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Zero-Effort-Miss Shaping Guidance Laws

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This paper suggests a new approach in designing homing guidance laws to enable direct shaping of the pattern of the zero-effort-miss (ZEM) as desired. The proposed approach uses the concept of weighted ZEM and its specific desired error dynamics: the former is to provide an additional degree of freedom in shaping actual ZEM and the latter is to guarantee a finite-time convergence. Utilization of these two concepts allows simple determination of the guidance law that can achieve the desired pattern of ZEM. The resultant guidance law is shown a type of proportional navigation guidance (PNG) law with the specific form of time-varying gain not revealed in previous studies. It provides unique information on how the time-varying gain should be shaped to obtain the desired pattern of ZEM. Accordingly, the resultant guidance laws can cope with various operational objectives in a more direct way compared with the previously existing approaches. This paper also performs theoretical analysis to investigate the properties of designed guidance laws including the closed-loop solutions of ZEM and acceleration command. Also, we determine the feasible set of desired ZEM patterns that can be achieved in the proposed framework. Two illustrative examples are considered to show how to design guidance laws using the proposed approach. Moreover, the characteristics of the guidance laws designed are validated and demonstrated via numerical simulations.

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Index Terms

Homing guidance law, Zero-effort miss (ZEM), Trajectory shaping, Feedback linearization

I. Introduction

Proportional navigation guidance (PNG) law [1-2] has been widely applied to many missile systems for the terminal homing guidance law and proven its effectiveness in real applications. The most attractive aspect of PNG is practicality and simplicity for implementation to real missile systems. Note that the guidance command of PNG is expressed only by the multiplications of the navigation constant and two variables, i.e. the line-of-sight (LOS) rate and the closing velocity. Those variables can be readily measured or estimated from an on-board seeker. Extensive studies on PNG have revealed that specific choice of the navigation constant, $N = 3$, in PNG is the energy optimal solution [1-2]. Recently, it has been shown that PNG with the other navigation constants $N \neq 3$ are also optimal solutions of different energy cost functions [3].

As battlefields have been ever evolving, missile systems have been continuously demanded to accommodate the changes in the battlefields. Operational goals of guidance have been naturally adapted according to these demands. In general, the operational goals are expressed as satisfaction of additional terminal constraints or desired patterns of important guidance parameters. For instance, anti-ship missile systems are additionally demanded to meet the terminal impact time constraint [4-5]. Satisfaction of a desired terminal impact angle is an additional important requirement for the anti-tank missile systems to increase the lethality [6-8]. If the interception condition is only consideration for the missile system, additional guidance goals can be typically represented as achieving desired patterns of zero-effort-miss (ZEM) or guidance commands: for the surface-to-air missile systems controlled by the aerodynamic force only, the decreasing pattern of the guidance command is desirable according to the change of the dynamic pressure. For the anti-ballistic missile systems, a rapid convergence of ZEM is important to effectively intercept a target within an extremely short engagement time.

It is found to be difficult to achieve these various operational goals of guidance using the conventional PNG (i.e., with a constant gain). Due to its essential property, the patterns of ZEM or guidance command in the conventional PNG are standardized depending on the initial engagement condition. Although adjusting navigation constant can allow certain changes in the patterns, it is well known that their variations in the conventional PNG are restricted.

In order to mitigate this issue, there have been extensive research activities in which new guidance laws were suggested. Most of those studies have been based on the optimal control framework. The key focus of this approach is how to formulate different cost functions and constraints in a way best accomplishing various guidance operational goals. Examples of such formulation include a time-to-go weighted cost function [9], an exponential weighted cost function [10-12], a penalty term in cost functions [13], and a cost function weighted by arbitrary functions [14]. Additionally, those guidance laws are realized as type of PNG with various forms of time-varying guidance gains according to cost functions.

Despite the reported performance superiority over the conventional PNG, this approach can significantly suffer from inherent limitation: the pattern of guidance command can be shaped only by adjusting weighting parameters in the cost function and constraints. This implies that shaping the guidance command using the optimal control framework is indirect and consequently it is difficult to precisely shape the guidance command as required. In other words, it cannot provide a direct relationship between the time-varying gain in PNG and the desired guidance command.

To this end, this paper proposes a new paradigm in designing guidance laws. In the proposed approach, the desired pattern of ZEM is first designed in consideration of given operational goals. Note that the behaviors and constraints of important parameters for the guidance operation are generally expressed by the terms of ZEM. Therefore, mapping from the operational goals to the ZEM pattern is more straightforward and insightful, compared with that to weighting parameters in the cost. The proposed approach then determines the guidance command that can accomplish the desired ZEM pattern. To enable analytical determination of the guidance command, this paper suggests the concept of weighted ZEM, which is the actual ZEM weighted by a class of functions. This weighted ZEM provides an additional degree of freedom in shaping the pattern of the actual ZEM. Then, the proposed approach chooses a specific desired error dynamics with respect to the weighted ZEM to guarantee a finite time convergence of its value. Finally, applying the feedback linearization methodology to the predetermined error dynamics completes analytical determination of the guidance command. The obtained guidance law is also represented as a type of PNG as similar in the command shaping guidance laws [9-14], however the time-varying gain uncovered in this paper is given by the specific form which is not revealed in previous studies. It reveals the direct relationship between the time-varying guidance gain and the desired pattern of ZEM, that is, important information about how the time-varying gain should change to achieve the desired pattern of ZEM.

The properties of the proposed approach are investigated through theoretical analysis. Since the boundedness of the commanded acceleration is of paramount in practice, the analysis finds the conditions of the ZEM weighting functions that guarantee the bounded guidance command. Also, based on this result, we examine the feasible set of desired ZEM patterns that are available. The closed-form solutions of ZEM and guidance command for a given weighting function are also obtained. These solutions enable identification of the weighting function of ZEM, which fulfils the desired pattern of ZEM designed. The solution of the ZEM weighting functions bridges the desired pattern of ZEM to the guidance command, i.e. we can simply determine the guidance command, which achieves the desired pattern of ZEM, using the weighting functions. As a part of validation, this paper introduces two illustrative application examples and finds their corresponding guidance laws following the proposed design procedure. Numerical simulations are conducted to demonstrate the characteristics of the new approach developed in this paper.

The potential significances of the proposed approach are given as follows. First and clear, it offers the new paradigm of designing guidance laws in which the pattern of ZEM can be directly shaped. Second, it allows guidance designers to develop new PN-type guidance laws that satisfy the guidance operational goals specific to their systems. This is because there exist an extensive number of candidate desired ZEM pattern. Third, as the resultant guidance law is a type of PNG, it preserves the most powerful advantages of PNG such as practicability and simplicity. This also implies that the time-varying guidance gain in PNG can be interpreted in terms of ZEM. Therefore, the proposed approach provides an additional and critical insight to understand the characteristics of PNG with a time-varying guidance gain.

The rest of this paper consists of six sections. In section II, the homing geometry and the concept of weighted ZEM are addressed. Section III provides the detailed derivation of the ZEM shaping guidance laws based on the proposed approach and their characteristics are analyzed in Section IV. Numerical simulation results are given in Section V. Finally, the conclusion of this paper is summarized in Section VI.

II. Problem Formulation

A. Engagement Kinematics

Fig. 1 shows the 2D homing engagement geometry considered in this paper. As shown in the geometry, the inertial reference frame is denoted as (X_I, Y_I). For the purpose of deriving the linearized kinematics, a new frame called the reference frame (x_R, y_R) is also defined. This frame is rotated from the inertial frame by σ_0, which is the

initial line-of-sight (LOS) angle. Variables with subscripts of M and T represent those of the missile and target, respectively. The notations of σ and γ_M represent the LOS angle and flight path angle, respectively. R denotes the relative distance between the target and the missile. y is the relative distance between the target and the missile along Y_R-direction. a_M and a_T are the missile and target accelerations normal to the velocity vectors, respectively. The variables of $a_{M\sigma}$ and $a_{T\sigma}$ denotes the missile and target accelerations normal to the LOS direction, respectively.

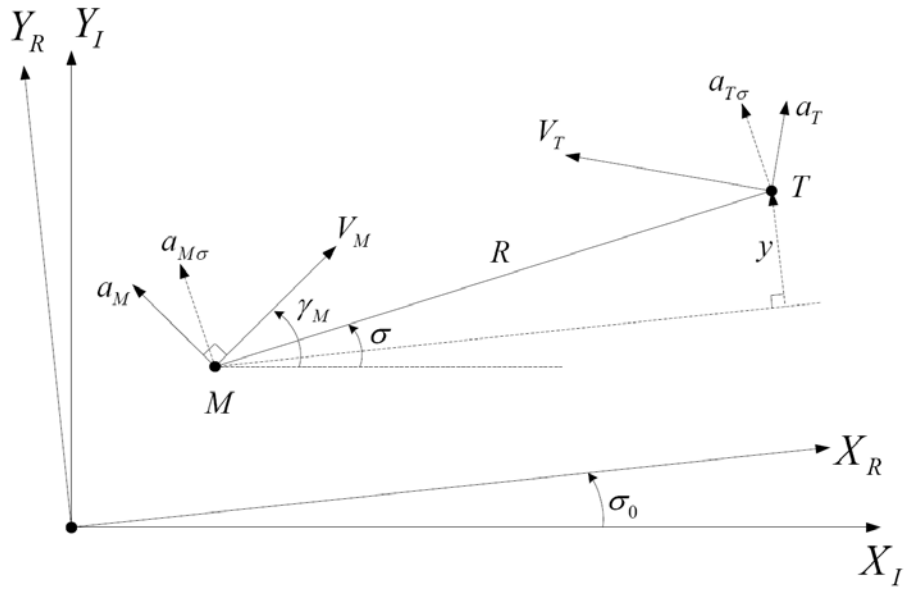


Fig. 1 The homing engagement geometry and parameter definitions.

In the reference frame, the engagement kinematics can be expressed as follows:

$$\begin{aligned} \dot{y} &= v \\ \dot{v} &= a_{T\sigma}\cos(\sigma-\sigma_0) - a_{M\sigma}\cos(\sigma-\sigma_0) \end{aligned} \tag{1}$$

where v is the relative velocity between the target and the missile perpendicular to X_R-direction. Under the small angle assumption of $\sigma-\sigma_0$, we have the linearized engagement kinematics as follows:

$$\begin{aligned} \dot{y} &= v \\ \dot{v} &= a_{T\sigma} - a_{M\sigma} \end{aligned} \tag{2}$$

In the presented engagement kinematics, ZEM can be expressed from its definition (the achieved miss distance at the terminal time when the missile does not control anymore), as follows

$$z = y + vt_{go} + \frac{1}{2}a_{T\sigma}t_{go}^2 \tag{3}$$

where $t_{go} = t_f - t$ represents the remaining time of interception, which can be approximated as follows:

$$t_{go} = R / V_c \tag{4}$$

Here, V_c denotes the closing velocity. From Eq. (3) with Eq. (2), the time-derivative of ZEM can be determined as

$$\dot{z} = -a_{M\sigma} t_{go} \tag{5}$$

$$\ddot{z} = -\dot{a}_{M\sigma} t_{go} + a_{M\sigma} \tag{6}$$

Note that as shown in Eq. (5) the rate of ZEM is directly proportional to the missile acceleration and the time-to-go. Therefore, ZEM can be controlled by the missile acceleration. In order to successfully intercept a target, the following condition should be achieved.

$$z\left(t_f\right) = 0 \tag{7}$$

In the next section, the proposed guidance law will be derived based on this kinematic equation.

B. Main Concept

The purpose of this paper is to devise a homing guidance law that can directly shape the decreasing pattern of ZEM as desired. The main idea to accomplish this goal is to introduce the concept of weighted ZEM as follows:

$$z_w = w(t) z \tag{8}$$

where $w(t)$ is an arbitrary weighting function and it has a positive value as

$$w(t) > 0 \quad \text{for } t \in \left[t_0, t_f\right] \tag{9}$$

According to Proposition 1, for a certain class of weighting functions, the intercept condition in the concept of weighted ZEM is equivalently given by

$$z_w\left(t_f\right) = 0 \tag{10}$$

Proposition 1. If the inverse of the weighting function is bounded during the flight as:

$$w^{-1}(t) < \infty \quad \text{for} \quad t \in \left[t_0, t_f\right] \tag{11}$$

the condition of Eq. (10) holds the interception condition given by Eq. (7).

Proof. From Eq. (8), it is readily obtained as $z = w(t)^{-1} z_w$. Here, it is trivial that z converges to zero as z_w approaches zero when $w^{-1}(t)$ is bounded as in Eq. (11). This completes the proof. ■

The purpose of introducing this weighting function is to provide an additional degree of freedom in shaping the pattern of ZEM. Namely, if we make z_w decreases with some pattern as $z_w \to g(t)$, then the pattern of actual ZEM changes as

$$z = w(t)^{-1} z_w = w(t)^{-1} g(t) \tag{12}$$

Therefore, through an appropriate selection of weighting functions, we can achieve a desired pattern of ZEM in this framework. Suppose that a desired pattern of ZEM to accomplish specific guidance objectives is given by z_d, then a desired weighting function is readily determined from Eq. (12) as

$$w(t) = g(t) z_d^{-1} \tag{13}$$

Then, in this framework, a guidance command is determined to make z_w decreases with a specific pattern as $g(t)$ and becomes zero in a finite time.

III. Proposed Guidance Law

A. Derivation of Proposed Guidance Law

In this section, the proposed guidance law that satisfies aforementioned conditions is derived using the feedback linearization technique [15]. Taking the time-derivative of the weighted ZEM yields

$$\dot{z}_w = \dot{w}(t) z + w(t) \dot{z} \tag{14}$$

Substituting Eq. (5) into Eq. (14) gives the following result.

$$\dot{z}_w = \dot{w}(t) z - w(t) t_{go} a_{M\sigma} \tag{15}$$

As shown in Eq. (15), the acceleration command, which is assumed to be the control input in this system equation, appears in the first time derivative of the weighted ZEM. Note that there is no internal dynamics because the relative degree is equal to the degree of the system [15]. Eq. (15) can be rewritten in the general form of nonlinear system as

$$\dot{x} = f + bu \tag{16}$$

where

$$x \to z_w, \qquad u \to a_{M\sigma} \tag{17}$$

$$f \to \dot{w}(t) z, \quad b \to -w(t) t_{go} \tag{18}$$

Since this equation gives a direct relationship between the weighted ZEM and the control input, we can directly apply the feedback linearization technique to this system equation.

For the nonlinear system represented as Eq. (16), previous studies on the guidance law design using nonlinear methodologies [16-18] widely used the following desired error dynamics for x

$$\dot{x} + kx = 0 \tag{19}$$

where k is a positive constant. This error dynamics enforces that x exponentially converges to zero:

$$x = x_0 e^{-kx} \tag{20}$$

However, this does not guarantee a finite time convergence. Unlike in the control law design, guarantee on the finite time convergence is an important requirement in the guidance law design. Accordingly, some modified versions of desired error dynamics to guarantee finite time convergence have also been suggested in [19-21]. However, not only those are generally given by complicated forms, but they also include many design parameters which could make proper tuning of guidance laws intricate.

As a remedy, this study suggests desired error dynamics for x as

$$\dot{x} + \frac{k}{t_{go}} x = 0 \tag{21}$$

where $k > 0$. It can be regarded as a parameter that determines the convergence speed of weighted ZEM. As shown in Lemma 1, note that the proposed error dynamics is to guarantee a finite time convergence. Additionally, the decreasing pattern of error is given by the closed-form function as shown in Eq. (24).

Lemma 1. Suppose $k > 0$. Then, the system presented in Eq. (21) is stable and x converges to zero as $t_{go} \to 0$.

Proof. The stability and finite time convergence of x will be proven by directly obtaining the closed-loop solution of x whose dynamics is given by Eq. (21). As this differential equation is one of Cauchy-Euler equation, it can be converted to an ordinary differential equation by replacing $t_{go} = e^{\tau}$ as

$$\frac{dx}{d\tau} - kx = 0 \tag{22}$$

If we assume a solution of Eq. (22) as $x = e^{\alpha\tau}$, then we have the following characteristics equation:

$$\alpha - k = 0 \tag{23}$$

Therefore, by using the result of Eq. (23) and the relation $t_{go} = e^{\tau}$, the closed-loop solution of Eq. (21) can be determined as

$$x = \left(\frac{x(t_0)}{t_f^k} \right) t_{go}^k \tag{24}$$

where $x(t_0)$ denotes the initial value and t_f is the final time. From Eq. (24), for $k > 0$, it is clear that x converges to zero as t_{go} approaches to zero, which completes the proof. ■

Then, a straightforward application of Eq. (16) to Eq. (21) gives

$$u = \frac{1}{b}\left(-\frac{k}{t_{go}} x - f \right) \tag{25}$$

Using the notational definitions in Eqs. (17) and (18), the proposed guidance command is obtained as

$$a_{M\sigma} = N(t)\frac{z}{t_{go}^2} \tag{26}$$

where

$$N(t) \equiv \frac{\dot{w}(t) t_{go}}{w(t)} + k \tag{27}$$

In Eq. (27), $N(t)$ is the specific form of time-varying guidance gain and given by the function of time-to-go, weighting function, time-derivative of weighting function, and value k.

In the proposed framework, the weighting function and the value k can be considered as the design parameters. As mentioned above, the value k is chosen to determine the convergence speed of weighted ZEM. For a chosen k, the weighting function is then selected to achieve the desired ZEM pattern. The solution of weighted ZEM bridges the desired pattern of ZEM to the weighting functions. Under the proposed guidance law, the decreasing pattern of weighted ZEM $g(t)$ is determined from Eqs. (8) and (24), as follows.

$$z_w = g(t) = \left(\frac{w(t_0) z(t_0)}{t_f^k} \right) t_{go}^k \tag{28}$$

where $w(t_0)$ and $z(t_0)$ are the initial values of weighting function and ZEM, respectively. From Eq. (13), the desired weighting function is determined as

$$w(t) = \left(\frac{w(t_0) z(t_0)}{t_f^k} \right) t_{go}^k z_d^{-1} \tag{29}$$

This is given by the polynomial function of time-to-go with k and inverse of desired ZEM pattern z_d. Accordingly, the desired time-varying gain is determined by substituting Eq. (29) into Eq. (27). It is given by the function of time-to-go and desired pattern of ZEM, and value k, respectively.

Remark 1. Although the proposed guidance law is formulated based on the 2D homing engagement geometry, this approach can be easily extended to the 3D scenario by using the well-known separation design concept. By introducing the LOS frame (it is rotated from the inertial reference frame by the LOS angles), we can decouple the homing engagement geometry into two identical and perpendicular planes. Then, we can apply the proposed approach to each engagement plane.

B. Implementation Issues

Let us now discuss an alternative representation of the proposed guidance law and implementation issues. From Fig. 1, the LOS angle can be expressed as follows:

$$\sigma = \sigma_0 + \frac{y}{R} \tag{30}$$

Then, the LOS rate can be obtained by taking time-derivative of Eq. (30) as

$$\dot{\sigma} = \frac{\dot{y}R - \dot{R}y}{R^2} \tag{31}$$

Substituting Eq. (4) into Eq. (31) yields

$$\dot{\sigma} = \frac{y + v t_{go}}{V_c t_{go}^2} \tag{32}$$

Putting Eq. (32) into Eq. (3) gives

$$z = V_c \dot{\sigma} t_{go}^2 + \frac{1}{2} a_{T\sigma} t_{go}^2 \tag{33}$$

Finally, combining Eqs. (26) and (33) yields

$$a_{M\sigma} = N(t) V_c \dot{\sigma} + \frac{1}{2} N(t) a_{T_\sigma} \tag{34}$$

As shown in Eq. (34), the alternative form of proposed guidance law becomes a type of augmented proportional navigation guidance (APNG) with the specific form of time-varying gain Eq. (27) with Eq. (29). Therefore, it holds

the most powerful benefits of APNG such as practicability and simplicity: the LOS rate, the closing velocity and the target acceleration are only required for implementing it. Those parameters can be estimated by a well-tuned homing filter using seeker measurement and radar measurement. If the estimated target acceleration is not accurate enough, as commonly applied in PNG, the proposed guidance law can also be implemented without the target acceleration term, i.e.

$$a_{M\sigma} = N(t)V_c\dot{\sigma} \tag{35}$$

In addition, since $a_{M\sigma}$ represents the missile acceleration normal to the LOS vector, for implementation, it should be converted as follows:

$$a_M = a_{M\sigma} / \cos(\gamma_M - \sigma) \tag{36}$$

where a_M is the missile acceleration perpendicular to the missile velocity vector. In Eq. (33), the flight path angle and the LOS angle can be measured or estimated from a built-in inertial navigation system (INS) and seeker.

Remark 2. In the proposed guidance law, the time-to-go information is also required to compute the time-varying guidance gain Eq. (27) with Eq.(29). However, it is used only for shaping the pattern of ZEM, so the homing performance is insensitive to small errors. Therefore, this study uses a simple time-to-go calculation shown in Eq. (4) from a practical point of view.

Remark 3. Depending on the accuracy of estimated target acceleration in the proposed method, the deviation of ZEM pattern between the desired one and achieved one is determined as

$$\Delta z = z_d - z = \frac{1}{2}\left(a_{T_\sigma} - \hat{a}_{T_\sigma}\right)t_{go}^2 \tag{37}$$

where $\hat{a}_{T_\sigma}$ is the estimated target acceleration and used to implement the proposed guidance law. Therefore, as the estimation error of target acceleration is made smaller, this deviation decreases. Additionally, if the proposed guidance law is implemented without the target acceleration term shown in Eq. (35), then the deviation $\Delta z = (1/2)a_{T_\sigma}t_{go}^2$ is expected according to the target acceleration a_{T_σ}.

IV. Discussion of ZEM Shaping Guidance Laws

A. Characteristics of Proposed Guidance Law

As shown in Eq. (34), the proposed guidance law can be characterized as a type of PNG with the specific form of time-varying guidance gain which is given by Eq. (27) with Eq. (29). During the course of years, some studies on PNG with a time-varying guidance gain have been reported [10-13]. Also, it has been shown that the generalized command shaping guidance law is realized as PNG with time-varying guidance gains [14]. Although the proposed guidance law is also represented as a type of PNG with a time-varying gain, a new form of time-varying gain not revealed in other studies is uncovered in this paper. In other words, the proposed results as shown in Eq. (27) with Eq. (29) provide information on how the time-varying gain should be shaped to obtain the desired pattern of ZEM, whereas other studies not easily allow this. This is one of key contributions of this study.

Next, let us discuss the feasible set of weighting functions that can be used for ZEM shaping. The weighting function should be designed in consideration of not only guidance operational objectives, but also physical and operational constraints of the missile system. The essential conditions should be met are the interception condition and the bounded navigation gain condition: the goal of the missile guidance is to intercept the target; the time-varying guidance gain should be bounded to avoid the introduction of an abrupt guidance command during the flight in practice. Following Proposition gives the condition on the weighting function to guarantee the bounded navigation gain. Then, Lemma 2 provides the set of weighting functions satisfying the two aforementioned essential conditions.

Proposition 2. Suppose $w(t)$ satisfies the following condition

$$\left| \frac{\dot{w}(t) t_{go}}{w(t)} + k \right| < \infty \text{ for } t = \left[t_0, t_f \right] \tag{38}$$

Then, the time-varying navigation gain guarantees the boundedness of navigation gain in the proposed guidance law.

Proof. If $w(t)$ satisfies Eq. (38), it is trivial that the time-varying gain is bounded because the time-varying navigation gain is expressed as Eq. (27). This completes the proof. ■

Lemma 2. Suppose $w(t)$ is an element of the following set:

$$S_{w(t)} \equiv \left\{ w(t) \mid \ w^{-1}(t) < \infty \quad \text{and} \quad \left| \frac{\dot{w}(t) t_{go}}{w(t)} + k \right| < \infty \quad \text{for} \quad t \in \left[t_0, t_f \right] \right\} \tag{39}$$

Then, the proposed guidance law whose navigation gain is given by Eq. (27) guarantees the interception of the target and bounded navigation gain.

Proof. From Propositions 1, it is trivial that the proposed guidance law with the weighting function holding the condition Eq. (39) guarantees the interception of the target. Moreover, from Propositions 2, it is obvious that the weighting function satisfying the condition in Eq. (39) ensures the bounded navigation gain. This completes the proof. ■

Based on this result, we can finally determine the feasible set of desired ZEM patterns that can be achieved in the proposed approach. The relationship between the desired weighting function and the desired ZEM pattern is given by Eq. (29). Therefore, from Eqs. (29) and (39), the feasible set of desired ZEM patterns z_d that satisfy the interception condition is determined as

$$S_{z_d} \triangleq \left\{ z_d \mid \left| t_{go}^{-k} z_d \right| < \infty \quad \text{and} \quad \left| \frac{\dot{z}_d t_{go}}{z_d} \right| < \infty \quad \text{for} \quad t \in \left[t_0, t_f \right] \right\} \tag{40}$$

Accordingly, any desired ZEM patterns that satisfy Eq. (40) can be achieved in the proposed approach.

When a weighting function satisfies the condition presented in Eq. (39), Eq. (27) indicates that the time-varying guidance gain converges to some constant value N_f as the missile approaches a target, i.e.

$$\lim_{t_{go} \to 0} N(t) = \lim_{t_{go} \to 0} \frac{\dot{w}(t) t_{go}}{w(t)} + k = N_f \tag{41}$$

with

$$N_f = \begin{cases} k+m & \text{if } \dot{w}(t) t_{go} / w(t) = m \\ k & \text{if } \dot{w}(t) t_{go} / w(t) \neq m \end{cases} \tag{42}$$

where m is a constant. This means that the proposed guidance law will behave as PNG with a navigation constant $k+m$ or k and provide similar terminal performance compared to PNG as the missile approaches to the target.

Let us now discuss about the closed-form solutions of ZEM and guidance command under the proposed method. From Eqs. (12) and (28), it is trivial that the closed-form solution of ZEM is given by

$$z = \left(\frac{w(t_0) z(t_0)}{t_f^k} \right) \frac{t_{go}^k}{w(t)} \tag{43}$$

This provides the relationship between the weighting function and the actual ZEM. By substituting Eq. (29) into Eq. (43), we can readily observe that the actual ZEM achieves the desired ZEM as $z = z_d$ in the proposed approach.

Next, substituting Eq. (43) into Eq. (26) provides the closed-form solution of guidance command under the proposed guidance law.

$$a_{M\sigma} = \left(\frac{w(t_0) z(t_0)}{t_f^k} \right) \left[\frac{\dot{w}(t) t_{go}}{w(t)} + k \right] \frac{t_{go}^{k-2}}{w(t)} \tag{44}$$

Note that the closed-form solution is given by multiplication of polynomial function of time-to-go with order of $k-2$ and inverse of $w(t)$. Eq. (44) enables the prediction of guidance command profile and consequently identification of useful information, e.g. where the maximum command occurs during the flight.

Now, let us investigate conditions on k to guarantee certain properties of proposed approach. In order to achieve the intercept condition, ZEM obtained in Eq. (43) should be zero at the terminal time. Proposition 3 provides the condition k that makes ZEM to be zero at the terminal time. In practice, it is of paramount to guarantee the bounded guidance command during the flight. Under this background, Proposition 4 finds the condition for k that guarantees the bounded guidance command. Also as shown in Proposition 4, for $k > 2$, the guidance command converges to zero as the missile approaches the target. This condition is important since it could provide capability to cope with unwanted situations and/or uncertainties near the interception.

Proposition 3. For $k > 0$ and $w(t)$ satisfying the condition in Eq. (39), ZEM always converges to zero as $t_{go} \to 0$.

Proof. As shown in Eq. (43), ZEM is given by the function of t_{go}^k multiplied by $w^{-1}(t)$. If $w(t)$ satisfies the condition in Eq. (39), then it is clear that $w^{-1}(t)$ is bounded because of its definition. Then, Eq. (43) implies that the convergence of ZEM is determined by the pattern of t_{go}^k. From Eq. (43), it is obvious that for $k > 0$, the term of t_{go}^k approaches zero as $t_{go} \to 0$. Therefore, $z \to 0$ as $t_{go} \to 0$, which completes the proof. ■

Proposition 4. Suppose that $w(t)$ holds the condition in Eq. (39) and the design parameter k is selected as

$$k >= 2 \tag{45}$$

Then, the acceleration command is bounded.

Proof. From Eq. (44), it is observed that the acceleration command is given by the function of t_{go}^{k-2} multiplied by $w^{-1}(t)$ and the term of $\dot{w}(t) t_{go} w^{-1}(t) + k$. If $w(t)$ is naturally chosen to satisfy Eq. (39), then $w^{-1}(t)$ and

$\dot{w}(t)t_{go}w^{-1}(t)+k$ are bounded according to the definition of $w(t)$. Therefore, the bound of acceleration command is governed by the term, t_{go}^{k-2}. Since for $k>2$ the term of t_{go}^{k-2} gradually approaches zero as $t_{go}\to 0$, the acceleration command converges to zero as $t_{go}\to 0$, which means that the acceleration command is bounded for $k>2$.

$$\lim_{t_{go}\to 0} a_{M\sigma} = \lim_{t_{go}\to 0}\left(\frac{w(t_0)z(t_0)}{t_f^k}\right)\left[\frac{\dot{w}(t)t_{go}}{w(t)}+k\right]\frac{t_{go}^{k-2}}{w(t)} = 0 \tag{46}$$

For $k=2$, the magnitude of acceleration command is given by

$$\left|a_{M\sigma}\right| = \left|\left(\frac{w(t_0)z(t_0)}{t_f^k}\right)\left[\frac{\dot{w}(t)t_{go}}{w(t)}+2\right]\frac{1}{w(t)}\right| < \infty \tag{47}$$

Since $w^{-1}(t)$ and $\dot{w}(t)t_{go}w^{-1}(t)+k$ are bounded from the definition of $w(t)$ and Lemma 2, Eq. (47) is also bounded. This completes the proof. ■

The behavior of ZEM under the proposed approach can be assessed in terms of the natural frequency and the damping ratio. Taking the time derivative of Eq. (26) gives

$$\dot{a}_{M\sigma} = \left[\frac{\dot{N}(t)}{t_{go}^2}+\frac{2N(t)}{t_{go}^3}\right]z+\frac{N(t)}{t_{go}^2}\dot{z} \tag{48}$$

Substituting Eqs. (26) and (48) into Eq. (6) yields

$$\ddot{z}+\frac{N(t)}{t_{go}}\dot{z}+\left[\frac{\dot{N}(t)t_{go}+N(t)}{t_{go}^2}\right]z = 0 \tag{49}$$

This equation represents the motion of ZEM under the proposed guidance law. For the analysis purpose, we regard the above time varying equation as an instantaneous linear time-invariant system at each time instance. Then, it is possible to investigate the tendency of ZEM motion at each time step [22]. Based on this approximation, the motion of ZEM can be expressed in terms of ω_n and ζ, i.e.

$$\ddot{z}+2\zeta\omega_n\dot{z}+\omega_n^2 z = 0 \tag{50}$$

where

$$\zeta = \frac{N(t)}{2\sqrt{N(t)+\dot{N}(t)t_{go}}}, \qquad \omega_n = \frac{\sqrt{N(t)+\dot{N}(t)t_{go}}}{t_{go}} \tag{51}$$

Here, ζ and ω_n are regarded as the damping ratio and the natural frequency of ZEM motion, respectively. Note that these two parameters are expressed as functions of time-to-go and time-varying guidance gain. Furthermore, the navigation gain is given as the function of weighting function and time-to-go. This implies that the motion of ZEM is a time-varying system. Therefore, the analysis based on the instantaneous linear time-invariant system is not rigorous enough. However, it could at least provide some insight about the behavior of ZEM motion. The reason behind performing this type of analysis is due to the well-known difficulties in the analysis of time-varying systems.

At the initial time, these values are given by

$$\zeta_0 = \frac{N(t_0)}{2\sqrt{N(t_0)+\dot{N}(t_0)t_f}} \qquad \omega_{n,0} = \frac{\sqrt{N(t_0)+\dot{N}(t_0)t_f}}{t_f} \tag{52}$$

where $N(t_0)$ and $\dot{N}(t_0)$ are the time-varying guidance gain and its derivative at the initial time. Also, as the time-to-go approaches to zero, these values converge to the following values.

$$\zeta_f = \lim_{t_{go}\to 0} \frac{N(t)}{2\sqrt{N(t)+\dot{N}(t)t_{go}}} = \frac{\sqrt{N_f}}{2}, \qquad \omega_{n,f} = \lim_{t_{go}\to 0} \frac{\sqrt{N(t)+\dot{N}(t)t_{go}}}{t_{go}} = \infty \tag{53}$$

The final values of damping and natural frequency are same regardless of weighting function chosen. On the other hand, the initial values are significantly affected by the initial values of time-varying guidance gain and its derivative. From Eq. (52) to Eq. (53), it is trivial that the behavior of ZEM motion is greatly affected by the time-varying-gain and its time derivative, which are functions of weighting function. This confirms that the pattern of ZEM could be shaped by appropriate selection of weighting function.

B. Illustrative Applications of Proposed Guidance Law

As a part of validation, this section examines two illustrative application examples of proposed approach. The guidance problems considered in the examples require specific patterns of ZEM to accomplish their guidance objectives.

Before introducing the two examples, let us first examine a special case of ZEM pattern, which will reveal the characteristic of conventional fixed gain PNG in reducing ZEM. In this case, it is assumed that the pattern of ZEM needs to be reduced as a polynomial function of time-to-go. Then, the desired pattern of ZEM can be desired as

$$z_d(t) = C_1 t_{go}^m \tag{54}$$

where C_1 is an initial condition and $m > 0$ is a design parameter. Using Eq. (29), the desired weighting function is determined as

$$w(t) = t_{go}^{-(m-k)} \tag{55}$$

In the above equation, m should be equal to or greater than k, i.e. $m \geq k$, to satisfy the condition of Eq. (39). Finally, from Eq. (27), the desired time-varying guidance gain is computed as

$$N(t) = m, \quad \text{for} \quad m \geq k \tag{56}$$

The navigation gain is time-invariant and the resultant guidance of proposed approach becomes the well-known PNG with a navigation constant m in the first example. This means that the conventional PNG enforces ZEM to decrease as the polynomial function of time-to-go. Therefore, the decrease of ZEM is only monotone and thus the shaping of ZEM is restricted in the conventional PNG.

Now, let us consider the first application example in which the guidance problem for the kill vehicle systems is considered. This example will show that the pattern of ZEM can be further shaped in the proposed approach to cope with specific guidance operational objectives. For kill vehicle systems, the guidance strategy, so called the early correction, is generally recommended to effectively intercept tactical ballistic missile (TBM) over an extremely short engagement time. Following the early correction strategy, it is desirable to actively reduce ZEM rather than actively save the control energy. We can thus design the desired pattern of ZEM z_d by additionally introducing an exponential decreasing term from Eq. (54) as

$$z_d(t) = C_1 e^{\mu t_{go}} t_{go}^k, \quad \text{for} \quad \mu \geq 0 \tag{57}$$

where C_1 is an initial condition and μ represents a parameter deciding a decreasing speed of exponential term. By substituting Eq. (29) and Eq. (57), the corresponding weighting function is obtained as

$$w(t) = e^{-\mu t_{go}}, \quad \text{for} \quad \mu \geq 0 \tag{58}$$

Since this weighting function is an element of set $S_{w(t)}$ given in Eq. (39), it holds the essential conditions to be met. Under this weighting function, the time-varying guidance is given by:

$$N(t) = \mu t_{go} + k \tag{59}$$

The values of μ and k represent the slope of guidance gain and its terminal value, respectively. In this case, the guidance gain linearly decreases from a large value $\mu t_f + k$ to a small value k.

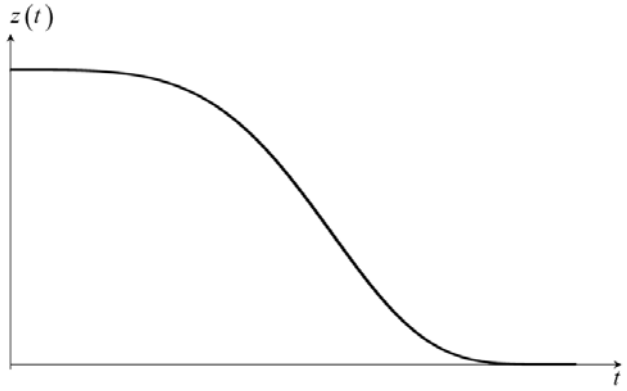


Fig. 2 An example of desired pattern of ZEM

For some missile systems, the quality of seeker measurement at the beginning of the homing phase is insufficient to provide the actual target position. The guidance strategy to rapidly reduce ZEM using incorrect ZEM might cause increase in actual ZEM. Therefore, in such a case, the guidance strategy so called the delayed correction is suitable. The delayed correction introduces ZEM slowly decreasing at the beginning and ZEM rapidly decreasing when the seeker measurement is good enough. To achieve the delayed correction, the desired pattern of ZEM can be designed as

$$z_d(t) = C_1\left[\cos\left(\pi\left(t/t_f\right)^{\eta}\right)+1\right]^k, \quad \text{for} \quad \eta \geq 1 \tag{60}$$

The general pattern of this function is shown as Fig. 2. To accomplish this ZEM pattern, the desired weighting function is defined as

$$w(t) = \frac{t_{go}^k}{\left[\cos\left(\pi\left(t/t_f\right)^{\eta}\right)+1\right]^k} \tag{61}$$

This weighting function is also valid since it satisfies the condition represented in Eq. (39). Substituting Eq. (61) into Eq. (27) yields

$$N(t) = \pi k\eta \frac{\sin\left(\pi\left(t/t_f\right)^{\eta}\right)}{\left[\cos\left(\pi\left(t/t_f\right)^{\eta}\right)+1\right]}\left(1-\frac{t}{t_f}\right)\left(\frac{t}{t_f}\right)^{\eta-1} \tag{62}$$

C. Potential significances of Proposed Guidance Law

This paper proposes a new paradigm in designing guidance laws (i.e., time-varying guidance gains of PNG). Different guidance problems could have their own distinct objectives besides the common operational objectives. We showed that these objectives can be directly considered in the design of an appropriate desired pattern of ZEM

and formulation of a corresponding weighting function. Moreover, it is shown that the guidance command obtained from the weighted ZEM enables the accomplishment of the guidance objectives considered. This implies that given the guidance problem and its distinctive operational objectives, one could follow the guidance design procedure detailed in section III.A and consequently develop a new guidance law specific to their guidance problem.

This paper also provides a significant and meaningful clue in understanding the behavior of important guidance parameters of PNG with a time-varying guidance gain: these guidance parameters include, but not being limited to, ZEM and guidance command. In practice, time-varying guidance gains in PNG are generally used without a strict guarantee on finite time convergence or a clear understanding of guidance loop motion. This is because it is hard to obtain the closed-form solutions of ZEM and guidance command when guidance gains are time-varying. The analysis results showed that we can readily identify the connection between a time-varying guidance gain and those closed-form solutions under the proposed guidance framework. This allows determination of closed-form solutions of ZEM and guidance command in case of time-varying guidance gains. Let us suppose a time-varying guidance gain $N(t)$ is given from a guidance law. From Eq. (27), it is possible to obtain the differential equation with respect to the weighting function as follows:

$$\dot{w}(t)+\frac{\left[k-N(t)\right]}{t_{go}}w(t)=0 \tag{63}$$

Then, we could obtain the solution of in Eq. (63), denoted by $w^*(t)$. Now, the closed-form solutions of ZEM and guidance command can be simply determined by substituting $w^*(t)$ into Eqs. (43) and (44). This indicates that a given time-varying guidance gain $N(t)$, finding the weighting function satisfying the condition in Eq. (63) should enable determination of the analytical solutions of ZEM and guidance command. Furthermore, examining whether $w^*(t)$ holds the conditions given in Proposition 3 and 4 will readily determine the finite convergence and the boundedness of guidance command, respectively.

V. Numerical Results

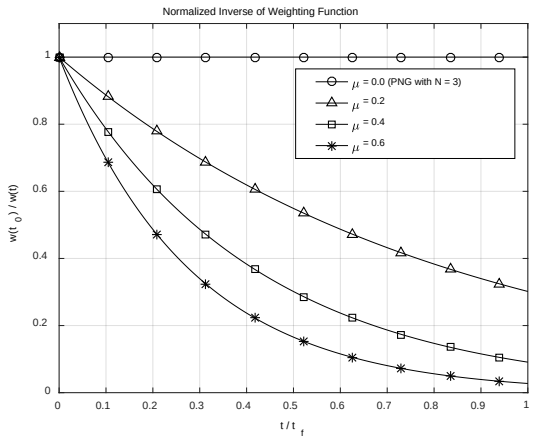
This section demonstrates the characteristics of guidance laws under the proposed approach through numerical simulations. The example guidance laws selected are the two laws designed for the two illustrative application examples in Section IV. C. The homing conditions presented in [11] are considered, that is, the initial relative

distance $R = 18\,\text{km}$, the closing velocity $V_c = 3000\,\text{m/s}$, the initial heading error $\varepsilon_h = 2°$, and the flight time $t_f = 6\,\text{s}$, respectively. In all simulations, the design parameter k is set to be equal to 3: if a chosen weighting function holds the second condition in Eq. (42) (that is $\dot{w}(t)t_{go} / w(t) \neq m$) the time-varying guidance gain converges to $N_f = k$; otherwise it converges to $N_f = k + m$. The performance of the two guidance laws is compared with that of the conventional PNG with the navigation gain equal to 3.

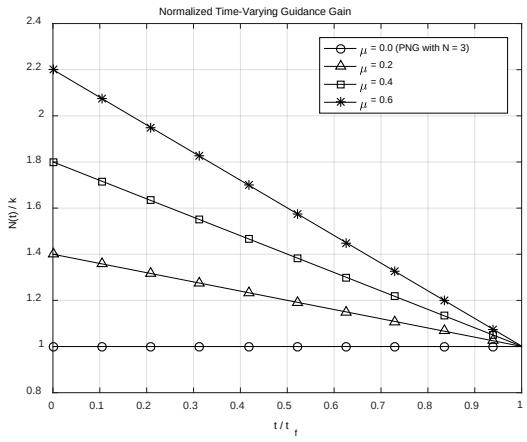
A. Basic characteristics of proposed method

The focus of the performance examination in this simulation is to investigate the capability in shaping the pattern of ZEM and the behavior of the corresponding guidance command, which will validate the analysis results in Section IV. Therefore, the simulation results include responses of the weighting function, time-varying guidance gain, pattern of ZEM, and acceleration command. Note that all responses are normalized: the time is normalized by t_f; the weighting function and ZEM by their initial values; the time-varying guidance gain by the design parameter k; and the acceleration command by $z(t_0) / t_f^2$.

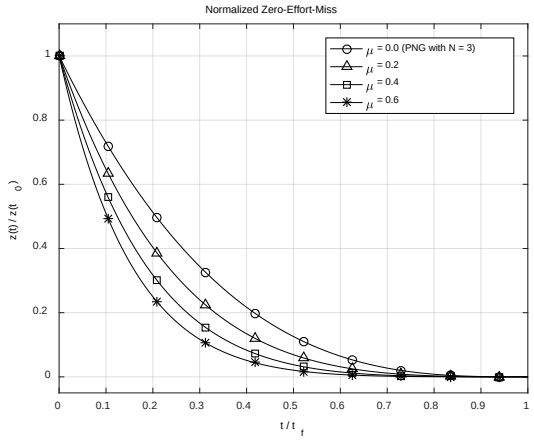
Fig. 3 depicts the simulation results of the first example guidance law whose weighting function is given in Eq. (58). As discussed in Section IV. B, this guidance law is designed for the engagement situations where early correction of ZEM is required. An additional design parameter of this guidance law is μ in the weighting function and various values of μ are tested in the simulations. In case where $\mu = 0$, the weighting function becomes identity and thus the first example guidance law is identical to PNG with $N = 3$, which is confirmed by the simulation results. As μ increases, value of the inverse weighting function more decreases exponentially as shown in in Fig. 3 (a). From Fig. 3 (b), it is evident that this type of guidance law results in linearly decreasing patterns of the time-varying guidance gain. Also, we can observe that its terminal value approaches to the value of k. These results validate the analysis results in Section IV. B. As shown in Fig. 3 (c), the guidance law considered leads to ZEM that is more exponentially decreasing than that of PNG, which meets the guidance operational objective. Regarding the acceleration command, the proposed guidance law requires more control effort at the beginning of homing phase compared with PNG, but it rapidly decreases in the vicinity of a target. Also, it is observed that the acceleration command for all cases converges to zero, which agrees with the analysis result in Proposition 4.



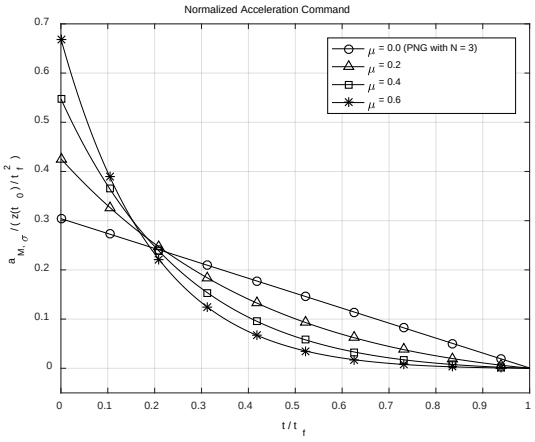
(a) Normalized inverse of weighting function



(b) Normalized time-varying guidance gain



(c) Normalized ZEM



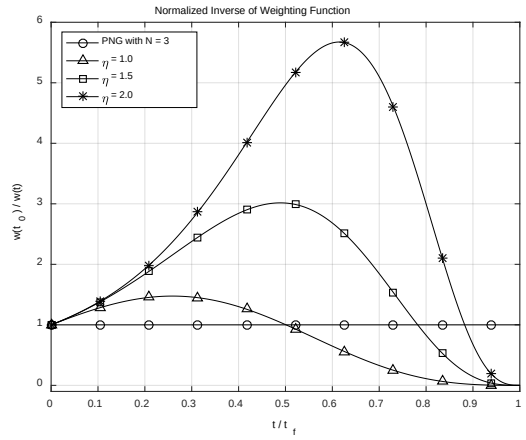
(d) Normalized Acceleration Command

Fig. 3 Simulation results for an early correction pattern of ZEM

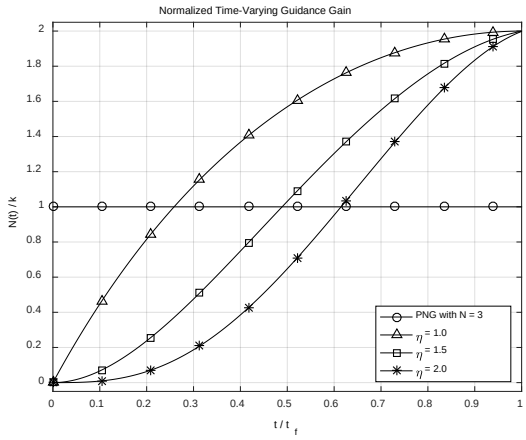
Fig. 4 demonstrates the simulation results of the second example guidance law of which the pattern of ZEM is represented by Eq. (61). This guidance law is designed to meet the delayed correction pattern of ZEM. Since k is set to be equal to 3, the remaining design parameter of this guidance is η. In the simulations, various values of η are tested. As shown in Fig. 4 (a), the pattern of inversed weighting functions initially increases and then converges to zero for all values of η examined. Its peak value and the occurrence time increase as η increases. Fig. 4 (b) also shows that the guidance gain is time-varying: it starts from zero and converges to $2k$ for all cases. The analysis results explain the reason behind the convergence of the guidance gain to $2k$ in this case. We have the condition where $\dot{w}(t)t_{go} / w(t) = k$ in the second example guidance law, so it is clear the gain converges to $2k$ from Eq. (42). The results depicted in Fig. 4 (c) verifies that the achieved patterns of ZEM are the same as the desired pattern

shown in Fig. 2. This confirms that it is possible to actively control the pattern of ZEM as desired, in this case to delay the correction of ZEM at the initial time and accelerate it after a certain point. Accordingly, the acceleration command starts to increase from zero at the beginning and converges to zero at the end. Moreover, the peak value of the guidance command and its occurrence moment can be controlled by the design parameter η: the peak value and its occurrence time increases as η increases.

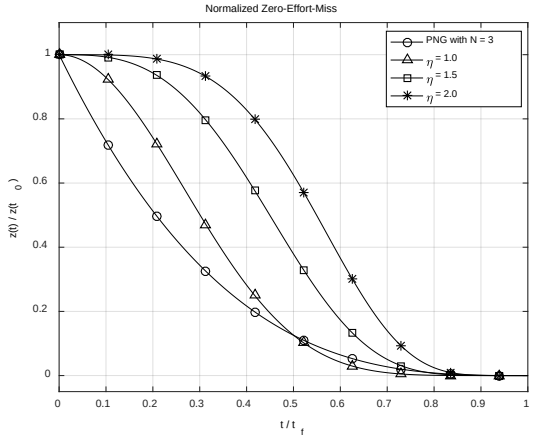
In conclusion, the simulation results demonstrate that the proposed approach enables simple design of new guidance laws that can achieve desired patterns of ZEM reflecting various guidance operational goals. These results also confirm the validity of the analysis results in Section IV.



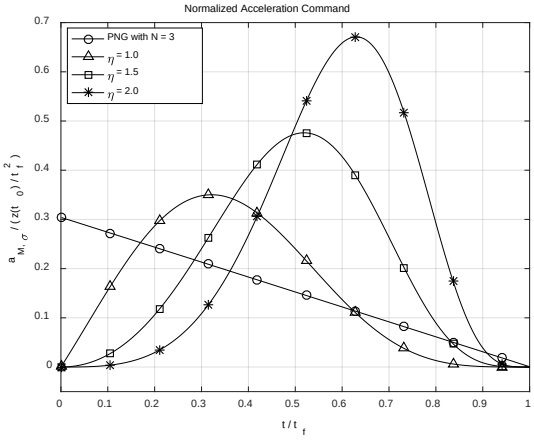
(a) Normalized inverse of weighting function



(b) Normalized time-varying guidance gain



(c) Normalized ZEM

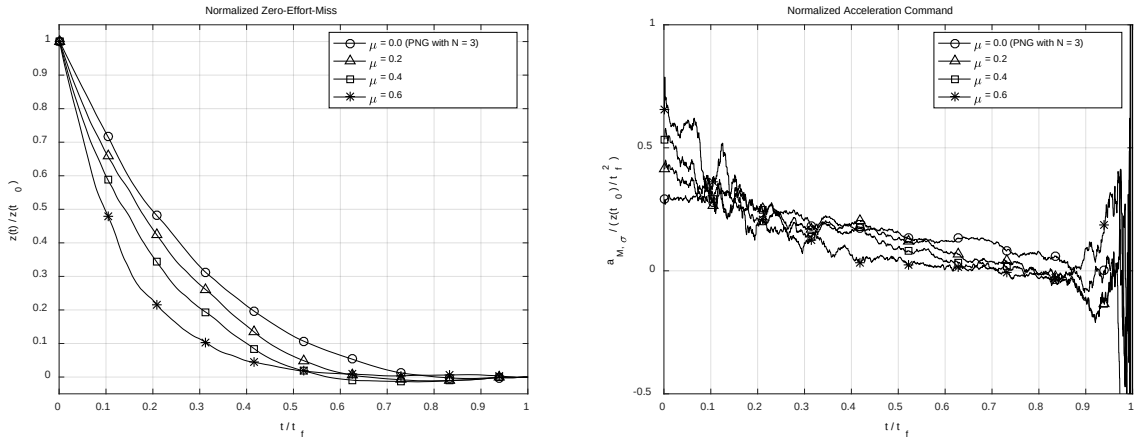


(d) Normalized Acceleration Command

Fig. 4 Simulation results for a delayed correction pattern of ZEM

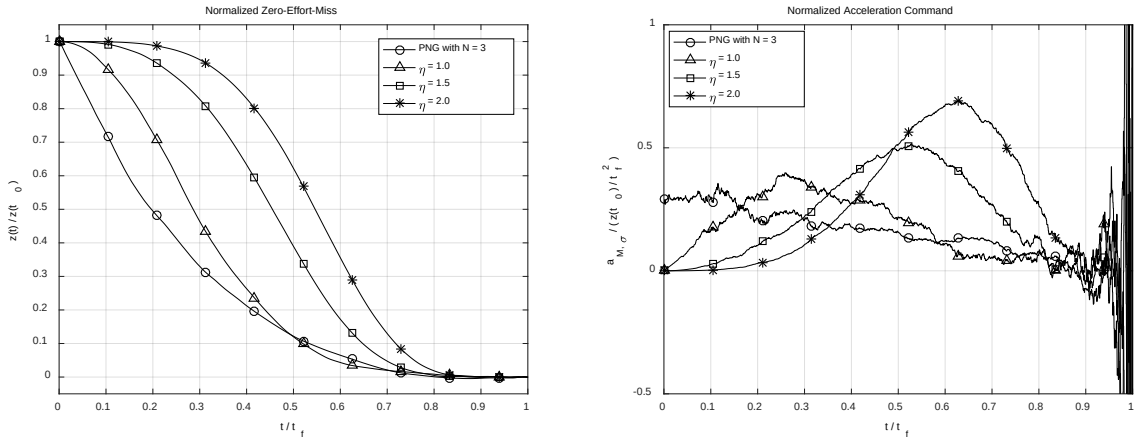
B. Application to realistic case

In this simulation, the proposed guidance laws are tested under the presence of seeker noise and autopilot lag in order to demonstrate the finite convergence of guidance command. We assume that the LOS angle and the relative distance are measured at 200 Hz from the seeker, and those are corrupted by a white Gaussian noise with zero-mean variance $v_\sigma^2 = 9(\text{mrad})^2$ and $v_R^2 = 25\,\text{m}^2$. A homing filter based on the modified-gain pseudo measurement filter (MGPMF) [23] is used to estimate all parameters for implementing the proposed guidance laws. Additionally, the autopilot lag is modelled as the first order lag system with the time constant $\tau = 0.1$.



(a) Normalized ZEM (b) Normalized Acceleration Command

Fig. 5 Simulation results for an early correction pattern of ZEM in presence of seeker noise and autopilot lag



(a) Normalized ZEM (b) Normalized Acceleration Command

Fig. 6 Simulation results for a delayed correction pattern of ZEM in presence of seeker noise and autopilot lag

Figs. 5 and 6 represent sample runs of proposed guidance laws in the noisy environment and the autopilot lag in order to show the tendency of performance. In all simulation cases, the proposed guidance laws can meet the finite convergence. As shown in Figs. 3, 4, 5, and 6, the proposed guidance laws have the similar performance tendency in shaping the pattern of ZEM. These results indicate that the capability in shaping the pattern of ZEM under the proposed method is satisfactory in the presence of seeker noise and autopilot lag.

VI. Conclusion

This paper proposes a new approach in designing guidance laws. The underlining idea behind this approach is to obtain the guidance command that can directly shape the pattern of ZEM depending on guidance operational objectives given. To achieve this idea, this paper first introduces the concept of weighted ZEM, which provides an additional degree of freedom in shaping the true ZEM. Then, this paper proposes the specific error dynamics to guarantee a finite-time convergence of the weighted ZEM. Next, the proposed approach devises a guidance law under the feedback linearization framework in which the weighted ZEM and the proposed error dynamics are utilized. It is shown that the resultant guidance has the form of PNG with the specific form of time-varying guidance gain, which is not revealed in previous studies. It provides important information about how the time-varying guidance should be behaved to obtain the desired pattern of ZEM. Accordingly, the design procedure in the proposed approach is as follows: first determine a desired pattern of ZEM to accomplish the given guidance objectives; find a desired weighting function reflecting the given pattern of ZEM; then the guidance command with the corresponding time-varying guidance gain can be simply determined.

This paper analyses the finite time converge of guidance laws designed by the proposed approach. We also theoretically investigate the conditions on the weighting function for which the interception of the target in a finite time and bounded guidance command are guaranteed. Also, it allows us the feasible set of desired patterns of ZEM that can be achieved in the proposed approach. The closed-form solutions of ZEM and guidance command under arbitrary weighting functions are also obtained in this paper. As illustrative examples, two ZEM shaping guidance laws are designed for different guidance objectives. The performance of the guidance laws designed and the validity of the analysis results are demonstrated through numerical simulations.

The potential significances of the proposed results are given as follows: First, the proposed approach allows guidance designers to devise a new PN-type guidance law perfectly fitting to their own specific guidance problems.

Second, the closed-form solution of ZEM, guidance command, and corresponding a time-varying gain can provide an useful insight on how the pattern of ZEM behaves with respect to the change in guidance command. Furthermore, it provides a significant clue in understanding the properties of PN-type guidance with time-varying guidance gains.

References

[1] Zarchan, P., *Tactical and Strategic Missile Guidance*, 5th ed. Washington, DC: AIAA, 2007, pp. 541-547.

[2] Ben-Asher, J. Z., and Yaesh, I., *Advances in Missile Guidance Theory*, Progress in Astronautics and Aeroautics, Vol. 180, AIAA, Reston, VA, 1998, pp. 25-32.

[3] Kreindler, E., "Optimality of Proportional Navigation," *AIAA Journal*, Vol. 11, No. 6, pp. 878-880, 1973.

[4] Jeon, I. S., Lee, J. I., and Tahk, M. J., "Impact-Time-Control Guidance Law for Anti-Ship Missiles," *IEEE Transactions on Control System Technology*, Vol. 14, No. 2, 2006, pp. 260-266.

[5] Harl, N., and Balakrishnan, S. N., "Impact Time and Angle Guidance with Sliding Mode Control," *IEEE Transactions on Control System Technology*, Vol. 20, No. 6, 2012, pp. 1436-1449.

[6] Kim, B. S., Lee, J. G., and Han, H. S., "Biased PNG Law for Impact with Angular Constraint," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 34, No. 1, 1998, pp. 277-288.

[7] Yoon, M. G., "Relative Circular Navigation Guidance for the Impact Angle Control Problem," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 44, No. 4, 2008, pp. 1449-1463.

[8] Lee, C. H., Kim, T. H., Tahk, M. J., and Whang, I. H., "Polynomial Guidance Law Considering Terminal Impact Angle and Acceleration Constraints," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 49, No. 1, 2013, pp. 55-73.

[9] Ryoo, C. K., Cho, H., and Tahk, M. J., "Time-to-go Weighted Optimal Guidance with Impact Angle Constraints," *IEEE Transactions on Control System Technology*, Vol. 14, No. 3, 2006, pp. 483-492.

[10] Rusnak, I., "Guidance Law Based on an Exponential Criterion for Acceleration Constrained Missile and a Maneuvering Target," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 3, 1996, pp.718-721.

[11] Ben-Asher, J. Z., Farber, N., and Levinson, S., "New Proportional Navigation Law for Ground-to-Air System," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 5, pp. 822-825.

[12] Ryu, M. Y., Lee, C. H., and Tahk, M. J., "Command Shaping Optimal Guidance Laws Against High-Speed Incoming Targets," *Journal of Guidance, Control, and Dynamics*, Vol. 38, No. 10, 2015, pp. 2025-2032.

[13] Weiss, M., and Shima, T., "Optimal Linear-Quadratic Missile Guidance Laws with Penalty on Command Variability," *Journal of Guidance, Control, and Dynamics*, Vol. 38, No. 2, 2015, pp. 226-237.

[14] Lee, C. H., "Linear Quadratic Optimal Guidance with Arbitrary Weighting Functions," The Journal of the Korean Society for Industrial and Applied Mathematics, Vol. 19, No. 3, 2015, pp. 365-386.

[15] Slotin, J. E., and Li, W., *Applied Nonlinear Control*, Prentice-Hall, Upper Saddle River, 1991.

[16] Koren, A., Idan, M., and Golan, O. M., "Integrated Sliding Mode Guidance and Control for a Missile with On-Off Actuators," *Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 1, 2008, pp. 204-214.

[17] Idan, M., Shima, T., and Golan, O. M., "Integrated Sliding Mode Autopilot-Guidance for Dual-Control Missiles," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 4, 2007, pp. 1081-1089.

[18] Moon, J., and Kim, Y., "Design of Missile Guidance Law via Variable Structure Control," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 6, 2001, pp. 659-664.

[19] Shtessel, Y. B., Shkolnikov, I. A. and Levant, A., "Guidance and Control of Missile Interceptor using Second-Order Sliding Modes," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 45, No. 1, 2009, pp. 110-124.

[20] Kumar, S. R., Rao, S., and Ghose, D., "Nonsingular Terminal Sliding Mode Guidance with Impact Angle Constraints," *Journal of Guidance, Control, and Dynamics*, Vol. 37, No. 4, 2014, pp. 1114-1130.

[21] Cho, D., Kim, H. J., and Tahk, M. J., "Impact Angle Constrained Sliding Mode Guidance against Maneuvering Target with Unknown Acceleration," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 51, No. 2, 2015, pp. 1310-1323.

[22] Lee, C. H., Kim, T. H., and Tahk, M. J., "Biased PNG for Target Observability Enhancement against Nonmaneuvering Targets," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 51, No. 1, 2015, pp. 2-17.

[23] Song, T. L., Ahn, J. Y., and Park, C., "Suboptimal Filter Design with Pseudomeasurements for Target Tracking," *IEEE Transactions on Aerospace and Electronics Systems*, Vol. 24, No. 1, 1988, pp. 28-39.

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Zero-effort-miss shaping guidance laws

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