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Assessing maintenance contracts when preventive maintenance is outsourced

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Abstract

In some companies, corrective maintenance is conducted in-house but preventive maintenance might be outsourced. This raises a need to optimise some parameters such as the number of contracts from a perspective of the equipment owner. This paper considers a maintenance policy for such a situation, analyses the roles of the parameters in a PM model, proposes approaches to defining bonus functions, and finally discusses special cases of both the PM policy and the bonus function. Numerical examples are also given to explore the impact of parameters on the expected lifecycle cost rate.

Keywords: maintenance outsourcing, preventive maintenance (PM); corrective maintenance (CM); minimal repair; bonus function.

1 Introduction

The issues of the optimum strategies of maintenance outsourcing have been studied by a number of authors[1–13]. Existing research has been focused on either outsourcing both preventive maintenance (PM) and corrective maintenance (CM) or outsourcing CM only. Little attention, however, has been paid to the problem of outsourcing PM, which is discussed in this paper.

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1.1 Prior work

In the literature, the types of maintenance outsourcing that have been studied so far include: Type-1 outsourcing: outsourcing both CM and PM[1–9], and Type-2 outsourcing: outsourcing CM[10–13] only.

Type-1 outsourcing: both CM and PM are outsourced. PM policies in the context of outsourcing both PM and CM have been discussed by a number of authors (see [1–6], for example). [7,8] use incentive contracts to induce the contractor to select the maintenance policy that optimises the total profit of the manufacturer and the contractor. In addition to the consideration of CM and PM, [9] considers inspection policies and optimises the contract parameter under different scenarios.

Type-2 outsourcing: Only CM is outsourced. Assuming that a sequence of CM contracts will be made to maintain a piece of equipment and that the service market can provide different kinds of CM contracts to the equipment owner [2], [10,11] propose methods for determining the optimal series of CM contracts for the equipment's lifetime. [12,13] present decision models for selecting CM contracts based on multi-criteria decision making theory, taking into account different variables such as cost and downtime.

Existing work can be categorised with Table 1.

Here: Table 1

1.2 Problems

In practice, however, there is a possibility that only PM is outsourced but CM is conducted in house. This has been reported from time to time, for example, in [14] where a case about outsourcing PM on power cables is presented.

When outsourcing PM, one might consider the following two options:

Option 1: PM on a piece of equipment will be outsourced to one agent within the lifecyle of the equipment;

[2] In this paper, we call the provider of maintenance service as an *agent*, and the recipient of the maintenance service as an *equipment owner*.

Option 2: PM on a piece of equipment will be outsourced to a number of agents within the lifecyle of the equipment and the quality of PM actions conducted within a contract period can be different.

Remarks on Option 2. Most companies periodically review (common periodicity is 3-5 years) their maintenance procedures, and make contracts with agents within these periods. The maintenance levels can be different from period to period [15]. If we assume that a PM contract starts from a PM action and ends with a PM, there can exist a time interval between two adjacent contracts. Hence, this option will have a time line containing a series events such as (also see Figure 1): *a new piece of equipment to start*$\rightarrow \tau_0 \rightarrow T_1 \rightarrow \tau_1 \rightarrow T_2 \rightarrow \tau_2 \rightarrow \cdots \rightarrow \tau_{N-1} \rightarrow T_N \rightarrow \tau_N \rightarrow$ *replacement*, where τ_k $(k = 0, 1, 2, ...)$ is the time interval between the end of the k-th contract and the start of the $(k+1)$-th contract, and T_k represents the length of the kth contract. Within different T_k, different agents are contracted to undertake PM. Any failures between PM actions are rectified by the equipment owner himself, but a penalty might be incurred on the failures to the agents. The quality of the last PM conducted by an agent in his contract period can be vitally important, as it can affect the remaining life of the equipment being maintained and therefore the optimum choice of maintenance schedules within its subsequent periods. That is, a piece of equipment maintained with good quality of the last PM conducted in a contract period might need fewer PM actions within its remaining lifetime and also fewer failures might occur, whereas a piece of equipment maintained with poor PM quality within a contract period might need more PM actions within its remaining lifetime and more failures might occur. Hence, good quality of the last PM in each contract period can be regarded as a profit to the owner as he will pay less on maintenance within the remaining lifetime of the equipment. In such a case, the owner of the equipment might be willing to pay a bonus to agents for encouraging good PM quality.

As one can see in the following sections, mathematically, Option 1 is a special case of Option 2. Hence, this paper will only discuss Option 2.

Here: Fig 1

Compared to the first three variables listed in Table 1, Option 2 differs from existing research in the following respects:

Type of outsourcing: Only PM is outsourced;

Type of penalty: Different penalty schemes can be used.

- The quality of the last PM will be assessed and a bonus might be paid for good quality.
- Penalty for repair not being carried out within specified time limits is not applicable here, as the agent does not undertake any CM upon equipment failures.

Decision variables: The objective is to seek the optimum series of PM contracts with respect to the length T_i of a contract, the time interval τ_i between two adjacent contracts, Penalty-2 and bonus schemes.

Hence, if a series of agents are contracted with the equipment owner, it is possible to introduce a new penalty scheme in following:

- failures between PMs can incur Penalty-2 to the agents due to the reliability performance specified in the maintenance contract being violated; and
- a bonus can be offered to the agent if the quality of the last PM in a contract is good.

It should be noted that [7,8] have already used the concept of bonus functions to encourage good maintenance quality in the context of maintenance outsourcing. In [7,8], however, both PM and CM are outsourced, which makes it different from the cases discussed in this paper.

This paper discusses PM policies used in Option 2. The equipment owner outsources PM within fixed periods, in which periodic or sequential PM's might be conducted. He might pay a bonus to agents for their good PM quality. The paper investigates the roles of the parameters in a typical PM model, proposes approaches to defining the bonus functions, derives algorithms to optimise the expected lifecycle cost rate, and discusses special cases of both the PM policy and the bonus function.

The paper considers optimising policies from a perspective of the equipment owner and assumes that costs on CM, PM, and replacement, and the levels of PM are known. This assumption might be rigorous for some cases as some values such as the levels of PM might only be determined by the agents, and unknown to the equipment owner. However, from a perspective of the equipment owner, he needs to assess the optimum values, including contract length, time intervals between adjacent contracts, penalty and bonus, respectively. Hence, such assumptions are necessary.

The rest of the paper is structured as follows. Section 2 presents notation and assumptions. Section 3 derives the lifecycle cost rate, investigates the roles of the parameters in a typical

PM model, proposes approaches to defining the bonus on PM actions, derives an algorithm to search the optimal solution, and compares two special PM policies. Section 4 presents data examples to look into the validity of the proposed models. Section 5 concludes the findings.

2 Notation and Assumptions

2.1 Notation

Here: Table 2

2.2 Assumptions

A typical lifecycle of the equipment under study is shown in Fig 1. Suppose the following assumptions hold.

A.1 The planning horizon is infinite.

A.2 The failure intensity function, $h_{0,0}(t)$, of the equipment before any maintenance intervention is conducted is continuous and strictly increasing.

A.3 The lifecycle of the equipment is defined as the time interval between two adjacent replacements. That is, the time in a lifecycle progresses as: *a new piece of equipment to start* $\rightarrow \tau_0 \rightarrow T_1 \rightarrow \tau_1 \rightarrow T_2 \rightarrow \tau_2 \rightarrow \cdots \rightarrow \tau_{N-1} \rightarrow T_N \rightarrow \tau_N \rightarrow$ *replacement*. T_k is called a the k-th *contract period*. For simplicity, we assume that $T_k = T$ in this paper.

A.4 Within the k-th contract period, m_k PM actions are conducted at time points $(k-1)T + \sum_{i=0}^{k-1} \tau_i + t_{k,1}, (k-1)T + \sum_{i=0}^{k-1} \tau_i + t_{k,1} + t_{k,2}, ..., (k-1)T + \sum_{i=0}^{k-1} \tau_i + \sum_{i=1}^{m_k} t_{k,i}$, respectively, where $k = 1, ..., N$.

A.5 The quality of the last PM in the k-th contract period might be different from the first $(m_k - 1)$th PM's in the same period [3].

A.6 Repair between PM actions is assumed to be minimal repair. A replacement is carried out at the end of a lifecycle. The equipment used in the replacement is a new, identical

[3] This assumption holds as only the maintenance quality of the last PM in each contract period will affect the reliability of the equipment in its subsequent time periods, and moreover, the case that PM's have the same quality within a contract period is a special case of this assumption.

item.

A.7 The time on any PM, minimal repair, and replacement are negligible.

3 Model formulation and analysis

3.1 Option 1

In Option 1, PM is outsourced to one agent.

3.2 Option 2

In Option 2, a series of PM contracts are outsourced to different agents. In this case, the equipment owner might be interested in optimising the following parameters:

- time lengths including T and τ_i, where $i = 0, 1, 2, ...$;
- value of Penalty-2 and bonus value.

Following the above assumptions, we can derive the expected lifecycle cost rate as follows

$$C(N, T, \tau_0, ..., \tau_N, t_{1,1}, ..., t_{N,m_N}) = \frac{G(N, T, \tau_0, ..., \tau_N, t_{1,1}, ..., t_{N,m_N})}{\sum_{k=0}^{N} \tau_k + NT}, \tag{1}$$

subject to

$$\sum_{i=1}^{m_k-1} t_{k,i} = T, \tag{2}$$

where

$$\begin{aligned} &G(N, T, \tau_0, ..., \tau_N, t_{1,1}, ..., t_{N,m_N}) \\ =& c_{o,1} \int_0^{\tau_0} h_{0,0}(t)dt + \sum_{k=1}^{N} \left[c_{c,k} \sum_{i=1}^{m_k-1} \int_0^{t_{k,i}} h_{k,i}(t)dt + c_{o,k} \int_0^{\tau_k} h_{k,m_k}(t)dt + c_k \right] \\ &+ \sum_{k=1}^{N} Q_k(h_{k,m_k}(\tau_k)) + c_r, \end{aligned} \tag{3}$$

$k = 1, 2, ..., N$, and $m_k > 2$.

In Eq. (1), $\sum_{k=0}^{N} \tau_k + NT$ is the expected time length of a lifecycle. In Eq. (3),

- $c_{o,1}\int_0^{\tau_0} h_{0,0}(t)dt$ is the expected cost on failures within time interval $(0, \tau_0)$,
- $c_{c,k}\sum_{i=1}^{m_k-1}\int_0^{t_{k,i}} h_{k,i}(t)dt$ is the expected cost on failures within time interval $((k-1)T + \sum_{i=0}^{k-1}\tau_i + \sum_{j=1}^{i-1} t_{k,j}, (k-1)T + \sum_{i=0}^{k-1}\tau_i + \sum_{j=1}^{i} t_{k,j})$, where $k \geq 1$;
- $c_{o,k}\int_0^{\tau_k} h_{k,m_k}(t)dt$ is the expected cost on failures within time interval $(kT + \sum_{i=0}^{k-1}\tau_i, kT + \sum_{i=0}^{k-1}\tau_i + \tau_k)$;
- $Q_k(h_{k,m_k}(\tau_k))$ is the bonus on the last PM in the k-th contract period, and
- c_k is the cost paid for the k-th contract,
- c_r is the replacement cost in a lifecycle.

$Q_k(h_{k,m_k}(\tau_k))$ is the bonus paid to the k-th agent who is responsible for the last PM in the k-th contract period, which considers PM quality that can affect PM frequency needed in its subsequent contract period(s). This makes Eq. (1) different from the problem of traditional PM policy optimisation (see [16–19], for example), in which the quality of the last PM has not been distinguished. Further discussion on $Q_k(h_{k,m_k}(\tau_k))$ will be carried out in Sections 3.3.

In Eq. (1), the equipment owner needs to optimise $N, t_{k,i}$ and τ_k. He might also need to choose the bonus function $Q_k(h_{k,m_k}(\tau_k))$. But before doing so, the bonus function $Q_k(h_{k,m_k}(\tau_k))$ needs to be discussed.

3.3 The bonus function

In the section, we discuss the choice of the bonus function $Q_k(h_{k,m_k}(\tau_k))$.

3.3.1 PM models

In Eq. (1), defining the function $Q_k(h_{k,m_k}(\tau_k))$ is important. $h_{k,m_k}(\tau_k)$ usually defines a PM model. In the reliability literature, PM models are introduced to assess PM quality (see [20,21], or Eq. (4) in the following, for example). The parameters in a PM model play important roles in measuring PM quality. In this section, we analyse the roles of those parameters, then in the following section we define the bonus function $Q_k(h_{k,m_k}(\tau_k))$.

PM models are developed based on adjusting either the failure intensity function of a piece of equipment or its age. After a maintenance action is carried out on a, the failure intensity

of the equipment is assumed to change from $h_{k,i-1}(t)$ to $h_{k,i}(t)$. If we assume that the quality of the first $(m_k - 1)$th PM actions in the k-th contract period are identical and the quality of the last PM is different, a typical case of $h_{k,i}(t)$ associated with $h_{k,i-1}(t)$ given by [20] is

$$h_{k,i}(t) = a_{k,i} h_{k,i-1}(\phi_{k,i} t + \psi_{k,i}) + b_{k,i}, \tag{4}$$

where $a_{k,i}(>)0$, $\phi_{k,i}(> 0)$, $b_{k,i}$ and $\psi_{k,i}$ are parameters.

If we consider Assumption (A.5), we have

- if $1 \leq i \leq m_k - 1$, then let $a_{k,i} = a'_{k,0}, b_{k,i} = b'_{k,0}, \phi_{k,i} = \phi'_{k,0}$, and $\psi_{k,i} = \psi'_{k,0}$;
- if $i = m_k$, then let $a_{k,m_k} = a'_{k,1}, b_{k,m_k} = b'_{k,1}, \phi_{k,m_k} = \phi'_{k,1}$, and $\psi_{k,m_k} = \psi'_{k,1}$.

Then, Eq. (4) can also be re-written as

$$h_{k,i}(t) = A_{ki} h_{0,0}(\Phi_{ki} t + \Psi_{ki}) + B_{ki}, \tag{5}$$

where $A_{00} = 1, \Phi_{00} = 1, \Psi_{00} = 0, B_{00} = 0$, $m_0 = 0$, for $k \geq 1$,

$$A_{k,i} = \begin{cases} a_{0k} A_{k-1,m_{k-1}} & \text{if } i = 1 \\ a_{0k} A_{k,i-1} & \text{if } 2 \leq i \leq m_k - 1, \\ a_{1k} A_{k,i-1} & \text{if } i = m_k \end{cases} \quad \Phi_{k,i} = \begin{cases} \phi_{0k} \Phi_{k-1,m_{k-1}} & \text{if } i = 1 \\ \phi_{0k} \Phi_{k,i-1} & \text{if } 2 \leq m_k - 1, \\ \phi_{1k} \Phi_{k,i-1} & \text{if } i = m_k \end{cases}$$

$$\Psi_{k,i} = \begin{cases} \phi_{0k} \Psi_{k-1,m_{k-1}} + \psi_{0k} & \text{if } i = 1 \\ \phi_{0k} \Psi_{k,i-1} + \psi_{0k} & \text{if } 2 \leq m_k - 1, \\ \phi_{1k} \Psi_{k,i-1} + \psi_{1k} & \text{if } i = m_k \end{cases} \text{ and } B_{k,i} = \begin{cases} a_{0k} B_{k-1,m_{k-1}} + b_{0k} & \text{if } i = 1 \\ a_{0k} B_{k,i-1} + b_{0k} & \text{if } 2 \leq m_k - 1. \\ a_{1k} B_{k,i-1} + b_{1k} & \text{if } i = m_k \end{cases}$$

The PM model shown in Eq. (4) can be seen as an extension of a wide spectrum of PM models such as those introduced in [21–23], etc.

Estimating the parameters in the bonus function as shown in Eq. (4) is vital. In practise, for example, we might estimate the parameters in Eq. (4) based on data collected from the equipment owner who might have a large number of pieces equipment of the same type maintained and have enough field data, or based on expert elicitation to estimate the parameters.

A simpler situation might be to count the number of failures within the time period τ_k to assess the quality of the last PM conducted by an agent.

3.3.2 Some bonus functions

From Eq. (4), we have

- Assume $h_{k,i-1}(\phi_{k,i}t + \psi_{k,i})$ is smaller than 1. Then we have $\frac{\partial h_{k,i}(t)}{\partial a_{k,i}} = h_{k,i-1}(\phi_{k,i}t + \psi_{k,i}) \leq \frac{\partial h_{k,i}(t)}{\partial b_{k,i}} = 1$, which implies that an improvement in $b_{k,i}$ can affect $h_{k,i-1}(\phi_{k,i}t + \psi_{k,i})$ more effectively than $a_{k,i}$ can; and
- As $h_{k,i-1}(\phi_{k,i}t+\psi_{k,i})$ is an increasing function, $h'_{k,i-1}(\phi_{k,i}t+\psi_{k,i}) > 0$. If $t > 1$, then $\frac{\partial h_{k,i}(t)}{\partial \phi_{k,i}} = h'_{k,i-1}(\phi_{k,i}t + \psi_{k,i})t > \frac{\partial h_{k,i}(t)}{\partial \psi_{k,i}} = h'_{k,i-1}(\phi_{k,i}t + \psi_{k,i})$, which implies that an improvement in $\phi_{k,i}$ can affect $h_{k,i-1}(\phi_{k,i}t + \psi_{k,i})$ more effectively than $\psi_{k,i}$ can.

Remark. *The above two bulleted points show that $\phi_{k,i}$ and $b_{k,i}$ in Eq. (4) can play more important roles than parameters $a_{k,i}$ and $\psi_{k,i}$.*

From the above discussion, we can define $Q_k(h_{k,m_k}(\tau_k))$, if we assume that it only depends on the parameters, $a_{k,i}$, $b_{k,i}$, $\phi_{k,i}$, and $\psi_{k,i}$, in Eq. (4). It might be reasonable to assume that $Q_k(h_{k,m_k}(\tau_k))$ is in inverse proportion to the values $a_{k,i}$ and $\phi_{k,i}$, and in direct proportion to the values $\psi_{k,i}$ and $b_{k,i}$. For example, if PM only changes the parameter $a_{k,i}$ in Eq. (4), then one might estimate the bonus using the following equation.

$$Q_k(h_{k,m_k}(\tau_k)) = \begin{cases} \exp(-\theta_1 a_{k,i}) \text{ if } a_{k,i} < 1 \\ 0 \qquad\qquad\quad \text{otherwise,} \end{cases} \tag{6}$$

where θ_1 is a positive parameter.

$a_{k,i}$, $b_{k,i}$, $\phi_{k,i}$, and $\psi_{k,i}$ play different roles, according to the above remark, which should be noted when we define/estimate the parameter θ_1.

We might also consider the following scenarios on the relationship between $Q_k(h_{k,m_k}(\tau_k))$ and the m_kth PM in the k-th contract period.

- *NoF (number of failures) based approach.* $Q_k(.)$ is a function of $\int_0^x h_{k-1,m_{k-1}}(t|\tau_{k-1}+T)dt - \int_0^x h_{k,m_k}(t)dt$: $\int_0^x h_{k,m_{k-1}}(t|\tau_{k-1}+T)dt$ is the number of failures of the equipment within the next x time units, given that the equipment has survived $\tau_{k-1}+T$ time units after the

last PM in the $(k-1)$-th contract period; $\int_0^x h_{k,m_k}(t)dt$ is the number of failures of the equipment within a given time, $(0,x)$ say, after the last PM in the k-th contract period.

$$Q_k(h_{k,m_k}(\tau_k)) = \begin{cases} e^{\theta_2(\int_0^x h_{k-1,m_{k-1}}(t|\tau_{k-1}+T)dt - \int_0^x h_{k,m_k}(t)dt)} & \text{if } \int_0^x h_{k-1,m_{k-1}}(t|\tau_{k-1}+T)dt > \int_0^x h_{k,m_k}(t)dt \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

where θ_2 is a positive parameter.

Using this bonus function, we aim to measure the reduction of the number of failures after the last PM is conducted in the k-th contract period.

- *Remaining life based approach.* $Q_k(.)$ might also be defined as a function of $\int_0^\infty udF_{k,m_k}(u) - \int_0^\infty udF_{k-1,m_{k-1}}(u|\tau_{k-1}+T)$: $\int_0^\infty udF_{k,m_k}(u)$ is the expected life of the equipment just after the last PM in the k-th contract period, and $\int_0^\infty udF_{k-1,m_{k-1}}(u|\tau_{k-1}+T)$ is the remaining life of the equipment, given that it has survived for $\tau_{k-1}+T$ time units since the last PM in the $(k-1)$-th contract period. For example, one might define that

$$Q_k(h_{k,m_k}(\tau_k)) = \begin{cases} e^{\theta_3(\int_0^\infty udF_{k,m_k}(u) - \int_0^\infty udF_{k-1,m_{k-1}}(u|\tau_{k-1}+T))} & \text{if } \int_0^\infty udF_{k,m_k}(u) > \int_0^\infty udF_{k-1,m_{k-1}}(u|\tau_{k-1}+T) \\ 0 & \text{otherwise,} \end{cases} \quad (8)$$

where θ_3 is a positive parameter.

Using this approach will encourage the $(k-1)$-th maintenance company to conduct better maintenance to prolong the time length before the k-th company is contracted.

3.4 Maintenance policies

We assume that $h_{k,i}(\bullet)$ is defined by Eq. (4), and $Q_k(h_{k,m_k}(\tau_k)$ is defined by Eq. (6). To find the optimal solution that minimises $C(N,\tau_0,\tau_1,...,\tau_N,t_{1,1},...,t_{N,m_N})$, we can set the partial derivatives of $C(N,\tau_0,\tau_1,...,\tau_N,t_{1,1},...,t_{N,m_N})$ with respect to $\tau_0,\tau_1,...,\tau_N,t_{1,1},...,t_{N,m_N}$ to 0, respectively, which gives

$$c_{o,1}h_{0,0}(\tau_0)\Big(\sum_{k=0}^{N}\tau_k + NT\Big) = G(N,T,\tau_0,\tau_1,...,\tau_N,t_{1,1},...,t_{N,m_N}), \quad (9)$$

$$\Big(c_{o,k}A_{k,m_k}h_{0,0}(\Phi_{k,m_k}\tau_k+\Psi_{k,m_k})+\frac{\partial Q_k(h_{k,m_k}(\tau_k))}{\partial\tau_k}\Big)\Big(\sum_{k=0}^{N}\tau_k+NT\Big) \tag{10}$$
$$=G(N,T,\tau_0,\tau_1,...,\tau_N,t_{1,1},...,t_{N,m_N}),$$

and

$$c_{c,k}A_{k,i}h_{0,0}(\Phi_{k,i}t_{k,i}+\Psi_{k,i})-c_{c,k}A_{k,m_k}h_{0,0}(\Phi_{k,m_k}(T-\sum_{i=1}^{m_k-1}t_{k,i}) \tag{11}$$
$$+\Psi_{k,m_k})+\frac{\partial Q_k(h_{k,m_k}(\tau_k))}{\partial t_{k,i}}=0.$$

For generality, we have not replaced $\frac{\partial Q_k(h_{k,m_k}(\tau_k))}{\partial t_{k,i}}$ with the one in Eq. (6).

From equations (9) and (11), we have

$$c_{o,k}A_{k,m_k}h_{0,0}(\Phi_{k,m_k}\tau_k+\Psi_{k,m_k})+\frac{\partial Q_k(h_{k,m_k}(\tau_k))}{\partial\tau_k}=c_{o,1}h_{0,0}(\tau_0). \tag{12}$$

The computing procedure to obtain an optimal schedule can be specified as follows: let N be fixed, then obtain the solutions based on the following steps, after that compare $C(N,T,\tau_0,\tau_1,...,\tau_N,t_{1,1},...,t_{N,m_N})$ on the solutions, and then choose those solutions with the minimal

We can specify the computing procedure to obtain an optimal schedule as follows: we can fix N, then obtain the solutions based on the following steps, then compare $C(N,T,\tau_0,\tau_1,...,\tau_N,t_{1,1},...,t_{N,m_N})$ on the solutions, and then choose those solutions with the minimal $G_3(N,T,\tau_0,\tau_1,...,\tau_N,t_{1,1},...,t_{N,m_N})$. In the following algorithm, we assume $Q_k(h_{k,m_k}(\tau_k))$ is simply related to τ_k $(k=1,2,...,N)$.

- Let $N=1,2,3,...,n$, where n is a given upper limit;
- Solve Eq. (12), and express τ_k $(k=0,...,N)$ by a function of τ_0;
- Solve Eq. (12) with respect to $t_{k,i}$;
- Substitute τ_k and $t_{k,i}$ into Eq. (9) to obtain τ_0;
- Substitute τ_0 into Eq. (12) to obtain τ_k; and
- determine N which minimises Eq. (1).

The above algorithm has simplified the optimisation problem of a complex mixed integer programming to the optimisation problem of an integer programming.

The above algorithm only considers to optimise the frequency of the PM. Similarly, we can

also optimise the bonus function and penalty costs in each contract periods: he can find the optimum parameters through minimising the expected cost in Eq. (1).

3.4.1 Special cases of the PM policy

PM can be conducted either periodically or sequentially. The periodic PM policy is more frequently used due to its simplicity in scheduling, and the sequential PM policy is less frequently used. An interesting PM policy in Option 2 might be a hybrid of the periodic PM policy and the sequential PM policy. In this section, the following two special maintenance policies are investigated (see Figure 1).

- *Periodic–sequential (PS) PM policy.* Within a given time period T, periodic PM is performed, but the values τ_k are unequal for $k = 1, 2, ...$, ie., $t_{k,1} = t_{k,2} = ... = t_{k,m_k}$ and $\tau_1 \neq \tau_2 \neq ..., \neq \tau_N$. The quality of PM's within a time period T are the same but are different among different time periods T's. Denote $C_{PS} = C(N, T, \tau_0, \tau_1, ..., \tau_N, t_{1,1}, ..., t_{N,m_N})$ as the expected lifecycle cost rate of this case.
- *Sequential–periodic (SP) PM policy.* Within a given time period T, sequential PM is performed. The values τ_k are equal for $k = 1, 2, ...$, ie, $t_{k,1} \neq t_{k,2} \neq ... \neq t_{k,m_k}$ and $\tau_1 = \tau_2 = ..., = \tau_N$. The quality of PM's within a time period T are the same but are different among different time periods T's. Denote $C_{SP} = C(N, T, \tau_0, \tau_1, ..., \tau_N, t_{1,1}, ..., t_{N,m_N})$ as the expected lifecycle cost rate of this case.

An interesting question is which PM policy, periodic–sequential PM policy or sequential-periodic PM policy, is better regarding the expected cost rate. The following two Lemmas list sufficient conditions for comparing the two policies.

Lemma 1 *Denote* $\tau = \dfrac{1}{N+1}\sum_{k=0}^{N}\tau_k$, *and assume that* $\tau_0 \geq \tau_1 \geq \cdots \geq \tau_{k_0} \geq \tau \geq \tau_{k_0+1} \geq \ldots \tau_N$, *then*

- $\int_0^{\tau} h_{0,0}(t)dt + \sum_{k=1}^{N}\int_0^{\tau} h_{k,m_k}(t)dt \geq \int_0^{\tau_0} h_{0,0}(t)dt + \sum_{k=1}^{N}\int_0^{\tau_k} h_{k,m_k}(t)dt$ *if*

$$\max\{h_{0,0}(\tau_0), h_{1,m_1}(\tau_1)..., h_{k_0,m_{k_0}}(\tau_{k_0})\} \leq \min\{h_{k_0+1,m_{k_0+1}}(\tau_{k_0+1}), ..., h_{N,m_N}(\tau_N)\}.$$

- $\int_0^{\tau} h_{0,0}(t)dt + \sum_{k=1}^{N}\int_0^{\tau} h_{k,m_k}(t)dt \leq \int_0^{\tau_0} h_{0,0}(t)dt + \sum_{k=1}^{N}\int_0^{\tau_k} h_{k,m_k}(t)dt$ *if*

$$\min\{h_{0,0}(\tau_0), h_{1,m_1}(\tau_1)..., h_{k_0,m_{k_0}}(\tau_{k_0})\} \geq \max\{h_{k_0+1,m_{k_0+1}}(\tau_{k_0+1}), ..., h_{N,m_N}(\tau_N)\}.$$

The proofs for the Lemmas in this paper can be found in Appendix.

Lemma 2 *Denote* $t_{k,0} = \dfrac{1}{m_k - 1} \sum_{i=1}^{m_k - 1} t_{k,i}$, $Q_k(h_{k,m_k}(\tau_k))$ *is measured with Eq. (6), and assume that* $t_{k,1} \geq t_{k,2} \geq \cdots \geq t_{k,i_k} \geq t_{k,0} \geq t_{k,i_k+1} \geq \ldots t_{k,m_{k-1}}$ *for* $k = 1, ..., N$. *Then,*

- $C_{SP} > C_{SP}$ *if* $\max\{h_{0,0}(\tau_0), h_{1,m_1}(\tau_1)..., h_{k_0,m_{k_0}}(\tau_{k_0})\} \leq \min\{h_{k_0+1,m_{k_0+1}}(\tau_{k_0+1}), ..., h_{N,m_N}(\tau_N)\}$, *and*
  $\min\{h_{k,1}(t_{k,1}), ..., h_{k,i_k}(t_{k,i_k})\} \geq \max\{h_{k,i_k+1}(t_{k,i_k+1}), ..., h_{k,m_k}(t_{k,m_{k-1}})\}$.
- $C_{SP} < C_{SP}$ *if* $\max\{h_{0,0}(\tau_0), h_{1,m_1}(\tau_1)..., h_{k_0,m_{k_0}}(\tau_{k_0})\} \geq \min\{h_{k_0+1,m_{k_0+1}}(\tau_{k_0+1}), ..., h_{N,m_N}(\tau_N)\}$, *and*
  $\min\{h_{k,1}(t_{k,1}), ..., h_{k,i_k}(t_{k,i_k})\} \leq \max\{h_{k,i_k+1}(t_{k,i_k+1}), ..., h_{k,m_k}(t_{k,m_{k-1}})\}$.

Lemma 2 gives sufficient conditions for comparing the two PM policies with respect to their expected lifecycle cost rate.

4 Illustrative examples

Consider a case where the life distribution of a piece of equipment under consideration is a 2-parameter Weibull distribution at operating state. The failure intensity is assumed to be $h_0(t) = \frac{1}{8}(\frac{t}{24})^2$. That is, its corresponding lifetime distributions is given $F(t) = 1 - \exp\{-(\frac{t}{24})^3\}$. We set $N = 3$, $T = 36$, $c_r = 5000$, $c_{o,k} = c_{c,k} = 100e^{0.5k}$, $Q_k(h_{k,m_k}(\tau_k)) = 0.1e^{\frac{0.8}{a_k - 1}}$. In Eq. (4), without loss of generality, we set $a_{k,i} = \frac{k+5.2}{k+4}$, $\phi_{k,i} = 1$, $\psi_{k,i} = 0$, and $b_{k,i} = 0$.

4.1 Comparison between the PS and the SP policies

Using the above parameters, we obtain the optimal PS and SP policies as shown in Tables 3 and 4, from which one can find that the two policies reach their optimal values when $N = 3$. However, the expected lifecycle cost rate of the SP policy is greater than that of the PS policy.

For the optimal policies $N = 3$, if we further prolong the length of τ in the SP policy to let $\tau = \dfrac{1}{N+1} \sum_{k=0}^{N} \tau_k$, we can obtain Table 5, from which one can find that the expected lifecycle

cost rate of the SP policy (67.12) is more than 1.4 times as big as that of the PS policy (47.73).

Here: Figure 2.

Here: Table 3

Here: Table 4

Here: Table 5

4.2 Impact of PM quality

Changing parameter Δ in $a_{k,i} = \frac{k+\Delta}{k+4}$, we can investigate the impact of the parameters $a_{k,i}$ on the expected lifecycle cost rate for the two policies, ie., the PS policy and the SP policy (as shown in Table 6). For example, the changes of C_{PS} against Δ are shown in Figure 2, which is a concave curve and the expected long-run cost rate reaches the optimal value when $\Delta = 5.2$.

Here: Table 6

4.3 Impact of CM cost

Now, we re-set $c_{o,k} = c_{c,k} = 100e^{c_m k}$. That is, we set $N = 3$, $T = 36$, $c_r = 5000$, $Q_k(h_{k,m_k}(\tau_k)) = 0.1e^{\frac{0.8}{a_k - 1}}$, and $a_k = \frac{k+5.2}{k+4}$. We change c_m from 0.1 to 1, the expected lifecycle cost rate shows a linear relationship with c_m, as shown in Table 7.

Here: Table 7

5 Conclusions

This paper considers providing equipment owners with approaches to benchmarking various parameters for the case where PM is outsourced. It investigated the roles of the parameters in a PM (PM) model in respect to PM quality, derived an approach to measuring the profit

of good PM actions, and derived an approach to searching the optimal solutions.

Future research includes:

- The planning horizon is assumed infinite in this paper. Another consideration that can be taken is the scenario of finite planning horizon.
- The contract periods (i.e., T) in a lifecycle are assumed to be identical in the paper. A reasonable extension can be to assume that they are varying.

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Appendix

Proof of Lemma 1

$$
\begin{aligned}
&\int_0^{\tau} h_{0,0}(t)dt + \sum_{k=1}^{N} \int_0^{\tau} h_{k,m_k}(t)dt - \int_0^{\tau_0} h_{0,0}(t)dt - \sum_{k=1}^{N} \int_0^{\tau_k} h_{k,m_k}(t)dt \\
&= \int_{\tau_0}^{\tau} h_{0,0}(t)dt - \sum_{k=1}^{k_0} \int_{\tau}^{\tau_k} h_{k,m_k}(t)dt + \sum_{k=k_0+1}^{N} \int_{\tau_k}^{\tau} h_{k,m_k}(t)dt \\
&\geq -\max\{h_{0,0}(\tau_0), h_{1,m_1}(\tau_1)..., h_{k_0,m_{k_0}}(\tau_{k_0})\} \sum_{k=0}^{k_0} (\tau_k - \tau) \\
&+ \min\{h_{k_0+1,m_{k_0+1}}(\tau_{k_0+1}), ..., h_{N,m_N}(\tau_N)\} \sum_{k=k_0+1}^{N} (\tau - \tau_k) \\
&\geq -\min\{h_{k_0+1,m_{k_0+1}}(\tau_{k_0+1}), ..., h_{N,m_N}(\tau_N)\} \sum_{k=0}^{k_0} (\tau_k - \tau) \\
&+ \min\{h_{k_0+1,m_{k_0+1}}(\tau_{k_0+1}), ..., h_{N,m_N}(\tau_N)\} \sum_{k=k_0+1}^{N} (\tau - \tau_k) \\
&= \min\{h_{k_0+1,m_{k_0+1}}(\tau_{k_0+1}), ..., h_{N,m_N}(\tau_N)\} \sum_{k=0}^{N} (\tau - \tau_k) \geq 0
\end{aligned}
$$

This proves the 1st bulleted item in Lemma 1. ■

Similarly, we can prove the 2nd bulleted item in Lemma 1.

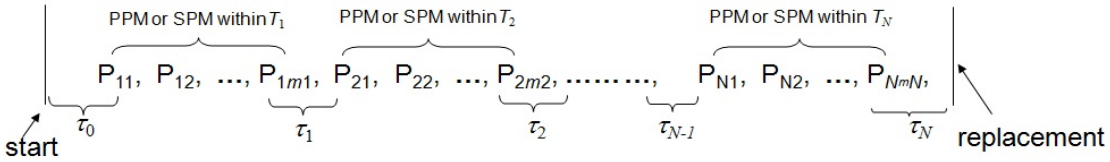

Fig. 1. A PM model (T_k=the length of the k-th contract, PPM=periodic PM, SPM=Sequential PM, P_{ki}=the i-th PM action in the k-th contract).

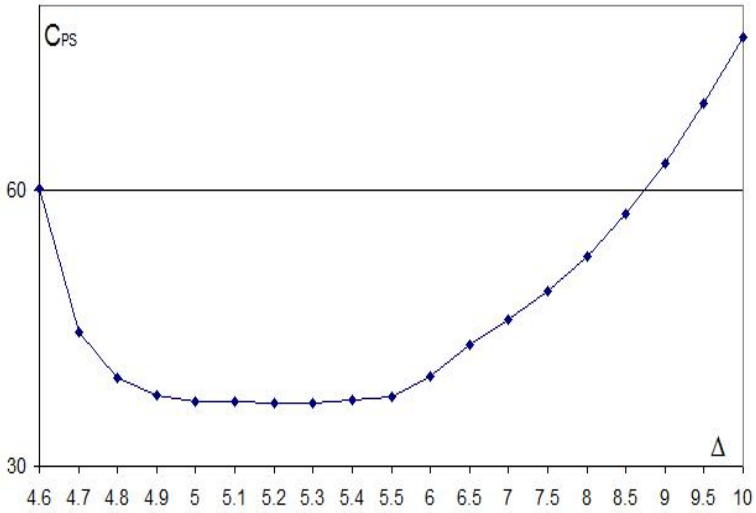

Fig. 2. Impact of PM quality on the expected long-run cost rate.

Table 1

A summary of some existing papers.

Type of outsourcing	Type of penalty [a]	Decision variables	References
PM & CM	Penalty-1 & Penalty-2	PM policy	[1–3]
PM & CM	Penalty-1 or Penalty-2	PM policy or else	[4–6,9]
CM	Penalty-1	CM contracts	[10–13]

[a] Penalty-1 : Penalty for repair not being carried out within specified time limits; Penalty-2: Penalty for equipment failures.

Table 2

Notation.

N	number of contracts before a replacement is carried out;
k	number of the contracts that the equipment has been passed, $k = 0, 1, 2, ..., N$;
i	number of the PM actions that an agent has carried out;
m_i	number of PM actions performed by the i-th contractor;
τ_k	time interval between the end of the (k)-th contract period; and the start of the $(k+1)$-th contract period;
$t_{k,i}$	time interval between the i-th and the $(i+1)$-th PM in the k-th contract;
T	length of a contract period;
$F_{k,i}(t)$	failure distribution function after the ith PM in the k-th contract period;
$h_{k,i}(t)$	failure intensity function after the ith PM in the k-th contract period;
m_k	number of PM's within the k-th contract period;
$c_{c,k}$	cost per CM within the k-th period after the penalty cost due to the failure has been paid. $c_{c,k}$ can be negative or positive;
$c_{o,k}$	cost per CM during the period of the τ_k after the penalty cost due to the failure has been paid. $c_{o,k}$ can be negative or positive;
c_k	value of the k-th contract;
c_r	cost per replacement;
$Q_k(.)$	bonus function.

Table 3

The expected lifecycle cost rate of the PS policy.

N	τ_1	m_1	τ_2	m_2	τ_3	m_3	τ_4	m_4	τ_5	m_5	τ_6	m_6	τ_7	C_{PS}
1	34.13	3	12.26											75.98
2	28.71	3	11.99	10	2.78									53.75
3	27.06	2	12.59	4	5.04	15	1.00							47.73
4	30.04	1	15.57	3	6.83	5	2.65	18	1.01					58.85
5	43.59	3	18.21	4	8.76	5	3.98	6	1.51	20	1.15			123.89
6	63.79	2	29.68	2	14.27	3	6.49	3	3.04	6	1.55	20	1.27	265.34

Table 4

The expected lifecycle cost of the SP policy.

N	τ_1	m_1	τ_2	m_2	τ_3	m_3	τ_4	m_4	τ_5	m_5	τ_6	m_6	τ_7	C_{PS}
1	21.31	2	21.31											83.17
2	8.64	2	8.64	5	8.64									65.15
3	3.85	2	3.85	4	3.85	6	3.85							62.34
4	2.32	2	2.32	2	2.32	4	2.32	7	2.32					83.62
5	1.13	2	1.13	3	1.13	4	1.13	6	1.13	18	1.13			235.17
6	1.10	2	1.10	3	1.10	3	1.10	5	1.10	5	1.10	24	1.10	358.79

Table 5

Comparison between the PS and the SP policies.

Policies	τ, τ_1	t_1, t_{1i}		τ, τ_2	t_2, t_{2i}				τ, τ_3	t_3, t_{3i}						τ, τ_3	C_{PS}, C_{SP}
PS	32	18	18	20	9	9	9	9	11	6	6	6	6	6	6	6	47.73
SP	11	19	17	11	10	9	9	8	11	7	7	6	6	5	5	11	62.34
SP	17.25	19	17	17.25	10	9	9	8	17.25	8	6	6	6	5	5	17.25	67.12

Table 6

Impact of PM quality on the expected lifecycle cost rate.

Δ	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5
C_{PS}	60.13	44.56	39.53	37.72	37.02	37.02	36.77	36.92	37.16	37.48
C_{SP}	325.59	82.58	69.45	66.74	63.64	63.89	64.15	66.30	66.91	68.88
Δ	6	6.5	7	7.5	8	8.5	9	9.5	10	
C_{PS}	39.81	43.22	46.00	49.06	52.78	57.41	62.84	69.42	76.62	
C_{SP}	87.32	110.15	143.54	187.78	246.64	314.86	416.21	481.74	578.23	

Table 7

Impact of CM cost on the expected lifecycle cost rate.

$c_{c,k}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
C_{PS}	31.34	32.67	34.03	35.42	36.77	38.17	39.68	41.32	43.15	45.25

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