

CRANFIELD INSTITUTE OF TECHNOLOGY

DEPARTMENT OF AERODYNAMICS

Ph.D. THESIS

Academic Year 1982-83

S.R. MOHAN

STUDIES IN TRANSONIC FLOW

Supervisor:

Professor J.L. Stollery

June 1983

DEDICATED TO THE MEMORY

OF MY FATHER

Abstract

This thesis is divided into two distinct parts. Part I describes the design and development of an intermittent cryogenic wind-tunnel, in which the cold conditions are generated by the expansion of high pressure gas. The device uses a light piston moving in a tube and conditions during the running time are maintained constant by 'tuning' the piston motion, i.e. by 'matching' the volumetric flow rate entering and leaving the tube. The results of the pilot tunnel (running time 0.3 secs.) show that gas temperatures of about 110K can be obtained with a pressure ratio of 35.

Part II describes the flow at transonic speeds on five aerofoil sections (thickness-chord ratios 6 to 14%). The aerofoil sections were 6 & 14% biconvex, an NACA 0012, a supercritical aerofoil CAST 7 and an 11.8% Joukowski profile. The tests were made in an intermittent, perforated wall wind-tunnel which was developed from an existing supersonic wind-tunnel.

A periodic flow due to shock-induced separation occurred over a narrow range of Mach numbers (from 0.82 to 0.90) on the 14% biconvex and 0012 sections at zero incidence, for both laminar and turbulent boundary layers. The frequency parameter (wc/U_∞) was about 1. Tests were also made with the aerofoils at incidence. Periodic flows occurred on the 14% biconvex, 0012 and Joukowski profiles. The instability Mach numbers ranged from 0.84 to 0.90 and the frequency parameters from 0.40 to 1.36.

A detailed study was made to determine the frequency spectrum of the tunnel noise and its influence on the periodic flow. Also, experiments were made to determine the influence of the aerofoil geometry on the periodic flow. The experimental results on the 14% biconvex aerofoil have been compared with the numerical computations of the full Navier-Stokes equations performed at NASA, Ames.

CONTENTS

<u>PART I</u>	<u>Page No.</u>
1. <u>INTRODUCTION</u>	
1.1 The Need for High Reynolds Number Testing	1
1.2 Methods for increasing the Reynolds Number	2
1.3 Cryogenic Wind-Tunnels	3
1.4 Light Piston Tunnel	3
1.5 The Present Work	4
2. <u>TEMPERATURES ACHIEVABLE BY THE EXPANSION OF HIGH PRESSURE GAS</u>	5
2.1 Calculation of Temperatures achievable by the expansion of High Pressure Gas	5
2.2 Expansion Experiments	5
2.3 Experimental Results and comparison with Theory	8
3. <u>CRYOGENIC PISTON TUNNEL</u>	10
3.1 Principle of Operation	10
3.2 The Expansion Process	10
3.3 Matching Conditions	11
3.4 Piston Oscillations	13
3.5 Departures from Ideal Performance	14
3.6 Transit Time Effects	15
3.7 Sizing of Reservoir	17
4. <u>DESIGN OF A PILOT CRYOGENIC WIND-TUNNEL</u>	18
4.1 Specification	18
4.2 Design Details	18
4.2.1 Charge Tube	18
4.2.2 Reservoir and Reservoir Throat	19
4.2.3 Piston Design	20
4.2.4 Fast Acting Valves	21
4.2.5 Contraction	22
4.2.6 Test-Section	22
4.2.7 Dump Chamber	22

CONTENTS contd.....

	<u>Page No.</u>
4.3 Instrumentation and Control System	22
4.4 Operation of the Pilot Tunnel	23
4.5 Temperature Measurement	23
5. <u>EXPERIMENTAL RESULTS AND DISCUSSION</u>	25
6. <u>CONCLUSIONS</u>	26
<u>REFERENCES</u>	27
<u>APPENDICES</u>	30
Appendix A Expressions for Reynolds Number	30
Appendix B The Calculation of Temperatures achieved by expansion of High Pressure Gas	32
Appendix C Table 2	36
<u>FIGURES</u>	37
 <u>PART II</u>	 <u>Page No.</u>
1. <u>INTRODUCTION</u>	1
2. <u>EXPERIMENTAL DETAILS</u>	4
2.1 Development of the 2.5 inch Transonic Wind-Tunnel	4
2.2 Mach Number Measurement	6
2.3 Aerofoil Models	7
2.4 Test Conditions	7
2.5 Test Techniques	8

	<u>Page No.</u>
3. <u>EXPERIMENTAL RESULTS AND ANALYSIS</u>	10
3.1 6% Biconvex Aerofoil	10
3.2 14% Biconvex Aerofoil	10
3.2.1 Development of Periodic Flow	11
3.2.2 Type of Shock-Wave Motion	12
3.2.3 Frequency Parameter	13
3.2.4 Instability Mach Number Range	14
3.2.5 Power Spectra	14
3.2.6 Resonant Frequencies of Ventilated Wind-Tunnels	15
3.2.7 Magnitude of Side-Wall Pressure Fluctuations	16
3.2.8 Pressure Distributions	18
3.2.9 Oil Flow Visualisation	18
3.2.10 Origin of the Periodic Flow	18
3.2.11 Aerofoil Tests with blunt Trailing-Edge	20
3.2.12 Aerofoil Tests with Splitter Plate	21
3.2.13 Aerofoil Tests with blunt Nose	22
3.2.14 Effect of Reynolds Number on Periodic Flow	22
3.2.15 Wind-Tunnel Interference	22
3.3 NACA 0012 Aerofoil	23
3.3.1 Aerofoil at Zero Incidence	23
3.3.1.1 Shock Positions	24
3.3.1.2 Frequency Parameter	25
3.3.1.3 Magnitude of Side-Wall Pressure Fluctuations	25
3.3.2 Aerofoil at an Incidence of 3°	26
3.3.3 Aerofoil at an Incidence of 6°	27
3.3.4 Some Comments	27

CONTENTS contd.....

Page No.

3.4	CAST 7 'Supercritical' Aerofoil	28
3.4.1	Aerofoil at Zero Incidence	29
3.4.2	Aerofoil at an Incidence of 3°	30
3.4.3	Aerofoil at an Incidence of 6°	31
3.5	11.8% Joukowski Aerofoil	31
3.5.1	Aerofoil at Zero Incidence	32
3.5.2	Aerofoil at an Incidence of 3°	33
3.5.3	Aerofoil at an Incidence of 6°	33
4.	<u>COMPARISON OF EXPERIMENTS WITH THEORY</u>	34
4.1	Laminar Boundary-Layer - Shock-Wave Interaction	34
4.2	Turbulent Boundary-Layer - Shock-Wave Interaction	34
4.3	Theories for Aileron Buzz	36
5.	<u>DISCUSSION</u>	38
5.1	Parameters for Periodic Flow	38
5.2	The Physical Mechanism causing the Periodic Flow	39
6.	<u>CONCLUSIONS</u>	42
	<u>REFERENCES</u>	44
	<u>APPENDICES</u>	51
Appendix A	Power Spectral Analysis	51
Appendix B	Criteria for Shock-Induced Separation on a Transonic Aerofoil	56
Appendix C	Wind-Tunnel Blockage Corrections	57
Appendix D	Tables	58

CONTENTS contd.....

Page No.

APPENDIX E The reasons for the location of
 shocks in steady conditions

69

APPENDIX F Shock-wave-boundary-layer
 interaction in supersonic and
 transonic flow

72

ACKNOWLEDGEMENT

74

FIGURES

75

PART I

LIST OF FIGURES

FIGURE

- 1 Differences between wind-tunnel tests and flight tests on a C141 aeroplane.
- 2 Variation of test-section parameters with stagnation temperature in air.
- 3 Continuous cryogenic tunnel.
- 4 Isentropic light piston tunnel.
- 5a Cryogenic isentropic light piston tunnel.
- 5b Cryogenic isentropic light piston tunnel (modified).
- 6 Effect of L/D ratio on final temperature achieved.
- 7 Effect of exhaust valve bore on final temperature achieved.
- 8 Effect of initial pressure on final temperature achieved.
- 9 Variation of expansion time with exhaust valve diameter.
- 10 Polytropic constants for different L/D ratios.
- 11 Experimental set-up for expansion experiments.
- 12 Typical pressure and temperature time record of an expansion experiment.
- 13 Temperature ratio vs pressure ratio

FIGURES contd.....

- 14 Temperature ratio vs pressure ratio in logarithmic co-ordinates.
- 15 Comparison of present work with literature.
- 16 Comparison of theory with experiments for large L/D ratios.
- 17 Final temperatures achieved for different L/D ratios and
 exhaust valve diameters.
- 18 Cryogenic piston tunnel.
- 19 Pressure-time history for an expansion process.
- 20a-c The variation of tube pressure with time under 'off matching'
 conditions.
- 21 x-t diagram for the cryogenic piston tunnel.
- 22 General arrangement of the pilot cryogenic tunnel.
- 23 Decrease in total pressure due to finite reservoir volume
 for different total pressures.
- 24 Photograph of aluminium piston.
- 25 Diagram of 'Nylatron' piston.
- 26a-c Tunnel static pressure traces illustrating piston
 oscillations.
- 27a Schematic to show method of valve opening.
- 27b A view of the nozzle valve.

FIGURES contd.....

- 28 A view of the contraction.
- 29 A view of the pilot cryogenic wind-tunnel.
- 30 Instrumentation and control system diagram.
- 31 'Ultraminiature' thermocouple needle.
- 32 A typical record showing the poor response of the thermocouple.
- 33a Tungsten wire resistance thermometer.
- 33b Tungsten wire resistance vs temperature.
- 34 Typical results of the pilot cryogenic tunnel.
- 35 Pressure ratio vs temperature ratio for the cryogenic
 tunnel.

PART II

FIGURE

- A1 Concept of a power spectral density analyser.
- B1 Osborne's criterion for shock-induced separation
 - critical value of the shock upstream Mach number
 for the occurrence of significant separation.
- C1 Blockage correction, 14% biconvex aerofoil at $\alpha = 0^\circ$.
- C2 Blockage correction, CAST 7 aerofoil at $\alpha = 0^\circ$.

FIGURES contd.....

- 1a Layout of the 2.5 inch transonic wind-tunnel.
- 1b A view of the transonic test-section.
- 1c A view of the perforated liner showing the entrance region.
- 2 Mach number distribution along transonic nozzle.
- 3 Centreline pressure distribution.
- 4 Aerofoils tested in the 2.5 inch transonic wind-tunnel.
- 5 Reynolds number vs Mach number.
- 6 Pressure port locations on 14% biconvex aerofoil.
- 7 Pressure transducer location.
- 8 Schlieren pictures of flow on a 6% thick biconvex aerofoil
(free-transition).
- 9 Schlieren pictures of flow on a 6% thick biconvex aerofoil
(fixed-transition).
- 10 Schlieren pictures of flow on a 14% thick biconvex aerofoil,
 $\alpha = 0^{\circ}$, (free-transition).
- 11 Schlieren pictures of flow on a 14% thick biconvex aerofoil
(c = 32 mm) at zero incidence (fixed-transition).

FIGURES contd.....

- 12 Schlieren pictures of flow on a 14% biconvex aerofoil
 at zero incidence, $c = 16$ mm (free-transition).

- 13 Schlieren pictures of flow on a 14% biconvex aerofoil
 ($c = 16$ mm) at zero incidence (fixed-transition).

- 14 Critical Mach number.

- 15 Shock position vs transonic similarity parameter.

- 16 High speed photographs of flow on a 14% biconvex aerofoil,
 $\alpha = 4^\circ$, $t = 160$ microsecs.

- 17 High speed photographs of flow on a 14% biconvex aerofoil,
 $\alpha = 4^\circ$, $t = 160$ microsecs.

- 18 Development of periodic flow.

- 19A Side-wall pressure traces, 14% biconvex aerofoil at zero
 incidence (free-transition).

- 19B Side-wall pressure traces, 14% biconvex aerofoil at zero
 incidence (fixed-transition).

- 20 Side-wall pressure traces (empty tunnel).

- 21 Side-wall pressure traces, 14% biconvex ($c = 16$ mm) aerofoil
 at zero incidence (fixed-transition).

- 22 Frequency parameter.

- 23 P.S.D. vs frequency for the empty tunnel.

FIGURES contd.....

- 24 P.S.D. vs frequency for flow on 14% biconvex aerofoil
(free-transition).
- 25 P.S.D. vs frequency for flow on 14% biconvex aerofoil
(fixed-transition).
- 26 P.S.D. vs frequency for the empty tunnel with perforations
sealed.
- 27 Resonance frequencies.
- 28 Magnitude of side-wall pressure fluctuations.
- 29 2.5 x 2.5 in. perforated tunnel - rms pressure fluctuations.
- 30 Tunnel unsteadiness, $\sqrt{NF(N)}$ vs $\log_e N$.
- 31 Pressure distribution on 14% biconvex aerofoil,
free-transition.
- 32 Local Mach number distribution on 14% biconvex aerofoil,
free-transition.
- 33 Pressure distribution on 14% biconvex aerofoil,
(fixed-transition).
- 34 Local Mach number distribution on 14% biconvex aerofoil,
(fixed-transition).
- 35 Oil flow photographs on 14% biconvex aerofoil, $\alpha = 0^\circ$,
(fixed transition).
- 36 Thickness chord ratio vs transonic similarity parameter.

FIGURES contd.....

- 37 Frequency parameter vs transonic similarity parameter.
- 38 Modified frequency parameter.
- 39 14% biconvex aerofoil configurations.
- 40 Schlieren pictures, 14% thick biconvex aerofoil with blunt trailing-edge ($c_1 = 0.976c$).
- 41 Schlieren pictures, 14% thick biconvex aerofoil with blunt trailing-edge ($c_1 = 0.825c$).
- 42 Schlieren pictures, 14% biconvex aerofoil with splitter plate ($c_1 = 0.826c$).
- 43 Schlieren pictures, 14% biconvex aerofoil with blunt leading edge ($c_1 = 0.825c$).
- 44 NACA 0012 at zero incidence (fixed-transition).
- 45 NACA 0012 at zero incidence (free-transition).
- 46 Schlieren pictures to illustrate shock oscillations on NACA 0012 aerofoil at zero incidence.
- 47 Shock position vs Mach number - low Reynolds number.
- 48 Shock position vs Mach number - high Reynolds number.
- 49 Frequency parameter.
- 50 Magnitude of side-wall pressure fluctuations.

FIGURES contd.....

- 51 Schlieren pictures and pressure traces.
NACA 0012, $\alpha = 3^\circ$ (fixed-transition).

- 52 Schlieren pictures and pressure traces.
NACA 0012, $\alpha = 3^\circ$ (free-transition).

- 53 NACA 0012 aerofoil, $\alpha = 6^\circ$ (fixed-transition).

- 54 NACA 0012 aerofoil, $\alpha = 6^\circ$ (free-transition).

- 55 C_L vs α curves.

- 56 Pressure distribution: Low and high Reynolds numbers.

- 57 Schlieren pictures and side-wall pressure traces.
CAST 7 aerofoil, $\alpha = 0^\circ$ (fixed-transition).

- 58 Pressure distribution.

- 59 CAST 7 aerofoil at zero incidence (free-transition).

- 60 CAST 7 aerofoil, $\alpha = 3^\circ$ (fixed-transition).

- 61 Pressure distribution.

- 62 CAST 7 aerofoil, $\alpha = 3^\circ$ (free-transition).
Side-wall pressure traces.

- 63 CAST 7 aerofoil, $\alpha = 6^\circ$ (fixed-transition).

- 64 CAST 7 aerofoil, $\alpha = 6^\circ$ (free-transition).

FIGURES contd.....

- 65 11.8% Joukowski aerofoil at zero incidence, fixed-transition.
Schlieren pictures and side-wall pressure traces.

- 66 Comparison of shock positions on aerofoils.

- 67 Comparison with theory.

- 68 11.8% Joukowski aerofoil at zero incidence, free-transition.
Schlieren pictures and side-wall pressure traces.

- 69 Side-wall pressure fluctuations on a 11.8% Joukowski
aerofoil at 3° incidence (fixed-transition).

- 70 Side-wall pressure fluctuations on a 11.8% Joukowski
aerofoil at 3° incidence (free-transition).

- 71 Side-wall pressure fluctuations on a 11.8% Joukowski
aerofoil at 6° incidence (fixed-transition).

- 72 Side-wall pressure fluctuations on a 11.8% Joukowski
aerofoil at 6° incidence (free-transition).

- 73 Frequency parameter vs Mach number.

- 74 Comparison of experiments with numerical computations
on 14% biconvex aerofoil at zero incidence.

- 75 Time history of shock-wave position.

- 76 Theory for aileron buzz.

- 77 Steady shock patterns for zero incidence and various
flap deflections.

FIGURES contd.....

- 78 Comparison of flows on aerofoil with oscillating flap
 and a 14% biconvex aerofoil.
- 79 Relation between shock position and shock strength.

PART I

DESIGN AND DEVELOPMENT OF AN INTERMITTENT
CRYOGENIC WIND-TUNNEL

PART INOTATION

a	Accoustic velocity
A	Cross sectional area of charge tube
A_a	C/s area of working section nozzle
A_e	C/s area of exhaust valve
A_r	C/s area of reservoir throat
D	Diameter of charge tube
h	Coefficient of convective heat transfer
L	Length of charge tube
m	Mass of test gas
$\dot{m}$	Mass flow rate through the nozzle
M_∞	Mach number
n	Polytropic constant
P	Pressure
p	Matching pressure
R	Gas constant
T	Temperature
U	Flow speed
$\bar{U}_p$	Piston velocity at matching conditions
V	Volume of charge tube
V_r	Reservoir volume
$\dot{V}$	Volumetric flow rate
t	Time
ρ	Density
μ	Viscosity

NOTATION contd.....

γ	Ratio of specific heats
M	Piston mass

Suffices

i	Initial conditions
f	Final conditions
o	Stagnation conditions
a	Conditions in a (Fig. 18)
b	Conditions in b (Fig. 18)
*	Critical conditions
r	Conditions in reservoir
$\bar{T}$	'Matching' value of T

1. INTRODUCTION

1.1 The Need for High Reynolds Number Testing

Flight tests on a transport and a fighter aeroplane have shown that a critical shock-wave boundary-layer interaction problem exists at high subsonic speeds (Ref. 1,2). Under turbulent-flow, low-Reynolds-number test conditions, the shock is located forward on the aerofoil surface with a large area of separation behind the shock. As the test Reynolds number is increased, the shock moves rearward reducing the area of separation and shifting the centre of pressure position rearwards. The effects of a change in location of the shock are significant and important for the prediction of aircraft performance at transonic speeds. Not only is the performance affected by the shock-wave boundary-layer interaction, but other characteristics such as loads and load distribution, stability, buffeting and maneuverability are also affected (Ref. 4). Although many devices for causing boundary-layer transition have been successful for simulating full scale conditions for two dimensional aerofoils, their applicability is questionable for accurate performance prediction of complete three dimensional configurations (Ref. 1).

Fig. 1 from Ref. 5 shows the large differences in pressure distribution which occur between low Reynolds number wind-tunnel test results and full scale flight data, for a C141 aeroplane. The differences in pressure distribution lead to large differences between measured and predicted airloads for the C141 aeroplane and the chordwise shift in the centre of pressure position was as much as 10%.

These factors as well as the current interest in the development of large transports to operate efficiently at transonic speeds, point to the need for wind-tunnel testing at increased Reynolds numbers (Refs. 3,4).

1.2 Methods for increasing the Reynolds Number

For a given Mach number the Reynolds number may be increased by:-

- (1) Using a heavy gas instead of air: Freon-12 with a density four times that of air is one of the most suitable of the heavy gases for use in a wind-tunnel. The Reynolds number can be increased by a factor of 3 over that with air as the medium, (at 288K) (Ref. 6). However, the ratio of specific-heats is different from that of air (1.13 for Freon-12 against 1.4 for air). This difference in specific-heat ratio is important when compressibility effects become significant, thus making Freon-12 a questionable test medium for transonic wind-tunnels. Chapman (Ref. 7) has reported that by using a mixture of gases in the correct proportion the specific-heat ratio can be made equal to 1.4. However, these gases are either too expensive or are toxic.
- (2) Increasing the size of the tunnel and model and/or increasing the stagnation pressure: these methods involve serious problems related to construction and operating costs and model and support loads.
- (3) Reducing the test temperature: In a theoretical investigation reported in 1945, Smelt (Ref. 8) pointed out that Reynolds number could be increased by the use of air at temperatures in the cryogenic range (150K or below). Not only is the Reynolds number increased but the power to operate the tunnel is reduced, thus making the use of low temperatures very attractive. Fig. 2 shows the benefits of using reduced temperatures. Expressions for the Reynolds number which show the influence of the various parameters are given in Appendix A. As the test temperature is reduced there is no change in dynamic pressure. Therefore by reducing temperature an increase in Reynolds number is achieved without any increase in aerodynamic loads except that introduced by the effect of Reynolds number on the aerodynamic coefficients. This

constant dynamic pressure feature of the cryogenic tunnel is important particularly with regard to isolating Reynolds number effects from aeroelastic effects. Also the Mach number effects can be separately studied. In addition, the use of a magnetic suspension system becomes attractive in a cryogenic wind-tunnel since the coil power needed is directly proportional to a power (approx. 3) of temperature.

1.3 Cryogenic Wind-Tunnels

One method of cooling air or nitrogen to cryogenic temperatures is to spray liquid nitrogen directly into the tunnel circuit (Fig. 3). One large continuous, cryogenic transonic tunnel using liquid nitrogen injection is being built at NASA Langley, U.S.A. and another tunnel is being planned for Western Europe. The specification of these two tunnels are shown in Fig. 3. These tunnels are obviously very expensive to build and quite costly to run. They are essentially for the aircraft industry and for applied research and development of new aircraft designs. For University research alternative wind-tunnel designs which are much simpler, cheaper and easier to operate are obviously needed for trying out new ideas and to undertake longer term research programmes. One device which satisfies the above requirements is the isentropic light piston tunnel.

1.4 Light Piston Tunnel

For measuring heat transfer rates at transonic speeds on gas turbine blades, Schultz and his co-workers at Oxford (Refs. 10,11) proposed and built an isentropic light piston tunnel (Fig. 4). To measure the heat transfer rate on gas turbine blades a hot, low-Reynolds number flow is required, (eg. $T_0 = 430K$, $Re = 1.2 \times 10^6$) for a short duration. This is obtained as follows:- opening the valve between the reservoir and the charge tube (Fig. 4) allows the high pressure air in the reservoir to drive the light piston down the tube so compressing and heating the test gas almost isentropically to the required conditions. Once the required conditions are achieved and with the piston still moving, the nozzle diaphragm (or valve) is opened. The test gas is

pushed out through the test-section past the model and into the dump tank. By matching the volumetric flow rates entering and leaving the tube, the piston motion is tuned such that pressure can be maintained constant during the running time.

Stollery & Murthy (Ref. 12) suggested that the oxford type of tunnel could be operated in reverse in order to generate low-temperature, high-Reynolds-number flows instead of the high temperature, low-Reynolds-number conditions already produced. A scheme for the cryogenic light piston tunnel is shown in Fig. 5a. Here the light piston is used to expand the gas isentropically to achieve the cryogenic reservoir conditions. A modification of the configuration Fig. 5a is shown in Fig. 5b. In this simpler scheme the piston is used only to push the test gas through the test-section and the expansion is performed separately with the use of valve 1. When the high pressure gas in 'A' has expanded and cooled to the required conditions, the nozzle valve and valve 2 are opened simultaneously, the piston set in motion and tuned to keep the pressure constant during the test time.

1.5 The Present Work

The present work describes the design and development of a pilot intermittent cryogenic wind-tunnel (Fig. 5b). The parameters which are important for the generation of low temperatures are discussed in Chapter 2 and calculations are presented for determining the temperatures achievable in a high pressure gas expansion. Expansion experiments are also described and the results compared with theory. It is shown that temperatures in the cryogenic range are achievable in a high pressure gas expansion. The equations governing the operation of a cryogenic light piston tunnel are described in Chapter 3. The design details of a pilot tunnel with a test-section 2.86 cm x 2.86 cm (1.125 inch x 1.125 inch) and running time of 0.3 seconds, are given in Chapter 4. The experimental results from the pilot tunnel are presented in Chapter 5.

2. TEMPERATURES ACHIEVABLE BY THE EXPANSION OF HIGH PRESSURE GAS

The expansion process in the ideal case is isentropic. To achieve reservoir conditions of $T_0 = 120\text{K}$ starting with test gas at ambient temperature, the ideal pressure ratio through which the test gas has to expand is 25. The departures from this ideal situation, caused by heat transfer from the walls of the charge tube, could be significant, causing the required pressure ratio to rise and reducing the efficiency of the process. Taking the same reservoir conditions as before the required pressure ratios for varying polytropic constants are:-

Polytropic Constant	Pressure Ratio
1.4	25
1.3	53
1.2	244
1.15	1124

Hence if the process is to be efficient, then the polytropic constant should be as close as possible to the ideal value.

2.1 Calculation of Temperatures achievable by the expansion of High Pressure Gas

The main parameters which determine the magnitude of the heat transfer from the walls of the charge tube are:-

- (1) The surface area of the charge tube; the lower the surface area, the nearer the expansion process will be to the ideal. Hence the use of a tube of low length to diameter ratio (L/D) is of fundamental importance.
- (2) The rate of expansion: this is controlled by the bore of the exhaust valve (Valve 1 in Fig. 5). The faster the expansion the lower the final temperature achieved for a given pressure ratio.

- (3) The initial pressure of the gas: the heat transfer rate is increased at higher pressures due to the greater density. However this will not be very significant since high expansion rates are contemplated.

The details of the theoretical model to calculate the temperature after expansion is given in Appendix B. The theoretical model assumes fully developed turbulent pipe flow and uses an empirical heat transfer relation for forced convection, i.e.

$$Nu = 0.0265 (Re_D)^{0.8} (Pr)^{0.3} \dots\dots (1)$$

The results of the calculations are shown in Figs. 6 to 10. Fig. 6 shows the importance of using a tube with a low aspect ratio (5 or less) to achieve the lowest temperature possible for a given pressure ratio. Fig. 7 shows the effect of the bore of the exhaust valve on the final temperatures achieved. The effect of the valve size is significant but not as much as the length to diameter ratio. Fig. 8 shows the effect of the initial pressure on the final temperature. The effect is insignificant (note the exaggerated temperature ordinate). The time needed for expansion is nearly independent of aspect ratio and is almost totally controlled by the bore of the exhaust valve (Fig. 9). Fig. 10 shows the polytropic constants obtained for tubes of various L/D ratios and for different pressure ratios. This again illustrates the superiority of the short, fat tube. With increase in pressure ratio the polytropic constants are lower because the expansion times are greater, for constant bore exhaust valve.

From the results of the calculations presented above, it is clear that deviations from the ideal, isentropic expansion are significant but can be minimised by the proper choice of dimensions for the charge tube.

The experiments to support these results are described in the next section.

2.2 Expansion Experiments

The expansion experiments were performed using a seamless steel cylinder rated for 300 bars (Fig. 11b). The cylinder has a length to diameter ratio of approximately 5. The cylinder is fitted with a fast acting valve (of bore 3.8 cm) at one end. The port at the other end is used for instrumentation purposes.

A typical experiment consists of charging the cylinder with nitrogen gas to the required pressure. The fast acting valve is then opened to atmosphere and the temperature and pressure of the gas in the cylinder are recorded as a function of time.

A copper-constantan micro thermocouple (Fig. 11a) was used for the temperature measurements. The wires were typically 20 microns diameter with a small response time (Refs. 15,16). The thermocouple was calibrated against a calibrated platinum resistance thermometer. The thermocouple and the resistance thermometer were mounted side by side in a dewar flask containing liquid nitrogen. By moving from the surface of the liquid to the atmosphere above the dewar, temperatures ranging from the boiling point of nitrogen to room temperature were obtained. Before the experiments, checks were made using liquid nitrogen and measuring the millivolts output of the thermocouple with a digital voltmeter and comparing with the NBS reference tables (Ref. 19). In the measurements the thermocouple output was amplified using a DC amplifier and then fed to a UV recorder.

Pressure measurements were made using a 0-2000 psi 'Druck'* diaphragm type pressure transducer. The output signal of the pressure transducer was amplified by a strain gauge bridge amplifier before being fed to a UV recorder.

* Druck Ltd., Fir Tree Lane, Groby, Leics. LE6 0FH.

2.3 Experimental Results and comparison with Theory

The principal dimensions of the expansion tube as well as the range of pressures covered are shown in Table 1. A typical pressure-time and temperature-time record are shown in Fig. 12.

TABLE 1

Length/Diameter Ratio	5
Diameter	23.5 cm
Range of Initial Pressures	10 to 93 bars
Final Pressure	1 bar
Temperatures achieved	160K to 93K

Fig. 13 shows the experimental results plotted as pressure ratio vs temperature ratio and compared with the isentropic case and polytropic expansion with $n = 1.3$. The experimental points lie within these two limits. In order to obtain a more accurate estimate of the polytropic exponents, the results were plotted on a log-log scale for each experiment (Fig. 14). The results show that the polytropic exponent ' n ' lies between 1.33 and 1.38. The theoretical model predicts a value of ' n ' of 1.341 to 1.373 for the configuration used in the experiments (Fig. 10).

The theoretical model is thus in good agreement with experiments for a charge tube with an L/D ratio of 5.

Fig. 15 is a comparison of our results with others reported in the literature (Refs. 17,18). The results of Bellet (Ref. 18) are for a charge tube of L/D ratio 4.4, but the expansion times are much longer (20 to 40 secs compared to 1.5 secs for our experiments at the highest pressure ratio). Hence the polytropic constant obtained ($n = 1.25$) is lower than that obtained in our experiments. The

results of Hutt & East (Ref. 17) give a value of $n = 1.3$ for a tube of low L/D ratio. For large L/D ratios the results of Ref. 17 show that the theoretical model fails to predict the temperature achieved (Fig. 16). To achieve matching between theory and experiment it becomes necessary to increase the convective heat transfer coefficient by a factor of 2.5,

$$\text{i.e. } Nu = 0.0663 (Re_D)^{0.8} (Pr)^{0.3} \text{ for large } (L/D) \text{ tubes}$$

..... (1a)

Using the values of the heat transfer coefficient for tubes of small L/D ratio (Eqn. 1) and large L/D ratio (Eqn. 1a), calculations are made to determine the temperature achieved after expansion. The results are shown in Fig. 17. To achieve reservoir conditions of $T_o = 120K$, a pressure ratio of 35 would be needed for a charge tube of L/D ratio ≤ 5 .

3. CRYOGENIC PISTON TUNNEL

3.1 Principle of Operation

The principle of the device may be outlined with reference to Fig. 18. The system consists of a medium pressure reservoir connected via a throat and a valve to the charge tube. The other end of the charge tube is connected to a working section nozzle through a nozzle valve. The nozzle exhausts into a dump tank. An exhaust valve is also provided at the end of the charge tube. A smooth piston runs in the tube, starting its travel at the reservoir end and is driven by the gas entering from the reservoir. High pressure test gas is contained in the charge tube and expanded by means of the exhaust valve. When the test gas has been expanded and hence cooled to the required temperature, it is 'pushed out' through the test-section. The pressure within the tube may be maintained constant if the volumetric flow rate of gas into the tube from the medium pressure reservoir is the same as that leaving through the working section. The gas is in equilibrium at all stages, times being large compared to the transit time of sound waves along the tube. The piston is considered merely as a 'divider' between the cold test gas and that coming from the medium pressure reservoir and the inertia of the piston is small. The reservoir drives the piston and provides constant pressure conditions during the run time. When the gas flows through the working section under matched conditions, the work done in ejecting the gas through the nozzle equals the work done by the reservoir and conditions within the test gas remain constant.

3.2 The Expansion Process

The temperatures achievable in an expansion process have already been treated in Chapter 2. The variation in pressure with time measured from the instant of exhaust valve opening (assuming isentropic expansion) is given by

$$P = \left(K \frac{t}{t_o} + 1\right)^{2\gamma/(1-\gamma)} P_i \quad \dots\dots (2)$$

where t_o is arbitrarily taken to be the time required to expand the gas to test conditions ($t/t_o = 1$ at the end of the expansion) (Ref. 17).

The constant K is given by,

$$K = A_e \left(\frac{\gamma-1}{2}\right) \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1}\right) \frac{\gamma+1}{\gamma-1}} \frac{P_i}{m_i \sqrt{T_i}} t_o \quad \dots\dots (3)$$

Fig. 19 is a plot of pressure time history for an expansion process obtained using equations 2 and 3 and compared with the theoretical model (Chapter 2) and an expansion experiment. In the experiment the opening of the valve is not instantaneous; hence the initial drop in pressure is not as rapid as shown by theory. The experiment gives an expansion time of 1.20 secs in close agreement with the the theoretical model which gives an expansion time of 1.27 secs. The differences between these two results and isentropic expansion is attributed to the effects of heat transfer from the charge tube to the gas.

3.3 Matching Conditions

The volumetric flow rate from the reservoir into the charge tube needed to maintain constant pressure during a run, may be found from the condition that the volumetric flow rate into the tube from the reservoir must equal that flowing out through the working section nozzle. Assuming a sonic working section nozzle, the volumetric flow rate out of the nozzle is given by,

$$\dot{V}_a = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} A_a^* a_a \quad \dots\dots (4)$$

The volumetric flow rate entering the charge tube from the reservoir is given by,

$$\dot{V}_b = \frac{\dot{m}_r RT_r}{P_b} \quad \dots\dots (5)$$

The assumption is made here that the reservoir gas is brought to rest on entering the tube and hence recovers the reservoir temperature (Ref. 11).

The 'matching condition' is,

$$\dot{V}_a = \dot{V}_b$$

$$\text{and this occurs for } P_a = P_b = \bar{P} \quad \dots\dots (6)$$

$$\therefore \dot{m}_r RT_r = \bar{P} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} A_a^* a_a \quad \dots\dots (7)$$

Assuming the reservoir throat is choked,

$$\begin{aligned} \dot{m}_r &= A_r^* \rho_r^* a_r^* \\ &= \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} A_r^* \rho_r a_r \quad \dots\dots (8) \end{aligned}$$

substituting (8) in (7) and eliminating ρ_r and T_r by the use of the relation for acoustic velocity, we get,

$$P_r A_r^* a_r = \bar{P} A_a^* a_a \quad \dots\dots (9)$$

From this relation, the reservoir throat can be sized for required tunnel stagnation conditions and available reservoir pressure.

The piston velocity at matching conditions is given by,

$$\bar{U}_p = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} a_a \cdot A_a^*/A \quad \dots\dots (10)$$

3.4 Piston Oscillations

The main effect of a finite piston mass is that the piston does not respond instantaneously to gas velocity changes when the valves are opened. The variation of velocity of a massless piston with time is shown by the solid line (Fig. 18c) where the piston instantaneously assumes a velocity determined by the flow of reservoir gas. If the piston had a finite mass its velocity would vary as shown by the dotted line, performing oscillations around the previous curve. The pressure fluctuations caused by the velocity oscillations are also shown by the dotted line on the pressure curve (Fig. 18c). An analysis has been made in Ref. 10 to determine the time period of these piston oscillations. The period of oscillation of the piston is given by,

$$T = 2\pi \left(\frac{V_a V_b}{V A^2} \right)^{\frac{1}{2}} \left(\frac{M}{P} \right)^{\frac{1}{2}} \quad \dots\dots (11)$$

where $V_a + V_b = V$, the volume of the charge tube (Fig. 18).

This equation gives the period of oscillation of the piston for any given position during the piston motion. However, the oscillations occurring when the valves are opened cannot be determined from the above equation since the piston starts impulsively from rest. Equation 11 indicates the importance of a low piston mass to keep the period of oscillation as small as possible.

3.5 Departures from Ideal Performance

When the tube is operated at 'matching' conditions there is no rise or fall in pressure during the tunnel running time. A change in pressure during the running time could occur due to the reservoir not being set to the correct pressure or due to the reservoir volume being finite resulting in a loss of reservoir pressure during the run. These two cases have been analysed in Ref. 10 and are discussed below:-

Case 1: For a finite reservoir driving a tube which is 'matched' at the instant of valve opening but subsequently falls below matching, the change in tube pressure follows the change in 'matching' pressure with a delay as shown in Fig. 20b (Ref. 10).

The variation of the change in tube pressure p with time is given by

$$p = -K_1(t - \xi) - K_1 \xi e^{-t/\xi} \quad \dots\dots (12)$$

where

$$K_1 = \gamma \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} A_r^* a_r \bar{p} / V_r$$

and

$$\xi = V / \left[\left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} A_a^* a_a \left(\frac{3\gamma - 1}{2} \right) \right]$$

The change in matching pressure is given by

$$\bar{p} = -K_1 t \quad \dots\dots (13)$$

Case 2: Suppose an infinite reservoir is used, but conditions are not matched at the instant of valve opening, then the change in tube pressure p is given by

$$p = \bar{p} (1 - e^{-t/\xi}) \quad \dots\dots (14)$$

The variation of pressure p with time is shown in Fig. 20a.

3.6 Transit Time Effects

The expansion process has been considered to occur instantaneously, the pressure being related to the volume by the isentropic relation. Actually, the change in pressure within the gas is due to compression or rarefaction waves which travel through the gas transmitting information about varying conditions at the ends of the tube. The expansion process is obtained as a rarefaction wave, generated when the exhaust valve is opened (Fig. 18), reflecting repeatedly from the end of the tube and the face of the stationary piston causing the pressure to vary stepwise, with an average pressure as in Fig. 21.

A wave diagram demonstrating this process is shown in Fig. 21a. The form of the variation in pressure is determined by the gas velocity at the exhaust end. The amplitude of the final rarefaction wave that is generated (i.e. at the end of expansion) is given by

$$\frac{\Delta P}{P} = \frac{\gamma U}{A_a} \quad \dots\dots (15)$$

where U is the gas velocity in the chamber. (The process is similar to an expansion process using a piston, U being equivalent to the piston velocity). Since the charge tube diameter is much larger than the exhaust valve diameter, the gas velocity in the tube would be very small, especially at the end of the expansion. Hence the magnitude of the disturbance given by eqn. (15) would be small. When the nozzle valve is opened a rarefaction wave is generated which

proceeds as described above. When the reservoir valve is opened, the piston is driven which generates a compression wave. If the tube is matched the rarefaction wave exactly cancels out the compression wave and the net change in pressure is afterwards zero.

The amplitude of the rarefaction wave produced when the nozzle valve is opened is given by the acoustic approximation (Ref. 10,20) as

$$\frac{\Delta p}{\bar{p}} = \frac{\gamma \bar{U}_p}{a_a} \quad \dots\dots (16)$$

where $\bar{U}_p$ is the piston velocity at matching conditions, given by eqn. 10.

$$\therefore \frac{\Delta p}{\bar{p}} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} \frac{A_a^*}{A} \quad \dots\dots (17)$$

There is pressure doubling because the rarefaction wave is reflected from the piston face back to the end wall. Therefore the resulting change in nozzle stagnation pressure is given by

$$\frac{\Delta p}{\bar{p}} = 2 \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} \frac{A_a^*}{A} \quad \dots\dots (18)$$

This is a conservative estimate since the piston starts from rest. Thus to keep this pressure disturbance to a minimum the area of the charge tube must be large in comparison with the nozzle area.

3.7 Sizing of Reservoir

The reservoir size may be found by using the analysis of Section 3.5 where the change in pressure from 'matching' conditions due to a finite reservoir volume was given by equation (12). For small changes in p , equation (12) may be approximated as (from Ref. 10)

$$p = \frac{-K_1 t_{\text{run}}^2}{2\xi} \quad \dots\dots (19)$$

where p is the pressure drop during the running time t_{run} . The running time is given by

$$t_{\text{run}} = \frac{\text{mass of test gas initially in charge tube}}{\text{mass flow rate through nozzle}}$$

$$= \rho_i V / \left[\left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} A_a^* \bar{a}_a \rho_i \right] \quad \dots\dots (20)$$

Using the expressions for K_1 , ξ , t_{run} and the matching conditions (eqn. 9), we get

$$\frac{p}{\bar{p}} = \frac{\gamma(3\gamma - 1)}{4} \left(\frac{\bar{p}}{p_r} \right) \left(\frac{V}{V_r} \right) \quad \dots\dots (21)$$

Equation (22) enables the required reservoir volume V_r to be determined for a certain tube pressure drop within the running time.

4. DESIGN OF A PILOT CRYOGENIC WIND-TUNNEL

4.1 Specification

It was decided to construct a small scale pilot tunnel to test the ideas given in the previous sections. It was also decided to use some components already available in the laboratory, for example a 2.86 cm x 2.86 cm (1.125 inch x 1.125 inch) test-section belonging to a compression shock tube. A run duration of 0.3 secs was thought to be long enough for making measurements. The specification for the pilot cryogenic wind-tunnel was:-

Test-section	2.86 cm x 2.86 cm (1.125 inch x 1.125 inch)
Stagnation pressure	1 bar
Stagnation temperature	120K
Running time	0.30 secs
Reynolds number (based on chord = H/2)	0.6×10^6
Mach number	Transonic

The stagnation temperature of 120K was chosen as the lowest consistent with the avoidance of condensation.

The general arrangement of the pilot tunnel is shown in Fig. 22.

4.2 Design Details

4.2.1 Charge Tube

Based on eqn. 20 (Section 3.7) the volume of the charge tube for a 0.3 sec. run time at a Mach number of unity, is 0.0279m^3 . The need to keep the heat losses to a minimum limits the length to diameter ratio of the tube to 5 (Chapter 2). With this requirement, the diameter of the charge tube would be 0.1954 m (7.694 inches).

The need to keep the disturbances small when the valves are open requires the charge tube area to be large in comparison with the nozzle area (Section 3.6). For an allowable variation of 5% of matching pressure during the run, the charge tube area must be 32 times the nozzle area (Eqn. 17, Section 3.6). This sets the minimum diameter of the charge tube to be 0.1834 m (7.22 inches). Choosing the larger of the 2 values, gives a value of 0.1956 m (7.7 inches). A stainless steel tube, bored and honed to an internal diameter of 7.7 inches and finished to an accuracy of 0.001 inches (25 microns) on radius, was used for the charge tube. The circularity and the smooth finish are considered essential for the attainment of constant flow properties (Ref. 10). The tube has a thickness of 12 mm (0.5 inch) and can withstand a maximum pressure of 100 bars.

4.2.2 Reservoir and Reservoir Throat

A tube 0.23 m dia and length 1.25 m, which formed part of an existing compression shock tube was utilised to serve as a reservoir for the cryogenic tunnel. This reservoir was connected to a bank of nitrogen cylinders through a manifold. It was estimated that this reservoir together with the bank of nitrogen cylinders supplying continuously would give the required reservoir volumetric flow rate. However, the pressure regulators which were used with the nitrogen gas bottles had very small nozzles which restricted the flow out of the bottles. Initial tests showed that the reservoir pressure dropped considerably during the running time. Hence an additional chamber 0.4 m dia and 2.16 m length was added on to the existing reservoir. This helped maintain the volumetric flow rate during the running time. The maximum pressure to which these reservoirs could be stressed was limited to 7 bars. Based on this limit, two reservoir throats 1.459 cm and 1.032 cm dia were manufactured for use at pressure of 3 bars and 6 bars respectively (Eqn. 9, Section 3.3). With these 2 values of reservoir pressure, the pressure drop during the running time due to a finite reservoir as given by eqn. 21 (Section 3.7) is equal to 3.2 and 1.8% of total pressure, respectively (Fig. 23). These values are within acceptable limits.

4.2.3 Piston Design

The piston plays an important part in the overall performance of the tunnel. Ideally it should be of the minimum mass so that it acts as a barrier between the reservoir gas and the cold test gas in the charge tube. It should move freely in the bore of the tube and provide a good seal between the two gases. The piston must be rigid enough to withstand the initial high pressure in the charge tube and the stresses which arise when it strikes the end of the charge tube.

A light high strength alloy of aluminium was used for making the piston. This piston has a face plate 3.8 mm (0.15 inch) thick, strengthened by 6 ribs at the rear (Fig. 24). Two PTFE rings provided the seal. A ring of cork was stuck onto the face of the piston to reduce the impact when the piston hit the face of the flange at the end of the run. This piston was used for the initial experiments. However, after several tens of runs it was found that the face of the piston was dished and the ribs had developed cracks. Hence another piston was made, this one of 'nylatron' (Fig. 25). This piston has a face thickness of 3.43 cm and rim thickness of 1.25 cm and was deliberately designed with a larger factor of safety to discount any possibility of piston damage again (Fig. 25). However, later experiments showed that the piston developed oscillations which resulted in a pressure fluctuation of amplitude approximately 3% of total pressure (Fig. 26b) and a frequency of 18 cps. Calculation of the frequency using eqn. 11 (Section 3.4) gives an average frequency of about 21 cps in fair agreement with experiments and indicating that the piston oscillations were due to piston mass. To prove that this was indeed the case, the piston mass was increased by 0.45 kgs and experiments repeated. The frequency dropped to 14 cps (Fig. 26a) as also indicated by eqn. 11 ($f = 17$ cps), thus confirming that the piston mass was responsible for the oscillations. The piston mass was then reduced from 1.97 kgs to 1.55 kgs by machining off a portion of the piston face. The level of pressure fluctuation dropped considerably as can be seen from Fig. 26c.

4.2.4 Fast Acting Valves

The expansion of the high pressure gas must be rapid in order to accomplish the necessary rate of expansion for cooling the gas (Section 2.1). A 5 cm 'Worcester' fast acting ball valve (bore 3.81 cm) was chosen for the exhausting operation. The opening of the valve is accomplished manually whereas the closing of the exhaust port when the expansion has taken place is accomplished automatically using the nozzle valve and relay circuit (see later sections).

The accurate timing of the reservoir valve and the nozzle valve is necessary for the maintenance of constant test conditions during the running time. A schematic diagram of the technique chosen to open the valve is shown in Fig. 27a. The drop in pressure during the expansion is measured with a 0-500 psi 'Solartron' pressure transducer located in the charge tube. When the output from its associated amplifier falls to the required value, a relay (Section 4.3) switches over rapidly and de-energises the solenoid valve thereby exhausting one side of the pneumatic cylinder. Thus the ram moves to the right, opening the nozzle valve and closing the exhaust port in approximately 30 msec. The reservoir valve operates similarly but in this case there is only one port to be uncovered. The opening of the reservoir valve may be delayed or advanced with respect to the opening of the nozzle valve by the use of the relay circuit. The operating time of the two valves can also be controlled by means of the pressure applied to the pneumatic cylinder. Usually the pressure of the gas in the cylinder is set to 5 to 7 bars. Linear transducers are mechanically coupled to the valves and allow each valve position to be continuously monitored on the UV recorder. PTFE seats are used on the valves for sealing. Fig. 27b is a photograph of the nozzle valve.

4.2.5 Contraction

This is basically a sonic nozzle and is illustrated in Fig. 28. The area ratio is 6:1. The nozzle plates have been designed so that they can be easily dismantled and replaced with a supersonic nozzle if required.

4.2.6 Test-Section

The test-section is 2.86 cm x 2.86 cm in cross section and has solid walls. It is provided with 2 circular windows 2.5 cm dia. which can be used as instrumentation ports or for flow visualisation. A pressure transducer ('Druck' 0-15 psi differential silicon strain gauge type) is flush mounted on the side-wall to measure the test-section static pressure. A straight pipe with an area of 2.5 times the test-section area is used as a diffuser. A 5 cm gate valve is used as a second throat to vary the Mach number.

4.2.7 Dump Chamber

A stainless steel chamber of capacity 0.21 cu.m coupled with a portion of an existing 18 inch shock tube of capacity 0.63 cu.m was used as a dump chamber for the pilot tunnel. The dump chamber is evacuated to a low pressure by the use of a 0.65 kw electrically driven vacuum pump. A 'Pirani' vacuum gauge is used to measure the pressure in the chamber.

4.3 Instrumentation and Control System

The locations of the transducers are indicated in Fig. 22. The instrumentation and control system diagram is shown in Fig. 30. The operation of the relays is as follows:- the nozzle valve operating relay is a voltage comparator transistor device with a variable delay facility included. The input of this relay is connected to the amplified output of the charge tube pressure transducer. When the voltage level of this output falls to the required value, the relay

switches over rapidly and operates the solenoid valves. The accurate and rapid switching is achieved by an internal 'Schmitt trigger' device integral with the relay.

The reservoir valve relay is a slave to the nozzle valve relay and may be delayed in operation with respect to this relay. The circuit may also be arranged such that the nozzle valve relay is a slave to the reservoir valve relay. The predetermined delay time can be varied from 0 to 5 seconds.

4.4 Operation of the Pilot Tunnel

The exhaust valve is manually opened to commence the run sequence. When the chamber pressure falls to the desired level (which is predetermined depending on the stagnation pressure), the nozzle valve relay switches over and de-energises the solenoid valve of the pneumatic cylinder thus allowing the nozzle valve to open. The reservoir valve would then operate in the same way at the predetermined delay interval.

4.5 Temperature Measurement

The temperature measurements were initially made using a copper-constantan 'ultraminiature' thermocouple needle (Fig. 31). The wire is typically 25 microns dia. and is encased in a hypodermic tube with quartz as insulator. However, initial tests showed that the thermocouple did not faithfully follow the temperature of the gas, as can be seen in Fig. 32, probably because the response of the thermocouple was inadequate due to thermal inertia and heat transfer from the support. It was suggested by Hutt (Ref. 21) that a fine tungsten resistance wire had the necessary response to follow rapid gas temperature changes in a cold environment. Hence a 7.5 micron dia. tungsten wire resistance thermometer was constructed. Fig. 33a shows the wire resistance thermometer and its mounting and Fig. 33b shows the variation of resistance with temperature obtained using the resistivity data for tungsten (Ref. 22). The resistance thermometer was calibrated against a copper-constantan thermocouple using liquid nitrogen. The output of the bridge circuit (Fig. 30) was amplified by a factor of 1000

before being recorded on the UV recorder. The tungsten wire resistance thermometer successfully followed the gas temperature changes as can be seen from Fig. 34.

5. EXPERIMENTAL RESULTS AND DISCUSSION

Experiments were conducted with different pressure ratios to determine the final temperatures achieved and the flow conditions during the running time of the tunnel. The results are tabulated in Table 2. The final temperatures achieved are plotted in Fig. 35 and compared with the isentropic values. The expansion process is polytropic with an average value of the polytropic constant of 1.35.

Typical traces obtained with the pilot tunnel are shown in Fig. 34. The expansion and running times are typically 0.65 secs and 0.3 secs respectively. The valves open in 40-50 msec. and in this case the delay circuit has been set so that the reservoir valve opens 10 msec. after the nozzle valve has opened. This has been done to reduce the magnitude of the disturbance when the reservoir valve opens. The test-section static pressure shows a variation during the running time of 1.4% of the total pressure. This is considered to be within acceptable limits. The output of the resistance thermometer shows that the gas temperature is higher at the beginning of the run than at the end. The reason for this behaviour is not clear. During the running time, the temperature varies by about 10 to 15K. Temperatures are repeatable to within ± 5 K. The stagnation temperatures reported have not been corrected for the recovery factor of the probe. However, the recovery factor for a fine wire probe would be close to 1 (Ref. 24).

6. CONCLUSIONS

Experiments in a pilot intermittent cryogenic wind-tunnel have shown that,

- (1) Temperatures in the cryogenic range ($T_0 \leq 150K$) can be achieved with pressure ratios of 15 or greater with a tube of length to diameter ratio of 5.
- (2) The expansion process is polytropic with an average value of polytropic constant of 1.35.
- (3) The low temperatures obtained are repeatable to within $\pm 5K$.
- (4) Temperature variations are of the order of 10 to 15K during the running time of 0.3 secs.

REFERENCES

1. BAALS, D.D. & STOKES, G.M. A facility concept for high Reynolds number testing at transonic speeds. Paper 28, AGARD Conference proceedings No. 83, April 1971.
2. HEPPE, R.R., O'LAUGHLIN, B.D. & CELNIKER, L. New aeronautical facilities - we need them now. Astronautics & Aeronautics, Vol. 6, No. 3, March 1968, pp. 42-54.
3. JONES, J.L. Problems of flow simulation in wind-tunnels. AIAA preprint Paper No. 69-660, June 1969.
4. KILGORE, R.A. et al The cryogenic wind-tunnel concept for high Reynolds number testing. NASA TN D-7762, Nov. 1974.
5. LOVING, D.L. Wind-tunnel-flight correlation of shock-induced separated flow. NASA TN D-3580, Sept. 1966.
6. TREON, S.L., HOFSETTER, W.R. & ABBOTT, F.T. On the use of Freon 12 for increasing Reynolds number in wind-tunnel testing of three dimensional aircraft models at subcritical and supercritical Mach numbers, Paper 27, AGARD conference proceedings No. 83, April 1971.
7. CHAPMAN, D.R. Some possibility of using gas mixtures other than air in aerodynamic research. NACA TN 3226, 1954.
8. SMELT, R. Power economy in high speed wind-tunnels by choice of working fluid and temperature. RAE report Aero 2081, 1945.
9. HILSENATH, J. & BECKETT, C.W. Tables of thermal properties of gases. NBS. Circ. 564, U.S. Dept. of Comm. 1955.
10. JONES, T.V., SCHULTZ, D.L. & HENDLEY, A.D. On the flow in an isentropic light piston tunnel. R & M 3731, 1973.
11. SCHULTZ, D.L. et al A new transient cascade facility for the measurement of heat transfer rates. Paper 31, AGARD conference proceedings No. 229, July 1977.

REFERENCES contd.....

12. STOLLERY, J.L. & MURTHY, A.V. An intermittent high Reynolds number wind-tunnel.
Aero. quar., Vol. 28. Nov. 1977.
13. BRITCHER, C.P. & GOODYER, M.J. The Southampton University magnetic suspension/cryogenic wind-tunnel facility.
Paper 10, Proceedings of the first international symposium on cryogenic wind-tunnels, April 1979.
14. EDMUNDSON, I.C. Unpublished work, 1979.
15. - High temperature instruments.
Bulletin 819 TN, Robinson Halpern Co., Pa, U.S.A.
16. CARBON, M.W., KUTSCH, H.J. & HAWKINS, G.A. The response of thermocouples to rapid gas temperature changes.
Transactions ASME, Vol. 72, pp. 655, July 1950.
17. HUTT, G.R. & EAST, R.A. Preliminary studies of a free-piston expander for an intermittent cryogenic wind-tunnel.
Proceedings, First international symposium on cryogenic wind-tunnels, Paper No. 8, April 1979.
18. BELLET, J.C. & MANSON, N. Contribution a l'etude de l'air dans un reservoir.
Publications scientifiques et techniques du ministere de l'air, 1962.
19. SCOTT, R.B. The calibration of thermocouples at low temperatures in 'Temperature, its measurement and control in industry.'
Vol. 1, pp. 206, Reinhold publishing corporation, 1941.
20. LEIPMAN, H.W. & ROSHKO, A. Elements of gas dynamics.
Wiley & Sons, 1957.
21. HUTT, G.R. Private communication, 6 Dec. 1982.
22. KAYE, G.W.C. & LABY, T.H. Tables of physical and chemical constants.
Longmans, 1959.

REFERENCES contd.....

23. TUTTLE, M.H. & KILGORE, R.A. Cryogenic wind-tunnels - a selected, annotated bibliography. NASA TM 84474, Sept. 1982.
24. MEIER, H.U. Measuring techniques for compressible turbulent boundary-layers. DLR-FB 77-49, 1977.

APPENDIX AExpressions for Reynolds Number

$$\text{Reynolds number } Re = \frac{\rho U_{\infty} l}{\mu} \quad \dots\dots (1)$$

where ρ = density of the medium
 U_{∞} = flow speed
 l = characteristic length
 μ = viscosity of the medium

$$\rho = \frac{p}{RT} \quad \dots\dots (2)$$

where p = static pressure
 T = static temperature
 R = gas constant

$$\text{and } U_{\infty} = M_{\infty} a \quad \dots\dots (3)$$

where a = acoustic velocity

$$= \sqrt{\gamma RT}$$

γ = ratio of specific heats

$$\therefore Re = \frac{\rho M_{\infty} l}{\mu} \sqrt{\frac{\gamma}{RT}} \quad \dots\dots (4)$$

Using isentropic relations,

$$\frac{T}{T_0} = \frac{1}{1 + \frac{\gamma - 1}{2} M_{\infty}^2} \quad \dots\dots (5)$$

$$\text{and } \frac{p}{P_o} = \left(\frac{1}{1 + \frac{\gamma - 1}{2} M^2} \right)^{\frac{\gamma}{\gamma - 1}}$$

where P_o and T_o are the stagnation pressure and temperature respectively.

$$\therefore Re \propto \frac{P_o}{\mu} \sqrt{\frac{1}{T_o}} \quad \text{for a given gas and Mach number} \quad \dots\dots (6)$$

$$\mu \propto \mu_o \propto T_o^{0.9} \quad \dots\dots (7)$$

$$\therefore Re \propto P_o / T_o^{1.4} \quad \dots\dots (8)$$

$$\text{Dynamic pressure } q = \frac{1}{2} \rho U_\infty^2$$

$$\therefore q \propto P_o \quad \text{for a given gas and Mach number} \quad \dots\dots (9)$$

Power to operate the wind-tunnel

$$= \eta q U_\infty A$$

where η is the power factor

A is the test-section area

$$\therefore \text{Power} \propto P_o \sqrt{T_o} \quad \text{for a given gas and Mach number} \quad \dots\dots (10)$$

APPENDIX B

The Calculation of Temperatures achieved by

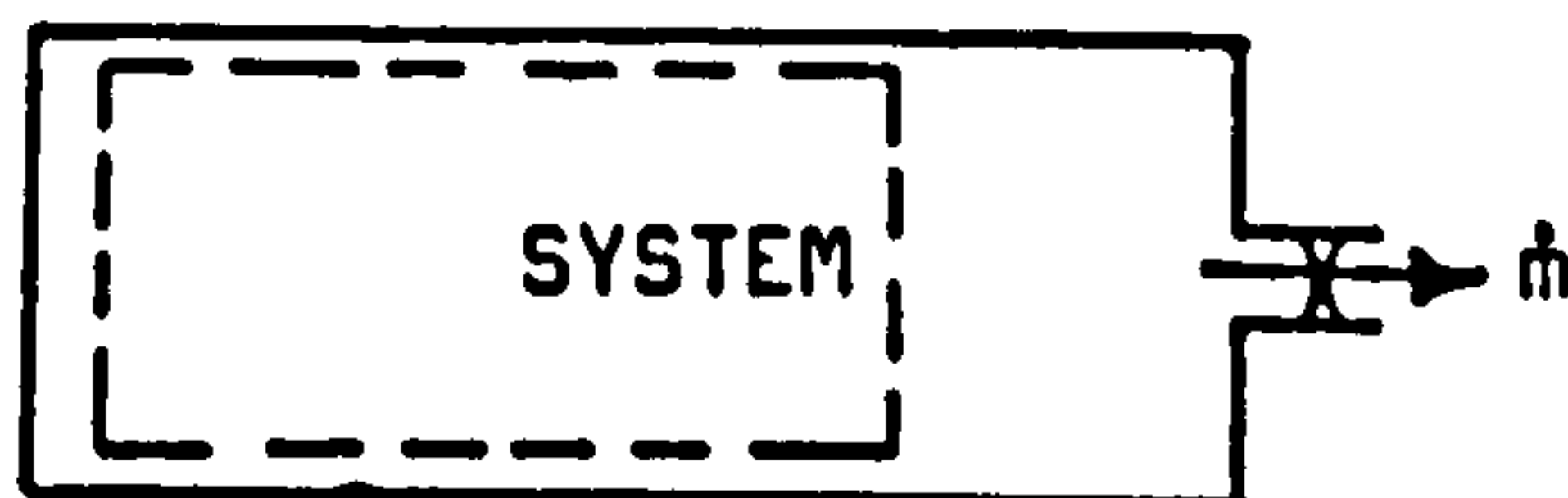
Expansion of High Pressure Gas

(from Ref. 14)

Consider a vessel filled with nitrogen at high pressure, fitted with a constant diameter, fast acting valve. For small time steps the following assumptions are made:-

- (1) Mass flow rate through the valve is constant over a time step and determined by the pressure at the start of the step.
- (2) The temperature of the wall of the vessel, T_w , is constant throughout the expansion process. This would be true since the thermal inertia of the vessel is large.

The gas remaining in the vessel at the end of the step is taken as a system



Applying the first law of thermodynamics,

$$Q = W + (E_2 - E_1) \quad \dots\dots (1)$$

The work done by the system in ejecting the gas

$$\begin{aligned}
 W &= \dot{m} \int_t^{t+\delta t} p.v. dt \\
 &= \dot{m} R \int T dt \quad \dots\dots (2)
 \end{aligned}$$

The initial and final internal energies of the gas are

$$E_1 = C_v \cdot m \cdot T - C_v \dot{m} \int T dt \quad \dots\dots (3)$$

i.e. the energy of the whole of the gas initially in the vessel less that of the gas ejected

$$E_2 = C_v T(t + \delta t) (m - \dot{m} \delta t) \quad \dots\dots (4)$$

$$\begin{aligned}
 \text{Hence } Q &= \dot{m} R \int T dt + C_v T(t + \delta t) (m - \dot{m} \delta t) \\
 &\quad - C_v \cdot m \cdot T + C_v \dot{m} \int T dt \quad \dots\dots (5)
 \end{aligned}$$

Assuming that the heat transfer takes place by forced convection

$$q = -hA_s(T - T_w) \quad \dots\dots (6)$$

where A_s is the surface area of the vessel.

The heat transfer coefficient is estimated using the fully developed turbulent pipe flow formula

$$\begin{aligned}
 NU_D &= 0.0265 (Re_D)^{0.8} (Pr)^{0.3} \\
 &= 0.0238 (Re_D)^{0.8} \text{ for } Pr = 0.7 \quad \dots\dots (7)
 \end{aligned}$$

$$\therefore q = -0.0238 \left(\frac{A_s}{D} \right) (Re_D)^{0.8} (T - T_w) \quad \dots\dots (8)$$

If the further assumption is made that Reynolds number is constant over a time step, eqn. (5) is differentiated and equated to (8).

$$\begin{aligned}
 q &= \dot{m}RT + C_v \frac{dT}{dt} (m - \dot{m} \delta t) - \dot{m}C_v T + \dot{m}C_v T \\
 &= \dot{m}RT + C_v \frac{dT}{dt} (m - \dot{m} \delta t) \quad \dots\dots (9)
 \end{aligned}$$

Equating (8) and (9) and re-arranging gives the following equation

$$\begin{aligned}
 \frac{dT}{dt} &= -T \frac{\dot{m}R + 0.0238 \frac{A_s}{D} (Re_D)^{0.8}}{C_v (m - \dot{m} \delta t)} \\
 &\quad + \frac{0.0238 (Re_D)^{0.8} \frac{A_s}{D} T_w}{C_v (m - \dot{m} \delta t)} \quad \dots\dots (10)
 \end{aligned}$$

The solution of (10) with the proper initial conditions is

$$T = (T_1 - B) e^{-At} + B \quad \dots\dots (11)$$

where T_1 = temperature at the start of the step and the constants

$$A = \frac{\dot{m}R + 0.0238 \frac{(A_s)}{D} (Re_D)^{0.8}}{C_v (\dot{m} - \dot{m} \delta t)}$$

$$\text{and } B = \frac{0.0238 \frac{(A_s)}{D} (Re_D)^{0.8}}{(\dot{m}R + 0.0238 \frac{(A_s)}{D} (Re_D)^{0.8})} \dots\dots (12)$$

The results agree well with experiments for tubes of low L/D ratios. For large L/D ratios, the predictions of realisable temperatures are always optimistic. Agreement with the experimental evidence (Ref. 17) was achieved by multiplying the Nusselt number used by 2.5. This is purely an empirical result.

	Ambient Temperature Deg, K.	Initial Pressure of Gas P_i (psig)	Reservoir Nozzle Diameter (cm)	Reservoir Pressure (psig)	Expansion Time (secs)	Running Time (secs)	Tunnel Stagnation Pressure Bars	Mach Number	Stagnation Temperature Deg K		Static Pressure Variation during Run $\Delta P_{st}/P\%$	Temperature Variation during Run Deg, K.
									Obtained (Average)	Isentropic		
1	288	500	1.459	30	0.65	0.294	1	0.985	110	104	1.36	± 6.0
2	286	500	1.032	68	0.65	0.297	1	0.958	115.5	104	2.50	± 5.5
3	286	500	1.032	66	0.65	0.290	1	0.964	111.5	104	1.36	± 7.0
4	287	400	1.459	29	0.60	0.272	1	0.88	119	111	1.70	± 5.0
5	286	300	1.032	72	0.50	0.265	1	0.962	135	120	2.38	± 8.0
6	286	300	1.032	74	0.50	0.265	1	0.962	133.5	120	1.7	± 6.5
7	288	200	1.032	77	0.42	0.260	1	0.93	143.5	134	1.7	± 10
8	288	100	1.032	84	0.30	0.250	1	0.962	174.5	160	1.7	± 5.5

TABLE 2

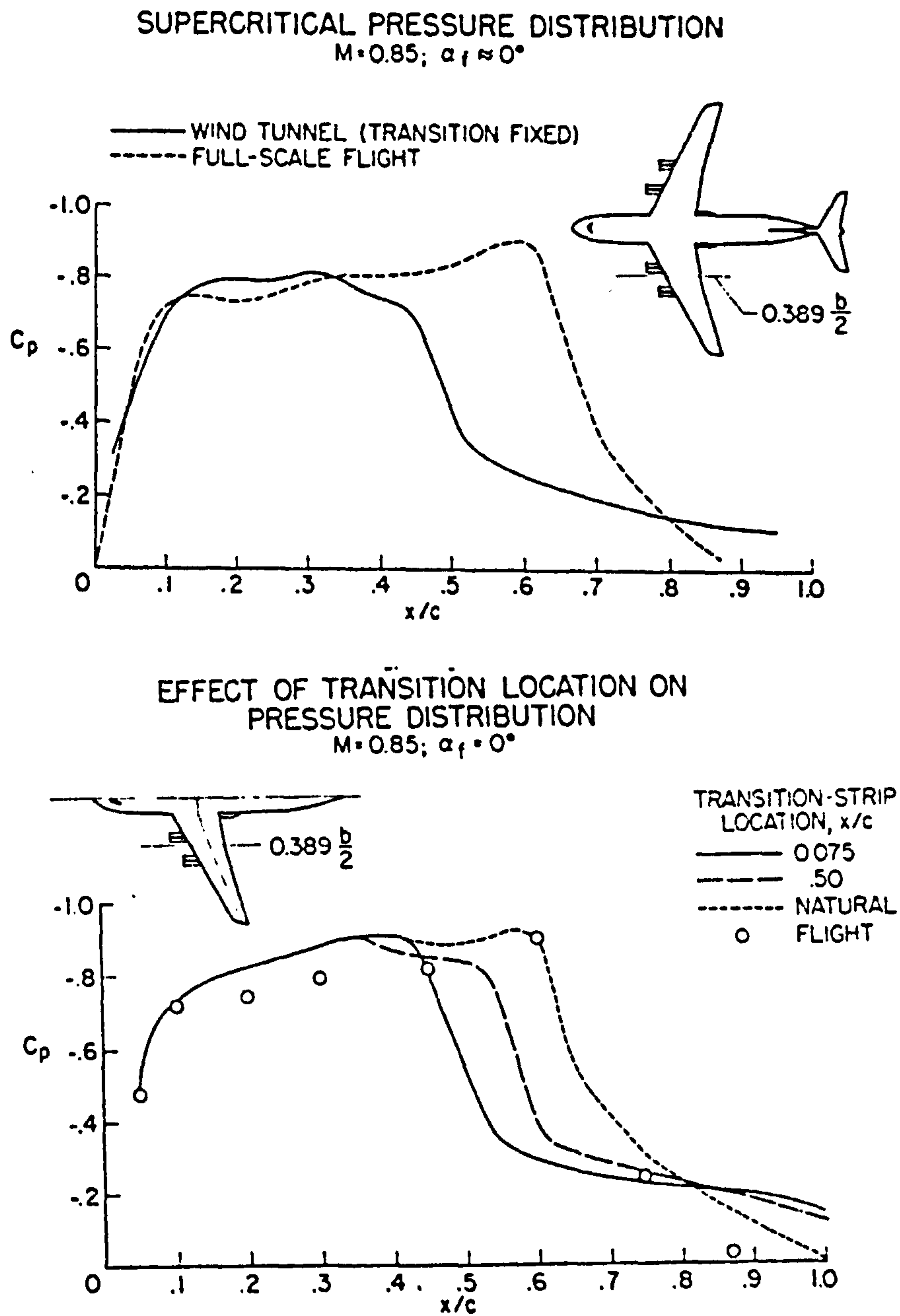


Figure 1. Differences between wind tunnel tests and flight tests on a C141 aeroplane. (ref. 5)

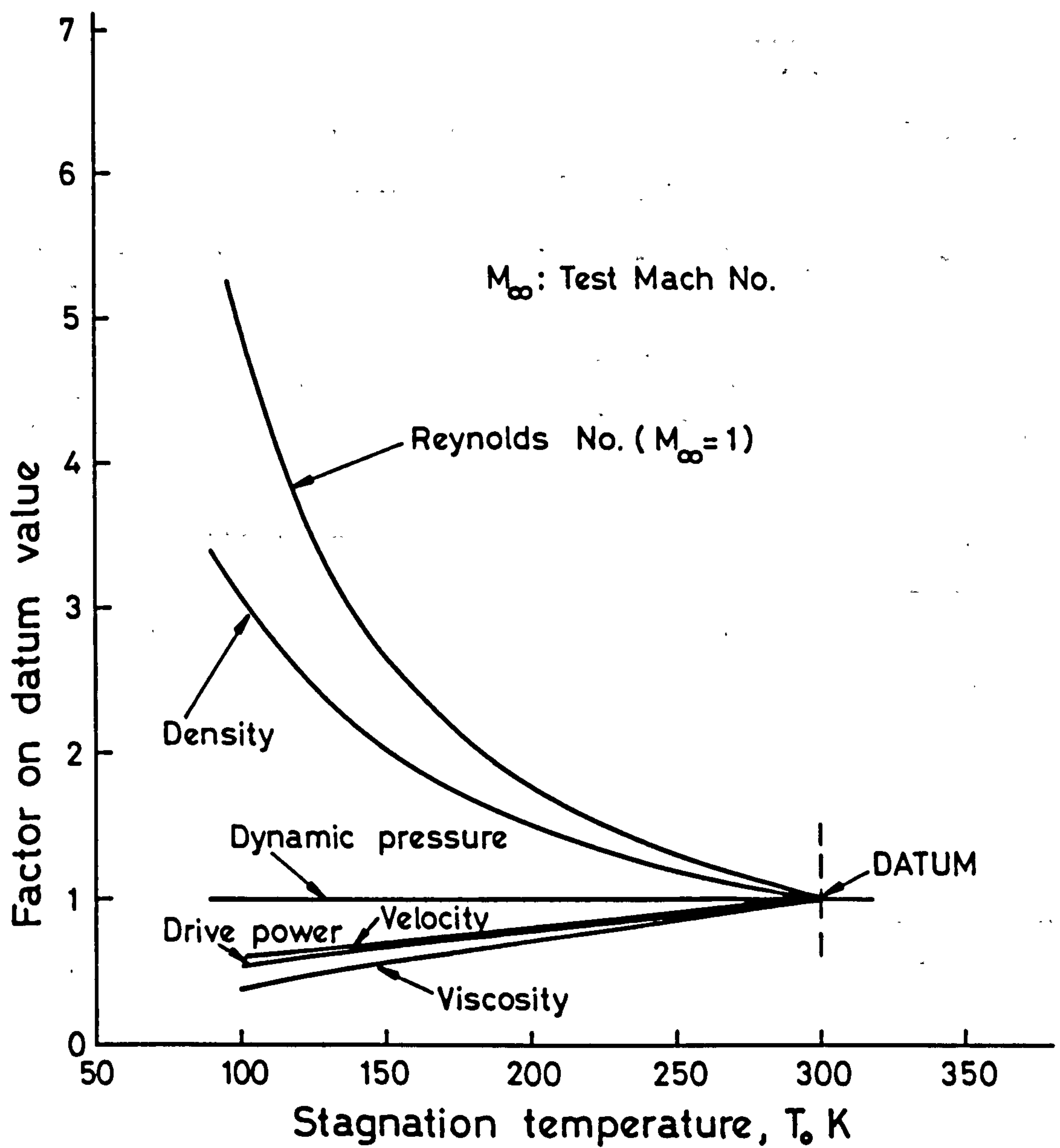
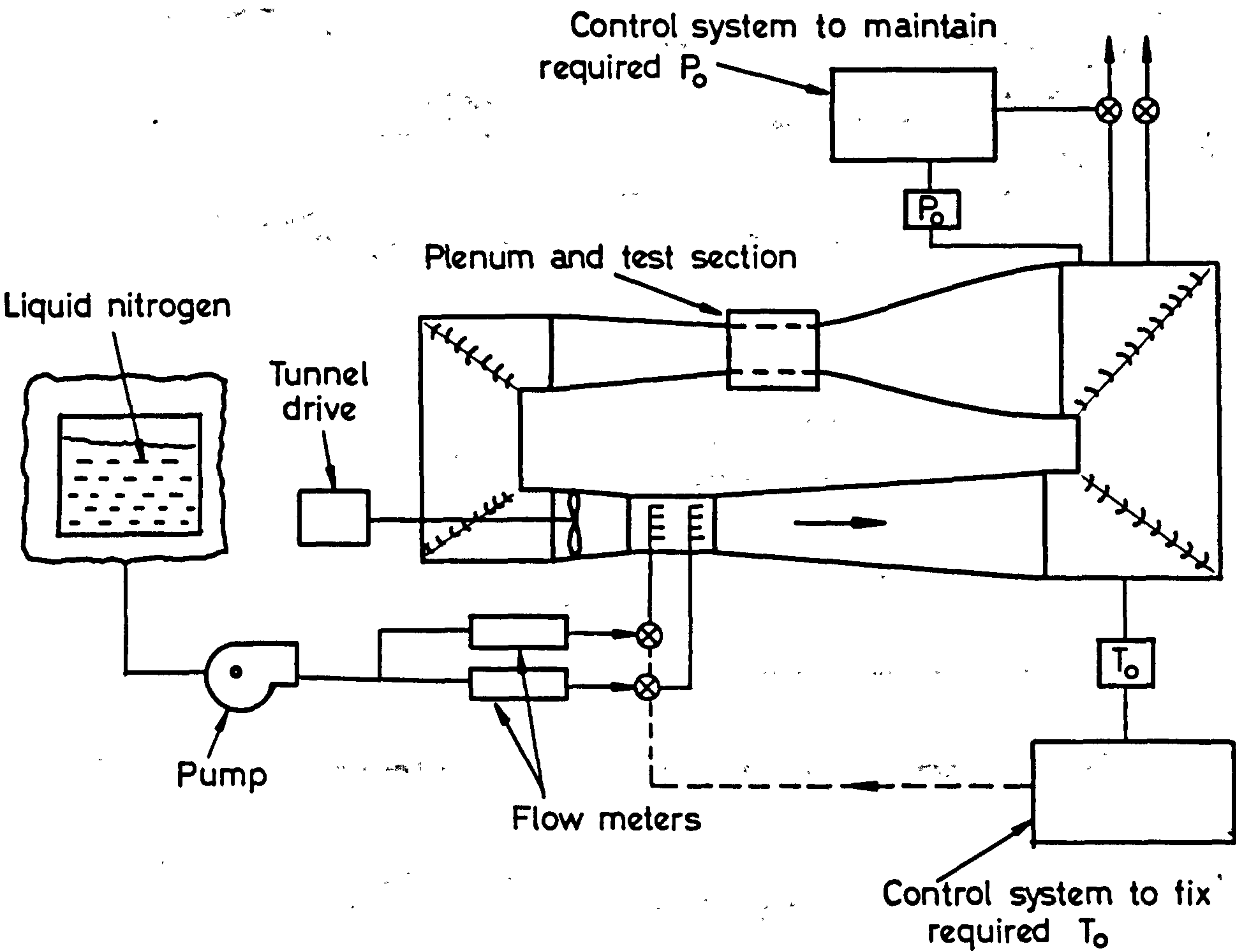


Figure 2. Variation of test-section parameters with stagnation temperature in air.

Typical Layout of a Continuous Cryogenic tunnel



Specifications

	American Tunnel	European Tunnel
SIZE	2.5m x 2.5m	2m x 2m
STAGNATION TEMPERATURE	120K	90 - 120K
REYNOLDS NUMBER	120×10^6	60×10^6

Figure 3. Continuous cryogenic tunnel .

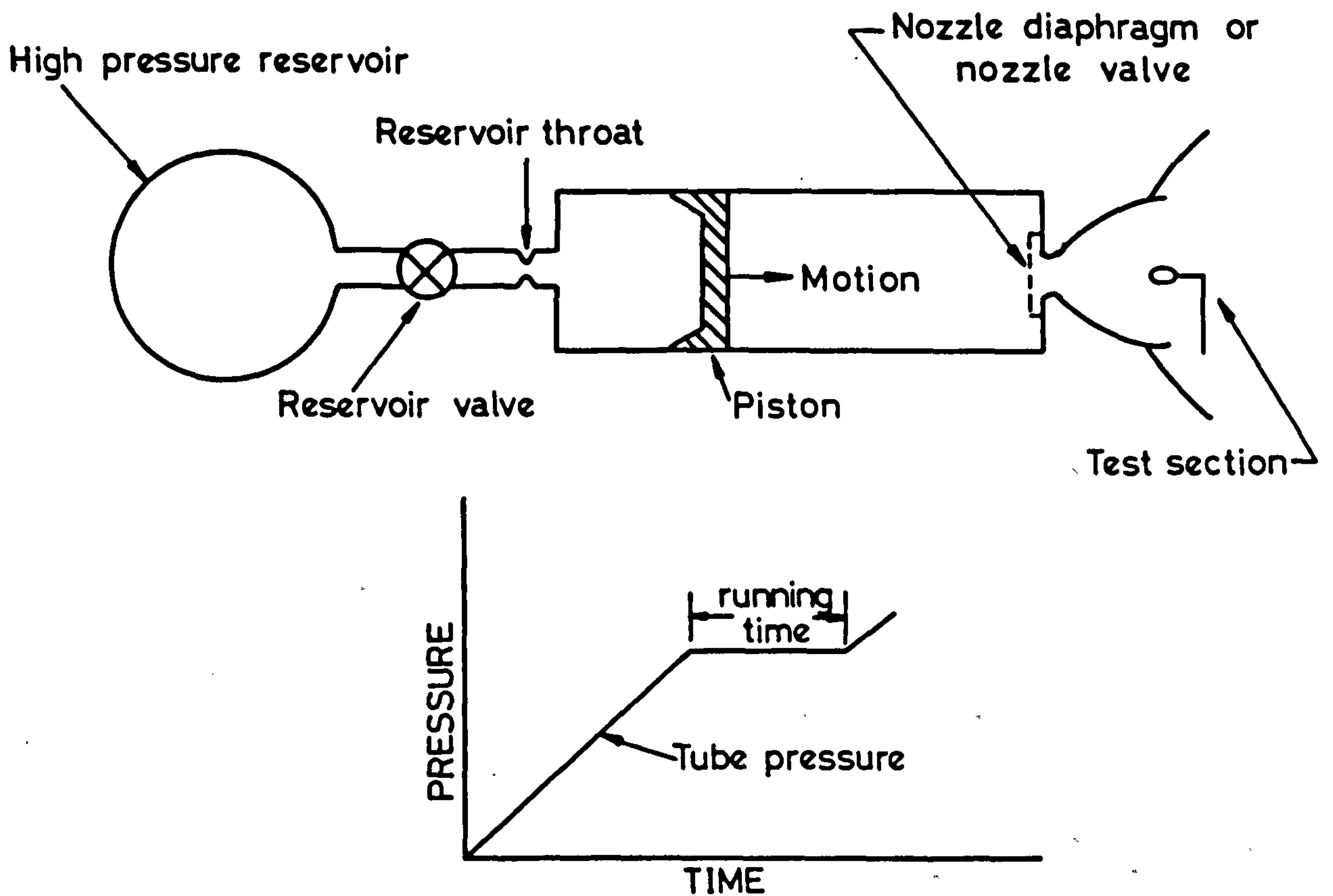


Figure 4. Isentropic light piston tunnel (ref.10).

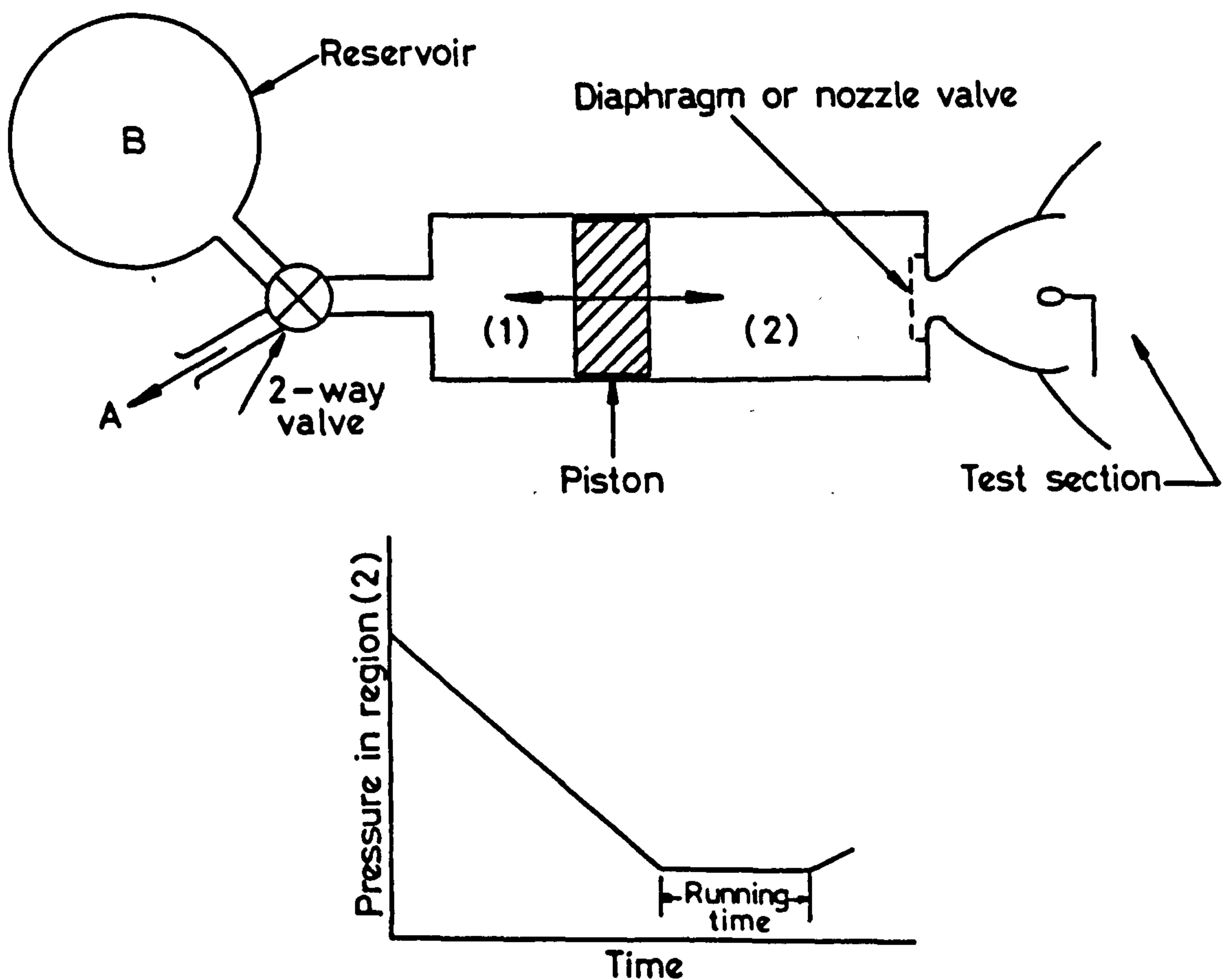


Figure 5.a. Cryogenic isentropic light piston tunnel. (ret.12)

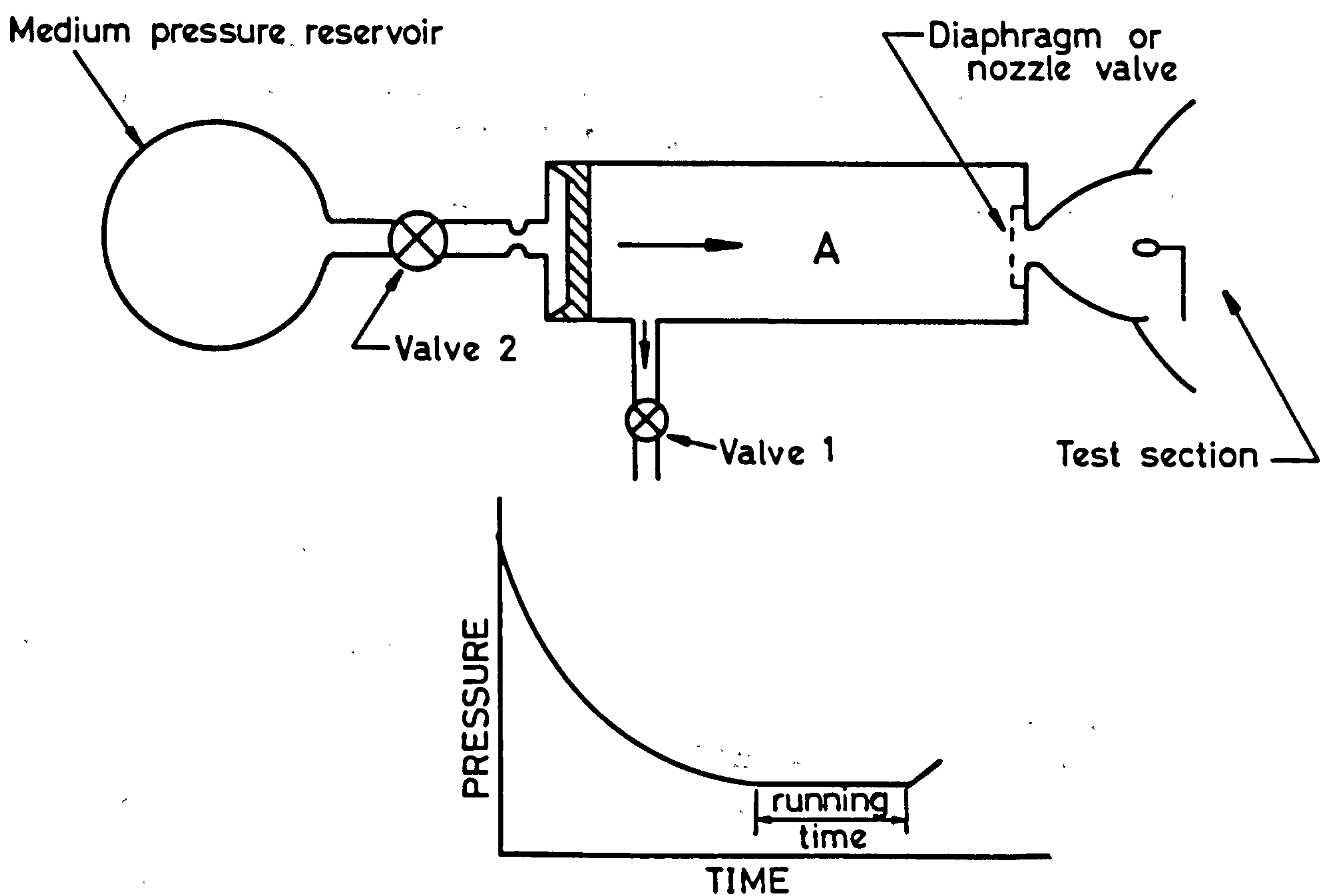


Figure 5.b.Cryogenic isentropic light piston tunnel (modified)

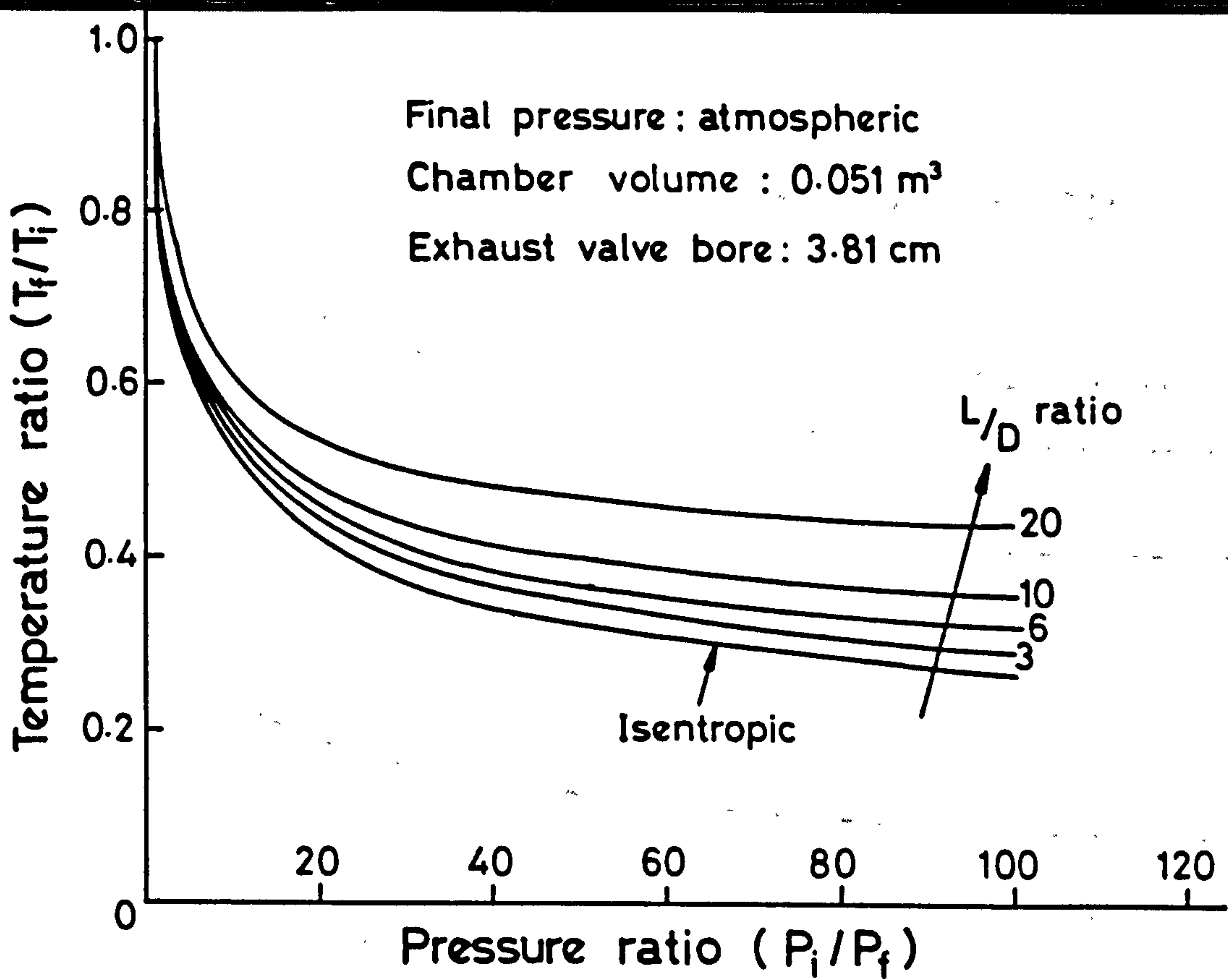


Figure 6. Effect of L/D ratio on final temperature achieved

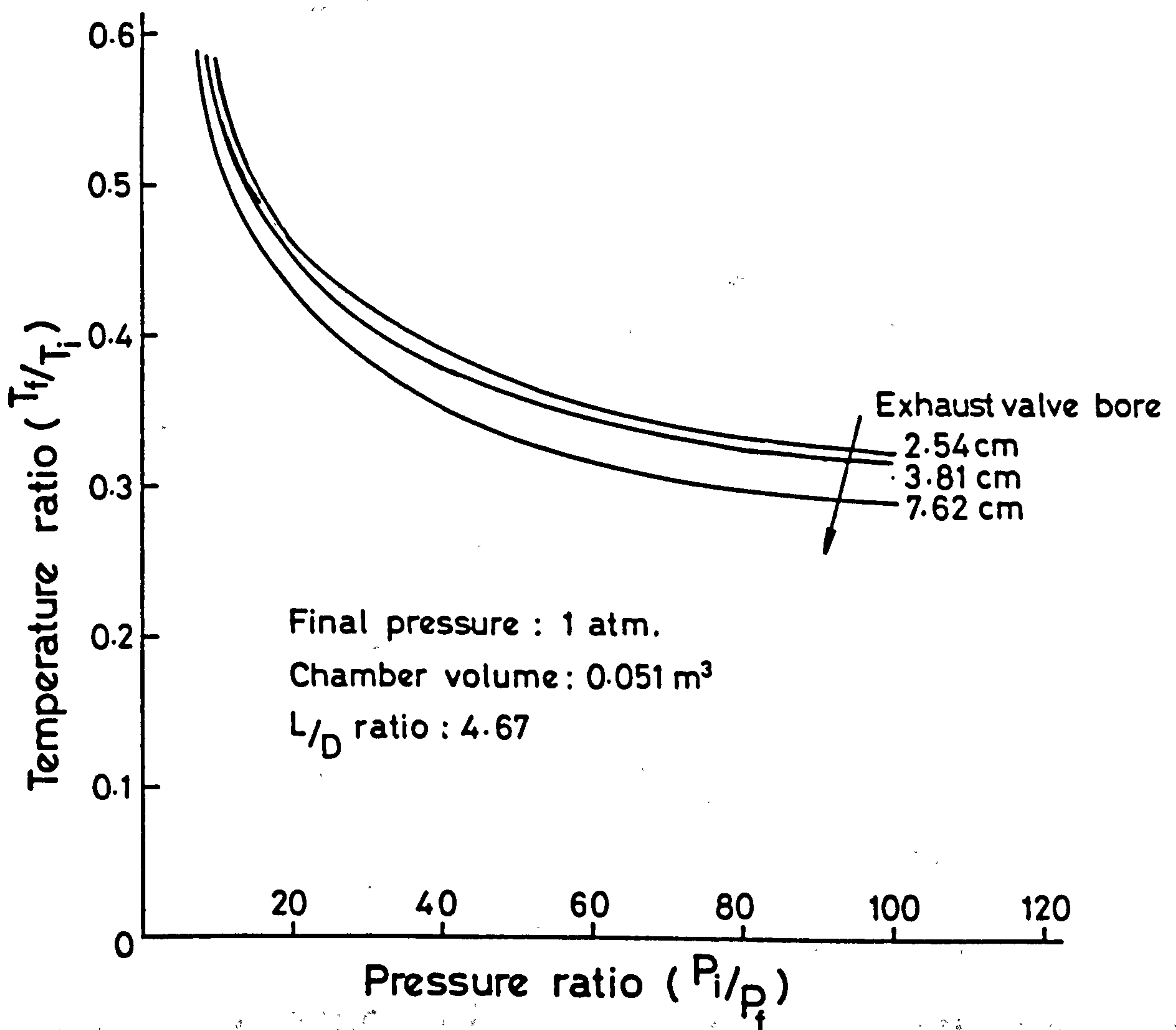


Figure 7. Effect of exhaust valve bore on final temperature achieved.

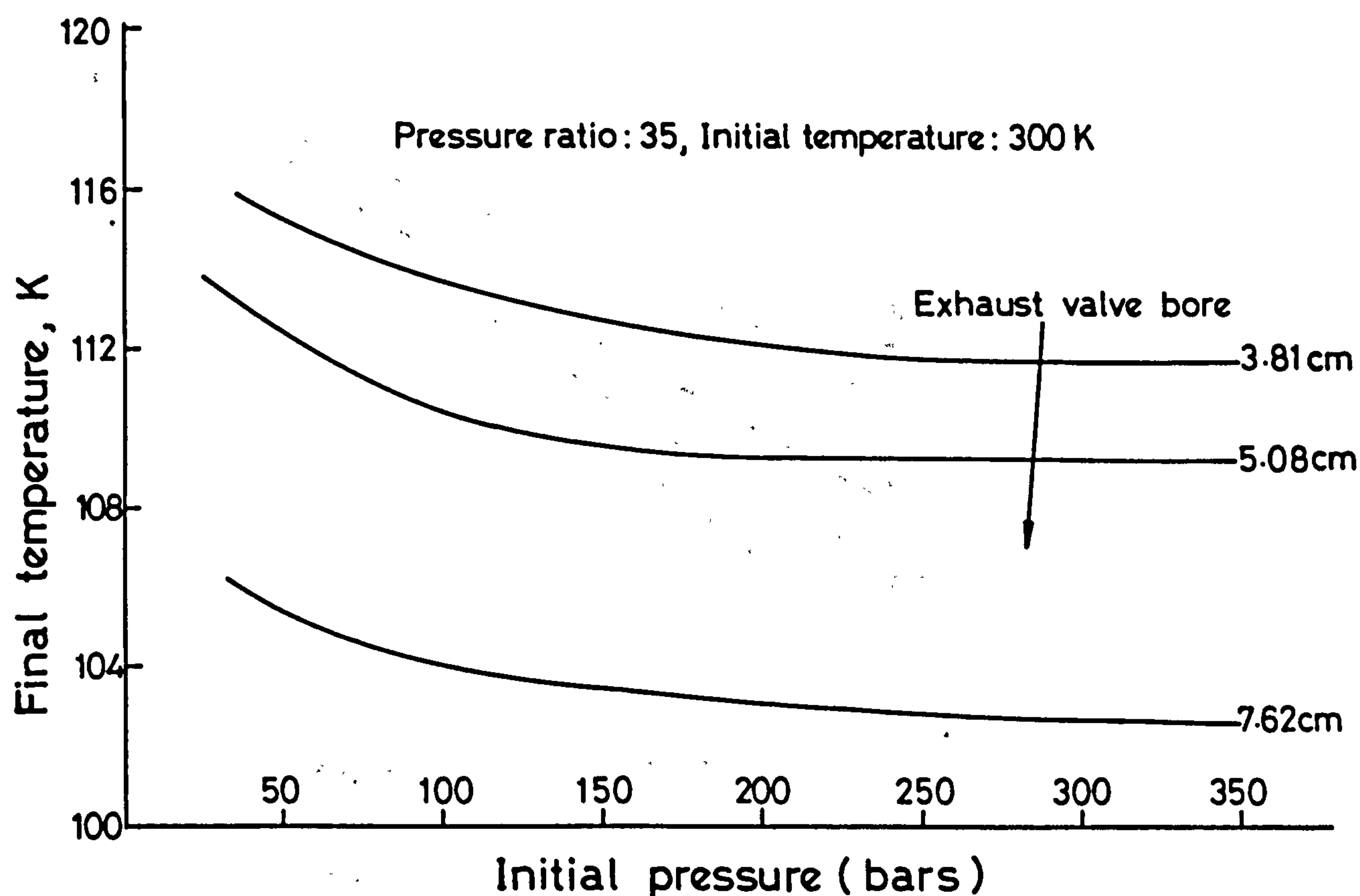


Figure 8. Effect of initial pressure on final temperature achieved.

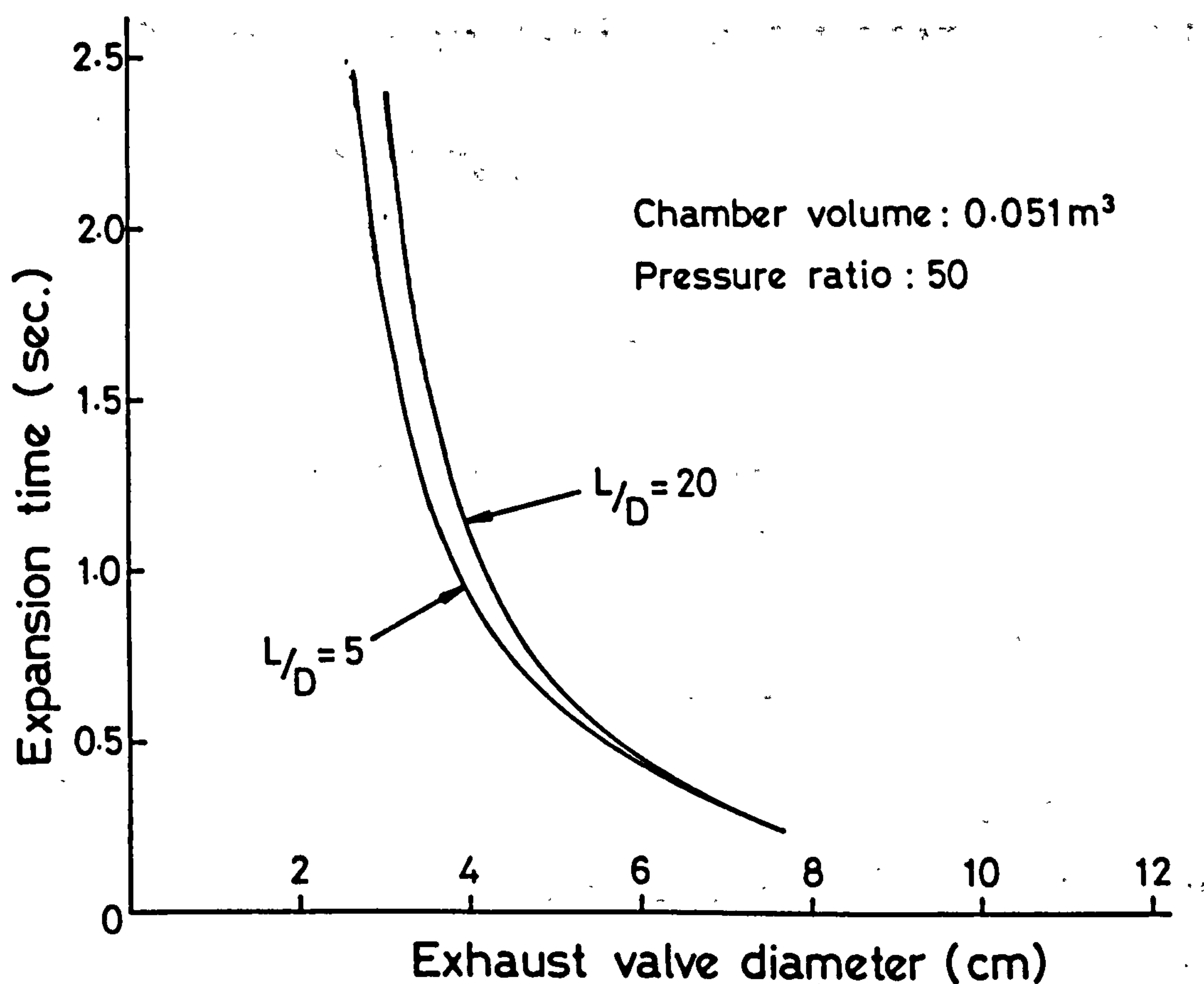


Figure 9. Variation of expansion time with exhaust valve diameter.

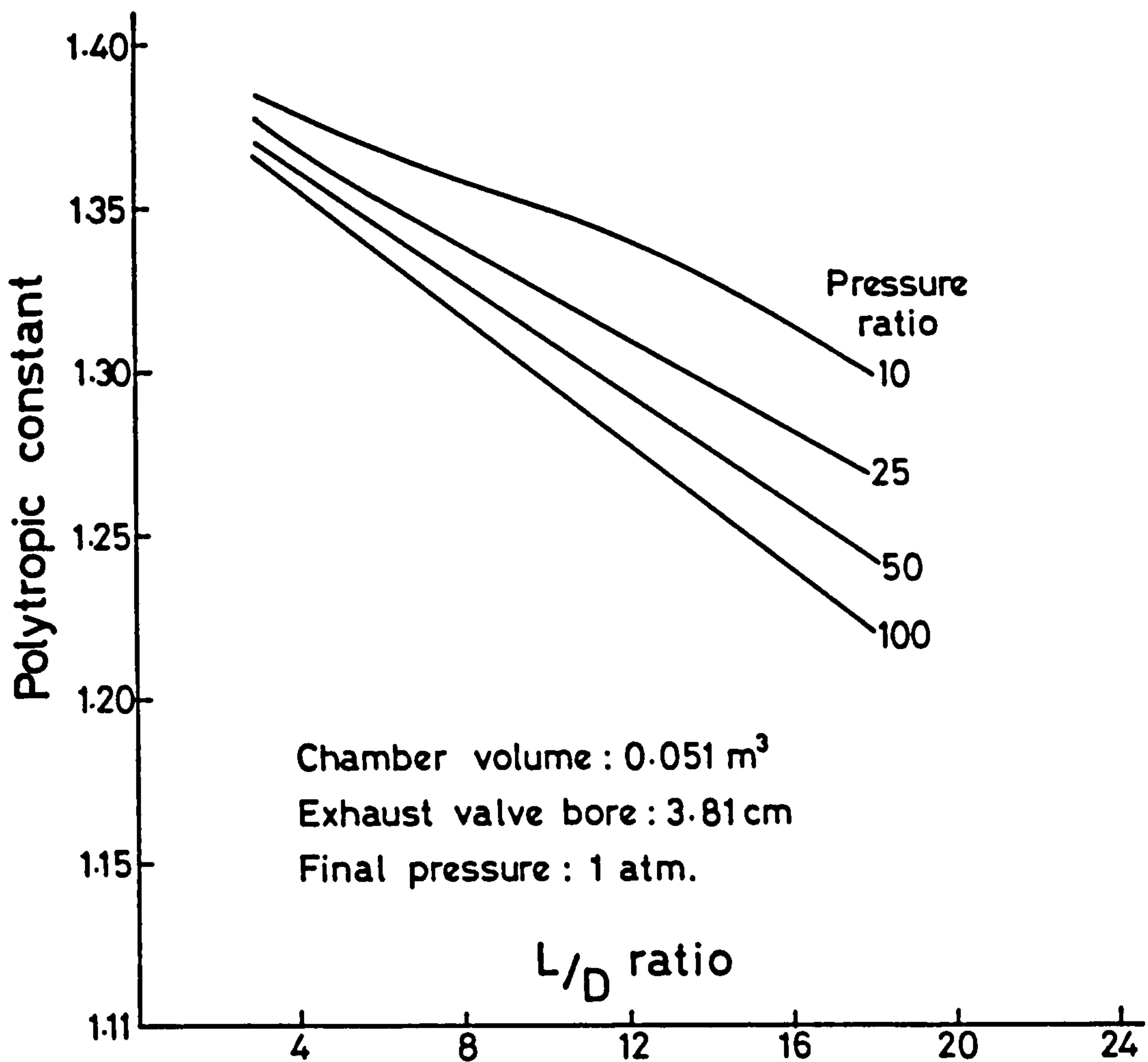


Figure 10. Polytropic constants for different (L/D) ratios.

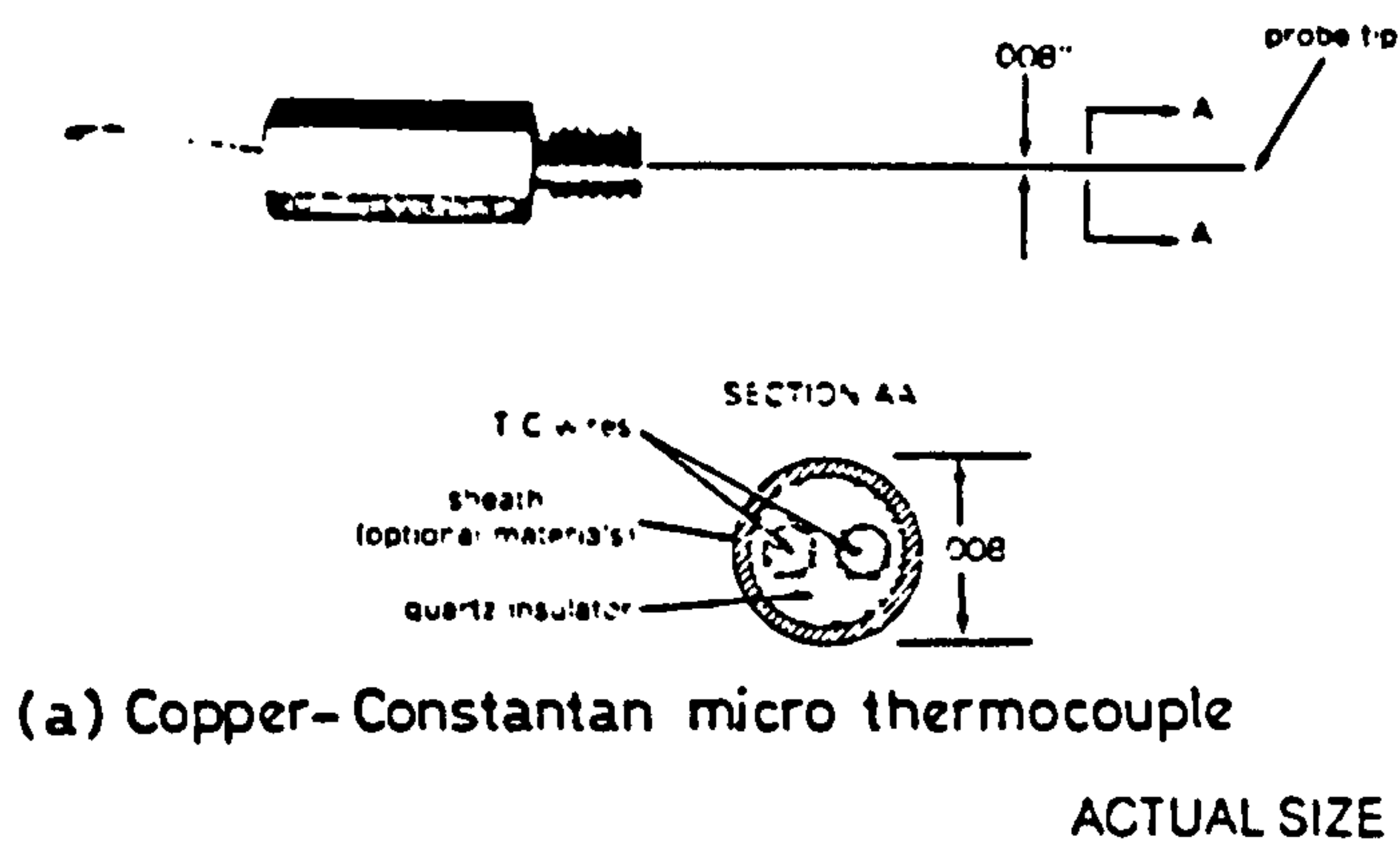
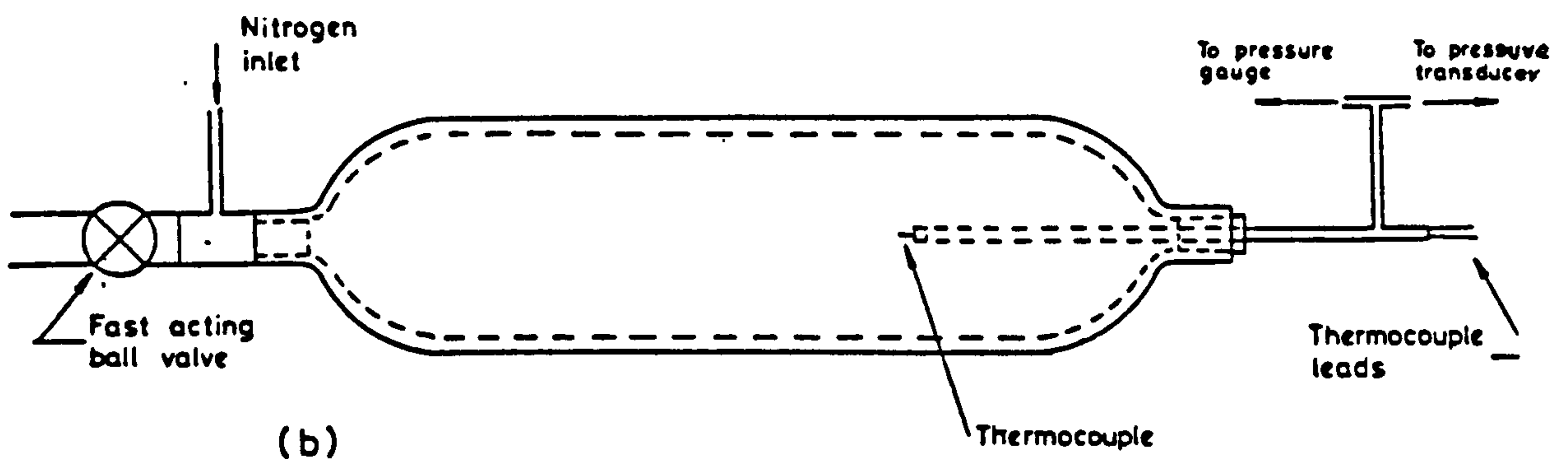


Figure 11. Experimental set up for expansion experiments.

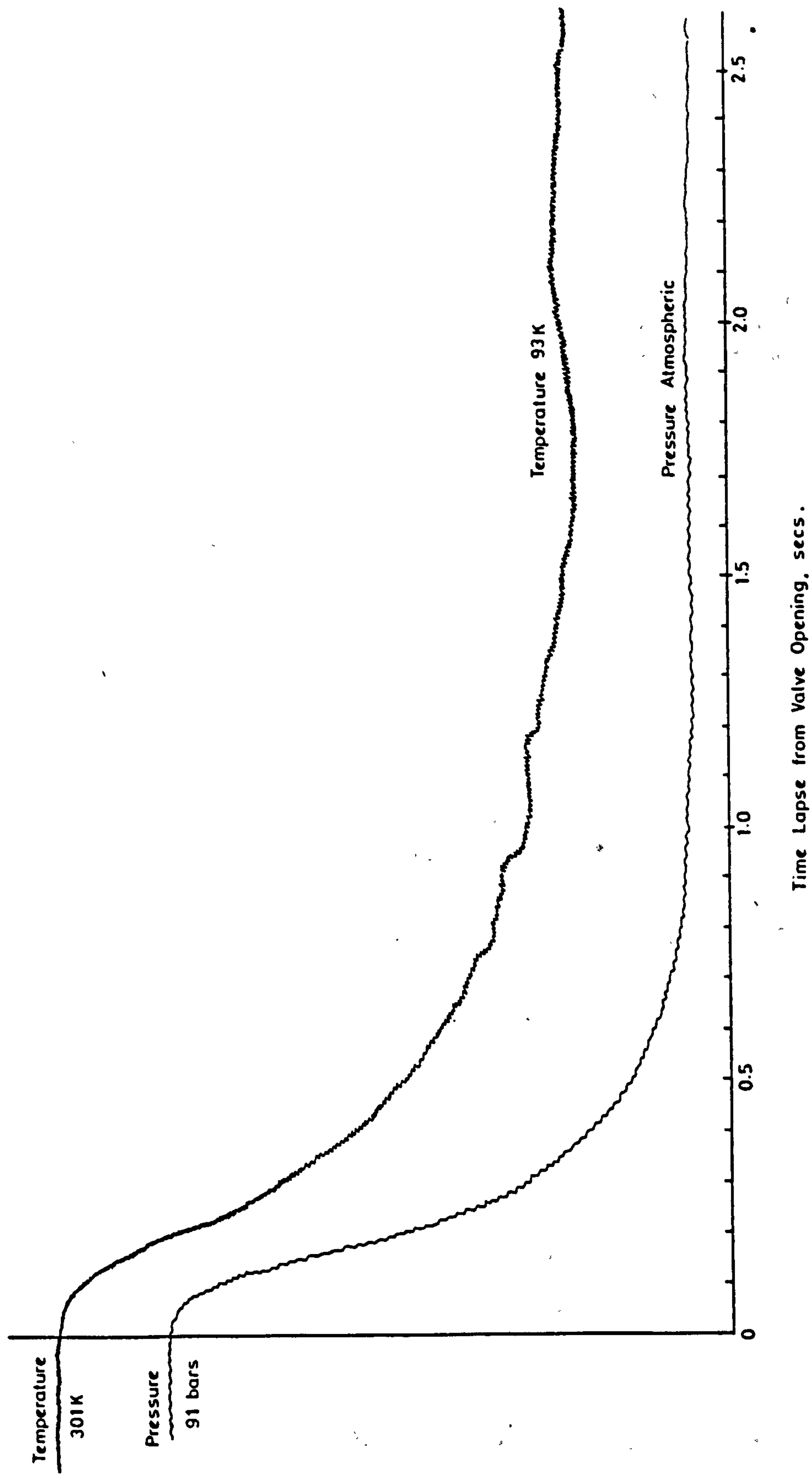


Fig.12. Typical Pressure & Temperature Time Record of an Expansion Expt. (present work)

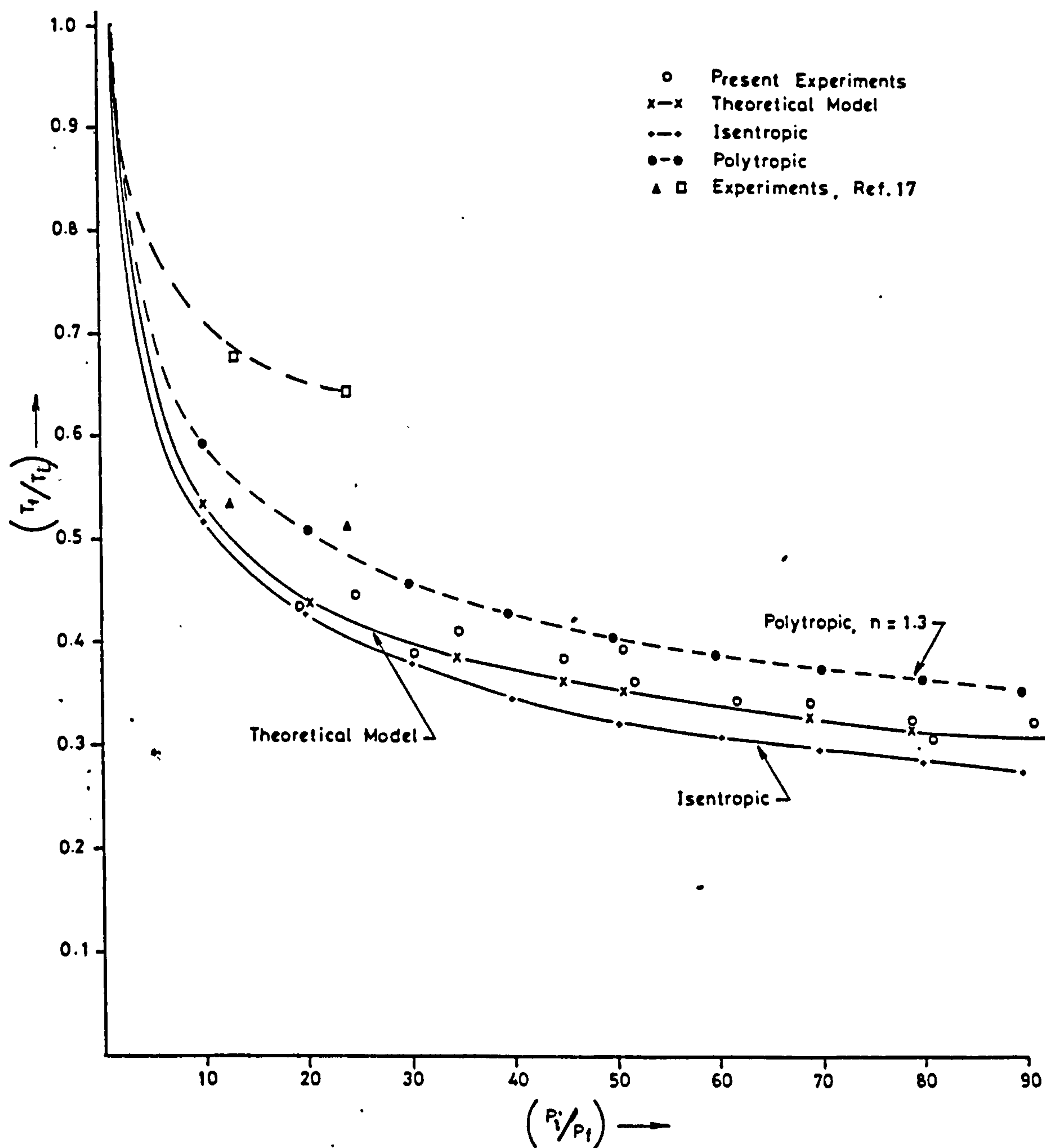


Fig 13. Temperature Ratio vs Pressure Ratio

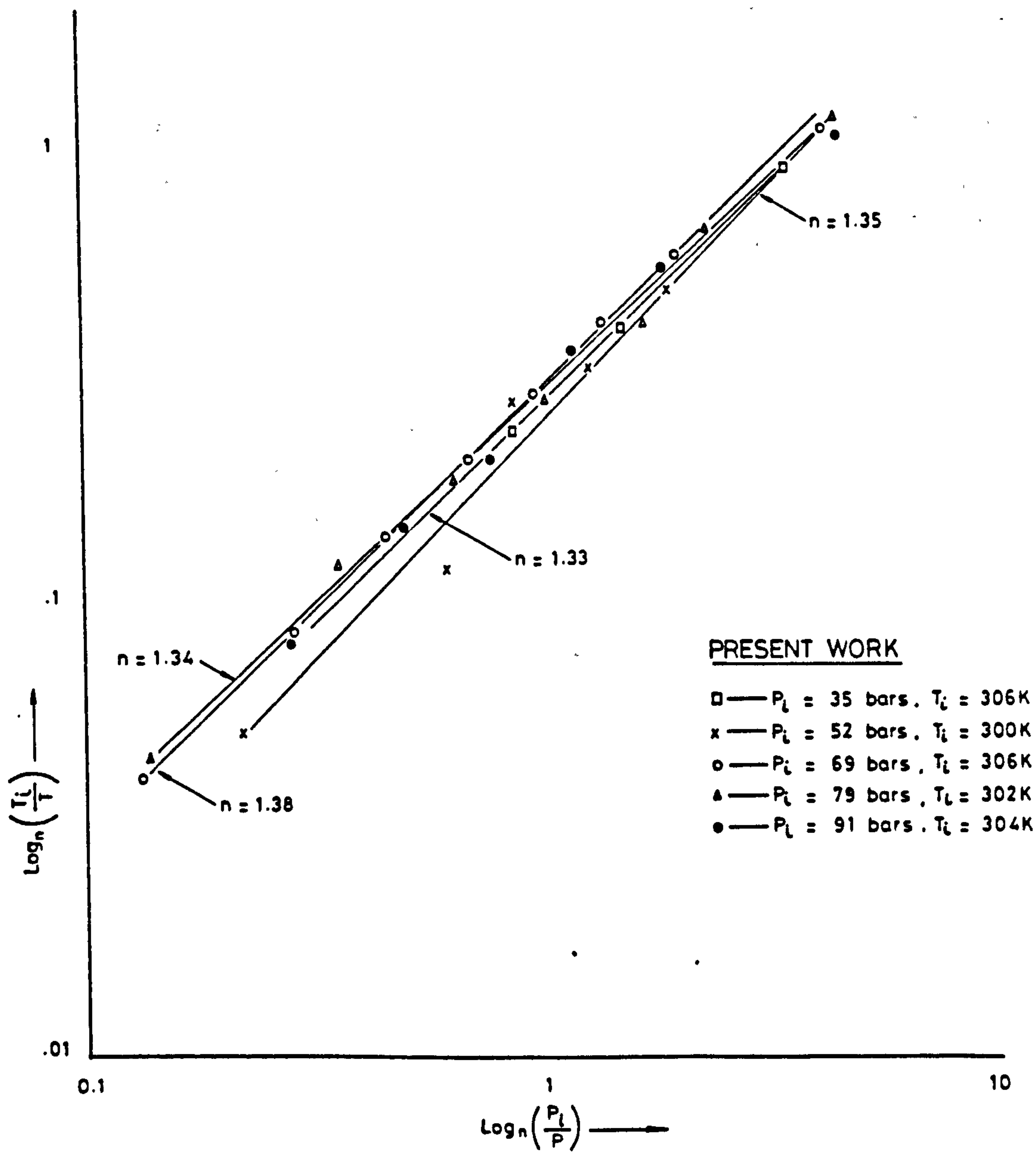


Fig 14 Temperature Ratio vs Pressure Ratio in Logarithmic Coordinates

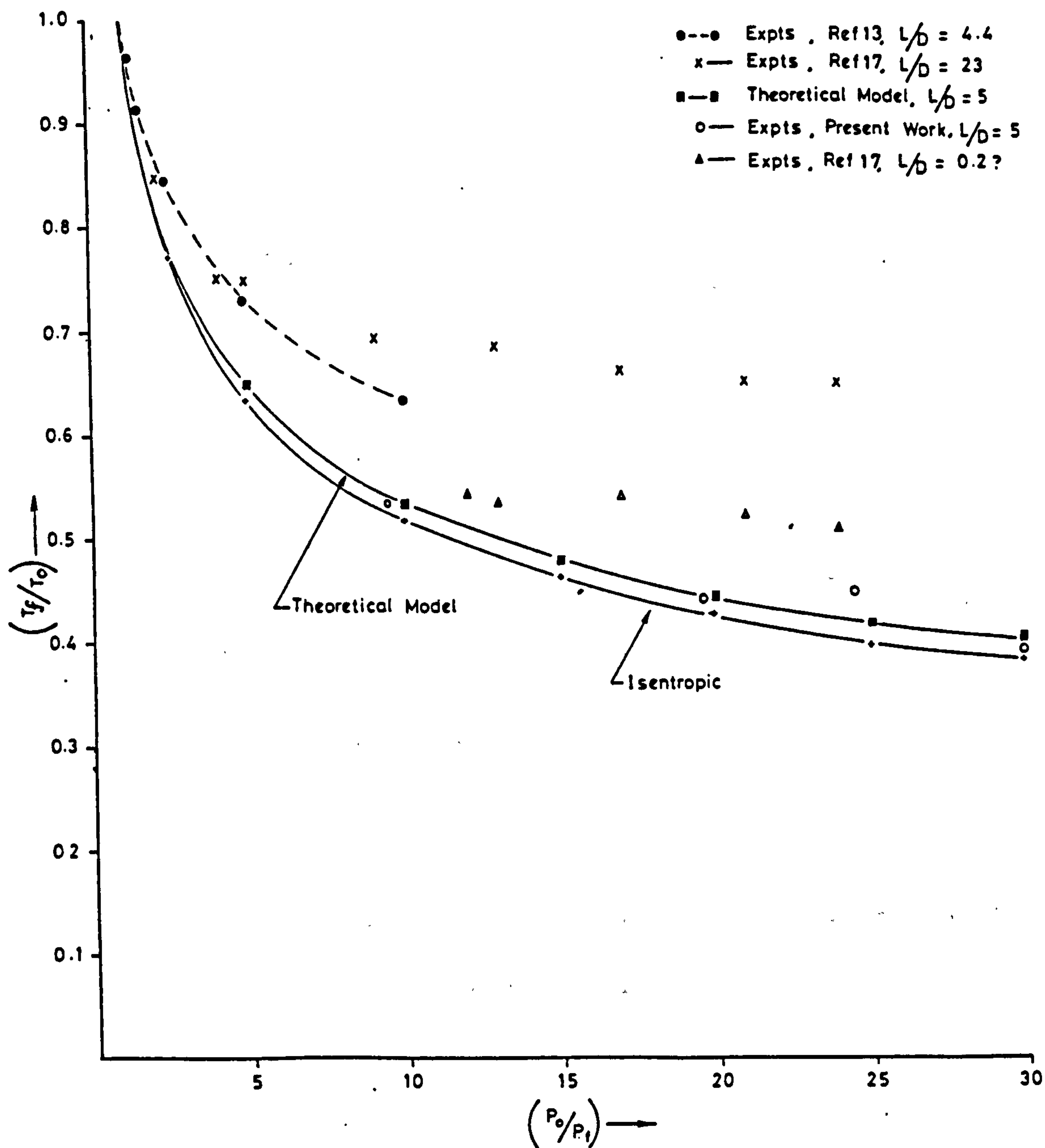


Fig 15 Comparison of Present Work with Literature

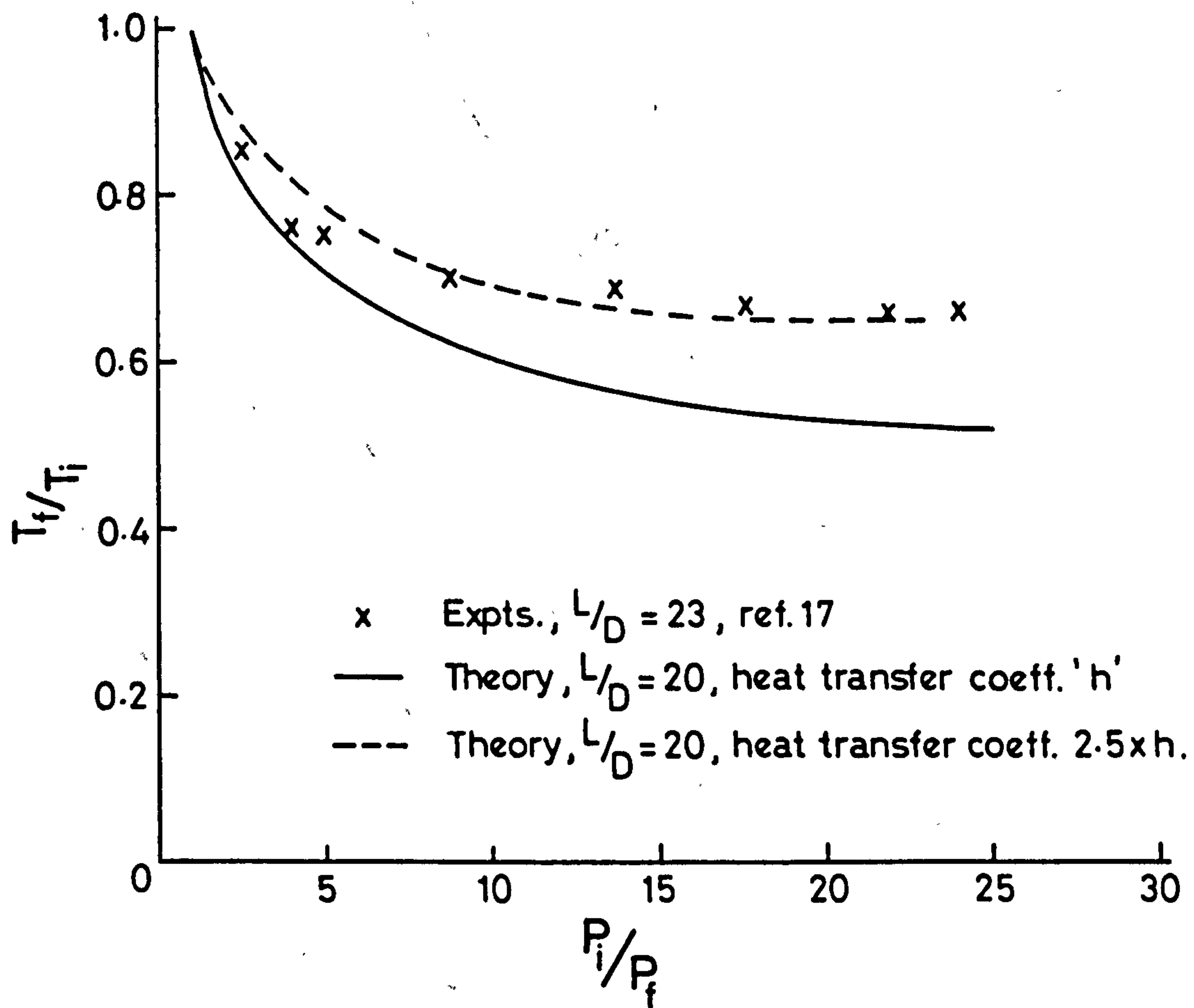


Figure 16. Comparison of theory with experiments for large (L/D) ratios.

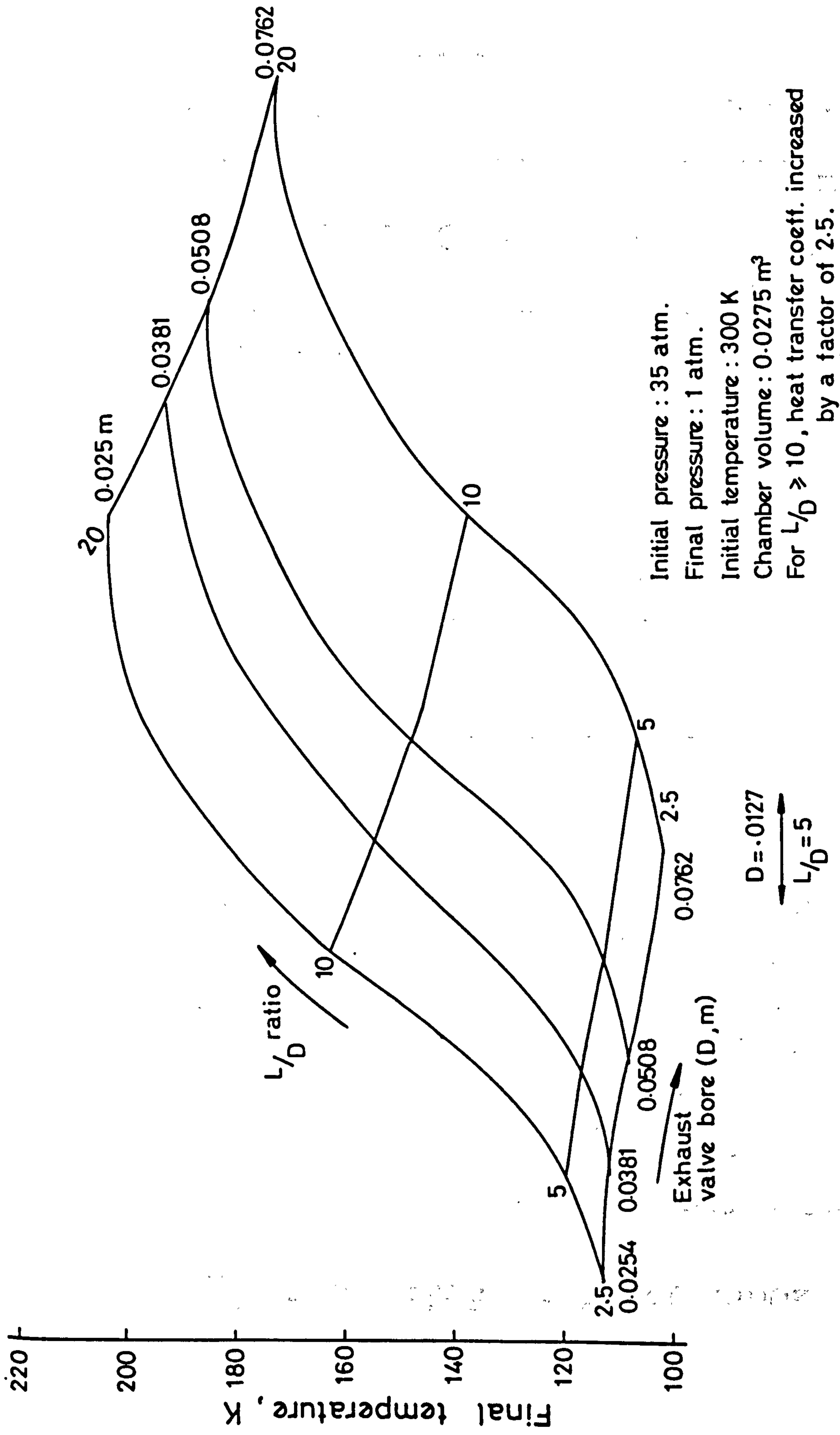


Figure 17. Final temperatures achieved for different (L/D) ratios and exhaust valve diameters.

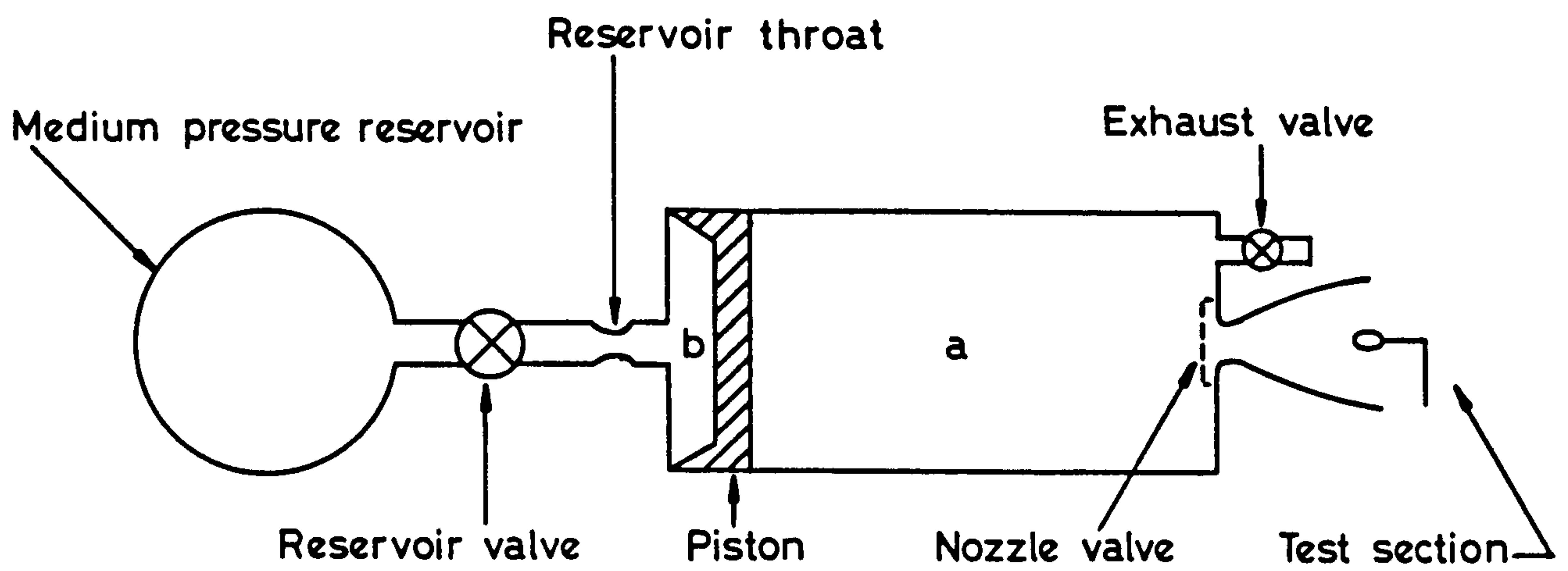
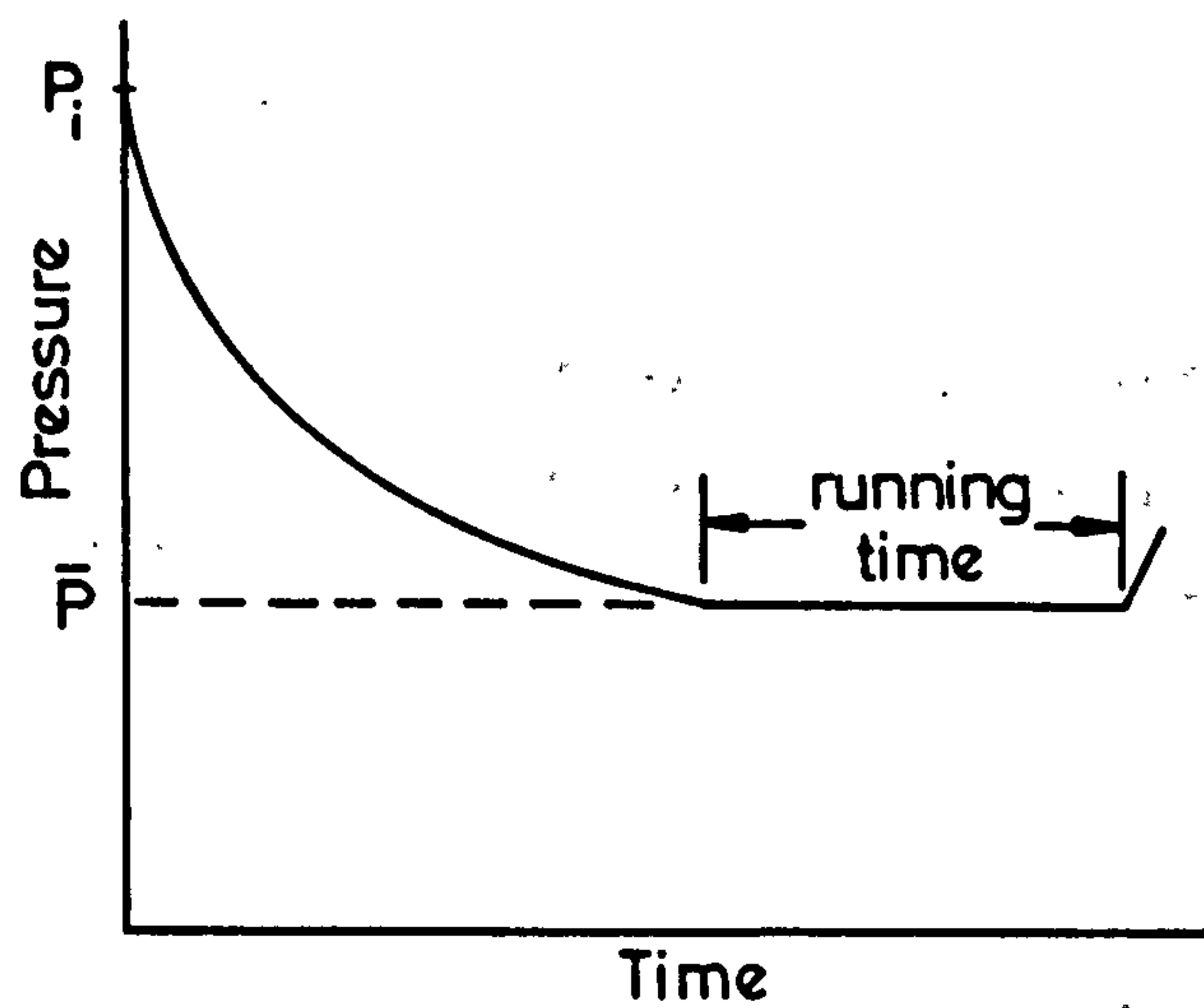
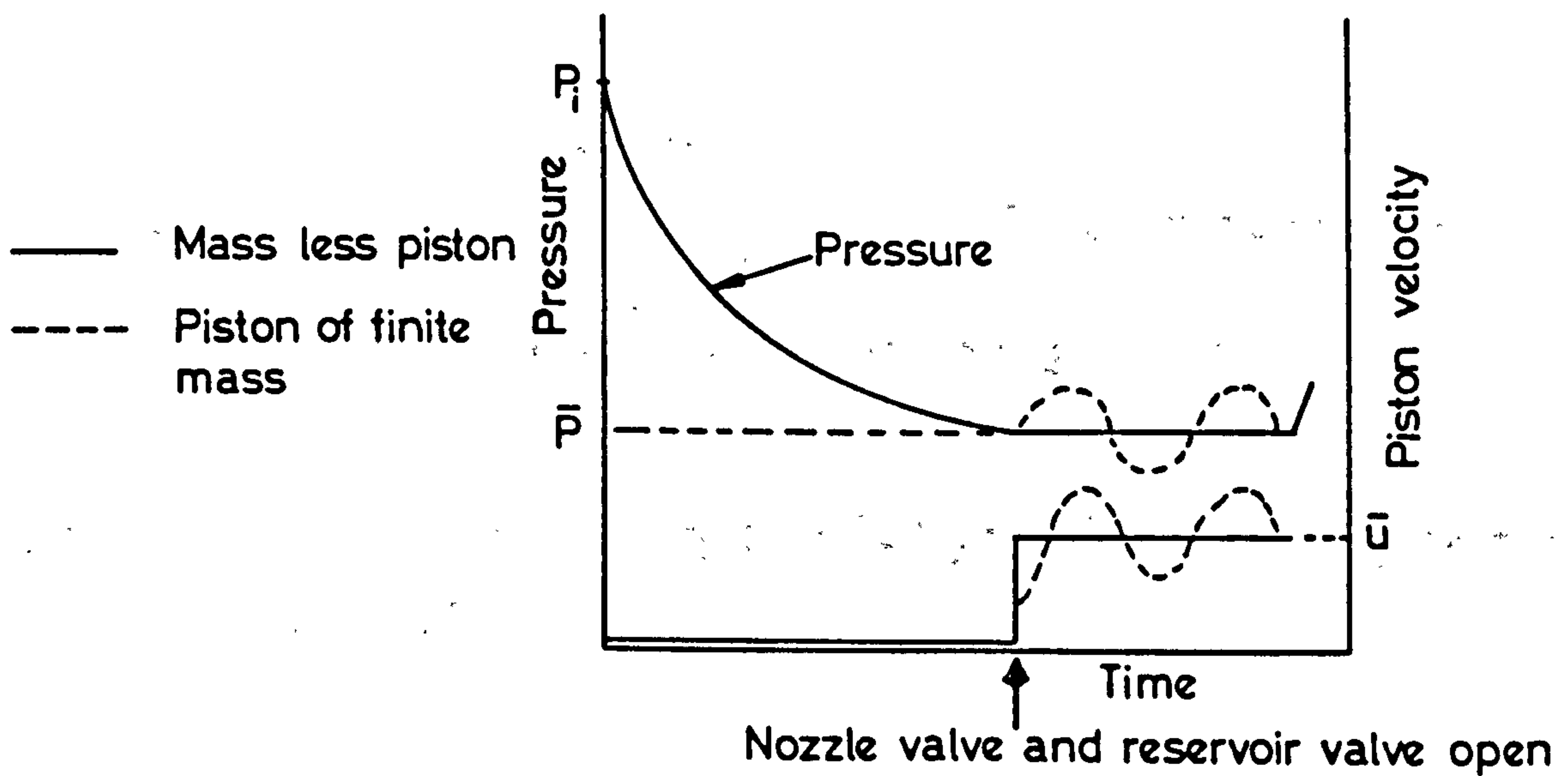


Figure 18. Cryogenic piston tunnel .

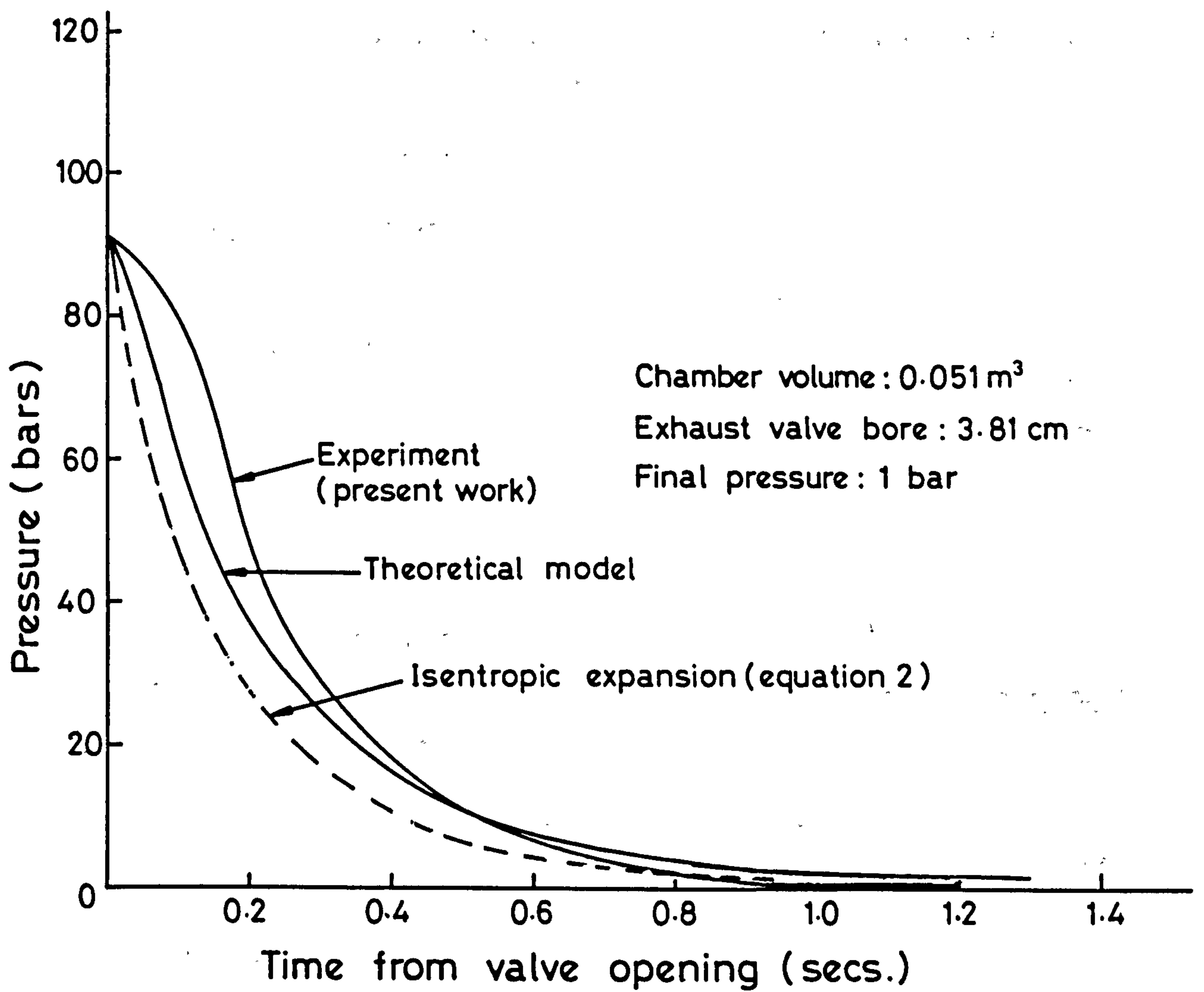
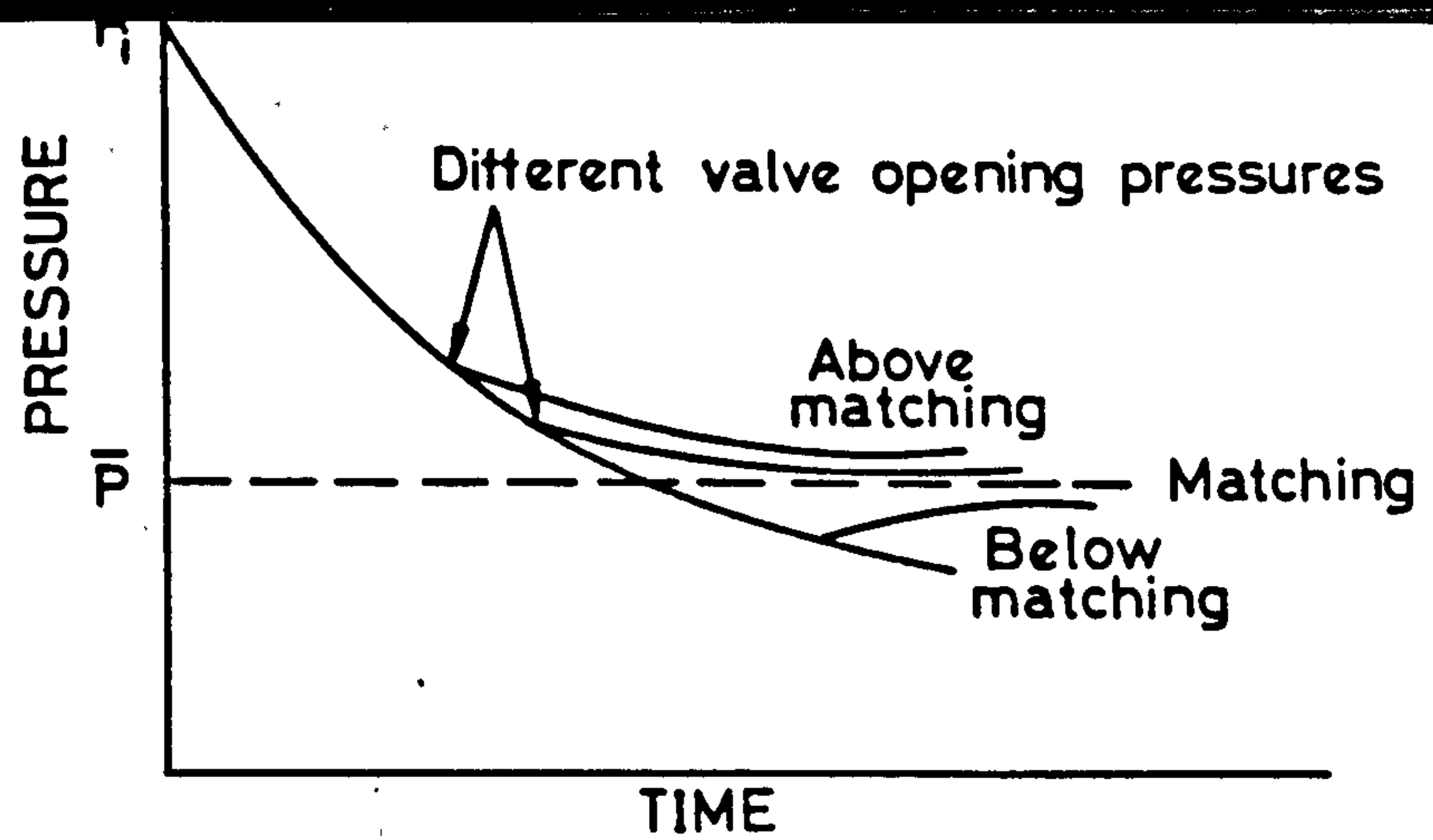
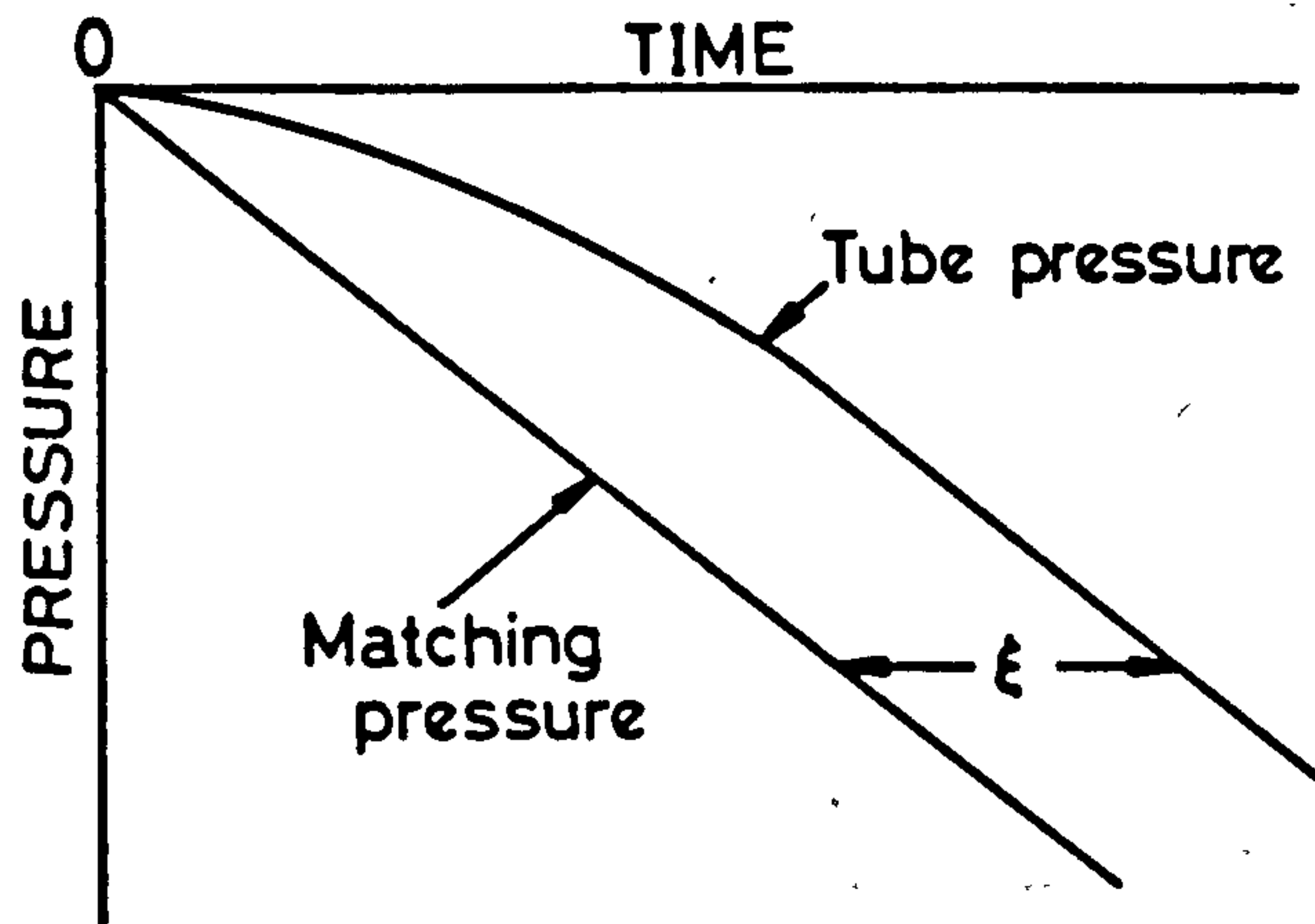


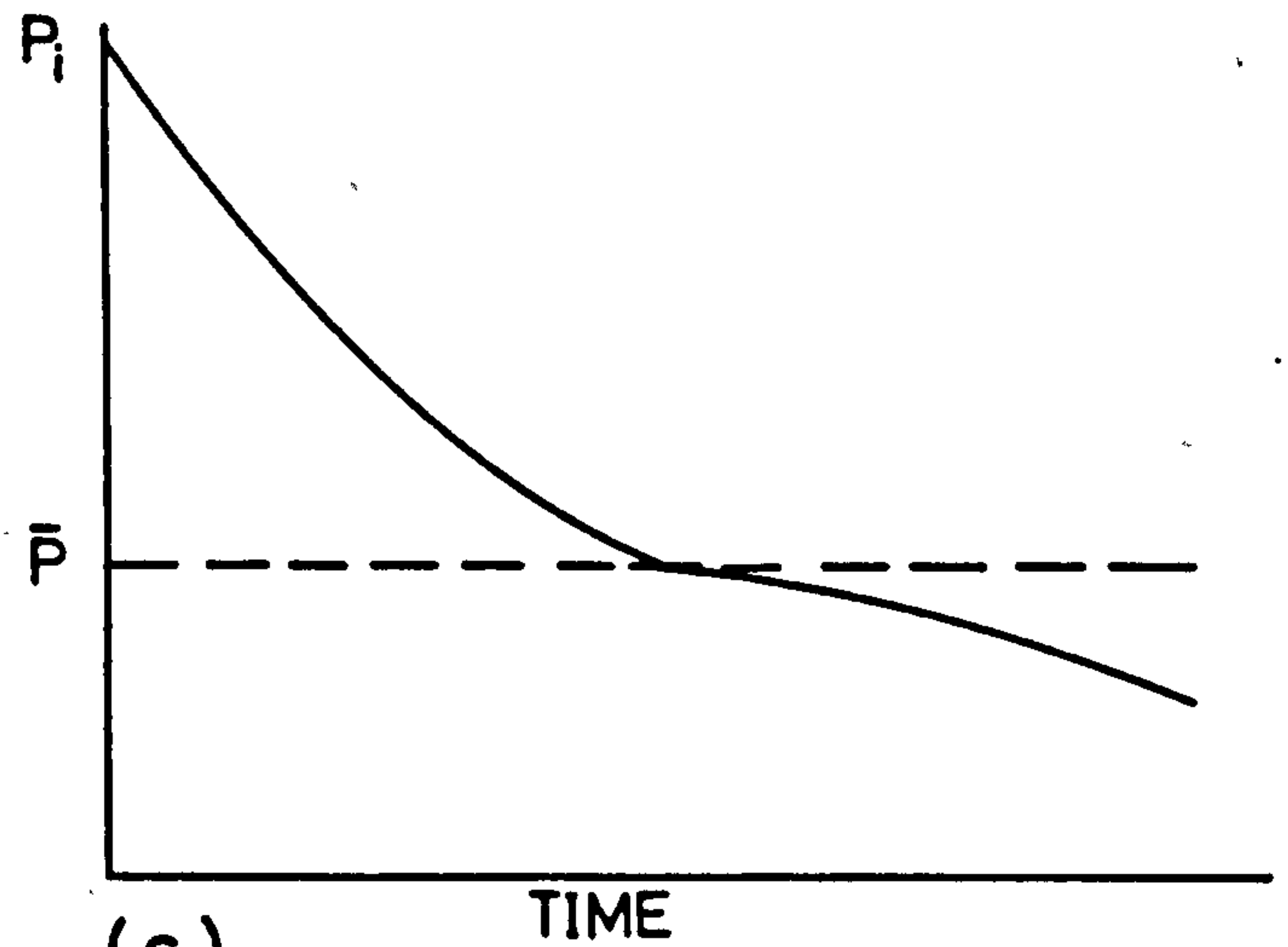
Figure 19. Pressure – time history for an expansion process.



(a) Infinite reservoir, but conditions not matched at the instant of valve opening.



(b)



(c)

(b) and (c) Finite reservoir, initially matched.

Figure 20. The variation of tube pressure with time under 'off matching' conditions.

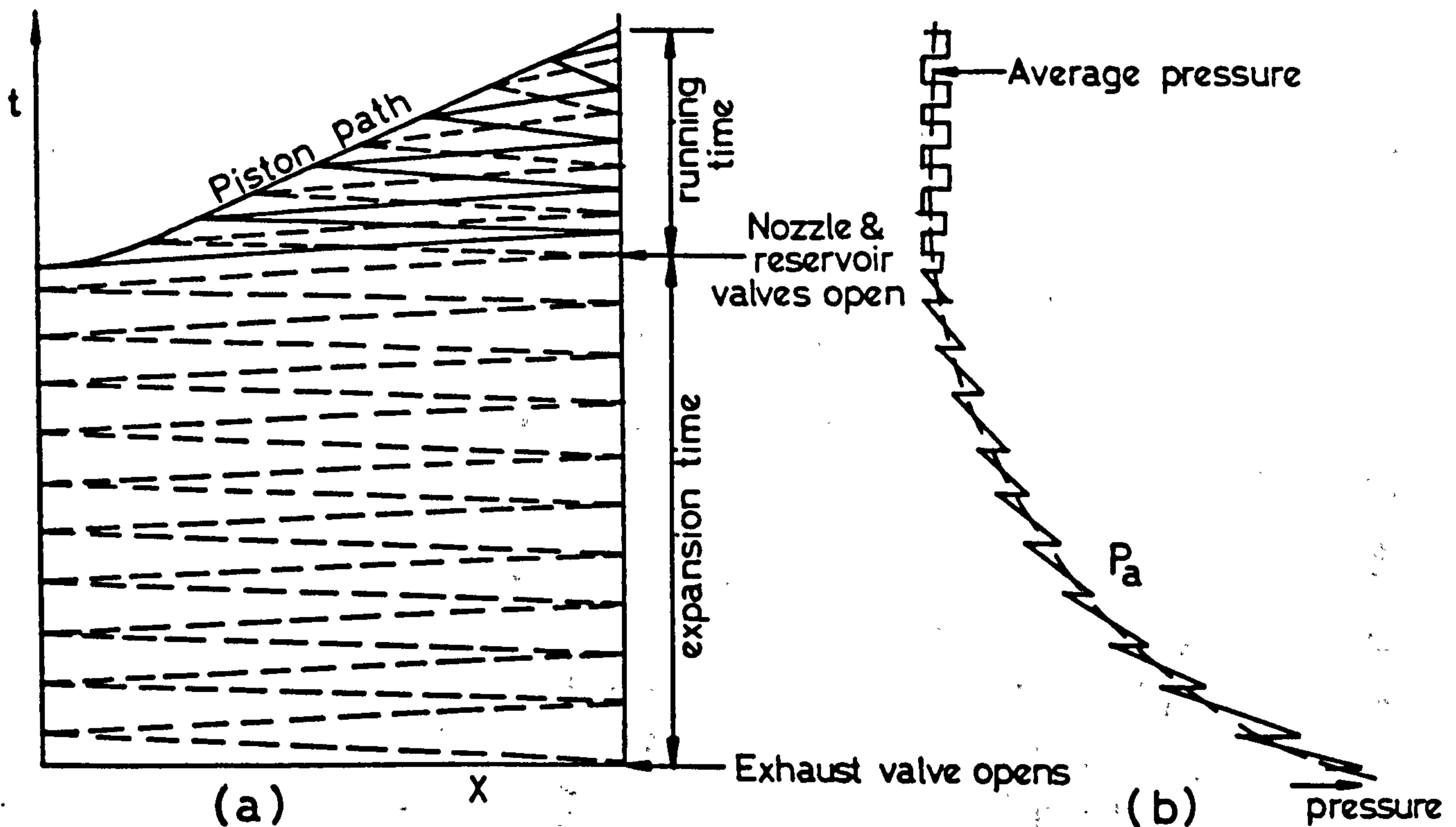


Figure 21. X-t diagram for the cryogenic piston tunnel.

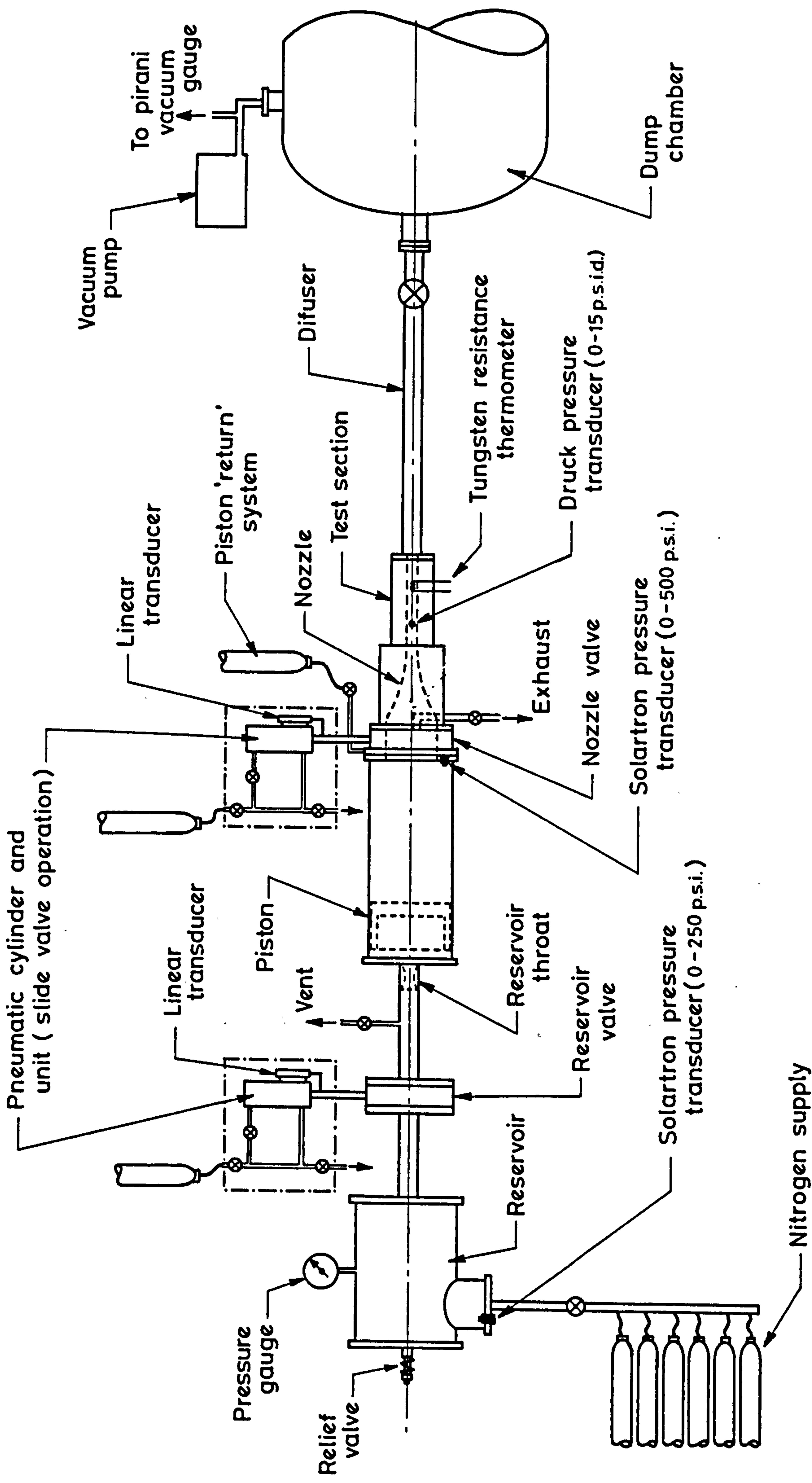


Figure 22. General arrangement of the pilot cryogenic tunnel.

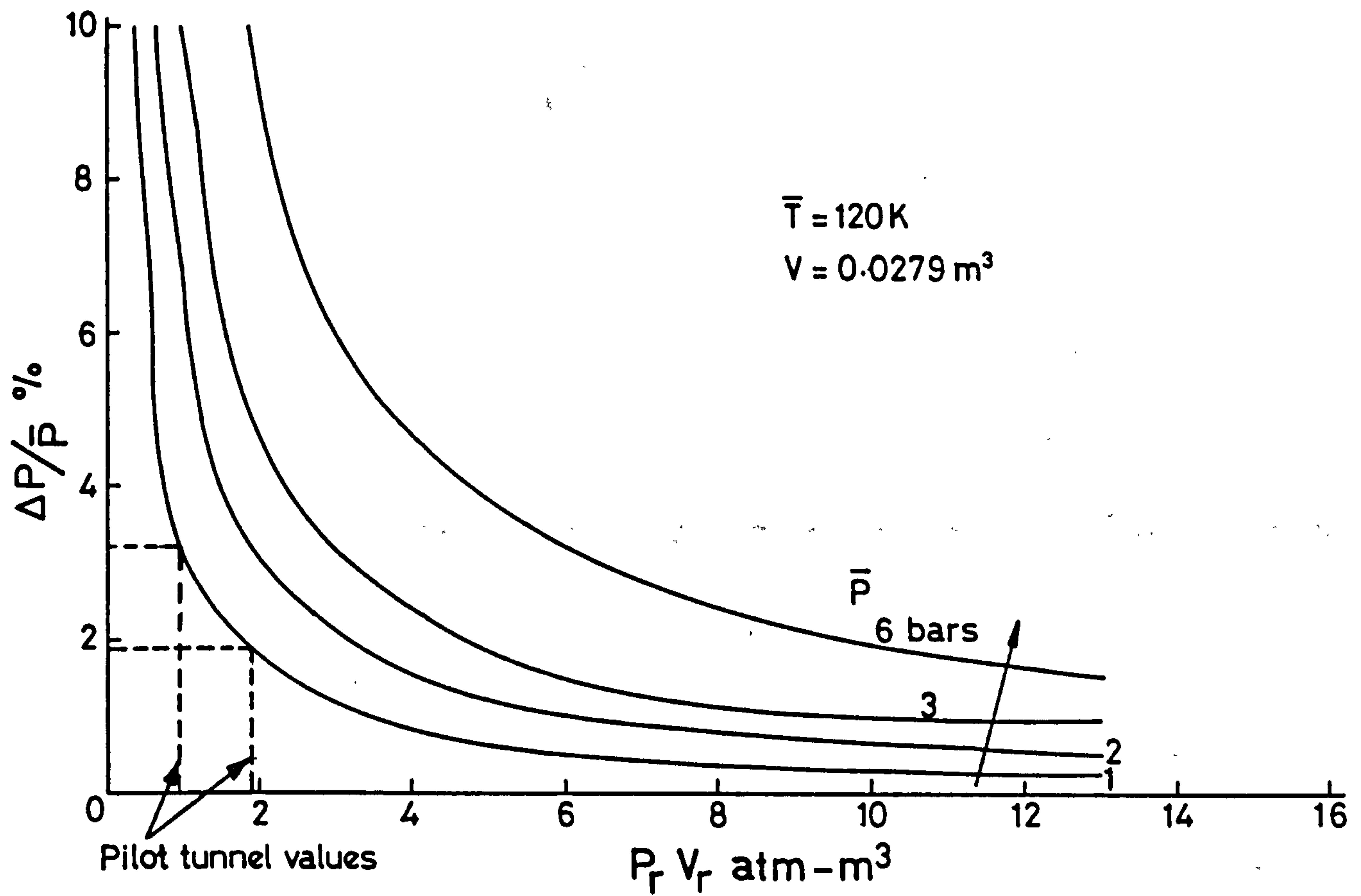


Figure 23. Decrease in total pressure due to finite reservoir volume for different total pressures.

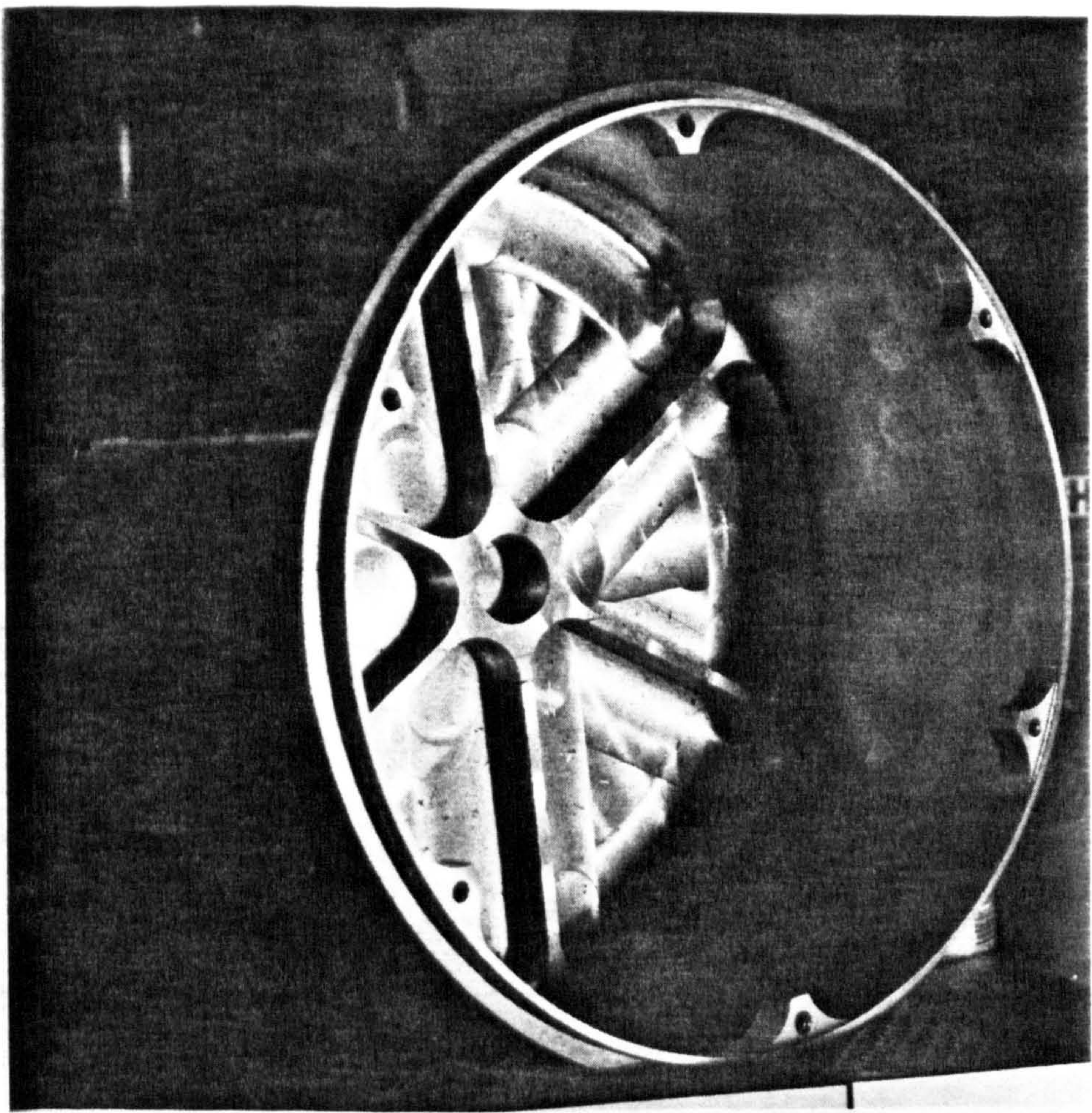


Figure 24. Photograph of aluminium piston (mass=1.14 kgs).

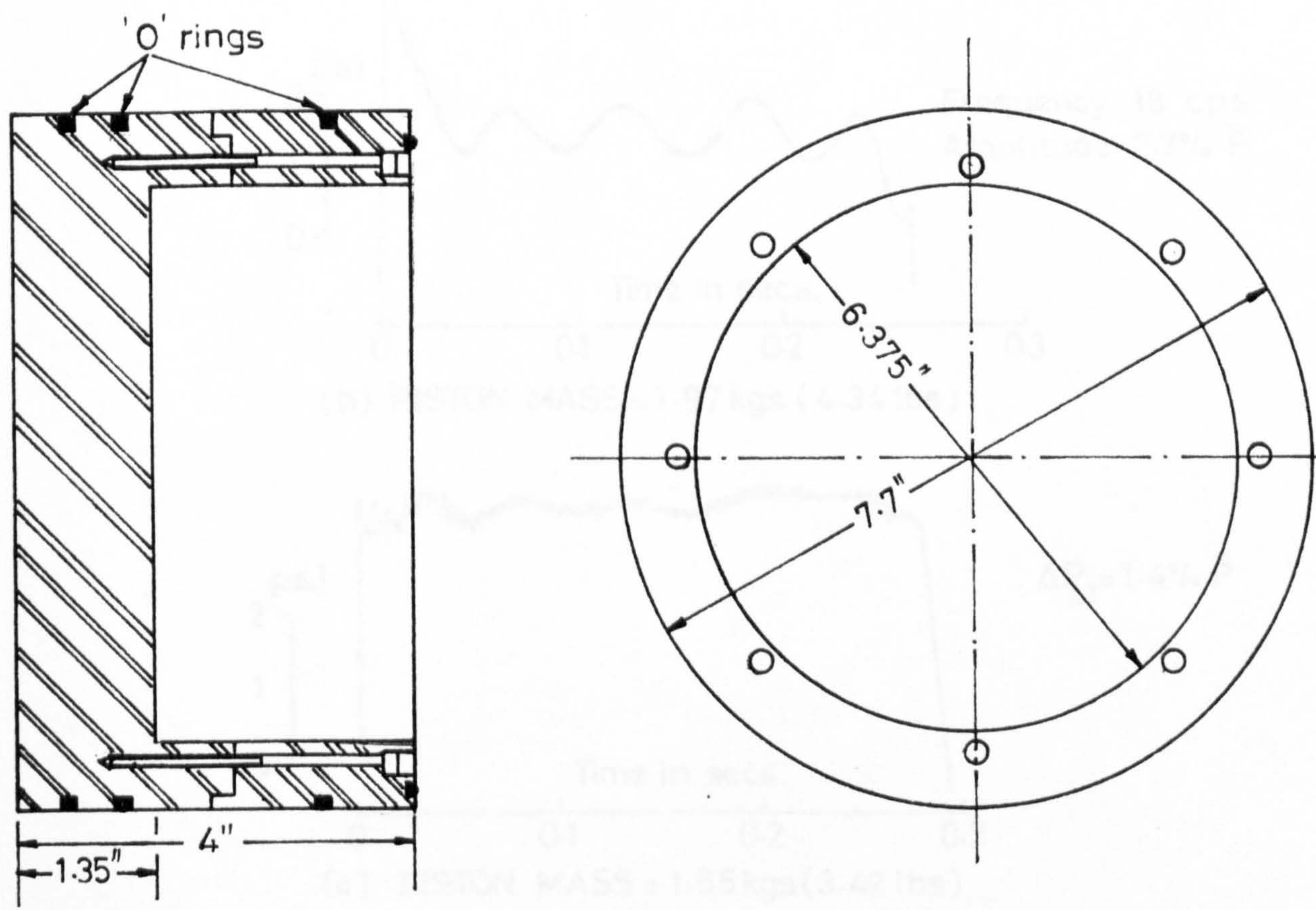
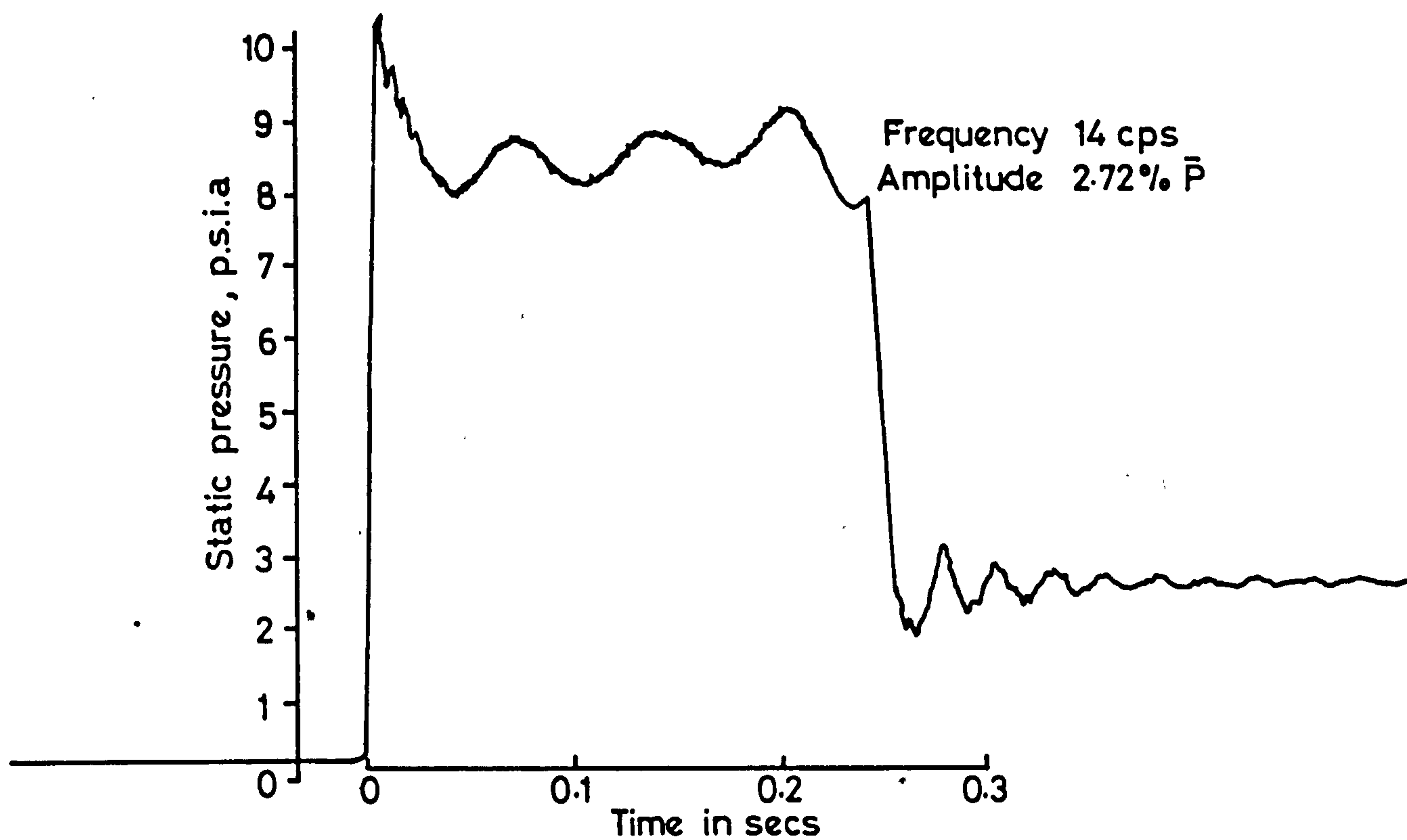
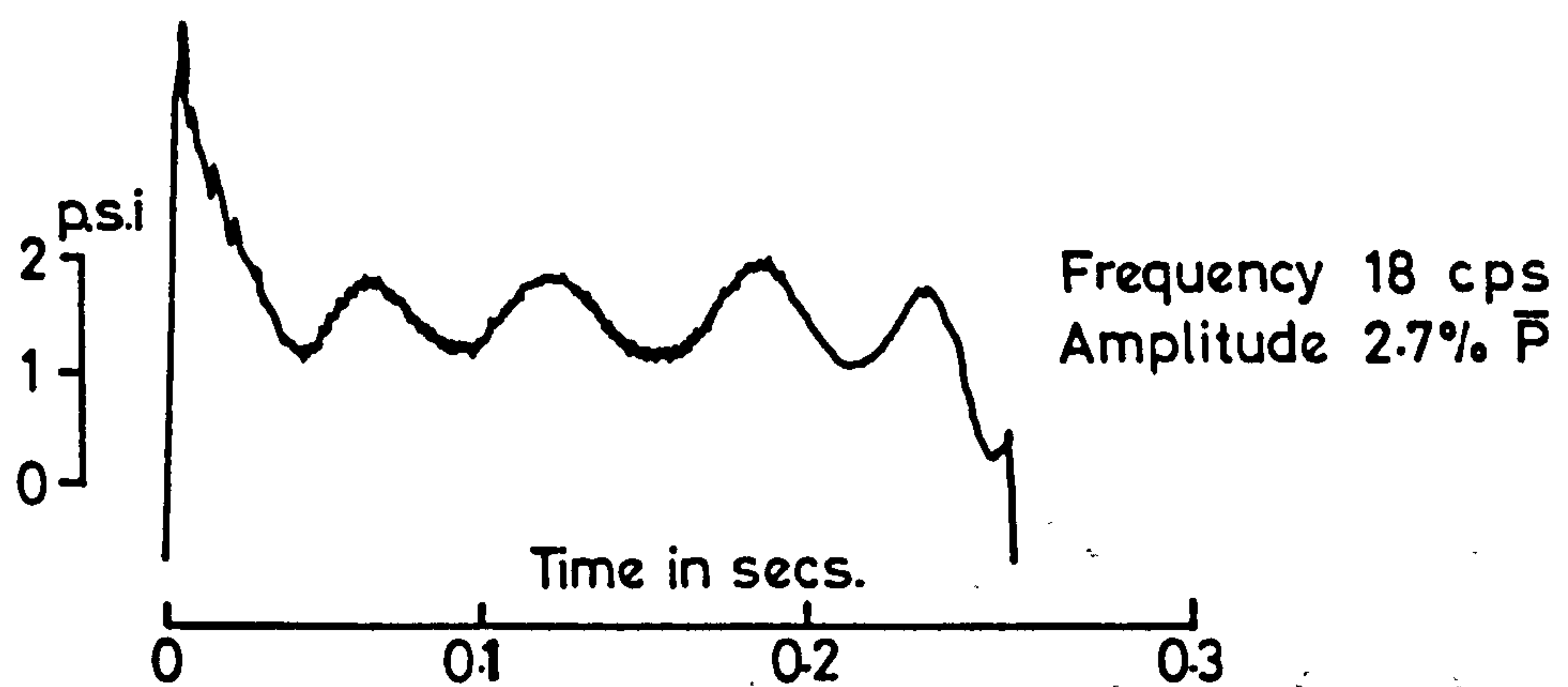


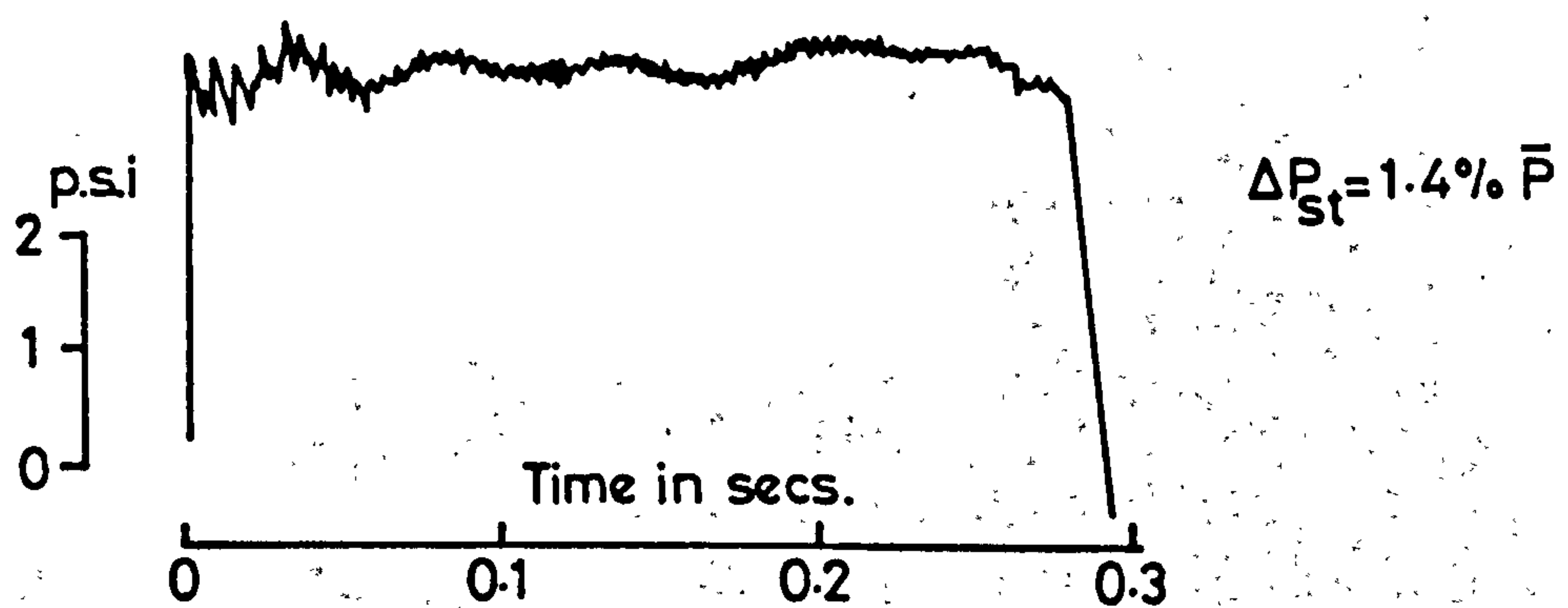
Figure 25. Diagram of 'Nylatron' piston (mass=1.97 kgs).



(a) PISTON MASS = 2.43 kgs. (5.34 lbs.)



(b) PISTON MASS = 1.97 kgs (4.34 lbs)



(c) PISTON MASS = 1.55 kgs (3.42 lbs)

Figure 26. Tunnel static pressure traces illustrating piston oscillations.

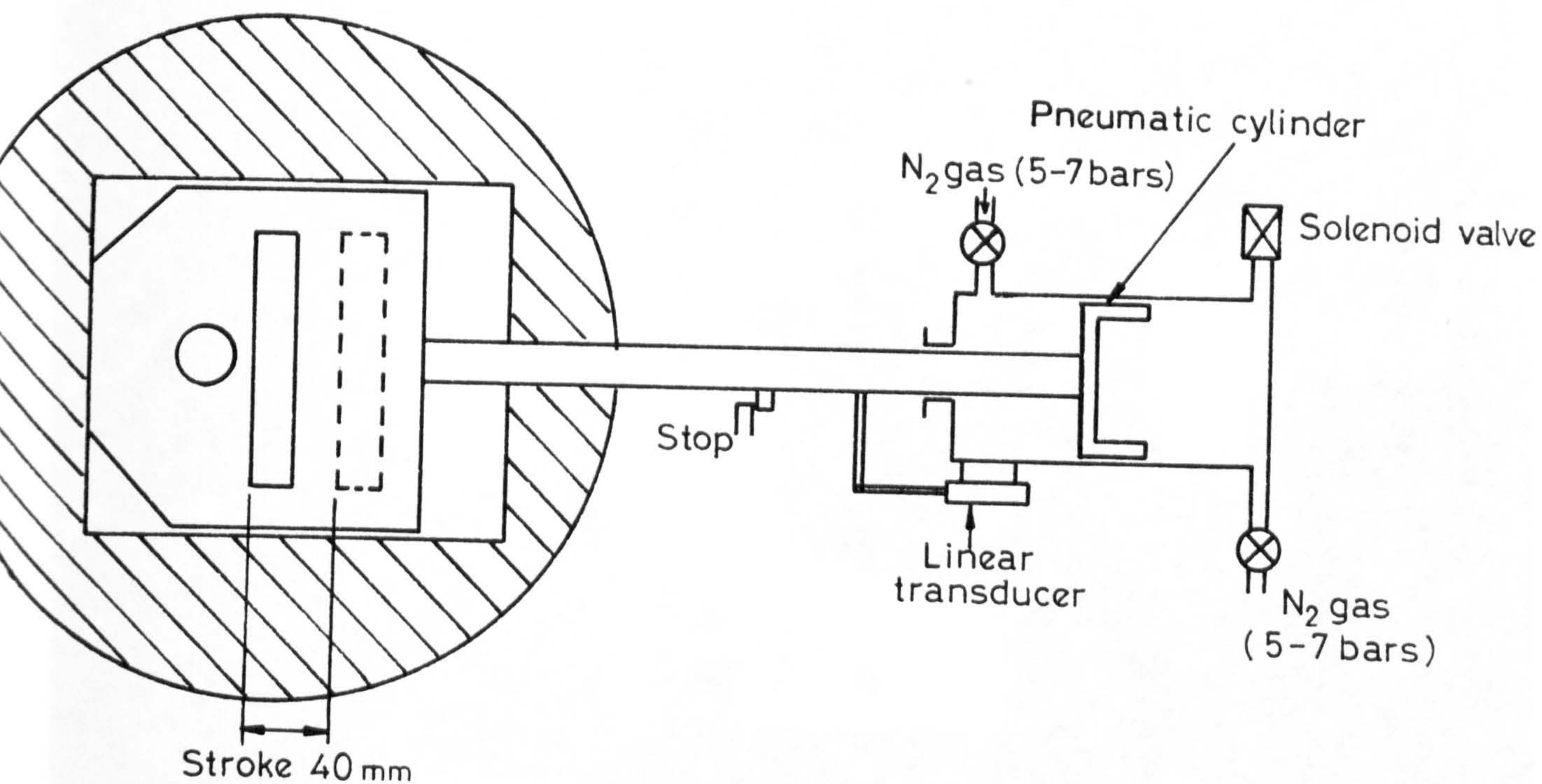


Figure 27.a. Schematic to show method of valve opening.

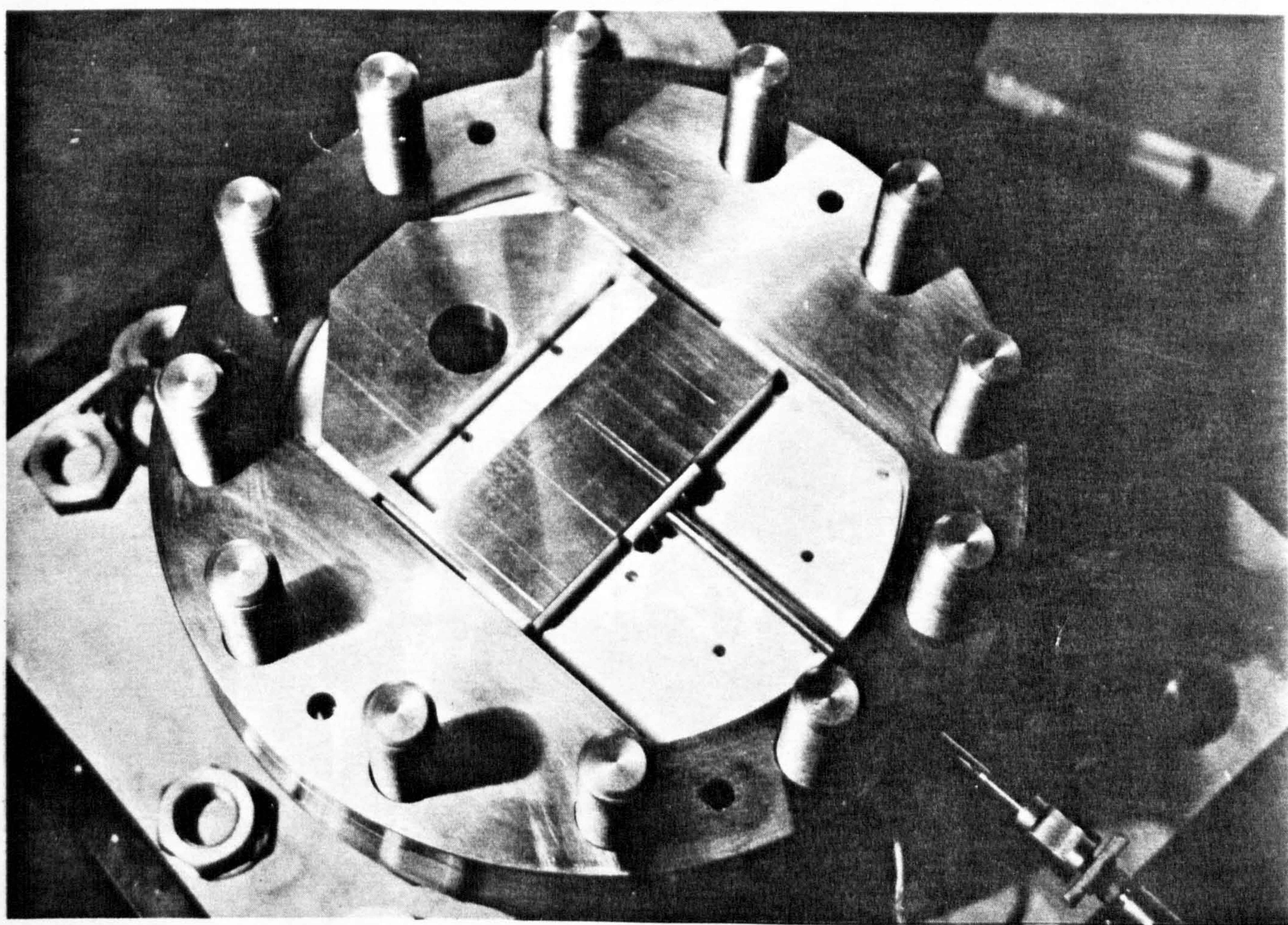


Figure 27.b. A view of the nozzle valve.

Figure 29. A view of the pilot cryogenic wind-tunnel.

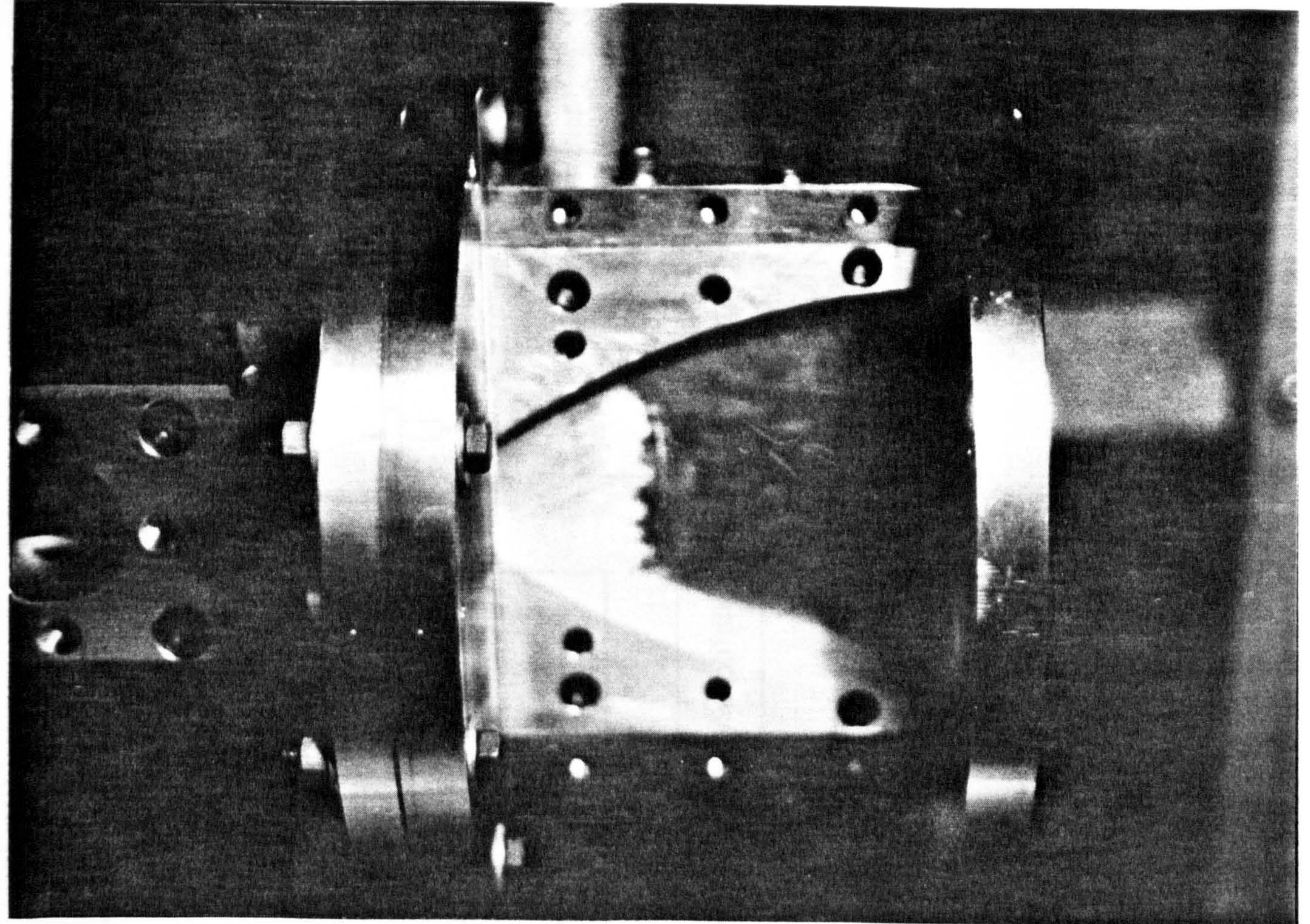


Figure 28. A view of the contraction.

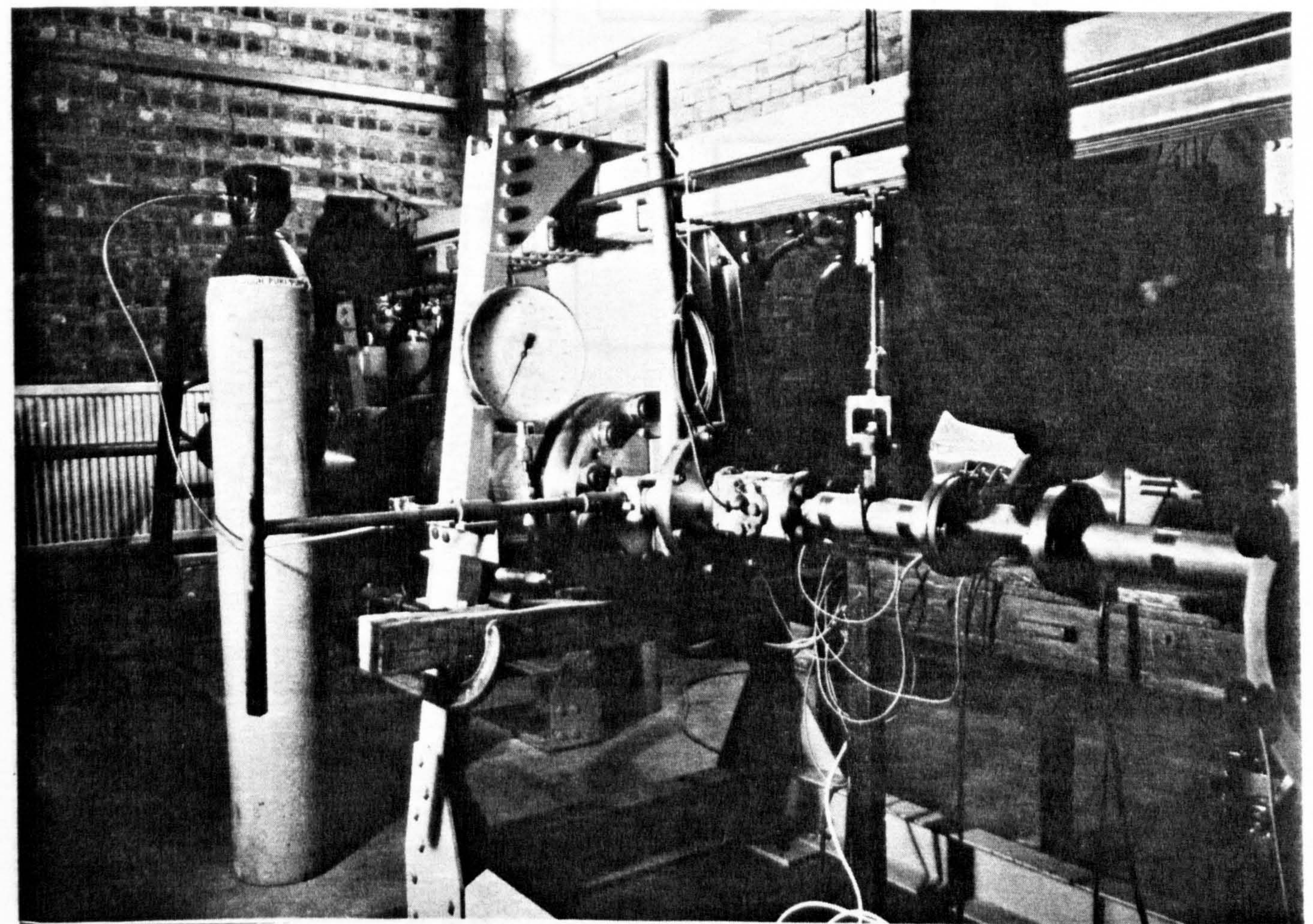


Figure 29. A view of the pilot cryogenic wind-tunnel.

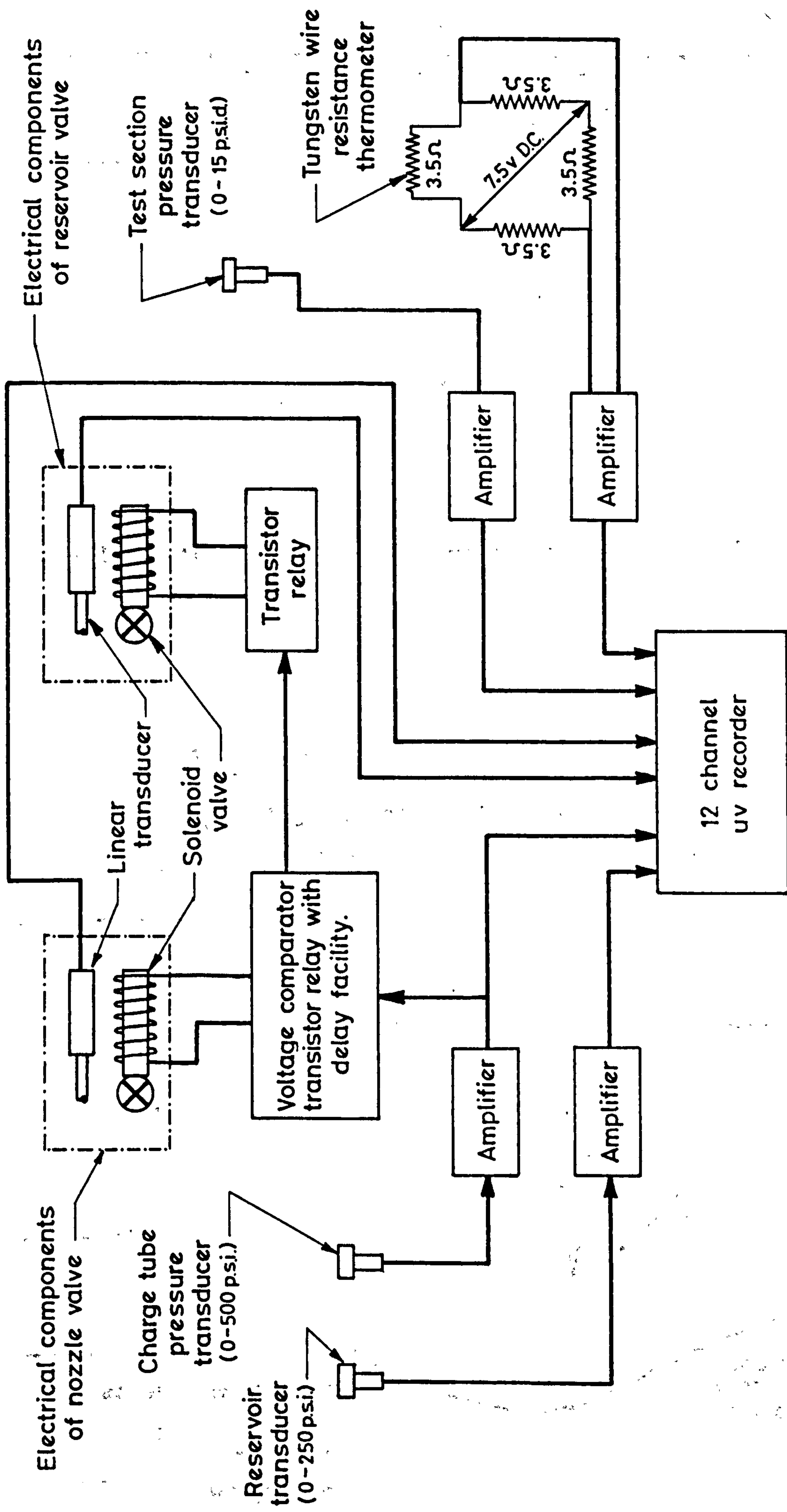


Figure 30. Instrumentation and control system diagram.

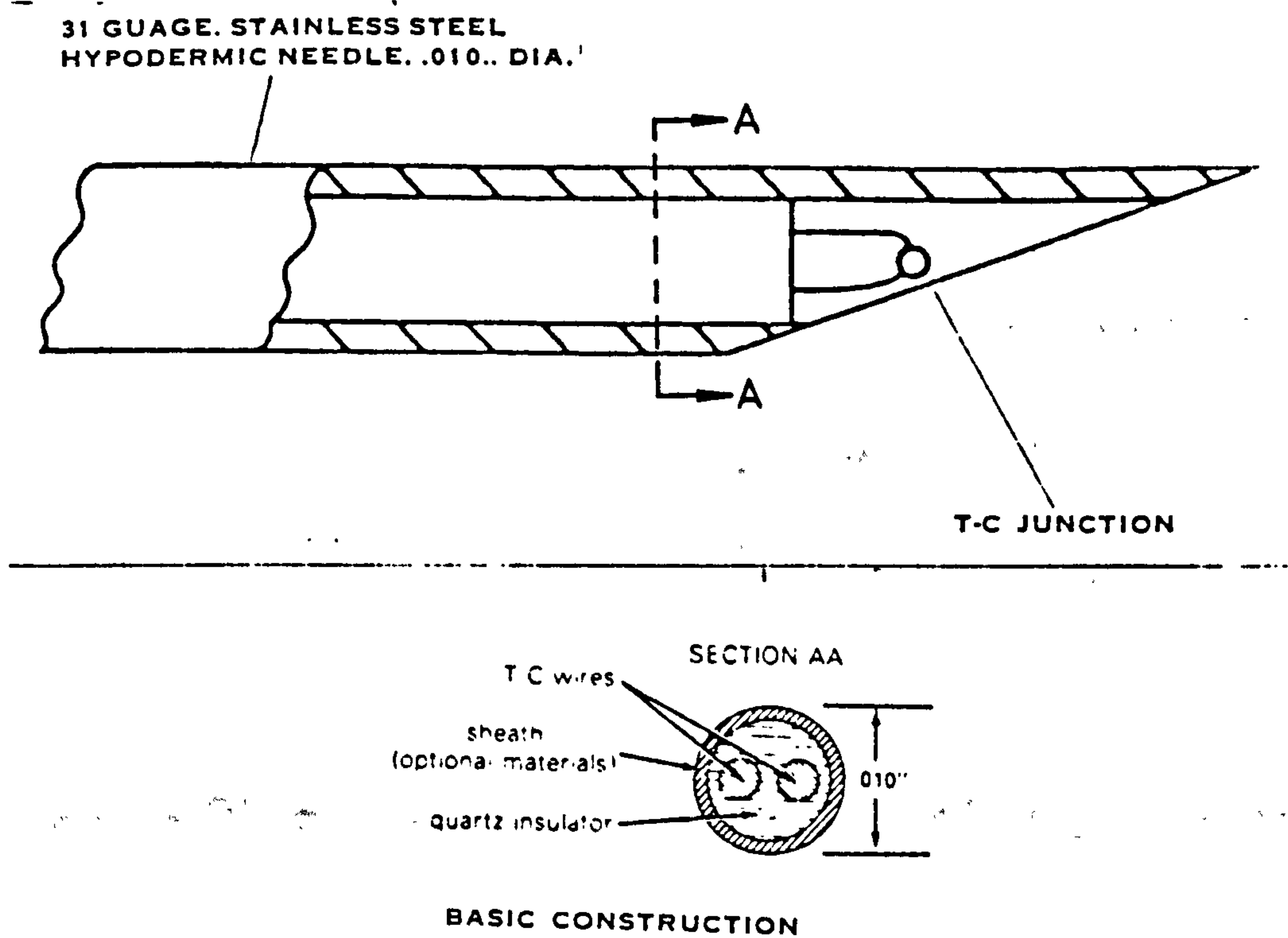


Figure 31. 'Ultraminiature' thermocouple needle.

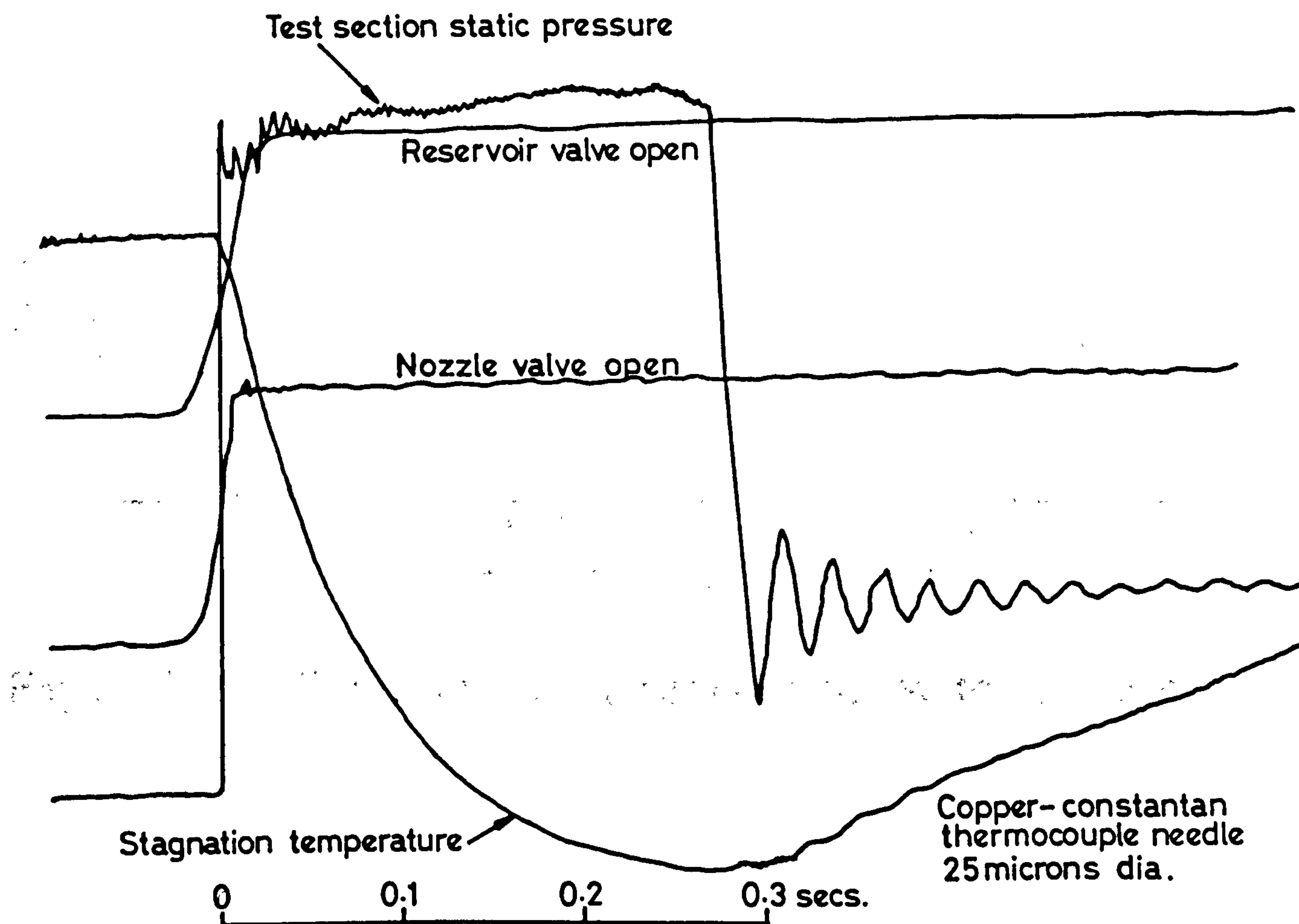


Figure 32. A typical record showing the poor response of the thermocouple.

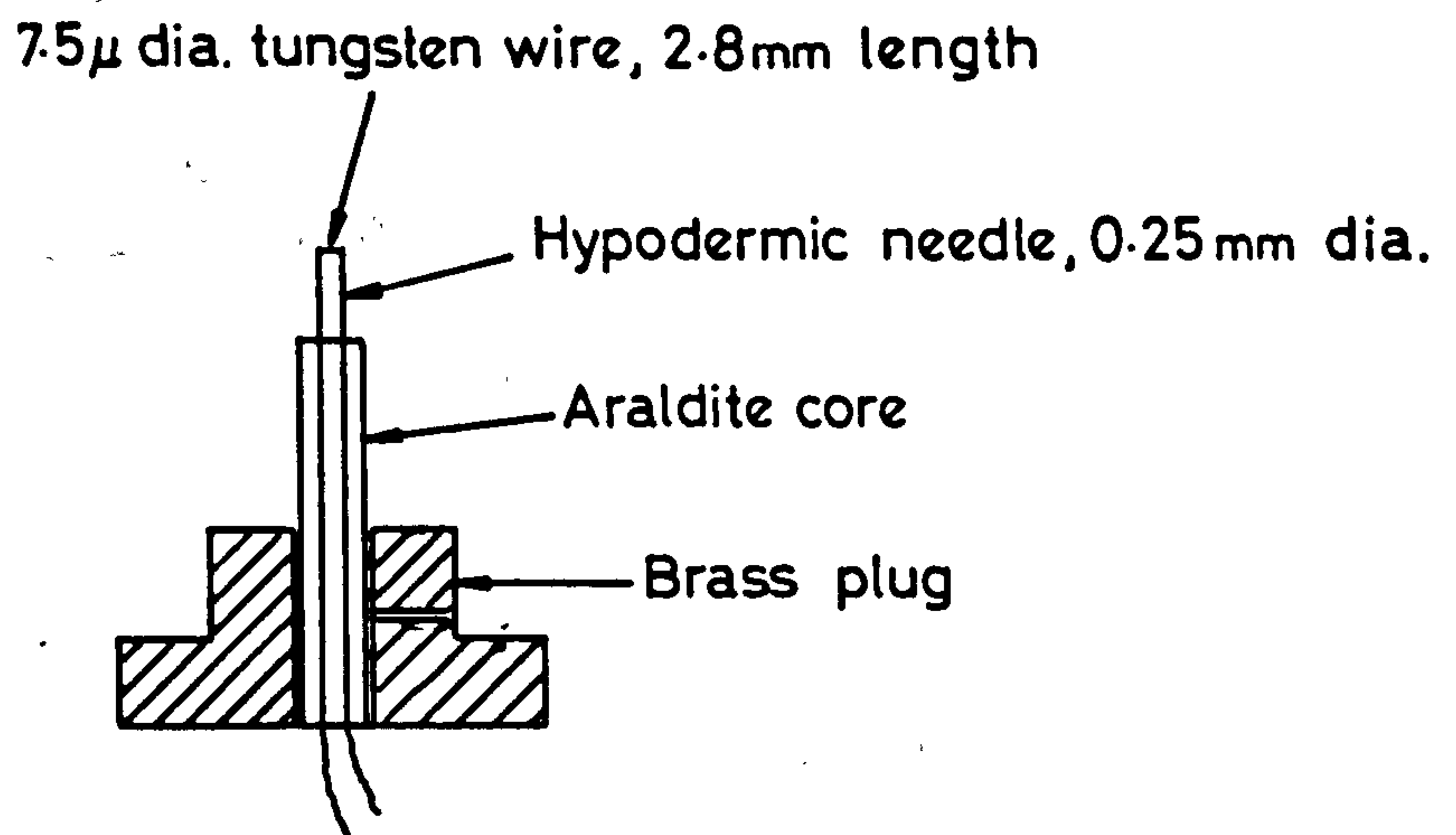


Figure 33.a. Tungsten wire resistance thermometer.

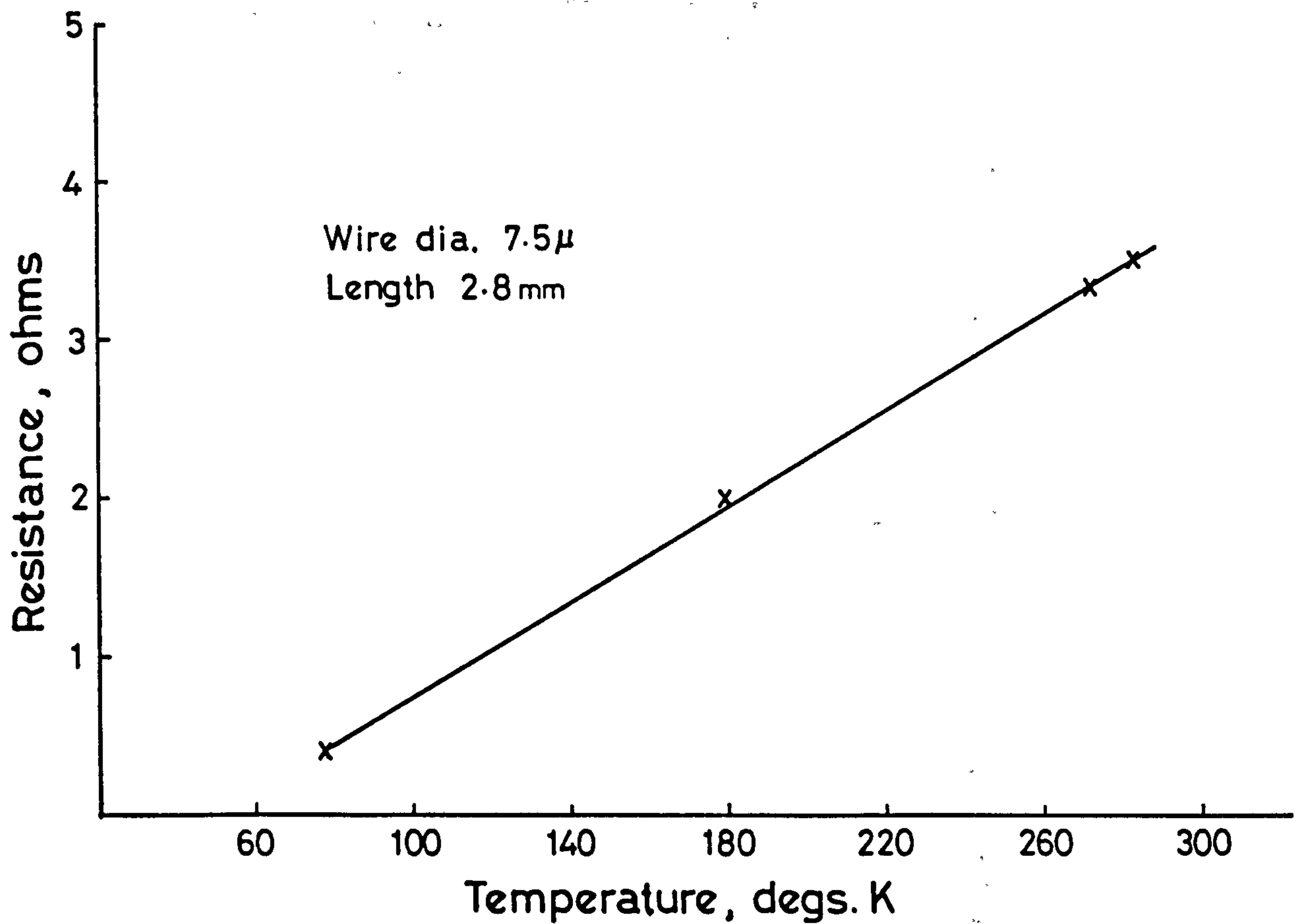


Figure 33.b. Tungsten wire resistance vs. temperature.

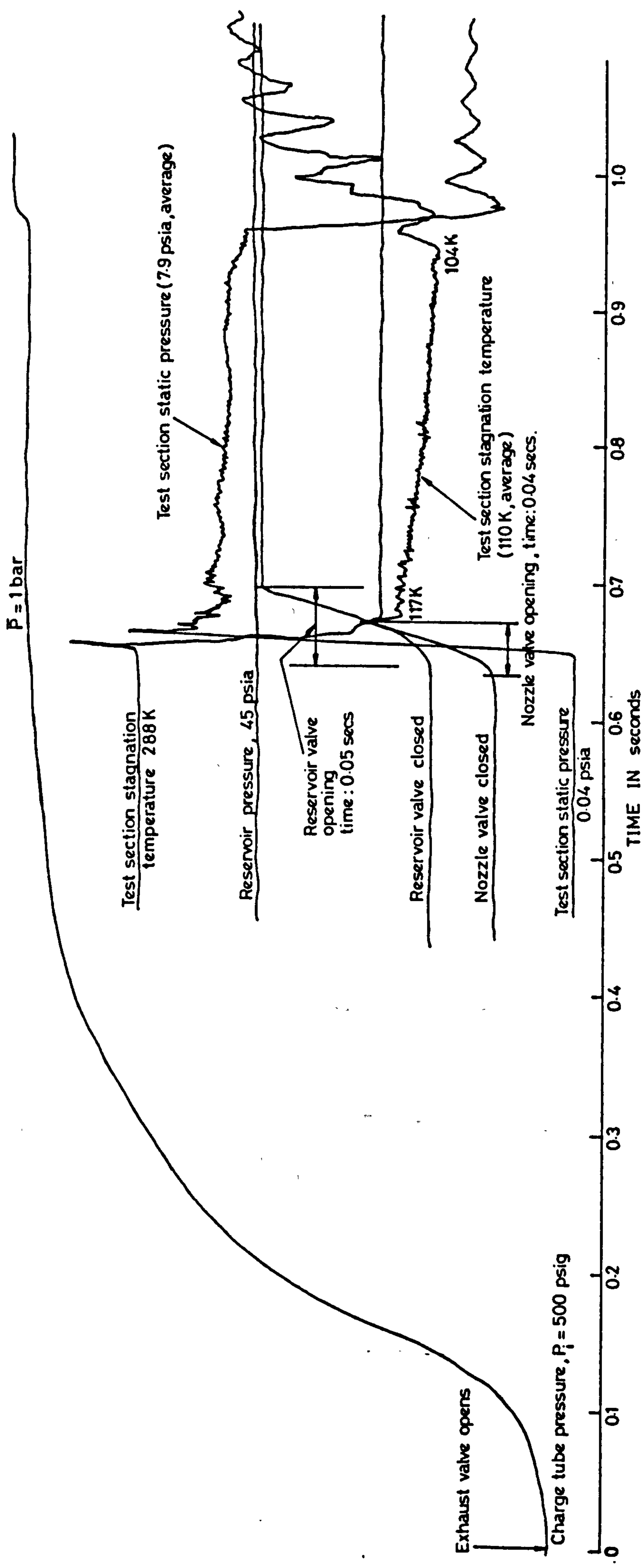


Figure 34. Typical results of the pilot cryogenic tunnel (running time = 0.29 sec., Mach No. = 0.985).

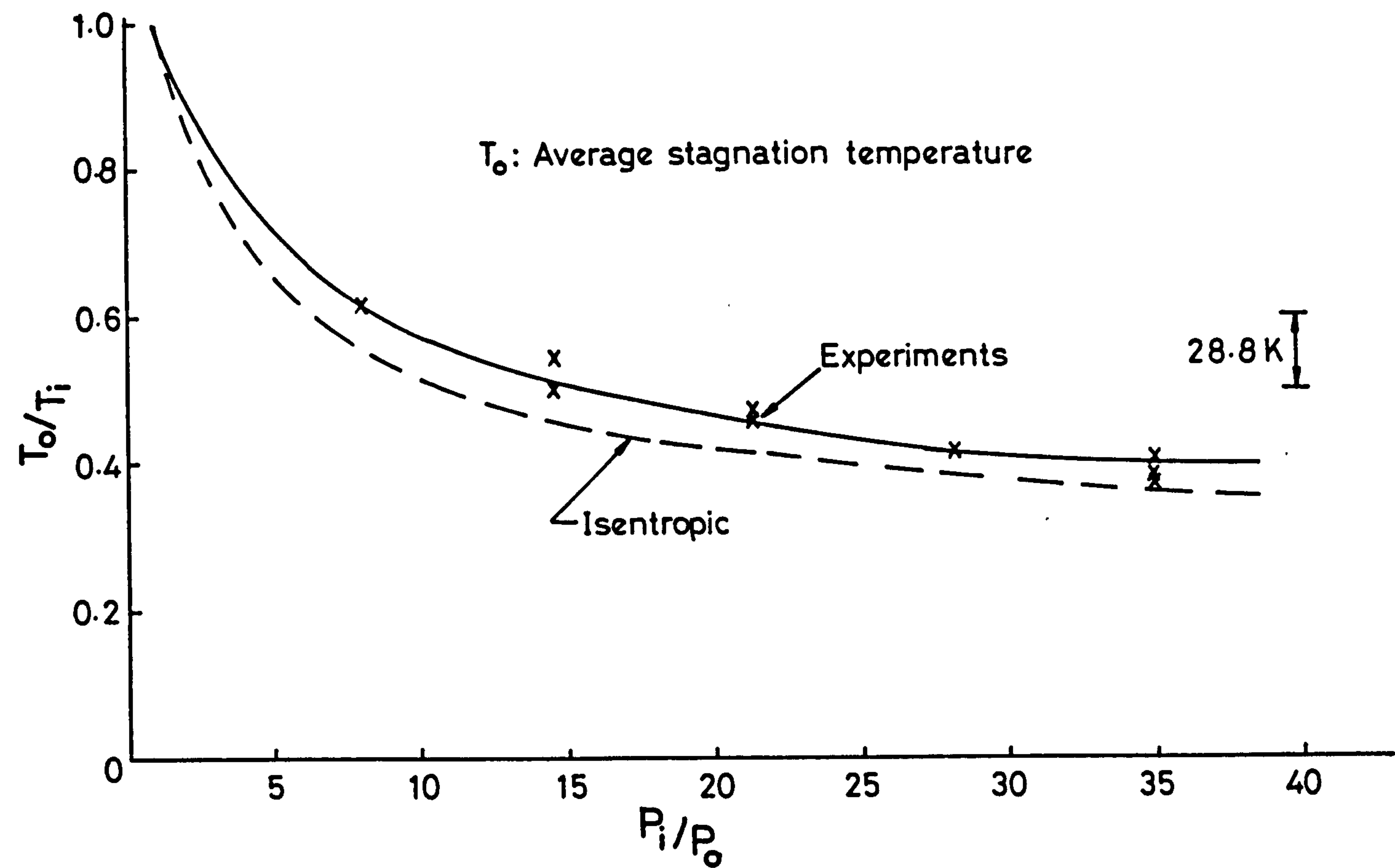


Figure 35. Pressure ratios vs. temperature ratio for the pilot cryogenic tunnel.

PART II

PERIODIC FLOWS ON RIGID AEROFOILS AT
TRANSONIC SPEEDS

PART IINOTATION

c	Aerofoil chord
C_p	Pressure coefficient
d	Total plenum chamber depth/test-section height
f	Frequency, Hz
H_T	Test-section height
K	Transonic similarity parameter
K_1	Constant (Section 3.2.6)
l	Length of separated region
M_∞	Freestream Mach number (uncorrected)
M_1	Local Mach number upstream of shock
M_{cr}	Critical Mach number
$N, F(N)$	Tunnel unsteadiness parameters (Section 3.2.7)
p	Local static pressure
P_∞	Freestream static pressure
ΔP_{max}	Maximum amplitude of pressure fluctuations
$\bar{p}$	r.m.s. value of pressure fluctuations
P_1	Plenum chamber static pressure
Re	Reynolds number based on chord
T	Perforated wall thickness
x	Distance from leading edge, Distance from nozzle throat
x_s	Distance of shock from leading edge
α	Incidence degs (uncorrected)
γ	Ratio of specific heats

NOTATION contd.....

τ Thickness-chord ratio

ω Circular frequency, $2\pi f$ rads/sec

1. INTRODUCTION

Shock-waves forming on wings in high speed flight interact with the boundary-layer flow and often lead to the flow separating from the surface. The flow separation caused by a shock-wave can lead to flow unsteadiness involving fluctuations in pressure over the aerofoil and in the wake, sometimes accompanied by considerable movements of the shock. There is evidence from flight tests (Refs. 6,10) that buffeting is associated with shock-induced separation. Other unsteady flow phenomena initiated by shock-induced separation are control surface oscillations ('aileron buzz', Refs. 10, 23-26), oscillatory flows in supersonic intakes ('intake buzz', Ref. 27) and wing-drop (Ref. 6).

The earliest example of an instability arising from shock-induced separation was reported by Daley & Humphreys (Ref. 9) who observed flow oscillations on thick aerofoils for propeller-shank sections (15 - 40% thickness-chord ratio) at Mach numbers 0.05 above the critical Mach number. Leipmann (Ref. 2) observed similar shock-wave oscillations on profiles in transonic flow but showed that the tunnel itself was a source of disturbance which produced the unstable flow unless it had a sonic throat downstream of the model. Holder & North (Ref. 1) noticed shock oscillations on an EC 1250 aerofoil with a laminar boundary-layer. Surface pressure and normal-force fluctuation measurements on NACA 65A series aerofoils with thicknesses ranging from 4 to 12% chord showed that many of the aerofoils exhibited random fluctuations, but one aerofoil (12% thick) with a laminar boundary-layer produced a periodic type fluctuation (Refs. 3,4). Polentz et al (Ref. 5) measured the unsteady normal-force coefficients on several profiles and found that an NACA 65-213 profile experienced periodic forces at high lift coefficients, under conditions for which shock-induced separation would be expected. From these and other investigations at NPL, Lambourne (Ref. 10) concluded that large scale flow oscillations can occur for aerofoils with high thickness-chord ratios and laminar boundary-layers and that definite frequencies are only associated with aerofoils with thicknesses greater than 10%.

Wind-tunnel noise and turbulence can also cause unsteady effects in transonic flows. Freestone & Cox (Ref. 8) noticed upstream travelling sound waves in a perforated test-section (without a model) at Mach numbers of 0.8 and above. In a study of transonic channel flow with one wall perforated, Meyer & Hiller (Ref. 7) observed shock oscillations in the absence of a model in the test-section. The instability was associated with the separation of the boundary-layer on the perforated wall.

Thick wings are of interest since they allow reduced wing structural weight or increased aspect ratio. Consequently there is considerable interest in the use of thick supercritical wings for civil aircraft operating at high subsonic speeds. A difficulty with the use of these thick wings could be the existence of periodic flows at transonic speeds and low angles of attack. With a periodic flow the structural implications for a wing could be very serious particularly if the frequency of periodic flow coincides with a structural frequency.

Transonic flows past thick biconvex aerofoils have been reported by Mabey (Refs. 13,14), Finke (Refs. 11,12), McDevitt (Ref. 15) and Levy (Refs. 16,17). These investigations have established that over a narrow range of Mach numbers, periodic flows occur involving shock oscillations in antiphase on opposite surfaces of the aerofoil at a frequency parameter ($\omega C/U_\infty$) about 1.

In this experimental investigation the flow at transonic speeds on five representative aerofoil sections were studied to examine the existence of periodic flows and to determine the conditions associated with them. An attempt has been made to explain the physical mechanism causing the periodic flow by drawing an analogy with the flow on an aerofoil with an oscillating flap (Ref. 40).

The tests were made in a 63.5 mm x 57 mm (2.5 inch x 2.24 inch) perforated wall transonic wind-tunnel which was developed from an existing supersonic wind-tunnel. The five aerofoil sections chosen were 6 and 14% biconvex aerofoils, an NACA 0012 section, a supercritical aerofoil CAST 7 and a symmetrical 11.8% Joukowski aerofoil. Tests on

the 6% biconvex aerofoil were made to demonstrate steady flows on a thin aerofoil. The results on the 14% biconvex aerofoil provided a bench mark reference for comparison of periodic flows with other aerofoils and with the theories currently being developed for unsteady transonic flows (Refs. 14-19). Tests on the NACA 0012 aerofoil were made since it is a standard aerofoil chosen by AGARD for testing transonic aerofoil computations. The CAST 7 aerofoil was chosen as a representative of the supercritical group of aerofoils. Tests on the Joukowski aerofoil were made to check potential flow theory (Ref. 67).

2. EXPERIMENTAL DETAILS

2.1 Development of the 2.5 inch Transonic Wind-Tunnel

The problems encountered in transonic testing in a solid wall wind-tunnel or an open jet wind-tunnel are well known (Refs. 42,43). At transonic speeds shocks and expansion waves developing in the vicinity of the model will be nearly normal to the flow. If shocks are allowed to strike a solid wall, they will reflect onto the model giving unrepresentative test conditions. Tests at Mach numbers close to sonic are limited in solid wall tunnels because of the choking phenomenon (Ref. 43). In an open jet wind-tunnel shocks will be reflected from the free boundary as expansion waves and will strike the model, again leading to false flow. A wind-tunnel with a proper arrangement of solid and open wall elements overcomes the problem of choking and minimises the effects of reflections of shock-waves and expansion waves.

A perforated wall transonic wind-tunnel was developed from an existing 6.35 cm x 6.35 cm (2.5 inch x 2.5 inch) wind-tunnel. This intermittent wind-tunnel is 'driven' by the suction of air at atmospheric pressure into two evacuated tanks. These tanks have a volume of 91 cubic m (3200 cubic ft) and are evacuated by a 50 kW electrically driven vacuum pump. A silica-gel bed at the beginning of the inlet duct dries the air and prevents condensation. The contraction in the wind-tunnel has an area ratio of 144 : 1. The tunnel running time is about 30 secs.

Some general guidelines for the design of a perforated wall test-section are:-

- (1) The minimum hole diameter should be greater than or equal to twice the boundary-layer displacement thickness. This criterion is based on the need for perforated walls having linear flow characteristics (Ref. 44).

- (2) The maximum diameter of the holes should not exceed 1/50th of the test-section height in order to keep the amplitude of disturbances caused by the perforations at the model location to a minimum (Ref. 44).
- (3) The partially open walls must be extended far enough upstream of the model to produce the required outflow of air into the plenum chamber ahead of the model. Generally this length is reckoned as three test-section heights (Ref. 42).
- (4) In the entrance region the open area must be gradually increased to allow a gradual expansion of the nozzle flow (Ref. 42).
- (5) Open-area ratios from 6 to 30% have been used in various tunnels. With the smaller open-area ratios it is difficult to generate the higher transonic Mach numbers because the pressure drop through the ventilated area would be higher (Ref. 42). With the higher open-area ratios a uniform pressure distribution in the test-section is difficult to maintain (Ref. 43).
- (6) Plenum chambers for wind-tunnel test-sections are usually of the order of the test-section volume (Ref. 45).

The turbulent boundary-layer on the test-section wall of the wind-tunnel is about 4 mm thick due to the long length of the inlet duct. An approximate estimate (Ref. 47) gives a value of 0.5 mm for the displacement thickness. Based on the requirements (1) and (2), 'Veroboard' liners (used for electronic circuit boards) with 1 mm diameter normal holes and an open-area ratio of 12% were chosen for the top and bottom walls. Plenum chambers with a total volume equal to test-section volume are located on the top and bottom walls. The side-walls of the tunnel are solid and are provided with 10 cm diameter glass windows for schlieren flow visualisation. At the end of the test-section the jet expands into a rectangular chamber 10 cm x 6.35 cm which leads into a 10 cm diameter diffuser pipe, 0.9 m long. A variable second throat (gate valve) located at the end of the diffuser pipe is used to vary the Mach number. The wind-tunnel details are shown

in Figs. 1 (a-c).

2.2 Mach Number Measurement

The stagnation pressure was measured using a pitot probe set up in the settling chamber and connected to a mercury manometer. This pressure was approximately 1.5 psi lower than atmospheric pressure due to the pressure drop across the silica-gel drier bed. The static pressures were measured via tappings suitably placed in the nozzle liner and plenum chambers.

The measured stagnation and static pressures determined the Mach number using the isentropic flow relations. With a model in the test-section the plenum chamber static pressure was used as a reference for Mach number measurement (Fig. 1a).

Figs. 2 (a-d) show the Mach number distribution along the nozzle for various divergence angles of the bottom liner. The angle of the bottom wall was increased in 0.5° steps to determine the position for the best Mach number distribution. It is seen that the flow is uniform upto a Mach number of 1.2 with a wall divergence angle of 2° . This wall angle was fixed for all later tests.

Pressure traverses across the test-section were made with the use of a static pressure probe and a traverse gear. As can be seen from Fig. 3 the flow is reasonably uniform upto a Mach number of 1.25.

Most of the pressure measurements were made using a multi-tube mercury manometer, but some measurements, especially the plenum chamber pressure (which is used to determine the Mach number) were made using a 'Texas Instruments' fused quartz precision pressure gauge. This instrument is accurate to a pressure of 0.002 psi.

Mach number corrections due to blockage are discussed in Appendix C.

2.3 Aerofoil Models

Five aerofoil shapes machined from brass plate were tested in the transonic tunnel. The models were:-

- (1) 6% biconvex aerofoil with a chord of 32 mm.
- (2) 14% biconvex aerofoil with chords of 16 mm and 32 mm.
- (3) NACA 0012 aerofoil with a chord of 25.4 mm.
- (4) CAST 7 supercritical aerofoil (11.8% thick) with a chord of 32 mm.
- (5) 11.8% thick Joukowski aerofoil with a chord of 32 mm.

The aerofoils are shown in Fig. 4 and their co-ordinates are tabulated in Tables 1-3 (Appendix D). The models spanned the tunnel and were held between circular windows by friction. The model incidence is set accurately by a travelling microscope.

2.4 Test Conditions

The aerofoils were tested over the high subsonic range of Mach numbers. The tests were made with both free transition and with transition fixed by a roughness band. The roughness band was Carborundum particles 180 microns dia., stuck on the aerofoil surface by a thin layer of durafix.

Fig. 5 shows the variation of Reynolds number based on the chord of the longer aerofoil (32 mm) versus Mach number. The range of Reynolds number is 0.4 to 0.5 million.

2.5 Test Techniques

The following test techniques were used:-

(1) Schlieren Photography

Schlieren flow visualisation was used for observing the flow past the aerofoil. The pictures were taken normal to the tunnel side-walls and the knife edge was adjusted to give good contrast. An argon-jet-stabilised spark source with a spark duration of 1 microsecond was used. Photographs were taken using 3000 ASA polaroid black and white film and the exposure time was equal to the spark duration.

(2) High Speed Schlieren Cinematography

To observe the shock movements on the aerofoil surface, schlieren pictures were taken with an 8 mm 'Hycam' high speed cine-camera. The camera was operated at 6300 frames per second. Ilford 200 ASA film was used and the effective exposure time was 60 microseconds. For these tests a mercury vapour lamp powered by a DC source (220 V DC, 150 W) was used. This system successfully recorded shock-oscillations at frequencies upto 1200 Hz.

(3) Surface Pressure Measurements

Surface pressure measurements were made only on the 14% biconvex aerofoil (32 mm chord). The size of the model permitted only seven pressure ports on the surface. The locations of the pressure ports are shown in Fig. 6. The pressure measurements were made using a 'Texas Instruments' fused quartz precision pressure gauge.

(4) Side-wall Pressure Measurements

Side-wall pressure measurements were made to detect any periodic flow over aerofoils and to measure the frequency and amplitude of pressure fluctuations. The measurements were made with a 'Kulite'

subminiature silicon diaphragm pressure transducer (5 psi differential) mounted on the test-section side-wall with its diaphragm flush with the wall as shown in Fig. 7. The transducer has a natural frequency of 70 kHz. The signal from the transducer was recorded on a 'Datalab' transient recorder. The signal was sampled for 40 msec at the rate of 1 point every 10 microseconds and then replayed onto a pen recorder for permanent record. In some of the tests the signal from the pressure transducer was amplified and recorded on a storage oscilloscope. The oscilloscope screen was then photographed using polaroid film.

For further data analysis the signal was recorded on an 'Ampex' tape-recorder (0 - 20 kHz) at a tape speed of 60 inches/sec. The sampling time was 10 secs. Static calibrations were made using a quartz pressure gauge.

(5) Oil-flow Visualisation

For observation of the surface flow, a mixture of motor oil, titanium dioxide and oleic acid was applied to the model. The tunnel had to be run about 5 times to produce a good flow pattern. The aerofoil surface was coated with black paint to produce a good contrast.

Table 4 (Appendix D) shows the various techniques used on the aerofoils.

3. EXPERIMENTAL RESULTS AND ANALYSIS

3.1 6% Biconvex Aerofoil

Figs. 8 (a-f) are schlieren pictures of the flow past a smooth aerofoil. A weak shock first appears at a Mach number of 0.84 at 44% chord. Spreiter's theory predicts a critical Mach number of 0.82 for this aerofoil (Fig. 14). The shock moves downstream with increasing Mach number and at a Mach number of 0.96 a lambda shock pattern is formed (Fig. 8f).

Figs. 9 (a-f) are the schlieren pictures for the case of the aerofoil with transition fixed at 25% chord. A weak shock appears at a Mach number of 0.84 at 56% chord. The oblique shock at 25% chord is due to the roughness. The shock moves downstream to 65% chord at a Mach number of 0.92.

In both cases, the flow is steady at all Mach numbers and separation occurs at or near the trailing-edge.

3.2 14% Biconvex Aerofoil

Typical schlieren pictures are shown in Fig. 10 for the free transition case. For this aerofoil the critical Mach number is about 0.73 (see Fig. 14). At a Mach number of 0.850, the top and bottom surface lambda shocks are antisymmetric suggesting a shock oscillation. The separation occurs alternately at the base of the shock and trailing edge on the 2 surfaces (Fig. 10c). The oscillation still persists at a Mach number of 0.900 (Fig. 10d). The shocks are symmetric again at a Mach number of 0.950 but the separation occurs at the foot of the shock. The wake is large in extent (Fig. 10e). A few weak shocks are also seen behind the main lambda shock.

Figs. 11 (a-m) illustrate the flow past the aerofoil with fixed transition at 25% chord. A weak shock is seen on the aerofoil at 0.58c at $M_\infty = 0.82$. Also there is a small separation at 0.86c. The shock intensifies and moves rearwards with increase in Mach number, to 0.67c

at $M_\infty = 0.85$. The separation point moves slightly forward (Fig. 11 f,g). Two flow pictures at $M_\infty = 0.86$ taken an arbitrary time apart (Fig. 11 h,i) suggest a shock oscillation, with the shock vanishing for a part of the cycle alternately on the top and bottom surfaces of the aerofoil. The separation alternates between the trailing-edge and the foot of the shock on the opposite surfaces. The unsteady flow persists at $M_\infty = 0.90$ (Fig. 11 j,k). The flow is steady again at $M_\infty = 0.95$ (Fig. 11 l,m) and the separation occurs at the foot of the shock.

Figs. 12 and 13 are the flow pictures for the 14% biconvex aerofoil of smaller chord ($c = 16$ mm). The unsteady flow is again observed at Mach numbers 0.81 - 0.87 for the laminar case and 0.82 - 0.87 for the turbulent case. The same type of flow using a smaller model suggests that the unsteady flow is a feature of the aerofoil and is not the result of any tunnel interference effect.

Fig. 15 shows the measured steady shock position plotted against the transonic similarity parameter. The experimental points for the thin and thick aerofoils correlate well, indicating that blockage effects are not serious at these Mach numbers. Also shown is the theoretical curve due to Spreiter (Ref. 28) which uses the inviscid shock-jump relations and predicts shock positions to be downstream of the experimental values.

3.2.1 Development of Periodic Flow

High speed photographs (Figs. 16 and 17) illustrate the flow development past the aerofoil with fixed transition and at an incidence of 4° . The flow is steady at Mach numbers of 0.81 and 0.85 (Fig. 16 a and b). The periodic flow occurs at $M_\infty = 0.90$ as seen in Fig. 17. The shocks are displaced by about $0.10c$ to $0.15c$ and vanish for a part of the cycle. Fig. 18 is a plot of the shock position versus time, obtained from the high speed pictures. The top surface shock moves downstream from $0.6c$ to $0.69c$ and vanishes for a part of the cycle. The bottom surface shock moves upstream from $0.90c$ to $0.74c$ and also vanishes for a part of the cycle but with a phase lag with respect to

the top shock. The frequency of shock oscillation is 1.25 kHz and the frequency parameter ($\omega c/U_\infty$) is 0.85.

3.2.2 Type of Shock-Wave Motion

Tijdeman (Ref. 29) has classified the periodic motion of shock-waves into three types:-

(i) Sinusoidal Shock-Wave Motion (Type A)

The shock moves almost sinusoidally and remains present during the complete cycle of oscillation, although its strength varies.

(ii) Interrupted Shock-Wave Motion (Type B)

This motion is similar to type A, but now the magnitude of the periodic change in shock strength becomes larger than the mean steady shock strength and as a consequence the shock-wave disappears during a part of its backward motion.

(iii) Upstream-Propagated Shock-Waves (Type C)

Periodically a shock-wave is formed on one surface of an aerofoil. This shock moves upstream while increasing its strength. The shock-wave weakens again, but continues its upstream motion, leaves the aerofoil from the leading edge and propagates upstream into the incoming flow as a (weak) free shock-wave. This phenomenon is repeated periodically and alternates between upper and lower surface.

According to this classification, the periodic motion of shock-waves on the 14% biconvex aerofoil should be classified as type B.

3.2.3 Frequency Parameter

The frequency of the oscillatory flow was accurately measured by making side-wall pressure measurements (described in Section 2.4). Figs. 19 a and b show the pressure traces for the case of free-transition and fixed-transition respectively, at Mach numbers of 0.82 to 0.95. These figures show that the flow is oscillatory rather than unsteady. Fig. 20 shows the pressure traces for the case of the test-section without the aerofoil model, for comparison. The frequency measurements are expressed as a dimensionless frequency parameter ($\omega c/U_\infty$). Fig. 22 shows the frequency parameter plotted against Mach number. The frequency parameter is of the order 1.0.

The frequency parameter is probably fixed by the time taken for the disturbance to travel from the trailing edge to the mean shock position (frequency being inversely proportional to the distance 'l' from the trailing-edge to the mean shock position) (Ref. 13). As the Mach number increases the shock moves rearward and the frequency parameter increases slightly. This is seen in Fig. 22, but there is a minor anomaly at the Mach number of 0.88 for the laminar interaction (for explanation see Section 3.7). The shock position is rearward for the laminar interaction (thinner boundary-layer) than for the turbulent interaction at the same Reynolds number. Hence the frequency parameter is higher for the laminar interaction.

For the smaller chord aerofoil ($c = 16$ mm) periodic flow occurred at a somewhat lower Mach number 0.83 - 0.87 (Figs. 21 and 22) since separation occurs earlier at a lower Reynolds number. At the same Mach number the frequency parameter is lower for the shorter aerofoil than for the longer aerofoil ($c = 32$ mm). This is reasonable since at the higher Reynolds number the relative thickness of the turbulent boundary-layer is lower, so the shock is located more towards the rear thereby increasing the frequency. For the smaller aerofoil, the frequency increases first and then drops with increasing mach number. The roughness used for fixing transition is relatively large for the smaller aerofoil and gives rise to a strong oblique shock which interacts with the main shock. This probably has an effect on the

frequency of shock oscillation.

3.2.4 Instability Mach Number Range

Three types of flow were observed on the 14% thick biconvex aerofoil at high subsonic speeds:-

- (i) Steady flow with trailing edge separation at Mach numbers lower than 0.83 for laminar interaction and lower than 0.86 for turbulent interaction.
- (ii) Unsteady flow with separation alternating between the trailing-edge position and the base of the shock. This occurs at Mach numbers between 0.83 - 0.91 for the laminar interaction and between 0.86 - 0.90 for the turbulent interaction.
- (iii) Steady flow with shock-induced separation at Mach numbers greater than 0.91.

3.2.5 Power Spectra

The pressure traces (Figs. 19-21) have frequency components due to the tunnel noise in addition to the discrete frequency due to the aerofoil flow instability.

A frequency analysis was performed to determine the frequency composition of the data. The voltage signal from the pressure transducer was recorded on magnetic tape for a run duration of 10 secs at each Mach number and subsequently analysed to determine the frequency spectrum. The results are expressed in terms of the power spectral density function (p.s.d.) versus frequency (see Appendix A for details).

Fig. 23 (a-d) shows the p.s.d. vs. frequency for the empty tunnel at Mach numbers 0.81 - 0.94. The tunnel noise has a peak at 1600 Hz which is the resonance frequency of the perforated test-section plenum chamber combination (see Section 3.2.6).

Fig. 24 (a-c) is the power spectra for the case of flows past the aerofoil with free transition. At $M_\infty = 0.84$ the power spectra (Fig. 24a) shows 2 peaks at 1200 Hz and 1500 Hz. However the pressure traces obtained on a transient recorder show only one frequency at approximately 1400 Hz (Fig. 19 a-c). The unsteadiness probably occurs in bursts of 2 frequencies (1200 and 1500 Hz) and since the data has been sampled over a short duration of 40 msec on a transient recorder, only one burst of frequencies at approximately 1400 Hz has been captured. The power spectrum is calculated from data sampled over a longer time (approximately 10 secs) and hence represents a better average. At $M_\infty = 0.87$ and 0.90 the power spectrum shows that the shock-oscillation is locked into the tunnel resonance frequency (1600 Hz) and must be regarded as a forced oscillation (Fig. 24 b-c).

With transition fixed, the power spectrum indicates a peak at 1150 Hz at Mach numbers 0.86 to 0.899 (Fig. 25 a-c), corresponding to the shock-oscillation.

Some tests were also conducted to determine the frequency spectrum of the empty tunnel with the perforations sealed off with sellotape (Fig. 26 a-d). The r.m.s. value of pressure fluctuations with the perforations open and closed are of the same order of magnitude suggesting that most of the pressure fluctuations originate upstream of the test-section. The frequency peak at 1600 Hz is more dominant with the perforations open (for eg. Figs. 23a and 26a).

3.2.6 Resonant Frequencies of Ventilated Wind-Tunnels

The resonance frequency of a ventilated wind-tunnel depends on the freestream Mach number, the plenum chamber depth and a wall porosity factor $2K_1T/H_T$ (where K_1 is a function of the open area ratio,

T is the wall thickness and H_T is the tunnel height). Fig. 27 is a plot of resonance frequency versus Mach number for a perforated tunnel with normal holes ($2K_1T/H_T = 0.3$) for different plenum chamber depths, obtained by theory (Ref. 30) and experiment. For the 2.5 inch tunnel $2K_1T/H_T = 0.3$ and $d = 1.16$, the resonance frequencies obtained from the frequency spectrum (Figs. 23 a-d) are shown in Fig. 27. There is general agreement between our results and the trends indicated by the experiments of Ref. 30.

3.2.7 Magnitude of Side-Wall Pressure Fluctuations

The r.m.s. intensity of pressure fluctuations as measured on the side-wall (Fig. 7) are shown in Fig. 28 for the laminar and turbulent unsteady flow regions. Also shown is the empty tunnel pressure fluctuation level. The maximum pressure fluctuations occur close to the middle of the unsteady flow regime. The pressure fluctuations are significantly higher for the laminar interaction than the turbulent interaction. This is to be expected since turbulent boundary-layers are more resistant to separation than laminar boundary-layers.

The transducer being mounted on the side-wall of the tunnel, also receives excitation from the fully developed wall turbulent boundary-layer. This is a small, nearly-constant correction which is usually taken as $\bar{P}/q = 0.006$ (Ref. 33). The present results are however left uncorrected.

Fig. 29 compares the r.m.s. pressure fluctuations of the empty test section with the perforations open and closed. The pressure fluctuations with the perforations sealed are slightly higher, indicating that the tunnel unsteadiness is not associated with the perforated test section.

Owen (Ref. 33) and Mabey (Ref. 34) describe a method of presenting the tunnel unsteadiness. The results are expressed in non-dimensional

form by dividing the r.m.s. intensity of pressure fluctuations, $\bar{p}$ by the wind-tunnel dynamic pressure q . A non-dimensional frequency parameter $N = f_1 W / U_\infty$ is used where f_1 = frequency Hz, W = width of tunnel m, and U_∞ = velocity m/sec and a non-dimensional spectrum function $F(N)$ such that

$$\frac{\bar{p}^2}{q^2} = \int_{N=0}^{N=\infty} F(N) dN = \int_{\text{LogN}=-\infty}^{\text{LogN}=+\infty} N F(N) d(\text{logN}) \quad \dots\dots (1)$$

$F(N)$ represents the intensity of pressure fluctuations over the interval dN

Integration over any range of frequencies gives the corresponding mean square value of the pressure fluctuations. A modified form of presentation is used here to compare the present results with those measured in other tunnels. The tunnel unsteadiness is expressed as $\sqrt{NF(N)}$ vs. logN . The same data that were used to determine the power spectra are used to determine the value of $\sqrt{NF(N)}$ which is given by

$$\sqrt{NF(N)} = (\text{voltage}) \times \frac{K}{q \sqrt{\epsilon}} \quad \dots\dots (2)$$

where K = Static calibration (psi/volt),
 ϵ = Analyser bandwidth ratio $\Delta f/f$,
 q = Dynamic pressure (psi).

A constant value of $\epsilon = 0.1$ has been used in this analysis.

Figs. 30 (a-d) show the tunnel unsteadiness for Mach numbers 0.81 - 0.94. Also shown for comparison are the $\sqrt{NF(N)}$ values for the RAE 4 inch x 4 inch wind-tunnel with slotted and perforated working sections. The unsteadiness in the 2.5 inch tunnel is higher than the RAE model tunnel. To determine the causes would require more extensive measurements in the test-section, plenum chamber and diffuser and is beyond the scope of the present work.

3.2.8 Pressure Distributions

Pressure and Mach number distributions on the 14% biconvex aerofoil are shown in Figs. 31-34. Considering first the case of free-transition (Figs. 31,32), the flow is accelerated almost linearly upto 40% chord. For comparison the results of linearised theory at $M_\infty = 0.7$ and 0.817 (Ref. 39) are shown in Fig. 31. The decrease in the Mach number downstream of the maximum Mach number position is due to the impingement of the front shock on the boundary-layer.

Figs. 33 and 34 are the pressure distribution and local Mach number distribution for the case of fixed-transition at freestream Mach numbers of 0.85 and 0.91 which are just outside the instability region. The upstream influence of the shock is smaller compared with the free-transition case.

In the instability region, the Mach number upstream of the shock lies within a narrow range of 1.22 to 1.26 for fixed-transition and in the range 1.1 to 1.2 for free-transition.

3.2.9 Oil Flow Visualisation

Figs. 35 (a and b) are the oil flow pictures of the aerofoil surface, for the case of fixed-transition at Mach numbers 0.80 and 0.85 respectively. The flow looks quasi-two-dimensional for the central 70% of the span. At $M_\infty = 0.80$ there is separation at 89% chord. At $M_\infty = 0.85$ the oil flow indicates a shock at 69% chord and separation at 87% chord. These two pictures are for steady flow and the shock and separation positions compare well with the schlieren pictures.

3.2.10 Origin of the Periodic Flow

The experiments illustrate a periodic flow occurring on a rigid 14% thick biconvex aerofoil at transonic speeds. The frequency of the shock-oscillation is inversely proportional to the chord length (Section 3.2) indicating that the periodic flow is due to the aerofoil

and is not caused by any tunnel effects.

The instability is basically a viscous effect. If it were inherent in the transonic inviscid flow, then the instability would start and end at particular values of the transonic similarity parameter. Periodic flows on other biconvex aerofoils have been reported by Karashima (10% thick, Ref. 22), Mabey (12-16% thick, Ref. 13), McDevitt (18% thick, Ref. 15) and Finke (20% thick, Ref. 11). The instability Mach numbers in terms of the transonic similarity parameter (K) are shown in Fig. 36 as a function of thickness-chord ratio. The critical Mach number line due to Spreiter (Ref. 28) is also shown. The instability does not start and end at particular values of the similarity parameter, thereby indicating that it is not due to the transonic inviscid flow. Also the results of computations on an 18% thick biconvex aerofoil (McDevitt, Ref. 15) showed that when the viscosity was eliminated, the solutions showed a steady flow.

The frequency parameter is probably determined by the time taken for the disturbance to travel a length ' l ', from the trailing-edge to the mean shock position (l = length of separated region). Fig. 37 shows the frequency parameter based on ' l ' plotted against the similarity parameter. As can be seen, the frequency measurements for different thickness-chord ratios are well correlated. Fig. 38 is a plot of the ratio of frequency parameter to the empirical factor $A = \pi (1 - M_\infty) / 2M_\infty$ derived by Erickson and Stephenson (' A ' is inversely proportional to the time taken for the disturbance to travel length ' l '), versus the similarity parameter. Although the experimental values do not exactly agree with the empirical result, the use of the parameter $(\omega l / U_\infty A)$ does reduce the variation in the frequency parameter with the similarity parameter K (compare Figs. 37 and 38).

A viscous solution (Ref. 15) for an 18% thick biconvex half model (for which unsteady flow occurred on the full aerofoil at $M_\infty = 0.754$) produced a steady flow solution. The half model boundary conditions preclude communication of pressure waves across the aerofoil wake and provide a solid boundary upon which the separated boundary-layer can re-attach. It was also determined experimentally

that a one quarter chord trailing-edge splitter plate stopped the unsteady flow at all Reynolds numbers. Viscous computations for the same aerofoil and splitter plate at the same conditions as above, produced a steady state solution.

The shock oscillation mechanism is related to the oscillation of the wake starting at the trailing-edge (Ref. 13). This is discussed in Section 5.2.

Over the observed instability region the Mach number ahead of the shock (M_1) lies in a narrow range (for turbulent boundary-layers)

$$1.22 < M_1 < 1.26 .$$

The lower value of M_1 is nearly the Mach number at which shock-induced separation begins. Our schlieren pictures at $M_\infty = 0.85$ (Fig. 11 f,g) show the separation to occur at the base of the shock. Theoretical predictions for the separation of a turbulent boundary-layer on a curved wall by a weak normal shock (Ref. 35) give a value for M_1 of 1.18 to 1.37 depending on Reynolds number. The instability Mach numbers lie in a range where separation effects are significant. Osborne's criterion (Ref. 36) for the critical Mach number for the onset of significant separation gives a value for $M_1 = 1.27$ (see Appendix B for details).

Two configurations of the 14% thick biconvex aerofoil were tested to study the effect of the conditions at the trailing-edge on the periodic flow. Also one set of tests were made with a blunt nose. The results of these tests are described in the following sections:-

3.2.11 Aerofoil Tests with blunt Trailing-Edge

The tail portion of the aerofoil was removed in stages, a small amount at a time, (Fig. 39), and tests performed in the original instability Mach number range. The results are shown in Figs. 40 and 41. For the case of fixed-transition, the frequency of shock oscillation increased from 1.15 kHz to 1.6 kHz as the aerofoil chord c_1 decreased

from $0.976c$ (where c is the chord of the full aerofoil) to $0.952c$. This suggests that the frequency of shock oscillation depends on the distance between the trailing-edge and mean shock position. However, as the aerofoil tail was cut further, the shock oscillation persisted at the same frequency of 1.6 kHz (forced oscillation) until the aerofoil was $0.825c$ at which point the shock oscillations stopped.

In the case of free-transition, the shock oscillations persisted at the same frequency of 1.6 kHz as that of the full aerofoil, on reduction of the tail end of the aerofoil. The shock-oscillations stopped when 14.3% of the aerofoil was taken off the trailing-edge (see Tables 5 and 6).

3.2.12 Aerofoil Tests with Splitter Plate

This configuration is shown in Fig. 39. The aerofoil chord ' c ' was maintained at the same value as that of the complete aerofoil (32 mm). The aerofoil portion at the tail was taken off in stages as before, but leaving the splitter plate intact. The results of these tests are shown in Tables 7 and 8. Schlieren pictures are shown in Fig. 42. For the case of fixed-transition, the shock oscillations occurred at the same frequency as that of the full aerofoil (1.1 to 1.2 kHz) until the aerofoil chord c_1 was reduced to $0.852c$ at which point the shock oscillations stopped (see Table 7). This indicates the importance of a definite length between the trailing-edge and the shock position which determines the timing of the feedback of signals establishing the periodic flow.

For the case of free-transition also, the shock oscillations occurred in the same way as with the full aerofoil (frequency 1.6 kHz forced oscillation) and persisted even when the chord c_1 was reduced to $0.826c$ (see Table 8).

3.2.13 Aerofoil Tests with Blunt Nose

Tests were repeated at the instability Mach numbers, with an aerofoil with blunt nose (Fig. 39). The shock oscillations occurred again at the same frequency as with the original aerofoil (Fig. 43). This test suggests that the periodic flow is unaffected by conditions at the leading edge. This is a very useful result since computations usually have to assume a nose-bluntness to avoid the singularity at the sharp leading edge. The tests also indicate that wind-tunnel interference is small (see Section 3.2.15).

3.2.14 Effect of Reynolds Number on Periodic Flow

A comparison of our low Reynolds number (0.4×10^6) results with the high Reynolds number (7×10^6) results (turbulent boundary-layer) shows that the periodic flow occurs at nearly the same Mach number and frequency parameter. This shows that with a turbulent boundary-layer, scale effects are small upto a Reynolds number of 7×10^6 .

With a laminar boundary-layer, scale effects are large and the periodic flow disappears over a Reynolds number range of 3.6×10^6 to 4.8×10^6 (Ref. 14). This is due to the onset of and completion of transition at the above Reynolds numbers and is discussed in Ref. 14.

3.2.15 Wind-Tunnel Interference

In our tests the blockage ratio was high (7.85%). However the periodic flow occurred at nearly the same Mach number as that in the RAE tests (blockage 4.3%, Ref. 14). Also the periodic flow was not sensitive to a large nose-bluntness (Section 3.2.13). This shows that the static interference of the tunnel is small.

As already shown (Sections 3.2.5 and 3.2.6) the resonance frequency of the perforated hard wall and plenum chamber combination is 1600 Hz. For shock oscillation frequencies close to this frequency, the dynamic interference of the tunnel wall can alter the frequency parameter of the periodic flow. However it cannot start or stop a real

instability. This dynamic interference can be reduced by the use of a sound-absorbing wall.

3.3 NACA 0012 Aerofoil

An NACA 0012 aerofoil has been chosen for investigation because it is a standard aerofoil selected by AGARD for testing numerical solutions (Ref. 58); this section is used for helicopter blades (Ref. 57) and experimental and theoretical results for the steady flow characteristics of this aerofoil are readily available (Refs. 51-58, 60).

3.3.1 Aerofoil at Zero Incidence

Fig. 44a shows the schlieren pictures of the flow on the aerofoil with a turbulent boundary-layer and Fig. 44b shows the side-wall pressure fluctuation traces. The critical Mach number for this aerofoil is 0.725 (Ref. 53). The shock is seen on the aerofoil at $M_\infty = 0.744$ at 0.24c. The pressure fluctuation is random in nature at Mach numbers 0.7 to 0.762. The shock moves rearward with increase in Mach number and at $M_\infty = 0.839$ the top and bottom surface shocks are slightly symmetric and separation extends from the foot of the shock towards the trailing-edge. Pressure traces at $M_\infty = 0.816$ and 0.843 show that the flow is periodic. At $M_\infty = 0.898$ the flow is quasi-steady again (with random pressure fluctuations). The schlieren picture at $M_\infty = 0.928$ shows that the shocks are asymmetric probably due to flow angularity at Mach numbers approaching sonic. The pressure trace at $M_\infty = 0.921$ indicates no periodic pressure fluctuations.

Fig. 46a shows three schlieren pictures of the flow at $M_\infty = 0.835$ taken in 3 tunnel runs. These illustrate the shock-oscillations and show that the shock is displaced by as much as 15% chord.

Figs. 45a and b are the schlieren pictures and side-wall pressure traces for the flow on the aerofoil with a laminar boundary-layer. A number of weak shocks are seen on the aerofoil surface at $M_\infty = 0.807$. The pressure traces show random pressure fluctuations at $M_\infty = 0.693$

and 0.783. At $M_\infty = 0.827$ the flow turns periodic as can be seen from the pressure trace. The schlieren picture at $M_\infty = 0.844$ shows a lambda shock followed by a weak shock. The periodic flow persists at $M_\infty = 0.852$. At $M_\infty = 0.895$ the pressure fluctuations have dropped in intensity, but the schlieren picture at $M_\infty = 0.904$ shows the shocks to be slightly asymmetric. With further increase in Mach number the periodic flow occurs again at $M_\infty = 0.927$. The top and bottom surface shocks are displaced by as much as 0.30c.

The set of schlieren pictures, Fig. 46b, illustrate the shock-oscillations at $M_\infty = 0.94$. The pictures show the existence of two lambda shocks followed by a weak nearly normal shock-wave on each surface.

3.3.1.1 Shock Positions

Fig. 47 shows the shock positions at different Mach numbers obtained from the schlieren pictures of the present tests and compared with other published literature (Refs. 51-54). The shock positions agree well with other experiments upto a Mach number of 0.85. Beyond $M_\infty = 0.85$ Vidal's results (Ref. 53) show the shock positions to be 10 to 22% rearward of that observed in the present tests. Vidal's tests were performed in a large tunnel with a H/C of 16 (H = tunnel height) whereas our tests have been made in a tunnel with H/C = 2.15. The schlieren picture at $M_\infty = 0.928$ shows that the shocks reach upto the tunnel walls (Fig. 45a). Hence tunnel interference probably inhibits the rearward movement of the shock at the higher Mach numbers (Ref. 59).

Fig. 48 shows the shock positions observed on the aerofoil in different tunnels at high Reynolds numbers (Refs. 52,60). The DFVLR tests (Ref. 52) show that the shock moves rearward with increase in Mach number but suddenly moves forward between $M_\infty = 0.84$ and 0.865, before it moves rearward again with increase in Mach number. It is interesting to note that our experiments show a shock-oscillation at $M_\infty = 0.84$. According to Pearcey (Refs. 41,61) the divergence of

the shock position vs. Mach number curve indicates the onset of shock-induced separation. Hence the shock oscillation occurs at the beginning of shock-induced separation. In the opinion of the author, the forward movement of the shock in the DFVLR tests is due to shock-oscillation.

Most of the results of the tests in the different tunnels agree very well (Fig. 48). The shock positions observed in our low Reynolds number tests are 2 - 8% chord forward of those observed in the high Reynolds number tests. This is to be expected since the relative boundary-layer thickness would be lower at the higher Reynolds numbers thereby moving the shock rearwards.

3.3.1.2 Frequency Parameter

The frequency parameter of the oscillatory flow is nearly 1 for the case of the aerofoil with a turbulent boundary-layer and increases slightly with Mach number (Fig. 49). The frequency of shock oscillation is near the tunnel resonance frequency of 1.6 kHz (Ref. 12) and hence the oscillation must be considered as a forced oscillation. Though the oscillation is forced, the result still indicates that shock oscillations are due to the flow on the aerofoil and such an oscillation would occur, at the same frequency parameter on an aerofoil of different chord.

With a laminar boundary-layer there are two ranges of periodic flow, i.e. at $M_\infty = 0.83$ to 0.85 and 0.93 to 0.95 . The frequency parameter drops from a value of 0.95 at the lower Mach number range to 0.65 at the higher Mach number range. The oscillation is forced at the lower Mach numbers. Although the oscillation is forced the results are indicative of periodic flow.

3.3.1.3 Magnitude of Side-Wall Pressure Fluctuations

Fig. 50 shows the magnitude of the side-wall pressure fluctuations for the empty tunnel and with the aerofoil in the test-section, at the instability Mach numbers. The maximum value of the pressure fluctuation ($\Delta P_{\max}$) increases from the empty tunnel

value of 1.5% of dynamic pressure (q) to 3% at the instability Mach number for the aerofoil with a turbulent boundary-layer and decreases slightly with Mach number.

In the case of the aerofoil with a laminar boundary-layer the pressure fluctuation level is somewhat lower at the lower instability Mach number range, but increases to 4.5% of dynamic pressure at the higher instability Mach number range 0.93 - 0.95. The chordwise extent of the laminar interaction is larger at the higher Mach numbers as can be seen from Figs. 45a and 46b. Hence the pressure fluctuation level is higher.

3.3.2 Aerofoil at an Incidence of 3°

Fig. 51 shows the schlieren pictures and side-wall pressure traces of the flow on the aerofoil with fixed-transition at 10% chord. The results show that periodic flow occurs between Mach numbers 0.839 and 0.871. At $M_\infty = 0.906$ the aerofoil develops steady shock-induced separation. The schlieren picture shows the shear layer to be deflected upwards at the foot of the upper surface shock.

The frequency of the oscillatory flow is 1.63 kHz and is a forced oscillation. The frequency parameter is nearly 1 (Fig. 49). The amplitude of pressure fluctuations is higher with the aerofoil at incidence, and ranges from 5 to 6% of dynamic pressure (Fig. 50).

Fig. 52 shows the results of the tests on the aerofoil with a laminar boundary-layer. A weak shock appears first at $M_\infty = 0.723$ on the upper surface of the aerofoil. Two lambda shocks are observed at $M_\infty = 0.750$. With an increase in Mach number to 0.796 the 2 shocks coalesce to form a single lambda shock and weak shocks appear on the lower surface. Separation occurs at the foot of the shock on the upper surface. The pressure fluctuations are random in nature. At $M_\infty = 0.919$ the separation is larger in extent and the pressure trace shows a periodic flow. Waves of an acoustic type are seen behind the shock. The frequency of shock-oscillation is 1.67 kHz and is a forced

oscillation. The pressure fluctuation level is lower (2.8% of dynamic pressure) (Fig. 50).

3.3.3 Aerofoil at an Incidence of 6°

Fig. 53 shows the development of the flow with increasing Mach number over the aerofoil with fixed-transition. Shock-induced separation occurs on the upper surface of the aerofoil at $M_\infty = 0.748$. A weak shock appears on the lower surface of the aerofoil at $M_\infty = 0.859$. The pressure trace at $M_\infty = 0.842$ shows a periodic flow at a somewhat higher frequency of 2.13 kHz. The periodic flow persists at $M_\infty = 0.879$, but the frequency is lower (1.18 kHz). At $M_\infty = 0.922$ the flow is quasi-steady again with shock-induced separation. The magnitude of side-wall pressure fluctuations at the instability Mach numbers range from 5 to 6% of dynamic pressure.

With a laminar boundary-layer, shock-induced separation occurs at $M_\infty = 0.725$. The periodic flow occurs over a range of Mach numbers 0.796 to 0.927 (Fig. 54). The frequency of shock-oscillation increases from 1.1 kHz at $M_\infty = 0.796$ to 1.6 kHz at $M_\infty = 0.927$. The oscillation is forced at $M_\infty = 0.927$. The pressure fluctuation level ranges from 3 to 6% of dynamic pressure (Fig. 50).

Table 9 summarises the results of the tests.

3.3.4 Some Comments

The experiments illustrate an oscillatory flow due to shock-induced separation, occurring on a rigid NACA 0012 aerofoil at zero and positive incidences. A periodic flow of this type has not been reported for this aerofoil. However, Vidal (Ref. 53) has reported that this aerofoil has unusual characteristics at small angles of attack for Mach numbers between 0.8 and 0.925, namely that the slope of the $C_L - \alpha$ curve is very small (Fig. 55).

For the periodic flow with a turbulent boundary-layer, ($\alpha = 0^\circ$)

the Mach number upstream of the shock lies in a narrow range

$$1.24 < M_1 < 1.28 .$$

This is based on the pressure distribution data of Vidal (Ref. 53).

Comparison of the pressure distribution data (Refs. 53,60) at low and high Reynolds numbers (25×10^6) (Fig. 56) show good agreement suggesting that scale effects are small provided the boundary-layer is turbulent.

The frequency of the periodic flow is probably determined by the distance between the trailing-edge and the mean shock position. (Section 3.2.3). Accordingly, as the Mach number increases the shock moves rearwards and the frequency parameter should increase. The results show that the frequency parameter is nearly constant or increases, but there is an anomaly with the results of the tests at an incidence of 6° (Fig. 49).

3.4 CAST 7 'Supercritical' Aerofoil

The basic idea in the use of supercritical wing technology is the suppression of boundary-layer separation by shifting the shock-waves that occur on the wing towards the trailing-edge and making them as weak as possible (Ref. 64). An aerofoil designed for shock-free transonic flow would remain shockless only for a specific Mach number and angle of attack. Aerofoil sections that permit shock-free transonic flow are computed by the determination of smooth, transonic steady solutions to the equations of irrotational motion of an inviscid, compressible fluid.

CAST 7 is a supercritical aerofoil designed by Dornier, having a thickness-chord ratio of 11.8%. At its design condition ($M_\infty = 0.76$, $C_L = 0.6$, $\alpha = 0^\circ$) the aerofoil has a supersonic 'roof top' on the upper surface terminated by a shock-wave at 60% chord (Refs. 63,66).

Experiments on a Whitcomb supercritical aerofoil (Ref. 65) showed that under 'off design' conditions unsteady flows can occur. Experiments on the CAST 7 aerofoil were performed with the objective of determining the existence of any unsteady flows on this supercritical aerofoil.

3.4.1 Aerofoil at Zero Incidence

Fig. 57 shows the results of the tests on the aerofoil with fixed-transition at 10% chord. A characteristic feature of the aerofoil flow is the existence of two shocks on the upper surface of the aerofoil. At $M_\infty = 0.755$, two weak shocks exist at 0.24c and 0.56c on the upper surface and a weak shock on the bottom surface.

With increase in Mach number the shock on the lower surface moves rearwards and at $M_\infty = 0.811$, the flow separates at the foot of the shock. On the upper surface the rear shock moves rearwards and strengthens with increasing Mach number, while the weak shock at the front moves rearwards until $M_\infty = 0.811$. At $M_\infty = 0.842$ a new front shock appears at 20% chord, which moves to 32% chord at $M_\infty = 0.876$. There are however no published results at these Mach numbers to compare our results with.

The pressure traces show random pressure fluctuations at $M_\infty = 0.809$ and 0.839. At $M_\infty = 0.873$ the pressure trace indicates pressure fluctuations of the periodic type but the amplitude of pressure fluctuations is of the order of the empty tunnel pressure fluctuations ($\Delta P_{\max}/q = 1.5\%$).

Fig. 59 shows the results of the tests with a laminar boundary-layer. The salient features of the flow are the presence of weak shocks at Mach numbers of 0.758 and 0.789. At the higher Mach numbers multiple lambda shocks appear on the two surfaces. Shock-induced separation occurs on the lower surface at $M_\infty = 0.847$. The pressure traces show low level random pressure fluctuations.

A comparison of our results with the pressure distribution data obtained at DFVLR (Ref. 66) at $M_\infty = 0.76$ is shown in Fig. 58. The shocks are about 6% forward in our experiments. The differences between the two results are probably due to the difference in Reynolds number and small differences in Mach number. There are no published results at the higher Mach numbers. The flow is steady at all Mach numbers upto 0.88 due to the presence of only weak shocks.

3.4.2 Aerofoil at an Incidence of 3°

Fig. 60 shows the results of the tests on the aerofoil with a turbulent boundary-layer. The schlieren pictures show the existence of two shocks on the upper surface of the aerofoil upto a Mach number of 0.839 and a single strong shock at higher Mach numbers.

Fig. 61 compares our results with the pressure distribution data obtained in the DFVLR tunnel (Ref. 66). The Mach number and incidence are slightly different in the two experiments and the Reynolds number is higher in the DFVLR tunnel. The DFVLR results show the existence of only one shock at 50% chord whereas our results show that there are two shocks, one at 24% chord and the other at 54% chord. The reasons for the differences are not understood.

The pressure traces show that the flows are steady at all Mach numbers upto 0.90.

Fig. 62 shows the side-wall pressure traces for the flow on the aerofoil with a laminar boundary-layer. Random pressure fluctuations are observed at Mach numbers of 0.757 and 0.822 and periodic pressure fluctuations at $M_\infty = 0.853$ and 0.887. The frequency parameter is about 0.50. The amplitude of pressure fluctuation is low (about 2% of dynamic pressure).

3.4.3 Aerofoil at an Incidence of 6°

The results of the tests on the aerofoil with a turbulent boundary-layer are shown in Fig. 63. The characteristics of the flow are the existence of a single shock only on the upper surface of the aerofoil which moves rearward with increasing Mach number. A few weak shocks appear on the lower surface at $M_\infty = 0.872$. Separation occurs at the foot of the shock at Mach numbers of 0.811 and greater. The pressure traces show the flow to be steady at all Mach numbers upto 0.87.

With a laminar boundary-layer, multiple lambda shocks occur at Mach numbers of 0.724 and 0.754 and at $M_\infty = 0.785$ to 0.822 the lambda shock is followed by a weak nearly normal shock-wave (Fig. 64). At $M_\infty = 0.822$ separation occurs at the foot of the shock and the pressure trace indicates a periodic flow. However the amplitude of pressure fluctuations is of the same order as the empty tunnel pressure fluctuations ($\approx 1.5\%$ of dynamic pressure). At $M_\infty = 0.851$ only a single lambda shock exists and the pressure trace shows a periodic flow. The frequency parameter is 0.65 and the amplitude of pressure fluctuations is about 4% of dynamic pressure. At $M_\infty = 0.883$ the shock has moved rearwards and weak shocks occur on the lower surface. The pressure trace shows random pressure fluctuations.

The results are tabulated in Table 10.

3.5 11.8% Joukowski Aerofoil

The tests on a symmetric 11.8% Joukowski aerofoil were prompted by the theoretical work of Steinhoff & Jameson (Ref. 67) who reported multiple solutions of the steady transonic potential flow equation at Mach numbers between 0.82 - 0.85 at zero incidence for this aerofoil. The tests were made to check whether these multiple solutions have a physical counterpart.

This aerofoil has its maximum thickness at 23% chord and has a

cusped trailing-edge (Fig. 4).

3.5.1 Aerofoil at Zero Incidence

Fig. 65 shows the results of the tests on the aerofoil with a turbulent boundary-layer. The critical Mach number calculated by the solution of the equations of motion for a compressible, steady, irrotational flow past the aerofoil (Ref. 68) is 0.746. The schlieren picture shows a weak shock at $M_\infty = 0.747$ at 0.18c. With increase in Mach number the shock moves slowly rearwards and at $M_\infty = 0.902$ the shock is at 0.46c. Shock-induced separation occurs at $M_\infty = 0.836$. The shock positions are seen to be forward for this aerofoil as compared with the other aerofoils (Fig. 66) due to the maximum thickness position being forward. The pressure traces indicate only random pressure fluctuations and the flow is steady at Mach numbers upto 0.90.

The theoretical work of Ref. 67 solves the transonic potential flow equation numerically, neglecting effects due to viscosity, vorticity and entropy production, but uses an artificial viscosity term to stabilise the calculation of the supersonic zone. The results for a 11.8% Joukowski aerofoil at zero incidence, indicate a symmetric solution at Mach numbers below 0.82 and above 0.85. At $M_\infty = 0.832$ there are two solutions - one solution is symmetric and the other is unsymmetric with the shock-wave on the upper surface near the trailing-edge and the shock-wave on the lower surface displaced forward (Fig. 67). Our experiments show the shocks are symmetric at $M_\infty = 0.836$ as well as at higher Mach numbers. Mabey (Ref. 69) has also experimentally confirmed that the flow on the Joukowski aerofoil is steady at these Mach numbers. Comparing the symmetric solution with our experiments we find that theory predicts the shocks to be at 70% chord whereas the experiments show the shocks to be at 35% chord (Fig. 67). The schlieren picture at $M_\infty = 0.836$ (Fig. 65) shows that the shocks extend to only 0.56 times the height of the test-section and hence the result can be considered interference free. There must, obviously, be some discrepancy in the theory particularly in view of the number of assumptions made. In our view, any theory which does not take the separated boundary-layer into consideration, cannot possibly predict the unsteady flow.

Fig. 68 shows the results of the tests on the aerofoil with a laminar boundary-layer. Weak shocks exist upto $M_\infty = 0.80$ and lambda shocks exist at the higher Mach numbers. The pressure traces indicate only random pressure fluctuations.

3.5.2 Aerofoil at an Incidence of 3°

Only the side-wall pressure measurements were made and these are shown in Fig. 69 for the case of the aerofoil with fixed-transition. The pressure traces show random pressure fluctuations upto $M_\infty = 0.831$. Between $M_\infty = 0.85$ and 0.887 the flow is periodic, with a frequency parameter ranging from 0.41 to 0.47 and a pressure fluctuation level of 4 to 7% of dynamic pressure.

With free-transition, periodic flow occurs over the Mach number range 0.877 to 0.89 (Fig. 70). The frequency parameter is 0.35 and the amplitude of pressure fluctuation ranges from 5 to 7.5% of dynamic pressure.

3.5.3 Aerofoil at an Incidence of 6°

The pressure traces show that periodic flow occurs for $M_\infty = 0.866 - 0.89$ for the case of the aerofoil with fixed-transition. The frequency parameter ranges from 0.40 to 0.47 and the amplitude of pressure fluctuations from 7 to 11% of dynamic pressure (Fig. 71).

With a laminar boundary-layer the periodic flow occurs at $M_\infty = 0.866$ with a frequency parameter of 0.64 and a pressure fluctuation level of 7.4% of dynamic pressure (Fig. 72).

The results are tabulated in Table 11.

4. COMPARISON OF EXPERIMENTS WITH THEORY

4.1 Laminar Boundary-Layer - Shock-Wave Interaction*

Trilling (Ref. 20) has considered analytically the interaction of a shock-wave incident on the laminar boundary-layer on a flat plate at zero incidence and supersonic speeds and has shown that it is capable of originating self-supported oscillations. The theory predicts that for certain values of shock strength and frequency, self-supported oscillations of the flow pattern occur. However there are no experiments to either confirm or invalidate the details of the theory.

Karashima (Ref. 22) has extended Trilling's approach for the case of a thin aerofoil in high subsonic flow. The theoretical model is based on a number of assumptions, some of which are unrealistic. Also the theory has the disadvantage that the Mach number and the Mach number gradient upstream of the shock and the length of the separated region must be known from the experiments. The theory is in good agreement with the experimental results of Karashima (Ref. 22) for a 10% biconvex aerofoil and predicts the frequency and instability Mach number correctly. For the 14% biconvex aerofoil the theory predicts roughly the right order of magnitude for the frequency parameter but the variation with Mach number is not predicted (Fig. 73). Given a particular aerofoil the theory is incapable of predicting whether the flow would be steady or unsteady. Thus for a 6% biconvex aerofoil the theory predicts a shock oscillation at $M_\infty = 0.96$ whereas in actual fact the flow is steady at that Mach number (Fig. 8).

4.2 Turbulent Boundary-Layer-Shock-Wave Interaction*

The transonic flow about a 14% thick biconvex aerofoil has been computed at NASA, Ames (Refs. 15-19) for both steady and unsteady flows by a numerical solution of the two dimensional compressible Navier-Stokes equations. The computer code employs algebraic eddy-viscosity turbulence models with separate formulations for the inner and outer regions of the boundary-layer and for the separated region and wake. The computations were made using free flight

* See Appendix E and F

boundary conditions and zero-gradient downstream boundary conditions and a Reynolds number of 7×10^6 .

Fig. 74a provides a comparison between the predictions of the computer simulation (Ref. 17), the present experiments, and those of Ref. 14. The freestream Mach number is 0.81 and the flow is steady. The predicted shock is located 2% chord downstream of the experimental value reported in Ref. 14. The shock location indicated by the present experiment is about 8% chord upstream of that indicated by the experiments, of Ref. 14. This is to be expected since the relative thickness of the lower Reynolds number boundary-layer is higher and is more susceptible to separation. The other factor could be tunnel interference, which could restrict the rearward movement of the shock. However the predicted sonic lines do not extend to the tunnel roof or floor and hence the shock position should not be greatly affected. Both sets of experiments show separations near the trailing-edge (0.87c) whereas theory predicts separation to occur first at the base of the shock-wave (0.7c) followed by reattachment and subsequent separation extending into the wake. The locations of reattachment and subsequent separation vary with time. These differences between experiment and predictions are attributed to deficiencies in the turbulence model and a mismatch in the downstream boundary conditions.

Fig. 74b compares the predictions with experiments for steady flow again but involving shock-induced separation at a freestream Mach number of 0.88. The computed shock is located 13%c downstream of the experimental values of Ref. 14. At this Mach number the predicted sonic lines extend much farther than the experimental tunnel roof and floor. Hence tunnel interference could explain the forward location of the shock in the experiments. However experimental and computed surface pressures upstream of the shock (Fig. 2, Ref. 17) agree well indicating that tunnel interference is small. Separation occurs at the foot of the shock in both cases. Our results also show the same pattern but at a slightly higher Mach number 0.93.

Fig. 75 compares the predictions (Ref. 17) and experiments for the case of unsteady flow in terms of the time history of the shock-wave (top surface shock) motion. The shock positions are compared on the basis of chords travelled by the mean flow ($U_{\infty}t/c$). The experiments show that the shock vanishes during a part of the cycle. The computations also indicate that the shock strength diminishes as the shock moves rearwards and probably vanishes, but it depends on the method used to define the shock in the theory. (This is discussed in Ref. 14). The computations take about 4 hours on a CDC 7600 computer (Ref. 18).

Though there is good agreement between theory and experiments, Horstmann (Ref. 70) has indicated that the use of the algebraic eddy-viscosity turbulence models are rather 'ad hoc' and that improved turbulence models are required before existing computer codes can correctly predict the flow features characteristic of strong shock-wave boundary-layer interactions with relatively large regions of separated flow. Thus any agreement with the computed results of Levy (Ref. 17) is entirely fortuitous.

4.3 Theories for Aileron Buzz

Eckhaus (Ref. 23) has developed an inviscid linearised theory for the perturbations due to harmonic oscillations of an aileron. Finke (Ref. 11) has suggested an analogy between wake oscillation on rigid aerofoils with the theory of Eckhaus. The theory predicts a wide range of frequencies and Mach numbers at which the instability could occur (Fig. 76). There is some uncertainty as to the Mach number of the flow behind the shock-wave. Eckhaus suggests the use of a mean value of the Mach number distribution behind the shock. The theory is in disagreement with our experimental results as well as those of Ref. 14.

Steger and Bailey (Ref. 25) have calculated transonic aileron buzz by a numerical solution of the 'thin layer' Navier-Stokes equations with an algebraic eddy-viscosity model. A calculation for a P-80 wing

section NACA 65₁-213 (the aircraft P80/F80 experienced severe control problems due to buzz, Ref. 25) predicts a frequency parameter of 0.76 at $M_\infty = 0.82$ and $\alpha = -1^\circ$. The experiments (Ref. 25) indicate that buzz occurs when the aileron is held at zero incidence and freed, whereas the predictions show the buzz to occur when the aileron is initially deflected to 4° and then released. Both sets of data show a phase difference between the shock motion and the aileron motion. The prediction also shows that the shock vanishes for a part of the cycle.

5. DISCUSSION

5.1 Parameters for Periodic Flow

Based on the tests at transonic speeds on five aerofoil shapes with a turbulent boundary-layer and zero incidence, several parameters important for the occurrence of periodic flow have been identified.

They are:-

- (1) Thickness-chord ratio: must be approximately 12% or greater.
- (2) Shock-upstream Mach number: the local Mach number M_1 upstream of the terminal shock must lie in a narrow range

$$1.2 < M_1 < 1.3$$

which just induces separation.

- (3) Aerofoil shape: this determines the position of the shock on the aerofoil surface. We have already indicated the importance of the mean shock position in determining the frequency of the periodic flow (Section 3.2.3).
- (4) Trailing-edge angle: a large trailing-edge angle increases the extent of the separation bubble and the possibility of a periodic flow.

Each of these conditions are necessary but by themselves are not sufficient for the occurrence of periodic flows. Table 12 lists the parameters for the five aerofoils.

5.2 The Physical Mechanism causing the Periodic Flow

The periodical shock motion on a rigid aerofoil shows some similarities with the shock movements observed on an aerofoil with an oscillating flap (Ref. 40). Fig. 77 illustrates the steady shock patterns on an aerofoil at zero incidence and various flap deflections. When the flap is deflected downwards the speed along the upper surface is increased and the shock is displaced downstream while both the shock strength and the size of the supersonic region are increased. At the same time the flow speed along the lower surface is reduced and the development of the flow pattern is reversed: the size of the supersonic region decreases and the shock wave becomes weaker. Beyond a certain flap deflection the lower surface flow is completely subcritical.

However, if the flap oscillates with a certain frequency, it is found that there is a time lag between flap deflection and shock movement (Fig. 78) and also between shock motion and shock strength (Ref. 40). These phase shifts are attributed to the dynamic character of the phenomenon.

Fig. 78 compares the flows on an aerofoil with an oscillatory flap (Ref. 40) and on a rigid 14% biconvex aerofoil (Ref. 14). It is seen that the flows are similar, with the wake taking the place of a flap. The mechanism of shock oscillation could be as follows:- If the local Mach number ahead of the shock is near the critical value for shock-induced separation, and if shock-induced separation suddenly occurs on the bottom surface while the upper surface separation is near the trailing-edge, the effective aerofoil profile is no longer symmetrical and we have a situation as in Stage 1 (Fig. 78), with the wake deflected downwards. The trailing-edge pressure is then lower on the lower surface than on the top surface. According to Pearcey (Ref. 41), the wake cannot support any pressure difference between its two edges and the interrelation between the two surfaces is governed by the equality of pressure at the two edges of the wake at the trailing-edge position. To cause a reduction in pressure at the trailing-edge on the top surface, the flow has to accelerate there (Stages 2-4). At the same time the downward deflection of the wake retards the flow on the lower surface

causing the shock to move forward and weaken (Stages 2 and 3). The acceleration of the flow on the top surface pushes the weak shock backwards and intensifies it (Stage 4). The strong shock on the top surface now tends to separate the flow upstream of the trailing-edge (Stage 5). This together with the forward movement of the shock on the bottom surface causes the wake to straighten (zero deflection) (Stages 4 and 5). This has the effect of accelerating the flow on the bottom surface (Stage 6) pushing the weak shock and the separation point rearwards, but the separation point on the top surface is forward at the foot of the shock (Stage 6). The difference in the position of the separation points results in the wake deflecting upwards (Stages 6 and 7). The upward deflection of the wake would tend to suppress the shock-induced separation on the top surface while accelerating the flow on the lower surface and exactly the reverse process now proceeds. As can be seen there is a time lag between the wake deflection and the development of the flow on the two surfaces analogous to the flow on the aerofoil with an oscillating flap.

Tijdeman (Ref. 40) has developed a simplified theory to determine the relation between shock position and shock strength by considering a shock-wave with prescribed amplitude and frequency in a given steady flow field. The instantaneous shock-strength p_2/p_1 across a shock-wave which oscillates sinusoidally with a small amplitude ' x_0 ' and frequency ' ω ' can be expressed as -

$$\frac{\bar{p}_2}{\bar{p}_1} = \left\{ \frac{p_2}{p_1} \right\}_{x=x_s} + \frac{4\gamma M_1}{(\gamma + 1)} \left(\frac{\delta M_1}{\delta x} - \frac{i\omega}{a_1} \right) x_0 e^{i\omega t}$$

where M_1 , $\frac{\delta M_1}{\delta x}$ and a_1 are the Mach number, Mach number gradient and the acoustic velocity just upstream of the shock-wave in its steady mean position ($x = x_s$). $(p_2/p_1)_{x=x_s}$ the pressure ratio across the shock-wave in its steady mean position.

Calculations using representative values from the experimental results for the 14% biconvex aerofoil are shown in Fig. 79. The shock strength varies with shock position in such a way that during a part of the cycle ($1.16\pi < \omega t < 1.48\pi$) (i.e. during a part of the backward motion of the shock), the pressure ratio p_2/p_1 is lower than 1.0. This means that the shock vanishes during that part of the cycle. This is in qualitative agreement with the experiments.

6. CONCLUSIONS

Tests on five aerofoils at transonic speeds suggest the following conclusions:-

1. Periodic flows due to shock-induced separation occur on a 14% biconvex and an NACA 0012 aerofoil at zero incidence over a narrow range of Mach numbers for both laminar and turbulent boundary-layers.
2. The frequency parameter is nearly 1 and the oscillatory flow involves shock movements in antiphase on opposite surfaces of the aerofoil.
3. The aerofoils which did not have any oscillatory flow were the 6% biconvex, the CAST 7 aerofoil at incidences from 0 to 6° and the 11.8% Joukowski aerofoil at zero incidence. (With turbulent boundary-layers).
4. The important parameters that control the periodic flow are the aerofoil shape, thickness-chord ratio, trailing-edge angle and the Mach number just upstream of the shock. Periodic flows can be expected to occur on aerofoils (with turbulent boundary-layers) at zero incidence with thickness-chord ratios approximately equal to or greater than 12% and with large trailing-edge angles when the Mach number just upstream of the shock (M_1) lies in a range $1.2 < M_1 < 1.3$ which just induces separation.
5. Periodic flows are widespread in their occurrence on aerofoils at incidence. With a turbulent boundary-layer, periodic flows occur on the 14% biconvex, NACA 0012 and Joukowski aerofoils at Mach numbers ranging from 0.84 to 0.90 and frequency parameters ranging from 0.40 to 1.36.
6. With the aerofoils at incidence and with a laminar boundary-layer the periodic flows occur on the 14% biconvex, the 0012 aerofoil,

CAST 7 and Joukowski profiles at Mach numbers 0.80 to 0.93 and the range of frequency parameters is 0.35 to 1.0.

7. Scale effects are small provided the boundary-layer is turbulent at the shock.

REFERENCES

1. HOLDER, D.W. & NORTH, R.J. An oscillatory flow resulting from the interaction of shock-waves with the boundary-layer on a rigid aerofoil. ARC Report 12,400, 1949.
2. LEIPMANN, H.W. Shock-wave oscillations in wind-tunnels. Journal of the Aeronautical Sciences, Vol. 14, 1947, pp 295-302.
3. HUMPHREYS, M.D. Pressure pulsations on rigid airfoils at transonic speeds. NACA RM L51I12, 1951.
4. HUMPHREYS, M.D. Measurements of normal-force-coefficient fluctuation on four 9% thick airfoils having different locations of maximum thickness. NACA RM L54B22, 1954.
5. POLENTZ, P.P.
PAGE, W.A. &
LEVY, L.L. The unsteady normal-force characteristics of selected NACA profiles at high subsonic Mach numbers. NACA RM A55C02, 1954.
6. PEARCEY, H.H. & HOLDER, D.W. Examples of the effects of shock-induced boundary-layer separation in transonic flight. R & M No. 3510, 1967.
7. MEYER, G. & HILLER, W. An experimental investigation of unsteady transonic flow by high speed interferometric photography. AGARD CP No. 35, 1968, Paper 9.
8. FREESTONE, M.M. & COX, R.N. Sound fields generated by transonic flows over surfaces having circular perforations. AGARD CP-83-71, 1971, Paper 24.
9. DALEY, B.N. & HUMPHREYS, M.D. Effects of compressibility on the flow past thick airfoil sections. NACA TN1657, 1948.
10. LAMBOURNE, N.C. Some instabilities arising from the interactions between shock-waves and boundary-layers. NPL/AERO/348, 1958.
11. FINKE, K. Unsteady shock-wave - boundary-layer interactions on profiles in transonic flow. AGARD CP 168, Paper 28, 1975.

REFERENCES contd.....

12. FINKE, K. Shock oscillations in transonic flows.
VDI Forschungsheft 580, 1977.
13. MABEY, D.G. Oscillatory flows from shock-induced
separations on biconvex airfoils of
varying thickness in ventilated
wind-tunnels.
RAE TM STRUCTURES 969, 1980.
14. MABEY, D.G.
WELSH, B.L. &
CRIPPS, B.E. Periodic flows on a rigid 14% thick
biconvex wing at transonic speeds.
RAE TR 81059 1981.
15. McDEVITT, J.B.
LEVY, L.L. &
DEIWART, G.S. Transonic flow past a thick circular
arc airfoil.
AIAA Journal, Vol. 14, No. 5, 1976,
pp 606-613.
16. LEVY, L.L. Experimental and computational steady and
unsteady transonic flows about a thick
airfoil.
AIAA Journal, Vol. 16, No. 6, 1978,
pp 564-572.
17. LEVY, L.L. Predicted and experimental steady and
unsteady transonic flows about a biconvex
airfoil.
NASA TM 81262, 1981.
18. DEIWART, G.S. Numerical simulation of high Reynolds
number transonic flows.
AIAA Journal, Vol. 13, No. 10, 1975,
pp 1354-59.
19. DEIWART, G.S. Computation of separated transonic
turbulent flows.
AIAA Journal, Vol. 14, No. 6, 1976,
pp 735-40.
20. TRILLING, L. Oscillating shock boundary-layer interaction.
Journal of the Aeronautical Sciences,
Vol. 33, No. 5, 1958, pp 301-304.
21. KNECHTEL, E.D. Experimental investigations at transonic
speeds of pressure distributions over
wedge and circular arc sections and
evaluation of perforated wall interference.
NASA TN D-15, 1959.

REFERENCES contd.....

22. KARASHIMA, K. Instability of shock-wave on thin airfoil in high subsonic flow. Aeronautical Research Institute, Univ. of Tokyo, Report No. 363, March 1961, pp 39-118.
23. ECKHAUS, W. A theory of transonic aileron buzz, neglecting viscous effects. Journal of the Aerospace Sciences, June 1962, pp 712-718.
24. NAKAMURA, Y. Some contributions on a control-surface buzz at high subsonic speeds. Journal of Aircraft, Vol. 5, No. 2, 1968, pp 118-125.
25. STEGER, J.L. & Calculation of transonic aileron buzz. BAILEY, H.E. AIAA Journal, Vol. 18, No. 3, 1980, pp 249-255.
26. SAITO, H. On the aileron buzz in the transonic flow. Aero. Res. Ins., Univ. of Tokyo, Report No. 346.
27. NAGASHIMA, T. et al Experiment of supersonic air intake buzz. ISAS Report No. 481, Tokyo, 1972.
28. SPREITER, J.R. et al Theoretical pressure distributions for several related non-lifting airfoils at high subsonic speeds. NACA TN 4148, 1958.
29. TIJDEMAN, H. & Transonic flow past oscillating airfoils. SEEBASS, R. Annual Review of Fluid Mechanics, 1980, pp 181-222.
30. MABEY, D.G. The resonance frequencies of ventilated wind-tunnels. R & M 3841, April 1978.
31. WILLIAMS, D.A. The analysis of random data. AGARD-AG-160, Vol. 14, 1981.
32. BENDAT, J.S. & Random data. Analysis and measurement PIERSOL, A.S. procedures. Wiley - Interscience, 1971.

REFERENCES contd.....

33. MABEY, D.G. Flow unsteadiness and model vibration in wind-tunnels at subsonic and transonic speeds.
ARC C.P. No. 1155, 1971.
34. OWEN, T.B. Techniques of pressure-fluctuation measurements employed in the RAE low speed wind-tunnels.
AGARD Report No. 172, March 1958.
35. BOHNING, R. & ZIEREP, J. Shock boundary layer interaction in turbulent flow at curved walls with separation.
Z. Flugwiss Weltramforsch, Vol. 6, No. 2, 1982.
36. PEARCEY, H.H. Shock-induced separation and its prevention by design and boundary-layer control in "Boundary Layer and Flow Control", Vol. 2, 1961, Lachmann, G.V. Ed.
37. ABBOT, I.H. & DOENHOFF, A.E.V. Theory of wing sections.
Dover, 1959.
38. MOHAN, S.R. Wind-tunnel tests on a 14% thick biconvex airfoil.
Data Analysis Report, C. of A. DA-781, 1981.
39. BRYSON, A.E. An experimental investigation of transonic flow past two-dimensional wedge and circular-arc sections using a Mach-Zehnder interferometer.
NACA Report No. 1094, 1952.
40. TIJDEMAN, H. Investigations of the transonic flow around oscillating airfoils.
NLR TR 77090U, 1977.
41. PEARCEY, H.H. Some effects of shock-induced separation of turbulent boundary layers in transonic flow past aerofoils.
ARC R & M No. 3108, 1959.
42. POPE, A. & GOIN, K.L. High speed wind-tunnel testing.
John Wiley, 1956.
43. GOETHERT, B.H. Transonic wind-tunnel testing.
Pergamon Press, 1961.

REFERENCES contd.....

44. LUKASIEWICZ, J. Effects of boundary-layer and geometry on characteristics of perforated walls for transonic wind-tunnels.
Aerospace Engg., Vol. 20, April 1961.
45. COOK, W.J. et al The shock tube as a device for testing transonic airfoils at high Reynolds numbers.
AIAA Paper 78-769, 1978.
46. BATES, G.P. Preliminary investigation of 3 inch slotted transonic wind-tunnel test-sections.
NACA RM L9D18, 1949.
47. SCHLICHTING, H. Boundary-layer theory.
McGraw Hill, 7th Edition, 1979, p 638.
48. GARNER, H.C. et al Subsonic wind-tunnel wall corrections.
AGARDograph 109, 1966.
49. RAMASWAMY, M.A. Transonic wind-tunnel testing (an overview).
Lecture notes in short course on "Aerospace ground test facilities and flight testing", UTSI., 1979.
50. BLACKWELL, J.A. Wind-tunnel blockage correction for two-dimensional transonic flow.
Journal of Aircraft, Vol. 16, No. 4, 1979, pp 256-263.
51. OSBORNE, J. A selection of measured transonic flow pressure distributions for the NACA 0012 aerofoil.
RAE Provisional Report, 1971.
52. STANEWSKI, E. et al The transonic wind-tunnel Braunschweig of DFVLR. Z. Flugwiss Weltramforsch, 6 (1982), Heft 6.
53. VIDAL, R.J. et al Two-dimensional subsonic experiments with a NACA 0012 aerofoil.
Calspan Report No. RK-5070-A-3, 1973.
54. DE PAUL, M.V. & DYMENT, A. Research on aerofoil sections in high subsonic flows.
L'Aeronautique et l'Astronautic, Vol. 19, No. 3, 1970, pp 15-30.

REFERENCES contd.....

55. HALL, M.G. "Transonic Flows" in Proc. Conf. on Computational methods and problems in aero. and fluid mechanics. Ed. Hewitt et al, September 1974.
56. KRUPP, J.A. & MURMAN, E.M. Computation of transonic flows past lifting aerofoils and slender bodies. AIAA Journal, Vol. 10, No. 5, 1972, pp 880-886.
57. STEGER, J.L. & LOMAX, H. Transonic flow about two-dimensional aerofoils by relaxation procedures. AIAA Journal, Vol. 10, No. 1, 1972, pp 49-54.
58. LOCK, R.C. Test cases for numerical methods in two-dimensional transonic flows. AGARD Report No. 575, 1970.
59. ROGERS, E.W.E. & HALL, I.M. Some experiments with static tubes at transonic speeds in a slotted wall wind-tunnel. ARC 20306, July 1958.
60. THIBERT, J.J. et al NACA 0012 aerofoil, in experimental data base for computer program assessment. AGARD AR 138, 1979.
61. PEARCEY, H.H. The occurrence and development of boundary-layer separations at high incidences and high speeds. ARC R & M 3109, 1955.
62. PEARCEY, H.H. et al The interaction between local effects at the shock and rear separation - a source of significant scale effects in wind-tunnel tests on aerofoils and wings. ARC 30477, September 1968.
63. LOCK, R.C. Report on a combined experimental and theoretical investigation of the aerofoil CAST 7. RAE TR 79073, June 1979.
64. BAUER, F. et al 'Supercritical Wing-Sections', Springer-verlag, 1972.

REFERENCES contd.....

65. ROOS, F.W. & RIDDLE, D.W. Measurements of surface pressure and wake flow fluctuations in the flow field of a Whitcomb supercritical aerofoil. NASA TN D-8443, 1977.
66. STANEWSKY, E. et al Supercritical aerofoil CAST 7 - surface pressure, wake and boundary-layer measurements in AGARD AR 138, 1979.
67. STEINHOFF, J. & JAMESON, A. Multiple solutions of the transonic potential flow equation. AIAA Paper 81-1019, 1981.
68. TOMOTIKA, S. & UMEMOTO, H. On the subsonic flow of a compressible fluid past a symmetrical Joukowski aerofoil. Report No. 205, 1941, Aeronautical Research Institute, Tokyo Imperial University.
69. MABEY, D.G. Private Communication, 16th March 1982.
70. HORSTMANN, C.C. Private Communication, 24th January 1983.
71. KAWAMURA, R. & KARASHIMA, K. Experimental investigation of transonic flow past two-dimensional biconvex circular-arc airfoils at small angles of attack. Aeronautical Research Institute, University of Tokyo, Report No. 342, 1959.
72. HOLDER, D.W. The transonic flow past two-dimensional aerofoils. Jour. Roy. Aero. Soc., 68, 644, Aug. 1964.
73. SHAPIRO, A.H. The dynamics and thermodynamics of compressible fluid flow. Vol. 2, The Ronald Press Co., 1954.

APPENDIX A

Power Spectral Analysis

The power spectral density function describes the frequency composition of the data (Ref. 32). The power spectral density of a random signal can be defined as the rate of change of mean square value of the signal with respect to frequency (Ref. 31). The concept of a power spectral density analyser is illustrated in Fig. A.1. The signal is input to a narrow band unity-gain, band-pass filter. The output of the filter may be squared and averaged. The mean square value so obtained is then divided by the bandwidth of the filter. The rate of change of mean square value (power spectral density) at the centre frequency of the filter is then obtained conceptually by reducing the bandwidth of the filter to zero. The complete PSD function for the signal may be obtained from such a circuit by varying the centre frequency of the band-pass filter. The data must be re-analysed for each new selected value of centre frequency.

In this analysis, a digital spectral analyser was used. The principle used is the same as the analogue analyser. Here the band-pass filter is simulated by computing the Fourier series of a fixed length portion of the sampled data sequence, calculating the mean square value of the modulus of the Fourier coefficients obtained from a portion of the data sequence and dividing by the effective bandwidth of the 'filter' represented by the Fourier series estimator.

An estimate of the power spectral density function is obtained as follows:- If the signal $y(t)$ is represented by an N -length sampled sequence such that $y_n = y(nh)$, $n = 0, 1, 2, \dots, N - 1$ where $h =$ interval between successive samples, then the discrete version of Fourier transform for arbitrary frequency f is

$$Y(f, T) = h \sum_{n=0}^{N-1} y_n \exp(-i 2\pi f n h) \quad \dots\dots (1)$$

The usual selection of discrete frequency values for the computation of $Y(f,T)$ is

$$f_k = kf = \frac{k}{T} = \frac{k}{Nh} \quad k = 0,1,2 \dots N-1 \quad \dots\dots (2)$$

At these frequencies, the transformed values give the Fourier components defined by

$$C_k = \frac{Y(f_k, T)}{h} = \sum_{n=0}^{N-1} y_n \exp \left(-i \frac{2\pi \cdot kn}{N} \right) \quad \dots\dots (3)$$

$$k = 0,1,2 \dots \frac{N}{2} - 1$$

The results are unique only upto $k = N/2$, (Ref. 31).

The power spectral density (PSD) is estimated from

$$G_{yy}(k \cdot \Delta f) = \frac{2h}{N} |C_k|^2, \quad k = 0,1,2 \dots \frac{N}{2} - 1 \quad \dots\dots (4)$$

The equation (3) requires a total of approximately N^2 complex multiply-add operations (where 1 complex multiply-add = 4 real multiply-adds) to compute all the C_k terms involved. For large N these computations can be time and cost consuming. To greatly reduce these computational times, Fast Fourier Transform (FFT) procedures have been developed. The FFT procedures are based upon decomposing N into its composite (non-unity) factors and carrying out Fourier transforms over the smaller number of terms in each of the composite factors. If N is the product of p factors such that

$$N = \prod_{i=1}^p r_i = r_1, r_2 \dots r_p \quad \dots\dots (5)$$

then C_k in eqn. (3) can be found by computing in an iterative way the sum of p terms

$$\left(\frac{N}{r_1}\right) \text{ Fourier transforms requiring } 4r_1^2 \text{ real operations each.}$$

$$\left(\frac{N}{r_2}\right) \text{ Fourier transforms requiring } 4r_2^2 \text{ real operations each.}$$

$$\left(\frac{N}{r_p}\right) \text{ Fourier transforms requiring } 4r_p^2 \text{ real operations each.}$$

$\dots\dots (6)$

Hence the total number of real operations becomes

$$4(Nr_1 + Nr_2 + \dots + Nr_p) = 4N \sum_{i=1}^p r_i \quad \dots\dots (7)$$

The resulting speed of these FFT procedures to the standard method is given by

$$\text{Speed ratio} = \frac{N^2}{4N \sum_{i=1}^p r_i} = \frac{N}{4 \sum_{i=1}^p r_i} \quad \dots\dots (8)$$

The majority of FFT algorithms make assumptions on the length of a sequence. In this analysis the Cooley-Tukey algorithm has been used. This algorithm is based upon "binary" or "power of two" lengths, i.e. $N = 2^P$, and uses the properties of the discrete Fourier transform based on the doubling theorem (Ref. 31 p 126) i.e.

$$\text{If } y_1(j) = y(2j) \langle \Rightarrow \rangle Y_1(n) \quad j, n = 0, 1, \dots, \frac{N}{2} - 1$$

..... (9)

$$\text{and } y_2(j) = y(2j + 1) \langle \Rightarrow \rangle Y_2(n) \quad j, n = 0, 1, \dots, \frac{N}{2} - 1$$

..... (10)

$$\text{then } Y(n) = \frac{1}{2}(Y_1(n) + Y_2(n) \cdot \exp(\frac{-2\pi i n}{N}))$$

..... (11)

$$\text{and } Y(\frac{N}{2} + n) = \frac{1}{2} \cdot (Y_1(n) - Y_2(n) \cdot \exp(\frac{-2\pi i n}{N}))$$

..... (12)

Before the transforms $Y_1(n)$ and $Y_2(n)$ can be calculated the series $y(j)$ must be partitioned into two series $y_1(j)$ and $y_2(j)$ containing the even and odd terms respectively as defined in eqns. (9) and (10).

The transforms $Y_1(n)$ and $Y_2(n)$ may be calculated separately by again applying eqns. (11) and (12), but this time replacing N by $N/2$. Before this can be effected, the two series $y_1(j)$ and $y_2(j)$ must be further partitioned as defined in eqns. (9) and (10), (with $N/2$ replaced by $N/4$).

The partitioning can be continued until N single element partitions have been obtained. At this point the argument is reversed, and the required discrete Fourier transform is obtained using

successive applications of eqns. (11) and (12), it being noted that the discrete Fourier transform of a single element sequence is equal to the value of the element, i.e.

$$y(o) = Y(o) \quad N = 1 \quad \dots\dots (13)$$

The partitioning process alters the order of the data sequence $y(j)$. Details of the algorithms are in Ref. 31.

The PSD estimate given by eqn. (4) is usually averaged over a number of independent estimates. This averaging process is equivalent to the integrator shown in the Fig. A.1.1.

APPENDIX BCriteria for Shock-Induced Separation on a Transonic Aerofoil

The qualitative features of shock-induced separation in a transonic flow are described by Pearcey (Refs. 36,62). A separation bubble at the foot of a shock on an aerofoil grows with increasing Mach number or incidence. Eventually it either spreads to the aerofoil trailing-edge or interacts with the rear adverse pressure-gradient causing separation downstream of the bubble. This situation is referred to as significant separation. The first appearance of a supersonic region downstream of the shock is closely associated with the beginning of significant separation. Significant separation has a pronounced effect on the total aerofoil flow field. A large number of experimental results for aerofoils were analysed by Osborne of NPL. Based on this data a purely empirical relationship was developed between the geometry of the aerofoil and the Mach number upstream of the shock at which significant separation first occurs. The parameter 'X' representing the shape of the leading edge and surface curvature just upstream of the crest of the aerofoil, accounts for most of the variation in the critical value of the shock upstream Mach number M_1 . In the Osborne criterion the detailed character of the boundary-layer or pressure rise in the interaction region is not required to predict the critical value of M_1 .

For the 14% biconvex aerofoil at zero incidence the value of X is 0.1961. Osborne's criterion predicts a value of 1.27 for M_1 which is close to the measured values of 1.22 to 1.26.

APPENDIX C

Wind-Tunnel Blockage Corrections

Blockage consists of a variation of the tunnel cross section due to model volume and to the model wake. It affects the velocity, static and dynamic pressures and the aerodynamic coefficients. The effect of blockage is to cause a change in velocity at the model centerline and variation of static pressure along the test-section.

The classical method, for correcting two-dimensional transonic flow results for blockage effects, is based on linearised subsonic theory (Refs. 48,49). However, there is considerable doubt in the determination of the porosity parameter used in the theory. Blackwell (Ref. 50) has developed an empirical method which uses linear theory in addition to wall pressure measurements to infer the blockage interference correction. According to this method the correction ΔM_B to the freestream Mach number is given by

$$\Delta M_B = \bar{M}_E - \bar{M}_T$$

where $\bar{M}_E$ is the average experimental Mach number measured near the wind-tunnel walls above and below the model and $\bar{M}_T$ is the average theoretical Mach number at a vertical position representative of the wind-tunnel walls using linear theory (eqn. 7, Ref. 50).

The method has been applied to determine the blockage corrections on 2 aerofoils - the 14% biconvex and 12% CAST 7 aerofoil. The results are shown in Figs. C.1 and C.2. The blockage corrections are seen to be quite low upto $M_\infty = 0.875$ for the 14% biconvex aerofoil and upto $M_\infty = 0.90$ for the 12% CAST 7 aerofoil. However there are limitations on the maximum Mach number at which linear theory would be applicable (Ref. 49).

APPENDIX DTABLE 1CO-ORDINATES OF CAST 7 AEROFOIL (DESIGN)

x/c	(y/c) upper	(y/c) lower
0	0	0
0.0004	0.00337	-0.00319
0.001	0.00531	-0.00506
0.002	0.00754	-0.00714
0.0035	0.01013	-0.00927
0.005	0.01230	-0.01082
0.0075	0.01534	-0.01270
0.0125	0.02011	-0.01506
0.0175	-	-0.01663
0.025	-	-0.01853
0.035	0.03273	-0.02085
0.0475	0.03770	-0.02363
0.065	0.04308	-0.02713
0.0875	0.04827	-0.03102
0.115	0.05304	-0.03518
0.155	0.05809	-0.04040
0.215	0.06313	-0.04632
0.275	0.06617	-0.04965
0.335	0.06787	-0.05027
0.395	0.06851	-0.04837
0.455	0.06809	-0.04431
0.515	0.06677	-0.03854
0.575	0.06416	-0.03153
0.635	0.06008	-0.02372
0.695	0.05423	-0.01566
0.755	0.04639	-0.00810
0.815	0.03662	-0.00209
0.875	0.02541	+0.00104
0.92	0.01651	+0.00079
0.95	0.01051	-0.00063
0.9775	0.00495	-0.00263
1.0	0.00036	-0.00465

Nose radius/c = 0.0115

Position of max. thickness/c = 0.3500

TABLE 2CO-ORDINATES OF 11.8% JOUKOWSKI AEROFOIL

x/c	y/c
0	0
0.0169	0.0231
0.0465	0.0366
0.0897	0.0474
0.1453	0.0548
0.2116	0.0585
0.2864	0.0586
0.3677	0.0552
0.4528	0.0492
0.5393	0.0413
0.6245	0.0325
0.7058	0.0238
0.7807	0.0159
0.8469	0.0094
0.9025	0.0047
0.9457	0.0018
0.9753	0.0004
0.9903	0.00002
1.0	0

Nose radius/c = 0.0375

Position of max. thickness/c = 0.2312

TABLE 3CO-ORDINATES OF NACA 0012 AEROFOIL

(Ref. 37)

x/c	y/c
0	0
0.0125	0.0189
0.0250	0.0262
0.0500	0.0356
0.0750	0.0420
0.1000	0.0468
0.1500	0.0535
0.2000	0.0574
0.2500	0.0594
0.3000	0.0600
0.4000	0.0580
0.5000	0.0529
0.6000	0.0456
0.7000	0.0366
0.8000	0.0262
0.9000	0.0145
0.9500	0.0081
1.0000	0.0013

Leading edge radius/c = 0.0158

Position of max. thickness/c = 0.3000

TABLE 4

TECHNIQUES USED FOR AEROFOIL TESTS

AEROFOIL	INCIDENCE (DEGS)	BOUNDARY- LAYER	SPARK SCHLIEREN	SIDE-WALL PRESSURE MEASURE- MENTS	SURFACE PRESSURE	HIGH SPEED SCHLIEREN	OIL FLOW
6% BICONVEX	0	LAM	X				
		TURB	X				
14% BICONVEX	0	LAM	X	X	X		
		TURB	X	X	X		X
	4	LAM	X				
		TURB	X			X	
NACA 0012	0	LAM	X	X			
		TURB	X	X			
	3	LAM	X	X			
		TURB	X	X			
	6	LAM	X	X			
		TURB	X	X			
CAST 7	0	LAM	X	X			
		TURB	X	X			
	3	LAM		X			
		TURB	X	X			
	6	LAM	X	X			
		TURB	X	X			
11.8% JOUKOWSKY	0	LAM	X	X			
		TURB	X	X			
	3	LAM		X			
		TURB					
	6	LAM		X			
		TURB		X			

X Denotes technique used

TABLE 5

14% BICONVEX AEROFOIL WITH BLUNT TRAILING-EDGE
(FIXED-TRANSITION)

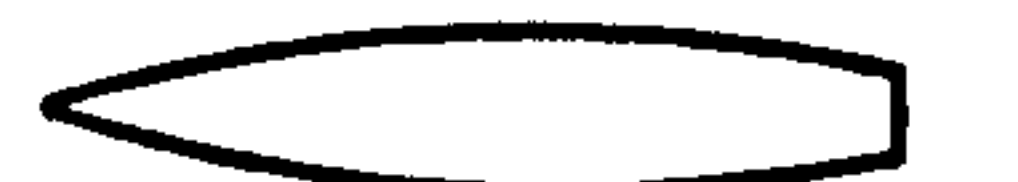
	AEROFOIL SIZE (% FULL)		MACH NUMBER	FREQUENCY OF SHOCK OSC.(kHz)	MAX. AMPL. OF PRESSURE FLUCTUATION $\Delta P/q\%$
1.	100		0.87 0.893	1.16 1.16	6.87 10.4
2.	99.95		0.869 0.891	1.15 1.15	6.87 9.96
3.	97.62		0.87 0.893	1.15 1.18	6.87 6.41
4.	95.20		0.891	1.6	3.10
5.	92.90		0.868 0.89	1.5 1.6	3.31 4.82
6.	90.50		0.868 0.891	1.5 1.5	4.03 3.10
7.	85.70		0.889	1.6	3.76
8.	82.50		0.884 0.915	NO SHOCK OSCILLATIONS	
MODEL REVERSED			(BLUNT LEADING EDGE)		
9.	82.5		0.915	1.15	5.75
10.	74.6		0.886 0.915	1.1 1.15	8.53 11.3

TABLE 6

14% BICONVEX AEROFOIL WITH BLUNT TRAILING-EDGE
(FREE-TRANSITION)

	AEROFOIL SIZE (% FULL)	MACH NUMBER	FREQUENCY OF SHOCK OSC.(kHz)	MAX. AMPL. OF PRESSURE FLUCTUATION $\Delta P/q$ (%)
1.	100	0.863	1.50	8.53
2.	99.95	0.849	1.50	5.45
3.	97.62	0.849	1.55	10.2
4.	95.2	0.844	1.60	8.53
5.	92.9	0.844	1.60	6.87
6.	90.5	0.844	1.60	6.16
7.	85.7	0.865	NO SHOCK OSCILLATIONS	
8.	82.5	0.886	NO SHOCK OSCILLATIONS	
AIRFOIL REVERSED (BLUNT LEADING EDGE				
9.	82.5	0.886	1.2	8.53

TABLE 7

14% BICONVEX AEROFOIL WITH SPLITTER PLATE
(FIXED-TRANSITION)

	AEROFOIL SIZE c_1 (%c)		MACH NUMBER	FREQUENCY OF SHOCK OSC. (kHz)	MAX. AMPL. OF PRESSURE FLUCTUATION ($\Delta P/q$) %
1.	100		0.895	1.136	8.52
2.	99.25		0.895	1.126	7.10
3.	98.50		0.895	1.136	7.10
4.	97.75		0.876 0.896	NO SHOCK OSCILLATIONS 1.136	7.10
5.	97.0		0.876 0.894	NO SHOCK OSCILLATIONS 1.110	7.10
6.	96.25		0.877 0.899	1.110 1.204	4.36 5.68
7.	95.75		0.879 0.894	1.136 1.136	6.25 5.68
8.	95.00		0.873 0.894	NO SHOCK OSCILLATIONS 1.110	5.68
9.	91.60		0.873 0.892	NO SHOCK OSCILLATIONS 1.123	7.38
10.	88.40		0.873 0.892	NO SHOCK OSCILLATIONS 1.10	4.26
11.	85.20		0.871 0.892	NO SHOCK OSCILLATIONS NO SHOCK OSCILLATIONS	
12.	82.60		0.869 0.887	NO SHOCK OSCILLATIONS NO SHOCK OSCILLATIONS	

TABLE 8

14% BICONVEX AEROFOIL WITH SPLITTER PLATE
(FREE-TRANSITION)

	AEROFOIL SIZE c_1 (%c)	MACH NUMBER	FREQUENCY OF SHOCK OSC. (kHz)	MAX. AMPL. OF PRESSURE FLUCTUATION $\Delta P/q$ (%)
1.	100	0.876 0.905	1.6 1.68	8.72 5.68
2.	99.25	0.866 0.895	1.55 1.60	7.28 5.68
3.	98.50	0.868 0.895	1.60 1.60	9.65 6.25
4.	97.75	0.884 0.893	1.60 1.60	7.38 12.8
5.	97.00	0.874 0.894	1.60 1.60	7.67 7.67
6.	96.25	0.878 0.894	1.59 1.60	7.09 7.67
7.	95.75	0.878 0.899	1.60 1.60	7.10 7.10
8.	95.00	0.873 0.892	1.60 1.60	8.16 7.10
9.	91.60	0.873 0.892	1.55 1.60	7.67 7.10
10.	88.40	0.871 0.892	1.60 1.60	7.28 7.10
11.	85.20	0.871 0.892	1.60 1.62	7.28 8.52
12.	82.60	0.871 0.892	1.60 1.60	8.74 9.37

TABLE 9PERIODIC FLOWS ON AN NACA 0012 AEROFOIL

	INSTABILITY MACH NUMBER (M_∞)	INCIDENCE	BOUNDARY- LAYER	FREQUENCY PARAMETER $(2\pi fc/U_\infty)$	MAX. AMPL. OF PRESSURE FLUCTUATIONS $\Delta P_{\max}/q\%$	REMARKS
1.	0.817	0°	TURBULENT	0.980	3.14	FORCED OSC.
2.	0.843	0°	TURBULENT	1.018	2.73	
3.	0.827	0°	LAMINAR	0.939	2.25	FORCED OSC.
4.	0.852	0°	LAMINAR	0.945	2.69	
5.	0.927	0°	LAMINAR	0.645	4.40	
6.	0.952	0°	LAMINAR	0.664	4.48	
7.	0.839	3°	TURBULENT	1.042	5.10	FORCED OSC.
8.	0.871	3°	TURBULENT	1.009	6.40	
9.	0.913	3°	LAMINAR	0.991	2.67	FORCED OSC.
10.	0.842	6°	TURBULENT	1.360	4.9	
11.	0.879	6°	TURBULENT	0.724	5.72	
12.	0.796	6°	LAMINAR	0.763	5.20	
13.	0.824	6°	LAMINAR	0.811	6.20	
14.	0.871	6°	LAMINAR	0.707	4.70	
15.	0.892	6°	LAMINAR	0.845	5.50	
16.	0.927	6°	LAMINAR	0.920	2.63	FORCED OSC.

TABLE 10PERIODIC FLOWS ON A CAST 7 AEROFOIL

	INSTABILITY MACH NUMBER (M_∞)	INCIDENCE	BOUNDARY- LAYER	FREQUENCY PARAMETER	MAX. AMPL. OF PRESSURE FLUCTUATIONS $(\Delta P/q)$	REMARKS
1.	0.853	3°	LAMINAR	0.506	2%	-
2.	0.887	3°	LAMINAR	0.495	2%	-
3.	0.855	6°	LAMINAR	0.65	4%	-

TABLE 11PERIODIC FLOWS ON A 11.8% JOUKOWSKI AEROFOIL

	INSTABILITY MACH NUMBER (M_∞)	INCIDENCE	BOUNDARY- LAYER	FREQUENCY PARAMETER	MAX. AMPL. OF PRESSURE FLUCTUATIONS $(\Delta P/q)$	REMARKS
1.	0.85	3°	TURBULENT	0.41	6.25%	-
2.	0.866	3°	TURBULENT	0.47	4.06%	-
3.	0.887	3°	TURBULENT	0.44	4.94%	-
4.	0.877	3°	LAMINAR	0.34	5.58%	-
5.	0.890	3°	LAMINAR	0.35	7.40%	-
6.	0.866	6°	TURBULENT	0.40	11.2%	-
7.	0.890	6°	TURBULENT	0.47	7.50%	-
8.	0.866	6°	LAMINAR	0.64	7.44%	-

TABLE 12

PARAMETERS FOR PERIODIC FLOW
(ZERO INCIDENCE)

	AEROFOIL	THICKNESS 12% OR GREATER?	SHOCK POSITION x_s/c	TRAILING- EDGE ANGLE	MACH NO. M_1	SHOCK OSCILLATION?
1.	14% BICONVEX	YES	0.62-0.71	LARGE	1.22-1.26	YES
2.	NACA 0012	YES	0.50-0.60	LARGE	1.24-1.28	YES
3.	6% BICONVEX	NO	0.75-0.90	LARGE	1.20-1.30 (Ref. 21)	NO
4.	CAST 7	YES	2 shocks 0.24-0.31 0.55-0.71	SMALL	1.25-1.28 at $M_\infty =$ 0.76	NO
5.	JOUKOWSKI	YES	0.17-0.46	SMALL	-	NO

APPENDIX E

The Reasons for the Location of Shocks in Steady Conditions

Consider the flow in a finite supersonic region as illustrated in Fig. E1. In general, the region of supersonic flow (assumed shock free in Fig. E1) will contain 2 families of waves. One of these originate at the surface and the other comes from the region where the first family meets the sonic line. The waves arising from the surface must be reflected at the sonic line as waves of equal strength but opposite sign. The waves springing from the surface are expansive and reflected waves are compressive. Unless the supersonic region breaks down previously, these reflected compression waves will again be reflected when they meet the surface as compressive waves and may coalesce to form a shock (Ref. 72). The other possible explanation for the occurrence of a shock could be the upstream motion of disturbance waves originating in the wake. These would be unable to propagate forwards into the finite supersonic region and hence might be expected to build up into a shock at its downstream end.

Fig. E2 illustrates the flow patterns on the upper surface of an aerofoil at incidence with increasing freestream Mach number. Experiments have shown that the local Mach numbers at fixed upstream points do not change greatly and over most of the surface are close to the pressure distribution achieved at a freestream Mach number of unity. By drawing an analogy with one dimensional nozzle flow, the shock position on the aerofoil can be considered to be determined by the downstream pressure by a mechanism illustrated in Fig. E5. Thus as the shock moves rearwards, the pressure p_1 immediately upstream traces out a certain locus and the downstream pressure p_2 traces out a corresponding locus determined by the pressure rise through the shock. If the pressure distribution in the subsonic flow behind the shock leading to the freestream static pressure far downstream could be determined, the shock must position itself at the point of intersection with the p_2 locus. This analogy shows that three features of the flow contribute strongly to the determination of the shock position.

These are the flow in the supersonic region ahead of the shock, the pressure rise across the shock and the subsonic flow behind the shock.

Analysis of a large number of experimental results have shown (Ref. 72) that the observed irregularities in the rearwards motion of the upper surface shocks (Fig. E4) with increasing Mach number are associated with the change of pressure rise through the shock and the downstream pressure distribution that occurs after the shock has become strong enough to provoke separation of the turbulent boundary-layer. The rapid rearwards movement of the lower surface shock (Fig. E4) occurs when the bubble of separated flow on the upper surface has extended rearwards to an extent as sketched in Fig. E2 (stage 4) where it affects the pressure changes along the wake and hence the pressure on the lower surface. Thus the wake cannot sustain any strong transverse pressure gradient so that the pressure near the trailing edge must be equal on the two surfaces, and if the trailing edge pressure changes as a result of separation on the upper surface, the lower surface must adjust itself to produce a similar change of trailing edge pressure.

APPENDIX F

Shock-Wave-Boundary-Layer Interaction in Supersonic and Transonic Flow

1. Supersonic Flow

When shock-waves appear in the neighbourhood of solid boundaries they impose large pressure gradients in the boundary-layer and distort them to a considerable extent. These effects on the boundary-layer can be propagated both upstream and downstream through the subsonic portion of the boundary-layer. The extent of the interaction region are different for laminar and turbulent boundary-layers. In a laminar interaction the disturbances propagate upstream over distances between 10 to 100 times the boundary-layer thickness. The length of the turbulent interaction is of the order of 10 times the boundary-layer thickness due to the subsonic part of the boundary-layer being smaller and to turbulent mixing in the interaction region. The changes in the boundary-layer simultaneously act in such a manner as to propagate compression and expansion waves into the external flow.

Laminar Boundary-Layer

Fig. F2 illustrates the features of the interaction between an oblique shock-wave with the boundary-layer on a flat plate. If the boundary-layer were not present the incident shock would be reflected as an oblique shock of equal turning angle as in Fig. F1. The pressure gradient imposed by the shock causes the pressure in the boundary-layer to rise, considerably upstream of the point of incidence. This pressure rise thickens the boundary-layer and the resulting streamline curvature generates compression waves which merge into a reflected shock whose origin is upstream of the original point of incidence. If the adverse pressure gradient is sufficiently large the boundary-layer will separate and this will increase the strength of the first reflected shock. This

would have an effect of modifying the incident shock at the point of intersection of the two. The separated zone is a region of constant pressure and hence the incident shock reflects as an expansion in this zone. The expansion turns the flow towards the surface, producing re-attachment of the boundary-layer. Usually the turning angle through the expansion is so large that the flow can align itself to the direction of the plate only through a second reflected shock.

Turbulent Boundary-Layer

The interaction between the shock and turbulent boundary-layer is much smaller than the laminar interaction. There is slight thickening of the boundary-layer, with some wavelets generated upstream and downstream of the interaction. The extent of upstream and downstream influence is small. The reflection of the shock corresponds to the regular reflection (Fig. F3).

2. Transonic Flow

By contrast with the supersonic interaction in which the shock impinges on the boundary-layer, in a transonic interaction the shock-wave "grows" up from the boundary-layer. The shock-wave is the rear boundary of the local supersonic region, on the upper side limited by a sonic line into which it continues smoothly and on the lower side by the effective profile boundary. The laminar and turbulent interactions differ again for the reasons mentioned in the previous section. Fig. F4 illustrates the laminar interaction on an aerofoil in transonic flow. The pressure disturbance introduced by the shock-wave propagates through the boundary-layer causing a wedgelike increase in the effective profile thickness. Near the wall there usually arises a region of separated flow with recirculation. The abrupt change of the effective profile thickness leads to a gradual compression region which together with the rear shock forms the typical lambda pattern.

A typical turbulent interaction pattern is shown in Fig. F5. In this case, the gradual compression due to the abrupt change in boundary-layer thickness is rather strong so that the oblique compression waves very soon coalesce into a strong shock, which at a certain distance from the profile is nearly normal. The flow patterns with increase in Mach number and their effects on the pressure distribution have been discussed in Appendix E.

Acknowledgements

The author wishes to thank Professor J.L. Stollery for the invaluable assistance and advice received during the course of this research programme.

My special thanks are extended to Mr. D.G. Mabey for useful and stimulating discussions.

The author is also indebted to Messrs. S.W. Clark, E.W. Osbourn, M.F. Goodridge and all the members of the Workshop Staff for the help received during the course of this project.

The author also wishes to thank Mr. T.A. Sills for assistance with the data analysis, Dr. K.C. Phan for assistance in the preparation of the diagrams, Miss P.M. Walsh for the excellent typing of the manuscript and all the Aerodynamics Department staff and students for their enthusiastic moral support.

Finally, the author wishes to express his utmost gratitude for the financial support provided by the Science and Engineering Research Council.

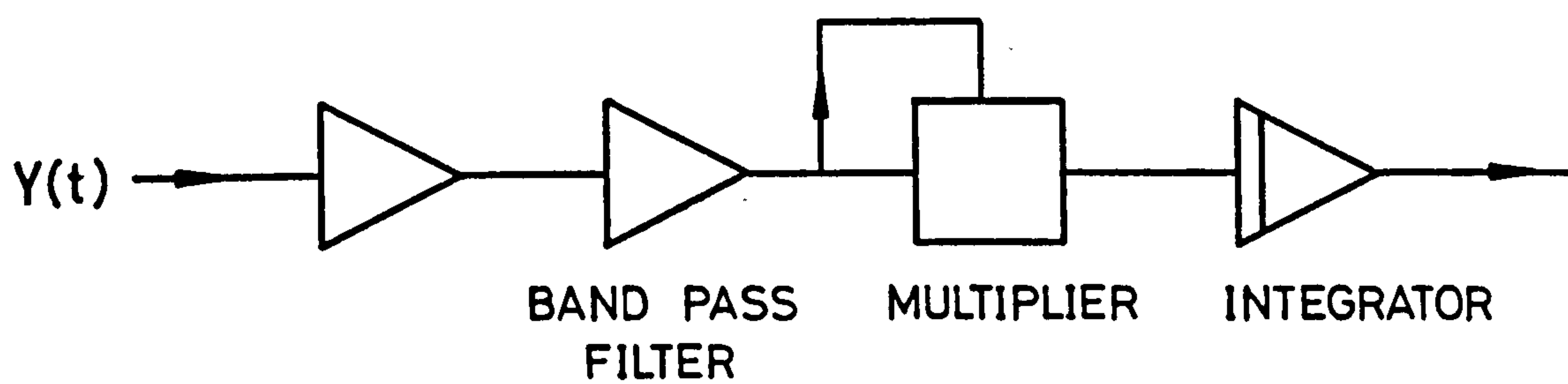


Figure A.1 Concept of a power spectral density analyser.

$$\chi = \frac{z}{x} = \frac{(z/c)_{x/c=0.2}}{(x/c)_{(\theta-\alpha)=2^\circ} - (x/c)_{(\theta-\alpha)=4^\circ}}$$

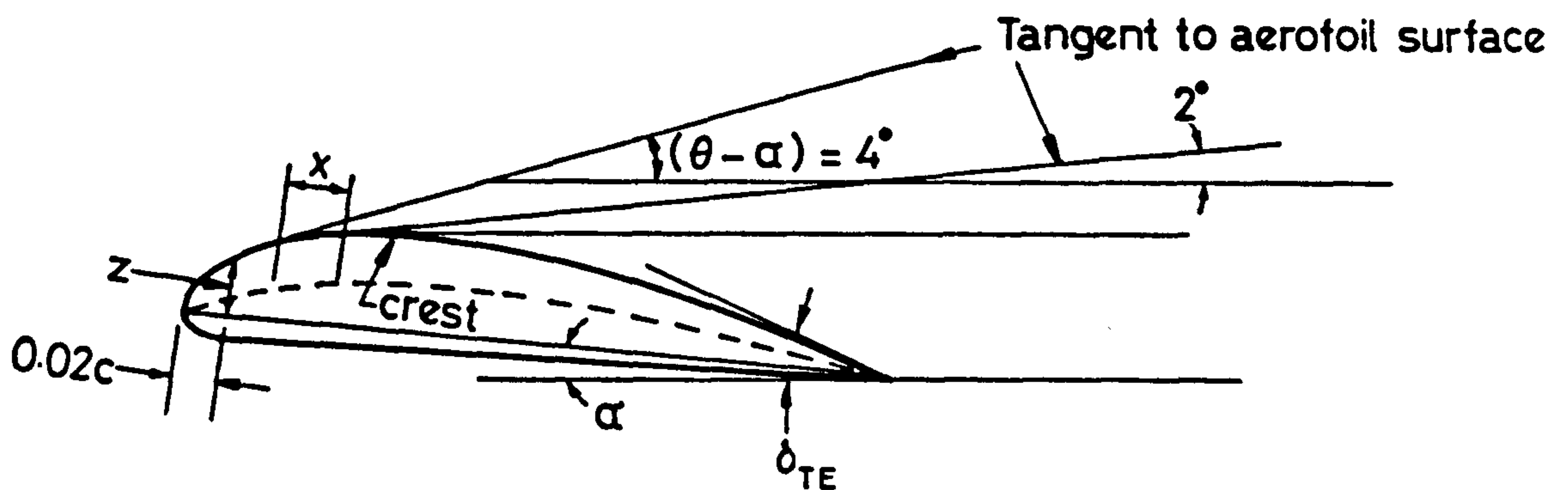
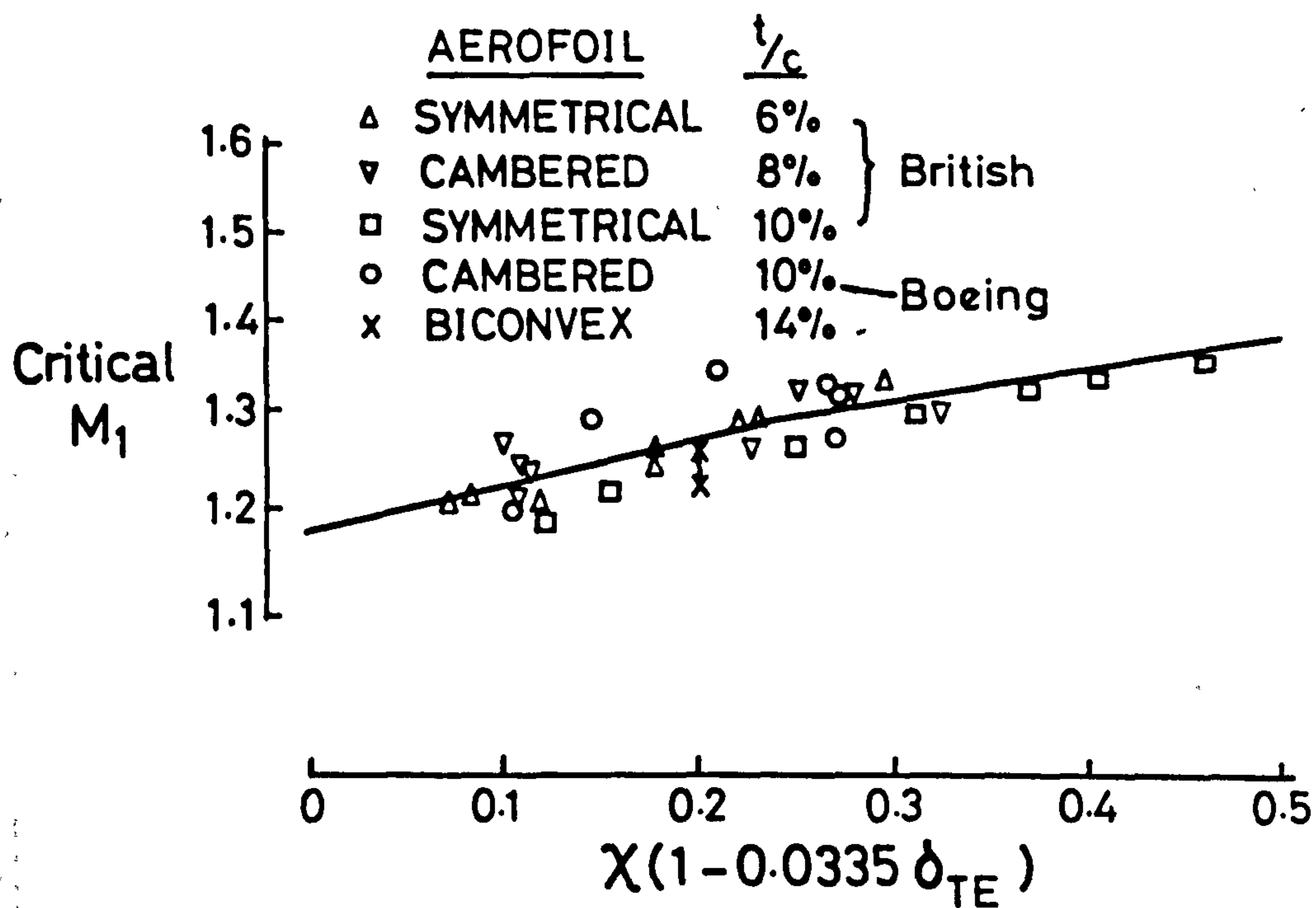
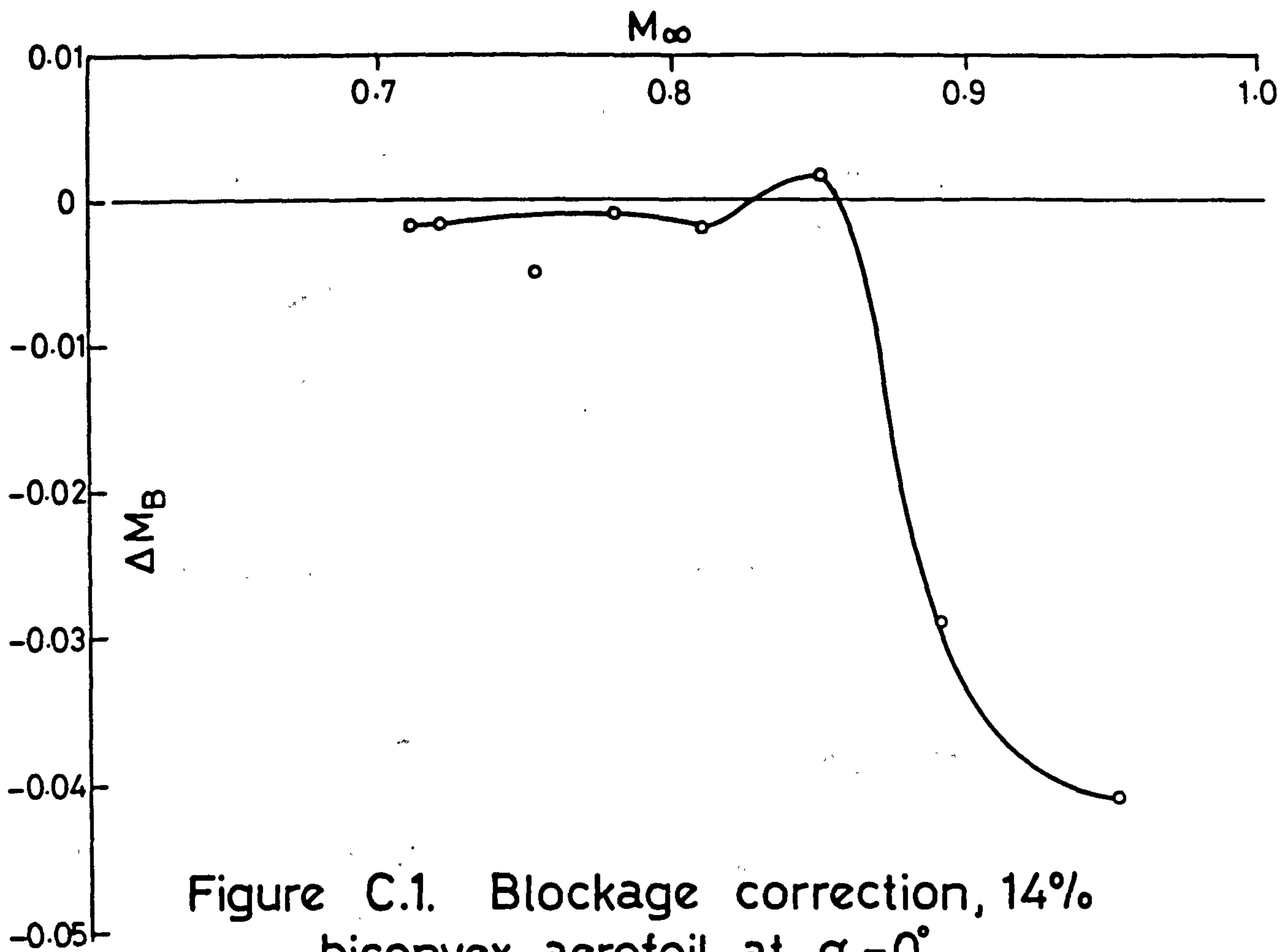
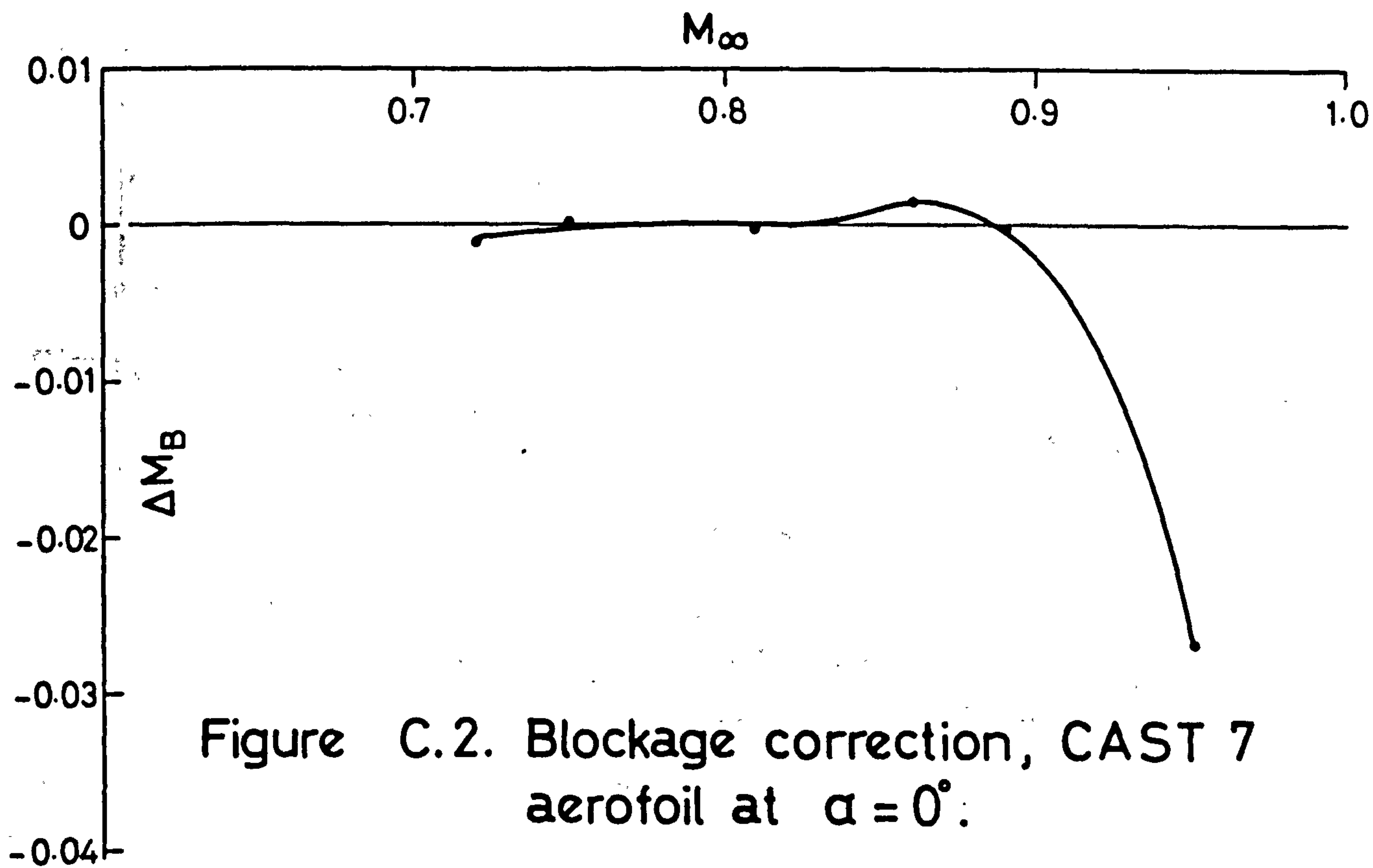


Figure B.1 Osborne's criterion for shock-induced separation – critical value of the shock upstream Mach Number for the occurrence of significant separation (ref. 37).



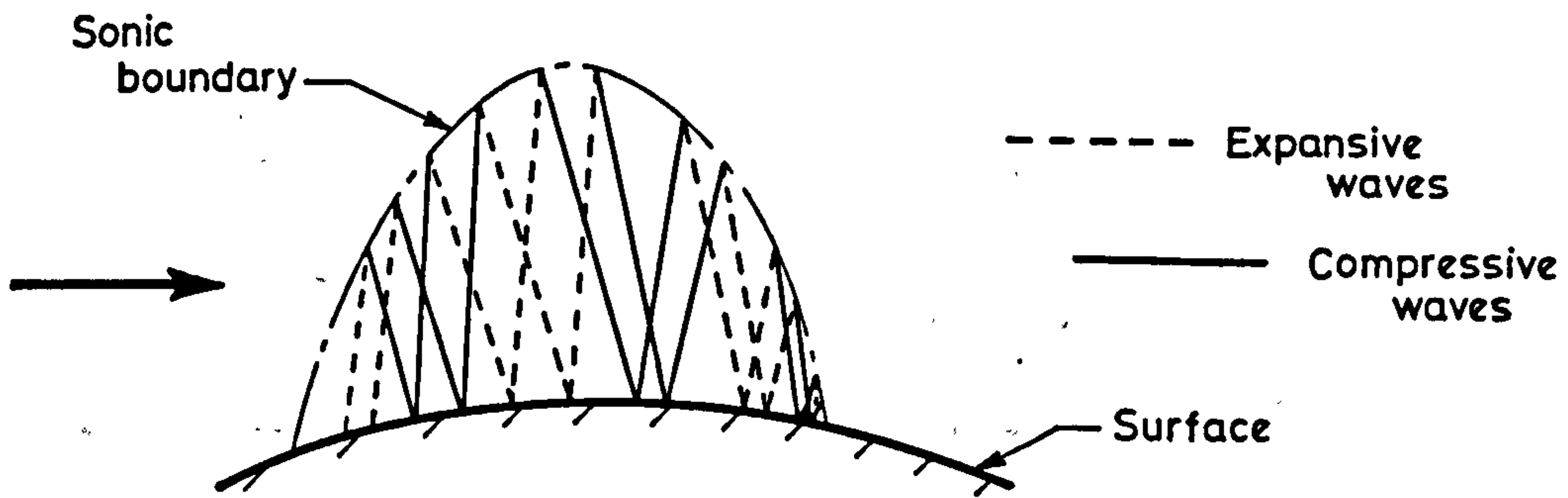


Figure E.1.

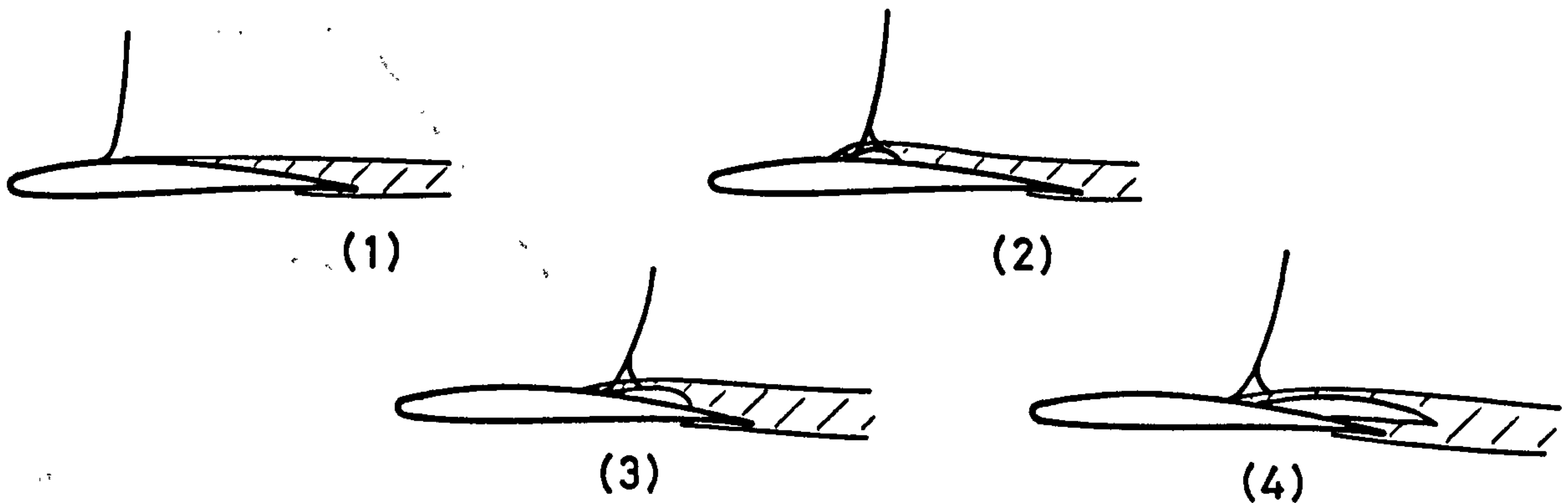


Figure E.2. Flow patterns on the upper surface of an aerofoil at incidence, (schematic)

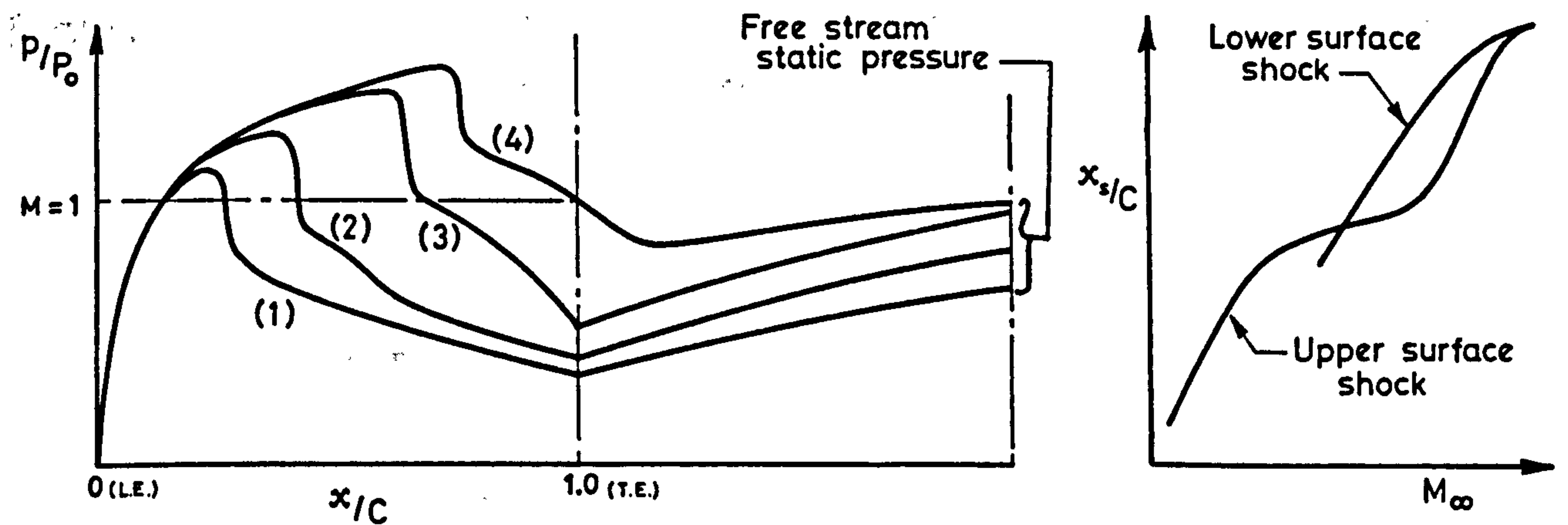


Figure E.3. Pressure distribution corresponding to stages (1)-(4), (Figure E.2)

Figure E.4. Shock positions

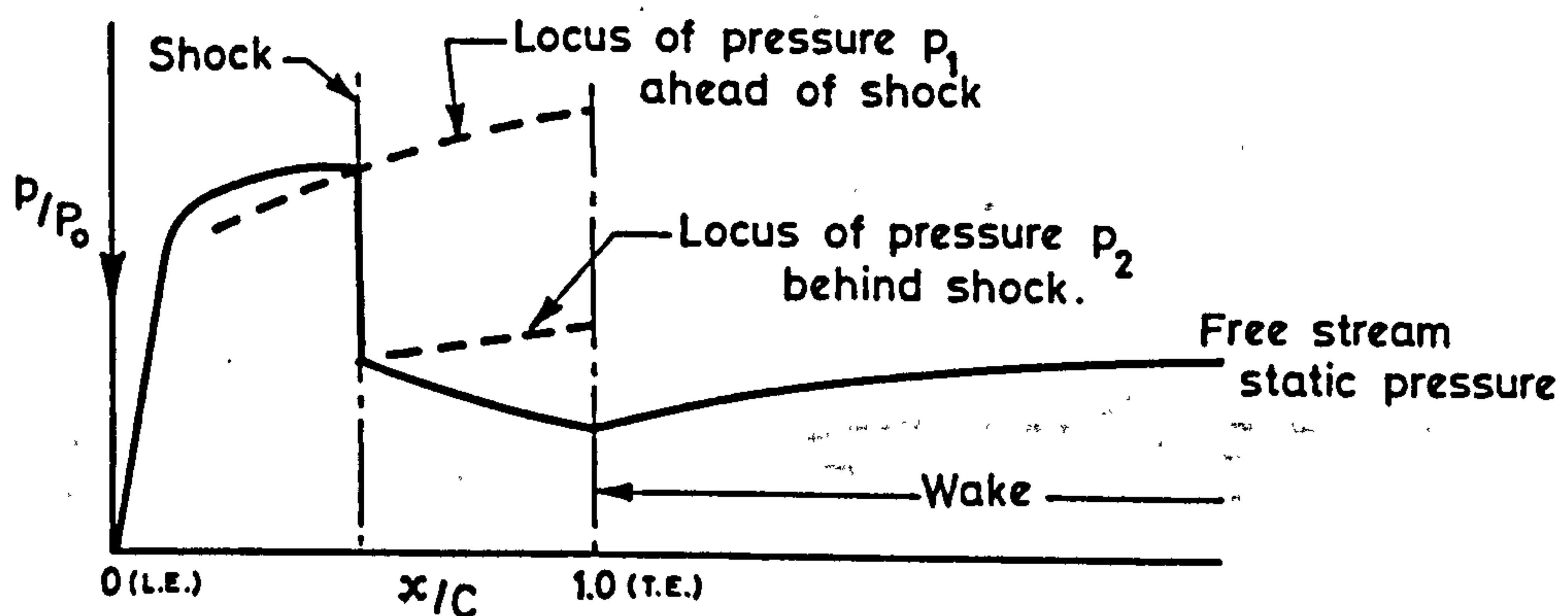


Figure E.5.

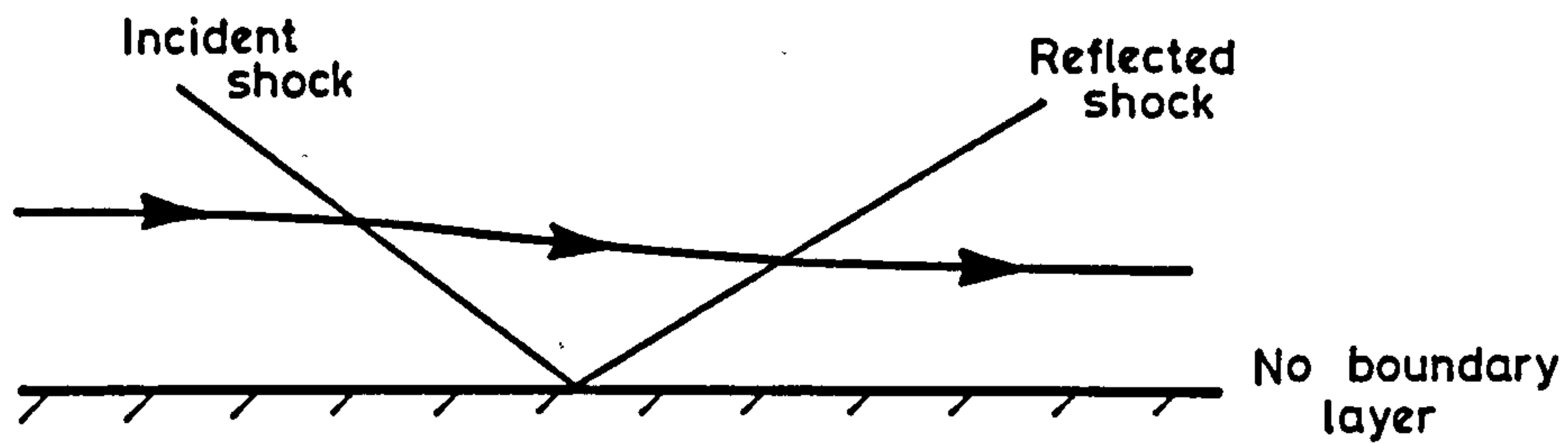


Figure F.1. Regular reflection

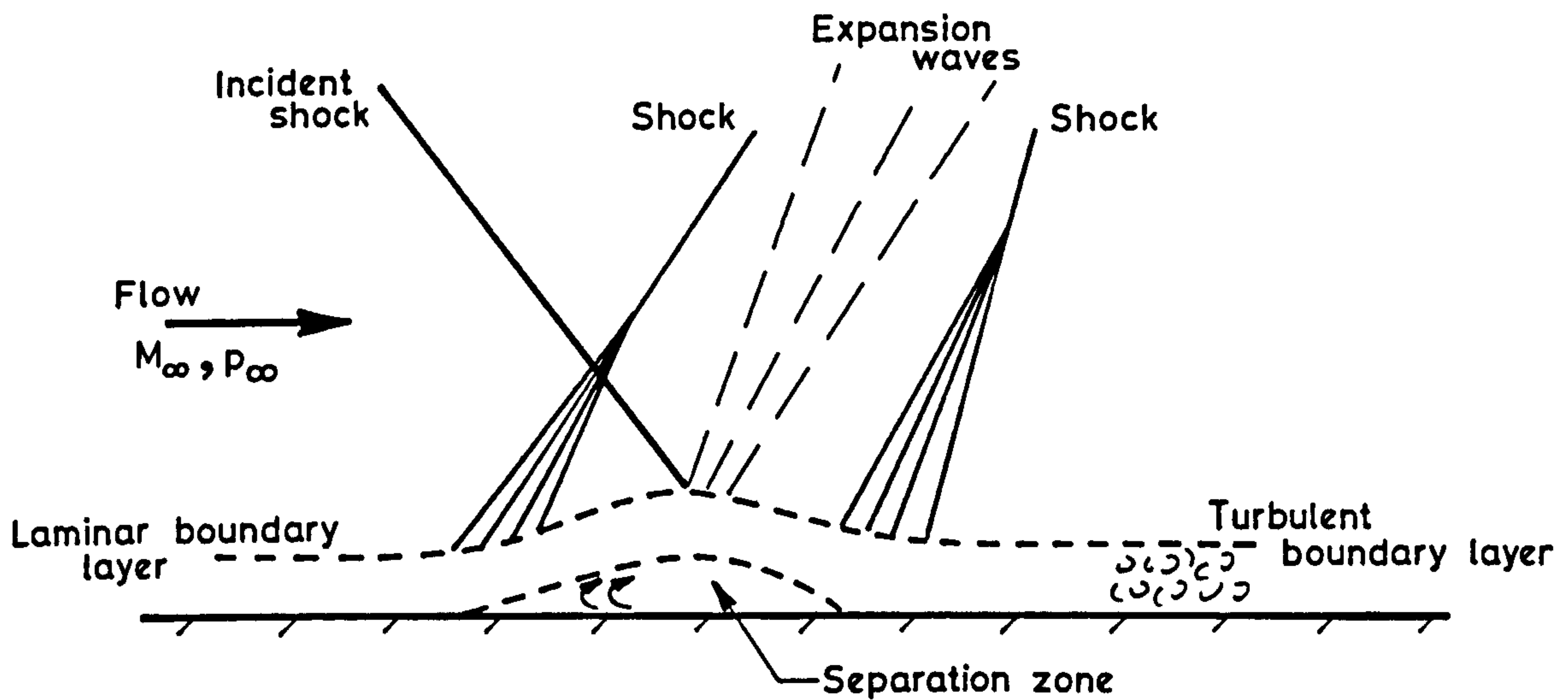


Figure F.2. Reflection of shocks from walls with laminar boundary layers. (Ref.73)

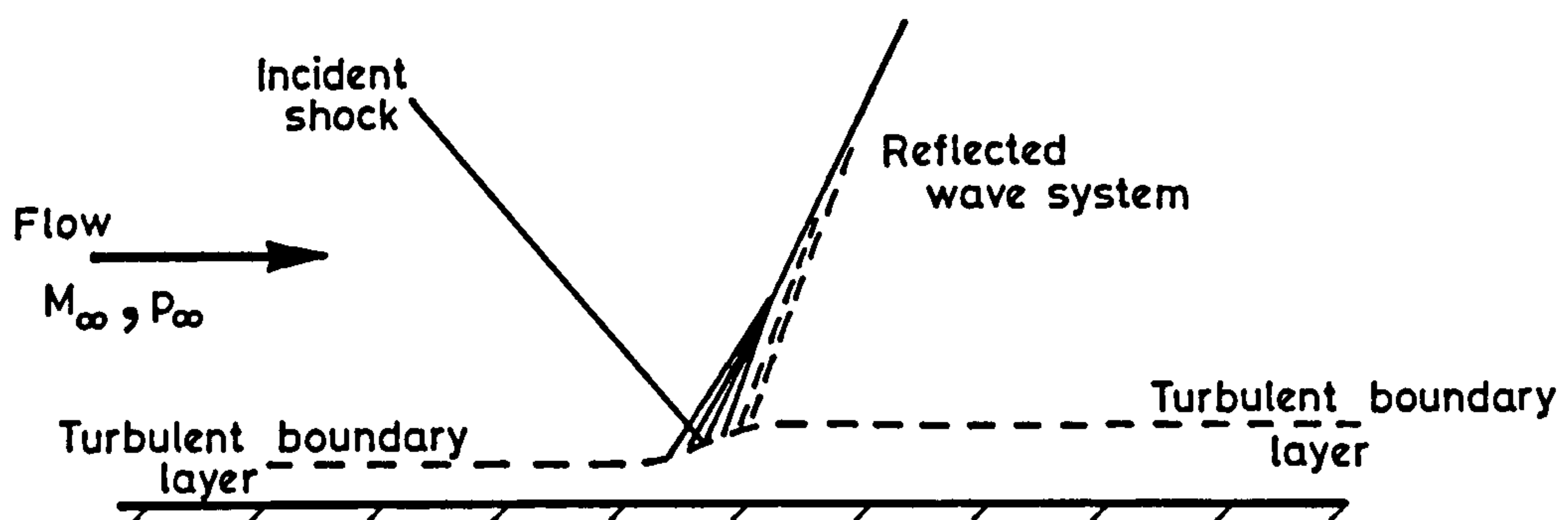


Figure F.3. Reflection of shocks from walls with turbulent boundary layers.

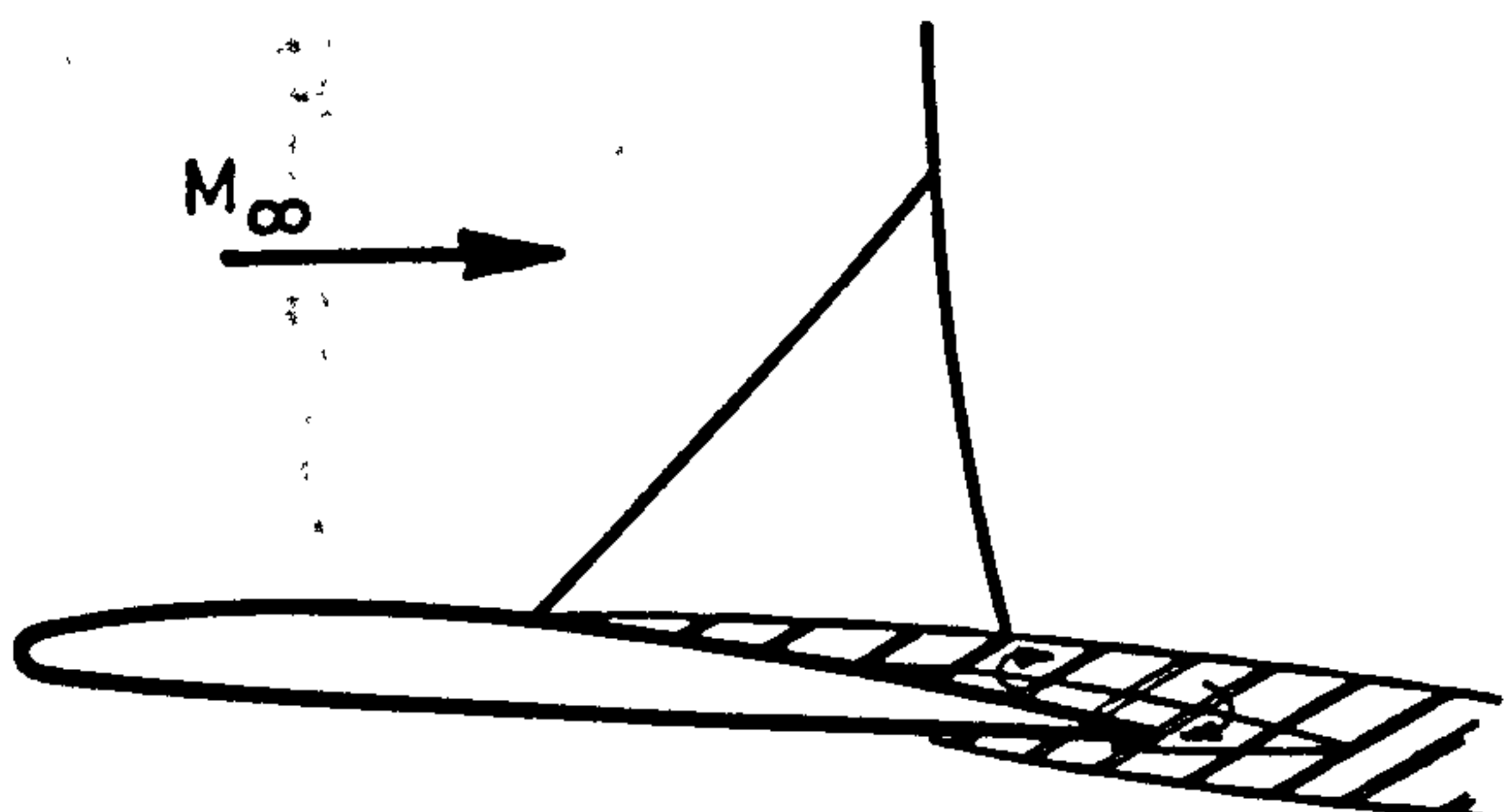
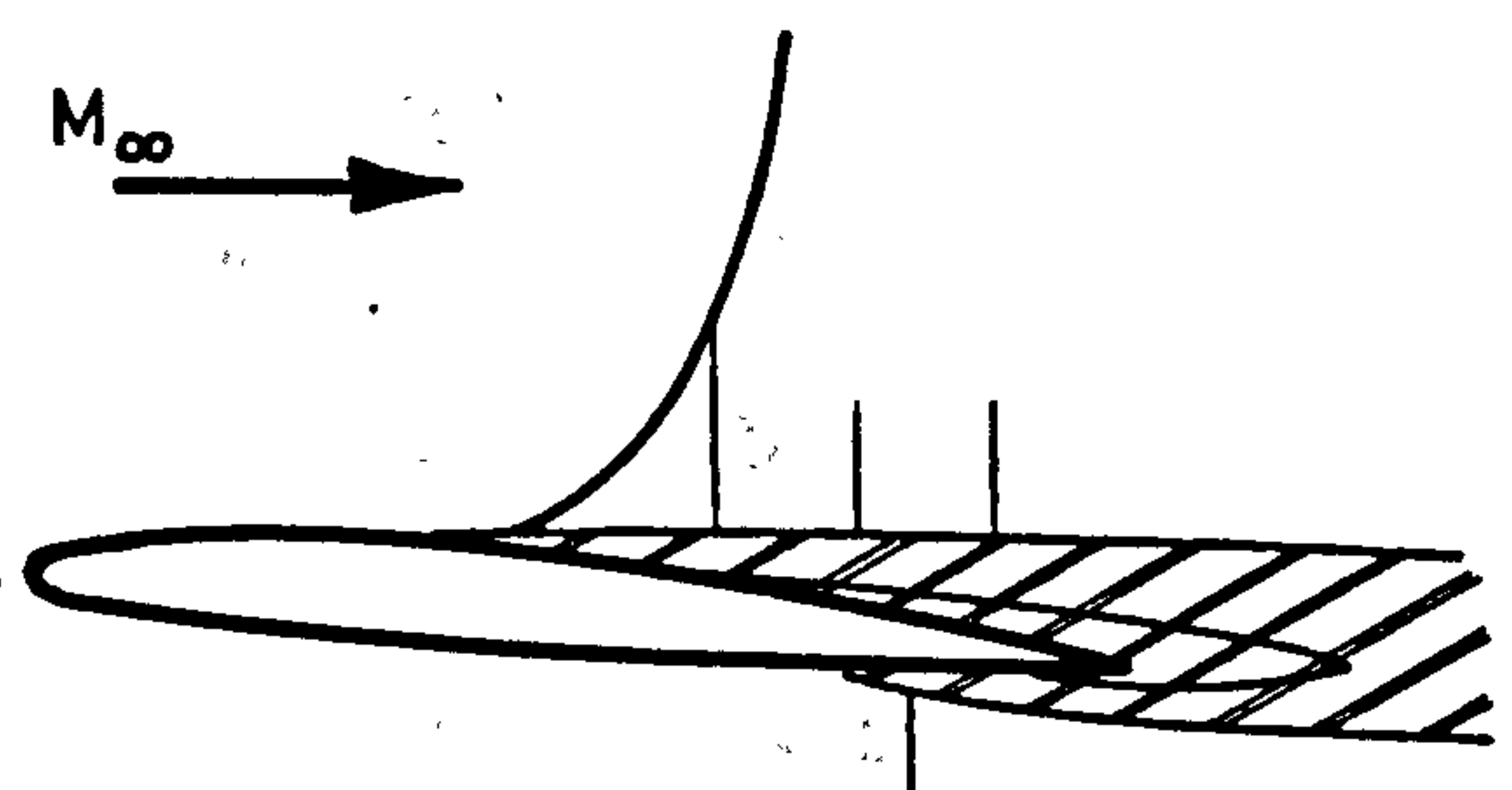
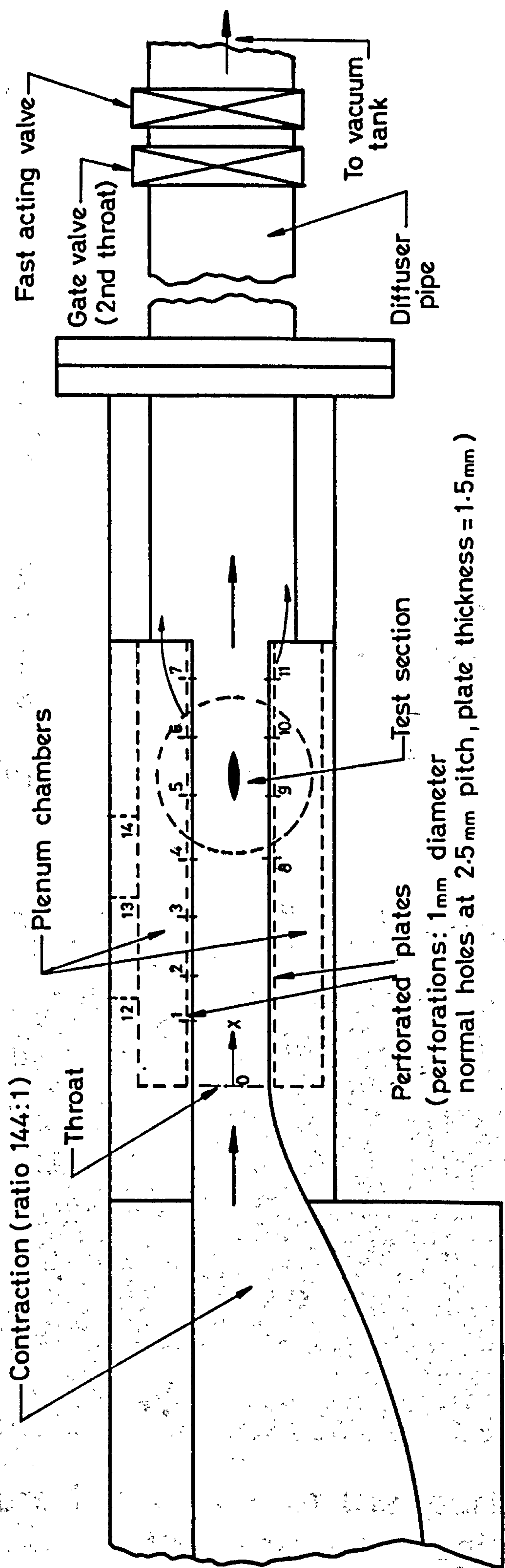


Figure F.4. Flow on an aerofoil with a laminar boundary layer.

Figure F.5. Flow on an aerofoil with a turbulent boundary layer.



Pressure Port Locations

PORT NO	1	2	3	4	5	6	7	8	9	10	11	12	13	14
x/H_T	0.84	1.41	2.17	2.96	3.81	4.58	5.35	2.96	3.81	4.58	5.35	1.11	2.44	3.51

Test section: height (H_T) = 57mm
width = 63.5mm

Nozzle open area ratio = 12%

Plenum chamber total depth = 1.16 x H_T

Diffuser area = 2.25 x test section area

Port no.13 (plenum chamber) used as reference for Mach number measurement.

Figure 1.a) Layout of the 2.5 inch transonic wind tunnel .

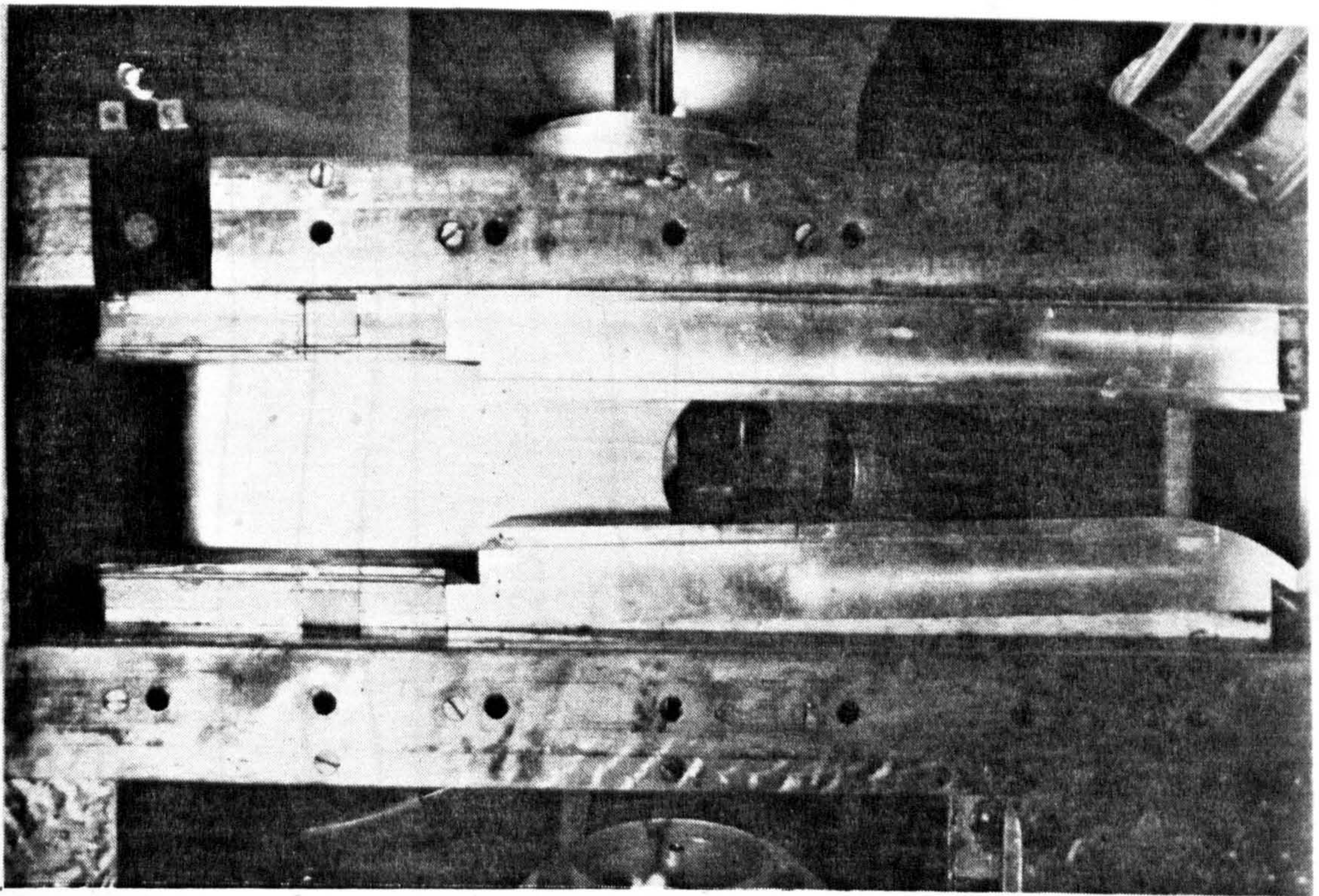


Figure 1 b) A view of the transonic test-section.

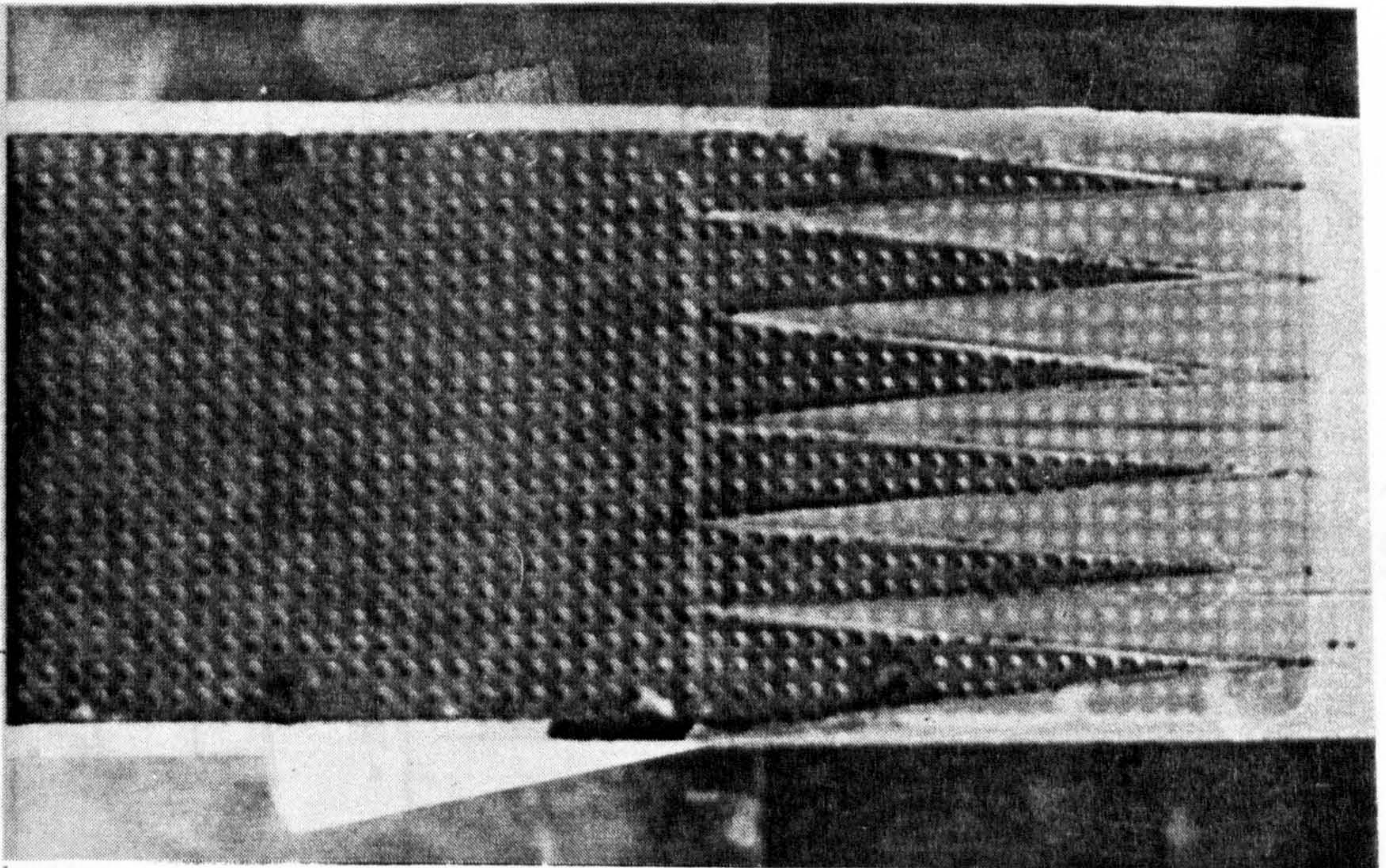
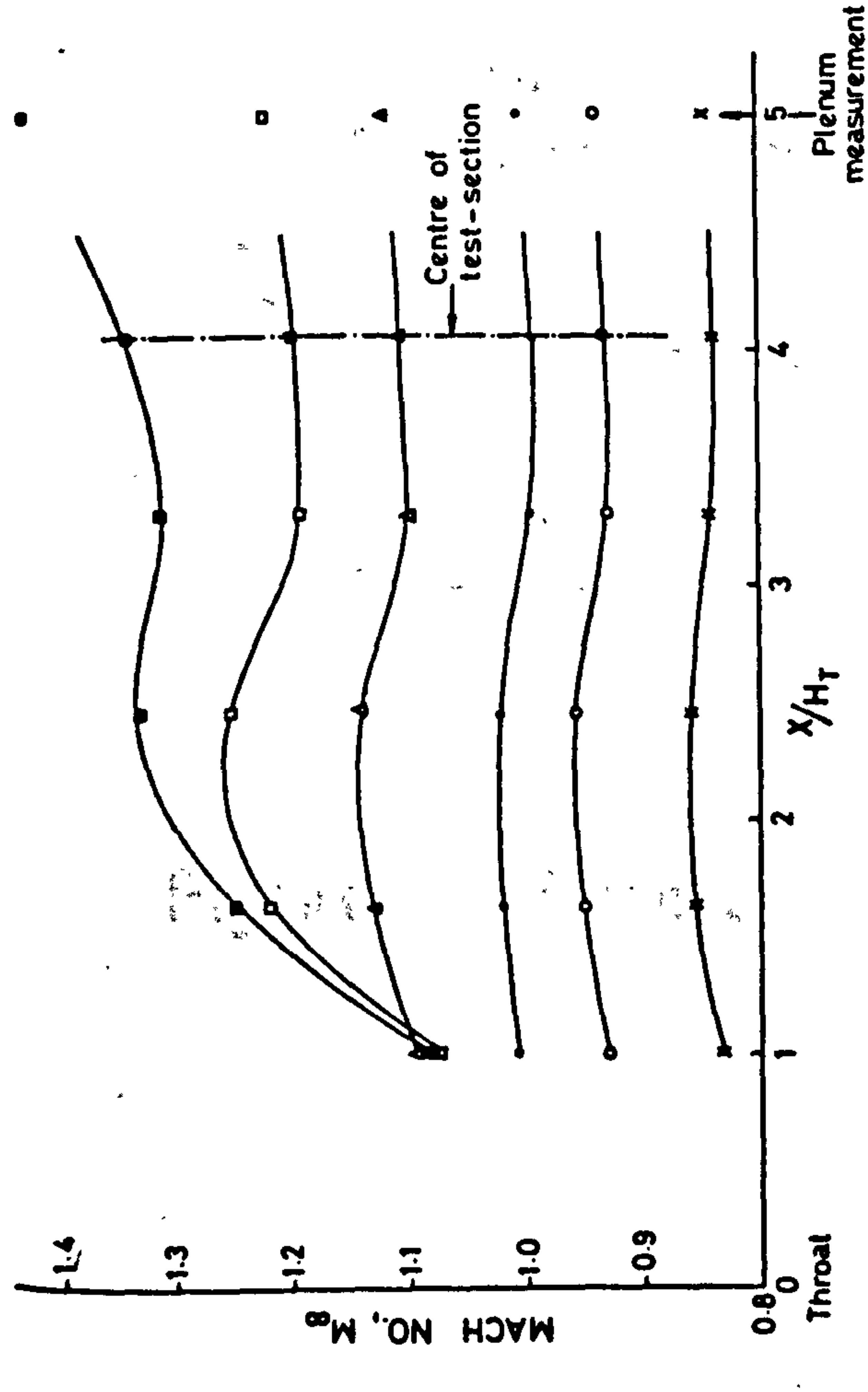
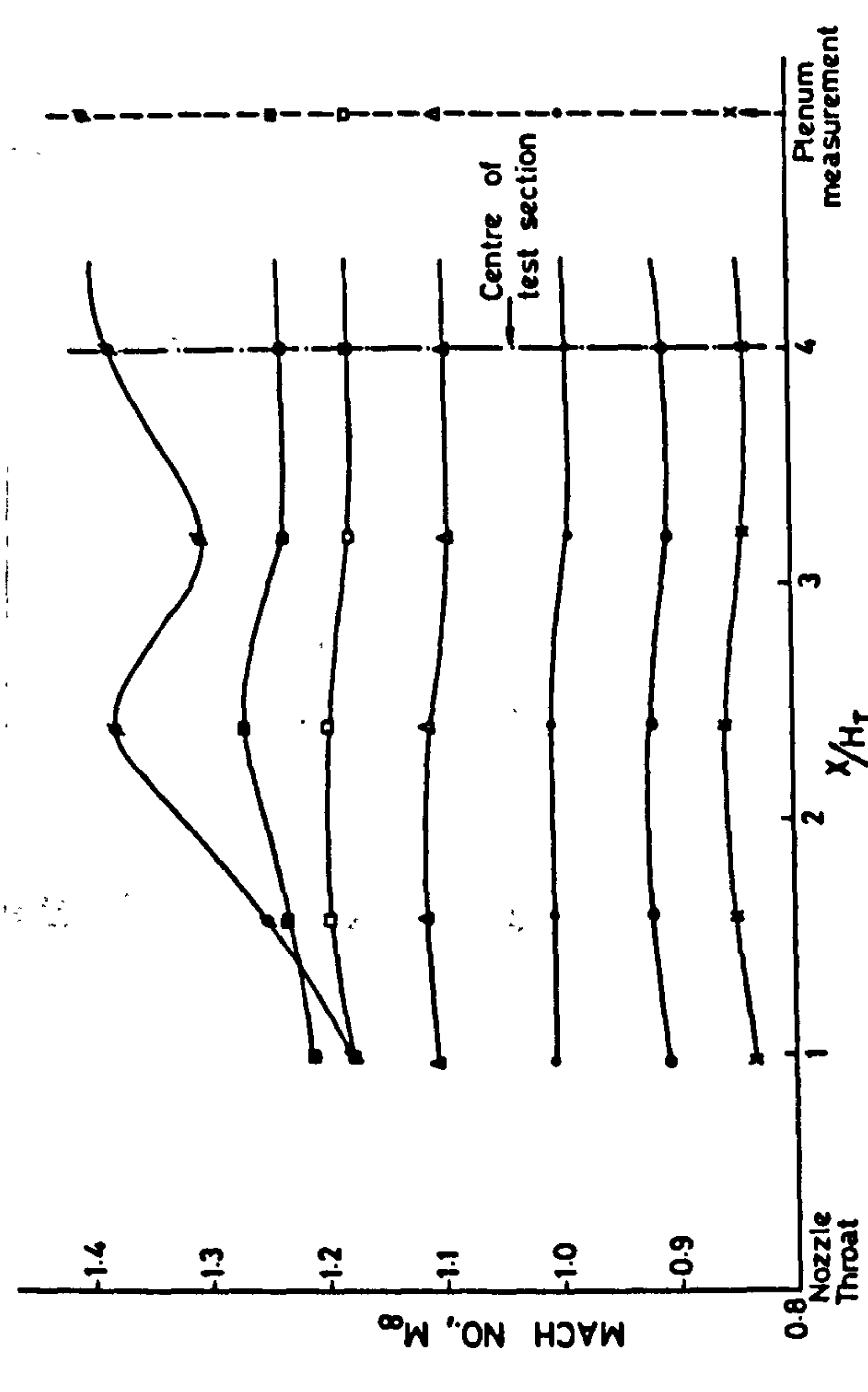


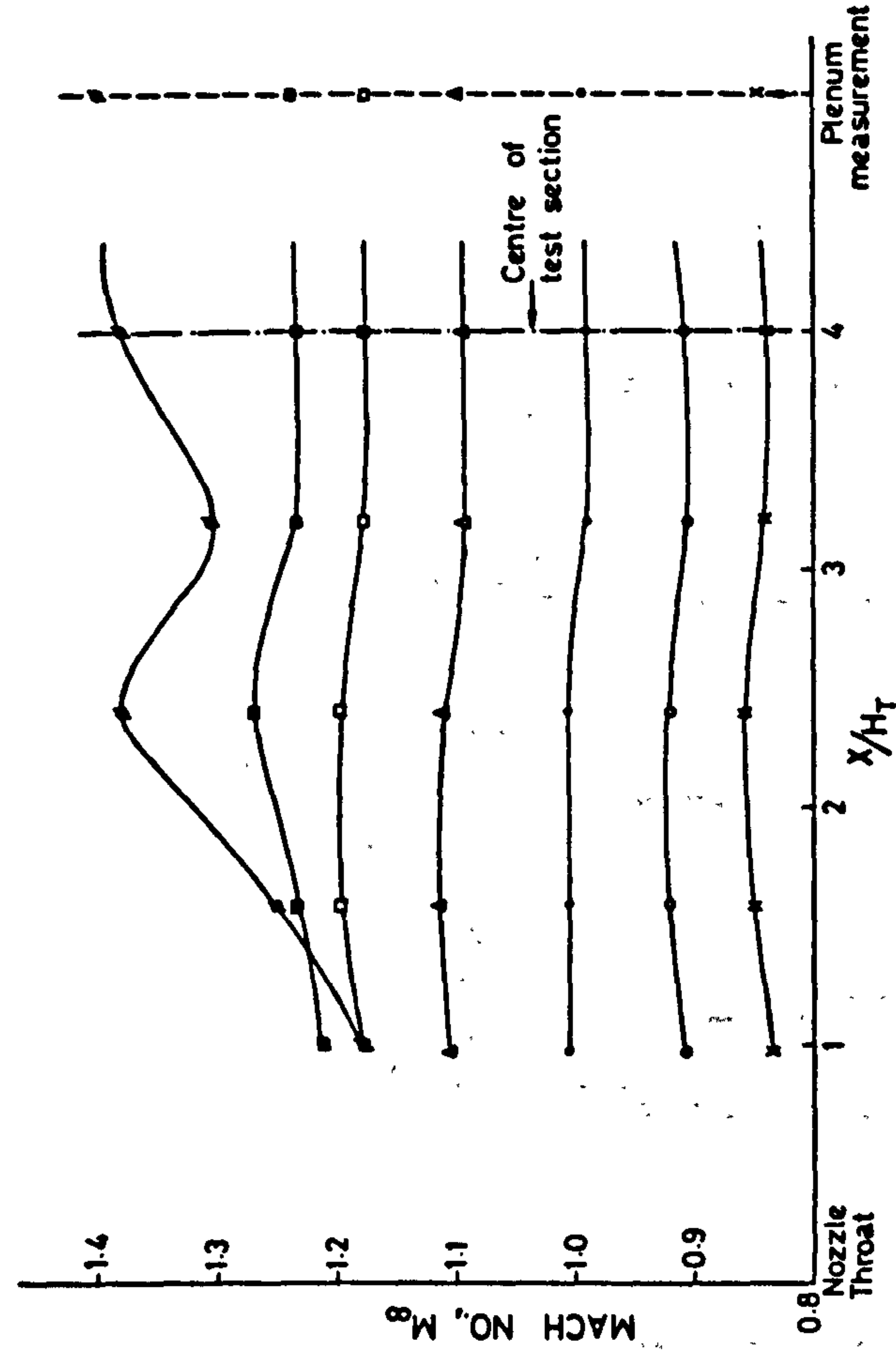
Figure 1 c) A view of the perforated liner showing the entrance region.



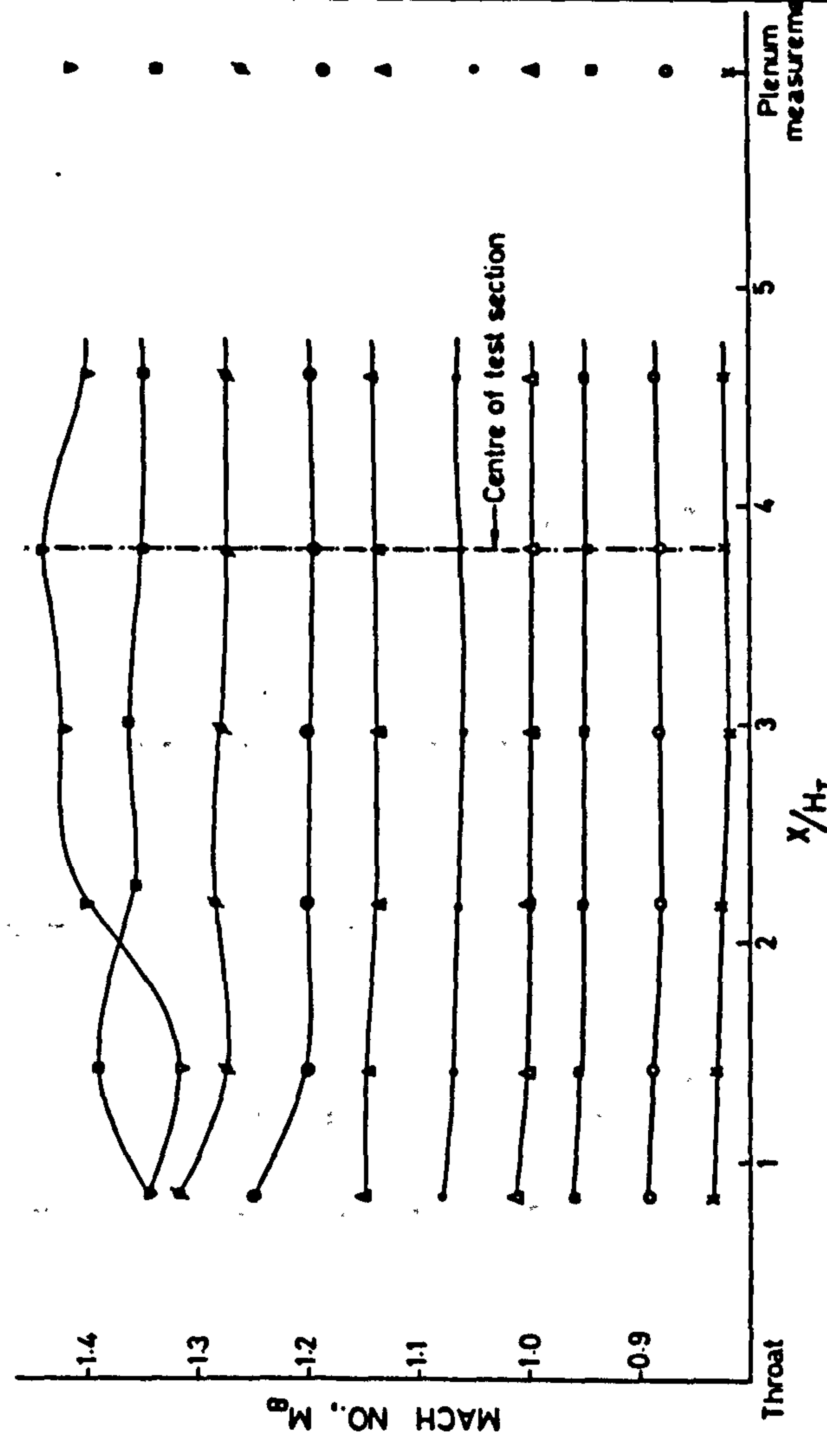
(2a)



(2c)



(2b)



(2d)

Figure 2. Mach No. distribution along transonic nozzle (bottom liner diverged by a) 0.5° , b) 1.0° , c) 1.5° , d) 2°).

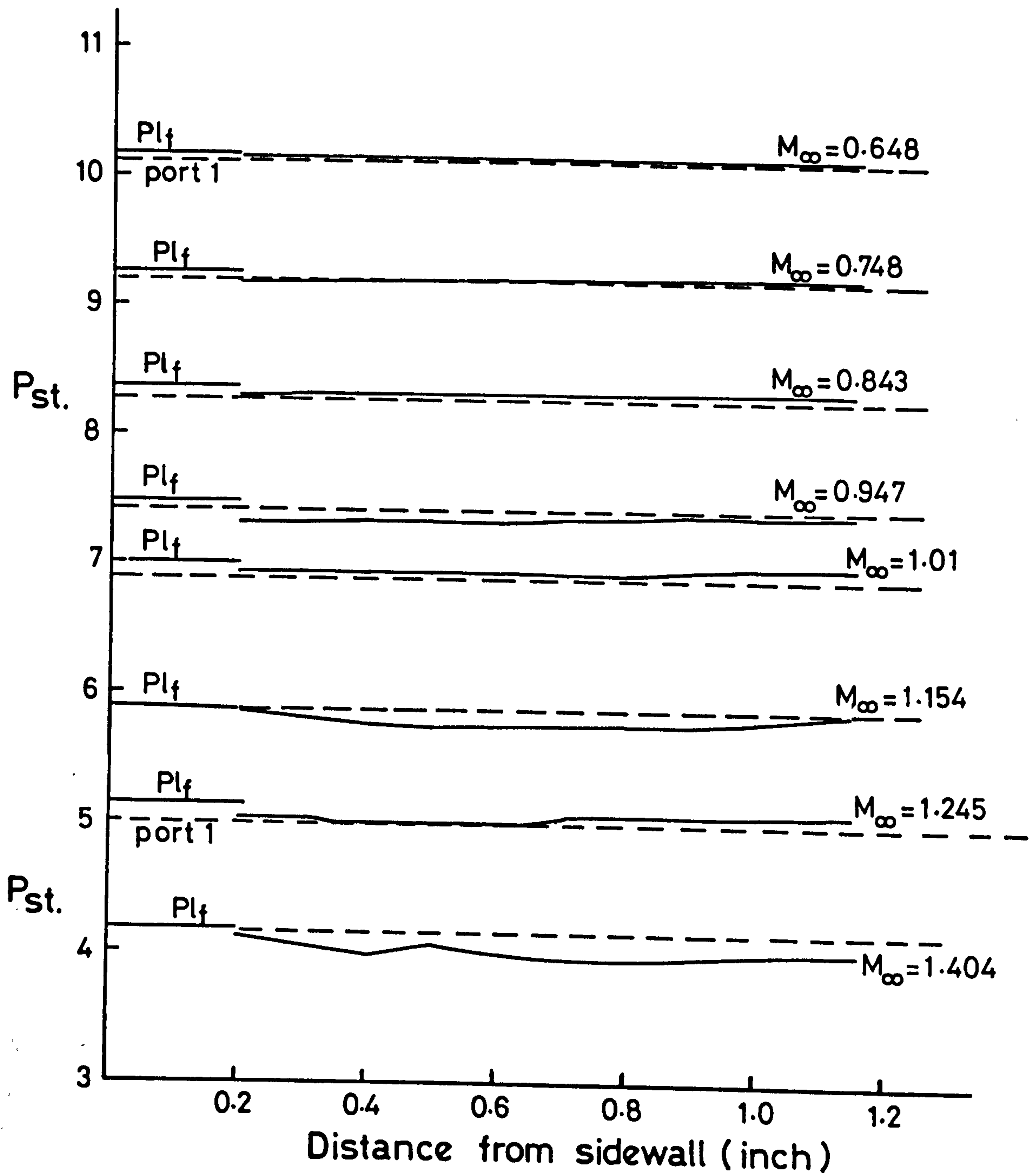
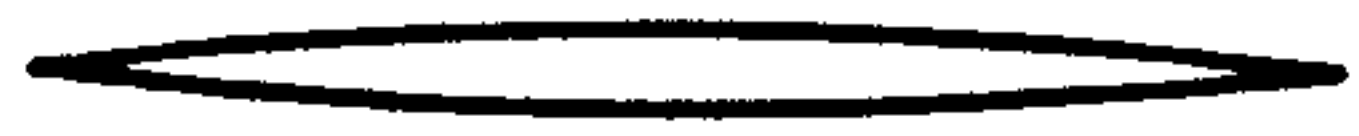


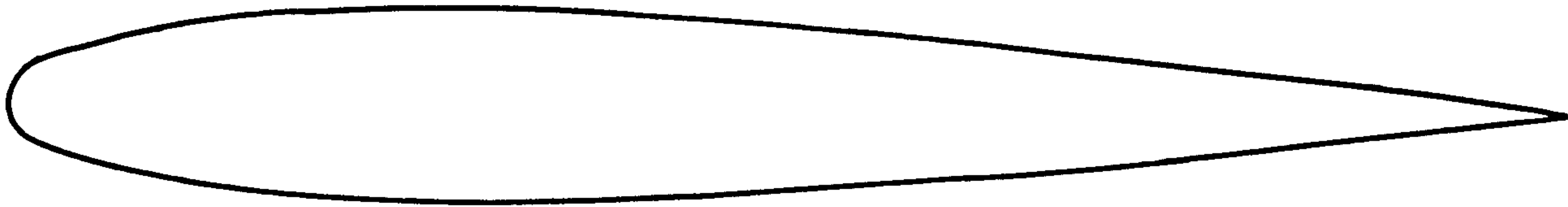
Figure 3. Centreline pressure distribution.



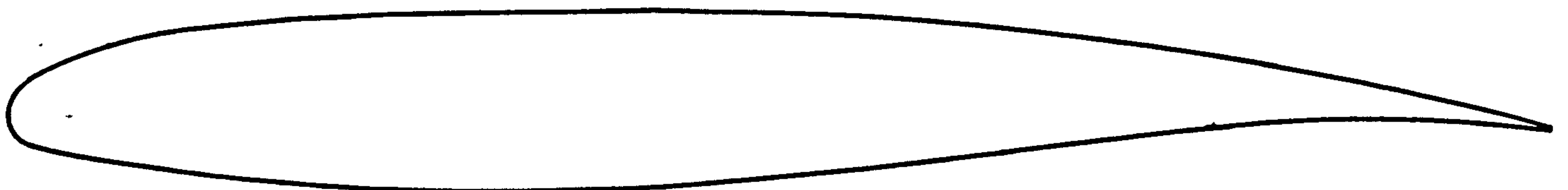
6% t/c BICONVEX AEROFOIL



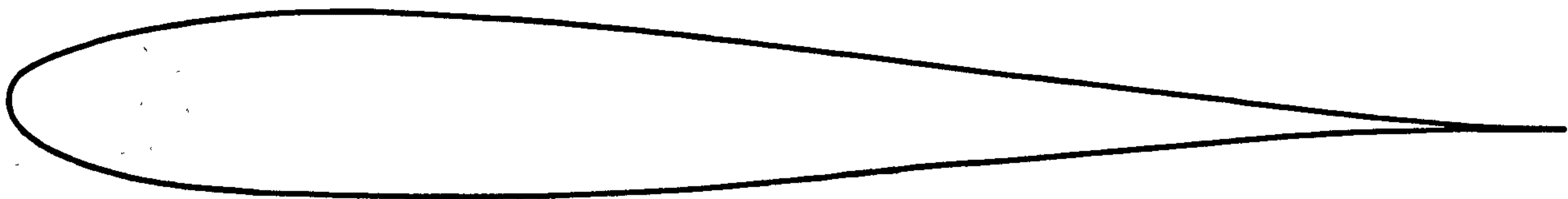
14% t/c BICONVEX AEROFOIL



NACA 0012



CAST 7 SUPERCRITICAL AEROFOIL



11.8% t/c JOUKOWSKY AEROFOIL

Figure 4.

AEROFOILS TESTED IN THE 2.5 inch TRANSONIC
WIND TUNNEL.

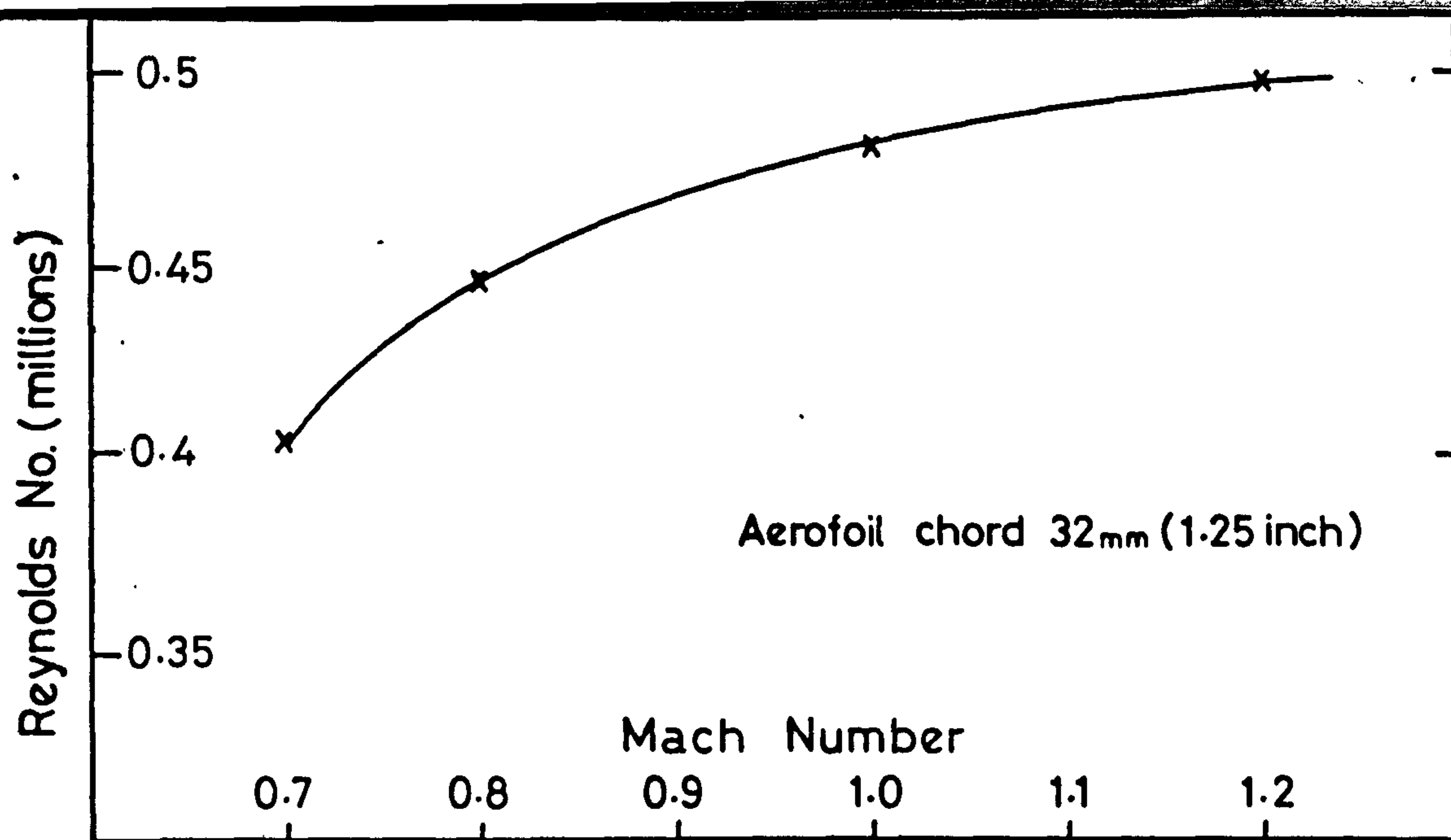
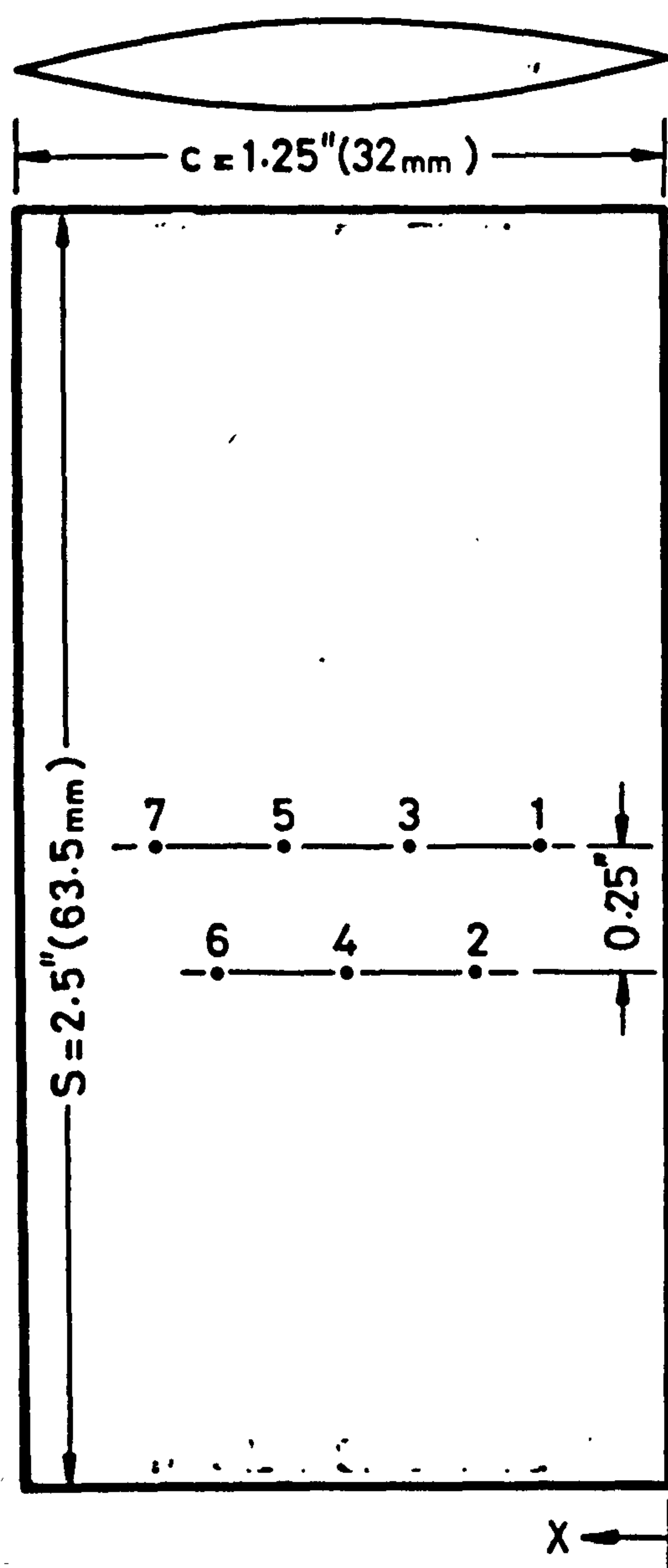


Figure 5 Reynolds No. vs Mach Number.



PORT NO.	X/c (%)
1	20
2	30
3	40
4	50
5	60
6	70
7	80

Figure 6 Pressure port locations on 14% biconvex aerofoil.

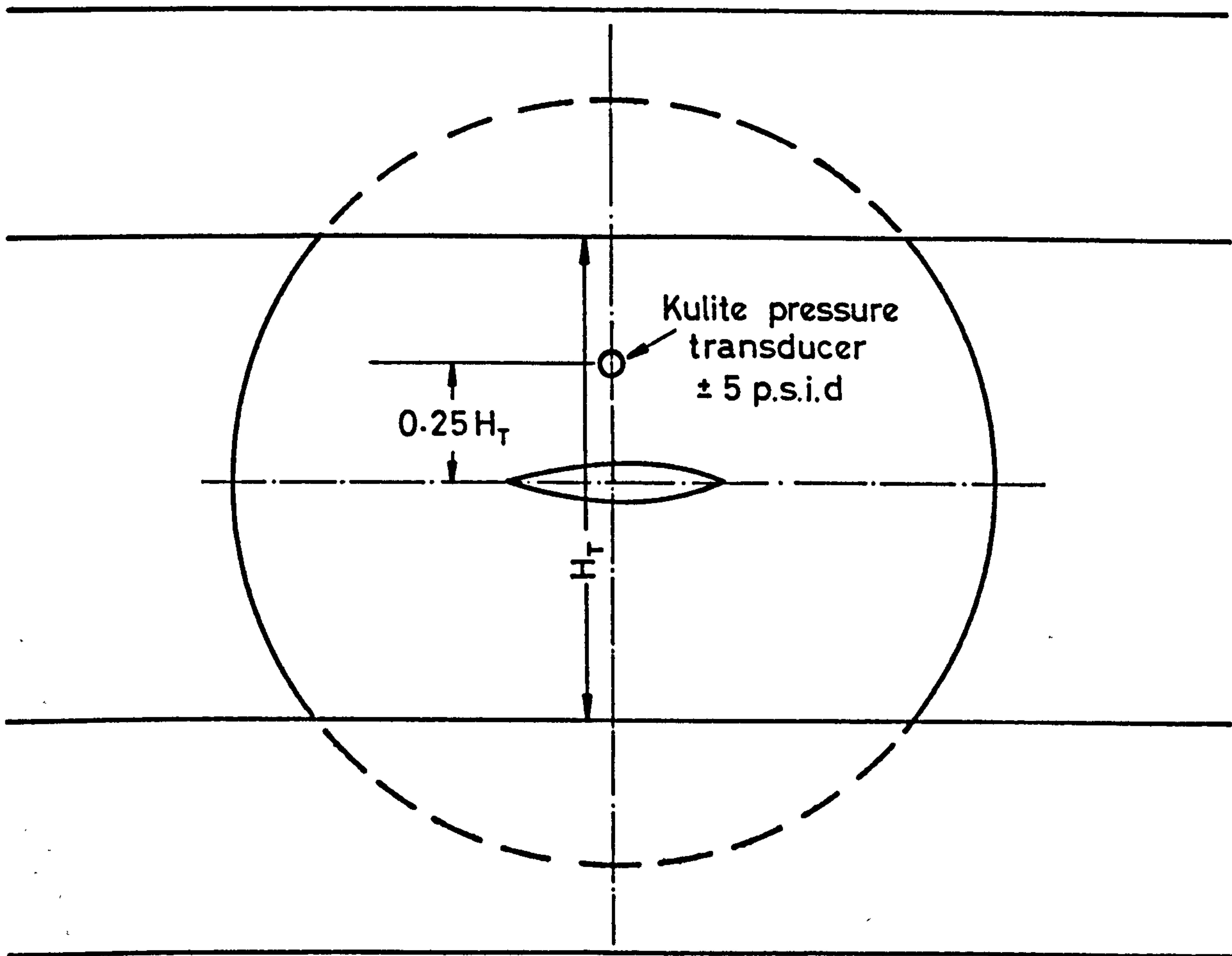
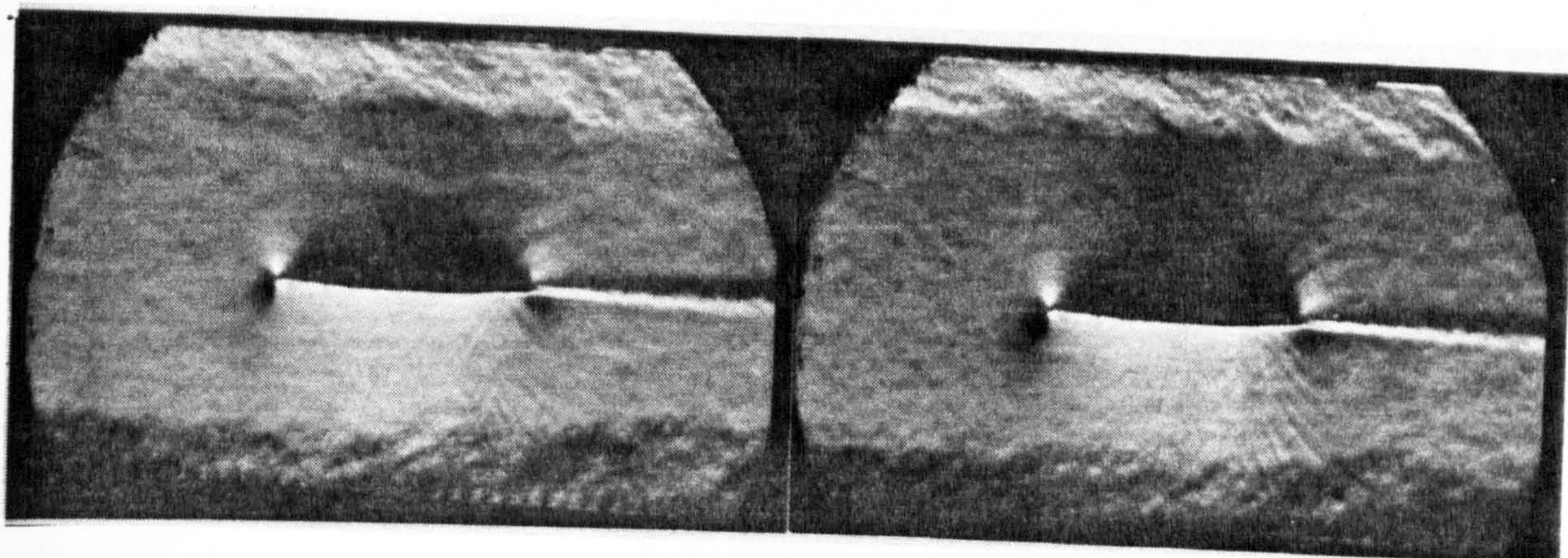
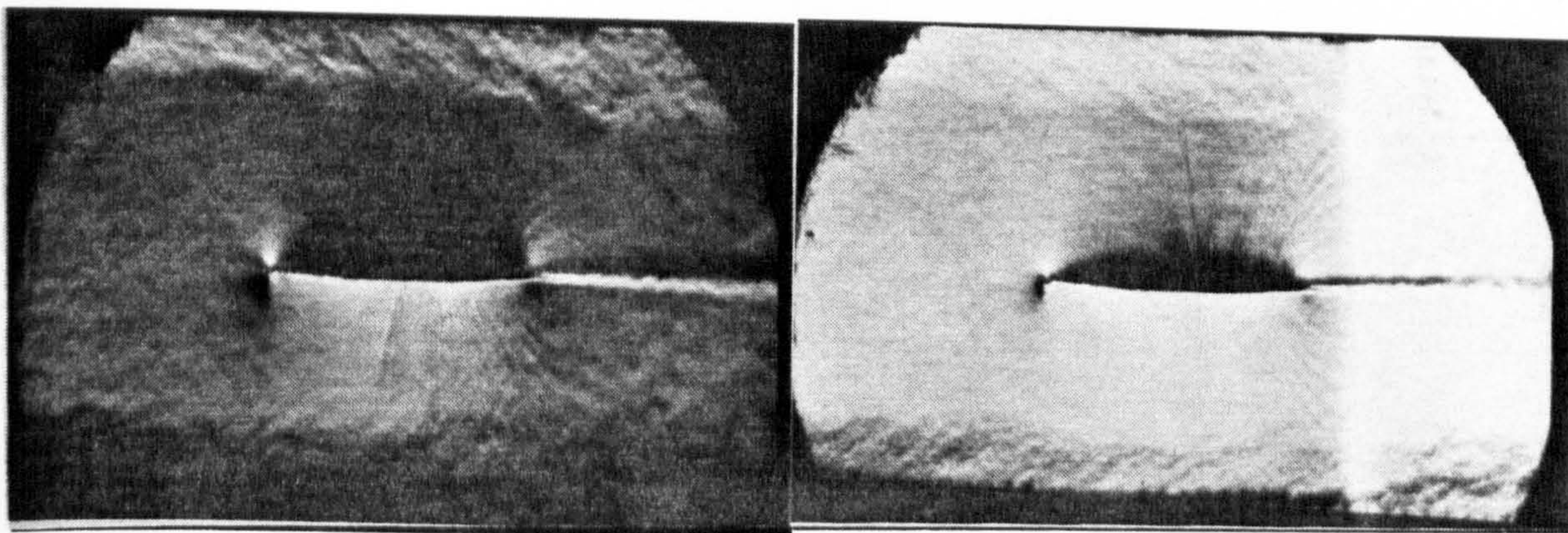


Figure 7. Pressure transducer location.



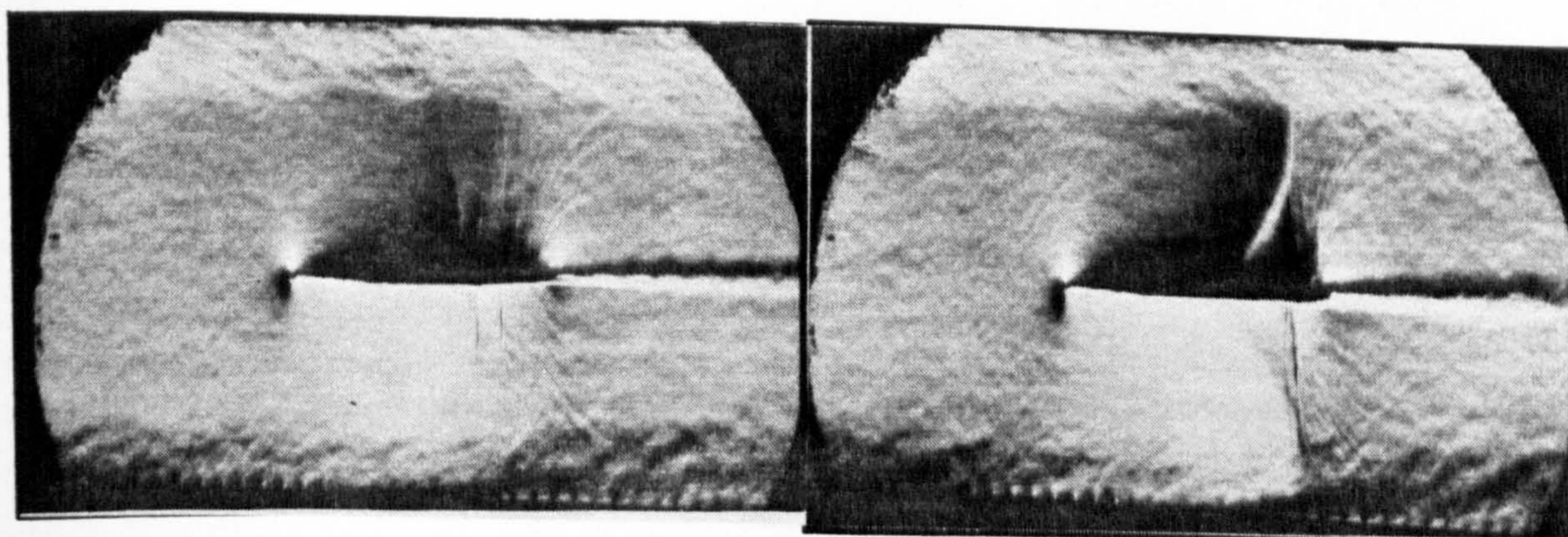
a) $M_\infty = 0.770$

b) 0.803



c) 0.840

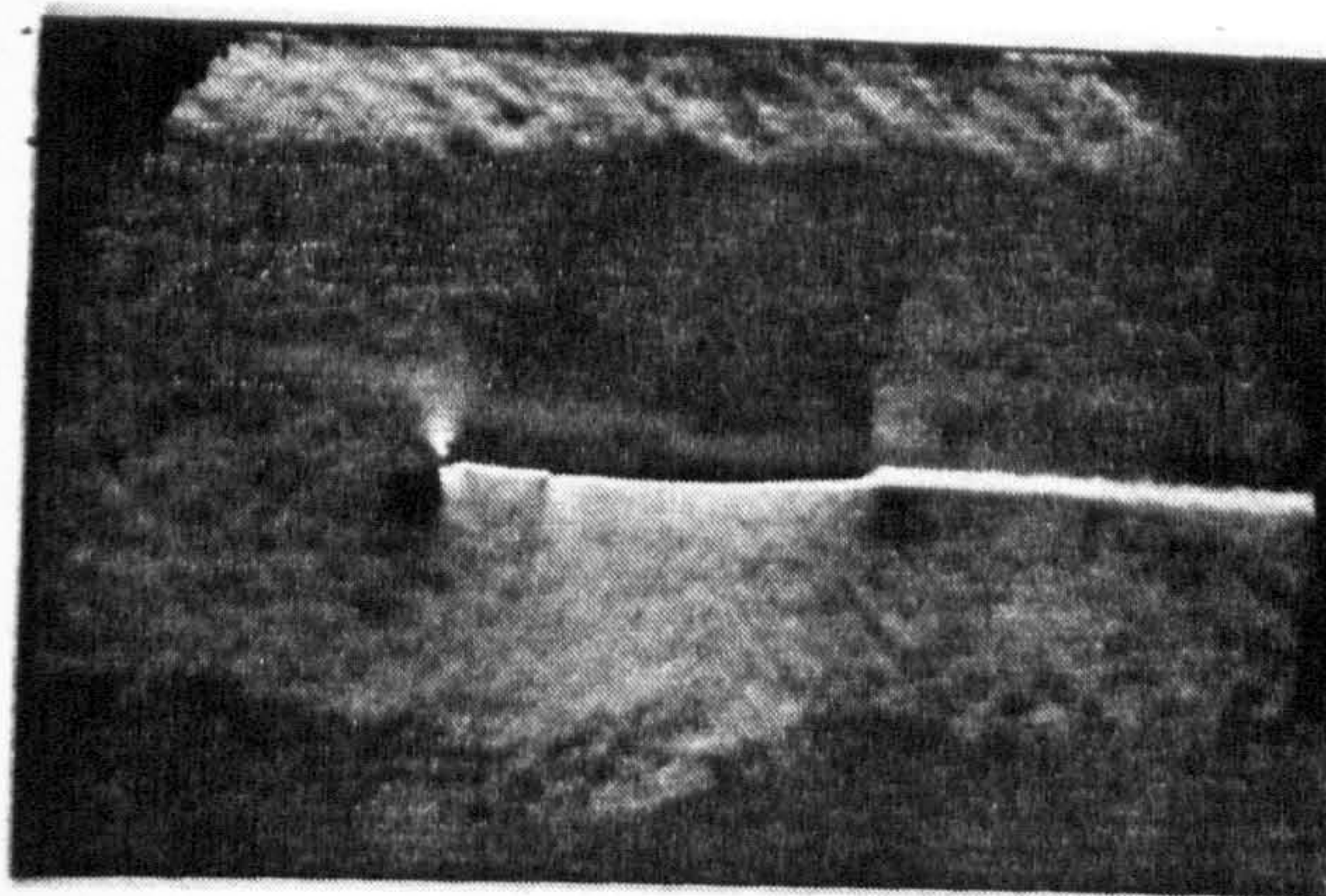
d) 0.870



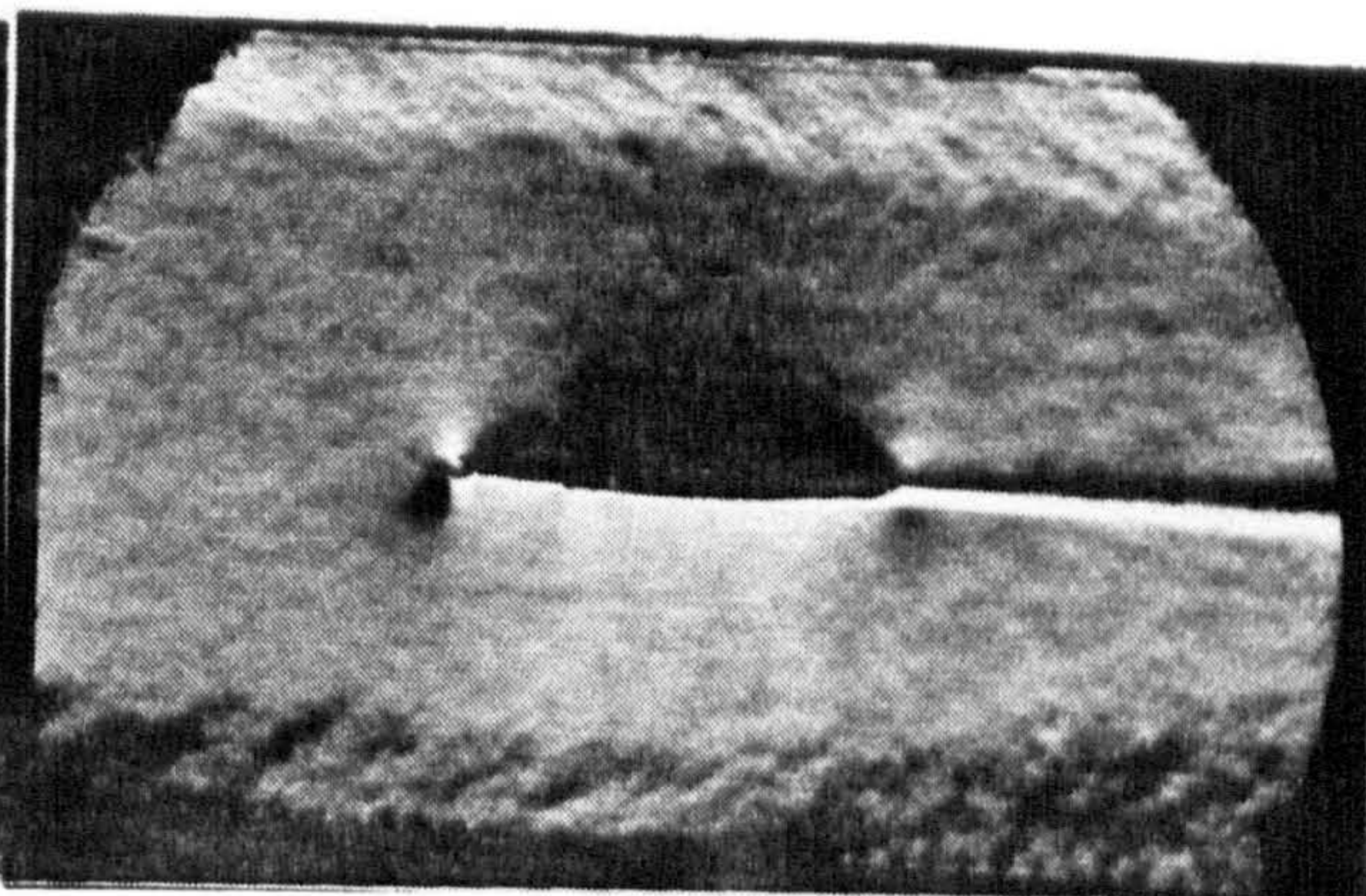
e) 0.910

f) 0.960

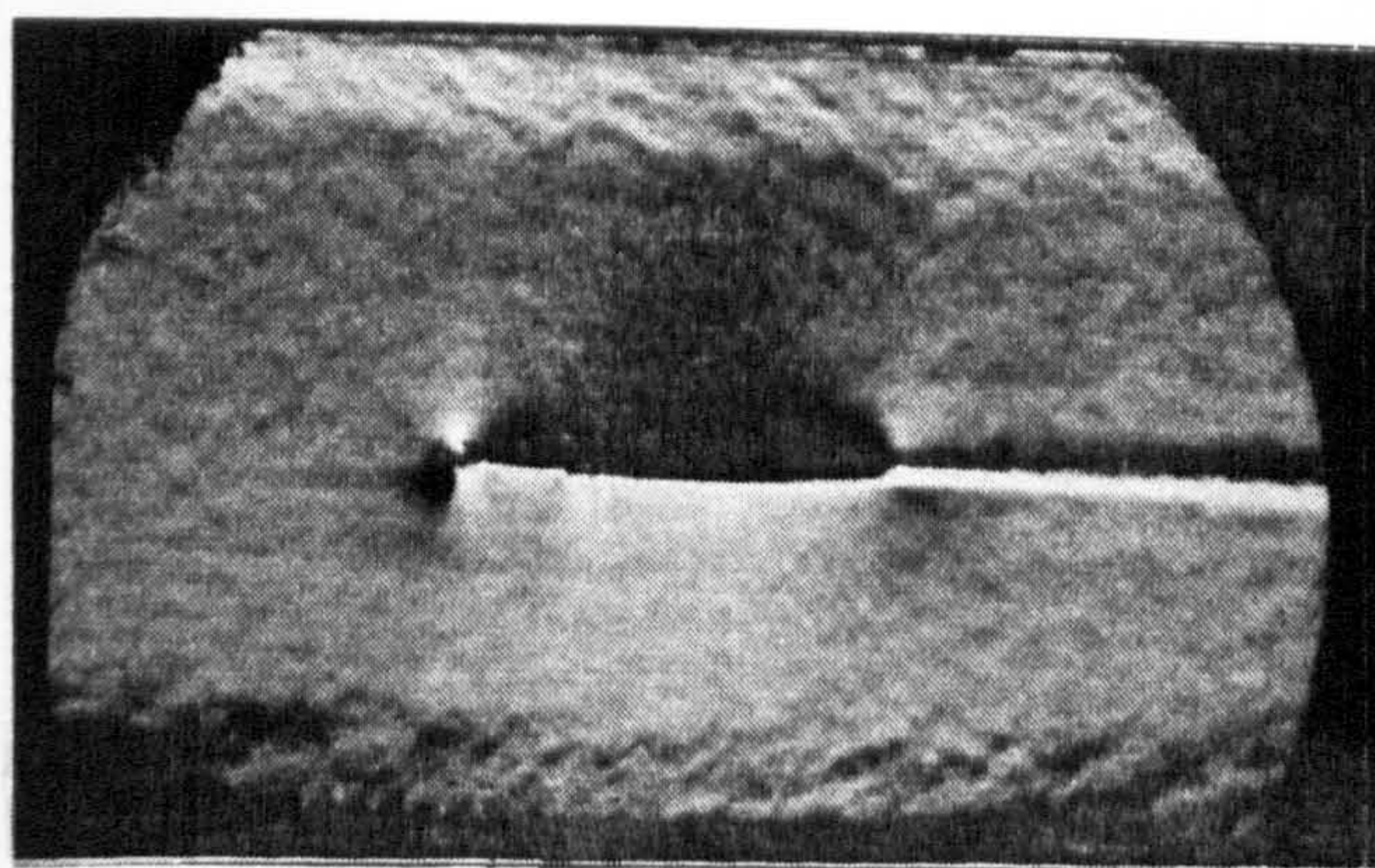
Figure 8. Schlieren pictures of flow on a 6% thick biconvex aerofoil (free transition).



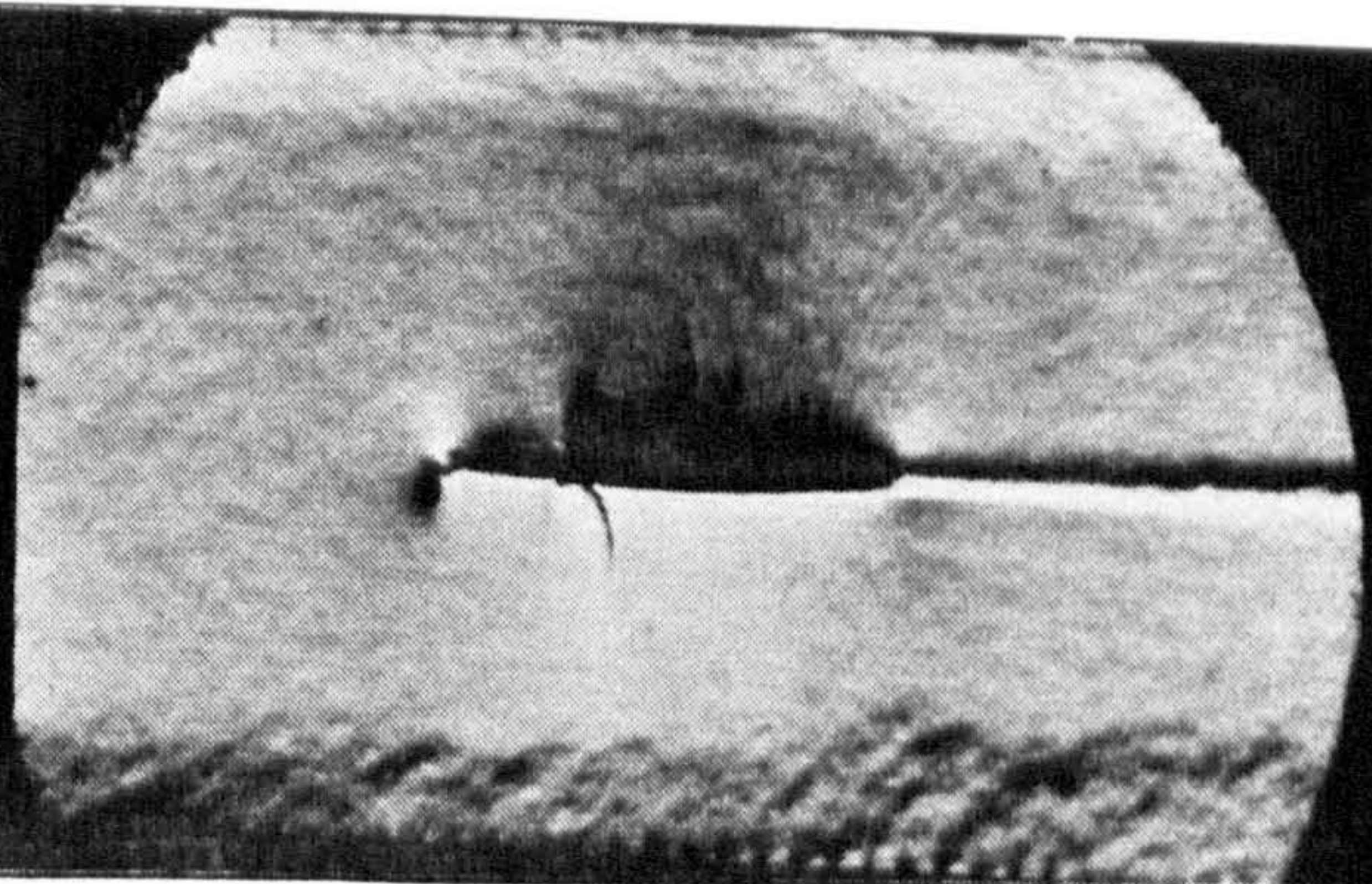
a) $M_{\infty} = 0.800$



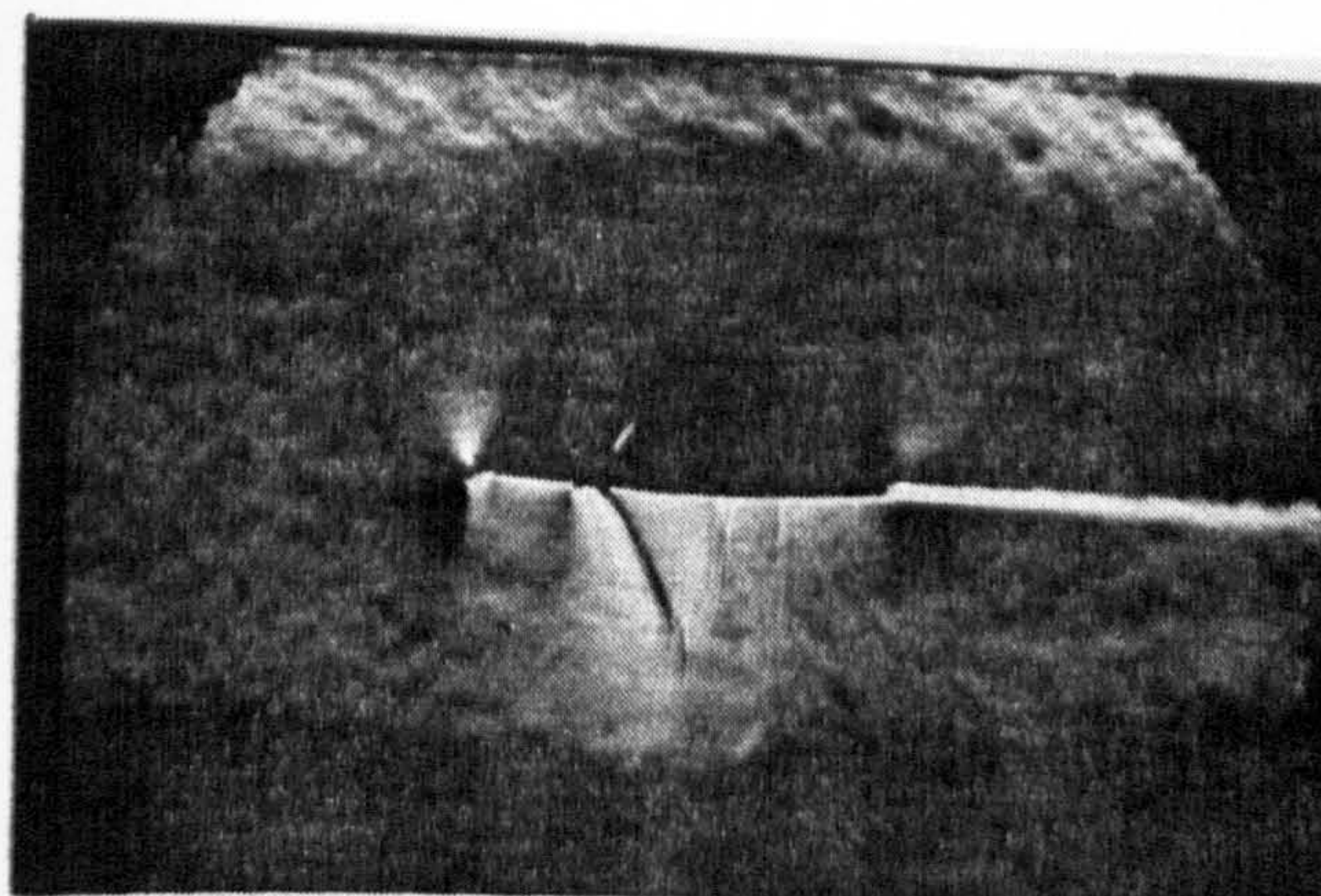
b) 0.800



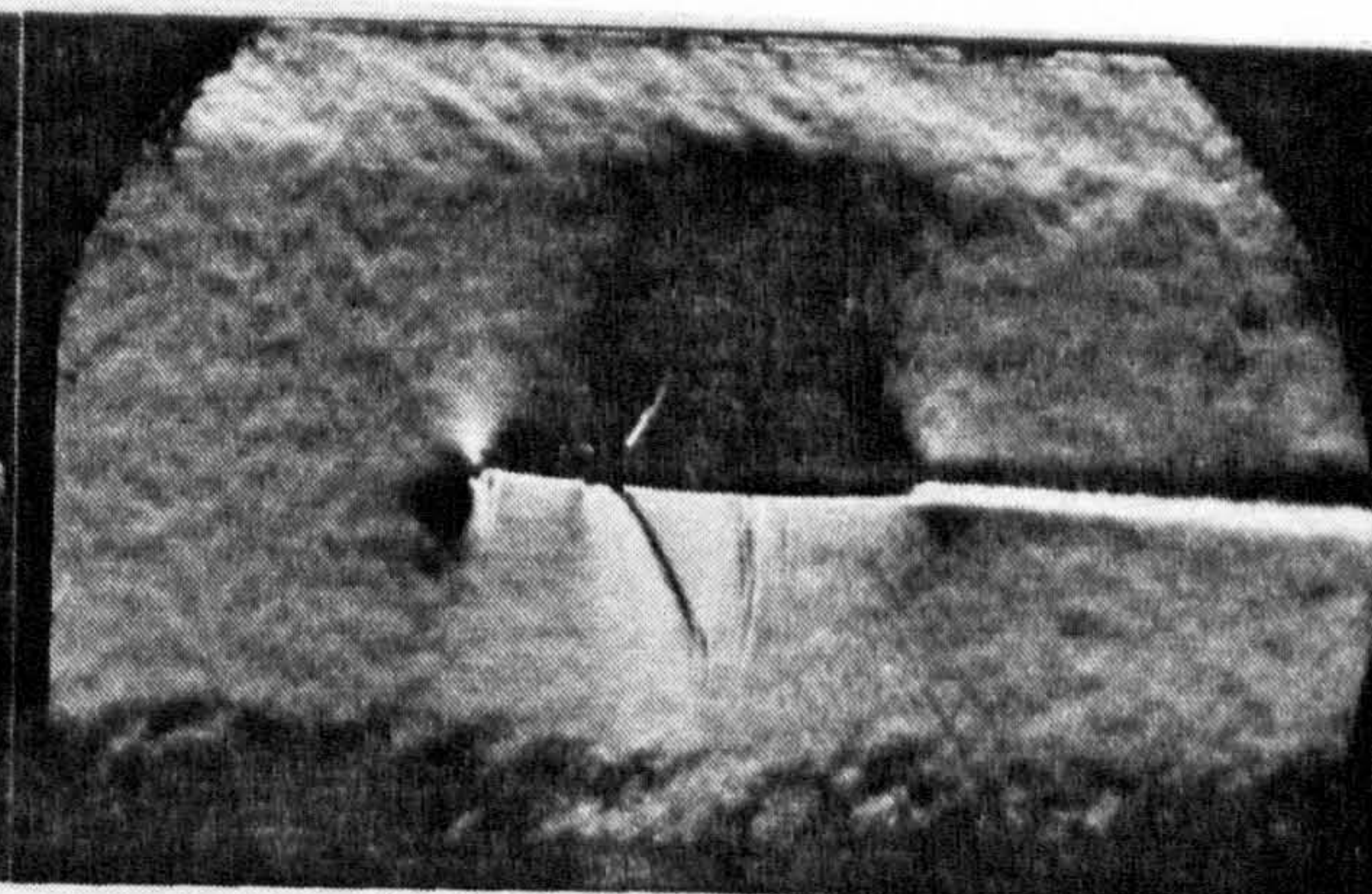
c) 0.840



d) 0.880

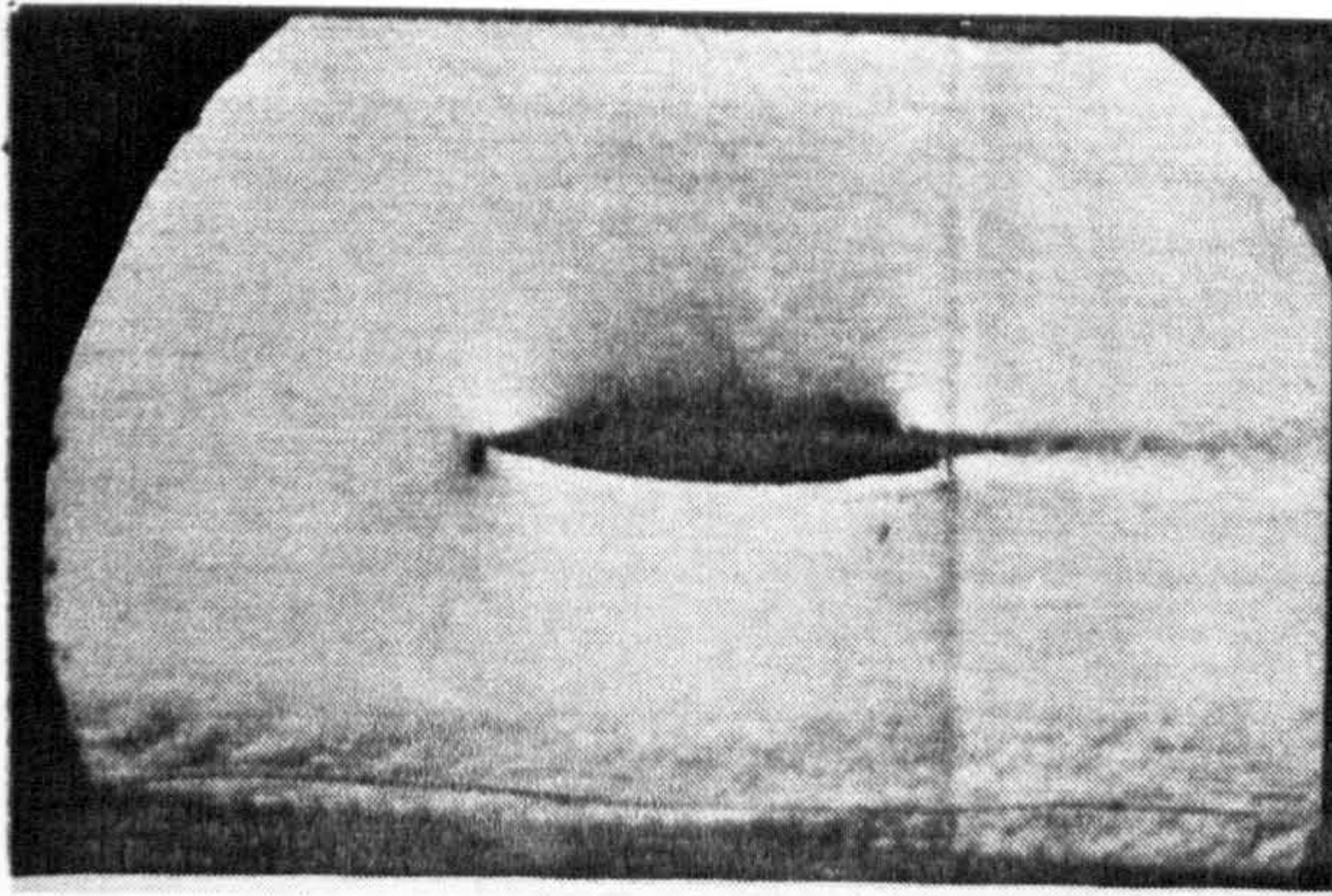


e) 0.920

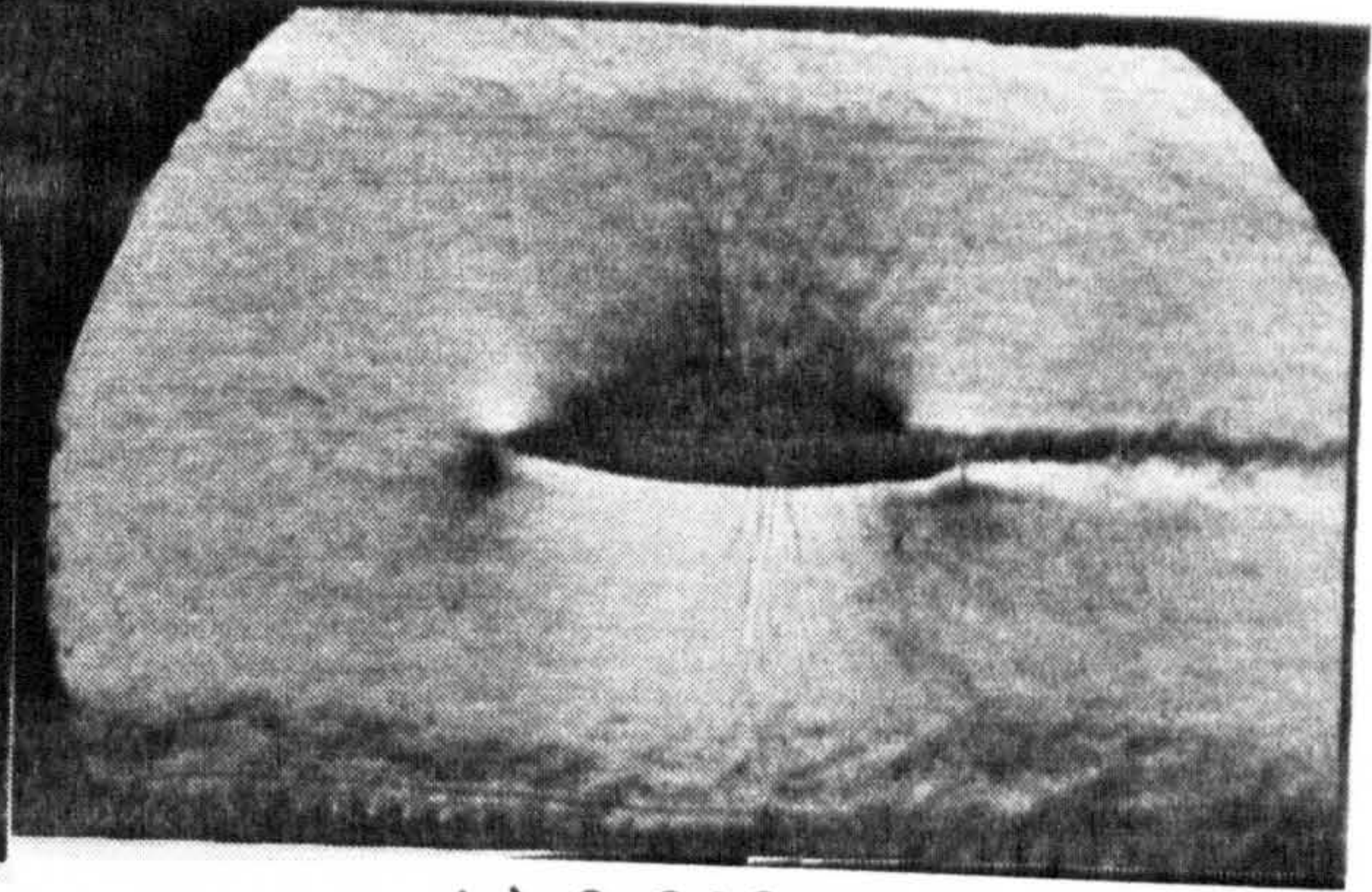


f) 0.920

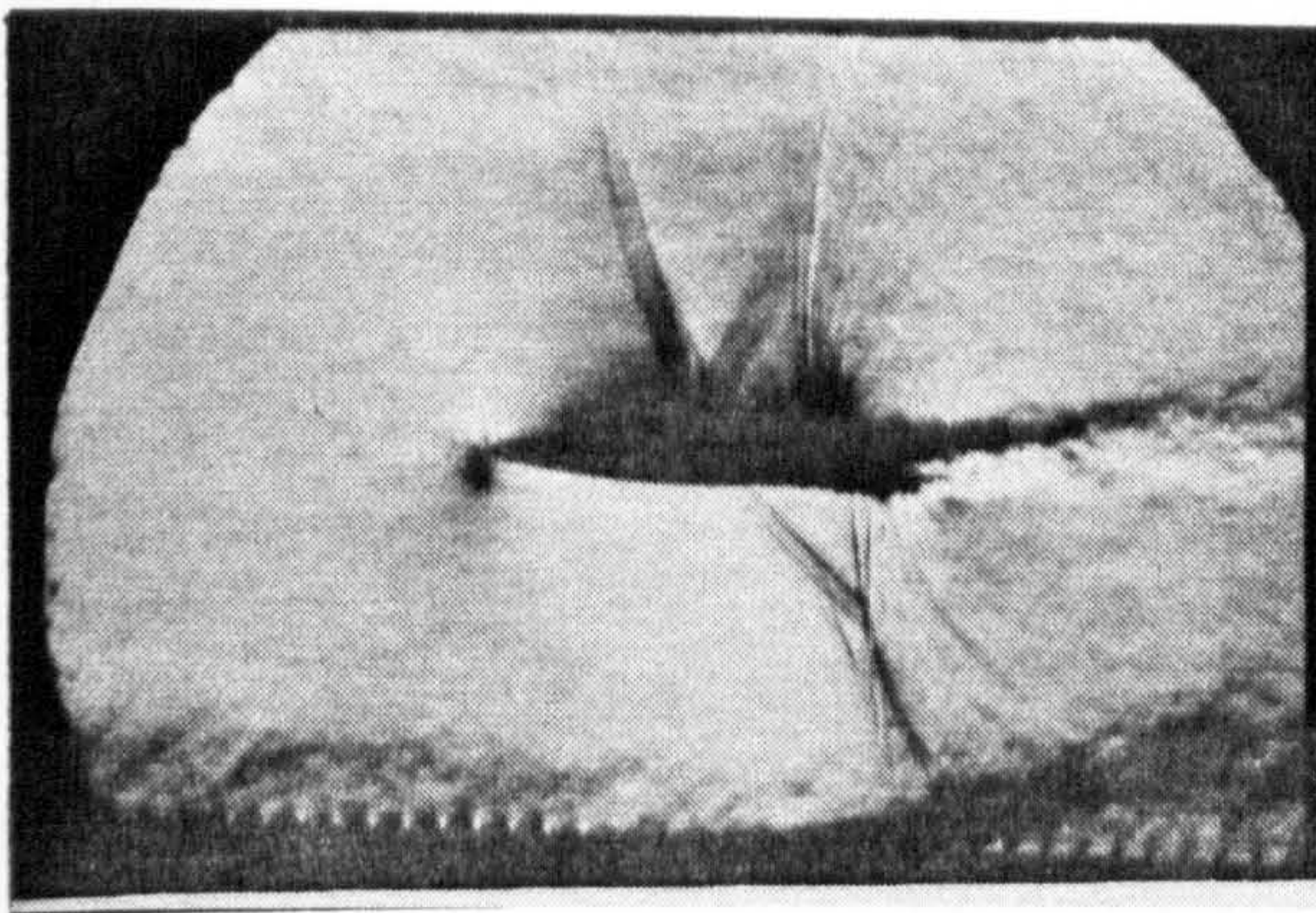
Figure 9. Schlieren pictures of flow on a 6% thick biconvex aerofoil, $\alpha = 0^\circ$, (fixed transition).



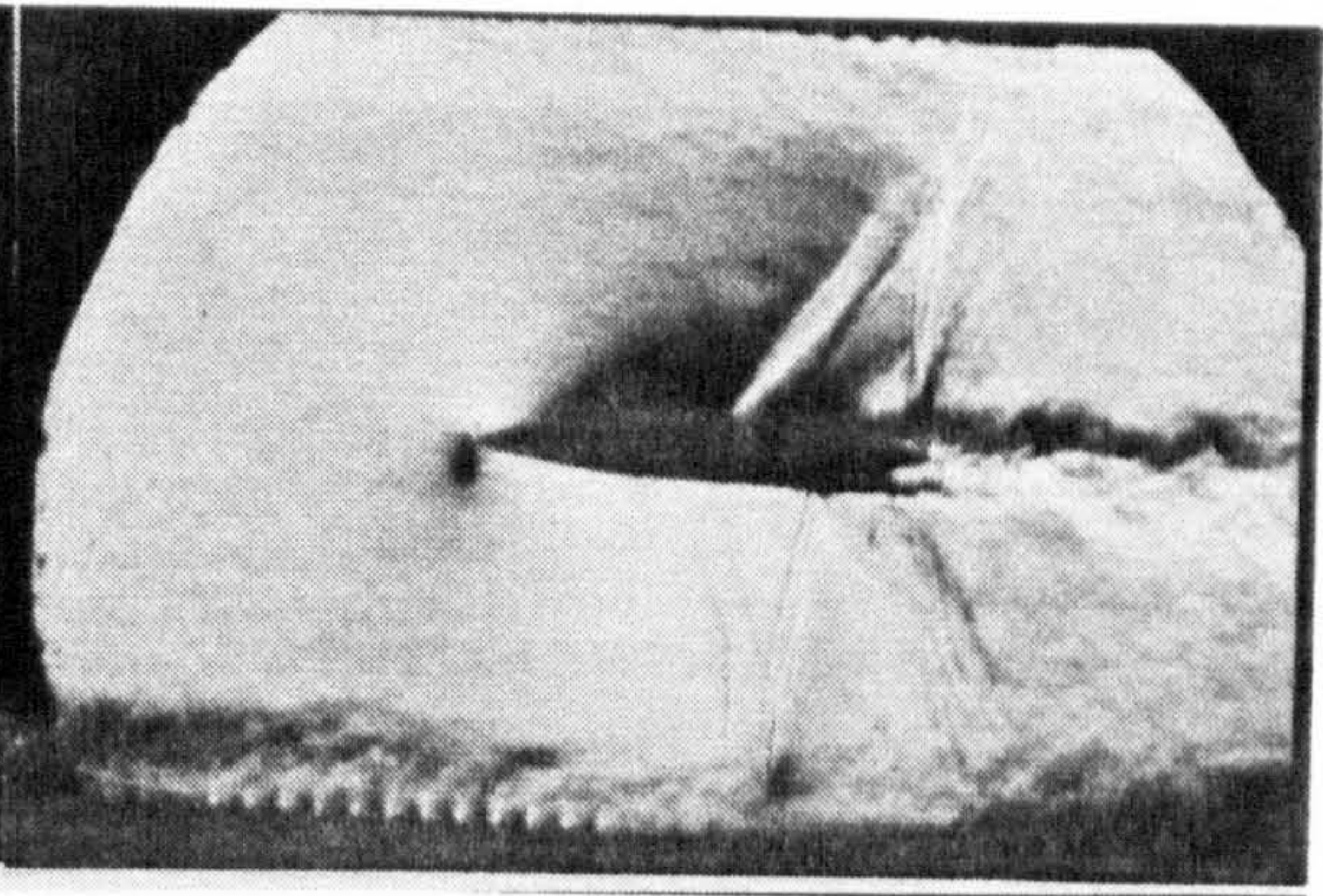
a) $M_\infty = 0.700$



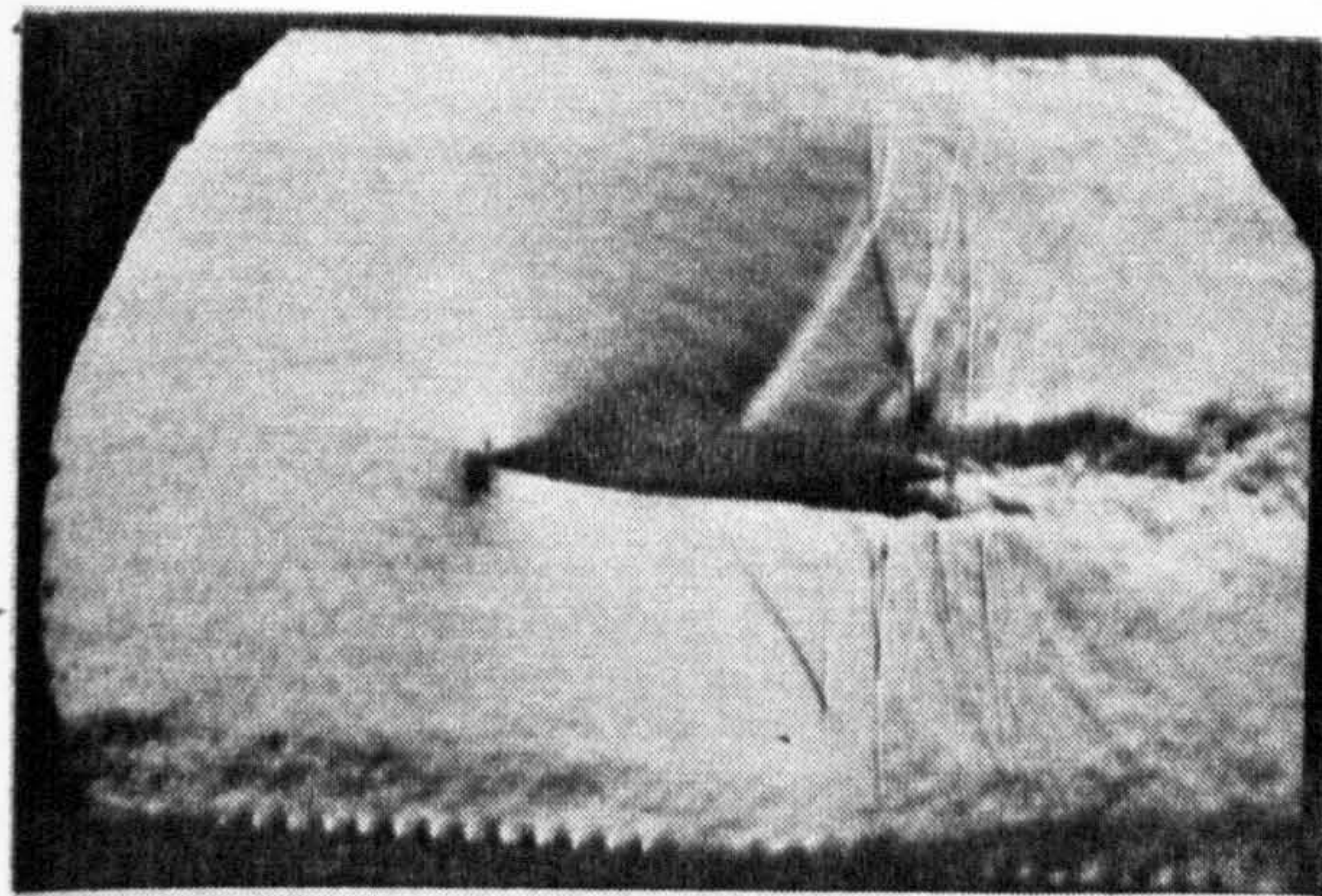
b) 0.810



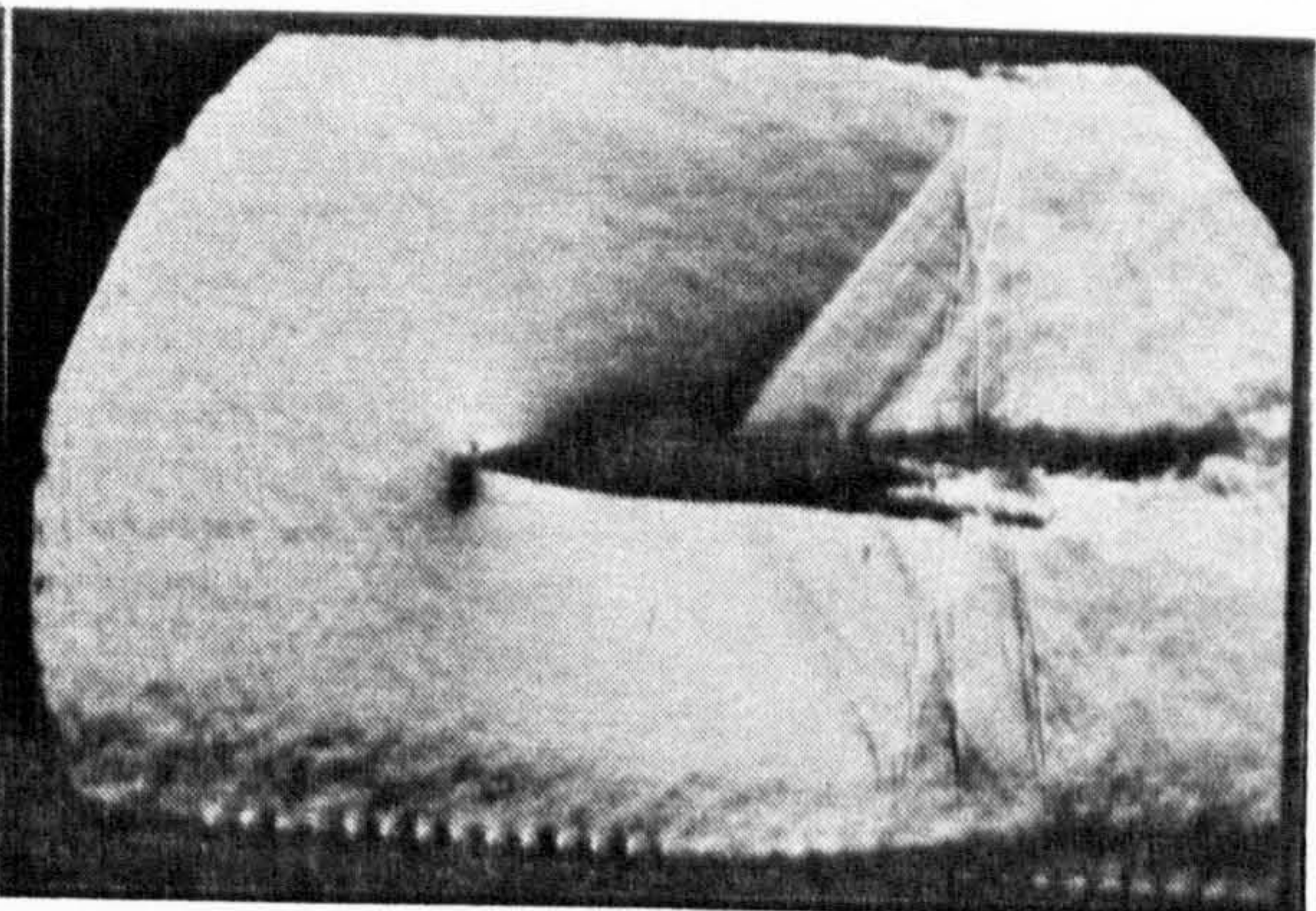
c) 0.850



d) 0.900

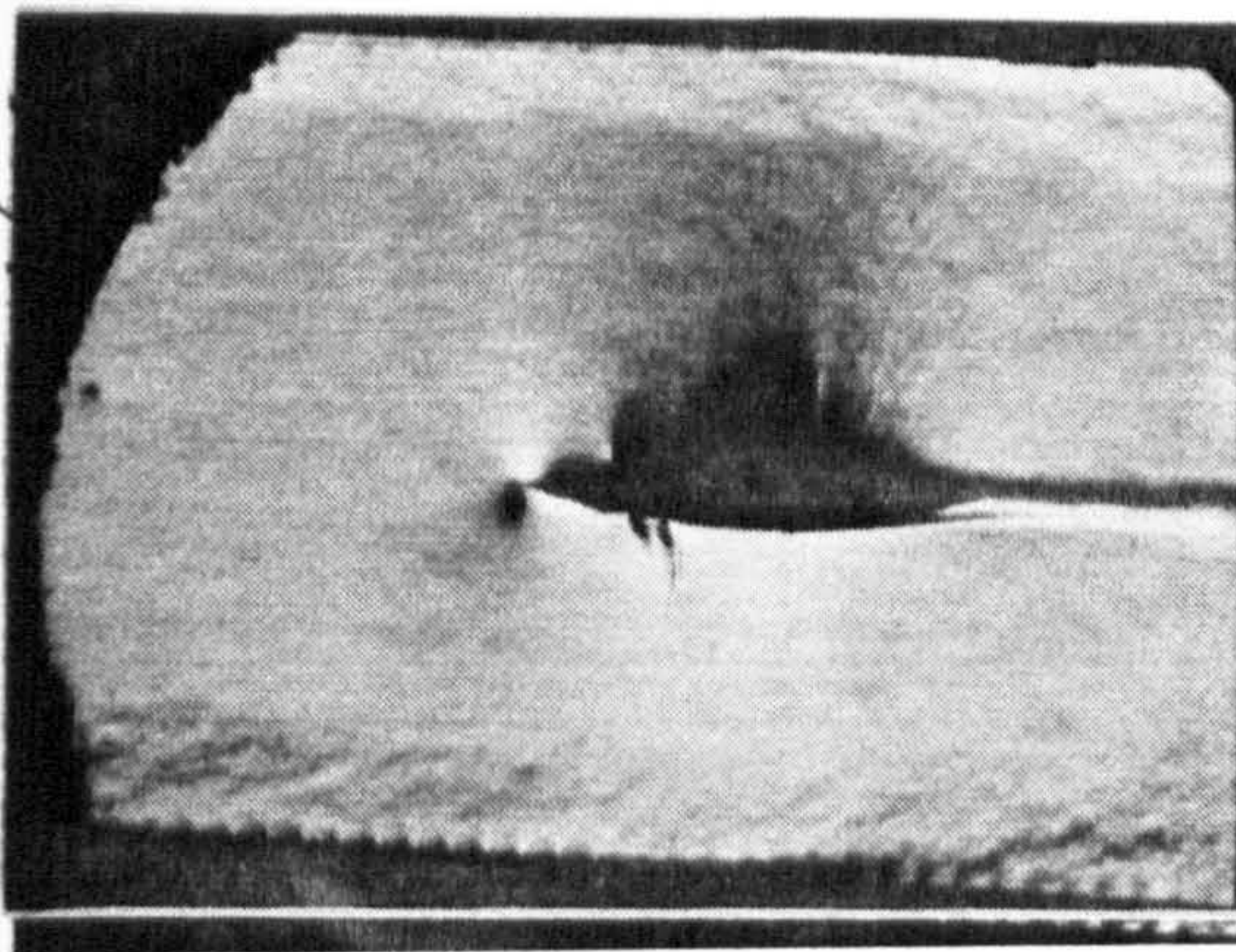


e) 0.950

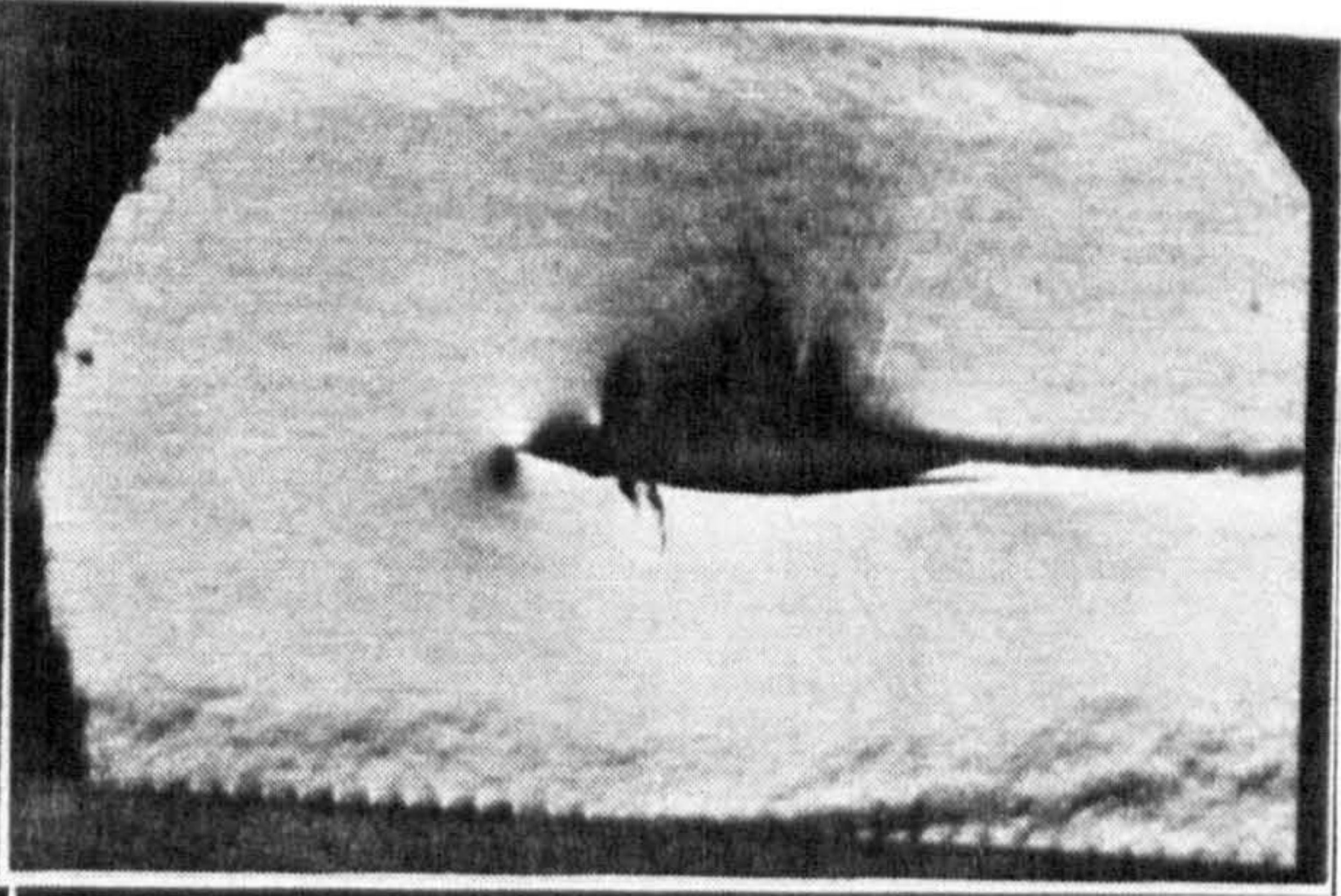


f) 0.990

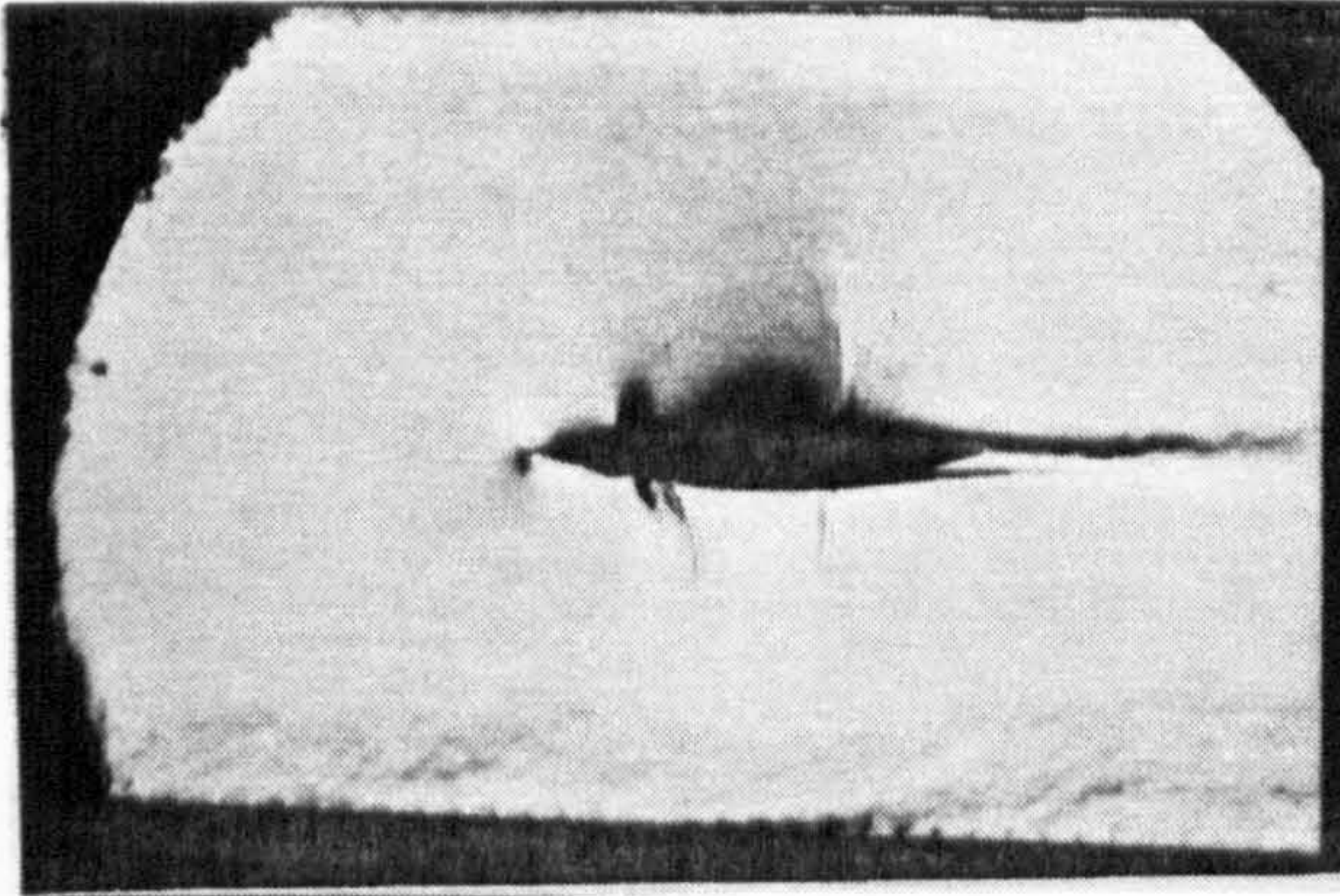
Figure 10. Schlieren pictures of flow on a 14% thick biconvex aerofoil, $\alpha = 0^\circ$, (free transition).



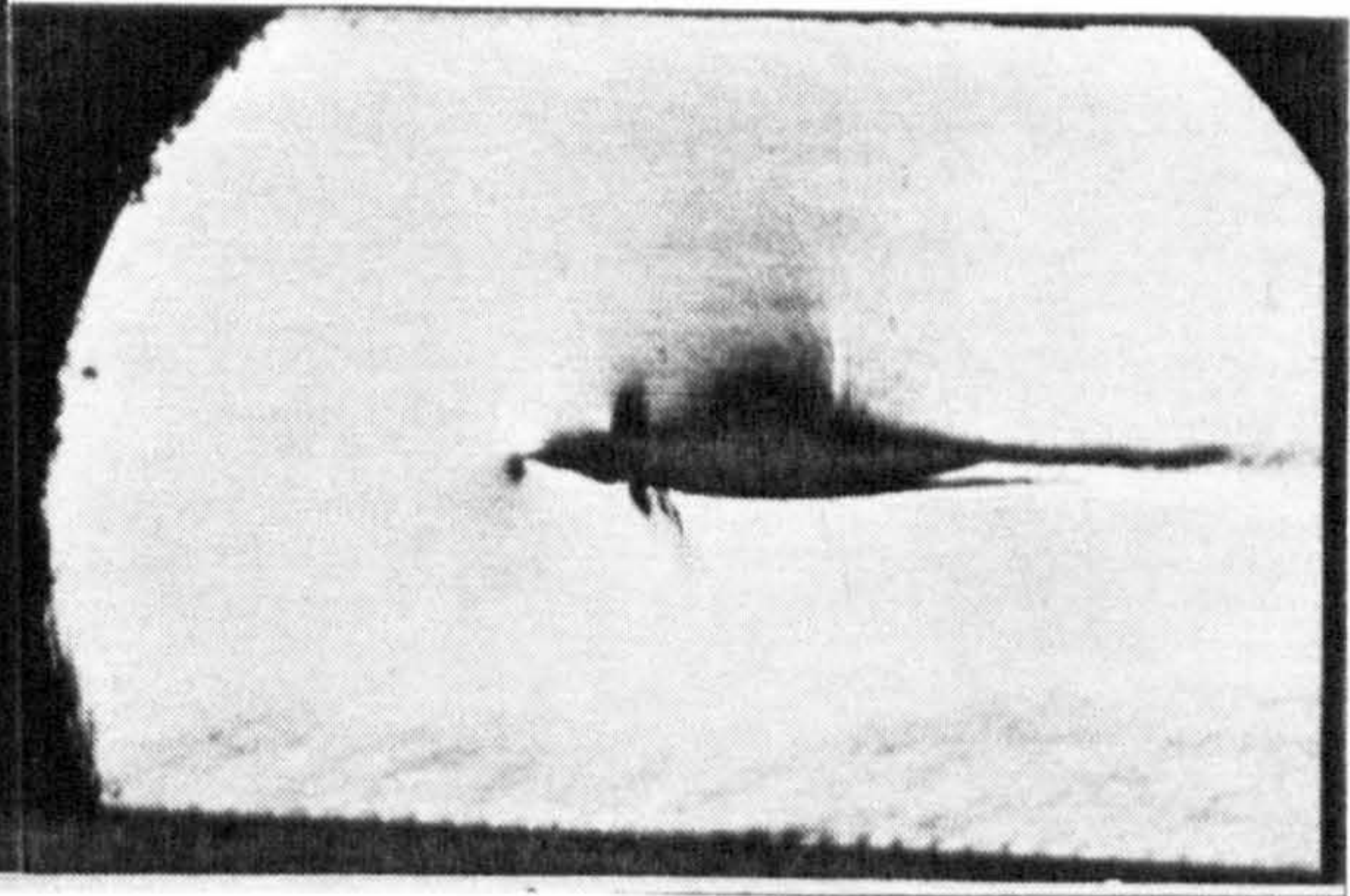
a) $M_\infty = 0.82$



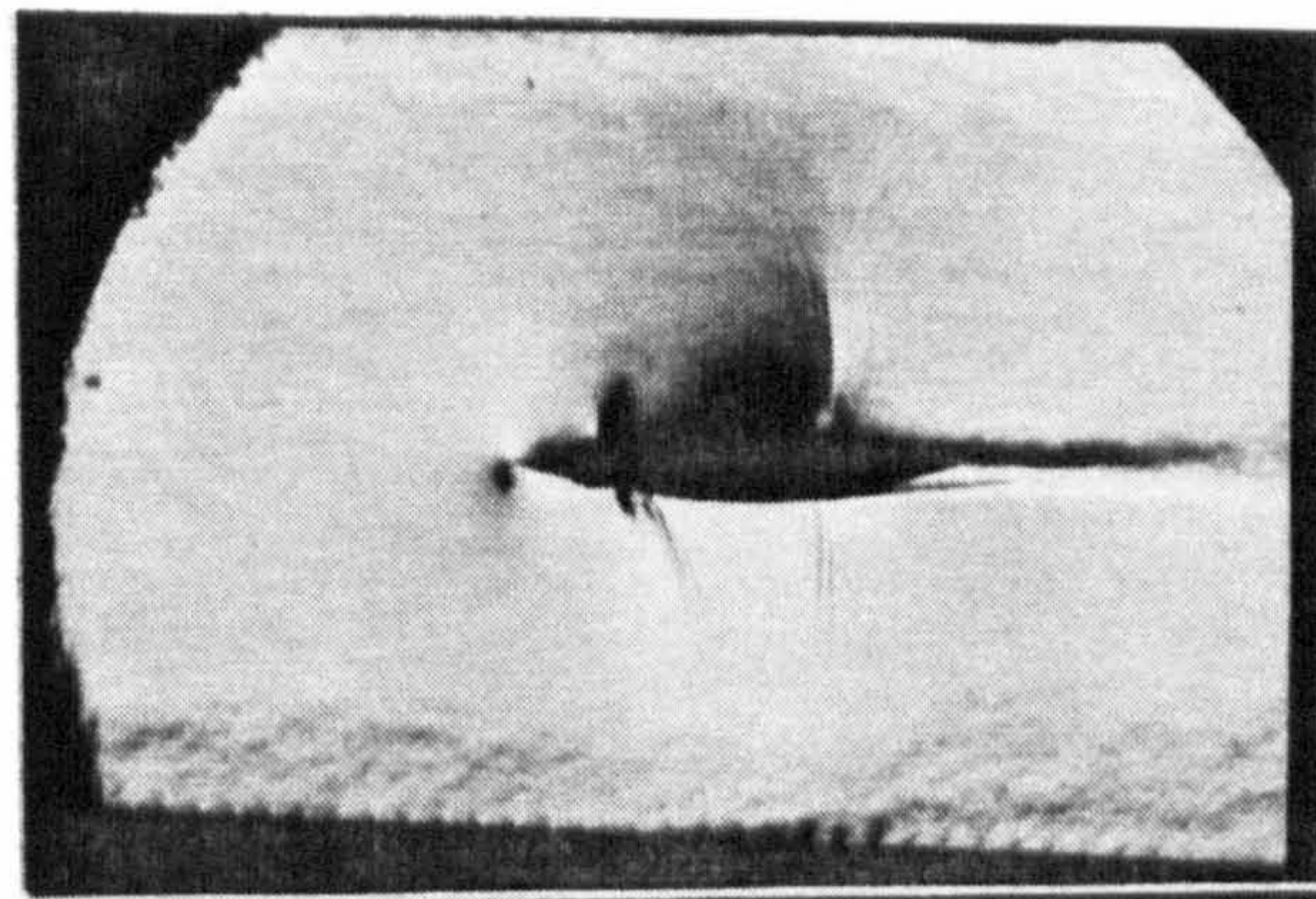
b) 0.82



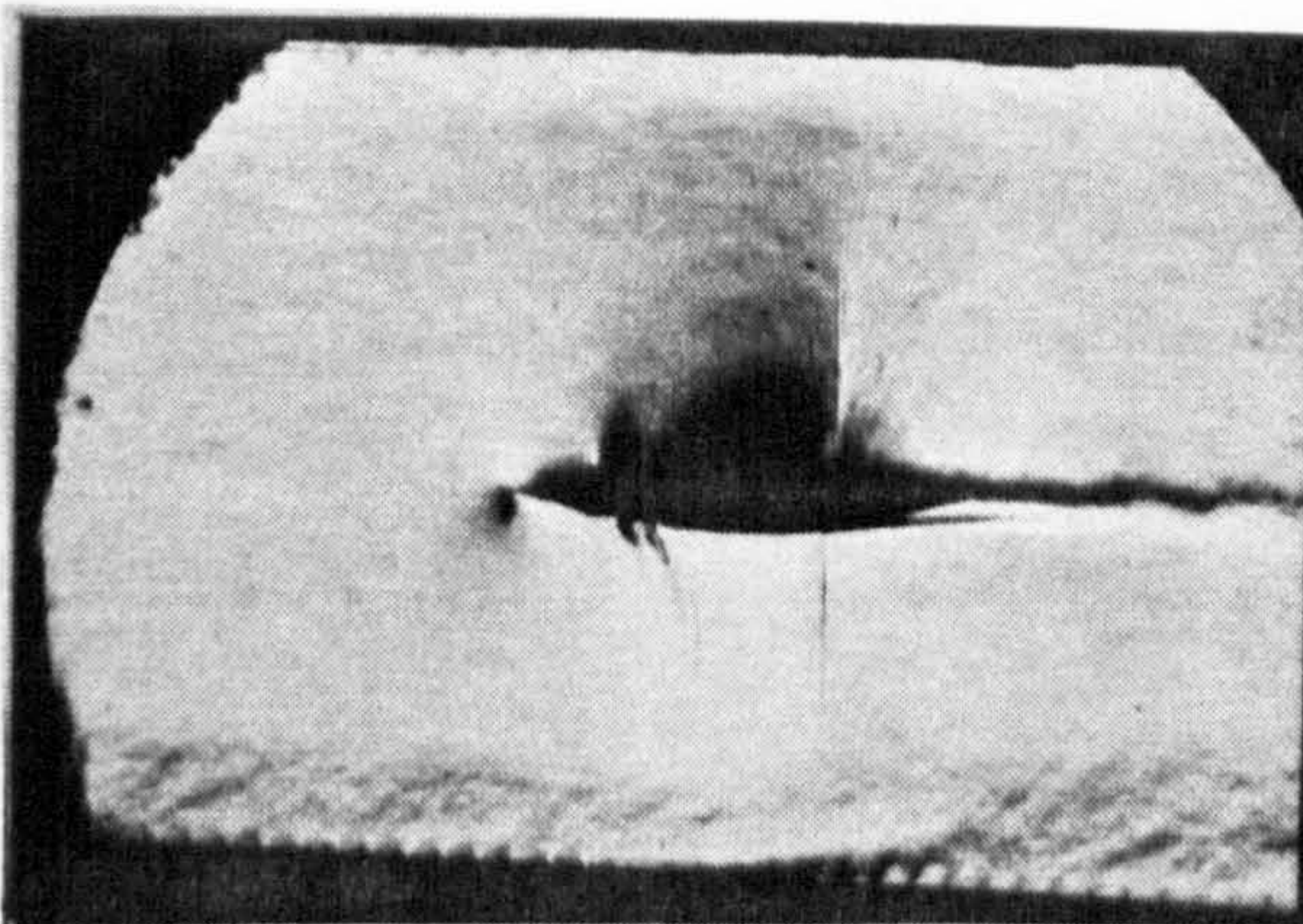
c) 0.83



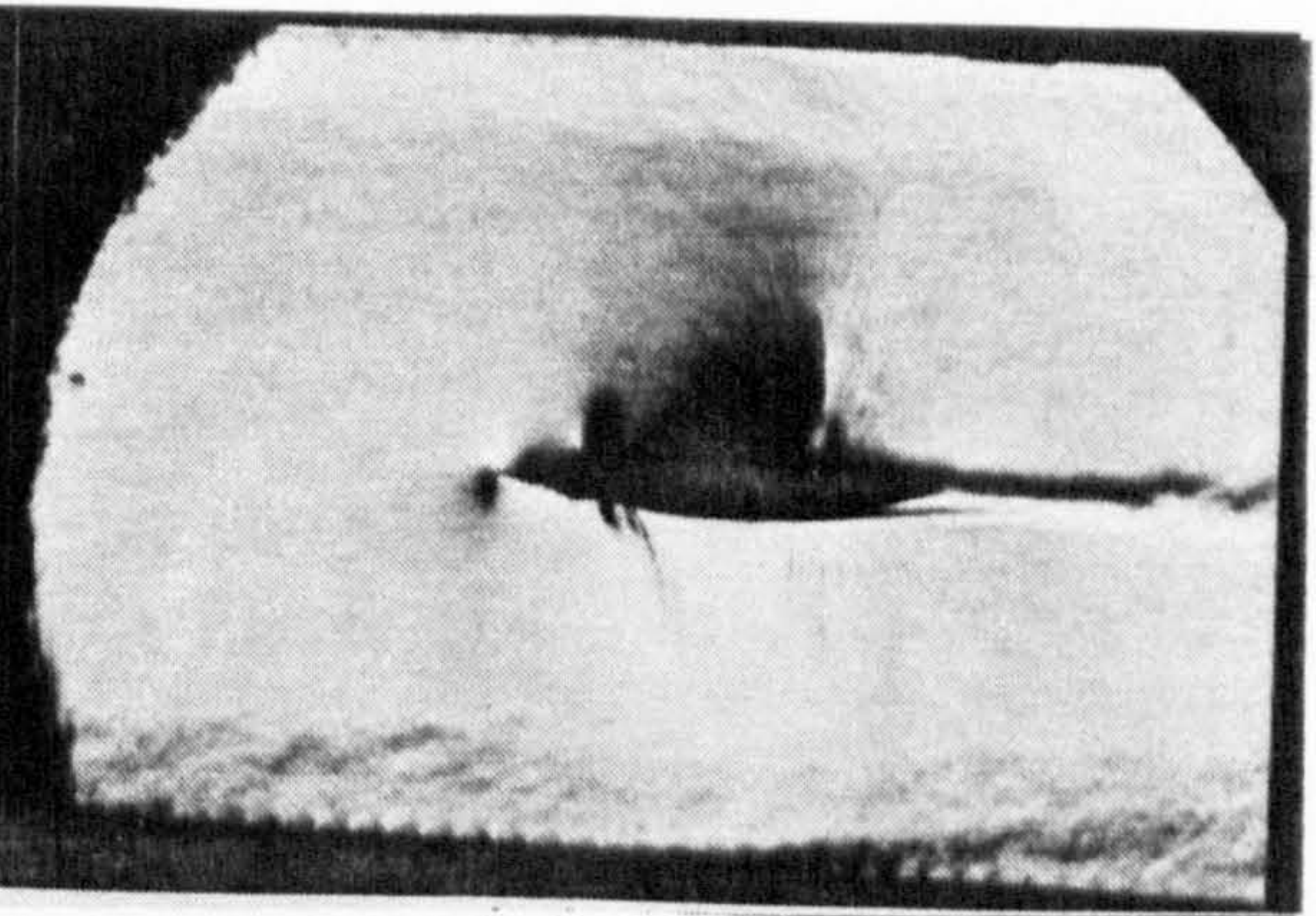
d) 0.83



e) 0.84

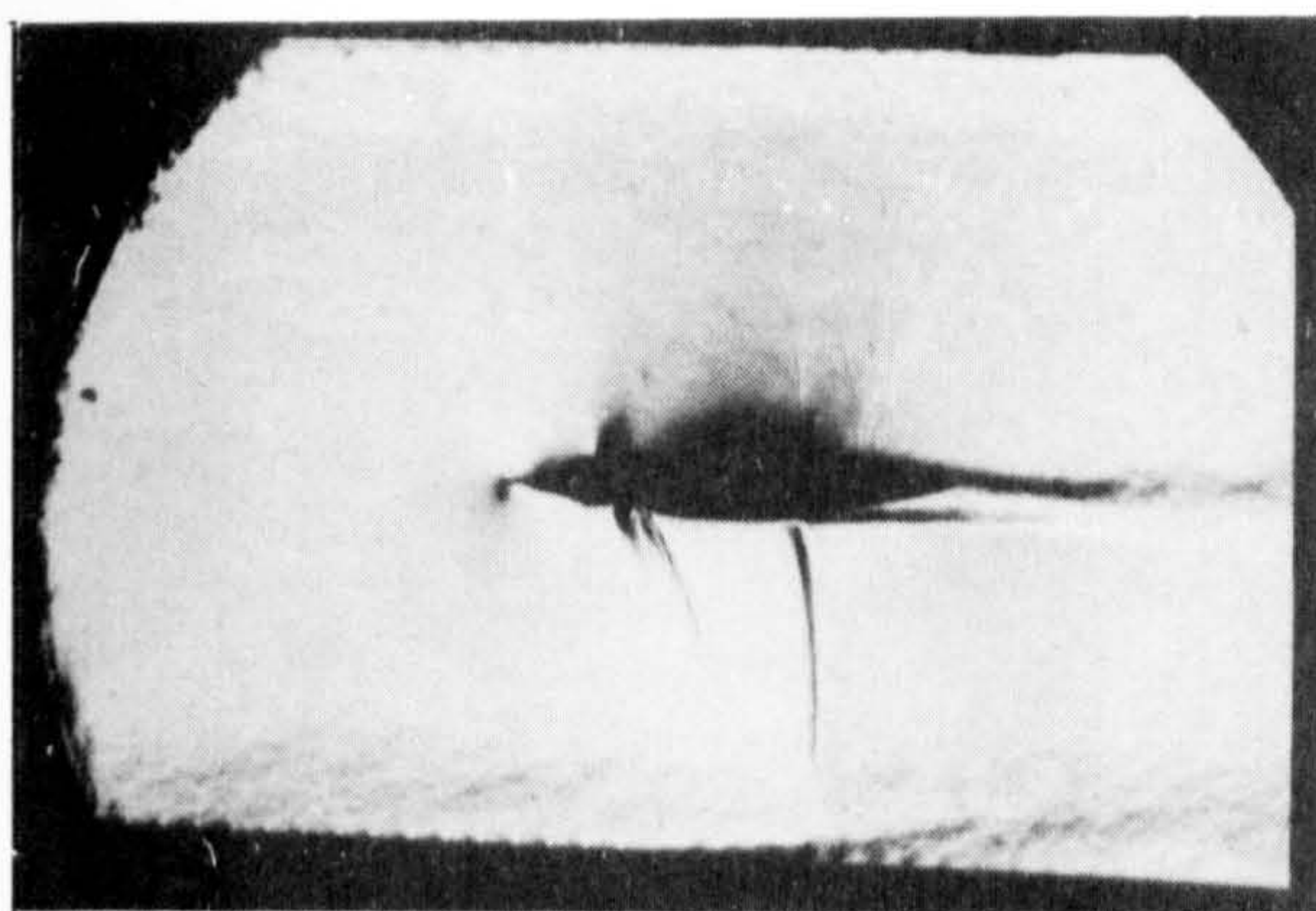


f) 0.85

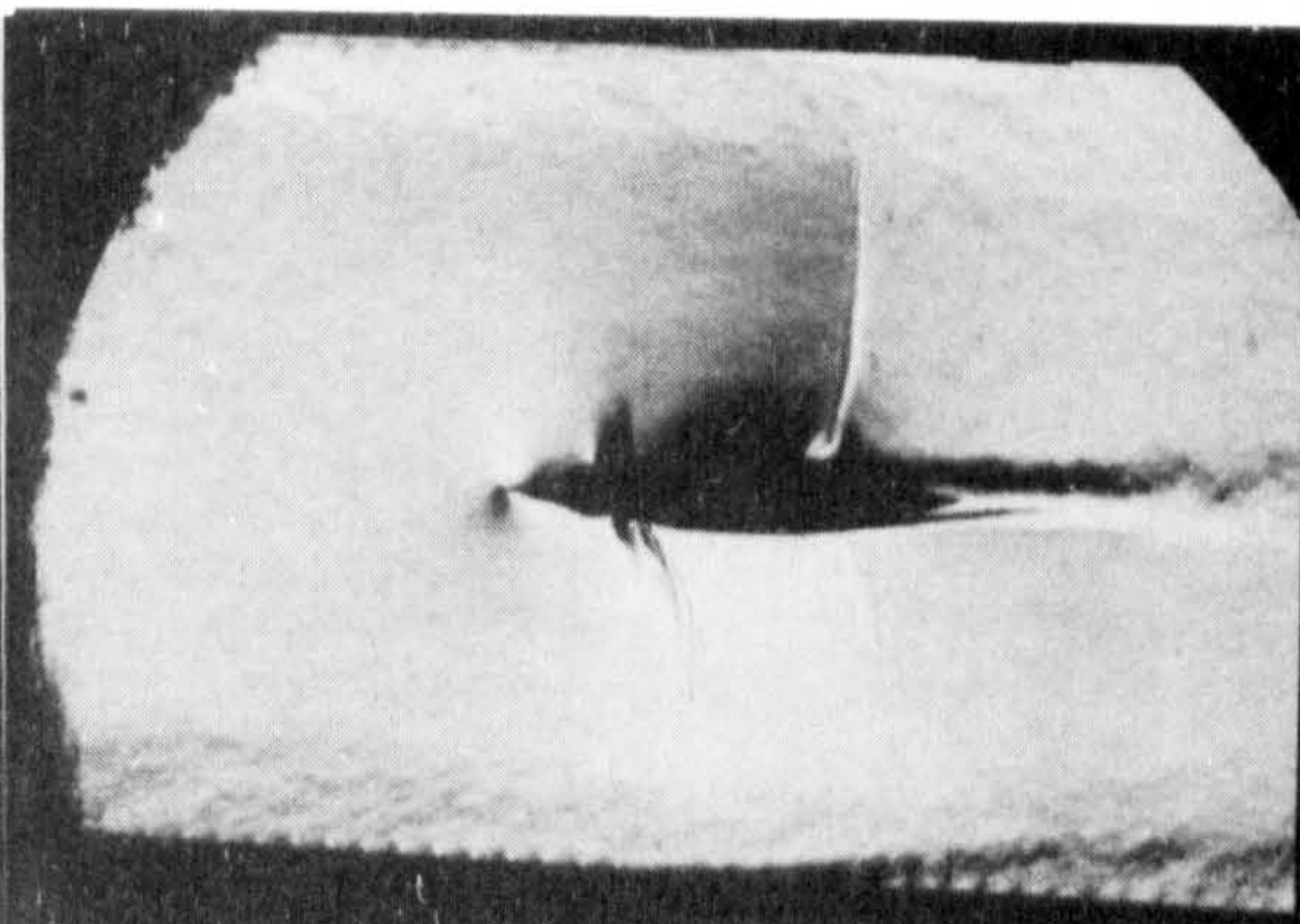


g) 0.85

STEADY FLOWS WITH TRAILING EDGE SEPARATION
Figure 11. Schlieren pictures of flow on a 14% thick biconvex aerofoil ($c=32\text{mm}$) at zero incidence (fixed transition).

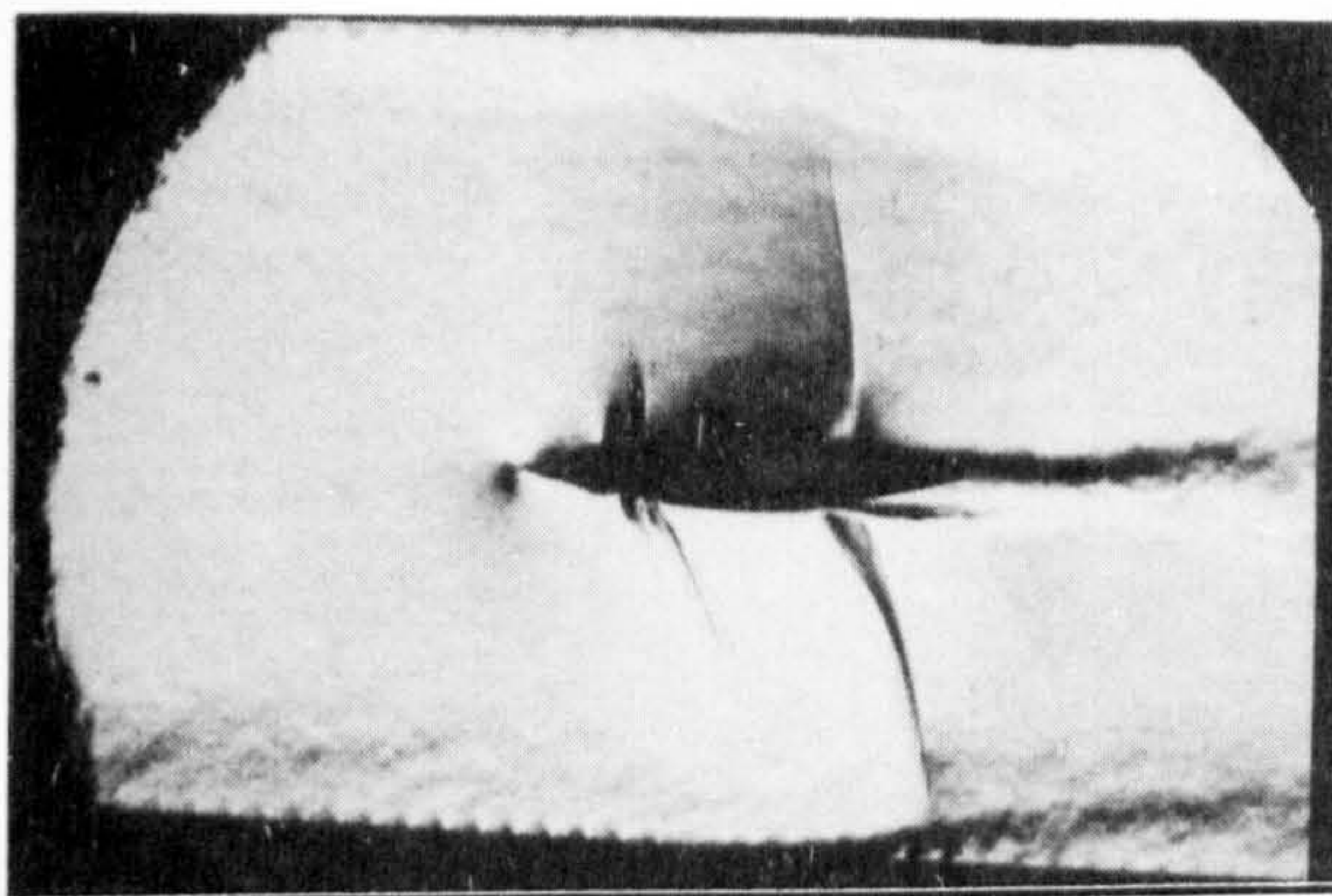


h) $M_{\infty} = 0.86$

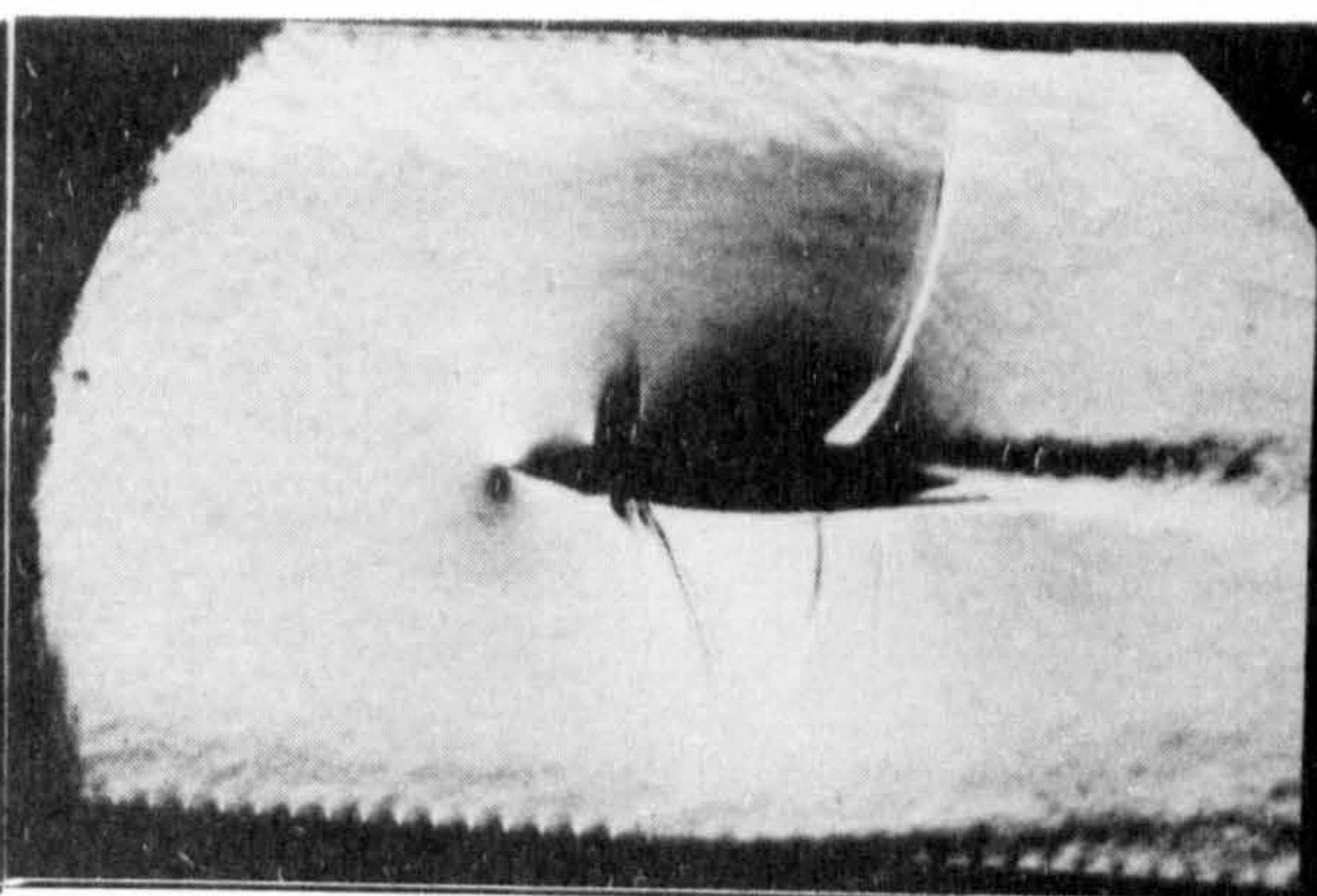


i) 0.86

UNSTEADY FLOW

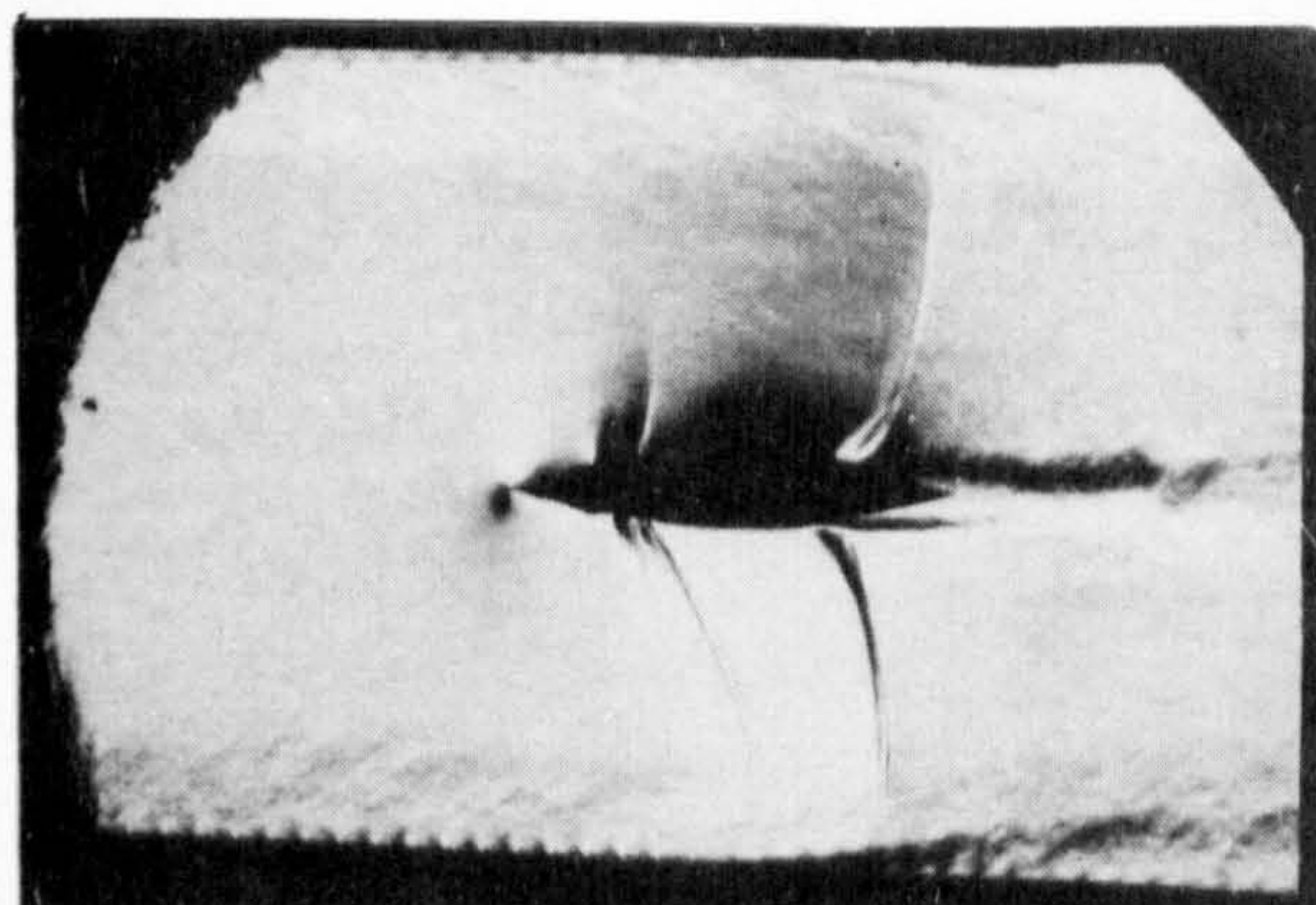


j) 0.90

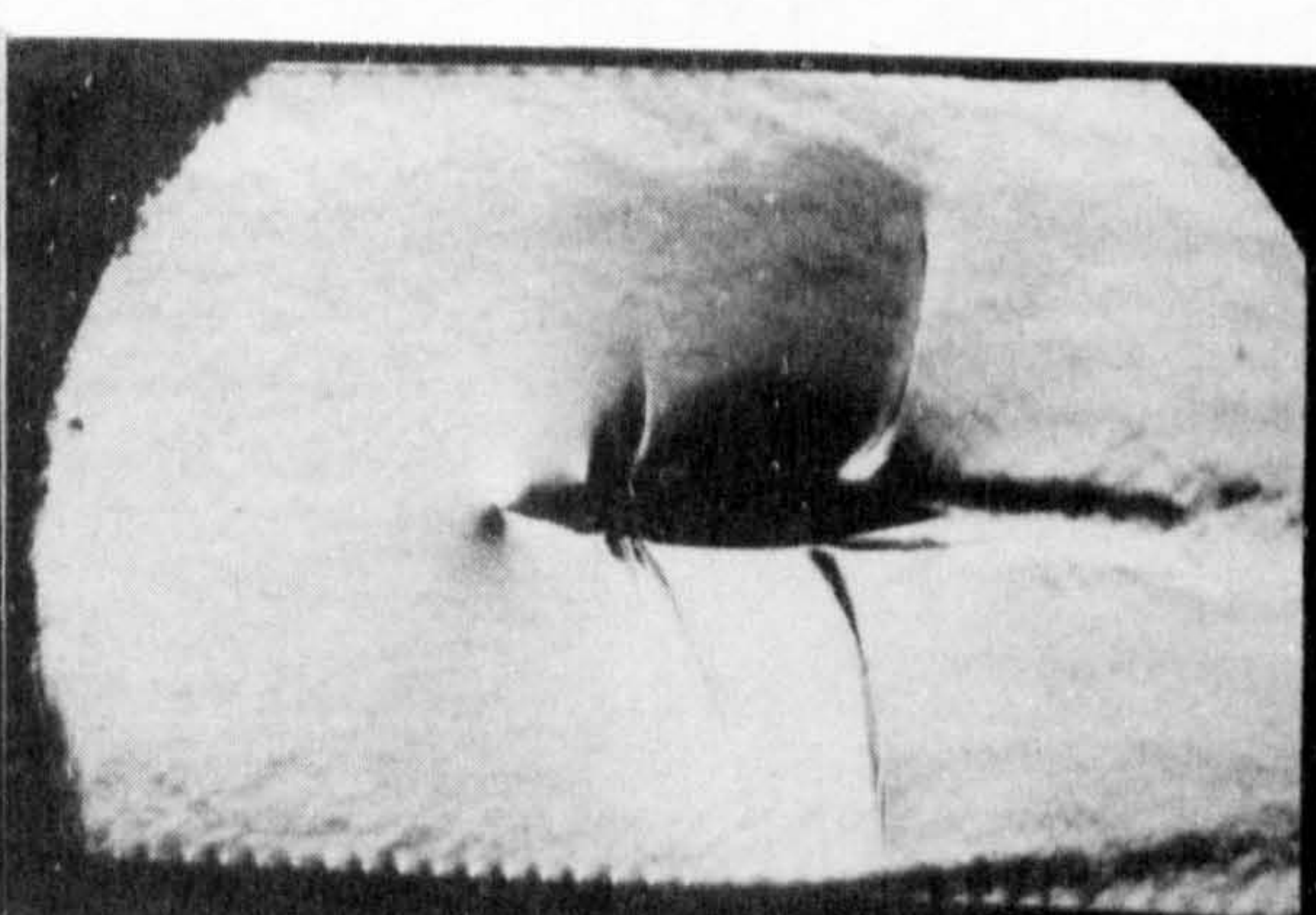


k) 0.90

UNSTEADY FLOW



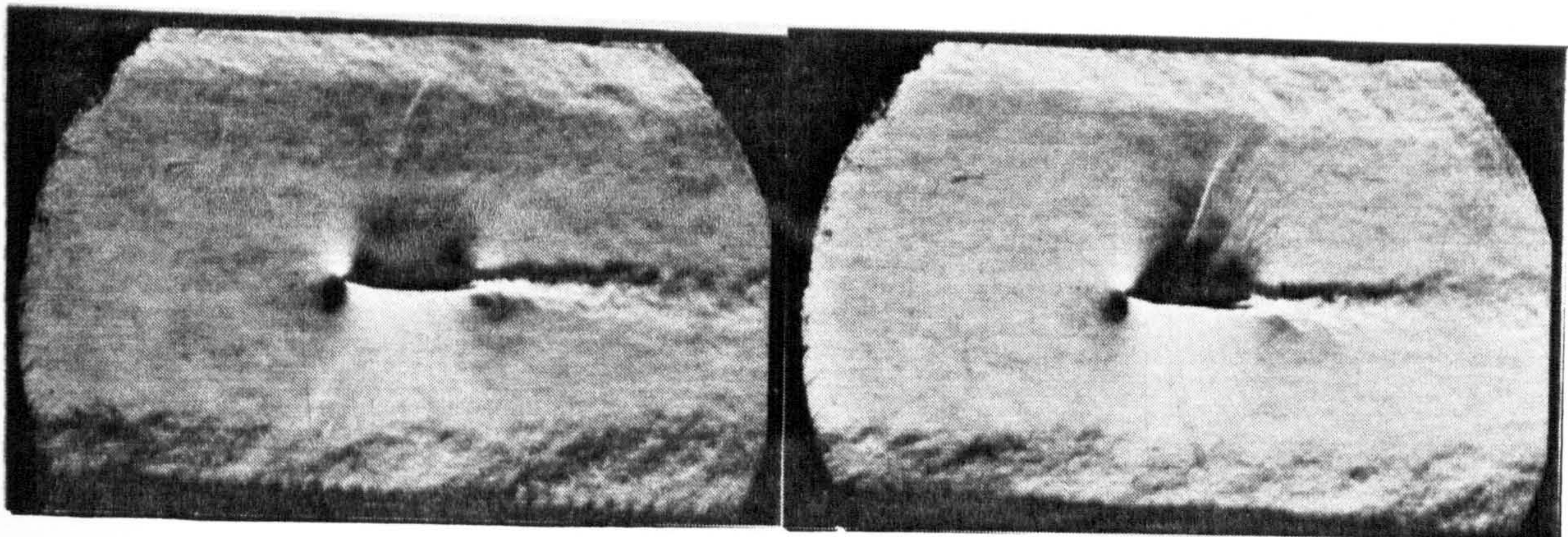
l) 0.95



m) 0.95

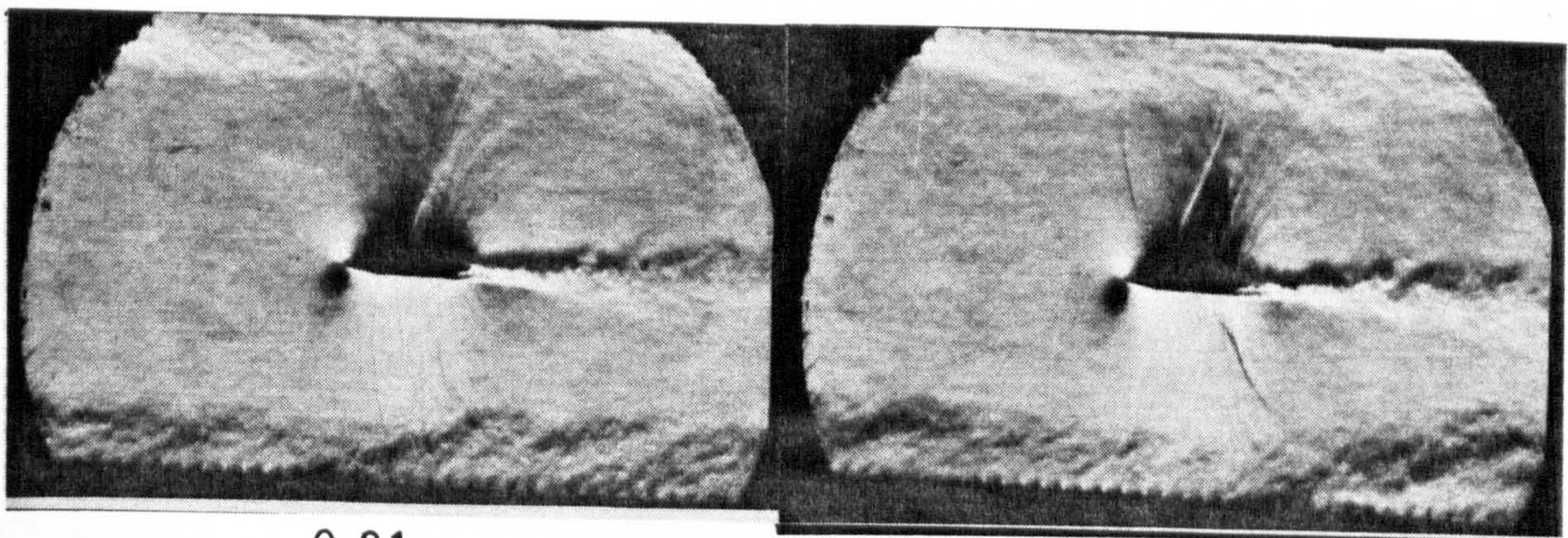
STEADY FLOW WITH SHOCK INDUCED SEPARATION

cont. Figure 11. Schlieren pictures of flow on a 14% thick biconvex aerofoil at zero incidence (fixed transition).



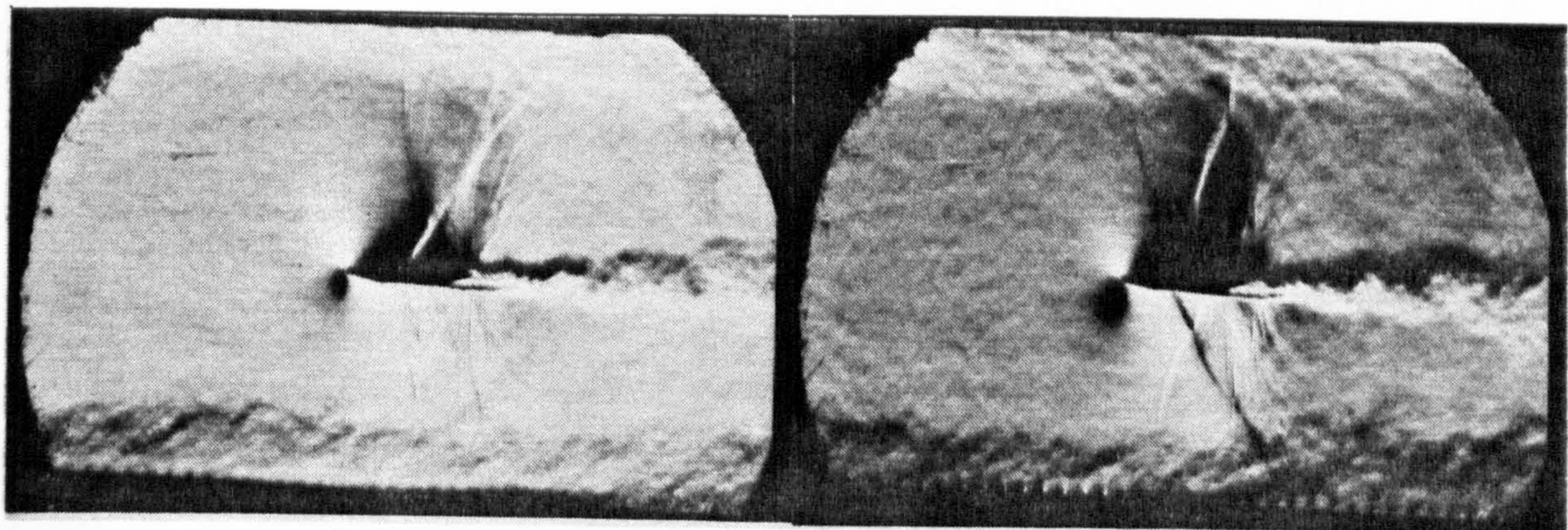
$M_\infty = 0.73$

0.78



0.81

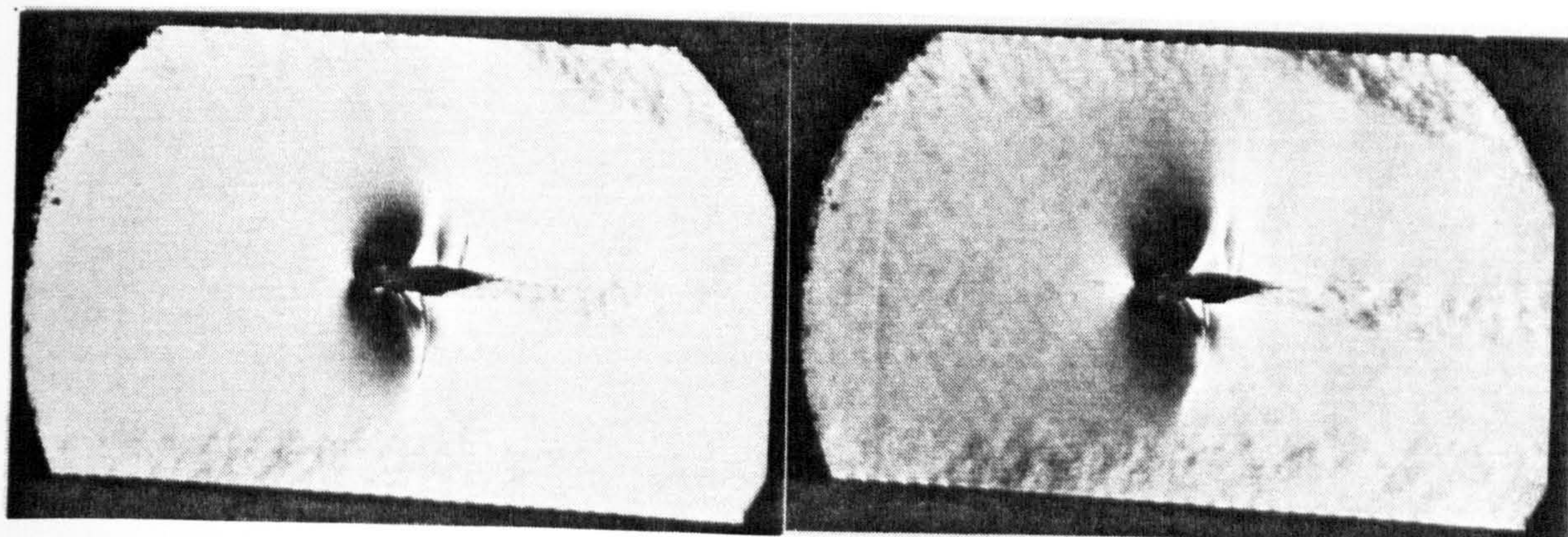
0.83



0.87

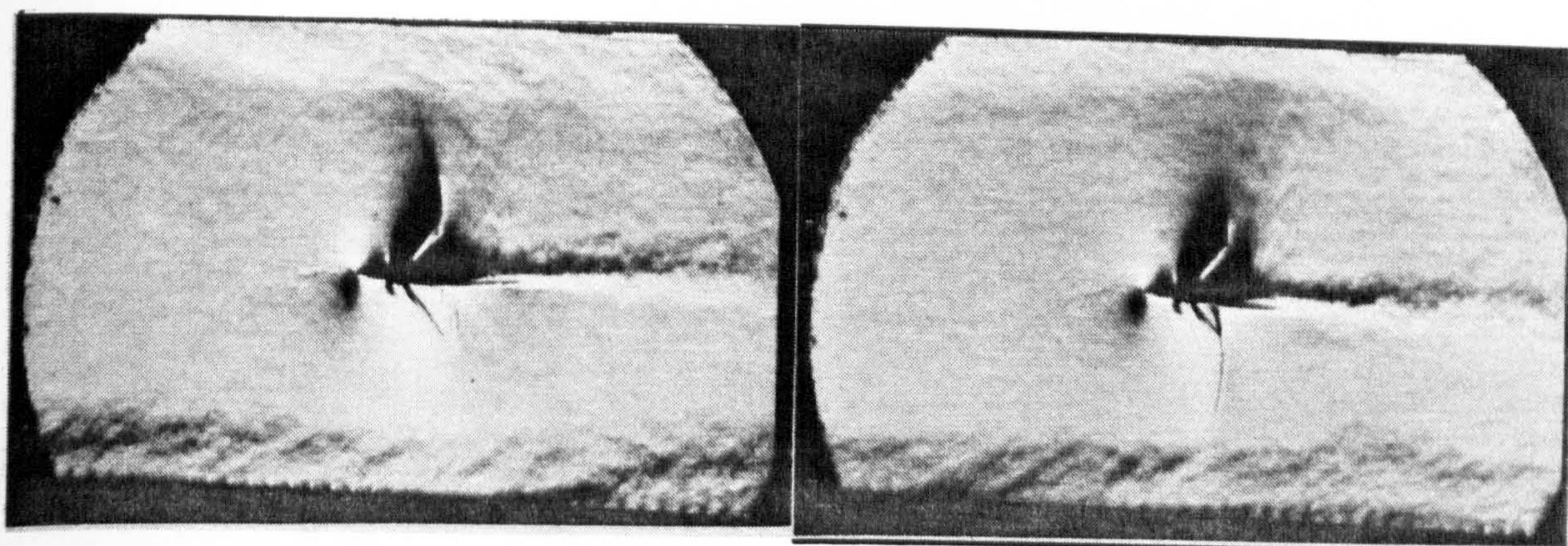
0.87

Figure 12. Schlieren pictures of flow on a 14% biconvex aerofoil at zero incidence, $c=16\text{mm}$ (free transition).



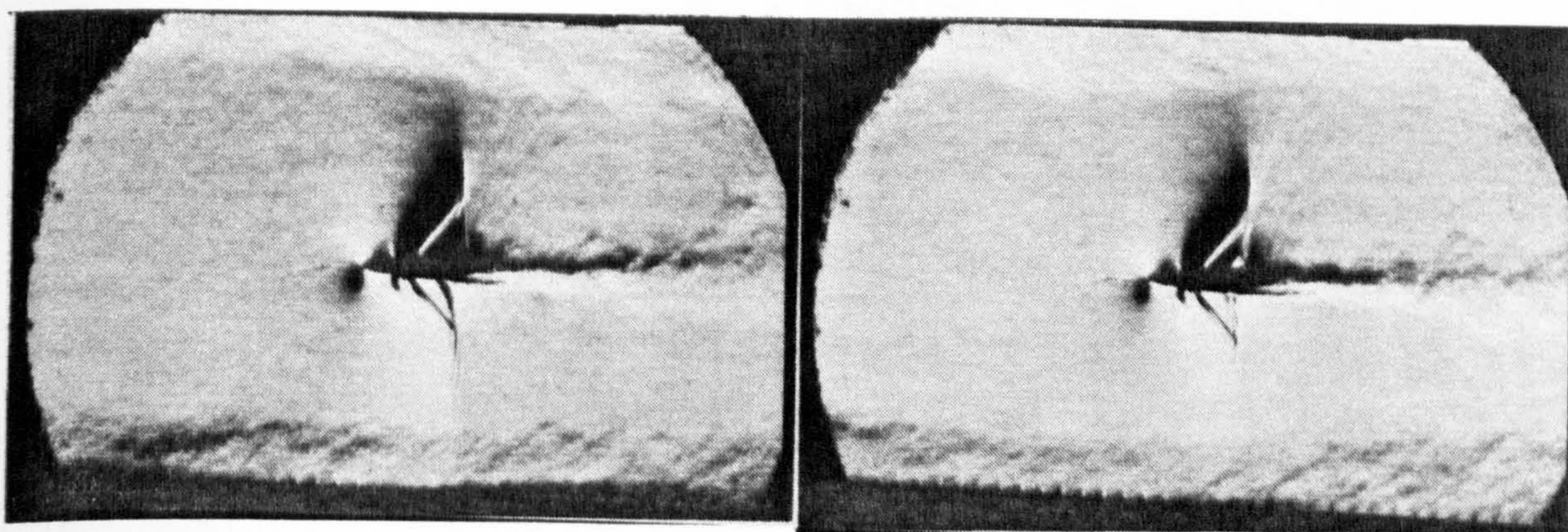
$M_\infty = 0.82$

0.82



0.84

0.84



0.87

0.87

Figure 13. Schlieren pictures of flow on a 14% biconvex aerofoil ($c=16\text{mm}$) at zero incidence (fixed transition).

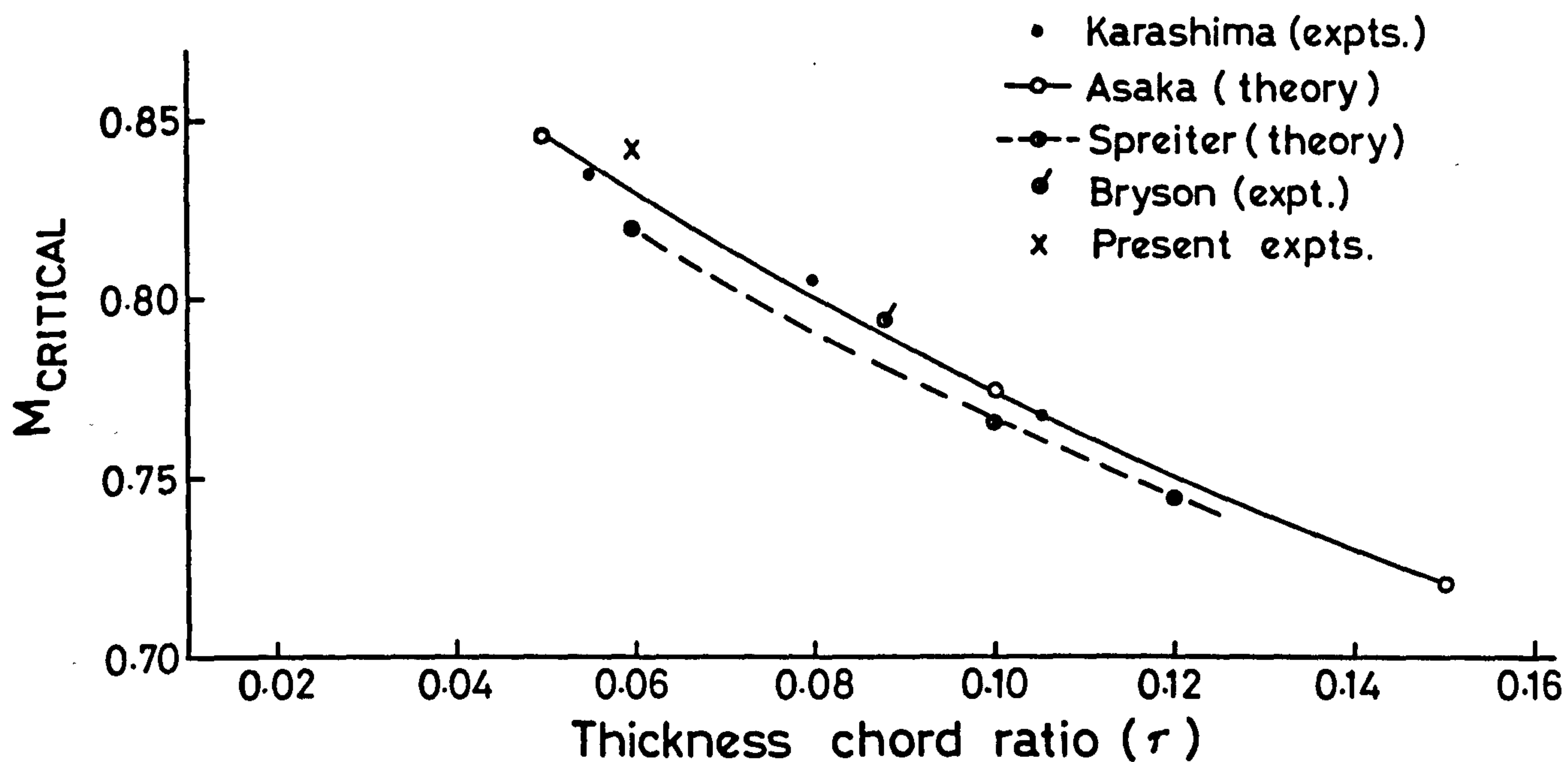


Figure 14. Critical Mach Number.

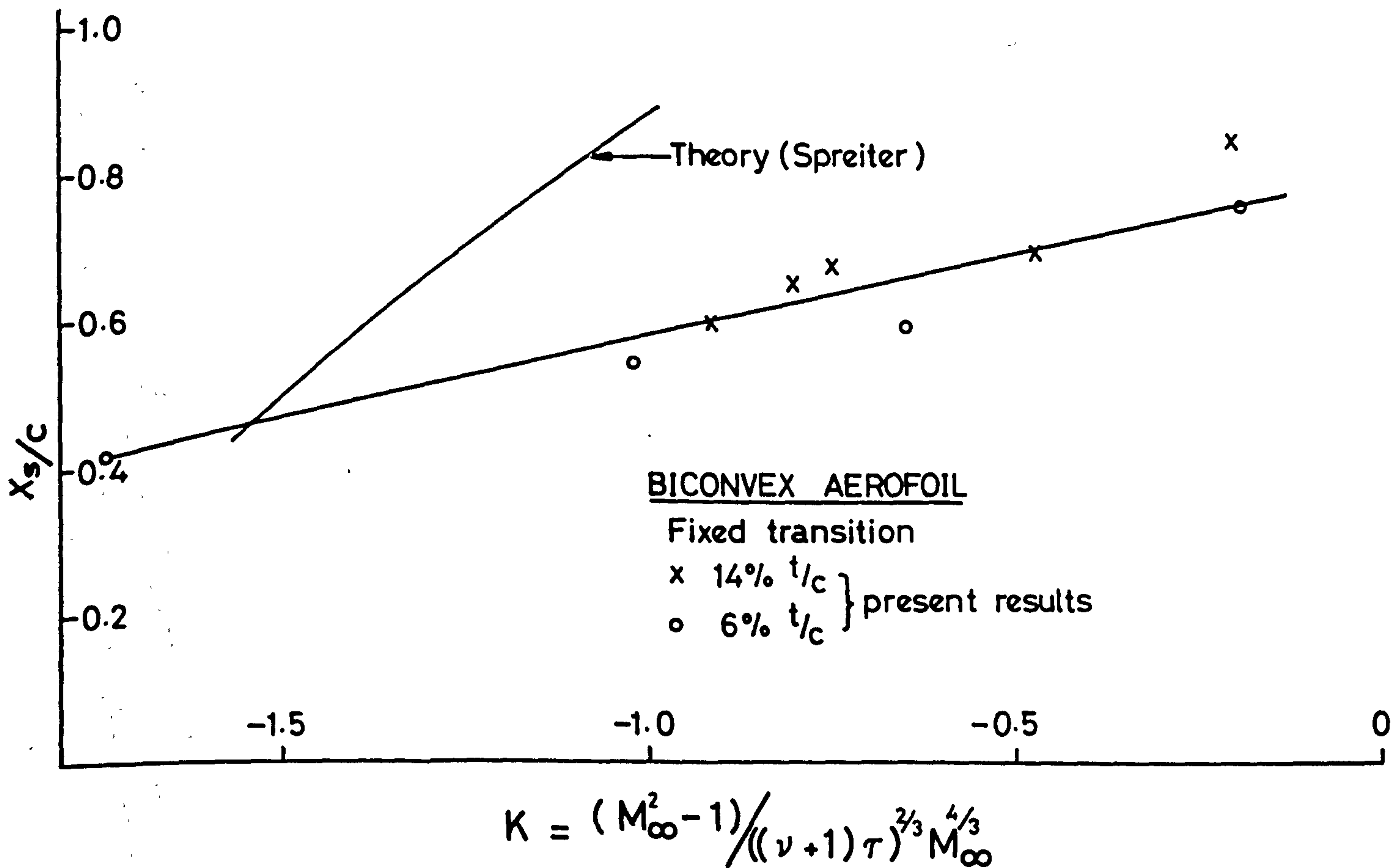
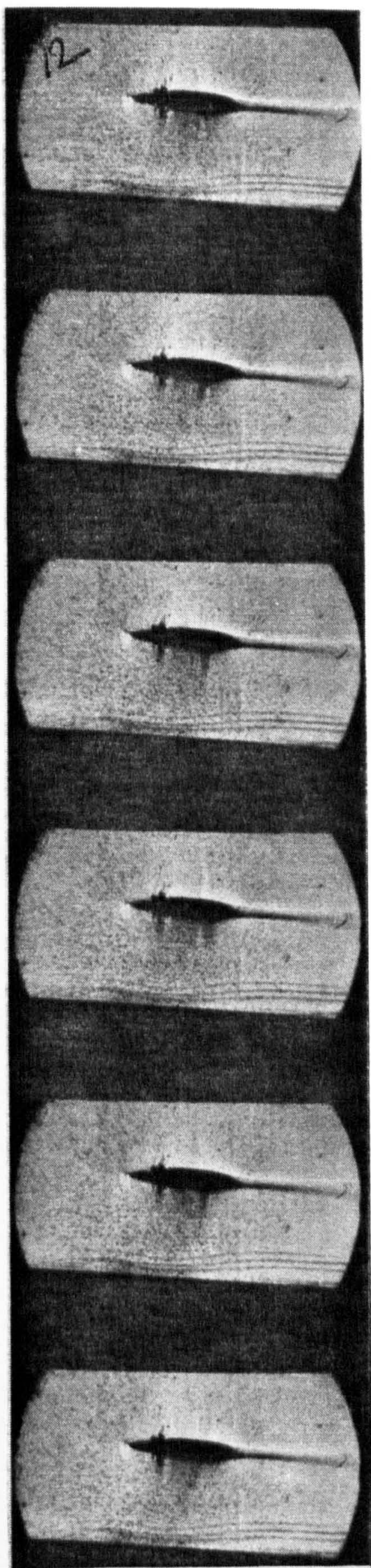
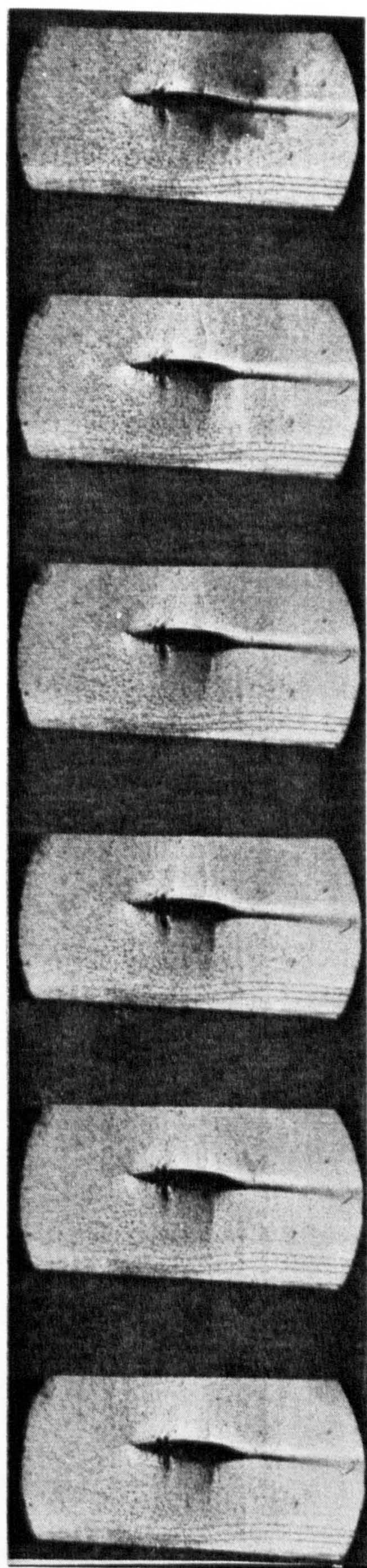


Figure 15. Shock position vs. transonic similarity parameter.



a) $M_\infty = 0.81$



b) $M_\infty = 0.85$

STEADY FLOW

Figure 16. High speed photographs of flow on a 14% biconvex aerofoil, $\alpha = 4^\circ$, $\Delta t = 160 \mu s$.

1)



2)



3)



4)



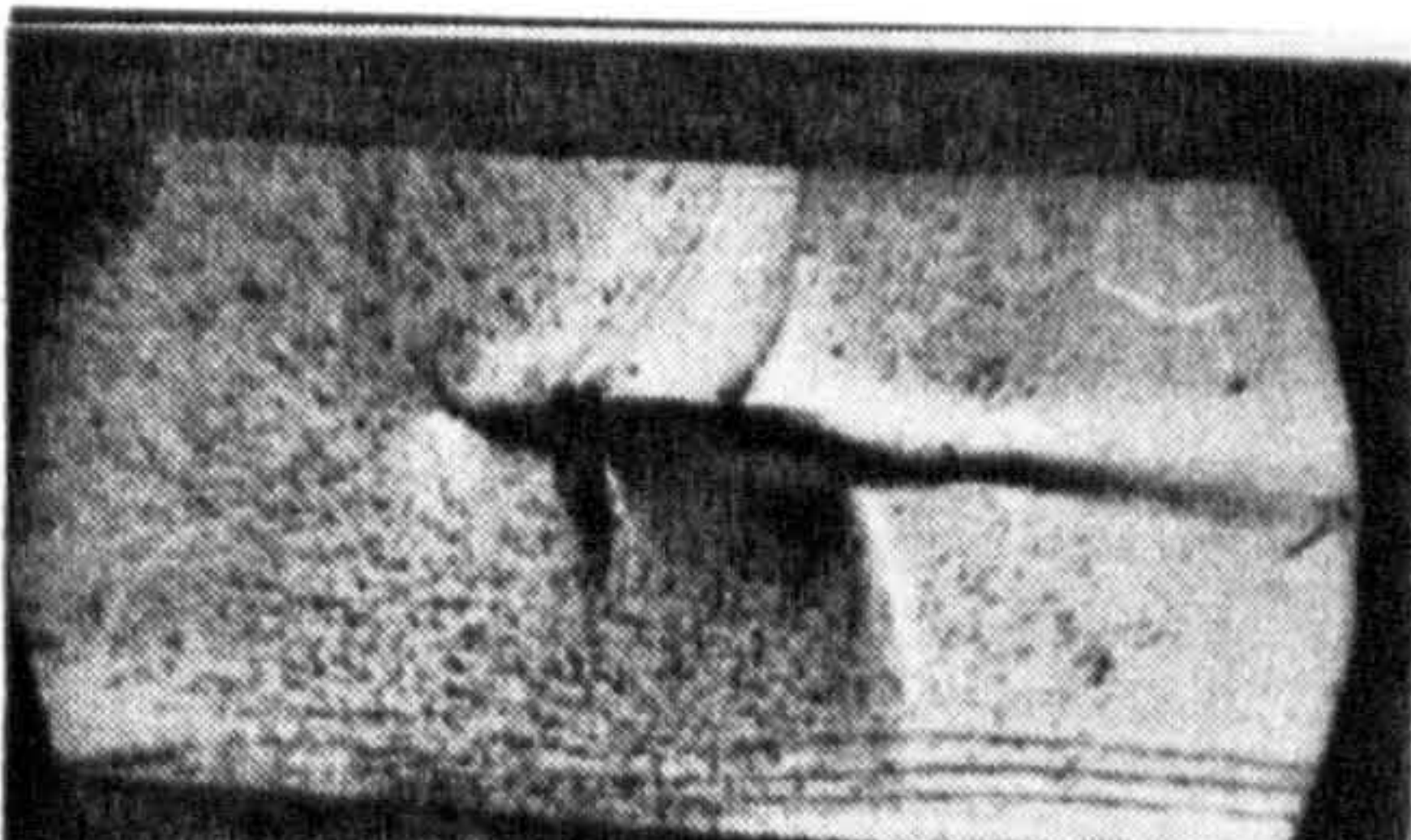
5)



6)



7)



8)



9)



10)



11)



12)



$M_{\infty} = 0.90$
PERIODIC FLOW

Figure 17. High speed photographs of flow on a 14% biconvex aerofoil, $\alpha = 4^\circ$, $\Delta t = 160 \mu s$.

14% t/c biconvex aerofoil

$$\alpha = 4^\circ$$

$$M_\infty = 0.90 \rightarrow$$

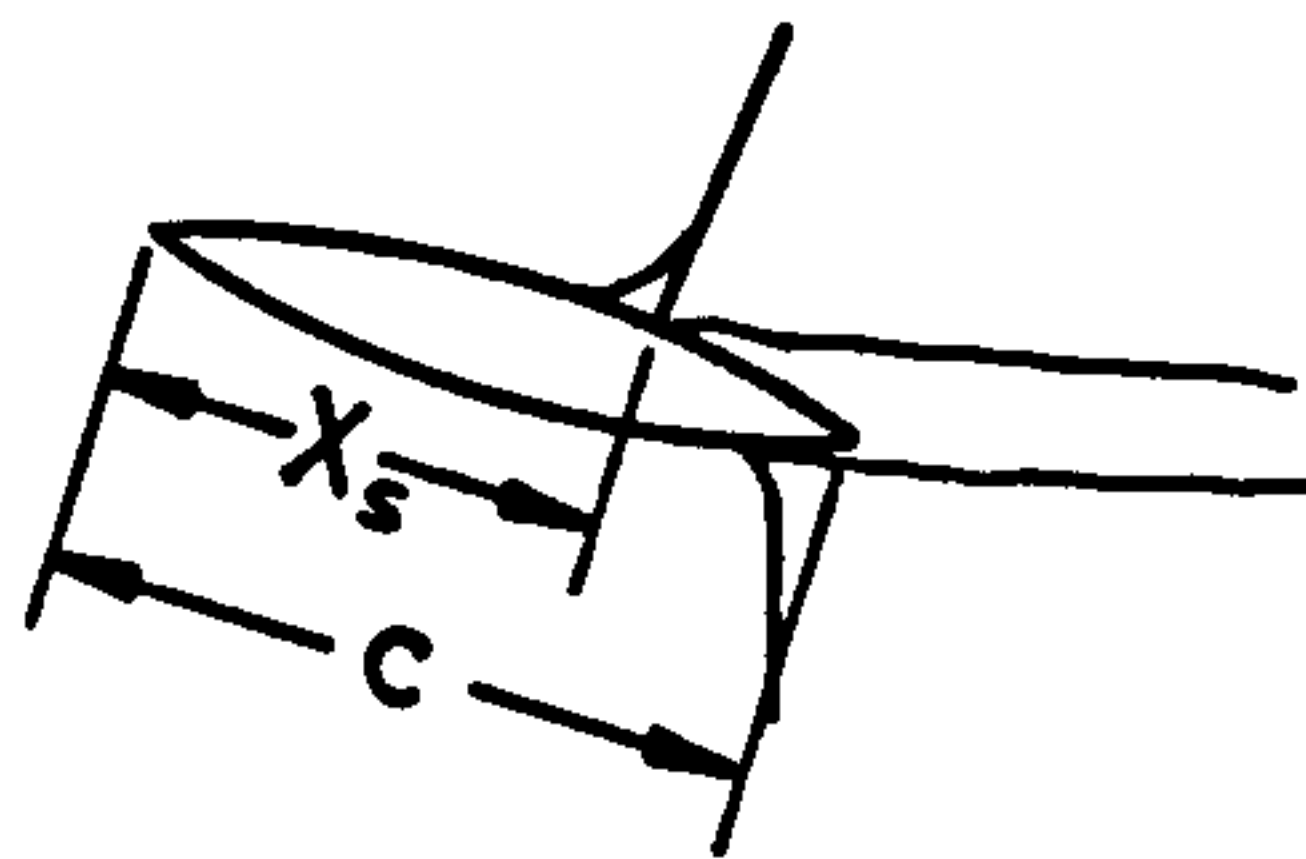
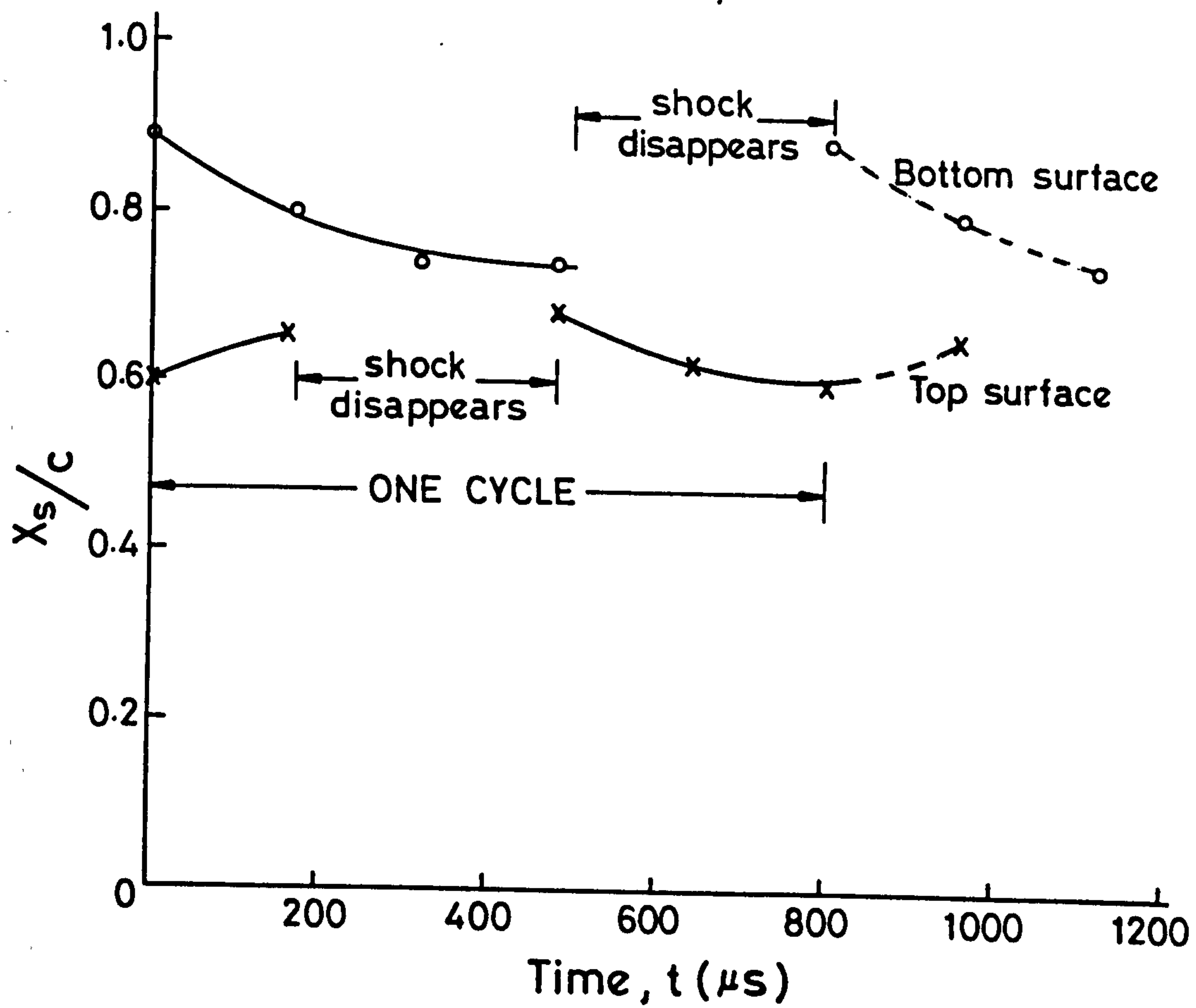
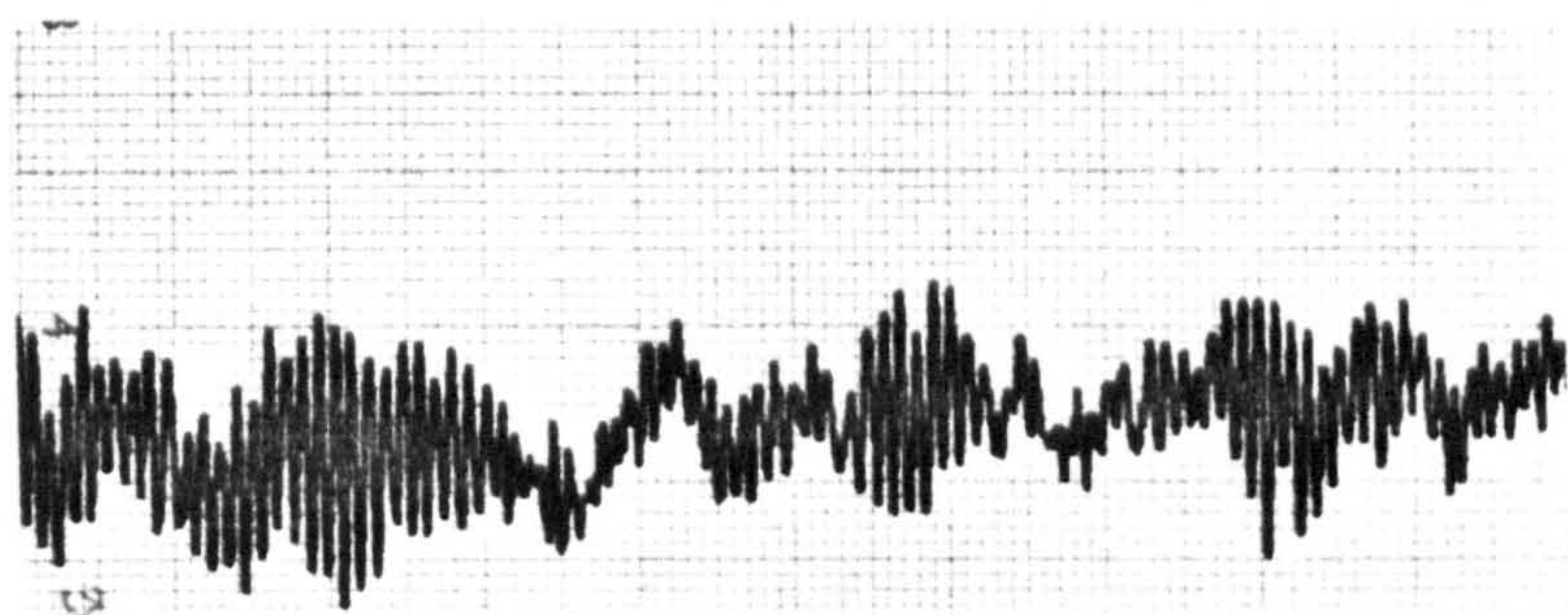
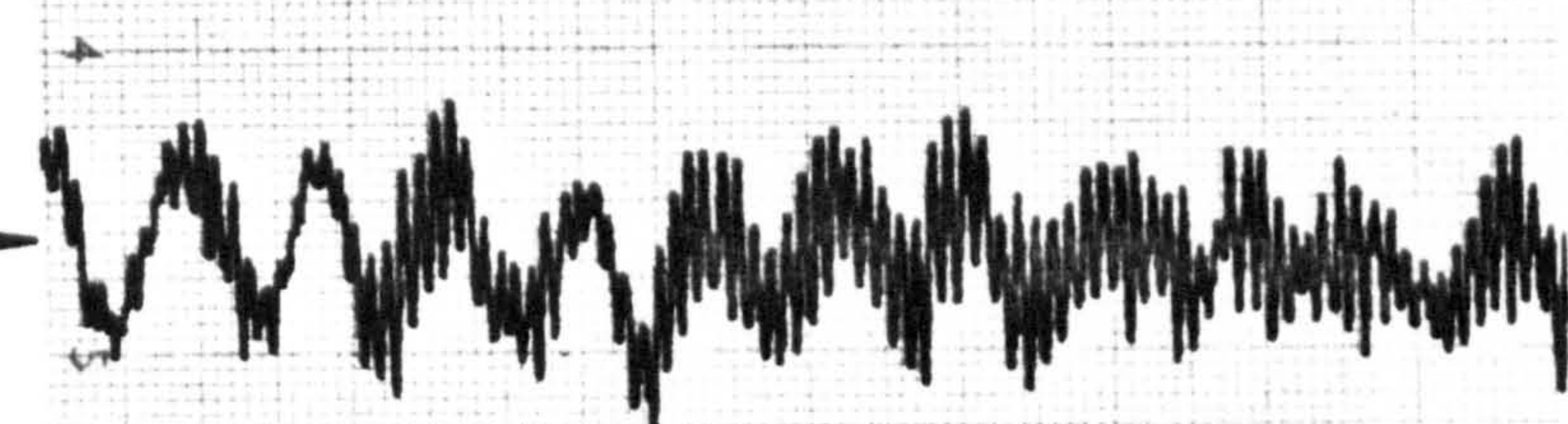


Figure 18. Development of periodic flow.

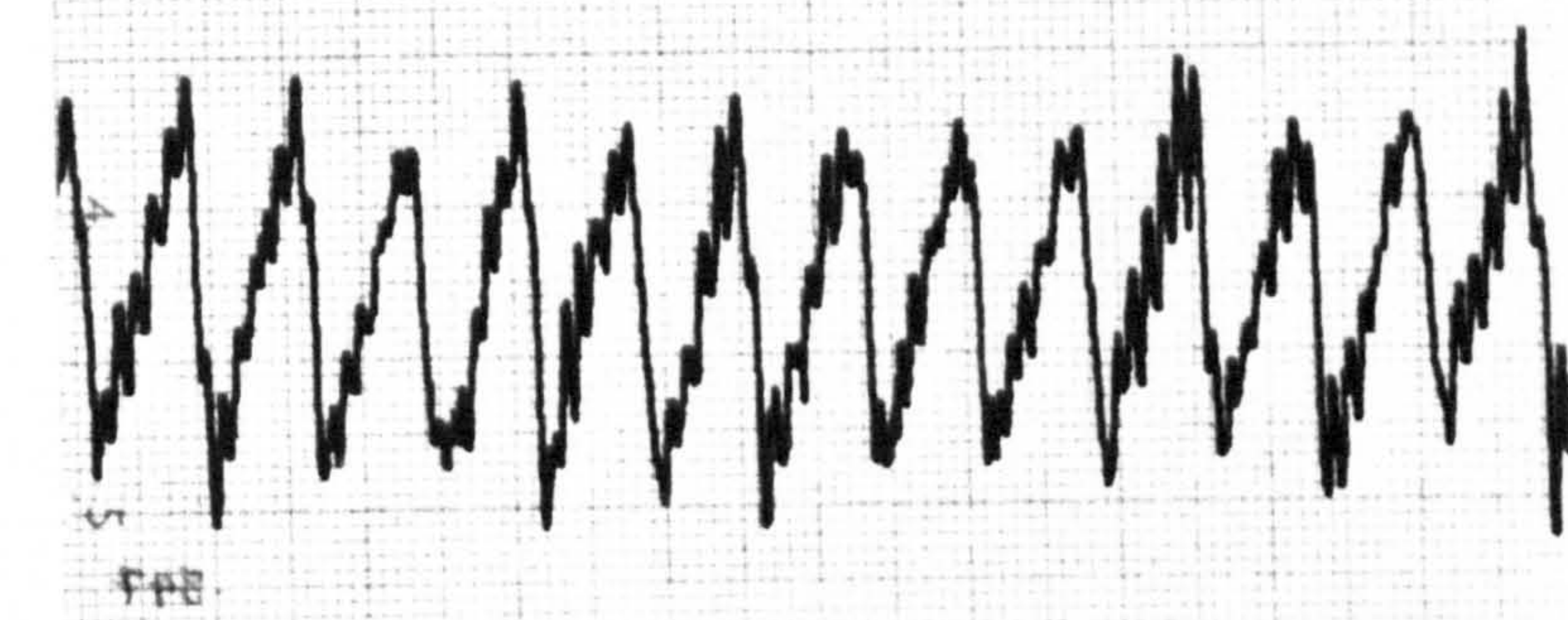
steady
flow



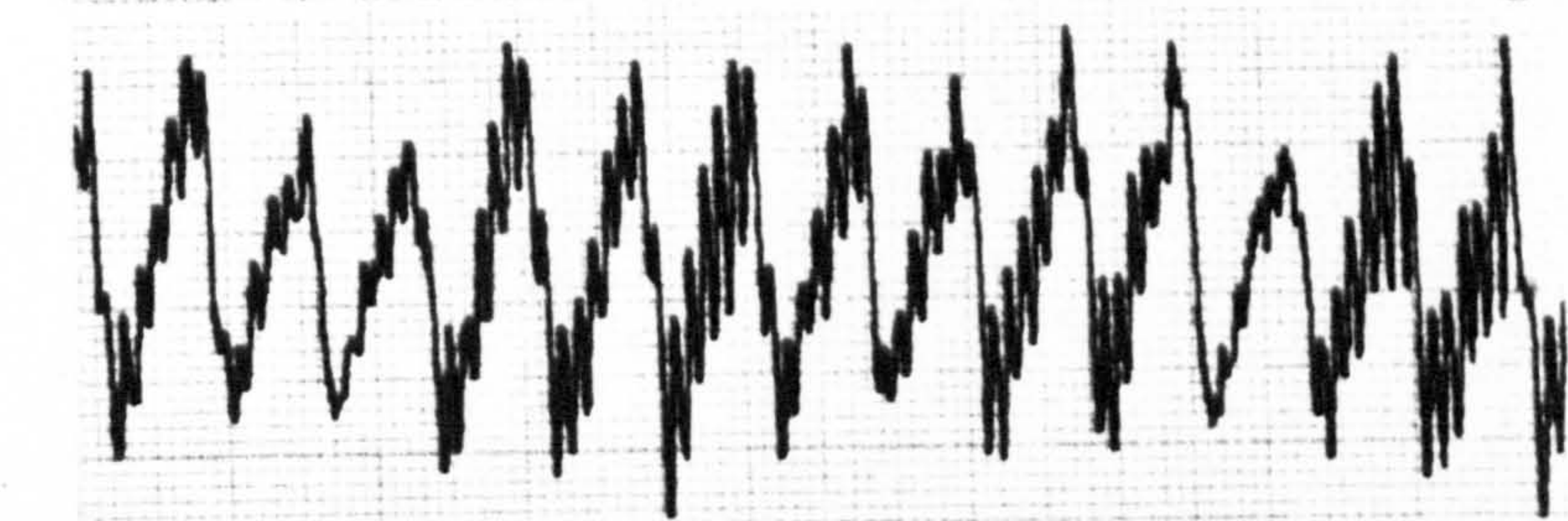
a) $M_\infty = 0.82$



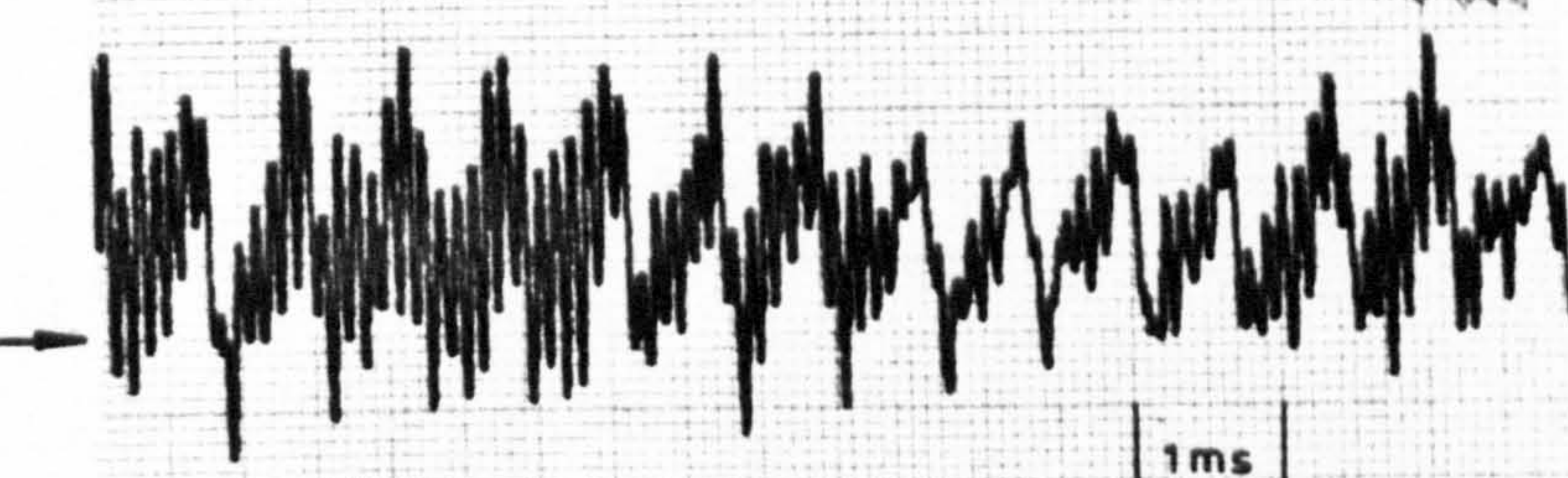
b) 0.83



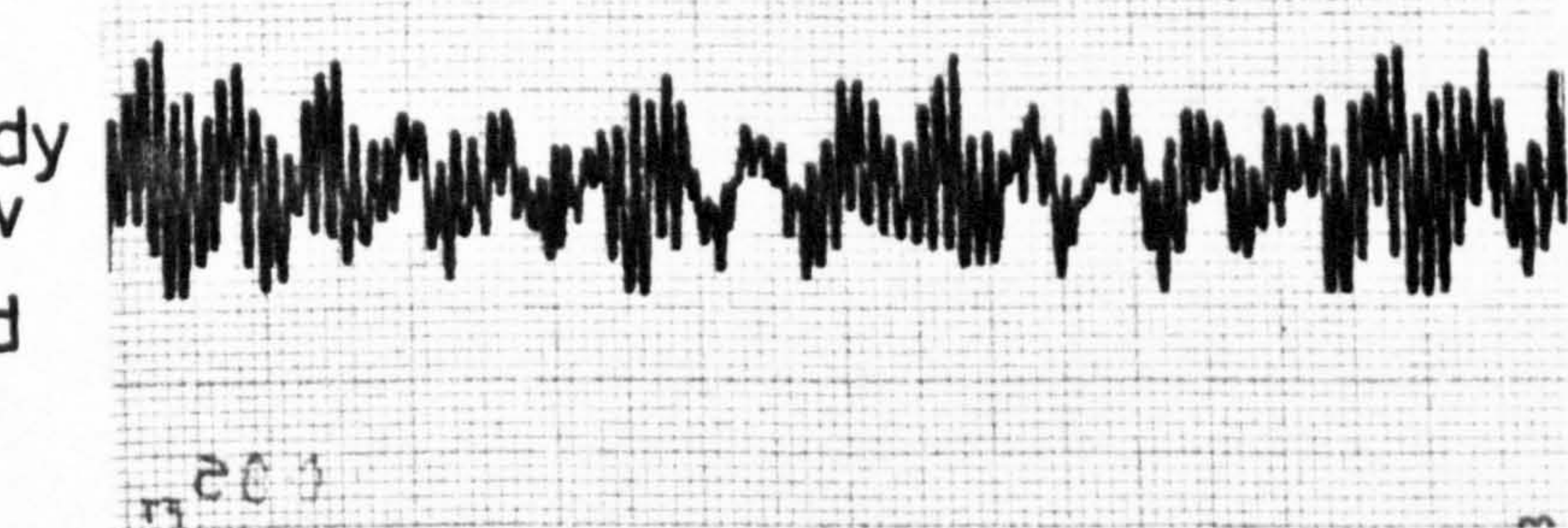
c) 0.85



d) 0.88



e) 0.91



f) 0.95

oscillatory flow

steady
flow
unfiltered
trace

1ms

0.25
p.s.i

Figure 19A). Sidewall pressure traces, 14% biconvex aerofoil, at zero incidence (free transition).

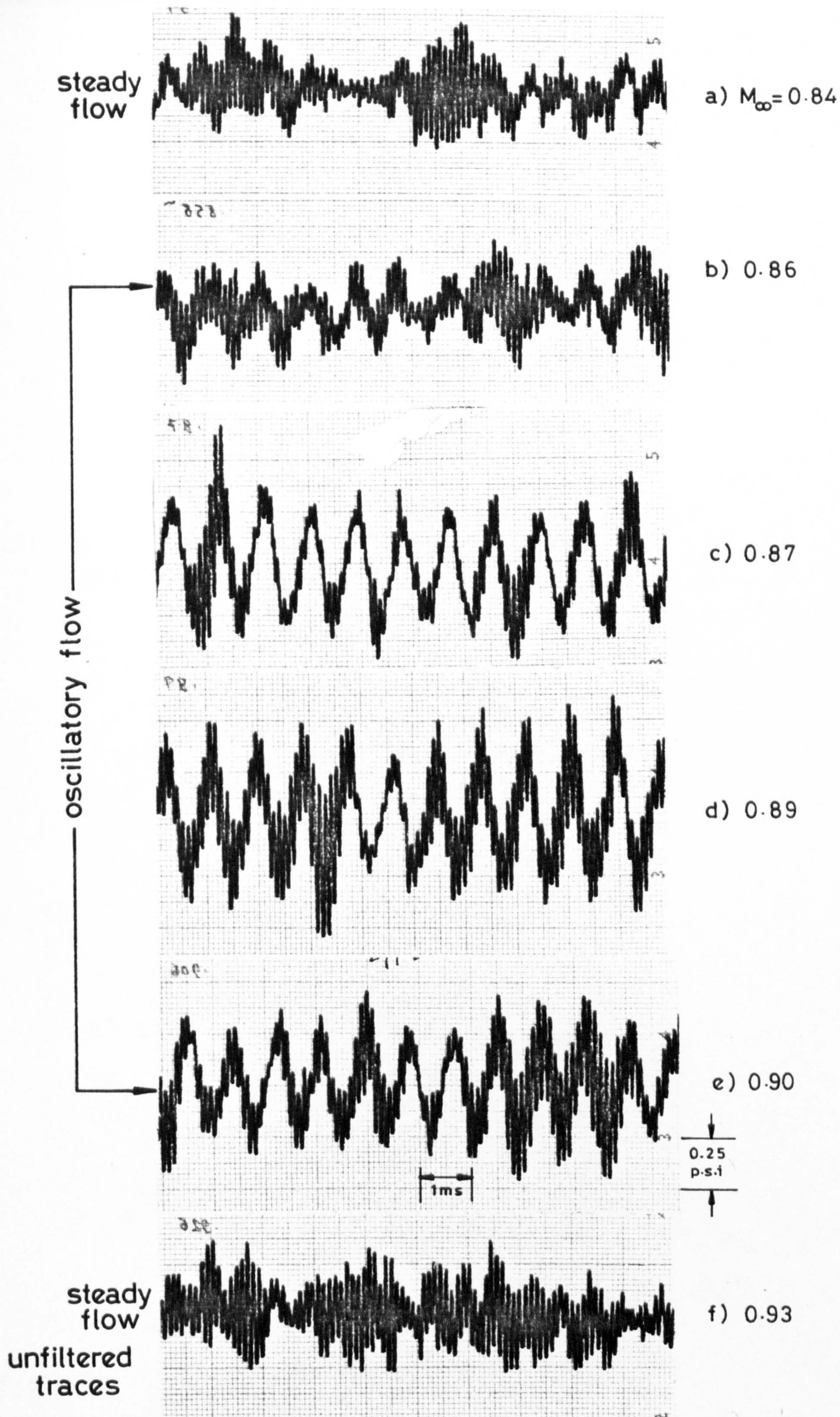


Figure 19B) Sidewall pressure traces, 14% biconvex aerofoil at zero incidence (fixed transition)

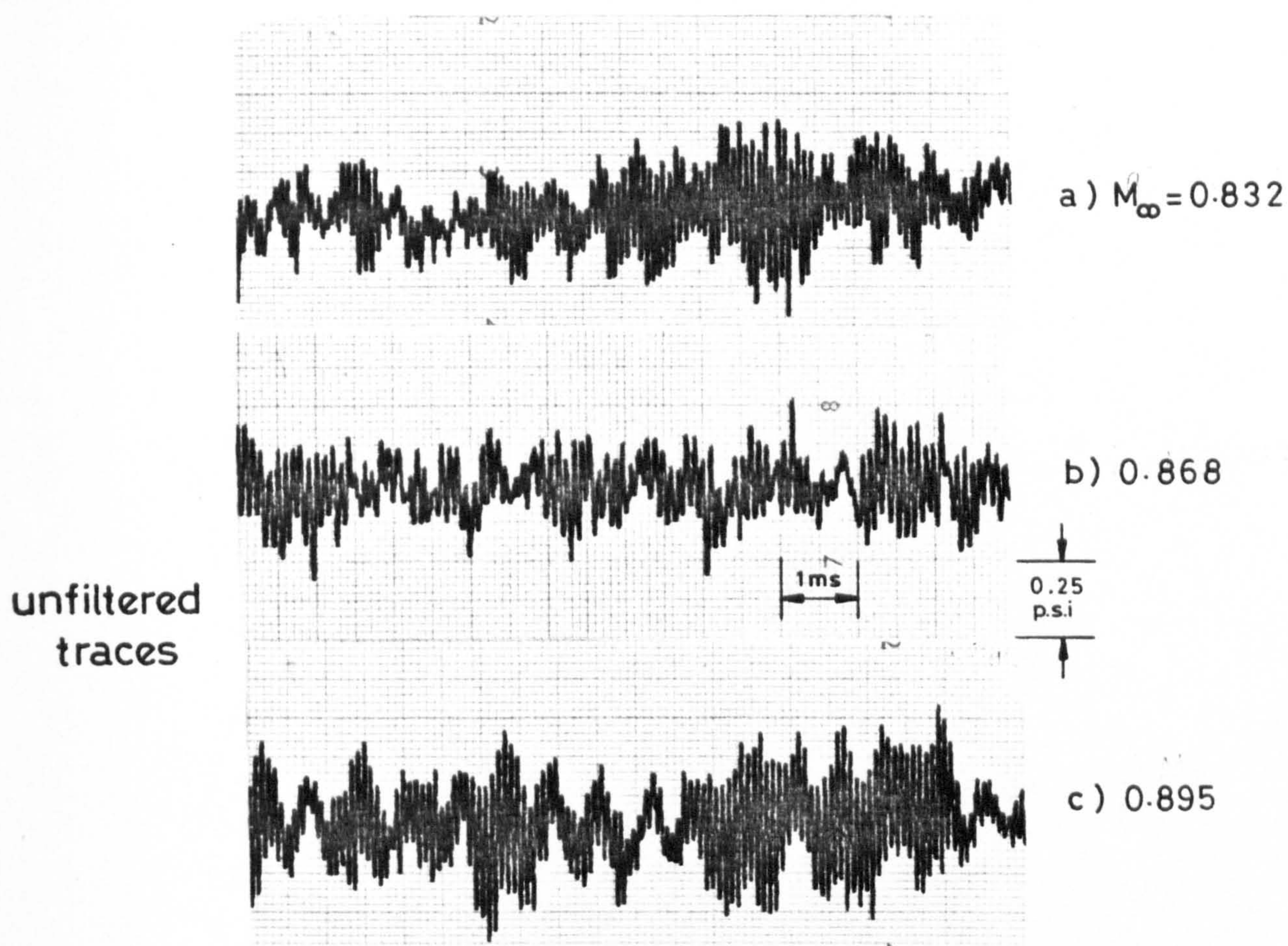


Figure 20. Sidewall pressure traces (empty tunnel)

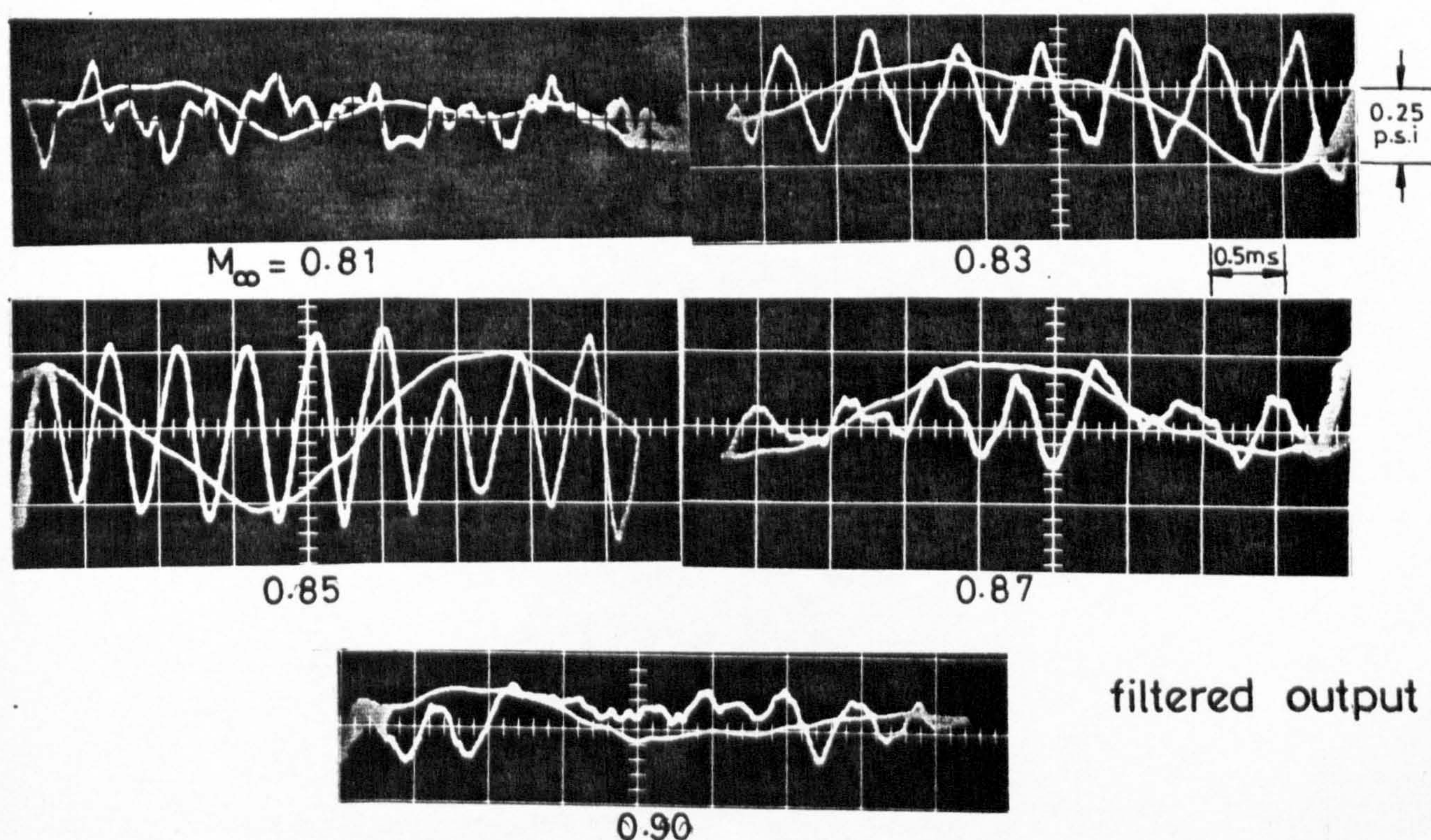


Figure 21. Sidewall pressure traces , 14% thick biconvex ($c=16\text{mm}$) aerofoil at zero incidence (fixed transition).

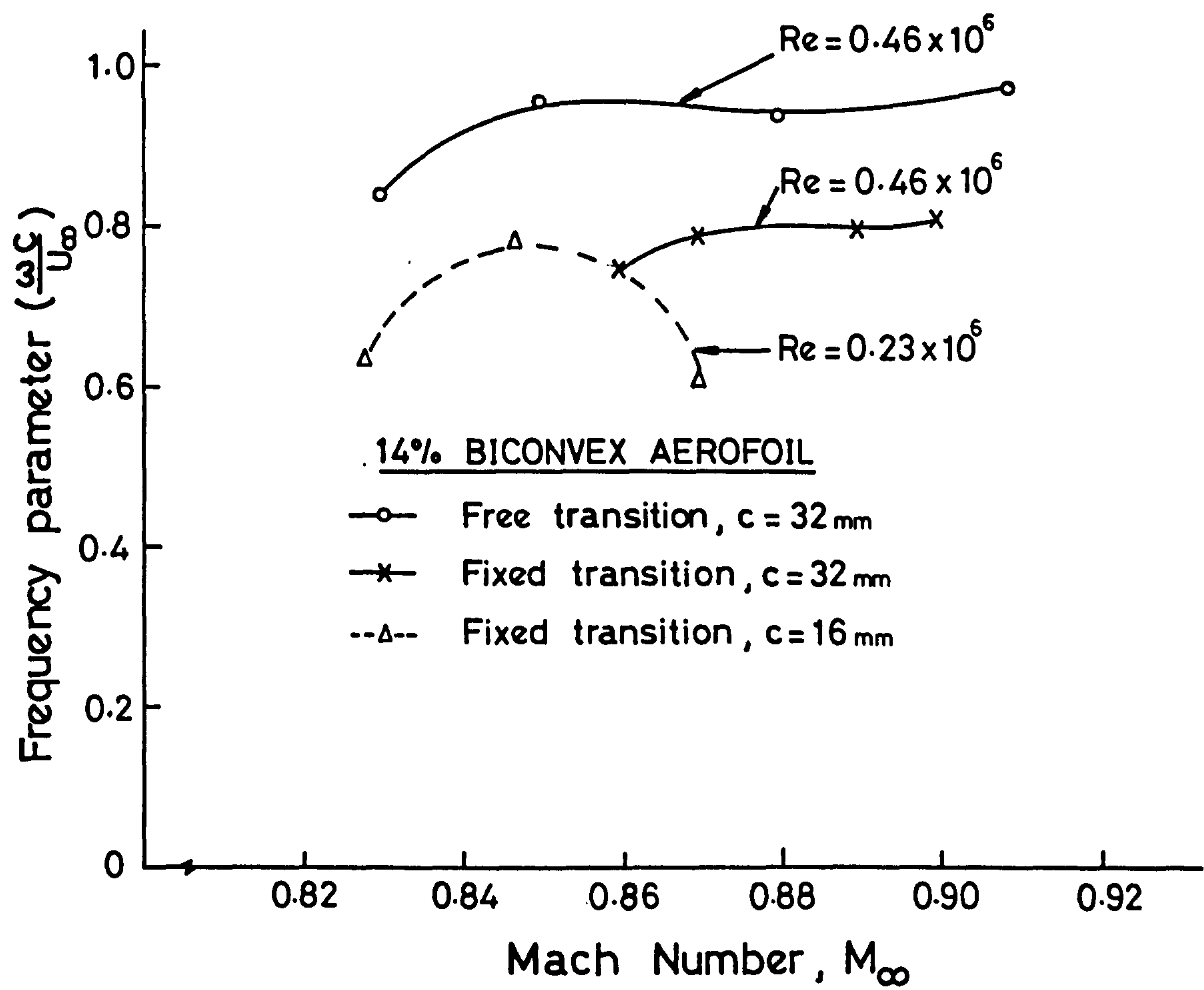
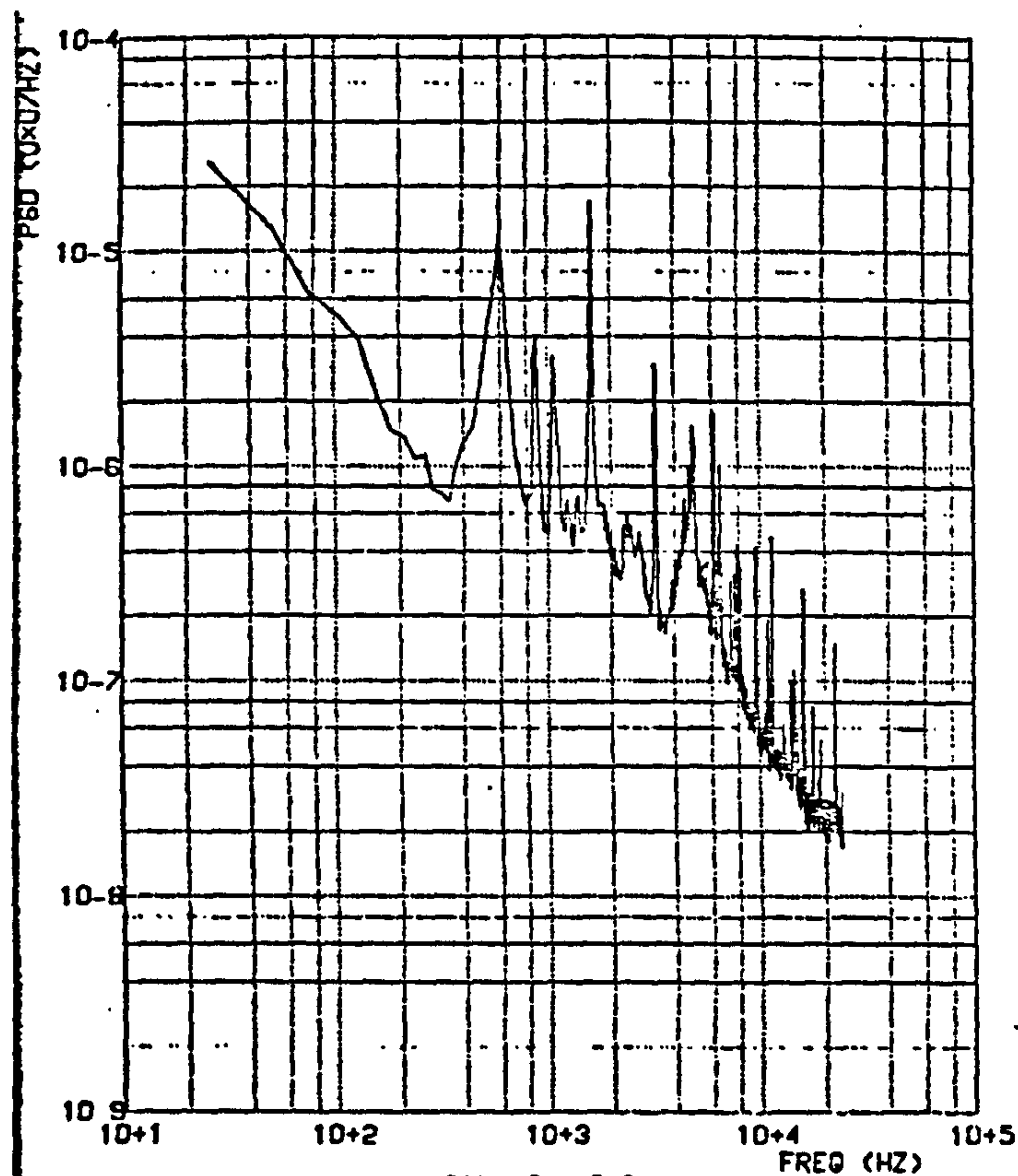
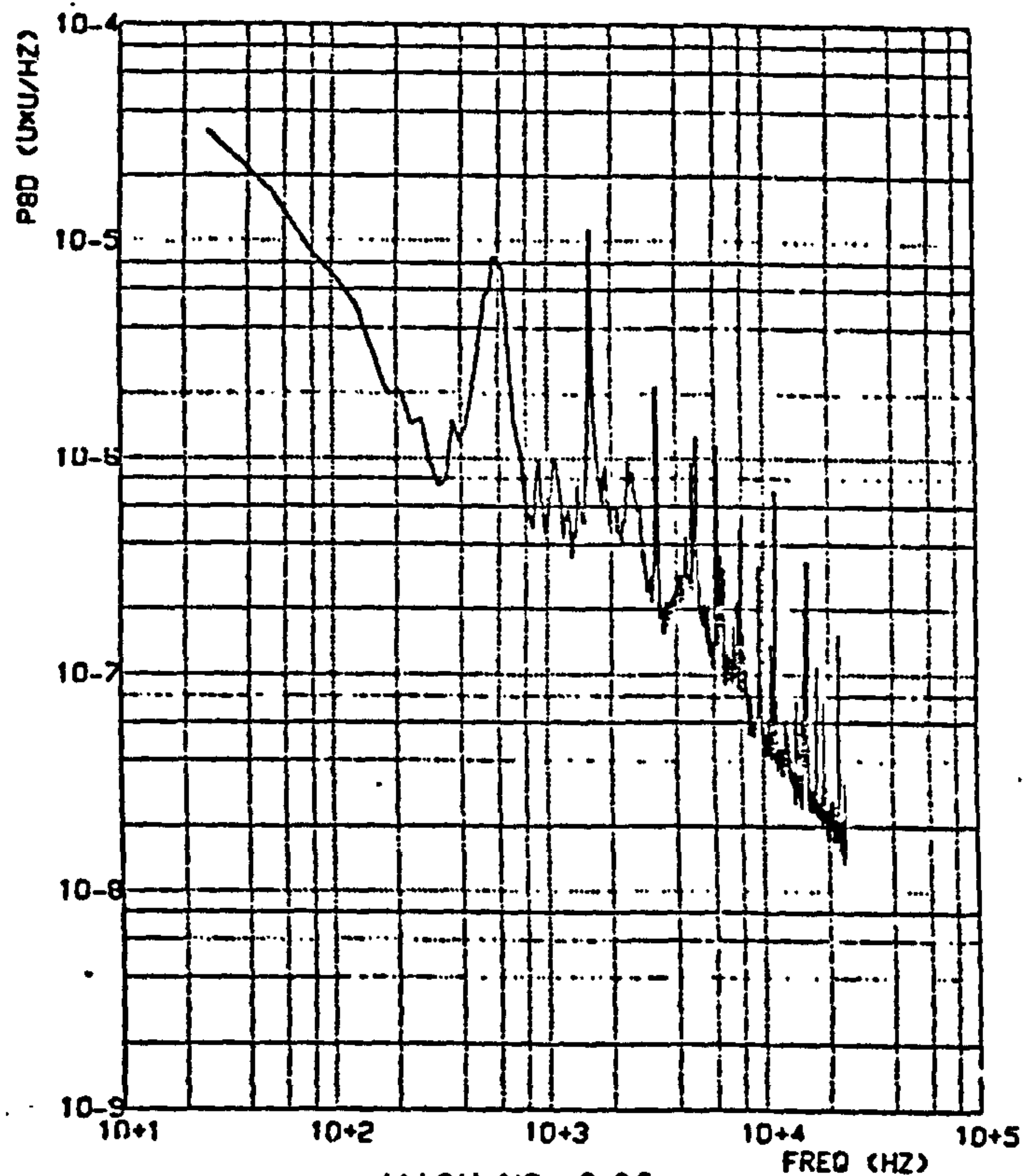


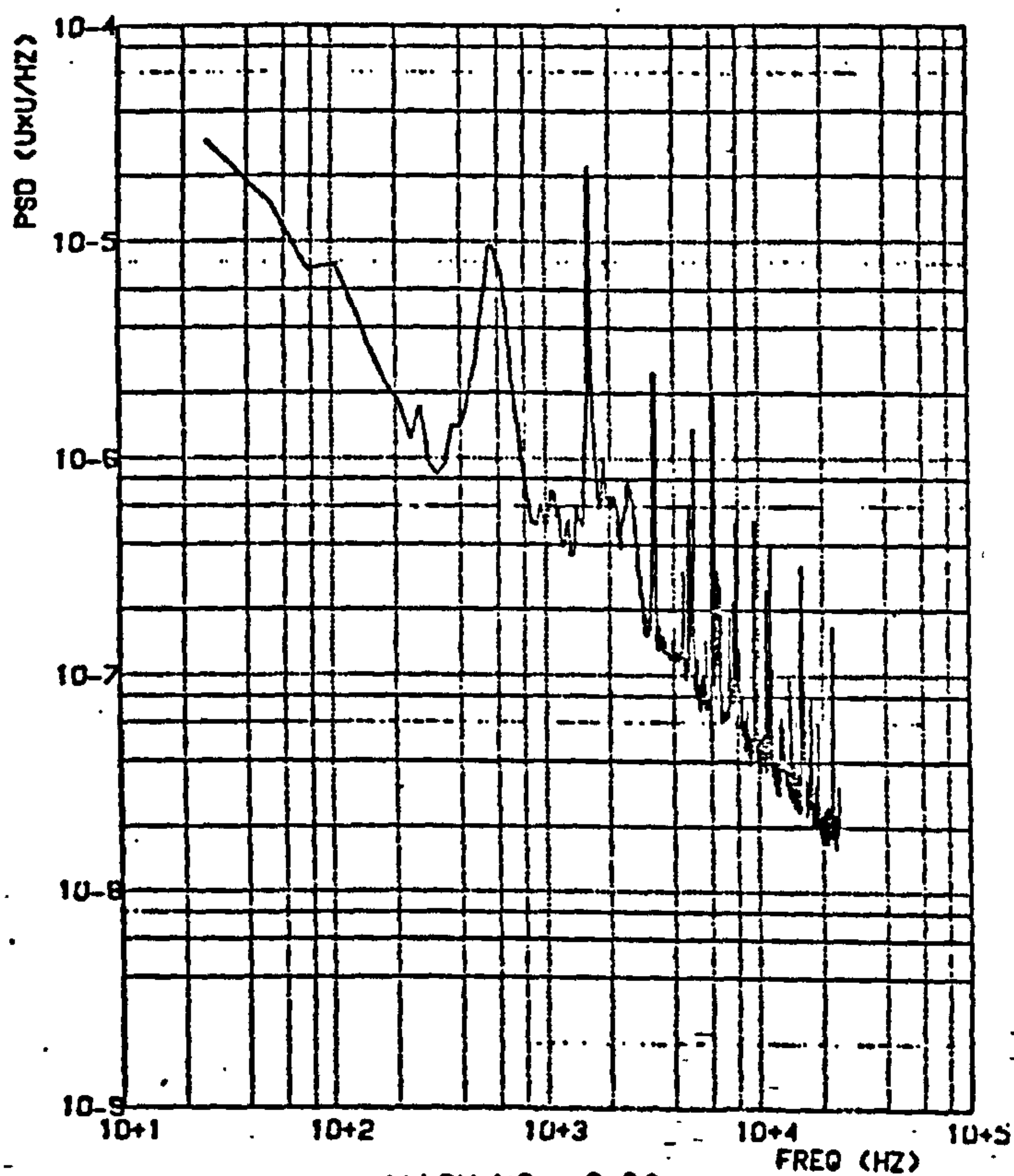
Figure 22. Frequency parameter.



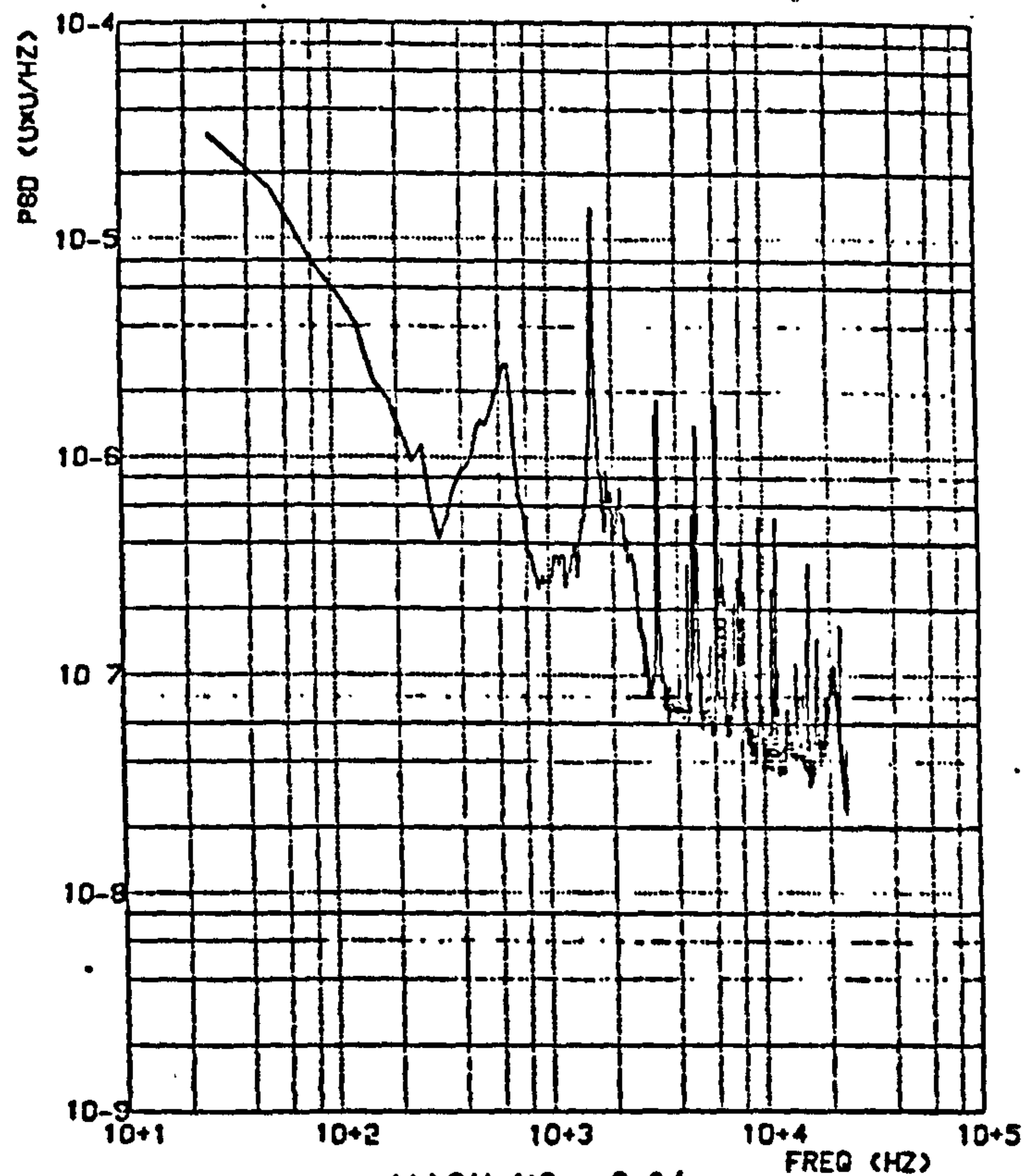
(a) MACH NO. = 0.81
 BANDWIDTH = 26.0417 HZ
 MEAN TIME = 7.99 SECS
 RMS VALUE = 8.637E-02 UNITS



(b) MACH NO. = 0.86
 BANDWIDTH = 26.0417 HZ
 MEAN TIME = 7.99 SECS
 RMS VALUE = 8.619E-02 UNITS

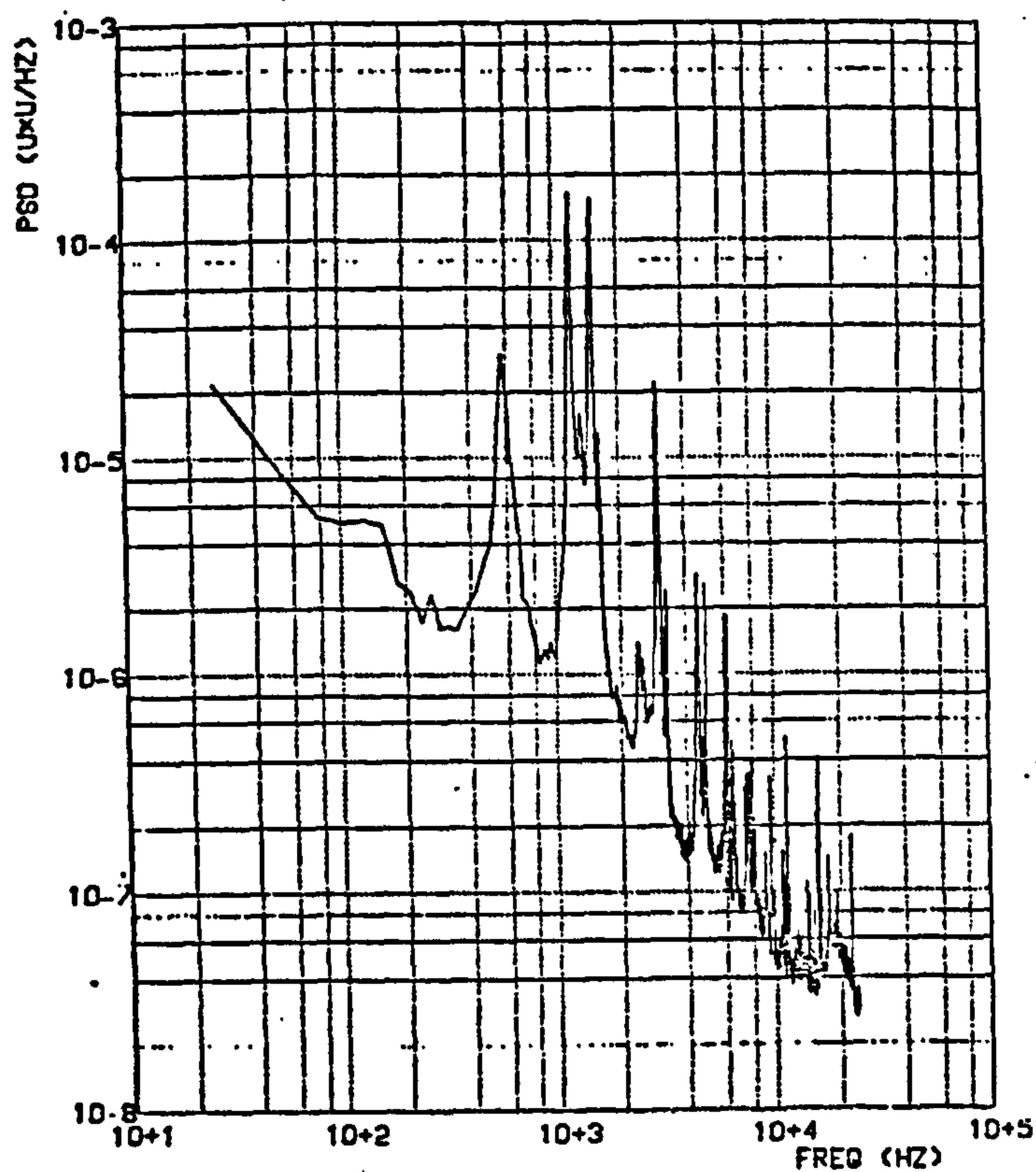


(c) MACH NO. = 0.89
 BANDWIDTH = 26.0417 HZ
 MEAN TIME = 9.98 SECS
 RMS VALUE = 8.726E-02 UNITS



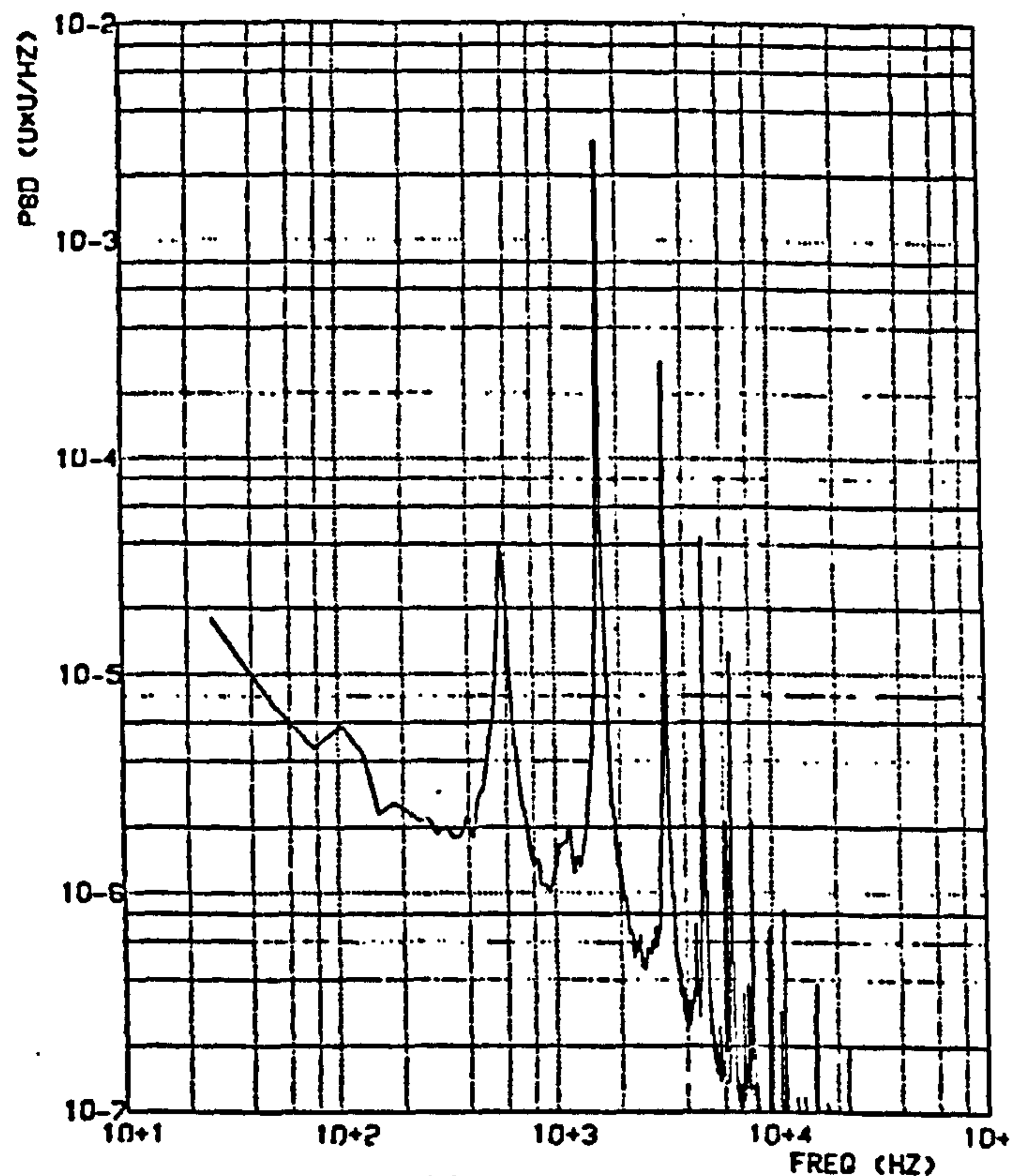
(d) MACH NO. = 0.94
 BANDWIDTH = 26.0417 HZ
 MEAN TIME = 9.98 SECS
 RMS VALUE = 7.543E-02 UNITS

Figure 23. P.S.D vs. frequency for the empty tunnel.



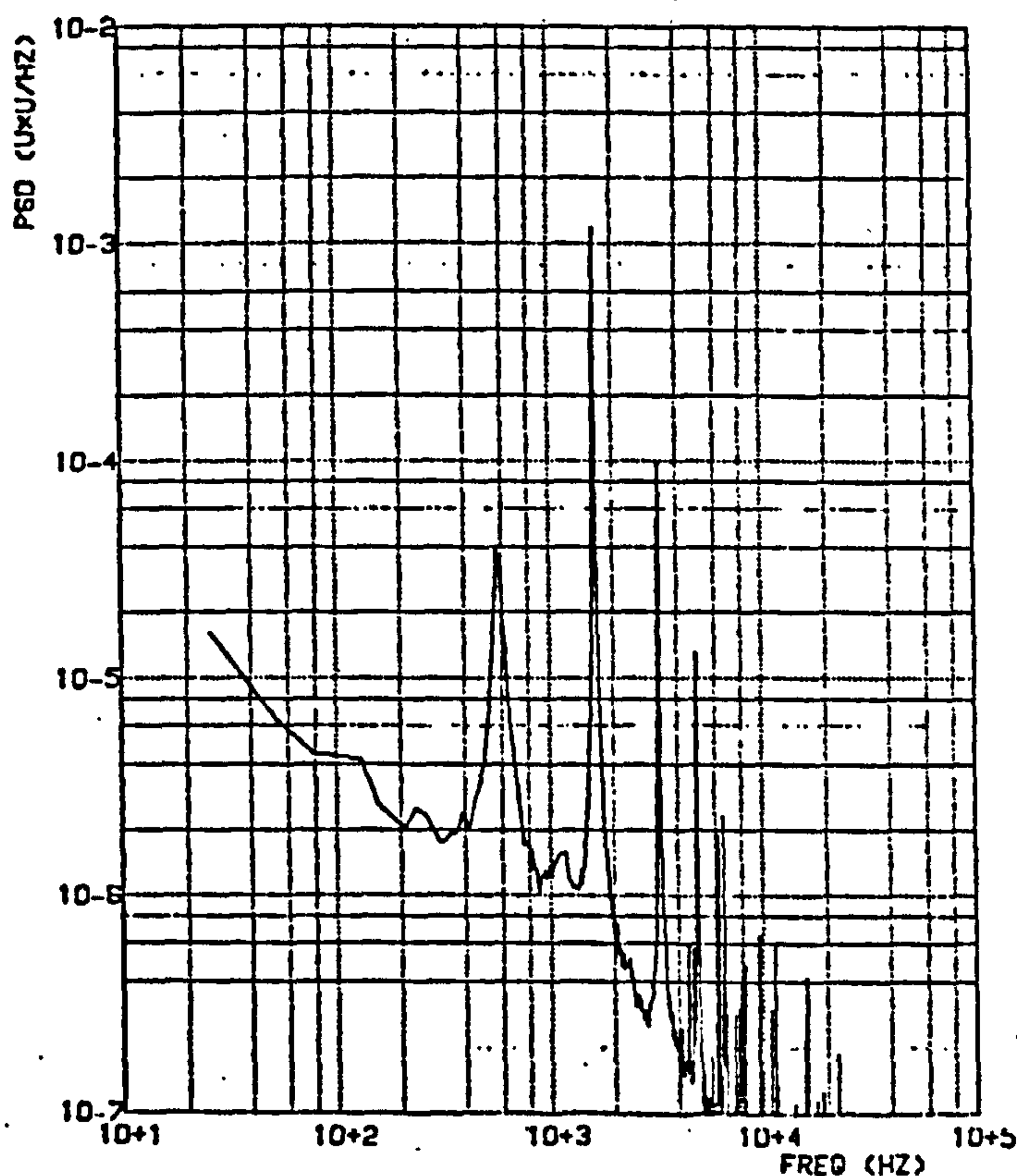
(a)

MACH NO. = 0.84
BANDWIDTH = 26.0417 HZ
MEAN TIME = 8.99 SECS
RMS VALUE = 1.721E-01 UNITS



(b)

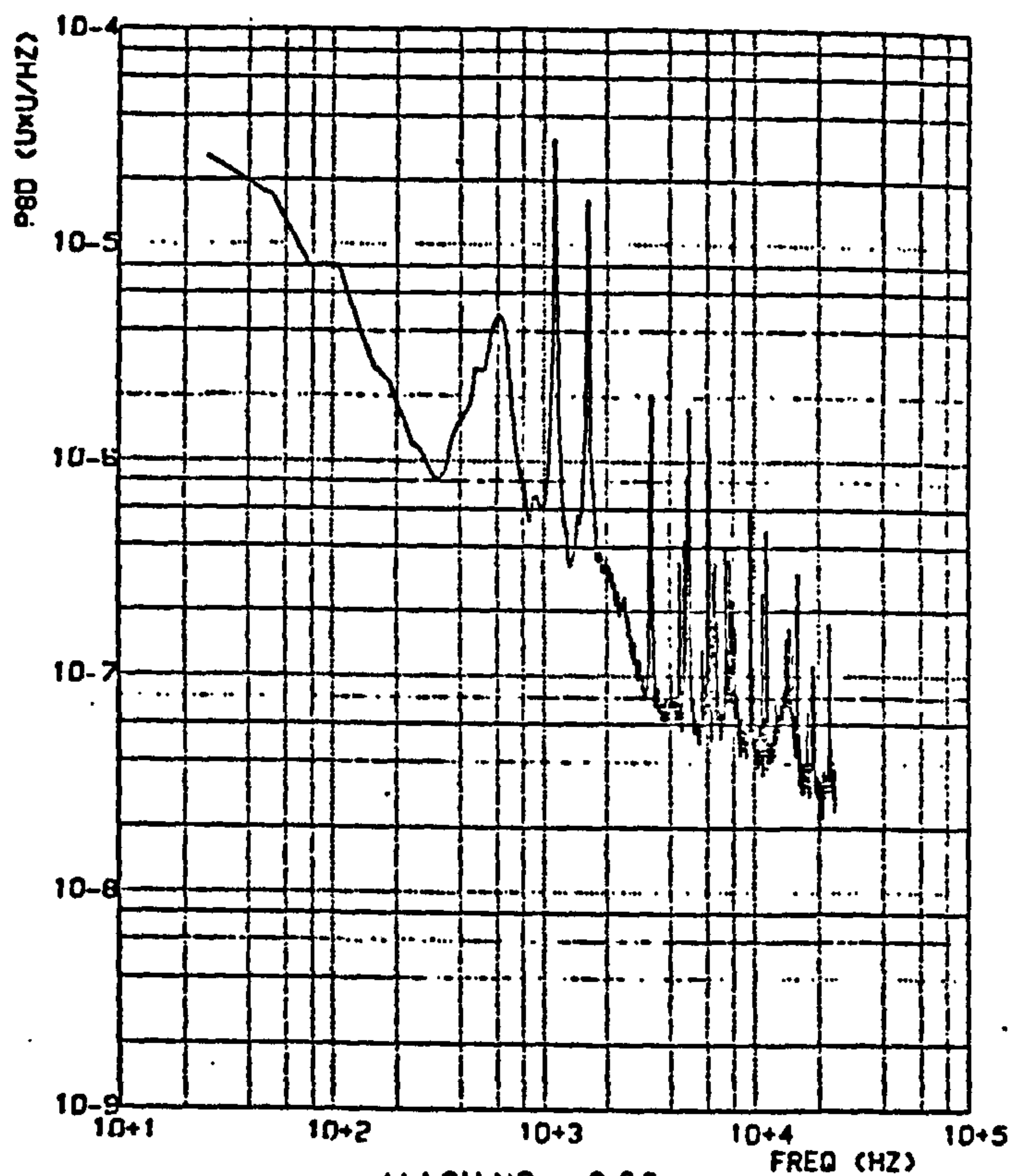
MACH NO. = 0.87
BANDWIDTH = 26.0417 HZ
MEAN TIME = 7.99 SECS
RMS VALUE = 3.316E-01 UNITS



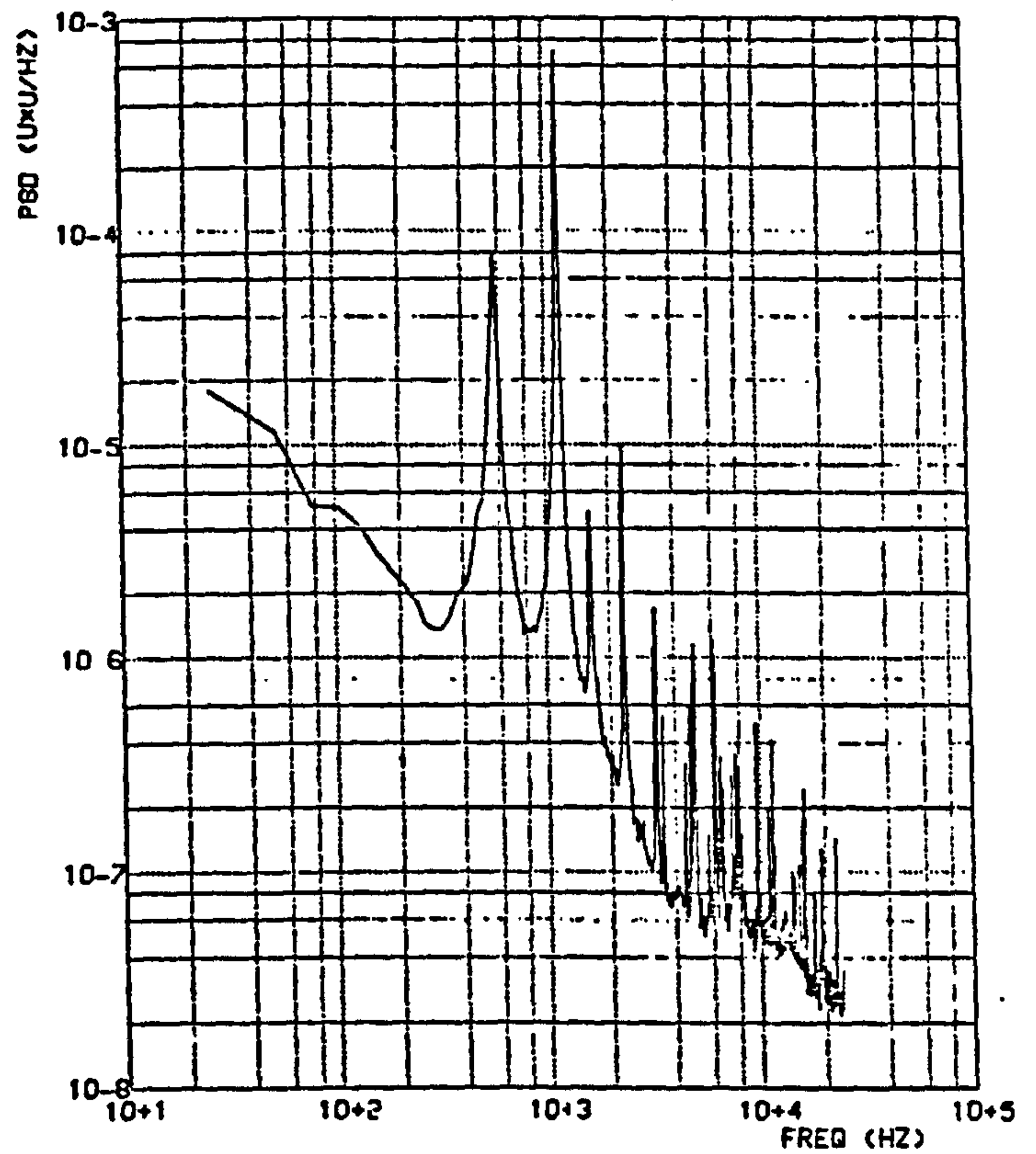
(c)

MACH NO. = 0.90
BANDWIDTH = 26.0417 HZ
MEAN TIME = 8.99 SECS
RMS VALUE = 2.237E-01 UNITS

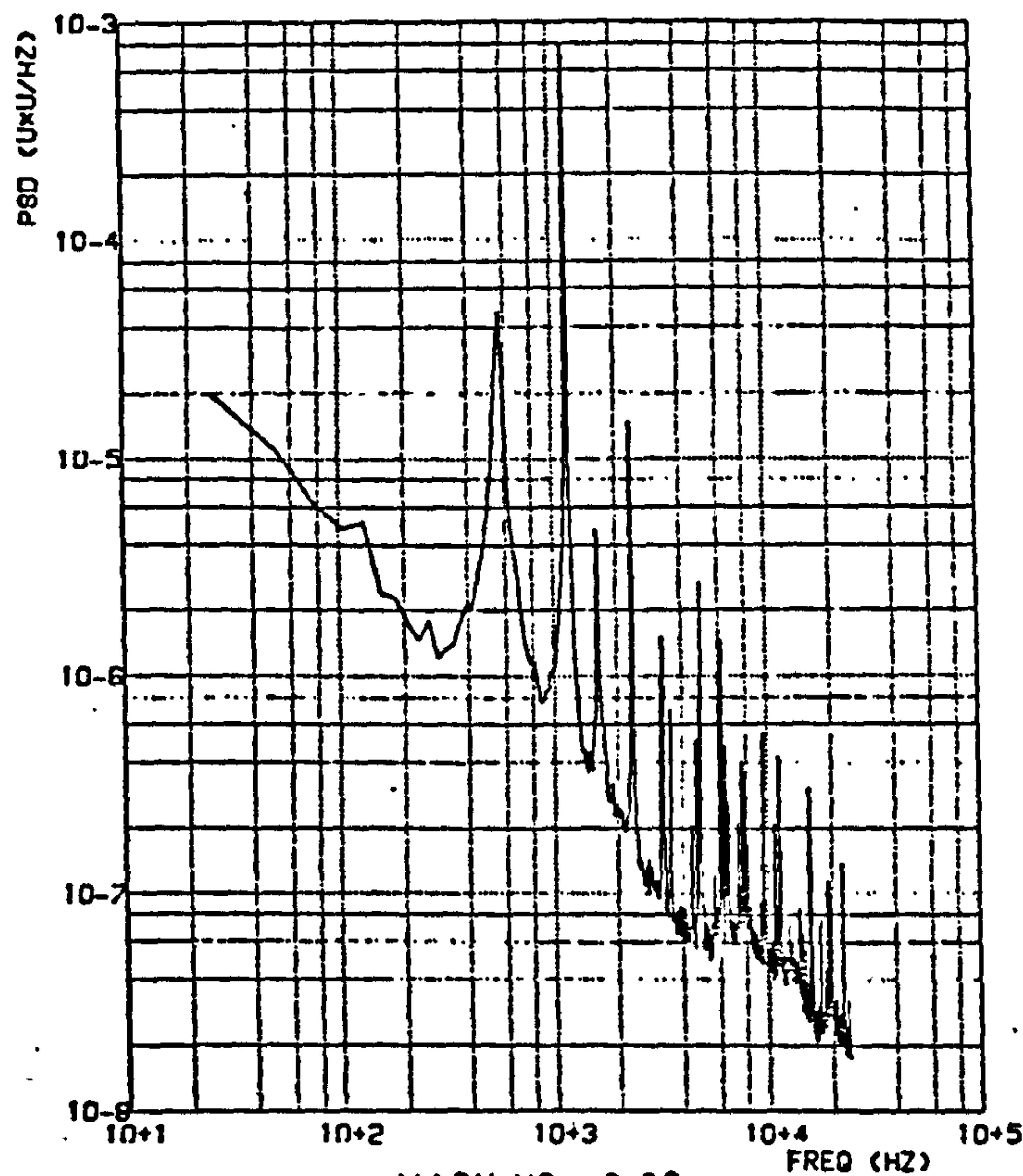
Figure 24. P. S. D. vs. frequency for flow on 14% biconvex aerofoil (free transition).



(a) MACH NO. = 0.86
BANDWIDTH = 26.0417 HZ
MEAN TIME = 7.99 SECS
RMS VALUF = 8.500E-02 UNITS

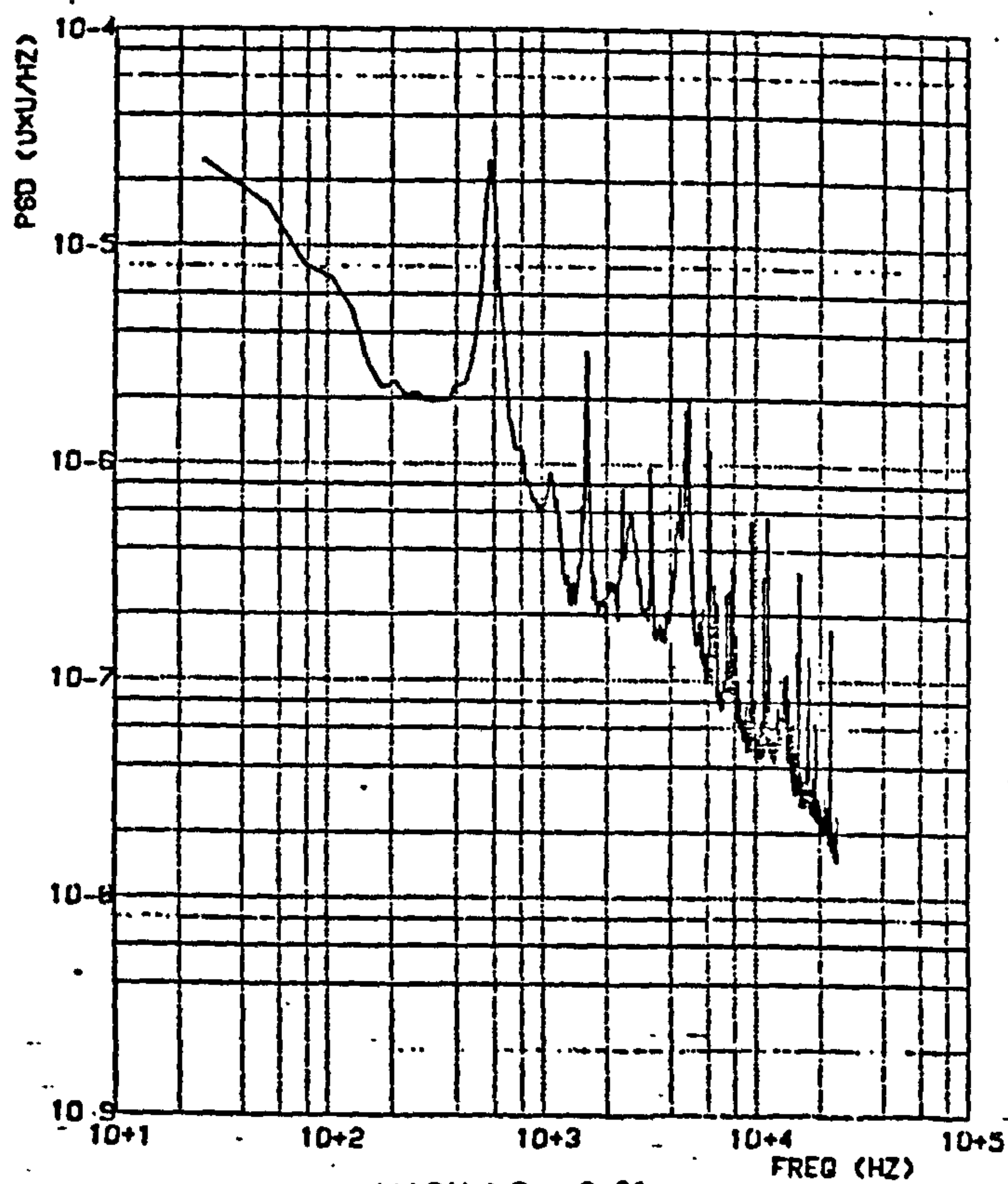


(b) MACH NO. = 0.88
BANDWIDTH = 26.0417 HZ
MEAN TIME = 9.98 SECS
RMS VALUF = 1.849E-01 UNITS

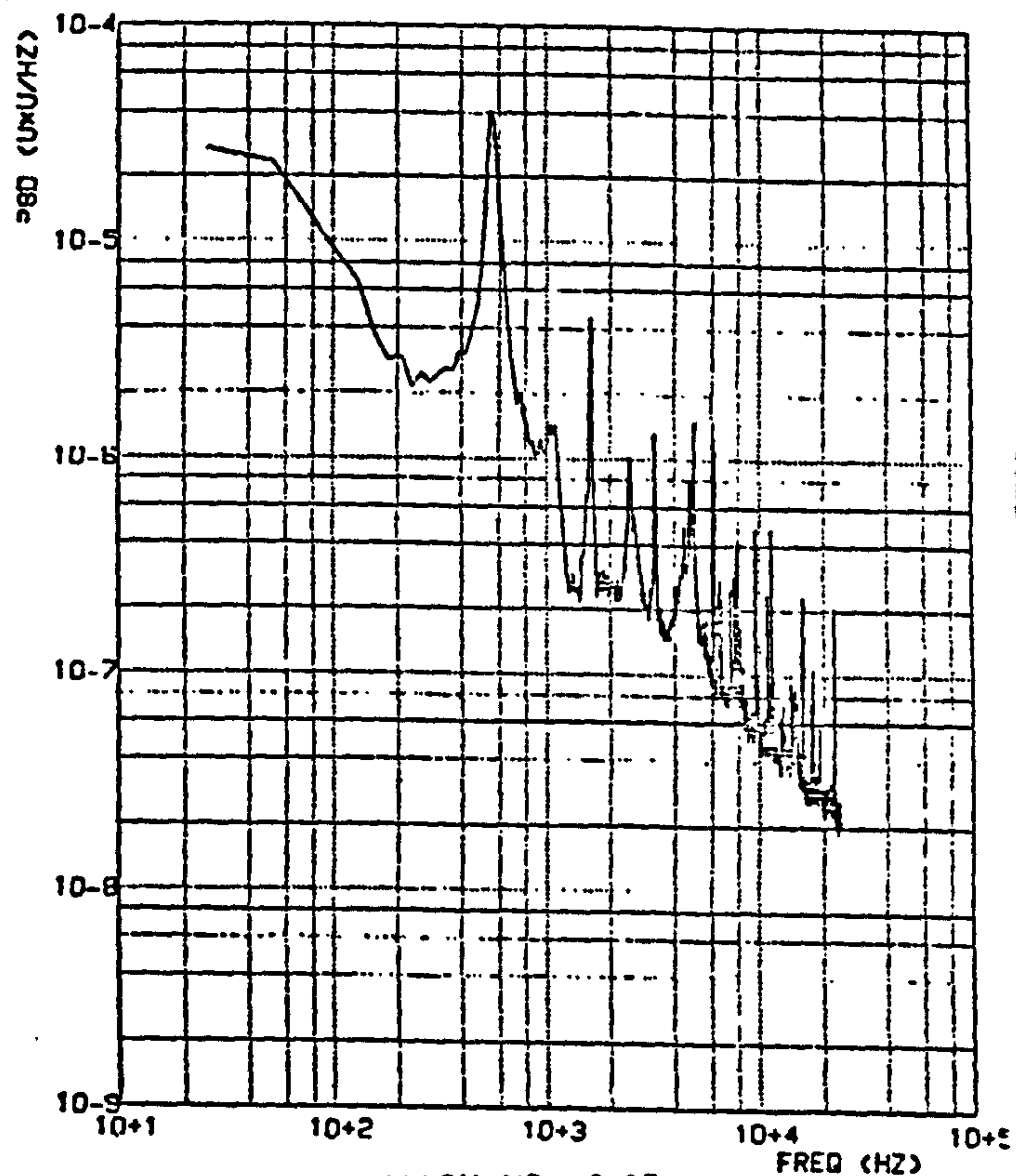


(c) MACH NO. = 0.90
BANDWIDTH = 26.0417 HZ
MEAN TIME = 7.99 SECS
RMS VALUF = 1.727E-01 UNITS

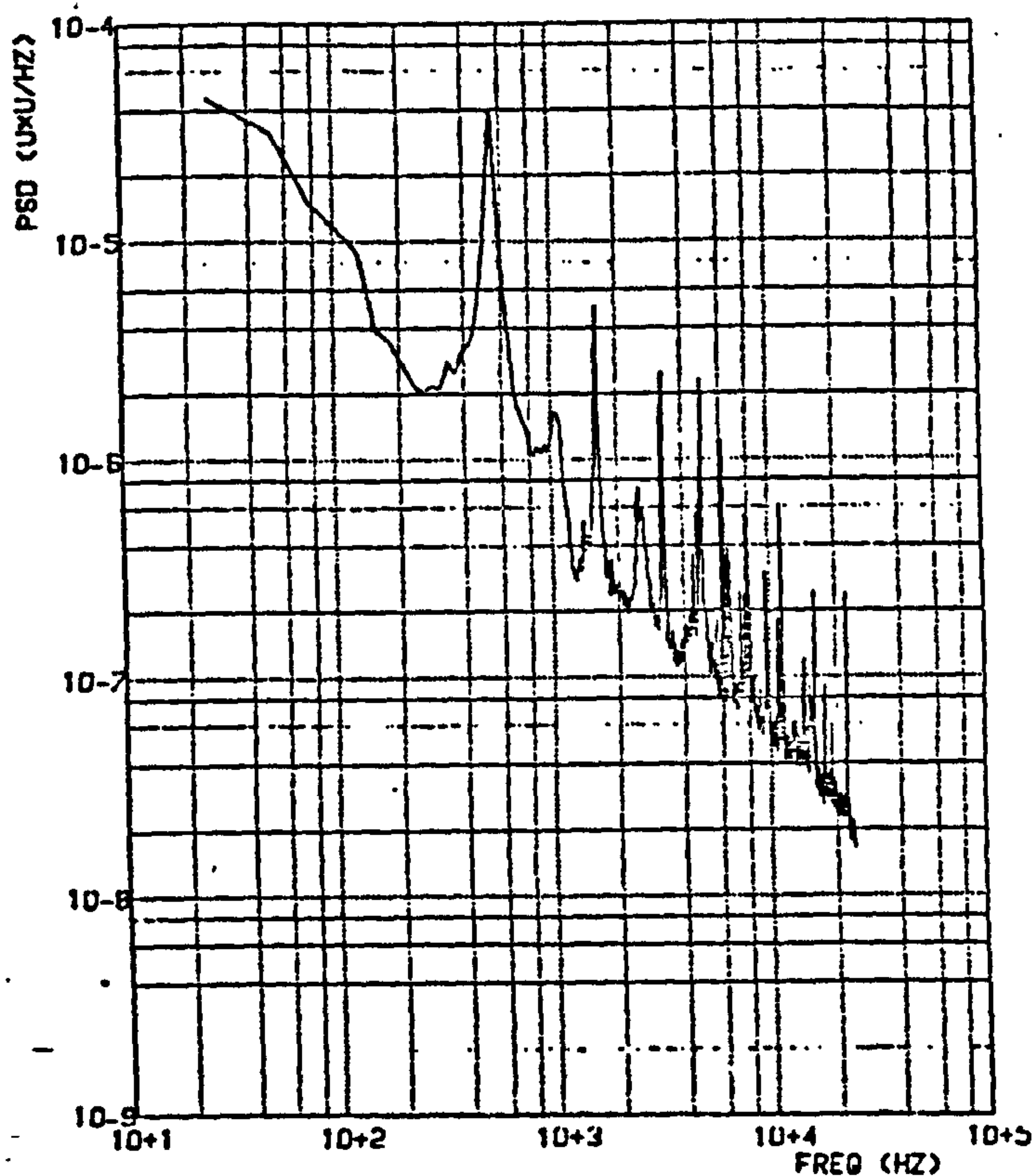
Figure 25. P. S. D vs. frequency for flow on 14% biconvex aerofoil (fixed transition).



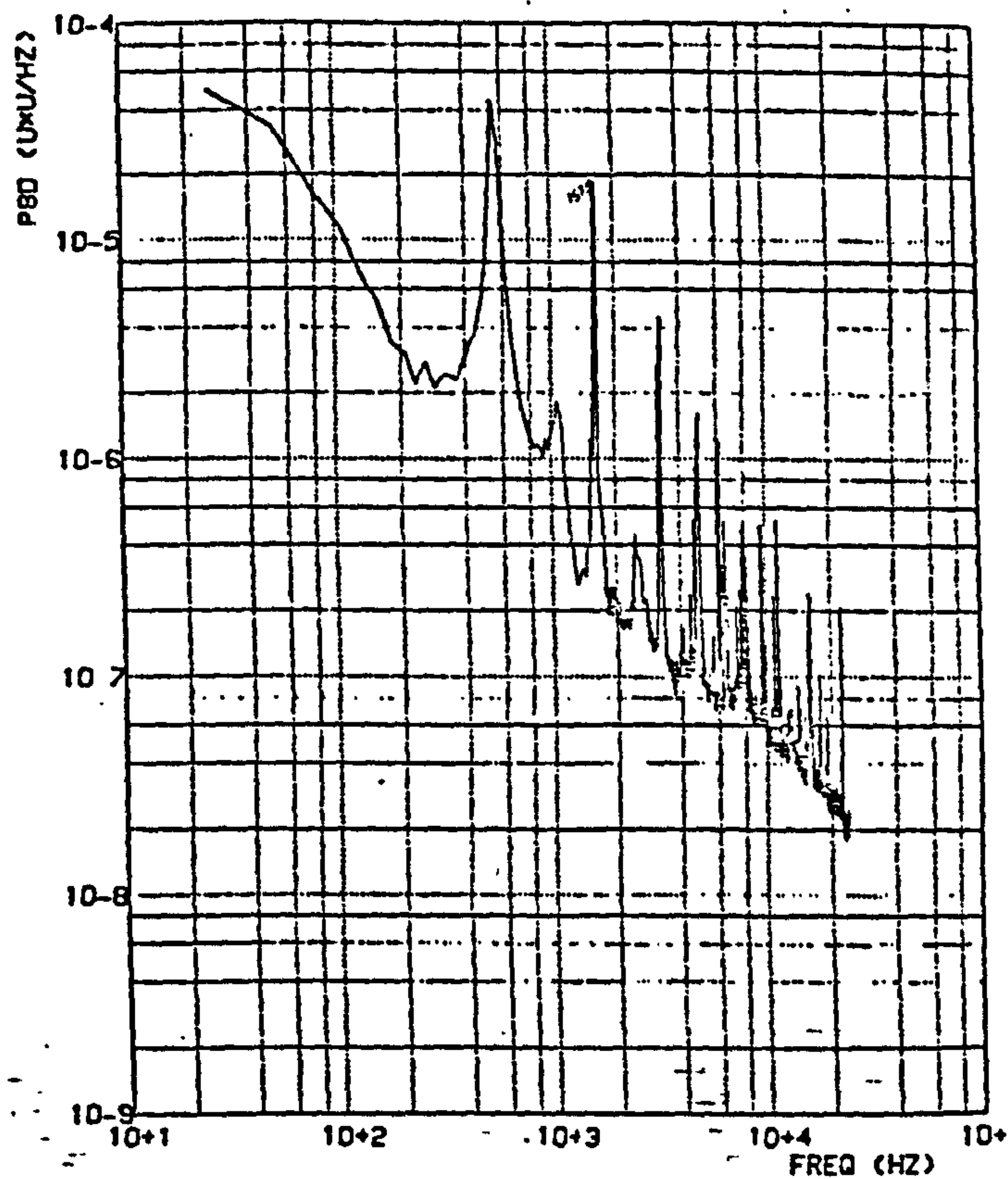
(a) MACH NO. = 0.81
BANDWIDTH = 26.0417 HZ
MEAN TIME = 8.99 SECS
RMS VALUE = 6.722E-02 UNITS



(b) MACH NO. = 0.85
BANDWIDTH = 26.0417 HZ
MEAN TIME = 8.99 SECS
RMS VALUE = 9.815E-02 UNITS



(c) MACH NO. = 0.88
BANDWIDTH = 26.0417 HZ
MEAN TIME = 9.98 SECS
RMS VALUE = 1.002E-01 UNITS



(d) MACH NO. = 0.91
BANDWIDTH = 26.0417 HZ
MEAN TIME = 8.99 SECS
RMS VALUE = 1.052E-01 UNITS

Figure 26. P. S. D vs. frequency for the empty tunnel with perforations sealed.

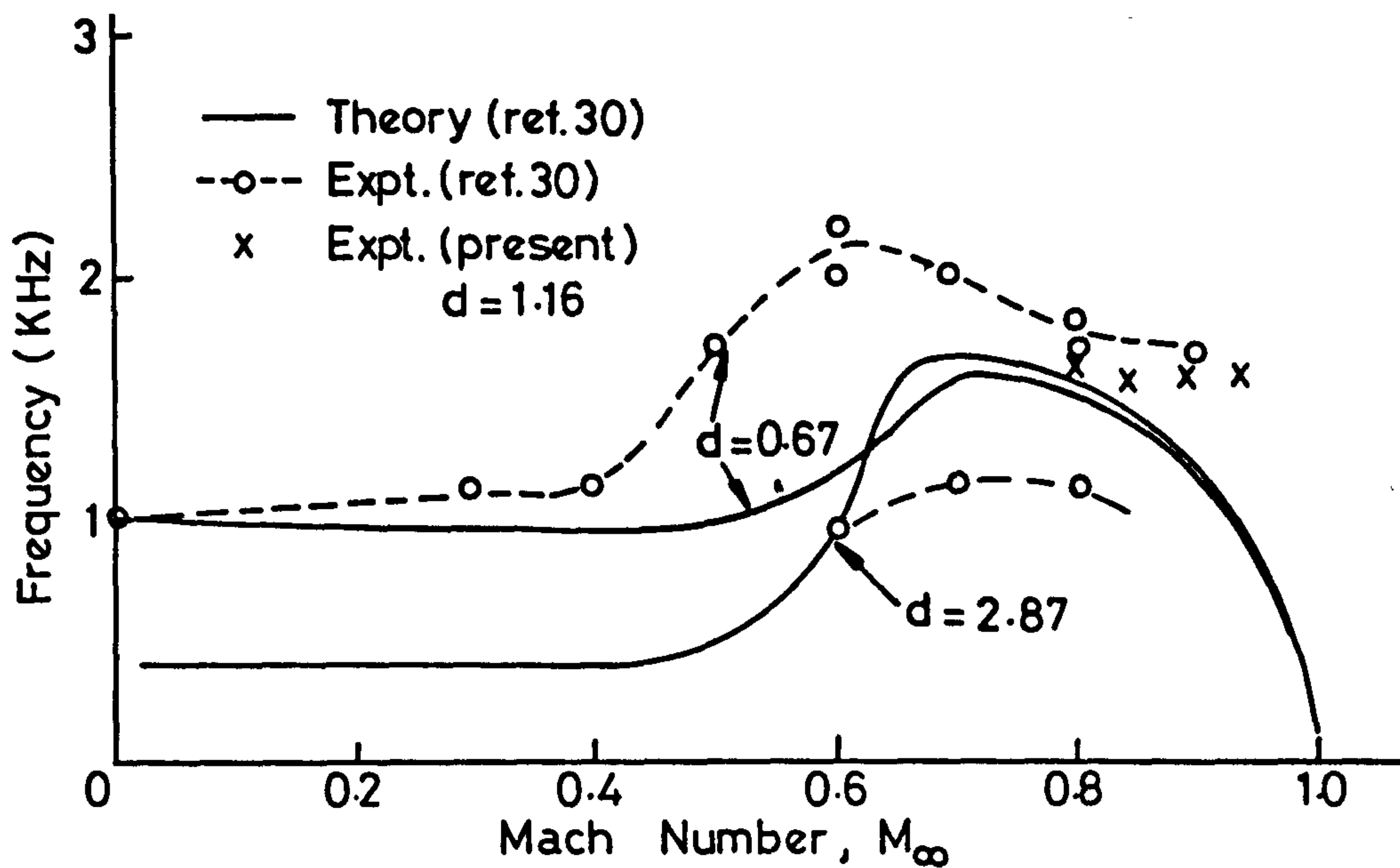


Figure 27. Resonance frequencies.

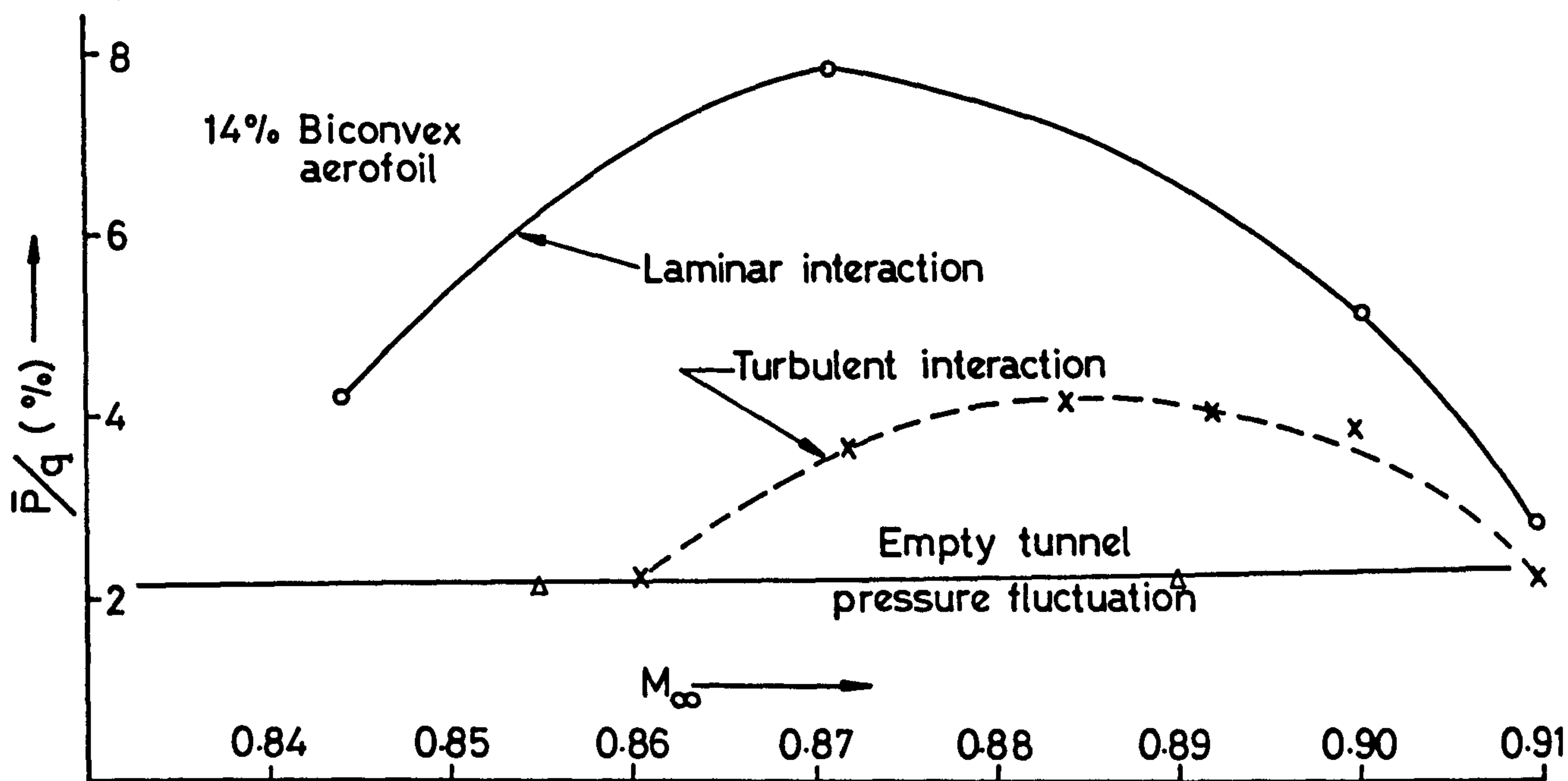


Figure 28. Magnitude of sidewall pressure fluctuations.

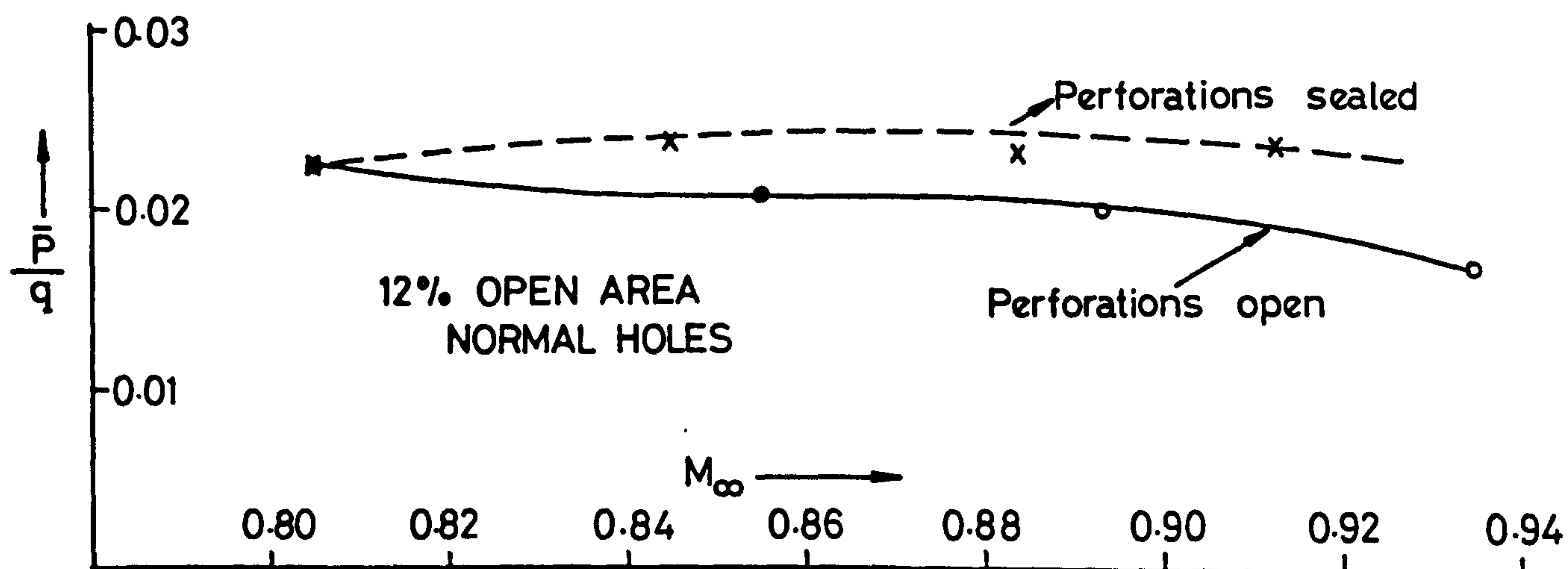
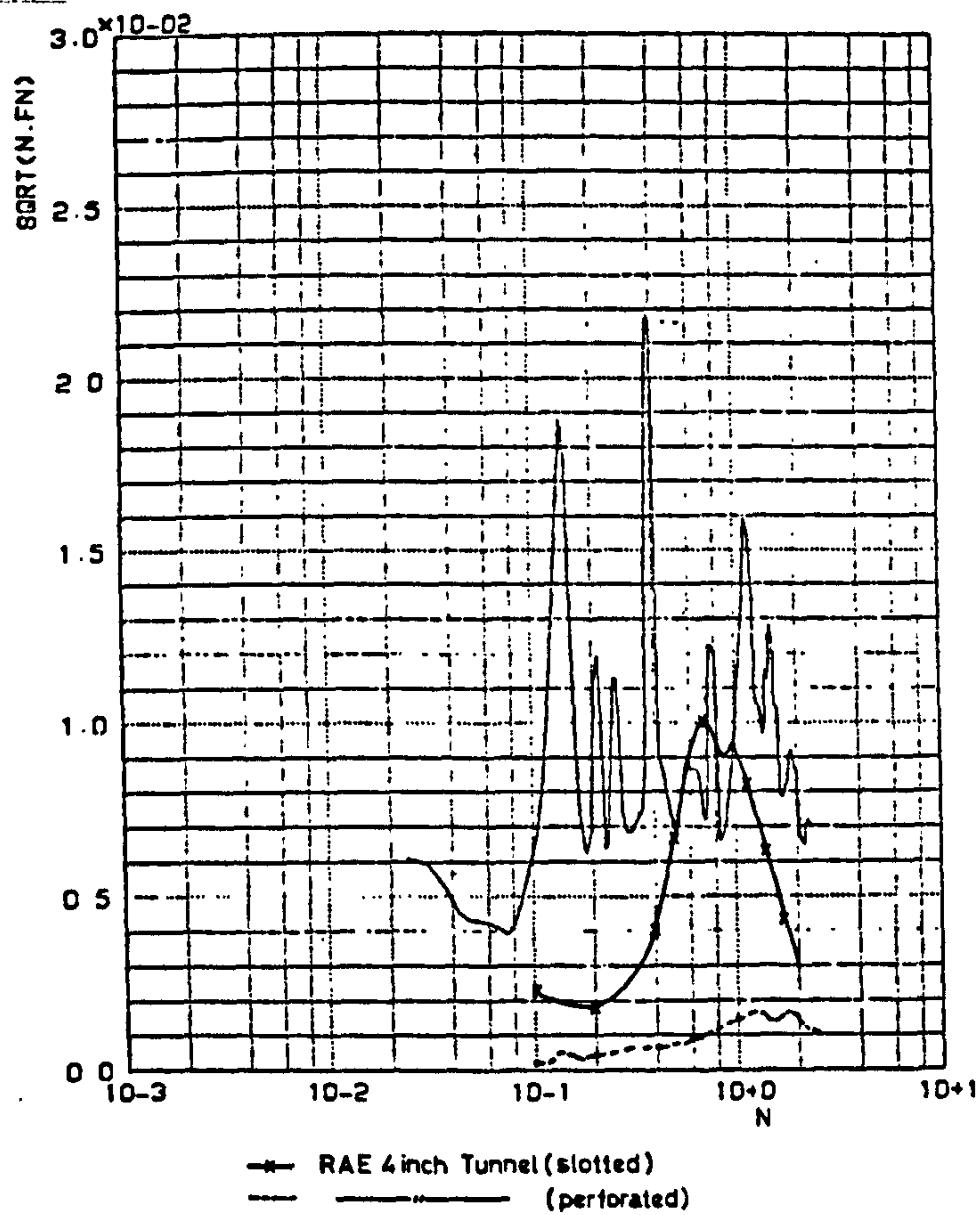
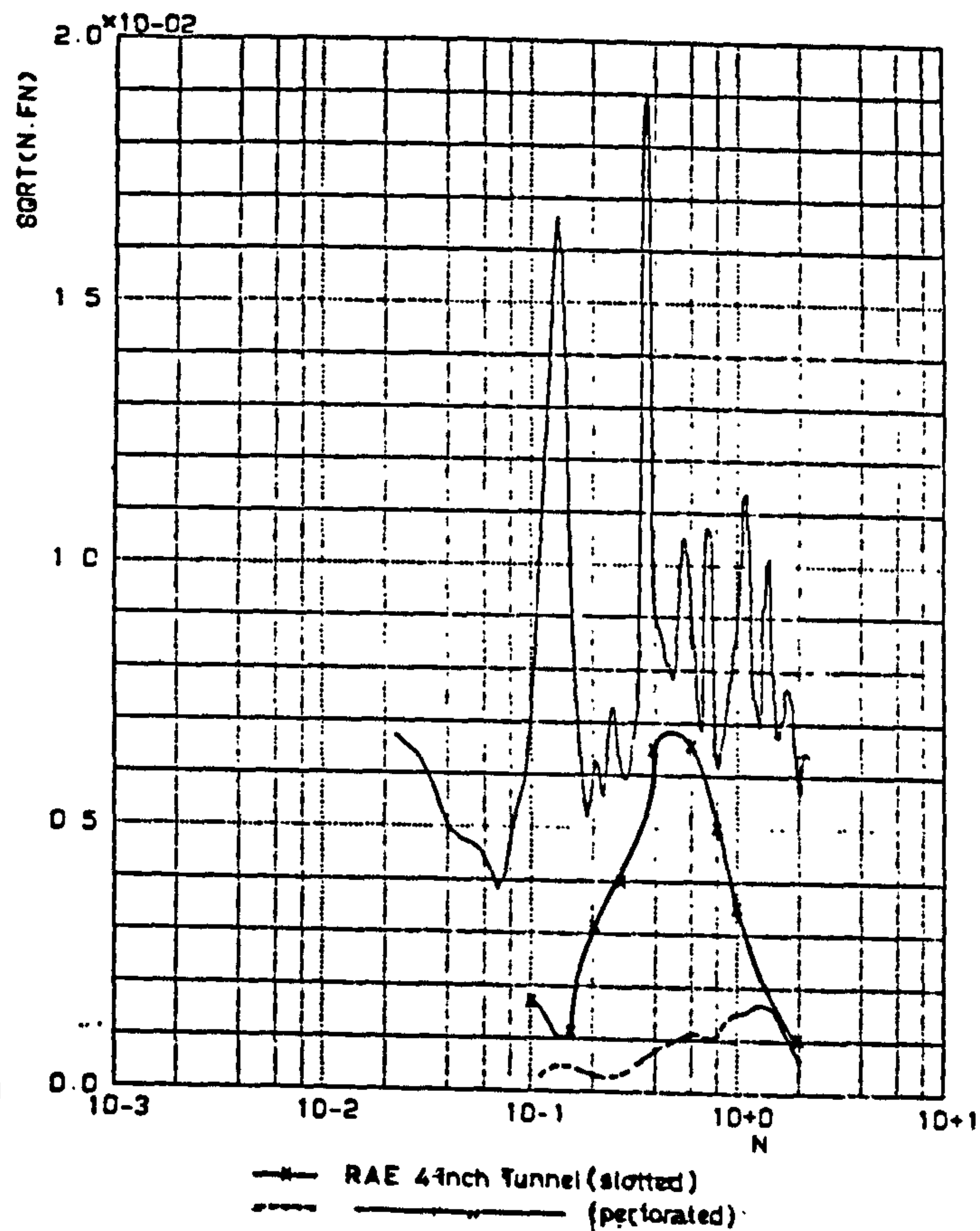


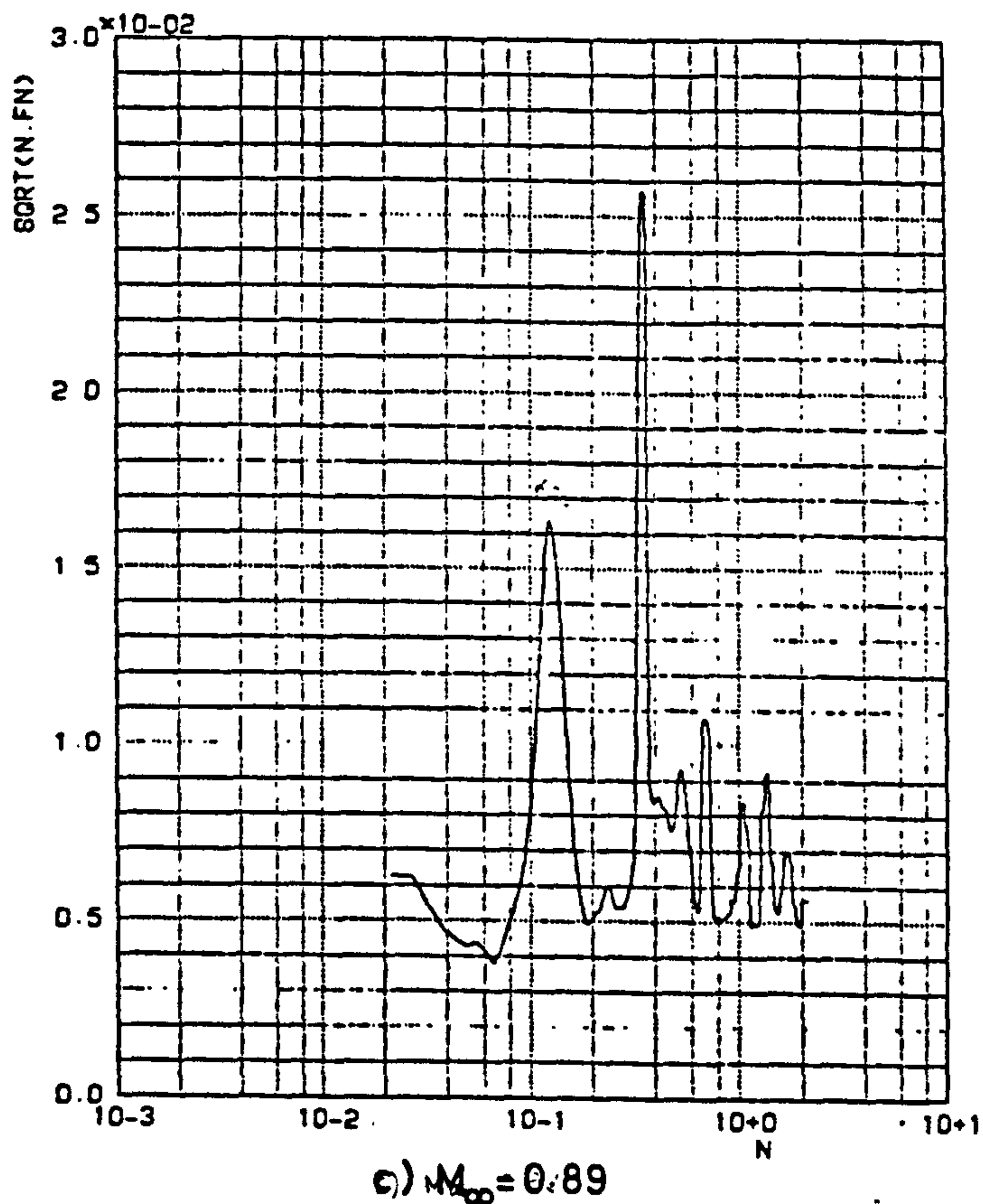
Figure 29. 2.5x2.5in perforated tunnel - r.m.s pressure fluctuations



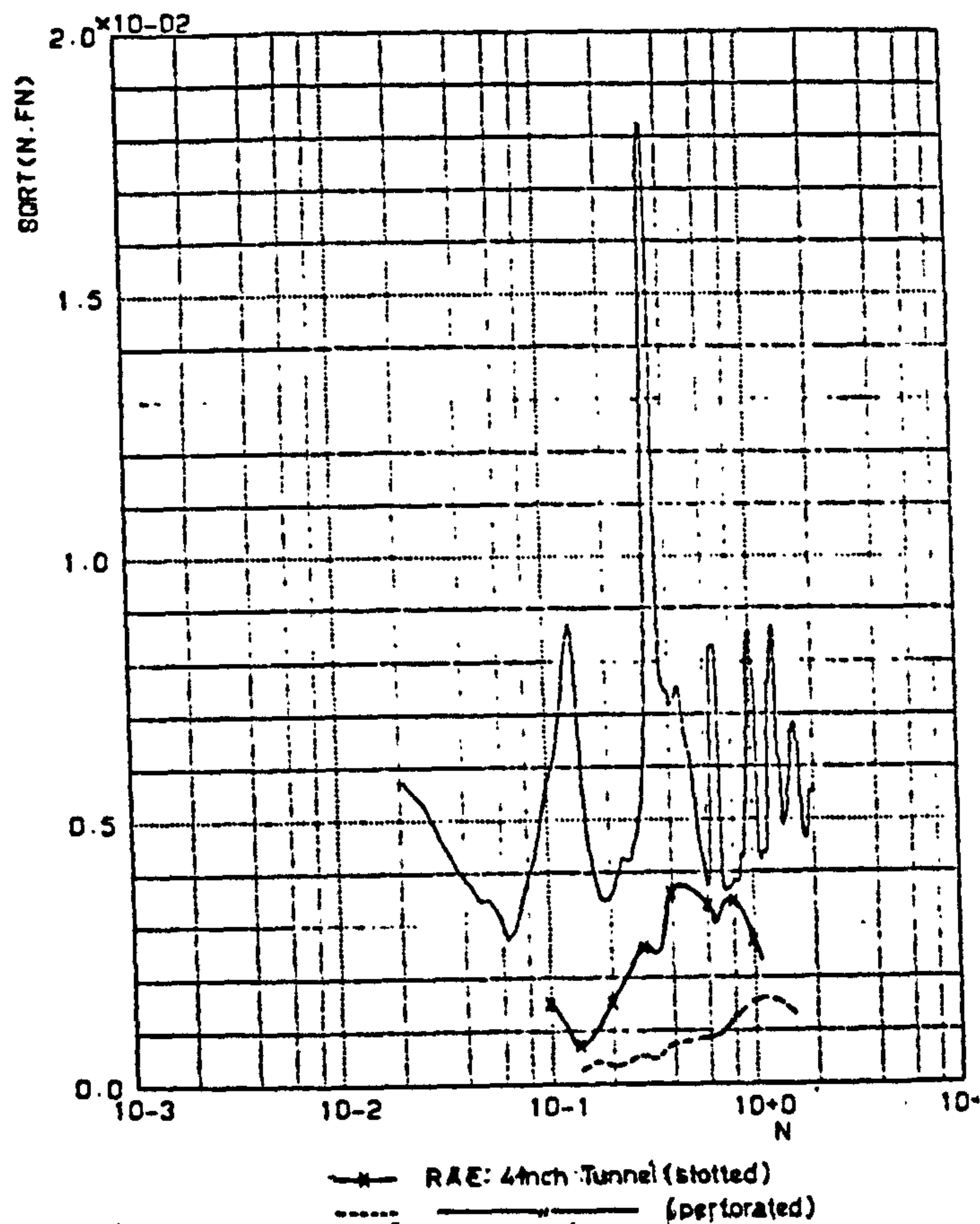
a) $M_{\infty} = 0.81$



b) $M_{\infty} = 0.86$



c) $M_{\infty} = 0.89$



d) $M_{\infty} = 0.94$

Figure 30. Tunnel unsteadiness, $\sqrt{N F(N)}$ vs. $\log_e N$
 (section 3.2.7)

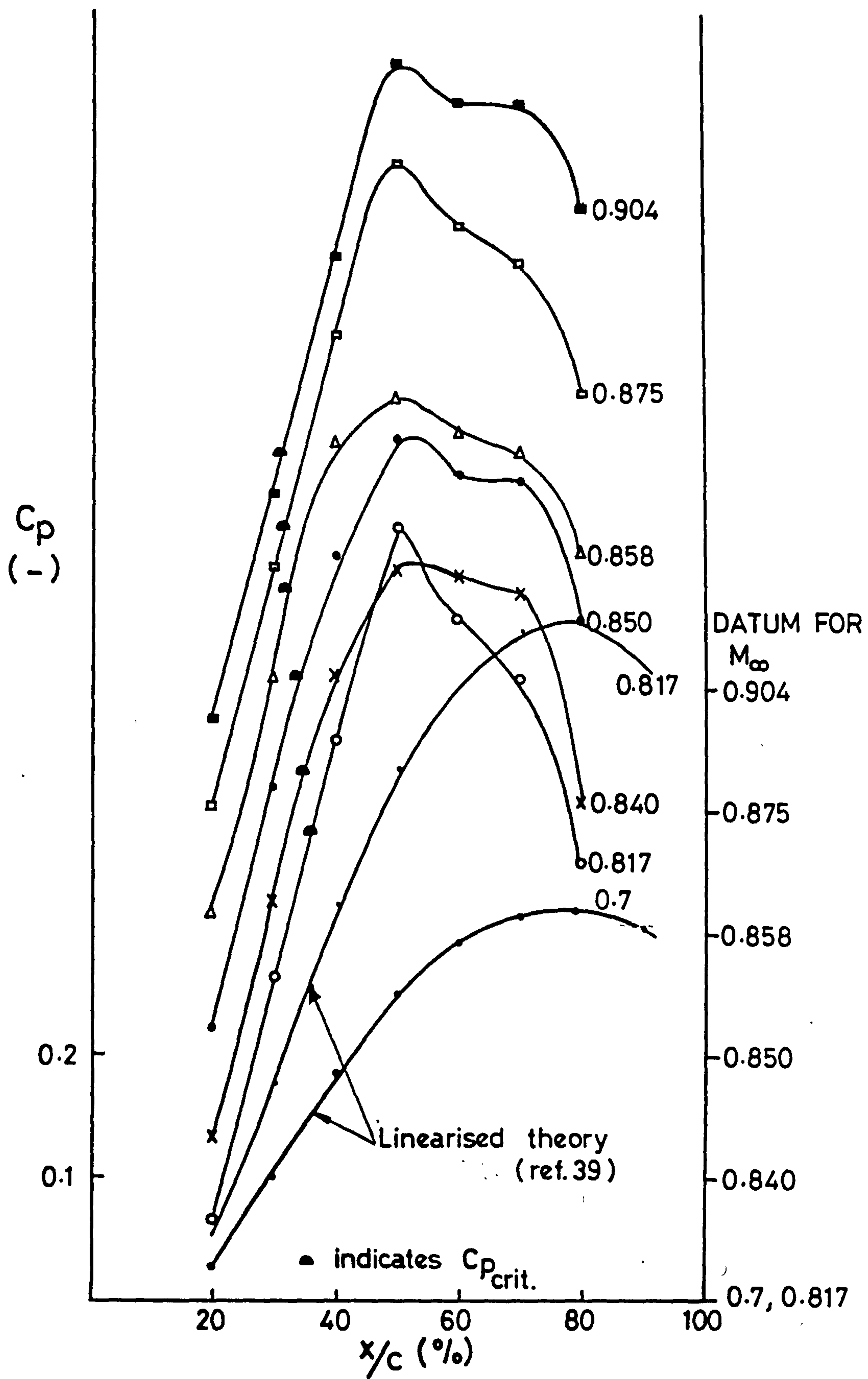


Figure 31. Pressure distribution on 14% biconvex aerofoil, free transition.

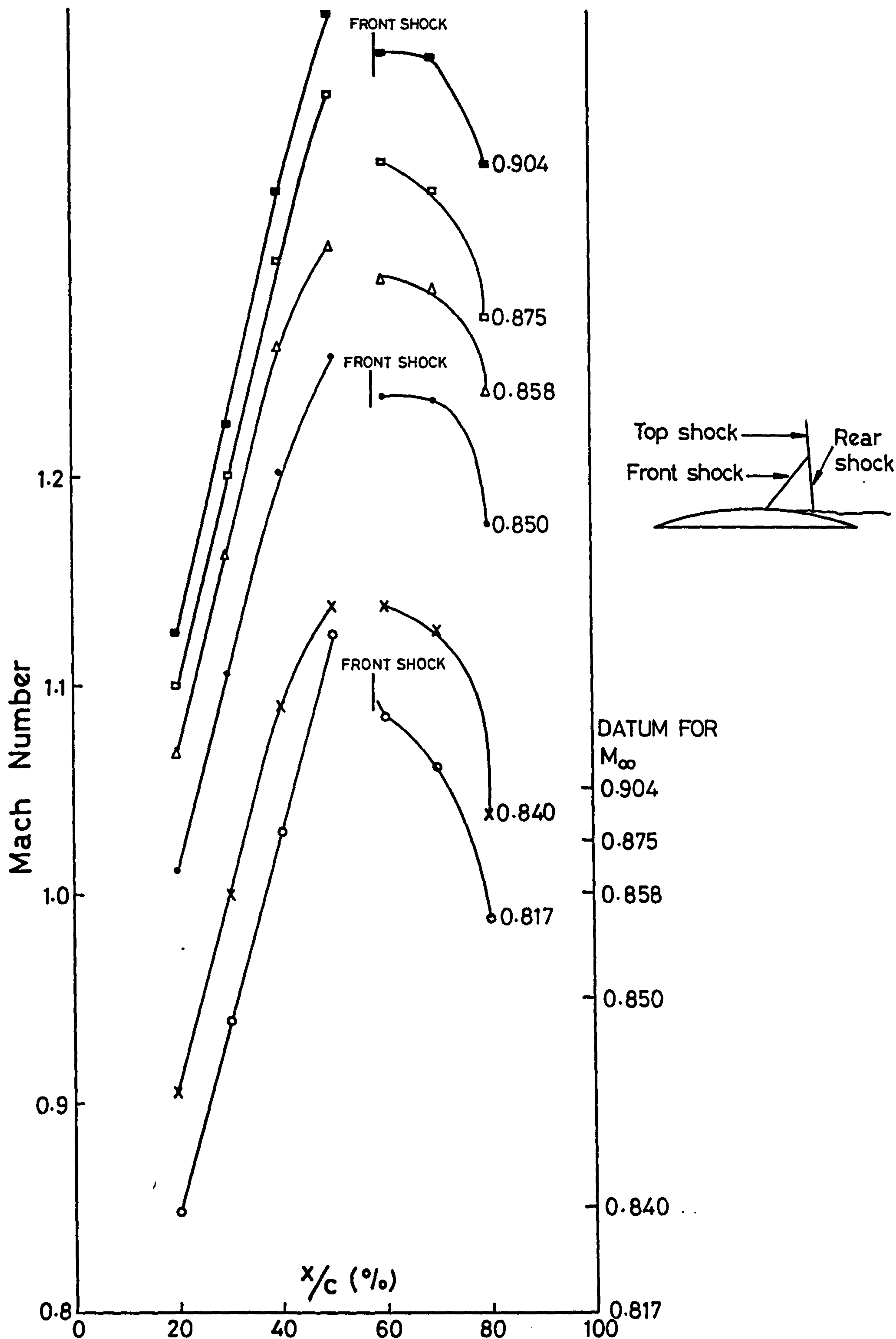


Figure 32. Local Mach No. distribution on 14% biconvex aerofoil, free transition.

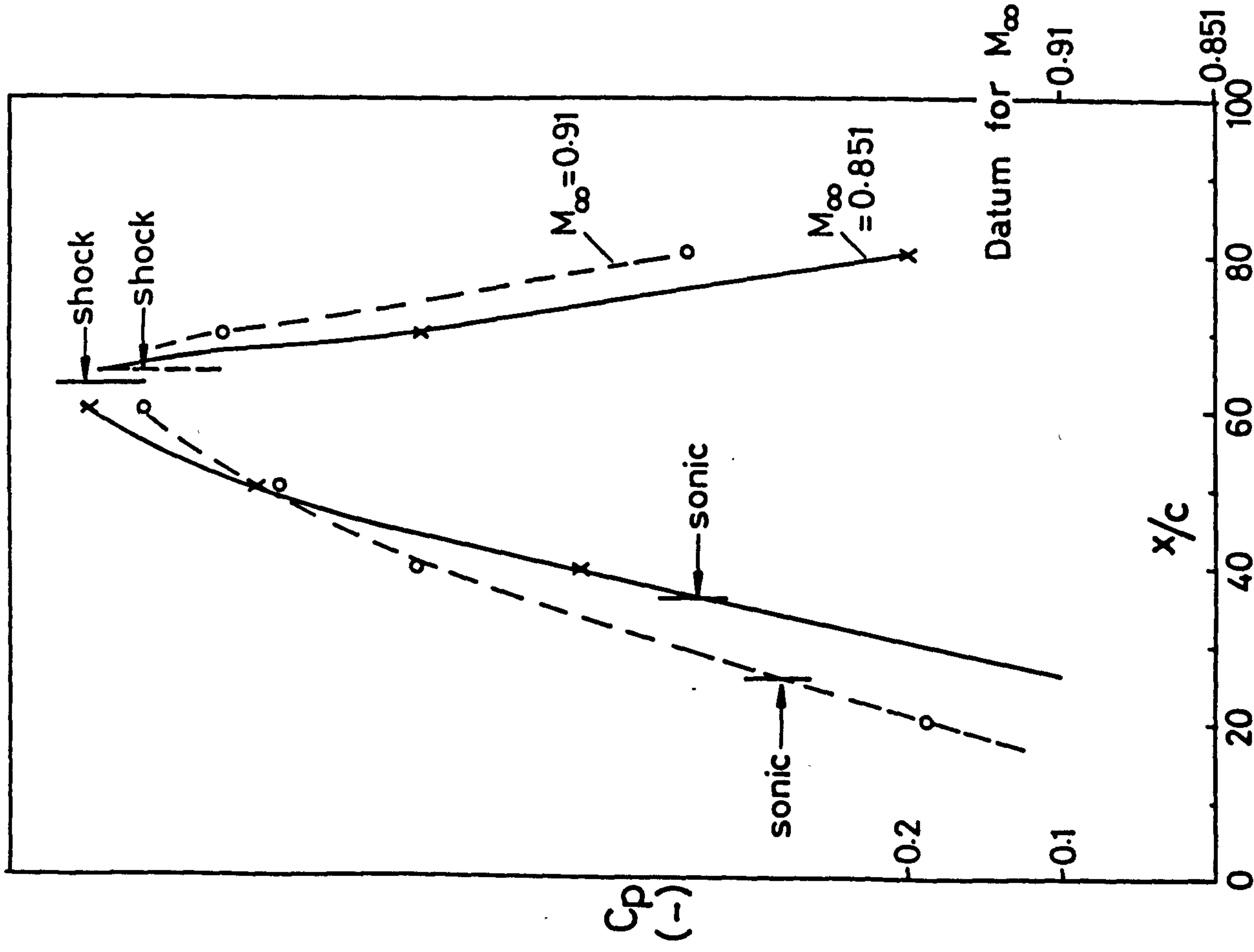


Figure 33. Pressure distribution on 14% biconvex aerofoil (fixed transition)

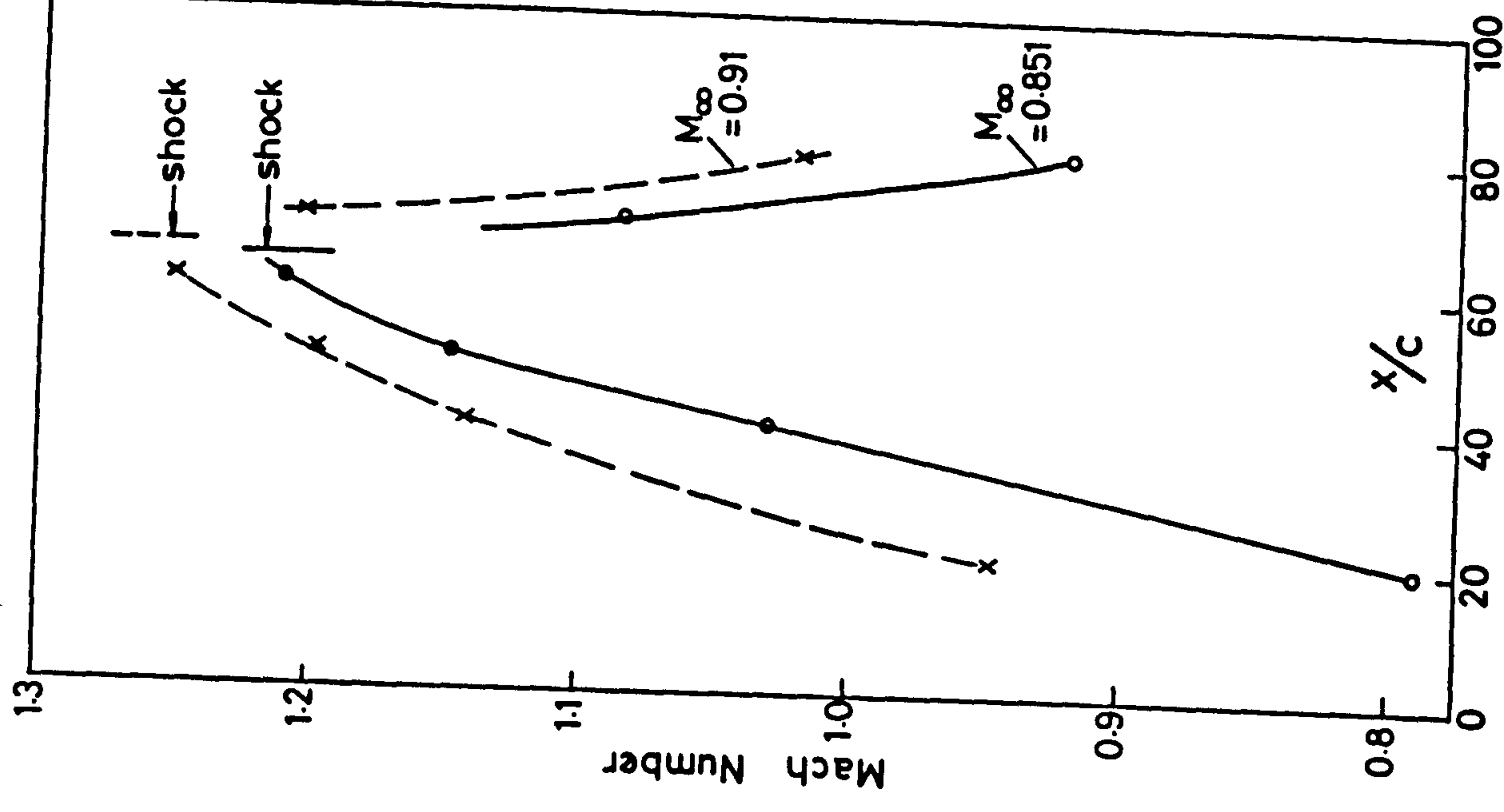
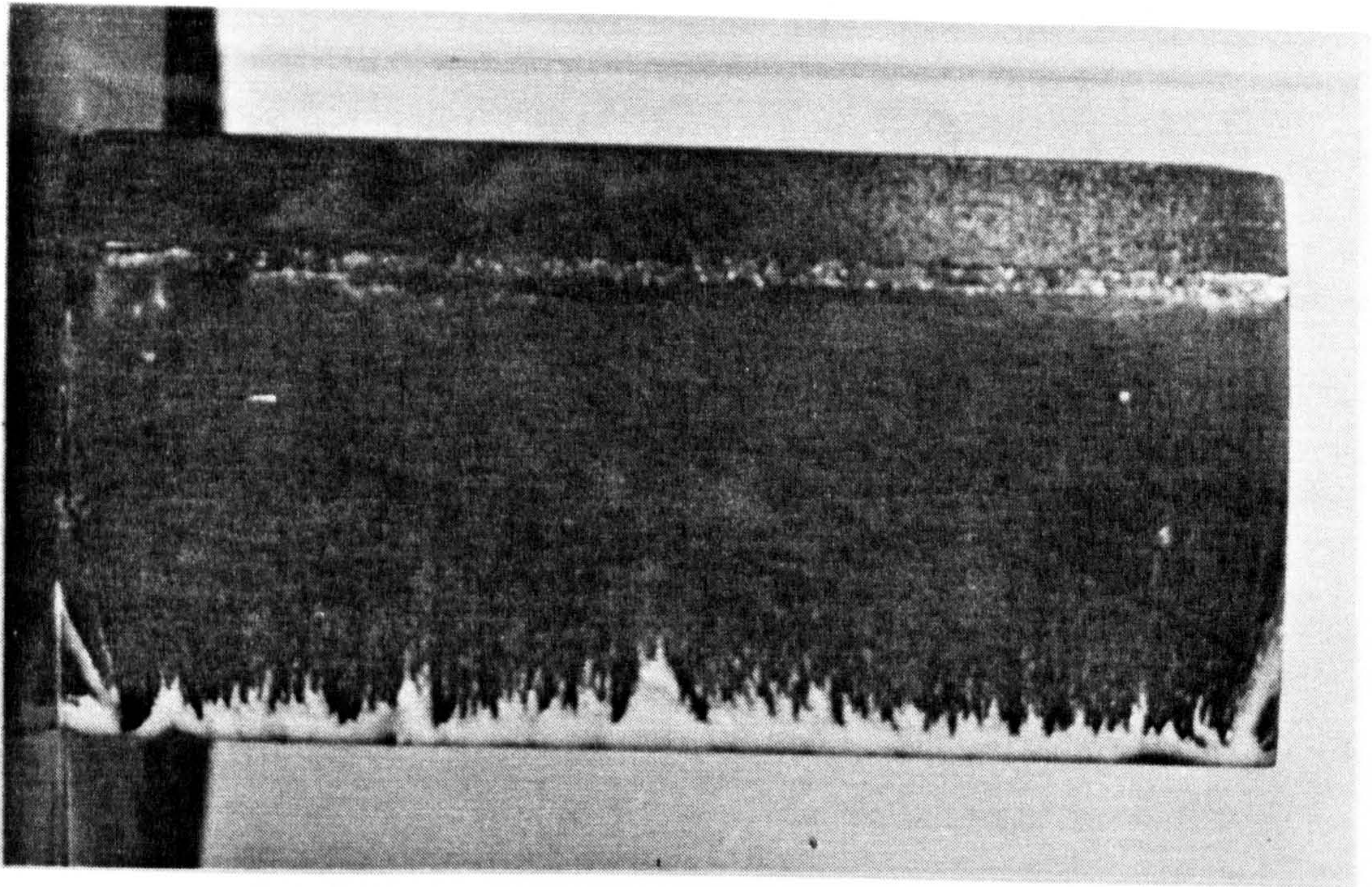
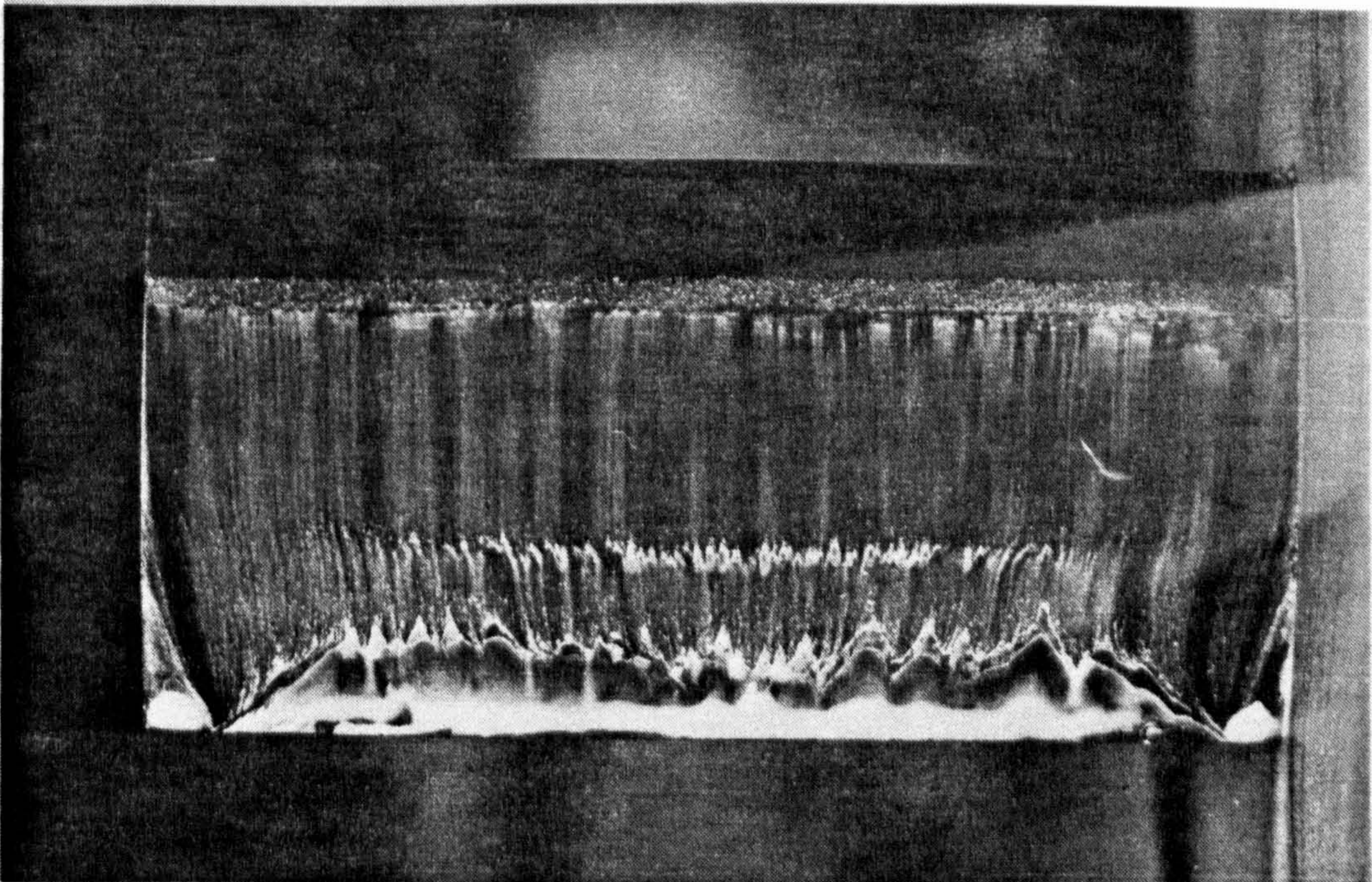


Figure 34. Local Mach No. distribution on 14% biconvex aerofoil (fixed transition)



a) $M_{\infty} = 0.80$



b) $M_{\infty} = 0.85$

Figure 35. Oil flow photographs on 14% biconvex aerofoil, $\alpha = 0^\circ$, fixed transition.

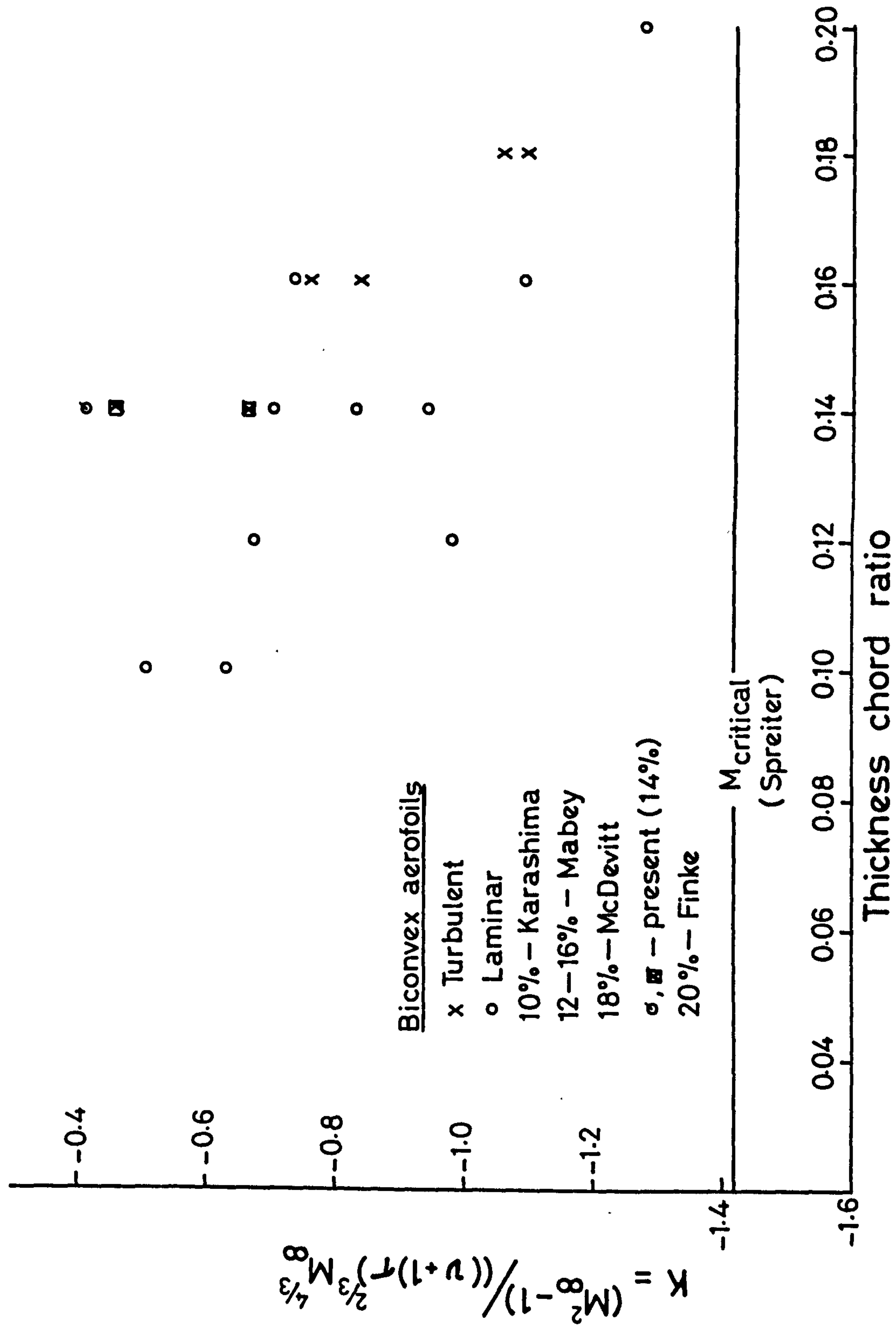


Figure 36. Thickness chord ratio vs transonic similarity parameter.

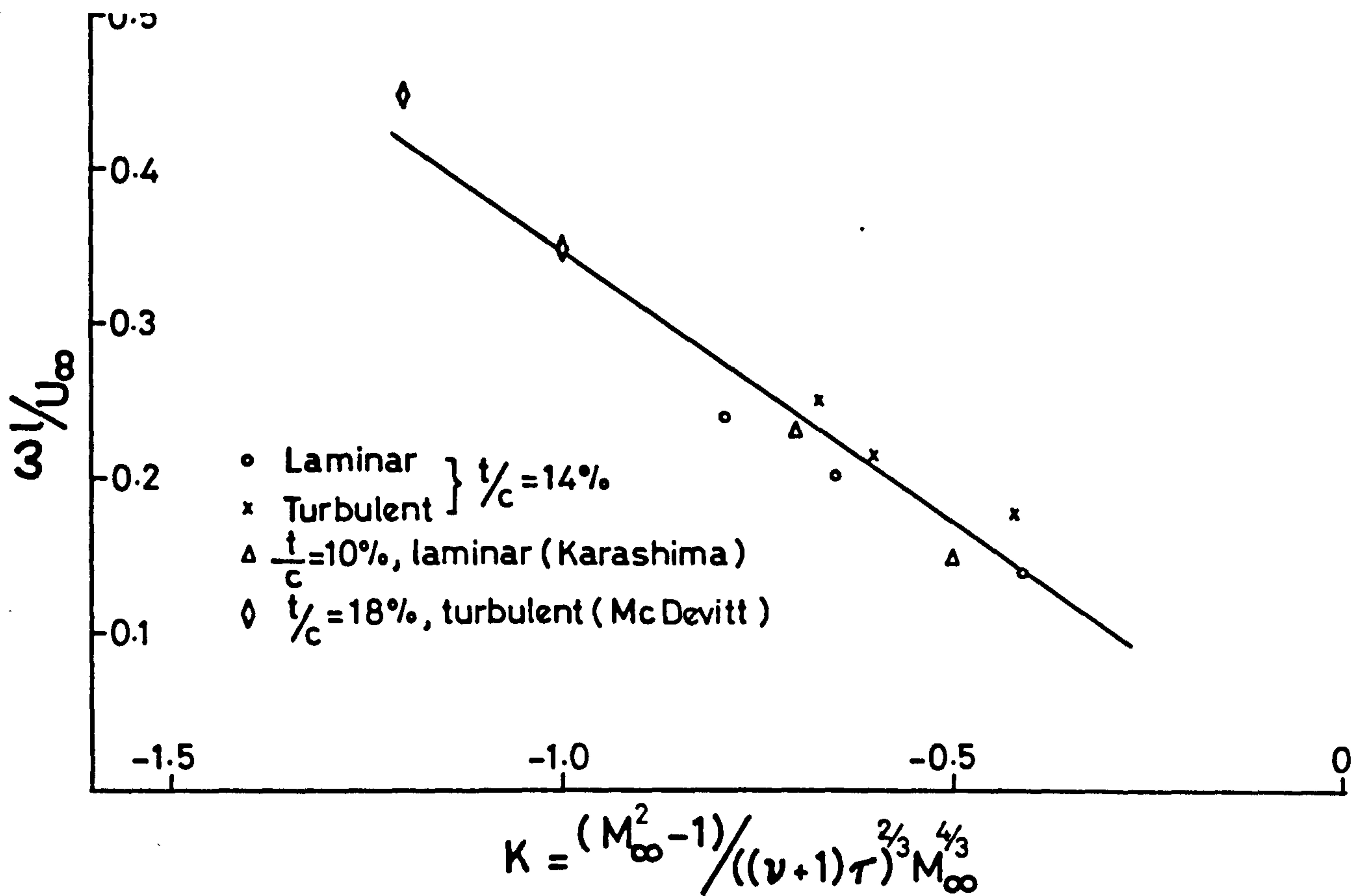


Figure 37. Frequency parameter vs. transonic similarity parameter.

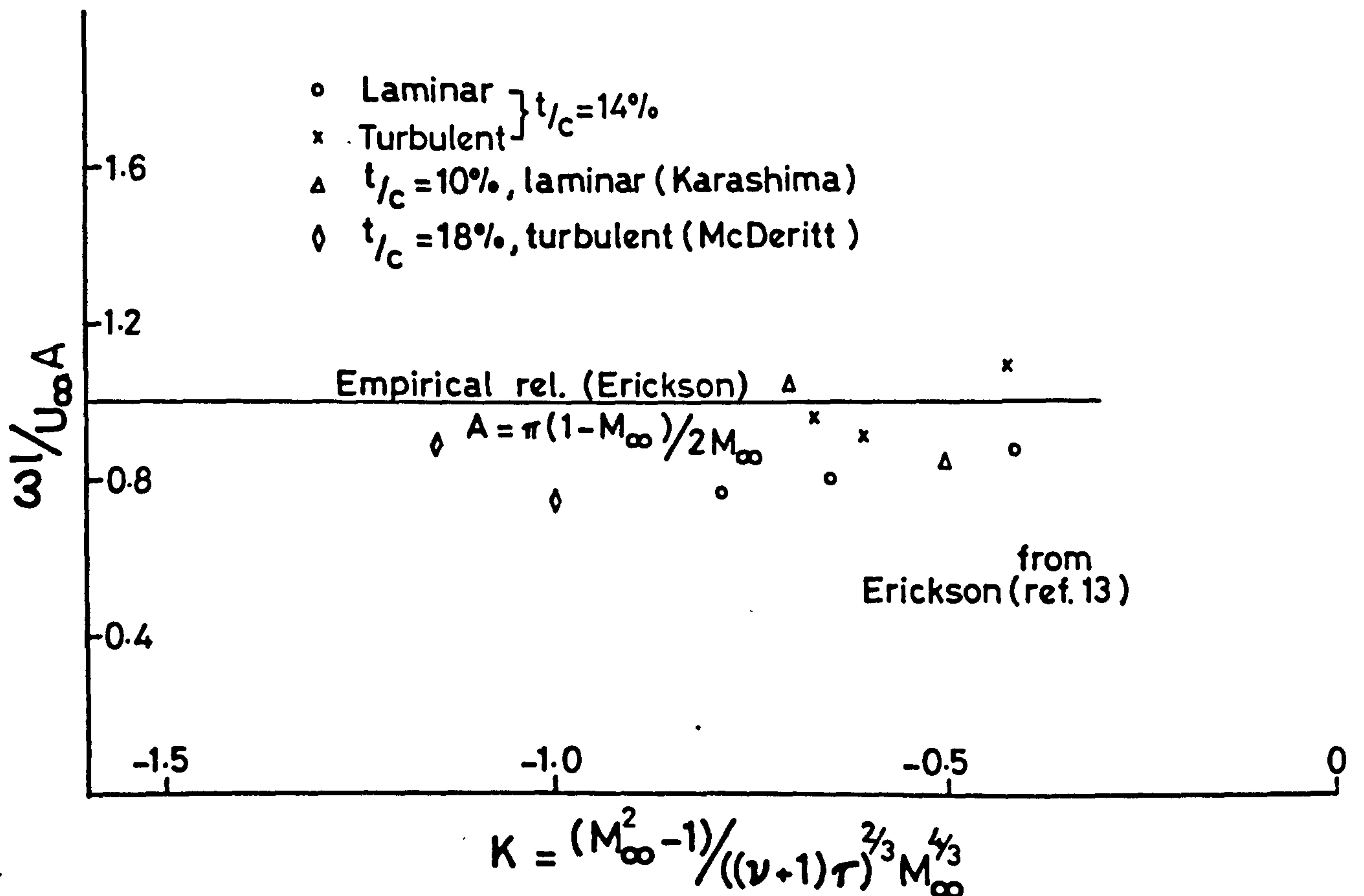
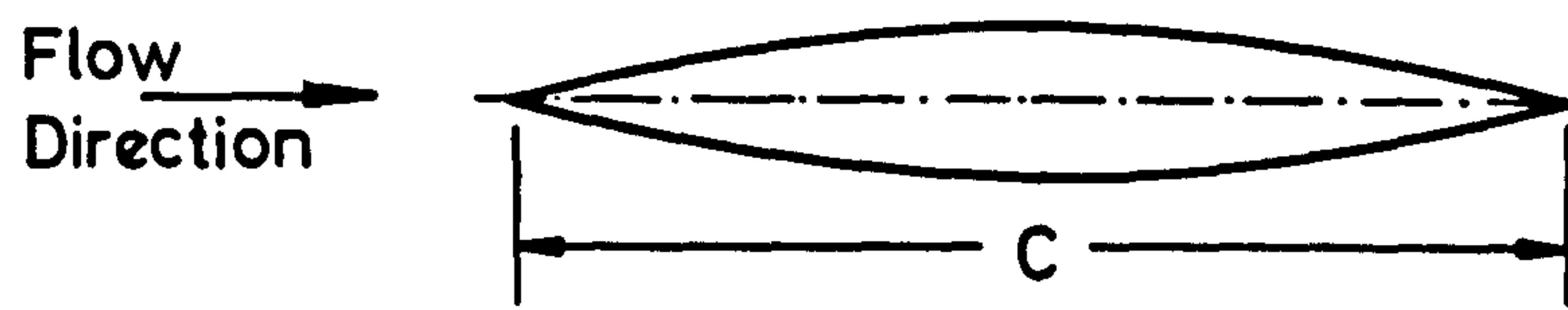
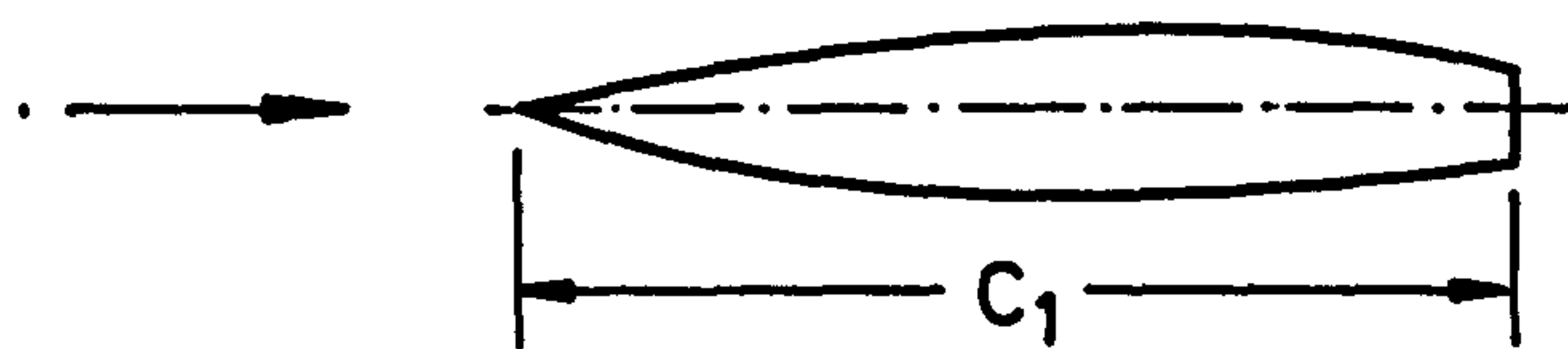


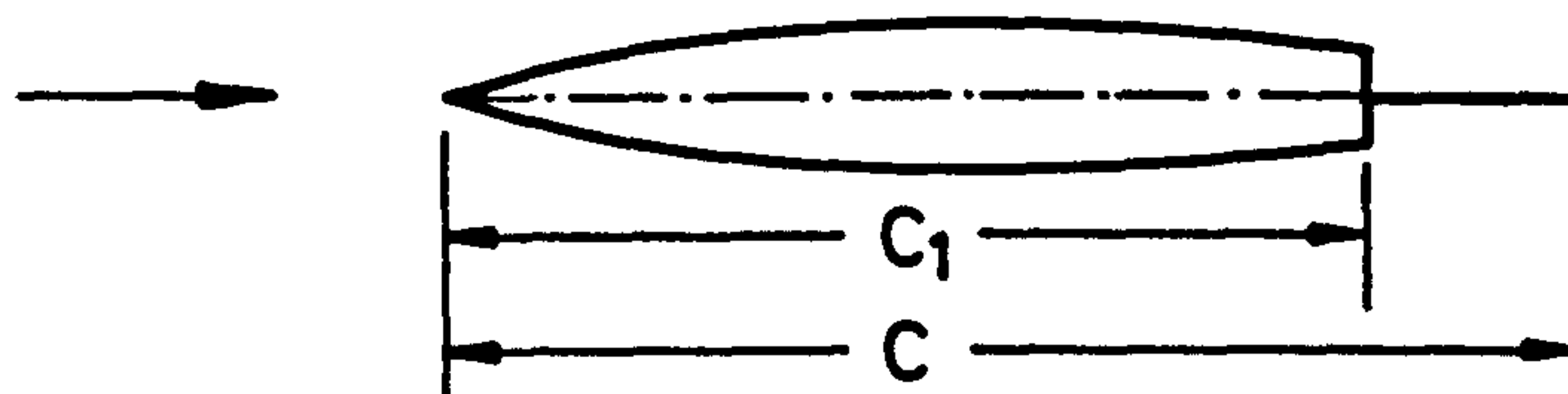
Figure 38. Modified frequency parameter vs transonic similarity parameter.



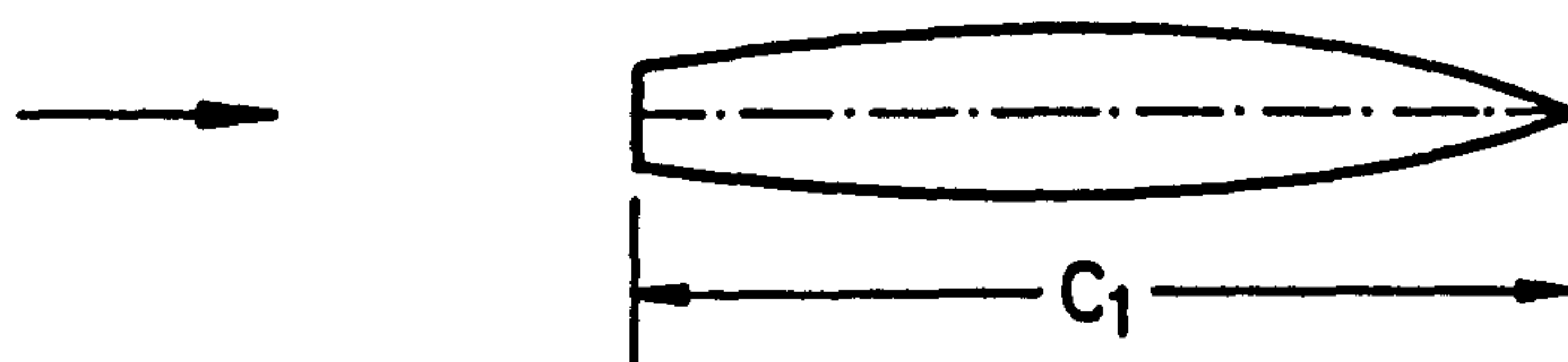
FULL AEROFOIL



AEROFOIL WITH BLUNT TRAILING EDGE

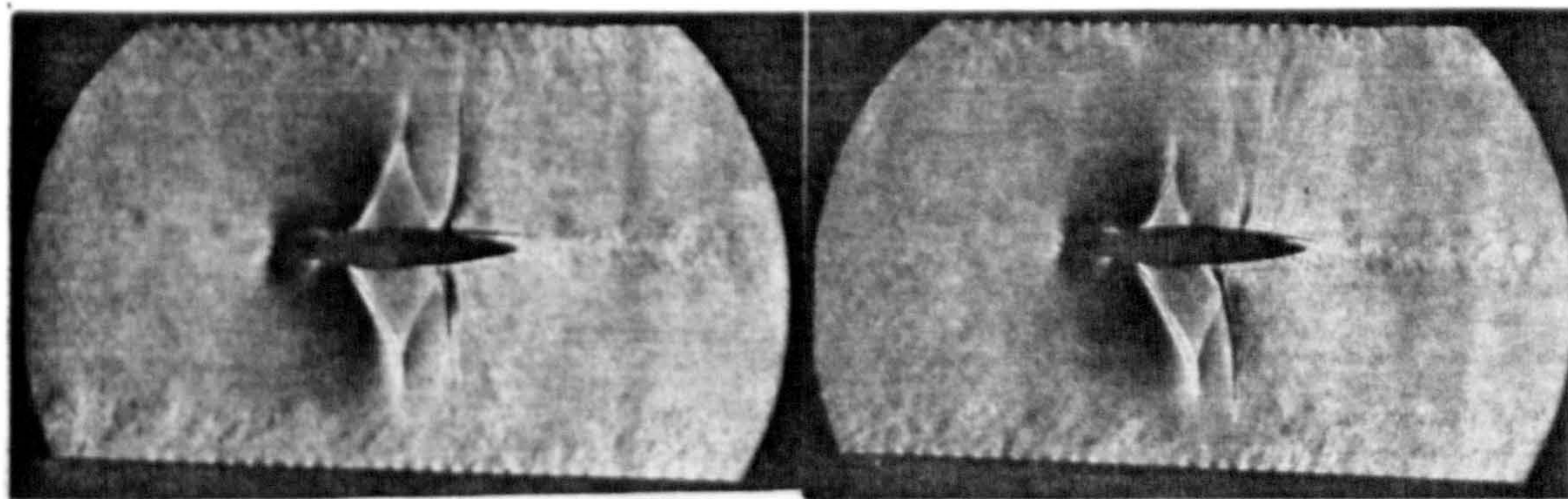


AEROFOIL WITH SPLITTER PLATE



AEROFOIL WITH BLUNT NOSE

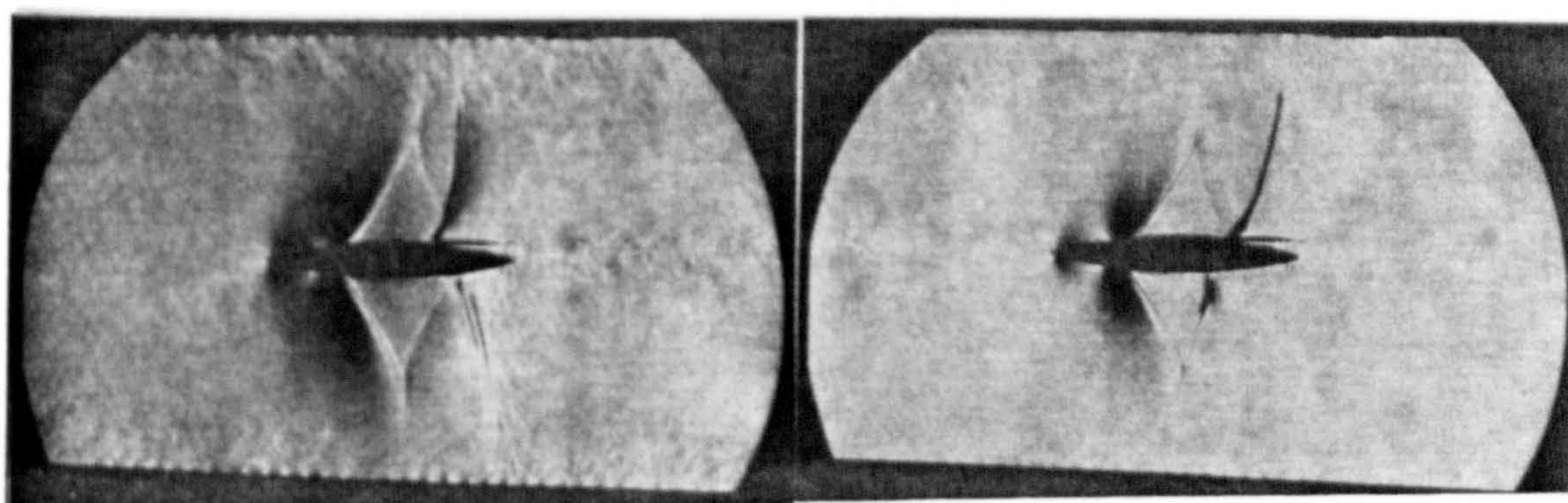
Figure 39.14% biconvex aerofoil configurations.



$M_\infty = 0.87$

0.87

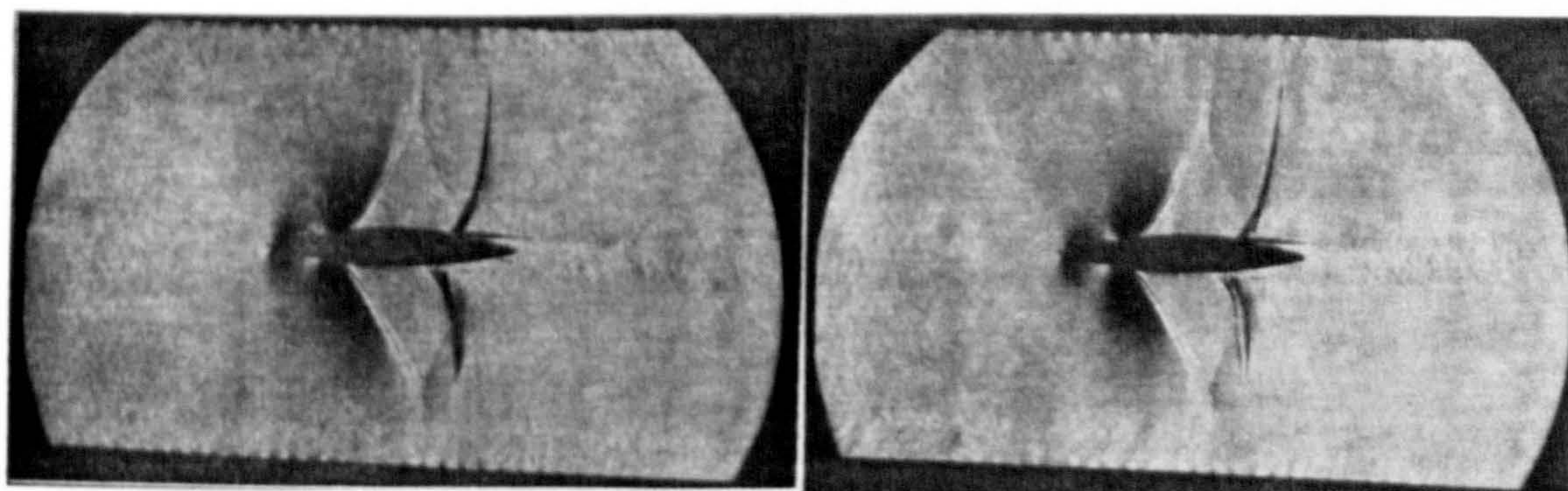
UNSTEADY FLOW



0.89

0.89

UNSTEADY FLOW

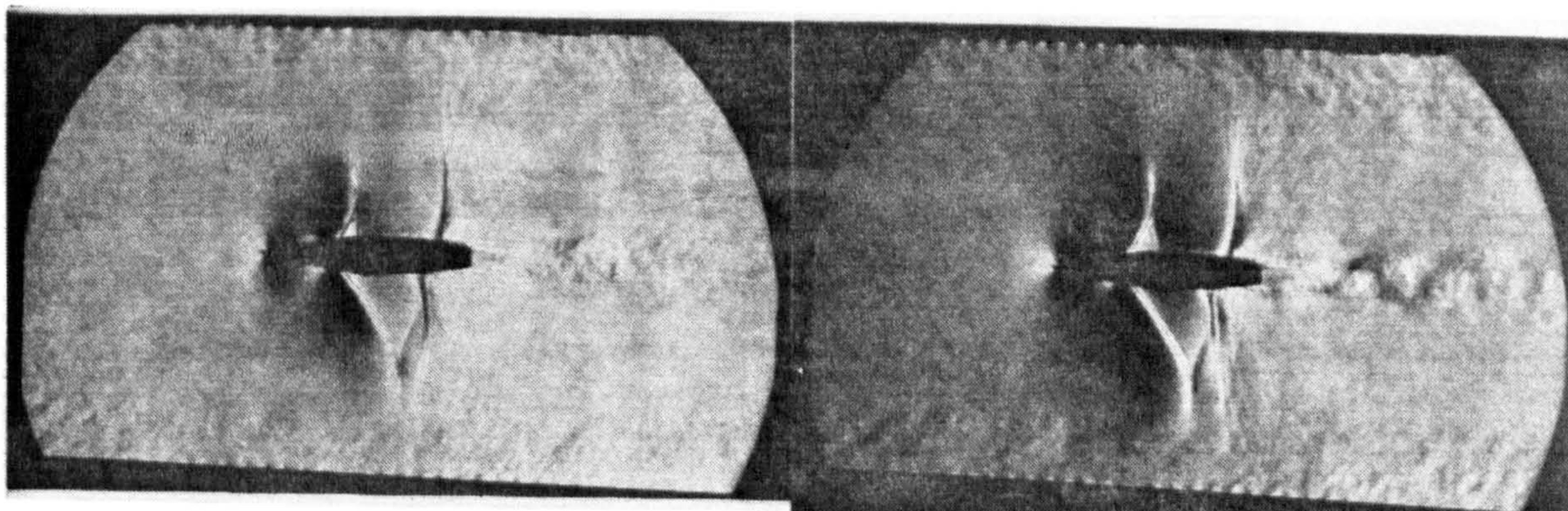


0.92

0.92

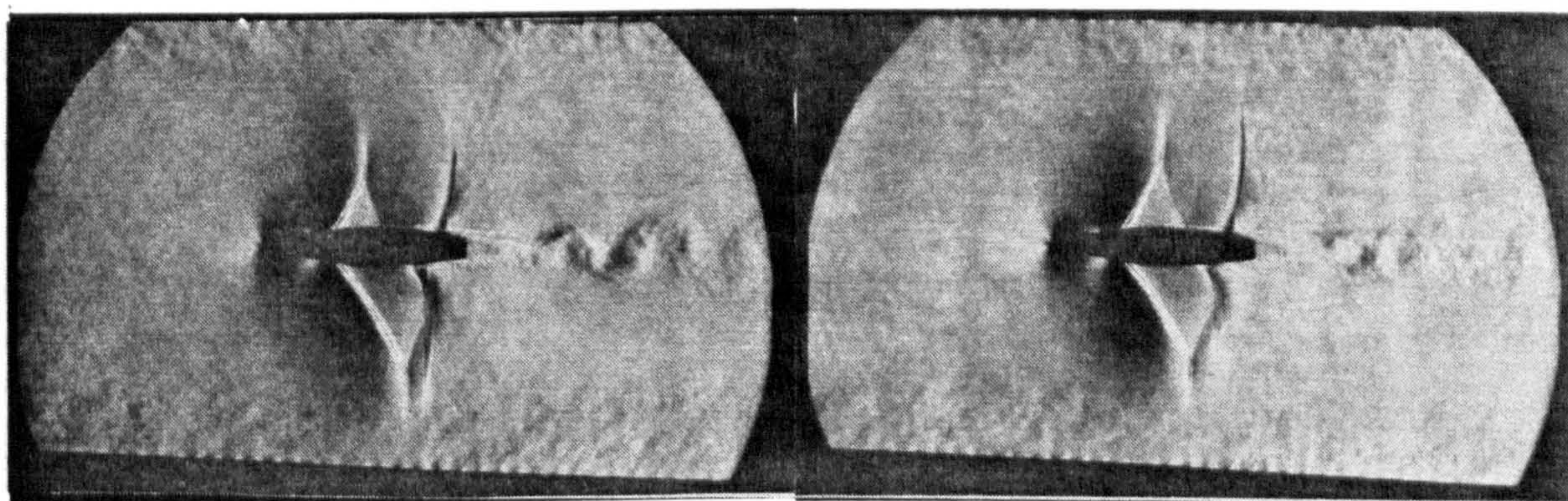
STEADY FLOW

Figure 40. Schlieren pictures, 14% thick biconvex aerofoil with blunt trailing edge ($c_1 = 0.976 c$)



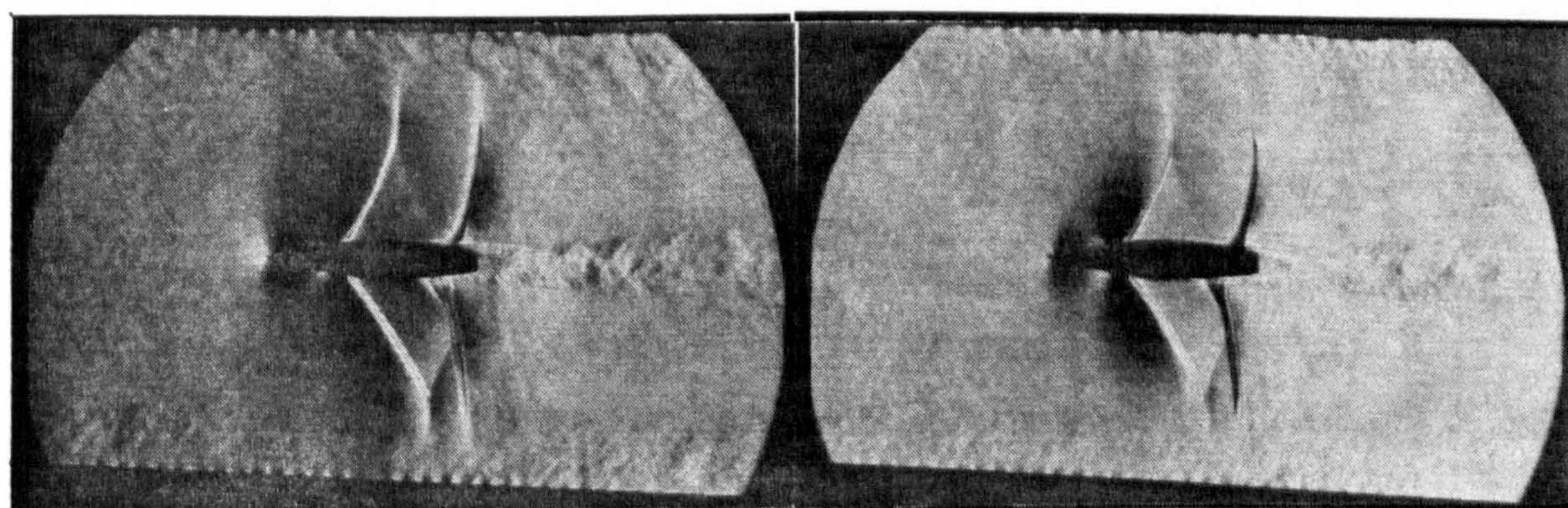
$M_\infty = 0.86$

0.86



0.88

0.88

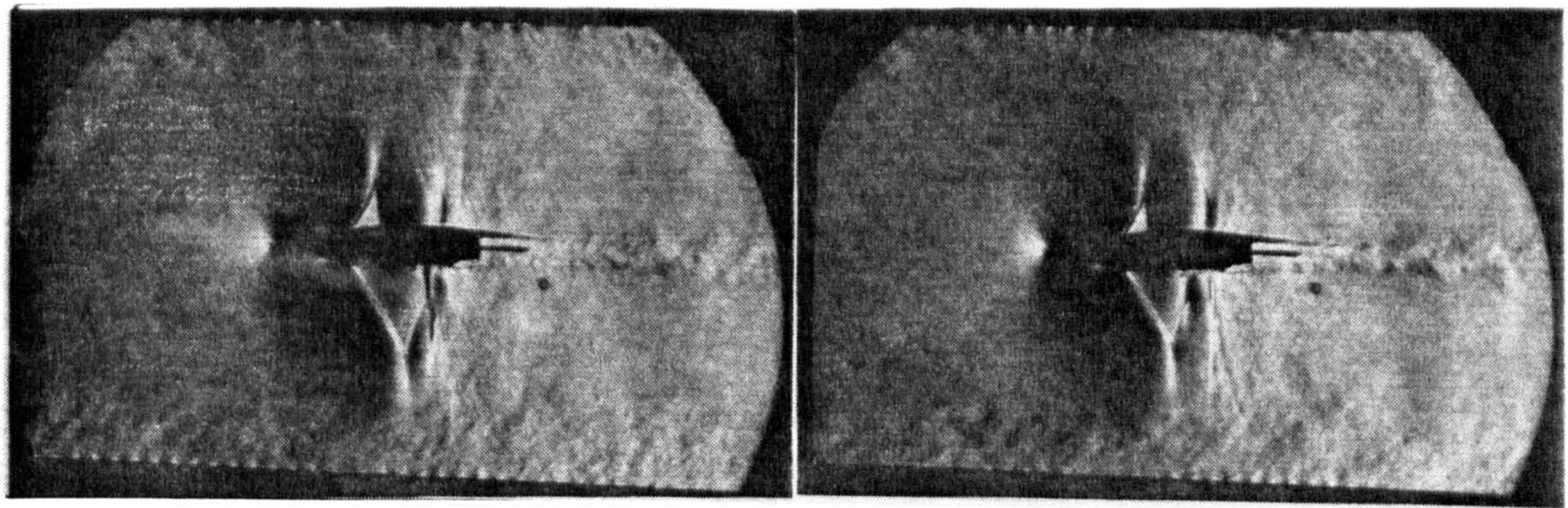


0.92

0.92

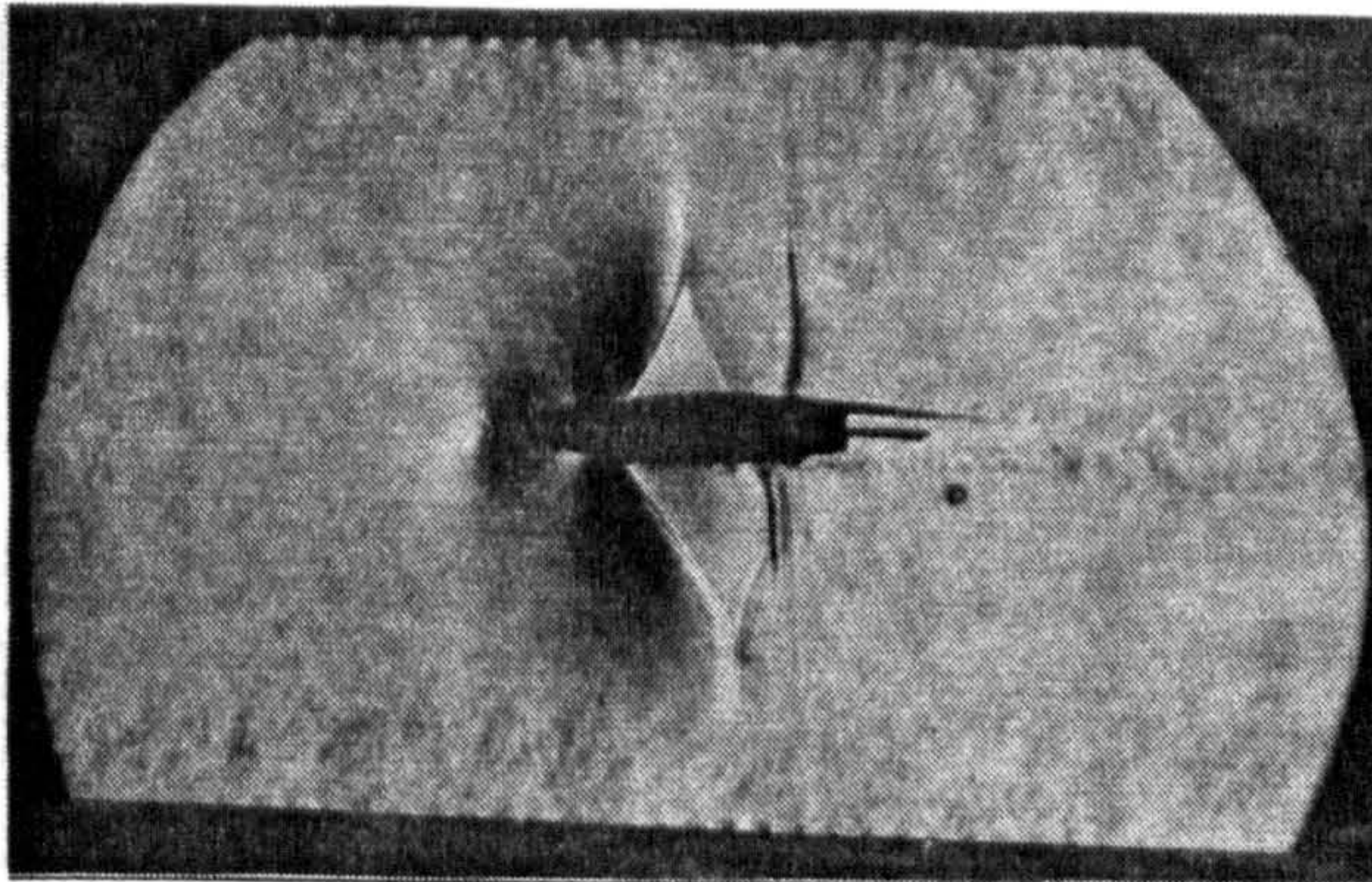
STEADY FLOWS

Figure 41. Schlieren pictures, 14% thick biconvex aerofoil with blunt trailing edge ($c_1 = 0.825c$).

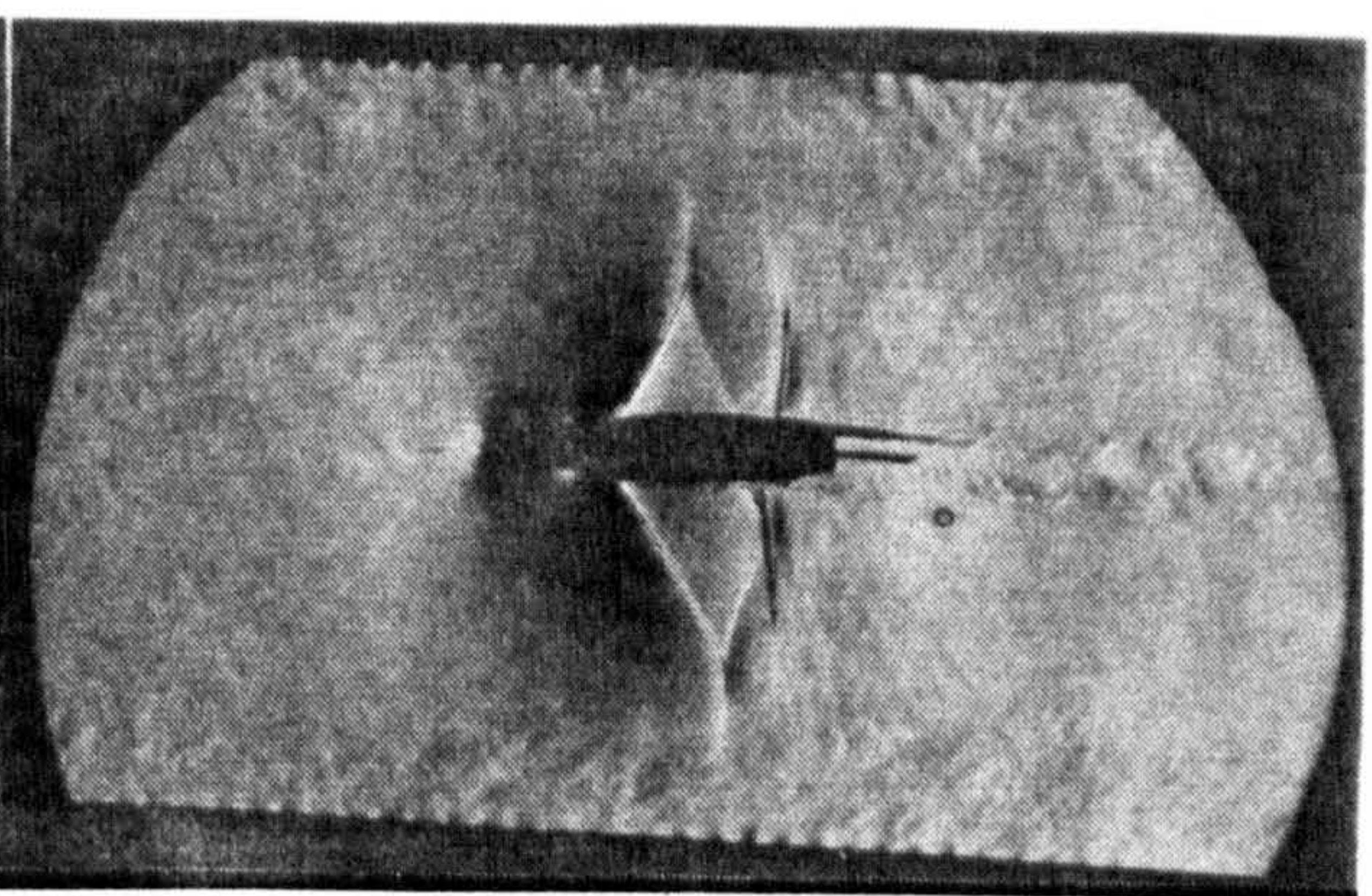


$M_{\infty} = 0.85$

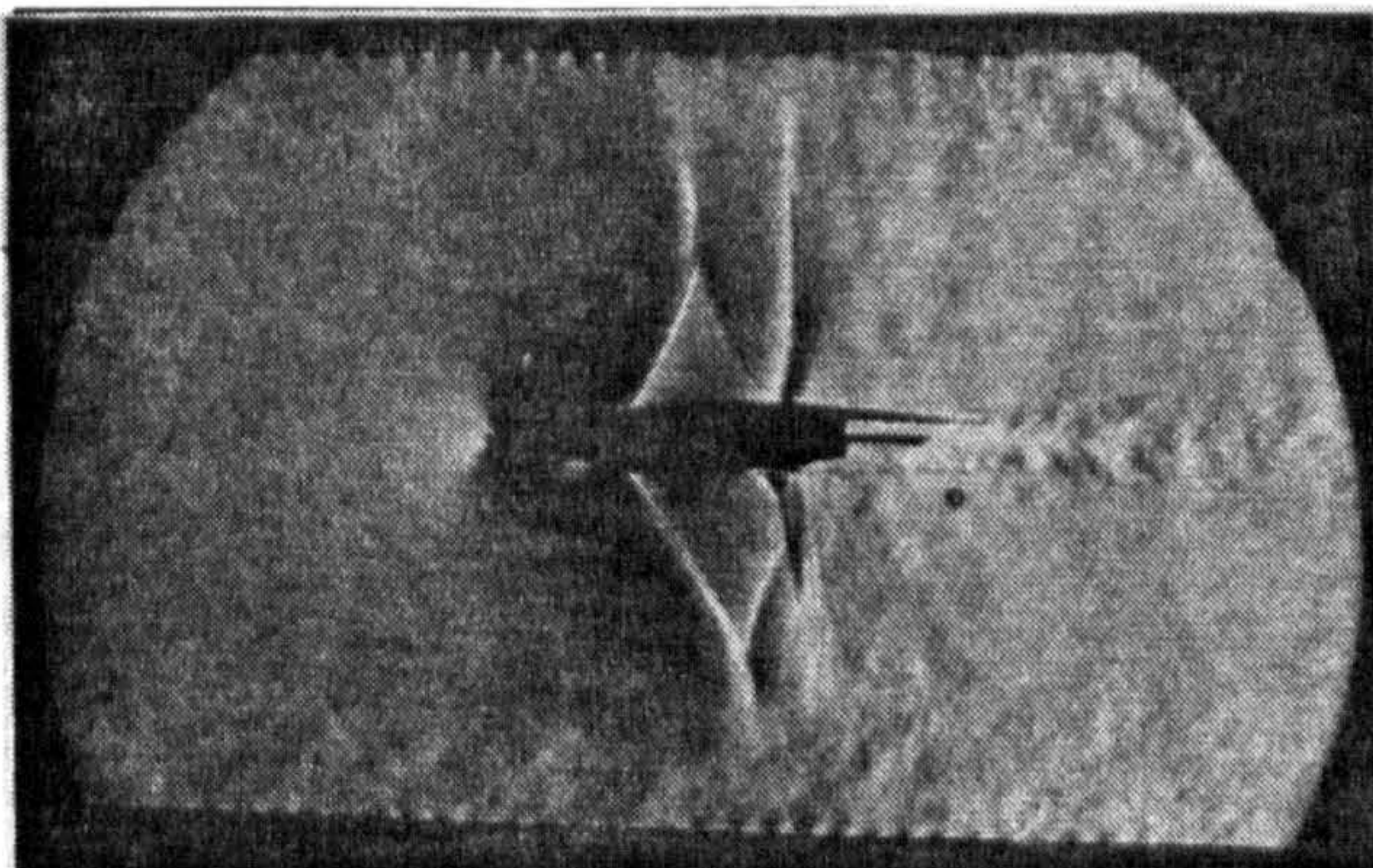
0.85



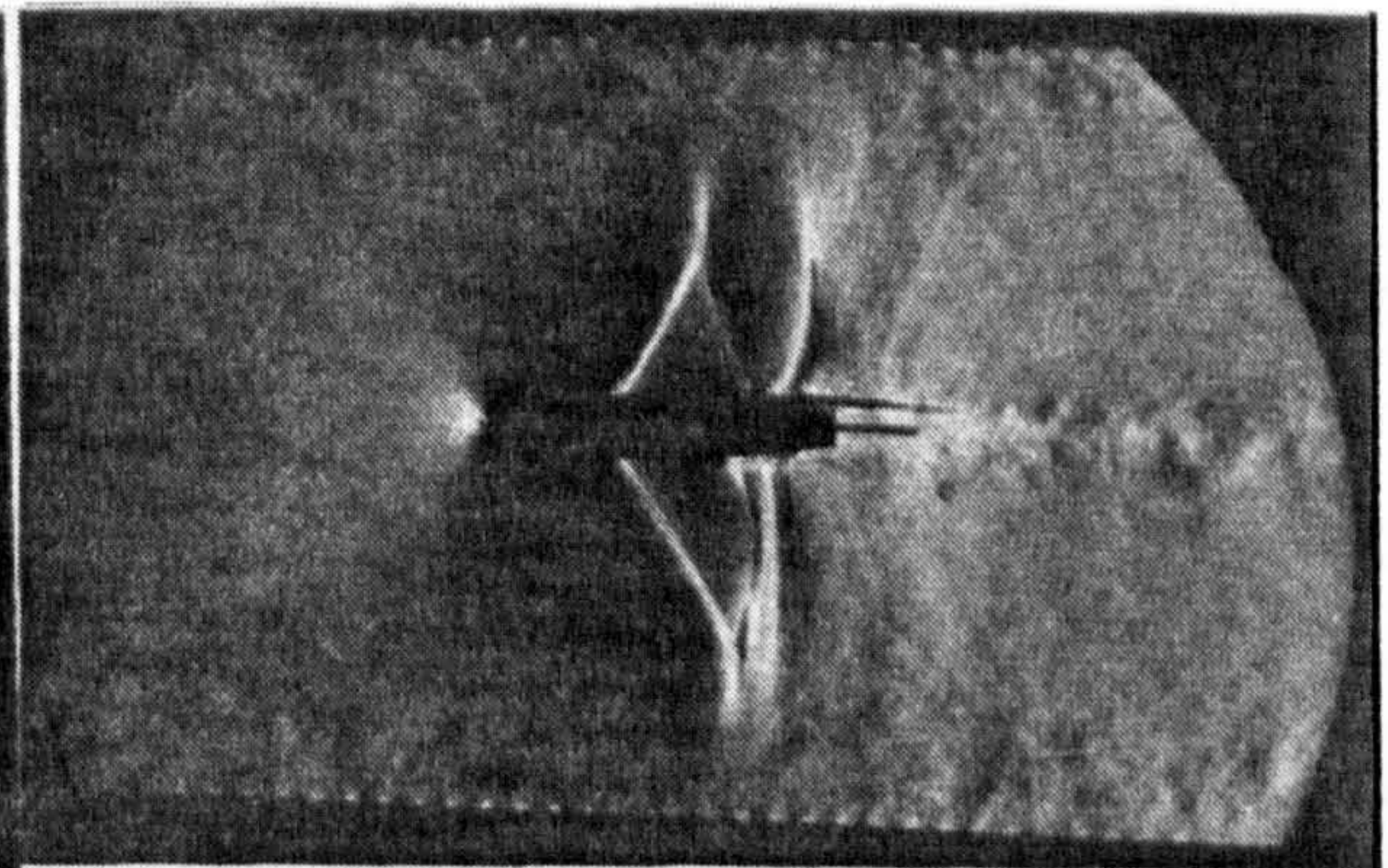
0.88



0.88



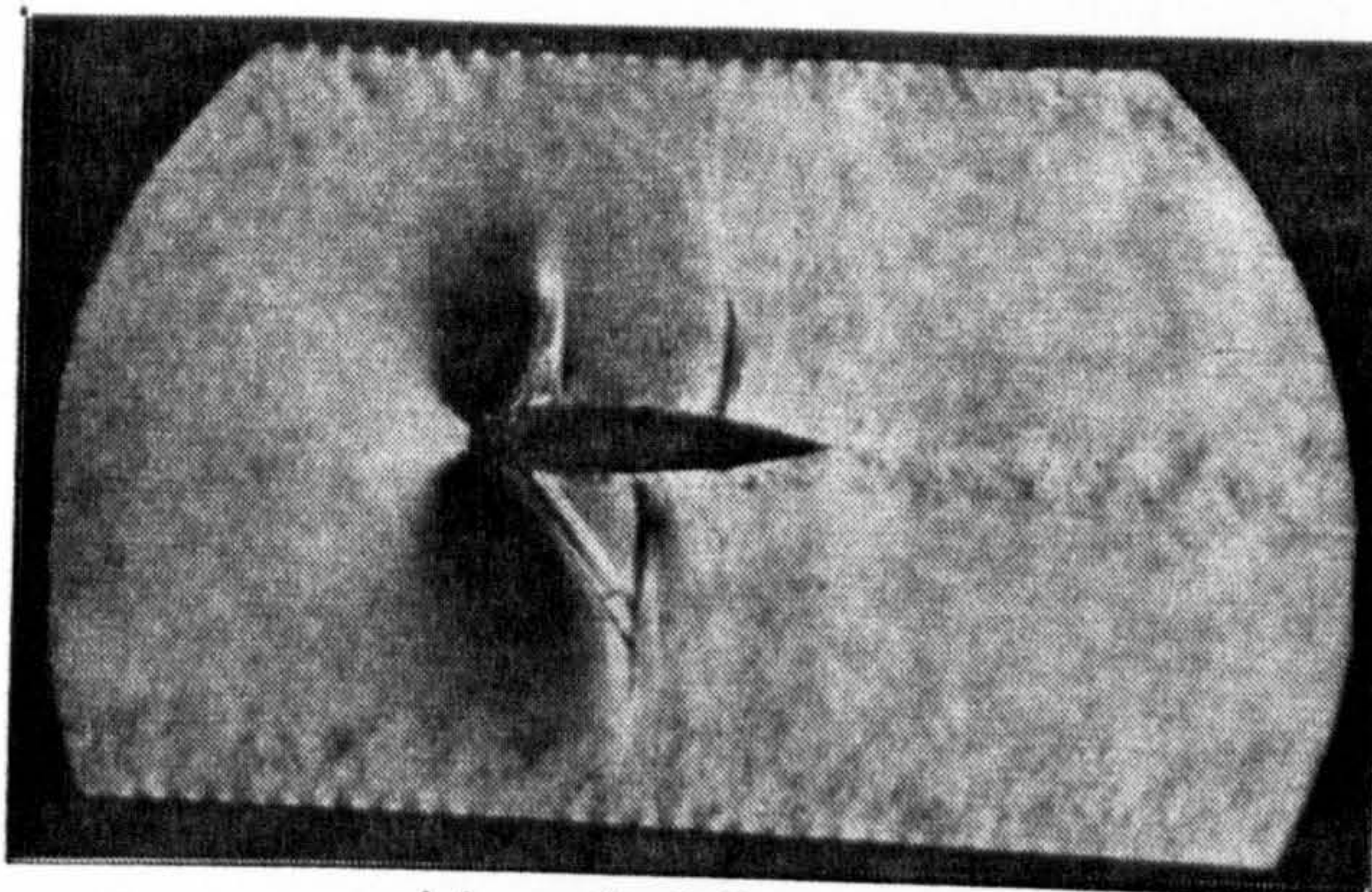
0.89



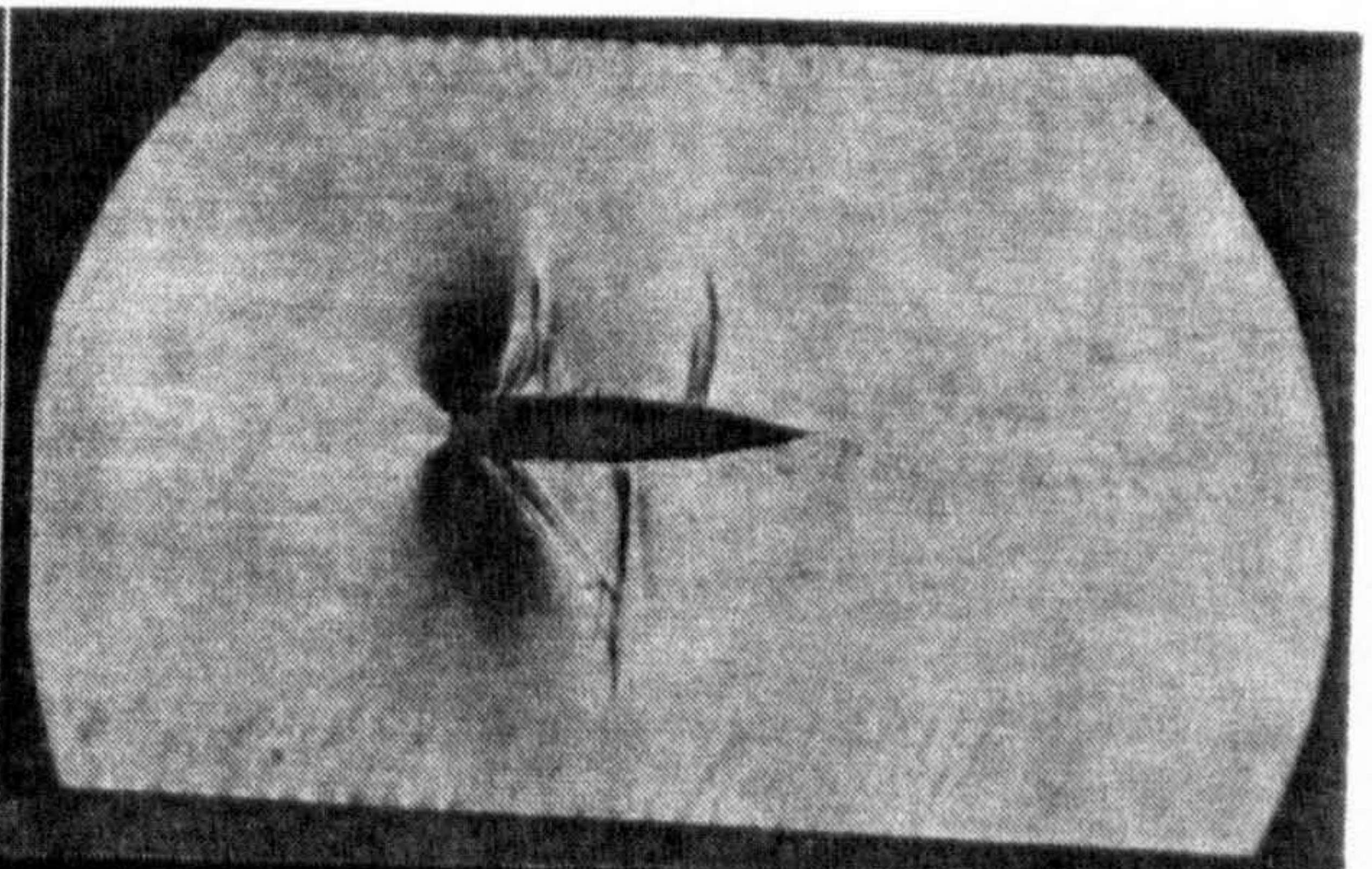
0.89

STEADY FLOWS

Figure 42 Schlieren pictures, 14% biconvex aerofoil with splitter plate ($c_1 = 0.826c$)



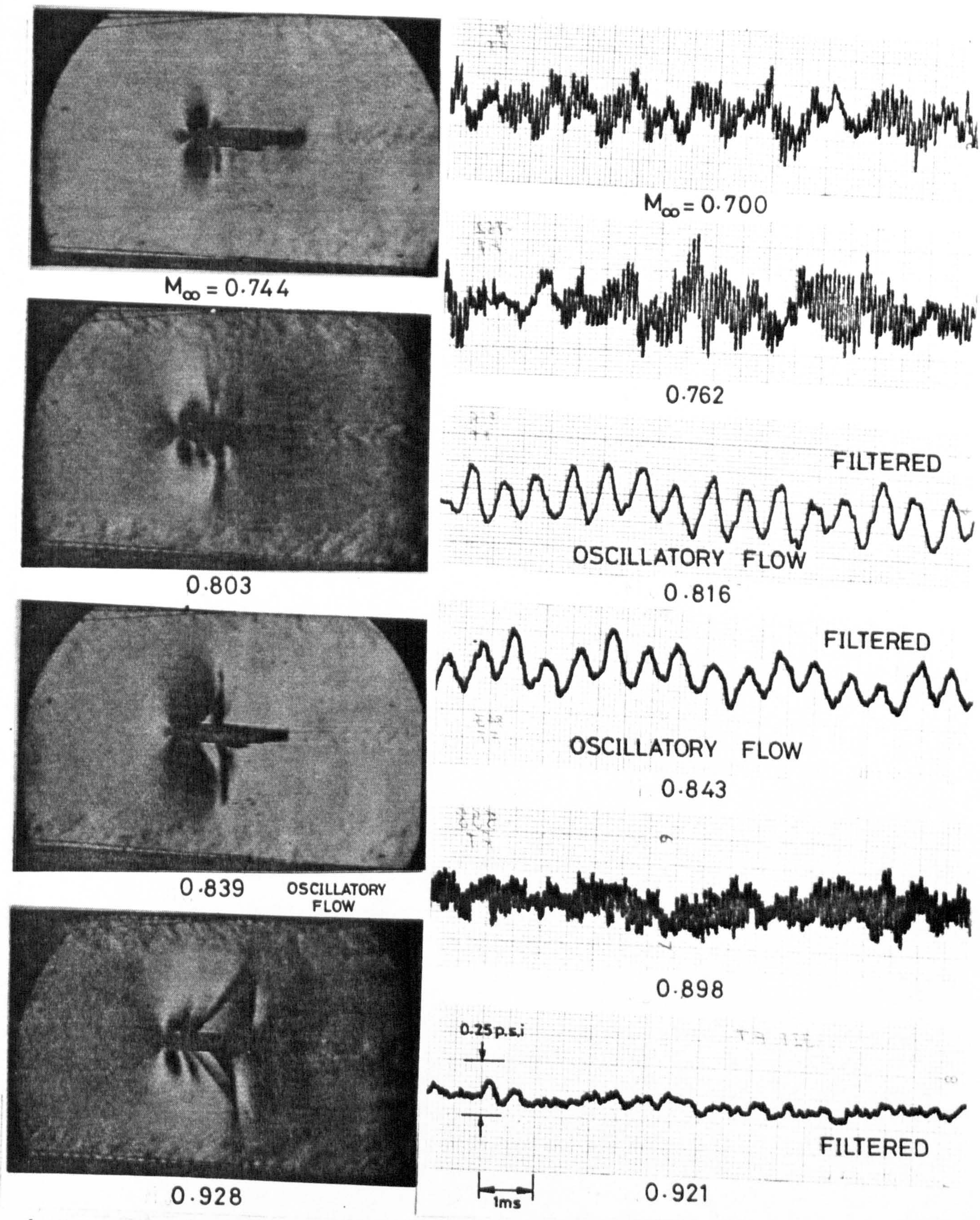
$M_{\infty} = 0.88$



0.88

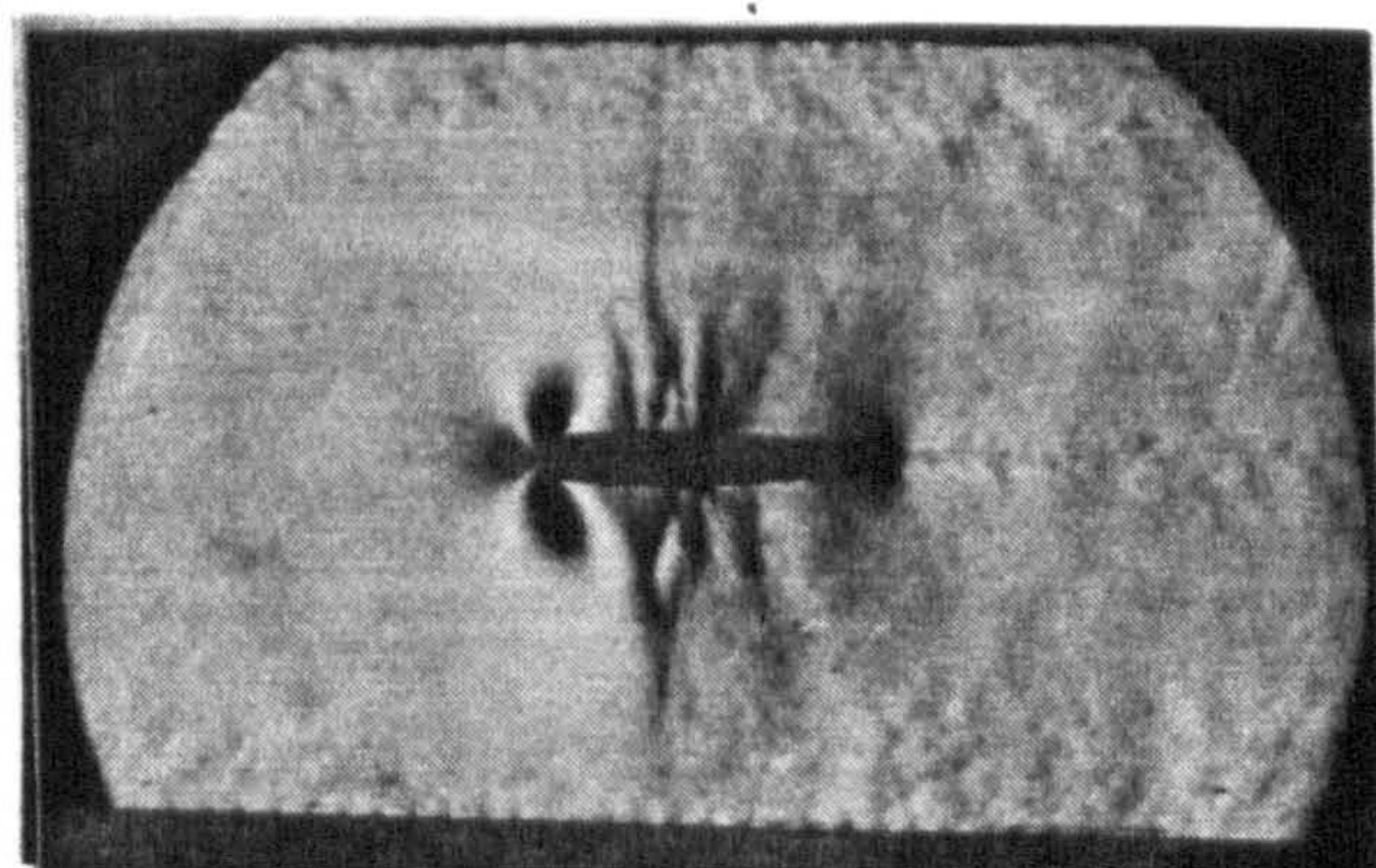
UNSTEADY FLOW

Figure 43. Schlieren pictures, 14% biconvex aerofoil with blunt leading edge ($c_1 = 0.825c$)

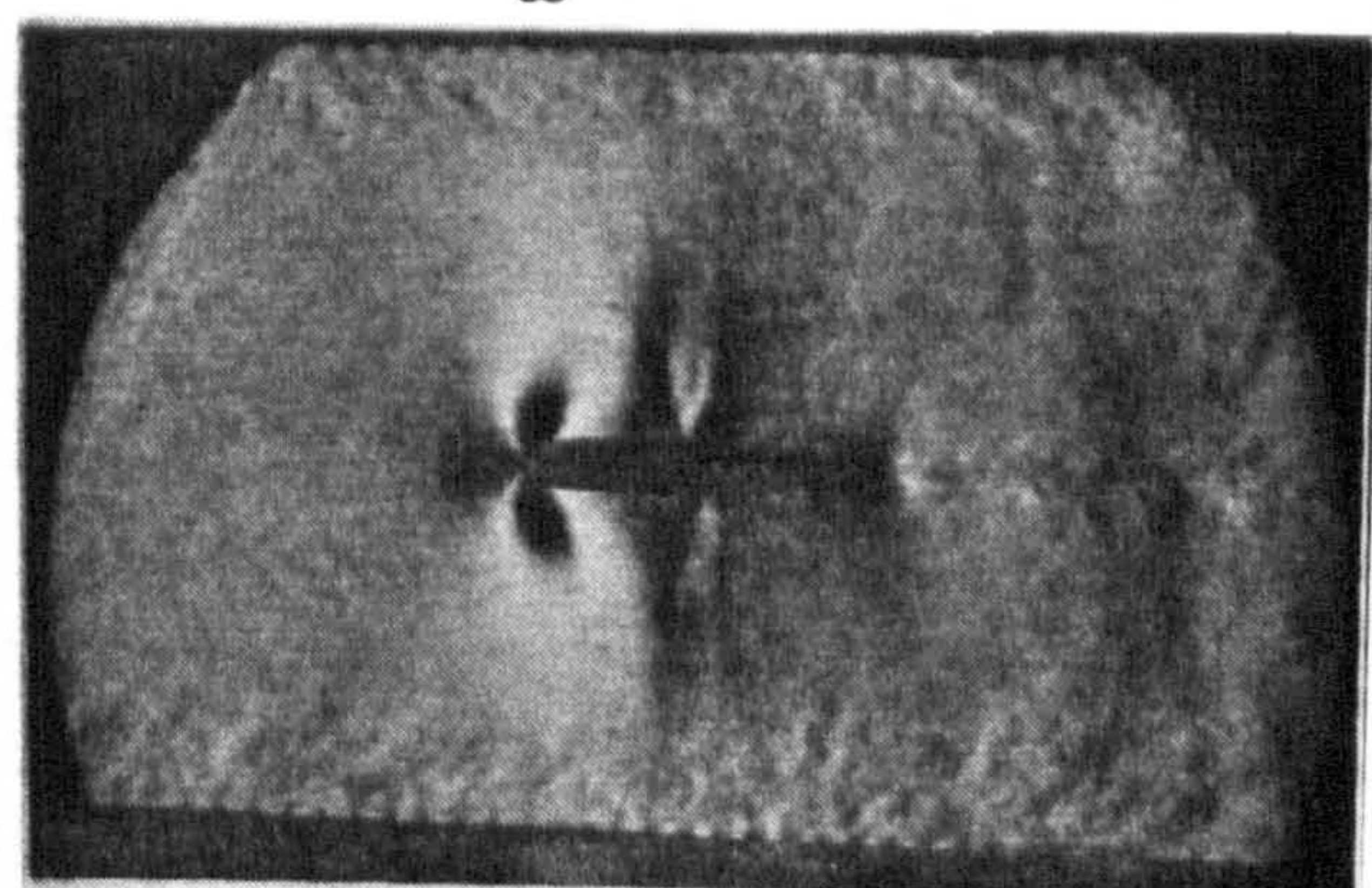


a) Schlieren pictures. b) Sidewall pressure traces.

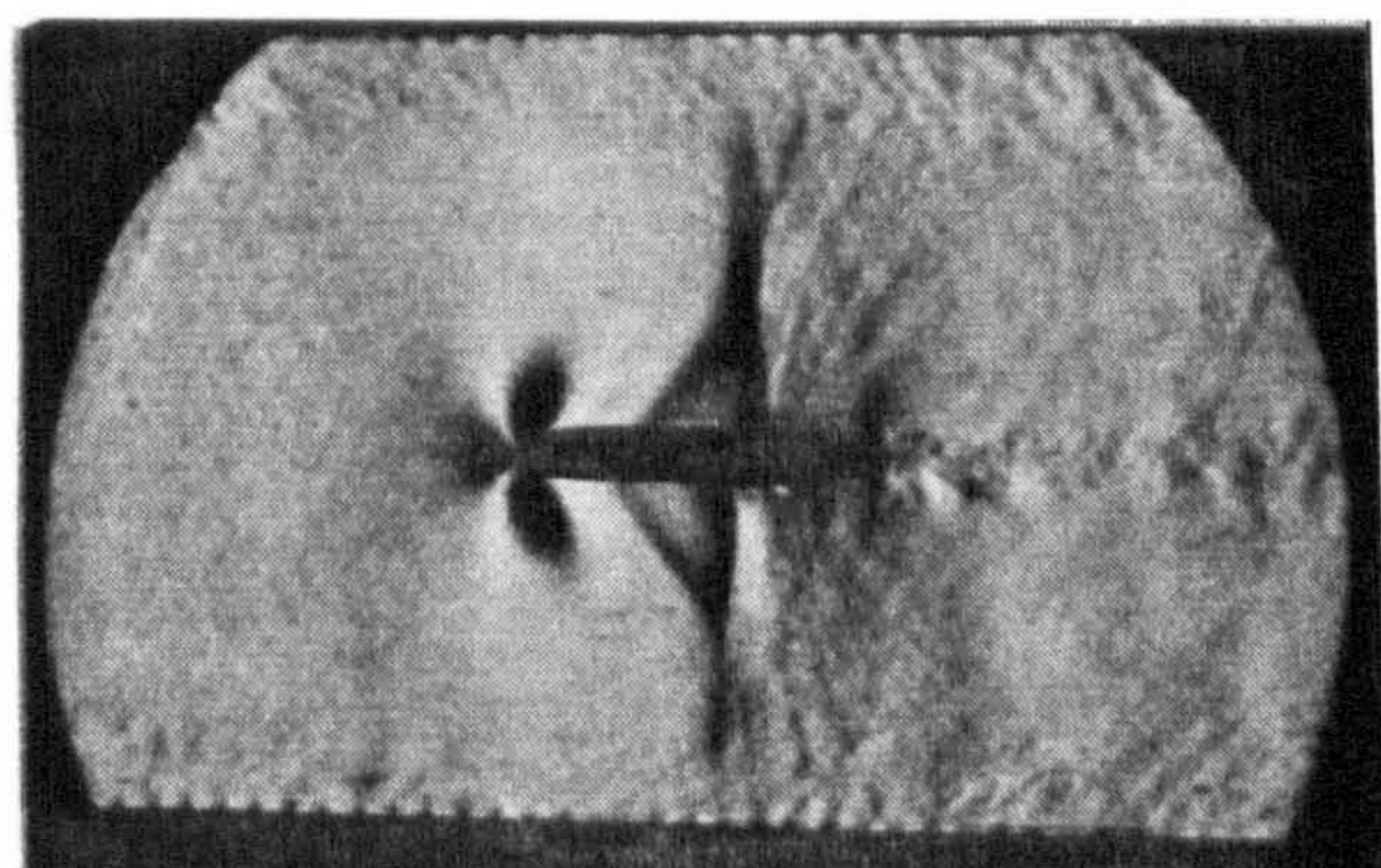
Figure 44. NACA 0012 at zero incidence
(fixed transition)



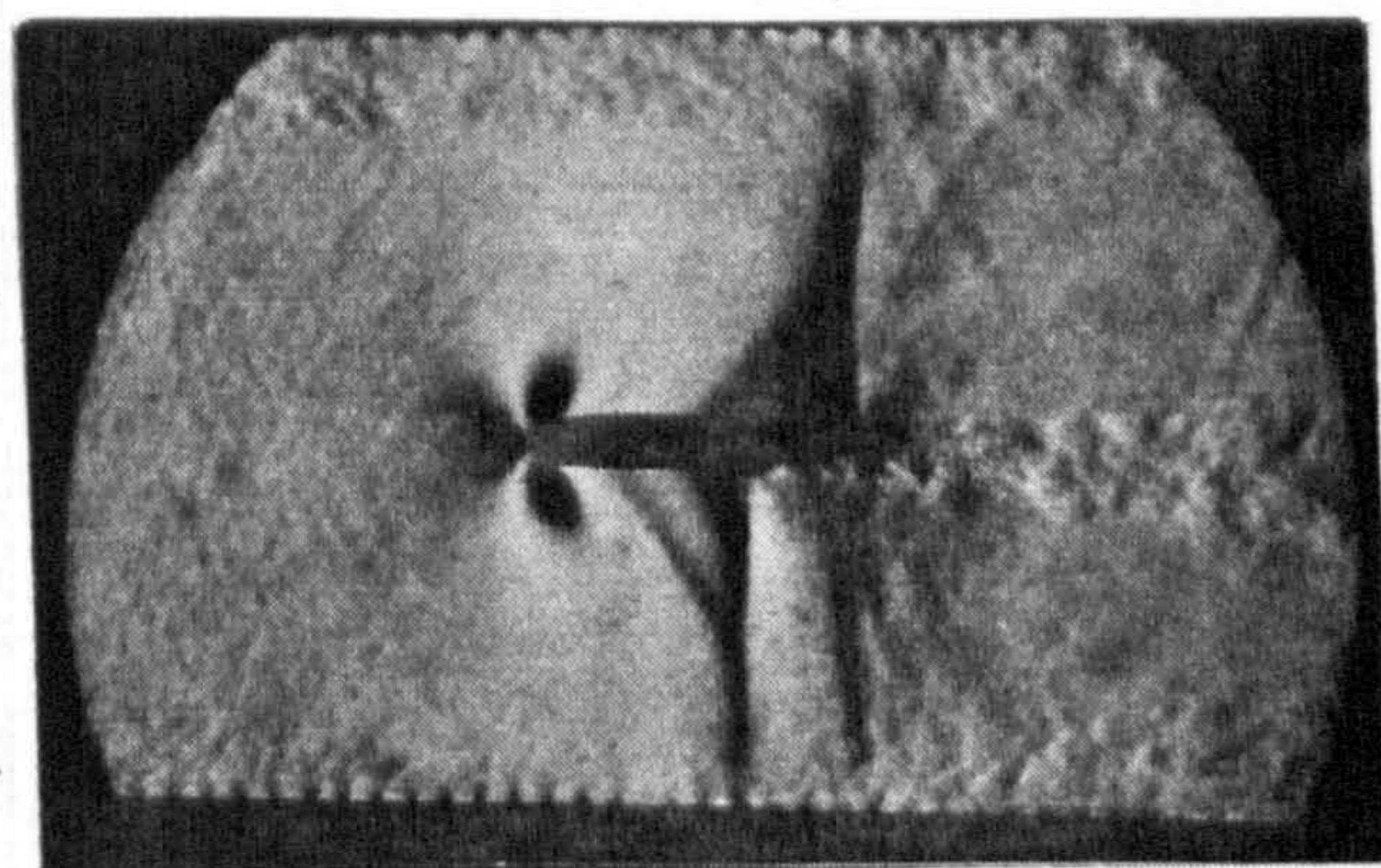
$M_{\infty} = 0.807$



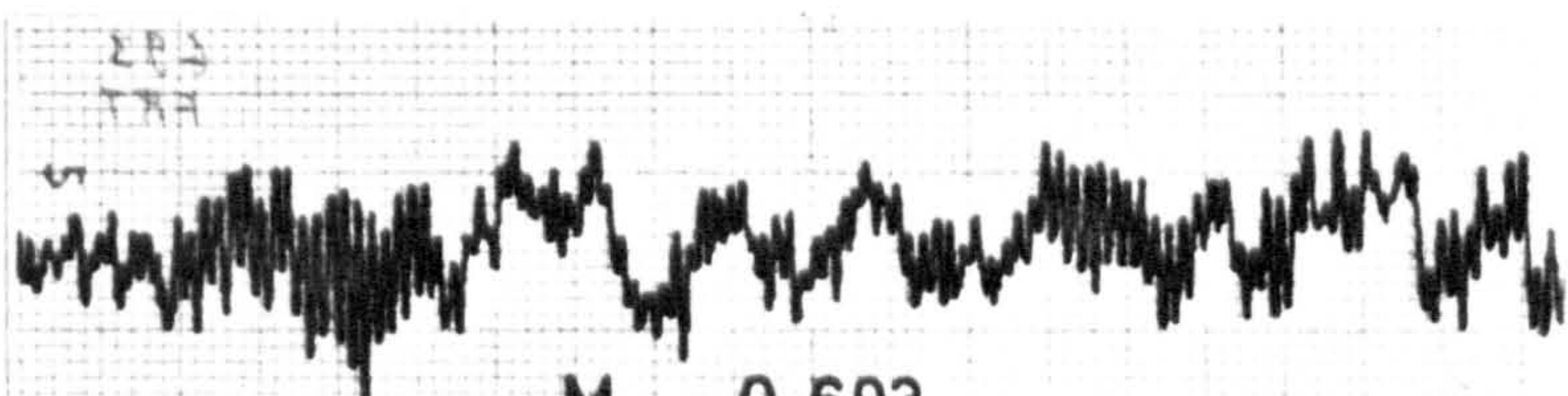
OSCILLATORY FLOW 0.844



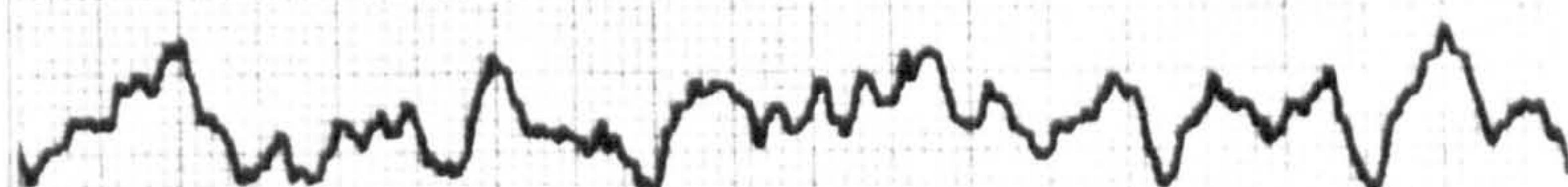
0.904



OSCILLATORY FLOW 0.948

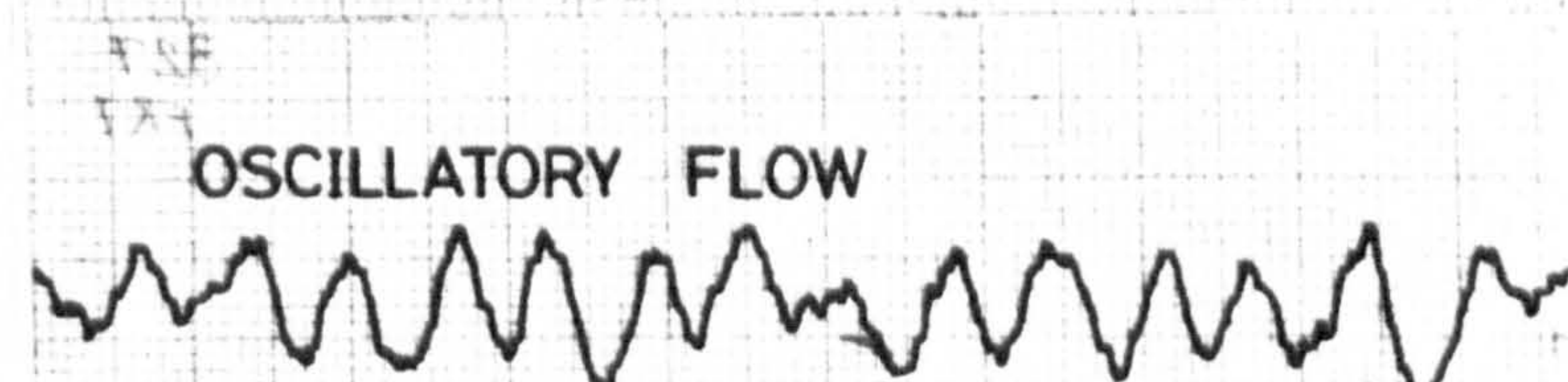


$M_{\infty} = 0.693$



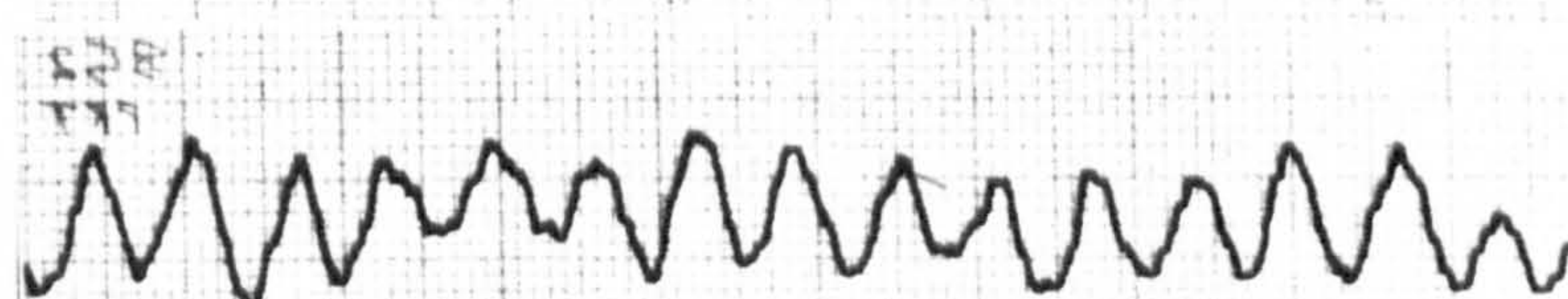
0.783

FILTERED



0.827

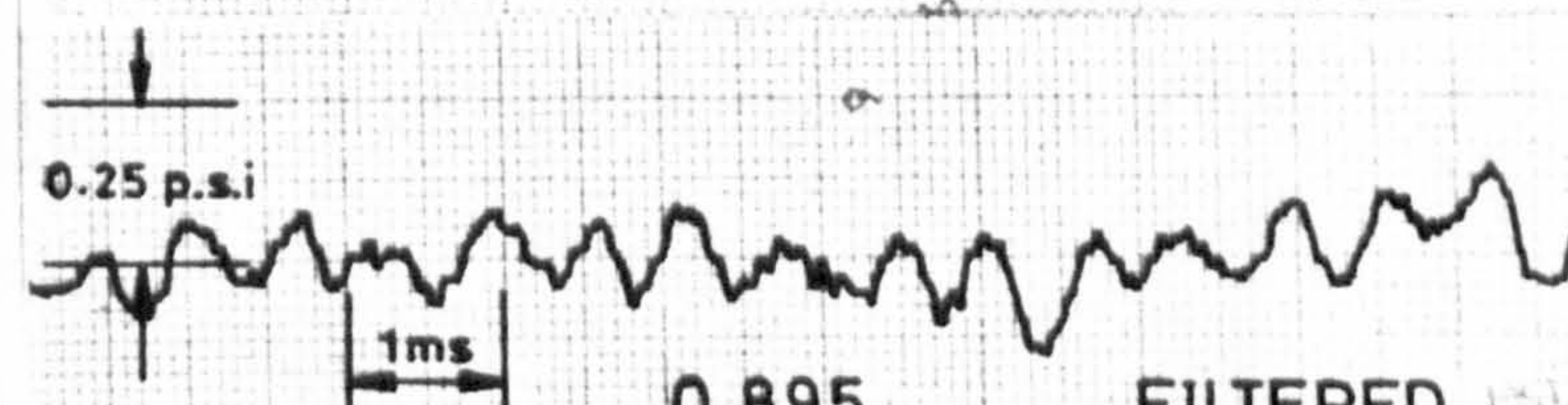
FILTERED



OSCILLATORY FLOW

0.852

FILTERED

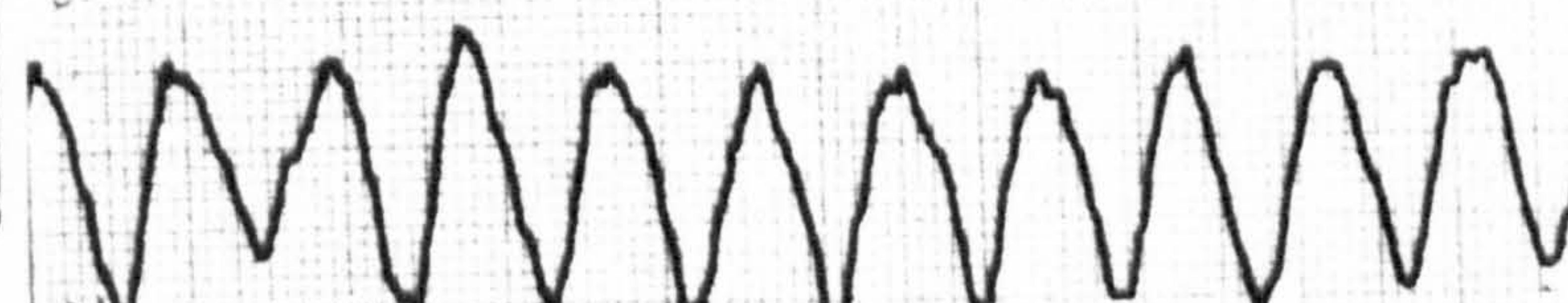


0.25 p.s.i.

1ms

0.895

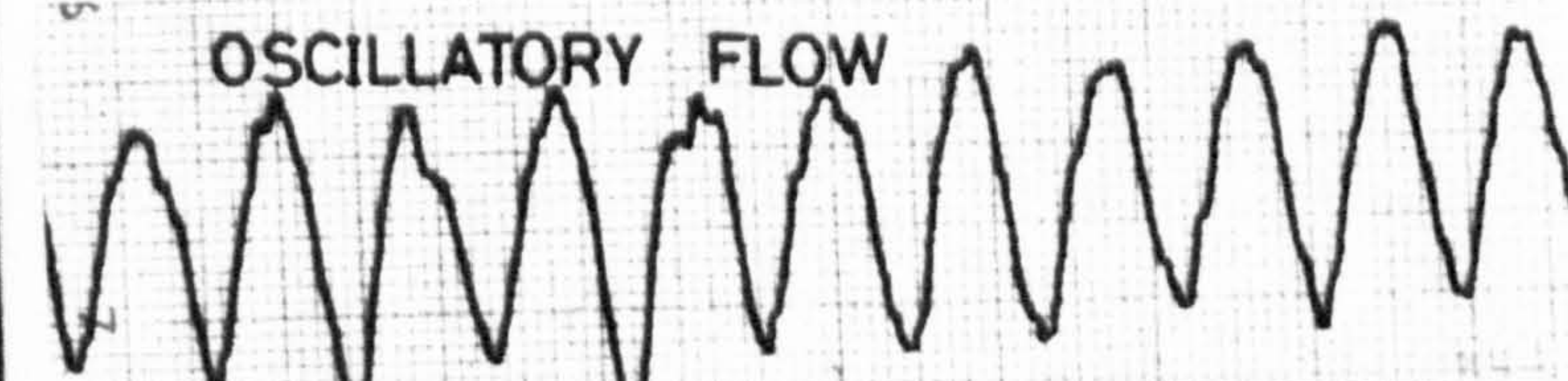
FILTERED



OSCILLATORY FLOW

0.927

FILTERED



OSCILLATORY FLOW

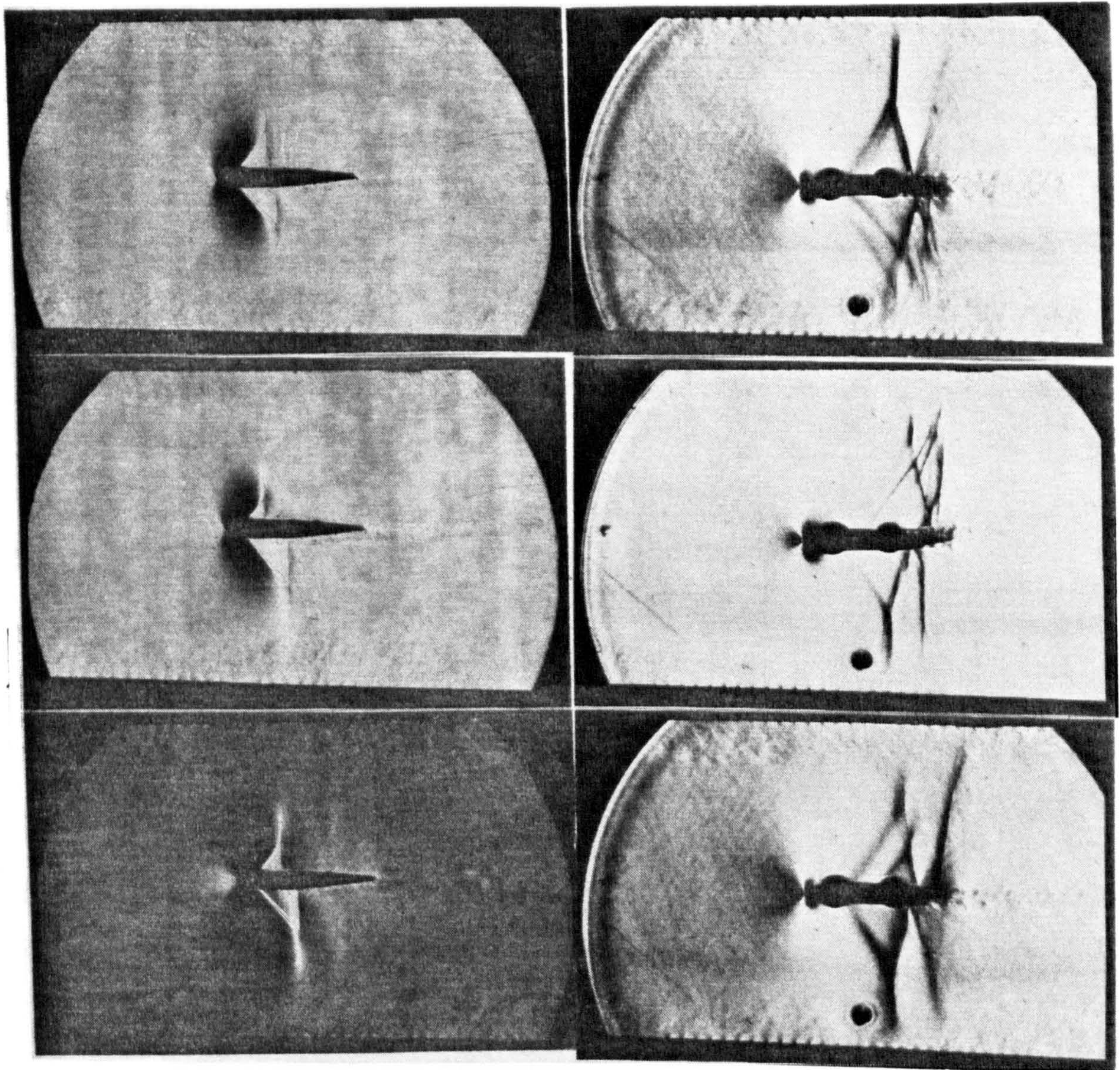
0.947

FILTERED

b) Sidewall pressure traces.

a) Schlieren pictures.

Figure 45. NACA 0012 at zero incidence
(free transition)



a) Turbulent Boundary Layer, $M_{\infty}=0.835$

b) Laminar Boundary Layer, $M_{\infty}=0.940$

Figure 46. Schlieren pictures to illustrate shock oscillations on NACA 0012 aerofoil at zero incidence.

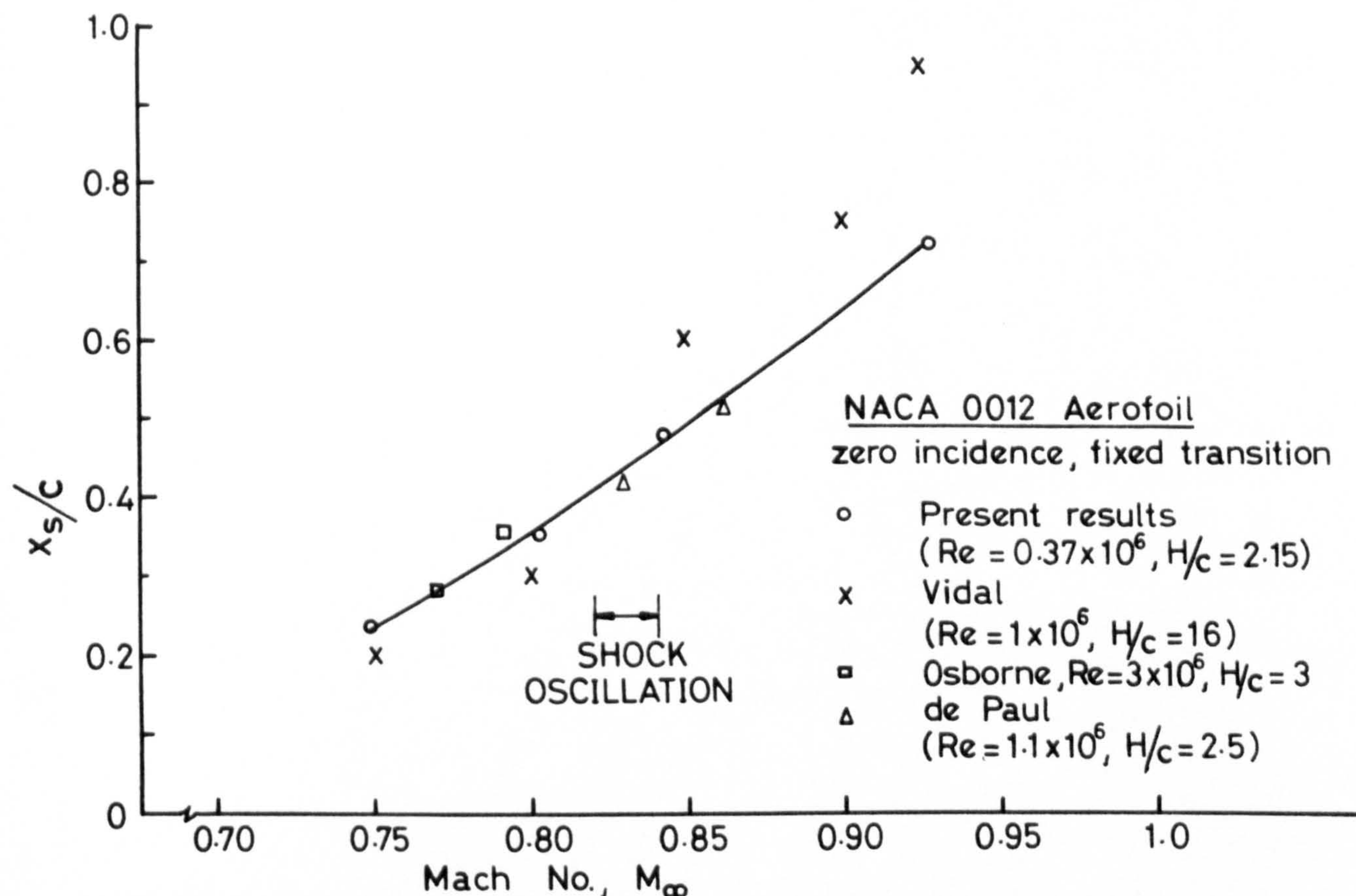


Figure 47. Shock position vs. Mach No.—low Reynolds No.

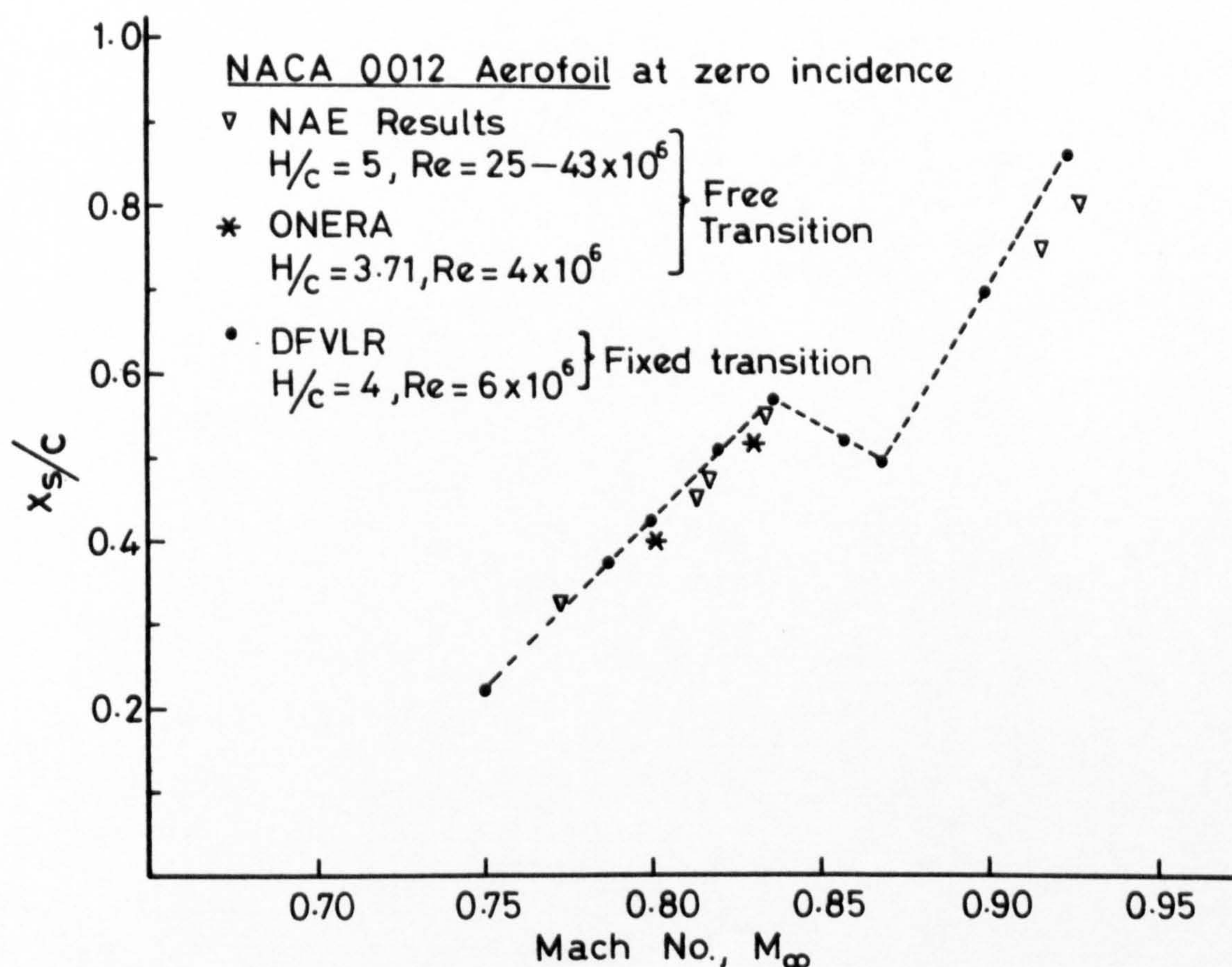


Figure 48. Shock position vs. Mach No.—high Reynolds No.

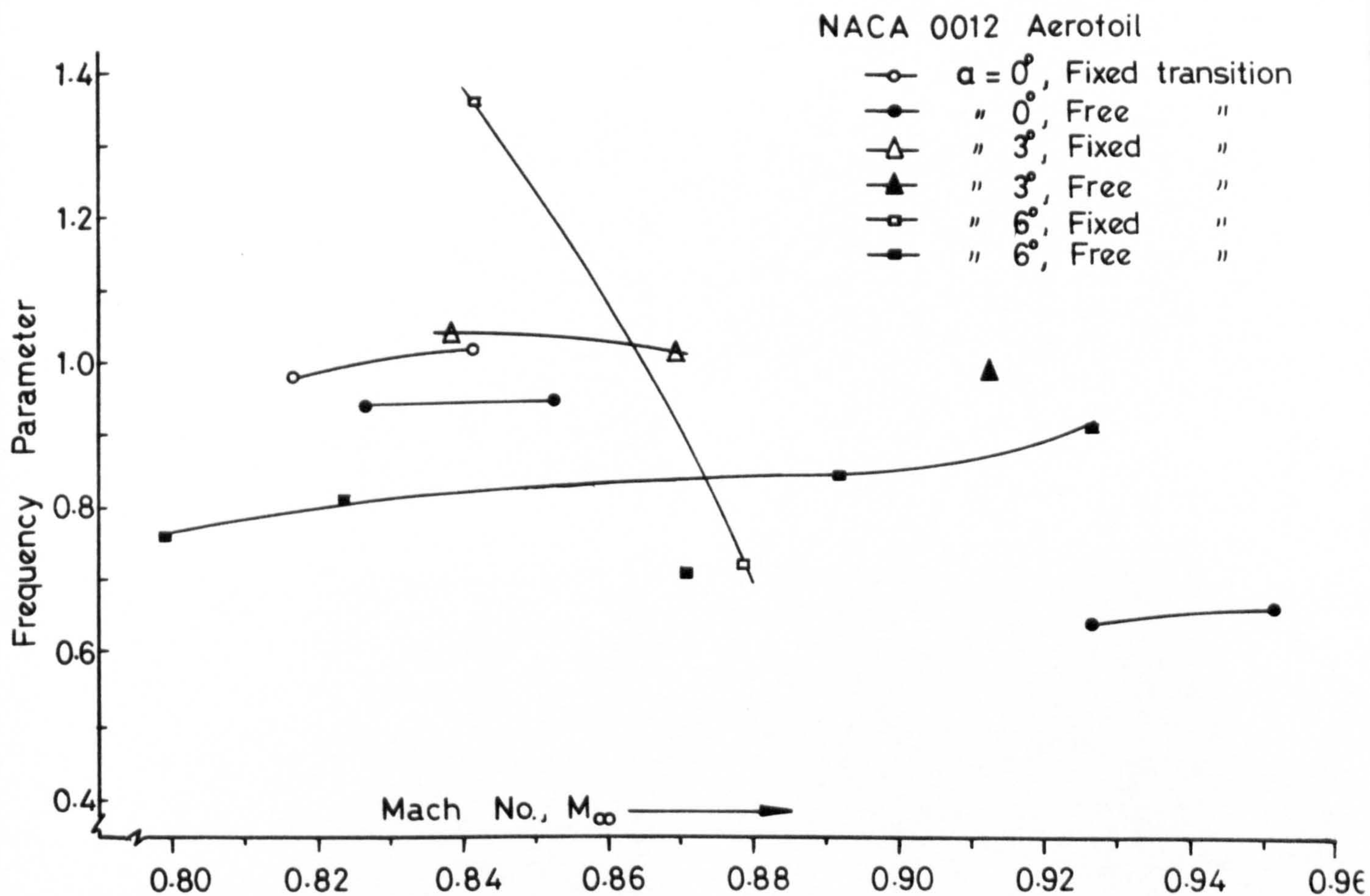


Figure 49. Frequency parameter.

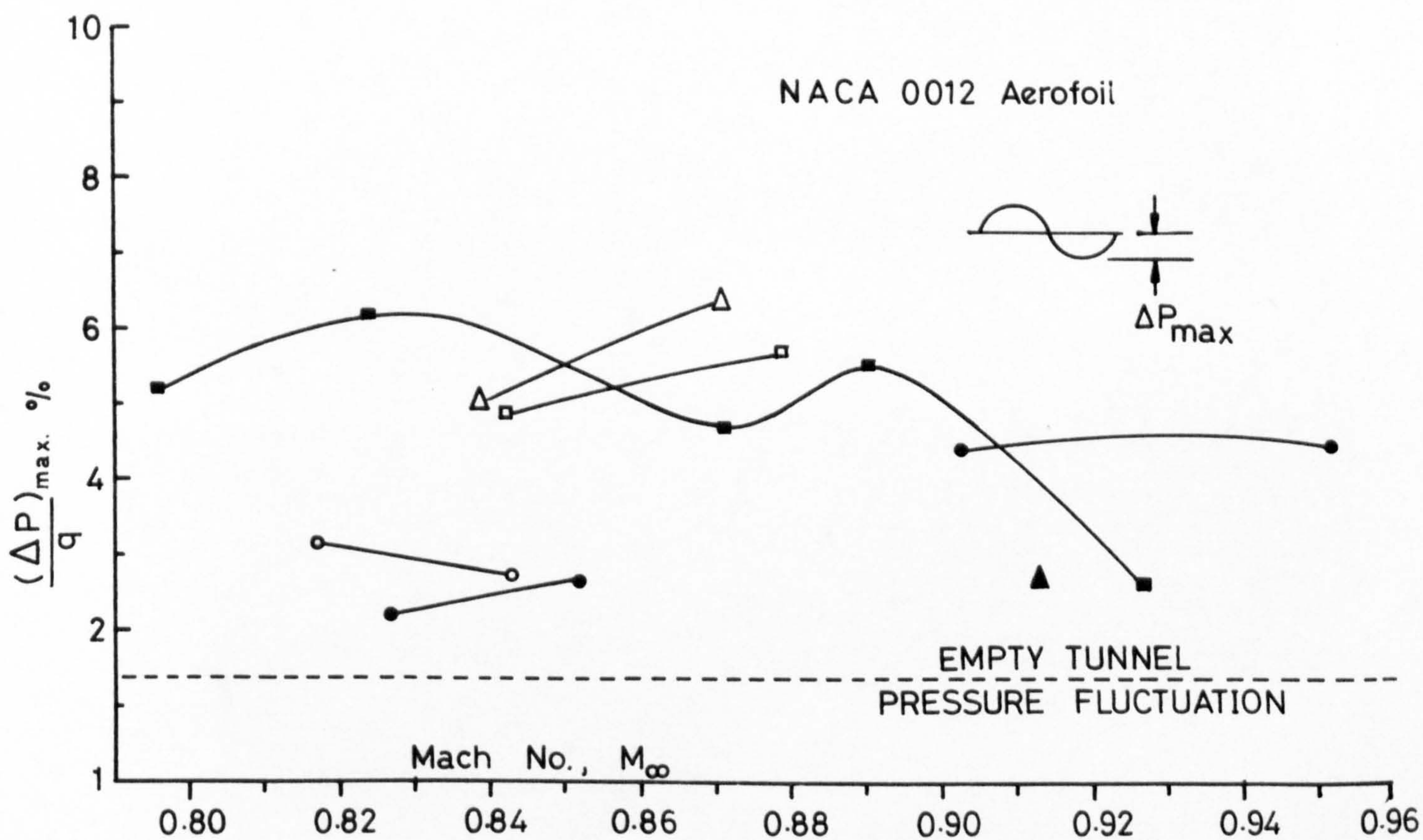
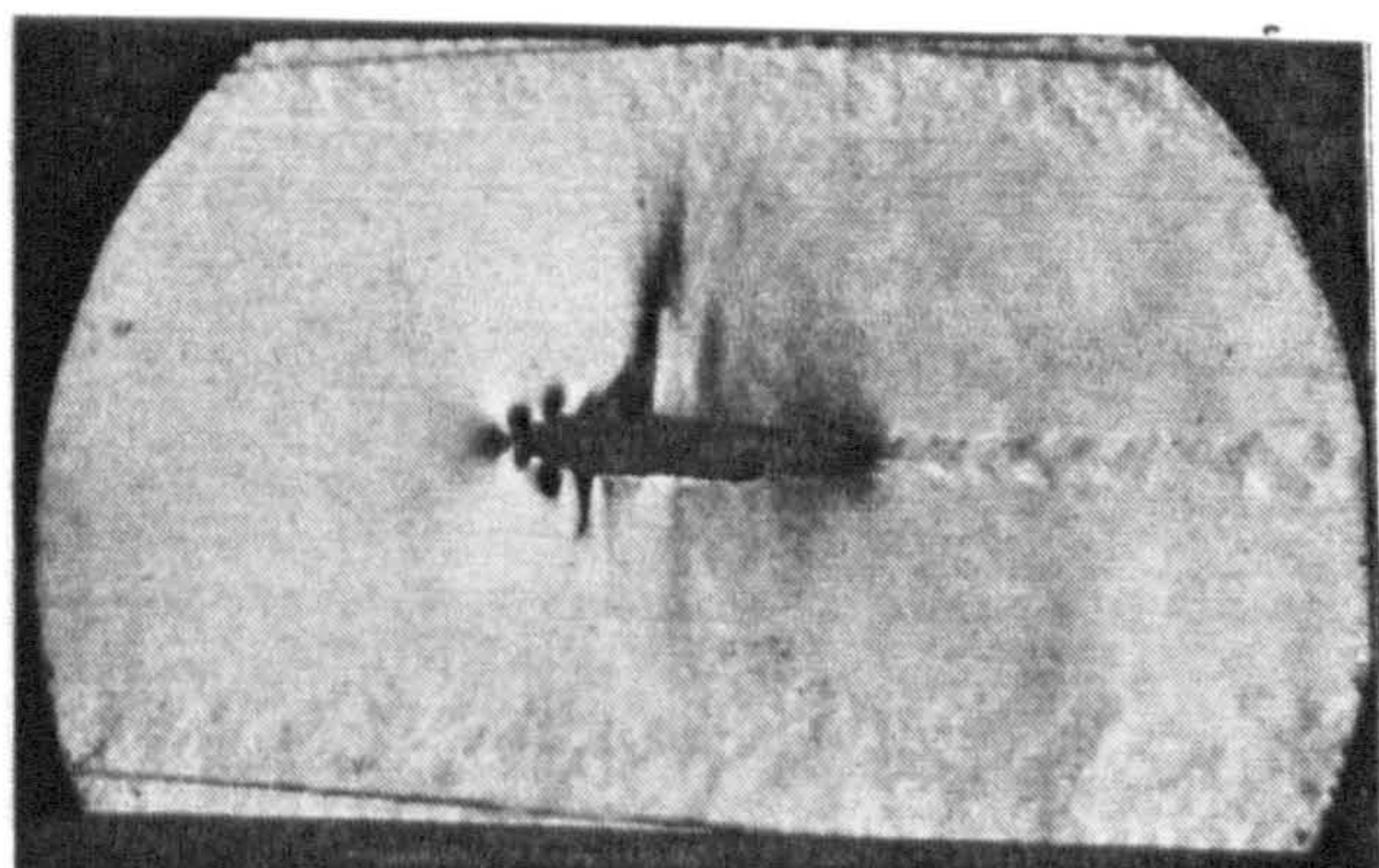
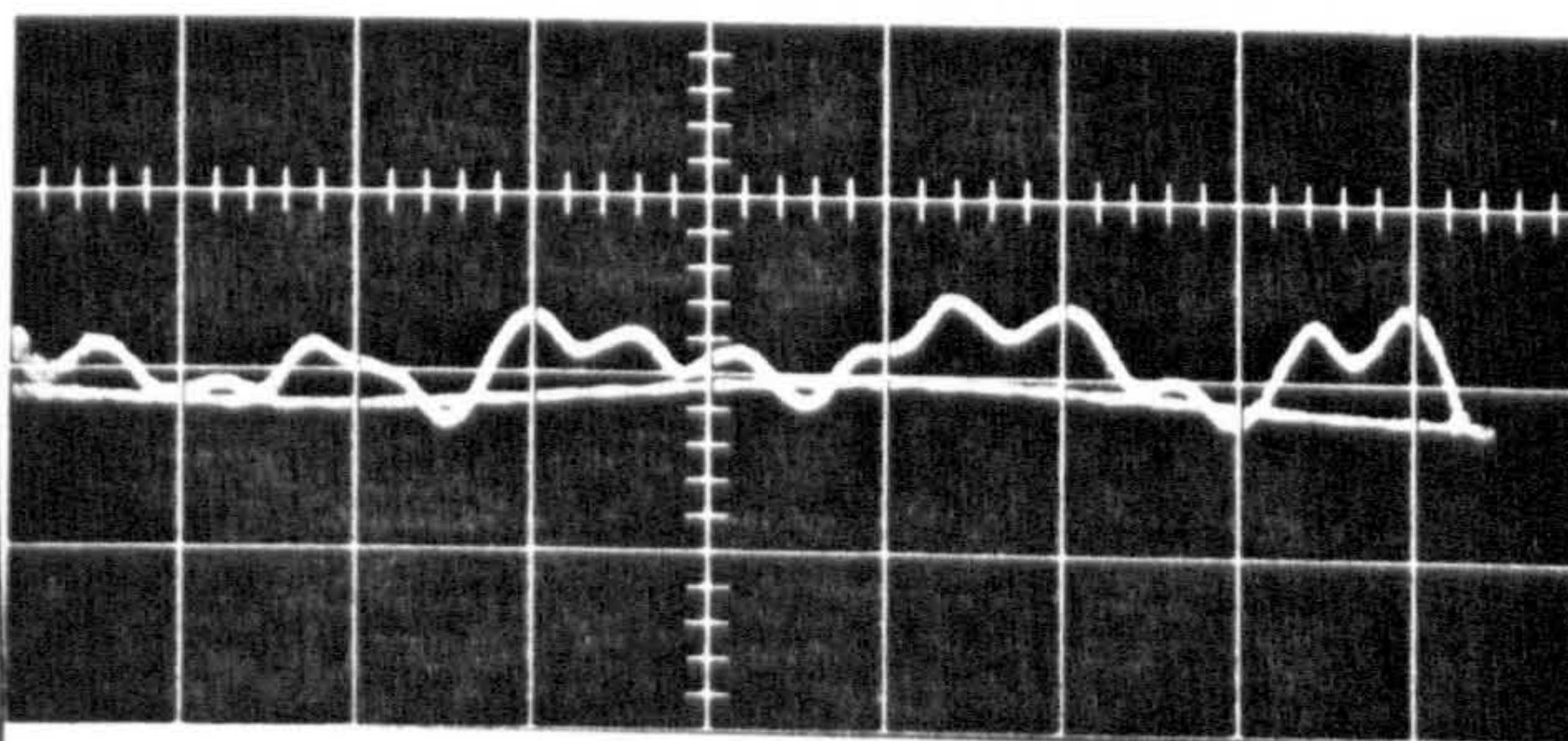


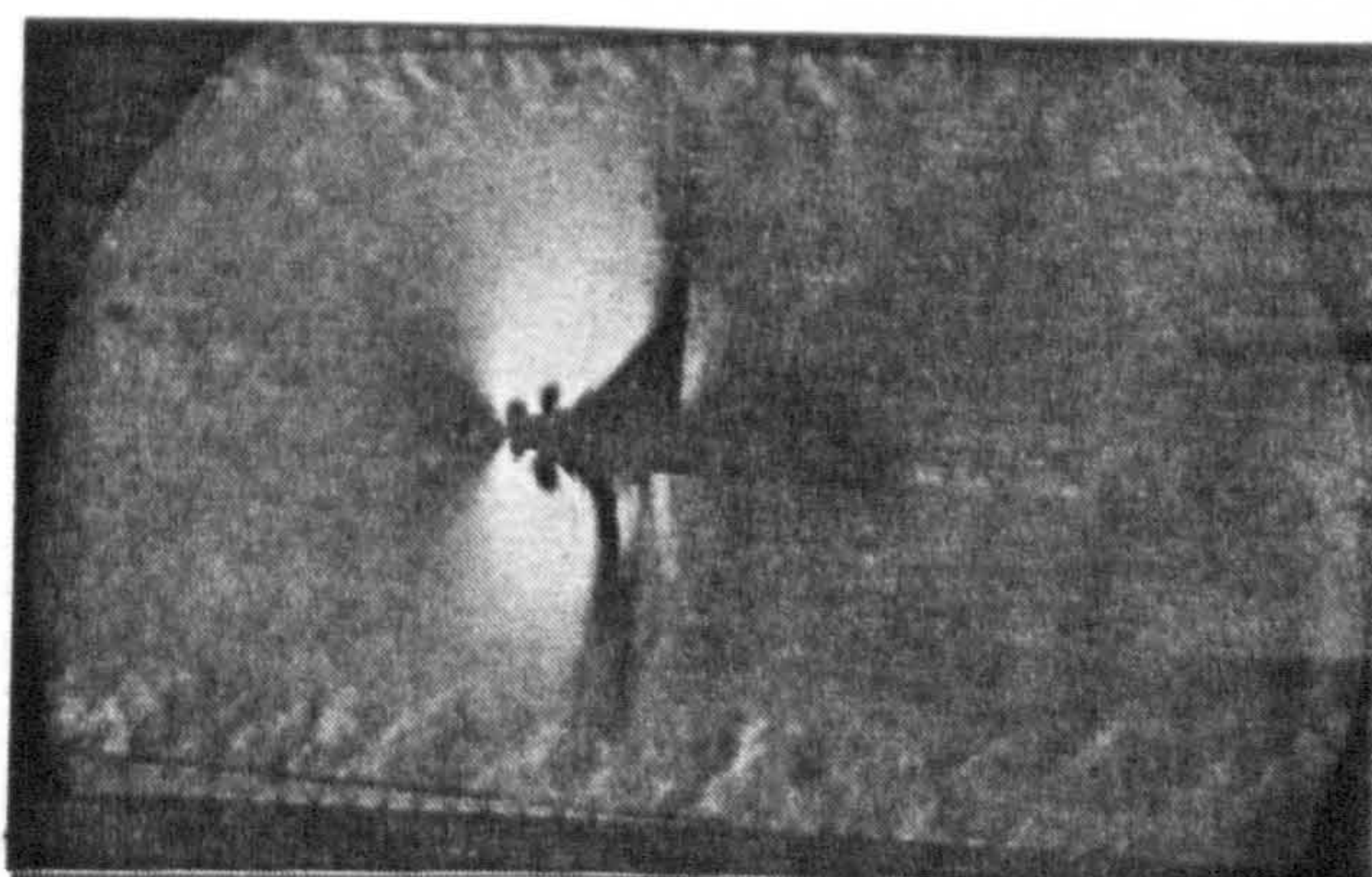
Figure 50. Magnitude of sidewall pressure fluctuations.



$M_\infty = 0.797$

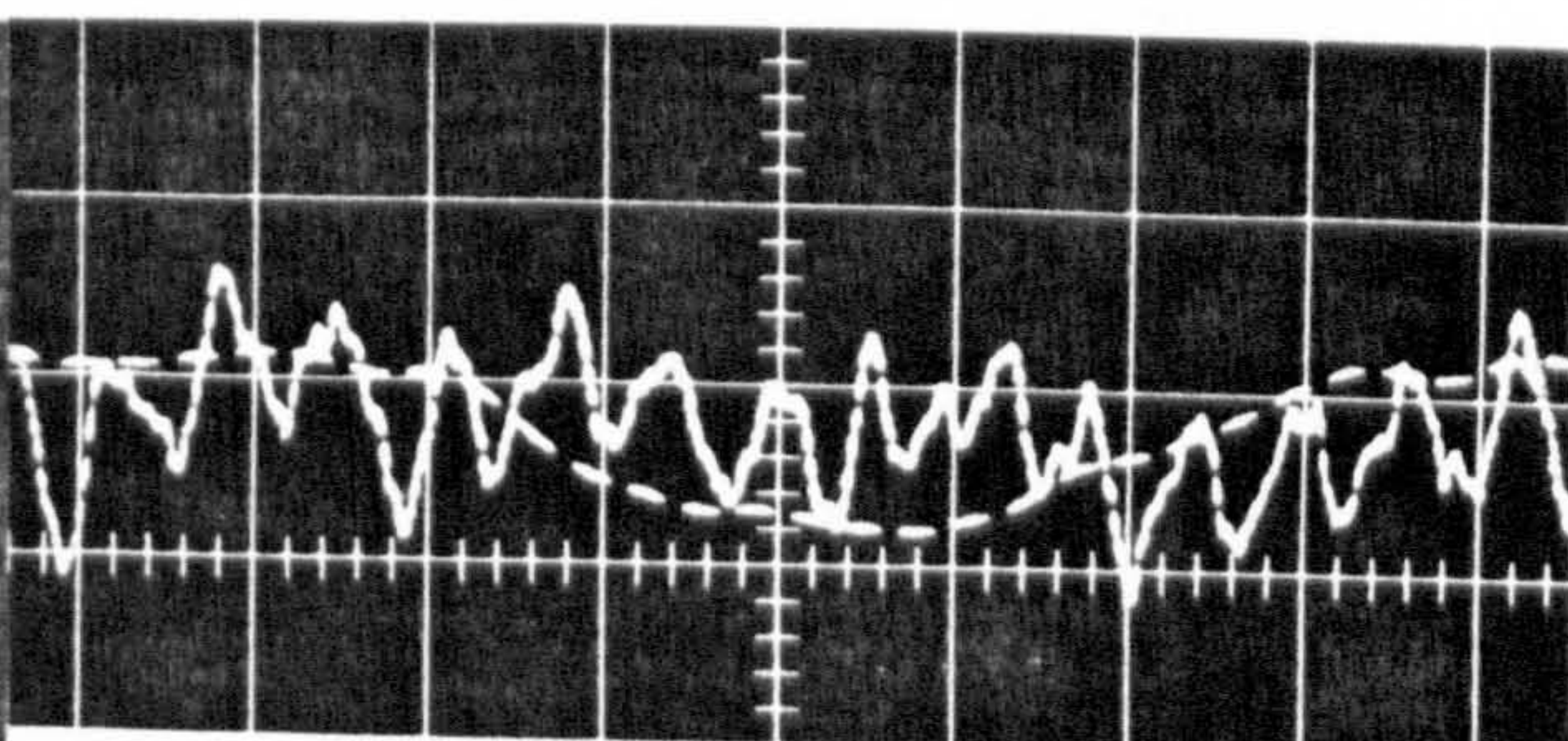


$M_\infty = 0.797$

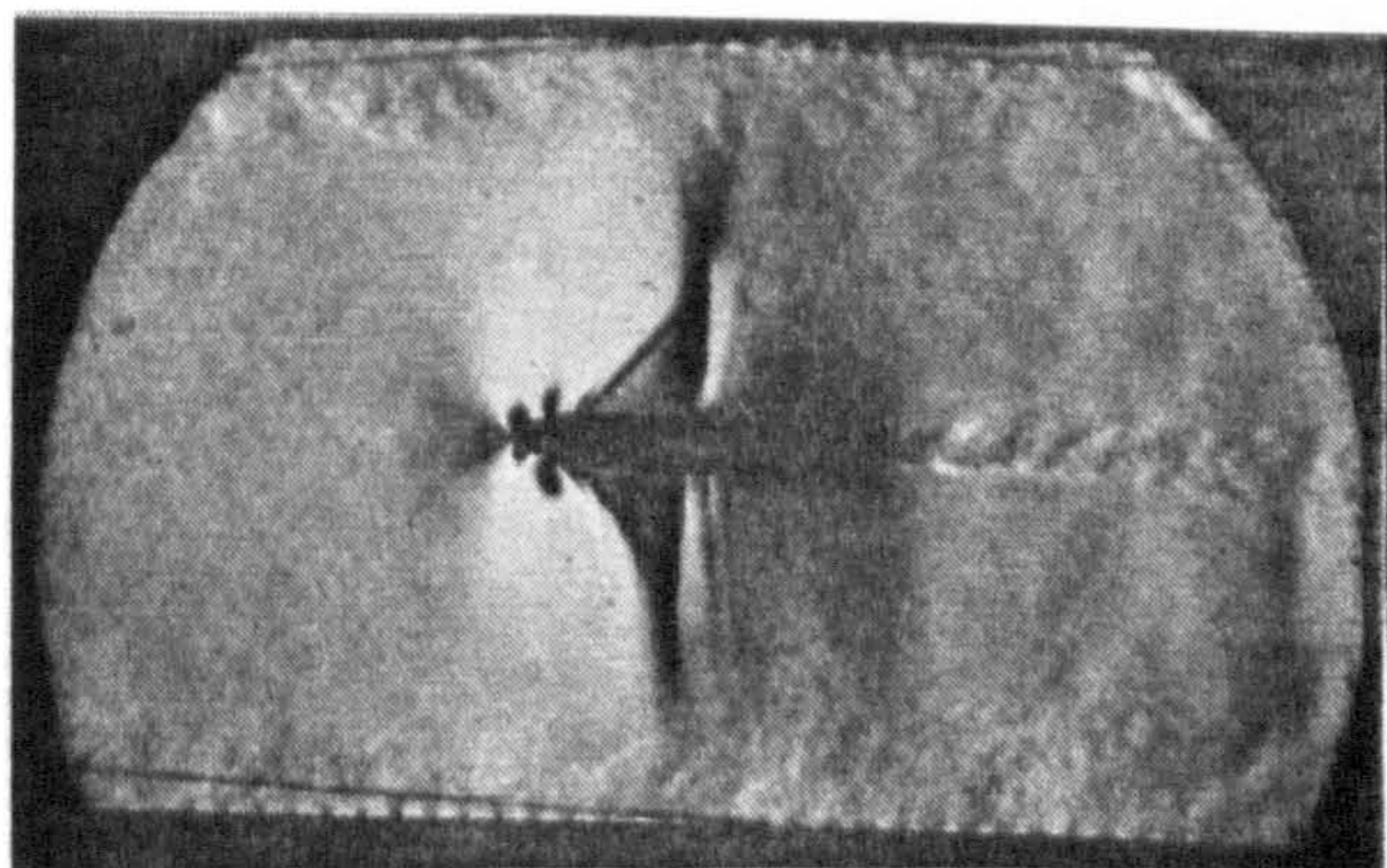


OSCILLATORY
FLOW

0.844

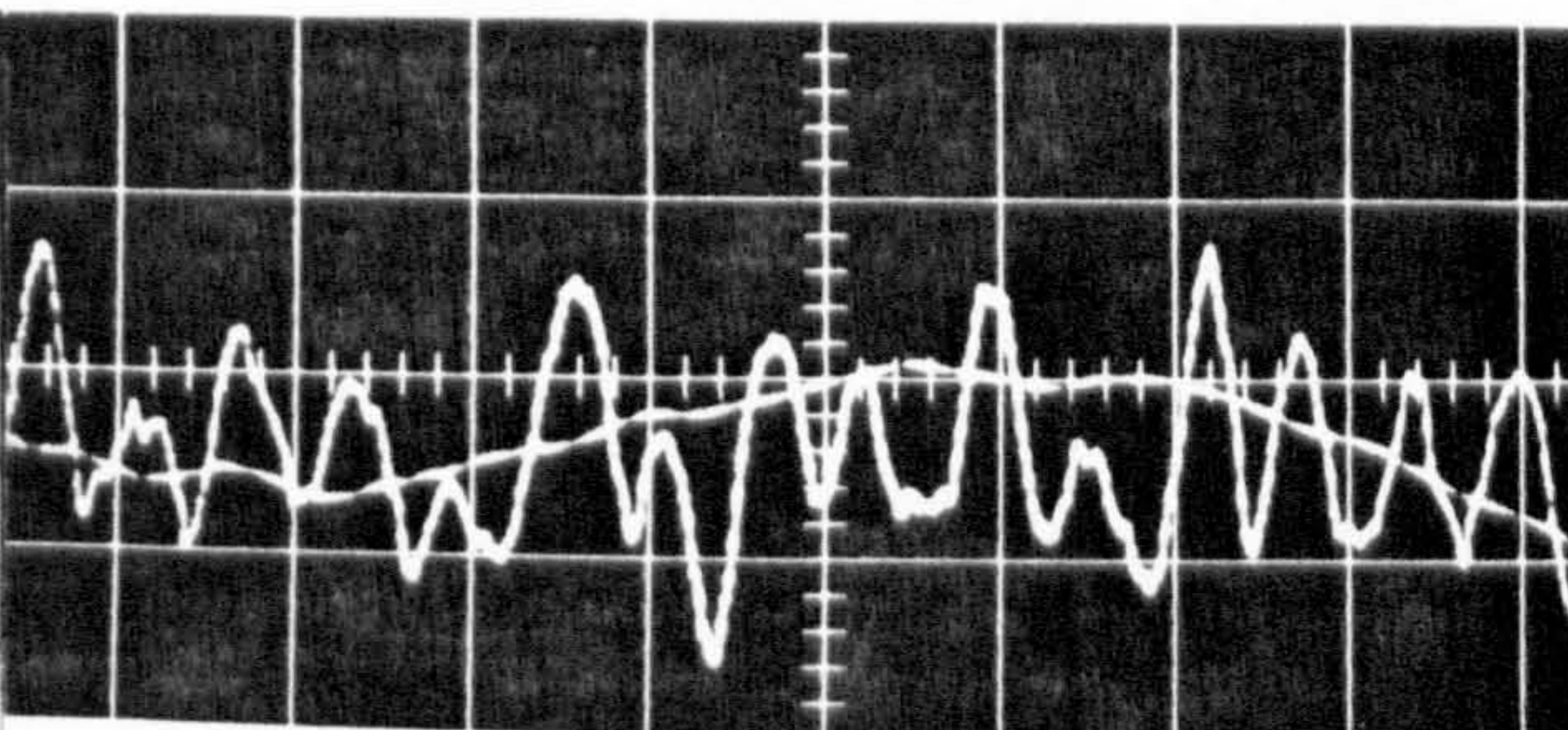


0.839 OSCILLATORY FLOW

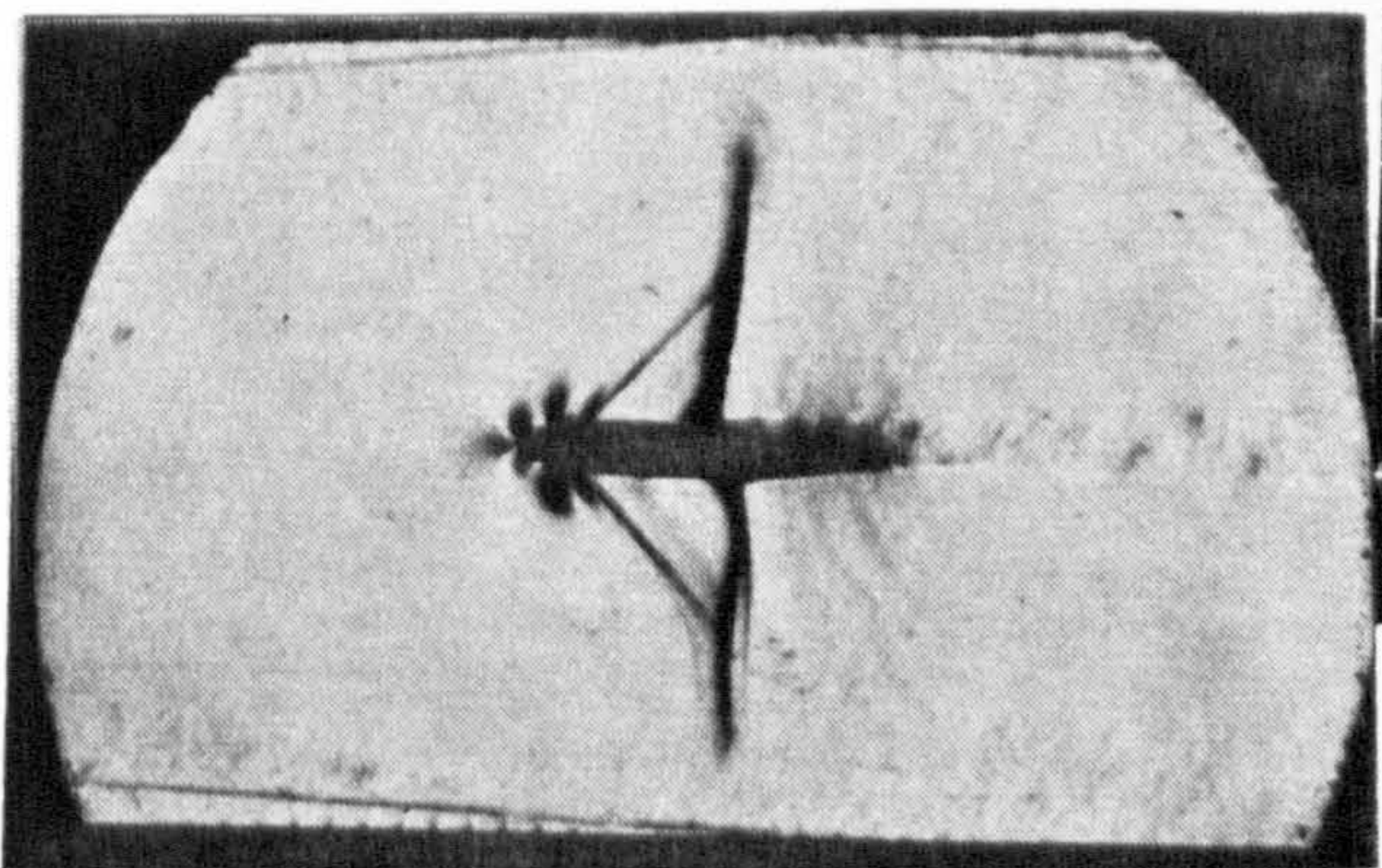


OSCILLATORY
FLOW

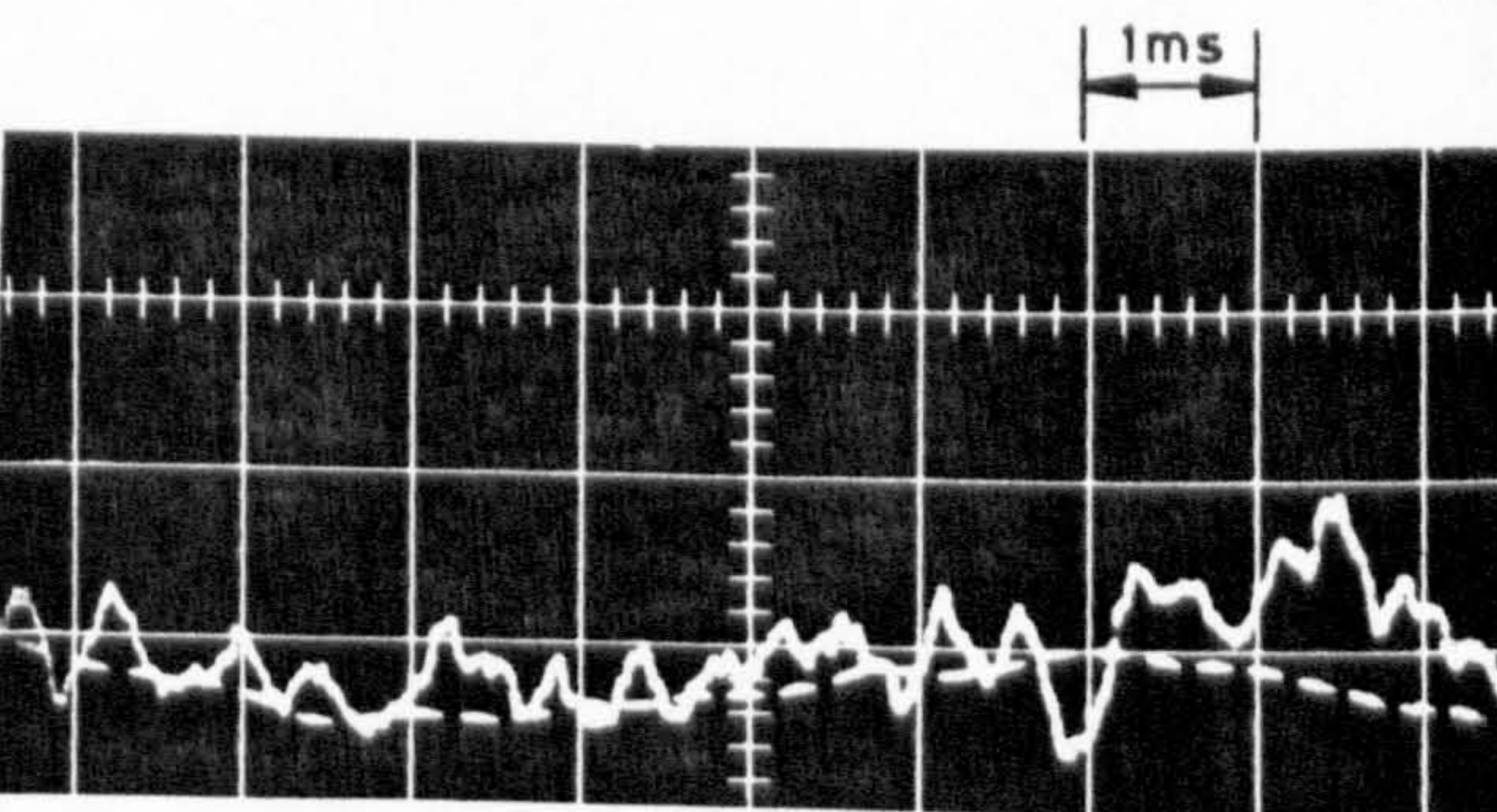
0.873



0.871 OSCILLATORY FLOW



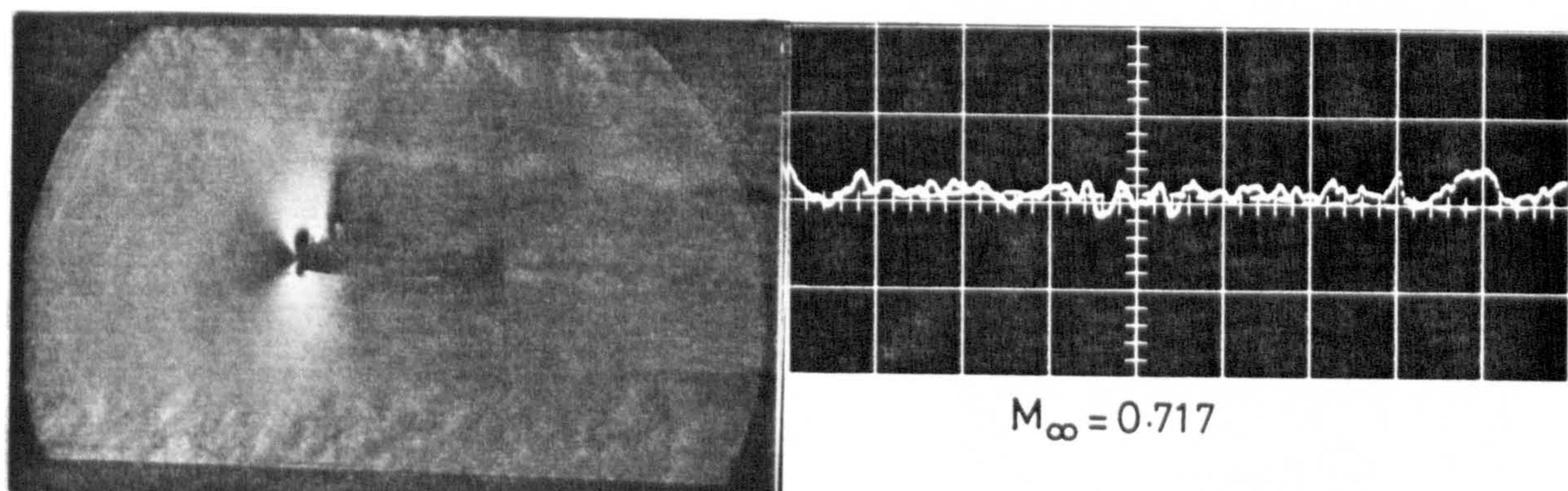
0.906



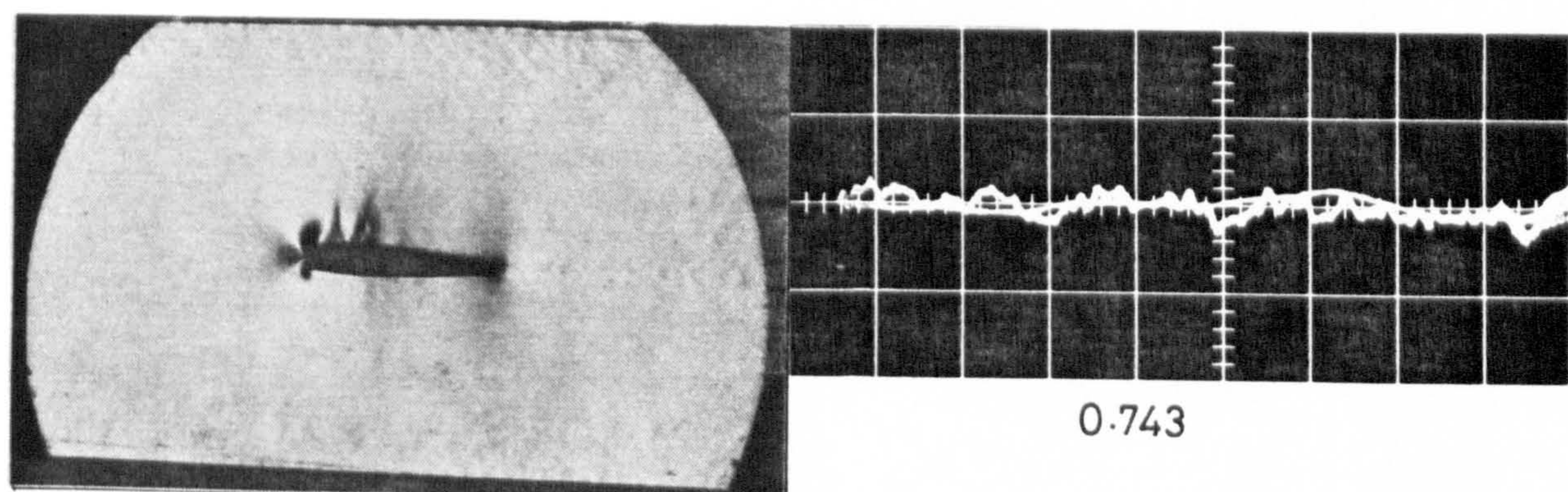
0.903

0.48
p.s.i.

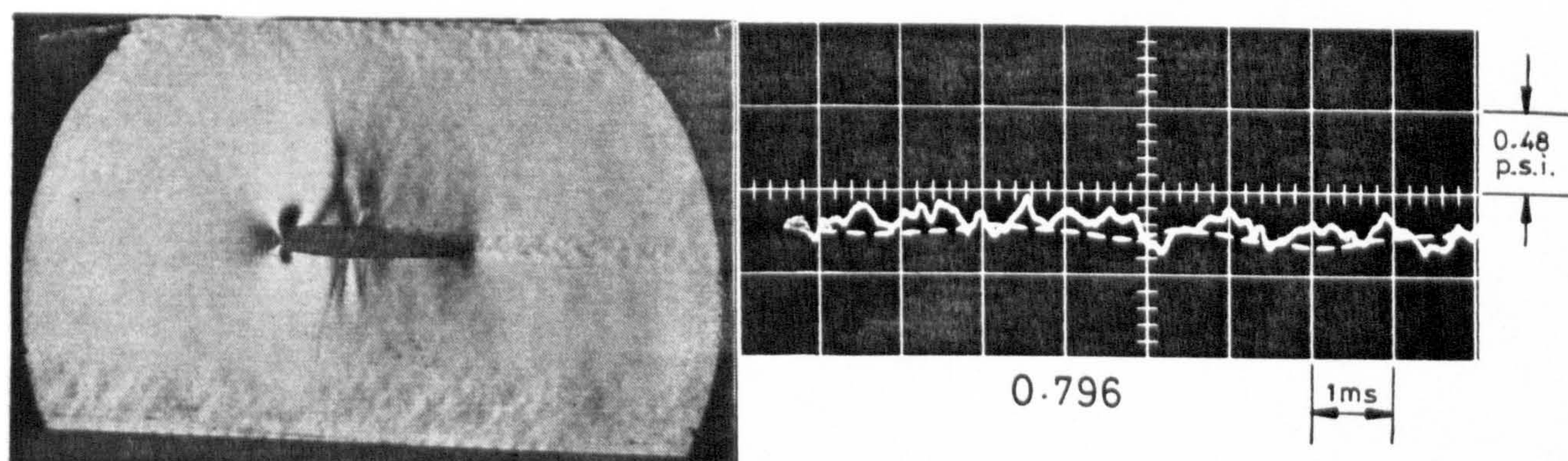
Figure 51. Schlieren pictures and pressure traces.
NACA 0012 aerofoil, $\alpha = 3^\circ$ (fixed transition).



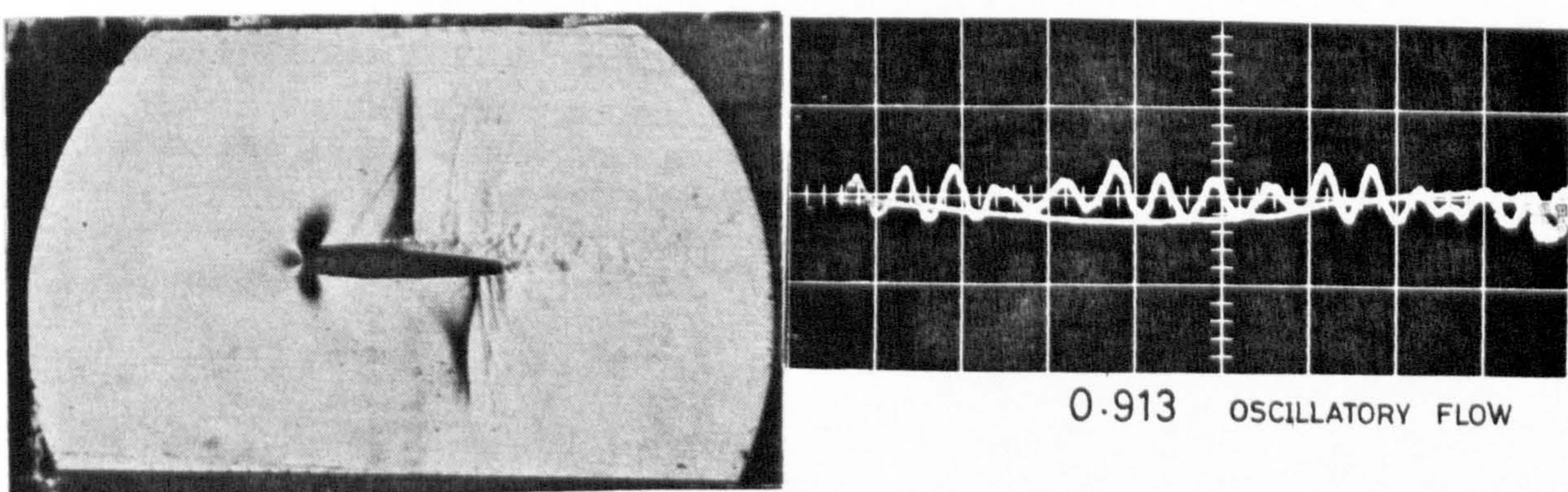
$M_{\infty} = 0.723$



0.750

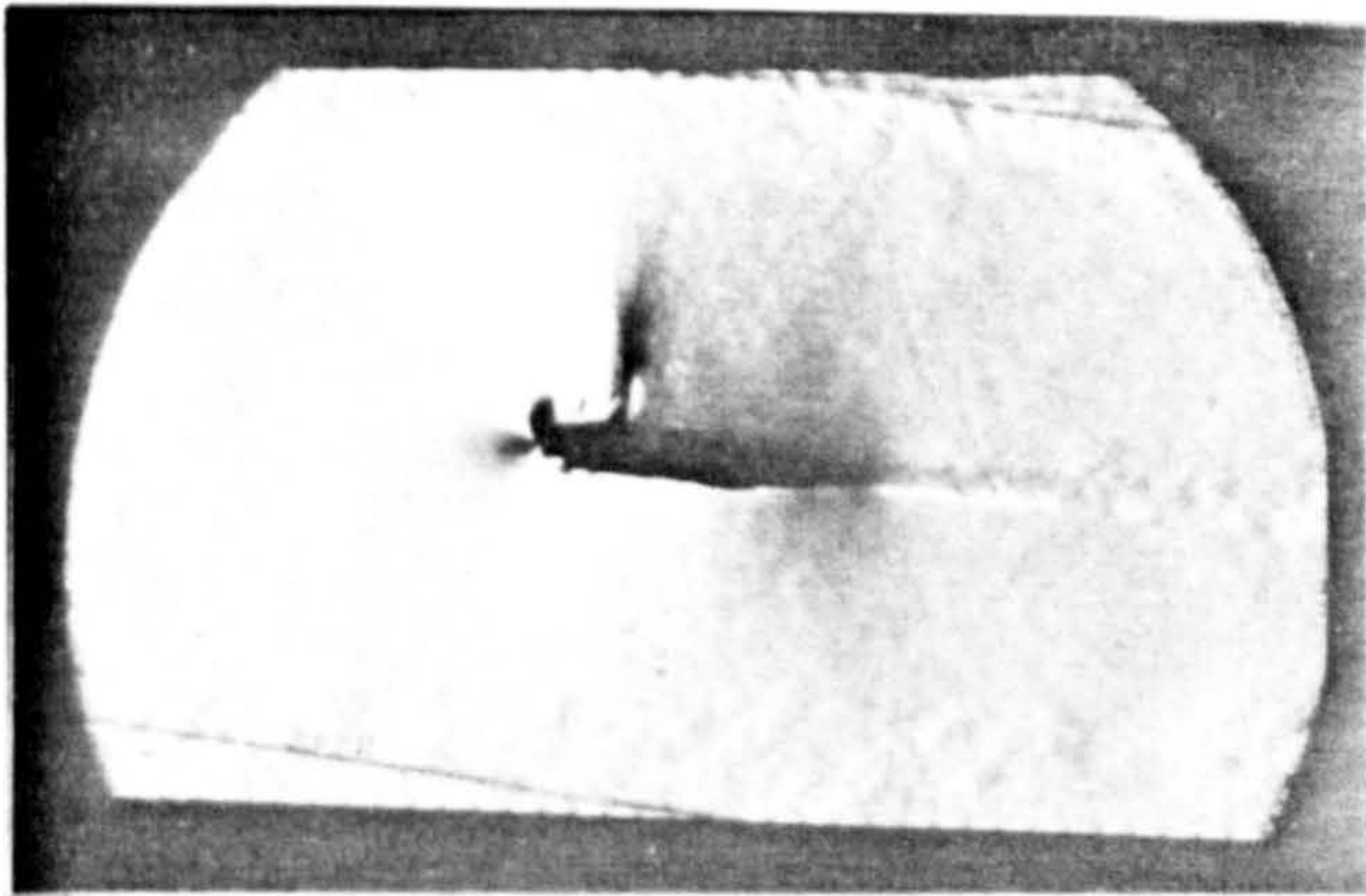


0.796

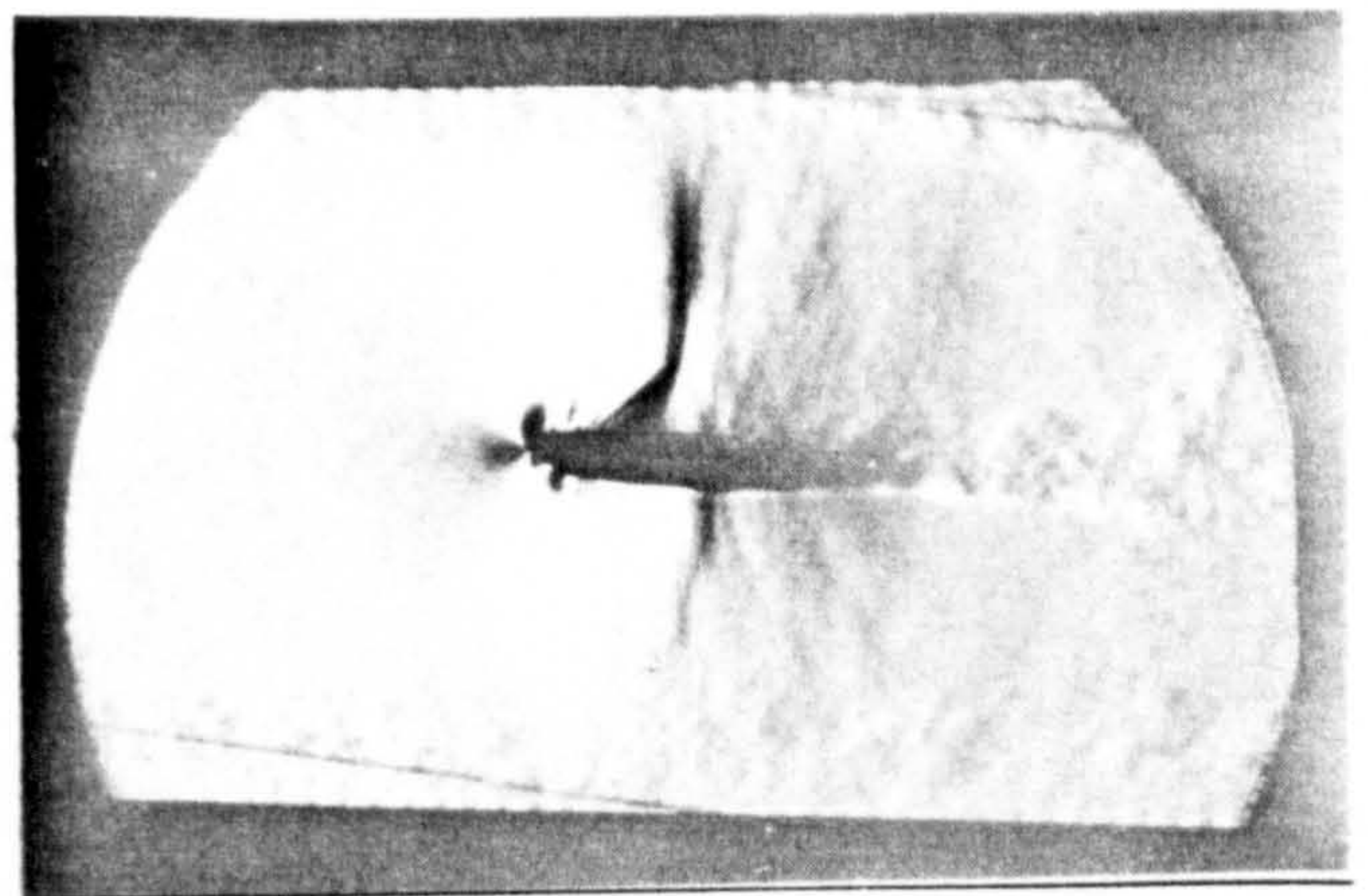


0.919 OSCILLATORY FLOW

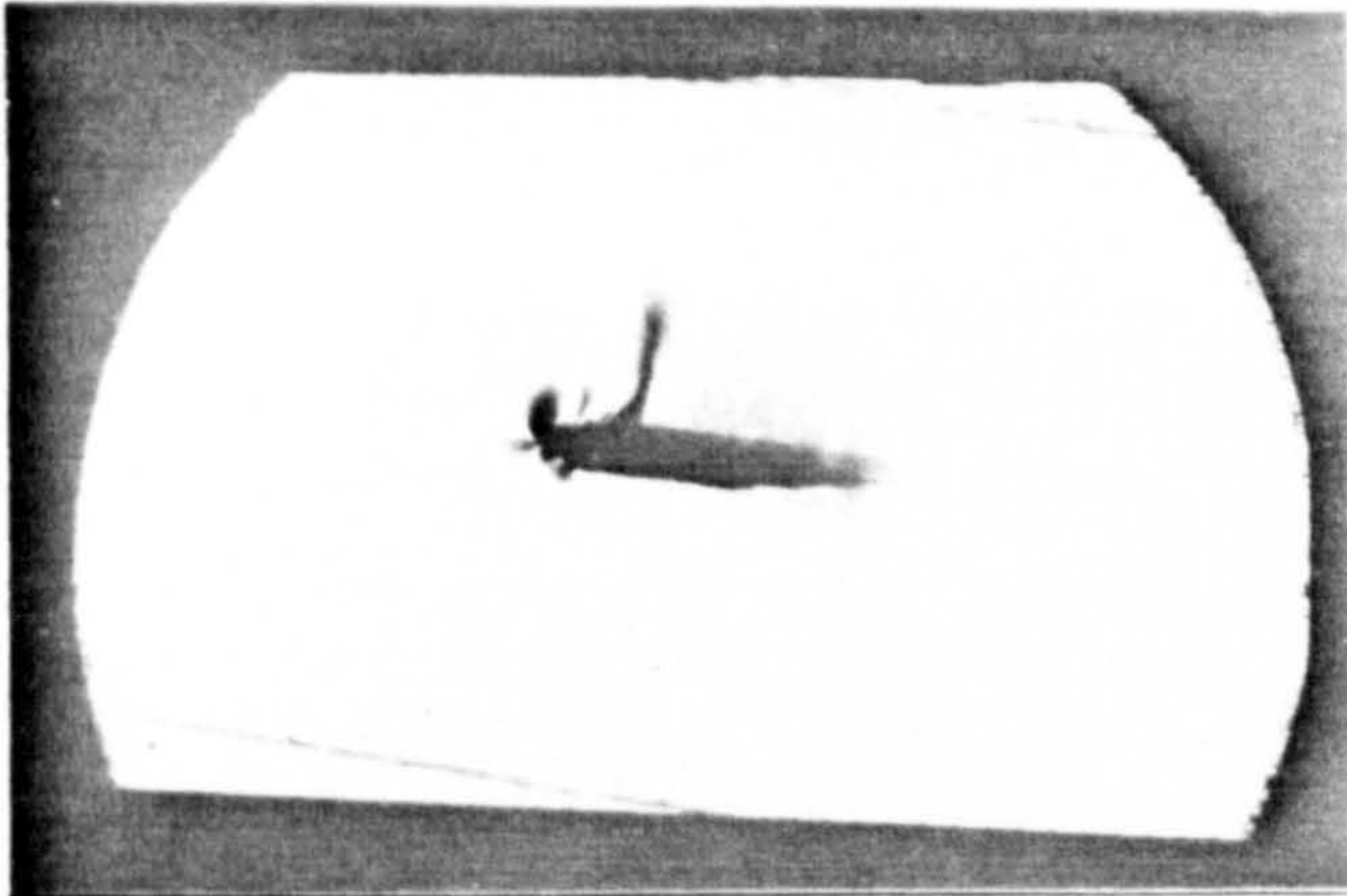
Figure 52. Schlieren pictures and pressure traces.
NACA 0012 aerofoil, $\alpha = 3^\circ$ (free transition).



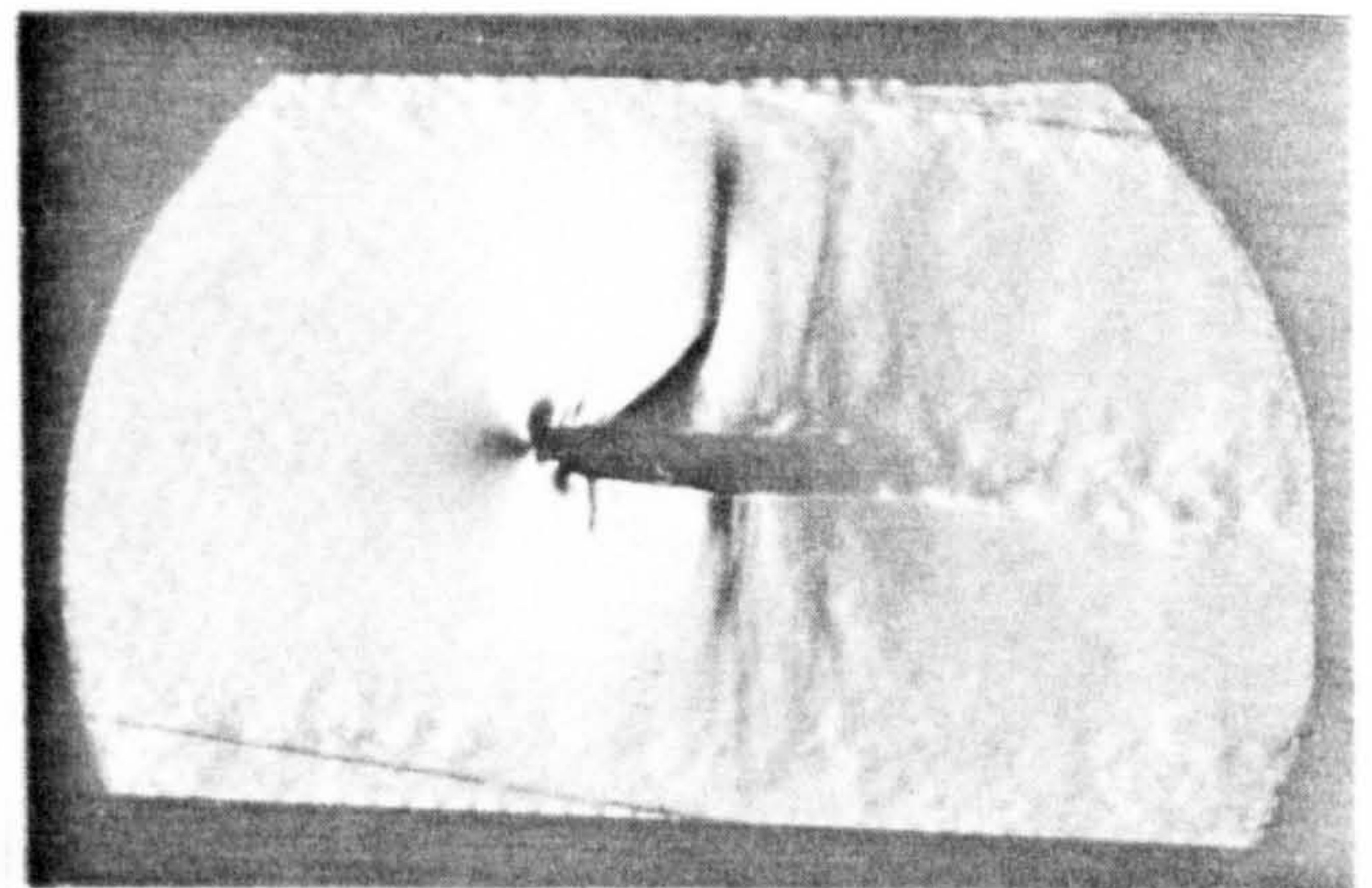
$M_{\infty} = 0.722$



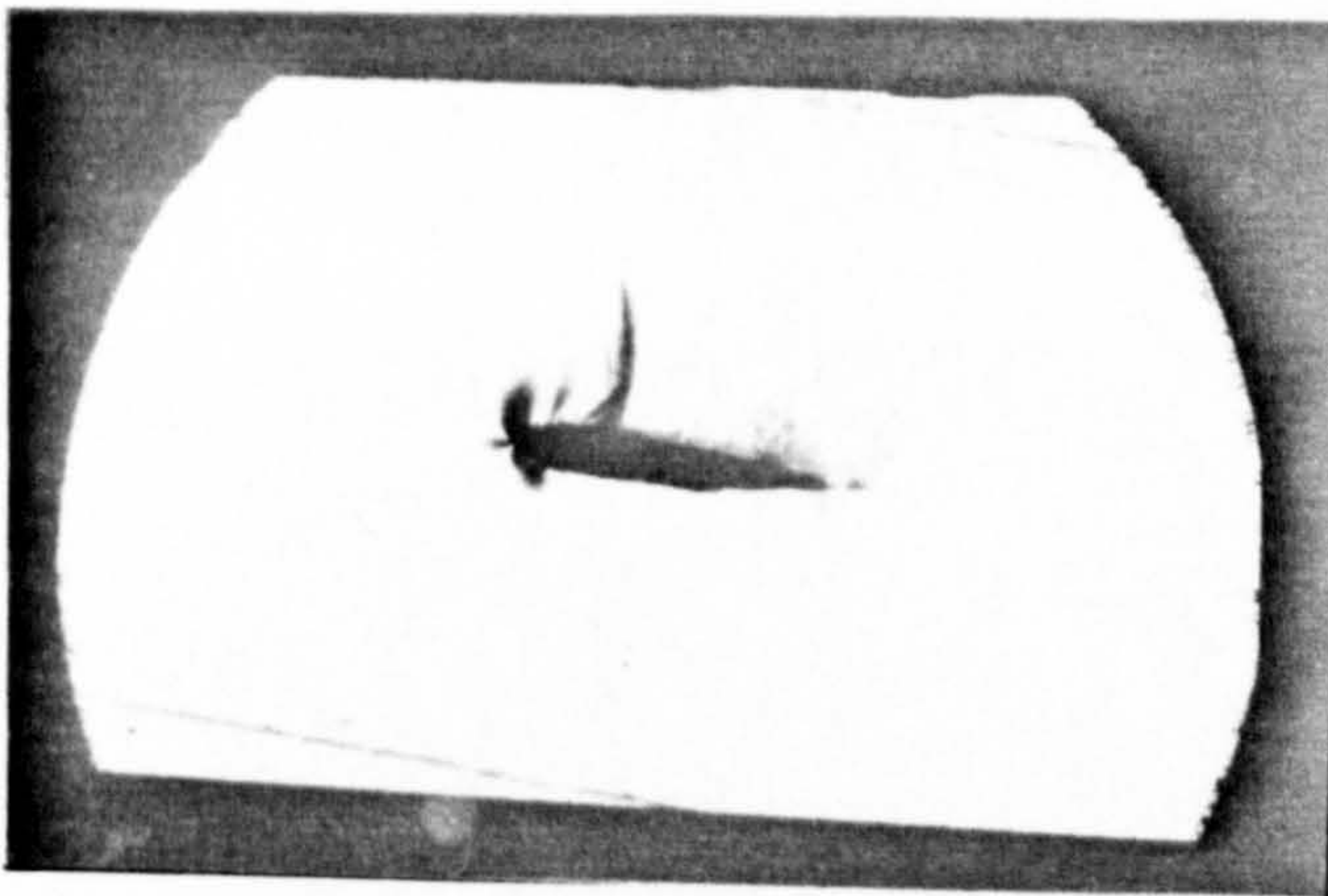
0.859 OSCILLATORY FLOW



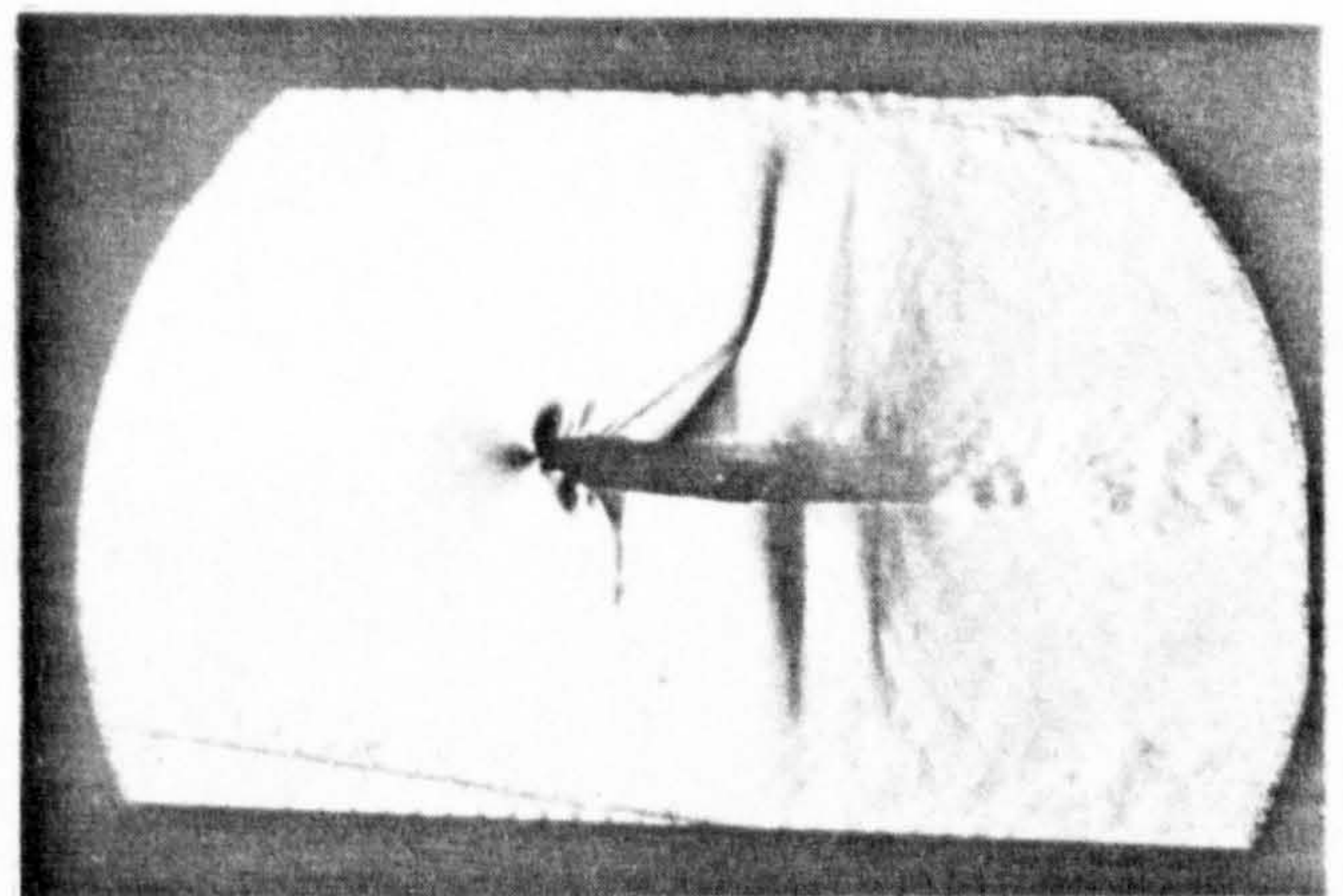
0.748



0.882 OSCILLATORY FLOW



0.798



0.922

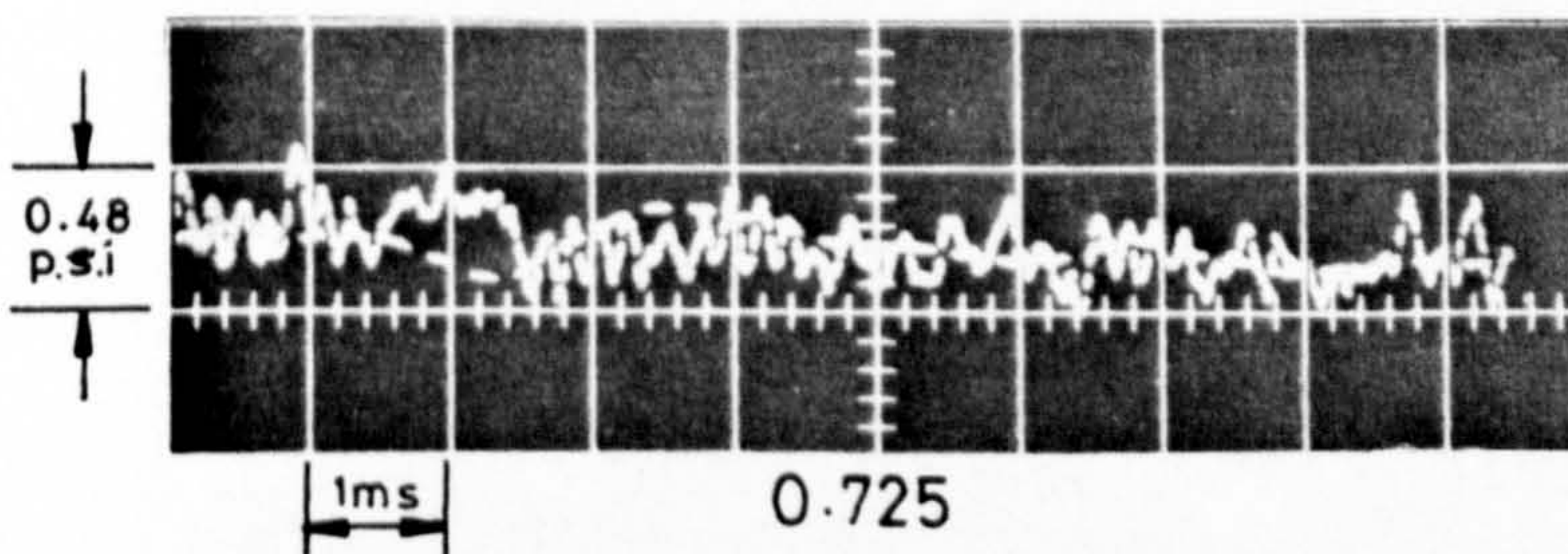
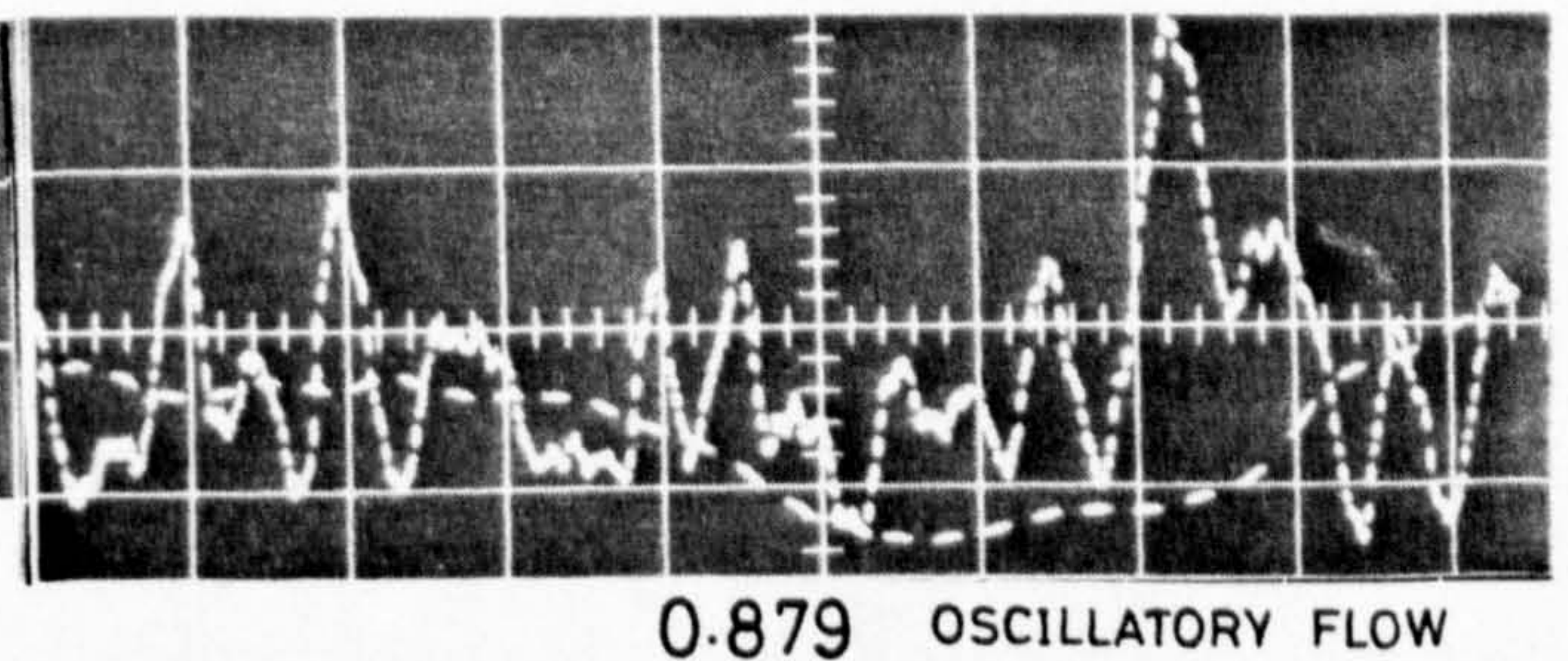
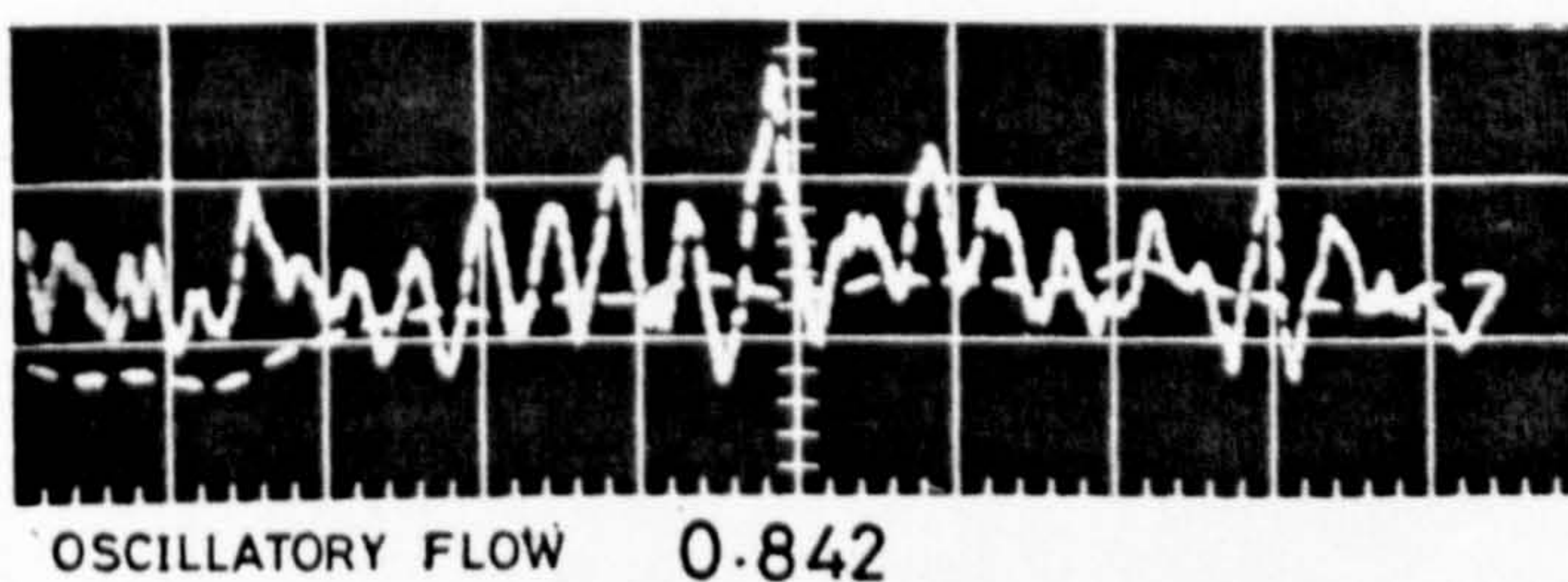
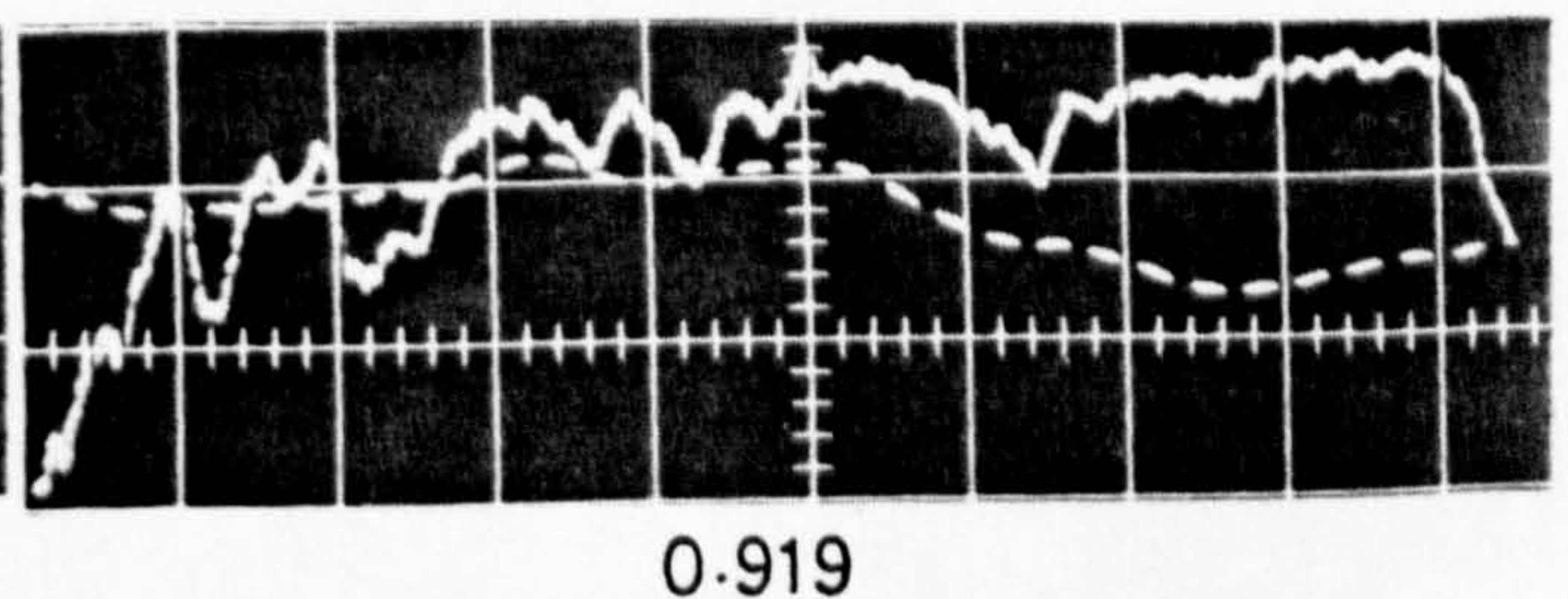


Figure 53. NACA 0012 aerofoil, $\alpha = 6^\circ$ (fixed transition)

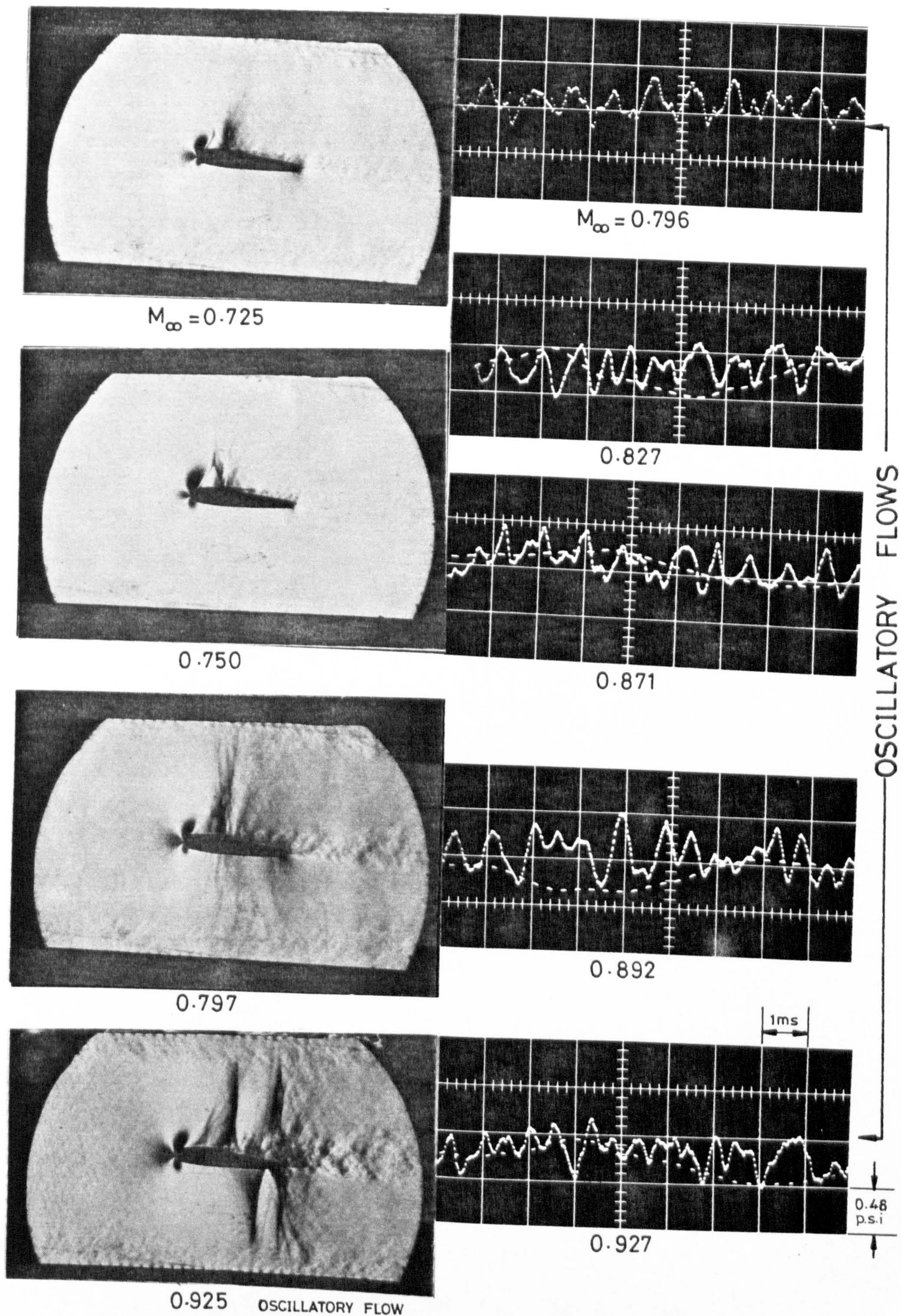


Figure 54. NACA 0012 aerofoil, $\alpha = 6^\circ$ (free transition).

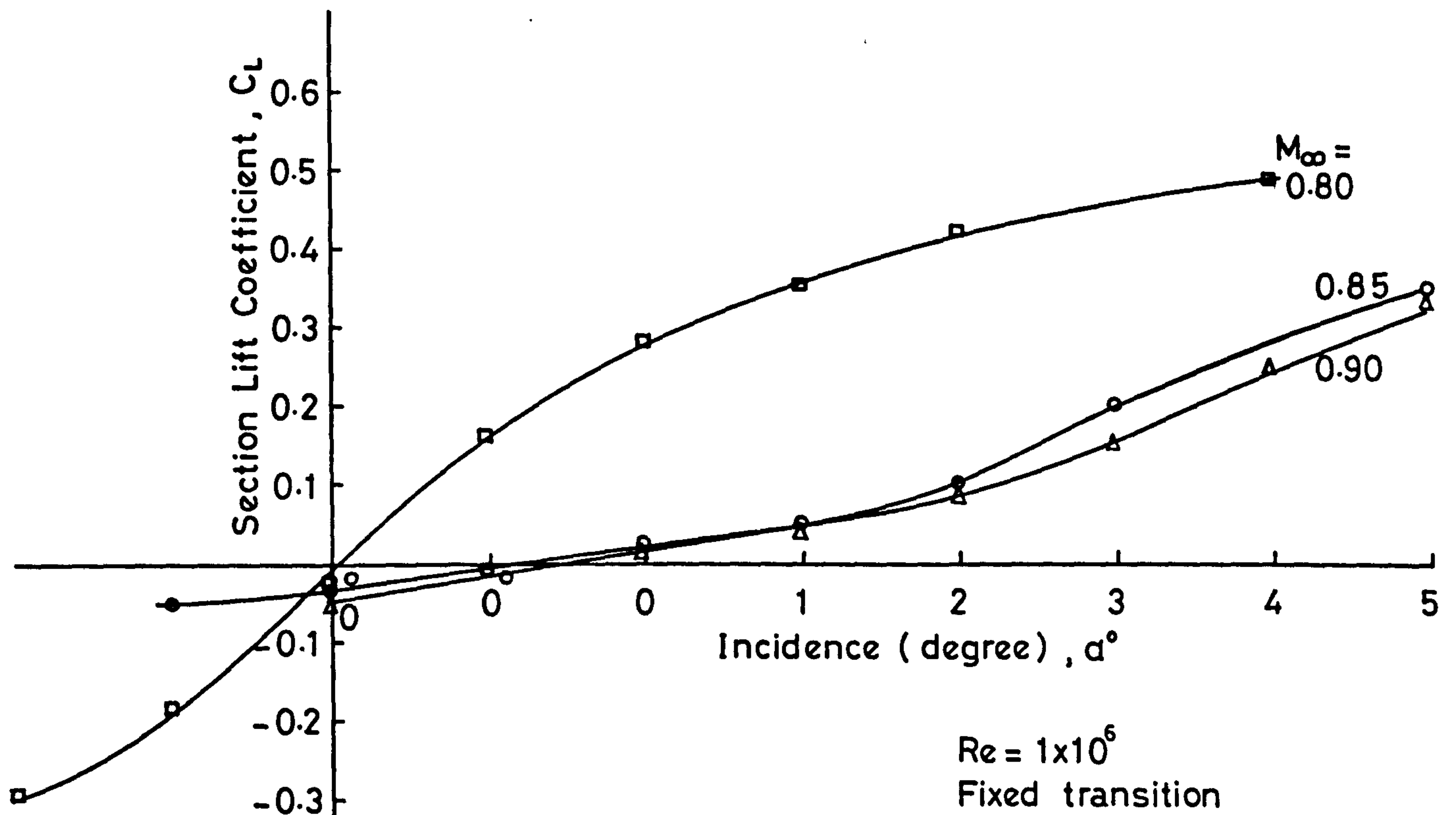


Figure 55. C_L vs α curves (ref. 53)

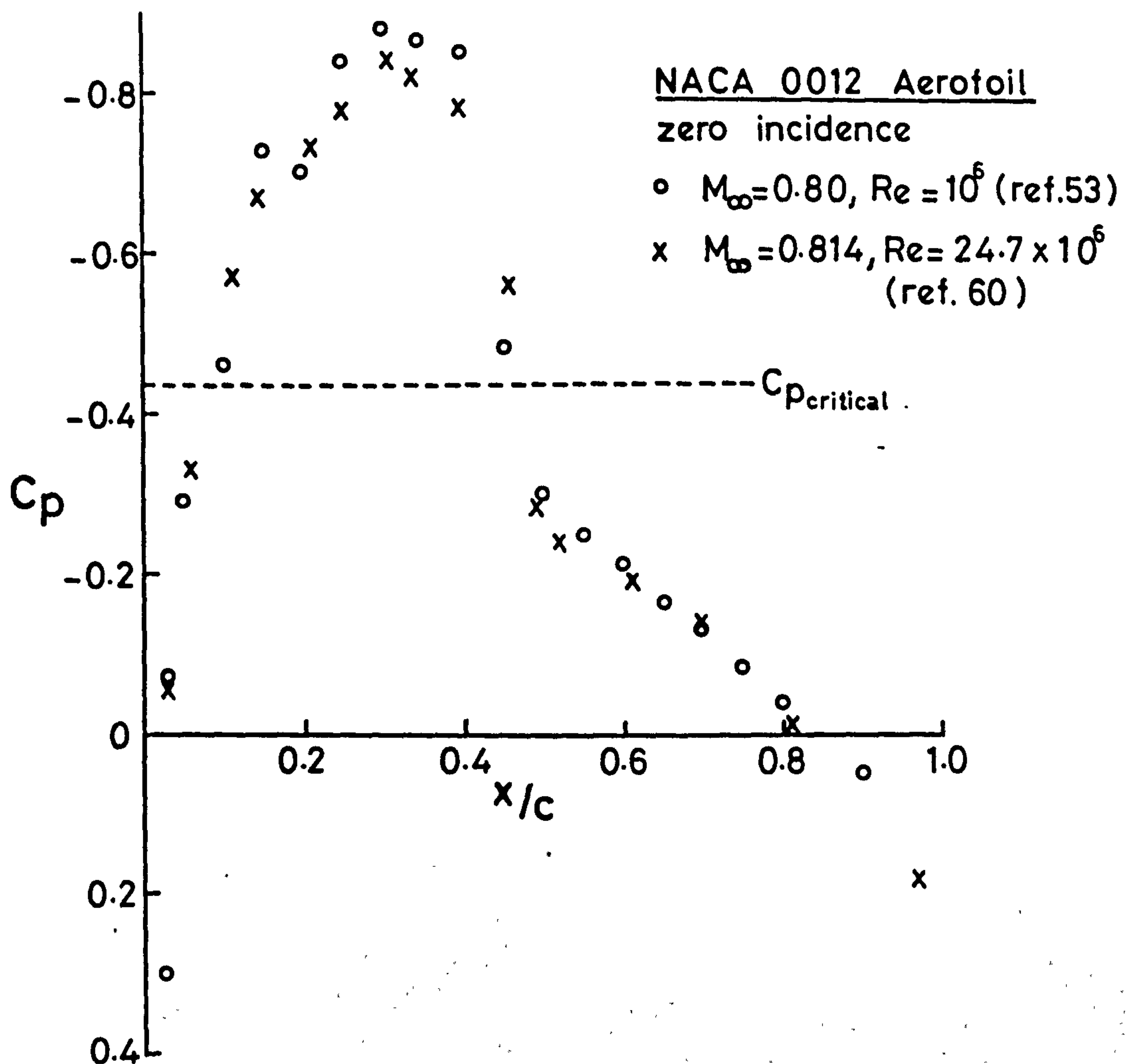


Figure 56. Pressure distribution: low and high Reynolds numbers.

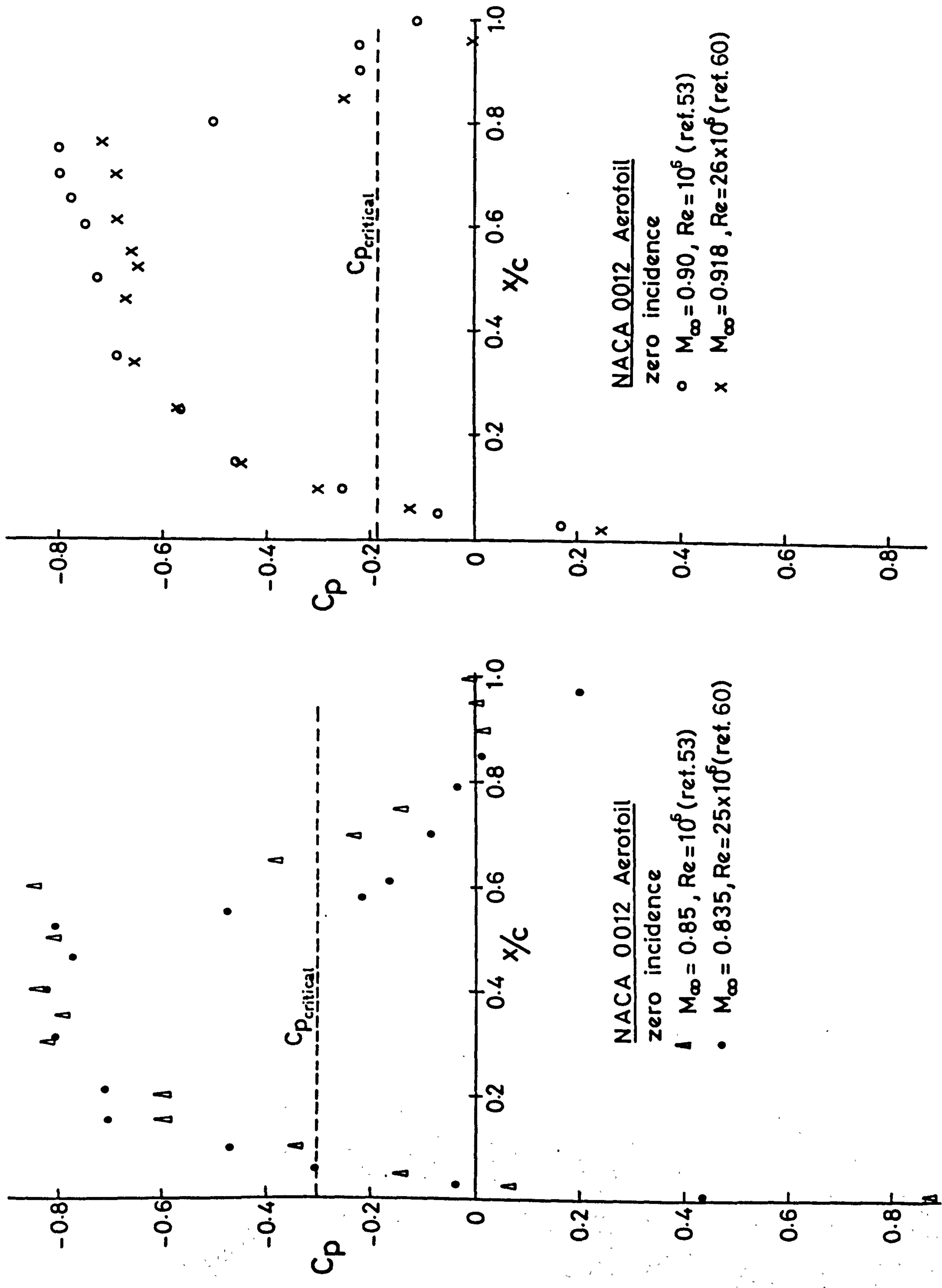
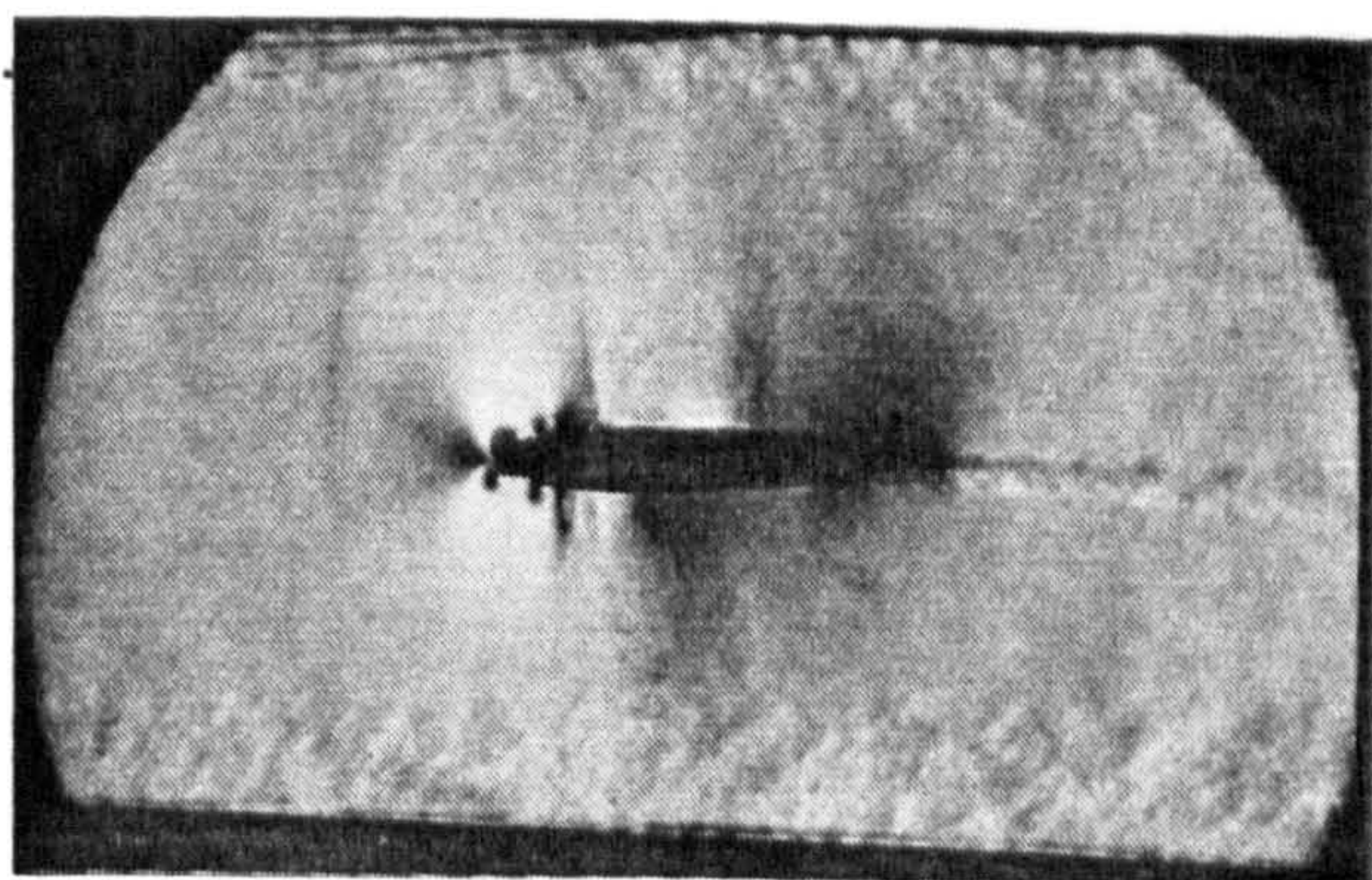
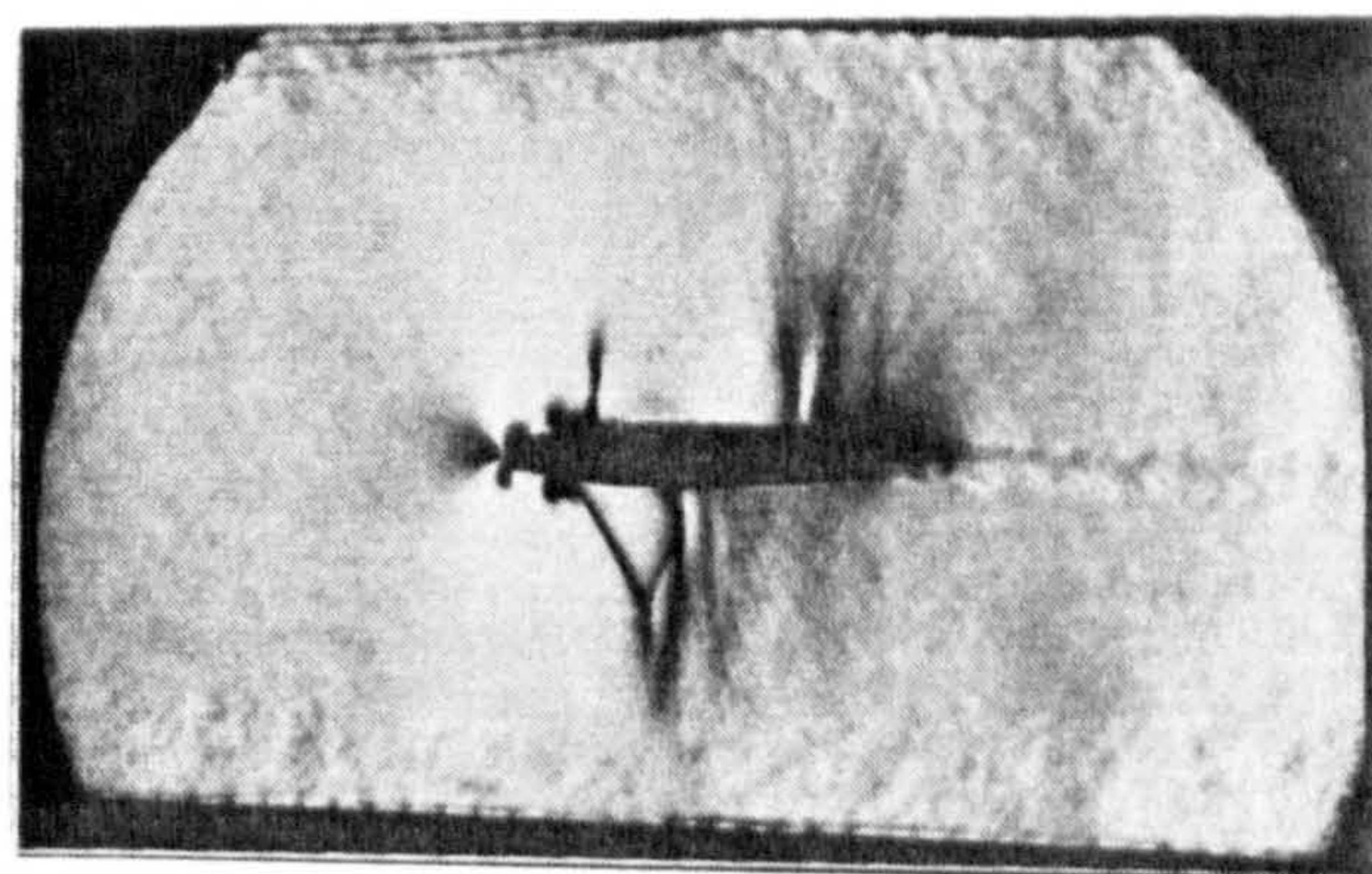


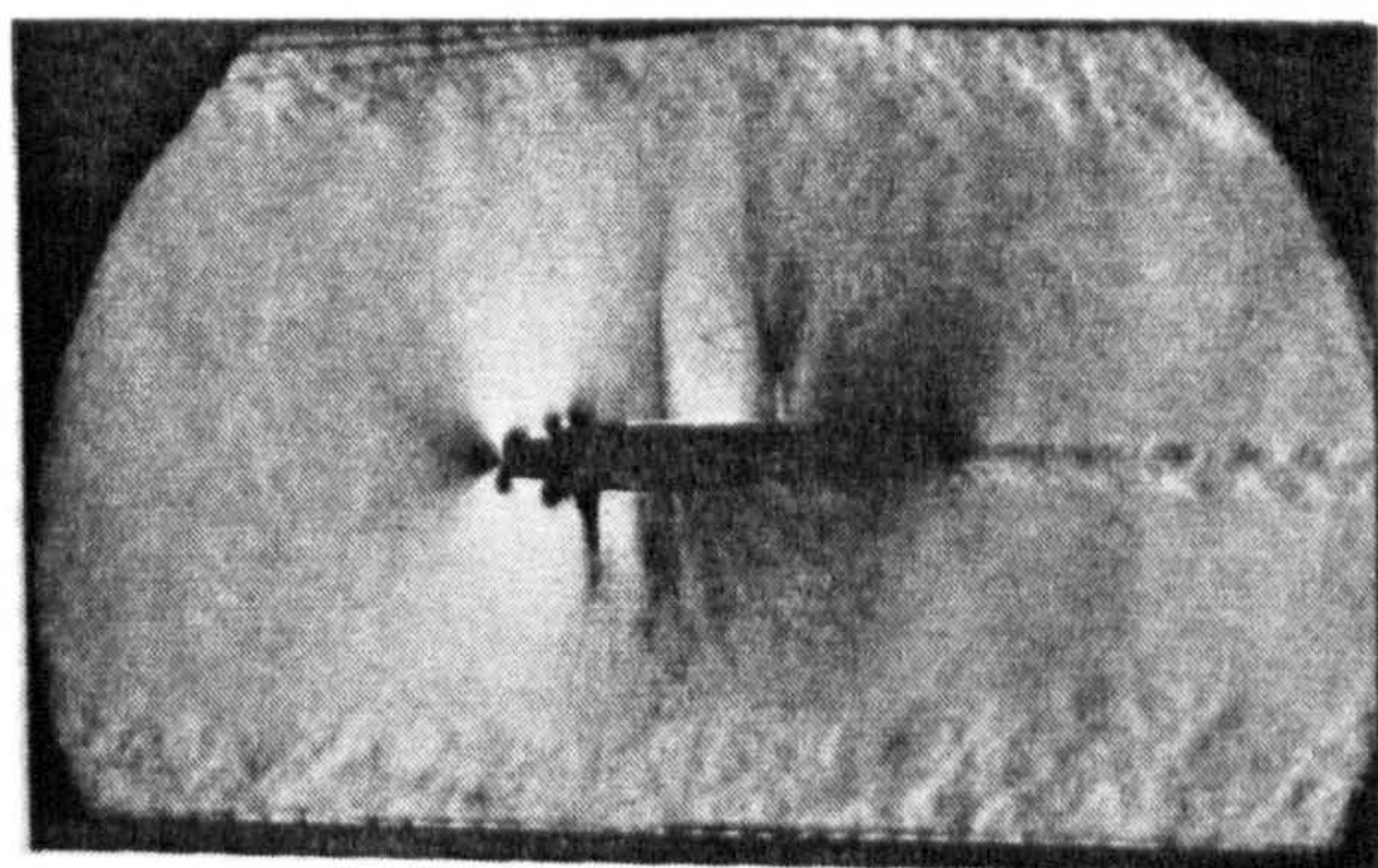
Figure 56 (contd.) Pressure distributions – low and high Reynolds numbers.



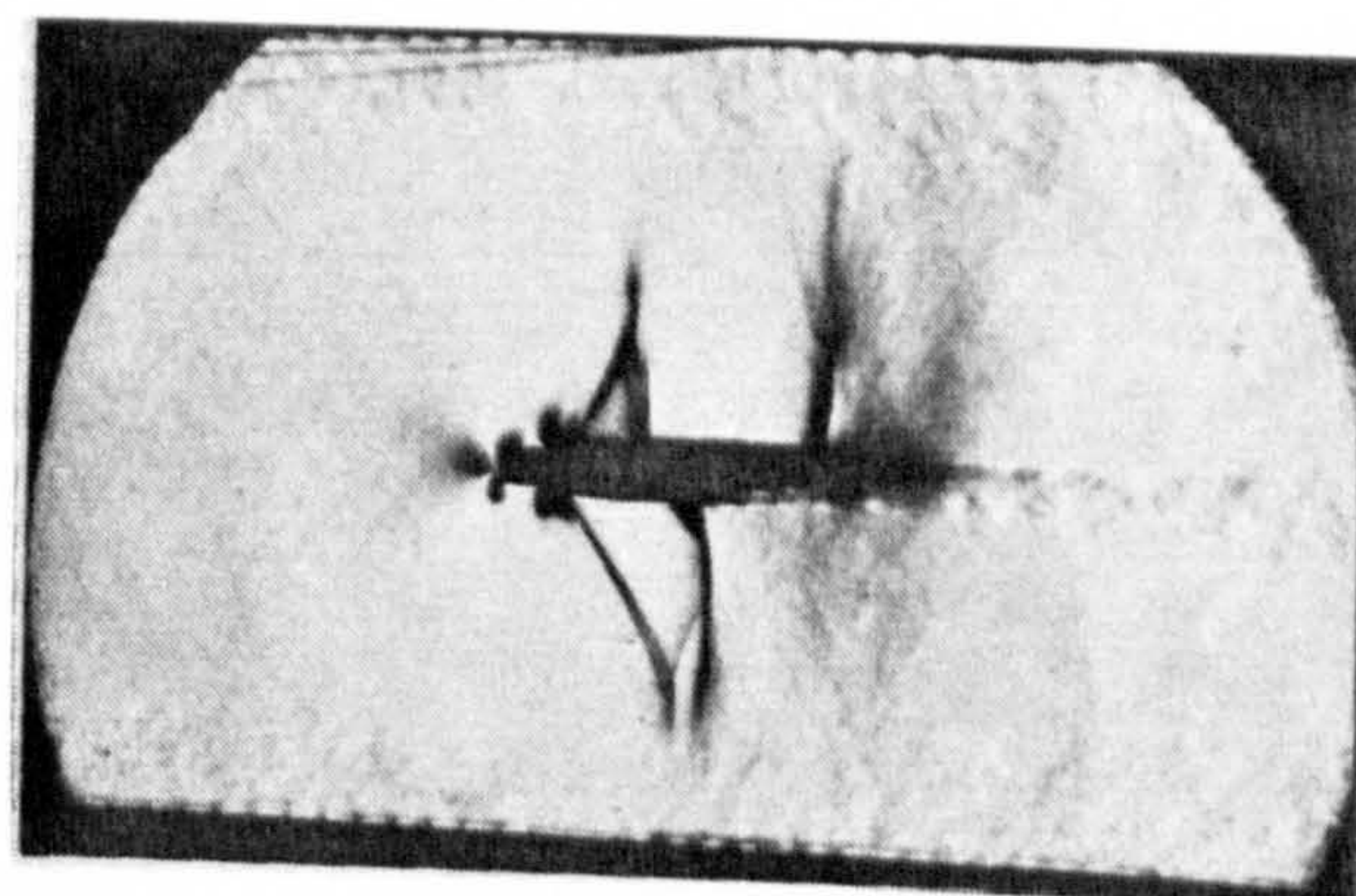
$M_{\infty} = 0.755$



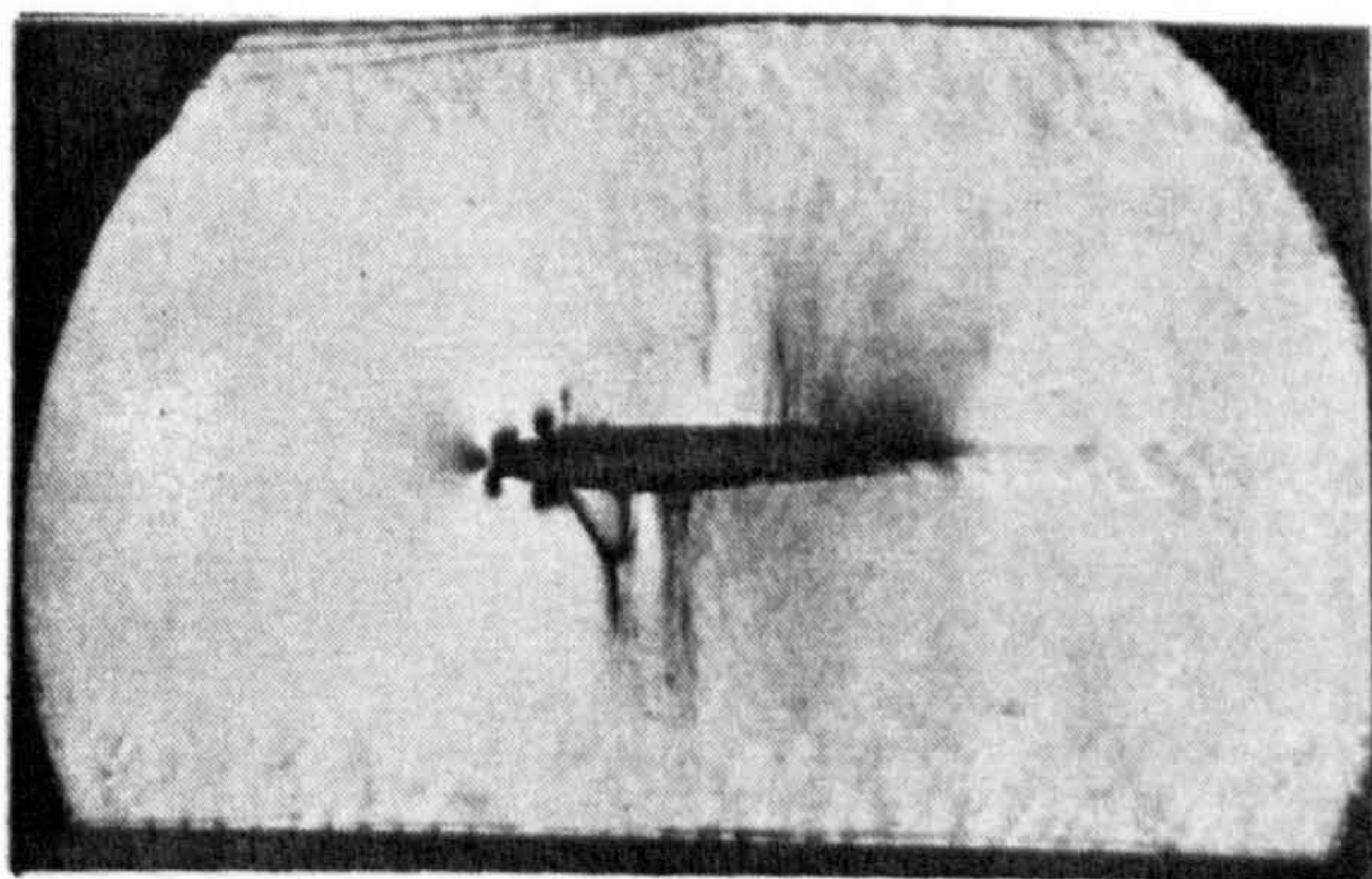
0.842



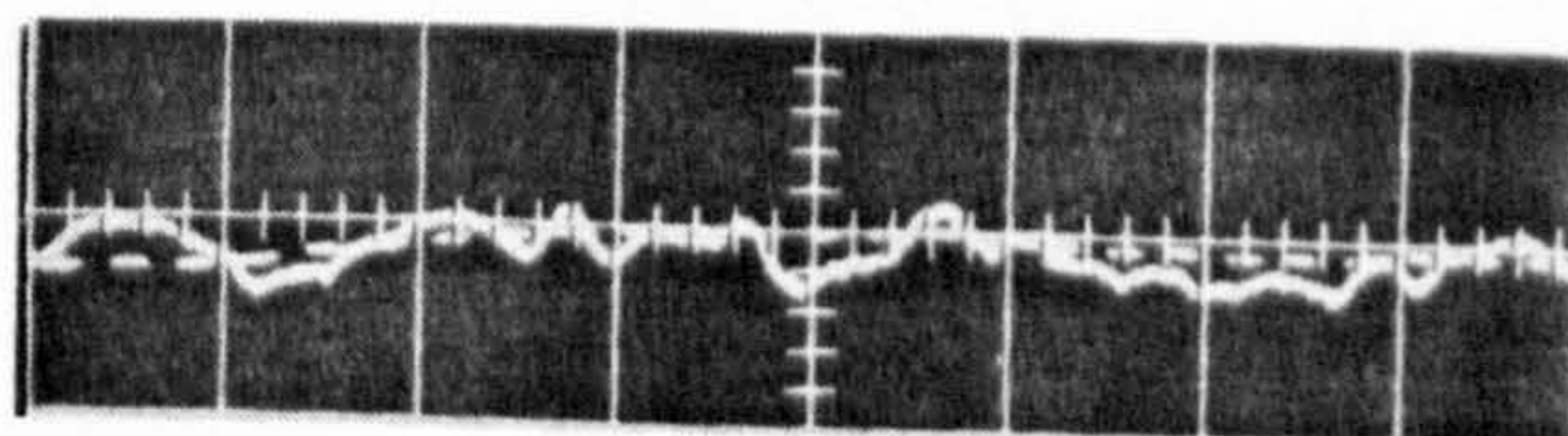
0.778



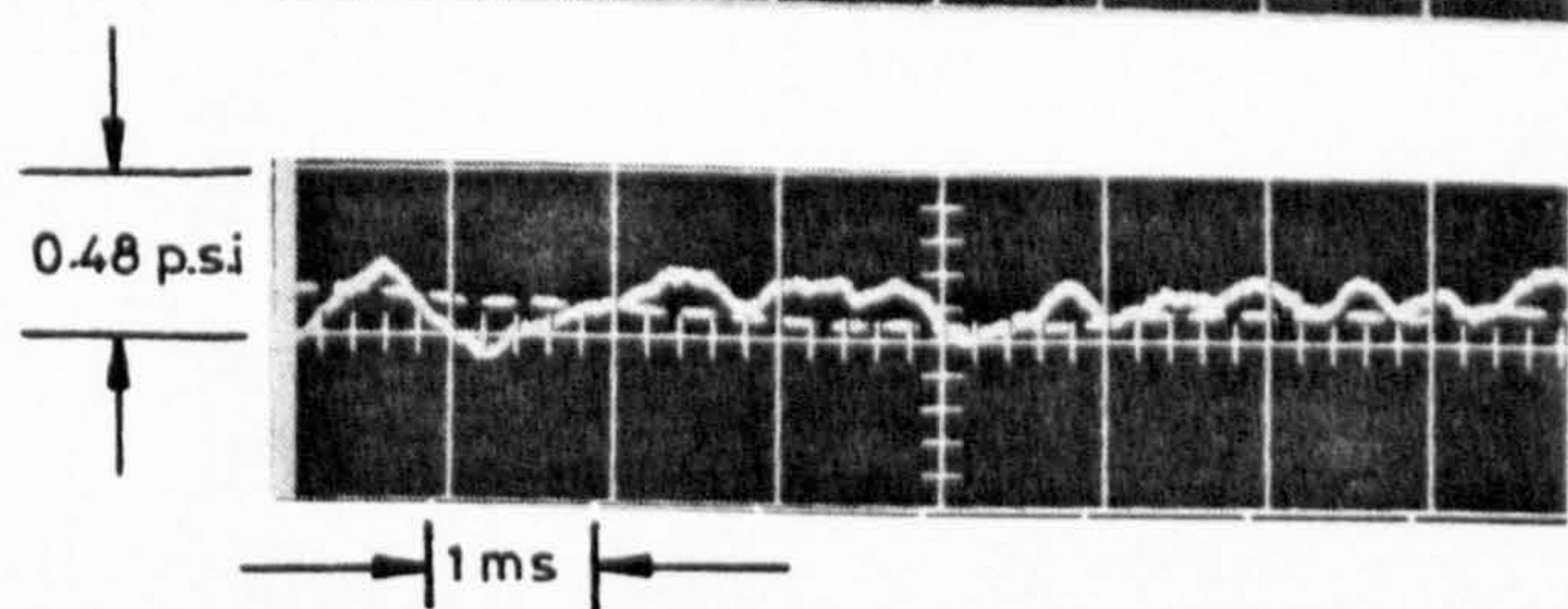
0.876



0.811

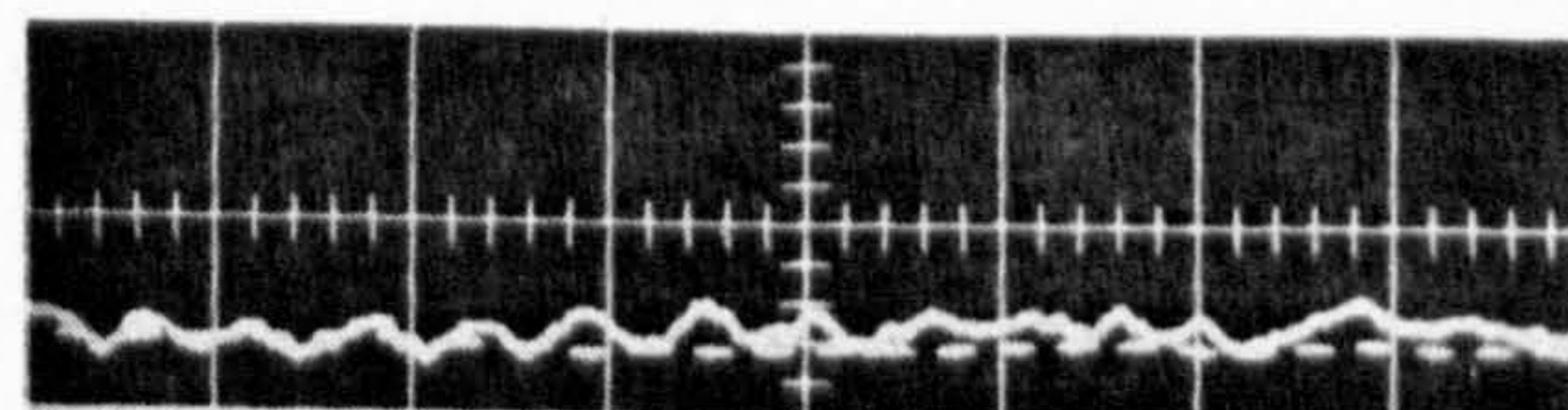


$M_{\infty} = 0.809$



0.839

filtered outputs



0.873

Figure 57. Schlieren pictures and sidewall pressure traces.
CAST 7 Aerofoil, $\alpha = 0^\circ$ (fixed transition)

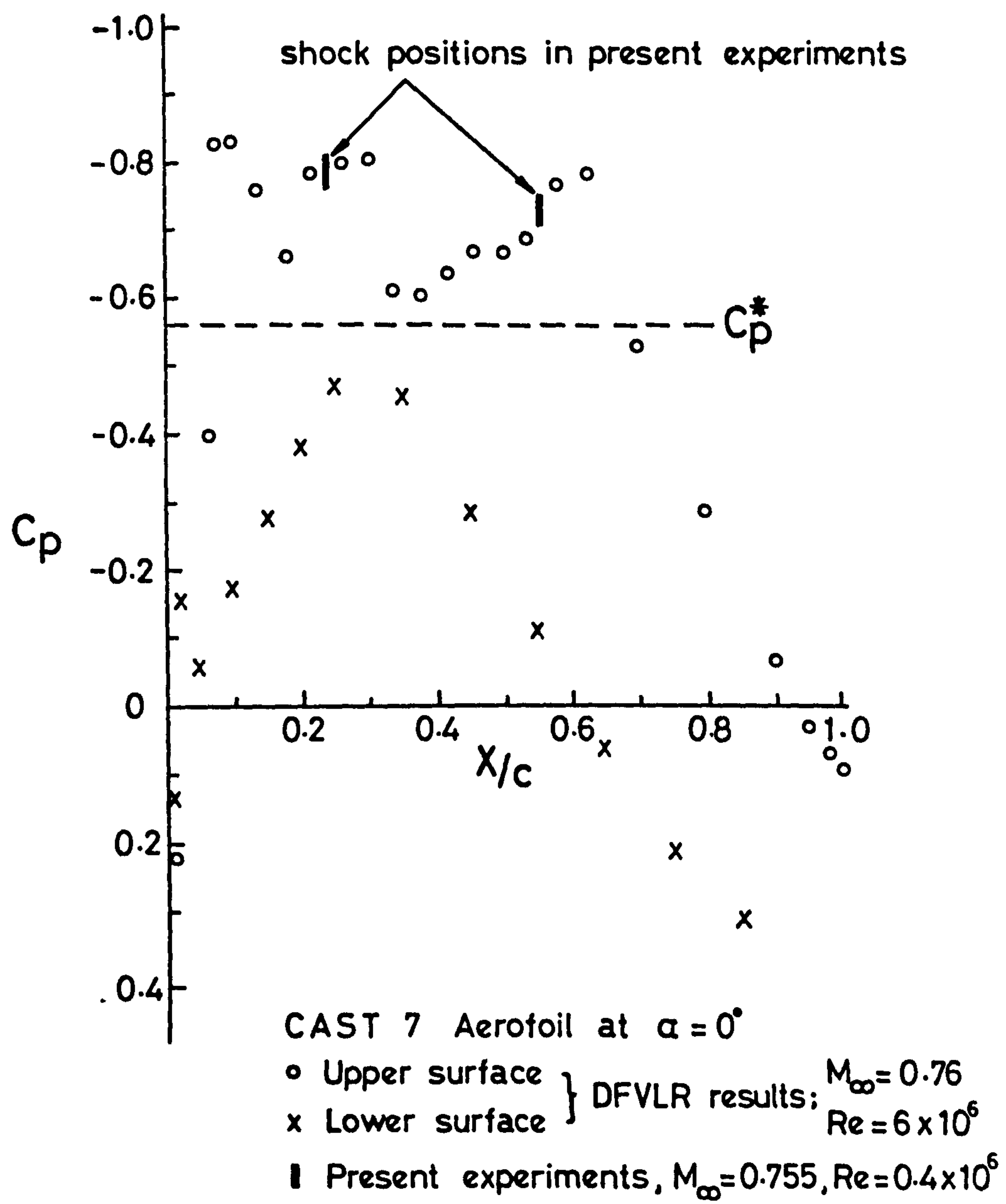
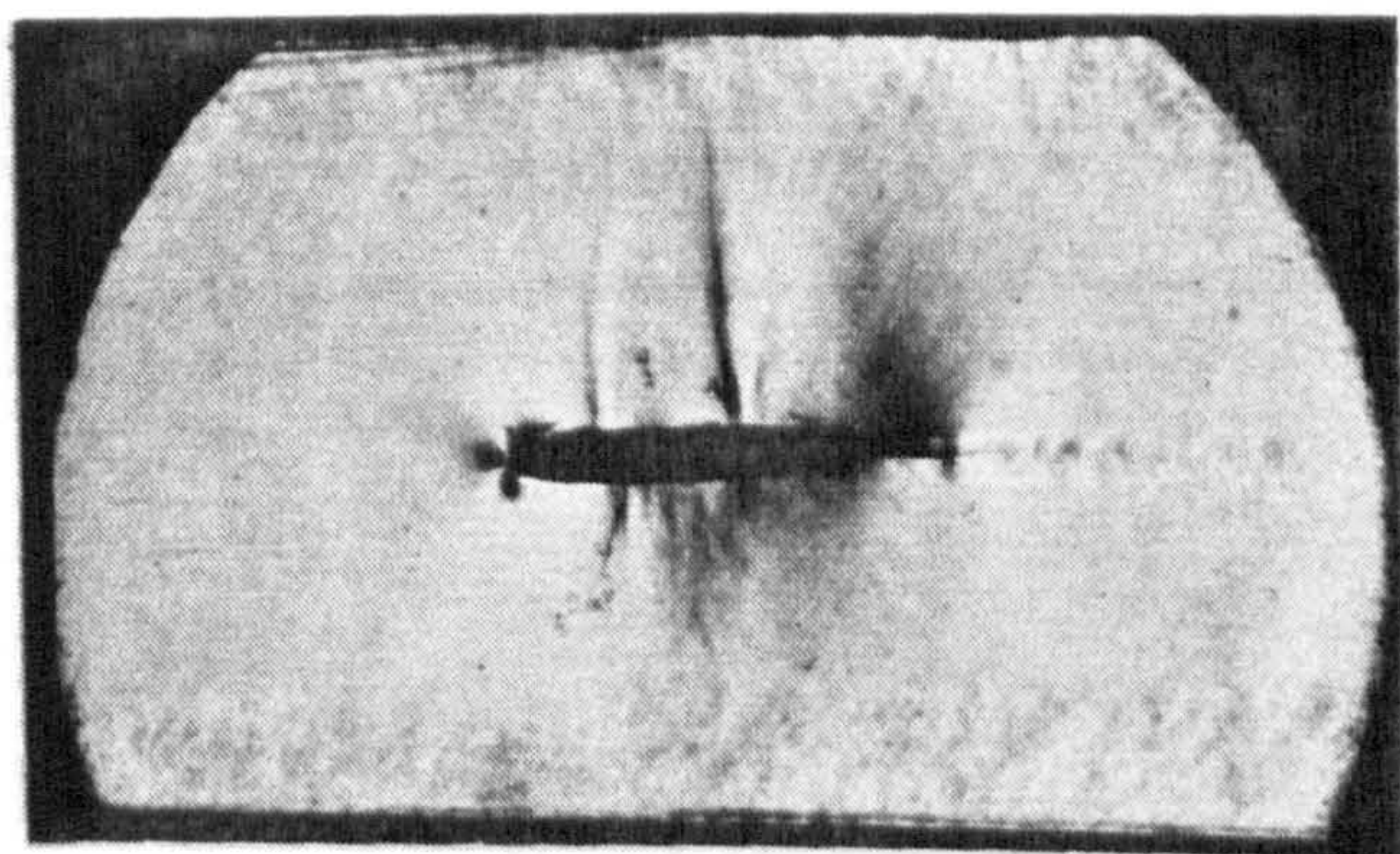
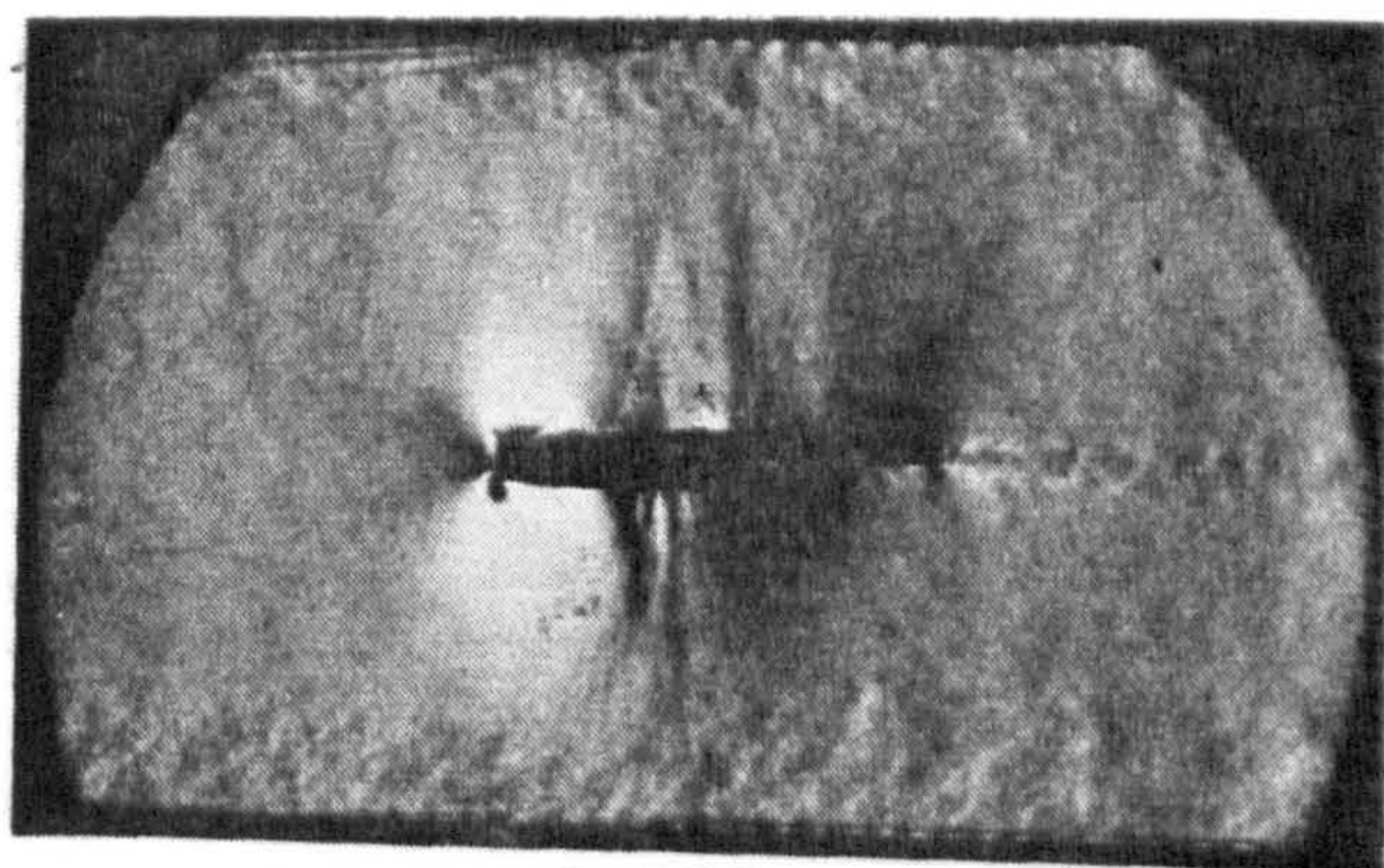


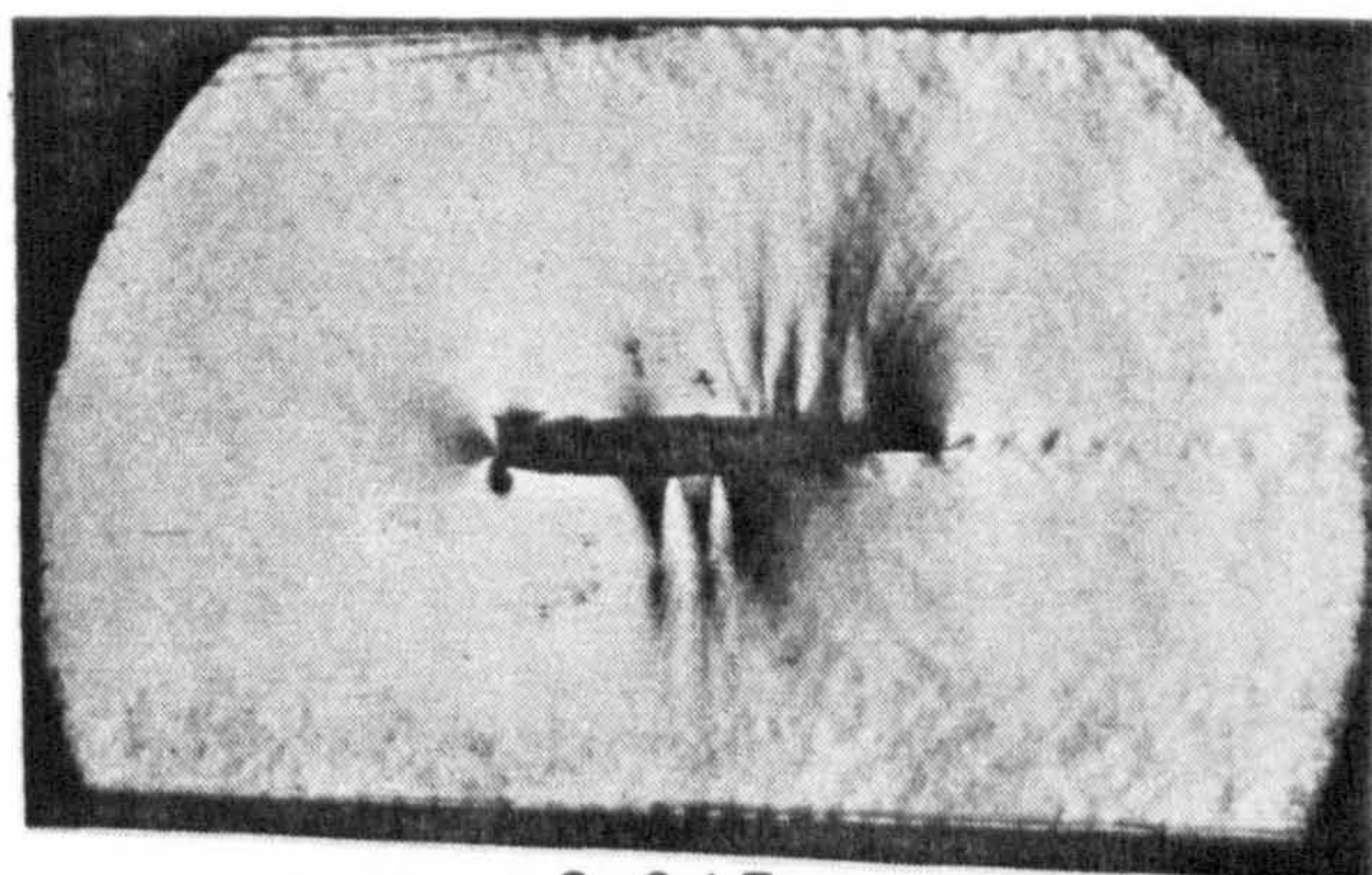
Figure 58. Pressure distribution (ref.1)



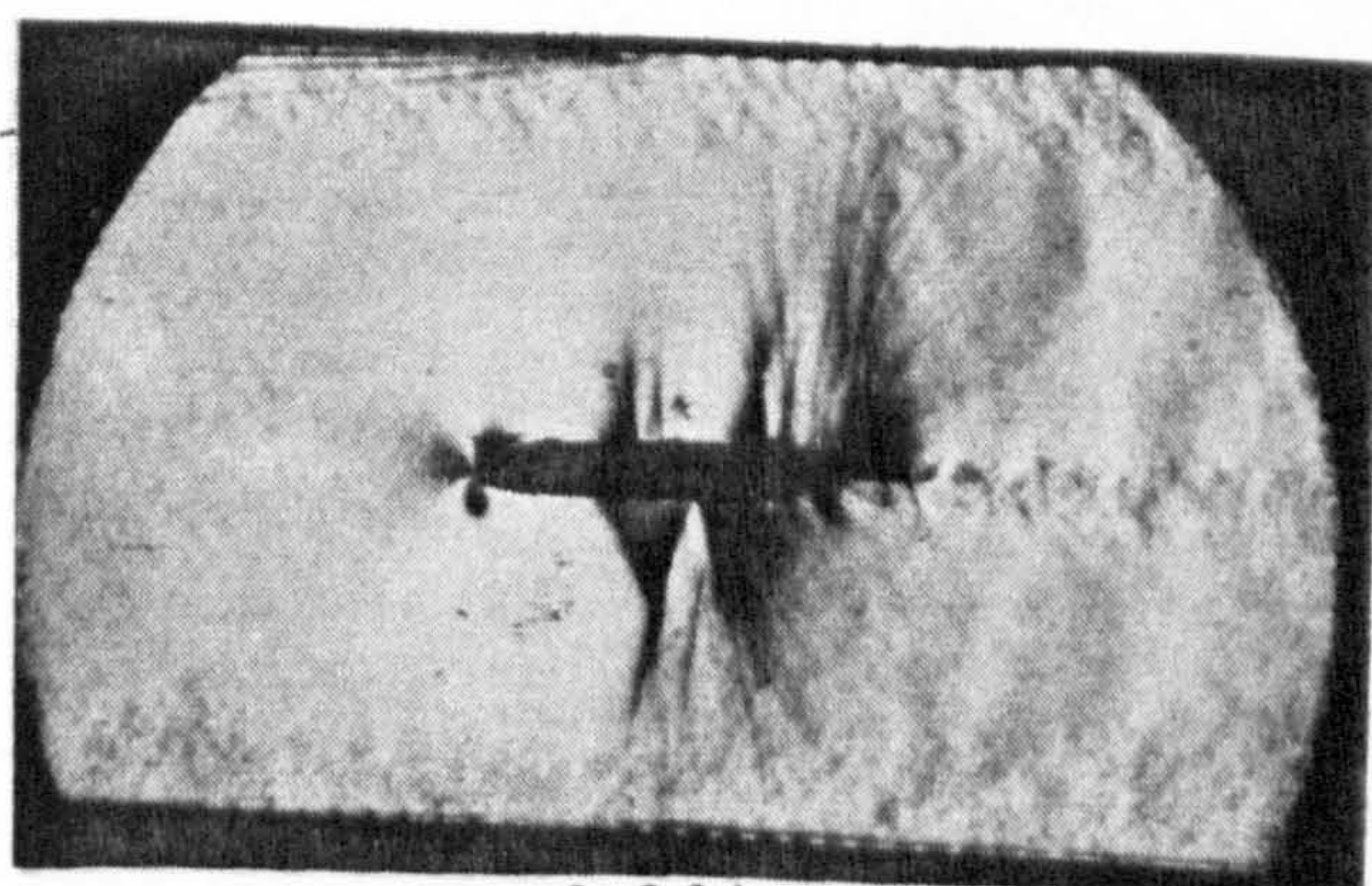
$M_{\infty} = 0.758$



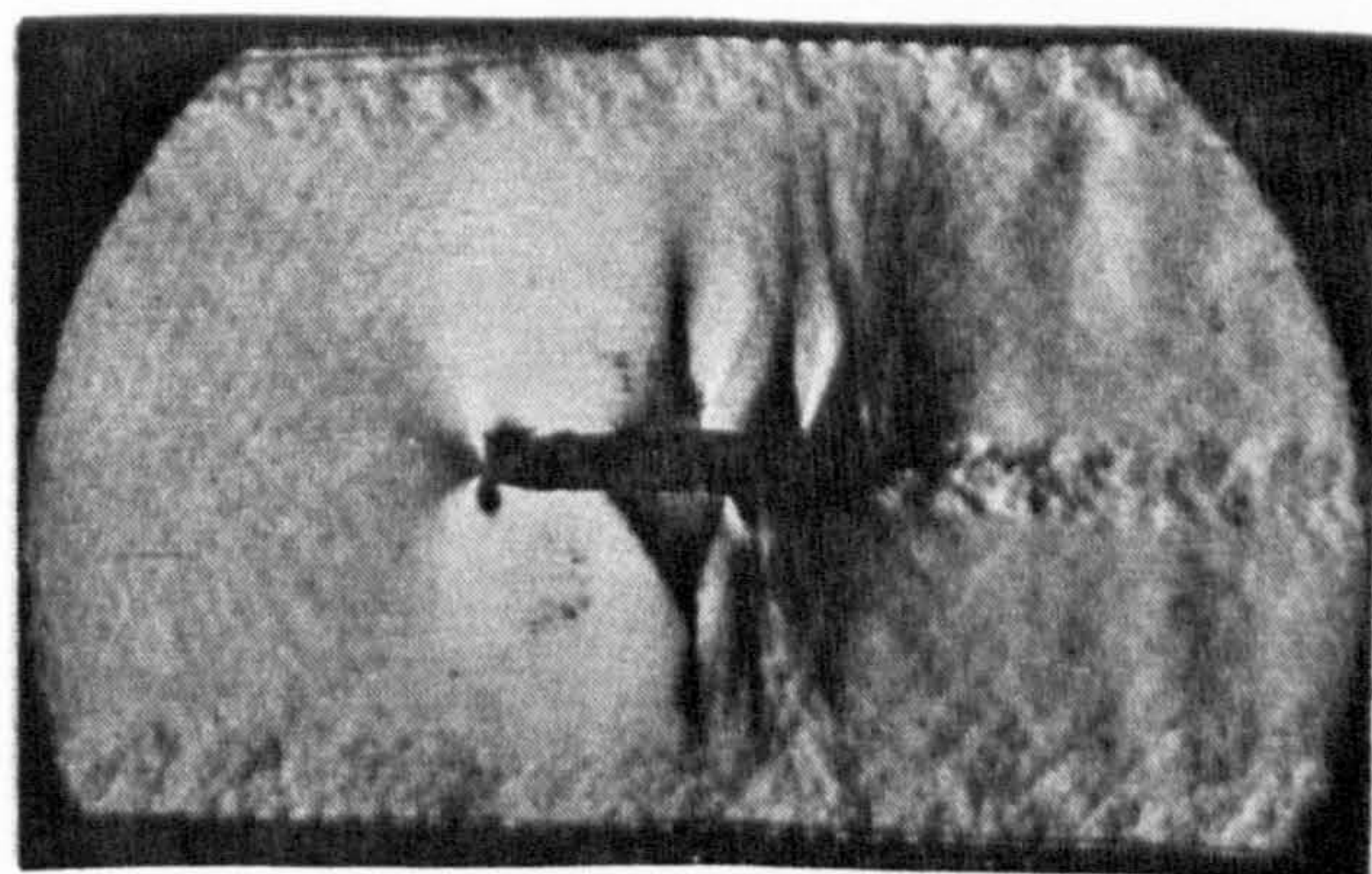
0.789



0.847



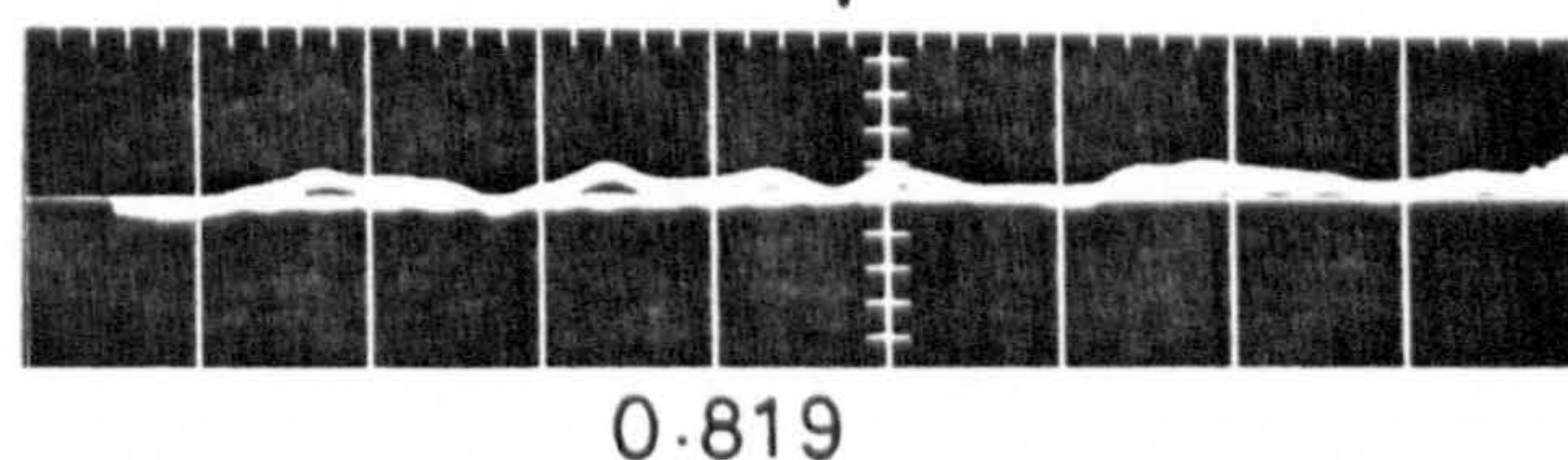
0.861



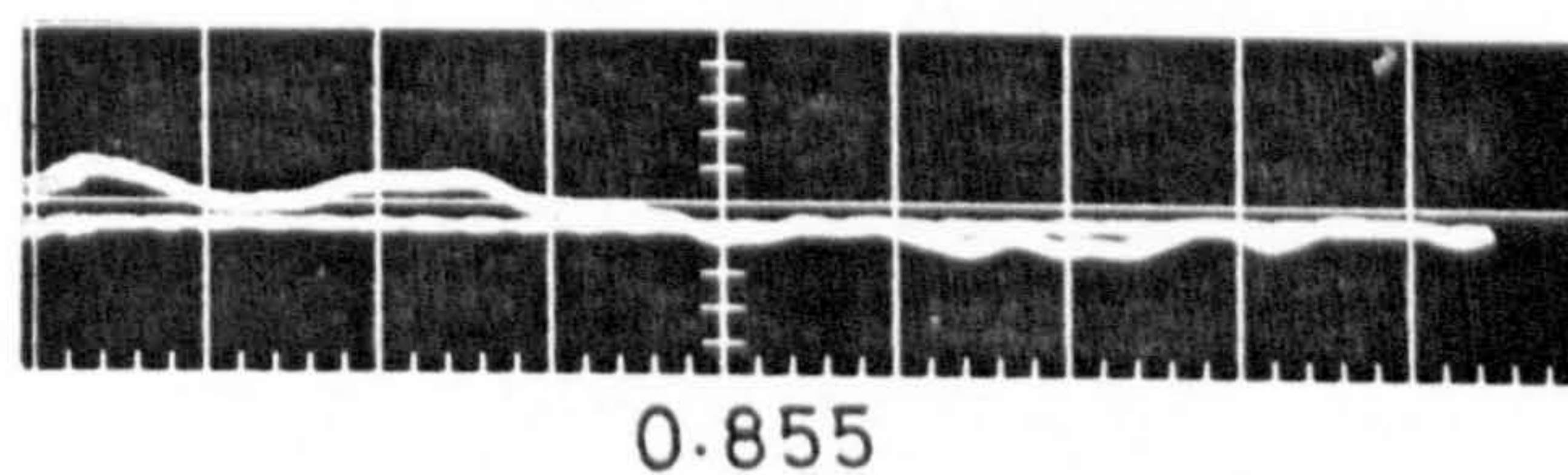
0.891

← Schlieren pictures.

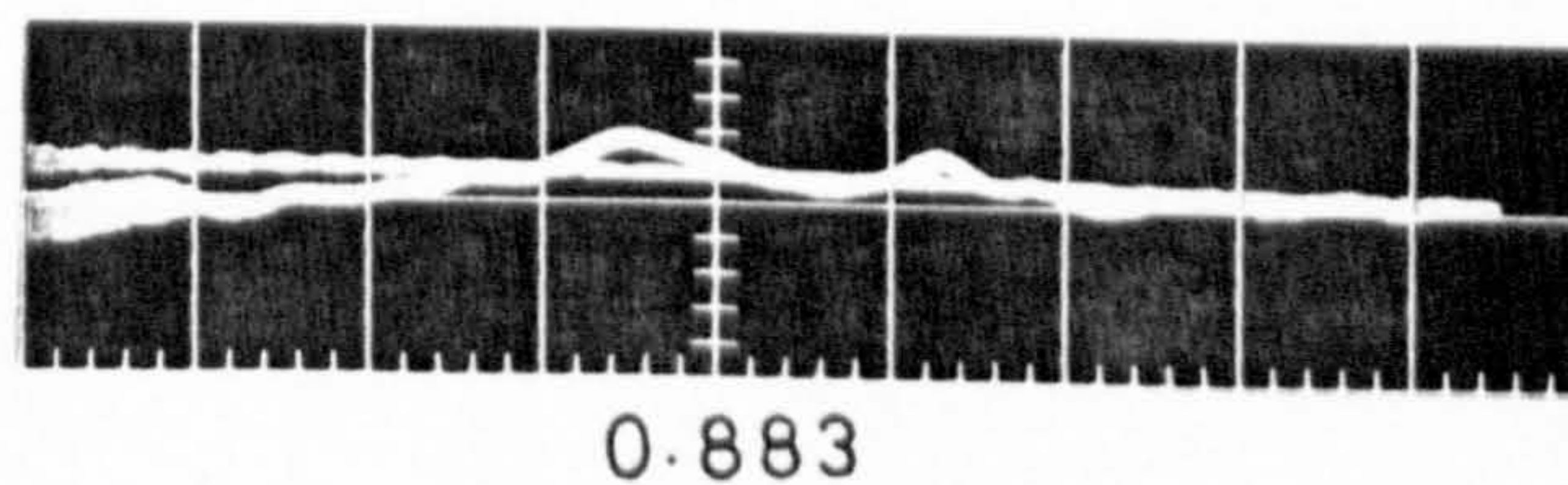
Sidewall pressure traces.



0.819

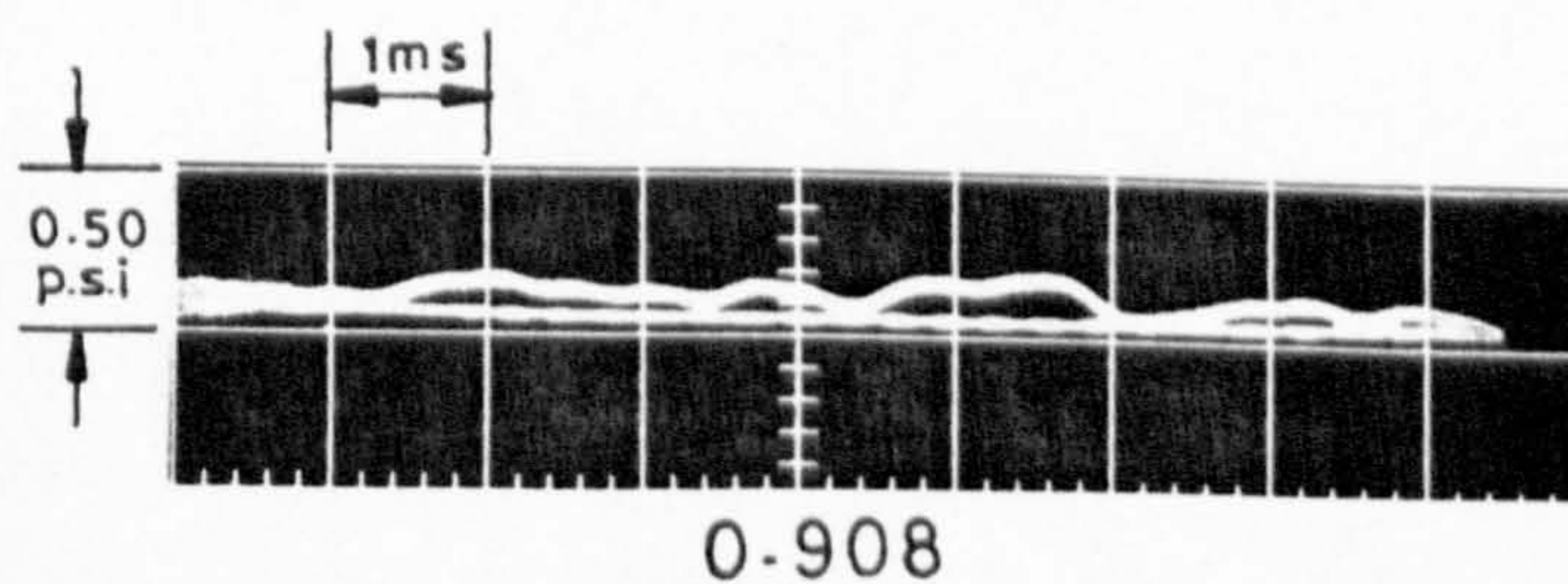


0.855



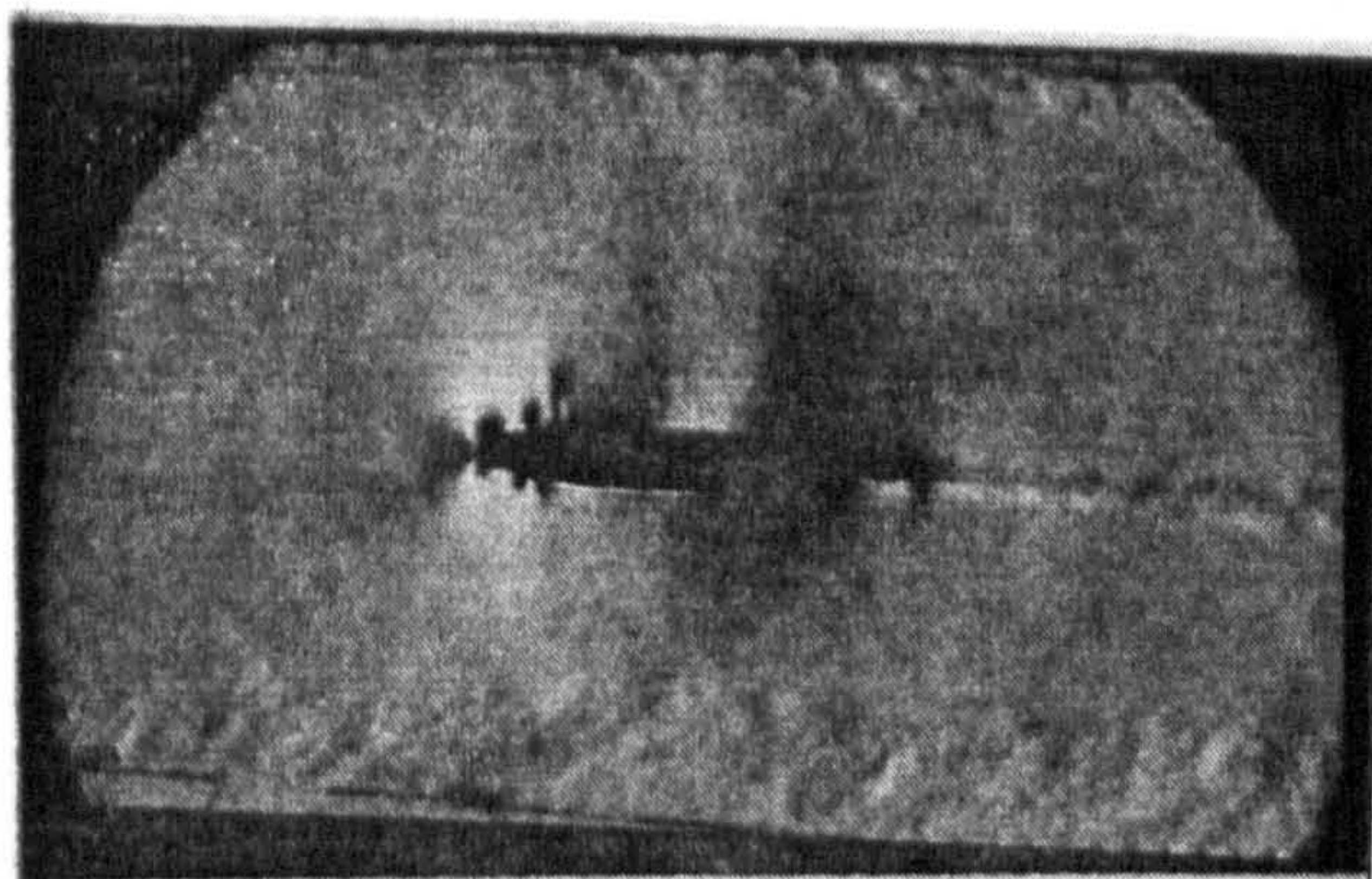
0.883

filtered output

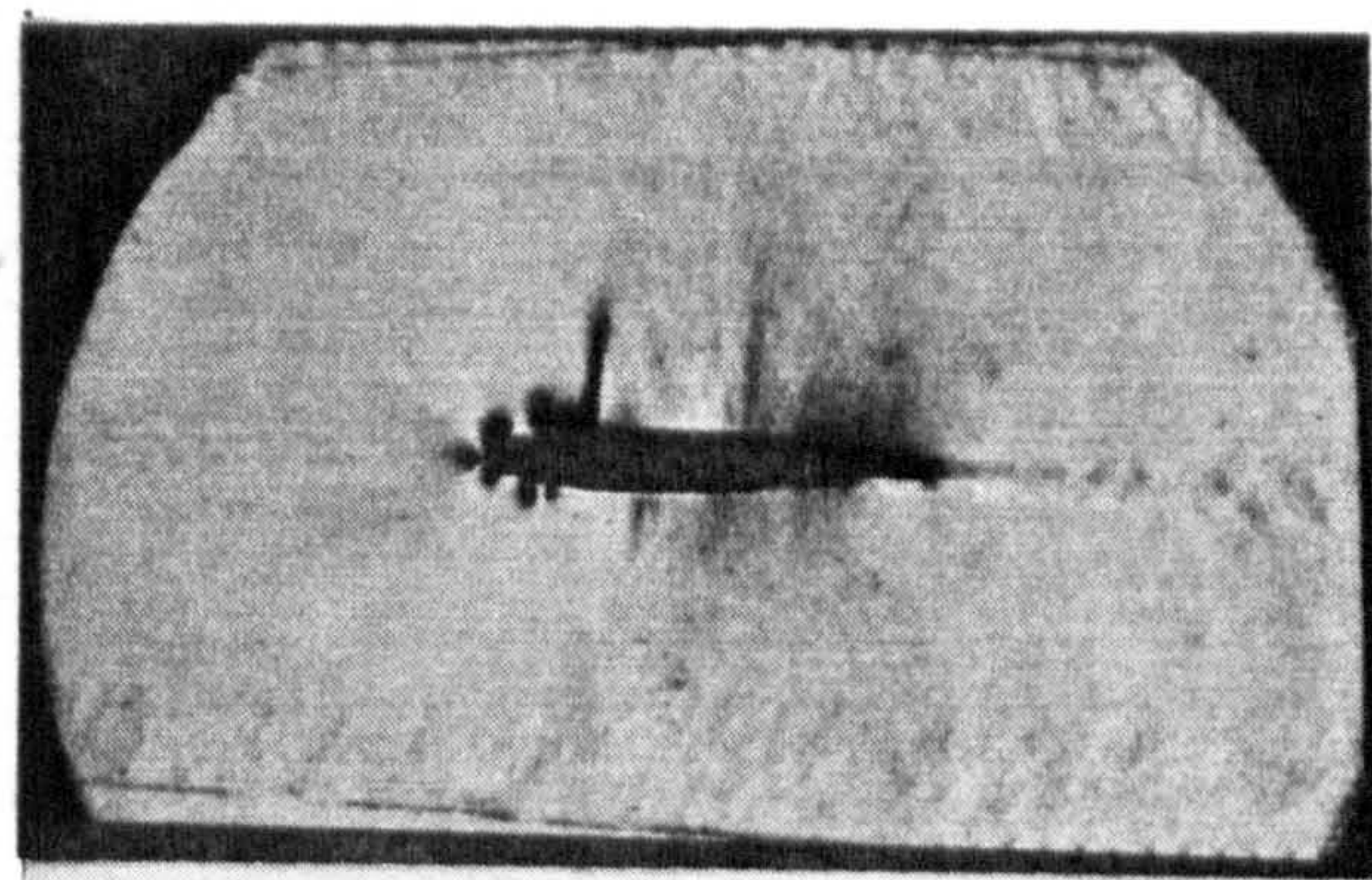


0.908

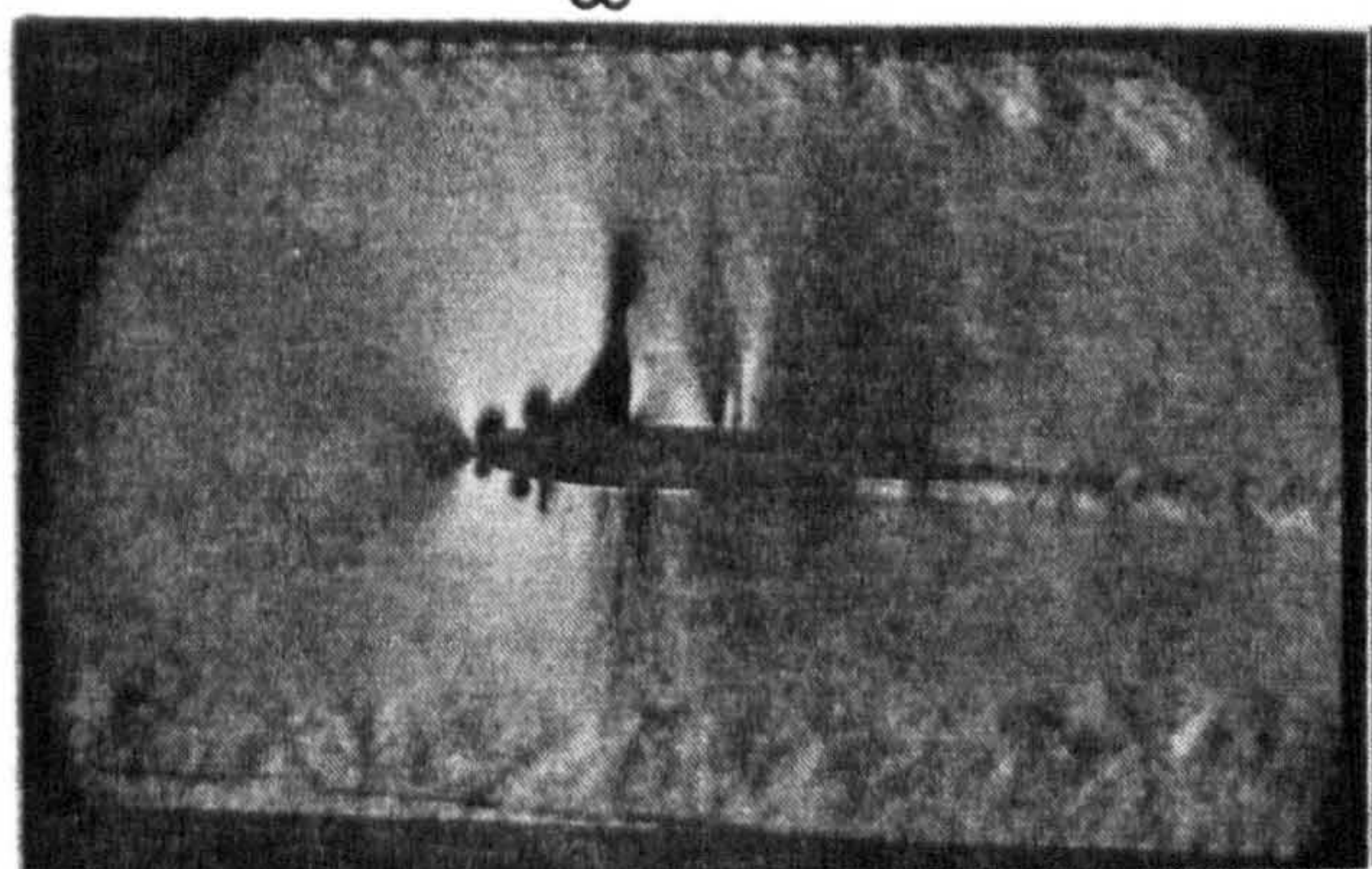
Figure 59. CAST 7 aerofoil at zero incidence (free transition).



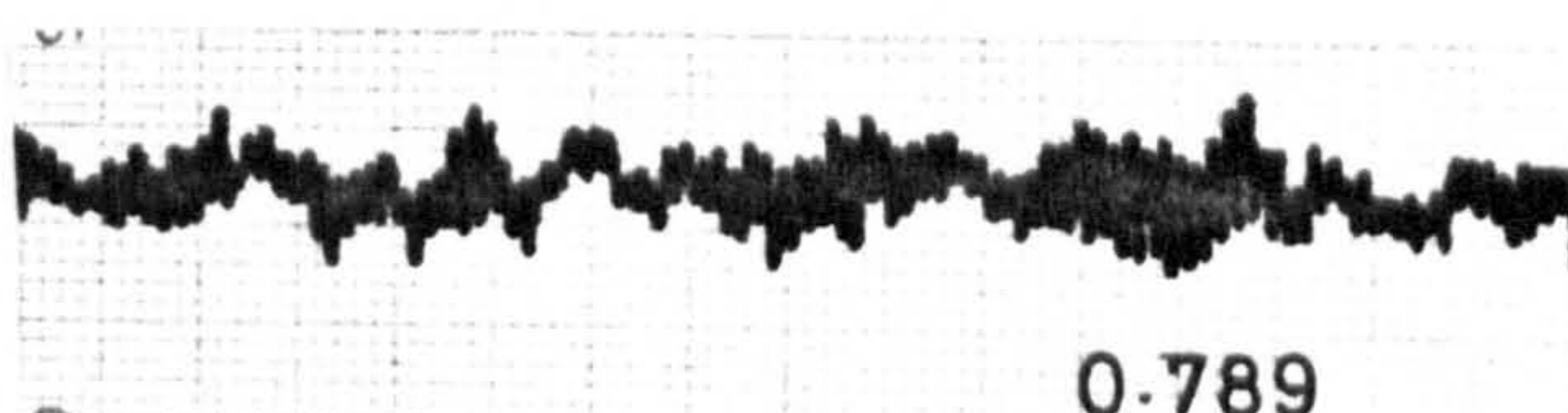
$M_{\infty} = 0.743$



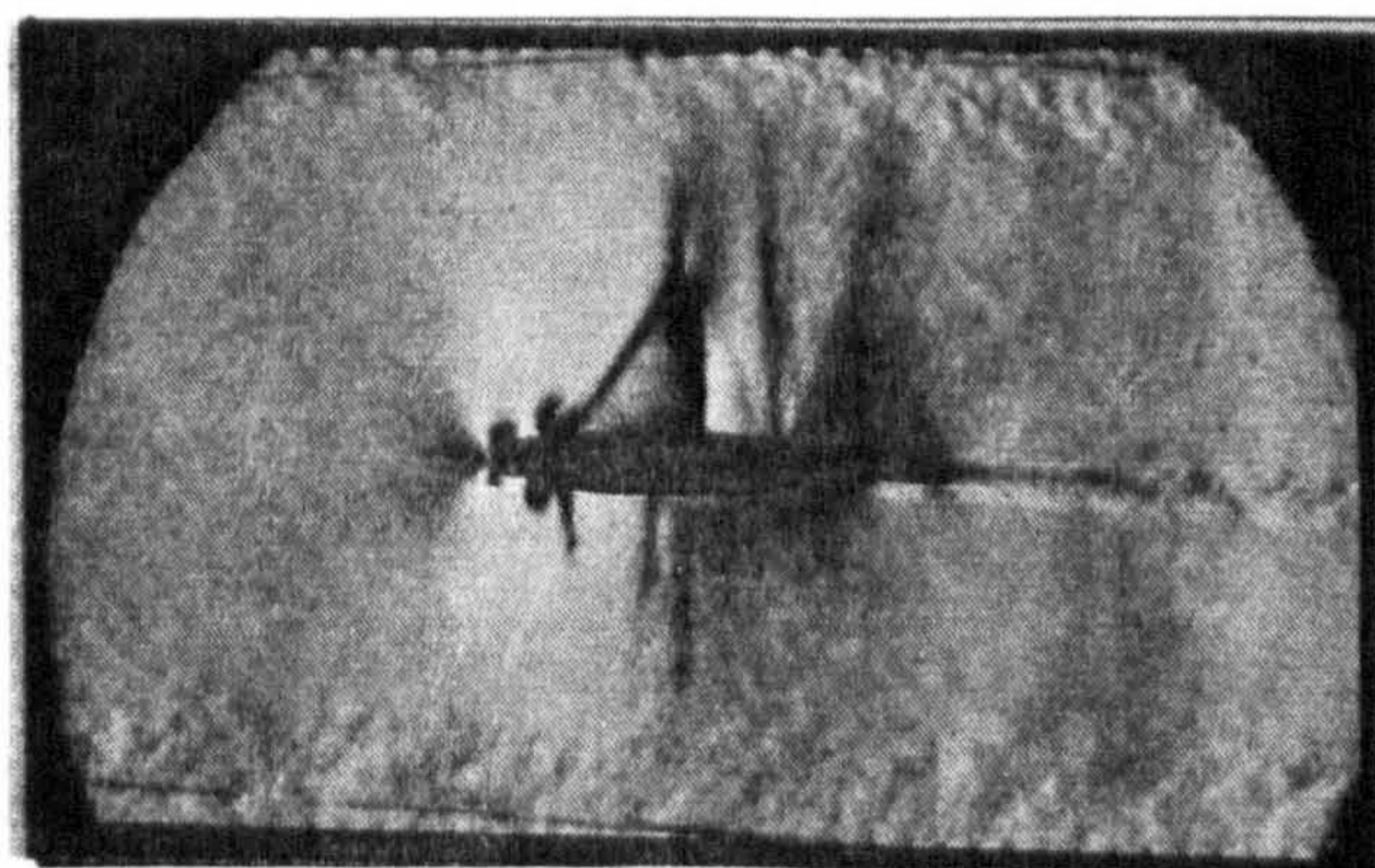
0.775



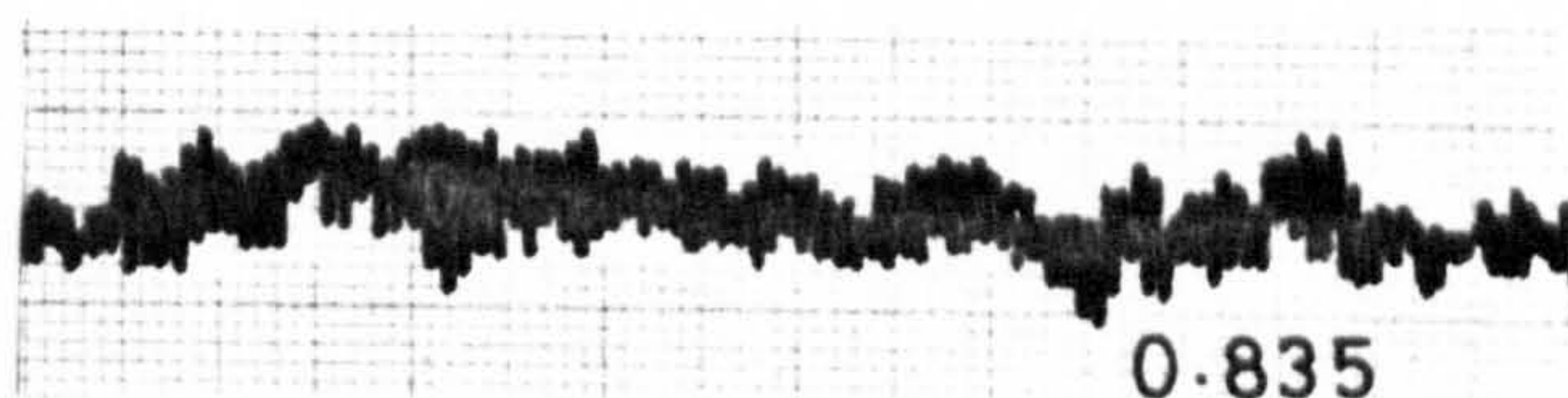
0.806



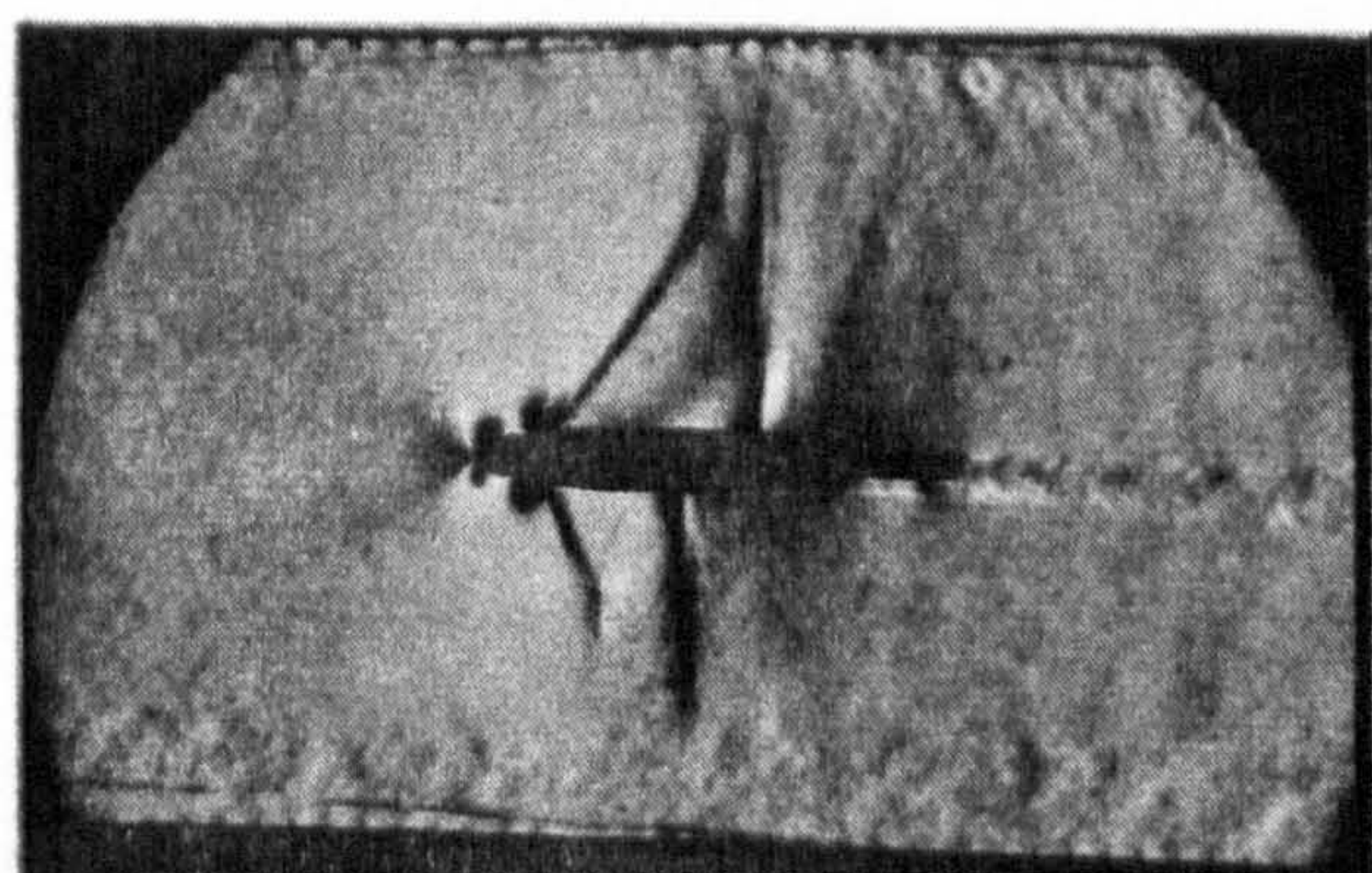
0.789



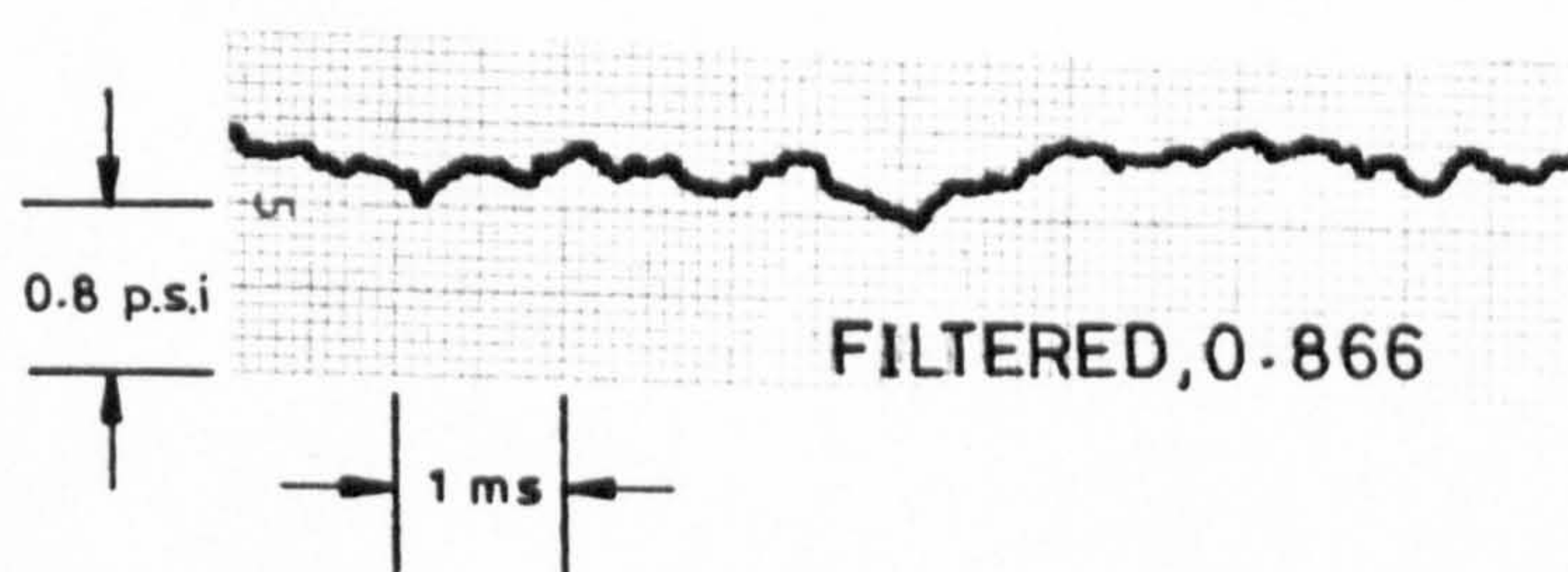
0.839



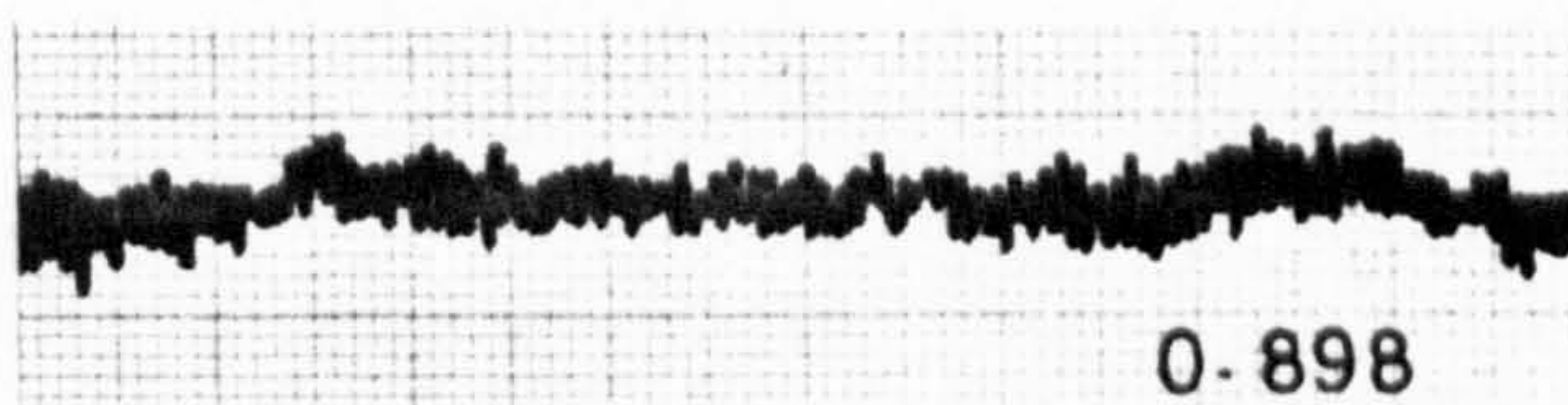
0.835



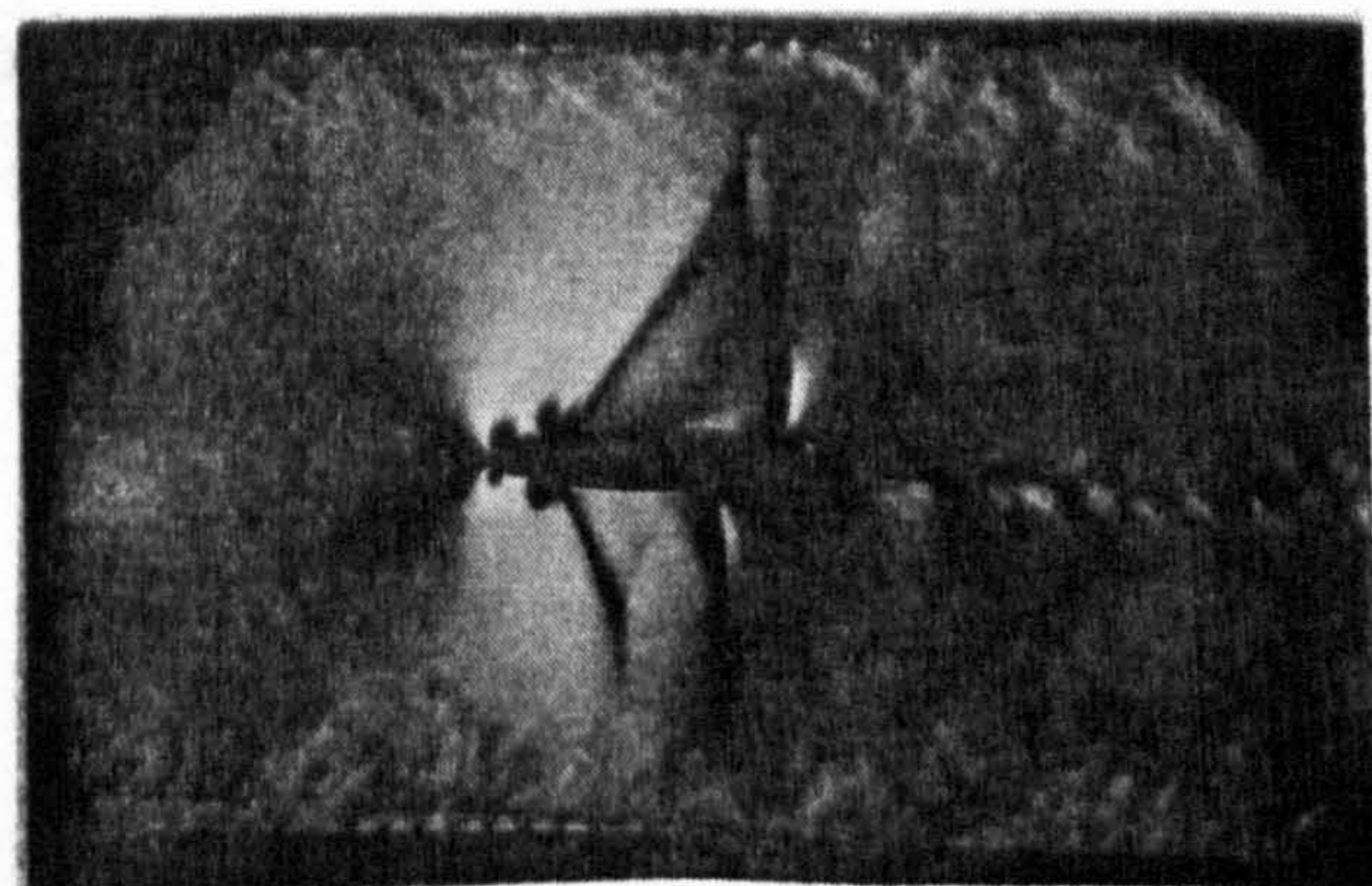
0.873



FILTERED, 0.866



0.898



0.902

STEADY FLOWS

Figure 60. CAST 7 Aerofoil,
 $\alpha = 3^\circ$ (fixed transition)

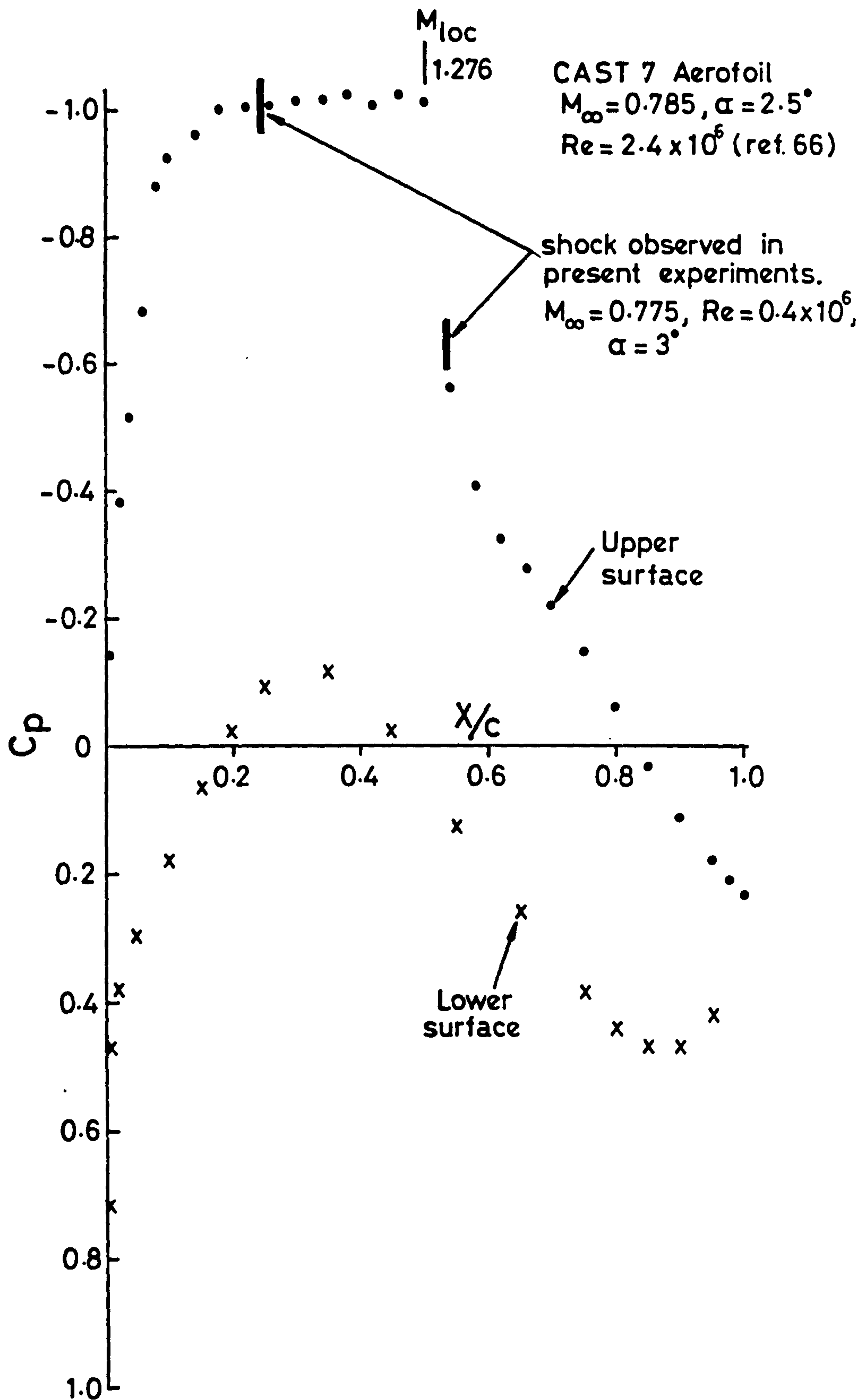
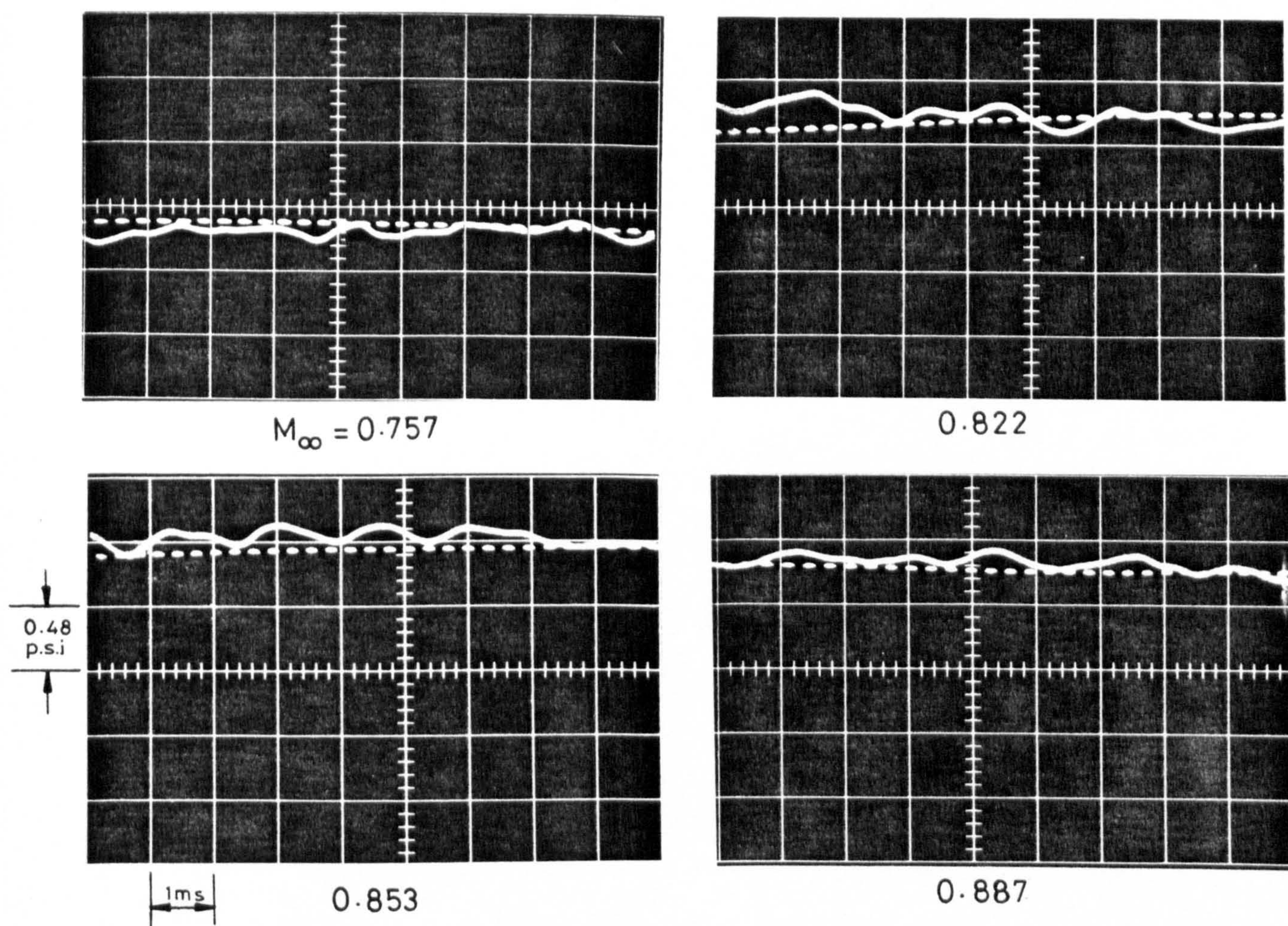
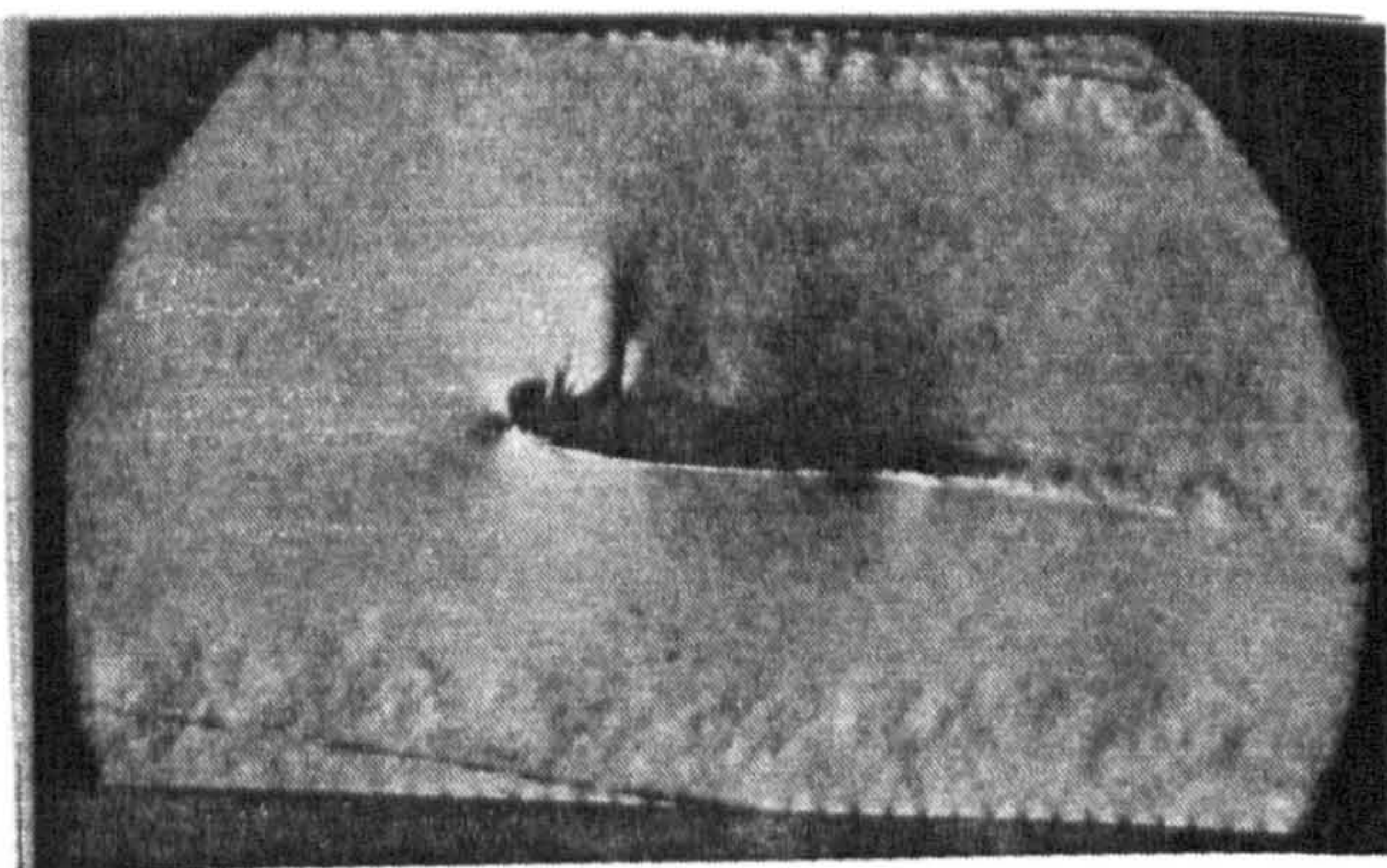


Figure 61. Pressure distribution (ref.66).

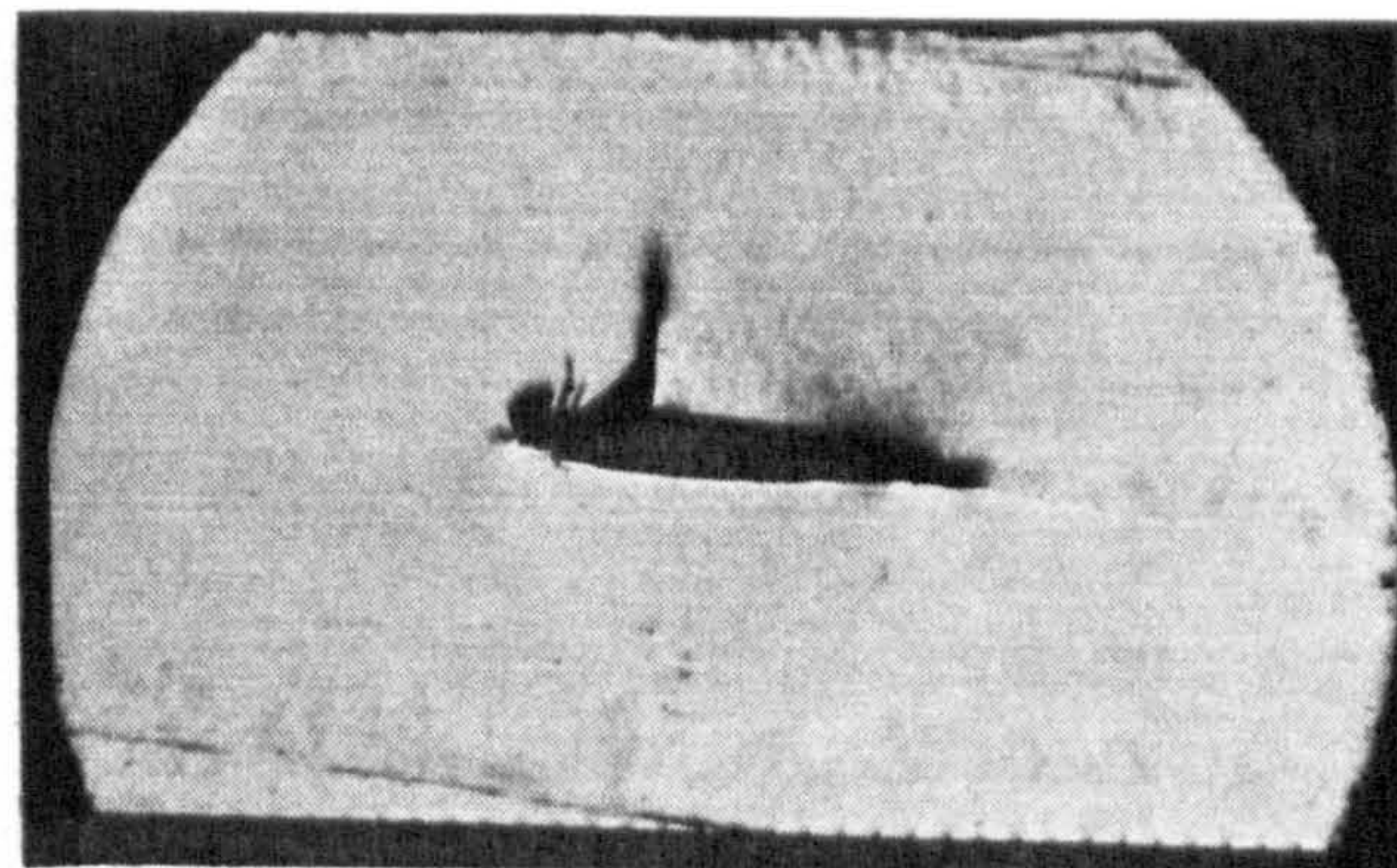


Sidewall pressure traces

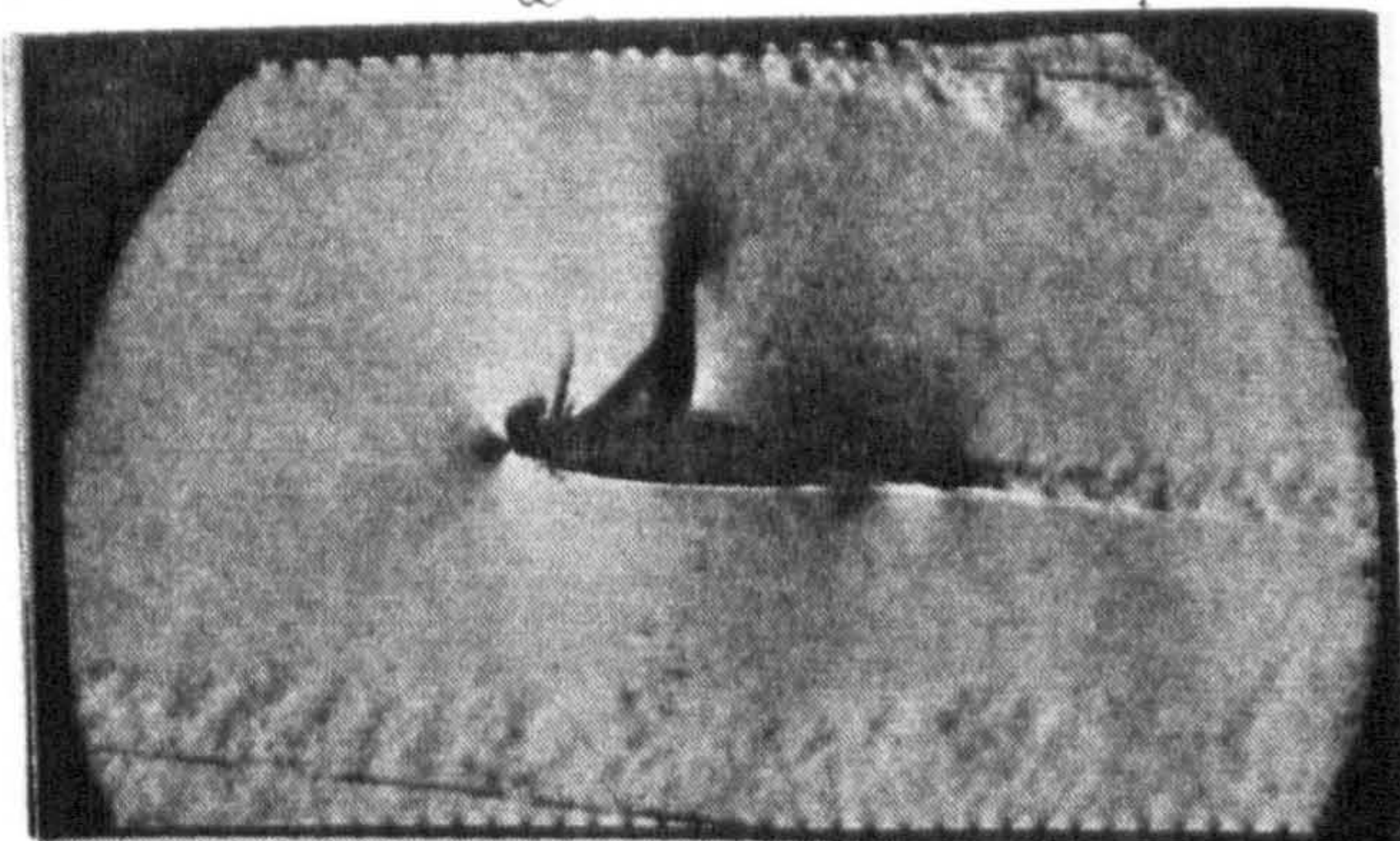
Figure 62. CAST 7 aerofoil, $\alpha = 3^\circ$ (free transition).



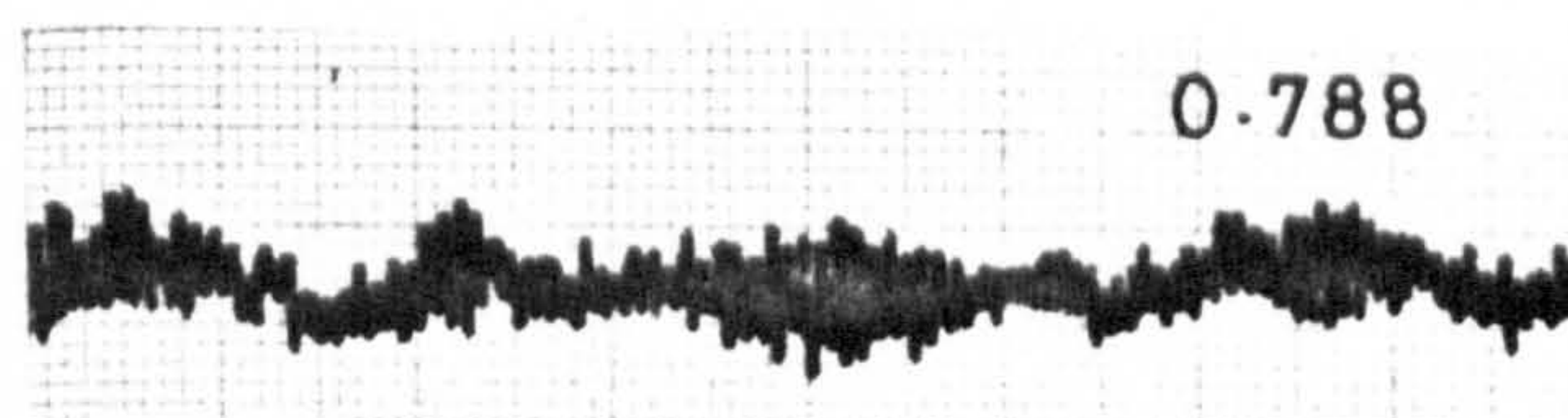
$M_\infty = 0.718$



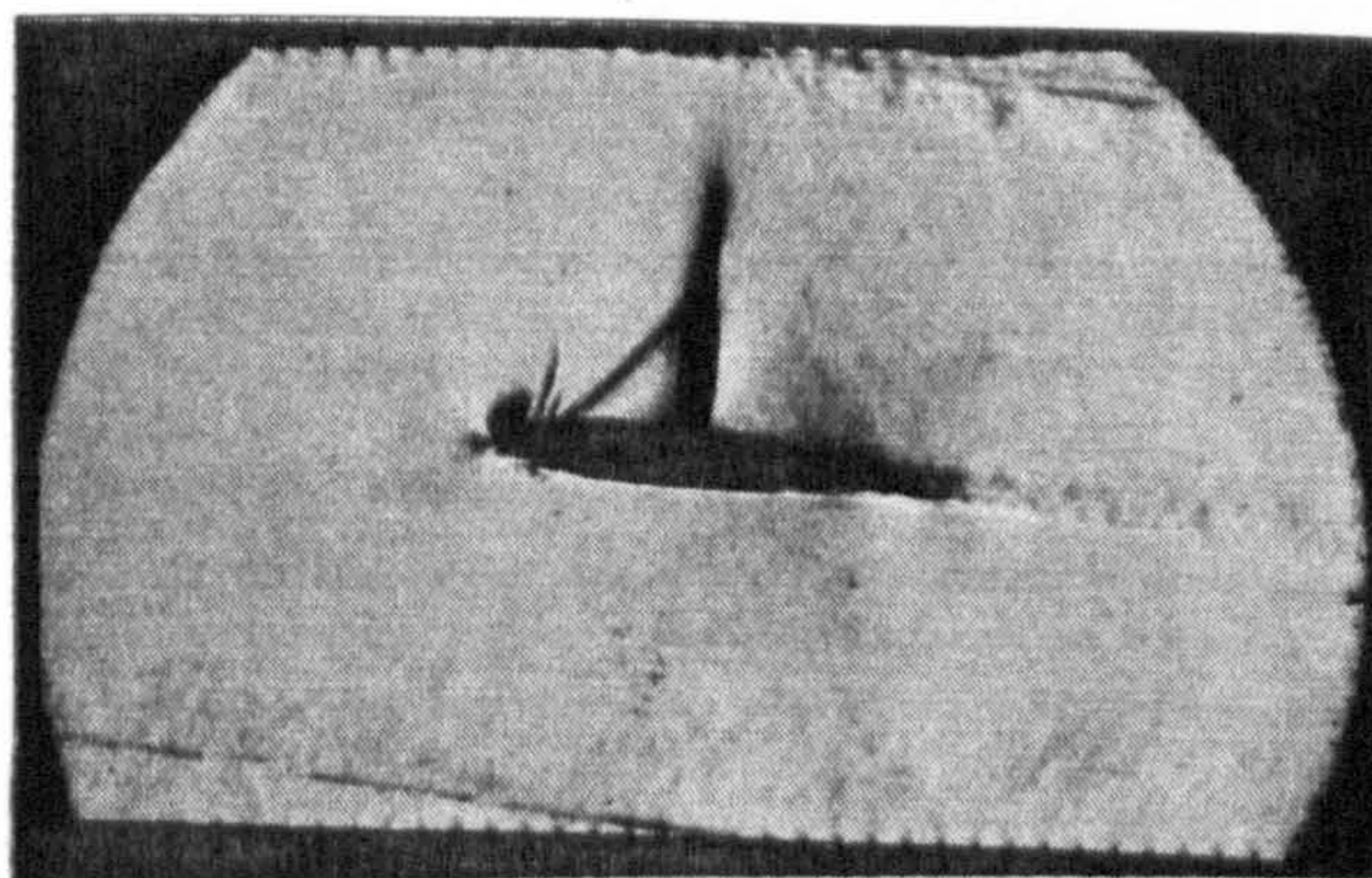
0.744



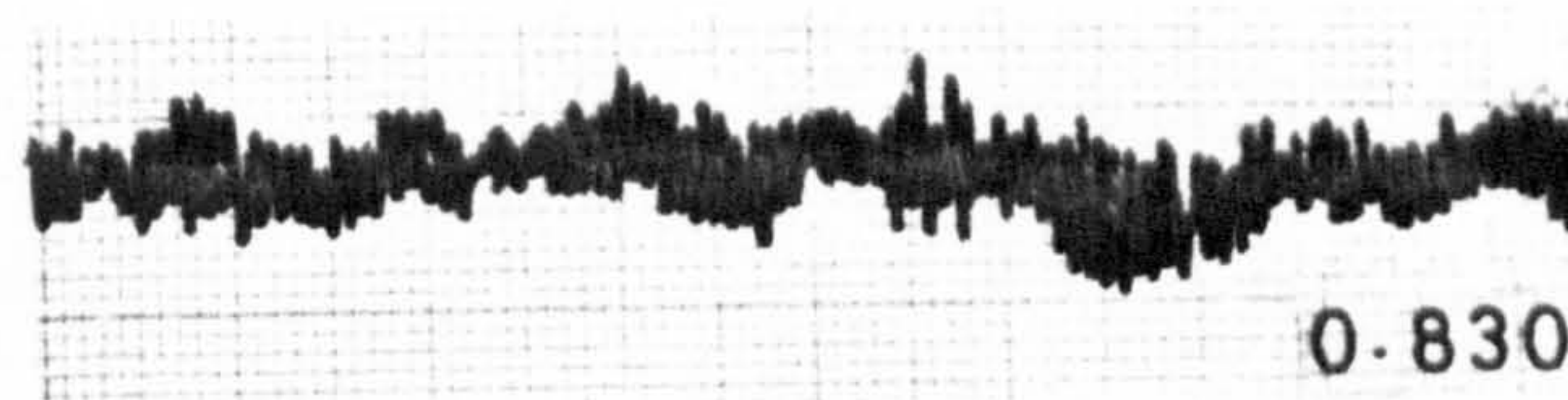
0.774



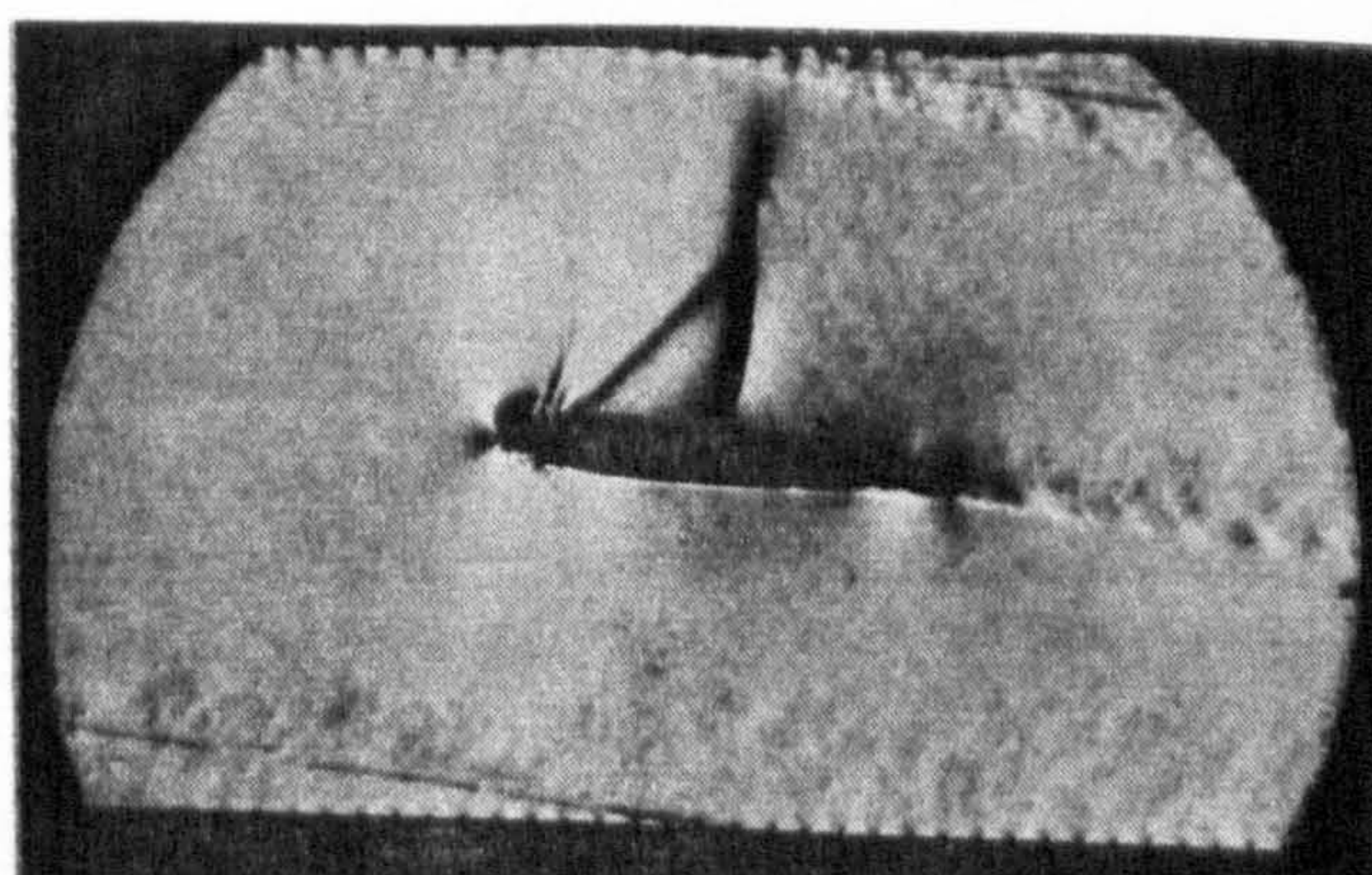
0.788



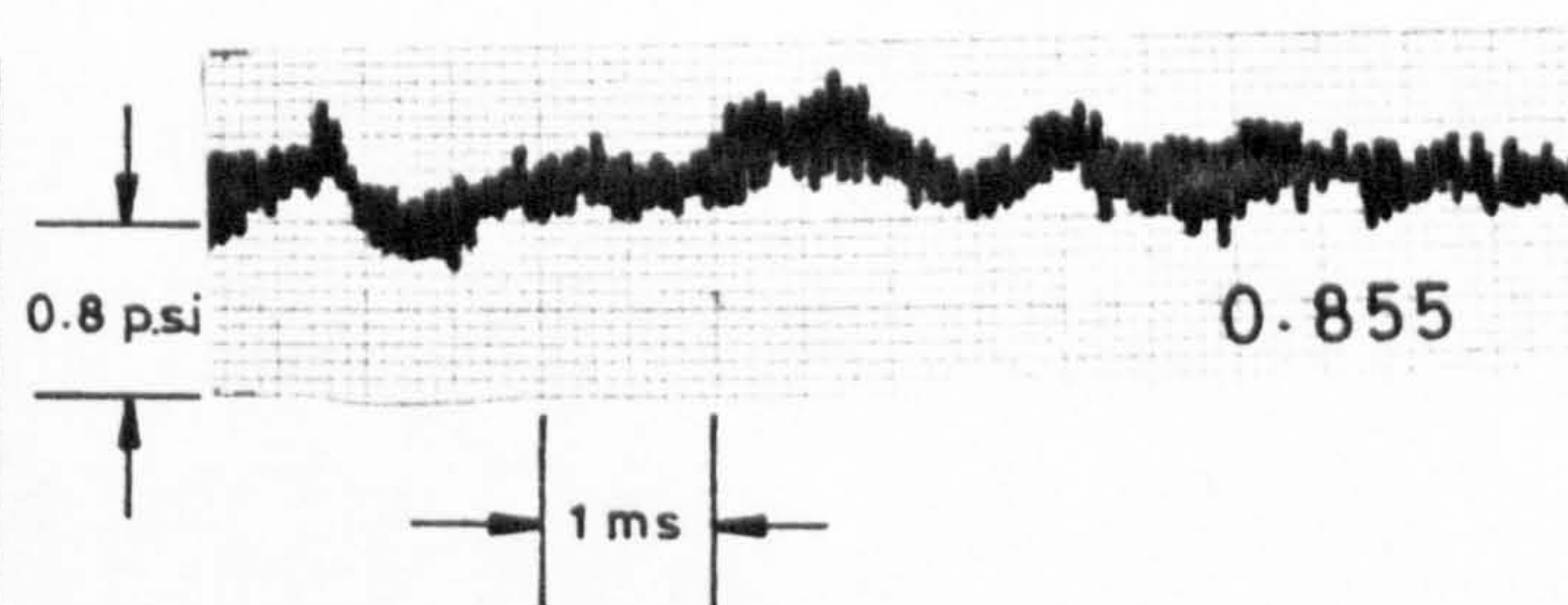
0.811



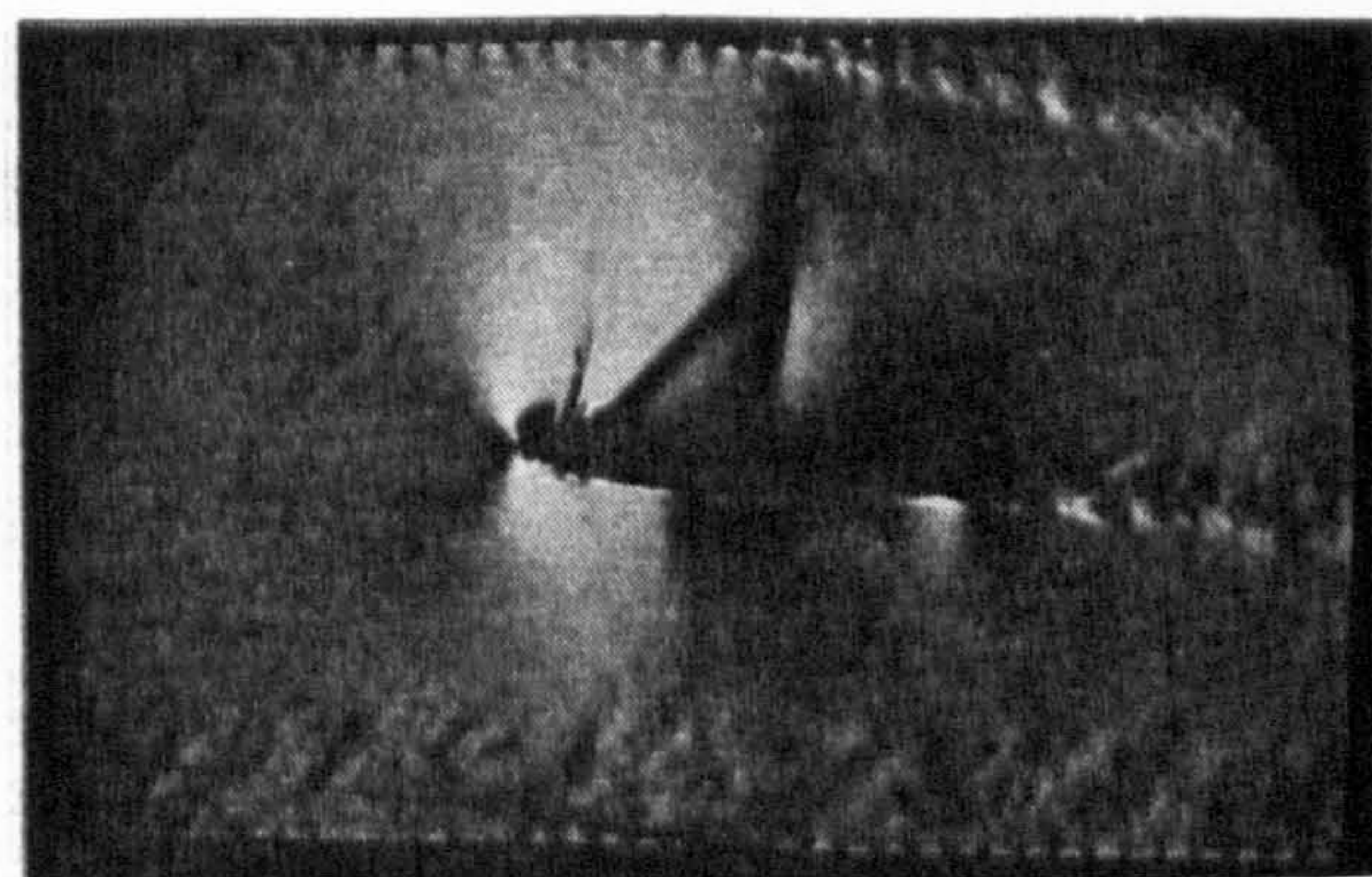
0.830



0.840



0.855



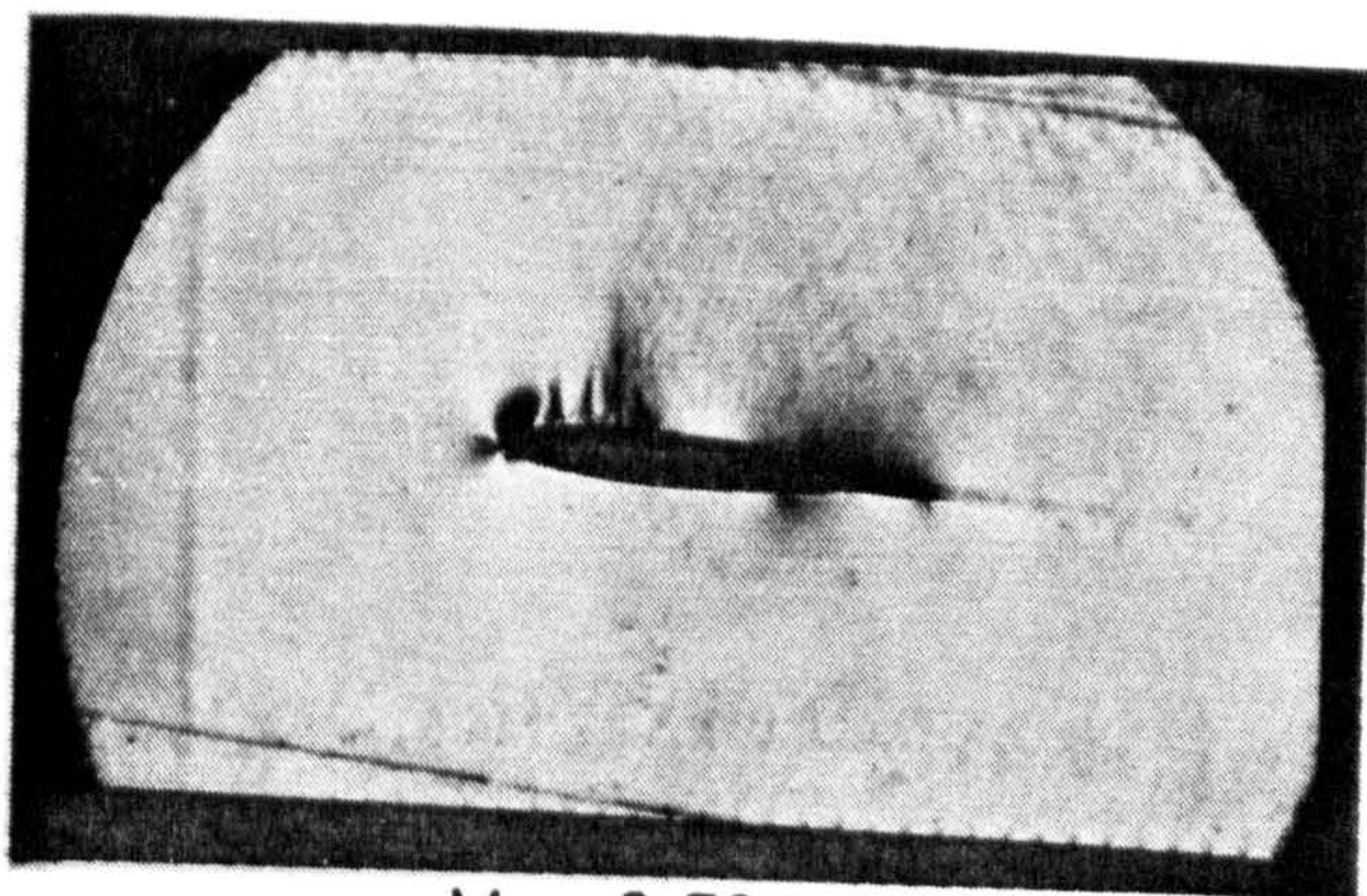
0.872



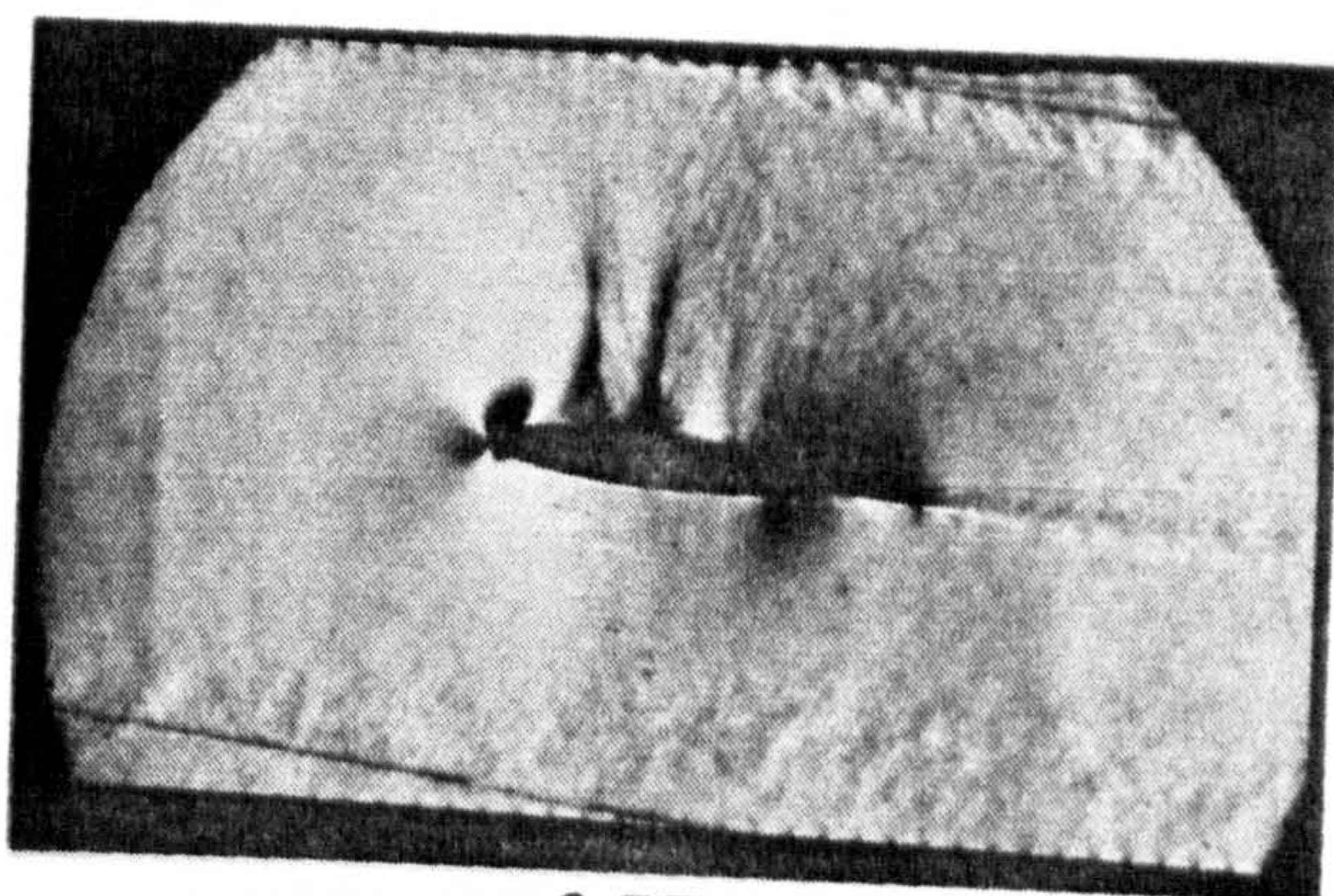
0.871

STEADY FLOWS

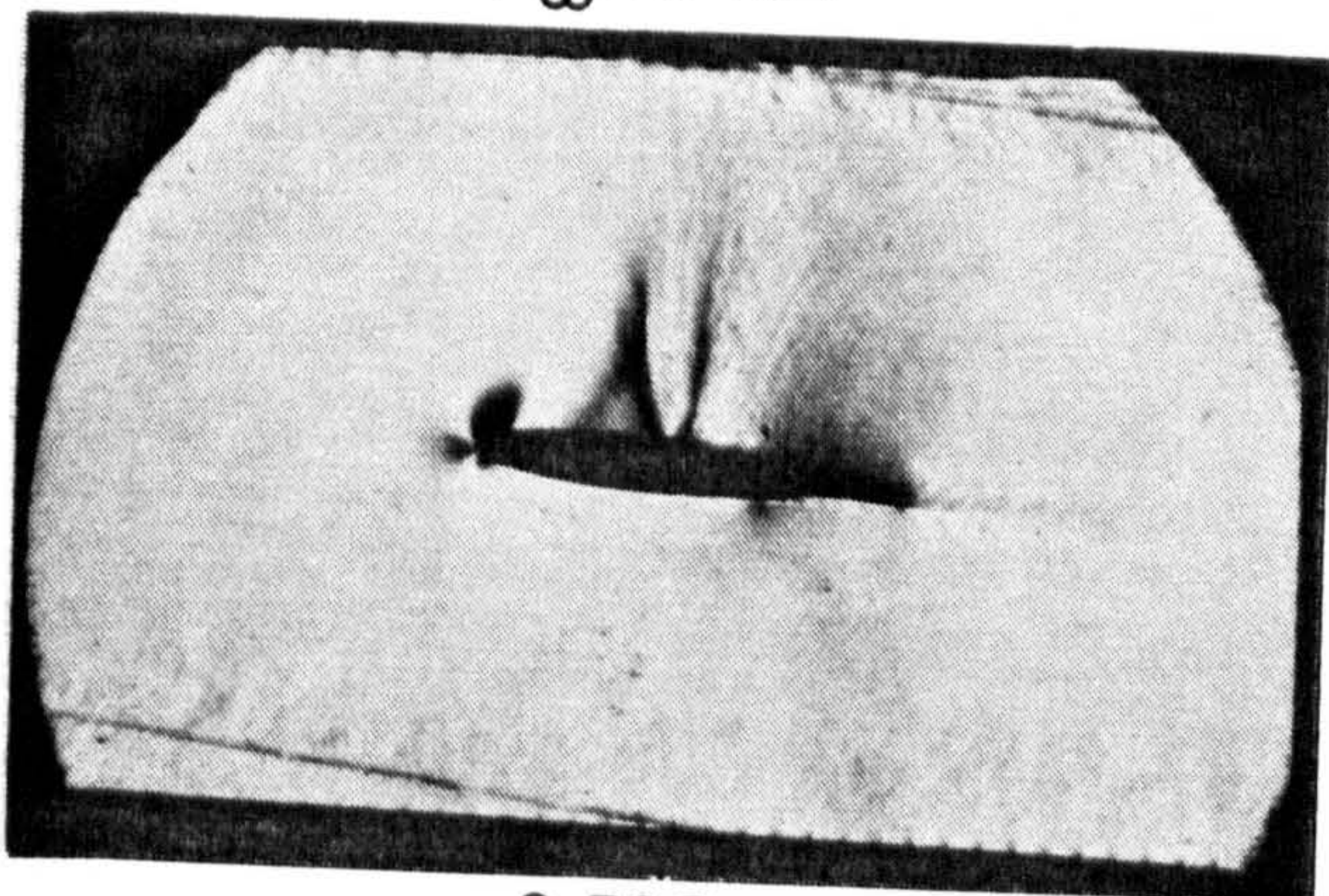
Figure 63. CAST 7 Aerofoil,
 $\alpha = 6^\circ$ (fixed transition)



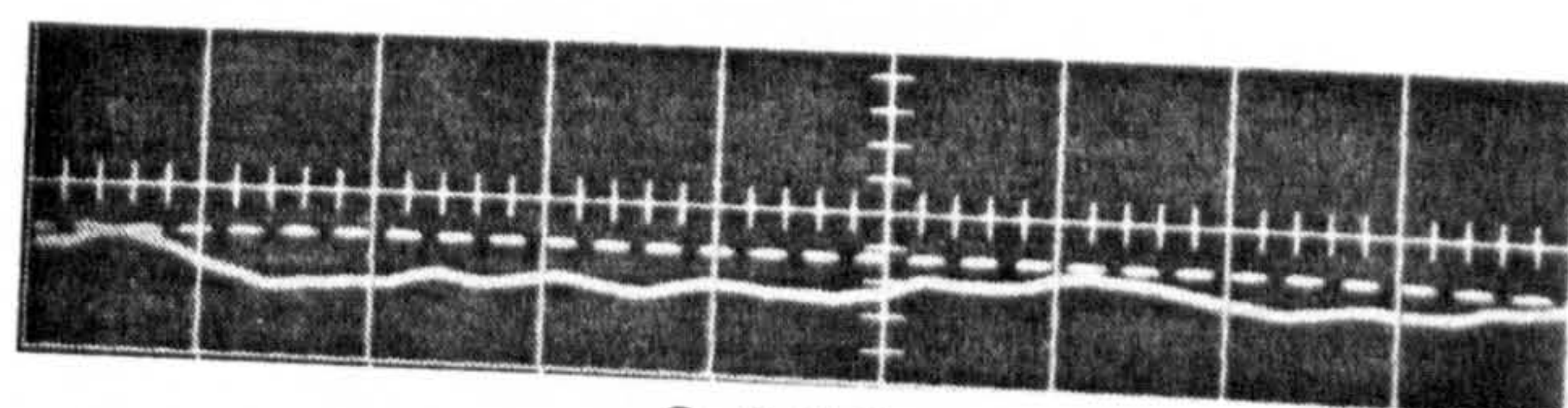
$M_{\infty} = 0.724$



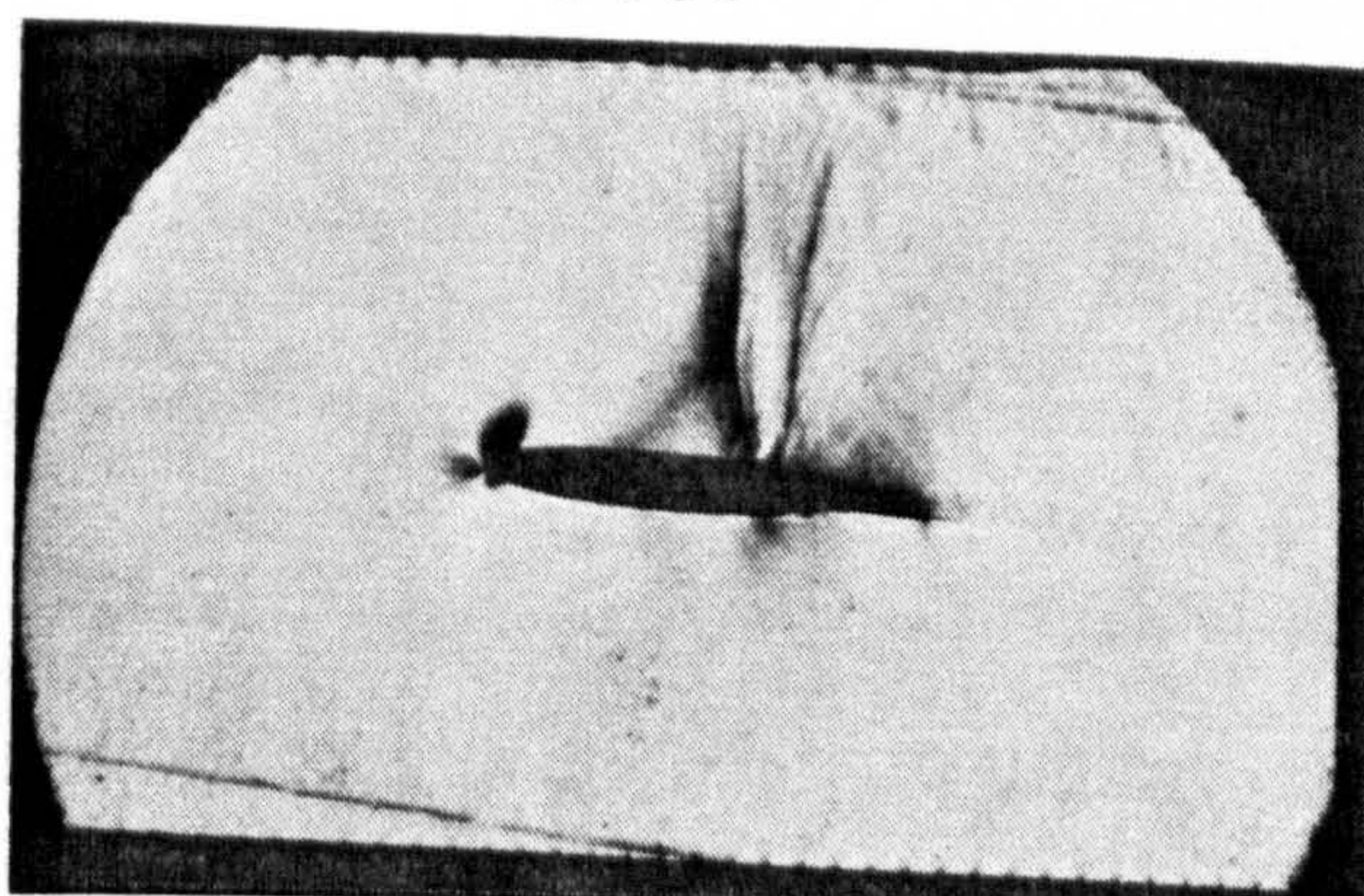
0.754



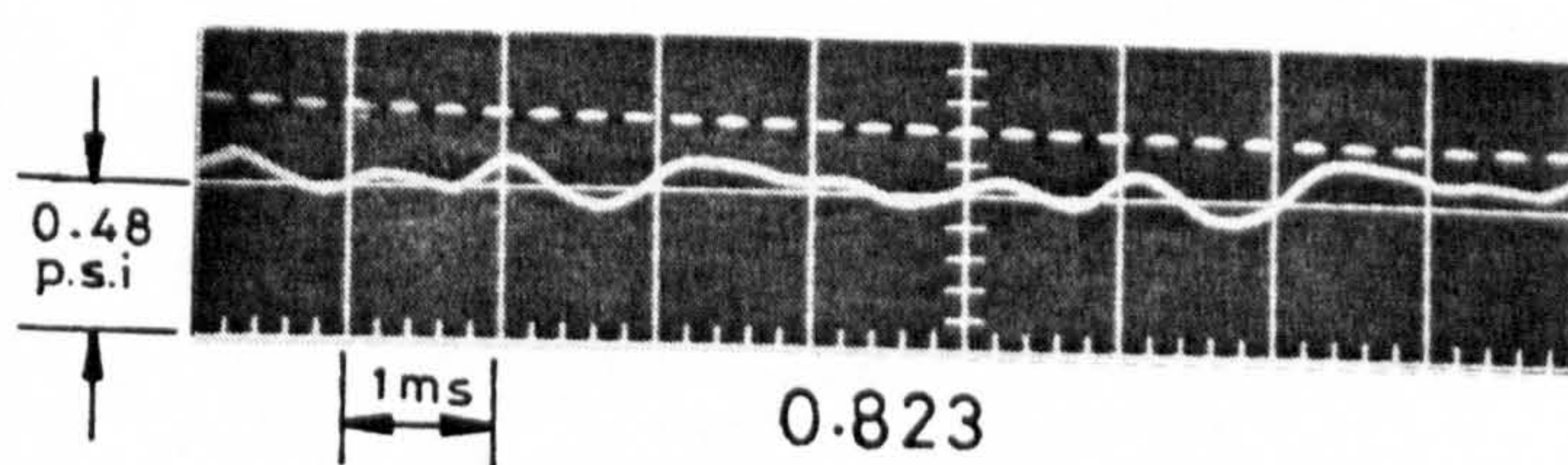
0.785



0.699

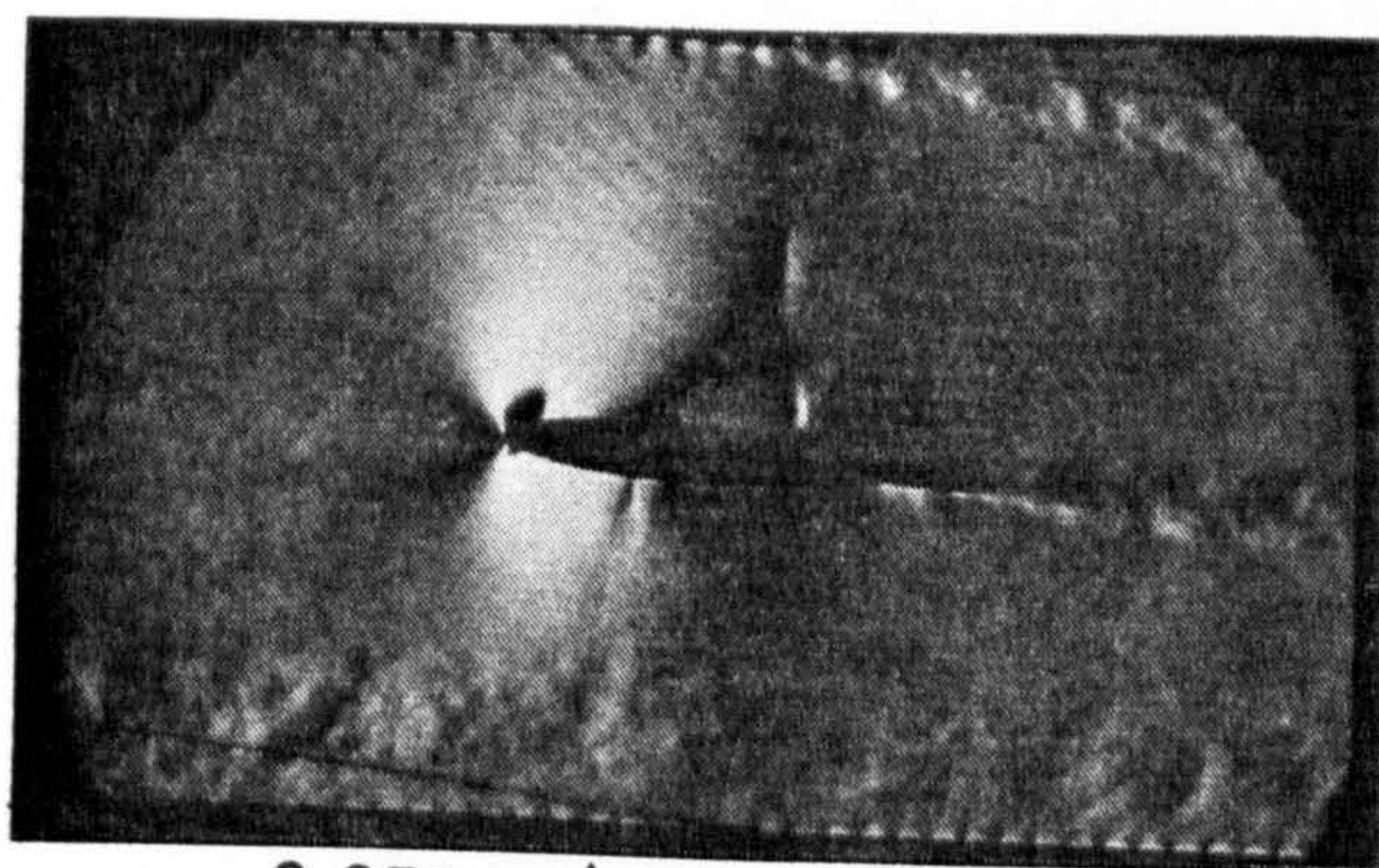


0.822

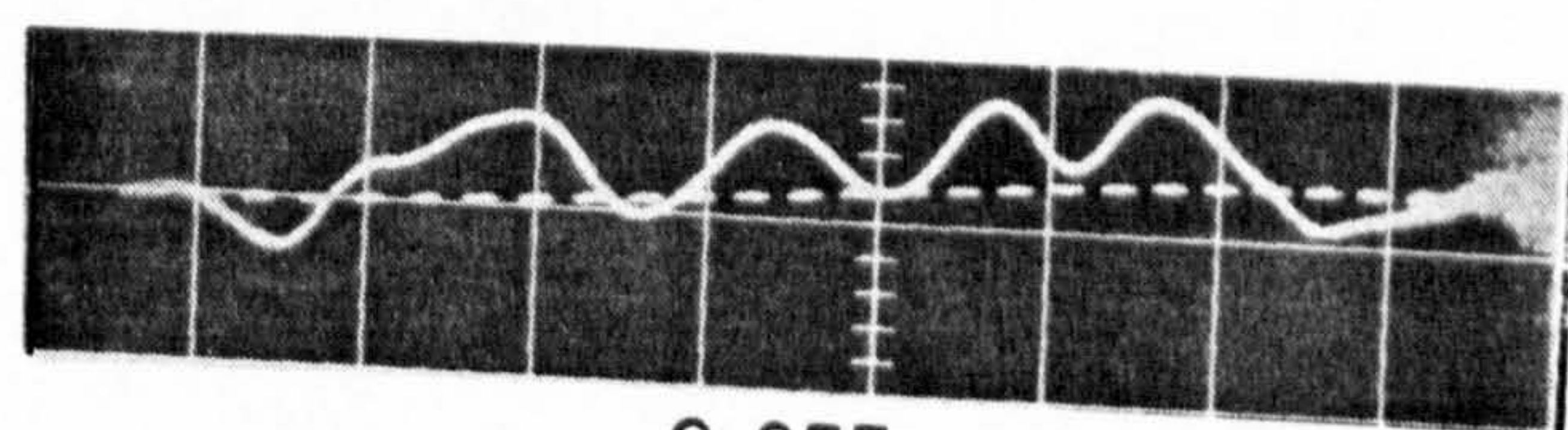


0.823

Pressure traces.

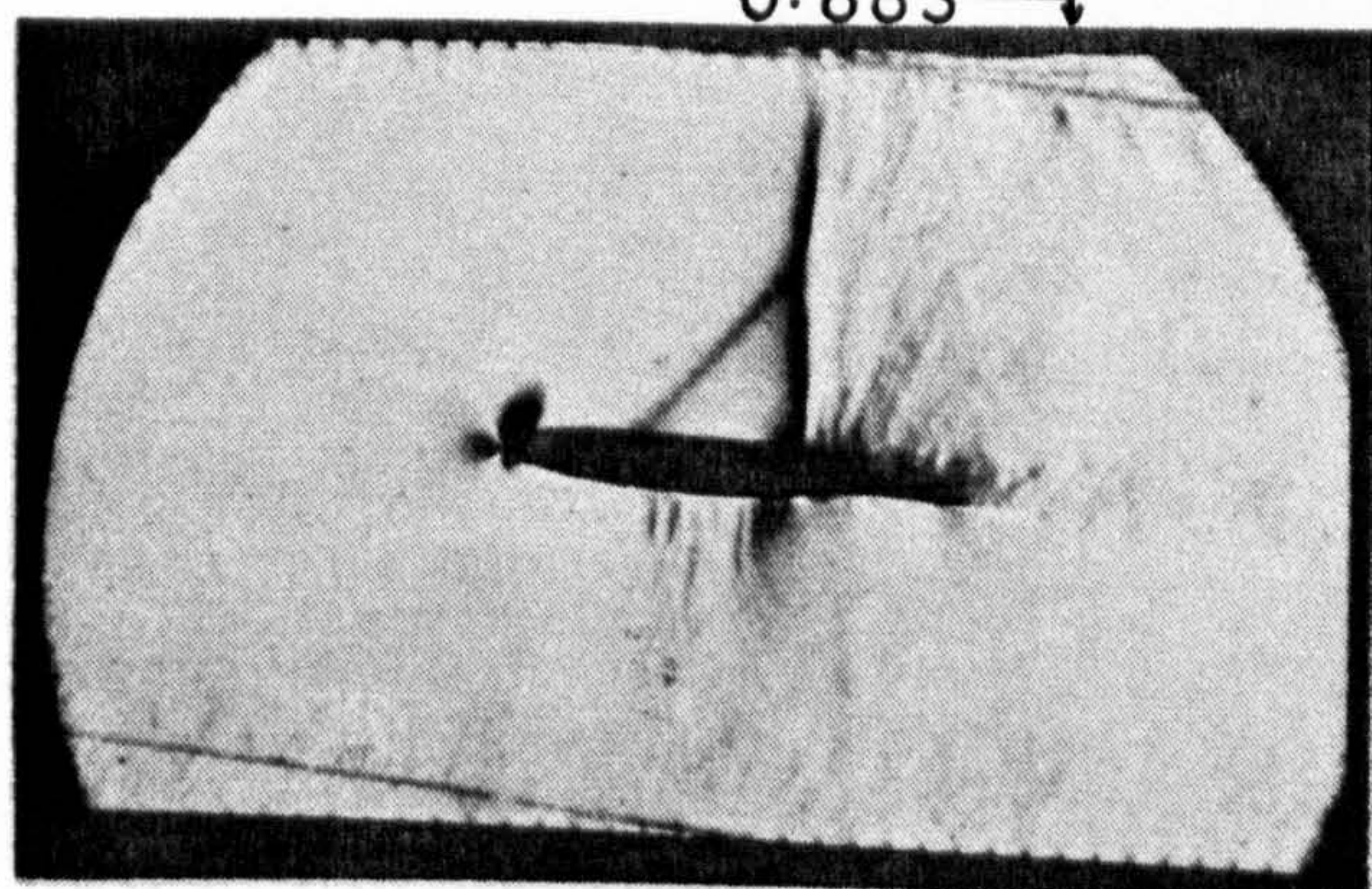


0.851

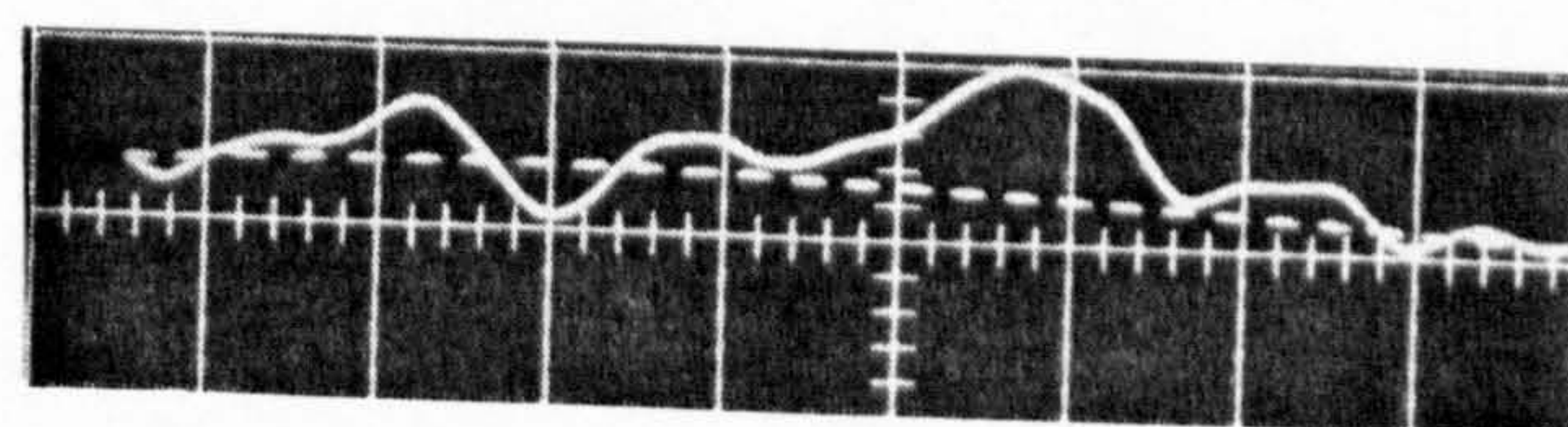


0.855

← OSCILLATORY FLOWS →



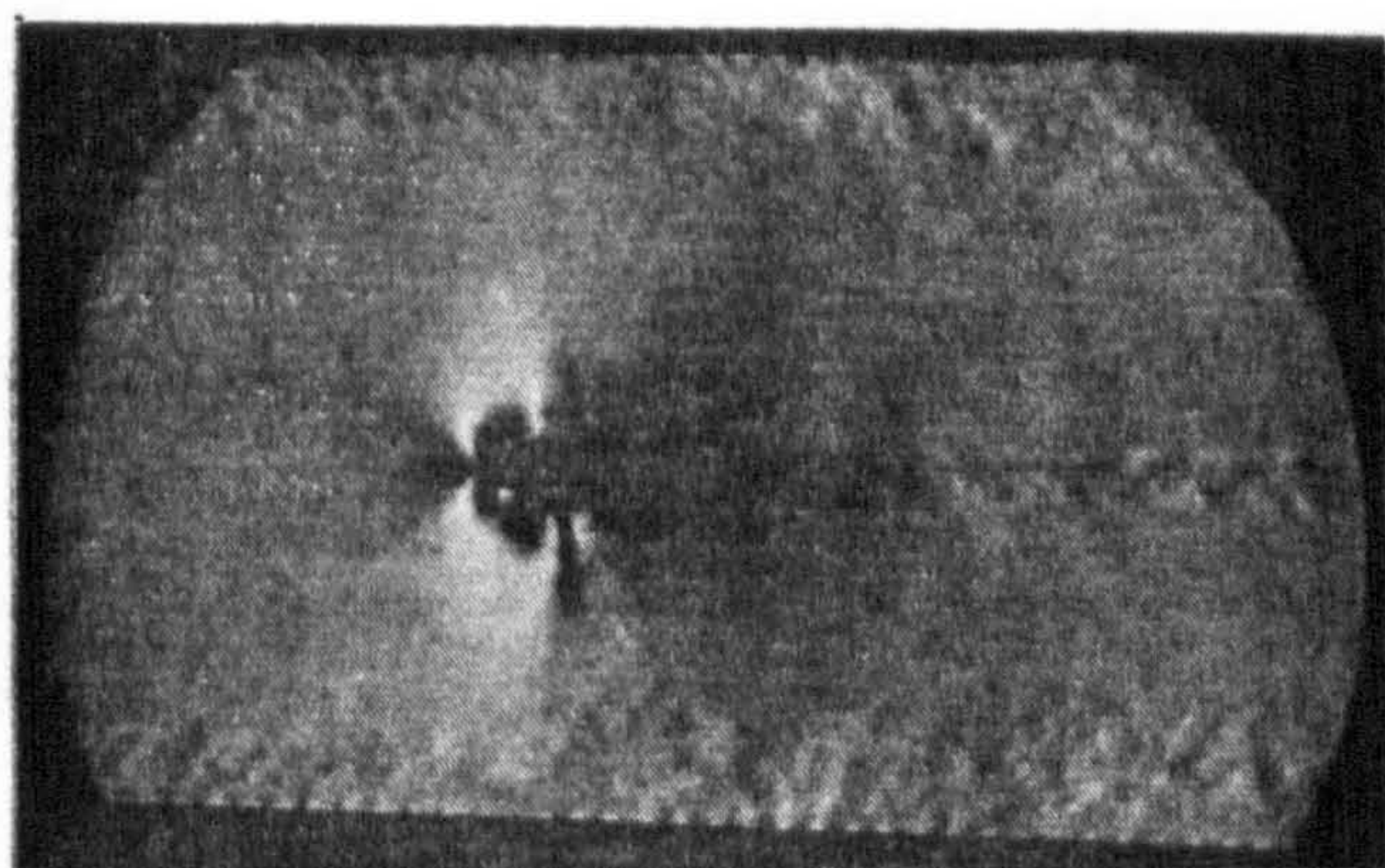
0.883



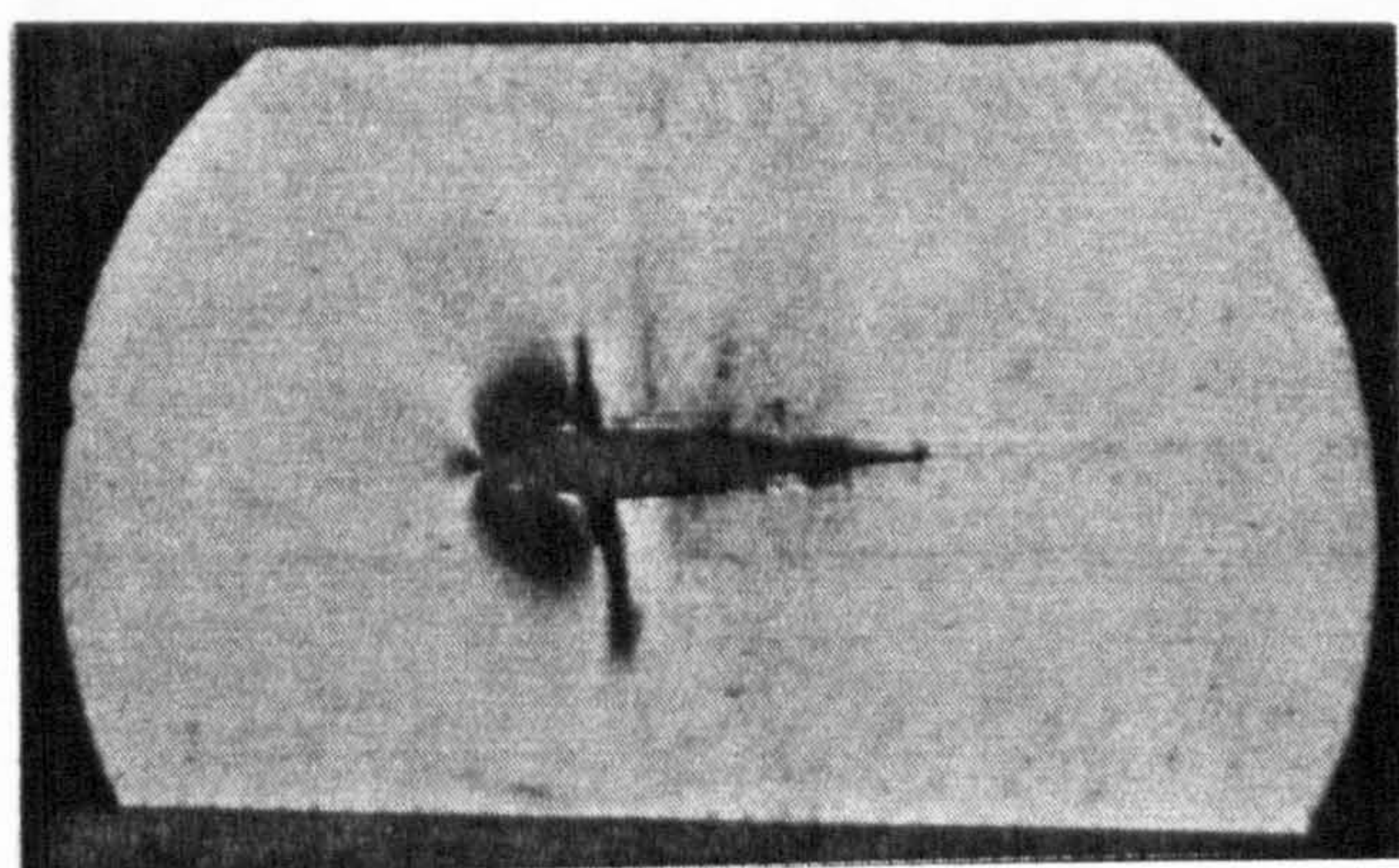
0.884

Figure 64. CAST 7 aerofoil,
 $\alpha = 6^\circ$ (free transition).

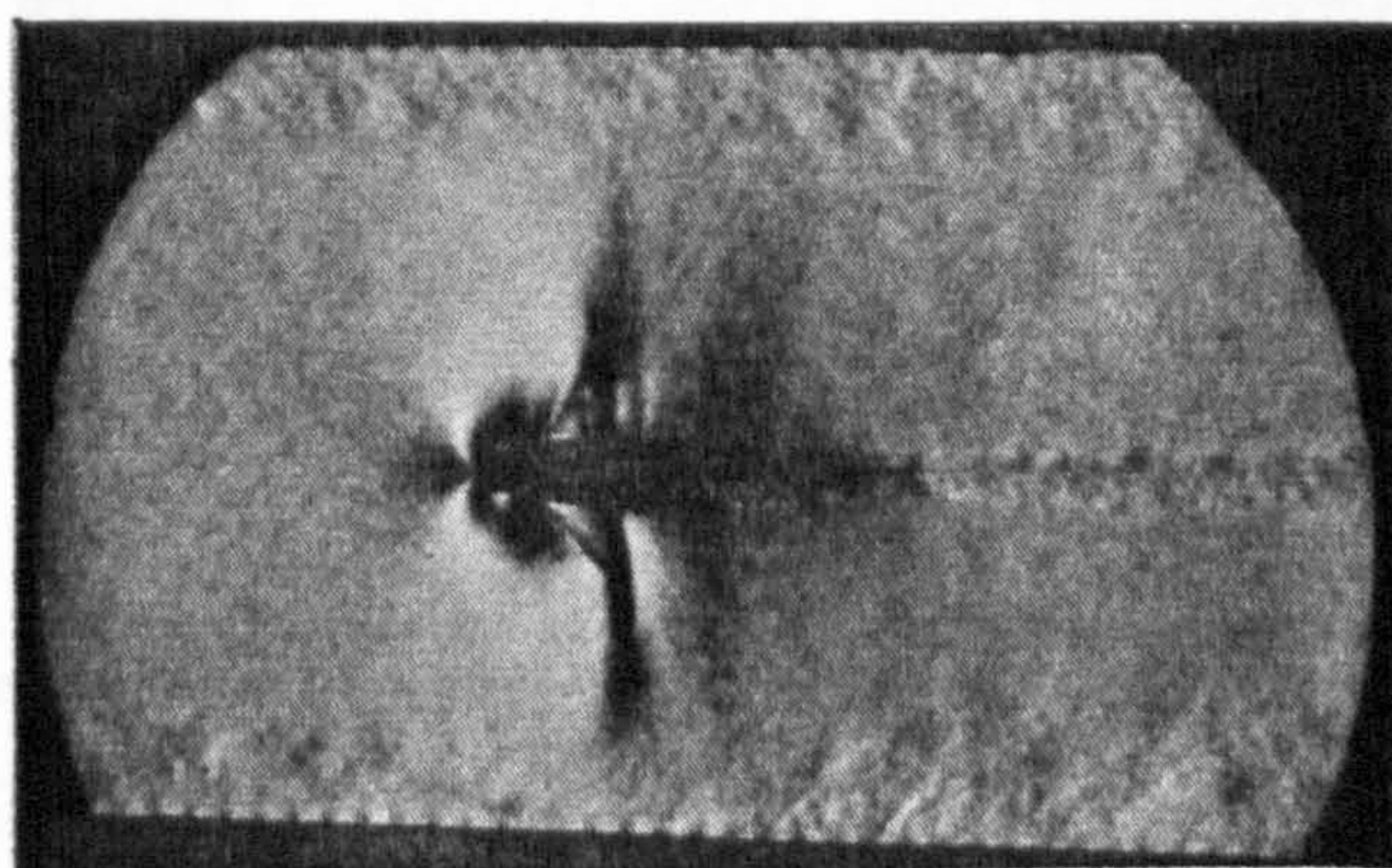
Schlieren pictures.



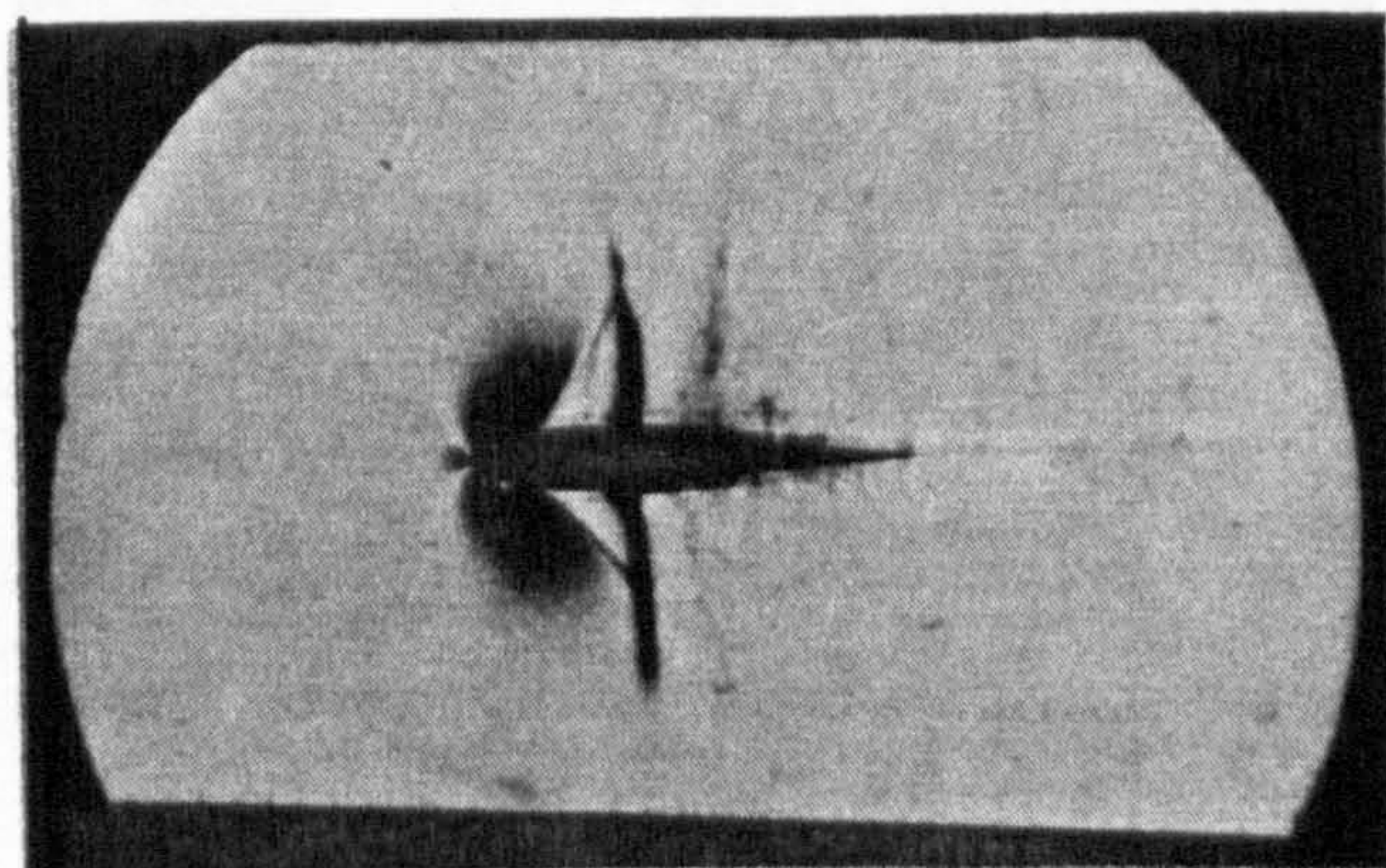
$M_\infty = 0.747$



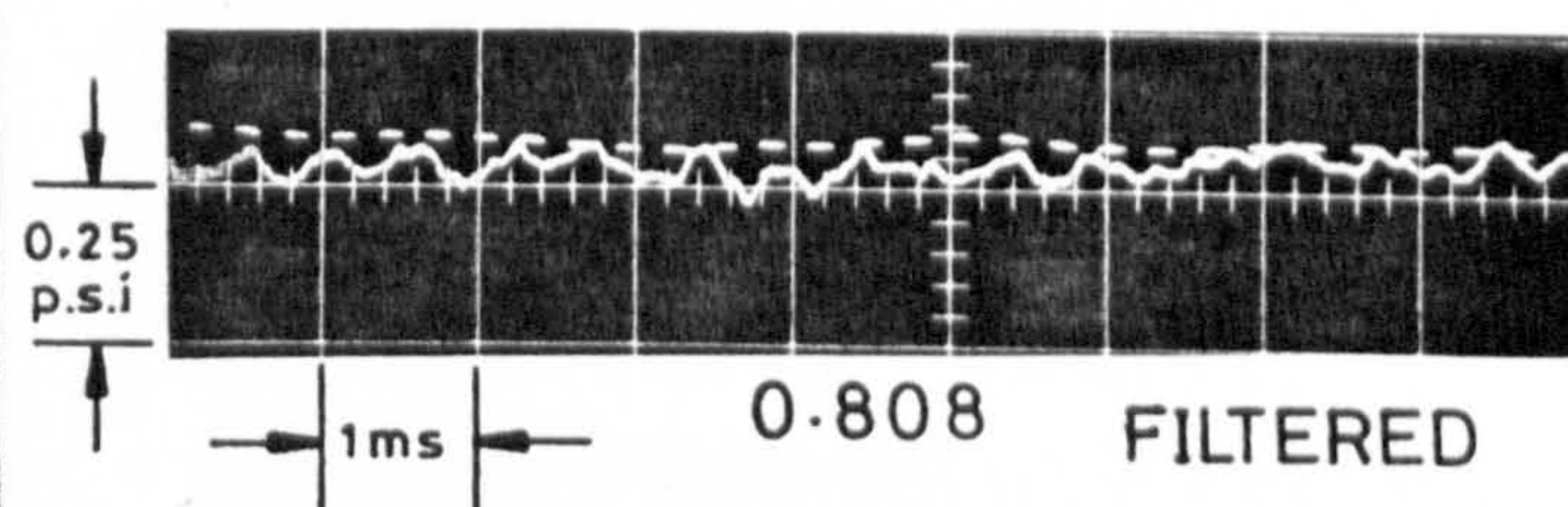
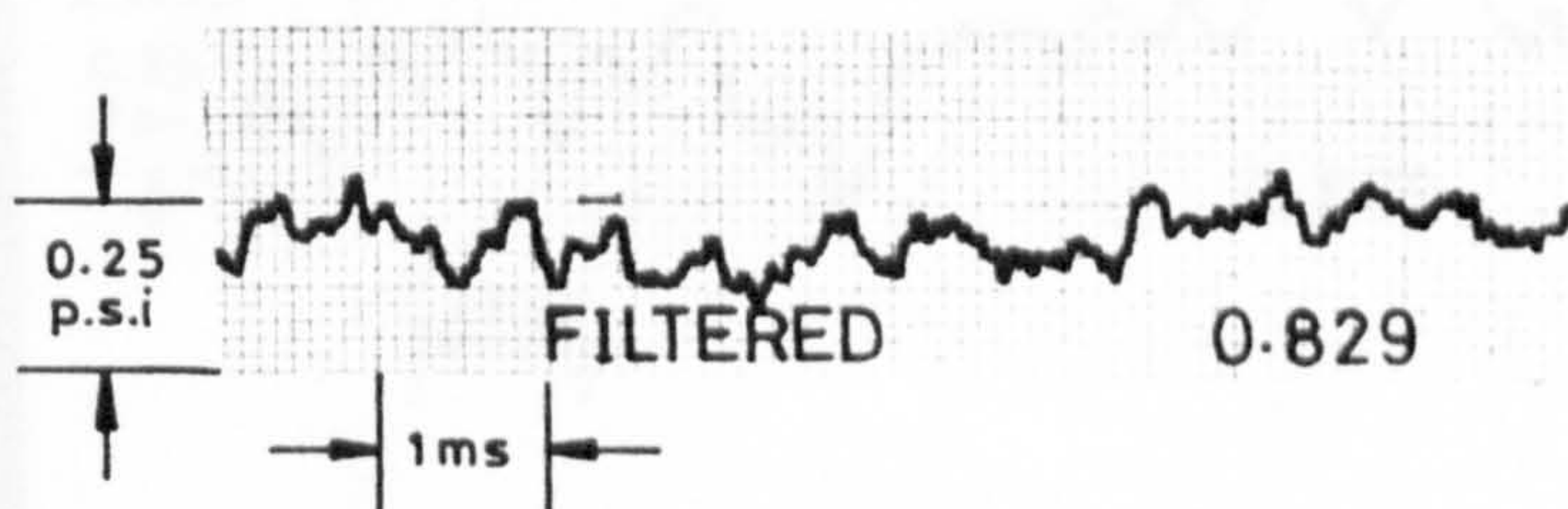
0.795



0.815

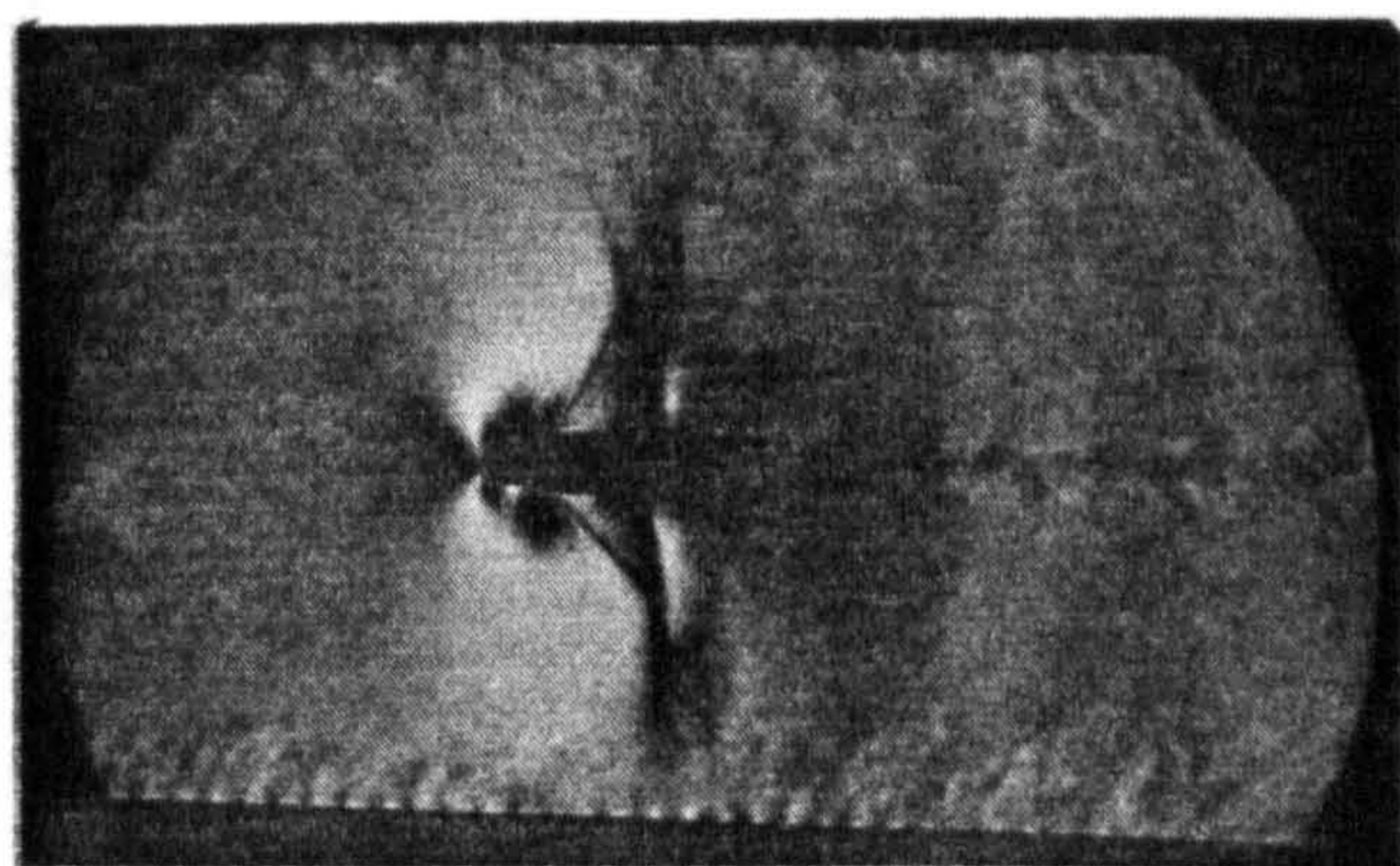


0.836

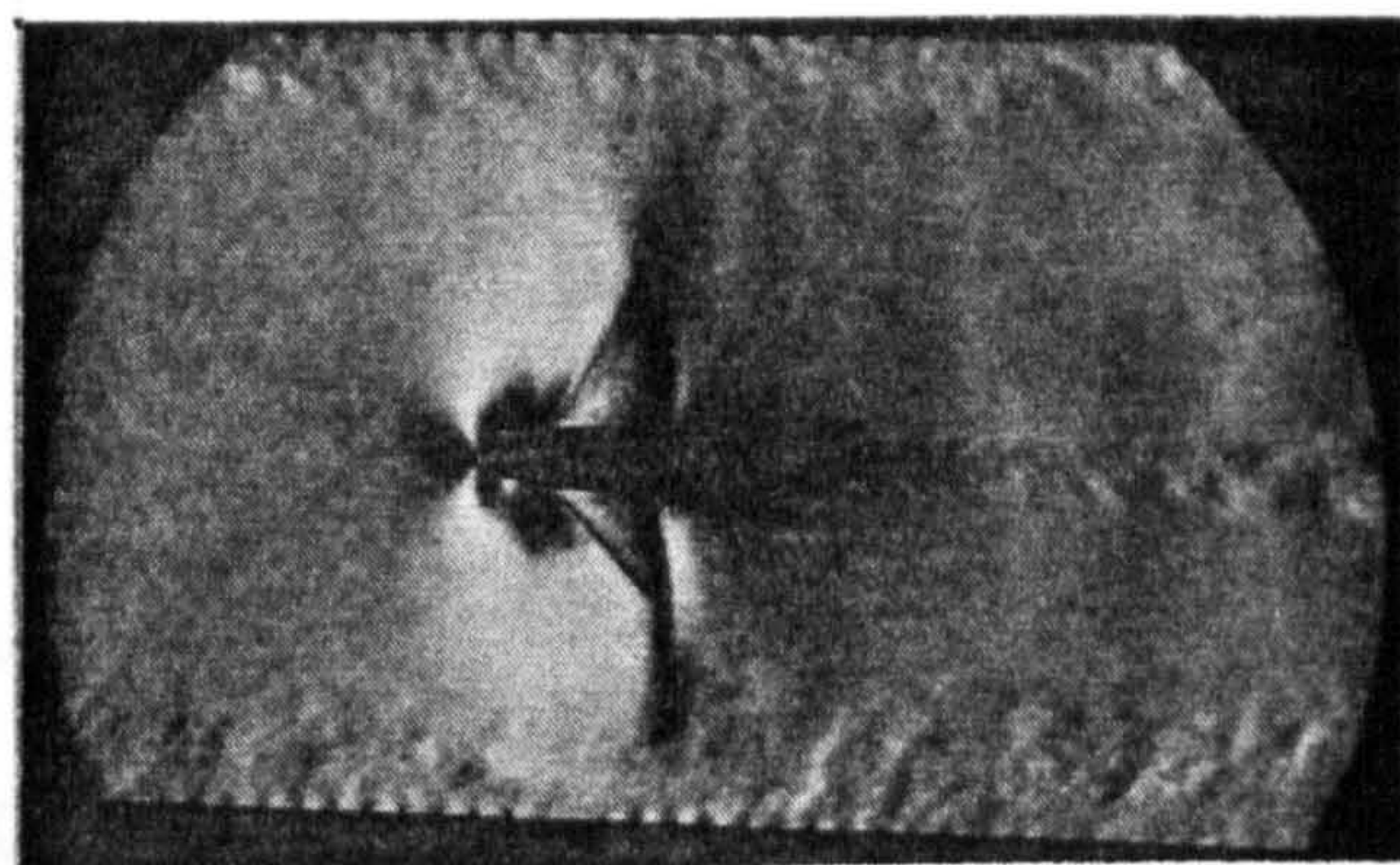


STEADY FLOWS

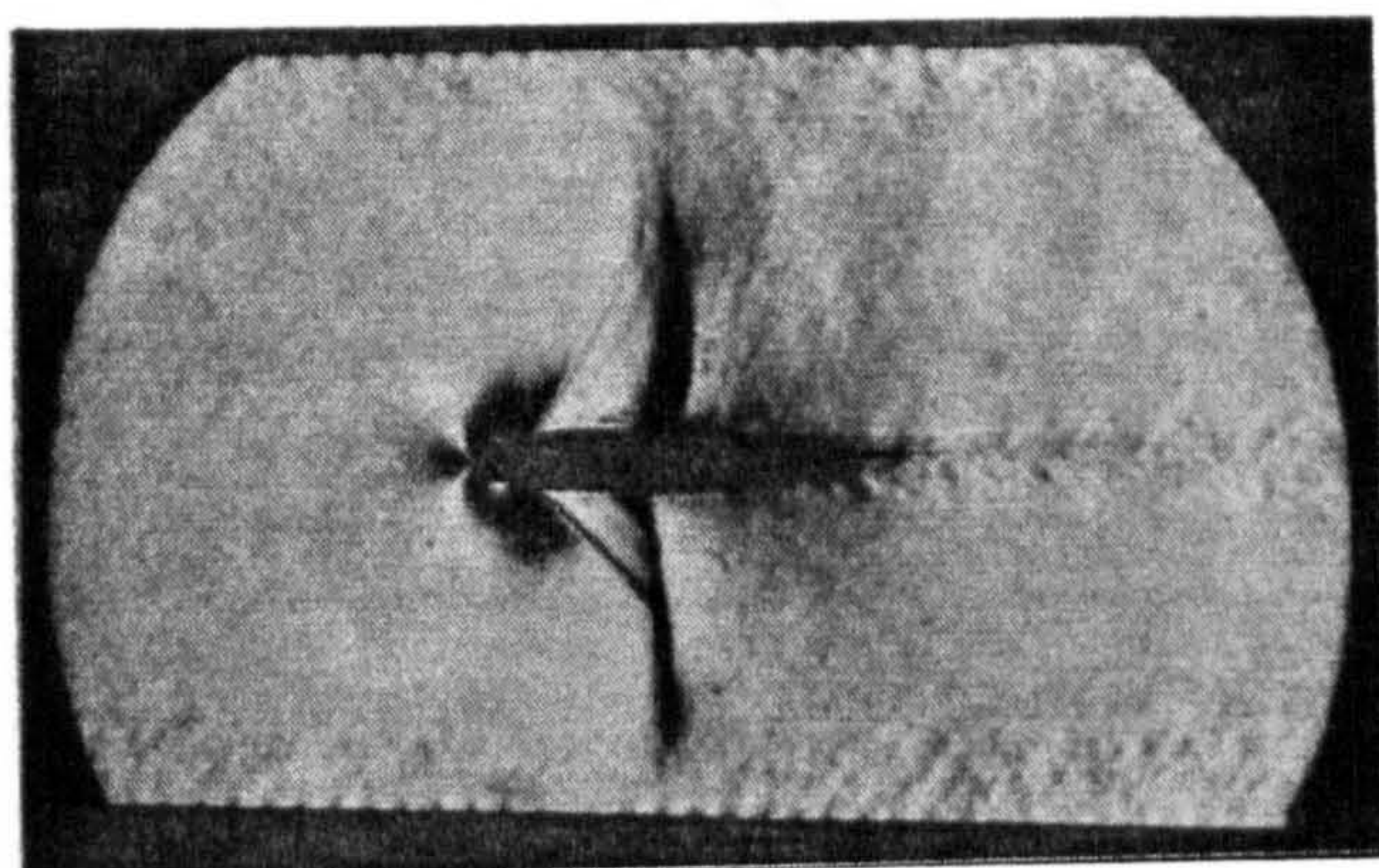
Figure 65. 11.8% Joukowski aerofoil at zero incidence, fixed transition. Schlieren pictures and sidewall pressure traces.



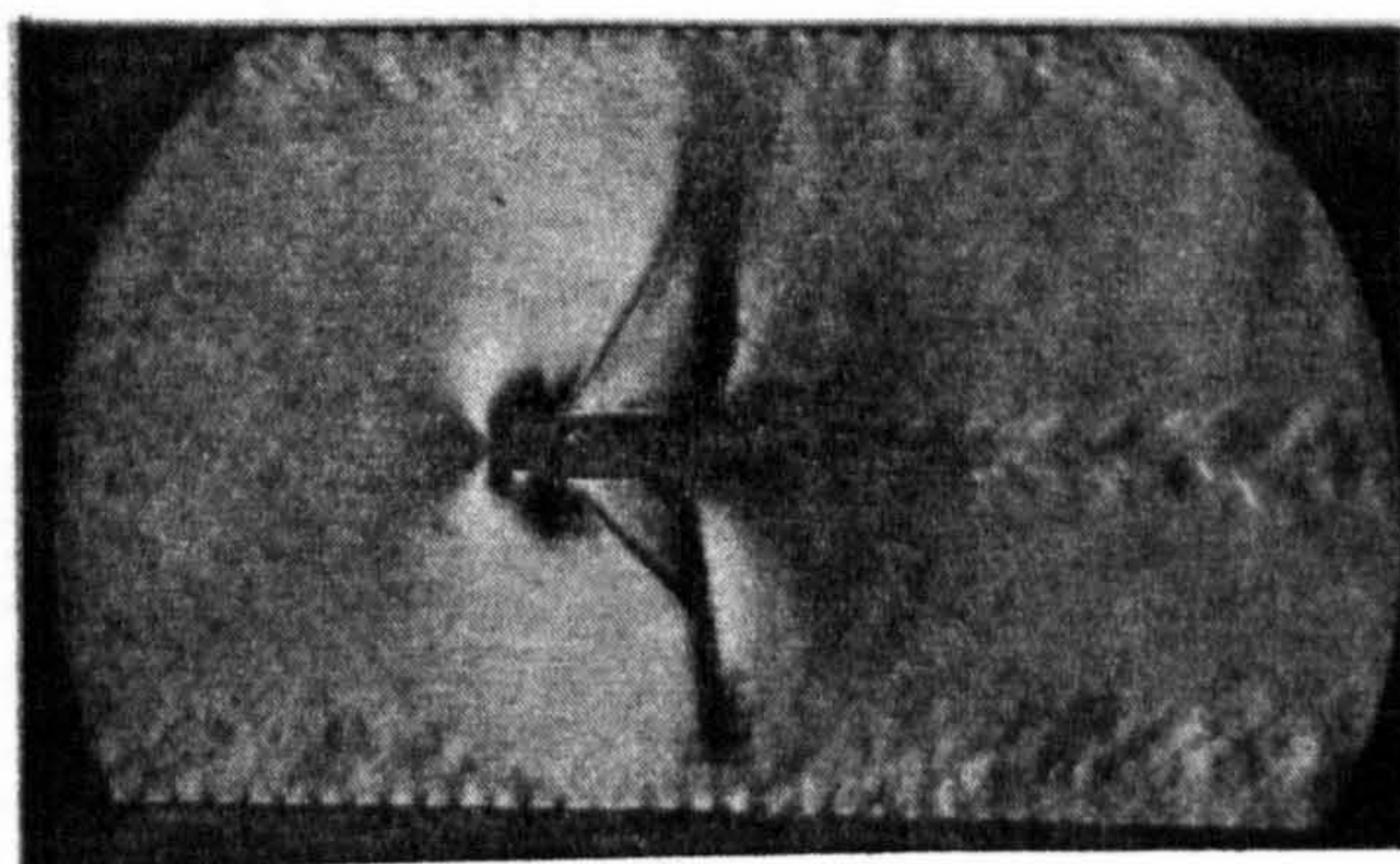
$M_\infty = 0.841$



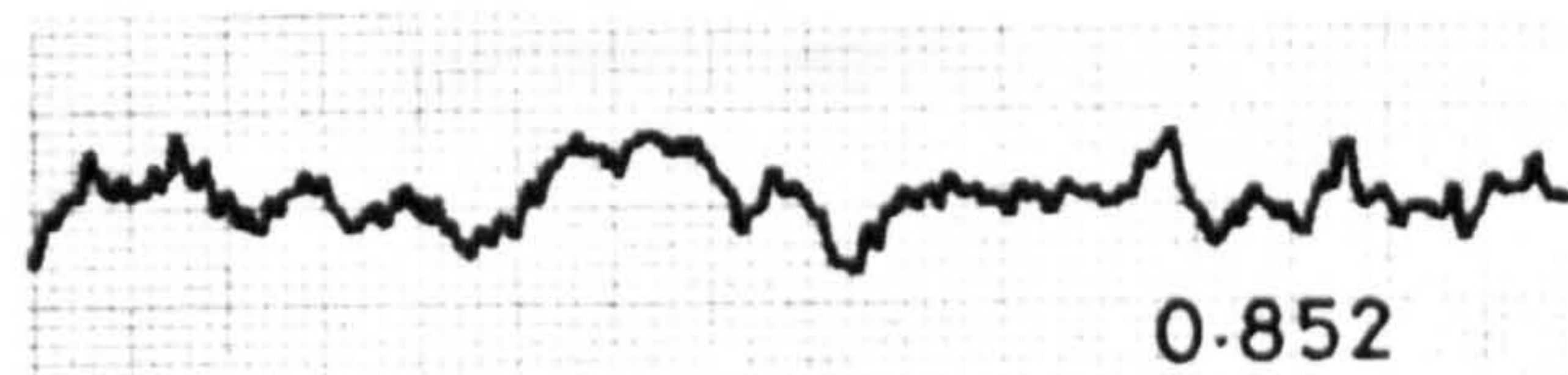
0.859



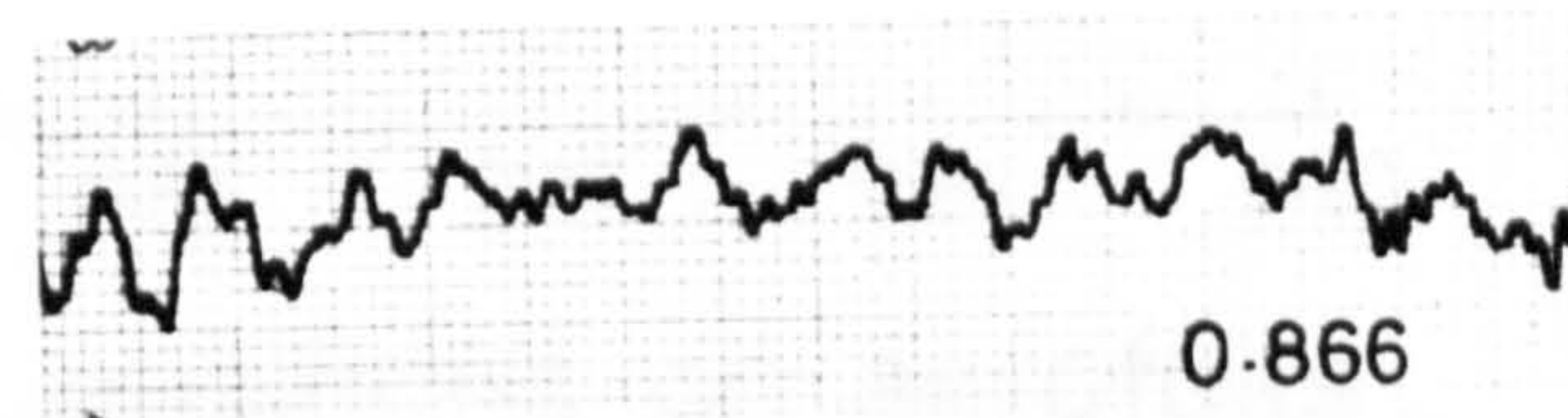
0.880



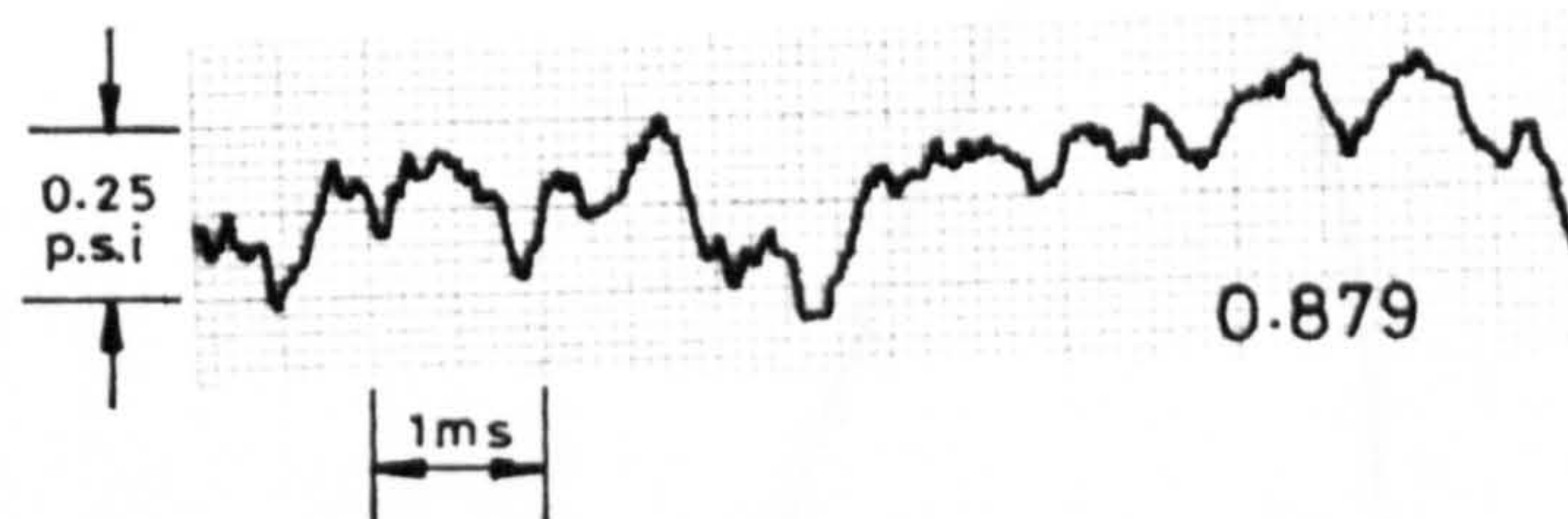
0.902



0.852

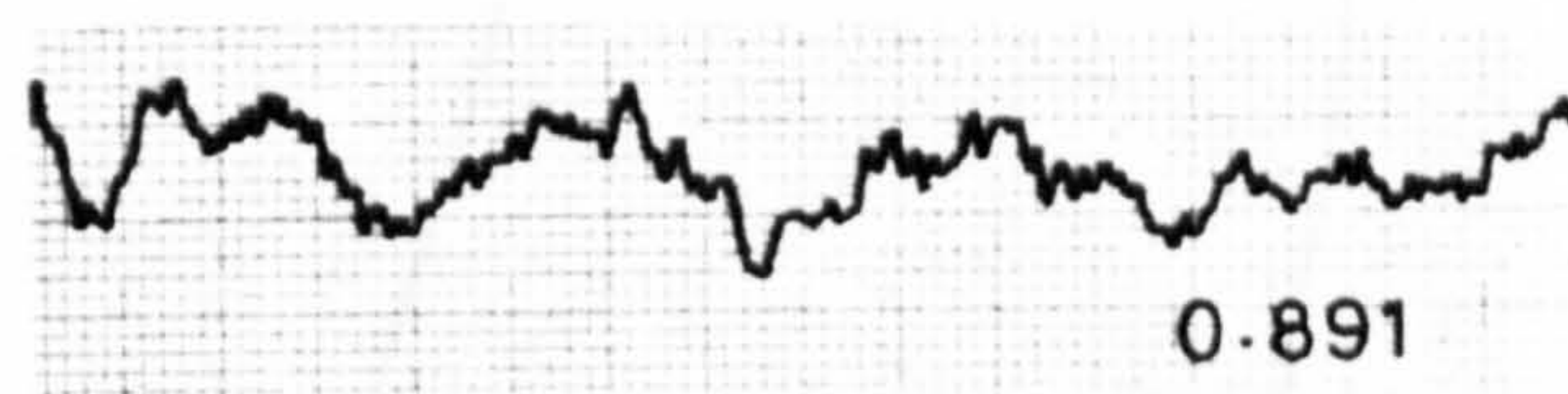


0.866



0.879

FILTERED OUTPUT



0.891

STEADY FLOWS

contd.

Figure 65.11.8% Joukowski aerofoil at zero incidence,
fixed transition.

Schlieren pictures and sidewall pressure traces.

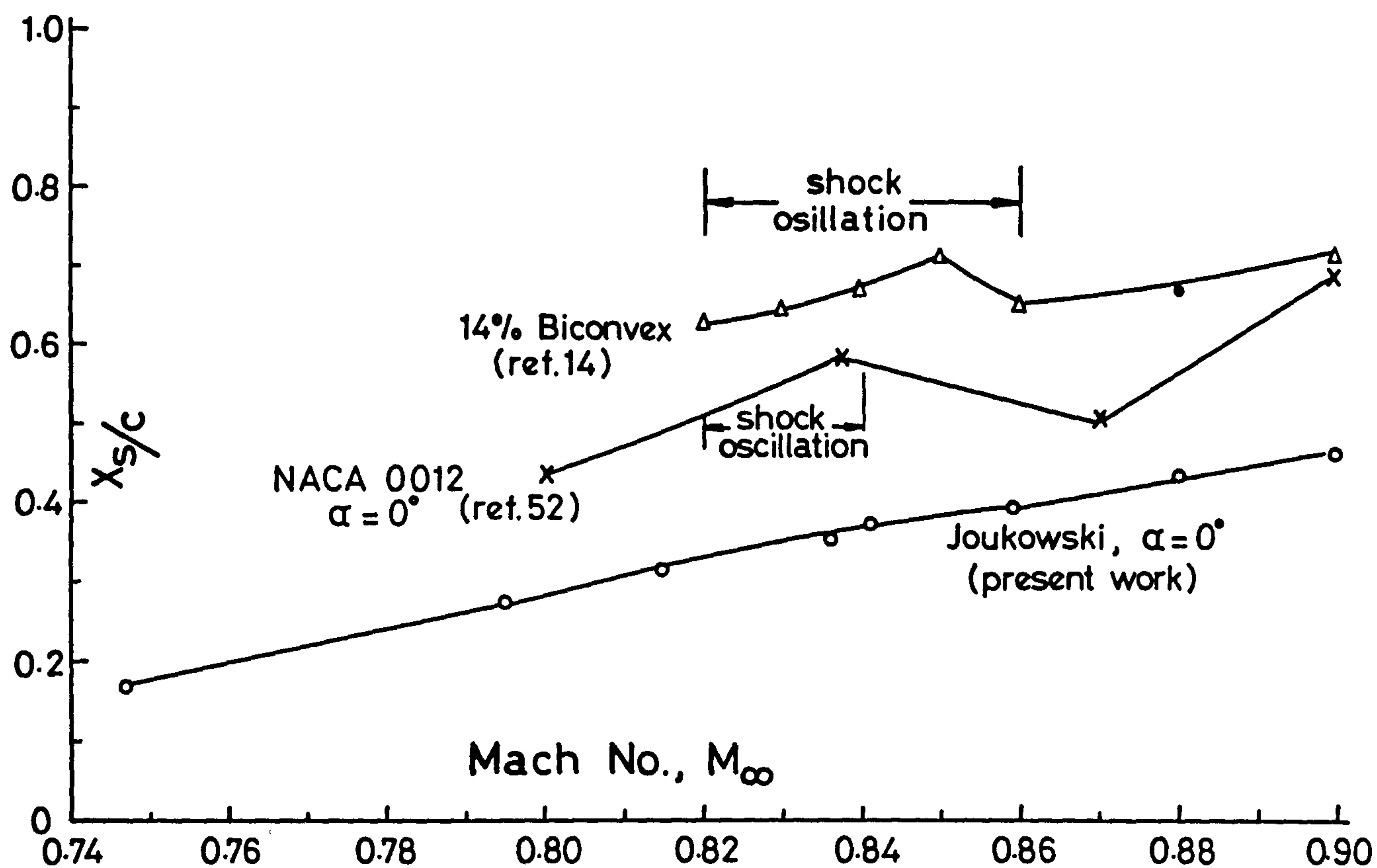


Figure 66. Comparison of shock positions on aerofoils.

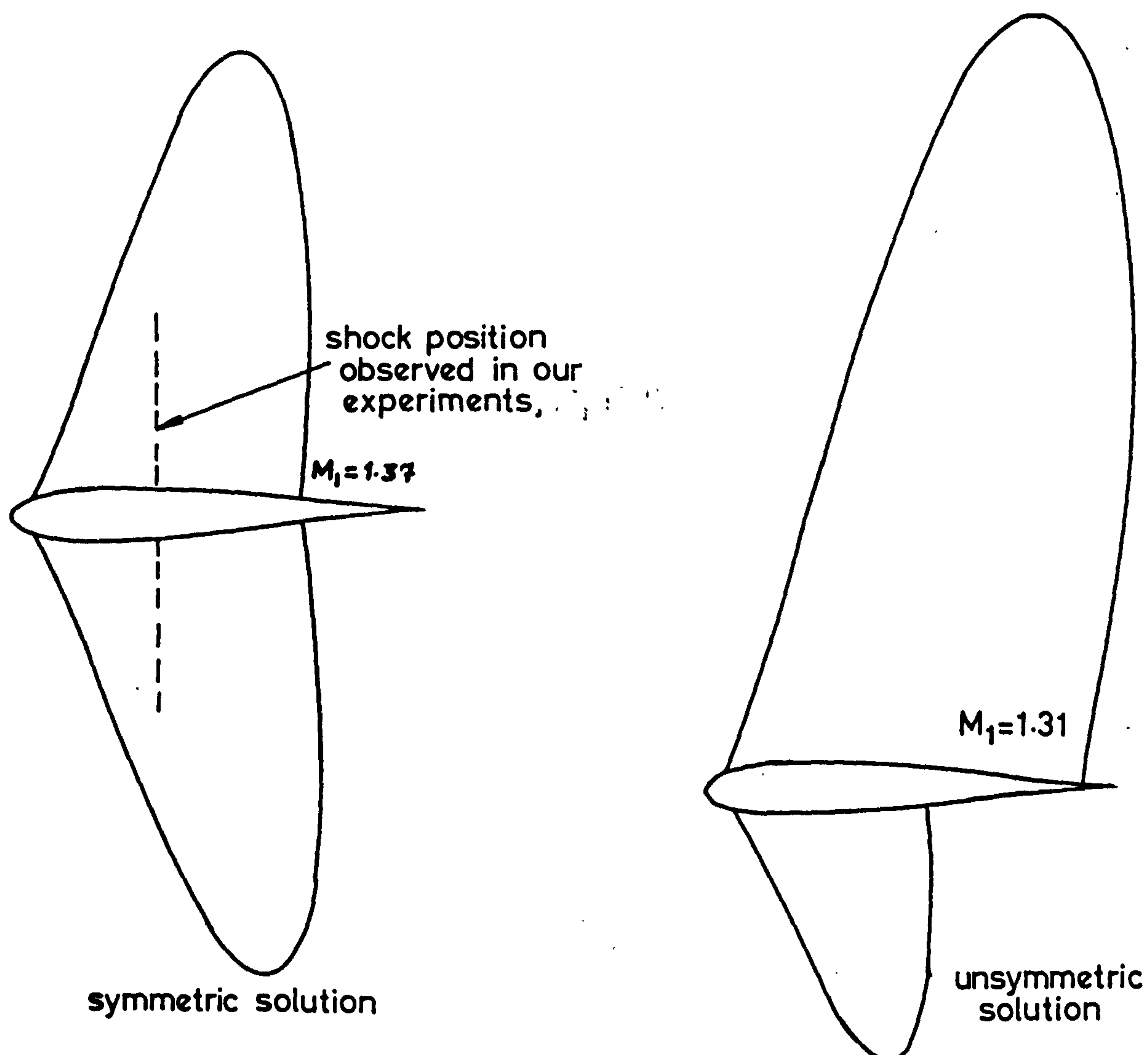
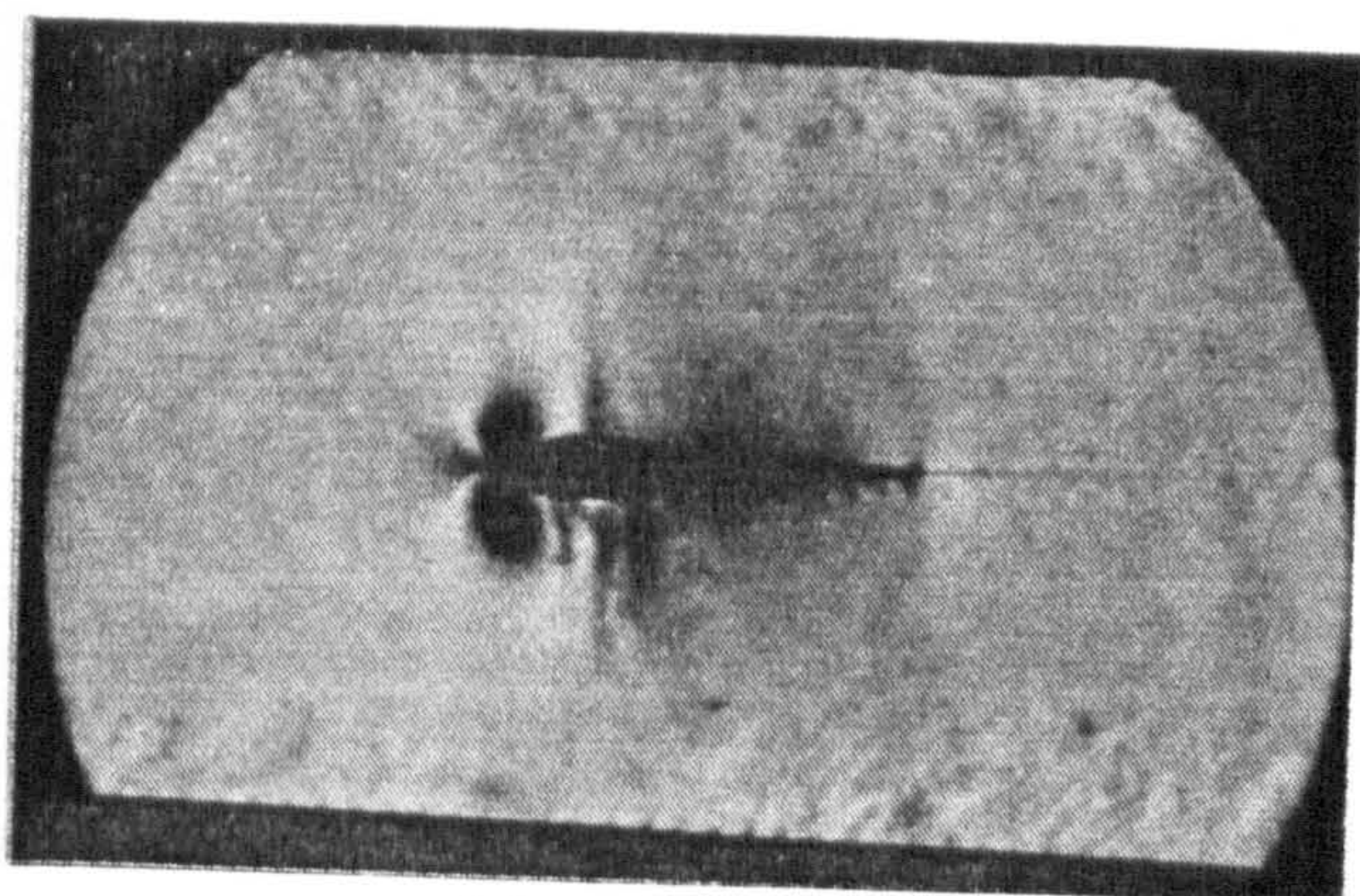
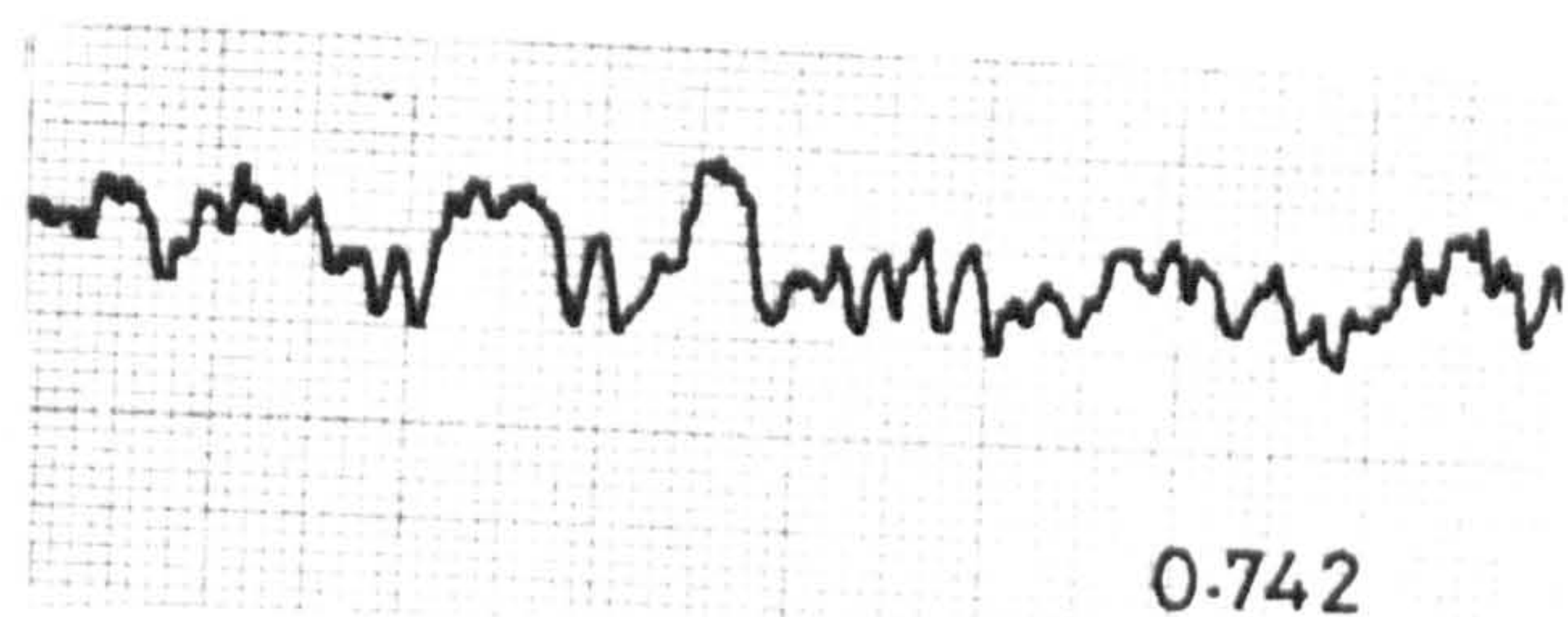


Figure 67. Comparison with theory (ref.67)

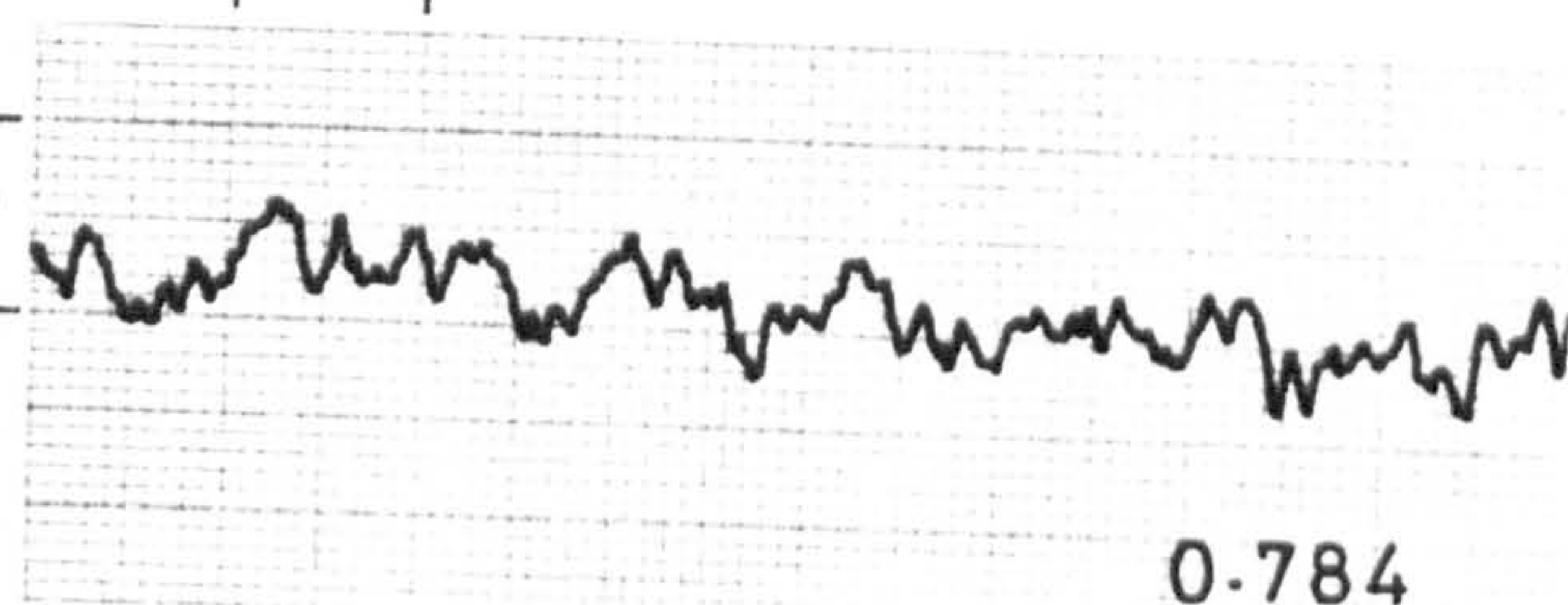
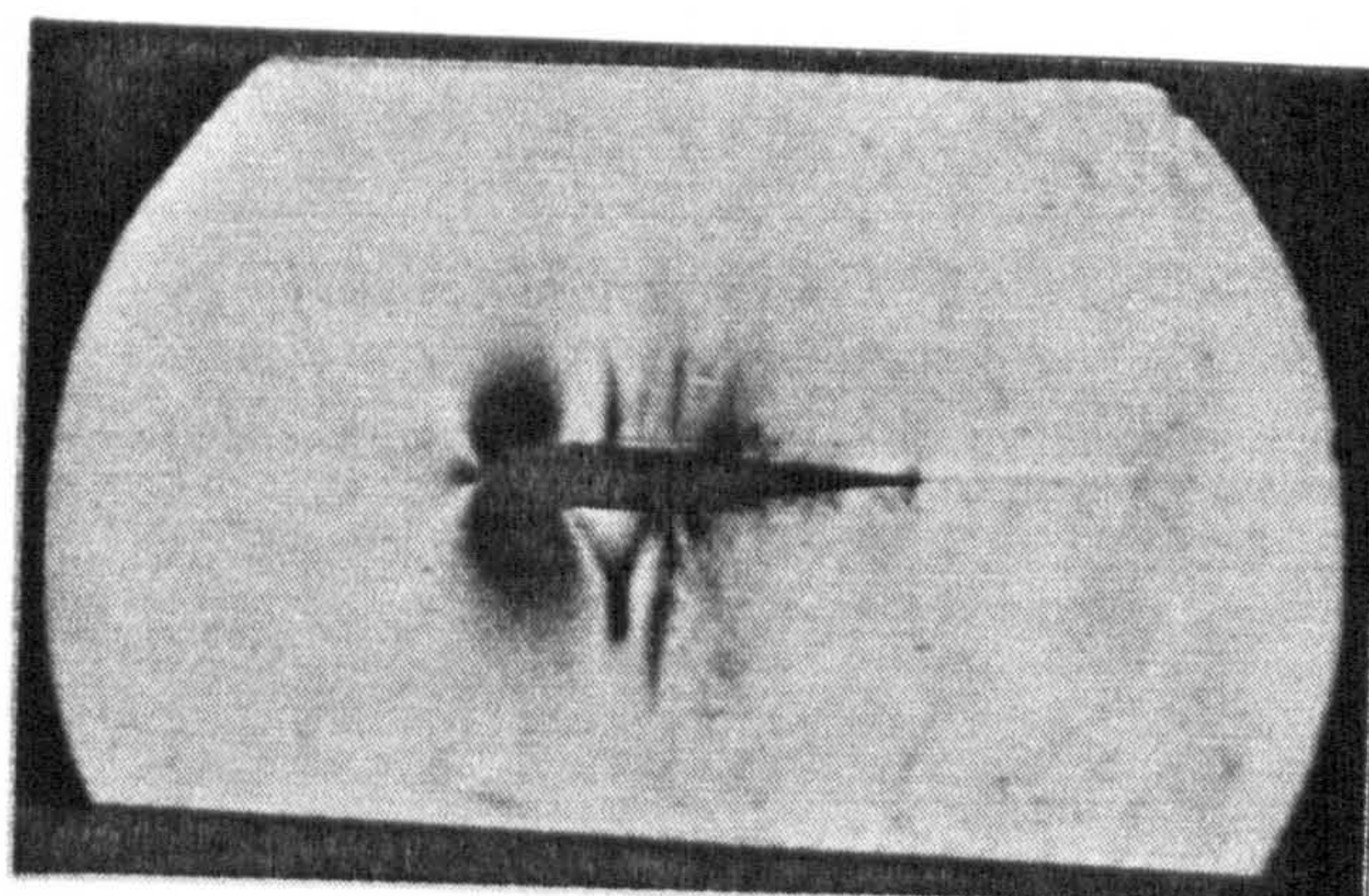


$M_{\infty} = 0.749$

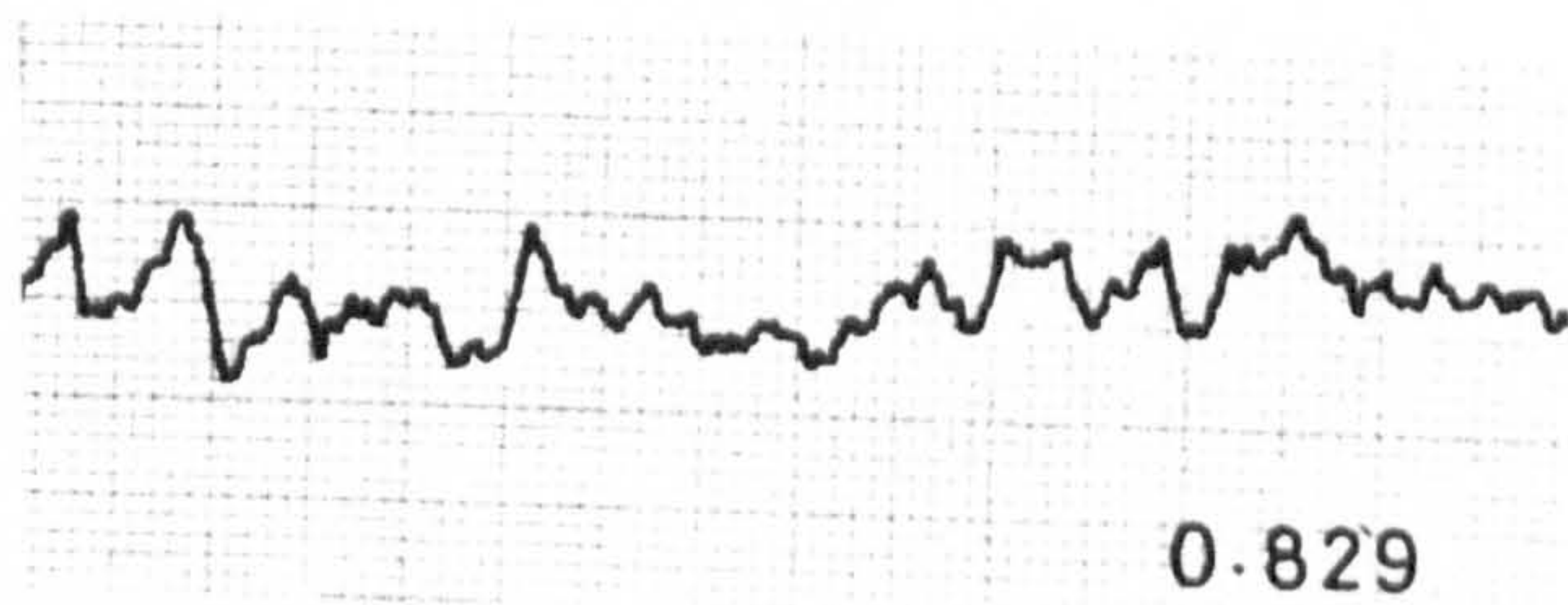
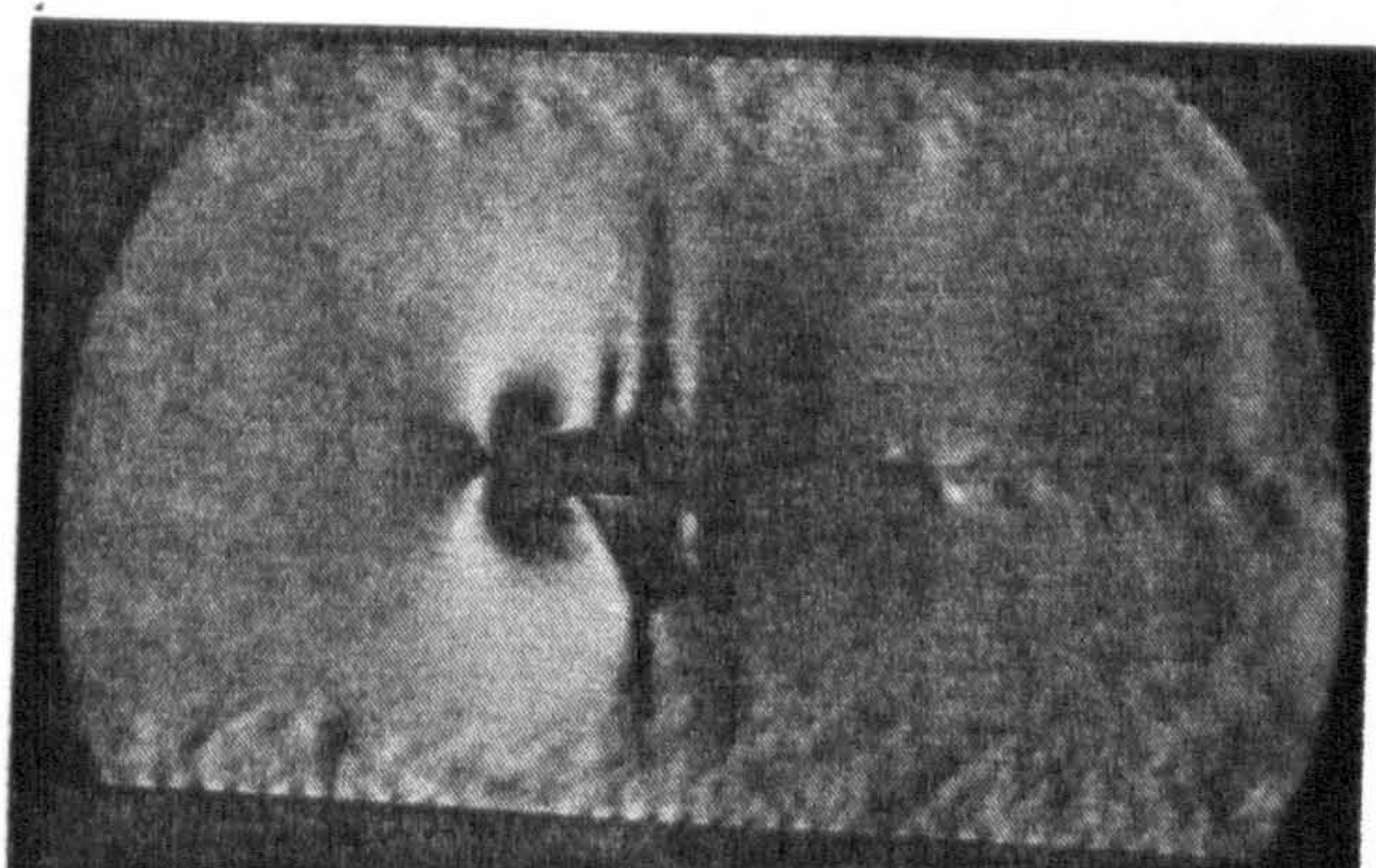


1ms

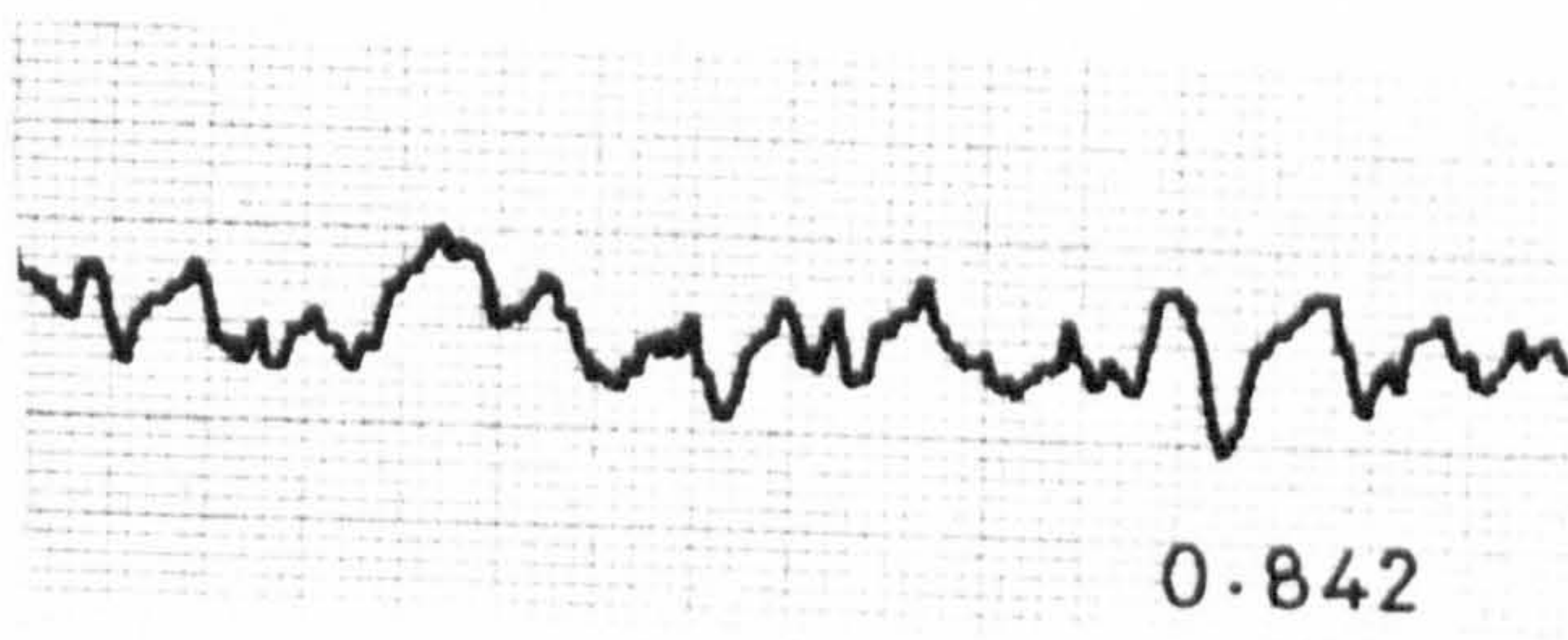
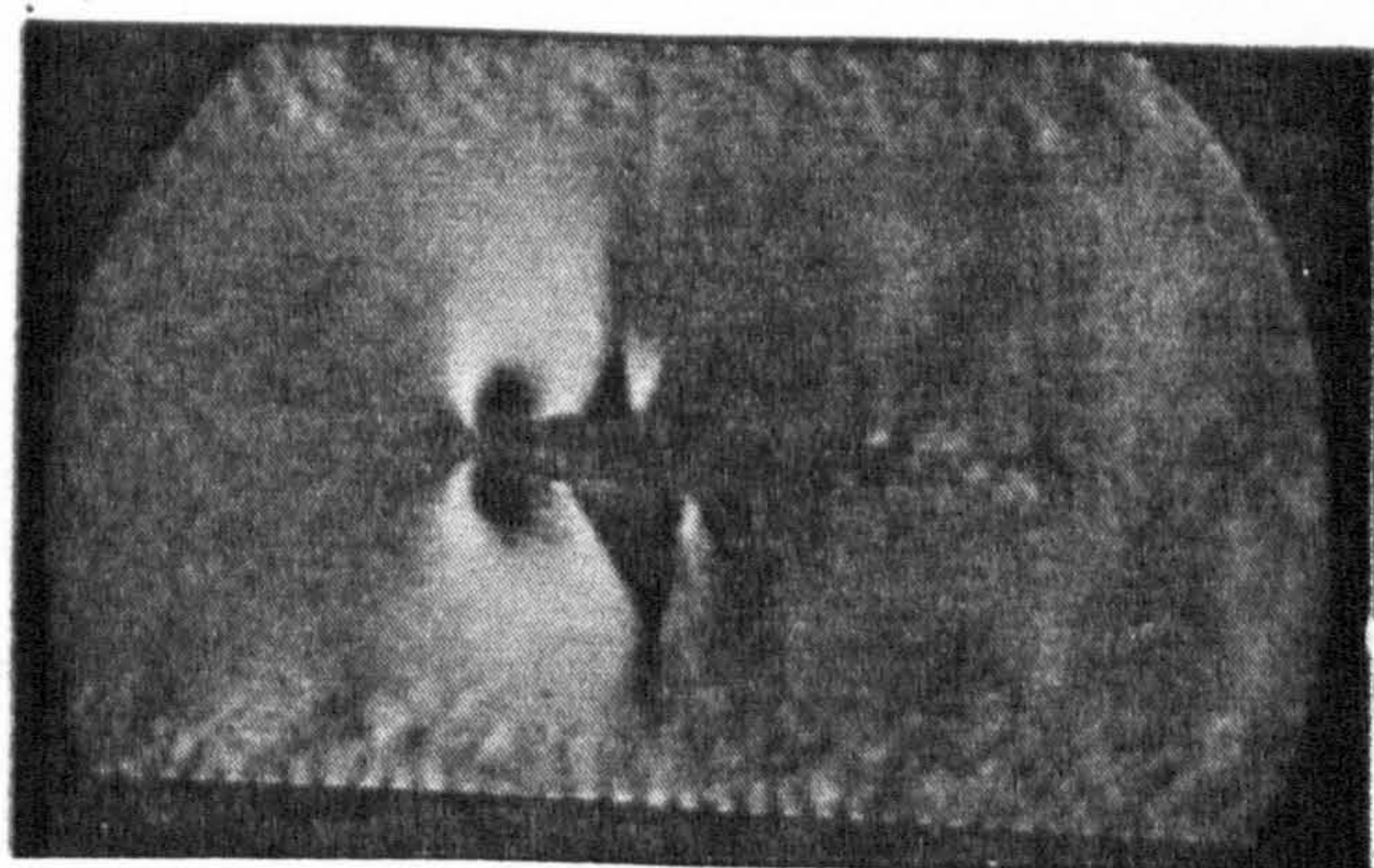
0.25
p.s.i



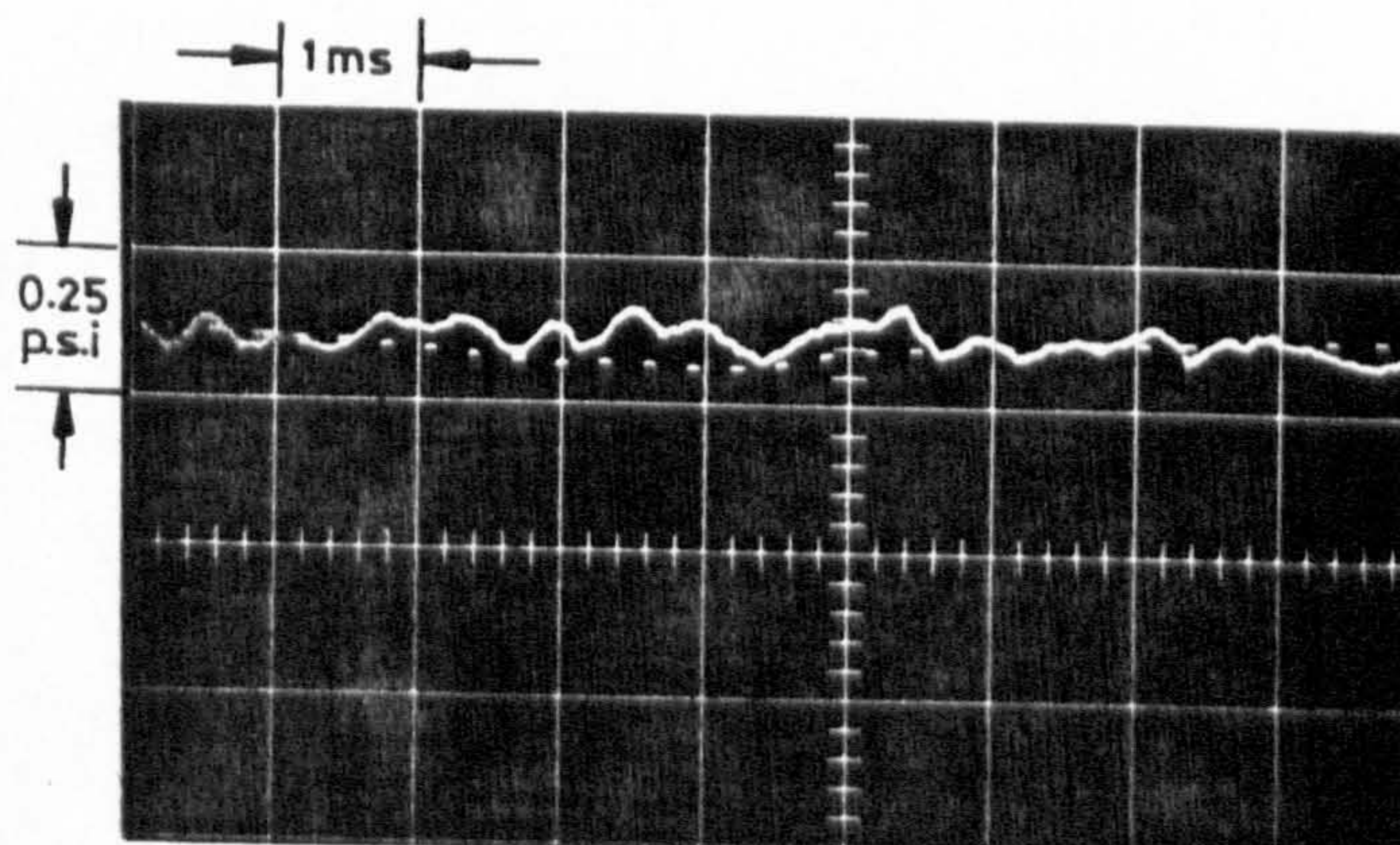
0.800



0.819



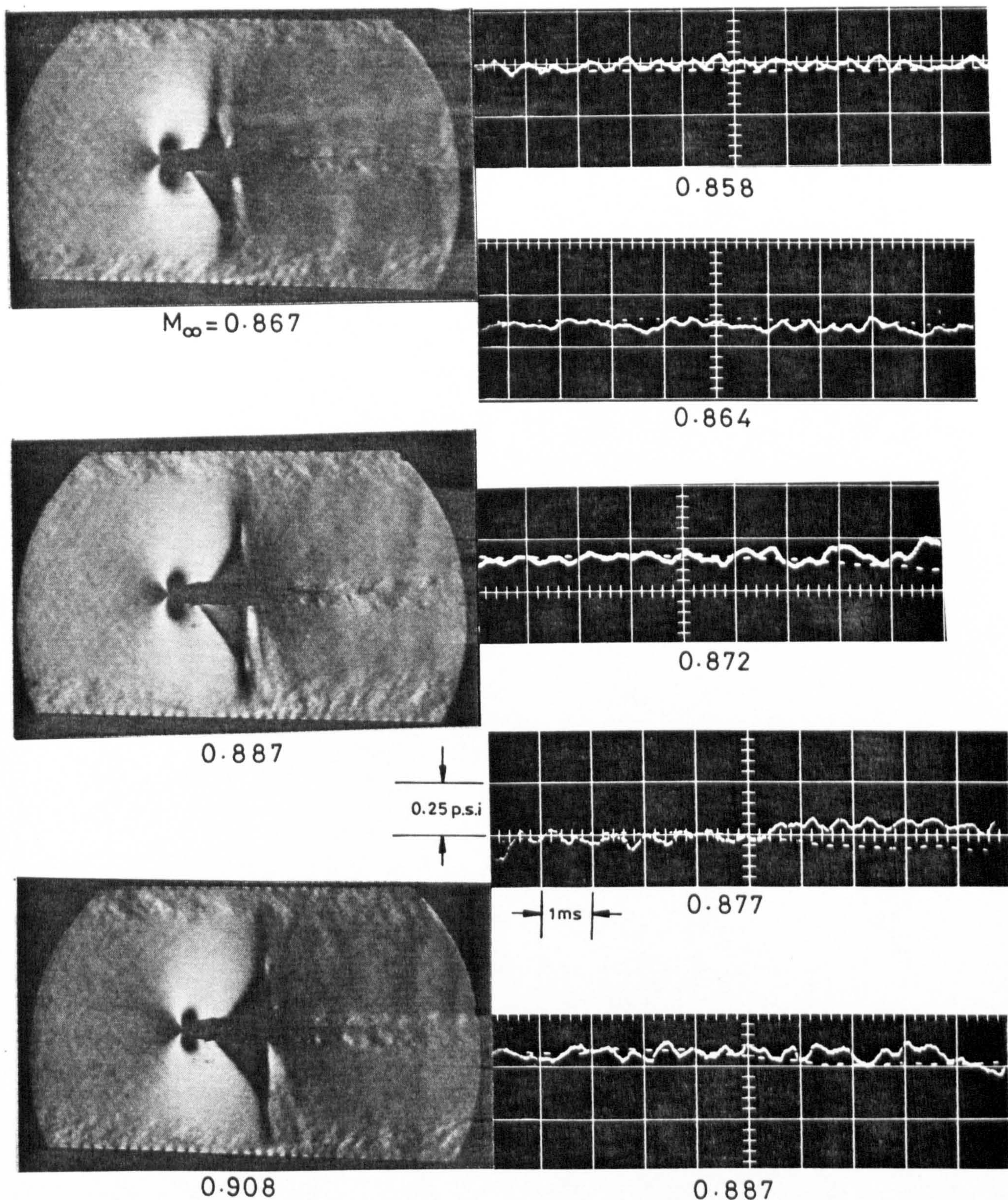
0.841



FILTERED TRACES

STEADY FLOWS

Figure 68.11-8% Joukowski aerofoil at zero incidence, free transition. Schlieren pictures and sidewall pressure traces.



contd.

STEADY FLOWS

FILTERED OUTPUTS

Figure 68. 11.8% Joukowski aerofoil at zero incidence, free transition.

Schlieren pictures and sidewall pressure traces.

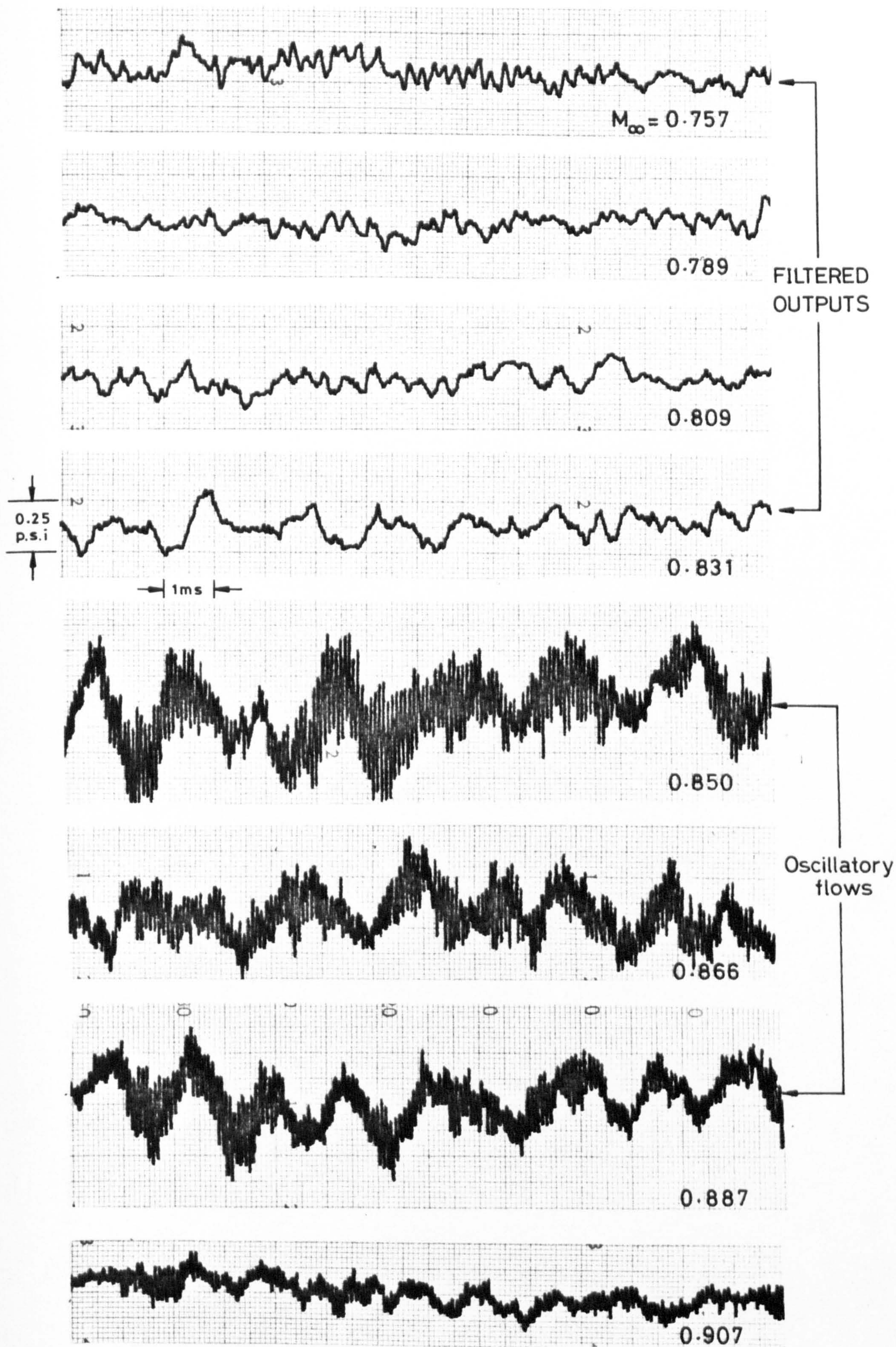


Figure 69. Sidewall pressure fluctuations on a 11.8% Joukowski aerofoil at 3° incidence (fixed transition).

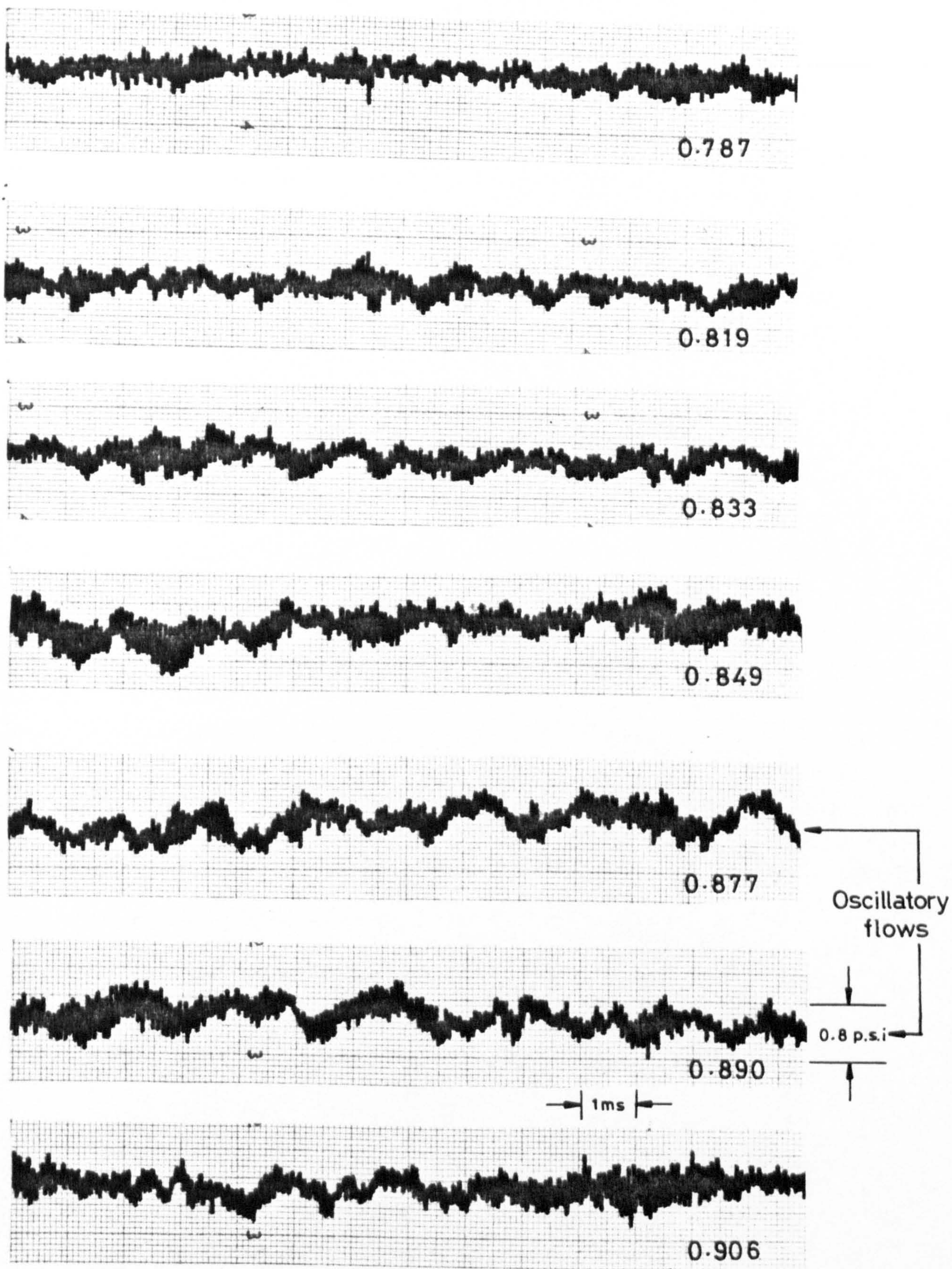


Figure 70. Sidewall pressure fluctuations on a 11.8% Joukowski aerofoil at 3° incidence (free transition).

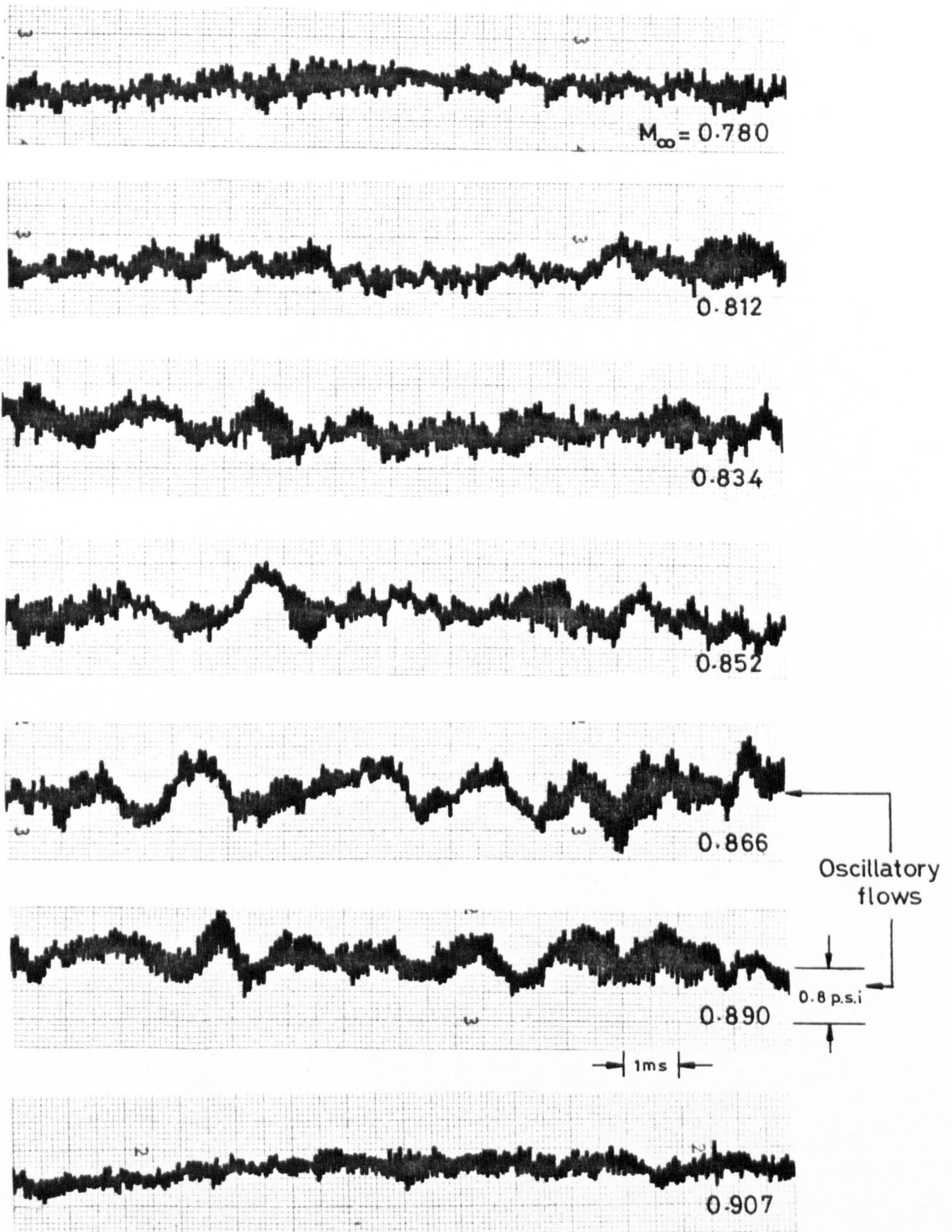


Figure 71. Sidewall pressure fluctuations on a 11.8% Joukowski aerofoil at 6° incidence (fixed transition).

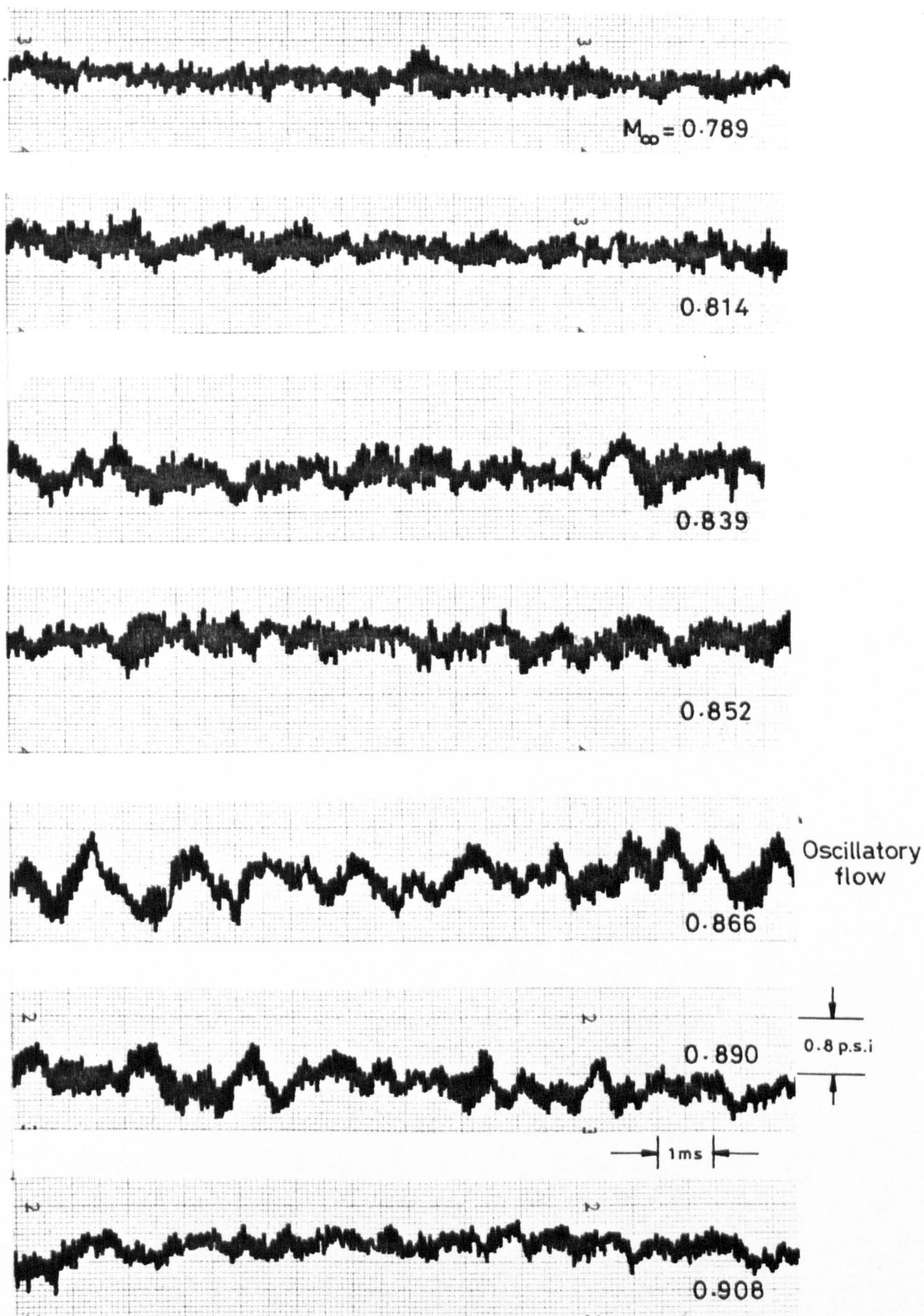


Figure 72. Sidewall pressure fluctuations on a 11.8% Joukowski aerofoil at 6° incidence (free transition).

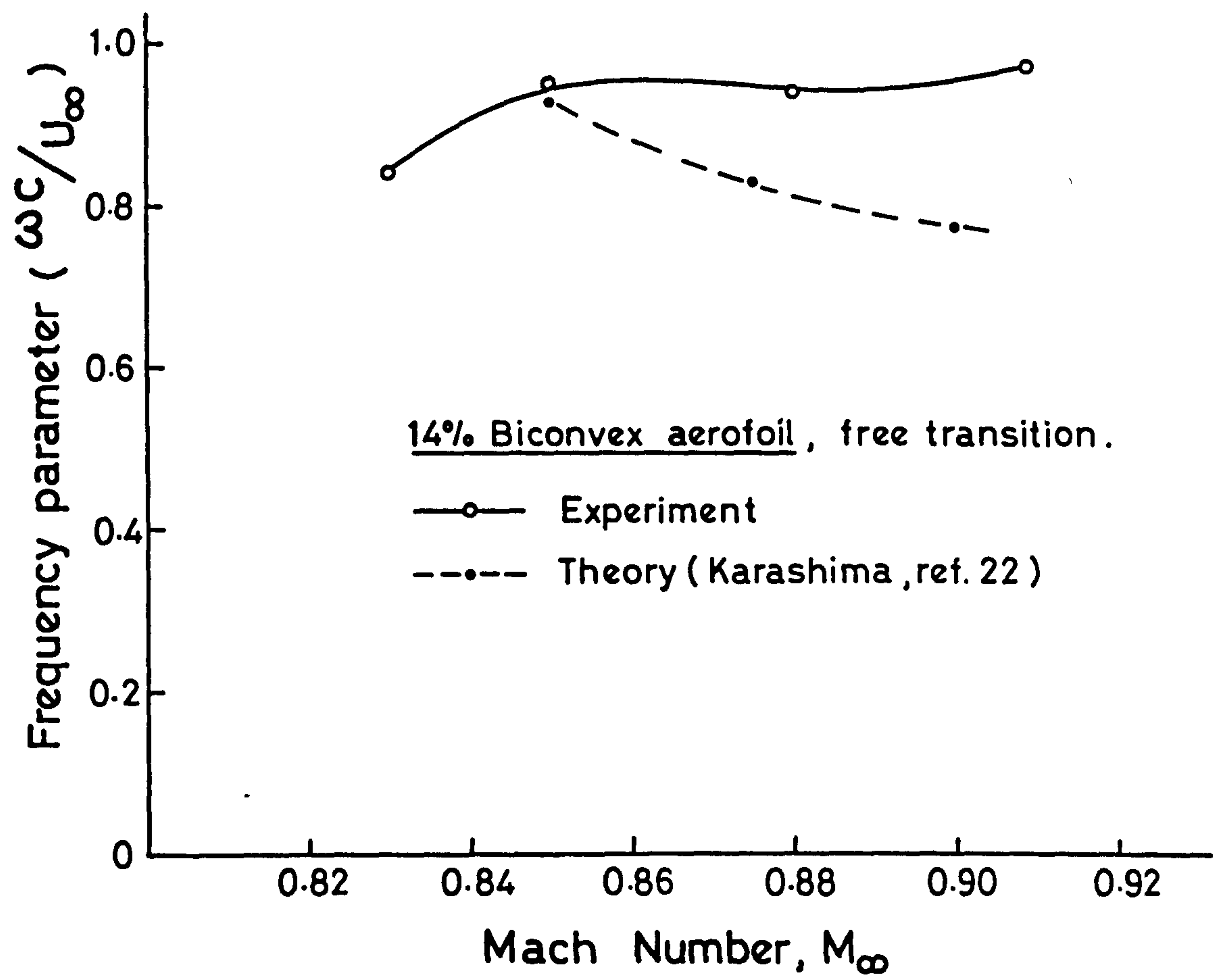
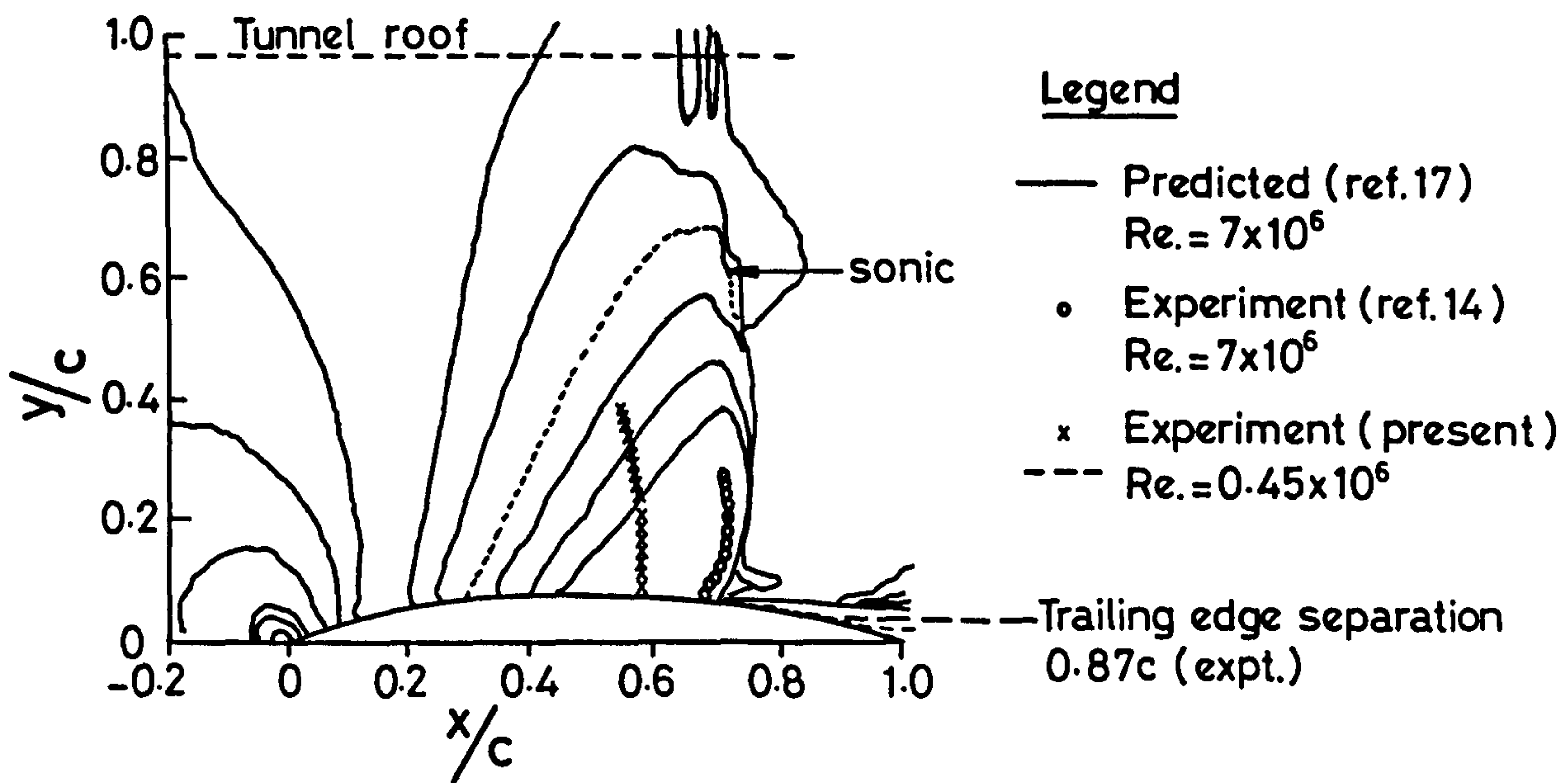


Figure 73. Frequency parameter vs. Mach No.

(a) $M_\infty = 0.81$



(b)

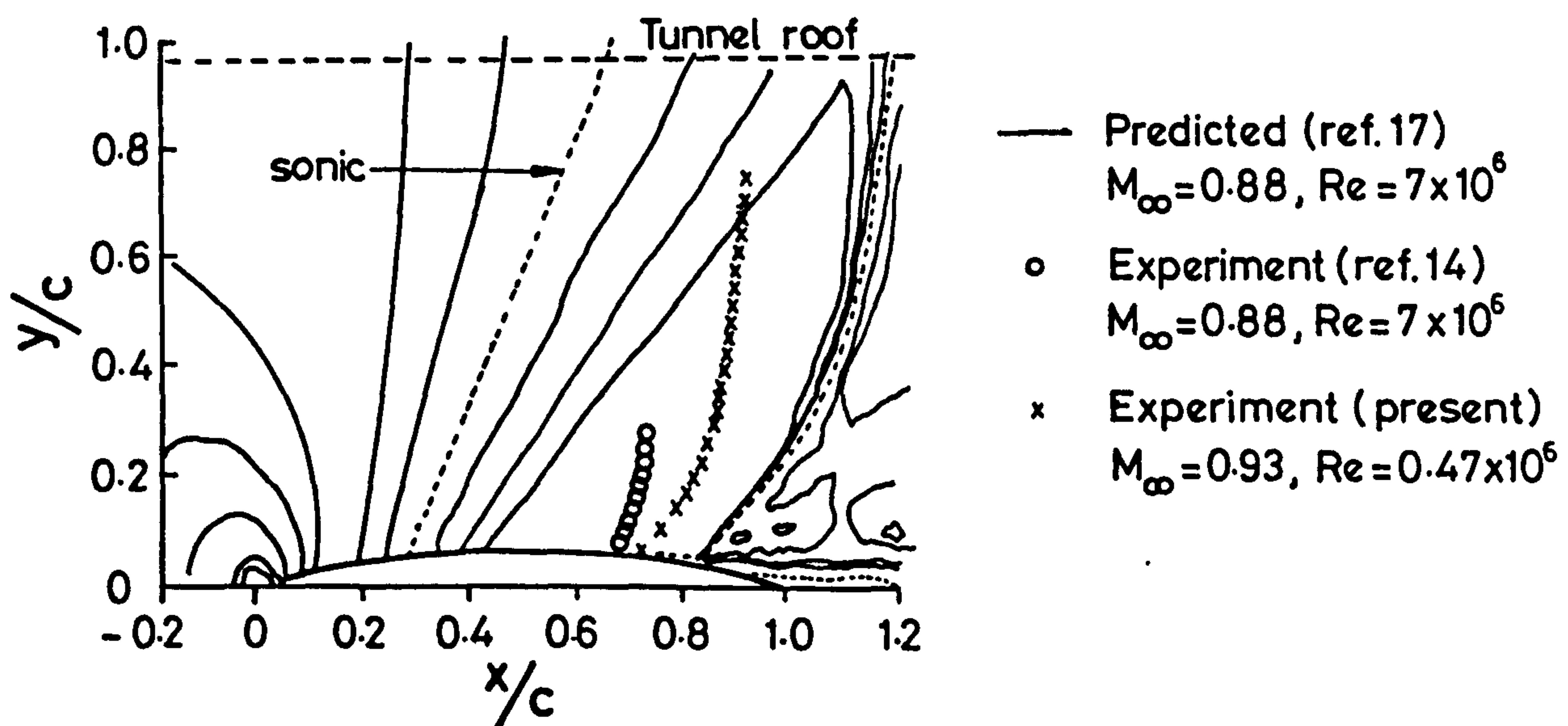


Figure 74. Comparison of experiments with numerical computations on 14% biconvex aerofoil at zero incidence.

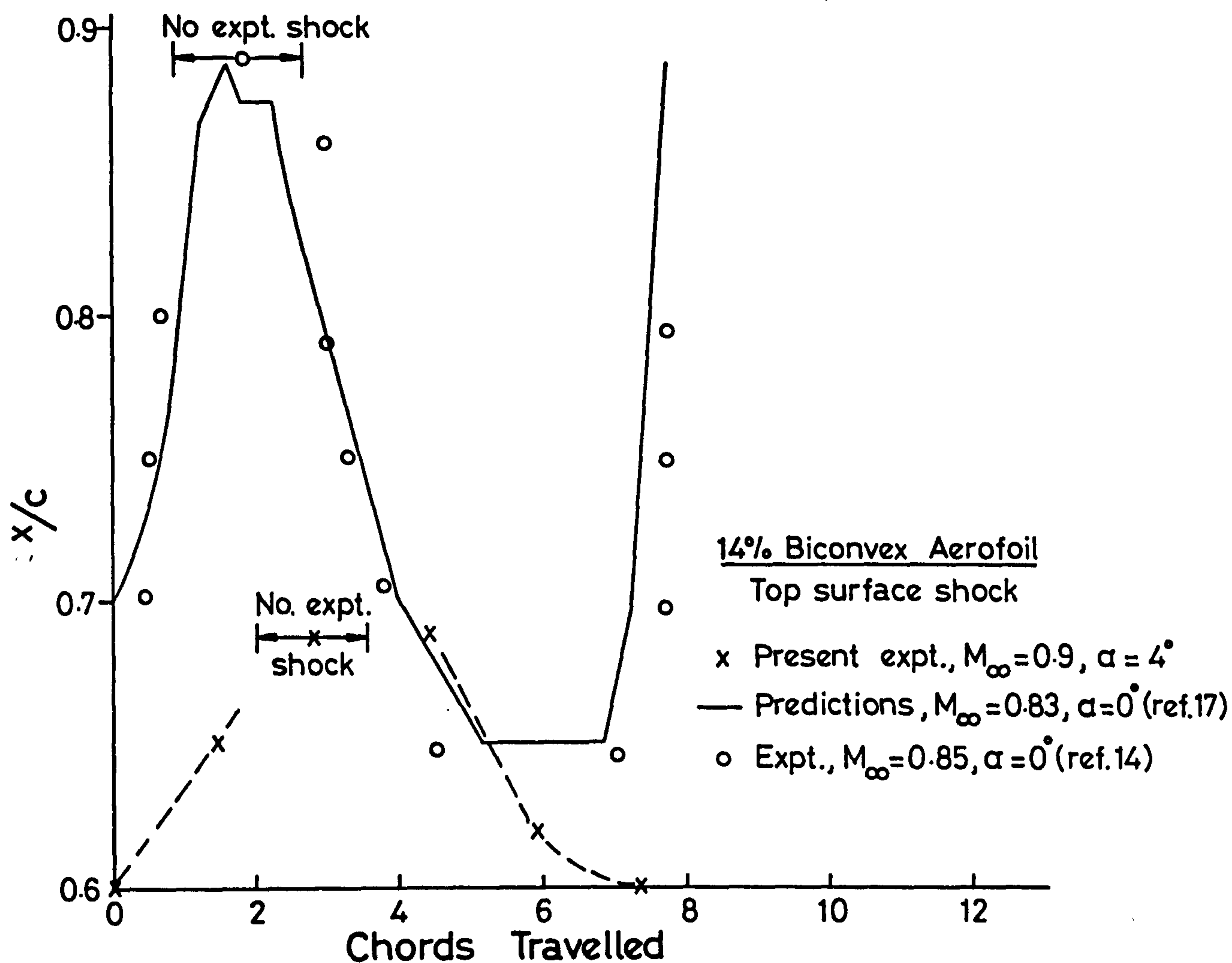


Figure 75. Time history of shock-wave position.

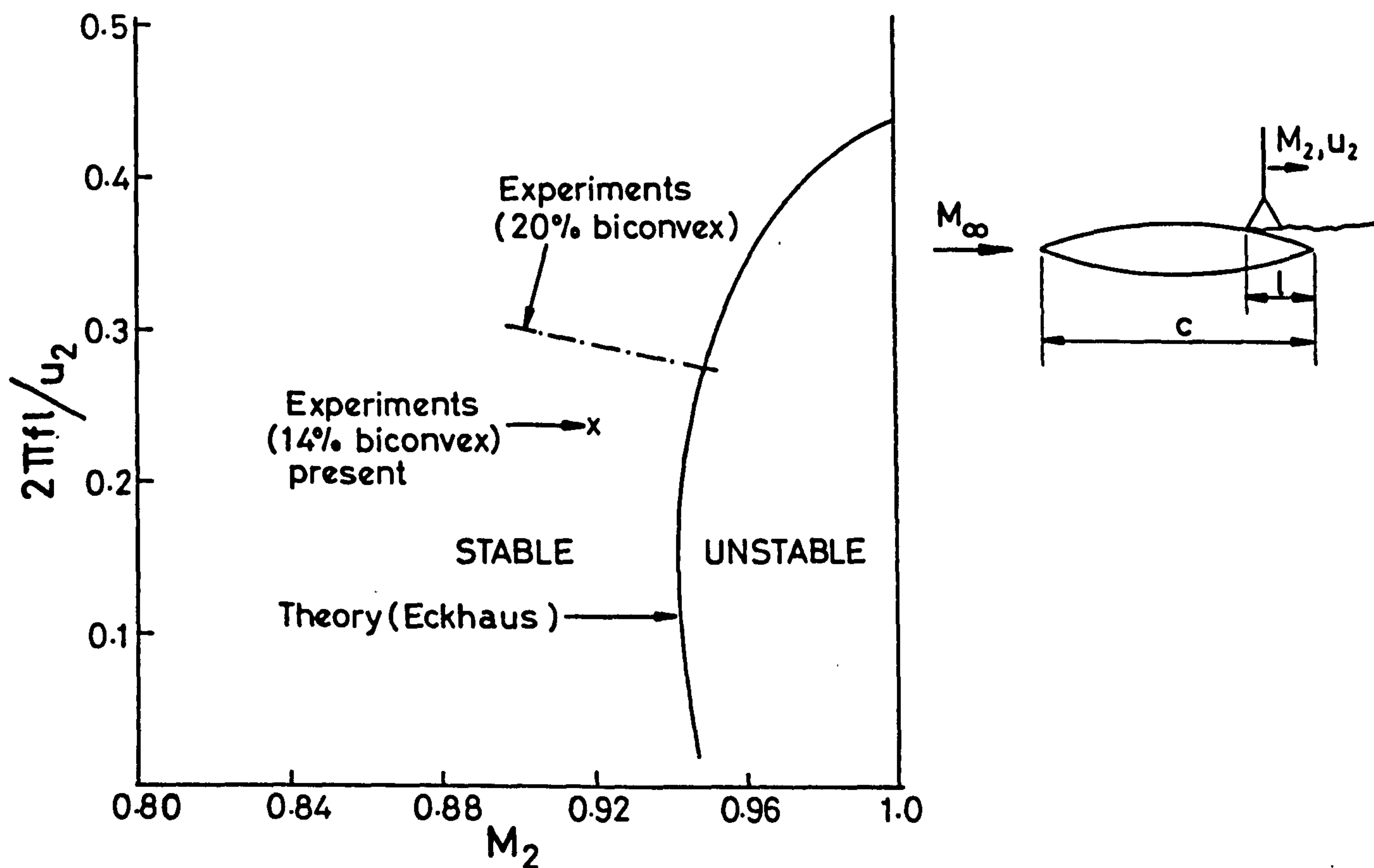


Figure 76. Theory for aileron buzz (Eckhaus, ref.23)

NACA 64A006 Aerofoil
 $M_\infty = 0.875$

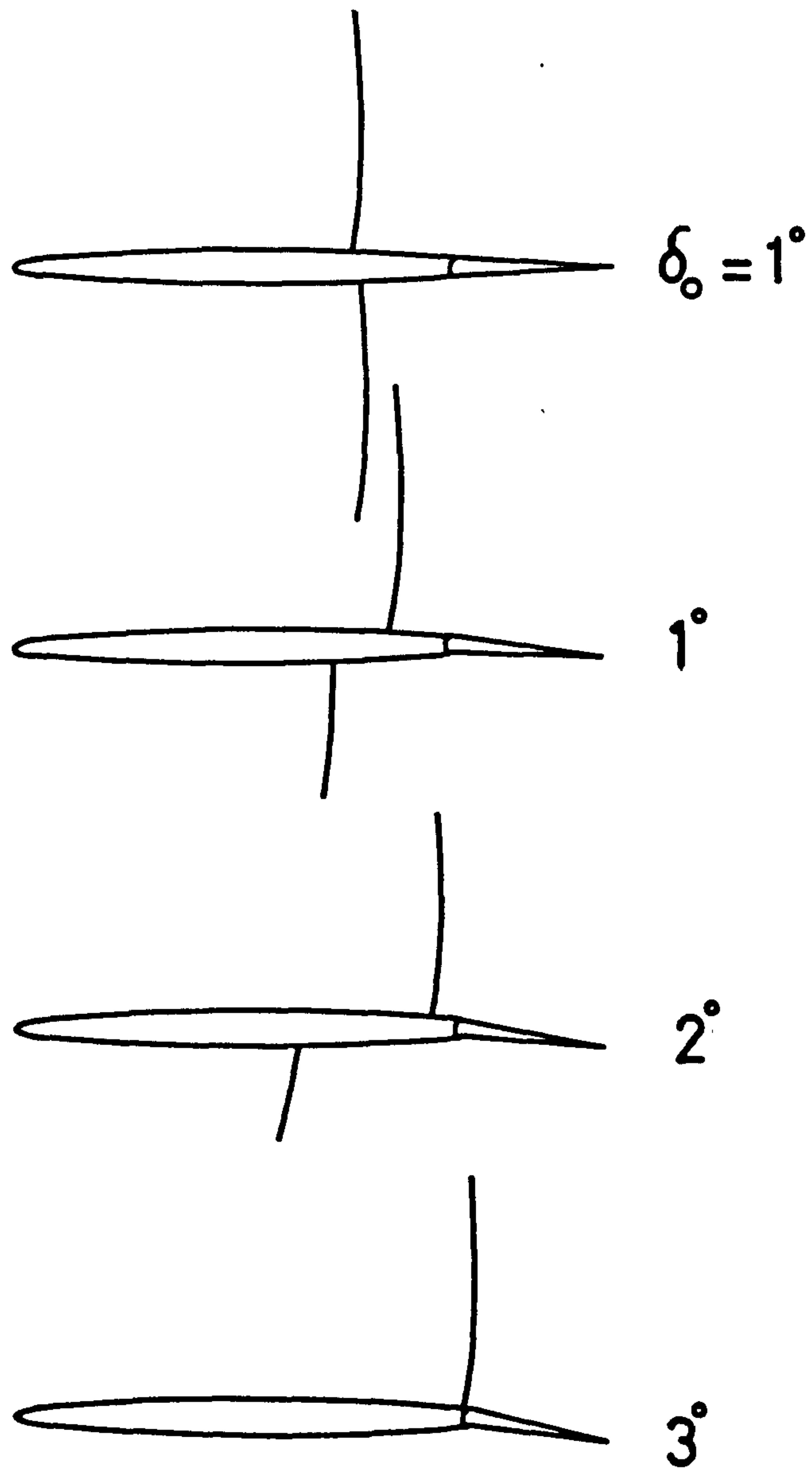
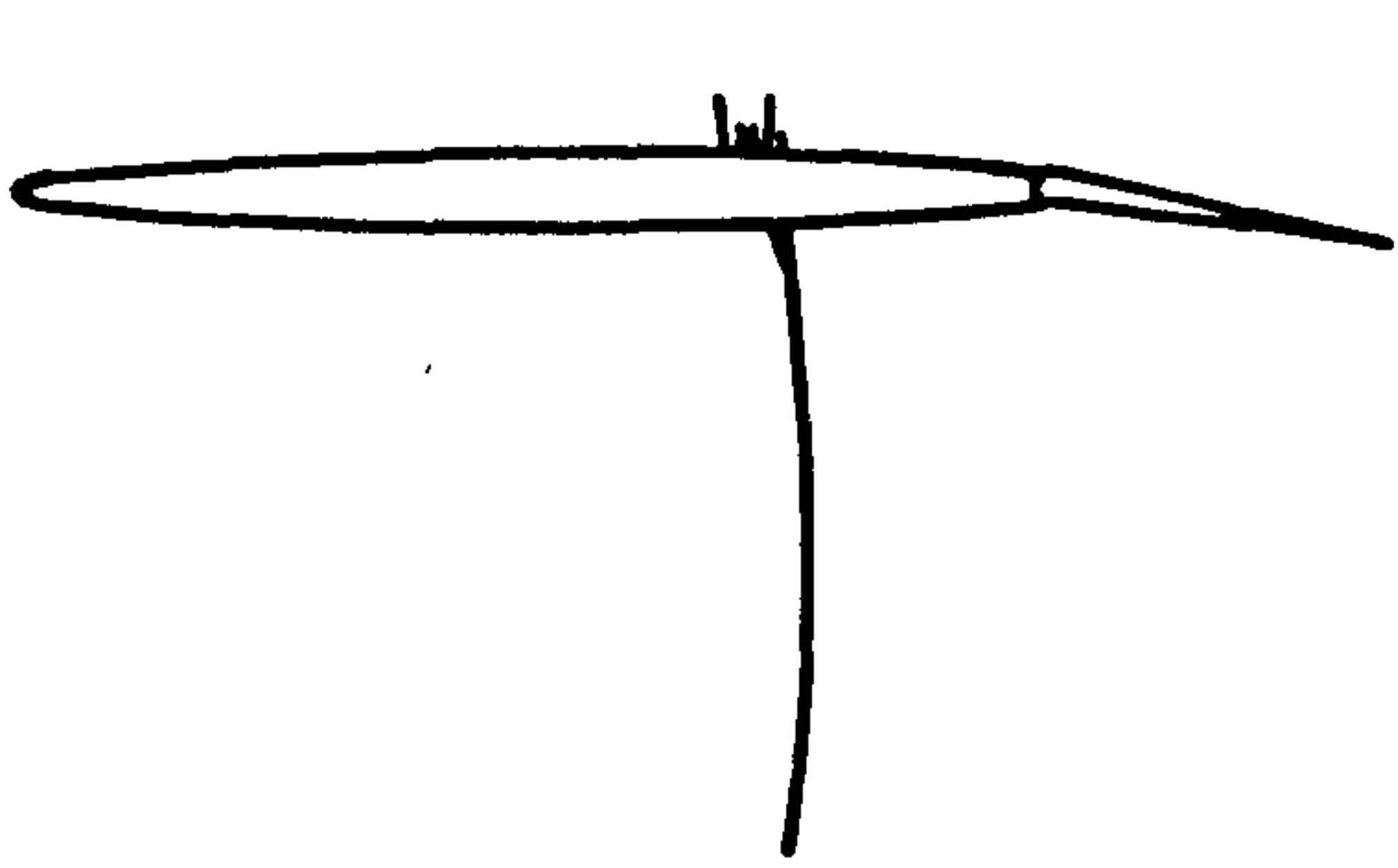


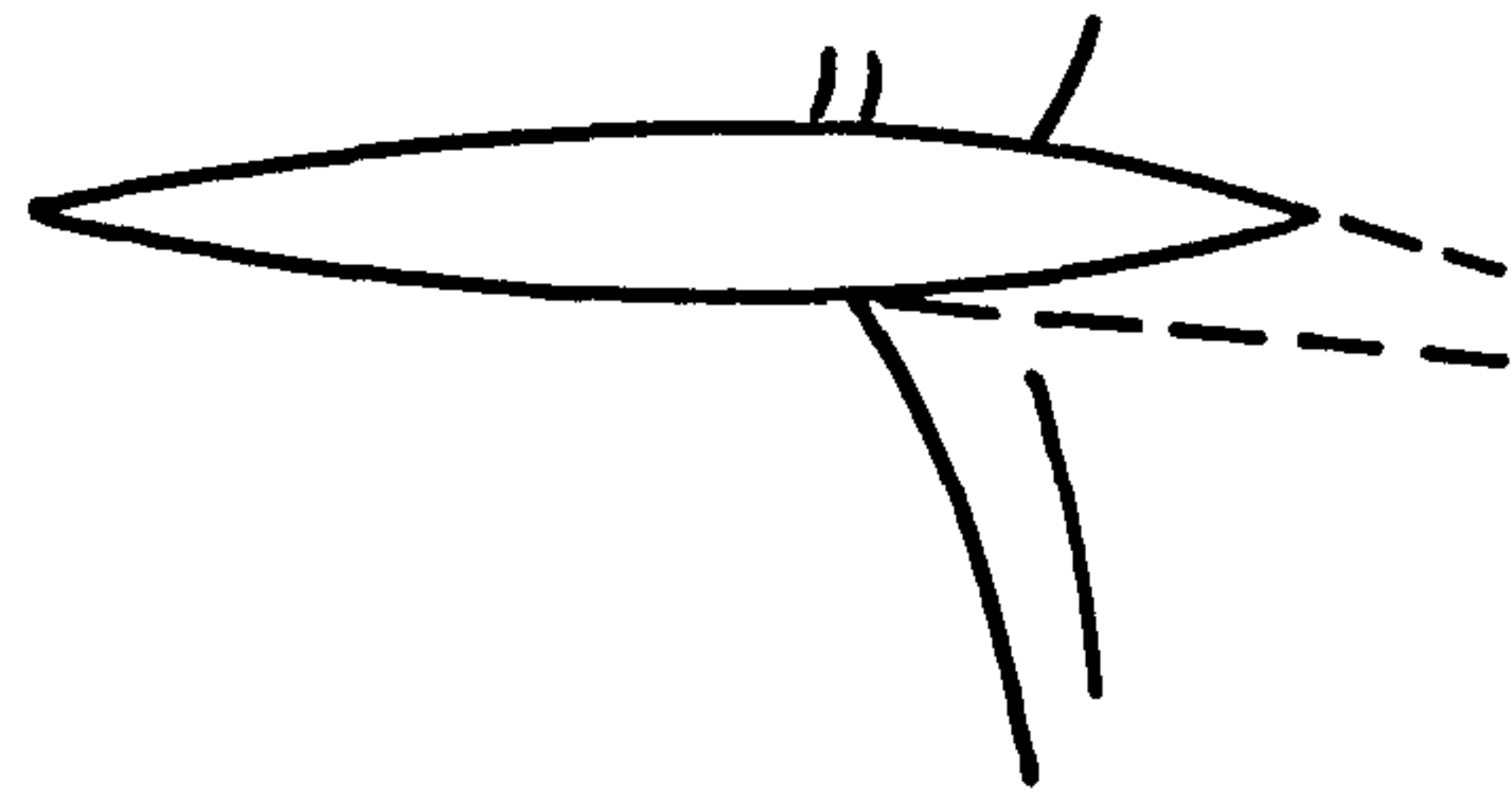
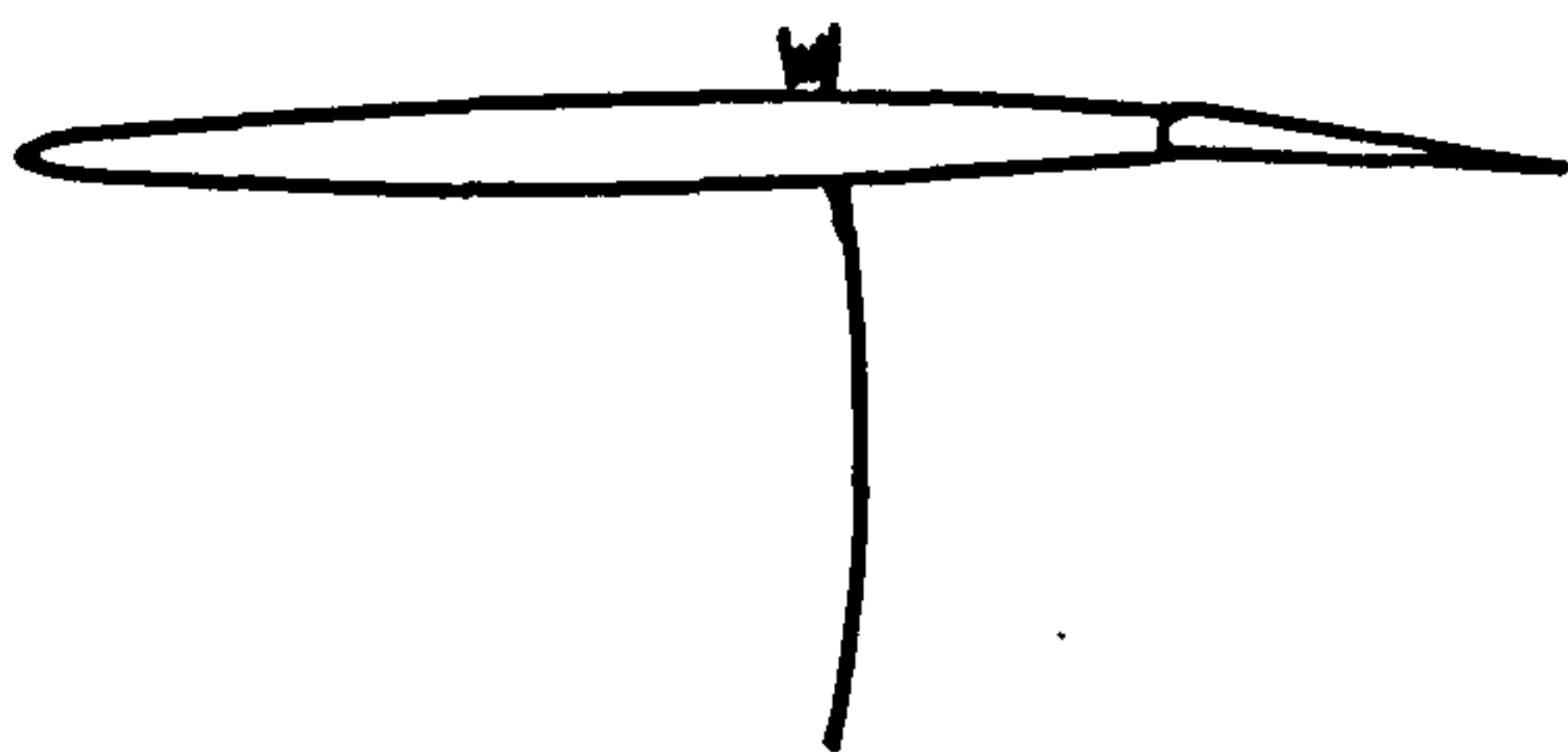
Figure 77 Steady shock patterns for zero incidence and various flap deflections (ref. 40, Tijdeman)

NACA 64A006 Aerofoil with
oscillating flap, $\alpha = 0^\circ$, $M_\infty = 0.875$

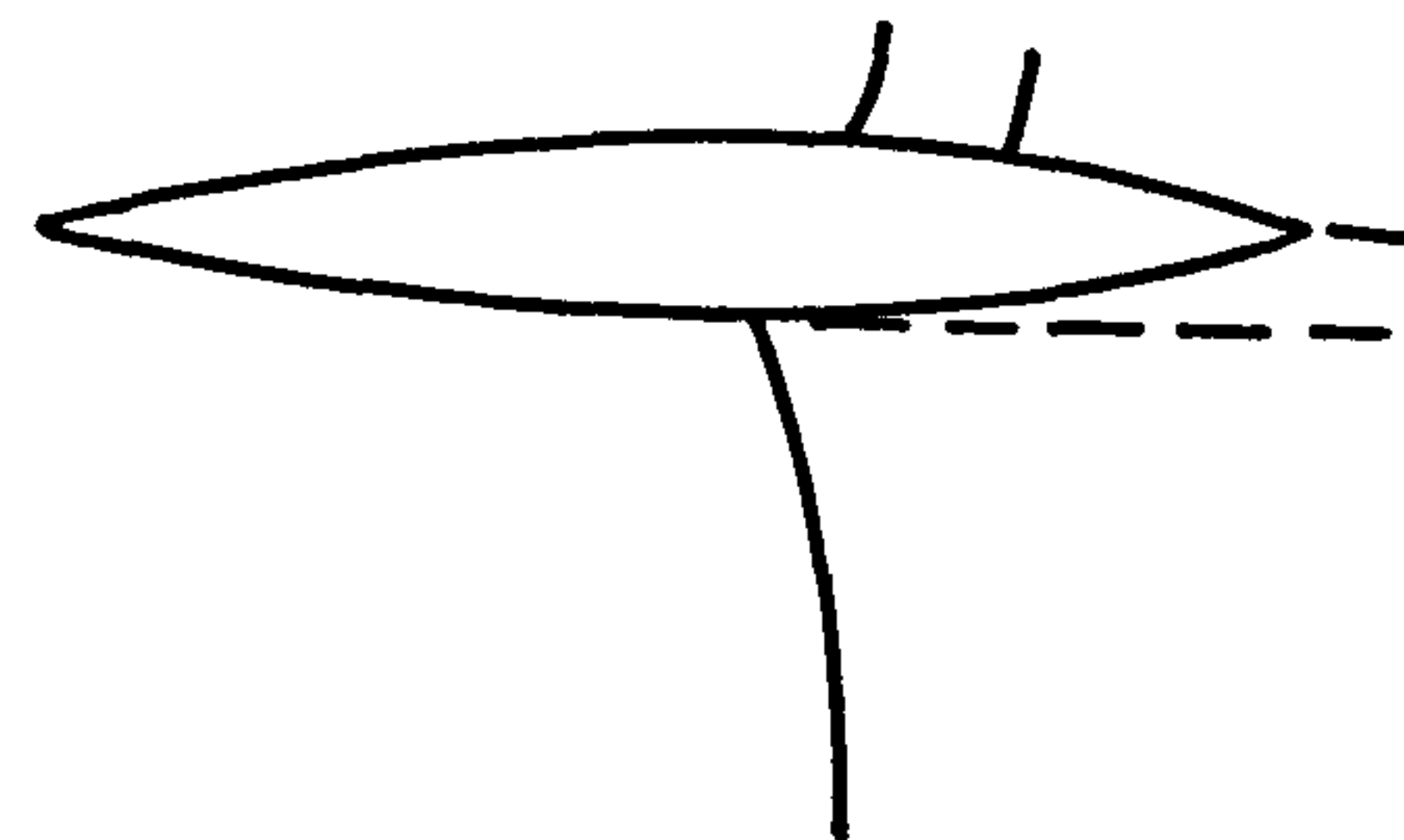
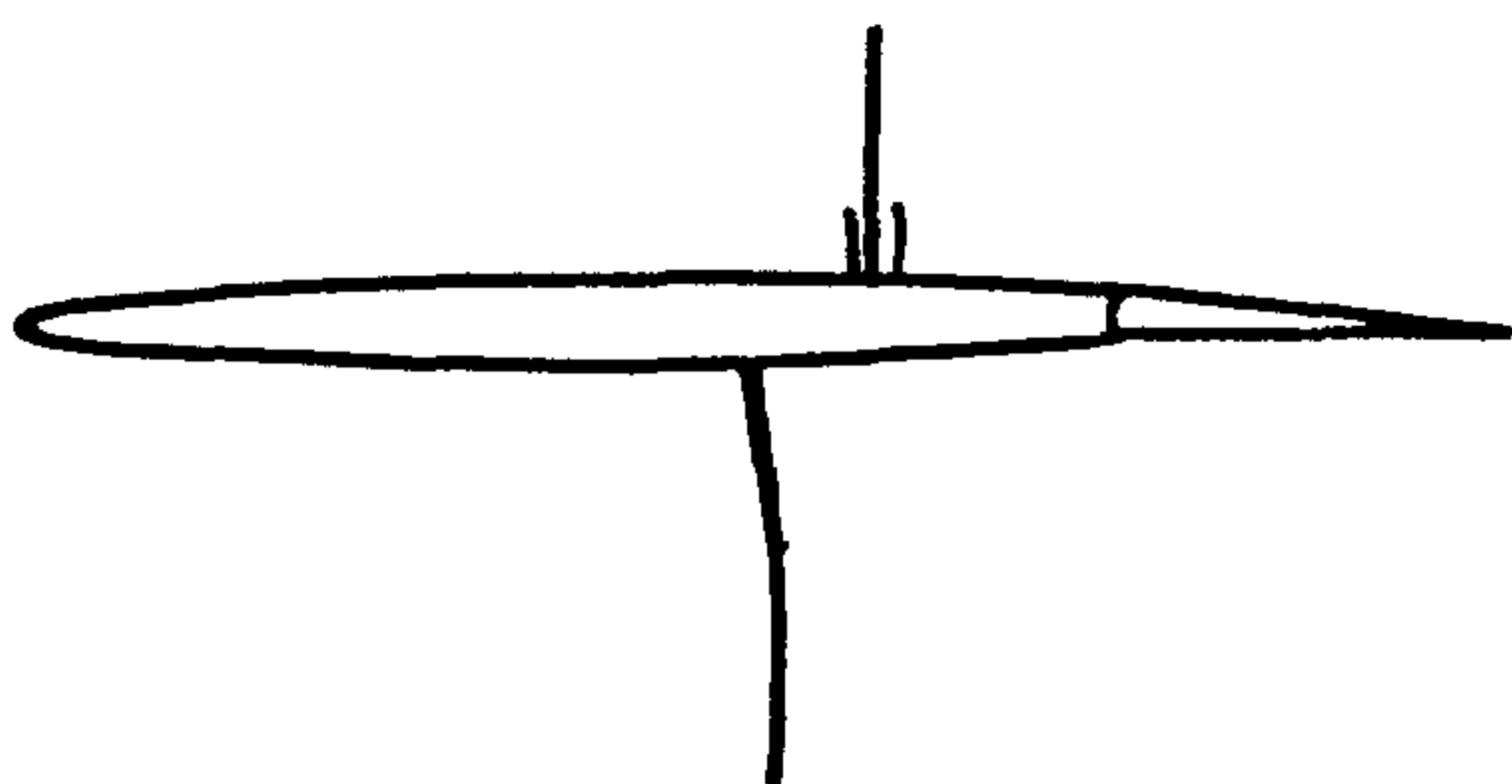
14% Biconvex aerofoil
 $\alpha = 0^\circ$, $M_\infty = 0.85$



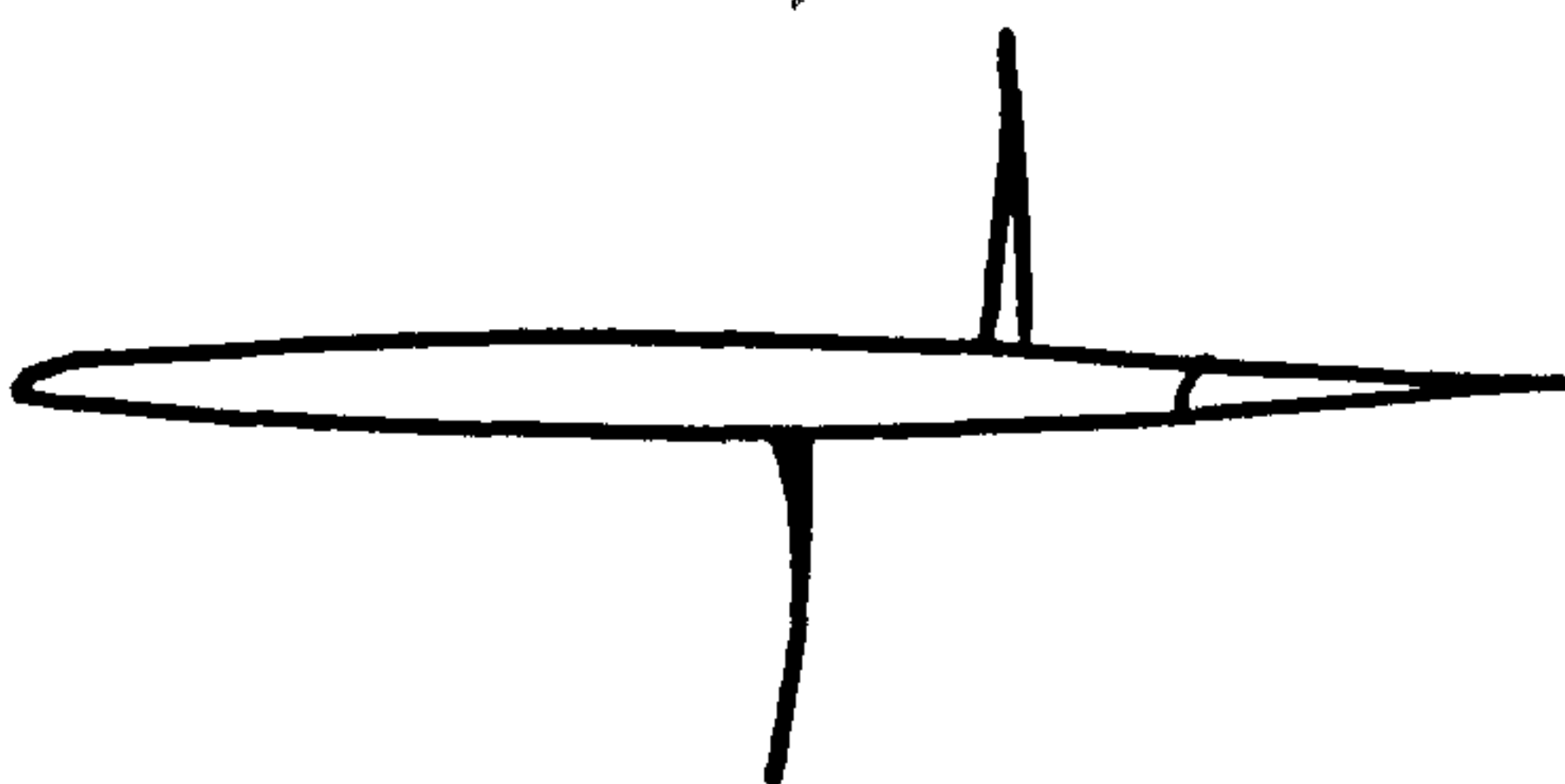
1



2



3



4

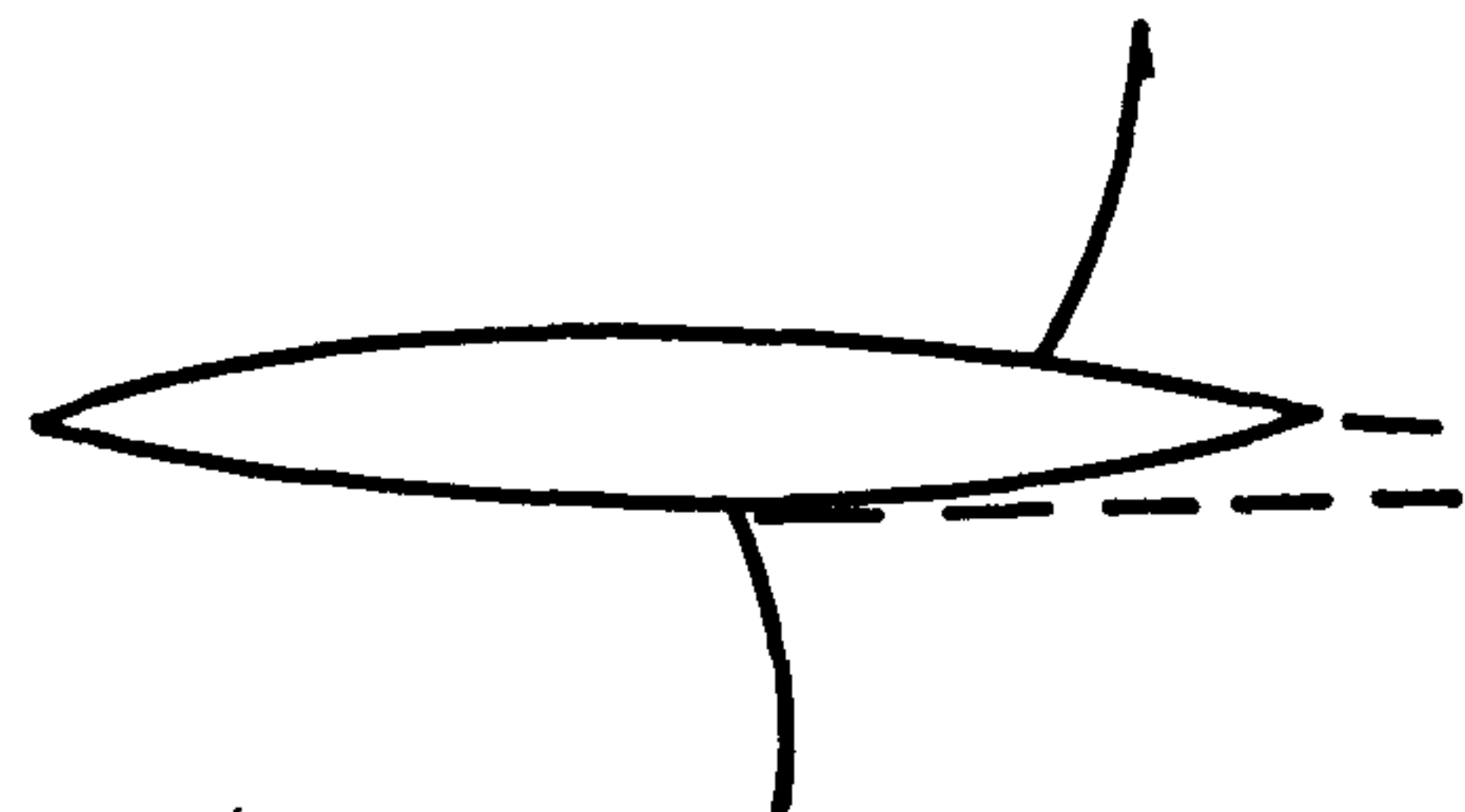
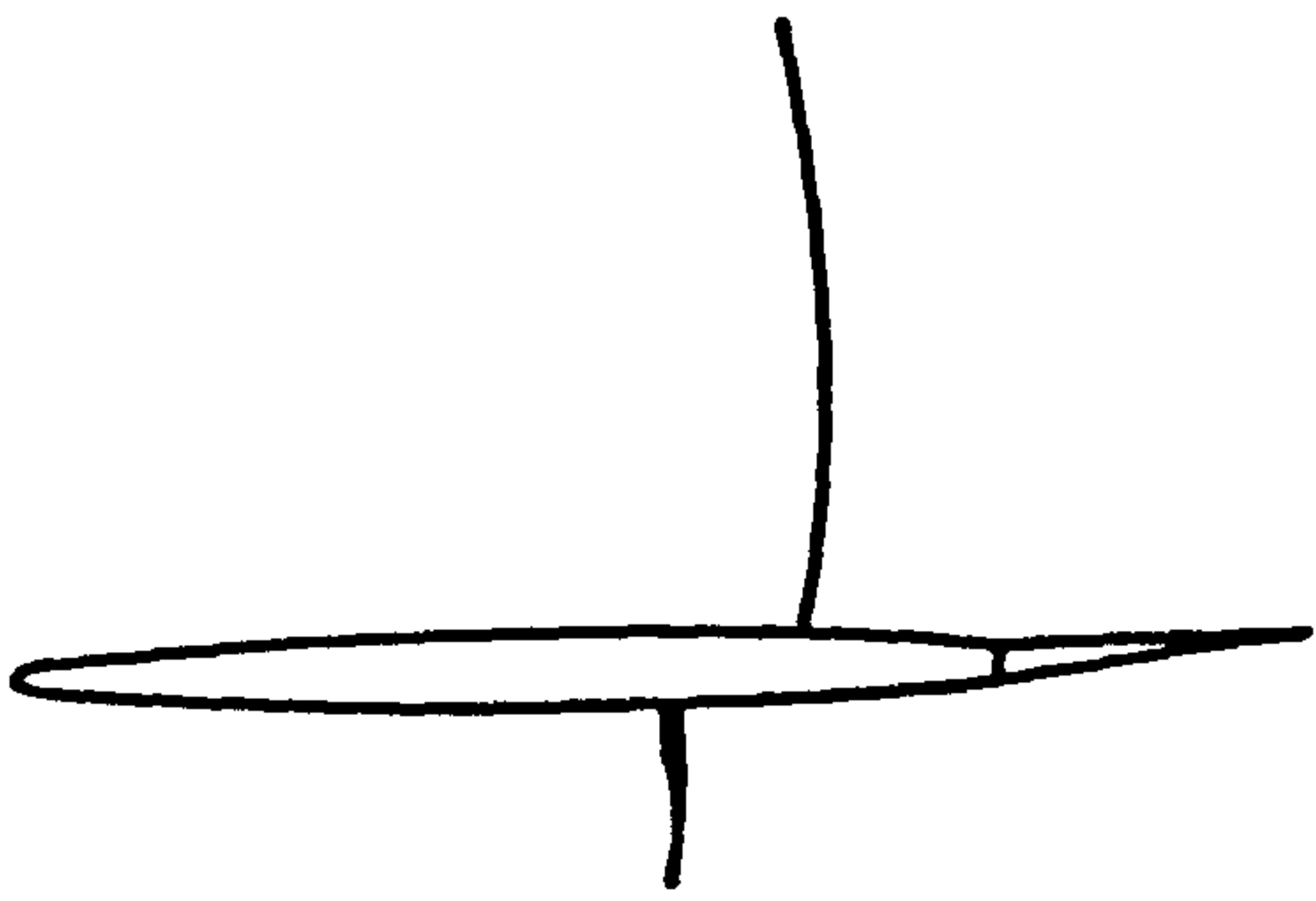
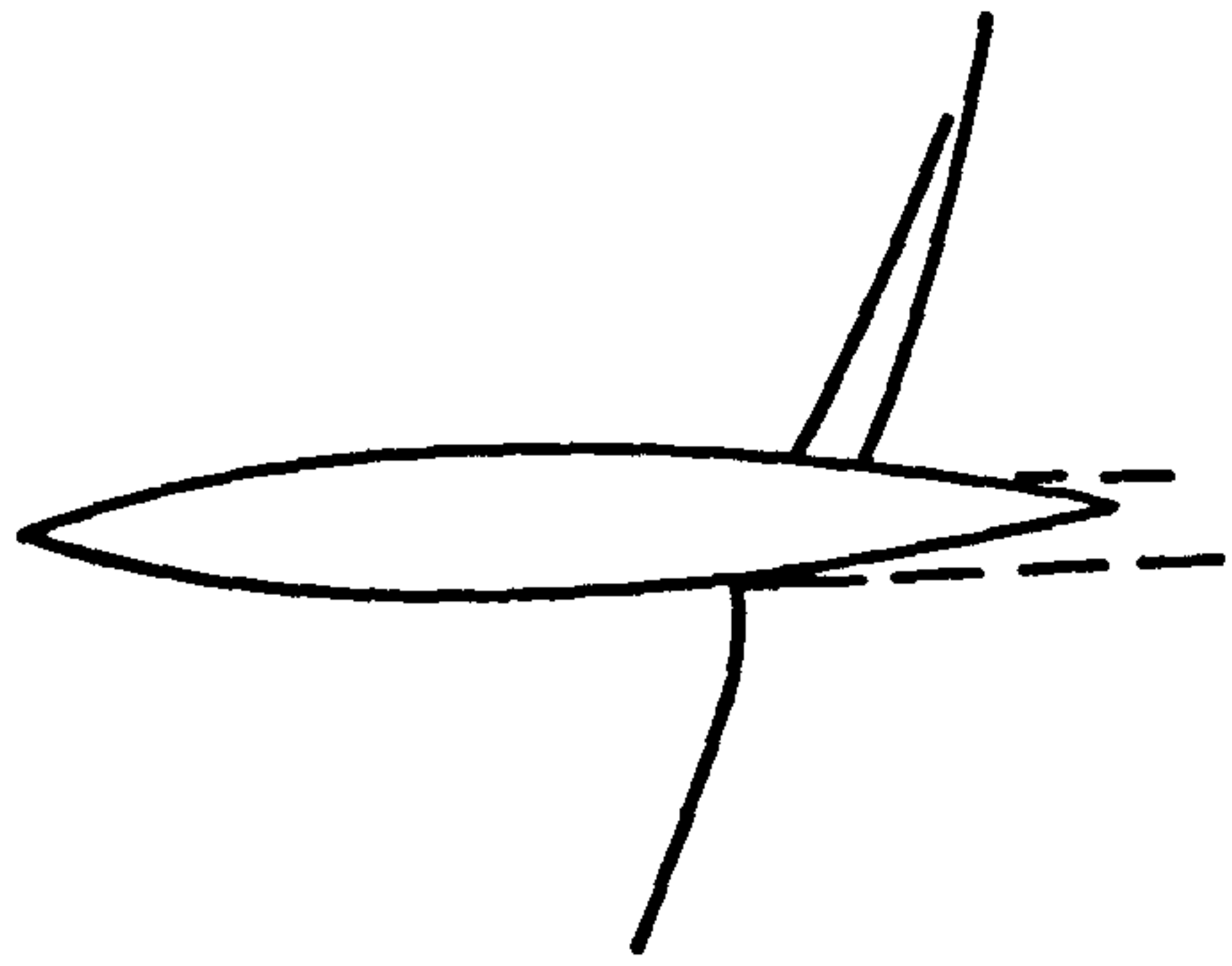
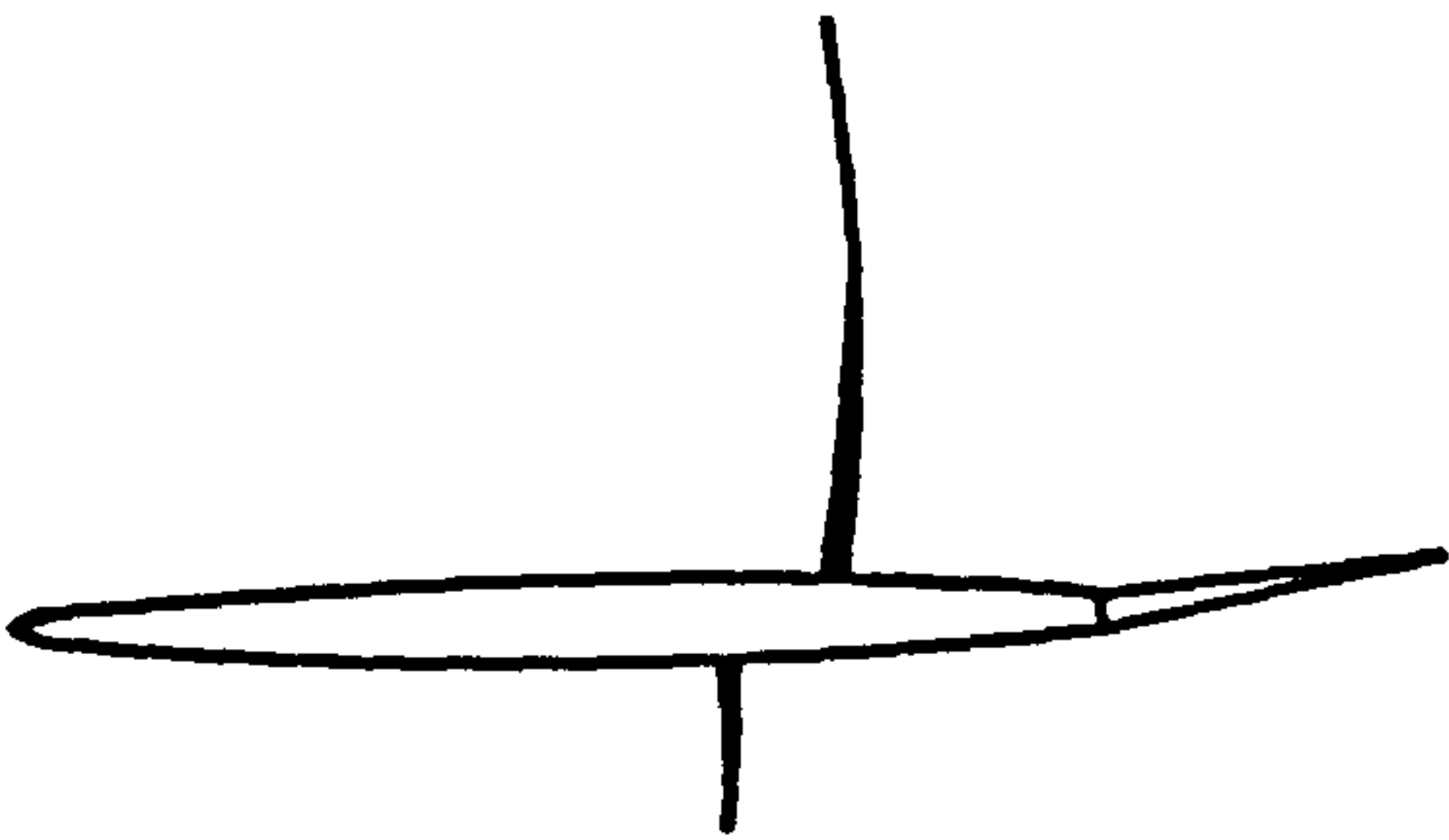


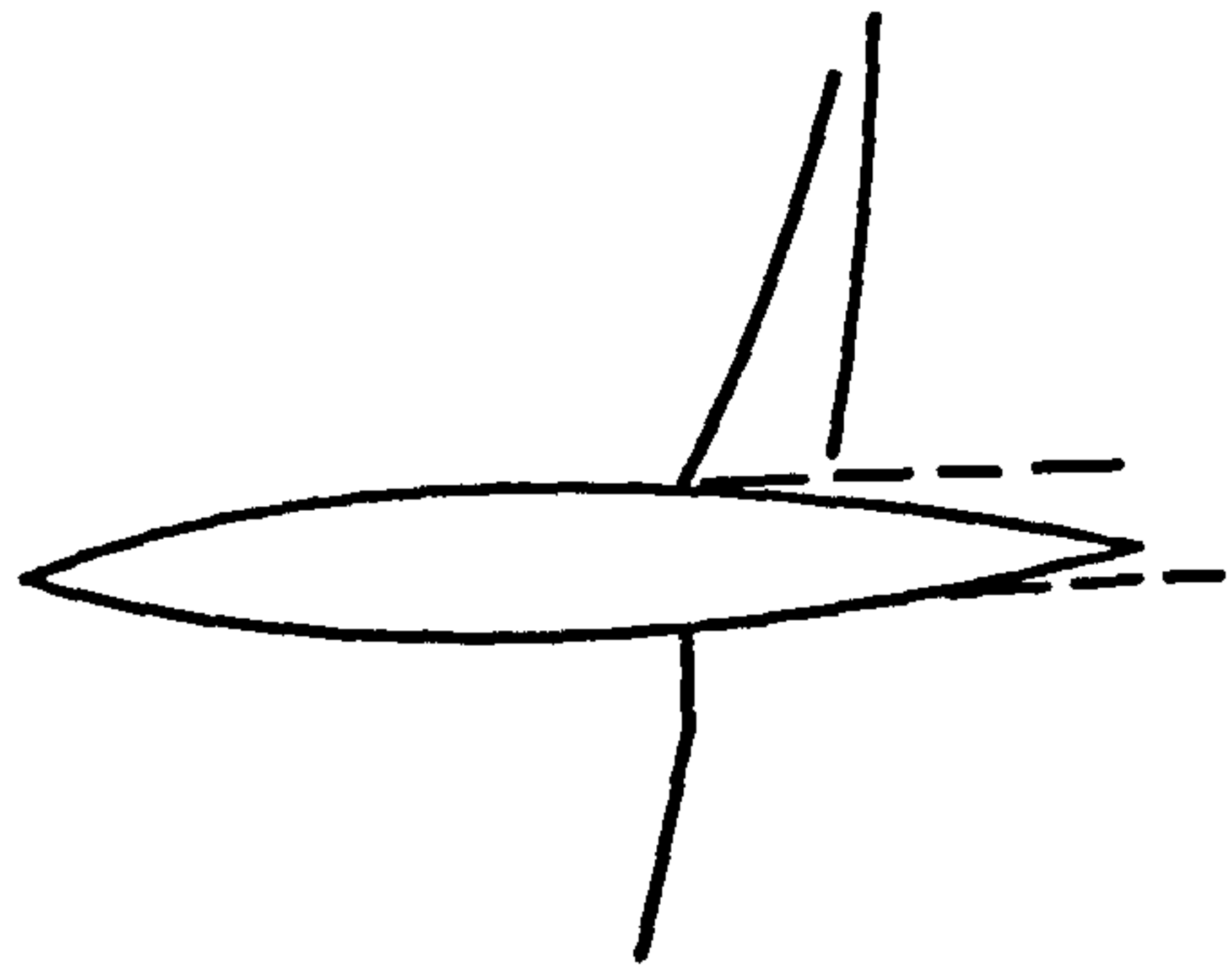
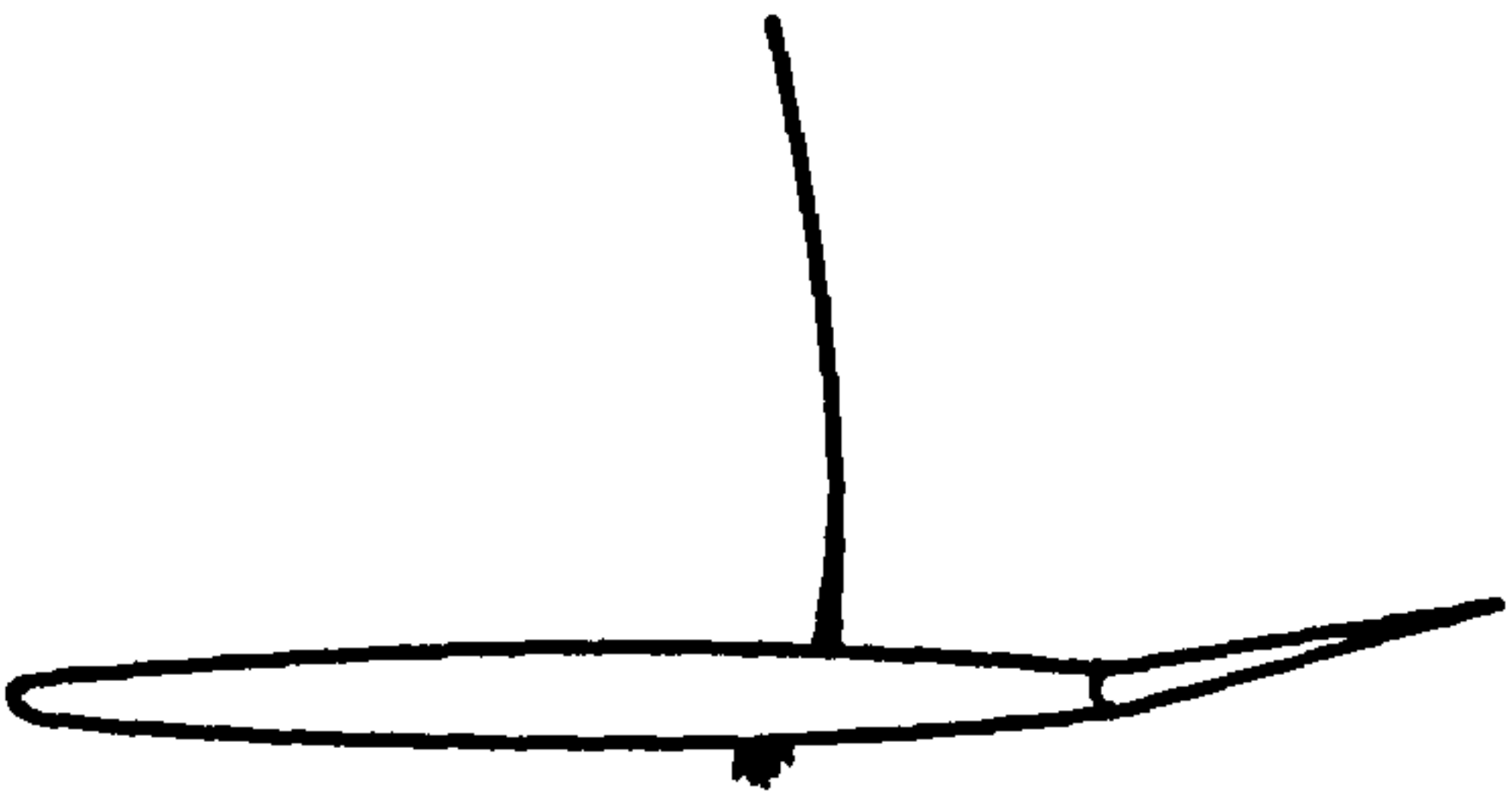
Figure 78 Comparison of flows on aerofoil with oscillating flap and a 14% biconvex aerofoil (refs.40,14)



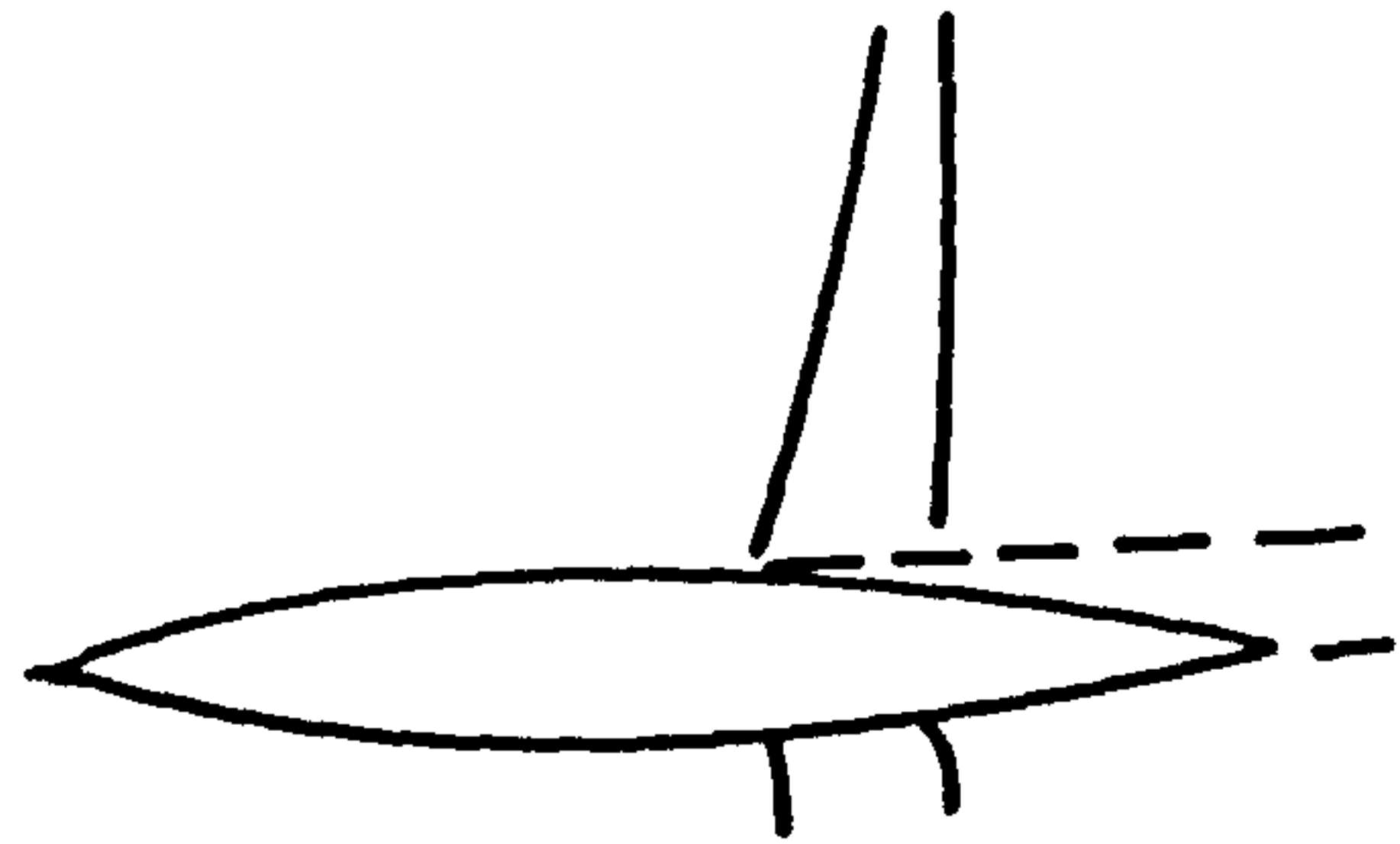
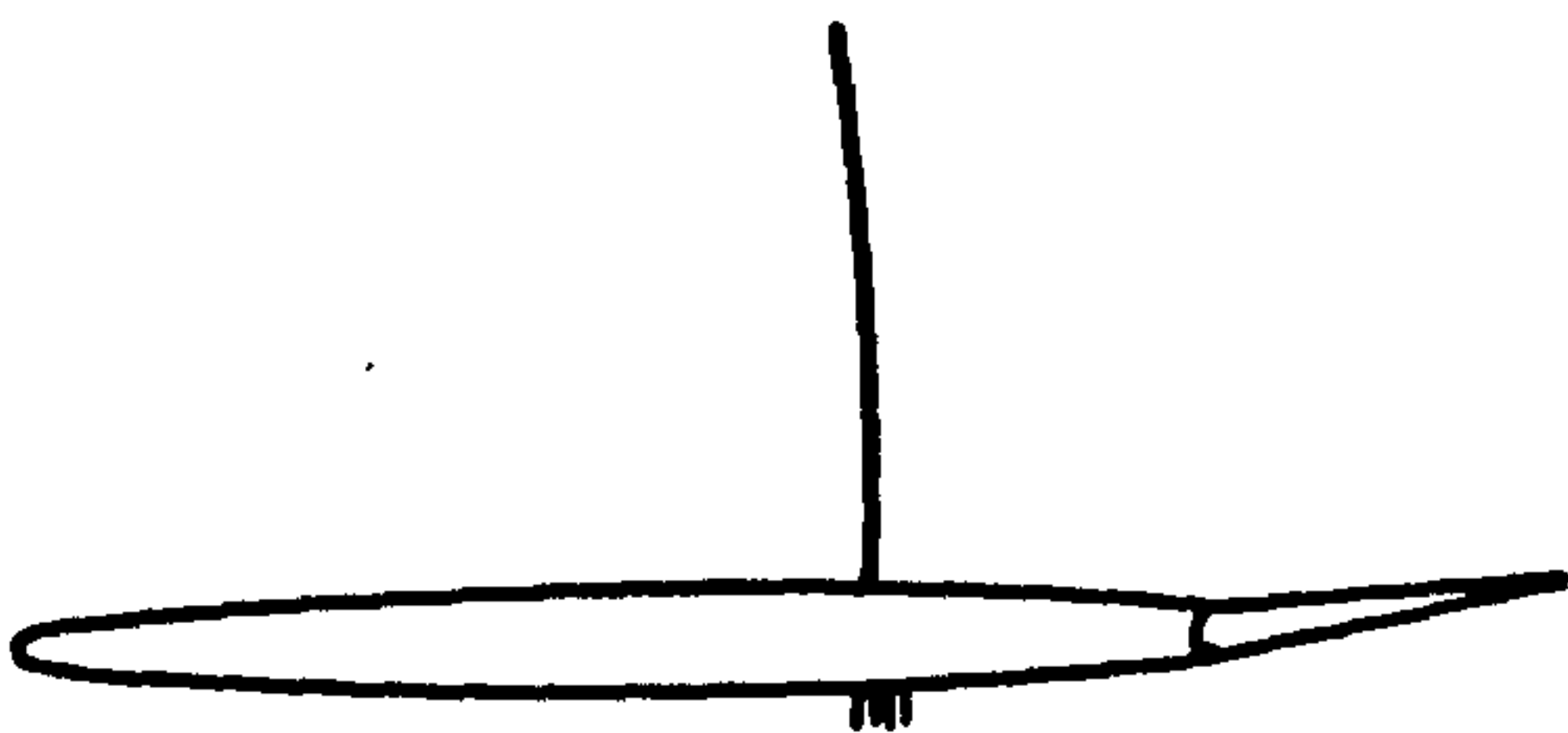
5



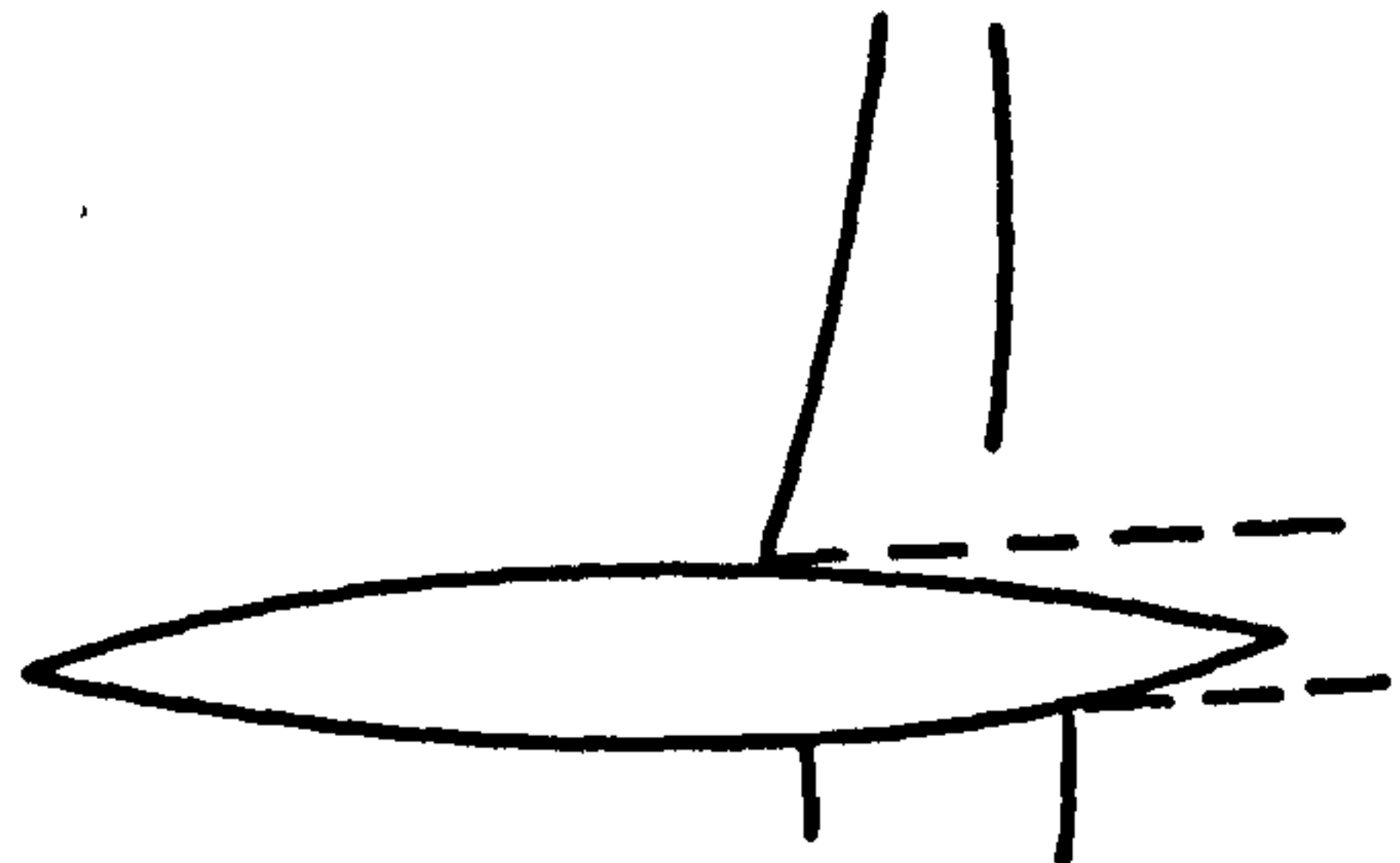
6



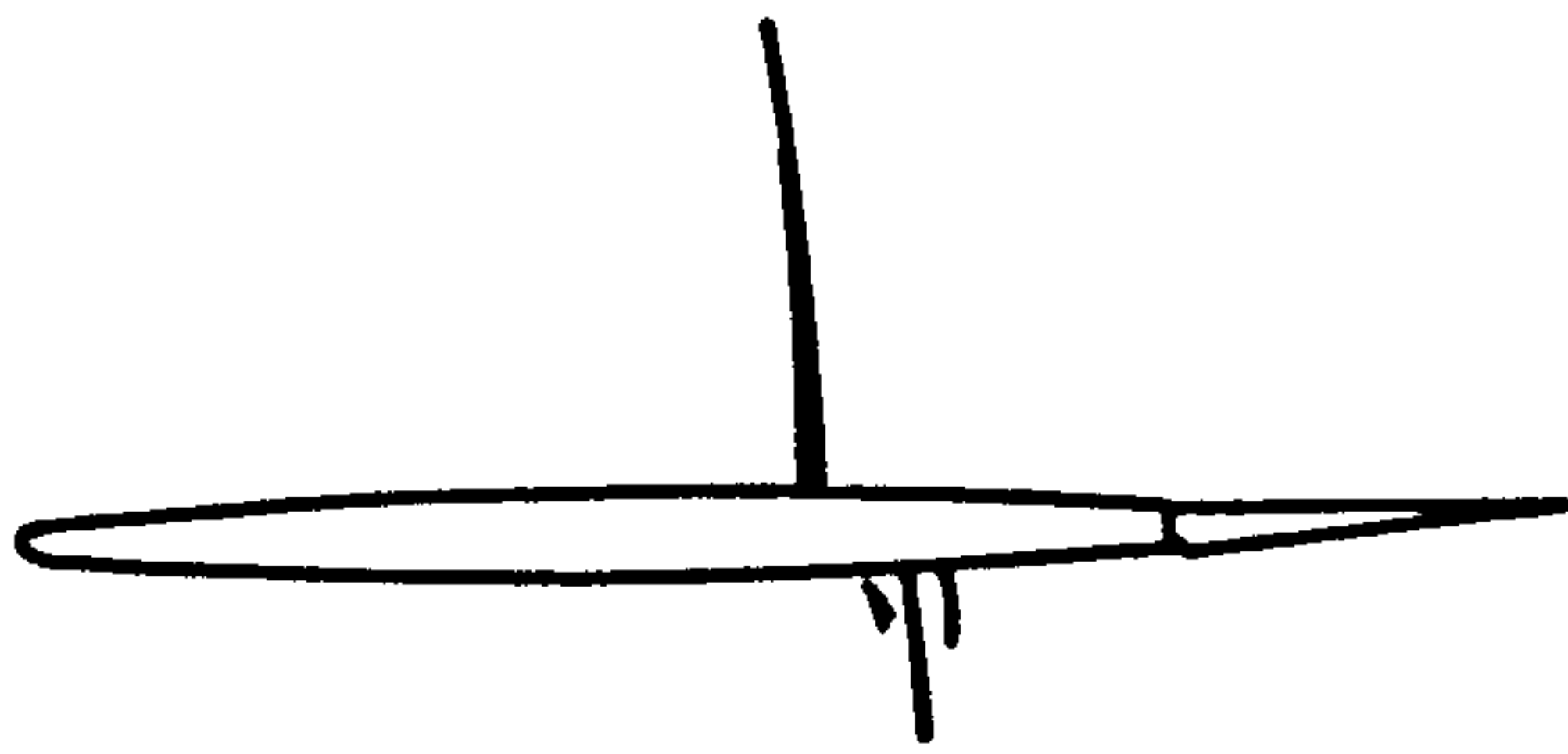
7



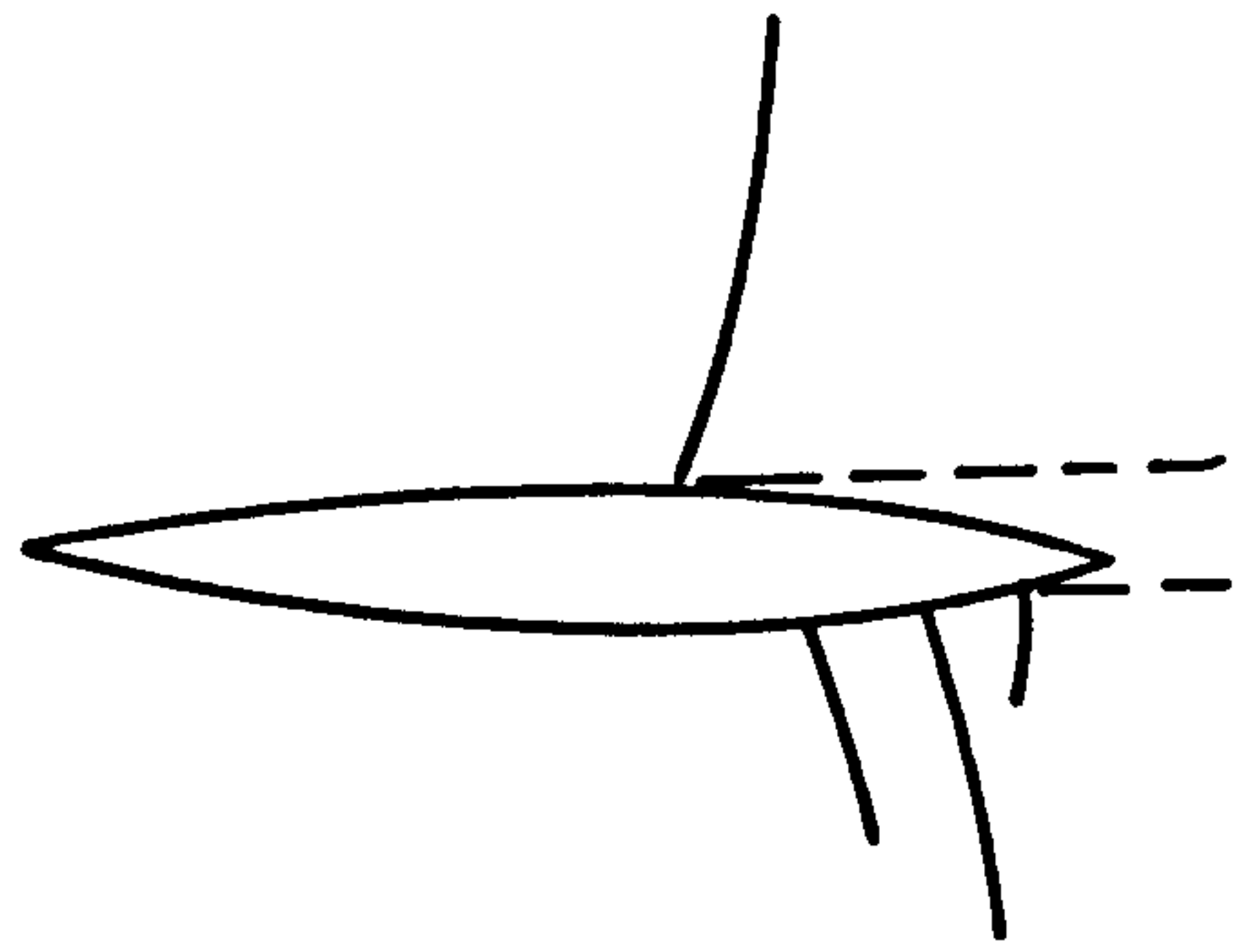
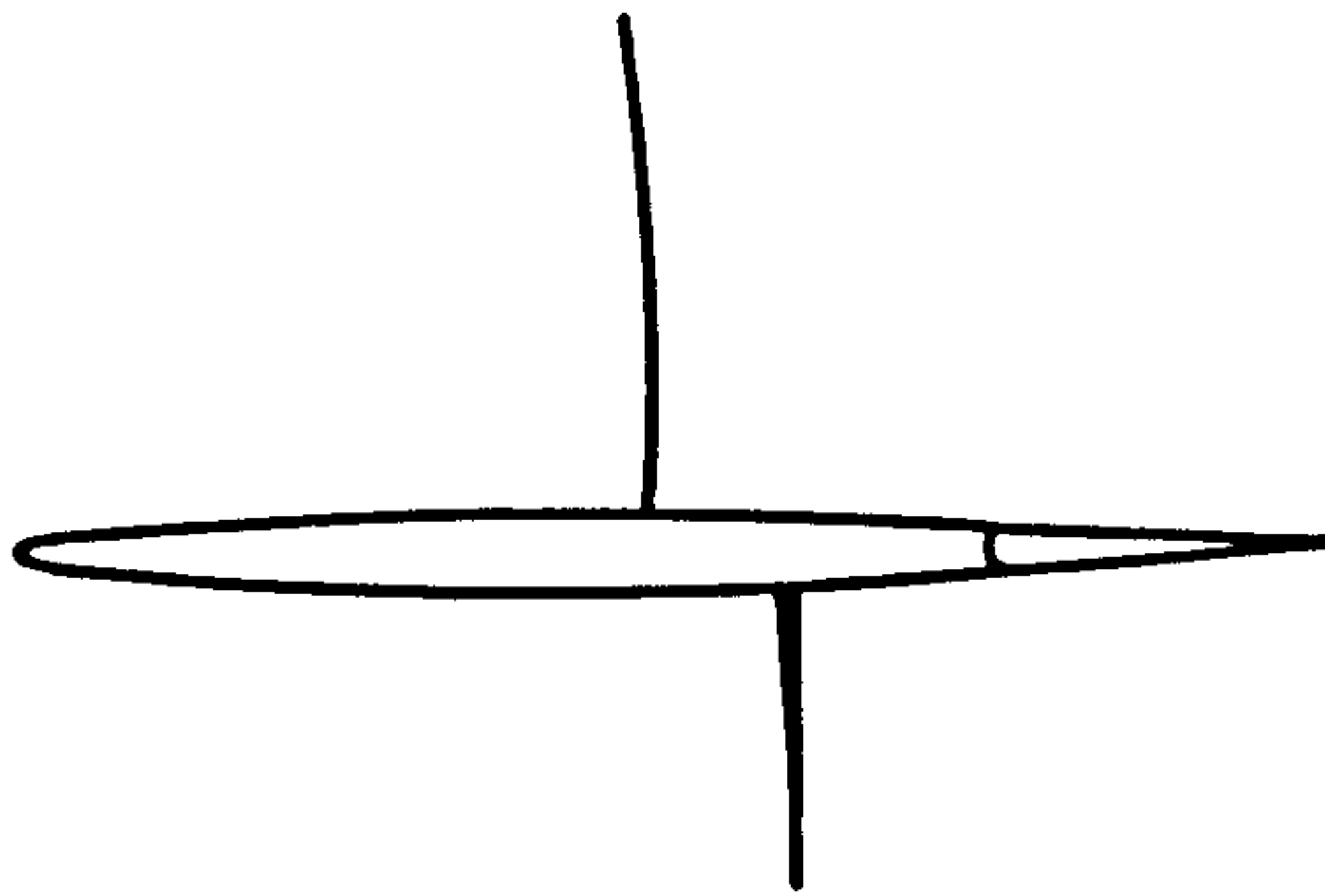
8



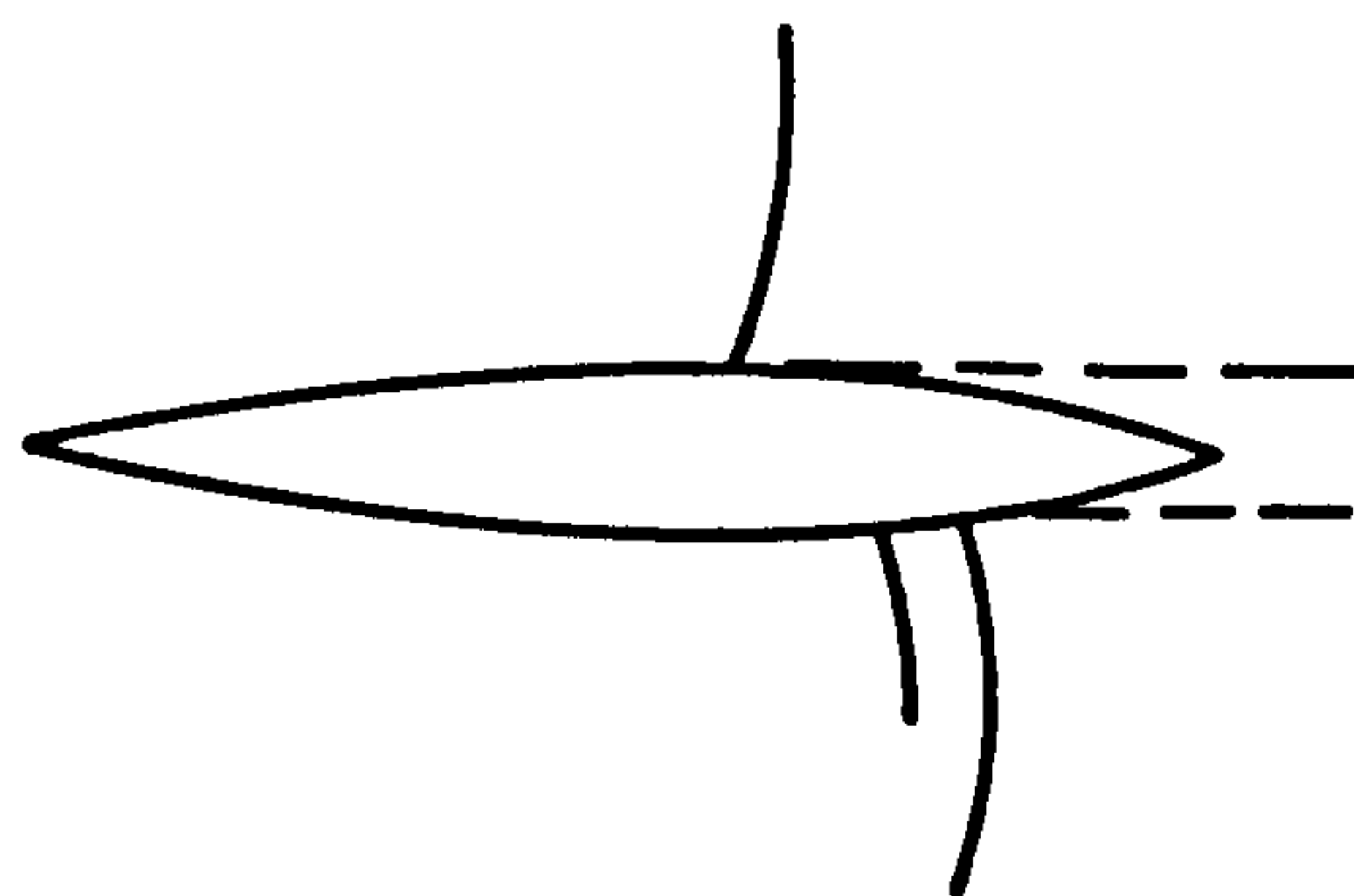
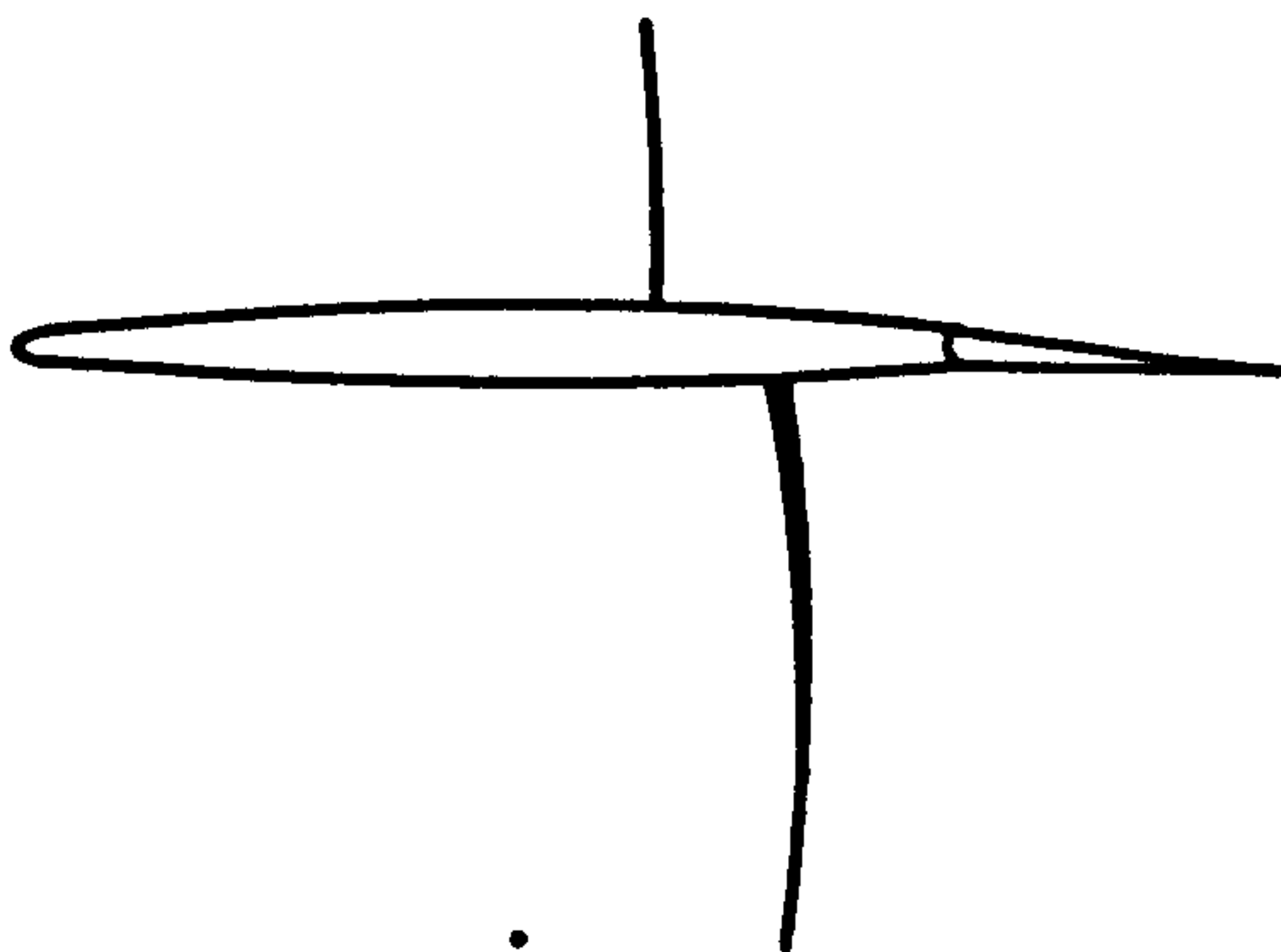
contd. Figure 78



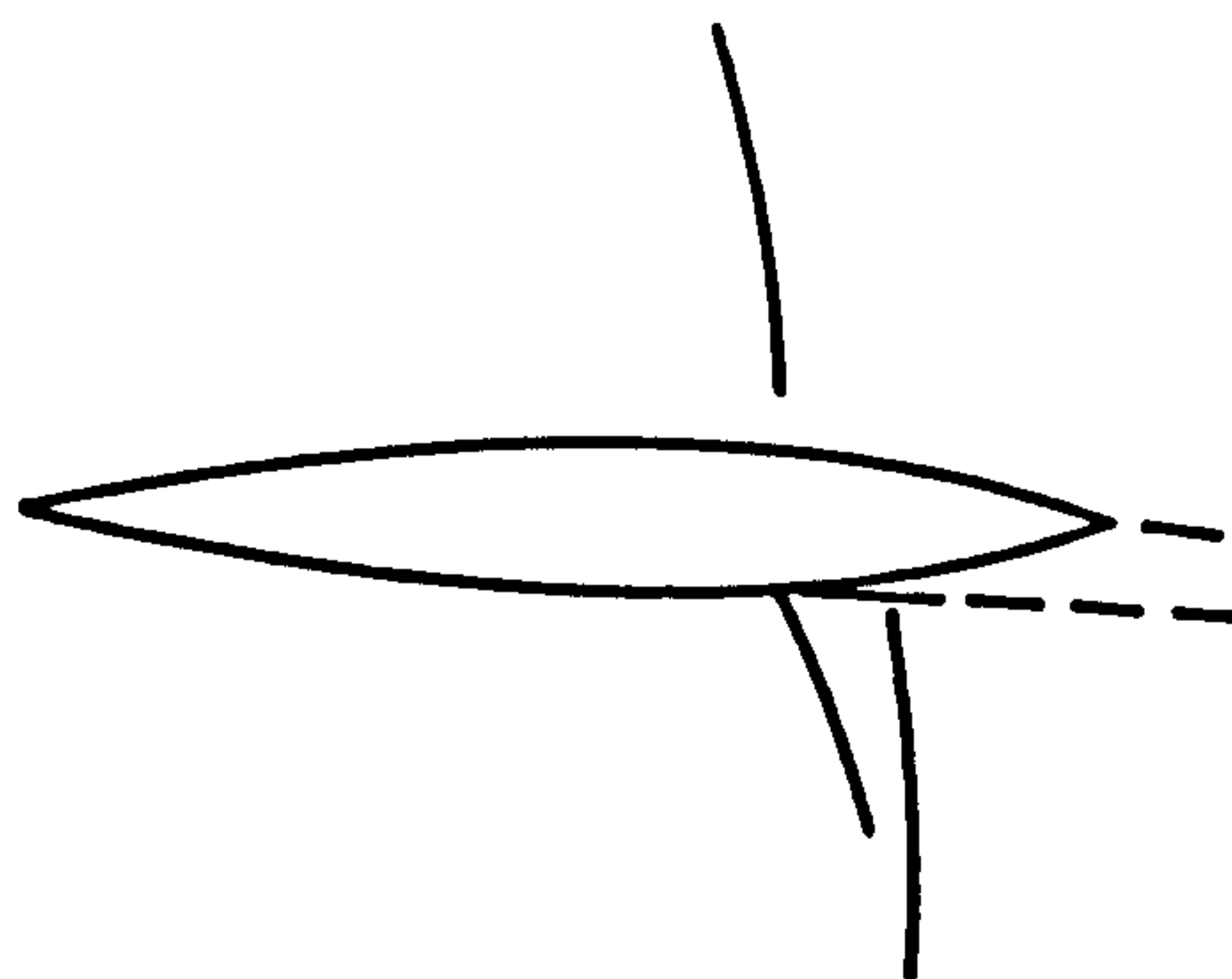
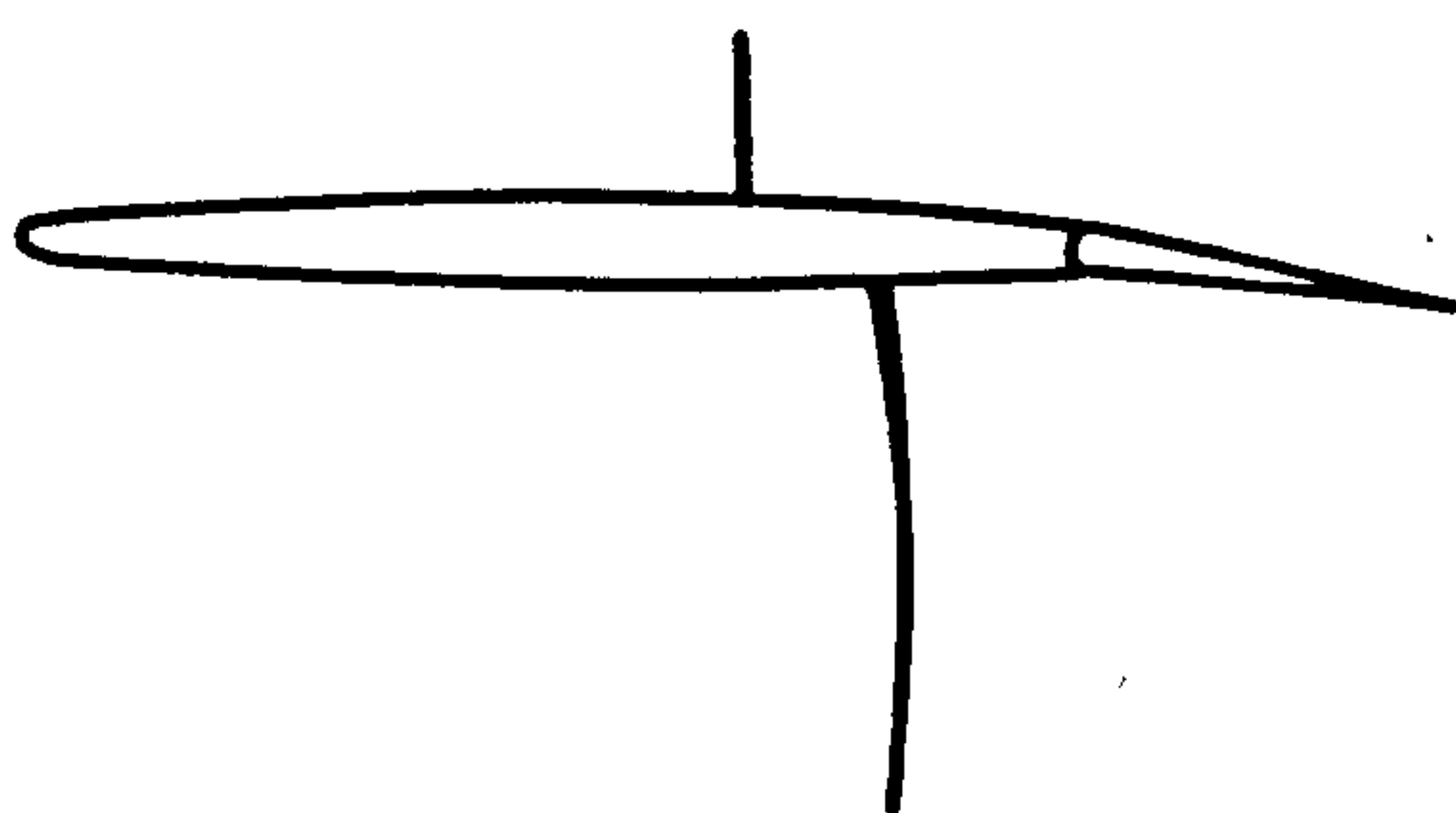
9



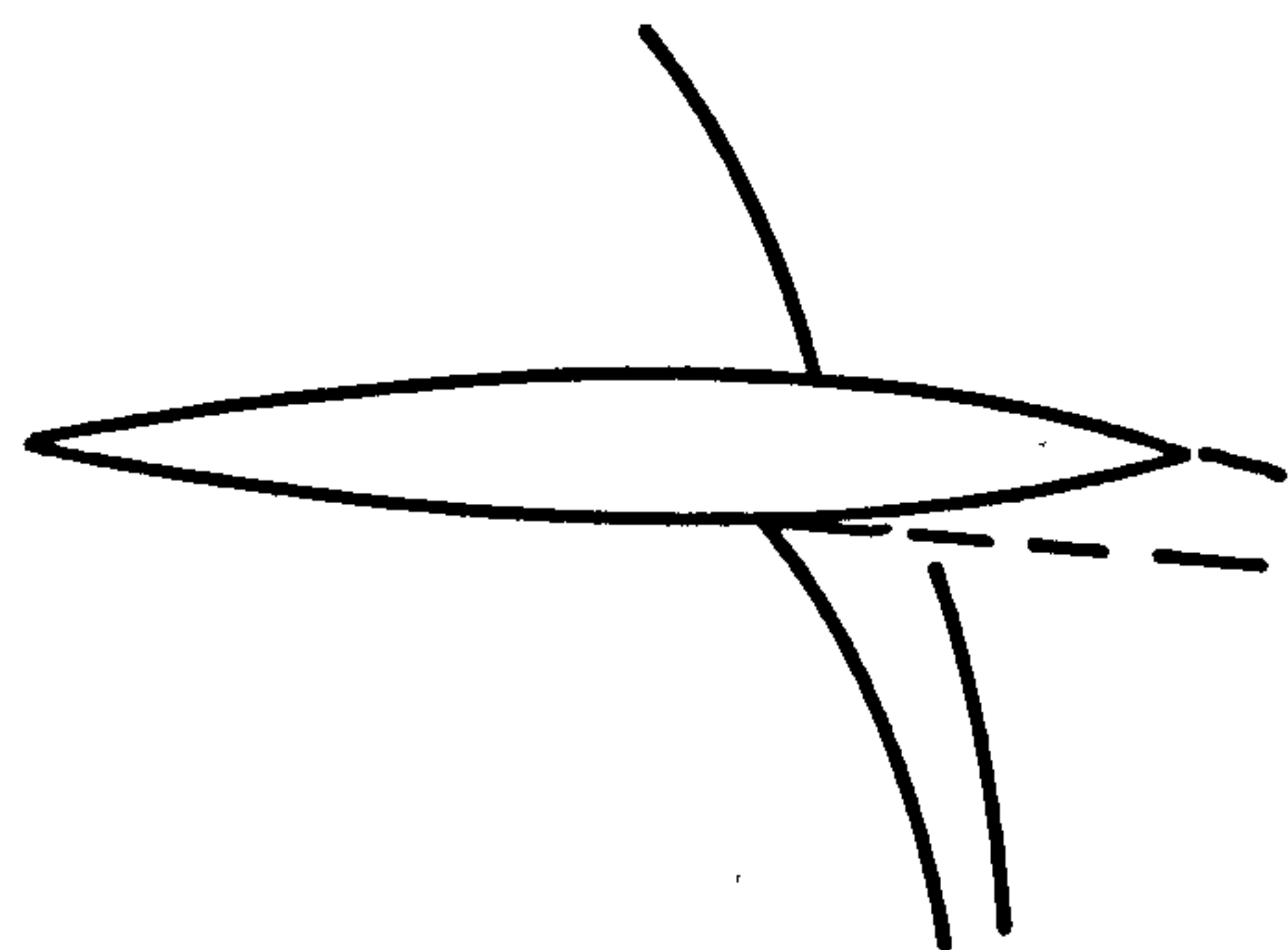
10



11



12



contd. Figure 78

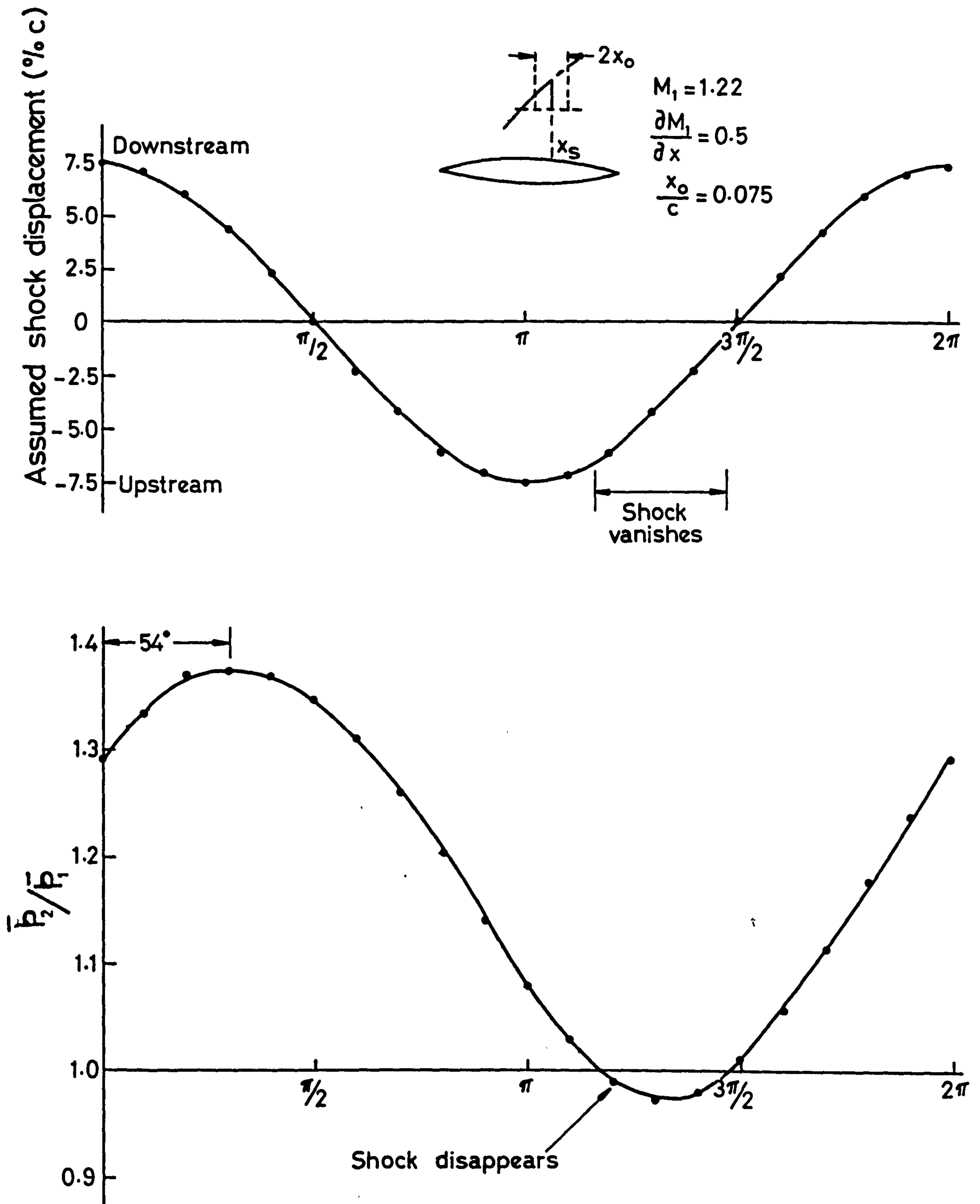


Figure 79 Relation between shock position and shock strength.