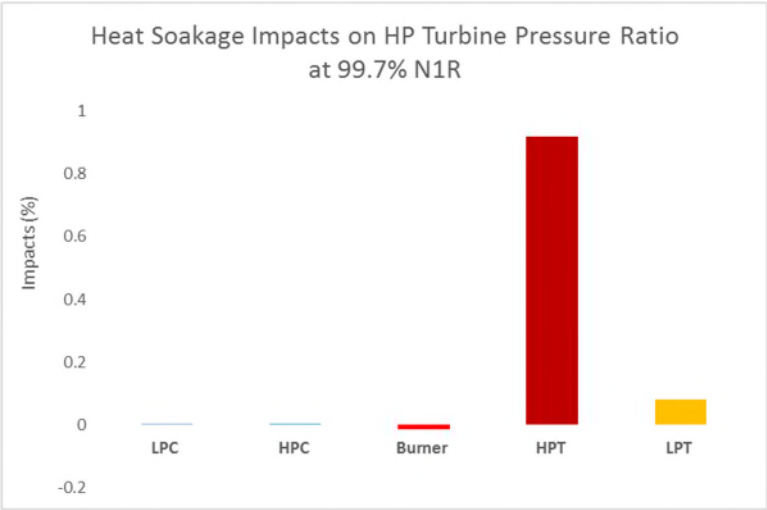


Figure 4-64 Individual Component Heat Soak Impact on HPT Pressure Ratio

The last figure 4-65 presents the heat soak impacts on LPT pressure ratio. It could be got that LPT heat soak would be the dominant factor in its pressure ratio increment, which is 1.14%. Heat soak impact of LPC, HPC and combustor are 0.04%, 0.07%, 0.12% and 0.14%, respectively.

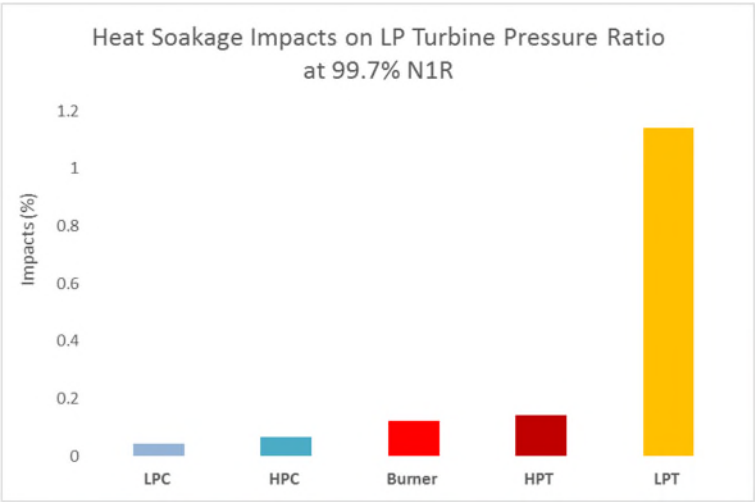


Figure 4-65 Individual Component Heat Soak Impact on LPT Pressure Ratio

5 Conclusion

The research of jet engine transient performance with heat soakage effects analysis has been concluded in this chapter.

Firstly, the rebuild method for engine transient performance simulation is presented below

- Imbalance Mass Flow (IMF) Method

The IMF method is based on the iterative thermodynamic matching method by introducing volume dynamics. Because of that, the IMF method could consider mass flow storage or release due to the mass flow non-continuity between different components for all kinds of transient process. Compared with the mass continuity method, the IMF could give results that are more realistic for a fast transient. Compared with inter-component volume method (ICV), it is learned that there are some certain advantages of IMF.

1. No limitation on interpolation of component maps especially for the vertical and horizontal part of speed line. This is because CMF method using iterative approach with beta format map. As each beta value would have only one group of parameters, then matching of mass flow and energy could be achieved through iterating beta values,
2. Potential faster computational time owing to the possible larger time step up to 1 second. However, it is normally 0.001 second for ICV, which could much slow down the computational speed.
3. Higher accuracy without oscillations on results. The ICV method could lead oscillations of working point due to the time lag between compressor outlet mass flow change and turbine inlet mass flow change. Nevertheless, there is no time lag in CMF method calculation.

IMF method has been verified against GasTurb in term of steady state and transient performance. The verification is carried out based on the two engine models shown below:

- Engine Model of Single Spool Turbojet without Afterburner

- Engine Model of Twin Spool Turbojet without Afterburner

According to the verification results, the maximum difference of steady state performance is only 2.5% while the maximum difference of transient performance is 3.9% compared with the engine model in GasTurb.

In addition, the IMF method is also compared with ICV method to validate the effect of volume dynamics. Based on the comparison results, the IMF could simulate the mass storage for a fast transient process, and the maximum of difference on the mass flow storage is less than 10% compared with ICV method.

Secondly, the novel simulation method for heat soakage effects simulation has been developed and shown below

- Generic Simplified Heat Soakage Method

The Generic heat soakage method contains three individual parts:

- Geometry Method
- Numerical Heat Transfer Method(1D)
- Tip Clearance Method

Geometry method is to estimate the engine component mass and area by using the engine design performance parameters.

Numerical heat transfer method calculates the heat exchange from inlet to outlet by integral method, which includes convection, conduction and radiation. The new heat transfer is through the surface-to-surface rather than traditional the point to point.

Tip Clearance method is to estimate tip clearance variation, and it would be taken into account on engine performance through a map correction. The effects of tip clearance variation have been successfully simulated and tested.

The numerical heat transfer method has also been verified against the traditional method. The verification is based on the twin-spool turbojet model at ISA take-off condition.

According to the comparison results, it is found that the maximum difference is less than 3K in term of HPC exit temperature. Additionally, the heat exchange power of different components is presented and discussed.

Finally, all methods are integrated into an in house developed performance code, TurboCycle and the case study of heat soakage effects have been implemented through the TurboCycle based on the two engine models mentioned previously.

From the results of the case study, it is known that heat soakage would have a tremendous impact on engine transient performance, such as delay of acceleration, reduction on thrust development. In terms of simulation results, it is known that the heat soakage of combustor and HP turbine would be the dominant impactors on engine transient performance. Impact of HPC is followed by Combustor and HPT. LPC and LPT have relatively small impacts on engine transient performance.

Furthermore, tip clearance variation is favourable at the beginning of acceleration, which could help to improve engine efficiency.

6 Future Work

Although jet engine transient performance with heat soakage effects has been successfully simulated and analysed. There are still many interesting aspects have been observed and recorded, which possibly could be subjected to prospective works.

6.1 Geometry Model Improvement

Heat soakage model already developed by the author contains the simplified geometry model, which can be improved further. Currently, the mass and area of the component are assumed as linear along the length for the simple cases. However, in reality, the distributions of mass and area probably are not linear. By Achieving more accurate results, the non-linear geometry model could be developed to address this issue.

6.2 Turbine Heat Transfer Model Improvement

In this paper, the heat transfer model for the turbine only considers the convective and conductive heat transfer between primary gas flow and engine metals. The heat radiation for the turbine is not included. For the LP turbine, current heat transfer may be probably good enough. However, the HP turbine is always facing very high temperature, which would result in a heat radiative heat transfer. Therefore, heat radiation should be applied to the turbine in the future. In addition, the heat transfer between the cooling air and primary gas in the turbine section is not under consideration either. In order to get more realistic results, the heat transfer for cooling air should be added for a turbine blade.

6.3 Radiative Heat Transfer Model Improvement

Current heat radiative model used in this paper is the very simple radiative model, which is to describe the black body. As black body is a perfect absorber as well as a perfect emitter, the two radiative properties are equal to one, which are emissivity and absorptivity, respectively. In practice, the engine wall cannot be

regarded as a black body; therefore, Eq. (2-18) is more accurate for heat radiative heat transfer by introducing the emissivity of wall and gas and absorptivity of gas.

6.4 Choice of Heat Transfer Coefficients

Heat transfer coefficients are used for heat convection. Different heat convection would result in different heat transfer coefficients. Typically, there are three modes of heat convection. The first one is the forced convection when the motion of flow is generated by external force, such as a fan or pump. The second one is called natural or free convection if it is driven by gravity force due to temperature gradients. The last one is the mixed convection. The heat transfer coefficients of the three modes of heat convection could be determine by using standard Nusselt correlations. In addition, flow types (laminar flow or turbulent flow) would also alter the heat transfer coefficients, which could be calculated by corresponding Nusselt correlations. Moreover, the surface of geometry would also have impact on the heat transfer coefficient, such as flat, duct, sphere, and cylinder. Those different cases should be further studied.

6.5 Thermal Properties

As the thermal properties of materials are changed with temperature variation, for example, the thermal conductivity is increasing when temperature goes up, the variation of the thermal properties should be considered rather than the constants.

6.6 Engine Performance Model Improvement

It will always be with the air bleeding during the engine transient process, such as the transient bleeds and variable bleeds. Those kinds of bleeding would have a significant impact on engine transient performance and engine heat soakage.

Therefore, it is necessary to add those functions and corresponding control schedules to current engine performance models. In addition, the Reynolds correction should also be added in case of flight condition simulation.

6.7 Control System Improvement

Fuel schedules would result in a very significant impact on engine transient performance and heat soakage. For different missions or flight envelopes, the physical fuel flow requirement is variable. Currently, the control system only includes a simple PID control with a direct fuel schedule. It is very difficult to control engine fuel to satisfy a variety of flight missions. Hence, the improved control system like NDOT control could be applied to fulfil different requirements.

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APPENDICES

A. Rolls Royce Heat Transfer Coefficient Correlations

Heat transfer coefficient correlation for forced convection of flow through a duct:

$$D = 4A/P$$

$$Re = W \cdot D / (A \cdot \text{Visc})$$

$$Pr = C_p \text{ Visc} / K$$

For $Re \leq 2100$

$$H = 1.86 \cdot Re^{1/3} Pr^{1/3} (D/L)^{1/3} (\text{Visc}/\text{Viscw})^{0.14} K/D$$

For $Re > 2100$

$$H = 0.036 \cdot Re^{0.8} Pr^{1/3} (D/L)^{1/18} (\text{Visc}/\text{Viscw})^{0.14} K/D$$

D	Hydraulic diameter of the duct
A	Flow cross sectional area of the duct
P	Wetted perimeter of the duct
Re	Reynolds number
W	Mass flow rate of the fluid
Visc	Bulk viscosity of the fluid
Pr	Prandtl number
C _p	Specific heat at constant pressure of the fluid
K	Thermal conductivity of the fluid
L	Starting length of the duct
Viscw	Viscosity of the fluid at the wall temperature
H	heat transfer coefficient

Heat transfer coefficient correlation for forced convection from a rotating disc:

$$Re = (\rho \cdot \Omega \cdot R^2) / \mu$$

$$Pr = C_p \cdot \mu / k$$

For $Re \leq 240000$

$$H = 0.616 \cdot Pr^{1/3} Re^{1/2} k / R$$

For $Re > 240000$

$$H = 0.0267 \cdot Pr^{0.6} Re^{0.8} k / R$$

Re	Reynolds number
ρ	Density of the fluid
Ω	Angular velocity of the disc
R	Radius of interest on the disc
μ	Viscosity of the fluid
Pr	Prandtl number
C_p	Specific heat at constant pressure of the fluid
k	Thermal conductivity of the fluid
H	Heat transfer coefficient