

Arbitrary-order unstructured finite-volume methods for implicit large eddy simulation of turbulent flows with adaptive dissipation/dispersion adjustment (ADDA)

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ABSTRACT

Implicit Large Eddy simulation (iLES) has gained substantial popularity for modelling high-Reynolds-number turbulent flows, found across several engineering and scientific fields. iLES on a first read represents a simple, computationally efficient, and straightforward way to model turbulent flows. The numerical dissipation/dispersion errors of high-resolution high-order numerical schemes employed in this context, can mimic the effects of the unresolved flow scales, and therefore acting like a subgrid-scale model, most often successfully. The numerical dissipation and dispersion of high-resolution methods are intertwined and controlling them to obtain physically meaningful results in under-resolved grid settings of compressible flows is challenging due to the presence of discontinuities and smooth flow features simultaneously. A numerical method should have the right amount of dissipation/dispersion, such that it can avoid the unphysical “build-up” of energy in the high modes or excessive diffusion, since this will render the method unsuitable for iLES. Several elegant approaches to master this delicate balance have been presented in the literature, including polynomial de-aliasing, Riemann solver dissipation adjustment, and adaptive blending of central and upwind schemes. This work presents an adaptive dissipation/dispersion adjustment (ADDA) algorithm that determines a well-resolved and under-resolved region and adjusts the numerical dissipation/dispersion of a high-order CWENOZ scheme, followed by further adjusting the flux-dissipation term depending on the presence of discontinuities. Implemented within an arbitrary-order finite-volume framework for unstructured meshes in compressible flows, the ADDA algorithm is put to the test across a range of under-resolved iLES simulations encompassing subsonic, transonic, and supersonic regimes. The developed framework exhibits enhanced robustness and scale-resolving capabilities, all while achieving physically meaningful results in a computationally efficient manner. All the methods have been implemented and have been made available to the research community in the open-source UCNS3D CFD solver to accelerate the improvement of the methods.

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1. Introduction

Many turbulent flows encountered by scientists and engineers exhibit high Reynolds numbers, encompassing a broad spectrum of spatial and temporal scales. This wide range of scales is the origin of the most important difficulty for the numerical simulation of turbulent flows. Higher the Reynolds number is, smaller are the smallest scales of the flow. When the size of these scales is smaller than the grid size (or more precisely, smaller than the resolution of the numerical method) the numerical scheme is not able to capture their effect on the dynamics of the flow. When this happens, it is said that the flow is under-resolved. Unfortunately, directly simulating all these scales using numerical methods like direct numerical simulation (DNS) of the Navier-Stokes equations is impractical due to computational limitations, despite the capabilities of today's supercomputers [1]. As an alternative, the Large Eddy Simulation (LES) family of techniques have been established as a suitable cost-effective alternative, wherein the large scales are resolved while the effects of the smaller unresolved scales are modeled using subgrid-scale (SGS) models [2–4]. In recent years, Implicit LES (iLES) has gained substantial momentum owing to its straightforward approach [5–35]. Here, the influence of the unresolved scales is implicitly incorporated via the numerical dissipation inherent in several elegant high-resolution non-oscillatory numerical methods, bypassing the need for explicit SGS models. iLES has become a preferred turbulence modeling method in various sophisticated numerical frameworks, including finite-volume (FV), finite-element (FE), finite-difference (FD) and a range of high-order non-oscillatory family of schemes such as WENO, MUSCL, TVD within them.

However, designing non-oscillatory schemes for compressible flows presents a challenge due to the simultaneous occurrence of smooth regions requiring low-dissipation methods and discontinuous areas necessitating non-oscillatory behaviour free of spurious oscillations within the computational domain. The non-linear nature of high-resolution schemes exhibit numerical dissipation that resembles SGS models, and therefore can be applied to a wide-range of practical engineering problems. It has been considered ideal to develop numerical schemes with very low-numerical dissipation and dispersion within this context, but this is far from true. Across all the numerical frameworks (FE, FD, FV) a method with very-low numerical dissipation although beneficial for resolving more features is unsuitable since this leads to an unphysical “build-up” of energy in the high modes, which could also be related to dispersive effects becoming more pronounced at this regime, a common observation in under-resolved grid resolutions. It might be more pronounced in some numerical frameworks such as DG, compared to FV methods, but even in FV implementations when the discretisation order increases (and dissipation is reduced), a similar behaviour is noticed. *A suitable numerical method for iLES should have “the right amount” of numerical dissipation and dispersion*, meaning that the numerical dissipation and dispersion should mimic the physics of the transfer of energy to the scales that are missing in the numerical grid. When the flow is under-resolved, the energy transfer from the larger scales to the smallest ones is truncated. When this truncation takes place in the inertial range (which is usual in practical turbulent flow computations), the energy transferred from the larger scales to the smaller ones must be dissipated at the smallest scales present in the grid, to avoid the “build-up” of energy. This dissipation is purely numeric, since physical dissipation is not the main mechanism that drives the dynamics in the inertial range.

There is a considerable body of work [5,7,9,15,25,32,34,36–38] focused on the adaptation of the numerical methods used to ensure that the “right amount” of dissipation is provided for iLES simulations, and to minimise the unphysical “build-up” of energy in the high modes, a campaign that can contribute to further improving the robustness and adoption of high-order numerical methods. Within this campaign there are approaches that focus on minimising the unphysical “build-up” of energy in the high modes by limiting the effect of high-order modes associated with the polynomials used as performed by Winters et al. [32], where a polynomial de-aliasing technique is employed for this purpose. Deng et al. [15] adapt the dissipation by blending an upwind and 2nd-order central scheme and have successfully deployed their framework in compressible isotropic turbulence, while Feng et al. [34] employ a Multi-objective Bayesian optimization (MOBO) to improve the performance of a WENO-iLES scheme and demonstrated the benefits of the approach using the Taylor-Green vortex flow problem, and Hickel et al. [38] employ a solution-adaptive local deconvolution and interpolation for the Taylor-Green vortex. Several approaches use the so-called automatic dissipation adjustment (ADA), or variants of them, using the energy ratio (ER) introduced by Tantikul and Domaradzki [37] to adjust the dissipation of their numerical framework through polynomial de-aliasing as done by Flad et al. [7], through the Riemann solver dissipation control of Li and Tsubokura [39], or through the adaptive blending of the central and the dissipation part of a WENO type discretisation [5,25]. In compressible flows, the modification of the intrinsic dissipation of the numerical scheme is not straightforward, since the possible presence of shock waves may lead the simulation to crash if the numerical dissipation is not enough to prevent spurious oscillations. A mechanism to allow the instantaneously recovering of the full dissipation of the numerical scheme is required for any of these methods to be successfully applied to compressible flows. The *a posteriori* approach proposed by Nogueira et al. [25] has demonstrated the stability of the numerical scheme in a framework of adaptive dissipation in compressible flows for structured meshes.

The *a posteriori* strategies however might be quite challenging to implement in an implicit time-stepping method, usually deployed for high-Reynolds number flows, since the cells contaminated with inadmissible solutions can influence the entire computational domain within the linear system solution. Obtaining analytical expressions for the numerical dissipation and dispersion for several non-linear high-order methods for practical problems where non-uniform meshes or unstructured-meshes are employed is not feasible, and the only alternative of approximate dispersion (ADR) is limited to unstructured meshes only for linear methods, that leads to results that are not understandable and therefore of unclear value at present. What complicates the situation is that for non-linear methods on unstructured meshes the dissipation and dispersion are entangled, and it is common for dispersion errors to becoming more pronounced when dissipation is reduced, while when increasing dissipation seems to overwhelm the dispersion errors and reduce their effect.

In this work, we are focused on developing an *a priori* approach that can adaptively adjust the dissipation and dispersion (ADDA) of high-order non-oscillatory methods for unstructured meshes that can be readily deployed for practical engineering problems. We want

our implementation to be generic, applicable to various non-oscillatory schemes of arbitrary order of accuracy, and easily extendable to other numerical frameworks, such as DG. Developing these methods are relying on utilising canonical well-established test problems (such as the Taylor-Green vortex) with available DNS results, to calibrate the developed methods to exhibit this favourable “right” amount of numerical dissipation. A key performance indicator of these methods is obtained by examining the 3D kinetic energy spectra across several wave numbers to understand if this “right” amount of dissipation is properly reflected by the way that the kinetic energy is dissipated. The selected numerical scheme for the present study is the high-order CWENOZ method [6] and the entirety of the present algorithms developed are implemented in the UCNS3D CFD code [40] which is written in object-oriented Fortran 2008, employing MPI message passing interface (MPI), and the Open Multi-Processing (OpenMP) application programming interface (API), and the reader is referred to [41] for more details on implementation and performance benchmarks. The computational times reported are obtained for the same hardware (and compilation settings) and normalised with respect to a reference setup for the same hardware so that the performance of various algorithms under identical hardware can be appreciated. All the meshes used for this study have been generated using the [Fidelity Pointwise 2023.1](#) mesh software.

The remainder of this paper is structured as follows. Section 2 describes the governing equations used, and section 3 is devoted to presenting the numerical framework used. Section 4 describes the building blocks of the ADDA algorithm, and provides a detailed description of the parameters used to control the dissipation/dispersion properties of the schemes. Section 5 follows with representative 3D iLES test problems used for assessing the performance of the ADDA algorithm. The cases considered include the iLES of the inviscid subsonic Taylor-Green vortex, the iLES of the subsonic SD7003 wing, the iLES of the viscous supersonic Taylor-Green vortex and the iLES of a viscous transonic cylinder. Finally, the conclusions drawn from this study are provided in Section 6.

2. Governing equations

The compressible Navier-Stokes equations are considered, written in conservative form as:

$$\frac{\partial \mathbf{U}(\mathbf{x}, t)}{\partial t} + \nabla \cdot (\bar{\mathbf{F}}_c(\mathbf{U}) - \bar{\mathbf{F}}_v(\mathbf{U}, \nabla \mathbf{U})) = 0, \quad (1)$$

where $\mathbf{U}$ is the vector of the conserved variables, $\bar{\mathbf{F}}_c$ and $\bar{\mathbf{F}}_v$ are the inviscid and viscous flux vectors, respectively, as

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E \end{bmatrix}, \bar{\mathbf{F}}_c = \begin{bmatrix} \rho u_n \\ \rho u u_n + n_x p \\ \rho v u_n + n_y p \\ \rho w u_n + n_z p \\ u_n(E + p) \end{bmatrix}, \bar{\mathbf{F}}_v = \begin{bmatrix} 0 \\ n_x \tau_{xx} + n_y \tau_{xy} + n_z \tau_{xz} \\ n_x \tau_{yx} + n_y \tau_{yy} + n_z \tau_{yz} \\ n_x \tau_{zx} + n_y \tau_{zy} + n_z \tau_{zz} \\ n_x \Theta_x + n_y \Theta_y + n_z \Theta_z \end{bmatrix}, \quad (2)$$

where ρ is the density; u, v, w are the velocity components in x, y and z Cartesian coordinates, respectively, and u_n is the velocity normal to the bounded surface area, defined by $u_n = n_x u + n_y v + n_z w$. Ideal gas is assumed where the total energy per unit mass is calculated by $E = p/(\gamma - 1) + (1/2)\rho(u^2 + v^2 + w^2)$, where p is the pressure, $\gamma = 1.4$ is the ratio of specific heats for air at normal atmospheric conditions; The laminar viscosity is related to the temperature through the Sutherland's law as

$$\frac{\mu_l}{\mu_0} = \left(\frac{T}{T_0} \right)^{\frac{3}{2}} \frac{T_0 + S}{T + S}, \quad (3)$$

S is the Sutherland temperature and the subscript 0 denotes a reference state for the corresponding variables. The work of viscous stresses and heat conduction, Θ , is given by:

$$\begin{aligned} \Theta_x &= u\tau_{xx} + v\tau_{xy} + w\tau_{xz} + \frac{\mu_l}{Pr} \frac{\gamma}{\gamma - 1} \frac{\partial T}{\partial x}, \\ \Theta_y &= u\tau_{yx} + v\tau_{yy} + w\tau_{yz} + \frac{\mu_l}{Pr} \frac{\gamma}{\gamma - 1} \frac{\partial T}{\partial y}, \\ \Theta_z &= u\tau_{zx} + v\tau_{zy} + w\tau_{zz} + \frac{\mu_l}{Pr} \frac{\gamma}{\gamma - 1} \frac{\partial T}{\partial z}. \end{aligned} \quad (4)$$

The viscous stress tensor τ_{ij} is defined by is

$$\tau_{ij} = \mu_l \left(\frac{\partial \mathbf{u}_i}{\partial \mathbf{x}_j} + \frac{\partial \mathbf{u}_j}{\partial \mathbf{x}_i} - \frac{2}{3} \frac{\partial \mathbf{u}_k}{\partial \mathbf{x}_k} \delta_{ij} \right), \quad (5)$$

where δ_{ij} is the Kronecker delta and the subscripts i, j, k refer to the Cartesian coordinate components $\mathbf{x} = (x, y, z)$. Unless otherwise stated, the reference values are taken at atmospheric conditions (sea level): dynamic viscosity $\mu_0 = 1.7894 \times 10^{-5} \text{ kg/(ms)}$; reference temperatures $T_0 = 288.16 \text{ K}$; $S = 110.4 \text{ K}$; and Prandtl number $Pr = 0.72$. The inviscid Euler equations are simply obtained by setting the viscous fluxes $\bar{\mathbf{F}}_v = 0$.

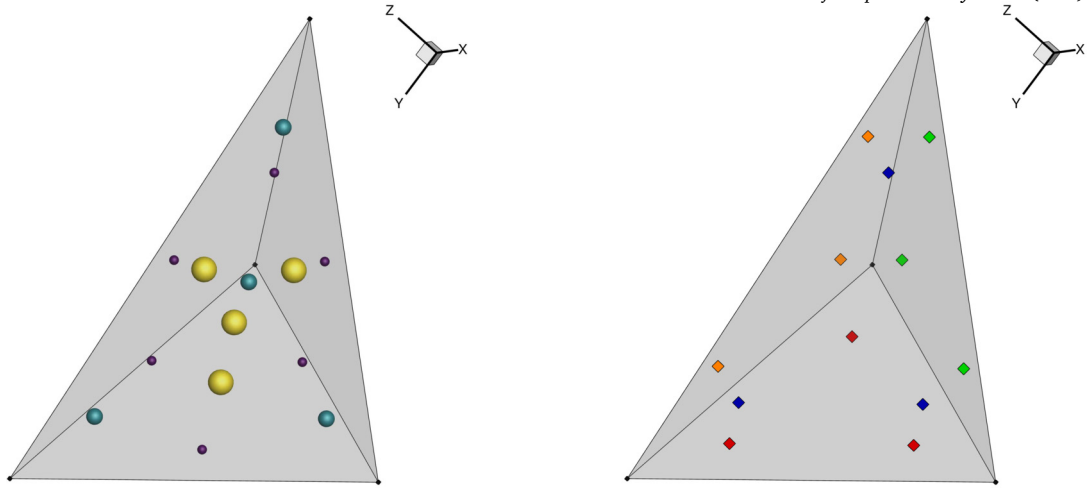


Fig. 1. Drawing illustrating the volume (left) and surface (right) quadrature points for a tetrahedral cell.

3. Numerical framework

We consider the Navier-Stokes Eq. (1) on a 3D domain Ω consisting of any combination of conforming tetrahedral, hexahedral, prism or pyramids in 3D. All the elements are indexed by a unique mono-index i . Integrating Eq. (1) over the mesh element i using a high-order explicit finite-volume formulation the following equation is obtained:

$$\begin{aligned} \frac{d\mathbf{U}_i}{dt} = & -\frac{1}{|V_i|} \sum_{l=1}^{N_f} \sum_{\alpha=1}^{N_{qp}} \tilde{\mathbf{F}}_{c_l} \left(\mathbf{U}_{l,L}^n(\mathbf{x}_{l,\alpha}, t), \mathbf{U}_{l,R}^n(\mathbf{x}_{l,\alpha}, t) \right) \omega_{\alpha} |S_l| \\ & + \frac{1}{|V_i|} \sum_{l=1}^{N_f} \sum_{\alpha=1}^{N_{qp}} \tilde{\mathbf{F}}_{v_l} \left(\mathbf{U}_{l,L}^n(\mathbf{x}_{l,\alpha}, t), \mathbf{U}_{l,R}^n(\mathbf{x}_{l,\alpha}, t), \nabla \mathbf{U}_{l,L}^n(\mathbf{x}_{l,\alpha}, t), \nabla \mathbf{U}_{l,R}^n(\mathbf{x}_{l,\alpha}, t) \right) \omega_{\alpha} |S_l|, \end{aligned} \quad (6)$$

where $\mathbf{U}_i$ are the volume averaged conserved variables

$$\mathbf{U}_i = \frac{1}{|V_i|} \int_{V_i} \mathbf{U}(x, y, z) dV, \quad (7)$$

and $\tilde{\mathbf{F}}_{c_l}$ and $\tilde{\mathbf{F}}_{v_l}$ are the numerical flux function in the direction normal to the cell interface between two cells as, N_f is the number of faces per element, N_{qp} is the number of quadrature points used for approximating the surface integrals, $|S_l|$ is the surface area of the corresponding face, and $\mathbf{U}_{l,L}^n(\mathbf{x}_{l,\alpha}, t)$ and $\mathbf{U}_{l,R}^n(\mathbf{x}_{l,\alpha}, t)$ are the high-order approximations of the solutions for the considered cell and its neighbour respectively and $\nabla \mathbf{U}_{l,L}^n(\mathbf{x}_{l,\alpha}, t)$ and $\nabla \mathbf{U}_{l,R}^n(\mathbf{x}_{l,\alpha}, t)$ are the high-order approximations of the gradients for the considered cell and its neighbour respectively; while α corresponds to different Gaussian integration points $\mathbf{x}_{\alpha}$ and weights ω_{α} over each face. The volume, surface and line integrals are numerically approximated by quadrature rules which are compatible to the polynomial order selected as shown in Fig. 1, the reader is referred to [42] for details on numerical approximations of multiple integrals.

3.1. Spatial discretisation

The reconstruction process employed in this work follows the approaches of Tsoutsanis et al. [6,43,44], Titarev et al. [45] that have been previously applied to smooth and discontinuous flow problems for single and multimaterial applications [41,46–59] and only the key elements are going to be described herein. For a cell i , a high-order polynomial $p_i(x, y, z)$ of order r is built in such a way that it provides $r+1$ order of accuracy, by requiring it to have the same average as a general quantity $\mathbf{U}_i$. This can be formulated as:

$$\mathbf{U}_i = \frac{1}{|V_i|} \int_{V_i} p_i(x, y, z) dV. \quad (8)$$

In this work we do not employ a transformation from physical to reference space as previous works [6,43,44] but we perform the reconstruction in physical coordinates as recently employed when hybridising DG-FV frameworks [60–63]. For computing the volume integrals, the non-tetrahedral elements are decomposed to numerous tetrahedral elements, following the strategy of Tsoutsanis et al. [44]. The reconstruction is performed by building a central stencil S^1 by recursively adding neighbouring elements as shown in Fig. 2, consisting of $M+1$ cells including the considered cell i . In this work, we employ the stencil-based compact algorithm (SBC) introduced in [64], where the reader is referred to for more details. For improved robustness, we employ $M=2K$ cells in the stencil as reported in several previous studies [64–70], where K is the total number of polynomial coefficients given by:

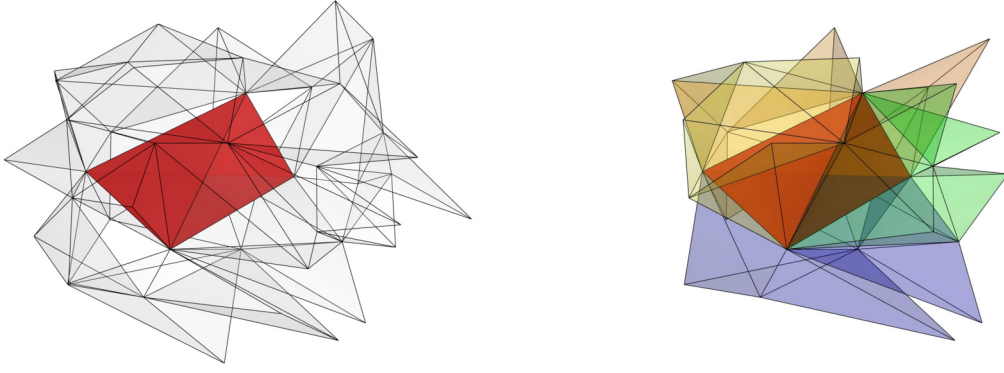


Fig. 2. Example of central reconstruction stencil (left) and directional reconstruction stencils (right), with the considered cell illustrated in red colour. (For interpretation of the colours in the figure(s), the reader is referred to the web version of this article.)

$$K(r, d) = \frac{1}{d!} \prod_{l=1}^d (r + l), \quad (9)$$

where $d \in [2, 3]$ is the number of space dimensions. The central stencil S^1 is given by

$$S_i^c = \bigcup_{m=0}^{M_c} V_m, \quad (10)$$

where the index m refers to the local numbering of the elements in the stencil with the element with index 0 being the considered cell i , and the index c referring to the stencil number (in case of multiple stencils) where for the central stencil $c = 1$. The r^{th} order reconstruction polynomial is an expansion over local polynomial basis functions $\phi_k(x, y, z)$ given by:

$$p(x, y, z) = \sum_{k=0}^K a_k \phi_k(x, y, z) = \mathbf{U}_0 + \sum_{k=1}^K a_k \phi_k(x, y, z), \quad (11)$$

where $\mathbf{U}_0$ corresponds to the vector of conserved variables at the considered cell i , and a_k are the degrees of freedom of the polynomial. The degrees of freedom a_k for the polynomial for each cell m are obtained by satisfying the condition that the cell average of the reconstruction polynomial $p(x, y, z)$ must be equal to the cell average of the solution $\mathbf{U}_m$:

$$\int_{V_m} p(x, y, z) dx dy dz = |\mathbf{U}_m| \mathbf{U}_0 + \sum_{k=1}^K \int_{V_m} a_k \phi_k dx dy dz = |\mathbf{U}_m| \mathbf{U}_m, \quad m = 1, \dots, M. \quad (12)$$

The basis functions ϕ_k should be formulated in such a fashion that they also satisfy equation (8), and they are defined as follows:

$$\phi_k(x, y, z) \equiv \psi_k(x, y, z) - \frac{1}{|V_0|} \int_{V_0} \psi_k dx dy dz \quad k = 1, 2, \dots, K, \quad (13)$$

and in the present study ψ_k are Taylor polynomials basis functions given by

$$\psi_k(x, y, z)|_{V_0} = \frac{(x - x_{c_0})^{p_k}}{h_0^{p_k}} \frac{(y - y_{c_0})^{q_k}}{h_0^{q_k}} \frac{(z - z_{c_0})^{r_k}}{h_0^{r_k}} \quad (14)$$

where $0 \leq p_k + q_k + r_k \leq K$, $k = 1, \dots, K$. The index 0 indicates the considered cell, and h_0 is the characteristic cell size defined as the square root of the volume of the considered cell, and $(x_{c_0}, y_{c_0}, z_{c_0})$ are the coordinates of the cell centre of the considered element i . Denoting the integrals of the basis function k over the cell m in the stencil, and the vector of right-hand side by A_{mk} and b respectively as given by

$$A_{mk} = \int_{V_m} \phi_k dx dy dz, \quad b_m = |\mathbf{U}_m| (\mathbf{U}_m - \mathbf{U}_0), \quad (15)$$

the equations for degrees of freedom a_k can be rewritten in a matrix form as:

$$\sum_{k=1}^K A_{mk} a_k = b_m, \quad m = 1, 2, \dots, M. \quad (16)$$

Finally, the resulting linear system is solved by a QR decomposition [71] while using a Moore-Penrose pseudo-inverse of A_{mk} which is stored in memory for improving computational speed as detailed in [41].

3.1.1. CWENOZ

The CWENOZ scheme developed by Tsoutsanis and Dumbser [6] is employed in this study, that has demonstrated improved computational efficiency and robustness compared to the original WENO schemes. The CWENOZ scheme combines the strength of an optimal (high-order) p_{opt} polynomial from the central stencil with lower-order polynomials from directional stencils. When encountering smooth data, the scheme effectively retrieves the optimal polynomial, achieving the desired accuracy. However, in the presence of discontinuous data, at least one lower-order polynomial from the directional stencils may capture smooth regions, thereby dampening oscillations in the computed solution. All polynomials adhere to the same requirement of matching the cell averages of the solution. Computational efficiency relative to WENO schemes stems from smaller directional stencil sizes and the containment of directional stencils within the central stencil, and the reader is referred to [6] for further details and comparisons. The definition of an optimal polynomial given by:

$$p_{opt}(x, y, z) = \sum_{s=1}^{s_t} \lambda_s p_s(z, y, z), \quad (17)$$

where s is the stencil index, with $s = 1$ being the central, $s = (2, 3, \dots, s_t)$ being the directional, s_t being the total number of stencils, and λ_s being the linear coefficient for each stencil, whose sum is equal to 1. The p_1 polynomial is computed by subtracting the lower-order polynomials from the optimum polynomial as follows:

$$p_1(x, y, z) = \frac{1}{\lambda_1} \left(p_{opt}(x, y, z) - \sum_{s=2}^{s_t} \lambda_s p_s(x, y, z) \right). \quad (18)$$

The CWENOZ reconstruction polynomial is a non-linear combination of all the polynomials as follows:

$$p(x, y, z)^{cwenoZ} = \sum_{s=1}^{s_t} \omega_s p_s(x, y, z) \equiv U_0 + \sum_{k=1}^K \tilde{a}_k \phi_k(x, y, z), \quad (19)$$

where ω_s corresponds to the non-linear weights assigned to each reconstruction polynomial, and in regions with smooth data $\omega_s \approx \lambda_s$, therefore achieving the high-order approximation provided from the central stencil, and in regions of discontinuous solutions the reconstructed solution will be more dependent on the lower-order reconstruction polynomials of the directional stencils, where $\tilde{a}_k$ are the reconstructed degrees of freedom; and the non-linear weight ω_s is defined as: The non-linear weights are now defined as:

$$\omega_s = \frac{\tilde{\omega}_s}{\sum_{s=1}^{s_t} \tilde{\omega}_s} \quad \text{where} \quad \tilde{\omega}_s = \lambda_s \left(1 + \left(\frac{\tau}{\epsilon + SI_s} \right)^b \right). \quad (20)$$

With τ being the universal oscillation indicator and taken as the absolute difference between the smoothness indicators as follows:

$$\tau = \left(\frac{\sum_{s=2}^{s_t} |SI_s - SI_1|}{s_t - 1} \right). \quad (21)$$

In this study a cell-size dependent value for $\epsilon = h_c^4$ is used following the work of [72,73] and $b = 4$, which is better suited for computational domains with cell-sizes of several orders of magnitude differences. For the present study we employ $r = 1$ for the directional polynomials resulting in 2nd-order of accuracy, and any arbitrary order of accuracy for the polynomial associated with the central stencil. The linear weights are computed by firstly assigning the non-normalised linear weight for the central stencil λ'_1 an arbitrary value, and then normalising this as follows:

$$\lambda_1 = 1 - \frac{1}{\lambda'_1}, \quad (22)$$

with the linear weights associated with lower-order polynomials being assigned the same linear weights as follows:

$$\lambda_s = \frac{1 - \lambda_1}{s_t - 1}, \quad (23)$$

where s_t is the total number of stencils. The smoothness indicator SI_m is given by:

$$SI_s = \sum_{1 \leq |\beta| \leq r} \int_{V_0} |h_c|^{|\beta|-1} (D^\beta p_s(x, y, z))^2 (dx, dy, dz), \quad (24)$$

where β is a multi-index, r is the polynomial's order, λ_s is the linear weight and h_c is the characteristic cell size of the considered cell. The central stencil is assigned a large linear weight λ_1 while the remaining directional stencils are assigned a value of $\lambda_s = 1$

and D is the derivative operator. The smoothness indicator is a quadratic function of the degrees of freedom (a_k^s) and Eq. (24) can be rewritten as:

$$SI_s = \sum_{k=1}^K a_k^s \left(\sum_{q=1}^K \mathcal{O}I_{kq} a_q^s \right), \quad (25)$$

where the oscillation indication matrix $\mathcal{O}I_{kq}$ is given by:

$$\mathcal{O}I_{kq} = \sum_{1 \leq |\beta| \leq r} \int_{V_0} |h_c|^{|\beta|-1} (D^\beta \phi_k(x, y, z)) (D^\beta \phi_q(x, y, z)) (dx, dy, dz), \quad (26)$$

and can be precomputed and stored at the beginning of the simulation. This reconstruction can be carried out with respect to the characteristic, primitive or conserved variables, and the reader is referred to [43,44,46,51,58,63] and references therein regarding the implementation.

3.2. Low-Mach number correction (LMC)

In low-Mach number regions the correction introduced by Simmonds et al. [74,75] is employed in this work, where the extrapolated reconstructed values at the cell interfaces are modified to take into account the correct flow physics of low speed flows. This treatment has demonstrated that significant improvements in terms of dissipation properties can be achieved in low-Mach number regions, particularly for hexahedral and quadrilateral elements in 3D and 2D respectively. In the LMC of Simmonds et al. [74,75] only the normal components $\hat{n}$ of the velocity at the interfaces are modified, and not their tangential $n_{||}$ as follows:

$$u_L^{\hat{n}*} = \frac{(1+z)u_L^{\hat{n}} + (1-z)u_R^{\hat{n}}}{2}, u_R^{\hat{n}*} = \frac{(1+z)u_R^{\hat{n}} + (1-z)u_L^{\hat{n}}}{2}, \quad z = \min(1, \max(M_L, M_R)). \quad (27)$$

M_L, M_R correspond to the Mach number of the left and right-states respectively, and $u_L^{\hat{n}*}, u_R^{\hat{n}*}$ are the modified extrapolated normal component of velocities for the left- and right-states respectively, and $u_L^{\hat{n}}, u_R^{\hat{n}}$ denote the reconstructed extrapolated values for the velocity in the direction normal to the face/edge for the left- and right-states respectively. The local Mach number for the left and right states M_L, M_R is calculated based on the velocity magnitude of all the velocity components independent of the normal direction.

3.3. Fluxes approximation

For the inviscid fluxes we employ the (Harten-Lax-van Leer-Contact) HLLC approximate Riemann solver of Toro et al. [76], or the Roe Riemann solver [77]. For the viscous fluxes, the gradients are computed using a constrained least-square reconstruction for the boundary condition enforcement, as detailed in [40,44]. The gradients of the discontinuous states for the approximation of the viscous fluxes are averaged by including the penalty terms similar to previous approaches [78,79] in the following manner:

$$\nabla \mathbf{U} = \frac{1}{2} (\nabla \mathbf{U}_L + \nabla \mathbf{U}_R) + \frac{\alpha}{L_{int}} (\mathbf{U}_R - \mathbf{U}_L) \vec{n}, \quad (28)$$

where L_{int} is the distance between the cell centres of adjacent cells, and we set $\alpha = 4/3$ similarly to previous approaches [65,78].

3.4. Temporal discretisation

For this study we employ the explicit 4th-order Strong-Stability-Preserving (SSP) Runge-Kutta of Spiteri and Ruuth [80], which is stable for ($CFL \approx 1.5$), where the readers are referred for further details [80]. For high-Reynolds number wall-resolved flow problems that we want to bypass the severe CFL limit of the explicit scheme, we employ the well established and widely used dual-time stepping strategy [81] where the transient problem is solved as a series of steady-state problems, and several techniques are used to accelerate the convergence to steady state including local time stepping, implicit approximate Jacobian factored implementation using the Rusanov flux [82] and the matrix-free lower-upper symmetric Gauss-Seidel (LU-SGS) similar to [83,84]. The reader is referred to the UCNS3D implementation [40] and references therein.

4. ADDA algorithm

The present ADDA algorithm has three procedures including measuring the energy ratio, CWENOZ adaptation and flux-dissipation adjustment. Considering an iLES simulation of the flow past a cylinder, then for every considered cell in the computational domain the ADDA algorithm will execute all these procedures in a finely-grained manner where every cell and cell interface can determine how to adjust its dissipation/dispersion characteristics. The ADDA algorithm flow chart is illustrated in Fig. 3 that follows, and is going to be useful for understanding how the ADDA algorithm operates at its entirety and for discussing the particular details of the implementation of each of the three distinct procedures.

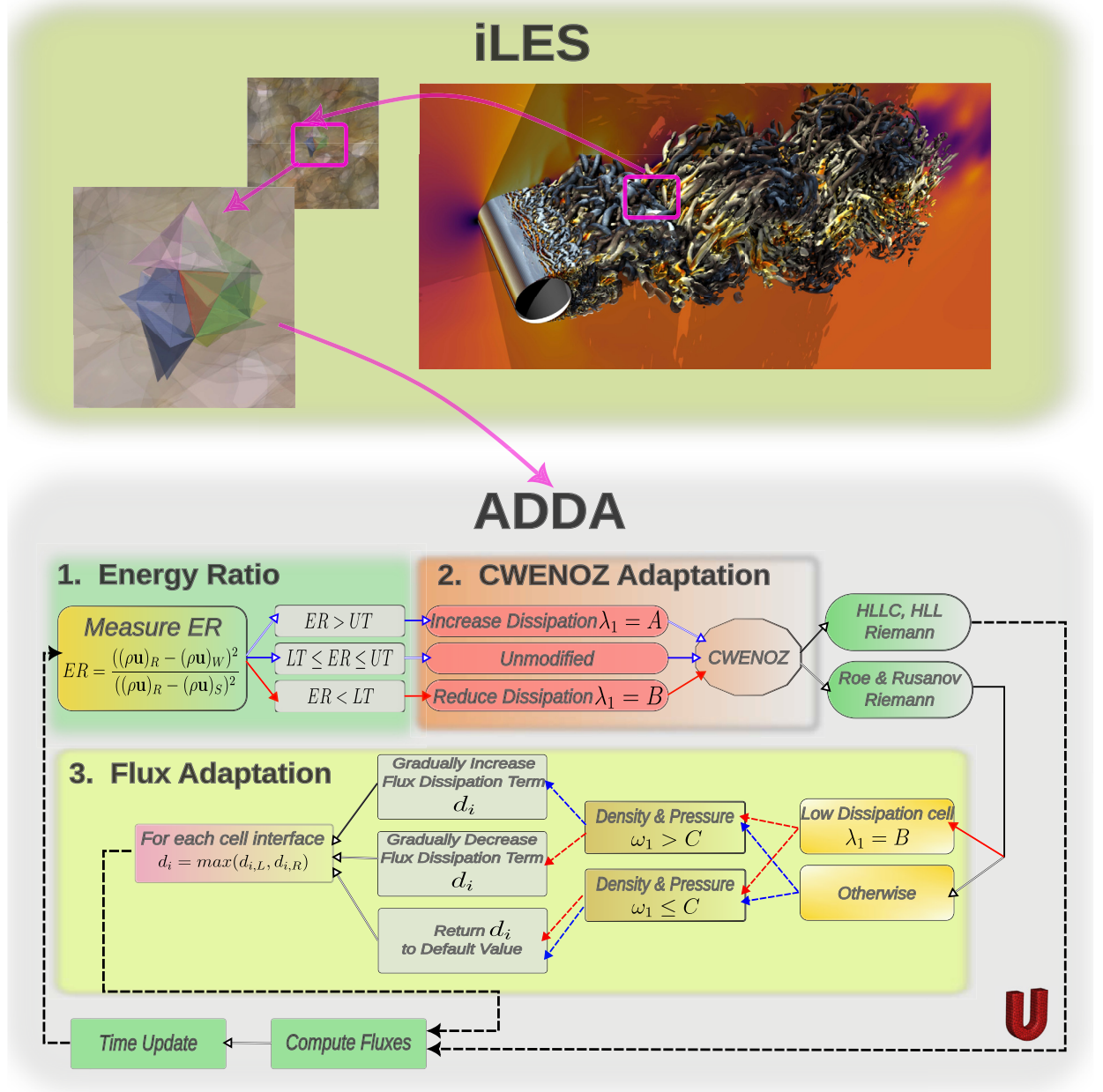


Fig. 3. Flow-chart of the ADDA-augmented UCNS3D code. In its current configuration, during an iLES simulation the ADDA operation is executed at every physical time step for every cell in the computational domain, through a series of procedures. The 1st procedure involves measuring the Energy Ratio that indicates if the field associated with a cell is well-resolved or under-resolved, and in turn dictates if this cell requires more or less dissipation. All the possible paths which are unique to the low dissipation cells are illustrated with red colour, and the alternatives with blue colour. The 2nd procedure is associated with the adaptation of CWENOZ reconstruction by adjusting the linear weight (λ_1) of the central high-order stencil. We can deploy a 3rd procedure of flux adaptation when the Roe or Rusanov Riemann solvers are used, by adjusting the dissipation (gradual increase, gradual decrease, or return to default value) through the flux dissipation control term (d_i) depending on the non-linear CWENOZ weight for the density and pressure, to ensure that the non-oscillatory properties of the framework are maintained. *It has to be noted that the gradual decrease of the flux dissipation term can only be performed for a low-dissipation cell in smooth regions of the flow, as indicated by the red coloured arrow lines.* This is then followed by using the maximum flux dissipation between two states $d_i = \max(d_{i,L}, d_{i,R})$ for ensuring conservation. Finally, the fluxes are computed, and the solution advanced in time.

4.1. Energy ratio

The first parameter employed in the context of ADDA is to measure how well-resolved the flow is at every cell. For this reason, we utilised the so-called local energy-ratio introduced in [8,37] and further adapted by [5] as follows:

$$ER = \frac{\int_{V_i} ((\rho \mathbf{u})_R - (\rho \mathbf{u})_W)^2 dV}{\int_{V_i} ((\rho \mathbf{u})_R - (\rho \mathbf{u})_S)^2 dV}, \quad (29)$$

Table 1

Filter parameters description, with **S** and **W** corresponding to strong and weak variations of the filters respectively.

filter number	type	a (S/W)	s (S/W)	η_c	η_{max} (S/W)
Filter 1	sharp	N/A	N/A	N/A	$r - 2/r - 1$
Filter 2	expon.	4/4	8/16	0	r/r
Filter 3	expon.	16/16	8/32	0	r/r
Filter 4	expon.	16/16	4/64	0	r/r
Filter 5	expon.	64/2	16/16	0	r/r
Filter 6	expon.	32/8	16/16	0	r/r
Filter 7	expon.	32/8	8/8	0	r/r

where the $(\rho\mathbf{u})_R$ represents the unfiltered reconstructed solution integrated within each cell, while the $(\rho\mathbf{u})_W$ and $(\rho\mathbf{u})_S$ represent the filtered reconstructed solution integrated within each cell with a weak- and strong-filter applied, respectively.

The choices of strong and weak filters are far from unique, with different elegant approaches available from exponential to sharp-filters that are widely used in the literature [5,7,37,85,86]. Determining optimum strong and weak filters with distinct scale separation, requires extensive testing and the calibration is usually performed on well-established turbulent flow problems such as the Taylor-Green vortex.

Remark 1. It is common practice that during this calibration, the numerical method parameters and the grid-type remain fixed. This calibration strategy can not be easily extended to a wide-range of numerical methods, with different settings, for unstructured meshes since these will require an excessive number of simulations for optimising these parameters, and of uncertain value especially when the unique-characteristics of mesh element shapes for realistic unstructured meshes are unknown.

We are also constrained by the fact that analytical expressions for dispersion/dissipation errors are limited to linear methods, and the only alternative of approximate dispersion relation (ADR) is limited to 1D and not accounting for non-linear interactions [87], while extension to unstructured meshes, even for linear methods, leads to results that are not understandable and therefore of unclear value at present [88]. Within this landscape we want to employ filters that are suitable for identifying the well-resolved and unresolved regions, that can be routinely used on unstructured meshes of various element types and to be deployed for CWENOZ class of schemes for various orders of spatial accuracy. For this purpose, we consider firstly an exponential type of filter with different parameters that is widely used in the DG/FR community for reducing the aliasing errors associated with the high-order polynomial modes in under-resolved settings [32,86,89] as shown in Eq. (30).

$$\sigma(\eta) = \begin{cases} 1, & \eta = 0 \\ \exp\left(-a\left(\frac{\eta - \eta_c}{\eta_{max} - \eta_c}\right)^s\right), & 1 \leq \eta \leq \eta_{max} \end{cases} \quad (30)$$

For the exponential filter a is the damping parameter, s is the strength parameter, $\eta = (p_k + q_k + r_k)$ is the summation of exponents of the Taylor-basis polynomial terms in the polynomial used as shown in Eq. (14), η_c is the cut-off degree, and η_{max} is the maximum summation of exponent of the Taylor-basis polynomial terms, and r is the desired polynomial order. Following the findings from previous studies [5,37], where it has been reported that the weak filter should not be too weak otherwise it's not useful at distinguishing well-resolved and under-resolved regions, we have selected to introduce a second type of filter by adopting a generic sharp filtering strategy that its weak variant will switch-off the Taylor-basis polynomial coefficients associated with the r -order terms, and its strong variant will additionally switch-off the Taylor-basis polynomial coefficients associated with the $r - 1$ -order terms. In this way, we have a weak filter corresponding to a one-polynomial order lower than the intended one, and the strong filter corresponding to a two-polynomial orders lower than the intended. The definition of the filter is shown in Eq. (31).

$$\sigma(\eta) = \begin{cases} 1, & \eta = 0 \\ 1, & 1 \leq \eta < \eta_{max} \\ 0, & \eta = \eta_{max} \end{cases} \quad (31)$$

We are not considering anisotropic filters in this study, since the filtering applied is the same for the all the Taylor-basis polynomial terms associated with polynomial order. We want to understand how the sensitive the energy ratio ER is to the choice of filters used, and which filters are good candidates to be used within this context. For this purpose, have selected the filters as detailed in Table 1.

Considering a generic variable Q , the reconstruction polynomial is an expansion over local polynomial basis functions $\phi_k(x, y, z)$ given by:

$$p(x, y, z) = \sum_{k=0}^K \sigma_k a_k \phi_k(x, y, z) = Q_0 \sigma_0 + \sum_{k=1}^K \sigma_k a_k \phi_k(x, y, z), \quad (32)$$

and the 0 degree of freedom is associated with Q_0 , and the parameter σ_k is associated with the filtering of the polynomial, and it can take any values between $[0, 1]$. The unfiltered variable is obtained with the following polynomial

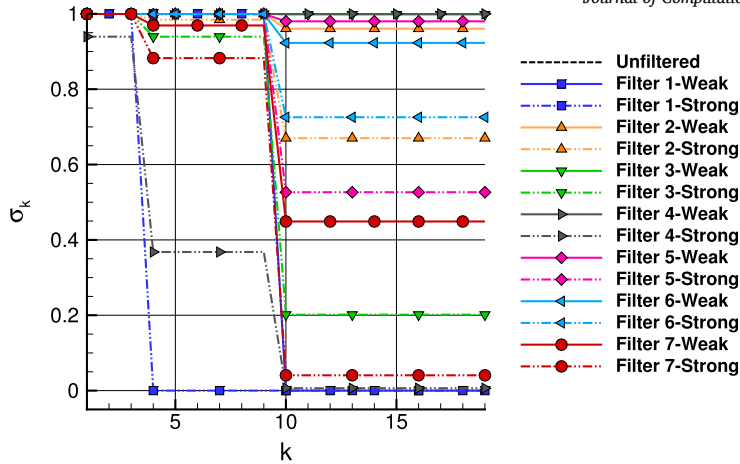


Fig. 4. Variation of σ_k for each polynomial degree of freedom K for different variants of filters used for the energy ratio definition, for a CWENOZ4 scheme.

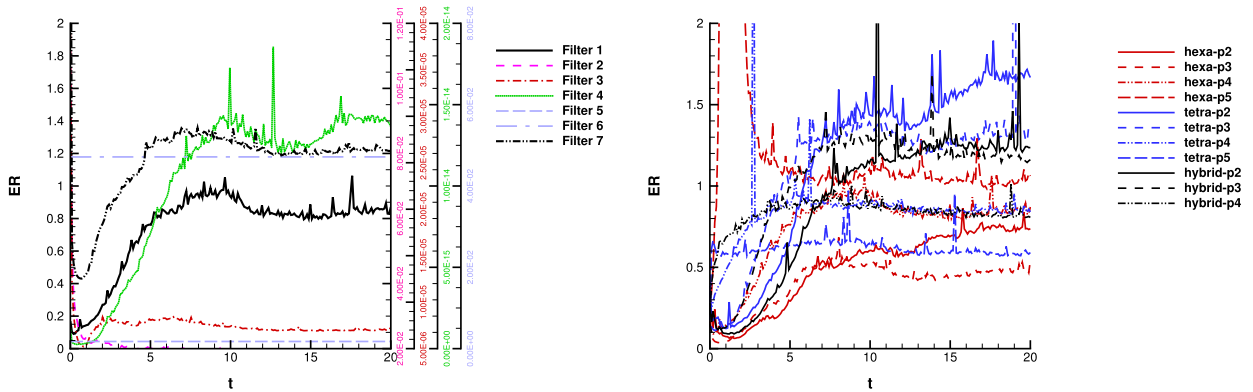


Fig. 5. Evolution of mean ER within the domain for the 3D inviscid Taylor-Green vortex for a hexahedral mesh with a CWENOZ4 scheme with different filters (left), and for various meshes and schemes with filter 1 (right). Different y-axis ranges are used (with different colours) when the range of ER variation of filters is different (left).

$$p^R(x, y, z) = \sum_{k=0}^K \sigma_k a_k \phi_k, \quad \text{with} \quad \sigma_k = 1, \quad (33)$$

the *weak filter* application to the variable is given by:

$$p^W(x, y, z) = \sum_{k=0}^K \sigma_k^W a_k \phi_k \quad \text{with} \quad \sigma_k^W = \sigma(\eta)^W, \quad (34)$$

and the *strong filter* application to the variable is given by:

$$p^S(x, y, z) = \sum_{k=0}^K \sigma_k^S a_k \phi_k \quad \text{with} \quad \sigma_k^S = \sigma(\eta)^S. \quad (35)$$

The variation of filtering parameter σ_k and degrees of freedom k for a 4th-order scheme can be seen in Fig. 4, where it can be noticed that the filters with the least-weak filtering are filter 1 and filter 7.

With these 7 filters, we simulated the inviscid 3D Taylor-Green vortex flow with a CWENOZ4 scheme on an under-resolved grid resolution of 64^3 . The filters were only used for monitoring purposes of the energy ratio ER, since we did not apply any filtering to our reconstructed solutions, but we used them for calculating the ER for each cell, since we wanted to understand how the ER varies during the simulation.

From the obtained results as shown in Fig. 5, it can be noticed that mean ER variation during the simulation is similar between filter 1 and filter 7, where the flow is well-resolved (low-ER) and as the Taylor-Green vortex flow structures interact with each other and transition to turbulence occurs we see a gradual increase of the ER (more regions are becoming under-resolved (high-ER), which we will explain later how this is manifested, and if any useful information can be drawn from this. All the other filters do not provide useful information regarding the ER, either because their variation is very close to machine precision (filter 4), or their ER remains flat due to the small differences between the weak and strong filters used. Any of the two filters (filter 1 and filter 7) seem as good

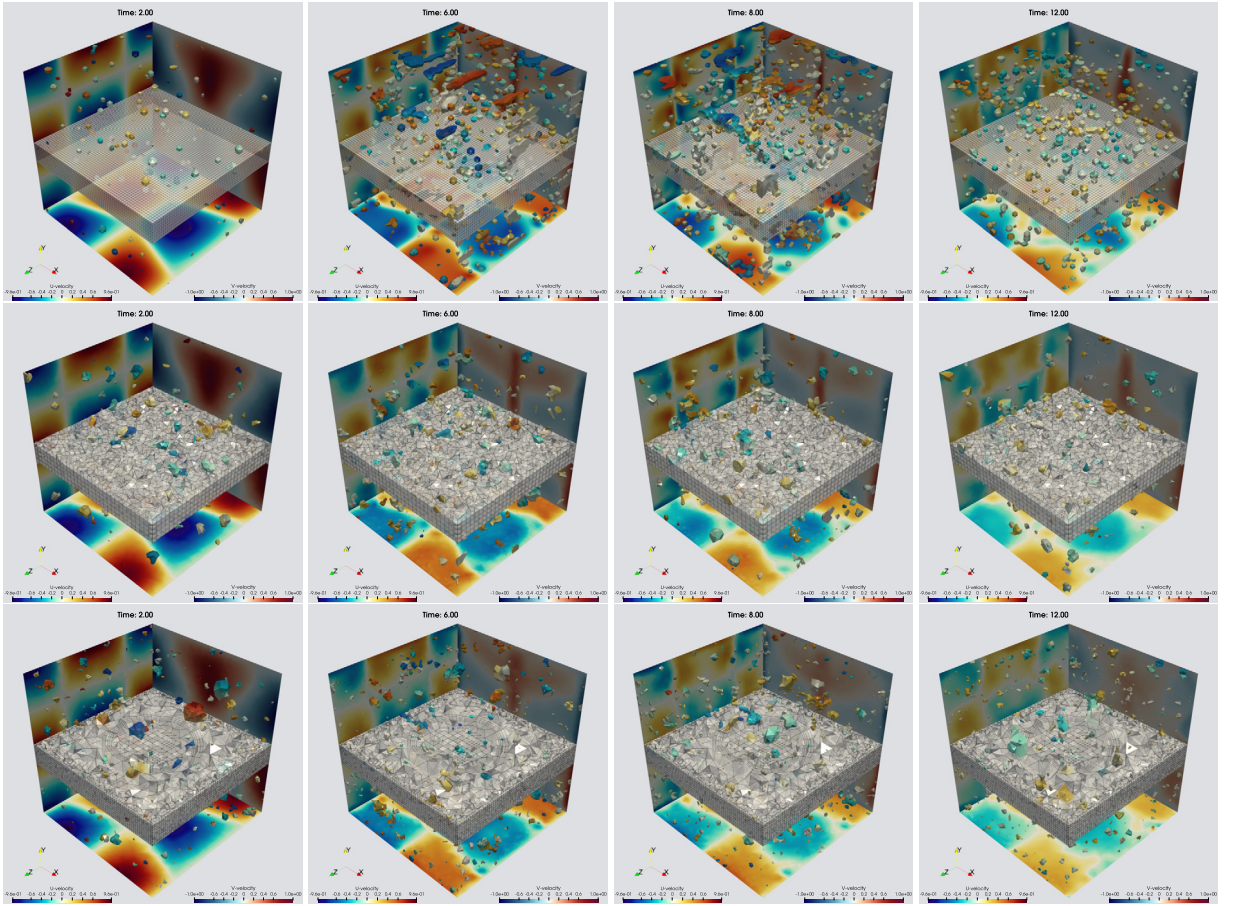


Fig. 6. Contour plots of U-velocity and V-velocity and the cells with ($ER > 1.2$) for the 3D inviscid Taylor-Green vortex flow for a CWENO4 schemes at different times, for a hexahedral (top), tetrahedral (medium), and hybrid (bottom) mesh.

candidates to be used since there is sufficient difference between their strong and weak variations, and their weak variants are the least-weak as previously described.

For this purpose we have selected the sharp-filter filter 1, to run the Taylor-Green vortex flow problem for three different types of meshes at the same resolution of 64^3 using a CWENO3, CWENO4, CWENO5 and CWENO6 schemes. From the obtained results as shown in Fig. 5 it can be noticed that the ER varies significantly throughout the simulation, with a common pattern which is associated with a rise of the ER at later times and stabilising until the end. The inviscid Taylor-Green vortex flow selected is severely under-resolved at late times, and therefore one would expect the mean ER to be increased as the simulation progresses. More and more cells become under-resolved as the simulation advances, however there are instances where ($ER \gg 1$), which is not only an indication that the flow is under-resolved for them, but rather that an unphysical “build-up” of energy in the high modes associated with the high-order polynomial terms has occurred. This can be seen in Fig. 6 where we have plotted only the cells with an ($ER > 1.2$) and it can be seen that we have a small number of cells at early time ($t = 2$), that increase at ($t = 8 - 6$) and become more dispersed at late times ($t = 12$). For the tetrahedral and hybrid meshes we notice that number of cells with ($ER > 1.2$) is increased compared to hexahedral mesh even at early times ($t = 2$), which is indicative of their dispersive properties as previously reported in [90], and a more pronounced accumulation of unphysical energy in the high modes.

Remark 2. The optimum filtering technique might be beyond reach for practical problems involving several numerical methods, and meshes. However, the two filters (filter 1 and filter 7) that are characterised with not “too-weak” variants and significant difference from their-strong variants, lead to an ER variation that can be useful in identifying instances of well-resolved, under-resolved regions and regions where there is an unphysical “build-up” of energy in the high modes.

The ER is measured on unlimited reconstruction polynomials using the central stencil prior to any limiting performed by the CWENOZ algorithm. In smooth regions of the flow, the CWENOZ procedure will recover the optimal reconstruction polynomial associated with the unlimited reconstruction polynomial, and is therefore representative of the optimal accuracy of the CWENOZ scheme. By subtracting from the unfiltered field the filtered ones, that have damped the high-frequency content, we obtain an

estimate of the high-frequency content at every cell, and therefore we get an indication from the ER if the flow is well-resolved or under-resolved for a particular cell.

Remark 3. When there is abundant high-frequency content, it is implied that the turbulent flow-structures are not properly captured by the current configuration of the numerical scheme and grid resolution. However, for compressible flows the high-frequency content might also be manifested by discontinuities (such as shock waves, rarefaction waves, contact waves), and therefore one should appreciate that these conditions are intertwined and therefore both should be employed as a metric regarding the adjustment of our numerical scheme.

The behaviour of the deployed numerical scheme should be adapted by increasing its dissipation when the flow is under-resolved and reduce its dissipation when the flow is well-resolved. What complicates the situation is that for non-linear methods on unstructured meshes the dissipation and dispersion are entangled, and it is common when increasing dissipation seems to overwhelm the dispersion errors and their effect. There are well established procedures to assess the dissipation and dispersion properties of several linear and non-linear schemes using the so-called approximate dispersion relation (ADR) as implemented by Pirozzoli [87] where several popular non-linear schemes were assessed, and further expanded by Cravero et al. [91] for CWENOZ schemes. Recently Ruano et al. [88] developed a new general method free of FFT for assessing the dissipation and dispersion properties of schemes even on stretched Cartesian meshes, where it has been highlighted that the extension of this general method or ADR for unstructured meshes even for linear methods leads to results that are not understandable and therefore of unclear value at present. Therefore, within this context the properties of the numerical schemes for unstructured meshes for multidimensional non-linear reconstructions are assessed against either analytical solutions, reference solutions from other computational studies and DNS results, since the presence of different element types present in unstructured meshes have different dissipation/dispersion behaviours. Dispersion errors are more evident in particular when deploying certain element types such as tetrahedra and prisms, as reported by [90]. They can in turn be manifested as high-frequency unresolved content in the ER metric, and a simple remedy would be to increase the dissipation of the numerical method, since these elements will present themselves with spikes in the ER.

It needs to be stressed that several parameters will be taken into account and several actions stem from each one of them depending on their values. The first ADDA operation is to measure the ER according to the equation Eq. (29) and depending on the values of the ER we take the following actions as shown below:

$$\text{Measure } ER \Rightarrow \begin{cases} ER > UT, & \text{Increase Dissipation,} \\ LT \leq ER \leq UT, & \text{Unmodified,} \\ ER < LT, & \text{Reduce Dissipation,} \end{cases} \quad (36)$$

where the low-threshold value used ($LT = 0.95$)-which corresponds to well-resolved regions, and an upper-threshold value of ($UT = 1.2$) is used for defining the under-resolved regions. We employ these set of values to distinguish between well-resolved and under-resolved regions, and they have been selected following the numerical experiments with filter 1 and filter 7 previously reported. The fine-tuning of these selected parameters is beyond the focus and scope of this study.

4.2. CWENOZ adaptation

The next step in our ADDA operation is to increase, decrease the dissipation or keep the dissipation unchanged according to Eq. (36) described previously. Our strategy is quite generic, and it can be applied to several high-order non-oscillatory schemes in this context such as WENO, WENOZ, TENO, TENOZ, TENO-A, CWENO, CWENOZ [72,73,92–101] by carefully selecting one or more parameters for controlling the dissipation/dispersion of the schemes. For this particular study, we note the CWENOZ non-linear reconstruction polynomial weights are given by the following equation:

$$\tilde{\omega}_s = \lambda_s \left(1 + \left(\frac{\tau}{\epsilon + SI_s} \right)^b \right), \quad (37)$$

where λ_s is the linear weight assigned to each stencil. One way to control the dissipation properties of the CWENO/CWENOZ type of schemes is through the linear weight assigned to the central stencil λ_1 , the one associated with the high-order approximation, as it has been previously documented [6,49,51,60,72,90].

When a cell is tagged for increasing its dissipation from the previous criterion we perform that by assigning a small value to the central stencil linear weight $\lambda_1 = A$, with ($A = 2$) for the present implementation, while when it is tagged for reducing its dissipation we assign a large value to it $\lambda_1 = B$, with ($B = 1000$). It has to be noted that this central stencil linear weight value is the non-normalised one found in (Eq. (22)). Each cell in the domain can set its own value for the central stencil linear weight λ_1 based on this criterion, prior to performing the CWENOZ reconstruction.

Remark 4. This process is executed at every explicit Runge-Kutta time-step or physical-time step of the implicit Dual-time stepping by computing the energy ratio, and deciding to increase or decrease the dissipation accordingly through the central stencil linear weight λ_1 prior to the CWENOZ reconstruction. The energy ratio of Eq. (29) is computed based on the current time-level values.

If the selected approximate Riemann solver is upwind such as HLLC, HLL the next steps involve computing the fluxes and advancing the solution in time as seen in the ADDA flow chart in Fig. 3.

4.3. Flux dissipation adaptation

If the selected approximate Riemann solver has a central structure such as Roe [77] or Rusanov [82] we can exploit additional control through the flux dissipation terms. Considering the Rusanov flux as follows:

$$\mathbf{F}_{c_l}^{Rusanov} = \frac{1}{2} (\mathbf{F}_L + \mathbf{F}_R) - \frac{1}{2} d_i \lambda_{max} (\mathbf{U}_R - \mathbf{U}_L) \quad (38)$$

where the left and right states are indicated by subscripts L , R , and the maximum eigenvalue associated with the Rusanov flux is λ_{max} . We have introduced the new flux-dissipation control parameter d_i that controls how much flux dissipation is included in the flux. When we assign $d_i = 0$, we end up with a simple flux averaging without dissipation terms, while when assign a value of $d_i = 1$, we recover the original formulation of the employed flux. For this study we vary the flux-dissipation term between $d_i \in [0.5, 1]$, since we have found this to be the lower-limit for Roe, and Rusanov Riemann solver to work robustly for the compressible flow problems that we have used. This strategy of introducing flux dissipation control terms in flux functions has been well-documented in the literature [102–106], with several elegant approaches improving significantly the characteristics of their numerical frameworks through this approach.

The value of the flux dissipation control term d_i is dynamically adjusted at each physical time-step for each cell for each face, and we smoothly vary its value in steps of 0.05 for the present work, similarly to [5]. Therefore, we gradually increase or decrease the flux dissipation control term up to its maximum or minimum value respectively, when the same conditions are met at each time step. So if a cell has been previously tagged for reducing its dissipation, through the increase of (λ_1) , it can also gradually decrease (d_i) up to the minimum value. In contrast, when a cell has been tagged for increasing its dissipation through the increase of (λ_1) , it can also gradually increase (d_i) up to the maximum value $(d_i = 1)$.

Remark 5. However, there are instances where the flux dissipation control term d_i needs to return to its default value $(d_i = 1)$ immediately when a strong discontinuity is present, independently if the cell has been previously tagged for low- or high-dissipation adjustment. This decision is taken based on the non-linear CWENOZ weights assigned to the central stencil for density and pressure.

When the non-linear CWENOZ weight is less than the value $(\omega_1 < C)$, with $(C < 0.999)$ for this study, then the flux dissipation control term returns to its default value $(d_i = 1)$, to prevent the spurious oscillations in the solution, otherwise it operates as previously described.

Remark 6. Once the appropriate flux dissipation control term d_i has been set for a cell according to the criteria previously defined, then we need to ensure the same value of d_i is used by both left and right states in order to ensure a conservative flux approximation. This is achieved by using the maximum value allowed from both states with $d_i = \max(d_{i,L}, d_{i,R})$ for robustness. This finding of the maximum value allowed between states translates to a small additional overhead in terms of the parallel communication when the left and right states lie in different parallel processes.

Finally, the ADDA algorithm proceeds with the computation of the fluxes and advancement of the solution in time, and the process is repeated at every time step.

5. Applications

5.1. iLES of subsonic inviscid Taylor-Green vortex

The iLES of the 3D inviscid subsonic Taylor-Green vortex test problem is employed since it presents an ideal-test problem for assessing several characteristics of numerical methods intended within the LES context, it is widely used by the community across several numerical frameworks and LES techniques [5,10,39,60,64,90,94,107–110] and therefore there is a wealth of DNS, LES, iLES computational data from the literature to compare against. We are particularly focused on “under-resolved” mesh resolutions since these are more representative of the environment that these methods are expected to operate, and we want to understand any benefits that the new ADDA technique can offer for reproducing the transition to turbulence in these situations. The computational domain is defined as $\Omega = [0, 2\pi]^3$ with periodic boundary conditions, and the present formulation of the Taylor-Green vortex problem is initialized with the following velocity, density and pressure fields:

$$u(x, y, z, 0) = \sin(kx) \cos(ky) \cos(kz), \quad (39)$$

$$v(x, y, z, 0) = -\cos(kx) \sin(ky) \cos(kz), \quad (40)$$

$$w(x, y, z, 0) = 0, \quad (41)$$

$$\rho(x, y, z, 0) = 1, \quad (42)$$

$$p(x, y, z, 0) = 100 + \frac{\rho}{16} [\cos(2z) + 2] \cdot [\cos(2x) + \cos(2y)]. \quad (43)$$

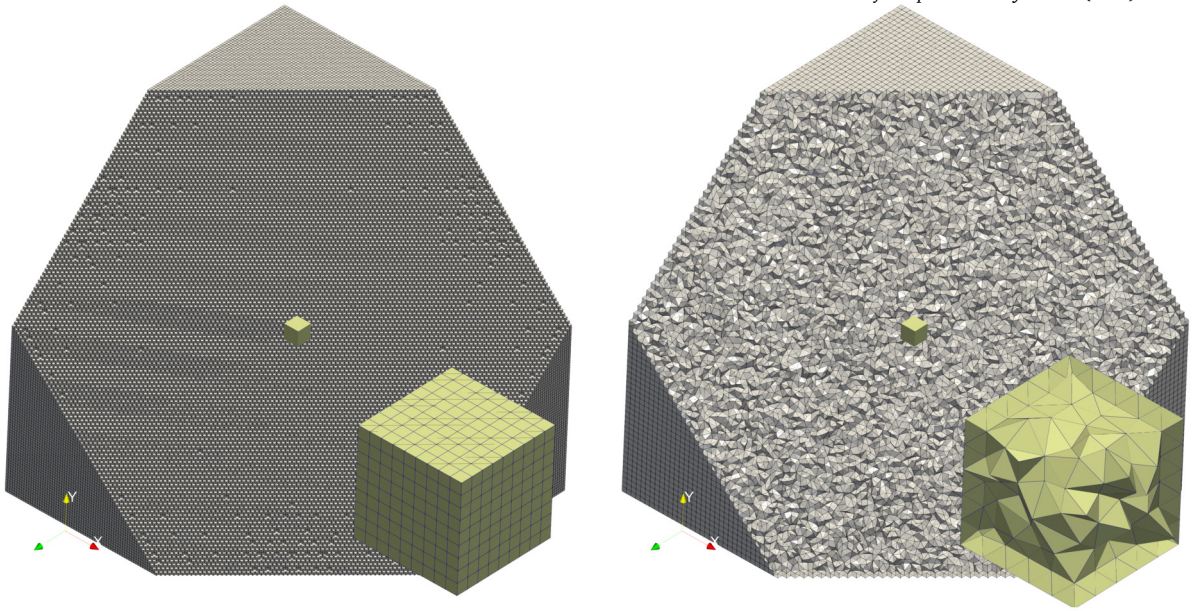


Fig. 7. Cutaway of the hexahedral (left) and arbitrary tetrahedral mesh (right) used for subsonic inviscid Taylor-Green vortex flow problem, with a zoomed-in region at the centre of their domain.

Table 2
Mesh characteristics for Taylor-Green vortex test problem.

mesh type	edges per domain Length L	Total number of cells
hexahedral	64	262,144
hexahedral	128	2,097,152
arbitrary tetrahedral	64	2,069,247

The initial condition corresponds to an initial Mach number $M \approx 0.08$, with wavenumber $k = 2\pi/\lambda = 1$. Simulations were carried out on hexahedral meshes of 64^3 and 128^3 , and an arbitrary tetrahedral-dominant (included pyramids at periodic boundaries) mesh with a cell edge resolution equivalent to 64^3 as shown in Fig. 7 and listed in Table 2. The primary reason that we are employing two resolutions for the hexahedral type is that the 128^3 resolution is equivalent to 64^3 resolution of a tetrahedral mesh, with respect to the number of elements, and their comparison might provide some useful insights on the selected numerical methods and the mesh elements.

For the present Taylor-Green vortex flow test problem we employ the CWENOZ schemes with different order of spatial accuracy and the 4th-order Runge-Kutta SSP with a CFL of 1.4, with the HLLC or Roe Riemann solver. Initially, we employ the coarsest hexahedral mesh 64^3 to assess the impact of the following parameters on the results:

- changing the polynomial order $\mathcal{P}$
- changing the central stencil linear weight λ_1
- using ADDA with Roe Riemann solver (adapts λ_1 and the flux dissipation term d_i)
- using ADDA with HLLC Riemann solver (acts only on λ_1)

We are also interested in the 3D spectra of the kinetic energy after the dissipation peak ($t = 10$), where we can assess if the dissipation follows the behaviour of Kolmogorov's ($k = -5/3$) law, at least for a limited range of wave numbers for the under-resolved resolution employed in this study. From the evolution of the kinetic energy Fig. 8, dissipation rate Fig. 9 and the kinetic energy spectra at $t = 10$ Fig. 10 it can be noticed that the increase of the polynomial order $\mathcal{P}$, and central stencil linear weight λ_1 reduces the numerical dissipation and provides a better agreement with the DNS results of Brachet et al. [107] and a better agreement with the Kolmogorov's slope for a wider-range of wave numbers. Regarding the ADDA technique, it provides a dissipation behaviour that follows better the Kolmogorov's slope and the DNS results, in particular when the Roe Riemann solver is deployed where the ADDA technique acts on the flux dissipation term d_i as well. The HLLC variant results are also improved with the ADDA technique, but due to the current implementation not acting on the flux components of HLLC Riemann solver it is inferior to the results obtained with the Roe Riemann solver due to the adjustment of the flux dissipation term of the Roe Riemann solver, that seems to make a significant improvement in the results. It has to be noted that the DNS study of Brachet et al. [107] is solving the viscous Navier-Stokes equations, however the comparison against the DNS results is valuable to understand the underlying dissipation mechanism of the deployed framework.

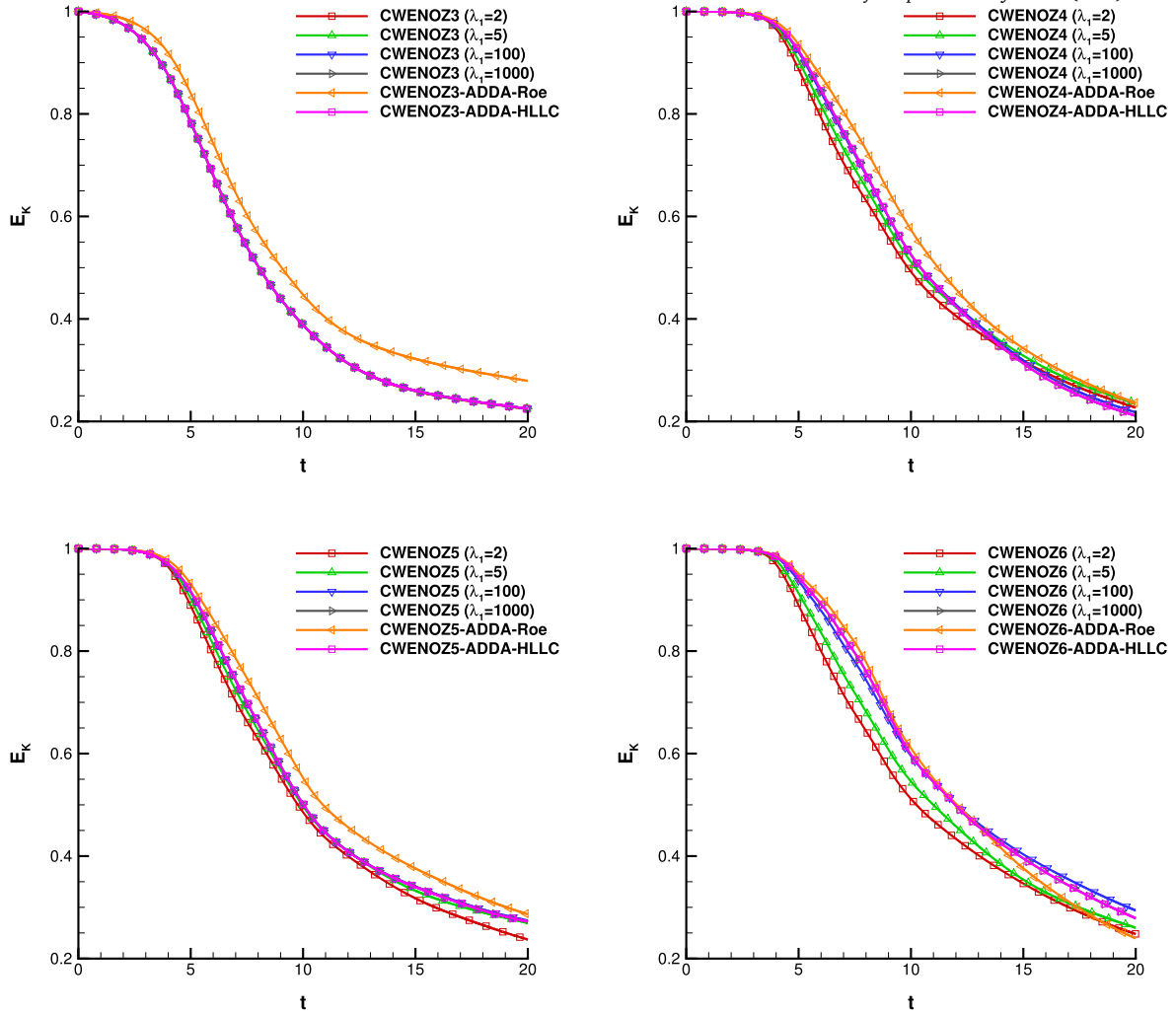


Fig. 8. Evolution of the normalised kinetic energy for the subsonic inviscid Taylor-Green vortex test problem with CWENOZ schemes of varying order of accuracy, varying central stencil linear weight λ_1 , and addition of the ADDA algorithm with the Roe [77] and HLLC [76] Riemann solver on the hexahedral 64^3 mesh.

We now shift our focus to understand how the different element-types can influence the performance of the ADDA technique, in particular when we use a very high-order numerical scheme CWENOZ5 with the Roe Riemann solver [77], since it enables acting on the flux dissipation term as well, in contrary to the HLLC Riemann solver [76]. For this purpose, we use a 128^3 resolution for the hexahedral mesh which is equivalent to the 64^3 resolution of a tetrahedral mesh with respect to the number of elements contained in each mesh being approximately equal to 2 million cells. Previous studies [74,90] have documented that arbitrary unstructured elements (prisms, tetras, pyramids) provide lower-dissipation, which on a first read might be beneficial, compared with hexahedral meshes. We will also complement our numerical framework with the low-Mach number correction (LMC) of Simmonds et al. [74], since it has been reported [74] that multidimensional finite-volume reconstructions using hexahedral elements benefit significantly from this LMC. From the normalised kinetic energy evolution shown in Fig. 11, initially it can be noticed that the LMC correction improves significantly the hexahedral mesh results and provides better agreement with the tetrahedral mesh prediction, and therefore reducing the differences between them. When deploying the ADDA technique, we obtain a much closer agreement across the different meshes with or without the LMC. This observation is repeated in the kinetic energy dissipation rate evolution plots as shown in Fig. 12, where a much closer agreement with the DNS results is observed for the ADDA enabled simulations. It has to be stressed that the amplitude of the double-peak feature at around $t \approx 9.5$ is greatly reduced in the ADDA-LMC combination for both meshes, where it has been reported [10] that this feature of the solution has been associated with dispersion errors of multi-dimensional reconstructions in iLES settings. In particular, it is the LMC component which seems to have the greatest influence of the reduction of the magnitude of the double peak structure, usually linked to dispersion effects as previously reported in [10]. Therefore, it is encouraging that we are able to reduce the dissipation and dispersion errors associated with this flow problem for both type of meshes, which is the intended goal of our approach to provide a transparent framework that provides the right amount of dissipation/dispersion across different mesh elements. It needs to be stressed that for the kinetic energy dissipation rate plots there are some high-frequencies

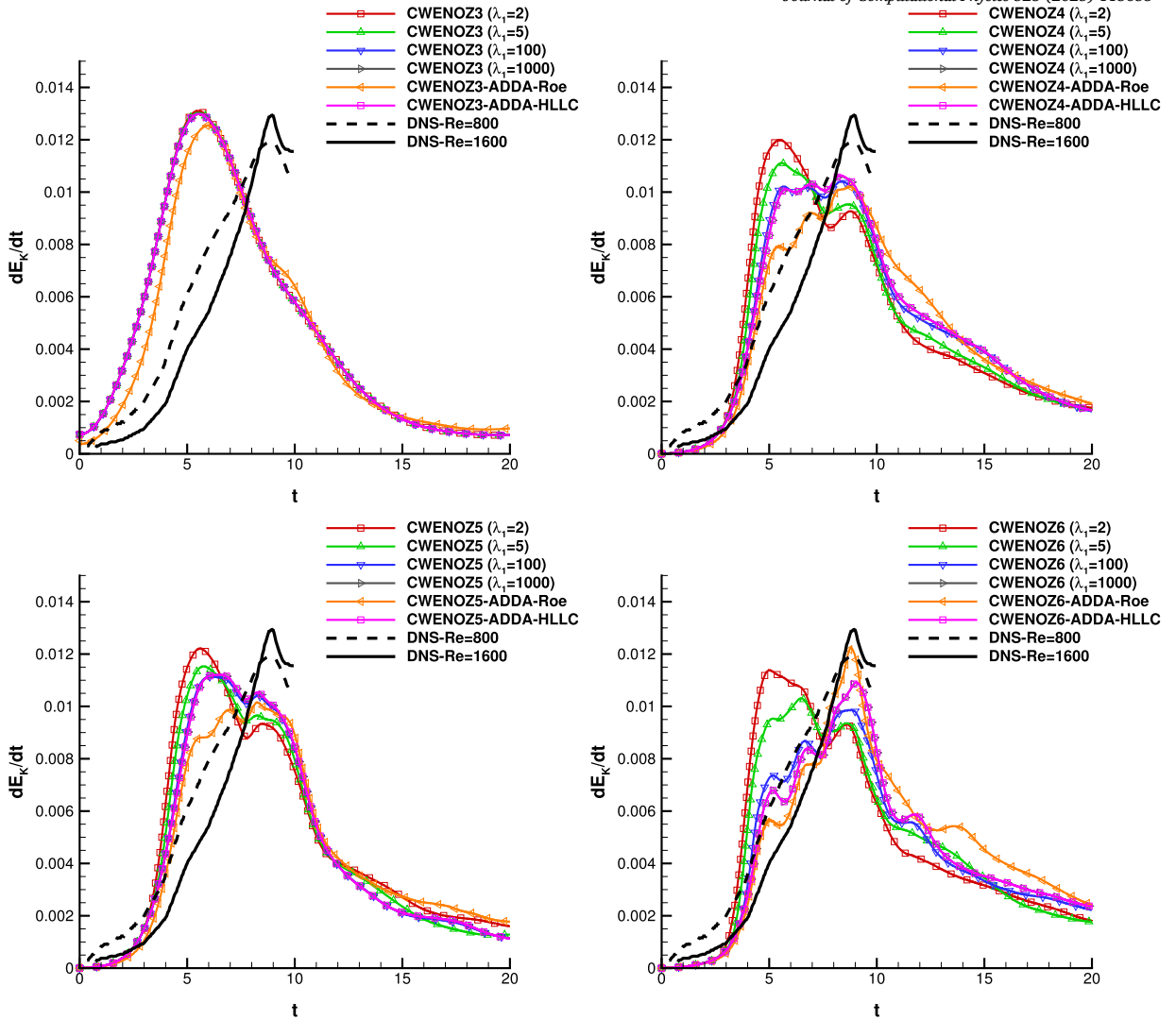


Fig. 9. Evolution of the normalised kinetic energy dissipation rate for the subsonic inviscid Taylor-Green vortex test problem with CWENOZ schemes of varying order of accuracy, varying central stencil linear weight λ_1 , and addition of the ADDA algorithm with the Roe [77] and HLLC [76] Riemann solver on the hexahedral 64^3 mesh, and comparison with the DNS results of Brachet et al. [107].

noticed and this expected for non-linear schemes. For the schemes without any ADDA the results without LMC seem to have more high frequencies, where with the addition of LMC they seem to be removed. This is expected due to the low-Mach number setting of the subsonic Taylor-Green vortex, where upwind type of methods do not perform really well due, as it has been previously reported [10,56,74]. However, when we move to the ADDA enabled schemes the high frequencies reappear for both the LMC and the no-LMC variants due to the flux adaptation strategy since we are increasing or decreasing this term by 0.05, or rapidly switching from the min to the max value depending on the non-linear CWENOZ weight of the central stencil for density and pressure (which rarely happens for the subsonic Taylor-Green vortex). It is therefore related to the flux adaptation step of 0.05 that is causing this. Resorting to a smaller step such as 0.01 will cause these high frequencies to be reduced, and using a higher step such as 0.1 to increase them.

Examining the kinetic energy spectra at $t = 10$ Fig. 13, the same pattern of behaviour is observed, where the LMC providing significant improvements in terms of resolving a wider-range of waves numbers to the hexahedral meshes for the non-ADDA variants, while with the inclusion of the ADDA the performance for both meshes is further improved. ADDA or LMC on their own are not sufficient to bring the hexahedral mesh results close to the tetrahedral mesh results, and therefore combining ADDA-LMC for hexahedral meshes is needed. It needs to be stresses that the tetrahedral mesh (with the same number of elements) outperforms the best hexahedral mesh results, since a closer agreement with the Kolmogorov's slope is achieved for a wider-range of wave numbers.

One of the most interesting features of the ADDA technique is that for some of the elements (depending on their energy ratio) it will increase their dissipation by reducing their central stencil linear weight λ_1 and by increasing their flux dissipation term d_i . This is better visualised in Fig. 14 where only the elements that had their dissipation increased are plotted. It can be seen that both

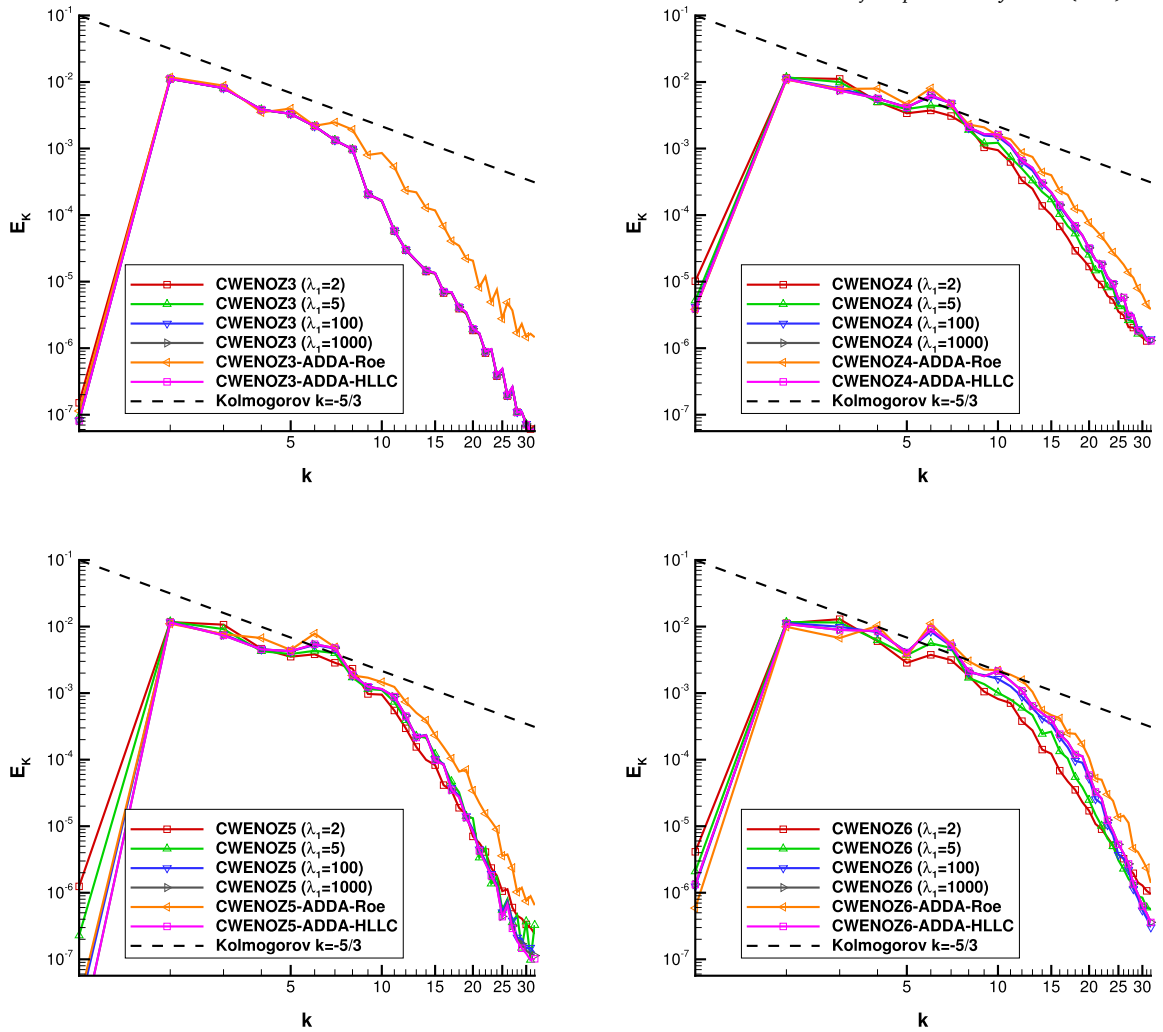


Fig. 10. Kinetic energy 3D spectra at $t = 10$ for the solution of the subsonic inviscid Taylor-Green vortex test problem with CWENOZ schemes of varying order of accuracy, varying central stencil linear weight λ_1 , and addition of the ADDA algorithm with the Roe [77] and HLLC [76] Riemann solver on the hexahedral 64^3 mesh.

the hexahedral and tetrahedral mesh have elements that required increased dissipation. It can be noticed firstly that the number of elements that require increased flux dissipation for both meshes seems to rapidly increase as we approach the dissipation peak, while being more clustered/organised in certain regions, and secondly as we move past the dissipation peak the elements requiring increased flux dissipation are more dispersed, as expected due to the transition to fully turbulent flow.

The computational overhead of the ADDA is approximately (1.8 – 8%) for this particular test problem, as listed in Table 3 that follows. The ADDA-Roe variant is marginally more expensive due to inter-processor communication requirement for ensuring a common flux dissipation term d_i applied to both sides of the cell face.

5.2. iLES of SD7003 wing

The turbulent flow over the SD7003 wing at Mach number of $M = 0.2$, angle of attack of $\alpha = 8^\circ$ and Reynolds number $Re = 60,000$ (based on the chord length) is examined to assess the performance of the developed methods in resolving transitional flow problems. This flow exhibits a laminar separation region, which reattaches further downstream forming a Laminar Separation Bubble (LSB), and along the separation bubble transition to turbulence occurs. This test case has been frequently used to benchmark several numerical methods within the LES family of methods for predicting separation, transition and turbulent flow [11,12,74,110,111]. The purpose of this selected flow problem is to assess any associated benefits of the new ADDA treatment for high-order schemes, compare with standard 2nd-order MUSCL methods and previous computational studies employing high-order methods.

Two hybrid unstructured meshes are considered consisting of 5.3 (mesh A) and 12.3 million cells (mesh B), consisting of hexahedral and prismatic cells, as illustrated in Fig. 15. The domain extends $50c$ upstream and downstream, and $0.2c$ in the span-wise direction, where c is the chord length. The grid resolution for mesh A at the boundary layer region gives a ($y^+ = 0.87, x^+ = 3.1, z^+ = 5.4$)

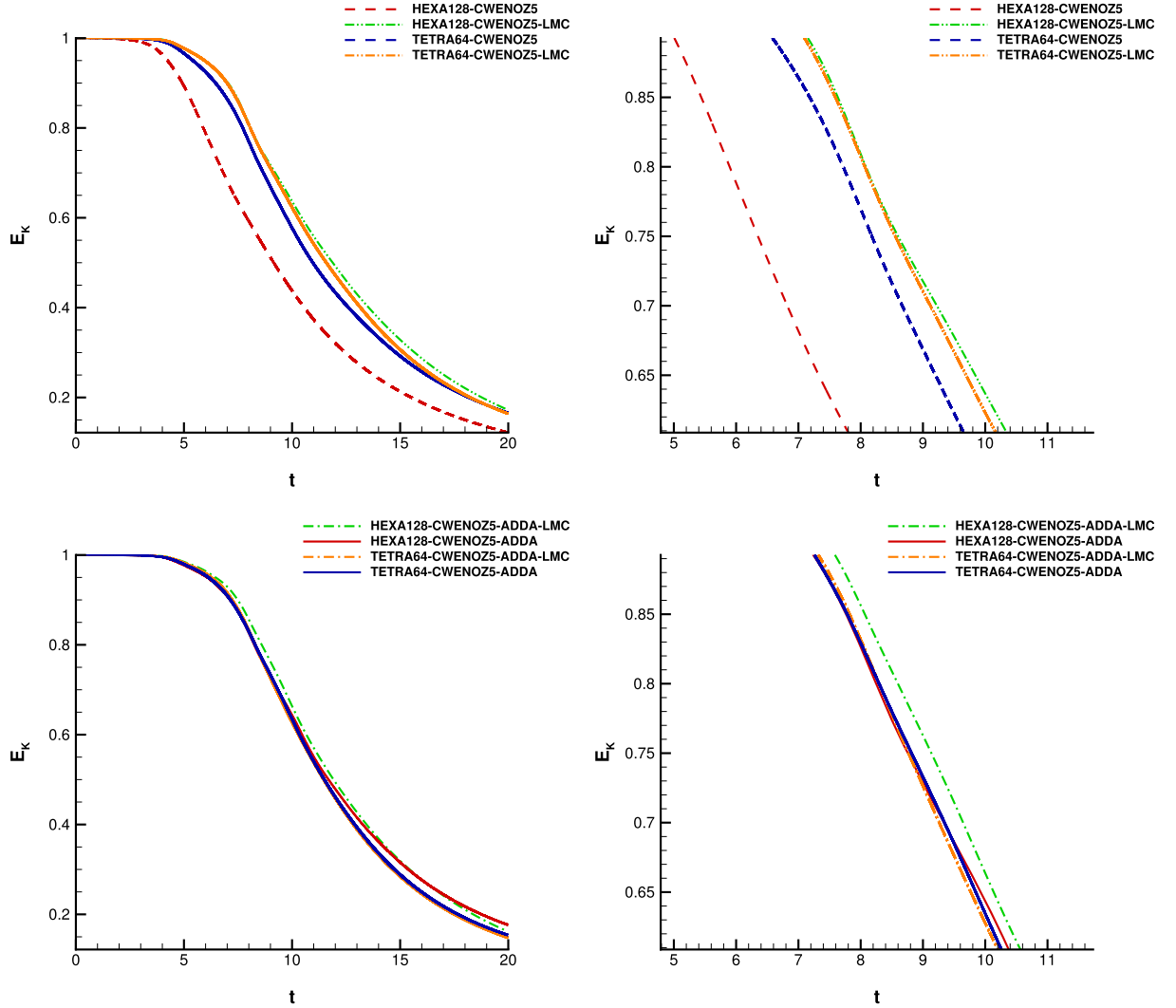


Fig. 11. Evolution of the normalised kinetic energy for the subsonic inviscid Taylor-Green vortex test problem with CWENOZ5 scheme and CWENOZ5-ADDA scheme with the Roe [77] Riemann solver on 64^3 tetrahedral mesh and 128^3 hexahedral mesh with and without the low-Mach number treatment of Simmonds et al. [74].

Table 3

Normalised CPU time per iteration for the Taylor-Green vortex flow problem for various schemes on ARCHER2-UK-HPC at 2 nodes (256 processes-executed using the optimum -8 MPI processes-16 threads per node for a total of 16 MPI processes, and 256 OpenMP threads) for the 64^3 hexahedral mesh. ADDA-Roe variant is marginally more expensive due to inter-processor communication requirement for the flux dissipation term d_i .

Scheme	Normalised CPU time per iteration	ADDA-overhead
CWENOZ3	1.00	-
CWENOZ3-ADDA-HLLC	1.06	6.1%
CWENOZ3-ADDA-Roe	1.08	8.2%
CWENOZ4	2.37	-
CWENOZ4-ADDA-HLLC	2.48	4.8%
CWENOZ4-ADDA-Roe	2.52	6.5%
CWENOZ5	4.21	-
CWENOZ5-ADDA-HLLC	4.32	2.7%
CWENOZ5-ADDA-Roe	4.34	3.1%
CWENOZ6	5.80	-
CWENOZ5-ADDA-HLLC	5.90	1.8%
CWENOZ5-ADDA-Roe	5.92	2.2%

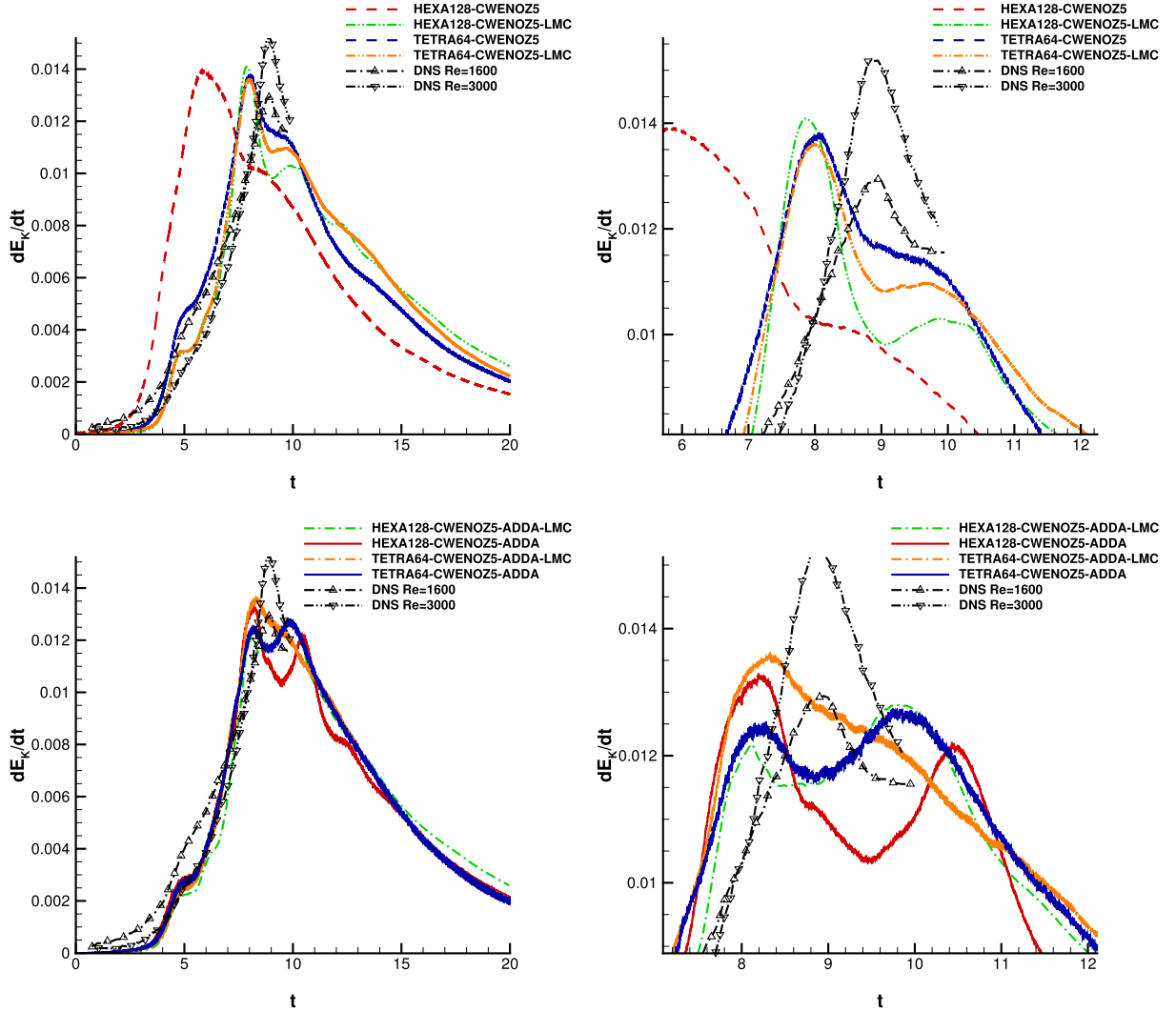


Fig. 12. Evolution of the normalised kinetic energy dissipation rate for the subsonic inviscid Taylor-Green vortex test problem with CWENOZ5 scheme and CWENOZ5-ADDA scheme with the Roe [77] Riemann solver on 64^3 tetrahedral mesh and 128^3 hexahedral mesh with and without the low-Mach number treatment of Simmonds et al. [74], and comparison with the DNS results of Brachet et al. [107].

and ($y^+ = 0.87, x^+ = 0.9, z^+ = 4.0$) for mesh B. Periodic boundary conditions are used in the span-wise direction, no-slip boundary conditions at the surface of the aerofoil and free-stream conditions at the farfield. For this problem, we deploy the CWENOZ-4th order method with and without ADDA algorithm, and a standard MUSCL2 method with the Barth and Jespersen limiter [112]. All the schemes employ the low-Mach number correction [74], the Roe Riemann solver [77] and the 2nd-order implicit dual-time stepping available in UCNS3D [40] with a timestep size of $\Delta t = \frac{t_c}{500}$ with the convection time being given by $t_c = c/U_\infty$. The simulations were run for $t = 20t_c$ to develop the flow, and an additional $20t_c$ for time averaging. The MUSCL2 method was deployed using both meshes, while the high-order CWENOZ-4th order scheme was deployed with the coarse (mesh A) to showcase its capabilities at coarse-grid resolutions.

From the obtained time-averaged solution as shown in Fig. 16 all the schemes produce similar results with respect to u-velocity profile, but the high-order CWENOZ4-ADDA-Mesh A produces a higher-turbulent kinetic energy immediately downstream the LSB indicative of the low-dissipation behaviour of this variant that is also evident by the increased number of resolved vortical structures further downstream the LSB as shown in Fig. 17. From the instantaneous iso-surfaces of Q criterion coloured by the ADDA flux dissipation term d_i as shown in Fig. 18 it can be appreciated how the ADDA is acting. As expected, low-dissipation is present at the LSB region, followed immediately by a rapid switching to high-dissipation dominant behaviour at the turbulent region that eventually switches to low-dissipation pattern further downstream.

CWENOZ4 schemes on the coarsest mesh exhibit improved agreement with computational studies obtained with several methods including flux-reconstruction PyFR (Vermeire et al. [111]), finite-volume Star-CCM (Vermeire et al. [111]), Discontinuous Galerkin Spectral Element Method (Frank et al. [12]), and a structured high-order finite-difference code FDLI3D (Galbraith & Visbal [13]) in

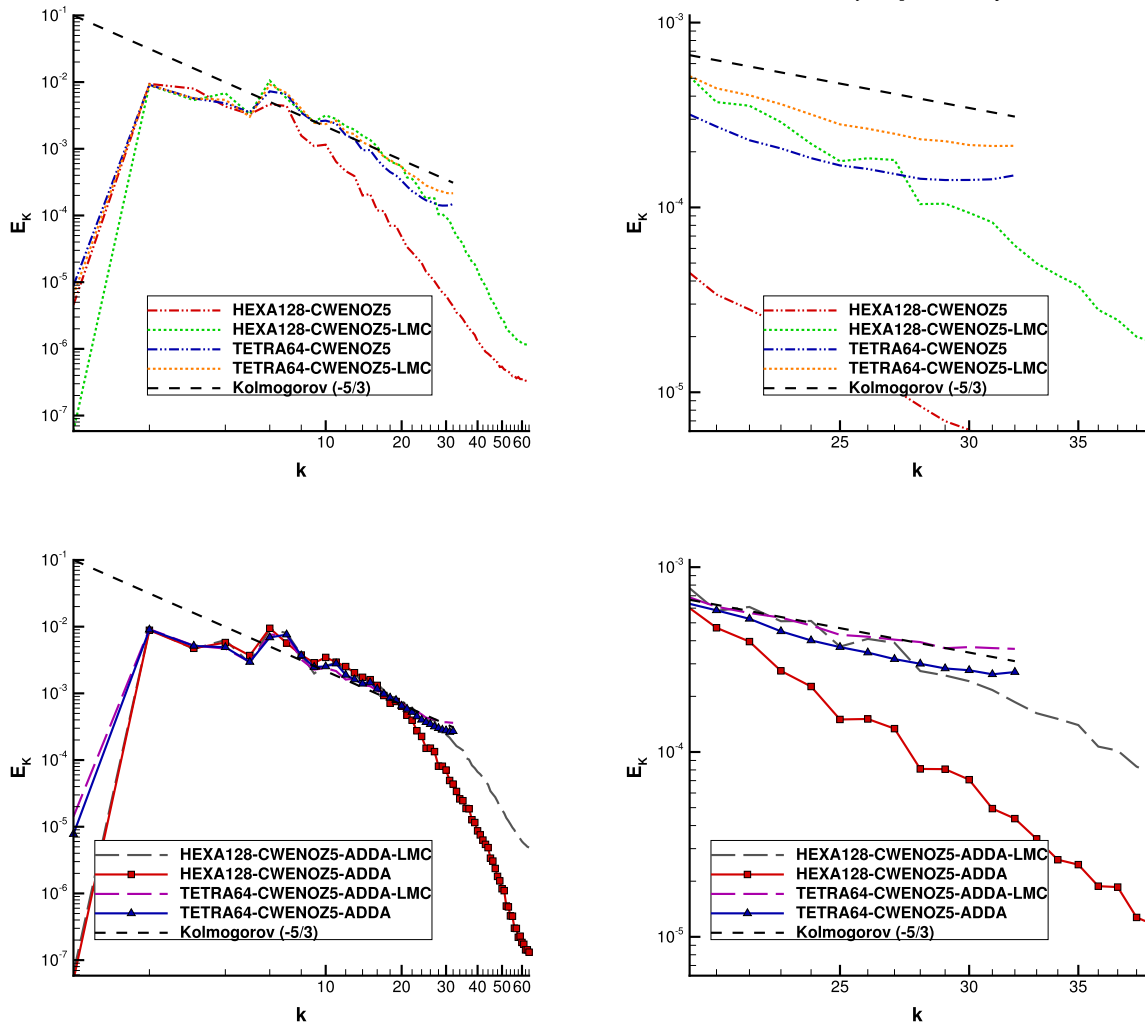


Fig. 13. Kinetic energy 3D spectra at $t = 10$ for the solution of the Taylor-Green vortex computed with CWENOZ5 scheme and CWENOZ5-ADDA scheme with the Roe [77] Riemann solver on 64^3 tetrahedral mesh and 128^3 hexahedral mesh with and without the low-Mach number treatment of Simmonds et al. [74].

terms of pressure coefficient C_p and skin friction coefficient C_f , and clearly outperform the standard MUSCL2 scheme even when the finer mesh B is used as shown in Fig. 19. The ADDA variant further improves the agreement, primarily for the C_f . From the time-averaged, span-wise averaged normalised velocity profiles at different stations on the suction side of the airfoil as seen in Fig. 20, the CWENOZ4 schemes provide a good agreement with the (Garmann & Visbal [11]) predictions at all the stations while a good agreement with the predictions of (Frank et al. [12]) are noticed aft ($x/c = 0.3$). The CWENOZ4-ADDA variant resolves a wider-range of frequencies, as shown in Fig. 21, for two probe position at the turbulent region downstream of LSB.

Finally, the CWENOZ4-schemes provide a closer agreement with the computational studies of Frank et al. [12] and Galbraith & Visbal [13] in terms of drag-coefficient C_D compared to MUSCL schemes, while marginal differences are noticed in the separation location and reattachment between all predictions. The MUSCL 2nd-order schemes on the other hand, being one of the most-widely used and robust numerical methods for compressible flows, are unsuitable for iLES simulations at under-resolved grid settings, due to their excessive dissipation characteristics which are not physically consistent. This can be seen in the results obtained for the skin friction, separation and reattachment, which an unpredictable behaviour is noticed when we move from mesh A to mesh B. (See Table 4.)

5.3. iLES of supersonic viscous Taylor-Green vortex

An interesting variation of the Taylor-Green vortex flow problem involves the supersonic viscous variant, which is mostly targeted towards assessing simultaneously the non-oscillatory properties and low-numerical dissipation of the developed framework. It was recently introduced by Lusher and Sandham [113], further investigated by Chapelier et al. [114] for assessing the performance of various CFD codes and is a well suited benchmark for numerical methods intended for compressible flow problems.

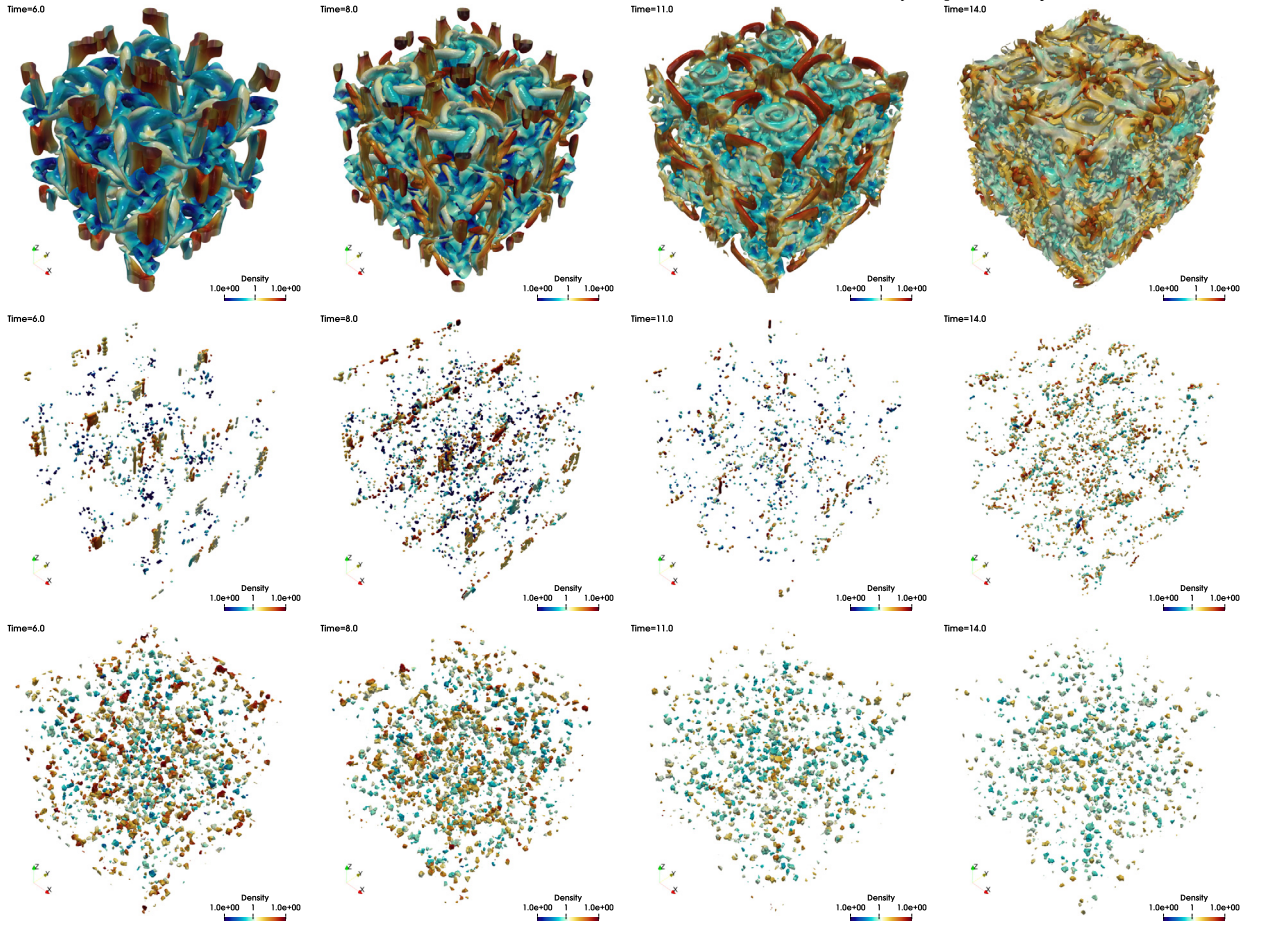


Fig. 14. Isosurface of Q-criterion coloured by density for the Taylor-Green vortex obtained on the 128^3 hexahedral mesh with CWENOZ5-ADDA-LMC scheme at different instants (top row), and cells with increased dissipation for the 128^3 hexahedral mesh (medium row) and 64^3 tetrahedral mesh (bottom row).

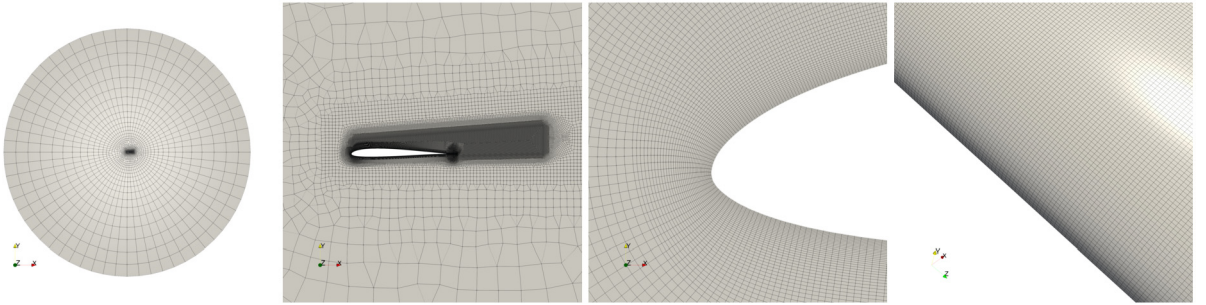


Fig. 15. Sections of the unstructured mesh used for the SD7003 airfoil test problem.

The computational domain and the velocity field remain the same as the subsonic variant previously described, and the parameters that change are the pressure field:

$$p(x, y, z, 0) = \frac{1}{\gamma M_{ref}^2} + \frac{1}{16} [\cos(2z) + 2] \cdot [\cos(2x) + \cos(2y)], \quad (44)$$

and the density, which is computed from the equation of state.

We only examine the variant with a Mach number $M_{ref} = 1.25$, $Re = 1600$ and the present viscous simulations were carried out on a hexahedral mesh of 128 cells per side (2097152 total number of cells), and on a tetrahedral mesh of 64 cells per side (2069247 total number of cells), and we employ a CWENOZ5- schemes with and without ADDA to assess the influence of the ADDA for this problem, and a CWENOZ6 with ADDA. A CFL number of 0.7 is used for the explicit Runge-Kutta 3rd-order scheme, the Roe Riemann

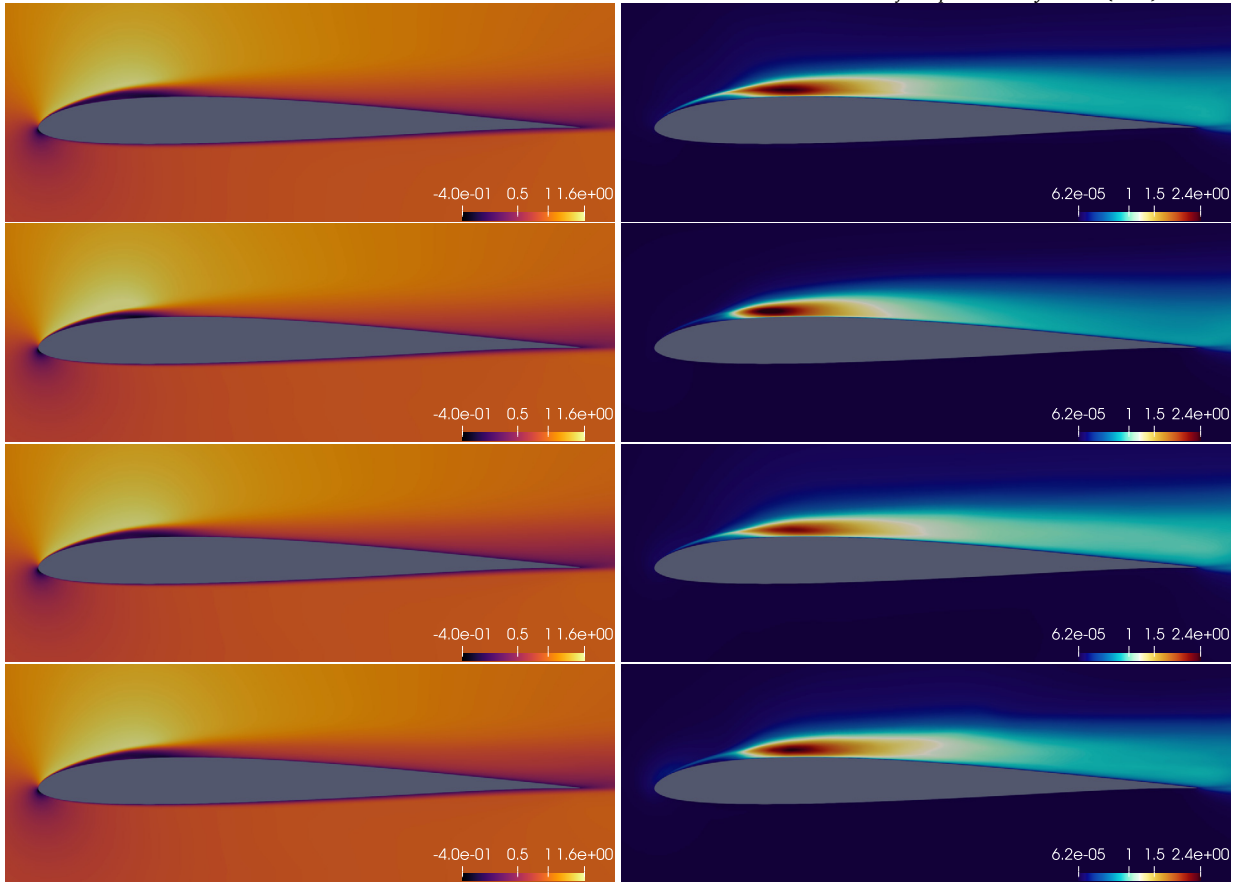


Fig. 16. Time-averaged, span-wise averaged contour plots of non-dimensional u -velocity (left) and turbulent kinetic energy (right) for the SD7003 flow problems with different schemes (MUSCL2-Mesh A, MUSCL2-Mesh B, CWENO4-Mesh A, CWENO4-ADDA-Mesh A) -from top to bottom. It can be noticed that all the schemes produce similar results in terms of u -velocity, but the higher order CWENO4-schemes provide higher-turbulent kinetic energy immediately downstream the LSB.

Table 4

Predictions for lift-coefficient C_L , drag-coefficient C_D , separation position x_s , and reattachment position x_r for the SD7003 airfoil test problem obtained with various schemes and comparison with computational studies in the literature.

Method	C_L	C_D	x_s	x_r
MUSCL2-mesh A	0.918	0.050	0.026	0.26
MUSCL2-mesh B	0.933	0.041	0.023	0.23
CWENO4-mesh A	0.926	0.044	0.039	0.28
CWENO4-ADDA-mesh A	0.927	0.045	0.041	0.27
Frank et al. [12]	0.923	0.045	0.027	0.310
Galbraith & Visbal [13]	0.917	0.044	0.031	0.303

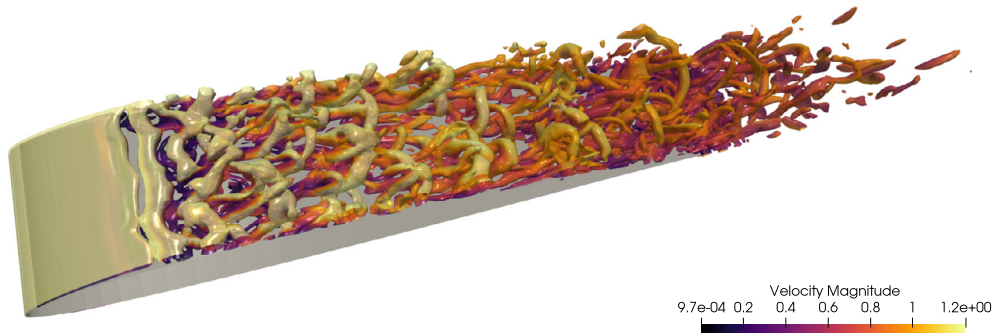
solver [77] and the LMC of Simmonds et al. [74] were employed, and the simulations were run up to $t = 20$ for obtaining the required statistics. The results of Lusher and Sandham [113] and the more recent comprehensive study of Chapelier et al. [114] are used for comparison against the computed solutions.

The schemes are assessed by examining the kinetic energy integrated over the domain and the total viscous dissipation rate, which consists of the solenoidal dissipation ε_s and dilatational dissipation ε_d given by Eq. (45) and Eq. (46) as:

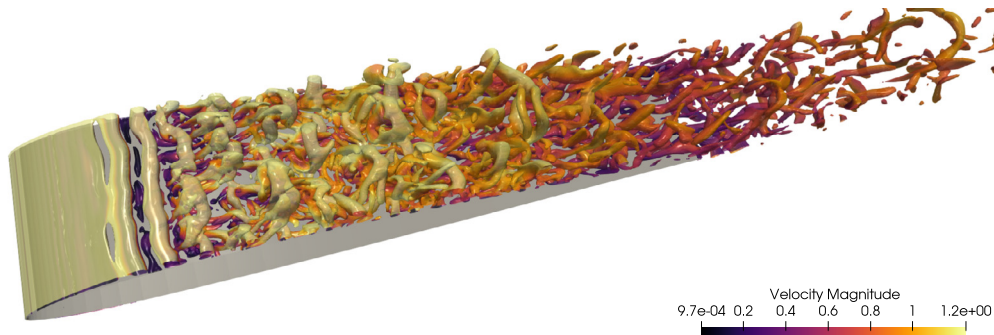
$$E_K^t = \frac{1}{\rho_{ref}\Omega} \int_{\Omega} \frac{1}{2} \rho u_j u_j d\Omega, \quad (45)$$

$$\varepsilon_t = \varepsilon_s + \varepsilon_d = \frac{1}{\rho_{ref}Re\Omega} \int_{\Omega} \mu (\omega \cdot \omega) d\Omega + \frac{4}{3\rho_{ref}Re\Omega} \int_{\Omega} \mu (\nabla \cdot \mathbf{u})^2 d\Omega. \quad (46)$$

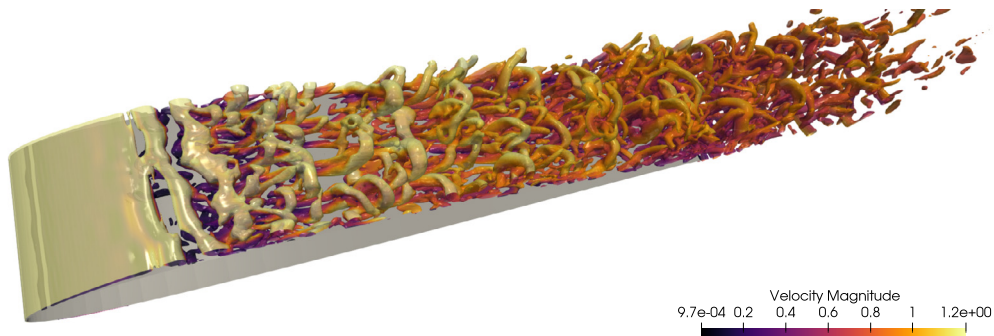
Where ω is the vorticity vector and $E_K = (E_K^t / E_K^0)$ is the normalised kinetic energy.



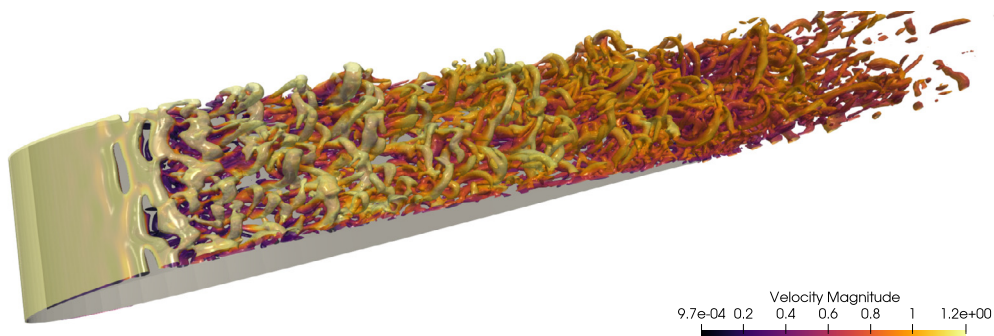
(a) MUSCL2-Mesh A



(b) MUSCL2-Mesh B



(c) CWENOZ4-Mesh A



(d) CWENOZ4-ADDA-Mesh A

Fig. 17. Instantaneous iso-surfaces of Q criterion ($Q=100$) coloured by the non-dimensional velocity magnitude for the SD7003 airfoil with different schemes at different grid resolutions. It can be seen that all the schemes capture the LSB and transition to turbulence, with the CWENOZ4 scheme -in particular the ADDA variant- resolving more vortical structures further downstream the LSB.

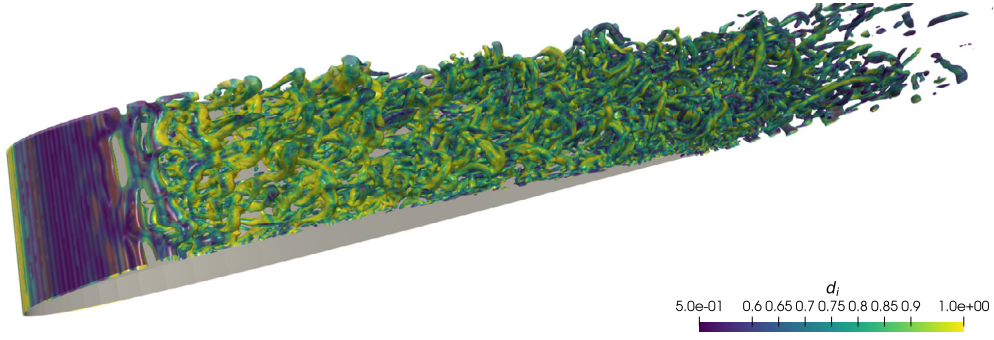


Fig. 18. Instantaneous iso-surfaces of Q criterion ($Q=100$) coloured by the ADDA flux dissipation d_i for the SD7003 airfoil with the CWENOZ4-ADDA scheme on mesh A. It can be seen that the ADDA algorithm switches to a low-dissipation d_i value in the LSB region, while it alternates between low-dissipation and high-dissipation profile in the turbulent region according to the energy ratio values.

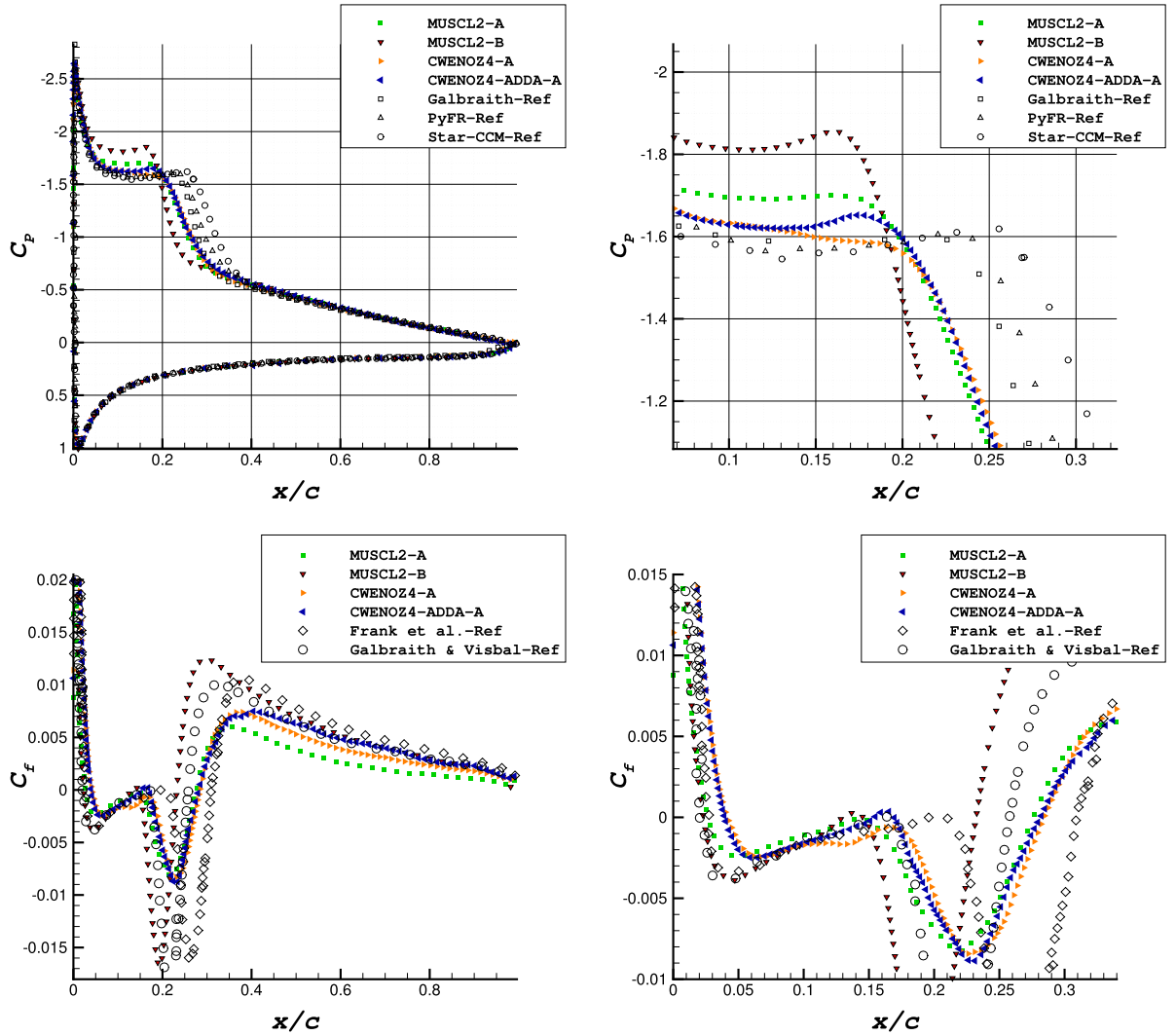


Fig. 19. Time-averaged, span-wise averaged profiles of pressure coefficient C_p and skin friction coefficient C_f at the upper side of the SD7003 airfoil, obtained with various schemes and compared with the predictions from computational studies obtained with several methods including flux-reconstruction PyFR (Vermeire et al. [111]), finite-volume Star-CCM (Vermeire et al. [111]), Discontinuous Galerkin Spectral Element Method (Frank et al. [12]), and a structured high-order finite-difference code FDLI3D (Galbraith & Visbal [13]). It can be noticed the MUSCL2 results over-predict C_p in the LSB region, and this is not resolved by using the finer Mesh B. On the other hand the CWENOZ4 scheme with and without the ADDA algorithm offer a better agreement with the predictions obtained with the other methods from the literature. Regarding the C_f the CWENOZ4 scheme and in particular the ADDA variant is able to provide better agreement with the results with the other methods.

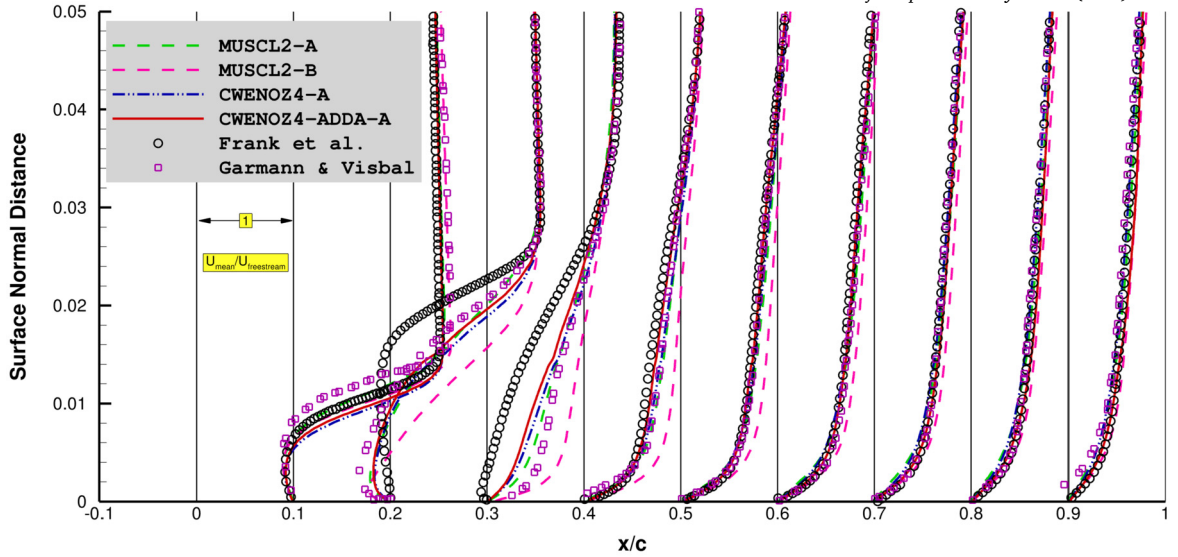


Fig. 20. Time-averaged, span-wise averaged normalised velocity profiles at different stations obtained with different methods and comparison with predictions from computational studies in the literature obtained with a Discontinuous Galerkin Spectral Element Method (Frank et al. [12]), and a structured high-order finite-difference code FDLI3D (Garmann & Visbal [11]). It can be noticed that the CWENOZ-variants offer good agreement with the (Garmann & Visbal [11]) predictions at all the stations, and a good agreement with the predictions of (Frank et al. [12]) only aft ($x/c = 0.3$).

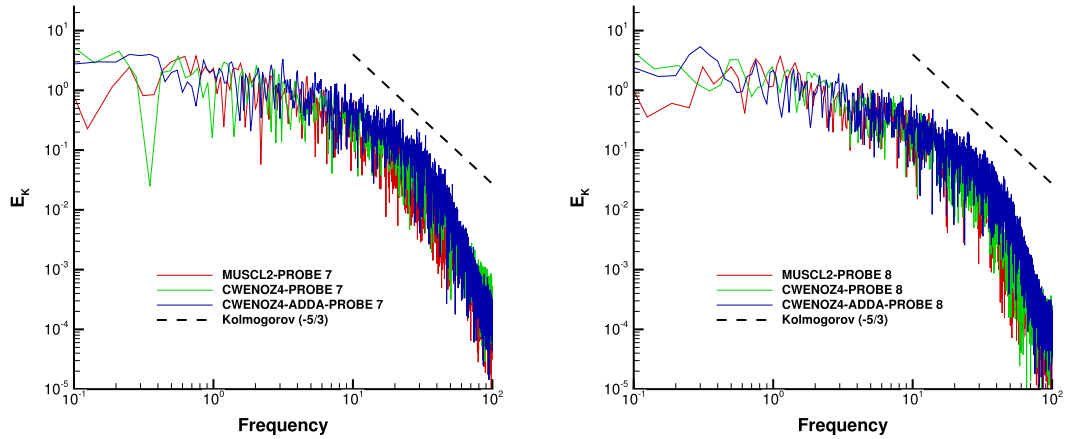


Fig. 21. 1D frequency spectra of the kinetic energy for several probe positions - probe 7 ($0.7c, 0.035c, 0.1c$), probe 8 ($0.8c, 0.026c, 0.1c$) - for the SD7003 turbulent flow test problem at, where it can be noticed that the CWENOZ4-ADDA scheme offers an improved agreement with the Kolmogorov slope compared to the MUSCL2 and CWENOZ4 scheme.

Investigating the evolution of the kinetic energy as seen in Fig. 22, it can be seen that all the schemes provide the expected results, with the CWENOZ5ADDA scheme and CWENOZ6ADDA scheme providing better agreement with the results obtained with very-low dissipation TENO 6th-order schemes of Lusher and Sandham [92,113] at (128^3) and (768^3) resolutions. Moving on to the total dissipation ε_t evolution we notice that the CWENOZ5 scheme without ADDA is characterised by the generation of lower total-dissipation due to the narrower range of scales being resolved for both meshes, although the tetrahedral mesh is able to resolve a wider range of scales and therefore generates higher dissipation level. Employing the ADDA technique the results are improved substantially for both meshes and when employing the CWENOZ6ADDA, even higher levels of total-dissipation are generated -while the difference in the obtained results from two different types of meshes is further reduced-, which on its own is an encouraging result, since the results from both mesh types converge to the same solution. As it has been reported in Lusher and Sandham [113], at very fine resolutions (768^3), a sudden increase and decrease of the total dissipation is noticed occurring between ($t = 2 - 4$), which is associated with a sudden increase and decrease of the dilatational dissipation ε_d increasing due to the generation of strong shocklets at that times. To better understand how our developed methods perform against very high-order schemes at the same resolution (128^3), we are comparing the obtained results of solenoidal dissipation ε_s and dilatational dissipation ε_d of our methods against the results obtained with a modal DG-4th order scheme of the (CODA) CFD software and a 6th-order TENO scheme of the (OPENSBLI) CFD software as provided by Chapelier et al. [114]. Firstly, it is evident that as we move from CWENOZ5 to CWENOZ5ADDA and then to CWENOZ6ADDA the agreement with the (CODA) and the (OPENSBLI) results improves significantly for both the solenoidal

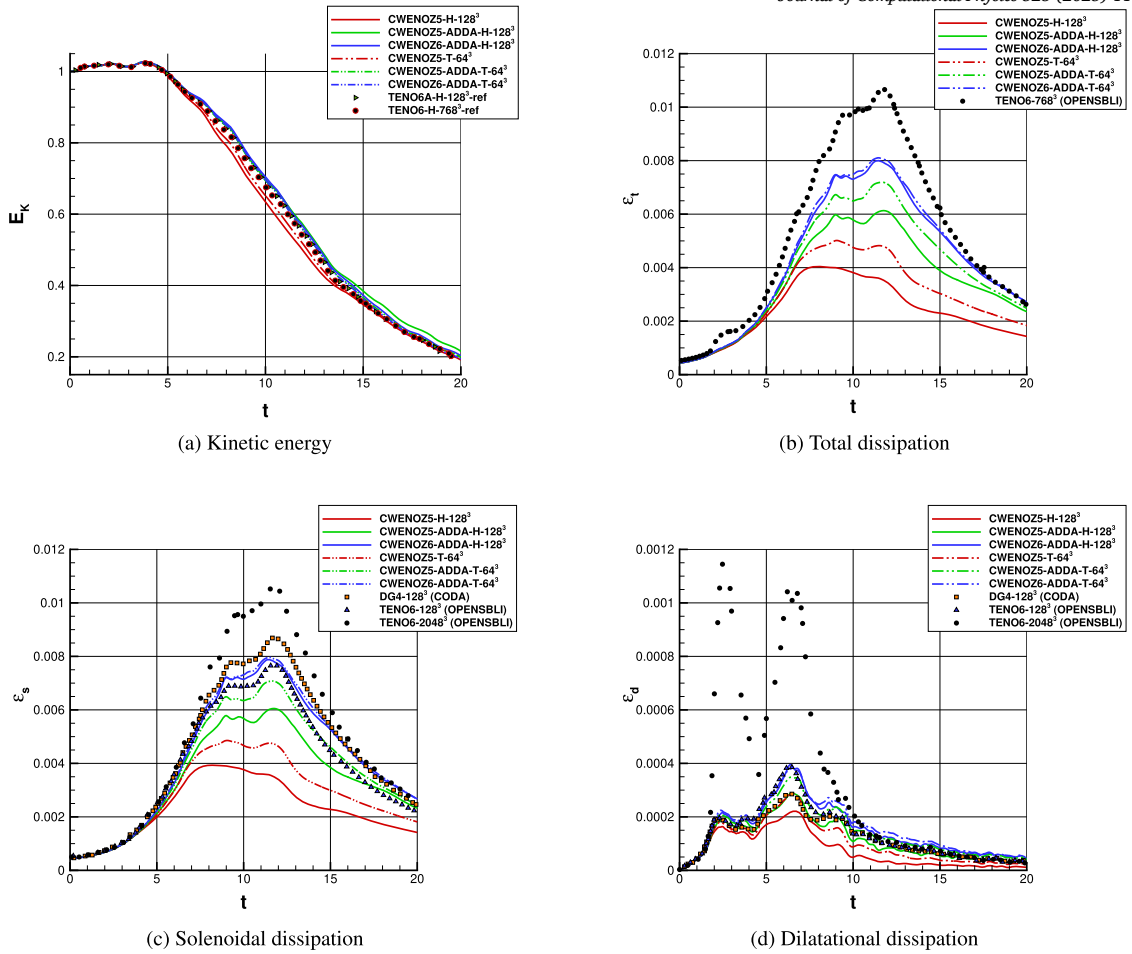


Fig. 22. Plots of the evolution of the normalised kinetic energy, total dissipation ϵ_t , solenoidal dissipation ϵ_s and dilatational dissipation ϵ_d for the solution of the supersonic viscous Taylor-Green vortex computed with several schemes on a hexahedral mesh 128³ and a tetrahedral mesh with a cell edge resolution equivalent to 64³. It can be noticed that the employment of ADDA improves significantly the results for the CWENO5 scheme, while the CWENO6-ADDA provides the best agreement with the modal DG-4th order scheme of the (CODA) CFD software and a 6th-order TENO scheme of the (OPENSBLI) CFD software as provided by Chapelier et al. [114] at the same (128³) resolution and the results of Lusher and Sandham [113], and a very close agreement between the tetrahedral and hexahedral meshes is obtained.

dissipation ϵ_s and dilatational dissipation ϵ_d . It is also evident that the sudden peak of the dilatational dissipation ϵ_d occurring at very fine mesh resolutions (2048³) at $(t = 2.8)$ is the underpinning mechanism that drives the sudden increase and decrease of the total dissipation occurring between $(t = 2 - 4)$.

Examining the kinetic energy spectra immediately after the total dissipation peak at $t = 12$ as seen in Fig. 23, it can be seen that the addition of ADDA for the CWENO5 scheme results in significant improvements in terms of resolving a wider-range of wave numbers and the CWENO6-ADDA further increases the number of wave numbers resolved, and a better agreement with the Kolmogorov slope. An interesting feature is that the tetrahedral mesh seems to be able to provide a better agreement with the Kolmogorov slope, due to the inherently lower-dissipation characterising tetra elements compared to hexahedral elements as previously reported [74,90].

A key component of the present ADDA scheme is the adaptation of dissipation through the linear weight λ_1 and flux dissipation term d_i . This adaptation is influenced by the non-linear CWENOZ weight assigned to the central stencil reconstruction for density, and therefore in the presence of strong gradients of density (as shown in Fig. 24) it will prevent the ADDA algorithm assigning very low-dissipation behaviour to these cells. This can be noticed in action, in Fig. 24 where the isosurface of Q criterion coloured by the Mach number and flux dissipation term d_i are presented. The ADDA algorithm progressively varies the flux dissipation term d_i in cells whose reconstruction stencils cross non-smooth regions of density to ensure a robust non-oscillatory behaviour and a representative distribution of low- and high-dissipation clusters is seen in Fig. 24.

5.4. iLES of transonic cylinder

This test problem involves the iLES of an infinite cylinder of diameter $D = 1$ at transonic flow conditions in accordance with the experimental setup of Murthy & Rose [115] $Re = 166000$ and $M_\infty = 0.8$. The subject flow conditions present an ideal environment

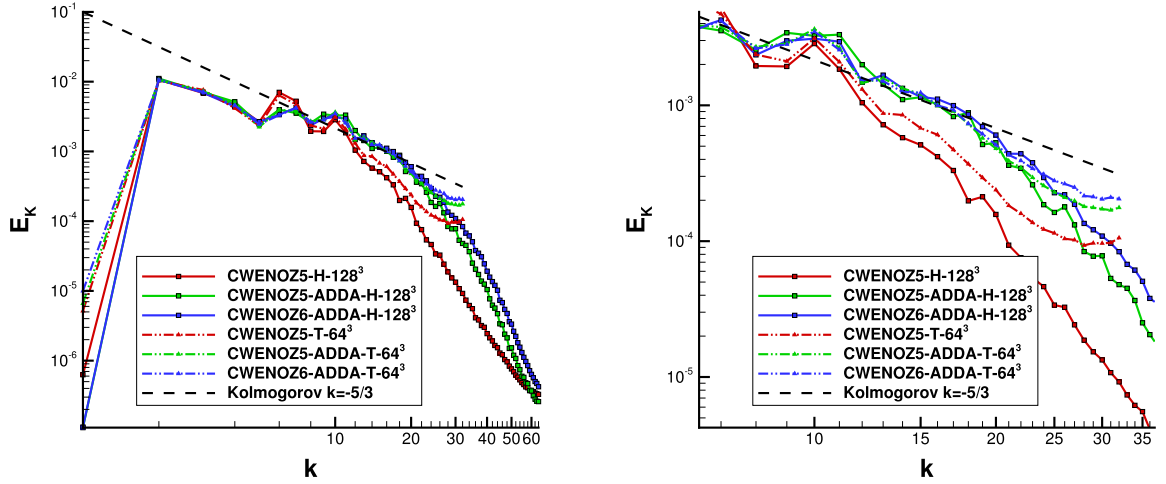


Fig. 23. Kinetic energy 3D spectra at $t = 12$ for the solution of the supersonic viscous Taylor-Green vortex computed with various schemes on 64^3 tetrahedral mesh and 128^3 hexahedral mesh.

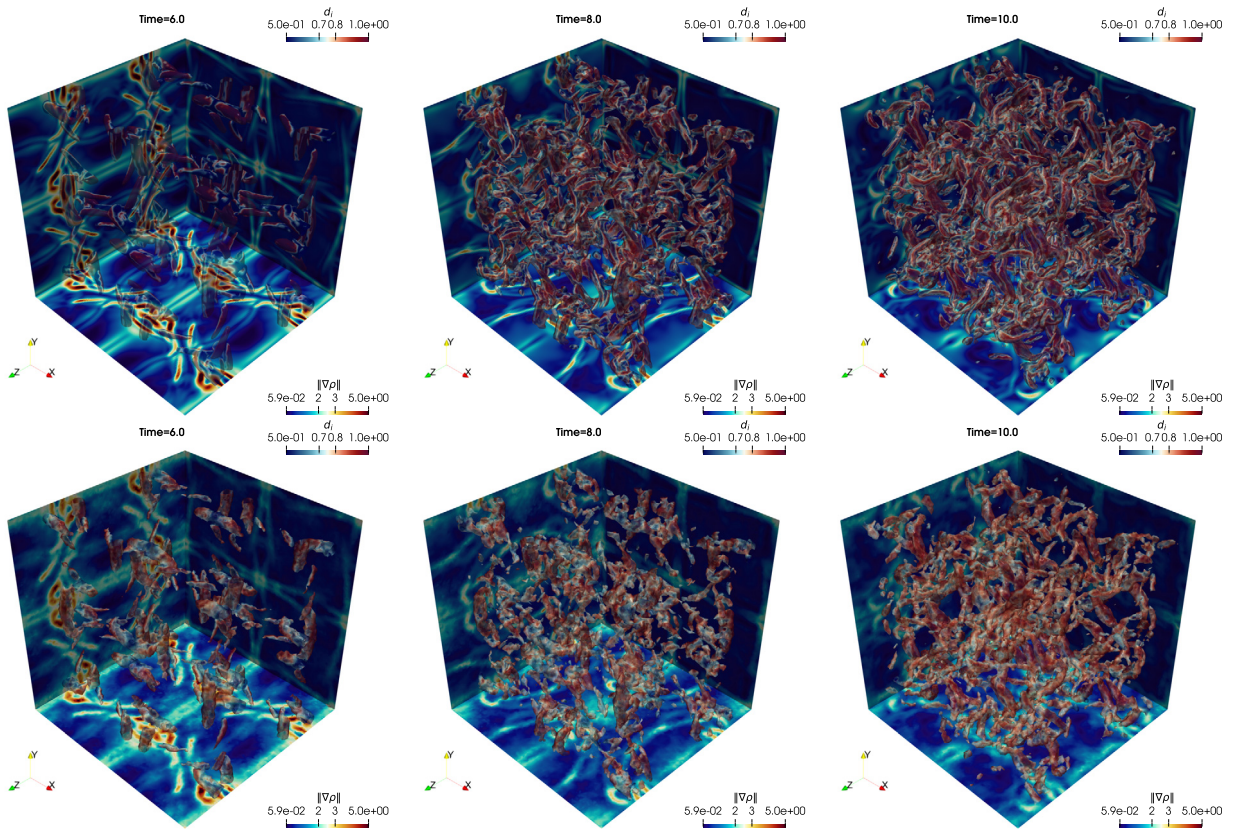


Fig. 24. Contour plots at different instants of the density gradient for the supersonic viscous Taylor-Green vortex flow at the $Y = \pi$ and $Z = \pi$ plane, and iso-surfaces of Q-criterion coloured by flux dissipation term d_i computed with the CWENOZ5-ADDA scheme on a hexahedral mesh of 128^3 (top) and the tetrahedral mesh with a cell edge resolution equivalent to 64^3 (bottom).

for assessing the robustness of the developed ADDA algorithm due to the presence of multitude phenomena such as shock wave shear layer and shocklet/turbulence interaction, or shock-wave/boundary-layer interaction.

A hybrid unstructured mesh is used consisting of approximately 12 million cells, with the computational domain extending $20D$ upstream and $50D$ downstream, and $\pi \cdot D$ in the z -direction. The grid resolution at the boundary layer region gives a $y^+ \approx 1$ at the first cell-centre off the surface, 240 equally-spaced cells are used in the z -direction, and a refinement region at the wake of the cylinder is used for $20D$ downstream with an average cell length of $0.04D$ as shown in Fig. 25. Periodic boundary conditions are used in the

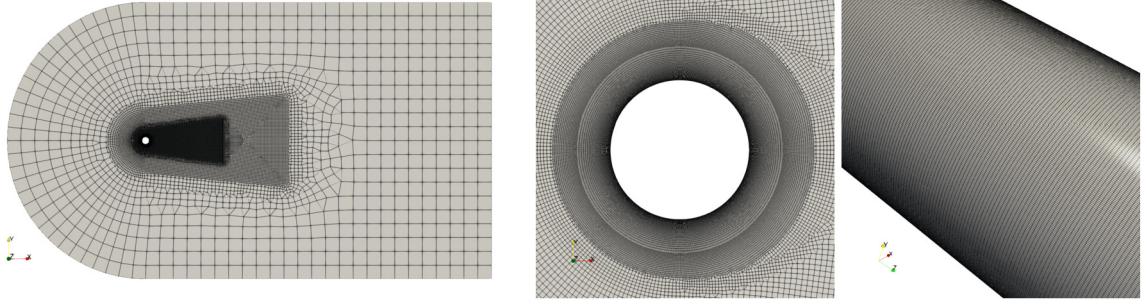


Fig. 25. Overview of the hybrid unstructured mesh used for the transonic flow past an infinite cylinder at $Re = 166000$ and $M_\infty = 0.8$.

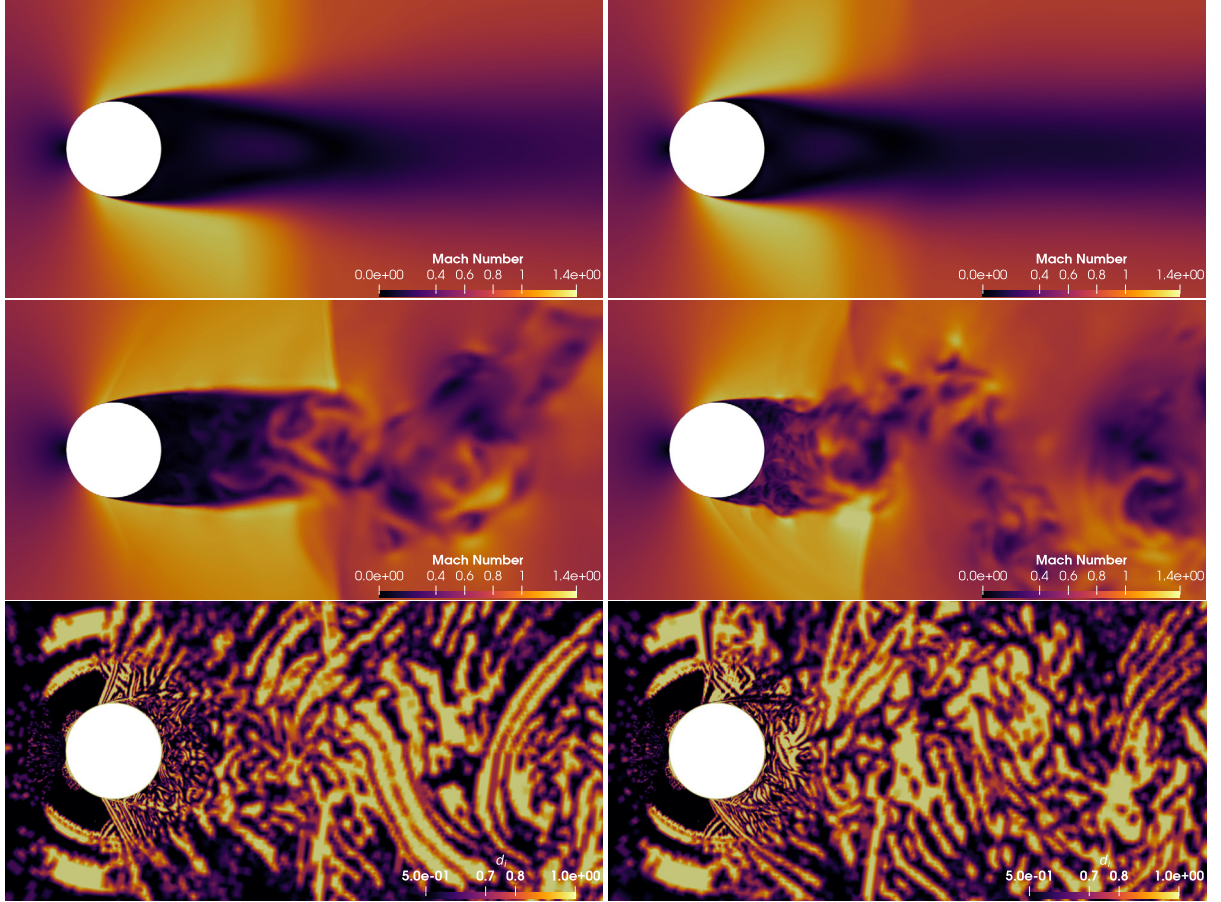
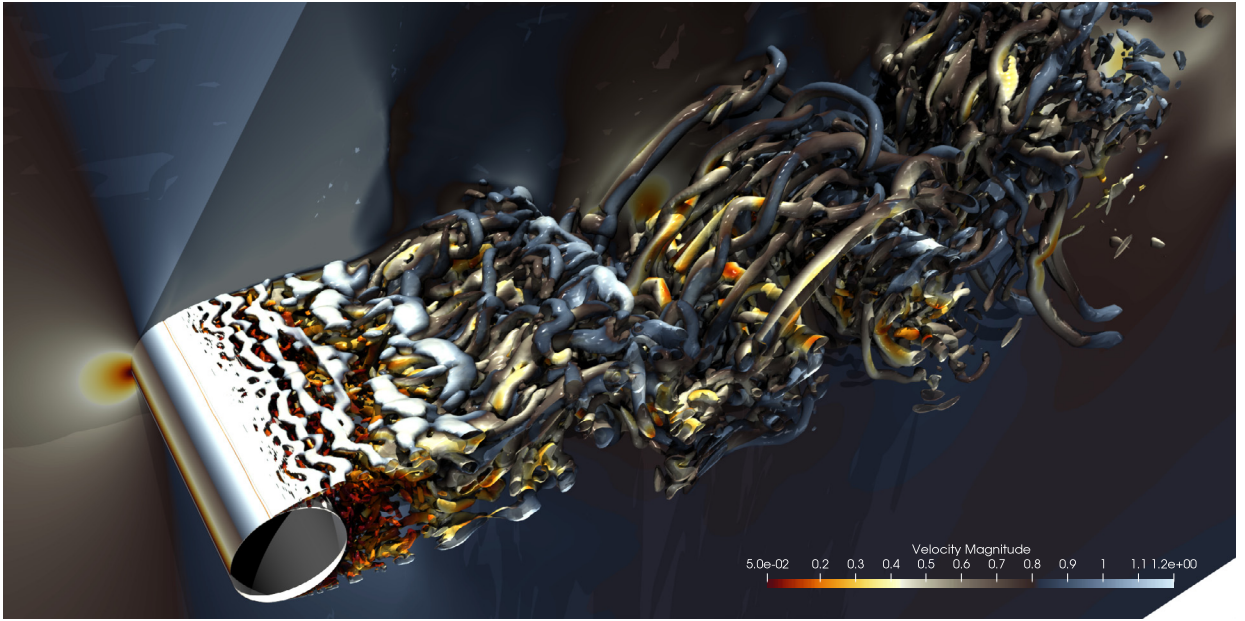


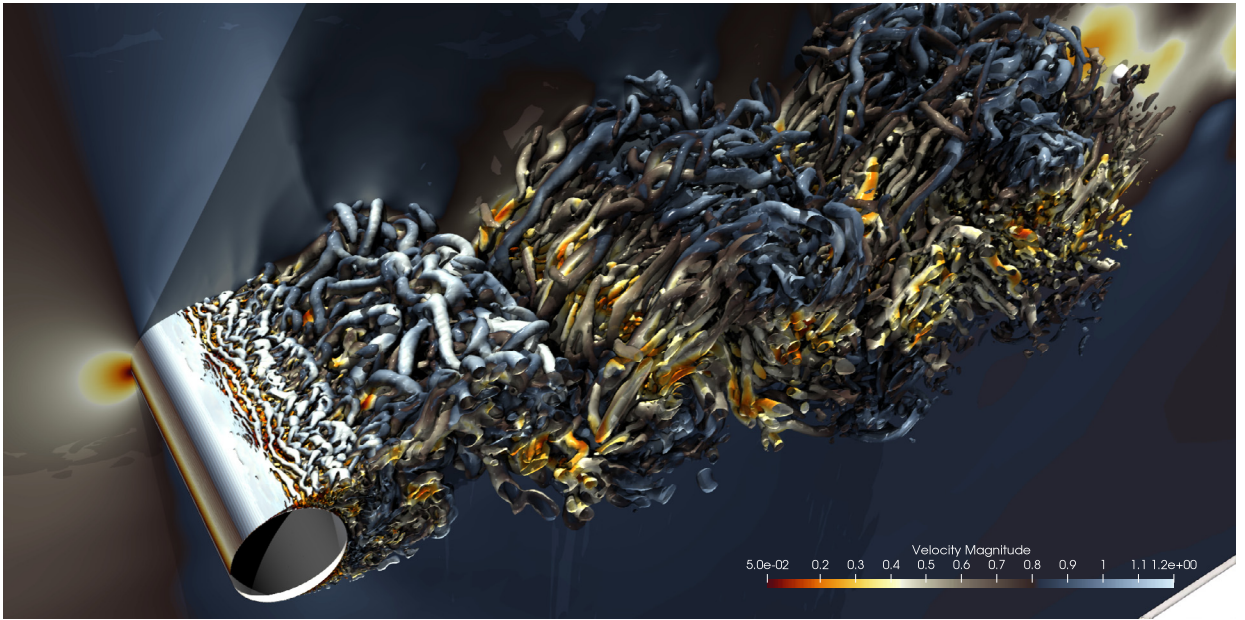
Fig. 26. Contour plots of the spanwise-averaged time-averaged Mach number (top) for the transonic flow past an infinite cylinder at $Re = 166000$ and $M_\infty = 0.8$ where it can be noticed that the CWENO4-ADDA scheme has a shorter wake compared to CWENO3-ADDA. Contour plots of the instantaneous Mach number (middle) and flux dissipation d_f term (at $z = \pi/2$) (bottom), where it can be seen that the CWENO4-ADDA schemes exhibits more regions with higher-dissipation compared to CWENO3-ADDA, while resolving more features. CWENO3-ADDA left column, CWENO4-ADDA right column.

z -direction, no-slip boundary conditions at the surface of the cylinder and free-stream conditions at the farfield. Two schemes are employed for this study, i.e., a CWENOZ 3rd-order scheme and a CWENOZ 4th-order scheme. The solution is advanced in time with an implicit-time stepping method of 2nd-order of accuracy and a $\Delta t = t_c/500$ where $t_c = D/U_\infty$. The simulation was run for $t = 40t_c$ for the initial transients to settle, and for an additional $90t_c$ for obtaining sufficiently converged statistical data.

Examining the time-averaged, and spanwise-averaged Mach number seen in Fig. 26, it is noticed that the wake is shorter for the CWENO4-ADDA scheme compared to the CWENO3-ADDA scheme, representative of the reduced dissipation of the higher-order approximation. This trend is continued in the instantaneous Mach number contour plots shown in Fig. 26, where the CWENO4-ADDA scheme demonstrated a shorter wake with more features resolved, while the flux-dissipation term indicates that more regions are associated with higher-dissipation compared to CWENO3-ADDA as expected.



(a) CWENOZ3-ADDA



(b) CWENOZ4-ADDA

Fig. 27. Isosurfaces of Q-criterion coloured by Mach number for the transonic flow past an infinite cylinder at $Re = 166000$ and $M_\infty = 0.8$, where the resolved vortical structures and transition to turbulence typical of this flow regime can be noticed.

The CWENOZ4-ADDA scheme resolves substantially more vortical features while capturing the shocks present as seen in Fig. 27 compared to the 3rd-order variant. The improved dissipation characteristics of the higher-order scheme are evident from the wider-range of frequencies resolved in the near field-wake region, as seen in Fig. 28 of the FFT of kinetic energy at different probe positions. The predictions of surface integrated pressure coefficient C_p are in good agreement with the results of Macha [116], and Murthy & Rose [115] up to 83° , and then an increased sensitivity is seen across all predictions as shown in Fig. 29.

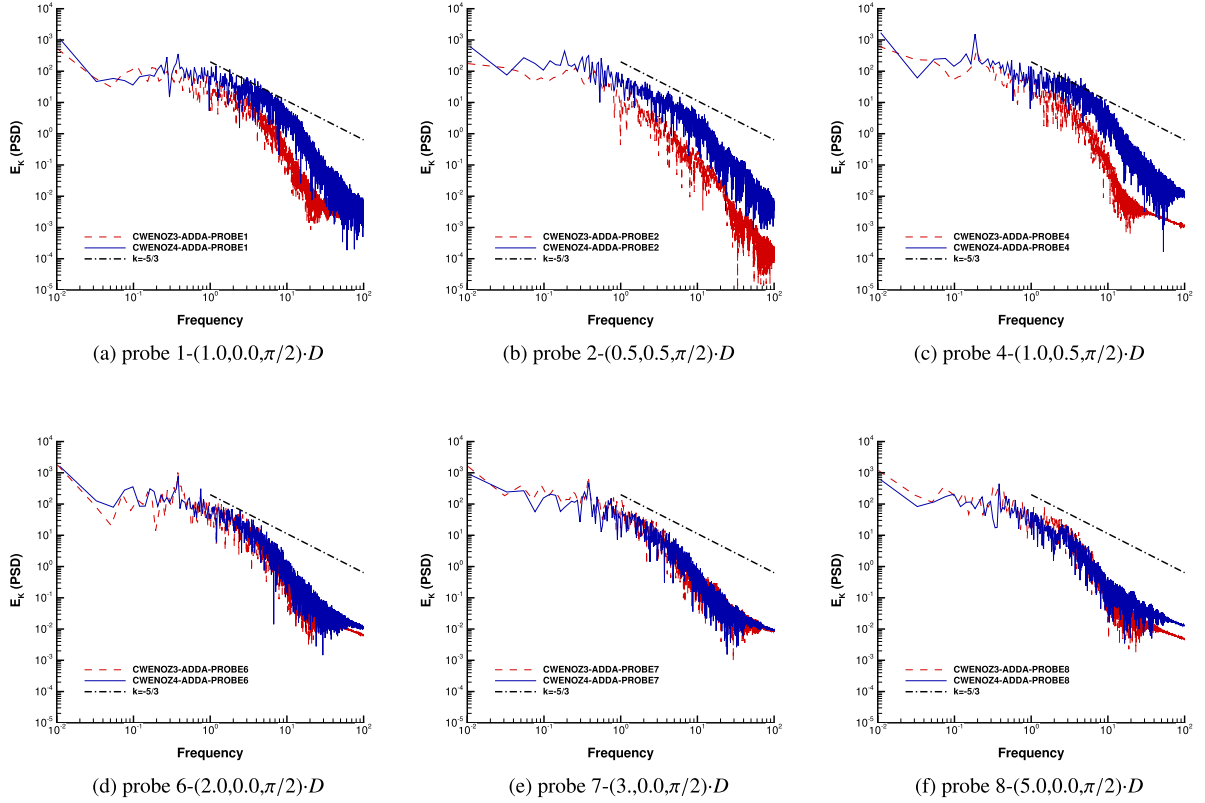


Fig. 28. 1D frequency spectra of the kinetic energy for several probe positions for the transonic flow past an infinite cylinder at $Re = 166000$ and $M_\infty = 0.8$, where it can be noticed that the CWENOZ4-ADDA scheme offers an improved agreement with the Kolmogorov slope in the near field-wake region (Up to $2 \cdot D$) compared to the CWENOZ3-ADDA scheme.

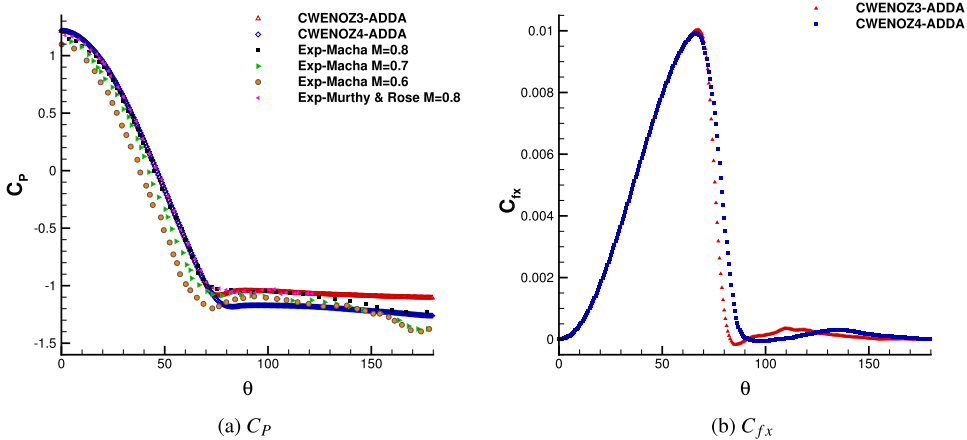


Fig. 29. Time-averaged and spanwise-averaged pressure coefficient C_p and skin friction coefficient C_{fx} for the transonic flow past an infinite cylinder at $Re = 166000$ and $M_\infty = 0.8$, and comparison with the experimental results of Macha [116], and Murthy & Rose [115].

6. Conclusions

This work presented the development of an adaptive dissipation/dispersion adjustment algorithm within a high order finite-volume framework and its application to iLES simulations of compressible turbulent flows in under-resolved settings. The ADDA algorithm initially measures the energy ratio to determine if the field is well- or under-resolved, then adjusts the linear central stencil weight of the CWENOZ reconstruction, and finally depending on the resulting non-linear weights can adjust the dissipation term of the Roe Riemann solver. This process is executed for every cell, at each time step during an iLES simulation, in a computationally efficient manner with a small overhead (2-8%). The framework was assessed in a series of well-established test problems and compared

against reference solutions. The tests included the iLES of inviscid subsonic Taylor-Green vortex, the viscous supersonic Taylor-Green vortex, the subsonic flow over the SD7003 wing, and the transonic flow past a circular cylinder. All the results obtained with several variations of the framework demonstrated an improved scale-resolving capability, while obtaining physically meaningful results, and maintaining the non-oscillatory properties of the CWENOZ schemes. Furthermore, the versatility of the ADDA algorithm extends beyond its current implementation, with potential applications in other numerical frameworks such as DG and hybrid DG-FV. Our future endeavours aim to explore its adaptability with different numerical schemes, including variants like TENO, CENO, and WENO. Moreover, we plan to extend the ADDA algorithm to wall-modeled large eddy simulations (WMLES), anticipating enhanced accuracy and robustness in turbulence modeling. It is worth noting that the ADDA algorithm has been successfully integrated into the open-source UCNS3D CFD solver, fostering accessibility for researchers and facilitating advancements in diverse fields of study. This work marks a significant step towards enhancing the efficiency and reliability of numerical simulations in turbulent flow regimes, with promising avenues for further exploration and development.

Declaration of competing interest

None.

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Appendix A. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.jcp.2024.113653>.

Data availability

Data relating to the results in this manuscript are available at the Cranfield Online Research Data Repository <https://doi.org/10.57996/cran.ceres-2638>.

References

- [1] P. Moin, K. Mahesh, Direct numerical simulation: a tool in turbulence research, *Annu. Rev. Fluid Mech.* 30 (1) (1998) 539–578.
- [2] M. Germano, U. Piomelli, P. Moin, W.H. Cabot, A dynamic subgrid-scale eddy viscosity model, *Phys. Fluids A, Fluid Dyn.* 3 (7) (1991) 1760–1765.
- [3] P. Moin, K. Squires, W. Cabot, S. Lee, A dynamic subgrid-scale model for compressible turbulence and scalar transport, *Phys. Fluids A, Fluid Dyn.* 3 (11) (1991) 2746–2757.
- [4] A. Vreman, An eddy-viscosity subgrid-scale model for turbulent shear flow: algebraic theory and applications, *Phys. Fluids* 16 (10) (2004) 3670–3681.
- [5] J. Fernandez-Fidalgo, L. Ramirez, P. Tsoutsanis, I. Colominas, X. Nogueira, A reduced-dissipation WENO scheme with automatic dissipation adjustment, *J. Comput. Phys.* 425 (2021), <https://doi.org/10.1016/j.jcp.2020.109749>.
- [6] P. Tsoutsanis, M. Dumbser, Arbitrary high order central non-oscillatory schemes on mixed-element unstructured meshes, *Comput. Fluids* 225 (2021), <https://doi.org/10.1016/j.compfluid.2021.104961>.
- [7] D. Flad, A. Beck, C. Munz, Simulation of underresolved turbulent flows by adaptive filtering using the high order discontinuous Galerkin spectral element method, *J. Comput. Phys.* 313 (2016) 1–12.
- [8] J. Domaradzki, K. Loh, P. Yee, Large eddy simulations using the subgrid-scale estimation model and truncated Navier-Stokes dynamics, *Theor. Comput. Fluid Dyn.* 15 (6) (2002) 421–450.
- [9] J. Domaradzki, Z. Xiao, P. Smolarkiewicz, Effective eddy viscosities in implicit large eddy simulations of turbulent flows, *Phys. Fluids* 15 (12) (2003) 3890–3893.
- [10] D. Drikakis, C. Fureby, F. Grinstein, D. Youngs, Simulation of transition and turbulence decay in the Taylor-Green vortex, *J. Turbul.* 8 (2007) 1–12.
- [11] D. Garmann, M. Visbal, P. Orkwis, Comparative study of implicit and subgrid-scale model large-eddy simulation techniques for low-Reynolds number airfoil applications, *Int. J. Numer. Methods Fluids* 71 (12) (2013) 1546–1565.
- [12] A. Beck, T. Bolemann, D. Flad, H. Frank, G. Gassner, F. Hindenlang, C.-D. Munz, High-order discontinuous Galerkin spectral element methods for transitional and turbulent flow simulations, *Int. J. Numer. Methods Fluids* 76 (8) (2014) 522–548.
- [13] M. Galbraith, M. Visbal, Implicit large eddy simulation of low Reynolds number flow past the SD7003 airfoil, in: 46th AIAA Aerospace Sciences Meeting and Exhibit, 2008.
- [14] L. Fu, Review of the high-order TENO schemes for compressible gas dynamics and turbulence, *Arch. Comput. Methods Eng.* 30 (4) (2023) 2493–2526.
- [15] X. Deng, Z.-h. Jiang, P. Vincent, F. Xiao, C. Yan, A new paradigm of dissipation-adjustable, multi-scale resolving schemes for compressible flows, *J. Comput. Phys.* 466 (2022), <https://doi.org/10.1016/j.jcp.2022.111287>.
- [16] G. Cao, W. Zhao, S. Chen, Quantitative analysis on implicit large eddy simulation, *Phys. Fluids* 34 (10) (2022), <https://doi.org/10.1063/5.0113291>.
- [17] X. Nogueira, L. Cueto-Felgueroso, I. Colominas, H. Gomez, Implicit large eddy simulation of non-wall-bounded turbulent flows based on the multiscale properties of a high-order finite volume method, *Comput. Methods Appl. Mech. Eng.* 199 (9–12) (2010) 615–624, <https://doi.org/10.1016/j.cma.2009.10.013>.
- [18] F. Witherden, A. Jameson, Impact of number representation for high-order implicit large-eddy simulations, *AIAA J.* 58 (1) (2020) 184–197, <https://doi.org/10.2514/1.J058434>.

- [19] C. Xie, J. Wang, E. Weinan, Modeling subgrid-scale forces by spatial artificial neural networks in large eddy simulation of turbulence, *Phys. Rev. Fluids* 5 (2020), <https://doi.org/10.1103/PhysRevFluids.5.054606>.
- [20] K. Ritos, I.W. Kokkinakis, D. Drikakis, Physical insight into the accuracy of finely-resolved files in turbulent boundary layers, *Comput. Fluids* 169 (2018) 309–316, <https://doi.org/10.1016/j.compfluid.2017.07.018>.
- [21] M. Karaca, N. Lardjane, I. Fedioun, Implicit large eddy simulation of high-speed non-reacting and reacting air/h 2 jets with a 5th order weno scheme, *Comput. Fluids* 62 (2012) 25–44, <https://doi.org/10.1016/j.compfluid.2012.03.013>.
- [22] S. Hickel, C.P. Egerer, J. Larsson, Subgrid-scale modeling for implicit large eddy simulation of compressible flows and shock-turbulence interaction, *Phys. Fluids* 26 (10) (2014), <https://doi.org/10.1063/1.4898641>.
- [23] K. Ritos, I.W. Kokkinakis, D. Drikakis, S.M. Spottswood, Implicit large eddy simulation of acoustic loading in supersonic turbulent boundary layers, *Phys. Fluids* 29 (4) (2017), <https://doi.org/10.1063/1.4979965>.
- [24] P. Fernandez, N. Nguyen, J. Peraire, The hybridized discontinuous Galerkin method for implicit large-eddy simulation of transitional turbulent flows, *J. Comput. Phys.* 336 (2017) 308–329, <https://doi.org/10.1016/j.jcp.2017.02.015>.
- [25] X. Nogueira, L. Ramirez, J. Fernandez-Fidalgo, M. Deligant, S. Khelladi, J.-C. Chassaing, F. Navarrina, An a posteriori-implicit turbulent model with automatic dissipation adjustment for large eddy simulation of compressible flows, *Comput. Fluids* 197 (2020), <https://doi.org/10.1016/j.compfluid.2019.104371>.
- [26] M. Latini, O. Schilling, A comparison of two- and three-dimensional single-mode reshocked Richtmyer-Meshkov instability growth, *Phys. D, Nonlinear Phenom.* 401 (2020), <https://doi.org/10.1016/j.physd.2019.132201>.
- [27] Y. Ma, Z.-G. Yan, H. Liu, Y. Min, H. Zhu, Improved weighted compact nonlinear scheme for implicit large-eddy simulations, *Comput. Fluids* 240 (2022), <https://doi.org/10.1016/j.compfluid.2022.105412>.
- [28] J. Park, F. Witherden, P. Vincent, High-order implicit large-eddy simulations of flow over a naca0021 aerofoil, *AIAA J.* 55 (7) (2017) 2186–2197, <https://doi.org/10.2514/1.J.055304>.
- [29] C. Cox, W. Trojak, T. Dzanic, F. Witherden, A. Jameson, Accuracy, stability, and performance comparison between the spectral difference and flux reconstruction schemes, *Comput. Fluids* 221 (2021), <https://doi.org/10.1016/j.compfluid.2021.104922>.
- [30] B. Huang, R. Wang, F. Wu, H. Xu, Applications of wall-models to implicit large eddy simulations in the spectral/ hp element method, *Phys. Fluids* 34 (10) (2022), <https://doi.org/10.1063/5.0117432>.
- [31] J.G. Pedersen, S.P. Malinowski, W.W. Grabowski, Resolution and domain-size sensitivity in implicit large-eddy simulation of the stratocumulus-topped boundary layer, *J. Adv. Model. Earth Syst.* 8 (2) (2016) 885–903, <https://doi.org/10.1002/2015MS000572>.
- [32] A.R. Winters, R.C. Moura, G. Mengaldo, G.J. Gassner, S. Walch, J. Peiro, S.J. Sherwin, A comparative study on polynomial dealiasing and split form discontinuous Galerkin schemes for under-resolved turbulence computations, *J. Comput. Phys.* 372 (2018) 1–21, <https://doi.org/10.1016/j.jcp.2018.06.016>.
- [33] A. Uranga, P.-O. Persson, M. Drela, J. Peraire, Implicit large eddy simulation of transition to turbulence at low Reynolds numbers using a discontinuous Galerkin method, *Int. J. Numer. Methods Eng.* 87 (1–5) (2011) 232–261, <https://doi.org/10.1002/nme.3036>.
- [34] Y. Feng, F.S. Schraner, J. Winter, N.A. Adams, A multi-objective Bayesian optimization environment for systematic design of numerical schemes for compressible flow, *J. Comput. Phys.* 468 (2022), <https://doi.org/10.1016/j.jcp.2022.111477>.
- [35] F.S. Schraner, J.A. Domaradzki, S. Hickel, N.A. Adams, Assessing the numerical dissipation rate and viscosity in numerical simulations of fluid flows, *Comput. Fluids* 114 (2015) 84–97, <https://doi.org/10.1016/j.compfluid.2015.02.011>.
- [36] J. Domaradzki, Effective eddy viscosities in implicit modeling of decaying high Reynolds number turbulence with and without rotation, *Fluid Dyn. Res.* 36 (4–6) (2005) 385–406.
- [37] T. Tantikul, J. Domaradzki, Large eddy simulations using truncated Navier-Stokes equations with the automatic filtering criterion, *J. Turbul.* 11 (2010) 1–24.
- [38] S. Hickel, N. Adams, J. Domaradzki, An adaptive local deconvolution method for implicit LES, *J. Comput. Phys.* 213 (1) (2006) 413–436, <https://doi.org/10.1016/j.jcp.2005.08.017>.
- [39] C.-G. Li, M. Tsubokura, An implicit turbulence model for low-Mach Roe scheme using truncated Navier-Stokes equations, *J. Comput. Phys.* 345 (2017) 462–474, <https://doi.org/10.1016/j.jcp.2017.05.032>.
- [40] A. Antoniadis, D. Drikakis, P. Farmakis, L. Fu, I. Kokkinakis, X. Nogueira, P. Silva, M. Skote, V. Titarev, P. Tsoutsanis, UCNS3D: an open-source high-order finite-volume unstructured CFD solver, *Comput. Phys. Commun.* 279 (2022), <https://doi.org/10.1016/j.cpc.2022.108453>.
- [41] P. Tsoutsanis, A. Antoniadis, K. Jenkins, Improvement of the computational performance of a parallel unstructured WENO finite volume cfd code for implicit large eddy simulation, *Comput. Fluids* 173 (2018) 157–170, <https://doi.org/10.1016/j.compfluid.2018.03.012>.
- [42] A. Stroud, Some fourth degree integration formulas for simplexes, *Math. Comput.* 30 (134) (1976) 291–294, <https://doi.org/10.1090/S0025-5718-1976-0391484-0>.
- [43] P. Tsoutsanis, V. Titarev, D. Drikakis, WENO schemes on arbitrary mixed-element unstructured meshes in three space dimensions, *J. Comput. Phys.* 230 (4) (2011) 1585–1601, <https://doi.org/10.1016/j.jcp.2010.11.023>.
- [44] P. Tsoutsanis, A. Antoniadis, D. Drikakis, WENO schemes on arbitrary unstructured meshes for laminar, transitional and turbulent flows, *J. Comput. Phys.* 256 (2014) 254–276, <https://doi.org/10.1016/j.jcp.2013.09.002>.
- [45] V. Titarev, P. Tsoutsanis, D. Drikakis, WENO schemes for mixed-element unstructured meshes, *Commun. Comput. Phys.* 8 (3) (2010) 585–609, <https://doi.org/10.4208/cicp.040909.080110a>.
- [46] P. Tsoutsanis, M. Pavan Kumar, P. Farmakis, A relaxed a posteriori MOOD algorithm for multicomponent compressible flows using high-order finite-volume methods on unstructured meshes, *Appl. Math. Comput.* 437 (2023), <https://doi.org/10.1016/j.amc.2022.127544>.
- [47] P.A.S.F. Silva, P. Tsoutsanis, A. Antoniadis, Numerical investigation of full helicopter with and without the ground effect, *Aerosp. Sci. Technol.* 122 (2022), <https://doi.org/10.1016/j.ast.2022.107401>.
- [48] P. Tsoutsanis, P. Farmakis, A posteriori mood limiting approach for multicomponent flows on unstructured meshes, in: *World Congress in Computational Mechanics and ECCOMAS Congress*, 2022.
- [49] E. Adebayo, P. Tsoutsanis, K. Jenkins, Implementation of cweno schemes for compressible multicomponent/multiphase flow using interface capturing models, in: *World Congress in Computational Mechanics and ECCOMAS Congress*, 2022.
- [50] P. Silva, P. Tsoutsanis, A. Antoniadis, K. Jenkins, Unstructured high-order solutions of hovering rotors with and without ground effect, in: *World Congress in Computational Mechanics and ECCOMAS Congress*, 2022.
- [51] P. Tsoutsanis, E. Adebayo, A. Merino, A. Arjona, M. Skote, CWENO finite-volume interface capturing schemes for multicomponent flows using unstructured meshes, *J. Sci. Comput.* 89 (3) (2021), <https://doi.org/10.1007/s10915-021-01673-y>.
- [52] P. Silva, P. Tsoutsanis, A. Antoniadis, Simple multiple reference frame for high-order solution of hovering rotors with and without ground effect, *Aerosp. Sci. Technol.* 111 (2021), <https://doi.org/10.1016/j.ast.2021.106518>.
- [53] P. Farmakis, P. Tsoutsanis, X. Nogueira, WENO schemes on unstructured meshes using a relaxed a posteriori MOOD limiting approach, *Comput. Methods Appl. Mech. Eng.* 363 (2020), <https://doi.org/10.1016/j.cma.2020.112921>.
- [54] F. Ricci, P. Silva, P. Tsoutsanis, A. Antoniadis, Hovering rotor solutions by high-order methods on unstructured grids, *Aerosp. Sci. Technol.* 97 (2020), <https://doi.org/10.1016/j.ast.2019.105648>.
- [55] A. Antoniadis, P. Tsoutsanis, D. Drikakis, Assessment of high-order finite volume methods on unstructured meshes for rans solutions of aeronautical configurations, *Comput. Fluids* 146 (2017) 86–104, <https://doi.org/10.1016/j.compfluid.2017.01.002>.
- [56] P. Tsoutsanis, I. Kokkinakis, L. Konozy, D. Drikakis, R. Williams, D. Youngs, Comparison of structured- and unstructured-grid, compressible and incompressible methods using the vortex pairing problem, *Comput. Methods Appl. Mech. Eng.* 293 (2015) 207–231, <https://doi.org/10.1016/j.cma.2015.04.010>.

- [57] P. Tsoutsanis, D. Drikakis, A high-order finite-volume method for atmospheric flows on unstructured grids, *J. Coupled Syst. Multiscale Dyn.* 4 (2016) 170–186, <https://doi.org/10.1166/jcsmd.2016.1104>.
- [58] E.M. Adebayo, P. Tsoutsanis, K.W. Jenkins, Application of central-weighted essentially non-oscillatory finite-volume interface-capturing schemes for modeling cavitation induced by an underwater explosion, *Fluids* 9 (2) (2024), <https://doi.org/10.3390/fluids9020033>.
- [59] T. Piovesan, M. Migliorini, P. Zachos, P. Tsoutsanis, Dynamic swirl distortion characteristics in s-shaped diffusers using UCNS3D and time-resolved, stereo piv methods, in: *AIAA SCITECH 2024 Forum*, 2024.
- [60] V. Maltsev, D. Yuan, K. Jenkins, M. Skote, P. Tsoutsanis, Hybrid discontinuous Galerkin-finite volume techniques for compressible flows on unstructured meshes, *J. Comput. Phys.* 473 (2023), <https://doi.org/10.1016/j.jcp.2022.111755>.
- [61] V. Maltsev, P. Tsoutsanis, M. Skote, Hybrid finite-volume/discontinuous Galerkin framework for the solution of multiphysics problems using unstructured meshes, in: *World Congress in Computational Mechanics and ECCOMAS Congress*, 2022.
- [62] D. Yuan, P. Tsoutsanis, K. Jenkins, Hybrid high-order finite volume discontinuous Galerkin methods for turbulent flows, in: *World Congress in Computational Mechanics and ECCOMAS Congress*, 2022.
- [63] V. Maltsev, M. Skote, P. Tsoutsanis, High-order hybrid DG-FV framework for compressible multi-fluid problems on unstructured meshes, *J. Comput. Phys.* 502 (2024) 112819, <https://doi.org/10.1016/j.jcp.2024.112819>.
- [64] P. Tsoutsanis, Stencil selection algorithms for WENO schemes on unstructured meshes, *J. Comput. Phys.* 475 (2023), <https://doi.org/10.1016/j.jcp.2019.07.039>.
- [65] A. Jalali, M. Sharbatdar, C. Ollivier-Gooch, Accuracy analysis of unstructured finite volume discretization schemes for diffusive fluxes, *Comput. Fluids* 101 (2014) 220–232.
- [66] M. Dumbser, W. Boscheri, M. Semplice, G. Russo, Central weighted ENO schemes for hyperbolic conservation laws on fixed and moving unstructured meshes, *SIAM J. Sci. Comput.* 39 (6) (2017) A2564–A2591, <https://doi.org/10.1137/17M1111036>.
- [67] M. Dumbser, O. Zanotti, R. Loubère, S. Diot, A posteriori subcell limiting of the discontinuous Galerkin finite element method for hyperbolic conservation laws, *J. Comput. Phys.* 278 (2014) 47–75.
- [68] M. Dumbser, M. Castro, C. Pares, E. Toro, ADER schemes on unstructured meshes for nonconservative hyperbolic systems: applications to geophysical flows, *Comput. Fluids* 38 (9) (2009) 1731–1748.
- [69] S. Clain, S. Diot, R. Loubère, A high-order finite volume method for systems of conservation laws-multi-dimensional optimal order detection (MOOD), *J. Comput. Phys.* 230 (10) (2011) 4028–4050, <https://doi.org/10.1016/j.jcp.2011.02.026>.
- [70] S. Diot, S. Clain, R. Loubère, Improved detection criteria for the multi-dimensional optimal order detection (MOOD) on unstructured meshes with very high-order polynomials, *Comput. Fluids* 64 (2012) 43–63, <https://doi.org/10.1016/j.compfluid.2012.05.004>.
- [71] G.W. Stewart, *Matrix Algorithms, Volume 1: Basic Decompositions*, Society for Industrial and Applied Mathematics SIAM, 1998.
- [72] M. Semplice, A. Coco, G. Russo, Adaptive mesh refinement for hyperbolic systems based on third-order compact WENO reconstruction, *J. Sci. Comput.* 66 (2) (2016) 692–724.
- [73] M. Semplice, G. Visconti, Efficient implementation of adaptive order reconstructions, *J. Sci. Comput.* 83 (1) (2020), <https://doi.org/10.1007/s10915-020-01156-6>.
- [74] N. Simmonds, P. Tsoutsanis, A. Antoniadis, K. Jenkins, A. Gaylard, Low-Mach number treatment for finite-volume schemes on unstructured meshes, *Appl. Math. Comput.* 336 (2018) 368–393, <https://doi.org/10.1016/j.amc.2018.04.076>.
- [75] N. Simmonds, P. Tsoutsanis, A. Gaylard, Implementation of a low-mach number modification for high-order finite-volume schemes for arbitrary hybrid unstructured meshes, in: *ECCOMAS Congress 2016 - Proceedings of the 7th European Congress on Computational Methods in Applied Sciences and Engineering 2*, 2016, pp. 2900–2915.
- [76] E. Toro, M. Spruce, W. Speares, Restoration of the contact surface in the HLL-Riemann solver, *Shock Waves* 4 (1) (1994) 25–34.
- [77] P. Roe, Approximate Riemann solvers, parameter vectors, and difference schemes, *J. Comput. Phys.* 43 (2) (1981) 357–372, [https://doi.org/10.1016/0021-9991\(81\)90128-5](https://doi.org/10.1016/0021-9991(81)90128-5).
- [78] H. Nishikawa, Robust and accurate viscous discretisation via upwind scheme-I: basic principle, *Comput. Fluids* 49 (2011) 62–86.
- [79] K. Nakahashi, D. Sharov, S. Kano, M. Koderu, Applications of unstructured hybrid grid method to high-Reynolds number viscous flows, *Int. J. Numer. Methods Fluids* 31 (1) (1999) 97–111.
- [80] R.J. Spiteri, S.J. Ruuth, A new class of optimal high-order strong-stability-preserving time-stepping schemes, *SIAM J. Numer. Anal.* 40 (2) (2002) 469–491.
- [81] A. Jameson, Time dependent calculations using multigrid, with applications to unsteady flows past airfoils and wings, *AIAA paper* 6 (1991-1596), <https://doi.org/10.2514/6.1991-1596>, 1991.
- [82] V. Rusanov, The calculation of the interaction of non-stationary shock waves and obstacles, *USSR Comput. Math. Math. Phys.* 1 (1962) 304–320, [https://doi.org/10.1016/0041-5553\(62\)90062-9](https://doi.org/10.1016/0041-5553(62)90062-9).
- [83] H. Luo, J. Baum, R. Lohner, A fast, matrix-free implicit method for compressible flows on unstructured grids, *J. Comput. Phys.* 146 (2) (1998) 664–690.
- [84] M. Petrov, A. Tambova, V. Titarev, S. Utyuzhnikov, A. Chikitkin, Flowmodellium software package for calculating high-speed flows of compressible fluid, *Comput. Math. Math. Phys.* 58 (11) (2018) 1865–1886.
- [85] T. Dzanic, W. Trojak, F. Witherden, On the anti-aliasing properties of entropy filtering for under-resolved turbulent flows, Preprint, <https://doi.org/10.48550/arXiv.2302.13359>, 2023.
- [86] M. Hamed, B. Vermeire, Optimized filters for stabilizing high-order large eddy simulation, *Comput. Fluids* 237 (2022) 105301, <https://doi.org/10.1016/j.compfluid.2021.105301>.
- [87] S. Pirozzoli, On the spectral properties of shock-capturing schemes, *J. Comput. Phys.* 219 (2) (2006) 489–497, <https://doi.org/10.1016/j.jcp.2006.07.009>.
- [88] J. Ruano, A. Baez Vidal, J. Rigola, F.X. Trias, A new general method to compute dispersion errors on Cartesian stretched meshes for both linear and non-linear operators, *Comput. Phys. Commun.* 271 (2022) 108192, <https://doi.org/10.1016/j.cpc.2021.108192>.
- [89] G. Gassner, A. Winters, A novel robust strategy for discontinuous Galerkin methods in computational fluid mechanics: Why? When? What? Where?, *Front. Phys.* 8 (2021), <https://doi.org/10.3389/fphy.2020.500690>.
- [90] P. Tsoutsanis, X. Nogueira, L. Fu, A short note on a 3D spectral analysis for turbulent flows on unstructured meshes, *J. Comput. Phys.* 474 (2023), <https://doi.org/10.1016/j.jcp.2022.111804>.
- [91] I. Cravero, G. Puppo, M. Semplice, G. Visconti, Cool weno schemes, *Comput. Fluids* 169 (2018) 71–86, <https://doi.org/10.1016/j.compfluid.2017.07.022>, Recent progress in nonlinear numerical methods for time-dependent flow & transport problems.
- [92] L. Fu, X.Y. Hu, N.A. Adams, Improved five- and six-point targeted essentially nonoscillatory schemes with adaptive dissipation, *AIAA J.* 57 (3) (2019) 1143–1158.
- [93] L. Fu, X. Hu, N.A. Adams, A targeted ENO scheme as implicit model for turbulent and genuine subgrid scales, *Commun. Comput. Phys.* 26 (2) (2019) 311–345.
- [94] L. Fu, A very-high-order TENO scheme for all-speed gas dynamics and turbulence, *Comput. Phys. Commun.* 244 (2019) 117–131, <https://doi.org/10.1016/j.cpc.2019.06.013>.
- [95] L. Fu, X. Hu, N. Adams, Targeted ENO schemes with tailored resolution property for hyperbolic conservation laws, *J. Comput. Phys.* 349 (2017) 97–121, <https://doi.org/10.1016/j.jcp.2017.07.054>.
- [96] L. Fu, X. Hu, N. Adams, A family of high-order targeted ENO schemes for compressible-fluid simulations, *J. Comput. Phys.* 305 (2016) 333–359, <https://doi.org/10.1016/j.jcp.2015.10.037>.
- [97] D. Levy, G. Puppo, G. Russo, Compact central WENO schemes for multidimensional conservation laws, *SIAM J. Sci. Comput.* 22 (2000) 656–672.
- [98] J. Zhu, J. Qiu, New finite volume weighted essentially nonoscillatory schemes on triangular meshes, *SIAM J. Sci. Comput.* 40 (2018) A903–A928.

- [99] J. Zhu, C.-W. Shu, A new type of third-order finite volume multi-resolution WENO schemes on tetrahedral meshes, *J. Comput. Phys.* 406 (2020) 109212.
- [100] D. Balsara, S. Garain, V. Florinski, W. Boscheri, An efficient class of WENO schemes with adaptive order for unstructured meshes, *J. Comput. Phys.* 404 (2020), <https://doi.org/10.1016/j.jcp.2019.109062>.
- [101] V. Maltsev, M. Skote, P. Tsoutsanis, High-order methods for diffuse-interface models in compressible multi-medium flows: a review, *Phys. Fluids* 34 (2) (2022), <https://doi.org/10.1063/5.0077314>.
- [102] A. Nigro, C. De Bartolo, A. Crivellini, M. Franciolini, A. Colombo, F. Bassi, A low-dissipation DG method for the under-resolved simulation of low Mach number turbulent flows, *Comput. Math. Appl.* 77 (6) (2019) 1739–1755, <https://doi.org/10.1016/j.camwa.2018.09.049>, 7th International Conference on Advanced Computational Methods in Engineering (ACOMEN 2017).
- [103] A. Gouasmi, S.M. Murman, K. Duraisamy, Entropy-stable schemes in the low-Mach-number regime: flux-preconditioning, entropy breakdowns, and entropy transfers, *J. Comput. Phys.* 456 (2022) 111036, <https://doi.org/10.1016/j.jcp.2022.111036>.
- [104] S.-sheng Chen, J.-ping Li, Z. Li, W. Yuan, Z.-hong Gao, Anti-dissipation pressure correction under low Mach numbers for Godunov-type schemes, *J. Comput. Phys.* 456 (2022) 111027, <https://doi.org/10.1016/j.jcp.2022.111027>.
- [105] J. Manzanero, E. Ferrer, G. Rubio, E. Valero, Design of a Smagorinsky spectral vanishing viscosity turbulence model for discontinuous Galerkin methods, *Comput. Fluids* 200 (2020) 104440, <https://doi.org/10.1016/j.compfluid.2020.104440>.
- [106] S. sheng Chen, C. Yan, X. hao Xiang, Effective low-Mach number improvement for upwind schemes, *Comput. Math. Appl.* 75 (10) (2018) 3737–3755, <https://doi.org/10.1016/j.camwa.2018.02.028>.
- [107] M. Brachet, D. Meiron, B. Nickel, R. Morf, U. Frisch, S. Orszag, Small-scale structure of the Taylor-Green vortex, *J. Fluid Mech.* 130 (1983) 411–452, <https://doi.org/10.1017/S0022112083001159>.
- [108] J. Bull, A. Jameson, Simulation of the Taylor-Green vortex using high-order flux reconstruction schemes, *AIAA J.* 53 (9) (2015) 2750–2761.
- [109] C.-W. Shu, W.-S. Don, D. Gottlieb, O. Schilling, L. Jameson, Numerical convergence study of nearly incompressible, inviscid Taylor-Green vortex flow, *J. Sci. Comput.* 24 (1) (2005) 569–595.
- [110] B. Vermeire, P. Vincent, On the properties of energy stable flux reconstruction schemes for implicit large eddy simulation, *J. Comput. Phys.* 327 (2016) 368–388.
- [111] B. Vermeire, F. Witherden, P. Vincent, On the utility of GPU accelerated high-order methods for unsteady flow simulations: a comparison with industry-standard tools, *J. Comput. Phys.* 334 (2017) 497–521.
- [112] P. Tsoutsanis, Extended bounds limiter for high-order finite-volume schemes on unstructured meshes, *J. Comput. Phys.* 362 (2018) 69–94, <https://doi.org/10.1016/j.jcp.2018.02.009>.
- [113] D. Lusher, N. Sandham, Assessment of low-dissipative shock-capturing schemes for the compressible Taylor-Green vortex, *AIAA J.* 59 (2) (2021) 533–545, <https://doi.org/10.2514/1.J059672>.
- [114] J.-B. Chapelier, D.J. Lusher, W. Van Noordt, C. Wenzel, T. Gibis, P. Mossier, A. Beck, G. Lodato, C. Brehm, M. Ruggeri, C. Scalo, N. Sandham, Comparison of high-order numerical methodologies for the simulation of the supersonic Taylor-Green vortex flow, *Phys. Fluids* 36 (5) (2024), <https://doi.org/10.1063/5.0206359>.
- [115] V. Murthy, W. Rose, Detailed measurements on a circular cylinder in cross flow, *AIAA J.* 16 (6) (1978) 549–550, <https://doi.org/10.2514/3.60930>.
- [116] J.M. Macha, Drag of circular cylinders at transonic mach numbers, *J. Aircr.* 14 (6) (1977) 605–607, <https://doi.org/10.2514/3.58828>.



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